Computer algebra independent integration tests

4-Trig-functions/4.1-Sine/4.1.2.1-a+b-sin-^m-c+d-sin-^n

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3.229 $\int (a+a\sin(e+fx))(c-c\sin(e+fx))^2\,dx$ 1255
3.230 $\int (a+a\sin(e+fx))(c-c\sin(e+fx))\,dx$ 1259
3.231 $\int \frac{a+a\sin(e+fx)}{c-c\sin(e+fx)}\,dx$ 1262
3.232 $\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^2}\,dx$ 1265
3.233 $\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^3}\,dx$ 1269
3.234 $\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^4}\,dx$ 1274
3.235 $\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^5}\,dx$ 1279
3.236 $\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^5\,dx$ .1284
3.237 $\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^4\,dx$ .1289
3.238 $\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^3\,dx$ .1294
3.239 $\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^2\,dx$ .1298
3.240 $\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$ 1302
3.241 $\int \frac{(a+a\sin(e+fx))^2}{c-c\sin(e+fx)}\,dx$ 1306
3.242 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^2}\,dx$ 1310
3.243 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^3}\,dx$ 1314
3.244 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^4}\,dx$ 1318

3.245 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^5}\,dx$ 1323
3.246 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^6}\,dx$ 1329
3.247 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^6\,dx$ 1336
3.248 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^5\,dx$ 1342
3.249 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^4\,dx$ 1347
3.250 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^3\,dx$ 1352
3.251 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^2\,dx$ 1356
3.252 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))\,dx$ 1360
3.253 $\int \frac{(a+a\sin(e+fx))^3}{c-c\sin(e+fx)}\,dx$ 1364
3.254 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^2}\,dx$ 1369
3.255 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^3}\,dx$ 1374
3.256 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^4}\,dx$ 1379
3.257 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^5}\,dx$ 1384
3.258 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^6}\,dx$ 1390
3.259 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^7}\,dx$ 1397
3.260 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^8}\,dx$ 1405
3.261 $\int \frac{(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx$ 1411
3.262 $\int \frac{(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx$ 1417
3.263 $\int \frac{(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$ 1422
3.264 $\int \frac{c-c\sin(e+fx)}{a+a\sin(e+fx)}\,dx$ 1426
3.265 $\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx$ 1429
3.266 $\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx$ 1432
3.267 $\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx$ 1436
3.268 $\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx$ 1441
3.269 $\int \frac{(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx$ 1446
3.270 $\int \frac{(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx$ 1454
3.271 $\int \frac{(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$ 1460
3.272 $\int \frac{(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$ 1465

3.273 $\int \frac{c-c\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx$ 1469
3.274 $\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx$ 1473
3.275 $\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx$ 1477
3.276 $\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx$ 1481
3.277 $\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx$ 1486
3.278 $\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx$ 1491
3.279 $\int \frac{(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx$ 1497
3.280 $\int \frac{(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx$ 1505
3.281 $\int \frac{(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx$ 1511
3.282 $\int \frac{(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$ 1516
3.283 $\int \frac{c-c\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx$ 1520
3.284 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx$ 1525
3.285 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx$ 1530
3.286 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx$ 1535
3.287 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx$ 1539
3.288 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx$ 1545
3.289 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx$ 1552
3.290 $\int (a+a\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$ 1560
3.291 $\int (a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$ 1564
3.292 $\int (a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$ 1568
3.293 $\int (a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$ 1572
3.294 $\int \frac{a+a\sin(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx$ 1575
3.295 $\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx$ 1579
3.296 $\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx$ 1583
3.297 $\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,dx$ 1588
3.298 $\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^{7/2}\,dx$ 1593
3.299 $\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}\,dx$ 1598
3.300 $\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}\,dx$ 1602
3.301 $\int (a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}\,dx$ 1606
3.302 $\int \frac{(a+a\sin(e+fx))^2}{\sqrt{c-c\sin(e+fx)}}\,dx$ 1609

3.303 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^{3/2}}\,dx$ 1614

3.304 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^{5/2}}\,dx$ 1619

3.305 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^{7/2}}\,dx$ 1624

3.306 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^{9/2}}\,dx$ 1629

3.307 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^{7/2}\,dx$ 1635

3.308 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}\,dx$ 1639

3.309 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}\,dx$ 1644

3.310 $\int (a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}\,dx$ 1648

3.311 $\int \frac{(a+a\sin(e+fx))^3}{\sqrt{c-c\sin(e+fx)}}\,dx$ 1652

3.312 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{3/2}}\,dx$ 1657

3.313 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{5/2}}\,dx$ 1662

3.314 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{7/2}}\,dx$ 1667

3.315 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{9/2}}\,dx$ 1672

3.316 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{11/2}}\,dx$ 1678

3.317 $\int \frac{(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx$ 1684

3.318 $\int \frac{(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx$ 1688

3.319 $\int \frac{(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx$ 1692

3.320 $\int \frac{\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx$ 1704

3.321 $\int \frac{1}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx$ 1707

3.322 $\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx$ 1711

3.323 $\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx$ 1716

3.324 $\int \frac{(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx$ 1721

3.325 $\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx$ 1725

3.326 $\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx$ 1729

3.327 $\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx$ 1733

3.328 $\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx$ 1737

3.329 $\int \frac{1}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx$ 1741

3.330 $\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx$ 1746
3.331 $\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx$ 1751
3.332 $\int \frac{(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx$ 1757
3.333 $\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx$ 1761
3.334 $\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx$ 1765
3.335 $\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx$ 1769
3.336 $\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx$ 1773
3.337 $\int \frac{1}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx$ 1777
3.338 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx$ 1782
3.339 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx$ 1788
3.340 $\int \sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}\,dx$ 1794
3.341 $\int \sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}\,dx$ 1797
3.342 $\int \sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}\,dx$ 1800
3.343 $\int \sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}\,dx$ 1803
3.344 $\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx$ 1806
3.345 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{3/2}}\,dx$ 1810
3.346 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{5/2}}\,dx$ 1813
3.347 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{7/2}}\,dx$ 1816
3.348 $\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}\,dx$ 1819
3.349 $\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}\,dx$ 1823
3.350 $\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}\,dx$ 1827
3.351 $\int (a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}\,dx$ 1831
3.352 $\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$ 1834
3.353 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$ 1839
3.354 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$ 1843
3.355 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$ 1846
3.356 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{9/2}}\,dx$ 1850
3.357 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$ 1854

3.358 $\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}\,dx$ 1858
3.359 $\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}\,dx$ 1862
3.360 $\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}\,dx$ 1866
3.361 $\int (a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}\,dx$ 1870
3.362 $\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$ 1873
3.363 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$ 1877
3.364 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$ 1881
3.365 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$ 1886
3.366 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{9/2}}\,dx$ 1889
3.367 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$ 1893
3.368 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{13/2}}\,dx$ 1897
3.369 $\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{9/2}\,dx$ 1901
3.370 $\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{7/2}\,dx$ 1906
3.371 $\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{5/2}\,dx$ 1910
3.372 $\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}\,dx$ 1914
3.373 $\int (a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}\,dx$ 1918
3.374 $\int \frac{(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$ 1921
3.375 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$ 1925
3.376 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$ 1930
3.377 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$ 1935
3.378 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx$ 1940
3.379 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$ 1943
3.380 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{13/2}}\,dx$ 1947
3.381 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{15/2}}\,dx$ 1951
3.382 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{17/2}}\,dx$ 1955
3.383 $\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$ 1959
3.384 $\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$ 1964
3.385 $\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx$ 1969

3.386 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx$ .1973

3.387 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx$ .1976

3.388 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx$ .1980

3.389 $\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$ .1984

3.390 $\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$ .1989

3.391 $\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$ .1993

3.392 $\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx$ .1997

3.393 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx$ .2000

3.394 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx$ .2004

3.395 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx$ .2008

3.396 $\int \frac{(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$ .2012

3.397 $\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$ .2017

3.398 $\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$ .2022

3.399 $\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$ .2027

3.400 $\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx$ .2030

3.401 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx$ .2033

3.402 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx$ .2037

3.403 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx$ .2041

3.404 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$ .2045

3.405 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3\,dx$ .2049

3.406 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^2\,dx$ .2053

3.407 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))\,dx$ .2057

3.408 $\int \frac{(a+a\sin(e+fx))^m}{c-c\sin(e+fx)}\,dx$ .2061

3.409 $\int \frac{(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^2}\,dx$ .2067

3.410 $\int \frac{(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^3}\,dx$ .2071

3.411 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{5/2}\,dx$ .2075

3.412 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3/2}\,dx$ .2079

3.413 $\int (a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}\,dx$ .2083

3.414 $\int \frac{(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx$ 2086
3.415 $\int \frac{(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{3/2}}\,dx$ 2090
3.416 $\int \frac{(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{5/2}}\,dx$ 2095
3.417 $\int \frac{(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx$ 2099
3.418 $\int \frac{(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx$ 2103
3.419 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}\,dx$ 2107
3.420 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}\,dx$ 2110
3.421 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}\,dx$ 2113
3.422 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-m}\,dx$ 2116
3.423 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1-m}\,dx$ 2120
3.424 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2-m}\,dx$ 2124
3.425 $\int (a+a\sin(e+fx))(c+d\sin(e+fx))^4\,dx$ 2128
3.426 $\int (a+a\sin(e+fx))(c+d\sin(e+fx))^3\,dx$ 2133
3.427 $\int (a+a\sin(e+fx))(c+d\sin(e+fx))^2\,dx$ 2137
3.428 $\int (a+a\sin(e+fx))(c+d\sin(e+fx))\,dx$ 2141
3.429 $\int (a+a\sin(e+fx))\,dx$ 2144
3.430 $\int \frac{a+a\sin(e+fx)}{c+d\sin(e+fx)}\,dx$ 2147
3.431 $\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^2}\,dx$ 2152
3.432 $\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^3}\,dx$ 2157
3.433 $\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^4}\,dx$ 2163
3.434 $\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^4\,dx$ 2171
3.435 $\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^3\,dx$ 2177
3.436 $\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^2\,dx$ 2182
3.437 $\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))\,dx$ 2187
3.438 $\int (a+a\sin(e+fx))^2\,dx$ 2191
3.439 $\int \frac{(a+a\sin(e+fx))^2}{c+d\sin(e+fx)}\,dx$ 2194
3.440 $\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^2}\,dx$ 2199
3.441 $\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^3}\,dx$ 2205
3.442 $\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^4}\,dx$ 2211
3.443 $\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^5}\,dx$ 2218
3.444 $\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^3\,dx$ 2225
3.445 $\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^2\,dx$ 2232

3.446 $\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))\,dx$ 2238
3.447 $\int (a+a\sin(e+fx))^3\,dx$ 2243
3.448 $\int \frac{(a+a\sin(e+fx))^3}{c+d\sin(e+fx)}\,dx$ 2247
3.449 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^2}\,dx$ 2254
3.450 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^3}\,dx$ 2262
3.451 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^4}\,dx$ 2271
3.452 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^5}\,dx$ 2278
3.453 $\int \frac{(c+d\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx$ 2285
3.454 $\int \frac{(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx$ 2294
3.455 $\int \frac{(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$ 2300
3.456 $\int \frac{c+d\sin(e+fx)}{a+a\sin(e+fx)}\,dx$ 2304
3.457 $\int \frac{1}{a+a\sin(e+fx)}\,dx$ 2307
3.458 $\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx$ 2310
3.459 $\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx$ 2315
3.460 $\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx$ 2321
3.461 $\int \frac{(c+d\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx$ 2328
3.462 $\int \frac{(c+d\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx$ 2335
3.463 $\int \frac{(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$ 2345
3.464 $\int \frac{(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$ 2352
3.465 $\int \frac{c+d\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx$ 2357
3.466 $\int \frac{1}{(a+a\sin(e+fx))^2}\,dx$ 2361
3.467 $\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx$ 2365
3.468 $\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx$ 2371
3.469 $\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx$ 2378
3.470 $\int \frac{(c+d\sin(e+fx))^6}{(a+a\sin(e+fx))^3}\,dx$ 2386
3.471 $\int \frac{(c+d\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx$ 2394
3.472 $\int \frac{(c+d\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx$ 2401

3.473 $\int \frac{(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx$ 2411
3.474 $\int \frac{(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$ 2418
3.475 $\int \frac{c+d\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx$ 2423
3.476 $\int \frac{1}{(a+a\sin(e+fx))^3}\,dx$ 2428
3.477 $\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx$ 2432
3.478 $\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx$ 2438
3.479 $\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx$ 2446
3.480 $\int \frac{A+B\sin(x)}{(1+\sin(x))^4}\,dx$ 2456
3.481 $\int \frac{A+B\sin(x)}{(1-\sin(x))^4}\,dx$ 2460
3.482 $\int (a+a\sin(e+fx))(c+d\sin(e+fx))^{5/2}\,dx$ 2464
3.483 $\int (a+a\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx$ 2471
3.484 $\int (a+a\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx$ 2478
3.485 $\int \frac{a+a\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx$ 2484
3.486 $\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^{3/2}}\,dx$ 2489
3.487 $\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^{5/2}}\,dx$ 2495
3.488 $\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^{7/2}}\,dx$ 2501
3.489 $\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^{5/2}\,dx$ 2508
3.490 $\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^{3/2}\,dx$ 2514
3.491 $\int (a+a\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}\,dx$ 2520
3.492 $\int \frac{(a+a\sin(e+fx))^2}{\sqrt{c+d\sin(e+fx)}}\,dx$ 2525
3.493 $\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^{3/2}}\,dx$ 2530
3.494 $\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^{5/2}}\,dx$ 2535
3.495 $\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^{7/2}}\,dx$ 2540
3.496 $\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^{5/2}\,dx$ 2546
3.497 $\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}\,dx$ 2553
3.498 $\int (a+a\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}\,dx$ 2560
3.499 $\int \frac{(a+a\sin(e+fx))^3}{\sqrt{c+d\sin(e+fx)}}\,dx$ 2566
3.500 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^{3/2}}\,dx$ 2572
3.501 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^{5/2}}\,dx$ 2578

3.502 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^{7/2}}\,dx$ 2584
3.503 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^{9/2}}\,dx$ 2591
3.504 $\int \frac{(c+d\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx$ 2598
3.505 $\int \frac{(c+d\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx$ 2604
3.506 $\int \frac{\sqrt{c+d\sin(e+fx)}}{a+a\sin(e+fx)}\,dx$ 2609
3.507 $\int \frac{1}{(a+a\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx$ 2614
3.508 $\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))^{3/2}}\,dx$ 2619
3.509 $\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))^{5/2}}\,dx$ 2624
3.510 $\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx$ 2630
3.511 $\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx$ 2636
3.512 $\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx$ 2642
3.513 $\int \frac{1}{(a+a\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}\,dx$ 2647
3.514 $\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^{3/2}}\,dx$ 2652
3.515 $\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^{5/2}}\,dx$ 2658
3.516 $\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx$ 2665
3.517 $\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx$ 2672
3.518 $\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx$ 2678
3.519 $\int \frac{1}{(a+a\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}}\,dx$ 2684
3.520 $\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}}\,dx$ 2690
3.521 $\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^{5/2}}\,dx$ 2697
3.522 $\int \sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^3\,dx$ 2704
3.523 $\int \sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2\,dx$ 2708
3.524 $\int \sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))\,dx$ 2712
3.525 $\int \sqrt{a+a\sin(e+fx)}\,dx$ 2715
3.526 $\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx$ 2718
3.527 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^2}\,dx$ 2722
3.528 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^3}\,dx$ 2728

3.529 $\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3\,dx$ 2734
3.530 $\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2\,dx$ 2740
3.531 $\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))\,dx$ 2744
3.532 $\int (a+a\sin(e+fx))^{3/2}\,dx$ 2748
3.533 $\int \frac{(a+a\sin(e+fx))^{3/2}}{c+d\sin(e+fx)}\,dx$ 2751
3.534 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^2}\,dx$ 2756
3.535 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^3}\,dx$ 2761
3.536 $\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3\,dx$ 2767
3.537 $\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2\,dx$ 2773
3.538 $\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))\,dx$ 2778
3.539 $\int (a+a\sin(e+fx))^{5/2}\,dx$ 2782
3.540 $\int \frac{(a+a\sin(e+fx))^{5/2}}{c+d\sin(e+fx)}\,dx$ 2786
3.541 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^2}\,dx$ 2791
3.542 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^3}\,dx$ 2796
3.543 $\int \frac{(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx$ 2801
3.544 $\int \frac{(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx$ 2807
3.545 $\int \frac{c+d\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$ 2812
3.546 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx$ 2816
3.547 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))}\,dx$ 2820
3.548 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2}\,dx$ 2826
3.549 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^3}\,dx$ 2832
3.550 $\int \frac{(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx$ 2842
3.551 $\int \frac{(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx$ 2848
3.552 $\int \frac{c+d\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$ 2853
3.553 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}}\,dx$ 2857
3.554 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx$ 2861
3.555 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx$ 2868
3.556 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx$ 2876
3.557 $\int \frac{(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx$ 2888

3.558 $\int \frac{(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx$ 2895
3.559 $\int \frac{c+d\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$ 2900
3.560 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}}\,dx$ 2905
3.561 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx$ 2909
3.562 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx$ 2917
3.563 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx$ 2927
3.564 $\int \sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}\,dx$ 2940
3.565 $\int \sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}\,dx$ 2945
3.566 $\int \sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}\,dx$ 2949
3.567 $\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx$ 2953
3.568 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^{3/2}}\,dx$ 2958
3.569 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^{5/2}}\,dx$ 2962
3.570 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^{7/2}}\,dx$ 2967
3.571 $\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}\,dx$ 2973
3.572 $\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}\,dx$ 2979
3.573 $\int (a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}\,dx$ 2984
3.574 $\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c+d\sin(e+fx)}}\,dx$ 2989
3.575 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{3/2}}\,dx$ 2994
3.576 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{5/2}}\,dx$ 2999
3.577 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{7/2}}\,dx$ 3004
3.578 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{9/2}}\,dx$ 3009
3.579 $\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{5/2}\,dx$ 3015
3.580 $\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}\,dx$ 3021
3.581 $\int (a+a\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}\,dx$ 3027
3.582 $\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c+d\sin(e+fx)}}\,dx$ 3032
3.583 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{3/2}}\,dx$ 3037
3.584 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{5/2}}\,dx$ 3042
3.585 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{7/2}}\,dx$ 3047

3.586 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{9/2}}\,dx$ 3052
3.587 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{11/2}}\,dx$ 3059
3.588 $\int \frac{(c+d\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$ 3067
3.589 $\int \frac{(c+d\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$ 3074
3.590 $\int \frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx$ 3081
3.591 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx$ 3088
3.592 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}\,dx$ 3092
3.593 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{5/2}}\,dx$ 3097
3.594 $\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$ 3104
3.595 $\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$ 3111
3.596 $\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx$ 3118
3.597 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx$ 3123
3.598 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx$ 3128
3.599 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx$ 3135
3.600 $\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$ 3142
3.601 $\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$ 3150
3.602 $\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx$ 3157
3.603 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}}\,dx$ 3164
3.604 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}}\,dx$ 3171
3.605 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{5/2}}\,dx$ 3180
3.606 $\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$ 3188
3.607 $\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^3\,dx$ 3192
3.608 $\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^2\,dx$ 3198
3.609 $\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))\,dx$ 3202
3.610 $\int (a+a\sin(e+fx))^m\,dx$ 3206
3.611 $\int \frac{(a+a\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx$ 3209
3.612 $\int \frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^2}\,dx$ 3213
3.613 $\int \frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^3}\,dx$ 3217

3.614 $\int(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{5/2}\,dx$ .3221
3.615 $\int(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{3/2}\,dx$ .3225
3.616 $\int(a+a\sin(e+fx))^m\sqrt{c+d\sin(e+fx)}\,dx$ .3229
3.617 $\int\frac{(a+a\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}}\,dx$ .3233
3.618 $\int\frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^{3/2}}\,dx$ .3237
3.619 $\int\frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^{5/2}}\,dx$ .3241
3.620 $\int(1+\sin(e+fx))^m(3+5\sin(e+fx))^{-1-m}\,dx$ 3245
3.621 $\int(1+\sin(e+fx))^m(3+4\sin(e+fx))^{-1-m}\,dx$ 3248
3.622 $\int(1+\sin(e+fx))^m(3+3\sin(e+fx))^{-1-m}\,dx$ 3251
3.623 $\int(1+\sin(e+fx))^m(3+2\sin(e+fx))^{-1-m}\,dx$ 3254
3.624 $\int(1+\sin(e+fx))^m(3+\sin(e+fx))^{-1-m}\,dx$ 3257
3.625 $\int 3^{-1-m}(1+\sin(e+fx))^m\,dx$ 3260
3.626 $\int(3-\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx$ 3263
3.627 $\int(3-2\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx$ 3266
3.628 $\int(3-3\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx$ 3269
3.629 $\int(3-4\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx$ 3272
3.630 $\int(3-5\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx$ 3275
3.631 $\int(3+5\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$ 3278
3.632 $\int(3+4\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$ 3281
3.633 $\int(3+3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$ 3284
3.634 $\int(3+2\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$ 3288
3.635 $\int(3+\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$ 3291
3.636 $\int 3^{-1-m}(a+a\sin(e+fx))^m\,dx$ 3294
3.637 $\int(3-\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$ 3297
3.638 $\int(3-2\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$ 3300
3.639 $\int(3-3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$ 3303
3.640 $\int(3-4\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$ 3306
3.641 $\int(3-5\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$ 3309
3.642 $\int(-3+5\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$ 3312
3.643 $\int(-3+4\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$ 3315
3.644 $\int(-3+3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$ 3318
3.645 $\int(-3+2\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$ 3321
3.646 $\int(-3+\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$ 3324
3.647 $\int(-3)^{-1-m}(a+a\sin(e+fx))^m\,dx$ 3327
3.648 $\int(-3-\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$ 3330
3.649 $\int(-3-2\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$ 3333
3.650 $\int(-3-3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$ 3336

3.651 $\int(-3-4\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$ 3340
3.652 $\int(-3-5\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$ 3343
3.653 $\int(d\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$ 3346
3.654 $\int(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx$ 3350
3.655 $\int(a+a\sin(e+fx))^3(c+d\sin(e+fx))^n\,dx$ 3353
3.656 $\int(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$ 3357
3.657 $\int(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx$ 3361
3.658 $\int(c+d\sin(e+fx))^n\,dx$ 3365
3.659 $\int\frac{(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$ 3369
3.660 $\int\frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$ 3373
3.661 $\int\frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx$ 3377
3.662 $\int(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^n\,dx$ 3381
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3.667 $\int\frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{5/2}}\,dx$ 3402
3.668 $\int(a+a\sin(e+fx))\sqrt[3]{c+d\sin(e+fx)}\,dx$ 3406
3.669 $\int\frac{a+a\sin(e+fx)}{\sqrt[3]{c+d\sin(e+fx)}}\,dx$ 3411
3.670 $\int\frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^{4/3}}\,dx$ 3416
3.671 $\int(a+b\sin(e+fx))(c+d\sin(e+fx))^3\,dx$ 3421
3.672 $\int(a+b\sin(e+fx))(c+d\sin(e+fx))^2\,dx$ 3425
3.673 $\int(a+b\sin(e+fx))(c+d\sin(e+fx))\,dx$ 3429
3.674 $\int(a+b\sin(e+fx))\,dx$ 3432
3.675 $\int\frac{a+b\sin(e+fx)}{c+d\sin(e+fx)}\,dx$ 3435
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3.677 $\int\frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^3}\,dx$ 3445
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3.681 $\int(a+b\sin(e+fx))^2\,dx$ 3464
3.682 $\int\frac{(a+b\sin(e+fx))^2}{c+d\sin(e+fx)}\,dx$ 3467
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3.685 $\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^4}\,dx$ 3488
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3.688 $\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))\,dx$ 3509
3.689 $\int (a+b\sin(e+fx))^3\,dx$ 3513
3.690 $\int \frac{(a+b\sin(e+fx))^3}{c+d\sin(e+fx)}\,dx$ 3517
3.691 $\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^2}\,dx$ 3525
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3.712 $\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx$ 3719
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3.714 $\int \frac{(c+d\sin(e+fx))^5}{(a+b\sin(e+fx))^3}\,dx$ 3834
3.715 $\int \frac{(c+d\sin(e+fx))^4}{(a+b\sin(e+fx))^3}\,dx$ 3857
3.716 $\int \frac{(c+d\sin(e+fx))^3}{(a+b\sin(e+fx))^3}\,dx$ 3874
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3.736 $\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^{7/2}}\,dx$ 4421
3.737 $\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))^{5/2}\,dx$ 4427
3.738 $\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}\,dx$ 4434
3.739 $\int (a+b\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}\,dx$ 4441
3.740 $\int \frac{(a+b\sin(e+fx))^3}{\sqrt{c+d\sin(e+fx)}}\,dx$ 4447

3.741 $\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^{3/2}}\,dx$ 4452
3.742 $\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^{5/2}}\,dx$ 4458
3.743 $\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^{7/2}}\,dx$ 4464
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3.747 $\int \frac{\sqrt{c+d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx$ 4490
3.748 $\int \frac{1}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx$ 4494
3.749 $\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))^{3/2}}\,dx$ 4497
3.750 $\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))^{5/2}}\,dx$ 4502
3.751 $\int \frac{(c+d\sin(e+fx))^{7/2}}{(a+b\sin(e+fx))^2}\,dx$ 4509
3.752 $\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+b\sin(e+fx))^2}\,dx$ 4516
3.753 $\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^2}\,dx$ 4523
3.754 $\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^2}\,dx$ 4530
3.755 $\int \frac{1}{(a+b\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}\,dx$ 4537
3.756 $\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))^{3/2}}\,dx$ 4543
3.757 $\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))^{5/2}}\,dx$ 4550
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3.759 $\int \frac{(c+d\sin(e+fx))^{7/2}}{(a+b\sin(e+fx))^3}\,dx$ 4566
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3.762 $\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^3}\,dx$ 4588
3.763 $\int \frac{1}{(a+b\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}}\,dx$ 4595
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3.766 $\int \sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}\,dx$ 4617
3.767 $\int \sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}\,dx$ 4624

3.768 $\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx$ 4629

3.769 $\int \frac{\sqrt{a+b\sin(e+fx)}}{(c+d\sin(e+fx))^{3/2}}\,dx$ 4633

3.770 $\int \frac{\sqrt{a+b\sin(e+fx)}}{(c+d\sin(e+fx))^{5/2}}\,dx$ 4637

3.771 $\int (a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}\,dx$ 4643

3.772 $\int (a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}\,dx$ 4651

3.773 $\int (a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}\,dx$ 4658

3.774 $\int \frac{(a+b\sin(e+fx))^{3/2}}{\sqrt{c+d\sin(e+fx)}}\,dx$ 4664

3.775 $\int \frac{(a+b\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{3/2}}\,dx$ 4670

3.776 $\int \frac{(a+b\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{5/2}}\,dx$ 4676

3.777 $\int (a+b\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{5/2}\,dx$ 4682

3.778 $\int (a+b\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}\,dx$ 4690

3.779 $\int (a+b\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}\,dx$ 4698

3.780 $\int \frac{(a+b\sin(e+fx))^{5/2}}{\sqrt{c+d\sin(e+fx)}}\,dx$ 4705

3.781 $\int \frac{(a+b\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{3/2}}\,dx$ 4712

3.782 $\int \frac{(a+b\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{5/2}}\,dx$ 4719

3.783 $\int \frac{(c+d\sin(e+fx))^{5/2}}{\sqrt{a+b\sin(e+fx)}}\,dx$ 4726

3.784 $\int \frac{(c+d\sin(e+fx))^{3/2}}{\sqrt{a+b\sin(e+fx)}}\,dx$ 4733

3.785 $\int \frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}}\,dx$ 4739

3.786 $\int \frac{1}{\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}\,dx$ 4743

3.787 $\int \frac{1}{\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}}\,dx$ 4747

3.788 $\int \frac{1}{\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}}\,dx$ 4751

3.789 $\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+b\sin(e+fx))^{3/2}}\,dx$ 4757

3.790 $\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx$ 4764

3.791 $\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx$ 4770

3.792 $\int \frac{1}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx$ 4774

3.793 $\int \frac{1}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx$ 4778

3.794 $\int \frac{1}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx$ 4784

3.795 $\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+b\sin(e+fx))^{5/2}}\,dx$ 4790

3.796 $\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{5/2}}\,dx$ 4797

3.797 $\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{5/2}}\,dx$ 4803

3.798 $\int \frac{1}{(a+b\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}}\,dx$ 4809

3.799 $\int \frac{1}{(a+b\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}}\,dx$ 4815

3.800 $\int \frac{1}{(a+b\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{5/2}}\,dx$ 4822

3.801 $\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$ 4829

3.802 $\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^2\,dx$ 4832

3.803 $\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))\,dx$ 4836

3.804 $\int (a+b\sin(e+fx))^m\,dx$ 4840

3.805 $\int \frac{(a+b\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx$ 4844

3.806 $\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^2}\,dx$ 4847

3.807 $\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^3}\,dx$ 4850

3.808 $\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{5/2}\,dx$ 4853

3.809 $\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{3/2}\,dx$ 4856

3.810 $\int (a+b\sin(e+fx))^m\sqrt{c+d\sin(e+fx)}\,dx$ 4859

3.811 $\int \frac{(a+b\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}}\,dx$ 4862

3.812 $\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{3/2}}\,dx$ 4865

3.813 $\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{5/2}}\,dx$ 4868

3.814 $\int (d\csc(e+fx))^n(a+a\sin(e+fx))^3\,dx$ 4871

3.815 $\int (d\csc(e+fx))^n(a+a\sin(e+fx))^2\,dx$ 4876

3.816 $\int (d\csc(e+fx))^n(a+a\sin(e+fx))\,dx$ 4880

3.817 $\int \frac{(d\csc(e+fx))^n}{a+a\sin(e+fx)}\,dx$ 4884

3.818 $\int \frac{(d\csc(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$ 4888

3.819 $\int \left(c(d\sin(e+fx))^p\right)^n(a+a\sin(e+fx))^m\,dx$ 4893

3.820 $\int \left(c(d\sin(e+fx))^p\right)^n(a+a\sin(e+fx))^3\,dx$ 4898

3.821 $\int \left(c(d\sin(e+fx))^p\right)^n(a+a\sin(e+fx))^2\,dx$ 4903

3.822 $\int \left(c(d\sin(e+fx))^p\right)^n(a+a\sin(e+fx))\,dx$ 4907

3.823 $\int \frac{\left(c(d\sin(e+fx))^p\right)^n}{a+a\sin(e+fx)}\,dx$ 4911

3.824 $\int \frac{\left(c(d\sin(e+fx))^p\right)^n}{(a+a\sin(e+fx))^2}\,dx$ 4915

3.825 $\int (d\csc(e+fx))^n(a+b\sin(e+fx))^3\,dx$ 4920
3.826 $\int (d\csc(e+fx))^n(a+b\sin(e+fx))^2\,dx$ 4925
3.827 $\int (d\csc(e+fx))^n(a+b\sin(e+fx))\,dx$ 4929
3.828 $\int \frac{(d\csc(e+fx))^n}{a+b\sin(e+fx)}\,dx$ 4933
3.829 $\int \frac{(d\csc(e+fx))^n}{(a+b\sin(e+fx))^2}\,dx$ 4938
3.830 $\int \frac{(d\csc(e+fx))^n}{(a+b\sin(e+fx))^3}\,dx$ 4944
3.831 $\int \left(c(d\sin(e+fx))^p\right)^n(a+b\sin(e+fx))^m\,dx$ 4950
3.832 $\int \left(c(d\sin(e+fx))^p\right)^n(a+b\sin(e+fx))^3\,dx$ 4953
3.833 $\int \left(c(d\sin(e+fx))^p\right)^n(a+b\sin(e+fx))^2\,dx$ 4958
3.834 $\int \left(c(d\sin(e+fx))^p\right)^n(a+b\sin(e+fx))\,dx$ 4962
3.835 $\int \frac{\left(c(d\sin(e+fx))^p\right)^n}{a+b\sin(e+fx)}\,dx$ 4966
3.836 $\int \frac{\left(c(d\sin(e+fx))^p\right)^n}{(a+b\sin(e+fx))^2}\,dx$ 4971
3.837 $\int \frac{\left(c(d\sin(e+fx))^p\right)^n}{(a+b\sin(e+fx))^3}\,dx$ 4977

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Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [837]. This is test number [73].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (837)	% 0.00 (0)
Mathematica	% 97.97 (820)	% 2.03 (17)
Maple	% 75.87 (635)	% 24.13 (202)
Maxima	% 25.93 (217)	% 74.07 (620)
Fricas	% 61.41 (514)	% 38.59 (323)
Sympy	% 18.28 (153)	% 81.72 (684)
Giac	% 36.08 (302)	% 63.92 (535)
Mupad	% 41.10 (344)	% 58.90 (493)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

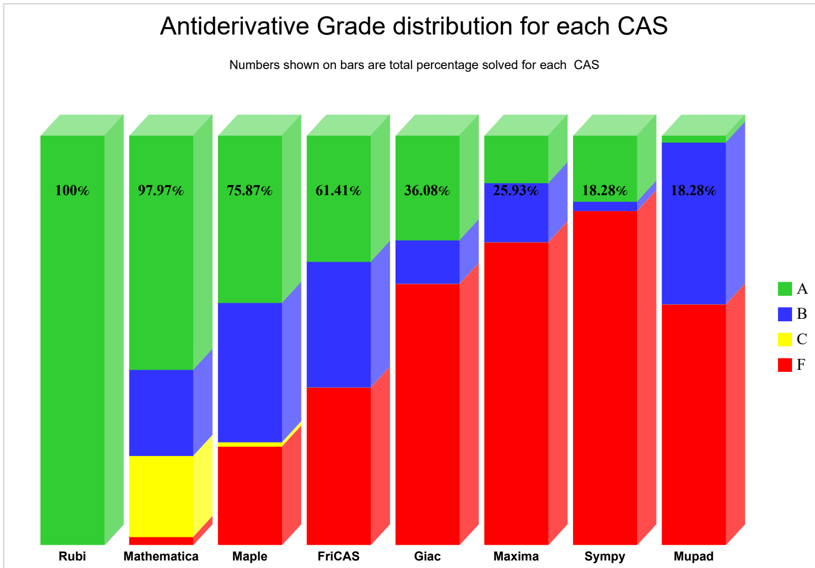
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

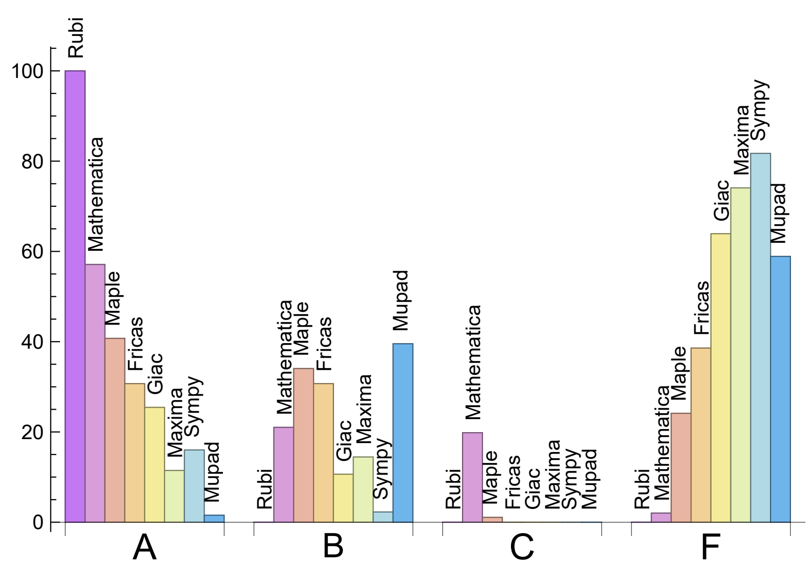
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	57.11	21.03	19.83	2.03
Maple	40.74	34.05	1.08	24.13
Maxima	11.47	14.46	0.00	74.07
Fricas	30.70	30.70	0.00	38.59
Sympy	16.01	2.27	0.00	81.72
Giac	25.45	10.63	0.00	63.92
Mupad	1.55	39.55	0.00	58.90

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	17	88.24 %	11.76 %	0.00 %
Maple	202	92.08 %	0.99 %	6.93 %
Maxima	620	81.61 %	4.03 %	14.35 %
Fricas	323	87.31 %	12.38 %	0.31 %
Sympy	684	49.85 %	49.71 %	0.44 %
Giac	535	63.55 %	13.64 %	22.80 %
Mupad	493	99.19 %	0.81 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

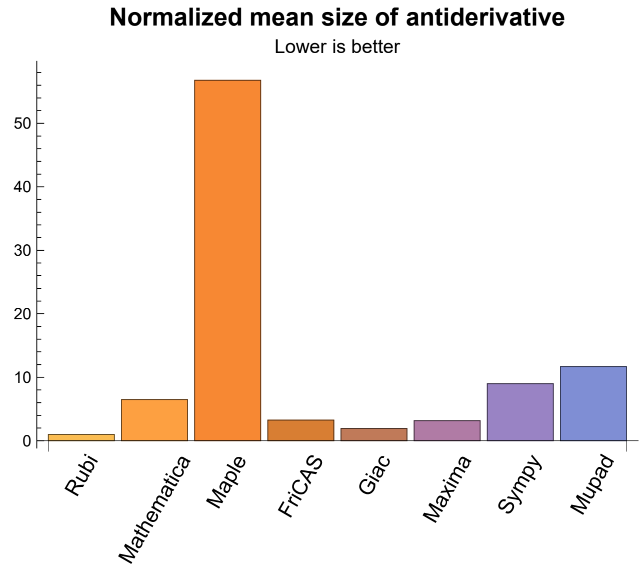
1.3 Performance

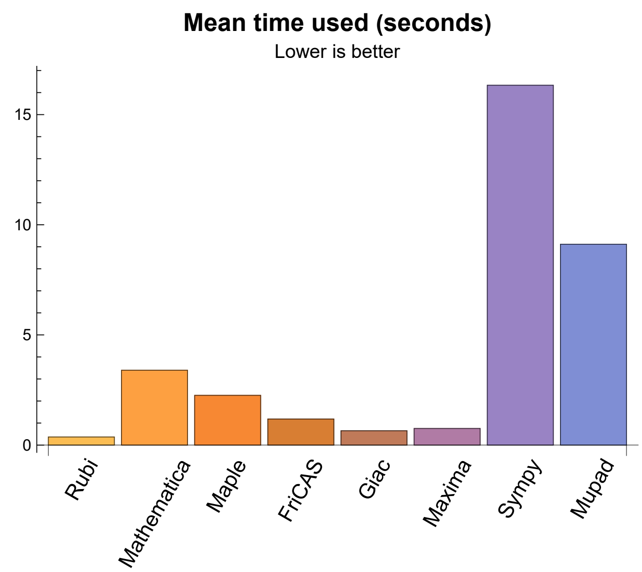
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.37	170.11	0.99	122.00	1.00
Mathematica	3.40	1685.92	6.49	174.00	1.46
Maple	2.26	37312.34	56.78	193.00	1.76
Maxima	0.75	307.66	3.16	175.00	2.23
Fricas	1.18	490.88	3.25	215.50	2.21
Sympy	16.33	947.22	8.98	398.00	4.72
Giac	0.65	271.22	1.93	133.50	1.44
Mupad	9.12	3943.13	11.69	149.50	2.01

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

$\{221, 226, 801, 805, 806, 807, 808, 809, 810, 811, 812, 813, 831\}$

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {665, 666, 667}

Mathematica {97, 102, 103, 104, 109, 115, 116, 117, 118, 119, 120, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 141, 142, 143, 145, 146, 148, 165, 174, 211, 216, 217, 218, 224, 225, 404, 406, 407, 408, 409, 410, 415, 416, 422, 423, 424, 482, 483, 484, 485, 486, 487, 488, 542, 563, 579, 580, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 624, 627, 629, 630, 631, 632, 635, 638, 640, 641, 642, 643, 645, 646, 648, 652, 653, 654, 658, 665, 666, 667, 668, 669, 670, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 787, 788, 789, 790, 792, 793, 794, 795, 796, 797, 798, 799, 800, 803, 804, 814, 816, 819, 822, 828, 829, 830, 835, 836, 837}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

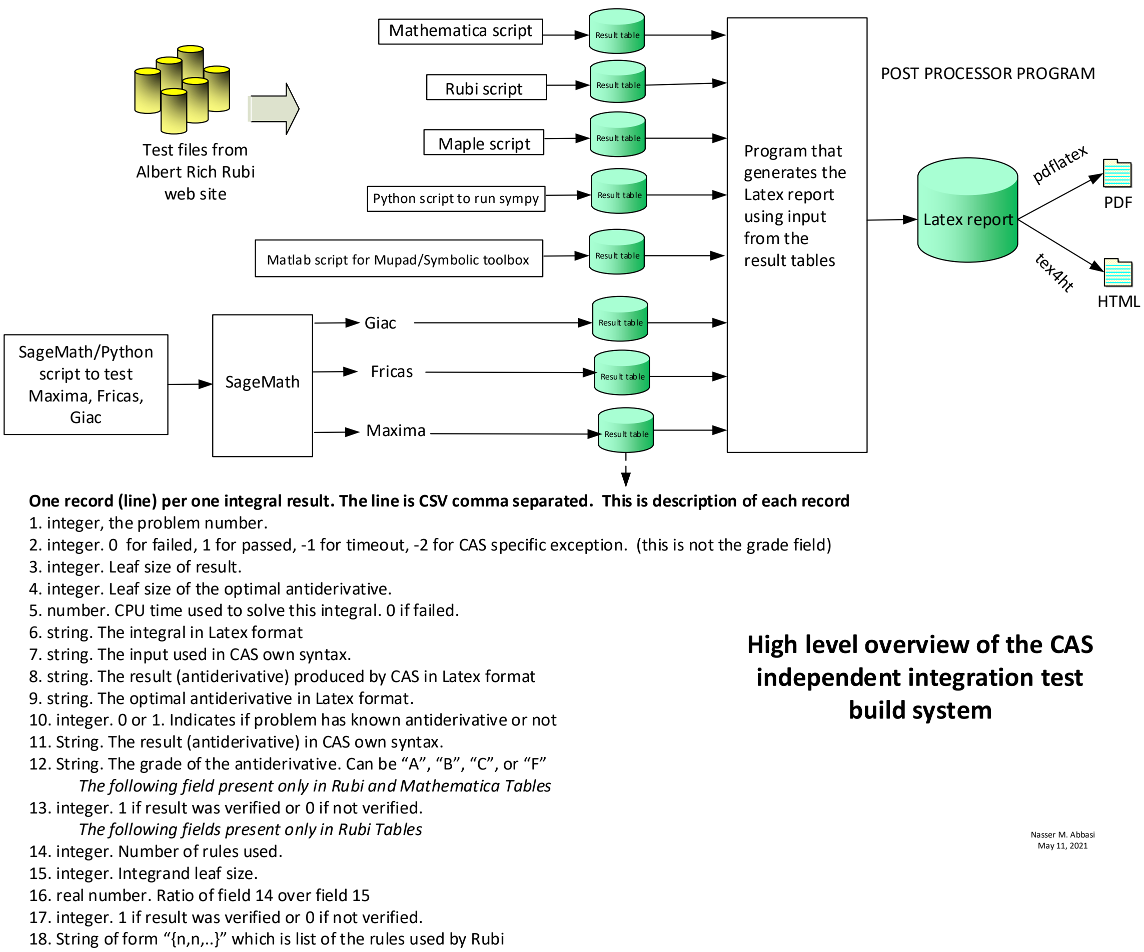
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 5, 10, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 27, 32, 33, 34, 35, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 59, 91, 92, 93, 94, 95, 104, 105, 106, 107, 110, 111, 112, 113, 144, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 160, 164, 166, 167, 168, 169, 170, 171, 172, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 207, 208, 209, 212, 213, 214, 215, 219, 220, 221, 224, 225, 226, 227, 228, 229, 230, 233, 234, 235, 236, 237, 238, 239, 240, 242, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 257, 258, 259, 260, 261, 262, 265, 266, 267, 268, 269, 270, 272, 274, 275, 276, 277, 278, 279, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 294, 295, 296, 297, 299, 300, 307, 309, 317, 318, 319, 320, 324, 325, 326, 327, 332, 333, 334, 335, 340, 341, 342, 343, 348, 349, 350, 351, 352, 353, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 381, 382, 383, 384, 386, 387, 388, 389, 390, 391, 393, 394, 395, 396, 397, 398, 401, 402, 403, 411, 412, 413, 419, 420, 425, 426, 427, 428, 429, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 458, 459, 460, 462, 463, 465, 466, 467, 468, 469, 474, 475, 476, 477, 478, 480, 481, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 529, 530, 531, 532, 535, 536, 537, 538, 539, 542, 568, 569, 570, 571, 572, 573, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 599, 604, 610, 621, 622, 623, 624, 625, 626, 627, 632, 636, 640, 643, 645, 646, 647, 648, 649, 651, 654, 658, 662, 663, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 717, 718, 719, 720, 721, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 747, 748, 768, 769, 771, 777, 778, 785, 786, 791, 801, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 820, 821, 823, 824, 825, 826, 827, 831, 832, 833, 834 }

B grade: { 4, 6, 7, 8, 9, 11, 17, 18, 19, 20, 28, 29, 30, 31, 36, 37, 38, 39, 40, 41, 42, 43, 49, 50, 58, 60, 118, 119, 122, 123, 126, 127, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 153, 161, 162, 163, 165, 173, 174, 216, 217, 218, 231, 232, 241, 243, 255, 256, 263, 264, 271, 273, 280, 281, 282, 293, 298, 301, 308, 310, 328, 336, 345, 346, 347, 354, 365, 378, 379, 380, 392, 399, 400, 414, 417, 418, 421, 456, 457, 461, 464, 471, 472, 473, 479, 525, 533, 534, 540, 541, 574, 575, 591, 592, 593, 596, 597, 598, 601, 602, 603, 605, 606, 607, 608, 611, 612, 613, 614, 615, 616, 617, 618, 619, 628, 629, 633, 634, 635, 637, 638, 639, 644, 650, 665, 666, 667, 668, 669, 670, 692, 715, 716, 722, 765, 766, 770, 772, 773, 775, 776, 779, 780, 781, 782, 783, 787, 788, 789, 790, 792, 793, 794, 795, 796, 797, 798, 799, 800, 819, 828, 829, 830, 835, 836, 837 }

C grade: { 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 97, 98, 99, 100, 101, 102, 103, 109, 115, 116, 117, 120, 121, 124, 125, 128, 129, 141, 142, 143, 145, 146, 206, 210, 211, 302, 303, 304, 305, 306, 311, 312, 313, 314, 315, 316, 321, 322, 323, 329, 330, 331, 337, 338, 339, 344, 385, 404, 406, 407, 408, 409, 410, 415, 416, 422, 423, 424, 430, 431, 432, 433, 470, 482, 483, 484, 485, 486, 487, 488, 526, 527, 528, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 588, 589, 590, 594, 595, 600, 609, 620, 630, 631, 641, 642, 652, 653, 714, 745, 746, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 767, 774, 784, 816, 822 }

F grade: { 96, 108, 114, 140, 222, 223, 405, 655, 656, 657, 659, 660, 661, 664, 802, 817, 818 }

2.1.3 Maple

A grade: { 1, 2, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 88, 90, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 205, 206, 207, 208, 209, 210, 221, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 239, 240, 241, 242, 244, 245, 246, 247, 250, 252, 254, 255, 258, 259, 260, 261, 263, 264, 266, 267, 268, 269, 270, 271, 272, 274, 276, 277, 278, 279, 280, 281, 283, 284, 285, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 342, 343, 345, 348, 349, 350, 351, 355, 358, 359, 360, 367, 369, 370, 371, 372, 381, 382, 387, 392, 393, 394, 395, 402, 403, 425, 426, 427, 428, 429, 431, 434, 435, 436, 437, 438, 446, 447, 456, 457, 458, 459, 465, 466, 467, 468, 474, 475, 476, 477, 478, 480, 481, 485, 486, 493, 506, 507, 513, 519, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 536, 537, 538, 539, 540, 543, 544, 545, 546, 547, 553, 671, 672, 673, 674, 675, 678, 679, 680, 681, 686, 687, 688, 689, 695, 697, 701, 702, 703, 710, 726, 746, 747, 748, 755, 763, 801, 805, 806, 807, 808, 809, 810, 811, 812, 813, 831 }

B grade: { 3, 4, 12, 37, 41, 42, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 176, 187, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 212, 238, 243, 248, 249, 251, 253, 256, 257, 262, 273, 282, 287, 340, 341, 344, 346, 347, 352, 353, 354, 356, 357, 361, 362, 363, 364, 365, 366, 368, 373, 374, 375, 376, 377, 378, 379, 380, 383, 384, 385, 386, 388, 389, 390, 391, 396, 397, 398, 399, 400, 401, 430, 432, 433, 439, 440, 441, 442, 443, 444, 445, 448, 449, 450, 451, 452, 453, 454, 455, 460, 461, 462, 463, 464, 469, 470, 471, 472, 473, 479, 482, 483, 484, 487, 488, 489, 490, 491, 492, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 508, 509, 510, 511, 512, 514, 515, 516, 517, 518, 520, 521, 534, 535, 541, 542, 548, 549, 550, 551, 552, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 567, 568, 569, 570, 575, 576, 577, 578, 584, 585, 586, 587, 590, 591, 592, 593, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 676, 677, 682, 683, 684, 685, 690, 691, 692, 693, 694, 696, 698, 699, 700, 704, 705, 706, 707, 708, 709, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 749, 750, 751, 752, 753, 754, 756, 757, 758, 759, 760, 761, 762, 764, 769, 770, 771, 774, 775, 776, 777, 778, 780, 781, 783, 784, 786, 787, 788, 789, 790, 791, 792, 793, 794, 796, 797, 798, 799, 800 }

C grade: { 211, 765, 766, 767, 768, 772, 773, 779, 785 }

F grade: { 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 265, 275, 286, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 564, 565, 566, 571, 572, 573, 574, 579, 580, 581, 582, 583, 588, 589, 594, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 782, 795, 802, 803, 804, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 832, 833, 834, 835, 836, 837 }

2.1.4 Maxima

A grade: { 1, 2, 6, 7, 8, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 221, 226, 227, 228, 229, 230, 236, 237, 239, 240, 247, 250, 252, 265, 275, 286, 344, 353, 364, 377, 385, 391, 398, 411, 425, 426, 427, 428, 429, 434, 435, 436, 437, 438, 445, 446, 447, 457, 622, 633, 650, 671, 672, 673, 674, 678, 679, 680, 681, 686, 687, 688, 689, 697, 801, 805, 806, 808, 809, 810, 811, 812, 813, 831 }

B grade: { 3, 4, 5, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 85, 231, 232, 233, 234, 235, 238, 241, 242, 243, 244, 245, 246, 248, 249, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 266, 267, 268, 269, 270, 271, 272, 273, 274, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 317, 318, 319, 320, 324, 325, 326, 327, 328, 332, 333, 334, 335, 336, 412, 413, 444, 453, 454, 455, 456, 461, 462, 463, 464, 465, 466, 470, 471, 472, 473, 474, 475, 476, 480, 481, 568, 569, 570, 576, 577, 578, 585, 586, 587 }

C grade: { }

F grade: { 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 321, 322, 323, 329, 330, 331, 337, 338, 339, 340, 341, 342, 343, 345, 346, 347, 348, 349, 350, 351, 352, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 378, 379, 380, 381, 382, 383, 384, 386, 387, 388, 389, 390, 392, 393, 394, 395, 396, 397, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 430, 431, 432, 433, 439, 440, 441, 442, 443, 448, 449, 450, 451, 452, 458, 459, 460, 467, 468, 469, 477, 478, 479, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 571, 572, 573, 574, 575, 579, 580, 581, 582, 583, 584, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 675, 676, 677, 682, 683, 684, 685, 690, 691, 692, 693, 694, 695, 696, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 802, 803, 804, 807, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 832, 833, 834, 835, 836, 837 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 12, 16, 21, 22, 26, 32, 33, 34, 35, 37, 41, 42, 43, 44, 45, 46, 47, 52, 53, 54, 55, 61, 64, 87, 88, 89, 90, 149, 150, 151, 152, 157, 158, 159, 160, 161, 164, 166, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 185, 186, 188, 189, 221, 226, 227, 228, 229, 230, 231, 236, 237, 238, 239, 240, 241, 247, 248, 249, 250, 251, 252, 253, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 274, 275, 276, 277, 278, 279, 284, 285, 286, 287, 288, 289, 290, 291, 292, 298, 317, 318, 319, 320, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 342, 343, 345, 346, 348, 349, 350, 351, 355, 356, 357, 358, 359, 360, 366, 367, 368, 369, 370, 371, 372, 380, 381, 382, 386, 387, 388, 392, 393, 394, 395, 400, 401, 402, 403, 411, 412, 413, 419, 420, 421, 425, 426, 427, 428, 429, 430, 431, 434, 435, 436, 437, 438, 439, 440, 444, 445, 446, 447, 448, 449, 453, 456, 457, 465, 466, 475, 476, 522, 523, 524, 526, 529, 530, 531, 532, 536, 537, 538, 539, 546, 591, 622, 628, 633, 639, 644, 650, 671, 672, 673, 674, 675, 676, 678, 679, 680, 681, 682, 686, 687, 688, 689, 690, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 709, 710, 801, 805, 806, 807, 808, 809, 810, 811, 812, 813, 831 }

B grade: { 8, 9, 10, 11, 13, 14, 15, 17, 18, 19, 20, 23, 24, 25, 27, 28, 29, 30, 31, 36, 38, 39, 40, 48, 49, 50, 51, 56, 57, 58, 59, 60, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 153, 154, 155, 156, 162, 163, 165, 172, 182, 183, 184, 187, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 232, 233, 234, 235, 242, 243, 244, 245, 246, 254, 255, 256, 257, 258, 259, 260, 271, 272, 273, 280, 281, 282, 283, 293, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 321, 322, 340, 341, 347, 354, 361, 365, 373, 378, 379, 399, 432, 433, 441, 442, 443, 450, 451, 452, 454, 455, 458, 459, 460, 461, 462, 463, 464, 467, 468, 469, 470, 471, 472, 473, 474, 477, 478, 479, 480, 481, 525, 527, 528, 533, 534, 535, 540, 541, 542, 543, 544, 545, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 677, 683, 684, 685, 691, 692, 693, 704, 706, 707, 708, 711, 714, 715, 716, 717, 718, 719 }

C grade: { }

F grade: { 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 344, 352, 353, 362, 363, 364, 374, 375, 376, 377, 383, 384, 385, 389, 390, 391, 396, 397, 398, 404, 405, 406, 407, 408, 409, 410, 414, 415, 416, 417, 418, 422, 423, 424, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 623, 624, 625, 626, 627, 629, 630, 631, 632, 634, 635, 636, 637, 638, 640, 641, 642, 643, 645, 646, 647, 648, 649, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 705, 712, 713, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 802, 803, 804, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 832, 833, 834, 835, 836, 837 }

2.1.6 Sympy

A grade: { 1, 2, 7, 149, 150, 151, 152, 157, 158, 159, 160, 166, 167, 168, 169, 175, 179, 180, 221, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 425, 426, 427, 428, 429, 430, 434, 435, 436, 437, 438, 444, 445, 446, 447, 453, 454, 455, 456, 457, 462, 463, 464, 465, 466, 472, 473, 474, 475, 476, 671, 672, 673, 674, 675, 678, 679, 680, 681, 686, 687, 688, 689, 694, 695, 697, 701, 702, 810, 811, 812, 831 }

B grade: { 3, 4, 5, 6, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 27, 153, 480, 481 }

C grade: { }

F grade: { 8, 9, 10, 11, 17, 18, 19, 20, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 154, 155, 156, 161, 162, 163, 164, 165, 170, 171, 172, 173, 174, 176, 177, 178, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 260, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 431, 432, 433, 439, 440, 441, 442, 443, 448, 449, 450, 451, 452, 458, 459, 460, 461, 467, 468, 469, 470, 471, 477, 478, 479, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 676, 677, 682, 683, 684, 685, 690, 691, 692, 693, 696, 698, 699, 700, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 832, 833, 834, 835, 836, 837 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 39, 40, 47, 149, 150, 151, 152, 157, 158, 159, 160, 166, 167, 168, 169, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 199, 200, 221, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 288, 289, 425, 426, 427, 428, 429, 430, 431, 434, 435, 436, 437, 438, 439, 440, 444, 445, 446, 447, 448, 453, 454, 456, 457, 458, 462, 463, 464, 465, 466, 467, 468, 474, 475, 476, 478, 480, 481, 622, 671, 672, 673, 674, 675, 676, 678, 679, 680, 681, 682, 686, 687, 688, 689, 690, 694, 695, 697, 699, 700, 701, 702, 703, 704, 706, 709, 710, 711, 801, 805, 806, 807, 808, 809, 810, 811, 812, 813, 831 }

B grade: { 9, 37, 38, 41, 42, 43, 44, 45, 46, 48, 52, 53, 54, 55, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 193, 197, 198, 201, 202, 256, 257, 287, 432, 433, 441, 442, 443, 449, 450, 451, 452, 455, 459, 460, 461, 469, 470, 471, 472, 473, 477, 479, 633, 650, 677, 683, 684, 685, 691, 692, 693, 696, 698, 705, 707, 708, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722 }

C grade: { }

F grade: { 49, 50, 51, 56, 57, 58, 59, 60, 65, 75, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 153, 154, 155, 156, 161, 162, 163, 164, 165, 170, 171, 172, 173, 174, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 802, 803, 804, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 832, 833, 834, 835, 836, 837 }

2.1.8 Mupad

A grade: { 221, 226, 801, 805, 806, 807, 808, 809, 810, 811, 812, 813, 831 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 36, 63, 64, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 204, 207, 208, 220, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 319, 320, 326, 327, 328, 333, 334, 335, 336, 340, 341, 342, 343, 346, 347, 348, 349, 350, 351, 355, 356, 357, 358, 359, 360, 361, 366, 367, 368, 369, 370, 371, 372, 373, 379, 380, 381, 382, 392, 399, 400, 411, 412, 413, 419, 420, 421, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 485, 525, 545, 546, 568, 569, 570, 576, 577, 578, 585, 586, 587, 622, 628, 633, 639, 644, 650, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 726 }

C grade: { }

F grade: { 32, 33, 34, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 203, 205, 206, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 222, 223, 224, 225, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 321, 322, 323, 324, 325, 329, 330, 331, 332, 337, 338, 339, 344, 345, 352, 353, 354, 362, 363, 364, 365, 374, 375, 376, 377, 378, 383, 384, 385, 386, 387, 388, 389, 390, 391, 393, 394, 395, 396, 397, 398, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 414, 415, 416, 417, 418, 422, 423, 424, 482, 483, 484, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 571, 572, 573, 574, 575, 579, 580, 581, 582, 583, 584, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 623, 624, 625, 626, 627, 629, 630, 631, 632, 634, 635, 636, 637, 638, 640, 641, 642, 643, 645, 646, 647, 648, 649, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 723, 724, 725, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 802, 803, 804, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 832, 833, 834, 835, 836, 837 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	105	96	95	83	221	94	225
normalized size	1	1.00	1.03	0.94	0.93	0.81	2.17	0.92	2.21
time (sec)	N/A	0.102	0.455	0.278	0.449	0.507	3.659	0.717	10.282

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	115	143	143	96	379	112	294
normalized size	1	1.00	0.89	1.11	1.11	0.74	2.94	0.87	2.28
time (sec)	N/A	0.145	0.529	0.343	0.428	0.473	6.816	1.929	10.326

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	101	121	180	70	1221	67	78
normalized size	1	1.00	1.91	2.28	3.40	1.32	23.04	1.26	1.47
time (sec)	N/A	0.069	0.130	0.077	0.834	0.451	5.910	0.912	6.818

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	87	100	128	53	665	56	59
normalized size	1	1.00	2.07	2.38	3.05	1.26	15.83	1.33	1.40
time (sec)	N/A	0.049	0.085	0.072	0.649	0.471	2.788	1.951	6.911

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	48	40	78	35	221	44	46
normalized size	1	1.00	1.78	1.48	2.89	1.30	8.19	1.63	1.70
time (sec)	N/A	0.066	0.065	0.070	0.492	0.482	1.347	0.288	6.770

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	42	25	32	28	34	19	19
normalized size	1	1.00	2.47	1.47	1.88	1.65	2.00	1.12	1.12
time (sec)	N/A	0.029	0.041	0.074	0.824	0.461	0.845	0.924	6.539

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	29	14	16	22	10	13	13
normalized size	1	1.00	2.42	1.17	1.33	1.83	0.83	1.08	1.08
time (sec)	N/A	0.010	0.026	0.055	0.542	0.436	0.409	0.320	0.021

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	74	24	31	53	0	24	23
normalized size	1	1.00	3.70	1.20	1.55	2.65	0.00	1.20	1.15
time (sec)	N/A	0.038	0.055	0.079	0.847	0.441	0.000	0.240	6.423

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	63	45	68	91	0	53	49
normalized size	1	1.00	2.42	1.73	2.62	3.50	0.00	2.04	1.88
time (sec)	N/A	0.060	0.161	0.099	0.942	0.483	0.000	0.688	6.714

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	83	67	97	134	0	73	69
normalized size	1	1.00	1.98	1.60	2.31	3.19	0.00	1.74	1.64
time (sec)	N/A	0.066	0.350	0.099	0.412	0.563	0.000	0.312	6.598

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	113	89	120	168	0	96	89
normalized size	1	1.00	2.05	1.62	2.18	3.05	0.00	1.75	1.62
time (sec)	N/A	0.070	0.813	0.099	0.569	0.458	0.000	0.308	6.497

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	100	126	198	105	1423	72	77
normalized size	1	1.00	1.52	1.91	3.00	1.59	21.56	1.09	1.17
time (sec)	N/A	0.121	0.253	0.102	1.241	0.444	10.918	0.335	6.813

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	84	66	144	95	779	51	62
normalized size	1	1.00	1.79	1.40	3.06	2.02	16.57	1.09	1.32
time (sec)	N/A	0.143	0.237	0.096	0.870	0.462	6.794	0.384	6.465

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	69	51	90	82	321	35	34
normalized size	1	1.00	1.97	1.46	2.57	2.34	9.17	1.00	0.97
time (sec)	N/A	0.073	0.129	0.099	0.968	0.448	4.264	0.825	6.482

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	27	62	60	87	21	21
normalized size	1	1.00	0.88	0.82	1.88	1.82	2.64	0.64	0.64
time (sec)	N/A	0.031	0.044	0.093	0.603	0.449	2.134	1.950	6.299

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	35	74	58	134	29	29
normalized size	1	1.00	0.94	1.06	2.24	1.76	4.06	0.88	0.88
time (sec)	N/A	0.021	0.028	0.078	0.634	0.470	1.019	1.789	6.306

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	129	50	89	117	0	40	38
normalized size	1	1.00	3.39	1.32	2.34	3.08	0.00	1.05	1.00
time (sec)	N/A	0.088	0.141	0.119	0.999	0.481	0.000	1.703	6.482

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	166	71	126	168	0	69	91
normalized size	1	1.00	3.69	1.58	2.80	3.73	0.00	1.53	2.02
time (sec)	N/A	0.135	0.375	0.129	0.922	0.463	0.000	0.303	6.540

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	203	92	155	220	0	93	111
normalized size	1	1.00	3.17	1.44	2.42	3.44	0.00	1.45	1.73
time (sec)	N/A	0.145	0.664	0.154	0.682	0.474	0.000	0.968	6.399

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	71	238	115	178	266	0	114	101
normalized size	1	1.09	3.66	1.77	2.74	4.09	0.00	1.75	1.55
time (sec)	N/A	0.150	3.594	0.145	1.040	0.487	0.000	0.284	6.396

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	191	174	306	158	3288	99	110
normalized size	1	1.00	1.89	1.72	3.03	1.56	32.55	0.98	1.09
time (sec)	N/A	0.226	0.105	0.105	0.844	0.508	50.536	0.966	7.022

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	170	152	252	145	2259	88	93
normalized size	1	1.00	1.89	1.69	2.80	1.61	25.10	0.98	1.03
time (sec)	N/A	0.208	0.080	0.138	0.772	0.509	33.591	0.378	6.670

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	140	79	198	132	1425	67	78
normalized size	1	1.00	1.97	1.11	2.79	1.86	20.07	0.94	1.10
time (sec)	N/A	0.221	0.080	0.104	0.931	0.488	19.640	0.884	6.891

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	112	77	144	119	777	51	50
normalized size	1	1.00	1.90	1.31	2.44	2.02	13.17	0.86	0.85
time (sec)	N/A	0.157	0.181	0.105	1.025	0.437	11.815	0.327	6.740

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	47	37	104	90	206	29	29
normalized size	1	1.00	0.94	0.74	2.08	1.80	4.12	0.58	0.58
time (sec)	N/A	0.076	0.065	0.093	0.644	0.419	6.921	0.268	6.637

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	41	45	116	88	277	37	37
normalized size	1	1.00	0.82	0.90	2.32	1.76	5.54	0.74	0.74
time (sec)	N/A	0.046	0.042	0.094	0.688	0.480	4.741	0.319	6.845

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	45	57	128	92	348	45	45
normalized size	1	1.00	0.90	1.14	2.56	1.84	6.96	0.90	0.90
time (sec)	N/A	0.035	0.061	0.082	0.457	0.425	2.226	0.495	6.633

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	160	76	143	168	0	56	54
normalized size	1	1.00	2.76	1.31	2.47	2.90	0.00	0.97	0.93
time (sec)	N/A	0.161	0.072	0.121	0.861	0.469	0.000	0.759	6.651

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	206	97	180	225	0	85	129
normalized size	1	1.00	3.17	1.49	2.77	3.46	0.00	1.31	1.98
time (sec)	N/A	0.229	0.152	0.141	0.750	0.513	0.000	0.638	6.725

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	247	119	209	276	0	109	97
normalized size	1	1.00	2.87	1.38	2.43	3.21	0.00	1.27	1.13
time (sec)	N/A	0.235	0.464	0.158	0.688	0.508	0.000	0.249	6.390

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	299	141	232	333	0	128	117
normalized size	1	1.00	2.90	1.37	2.25	3.23	0.00	1.24	1.14
time (sec)	N/A	0.245	0.928	0.154	0.674	0.506	0.000	0.281	6.687

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	165	83	0	132	0	159	-1
normalized size	1	1.00	1.04	0.53	0.00	0.84	0.00	1.01	-0.01
time (sec)	N/A	0.230	0.496	0.803	0.000	0.532	0.000	0.582	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	141	73	0	111	0	129	-1
normalized size	1	1.00	1.16	0.60	0.00	0.91	0.00	1.06	-0.01
time (sec)	N/A	0.169	0.288	0.675	0.000	0.518	0.000	0.946	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	117	63	0	92	0	99	-1
normalized size	1	1.00	1.36	0.73	0.00	1.07	0.00	1.15	-0.01
time (sec)	N/A	0.111	0.182	0.698	0.000	0.493	0.000	0.445	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	81	51	0	67	0	68	-1
normalized size	1	1.00	1.45	0.91	0.00	1.20	0.00	1.21	-0.02
time (sec)	N/A	0.045	0.117	0.628	0.000	0.477	0.000	1.392	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	65	43	0	50	0	36	33
normalized size	1	1.00	2.50	1.65	0.00	1.92	0.00	1.38	1.27
time (sec)	N/A	0.013	0.034	0.486	0.000	0.433	0.000	0.803	0.211

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	94	68	0	219	0	67	-1
normalized size	1	1.00	2.54	1.84	0.00	5.92	0.00	1.81	-0.03
time (sec)	N/A	0.055	0.095	0.494	0.000	0.488	0.000	0.460	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	178	104	0	258	0	122	-1
normalized size	1	1.00	2.78	1.62	0.00	4.03	0.00	1.91	-0.02
time (sec)	N/A	0.103	0.703	0.768	0.000	0.578	0.000	0.436	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	249	132	0	319	0	155	-1
normalized size	1	1.00	2.44	1.29	0.00	3.13	0.00	1.52	-0.01
time (sec)	N/A	0.156	0.750	0.918	0.000	0.573	0.000	1.119	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	285	158	0	361	0	184	-1
normalized size	1	1.00	2.07	1.14	0.00	2.62	0.00	1.33	-0.01
time (sec)	N/A	0.213	1.340	0.782	0.000	0.539	0.000	0.715	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	97	67	0	223	0	104	-1
normalized size	1	1.00	2.55	1.76	0.00	5.87	0.00	2.74	-0.03
time (sec)	N/A	0.051	0.101	0.730	0.000	0.574	0.000	1.425	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	96	70	0	223	0	106	-1
normalized size	1	1.00	2.46	1.79	0.00	5.72	0.00	2.72	-0.03
time (sec)	N/A	0.050	0.078	0.542	0.000	0.547	0.000	1.108	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	95	69	0	221	0	69	-1
normalized size	1	1.00	2.38	1.72	0.00	5.52	0.00	1.72	-0.02
time (sec)	N/A	0.053	0.084	0.491	0.000	0.508	0.000	0.683	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	165	85	0	145	0	288	-1
normalized size	1	1.00	1.02	0.52	0.00	0.90	0.00	1.78	-0.01
time (sec)	N/A	0.241	0.537	0.630	0.000	0.565	0.000	1.177	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	141	75	0	122	0	226	-1
normalized size	1	1.00	1.22	0.65	0.00	1.05	0.00	1.95	-0.01
time (sec)	N/A	0.135	0.360	0.610	0.000	0.478	0.000	0.597	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	115	63	0	99	0	164	-1
normalized size	1	1.00	1.34	0.73	0.00	1.15	0.00	1.91	-0.01
time (sec)	N/A	0.062	0.164	0.767	0.000	0.519	0.000	1.024	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	89	53	0	76	0	101	-1
normalized size	1	1.00	1.51	0.90	0.00	1.29	0.00	1.71	-0.02
time (sec)	N/A	0.028	0.138	0.582	0.000	0.449	0.000	0.753	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	118	84	0	239	0	2251	-1
normalized size	1	1.00	1.79	1.27	0.00	3.62	0.00	34.11	-0.02
time (sec)	N/A	0.098	0.153	0.643	0.000	0.450	0.000	5.434	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	180	103	0	268	0	0	-1
normalized size	1	1.00	2.73	1.56	0.00	4.06	0.00	0.00	-0.02
time (sec)	N/A	0.110	0.654	0.744	0.000	0.461	0.000	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	250	126	0	337	0	0	-1
normalized size	1	1.00	2.36	1.19	0.00	3.18	0.00	0.00	-0.01
time (sec)	N/A	0.166	0.614	0.934	0.000	0.515	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	286	144	0	380	0	0	-1
normalized size	1	1.00	1.99	1.00	0.00	2.64	0.00	0.00	-0.01
time (sec)	N/A	0.231	0.954	0.785	0.000	0.480	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	189	95	0	192	0	372	-1
normalized size	1	1.00	0.93	0.47	0.00	0.95	0.00	1.83	-0.00
time (sec)	N/A	0.352	1.257	0.618	0.000	0.515	0.000	0.979	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	165	85	0	167	0	306	-1
normalized size	1	1.00	1.13	0.58	0.00	1.14	0.00	2.10	-0.01
time (sec)	N/A	0.155	1.020	0.707	0.000	0.480	0.000	0.790	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	141	75	0	140	0	240	-1
normalized size	1	1.00	1.22	0.65	0.00	1.21	0.00	2.07	-0.01
time (sec)	N/A	0.083	0.613	0.612	0.000	0.475	0.000	0.732	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	117	65	0	115	0	174	-1
normalized size	1	1.00	1.31	0.73	0.00	1.29	0.00	1.96	-0.01
time (sec)	N/A	0.046	0.318	0.575	0.000	0.504	0.000	0.985	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	143	103	0	279	0	0	-1
normalized size	1	1.00	1.46	1.05	0.00	2.85	0.00	0.00	-0.01
time (sec)	N/A	0.192	0.412	0.820	0.000	0.502	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	182	123	0	308	0	0	-1
normalized size	1	1.00	1.94	1.31	0.00	3.28	0.00	0.00	-0.01
time (sec)	N/A	0.193	0.812	0.958	0.000	0.519	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	252	126	0	359	0	0	-1
normalized size	1	1.00	2.38	1.19	0.00	3.39	0.00	0.00	-0.01
time (sec)	N/A	0.218	0.739	0.786	0.000	0.509	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	288	144	0	408	0	0	-1
normalized size	1	1.00	2.00	1.00	0.00	2.83	0.00	0.00	-0.01
time (sec)	N/A	0.275	1.175	0.812	0.000	0.522	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	370	162	0	473	0	0	-1
normalized size	1	1.00	2.03	0.89	0.00	2.60	0.00	0.00	-0.01
time (sec)	N/A	0.335	1.703	0.836	0.000	0.521	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	150	130	0	234	0	314	-1
normalized size	1	1.00	1.08	0.94	0.00	1.68	0.00	2.26	-0.01
time (sec)	N/A	0.232	0.223	0.937	0.000	0.555	0.000	0.920	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	105	96	0	209	0	245	-1
normalized size	1	1.00	1.00	0.91	0.00	1.99	0.00	2.33	-0.01
time (sec)	N/A	0.119	0.210	0.766	0.000	0.536	0.000	2.279	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	98	94	0	191	0	182	99
normalized size	1	1.00	1.36	1.31	0.00	2.65	0.00	2.53	1.38
time (sec)	N/A	0.048	0.101	0.793	0.000	0.506	0.000	5.688	0.745

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	73	75	0	167	0	111	49
normalized size	1	1.00	1.55	1.60	0.00	3.55	0.00	2.36	1.04
time (sec)	N/A	0.020	0.054	0.634	0.000	0.512	0.000	0.700	6.436

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	128	96	0	290	0	0	-1
normalized size	1	1.00	1.52	1.14	0.00	3.45	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.098	0.781	0.000	0.480	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	168	133	0	412	0	465	-1
normalized size	1	1.00	1.54	1.22	0.00	3.78	0.00	4.27	-0.01
time (sec)	N/A	0.204	1.263	0.811	0.000	0.511	0.000	0.813	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	307	162	0	492	0	615	-1
normalized size	1	1.00	2.10	1.11	0.00	3.37	0.00	4.21	-0.01
time (sec)	N/A	0.343	3.404	1.002	0.000	0.482	0.000	3.049	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	178	183	0	314	0	469	-1
normalized size	1	1.00	0.97	1.00	0.00	1.72	0.00	2.56	-0.01
time (sec)	N/A	0.382	0.453	0.724	0.000	0.509	0.000	0.838	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	156	183	0	295	0	409	-1
normalized size	1	1.00	1.08	1.26	0.00	2.03	0.00	2.82	-0.01
time (sec)	N/A	0.253	0.263	0.684	0.000	0.515	0.000	0.743	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	134	143	0	274	0	360	-1
normalized size	1	1.00	1.28	1.36	0.00	2.61	0.00	3.43	-0.01
time (sec)	N/A	0.128	0.274	0.604	0.000	0.528	0.000	0.780	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	108	123	0	253	0	294	-1
normalized size	1	1.00	1.40	1.60	0.00	3.29	0.00	3.82	-0.01
time (sec)	N/A	0.057	0.196	0.724	0.000	0.524	0.000	1.432	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	108	125	0	252	0	293	-1
normalized size	1	1.00	1.40	1.62	0.00	3.27	0.00	3.81	-0.01
time (sec)	N/A	0.039	0.148	0.549	0.000	0.520	0.000	1.964	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	223	172	0	453	0	412	-1
normalized size	1	1.00	1.96	1.51	0.00	3.97	0.00	3.61	-0.01
time (sec)	N/A	0.213	0.199	0.800	0.000	0.533	0.000	3.095	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	449	219	0	539	0	507	-1
normalized size	1	1.00	3.12	1.52	0.00	3.74	0.00	3.52	-0.01
time (sec)	N/A	0.358	0.626	0.683	0.000	0.544	0.000	0.977	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	620	299	0	626	0	0	-1
normalized size	1	1.00	3.33	1.61	0.00	3.37	0.00	0.00	-0.01
time (sec)	N/A	0.486	4.747	0.926	0.000	0.528	0.000	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	221	323	0	381	0	626	-1
normalized size	1	1.00	1.00	1.46	0.00	1.72	0.00	2.83	-0.00
time (sec)	N/A	0.521	0.556	0.944	0.000	0.566	0.000	1.524	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	197	269	0	360	0	584	-1
normalized size	1	1.00	1.08	1.47	0.00	1.97	0.00	3.19	-0.01
time (sec)	N/A	0.385	0.478	0.968	0.000	0.533	0.000	8.722	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	173	233	0	341	0	523	-1
normalized size	1	1.00	1.19	1.61	0.00	2.35	0.00	3.61	-0.01
time (sec)	N/A	0.268	0.322	1.012	0.000	0.518	0.000	0.935	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	196	193	0	320	0	457	-1
normalized size	1	1.00	1.83	1.80	0.00	2.99	0.00	4.27	-0.01
time (sec)	N/A	0.128	0.202	0.818	0.000	0.537	0.000	0.832	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	196	193	0	318	0	457	-1
normalized size	1	1.00	1.83	1.80	0.00	2.97	0.00	4.27	-0.01
time (sec)	N/A	0.074	0.189	0.997	0.000	0.505	0.000	0.791	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	196	195	0	320	0	457	-1
normalized size	1	1.00	1.83	1.82	0.00	2.99	0.00	4.27	-0.01
time (sec)	N/A	0.058	0.162	0.849	0.000	0.593	0.000	1.324	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	296	262	0	539	0	575	-1
normalized size	1	1.00	2.06	1.82	0.00	3.74	0.00	3.99	-0.01
time (sec)	N/A	0.325	0.268	0.921	0.000	0.589	0.000	1.267	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	509	356	0	631	0	671	-1
normalized size	1	1.00	2.93	2.05	0.00	3.63	0.00	3.86	-0.01
time (sec)	N/A	0.507	0.626	1.120	0.000	0.542	0.000	3.135	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	680	404	0	715	0	817	-1
normalized size	1	1.00	3.04	1.80	0.00	3.19	0.00	3.65	-0.00
time (sec)	N/A	0.660	1.134	1.283	0.000	0.599	0.000	1.359	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	164	320	210	330	0	0	-1
normalized size	1	1.00	4.43	8.65	5.68	8.92	0.00	0.00	-0.03
time (sec)	N/A	0.056	0.512	0.316	0.930	0.669	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	119	271	0	341	0	0	-1
normalized size	1	1.00	3.13	7.13	0.00	8.97	0.00	0.00	-0.03
time (sec)	N/A	0.070	0.473	0.225	0.000	0.621	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	123	52	0	28	0	0	-1
normalized size	1	1.00	7.24	3.06	0.00	1.65	0.00	0.00	-0.06
time (sec)	N/A	0.039	2.571	0.129	0.000	0.495	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	125	54	0	163	0	0	-1
normalized size	1	1.00	2.98	1.29	0.00	3.88	0.00	0.00	-0.02
time (sec)	N/A	0.057	0.093	0.161	0.000	0.597	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	125	52	0	31	0	0	-1
normalized size	1	1.00	4.03	1.68	0.00	1.00	0.00	0.00	-0.03
time (sec)	N/A	0.046	2.518	0.160	0.000	0.494	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	128	53	0	168	0	0	-1
normalized size	1	1.00	3.05	1.26	0.00	4.00	0.00	0.00	-0.02
time (sec)	N/A	0.064	0.100	0.152	0.000	0.620	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	121	0	0	0	0	0	-1
normalized size	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.213	0.402	0.775	0.000	0.490	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	160	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.278	0.692	0.651	0.000	0.501	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	151	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.439	0.649	0.000	0.494	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	138	0	0	0	0	0	-1
normalized size	1	1.00	1.44	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.233	0.160	0.000	0.485	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	124	0	0	0	0	0	-1
normalized size	1	1.00	1.88	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.205	0.006	0.000	0.560	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	2.791	0.210	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	143	0	0	0	0	0	-1
normalized size	1	1.00	1.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	14.749	0.178	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	373	0	0	0	0	0	-1
normalized size	1	1.00	2.30	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.279	2.645	0.593	0.000	0.637	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	363	0	0	0	0	0	-1
normalized size	1	1.00	2.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.143	2.619	0.602	0.000	0.659	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	351	0	0	0	0	0	-1
normalized size	1	1.00	3.62	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	1.995	0.156	0.000	0.757	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	341	0	0	0	0	0	-1
normalized size	1	1.00	5.09	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.030	1.722	0.006	0.000	0.498	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	2791	0	0	0	0	0	-1
normalized size	1	1.00	35.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	9.609	0.204	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	2800	0	0	0	0	0	-1
normalized size	1	1.00	35.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.130	10.569	0.178	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	110	0	0	0	0	0	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.253	0.461	0.830	0.000	1.039	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	95	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.295	0.489	0.000	0.711	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	84	0	0	0	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.147	0.342	0.000	0.895	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	70	0	0	0	0	0	-1
normalized size	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.098	0.005	0.000	0.629	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	3.474	0.235	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	184	0	0	0	0	0	-1
normalized size	1	1.00	2.39	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	8.839	0.226	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	116	0	0	0	0	0	-1
normalized size	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.270	0.496	0.857	0.000	0.758	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	108	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.333	0.433	0.000	0.735	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	130	0	0	0	0	0	-1
normalized size	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.290	0.170	0.000	0.506	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	130	0	0	0	0	0	-1
normalized size	1	1.00	1.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.205	0.005	0.000	0.545	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	10.316	0.204	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	230	0	0	0	0	0	-1
normalized size	1	1.00	2.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	14.176	0.207	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	5109	0	0	0	0	0	-1
normalized size	1	1.00	53.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	22.545	0.240	0.000	0.952	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	186	0	0	0	0	0	-1
normalized size	1	1.00	4.33	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.048	0.445	0.166	0.000	0.465	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	225	0	0	0	0	0	-1
normalized size	1	1.00	3.88	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.068	1.582	0.156	0.000	0.484	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	263	0	0	0	0	0	-1
normalized size	1	1.00	4.38	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.072	3.792	0.155	0.000	0.520	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	5111	0	0	0	0	0	-1
normalized size	1	1.00	48.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.140	6.335	0.216	0.000	0.479	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	264	0	0	0	0	0	-1
normalized size	1	1.00	5.74	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.062	4.295	0.221	0.000	0.476	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	234	0	0	0	0	0	-1
normalized size	1	1.00	3.90	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.124	1.338	0.205	0.000	0.522	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	274	0	0	0	0	0	-1
normalized size	1	1.00	4.22	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.136	2.270	0.204	0.000	0.479	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	5129	0	0	0	0	0	-1
normalized size	1	1.00	39.45	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	6.307	0.220	0.000	0.549	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	215	0	0	0	0	0	-1
normalized size	1	1.00	2.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.354	0.188	0.000	0.487	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	227	0	0	0	0	0	-1
normalized size	1	1.00	2.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.411	0.181	0.000	0.458	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	265	0	0	0	0	0	-1
normalized size	1	1.00	3.31	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.933	0.191	0.000	0.500	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	5131	0	0	0	0	0	-1
normalized size	1	1.00	39.17	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.170	6.337	0.246	0.000	0.511	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	266	0	0	0	0	0	-1
normalized size	1	1.00	4.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.075	0.429	0.244	0.000	0.464	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	242	0	0	0	0	0	-1
normalized size	1	1.00	3.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.180	0.711	0.258	0.000	0.490	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	276	0	0	0	0	0	-1
normalized size	1	1.00	3.25	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.202	1.264	0.215	0.000	0.527	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	2805	0	0	0	0	0	-1
normalized size	1	1.00	39.51	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	14.923	0.855	0.000	0.494	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	300	0	0	0	0	0	-1
normalized size	1	1.00	4.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	2.324	1.055	0.000	0.470	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	2813	0	0	0	0	0	-1
normalized size	1	1.00	30.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	6.223	0.966	0.000	0.491	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	300	0	0	0	0	0	-1
normalized size	1	1.00	3.33	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.725	0.979	0.000	0.511	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	2807	0	0	0	0	0	-1
normalized size	1	1.00	32.26	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	6.307	1.013	0.000	0.485	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	301	0	0	0	0	0	-1
normalized size	1	1.00	3.54	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.595	1.168	0.000	0.490	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	2815	0	0	0	0	0	-1
normalized size	1	1.00	26.31	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.150	6.239	1.063	0.000	0.473	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	301	0	0	0	0	0	-1
normalized size	1	1.00	2.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.528	1.054	0.000	0.492	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.510	180.005	3.039	0.000	0.462	0.000	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	60244	0	0	0	0	0	-1
normalized size	1	1.00	280.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.294	128.810	2.210	0.000	0.489	0.000	0.000	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	28439	0	0	0	0	0	-1
normalized size	1	1.00	182.30	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.143	54.578	1.817	0.000	0.475	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	178	0	0	0	0	0	-1
normalized size	1	1.00	1.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.441	1.526	0.000	0.489	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	90	0	0	0	0	0	-1
normalized size	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.166	0.006	0.000	0.487	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	2560	0	0	0	0	0	-1
normalized size	1	1.00	30.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	16.107	1.275	0.000	0.470	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	4206	0	0	0	0	0	-1
normalized size	1	1.00	49.48	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	26.563	0.602	0.000	0.495	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	88	0	0	0	0	0	-1
normalized size	1	1.00	1.52	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.014	0.151	0.468	0.000	0.452	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	90	0	0	0	0	0	-1
normalized size	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.110	0.527	0.000	0.467	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	76	60	57	60	144	66	111
normalized size	1	1.00	0.99	0.78	0.74	0.78	1.87	0.86	1.44
time (sec)	N/A	0.058	0.165	0.225	1.321	0.466	1.341	0.144	10.285

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	60	49	48	46	92	50	68
normalized size	1	1.00	1.09	0.89	0.87	0.84	1.67	0.91	1.24
time (sec)	N/A	0.045	0.060	0.158	0.649	0.448	0.741	0.146	8.575

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	39	36	34	66	34	68
normalized size	1	1.00	0.90	1.00	0.92	0.87	1.69	0.87	1.74
time (sec)	N/A	0.014	0.096	0.049	0.383	0.478	0.308	0.145	7.191

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	27	17	16	18	19	17	25
normalized size	1	1.00	1.69	1.06	1.00	1.12	1.19	1.06	1.56
time (sec)	N/A	0.008	0.006	0.009	0.876	0.486	0.157	0.151	6.511

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	43	32	29	38	51	0	85
normalized size	1	1.00	2.53	1.88	1.71	2.24	3.00	0.00	5.00
time (sec)	N/A	0.022	0.015	0.109	0.636	0.452	6.758	0.000	6.797

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	52	35	40	62	0	0	28
normalized size	1	1.00	2.00	1.35	1.54	2.38	0.00	0.00	1.08
time (sec)	N/A	0.037	0.025	0.167	0.607	0.490	0.000	0.000	6.759

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	91	54	60	96	0	0	81
normalized size	1	1.00	1.90	1.12	1.25	2.00	0.00	0.00	1.69
time (sec)	N/A	0.047	0.032	0.288	0.823	0.512	0.000	0.000	6.723

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	115	74	73	128	0	0	111
normalized size	1	1.00	1.80	1.16	1.14	2.00	0.00	0.00	1.73
time (sec)	N/A	0.051	0.029	0.339	0.303	0.509	0.000	0.000	6.741

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	91	95	94	90	221	128	157
normalized size	1	1.00	0.81	0.85	0.84	0.80	1.97	1.14	1.40
time (sec)	N/A	0.104	0.350	0.283	0.722	0.488	3.105	0.223	10.366

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	117	89	84	84	211	86	85
normalized size	1	1.00	1.16	0.88	0.83	0.83	2.09	0.85	0.84
time (sec)	N/A	0.089	0.162	0.210	1.001	0.535	1.732	0.882	6.927

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	59	64	62	55	107	76	103
normalized size	1	1.00	0.83	0.90	0.87	0.77	1.51	1.07	1.45
time (sec)	N/A	0.049	0.220	0.163	0.643	0.486	0.718	0.281	8.993

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	46	51	46	45	78	45	44
normalized size	1	1.00	0.92	1.02	0.92	0.90	1.56	0.90	0.88
time (sec)	N/A	0.015	0.104	0.084	0.301	0.483	0.400	0.161	6.790

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	76	52	44	54	0	0	125
normalized size	1	1.00	2.17	1.49	1.26	1.54	0.00	0.00	3.57
time (sec)	N/A	0.059	0.024	0.195	0.641	0.494	0.000	0.000	6.480

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	76	52	52	77	0	0	105
normalized size	1	1.00	2.24	1.53	1.53	2.26	0.00	0.00	3.09
time (sec)	N/A	0.066	0.239	0.250	0.629	0.523	0.000	0.000	6.834

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	133	82	89	129	0	0	92
normalized size	1	1.00	2.25	1.39	1.51	2.19	0.00	0.00	1.56
time (sec)	N/A	0.076	0.474	0.370	0.307	0.471	0.000	0.000	6.494

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	132	93	89	149	0	0	136
normalized size	1	1.00	1.61	1.13	1.09	1.82	0.00	0.00	1.66
time (sec)	N/A	0.087	0.042	0.402	0.688	0.519	0.000	0.000	6.783

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	255	146	147	229	0	0	178
normalized size	1	1.00	2.32	1.33	1.34	2.08	0.00	0.00	1.62
time (sec)	N/A	0.094	0.042	0.428	1.013	0.468	0.000	0.000	6.876

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	193	147	145	145	138	393	180	417
normalized size	1	1.13	0.86	0.85	0.85	0.81	2.30	1.05	2.44
time (sec)	N/A	0.209	0.736	0.352	0.777	0.521	5.552	0.198	8.401

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	180	117	124	121	118	284	129	328
normalized size	1	1.12	0.73	0.78	0.76	0.74	1.78	0.81	2.05
time (sec)	N/A	0.216	0.651	0.273	1.617	0.485	2.852	0.176	8.171

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	100	104	97	93	233	116	313
normalized size	1	1.00	0.83	0.86	0.80	0.77	1.93	0.96	2.59
time (sec)	N/A	0.114	0.353	0.227	0.510	0.515	1.659	0.394	8.085

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	71	76	74	71	128	75	127
normalized size	1	1.00	0.79	0.84	0.82	0.79	1.42	0.83	1.41
time (sec)	N/A	0.066	0.170	0.161	0.648	0.470	0.737	0.169	6.738

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	81	92	71	79	0	0	259
normalized size	1	1.00	1.09	1.24	0.96	1.07	0.00	0.00	3.50
time (sec)	N/A	0.115	0.166	0.207	0.671	0.488	0.000	0.000	6.791

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	87	72	68	99	0	0	194
normalized size	1	1.00	1.28	1.06	1.00	1.46	0.00	0.00	2.85
time (sec)	N/A	0.120	0.542	0.266	0.307	0.500	0.000	0.000	6.720

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	152	99	102	155	0	0	234
normalized size	1	1.00	1.92	1.25	1.29	1.96	0.00	0.00	2.96
time (sec)	N/A	0.133	0.675	0.365	1.283	0.504	0.000	0.000	6.963

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	525	122	118	191	0	0	150
normalized size	1	1.00	4.82	1.12	1.08	1.75	0.00	0.00	1.38
time (sec)	N/A	0.181	6.198	0.396	0.684	0.486	0.000	0.000	6.785

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	322	166	162	238	0	0	203
normalized size	1	1.00	2.40	1.24	1.21	1.78	0.00	0.00	1.51
time (sec)	N/A	0.205	6.181	0.478	1.165	0.507	0.000	0.000	6.858

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	106	116	113	106	240	112	114
normalized size	1	1.00	0.77	0.85	0.82	0.77	1.75	0.82	0.83
time (sec)	N/A	0.146	0.367	0.220	1.068	0.481	1.670	0.168	6.952

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	98	213	0	333	0	149	1075
normalized size	1	1.00	0.89	1.94	0.00	3.03	0.00	1.35	9.77
time (sec)	N/A	0.278	0.260	0.076	0.000	0.541	0.000	0.198	7.221

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	78	142	0	291	0	112	1004
normalized size	1	1.00	0.95	1.73	0.00	3.55	0.00	1.37	12.24
time (sec)	N/A	0.164	0.111	0.072	0.000	0.531	0.000	0.230	7.080

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	56	72	0	231	0	77	623
normalized size	1	1.00	0.92	1.18	0.00	3.79	0.00	1.26	10.21
time (sec)	N/A	0.104	0.091	0.068	0.000	0.503	0.000	0.359	6.911

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	47	54	0	192	236	58	101
normalized size	1	1.00	0.94	1.08	0.00	3.84	4.72	1.16	2.02
time (sec)	N/A	0.055	0.042	0.059	0.000	0.780	80.939	0.319	6.704

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	39	0	148	133	48	45
normalized size	1	1.00	1.00	0.98	0.00	3.70	3.32	1.20	1.12
time (sec)	N/A	0.031	0.022	0.048	0.000	0.473	9.661	0.169	6.883

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	62	53	0	239	0	63	122
normalized size	1	1.00	1.17	1.00	0.00	4.51	0.00	1.19	2.30
time (sec)	N/A	0.066	0.055	0.097	0.000	0.576	0.000	0.183	6.857

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	91	77	0	302	0	98	179
normalized size	1	1.00	1.47	1.24	0.00	4.87	0.00	1.58	2.89
time (sec)	N/A	0.111	0.248	0.105	0.000	0.564	0.000	0.162	7.006

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	144	112	0	490	0	141	531
normalized size	1	1.00	1.71	1.33	0.00	5.83	0.00	1.68	6.32
time (sec)	N/A	0.271	0.518	0.119	0.000	0.694	0.000	0.263	7.388

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	125	162	0	577	0	194	586
normalized size	1	1.00	1.12	1.45	0.00	5.15	0.00	1.73	5.23
time (sec)	N/A	0.433	1.697	0.115	0.000	0.708	0.000	0.239	7.517

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	115	266	0	580	0	184	4368
normalized size	1	1.00	0.68	1.57	0.00	3.43	0.00	1.09	25.85
time (sec)	N/A	0.383	0.602	0.113	0.000	0.587	0.000	0.170	11.883

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	94	196	0	483	0	204	2578
normalized size	1	1.00	0.76	1.58	0.00	3.90	0.00	1.65	20.79
time (sec)	N/A	0.218	0.427	0.106	0.000	0.555	0.000	0.167	9.983

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	83	170	0	403	0	124	2562
normalized size	1	1.00	0.95	1.95	0.00	4.63	0.00	1.43	29.45
time (sec)	N/A	0.125	0.227	0.102	0.000	0.546	0.000	0.162	9.953

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	67	99	0	266	0	90	123
normalized size	1	1.00	1.02	1.50	0.00	4.03	0.00	1.36	1.86
time (sec)	N/A	0.063	0.109	0.093	0.000	0.523	0.000	0.173	6.494

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	66	98	0	268	0	95	148
normalized size	1	1.00	1.02	1.51	0.00	4.12	0.00	1.46	2.28
time (sec)	N/A	0.051	0.098	0.084	0.000	0.563	0.000	0.171	6.821

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	99	174	0	511	0	134	1356
normalized size	1	1.00	1.06	1.87	0.00	5.49	0.00	1.44	14.58
time (sec)	N/A	0.180	0.250	0.127	0.000	0.860	0.000	0.252	7.819

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	127	201	0	784	0	234	1471
normalized size	1	1.00	1.03	1.63	0.00	6.37	0.00	1.90	11.96
time (sec)	N/A	0.335	0.725	0.141	0.000	0.844	0.000	0.152	7.634

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	171	236	0	1174	0	215	1576
normalized size	1	1.00	1.02	1.40	0.00	6.99	0.00	1.28	9.38
time (sec)	N/A	0.574	1.009	0.147	0.000	1.265	0.000	0.184	7.669

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	164	712	0	1090	0	516	6640
normalized size	1	1.00	0.67	2.93	0.00	4.49	0.00	2.12	27.33
time (sec)	N/A	0.661	1.039	0.128	0.000	0.640	0.000	0.423	15.806

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	144	634	0	945	0	256	5945
normalized size	1	1.00	0.80	3.54	0.00	5.28	0.00	1.43	33.21
time (sec)	N/A	0.411	0.839	0.120	0.000	0.631	0.000	0.262	14.874

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	136	612	0	819	0	234	5756
normalized size	1	1.00	0.94	4.25	0.00	5.69	0.00	1.62	39.97
time (sec)	N/A	0.245	0.572	0.119	0.000	0.549	0.000	0.221	14.903

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	94	265	0	516	0	182	318
normalized size	1	1.00	0.80	2.25	0.00	4.37	0.00	1.54	2.69
time (sec)	N/A	0.144	0.377	0.108	0.000	0.516	0.000	0.155	7.233

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	94	221	0	490	0	189	310
normalized size	1	1.00	0.91	2.15	0.00	4.76	0.00	1.83	3.01
time (sec)	N/A	0.103	0.299	0.106	0.000	0.515	0.000	0.422	7.509

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	93	300	0	516	0	215	349
normalized size	1	1.00	0.91	2.94	0.00	5.06	0.00	2.11	3.42
time (sec)	N/A	0.095	0.203	0.096	0.000	0.520	0.000	0.165	7.323

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	140	614	0	1027	0	246	2191
normalized size	1	1.00	0.97	4.23	0.00	7.08	0.00	1.70	15.11
time (sec)	N/A	0.370	0.905	0.142	0.000	1.394	0.000	0.177	11.385

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	174	641	0	1436	0	280	2295
normalized size	1	1.00	0.93	3.43	0.00	7.68	0.00	1.50	12.27
time (sec)	N/A	0.639	1.382	0.162	0.000	1.419	0.000	0.229	9.016

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	220	686	0	2005	0	514	2405
normalized size	1	1.00	0.91	2.85	0.00	8.32	0.00	2.13	9.98
time (sec)	N/A	0.873	2.085	0.167	0.000	2.546	0.000	0.457	9.277

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	157	1733	0	965	0	510	708
normalized size	1	1.00	0.86	9.52	0.00	5.30	0.00	2.80	3.89
time (sec)	N/A	0.225	1.025	0.207	0.000	0.592	0.000	0.360	10.384

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	143	460	0	0	0	0	-1
normalized size	1	1.00	0.83	2.67	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.170	3.140	1.166	0.000	0.487	0.000	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	61	239	0	0	0	0	55
normalized size	1	1.00	0.98	3.85	0.00	0.00	0.00	0.00	0.89
time (sec)	N/A	0.037	0.072	1.423	0.000	0.503	0.000	0.000	6.871

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	89	169	0	0	0	0	-1
normalized size	1	1.00	0.70	1.32	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.235	16.179	1.209	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	312	457	0	0	0	0	-1
normalized size	1	1.00	1.46	2.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.481	8.872	1.348	0.000	0.896	0.000	0.000	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	94	202	0	0	0	0	118
normalized size	1	1.00	0.71	1.53	0.00	0.00	0.00	0.00	0.89
time (sec)	N/A	0.107	2.396	1.061	0.000	0.689	0.000	0.000	7.207

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	61	126	0	0	0	0	55
normalized size	1	1.00	0.98	2.03	0.00	0.00	0.00	0.00	0.89
time (sec)	N/A	0.037	0.054	0.709	0.000	0.853	0.000	0.000	6.913

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	62	135	0	0	0	0	-1
normalized size	1	1.00	0.98	2.14	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.130	0.081	0.986	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	315	412	0	0	0	0	-1
normalized size	1	1.00	1.42	1.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.495	10.049	2.923	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	10847	9823	0	0	0	0	-1
normalized size	1	1.00	29.24	26.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.563	26.830	0.761	0.000	2.701	0.000	0.000	0.000

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	172	318	0	0	0	0	-1
normalized size	1	1.00	1.58	2.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	3.343	0.295	0.000	0.745	0.000	0.000	0.000

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	199	0	0	0	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.380	0.805	5.107	0.000	2.590	0.000	0.000	0.000

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	144	0	0	0	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.329	7.288	0.000	0.531	0.000	0.000	0.000

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	111	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.160	1.872	0.000	0.790	0.000	0.000	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	1590	0	0	0	0	0	-1
normalized size	1	1.00	8.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.248	17.687	0.753	0.000	0.554	0.000	0.000	0.000

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	1790	0	0	0	0	0	-1
normalized size	1	1.00	5.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.420	18.876	1.463	0.000	0.464	0.000	0.000	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	2298	0	0	0	0	0	-1
normalized size	1	1.00	5.66	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.549	18.687	1.773	0.000	0.486	0.000	0.000	0.000

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	188	0	0	0	0	0	-1
normalized size	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.303	10.978	0.000	0.487	0.000	0.000	0.000

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	0	0	0	0	127
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.74
time (sec)	N/A	0.074	0.101	0.756	0.000	0.513	0.000	0.000	7.473

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	2.374	1.189	0.000	0.486	0.000	0.000	0.000

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.486	4.144	0.784	0.000	0.492	0.000	0.000	0.000

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.298	6.814	0.730	0.000	0.476	0.000	0.000	0.000

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	193	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.188	0.489	0.225	0.000	0.466	0.000	0.000	0.000

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	120	0	0	0	0	0	-1
normalized size	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.233	0.008	0.000	0.453	0.000	0.000	0.000

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.031	2.444	1.372	0.000	0.445	0.000	0.000	0.000

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	64	149	146	77	314	100	292
normalized size	1	1.00	0.55	1.28	1.26	0.66	2.71	0.86	2.52
time (sec)	N/A	0.162	0.561	0.283	0.872	0.467	3.638	0.433	8.859

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	54	89	86	63	196	81	250
normalized size	1	1.00	0.65	1.07	1.04	0.76	2.36	0.98	3.01
time (sec)	N/A	0.113	0.368	0.230	1.714	0.465	1.369	0.188	9.004

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	42	77	77	46	133	62	125
normalized size	1	1.00	0.81	1.48	1.48	0.88	2.56	1.19	2.40
time (sec)	N/A	0.065	0.277	0.165	0.592	0.438	0.904	0.192	8.963

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	40	37	26	70	23	54
normalized size	1	1.00	0.86	1.38	1.28	0.90	2.41	0.79	1.86
time (sec)	N/A	0.018	0.023	0.048	0.513	0.430	0.300	0.172	7.154

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	83	43	82	66	88	37	46
normalized size	1	1.00	2.52	1.30	2.48	2.00	2.67	1.12	1.39
time (sec)	N/A	0.048	0.191	0.181	0.919	0.441	1.605	0.159	6.809

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	74	56	217	104	158	39	56
normalized size	1	1.00	2.47	1.87	7.23	3.47	5.27	1.30	1.87
time (sec)	N/A	0.073	0.279	0.223	0.739	0.415	3.713	0.197	6.726

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	96	86	389	154	571	84	136
normalized size	1	1.00	1.60	1.43	6.48	2.57	9.52	1.40	2.27
time (sec)	N/A	0.117	0.338	0.254	1.050	0.466	7.491	0.240	7.104

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	109	116	561	206	1061	114	97
normalized size	1	1.00	1.18	1.26	6.10	2.24	11.53	1.24	1.05
time (sec)	N/A	0.170	0.482	0.225	0.551	0.430	16.672	0.212	7.353

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	124	146	733	256	1700	144	119
normalized size	1	1.00	0.98	1.16	5.82	2.03	13.49	1.14	0.94
time (sec)	N/A	0.220	0.619	0.274	0.610	0.469	33.029	0.600	8.769

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	89	255	256	103	629	154	452
normalized size	1	1.00	0.59	1.68	1.68	0.68	4.14	1.01	2.97
time (sec)	N/A	0.197	1.111	0.399	0.824	0.485	10.577	0.232	9.229

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	79	211	209	87	530	133	284
normalized size	1	1.00	0.67	1.79	1.77	0.74	4.49	1.13	2.41
time (sec)	N/A	0.145	0.727	0.343	0.680	0.458	6.932	0.220	8.901

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	69	159	158	71	340	112	220
normalized size	1	1.00	0.81	1.87	1.86	0.84	4.00	1.32	2.59
time (sec)	N/A	0.097	1.643	0.286	0.998	0.475	3.039	0.198	10.004

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	39	88	81	54	206	52	36
normalized size	1	1.00	0.61	1.38	1.27	0.84	3.22	0.81	0.56
time (sec)	N/A	0.069	0.044	0.174	0.310	0.476	1.469	1.252	6.742

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	43	78	77	46	133	62	125
normalized size	1	1.00	0.83	1.50	1.48	0.88	2.56	1.19	2.40
time (sec)	N/A	0.077	0.329	0.165	0.675	0.448	0.893	0.221	9.017

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	130	73	218	105	454	103	118
normalized size	1	1.00	2.28	1.28	3.82	1.84	7.96	1.81	2.07
time (sec)	N/A	0.141	0.380	0.230	1.921	0.438	4.197	0.238	6.905

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	121	71	364	158	473	60	90
normalized size	1	1.00	1.68	0.99	5.06	2.19	6.57	0.83	1.25
time (sec)	N/A	0.137	0.619	0.243	1.182	0.436	8.243	0.242	6.873

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	81	88	557	168	354	60	92
normalized size	1	1.00	2.38	2.59	16.38	4.94	10.41	1.76	2.71
time (sec)	N/A	0.094	0.401	0.273	2.027	0.459	14.876	0.271	6.993

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	117	118	816	222	1074	128	99
normalized size	1	1.00	1.75	1.76	12.18	3.31	16.03	1.91	1.48
time (sec)	N/A	0.136	0.628	0.273	0.573	0.470	25.118	0.219	7.296

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	121	148	1073	278	1717	162	121
normalized size	1	1.00	1.23	1.51	10.95	2.84	17.52	1.65	1.23
time (sec)	N/A	0.182	0.580	0.296	1.193	0.438	50.690	0.272	8.810

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	133	178	1332	332	2509	196	143
normalized size	1	1.00	1.01	1.35	10.09	2.52	19.01	1.48	1.08
time (sec)	N/A	0.233	0.743	0.295	2.362	0.444	84.941	0.264	9.357

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	109	297	301	119	838	196	403
normalized size	1	1.00	0.61	1.65	1.67	0.66	4.66	1.09	2.24
time (sec)	N/A	0.206	2.135	0.410	0.330	0.488	26.386	0.294	9.307

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	89	276	282	103	740	154	372
normalized size	1	1.00	0.61	1.90	1.94	0.71	5.10	1.06	2.57
time (sec)	N/A	0.157	1.212	0.429	0.769	0.471	15.629	0.259	9.123

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	89	255	256	87	631	154	301
normalized size	1	1.00	0.79	2.28	2.29	0.78	5.63	1.38	2.69
time (sec)	N/A	0.108	1.067	0.421	0.532	0.463	9.274	0.231	10.454

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	49	140	132	70	398	73	143
normalized size	1	1.00	0.54	1.54	1.45	0.77	4.37	0.80	1.57
time (sec)	N/A	0.079	0.049	0.242	0.781	0.458	5.417	0.241	10.139

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	69	160	158	71	340	112	220
normalized size	1	1.00	0.81	1.88	1.86	0.84	4.00	1.32	2.59
time (sec)	N/A	0.092	1.650	0.293	1.100	0.467	3.324	0.172	10.318

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	54	89	86	63	196	81	250
normalized size	1	1.00	0.66	1.09	1.05	0.77	2.39	0.99	3.05
time (sec)	N/A	0.098	0.383	0.241	0.834	0.476	1.492	0.424	8.812

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	153	181	433	129	1168	117	219
normalized size	1	1.00	1.63	1.93	4.61	1.37	12.43	1.24	2.33
time (sec)	N/A	0.183	0.531	0.270	1.777	0.442	7.425	0.288	9.123

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	149	121	594	184	1282	101	218
normalized size	1	1.00	1.62	1.32	6.46	2.00	13.93	1.10	2.37
time (sec)	N/A	0.186	1.002	0.282	1.500	0.448	13.649	0.382	9.752

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	249	143	785	235	1282	111	203
normalized size	1	1.00	2.35	1.35	7.41	2.22	12.09	1.05	1.92
time (sec)	N/A	0.189	0.458	0.314	1.761	0.440	26.030	0.203	8.480

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	93	118	1045	222	619	77	116
normalized size	1	1.00	2.74	3.47	30.74	6.53	18.21	2.26	3.41
time (sec)	N/A	0.090	0.803	0.280	0.889	0.446	41.606	0.225	7.010

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	135	148	1389	276	1717	162	121
normalized size	1	1.00	1.96	2.14	20.13	4.00	24.88	2.35	1.75
time (sec)	N/A	0.139	0.739	0.338	0.810	0.449	72.339	0.260	8.578

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	145	178	1734	332	2509	196	143
normalized size	1	1.00	1.44	1.76	17.17	3.29	24.84	1.94	1.42
time (sec)	N/A	0.182	0.901	0.319	1.250	0.495	118.535	0.366	9.339

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	157	208	2078	386	3451	230	165
normalized size	1	1.00	1.19	1.58	15.74	2.92	26.14	1.74	1.25
time (sec)	N/A	0.230	1.985	0.372	0.878	0.494	178.922	0.326	10.171

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	209	238	2422	440	0	264	187
normalized size	1	1.00	1.26	1.43	14.59	2.65	0.00	1.59	1.13
time (sec)	N/A	0.287	1.925	0.449	0.936	0.489	0.000	0.331	11.198

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	175	219	720	156	2108	135	290
normalized size	1	1.00	1.48	1.86	6.10	1.32	17.86	1.14	2.46
time (sec)	N/A	0.196	1.436	0.322	0.912	0.439	14.000	0.218	10.530

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	155	181	424	128	1170	117	216
normalized size	1	1.00	1.68	1.97	4.61	1.39	12.72	1.27	2.35
time (sec)	N/A	0.176	0.511	0.246	1.130	0.457	7.444	0.198	8.709

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	129	73	210	101	456	100	118
normalized size	1	1.00	2.30	1.30	3.75	1.80	8.14	1.79	2.11
time (sec)	N/A	0.136	0.367	0.210	0.874	0.456	3.894	0.204	6.994

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	79	43	77	64	90	37	45
normalized size	1	1.00	2.47	1.34	2.41	2.00	2.81	1.16	1.41
time (sec)	N/A	0.044	0.186	0.181	0.858	0.464	1.818	0.226	6.645

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	0	16	24	49	17	35
normalized size	1	1.00	1.00	0.00	1.00	1.50	3.06	1.06	2.19
time (sec)	N/A	0.067	0.012	180.000	0.305	0.432	1.650	0.163	6.849

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	87	73	142	56	328	77	74
normalized size	1	1.00	1.64	1.38	2.68	1.06	6.19	1.45	1.40
time (sec)	N/A	0.110	0.421	0.181	0.711	0.438	4.020	0.190	7.012

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	111	103	211	83	614	105	89
normalized size	1	1.00	1.31	1.21	2.48	0.98	7.22	1.24	1.05
time (sec)	N/A	0.158	0.667	0.251	1.581	0.443	8.434	0.304	7.158

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	131	133	319	111	1307	133	96
normalized size	1	1.00	1.11	1.13	2.70	0.94	11.08	1.13	0.81
time (sec)	N/A	0.207	0.741	0.246	0.676	0.445	13.733	0.227	7.273

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	276	267	1304	238	3641	203	372
normalized size	1	1.00	1.86	1.80	8.81	1.61	24.60	1.37	2.51
time (sec)	N/A	0.240	0.724	0.306	1.117	0.557	41.194	0.247	10.855

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	243	229	903	210	2312	152	291
normalized size	1	1.00	1.80	1.70	6.69	1.56	17.13	1.13	2.16
time (sec)	N/A	0.224	0.494	0.291	0.873	0.510	23.377	0.193	10.091

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	210	121	590	185	1282	101	217
normalized size	1	1.00	2.33	1.34	6.56	2.06	14.24	1.12	2.41
time (sec)	N/A	0.175	0.379	0.262	0.925	0.474	12.757	0.198	9.623

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	119	71	361	158	473	58	89
normalized size	1	1.00	1.70	1.01	5.16	2.26	6.76	0.83	1.27
time (sec)	N/A	0.132	0.619	0.263	0.754	0.519	7.688	0.170	7.067

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	70	56	215	104	158	39	54
normalized size	1	1.00	2.41	1.93	7.41	3.59	5.45	1.34	1.86
time (sec)	N/A	0.067	0.271	0.224	0.704	0.517	3.857	0.179	7.029

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	87	73	142	55	328	77	74
normalized size	1	1.00	1.67	1.40	2.73	1.06	6.31	1.48	1.42
time (sec)	N/A	0.107	0.479	0.264	0.722	0.426	3.920	0.206	6.994

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	29	0	28	37	286	30	63
normalized size	1	1.00	0.76	0.00	0.74	0.97	7.53	0.79	1.66
time (sec)	N/A	0.063	0.052	180.000	0.671	0.421	5.070	0.436	6.839

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	131	133	335	86	1418	133	128
normalized size	1	1.00	1.72	1.75	4.41	1.13	18.66	1.75	1.68
time (sec)	N/A	0.113	0.894	0.259	0.975	0.435	15.884	0.231	7.827

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	151	163	427	113	2213	161	119
normalized size	1	1.00	1.36	1.47	3.85	1.02	19.94	1.45	1.07
time (sec)	N/A	0.159	0.950	0.266	0.942	0.431	30.149	0.323	6.979

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	193	193	519	143	3186	189	180
normalized size	1	1.00	1.34	1.34	3.60	0.99	22.12	1.31	1.25
time (sec)	N/A	0.215	1.162	0.280	0.814	0.457	56.787	0.243	9.290

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	303	277	1496	285	3643	186	364
normalized size	1	1.00	1.88	1.72	9.29	1.77	22.63	1.16	2.26
time (sec)	N/A	0.276	0.857	0.336	1.486	0.479	72.501	0.241	11.145

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	270	145	1096	256	2314	135	290
normalized size	1	1.00	2.18	1.17	8.84	2.06	18.66	1.09	2.34
time (sec)	N/A	0.221	0.618	0.312	0.970	0.442	44.740	0.211	10.904

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	239	143	781	233	1284	111	200
normalized size	1	1.00	2.32	1.39	7.58	2.26	12.47	1.08	1.94
time (sec)	N/A	0.179	0.441	0.297	1.460	0.434	26.241	0.232	8.939

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	81	88	554	168	354	60	90
normalized size	1	1.00	2.45	2.67	16.79	5.09	10.73	1.82	2.73
time (sec)	N/A	0.086	0.409	0.273	0.713	0.423	14.473	0.258	7.215

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	92	86	387	154	573	84	134
normalized size	1	1.00	1.59	1.48	6.67	2.66	9.88	1.45	2.31
time (sec)	N/A	0.107	0.344	0.264	0.983	0.487	8.426	0.701	7.238

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	111	101	211	82	614	105	89
normalized size	1	1.00	1.34	1.22	2.54	0.99	7.40	1.27	1.07
time (sec)	N/A	0.148	0.670	0.255	0.652	0.439	8.243	0.198	7.376

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	131	133	335	85	1418	133	128
normalized size	1	1.00	1.75	1.77	4.47	1.13	18.91	1.77	1.71
time (sec)	N/A	0.110	0.795	0.227	1.117	0.432	15.764	0.294	8.425

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	41	0	40	47	687	43	89
normalized size	1	1.00	0.69	0.00	0.68	0.80	11.64	0.73	1.51
time (sec)	N/A	0.073	0.129	180.000	0.668	0.422	14.794	0.220	8.343

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	193	193	519	106	3186	189	180
normalized size	1	1.00	1.99	1.99	5.35	1.09	32.85	1.95	1.86
time (sec)	N/A	0.120	1.190	0.286	1.019	0.474	55.959	1.433	9.416

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	213	223	610	133	4335	217	190
normalized size	1	1.00	1.63	1.70	4.66	1.02	33.09	1.66	1.45
time (sec)	N/A	0.171	1.544	0.286	0.440	0.459	98.598	0.252	8.198

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	233	253	703	163	5661	245	185
normalized size	1	1.00	1.40	1.51	4.21	0.98	33.90	1.47	1.11
time (sec)	N/A	0.217	1.625	0.315	0.731	0.462	160.148	0.262	8.616

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	104	79	0	179	0	0	-1
normalized size	1	1.00	0.76	0.58	0.00	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.295	0.860	0.714	0.000	0.465	0.000	0.000	0.000

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	94	69	0	152	0	0	-1
normalized size	1	1.00	0.91	0.67	0.00	1.48	0.00	0.00	-0.01
time (sec)	N/A	0.222	0.572	0.876	0.000	0.433	0.000	0.000	0.000

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	82	59	0	109	0	0	-1
normalized size	1	1.00	1.19	0.86	0.00	1.58	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.365	0.757	0.000	0.456	0.000	0.000	0.000

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	71	47	0	79	0	0	-1
normalized size	1	1.00	2.09	1.38	0.00	2.32	0.00	0.00	-0.03
time (sec)	N/A	0.092	0.125	0.540	0.000	0.440	0.000	0.000	0.000

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	135	94	0	196	0	0	-1
normalized size	1	1.00	1.75	1.22	0.00	2.55	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.653	0.716	0.000	0.454	0.000	0.000	0.000

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	107	120	0	255	0	0	-1
normalized size	1	1.00	1.41	1.58	0.00	3.36	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.708	0.632	0.000	0.457	0.000	0.000	0.000

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	176	189	0	336	0	0	-1
normalized size	1	1.00	1.56	1.67	0.00	2.97	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.968	0.815	0.000	0.463	0.000	0.000	0.000

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	189	243	0	408	0	0	-1
normalized size	1	1.00	1.30	1.68	0.00	2.81	0.00	0.00	-0.01
time (sec)	N/A	0.193	1.161	1.060	0.000	0.463	0.000	0.000	0.000

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	1105	81	0	234	0	0	-1
normalized size	1	1.00	7.62	0.56	0.00	1.61	0.00	0.00	-0.01
time (sec)	N/A	0.327	6.436	0.872	0.000	0.477	0.000	0.000	0.000

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	96	71	0	201	0	0	-1
normalized size	1	1.00	0.88	0.65	0.00	1.84	0.00	0.00	-0.01
time (sec)	N/A	0.260	5.693	0.840	0.000	0.457	0.000	0.000	0.000

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	84	61	0	152	0	0	-1
normalized size	1	1.00	1.15	0.84	0.00	2.08	0.00	0.00	-0.01
time (sec)	N/A	0.195	1.388	0.839	0.000	0.442	0.000	0.000	0.000

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	73	49	0	114	0	0	-1
normalized size	1	1.00	2.03	1.36	0.00	3.17	0.00	0.00	-0.03
time (sec)	N/A	0.125	0.233	0.641	0.000	0.428	0.000	0.000	0.000

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	130	112	0	238	0	0	-1
normalized size	1	1.00	1.13	0.97	0.00	2.07	0.00	0.00	-0.01
time (sec)	N/A	0.244	0.473	0.924	0.000	0.453	0.000	0.000	0.000

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	149	145	0	299	0	0	-1
normalized size	1	1.00	1.30	1.26	0.00	2.60	0.00	0.00	-0.01
time (sec)	N/A	0.244	0.685	0.776	0.000	0.462	0.000	0.000	0.000

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	163	191	0	362	0	0	-1
normalized size	1	1.00	1.34	1.57	0.00	2.97	0.00	0.00	-0.01
time (sec)	N/A	0.240	0.974	0.895	0.000	0.461	0.000	0.000	0.000

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	307	245	0	440	0	0	-1
normalized size	1	1.00	1.97	1.57	0.00	2.82	0.00	0.00	-0.01
time (sec)	N/A	0.273	0.983	1.016	0.000	0.486	0.000	0.000	0.000

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	371	299	0	523	0	0	-1
normalized size	1	1.00	1.95	1.57	0.00	2.75	0.00	0.00	-0.01
time (sec)	N/A	0.303	1.456	1.299	0.000	0.469	0.000	0.000	0.000

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	112	81	0	265	0	0	-1
normalized size	1	1.00	0.77	0.56	0.00	1.83	0.00	0.00	-0.01
time (sec)	N/A	0.330	6.267	0.894	0.000	0.445	0.000	0.000	0.000

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	1105	71	0	234	0	0	-1
normalized size	1	1.00	10.14	0.65	0.00	2.15	0.00	0.00	-0.01
time (sec)	N/A	0.268	6.477	0.759	0.000	0.452	0.000	0.000	0.000

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	84	61	0	179	0	0	-1
normalized size	1	1.00	1.15	0.84	0.00	2.45	0.00	0.00	-0.01
time (sec)	N/A	0.203	3.094	0.832	0.000	0.465	0.000	0.000	0.000

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	73	49	0	141	0	0	-1
normalized size	1	1.00	2.03	1.36	0.00	3.92	0.00	0.00	-0.03
time (sec)	N/A	0.127	0.376	0.729	0.000	0.431	0.000	0.000	0.000

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	156	129	0	265	0	0	-1
normalized size	1	1.00	1.03	0.85	0.00	1.75	0.00	0.00	-0.01
time (sec)	N/A	0.316	0.753	1.132	0.000	0.454	0.000	0.000	0.000

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	173	189	0	326	0	0	-1
normalized size	1	1.00	1.15	1.26	0.00	2.17	0.00	0.00	-0.01
time (sec)	N/A	0.320	0.841	1.017	0.000	0.461	0.000	0.000	0.000

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	187	239	0	388	0	0	-1
normalized size	1	1.00	1.19	1.52	0.00	2.47	0.00	0.00	-0.01
time (sec)	N/A	0.325	1.051	1.168	0.000	0.465	0.000	0.000	0.000

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	307	245	0	440	0	0	-1
normalized size	1	1.00	1.96	1.56	0.00	2.80	0.00	0.00	-0.01
time (sec)	N/A	0.323	1.574	1.004	0.000	0.463	0.000	0.000	0.000

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	371	299	0	523	0	0	-1
normalized size	1	1.00	1.94	1.57	0.00	2.74	0.00	0.00	-0.01
time (sec)	N/A	0.353	2.655	1.123	0.000	0.473	0.000	0.000	0.000

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	435	353	0	600	0	0	-1
normalized size	1	1.00	1.93	1.57	0.00	2.67	0.00	0.00	-0.00
time (sec)	N/A	0.388	4.110	1.117	0.000	0.513	0.000	0.000	0.000

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	112	69	238	74	0	0	-1
normalized size	1	1.00	0.85	0.52	1.80	0.56	0.00	0.00	-0.01
time (sec)	N/A	0.345	2.185	0.637	1.802	0.463	0.000	0.000	0.000

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	102	59	192	58	0	0	-1
normalized size	1	1.00	1.04	0.60	1.96	0.59	0.00	0.00	-0.01
time (sec)	N/A	0.269	0.753	0.705	0.661	0.439	0.000	0.000	0.000

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	88	49	146	41	0	0	90
normalized size	1	1.00	1.47	0.82	2.43	0.68	0.00	0.00	1.50
time (sec)	N/A	0.204	0.307	0.633	0.920	0.441	0.000	0.000	7.290

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	39	76	29	0	0	40
normalized size	1	1.00	1.00	1.34	2.62	1.00	0.00	0.00	1.38
time (sec)	N/A	0.128	0.097	0.612	0.778	0.445	0.000	0.000	0.197

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	97	86	0	154	0	0	-1
normalized size	1	1.00	1.17	1.04	0.00	1.86	0.00	0.00	-0.01
time (sec)	N/A	0.159	0.332	0.963	0.000	0.477	0.000	0.000	0.000

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	125	134	0	207	0	0	-1
normalized size	1	1.00	1.07	1.15	0.00	1.77	0.00	0.00	-0.01
time (sec)	N/A	0.192	0.625	0.822	0.000	0.456	0.000	0.000	0.000

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	162	210	0	241	0	0	-1
normalized size	1	1.00	1.04	1.35	0.00	1.54	0.00	0.00	-0.01
time (sec)	N/A	0.267	0.839	1.525	0.000	0.469	0.000	0.000	0.000

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	124	91	380	104	0	0	-1
normalized size	1	1.00	0.70	0.52	2.16	0.59	0.00	0.00	-0.01
time (sec)	N/A	0.420	3.123	0.671	1.021	0.473	0.000	0.000	0.000

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	112	79	334	91	0	0	-1
normalized size	1	1.00	0.82	0.58	2.46	0.67	0.00	0.00	-0.01
time (sec)	N/A	0.332	1.247	0.843	0.823	0.441	0.000	0.000	0.000

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	104	71	288	76	0	0	360
normalized size	1	1.00	1.04	0.71	2.88	0.76	0.00	0.00	3.60
time (sec)	N/A	0.264	0.801	0.822	1.177	0.429	0.000	0.000	11.683

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	92	61	239	57	0	0	120
normalized size	1	1.00	1.35	0.90	3.51	0.84	0.00	0.00	1.76
time (sec)	N/A	0.196	0.366	0.914	0.427	0.420	0.000	0.000	10.323

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	73	49	149	46	0	0	227
normalized size	1	1.00	2.03	1.36	4.14	1.28	0.00	0.00	6.31
time (sec)	N/A	0.136	0.133	0.747	0.984	0.423	0.000	0.000	9.347

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	109	109	0	204	0	0	-1
normalized size	1	1.00	0.88	0.88	0.00	1.65	0.00	0.00	-0.01
time (sec)	N/A	0.224	0.514	1.251	0.000	0.450	0.000	0.000	0.000

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	164	157	0	186	0	0	-1
normalized size	1	1.00	1.06	1.01	0.00	1.20	0.00	0.00	-0.01
time (sec)	N/A	0.250	0.789	1.246	0.000	0.450	0.000	0.000	0.000

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	156	233	0	241	0	0	-1
normalized size	1	1.00	0.81	1.21	0.00	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.331	1.173	1.187	0.000	0.466	0.000	0.000	0.000

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	124	91	472	119	0	0	-1
normalized size	1	1.00	0.71	0.52	2.71	0.68	0.00	0.00	-0.01
time (sec)	N/A	0.403	3.151	0.790	0.837	0.431	0.000	0.000	0.000

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	114	81	426	106	0	0	542
normalized size	1	1.00	0.85	0.60	3.18	0.79	0.00	0.00	4.04
time (sec)	N/A	0.330	1.275	1.359	1.002	0.429	0.000	0.000	13.874

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	71	380	91	0	0	453
normalized size	1	1.00	1.00	0.68	3.65	0.88	0.00	0.00	4.36
time (sec)	N/A	0.269	0.868	0.839	1.086	0.425	0.000	0.000	11.949

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	92	61	331	74	0	0	355
normalized size	1	1.00	1.26	0.84	4.53	1.01	0.00	0.00	4.86
time (sec)	N/A	0.196	0.392	0.816	0.853	0.464	0.000	0.000	10.912

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	73	49	218	61	0	0	90
normalized size	1	1.00	2.03	1.36	6.06	1.69	0.00	0.00	2.50
time (sec)	N/A	0.122	0.152	0.656	1.134	0.432	0.000	0.000	9.882

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	189	122	0	241	0	0	-1
normalized size	1	1.00	1.18	0.76	0.00	1.51	0.00	0.00	-0.01
time (sec)	N/A	0.294	0.615	1.184	0.000	0.483	0.000	0.000	0.000

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	174	170	0	238	0	0	-1
normalized size	1	1.00	0.91	0.89	0.00	1.25	0.00	0.00	-0.01
time (sec)	N/A	0.328	1.192	1.250	0.000	0.465	0.000	0.000	0.000

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	443	246	0	208	0	0	-1
normalized size	1	1.00	1.94	1.08	0.00	0.91	0.00	0.00	-0.00
time (sec)	N/A	0.407	1.537	1.298	0.000	0.470	0.000	0.000	0.000

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	83	103	0	95	0	0	99
normalized size	1	1.00	1.93	2.40	0.00	2.21	0.00	0.00	2.30
time (sec)	N/A	0.083	0.397	0.441	0.000	0.446	0.000	0.000	8.250

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	74	78	0	83	0	0	88
normalized size	1	1.00	1.72	1.81	0.00	1.93	0.00	0.00	2.05
time (sec)	N/A	0.082	0.304	0.338	0.000	0.436	0.000	0.000	7.707

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	60	61	0	61	0	0	71
normalized size	1	1.00	1.40	1.42	0.00	1.42	0.00	0.00	1.65
time (sec)	N/A	0.081	0.217	0.312	0.000	0.437	0.000	0.000	0.845

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	39	44	0	43	0	0	47
normalized size	1	1.00	0.95	1.07	0.00	1.05	0.00	0.00	1.15
time (sec)	N/A	0.073	0.088	0.306	0.000	0.431	0.000	0.000	7.010

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	119	106	63	0	0	0	-1
normalized size	1	1.00	2.29	2.04	1.21	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.101	0.929	0.263	0.917	0.629	0.000	0.000	0.000

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	84	68	0	59	0	0	-1
normalized size	1	1.00	2.10	1.70	0.00	1.48	0.00	0.00	-0.02
time (sec)	N/A	0.082	0.207	0.272	0.000	0.439	0.000	0.000	0.000

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	87	96	0	73	0	0	142
normalized size	1	1.00	2.02	2.23	0.00	1.70	0.00	0.00	3.30
time (sec)	N/A	0.085	0.225	0.275	0.000	0.440	0.000	0.000	8.895

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	87	120	0	90	0	0	190
normalized size	1	1.00	2.02	2.79	0.00	2.09	0.00	0.00	4.42
time (sec)	N/A	0.084	0.283	0.299	0.000	0.435	0.000	0.000	10.982

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	146	106	0	101	0	0	111
normalized size	1	1.00	1.64	1.19	0.00	1.13	0.00	0.00	1.25
time (sec)	N/A	0.188	1.044	0.326	0.000	0.452	0.000	0.000	8.917

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	137	90	0	87	0	0	100
normalized size	1	1.00	1.54	1.01	0.00	0.98	0.00	0.00	1.12
time (sec)	N/A	0.179	0.646	0.289	0.000	0.448	0.000	0.000	1.750

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	70	55	0	60	0	0	66
normalized size	1	1.00	0.79	0.62	0.00	0.67	0.00	0.00	0.74
time (sec)	N/A	0.178	0.431	0.258	0.000	0.435	0.000	0.000	0.889

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	60	63	0	61	0	0	71
normalized size	1	1.00	1.40	1.47	0.00	1.42	0.00	0.00	1.65
time (sec)	N/A	0.082	0.212	0.298	0.000	0.422	0.000	0.000	7.348

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	113	252	0	0	0	0	-1
normalized size	1	1.00	1.18	2.62	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.192	0.336	0.266	0.000	0.636	0.000	0.000	0.000

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	153	375	137	0	0	0	-1
normalized size	1	1.00	1.58	3.87	1.41	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.195	0.470	0.241	0.898	0.846	0.000	0.000	0.000

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	99	90	0	80	0	0	-1
normalized size	1	1.00	2.36	2.14	0.00	1.90	0.00	0.00	-0.02
time (sec)	N/A	0.091	0.481	0.228	0.000	0.439	0.000	0.000	0.000

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	106	141	0	101	0	0	124
normalized size	1	1.00	1.20	1.60	0.00	1.15	0.00	0.00	1.41
time (sec)	N/A	0.184	0.584	0.257	0.000	0.441	0.000	0.000	10.650

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	106	169	0	114	0	0	195
normalized size	1	1.00	1.15	1.84	0.00	1.24	0.00	0.00	2.12
time (sec)	N/A	0.177	1.071	0.283	0.000	0.448	0.000	0.000	11.814

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	106	196	0	131	0	0	225
normalized size	1	1.00	1.15	2.13	0.00	1.42	0.00	0.00	2.45
time (sec)	N/A	0.176	1.495	0.279	0.000	0.481	0.000	0.000	12.589

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	156	116	0	112	0	0	124
normalized size	1	1.00	1.16	0.87	0.00	0.84	0.00	0.00	0.93
time (sec)	N/A	0.270	1.378	0.321	0.000	0.450	0.000	0.000	9.943

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	77	67	0	85	0	0	83
normalized size	1	1.00	0.57	0.50	0.00	0.63	0.00	0.00	0.62
time (sec)	N/A	0.271	0.543	0.278	0.000	0.457	0.000	0.000	1.501

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	133	91	0	87	0	0	100
normalized size	1	1.00	1.49	1.02	0.00	0.98	0.00	0.00	1.12
time (sec)	N/A	0.172	0.689	0.284	0.000	0.461	0.000	0.000	8.271

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	72	80	0	82	0	0	86
normalized size	1	1.00	1.67	1.86	0.00	1.91	0.00	0.00	2.00
time (sec)	N/A	0.080	0.271	0.316	0.000	0.438	0.000	0.000	7.747

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	127	315	0	0	0	0	-1
normalized size	1	1.00	0.90	2.23	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.279	0.593	0.299	0.000	0.687	0.000	0.000	0.000

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	169	439	0	0	0	0	-1
normalized size	1	1.00	1.17	3.05	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.290	0.808	0.269	0.000	0.974	0.000	0.000	0.000

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	190	553	184	0	0	0	-1
normalized size	1	1.00	1.29	3.76	1.25	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.293	1.162	0.263	1.923	1.267	0.000	0.000	0.000

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	110	130	0	109	0	0	-1
normalized size	1	1.00	2.62	3.10	0.00	2.60	0.00	0.00	-0.02
time (sec)	N/A	0.092	0.983	0.275	0.000	0.431	0.000	0.000	0.000

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	118	199	0	133	0	0	242
normalized size	1	1.00	1.34	2.26	0.00	1.51	0.00	0.00	2.75
time (sec)	N/A	0.193	2.251	0.283	0.000	0.448	0.000	0.000	11.575

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	118	226	0	148	0	0	273
normalized size	1	1.00	0.89	1.70	0.00	1.11	0.00	0.00	2.05
time (sec)	N/A	0.283	3.424	0.285	0.000	0.473	0.000	0.000	12.076

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	118	252	0	162	0	0	287
normalized size	1	1.00	0.84	1.80	0.00	1.16	0.00	0.00	2.05
time (sec)	N/A	0.273	4.909	0.323	0.000	0.477	0.000	0.000	12.240

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	127	143	0	128	0	0	376
normalized size	1	1.00	0.71	0.80	0.00	0.72	0.00	0.00	2.10
time (sec)	N/A	0.366	5.289	0.370	0.000	0.476	0.000	0.000	11.301

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	87	77	0	101	0	0	179
normalized size	1	1.00	0.49	0.43	0.00	0.56	0.00	0.00	1.00
time (sec)	N/A	0.366	0.987	0.308	0.000	0.443	0.000	0.000	10.648

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	107	116	0	112	0	0	124
normalized size	1	1.00	0.80	0.87	0.00	0.84	0.00	0.00	0.93
time (sec)	N/A	0.264	1.262	0.314	0.000	0.460	0.000	0.000	10.287

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	93	106	0	101	0	0	109
normalized size	1	1.00	1.04	1.19	0.00	1.13	0.00	0.00	1.22
time (sec)	N/A	0.170	0.963	0.308	0.000	0.435	0.000	0.000	9.187

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	82	103	0	95	0	0	99
normalized size	1	1.00	1.91	2.40	0.00	2.21	0.00	0.00	2.30
time (sec)	N/A	0.082	0.335	0.342	0.000	0.435	0.000	0.000	8.294

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	150	367	0	0	0	0	-1
normalized size	1	1.00	0.82	1.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.376	1.035	0.315	0.000	0.597	0.000	0.000	0.000

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	179	491	0	0	0	0	-1
normalized size	1	1.00	0.93	2.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.396	1.633	0.289	0.000	1.182	0.000	0.000	0.000

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	207	618	0	0	0	0	-1
normalized size	1	1.00	1.06	3.17	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.402	1.937	0.300	0.000	1.541	0.000	0.000	0.000

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	232	748	336	0	0	0	-1
normalized size	1	1.00	1.20	3.88	1.74	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.408	2.318	0.294	1.005	2.250	0.000	0.000	0.000

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	115	154	0	127	0	0	-1
normalized size	1	1.00	2.74	3.67	0.00	3.02	0.00	0.00	-0.02
time (sec)	N/A	0.093	4.501	0.255	0.000	0.475	0.000	0.000	0.000

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	331	247	0	163	0	0	317
normalized size	1	1.00	3.76	2.81	0.00	1.85	0.00	0.00	3.60
time (sec)	N/A	0.186	6.587	0.303	0.000	0.443	0.000	0.000	12.452

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	335	276	0	178	0	0	330
normalized size	1	1.00	2.52	2.08	0.00	1.34	0.00	0.00	2.48
time (sec)	N/A	0.294	6.658	0.313	0.000	0.456	0.000	0.000	12.057

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	333	302	0	192	0	0	647
normalized size	1	1.00	1.87	1.70	0.00	1.08	0.00	0.00	3.63
time (sec)	N/A	0.380	6.691	0.346	0.000	0.488	0.000	0.000	13.324

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	329	328	0	204	0	0	673
normalized size	1	1.00	1.75	1.74	0.00	1.09	0.00	0.00	3.58
time (sec)	N/A	0.384	6.758	0.384	0.000	0.496	0.000	0.000	13.981

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	136	320	0	0	0	0	-1
normalized size	1	1.00	0.98	2.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.281	0.553	0.292	0.000	0.699	0.000	0.000	0.000

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	119	261	0	0	0	0	-1
normalized size	1	1.00	1.28	2.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.309	0.273	0.000	0.628	0.000	0.000	0.000

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	118	106	64	0	0	0	-1
normalized size	1	1.00	2.41	2.16	1.31	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.100	0.984	0.228	0.602	0.631	0.000	0.000	0.000

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	89	92	0	160	0	0	-1
normalized size	1	1.00	1.93	2.00	0.00	3.48	0.00	0.00	-0.02
time (sec)	N/A	0.086	0.233	0.250	0.000	0.507	0.000	0.000	0.000

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	161	165	0	311	0	0	-1
normalized size	1	1.00	1.69	1.74	0.00	3.27	0.00	0.00	-0.01
time (sec)	N/A	0.175	0.401	0.269	0.000	0.527	0.000	0.000	0.000

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	224	252	0	376	0	0	-1
normalized size	1	1.00	1.60	1.80	0.00	2.69	0.00	0.00	-0.01
time (sec)	N/A	0.269	0.633	0.273	0.000	0.517	0.000	0.000	0.000

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	162	501	0	0	0	0	-1
normalized size	1	1.00	0.85	2.62	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.377	1.609	0.296	0.000	1.110	0.000	0.000	0.000

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	153	446	0	0	0	0	-1
normalized size	1	1.00	1.07	3.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.288	0.766	0.274	0.000	0.955	0.000	0.000	0.000

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	134	388	136	0	0	0	-1
normalized size	1	1.00	1.38	4.00	1.40	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.197	0.440	0.284	1.001	0.858	0.000	0.000	0.000

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	85	68	0	58	0	0	52
normalized size	1	1.00	2.07	1.66	0.00	1.41	0.00	0.00	1.27
time (sec)	N/A	0.085	0.192	0.271	0.000	0.431	0.000	0.000	7.519

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	148	167	0	305	0	0	-1
normalized size	1	1.00	1.56	1.76	0.00	3.21	0.00	0.00	-0.01
time (sec)	N/A	0.176	0.405	0.267	0.000	0.523	0.000	0.000	0.000

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	170	115	0	262	0	0	-1
normalized size	1	1.00	1.19	0.80	0.00	1.83	0.00	0.00	-0.01
time (sec)	N/A	0.278	0.617	0.243	0.000	0.484	0.000	0.000	0.000

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	287	229	0	377	0	0	-1
normalized size	1	1.00	1.50	1.20	0.00	1.97	0.00	0.00	-0.01
time (sec)	N/A	0.374	0.767	0.273	0.000	0.533	0.000	0.000	0.000

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	202	685	0	0	0	0	-1
normalized size	1	1.00	0.85	2.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.493	5.120	0.319	0.000	1.931	0.000	0.000	0.000

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	187	633	0	0	0	0	-1
normalized size	1	1.00	0.97	3.28	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.387	1.998	0.308	0.000	1.496	0.000	0.000	0.000

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	172	567	183	0	0	0	-1
normalized size	1	1.00	1.20	3.97	1.28	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.303	1.188	0.259	0.436	1.172	0.000	0.000	0.000

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	86	93	0	80	0	0	118
normalized size	1	1.00	2.05	2.21	0.00	1.90	0.00	0.00	2.81
time (sec)	N/A	0.092	0.458	0.291	0.000	0.436	0.000	0.000	8.214

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	87	92	0	73	0	0	103
normalized size	1	1.00	2.02	2.14	0.00	1.70	0.00	0.00	2.40
time (sec)	N/A	0.085	0.220	0.280	0.000	0.423	0.000	0.000	7.696

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	211	252	0	376	0	0	-1
normalized size	1	1.00	1.51	1.80	0.00	2.69	0.00	0.00	-0.01
time (sec)	N/A	0.271	0.648	0.274	0.000	0.530	0.000	0.000	0.000

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	287	229	0	371	0	0	-1
normalized size	1	1.00	1.53	1.22	0.00	1.97	0.00	0.00	-0.01
time (sec)	N/A	0.383	0.746	0.268	0.000	0.503	0.000	0.000	0.000

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	237	134	0	288	0	0	-1
normalized size	1	1.00	1.00	0.57	0.00	1.22	0.00	0.00	-0.00
time (sec)	N/A	0.479	0.971	0.281	0.000	0.533	0.000	0.000	0.000

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	365	0	0	0	0	0	-1
normalized size	1	1.00	3.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.155	2.957	1.716	0.000	0.454	0.000	0.000	0.000

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.143	180.011	5.568	0.000	0.457	0.000	0.000	0.000

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	88512	0	0	0	0	0	-1
normalized size	1	1.00	1029.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.140	153.514	6.569	0.000	0.442	0.000	0.000	0.000

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	261	0	0	0	0	0	-1
normalized size	1	1.00	3.11	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	1.664	1.798	0.000	0.472	0.000	0.000	0.000

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	3844	0	0	0	0	0	-1
normalized size	1	1.00	50.58	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	6.392	0.400	0.000	0.466	0.000	0.000	0.000

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	5391	0	0	0	0	0	-1
normalized size	1	1.00	62.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	6.424	1.126	0.000	0.476	0.000	0.000	0.000

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	7184	0	0	0	0	0	-1
normalized size	1	1.00	83.53	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	22.776	1.086	0.000	0.488	0.000	0.000	0.000

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	149	0	290	267	0	0	163
normalized size	1	1.00	0.93	0.00	1.81	1.67	0.00	0.00	1.02
time (sec)	N/A	0.252	2.508	0.365	0.466	0.468	0.000	0.000	10.048

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	110	0	193	145	0	0	94
normalized size	1	1.00	1.10	0.00	1.93	1.45	0.00	0.00	0.94
time (sec)	N/A	0.149	0.478	0.301	0.445	0.471	0.000	0.000	1.188

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	85	0	116	76	0	0	53
normalized size	1	1.00	1.85	0.00	2.52	1.65	0.00	0.00	1.15
time (sec)	N/A	0.068	0.166	0.300	0.450	0.487	0.000	0.000	0.451

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	157	0	0	0	0	0	-1
normalized size	1	1.00	2.31	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	1.456	0.280	0.000	0.451	0.000	0.000	0.000

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	3006	0	0	0	0	0	-1
normalized size	1	1.00	40.62	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.157	17.234	0.277	0.000	0.455	0.000	0.000	0.000

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	5136	0	0	0	0	0	-1
normalized size	1	1.00	69.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.160	21.982	0.247	0.000	0.488	0.000	0.000	0.000

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	157	0	0	0	0	0	-1
normalized size	1	1.00	2.31	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.527	0.008	0.000	0.461	0.000	0.000	0.000

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	157	0	0	0	0	0	-1
normalized size	1	1.00	2.31	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.438	0.358	0.000	0.459	0.000	0.000	0.000

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	174	0	0	101	0	0	149
normalized size	1	1.00	1.06	0.00	0.00	0.62	0.00	0.00	0.91
time (sec)	N/A	0.223	8.533	1.021	0.000	0.488	0.000	0.000	8.249

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	136	0	0	72	0	0	111
normalized size	1	1.00	1.35	0.00	0.00	0.71	0.00	0.00	1.10
time (sec)	N/A	0.135	3.044	0.793	0.000	0.465	0.000	0.000	0.880

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	107	0	0	44	0	0	58
normalized size	1	1.00	2.33	0.00	0.00	0.96	0.00	0.00	1.26
time (sec)	N/A	0.067	1.527	0.606	0.000	0.464	0.000	0.000	0.409

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	388	0	0	0	0	0	-1
normalized size	1	1.00	3.46	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.158	2.915	0.707	0.000	0.452	0.000	0.000	0.000

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	602	0	0	0	0	0	-1
normalized size	1	1.00	5.28	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.186	7.180	0.555	0.000	0.462	0.000	0.000	0.000

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	1201	0	0	0	0	0	-1
normalized size	1	1.00	10.54	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.182	12.711	0.806	0.000	0.469	0.000	0.000	0.000

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	207	259	250	204	580	272	559
normalized size	1	1.00	0.91	1.14	1.10	0.90	2.56	1.20	2.46
time (sec)	N/A	0.282	1.362	0.329	0.386	0.473	6.569	0.177	9.941

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	124	182	175	145	386	191	460
normalized size	1	1.00	0.77	1.12	1.08	0.90	2.38	1.18	2.84
time (sec)	N/A	0.189	0.801	0.288	0.330	0.460	2.680	0.471	8.175

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	89	115	112	90	199	117	108
normalized size	1	1.00	0.90	1.16	1.13	0.91	2.01	1.18	1.09
time (sec)	N/A	0.093	0.396	0.188	0.309	0.442	1.331	0.160	6.987

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	45	59	57	48	94	55	100
normalized size	1	1.00	0.94	1.23	1.19	1.00	1.96	1.15	2.08
time (sec)	N/A	0.024	0.108	0.104	0.310	0.427	0.721	0.225	6.994

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	27	17	16	18	19	17	25
normalized size	1	1.00	1.69	1.06	1.00	1.12	1.19	1.06	1.56
time (sec)	N/A	0.008	0.007	0.013	0.318	0.436	0.233	0.169	6.718

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	182	119	0	228	537	86	449
normalized size	1	1.00	2.89	1.89	0.00	3.62	8.52	1.37	7.13
time (sec)	N/A	0.091	0.323	0.200	0.000	0.483	119.906	0.223	7.392

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	220	147	0	362	0	129	140
normalized size	1	1.00	2.65	1.77	0.00	4.36	0.00	1.55	1.69
time (sec)	N/A	0.092	0.589	0.238	0.000	0.490	0.000	0.340	6.946

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	242	1104	0	803	0	384	445
normalized size	1	1.00	1.81	8.24	0.00	5.99	0.00	2.87	3.32
time (sec)	N/A	0.184	1.201	0.281	0.000	0.484	0.000	0.362	8.967

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	428	3104	0	1344	0	808	877
normalized size	1	1.00	2.23	16.17	0.00	7.00	0.00	4.21	4.57
time (sec)	N/A	0.335	2.706	0.317	0.000	0.565	0.000	1.382	9.706

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	262	462	451	299	1136	458	865
normalized size	1	1.00	0.82	1.45	1.42	0.94	3.57	1.44	2.72
time (sec)	N/A	0.463	1.399	0.394	0.347	0.469	9.416	0.215	9.924

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	204	329	318	217	729	324	611
normalized size	1	1.00	0.88	1.41	1.36	0.93	3.13	1.39	2.62
time (sec)	N/A	0.309	0.940	0.332	0.332	0.464	4.894	0.212	8.420

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	148	219	211	145	459	208	440
normalized size	1	1.00	0.95	1.40	1.35	0.93	2.94	1.33	2.82
time (sec)	N/A	0.202	0.545	0.256	0.330	0.449	2.359	0.352	8.340

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	106	117	114	82	199	109	91
normalized size	1	1.00	1.13	1.24	1.21	0.87	2.12	1.16	0.97
time (sec)	N/A	0.062	0.331	0.188	0.312	0.437	0.983	0.171	6.923

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	34	52	47	41	78	40	123
normalized size	1	1.00	0.76	1.16	1.04	0.91	1.73	0.89	2.73
time (sec)	N/A	0.015	0.186	0.092	0.303	0.423	0.367	0.160	6.871

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	130	228	0	296	0	136	940
normalized size	1	1.00	1.41	2.48	0.00	3.22	0.00	1.48	10.22
time (sec)	N/A	0.202	0.412	0.247	0.000	0.476	0.000	0.209	7.678

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	115	139	389	0	476	0	205	2836
normalized size	1	1.03	1.24	3.47	0.00	4.25	0.00	1.83	25.32
time (sec)	N/A	0.182	0.485	0.294	0.000	0.483	0.000	0.220	11.241

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	140	799	0	679	0	348	362
normalized size	1	1.00	1.01	5.79	0.00	4.92	0.00	2.52	2.62
time (sec)	N/A	0.182	0.620	0.324	0.000	0.500	0.000	0.261	9.427

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	196	2425	0	1366	0	781	735
normalized size	1	1.00	0.95	11.71	0.00	6.60	0.00	3.77	3.55
time (sec)	N/A	0.315	2.561	0.363	0.000	0.559	0.000	0.589	10.316

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	269	6466	0	2151	0	1555	1411
normalized size	1	1.00	0.94	22.61	0.00	7.52	0.00	5.44	4.93
time (sec)	N/A	0.507	4.888	0.413	0.000	0.616	0.000	2.323	10.347

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	326	233	481	469	261	1176	373	773
normalized size	1	1.52	1.08	2.24	2.18	1.21	5.47	1.73	3.60
time (sec)	N/A	0.545	1.393	0.398	0.361	0.475	9.291	0.272	8.479

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	189	177	319	308	180	702	251	493
normalized size	1	1.15	1.08	1.95	1.88	1.10	4.28	1.53	3.01
time (sec)	N/A	0.260	0.740	0.317	0.353	0.461	4.707	0.206	8.359

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	117	120	178	171	108	371	138	330
normalized size	1	1.06	1.09	1.62	1.55	0.98	3.37	1.25	3.00
time (sec)	N/A	0.097	0.518	0.251	0.345	0.450	2.001	0.201	8.067

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	44	74	72	54	121	58	156
normalized size	1	1.00	0.70	1.17	1.14	0.86	1.92	0.92	2.48
time (sec)	N/A	0.050	0.354	0.184	0.317	0.468	0.744	0.510	9.120

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	162	480	0	404	0	239	3382
normalized size	1	1.00	1.13	3.36	0.00	2.83	0.00	1.67	23.65
time (sec)	N/A	0.389	0.658	0.258	0.000	0.506	0.000	0.261	9.220

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	162	600	0	645	0	395	5079
normalized size	1	1.00	1.01	3.73	0.00	4.01	0.00	2.45	31.55
time (sec)	N/A	0.381	0.676	0.314	0.000	0.503	0.000	0.246	12.955

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	196	1400	0	1064	0	522	6246
normalized size	1	1.00	1.05	7.49	0.00	5.69	0.00	2.79	33.40
time (sec)	N/A	0.477	1.008	0.340	0.000	0.544	0.000	0.410	14.274

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	178	1924	0	1106	0	667	649
normalized size	1	1.00	0.86	9.29	0.00	5.34	0.00	3.22	3.14
time (sec)	N/A	0.475	2.353	0.378	0.000	0.536	0.000	0.554	10.363

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	240	5149	0	2009	0	1338	1231
normalized size	1	1.00	0.83	17.82	0.00	6.95	0.00	4.63	4.26
time (sec)	N/A	0.696	3.206	0.418	0.000	0.594	0.000	2.222	10.104

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	234	673	725	349	8605	306	451
normalized size	1	1.00	1.24	3.56	3.84	1.85	45.53	1.62	2.39
time (sec)	N/A	0.227	0.412	0.233	0.444	0.476	15.638	0.221	9.786

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	192	364	425	236	3602	172	282
normalized size	1	1.00	1.59	3.01	3.51	1.95	29.77	1.42	2.33
time (sec)	N/A	0.125	0.586	0.263	0.439	0.539	8.350	0.194	9.435

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	122	140	209	142	940	143	124
normalized size	1	1.00	1.97	2.26	3.37	2.29	15.16	2.31	2.00
time (sec)	N/A	0.137	0.464	0.245	0.447	0.476	3.667	0.151	7.380

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	79	65	78	66	109	40	35
normalized size	1	1.00	2.26	1.86	2.23	1.89	3.11	1.14	1.00
time (sec)	N/A	0.047	0.165	0.154	0.429	0.492	1.825	0.154	6.826

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	48	22	27	42	27	22	21
normalized size	1	1.00	2.09	0.96	1.17	1.83	1.17	0.96	0.91
time (sec)	N/A	0.012	0.042	0.115	0.308	0.434	0.931	0.409	6.975

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	114	87	0	489	0	100	121
normalized size	1	1.00	1.28	0.98	0.00	5.49	0.00	1.12	1.36
time (sec)	N/A	0.130	0.357	0.291	0.000	0.475	0.000	1.213	6.986

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	162	273	0	1120	0	311	309
normalized size	1	1.00	1.08	1.82	0.00	7.47	0.00	2.07	2.06
time (sec)	N/A	0.178	0.647	0.336	0.000	0.486	0.000	0.344	8.566

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	230	1224	0	2365	0	482	753
normalized size	1	1.00	1.08	5.75	0.00	11.10	0.00	2.26	3.54
time (sec)	N/A	0.323	1.983	0.332	0.000	0.573	0.000	1.861	10.203

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	837	982	1312	578	0	977	692
normalized size	1	1.00	3.22	3.78	5.05	2.22	0.00	3.76	2.66
time (sec)	N/A	0.495	1.697	0.318	0.475	0.464	0.000	0.244	9.426

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	378	618	908	440	8950	338	478
normalized size	1	1.00	1.94	3.17	4.66	2.26	45.90	1.73	2.45
time (sec)	N/A	0.362	1.947	0.303	0.454	0.451	28.138	0.245	9.207

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	212	340	591	308	3585	209	298
normalized size	1	1.00	1.77	2.83	4.92	2.57	29.88	1.74	2.48
time (sec)	N/A	0.367	0.354	0.267	0.436	0.450	14.917	0.955	8.385

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	172	213	360	197	915	132	93
normalized size	1	1.00	2.02	2.51	4.24	2.32	10.76	1.55	1.09
time (sec)	N/A	0.142	0.281	0.253	0.430	0.449	8.060	0.173	7.433

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	43	70	214	117	372	68	97
normalized size	1	1.00	0.66	1.08	3.29	1.80	5.72	1.05	1.49
time (sec)	N/A	0.052	0.060	0.217	0.332	0.412	3.649	0.218	7.213

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	54	53	117	95	221	50	76
normalized size	1	1.00	0.98	0.96	2.13	1.73	4.02	0.91	1.38
time (sec)	N/A	0.028	0.100	0.141	0.319	0.437	1.795	0.163	6.981

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	204	175	0	989	0	195	250
normalized size	1	1.00	1.56	1.34	0.00	7.55	0.00	1.49	1.91
time (sec)	N/A	0.282	0.351	0.310	0.000	0.485	0.000	0.188	8.019

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	267	361	0	2297	0	320	625
normalized size	1	1.00	1.21	1.63	0.00	10.39	0.00	1.45	2.83
time (sec)	N/A	0.417	1.372	0.319	0.000	0.558	0.000	0.210	10.487

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	338	1313	0	3540	0	614	1199
normalized size	1	1.00	1.15	4.47	0.00	12.04	0.00	2.09	4.08
time (sec)	N/A	0.615	1.203	0.353	0.000	0.628	0.000	0.444	10.813

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	560	1340	1993	823	0	776	898
normalized size	1	1.00	1.58	3.79	5.63	2.32	0.00	2.19	2.54
time (sec)	N/A	0.790	2.863	0.286	0.837	0.492	0.000	0.292	9.713

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	992	924	1504	653	0	564	652
normalized size	1	1.00	3.57	3.32	5.41	2.35	0.00	2.03	2.35
time (sec)	N/A	0.614	7.899	0.292	0.486	0.496	0.000	0.250	9.536

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	683	593	1101	494	7373	395	440
normalized size	1	1.00	3.50	3.04	5.65	2.53	37.81	2.03	2.26
time (sec)	N/A	0.613	1.455	0.288	0.454	0.475	48.331	0.258	9.028

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	408	438	784	352	2640	280	240
normalized size	1	1.00	2.87	3.08	5.52	2.48	18.59	1.97	1.69
time (sec)	N/A	0.333	5.610	0.252	0.438	0.441	25.969	0.228	9.904

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	84	139	553	242	1365	181	218
normalized size	1	1.00	0.67	1.11	4.42	1.94	10.92	1.45	1.74
time (sec)	N/A	0.182	0.126	0.235	0.341	0.436	15.740	2.209	7.486

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	63	114	387	190	1015	130	150
normalized size	1	1.00	0.62	1.12	3.79	1.86	9.95	1.27	1.47
time (sec)	N/A	0.075	0.083	0.194	0.330	0.421	9.061	0.264	7.251

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	76	85	203	147	558	78	133
normalized size	1	1.00	0.92	1.02	2.45	1.77	6.72	0.94	1.60
time (sec)	N/A	0.047	0.123	0.158	0.336	0.440	3.537	0.388	6.957

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	301	325	0	1744	0	364	466
normalized size	1	1.00	1.62	1.75	0.00	9.38	0.00	1.96	2.51
time (sec)	N/A	0.522	0.706	0.327	0.000	0.531	0.000	0.337	10.069

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	361	511	0	3235	0	517	987
normalized size	1	1.00	1.21	1.71	0.00	10.86	0.00	1.73	3.31
time (sec)	N/A	0.730	2.553	0.352	0.000	0.599	0.000	1.313	10.302

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	1066	1462	0	5226	0	794	1660
normalized size	1	1.00	2.82	3.87	0.00	13.83	0.00	2.10	4.39
time (sec)	N/A	0.962	6.324	0.370	0.000	0.743	0.000	2.039	11.645

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	55	115	309	150	889	112	94
normalized size	1	1.00	0.73	1.53	4.12	2.00	11.85	1.49	1.25
time (sec)	N/A	0.057	0.065	0.096	0.325	0.444	7.702	0.198	7.065

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	54	115	309	150	887	112	97
normalized size	1	1.00	0.67	1.42	3.81	1.85	10.95	1.38	1.20
time (sec)	N/A	0.066	0.069	0.098	0.327	0.429	7.869	0.272	7.059

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	3531	1315	0	0	0	0	-1
normalized size	1	1.00	12.18	4.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.443	6.764	1.720	0.000	0.471	0.000	0.000	0.000

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	2625	1034	0	0	0	0	-1
normalized size	1	1.00	11.36	4.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.317	6.397	1.459	0.000	0.487	0.000	0.000	0.000

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	1736	657	0	0	0	0	-1
normalized size	1	1.00	9.70	3.67	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.198	6.277	1.266	0.000	0.456	0.000	0.000	0.000

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	880	203	0	0	0	0	176
normalized size	1	1.00	6.38	1.47	0.00	0.00	0.00	0.00	1.28
time (sec)	N/A	0.122	6.247	1.307	0.000	0.455	0.000	0.000	7.619

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	938	246	0	0	0	0	-1
normalized size	1	1.00	5.55	1.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.206	6.388	1.089	0.000	0.442	0.000	0.000	0.000

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	1870	884	0	0	0	0	-1
normalized size	1	1.00	7.89	3.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.336	6.774	4.678	0.000	0.463	0.000	0.000	0.000

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	2815	1046	0	0	0	0	-1
normalized size	1	1.00	8.85	3.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.507	7.154	6.395	0.000	0.485	0.000	0.000	0.000

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	322	1614	0	0	0	0	-1
normalized size	1	1.00	0.85	4.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.670	1.978	1.593	0.000	0.531	0.000	0.000	0.000

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	262	1316	0	0	0	0	-1
normalized size	1	1.00	0.88	4.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.479	2.111	1.626	0.000	0.478	0.000	0.000	0.000

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	244	1035	0	0	0	0	-1
normalized size	1	1.00	1.02	4.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.336	1.414	1.447	0.000	0.457	0.000	0.000	0.000

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	193	758	0	0	0	0	-1
normalized size	1	1.00	1.02	4.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.246	1.082	1.465	0.000	0.472	0.000	0.000	0.000

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	175	463	0	0	0	0	-1
normalized size	1	1.00	0.93	2.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.239	0.912	1.398	0.000	0.489	0.000	0.000	0.000

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	207	1221	0	0	0	0	-1
normalized size	1	1.00	0.84	4.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.369	1.769	1.519	0.000	0.480	0.000	0.000	0.000

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	283	1436	0	0	0	0	-1
normalized size	1	1.00	0.88	4.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.578	2.026	6.392	0.000	0.491	0.000	0.000	0.000

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	467	467	377	1926	0	0	0	0	-1
normalized size	1	1.00	0.81	4.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.032	1.903	1.705	0.000	0.565	0.000	0.000	0.000

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	390	390	318	1613	0	0	0	0	-1
normalized size	1	1.00	0.82	4.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.775	2.274	1.483	0.000	0.558	0.000	0.000	0.000

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	266	1316	0	0	0	0	-1
normalized size	1	1.00	0.84	4.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.580	2.723	1.378	0.000	0.475	0.000	0.000	0.000

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	246	1035	0	0	0	0	-1
normalized size	1	1.00	0.95	4.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.476	1.619	1.569	0.000	0.502	0.000	0.000	0.000

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	234	1031	0	0	0	0	-1
normalized size	1	1.00	0.87	3.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.481	1.448	1.406	0.000	0.507	0.000	0.000	0.000

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	232	1257	0	0	0	0	-1
normalized size	1	1.00	0.83	4.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.577	1.548	5.408	0.000	0.513	0.000	0.000	0.000

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	298	1589	0	0	0	0	-1
normalized size	1	1.00	0.89	4.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.720	2.121	6.597	0.000	0.481	0.000	0.000	0.000

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	419	419	351	2079	0	0	0	0	-1
normalized size	1	1.00	0.84	4.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.928	3.701	9.864	0.000	0.532	0.000	0.000	0.000

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	298	1372	0	0	0	0	-1
normalized size	1	1.00	1.21	5.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.381	1.385	1.649	0.000	0.463	0.000	0.000	0.000

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	223	925	0	0	0	0	-1
normalized size	1	1.00	1.20	4.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.257	1.529	1.605	0.000	0.458	0.000	0.000	0.000

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	201	382	0	0	0	0	-1
normalized size	1	1.00	1.18	2.25	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.207	1.123	1.586	0.000	0.438	0.000	0.000	0.000

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	210	443	0	0	0	0	-1
normalized size	1	1.00	1.16	2.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.214	1.146	3.467	0.000	0.454	0.000	0.000	0.000

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	264	925	0	0	0	0	-1
normalized size	1	1.00	1.08	3.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.327	2.061	1.968	0.000	0.463	0.000	0.000	0.000

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	367	1291	0	0	0	0	-1
normalized size	1	1.00	1.10	3.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.488	4.263	6.546	0.000	0.508	0.000	0.000	0.000

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	310	1372	0	0	0	0	-1
normalized size	1	1.00	1.21	5.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.553	2.606	5.559	0.000	0.488	0.000	0.000	0.000

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	283	1049	0	0	0	0	-1
normalized size	1	1.00	1.19	4.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.538	2.771	5.278	0.000	0.496	0.000	0.000	0.000

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	256	906	0	0	0	0	-1
normalized size	1	1.00	1.10	3.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.410	2.549	5.179	0.000	0.511	0.000	0.000	0.000

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	290	507	0	0	0	0	-1
normalized size	1	1.00	1.13	1.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.441	2.472	3.720	0.000	0.466	0.000	0.000	0.000

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	405	1299	0	0	0	0	-1
normalized size	1	1.00	1.24	3.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.642	4.753	5.979	0.000	0.495	0.000	0.000	0.000

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	674	1758	0	0	0	0	-1
normalized size	1	1.00	1.66	4.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.834	6.676	8.658	0.000	0.606	0.000	0.000	0.000

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	385	1615	0	0	0	0	-1
normalized size	1	1.00	1.20	5.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.833	5.813	7.626	0.000	0.526	0.000	0.000	0.000

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	441	1462	0	0	0	0	-1
normalized size	1	1.00	1.37	4.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.877	6.206	7.637	0.000	0.532	0.000	0.000	0.000

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	449	1056	0	0	0	0	-1
normalized size	1	1.00	1.34	3.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.768	5.887	6.802	0.000	0.475	0.000	0.000	0.000

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	445	593	0	0	0	0	-1
normalized size	1	1.00	1.29	1.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.769	6.319	4.631	0.000	0.454	0.000	0.000	0.000

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	423	423	745	1851	0	0	0	0	-1
normalized size	1	1.00	1.76	4.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.025	6.544	8.769	0.000	0.548	0.000	0.000	0.000

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	518	518	828	2311	0	0	0	0	-1
normalized size	1	1.00	1.60	4.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.215	6.921	11.455	0.000	0.722	0.000	0.000	0.000

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	146	141	0	240	0	0	-1
normalized size	1	1.00	0.91	0.88	0.00	1.49	0.00	0.00	-0.01
time (sec)	N/A	0.279	0.519	0.920	0.000	0.438	0.000	0.000	0.000

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	111	92	0	157	0	0	-1
normalized size	1	1.00	0.99	0.82	0.00	1.40	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.291	0.806	0.000	0.468	0.000	0.000	0.000

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	82	58	0	85	0	0	-1
normalized size	1	1.00	1.32	0.94	0.00	1.37	0.00	0.00	-0.02
time (sec)	N/A	0.055	0.124	0.640	0.000	0.463	0.000	0.000	0.000

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	65	43	0	50	0	0	33
normalized size	1	1.00	2.50	1.65	0.00	1.92	0.00	0.00	1.27
time (sec)	N/A	0.014	0.036	0.507	0.000	0.427	0.000	0.000	7.416

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	657	80	0	464	0	0	-1
normalized size	1	1.00	10.77	1.31	0.00	7.61	0.00	0.00	-0.02
time (sec)	N/A	0.115	5.485	0.793	0.000	0.548	0.000	0.000	0.000

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	871	155	0	786	0	0	-1
normalized size	1	1.00	8.30	1.48	0.00	7.49	0.00	0.00	-0.01
time (sec)	N/A	0.186	6.018	1.273	0.000	0.565	0.000	0.000	0.000

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	920	254	0	1250	0	0	-1
normalized size	1	1.00	5.97	1.65	0.00	8.12	0.00	0.00	-0.01
time (sec)	N/A	0.269	7.546	1.450	0.000	0.633	0.000	0.000	0.000

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	203	195	0	339	0	0	-1
normalized size	1	1.00	0.88	0.84	0.00	1.47	0.00	0.00	-0.00
time (sec)	N/A	0.384	1.693	0.923	0.000	0.455	0.000	0.000	0.000

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	136	130	0	229	0	0	-1
normalized size	1	1.00	0.87	0.83	0.00	1.46	0.00	0.00	-0.01
time (sec)	N/A	0.229	0.876	0.886	0.000	0.453	0.000	0.000	0.000

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	101	77	0	136	0	0	-1
normalized size	1	1.00	1.00	0.76	0.00	1.35	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.395	0.709	0.000	0.453	0.000	0.000	0.000

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	89	53	0	76	0	0	-1
normalized size	1	1.00	1.51	0.90	0.00	1.29	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.142	0.664	0.000	0.423	0.000	0.000	0.000

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	233	137	0	651	0	0	-1
normalized size	1	1.00	2.38	1.40	0.00	6.64	0.00	0.00	-0.01
time (sec)	N/A	0.200	2.293	1.122	0.000	0.567	0.000	0.000	0.000

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	268	233	0	970	0	0	-1
normalized size	1	1.00	2.25	1.96	0.00	8.15	0.00	0.00	-0.01
time (sec)	N/A	0.193	2.351	1.419	0.000	0.598	0.000	0.000	0.000

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	313	429	0	1558	0	0	-1
normalized size	1	1.00	1.75	2.40	0.00	8.70	0.00	0.00	-0.01
time (sec)	N/A	0.290	4.061	1.497	0.000	0.663	0.000	0.000	0.000

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	246	249	0	493	0	0	-1
normalized size	1	1.00	0.75	0.76	0.00	1.50	0.00	0.00	-0.00
time (sec)	N/A	0.656	6.267	0.888	0.000	0.486	0.000	0.000	0.000

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	180	168	0	339	0	0	-1
normalized size	1	1.00	0.89	0.83	0.00	1.68	0.00	0.00	-0.00
time (sec)	N/A	0.274	3.299	0.943	0.000	0.479	0.000	0.000	0.000

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	119	99	0	206	0	0	-1
normalized size	1	1.00	0.86	0.72	0.00	1.49	0.00	0.00	-0.01
time (sec)	N/A	0.108	1.499	0.782	0.000	0.441	0.000	0.000	0.000

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	117	65	0	115	0	0	-1
normalized size	1	1.00	1.31	0.73	0.00	1.29	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.303	0.772	0.000	0.438	0.000	0.000	0.000

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	330	229	0	868	0	0	-1
normalized size	1	1.00	2.32	1.61	0.00	6.11	0.00	0.00	-0.01
time (sec)	N/A	0.411	3.597	1.431	0.000	0.594	0.000	0.000	0.000

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	350	393	0	1322	0	0	-1
normalized size	1	1.00	2.11	2.37	0.00	7.96	0.00	0.00	-0.01
time (sec)	N/A	0.389	4.086	1.482	0.000	0.627	0.000	0.000	0.000

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	379	567	0	1994	0	0	-1
normalized size	1	1.00	1.95	2.92	0.00	10.28	0.00	0.00	-0.01
time (sec)	N/A	0.443	4.917	1.807	0.000	0.708	0.000	0.000	0.000

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	155	285	0	387	0	0	-1
normalized size	1	1.00	0.87	1.60	0.00	2.17	0.00	0.00	-0.01
time (sec)	N/A	0.440	0.591	1.234	0.000	0.480	0.000	0.000	0.000

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	125	185	0	286	0	0	-1
normalized size	1	1.00	1.02	1.50	0.00	2.33	0.00	0.00	-0.01
time (sec)	N/A	0.201	0.388	1.250	0.000	0.460	0.000	0.000	0.000

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	106	128	0	214	0	0	151
normalized size	1	1.00	1.34	1.62	0.00	2.71	0.00	0.00	1.91
time (sec)	N/A	0.070	0.212	0.997	0.000	0.474	0.000	0.000	8.962

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	73	75	0	167	0	0	49
normalized size	1	1.00	1.55	1.60	0.00	3.55	0.00	0.00	1.04
time (sec)	N/A	0.023	0.046	0.520	0.000	0.476	0.000	0.000	7.718

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	215	131	0	685	0	0	-1
normalized size	1	1.00	1.75	1.07	0.00	5.57	0.00	0.00	-0.01
time (sec)	N/A	0.214	1.801	1.237	0.000	0.583	0.000	0.000	0.000

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	324	449	0	1494	0	0	-1
normalized size	1	1.00	1.85	2.57	0.00	8.54	0.00	0.00	-0.01
time (sec)	N/A	0.425	3.561	1.865	0.000	0.839	0.000	0.000	0.000

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	414	1065	0	2903	0	0	-1
normalized size	1	1.00	1.68	4.31	0.00	11.75	0.00	0.00	-0.00
time (sec)	N/A	0.732	4.979	2.196	0.000	1.199	0.000	0.000	0.000

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	328	490	0	494	0	0	-1
normalized size	1	1.00	1.71	2.55	0.00	2.57	0.00	0.00	-0.01
time (sec)	N/A	0.463	0.550	1.092	0.000	0.463	0.000	0.000	0.000

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	239	316	0	380	0	0	-1
normalized size	1	1.00	1.73	2.29	0.00	2.75	0.00	0.00	-0.01
time (sec)	N/A	0.216	0.339	1.010	0.000	0.448	0.000	0.000	0.000

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	150	176	0	293	0	0	-1
normalized size	1	1.00	1.72	2.02	0.00	3.37	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.199	0.861	0.000	0.443	0.000	0.000	0.000

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	108	125	0	252	0	0	-1
normalized size	1	1.00	1.40	1.62	0.00	3.27	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.175	0.809	0.000	0.451	0.000	0.000	0.000

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	385	338	0	1299	0	0	-1
normalized size	1	1.00	2.35	2.06	0.00	7.92	0.00	0.00	-0.01
time (sec)	N/A	0.417	1.808	1.322	0.000	0.803	0.000	0.000	0.000

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	491	978	0	2515	0	0	-1
normalized size	1	1.00	2.02	4.02	0.00	10.35	0.00	0.00	-0.00
time (sec)	N/A	0.743	4.559	1.809	0.000	1.232	0.000	0.000	0.000

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	570	2219	0	4133	0	0	-1
normalized size	1	1.00	1.79	6.98	0.00	13.00	0.00	0.00	-0.00
time (sec)	N/A	1.108	6.524	2.432	0.000	2.095	0.000	0.000	0.000

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	400	688	0	608	0	0	-1
normalized size	1	1.00	2.06	3.55	0.00	3.13	0.00	0.00	-0.01
time (sec)	N/A	0.469	0.749	1.317	0.000	0.463	0.000	0.000	0.000

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	252	378	0	492	0	0	-1
normalized size	1	1.00	1.71	2.57	0.00	3.35	0.00	0.00	-0.01
time (sec)	N/A	0.230	0.543	1.433	0.000	0.474	0.000	0.000	0.000

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	227	279	0	392	0	0	-1
normalized size	1	1.00	1.80	2.21	0.00	3.11	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.354	1.177	0.000	0.470	0.000	0.000	0.000

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	196	195	0	320	0	0	-1
normalized size	1	1.00	1.83	1.82	0.00	2.99	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.154	0.911	0.000	0.493	0.000	0.000	0.000

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	501	732	0	2015	0	0	-1
normalized size	1	1.00	2.30	3.36	0.00	9.24	0.00	0.00	-0.00
time (sec)	N/A	0.741	3.289	1.666	0.000	1.057	0.000	0.000	0.000

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	570	1972	0	3719	0	0	-1
normalized size	1	1.00	1.82	6.30	0.00	11.88	0.00	0.00	-0.00
time (sec)	N/A	1.100	5.838	2.394	0.000	1.955	0.000	0.000	0.000

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	400	400	958	3535	0	5999	0	0	-1
normalized size	1	1.00	2.40	8.84	0.00	15.00	0.00	0.00	-0.00
time (sec)	N/A	1.515	9.360	3.264	0.000	3.537	0.000	0.000	0.000

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-2)	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	391	0	0	1257	0	0	-1
normalized size	1	1.00	1.93	0.00	0.00	6.19	0.00	0.00	-0.00
time (sec)	N/A	0.415	3.725	180.000	0.000	1.066	0.000	0.000	0.000

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-2)	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	365	0	0	1069	0	0	-1
normalized size	1	1.00	2.34	0.00	0.00	6.85	0.00	0.00	-0.01
time (sec)	N/A	0.290	2.046	180.000	0.000	0.907	0.000	0.000	0.000

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-2)	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	350	0	0	945	0	0	-1
normalized size	1	1.00	3.33	0.00	0.00	9.00	0.00	0.00	-0.01
time (sec)	N/A	0.182	1.313	180.000	0.000	0.859	0.000	0.000	0.000

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	305	2707	0	777	0	0	-1
normalized size	1	1.00	5.00	44.38	0.00	12.74	0.00	0.00	-0.02
time (sec)	N/A	0.093	1.155	0.539	0.000	0.837	0.000	0.000	0.000

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	84	99	179	129	0	0	145
normalized size	1	1.00	1.87	2.20	3.98	2.87	0.00	0.00	3.22
time (sec)	N/A	0.092	0.193	0.312	1.030	0.450	0.000	0.000	9.004

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	100	222	340	300	0	0	353
normalized size	1	1.00	1.05	2.34	3.58	3.16	0.00	0.00	3.72
time (sec)	N/A	0.192	0.275	0.341	1.053	0.463	0.000	0.000	13.902

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	128	430	544	556	0	0	501
normalized size	1	1.00	0.90	3.03	3.83	3.92	0.00	0.00	3.53
time (sec)	N/A	0.295	0.394	0.362	1.335	0.520	0.000	0.000	16.352

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	318	0	0	1579	0	0	-1
normalized size	1	1.00	1.12	0.00	0.00	5.54	0.00	0.00	-0.00
time (sec)	N/A	0.567	1.367	180.000	0.000	1.595	0.000	0.000	0.000

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	281	0	0	1337	0	0	-1
normalized size	1	1.00	1.23	0.00	0.00	5.86	0.00	0.00	-0.00
time (sec)	N/A	0.435	0.884	180.000	0.000	1.070	0.000	0.000	0.000

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	247	0	0	1127	0	0	-1
normalized size	1	1.00	1.44	0.00	0.00	6.59	0.00	0.00	-0.01
time (sec)	N/A	0.311	0.593	180.000	0.000	0.996	0.000	0.000	0.000

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	301	0	0	989	0	0	-1
normalized size	1	1.00	2.71	0.00	0.00	8.91	0.00	0.00	-0.01
time (sec)	N/A	0.209	0.578	0.490	0.000	0.930	0.000	0.000	0.000

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	377	6630	0	1297	0	0	-1
normalized size	1	1.00	3.22	56.67	0.00	11.09	0.00	0.00	-0.01
time (sec)	N/A	0.214	7.405	0.559	0.000	0.872	0.000	0.000	0.000

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	104	345	307	323	0	0	387
normalized size	1	1.00	0.90	3.00	2.67	2.81	0.00	0.00	3.37
time (sec)	N/A	0.215	0.594	0.313	1.269	0.496	0.000	0.000	14.401

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	140	625	505	598	0	0	541
normalized size	1	1.00	0.81	3.63	2.94	3.48	0.00	0.00	3.15
time (sec)	N/A	0.324	0.904	0.364	1.223	0.493	0.000	0.000	17.483

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	193	979	750	937	0	0	807
normalized size	1	1.00	0.84	4.28	3.28	4.09	0.00	0.00	3.52
time (sec)	N/A	0.448	1.528	0.478	1.327	0.557	0.000	0.000	20.213

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	395	0	0	2083	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	5.53	0.00	0.00	-0.00
time (sec)	N/A	0.855	2.804	180.000	0.000	2.056	0.000	0.000	0.000

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	327	0	0	1751	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	5.61	0.00	0.00	-0.00
time (sec)	N/A	0.726	1.730	180.000	0.000	1.691	0.000	0.000	0.000

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	285	0	0	1455	0	0	-1
normalized size	1	1.00	1.18	0.00	0.00	6.04	0.00	0.00	-0.00
time (sec)	N/A	0.576	0.914	180.000	0.000	1.138	0.000	0.000	0.000

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	256	0	0	1219	0	0	-1
normalized size	1	1.00	1.44	0.00	0.00	6.85	0.00	0.00	-0.01
time (sec)	N/A	0.433	0.796	180.000	0.000	1.085	0.000	0.000	0.000

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	263	0	0	1665	0	0	-1
normalized size	1	1.00	1.46	0.00	0.00	9.25	0.00	0.00	-0.01
time (sec)	N/A	0.438	1.012	0.374	0.000	0.996	0.000	0.000	0.000

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	261	16223	0	2297	0	0	-1
normalized size	1	1.00	1.43	88.65	0.00	12.55	0.00	0.00	-0.01
time (sec)	N/A	0.444	8.156	0.615	0.000	0.985	0.000	0.000	0.000

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	152	793	464	654	0	0	590
normalized size	1	1.00	0.80	4.20	2.46	3.46	0.00	0.00	3.12
time (sec)	N/A	0.485	2.114	0.397	1.140	0.516	0.000	0.000	17.457

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	216	1223	703	1033	0	0	862
normalized size	1	1.00	0.85	4.81	2.77	4.07	0.00	0.00	3.39
time (sec)	N/A	0.625	4.309	0.513	1.211	0.587	0.000	0.000	20.609

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	616	1730	984	1492	0	0	1155
normalized size	1	1.00	1.94	5.46	3.10	4.71	0.00	0.00	3.64
time (sec)	N/A	0.776	6.553	0.563	1.946	0.664	0.000	0.000	26.142

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-2)	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	1893	0	0	2925	0	0	-1
normalized size	1	1.00	7.60	0.00	0.00	11.75	0.00	0.00	-0.00
time (sec)	N/A	0.929	17.609	180.000	0.000	1.225	0.000	0.000	0.000

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	1639	0	0	2525	0	0	-1
normalized size	1	1.00	8.72	0.00	0.00	13.43	0.00	0.00	-0.01
time (sec)	N/A	0.598	17.212	0.481	0.000	1.069	0.000	0.000	0.000

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	1251	3359	0	1944	0	0	-1
normalized size	1	1.00	8.87	23.82	0.00	13.79	0.00	0.00	-0.01
time (sec)	N/A	0.296	15.147	0.444	0.000	0.911	0.000	0.000	0.000

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	283	191	0	475	0	0	-1
normalized size	1	1.00	3.58	2.42	0.00	6.01	0.00	0.00	-0.01
time (sec)	N/A	0.107	4.102	0.233	0.000	0.680	0.000	0.000	0.000

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	306	874	0	1016	0	0	-1
normalized size	1	1.00	2.34	6.67	0.00	7.76	0.00	0.00	-0.01
time (sec)	N/A	0.243	6.482	0.343	0.000	0.735	0.000	0.000	0.000

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	387	2572	0	1855	0	0	-1
normalized size	1	1.00	2.03	13.47	0.00	9.71	0.00	0.00	-0.01
time (sec)	N/A	0.476	6.451	0.366	0.000	0.916	0.000	0.000	0.000

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	1844	0	0	3420	0	0	-1
normalized size	1	1.00	7.35	0.00	0.00	13.63	0.00	0.00	-0.00
time (sec)	N/A	0.903	17.020	0.553	0.000	1.224	0.000	0.000	0.000

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	1625	6681	0	2883	0	0	-1
normalized size	1	1.00	8.38	34.44	0.00	14.86	0.00	0.00	-0.01
time (sec)	N/A	0.564	17.117	0.474	0.000	1.078	0.000	0.000	0.000

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	372	1373	0	896	0	0	-1
normalized size	1	1.00	2.95	10.90	0.00	7.11	0.00	0.00	-0.01
time (sec)	N/A	0.221	5.371	0.332	0.000	0.729	0.000	0.000	0.000

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	381	1268	0	1008	0	0	-1
normalized size	1	1.00	2.82	9.39	0.00	7.47	0.00	0.00	-0.01
time (sec)	N/A	0.237	5.671	0.331	0.000	0.826	0.000	0.000	0.000

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	401	2246	0	1954	0	0	-1
normalized size	1	1.00	2.04	11.40	0.00	9.92	0.00	0.00	-0.01
time (sec)	N/A	0.494	6.237	0.373	0.000	1.070	0.000	0.000	0.000

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	478	5040	0	3182	0	0	-1
normalized size	1	1.00	1.76	18.60	0.00	11.74	0.00	0.00	-0.00
time (sec)	N/A	0.855	9.453	0.411	0.000	1.376	0.000	0.000	0.000

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	1845	10738	0	3855	0	0	-1
normalized size	1	1.00	7.10	41.30	0.00	14.83	0.00	0.00	-0.00
time (sec)	N/A	0.864	17.435	0.616	0.000	1.288	0.000	0.000	0.000

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	396	3050	0	1304	0	0	-1
normalized size	1	1.00	2.15	16.58	0.00	7.09	0.00	0.00	-0.01
time (sec)	N/A	0.538	7.219	0.361	0.000	0.829	0.000	0.000	0.000

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	412	3050	0	1474	0	0	-1
normalized size	1	1.00	2.16	15.97	0.00	7.72	0.00	0.00	-0.01
time (sec)	N/A	0.488	7.599	0.364	0.000	0.900	0.000	0.000	0.000

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	411	2805	0	1644	0	0	-1
normalized size	1	1.00	2.04	13.96	0.00	8.18	0.00	0.00	-0.00
time (sec)	N/A	0.494	7.133	0.357	0.000	1.141	0.000	0.000	0.000

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	462	4262	0	2984	0	0	-1
normalized size	1	1.00	1.71	15.79	0.00	11.05	0.00	0.00	-0.00
time (sec)	N/A	0.861	8.964	0.373	0.000	1.547	0.000	0.000	0.000

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	717	8035	0	4858	0	0	-1
normalized size	1	1.00	2.02	22.63	0.00	13.68	0.00	0.00	-0.00
time (sec)	N/A	1.264	10.428	0.404	0.000	3.172	0.000	0.000	0.000

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	373	0	0	0	0	0	-1
normalized size	1	1.00	2.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	1.415	2.018	0.000	0.483	0.000	0.000	0.000

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	3599	0	0	0	0	0	-1
normalized size	1	1.00	11.25	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.663	57.573	6.748	0.000	0.460	0.000	0.000	0.000

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	1774	0	0	0	0	0	-1
normalized size	1	1.00	9.19	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.269	65.648	6.609	0.000	0.461	0.000	0.000	0.000

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	275	0	0	0	0	0	-1
normalized size	1	1.00	2.35	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	1.862	2.117	0.000	0.471	0.000	0.000	0.000

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	90	0	0	0	0	0	-1
normalized size	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.141	0.009	0.000	0.463	0.000	0.000	0.000

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	363	0	0	0	0	0	-1
normalized size	1	1.00	3.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	1.132	1.412	0.000	0.437	0.000	0.000	0.000

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	363	0	0	0	0	0	-1
normalized size	1	1.00	3.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	1.265	1.845	0.000	0.486	0.000	0.000	0.000

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	363	0	0	0	0	0	-1
normalized size	1	1.00	3.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	1.415	2.003	0.000	0.476	0.000	0.000	0.000

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	365	0	0	0	0	0	-1
normalized size	1	1.00	2.64	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.187	1.896	0.295	0.000	0.529	0.000	0.000	0.000

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	365	0	0	0	0	0	-1
normalized size	1	1.00	2.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.184	1.289	0.294	0.000	0.473	0.000	0.000	0.000

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	365	0	0	0	0	0	-1
normalized size	1	1.00	2.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.165	1.103	0.285	0.000	0.459	0.000	0.000	0.000

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	373	0	0	0	0	0	-1
normalized size	1	1.00	2.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.164	1.166	0.286	0.000	0.464	0.000	0.000	0.000

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	373	0	0	0	0	0	-1
normalized size	1	1.00	2.70	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.182	1.312	0.237	0.000	0.530	0.000	0.000	0.000

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	373	0	0	0	0	0	-1
normalized size	1	1.00	2.70	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.180	1.532	0.250	0.000	0.535	0.000	0.000	0.000

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	111	238	0	0	0	0	0	-1
normalized size	1	1.79	3.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.106	1.404	0.622	0.000	0.460	0.000	0.000	0.000

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	122	88	0	0	0	0	0	-1
normalized size	1	1.91	1.38	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.116	0.482	0.668	0.000	0.460	0.000	0.000	0.000

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	45	0	35	54	0	45	41
normalized size	1	1.00	1.61	0.00	1.25	1.93	0.00	1.61	1.46
time (sec)	N/A	0.020	0.037	0.513	0.835	0.429	0.000	0.613	0.432

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	131	0	0	0	0	0	-1
normalized size	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.539	0.675	0.000	0.461	0.000	0.000	0.000

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	167	0	0	0	0	0	-1
normalized size	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.571	0.654	0.000	0.465	0.000	0.000	0.000

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	95	0	0	0	0	0	-1
normalized size	1	1.00	1.46	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.021	0.134	0.533	0.000	0.444	0.000	0.000	0.000

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	182	0	0	0	0	0	-1
normalized size	1	1.00	1.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	1.042	0.869	0.000	0.466	0.000	0.000	0.000

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	177	0	0	0	0	0	-1
normalized size	1	1.00	1.55	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.874	0.745	0.000	0.467	0.000	0.000	0.000

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	97	0	0	41	0	0	43
normalized size	1	1.00	2.26	0.00	0.00	0.95	0.00	0.00	1.00
time (sec)	N/A	0.054	0.521	0.779	0.000	0.451	0.000	0.000	7.849

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	113	176	0	0	0	0	0	-1
normalized size	1	1.36	2.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.873	0.780	0.000	0.459	0.000	0.000	0.000

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	111	246	0	0	0	0	0	-1
normalized size	1	1.42	3.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	1.765	0.685	0.000	0.466	0.000	0.000	0.000

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	115	240	0	0	0	0	0	-1
normalized size	1	1.42	2.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.554	0.575	0.000	0.461	0.000	0.000	0.000

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	118	90	0	0	0	0	0	-1
normalized size	1	1.42	1.08	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.472	0.617	0.000	0.467	0.000	0.000	0.000

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	104	0	38	43	0	827	52
normalized size	1	1.00	2.67	0.00	0.97	1.10	0.00	21.21	1.33
time (sec)	N/A	0.017	5.240	0.530	0.884	0.432	0.000	2.575	0.426

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	118	179	0	0	0	0	0	-1
normalized size	1	1.42	2.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.622	0.557	0.000	0.466	0.000	0.000	0.000

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	117	166	0	0	0	0	0	-1
normalized size	1	1.44	2.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.676	0.598	0.000	0.461	0.000	0.000	0.000

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	97	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.158	0.597	0.000	0.457	0.000	0.000	0.000

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	118	184	0	0	0	0	0	-1
normalized size	1	1.64	2.56	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.810	0.645	0.000	0.454	0.000	0.000	0.000

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	77	116	179	0	0	0	0	0	-1
normalized size	1	1.51	2.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.631	0.623	0.000	0.474	0.000	0.000	0.000

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	99	0	0	43	0	0	45
normalized size	1	1.00	2.20	0.00	0.00	0.96	0.00	0.00	1.00
time (sec)	N/A	0.062	0.605	0.636	0.000	0.469	0.000	0.000	0.371

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	178	0	0	0	0	0	-1
normalized size	1	1.00	1.55	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.646	0.639	0.000	0.474	0.000	0.000	0.000

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	248	0	0	0	0	0	-1
normalized size	1	1.00	2.19	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.573	0.634	0.000	0.475	0.000	0.000	0.000

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	72	113	247	0	0	0	0	0	-1
normalized size	1	1.57	3.43	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	1.533	0.567	0.000	0.484	0.000	0.000	0.000

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	77	116	154	0	0	0	0	0	-1
normalized size	1	1.51	2.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.618	0.596	0.000	0.467	0.000	0.000	0.000

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	110	0	0	43	0	0	45
normalized size	1	1.00	2.44	0.00	0.00	0.96	0.00	0.00	1.00
time (sec)	N/A	0.063	0.664	0.633	0.000	0.462	0.000	0.000	7.829

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	155	0	0	0	0	0	-1
normalized size	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.629	0.585	0.000	0.493	0.000	0.000	0.000

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	155	0	0	0	0	0	-1
normalized size	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.705	0.597	0.000	0.473	0.000	0.000	0.000

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	97	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.168	0.666	0.000	0.468	0.000	0.000	0.000

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	131	0	0	0	0	0	-1
normalized size	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.912	0.733	0.000	0.471	0.000	0.000	0.000

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	186	0	0	0	0	0	-1
normalized size	1	1.00	1.56	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	1.152	0.714	0.000	0.529	0.000	0.000	0.000

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	106	0	45	43	0	827	52
normalized size	1	1.00	2.72	0.00	1.15	1.10	0.00	21.21	1.33
time (sec)	N/A	0.016	0.521	0.641	1.234	0.479	0.000	1.257	0.394

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	187	0	0	0	0	0	-1
normalized size	1	1.00	1.60	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	1.162	0.740	0.000	0.504	0.000	0.000	0.000

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	241	0	0	0	0	0	-1
normalized size	1	1.00	2.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	1.559	0.682	0.000	0.505	0.000	0.000	0.000

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	194	0	0	0	0	0	-1
normalized size	1	1.00	1.67	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.189	1.490	0.563	0.000	0.503	0.000	0.000	0.000

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	187	0	0	0	0	0	-1
normalized size	1	1.00	1.45	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.149	1.399	0.635	0.000	0.523	0.000	0.000	0.000

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	36.056	1.286	0.000	0.545	0.000	0.000	0.000

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	16.631	1.382	0.000	0.523	0.000	0.000	0.000

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	5.663	0.523	0.000	0.503	0.000	0.000	0.000

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	120	0	0	0	0	0	-1
normalized size	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.280	0.727	0.000	0.465	0.000	0.000	0.000

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	2.696	0.734	0.000	0.508	0.000	0.000	0.000

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	7.012	1.471	0.000	0.501	0.000	0.000	0.000

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	13.132	1.470	0.000	0.550	0.000	0.000	0.000

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	190	0	0	0	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.481	32.777	0.354	0.000	0.537	0.000	0.000	0.000

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	133	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.223	7.638	0.267	0.000	0.499	0.000	0.000	0.000

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	4.629	0.314	0.000	0.459	0.000	0.000	0.000

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	99	129	236	0	0	0	0	0	-1
normalized size	1	1.30	2.38	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.169	2.817	0.257	0.000	0.455	0.000	0.000	0.000

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	130	319	0	0	0	0	0	-1
normalized size	1	1.25	3.07	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.161	4.810	0.254	0.000	0.510	0.000	0.000	0.000

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	137	414	0	0	0	0	0	-1
normalized size	1	1.32	3.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.178	9.543	0.249	0.000	0.481	0.000	0.000	0.000

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	1736	0	0	0	0	0	-1
normalized size	1	1.00	16.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	6.431	0.419	0.000	0.470	0.000	0.000	0.000

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	886	0	0	0	0	0	-1
normalized size	1	1.00	8.28	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	6.272	0.810	0.000	0.476	0.000	0.000	0.000

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	942	0	0	0	0	0	-1
normalized size	1	1.00	8.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	6.418	0.359	0.000	0.506	0.000	0.000	0.000

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	143	182	175	145	386	152	183
normalized size	1	1.00	0.84	1.06	1.02	0.85	2.26	0.89	1.07
time (sec)	N/A	0.211	0.679	0.262	1.172	0.475	1.857	0.193	8.080

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	90	115	112	90	199	96	108
normalized size	1	1.00	0.85	1.08	1.06	0.85	1.88	0.91	1.02
time (sec)	N/A	0.101	0.307	0.190	0.403	0.439	0.830	0.195	7.894

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	52	59	57	48	94	48	52
normalized size	1	1.00	0.98	1.11	1.08	0.91	1.77	0.91	0.98
time (sec)	N/A	0.022	0.087	0.089	0.667	0.441	0.324	0.820	7.720

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	27	17	16	18	19	17	25
normalized size	1	1.00	1.69	1.06	1.00	1.12	1.19	1.06	1.56
time (sec)	N/A	0.008	0.006	0.012	0.628	0.445	0.142	0.236	7.636

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	67	119	0	255	537	86	342
normalized size	1	1.00	1.03	1.83	0.00	3.92	8.26	1.32	5.26
time (sec)	N/A	0.088	0.134	0.131	0.000	0.472	85.453	0.179	9.683

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	96	309	0	394	0	158	214
normalized size	1	1.00	0.98	3.15	0.00	4.02	0.00	1.61	2.18
time (sec)	N/A	0.096	0.301	0.231	0.000	0.472	0.000	0.482	7.920

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	157	1291	0	793	0	428	477
normalized size	1	1.00	0.96	7.87	0.00	4.84	0.00	2.61	2.91
time (sec)	N/A	0.198	0.617	0.276	0.000	0.498	0.000	0.259	9.608

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	249	325	314	247	729	274	358
normalized size	1	1.00	0.79	1.04	1.00	0.79	2.32	0.87	1.14
time (sec)	N/A	0.546	1.348	0.312	0.853	0.495	4.329	0.247	8.444

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	160	216	208	163	459	176	221
normalized size	1	1.00	0.74	1.00	0.96	0.75	2.12	0.81	1.02
time (sec)	N/A	0.281	0.782	0.263	0.747	0.443	1.943	0.215	8.140

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	90	115	112	89	199	96	108
normalized size	1	1.00	0.84	1.07	1.05	0.83	1.86	0.90	1.01
time (sec)	N/A	0.094	0.297	0.197	0.444	0.448	0.831	1.370	7.765

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	46	51	46	45	78	45	44
normalized size	1	1.00	0.92	1.02	0.92	0.90	1.56	0.90	0.88
time (sec)	N/A	0.016	0.097	0.097	0.626	0.462	0.313	0.165	7.611

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	89	226	0	375	0	134	2629
normalized size	1	1.00	0.96	2.43	0.00	4.03	0.00	1.44	28.27
time (sec)	N/A	0.182	0.212	0.199	0.000	0.495	0.000	0.337	12.432

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	134	556	0	677	0	249	5776
normalized size	1	1.00	1.04	4.31	0.00	5.25	0.00	1.93	44.78
time (sec)	N/A	0.231	0.540	0.257	0.000	0.505	0.000	0.224	15.469

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	202	1923	0	1027	0	609	641
normalized size	1	1.00	1.03	9.81	0.00	5.24	0.00	3.11	3.27
time (sec)	N/A	0.279	0.940	0.276	0.000	0.531	0.000	0.290	10.265

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	346	4818	0	1724	0	1316	1220
normalized size	1	1.00	1.13	15.80	0.00	5.65	0.00	4.31	4.00
time (sec)	N/A	0.563	1.443	0.330	0.000	0.587	0.000	3.153	11.155

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	493	552	489	477	375	1217	416	574
normalized size	1	1.23	1.38	1.22	1.19	0.94	3.04	1.04	1.44
time (sec)	N/A	0.948	1.176	0.328	0.726	0.479	8.027	0.217	8.916

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	246	325	314	253	729	274	358
normalized size	1	1.00	0.78	1.03	1.00	0.80	2.31	0.87	1.14
time (sec)	N/A	0.460	1.585	0.324	0.523	0.457	4.292	1.136	8.521

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	142	182	175	148	386	152	183
normalized size	1	1.00	0.83	1.06	1.02	0.87	2.26	0.89	1.07
time (sec)	N/A	0.197	0.657	0.267	0.967	0.449	1.872	0.191	7.953

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	71	76	74	71	128	75	127
normalized size	1	1.00	0.79	0.84	0.82	0.79	1.42	0.83	1.41
time (sec)	N/A	0.069	0.172	0.184	0.672	0.433	0.673	0.148	7.686

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	137	506	0	578	0	252	5902
normalized size	1	1.00	0.88	3.24	0.00	3.71	0.00	1.62	37.83
time (sec)	N/A	0.378	0.372	0.240	0.000	0.506	0.000	0.601	14.561

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	152	842	0	1062	0	585	8953
normalized size	1	1.00	0.73	4.05	0.00	5.11	0.00	2.81	43.04
time (sec)	N/A	0.495	1.100	0.286	0.000	0.555	0.000	0.206	17.637

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	521	2785	0	1707	0	889	11848
normalized size	1	1.00	2.04	10.92	0.00	6.69	0.00	3.49	46.46
time (sec)	N/A	0.631	2.351	0.299	0.000	0.604	0.000	0.293	20.458

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	345	6128	0	2136	0	1677	1423
normalized size	1	1.00	1.06	18.86	0.00	6.57	0.00	5.16	4.38
time (sec)	N/A	0.725	5.267	0.333	0.000	0.666	0.000	0.379	11.797

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	52	99	0	163	87	73	94
normalized size	1	1.00	0.96	1.83	0.00	3.02	1.61	1.35	1.74
time (sec)	N/A	0.082	0.075	0.085	0.000	0.461	50.124	0.180	8.125

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	0	6	3	6	6
normalized size	1	1.00	1.00	1.17	0.00	1.00	0.50	1.00	1.00
time (sec)	N/A	0.001	0.000	0.007	0.000	0.407	0.332	0.232	7.701

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	34	0	12	0	32	24
normalized size	1	1.00	1.00	2.83	0.00	1.00	0.00	2.67	2.00
time (sec)	N/A	0.029	0.031	0.109	0.000	0.415	0.000	0.290	7.866

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	24	36	27	42	51	36
normalized size	1	1.00	0.82	0.71	1.06	0.79	1.24	1.50	1.06
time (sec)	N/A	0.033	0.030	0.096	1.225	0.440	0.879	0.358	7.830

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	203	948	0	795	0	465	8720
normalized size	1	1.00	0.86	4.03	0.00	3.38	0.00	1.98	37.11
time (sec)	N/A	0.648	0.582	0.237	0.000	0.509	0.000	0.234	16.486

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	138	506	0	566	0	252	5902
normalized size	1	1.00	0.88	3.24	0.00	3.63	0.00	1.62	37.83
time (sec)	N/A	0.363	0.350	0.215	0.000	0.530	0.000	0.197	14.771

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	90	226	0	368	0	134	2628
normalized size	1	1.00	0.97	2.43	0.00	3.96	0.00	1.44	28.26
time (sec)	N/A	0.167	0.158	0.189	0.000	0.482	0.000	0.304	12.623

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	67	119	0	253	537	86	343
normalized size	1	1.00	1.03	1.83	0.00	3.89	8.26	1.32	5.28
time (sec)	N/A	0.072	0.102	0.119	0.000	0.486	84.853	0.322	10.009

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	47	0	190	177	62	42
normalized size	1	1.00	1.00	1.00	0.00	4.04	3.77	1.32	0.89
time (sec)	N/A	0.034	0.032	0.103	0.000	0.451	10.631	0.421	7.793

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	104	116	0	1057	0	145	3281
normalized size	1	1.00	0.89	0.99	0.00	9.03	0.00	1.24	28.04
time (sec)	N/A	0.153	0.183	0.296	0.000	1.854	0.000	0.219	9.989

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	165	514	0	2882	0	308	24122
normalized size	1	1.00	0.89	2.78	0.00	15.58	0.00	1.66	130.39
time (sec)	N/A	0.468	0.927	0.335	0.000	174.523	0.000	0.223	22.739

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	263	2644	0	0	0	786	62873
normalized size	1	1.00	0.93	9.31	0.00	0.00	0.00	2.77	221.38
time (sec)	N/A	1.080	2.217	0.376	0.000	0.000	0.000	1.418	30.300

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	199	1303	0	1451	0	517	13700
normalized size	1	1.00	0.65	4.26	0.00	4.74	0.00	1.69	44.77
time (sec)	N/A	0.941	2.016	0.318	0.000	0.826	0.000	0.410	20.452

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	151	842	0	1006	0	579	8953
normalized size	1	1.00	0.74	4.11	0.00	4.91	0.00	2.82	43.67
time (sec)	N/A	0.459	1.103	0.293	0.000	0.683	0.000	0.251	17.831

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	133	556	0	665	0	249	5776
normalized size	1	1.00	1.03	4.31	0.00	5.16	0.00	1.93	44.78
time (sec)	N/A	0.218	0.561	0.323	0.000	0.808	0.000	0.255	15.469

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	96	309	0	399	0	157	215
normalized size	1	1.00	0.99	3.19	0.00	4.11	0.00	1.62	2.22
time (sec)	N/A	0.092	0.298	0.245	0.000	0.489	0.000	0.217	8.021

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	82	155	0	336	0	127	174
normalized size	1	1.00	0.99	1.87	0.00	4.05	0.00	1.53	2.10
time (sec)	N/A	0.060	0.184	0.204	0.000	0.488	0.000	0.324	8.240

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	178	514	0	2871	0	304	24123
normalized size	1	1.00	0.98	2.84	0.00	15.86	0.00	1.68	133.28
time (sec)	N/A	0.438	0.867	0.380	0.000	140.159	0.000	0.240	22.797

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	227	886	0	0	0	1005	71320
normalized size	1	1.00	0.78	3.06	0.00	0.00	0.00	3.47	245.93
time (sec)	N/A	1.197	2.909	0.380	0.000	0.000	0.000	4.780	30.890

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	458	458	346	4023	0	0	0	1109	137274
normalized size	1	1.00	0.76	8.78	0.00	0.00	0.00	2.42	299.72
time (sec)	N/A	2.439	6.586	0.439	0.000	0.000	0.000	0.417	45.339

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	534	534	341	4767	0	3174	0	3162	23910
normalized size	1	1.00	0.64	8.93	0.00	5.94	0.00	5.92	44.78
time (sec)	N/A	2.155	3.823	0.355	0.000	1.248	0.000	0.377	25.760

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	894	3683	0	2335	0	1159	16958
normalized size	1	1.00	2.81	11.58	0.00	7.34	0.00	3.64	53.33
time (sec)	N/A	0.972	4.161	0.345	0.000	1.038	0.000	2.209	21.743

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	524	2785	0	1631	0	887	11848
normalized size	1	1.00	2.11	11.23	0.00	6.58	0.00	3.58	47.77
time (sec)	N/A	0.806	2.355	0.306	0.000	0.783	0.000	1.188	20.932

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	204	1923	0	1025	0	609	641
normalized size	1	1.00	1.04	9.81	0.00	5.23	0.00	3.11	3.27
time (sec)	N/A	0.285	0.946	0.267	0.000	1.038	0.000	0.274	10.443

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	157	1291	0	799	0	429	477
normalized size	1	1.00	0.97	7.97	0.00	4.93	0.00	2.65	2.94
time (sec)	N/A	0.174	0.626	0.243	0.000	0.914	0.000	0.237	9.793

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	114	705	0	618	0	284	395
normalized size	1	1.00	0.87	5.38	0.00	4.72	0.00	2.17	3.02
time (sec)	N/A	0.114	0.397	0.200	0.000	0.705	0.000	0.175	10.150

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	275	2644	0	0	0	788	62873
normalized size	1	1.00	0.96	9.28	0.00	0.00	0.00	2.76	220.61
time (sec)	N/A	1.037	2.353	0.367	0.000	0.000	0.000	1.345	30.173

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	454	454	346	3241	0	0	0	1111	137273
normalized size	1	1.00	0.76	7.14	0.00	0.00	0.00	2.45	302.36
time (sec)	N/A	2.439	5.472	0.442	0.000	0.000	0.000	1.377	43.673

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	669	669	1815	7348	0	0	0	7128	571173
normalized size	1	1.00	2.71	10.98	0.00	0.00	0.00	10.65	853.77
time (sec)	N/A	3.309	8.528	0.520	0.000	0.000	0.000	16.260	80.303

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	275	1839	0	0	0	0	-1
normalized size	1	1.00	0.92	6.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.481	1.105	1.506	0.000	0.809	0.000	0.000	0.000

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	218	1449	0	0	0	0	-1
normalized size	1	1.00	0.93	6.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.359	0.761	1.536	0.000	0.954	0.000	0.000	0.000

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	152	862	0	0	0	0	-1
normalized size	1	1.00	0.84	4.76	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.213	0.633	1.249	0.000	0.821	0.000	0.000	0.000

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	101	243	0	0	0	0	176
normalized size	1	1.00	0.72	1.74	0.00	0.00	0.00	0.00	1.26
time (sec)	N/A	0.124	2.557	1.328	0.000	0.730	0.000	0.000	8.580

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	159	567	0	0	0	0	-1
normalized size	1	1.00	0.82	2.91	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.216	0.558	3.274	0.000	0.463	0.000	0.000	0.000

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	199	887	0	0	0	0	-1
normalized size	1	1.00	0.70	3.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.395	1.553	5.127	0.000	0.462	0.000	0.000	0.000

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	297	1049	0	0	0	0	-1
normalized size	1	1.00	0.80	2.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.526	3.063	7.602	0.000	0.532	0.000	0.000	0.000

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	451	451	382	2112	0	0	0	0	-1
normalized size	1	1.00	0.85	4.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.945	1.802	7.077	0.000	0.523	0.000	0.000	0.000

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	292	1575	0	0	0	0	-1
normalized size	1	1.00	0.84	4.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.634	1.202	5.135	0.000	0.491	0.000	0.000	0.000

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	214	1100	0	0	0	0	-1
normalized size	1	1.00	0.84	4.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.411	0.917	4.434	0.000	0.477	0.000	0.000	0.000

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	173	695	0	0	0	0	-1
normalized size	1	1.00	0.85	3.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.285	0.903	2.998	0.000	0.489	0.000	0.000	0.000

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	172	888	0	0	0	0	-1
normalized size	1	1.00	0.75	3.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.308	0.866	3.989	0.000	0.481	0.000	0.000	0.000

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	302	1043	0	0	0	0	-1
normalized size	1	1.00	0.92	3.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.499	2.482	6.034	0.000	0.499	0.000	0.000	0.000

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	460	460	424	1450	0	0	0	0	-1
normalized size	1	1.00	0.92	3.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.863	4.988	8.367	0.000	0.519	0.000	0.000	0.000

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	642	642	545	2728	0	0	0	0	-1
normalized size	1	1.00	0.85	4.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.401	2.607	9.399	0.000	0.588	0.000	0.000	0.000

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	496	496	410	2112	0	0	0	0	-1
normalized size	1	1.00	0.83	4.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.026	2.434	7.027	0.000	0.529	0.000	0.000	0.000

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	306	1561	0	0	0	0	-1
normalized size	1	1.00	0.82	4.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.678	1.449	5.820	0.000	0.532	0.000	0.000	0.000

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	219	1085	0	0	0	0	-1
normalized size	1	1.00	0.73	3.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.479	1.198	4.182	0.000	0.473	0.000	0.000	0.000

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	311	1398	0	0	0	0	-1
normalized size	1	1.00	0.86	3.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.622	2.046	5.200	0.000	0.563	0.000	0.000	0.000

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	357	1379	0	0	0	0	-1
normalized size	1	1.00	0.91	3.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.748	3.727	6.790	0.000	0.555	0.000	0.000	0.000

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	532	532	584	1621	0	0	0	0	-1
normalized size	1	1.00	1.10	3.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.109	5.297	9.591	0.000	0.596	0.000	0.000	0.000

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	716	716	1127	2111	0	0	0	0	-1
normalized size	1	1.00	1.57	2.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.436	6.980	14.084	0.000	0.590	0.000	0.000	0.000

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	606	1190	0	0	0	0	-1
normalized size	1	1.00	2.05	4.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.088	5.772	4.416	0.000	0.000	0.000	0.000	0.000

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	242	391	0	0	0	0	-1
normalized size	1	1.00	1.06	1.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.491	3.963	1.515	0.000	24.498	0.000	0.000	0.000

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	114	181	0	0	0	0	-1
normalized size	1	1.00	0.75	1.18	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.329	2.848	1.422	0.000	0.000	0.000	0.000	0.000

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	74	151	0	0	0	0	-1
normalized size	1	1.00	0.99	2.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.214	0.113	1.554	0.000	0.000	0.000	0.000	0.000

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	617	610	0	0	0	0	-1
normalized size	1	1.00	2.80	2.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.630	6.888	3.766	0.000	0.000	0.000	0.000	0.000

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	1079	1072	0	0	0	0	-1
normalized size	1	1.00	2.70	2.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.653	7.169	6.198	0.000	0.000	0.000	0.000	0.000

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	534	534	1109	1886	0	0	0	0	-1
normalized size	1	1.00	2.08	3.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.040	8.117	7.199	0.000	0.000	0.000	0.000	0.000

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	390	390	986	1363	0	0	0	0	-1
normalized size	1	1.00	2.53	3.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.261	8.184	6.697	0.000	177.630	0.000	0.000	0.000

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	891	1027	0	0	0	0	-1
normalized size	1	1.00	2.54	2.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.037	7.158	5.111	0.000	0.000	0.000	0.000	0.000

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	846	872	0	0	0	0	-1
normalized size	1	1.00	2.76	2.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.867	6.947	4.823	0.000	0.000	0.000	0.000	0.000

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	871	690	0	0	0	0	-1
normalized size	1	1.00	2.68	2.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.968	7.475	3.824	0.000	0.000	0.000	0.000	0.000

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	1057	1266	0	0	0	0	-1
normalized size	1	1.00	2.35	2.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.627	8.066	6.921	0.000	0.000	0.000	0.000	0.000

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	661	661	1319	1731	0	0	0	0	-1
normalized size	1	1.00	2.00	2.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.799	9.015	11.970	0.000	0.000	0.000	0.000	0.000

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	816	816	1526	2775	0	0	0	0	-1
normalized size	1	1.00	1.87	3.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.179	8.783	13.686	0.000	0.000	0.000	0.000	0.000

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	605	605	1323	2237	0	0	0	0	-1
normalized size	1	1.00	2.19	3.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.201	8.351	11.597	0.000	0.000	0.000	0.000	0.000

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	549	549	1149	1888	0	0	0	0	-1
normalized size	1	1.00	2.09	3.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.076	8.069	10.042	0.000	0.000	0.000	0.000	0.000

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	472	472	1001	1718	0	0	0	0	-1
normalized size	1	1.00	2.12	3.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.809	7.431	9.957	0.000	0.000	0.000	0.000	0.000

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	1038	1525	0	0	0	0	-1
normalized size	1	1.00	2.13	3.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.605	7.740	9.439	0.000	0.000	0.000	0.000	0.000

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	503	503	1069	867	0	0	0	0	-1
normalized size	1	1.00	2.13	1.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.675	7.816	6.336	0.000	0.000	0.000	0.000	0.000

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	682	682	1318	2099	0	0	0	0	-1
normalized size	1	1.00	1.93	3.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.768	9.410	13.150	0.000	0.000	0.000	0.000	0.000

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	888	888	1978	404501	0	0	0	0	-1
normalized size	1	1.00	2.23	455.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.386	7.138	17.169	0.000	0.000	0.000	0.000	0.000

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	784	784	1879	277000	0	0	0	0	-1
normalized size	1	1.00	2.40	353.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.516	9.541	6.806	0.000	100.085	0.000	0.000	0.000

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	628	628	228392	146762	0	0	0	0	-1
normalized size	1	1.00	363.68	233.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.327	31.707	1.850	0.000	73.826	0.000	0.000	0.000

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	197	248299	0	0	0	0	-1
normalized size	1	1.00	0.99	1254.04	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.216	5.864	0.000	25.183	0.000	0.000	0.000

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	409	263	46827	0	0	0	0	-1
normalized size	1	1.00	0.64	114.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.494	7.424	1.060	0.000	0.460	0.000	0.000	0.000

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	489	489	2067	196704	0	0	0	0	-1
normalized size	1	1.00	4.23	402.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.865	6.416	4.500	0.000	0.525	0.000	0.000	0.000

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1080	1080	2091	577718	0	0	0	0	-1
normalized size	1	1.00	1.94	534.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	5.238	7.528	27.128	0.000	81.491	0.000	0.000	0.000

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	870	870	1952	409354	0	0	0	0	-1
normalized size	1	1.00	2.24	470.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.359	6.309	17.433	0.000	24.377	0.000	0.000	0.000

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	740	740	1879	278658	0	0	0	0	-1
normalized size	1	1.00	2.54	376.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.294	9.481	8.967	0.000	7.554	0.000	0.000	0.000

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	644	644	222963	529691	0	0	0	0	-1
normalized size	1	1.00	346.22	822.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.538	32.693	10.601	0.000	2.915	0.000	0.000	0.000

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	600	600	1896	2626418	0	0	0	0	-1
normalized size	1	1.00	3.16	4377.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.934	9.562	133.542	0.000	1.824	0.000	0.000	0.000

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	497	497	2012	190874	0	0	0	0	-1
normalized size	1	1.00	4.05	384.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.954	6.337	3.432	0.000	0.531	0.000	0.000	0.000

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1295	1295	2276	755109	0	0	0	0	-1
normalized size	1	1.00	1.76	583.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	7.844	8.471	79.797	0.000	0.000	0.000	0.000	0.000

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1071	1071	2091	577725	0	0	0	0	-1
normalized size	1	1.00	1.95	539.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.995	7.603	28.455	0.000	156.246	0.000	0.000	0.000

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	894	894	1979	410016	0	0	0	0	-1
normalized size	1	1.00	2.21	458.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.269	7.105	19.753	0.000	35.756	0.000	0.000	0.000

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	745	745	1894	730813	0	0	0	0	-1
normalized size	1	1.00	2.54	980.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.189	10.246	22.141	0.000	13.733	0.000	0.000	0.000

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	780	780	2006	3436958	0	0	0	0	-1
normalized size	1	1.00	2.57	4406.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.468	6.814	51.243	0.000	4.842	0.000	0.000	0.000

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F(-1)	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	737	737	2169	0	0	0	0	0	-1
normalized size	1	1.00	2.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.781	6.944	180.000	0.000	43.710	0.000	0.000	0.000

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	772	772	1894	731601	0	0	0	0	-1
normalized size	1	1.00	2.45	947.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.303	10.393	24.280	0.000	0.000	0.000	0.000	0.000

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	644	644	222963	544151	0	0	0	0	-1
normalized size	1	1.00	346.22	844.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.580	32.456	10.947	0.000	0.000	0.000	0.000	0.000

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	197	248841	0	0	0	0	-1
normalized size	1	1.00	0.99	1256.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.275	4.645	0.000	1.420	0.000	0.000	0.000

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	191	1233	0	0	0	0	-1
normalized size	1	1.00	0.99	6.42	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.236	0.694	0.000	0.750	0.000	0.000	0.000

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	90261	41868	0	0	0	0	-1
normalized size	1	1.00	222.87	103.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.459	32.659	1.134	0.000	0.984	0.000	0.000	0.000

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	521	521	2102	218898	0	0	0	0	-1
normalized size	1	1.00	4.03	420.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.019	6.782	5.595	0.000	0.787	0.000	0.000	0.000

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	822	822	2005	3904542	0	0	0	0	-1
normalized size	1	1.00	2.44	4750.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.652	6.843	59.718	0.000	5.698	0.000	0.000	0.000

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	600	600	1896	2948827	0	0	0	0	-1
normalized size	1	1.00	3.16	4914.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.928	9.451	142.722	0.000	0.000	0.000	0.000	0.000

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	409	226	47019	0	0	0	0	-1
normalized size	1	1.00	0.55	114.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.455	4.164	0.969	0.000	1.065	0.000	0.000	0.000

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	90261	40621	0	0	0	0	-1
normalized size	1	1.00	222.87	100.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.451	32.737	1.021	0.000	1.038	0.000	0.000	0.000

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	495	495	2082	119964	0	0	0	0	-1
normalized size	1	1.00	4.21	242.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.908	6.918	2.083	0.000	1.292	0.000	0.000	0.000

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	681	681	2350	415383	0	0	0	0	-1
normalized size	1	1.00	3.45	609.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.680	7.622	10.108	0.000	1.699	0.000	0.000	0.000

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F(-1)	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	736	736	2172	0	0	0	0	0	-1
normalized size	1	1.00	2.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.843	7.090	180.000	0.000	0.000	0.000	0.000	0.000

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	497	497	2012	195220	0	0	0	0	-1
normalized size	1	1.00	4.05	392.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.001	6.349	4.101	0.000	0.892	0.000	0.000	0.000

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	489	489	2067	212259	0	0	0	0	-1
normalized size	1	1.00	4.23	434.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.860	6.428	4.683	0.000	0.917	0.000	0.000	0.000

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	516	516	2102	242318	0	0	0	0	-1
normalized size	1	1.00	4.07	469.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.994	6.453	5.832	0.000	1.008	0.000	0.000	0.000

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	688	688	2352	438748	0	0	0	0	-1
normalized size	1	1.00	3.42	637.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.847	7.816	10.676	0.000	1.270	0.000	0.000	0.000

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	941	941	2669	1123217	0	0	0	0	-1
normalized size	1	1.00	2.84	1193.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.472	8.985	30.727	0.000	1.176	0.000	0.000	0.000

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.050	3.253	1.625	0.000	0.767	0.000	0.000	0.000

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.439	18.727	1.309	0.000	0.483	0.000	0.000	0.000

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	200	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.203	0.571	0.412	0.000	0.457	0.000	0.000	0.000

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	120	0	0	0	0	0	-1
normalized size	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.255	0.649	0.000	0.449	0.000	0.000	0.000

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.057	2.648	1.422	0.000	0.450	0.000	0.000	0.000

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.056	4.848	1.578	0.000	0.587	0.000	0.000	0.000

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.055	15.870	1.908	0.000	0.866	0.000	0.000	0.000

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.078	34.711	0.314	0.000	0.924	0.000	0.000	0.000

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.076	13.616	0.257	0.000	0.698	0.000	0.000	0.000

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.069	0.442	0.244	0.000	0.634	0.000	0.000	0.000

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.069	3.334	0.228	0.000	0.630	0.000	0.000	0.000

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.076	4.598	0.220	0.000	0.874	0.000	0.000	0.000

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.078	9.691	0.239	0.000	0.973	0.000	0.000	0.000

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	493	0	0	0	0	0	-1
normalized size	1	1.00	1.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.459	11.573	5.857	0.000	0.469	0.000	0.000	0.000

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	342	0	0	0	0	0	-1
normalized size	1	1.00	1.68	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.257	6.343	8.406	0.000	0.443	0.000	0.000	0.000

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	280	0	0	0	0	0	-1
normalized size	1	1.00	1.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.150	1.657	1.866	0.000	0.456	0.000	0.000	0.000

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.238	2.800	0.852	0.000	0.506	0.000	0.000	0.000

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.442	4.794	1.946	0.000	0.464	0.000	0.000	0.000

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	2967	0	0	0	0	0	-1
normalized size	1	1.00	26.26	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.232	15.158	0.878	0.000	0.484	0.000	0.000	0.000

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	297	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.495	1.366	1.418	0.000	0.460	0.000	0.000	0.000

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	222	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.249	0.664	1.435	0.000	0.464	0.000	0.000	0.000

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	270	0	0	0	0	0	-1
normalized size	1	1.00	1.66	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	1.515	0.503	0.000	0.639	0.000	0.000	0.000

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	157	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.230	0.272	0.336	0.000	0.808	0.000	0.000	0.000

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	195	0	0	0	0	0	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.487	2.904	1.426	0.000	0.819	0.000	0.000	0.000

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	167	0	0	0	0	0	-1
normalized size	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.566	0.565	6.323	0.000	0.763	0.000	0.000	0.000

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	135	0	0	0	0	0	-1
normalized size	1	1.00	0.63	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.269	0.385	7.819	0.000	0.580	0.000	0.000	0.000

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	105	0	0	0	0	0	-1
normalized size	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.235	2.382	0.000	0.457	0.000	0.000	0.000

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	1665	0	0	0	0	0	-1
normalized size	1	1.00	8.16	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.399	16.849	0.877	0.000	0.435	0.000	0.000	0.000

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	1872	0	0	0	0	0	-1
normalized size	1	1.00	5.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.544	19.061	1.917	0.000	0.507	0.000	0.000	0.000

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	432	432	2406	0	0	0	0	0	-1
normalized size	1	1.00	5.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.703	20.220	2.061	0.000	0.488	0.000	0.000	0.000

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.112	2.571	0.365	0.000	0.564	0.000	0.000	0.000

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	303	230	0	0	0	0	0	-1
normalized size	1	0.94	0.71	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.548	1.039	1.269	0.000	0.456	0.000	0.000	0.000

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	221	152	0	0	0	0	0	-1
normalized size	1	0.96	0.66	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.233	0.350	1.422	0.000	0.450	0.000	0.000	0.000

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	129	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.241	0.400	0.000	0.434	0.000	0.000	0.000

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	1808	0	0	0	0	0	-1
normalized size	1	1.00	8.86	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.338	18.003	0.312	0.000	0.457	0.000	0.000	0.000

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	1970	0	0	0	0	0	-1
normalized size	1	1.00	6.12	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.508	19.017	1.144	0.000	0.481	0.000	0.000	0.000

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	428	428	2570	0	0	0	0	0	-1
normalized size	1	1.00	6.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.652	20.507	1.270	0.000	0.520	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [198] had the largest ratio of [.7500]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	9	4	1.00	21	0.190
2	A	13	4	1.00	21	0.190
3	A	6	5	1.00	13	0.385
4	A	2	2	1.00	13	0.154
5	A	4	4	1.00	13	0.308
6	A	2	2	1.00	11	0.182
7	A	1	1	1.00	8	0.125
8	A	3	3	1.00	11	0.273
9	A	5	5	1.00	13	0.385
10	A	6	6	1.00	13	0.462
11	A	6	5	1.00	13	0.385
12	A	3	3	1.00	13	0.231
13	A	6	6	1.00	13	0.462
14	A	3	3	1.00	13	0.231
15	A	2	2	1.00	11	0.182
16	A	2	2	1.00	8	0.250
17	A	4	4	1.00	11	0.364
18	A	6	6	1.00	13	0.462
19	A	7	7	1.00	13	0.538
20	A	7	6	1.09	13	0.462
21	A	8	6	1.00	13	0.462
22	A	4	3	1.00	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	7	7	1.00	13	0.538
24	A	5	5	1.00	13	0.385
25	A	3	3	1.00	13	0.231
26	A	3	3	1.00	11	0.273
27	A	3	2	1.00	8	0.250
28	A	5	4	1.00	11	0.364
29	A	7	6	1.00	13	0.462
30	A	8	7	1.00	13	0.538
31	A	8	6	1.00	13	0.462
32	A	5	4	1.00	23	0.174
33	A	4	4	1.00	23	0.174
34	A	3	3	1.00	23	0.130
35	A	2	2	1.00	21	0.095
36	A	1	1	1.00	14	0.071
37	A	2	2	1.00	21	0.095
38	A	3	3	1.00	23	0.130
39	A	4	3	1.00	23	0.130
40	A	5	3	1.00	23	0.130
41	A	2	2	1.00	22	0.091
42	A	2	2	1.00	23	0.087
43	A	2	2	1.00	24	0.083
44	A	6	6	1.00	23	0.261
45	A	4	4	1.00	23	0.174
46	A	3	3	1.00	21	0.143
47	A	2	2	1.00	14	0.143
48	A	4	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
49	A	4	4	1.00	23	0.174
50	A	5	5	1.00	23	0.217
51	A	6	5	1.00	23	0.217
52	A	6	6	1.00	23	0.261
53	A	5	4	1.00	23	0.174
54	A	4	3	1.00	21	0.143
55	A	3	2	1.00	14	0.143
56	A	4	4	1.00	21	0.190
57	A	4	4	1.00	23	0.174
58	A	4	4	1.00	23	0.174
59	A	5	5	1.00	23	0.217
60	A	6	5	1.00	23	0.217
61	A	6	6	1.00	23	0.261
62	A	4	4	1.00	23	0.174
63	A	3	3	1.00	21	0.143
64	A	2	2	1.00	14	0.143
65	A	5	4	1.00	21	0.190
66	A	6	5	1.00	23	0.217
67	A	7	6	1.00	23	0.261
68	A	7	7	1.00	23	0.304
69	A	6	6	1.00	23	0.261
70	A	4	4	1.00	23	0.174
71	A	3	3	1.00	21	0.143
72	A	3	3	1.00	14	0.214
73	A	6	5	1.00	21	0.238
74	A	7	6	1.00	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
75	A	8	6	1.00	23	0.261
76	A	8	8	1.00	23	0.348
77	A	7	7	1.00	23	0.304
78	A	6	6	1.00	23	0.261
79	A	4	4	1.00	23	0.174
80	A	4	4	1.00	21	0.190
81	A	4	3	1.00	14	0.214
82	A	7	6	1.00	21	0.286
83	A	8	7	1.00	23	0.304
84	A	9	7	1.00	23	0.304
85	A	2	2	1.00	25	0.080
86	A	2	2	1.00	28	0.071
87	A	2	2	1.00	15	0.133
88	A	2	2	1.00	17	0.118
89	A	2	2	1.00	17	0.118
90	A	2	2	1.00	18	0.111
91	A	5	4	1.00	23	0.174
92	A	6	6	1.00	23	0.261
93	A	4	4	1.00	23	0.174
94	A	3	3	1.00	21	0.143
95	A	2	2	1.00	14	0.143
96	A	4	4	1.00	21	0.190
97	A	4	4	1.00	23	0.174
98	A	6	6	1.00	23	0.261
99	A	4	4	1.00	23	0.174
100	A	3	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
101	A	2	2	1.00	14	0.143
102	A	4	4	1.00	21	0.190
103	A	4	4	1.00	23	0.174
104	A	6	6	1.00	23	0.261
105	A	4	4	1.00	23	0.174
106	A	3	3	1.00	21	0.143
107	A	2	2	1.00	14	0.143
108	A	4	4	1.00	21	0.190
109	A	4	4	1.00	23	0.174
110	A	6	6	1.00	23	0.261
111	A	4	4	1.00	23	0.174
112	A	3	3	1.00	21	0.143
113	A	2	2	1.00	14	0.143
114	A	4	4	1.00	21	0.190
115	A	4	4	1.00	23	0.174
116	A	4	4	1.00	21	0.190
117	A	2	2	1.00	21	0.095
118	A	3	3	1.00	21	0.143
119	A	3	3	1.00	21	0.143
120	A	4	4	1.00	23	0.174
121	A	2	2	1.00	23	0.087
122	A	4	4	1.00	23	0.174
123	A	4	4	1.00	23	0.174
124	A	4	4	1.00	23	0.174
125	A	2	2	1.00	23	0.087
126	A	4	4	1.00	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
127	A	4	4	1.00	23	0.174
128	A	5	5	1.00	25	0.200
129	A	3	3	1.00	25	0.120
130	A	5	5	1.00	25	0.200
131	A	5	5	1.00	25	0.200
132	A	2	2	1.00	19	0.105
133	A	2	2	1.00	23	0.087
134	A	3	3	1.00	21	0.143
135	A	3	3	1.00	23	0.130
136	A	3	3	1.00	21	0.143
137	A	3	3	1.00	24	0.125
138	A	4	4	1.00	23	0.174
139	A	4	4	1.00	24	0.167
140	A	7	7	1.00	21	0.333
141	A	6	6	1.00	21	0.286
142	A	4	4	1.00	21	0.190
143	A	3	3	1.00	19	0.158
144	A	2	2	1.00	12	0.167
145	A	4	4	1.00	19	0.210
146	A	4	4	1.00	21	0.190
147	A	1	1	1.00	10	0.100
148	A	1	1	1.00	12	0.083
149	A	6	4	1.00	19	0.210
150	A	5	4	1.00	19	0.210
151	A	1	1	1.00	17	0.059
152	A	2	1	1.00	10	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
153	A	2	2	1.00	17	0.118
154	A	4	4	1.00	19	0.210
155	A	5	5	1.00	19	0.263
156	A	5	4	1.00	19	0.210
157	A	7	5	1.00	21	0.238
158	A	6	5	1.00	21	0.238
159	A	2	2	1.00	19	0.105
160	A	1	1	1.00	12	0.083
161	A	3	3	1.00	19	0.158
162	A	4	4	1.00	21	0.190
163	A	5	5	1.00	21	0.238
164	A	6	6	1.00	21	0.286
165	A	6	5	1.00	21	0.238
166	A	8	6	1.13	21	0.286
167	A	4	3	1.12	21	0.143
168	A	3	2	1.00	19	0.105
169	A	2	2	1.00	12	0.167
170	A	4	4	1.00	19	0.210
171	A	4	4	1.00	21	0.190
172	A	4	4	1.00	21	0.190
173	A	6	6	1.00	21	0.286
174	A	7	7	1.00	21	0.333
175	A	3	3	1.00	12	0.250
176	A	7	7	1.00	13	0.538
177	A	6	6	1.00	13	0.462
178	A	6	6	1.00	13	0.462

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
179	A	4	4	1.00	11	0.364
180	A	3	3	1.00	8	0.375
181	A	5	5	1.00	11	0.454
182	A	7	7	1.00	13	0.538
183	A	7	7	1.00	13	0.538
184	A	8	7	1.00	13	0.538
185	A	7	7	1.00	13	0.538
186	A	6	6	1.00	13	0.462
187	A	5	5	1.00	13	0.385
188	A	5	5	1.00	11	0.454
189	A	5	5	1.00	8	0.625
190	A	6	6	1.00	11	0.546
191	A	7	7	1.00	13	0.538
192	A	8	7	1.00	13	0.538
193	A	8	8	1.00	13	0.615
194	A	7	7	1.00	13	0.538
195	A	6	6	1.00	13	0.462
196	A	6	6	1.00	13	0.462
197	A	6	5	1.00	11	0.454
198	A	6	6	1.00	8	0.750
199	A	7	7	1.00	11	0.636
200	A	8	7	1.00	13	0.538
201	A	9	7	1.00	13	0.538
202	A	7	6	1.00	12	0.500
203	A	6	6	1.00	21	0.286
204	A	2	2	1.00	14	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
205	A	5	5	1.00	21	0.238
206	A	9	9	1.00	23	0.391
207	A	5	5	1.00	21	0.238
208	A	2	2	1.00	14	0.143
209	A	2	2	1.00	21	0.095
210	A	9	9	1.00	23	0.391
211	A	7	7	1.00	25	0.280
212	A	1	1	1.00	25	0.040
213	A	5	4	1.00	23	0.174
214	A	4	3	1.00	23	0.130
215	A	3	2	1.00	21	0.095
216	A	5	3	1.00	23	0.130
217	A	10	4	1.00	23	0.174
218	A	13	4	1.00	23	0.174
219	A	3	3	1.00	36	0.083
220	A	3	3	1.00	23	0.130
221	A	0	0	0.00	0	0.000
222	A	9	6	1.00	21	0.286
223	A	8	5	1.00	21	0.238
224	A	7	4	1.00	19	0.210
225	A	3	3	1.00	12	0.250
226	A	0	0	0.00	0	0.000
227	A	6	5	1.00	24	0.208
228	A	5	5	1.00	24	0.208
229	A	4	4	1.00	24	0.167
230	A	1	1	1.00	22	0.045

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
231	A	2	2	1.00	24	0.083
232	A	2	2	1.00	24	0.083
233	A	3	3	1.00	24	0.125
234	A	4	3	1.00	24	0.125
235	A	5	3	1.00	24	0.125
236	A	7	5	1.00	26	0.192
237	A	6	5	1.00	26	0.192
238	A	5	4	1.00	26	0.154
239	A	4	3	1.00	26	0.115
240	A	4	4	1.00	24	0.167
241	A	4	4	1.00	26	0.154
242	A	4	3	1.00	26	0.115
243	A	2	2	1.00	26	0.077
244	A	3	3	1.00	26	0.115
245	A	4	3	1.00	26	0.115
246	A	5	3	1.00	26	0.115
247	A	8	5	1.00	26	0.192
248	A	7	5	1.00	26	0.192
249	A	6	4	1.00	26	0.154
250	A	5	3	1.00	26	0.115
251	A	5	4	1.00	26	0.154
252	A	5	5	1.00	24	0.208
253	A	5	5	1.00	26	0.192
254	A	5	4	1.00	26	0.154
255	A	5	3	1.00	26	0.115
256	A	2	2	1.00	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
257	A	3	3	1.00	26	0.115
258	A	4	3	1.00	26	0.115
259	A	5	3	1.00	26	0.115
260	A	6	3	1.00	26	0.115
261	A	6	5	1.00	26	0.192
262	A	5	5	1.00	26	0.192
263	A	4	4	1.00	26	0.154
264	A	2	2	1.00	24	0.083
265	A	3	3	1.00	26	0.115
266	A	4	4	1.00	26	0.154
267	A	5	4	1.00	26	0.154
268	A	6	4	1.00	26	0.154
269	A	7	5	1.00	26	0.192
270	A	6	5	1.00	26	0.192
271	A	5	4	1.00	26	0.154
272	A	4	3	1.00	26	0.115
273	A	2	2	1.00	24	0.083
274	A	4	4	1.00	26	0.154
275	A	3	2	1.00	26	0.077
276	A	4	3	1.00	26	0.115
277	A	5	3	1.00	26	0.115
278	A	6	3	1.00	26	0.115
279	A	7	5	1.00	26	0.192
280	A	6	4	1.00	26	0.154
281	A	5	3	1.00	26	0.115
282	A	2	2	1.00	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
283	A	3	3	1.00	24	0.125
284	A	5	4	1.00	26	0.154
285	A	4	3	1.00	26	0.115
286	A	3	2	1.00	26	0.077
287	A	4	3	1.00	26	0.115
288	A	5	3	1.00	26	0.115
289	A	6	3	1.00	26	0.115
290	A	5	3	1.00	26	0.115
291	A	4	3	1.00	26	0.115
292	A	3	3	1.00	26	0.115
293	A	2	2	1.00	26	0.077
294	A	4	4	1.00	26	0.154
295	A	4	4	1.00	26	0.154
296	A	5	5	1.00	26	0.192
297	A	6	5	1.00	26	0.192
298	A	5	3	1.00	28	0.107
299	A	4	3	1.00	28	0.107
300	A	3	3	1.00	28	0.107
301	A	2	2	1.00	28	0.071
302	A	5	4	1.00	28	0.143
303	A	5	5	1.00	28	0.179
304	A	5	4	1.00	28	0.143
305	A	6	5	1.00	28	0.179
306	A	7	5	1.00	28	0.179
307	A	5	3	1.00	28	0.107
308	A	4	3	1.00	28	0.107

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
309	A	3	3	1.00	28	0.107
310	A	2	2	1.00	28	0.071
311	A	6	4	1.00	28	0.143
312	A	6	5	1.00	28	0.179
313	A	6	5	1.00	28	0.179
314	A	6	4	1.00	28	0.143
315	A	7	5	1.00	28	0.179
316	A	8	5	1.00	28	0.179
317	A	5	3	1.00	28	0.107
318	A	4	3	1.00	28	0.107
319	A	3	3	1.00	28	0.107
320	A	2	2	1.00	28	0.071
321	A	4	4	1.00	28	0.143
322	A	5	5	1.00	28	0.179
323	A	6	6	1.00	28	0.214
324	A	6	3	1.00	28	0.107
325	A	5	3	1.00	28	0.107
326	A	4	3	1.00	28	0.107
327	A	3	3	1.00	28	0.107
328	A	2	2	1.00	28	0.071
329	A	5	4	1.00	28	0.143
330	A	6	6	1.00	28	0.214
331	A	7	6	1.00	28	0.214
332	A	6	3	1.00	28	0.107
333	A	5	3	1.00	28	0.107
334	A	4	3	1.00	28	0.107

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
335	A	3	3	1.00	28	0.107
336	A	2	2	1.00	28	0.071
337	A	6	4	1.00	28	0.143
338	A	7	6	1.00	28	0.214
339	A	8	7	1.00	28	0.250
340	A	1	1	1.00	30	0.033
341	A	1	1	1.00	30	0.033
342	A	1	1	1.00	30	0.033
343	A	1	1	1.00	30	0.033
344	A	3	3	1.00	30	0.100
345	A	1	1	1.00	30	0.033
346	A	1	1	1.00	30	0.033
347	A	1	1	1.00	30	0.033
348	A	2	2	1.00	30	0.067
349	A	2	2	1.00	30	0.067
350	A	2	2	1.00	30	0.067
351	A	1	1	1.00	30	0.033
352	A	4	4	1.00	30	0.133
353	A	4	4	1.00	30	0.133
354	A	1	1	1.00	30	0.033
355	A	2	2	1.00	30	0.067
356	A	2	2	1.00	30	0.067
357	A	2	2	1.00	30	0.067
358	A	3	2	1.00	30	0.067
359	A	3	2	1.00	30	0.067
360	A	2	2	1.00	30	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
361	A	1	1	1.00	30	0.033
362	A	5	4	1.00	30	0.133
363	A	5	5	1.00	30	0.167
364	A	5	4	1.00	30	0.133
365	A	1	1	1.00	30	0.033
366	A	2	2	1.00	30	0.067
367	A	3	2	1.00	30	0.067
368	A	3	2	1.00	30	0.067
369	A	4	2	1.00	30	0.067
370	A	4	2	1.00	30	0.067
371	A	3	2	1.00	30	0.067
372	A	2	2	1.00	30	0.067
373	A	1	1	1.00	30	0.033
374	A	6	4	1.00	30	0.133
375	A	6	5	1.00	30	0.167
376	A	6	5	1.00	30	0.167
377	A	6	4	1.00	30	0.133
378	A	1	1	1.00	30	0.033
379	A	2	2	1.00	30	0.067
380	A	3	2	1.00	30	0.067
381	A	4	2	1.00	30	0.067
382	A	4	2	1.00	30	0.067
383	A	5	4	1.00	30	0.133
384	A	4	4	1.00	30	0.133
385	A	3	3	1.00	30	0.100
386	A	2	2	1.00	30	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
387	A	3	3	1.00	30	0.100
388	A	4	3	1.00	30	0.100
389	A	6	5	1.00	30	0.167
390	A	5	5	1.00	30	0.167
391	A	4	4	1.00	30	0.133
392	A	1	1	1.00	30	0.033
393	A	3	3	1.00	30	0.100
394	A	4	3	1.00	30	0.100
395	A	5	3	1.00	30	0.100
396	A	7	5	1.00	30	0.167
397	A	6	5	1.00	30	0.167
398	A	5	4	1.00	30	0.133
399	A	1	1	1.00	30	0.033
400	A	1	1	1.00	30	0.033
401	A	4	3	1.00	30	0.100
402	A	5	3	1.00	30	0.100
403	A	6	3	1.00	30	0.100
404	A	4	4	1.00	26	0.154
405	A	4	4	1.00	26	0.154
406	A	4	4	1.00	26	0.154
407	A	4	4	1.00	24	0.167
408	A	4	4	1.00	26	0.154
409	A	4	4	1.00	26	0.154
410	A	4	4	1.00	26	0.154
411	A	3	2	1.00	28	0.071
412	A	2	2	1.00	28	0.071

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
413	A	1	1	1.00	28	0.036
414	A	3	3	1.00	28	0.107
415	A	3	3	1.00	28	0.107
416	A	3	3	1.00	28	0.107
417	A	3	3	1.00	28	0.107
418	A	3	3	1.00	28	0.107
419	A	3	2	1.00	30	0.067
420	A	2	2	1.00	30	0.067
421	A	1	1	1.00	30	0.033
422	A	4	4	1.00	28	0.143
423	A	4	4	1.00	30	0.133
424	A	4	4	1.00	30	0.133
425	A	4	2	1.00	23	0.087
426	A	3	2	1.00	23	0.087
427	A	2	2	1.00	23	0.087
428	A	1	1	1.00	21	0.048
429	A	2	1	1.00	10	0.100
430	A	4	4	1.00	23	0.174
431	A	5	5	1.00	23	0.217
432	A	6	5	1.00	23	0.217
433	A	7	5	1.00	23	0.217
434	A	5	3	1.00	25	0.120
435	A	4	3	1.00	25	0.120
436	A	3	3	1.00	25	0.120
437	A	2	2	1.00	23	0.087
438	A	1	1	1.00	12	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
439	A	5	5	1.00	25	0.200
440	A	5	5	1.03	25	0.200
441	A	6	6	1.00	25	0.240
442	A	7	6	1.00	25	0.240
443	A	8	6	1.00	25	0.240
444	A	6	5	1.52	25	0.200
445	A	9	7	1.15	25	0.280
446	A	8	6	1.06	23	0.261
447	A	7	5	1.00	12	0.417
448	A	7	7	1.00	25	0.280
449	A	7	7	1.00	25	0.280
450	A	7	7	1.00	25	0.280
451	A	8	8	1.00	25	0.320
452	A	9	8	1.00	25	0.320
453	A	3	3	1.00	25	0.120
454	A	2	2	1.00	25	0.080
455	A	3	3	1.00	25	0.120
456	A	2	2	1.00	23	0.087
457	A	1	1	1.00	12	0.083
458	A	5	5	1.00	25	0.200
459	A	6	6	1.00	25	0.240
460	A	7	6	1.00	25	0.240
461	A	4	4	1.00	25	0.160
462	A	3	3	1.00	25	0.120
463	A	5	5	1.00	25	0.200
464	A	3	3	1.00	25	0.120

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
465	A	2	2	1.00	23	0.087
466	A	2	2	1.00	12	0.167
467	A	6	6	1.00	25	0.240
468	A	7	7	1.00	25	0.280
469	A	8	7	1.00	25	0.280
470	A	5	4	1.00	25	0.160
471	A	4	3	1.00	25	0.120
472	A	6	6	1.00	25	0.240
473	A	5	5	1.00	25	0.200
474	A	3	3	1.00	25	0.120
475	A	3	3	1.00	23	0.130
476	A	3	2	1.00	12	0.167
477	A	7	6	1.00	25	0.240
478	A	8	7	1.00	25	0.280
479	A	9	7	1.00	25	0.280
480	A	4	3	1.00	13	0.231
481	A	4	3	1.00	15	0.200
482	A	8	6	1.00	25	0.240
483	A	7	6	1.00	25	0.240
484	A	6	6	1.00	25	0.240
485	A	5	5	1.00	25	0.200
486	A	6	6	1.00	25	0.240
487	A	7	6	1.00	25	0.240
488	A	8	6	1.00	25	0.240
489	A	9	7	1.00	27	0.259
490	A	8	7	1.00	27	0.259

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
491	A	7	7	1.00	27	0.259
492	A	6	6	1.00	27	0.222
493	A	6	6	1.00	27	0.222
494	A	7	7	1.00	27	0.259
495	A	8	7	1.00	27	0.259
496	A	11	9	1.00	27	0.333
497	A	10	9	1.00	27	0.333
498	A	9	9	1.00	27	0.333
499	A	8	8	1.00	27	0.296
500	A	8	8	1.00	27	0.296
501	A	8	8	1.00	27	0.296
502	A	9	9	1.00	27	0.333
503	A	10	9	1.00	27	0.333
504	A	7	7	1.00	27	0.259
505	A	6	6	1.00	27	0.222
506	A	6	6	1.00	27	0.222
507	A	6	6	1.00	27	0.222
508	A	7	7	1.00	27	0.259
509	A	8	7	1.00	27	0.259
510	A	7	7	1.00	27	0.259
511	A	7	7	1.00	27	0.259
512	A	7	7	1.00	27	0.259
513	A	7	7	1.00	27	0.259
514	A	8	8	1.00	27	0.296
515	A	9	8	1.00	27	0.296
516	A	8	8	1.00	27	0.296

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
517	A	8	7	1.00	27	0.259
518	A	8	7	1.00	27	0.259
519	A	8	7	1.00	27	0.259
520	A	9	8	1.00	27	0.296
521	A	10	8	1.00	27	0.296
522	A	4	4	1.00	27	0.148
523	A	3	3	1.00	27	0.111
524	A	2	2	1.00	25	0.080
525	A	1	1	1.00	14	0.071
526	A	2	2	1.00	27	0.074
527	A	3	3	1.00	27	0.111
528	A	4	3	1.00	27	0.111
529	A	6	6	1.00	27	0.222
530	A	4	4	1.00	27	0.148
531	A	3	3	1.00	25	0.120
532	A	2	2	1.00	14	0.143
533	A	4	4	1.00	27	0.148
534	A	4	4	1.00	27	0.148
535	A	5	5	1.00	27	0.185
536	A	6	6	1.00	27	0.222
537	A	5	4	1.00	27	0.148
538	A	4	3	1.00	25	0.120
539	A	3	2	1.00	14	0.143
540	A	4	4	1.00	27	0.148
541	A	4	4	1.00	27	0.148
542	A	4	4	1.00	27	0.148

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
543	A	6	6	1.00	27	0.222
544	A	4	4	1.00	27	0.148
545	A	3	3	1.00	25	0.120
546	A	2	2	1.00	14	0.143
547	A	5	5	1.00	27	0.185
548	A	6	6	1.00	27	0.222
549	A	7	7	1.00	27	0.259
550	A	6	6	1.00	27	0.222
551	A	4	4	1.00	27	0.148
552	A	3	3	1.00	25	0.120
553	A	3	3	1.00	14	0.214
554	A	6	6	1.00	27	0.222
555	A	7	7	1.00	27	0.259
556	A	8	7	1.00	27	0.259
557	A	6	6	1.00	27	0.222
558	A	4	4	1.00	27	0.148
559	A	4	4	1.00	25	0.160
560	A	4	3	1.00	14	0.214
561	A	7	7	1.00	27	0.259
562	A	8	8	1.00	27	0.296
563	A	9	8	1.00	27	0.296
564	A	5	3	1.00	29	0.103
565	A	4	3	1.00	29	0.103
566	A	3	3	1.00	29	0.103
567	A	2	2	1.00	29	0.069
568	A	1	1	1.00	29	0.034

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
569	A	2	2	1.00	29	0.069
570	A	3	2	1.00	29	0.069
571	A	7	5	1.00	29	0.172
572	A	6	5	1.00	29	0.172
573	A	5	5	1.00	29	0.172
574	A	4	4	1.00	29	0.138
575	A	4	4	1.00	29	0.138
576	A	3	3	1.00	29	0.103
577	A	4	4	1.00	29	0.138
578	A	5	4	1.00	29	0.138
579	A	7	5	1.00	29	0.172
580	A	6	5	1.00	29	0.172
581	A	5	5	1.00	29	0.172
582	A	4	4	1.00	29	0.138
583	A	4	4	1.00	29	0.138
584	A	4	4	1.00	29	0.138
585	A	3	3	1.00	29	0.103
586	A	4	4	1.00	29	0.138
587	A	5	4	1.00	29	0.138
588	A	7	7	1.00	29	0.241
589	A	6	6	1.00	29	0.207
590	A	5	5	1.00	29	0.172
591	A	2	2	1.00	29	0.069
592	A	4	4	1.00	29	0.138
593	A	5	5	1.00	29	0.172
594	A	7	7	1.00	29	0.241

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
595	A	6	6	1.00	29	0.207
596	A	4	4	1.00	29	0.138
597	A	4	4	1.00	29	0.138
598	A	5	5	1.00	29	0.172
599	A	6	5	1.00	29	0.172
600	A	7	7	1.00	29	0.241
601	A	5	5	1.00	29	0.172
602	A	5	5	1.00	29	0.172
603	A	5	5	1.00	29	0.172
604	A	6	6	1.00	29	0.207
605	A	7	6	1.00	29	0.207
606	A	4	4	1.00	25	0.160
607	A	6	6	1.00	25	0.240
608	A	4	4	1.00	25	0.160
609	A	3	3	1.00	23	0.130
610	A	2	2	1.00	12	0.167
611	A	3	3	1.00	25	0.120
612	A	3	3	1.00	25	0.120
613	A	3	3	1.00	25	0.120
614	A	4	4	1.00	27	0.148
615	A	4	4	1.00	27	0.148
616	A	4	4	1.00	27	0.148
617	A	4	4	1.00	27	0.148
618	A	4	4	1.00	27	0.148
619	A	4	4	1.00	27	0.148
620	A	2	2	1.79	27	0.074

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
621	A	2	2	1.91	27	0.074
622	A	2	2	1.00	27	0.074
623	A	2	2	1.00	27	0.074
624	A	2	2	1.00	25	0.080
625	A	2	2	1.00	18	0.111
626	A	2	2	1.00	27	0.074
627	A	2	2	1.00	27	0.074
628	A	1	1	1.00	27	0.037
629	A	2	2	1.36	27	0.074
630	A	2	2	1.42	27	0.074
631	A	2	2	1.42	29	0.069
632	A	2	2	1.42	29	0.069
633	A	2	2	1.00	29	0.069
634	A	2	2	1.42	29	0.069
635	A	2	2	1.44	27	0.074
636	A	3	3	1.00	20	0.150
637	A	2	2	1.64	29	0.069
638	A	2	2	1.51	29	0.069
639	A	1	1	1.00	29	0.034
640	A	2	2	1.00	29	0.069
641	A	2	2	1.00	29	0.069
642	A	2	2	1.57	29	0.069
643	A	2	2	1.51	29	0.069
644	A	1	1	1.00	29	0.034
645	A	2	2	1.00	29	0.069
646	A	2	2	1.00	27	0.074

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
647	A	3	3	1.00	20	0.150
648	A	2	2	1.00	29	0.069
649	A	2	2	1.00	29	0.069
650	A	2	2	1.00	29	0.069
651	A	2	2	1.00	29	0.069
652	A	2	2	1.00	29	0.069
653	A	4	4	1.00	27	0.148
654	A	2	2	1.00	29	0.069
655	A	3	3	1.00	25	0.120
656	A	3	3	1.00	25	0.120
657	A	3	3	1.00	23	0.130
658	A	3	3	1.00	12	0.250
659	A	3	3	1.00	25	0.120
660	A	3	3	1.00	25	0.120
661	A	3	3	1.00	25	0.120
662	A	5	5	1.00	27	0.185
663	A	5	5	1.00	27	0.185
664	A	3	3	1.00	27	0.111
665	A	3	3	1.30	27	0.111
666	A	3	3	1.25	27	0.111
667	A	3	3	1.32	27	0.111
668	A	3	3	1.00	25	0.120
669	A	3	3	1.00	25	0.120
670	A	3	3	1.00	25	0.120
671	A	3	2	1.00	23	0.087
672	A	2	2	1.00	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
673	A	1	1	1.00	21	0.048
674	A	2	1	1.00	10	0.100
675	A	4	4	1.00	23	0.174
676	A	5	5	1.00	23	0.217
677	A	6	5	1.00	23	0.217
678	A	4	3	1.00	25	0.120
679	A	3	3	1.00	25	0.120
680	A	2	2	1.00	23	0.087
681	A	1	1	1.00	12	0.083
682	A	5	5	1.00	25	0.200
683	A	5	5	1.00	25	0.200
684	A	6	6	1.00	25	0.240
685	A	7	6	1.00	25	0.240
686	A	5	4	1.23	25	0.160
687	A	4	3	1.00	25	0.120
688	A	3	2	1.00	23	0.087
689	A	2	2	1.00	12	0.167
690	A	6	6	1.00	25	0.240
691	A	6	6	1.00	25	0.240
692	A	6	6	1.00	25	0.240
693	A	7	7	1.00	25	0.280
694	A	4	4	1.00	20	0.200
695	A	2	2	1.00	20	0.100
696	A	2	2	1.00	15	0.133
697	A	2	2	1.00	13	0.154
698	A	7	7	1.00	25	0.280

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
699	A	6	6	1.00	25	0.240
700	A	5	5	1.00	25	0.200
701	A	4	4	1.00	23	0.174
702	A	3	3	1.00	12	0.250
703	A	7	4	1.00	25	0.160
704	A	8	5	1.00	25	0.200
705	A	9	6	1.00	25	0.240
706	A	7	7	1.00	25	0.280
707	A	6	6	1.00	25	0.240
708	A	5	5	1.00	25	0.200
709	A	5	5	1.00	23	0.217
710	A	5	5	1.00	12	0.417
711	A	8	5	1.00	25	0.200
712	A	9	6	1.00	25	0.240
713	A	10	6	1.00	25	0.240
714	A	8	8	1.00	25	0.320
715	A	7	7	1.00	25	0.280
716	A	6	6	1.00	25	0.240
717	A	6	6	1.00	25	0.240
718	A	6	5	1.00	23	0.217
719	A	6	6	1.00	12	0.500
720	A	9	6	1.00	25	0.240
721	A	10	6	1.00	25	0.240
722	A	11	6	1.00	25	0.240
723	A	8	6	1.00	25	0.240
724	A	7	6	1.00	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
725	A	6	6	1.00	25	0.240
726	A	5	5	1.00	25	0.200
727	A	6	6	1.00	25	0.240
728	A	7	6	1.00	25	0.240
729	A	8	6	1.00	25	0.240
730	A	9	7	1.00	27	0.259
731	A	8	7	1.00	27	0.259
732	A	7	7	1.00	27	0.259
733	A	6	6	1.00	27	0.222
734	A	6	6	1.00	27	0.222
735	A	7	7	1.00	27	0.259
736	A	8	7	1.00	27	0.259
737	A	10	8	1.00	27	0.296
738	A	9	8	1.00	27	0.296
739	A	8	8	1.00	27	0.296
740	A	7	7	1.00	27	0.259
741	A	7	7	1.00	27	0.259
742	A	7	7	1.00	27	0.259
743	A	8	8	1.00	27	0.296
744	A	9	8	1.00	27	0.296
745	A	9	9	1.00	27	0.333
746	A	8	8	1.00	27	0.296
747	A	5	5	1.00	27	0.185
748	A	2	2	1.00	27	0.074
749	A	7	7	1.00	27	0.259
750	A	10	10	1.00	27	0.370

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
751	A	10	10	1.00	27	0.370
752	A	9	9	1.00	27	0.333
753	A	9	9	1.00	27	0.333
754	A	9	9	1.00	27	0.333
755	A	9	9	1.00	27	0.333
756	A	10	10	1.00	27	0.370
757	A	11	10	1.00	27	0.370
758	A	11	11	1.00	27	0.407
759	A	10	10	1.00	27	0.370
760	A	10	10	1.00	27	0.370
761	A	10	10	1.00	27	0.370
762	A	10	10	1.00	27	0.370
763	A	10	10	1.00	27	0.370
764	A	11	10	1.00	27	0.370
765	A	8	8	1.00	29	0.276
766	A	8	8	1.00	29	0.276
767	A	7	7	1.00	29	0.241
768	A	1	1	1.00	29	0.034
769	A	3	3	1.00	29	0.103
770	A	4	4	1.00	29	0.138
771	A	9	8	1.00	29	0.276
772	A	8	8	1.00	29	0.276
773	A	7	7	1.00	29	0.241
774	A	6	6	1.00	29	0.207
775	A	5	5	1.00	29	0.172
776	A	4	4	1.00	29	0.138

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
777	A	10	8	1.00	29	0.276
778	A	9	8	1.00	29	0.276
779	A	8	8	1.00	29	0.276
780	A	7	7	1.00	29	0.241
781	A	7	7	1.00	29	0.241
782	A	6	6	1.00	29	0.207
783	A	7	7	1.00	29	0.241
784	A	6	6	1.00	29	0.207
785	A	1	1	1.00	29	0.034
786	A	1	1	1.00	29	0.034
787	A	3	3	1.00	29	0.103
788	A	4	4	1.00	29	0.138
789	A	7	7	1.00	29	0.241
790	A	5	5	1.00	29	0.172
791	A	3	3	1.00	29	0.103
792	A	3	3	1.00	29	0.103
793	A	4	4	1.00	29	0.138
794	A	5	5	1.00	29	0.172
795	A	6	6	1.00	29	0.207
796	A	4	4	1.00	29	0.138
797	A	4	4	1.00	29	0.138
798	A	4	4	1.00	29	0.138
799	A	5	5	1.00	29	0.172
800	A	6	5	1.00	29	0.172
801	A	0	0	0.00	0	0.000
802	A	8	5	1.00	25	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
803	A	7	4	1.00	23	0.174
804	A	3	3	1.00	12	0.250
805	A	0	0	0.00	0	0.000
806	A	0	0	0.00	0	0.000
807	A	0	0	0.00	0	0.000
808	A	0	0	0.00	0	0.000
809	A	0	0	0.00	0	0.000
810	A	0	0	0.00	0	0.000
811	A	0	0	0.00	0	0.000
812	A	0	0	0.00	0	0.000
813	A	0	0	0.00	0	0.000
814	A	8	6	1.00	23	0.261
815	A	7	5	1.00	23	0.217
816	A	6	4	1.00	21	0.190
817	A	7	5	1.00	23	0.217
818	A	8	6	1.00	23	0.261
819	A	5	5	1.00	27	0.185
820	A	7	6	1.00	27	0.222
821	A	5	4	1.00	27	0.148
822	A	4	3	1.00	25	0.120
823	A	5	4	1.00	27	0.148
824	A	6	5	1.00	27	0.185
825	A	8	6	1.00	23	0.261
826	A	7	5	1.00	23	0.217
827	A	6	4	1.00	21	0.190
828	A	7	5	1.00	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
829	A	10	5	1.00	23	0.217
830	A	12	5	1.00	23	0.217
831	A	0	0	0.00	0	0.000
832	A	6	5	0.94	27	0.185
833	A	5	4	0.96	27	0.148
834	A	4	3	1.00	25	0.120
835	A	6	4	1.00	27	0.148
836	A	11	5	1.00	27	0.185
837	A	14	5	1.00	27	0.185

Chapter 3

Listing of integrals

3.1 $\int \sin^3(e+fx)(a+a\sin(e+fx))^2\,dx$

Optimal. Leaf size=102

$$-\frac{a^2\cos^5(e+fx)}{5f}+\frac{a^2\cos^3(e+fx)}{f}-\frac{2a^2\cos(e+fx)}{f}-\frac{a^2\sin^3(e+fx)\cos(e+fx)}{2f}-\frac{3a^2\sin(e+fx)\cos(e+fx)}{4f}+$$

```
[Out] 3/4*a^2*x-2*a^2*cos(f*x+e)/f+a^2*cos(f*x+e)^3/f-1/5*a^2*cos(f*x+e)^5/f-3/4*
a^2*cos(f*x+e)*sin(f*x+e)/f-1/2*a^2*cos(f*x+e)*sin(f*x+e)^3/f
```

Rubi [A] time = 0.10, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2757, 2633, 2635, 8}

$$-\frac{a^2\cos^5(e+fx)}{5f}+\frac{a^2\cos^3(e+fx)}{f}-\frac{2a^2\cos(e+fx)}{f}-\frac{a^2\sin^3(e+fx)\cos(e+fx)}{2f}-\frac{3a^2\sin(e+fx)\cos(e+fx)}{4f}+$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (3*a^2*x)/4 - (2*a^2*Cos[e + f*x])/f + (a^2*Cos[e + f*x]^3)/f - (a^2*Cos[e
+ f*x]^5)/(5*f) - (3*a^2*Cos[e + f*x]*Sin[e + f*x])/(4*f) - (a^2*Cos[e + f*
x]*Sin[e + f*x]^3)/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}\int \sin^3(e+fx)(a+a\sin(e+fx))^2\,dx &= \int \left(a^2\sin^3(e+fx)+2a^2\sin^4(e+fx)+a^2\sin^5(e+fx)\right)dx\\ &= a^2\int \sin^3(e+fx)\,dx+a^2\int \sin^5(e+fx)\,dx+\left(2a^2\right)\int \sin^4(e+fx)\,dx\\ &= -\frac{a^2\cos(e+fx)\sin^3(e+fx)}{2f}+\frac{1}{2}\left(3a^2\right)\int \sin^2(e+fx)\,dx-\frac{a^2\,\mathrm{Subst}\left(\int\right.}{}\\ &= -\frac{2a^2\cos(e+fx)}{f}+\frac{a^2\cos^3(e+fx)}{f}-\frac{a^2\cos^5(e+fx)}{5f}-\frac{3a^2\cos(e+fx}{4f}\\ &= \frac{3a^2x}{4}-\frac{2a^2\cos(e+fx)}{f}+\frac{a^2\cos^3(e+fx)}{f}-\frac{a^2\cos^5(e+fx)}{5f}-\frac{3a^2\cos(}{}\end{aligned}$$

Mathematica [A] time = 0.46, size = 105, normalized size = 1.03

$$-\frac{a^2\cos(e+fx)\left(30\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\left(4\sin^4(e+fx)+10\sin^3(e+fx)+12\sin^2(e+fx)+15\sin(e+fx)+\right.\right.}{20f\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -1/20*(a^2*Cos[e + f*x]*(30*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[C
os[e + f*x]^2]*(24 + 15*Sin[e + f*x] + 12*Sin[e + f*x]^2 + 10*Sin[e + f*x]^
3 + 4*Sin[e + f*x]^4)))/(f*Sqrt[Cos[e + f*x]^2])
```

fricas [A] time = 0.51, size = 83, normalized size = 0.81

$$-\frac{4\,a^2\cos\left(fx+e\right)^5-20\,a^2\cos\left(fx+e\right)^3-15\,a^2fx+40\,a^2\cos\left(fx+e\right)-5\left(2\,a^2\cos\left(fx+e\right)^3-5\,a^2\cos\left(fx+\right.\right.}{20\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/20*(4*a^2*cos(f*x + e)^5 - 20*a^2*cos(f*x + e)^3 - 15*a^2*f*x + 40*a^2*c
os(f*x + e) - 5*(2*a^2*cos(f*x + e)^3 - 5*a^2*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.72, size = 94, normalized size = 0.92

$$\frac{3}{4}\,a^2x-\frac{a^2\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{3\,a^2\cos\left(3\,fx+3\,e\right)}{16\,f}-\frac{11\,a^2\cos\left(fx+e\right)}{8\,f}+\frac{a^2\sin\left(4\,fx+4\,e\right)}{16\,f}-\frac{a^2\sin\left(2\,fx+2\,e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 3/4*a^2*x - 1/80*a^2*cos(5*f*x + 5*e)/f + 3/16*a^2*cos(3*f*x + 3*e)/f - 11/
8*a^2*cos(f*x + e)/f + 1/16*a^2*sin(4*f*x + 4*e)/f - 1/2*a^2*sin(2*f*x + 2*
e)/f
```

maple [A] time = 0.28, size = 96, normalized size = 0.94

$$\frac{-\frac{a^2\left(\frac{8}{3}+\sin^4\left(fx+e\right)+\frac{4\left(\sin^2\left(fx+e\right)\right)}{3}\right)\cos\left(fx+e\right)}{5}+2a^2\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{a^2\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+a*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(-1/5*a^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+2*a^2*(-1/4*(s
in(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/3*a^2*(2+sin(f*x+e)
^2)*cos(f*x+e))
```

maxima [A] time = 0.45, size = 95, normalized size = 0.93

$$-\frac{16\left(3\cos\left(fx+e\right)^{5}-10\cos\left(fx+e\right)^{3}+15\cos\left(fx+e\right)\right)a^{2}-80\left(\cos\left(fx+e\right)^{3}-3\cos\left(fx+e\right)\right)a^{2}-15\left(12f\right.}{240f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/240*(16*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a^2 - 8
0*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2 - 15*(12*f*x + 12*e + sin(4*f*x + 4
*e) - 8*sin(2*f*x + 2*e))*a^2)/f
```

mupad [B] time = 10.28, size = 225, normalized size = 2.21

$$\frac{3a^2x}{4}-\frac{\frac{3a^2(e+fx)}{4}+7a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3-7a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7-\frac{3a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^9}{2}-\frac{a^2(15e+15fx-48)}{20}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6\left(\frac{1}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + a*sin(e + f*x))^2,x)
```

```
[Out] (3*a^2*x)/4 - ((3*a^2*(e + f*x))/4 + 7*a^2*tan(e/2 + (f*x)/2)^3 - 7*a^2*tan
(e/2 + (f*x)/2)^7 - (3*a^2*tan(e/2 + (f*x)/2)^9)/2 - (a^2*(15*e + 15*f*x -
48))/20 + tan(e/2 + (f*x)/2)^6*((15*a^2*(e + f*x))/2 - (a^2*(150*e + 150*f*
x - 80))/20) + tan(e/2 + (f*x)/2)^2*((15*a^2*(e + f*x))/4 - (a^2*(75*e + 75
*f*x - 240))/20) + tan(e/2 + (f*x)/2)^4*((15*a^2*(e + f*x))/2 - (a^2*(150*e
 + 150*f*x - 400))/20) + (3*a^2*tan(e/2 + (f*x)/2))/2)/(f*(tan(e/2 + (f*x)/
2)^2 + 1)^5)
```

sympy [A] time = 3.66, size = 221, normalized size = 2.17

$$\begin{cases}\frac{3a^2x\sin^4(e+fx)}{4}+\frac{3a^2x\sin^2(e+fx)\cos^2(e+fx)}{2}+\frac{3a^2x\cos^4(e+fx)}{4}-\frac{a^2\sin^4(e+fx)\cos(e+fx)}{f}-\frac{5a^2\sin^3(e+fx)\cos(e+fx)}{4f}-\frac{4a^2\sin^2(e\ldots}{} \\ x(a\sin(e)+a)^2\sin^3(e)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(e + f*x)**4/4 + 3*a**2*x*sin(e + f*x)**2*cos(e + f*
x)**2/2 + 3*a**2*x*cos(e + f*x)**4/4 - a**2*sin(e + f*x)**4*cos(e + f*x)/f
```

```
- 5*a**2*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 4*a**2*sin(e + f*x)**2*cos(e
+ f*x)**3/(3*f) - a**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*a**2*sin(e + f*x)
*cos(e + f*x)**3/(4*f) - 8*a**2*cos(e + f*x)**5/(15*f) - 2*a**2*cos(e + f*x
)**3/(3*f), Ne(f, 0)), (x*(a*sin(e) + a)**2*sin(e)**3, True))
```

3.2 $\int \sin^3(e+fx)(a+a\sin(e+fx))^3\,dx$

Optimal. Leaf size=129

$$-\frac{3a^3\cos^5(e+fx)}{5f}+\frac{7a^3\cos^3(e+fx)}{3f}-\frac{4a^3\cos(e+fx)}{f}-\frac{a^3\sin^5(e+fx)\cos(e+fx)}{6f}-\frac{23a^3\sin^3(e+fx)\cos(e+f}{24f}$$

```
[Out] 23/16*a^3*x-4*a^3*cos(f*x+e)/f+7/3*a^3*cos(f*x+e)^3/f-3/5*a^3*cos(f*x+e)^5/
f-23/16*a^3*cos(f*x+e)*sin(f*x+e)/f-23/24*a^3*cos(f*x+e)*sin(f*x+e)^3/f-1/6
*a^3*cos(f*x+e)*sin(f*x+e)^5/f
```

Rubi [A] time = 0.14, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2757, 2633, 2635, 8}

$$-\frac{3a^3\cos^5(e+fx)}{5f}+\frac{7a^3\cos^3(e+fx)}{3f}-\frac{4a^3\cos(e+fx)}{f}-\frac{a^3\sin^5(e+fx)\cos(e+fx)}{6f}-\frac{23a^3\sin^3(e+fx)\cos(e+f}{24f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (23*a^3*x)/16 - (4*a^3*Cos[e + f*x])/f + (7*a^3*Cos[e + f*x]^3)/(3*f) - (3*
a^3*Cos[e + f*x]^5)/(5*f) - (23*a^3*Cos[e + f*x]*Sin[e + f*x])/(16*f) - (23
*a^3*Cos[e + f*x]*Sin[e + f*x]^3)/(24*f) - (a^3*Cos[e + f*x]*Sin[e + f*x]^5
)/(6*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \sin^3(e+fx)(a+a\sin(e+fx))^3\,dx = \int \left(a^3\sin^3(e+fx)+3a^3\sin^4(e+fx)+3a^3\sin^5(e+fx)+a^3\sin^6(e+\right.$$

$$= a^3\int \sin^3(e+fx)\,dx + a^3\int \sin^6(e+fx)\,dx + \left(3a^3\right)\int \sin^4(e+fx)\,dx$$

$$= -\frac{3a^3\cos(e+fx)\sin^3(e+fx)}{4f} - \frac{a^3\cos(e+fx)\sin^5(e+fx)}{6f} + \frac{1}{6}\left(5a^3\right)$$

$$= -\frac{4a^3\cos(e+fx)}{f} + \frac{7a^3\cos^3(e+fx)}{3f} - \frac{3a^3\cos^5(e+fx)}{5f} - \frac{9a^3\cos(e+}{}$$

$$= \frac{9a^3x}{8} - \frac{4a^3\cos(e+fx)}{f} + \frac{7a^3\cos^3(e+fx)}{3f} - \frac{3a^3\cos^5(e+fx)}{5f} - \frac{23a^3}{}$$

$$= \frac{23a^3x}{16} - \frac{4a^3\cos(e+fx)}{f} + \frac{7a^3\cos^3(e+fx)}{3f} - \frac{3a^3\cos^5(e+fx)}{5f} - \frac{23a}{}$$

Mathematica [A] time = 0.53, size = 115, normalized size = 0.89

$$-\frac{a^3\cos(e+fx)\left(690\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\left(40\sin^5(e+fx)+144\sin^4(e+fx)+230\sin^3(e+fx)+272\sin^2(e\right.\right.}{240f\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -1/240*(a^3*Cos[e + f*x]*(690*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt
[Cos[e + f*x]^2]*(544 + 345*Sin[e + f*x] + 272*Sin[e + f*x]^2 + 230*Sin[e +
 f*x]^3 + 144*Sin[e + f*x]^4 + 40*Sin[e + f*x]^5)))/(f*Sqrt[Cos[e + f*x]^2]
)
```

fricas [A] time = 0.47, size = 96, normalized size = 0.74

$$-\frac{144\,a^3\cos\left(fx+e\right)^5-560\,a^3\cos\left(fx+e\right)^3-345\,a^3fx+960\,a^3\cos\left(fx+e\right)+5\left(8\,a^3\cos\left(fx+e\right)^5-62\,a^3\mathrm{co}\right.}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/240*(144*a^3*cos(f*x + e)^5 - 560*a^3*cos(f*x + e)^3 - 345*a^3*f*x + 960
*a^3*cos(f*x + e) + 5*(8*a^3*cos(f*x + e)^5 - 62*a^3*cos(f*x + e)^3 + 123*a
^3*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 1.93, size = 112, normalized size = 0.87

$$\frac{23}{16}\,a^3x - \frac{3\,a^3\cos\left(5\,fx+5\,e\right)}{80\,f} + \frac{19\,a^3\cos\left(3\,fx+3\,e\right)}{48\,f} - \frac{21\,a^3\cos\left(fx+e\right)}{8\,f} - \frac{a^3\sin\left(6\,fx+6\,e\right)}{192\,f} + \frac{9\,a^3\sin\left(4\,fx+4\,e\right)}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 23/16*a^3*x - 3/80*a^3*cos(5*f*x + 5*e)/f + 19/48*a^3*cos(3*f*x + 3*e)/f -
21/8*a^3*cos(f*x + e)/f - 1/192*a^3*sin(6*f*x + 6*e)/f + 9/64*a^3*sin(4*f*x
 + 4*e)/f - 63/64*a^3*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.34, size = 143, normalized size = 1.11

$$\frac{a^3\left(-\frac{\left(\sin^5\left(fx+e\right)+\frac{5\left(\sin^3\left(fx+e\right)\right)}{4}+\frac{15\sin\left(fx+e\right)}{8}\right)\cos\left(fx+e\right)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)-\frac{3a^3\left(\frac{8}{3}+\sin^4\left(fx+e\right)+\frac{4\left(\sin^2\left(fx+e\right)\right)}{3}\right)\cos\left(fx+e\right)}{5}+3a^3\left(-\frac{\left(\sin^3\left(fx+e\right)\right.}{}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+a*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(a^3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5
/16*f*x+5/16*e)-3/5*a^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+3*a^
3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/3*a^3*(2+
sin(f*x+e)^2)*cos(f*x+e))
```

maxima [A] time = 0.43, size = 143, normalized size = 1.11

$$-\frac{192\left(3\,\cos\left(fx+e\right)^5-10\,\cos\left(fx+e\right)^3+15\,\cos\left(fx+e\right)\right)a^3-320\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)a^3-5\left(4\,\mathrm{s}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/960*(192*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a^3 -
320*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3 - 5*(4*sin(2*f*x + 2*e)^3 + 60*f*
```

```
x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*a^3 - 90*(12*f*x + 12*
e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^3)/f
```

mupad [B] time = 10.33, size = 294, normalized size = 2.28

$$\frac{23\,a^3\,x}{16}-\frac{\frac{23\,a^3\,\left(e+f\,x\right)}{16}+\frac{391\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{24}+\frac{75\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{4}-\frac{75\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{4}-\frac{391\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{24}-\frac{23\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{8}-\frac{a^3}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + a*sin(e + f*x))^3,x)
```

```
[Out] (23*a^3*x)/16 - ((23*a^3*(e + f*x))/16 + (391*a^3*tan(e/2 + (f*x)/2)^3)/24
+ (75*a^3*tan(e/2 + (f*x)/2)^5)/4 - (75*a^3*tan(e/2 + (f*x)/2)^7)/4 - (391*
a^3*tan(e/2 + (f*x)/2)^9)/24 - (23*a^3*tan(e/2 + (f*x)/2)^11)/8 - (a^3*(345
*e + 345*f*x - 1088))/240 + tan(e/2 + (f*x)/2)^2*((69*a^3*(e + f*x))/8 - (a
^3*(2070*e + 2070*f*x - 6528))/240) + tan(e/2 + (f*x)/2)^8*((345*a^3*(e + f
*x))/16 - (a^3*(5175*e + 5175*f*x - 960))/240) + tan(e/2 + (f*x)/2)^6*((115
*a^3*(e + f*x))/4 - (a^3*(6900*e + 6900*f*x - 10880))/240) + tan(e/2 + (f*x
)/2)^4*((345*a^3*(e + f*x))/16 - (a^3*(5175*e + 5175*f*x - 15360))/240) + (
23*a^3*tan(e/2 + (f*x)/2))/8)/(f*(tan(e/2 + (f*x)/2)^2 + 1)^6)
```

sympy [A] time = 6.82, size = 379, normalized size = 2.94

$$\begin{cases} \frac{5a^3x\sin^6\left(e+fx\right)}{16}+\frac{15a^3x\sin^4\left(e+fx\right)\cos^2\left(e+fx\right)}{16}+\frac{9a^3x\sin^4\left(e+fx\right)}{8}+\frac{15a^3x\sin^2\left(e+fx\right)\cos^4\left(e+fx\right)}{16}+\frac{9a^3x\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4}+ \\ x\left(a\sin\left(e\right)+a\right)^3\sin^3\left(e\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((5*a**3*x*sin(e + f*x)**6/16 + 15*a**3*x*sin(e + f*x)**4*cos(e +
f*x)**2/16 + 9*a**3*x*sin(e + f*x)**4/8 + 15*a**3*x*sin(e + f*x)**2*cos(e +
 f*x)**4/16 + 9*a**3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 5*a**3*x*cos(e +
 f*x)**6/16 + 9*a**3*x*cos(e + f*x)**4/8 - 11*a**3*sin(e + f*x)**5*cos(e +
f*x)/(16*f) - 3*a**3*sin(e + f*x)**4*cos(e + f*x)/f - 5*a**3*sin(e + f*x)**
3*cos(e + f*x)**3/(6*f) - 15*a**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*a*
*3*sin(e + f*x)**2*cos(e + f*x)**3/f - a**3*sin(e + f*x)**2*cos(e + f*x)/f
- 5*a**3*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 9*a**3*sin(e + f*x)*cos(e +
f*x)**3/(8*f) - 8*a**3*cos(e + f*x)**5/(5*f) - 2*a**3*cos(e + f*x)**3/(3*f)
, Ne(f, 0)), (x*(a*sin(e) + a)**3*sin(e)**3, True))
```

3.3 $\int \frac{\sin^4(x)}{a+a\sin(x)}\,dx$

Optimal. Leaf size=53

$$-\frac{3x}{2a}+\frac{4\cos^3(x)}{3a}-\frac{4\cos(x)}{a}+\frac{\sin^3(x)\cos(x)}{a\sin(x)+a}+\frac{3\sin(x)\cos(x)}{2a}$$

```
[Out] -3/2*x/a-4*cos(x)/a+4/3*cos(x)^3/a+3/2*cos(x)*sin(x)/a+cos(x)*sin(x)^3/(a+a
*sin(x))
```

Rubi [A] time = 0.07, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2767, 2748, 2635, 8, 2633}

$$-\frac{3x}{2a}+\frac{4\cos^3(x)}{3a}-\frac{4\cos(x)}{a}+\frac{\sin^3(x)\cos(x)}{a\sin(x)+a}+\frac{3\sin(x)\cos(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4/(a + a*Sin[x]),x]
```

```
[Out] (-3*x)/(2*a) - (4*Cos[x])/a + (4*Cos[x]^3)/(3*a) + (3*Cos[x]*Sin[x])/(2*a)
+ (Cos[x]*Sin[x]^3)/(a + a*Sin[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^4(x)}{a+a\sin(x)}\,dx &= \frac{\cos(x)\sin^3(x)}{a+a\sin(x)} - \frac{\int \sin^2(x)(3a-4a\sin(x))\,dx}{a^2}\\
&= \frac{\cos(x)\sin^3(x)}{a+a\sin(x)} - \frac{3\int \sin^2(x)\,dx}{a} + \frac{4\int \sin^3(x)\,dx}{a}\\
&= \frac{3\cos(x)\sin(x)}{2a} + \frac{\cos(x)\sin^3(x)}{a+a\sin(x)} - \frac{3\int 1\,dx}{2a} - \frac{4\,\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos(x)\right)}{a}\\
&= -\frac{3x}{2a} - \frac{4\cos(x)}{a} + \frac{4\cos^3(x)}{3a} + \frac{3\cos(x)\sin(x)}{2a} + \frac{\cos(x)\sin^3(x)}{a+a\sin(x)}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 101, normalized size = 1.91

$$\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(-36x\sin\left(\frac{x}{2}\right)+69\sin\left(\frac{x}{2}\right)-18\sin\left(\frac{3x}{2}\right)+2\sin\left(\frac{5x}{2}\right)+\sin\left(\frac{7x}{2}\right)-3(12x+7)\cos\left(\frac{x}{2}\right)-18\cos\right.}{24a(\sin(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4/(a + a*Sin[x]),x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(-3*(7 + 12*x)*Cos[x/2] - 18*Cos[(3*x)/2] - 2*Cos[(5
*x)/2] + Cos[(7*x)/2] + 69*Sin[x/2] - 36*x*Sin[x/2] - 18*Sin[(3*x)/2] + 2*S
in[(5*x)/2] + Sin[(7*x)/2]))/(24*a*(1 + Sin[x]))
```

fricas [A] time = 0.45, size = 70, normalized size = 1.32

$$\frac{2\cos(x)^4-\cos(x)^3-3\,(3\,x+5)\cos(x)-12\,\cos(x)^2+\left(2\,\cos(x)^3+3\,\cos(x)^2-9\,x-9\,\cos(x)+6\right)\sin(x)-9\,}{6\,(a\cos(x)+a\sin(x)+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+a*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*cos(x)^4 - cos(x)^3 - 3*(3*x + 5)*cos(x) - 12*cos(x)^2 + (2*cos(x)^3
 + 3*cos(x)^2 - 9*x - 9*cos(x) + 6)*sin(x) - 9*x - 6)/(a*cos(x) + a*sin(x)
+ a)
```

giac [A] time = 0.91, size = 67, normalized size = 1.26

$$-\frac{3\,x}{2\,a}-\frac{2}{a\left(\tan\left(\frac{1}{2}\,x\right)+1\right)}-\frac{3\,\tan\left(\frac{1}{2}\,x\right)^{5}+6\,\tan\left(\frac{1}{2}\,x\right)^{4}+24\,\tan\left(\frac{1}{2}\,x\right)^{2}-3\,\tan\left(\frac{1}{2}\,x\right)+10}{3\left(\tan\left(\frac{1}{2}\,x\right)^{2}+1\right)^{3}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+a*sin(x)),x, algorithm="giac")
```

```
[Out] -3/2*x/a - 2/(a*(tan(1/2*x) + 1)) - 1/3*(3*tan(1/2*x)^5 + 6*tan(1/2*x)^4 +
24*tan(1/2*x)^2 - 3*tan(1/2*x) + 10)/((tan(1/2*x)^2 + 1)^3*a)
```

maple [B] time = 0.08, size = 121, normalized size = 2.28

$$-\frac{\tan^{5}\left(\frac{x}{2}\right)}{a\left(\tan^{2}\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{2\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{a\left(\tan^{2}\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{8\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{a\left(\tan^{2}\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{\tan\left(\frac{x}{2}\right)}{a\left(\tan^{2}\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{10}{3a\left(\tan^{2}\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{3\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a+a*sin(x)),x)
```

```
[Out] -1/a/(tan(1/2*x)^2+1)^3*tan(1/2*x)^5-2/a/(tan(1/2*x)^2+1)^3*tan(1/2*x)^4-8/
a/(tan(1/2*x)^2+1)^3*tan(1/2*x)^2+1/a/(tan(1/2*x)^2+1)^3*tan(1/2*x)-10/3/a/
(tan(1/2*x)^2+1)^3-3/a*arctan(tan(1/2*x))-2/a/(tan(1/2*x)+1)
```

maxima [B] time = 0.83, size = 180, normalized size = 3.40

$$-\frac{\frac{7\,\sin(x)}{\cos(x)+1}+\frac{39\,\sin(x)^{2}}{(\cos(x)+1)^{2}}+\frac{24\,\sin(x)^{3}}{(\cos(x)+1)^{3}}+\frac{24\,\sin(x)^{4}}{(\cos(x)+1)^{4}}+\frac{9\,\sin(x)^{5}}{(\cos(x)+1)^{5}}+\frac{9\,\sin(x)^{6}}{(\cos(x)+1)^{6}}+16}{3\left(a+\frac{a\sin(x)}{\cos(x)+1}+\frac{3\,a\sin(x)^{2}}{(\cos(x)+1)^{2}}+\frac{3\,a\sin(x)^{3}}{(\cos(x)+1)^{3}}+\frac{3\,a\sin(x)^{4}}{(\cos(x)+1)^{4}}+\frac{3\,a\sin(x)^{5}}{(\cos(x)+1)^{5}}+\frac{a\sin(x)^{6}}{(\cos(x)+1)^{6}}+\frac{a\sin(x)^{7}}{(\cos(x)+1)^{7}}\right)}-\frac{3\,\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+a*sin(x)),x, algorithm="maxima")
```

```
[Out] -1/3*(7*sin(x)/(cos(x) + 1) + 39*sin(x)^2/(cos(x) + 1)^2 + 24*sin(x)^3/(cos
(x) + 1)^3 + 24*sin(x)^4/(cos(x) + 1)^4 + 9*sin(x)^5/(cos(x) + 1)^5 + 9*sin
(x)^6/(cos(x) + 1)^6 + 16)/(a + a*sin(x)/(cos(x) + 1) + 3*a*sin(x)^2/(cos(x
) + 1)^2 + 3*a*sin(x)^3/(cos(x) + 1)^3 + 3*a*sin(x)^4/(cos(x) + 1)^4 + 3*a*
sin(x)^5/(cos(x) + 1)^5 + a*sin(x)^6/(cos(x) + 1)^6 + a*sin(x)^7/(cos(x) +
1)^7) - 3*arctan(sin(x)/(cos(x) + 1))/a
```

mupad [B] time = 6.82, size = 78, normalized size = 1.47

$$-\frac{3\,x}{2\,a}-\frac{3\tan\left(\frac{x}{2}\right)^6+3\tan\left(\frac{x}{2}\right)^5+8\tan\left(\frac{x}{2}\right)^4+8\tan\left(\frac{x}{2}\right)^3+13\tan\left(\frac{x}{2}\right)^2+\frac{7\tan\left(\frac{x}{2}\right)}{3}+\frac{16}{3}}{a\left(\tan\left(\frac{x}{2}\right)^2+1\right)^3\,\left(\tan\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a + a*sin(x)),x)
```

```
[Out] - (3*x)/(2*a) - ((7*tan(x/2))/3 + 13*tan(x/2)^2 + 8*tan(x/2)^3 + 8*tan(x/2)
^4 + 3*tan(x/2)^5 + 3*tan(x/2)^6 + 16/3)/(a*(tan(x/2)^2 + 1)^3*(tan(x/2) +
1))
```

sympy [B] time = 5.91, size = 1221, normalized size = 23.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4/(a+a*sin(x)),x)
```

```
[Out] -9*x*tan(x/2)**7/(6*a*tan(x/2)**7 + 6*a*tan(x/2)**6 + 18*a*tan(x/2)**5 + 18
*a*tan(x/2)**4 + 18*a*tan(x/2)**3 + 18*a*tan(x/2)**2 + 6*a*tan(x/2) + 6*a)
- 9*x*tan(x/2)**6/(6*a*tan(x/2)**7 + 6*a*tan(x/2)**6 + 18*a*tan(x/2)**5 + 1
8*a*tan(x/2)**4 + 18*a*tan(x/2)**3 + 18*a*tan(x/2)**2 + 6*a*tan(x/2) + 6*a)
 - 27*x*tan(x/2)**5/(6*a*tan(x/2)**7 + 6*a*tan(x/2)**6 + 18*a*tan(x/2)**5 +
 18*a*tan(x/2)**4 + 18*a*tan(x/2)**3 + 18*a*tan(x/2)**2 + 6*a*tan(x/2) + 6*
a) - 27*x*tan(x/2)**4/(6*a*tan(x/2)**7 + 6*a*tan(x/2)**6 + 18*a*tan(x/2)**5
 + 18*a*tan(x/2)**4 + 18*a*tan(x/2)**3 + 18*a*tan(x/2)**2 + 6*a*tan(x/2) +
6*a) - 27*x*tan(x/2)**3/(6*a*tan(x/2)**7 + 6*a*tan(x/2)**6 + 18*a*tan(x/2)*
*5 + 18*a*tan(x/2)**4 + 18*a*tan(x/2)**3 + 18*a*tan(x/2)**2 + 6*a*tan(x/2)
+ 6*a) - 27*x*tan(x/2)**2/(6*a*tan(x/2)**7 + 6*a*tan(x/2)**6 + 18*a*tan(x/2
)**5 + 18*a*tan(x/2)**4 + 18*a*tan(x/2)**3 + 18*a*tan(x/2)**2 + 6*a*tan(x/2
) + 6*a) - 9*x*tan(x/2)/(6*a*tan(x/2)**7 + 6*a*tan(x/2)**6 + 18*a*tan(x/2)*
*5 + 18*a*tan(x/2)**4 + 18*a*tan(x/2)**3 + 18*a*tan(x/2)**2 + 6*a*tan(x/2)
+ 6*a) - 9*x/(6*a*tan(x/2)**7 + 6*a*tan(x/2)**6 + 18*a*tan(x/2)**5 + 18*a*t
an(x/2)**4 + 18*a*tan(x/2)**3 + 18*a*tan(x/2)**2 + 6*a*tan(x/2) + 6*a) - 18
*tan(x/2)**6/(6*a*tan(x/2)**7 + 6*a*tan(x/2)**6 + 18*a*tan(x/2)**5 + 18*a*t
```

```
an(x/2)**4 + 18*a*tan(x/2)**3 + 18*a*tan(x/2)**2 + 6*a*tan(x/2) + 6*a) - 18
*tan(x/2)**5/(6*a*tan(x/2)**7 + 6*a*tan(x/2)**6 + 18*a*tan(x/2)**5 + 18*a*t
an(x/2)**4 + 18*a*tan(x/2)**3 + 18*a*tan(x/2)**2 + 6*a*tan(x/2) + 6*a) - 48
*tan(x/2)**4/(6*a*tan(x/2)**7 + 6*a*tan(x/2)**6 + 18*a*tan(x/2)**5 + 18*a*t
an(x/2)**4 + 18*a*tan(x/2)**3 + 18*a*tan(x/2)**2 + 6*a*tan(x/2) + 6*a) - 48
*tan(x/2)**3/(6*a*tan(x/2)**7 + 6*a*tan(x/2)**6 + 18*a*tan(x/2)**5 + 18*a*t
an(x/2)**4 + 18*a*tan(x/2)**3 + 18*a*tan(x/2)**2 + 6*a*tan(x/2) + 6*a) - 78
*tan(x/2)**2/(6*a*tan(x/2)**7 + 6*a*tan(x/2)**6 + 18*a*tan(x/2)**5 + 18*a*t
an(x/2)**4 + 18*a*tan(x/2)**3 + 18*a*tan(x/2)**2 + 6*a*tan(x/2) + 6*a) - 14
*tan(x/2)/(6*a*tan(x/2)**7 + 6*a*tan(x/2)**6 + 18*a*tan(x/2)**5 + 18*a*tan(
x/2)**4 + 18*a*tan(x/2)**3 + 18*a*tan(x/2)**2 + 6*a*tan(x/2) + 6*a) - 32/(6
*a*tan(x/2)**7 + 6*a*tan(x/2)**6 + 18*a*tan(x/2)**5 + 18*a*tan(x/2)**4 + 18
*a*tan(x/2)**3 + 18*a*tan(x/2)**2 + 6*a*tan(x/2) + 6*a)
```

3.4 $\int \frac{\sin^3(x)}{a+a\sin(x)}\,dx$

Optimal. Leaf size=42

$$\frac{3x}{2a}+\frac{2\cos(x)}{a}+\frac{\sin^2(x)\cos(x)}{a\sin(x)+a}-\frac{3\sin(x)\cos(x)}{2a}$$

```
[Out] 3/2*x/a+2*cos(x)/a-3/2*cos(x)*sin(x)/a+cos(x)*sin(x)^2/(a+a*sin(x))
```

Rubi [A] time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2767, 2734}

$$\frac{3x}{2a}+\frac{2\cos(x)}{a}+\frac{\sin^2(x)\cos(x)}{a\sin(x)+a}-\frac{3\sin(x)\cos(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a + a*Sin[x]),x]
```

```
[Out] (3*x)/(2*a) + (2*Cos[x])/a - (3*Cos[x]*Sin[x])/(2*a) + (Cos[x]*Sin[x]^2)/(a
 + a*Sin[x])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rubi steps

$$\int \frac{\sin^3(x)}{a+a\sin(x)}\,dx = \frac{\cos(x)\sin^2(x)}{a+a\sin(x)} - \frac{\int \sin(x)(2a-3a\sin(x))\,dx}{a^2}$$
$$= \frac{3x}{2a} + \frac{2\cos(x)}{a} - \frac{3\cos(x)\sin(x)}{2a} + \frac{\cos(x)\sin^2(x)}{a+a\sin(x)}$$

Mathematica [B] time = 0.09, size = 87, normalized size = 2.07

$$\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(12x\sin\left(\frac{x}{2}\right)-20\sin\left(\frac{x}{2}\right)+3\sin\left(\frac{3x}{2}\right)-\sin\left(\frac{5x}{2}\right)+4(3x+1)\cos\left(\frac{x}{2}\right)+3\cos\left(\frac{3x}{2}\right)+\cos\left(\frac{5x}{2}\right)\right)}{8a(\sin(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a + a*Sin[x]),x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(4*(1 + 3*x)*Cos[x/2] + 3*Cos[(3*x)/2] + Cos[(5*x)/2
] - 20*Sin[x/2] + 12*x*Sin[x/2] + 3*Sin[(3*x)/2] - Sin[(5*x)/2]))/(8*a*(1 +
 Sin[x]))
```

fricas [A] time = 0.47, size = 53, normalized size = 1.26

$$\frac{\cos(x)^3 + 3\,(x+1)\cos(x) + 2\,\cos(x)^2 - \left(\cos(x)^2 - 3\,x - \cos(x) + 2\right)\sin(x) + 3\,x + 2}{2\,(a\cos(x) + a\sin(x) + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(cos(x)^3 + 3*(x + 1)*cos(x) + 2*cos(x)^2 - (cos(x)^2 - 3*x - cos(x) +
2)*sin(x) + 3*x + 2)/(a*cos(x) + a*sin(x) + a)
```

giac [A] time = 1.95, size = 56, normalized size = 1.33

$$\frac{3\,x}{2\,a} + \frac{\tan\left(\frac{1}{2}\,x\right)^3 + 2\,\tan\left(\frac{1}{2}\,x\right)^2 - \tan\left(\frac{1}{2}\,x\right) + 2}{\left(\tan\left(\frac{1}{2}\,x\right)^2 + 1\right)^2 a} + \frac{2}{a\left(\tan\left(\frac{1}{2}\,x\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*sin(x)),x, algorithm="giac")
```

```
[Out] 3/2*x/a + (tan(1/2*x)^3 + 2*tan(1/2*x)^2 - tan(1/2*x) + 2)/((tan(1/2*x)^2 +
 1)^2*a) + 2/(a*(tan(1/2*x) + 1))
```

maple [B] time = 0.07, size = 100, normalized size = 2.38

$$\frac{\tan^3\left(\frac{x}{2}\right)}{a\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}+\frac{2\left(\tan^2\left(\frac{x}{2}\right)\right)}{a\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}-\frac{\tan\left(\frac{x}{2}\right)}{a\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}+\frac{2}{a\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}+\frac{3\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a}+\frac{2}{a\left(\tan\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a+a*sin(x)),x)
```

```
[Out] 1/a/(tan(1/2*x)^2+1)^2*tan(1/2*x)^3+2/a/(tan(1/2*x)^2+1)^2*tan(1/2*x)^2-1/a
/(tan(1/2*x)^2+1)^2*tan(1/2*x)+2/a/(tan(1/2*x)^2+1)^2+3/a*arctan(tan(1/2*x)
)+2/a/(tan(1/2*x)+1)
```

maxima [B] time = 0.65, size = 128, normalized size = 3.05

$$\frac{\frac{\sin(x)}{\cos(x)+1}+\frac{5\sin(x)^2}{(\cos(x)+1)^2}+\frac{3\sin(x)^3}{(\cos(x)+1)^3}+\frac{3\sin(x)^4}{(\cos(x)+1)^4}+4}{a+\frac{a\sin(x)}{\cos(x)+1}+\frac{2a\sin(x)^2}{(\cos(x)+1)^2}+\frac{2a\sin(x)^3}{(\cos(x)+1)^3}+\frac{a\sin(x)^4}{(\cos(x)+1)^4}+\frac{a\sin(x)^5}{(\cos(x)+1)^5}}+\frac{3\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*sin(x)),x, algorithm="maxima")
```

```
[Out] (sin(x)/(cos(x) + 1) + 5*sin(x)^2/(cos(x) + 1)^2 + 3*sin(x)^3/(cos(x) + 1)^
3 + 3*sin(x)^4/(cos(x) + 1)^4 + 4)/(a + a*sin(x)/(cos(x) + 1) + 2*a*sin(x)^
2/(cos(x) + 1)^2 + 2*a*sin(x)^3/(cos(x) + 1)^3 + a*sin(x)^4/(cos(x) + 1)^4
+ a*sin(x)^5/(cos(x) + 1)^5) + 3*arctan(sin(x)/(cos(x) + 1))/a
```

mupad [B] time = 6.91, size = 59, normalized size = 1.40

$$\frac{3\,x}{2\,a}+\frac{3\tan\left(\frac{x}{2}\right)^4+3\tan\left(\frac{x}{2}\right)^3+5\tan\left(\frac{x}{2}\right)^2+\tan\left(\frac{x}{2}\right)+4}{a\left(\tan\left(\frac{x}{2}\right)^2+1\right)^2\left(\tan\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a + a*sin(x)),x)
```

```
[Out] (3*x)/(2*a) + (tan(x/2) + 5*tan(x/2)^2 + 3*tan(x/2)^3 + 3*tan(x/2)^4 + 4)/(
a*(tan(x/2)^2 + 1)^2*(tan(x/2) + 1))
```

<u>**sympy**</u> **[B]** time = 2.79, size = 665, normalized size = 15.83

$$\frac{3x\tan^{5}\left(\frac{x}{2}\right)}{2a\tan^{5}\left(\frac{x}{2}\right)+2a\tan^{4}\left(\frac{x}{2}\right)+4a\tan^{3}\left(\frac{x}{2}\right)+4a\tan^{2}\left(\frac{x}{2}\right)+2a\tan\left(\frac{x}{2}\right)+2a}+\frac{3x\tan^{4}\left(\right.}{2a\tan^{5}\left(\frac{x}{2}\right)+2a\tan^{4}\left(\frac{x}{2}\right)+4a\tan^{3}\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a+a*sin(x)),x)
```

```
[Out] 3*x*tan(x/2)**5/(2*a*tan(x/2)**5 + 2*a*tan(x/2)**4 + 4*a*tan(x/2)**3 + 4*a*
tan(x/2)**2 + 2*a*tan(x/2) + 2*a) + 3*x*tan(x/2)**4/(2*a*tan(x/2)**5 + 2*a*
tan(x/2)**4 + 4*a*tan(x/2)**3 + 4*a*tan(x/2)**2 + 2*a*tan(x/2) + 2*a) + 6*x
*tan(x/2)**3/(2*a*tan(x/2)**5 + 2*a*tan(x/2)**4 + 4*a*tan(x/2)**3 + 4*a*tan
(x/2)**2 + 2*a*tan(x/2) + 2*a) + 6*x*tan(x/2)**2/(2*a*tan(x/2)**5 + 2*a*tan
(x/2)**4 + 4*a*tan(x/2)**3 + 4*a*tan(x/2)**2 + 2*a*tan(x/2) + 2*a) + 3*x*ta
n(x/2)/(2*a*tan(x/2)**5 + 2*a*tan(x/2)**4 + 4*a*tan(x/2)**3 + 4*a*tan(x/2)*
*2 + 2*a*tan(x/2) + 2*a) + 3*x/(2*a*tan(x/2)**5 + 2*a*tan(x/2)**4 + 4*a*tan
(x/2)**3 + 4*a*tan(x/2)**2 + 2*a*tan(x/2) + 2*a) + 6*tan(x/2)**4/(2*a*tan(x
/2)**5 + 2*a*tan(x/2)**4 + 4*a*tan(x/2)**3 + 4*a*tan(x/2)**2 + 2*a*tan(x/2)
 + 2*a) + 6*tan(x/2)**3/(2*a*tan(x/2)**5 + 2*a*tan(x/2)**4 + 4*a*tan(x/2)**
3 + 4*a*tan(x/2)**2 + 2*a*tan(x/2) + 2*a) + 10*tan(x/2)**2/(2*a*tan(x/2)**5
 + 2*a*tan(x/2)**4 + 4*a*tan(x/2)**3 + 4*a*tan(x/2)**2 + 2*a*tan(x/2) + 2*a
) + 2*tan(x/2)/(2*a*tan(x/2)**5 + 2*a*tan(x/2)**4 + 4*a*tan(x/2)**3 + 4*a*t
an(x/2)**2 + 2*a*tan(x/2) + 2*a) + 8/(2*a*tan(x/2)**5 + 2*a*tan(x/2)**4 + 4
*a*tan(x/2)**3 + 4*a*tan(x/2)**2 + 2*a*tan(x/2) + 2*a)
```

3.5 $\int \frac{\sin^2(x)}{a+a\sin(x)}\,dx$

Optimal. Leaf size=27

$$-\frac{x}{a}-\frac{\cos(x)}{a}-\frac{\cos(x)}{a(\sin(x)+1)}$$

```
[Out] -x/a-cos(x)/a-cos(x)/a/(1+sin(x))
```

Rubi [A] time = 0.07, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2746, 12, 2735, 2648}

$$-\frac{x}{a}-\frac{\cos(x)}{a}-\frac{\cos(x)}{a(\sin(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a + a*Sin[x]),x]
```

```
[Out] -(x/a) - Cos[x]/a - Cos[x]/(a*(1 + Sin[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^2(x)}{a+a\sin(x)}\,dx &= -\frac{\cos(x)}{a}-\frac{\int \frac{a\sin(x)}{a+a\sin(x)}\,dx}{a}\\
&= -\frac{\cos(x)}{a}-\int \frac{\sin(x)}{a+a\sin(x)}\,dx\\
&= -\frac{x}{a}-\frac{\cos(x)}{a}+\int \frac{1}{a+a\sin(x)}\,dx\\
&= -\frac{x}{a}-\frac{\cos(x)}{a}-\frac{\cos(x)}{a+a\sin(x)}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 48, normalized size = 1.78

$$
-\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(\cos\left(\frac{x}{2}\right)(x+\cos(x))+\sin\left(\frac{x}{2}\right)(x+\cos(x)-2)\right)}{a(\sin(x)+1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a + a*Sin[x]),x]
```

```
[Out] -(((Cos[x/2] + Sin[x/2])*(Cos[x/2]*(x + Cos[x]) + (-2 + x + Cos[x])*Sin[x/2
]))/(a*(1 + Sin[x])))
```

fricas [A] time = 0.48, size = 35, normalized size = 1.30

$$
-\frac{(x+2)\cos(x)+\cos(x)^2+(x+\cos(x)-1)\sin(x)+x+1}{a\cos(x)+a\sin(x)+a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*sin(x)),x, algorithm="fricas")
```

```
[Out] -((x + 2)*cos(x) + cos(x)^2 + (x + cos(x) - 1)*sin(x) + x + 1)/(a*cos(x) +
a*sin(x) + a)
```

giac [A] time = 0.29, size = 44, normalized size = 1.63

$$
-\frac{x}{a}-\frac{2\left(\tan\left(\frac{1}{2}x\right)^2+\tan\left(\frac{1}{2}x\right)+2\right)}{\left(\tan\left(\frac{1}{2}x\right)^3+\tan\left(\frac{1}{2}x\right)^2+\tan\left(\frac{1}{2}x\right)+1\right)a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*sin(x)),x, algorithm="giac")
```

```
[Out] -x/a - 2*(tan(1/2*x)^2 + tan(1/2*x) + 2)/((tan(1/2*x)^3 + tan(1/2*x)^2 + ta
n(1/2*x) + 1)*a)
```

maple [A] time = 0.07, size = 40, normalized size = 1.48

$$-\frac{2}{a\left(\tan^{2}\left(\frac{x}{2}\right)+1\right)}-\frac{2\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a}-\frac{2}{a\left(\tan\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a+a*sin(x)),x)
```

```
[Out] -2/a/(tan(1/2*x)^2+1)-2/a*arctan(tan(1/2*x))-2/a/(tan(1/2*x)+1)
```

maxima [B] time = 0.49, size = 78, normalized size = 2.89

$$-\frac{2\left(\frac{\sin(x)}{\cos(x)+1}+\frac{\sin(x)^{2}}{(\cos(x)+1)^{2}}+2\right)}{a+\frac{a\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^{2}}{(\cos(x)+1)^{2}}+\frac{a\sin(x)^{3}}{(\cos(x)+1)^{3}}}-\frac{2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*sin(x)),x, algorithm="maxima")
```

```
[Out] -2*(sin(x)/(cos(x) + 1) + sin(x)^2/(cos(x) + 1)^2 + 2)/(a + a*sin(x)/(cos(x
) + 1) + a*sin(x)^2/(cos(x) + 1)^2 + a*sin(x)^3/(cos(x) + 1)^3) - 2*arctan(
sin(x)/(cos(x) + 1))/a
```

mupad [B] time = 6.77, size = 46, normalized size = 1.70

$$-\frac{x}{a}-\frac{2\tan\left(\frac{x}{2}\right)^{2}+2\tan\left(\frac{x}{2}\right)+4}{a\left(\tan\left(\frac{x}{2}\right)^{2}+1\right)\left(\tan\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a + a*sin(x)),x)
```

```
[Out] - x/a - (2*tan(x/2) + 2*tan(x/2)^2 + 4)/(a*(tan(x/2)^2 + 1)*(tan(x/2) + 1))
```

sympy [B] time = 1.35, size = 221, normalized size = 8.19

$$-\frac{x\tan^{3}\left(\frac{x}{2}\right)}{a\tan^{3}\left(\frac{x}{2}\right)+a\tan^{2}\left(\frac{x}{2}\right)+a\tan\left(\frac{x}{2}\right)+a}-\frac{x\tan^{2}\left(\frac{x}{2}\right)}{a\tan^{3}\left(\frac{x}{2}\right)+a\tan^{2}\left(\frac{x}{2}\right)+a\tan\left(\frac{x}{2}\right)+a}-\frac{x\tan\left(\frac{x}{2}\right)}{a\tan^{3}\left(\frac{x}{2}\right)+a\tan^{2}\left(\frac{x}{2}\right)+a\ta$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+a*sin(x)),x)
```

```
[Out] -x*tan(x/2)**3/(a*tan(x/2)**3 + a*tan(x/2)**2 + a*tan(x/2) + a) - x*tan(x/2
)**2/(a*tan(x/2)**3 + a*tan(x/2)**2 + a*tan(x/2) + a) - x*tan(x/2)/(a*tan(x
/2)**3 + a*tan(x/2)**2 + a*tan(x/2) + a) - x/(a*tan(x/2)**3 + a*tan(x/2)**2
 + a*tan(x/2) + a) - 2*tan(x/2)**2/(a*tan(x/2)**3 + a*tan(x/2)**2 + a*tan(x
/2) + a) - 2*tan(x/2)/(a*tan(x/2)**3 + a*tan(x/2)**2 + a*tan(x/2) + a) - 4/
(a*tan(x/2)**3 + a*tan(x/2)**2 + a*tan(x/2) + a)
```

3.6 $\int \frac{\sin(x)}{a+a\sin(x)}\,dx$

Optimal. Leaf size=17

$$\frac{x}{a}+\frac{\cos(x)}{a\sin(x)+a}$$

```
[Out] x/a+cos(x)/(a+a*sin(x))
```

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2735, 2648}

$$\frac{x}{a}+\frac{\cos(x)}{a\sin(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a + a*Sin[x]),x]
```

```
[Out] x/a + Cos[x]/(a + a*Sin[x])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{a+a\sin(x)}\,dx &= \frac{x}{a}-\int \frac{1}{a+a\sin(x)}\,dx\\ &= \frac{x}{a}+\frac{\cos(x)}{a+a\sin(x)}\end{aligned}$$

Mathematica [B] time = 0.04, size = 42, normalized size = 2.47

$$\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left((x-2)\sin\left(\frac{x}{2}\right)+x\cos\left(\frac{x}{2}\right)\right)}{a(\sin(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a + a*Sin[x]),x]
[Out] ((Cos[x/2] + Sin[x/2])*(x*Cos[x/2] + (-2 + x)*Sin[x/2]))/(a*(1 + Sin[x]))
```

fricas [A] time = 0.46, size = 28, normalized size = 1.65

$$\frac{(x+1)\cos(x)+(x-1)\sin(x)+x+1}{a\cos(x)+a\sin(x)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x)),x, algorithm="fricas")
[Out] ((x + 1)*cos(x) + (x - 1)*sin(x) + x + 1)/(a*cos(x) + a*sin(x) + a)
```

giac [A] time = 0.92, size = 19, normalized size = 1.12

$$\frac{x}{a}+\frac{2}{a\left(\tan\left(\frac{1}{2}x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x)),x, algorithm="giac")
[Out] x/a + 2/(a*(tan(1/2*x) + 1))
```

maple [A] time = 0.07, size = 25, normalized size = 1.47

$$\frac{2\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a}+\frac{2}{a\left(\tan\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a+a*sin(x)),x)
[Out] 2/a*arctan(tan(1/2*x))+2/a/(tan(1/2*x)+1)
```

maxima [A] time = 0.82, size = 32, normalized size = 1.88

$$\frac{2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}+\frac{2}{a+\frac{a\sin(x)}{\cos(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x)),x, algorithm="maxima")
[Out] 2*arctan(sin(x)/(cos(x) + 1))/a + 2/(a + a*sin(x)/(cos(x) + 1))
```

mupad [B] time = 6.54, size = 19, normalized size = 1.12

$$\frac{2}{a\left(\tan\left(\frac{x}{2}\right)+1\right)}+\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a + a*sin(x)),x)
[Out] 2/(a*(tan(x/2) + 1)) + x/a
```

sympy [B] time = 0.84, size = 34, normalized size = 2.00

$$\frac{x\tan\left(\frac{x}{2}\right)}{a\tan\left(\frac{x}{2}\right)+a}+\frac{x}{a\tan\left(\frac{x}{2}\right)+a}+\frac{2}{a\tan\left(\frac{x}{2}\right)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x)),x)
[Out] x*tan(x/2)/(a*tan(x/2) + a) + x/(a*tan(x/2) + a) + 2/(a*tan(x/2) + a)
```

3.7 $\int \frac{1}{a+a\sin(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\cos(x)}{a\sin(x)+a}$$

```
[Out] -cos(x)/(a+a*sin(x))
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2648}

$$-\frac{\cos(x)}{a\sin(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[x])^(-1),x]
```

```
[Out] -(Cos[x]/(a + a*Sin[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{a+a\sin(x)}\,dx = -\frac{\cos(x)}{a+a\sin(x)}$$

Mathematica [B] time = 0.03, size = 29, normalized size = 2.42

$$\frac{2\sin\left(\frac{x}{2}\right)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)}{a\sin(x)+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[x])^(-1),x]
```

```
[Out] (2*Sin[x/2]*(Cos[x/2] + Sin[x/2]))/(a + a*Sin[x])
```

fricas [A] time = 0.44, size = 22, normalized size = 1.83

$$-\frac{\cos(x) - \sin(x) + 1}{a\cos(x) + a\sin(x) + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x)),x, algorithm="fricas")
[Out] -(cos(x) - sin(x) + 1)/(a*cos(x) + a*sin(x) + a)
```

giac [A] time = 0.32, size = 13, normalized size = 1.08

$$-\frac{2}{a\left(\tan\left(\frac{1}{2}x\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x)),x, algorithm="giac")
[Out] -2/(a*(tan(1/2*x) + 1))
```

maple [A] time = 0.06, size = 14, normalized size = 1.17

$$-\frac{2}{a\left(\tan\left(\frac{x}{2}\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(x)),x)
[Out] -2/a/(tan(1/2*x)+1)
```

maxima [A] time = 0.54, size = 16, normalized size = 1.33

$$-\frac{2}{a + \frac{a\sin(x)}{\cos(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x)),x, algorithm="maxima")
[Out] -2/(a + a*sin(x)/(cos(x) + 1))
```

mupad [B] time = 0.02, size = 13, normalized size = 1.08

$$-\frac{2}{a\left(\tan\left(\frac{x}{2}\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sin(x)),x)
```

```
[Out] -2/(a*(tan(x/2) + 1))
```

sympy [A] time = 0.41, size = 10, normalized size = 0.83

$$-\frac{2}{a\tan\left(\frac{x}{2}\right)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x)),x)
```

```
[Out] -2/(a*tan(x/2) + a)
```

3.8 $\int \frac{\csc(x)}{a+a\sin(x)}\,dx$

Optimal. Leaf size=20

$$\frac{\cos(x)}{a\sin(x)+a}-\frac{\tanh^{-1}(\cos(x))}{a}$$

```
[Out] -arctanh(cos(x))/a+cos(x)/(a+a*sin(x))
```

Rubi [A] time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2747, 3770, 2648}

$$\frac{\cos(x)}{a\sin(x)+a}-\frac{\tanh^{-1}(\cos(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a + a*Sin[x]),x]
```

```
[Out] -(ArcTanh[Cos[x]]/a) + Cos[x]/(a + a*Sin[x])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc(x)}{a+a\sin(x)}\,dx = \frac{\int \csc(x)\,dx}{a} - \int \frac{1}{a+a\sin(x)}\,dx$$
$$= -\frac{\tanh^{-1}(\cos(x))}{a} + \frac{\cos(x)}{a+a\sin(x)}$$

Mathematica [B] time = 0.06, size = 74, normalized size = 3.70

$$-\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(\cos\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)+\sin\left(\frac{x}{2}\right)\left(-\log\left(\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)\right)+2\right)\right)}{a(\sin(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a + a*Sin[x]),x]
```

```
[Out] -(((Cos[x/2] + Sin[x/2])*(Cos[x/2]*(Log[Cos[x/2]] - Log[Sin[x/2]]) + (2 + L
og[Cos[x/2]] - Log[Sin[x/2]])*Sin[x/2]))/(a*(1 + Sin[x])))
```

fricas [B] time = 0.44, size = 53, normalized size = 2.65

$$-\frac{(\cos(x)+\sin(x)+1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-(\cos(x)+\sin(x)+1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-2\cos(x)+2\sin(x)-2}{2\left(a\cos(x)+a\sin(x)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*((cos(x) + sin(x) + 1)*log(1/2*cos(x) + 1/2) - (cos(x) + sin(x) + 1)*l
og(-1/2*cos(x) + 1/2) - 2*cos(x) + 2*sin(x) - 2)/(a*cos(x) + a*sin(x) + a)
```

giac [A] time = 0.24, size = 24, normalized size = 1.20

$$\frac{\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{a} + \frac{2}{a\left(\tan\left(\frac{1}{2}x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x)),x, algorithm="giac")
```

```
[Out] log(abs(tan(1/2*x)))/a + 2/(a*(tan(1/2*x) + 1))
```

maple [A] time = 0.08, size = 24, normalized size = 1.20

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a}+\frac{2}{a\left(\tan\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a+a*sin(x)),x)
[Out] 1/a*ln(tan(1/2*x))+2/a/(tan(1/2*x)+1)
```

maxima [A] time = 0.85, size = 31, normalized size = 1.55

$$\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}+\frac{2}{a+\frac{a\sin(x)}{\cos(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x)),x, algorithm="maxima")
[Out] log(sin(x)/(cos(x) + 1))/a + 2/(a + a*sin(x)/(cos(x) + 1))
```

mupad [B] time = 6.42, size = 23, normalized size = 1.15

$$\frac{2}{a\left(\tan\left(\frac{x}{2}\right)+1\right)}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(a + a*sin(x))),x)
[Out] 2/(a*(tan(x/2) + 1)) + log(tan(x/2))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\csc(x)}{\sin(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x)),x)
[Out] Integral(csc(x)/(sin(x) + 1), x)/a
```

3.9 $\int \frac{\csc^2(x)}{a+a\sin(x)}\,dx$

Optimal. Leaf size=26

$$-\frac{2\cot(x)}{a}+\frac{\tanh^{-1}(\cos(x))}{a}+\frac{\cot(x)}{a\sin(x)+a}$$

[Out] arctanh(cos(x))/a-2*cot(x)/a+cot(x)/(a+a*sin(x))

Rubi [A] time = 0.06, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2768, 2748, 3767, 8, 3770}

$$-\frac{2\cot(x)}{a}+\frac{\tanh^{-1}(\cos(x))}{a}+\frac{\cot(x)}{a\sin(x)+a}$$

Antiderivative was successfully verified.

[In] Int[Csc[x]^2/(a + a*Sin[x]),x]

[Out] ArcTanh[Cos[x]]/a - (2*Cot[x])/a + Cot[x]/(a + a*Sin[x])

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(x)}{a+a\sin(x)}\,dx &= \frac{\cot(x)}{a+a\sin(x)}-\frac{\int \csc^2(x)(-2a+a\sin(x))\,dx}{a^2}\\ &= \frac{\cot(x)}{a+a\sin(x)}-\frac{\int \csc(x)\,dx}{a}+\frac{2\int \csc^2(x)\,dx}{a}\\ &= \frac{\tanh^{-1}(\cos(x))}{a}+\frac{\cot(x)}{a+a\sin(x)}-\frac{2\,\mathrm{Subst}(\int 1\,dx,x,\cot(x))}{a}\\ &= \frac{\tanh^{-1}(\cos(x))}{a}-\frac{2\cot(x)}{a}+\frac{\cot(x)}{a+a\sin(x)}\end{aligned}$$

Mathematica [B] time = 0.16, size = 63, normalized size = 2.42

$$\frac{\tan\left(\frac{x}{2}\right)-\cot\left(\frac{x}{2}\right)-2\log\left(\sin\left(\frac{x}{2}\right)\right)+2\log\left(\cos\left(\frac{x}{2}\right)\right)+\frac{4\sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a + a*Sin[x]),x]
```

```
[Out] (-Cot[x/2] + 2*Log[Cos[x/2]] - 2*Log[Sin[x/2]] + (4*Sin[x/2])/(Cos[x/2] + S
in[x/2]) + Tan[x/2])/(2*a)
```

fricas [B] time = 0.48, size = 91, normalized size = 3.50

$$\frac{4\cos(x)^2+\left(\cos(x)^2-(\cos(x)+1)\sin(x)-1\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-\left(\cos(x)^2-(\cos(x)+1)\sin(x)-1\right)\log\left(-\frac{1}{2}\right.}{2\left(a\cos(x)^2-(a\cos(x)+a)\sin(x)-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(4*cos(x)^2 + (cos(x)^2 - (cos(x) + 1)*sin(x) - 1)*log(1/2*cos(x) + 1/2
) - (cos(x)^2 - (cos(x) + 1)*sin(x) - 1)*log(-1/2*cos(x) + 1/2) + 2*(2*cos(
x) + 1)*sin(x) + 2*cos(x) - 2)/(a*cos(x)^2 - (a*cos(x) + a)*sin(x) - a)
```

giac [B] time = 0.69, size = 53, normalized size = 2.04

$$-\frac{\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{a}+\frac{\tan\left(\frac{1}{2}x\right)}{2a}+\frac{\tan\left(\frac{1}{2}x\right)^2-4\tan\left(\frac{1}{2}x\right)-1}{2\left(\tan\left(\frac{1}{2}x\right)^2+\tan\left(\frac{1}{2}x\right)\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*sin(x)),x, algorithm="giac")
```

```
[Out] -log(abs(tan(1/2*x)))/a + 1/2*tan(1/2*x)/a + 1/2*(tan(1/2*x)^2 - 4*tan(1/2*
x) - 1)/((tan(1/2*x)^2 + tan(1/2*x))*a)
```

maple [A] time = 0.10, size = 45, normalized size = 1.73

$$\frac{\tan\left(\frac{x}{2}\right)}{2a}-\frac{1}{2a\tan\left(\frac{x}{2}\right)}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a}-\frac{2}{a\left(\tan\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a+a*sin(x)),x)
```

```
[Out] 1/2/a*tan(1/2*x)-1/2/a/tan(1/2*x)-1/a*ln(tan(1/2*x))-2/a/(tan(1/2*x)+1)
```

maxima [B] time = 0.94, size = 68, normalized size = 2.62

$$-\frac{\frac{5\sin(x)}{\cos(x)+1}+1}{2\left(\frac{a\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}-\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}+\frac{\sin(x)}{2a(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*sin(x)),x, algorithm="maxima")
```

```
[Out] -1/2*(5*sin(x)/(cos(x) + 1) + 1)/(a*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x
) + 1)^2) - log(sin(x)/(cos(x) + 1))/a + 1/2*sin(x)/(a*(cos(x) + 1))
```

mupad [B] time = 6.71, size = 49, normalized size = 1.88

$$\frac{\tan\left(\frac{x}{2}\right)}{2a}-\frac{5\tan\left(\frac{x}{2}\right)+1}{2a\tan\left(\frac{x}{2}\right)^2+2a\tan\left(\frac{x}{2}\right)}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(a + a*sin(x))),x)
```

```
[Out] tan(x/2)/(2*a) - (5*tan(x/2) + 1)/(2*a*tan(x/2) + 2*a*tan(x/2)^2) - log(tan
(x/2))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^2(x)}{\sin(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a+a*sin(x)),x)
```

```
[Out] Integral(csc(x)**2/(sin(x) + 1), x)/a
```

3.10 $\int \frac{\csc^3(x)}{a+a\sin(x)}\,dx$

Optimal. Leaf size=42

$$\frac{2\cot(x)}{a}-\frac{3\tanh^{-1}(\cos(x))}{2a}-\frac{3\cot(x)\csc(x)}{2a}+\frac{\cot(x)\csc(x)}{a\sin(x)+a}$$

```
[Out] -3/2*arctanh(cos(x))/a+2*cot(x)/a-3/2*cot(x)*csc(x)/a+cot(x)*csc(x)/(a+a*si
n(x))
```

Rubi [A] time = 0.07, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2768, 2748, 3768, 3770, 3767, 8}

$$\frac{2\cot(x)}{a}-\frac{3\tanh^{-1}(\cos(x))}{2a}-\frac{3\cot(x)\csc(x)}{2a}+\frac{\cot(x)\csc(x)}{a\sin(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a + a*Sin[x]),x]
```

```
[Out] (-3*ArcTanh[Cos[x]])/(2*a) + (2*Cot[x])/a - (3*Cot[x]*Csc[x])/(2*a) + (Cot[
x]*Csc[x])/(a + a*Sin[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^3(x)}{a+a\sin(x)}\,dx &= \frac{\cot(x)\csc(x)}{a+a\sin(x)}-\frac{\int \csc^3(x)(-3a+2a\sin(x))\,dx}{a^2}\\
&= \frac{\cot(x)\csc(x)}{a+a\sin(x)}-\frac{2\int \csc^2(x)\,dx}{a}+\frac{3\int \csc^3(x)\,dx}{a}\\
&= -\frac{3\cot(x)\csc(x)}{2a}+\frac{\cot(x)\csc(x)}{a+a\sin(x)}+\frac{3\int \csc(x)\,dx}{2a}+\frac{2\,\mathrm{Subst}\left(\int 1\,dx,x,\cot(x)\right)}{a}\\
&= -\frac{3\tanh^{-1}(\cos(x))}{2a}+\frac{2\cot(x)}{a}-\frac{3\cot(x)\csc(x)}{2a}+\frac{\cot(x)\csc(x)}{a+a\sin(x)}
\end{aligned}
$$

Mathematica [A] time = 0.35, size = 83, normalized size = 1.98

$$
\frac{-4\tan\left(\frac{x}{2}\right)+4\cot\left(\frac{x}{2}\right)-\csc^2\left(\frac{x}{2}\right)+\sec^2\left(\frac{x}{2}\right)+12\log\left(\sin\left(\frac{x}{2}\right)\right)-12\log\left(\cos\left(\frac{x}{2}\right)\right)-\frac{16\sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)}}{8a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a + a*Sin[x]),x]
```

```
[Out] (4*Cot[x/2] - Csc[x/2]^2 - 12*Log[Cos[x/2]] + 12*Log[Sin[x/2]] + Sec[x/2]^2
 - (16*Sin[x/2])/(Cos[x/2] + Sin[x/2]) - 4*Tan[x/2])/(8*a)
```

fricas [B] time = 0.56, size = 134, normalized size = 3.19

$$\frac{8\cos(x)^3+6\cos(x)^2-3\left(\cos(x)^3+\cos(x)^2+\left(\cos(x)^2-1\right)\sin(x)-\cos(x)-1\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+3\left(\cos(x)^3\right.}{4\left(a\cos(x)^3+a\cos(x)^2-a\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/4*(8*cos(x)^3 + 6*cos(x)^2 - 3*(cos(x)^3 + cos(x)^2 + (cos(x)^2 - 1)*sin(
x) - cos(x) - 1)*log(1/2*cos(x) + 1/2) + 3*(cos(x)^3 + cos(x)^2 + (cos(x)^2
 - 1)*sin(x) - cos(x) - 1)*log(-1/2*cos(x) + 1/2) - 2*(4*cos(x)^2 + cos(x)
- 2)*sin(x) - 6*cos(x) - 4)/(a*cos(x)^3 + a*cos(x)^2 - a*cos(x) + (a*cos(x)
^2 - a)*sin(x) - a)
```

giac [A] time = 0.31, size = 73, normalized size = 1.74

$$\frac{3\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{2a}+\frac{a\tan\left(\frac{1}{2}x\right)^2-4a\tan\left(\frac{1}{2}x\right)}{8a^2}+\frac{2}{a\left(\tan\left(\frac{1}{2}x\right)+1\right)}-\frac{18\tan\left(\frac{1}{2}x\right)^2-4\tan\left(\frac{1}{2}x\right)+1}{8a\tan\left(\frac{1}{2}x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*sin(x)),x, algorithm="giac")
```

```
[Out] 3/2*log(abs(tan(1/2*x)))/a + 1/8*(a*tan(1/2*x)^2 - 4*a*tan(1/2*x))/a^2 + 2/
(a*(tan(1/2*x) + 1)) - 1/8*(18*tan(1/2*x)^2 - 4*tan(1/2*x) + 1)/(a*tan(1/2*
x)^2)
```

maple [A] time = 0.10, size = 67, normalized size = 1.60

$$\frac{\tan^2\left(\frac{x}{2}\right)}{8a}-\frac{\tan\left(\frac{x}{2}\right)}{2a}-\frac{1}{8a\tan\left(\frac{x}{2}\right)^2}+\frac{1}{2a\tan\left(\frac{x}{2}\right)}+\frac{3\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a}+\frac{2}{a\left(\tan\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a+a*sin(x)),x)
```

```
[Out] 1/8/a*tan(1/2*x)^2-1/2/a*tan(1/2*x)-1/8/a/tan(1/2*x)^2+1/2/a/tan(1/2*x)+3/2
/a*ln(tan(1/2*x))+2/a/(tan(1/2*x)+1)
```

maxima [B] time = 0.41, size = 97, normalized size = 2.31

$$-\frac{\frac{4\sin(x)}{\cos(x)+1}-\frac{\sin(x)^2}{(\cos(x)+1)^2}}{8a}+\frac{\frac{3\sin(x)}{\cos(x)+1}+\frac{20\sin(x)^2}{(\cos(x)+1)^2}-1}{8\left(\frac{a\sin(x)^2}{(\cos(x)+1)^2}+\frac{a\sin(x)^3}{(\cos(x)+1)^3}\right)}+\frac{3\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*sin(x)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*sin(x)/(cos(x) + 1) - sin(x)^2/(cos(x) + 1)^2)/a + 1/8*(3*sin(x)/(c
os(x) + 1) + 20*sin(x)^2/(cos(x) + 1)^2 - 1)/(a*sin(x)^2/(cos(x) + 1)^2 + a
*sin(x)^3/(cos(x) + 1)^3) + 3/2*log(sin(x)/(cos(x) + 1))/a
```

mupad [B] time = 6.60, size = 69, normalized size = 1.64

$$\frac{10\tan\left(\frac{x}{2}\right)^2+\frac{3\tan\left(\frac{x}{2}\right)}{2}-\frac{1}{2}}{4\,a\tan\left(\frac{x}{2}\right)^3+4\,a\tan\left(\frac{x}{2}\right)^2}-\frac{\tan\left(\frac{x}{2}\right)}{2\,a}+\frac{\tan\left(\frac{x}{2}\right)^2}{8\,a}+\frac{3\,\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^3*(a + a*sin(x))),x)
```

```
[Out] ((3*tan(x/2))/2 + 10*tan(x/2)^2 - 1/2)/(4*a*tan(x/2)^2 + 4*a*tan(x/2)^3) -
tan(x/2)/(2*a) + tan(x/2)^2/(8*a) + (3*log(tan(x/2)))/(2*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^3(x)}{\sin(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a+a*sin(x)),x)
```

```
[Out] Integral(csc(x)**3/(sin(x) + 1), x)/a
```

3.11 $\int \frac{\csc^4(x)}{a+a\sin(x)}\,dx$

Optimal. Leaf size=55

$$-\frac{4\cot^3(x)}{3a}-\frac{4\cot(x)}{a}+\frac{3\tanh^{-1}(\cos(x))}{2a}+\frac{3\cot(x)\csc(x)}{2a}+\frac{\cot(x)\csc^2(x)}{a\sin(x)+a}$$

```
[Out] 3/2*arctanh(cos(x))/a-4*cot(x)/a-4/3*cot(x)^3/a+3/2*cot(x)*csc(x)/a+cot(x)*
csc(x)^2/(a+a*sin(x))
```

Rubi [A] time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2768, 2748, 3767, 3768, 3770}

$$-\frac{4\cot^3(x)}{3a}-\frac{4\cot(x)}{a}+\frac{3\tanh^{-1}(\cos(x))}{2a}+\frac{3\cot(x)\csc(x)}{2a}+\frac{\cot(x)\csc^2(x)}{a\sin(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^4/(a + a*Sin[x]),x]
```

```
[Out] (3*ArcTanh[Cos[x]])/(2*a) - (4*Cot[x])/a - (4*Cot[x]^3)/(3*a) + (3*Cot[x]*C
sc[x])/(2*a) + (Cot[x]*Csc[x]^2)/(a + a*Sin[x])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^4(x)}{a+a\sin(x)}\,dx &= \frac{\cot(x)\csc^2(x)}{a+a\sin(x)}-\frac{\int \csc^4(x)(-4a+3a\sin(x))\,dx}{a^2}\\ &= \frac{\cot(x)\csc^2(x)}{a+a\sin(x)}-\frac{3\int \csc^3(x)\,dx}{a}+\frac{4\int \csc^4(x)\,dx}{a}\\ &= \frac{3\cot(x)\csc(x)}{2a}+\frac{\cot(x)\csc^2(x)}{a+a\sin(x)}-\frac{3\int \csc(x)\,dx}{2a}-\frac{4\,\mathrm{Subst}\left(\int \left(1+x^2\right)\,dx,x,\cot(x)\right)}{a}\\ &= \frac{3\tanh^{-1}(\cos(x))}{2a}-\frac{4\cot(x)}{a}-\frac{4\cot^3(x)}{3a}+\frac{3\cot(x)\csc(x)}{2a}+\frac{\cot(x)\csc^2(x)}{a+a\sin(x)}\end{aligned}$$

Mathematica [B] time = 0.81, size = 113, normalized size = 2.05

$$\frac{20\tan\left(\frac{x}{2}\right)-20\cot\left(\frac{x}{2}\right)+3\csc^2\left(\frac{x}{2}\right)-3\sec^2\left(\frac{x}{2}\right)-36\log\left(\sin\left(\frac{x}{2}\right)\right)+36\log\left(\cos\left(\frac{x}{2}\right)\right)+\frac{48\sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)}-\frac{1}{2}\sin(x)}{24a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^4/(a + a*Sin[x]),x]
```

```
[Out] (-20*Cot[x/2] + 3*Csc[x/2]^2 + 36*Log[Cos[x/2]] - 36*Log[Sin[x/2]] - 3*Sec[
x/2]^2 + 8*Csc[x]^3*Sin[x/2]^4 + (48*Sin[x/2])/(Cos[x/2] + Sin[x/2]) - (Csc
[x/2]^4*Sin[x])/2 + 20*Tan[x/2])/(24*a)
```

fricas [B] time = 0.46, size = 168, normalized size = 3.05

$$\frac{32\,\cos(x)^4+14\,\cos(x)^3-48\,\cos(x)^2+9\left(\cos(x)^4-2\,\cos(x)^2-\left(\cos(x)^3+\cos(x)^2-\cos(x)-1\right)\sin(x)+1\right)\mathrm{lo}}{12\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/12*(32*cos(x)^4 + 14*cos(x)^3 - 48*cos(x)^2 + 9*(cos(x)^4 - 2*cos(x)^2 -
(cos(x)^3 + cos(x)^2 - cos(x) - 1)*sin(x) + 1)*log(1/2*cos(x) + 1/2) - 9*(c
os(x)^4 - 2*cos(x)^2 - (cos(x)^3 + cos(x)^2 - cos(x) - 1)*sin(x) + 1)*log(-
1/2*cos(x) + 1/2) + 2*(16*cos(x)^3 + 9*cos(x)^2 - 15*cos(x) - 6)*sin(x) - 1
8*cos(x) + 12)/(a*cos(x)^4 - 2*a*cos(x)^2 - (a*cos(x)^3 + a*cos(x)^2 - a*co
s(x) - a)*sin(x) + a)
```

giac [A] time = 0.31, size = 96, normalized size = 1.75

$$-\frac{3\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{2\,a}+\frac{a^{2}\tan\left(\frac{1}{2}\,x\right)^{3}-3\,a^{2}\tan\left(\frac{1}{2}\,x\right)^{2}+21\,a^{2}\tan\left(\frac{1}{2}\,x\right)}{24\,a^{3}}-\frac{2}{a\left(\tan\left(\frac{1}{2}\,x\right)+1\right)}+\frac{66\,\tan\left(\frac{1}{2}\,x\right)^{3}-21\,\tan}{24\,a\,\mathrm{ta}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*sin(x)),x, algorithm="giac")
```

```
[Out] -3/2*log(abs(tan(1/2*x)))/a + 1/24*(a^2*tan(1/2*x)^3 - 3*a^2*tan(1/2*x)^2 +
 21*a^2*tan(1/2*x))/a^3 - 2/(a*(tan(1/2*x) + 1)) + 1/24*(66*tan(1/2*x)^3 -
21*tan(1/2*x)^2 + 3*tan(1/2*x) - 1)/(a*tan(1/2*x)^3)
```

maple [A] time = 0.10, size = 89, normalized size = 1.62

$$\frac{\tan^{3}\left(\frac{x}{2}\right)}{24a}-\frac{\tan^{2}\left(\frac{x}{2}\right)}{8a}+\frac{7\tan\left(\frac{x}{2}\right)}{8a}-\frac{1}{24a\tan\left(\frac{x}{2}\right)^{3}}+\frac{1}{8a\tan\left(\frac{x}{2}\right)^{2}}-\frac{7}{8a\tan\left(\frac{x}{2}\right)}-\frac{3\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a}-\frac{2}{a\left(\tan\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^4/(a+a*sin(x)),x)
```

```
[Out] 1/24/a*tan(1/2*x)^3-1/8/a*tan(1/2*x)^2+7/8/a*tan(1/2*x)-1/24/a/tan(1/2*x)^3
+1/8/a/tan(1/2*x)^2-7/8/a/tan(1/2*x)-3/2/a*ln(tan(1/2*x))-2/a/(tan(1/2*x)+1
)
```

maxima [B] time = 0.57, size = 120, normalized size = 2.18

$$\frac{\frac{21\,\sin(x)}{\cos(x)+1}-\frac{3\,\sin(x)^{2}}{(\cos(x)+1)^{2}}+\frac{\sin(x)^{3}}{(\cos(x)+1)^{3}}}{24\,a}+\frac{\frac{2\,\sin(x)}{\cos(x)+1}-\frac{18\,\sin(x)^{2}}{(\cos(x)+1)^{2}}-\frac{69\,\sin(x)^{3}}{(\cos(x)+1)^{3}}-1}{24\left(\frac{a\sin(x)^{3}}{(\cos(x)+1)^{3}}+\frac{a\sin(x)^{4}}{(\cos(x)+1)^{4}}\right)}-\frac{3\,\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/24*(21*sin(x)/(cos(x) + 1) - 3*sin(x)^2/(cos(x) + 1)^2 + sin(x)^3/(cos(x)
 + 1)^3)/a + 1/24*(2*sin(x)/(cos(x) + 1) - 18*sin(x)^2/(cos(x) + 1)^2 - 69*
sin(x)^3/(cos(x) + 1)^3 - 1)/(a*sin(x)^3/(cos(x) + 1)^3 + a*sin(x)^4/(cos(x
) + 1)^4) - 3/2*log(sin(x)/(cos(x) + 1))/a
```

mupad [B] time = 6.50, size = 89, normalized size = 1.62

$$\frac{7\tan\left(\frac{x}{2}\right)}{8\,a}-\frac{23\tan\left(\frac{x}{2}\right)^3+6\tan\left(\frac{x}{2}\right)^2-\frac{2\tan\left(\frac{x}{2}\right)}{3}+\frac{1}{3}}{8\,a\tan\left(\frac{x}{2}\right)^4+8\,a\tan\left(\frac{x}{2}\right)^3}-\frac{\tan\left(\frac{x}{2}\right)^2}{8\,a}+\frac{\tan\left(\frac{x}{2}\right)^3}{24\,a}-\frac{3\,\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^4*(a + a*sin(x))),x)
```

```
[Out] (7*tan(x/2))/(8*a) - (6*tan(x/2)^2 - (2*tan(x/2))/3 + 23*tan(x/2)^3 + 1/3)/
(8*a*tan(x/2)^3 + 8*a*tan(x/2)^4) - tan(x/2)^2/(8*a) + tan(x/2)^3/(24*a) -
(3*log(tan(x/2)))/(2*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^4(x)}{\sin(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**4/(a+a*sin(x)),x)
```

```
[Out] Integral(csc(x)**4/(sin(x) + 1), x)/a
```

3.12 $\int \frac{\sin^4(x)}{(a+a\sin(x))^2}\,dx$

Optimal. Leaf size=66

$$\frac{7x}{2a^2}+\frac{16\cos(x)}{3a^2}+\frac{8\sin^2(x)\cos(x)}{3a^2(\sin(x)+1)}-\frac{7\sin(x)\cos(x)}{2a^2}+\frac{\sin^3(x)\cos(x)}{3(a\sin(x)+a)^2}$$

```
[Out] 7/2*x/a^2+16/3*cos(x)/a^2-7/2*cos(x)*sin(x)/a^2+8/3*cos(x)*sin(x)^2/a^2/(1+
sin(x))+1/3*cos(x)*sin(x)^3/(a+a*sin(x))^2
```

Rubi [A] time = 0.12, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2765, 2977, 2734}

$$\frac{7x}{2a^2}+\frac{16\cos(x)}{3a^2}+\frac{8\sin^2(x)\cos(x)}{3a^2(\sin(x)+1)}-\frac{7\sin(x)\cos(x)}{2a^2}+\frac{\sin^3(x)\cos(x)}{3(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4/(a + a*Sin[x])^2,x]
```

```
[Out] (7*x)/(2*a^2) + (16*Cos[x])/(3*a^2) - (7*Cos[x]*Sin[x])/(2*a^2) + (8*Cos[x]
*Sin[x]^2)/(3*a^2*(1 + Sin[x])) + (Cos[x]*Sin[x]^3)/(3*(a + a*Sin[x])^2)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^4(x)}{(a+a\sin(x))^2}\,dx &= \frac{\cos(x)\sin^3(x)}{3(a+a\sin(x))^2} - \frac{\int \frac{\sin^2(x)(3a-5a\sin(x))}{a+a\sin(x)}\,dx}{3a^2} \\
&= \frac{8\cos(x)\sin^2(x)}{3a^2(1+\sin(x))} + \frac{\cos(x)\sin^3(x)}{3(a+a\sin(x))^2} - \frac{\int \sin(x)\left(16a^2-21a^2\sin(x)\right)\,dx}{3a^4} \\
&= \frac{7x}{2a^2} + \frac{16\cos(x)}{3a^2} - \frac{7\cos(x)\sin(x)}{2a^2} + \frac{8\cos(x)\sin^2(x)}{3a^2(1+\sin(x))} + \frac{\cos(x)\sin^3(x)}{3(a+a\sin(x))^2}
\end{aligned}
$$

Mathematica [A] time = 0.25, size = 100, normalized size = 1.52

$$
\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(21(12x-7)\cos\left(\frac{x}{2}\right)+(239-84x)\cos\left(\frac{3x}{2}\right)+3\left(-5\cos\left(\frac{5x}{2}\right)+\cos\left(\frac{7x}{2}\right)+2\sin\left(\frac{x}{2}\right)(56x+\cdots\right.\right.}{48a^2(\sin(x)+1)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4/(a + a*Sin[x])^2,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(21*(-7 + 12*x)*Cos[x/2] + (239 - 84*x)*Cos[(3*x)/2]
 + 3*(-5*Cos[(5*x)/2] + Cos[(7*x)/2] + 2*(-50 + 56*x + (27 + 28*x)*Cos[x] +
 6*Cos[2*x] + Cos[3*x])*Sin[x/2])))/(48*a^2*(1 + Sin[x])^2)
```

fricas [A] time = 0.44, size = 105, normalized size = 1.59

$$
-\frac{3\cos(x)^4-(21x-31)\cos(x)^2-6\cos(x)^3+(21x+38)\cos(x)+\left(3\cos(x)^3+(21x+40)\cos(x)+9\cos(x)^2\cdots\right.}{6\left(a^2\cos(x)^2-a^2\cos(x)-2a^2-\left(a^2\cos(x)+2a^2\right)\sin(x)\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+a*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*cos(x)^4 - (21*x - 31)*cos(x)^2 - 6*cos(x)^3 + (21*x + 38)*cos(x) +
 (3*cos(x)^3 + (21*x + 40)*cos(x) + 9*cos(x)^2 + 42*x + 2)*sin(x) + 42*x -
2)/(a^2*cos(x)^2 - a^2*cos(x) - 2*a^2 - (a^2*cos(x) + 2*a^2)*sin(x))
```

giac [A] time = 0.33, size = 72, normalized size = 1.09

$$\frac{7\,x}{2\,a^2} + \frac{\tan\left(\frac{1}{2}\,x\right)^3 + 4\,\tan\left(\frac{1}{2}\,x\right)^2 - \tan\left(\frac{1}{2}\,x\right) + 4}{\left(\tan\left(\frac{1}{2}\,x\right)^2 + 1\right)^2 a^2} + \frac{2\left(9\,\tan\left(\frac{1}{2}\,x\right)^2 + 21\,\tan\left(\frac{1}{2}\,x\right) + 10\right)}{3\,a^2\left(\tan\left(\frac{1}{2}\,x\right) + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+a*sin(x))^2,x, algorithm="giac")
```

```
[Out] 7/2*x/a^2 + (tan(1/2*x)^3 + 4*tan(1/2*x)^2 - tan(1/2*x) + 4)/((tan(1/2*x)^2
 + 1)^2*a^2) + 2/3*(9*tan(1/2*x)^2 + 21*tan(1/2*x) + 10)/(a^2*(tan(1/2*x) +
 1)^3)
```

maple [B] time = 0.10, size = 126, normalized size = 1.91

$$\frac{\tan^3\left(\frac{x}{2}\right)}{a^2\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}+\frac{4\left(\tan^2\left(\frac{x}{2}\right)\right)}{a^2\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}-\frac{\tan\left(\frac{x}{2}\right)}{a^2\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}+\frac{4}{a^2\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}+\frac{7\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}-\frac{4}{3a^2\left(\tan\left(\frac{x}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a+a*sin(x))^2,x)
```

```
[Out] 1/a^2/(tan(1/2*x)^2+1)^2*tan(1/2*x)^3+4/a^2/(tan(1/2*x)^2+1)^2*tan(1/2*x)^2
-1/a^2/(tan(1/2*x)^2+1)^2*tan(1/2*x)+4/a^2/(tan(1/2*x)^2+1)^2+7/a^2*arctan(
tan(1/2*x))-4/3/a^2/(tan(1/2*x)+1)^3+2/a^2/(tan(1/2*x)+1)^2+6/a^2/(tan(1/2*
x)+1)
```

maxima [B] time = 1.24, size = 198, normalized size = 3.00

$$\frac{\frac{75\,\sin(x)}{\cos(x)+1} + \frac{97\,\sin(x)^2}{(\cos(x)+1)^2} + \frac{126\,\sin(x)^3}{(\cos(x)+1)^3} + \frac{98\,\sin(x)^4}{(\cos(x)+1)^4} + \frac{63\,\sin(x)^5}{(\cos(x)+1)^5} + \frac{21\,\sin(x)^6}{(\cos(x)+1)^6} + 32}{3\left(a^2 + \frac{3\,a^2\sin(x)}{\cos(x)+1} + \frac{5\,a^2\sin(x)^2}{(\cos(x)+1)^2} + \frac{7\,a^2\sin(x)^3}{(\cos(x)+1)^3} + \frac{7\,a^2\sin(x)^4}{(\cos(x)+1)^4} + \frac{5\,a^2\sin(x)^5}{(\cos(x)+1)^5} + \frac{3\,a^2\sin(x)^6}{(\cos(x)+1)^6} + \frac{a^2\sin(x)^7}{(\cos(x)+1)^7}\right)} + \frac{7\,\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+a*sin(x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(75*sin(x)/(cos(x) + 1) + 97*sin(x)^2/(cos(x) + 1)^2 + 126*sin(x)^3/(co
s(x) + 1)^3 + 98*sin(x)^4/(cos(x) + 1)^4 + 63*sin(x)^5/(cos(x) + 1)^5 + 21*
```

```
sin(x)^6/(cos(x) + 1)^6 + 32)/(a^2 + 3*a^2*sin(x)/(cos(x) + 1) + 5*a^2*sin(
x)^2/(cos(x) + 1)^2 + 7*a^2*sin(x)^3/(cos(x) + 1)^3 + 7*a^2*sin(x)^4/(cos(x
) + 1)^4 + 5*a^2*sin(x)^5/(cos(x) + 1)^5 + 3*a^2*sin(x)^6/(cos(x) + 1)^6 +
a^2*sin(x)^7/(cos(x) + 1)^7) + 7*arctan(sin(x)/(cos(x) + 1))/a^2
```

mupad [B] time = 6.81, size = 77, normalized size = 1.17

$$\frac{7\,x}{2\,a^2}+\frac{7\tan\left(\frac{x}{2}\right)^6+21\tan\left(\frac{x}{2}\right)^5+\frac{98\tan\left(\frac{x}{2}\right)^4}{3}+42\tan\left(\frac{x}{2}\right)^3+\frac{97\tan\left(\frac{x}{2}\right)^2}{3}+25\tan\left(\frac{x}{2}\right)+\frac{32}{3}}{a^2\left(\tan\left(\frac{x}{2}\right)^2+1\right)^2\left(\tan\left(\frac{x}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a + a*sin(x))^2,x)
```

```
[Out] (7*x)/(2*a^2) + (25*tan(x/2) + (97*tan(x/2)^2)/3 + 42*tan(x/2)^3 + (98*tan(
x/2)^4)/3 + 21*tan(x/2)^5 + 7*tan(x/2)^6 + 32/3)/(a^2*(tan(x/2)^2 + 1)^2*(t
an(x/2) + 1)^3)
```

sympy [B] time = 10.92, size = 1423, normalized size = 21.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4/(a+a*sin(x))**2,x)
```

```
[Out] 21*x*tan(x/2)**7/(6*a**2*tan(x/2)**7 + 18*a**2*tan(x/2)**6 + 30*a**2*tan(x/
2)**5 + 42*a**2*tan(x/2)**4 + 42*a**2*tan(x/2)**3 + 30*a**2*tan(x/2)**2 + 1
8*a**2*tan(x/2) + 6*a**2) + 63*x*tan(x/2)**6/(6*a**2*tan(x/2)**7 + 18*a**2*
tan(x/2)**6 + 30*a**2*tan(x/2)**5 + 42*a**2*tan(x/2)**4 + 42*a**2*tan(x/2)*
*3 + 30*a**2*tan(x/2)**2 + 18*a**2*tan(x/2) + 6*a**2) + 105*x*tan(x/2)**5/(
6*a**2*tan(x/2)**7 + 18*a**2*tan(x/2)**6 + 30*a**2*tan(x/2)**5 + 42*a**2*ta
n(x/2)**4 + 42*a**2*tan(x/2)**3 + 30*a**2*tan(x/2)**2 + 18*a**2*tan(x/2) +
6*a**2) + 147*x*tan(x/2)**4/(6*a**2*tan(x/2)**7 + 18*a**2*tan(x/2)**6 + 30*
a**2*tan(x/2)**5 + 42*a**2*tan(x/2)**4 + 42*a**2*tan(x/2)**3 + 30*a**2*tan(
x/2)**2 + 18*a**2*tan(x/2) + 6*a**2) + 147*x*tan(x/2)**3/(6*a**2*tan(x/2)**
7 + 18*a**2*tan(x/2)**6 + 30*a**2*tan(x/2)**5 + 42*a**2*tan(x/2)**4 + 42*a*
*2*tan(x/2)**3 + 30*a**2*tan(x/2)**2 + 18*a**2*tan(x/2) + 6*a**2) + 105*x*t
an(x/2)**2/(6*a**2*tan(x/2)**7 + 18*a**2*tan(x/2)**6 + 30*a**2*tan(x/2)**5
+ 42*a**2*tan(x/2)**4 + 42*a**2*tan(x/2)**3 + 30*a**2*tan(x/2)**2 + 18*a**2
*tan(x/2) + 6*a**2) + 63*x*tan(x/2)/(6*a**2*tan(x/2)**7 + 18*a**2*tan(x/2)*
*6 + 30*a**2*tan(x/2)**5 + 42*a**2*tan(x/2)**4 + 42*a**2*tan(x/2)**3 + 30*a
**2*tan(x/2)**2 + 18*a**2*tan(x/2) + 6*a**2) + 21*x/(6*a**2*tan(x/2)**7 + 1
8*a**2*tan(x/2)**6 + 30*a**2*tan(x/2)**5 + 42*a**2*tan(x/2)**4 + 42*a**2*ta
```

```
n(x/2)**3 + 30*a**2*tan(x/2)**2 + 18*a**2*tan(x/2) + 6*a**2) + 42*tan(x/2)*
*6/(6*a**2*tan(x/2)**7 + 18*a**2*tan(x/2)**6 + 30*a**2*tan(x/2)**5 + 42*a**
2*tan(x/2)**4 + 42*a**2*tan(x/2)**3 + 30*a**2*tan(x/2)**2 + 18*a**2*tan(x/2
) + 6*a**2) + 126*tan(x/2)**5/(6*a**2*tan(x/2)**7 + 18*a**2*tan(x/2)**6 + 3
0*a**2*tan(x/2)**5 + 42*a**2*tan(x/2)**4 + 42*a**2*tan(x/2)**3 + 30*a**2*ta
n(x/2)**2 + 18*a**2*tan(x/2) + 6*a**2) + 196*tan(x/2)**4/(6*a**2*tan(x/2)**
7 + 18*a**2*tan(x/2)**6 + 30*a**2*tan(x/2)**5 + 42*a**2*tan(x/2)**4 + 42*a*
*2*tan(x/2)**3 + 30*a**2*tan(x/2)**2 + 18*a**2*tan(x/2) + 6*a**2) + 252*tan
(x/2)**3/(6*a**2*tan(x/2)**7 + 18*a**2*tan(x/2)**6 + 30*a**2*tan(x/2)**5 +
42*a**2*tan(x/2)**4 + 42*a**2*tan(x/2)**3 + 30*a**2*tan(x/2)**2 + 18*a**2*t
an(x/2) + 6*a**2) + 194*tan(x/2)**2/(6*a**2*tan(x/2)**7 + 18*a**2*tan(x/2)*
*6 + 30*a**2*tan(x/2)**5 + 42*a**2*tan(x/2)**4 + 42*a**2*tan(x/2)**3 + 30*a
**2*tan(x/2)**2 + 18*a**2*tan(x/2) + 6*a**2) + 150*tan(x/2)/(6*a**2*tan(x/2
)**7 + 18*a**2*tan(x/2)**6 + 30*a**2*tan(x/2)**5 + 42*a**2*tan(x/2)**4 + 42
*a**2*tan(x/2)**3 + 30*a**2*tan(x/2)**2 + 18*a**2*tan(x/2) + 6*a**2) + 64/(
6*a**2*tan(x/2)**7 + 18*a**2*tan(x/2)**6 + 30*a**2*tan(x/2)**5 + 42*a**2*ta
n(x/2)**4 + 42*a**2*tan(x/2)**3 + 30*a**2*tan(x/2)**2 + 18*a**2*tan(x/2) +
6*a**2)
```

3.13 $\int \frac{\sin^3(x)}{(a+a\sin(x))^2}\,dx$

Optimal. Leaf size=47

$$-\frac{2x}{a^2}-\frac{4\cos(x)}{3a^2}-\frac{2\cos(x)}{a^2(\sin(x)+1)}+\frac{\sin^2(x)\cos(x)}{3(a\sin(x)+a)^2}$$

```
[Out] -2*x/a^2-4/3*cos(x)/a^2-2*cos(x)/a^2/(1+sin(x))+1/3*cos(x)*sin(x)^2/(a+a*si
n(x))^2
```

Rubi [A] time = 0.14, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2765, 2968, 3023, 12, 2735, 2648}

$$-\frac{2x}{a^2}-\frac{4\cos(x)}{3a^2}-\frac{2\cos(x)}{a^2(\sin(x)+1)}+\frac{\sin^2(x)\cos(x)}{3(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a + a*Sin[x])^2,x]
```

```
[Out] (-2*x)/a^2 - (4*Cos[x])/(3*a^2) - (2*Cos[x])/(a^2*(1 + Sin[x])) + (Cos[x]*S
in[x]^2)/(3*(a + a*Sin[x])^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
```

```
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^3(x)}{(a+a\sin(x))^2}\,dx &= \frac{\cos(x)\sin^2(x)}{3(a+a\sin(x))^2} - \frac{\int \frac{\sin(x)(2a-4a\sin(x))}{a+a\sin(x)}\,dx}{3a^2}\\
&= \frac{\cos(x)\sin^2(x)}{3(a+a\sin(x))^2} - \frac{\int \frac{2a\sin(x)-4a\sin^2(x)}{a+a\sin(x)}\,dx}{3a^2}\\
&= -\frac{4\cos(x)}{3a^2} + \frac{\cos(x)\sin^2(x)}{3(a+a\sin(x))^2} - \frac{\int \frac{6a^2\sin(x)}{a+a\sin(x)}\,dx}{3a^3}\\
&= -\frac{4\cos(x)}{3a^2} + \frac{\cos(x)\sin^2(x)}{3(a+a\sin(x))^2} - \frac{2\int \frac{\sin(x)}{a+a\sin(x)}\,dx}{a}\\
&= -\frac{2x}{a^2} - \frac{4\cos(x)}{3a^2} + \frac{\cos(x)\sin^2(x)}{3(a+a\sin(x))^2} + \frac{2\int \frac{1}{a+a\sin(x)}\,dx}{a}\\
&= -\frac{2x}{a^2} - \frac{4\cos(x)}{3a^2} + \frac{\cos(x)\sin^2(x)}{3(a+a\sin(x))^2} - \frac{2\cos(x)}{a^2+a^2\sin(x)}
\end{aligned}
$$

Mathematica [A] time = 0.24, size = 84, normalized size = 1.79

$$-\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(6(6x-5)\cos\left(\frac{x}{2}\right)+(41-12x)\cos\left(\frac{3x}{2}\right)-3\cos\left(\frac{5x}{2}\right)+6\sin\left(\frac{x}{2}\right)(8x+4(x+1)\cos(x)+\cos\ldots}{12a^2(\sin(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a + a*Sin[x])^2,x]
```

```
[Out] -1/12*((Cos[x/2] + Sin[x/2])*(6*(-5 + 6*x)*Cos[x/2] + (41 - 12*x)*Cos[(3*x)
/2] - 3*Cos[(5*x)/2] + 6*(-9 + 8*x + 4*(1 + x)*Cos[x] + Cos[2*x])*Sin[x/2])
)/(a^2*(1 + Sin[x])^2)
```

fricas [B] time = 0.46, size = 95, normalized size = 2.02

$$-\frac{(6\,x-11)\cos(x)^2+3\,\cos(x)^3-(6\,x+13)\cos(x)-\left(2\,(3\,x+7)\cos(x)+3\,\cos(x)^2+12\,x+1\right)\sin(x)-12\,x+\ldots}{3\left(a^2\cos(x)^2-a^2\cos(x)-2\,a^2-\left(a^2\cos(x)+2\,a^2\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/3*((6*x - 11)*cos(x)^2 + 3*cos(x)^3 - (6*x + 13)*cos(x) - (2*(3*x + 7)*c
os(x) + 3*cos(x)^2 + 12*x + 1)*sin(x) - 12*x + 1)/(a^2*cos(x)^2 - a^2*cos(x
) - 2*a^2 - (a^2*cos(x) + 2*a^2)*sin(x))
```

giac [A] time = 0.38, size = 51, normalized size = 1.09

$$-\frac{2\,x}{a^2}-\frac{2}{\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)a^2}-\frac{2\left(6\,\tan\left(\frac{1}{2}\,x\right)^2+15\,\tan\left(\frac{1}{2}\,x\right)+7\right)}{3\,a^2\left(\tan\left(\frac{1}{2}\,x\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*x/a^2 - 2/((tan(1/2*x)^2 + 1)*a^2) - 2/3*(6*tan(1/2*x)^2 + 15*tan(1/2*x)
 + 7)/(a^2*(tan(1/2*x) + 1)^3)
```

maple [A] time = 0.10, size = 66, normalized size = 1.40

$$-\frac{2}{a^2\left(\tan^2\left(\frac{x}{2}\right)+1\right)}-\frac{4\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}+\frac{4}{3a^2\left(\tan\left(\frac{x}{2}\right)+1\right)^3}-\frac{2}{a^2\left(\tan\left(\frac{x}{2}\right)+1\right)^2}-\frac{4}{a^2\left(\tan\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a+a*sin(x))^2,x)
```

```
[Out] -2/a^2/(tan(1/2*x)^2+1)-4/a^2*arctan(tan(1/2*x))+4/3/a^2/(tan(1/2*x)+1)^3-2
/a^2/(tan(1/2*x)+1)^2-4/a^2/(tan(1/2*x)+1)
```

maxima [B] time = 0.87, size = 144, normalized size = 3.06

$$-\frac{4\left(\frac{12\sin(x)}{\cos(x)+1}+\frac{11\sin(x)^2}{(\cos(x)+1)^2}+\frac{9\sin(x)^3}{(\cos(x)+1)^3}+\frac{3\sin(x)^4}{(\cos(x)+1)^4}+5\right)}{3\left(a^2+\frac{3a^2\sin(x)}{\cos(x)+1}+\frac{4a^2\sin(x)^2}{(\cos(x)+1)^2}+\frac{4a^2\sin(x)^3}{(\cos(x)+1)^3}+\frac{3a^2\sin(x)^4}{(\cos(x)+1)^4}+\frac{a^2\sin(x)^5}{(\cos(x)+1)^5}\right)}-\frac{4\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -4/3*(12*sin(x)/(cos(x) + 1) + 11*sin(x)^2/(cos(x) + 1)^2 + 9*sin(x)^3/(cos
(x) + 1)^3 + 3*sin(x)^4/(cos(x) + 1)^4 + 5)/(a^2 + 3*a^2*sin(x)/(cos(x) + 1
) + 4*a^2*sin(x)^2/(cos(x) + 1)^2 + 4*a^2*sin(x)^3/(cos(x) + 1)^3 + 3*a^2*s
in(x)^4/(cos(x) + 1)^4 + a^2*sin(x)^5/(cos(x) + 1)^5) - 4*arctan(sin(x)/(co
s(x) + 1))/a^2
```

mupad [B] time = 6.46, size = 62, normalized size = 1.32

$$-\frac{2x}{a^2}-\frac{4\tan\left(\frac{x}{2}\right)^4+12\tan\left(\frac{x}{2}\right)^3+\frac{44\tan\left(\frac{x}{2}\right)^2}{3}+16\tan\left(\frac{x}{2}\right)+\frac{20}{3}}{a^2\left(\tan\left(\frac{x}{2}\right)^2+1\right)\left(\tan\left(\frac{x}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a + a*sin(x))^2,x)
```

```
[Out] - (2*x)/a^2 - (16*tan(x/2) + (44*tan(x/2)^2)/3 + 12*tan(x/2)^3 + 4*tan(x/2)
^4 + 20/3)/(a^2*(tan(x/2)^2 + 1)*(tan(x/2) + 1)^3)
```

sympy [B] time = 6.79, size = 779, normalized size = 16.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a+a*sin(x))**2,x)
```

```
[Out] -6*x*tan(x/2)**5/(3*a**2*tan(x/2)**5 + 9*a**2*tan(x/2)**4 + 12*a**2*tan(x/2
)**3 + 12*a**2*tan(x/2)**2 + 9*a**2*tan(x/2) + 3*a**2) - 18*x*tan(x/2)**4/(
```

```
3*a**2*tan(x/2)**5 + 9*a**2*tan(x/2)**4 + 12*a**2*tan(x/2)**3 + 12*a**2*tan
(x/2)**2 + 9*a**2*tan(x/2) + 3*a**2) - 24*x*tan(x/2)**3/(3*a**2*tan(x/2)**5
 + 9*a**2*tan(x/2)**4 + 12*a**2*tan(x/2)**3 + 12*a**2*tan(x/2)**2 + 9*a**2*
tan(x/2) + 3*a**2) - 24*x*tan(x/2)**2/(3*a**2*tan(x/2)**5 + 9*a**2*tan(x/2)
**4 + 12*a**2*tan(x/2)**3 + 12*a**2*tan(x/2)**2 + 9*a**2*tan(x/2) + 3*a**2)
 - 18*x*tan(x/2)/(3*a**2*tan(x/2)**5 + 9*a**2*tan(x/2)**4 + 12*a**2*tan(x/2
)**3 + 12*a**2*tan(x/2)**2 + 9*a**2*tan(x/2) + 3*a**2) - 6*x/(3*a**2*tan(x/
2)**5 + 9*a**2*tan(x/2)**4 + 12*a**2*tan(x/2)**3 + 12*a**2*tan(x/2)**2 + 9*
a**2*tan(x/2) + 3*a**2) - 12*tan(x/2)**4/(3*a**2*tan(x/2)**5 + 9*a**2*tan(x
/2)**4 + 12*a**2*tan(x/2)**3 + 12*a**2*tan(x/2)**2 + 9*a**2*tan(x/2) + 3*a*
*2) - 36*tan(x/2)**3/(3*a**2*tan(x/2)**5 + 9*a**2*tan(x/2)**4 + 12*a**2*tan
(x/2)**3 + 12*a**2*tan(x/2)**2 + 9*a**2*tan(x/2) + 3*a**2) - 44*tan(x/2)**2
/(3*a**2*tan(x/2)**5 + 9*a**2*tan(x/2)**4 + 12*a**2*tan(x/2)**3 + 12*a**2*t
an(x/2)**2 + 9*a**2*tan(x/2) + 3*a**2) - 48*tan(x/2)/(3*a**2*tan(x/2)**5 +
9*a**2*tan(x/2)**4 + 12*a**2*tan(x/2)**3 + 12*a**2*tan(x/2)**2 + 9*a**2*tan
(x/2) + 3*a**2) - 20/(3*a**2*tan(x/2)**5 + 9*a**2*tan(x/2)**4 + 12*a**2*tan
(x/2)**3 + 12*a**2*tan(x/2)**2 + 9*a**2*tan(x/2) + 3*a**2)
```

3.14 $\int \frac{\sin^2(x)}{(a+a\sin(x))^2}\,dx$

Optimal. Leaf size=35

$$\frac{x}{a^2}+\frac{5\cos(x)}{3a^2(\sin(x)+1)}-\frac{\cos(x)}{3(a\sin(x)+a)^2}$$

```
[Out] x/a^2+5/3*cos(x)/a^2/(1+sin(x))-1/3*cos(x)/(a+a*sin(x))^2
```

Rubi [A] time = 0.07, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2758, 2735, 2648}

$$\frac{x}{a^2}+\frac{5\cos(x)}{3a^2(\sin(x)+1)}-\frac{\cos(x)}{3(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a + a*Sin[x])^2,x]
```

```
[Out] x/a^2 + (5*Cos[x])/(3*a^2*(1 + Sin[x])) - Cos[x]/(3*(a + a*Sin[x])^2)
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2758

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sin^2(x)}{(a+a\sin(x))^2}\,dx = -\frac{\cos(x)}{3(a+a\sin(x))^2} + \frac{\int \frac{-2a+3a\sin(x)}{a+a\sin(x)}\,dx}{3a^2}$$
$$= \frac{x}{a^2} - \frac{\cos(x)}{3(a+a\sin(x))^2} - \frac{5\int \frac{1}{a+a\sin(x)}\,dx}{3a}$$
$$= \frac{x}{a^2} - \frac{\cos(x)}{3(a+a\sin(x))^2} + \frac{5\cos(x)}{3\left(a^2+a^2\sin(x)\right)}$$

Mathematica [A] time = 0.13, size = 69, normalized size = 1.97

$$\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(3(3x-4)\cos\left(\frac{x}{2}\right)+(10-3x)\cos\left(\frac{3x}{2}\right)+6\sin\left(\frac{x}{2}\right)(2x+x\cos(x)-3)\right)}{6a^2(\sin(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a + a*Sin[x])^2,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(3*(-4 + 3*x)*Cos[x/2] + (10 - 3*x)*Cos[(3*x)/2] + 6
*(-3 + 2*x + x*Cos[x])*Sin[x/2]))/(6*a^2*(1 + Sin[x])^2)
```

fricas [B] time = 0.45, size = 82, normalized size = 2.34

$$\frac{(3x-5)\cos(x)^2-(3x+4)\cos(x)-((3x+5)\cos(x)+6x+1)\sin(x)-6x+1}{3\left(a^2\cos(x)^2-a^2\cos(x)-2a^2-\left(a^2\cos(x)+2a^2\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/3*((3*x - 5)*cos(x)^2 - (3*x + 4)*cos(x) - ((3*x + 5)*cos(x) + 6*x + 1)*s
in(x) - 6*x + 1)/(a^2*cos(x)^2 - a^2*cos(x) - 2*a^2 - (a^2*cos(x) + 2*a^2)*
sin(x))
```

giac [A] time = 0.83, size = 35, normalized size = 1.00

$$\frac{x}{a^2} + \frac{2\left(3\tan\left(\frac{1}{2}x\right)^2+9\tan\left(\frac{1}{2}x\right)+4\right)}{3a^2\left(\tan\left(\frac{1}{2}x\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*sin(x))^2,x, algorithm="giac")
```

```
[Out] x/a^2 + 2/3*(3*tan(1/2*x)^2 + 9*tan(1/2*x) + 4)/(a^2*(tan(1/2*x) + 1)^3)
```

maple [A] time = 0.10, size = 51, normalized size = 1.46

$$\frac{2\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}-\frac{4}{3a^2\left(\tan\left(\frac{x}{2}\right)+1\right)^3}+\frac{2}{a^2\left(\tan\left(\frac{x}{2}\right)+1\right)^2}+\frac{2}{a^2\left(\tan\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a+a*sin(x))^2,x)
```

```
[Out] 2/a^2*arctan(tan(1/2*x))-4/3/a^2/(tan(1/2*x)+1)^3+2/a^2/(tan(1/2*x)+1)^2+2/
a^2/(tan(1/2*x)+1)
```

maxima [B] time = 0.97, size = 90, normalized size = 2.57

$$\frac{2\left(\frac{9\,\sin(x)}{\cos(x)+1}+\frac{3\,\sin(x)^2}{(\cos(x)+1)^2}+4\right)}{3\left(a^2+\frac{3\,a^2\sin(x)}{\cos(x)+1}+\frac{3\,a^2\sin(x)^2}{(\cos(x)+1)^2}+\frac{a^2\sin(x)^3}{(\cos(x)+1)^3}\right)}+\frac{2\,\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*sin(x))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(9*sin(x)/(cos(x) + 1) + 3*sin(x)^2/(cos(x) + 1)^2 + 4)/(a^2 + 3*a^2*si
n(x)/(cos(x) + 1) + 3*a^2*sin(x)^2/(cos(x) + 1)^2 + a^2*sin(x)^3/(cos(x) +
1)^3) + 2*arctan(sin(x)/(cos(x) + 1))/a^2
```

mupad [B] time = 6.48, size = 34, normalized size = 0.97

$$\frac{x}{a^2}+\frac{2\tan\left(\frac{x}{2}\right)^2+6\tan\left(\frac{x}{2}\right)+\frac{8}{3}}{a^2\left(\tan\left(\frac{x}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a + a*sin(x))^2,x)
```

```
[Out] x/a^2 + (6*tan(x/2) + 2*tan(x/2)^2 + 8/3)/(a^2*(tan(x/2) + 1)^3)
```

sympy [B] time = 4.26, size = 321, normalized size = 9.17

$$\frac{3x\tan^3\left(\frac{x}{2}\right)}{3a^2\tan^3\left(\frac{x}{2}\right)+9a^2\tan^2\left(\frac{x}{2}\right)+9a^2\tan\left(\frac{x}{2}\right)+3a^2}+\frac{9x\tan^2\left(\frac{x}{2}\right)}{3a^2\tan^3\left(\frac{x}{2}\right)+9a^2\tan^2\left(\frac{x}{2}\right)+9a^2\tan\left(\frac{x}{2}\right)+3a^2}+\frac{}{3a^2\tan^3\left(\frac{x}{2}\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+a*sin(x))**2,x)
```

```
[Out] 3*x*tan(x/2)**3/(3*a**2*tan(x/2)**3 + 9*a**2*tan(x/2)**2 + 9*a**2*tan(x/2)
+ 3*a**2) + 9*x*tan(x/2)**2/(3*a**2*tan(x/2)**3 + 9*a**2*tan(x/2)**2 + 9*a*
*2*tan(x/2) + 3*a**2) + 9*x*tan(x/2)/(3*a**2*tan(x/2)**3 + 9*a**2*tan(x/2)*
*2 + 9*a**2*tan(x/2) + 3*a**2) + 3*x/(3*a**2*tan(x/2)**3 + 9*a**2*tan(x/2)*
*2 + 9*a**2*tan(x/2) + 3*a**2) + 6*tan(x/2)**2/(3*a**2*tan(x/2)**3 + 9*a**2
*tan(x/2)**2 + 9*a**2*tan(x/2) + 3*a**2) + 18*tan(x/2)/(3*a**2*tan(x/2)**3
+ 9*a**2*tan(x/2)**2 + 9*a**2*tan(x/2) + 3*a**2) + 8/(3*a**2*tan(x/2)**3 +
9*a**2*tan(x/2)**2 + 9*a**2*tan(x/2) + 3*a**2)
```

3.15 $\int \frac{\sin(x)}{(a+a\sin(x))^2}\,dx$

Optimal. Leaf size=33

$$\frac{\cos(x)}{3(a\sin(x)+a)^2}-\frac{2\cos(x)}{3\left(a^2\sin(x)+a^2\right)}$$

[Out] 1/3*cos(x)/(a+a*sin(x))^2-2/3*cos(x)/(a^2+a^2*sin(x))

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2750, 2648}

$$\frac{\cos(x)}{3(a\sin(x)+a)^2}-\frac{2\cos(x)}{3\left(a^2\sin(x)+a^2\right)}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]/(a + a*Sin[x])^2,x]

[Out] Cos[x]/(3*(a + a*Sin[x])^2) - (2*Cos[x])/(3*(a^2 + a^2*Sin[x]))

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{(a+a\sin(x))^2}\,dx &= \frac{\cos(x)}{3(a+a\sin(x))^2}+\frac{2\int \frac{1}{a+a\sin(x)}\,dx}{3a}\\ &= \frac{\cos(x)}{3(a+a\sin(x))^2}-\frac{2\cos(x)}{3\left(a^2+a^2\sin(x)\right)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 29, normalized size = 0.88

$$-\frac{-4\sin(x)+\sin(2x)+\cos(x)+\cos(2x)-3}{3a^2(\sin(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a + a*Sin[x])^2,x]
```

```
[Out] -1/3*(-3 + Cos[x] + Cos[2*x] - 4*Sin[x] + Sin[2*x])/(a^2*(1 + Sin[x])^2)
```

fricas [B] time = 0.45, size = 60, normalized size = 1.82

$$\frac{2\cos(x)^2+(2\cos(x)+1)\sin(x)+\cos(x)-1}{3\left(a^2\cos(x)^2-a^2\cos(x)-2a^2-\left(a^2\cos(x)+2a^2\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*cos(x)^2 + (2*cos(x) + 1)*sin(x) + cos(x) - 1)/(a^2*cos(x)^2 - a^2*c
os(x) - 2*a^2 - (a^2*cos(x) + 2*a^2)*sin(x))
```

giac [A] time = 1.95, size = 21, normalized size = 0.64

$$-\frac{2\left(3\tan\left(\frac{1}{2}x\right)+1\right)}{3a^2\left(\tan\left(\frac{1}{2}x\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*tan(1/2*x) + 1)/(a^2*(tan(1/2*x) + 1)^3)
```

maple [A] time = 0.09, size = 27, normalized size = 0.82

$$\frac{\frac{4}{3\left(\tan\left(\frac{x}{2}\right)+1\right)^3}-\frac{2}{\left(\tan\left(\frac{x}{2}\right)+1\right)^2}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a+a*sin(x))^2,x)
```

```
[Out] 4/a^2*(1/3/(tan(1/2*x)+1)^3-1/2/(tan(1/2*x)+1)^2)
```

maxima [B] time = 0.60, size = 62, normalized size = 1.88

$$-\frac{2\left(\frac{3\sin(x)}{\cos(x)+1}+1\right)}{3\left(a^2+\frac{3a^2\sin(x)}{\cos(x)+1}+\frac{3a^2\sin(x)^2}{(\cos(x)+1)^2}+\frac{a^2\sin(x)^3}{(\cos(x)+1)^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(3*sin(x)/(cos(x) + 1) + 1)/(a^2 + 3*a^2*sin(x)/(cos(x) + 1) + 3*a^2*s
in(x)^2/(cos(x) + 1)^2 + a^2*sin(x)^3/(cos(x) + 1)^3)
```

mupad [B] time = 6.30, size = 21, normalized size = 0.64

$$-\frac{2\left(3\tan\left(\frac{x}{2}\right)+1\right)}{3a^2\left(\tan\left(\frac{x}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a + a*sin(x))^2,x)
```

```
[Out] -(2*(3*tan(x/2) + 1))/(3*a^2*(tan(x/2) + 1)^3)
```

sympy [B] time = 2.13, size = 87, normalized size = 2.64

$$-\frac{6\tan\left(\frac{x}{2}\right)}{3a^2\tan^3\left(\frac{x}{2}\right)+9a^2\tan^2\left(\frac{x}{2}\right)+9a^2\tan\left(\frac{x}{2}\right)+3a^2}-\frac{2}{3a^2\tan^3\left(\frac{x}{2}\right)+9a^2\tan^2\left(\frac{x}{2}\right)+9a^2\tan\left(\frac{x}{2}\right)+3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x))**2,x)
```

```
[Out] -6*tan(x/2)/(3*a**2*tan(x/2)**3 + 9*a**2*tan(x/2)**2 + 9*a**2*tan(x/2) + 3*
a**2) - 2/(3*a**2*tan(x/2)**3 + 9*a**2*tan(x/2)**2 + 9*a**2*tan(x/2) + 3*a*
*2)
```

3.16 $\int \frac{1}{(a+a\sin(x))^2}\,dx$

Optimal. Leaf size=33

$$-\frac{\cos(x)}{3\left(a^2\sin(x)+a^2\right)}-\frac{\cos(x)}{3(a\sin(x)+a)^2}$$

```
[Out] -1/3*cos(x)/(a+a*sin(x))^2-1/3*cos(x)/(a^2+a^2*sin(x))
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2650, 2648}

$$-\frac{\cos(x)}{3\left(a^2\sin(x)+a^2\right)}-\frac{\cos(x)}{3(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[x])^(-2),x]
```

```
[Out] -Cos[x]/(3*(a + a*Sin[x])^2) - Cos[x]/(3*(a^2 + a^2*Sin[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\sin(x))^2}\,dx &= -\frac{\cos(x)}{3(a+a\sin(x))^2}+\frac{\int \frac{1}{a+a\sin(x)}\,dx}{3a}\\ &= -\frac{\cos(x)}{3(a+a\sin(x))^2}-\frac{\cos(x)}{3\left(a^2+a^2\sin(x)\right)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 31, normalized size = 0.94

$$-\frac{-4\sin(x)+\sin(2x)+4\cos(x)+\cos(2x)-3}{6a^2(\sin(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[x])^(-2),x]
[Out] -1/6*(-3 + 4*Cos[x] + Cos[2*x] - 4*Sin[x] + Sin[2*x])/(a^2*(1 + Sin[x])^2)
```

fricas [A] time = 0.47, size = 58, normalized size = 1.76

$$\frac{\cos(x)^2+(\cos(x)-1)\sin(x)+2\cos(x)+1}{3\left(a^2\cos(x)^2-a^2\cos(x)-2a^2-\left(a^2\cos(x)+2a^2\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x))^2,x, algorithm="fricas")
[Out] 1/3*(cos(x)^2 + (cos(x) - 1)*sin(x) + 2*cos(x) + 1)/(a^2*cos(x)^2 - a^2*cos
(x) - 2*a^2 - (a^2*cos(x) + 2*a^2)*sin(x))
```

giac [A] time = 1.79, size = 29, normalized size = 0.88

$$-\frac{2\left(3\tan\left(\frac{1}{2}x\right)^2+3\tan\left(\frac{1}{2}x\right)+2\right)}{3a^2\left(\tan\left(\frac{1}{2}x\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x))^2,x, algorithm="giac")
[Out] -2/3*(3*tan(1/2*x)^2 + 3*tan(1/2*x) + 2)/(a^2*(tan(1/2*x) + 1)^3)
```

maple [A] time = 0.08, size = 35, normalized size = 1.06

$$\frac{\frac{2}{\left(\tan\left(\frac{x}{2}\right)+1\right)^2}-\frac{4}{3\left(\tan\left(\frac{x}{2}\right)+1\right)^3}-\frac{2}{\tan\left(\frac{x}{2}\right)+1}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(x))^2,x)
[Out] 2/a^2*(1/(tan(1/2*x)+1)^2-2/3/(tan(1/2*x)+1)^3-1/(tan(1/2*x)+1))
```

maxima [B] time = 0.63, size = 74, normalized size = 2.24

$$-\frac{2\left(\frac{3\sin(x)}{\cos(x)+1}+\frac{3\sin(x)^2}{(\cos(x)+1)^2}+2\right)}{3\left(a^2+\frac{3a^2\sin(x)}{\cos(x)+1}+\frac{3a^2\sin(x)^2}{(\cos(x)+1)^2}+\frac{a^2\sin(x)^3}{(\cos(x)+1)^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(3*sin(x)/(cos(x) + 1) + 3*sin(x)^2/(cos(x) + 1)^2 + 2)/(a^2 + 3*a^2*s
in(x)/(cos(x) + 1) + 3*a^2*sin(x)^2/(cos(x) + 1)^2 + a^2*sin(x)^3/(cos(x) +
 1)^3)
```

mupad [B] time = 6.31, size = 29, normalized size = 0.88

$$-\frac{2\left(3\tan\left(\frac{x}{2}\right)^2+3\tan\left(\frac{x}{2}\right)+2\right)}{3a^2\left(\tan\left(\frac{x}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sin(x))^2,x)
```

```
[Out] -(2*(3*tan(x/2) + 3*tan(x/2)^2 + 2))/(3*a^2*(tan(x/2) + 1)^3)
```

sympy [B] time = 1.02, size = 134, normalized size = 4.06

$$-\frac{6\tan^2\left(\frac{x}{2}\right)}{3a^2\tan^3\left(\frac{x}{2}\right)+9a^2\tan^2\left(\frac{x}{2}\right)+9a^2\tan\left(\frac{x}{2}\right)+3a^2}-\frac{6\tan\left(\frac{x}{2}\right)}{3a^2\tan^3\left(\frac{x}{2}\right)+9a^2\tan^2\left(\frac{x}{2}\right)+9a^2\tan\left(\frac{x}{2}\right)+3a^2}-\frac{}{3a^2\tan^3\left(\frac{x}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x))**2,x)
```

```
[Out] -6*tan(x/2)**2/(3*a**2*tan(x/2)**3 + 9*a**2*tan(x/2)**2 + 9*a**2*tan(x/2) +
 3*a**2) - 6*tan(x/2)/(3*a**2*tan(x/2)**3 + 9*a**2*tan(x/2)**2 + 9*a**2*tan
(x/2) + 3*a**2) - 4/(3*a**2*tan(x/2)**3 + 9*a**2*tan(x/2)**2 + 9*a**2*tan(x
/2) + 3*a**2)
```

3.17 $\int \frac{\csc(x)}{(a+a\sin(x))^2}\,dx$

Optimal. Leaf size=38

$$\frac{4\cos(x)}{3a^2(\sin(x)+1)}-\frac{\tanh^{-1}(\cos(x))}{a^2}+\frac{\cos(x)}{3(a\sin(x)+a)^2}$$

```
[Out] -arctanh(cos(x))/a^2+4/3*cos(x)/a^2/(1+sin(x))+1/3*cos(x)/(a+a*sin(x))^2
```

Rubi [A] time = 0.09, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2766, 2978, 12, 3770}

$$\frac{4\cos(x)}{3a^2(\sin(x)+1)}-\frac{\tanh^{-1}(\cos(x))}{a^2}+\frac{\cos(x)}{3(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a + a*Sin[x])^2,x]
```

```
[Out] -(ArcTanh[Cos[x]]/a^2) + (4*Cos[x])/(3*a^2*(1 + Sin[x])) + Cos[x]/(3*(a + a
*Sin[x])^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
```

```
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(x)}{(a+a\sin(x))^2}\,dx &= \frac{\cos(x)}{3(a+a\sin(x))^2}+\frac{\int \frac{\csc(x)(3a-a\sin(x))}{a+a\sin(x)}\,dx}{3a^2}\\ &= \frac{4\cos(x)}{3a^2(1+\sin(x))}+\frac{\cos(x)}{3(a+a\sin(x))^2}+\frac{\int 3a^2\csc(x)\,dx}{3a^4}\\ &= \frac{4\cos(x)}{3a^2(1+\sin(x))}+\frac{\cos(x)}{3(a+a\sin(x))^2}+\frac{\int \csc(x)\,dx}{a^2}\\ &= -\frac{\tanh^{-1}(\cos(x))}{a^2}+\frac{4\cos(x)}{3a^2(1+\sin(x))}+\frac{\cos(x)}{3(a+a\sin(x))^2}\end{aligned}$$

Mathematica [B] time = 0.14, size = 129, normalized size = 3.39

$$\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(\cos\left(\frac{3x}{2}\right)\left(-3\log\left(\sin\left(\frac{x}{2}\right)\right)+3\log\left(\cos\left(\frac{x}{2}\right)\right)+8\right)+\cos\left(\frac{x}{2}\right)\left(9\log\left(\sin\left(\frac{x}{2}\right)\right)-9\log\left(\cos\left(\frac{x}{2}\right.\right.\right.\right.}{6a^2(\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a + a*Sin[x])^2,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(Cos[(3*x)/2]*(8 + 3*Log[Cos[x/2]] - 3*Log[Sin[x/2]]
) + Cos[x/2]*(-6 - 9*Log[Cos[x/2]] + 9*Log[Sin[x/2]]) - 6*(3 + 2*Log[Cos[x/
2]] + Cos[x]*(Log[Cos[x/2]] - Log[Sin[x/2]]) - 2*Log[Sin[x/2]])*Sin[x/2]))/
(6*a^2*(1 + Sin[x])^2)
```

fricas [B] time = 0.48, size = 117, normalized size = 3.08

$$-\frac{8\cos(x)^2+3\left(\cos(x)^2-(\cos(x)+2)\sin(x)-\cos(x)-2\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-3\left(\cos(x)^2-(\cos(x)+2)\sin(x\right.}{6\left(a^2\cos(x)^2-a^2\cos(x)-2a^2-\left(a^2\cos(x)+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(8*cos(x)^2 + 3*(cos(x)^2 - (cos(x) + 2)*sin(x) - cos(x) - 2)*log(1/2*
cos(x) + 1/2) - 3*(cos(x)^2 - (cos(x) + 2)*sin(x) - cos(x) - 2)*log(-1/2*co
s(x) + 1/2) + 2*(4*cos(x) - 1)*sin(x) + 10*cos(x) + 2)/(a^2*cos(x)^2 - a^2*
cos(x) - 2*a^2 - (a^2*cos(x) + 2*a^2)*sin(x))
```

giac [A] time = 1.70, size = 40, normalized size = 1.05

$$\frac{\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{a^2} + \frac{2\left(6\tan\left(\frac{1}{2}x\right)^2 + 9\tan\left(\frac{1}{2}x\right) + 5\right)}{3a^2\left(\tan\left(\frac{1}{2}x\right) + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x))^2,x, algorithm="giac")
```

```
[Out] log(abs(tan(1/2*x)))/a^2 + 2/3*(6*tan(1/2*x)^2 + 9*tan(1/2*x) + 5)/(a^2*(ta
n(1/2*x) + 1)^3)
```

maple [A] time = 0.12, size = 50, normalized size = 1.32

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^2} + \frac{4}{3a^2\left(\tan\left(\frac{x}{2}\right) + 1\right)^3} - \frac{2}{a^2\left(\tan\left(\frac{x}{2}\right) + 1\right)^2} + \frac{4}{a^2\left(\tan\left(\frac{x}{2}\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a+a*sin(x))^2,x)
```

```
[Out] 1/a^2*ln(tan(1/2*x))+4/3/a^2/(tan(1/2*x)+1)^3-2/a^2/(tan(1/2*x)+1)^2+4/a^2/
(tan(1/2*x)+1)
```

maxima [B] time = 1.00, size = 89, normalized size = 2.34

$$\frac{2\left(\frac{9\sin(x)}{\cos(x)+1} + \frac{6\sin(x)^2}{(\cos(x)+1)^2} + 5\right)}{3\left(a^2 + \frac{3a^2\sin(x)}{\cos(x)+1} + \frac{3a^2\sin(x)^2}{(\cos(x)+1)^2} + \frac{a^2\sin(x)^3}{(\cos(x)+1)^3}\right)} + \frac{\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(9*sin(x)/(cos(x) + 1) + 6*sin(x)^2/(cos(x) + 1)^2 + 5)/(a^2 + 3*a^2*si
n(x)/(cos(x) + 1) + 3*a^2*sin(x)^2/(cos(x) + 1)^2 + a^2*sin(x)^3/(cos(x) +
1)^3) + log(sin(x)/(cos(x) + 1))/a^2
```

mupad [B] time = 6.48, size = 38, normalized size = 1.00

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}+\frac{4\tan\left(\frac{x}{2}\right)^2+6\tan\left(\frac{x}{2}\right)+\frac{10}{3}}{a^2\left(\tan\left(\frac{x}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(a + a*sin(x))^2),x)
```

```
[Out] log(tan(x/2))/a^2 + (6*tan(x/2) + 4*tan(x/2)^2 + 10/3)/(a^2*(tan(x/2) + 1)^
3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\csc(x)}{\sin^2(x)+2\sin(x)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x))**2,x)
```

```
[Out] Integral(csc(x)/(sin(x)**2 + 2*sin(x) + 1), x)/a**2
```

3.18 $\int \frac{\csc^2(x)}{(a+a\sin(x))^2}\,dx$

Optimal. Leaf size=45

$$-\frac{10\cot(x)}{3a^2}+\frac{2\tanh^{-1}(\cos(x))}{a^2}+\frac{2\cot(x)}{a^2(\sin(x)+1)}+\frac{\cot(x)}{3(a\sin(x)+a)^2}$$

```
[Out] 2*arctanh(cos(x))/a^2-10/3*cot(x)/a^2+2*cot(x)/a^2/(1+sin(x))+1/3*cot(x)/(a
+a*sin(x))^2
```

Rubi [A] time = 0.13, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2766, 2978, 2748, 3767, 8, 3770}

$$-\frac{10\cot(x)}{3a^2}+\frac{2\tanh^{-1}(\cos(x))}{a^2}+\frac{2\cot(x)}{a^2(\sin(x)+1)}+\frac{\cot(x)}{3(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a + a*Sin[x])^2,x]
```

```
[Out] (2*ArcTanh[Cos[x]])/a^2 - (10*Cot[x])/(3*a^2) + (2*Cot[x])/(a^2*(1 + Sin[x]
)) + Cot[x]/(3*(a + a*Sin[x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(x)}{(a+a\sin(x))^2}\,dx &= \frac{\cot(x)}{3(a+a\sin(x))^2}+\frac{\int \frac{\csc^2(x)(4a-2a\sin(x))}{a+a\sin(x)}\,dx}{3a^2}\\
&= \frac{2\cot(x)}{a^2(1+\sin(x))}+\frac{\cot(x)}{3(a+a\sin(x))^2}+\frac{\int \csc^2(x)\left(10a^2-6a^2\sin(x)\right)\,dx}{3a^4}\\
&= \frac{2\cot(x)}{a^2(1+\sin(x))}+\frac{\cot(x)}{3(a+a\sin(x))^2}-\frac{2\int \csc(x)\,dx}{a^2}+\frac{10\int \csc^2(x)\,dx}{3a^2}\\
&= \frac{2\tanh^{-1}(\cos(x))}{a^2}+\frac{2\cot(x)}{a^2(1+\sin(x))}+\frac{\cot(x)}{3(a+a\sin(x))^2}-\frac{10\,\mathrm{Subst}(\int 1\,dx,x,\cot(x))}{3a^2}\\
&= \frac{2\tanh^{-1}(\cos(x))}{a^2}-\frac{10\cot(x)}{3a^2}+\frac{2\cot(x)}{a^2(1+\sin(x))}+\frac{\cot(x)}{3(a+a\sin(x))^2}
\end{aligned}$$

Mathematica [B] time = 0.37, size = 166, normalized size = 3.69

$$\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(4\sin\left(\frac{x}{2}\right)+28\sin\left(\frac{x}{2}\right)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^2-2\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)+12\log\left(\cos\left(\frac{x}{2}\right)\right)\left(\sin\left(\right.\right.\right.}{6(a\,\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a + a*Sin[x])^2,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(4*Sin[x/2] - 2*(Cos[x/2] + Sin[x/2]) + 28*Sin[x/2]*
(Cos[x/2] + Sin[x/2])^2 - 3*Cot[x/2]*(Cos[x/2] + Sin[x/2])^3 + 12*Log[Cos[x
/2]]*(Cos[x/2] + Sin[x/2])^3 - 12*Log[Sin[x/2]]*(Cos[x/2] + Sin[x/2])^3 + 3
*(Cos[x/2] + Sin[x/2])^3*Tan[x/2]))/(6*(a + a*Sin[x])^2)
```

fricas [B] time = 0.46, size = 168, normalized size = 3.73

$$-\frac{10\,\cos(x)^3 - 4\,\cos(x)^2 - 3\left(\cos(x)^3 + 2\,\cos(x)^2 + \left(\cos(x)^2 - \cos(x) - 2\right)\sin(x) - \cos(x) - 2\right)\log\left(\frac{1}{2}\,\cos(x) + \frac{1}{2}\right.}{3\left(a^2\cos(x)^3 + 2\,a^2\cos(x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(10*cos(x)^3 - 4*cos(x)^2 - 3*(cos(x)^3 + 2*cos(x)^2 + (cos(x)^2 - cos
(x) - 2)*sin(x) - cos(x) - 2)*log(1/2*cos(x) + 1/2) + 3*(cos(x)^3 + 2*cos(x
)^2 + (cos(x)^2 - cos(x) - 2)*sin(x) - cos(x) - 2)*log(-1/2*cos(x) + 1/2) -
 (10*cos(x)^2 + 14*cos(x) + 1)*sin(x) - 13*cos(x) + 1)/(a^2*cos(x)^3 + 2*a^
2*cos(x)^2 - a^2*cos(x) - 2*a^2 + (a^2*cos(x)^2 - a^2*cos(x) - 2*a^2)*sin(x
))
```

giac [A] time = 0.30, size = 69, normalized size = 1.53

$$-\frac{2\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{a^2} + \frac{\tan\left(\frac{1}{2}\,x\right)}{2\,a^2} + \frac{4\,\tan\left(\frac{1}{2}\,x\right) - 1}{2\,a^2\tan\left(\frac{1}{2}\,x\right)} - \frac{2\left(9\,\tan\left(\frac{1}{2}\,x\right)^2 + 15\,\tan\left(\frac{1}{2}\,x\right) + 8\right)}{3\,a^2\left(\tan\left(\frac{1}{2}\,x\right) + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*log(abs(tan(1/2*x)))/a^2 + 1/2*tan(1/2*x)/a^2 + 1/2*(4*tan(1/2*x) - 1)/(
a^2*tan(1/2*x)) - 2/3*(9*tan(1/2*x)^2 + 15*tan(1/2*x) + 8)/(a^2*(tan(1/2*x)
 + 1)^3)
```

maple [A] time = 0.13, size = 71, normalized size = 1.58

$$\frac{\tan\left(\frac{x}{2}\right)}{2a^2} - \frac{1}{2a^2\tan\left(\frac{x}{2}\right)} - \frac{2\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^2} - \frac{4}{3a^2\left(\tan\left(\frac{x}{2}\right)+1\right)^3} + \frac{2}{a^2\left(\tan\left(\frac{x}{2}\right)+1\right)^2} - \frac{6}{a^2\left(\tan\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a+a*sin(x))^2,x)
```

```
[Out] 1/2/a^2*tan(1/2*x)-1/2/a^2/tan(1/2*x)-2/a^2*ln(tan(1/2*x))-4/3/a^2/(tan(1/2
*x)+1)^3+2/a^2/(tan(1/2*x)+1)^2-6/a^2/(tan(1/2*x)+1)
```

maxima [B] time = 0.92, size = 126, normalized size = 2.80

$$-\frac{\frac{41\sin(x)}{\cos(x)+1}+\frac{69\sin(x)^2}{(\cos(x)+1)^2}+\frac{39\sin(x)^3}{(\cos(x)+1)^3}+3}{6\left(\frac{a^2\sin(x)}{\cos(x)+1}+\frac{3a^2\sin(x)^2}{(\cos(x)+1)^2}+\frac{3a^2\sin(x)^3}{(\cos(x)+1)^3}+\frac{a^2\sin(x)^4}{(\cos(x)+1)^4}\right)}-\frac{2\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2}+\frac{\sin(x)}{2a^2(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(41*sin(x)/(cos(x) + 1) + 69*sin(x)^2/(cos(x) + 1)^2 + 39*sin(x)^3/(co
s(x) + 1)^3 + 3)/(a^2*sin(x)/(cos(x) + 1) + 3*a^2*sin(x)^2/(cos(x) + 1)^2 +
 3*a^2*sin(x)^3/(cos(x) + 1)^3 + a^2*sin(x)^4/(cos(x) + 1)^4) - 2*log(sin(x
)/(cos(x) + 1))/a^2 + 1/2*sin(x)/(a^2*(cos(x) + 1))
```

mupad [B] time = 6.54, size = 91, normalized size = 2.02

$$\frac{\tan\left(\frac{x}{2}\right)}{2a^2}-\frac{13\tan\left(\frac{x}{2}\right)^3+23\tan\left(\frac{x}{2}\right)^2+\frac{41\tan\left(\frac{x}{2}\right)}{3}+1}{2a^2\tan\left(\frac{x}{2}\right)^4+6a^2\tan\left(\frac{x}{2}\right)^3+6a^2\tan\left(\frac{x}{2}\right)^2+2a^2\tan\left(\frac{x}{2}\right)}-\frac{2\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(a + a*sin(x))^2),x)
```

```
[Out] tan(x/2)/(2*a^2) - ((41*tan(x/2))/3 + 23*tan(x/2)^2 + 13*tan(x/2)^3 + 1)/(2
*a^2*tan(x/2) + 6*a^2*tan(x/2)^2 + 6*a^2*tan(x/2)^3 + 2*a^2*tan(x/2)^4) - (
2*log(tan(x/2)))/a^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\csc^2(x)}{\sin^2(x)+2\sin(x)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a+a*sin(x))**2,x)
```

```
[Out] Integral(csc(x)**2/(sin(x)**2 + 2*sin(x) + 1), x)/a**2
```

3.19 $\int \frac{\csc^3(x)}{(a+a\sin(x))^2}\,dx$

Optimal. Leaf size=64

$$\frac{16\cot(x)}{3a^2}-\frac{7\tanh^{-1}(\cos(x))}{2a^2}-\frac{7\cot(x)\csc(x)}{2a^2}+\frac{8\cot(x)\csc(x)}{3a^2(\sin(x)+1)}+\frac{\cot(x)\csc(x)}{3(a\sin(x)+a)^2}$$

```
[Out] -7/2*arctanh(cos(x))/a^2+16/3*cot(x)/a^2-7/2*cot(x)*csc(x)/a^2+8/3*cot(x)*c
sc(x)/a^2/(1+sin(x))+1/3*cot(x)*csc(x)/(a+a*sin(x))^2
```

Rubi [A] time = 0.15, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2766, 2978, 2748, 3768, 3770, 3767, 8}

$$\frac{16\cot(x)}{3a^2}-\frac{7\tanh^{-1}(\cos(x))}{2a^2}-\frac{7\cot(x)\csc(x)}{2a^2}+\frac{8\cot(x)\csc(x)}{3a^2(\sin(x)+1)}+\frac{\cot(x)\csc(x)}{3(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a + a*Sin[x])^2,x]
```

```
[Out] (-7*ArcTanh[Cos[x]])/(2*a^2) + (16*Cot[x])/(3*a^2) - (7*Cot[x]*Csc[x])/(2*a
^2) + (8*Cot[x]*Csc[x])/(3*a^2*(1 + Sin[x])) + (Cot[x]*Csc[x])/(3*(a + a*Si
n[x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
```

```
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^3(x)}{(a+a\sin(x))^2}\,dx = \frac{\cot(x)\csc(x)}{3(a+a\sin(x))^2} + \frac{\int \frac{\csc^3(x)(5a-3a\sin(x))}{a+a\sin(x)}\,dx}{3a^2}$$
$$= \frac{8\cot(x)\csc(x)}{3a^2(1+\sin(x))} + \frac{\cot(x)\csc(x)}{3(a+a\sin(x))^2} + \frac{\int \csc^3(x)\left(21a^2-16a^2\sin(x)\right)\,dx}{3a^4}$$
$$= \frac{8\cot(x)\csc(x)}{3a^2(1+\sin(x))} + \frac{\cot(x)\csc(x)}{3(a+a\sin(x))^2} - \frac{16\int \csc^2(x)\,dx}{3a^2} + \frac{7\int \csc^3(x)\,dx}{a^2}$$
$$= -\frac{7\cot(x)\csc(x)}{2a^2} + \frac{8\cot(x)\csc(x)}{3a^2(1+\sin(x))} + \frac{\cot(x)\csc(x)}{3(a+a\sin(x))^2} + \frac{7\int \csc(x)\,dx}{2a^2} + \frac{16\,\mathrm{Subst}(\int 1\,dx,}{3a^2}$$
$$= -\frac{7\tanh^{-1}(\cos(x))}{2a^2} + \frac{16\cot(x)}{3a^2} - \frac{7\cot(x)\csc(x)}{2a^2} + \frac{8\cot(x)\csc(x)}{3a^2(1+\sin(x))} + \frac{\cot(x)\csc(x)}{3(a+a\sin(x))^2}$$

Mathematica [B] time = 0.66, size = 203, normalized size = 3.17

$$\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(-16\sin\left(\frac{x}{2}\right)-160\sin\left(\frac{x}{2}\right)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^2+8\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)+3\cos\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a + a*Sin[x])^2,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(-16*Sin[x/2] - 3*(1 + Cot[x/2])^3*Sin[x/2] + 8*(Cos
[x/2] + Sin[x/2]) - 160*Sin[x/2]*(Cos[x/2] + Sin[x/2])^2 + 24*Cot[x/2]*(Cos
[x/2] + Sin[x/2])^3 - 84*Log[Cos[x/2]]*(Cos[x/2] + Sin[x/2])^3 + 84*Log[Sin
[x/2]]*(Cos[x/2] + Sin[x/2])^3 - 24*(Cos[x/2] + Sin[x/2])^3*Tan[x/2] + 3*Co
s[x/2]*(1 + Tan[x/2])^3))/(24*a^2*(1 + Sin[x])^2)
```

fricas [B] time = 0.47, size = 220, normalized size = 3.44

$$-\frac{64\cos(x)^4+86\cos(x)^3-54\cos(x)^2+21\left(\cos(x)^4-\cos(x)^3-3\cos(x)^2-\left(\cos(x)^3+2\cos(x)^2-\cos(x)-2\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/12*(64*cos(x)^4 + 86*cos(x)^3 - 54*cos(x)^2 + 21*(cos(x)^4 - cos(x)^3 -
3*cos(x)^2 - (cos(x)^3 + 2*cos(x)^2 - cos(x) - 2)*sin(x) + cos(x) + 2)*log(
1/2*cos(x) + 1/2) - 21*(cos(x)^4 - cos(x)^3 - 3*cos(x)^2 - (cos(x)^3 + 2*co
```

```
s(x)^2 - cos(x) - 2)*sin(x) + cos(x) + 2)*log(-1/2*cos(x) + 1/2) + 2*(32*co
s(x)^3 - 11*cos(x)^2 - 38*cos(x) + 2)*sin(x) - 80*cos(x) - 4)/(a^2*cos(x)^4
 - a^2*cos(x)^3 - 3*a^2*cos(x)^2 + a^2*cos(x) + 2*a^2 - (a^2*cos(x)^3 + 2*a
^2*cos(x)^2 - a^2*cos(x) - 2*a^2)*sin(x))
```

giac [A] time = 0.97, size = 93, normalized size = 1.45

$$\frac{7 \log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{2a^2}+\frac{a^2\tan\left(\frac{1}{2}x\right)^2-8a^2\tan\left(\frac{1}{2}x\right)}{8a^4}-\frac{42\tan\left(\frac{1}{2}x\right)^2-8\tan\left(\frac{1}{2}x\right)+1}{8a^2\tan\left(\frac{1}{2}x\right)^2}+\frac{2\left(12\tan\left(\frac{1}{2}x\right)^2+21\tan\right.}{3a^2\left(\tan\left(\frac{1}{2}x\right)+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*sin(x))^2,x, algorithm="giac")
```

```
[Out] 7/2*log(abs(tan(1/2*x)))/a^2 + 1/8*(a^2*tan(1/2*x)^2 - 8*a^2*tan(1/2*x))/a^
4 - 1/8*(42*tan(1/2*x)^2 - 8*tan(1/2*x) + 1)/(a^2*tan(1/2*x)^2) + 2/3*(12*t
an(1/2*x)^2 + 21*tan(1/2*x) + 11)/(a^2*(tan(1/2*x) + 1)^3)
```

maple [A] time = 0.15, size = 92, normalized size = 1.44

$$\frac{\tan^2\left(\frac{x}{2}\right)}{8a^2}-\frac{\tan\left(\frac{x}{2}\right)}{a^2}-\frac{1}{8a^2\tan\left(\frac{x}{2}\right)^2}+\frac{1}{a^2\tan\left(\frac{x}{2}\right)}+\frac{7\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a^2}+\frac{4}{3a^2\left(\tan\left(\frac{x}{2}\right)+1\right)^3}-\frac{2}{a^2\left(\tan\left(\frac{x}{2}\right)+1\right)^2}+\frac{8}{a^2\left(\tan\left(\frac{x}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a+a*sin(x))^2,x)
```

```
[Out] 1/8/a^2*tan(1/2*x)^2-1/a^2*tan(1/2*x)-1/8/a^2/tan(1/2*x)^2+1/a^2/tan(1/2*x)
+7/2/a^2*ln(tan(1/2*x))+4/3/a^2/(tan(1/2*x)+1)^3-2/a^2/(tan(1/2*x)+1)^2+8/a
^2/(tan(1/2*x)+1)
```

maxima [B] time = 0.68, size = 155, normalized size = 2.42

$$\frac{\frac{15\sin(x)}{\cos(x)+1}+\frac{239\sin(x)^2}{(\cos(x)+1)^2}+\frac{405\sin(x)^3}{(\cos(x)+1)^3}+\frac{216\sin(x)^4}{(\cos(x)+1)^4}-3}{24\left(\frac{a^2\sin(x)^2}{(\cos(x)+1)^2}+\frac{3a^2\sin(x)^3}{(\cos(x)+1)^3}+\frac{3a^2\sin(x)^4}{(\cos(x)+1)^4}+\frac{a^2\sin(x)^5}{(\cos(x)+1)^5}\right)}-\frac{\frac{8\sin(x)}{\cos(x)+1}-\frac{\sin(x)^2}{(\cos(x)+1)^2}}{8a^2}+\frac{7\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*sin(x))^2,x, algorithm="maxima")
```

```
[Out] 1/24*(15*sin(x)/(cos(x) + 1) + 239*sin(x)^2/(cos(x) + 1)^2 + 405*sin(x)^3/(
cos(x) + 1)^3 + 216*sin(x)^4/(cos(x) + 1)^4 - 3)/(a^2*sin(x)^2/(cos(x) + 1)
```

```
^2 + 3*a^2*sin(x)^3/(cos(x) + 1)^3 + 3*a^2*sin(x)^4/(cos(x) + 1)^4 + a^2*si
n(x)^5/(cos(x) + 1)^5) - 1/8*(8*sin(x)/(cos(x) + 1) - sin(x)^2/(cos(x) + 1)
^2)/a^2 + 7/2*log(sin(x)/(cos(x) + 1))/a^2
```

mupad [B] time = 6.40, size = 111, normalized size = 1.73

$$\frac{36\tan\left(\frac{x}{2}\right)^4+\frac{135\tan\left(\frac{x}{2}\right)^3}{2}+\frac{239\tan\left(\frac{x}{2}\right)^2}{6}+\frac{5\tan\left(\frac{x}{2}\right)}{2}-\frac{1}{2}}{4\,a^2\tan\left(\frac{x}{2}\right)^5+12\,a^2\tan\left(\frac{x}{2}\right)^4+12\,a^2\tan\left(\frac{x}{2}\right)^3+4\,a^2\tan\left(\frac{x}{2}\right)^2}-\frac{\tan\left(\frac{x}{2}\right)}{a^2}+\frac{\tan\left(\frac{x}{2}\right)^2}{8\,a^2}+\frac{7\,\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^3*(a + a*sin(x))^2),x)
```

```
[Out] ((5*tan(x/2))/2 + (239*tan(x/2)^2)/6 + (135*tan(x/2)^3)/2 + 36*tan(x/2)^4 -
 1/2)/(4*a^2*tan(x/2)^2 + 12*a^2*tan(x/2)^3 + 12*a^2*tan(x/2)^4 + 4*a^2*tan
(x/2)^5) - tan(x/2)/a^2 + tan(x/2)^2/(8*a^2) + (7*log(tan(x/2)))/(2*a^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^3(x)}{\sin^2(x)+2\sin(x)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a+a*sin(x))**2,x)
```

```
[Out] Integral(csc(x)**3/(sin(x)**2 + 2*sin(x) + 1), x)/a**2
```

3.20 $\int \frac{\csc^4(x)}{(a+a\sin(x))^2}\,dx$

Optimal. Leaf size=65

$$-\frac{\cot^3(x)}{3a^2}-\frac{4\cot(x)}{a^2}-\frac{13\cos(x)}{3a^2(\sin(x)+1)}-\frac{\cos(x)}{3a^2(\sin(x)+1)^2}+\frac{5\tanh^{-1}(\cos(x))}{a^2}+\frac{\cot(x)\csc(x)}{a^2}$$

```
[Out] 5*arctanh(cos(x))/a^2-4*cot(x)/a^2-1/3*cot(x)^3/a^2+cot(x)*csc(x)/a^2-1/3*c
os(x)/a^2/(1+sin(x))^2-13/3*cos(x)/a^2/(1+sin(x))
```

Rubi [A] time = 0.15, antiderivative size = 71, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2766, 2978, 2748, 3767, 3768, 3770}

$$-\frac{4\cot^3(x)}{a^2}-\frac{12\cot(x)}{a^2}+\frac{5\tanh^{-1}(\cos(x))}{a^2}+\frac{5\cot(x)\csc(x)}{a^2}+\frac{10\cot(x)\csc^2(x)}{3a^2(\sin(x)+1)}+\frac{\cot(x)\csc^2(x)}{3(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^4/(a + a*Sin[x])^2,x]
```

```
[Out] (5*ArcTanh[Cos[x]])/a^2 - (12*Cot[x])/a^2 - (4*Cot[x]^3)/a^2 + (5*Cot[x]*Cs
c[x])/a^2 + (10*Cot[x]*Csc[x]^2)/(3*a^2*(1 + Sin[x])) + (Cot[x]*Csc[x]^2)/(
3*(a + a*Sin[x])^2)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^4(x)}{(a+a\sin(x))^2}\,dx = \frac{\cot(x)\csc^2(x)}{3(a+a\sin(x))^2} + \frac{\int \frac{\csc^4(x)(6a-4a\sin(x))}{a+a\sin(x)}\,dx}{3a^2}$$
$$= \frac{10\cot(x)\csc^2(x)}{3a^2(1+\sin(x))} + \frac{\cot(x)\csc^2(x)}{3(a+a\sin(x))^2} + \frac{\int \csc^4(x)\left(36a^2-30a^2\sin(x)\right)\,dx}{3a^4}$$
$$= \frac{10\cot(x)\csc^2(x)}{3a^2(1+\sin(x))} + \frac{\cot(x)\csc^2(x)}{3(a+a\sin(x))^2} - \frac{10\int \csc^3(x)\,dx}{a^2} + \frac{12\int \csc^4(x)\,dx}{a^2}$$
$$= \frac{5\cot(x)\csc(x)}{a^2} + \frac{10\cot(x)\csc^2(x)}{3a^2(1+\sin(x))} + \frac{\cot(x)\csc^2(x)}{3(a+a\sin(x))^2} - \frac{5\int \csc(x)\,dx}{a^2} - \frac{12\,\mathrm{Subst}\left(\int (1\right.}{}$$
$$= \frac{5\tanh^{-1}(\cos(x))}{a^2} - \frac{12\cot(x)}{a^2} - \frac{4\cot^3(x)}{a^2} + \frac{5\cot(x)\csc(x)}{a^2} + \frac{10\cot(x)\csc^2(x)}{3a^2(1+\sin(x))} + \frac{\cot(x}{3(a+}$$

Mathematica [B] time = 3.59, size = 238, normalized size = 3.66

$$\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(16\sin\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)+1\right)^3+208\sin\left(\frac{x}{2}\right)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^2-8\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^4/(a + a*Sin[x])^2,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(-(Cos[x/2]*(1 + Cot[x/2])^3) + 16*Sin[x/2] + 6*(1 +
 Cot[x/2])^3*Sin[x/2] - 8*(Cos[x/2] + Sin[x/2]) + 208*Sin[x/2]*(Cos[x/2] +
Sin[x/2])^2 - 44*Cot[x/2]*(Cos[x/2] + Sin[x/2])^3 + 120*Log[Cos[x/2]]*(Cos[
x/2] + Sin[x/2])^3 - 120*Log[Sin[x/2]]*(Cos[x/2] + Sin[x/2])^3 + 44*(Cos[x/
2] + Sin[x/2])^3*Tan[x/2] - 6*Cos[x/2]*(1 + Tan[x/2])^3 + Sin[x/2]*(1 + Tan
[x/2])^3))/(24*a^2*(1 + Sin[x])^2)
```

fricas [B] time = 0.49, size = 266, normalized size = 4.09

$$-\frac{48\cos(x)^5-18\cos(x)^4-108\cos(x)^3+22\cos(x)^2-15\left(\cos(x)^5+2\cos(x)^4-2\cos(x)^3-4\cos(x)^2+\left(\cos\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(48*cos(x)^5 - 18*cos(x)^4 - 108*cos(x)^3 + 22*cos(x)^2 - 15*(cos(x)^5
 + 2*cos(x)^4 - 2*cos(x)^3 - 4*cos(x)^2 + (cos(x)^4 - cos(x)^3 - 3*cos(x)^2
```

```
 + cos(x) + 2)*sin(x) + cos(x) + 2)*log(1/2*cos(x) + 1/2) + 15*(cos(x)^5 +
2*cos(x)^4 - 2*cos(x)^3 - 4*cos(x)^2 + (cos(x)^4 - cos(x)^3 - 3*cos(x)^2 +
cos(x) + 2)*sin(x) + cos(x) + 2)*log(-1/2*cos(x) + 1/2) - 2*(24*cos(x)^4 +
33*cos(x)^3 - 21*cos(x)^2 - 32*cos(x) - 1)*sin(x) + 62*cos(x) - 2)/(a^2*cos
(x)^5 + 2*a^2*cos(x)^4 - 2*a^2*cos(x)^3 - 4*a^2*cos(x)^2 + a^2*cos(x) + 2*a
^2 + (a^2*cos(x)^4 - a^2*cos(x)^3 - 3*a^2*cos(x)^2 + a^2*cos(x) + 2*a^2)*si
n(x))
```

giac [A] time = 0.28, size = 114, normalized size = 1.75

$$-\frac{5\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{a^2}+\frac{110\tan\left(\frac{1}{2}x\right)^6+45\tan\left(\frac{1}{2}x\right)^5-231\tan\left(\frac{1}{2}x\right)^4-232\tan\left(\frac{1}{2}x\right)^3-30\tan\left(\frac{1}{2}x\right)^2+3\tan}{24\left(\tan\left(\frac{1}{2}x\right)^2+\tan\left(\frac{1}{2}x\right)\right)^3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*sin(x))^2,x, algorithm="giac")
```

```
[Out] -5*log(abs(tan(1/2*x)))/a^2 + 1/24*(110*tan(1/2*x)^6 + 45*tan(1/2*x)^5 - 23
1*tan(1/2*x)^4 - 232*tan(1/2*x)^3 - 30*tan(1/2*x)^2 + 3*tan(1/2*x) - 1)/((t
an(1/2*x)^2 + tan(1/2*x))^3*a^2) + 1/24*(a^4*tan(1/2*x)^3 - 6*a^4*tan(1/2*x
)^2 + 45*a^4*tan(1/2*x))/a^6
```

maple [A] time = 0.14, size = 115, normalized size = 1.77

$$\frac{\tan^3\left(\frac{x}{2}\right)}{24a^2}-\frac{\tan^2\left(\frac{x}{2}\right)}{4a^2}+\frac{15\tan\left(\frac{x}{2}\right)}{8a^2}-\frac{1}{24a^2\tan\left(\frac{x}{2}\right)^3}+\frac{1}{4a^2\tan\left(\frac{x}{2}\right)^2}-\frac{15}{8a^2\tan\left(\frac{x}{2}\right)}-\frac{5\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}-\frac{4}{3a^2\left(\tan\left(\frac{x}{2}\right)+1\right)^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^4/(a+a*sin(x))^2,x)
```

```
[Out] 1/24/a^2*tan(1/2*x)^3-1/4/a^2*tan(1/2*x)^2+15/8/a^2*tan(1/2*x)-1/24/a^2/tan
(1/2*x)^3+1/4/a^2/tan(1/2*x)^2-15/8/a^2/tan(1/2*x)-5/a^2*ln(tan(1/2*x))-4/3
/a^2/(tan(1/2*x)+1)^3+2/a^2/(tan(1/2*x)+1)^2-10/a^2/(tan(1/2*x)+1)
```

maxima [B] time = 1.04, size = 178, normalized size = 2.74

$$\frac{\frac{3\sin(x)}{\cos(x)+1}-\frac{30\sin(x)^2}{(\cos(x)+1)^2}-\frac{342\sin(x)^3}{(\cos(x)+1)^3}-\frac{561\sin(x)^4}{(\cos(x)+1)^4}-\frac{285\sin(x)^5}{(\cos(x)+1)^5}-1}{24\left(\frac{a^2\sin(x)^3}{(\cos(x)+1)^3}+\frac{3a^2\sin(x)^4}{(\cos(x)+1)^4}+\frac{3a^2\sin(x)^5}{(\cos(x)+1)^5}+\frac{a^2\sin(x)^6}{(\cos(x)+1)^6}\right)}+\frac{\frac{45\sin(x)}{\cos(x)+1}-\frac{6\sin(x)^2}{(\cos(x)+1)^2}+\frac{\sin(x)^3}{(\cos(x)+1)^3}}{24a^2}-\frac{5\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*sin(x))^2,x, algorithm="maxima")
```

```
[Out] 1/24*(3*sin(x)/(cos(x) + 1) - 30*sin(x)^2/(cos(x) + 1)^2 - 342*sin(x)^3/(co
s(x) + 1)^3 - 561*sin(x)^4/(cos(x) + 1)^4 - 285*sin(x)^5/(cos(x) + 1)^5 - 1
)/(a^2*sin(x)^3/(cos(x) + 1)^3 + 3*a^2*sin(x)^4/(cos(x) + 1)^4 + 3*a^2*sin(
x)^5/(cos(x) + 1)^5 + a^2*sin(x)^6/(cos(x) + 1)^6) + 1/24*(45*sin(x)/(cos(x
) + 1) - 6*sin(x)^2/(cos(x) + 1)^2 + sin(x)^3/(cos(x) + 1)^3)/a^2 - 5*log(s
in(x)/(cos(x) + 1))/a^2
```

mupad [B] time = 6.40, size = 101, normalized size = 1.55

$$\frac{15\tan\left(\frac{x}{2}\right)}{8\,a^2}-\frac{\tan\left(\frac{x}{2}\right)^2}{4\,a^2}+\frac{\tan\left(\frac{x}{2}\right)^3}{24\,a^2}-\frac{5\,\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}-\frac{\frac{95\tan\left(\frac{x}{2}\right)^5}{8}+\frac{187\tan\left(\frac{x}{2}\right)^4}{8}+\frac{57\tan\left(\frac{x}{2}\right)^3}{4}+\frac{5\tan\left(\frac{x}{2}\right)^2}{4}-\frac{\tan\left(\frac{x}{2}\right)}{8}+\frac{1}{24}}{a^2\tan\left(\frac{x}{2}\right)^3\left(\tan\left(\frac{x}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^4*(a + a*sin(x))^2),x)
```

```
[Out] (15*tan(x/2))/(8*a^2) - tan(x/2)^2/(4*a^2) + tan(x/2)^3/(24*a^2) - (5*log(t
an(x/2)))/a^2 - ((5*tan(x/2)^2)/4 - tan(x/2)/8 + (57*tan(x/2)^3)/4 + (187*t
an(x/2)^4)/8 + (95*tan(x/2)^5)/8 + 1/24)/(a^2*tan(x/2)^3*(tan(x/2) + 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^4(x)}{\sin^2(x)+2\sin(x)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**4/(a+a*sin(x))**2,x)
```

```
[Out] Integral(csc(x)**4/(sin(x)**2 + 2*sin(x) + 1), x)/a**2
```

3.21 $\int \frac{\sin^6(x)}{(a+a\sin(x))^3}\,dx$

Optimal. Leaf size=101

$$-\frac{23x}{2a^3}+\frac{136\cos^3(x)}{15a^3}-\frac{136\cos(x)}{5a^3}+\frac{23\sin^3(x)\cos(x)}{3\left(a^3\sin(x)+a^3\right)}+\frac{23\sin(x)\cos(x)}{2a^3}+\frac{\sin^5(x)\cos(x)}{5(a\sin(x)+a)^3}+\frac{13\sin^4(x)\cos(x)}{15a(a\sin(x)+a)^2}$$

```
[Out] -23/2*x/a^3-136/5*cos(x)/a^3+136/15*cos(x)^3/a^3+23/2*cos(x)*sin(x)/a^3+1/5
*cos(x)*sin(x)^5/(a+a*sin(x))^3+13/15*cos(x)*sin(x)^4/a/(a+a*sin(x))^2+23/3
*cos(x)*sin(x)^3/(a^3+a^3*sin(x))
```

Rubi [A] time = 0.23, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2765, 2977, 2748, 2635, 8, 2633}

$$-\frac{23x}{2a^3}+\frac{136\cos^3(x)}{15a^3}-\frac{136\cos(x)}{5a^3}+\frac{23\sin^3(x)\cos(x)}{3\left(a^3\sin(x)+a^3\right)}+\frac{23\sin(x)\cos(x)}{2a^3}+\frac{\sin^5(x)\cos(x)}{5(a\sin(x)+a)^3}+\frac{13\sin^4(x)\cos(x)}{15a(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^6/(a + a*Sin[x])^3,x]
```

```
[Out] (-23*x)/(2*a^3) - (136*Cos[x])/(5*a^3) + (136*Cos[x]^3)/(15*a^3) + (23*Cos[
x]*Sin[x])/(2*a^3) + (Cos[x]*Sin[x]^5)/(5*(a + a*Sin[x])^3) + (13*Cos[x]*Si
n[x]^4)/(15*a*(a + a*Sin[x])^2) + (23*Cos[x]*Sin[x]^3)/(3*(a^3 + a^3*Sin[x]
))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\sin^6(x)}{(a+a\sin(x))^3}\,dx = \frac{\cos(x)\sin^5(x)}{5(a+a\sin(x))^3} - \frac{\int \frac{\sin^4(x)(5a-8a\sin(x))}{(a+a\sin(x))^2}\,dx}{5a^2}$$

$$= \frac{\cos(x)\sin^5(x)}{5(a+a\sin(x))^3} + \frac{13\cos(x)\sin^4(x)}{15a(a+a\sin(x))^2} - \frac{\int \frac{\sin^3(x)\left(52a^2-63a^2\sin(x)\right)}{a+a\sin(x)}\,dx}{15a^4}$$

$$= \frac{\cos(x)\sin^5(x)}{5(a+a\sin(x))^3} + \frac{13\cos(x)\sin^4(x)}{15a(a+a\sin(x))^2} + \frac{23\cos(x)\sin^3(x)}{3\left(a^3+a^3\sin(x)\right)} - \frac{\int \sin^2(x)\left(345a^3 - 408a^3 \sin \ldots\right.}{15a^6}$$

$$= \frac{\cos(x)\sin^5(x)}{5(a+a\sin(x))^3} + \frac{13\cos(x)\sin^4(x)}{15a(a+a\sin(x))^2} + \frac{23\cos(x)\sin^3(x)}{3\left(a^3+a^3\sin(x)\right)} - \frac{23\int \sin^2(x)\,dx}{a^3} + \frac{136\int \sin^3 \ldots}{5a^3}$$

$$= \frac{23\cos(x)\sin(x)}{2a^3} + \frac{\cos(x)\sin^5(x)}{5(a+a\sin(x))^3} + \frac{13\cos(x)\sin^4(x)}{15a(a+a\sin(x))^2} + \frac{23\cos(x)\sin^3(x)}{3\left(a^3+a^3\sin(x)\right)} - \frac{23\int 1\,dx \ldots}{2a^3}$$

$$= -\frac{23x}{2a^3} - \frac{136\cos(x)}{5a^3} + \frac{136\cos^3(x)}{15a^3} + \frac{23\cos(x)\sin(x)}{2a^3} + \frac{\cos(x)\sin^5(x)}{5(a+a\sin(x))^3} + \frac{13\cos(x)\sin^4 \ldots}{15a(a+a\sin \ldots}$$

Mathematica [A] time = 0.10, size = 191, normalized size = 1.89

$$\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(24\sin\left(\frac{x}{2}\right) - 690x\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^5 - 405\cos(x)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^5 + 5\cos(3x)\left(\sin\left(\frac{x}{2} \ldots\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^6/(a + a*Sin[x])^3,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(24*Sin[x/2] - 12*(Cos[x/2] + Sin[x/2]) - 224*Sin[x/
2]*(Cos[x/2] + Sin[x/2])^2 + 112*(Cos[x/2] + Sin[x/2])^3 + 1576*Sin[x/2]*(C
os[x/2] + Sin[x/2])^4 - 690*x*(Cos[x/2] + Sin[x/2])^5 - 405*Cos[x]*(Cos[x/2
] + Sin[x/2])^5 + 5*Cos[3*x]*(Cos[x/2] + Sin[x/2])^5 + 45*(Cos[x/2] + Sin[x
/2])^5*Sin[2*x]))/(60*(a + a*Sin[x])^3)
```

fricas [A] time = 0.51, size = 158, normalized size = 1.56

$$\frac{10\cos(x)^6 - 15\cos(x)^5 - (345x+839)\cos(x)^3 - 140\cos(x)^4 - (1035x-668)\cos(x)^2 + 6(115x+233)\cos(x) \ldots}{30\left(a^3\cos(x)^3 + 3a^3\cos(x)^2 - 2a^3\cos \ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/30*(10*cos(x)^6 - 15*cos(x)^5 - (345*x + 839)*cos(x)^3 - 140*cos(x)^4 - (
1035*x - 668)*cos(x)^2 + 6*(115*x + 233)*cos(x) + (10*cos(x)^5 + 25*cos(x)^
4 - (345*x - 724)*cos(x)^2 - 115*cos(x)^3 + 6*(115*x + 232)*cos(x) + 1380*x
 - 6)*sin(x) + 1380*x + 6)/(a^3*cos(x)^3 + 3*a^3*cos(x)^2 - 2*a^3*cos(x) -
4*a^3 + (a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3)*sin(x))
```

giac [A] time = 0.97, size = 99, normalized size = 0.98

$$-\frac{23\,x}{2\,a^3}-\frac{9\,\tan\left(\frac{1}{2}\,x\right)^5+36\,\tan\left(\frac{1}{2}\,x\right)^4+84\,\tan\left(\frac{1}{2}\,x\right)^2-9\,\tan\left(\frac{1}{2}\,x\right)+40}{3\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)^3a^3}-\frac{4\left(75\,\tan\left(\frac{1}{2}\,x\right)^4+330\,\tan\left(\frac{1}{2}\,x\right)^3+5\cdots\right.}{15\,a^3\left(\tan\left(\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] -23/2*x/a^3 - 1/3*(9*tan(1/2*x)^5 + 36*tan(1/2*x)^4 + 84*tan(1/2*x)^2 - 9*t
an(1/2*x) + 40)/((tan(1/2*x)^2 + 1)^3*a^3) - 4/15*(75*tan(1/2*x)^4 + 330*ta
n(1/2*x)^3 + 530*tan(1/2*x)^2 + 355*tan(1/2*x) + 86)/(a^3*(tan(1/2*x) + 1)^
5)
```

maple [A] time = 0.10, size = 174, normalized size = 1.72

$$-\frac{3\left(\tan^5\left(\frac{x}{2}\right)\right)}{a^3\left(\tan^2\left(\frac{x}{2}\right)+1\right)^3}-\frac{12\left(\tan^4\left(\frac{x}{2}\right)\right)}{a^3\left(\tan^2\left(\frac{x}{2}\right)+1\right)^3}-\frac{28\left(\tan^2\left(\frac{x}{2}\right)\right)}{a^3\left(\tan^2\left(\frac{x}{2}\right)+1\right)^3}+\frac{3\tan\left(\frac{x}{2}\right)}{a^3\left(\tan^2\left(\frac{x}{2}\right)+1\right)^3}-\frac{40}{3a^3\left(\tan^2\left(\frac{x}{2}\right)+1\right)^3}-\frac{23\arctan\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^6/(a+a*sin(x))^3,x)
```

```
[Out] -3/a^3/(tan(1/2*x)^2+1)^3*tan(1/2*x)^5-12/a^3/(tan(1/2*x)^2+1)^3*tan(1/2*x)
^4-28/a^3/(tan(1/2*x)^2+1)^3*tan(1/2*x)^2+3/a^3/(tan(1/2*x)^2+1)^3*tan(1/2*
x)-40/3/a^3/(tan(1/2*x)^2+1)^3-23/a^3*arctan(tan(1/2*x))-8/5/a^3/(tan(1/2*x
)+1)^5+4/a^3/(tan(1/2*x)+1)^4+8/3/a^3/(tan(1/2*x)+1)^3-8/a^3/(tan(1/2*x)+1)
^2-20/a^3/(tan(1/2*x)+1)
```

maxima [B] time = 0.84, size = 306, normalized size = 3.03

$$-\frac{\frac{2375\,\sin(x)}{\cos(x)+1}+\frac{5347\,\sin(x)^2}{(\cos(x)+1)^2}+\frac{9230\,\sin(x)^3}{(\cos(x)+1)^3}+\frac{12622\,\sin(x)^4}{(\cos(x)+1)^4}+\frac{13340\,\sin(x)^5}{(\cos(x)+1)^5}+\frac{11684\,\sin(x)^6}{(\cos(x)+1)^6}+\frac{8050\,\sin(x)^7}{(\cos(x)+1)^7}+\frac{4370\,\sin(x)^8}{(\cos(x)+1)^8}+\cdots}{15\left(a^3+\frac{5\,a^3\sin(x)}{\cos(x)+1}+\frac{13\,a^3\sin(x)^2}{(\cos(x)+1)^2}+\frac{25\,a^3\sin(x)^3}{(\cos(x)+1)^3}+\frac{38\,a^3\sin(x)^4}{(\cos(x)+1)^4}+\frac{46\,a^3\sin(x)^5}{(\cos(x)+1)^5}+\frac{46\,a^3\sin(x)^6}{(\cos(x)+1)^6}+\frac{38\,a^3\sin(x)^7}{(\cos(x)+1)^7}+\frac{25\,a^3\sin(x)^8}{(\cos(x)+1)^8}+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -1/15*(2375*sin(x)/(cos(x) + 1) + 5347*sin(x)^2/(cos(x) + 1)^2 + 9230*sin(x
)^3/(cos(x) + 1)^3 + 12622*sin(x)^4/(cos(x) + 1)^4 + 13340*sin(x)^5/(cos(x)
 + 1)^5 + 11684*sin(x)^6/(cos(x) + 1)^6 + 8050*sin(x)^7/(cos(x) + 1)^7 + 43
70*sin(x)^8/(cos(x) + 1)^8 + 1725*sin(x)^9/(cos(x) + 1)^9 + 345*sin(x)^10/(
cos(x) + 1)^10 + 544)/(a^3 + 5*a^3*sin(x)/(cos(x) + 1) + 13*a^3*sin(x)^2/(c
os(x) + 1)^2 + 25*a^3*sin(x)^3/(cos(x) + 1)^3 + 38*a^3*sin(x)^4/(cos(x) + 1
)^4 + 46*a^3*sin(x)^5/(cos(x) + 1)^5 + 46*a^3*sin(x)^6/(cos(x) + 1)^6 + 38*
a^3*sin(x)^7/(cos(x) + 1)^7 + 25*a^3*sin(x)^8/(cos(x) + 1)^8 + 13*a^3*sin(x
)^9/(cos(x) + 1)^9 + 5*a^3*sin(x)^10/(cos(x) + 1)^10 + a^3*sin(x)^11/(cos(x
) + 1)^11) - 23*arctan(sin(x)/(cos(x) + 1))/a^3
```

mupad [B] time = 7.02, size = 110, normalized size = 1.09

$$-\frac{23\,x}{2\,a^3}-\frac{23\tan\left(\frac{x}{2}\right)^{10}+115\tan\left(\frac{x}{2}\right)^{9}+\frac{874\tan\left(\frac{x}{2}\right)^{8}}{3}+\frac{1610\tan\left(\frac{x}{2}\right)^{7}}{3}+\frac{11684\tan\left(\frac{x}{2}\right)^{6}}{15}+\frac{2668\tan\left(\frac{x}{2}\right)^{5}}{3}+\frac{12622\tan\left(\frac{x}{2}\right)^{4}}{15}+\frac{1846\tan(}{3}}{a^3\left(\tan\left(\frac{x}{2}\right)^2+1\right)^3\left(\tan\left(\frac{x}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^6/(a + a*sin(x))^3,x)
```

```
[Out] - (23*x)/(2*a^3) - ((475*tan(x/2))/3 + (5347*tan(x/2)^2)/15 + (1846*tan(x/2
)^3)/3 + (12622*tan(x/2)^4)/15 + (2668*tan(x/2)^5)/3 + (11684*tan(x/2)^6)/1
5 + (1610*tan(x/2)^7)/3 + (874*tan(x/2)^8)/3 + 115*tan(x/2)^9 + 23*tan(x/2)
^10 + 544/15)/(a^3*(tan(x/2)^2 + 1)^3*(tan(x/2) + 1)^5)
```

sympy [B] time = 50.54, size = 3288, normalized size = 32.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**6/(a+a*sin(x))**3,x)
```

```
[Out] -345*x*tan(x/2)**11/(30*a**3*tan(x/2)**11 + 150*a**3*tan(x/2)**10 + 390*a**
3*tan(x/2)**9 + 750*a**3*tan(x/2)**8 + 1140*a**3*tan(x/2)**7 + 1380*a**3*ta
n(x/2)**6 + 1380*a**3*tan(x/2)**5 + 1140*a**3*tan(x/2)**4 + 750*a**3*tan(x/
2)**3 + 390*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) - 1725*x*tan(x/
2)**10/(30*a**3*tan(x/2)**11 + 150*a**3*tan(x/2)**10 + 390*a**3*tan(x/2)**9
 + 750*a**3*tan(x/2)**8 + 1140*a**3*tan(x/2)**7 + 1380*a**3*tan(x/2)**6 + 1
380*a**3*tan(x/2)**5 + 1140*a**3*tan(x/2)**4 + 750*a**3*tan(x/2)**3 + 390*a
**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) - 4485*x*tan(x/2)**9/(30*a**
3*tan(x/2)**11 + 150*a**3*tan(x/2)**10 + 390*a**3*tan(x/2)**9 + 750*a**3*ta
n(x/2)**8 + 1140*a**3*tan(x/2)**7 + 1380*a**3*tan(x/2)**6 + 1380*a**3*tan(x
```

```
/2)**5 + 1140*a**3*tan(x/2)**4 + 750*a**3*tan(x/2)**3 + 390*a**3*tan(x/2)**
2 + 150*a**3*tan(x/2) + 30*a**3) - 8625*x*tan(x/2)**8/(30*a**3*tan(x/2)**11
 + 150*a**3*tan(x/2)**10 + 390*a**3*tan(x/2)**9 + 750*a**3*tan(x/2)**8 + 11
40*a**3*tan(x/2)**7 + 1380*a**3*tan(x/2)**6 + 1380*a**3*tan(x/2)**5 + 1140*
a**3*tan(x/2)**4 + 750*a**3*tan(x/2)**3 + 390*a**3*tan(x/2)**2 + 150*a**3*t
an(x/2) + 30*a**3) - 13110*x*tan(x/2)**7/(30*a**3*tan(x/2)**11 + 150*a**3*t
an(x/2)**10 + 390*a**3*tan(x/2)**9 + 750*a**3*tan(x/2)**8 + 1140*a**3*tan(x
/2)**7 + 1380*a**3*tan(x/2)**6 + 1380*a**3*tan(x/2)**5 + 1140*a**3*tan(x/2)
**4 + 750*a**3*tan(x/2)**3 + 390*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*
a**3) - 15870*x*tan(x/2)**6/(30*a**3*tan(x/2)**11 + 150*a**3*tan(x/2)**10 +
 390*a**3*tan(x/2)**9 + 750*a**3*tan(x/2)**8 + 1140*a**3*tan(x/2)**7 + 1380
*a**3*tan(x/2)**6 + 1380*a**3*tan(x/2)**5 + 1140*a**3*tan(x/2)**4 + 750*a**
3*tan(x/2)**3 + 390*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) - 15870
*x*tan(x/2)**5/(30*a**3*tan(x/2)**11 + 150*a**3*tan(x/2)**10 + 390*a**3*tan
(x/2)**9 + 750*a**3*tan(x/2)**8 + 1140*a**3*tan(x/2)**7 + 1380*a**3*tan(x/2
)**6 + 1380*a**3*tan(x/2)**5 + 1140*a**3*tan(x/2)**4 + 750*a**3*tan(x/2)**3
 + 390*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) - 13110*x*tan(x/2)**
4/(30*a**3*tan(x/2)**11 + 150*a**3*tan(x/2)**10 + 390*a**3*tan(x/2)**9 + 75
0*a**3*tan(x/2)**8 + 1140*a**3*tan(x/2)**7 + 1380*a**3*tan(x/2)**6 + 1380*a
**3*tan(x/2)**5 + 1140*a**3*tan(x/2)**4 + 750*a**3*tan(x/2)**3 + 390*a**3*t
an(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) - 8625*x*tan(x/2)**3/(30*a**3*tan
(x/2)**11 + 150*a**3*tan(x/2)**10 + 390*a**3*tan(x/2)**9 + 750*a**3*tan(x/2
)**8 + 1140*a**3*tan(x/2)**7 + 1380*a**3*tan(x/2)**6 + 1380*a**3*tan(x/2)**
5 + 1140*a**3*tan(x/2)**4 + 750*a**3*tan(x/2)**3 + 390*a**3*tan(x/2)**2 + 1
50*a**3*tan(x/2) + 30*a**3) - 4485*x*tan(x/2)**2/(30*a**3*tan(x/2)**11 + 15
0*a**3*tan(x/2)**10 + 390*a**3*tan(x/2)**9 + 750*a**3*tan(x/2)**8 + 1140*a*
*3*tan(x/2)**7 + 1380*a**3*tan(x/2)**6 + 1380*a**3*tan(x/2)**5 + 1140*a**3*
tan(x/2)**4 + 750*a**3*tan(x/2)**3 + 390*a**3*tan(x/2)**2 + 150*a**3*tan(x/
2) + 30*a**3) - 1725*x*tan(x/2)/(30*a**3*tan(x/2)**11 + 150*a**3*tan(x/2)**
10 + 390*a**3*tan(x/2)**9 + 750*a**3*tan(x/2)**8 + 1140*a**3*tan(x/2)**7 +
1380*a**3*tan(x/2)**6 + 1380*a**3*tan(x/2)**5 + 1140*a**3*tan(x/2)**4 + 750
*a**3*tan(x/2)**3 + 390*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) - 3
45*x/(30*a**3*tan(x/2)**11 + 150*a**3*tan(x/2)**10 + 390*a**3*tan(x/2)**9 +
 750*a**3*tan(x/2)**8 + 1140*a**3*tan(x/2)**7 + 1380*a**3*tan(x/2)**6 + 138
0*a**3*tan(x/2)**5 + 1140*a**3*tan(x/2)**4 + 750*a**3*tan(x/2)**3 + 390*a**
3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) - 690*tan(x/2)**10/(30*a**3*ta
n(x/2)**11 + 150*a**3*tan(x/2)**10 + 390*a**3*tan(x/2)**9 + 750*a**3*tan(x/
2)**8 + 1140*a**3*tan(x/2)**7 + 1380*a**3*tan(x/2)**6 + 1380*a**3*tan(x/2)*
*5 + 1140*a**3*tan(x/2)**4 + 750*a**3*tan(x/2)**3 + 390*a**3*tan(x/2)**2 +
150*a**3*tan(x/2) + 30*a**3) - 3450*tan(x/2)**9/(30*a**3*tan(x/2)**11 + 150
*a**3*tan(x/2)**10 + 390*a**3*tan(x/2)**9 + 750*a**3*tan(x/2)**8 + 1140*a**
3*tan(x/2)**7 + 1380*a**3*tan(x/2)**6 + 1380*a**3*tan(x/2)**5 + 1140*a**3*t
an(x/2)**4 + 750*a**3*tan(x/2)**3 + 390*a**3*tan(x/2)**2 + 150*a**3*tan(x/2
) + 30*a**3) - 8740*tan(x/2)**8/(30*a**3*tan(x/2)**11 + 150*a**3*tan(x/2)**
10 + 390*a**3*tan(x/2)**9 + 750*a**3*tan(x/2)**8 + 1140*a**3*tan(x/2)**7 +
```

```
1380*a**3*tan(x/2)**6 + 1380*a**3*tan(x/2)**5 + 1140*a**3*tan(x/2)**4 + 750
*a**3*tan(x/2)**3 + 390*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) - 1
6100*tan(x/2)**7/(30*a**3*tan(x/2)**11 + 150*a**3*tan(x/2)**10 + 390*a**3*t
an(x/2)**9 + 750*a**3*tan(x/2)**8 + 1140*a**3*tan(x/2)**7 + 1380*a**3*tan(x
/2)**6 + 1380*a**3*tan(x/2)**5 + 1140*a**3*tan(x/2)**4 + 750*a**3*tan(x/2)*
*3 + 390*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) - 23368*tan(x/2)**
6/(30*a**3*tan(x/2)**11 + 150*a**3*tan(x/2)**10 + 390*a**3*tan(x/2)**9 + 75
0*a**3*tan(x/2)**8 + 1140*a**3*tan(x/2)**7 + 1380*a**3*tan(x/2)**6 + 1380*a
**3*tan(x/2)**5 + 1140*a**3*tan(x/2)**4 + 750*a**3*tan(x/2)**3 + 390*a**3*t
an(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) - 26680*tan(x/2)**5/(30*a**3*tan(
x/2)**11 + 150*a**3*tan(x/2)**10 + 390*a**3*tan(x/2)**9 + 750*a**3*tan(x/2)
**8 + 1140*a**3*tan(x/2)**7 + 1380*a**3*tan(x/2)**6 + 1380*a**3*tan(x/2)**5
 + 1140*a**3*tan(x/2)**4 + 750*a**3*tan(x/2)**3 + 390*a**3*tan(x/2)**2 + 15
0*a**3*tan(x/2) + 30*a**3) - 25244*tan(x/2)**4/(30*a**3*tan(x/2)**11 + 150*
a**3*tan(x/2)**10 + 390*a**3*tan(x/2)**9 + 750*a**3*tan(x/2)**8 + 1140*a**3
*tan(x/2)**7 + 1380*a**3*tan(x/2)**6 + 1380*a**3*tan(x/2)**5 + 1140*a**3*ta
n(x/2)**4 + 750*a**3*tan(x/2)**3 + 390*a**3*tan(x/2)**2 + 150*a**3*tan(x/2)
 + 30*a**3) - 18460*tan(x/2)**3/(30*a**3*tan(x/2)**11 + 150*a**3*tan(x/2)**
10 + 390*a**3*tan(x/2)**9 + 750*a**3*tan(x/2)**8 + 1140*a**3*tan(x/2)**7 +
1380*a**3*tan(x/2)**6 + 1380*a**3*tan(x/2)**5 + 1140*a**3*tan(x/2)**4 + 750
*a**3*tan(x/2)**3 + 390*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) - 1
0694*tan(x/2)**2/(30*a**3*tan(x/2)**11 + 150*a**3*tan(x/2)**10 + 390*a**3*t
an(x/2)**9 + 750*a**3*tan(x/2)**8 + 1140*a**3*tan(x/2)**7 + 1380*a**3*tan(x
/2)**6 + 1380*a**3*tan(x/2)**5 + 1140*a**3*tan(x/2)**4 + 750*a**3*tan(x/2)*
*3 + 390*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) - 4750*tan(x/2)/(3
0*a**3*tan(x/2)**11 + 150*a**3*tan(x/2)**10 + 390*a**3*tan(x/2)**9 + 750*a*
*3*tan(x/2)**8 + 1140*a**3*tan(x/2)**7 + 1380*a**3*tan(x/2)**6 + 1380*a**3*
tan(x/2)**5 + 1140*a**3*tan(x/2)**4 + 750*a**3*tan(x/2)**3 + 390*a**3*tan(x
/2)**2 + 150*a**3*tan(x/2) + 30*a**3) - 1088/(30*a**3*tan(x/2)**11 + 150*a*
*3*tan(x/2)**10 + 390*a**3*tan(x/2)**9 + 750*a**3*tan(x/2)**8 + 1140*a**3*t
an(x/2)**7 + 1380*a**3*tan(x/2)**6 + 1380*a**3*tan(x/2)**5 + 1140*a**3*tan(
x/2)**4 + 750*a**3*tan(x/2)**3 + 390*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) +
 30*a**3)
```

3.22 $\int \frac{\sin^5(x)}{(a+a\sin(x))^3}\,dx$

Optimal. Leaf size=90

$$\frac{13x}{2a^3}+\frac{152\cos(x)}{15a^3}+\frac{76\sin^2(x)\cos(x)}{15\left(a^3\sin(x)+a^3\right)}-\frac{13\sin(x)\cos(x)}{2a^3}+\frac{\sin^4(x)\cos(x)}{5(a\sin(x)+a)^3}+\frac{11\sin^3(x)\cos(x)}{15a(a\sin(x)+a)^2}$$

```
[Out] 13/2*x/a^3+152/15*cos(x)/a^3-13/2*cos(x)*sin(x)/a^3+1/5*cos(x)*sin(x)^4/(a+
a*sin(x))^3+11/15*cos(x)*sin(x)^3/a/(a+a*sin(x))^2+76/15*cos(x)*sin(x)^2/(a
^3+a^3*sin(x))
```

Rubi [A] time = 0.21, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2765, 2977, 2734}

$$\frac{13x}{2a^3}+\frac{152\cos(x)}{15a^3}+\frac{76\sin^2(x)\cos(x)}{15\left(a^3\sin(x)+a^3\right)}-\frac{13\sin(x)\cos(x)}{2a^3}+\frac{\sin^4(x)\cos(x)}{5(a\sin(x)+a)^3}+\frac{11\sin^3(x)\cos(x)}{15a(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^5/(a + a*Sin[x])^3,x]
```

```
[Out] (13*x)/(2*a^3) + (152*Cos[x])/(15*a^3) - (13*Cos[x]*Sin[x])/(2*a^3) + (Cos[
x]*Sin[x]^4)/(5*(a + a*Sin[x])^3) + (11*Cos[x]*Sin[x]^3)/(15*a*(a + a*Sin[x
])^2) + (76*Cos[x]*Sin[x]^2)/(15*(a^3 + a^3*Sin[x]))
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\sin^5(x)}{(a+a\sin(x))^3}\,dx = \frac{\cos(x)\sin^4(x)}{5(a+a\sin(x))^3} - \frac{\int \frac{\sin^3(x)(4a-7a\sin(x))}{(a+a\sin(x))^2}\,dx}{5a^2}$$

$$= \frac{\cos(x)\sin^4(x)}{5(a+a\sin(x))^3} + \frac{11\cos(x)\sin^3(x)}{15a(a+a\sin(x))^2} - \frac{\int \frac{\sin^2(x)\left(33a^2-43a^2\sin(x)\right)}{a+a\sin(x)}\,dx}{15a^4}$$

$$= \frac{\cos(x)\sin^4(x)}{5(a+a\sin(x))^3} + \frac{11\cos(x)\sin^3(x)}{15a(a+a\sin(x))^2} + \frac{76\cos(x)\sin^2(x)}{15\left(a^3+a^3\sin(x)\right)} - \frac{\int \sin(x)\left(152a^3-195a^3\sin\right.}{15a^6}$$

$$= \frac{13x}{2a^3} + \frac{152\cos(x)}{15a^3} - \frac{13\cos(x)\sin(x)}{2a^3} + \frac{\cos(x)\sin^4(x)}{5(a+a\sin(x))^3} + \frac{11\cos(x)\sin^3(x)}{15a(a+a\sin(x))^2} + \frac{76\cos(x}{15\left(a^3+a\right.}$$

Mathematica [A] time = 0.08, size = 170, normalized size = 1.89

$$\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(-24\sin\left(\frac{x}{2}\right)+390x\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^5+180\cos(x)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^5-15\sin(2x)\left(\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^5/(a + a*Sin[x])^3,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(-24*Sin[x/2] + 12*(Cos[x/2] + Sin[x/2]) + 184*Sin[x
/2]*(Cos[x/2] + Sin[x/2])^2 - 92*(Cos[x/2] + Sin[x/2])^3 - 1016*Sin[x/2]*(C
os[x/2] + Sin[x/2])^4 + 390*x*(Cos[x/2] + Sin[x/2])^5 + 180*Cos[x]*(Cos[x/2
] + Sin[x/2])^5 - 15*(Cos[x/2] + Sin[x/2])^5*Sin[2*x]))/(60*(a + a*Sin[x])^
3)
```

fricas [A] time = 0.51, size = 145, normalized size = 1.61

$$\frac{15\cos(x)^5+(195x+449)\cos(x)^3+60\cos(x)^4+(585x-358)\cos(x)^2-6(65x+128)\cos(x)-\left(15\cos(x)^4\right.}{30\left(a^3\cos(x)^3+3a^3\cos(x)^2-2a^3\cos(x)-4a^3+\left(a^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/30*(15*cos(x)^5 + (195*x + 449)*cos(x)^3 + 60*cos(x)^4 + (585*x - 358)*co
s(x)^2 - 6*(65*x + 128)*cos(x) - (15*cos(x)^4 - (195*x - 404)*cos(x)^2 - 45
*cos(x)^3 + 6*(65*x + 127)*cos(x) + 780*x - 6)*sin(x) - 780*x - 6)/(a^3*cos
(x)^3 + 3*a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3 + (a^3*cos(x)^2 - 2*a^3*cos(x
) - 4*a^3)*sin(x))
```

giac [A] time = 0.38, size = 88, normalized size = 0.98

$$\frac{13x}{2a^3}+\frac{\tan\left(\frac{1}{2}x\right)^3+6\tan\left(\frac{1}{2}x\right)^2-\tan\left(\frac{1}{2}x\right)+6}{\left(\tan\left(\frac{1}{2}x\right)^2+1\right)^2a^3}+\frac{2\left(90\tan\left(\frac{1}{2}x\right)^4+405\tan\left(\frac{1}{2}x\right)^3+665\tan\left(\frac{1}{2}x\right)^2+445\tan\right.}{15a^3\left(\tan\left(\frac{1}{2}x\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] 13/2*x/a^3 + (tan(1/2*x)^3 + 6*tan(1/2*x)^2 - tan(1/2*x) + 6)/((tan(1/2*x)^
2 + 1)^2*a^3) + 2/15*(90*tan(1/2*x)^4 + 405*tan(1/2*x)^3 + 665*tan(1/2*x)^2
 + 445*tan(1/2*x) + 107)/(a^3*(tan(1/2*x) + 1)^5)
```

maple [A] time = 0.14, size = 152, normalized size = 1.69

$$\frac{\tan^3\left(\frac{x}{2}\right)}{a^3\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}+\frac{6\left(\tan^2\left(\frac{x}{2}\right)\right)}{a^3\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}-\frac{\tan\left(\frac{x}{2}\right)}{a^3\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}+\frac{6}{a^3\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}+\frac{13\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a^3}+\frac{8}{5a^3\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^5/(a+a*sin(x))^3,x)
```

```
[Out] 1/a^3/(tan(1/2*x)^2+1)^2*tan(1/2*x)^3+6/a^3/(tan(1/2*x)^2+1)^2*tan(1/2*x)^2
-1/a^3/(tan(1/2*x)^2+1)^2*tan(1/2*x)+6/a^3/(tan(1/2*x)^2+1)^2+13/a^3*arctan
(tan(1/2*x))+8/5/a^3/(tan(1/2*x)+1)^5-4/a^3/(tan(1/2*x)+1)^4-4/3/a^3/(tan(1
/2*x)+1)^3+6/a^3/(tan(1/2*x)+1)^2+12/a^3/(tan(1/2*x)+1)
```

maxima [B] time = 0.77, size = 252, normalized size = 2.80

$$\frac{\frac{1325 \sin(x)}{\cos(x)+1} + \frac{2673 \sin(x)^2}{(\cos(x)+1)^2} + \frac{3805 \sin(x)^3}{(\cos(x)+1)^3} + \frac{4329 \sin(x)^4}{(\cos(x)+1)^4} + \frac{3575 \sin(x)^5}{(\cos(x)+1)^5} + \frac{2275 \sin(x)^6}{(\cos(x)+1)^6} + \frac{975 \sin(x)^7}{(\cos(x)+1)^7} + \frac{195 \sin(x)^8}{(\cos(x)+1)^8} + 304}{15\left(a^3 + \frac{5a^3 \sin(x)}{\cos(x)+1} + \frac{12a^3 \sin(x)^2}{(\cos(x)+1)^2} + \frac{20a^3 \sin(x)^3}{(\cos(x)+1)^3} + \frac{26a^3 \sin(x)^4}{(\cos(x)+1)^4} + \frac{26a^3 \sin(x)^5}{(\cos(x)+1)^5} + \frac{20a^3 \sin(x)^6}{(\cos(x)+1)^6} + \frac{12a^3 \sin(x)^7}{(\cos(x)+1)^7} + \frac{5a^3 \sin(x)^8}{(\cos(x)+1)^8} + \frac{a^3 \sin}{(\cos}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] 1/15*(1325*sin(x)/(cos(x) + 1) + 2673*sin(x)^2/(cos(x) + 1)^2 + 3805*sin(x)
^3/(cos(x) + 1)^3 + 4329*sin(x)^4/(cos(x) + 1)^4 + 3575*sin(x)^5/(cos(x) +
1)^5 + 2275*sin(x)^6/(cos(x) + 1)^6 + 975*sin(x)^7/(cos(x) + 1)^7 + 195*sin
(x)^8/(cos(x) + 1)^8 + 304)/(a^3 + 5*a^3*sin(x)/(cos(x) + 1) + 12*a^3*sin(x
)^2/(cos(x) + 1)^2 + 20*a^3*sin(x)^3/(cos(x) + 1)^3 + 26*a^3*sin(x)^4/(cos(
x) + 1)^4 + 26*a^3*sin(x)^5/(cos(x) + 1)^5 + 20*a^3*sin(x)^6/(cos(x) + 1)^6
 + 12*a^3*sin(x)^7/(cos(x) + 1)^7 + 5*a^3*sin(x)^8/(cos(x) + 1)^8 + a^3*sin
(x)^9/(cos(x) + 1)^9) + 13*arctan(sin(x)/(cos(x) + 1))/a^3
```

mupad [B] time = 6.67, size = 93, normalized size = 1.03

$$\frac{13\,x}{2\,a^3}+\frac{13\tan\left(\frac{x}{2}\right)^8+65\tan\left(\frac{x}{2}\right)^7+\frac{455\tan\left(\frac{x}{2}\right)^6}{3}+\frac{715\tan\left(\frac{x}{2}\right)^5}{3}+\frac{1443\tan\left(\frac{x}{2}\right)^4}{5}+\frac{761\tan\left(\frac{x}{2}\right)^3}{3}+\frac{891\tan\left(\frac{x}{2}\right)^2}{5}+\frac{265\tan\left(\frac{x}{2}\right)}{3}+\frac{304}{15}}{a^3\left(\tan\left(\frac{x}{2}\right)^2+1\right)^2\left(\tan\left(\frac{x}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^5/(a + a*sin(x))^3,x)
```

```
[Out] (13*x)/(2*a^3) + ((265*tan(x/2))/3 + (891*tan(x/2)^2)/5 + (761*tan(x/2)^3)/
3 + (1443*tan(x/2)^4)/5 + (715*tan(x/2)^5)/3 + (455*tan(x/2)^6)/3 + 65*tan(
x/2)^7 + 13*tan(x/2)^8 + 304/15)/(a^3*(tan(x/2)^2 + 1)^2*(tan(x/2) + 1)^5)
```

sympy [B] time = 33.59, size = 2259, normalized size = 25.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**5/(a+a*sin(x))**3,x)
```

```
[Out] 195*x*tan(x/2)**9/(30*a**3*tan(x/2)**9 + 150*a**3*tan(x/2)**8 + 360*a**3*ta
n(x/2)**7 + 600*a**3*tan(x/2)**6 + 780*a**3*tan(x/2)**5 + 780*a**3*tan(x/2)
**4 + 600*a**3*tan(x/2)**3 + 360*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*
a**3) + 975*x*tan(x/2)**8/(30*a**3*tan(x/2)**9 + 150*a**3*tan(x/2)**8 + 360
```

```
*a**3*tan(x/2)**7 + 600*a**3*tan(x/2)**6 + 780*a**3*tan(x/2)**5 + 780*a**3*
tan(x/2)**4 + 600*a**3*tan(x/2)**3 + 360*a**3*tan(x/2)**2 + 150*a**3*tan(x/
2) + 30*a**3) + 2340*x*tan(x/2)**7/(30*a**3*tan(x/2)**9 + 150*a**3*tan(x/2)
**8 + 360*a**3*tan(x/2)**7 + 600*a**3*tan(x/2)**6 + 780*a**3*tan(x/2)**5 +
780*a**3*tan(x/2)**4 + 600*a**3*tan(x/2)**3 + 360*a**3*tan(x/2)**2 + 150*a*
*3*tan(x/2) + 30*a**3) + 3900*x*tan(x/2)**6/(30*a**3*tan(x/2)**9 + 150*a**3
*tan(x/2)**8 + 360*a**3*tan(x/2)**7 + 600*a**3*tan(x/2)**6 + 780*a**3*tan(x
/2)**5 + 780*a**3*tan(x/2)**4 + 600*a**3*tan(x/2)**3 + 360*a**3*tan(x/2)**2
 + 150*a**3*tan(x/2) + 30*a**3) + 5070*x*tan(x/2)**5/(30*a**3*tan(x/2)**9 +
 150*a**3*tan(x/2)**8 + 360*a**3*tan(x/2)**7 + 600*a**3*tan(x/2)**6 + 780*a
**3*tan(x/2)**5 + 780*a**3*tan(x/2)**4 + 600*a**3*tan(x/2)**3 + 360*a**3*ta
n(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) + 5070*x*tan(x/2)**4/(30*a**3*tan(
x/2)**9 + 150*a**3*tan(x/2)**8 + 360*a**3*tan(x/2)**7 + 600*a**3*tan(x/2)**
6 + 780*a**3*tan(x/2)**5 + 780*a**3*tan(x/2)**4 + 600*a**3*tan(x/2)**3 + 36
0*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) + 3900*x*tan(x/2)**3/(30*
a**3*tan(x/2)**9 + 150*a**3*tan(x/2)**8 + 360*a**3*tan(x/2)**7 + 600*a**3*t
an(x/2)**6 + 780*a**3*tan(x/2)**5 + 780*a**3*tan(x/2)**4 + 600*a**3*tan(x/2
)**3 + 360*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) + 2340*x*tan(x/2
)**2/(30*a**3*tan(x/2)**9 + 150*a**3*tan(x/2)**8 + 360*a**3*tan(x/2)**7 + 6
00*a**3*tan(x/2)**6 + 780*a**3*tan(x/2)**5 + 780*a**3*tan(x/2)**4 + 600*a**
3*tan(x/2)**3 + 360*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) + 975*x
*tan(x/2)/(30*a**3*tan(x/2)**9 + 150*a**3*tan(x/2)**8 + 360*a**3*tan(x/2)**
7 + 600*a**3*tan(x/2)**6 + 780*a**3*tan(x/2)**5 + 780*a**3*tan(x/2)**4 + 60
0*a**3*tan(x/2)**3 + 360*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) +
195*x/(30*a**3*tan(x/2)**9 + 150*a**3*tan(x/2)**8 + 360*a**3*tan(x/2)**7 +
600*a**3*tan(x/2)**6 + 780*a**3*tan(x/2)**5 + 780*a**3*tan(x/2)**4 + 600*a*
*3*tan(x/2)**3 + 360*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) + 390*
tan(x/2)**8/(30*a**3*tan(x/2)**9 + 150*a**3*tan(x/2)**8 + 360*a**3*tan(x/2)
**7 + 600*a**3*tan(x/2)**6 + 780*a**3*tan(x/2)**5 + 780*a**3*tan(x/2)**4 +
600*a**3*tan(x/2)**3 + 360*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3)
+ 1950*tan(x/2)**7/(30*a**3*tan(x/2)**9 + 150*a**3*tan(x/2)**8 + 360*a**3*t
an(x/2)**7 + 600*a**3*tan(x/2)**6 + 780*a**3*tan(x/2)**5 + 780*a**3*tan(x/2
)**4 + 600*a**3*tan(x/2)**3 + 360*a**3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30
*a**3) + 4550*tan(x/2)**6/(30*a**3*tan(x/2)**9 + 150*a**3*tan(x/2)**8 + 360
*a**3*tan(x/2)**7 + 600*a**3*tan(x/2)**6 + 780*a**3*tan(x/2)**5 + 780*a**3*
tan(x/2)**4 + 600*a**3*tan(x/2)**3 + 360*a**3*tan(x/2)**2 + 150*a**3*tan(x/
2) + 30*a**3) + 7150*tan(x/2)**5/(30*a**3*tan(x/2)**9 + 150*a**3*tan(x/2)**
8 + 360*a**3*tan(x/2)**7 + 600*a**3*tan(x/2)**6 + 780*a**3*tan(x/2)**5 + 78
0*a**3*tan(x/2)**4 + 600*a**3*tan(x/2)**3 + 360*a**3*tan(x/2)**2 + 150*a**3
*tan(x/2) + 30*a**3) + 8658*tan(x/2)**4/(30*a**3*tan(x/2)**9 + 150*a**3*tan
(x/2)**8 + 360*a**3*tan(x/2)**7 + 600*a**3*tan(x/2)**6 + 780*a**3*tan(x/2)*
*5 + 780*a**3*tan(x/2)**4 + 600*a**3*tan(x/2)**3 + 360*a**3*tan(x/2)**2 + 1
50*a**3*tan(x/2) + 30*a**3) + 7610*tan(x/2)**3/(30*a**3*tan(x/2)**9 + 150*a
**3*tan(x/2)**8 + 360*a**3*tan(x/2)**7 + 600*a**3*tan(x/2)**6 + 780*a**3*ta
n(x/2)**5 + 780*a**3*tan(x/2)**4 + 600*a**3*tan(x/2)**3 + 360*a**3*tan(x/2)
```

```
**2 + 150*a**3*tan(x/2) + 30*a**3) + 5346*tan(x/2)**2/(30*a**3*tan(x/2)**9
+ 150*a**3*tan(x/2)**8 + 360*a**3*tan(x/2)**7 + 600*a**3*tan(x/2)**6 + 780*
a**3*tan(x/2)**5 + 780*a**3*tan(x/2)**4 + 600*a**3*tan(x/2)**3 + 360*a**3*t
an(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) + 2650*tan(x/2)/(30*a**3*tan(x/2)
**9 + 150*a**3*tan(x/2)**8 + 360*a**3*tan(x/2)**7 + 600*a**3*tan(x/2)**6 +
780*a**3*tan(x/2)**5 + 780*a**3*tan(x/2)**4 + 600*a**3*tan(x/2)**3 + 360*a*
*3*tan(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3) + 608/(30*a**3*tan(x/2)**9 +
150*a**3*tan(x/2)**8 + 360*a**3*tan(x/2)**7 + 600*a**3*tan(x/2)**6 + 780*a*
*3*tan(x/2)**5 + 780*a**3*tan(x/2)**4 + 600*a**3*tan(x/2)**3 + 360*a**3*tan
(x/2)**2 + 150*a**3*tan(x/2) + 30*a**3)
```

3.23 $\int \frac{\sin^4(x)}{(a+a\sin(x))^3}\,dx$

Optimal. Leaf size=71

$$-\frac{3x}{a^3}-\frac{9\cos(x)}{5a^3}-\frac{3\cos(x)}{a^3\sin(x)+a^3}+\frac{\sin^3(x)\cos(x)}{5(a\sin(x)+a)^3}+\frac{3\sin^2(x)\cos(x)}{5a(a\sin(x)+a)^2}$$

```
[Out] -3*x/a^3-9/5*cos(x)/a^3+1/5*cos(x)*sin(x)^3/(a+a*sin(x))^3+3/5*cos(x)*sin(x
)^2/a/(a+a*sin(x))^2-3*cos(x)/(a^3+a^3*sin(x))
```

Rubi [A] time = 0.22, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2765, 2977, 2968, 3023, 12, 2735, 2648}

$$-\frac{3x}{a^3}-\frac{9\cos(x)}{5a^3}-\frac{3\cos(x)}{a^3\sin(x)+a^3}+\frac{\sin^3(x)\cos(x)}{5(a\sin(x)+a)^3}+\frac{3\sin^2(x)\cos(x)}{5a(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4/(a + a*Sin[x])^3,x]
```

```
[Out] (-3*x)/a^3 - (9*Cos[x])/(5*a^3) + (Cos[x]*Sin[x]^3)/(5*(a + a*Sin[x])^3) +
(3*Cos[x]*Sin[x]^2)/(5*a*(a + a*Sin[x])^2) - (3*Cos[x])/(a^3 + a^3*Sin[x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
```

```
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^4(x)}{(a+a\sin(x))^3}\,dx = \frac{\cos(x)\sin^3(x)}{5(a+a\sin(x))^3} - \frac{\int \frac{\sin^2(x)(3a-6a\sin(x))}{(a+a\sin(x))^2}\,dx}{5a^2}$$

$$= \frac{\cos(x)\sin^3(x)}{5(a+a\sin(x))^3} + \frac{3\cos(x)\sin^2(x)}{5a(a+a\sin(x))^2} - \frac{\int \frac{\sin(x)\left(18a^2-27a^2\sin(x)\right)}{a+a\sin(x)}\,dx}{15a^4}$$

$$= \frac{\cos(x)\sin^3(x)}{5(a+a\sin(x))^3} + \frac{3\cos(x)\sin^2(x)}{5a(a+a\sin(x))^2} - \frac{\int \frac{18a^2\sin(x)-27a^2\sin^2(x)}{a+a\sin(x)}\,dx}{15a^4}$$

$$= -\frac{9\cos(x)}{5a^3} + \frac{\cos(x)\sin^3(x)}{5(a+a\sin(x))^3} + \frac{3\cos(x)\sin^2(x)}{5a(a+a\sin(x))^2} - \frac{\int \frac{45a^3\sin(x)}{a+a\sin(x)}\,dx}{15a^5}$$

$$= -\frac{9\cos(x)}{5a^3} + \frac{\cos(x)\sin^3(x)}{5(a+a\sin(x))^3} + \frac{3\cos(x)\sin^2(x)}{5a(a+a\sin(x))^2} - \frac{3\int \frac{\sin(x)}{a+a\sin(x)}\,dx}{a^2}$$

$$= -\frac{3x}{a^3} - \frac{9\cos(x)}{5a^3} + \frac{\cos(x)\sin^3(x)}{5(a+a\sin(x))^3} + \frac{3\cos(x)\sin^2(x)}{5a(a+a\sin(x))^2} + \frac{3\int \frac{1}{a+a\sin(x)}\,dx}{a^2}$$

$$= -\frac{3x}{a^3} - \frac{9\cos(x)}{5a^3} + \frac{\cos(x)\sin^3(x)}{5(a+a\sin(x))^3} + \frac{3\cos(x)\sin^2(x)}{5a(a+a\sin(x))^2} - \frac{3\cos(x)}{a^3+a^3\sin(x)}$$

Mathematica [A] time = 0.08, size = 140, normalized size = 1.97

$$\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(\sin\left(\frac{x}{2}\right)-\cos\left(\frac{x}{2}\right)-15x\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^5-5\cos(x)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^5+48\sin\left(\frac{x}{2}\right)\left(s\right.}{5(a\sin(x)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4/(a + a*Sin[x])^3,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(-Cos[x/2] + Sin[x/2] - 12*Sin[x/2]*(Cos[x/2] + Sin[
x/2])^2 + 6*(Cos[x/2] + Sin[x/2])^3 + 48*Sin[x/2]*(Cos[x/2] + Sin[x/2])^4 -
 15*x*(Cos[x/2] + Sin[x/2])^5 - 5*Cos[x]*(Cos[x/2] + Sin[x/2])^5))/(5*(a +
a*Sin[x])^3)
```

fricas [B] time = 0.49, size = 132, normalized size = 1.86

$$-\frac{3\,(5\,x+13)\cos(x)^3+5\,\cos(x)^4+(45\,x-28)\cos(x)^2-3\,(10\,x+21)\cos(x)+\left((15\,x-34)\cos(x)^2+5\,\cos(x)^3\right.}{5\left(a^3\cos(x)^3+3\,a^3\cos(x)^2-2\,a^3\cos(x)-4\,a^3+\left(a^3\cos(x)^2-2\,a^3\cos(x)\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/5*(3*(5*x + 13)*cos(x)^3 + 5*cos(x)^4 + (45*x - 28)*cos(x)^2 - 3*(10*x +
 21)*cos(x) + ((15*x - 34)*cos(x)^2 + 5*cos(x)^3 - 2*(15*x + 31)*cos(x) - 6
0*x + 1)*sin(x) - 60*x - 1)/(a^3*cos(x)^3 + 3*a^3*cos(x)^2 - 2*a^3*cos(x) -
 4*a^3 + (a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3)*sin(x))
```

giac [A] time = 0.88, size = 67, normalized size = 0.94

$$-\frac{3x}{a^3}-\frac{2}{\left(\tan\left(\frac{1}{2}x\right)^2+1\right)a^3}-\frac{2\left(15\tan\left(\frac{1}{2}x\right)^4+70\tan\left(\frac{1}{2}x\right)^3+120\tan\left(\frac{1}{2}x\right)^2+80\tan\left(\frac{1}{2}x\right)+19\right)}{5a^3\left(\tan\left(\frac{1}{2}x\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] -3*x/a^3 - 2/((tan(1/2*x)^2 + 1)*a^3) - 2/5*(15*tan(1/2*x)^4 + 70*tan(1/2*x
)^3 + 120*tan(1/2*x)^2 + 80*tan(1/2*x) + 19)/(a^3*(tan(1/2*x) + 1)^5)
```

maple [A] time = 0.10, size = 79, normalized size = 1.11

$$-\frac{2}{a^3\left(\tan^2\left(\frac{x}{2}\right)+1\right)}-\frac{6\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a^3}-\frac{8}{5a^3\left(\tan\left(\frac{x}{2}\right)+1\right)^5}+\frac{4}{a^3\left(\tan\left(\frac{x}{2}\right)+1\right)^4}-\frac{4}{a^3\left(\tan\left(\frac{x}{2}\right)+1\right)^2}-\frac{6}{a^3\left(\tan\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a+a*sin(x))^3,x)
```

```
[Out] -2/a^3/(tan(1/2*x)^2+1)-6/a^3*arctan(tan(1/2*x))-8/5/a^3/(tan(1/2*x)+1)^5+4
/a^3/(tan(1/2*x)+1)^4-4/a^3/(tan(1/2*x)+1)^2-6/a^3/(tan(1/2*x)+1)
```

maxima [B] time = 0.93, size = 198, normalized size = 2.79

$$-\frac{2\left(\frac{105\sin(x)}{\cos(x)+1}+\frac{189\sin(x)^2}{(\cos(x)+1)^2}+\frac{200\sin(x)^3}{(\cos(x)+1)^3}+\frac{160\sin(x)^4}{(\cos(x)+1)^4}+\frac{75\sin(x)^5}{(\cos(x)+1)^5}+\frac{15\sin(x)^6}{(\cos(x)+1)^6}+24\right)}{5\left(a^3+\frac{5a^3\sin(x)}{\cos(x)+1}+\frac{11a^3\sin(x)^2}{(\cos(x)+1)^2}+\frac{15a^3\sin(x)^3}{(\cos(x)+1)^3}+\frac{15a^3\sin(x)^4}{(\cos(x)+1)^4}+\frac{11a^3\sin(x)^5}{(\cos(x)+1)^5}+\frac{5a^3\sin(x)^6}{(\cos(x)+1)^6}+\frac{a^3\sin(x)^7}{(\cos(x)+1)^7}\right)}-\frac{6\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -2/5*(105*sin(x)/(cos(x) + 1) + 189*sin(x)^2/(cos(x) + 1)^2 + 200*sin(x)^3/
(cos(x) + 1)^3 + 160*sin(x)^4/(cos(x) + 1)^4 + 75*sin(x)^5/(cos(x) + 1)^5 +
```

```
15*sin(x)^6/(cos(x) + 1)^6 + 24)/(a^3 + 5*a^3*sin(x)/(cos(x) + 1) + 11*a^3
*sin(x)^2/(cos(x) + 1)^2 + 15*a^3*sin(x)^3/(cos(x) + 1)^3 + 15*a^3*sin(x)^4
/(cos(x) + 1)^4 + 11*a^3*sin(x)^5/(cos(x) + 1)^5 + 5*a^3*sin(x)^6/(cos(x) +
 1)^6 + a^3*sin(x)^7/(cos(x) + 1)^7) - 6*arctan(sin(x)/(cos(x) + 1))/a^3
```

mupad [B] time = 6.89, size = 78, normalized size = 1.10

$$-\frac{3\,x}{a^3}-\frac{6\tan\left(\frac{x}{2}\right)^6+30\tan\left(\frac{x}{2}\right)^5+64\tan\left(\frac{x}{2}\right)^4+80\tan\left(\frac{x}{2}\right)^3+\frac{378\tan\left(\frac{x}{2}\right)^2}{5}+42\tan\left(\frac{x}{2}\right)+\frac{48}{5}}{a^3\left(\tan\left(\frac{x}{2}\right)^2+1\right)\left(\tan\left(\frac{x}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a + a*sin(x))^3,x)
```

```
[Out] - (3*x)/a^3 - (42*tan(x/2) + (378*tan(x/2)^2)/5 + 80*tan(x/2)^3 + 64*tan(x/
2)^4 + 30*tan(x/2)^5 + 6*tan(x/2)^6 + 48/5)/(a^3*(tan(x/2)^2 + 1)*(tan(x/2)
 + 1)^5)
```

sympy [B] time = 19.64, size = 1425, normalized size = 20.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4/(a+a*sin(x))**3,x)
```

```
[Out] -15*x*tan(x/2)**7/(5*a**3*tan(x/2)**7 + 25*a**3*tan(x/2)**6 + 55*a**3*tan(x
/2)**5 + 75*a**3*tan(x/2)**4 + 75*a**3*tan(x/2)**3 + 55*a**3*tan(x/2)**2 +
25*a**3*tan(x/2) + 5*a**3) - 75*x*tan(x/2)**6/(5*a**3*tan(x/2)**7 + 25*a**3
*tan(x/2)**6 + 55*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 75*a**3*tan(x/2)
**3 + 55*a**3*tan(x/2)**2 + 25*a**3*tan(x/2) + 5*a**3) - 165*x*tan(x/2)**5/
(5*a**3*tan(x/2)**7 + 25*a**3*tan(x/2)**6 + 55*a**3*tan(x/2)**5 + 75*a**3*t
an(x/2)**4 + 75*a**3*tan(x/2)**3 + 55*a**3*tan(x/2)**2 + 25*a**3*tan(x/2) +
 5*a**3) - 225*x*tan(x/2)**4/(5*a**3*tan(x/2)**7 + 25*a**3*tan(x/2)**6 + 55
*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 75*a**3*tan(x/2)**3 + 55*a**3*tan
(x/2)**2 + 25*a**3*tan(x/2) + 5*a**3) - 225*x*tan(x/2)**3/(5*a**3*tan(x/2)*
*7 + 25*a**3*tan(x/2)**6 + 55*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 75*a
**3*tan(x/2)**3 + 55*a**3*tan(x/2)**2 + 25*a**3*tan(x/2) + 5*a**3) - 165*x*
tan(x/2)**2/(5*a**3*tan(x/2)**7 + 25*a**3*tan(x/2)**6 + 55*a**3*tan(x/2)**5
 + 75*a**3*tan(x/2)**4 + 75*a**3*tan(x/2)**3 + 55*a**3*tan(x/2)**2 + 25*a**
3*tan(x/2) + 5*a**3) - 75*x*tan(x/2)/(5*a**3*tan(x/2)**7 + 25*a**3*tan(x/2)
**6 + 55*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 75*a**3*tan(x/2)**3 + 55*
a**3*tan(x/2)**2 + 25*a**3*tan(x/2) + 5*a**3) - 15*x/(5*a**3*tan(x/2)**7 +
25*a**3*tan(x/2)**6 + 55*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 75*a**3*t
an(x/2)**3 + 55*a**3*tan(x/2)**2 + 25*a**3*tan(x/2) + 5*a**3) - 30*tan(x/2)
```

```
**6/(5*a**3*tan(x/2)**7 + 25*a**3*tan(x/2)**6 + 55*a**3*tan(x/2)**5 + 75*a*
*3*tan(x/2)**4 + 75*a**3*tan(x/2)**3 + 55*a**3*tan(x/2)**2 + 25*a**3*tan(x/
2) + 5*a**3) - 150*tan(x/2)**5/(5*a**3*tan(x/2)**7 + 25*a**3*tan(x/2)**6 +
55*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 75*a**3*tan(x/2)**3 + 55*a**3*t
an(x/2)**2 + 25*a**3*tan(x/2) + 5*a**3) - 320*tan(x/2)**4/(5*a**3*tan(x/2)*
*7 + 25*a**3*tan(x/2)**6 + 55*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 75*a
**3*tan(x/2)**3 + 55*a**3*tan(x/2)**2 + 25*a**3*tan(x/2) + 5*a**3) - 400*ta
n(x/2)**3/(5*a**3*tan(x/2)**7 + 25*a**3*tan(x/2)**6 + 55*a**3*tan(x/2)**5 +
 75*a**3*tan(x/2)**4 + 75*a**3*tan(x/2)**3 + 55*a**3*tan(x/2)**2 + 25*a**3*
tan(x/2) + 5*a**3) - 378*tan(x/2)**2/(5*a**3*tan(x/2)**7 + 25*a**3*tan(x/2)
**6 + 55*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 75*a**3*tan(x/2)**3 + 55*
a**3*tan(x/2)**2 + 25*a**3*tan(x/2) + 5*a**3) - 210*tan(x/2)/(5*a**3*tan(x/
2)**7 + 25*a**3*tan(x/2)**6 + 55*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 7
5*a**3*tan(x/2)**3 + 55*a**3*tan(x/2)**2 + 25*a**3*tan(x/2) + 5*a**3) - 48/
(5*a**3*tan(x/2)**7 + 25*a**3*tan(x/2)**6 + 55*a**3*tan(x/2)**5 + 75*a**3*t
an(x/2)**4 + 75*a**3*tan(x/2)**3 + 55*a**3*tan(x/2)**2 + 25*a**3*tan(x/2) +
 5*a**3)
```

3.24 $\int \frac{\sin^3(x)}{(a+a\sin(x))^3}\,dx$

Optimal. Leaf size=59

$$\frac{x}{a^3}+\frac{29\cos(x)}{15\left(a^3\sin(x)+a^3\right)}+\frac{\sin^2(x)\cos(x)}{5(a\sin(x)+a)^3}-\frac{7\cos(x)}{15a(a\sin(x)+a)^2}$$

```
[Out] x/a^3+1/5*cos(x)*sin(x)^2/(a+a*sin(x))^3-7/15*cos(x)/a/(a+a*sin(x))^2+29/15
*cos(x)/(a^3+a^3*sin(x))
```

Rubi [A] time = 0.16, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2765, 2968, 3019, 2735, 2648}

$$\frac{x}{a^3}+\frac{29\cos(x)}{15\left(a^3\sin(x)+a^3\right)}+\frac{\sin^2(x)\cos(x)}{5(a\sin(x)+a)^3}-\frac{7\cos(x)}{15a(a\sin(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a + a*Sin[x])^3,x]
```

```
[Out] x/a^3 + (Cos[x]*Sin[x]^2)/(5*(a + a*Sin[x])^3) - (7*Cos[x])/(15*a*(a + a*Si
n[x])^2) + (29*Cos[x])/(15*(a^3 + a^3*Sin[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
```

```
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(x)}{(a+a\sin(x))^3}\,dx &= \frac{\cos(x)\sin^2(x)}{5(a+a\sin(x))^3} - \frac{\int \frac{\sin(x)(2a-5a\sin(x))}{(a+a\sin(x))^2}\,dx}{5a^2}\\
&= \frac{\cos(x)\sin^2(x)}{5(a+a\sin(x))^3} - \frac{\int \frac{2a\sin(x)-5a\sin^2(x)}{(a+a\sin(x))^2}\,dx}{5a^2}\\
&= \frac{\cos(x)\sin^2(x)}{5(a+a\sin(x))^3} - \frac{7\cos(x)}{15a(a+a\sin(x))^2} + \frac{\int \frac{-14a^2+15a^2\sin(x)}{a+a\sin(x)}\,dx}{15a^4}\\
&= \frac{x}{a^3} + \frac{\cos(x)\sin^2(x)}{5(a+a\sin(x))^3} - \frac{7\cos(x)}{15a(a+a\sin(x))^2} - \frac{29\int \frac{1}{a+a\sin(x)}\,dx}{15a^2}\\
&= \frac{x}{a^3} + \frac{\cos(x)\sin^2(x)}{5(a+a\sin(x))^3} - \frac{7\cos(x)}{15a(a+a\sin(x))^2} + \frac{29\cos(x)}{15\left(a^3+a^3\sin(x)\right)}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 112, normalized size = 1.90

$$\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(150x\sin\left(\frac{x}{2}\right)-370\sin\left(\frac{x}{2}\right)+75x\sin\left(\frac{3x}{2}\right)-90\sin\left(\frac{3x}{2}\right)-15x\sin\left(\frac{5x}{2}\right)+64\sin\left(\frac{5x}{2}\right)+30(5x\right.}{60a^3(\sin(x)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a + a*Sin[x])^3,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(30*(-9 + 5*x)*Cos[x/2] + (230 - 75*x)*Cos[(3*x)/2]
- 15*x*Cos[(5*x)/2] - 370*Sin[x/2] + 150*x*Sin[x/2] - 90*Sin[(3*x)/2] + 75*
x*Sin[(3*x)/2] + 64*Sin[(5*x)/2] - 15*x*Sin[(5*x)/2]))/(60*a^3*(1 + Sin[x])
^3)
```

fricas [B] time = 0.44, size = 119, normalized size = 2.02

$$\frac{(15\,x+32)\cos(x)^3+(45\,x-19)\cos(x)^2-6\,(5\,x+9)\cos(x)+\left((15\,x-32)\cos(x)^2-3\,(10\,x+17)\cos(x)-60\,x\right.}{15\left(a^3\cos(x)^3+3\,a^3\cos(x)^2-2\,a^3\cos(x)-4\,a^3+\left(a^3\cos(x)^2-2\,a^3\cos(x)-4\,a^3\right)\sin(x)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/15*((15*x + 32)*cos(x)^3 + (45*x - 19)*cos(x)^2 - 6*(5*x + 9)*cos(x) + ((
15*x - 32)*cos(x)^2 - 3*(10*x + 17)*cos(x) - 60*x + 3)*sin(x) - 60*x - 3)/(
a^3*cos(x)^3 + 3*a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3 + (a^3*cos(x)^2 - 2*a^
3*cos(x) - 4*a^3)*sin(x))
```

giac [A] time = 0.33, size = 51, normalized size = 0.86

$$\frac{x}{a^3}+\frac{2\left(15\,\tan\left(\frac{1}{2}\,x\right)^4+75\,\tan\left(\frac{1}{2}\,x\right)^3+145\,\tan\left(\frac{1}{2}\,x\right)^2+95\,\tan\left(\frac{1}{2}\,x\right)+22\right)}{15\,a^3\left(\tan\left(\frac{1}{2}\,x\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] x/a^3 + 2/15*(15*tan(1/2*x)^4 + 75*tan(1/2*x)^3 + 145*tan(1/2*x)^2 + 95*tan
(1/2*x) + 22)/(a^3*(tan(1/2*x) + 1)^5)
```

maple [A] time = 0.10, size = 77, normalized size = 1.31

$$\frac{2\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a^3}-\frac{4}{a^3\left(\tan\left(\frac{x}{2}\right)+1\right)^4}+\frac{8}{5a^3\left(\tan\left(\frac{x}{2}\right)+1\right)^5}+\frac{4}{3a^3\left(\tan\left(\frac{x}{2}\right)+1\right)^3}+\frac{2}{a^3\left(\tan\left(\frac{x}{2}\right)+1\right)^2}+\frac{2}{a^3\left(\tan\left(\frac{x}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a+a*sin(x))^3,x)
```

```
[Out] 2/a^3*arctan(tan(1/2*x))-4/a^3/(tan(1/2*x)+1)^4+8/5/a^3/(tan(1/2*x)+1)^5+4/
3/a^3/(tan(1/2*x)+1)^3+2/a^3/(tan(1/2*x)+1)^2+2/a^3/(tan(1/2*x)+1)
```

maxima [B] time = 1.02, size = 144, normalized size = 2.44

$$\frac{2\left(\frac{95\sin(x)}{\cos(x)+1}+\frac{145\sin(x)^2}{(\cos(x)+1)^2}+\frac{75\sin(x)^3}{(\cos(x)+1)^3}+\frac{15\sin(x)^4}{(\cos(x)+1)^4}+22\right)}{15\left(a^3+\frac{5a^3\sin(x)}{\cos(x)+1}+\frac{10a^3\sin(x)^2}{(\cos(x)+1)^2}+\frac{10a^3\sin(x)^3}{(\cos(x)+1)^3}+\frac{5a^3\sin(x)^4}{(\cos(x)+1)^4}+\frac{a^3\sin(x)^5}{(\cos(x)+1)^5}\right)}+\frac{2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] 2/15*(95*sin(x)/(cos(x) + 1) + 145*sin(x)^2/(cos(x) + 1)^2 + 75*sin(x)^3/(c
os(x) + 1)^3 + 15*sin(x)^4/(cos(x) + 1)^4 + 22)/(a^3 + 5*a^3*sin(x)/(cos(x)
 + 1) + 10*a^3*sin(x)^2/(cos(x) + 1)^2 + 10*a^3*sin(x)^3/(cos(x) + 1)^3 + 5
*a^3*sin(x)^4/(cos(x) + 1)^4 + a^3*sin(x)^5/(cos(x) + 1)^5) + 2*arctan(sin(
x)/(cos(x) + 1))/a^3
```

mupad [B] time = 6.74, size = 50, normalized size = 0.85

$$\frac{x}{a^3}+\frac{2\tan\left(\frac{x}{2}\right)^4+10\tan\left(\frac{x}{2}\right)^3+\frac{58\tan\left(\frac{x}{2}\right)^2}{3}+\frac{38\tan\left(\frac{x}{2}\right)}{3}+\frac{44}{15}}{a^3\left(\tan\left(\frac{x}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a + a*sin(x))^3,x)
```

```
[Out] x/a^3 + ((38*tan(x/2))/3 + (58*tan(x/2)^2)/3 + 10*tan(x/2)^3 + 2*tan(x/2)^4
 + 44/15)/(a^3*(tan(x/2) + 1)^5)
```

sympy [B] time = 11.82, size = 777, normalized size = 13.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a+a*sin(x))**3,x)
```

```
[Out] 15*x*tan(x/2)**5/(15*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*tan(
x/2)**3 + 150*a**3*tan(x/2)**2 + 75*a**3*tan(x/2) + 15*a**3) + 75*x*tan(x/2
)**4/(15*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*tan(x/2)**3 + 15
0*a**3*tan(x/2)**2 + 75*a**3*tan(x/2) + 15*a**3) + 150*x*tan(x/2)**3/(15*a*
*3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*tan(x/2)**3 + 150*a**3*tan(
x/2)**2 + 75*a**3*tan(x/2) + 15*a**3) + 150*x*tan(x/2)**2/(15*a**3*tan(x/2)
```

```
**5 + 75*a**3*tan(x/2)**4 + 150*a**3*tan(x/2)**3 + 150*a**3*tan(x/2)**2 + 7
5*a**3*tan(x/2) + 15*a**3) + 75*x*tan(x/2)/(15*a**3*tan(x/2)**5 + 75*a**3*t
an(x/2)**4 + 150*a**3*tan(x/2)**3 + 150*a**3*tan(x/2)**2 + 75*a**3*tan(x/2)
 + 15*a**3) + 15*x/(15*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*ta
n(x/2)**3 + 150*a**3*tan(x/2)**2 + 75*a**3*tan(x/2) + 15*a**3) + 30*tan(x/2
)**4/(15*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*tan(x/2)**3 + 15
0*a**3*tan(x/2)**2 + 75*a**3*tan(x/2) + 15*a**3) + 150*tan(x/2)**3/(15*a**3
*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*tan(x/2)**3 + 150*a**3*tan(x/
2)**2 + 75*a**3*tan(x/2) + 15*a**3) + 290*tan(x/2)**2/(15*a**3*tan(x/2)**5
+ 75*a**3*tan(x/2)**4 + 150*a**3*tan(x/2)**3 + 150*a**3*tan(x/2)**2 + 75*a*
*3*tan(x/2) + 15*a**3) + 190*tan(x/2)/(15*a**3*tan(x/2)**5 + 75*a**3*tan(x/
2)**4 + 150*a**3*tan(x/2)**3 + 150*a**3*tan(x/2)**2 + 75*a**3*tan(x/2) + 15
*a**3) + 44/(15*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*tan(x/2)*
*3 + 150*a**3*tan(x/2)**2 + 75*a**3*tan(x/2) + 15*a**3)
```

3.25 $\int \frac{\sin^2(x)}{(a+a\sin(x))^3} \, dx$

Optimal. Leaf size=50

$$-\frac{7\cos(x)}{15\left(a^3\sin(x)+a^3\right)}+\frac{8\cos(x)}{15a(a\sin(x)+a)^2}-\frac{\cos(x)}{5(a\sin(x)+a)^3}$$

```
[Out] -1/5*cos(x)/(a+a*sin(x))^3+8/15*cos(x)/a/(a+a*sin(x))^2-7/15*cos(x)/(a^3+a^
3*sin(x))
```

Rubi [A] time = 0.08, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2758, 2750, 2648}

$$-\frac{7\cos(x)}{15\left(a^3\sin(x)+a^3\right)}+\frac{8\cos(x)}{15a(a\sin(x)+a)^2}-\frac{\cos(x)}{5(a\sin(x)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a + a*Sin[x])^3,x]
```

```
[Out] -Cos[x]/(5*(a + a*Sin[x])^3) + (8*Cos[x])/(15*a*(a + a*Sin[x])^2) - (7*Cos[
x])/(15*(a^3 + a^3*Sin[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2758

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

```
&& LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(x)}{(a+a\sin(x))^3}\,dx &= -\frac{\cos(x)}{5(a+a\sin(x))^3}+\frac{\int \frac{-3a+5a\sin(x)}{(a+a\sin(x))^2}\,dx}{5a^2}\\ &= -\frac{\cos(x)}{5(a+a\sin(x))^3}+\frac{8\cos(x)}{15a(a+a\sin(x))^2}+\frac{7\int \frac{1}{a+a\sin(x)}\,dx}{15a^2}\\ &= -\frac{\cos(x)}{5(a+a\sin(x))^3}+\frac{8\cos(x)}{15a(a+a\sin(x))^2}-\frac{7\cos(x)}{15\left(a^3+a^3\sin(x)\right)}\end{aligned}$$

Mathematica [A] time = 0.07, size = 47, normalized size = 0.94

$$\frac{105\sin(x)-12\sin(2x)-7\sin(3x)-15\cos(x)-42\cos(2x)+7\cos(3x)+70}{60a^3(\sin(x)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a + a*Sin[x])^3,x]
[Out] (70 - 15*Cos[x] - 42*Cos[2*x] + 7*Cos[3*x] + 105*Sin[x] - 12*Sin[2*x] - 7*S
in[3*x])/(60*a^3*(1 + Sin[x])^3)
```

fricas [B] time = 0.42, size = 90, normalized size = 1.80

$$-\frac{7\cos(x)^3+\cos(x)^2-\left(7\cos(x)^2+6\cos(x)-3\right)\sin(x)-9\cos(x)-3}{15\left(a^3\cos(x)^3+3a^3\cos(x)^2-2a^3\cos(x)-4a^3+\left(a^3\cos(x)^2-2a^3\cos(x)-4a^3\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*sin(x))^3,x, algorithm="fricas")
[Out] -1/15*(7*cos(x)^3 + cos(x)^2 - (7*cos(x)^2 + 6*cos(x) - 3)*sin(x) - 9*cos(x
) - 3)/(a^3*cos(x)^3 + 3*a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3 + (a^3*cos(x)^
2 - 2*a^3*cos(x) - 4*a^3)*sin(x))
```

giac [A] time = 0.27, size = 29, normalized size = 0.58

$$-\frac{4\left(10\tan\left(\frac{1}{2}x\right)^2+5\tan\left(\frac{1}{2}x\right)+1\right)}{15a^3\left(\tan\left(\frac{1}{2}x\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] -4/15*(10*tan(1/2*x)^2 + 5*tan(1/2*x) + 1)/(a^3*(tan(1/2*x) + 1)^5)
```

maple [A] time = 0.09, size = 37, normalized size = 0.74

$$\frac{-\frac{8}{5\left(\tan\left(\frac{x}{2}\right)+1\right)^{5}}+\frac{4}{\left(\tan\left(\frac{x}{2}\right)+1\right)^{4}}-\frac{8}{3\left(\tan\left(\frac{x}{2}\right)+1\right)^{3}}}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a+a*sin(x))^3,x)
```

```
[Out] 8/a^3*(-1/5/(tan(1/2*x)+1)^5+1/2/(tan(1/2*x)+1)^4-1/3/(tan(1/2*x)+1)^3)
```

maxima [B] time = 0.64, size = 104, normalized size = 2.08

$$-\frac{4\left(\frac{5\,\sin(x)}{\cos(x)+1}+\frac{10\,\sin(x)^{2}}{(\cos(x)+1)^{2}}+1\right)}{15\left(a^{3}+\frac{5\,a^{3}\sin(x)}{\cos(x)+1}+\frac{10\,a^{3}\sin(x)^{2}}{(\cos(x)+1)^{2}}+\frac{10\,a^{3}\sin(x)^{3}}{(\cos(x)+1)^{3}}+\frac{5\,a^{3}\sin(x)^{4}}{(\cos(x)+1)^{4}}+\frac{a^{3}\sin(x)^{5}}{(\cos(x)+1)^{5}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -4/15*(5*sin(x)/(cos(x) + 1) + 10*sin(x)^2/(cos(x) + 1)^2 + 1)/(a^3 + 5*a^3
*sin(x)/(cos(x) + 1) + 10*a^3*sin(x)^2/(cos(x) + 1)^2 + 10*a^3*sin(x)^3/(co
s(x) + 1)^3 + 5*a^3*sin(x)^4/(cos(x) + 1)^4 + a^3*sin(x)^5/(cos(x) + 1)^5)
```

mupad [B] time = 6.64, size = 29, normalized size = 0.58

$$-\frac{4\left(10\tan\left(\frac{x}{2}\right)^{2}+5\tan\left(\frac{x}{2}\right)+1\right)}{15\,a^{3}\left(\tan\left(\frac{x}{2}\right)+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a + a*sin(x))^3,x)
```

```
[Out] -(4*(5*tan(x/2) + 10*tan(x/2)^2 + 1))/(15*a^3*(tan(x/2) + 1)^5)
```

sympy [B] time = 6.92, size = 206, normalized size = 4.12

$$-\frac{40\tan^{2}\left(\frac{x}{2}\right)}{15a^{3}\tan^{5}\left(\frac{x}{2}\right)+75a^{3}\tan^{4}\left(\frac{x}{2}\right)+150a^{3}\tan^{3}\left(\frac{x}{2}\right)+150a^{3}\tan^{2}\left(\frac{x}{2}\right)+75a^{3}\tan\left(\frac{x}{2}\right)+15a^{3}}-\frac{}{15a^{3}\tan^{5}\left(\frac{x}{2}\right)+75a^{3}\,\mathrm{t}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+a*sin(x))**3,x)
```

```
[Out] -40*tan(x/2)**2/(15*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*tan(x
/2)**3 + 150*a**3*tan(x/2)**2 + 75*a**3*tan(x/2) + 15*a**3) - 20*tan(x/2)/(
15*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*tan(x/2)**3 + 150*a**3
*tan(x/2)**2 + 75*a**3*tan(x/2) + 15*a**3) - 4/(15*a**3*tan(x/2)**5 + 75*a*
*3*tan(x/2)**4 + 150*a**3*tan(x/2)**3 + 150*a**3*tan(x/2)**2 + 75*a**3*tan(
x/2) + 15*a**3)
```

3.26 $\int \frac{\sin(x)}{(a+a\sin(x))^3}\,dx$

Optimal. Leaf size=50

$$-\frac{\cos(x)}{5\left(a^3\sin(x)+a^3\right)}-\frac{\cos(x)}{5a(a\sin(x)+a)^2}+\frac{\cos(x)}{5(a\sin(x)+a)^3}$$

```
[Out] 1/5*cos(x)/(a+a*sin(x))^3-1/5*cos(x)/a/(a+a*sin(x))^2-1/5*cos(x)/(a^3+a^3*s
in(x))
```

Rubi [A] time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2750, 2650, 2648}

$$-\frac{\cos(x)}{5\left(a^3\sin(x)+a^3\right)}-\frac{\cos(x)}{5a(a\sin(x)+a)^2}+\frac{\cos(x)}{5(a\sin(x)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a + a*Sin[x])^3,x]
```

```
[Out] Cos[x]/(5*(a + a*Sin[x])^3) - Cos[x]/(5*a*(a + a*Sin[x])^2) - Cos[x]/(5*(a^
3 + a^3*Sin[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin(x)}{(a+a\sin(x))^3}\,dx &= \frac{\cos(x)}{5(a+a\sin(x))^3}+\frac{3\int \frac{1}{(a+a\sin(x))^2}\,dx}{5a}\\
&= \frac{\cos(x)}{5(a+a\sin(x))^3}-\frac{\cos(x)}{5a(a+a\sin(x))^2}+\frac{\int \frac{1}{a+a\sin(x)}\,dx}{5a^2}\\
&= \frac{\cos(x)}{5(a+a\sin(x))^3}-\frac{\cos(x)}{5a(a+a\sin(x))^2}-\frac{\cos(x)}{5\left(a^3+a^3\sin(x)\right)}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 41, normalized size = 0.82

$$
\frac{\sin^2\left(\frac{x}{2}\right)(8\sin(x)+\sin(2x)+4\cos(x)-\cos(2x)+7)}{5a^3(\sin(x)+1)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a + a*Sin[x])^3,x]
```

```
[Out] (Sin[x/2]^2*(7 + 4*Cos[x] - Cos[2*x] + 8*Sin[x] + Sin[2*x]))/(5*a^3*(1 + Si
n[x])^3)
```

fricas [A] time = 0.48, size = 88, normalized size = 1.76

$$
-\frac{\cos(x)^3-2\cos(x)^2-\left(\cos(x)^2+3\cos(x)+1\right)\sin(x)-2\cos(x)+1}{5\left(a^3\cos(x)^3+3a^3\cos(x)^2-2a^3\cos(x)-4a^3+\left(a^3\cos(x)^2-2a^3\cos(x)-4a^3\right)\sin(x)\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/5*(cos(x)^3 - 2*cos(x)^2 - (cos(x)^2 + 3*cos(x) + 1)*sin(x) - 2*cos(x) +
 1)/(a^3*cos(x)^3 + 3*a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3 + (a^3*cos(x)^2 -
 2*a^3*cos(x) - 4*a^3)*sin(x))
```

giac [A] time = 0.32, size = 37, normalized size = 0.74

$$
-\frac{2\left(5\tan\left(\frac{1}{2}x\right)^3+5\tan\left(\frac{1}{2}x\right)^2+5\tan\left(\frac{1}{2}x\right)+1\right)}{5a^3\left(\tan\left(\frac{1}{2}x\right)+1\right)^5}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] -2/5*(5*tan(1/2*x)^3 + 5*tan(1/2*x)^2 + 5*tan(1/2*x) + 1)/(a^3*(tan(1/2*x)
+ 1)^5)
```

maple [A] time = 0.09, size = 45, normalized size = 0.90

$$\frac{\frac{8}{5\left(\tan\left(\frac{x}{2}\right)+1\right)^{5}}+\frac{4}{\left(\tan\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{2}{\left(\tan\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{4}{\left(\tan\left(\frac{x}{2}\right)+1\right)^{4}}}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a+a*sin(x))^3,x)
```

```
[Out] 4/a^3*(2/5/(tan(1/2*x)+1)^5+1/(tan(1/2*x)+1)^3-1/2/(tan(1/2*x)+1)^2-1/(tan(
1/2*x)+1)^4)
```

maxima [B] time = 0.69, size = 116, normalized size = 2.32

$$-\frac{2\left(\frac{5\sin(x)}{\cos(x)+1}+\frac{5\sin(x)^{2}}{(\cos(x)+1)^{2}}+\frac{5\sin(x)^{3}}{(\cos(x)+1)^{3}}+1\right)}{5\left(a^{3}+\frac{5a^{3}\sin(x)}{\cos(x)+1}+\frac{10a^{3}\sin(x)^{2}}{(\cos(x)+1)^{2}}+\frac{10a^{3}\sin(x)^{3}}{(\cos(x)+1)^{3}}+\frac{5a^{3}\sin(x)^{4}}{(\cos(x)+1)^{4}}+\frac{a^{3}\sin(x)^{5}}{(\cos(x)+1)^{5}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -2/5*(5*sin(x)/(cos(x) + 1) + 5*sin(x)^2/(cos(x) + 1)^2 + 5*sin(x)^3/(cos(x
) + 1)^3 + 1)/(a^3 + 5*a^3*sin(x)/(cos(x) + 1) + 10*a^3*sin(x)^2/(cos(x) +
1)^2 + 10*a^3*sin(x)^3/(cos(x) + 1)^3 + 5*a^3*sin(x)^4/(cos(x) + 1)^4 + a^3
*sin(x)^5/(cos(x) + 1)^5)
```

mupad [B] time = 6.85, size = 37, normalized size = 0.74

$$-\frac{2\left(5\tan\left(\frac{x}{2}\right)^{3}+5\tan\left(\frac{x}{2}\right)^{2}+5\tan\left(\frac{x}{2}\right)+1\right)}{5a^{3}\left(\tan\left(\frac{x}{2}\right)+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a + a*sin(x))^3,x)
```

```
[Out] -(2*(5*tan(x/2) + 5*tan(x/2)^2 + 5*tan(x/2)^3 + 1))/(5*a^3*(tan(x/2) + 1)^5
)
```

sympy [B] time = 4.74, size = 277, normalized size = 5.54

$$-\frac{10\tan^3\left(\frac{x}{2}\right)}{5a^3\tan^5\left(\frac{x}{2}\right)+25a^3\tan^4\left(\frac{x}{2}\right)+50a^3\tan^3\left(\frac{x}{2}\right)+50a^3\tan^2\left(\frac{x}{2}\right)+25a^3\tan\left(\frac{x}{2}\right)+5a^3}-\frac{}{5a^3\tan^5\left(\frac{x}{2}\right)+25a^3\tan^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*sin(x))**3,x)
```

```
[Out] -10*tan(x/2)**3/(5*a**3*tan(x/2)**5 + 25*a**3*tan(x/2)**4 + 50*a**3*tan(x/2
)**3 + 50*a**3*tan(x/2)**2 + 25*a**3*tan(x/2) + 5*a**3) - 10*tan(x/2)**2/(5
*a**3*tan(x/2)**5 + 25*a**3*tan(x/2)**4 + 50*a**3*tan(x/2)**3 + 50*a**3*tan
(x/2)**2 + 25*a**3*tan(x/2) + 5*a**3) - 10*tan(x/2)/(5*a**3*tan(x/2)**5 + 2
5*a**3*tan(x/2)**4 + 50*a**3*tan(x/2)**3 + 50*a**3*tan(x/2)**2 + 25*a**3*ta
n(x/2) + 5*a**3) - 2/(5*a**3*tan(x/2)**5 + 25*a**3*tan(x/2)**4 + 50*a**3*ta
n(x/2)**3 + 50*a**3*tan(x/2)**2 + 25*a**3*tan(x/2) + 5*a**3)
```

3.27 $\int \frac{1}{(a+a\sin(x))^3}\,dx$

Optimal. Leaf size=50

$$-\frac{2\cos(x)}{15\left(a^3\sin(x)+a^3\right)}-\frac{2\cos(x)}{15a(a\sin(x)+a)^2}-\frac{\cos(x)}{5(a\sin(x)+a)^3}$$

```
[Out] -1/5*cos(x)/(a+a*sin(x))^3-2/15*cos(x)/a/(a+a*sin(x))^2-2/15*cos(x)/(a^3+a^
3*sin(x))
```

Rubi [A] time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2650, 2648}

$$-\frac{2\cos(x)}{15\left(a^3\sin(x)+a^3\right)}-\frac{2\cos(x)}{15a(a\sin(x)+a)^2}-\frac{\cos(x)}{5(a\sin(x)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[x])^(-3),x]
```

```
[Out] -Cos[x]/(5*(a + a*Sin[x])^3) - (2*Cos[x])/(15*a*(a + a*Sin[x])^2) - (2*Cos[
x])/(15*(a^3 + a^3*Sin[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(x))^3}\,dx = -\frac{\cos(x)}{5(a+a\sin(x))^3} + \frac{2\int \frac{1}{(a+a\sin(x))^2}\,dx}{5a}$$
$$= -\frac{\cos(x)}{5(a+a\sin(x))^3} - \frac{2\cos(x)}{15a(a+a\sin(x))^2} + \frac{2\int \frac{1}{a+a\sin(x)}\,dx}{15a^2}$$
$$= -\frac{\cos(x)}{5(a+a\sin(x))^3} - \frac{2\cos(x)}{15a(a+a\sin(x))^2} - \frac{2\cos(x)}{15\left(a^3+a^3\sin(x)\right)}$$

Mathematica [A] time = 0.06, size = 45, normalized size = 0.90

$$-\frac{-10\sin\left(\frac{x}{2}\right)+\sin\left(\frac{5x}{2}\right)+5\cos\left(\frac{3x}{2}\right)}{15a^3\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[x])^(-3),x]
```

```
[Out] -1/15*(5*Cos[(3*x)/2] - 10*Sin[x/2] + Sin[(5*x)/2])/(a^3*(Cos[x/2] + Sin[x/
2])^5)
```

fricas [B] time = 0.42, size = 92, normalized size = 1.84

$$-\frac{2\cos(x)^3-4\cos(x)^2-\left(2\cos(x)^2+6\cos(x)-3\right)\sin(x)-9\cos(x)-3}{15\left(a^3\cos(x)^3+3a^3\cos(x)^2-2a^3\cos(x)-4a^3+\left(a^3\cos(x)^2-2a^3\cos(x)-4a^3\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(2*cos(x)^3 - 4*cos(x)^2 - (2*cos(x)^2 + 6*cos(x) - 3)*sin(x) - 9*cos
(x) - 3)/(a^3*cos(x)^3 + 3*a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3 + (a^3*cos(x
)^2 - 2*a^3*cos(x) - 4*a^3)*sin(x))
```

giac [A] time = 0.50, size = 45, normalized size = 0.90

$$-\frac{2\left(15\tan\left(\frac{1}{2}x\right)^4+30\tan\left(\frac{1}{2}x\right)^3+40\tan\left(\frac{1}{2}x\right)^2+20\tan\left(\frac{1}{2}x\right)+7\right)}{15a^3\left(\tan\left(\frac{1}{2}x\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*tan(1/2*x)^4 + 30*tan(1/2*x)^3 + 40*tan(1/2*x)^2 + 20*tan(1/2*x)
+ 7)/(a^3*(tan(1/2*x) + 1)^5)
```

maple [A] time = 0.08, size = 57, normalized size = 1.14

$$\frac{\frac{4}{\left(\tan\left(\frac{x}{2}\right)+1\right)^{4}}-\frac{8}{5\left(\tan\left(\frac{x}{2}\right)+1\right)^{5}}-\frac{16}{3\left(\tan\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{2}{\tan\left(\frac{x}{2}\right)+1}+\frac{4}{\left(\tan\left(\frac{x}{2}\right)+1\right)^{2}}}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(x))^3,x)
```

```
[Out] 2/a^3*(2/(tan(1/2*x)+1)^4-4/5/(tan(1/2*x)+1)^5-8/3/(tan(1/2*x)+1)^3-1/(tan(
1/2*x)+1)+2/(tan(1/2*x)+1)^2)
```

maxima [B] time = 0.46, size = 128, normalized size = 2.56

$$-\frac{2\left(\frac{20\sin(x)}{\cos(x)+1}+\frac{40\sin(x)^{2}}{\left(\cos(x)+1\right)^{2}}+\frac{30\sin(x)^{3}}{\left(\cos(x)+1\right)^{3}}+\frac{15\sin(x)^{4}}{\left(\cos(x)+1\right)^{4}}+7\right)}{15\left(a^{3}+\frac{5a^{3}\sin(x)}{\cos(x)+1}+\frac{10a^{3}\sin(x)^{2}}{\left(\cos(x)+1\right)^{2}}+\frac{10a^{3}\sin(x)^{3}}{\left(\cos(x)+1\right)^{3}}+\frac{5a^{3}\sin(x)^{4}}{\left(\cos(x)+1\right)^{4}}+\frac{a^{3}\sin(x)^{5}}{\left(\cos(x)+1\right)^{5}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(20*sin(x)/(cos(x) + 1) + 40*sin(x)^2/(cos(x) + 1)^2 + 30*sin(x)^3/(c
os(x) + 1)^3 + 15*sin(x)^4/(cos(x) + 1)^4 + 7)/(a^3 + 5*a^3*sin(x)/(cos(x)
+ 1) + 10*a^3*sin(x)^2/(cos(x) + 1)^2 + 10*a^3*sin(x)^3/(cos(x) + 1)^3 + 5*
a^3*sin(x)^4/(cos(x) + 1)^4 + a^3*sin(x)^5/(cos(x) + 1)^5)
```

mupad [B] time = 6.63, size = 45, normalized size = 0.90

$$-\frac{2\left(15\tan\left(\frac{x}{2}\right)^{4}+30\tan\left(\frac{x}{2}\right)^{3}+40\tan\left(\frac{x}{2}\right)^{2}+20\tan\left(\frac{x}{2}\right)+7\right)}{15a^{3}\left(\tan\left(\frac{x}{2}\right)+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sin(x))^3,x)
```

```
[Out] -(2*(20*tan(x/2) + 40*tan(x/2)^2 + 30*tan(x/2)^3 + 15*tan(x/2)^4 + 7))/(15*
a^3*(tan(x/2) + 1)^5)
```

sympy [B] time = 2.23, size = 348, normalized size = 6.96

$$-\frac{30\tan^4\left(\frac{x}{2}\right)}{15a^3\tan^5\left(\frac{x}{2}\right)+75a^3\tan^4\left(\frac{x}{2}\right)+150a^3\tan^3\left(\frac{x}{2}\right)+150a^3\tan^2\left(\frac{x}{2}\right)+75a^3\tan\left(\frac{x}{2}\right)+15a^3}-\frac{}{15a^3\tan^5\left(\frac{x}{2}\right)+75a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(x))**3,x)
```

```
[Out] -30*tan(x/2)**4/(15*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*tan(x
/2)**3 + 150*a**3*tan(x/2)**2 + 75*a**3*tan(x/2) + 15*a**3) - 60*tan(x/2)**
3/(15*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*tan(x/2)**3 + 150*a
**3*tan(x/2)**2 + 75*a**3*tan(x/2) + 15*a**3) - 80*tan(x/2)**2/(15*a**3*tan
(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*tan(x/2)**3 + 150*a**3*tan(x/2)**
2 + 75*a**3*tan(x/2) + 15*a**3) - 40*tan(x/2)/(15*a**3*tan(x/2)**5 + 75*a**
3*tan(x/2)**4 + 150*a**3*tan(x/2)**3 + 150*a**3*tan(x/2)**2 + 75*a**3*tan(x
/2) + 15*a**3) - 14/(15*a**3*tan(x/2)**5 + 75*a**3*tan(x/2)**4 + 150*a**3*t
an(x/2)**3 + 150*a**3*tan(x/2)**2 + 75*a**3*tan(x/2) + 15*a**3)
```

3.28 $\int \frac{\csc(x)}{(a+a\sin(x))^3}\,dx$

Optimal. Leaf size=58

$$\frac{22\cos(x)}{15\left(a^3\sin(x)+a^3\right)}-\frac{\tanh^{-1}(\cos(x))}{a^3}+\frac{7\cos(x)}{15a(a\sin(x)+a)^2}+\frac{\cos(x)}{5(a\sin(x)+a)^3}$$

```
[Out] -arctanh(cos(x))/a^3+1/5*cos(x)/(a+a*sin(x))^3+7/15*cos(x)/a/(a+a*sin(x))^2
+22/15*cos(x)/(a^3+a^3*sin(x))
```

Rubi [A] time = 0.16, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2766, 2978, 12, 3770}

$$\frac{22\cos(x)}{15\left(a^3\sin(x)+a^3\right)}-\frac{\tanh^{-1}(\cos(x))}{a^3}+\frac{7\cos(x)}{15a(a\sin(x)+a)^2}+\frac{\cos(x)}{5(a\sin(x)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a + a*Sin[x])^3,x]
```

```
[Out] -(ArcTanh[Cos[x]]/a^3) + Cos[x]/(5*(a + a*Sin[x])^3) + (7*Cos[x])/(15*a*(a
+ a*Sin[x])^2) + (22*Cos[x])/(15*(a^3 + a^3*Sin[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(x)}{(a+a\sin(x))^3}\,dx &= \frac{\cos(x)}{5(a+a\sin(x))^3}+\frac{\int \frac{\csc(x)(5a-2a\sin(x))}{(a+a\sin(x))^2}\,dx}{5a^2}\\
&= \frac{\cos(x)}{5(a+a\sin(x))^3}+\frac{7\cos(x)}{15a(a+a\sin(x))^2}+\frac{\int \frac{\csc(x)\left(15a^2-7a^2\sin(x)\right)}{a+a\sin(x)}\,dx}{15a^4}\\
&= \frac{\cos(x)}{5(a+a\sin(x))^3}+\frac{7\cos(x)}{15a(a+a\sin(x))^2}+\frac{22\cos(x)}{15\left(a^3+a^3\sin(x)\right)}+\frac{\int 15a^3\csc(x)\,dx}{15a^6}\\
&= \frac{\cos(x)}{5(a+a\sin(x))^3}+\frac{7\cos(x)}{15a(a+a\sin(x))^2}+\frac{22\cos(x)}{15\left(a^3+a^3\sin(x)\right)}+\frac{\int \csc(x)\,dx}{a^3}\\
&= -\frac{\tanh^{-1}(\cos(x))}{a^3}+\frac{\cos(x)}{5(a+a\sin(x))^3}+\frac{7\cos(x)}{15a(a+a\sin(x))^2}+\frac{22\cos(x)}{15\left(a^3+a^3\sin(x)\right)}
\end{aligned}$$

Mathematica [B] time = 0.07, size = 160, normalized size = 2.76

$$\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(-6\sin\left(\frac{x}{2}\right)-44\sin\left(\frac{x}{2}\right)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^4+7\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^3-14\sin\left(\frac{x}{2}\right)\left(\sin\left(\frac{x}{2}\right)+\right.}{15(a\sin}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a + a*Sin[x])^3,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(-6*Sin[x/2] + 3*(Cos[x/2] + Sin[x/2]) - 14*Sin[x/2]
*(Cos[x/2] + Sin[x/2])^2 + 7*(Cos[x/2] + Sin[x/2])^3 - 44*Sin[x/2]*(Cos[x/2
```

```
] + Sin[x/2])^4 - 15*Log[Cos[x/2]]*(Cos[x/2] + Sin[x/2])^5 + 15*Log[Sin[x/2
]]*(Cos[x/2] + Sin[x/2])^5))/(15*(a + a*Sin[x])^3)
```

fricas [B] time = 0.47, size = 168, normalized size = 2.90

$$\frac{44\cos(x)^3-58\cos(x)^2-15\left(\cos(x)^3+3\cos(x)^2+\left(\cos(x)^2-2\cos(x)-4\right)\sin(x)-2\cos(x)-4\right)\log\left(\frac{1}{2}\cos(x\right.}{30\left(a^3\cos(x)^3+3a^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/30*(44*cos(x)^3 - 58*cos(x)^2 - 15*(cos(x)^3 + 3*cos(x)^2 + (cos(x)^2 - 2
*cos(x) - 4)*sin(x) - 2*cos(x) - 4)*log(1/2*cos(x) + 1/2) + 15*(cos(x)^3 +
3*cos(x)^2 + (cos(x)^2 - 2*cos(x) - 4)*sin(x) - 2*cos(x) - 4)*log(-1/2*cos(
x) + 1/2) - 2*(22*cos(x)^2 + 51*cos(x) - 3)*sin(x) - 108*cos(x) - 6)/(a^3*c
os(x)^3 + 3*a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3 + (a^3*cos(x)^2 - 2*a^3*cos
(x) - 4*a^3)*sin(x))
```

giac [A] time = 0.76, size = 56, normalized size = 0.97

$$\frac{\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{a^3}+\frac{2\left(45\tan\left(\frac{1}{2}x\right)^4+135\tan\left(\frac{1}{2}x\right)^3+185\tan\left(\frac{1}{2}x\right)^2+115\tan\left(\frac{1}{2}x\right)+32\right)}{15a^3\left(\tan\left(\frac{1}{2}x\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] log(abs(tan(1/2*x)))/a^3 + 2/15*(45*tan(1/2*x)^4 + 135*tan(1/2*x)^3 + 185*t
an(1/2*x)^2 + 115*tan(1/2*x) + 32)/(a^3*(tan(1/2*x) + 1)^5)
```

maple [A] time = 0.12, size = 76, normalized size = 1.31

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^3}+\frac{8}{5a^3\left(\tan\left(\frac{x}{2}\right)+1\right)^5}-\frac{4}{a^3\left(\tan\left(\frac{x}{2}\right)+1\right)^4}+\frac{20}{3a^3\left(\tan\left(\frac{x}{2}\right)+1\right)^3}-\frac{6}{a^3\left(\tan\left(\frac{x}{2}\right)+1\right)^2}+\frac{6}{a^3\left(\tan\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a+a*sin(x))^3,x)
```

```
[Out] 1/a^3*ln(tan(1/2*x))+8/5/a^3/(tan(1/2*x)+1)^5-4/a^3/(tan(1/2*x)+1)^4+20/3/a
^3/(tan(1/2*x)+1)^3-6/a^3/(tan(1/2*x)+1)^2+6/a^3/(tan(1/2*x)+1)
```

maxima [B] time = 0.86, size = 143, normalized size = 2.47

$$\frac{2\left(\frac{115\sin(x)}{\cos(x)+1}+\frac{185\sin(x)^2}{(\cos(x)+1)^2}+\frac{135\sin(x)^3}{(\cos(x)+1)^3}+\frac{45\sin(x)^4}{(\cos(x)+1)^4}+32\right)}{15\left(a^3+\frac{5a^3\sin(x)}{\cos(x)+1}+\frac{10a^3\sin(x)^2}{(\cos(x)+1)^2}+\frac{10a^3\sin(x)^3}{(\cos(x)+1)^3}+\frac{5a^3\sin(x)^4}{(\cos(x)+1)^4}+\frac{a^3\sin(x)^5}{(\cos(x)+1)^5}\right)}+\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] 2/15*(115*sin(x)/(cos(x) + 1) + 185*sin(x)^2/(cos(x) + 1)^2 + 135*sin(x)^3/
(cos(x) + 1)^3 + 45*sin(x)^4/(cos(x) + 1)^4 + 32)/(a^3 + 5*a^3*sin(x)/(cos(
x) + 1) + 10*a^3*sin(x)^2/(cos(x) + 1)^2 + 10*a^3*sin(x)^3/(cos(x) + 1)^3 +
 5*a^3*sin(x)^4/(cos(x) + 1)^4 + a^3*sin(x)^5/(cos(x) + 1)^5) + log(sin(x)/
(cos(x) + 1))/a^3
```

mupad [B] time = 6.65, size = 54, normalized size = 0.93

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^3}+\frac{6\tan\left(\frac{x}{2}\right)^4+18\tan\left(\frac{x}{2}\right)^3+\frac{74\tan\left(\frac{x}{2}\right)^2}{3}+\frac{46\tan\left(\frac{x}{2}\right)}{3}+\frac{64}{15}}{a^3\left(\tan\left(\frac{x}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(a + a*sin(x))^3),x)
```

```
[Out] log(tan(x/2))/a^3 + ((46*tan(x/2))/3 + (74*tan(x/2)^2)/3 + 18*tan(x/2)^3 +
6*tan(x/2)^4 + 64/15)/(a^3*(tan(x/2) + 1)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc(x)}{\sin^3(x)+3\sin^2(x)+3\sin(x)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*sin(x))**3,x)
```

```
[Out] Integral(csc(x)/(sin(x)**3 + 3*sin(x)**2 + 3*sin(x) + 1), x)/a**3
```

3.29 $\int \frac{\csc^2(x)}{(a+a\sin(x))^3} dx$

Optimal. Leaf size=65

$$-\frac{24\cot(x)}{5a^3}+\frac{3\tanh^{-1}(\cos(x))}{a^3}+\frac{3\cot(x)}{a^3\sin(x)+a^3}+\frac{3\cot(x)}{5a(a\sin(x)+a)^2}+\frac{\cot(x)}{5(a\sin(x)+a)^3}$$

```
[Out] 3*arctanh(cos(x))/a^3-24/5*cot(x)/a^3+1/5*cot(x)/(a+a*sin(x))^3+3/5*cot(x)/
a/(a+a*sin(x))^2+3*cot(x)/(a^3+a^3*sin(x))
```

Rubi [A] time = 0.23, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2766, 2978, 2748, 3767, 8, 3770}

$$-\frac{24\cot(x)}{5a^3}+\frac{3\tanh^{-1}(\cos(x))}{a^3}+\frac{3\cot(x)}{a^3\sin(x)+a^3}+\frac{3\cot(x)}{5a(a\sin(x)+a)^2}+\frac{\cot(x)}{5(a\sin(x)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a + a*Sin[x])^3,x]
```

```
[Out] (3*ArcTanh[Cos[x]])/a^3 - (24*Cot[x])/(5*a^3) + Cot[x]/(5*(a + a*Sin[x])^3)
 + (3*Cot[x])/(5*a*(a + a*Sin[x])^2) + (3*Cot[x])/(a^3 + a^3*Sin[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^2(x)}{(a+a\sin(x))^3}\,dx &= \frac{\cot(x)}{5(a+a\sin(x))^3} + \frac{\int \frac{\csc^2(x)(6a-3a\sin(x))}{(a+a\sin(x))^2}\,dx}{5a^2} \\
&= \frac{\cot(x)}{5(a+a\sin(x))^3} + \frac{3\cot(x)}{5a(a+a\sin(x))^2} + \frac{\int \frac{\csc^2(x)\left(27a^2-18a^2\sin(x)\right)}{a+a\sin(x)}\,dx}{15a^4} \\
&= \frac{\cot(x)}{5(a+a\sin(x))^3} + \frac{3\cot(x)}{5a(a+a\sin(x))^2} + \frac{3\cot(x)}{a^3+a^3\sin(x)} + \frac{\int \csc^2(x)\left(72a^3-45a^3\sin(x)\right)}{15a^6} \\
&= \frac{\cot(x)}{5(a+a\sin(x))^3} + \frac{3\cot(x)}{5a(a+a\sin(x))^2} + \frac{3\cot(x)}{a^3+a^3\sin(x)} - \frac{3\int \csc(x)\,dx}{a^3} + \frac{24\int \csc^2(x)\,dx}{5a^3} \\
&= \frac{3\tanh^{-1}(\cos(x))}{a^3} + \frac{\cot(x)}{5(a+a\sin(x))^3} + \frac{3\cot(x)}{5a(a+a\sin(x))^2} + \frac{3\cot(x)}{a^3+a^3\sin(x)} - \frac{24\,\mathrm{Subst}(\int}{} \\
&= \frac{3\tanh^{-1}(\cos(x))}{a^3} - \frac{24\cot(x)}{5a^3} + \frac{\cot(x)}{5(a+a\sin(x))^3} + \frac{3\cot(x)}{5a(a+a\sin(x))^2} + \frac{3\cot(x)}{a^3+a^3\sin(x)}
\end{aligned}
$$

Mathematica [B] time = 0.15, size = 206, normalized size = 3.17

$$\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(4\sin\left(\frac{x}{2}\right)+76\sin\left(\frac{x}{2}\right)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^4-8\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^3+16\sin\left(\frac{x}{2}\right)\left(\sin\left(\frac{x}{2}\right)+\cos\ldots\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a + a*Sin[x])^3,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(4*Sin[x/2] - 2*(Cos[x/2] + Sin[x/2]) + 16*Sin[x/2]*
(Cos[x/2] + Sin[x/2])^2 - 8*(Cos[x/2] + Sin[x/2])^3 + 76*Sin[x/2]*(Cos[x/2]
 + Sin[x/2])^4 - 5*Cot[x/2]*(Cos[x/2] + Sin[x/2])^5 + 30*Log[Cos[x/2]]*(Cos
[x/2] + Sin[x/2])^5 - 30*Log[Sin[x/2]]*(Cos[x/2] + Sin[x/2])^5 + 5*(Cos[x/2
] + Sin[x/2])^5*Tan[x/2]))/(10*(a + a*Sin[x])^3)
```

fricas [B] time = 0.51, size = 225, normalized size = 3.46

$$48\cos(x)^4+114\cos(x)^3-60\cos(x)^2+15\left(\cos(x)^4-2\cos(x)^3-5\cos(x)^2-\left(\cos(x)^3+3\cos(x)^2-2\cos(x)\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/10*(48*cos(x)^4 + 114*cos(x)^3 - 60*cos(x)^2 + 15*(cos(x)^4 - 2*cos(x)^3
- 5*cos(x)^2 - (cos(x)^3 + 3*cos(x)^2 - 2*cos(x) - 4)*sin(x) + 2*cos(x) + 4
)*log(1/2*cos(x) + 1/2) - 15*(cos(x)^4 - 2*cos(x)^3 - 5*cos(x)^2 - (cos(x)^
3 + 3*cos(x)^2 - 2*cos(x) - 4)*sin(x) + 2*cos(x) + 4)*log(-1/2*cos(x) + 1/2
) + 2*(24*cos(x)^3 - 33*cos(x)^2 - 63*cos(x) - 1)*sin(x) - 124*cos(x) + 2)/
(a^3*cos(x)^4 - 2*a^3*cos(x)^3 - 5*a^3*cos(x)^2 + 2*a^3*cos(x) + 4*a^3 - (a
^3*cos(x)^3 + 3*a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3)*sin(x))
```

giac [A] time = 0.64, size = 85, normalized size = 1.31

$$-\frac{3\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{a^3}+\frac{\tan\left(\frac{1}{2}x\right)}{2a^3}+\frac{6\tan\left(\frac{1}{2}x\right)-1}{2a^3\tan\left(\frac{1}{2}x\right)}-\frac{4\left(15\tan\left(\frac{1}{2}x\right)^4+50\tan\left(\frac{1}{2}x\right)^3+70\tan\left(\frac{1}{2}x\right)^2+45\tan\left(\frac{1}{2}x\ldots\right.\right.}{5a^3\left(\tan\left(\frac{1}{2}x\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] -3*log(abs(tan(1/2*x)))/a^3 + 1/2*tan(1/2*x)/a^3 + 1/2*(6*tan(1/2*x) - 1)/(
a^3*tan(1/2*x)) - 4/5*(15*tan(1/2*x)^4 + 50*tan(1/2*x)^3 + 70*tan(1/2*x)^2
+ 45*tan(1/2*x) + 12)/(a^3*(tan(1/2*x) + 1)^5)
```

maple [A] time = 0.14, size = 97, normalized size = 1.49

$$\frac{\tan\left(\frac{x}{2}\right)}{2a^3}-\frac{1}{2a^3\tan\left(\frac{x}{2}\right)}-\frac{3\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^3}-\frac{8}{5a^3\left(\tan\left(\frac{x}{2}\right)+1\right)^5}+\frac{4}{a^3\left(\tan\left(\frac{x}{2}\right)+1\right)^4}-\frac{8}{a^3\left(\tan\left(\frac{x}{2}\right)+1\right)^3}+\frac{8}{a^3\left(\tan\left(\frac{x}{2}\right)+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a+a*sin(x))^3,x)
```

```
[Out] 1/2/a^3*tan(1/2*x)-1/2/a^3/tan(1/2*x)-3/a^3*ln(tan(1/2*x))-8/5/a^3/(tan(1/2
*x)+1)^5+4/a^3/(tan(1/2*x)+1)^4-8/a^3/(tan(1/2*x)+1)^3+8/a^3/(tan(1/2*x)+1)
^2-12/a^3/(tan(1/2*x)+1)
```

maxima [B] time = 0.75, size = 180, normalized size = 2.77

$$-\frac{\frac{121\sin(x)}{\cos(x)+1}+\frac{410\sin(x)^2}{(\cos(x)+1)^2}+\frac{610\sin(x)^3}{(\cos(x)+1)^3}+\frac{425\sin(x)^4}{(\cos(x)+1)^4}+\frac{125\sin(x)^5}{(\cos(x)+1)^5}+5}{10\left(\frac{a^3\sin(x)}{\cos(x)+1}+\frac{5a^3\sin(x)^2}{(\cos(x)+1)^2}+\frac{10a^3\sin(x)^3}{(\cos(x)+1)^3}+\frac{10a^3\sin(x)^4}{(\cos(x)+1)^4}+\frac{5a^3\sin(x)^5}{(\cos(x)+1)^5}+\frac{a^3\sin(x)^6}{(\cos(x)+1)^6}\right)}-\frac{3\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^3}+\frac{\sin(x)}{2a^3(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -1/10*(121*sin(x)/(cos(x) + 1) + 410*sin(x)^2/(cos(x) + 1)^2 + 610*sin(x)^3
/(cos(x) + 1)^3 + 425*sin(x)^4/(cos(x) + 1)^4 + 125*sin(x)^5/(cos(x) + 1)^5
 + 5)/(a^3*sin(x)/(cos(x) + 1) + 5*a^3*sin(x)^2/(cos(x) + 1)^2 + 10*a^3*sin
(x)^3/(cos(x) + 1)^3 + 10*a^3*sin(x)^4/(cos(x) + 1)^4 + 5*a^3*sin(x)^5/(cos
(x) + 1)^5 + a^3*sin(x)^6/(cos(x) + 1)^6) - 3*log(sin(x)/(cos(x) + 1))/a^3
+ 1/2*sin(x)/(a^3*(cos(x) + 1))
```

mupad [B] time = 6.72, size = 129, normalized size = 1.98

$$\frac{\tan\left(\frac{x}{2}\right)}{2a^3}-\frac{25\tan\left(\frac{x}{2}\right)^5+85\tan\left(\frac{x}{2}\right)^4+122\tan\left(\frac{x}{2}\right)^3+82\tan\left(\frac{x}{2}\right)^2+\frac{121\tan\left(\frac{x}{2}\right)}{5}+1}{2a^3\tan\left(\frac{x}{2}\right)^6+10a^3\tan\left(\frac{x}{2}\right)^5+20a^3\tan\left(\frac{x}{2}\right)^4+20a^3\tan\left(\frac{x}{2}\right)^3+10a^3\tan\left(\frac{x}{2}\right)^2+2a^3\tan\left(\frac{x}{2}\right)}-\frac{3\ln\left(t\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(a + a*sin(x))^3),x)
```

```
[Out] tan(x/2)/(2*a^3) - ((121*tan(x/2))/5 + 82*tan(x/2)^2 + 122*tan(x/2)^3 + 85*
tan(x/2)^4 + 25*tan(x/2)^5 + 1)/(2*a^3*tan(x/2) + 10*a^3*tan(x/2)^2 + 20*a^
3*tan(x/2)^3 + 20*a^3*tan(x/2)^4 + 10*a^3*tan(x/2)^5 + 2*a^3*tan(x/2)^6) -
(3*log(tan(x/2)))/a^3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^2(x)}{\sin^3(x)+3\sin^2(x)+3\sin(x)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a+a*sin(x))**3,x)
```

```
[Out] Integral(csc(x)**2/(sin(x)**3 + 3*sin(x)**2 + 3*sin(x) + 1), x)/a**3
```

3.30 $\int \frac{\csc^3(x)}{(a+a\sin(x))^3} \, dx$

Optimal. Leaf size=86

$$\frac{152\cot(x)}{15a^3}-\frac{13\tanh^{-1}(\cos(x))}{2a^3}-\frac{13\cot(x)\csc(x)}{2a^3}+\frac{76\cot(x)\csc(x)}{15\left(a^3\sin(x)+a^3\right)}+\frac{11\cot(x)\csc(x)}{15a(a\sin(x)+a)^2}+\frac{\cot(x)\csc(x)}{5(a\sin(x)+a)^3}$$

```
[Out] -13/2*arctanh(cos(x))/a^3+152/15*cot(x)/a^3-13/2*cot(x)*csc(x)/a^3+1/5*cot(
x)*csc(x)/(a+a*sin(x))^3+11/15*cot(x)*csc(x)/a/(a+a*sin(x))^2+76/15*cot(x)*
csc(x)/(a^3+a^3*sin(x))
```

Rubi [A] time = 0.24, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2766, 2978, 2748, 3768, 3770, 3767, 8}

$$\frac{152\cot(x)}{15a^3}-\frac{13\tanh^{-1}(\cos(x))}{2a^3}-\frac{13\cot(x)\csc(x)}{2a^3}+\frac{76\cot(x)\csc(x)}{15\left(a^3\sin(x)+a^3\right)}+\frac{11\cot(x)\csc(x)}{15a(a\sin(x)+a)^2}+\frac{\cot(x)\csc(x)}{5(a\sin(x)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a + a*Sin[x])^3,x]
```

```
[Out] (-13*ArcTanh[Cos[x]])/(2*a^3) + (152*Cot[x])/(15*a^3) - (13*Cot[x]*Csc[x])/
(2*a^3) + (Cot[x]*Csc[x])/(5*(a + a*Sin[x])^3) + (11*Cot[x]*Csc[x])/(15*a*(
a + a*Sin[x])^2) + (76*Cot[x]*Csc[x])/(15*(a^3 + a^3*Sin[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
```

```
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^3(x)}{(a+a\sin(x))^3}\,dx = \frac{\cot(x)\csc(x)}{5(a+a\sin(x))^3} + \frac{\int \frac{\csc^3(x)(7a-4a\sin(x))}{(a+a\sin(x))^2}\,dx}{5a^2}$$

$$= \frac{\cot(x)\csc(x)}{5(a+a\sin(x))^3} + \frac{11\cot(x)\csc(x)}{15a(a+a\sin(x))^2} + \frac{\int \frac{\csc^3(x)\left(43a^2-33a^2\sin(x)\right)}{a+a\sin(x)}\,dx}{15a^4}$$

$$= \frac{\cot(x)\csc(x)}{5(a+a\sin(x))^3} + \frac{11\cot(x)\csc(x)}{15a(a+a\sin(x))^2} + \frac{76\cot(x)\csc(x)}{15\left(a^3+a^3\sin(x)\right)} + \frac{\int \csc^3(x)\left(195a^3-152a^3\right.}{15a^6}$$

$$= \frac{\cot(x)\csc(x)}{5(a+a\sin(x))^3} + \frac{11\cot(x)\csc(x)}{15a(a+a\sin(x))^2} + \frac{76\cot(x)\csc(x)}{15\left(a^3+a^3\sin(x)\right)} - \frac{152\int \csc^2(x)\,dx}{15a^3} + \frac{13\int c}{}$$

$$= -\frac{13\cot(x)\csc(x)}{2a^3} + \frac{\cot(x)\csc(x)}{5(a+a\sin(x))^3} + \frac{11\cot(x)\csc(x)}{15a(a+a\sin(x))^2} + \frac{76\cot(x)\csc(x)}{15\left(a^3+a^3\sin(x)\right)} + \frac{13\int}{}$$

$$= -\frac{13\tanh^{-1}(\cos(x))}{2a^3} + \frac{152\cot(x)}{15a^3} - \frac{13\cot(x)\csc(x)}{2a^3} + \frac{\cot(x)\csc(x)}{5(a+a\sin(x))^3} + \frac{11\cot(x)\csc}{15a(a+a\sin}$$

Mathematica [B] time = 0.46, size = 247, normalized size = 2.87

$$\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(-48\sin\left(\frac{x}{2}\right)+15\cos^3\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)+1\right)^5-1712\sin\left(\frac{x}{2}\right)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^4+136\left(\sin\left(\frac{x}{2}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a + a*Sin[x])^3,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(-48*Sin[x/2] - 15*(1 + Cot[x/2])^5*Sin[x/2]^3 + 24*
(Cos[x/2] + Sin[x/2]) - 272*Sin[x/2]*(Cos[x/2] + Sin[x/2])^2 + 136*(Cos[x/2
] + Sin[x/2])^3 - 1712*Sin[x/2]*(Cos[x/2] + Sin[x/2])^4 + 180*Cot[x/2]*(Cos
[x/2] + Sin[x/2])^5 - 780*Log[Cos[x/2]]*(Cos[x/2] + Sin[x/2])^5 + 780*Log[S
in[x/2]]*(Cos[x/2] + Sin[x/2])^5 - 180*(Cos[x/2] + Sin[x/2])^5*Tan[x/2] + 1
5*Cos[x/2]^3*(1 + Tan[x/2])^5))/(120*a^3*(1 + Sin[x])^3)
```

fricas [B] time = 0.51, size = 276, normalized size = 3.21

$$\frac{608\cos(x)^5-826\cos(x)^4-2174\cos(x)^3+784\cos(x)^2-195\left(\cos(x)^5+3\cos(x)^4-3\cos(x)^3-7\cos(x)^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(608*cos(x)^5 - 826*cos(x)^4 - 2174*cos(x)^3 + 784*cos(x)^2 - 195*(cos
(x)^5 + 3*cos(x)^4 - 3*cos(x)^3 - 7*cos(x)^2 + (cos(x)^4 - 2*cos(x)^3 - 5*c
os(x)^2 + 2*cos(x) + 4)*sin(x) + 2*cos(x) + 4)*log(1/2*cos(x) + 1/2) + 195*
(cos(x)^5 + 3*cos(x)^4 - 3*cos(x)^3 - 7*cos(x)^2 + (cos(x)^4 - 2*cos(x)^3 -
 5*cos(x)^2 + 2*cos(x) + 4)*sin(x) + 2*cos(x) + 4)*log(-1/2*cos(x) + 1/2) -
 2*(304*cos(x)^4 + 717*cos(x)^3 - 370*cos(x)^2 - 762*cos(x) + 6)*sin(x) + 1
536*cos(x) + 12)/(a^3*cos(x)^5 + 3*a^3*cos(x)^4 - 3*a^3*cos(x)^3 - 7*a^3*co
s(x)^2 + 2*a^3*cos(x) + 4*a^3 + (a^3*cos(x)^4 - 2*a^3*cos(x)^3 - 5*a^3*cos(
x)^2 + 2*a^3*cos(x) + 4*a^3)*sin(x))
```

giac [A] time = 0.25, size = 109, normalized size = 1.27

$$\frac{13\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{2\,a^3}-\frac{78\,\tan\left(\frac{1}{2}\,x\right)^2-12\,\tan\left(\frac{1}{2}\,x\right)+1}{8\,a^3\tan\left(\frac{1}{2}\,x\right)^2}+\frac{a^3\tan\left(\frac{1}{2}\,x\right)^2-12\,a^3\tan\left(\frac{1}{2}\,x\right)}{8\,a^6}+\frac{2\left(150\,\tan\left(\frac{1}{2}\,x\right)^4+525\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] 13/2*log(abs(tan(1/2*x)))/a^3 - 1/8*(78*tan(1/2*x)^2 - 12*tan(1/2*x) + 1)/(
a^3*tan(1/2*x)^2) + 1/8*(a^3*tan(1/2*x)^2 - 12*a^3*tan(1/2*x))/a^6 + 2/15*(
150*tan(1/2*x)^4 + 525*tan(1/2*x)^3 + 745*tan(1/2*x)^2 + 485*tan(1/2*x) + 1
27)/(a^3*(tan(1/2*x) + 1)^5)
```

maple [A] time = 0.16, size = 119, normalized size = 1.38

$$\frac{\tan^2\left(\frac{x}{2}\right)}{8a^3}-\frac{3\tan\left(\frac{x}{2}\right)}{2a^3}-\frac{1}{8a^3\tan\left(\frac{x}{2}\right)^2}+\frac{3}{2a^3\tan\left(\frac{x}{2}\right)}+\frac{13\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a^3}+\frac{8}{5a^3\left(\tan\left(\frac{x}{2}\right)+1\right)^5}-\frac{4}{a^3\left(\tan\left(\frac{x}{2}\right)+1\right)^4}+\frac{}{3a^3\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a+a*sin(x))^3,x)
```

```
[Out] 1/8/a^3*tan(1/2*x)^2-3/2/a^3*tan(1/2*x)-1/8/a^3/tan(1/2*x)^2+3/2/a^3/tan(1/
2*x)+13/2/a^3*ln(tan(1/2*x))+8/5/a^3/(tan(1/2*x)+1)^5-4/a^3/(tan(1/2*x)+1)^
4+28/3/a^3/(tan(1/2*x)+1)^3-10/a^3/(tan(1/2*x)+1)^2+20/a^3/(tan(1/2*x)+1)
```

maxima [B] time = 0.69, size = 209, normalized size = 2.43

$$\frac{\frac{105\,\sin(x)}{\cos(x)+1}+\frac{2782\,\sin(x)^2}{(\cos(x)+1)^2}+\frac{9410\,\sin(x)^3}{(\cos(x)+1)^3}+\frac{13645\,\sin(x)^4}{(\cos(x)+1)^4}+\frac{9285\,\sin(x)^5}{(\cos(x)+1)^5}+\frac{2580\,\sin(x)^6}{(\cos(x)+1)^6}-15}{120\left(\frac{a^3\sin(x)^2}{(\cos(x)+1)^2}+\frac{5\,a^3\sin(x)^3}{(\cos(x)+1)^3}+\frac{10\,a^3\sin(x)^4}{(\cos(x)+1)^4}+\frac{10\,a^3\sin(x)^5}{(\cos(x)+1)^5}+\frac{5\,a^3\sin(x)^6}{(\cos(x)+1)^6}+\frac{a^3\sin(x)^7}{(\cos(x)+1)^7}\right)}-\frac{\frac{12\,\sin(x)}{\cos(x)+1}-\frac{\sin(x)^2}{(\cos(x)+1)^2}}{8\,a^3}+\frac{13\,\log\left(\frac{\mathrm{si}}{\cos}\right.}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] 1/120*(105*sin(x)/(cos(x) + 1) + 2782*sin(x)^2/(cos(x) + 1)^2 + 9410*sin(x)
^3/(cos(x) + 1)^3 + 13645*sin(x)^4/(cos(x) + 1)^4 + 9285*sin(x)^5/(cos(x) +
 1)^5 + 2580*sin(x)^6/(cos(x) + 1)^6 - 15)/(a^3*sin(x)^2/(cos(x) + 1)^2 + 5
*a^3*sin(x)^3/(cos(x) + 1)^3 + 10*a^3*sin(x)^4/(cos(x) + 1)^4 + 10*a^3*sin(
x)^5/(cos(x) + 1)^5 + 5*a^3*sin(x)^6/(cos(x) + 1)^6 + a^3*sin(x)^7/(cos(x)
+ 1)^7) - 1/8*(12*sin(x)/(cos(x) + 1) - sin(x)^2/(cos(x) + 1)^2)/a^3 + 13/2
*log(sin(x)/(cos(x) + 1))/a^3
```

mupad [B] time = 6.39, size = 97, normalized size = 1.13

$$\frac{\tan\left(\frac{x}{2}\right)^2}{8\,a^3}-\frac{3\tan\left(\frac{x}{2}\right)}{2\,a^3}+\frac{13\,\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2\,a^3}+\frac{\frac{43\tan\left(\frac{x}{2}\right)^6}{2}+\frac{619\tan\left(\frac{x}{2}\right)^5}{8}+\frac{2729\tan\left(\frac{x}{2}\right)^4}{24}+\frac{941\tan\left(\frac{x}{2}\right)^3}{12}+\frac{1391\tan\left(\frac{x}{2}\right)^2}{60}+\frac{7\tan\left(\frac{x}{2}\right)}{8}-}{a^3\tan\left(\frac{x}{2}\right)^2\left(\tan\left(\frac{x}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^3*(a + a*sin(x))^3),x)
```

```
[Out] tan(x/2)^2/(8*a^3) - (3*tan(x/2))/(2*a^3) + (13*log(tan(x/2)))/(2*a^3) + ((
7*tan(x/2))/8 + (1391*tan(x/2)^2)/60 + (941*tan(x/2)^3)/12 + (2729*tan(x/2)
^4)/24 + (619*tan(x/2)^5)/8 + (43*tan(x/2)^6)/2 - 1/8)/(a^3*tan(x/2)^2*(tan
(x/2) + 1)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^3(x)}{\sin^3(x)+3\sin^2(x)+3\sin(x)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a+a*sin(x))**3,x)
```

```
[Out] Integral(csc(x)**3/(sin(x)**3 + 3*sin(x)**2 + 3*sin(x) + 1), x)/a**3
```

3.31 $\int \frac{\csc^4(x)}{(a+a\sin(x))^3}\,dx$

Optimal. Leaf size=103

$$-\frac{136\cot^3(x)}{15a^3}-\frac{136\cot(x)}{5a^3}+\frac{23\tanh^{-1}(\cos(x))}{2a^3}+\frac{23\cot(x)\csc(x)}{2a^3}+\frac{23\cot(x)\csc^2(x)}{3\left(a^3\sin(x)+a^3\right)}+\frac{13\cot(x)\csc^2(x)}{15a(a\sin(x)+a)^2}+\frac{\cot(x)\csc^2(x)}{5(a\sin(x)+a)^3}$$

```
[Out] 23/2*arctanh(cos(x))/a^3-136/5*cot(x)/a^3-136/15*cot(x)^3/a^3+23/2*cot(x)*c
sc(x)/a^3+1/5*cot(x)*csc(x)^2/(a+a*sin(x))^3+13/15*cot(x)*csc(x)^2/a/(a+a*s
in(x))^2+23/3*cot(x)*csc(x)^2/(a^3+a^3*sin(x))
```

Rubi [A] time = 0.24, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2766, 2978, 2748, 3767, 3768, 3770}

$$-\frac{136\cot^3(x)}{15a^3}-\frac{136\cot(x)}{5a^3}+\frac{23\tanh^{-1}(\cos(x))}{2a^3}+\frac{23\cot(x)\csc(x)}{2a^3}+\frac{23\cot(x)\csc^2(x)}{3\left(a^3\sin(x)+a^3\right)}+\frac{13\cot(x)\csc^2(x)}{15a(a\sin(x)+a)^2}+\frac{\cot(x)\csc^2(x)}{5(a\sin(x)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^4/(a + a*Sin[x])^3,x]
```

```
[Out] (23*ArcTanh[Cos[x]])/(2*a^3) - (136*Cot[x])/(5*a^3) - (136*Cot[x]^3)/(15*a^
3) + (23*Cot[x]*Csc[x])/(2*a^3) + (Cot[x]*Csc[x]^2)/(5*(a + a*Sin[x])^3) +
(13*Cot[x]*Csc[x]^2)/(15*a*(a + a*Sin[x])^2) + (23*Cot[x]*Csc[x]^2)/(3*(a^3
 + a^3*Sin[x]))
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^4(x)}{(a+a\sin(x))^3}\,dx = \frac{\cot(x)\csc^2(x)}{5(a+a\sin(x))^3} + \frac{\int \frac{\csc^4(x)(8a-5a\sin(x))}{(a+a\sin(x))^2}\,dx}{5a^2}$$

$$= \frac{\cot(x)\csc^2(x)}{5(a+a\sin(x))^3} + \frac{13\cot(x)\csc^2(x)}{15a(a+a\sin(x))^2} + \frac{\int \frac{\csc^4(x)\left(63a^2-52a^2\sin(x)\right)}{a+a\sin(x)}\,dx}{15a^4}$$

$$= \frac{\cot(x)\csc^2(x)}{5(a+a\sin(x))^3} + \frac{13\cot(x)\csc^2(x)}{15a(a+a\sin(x))^2} + \frac{23\cot(x)\csc^2(x)}{3\left(a^3+a^3\sin(x)\right)} + \frac{\int \csc^4(x)\left(408a^3-345a^3\sin\right.}{15a^6}$$

$$= \frac{\cot(x)\csc^2(x)}{5(a+a\sin(x))^3} + \frac{13\cot(x)\csc^2(x)}{15a(a+a\sin(x))^2} + \frac{23\cot(x)\csc^2(x)}{3\left(a^3+a^3\sin(x)\right)} - \frac{23\int \csc^3(x)\,dx}{a^3} + \frac{136\int \csc}{5a^3}$$

$$= \frac{23\cot(x)\csc(x)}{2a^3} + \frac{\cot(x)\csc^2(x)}{5(a+a\sin(x))^3} + \frac{13\cot(x)\csc^2(x)}{15a(a+a\sin(x))^2} + \frac{23\cot(x)\csc^2(x)}{3\left(a^3+a^3\sin(x)\right)} - \frac{23\int \csc(}{2a^3}$$

$$= \frac{23\tanh^{-1}(\cos(x))}{2a^3} - \frac{136\cot(x)}{5a^3} - \frac{136\cot^3(x)}{15a^3} + \frac{23\cot(x)\csc(x)}{2a^3} + \frac{\cot(x)\csc^2(x)}{5(a+a\sin(x))^3} + \frac{13}{15}$$

Mathematica [B] time = 0.93, size = 299, normalized size = 2.90

$$\frac{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(48\sin\left(\frac{x}{2}\right)-45\cos^3\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)+1\right)^5+2752\sin\left(\frac{x}{2}\right)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^4-176\left(\sin\left(\frac{x}{2}\right)+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^4/(a + a*Sin[x])^3,x]
```

```
[Out] ((Cos[x/2] + Sin[x/2])*(48*Sin[x/2] - 5*Cos[x/2]*(1 + Cot[x/2])^5*Sin[x/2]^
2 + 45*(1 + Cot[x/2])^5*Sin[x/2]^3 - 24*(Cos[x/2] + Sin[x/2]) + 352*Sin[x/2
]*(Cos[x/2] + Sin[x/2])^2 - 176*(Cos[x/2] + Sin[x/2])^3 + 2752*Sin[x/2]*(Co
s[x/2] + Sin[x/2])^4 - 400*Cot[x/2]*(Cos[x/2] + Sin[x/2])^5 + 1380*Log[Cos[
x/2]]*(Cos[x/2] + Sin[x/2])^5 - 1380*Log[Sin[x/2]]*(Cos[x/2] + Sin[x/2])^5
+ 400*(Cos[x/2] + Sin[x/2])^5*Tan[x/2] - 45*Cos[x/2]^3*(1 + Tan[x/2])^5 + 5
*Cos[x/2]^2*Sin[x/2]*(1 + Tan[x/2])^5))/(120*a^3*(1 + Sin[x])^3)
```

fricas [B] time = 0.51, size = 333, normalized size = 3.23

$$\frac{1088\cos(x)^6+2574\cos(x)^5-2428\cos(x)^4-5338\cos(x)^3+1372\cos(x)^2+345\left(\cos(x)^6-2\cos(x)^5-6\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(1088*cos(x)^6 + 2574*cos(x)^5 - 2428*cos(x)^4 - 5338*cos(x)^3 + 1372*
cos(x)^2 + 345*(cos(x)^6 - 2*cos(x)^5 - 6*cos(x)^4 + 4*cos(x)^3 + 9*cos(x)^
2 - (cos(x)^5 + 3*cos(x)^4 - 3*cos(x)^3 - 7*cos(x)^2 + 2*cos(x) + 4)*sin(x)
 - 2*cos(x) - 4)*log(1/2*cos(x) + 1/2) - 345*(cos(x)^6 - 2*cos(x)^5 - 6*cos
(x)^4 + 4*cos(x)^3 + 9*cos(x)^2 - (cos(x)^5 + 3*cos(x)^4 - 3*cos(x)^3 - 7*c
os(x)^2 + 2*cos(x) + 4)*sin(x) - 2*cos(x) - 4)*log(-1/2*cos(x) + 1/2) + 2*(
544*cos(x)^5 - 743*cos(x)^4 - 1957*cos(x)^3 + 712*cos(x)^2 + 1398*cos(x) +
6)*sin(x) + 2784*cos(x) - 12)/(a^3*cos(x)^6 - 2*a^3*cos(x)^5 - 6*a^3*cos(x)
^4 + 4*a^3*cos(x)^3 + 9*a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3 - (a^3*cos(x)^5
 + 3*a^3*cos(x)^4 - 3*a^3*cos(x)^3 - 7*a^3*cos(x)^2 + 2*a^3*cos(x) + 4*a^3)
*sin(x))
```

giac [A] time = 0.28, size = 128, normalized size = 1.24

$$-\frac{23\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{2\,a^{3}}+\frac{506\,\tan\left(\frac{1}{2}\,x\right)^{3}-81\,\tan\left(\frac{1}{2}\,x\right)^{2}+9\,\tan\left(\frac{1}{2}\,x\right)-1}{24\,a^{3}\tan\left(\frac{1}{2}\,x\right)^{3}}-\frac{2\left(225\,\tan\left(\frac{1}{2}\,x\right)^{4}+810\,\tan\left(\frac{1}{2}\,x\right)^{3}+\cdots\right.}{15\,a^{3}\left(\tan\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*sin(x))^3,x, algorithm="giac")
```

```
[Out] -23/2*log(abs(tan(1/2*x)))/a^3 + 1/24*(506*tan(1/2*x)^3 - 81*tan(1/2*x)^2 +
 9*tan(1/2*x) - 1)/(a^3*tan(1/2*x)^3) - 2/15*(225*tan(1/2*x)^4 + 810*tan(1/
2*x)^3 + 1160*tan(1/2*x)^2 + 760*tan(1/2*x) + 197)/(a^3*(tan(1/2*x) + 1)^5)
 + 1/24*(a^6*tan(1/2*x)^3 - 9*a^6*tan(1/2*x)^2 + 81*a^6*tan(1/2*x))/a^9
```

maple [A] time = 0.15, size = 141, normalized size = 1.37

$$\frac{\tan^{3}\left(\frac{x}{2}\right)}{24a^{3}}-\frac{3\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{8a^{3}}+\frac{27\tan\left(\frac{x}{2}\right)}{8a^{3}}-\frac{1}{24a^{3}\tan\left(\frac{x}{2}\right)^{3}}+\frac{3}{8a^{3}\tan\left(\frac{x}{2}\right)^{2}}-\frac{27}{8a^{3}\tan\left(\frac{x}{2}\right)}-\frac{23\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a^{3}}-\frac{8}{5a^{3}\left(\tan\left(\frac{x}{2}\right)+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^4/(a+a*sin(x))^3,x)
```

```
[Out] 1/24/a^3*tan(1/2*x)^3-3/8/a^3*tan(1/2*x)^2+27/8/a^3*tan(1/2*x)-1/24/a^3/tan
(1/2*x)^3+3/8/a^3/tan(1/2*x)^2-27/8/a^3/tan(1/2*x)-23/2/a^3*ln(tan(1/2*x))-
8/5/a^3/(tan(1/2*x)+1)^5+4/a^3/(tan(1/2*x)+1)^4-32/3/a^3/(tan(1/2*x)+1)^3+1
2/a^3/(tan(1/2*x)+1)^2-30/a^3/(tan(1/2*x)+1)
```

maxima [B] time = 0.67, size = 232, normalized size = 2.25

$$\frac{\frac{20\sin(x)}{\cos(x)+1} - \frac{230\sin(x)^2}{(\cos(x)+1)^2} - \frac{4777\sin(x)^3}{(\cos(x)+1)^3} - \frac{15785\sin(x)^4}{(\cos(x)+1)^4} - \frac{22390\sin(x)^5}{(\cos(x)+1)^5} - \frac{14940\sin(x)^6}{(\cos(x)+1)^6} - \frac{4005\sin(x)^7}{(\cos(x)+1)^7} - 5}{120\left(\frac{a^3\sin(x)^3}{(\cos(x)+1)^3} + \frac{5a^3\sin(x)^4}{(\cos(x)+1)^4} + \frac{10a^3\sin(x)^5}{(\cos(x)+1)^5} + \frac{10a^3\sin(x)^6}{(\cos(x)+1)^6} + \frac{5a^3\sin(x)^7}{(\cos(x)+1)^7} + \frac{a^3\sin(x)^8}{(\cos(x)+1)^8}\right)} + \frac{\frac{81\sin(x)}{\cos(x)+1} - \frac{9\sin(x)^2}{(\cos(x)+1)^2} +}{24a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*sin(x))^3,x, algorithm="maxima")
```

```
[Out] 1/120*(20*sin(x)/(cos(x) + 1) - 230*sin(x)^2/(cos(x) + 1)^2 - 4777*sin(x)^3
/(cos(x) + 1)^3 - 15785*sin(x)^4/(cos(x) + 1)^4 - 22390*sin(x)^5/(cos(x) +
1)^5 - 14940*sin(x)^6/(cos(x) + 1)^6 - 4005*sin(x)^7/(cos(x) + 1)^7 - 5)/(a
^3*sin(x)^3/(cos(x) + 1)^3 + 5*a^3*sin(x)^4/(cos(x) + 1)^4 + 10*a^3*sin(x)^
5/(cos(x) + 1)^5 + 10*a^3*sin(x)^6/(cos(x) + 1)^6 + 5*a^3*sin(x)^7/(cos(x)
+ 1)^7 + a^3*sin(x)^8/(cos(x) + 1)^8) + 1/24*(81*sin(x)/(cos(x) + 1) - 9*si
n(x)^2/(cos(x) + 1)^2 + sin(x)^3/(cos(x) + 1)^3)/a^3 - 23/2*log(sin(x)/(cos
(x) + 1))/a^3
```

mupad [B] time = 6.69, size = 117, normalized size = 1.14

$$\frac{27\tan\left(\frac{x}{2}\right)}{8a^3} - \frac{3\tan\left(\frac{x}{2}\right)^2}{8a^3} + \frac{\tan\left(\frac{x}{2}\right)^3}{24a^3} - \frac{23\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a^3} - \frac{\frac{267\tan\left(\frac{x}{2}\right)^7}{8} + \frac{249\tan\left(\frac{x}{2}\right)^6}{2} + \frac{2239\tan\left(\frac{x}{2}\right)^5}{12} + \frac{3157\tan\left(\frac{x}{2}\right)^4}{24} + \frac{4777\tan\left(\frac{x}{2}\right)}{120}}{a^3\tan\left(\frac{x}{2}\right)^3\left(\tan\left(\frac{x}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^4*(a + a*sin(x))^3),x)
```

```
[Out] (27*tan(x/2))/(8*a^3) - (3*tan(x/2)^2)/(8*a^3) + tan(x/2)^3/(24*a^3) - (23*
log(tan(x/2)))/(2*a^3) - ((23*tan(x/2)^2)/12 - tan(x/2)/6 + (4777*tan(x/2)^
3)/120 + (3157*tan(x/2)^4)/24 + (2239*tan(x/2)^5)/12 + (249*tan(x/2)^6)/2 +
 (267*tan(x/2)^7)/8 + 1/24)/(a^3*tan(x/2)^3*(tan(x/2) + 1)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^4(x)}{\sin^3(x)+3\sin^2(x)+3\sin(x)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**4/(a+a*sin(x))**3,x)
```

```
[Out] Integral(csc(x)**4/(sin(x)**3 + 3*sin(x)**2 + 3*sin(x) + 1), x)/a**3
```

3.32 $\int \sin^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=158

$$-\frac{2a\sin^4(c+dx)\cos(c+dx)}{9d\sqrt{a\sin(c+dx)+a}}-\frac{16a\sin^3(c+dx)\cos(c+dx)}{63d\sqrt{a\sin(c+dx)+a}}-\frac{32\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{105ad}+\frac{64\cos(c+dx}{}$$

```
[Out] -32/105*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/a/d-32/45*a*cos(d*x+c)/d/(a+a*sin
(d*x+c))^(1/2)-16/63*a*cos(d*x+c)*sin(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)-2/9
*a*cos(d*x+c)*sin(d*x+c)^4/d/(a+a*sin(d*x+c))^(1/2)+64/315*cos(d*x+c)*(a+a*
sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.23, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2770, 2759, 2751, 2646}

$$-\frac{2a\sin^4(c+dx)\cos(c+dx)}{9d\sqrt{a\sin(c+dx)+a}}-\frac{16a\sin^3(c+dx)\cos(c+dx)}{63d\sqrt{a\sin(c+dx)+a}}-\frac{32\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{105ad}+\frac{64\cos(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-32*a*Cos[c + d*x])/(45*d*Sqrt[a + a*Sin[c + d*x]]) - (16*a*Cos[c + d*x]*S
in[c + d*x]^3)/(63*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]*Sin[c +
d*x]^4)/(9*d*Sqrt[a + a*Sin[c + d*x]]) + (64*Cos[c + d*x]*Sqrt[a + a*Sin[c
+ d*x]])/(315*d) - (32*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(105*a*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
```

```
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int \sin^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{2a\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} + \frac{8}{9}\int \sin^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$$

$$= -\frac{16a\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} + \frac{16}{21}\int \sin^2$$

$$= -\frac{16a\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} - \frac{32\cos(c+}{}$$

$$= -\frac{16a\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} + \frac{64\cos(c-}{}$$

$$= -\frac{32a\cos(c+dx)}{45d\sqrt{a+a\sin(c+dx)}} - \frac{16a\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos(c+dx)}{9d\sqrt{a+a\,\mathrm{s}}}$$

Mathematica [A] time = 0.50, size = 165, normalized size = 1.04

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(1890\sin\left(\frac{1}{2}(c+dx)\right) - 420\sin\left(\frac{3}{2}(c+dx)\right) - 252\sin\left(\frac{5}{2}(c+dx)\right) + 45\sin\left(\frac{7}{2}(c+dx)\right) + 35\right.}{2520d\left(\sin\left(\frac{1}{2}(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*(-1890*Cos[(c + d*x)/2] - 420*Cos[(3*(c + d*x))
/2] + 252*Cos[(5*(c + d*x))/2] + 45*Cos[(7*(c + d*x))/2] - 35*Cos[(9*(c + d
```

```
*x))/2] + 1890*Sin[(c + d*x)/2] - 420*Sin[(3*(c + d*x))/2] - 252*Sin[(5*(c
+ d*x))/2] + 45*Sin[(7*(c + d*x))/2] + 35*Sin[(9*(c + d*x))/2]))/(2520*d*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

fricas [A] time = 0.53, size = 132, normalized size = 0.84

$$-\frac{2\left(35\cos(dx+c)^5 - 5\cos(dx+c)^4 - 118\cos(dx+c)^3 + 26\cos(dx+c)^2 - \left(35\cos(dx+c)^4 + 40\cos(dx+\right.\right.}{315\left(d\cos(dx+c)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*(35*cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 118*cos(d*x + c)^3 + 26*cos(
d*x + c)^2 - (35*cos(d*x + c)^4 + 40*cos(d*x + c)^3 - 78*cos(d*x + c)^2 - 1
04*cos(d*x + c) + 107)*sin(d*x + c) + 211*cos(d*x + c) + 107)*sqrt(a*sin(d*
x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

giac [A] time = 0.58, size = 159, normalized size = 1.01

$$\frac{1}{2520}\sqrt{2}\sqrt{a}\left(\frac{45\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(\frac{1}{4}\pi+\frac{7}{2}dx+\frac{7}{2}c\right)}{d} - \frac{420\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2520*sqrt(2)*sqrt(a)*(45*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi +
 7/2*d*x + 7/2*c)/d - 420*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi +
3/2*d*x + 3/2*c)/d + 35*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 9
/2*d*x + 9/2*c)/d - 252*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 5
/2*d*x + 5/2*c)/d + 1890*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi +
1/2*d*x + 1/2*c)/d)
```

maple [A] time = 0.80, size = 83, normalized size = 0.53

$$\frac{2(1+\sin(dx+c))\,a(\sin(dx+c)-1)\left(35\left(\sin^4(dx+c)\right)+40\left(\sin^3(dx+c)\right)+48\left(\sin^2(dx+c)\right)+64\sin(dx+\right.}{315\cos(dx+c)\sqrt{a+a\sin(dx+c)}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/315*(1+sin(d*x+c))*a*(sin(d*x+c)-1)*(35*sin(d*x+c)^4+40*sin(d*x+c)^3+48*s
in(d*x+c)^2+64*sin(d*x+c)+128)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin(dx+c)+a}\,\sin(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*sin(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(c+d\,x)^4\,\sqrt{a+a\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4*(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(sin(c + d*x)^4*(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sin(c+dx)+1)}\,\sin^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sin(c + d*x)**4, x)
```

3.33 $\int \sin^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=122

$$-\frac{2a\sin^3(c+dx)\cos(c+dx)}{7d\sqrt{a\sin(c+dx)+a}}-\frac{12\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{35ad}+\frac{8\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{35d}-\frac{4a\cos}{5d\sqrt{a\sin}}$$

```
[Out] -12/35*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/a/d-4/5*a*cos(d*x+c)/d/(a+a*sin(d*
x+c))^(1/2)-2/7*a*cos(d*x+c)*sin(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)+8/35*cos
(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.17, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2770, 2759, 2751, 2646}

$$-\frac{2a\sin^3(c+dx)\cos(c+dx)}{7d\sqrt{a\sin(c+dx)+a}}-\frac{12\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{35ad}+\frac{8\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{35d}-\frac{4a\cos}{5d\sqrt{a\sin}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-4*a*Cos[c + d*x])/(5*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]*Sin[
c + d*x]^3)/(7*d*Sqrt[a + a*Sin[c + d*x]]) + (8*Cos[c + d*x]*Sqrt[a + a*Sin
[c + d*x]])/(35*d) - (12*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(35*a*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
```

```
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int \sin^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{2a\cos(c+dx)\sin^3(c+dx)}{7d\sqrt{a+a\sin(c+dx)}}+\frac{6}{7}\int \sin^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$$

$$= -\frac{2a\cos(c+dx)\sin^3(c+dx)}{7d\sqrt{a+a\sin(c+dx)}}-\frac{12\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{35ad}+\frac{12}{\ldots}$$

$$= -\frac{2a\cos(c+dx)\sin^3(c+dx)}{7d\sqrt{a+a\sin(c+dx)}}+\frac{8\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{35d}-\frac{12\cos\ldots}{\ldots}$$

$$= -\frac{4a\cos(c+dx)}{5d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos(c+dx)\sin^3(c+dx)}{7d\sqrt{a+a\sin(c+dx)}}+\frac{8\cos(c+dx)\sqrt{a\ldots}}{35\ldots}$$

Mathematica [A] time = 0.29, size = 141, normalized size = 1.16

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(105\sin\left(\frac{1}{2}(c+dx)\right)-35\sin\left(\frac{3}{2}(c+dx)\right)-7\sin\left(\frac{5}{2}(c+dx)\right)+5\sin\left(\frac{7}{2}(c+dx)\right)-105\cos\left(\ldots\right)\ldots\right)}{140d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*(-105*Cos[(c + d*x)/2] - 35*Cos[(3*(c + d*x))/2
] + 7*Cos[(5*(c + d*x))/2] + 5*Cos[(7*(c + d*x))/2] + 105*Sin[(c + d*x)/2]
- 35*Sin[(3*(c + d*x))/2] - 7*Sin[(5*(c + d*x))/2] + 5*Sin[(7*(c + d*x))/2]
))/(140*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

fricas [A] time = 0.52, size = 111, normalized size = 0.91

$$\frac{2\left(5\cos(dx+c)^4+6\cos(dx+c)^3-12\cos(dx+c)^2+\left(5\cos(dx+c)^3-\cos(dx+c)^2-13\cos(dx+c)+9\right)\right.}{35\left(d\cos(dx+c)+d\sin(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*cos(d*x + c)^4 + 6*cos(d*x + c)^3 - 12*cos(d*x + c)^2 + (5*cos(d*x
+ c)^3 - cos(d*x + c)^2 - 13*cos(d*x + c) + 9)*sin(d*x + c) - 22*cos(d*x +
c) - 9)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

giac [A] time = 0.95, size = 129, normalized size = 1.06

$$\frac{1}{140}\sqrt{2}\sqrt{a}\left(\frac{7\cos\left(\frac{1}{4}\pi+\frac{5}{2}dx+\frac{5}{2}c\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{d}-\frac{105\cos\left(\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\right.\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/140*sqrt(2)*sqrt(a)*(7*cos(1/4*pi + 5/2*d*x + 5/2*c)*sgn(cos(-1/4*pi + 1/
2*d*x + 1/2*c))/d - 105*cos(1/4*pi + 1/2*d*x + 1/2*c)*sgn(cos(-1/4*pi + 1/2
*d*x + 1/2*c))/d + 5*cos(-1/4*pi + 7/2*d*x + 7/2*c)*sgn(cos(-1/4*pi + 1/2*d
*x + 1/2*c))/d - 35*cos(-1/4*pi + 3/2*d*x + 3/2*c)*sgn(cos(-1/4*pi + 1/2*d*
x + 1/2*c))/d)
```

maple [A] time = 0.68, size = 73, normalized size = 0.60

$$\frac{2\left(1+\sin(dx+c)\right)a\left(\sin(dx+c)-1\right)\left(5\left(\sin^3(dx+c)\right)+6\left(\sin^2(dx+c)\right)+8\sin(dx+c)+16\right)}{35\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/35*(1+sin(d*x+c))*a*(sin(d*x+c)-1)*(5*sin(d*x+c)^3+6*sin(d*x+c)^2+8*sin(d
*x+c)+16)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sin(dx+c)+a}\,\sin(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sin(d*x + c) + a)*sin(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(c+d\,x)^3\,\sqrt{a+a\,\sin(c+d\,x)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + a*sin(c + d*x))^(1/2),x)
[Out] int(sin(c + d*x)^3*(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+a*sin(d*x+c))**(1/2),x)
[Out] Timed out
```

3.34 $\int \sin^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=86

$$-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5ad}+\frac{4\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{15d}-\frac{14a\cos(c+dx)}{15d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -2/5*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/a/d-14/15*a*cos(d*x+c)/d/(a+a*sin(d*
x+c))^(1/2)+4/15*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2759, 2751, 2646}

$$-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5ad}+\frac{4\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{15d}-\frac{14a\cos(c+dx)}{15d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-14*a*Cos[c + d*x])/(15*d*Sqrt[a + a*Sin[c + d*x]]) + (4*Cos[c + d*x]*Sqrt
[a + a*Sin[c + d*x]])/(15*d) - (2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/
(5*a*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
```

```
[m, -2^(-1)]
```

Rubi steps

$$\int \sin^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{2\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{5ad} + \frac{2\int\left(\frac{3a}{2}-a\sin(c+dx)\right)\sqrt{a+a\sin}}{5a}$$
$$= \frac{4\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{15d} - \frac{2\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{5ad} +$$
$$= -\frac{14a\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}} + \frac{4\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{15d} - \frac{2\cos(c+}{}$$

Mathematica [A] time = 0.18, size = 117, normalized size = 1.36

$$-\frac{\sqrt{a(\sin(c+dx)+1)}\left(-30\sin\left(\frac{1}{2}(c+dx)\right)+5\sin\left(\frac{3}{2}(c+dx)\right)+3\sin\left(\frac{5}{2}(c+dx)\right)+30\cos\left(\frac{1}{2}(c+dx)\right)+5\cos\left(\right.\right.}{30d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/30*(Sqrt[a*(1 + Sin[c + d*x])]*(30*Cos[(c + d*x)/2] + 5*Cos[(3*(c + d*x)
)/2] - 3*Cos[(5*(c + d*x))/2] - 30*Sin[(c + d*x)/2] + 5*Sin[(3*(c + d*x))/2
] + 3*Sin[(5*(c + d*x))/2]))/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

fricas [A] time = 0.49, size = 92, normalized size = 1.07

$$\frac{2\left(3\cos(dx+c)^3-\cos(dx+c)^2-\left(3\cos(dx+c)^2+4\cos(dx+c)-7\right)\sin(dx+c)-11\cos(dx+c)-7\right)\sqrt{a\,\mathrm{s}}}{15\left(d\cos(dx+c)+d\sin(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*cos(d*x + c)^3 - cos(d*x + c)^2 - (3*cos(d*x + c)^2 + 4*cos(d*x + c
) - 7)*sin(d*x + c) - 11*cos(d*x + c) - 7)*sqrt(a*sin(d*x + c) + a)/(d*cos(
d*x + c) + d*sin(d*x + c) + d)
```

giac [A] time = 0.44, size = 99, normalized size = 1.15

$$-\frac{1}{30}\sqrt{2}\sqrt{a}\left(\frac{5\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(\frac{1}{4}\pi+\frac{3}{2}dx+\frac{3}{2}c\right)}{d}+\frac{3\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/30*sqrt(2)*sqrt(a)*(5*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 3
/2*d*x + 3/2*c)/d + 3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 5/2
*d*x + 5/2*c)/d - 30*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*
d*x + 1/2*c)/d)
```

maple [A] time = 0.70, size = 63, normalized size = 0.73

$$\frac{2\left(1+\sin\left(dx+c\right)\right)a\left(\sin\left(dx+c\right)-1\right)\left(3\left(\sin^{2}\left(dx+c\right)\right)+4\sin\left(dx+c\right)+8\right)}{15\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/15*(1+sin(d*x+c))*a*(sin(d*x+c)-1)*(3*sin(d*x+c)^2+4*sin(d*x+c)+8)/cos(d*
x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sin\left(dx+c\right)+a}\,\sin\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*sin(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\sin\left(c+d\,x\right)^{2}\,\sqrt{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(sin(c + d*x)^2*(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\sin^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+a*sin(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sin(c + d*x)**2, x)
```

3.35 $\int \sin(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=56

$$-\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{2a\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -2/3*a*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-2/3*cos(d*x+c)*(a+a*sin(d*x+c))^
(1/2)/d
```

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2751, 2646}

$$-\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{2a\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*a*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]*Sqrt[a
 + a*Sin[c + d*x]])/(3*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \sin(c+dx)\sqrt{a+a\sin(c+dx)}\,dx &= -\frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}+\frac{1}{3}\int \sqrt{a+a\sin(c+dx)}\,dx\\ &= -\frac{2a\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}-\frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}\end{aligned}$$

Mathematica [A] time = 0.12, size = 81, normalized size = 1.45

$$-\frac{\sqrt{a(\sin(c+dx)+1)}\left(-4\sin^3\left(\frac{1}{2}(c+dx)\right)+3\cos\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{3}{2}(c+dx)\right)\right)}{3d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/3*((3*Cos[(c + d*x)/2] + Cos[(3*(c + d*x))/2] - 4*Sin[(c + d*x)/2]^3)*Sq
rt[a*(1 + Sin[c + d*x]])])/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

fricas [A] time = 0.48, size = 67, normalized size = 1.20

$$-\frac{2\left(\cos(dx+c)^2+(\cos(dx+c)-1)\sin(dx+c)+2\cos(dx+c)+1\right)\sqrt{a\sin(dx+c)+a}}{3\,(d\cos(dx+c)+d\sin(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(cos(d*x + c)^2 + (cos(d*x + c) - 1)*sin(d*x + c) + 2*cos(d*x + c) + 1
)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

giac [A] time = 1.39, size = 68, normalized size = 1.21

$$-\frac{1}{3}\sqrt{2}\sqrt{a}\left(\frac{3\cos\left(\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{d}+\frac{\cos\left(-\frac{1}{4}\pi+\frac{3}{2}dx+\frac{3}{2}c\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\right.\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(2)*sqrt(a)*(3*cos(1/4*pi + 1/2*d*x + 1/2*c)*sgn(cos(-1/4*pi + 1/2
*d*x + 1/2*c))/d + cos(-1/4*pi + 3/2*d*x + 3/2*c)*sgn(cos(-1/4*pi + 1/2*d*x
 + 1/2*c))/d)
```

maple [A] time = 0.63, size = 51, normalized size = 0.91

$$\frac{2\,(1+\sin(dx+c))\,a\,(\sin(dx+c)-1)\,(\sin(dx+c)+2)}{3\cos(dx+c)\,\sqrt{a+a\sin(dx+c)}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/3*(1+sin(d*x+c))*a*(sin(d*x+c)-1)*(sin(d*x+c)+2)/cos(d*x+c)/(a+a*sin(d*x+
c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin(dx+c)+a}\,\sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*sin(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sin(c+d\,x)\,\sqrt{a+a\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(sin(c + d*x)*(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sin(c+dx)+1)}\,\sin(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sin(c + d*x), x)
```

3.36 $\int \sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=26

$$-\frac{2a\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -2*a*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2646}

$$-\frac{2a\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*a*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sin(c+dx)}\,dx = -\frac{2a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [B] time = 0.03, size = 65, normalized size = 2.50

$$\frac{2\sqrt{a(\sin(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)-\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (2*(-Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*Sqrt[a*(1 + Sin[c + d*x])])/(d*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

fricas [B] time = 0.43, size = 50, normalized size = 1.92

$$-\frac{2\sqrt{a\sin(dx+c)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{d\cos(dx+c)+d\sin(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/(d*cos(d*x +
c) + d*sin(d*x + c) + d)
```

giac [A] time = 0.80, size = 36, normalized size = 1.38

$$\frac{2\sqrt{2}\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*sqrt(a)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x
 + 1/2*c)/d
```

maple [A] time = 0.49, size = 43, normalized size = 1.65

$$\frac{2(1+\sin(dx+c))\,a\,(\sin(dx+c)-1)}{\cos(dx+c)\sqrt{a+a\sin(dx+c)}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2*(1+sin(d*x+c))*a*(sin(d*x+c)-1)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sin(dx+c)+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a), x)
```

mupad [B] time = 0.21, size = 33, normalized size = 1.27

$$-\frac{2\,\cos\left(c+d\,x\right)\,\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}}{d\,\left(\sin\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2),x)
[Out] -(2*cos(c + d*x)*(a*(sin(c + d*x) + 1))^(1/2))/(d*(sin(c + d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sin\left(c+dx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*sin(c + d*x) + a), x)
```

3.37 $\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\, dx$

Optimal. Leaf size=37

$$-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

```
[Out] -2*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)/d
```

Rubi [A] time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2773, 206}

$$-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\, dx &= -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{a-x^2}\, dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}\\ &= -\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}\end{aligned}$$

Mathematica [B] time = 0.09, size = 94, normalized size = 2.54

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)-\cos\left(\frac{1}{2}(c+dx)\right)+1\right)-\log\left(-\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)+1\right)\right.}{d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((-Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[1 - Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]])*Sqrt[a*(1 + Sin[c + d*x])])/(d*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]))
```

fricas [A] time = 0.49, size = 219, normalized size = 5.92

$$\left[\frac{\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)\right.}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right.}{2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2
 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c
) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) -
a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1
)*sin(d*x + c) - cos(d*x + c) - 1))/d, -sqrt(-a)*arctan(1/2*sqrt(a*sin(d*x
+ c) + a)*sqrt(-a)*(sin(d*x + c) - 2)/(a*cos(d*x + c)))/d]
```

giac [B] time = 0.46, size = 67, normalized size = 1.81

$$-\frac{\sqrt{a}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(a)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c))/abs(2*sqrt(
2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))
/d
```

maple [B] time = 0.49, size = 68, normalized size = 1.84

$$-\frac{2\left(1+\sin\left(dx+c\right)\right)\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\sqrt{a}\;\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)}{\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*a^(1/2)*arctanh((-a*(sin(d*x+c)
-1))^(1/2)/a^(1/2))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin\left(dx+c\right)+a}\,\csc\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*csc(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{a+a\,\sin\left(c+d\,x\right)}}{\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/sin(c + d*x),x)
```

```
[Out] int((a + a*sin(c + d*x))^(1/2)/sin(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\csc\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*csc(c + d*x), x)
```

3.38 $\int \csc^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=64

$$-\frac{a\cot(c+dx)}{d\sqrt{a\sin(c+dx)+a}}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

```
[Out] -arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)/d-a*cot(d*x+c)/
d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2772, 2773, 206}

$$-\frac{a\cot(c+dx)}{d\sqrt{a\sin(c+dx)+a}}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d) - (
a*Cot[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \csc^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{a\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}+\frac{1}{2}\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$$
$$= -\frac{a\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}-\frac{a\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}$$
$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}-\frac{a\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [B] time = 0.70, size = 178, normalized size = 2.78

$$-\frac{\csc^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(-2\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)+\sin(c+dx)\left(\log\left(-\sin\left(\frac{1}{2}(c+\right.\right.\right.\right.}{d\left(\cot\left(\frac{1}{2}(c+dx)\right)+1\right)\left(\csc\left(\frac{1}{4}(c+dx)\right)-\sec\left(\frac{1}{4}(c+dx)\right)\right)\left(\csc\left(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((Csc[(c + d*x)/2]^4*Sqrt[a*(1 + Sin[c + d*x])]*(2*Cos[(c + d*x)/2] - 2*Si
n[(c + d*x)/2] + (Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[1 - Co
s[(c + d*x)/2] + Sin[(c + d*x)/2]])*Sin[c + d*x]))/(d*(1 + Cot[(c + d*x)/2]
)*(Csc[(c + d*x)/4] - Sec[(c + d*x)/4])*(Csc[(c + d*x)/4] + Sec[(c + d*x)/4
])))
```

fricas [B] time = 0.58, size = 258, normalized size = 4.03

$$\frac{\left(\cos(dx+c)^2-(\cos(dx+c)+1)\sin(dx+c)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+\right.}{\cos(dx+c)^3+\cos(\right.}}{4\left(d\cos(dx+c)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log((a*
cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3
```

```
)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a
*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)
/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos
(d*x + c) - 1)) + 4*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) +
 1))/(d*cos(d*x + c)^2 - (d*cos(d*x + c) + d)*sin(d*x + c) - d)
```

giac [B] time = 0.44, size = 122, normalized size = 1.91

$$-\frac{\sqrt{2}\left(\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)+\frac{4\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}{2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2-1}$$

$$4d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)
)/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c)) + 4*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x
 + 1/2*c)/(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1))*sqrt(a)/d
```

maple [A] time = 0.77, size = 104, normalized size = 1.62

$$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(\sqrt{a-a\sin(dx+c)}\,a^{\frac{3}{2}}+\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)a^2\sin(dx+c)\right)}{\sin(dx+c)\,a^{\frac{3}{2}}\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*((a-a*sin(d*x+c))^(1/2)*a^(3/2)+a
rctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2))*a^2*sin(d*x+c))/sin(d*x+c)/a^(3/2)/c
os(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sin(dx+c)+a}\,\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*csc(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+a\,\sin\left(c+d\,x\right)}}{{\sin\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/sin(c + d*x)^2,x)
[Out] int((a + a*sin(c + d*x))^(1/2)/sin(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(c+dx\right)+1\right)}\csc^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sin(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*csc(c + d*x)**2, x)
```

3.39 $\int \csc^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=102

$$-\frac{3a\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}-\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -3/4*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)/d-3/4*a*cot
(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-1/2*a*cot(d*x+c)*csc(d*x+c)/d/(a+a*sin(d*x
+c))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2772, 2773, 206}

$$-\frac{3a\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}-\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-3*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*d)
 - (3*a*Cot[c + d*x])/(4*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[
c + d*x])/(2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \csc^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{a\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{3}{4}\int \csc^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$$

$$= -\frac{3a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{3}{8}\int \csc(c+dx)\sqrt{\ldots}$$

$$= -\frac{3a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{(3a)\,\text{Subst}\left(\int \frac{1}{a-x\ldots}\right)}{\ldots}$$

$$= -\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{3a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\ldots}{2d\sqrt{a+a\sin\ldots}}$$

Mathematica [B] time = 0.75, size = 249, normalized size = 2.44

$$\frac{\csc^7\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)-6\sin\left(\frac{3}{2}(c+dx)\right)-2\cos\left(\frac{1}{2}(c+dx)\right)-6\cos\left(\frac{3}{2}(c+d\ldots\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/2]^7*Sqrt[a*(1 + Sin[c + d*x])]*(-2*Cos[(c + d*x)/2] - 6*Cos
[(3*(c + d*x))/2] - 3*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3*Cos[
2*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3*Log[1 - Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]] - 3*Cos[2*(c + d*x)]*Log[1 - Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]] + 2*Sin[(c + d*x)/2] - 6*Sin[(3*(c + d*x))/2]))/(4*d
*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^2)
```

fricas [B] time = 0.57, size = 319, normalized size = 3.13

$$\frac{3\left(\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx\ldots}{\ldots}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c
) - cos(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 -
4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*s
qrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*
a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (c
os(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(3*cos(d*x + c)^2
+ (3*cos(d*x + c) + 1)*sin(d*x + c) + 2*cos(d*x + c) - 1)*sqrt(a*sin(d*x +
c) + a))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x
 + c)^2 - d)*sin(d*x + c) - d)
```

giac [A] time = 1.12, size = 155, normalized size = 1.52

$$-\frac{\sqrt{2}\left(3\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)+\frac{4\left(6\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx\cdots\right.\right.}{\left(2\sin\left(-\cdots\right.\right.}\right.}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*(3*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2
*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*pi + 1
/2*d*x + 1/2*c)) + 4*(6*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1
/2*d*x + 1/2*c)^3 - 5*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2
*d*x + 1/2*c))/(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^2)*sqrt(a)/d
```

maple [A] time = 0.92, size = 132, normalized size = 1.29

$$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(3\sqrt{-a(\sin(dx+c)-1)}\,\sin(dx+c)\,a^{\frac{3}{2}}+3\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)\cdots\right.}{4\sin(dx+c)^{2}a^{\frac{3}{2}}\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/4*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(3*(-a*(sin(d*x+c)-1))^(1/2)*
sin(d*x+c)*a^(3/2)+3*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^
2*a^2+2*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2))/sin(d*x+c)^2/a^(3/2)/cos(d*x+c)/
(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sin\left(dx + c\right) + a} \csc\left(dx + c\right)^{3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sin(d*x + c) + a)*csc(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + a\,\sin\left(c + d\,x\right)}}{\sin\left(c + d\,x\right)^{3}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/sin(c + d*x)^3,x)
[Out] int((a + a*sin(c + d*x))^(1/2)/sin(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \left(\sin\left(c + dx\right) + 1\right)} \csc^{3}\left(c + dx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+a*sin(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*csc(c + d*x)**3, x)
```

3.40 $\int \csc^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=138

$$-\frac{5a\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}-\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{5a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -5/8*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)/d-5/8*a*cot
(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-5/12*a*cot(d*x+c)*csc(d*x+c)/d/(a+a*sin(d*
x+c))^(1/2)-1/3*a*cot(d*x+c)*csc(d*x+c)^2/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2772, 2773, 206}

$$-\frac{5a\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}-\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{5a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-5*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*d)
 - (5*a*Cot[c + d*x])/(8*d*Sqrt[a + a*Sin[c + d*x]]) - (5*a*Cot[c + d*x]*Cs
c[c + d*x])/(12*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c + d*x]^
2)/(3*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \csc^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{a\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{5}{6}\int \csc^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$$

$$= -\frac{5a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{5}{8}\int \csc^2(c+$$

$$= -\frac{5a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{5a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc}{3d\sqrt{a+a\sin(}$$

$$= -\frac{5a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{5a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc}{3d\sqrt{a+a\sin(}$$

$$= -\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{5a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{5a\cot(c+dx)}{12d\sqrt{a+a\,\mathrm{s}}}$$

Mathematica [B] time = 1.34, size = 285, normalized size = 2.07

$$\frac{\csc^{10}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(84\sin\left(\frac{1}{2}(c+dx)\right)-10\sin\left(\frac{3}{2}(c+dx)\right)-30\sin\left(\frac{5}{2}(c+dx)\right)-84\cos\left(\frac{1}{2}(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/2]^10*Sqrt[a*(1 + Sin[c + d*x])]*(-84*Cos[(c + d*x)/2] - 10*
Cos[(3*(c + d*x))/2] + 30*Cos[(5*(c + d*x))/2] + 84*Sin[(c + d*x)/2] - 45*L
og[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 45*Log[1 - Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] - 10*Sin[(3*(c + d*x))/2] - 30
*Sin[(5*(c + d*x))/2] + 15*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin
[3*(c + d*x)] - 15*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c +
d*x)]))/(24*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]
^2)^3)
```

fricas [B] time = 0.54, size = 361, normalized size = 2.62

$$\frac{15\left(\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)-1\right)\sin\left(dx+c\right)+1\right)\sqrt{a}\,\log\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(15*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c
)^2 - cos(d*x + c) - 1)*sin(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7
*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2
*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a
*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 +
 cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) +
4*(15*cos(d*x + c)^3 + 5*cos(d*x + c)^2 - (15*cos(d*x + c)^2 + 10*cos(d*x +
 c) - 13)*sin(d*x + c) - 23*cos(d*x + c) - 13)*sqrt(a*sin(d*x + c) + a))/(d
*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 - (d*cos(d*x + c)^3 + d*cos(d*x + c)^2
 - d*cos(d*x + c) - d)*sin(d*x + c) + d)
```

giac [A] time = 0.71, size = 184, normalized size = 1.33

$$-\frac{\sqrt{2}\left(15\,\sqrt{2}\,\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+\frac{4\left(60\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,\cdots\right.\right.}{\cdots}\right.}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/96*sqrt(2)*(15*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/
2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*pi +
1/2*d*x + 1/2*c)) + 4*(60*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi +
 1/2*d*x + 1/2*c)^5 - 80*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi +
1/2*d*x + 1/2*c)^3 + 33*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1
/2*d*x + 1/2*c))/(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^3)*sqrt(a)/d
```

maple [A] time = 0.78, size = 158, normalized size = 1.14

$$-\frac{\left(1+\sin\left(dx+c\right)\right)\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\left(15\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,a^{\frac{3}{2}}\left(\sin^{2}\left(dx+c\right)\right)+15\,\mathrm{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c\cdots}}{\sqrt{a}}\right.\right.}{24\sin\left(dx+c\right)^{3}a^{\frac{3}{2}}\cos\left(dx+c\right)\sqrt{a\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/24*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(15*(-a*(sin(d*x+c)-1))^(1/2
)*a^(3/2)*sin(d*x+c)^2+15*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*
x+c)^3*a^2+10*(-a*(sin(d*x+c)-1))^(1/2)*sin(d*x+c)*a^(3/2)+8*(-a*(sin(d*x+c
)-1))^(1/2)*a^(3/2))/sin(d*x+c)^3/a^(3/2)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)
/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sin(dx + c) + a} \csc(dx + c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*csc(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + a \sin(c + d x)}}{\sin(c + d x)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/sin(c + d*x)^4,x)
```

```
[Out] int((a + a*sin(c + d*x))^(1/2)/sin(c + d*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a (\sin(c + dx) + 1)} \csc^4(c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*csc(c + d*x)**4, x)
```

3.41 $\int \csc(c+dx)\sqrt{a-a\sin(c+dx)}\,dx$

Optimal. Leaf size=38

$$-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a-a\sin(c+dx)}}\right)}{d}$$

```
[Out] -2*arctanh(cos(d*x+c)*a^(1/2)/(a-a*sin(d*x+c))^(1/2))*a^(1/2)/d
```

Rubi [A] time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2773, 206}

$$-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a-a\sin(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sqrt[a - a*Sin[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a - a*Sin[c + d*x]]])/d
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)\sqrt{a-a\sin(c+dx)}\,dx &= \frac{(2a)\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\frac{a\cos(c+dx)}{\sqrt{a-a\sin(c+dx)}}\right)}{d}\\ &= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a-a\sin(c+dx)}}\right)}{d}\end{aligned}$$

Mathematica [B] time = 0.10, size = 97, normalized size = 2.55

$$\frac{\sqrt{a-a\sin(c+dx)}\left(\log\left(-\sin\left(\frac{1}{2}(c+dx)\right)-\cos\left(\frac{1}{2}(c+dx)\right)+1\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)+1\right)\right.}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sqrt[a - a*Sin[c + d*x]],x]
```

```
[Out] ((Log[1 - Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[1 + Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]])*Sqrt[a - a*Sin[c + d*x]])/(d*(Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]))
```

fricas [A] time = 0.57, size = 223, normalized size = 5.87

$$\left[\frac{\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2-(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{-a\sin(dx+c)+a}\sqrt{a}-9a\cos(dx+c)-\left(a\cos(dx\right.}{\cos(dx+c)^3+\cos(dx+c)^2-\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right.}{2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a-a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2
 - (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(-a*sin(d*x +
c) + a)*sqrt(a) - 9*a*cos(d*x + c) - (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) -
 a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 - (cos(d*x + c)^2 -
1)*sin(d*x + c) - cos(d*x + c) - 1))/d, sqrt(-a)*arctan(1/2*sqrt(-a*sin(d*x
 + c) + a)*sqrt(-a)*(sin(d*x + c) + 2)/(a*cos(d*x + c)))/d]
```

giac [B] time = 1.43, size = 104, normalized size = 2.74

$$\frac{\sqrt{a}\log\left(\frac{\left|-4\sqrt{2}-\frac{2\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1}-6\right|}{\left|4\sqrt{2}-\frac{2\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1}-6\right|}\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a-a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(a)*log(abs(-4*sqrt(2) - 2*(cos(-1/4*pi + 1/2*d*x + 1/2*c) - 1)/(cos(-1
/4*pi + 1/2*d*x + 1/2*c) + 1) - 6)/abs(4*sqrt(2) - 2*(cos(-1/4*pi + 1/2*d*x
```

```
+ 1/2*c) - 1)/(cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1) - 6))*sgn(sin(-1/4*pi +
1/2*d*x + 1/2*c))/d
```

maple [B] time = 0.73, size = 67, normalized size = 1.76

$$\frac{2 (\sin (dx+c)-1) \sqrt{a (1+\sin (dx+c))}\, \sqrt{a}\, \operatorname{arctanh}\left(\frac{\sqrt{a(1+\sin(dx+c))}}{\sqrt{a}}\right)}{\cos (dx+c) \sqrt{a-a \sin (dx+c)}\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a-a*sin(d*x+c))^(1/2),x)
[Out] 2*(sin(d*x+c)-1)*(a*(1+sin(d*x+c)))^(1/2)*a^(1/2)*arctanh((a*(1+sin(d*x+c))
)^(1/2)/a^(1/2))/cos(d*x+c)/(a-a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a \sin (dx+c)+a}\, \csc (dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a-a*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a*sin(d*x + c) + a)*csc(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{a-a\,\sin (c+d\,x)}}{\sin (c+d\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*sin(c + d*x))^(1/2)/sin(c + d*x),x)
[Out] int((a - a*sin(c + d*x))^(1/2)/sin(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a (\sin (c+dx)-1)}\, \csc (c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a-a*sin(d*x+c))**(1/2),x)
[Out] Integral(sqrt(-a*(sin(c + d*x) - 1))*csc(c + d*x), x)
```

3.42 $\int \csc(c+dx)\sqrt{-a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=39

$$\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)-a}}\right)}{d}$$

```
[Out] 2*arctan(cos(d*x+c)*a^(1/2)/(-a+a*sin(d*x+c))^(1/2))*a^(1/2)/d
```

Rubi [A] time = 0.05, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2773, 204}

$$\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)-a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sqrt[-a + a*Sin[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Cos[c + d*x])/Sqrt[-a + a*Sin[c + d*x]]])/d
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)\sqrt{-a+a\sin(c+dx)}\,dx &= -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{-a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{-a+a\sin(c+dx)}}\right)}{d}\\ &= \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{-a+a\sin(c+dx)}}\right)}{d}\end{aligned}$$

Mathematica [B] time = 0.08, size = 96, normalized size = 2.46

$$\frac{\sqrt{a(\sin(c+dx)-1)}\left(\log\left(-\sin\left(\frac{1}{2}(c+dx)\right)-\cos\left(\frac{1}{2}(c+dx)\right)+1\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)+1\right.\right.}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sqrt[-a + a*Sin[c + d*x]],x]
```

```
[Out] ((Log[1 - Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[1 + Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]])*Sqrt[a*(-1 + Sin[c + d*x])])/(d*(Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]))
```

fricas [A] time = 0.55, size = 223, normalized size = 5.72

$$\left[\frac{\sqrt{-a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2+4\left(\cos(dx+c)^2-(\cos(dx+c)+3)\sin(dx+c)-2\,\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)-a}\,\sqrt{-a}-9\,a\cos(dx+c)-\left(a\cos(dx\right.}{\cos(dx+c)^3+\cos(dx+c)^2-\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right.}{2\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(-a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^
2 - (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x +
c) - a)*sqrt(-a) - 9*a*cos(d*x + c) - (a*cos(d*x + c)^2 + 8*a*cos(d*x + c)
- a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 - (cos(d*x + c)^2 -
 1)*sin(d*x + c) - cos(d*x + c) - 1))/d, -sqrt(a)*arctan(1/2*sqrt(a*sin(d*x
 + c) - a)*(sin(d*x + c) + 2)/(sqrt(a)*cos(d*x + c)))/d]
```

giac [B] time = 1.11, size = 106, normalized size = 2.72

$$\frac{\sqrt{-a}\log\left(\frac{\left|-4\sqrt{2}-\frac{2\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1}-6\right|}{\left|4\sqrt{2}-\frac{2\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1}-6\right|}\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(-a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(-a)*log(abs(-4*sqrt(2) - 2*(cos(-1/4*pi + 1/2*d*x + 1/2*c) - 1)/(cos(-
1/4*pi + 1/2*d*x + 1/2*c) + 1) - 6)/abs(4*sqrt(2) - 2*(cos(-1/4*pi + 1/2*d*
```

```
x + 1/2*c) - 1)/(cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1) - 6))*sgn(sin(-1/4*pi
+ 1/2*d*x + 1/2*c))/d
```

maple [B] time = 0.54, size = 70, normalized size = 1.79

$$\frac{2\left(\sin\left(dx+c\right)-1\right)\sqrt{-a\left(1+\sin\left(dx+c\right)\right)}\,\sqrt{a}\,\arctan\left(\frac{\sqrt{-a(1+\sin(dx+c))}}{\sqrt{a}}\right)}{\cos\left(dx+c\right)\sqrt{a\sin\left(dx+c\right)-a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a*sin(d*x+c)-a)^(1/2),x)
[Out] 2*(sin(d*x+c)-1)*(-a*(1+sin(d*x+c)))^(1/2)*a^(1/2)*arctan((-a*(1+sin(d*x+c)
))^(1/2)/a^(1/2))/cos(d*x+c)/(a*sin(d*x+c)-a)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sin\left(dx+c\right)-a}\,\csc\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(-a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sin(d*x + c) - a)*csc(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{\sqrt{a\,\sin\left(c+d\,x\right)-a}}{\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) - a)^(1/2)/sin(c + d*x),x)
[Out] int((a*sin(c + d*x) - a)^(1/2)/sin(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sin\left(c+dx\right)-1\right)}\,\csc\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(-a+a*sin(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*(sin(c + d*x) - 1))*csc(c + d*x), x)
```

3.43 $\int \csc(c+dx)\sqrt{-a-a\sin(c+dx)}\,dx$

Optimal. Leaf size=40

$$\frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a(-\sin(c+dx))-a}}\right)}{d}$$

```
[Out] 2*arctan(cos(d*x+c)*a^(1/2)/(-a-a*sin(d*x+c))^(1/2))*a^(1/2)/d
```

Rubi [A] time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2773, 204}

$$\frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a(-\sin(c+dx))-a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sqrt[-a - a*Sin[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Cos[c + d*x])/Sqrt[-a - a*Sin[c + d*x]]])/d
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)\sqrt{-a-a\sin(c+dx)}\,dx &= \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-a-x^2}\,dx, x, -\frac{a\cos(c+dx)}{\sqrt{-a-a\sin(c+dx)}}\right)}{d}\\ &= \frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{-a-a\sin(c+dx)}}\right)}{d}\end{aligned}$$

Mathematica [B] time = 0.08, size = 95, normalized size = 2.38

$$\frac{\sqrt{-a(\sin(c+dx)+1)}\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)-\cos\left(\frac{1}{2}(c+dx)\right)+1\right)-\log\left(-\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)+\right.\right.}{d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sqrt[-a - a*Sin[c + d*x]],x]
```

```
[Out] ((-Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[1 - Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]])*Sqrt[-(a*(1 + Sin[c + d*x]))])/(d*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]))
```

fricas [A] time = 0.51, size = 221, normalized size = 5.52

$$\left[\frac{\sqrt{-a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2+4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{-a\sin(dx+c)-a}\,\sqrt{-a}-9\,a\cos(dx+c)+\left(a\cos\right.}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right.}{2\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(-a-a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^
2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(-a*sin(d*x +
 c) - a)*sqrt(-a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c)
 - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2
- 1)*sin(d*x + c) - cos(d*x + c) - 1))/d, sqrt(a)*arctan(1/2*sqrt(-a*sin(d*
x + c) - a)*(sin(d*x + c) - 2)/(sqrt(a)*cos(d*x + c)))/d]
```

giac [B] time = 0.68, size = 69, normalized size = 1.72

$$-\frac{\sqrt{-a}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(-a-a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-a)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c))/abs(2*sqrt
(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)
)/d
```

maple [A] time = 0.49, size = 69, normalized size = 1.72

$$-\frac{2\left(1+\sin\left(dx+c\right)\right)\sqrt{a\left(\sin\left(dx+c\right)-1\right)}\,\sqrt{a}\,\arctan\left(\frac{\sqrt{a(\sin(dx+c)-1)}}{\sqrt{a}}\right)}{\cos\left(dx+c\right)\sqrt{-a-a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(-a-a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2*(1+sin(d*x+c))*(a*(sin(d*x+c)-1))^(1/2)*a^(1/2)*arctan((a*(sin(d*x+c)-1)
)^(1/2)/a^(1/2))/cos(d*x+c)/(-a-a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a\sin\left(dx+c\right)-a}\,\csc\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(-a-a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*sin(d*x + c) - a)*csc(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{-a-a\,\sin\left(c+d\,x\right)}}{\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- a - a*sin(c + d*x))^(1/2)/sin(c + d*x),x)
```

```
[Out] int((- a - a*sin(c + d*x))^(1/2)/sin(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a\left(\sin\left(c+dx\right)+1\right)}\csc\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(-a-a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sin(c + d*x) + 1))*csc(c + d*x), x)
```

3.44 $\int \sin^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=162

$$-\frac{2a^2\sin^4(c+dx)\cos(c+dx)}{9d\sqrt{a\sin(c+dx)+a}}-\frac{34a^2\sin^3(c+dx)\cos(c+dx)}{63d\sqrt{a\sin(c+dx)+a}}-\frac{68a^2\cos(c+dx)}{45d\sqrt{a\sin(c+dx)+a}}-\frac{68\cos(c+dx)(a\sin(c+}{105d}$$

```
[Out] -68/105*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/d-68/45*a^2*cos(d*x+c)/d/(a+a*sin
(d*x+c))^(1/2)-34/63*a^2*cos(d*x+c)*sin(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)-2
/9*a^2*cos(d*x+c)*sin(d*x+c)^4/d/(a+a*sin(d*x+c))^(1/2)+136/315*a*cos(d*x+c
)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.24, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2763, 21, 2770, 2759, 2751, 2646}

$$-\frac{2a^2\sin^4(c+dx)\cos(c+dx)}{9d\sqrt{a\sin(c+dx)+a}}-\frac{34a^2\sin^3(c+dx)\cos(c+dx)}{63d\sqrt{a\sin(c+dx)+a}}-\frac{68a^2\cos(c+dx)}{45d\sqrt{a\sin(c+dx)+a}}-\frac{68\cos(c+dx)(a\sin(c+}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-68*a^2*Cos[c + d*x])/(45*d*Sqrt[a + a*Sin[c + d*x]]) - (34*a^2*Cos[c + d*
x]*Sin[c + d*x]^3)/(63*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a^2*Cos[c + d*x]*Si
n[c + d*x]^4)/(9*d*Sqrt[a + a*Sin[c + d*x]]) + (136*a*Cos[c + d*x]*Sqrt[a +
 a*Sin[c + d*x]])/(315*d) - (68*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(1
05*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = -\frac{2a^2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} + \frac{2}{9}\int \frac{\sin^3(c+dx)\left(\frac{17a^2}{2}+\frac{17}{2}a^2\sin(c\right.}{\sqrt{a+a\sin(c+dx)}}$$
$$= -\frac{2a^2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} + \frac{1}{9}(17a)\int \sin^3(c+dx)\sqrt{a+a\sin(c+}$$
$$= -\frac{34a^2\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}} - \frac{2a^2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} + \frac{1}{21}(3$$
$$= -\frac{34a^2\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}} - \frac{2a^2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} - \frac{68\,c}{}$$
$$= -\frac{34a^2\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}} - \frac{2a^2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} + \frac{136a}{}$$
$$= -\frac{68a^2\cos(c+dx)}{45d\sqrt{a+a\sin(c+dx)}} - \frac{34a^2\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}} - \frac{2a^2\cos(c}{9d\sqrt{a}}$$

Mathematica [A] time = 0.54, size = 165, normalized size = 1.02

$$\frac{(a(\sin(c+dx)+1))^{3/2}\left(3780\sin\left(\frac{1}{2}(c+dx)\right)-1050\sin\left(\frac{3}{2}(c+dx)\right)-378\sin\left(\frac{5}{2}(c+dx)\right)+135\sin\left(\frac{7}{2}(c+dx)\right)\right.}{2520d\left(\sin\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(3/2)*(-3780*Cos[(c + d*x)/2] - 1050*Cos[(3*(c + d*
x))/2] + 378*Cos[(5*(c + d*x))/2] + 135*Cos[(7*(c + d*x))/2] - 35*Cos[(9*(c
 + d*x))/2] + 3780*Sin[(c + d*x)/2] - 1050*Sin[(3*(c + d*x))/2] - 378*Sin[(
5*(c + d*x))/2] + 135*Sin[(7*(c + d*x))/2] + 35*Sin[(9*(c + d*x))/2]))/(252
0*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

fricas [A] time = 0.56, size = 145, normalized size = 0.90

$$-\frac{2\left(35\,a\cos\left(dx+c\right)^5-50\,a\cos\left(dx+c\right)^4-172\,a\cos\left(dx+c\right)^3+134\,a\cos\left(dx+c\right)^2+409\,a\cos\left(dx+c\right)-\left(35\right.\right.}{315\,(d\cos(d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/315*(35*a*cos(d*x + c)^5 - 50*a*cos(d*x + c)^4 - 172*a*cos(d*x + c)^3 +
134*a*cos(d*x + c)^2 + 409*a*cos(d*x + c) - (35*a*cos(d*x + c)^4 + 85*a*cos
(d*x + c)^3 - 87*a*cos(d*x + c)^2 - 221*a*cos(d*x + c) + 188*a)*sin(d*x + c
) + 188*a)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

giac [B] time = 1.18, size = 288, normalized size = 1.78

$$\frac{1}{2520}\sqrt{2}\left(\frac{126\,a\cos\left(\frac{1}{4}\,\pi+\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}-\frac{1890\,a\cos\left(\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\operatorname{sgn}\left(\cos\left(\right.\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2520*sqrt(2)*(126*a*cos(1/4*pi + 5/2*d*x + 5/2*c)*sgn(cos(-1/4*pi + 1/2*d
*x + 1/2*c))/d - 1890*a*cos(1/4*pi + 1/2*d*x + 1/2*c)*sgn(cos(-1/4*pi + 1/2
*d*x + 1/2*c))/d + 90*a*cos(-1/4*pi + 7/2*d*x + 7/2*c)*sgn(cos(-1/4*pi + 1/
2*d*x + 1/2*c))/d - 630*a*cos(-1/4*pi + 3/2*d*x + 3/2*c)*sgn(cos(-1/4*pi +
1/2*d*x + 1/2*c))/d + 45*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi +
 7/2*d*x + 7/2*c)/d - 420*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi
+ 3/2*d*x + 3/2*c)/d + 35*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi
 + 9/2*d*x + 9/2*c)/d - 252*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*
pi + 5/2*d*x + 5/2*c)/d + 1890*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1
/4*pi + 1/2*d*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.63, size = 85, normalized size = 0.52

$$\frac{2\left(1+\sin\left(dx+c\right)\right)a^{2}\left(\sin\left(dx+c\right)-1\right)\left(35\left(\sin^{4}\left(dx+c\right)\right)+85\left(\sin^{3}\left(dx+c\right)\right)+102\left(\sin^{2}\left(dx+c\right)\right)+136\sin\left(dx\right.\right.}{315\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/315*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)*(35*sin(d*x+c)^4+85*sin(d*x+c)^3+10
2*sin(d*x+c)^2+136*sin(d*x+c)+272)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\sin\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*sin(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(c+dx)^3 \left(a+a\sin(c+dx)\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + a*sin(c + d*x))^(3/2),x)
[Out] int(sin(c + d*x)^3*(a + a*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+a*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

3.45 $\int \sin^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=116

$$-\frac{152a^2\cos(c+dx)}{105d\sqrt{a\sin(c+dx)+a}}-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{5/2}}{7ad}+\frac{4\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{35d}-\frac{38a\cos(c+a}{}$$

```
[Out] 4/35*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/d-2/7*cos(d*x+c)*(a+a*sin(d*x+c))^(5
/2)/a/d-152/105*a^2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-38/105*a*cos(d*x+c)
*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.14, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2759, 2751, 2647, 2646}

$$-\frac{152a^2\cos(c+dx)}{105d\sqrt{a\sin(c+dx)+a}}-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{5/2}}{7ad}+\frac{4\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{35d}-\frac{38a\cos(c+a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-152*a^2*Cos[c + d*x])/(105*d*Sqrt[a + a*Sin[c + d*x]]) - (38*a*Cos[c + d*
x]*Sqrt[a + a*Sin[c + d*x]])/(105*d) + (4*Cos[c + d*x]*(a + a*Sin[c + d*x])
^(3/2))/(35*d) - (2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(5/2))/(7*a*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rubi steps

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = -\frac{2\cos(c+dx)(a+a\sin(c+dx))^{5/2}}{7ad}+\frac{2\int\left(\frac{5a}{2}-a\sin(c+dx)\right)(a+a$$

$$= \frac{4\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{35d}-\frac{2\cos(c+dx)(a+a\sin(c+dx))^{5}}{7ad}$$

$$= -\frac{38a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{105d}+\frac{4\cos(c+dx)(a+a\sin(c+dx}{35d}$$

$$= -\frac{152a^2\cos(c+dx)}{105d\sqrt{a+a\sin(c+dx)}}-\frac{38a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{105d}+\frac{4c}{}$$

Mathematica [A] time = 0.36, size = 141, normalized size = 1.22

$$\frac{(a(\sin(c+dx)+1))^{3/2}\left(735\sin\left(\frac{1}{2}(c+dx)\right)-175\sin\left(\frac{3}{2}(c+dx)\right)-63\sin\left(\frac{5}{2}(c+dx)\right)+15\sin\left(\frac{7}{2}(c+dx)\right)-73\right.}{420d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(3/2)*(-735*Cos[(c + d*x)/2] - 175*Cos[(3*(c + d*x)
)/2] + 63*Cos[(5*(c + d*x))/2] + 15*Cos[(7*(c + d*x))/2] + 735*Sin[(c + d*x
)/2] - 175*Sin[(3*(c + d*x))/2] - 63*Sin[(5*(c + d*x))/2] + 15*Sin[(7*(c +
d*x))/2]))/(420*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

fricas [A] time = 0.48, size = 122, normalized size = 1.05

$$\frac{2\left(15\,a\cos\left(dx+c\right)^4+39\,a\cos\left(dx+c\right)^3-43\,a\cos\left(dx+c\right)^2-143\,a\cos\left(dx+c\right)+\left(15\,a\cos\left(dx+c\right)^3-24\,a\cos\right.\right.}{105\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*a*cos(d*x + c)^4 + 39*a*cos(d*x + c)^3 - 43*a*cos(d*x + c)^2 - 14
3*a*cos(d*x + c) + (15*a*cos(d*x + c)^3 - 24*a*cos(d*x + c)^2 - 67*a*cos(d*
x + c) + 76*a)*sin(d*x + c) - 76*a)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c
) + d*sin(d*x + c) + d)
```

giac [B] time = 0.60, size = 226, normalized size = 1.95

$$\frac{1}{420}\sqrt{2}\left(\frac{21\,a\cos\left(\frac{1}{4}\,\pi+\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}-\frac{315\,a\cos\left(\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi\right.\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/420*sqrt(2)*(21*a*cos(1/4*pi + 5/2*d*x + 5/2*c)*sgn(cos(-1/4*pi + 1/2*d*x
 + 1/2*c))/d - 315*a*cos(1/4*pi + 1/2*d*x + 1/2*c)*sgn(cos(-1/4*pi + 1/2*d*
x + 1/2*c))/d + 15*a*cos(-1/4*pi + 7/2*d*x + 7/2*c)*sgn(cos(-1/4*pi + 1/2*d
*x + 1/2*c))/d - 105*a*cos(-1/4*pi + 3/2*d*x + 3/2*c)*sgn(cos(-1/4*pi + 1/2
*d*x + 1/2*c))/d - 70*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 3/
2*d*x + 3/2*c)/d - 42*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 5
/2*d*x + 5/2*c)/d + 420*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi +
 1/2*d*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.61, size = 75, normalized size = 0.65

$$\frac{2\left(1+\sin\left(dx+c\right)\right)a^{2}\left(\sin\left(dx+c\right)-1\right)\left(15\left(\sin^{3}\left(dx+c\right)\right)+39\left(\sin^{2}\left(dx+c\right)\right)+52\sin\left(dx+c\right)+104\right)}{105\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/105*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)*(15*sin(d*x+c)^3+39*sin(d*x+c)^2+52
*sin(d*x+c)+104)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\sin\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*sin(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(c+d\,x)^2\,(a+a\,\sin(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + a*sin(c + d*x))^(3/2),x)
[Out] int(sin(c + d*x)^2*(a + a*sin(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^{\frac{3}{2}}\sin^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+a*sin(d*x+c))**(3/2),x)
[Out] Integral((a*(sin(c + d*x) + 1))**(3/2)*sin(c + d*x)**2, x)
```

3.46 $\int \sin(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=86

$$-\frac{8a^2\cos(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}-\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{5d}-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5d}$$

```
[Out] -2/5*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/d-8/5*a^2*cos(d*x+c)/d/(a+a*sin(d*x+
c))^(1/2)-2/5*a*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2751, 2647, 2646}

$$-\frac{8a^2\cos(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}-\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{5d}-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-8*a^2*Cos[c + d*x])/(5*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]*Sq
rt[a + a*Sin[c + d*x]])/(5*d) - (2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))
/(5*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \sin(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = -\frac{2\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{5d}+\frac{3}{5}\int (a+a\sin(c+dx))^{3/2}\,dx$$
$$= -\frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{5d}-\frac{2\cos(c+dx)(a+a\sin(c+dx))^{3}}{5d}$$
$$= -\frac{8a^2\cos(c+dx)}{5d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{5d}-\frac{2\cos(c}{}$$

Mathematica [A] time = 0.16, size = 115, normalized size = 1.34

$$-\frac{(a(\sin(c+dx)+1))^{3/2}\left(-20\sin\left(\frac{1}{2}(c+dx)\right)+5\sin\left(\frac{3}{2}(c+dx)\right)+\sin\left(\frac{5}{2}(c+dx)\right)+20\cos\left(\frac{1}{2}(c+dx)\right)+5\cos\right.}{10d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/10*((a*(1 + Sin[c + d*x]))^(3/2)*(20*Cos[(c + d*x)/2] + 5*Cos[(3*(c + d*
x))/2] - Cos[(5*(c + d*x))/2] - 20*Sin[(c + d*x)/2] + 5*Sin[(3*(c + d*x))/2
] + Sin[(5*(c + d*x))/2]))/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

fricas [A] time = 0.52, size = 99, normalized size = 1.15

$$\frac{2\left(a\cos(dx+c)^3-2a\cos(dx+c)^2-7a\cos(dx+c)-\left(a\cos(dx+c)^2+3a\cos(dx+c)-4a\right)\sin(dx+c)-\right.}{5\left(d\cos(dx+c)+d\sin(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(a*cos(d*x + c)^3 - 2*a*cos(d*x + c)^2 - 7*a*cos(d*x + c) - (a*cos(d*x
+ c)^2 + 3*a*cos(d*x + c) - 4*a)*sin(d*x + c) - 4*a)*sqrt(a*sin(d*x + c) +
a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

giac [B] time = 1.02, size = 164, normalized size = 1.91

$$-\frac{1}{30}\sqrt{2}\left(\frac{30a\cos\left(\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{d}+\frac{10a\cos\left(-\frac{1}{4}\pi+\frac{3}{2}dx+\frac{3}{2}c\right)\mathrm{sgn}\left(\cos\left(-\right.\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/30*sqrt(2)*(30*a*cos(1/4*pi + 1/2*d*x + 1/2*c)*sgn(cos(-1/4*pi + 1/2*d*x
 + 1/2*c))/d + 10*a*cos(-1/4*pi + 3/2*d*x + 3/2*c)*sgn(cos(-1/4*pi + 1/2*d*
x + 1/2*c))/d + 5*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi + 3/2*d*
x + 3/2*c)/d + 3*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 5/2*d*
x + 5/2*c)/d - 30*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d
*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.77, size = 63, normalized size = 0.73

$$\frac{2 \left(1+\sin \left(d x+c\right)\right) a^{2} \left(\sin \left(d x+c\right)-1\right) \left(\sin^{2}\left(d x+c\right)+3 \sin \left(d x+c\right)+6\right)}{5 \cos \left(d x+c\right) \sqrt{a+a \sin \left(d x+c\right)}\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/5*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)*(sin(d*x+c)^2+3*sin(d*x+c)+6)/cos(d*x
+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin \left(dx+c\right)+a\right)^{\frac{3}{2}} \sin \left(dx+c\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*sin(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(c+d\,x\right)\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(sin(c + d*x)*(a + a*sin(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\sin{\left(c + dx \right)} + 1\right)\right)^{\frac{3}{2}} \sin{\left(c + dx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**(3/2)*sin(c + d*x), x)
```

3.47 $\int (a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=59

$$-\frac{8a^2\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}$$

```
[Out] -8/3*a^2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-2/3*a*cos(d*x+c)*(a+a*sin(d*x+
c))^(1/2)/d
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2647, 2646}

$$-\frac{8a^2\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-8*a^2*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]*Sq
rt[a + a*Sin[c + d*x]])/(3*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))^{3/2}\,dx &= -\frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}+\frac{1}{3}(4a)\int \sqrt{a+a\sin(c+dx)}\,dx\\ &= -\frac{8a^2\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}\end{aligned}$$

Mathematica [A] time = 0.14, size = 89, normalized size = 1.51

$$-\frac{(a(\sin(c+dx)+1))^{3/2}\left(-9\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)+9\cos\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{3}{2}(c+dx)\right)\right)}{3d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/3*((a*(1 + Sin[c + d*x]))^(3/2)*(9*Cos[(c + d*x)/2] + Cos[(3*(c + d*x))/
2] - 9*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/(d*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])^3)
```

fricas [A] time = 0.45, size = 76, normalized size = 1.29

$$-\frac{2\left(a\cos\left(dx+c\right)^2+5\,a\cos\left(dx+c\right)+\left(a\cos\left(dx+c\right)-4\,a\right)\sin\left(dx+c\right)+4\,a\right)\sqrt{a\sin\left(dx+c\right)+a}}{3\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(a*cos(d*x + c)^2 + 5*a*cos(d*x + c) + (a*cos(d*x + c) - 4*a)*sin(d*x
+ c) + 4*a)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

giac [A] time = 0.75, size = 101, normalized size = 1.71

$$-\frac{1}{3}\sqrt{2}\left(\frac{3\,a\cos\left(\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}+\frac{a\cos\left(-\frac{1}{4}\,\pi+\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\right.\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(2)*(3*a*cos(1/4*pi + 1/2*d*x + 1/2*c)*sgn(cos(-1/4*pi + 1/2*d*x +
 1/2*c))/d + a*cos(-1/4*pi + 3/2*d*x + 3/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1
/2*c))/d - 6*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x +
1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.58, size = 53, normalized size = 0.90

$$\frac{2\left(1+\sin\left(dx+c\right)\right)a^2\left(\sin\left(dx+c\right)-1\right)\left(\sin\left(dx+c\right)+5\right)}{3\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/3*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)*(sin(d*x+c)+5)/cos(d*x+c)/(a+a*sin(d*
x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (a+a\,\sin(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(c+dx)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*sin(c + d*x) + a)**(3/2), x)
```

3.48 $\int \csc(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=66

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}-\frac{2a^2\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -2*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-2*a^2*cos(d
*x+c)/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2763, 21, 2773, 206}

$$-\frac{2a^2\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d - (
2*a^2*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
```

```
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \csc(c+dx)(a+a\sin(c+dx))^{3/2}\,dx &= -\frac{2a^2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}+2\int \frac{\csc(c+dx)\left(\frac{a^2}{2}+\frac{1}{2}a^2\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx\\
&= -\frac{2a^2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}+a\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx\\
&= -\frac{2a^2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}-\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,\frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}\\
&= -\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}-\frac{2a^2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 118, normalized size = 1.79

$$\frac{(a(\sin(c+dx)+1))^{3/2}\left(2\sin\left(\frac{1}{2}(c+dx)\right)-2\cos\left(\frac{1}{2}(c+dx)\right)-\log\left(-\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)+1\right)+\text{lo}}{d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((-2*Cos[(c + d*x)/2] - Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[
1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 2*Sin[(c + d*x)/2])*(a*(1 + Sin[
c + d*x]))^(3/2))/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

fricas [B] time = 0.45, size = 239, normalized size = 3.62

$$\frac{(a\cos(dx+c)+a\sin(dx+c)+a)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\,\cos(dx+c)-3\right)\sqrt{}}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2\right.}\right.}{2\,(d\cos(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((a*cos(d*x + c) + a*sin(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c)^3 -
7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) -
2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (
a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3
+ cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) -
 4*(a*cos(d*x + c) - a*sin(d*x + c) + a)*sqrt(a*sin(d*x + c) + a))/(d*cos(d
*x + c) + d*sin(d*x + c) + d)
```

giac [B] time = 5.43, size = 2251, normalized size = 34.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*sqrt(a)*((sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2
*c)^3*tan(1/4*c)^6 - 6*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/
2*c)^3*tan(1/4*c)^5 + 3*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1
/2*c)^2*tan(1/4*c)^6 - 15*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan
(1/2*c)^3*tan(1/4*c)^4 + 18*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*t
an(1/2*c)^2*tan(1/4*c)^5 - 3*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*
tan(1/2*c)*tan(1/4*c)^6 + 20*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*
tan(1/2*c)^3*tan(1/4*c)^3 - 45*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)
)*tan(1/2*c)^2*tan(1/4*c)^4 + 18*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*
c))*tan(1/2*c)*tan(1/4*c)^5 - sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))
*tan(1/4*c)^6 + 15*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)
^3*tan(1/4*c)^2 - 60*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*
c)^2*tan(1/4*c)^3 + 45*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/
2*c)*tan(1/4*c)^4 - 6*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4
*c)^5 - 6*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/
4*c) + 45*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/
4*c)^2 - 60*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/
4*c)^3 + 15*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^4 - sq
rt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^3 + 18*sqrt(2)*a*sgn
(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c) - 45*sqrt(2)*a*sgn
```

(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^2 + 20*sqrt(2)*a*sgn
(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^3 - 3*sqrt(2)*a*sgn(cos(-1/4*pi
 + 1/2*d*x + 1/2*c))*tan(1/2*c)^2 + 18*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x
+ 1/2*c))*tan(1/2*c)*tan(1/4*c) - 15*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x +
1/2*c))*tan(1/4*c)^2 + 3*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(
1/2*c) - 6*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c) + sqrt(
2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))*log(abs(-2*tan(1/4*d*x + c)*tan(1
/2*c)^3 + 6*tan(1/4*d*x + c)*tan(1/2*c) - 6*tan(1/2*c)^2 - 2*(tan(1/2*c)^2
+ 1)^(3/2) + 2)/abs(-2*tan(1/4*d*x + c)*tan(1/2*c)^3 + 6*tan(1/4*d*x + c)*t
an(1/2*c) - 6*tan(1/2*c)^2 + 2*(tan(1/2*c)^2 + 1)^(3/2) + 2))/((tan(1/4*c)^
6 + 3*tan(1/4*c)^4 + 3*tan(1/4*c)^2 + 1)*(tan(1/2*c)^2 + 1)^(3/2)) + (sqrt(
2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^6 + 6*sqrt
(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^5 - 3*sqr
t(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^6 - 15*s
qrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^4 + 18
*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^5 -
3*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^6 - 2
0*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^3 +
 45*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^4
 - 18*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^5
 + sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^6 + 15*sqrt(2)*
a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^2 - 60*sqrt(2
)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^3 + 45*sqrt
(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^4 - 6*sqrt(
2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^5 + 6*sqrt(2)*a*sgn(cos
(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c) - 45*sqrt(2)*a*sgn(cos
(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^2 + 60*sqrt(2)*a*sgn(c
os(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^3 - 15*sqrt(2)*a*sgn(c
os(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^4 - sqrt(2)*a*sgn(cos(-1/4*pi + 1
/2*d*x + 1/2*c))*tan(1/2*c)^3 + 18*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/
2*c))*tan(1/2*c)^2*tan(1/4*c) - 45*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/
2*c))*tan(1/2*c)*tan(1/4*c)^2 + 20*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/
2*c))*tan(1/4*c)^3 + 3*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/
2*c)^2 - 18*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/
4*c) + 15*sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^2 + 3*sq
rt(2)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/2*c) - 6*sqrt(2)*a*sgn(co
s(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c) - sqrt(2)*a*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c)))*log(abs(-6*tan(1/4*d*x + c)*tan(1/2*c)^2 + 2*tan(1/2*c)^3 -
2*(tan(1/2*c)^2 + 1)^(3/2) + 2*tan(1/4*d*x + c) - 6*tan(1/2*c))/abs(-6*tan(
1/4*d*x + c)*tan(1/2*c)^2 + 2*tan(1/2*c)^3 + 2*(tan(1/2*c)^2 + 1)^(3/2) + 2
*tan(1/4*d*x + c) - 6*tan(1/2*c)))/((tan(1/4*c)^6 + 3*tan(1/4*c)^4 + 3*tan(
1/4*c)^2 + 1)*(tan(1/2*c)^2 + 1)^(3/2)) - 8*(a*sgn(cos(-1/4*pi + 1/2*d*x +
1/2*c))*tan(1/4*d*x + c)*tan(1/4*c)^6 + 6*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2
*c))*tan(1/4*d*x + c)*tan(1/4*c)^5 - a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*
tan(1/4*c)^6 - 15*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*ta

```
n(1/4*c)^4 + 6*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^5 - 20*a*sg
n(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/4*c)^3 + 15*a*sgn(
cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^4 + 15*a*sgn(cos(-1/4*pi + 1/2*d
*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/4*c)^2 - 20*a*sgn(cos(-1/4*pi + 1/2*d*x
 + 1/2*c))*tan(1/4*c)^3 + 6*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d
*x + c)*tan(1/4*c) - 15*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*c)^2
- a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*tan(1/4*d*x + c) + 6*a*sgn(cos(-1/4
*pi + 1/2*d*x + 1/2*c))*tan(1/4*c) + a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))
/((sqrt(2)*tan(1/4*c)^6 + 3*sqrt(2)*tan(1/4*c)^4 + 3*sqrt(2)*tan(1/4*c)^2 +
 sqrt(2))*(tan(1/4*d*x + c)^2 + 1)))/d
```

maple [A] time = 0.64, size = 84, normalized size = 1.27

$$-\frac{2\left(1+\sin\left(dx+c\right)\right)\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,a\left(\sqrt{a-a\sin\left(dx+c\right)}+\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)\right)}{\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*a*((a-a*sin(d*x+c))^(1/2)+a^(1/
2)*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2)))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/
2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\csc\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*csc(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\left(a+a\,\sin\left(c+d\,x\right)\right)^{3/2}}{\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2)/sin(c + d*x),x)
```

```
[Out] int((a + a*sin(c + d*x))^(3/2)/sin(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^{\frac{3}{2}}\csc(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))**(3/2),x)
[Out] Integral((a*(sin(c + d*x) + 1))**(3/2)*csc(c + d*x), x)
```

3.49 $\int \csc^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=66

$$-\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}-\frac{a^2\cot(c+dx)}{d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -3*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-a^2*cot(d*x
+c)/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2762, 21, 2773, 206}

$$-\frac{a^2\cot(c+dx)}{d\sqrt{a\sin(c+dx)+a}}-\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-3*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d - (
a^2*Cot[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
```

```
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \csc^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx &= -\frac{a^2\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - a\int \frac{\csc(c+dx)\left(-\frac{3a}{2}-\frac{3}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx \\
&= -\frac{a^2\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{1}{2}(3a)\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx \\
&= -\frac{a^2\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} \\
&= -\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{a^2\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}
\end{aligned}$$

Mathematica [B] time = 0.65, size = 180, normalized size = 2.73

$$-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(-2\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)+3\sin(c+dx)\left(\log\left(-\sin\left(\frac{1}{2}(c\right.\right.\right.\right.}{d\left(\cot\left(\frac{1}{2}(c+dx)\right)+1\right)\left(\csc\left(\frac{1}{4}(c+dx)\right)-\sec\left(\frac{1}{4}(c+dx)\right)\right)\left(\csc\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -((a*Csc[(c + d*x)/2]^4*Sqrt[a*(1 + Sin[c + d*x])]*(2*Cos[(c + d*x)/2] - 2*
Sin[(c + d*x)/2] + 3*(Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[1
- Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*Sin[c + d*x]))/(d*(1 + Cot[(c + d*x
)/2])*(Csc[(c + d*x)/4] - Sec[(c + d*x)/4])*(Csc[(c + d*x)/4] + Sec[(c + d*
x)/4])))
```

fricas [B] time = 0.46, size = 268, normalized size = 4.06

$$\frac{3\left(a\cos\left(dx+c\right)^{2}-\left(a\cos\left(dx+c\right)+a\right)\sin\left(dx+c\right)-a\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\,\mathrm{s}\right.}{\cos(dx+c)}\right.}{4\left(d\cos\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(3*(a*cos(d*x + c)^2 - (a*cos(d*x + c) + a)*sin(d*x + c) - a)*sqrt(a)*l
og((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x +
c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a)
 - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c
) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c)
 - cos(d*x + c) - 1)) + 4*(a*cos(d*x + c) - a*sin(d*x + c) + a)*sqrt(a*sin(
d*x + c) + a))/(d*cos(d*x + c)^2 - (d*cos(d*x + c) + d)*sin(d*x + c) - d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.74, size = 103, normalized size = 1.56

$$-\frac{\left(1+\sin\left(dx+c\right)\right)\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\sqrt{a}\,\left(3\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)a\sin\left(dx+c\right)+\sqrt{a-a\sin\left(dx+c\right)}\,\sqrt{a}\right.}{\sin\left(dx+c\right)\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*a^(1/2)*(3*arctanh((a-a*sin(d*x+c
))^(1/2)/a^(1/2))*a*sin(d*x+c)+(a-a*sin(d*x+c))^(1/2)*a^(1/2))/sin(d*x+c)/c
os(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\csc\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*csc(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a + a \sin(c + d x))^{3/2}}{\sin(c + d x)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2)/sin(c + d*x)^2,x)
```

```
[Out] int((a + a*sin(c + d*x))^(3/2)/sin(c + d*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.50 $\int \csc^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=106

$$-\frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}-\frac{7a^2\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -7/4*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-7/4*a^2*c
ot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-1/2*a^2*cot(d*x+c)*csc(d*x+c)/d/(a+a*sin
(d*x+c))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2762, 21, 2772, 2773, 206}

$$-\frac{7a^2\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}-\frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-7*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*d)
 - (7*a^2*Cot[c + d*x])/(4*d*Sqrt[a + a*Sin[c + d*x]]) - (a^2*Cot[c + d*x]*
Csc[c + d*x])/(2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
```

```
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \csc^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = -\frac{a^2\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{1}{2}a\int \frac{\csc^2(c+dx)\left(-\frac{7a}{2}-\frac{7}{2}a\sin(c+dx)\right.}{\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{a^2\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{1}{4}(7a)\int \csc^2(c+dx)\sqrt{a+a\sin(c+dx)}$$

$$= -\frac{7a^2\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{1}{8}(7a)\int \csc(c+$$

$$= -\frac{7a^2\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{\left(7a^2\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{7a^2\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx}{2d\sqrt{a+a\sin}}$$

Mathematica [B] time = 0.61, size = 250, normalized size = 2.36

$$a\csc^7\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(-6\sin\left(\frac{1}{2}(c+dx)\right)-14\sin\left(\frac{3}{2}(c+dx)\right)+6\cos\left(\frac{1}{2}(c+dx)\right)-14\cos\left(\frac{3}{2}(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a*Csc[(c + d*x)/2]^7*Sqrt[a*(1 + Sin[c + d*x])]*(6*Cos[(c + d*x)/2] - 14*C
os[(3*(c + d*x))/2] - 7*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 7*Co
s[2*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 7*Log[1 - Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]] - 7*Cos[2*(c + d*x)]*Log[1 - Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]] - 6*Sin[(c + d*x)/2] - 14*Sin[(3*(c + d*x))/2]))/(
4*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^2)
```

fricas [B] time = 0.51, size = 337, normalized size = 3.18

$$7\left(a\cos(dx+c)^3+a\cos(dx+c)^2-a\cos(dx+c)+\left(a\cos(dx+c)^2-a\right)\sin(dx+c)-a\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/16*(7*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2 - a*cos(d*x + c) + (a*cos(d*x
+ c)^2 - a)*sin(d*x + c) - a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x +
 c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c
) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c
)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c
)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(7*a*cos(d
*x + c)^2 + 2*a*cos(d*x + c) + (7*a*cos(d*x + c) + 5*a)*sin(d*x + c) - 5*a)
*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x
 + c) + (d*cos(d*x + c)^2 - d)*sin(d*x + c) - d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.93, size = 126, normalized size = 1.19

$$\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(7(-a(\sin(dx+c)-1))^{\frac{3}{2}}a^{\frac{3}{2}}-7\,\mathrm{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)a^3\left(\sin^2(dx\right.\right.}{4\sin(dx+c)^2 a^{\frac{3}{2}}\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/4*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(7*(-a*(sin(d*x+c)-1))^(3/2)*a
^(3/2)-7*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^3*sin(d*x+c)^2-9*(-a*
(sin(d*x+c)-1))^(1/2)*a^(5/2))/sin(d*x+c)^2/a^(3/2)/cos(d*x+c)/(a+a*sin(d*x
+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\csc(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*csc(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^{3/2}}{\sin(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2)/sin(c + d*x)^3,x)
```

```
[Out] int((a + a*sin(c + d*x))^(3/2)/sin(c + d*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.51 $\int \csc^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=144

$$-\frac{11a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{11a^2\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}-\frac{a^2\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{11a^2\cot(c+dx)\csc(c+dx)}{12d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -11/8*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-11/8*a^2
*cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-11/12*a^2*cot(d*x+c)*csc(d*x+c)/d/(a+a
*sin(d*x+c))^(1/2)-1/3*a^2*cot(d*x+c)*csc(d*x+c)^2/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2762, 21, 2772, 2773, 206}

$$-\frac{11a^2\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}-\frac{11a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{a^2\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{11a^2\cot(c+dx)\csc(c+dx)}{12d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-11*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*d
) - (11*a^2*Cot[c + d*x])/(8*d*Sqrt[a + a*Sin[c + d*x]]) - (11*a^2*Cot[c +
d*x]*Csc[c + d*x])/(12*d*Sqrt[a + a*Sin[c + d*x]]) - (a^2*Cot[c + d*x]*Csc[
c + d*x]^2)/(3*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
```

```
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \csc^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = -\frac{a^2\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{1}{3}a\int \frac{\csc^3(c+dx)\left(-\frac{11a}{2}-\frac{11}{2}a\sin(c+\right.}{\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{a^2\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{1}{6}(11a)\int \csc^3(c+dx)\sqrt{a+a\sin(c+}$$

$$= -\frac{11a^2\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{1}{8}(11a)$$

$$= -\frac{11a^2\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{11a^2\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+d}{3d\sqrt{a+a}}$$

$$= -\frac{11a^2\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{11a^2\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+d}{3d\sqrt{a+a}}$$

$$= -\frac{11a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{11a^2\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{11a^2\cot(c+}{12d\sqrt{a+}}$$

Mathematica [A] time = 0.95, size = 286, normalized size = 1.99

$$a\csc^{10}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(108\sin\left(\frac{1}{2}(c+dx)\right)-22\sin\left(\frac{3}{2}(c+dx)\right)-66\sin\left(\frac{5}{2}(c+dx)\right)-108\cos\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a*Csc[(c + d*x)/2]^10*Sqrt[a*(1 + Sin[c + d*x])]*(-108*Cos[(c + d*x)/2] -
22*Cos[(3*(c + d*x))/2] + 66*Cos[(5*(c + d*x))/2] + 108*Sin[(c + d*x)/2] -
99*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 99*Log[1 - C
os[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] - 22*Sin[(3*(c + d*x))/2]
- 66*Sin[(5*(c + d*x))/2] + 33*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
*Sin[3*(c + d*x)] - 33*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(
c + d*x)]))/(24*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x
)/4]^2)^3)
```

fricas [B] time = 0.48, size = 380, normalized size = 2.64

$$33\left(a\cos(dx+c)^4-2a\cos(dx+c)^2-\left(a\cos(dx+c)^3+a\cos(dx+c)^2-a\cos(dx+c)-a\right)\sin(dx+c)+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/96*(33*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 - (a*cos(d*x + c)^3 + a*cos
(d*x + c)^2 - a*cos(d*x + c) - a)*sin(d*x + c) + a)*sqrt(a)*log((a*cos(d*x
+ c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*
x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x
 + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*
x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c
) - 1)) + 4*(33*a*cos(d*x + c)^3 + 11*a*cos(d*x + c)^2 - 41*a*cos(d*x + c)
- (33*a*cos(d*x + c)^2 + 22*a*cos(d*x + c) - 19*a)*sin(d*x + c) - 19*a)*sqr
t(a*sin(d*x + c) + a))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 - (d*cos(d*x
+ c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) - d)*sin(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.78, size = 144, normalized size = 1.00

$$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(33(-a(\sin(dx+c)-1))^{\frac{5}{2}}a^{\frac{5}{2}}-88(-a(\sin(dx+c)-1))^{\frac{3}{2}}a^{\frac{7}{2}}+33a^5 a\right.}{24a^{\frac{7}{2}}\sin(dx+c)^3\cos(dx+c)\sqrt{a+a\sin(dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/24*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(7/2)*(33*(-a*(sin(d*x+c)-
1))^(5/2)*a^(5/2)-88*(-a*(sin(d*x+c)-1))^(3/2)*a^(7/2)+33*a^5*arctanh((-a*(
sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^3+63*(-a*(sin(d*x+c)-1))^(1/2)*a^(
9/2))/sin(d*x+c)^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\csc(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*csc(d*x + c)^4, x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + a\sin\left(c + d\,x\right)\right)^{3/2}}{\sin\left(c + d\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2)/sin(c + d*x)^4,x)
[Out] int((a + a*sin(c + d*x))^(3/2)/sin(c + d*x)^4, x)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+a*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

3.52 $\int \sin^3(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=203

$$-\frac{46a^3\sin^4(c+dx)\cos(c+dx)}{99d\sqrt{a\sin(c+dx)+a}}-\frac{710a^3\sin^3(c+dx)\cos(c+dx)}{693d\sqrt{a\sin(c+dx)+a}}-\frac{284a^3\cos(c+dx)}{99d\sqrt{a\sin(c+dx)+a}}-\frac{2a^2\sin^4(c+dx)\cos(c+\cdots}{1\cdots}$$

```
[Out] -284/231*a*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/d-284/99*a^3*cos(d*x+c)/d/(a+a
*sin(d*x+c))^(1/2)-710/693*a^3*cos(d*x+c)*sin(d*x+c)^3/d/(a+a*sin(d*x+c))^(
1/2)-46/99*a^3*cos(d*x+c)*sin(d*x+c)^4/d/(a+a*sin(d*x+c))^(1/2)+568/693*a^2
*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d-2/11*a^2*cos(d*x+c)*sin(d*x+c)^4*(a+a*
sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.35, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2763, 2981, 2770, 2759, 2751, 2646}

$$-\frac{2a^2\sin^4(c+dx)\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{11d}-\frac{46a^3\sin^4(c+dx)\cos(c+dx)}{99d\sqrt{a\sin(c+dx)+a}}-\frac{710a^3\sin^3(c+dx)\cos(c+dx)}{693d\sqrt{a\sin(c+dx)+a}}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-284*a^3*Cos[c + d*x])/(99*d*Sqrt[a + a*Sin[c + d*x]]) - (710*a^3*Cos[c +
d*x]*Sin[c + d*x]^3)/(693*d*Sqrt[a + a*Sin[c + d*x]]) - (46*a^3*Cos[c + d*x
]*Sin[c + d*x]^4)/(99*d*Sqrt[a + a*Sin[c + d*x]]) + (568*a^2*Cos[c + d*x]*S
qrt[a + a*Sin[c + d*x]])/(693*d) - (2*a^2*Cos[c + d*x]*Sin[c + d*x]^4*Sqrt[
a + a*Sin[c + d*x]])/(11*d) - (284*a*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2
))/(231*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = -\frac{2a^2\cos(c+dx)\sin^4(c+dx)\sqrt{a+a\sin(c+dx)}}{11d} + \frac{2}{11}\int \sin^3(c+dx)\sqrt{}$$
$$= -\frac{46a^3\cos(c+dx)\sin^4(c+dx)}{99d\sqrt{a+a\sin(c+dx)}} - \frac{2a^2\cos(c+dx)\sin^4(c+dx)\sqrt{a+a\sin}}{11d}$$
$$= -\frac{710a^3\cos(c+dx)\sin^3(c+dx)}{693d\sqrt{a+a\sin(c+dx)}} - \frac{46a^3\cos(c+dx)\sin^4(c+dx)}{99d\sqrt{a+a\sin(c+dx)}} - \frac{2a^2}{}$$
$$= -\frac{710a^3\cos(c+dx)\sin^3(c+dx)}{693d\sqrt{a+a\sin(c+dx)}} - \frac{46a^3\cos(c+dx)\sin^4(c+dx)}{99d\sqrt{a+a\sin(c+dx)}} - \frac{2a^2}{}$$
$$= -\frac{710a^3\cos(c+dx)\sin^3(c+dx)}{693d\sqrt{a+a\sin(c+dx)}} - \frac{46a^3\cos(c+dx)\sin^4(c+dx)}{99d\sqrt{a+a\sin(c+dx)}} + \frac{568}{}$$
$$= -\frac{284a^3\cos(c+dx)}{99d\sqrt{a+a\sin(c+dx)}} - \frac{710a^3\cos(c+dx)\sin^3(c+dx)}{693d\sqrt{a+a\sin(c+dx)}} - \frac{46a^3\cos(}{99d\sqrt{a}}$$

Mathematica [A] time = 1.26, size = 189, normalized size = 0.93

$$-\frac{(a(\sin(c+dx)+1))^{5/2}\left(-31878\sin\left(\frac{1}{2}(c+dx)\right)+8778\sin\left(\frac{3}{2}(c+dx)\right)+3465\sin\left(\frac{5}{2}(c+dx)\right)-1287\sin\left(\frac{7}{2}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/11088*((a*(1 + Sin[c + d*x]))^(5/2)*(31878*Cos[(c + d*x)/2] + 8778*Cos[(
3*(c + d*x))/2] - 3465*Cos[(5*(c + d*x))/2] - 1287*Cos[(7*(c + d*x))/2] + 3
85*Cos[(9*(c + d*x))/2] + 63*Cos[(11*(c + d*x))/2] - 31878*Sin[(c + d*x)/2]
 + 8778*Sin[(3*(c + d*x))/2] + 3465*Sin[(5*(c + d*x))/2] - 1287*Sin[(7*(c +
 d*x))/2] - 385*Sin[(9*(c + d*x))/2] + 63*Sin[(11*(c + d*x))/2]))/(d*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

fricas [A] time = 0.52, size = 192, normalized size = 0.95

$$-\frac{2\left(63\,a^2\cos(dx+c)^6+224\,a^2\cos(dx+c)^5-320\,a^2\cos(dx+c)^4-874\,a^2\cos(dx+c)^3+593\,a^2\cos(dx+c)^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/693*(63*a^2*cos(d*x + c)^6 + 224*a^2*cos(d*x + c)^5 - 320*a^2*cos(d*x +
c)^4 - 874*a^2*cos(d*x + c)^3 + 593*a^2*cos(d*x + c)^2 + 1786*a^2*cos(d*x +
 c) + 800*a^2 + (63*a^2*cos(d*x + c)^5 - 161*a^2*cos(d*x + c)^4 - 481*a^2*c
os(d*x + c)^3 + 393*a^2*cos(d*x + c)^2 + 986*a^2*cos(d*x + c) - 800*a^2)*si
n(d*x + c))*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

giac [B] time = 0.98, size = 372, normalized size = 1.83

$$-\frac{1}{55440}\sqrt{2}\left(\frac{385\,a^2\cos\left(\frac{1}{4}\,\pi+\frac{9}{2}\,dx+\frac{9}{2}\,c\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}-\frac{6237\,a^2\cos\left(\frac{1}{4}\,\pi+\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\operatorname{sgn}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/55440*sqrt(2)*(385*a^2*cos(1/4*pi + 9/2*d*x + 9/2*c)*sgn(cos(-1/4*pi + 1
/2*d*x + 1/2*c))/d - 6237*a^2*cos(1/4*pi + 5/2*d*x + 5/2*c)*sgn(cos(-1/4*pi
 + 1/2*d*x + 1/2*c))/d + 76230*a^2*cos(1/4*pi + 1/2*d*x + 1/2*c)*sgn(cos(-1
/4*pi + 1/2*d*x + 1/2*c))/d + 315*a^2*cos(-1/4*pi + 11/2*d*x + 11/2*c)*sgn(
cos(-1/4*pi + 1/2*d*x + 1/2*c))/d - 4455*a^2*cos(-1/4*pi + 7/2*d*x + 7/2*c)
*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d + 25410*a^2*cos(-1/4*pi + 3/2*d*x +
3/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d - 1980*a^2*sgn(cos(-1/4*pi + 1
/2*d*x + 1/2*c))*sin(1/4*pi + 7/2*d*x + 7/2*c)/d + 18480*a^2*sgn(cos(-1/4*p
i + 1/2*d*x + 1/2*c))*sin(1/4*pi + 3/2*d*x + 3/2*c)/d - 1540*a^2*sgn(cos(-1
/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 9/2*d*x + 9/2*c)/d + 11088*a^2*sgn(
cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 5/2*d*x + 5/2*c)/d - 83160*a^
2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)/d)*sqr
t(a)
```

maple [A] time = 0.62, size = 95, normalized size = 0.47

$$\frac{2\left(1+\sin\left(dx+c\right)\right)a^3\left(\sin\left(dx+c\right)-1\right)\left(63\left(\sin^5\left(dx+c\right)\right)+224\left(\sin^4\left(dx+c\right)\right)+355\left(\sin^3\left(dx+c\right)\right)+426\left(\sin\right.\right.}{693\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/693*(1+sin(d*x+c))*a^3*(sin(d*x+c)-1)*(63*sin(d*x+c)^5+224*sin(d*x+c)^4+3
55*sin(d*x+c)^3+426*sin(d*x+c)^2+568*sin(d*x+c)+1136)/cos(d*x+c)/(a+a*sin(d
*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{5}{2}}\sin(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*sin(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(c+d\,x\right)^3\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(sin(c + d*x)^3*(a + a*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.53 $\int \sin^2(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=146

$$-\frac{832a^3\cos(c+dx)}{315d\sqrt{a\sin(c+dx)+a}}-\frac{208a^2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{315d}-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{7/2}}{9ad}+\frac{4\cos(c+ \cdots}{}$$

```
[Out] -26/105*a*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/d+4/63*cos(d*x+c)*(a+a*sin(d*x+
c))^(5/2)/d-2/9*cos(d*x+c)*(a+a*sin(d*x+c))^(7/2)/a/d-832/315*a^3*cos(d*x+c
)/d/(a+a*sin(d*x+c))^(1/2)-208/315*a^2*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.15, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2759, 2751, 2647, 2646}

$$-\frac{208a^2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{315d}-\frac{832a^3\cos(c+dx)}{315d\sqrt{a\sin(c+dx)+a}}-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{7/2}}{9ad}+\frac{4\cos(c+ \cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-832*a^3*Cos[c + d*x])/(315*d*Sqrt[a + a*Sin[c + d*x]]) - (208*a^2*Cos[c +
 d*x]*Sqrt[a + a*Sin[c + d*x]])/(315*d) - (26*a*Cos[c + d*x]*(a + a*Sin[c +
 d*x])^(3/2))/(105*d) + (4*Cos[c + d*x]*(a + a*Sin[c + d*x])^(5/2))/(63*d)
- (2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(7/2))/(9*a*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
```

```
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rubi steps

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = -\frac{2\cos(c+dx)(a+a\sin(c+dx))^{7/2}}{9ad} + \frac{2\int\left(\frac{7a}{2}-a\sin(c+dx)\right)(a+a\sin}{9a}$$
$$= \frac{4\cos(c+dx)(a+a\sin(c+dx))^{5/2}}{63d} - \frac{2\cos(c+dx)(a+a\sin(c+dx))^{7/2}}{9ad}$$
$$= -\frac{26a\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{105d} + \frac{4\cos(c+dx)(a+a\sin(c+dx}{63d}$$
$$= -\frac{208a^2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{315d} - \frac{26a\cos(c+dx)(a+a\sin(c+}{105d}$$
$$= -\frac{832a^3\cos(c+dx)}{315d\sqrt{a+a\sin(c+dx)}} - \frac{208a^2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{315d} - \frac{26}{}$$

Mathematica [A] time = 1.02, size = 165, normalized size = 1.13

$$\frac{(a(\sin(c+dx)+1))^{5/2}\left(8190\sin\left(\frac{1}{2}(c+dx)\right)-2100\sin\left(\frac{3}{2}(c+dx)\right)-756\sin\left(\frac{5}{2}(c+dx)\right)+225\sin\left(\frac{7}{2}(c+dx)\right)+\right.}{2520d\left(\sin\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(5/2)*(-8190*Cos[(c + d*x)/2] - 2100*Cos[(3*(c + d*
x))/2] + 756*Cos[(5*(c + d*x))/2] + 225*Cos[(7*(c + d*x))/2] - 35*Cos[(9*(c
 + d*x))/2] + 8190*Sin[(c + d*x)/2] - 2100*Sin[(3*(c + d*x))/2] - 756*Sin[(
5*(c + d*x))/2] + 225*Sin[(7*(c + d*x))/2] + 35*Sin[(9*(c + d*x))/2]))/(252
0*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

fricas [A] time = 0.48, size = 167, normalized size = 1.14

$$-\frac{2\left(35\,a^2\cos\left(dx+c\right)^5-95\,a^2\cos\left(dx+c\right)^4-289\,a^2\cos\left(dx+c\right)^3+263\,a^2\cos\left(dx+c\right)^2+838\,a^2\cos\left(dx+c\right)\cdots}{315\,(\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/315*(35*a^2*cos(d*x + c)^5 - 95*a^2*cos(d*x + c)^4 - 289*a^2*cos(d*x + c
)^3 + 263*a^2*cos(d*x + c)^2 + 838*a^2*cos(d*x + c) + 416*a^2 - (35*a^2*cos
(d*x + c)^4 + 130*a^2*cos(d*x + c)^3 - 159*a^2*cos(d*x + c)^2 - 422*a^2*cos
(d*x + c) + 416*a^2)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c)
 + d*sin(d*x + c) + d)
```

giac [B] time = 0.79, size = 306, normalized size = 2.10

$$\frac{1}{2520}\sqrt{2}\left(\frac{252\,a^2\cos\left(\frac{1}{4}\,\pi+\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}-\frac{3780\,a^2\cos\left(\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\mathrm{sgn}\left(\mathrm{co}\cdots\right.}{d}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/2520*sqrt(2)*(252*a^2*cos(1/4*pi + 5/2*d*x + 5/2*c)*sgn(cos(-1/4*pi + 1/2
*d*x + 1/2*c))/d - 3780*a^2*cos(1/4*pi + 1/2*d*x + 1/2*c)*sgn(cos(-1/4*pi +
 1/2*d*x + 1/2*c))/d + 180*a^2*cos(-1/4*pi + 7/2*d*x + 7/2*c)*sgn(cos(-1/4*
pi + 1/2*d*x + 1/2*c))/d - 1260*a^2*cos(-1/4*pi + 3/2*d*x + 3/2*c)*sgn(cos(
-1/4*pi + 1/2*d*x + 1/2*c))/d + 45*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*
sin(1/4*pi + 7/2*d*x + 7/2*c)/d - 840*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c
))*sin(1/4*pi + 3/2*d*x + 3/2*c)/d + 35*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2
*c))*sin(-1/4*pi + 9/2*d*x + 9/2*c)/d - 504*a^2*sgn(cos(-1/4*pi + 1/2*d*x +
 1/2*c))*sin(-1/4*pi + 5/2*d*x + 5/2*c)/d + 4410*a^2*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.71, size = 85, normalized size = 0.58

$$\frac{2\left(1+\sin\left(dx+c\right)\right)a^3\left(\sin\left(dx+c\right)-1\right)\left(35\left(\sin^4\left(dx+c\right)\right)+130\left(\sin^3\left(dx+c\right)\right)+219\left(\sin^2\left(dx+c\right)\right)+292\sin\cdots\right.}{315\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/315*(1+sin(d*x+c))*a^3*(sin(d*x+c)-1)*(35*sin(d*x+c)^4+130*sin(d*x+c)^3+2
19*sin(d*x+c)^2+292*sin(d*x+c)+584)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{5}{2}}\sin(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*sin(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(c+d\,x)^2\,(a+a\,\sin(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + a*sin(c + d*x))^(5/2),x)
[Out] int(sin(c + d*x)^2*(a + a*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+a*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

3.54 $\int \sin(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=116

$$-\frac{64a^3\cos(c+dx)}{21d\sqrt{a\sin(c+dx)+a}}-\frac{16a^2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{21d}-\frac{2a\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{7d}-\frac{2\cos(c+}{}$$

```
[Out] -2/7*a*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/d-2/7*cos(d*x+c)*(a+a*sin(d*x+c))^
(5/2)/d-64/21*a^3*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-16/21*a^2*cos(d*x+c)*
(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2751, 2647, 2646}

$$-\frac{64a^3\cos(c+dx)}{21d\sqrt{a\sin(c+dx)+a}}-\frac{16a^2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{21d}-\frac{2a\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{7d}-\frac{2\cos(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-64*a^3*Cos[c + d*x])/(21*d*Sqrt[a + a*Sin[c + d*x]]) - (16*a^2*Cos[c + d*
x]*Sqrt[a + a*Sin[c + d*x]])/(21*d) - (2*a*Cos[c + d*x]*(a + a*Sin[c + d*x]
)^(3/2))/(7*d) - (2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(5/2))/(7*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \sin(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = -\frac{2\cos(c+dx)(a+a\sin(c+dx))^{5/2}}{7d} + \frac{5}{7}\int (a+a\sin(c+dx))^{5/2}\,dx$$
$$= -\frac{2a\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{7d} - \frac{2\cos(c+dx)(a+a\sin(c+dx))^{5/2}}{7d}$$
$$= -\frac{16a^2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{21d} - \frac{2a\cos(c+dx)(a+a\sin(c+dx)}{7d}$$
$$= -\frac{64a^3\cos(c+dx)}{21d\sqrt{a+a\sin(c+dx)}} - \frac{16a^2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{21d} - \frac{2a\cos}{}$$

Mathematica [A] time = 0.61, size = 141, normalized size = 1.22

$$\frac{(a(\sin(c+dx)+1))^{5/2}\left(315\sin\left(\frac{1}{2}(c+dx)\right)-77\sin\left(\frac{3}{2}(c+dx)\right)-21\sin\left(\frac{5}{2}(c+dx)\right)+3\sin\left(\frac{7}{2}(c+dx)\right)-315\cos\right.}{84d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(5/2)*(-315*Cos[(c + d*x)/2] - 77*Cos[(3*(c + d*x))
/2] + 21*Cos[(5*(c + d*x))/2] + 3*Cos[(7*(c + d*x))/2] + 315*Sin[(c + d*x)/
2] - 77*Sin[(3*(c + d*x))/2] - 21*Sin[(5*(c + d*x))/2] + 3*Sin[(7*(c + d*x)
)/2]))/(84*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

fricas [A] time = 0.47, size = 140, normalized size = 1.21

$$\frac{2\left(3\,a^2\cos\left(dx+c\right)^4+12\,a^2\cos\left(dx+c\right)^3-17\,a^2\cos\left(dx+c\right)^2-58\,a^2\cos\left(dx+c\right)-32\,a^2+\left(3\,a^2\cos\left(dx+c\right)^3-\right.\right.}{21\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*a^2*cos(d*x + c)^4 + 12*a^2*cos(d*x + c)^3 - 17*a^2*cos(d*x + c)^2
- 58*a^2*cos(d*x + c) - 32*a^2 + (3*a^2*cos(d*x + c)^3 - 9*a^2*cos(d*x + c)
^2 - 26*a^2*cos(d*x + c) + 32*a^2)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)/(
d*cos(d*x + c) + d*sin(d*x + c) + d)
```

giac [B] time = 0.73, size = 240, normalized size = 2.07

$$\frac{1}{420}\sqrt{2}\left(\frac{21\,a^2\cos\left(\frac{1}{4}\pi+\frac{5}{2}dx+\frac{5}{2}c\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{d}-\frac{735\,a^2\cos\left(\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\operatorname{sgn}\left(\cos\left(-\right.\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/420*sqrt(2)*(21*a^2*cos(1/4*pi + 5/2*d*x + 5/2*c)*sgn(cos(-1/4*pi + 1/2*d
*x + 1/2*c))/d - 735*a^2*cos(1/4*pi + 1/2*d*x + 1/2*c)*sgn(cos(-1/4*pi + 1/
2*d*x + 1/2*c))/d + 15*a^2*cos(-1/4*pi + 7/2*d*x + 7/2*c)*sgn(cos(-1/4*pi +
 1/2*d*x + 1/2*c))/d - 245*a^2*cos(-1/4*pi + 3/2*d*x + 3/2*c)*sgn(cos(-1/4*
pi + 1/2*d*x + 1/2*c))/d - 140*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(
1/4*pi + 3/2*d*x + 3/2*c)/d - 84*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*si
n(-1/4*pi + 5/2*d*x + 5/2*c)/d + 840*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)
)*sin(-1/4*pi + 1/2*d*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.61, size = 75, normalized size = 0.65

$$\frac{2\left(1+\sin\left(dx+c\right)\right)a^3\left(\sin\left(dx+c\right)-1\right)\left(3\left(\sin^3\left(dx+c\right)\right)+12\left(\sin^2\left(dx+c\right)\right)+23\sin\left(dx+c\right)+46\right)}{21\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/21*(1+sin(d*x+c))*a^3*(sin(d*x+c)-1)*(3*sin(d*x+c)^3+12*sin(d*x+c)^2+23*s
in(d*x+c)+46)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}\sin\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*sin(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\sin\left(c+d\,x\right)\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(sin(c + d*x)*(a + a*sin(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a (\sin (c + dx) + 1))^{\frac{5}{2}} \sin (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**(5/2)*sin(c + d*x), x)
```

3.55 $\int (a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=89

$$-\frac{64a^3\cos(c+dx)}{15d\sqrt{a\sin(c+dx)+a}}-\frac{16a^2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{15d}-\frac{2a\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5d}$$

```
[Out] -2/5*a*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/d-64/15*a^3*cos(d*x+c)/d/(a+a*sin(
d*x+c))^(1/2)-16/15*a^2*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2647, 2646}

$$-\frac{64a^3\cos(c+dx)}{15d\sqrt{a\sin(c+dx)+a}}-\frac{16a^2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{15d}-\frac{2a\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-64*a^3*Cos[c + d*x])/(15*d*Sqrt[a + a*Sin[c + d*x]]) - (16*a^2*Cos[c + d*
x]*Sqrt[a + a*Sin[c + d*x]])/(15*d) - (2*a*Cos[c + d*x]*(a + a*Sin[c + d*x]
)^(3/2))/(5*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rubi steps

$$\int (a + a\sin(c+dx))^{5/2}\,dx = -\frac{2a\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{5d} + \frac{1}{5}(8a)\int (a+a\sin(c+dx))^{3/2}\,dx$$
$$= -\frac{16a^2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{15d} - \frac{2a\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{5d} + \frac{1}{15}\Big(3$$
$$= -\frac{64a^3\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}} - \frac{16a^2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{15d} - \frac{2a\cos(c+dx)(a+}{5}$$

Mathematica [A] time = 0.32, size = 117, normalized size = 1.31

$$-\frac{(a(\sin(c+dx)+1))^{5/2}\left(-150\sin\left(\frac{1}{2}(c+dx)\right)+25\sin\left(\frac{3}{2}(c+dx)\right)+3\sin\left(\frac{5}{2}(c+dx)\right)+150\cos\left(\frac{1}{2}(c+dx)\right)+25\right.}{30d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/30*((a*(1 + Sin[c + d*x]))^(5/2)*(150*Cos[(c + d*x)/2] + 25*Cos[(3*(c +
d*x))/2] - 3*Cos[(5*(c + d*x))/2] - 150*Sin[(c + d*x)/2] + 25*Sin[(3*(c + d
*x))/2] + 3*Sin[(5*(c + d*x))/2]))/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^5)
```

fricas [A] time = 0.50, size = 115, normalized size = 1.29

$$\frac{2\left(3\,a^2\cos(dx+c)^3 - 11\,a^2\cos(dx+c)^2 - 46\,a^2\cos(dx+c) - 32\,a^2 - \left(3\,a^2\cos(dx+c)^2 + 14\,a^2\cos(dx+c) - 3\right.\right.}{15\,(d\cos(dx+c) + d\sin(dx+c) + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*a^2*cos(d*x + c)^3 - 11*a^2*cos(d*x + c)^2 - 46*a^2*cos(d*x + c) -
32*a^2 - (3*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) - 32*a^2)*sin(d*x + c)
)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

giac [B] time = 0.99, size = 174, normalized size = 1.96

$$-\frac{1}{30}\sqrt{2}\left(\frac{60\,a^2\cos\left(\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{d} + \frac{20\,a^2\cos\left(-\frac{1}{4}\pi+\frac{3}{2}dx+\frac{3}{2}c\right)\mathrm{sgn}\left(\cos\left(-\right.\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/30*sqrt(2)*(60*a^2*cos(1/4*pi + 1/2*d*x + 1/2*c)*sgn(cos(-1/4*pi + 1/2*d
*x + 1/2*c))/d + 20*a^2*cos(-1/4*pi + 3/2*d*x + 3/2*c)*sgn(cos(-1/4*pi + 1/
2*d*x + 1/2*c))/d + 5*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(1/4*pi +
3/2*d*x + 3/2*c)/d + 3*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi
+ 5/2*d*x + 5/2*c)/d - 90*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*
pi + 1/2*d*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.58, size = 65, normalized size = 0.73

$$\frac{2\left(1+\sin\left(dx+c\right)\right)a^{3}\left(\sin\left(dx+c\right)-1\right)\left(3\left(\sin^{2}\left(dx+c\right)\right)+14\sin\left(dx+c\right)+43\right)}{15\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/15*(1+sin(d*x+c))*a^3*(sin(d*x+c)-1)*(3*sin(d*x+c)^2+14*sin(d*x+c)+43)/co
s(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(c+dx\right)+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(5/2),x)
[Out] Integral((a*sin(c + d*x) + a)**(5/2), x)
```

3.56 $\int \csc(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=98

$$-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}-\frac{14a^3\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{2a^2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}$$

```
[Out] -2*a^(5/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-14/3*a^3*co
s(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-2/3*a^2*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)
/d
```

Rubi [A] time = 0.19, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2763, 2981, 2773, 206}

$$-\frac{14a^3\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{2a^2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d - (
14*a^3*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a^2*Cos[c + d*x]*S
qrt[a + a*Sin[c + d*x]])/(3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \csc(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = -\frac{2a^2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} + \frac{2}{3}\int \csc(c+dx)\sqrt{a+a\sin(c+d$$

$$= -\frac{14a^3\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{2a^2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} + a^2\int \mathrm{cs}$$

$$= -\frac{14a^3\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{2a^2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} - \frac{\left(2a^3\right)\mathrm{Su}}{}$$

$$= -\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{14a^3\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{2a^2\cos(c+dx}{}$$

Mathematica [A] time = 0.41, size = 143, normalized size = 1.46

$$-\frac{(a(\sin(c+dx)+1))^{5/2}\left(-15\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)+15\cos\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{3}{2}(c+dx)\right)+3\log\left(-\right.\right.}{3d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/3*((a*(1 + Sin[c + d*x]))^(5/2)*(15*Cos[(c + d*x)/2] + Cos[(3*(c + d*x))
/2] + 3*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 3*Log[1 - Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]] - 15*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/(
d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

fricas [B] time = 0.50, size = 279, normalized size = 2.85

$$\frac{3\left(a^2\cos(dx+c)+a^2\sin(dx+c)+a^2\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\,\cos(dx+c}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(a^2*cos(d*x + c) + a^2*sin(d*x + c) + a^2)*sqrt(a)*log((a*cos(d*x +
 c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x
 + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x
+ c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x
 + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c)
 - 1)) - 4*(a^2*cos(d*x + c)^2 + 8*a^2*cos(d*x + c) + 7*a^2 + (a^2*cos(d*x
+ c) - 7*a^2)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c) + d*s
in(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.82, size = 103, normalized size = 1.05

$$-\frac{2\left(1+\sin(dx+c)\right)\sqrt{-a\left(\sin(dx+c)-1\right)}\;a\left(3a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)-\left(a-a\sin(dx+c)\right)^{\frac{3}{2}}+9a\sqrt{a-a\,\mathrm{si}}\right.}{3\cos(dx+c)\sqrt{a+a\sin(dx+c)}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/3*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*a*(3*a^(3/2)*arctanh((a-a*sin
(d*x+c))^(1/2)/a^(1/2))-(a-a*sin(d*x+c))^(3/2)+9*a*(a-a*sin(d*x+c))^(1/2))/
cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{5}{2}}\csc(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*csc(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^{5/2}}{\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/sin(c + d*x),x)
[Out] int((a + a*sin(c + d*x))^(5/2)/sin(c + d*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

3.57 $\int \csc^2(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=94

$$-\frac{5a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}-\frac{a^3\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}-\frac{a^2\cot(c+dx)\sqrt{a\sin(c+dx)+a}}{d}$$

```
[Out] -5*a^(5/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-a^3*cos(d*x
+c)/d/(a+a*sin(d*x+c))^(1/2)-a^2*cot(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.19, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2762, 2981, 2773, 206}

$$-\frac{a^3\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}-\frac{a^2\cot(c+dx)\sqrt{a\sin(c+dx)+a}}{d}-\frac{5a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-5*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d - (
a^3*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]]) - (a^2*Cot[c + d*x]*Sqrt[a +
 a*Sin[c + d*x]])/d
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = -\frac{a^2\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{d} - a\int \csc(c+dx)\left(-\frac{5a}{2}-\frac{1}{2}a\sin(c\right.$$
$$= -\frac{a^3\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{d} + \frac{1}{2}\left(5a^2\right)\int$$
$$= -\frac{a^3\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{d} - \frac{\left(5a^3\right)\mathrm{Sub}}{}$$
$$= -\frac{5a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{a^3\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\sqrt{}}{}$$

Mathematica [A] time = 0.81, size = 182, normalized size = 1.94

$$-\frac{a^2\csc^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(2\sin\left(\frac{3}{2}(c+dx)\right)+2\cos\left(\frac{3}{2}(c+dx)\right)+5\sin(c+dx)\left(\log\left(-\sin\left(\frac{1}{2}(c+\right.\right.\right.\right.}{d\left(\cot\left(\frac{1}{2}(c+dx)\right)+1\right)\left(\csc\left(\frac{1}{4}(c+dx)\right)-\sec\left(\frac{1}{4}(c+dx)\right)\right)\left(\csc\left(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((a^2*Csc[(c + d*x)/2]^4*Sqrt[a*(1 + Sin[c + d*x])]*(2*Cos[(3*(c + d*x))/2
] + 5*(Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[1 - Cos[(c + d*x)
```

```
/2] + Sin[(c + d*x)/2]])*Sin[c + d*x] + 2*Sin[(3*(c + d*x))/2]))/(d*(1 + Co
t[(c + d*x)/2])*(Csc[(c + d*x)/4] - Sec[(c + d*x)/4])*(Csc[(c + d*x)/4] + S
ec[(c + d*x)/4]))
```

fricas [B] time = 0.52, size = 308, normalized size = 3.28

$$\frac{5\left(a^2\cos(dx+c)^2 - a^2 - \left(a^2\cos(dx+c) + a^2\right)\sin(dx+c)\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3 - 7\,a\cos(dx+c)^2 - 4\left(\cos(dx+c)^2 + (\cos(dx+c)\right.}{\cos(}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(5*(a^2*cos(d*x + c)^2 - a^2 - (a^2*cos(d*x + c) + a^2)*sin(d*x + c))*s
qrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (co
s(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)
*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin
(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(
d*x + c) - cos(d*x + c) - 1)) + 4*(2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c)
- a^2 + (2*a^2*cos(d*x + c) + a^2)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a))/
(d*cos(d*x + c)^2 - (d*cos(d*x + c) + d)*sin(d*x + c) - d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.96, size = 123, normalized size = 1.31

$$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\;a^{\frac{3}{2}}\left(\sin(dx+c)\left(2\sqrt{a-a\sin(dx+c)}\;\sqrt{a}\;+5\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right.\right.\right.}{\sin(dx+c)\cos(dx+c)\sqrt{a+a\sin(dx+c)}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2)*(sin(d*x+c)*(2*(a-a*sin(d
*x+c))^(1/2)*a^(1/2)+5*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2))*a)+(a-a*sin(
d*x+c))^(1/2)*a^(1/2))/sin(d*x+c)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{5}{2}}\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*csc(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^{5/2}}{\sin(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/sin(c + d*x)^2,x)
```

```
[Out] int((a + a*sin(c + d*x))^(5/2)/sin(c + d*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.58 $\int \csc^3(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=106

$$-\frac{19a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}-\frac{9a^3\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}-\frac{a^2\cot(c+dx)\csc(c+dx)\sqrt{a\sin(c+dx)+a}}{2d}$$

```
[Out] -19/4*a^(5/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-9/4*a^3*
cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-1/2*a^2*cot(d*x+c)*csc(d*x+c)*(a+a*sin(
d*x+c))^(1/2)/d
```

Rubi [A] time = 0.22, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2762, 2980, 2773, 206}

$$-\frac{9a^3\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}-\frac{19a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}-\frac{a^2\cot(c+dx)\csc(c+dx)\sqrt{a\sin(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-19*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*d
) - (9*a^3*Cot[c + d*x])/(4*d*Sqrt[a + a*Sin[c + d*x]]) - (a^2*Cot[c + d*x]
*Csc[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \csc^3(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = -\frac{a^2\cot(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}}{2d} - \frac{1}{2}a\int \csc^2(c+dx)\Big(-$$

$$= -\frac{9a^3\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}}{2d}$$

$$= -\frac{9a^3\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}}{2d}$$

$$= -\frac{19a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{9a^3\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx}{}$$

Mathematica [B] time = 0.74, size = 252, normalized size = 2.38

$$\frac{a^2\csc^7\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(-14\sin\left(\frac{1}{2}(c+dx)\right)-22\sin\left(\frac{3}{2}(c+dx)\right)+14\cos\left(\frac{1}{2}(c+dx)\right)-22\cos\Big(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Csc[(c + d*x)/2]^7*Sqrt[a*(1 + Sin[c + d*x])]*(14*Cos[(c + d*x)/2] - 2
2*Cos[(3*(c + d*x))/2] - 19*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
19*Cos[2*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 19*Log[1
 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 19*Cos[2*(c + d*x)]*Log[1 - Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]] - 14*Sin[(c + d*x)/2] - 22*Sin[(3*(c + d*x)
)/2]))/(4*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2
)^2)
```

fricas [B] time = 0.51, size = 359, normalized size = 3.39

$$\frac{19 \left(a^2 \cos(dx+c)^3 + a^2 \cos(dx+c)^2 - a^2 \cos(dx+c) - a^2 + \left(a^2 \cos(dx+c)^2 - a^2\right) \sin(dx+c)\right)\sqrt{a} \log\left(\frac{a \cos(\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/16*(19*(a^2*cos(d*x + c)^3 + a^2*cos(d*x + c)^2 - a^2*cos(d*x + c) - a^2
+ (a^2*cos(d*x + c)^2 - a^2)*sin(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 -
7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) -
2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (
a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3
+ cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) +
 4*(11*a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) - 9*a^2 + (11*a^2*cos(d*x +
c) + 9*a^2)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^3 + d*c
os(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c)^2 - d)*sin(d*x + c) - d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.79, size = 126, normalized size = 1.19

$$\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\,\sqrt{a}\,\left(-19 \operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)\left(\sin^2(dx+c)\right)a^2 + 11\left(-a\left(\sin(dx\cdots\right.\right.\right.}{4 \sin(dx+c)^2 \cos(dx+c)\sqrt{a+a\sin(dx+c)}\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x)
```

[Out] 1/4*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*a^(1/2)*(-19*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^2*a^2+11*(-a*(sin(d*x+c)-1))^(3/2)*a^(1/2)-13*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2))/sin(d*x+c)^2/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{5}{2}}\csc(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((a*sin(d*x + c) + a)^(5/2)*csc(d*x + c)^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^{5/2}}{\sin(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*sin(c + d*x))^(5/2)/sin(c + d*x)^3,x)

[Out] int((a + a*sin(c + d*x))^(5/2)/sin(c + d*x)^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**3*(a+a*sin(d*x+c))**(5/2),x)

[Out] Timed out

3.59 $\int \csc^4(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=144

$$-\frac{25a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{25a^3\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}-\frac{13a^3\cot(c+dx)\csc(c+dx)}{12d\sqrt{a\sin(c+dx)+a}}-\frac{a^2\cot(c+dx)\csc^2(c+dx}{3d}$$

```
[Out] -25/8*a^(5/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-25/8*a^3
*cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-13/12*a^3*cot(d*x+c)*csc(d*x+c)/d/(a+a
*sin(d*x+c))^(1/2)-1/3*a^2*cot(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.28, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2762, 2980, 2772, 2773, 206}

$$-\frac{25a^3\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}-\frac{25a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{a^2\cot(c+dx)\csc^2(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{13a^3\,\mathrm{co}}{12d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-25*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*d
) - (25*a^3*Cot[c + d*x])/(8*d*Sqrt[a + a*Sin[c + d*x]]) - (13*a^3*Cot[c +
d*x]*Csc[c + d*x])/(12*d*Sqrt[a + a*Sin[c + d*x]]) - (a^2*Cot[c + d*x]*Csc[
c + d*x]^2*Sqrt[a + a*Sin[c + d*x]])/(3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \csc^4(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = -\frac{a^2\cot(c+dx)\csc^2(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} - \frac{1}{3}a\int \csc^3(c+dx)\Big(-$$

$$= -\frac{13a^3\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc^2(c+dx)\sqrt{a+a\sin(}}{3d}$$

$$= -\frac{25a^3\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{13a^3\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)}{}$$

$$= -\frac{25a^3\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{13a^3\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)}{}$$

$$= -\frac{25a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{25a^3\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{13a^3\cot(c+}{12d\sqrt{a+}}$$

Mathematica [A] time = 1.18, size = 288, normalized size = 2.00

$$a^2\csc^{10}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(228\sin\left(\frac{1}{2}(c+dx)\right)+14\sin\left(\frac{3}{2}(c+dx)\right)-150\sin\left(\frac{5}{2}(c+dx)\right)-228\text{c}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Csc[(c + d*x)/2]^10*Sqrt[a*(1 + Sin[c + d*x])]*(-228*Cos[(c + d*x)/2]
+ 14*Cos[(3*(c + d*x))/2] + 150*Cos[(5*(c + d*x))/2] + 228*Sin[(c + d*x)/2]
 - 225*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 225*Log[
1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] + 14*Sin[(3*(c + d*x)
)/2] - 150*Sin[(5*(c + d*x))/2] + 75*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]]*Sin[3*(c + d*x)] - 75*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*S
in[3*(c + d*x)]))/(24*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c
 + d*x)/4]^2)^3)
```

fricas [B] time = 0.52, size = 408, normalized size = 2.83

$$75\left(a^2\cos\left(dx+c\right)^4-2\,a^2\cos\left(dx+c\right)^2+a^2-\left(a^2\cos\left(dx+c\right)^3+a^2\cos\left(dx+c\right)^2-a^2\cos\left(dx+c\right)-a^2\right)\sin\left(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/96*(75*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2 - (a^2*cos(d*x +
c)^3 + a^2*cos(d*x + c)^2 - a^2*cos(d*x + c) - a^2)*sin(d*x + c))*sqrt(a)*l
og((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x +
c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a)
 - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c
) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c)
 - cos(d*x + c) - 1)) + 4*(75*a^2*cos(d*x + c)^3 + 41*a^2*cos(d*x + c)^2 -
83*a^2*cos(d*x + c) - 49*a^2 - (75*a^2*cos(d*x + c)^2 + 34*a^2*cos(d*x + c)
 - 49*a^2)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^4 - 2*d*
cos(d*x + c)^2 - (d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) - d)
*sin(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")

[Out] Timed out

maple [A] time = 0.81, size = 144, normalized size = 1.00

$$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(75(-a(\sin(dx+c)-1))^{\frac{5}{2}}a^{\frac{3}{2}}+75\,\mathrm{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)a^4\left(\sin^3\right.\right.}{24\sin(dx+c)^3a^{\frac{3}{2}}\cos(dx+c)\sqrt{a+a\sin(d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x)

[Out] -1/24*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(75*(-a*(sin(d*x+c)-1))^(5/2)*a^(3/2)+75*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^4*sin(d*x+c)^3-184*(-a*(sin(d*x+c)-1))^(3/2)*a^(5/2)+117*(-a*(sin(d*x+c)-1))^(1/2)*a^(7/2))/sin(d*x+c)^3/a^(3/2)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{5}{2}}\csc(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((a*sin(d*x + c) + a)^(5/2)*csc(d*x + c)^4, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^{5/2}}{\sin(c+d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*sin(c + d*x))^(5/2)/sin(c + d*x)^4,x)

[Out] int((a + a*sin(c + d*x))^(5/2)/sin(c + d*x)^4, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**4*(a+a*sin(d*x+c))**(5/2),x)

[Out] Timed out

3.60 $\int \csc^5(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=182

$$-\frac{163a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{64d}-\frac{163a^3\cot(c+dx)}{64d\sqrt{a\sin(c+dx)+a}}-\frac{17a^3\cot(c+dx)\csc^2(c+dx)}{24d\sqrt{a\sin(c+dx)+a}}-\frac{163a^3\cot(c+dx)\csc(}{96d\sqrt{a\sin(c+dx)}}$$

```
[Out] -163/64*a^(5/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-163/64
*a^3*cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-163/96*a^3*cot(d*x+c)*csc(d*x+c)/d
/(a+a*sin(d*x+c))^(1/2)-17/24*a^3*cot(d*x+c)*csc(d*x+c)^2/d/(a+a*sin(d*x+c)
)^(1/2)-1/4*a^2*cot(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.34, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2762, 2980, 2772, 2773, 206}

$$-\frac{163a^3\cot(c+dx)}{64d\sqrt{a\sin(c+dx)+a}}-\frac{163a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{64d}-\frac{17a^3\cot(c+dx)\csc^2(c+dx)}{24d\sqrt{a\sin(c+dx)+a}}-\frac{a^2\cot(c+dx)\csc^3(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-163*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(64
*d) - (163*a^3*Cot[c + d*x])/(64*d*Sqrt[a + a*Sin[c + d*x]]) - (163*a^3*Cot
[c + d*x]*Csc[c + d*x])/(96*d*Sqrt[a + a*Sin[c + d*x]]) - (17*a^3*Cot[c + d
*x]*Csc[c + d*x]^2)/(24*d*Sqrt[a + a*Sin[c + d*x]]) - (a^2*Cot[c + d*x]*Csc
[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]])/(4*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

```
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \csc^5(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = -\frac{a^2\cot(c+dx)\csc^3(c+dx)\sqrt{a+a\sin(c+dx)}}{4d} - \frac{1}{4}a\int \csc^4(c+dx)$$

$$= -\frac{17a^3\cot(c+dx)\csc^2(c+dx)}{24d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc^3(c+dx)\sqrt{a+a\,\mathrm{si}}}{4d}$$

$$= -\frac{163a^3\cot(c+dx)\csc(c+dx)}{96d\sqrt{a+a\sin(c+dx)}} - \frac{17a^3\cot(c+dx)\csc^2(c+dx)}{24d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\,\mathrm{c}}{}$$

$$= -\frac{163a^3\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}} - \frac{163a^3\cot(c+dx)\csc(c+dx)}{96d\sqrt{a+a\sin(c+dx)}} - \frac{17a^3\cot(}{24d\sqrt{a}}$$

$$= -\frac{163a^3\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}} - \frac{163a^3\cot(c+dx)\csc(c+dx)}{96d\sqrt{a+a\sin(c+dx)}} - \frac{17a^3\cot(}{24d\sqrt{a}}$$

$$= -\frac{163a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64d} - \frac{163a^3\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}} - \frac{163a^3\,\mathrm{cc}}{96d\sqrt{}}$$

Mathematica [B] time = 1.70, size = 370, normalized size = 2.03

$$-\frac{a^2\csc^{13}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sin(c+dx)+1)}\left(1030\sin\left(\frac{1}{2}(c+dx)\right)+3102\sin\left(\frac{3}{2}(c+dx)\right)+326\sin\left(\frac{5}{2}(c+dx)\right)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/192*(a^2*Csc[(c + d*x)/2]^13*Sqrt[a*(1 + Sin[c + d*x])]*(-1030*Cos[(c +
d*x)/2] + 3102*Cos[(3*(c + d*x))/2] - 326*Cos[(5*(c + d*x))/2] - 978*Cos[(7
*(c + d*x))/2] + 1467*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 1956*C
os[2*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 489*Cos[4*(c
 + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 1467*Log[1 - Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]] + 1956*Cos[2*(c + d*x)]*Log[1 - Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]] - 489*Cos[4*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]] + 1030*Sin[(c + d*x)/2] + 3102*Sin[(3*(c + d*x))/2] + 326
*Sin[(5*(c + d*x))/2] - 978*Sin[(7*(c + d*x))/2]))/(d*(1 + Cot[(c + d*x)/2]
)*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^4)
```

fricas [B] time = 0.52, size = 473, normalized size = 2.60

$$\frac{489\left(a^2\cos(dx+c)^5+a^2\cos(dx+c)^4-2a^2\cos(dx+c)^3-2a^2\cos(dx+c)^2+a^2\cos(dx+c)+a^2+\left(a^2\cos\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/768*(489*(a^2*cos(d*x + c)^5 + a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^3
- 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c) + a^2 + (a^2*cos(d*x + c)^4 - 2*a
^2*cos(d*x + c)^2 + a^2)*sin(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*
cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*co
s(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*co
s(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + co
s(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(
489*a^2*cos(d*x + c)^4 + 326*a^2*cos(d*x + c)^3 - 836*a^2*cos(d*x + c)^2 -
374*a^2*cos(d*x + c) + 299*a^2 + (489*a^2*cos(d*x + c)^3 + 163*a^2*cos(d*x
+ c)^2 - 673*a^2*cos(d*x + c) - 299*a^2)*sin(d*x + c))*sqrt(a*sin(d*x + c)
+ a))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 - 2*d*cos(d
*x + c)^2 + d*cos(d*x + c) + (d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*si
n(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.84, size = 162, normalized size = 0.89

$$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(1047\sqrt{-a(\sin(dx+c)-1)}\,a^{\frac{11}{2}}-2303(-a(\sin(dx+c)-1))^{\frac{3}{2}}a^{\frac{9}{2}}+1\right.}{192a^{\frac{7}{2}}\sin(dx+c)^{4}\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/192*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(1047*(-a*(sin(d*x+c)-1))^(
1/2)*a^(11/2)-2303*(-a*(sin(d*x+c)-1))^(3/2)*a^(9/2)+1793*(-a*(sin(d*x+c)-1
))^(5/2)*a^(7/2)-489*(-a*(sin(d*x+c)-1))^(7/2)*a^(5/2)+489*arctanh((-a*(sin
(d*x+c)-1))^(1/2)/a^(1/2))*a^6*sin(d*x+c)^4)/a^(7/2)/sin(d*x+c)^4/cos(d*x+c
)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{5}{2}}\csc(dx+c)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*csc(d*x + c)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^{5/2}}{\sin(c+d\,x)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/sin(c + d*x)^5,x)
```

```
[Out] int((a + a*sin(c + d*x))^(5/2)/sin(c + d*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.61 $\int \frac{\sin^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=139

$$-\frac{2\sin^2(c+dx)\cos(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}+\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{15ad}-\frac{28\cos(c+dx)}{15d\sqrt{a\sin(c+dx)+a}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/d/a^
(1/2)-28/15*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-2/5*cos(d*x+c)*sin(d*x+c)^2
/d/(a+a*sin(d*x+c))^(1/2)+2/15*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/a/d
```

Rubi [A] time = 0.23, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2778, 2968, 3023, 2751, 2649, 206}

$$-\frac{2\sin^2(c+dx)\cos(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}+\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{15ad}-\frac{28\cos(c+dx)}{15d\sqrt{a\sin(c+dx)+a}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])]
)/(Sqrt[a]*d) - (28*Cos[c + d*x])/(15*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[
c + d*x]*Sin[c + d*x]^2)/(5*d*Sqrt[a + a*Sin[c + d*x]]) + (2*Cos[c + d*x]*S
qrt[a + a*Sin[c + d*x]])/(15*a*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{2\cos(c+dx)\sin^2(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} - \frac{\int \frac{\sin(c+dx)(-4a+a\sin(c+dx))}{\sqrt{a+a\sin(c+dx)}}\,dx}{5a}$$

$$= -\frac{2\cos(c+dx)\sin^2(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} - \frac{\int \frac{-4a\sin(c+dx)+a\sin^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{5a}$$

$$= -\frac{2\cos(c+dx)\sin^2(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{15ad} - \frac{2\int \frac{\frac{a^2}{2}-7a^2\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}}{15a^2}$$

$$= -\frac{28\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^2(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sqrt{a+a\sin(c+}}{15ad}$$

$$= -\frac{28\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^2(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sqrt{a+a\sin(c+}}{15ad}$$

$$= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{28\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^2(c+dx)}{5d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [C] time = 0.22, size = 150, normalized size = 1.08

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(60\sin\left(\frac{1}{2}(c+dx)\right)+5\sin\left(\frac{3}{2}(c+dx)\right)-3\sin\left(\frac{5}{2}(c+dx)\right)-60\cos\left(\frac{1}{2}(c+dx\right.\right.}{30d\sqrt{a(\sin(c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*((-60 - 60*I)*(-1)^(3/4)*ArcTanh[(1/
2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] - 60*Cos[(c + d*x)/2] + 5*Cos[
(3*(c + d*x))/2] + 3*Cos[(5*(c + d*x))/2] + 60*Sin[(c + d*x)/2] + 5*Sin[(3*
(c + d*x))/2] - 3*Sin[(5*(c + d*x))/2]))/(30*d*Sqrt[a*(1 + Sin[c + d*x])])
```

fricas [A] time = 0.56, size = 234, normalized size = 1.68

$$\frac{\frac{15\sqrt{2}\left(a\cos(dx+c)+a\sin(dx+c)+a\right)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)+\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}+4\left(3\cos(dx\right.}{30\left(ad\cos(dx+c)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(15*sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)^2
 - (cos(d*x + c) - 2)*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(co
s(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^
2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) + 4*(3*cos
(d*x + c)^3 + 4*cos(d*x + c)^2 - (3*cos(d*x + c)^2 - cos(d*x + c) - 17)*sin
(d*x + c) - 16*cos(d*x + c) - 17)*sqrt(a*sin(d*x + c) + a))/(a*d*cos(d*x +
c) + a*d*sin(d*x + c) + a*d)
```

giac [B] time = 0.92, size = 314, normalized size = 2.26

$$2\left(\frac{\left(15\sqrt{2}\,a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+17\sqrt{2}\sqrt{-a}\sqrt{a}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\sqrt{-a}\,a}-\frac{15\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}+\frac{\left(\left(\left(\left(\frac{13}{\mathrm{sgn}}\right.\right.\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*((15*sqrt(2)*a*arctan(sqrt(a)/sqrt(-a)) + 17*sqrt(2)*sqrt(-a)*sqrt(a))
*sgn(tan(1/2*d*x + 1/2*c) + 1)/(sqrt(-a)*a) - 15*sqrt(2)*arctan(-1/2*sqrt(2
)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt
(a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + (((((13*a^2*tan(1
/2*d*x + 1/2*c)/sgn(tan(1/2*d*x + 1/2*c) + 1) - 15*a^2/sgn(tan(1/2*d*x + 1/
2*c) + 1))*tan(1/2*d*x + 1/2*c) + 40*a^2/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan
(1/2*d*x + 1/2*c) - 40*a^2/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2
*c) + 15*a^2/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) - 13*a^2/s
gn(tan(1/2*d*x + 1/2*c) + 1))/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(5/2))/d
```

maple [A] time = 0.94, size = 130, normalized size = 0.94

$$\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(15a^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{2\sqrt{a}}\right)-6(a-a\sin(dx+c))^{\frac{5}{2}}+10(a-\right.}{15a^3\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/15*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(15*a^(5/2)*2^(1/2)*arctanh(1
/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-6*(a-a*sin(d*x+c))^(5/2)+10*(a-a
```

```
*sin(d*x+c))^(3/2)*a-30*a^2*(a-a*sin(d*x+c))^(1/2))/a^3/cos(d*x+c)/(a+a*sin
(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^3}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sin(d*x + c)^3/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(c+d\,x)^3}{\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + a*sin(c + d*x))^(1/2),x)
[Out] int(sin(c + d*x)^3/(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+a*sin(d*x+c))**(1/2),x)
[Out] Timed out
```

3.62 $\int \frac{\sin^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=105

$$-\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3ad}+\frac{4\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] -arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/d/a
^(1/2)+4/3*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-2/3*cos(d*x+c)*(a+a*sin(d*x+
c))^(1/2)/a/d
```

Rubi [A] time = 0.12, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2759, 2751, 2649, 206}

$$-\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3ad}+\frac{4\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)]])/(Sqrt[a]*d)) + (4*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos
[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(3*a*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
```

```
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3ad} + \frac{2\int \frac{\frac{a}{2}-a\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{3a}$$
$$= \frac{4\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3ad} + \int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,d$$
$$= \frac{4\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3ad} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\right.}{d}$$
$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{4\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+a\sin(c}{3ad}$$

Mathematica [C] time = 0.21, size = 105, normalized size = 1.00

$$-\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3-(6+6i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\right.\right.}{3d\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/3*(((-6 - 6*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c +
d*x)/4])] - 2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3)*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]))/(d*Sqrt[a*(1 + Sin[c + d*x])])
```

fricas [B] time = 0.54, size = 209, normalized size = 1.99

$$\frac{\frac{3\,\sqrt{2}\,(a\cos(dx+c)+a\sin(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)-\frac{2\,\sqrt{2}\,\sqrt{a\sin(dx+c)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\,\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}-4\left(\cos\left(dx+\right.\right.}{6\left(ad\cos\left(dx+c\right)+ad\sin\left(dx+c\right)+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)^2 -
 (cos(d*x + c) - 2)*sin(d*x + c) - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos(
d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2
- (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) - 4*(cos(d*x
 + c)^2 + (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)*sqrt(a*sin(d*
x + c) + a))/(a*d*cos(d*x + c) + a*d*sin(d*x + c) + a*d)
```

giac [B] time = 2.28, size = 245, normalized size = 2.33

$$-\frac{2\left(\frac{\left(3\,\sqrt{2}\,a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+2\,\sqrt{2}\,\sqrt{-a}\,\sqrt{a}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{\sqrt{-a}\,a}-\frac{3\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}+\sqrt{a}\right)}{2\,\sqrt{-a}}\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\frac{\left(\left(\frac{a\tan}{\mathrm{sgn}(\tan}\right.\right.}{}\right.}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*((3*sqrt(2)*a*arctan(sqrt(a)/sqrt(-a)) + 2*sqrt(2)*sqrt(-a)*sqrt(a))*s
gn(tan(1/2*d*x + 1/2*c) + 1)/(sqrt(-a)*a) - 3*sqrt(2)*arctan(-1/2*sqrt(2)*(
sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a)
)/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + (((a*tan(1/2*d*x + 1
/2*c)/sgn(tan(1/2*d*x + 1/2*c) + 1) - 3*a/sgn(tan(1/2*d*x + 1/2*c) + 1))*ta
n(1/2*d*x + 1/2*c) + 3*a/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c
) - a/sgn(tan(1/2*d*x + 1/2*c) + 1))/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2))/
d
```

maple [A] time = 0.77, size = 96, normalized size = 0.91

$$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\,\left(3a^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)-2\left(a-a\sin(dx+c)\right)^{\frac{3}{2}}\right)}{3a^2\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/3*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(3*a^(3/2)*2^(1/2)*arctanh(1/
2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-2*(a-a*sin(d*x+c))^(3/2))/a^2/cos
(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^2}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^2/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(c+d\,x)^2}{\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(sin(c + d*x)^2/(a + a*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sin(c + d*x)**2/sqrt(a*(sin(c + d*x) + 1)), x)
```

3.63 $\int \frac{\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=72

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{2\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/d/a^
(1/2)-2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2751, 2649, 206}

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{2\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])]
)/(Sqrt[a]*d) - (2*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx$$
$$= -\frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}$$
$$= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [C] time = 0.10, size = 98, normalized size = 1.36

$$-\frac{2\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)+(1+i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)\right.\right.}{d\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*((1 + I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/
4])] + Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]))/(d*Sqrt[a*(1 + Sin[c + d*x])])
```

fricas [B] time = 0.51, size = 191, normalized size = 2.65

$$\frac{\frac{\sqrt{2}\left(a\cos(dx+c)+a\sin(dx+c)+a\right)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)+\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}} - 4\sqrt{a\sin(dx+}}{2\left(ad\cos(dx+c)+ad\sin(dx+c)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)^2 - (
cos(d*x + c) - 2)*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*
x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2 -
(cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) - 4*sqrt(a*sin
(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1))/(a*d*cos(d*x + c) + a*d*s
in(d*x + c) + a*d)
```

giac [B] time = 5.69, size = 182, normalized size = 2.53

$$\frac{2\left(\frac{\sqrt{2}\left(a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\sqrt{a}\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\sqrt{-a}\,a}+\frac{\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{1}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}}-\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2*(sqrt(2)*(a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(tan(1/2*d*x
+ 1/2*c) + 1)/(sqrt(-a)*a) + (tan(1/2*d*x + 1/2*c)/sgn(tan(1/2*d*x + 1/2*c)
 + 1) - 1/sgn(tan(1/2*d*x + 1/2*c) + 1))/sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)
 - sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2
*c) + 1)))/d
```

maple [A] time = 0.79, size = 94, normalized size = 1.31

$$\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(\sqrt{a}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)-2\sqrt{a-a\sin(dx+c)}\right)}{a\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] (1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(a^(1/2)*2^(1/2)*arctanh(1/2*(a-a*
sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-2*(a-a*sin(d*x+c))^(1/2))/a/cos(d*x+c)/(
a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin(dx+c)}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/sqrt(a*sin(d*x + c) + a), x)
```

mupad [B] time = 0.75, size = 99, normalized size = 1.38

$$-\frac{\left(4\,\mathrm{E}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin(c+d\,x)}}{2}\right)\middle|1\right)-2\,\mathrm{F}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin(c+d\,x)}}{2}\right)\middle|1\right)\right)\,\sqrt{\cos\left(c+d\,x\right)^2}\,\sqrt{\frac{a+a\,\sin(c+d\,x)}{2\,a}}}{d\,\cos\left(c+d\,x\right)\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] -((4*ellipticE(asin((2^(1/2)*(1 - sin(c + d*x))^(1/2))/2), 1) - 2*ellipticF
(asin((2^(1/2)*(1 - sin(c + d*x))^(1/2))/2), 1))*(cos(c + d*x)^2)^(1/2)*((a
 + a*sin(c + d*x))/(2*a))^(1/2))/(d*cos(c + d*x)*(a + a*sin(c + d*x))^(1/2)
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(c+dx\right)}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sin(c + d*x)/sqrt(a*(sin(c + d*x) + 1)), x)
```

3.64 $\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=47

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] -arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/d/a
^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2649, 206}

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)])/(Sqrt[a]*d))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}$$
$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [C] time = 0.05, size = 73, normalized size = 1.55

$$\frac{(2+2i)(-1)^{3/4}\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(\tan\left(\frac{1}{4}(c+dx)\right)-1\right)\right)}{d\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((2 + 2*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4]
)]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(d*Sqrt[a*(1 + Sin[c + d*x])])
```

fricas [A] time = 0.51, size = 167, normalized size = 3.55

$$\left[\frac{\sqrt{2}\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)-\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{2\sqrt{a}\,d}, \frac{\sqrt{2}\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{a\sin(}{\cos}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*log(-(cos(d*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x + c) - 2*sqr
t(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3
*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(
d*x + c) - 2))/(sqrt(a)*d), sqrt(2)*sqrt(-1/a)*arctan(sqrt(2)*sqrt(a*sin(d*
x + c) + a)*sqrt(-1/a)/cos(d*x + c))/d]
```

giac [B] time = 0.70, size = 111, normalized size = 2.36

$$\frac{\frac{\sqrt{2}\log\left(\left|\frac{1}{\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}+\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+2\right|\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{\sqrt{2}\log\left(\left|\frac{1}{\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}+\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)-2\right|\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*log(abs(1/sin(-1/4*pi + 1/2*d*x + 1/2*c) + sin(-1/4*pi + 1/2*d
*x + 1/2*c) + 2))/(sqrt(a)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - sqrt(2)*l
og(abs(1/sin(-1/4*pi + 1/2*d*x + 1/2*c) + sin(-1/4*pi + 1/2*d*x + 1/2*c) -
2))/(sqrt(a)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

maple [A] time = 0.63, size = 75, normalized size = 1.60

$$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}\,\sqrt{2}}{2\sqrt{a}}\right)}{\sqrt{a}\,\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2)*arctanh(1/2*(-a*(
sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*sin(d*x + c) + a), x)
```

mupad [B] time = 6.44, size = 49, normalized size = 1.04

$$-\frac{\mathrm{F}\left(\frac{\pi}{4}-\frac{c}{2}-\frac{d\,x}{2}\middle|1\right)\sqrt{\frac{2\,(a+a\,\sin(c+d\,x))}{a}}}{d\sqrt{a+a\,\sin(c+d\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] -(ellipticF(pi/4 - c/2 - (d*x)/2, 1)*((2*(a + a*sin(c + d*x)))/a)^(1/2))/(d
*(a + a*sin(c + d*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sin(c+dx)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sin(c + d*x) + a), x)
```

3.65 $\int \frac{\csc(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=84

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] -2*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d/a^(1/2)+arctanh(1/2
*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2780, 2649, 206, 2773}

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(Sqrt[a]*d) +
 (Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])
])/(Sqrt[a]*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2780

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e +
f*x]], x], x] - Dist[d/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx &= \frac{\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{a} - \int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx \\
&= -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{2\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} \\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d}
\end{aligned}
$$

Mathematica [C] time = 0.10, size = 128, normalized size = 1.52

$$
-\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left((2+2i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(\tan\left(\frac{1}{4}(c+dx)\right)-1\right)\right)+\log\left(-\sin\right.\right.}{d\sqrt{a(\sin(c+dx)+1)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((((2 + 2*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)
/4])] + Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[1 - Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(d*Sqrt[a*
(1 + Sin[c + d*x])]))
```

fricas [B] time = 0.48, size = 290, normalized size = 3.45

$$
\frac{\sqrt{2}\sqrt{a}\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)+\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)+\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\,\mathrm{c}}{}\right.}{2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*sqrt(a)*log(-(cos(d*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x + c)
 + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/sqr
t(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c
) - cos(d*x + c) - 2)) + sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2
 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3
)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 +
 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 +
 (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)))/(a*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integrati
on of abs or sign assumes constant sign by intervals (correct if the argume
nt is real):Check [abs(cos((d*t_nostep+c)/2-pi/4))]Discontinuities at zeroe
s of cos((d*t_nostep+c)/2-pi/4) were not checkedWarning, integration of abs
 or sign assumes constant sign by intervals (correct if the argument is rea
l):Check [abs(t_nostep+1)]Error: Bad Argument Type
```

maple [A] time = 0.78, size = 96, normalized size = 1.14

$$\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)-2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)\right)}{\sqrt{a}\,\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] (1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(2^(1/2)*arctanh(1/2*(a-a*sin(d*x+
c))^(1/2)*2^(1/2)/a^(1/2))-2*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2)))/a^(1/
2)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(dx+c)}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(csc(d*x + c)/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(c+d\,x\right)\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + a*sin(c + d*x))^(1/2)),x)
[Out] int(1/(sin(c + d*x)*(a + a*sin(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(c+dx\right)}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))**(1/2),x)
[Out] Integral(csc(c + d*x)/sqrt(a*(sin(c + d*x) + 1)), x)
```

3.66 $\int \frac{\csc^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=109

$$-\frac{\cot(c+dx)}{d\sqrt{a\sin(c+dx)+a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d/a^(1/2)-arctanh(1/2*co
s(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)-cot(d*x+
c)/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2779, 2985, 2649, 206, 2773}

$$-\frac{\cot(c+dx)}{d\sqrt{a\sin(c+dx)+a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]]/(Sqrt[a]*d) - (Sqr
t[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(S
qrt[a]*d) - Cot[c + d*x]/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\int \frac{\csc(c+dx)(a-a\sin(c+dx))}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a}$$

$$= -\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{2a} + \int \frac{1}{\sqrt{a+a\sin(c+dx}}$$

$$= -\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{2\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x\right.}{d}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [C] time = 1.26, size = 168, normalized size = 1.54

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-\tan\left(\frac{1}{4}(c+dx)\right)-\cot\left(\frac{1}{4}(c+dx)\right)+2\sec\left(\frac{1}{2}(c+dx)\right)+(8+8i)(-1)^{3/4}\tan\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*((8 + 8*I)*(-1)^(3/4)*ArcTanh[(1/2 +
 I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] - Cot[(c + d*x)/4] + 2*Log[1 + Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2*Log[1 - Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + 2*Sec[(c + d*x)/2] - Tan[(c + d*x)/4]))/(4*d*Sqrt[a*(1 + Sin[c +
 d*x])])
```

fricas [B] time = 0.51, size = 412, normalized size = 3.78

$$\frac{\left(\cos\left(dx+c\right)^2-\left(\cos\left(dx+c\right)+1\right)\sin\left(dx+c\right)-1\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2+4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+\cdots\right.}{\cos(dx+c)^3+\cos(\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log((a*
cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3
)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a
*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)
/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos
(d*x + c) - 1)) + 2*sqrt(2)*(a*cos(d*x + c)^2 - (a*cos(d*x + c) + a)*sin(d*
x + c) - a)*log(-(cos(d*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x + c) - 2*sqrt
(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*
cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d
*x + c) - 2))/sqrt(a) + 4*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x
+ c) + 1))/(a*d*cos(d*x + c)^2 - a*d - (a*d*cos(d*x + c) + a*d)*sin(d*x + c
))
```

giac [B] time = 0.81, size = 465, normalized size = 4.27

$$\frac{\frac{\left(2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)-8\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)-\sqrt{2}\sqrt{-a}\log\left(\sqrt{2}\sqrt{a}+\sqrt{a}\right)+4\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)-8\sqrt{a}\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)-2\sqrt{-a}\log\left(\sqrt{2}\cdots\right.\right.}{\sqrt{2}\sqrt{-a}\sqrt{a}+2\sqrt{-a}\sqrt{a}}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*((2*sqrt(2)*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 8*sq
rt(2)*sqrt(a)*arctan(sqrt(a)/sqrt(-a)) - sqrt(2)*sqrt(-a)*log(sqrt(2)*sqrt(
a) + sqrt(a)) + 4*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 8*
sqrt(a)*arctan(sqrt(a)/sqrt(-a)) - 2*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)
) - 3*sqrt(2)*sqrt(-a) - 2*sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(sqrt(2)
*sqrt(-a)*sqrt(a) + 2*sqrt(-a)*sqrt(a)) + 4*sqrt(2)*arctan(-1/2*sqrt(2)*(sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/
sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 2*arctan(-(sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)
*sgn(tan(1/2*d*x + 1/2*c) + 1)) + log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(a)*sgn(tan(1/2*d*x + 1/2*c) + 1))
 + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 2
*sqrt(a)/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a))^2 - a)*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 0.81, size = 133, normalized size = 1.22

$$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(\sin(dx+c)\,a^3\left(\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\ \sqrt{2}}{2\sqrt{a}}\right)-\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right.\right.\right.}{a^{\frac{7}{2}}\sin(dx+c)\cos(dx+c)\sqrt{a+a\sin(dx+c)}\ d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(7/2)*(sin(d*x+c)*a^3*(2^(1/2)*
arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-arctanh((a-a*sin(d*x+c)
)^(1/2)/a^(1/2)))+(a-a*sin(d*x+c))^(1/2)*a^(5/2))/sin(d*x+c)/cos(d*x+c)/(a+
a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(dx+c)^2}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)^2/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(c+d\,x)^2\,\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(csc(c + d*x)**2/sqrt(a*(sin(c + d*x) + 1)), x)
```

3.67 $\int \frac{\csc^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=146

$$\frac{\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4\sqrt{a}\,d}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -7/4*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d/a^(1/2)+arctanh(1
/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)+1/4
*cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-1/2*cot(d*x+c)*csc(d*x+c)/d/(a+a*sin(d
*x+c))^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2779, 2984, 2985, 2649, 206, 2773}

$$\frac{\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4\sqrt{a}\,d}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-7*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*Sqrt[a]*d)
 + (Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(Sqrt[a]*d) + Cot[c + d*x]/(4*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d
*x]*Csc[c + d*x])/(2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{\int \frac{\csc^2(c+dx)(a-3a\sin(c+dx))}{\sqrt{a+a\sin(c+dx)}}\,dx}{4a}$$

$$= \frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{\int \frac{\csc(c+dx)\left(-\frac{7a^2}{2}+\frac{1}{2}a^2\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{4a^2}$$

$$= \frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{7\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}}{8a}$$

$$= \frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{7\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx}}\right.}{4d}$$

$$= -\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4\sqrt{a}\,d} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [C] time = 3.40, size = 307, normalized size = 2.10

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(4\tan\left(\frac{1}{4}(c+dx)\right)+4\cot\left(\frac{1}{4}(c+dx)\right)-\csc^2\left(\frac{1}{4}(c+dx)\right)+\sec^2\left(\frac{1}{4}(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-8 - (64 + 64*I)*(-1)^(3/4)*ArcTanh
[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] + 4*Cot[(c + d*x)/4] - Csc
[(c + d*x)/4]^2 - 28*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 28*Log[
1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sec[(c + d*x)/4]^2 + 2/(Cos[(c +
 d*x)/4] - Sin[(c + d*x)/4])^2 - (8*Sin[(c + d*x)/4])/(Cos[(c + d*x)/4] - S
in[(c + d*x)/4]) - 2/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^2 + (8*Sin[(c +
d*x)/4])/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4]) + 4*Tan[(c + d*x)/4]))/(32*d
*Sqrt[a*(1 + Sin[c + d*x])])
```

fricas [B] time = 0.48, size = 492, normalized size = 3.37

$$\frac{7\left(\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(7*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c
) - cos(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 -
4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*s
qrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*
a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (c
os(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 8*sqrt(2)*(a*cos(d*x
 + c)^3 + a*cos(d*x + c)^2 - a*cos(d*x + c) + (a*cos(d*x + c)^2 - a)*sin(d*
x + c) - a)*log(-(cos(d*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x + c) + 2*sqrt
(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*
cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d
*x + c) - 2))/sqrt(a) - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c)
 - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a))/(a*d*cos(d*x + c)^3 + a*d*
cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d + (a*d*cos(d*x + c)^2 - a*d)*sin(d*
x + c))
```

giac [B] time = 3.05, size = 615, normalized size = 4.21

$$\sqrt{a \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{2} + a}\left(\frac{\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right)} - \frac{2}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right)}\right) - \frac{\left(42\,\sqrt{2}\,a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\,\sqrt{a} + \sqrt{a}}{\sqrt{-a}}\right) - 64\,\sqrt{2}\,a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right) \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(tan(1/2*d*x + 1/2*c)/(a*sgn(tan(1/
2*d*x + 1/2*c) + 1)) - 2/(a*sgn(tan(1/2*d*x + 1/2*c) + 1))) - (42*sqrt(2)*a
^(3/2)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 64*sqrt(2)*a^(3/2)*ar
ctan(sqrt(a)/sqrt(-a)) - 21*sqrt(2)*sqrt(-a)*a*log(sqrt(2)*sqrt(a) + sqrt(a
)) + 56*a^(3/2)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 96*a^(3/2)*a
rctan(sqrt(a)/sqrt(-a)) - 28*sqrt(-a)*a*log(sqrt(2)*sqrt(a) + sqrt(a)) - 18
*sqrt(2)*sqrt(-a)*a - 28*sqrt(-a)*a)*sgn(tan(1/2*d*x + 1/2*c) + 1)/(3*sqrt(
2)*sqrt(-a)*a^(3/2) + 4*sqrt(-a)*a^(3/2)) - 16*sqrt(2)*arctan(-1/2*sqrt(2)*
(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a
))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 14*arctan(-(sqrt(a)
*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt
(-a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 7*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*
c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(a)*sgn(tan(1/2*d*x + 1/2*c)
```

```
 + 1)) + 2*((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
 a))^3 - 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a))^2*sqrt(a) + (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)
^2 + a))*a + 2*a^(3/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a))^2 - a)^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 1.00, size = 162, normalized size = 1.11

$$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\,\left(7a^5\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)\left(\sin^2(dx+c)\right)+(-a(\sin(dx+c)-1))\right.}{4a^{\frac{11}{2}}\sin(dx+c)^2\cos(dx+c)\sqrt{a+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/4*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(11/2)*(7*a^5*arctanh((-a*(
sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^2+(-a*(sin(d*x+c)-1))^(3/2)*a^(7/2
)+(-a*(sin(d*x+c)-1))^(1/2)*a^(9/2)-4*2^(1/2)*arctanh(1/2*(-a*(sin(d*x+c)-1
))^(1/2)*2^(1/2)/a^(1/2))*a^5*sin(d*x+c)^2)/sin(d*x+c)^2/cos(d*x+c)/(a+a*si
n(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc(dx+c)^3}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)^3/sqrt(a*sin(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\sin(c+d\,x)^3\,\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^3(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(csc(c + d*x)**3/sqrt(a*(sin(c + d*x) + 1)), x)
```

3.68 $\int \frac{\sin^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=183

$$\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{13\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{10a^2d}+\frac{\sin^3(c+dx)\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}-\frac{9\sin^2(c+dx)\cos(c+dx)}{10ad\sqrt{a\sin(c+dx)}}$$

```
[Out] 1/2*cos(d*x+c)*sin(d*x+c)^3/d/(a+a*sin(d*x+c))^(3/2)+15/4*arctanh(1/2*cos(d
*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)-31/5*cos(d*
x+c)/a/d/(a+a*sin(d*x+c))^(1/2)-9/10*cos(d*x+c)*sin(d*x+c)^2/a/d/(a+a*sin(d
*x+c))^(1/2)+13/10*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/a^2/d
```

Rubi [A] time = 0.38, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2765, 2983, 2968, 3023, 2751, 2649, 206}

$$\frac{13\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{10a^2d}+\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{\sin^3(c+dx)\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}-\frac{9\sin^2(c+dx)\cos(c+dx)}{10ad\sqrt{a\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (15*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(2*
Sqrt[2]*a^(3/2)*d) + (Cos[c + d*x]*Sin[c + d*x]^3)/(2*d*(a + a*Sin[c + d*x]
)^(3/2)) - (31*Cos[c + d*x])/(5*a*d*Sqrt[a + a*Sin[c + d*x]]) - (9*Cos[c +
d*x]*Sin[c + d*x]^2)/(10*a*d*Sqrt[a + a*Sin[c + d*x]]) + (13*Cos[c + d*x]*S
qrt[a + a*Sin[c + d*x]])/(10*a^2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\cos(c+dx)\sin^3(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{\int \frac{\sin^2(c+dx)\left(3a-\frac{9}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^2}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{9\cos(c+dx)\sin^2(c+dx)}{10ad\sqrt{a+a\sin(c+dx)}} - \frac{\int \frac{\sin(c+dx)\left(-9a^2+\frac{39}{4}a^2\sin(c+dx\right.}{\sqrt{a+a\sin(c+dx)}}}{5a^3}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{9\cos(c+dx)\sin^2(c+dx)}{10ad\sqrt{a+a\sin(c+dx)}} - \frac{\int \frac{-9a^2\sin(c+dx)+\frac{39}{4}a^2\sin^2(c+dx}{\sqrt{a+a\sin(c+dx)}}}{5a^3}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{9\cos(c+dx)\sin^2(c+dx)}{10ad\sqrt{a+a\sin(c+dx)}} + \frac{13\cos(c+dx)\sqrt{a+a\sin(}}{10a^2d}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{31\cos(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}} - \frac{9\cos(c+dx)\sin^2(c+dx)}{10ad\sqrt{a+a\sin(c+dx)}} +$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{31\cos(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}} - \frac{9\cos(c+dx)\sin^2(c+dx)}{10ad\sqrt{a+a\sin(c+dx)}} +$$

$$= \frac{15\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{\cos(c+dx)\sin^3(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{31\cos(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}}$$

Mathematica [C] time = 0.45, size = 178, normalized size = 0.97

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(55\sin\left(\frac{1}{2}(c+dx)\right)-41\sin\left(\frac{3}{2}(c+dx)\right)+3\sin\left(\frac{5}{2}(c+dx)\right)+\sin\left(\frac{7}{2}(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-55*Cos[(c + d*x)/2] - 41*Cos[(3*(c
 + d*x))/2] - 3*Cos[(5*(c + d*x))/2] + Cos[(7*(c + d*x))/2] + 55*Sin[(c + d
*x)/2] - (150 + 150*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[
(c + d*x)/4])]*(1 + Sin[c + d*x]) - 41*Sin[(3*(c + d*x))/2] + 3*Sin[(5*(c +
 d*x))/2] + Sin[(7*(c + d*x))/2]))/(20*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

fricas [B] time = 0.51, size = 314, normalized size = 1.72

$$\frac{75\sqrt{2}\left(\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\,\sqrt{}}{\cos(dx}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/40*(75*sqrt(2)*(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*
x + c) - 2)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*sin(d*x + c)
+ a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(
d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*si
n(d*x + c) - cos(d*x + c) - 2)) - 4*(4*cos(d*x + c)^4 - 4*cos(d*x + c)^3 -
48*cos(d*x + c)^2 + (4*cos(d*x + c)^3 + 8*cos(d*x + c)^2 - 40*cos(d*x + c)
+ 5)*sin(d*x + c) - 45*cos(d*x + c) - 5)*sqrt(a*sin(d*x + c) + a))/(a^2*d*c
os(d*x + c)^2 - a^2*d*cos(d*x + c) - 2*a^2*d - (a^2*d*cos(d*x + c) + 2*a^2*
d)*sin(d*x + c))
```

giac [B] time = 0.84, size = 469, normalized size = 2.56

$$-\frac{\frac{75\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{4\left(\left(\left(\left(\left(\frac{11a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{15a}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{30}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\right.\right.}\right.\right.\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/10*(75*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(
a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*
d*x + 1/2*c) + 1)) - 4*(((((11*a*tan(1/2*d*x + 1/2*c)/sgn(tan(1/2*d*x + 1/2
*c) + 1) - 15*a/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 30*a/
sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) - 30*a/sgn(tan(1/2*d*x
+ 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 15*a/sgn(tan(1/2*d*x + 1/2*c) + 1))*t
an(1/2*d*x + 1/2*c) - 11*a/sgn(tan(1/2*d*x + 1/2*c) + 1))/(a*tan(1/2*d*x +
1/2*c)^2 + a)^(5/2) - 10*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*
d*x + 1/2*c)^2 + a))^3 + (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
 + 1/2*c)^2 + a))^2*sqrt(a) - (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))*a + a^(3/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(
```

```
a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a
*tan(1/2*d*x + 1/2*c)^2 + a))*sqrt(a) - a)^2*a*sgn(tan(1/2*d*x + 1/2*c) + 1
)))/d
```

maple [A] time = 0.72, size = 183, normalized size = 1.00

$$\frac{\left(\sin(dx+c)\left(-8\left(a-a\sin(dx+c)\right)^{\frac{5}{2}}\sqrt{a}-80\sqrt{a-a\sin(dx+c)}\,a^{\frac{5}{2}}+75\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a^{3}\right)-8}{20a^{\frac{9}{2}}\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/20*(sin(d*x+c)*(-8*(a-a*sin(d*x+c))^(5/2)*a^(1/2)-80*(a-a*sin(d*x+c))^(1/
2)*a^(5/2)+75*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a
^3)-8*(a-a*sin(d*x+c))^(5/2)*a^(1/2)-90*(a-a*sin(d*x+c))^(1/2)*a^(5/2)+75*2
^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3)*(-a*(sin(d*
x+c)-1))^(1/2)/a^(9/2)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^{4}}{\left(a\sin(dx+c)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^4/(a*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(c+d\,x)^{4}}{\left(a+a\,\sin(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(sin(c + d*x)^4/(a + a*sin(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{4}(c+dx)}{\left(a\left(\sin(c+dx)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sin(c + d*x)**4/(a*(sin(c + d*x) + 1))**(3/2), x)
```

3.69 $\int \frac{\sin^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{7\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{6a^2d}+\frac{\sin^2(c+dx)\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}+\frac{13\cos(c+dx)}{3ad\sqrt{a\sin(c+dx)+}}$$

```
[Out] 1/2*cos(d*x+c)*sin(d*x+c)^2/d/(a+a*sin(d*x+c))^(3/2)-11/4*arctanh(1/2*cos(d
*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)+13/3*cos(d*
x+c)/a/d/(a+a*sin(d*x+c))^(1/2)-7/6*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/a^2/d
```

Rubi [A] time = 0.25, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2765, 2968, 3023, 2751, 2649, 206}

$$-\frac{7\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{6a^2d}-\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{\sin^2(c+dx)\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}+\frac{13\cos(c+dx)}{3ad\sqrt{a\sin(c+dx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-11*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(2
*Sqrt[2]*a^(3/2)*d) + (Cos[c + d*x]*Sin[c + d*x]^2)/(2*d*(a + a*Sin[c + d*x
])^(3/2)) + (13*Cos[c + d*x])/(3*a*d*Sqrt[a + a*Sin[c + d*x]]) - (7*Cos[c +
 d*x]*Sqrt[a + a*Sin[c + d*x]])/(6*a^2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\cos(c+dx)\sin^2(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{\int \frac{\sin(c+dx)\left(2a-\frac{7}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^2}$$

$$= \frac{\cos(c+dx)\sin^2(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{\int \frac{2a\sin(c+dx)-\frac{7}{2}a\sin^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^2}$$

$$= \frac{\cos(c+dx)\sin^2(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{7\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{6a^2d} - \frac{\int \frac{-\frac{7a^2}{4}+\frac{13}{2}a^2\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}$$

$$= \frac{\cos(c+dx)\sin^2(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{13\cos(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} - \frac{7\cos(c+dx)\sqrt{a+a\sin(c+}$$

$$= \frac{\cos(c+dx)\sin^2(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{13\cos(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} - \frac{7\cos(c+dx)\sqrt{a+a\sin(c+}$$

$$= -\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} + \frac{\cos(c+dx)\sin^2(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{13\cos(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}}$$

Mathematica [C] time = 0.26, size = 156, normalized size = 1.08

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-11\sin\left(\frac{1}{2}(c+dx)\right)+7\sin\left(\frac{3}{2}(c+dx)\right)-\sin\left(\frac{5}{2}(c+dx)\right)+11\cos\left(\frac{1}{2}(c+dx\right.\right.}{6d(a(s}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(11*Cos[(c + d*x)/2] + 7*Cos[(3*(c +
 d*x))/2] + Cos[(5*(c + d*x))/2] - 11*Sin[(c + d*x)/2] + (33 + 33*I)*(-1)^(
3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(1 + Sin[c + d
*x]) + 7*Sin[(3*(c + d*x))/2] - Sin[(5*(c + d*x))/2]))/(6*d*(a*(1 + Sin[c +
 d*x]))^(3/2))
```

fricas [B] time = 0.51, size = 295, normalized size = 2.03

$$\frac{33\sqrt{2}\left(\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}\,(}{\cos(dx+c}\right.}{24\,(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(33*sqrt(2)*(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*
x + c) - 2)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c)
+ a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(
d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*si
n(d*x + c) - cos(d*x + c) - 2)) - 4*(4*cos(d*x + c)^3 + 16*cos(d*x + c)^2 -
 (4*cos(d*x + c)^2 - 12*cos(d*x + c) + 3)*sin(d*x + c) + 15*cos(d*x + c) +
3)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - 2
*a^2*d - (a^2*d*cos(d*x + c) + 2*a^2*d)*sin(d*x + c))
```

giac [B] time = 0.74, size = 409, normalized size = 2.82

$$-\frac{\frac{8\left(\left(\left(\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{3}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{3}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{2}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}}-\frac{33\sqrt{2}\arctan\left(\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/6*(8*(((2*tan(1/2*d*x + 1/2*c)/sgn(tan(1/2*d*x + 1/2*c) + 1) - 3/sgn(tan
(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 3/sgn(tan(1/2*d*x + 1/2*c) +
 1))*tan(1/2*d*x + 1/2*c) - 2/sgn(tan(1/2*d*x + 1/2*c) + 1))/(a*tan(1/2*d*x
 + 1/2*c)^2 + a)^(3/2) - 33*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*
x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(
-a)*a*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 6*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
 sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3 + (sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(a) - (sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a + a^(3/2))/(((sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*d*x +
 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*sqrt(a) - a)^2*a*sgn(tan(1/2*
d*x + 1/2*c) + 1)))/d
```

maple [A] time = 0.68, size = 183, normalized size = 1.26

$$-\frac{\left(\sin\left(dx+c\right)\left(33\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(dx+c\right)}\ \sqrt{2}}{2\sqrt{a}}\right)a^{2}-8\left(a-a\sin\left(dx+c\right)\right)^{\frac{3}{2}}\sqrt{a}\ -24\sqrt{a-a\sin\left(dx+c\right)}\ a^{\frac{3}{2}}\right)+\cdots}{12a^{\frac{7}{2}}\cos\left(dx+c\right.\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/12*(sin(d*x+c)*(33*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^
(1/2))*a^2-8*(a-a*sin(d*x+c))^(3/2)*a^(1/2)-24*(a-a*sin(d*x+c))^(1/2)*a^(3/
2))+33*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2-8*(a
-a*sin(d*x+c))^(3/2)*a^(1/2)-30*(a-a*sin(d*x+c))^(1/2)*a^(3/2))*(-a*(sin(d*
x+c)-1))^(1/2)/a^(7/2)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin{(dx+c)}^3}{(a\sin{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^3/(a*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin{(c+d\,x)}^3}{(a+a\,\sin{(c+d\,x)})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(sin(c + d*x)^3/(a + a*sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.70 $\int \frac{\sin^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=105

$$\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{2\cos(c+dx)}{ad\sqrt{a\sin(c+dx)+a}}-\frac{\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] -1/2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(3/2)+7/4*arctanh(1/2*cos(d*x+c)*a^(1/2)
*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)-2*cos(d*x+c)/a/d/(a+a*si
n(d*x+c))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2758, 2751, 2649, 206}

$$\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{2\cos(c+dx)}{ad\sqrt{a\sin(c+dx)+a}}-\frac{\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (7*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(2*S
qrt[2]*a^(3/2)*d) - Cos[c + d*x]/(2*d*(a + a*Sin[c + d*x])^(3/2)) - (2*Cos[
c + d*x])/(a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
```

```
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2758

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{\int \frac{-\frac{3a}{2}+2a\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{2\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} - \frac{7\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{4a}$$

$$= -\frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{2\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} + \frac{7\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(c}{\sqrt{a+a\sin}}\right.}{2ad}$$

$$= \frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{2\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}}$$

Mathematica [C] time = 0.27, size = 134, normalized size = 1.28

$$-\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-3\sin\left(\frac{1}{2}(c+dx)\right)+2\sin\left(\frac{3}{2}(c+dx)\right)+3\cos\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{3}{2}(c+a\right.\right.}{2d(a(\sin(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/2*((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(3*Cos[(c + d*x)/2] + 2*Cos[(3*
(c + d*x))/2] - 3*Sin[(c + d*x)/2] + (7 + 7*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/
2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(1 + Sin[c + d*x]) + 2*Sin[(3*(c + d
*x))/2]))/(d*(a*(1 + Sin[c + d*x]))^(3/2))
```

fricas [B] time = 0.53, size = 274, normalized size = 2.61

$$\frac{7\,\sqrt{2}\left(\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)-\cos\left(dx+c\right)-2\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^{2}+2\,\sqrt{2}\,\sqrt{a\sin(dx+c)+a}\,\sqrt{a}\cdots}{\cos(dx+\cdots}\right.}{8\left(a^{2}d\cos\left(dx+c\right)^{2}-\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(7*sqrt(2)*(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x
+ c) - 2)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*sin(d*x + c) +
a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*
x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(
d*x + c) - cos(d*x + c) - 2)) + 4*(4*cos(d*x + c)^2 + (4*cos(d*x + c) - 1)*
sin(d*x + c) + 5*cos(d*x + c) + 1)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x
 + c)^2 - a^2*d*cos(d*x + c) - 2*a^2*d - (a^2*d*cos(d*x + c) + 2*a^2*d)*sin
(d*x + c))
```

giac [B] time = 0.78, size = 360, normalized size = 3.43

$$\frac{\frac{4\left(\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\frac{1}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}\right)}{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}-\frac{7\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}+\sqrt{a}\right)}{2\,\sqrt{-a}}\right)}{\sqrt{-a}\,a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\frac{2\left(3\left(\sqrt{a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{\cdots}\right.\right.}{\left(\left(\sqrt{a}\,\tan\left(\frac{1}{2}\,dx\cdots\right.\right.\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(4*(tan(1/2*d*x + 1/2*c)/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 1/(a*sgn(t
an(1/2*d*x + 1/2*c) + 1)))/sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) - 7*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2
*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c) + 1))
+ 2*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^
3 + (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*s
qrt(a) - (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)
)*a + a^(3/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c
)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)
^2 + a))*sqrt(a) - a)^2*a*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 0.60, size = 143, normalized size = 1.36

$$-\frac{\left(\sin \left(dx+c\right)\left(-7\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\ \sqrt{2}}{2\sqrt{a}}\right)a+8\sqrt{a-a\sin \left(dx+c\right)}\ \sqrt{a}\right)-7\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\ \sqrt{2}}{2\sqrt{a}}\right.\right.}{4a^{\frac{5}{2}}\cos \left(dx+c\right)\sqrt{a+a\sin \left(dx+c\right)}\ d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/4/a^(5/2)*(sin(d*x+c)*(-7*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(
1/2)/a^(1/2))*a+8*(a-a*sin(d*x+c))^(1/2)*a^(1/2))-7*2^(1/2)*arctanh(1/2*(a-
a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a+10*(a-a*sin(d*x+c))^(1/2)*a^(1/2))*(
-a*(sin(d*x+c)-1))^(1/2)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin \left(dx+c\right)^{2}}{\left(a\sin \left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^2/(a*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin \left(c+d\,x\right)^{2}}{\left(a+a\,\sin \left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(sin(c + d*x)^2/(a + a*sin(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{2}\left(c+dx\right)}{\left(a\left(\sin \left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sin(c + d*x)**2/(a*(sin(c + d*x) + 1))**(3/2), x)
```

3.71 $\int \frac{\sin(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}$$

```
[Out] 1/2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(3/2)-3/4*arctanh(1/2*cos(d*x+c)*a^(1/2)*
2^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2750, 2649, 206}

$$\frac{\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(2*
Sqrt[2]*a^(3/2)*d) + Cos[c + d*x]/(2*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{4a}$$
$$= \frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{3\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{2ad}$$
$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} + \frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}$$

Mathematica [C] time = 0.20, size = 108, normalized size = 1.40

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)+(3+3i)(-1)^{3/4}(\sin(c+dx)+1)\tanh^{-}\right.}{2d(a(\sin(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
 + (3 + 3*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/
4])]*(1 + Sin[c + d*x])))/(2*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

fricas [B] time = 0.52, size = 253, normalized size = 3.29

$$\frac{3\sqrt{2}\left(\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}(c}{\cos(dx+c)^2}\right.}{8\left(a^2d\cos(dx+c)^2-a^2d\cos(dx+c)-2a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(3*sqrt(2)*(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x
+ c) - 2)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) +
a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*
x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(
d*x + c) - cos(d*x + c) - 2)) - 4*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) -
sin(d*x + c) + 1))/(a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - 2*a^2*d - (
a^2*d*cos(d*x + c) + 2*a^2*d)*sin(d*x + c))
```

giac [B] time = 1.43, size = 294, normalized size = 3.82

$$\frac{\frac{3\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{2\left(3\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^3+\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\cdots}\right.}{\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\cdots\right.\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(3*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x
 + 1/2*c) + 1)) - 2*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
 1/2*c)^2 + a))^3 + (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^2*sqrt(a) - (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
 + 1/2*c)^2 + a))*a + a^(3/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan
(1/2*d*x + 1/2*c)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a))*sqrt(a) - a)^2*a*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 0.72, size = 123, normalized size = 1.60

$$-\frac{\left(3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a\sin(dx+c)+3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a-2\sqrt{a-a\sin(dx+c)}\,\sqrt{\cdots}\right.}{4a^{\frac{5}{2}}\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/4*(3*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a*sin(d
*x+c)+3*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a-2*(a-
a*sin(d*x+c))^(1/2)*a^(1/2))*(-a*(sin(d*x+c)-1))^(1/2)/a^(5/2)/cos(d*x+c)/(
a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/(a*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin (c + d\, x)}{\left(a + a\, \sin (c + d\, x)\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + a*sin(c + d*x))^(3/2),x)
[Out] int(sin(c + d*x)/(a + a*sin(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin (c + dx)}{\left(a \left(\sin (c + dx) + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))**(3/2),x)
[Out] Integral(sin(c + d*x)/(a*(sin(c + d*x) + 1))**(3/2), x)
```

3.72 $\int \frac{1}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] -1/2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(3/2)-1/4*arctanh(1/2*cos(d*x+c)*a^(1/2)
*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2650, 2649, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(-3/2),x]
```

```
[Out] -ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])]/(2*Sqrt
[2]*a^(3/2)*d) - Cos[c + d*x]/(2*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}+\frac{\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{4a}$$
$$= -\frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}-\frac{\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,\frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{2ad}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}$$

Mathematica [C] time = 0.15, size = 108, normalized size = 1.40

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)-\cos\left(\frac{1}{2}(c+dx)\right)+(1+i)(-1)^{3/4}(\sin(c+dx)+1)\tanh^{-1}\left(\left(\right.\right.\right.}{2d(a(\sin(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(-3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-Cos[(c + d*x)/2] + Sin[(c + d*x)/2
] + (1 + I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4
])]*(1 + Sin[c + d*x])))/(2*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

fricas [B] time = 0.52, size = 252, normalized size = 3.27

$$\frac{\sqrt{2}\left(\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}(\cos}{\cos(dx+c)^2-}\right.}{8\left(a^2d\cos(dx+c)^2-a^2d\cos(dx+c)-2a^2c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x +
c) - 2)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)
*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x
+ c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*
x + c) - cos(d*x + c) - 2)) + 4*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - si
n(d*x + c) + 1))/(a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - 2*a^2*d - (a^
2*d*cos(d*x + c) + 2*a^2*d)*sin(d*x + c))
```

giac [B] time = 1.96, size = 293, normalized size = 3.81

$$\frac{\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}+\frac{2\left(3\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{3}+\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan(\right.\right.}{\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}+2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.\right.\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan
(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x +
 1/2*c) + 1)) + 2*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1
/2*c)^2 + a))^3 + (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a))^2*sqrt(a) - (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
 1/2*c)^2 + a))*a + a^(3/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))*sqrt(a) - a)^2*a*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 0.55, size = 125, normalized size = 1.62

$$-\frac{\left(\sqrt{2}\ \mathrm{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\ \sqrt{2}}{2\sqrt{a}}\right)a^{2}\sin\left(dx+c\right)+2\sqrt{a-a\sin\left(dx+c\right)}\ a^{\frac{3}{2}}+\sqrt{2}\ \mathrm{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\ \sqrt{2}}{2\sqrt{a}}\right)a^{2}\right.}{4a^{\frac{7}{2}}\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\ d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/4/a^(7/2)*(2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a
^2*sin(d*x+c)+2*(a-a*sin(d*x+c))^(1/2)*a^(3/2)+2^(1/2)*arctanh(1/2*(a-a*sin
(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2)*(-a*(sin(d*x+c)-1))^(1/2)/cos(d*x+c)/(
a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + a\,\sin(c + d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sin(c + d*x))^(3/2),x)
[Out] int(1/(a + a*sin(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sin(c + dx) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))**(3/2),x)
[Out] Integral((a*sin(c + d*x) + a)**(-3/2), x)
```

3.73 $\int \frac{\csc(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=114

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}+\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] -2*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d+1/2*cos(d*x
+c)/d/(a+a*sin(d*x+c))^(3/2)+5/4*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+
a*sin(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2766, 2985, 2649, 206, 2773}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}+\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{\cos(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(3/2)*d) +
 (5*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(2*
Sqrt[2]*a^(3/2)*d) + Cos[c + d*x]/(2*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
```

```
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx &= \frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{\int \frac{\csc(c+dx)\left(2a-\frac{1}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^2} \\
&= \frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{a^2} - \frac{5\int \frac{1}{\sqrt{a+a\sin(c+dx)}}}{4a} \\
&= \frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{ad} + \frac{5\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\right.}{} \\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} + \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} + \frac{\cos(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.20, size = 223, normalized size = 1.96

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)-2\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
 - (5 + 5*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/
4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - 2*Log[1 + Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + 2*Log[1 - Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2))/
(2*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

fricas [B] time = 0.53, size = 453, normalized size = 3.97

$$\frac{5\sqrt{2}\left(\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\,\sqrt{a}\cdots}{\cos(dx+\cdots}\right.\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(5*sqrt(2)*(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x
+ c) - 2)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*sin(d*x + c) +
a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*
x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(
d*x + c) - cos(d*x + c) - 2)) + 4*(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(
d*x + c) - cos(d*x + c) - 2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x +
c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c)
 - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)
^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)
^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4*sqrt(a*sin(
d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1))/(a^2*d*cos(d*x + c)^2 - a^
2*d*cos(d*x + c) - 2*a^2*d - (a^2*d*cos(d*x + c) + 2*a^2*d)*sin(d*x + c))
```

giac [B] time = 3.09, size = 412, normalized size = 3.61

$$-\frac{\frac{5\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{4\arctan\left(-\frac{\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}+\frac{2\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}\cdots\right.\right.}{a^{\frac{3}{2}}\mathrm{sgn}\left(\tan\cdots\right.}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(5*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*
tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*
x + 1/2*c) + 1)) - 4*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2
*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c) + 1))
+ 2*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
 a)))/(a^(3/2)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 2*(3*(sqrt(a)*tan(1/2*d*x +
 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3 + (sqrt(a)*tan(1/2*d*x + 1/
2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(a) - (sqrt(a)*tan(1/2*d*x
 + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a + a^(3/2))/(((sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 2*(sqrt(a)*tan(
1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*sqrt(a) - a)^2*a*sgn
(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 0.80, size = 172, normalized size = 1.51

$$\frac{\left(\sin\left(dx+c\right)a^{3}\left(5\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\ \sqrt{2}}{2\sqrt{a}}\right)-8\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)\right)+2\sqrt{a-a\sin\left(dx+c\right)}\ a^{\frac{5}{2}}+5\sqrt{2}\right.}{4a^{\frac{9}{2}}\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/4/a^(9/2)*(sin(d*x+c)*a^3*(5*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2
^(1/2)/a^(1/2))-8*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2)))+2*(a-a*sin(d*x+c
))^(1/2)*a^(5/2)+5*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/
2))*a^3-8*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2))*a^3)*(-a*(sin(d*x+c)-1))^
(1/2)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc\left(dx+c\right)}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)/(a*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\sin\left(c+d\,x\right)\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + a*sin(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(sin(c + d*x)*(a + a*sin(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(csc(c + d*x)/(a*(sin(c + d*x) + 1))**(3/2), x)
```

3.74 $\int \frac{\csc^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=144

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{3\cot(c+dx)}{2ad\sqrt{a\sin(c+dx)+a}}+\frac{\cot(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] 3*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d+1/2*cot(d*x+
c)/d/(a+a*sin(d*x+c))^(3/2)-9/4*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a
*sin(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)-3/2*cot(d*x+c)/a/d/(a+a*sin(d*x+c))^(
1/2)
```

Rubi [A] time = 0.36, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2766, 2984, 2985, 2649, 206, 2773}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{3\cot(c+dx)}{2ad\sqrt{a\sin(c+dx)+a}}+\frac{\cot(c+dx)}{2d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(3/2)*d) -
(9*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(2*S
qrt[2]*a^(3/2)*d) + Cot[c + d*x]/(2*d*(a + a*Sin[c + d*x])^(3/2)) - (3*Cot[
c + d*x])/(2*a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
```

```
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\cot(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{\int \frac{\csc^2(c+dx)\left(3a-\frac{3}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^2}$$

$$= \frac{\cot(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{3\cot(c+dx)}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc(c+dx)\left(-3a^2+\frac{3}{2}a^2\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^3}$$

$$= \frac{\cot(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{3\cot(c+dx)}{2ad\sqrt{a+a\sin(c+dx)}} - \frac{3\int \csc(c+dx)\sqrt{a+a\sin(c+}}{2a^2}$$

$$= \frac{\cot(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{3\cot(c+dx)}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos(c+}{\sqrt{a+a\sin(}}\right.}{ad}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} - \frac{9\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} + \frac{\cot(c+dx)}{2d(a+a\sin(c+dx))^{3/2}}$$

Mathematica [C] time = 0.63, size = 449, normalized size = 3.12

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(4\sin\left(\frac{1}{2}(c+dx)\right)+\frac{2\sin\left(\frac{1}{4}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}{\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)} - \frac{2\sin\left(\frac{1}{4}(c+dx)\right)\left(\sin\left(\frac{1}{2}(\right.\right.}{\sin\left(\frac{1}{4}(c+dx)\right)}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(4*Sin[(c + d*x)/2] - 2*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]) + 2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (18
 + 18*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - Cot[(c + d*x)/4]*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2])^2 + 6*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - 6*Log[1 - Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (2*Sin[(c + d*x)/4]*
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/4] - Sin[(c + d*x)/
4]) - (2*Sin[(c + d*x)/4]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(Cos[(c
+ d*x)/4] + Sin[(c + d*x)/4]) - (Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*Tan
[(c + d*x)/4]))/(4*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

fricas [B] time = 0.54, size = 539, normalized size = 3.74

$$9\sqrt{2}\left(\cos\left(dx+c\right)^{3}+2\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)-2\right)\sin\left(dx+c\right)-\cos\left(dx+c\right)-2\right)\sqrt{a}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(9*sqrt(2)*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + (cos(d*x + c)^2 - cos(d
*x + c) - 2)*sin(d*x + c) - cos(d*x + c) - 2)*sqrt(a)*log(-(a*cos(d*x + c)^
2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c)
 + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(
d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 6*(cos(
d*x + c)^3 + 2*cos(d*x + c)^2 + (cos(d*x + c)^2 - cos(d*x + c) - 2)*sin(d*x
 + c) - cos(d*x + c) - 2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^
2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) -
3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2
+ 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2
+ (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(3*cos(d*x + c
)^2 + (3*cos(d*x + c) + 1)*sin(d*x + c) + 2*cos(d*x + c) - 1)*sqrt(a*sin(d*
x + c) + a))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x
 + c) - 2*a^2*d + (a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - 2*a^2*d)*sin
(d*x + c))
```

giac [B] time = 0.98, size = 507, normalized size = 3.52

$$\frac{9\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{6\arctan\left(-\frac{\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}+\frac{3\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.\right.\right.}{a^{\frac{3}{2}}\mathrm{sgn}\left(\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(9*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x
 + 1/2*c) + 1)) - 6*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*
d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c) + 1)) +
 3*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
```

```
a)))/(a^(3/2)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + sqrt(a*tan(1/2*d*x + 1/2*c)^
2 + a)/(a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 2/(((sqrt(a)*tan(1/2*d*x + 1/2
*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)*sqrt(a)*sgn(tan(1/2*d*x +
1/2*c) + 1)) + 2*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^3 + (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c
)^2 + a))^2*sqrt(a) - (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a))*a + a^(3/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2
*d*x + 1/2*c)^2 + a))*sqrt(a) - a)^2*a*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 0.68, size = 219, normalized size = 1.52

$$-\frac{\left(9\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}\ \sqrt{2}}{2\sqrt{a}}\right)\left(\sin^2(dx+c)\right)a+9\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}\ \sqrt{2}}{2\sqrt{a}}\right)a\sin(dx+c)-12\operatorname{arc}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/4/a^(5/2)*(9*2^(1/2)*arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/
2))*sin(d*x+c)^2*a+9*2^(1/2)*arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/
a^(1/2))*a*sin(d*x+c)-12*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x
+c)^2*a+6*(-a*(sin(d*x+c)-1))^(1/2)*a^(1/2)*sin(d*x+c)-12*arctanh((-a*(sin(
d*x+c)-1))^(1/2)/a^(1/2))*a*sin(d*x+c)+4*(-a*(sin(d*x+c)-1))^(1/2)*a^(1/2))
*(-a*(sin(d*x+c)-1))^(1/2)/sin(d*x+c)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(dx+c)^2}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)^2/(a*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(c+d\,x)^2\,(a+a\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+a*sin(d*x+c))**(3/2),x)
[Out] Integral(csc(c + d*x)**2/(a*(sin(c + d*x) + 1))**(3/2), x)
```

3.75 $\int \frac{\csc^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=186

$$-\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4a^{3/2}d}+\frac{13\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{7\cot(c+dx)}{4ad\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\csc(c+dx)}{ad\sqrt{a\sin(c+dx)+a}}+\frac{\cot}{2d(}$$

```
[Out] -19/4*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d+1/2*cot(
d*x+c)*csc(d*x+c)/d/(a+a*sin(d*x+c))^(3/2)+13/4*arctanh(1/2*cos(d*x+c)*a^(1
/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)+7/4*cot(d*x+c)/a/d/(a
+a*sin(d*x+c))^(1/2)-cot(d*x+c)*csc(d*x+c)/a/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2766, 2984, 2985, 2649, 206, 2773}

$$-\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4a^{3/2}d}+\frac{13\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{7\cot(c+dx)}{4ad\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\csc(c+dx)}{ad\sqrt{a\sin(c+dx)+a}}+\frac{\cot}{2d(}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-19*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*a^(3/2)*d
) + (13*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])
/(2*Sqrt[2]*a^(3/2)*d) + (Cot[c + d*x]*Csc[c + d*x])/(2*d*(a + a*Sin[c + d*
x])^(3/2)) + (7*Cot[c + d*x])/(4*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d
*x]*Csc[c + d*x])/(a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\cot(c+dx)\csc(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{\int \frac{\csc^3(c+dx)\left(4a-\frac{5}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^2}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^2(c+dx)\left(-7a^2+6a^2\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{4a^3}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{7\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\text{cs}}{-}}{}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{7\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} + \frac{19\int}{}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} + \frac{7\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} - \frac{19\,\mathrm{S}}{}$$

$$= -\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4a^{3/2}d} + \frac{13\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} + \frac{\cot(c+dx)\csc(c+d}{2d(a+a\sin(c+dx))}$$

Mathematica [C] time = 4.75, size = 620, normalized size = 3.33

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-32\sin\left(\frac{1}{2}(c+dx)\right) - \frac{24\sin\left(\frac{1}{4}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}{\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)} + \frac{24\sin\left(\frac{1}{4}(c+dx)\right)(\text{s}}{\sin\left(\frac{1}{4}(c\right.}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-32*Sin[(c + d*x)/2] + 16*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]) - 24*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 -
 (208 + 208*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x
)/4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + 12*Cot[(c + d*x)/4]*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])^2 - Csc[(c + d*x)/4]^2*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])^2 - 76*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2])^2 + 76*Log[1 - Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + Sec[(c + d*x)/4]^2*(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^2)/(Cos[(c + d*x)/4] - Sin[(c + d*x)/4])^2 - (24*Sin[(c + d*x)/4]*(Cos[
```

```
(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/4] - Sin[(c + d*x)/4]) -
 (2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/4] + Sin[(c + d
*x)/4])^2 + (24*Sin[(c + d*x)/4]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(
Cos[(c + d*x)/4] + Sin[(c + d*x)/4]) + 12*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])^2*Tan[(c + d*x)/4]))/(32*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

fricas [B] time = 0.53, size = 626, normalized size = 3.37

$$\frac{26\sqrt{2}\left(\cos\left(dx+c\right)^{4}-\cos\left(dx+c\right)^{3}-3\,\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)^{3}+2\,\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)-2\right)\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/16*(26*sqrt(2)*(cos(d*x + c)^4 - cos(d*x + c)^3 - 3*cos(d*x + c)^2 - (cos
(d*x + c)^3 + 2*cos(d*x + c)^2 - cos(d*x + c) - 2)*sin(d*x + c) + cos(d*x +
 c) + 2)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a
)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x
 + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d
*x + c) - cos(d*x + c) - 2)) + 19*(cos(d*x + c)^4 - cos(d*x + c)^3 - 3*cos(
d*x + c)^2 - (cos(d*x + c)^3 + 2*cos(d*x + c)^2 - cos(d*x + c) - 2)*sin(d*x
 + c) + cos(d*x + c) + 2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^
2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) -
3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2
+ 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2
+ (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4*(7*cos(d*x + c
)^3 + 4*cos(d*x + c)^2 - (7*cos(d*x + c)^2 + 3*cos(d*x + c) - 2)*sin(d*x +
c) - 5*cos(d*x + c) - 2)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x + c)^4 -
a^2*d*cos(d*x + c)^3 - 3*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c) + 2*a^2*
d - (a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - 2
*a^2*d)*sin(d*x + c))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integrati
```

```
on of abs or sign assumes constant sign by intervals (correct if the argume
nt is real):Check [abs(cos((d*t_nostep+c)/2-pi/4))]Discontinuities at zeroe
s of cos((d*t_nostep+c)/2-pi/4) were not checkedWarning, integration of abs
 or sign assumes constant sign by intervals (correct if the argument is rea
l):Check [abs(t_nostep+1)]Evaluation time: 0.42Error: Bad Argument Type
```

maple [A] time = 0.93, size = 299, normalized size = 1.61

$$\frac{\left(13\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}\ \sqrt{2}}{2\sqrt{a}}\right)\left(\sin^3(dx+c)\right)a^2+2\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\ a^{\frac{3}{2}}\left(\sin^2(dx+c)\right)+13\sqrt{2}\ \operatorname{arcta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/4*(13*2^(1/2)*arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(
d*x+c)^3*a^2+2*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2)*sin(d*x+c)^2+13*2^(1/2)*ar
ctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(d*x+c)^2*a^2-19*ar
ctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^3*a^2+3*(-a*(sin(d*x+c)
-1))^(1/2)*sin(d*x+c)*a^(3/2)-5*(-a*(sin(d*x+c)-1))^(3/2)*a^(1/2)*sin(d*x+c
)-19*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^2*a^2+3*(-a*(sin
(d*x+c)-1))^(1/2)*a^(3/2)-5*(-a*(sin(d*x+c)-1))^(3/2)*a^(1/2))*(-a*(sin(d*x
+c)-1))^(1/2)/a^(7/2)/sin(d*x+c)^2/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(dx+c\right)^3}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)^3/(a*sin(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(c+d\,x\right)^3\,\left(a+a\,\sin\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(csc(c + d*x)**3/(a*(sin(c + d*x) + 1))**(3/2), x)
```

3.76 $\int \frac{\sin^5(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=221

$$\frac{283\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{787\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{240a^3d}-\frac{157\sin^2(c+dx)\cos(c+dx)}{80a^2d\sqrt{a\sin(c+dx)+a}}-\frac{1729\cos(\ldots}{120a^2d\sqrt{a\sin(\ldots}}$$

```
[Out] 1/4*cos(d*x+c)*sin(d*x+c)^4/d/(a+a*sin(d*x+c))^(5/2)+21/16*cos(d*x+c)*sin(d
*x+c)^3/a/d/(a+a*sin(d*x+c))^(3/2)+283/32*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^
(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)-1729/120*cos(d*x+c)/a^2/d/(
a+a*sin(d*x+c))^(1/2)-157/80*cos(d*x+c)*sin(d*x+c)^2/a^2/d/(a+a*sin(d*x+c))
^(1/2)+787/240*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/a^3/d
```

Rubi [A] time = 0.52, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2765, 2977, 2983, 2968, 3023, 2751, 2649, 206}

$$-\frac{157\sin^2(c+dx)\cos(c+dx)}{80a^2d\sqrt{a\sin(c+dx)+a}}+\frac{787\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{240a^3d}-\frac{1729\cos(c+dx)}{120a^2d\sqrt{a\sin(c+dx)+a}}+\frac{283\tanh^{-1}\left(\frac{\ldots}{\sqrt{2}\ldots}\right)}{16\sqrt{2}\,a\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (283*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(1
6*Sqrt[2]*a^(5/2)*d) + (Cos[c + d*x]*Sin[c + d*x]^4)/(4*d*(a + a*Sin[c + d*
x])^(5/2)) + (21*Cos[c + d*x]*Sin[c + d*x]^3)/(16*a*d*(a + a*Sin[c + d*x])^
(3/2)) - (1729*Cos[c + d*x])/(120*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (157*Co
s[c + d*x]*Sin[c + d*x]^2)/(80*a^2*d*Sqrt[a + a*Sin[c + d*x]]) + (787*Cos[c
 + d*x]*Sqrt[a + a*Sin[c + d*x]])/(240*a^3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
```

```
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\cos(c+dx)\sin^4(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{\int \frac{\sin^3(c+dx)\left(4a-\frac{13}{2}a\sin(c+dx)\right)}{(a+a\sin(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{21\cos(c+dx)\sin^3(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{\int \frac{\sin^2(c+dx)\left(\frac{63a^2}{2}-\frac{157}{4}a^2\sin(\right.}{\sqrt{a+a\sin(c+dx)}}}{8a^4}$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{21\cos(c+dx)\sin^3(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{157\cos(c+dx)\sin^2(c+}{80a^2d\sqrt{a+a\sin(c+d}}$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{21\cos(c+dx)\sin^3(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{157\cos(c+dx)\sin^2(c+}{80a^2d\sqrt{a+a\sin(c+d}}$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{21\cos(c+dx)\sin^3(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{157\cos(c+dx)\sin^2(c+}{80a^2d\sqrt{a+a\sin(c+d}}$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{21\cos(c+dx)\sin^3(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{1729\cos(c+dx)}{120a^2d\sqrt{a+a\sin(c+d}}$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{21\cos(c+dx)\sin^3(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{1729\cos(c+dx)}{120a^2d\sqrt{a+a\sin(c+d}}$$

$$= \frac{283\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{\cos(c+dx)\sin^4(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{21\cos(c+dx)\sin^3(c}{16ad(a+a\sin(c+d}$$

Mathematica [C] time = 0.56, size = 221, normalized size = 1.00

$$-\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-2547\sin\left(\frac{1}{2}(c+dx)\right)+3603\sin\left(\frac{3}{2}(c+dx)\right)+872\sin\left(\frac{5}{2}(c+dx)\right)+52\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/480*((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(2547*Cos[(c + d*x)/2] + 3603
*Cos[(3*(c + d*x))/2] - 872*Cos[(5*(c + d*x))/2] + 52*Cos[(7*(c + d*x))/2]
+ 12*Cos[(9*(c + d*x))/2] - 2547*Sin[(c + d*x)/2] + (8490 + 8490*I)*(-1)^(3
```

```
/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2])^4 + 3603*Sin[(3*(c + d*x))/2] + 872*Sin[(5*(c + d*x)
)/2] + 52*Sin[(7*(c + d*x))/2] - 12*Sin[(9*(c + d*x))/2]))/(d*(a*(1 + Sin[c
 + d*x]))^(5/2))
```

fricas [B] time = 0.57, size = 381, normalized size = 1.72

$$4245\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{2}-2\cos\left(dx+c\right)-4\right)\sin\left(dx+c\right)-2\cos\left(dx+c\right)-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/960*(4245*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + (cos(d*x + c)^2 -
2*cos(d*x + c) - 4)*sin(d*x + c) - 2*cos(d*x + c) - 4)*sqrt(a)*log(-(a*cos(
d*x + c)^2 + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin
(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2
*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))
+ 4*(96*cos(d*x + c)^5 + 256*cos(d*x + c)^4 - 1760*cos(d*x + c)^3 + 2475*co
s(d*x + c)^2 - (96*cos(d*x + c)^4 - 160*cos(d*x + c)^3 - 1920*cos(d*x + c)^
2 - 4395*cos(d*x + c) - 60)*sin(d*x + c) + 4335*cos(d*x + c) - 60)*sqrt(a*s
in(d*x + c) + a))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*
cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a
^3*d)*sin(d*x + c))
```

giac [B] time = 1.52, size = 626, normalized size = 2.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/240*(64*(((((34*tan(1/2*d*x + 1/2*c)/sgn(tan(1/2*d*x + 1/2*c) + 1) - 45/s
gn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) + 85/sgn(tan(1/2*d*x + 1
/2*c) + 1))*tan(1/2*d*x + 1/2*c) - 85/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/
2*d*x + 1/2*c) + 45/sgn(tan(1/2*d*x + 1/2*c) + 1))*tan(1/2*d*x + 1/2*c) - 3
4/sgn(tan(1/2*d*x + 1/2*c) + 1))/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(5/2) - 424
5*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x +
1/2*c) + 1)) + 30*(91*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a))^7 + 445*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
 1/2*c)^2 + a))^6*sqrt(a) + 305*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a))^5*a - 429*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
```

```
an(1/2*d*x + 1/2*c)^2 + a))^4*a^(3/2) + 41*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3*a^2 + 215*(sqrt(a)*tan(1/2*d*x + 1/2*
c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(5/2) - 173*(sqrt(a)*tan(1/2*d
*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^3 + 33*a^(7/2))/(((sqrt
(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 2*(sqrt(
a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*sqrt(a) - a)^
4*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 0.94, size = 323, normalized size = 1.46

$$-\frac{\left(\sin \left(dx+c\right)\left(384\left(a-a\sin \left(dx+c\right)\right)^{\frac{5}{2}}\sqrt{a}+640\left(a-a\sin \left(dx+c\right)\right)^{\frac{3}{2}}a^{\frac{3}{2}}+7680\sqrt{a-a\sin \left(dx+c\right)}\,a^{\frac{5}{2}}-8490\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/480/a^(11/2)*(sin(d*x+c)*(384*(a-a*sin(d*x+c))^(5/2)*a^(1/2)+640*(a-a*si
n(d*x+c))^(3/2)*a^(3/2)+7680*(a-a*sin(d*x+c))^(1/2)*a^(5/2)-8490*2^(1/2)*ar
ctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3)+(-192*(a-a*sin(d*x+c
))^(5/2)*a^(1/2)-320*(a-a*sin(d*x+c))^(3/2)*a^(3/2)-3840*(a-a*sin(d*x+c))^(
1/2)*a^(5/2)+4245*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2
))*a^3)*cos(d*x+c)^2+384*(a-a*sin(d*x+c))^(5/2)*a^(1/2)-470*(a-a*sin(d*x+c)
)^(3/2)*a^(3/2)+9780*(a-a*sin(d*x+c))^(1/2)*a^(5/2)-8490*2^(1/2)*arctanh(1/
2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3)*(-a*(sin(d*x+c)-1))^(1/2)/(1
+sin(d*x+c))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin \left(dx+c\right)^{5}}{\left(a\sin \left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^5/(a*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\sin \left(c+d\,x\right)}^5}{{\left(a+a\,\sin \left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a + a*sin(c + d*x))^(5/2),x)
[Out] int(sin(c + d*x)^5/(a + a*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+a*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

3.77 $\int \frac{\sin^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=183

$$-\frac{163\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{95\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{48a^3d}+\frac{197\cos(c+dx)}{24a^2d\sqrt{a\sin(c+dx)+a}}+\frac{\sin^3(c+dx)\cos}{4d(a\sin(c+dx}$$

```
[Out] 1/4*cos(d*x+c)*sin(d*x+c)^3/d/(a+a*sin(d*x+c))^(5/2)+17/16*cos(d*x+c)*sin(d
*x+c)^2/a/d/(a+a*sin(d*x+c))^(3/2)-163/32*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^
(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+197/24*cos(d*x+c)/a^2/d/(a+
a*sin(d*x+c))^(1/2)-95/48*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/a^3/d
```

Rubi [A] time = 0.39, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2765, 2977, 2968, 3023, 2751, 2649, 206}

$$-\frac{95\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{48a^3d}+\frac{197\cos(c+dx)}{24a^2d\sqrt{a\sin(c+dx)+a}}-\frac{163\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{\sin^3(c+dx)\cos}{4d(a\sin(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-163*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(
16*Sqrt[2]*a^(5/2)*d) + (Cos[c + d*x]*Sin[c + d*x]^3)/(4*d*(a + a*Sin[c + d
*x])^(5/2)) + (17*Cos[c + d*x]*Sin[c + d*x]^2)/(16*a*d*(a + a*Sin[c + d*x])
^(3/2)) + (197*Cos[c + d*x])/(24*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (95*Cos[
c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(48*a^3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\cos(c+dx)\sin^3(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{\int \frac{\sin^2(c+dx)\left(3a-\frac{11}{2}a\sin(c+dx)\right)}{(a+a\sin(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{17\cos(c+dx)\sin^2(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{\int \frac{\sin(c+dx)\left(17a^2-\frac{95}{4}a^2\sin(c+\cdots\right.}{\sqrt{a+a\sin(c+dx)}}}{8a^4}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{17\cos(c+dx)\sin^2(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{\int \frac{17a^2\sin(c+dx)-\frac{95}{4}a^2\sin^2(c\cdots}{\sqrt{a+a\sin(c+dx)}}}{8a^4}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{17\cos(c+dx)\sin^2(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{95\cos(c+dx)\sqrt{a+a\sin\cdots}}{48a^3d}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{17\cos(c+dx)\sin^2(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{197\cos(c+dx)}{24a^2d\sqrt{a+a\sin(c+dx\cdots}}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{17\cos(c+dx)\sin^2(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{197\cos(c+dx)}{24a^2d\sqrt{a+a\sin(c+dx\cdots}}$$

$$= -\frac{163\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} + \frac{\cos(c+dx)\sin^3(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{17\cos(c+dx)\sin^2(\cdots}{16ad(a+a\sin(c+\cdots}$$

Mathematica [C] time = 0.48, size = 197, normalized size = 1.08

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-279\sin\left(\frac{1}{2}(c+dx)\right)+399\sin\left(\frac{3}{2}(c+dx)\right)+88\sin\left(\frac{5}{2}(c+dx)\right)+8\sin\left(\frac{7}{2}\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(279*Cos[(c + d*x)/2] + 399*Cos[(3*(
c + d*x))/2] - 88*Cos[(5*(c + d*x))/2] + 8*Cos[(7*(c + d*x))/2] - 279*Sin[(
c + d*x)/2] + (978 + 978*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 +
 Tan[(c + d*x)/4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + 399*Sin[(3*(c
 + d*x))/2] + 88*Sin[(5*(c + d*x))/2] + 8*Sin[(7*(c + d*x))/2]))/(96*d*(a*(
1 + Sin[c + d*x]))^(5/2))
```

fricas [B] time = 0.53, size = 360, normalized size = 1.97

$$\frac{489\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{2}-2\cos\left(dx+c\right)-4\right)\sin\left(dx+c\right)-2\cos\left(dx+c\right)-4\right)\sqrt{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/192*(489*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 2
*cos(d*x + c) - 4)*sin(d*x + c) - 2*cos(d*x + c) - 4)*sqrt(a)*log(-(a*cos(d
*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(
d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*
a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) -
 4*(32*cos(d*x + c)^4 - 160*cos(d*x + c)^3 + 279*cos(d*x + c)^2 + (32*cos(d
*x + c)^3 + 192*cos(d*x + c)^2 + 471*cos(d*x + c) + 12)*sin(d*x + c) + 459*
cos(d*x + c) - 12)*sqrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*x + c)^3 + 3*a^3*
d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 -
 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

giac [B] time = 8.72, size = 584, normalized size = 3.19

$$-\frac{\frac{32\left(\left(\left(\frac{7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{9}{a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{9}{a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{7}{a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a\right)^{\frac{3}{2}}}-\frac{489\sqrt{2}\arctan\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/48*(32*(((7*tan(1/2*d*x + 1/2*c)/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 9/(
a*sgn(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/2*d*x + 1/2*c) + 9/(a*sgn(tan(1/2*d
*x + 1/2*c) + 1)))*tan(1/2*d*x + 1/2*c) - 7/(a*sgn(tan(1/2*d*x + 1/2*c) + 1
)))/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2) - 489*sqrt(2)*arctan(-1/2*sqrt(2)*
(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a
))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 6*(67*(sqrt(a)*
tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^7 + 341*(sqrt(a)
*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(a) + 233
*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^5*a -
325*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a
```

```
^(3/2) + 33*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
 a))^3*a^2 + 159*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c
)^2 + a))^2*a^(5/2) - 133*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a))*a^3 + 25*a^(7/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt
(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(
a*tan(1/2*d*x + 1/2*c)^2 + a))*sqrt(a) - a)^4*a^2*sgn(tan(1/2*d*x + 1/2*c)
+ 1)))/d
```

maple [A] time = 0.97, size = 269, normalized size = 1.47

$$\frac{\left(\sin\left(dx+c\right)\left(128\left(a-a\sin\left(dx+c\right)\right)^{\frac{3}{2}}\sqrt{a}+768\sqrt{a-a\sin\left(dx+c\right)}\,a^{\frac{3}{2}}-978\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a^{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/96/a^(9/2)*(sin(d*x+c)*(128*(a-a*sin(d*x+c))^(3/2)*a^(1/2)+768*(a-a*sin(d
*x+c))^(1/2)*a^(3/2)-978*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)
/a^(1/2))*a^2)+(-64*(a-a*sin(d*x+c))^(3/2)*a^(1/2)-384*(a-a*sin(d*x+c))^(1/
2)*a^(3/2)+489*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*
a^2)*cos(d*x+c)^2-46*(a-a*sin(d*x+c))^(3/2)*a^(1/2)+1092*(a-a*sin(d*x+c))^(
1/2)*a^(3/2)-978*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)
)*a^2)*(-a*(sin(d*x+c)-1))^(1/2)/(1+sin(d*x+c))/cos(d*x+c)/(a+a*sin(d*x+c))
^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(dx+c\right)^{4}}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^4/(a*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(c+d\,x\right)^{4}}{\left(a+a\,\sin\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(sin(c + d*x)^4/(a + a*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.78 $\int \frac{\sin^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=145

$$\frac{75\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{9\cos(c+dx)}{4a^2d\sqrt{a\sin(c+dx)+a}}+\frac{\sin^2(c+dx)\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}-\frac{13\cos(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] 1/4*cos(d*x+c)*sin(d*x+c)^2/d/(a+a*sin(d*x+c))^(5/2)-13/16*cos(d*x+c)/a/d/(
a+a*sin(d*x+c))^(3/2)+75/32*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin
(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)-9/4*cos(d*x+c)/a^2/d/(a+a*sin(d*x+c))^(1/
2)
```

Rubi [A] time = 0.27, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2765, 2968, 3019, 2751, 2649, 206}

$$-\frac{9\cos(c+dx)}{4a^2d\sqrt{a\sin(c+dx)+a}}+\frac{75\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{\sin^2(c+dx)\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}-\frac{13\cos(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (75*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) + (Cos[c + d*x]*Sin[c + d*x]^2)/(4*d*(a + a*Sin[c + d*x
])^(5/2)) - (13*Cos[c + d*x])/(16*a*d*(a + a*Sin[c + d*x])^(3/2)) - (9*Cos[
c + d*x])/(4*a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\cos(c+dx)\sin^2(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{\int \frac{\sin(c+dx)\left(2a-\frac{9}{2}a\sin(c+dx)\right)}{(a+a\sin(c+dx))^{3/2}}dx}{4a^2}$$

$$= \frac{\cos(c+dx)\sin^2(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{\int \frac{2a\sin(c+dx)-\frac{9}{2}a\sin^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}dx}{4a^2}$$

$$= \frac{\cos(c+dx)\sin^2(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{13\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{\int \frac{-\frac{39a^2}{4}+9a^2\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}dx}{8a^4}$$

$$= \frac{\cos(c+dx)\sin^2(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{13\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{9\cos(c+dx)}{4a^2d\sqrt{a+a\sin(c+dx)}}$$

$$= \frac{\cos(c+dx)\sin^2(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{13\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{9\cos(c+dx)}{4a^2d\sqrt{a+a\sin(c+dx)}}$$

$$= \frac{75\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{\cos(c+dx)\sin^2(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{13\cos(c+dx)}{16ad(a+a\sin(c+dx)}$$

Mathematica [C] time = 0.32, size = 173, normalized size = 1.19

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(45\sin\left(\frac{1}{2}(c+dx)\right)-69\sin\left(\frac{3}{2}(c+dx)\right)-16\sin\left(\frac{5}{2}(c+dx)\right)-45\cos\left(\frac{1}{2}(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-45*Cos[(c + d*x)/2] - 69*Cos[(3*(c
 + d*x))/2] + 16*Cos[(5*(c + d*x))/2] + 45*Sin[(c + d*x)/2] - (150 + 150*I)
*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2])^4 - 69*Sin[(3*(c + d*x))/2] - 16*Sin[(5*(c +
 d*x))/2]))/(32*d*(a*(1 + Sin[c + d*x]))^(5/2))
```

fricas [B] time = 0.52, size = 341, normalized size = 2.35

$$\frac{75\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+\left(\cos(dx+c)^2-2\cos(dx+c)-4\right)\sin(dx+c)-2\cos(dx+c)-4\right)\sqrt{}}{64\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 2*c
os(d*x + c) - 4)*sin(d*x + c) - 2*cos(d*x + c) - 4)*sqrt(a)*log(-(a*cos(d*x
 + c)^2 + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*
x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)
/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) - 4
*(32*cos(d*x + c)^3 - 53*cos(d*x + c)^2 - (32*cos(d*x + c)^2 + 85*cos(d*x +
 c) + 4)*sin(d*x + c) - 81*cos(d*x + c) + 4)*sqrt(a*sin(d*x + c) + a))/(a^3
*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d
 + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

giac [B] time = 0.93, size = 523, normalized size = 3.61

$$\frac{\frac{32\left(\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\frac{1}{a^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}\right)}{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}-\frac{75\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}+\sqrt{a}\right)}{2\,\sqrt{-a}}\right)}{\sqrt{-a}\,a^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\frac{2\left(43\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\cdots\right.\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*(32*(tan(1/2*d*x + 1/2*c)/(a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 1/(a^2
*sgn(tan(1/2*d*x + 1/2*c) + 1)))/sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) - 75*sq
rt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*
c) + 1)) + 2*(43*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c
)^2 + a))^7 + 237*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a))^6*sqrt(a) + 161*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a))^5*a - 221*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^4*a^(3/2) + 25*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(
a*tan(1/2*d*x + 1/2*c)^2 + a))^3*a^2 + 103*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(5/2) - 93*(sqrt(a)*tan(1/2*d*x + 1
/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^3 + 17*a^(7/2))/(((sqrt(a)*ta
n(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 2*(sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*sqrt(a) - a)^4*a^2*
sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [A] time = 1.01, size = 233, normalized size = 1.61

$$\left(\sin\left(dx+c\right)\left(150\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\ \sqrt{2}}{2\sqrt{a}}\right)a^2-128\sqrt{a-a\sin\left(dx+c\right)}\ a^{\frac{3}{2}}\right)+\left(-75\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(}}{2\sqrt{}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/32/a^(9/2)*(sin(d*x+c)*(150*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^
(1/2)/a^(1/2))*a^2-128*(a-a*sin(d*x+c))^(1/2)*a^(3/2))+(-75*2^(1/2)*arctanh
(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2+64*(a-a*sin(d*x+c))^(1/2)*
a^(3/2))*cos(d*x+c)^2+150*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2
)/a^(1/2))*a^2+42*(a-a*sin(d*x+c))^(3/2)*a^(1/2)-204*(a-a*sin(d*x+c))^(1/2)
*a^(3/2))*(-a*(sin(d*x+c)-1))^(1/2)/(1+sin(d*x+c))/cos(d*x+c)/(a+a*sin(d*x+
c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(dx+c\right)^3}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^3/(a*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\sin\left(c+d\,x\right)}^3}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(sin(c + d*x)^3/(a + a*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.79 $\int \frac{\sin^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=107

$$-\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{13\cos(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}-\frac{\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}$$

```
[Out] -1/4*cos(d*x+c)/d/(a+a*sin(d*x+c))^(5/2)+13/16*cos(d*x+c)/a/d/(a+a*sin(d*x+
c))^(3/2)-19/32*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/
2))/a^(5/2)/d*2^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2758, 2750, 2649, 206}

$$-\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{13\cos(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}-\frac{\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-19*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(1
6*Sqrt[2]*a^(5/2)*d) - Cos[c + d*x]/(4*d*(a + a*Sin[c + d*x])^(5/2)) + (13*
Cos[c + d*x])/(16*a*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
```

```
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2758

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{\int \frac{-\frac{5a}{2}+4a\sin(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{13\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{19\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{32a^2}$$

$$= -\frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{13\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{19\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \right.}{16a^2d}$$

$$= -\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{13\cos(c+dx)}{16ad(a+a\sin(c+dx}$$

Mathematica [C] time = 0.20, size = 196, normalized size = 1.83

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(8\sin\left(\frac{1}{2}(c+dx)\right)+13\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3-26\sin\left(\frac{1}{2}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(8*Sin[(c + d*x)/2] - 4*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]) - 26*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])^2 + 13*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + (19 + 19*I)*(-1
)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])^4))/(16*d*(a*(1 + Sin[c + d*x]))^(5/2))
```

fricas [B] time = 0.54, size = 320, normalized size = 2.99

$$\frac{19 \sqrt{2} \left(\cos \left(dx+c\right)^{3}+3 \cos \left(dx+c\right)^{2}+\left(\cos \left(dx+c\right)^{2}-2 \cos \left(dx+c\right)-4\right) \sin \left(dx+c\right)-2 \cos \left(dx+c\right)-4\right) \sqrt{a}}{64 \left(a^{3} d \cos \left(dx+c\right)^{3}+3 a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 2*c
os(d*x + c) - 4)*sin(d*x + c) - 2*cos(d*x + c) - 4)*sqrt(a)*log(-(a*cos(d*x
 + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*
x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)
/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) - 4
*(13*cos(d*x + c)^2 + (13*cos(d*x + c) + 4)*sin(d*x + c) + 9*cos(d*x + c) -
 4)*sqrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^
2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*
x + c) - 4*a^3*d)*sin(d*x + c))
```

giac [B] time = 0.83, size = 457, normalized size = 4.27

$$\frac{\frac{19 \sqrt{2} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{a} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-\sqrt{a \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}+a}+\sqrt{a}\right)}{2 \sqrt{-a}}\right)}{\sqrt{-a}\, a^{2} \mathrm{sgn}\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right)}-\frac{2\left(19\left(\sqrt{a} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-\sqrt{a \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}+a}\right)^{7}+133\left(\sqrt{a} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-\sqrt{\cdots}\right.\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*(19*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a
*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2
*d*x + 1/2*c) + 1)) - 2*(19*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*
d*x + 1/2*c)^2 + a))^7 + 133*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2
*d*x + 1/2*c)^2 + a))^6*sqrt(a) + 89*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a
*tan(1/2*d*x + 1/2*c)^2 + a))^5*a - 117*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(3/2) + 17*(sqrt(a)*tan(1/2*d*x + 1/2*
c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3*a^2 + 47*(sqrt(a)*tan(1/2*d*x +
1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(5/2) - 53*(sqrt(a)*tan(1/
2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^3 + 9*a^(7/2))/(((sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 2*(sqr
```

```
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*sqrt(a) - a
)^4*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [B] time = 0.82, size = 193, normalized size = 1.80

$$-\frac{\left(-19\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}\,a^2\left(\cos^2(dx+c)\right)+38\sqrt{2}\,\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2\sin(dx+c)+38}{32a^{\frac{9}{2}}\,(1+\sin(dx+c))\,\mathrm{co}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/32/a^(9/2)*(-19*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/
2))*a^2*cos(d*x+c)^2+38*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/
a^(1/2))*a^2*sin(d*x+c)+38*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/
2)/a^(1/2))*a^2-44*(a-a*sin(d*x+c))^(1/2)*a^(3/2)+26*(a-a*sin(d*x+c))^(3/2)
*a^(1/2))*(-a*(sin(d*x+c)-1))^(1/2)/(1+sin(d*x+c))/cos(d*x+c)/(a+a*sin(d*x+
c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^2}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^2/(a*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(c+d\,x)^2}{(a+a\,\sin(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(sin(c + d*x)^2/(a + a*sin(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(c+dx)}{(a(\sin(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(sin(c + d*x)**2/(a*(sin(c + d*x) + 1))**(5/2), x)
```

3.80 $\int \frac{\sin(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=107

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{5\cos(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}+\frac{\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}$$

```
[Out] 1/4*cos(d*x+c)/d/(a+a*sin(d*x+c))^(5/2)-5/16*cos(d*x+c)/a/d/(a+a*sin(d*x+c)
)^(3/2)-5/32*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))
/a^(5/2)/d*2^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2750, 2650, 2649, 206}

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{5\cos(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}+\frac{\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-5*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) + Cos[c + d*x]/(4*d*(a + a*Sin[c + d*x])^(5/2)) - (5*Co
s[c + d*x])/(16*a*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{5\int \frac{1}{(a+a\sin(c+dx))^{3/2}}\,dx}{8a}$$

$$= \frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{5\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{5\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{32a^2}$$

$$= \frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{5\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{5\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\,\mathrm{co}}{\sqrt{a+a}}\right.}{16a^2d}$$

$$= -\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} + \frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{5\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3}}$$

Mathematica [C] time = 0.19, size = 196, normalized size = 1.83

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-8\sin\left(\frac{1}{2}(c+dx)\right)-5\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3+10\sin\left(\frac{1}{2}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-8*Sin[(c + d*x)/2] + 4*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]) + 10*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2])^2 - 5*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + (5 + 5*I)*(-1)^
(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2])^4))/(16*d*(a*(1 + Sin[c + d*x]))^(5/2))
```

fricas [B] time = 0.51, size = 318, normalized size = 2.97

$$\frac{5\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{2}-2\cos\left(dx+c\right)-4\right)\sin\left(dx+c\right)-2\cos\left(dx+c\right)-4\right)\sqrt{a}}{64\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 2*co
s(d*x + c) - 4)*sin(d*x + c) - 2*cos(d*x + c) - 4)*sqrt(a)*log(-(a*cos(d*x
+ c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*x
 + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/
(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 4*
(5*cos(d*x + c)^2 + (5*cos(d*x + c) + 4)*sin(d*x + c) + cos(d*x + c) - 4)*s
qrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2
*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c
) - 4*a^3*d)*sin(d*x + c))
```

giac [B] time = 0.79, size = 457, normalized size = 4.27

$$\frac{\dfrac{5\sqrt{2}\arctan\left(-\dfrac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\dfrac{2\left(5\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{7}-29\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\cdots}\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*(5*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*
tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*
d*x + 1/2*c) + 1)) - 2*(5*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a))^7 - 29*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a))^6*sqrt(a) - 17*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))^5*a + 13*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*
tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(3/2) - 9*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3*a^2 + 9*(sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(5/2) + 13*(sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^3 - a^(7/2))/(((sqrt(a)*ta
n(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 2*(sqrt(a)*tan
```

```
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*sqrt(a) - a)^4*a^2*
sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [B] time = 1.00, size = 193, normalized size = 1.80

$$\frac{\left(5\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}\,a^{3}\left(\cos^{2}(dx+c)\right)-10\sqrt{2}\,\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a^{3}\sin(dx+c)+10\,(a-}{32a^{\frac{11}{2}}\,(1+\sin(dx+c))\cos(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/32/a^(11/2)*(5*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2
)*a^3*cos(d*x+c)^2-10*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^
(1/2))*a^3*sin(d*x+c)+10*(a-a*sin(d*x+c))^(3/2)*a^(3/2)-12*(a-a*sin(d*x+c))
^(1/2)*a^(5/2)-10*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2
))*a^3)*(-a*(sin(d*x+c)-1))^(1/2)/(1+sin(d*x+c))/cos(d*x+c)/(a+a*sin(d*x+c)
)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/(a*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(c+d\,x)}{{\left(a+a\,\sin(c+d\,x)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(sin(c + d*x)/(a + a*sin(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)}{(a(\sin(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(sin(c + d*x)/(a*(sin(c + d*x) + 1))**(5/2), x)
```

3.81 $\int \frac{1}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=107

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{3\cos(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}-\frac{\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}$$

```
[Out] -1/4*cos(d*x+c)/d/(a+a*sin(d*x+c))^(5/2)-3/16*cos(d*x+c)/a/d/(a+a*sin(d*x+c
))^(3/2)-3/32*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2)
)/a^(5/2)/d*2^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2650, 2649, 206}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{3\cos(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}-\frac{\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(-5/2),x]
```

```
[Out] (-3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) - Cos[c + d*x]/(4*d*(a + a*Sin[c + d*x])^(5/2)) - (3*Co
s[c + d*x])/(16*a*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{3\int \frac{1}{(a+a\sin(c+dx))^{3/2}}\,dx}{8a}$$

$$= -\frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{3\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{32a^2}$$

$$= -\frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{3\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{3\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\right.}{16a^2d}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} - \frac{3\cos(c+dx)}{16ad(a+a\sin(c+dx))}$$

Mathematica [C] time = 0.16, size = 196, normalized size = 1.83

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(8\sin\left(\frac{1}{2}(c+dx)\right)-3\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3+6\sin\left(\frac{1}{2}(c+d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(-5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(8*Sin[(c + d*x)/2] - 4*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]) + 6*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])^2 - 3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + (3 + 3*I)*(-1)^(3
/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2])^4))/(16*d*(a*(1 + Sin[c + d*x]))^(5/2))
```

fricas [B] time = 0.59, size = 320, normalized size = 2.99

$$\frac{3\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+\left(\cos(dx+c)^2-2\cos(dx+c)-4\right)\sin(dx+c)-2\cos(dx+c)-4\right)\sqrt{a}}{64\left(a^3d\cos(dx+c)^3+3a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 2*co
s(d*x + c) - 4)*sin(d*x + c) - 2*cos(d*x + c) - 4)*sqrt(a)*log(-(a*cos(d*x
+ c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*x
 + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/
(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 4*
(3*cos(d*x + c)^2 + (3*cos(d*x + c) - 4)*sin(d*x + c) + 7*cos(d*x + c) + 4)
*sqrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 -
 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x +
 c) - 4*a^3*d)*sin(d*x + c))
```

giac [B] time = 1.32, size = 457, normalized size = 4.27

$$\frac{\frac{3\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}+\frac{2\left(29\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^7+75\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\cdots\right.\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*(3*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*
tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*
d*x + 1/2*c) + 1)) + 2*(29*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a))^7 + 75*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a))^6*sqrt(a) + 55*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a))^5*a - 91*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a
*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(3/2) - (sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3*a^2 + 65*(sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(5/2) - 27*(sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^3 + 7*a^(7/2))/(((sqrt(a)*
tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 2*(sqrt(a)*t
an(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*sqrt(a) - a)^4*a^
2*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [B] time = 0.85, size = 195, normalized size = 1.82

$$-\frac{\left(\sin\left(dx+c\right)\left(6\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2+6\sqrt{a-a\sin\left(dx+c\right)}\,a^{\frac{3}{2}}\right)-3\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}\cdots\right.}{32a^{\frac{9}{2}}\left(1+\sin\left(dx+c\right)\right)\cos\left(dx\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/32/a^(9/2)*(sin(d*x+c)*(6*(a-a*sin(d*x+c))^(1/2)*a^(3/2)+6*2^(1/2)*arcta
nh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2)-3*2^(1/2)*arctanh(1/2*(
a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2*cos(d*x+c)^2+14*(a-a*sin(d*x+c))
^(1/2)*a^(3/2)+6*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)
)*a^2)*(-a*(sin(d*x+c)-1))^(1/2)/(1+sin(d*x+c))/cos(d*x+c)/(a+a*sin(d*x+c))
^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(1/(a + a*sin(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sin(c+dx)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral((a*sin(c + d*x) + a)**(-5/2), x)
```

3.82 $\int \frac{\csc(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=144

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}+\frac{43\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{11\cos(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}+\frac{\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}$$

```
[Out] -2*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/d+1/4*cos(d*x
+c)/d/(a+a*sin(d*x+c))^(5/2)+11/16*cos(d*x+c)/a/d/(a+a*sin(d*x+c))^(3/2)+43
/32*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/
d*2^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2766, 2978, 2985, 2649, 206, 2773}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}+\frac{43\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{11\cos(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}+\frac{\cos(c+dx)}{4d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(5/2)*d) +
 (43*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(1
6*Sqrt[2]*a^(5/2)*d) + Cos[c + d*x]/(4*d*(a + a*Sin[c + d*x])^(5/2)) + (11*
Cos[c + d*x])/(16*a*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
```

```
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{\int \frac{\csc(c+dx)\left(4a-\frac{3}{2}a\sin(c+dx)\right)}{(a+a\sin(c+dx))^{3/2}}dx}{4a^2}$$

$$= \frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{11\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{\int \frac{\csc(c+dx)\left(8a^2-\frac{11}{4}a^2\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}}{8a^4}$$

$$= \frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{11\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{\int \csc(c+dx)\sqrt{a+a\sin(c}}{a^3}$$

$$= \frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{11\cos(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{2\,\text{Subst}\left(\int \frac{1}{a-x^2}dx, x, \frac{a\cos}{\sqrt{a+a}}\right.}{a^2d}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} + \frac{43\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} + \frac{\cos(c+dx)}{4d(a+a\sin(c+dx))^{5}}$$

Mathematica [C] time = 0.27, size = 296, normalized size = 2.06

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-8\sin\left(\frac{1}{2}(c+dx)\right)+11\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3-22\sin\left(\frac{1}{2}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-8*Sin[(c + d*x)/2] + 4*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]) - 22*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2])^2 + 11*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - (43 + 43*I)*(-
1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])^4 - 16*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + 16*Log[1 - Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4))/(16*d*(a*(1 +
 Sin[c + d*x]))^(5/2))
```

fricas [B] time = 0.59, size = 539, normalized size = 3.74

$$\frac{43\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+\left(\cos(dx+c)^2-2\cos(dx+c)-4\right)\sin(dx+c)-2\cos(dx+c)-4\right)\sqrt{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 2*c
os(d*x + c) - 4)*sin(d*x + c) - 2*cos(d*x + c) - 4)*sqrt(a)*log(-(a*cos(d*x
 + c)^2 + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*
x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)
/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 3
2*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 2*cos(d*x + c) - 4
)*sin(d*x + c) - 2*cos(d*x + c) - 4)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*co
s(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(
d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(
d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(
d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4*(11
*cos(d*x + c)^2 + (11*cos(d*x + c) - 4)*sin(d*x + c) + 15*cos(d*x + c) + 4)
*sqrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 -
 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x +
 c) - 4*a^3*d)*sin(d*x + c))
```

giac [B] time = 1.27, size = 575, normalized size = 3.99

$$-\frac{\frac{43\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a^2\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{32\arctan\left(-\frac{\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}+\frac{16\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}d\right.\right.}{a^{\frac{5}{2}}\operatorname{sgn}\left(\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/16*(43*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(
a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/
2*d*x + 1/2*c) + 1)) - 32*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)
 + 1)) + 16*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a)))/(a^(5/2)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 2*(53*(sqrt(a)*tan(
1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^7 + 179*(sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(a) + 127*(sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^5*a - 195*
(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(3/
2) + 7*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^
3*a^2 + 121*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
 a))^2*a^(5/2) - 67*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))*a^3 + 15*a^(7/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan
```

```
(1/2*d*x + 1/2*c)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a))*sqrt(a) - a)^4*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))
/d
```

maple [B] time = 0.92, size = 262, normalized size = 1.82

$$\frac{\left(2\sin(dx+c)\,a^5\left(43\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\ \sqrt{2}}{2\sqrt{a}}\right)-64\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)\right)-a^5\left(43\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin}}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/32/a^(15/2)*(2*sin(d*x+c)*a^5*(43*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1
/2)*2^(1/2)/a^(1/2))-64*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2)))-a^5*(43*2^
(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-64*arctanh((a-a*s
in(d*x+c))^(1/2)/a^(1/2)))*cos(d*x+c)^2+52*(a-a*sin(d*x+c))^(1/2)*a^(9/2)-2
2*(a-a*sin(d*x+c))^(3/2)*a^(7/2)+86*a^5*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c)
)^(1/2)*2^(1/2)/a^(1/2))-128*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2))*a^5)*(
-a*(sin(d*x+c)-1))^(1/2)/(1+sin(d*x+c))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(dx+c)}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)/(a*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(c+d\,x)\,\left(a+a\,\sin(c+d\,x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + a*sin(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(sin(c + d*x)*(a + a*sin(c + d*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)}{(a(\sin(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(csc(c + d*x)/(a*(sin(c + d*x) + 1))**(5/2), x)
```

3.83 $\int \frac{\csc^2(c+dx)}{(a+a\sin(c+dx))^{5/2}} dx$

Optimal. Leaf size=174

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{115\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{35\cot(c+dx)}{16a^2d\sqrt{a\sin(c+dx)+a}}+\frac{15\cot(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}+$$

```
[Out] 5*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/d+1/4*cot(d*x+
c)/d/(a+a*sin(d*x+c))^(5/2)+15/16*cot(d*x+c)/a/d/(a+a*sin(d*x+c))^(3/2)-115
/32*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/
d*2^(1/2)-35/16*cot(d*x+c)/a^2/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2766, 2978, 2984, 2985, 2649, 206, 2773}

$$-\frac{35\cot(c+dx)}{16a^2d\sqrt{a\sin(c+dx)+a}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{115\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{15\cot(c+dx)}{16ad(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (5*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(5/2)*d) -
(115*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(1
6*Sqrt[2]*a^(5/2)*d) + Cot[c + d*x]/(4*d*(a + a*Sin[c + d*x])^(5/2)) + (15*
Cot[c + d*x])/(16*a*d*(a + a*Sin[c + d*x])^(3/2)) - (35*Cot[c + d*x])/(16*a
^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
```

```
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\cot(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{\int \frac{\csc^2(c+dx)\left(5a-\frac{5}{2}a\sin(c+dx)\right)}{(a+a\sin(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= \frac{\cot(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{15\cot(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{\int \frac{\csc^2(c+dx)\left(\frac{35a^2}{2}-\frac{45}{4}a^2\sin(c+dx\right.}{\sqrt{a+a\sin(c+dx)}}}{8a^4}$$

$$= \frac{\cot(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{15\cot(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{35\cot(c+dx)}{16a^2d\sqrt{a+a\sin(c+dx)}} +$$

$$= \frac{\cot(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{15\cot(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{35\cot(c+dx)}{16a^2d\sqrt{a+a\sin(c+dx)}} -$$

$$= \frac{\cot(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{15\cot(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{35\cot(c+dx)}{16a^2d\sqrt{a+a\sin(c+dx)}} +$$

$$= \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{115\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{\cot(c+dx)}{4d(a+a\sin(c+dx))^{5/2}}$$

Mathematica [C] time = 0.63, size = 509, normalized size = 2.93

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(8\sin\left(\frac{1}{2}(c+dx)\right)+\frac{8\sin\left(\frac{1}{4}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4}{\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)}-\frac{8\sin\left(\frac{1}{4}(c+dx)\right)\left(\sin\left(\frac{1}{2}(\right.\right.}{\sin\left(\frac{1}{4}(c+dx)\right)}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(8*Sin[(c + d*x)/2] - 4*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]) + 38*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])^2 - 19*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + 8*(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2])^4 + (115 + 115*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(
```

```
-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4
- 4*Cot[(c + d*x)/4]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + 40*Log[1 + C
os[(c + d*x)/2] - Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4
 - 40*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^4 + (8*Sin[(c + d*x)/4]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^4)/(Cos[(c + d*x)/4] - Sin[(c + d*x)/4]) - (8*Sin[(c + d*x)/4]*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])^4)/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4]) - 4*(Co
s[(c + d*x)/2] + Sin[(c + d*x)/2])^4*Tan[(c + d*x)/4]))/(16*d*(a*(1 + Sin[c
 + d*x]))^(5/2))
```

fricas [B] time = 0.54, size = 631, normalized size = 3.63

$$115\sqrt{2}\left(\cos\left(dx+c\right)^{4}-2\cos\left(dx+c\right)^{3}-5\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}-2\cos\left(dx+c\right)-4\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(115*sqrt(2)*(cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 5*cos(d*x + c)^2 - (
cos(d*x + c)^3 + 3*cos(d*x + c)^2 - 2*cos(d*x + c) - 4)*sin(d*x + c) + 2*co
s(d*x + c) + 4)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x +
 c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*
cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2
)*sin(d*x + c) - cos(d*x + c) - 2)) + 80*(cos(d*x + c)^4 - 2*cos(d*x + c)^3
 - 5*cos(d*x + c)^2 - (cos(d*x + c)^3 + 3*cos(d*x + c)^2 - 2*cos(d*x + c) -
 4)*sin(d*x + c) + 2*cos(d*x + c) + 4)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*
cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*co
s(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*co
s(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + co
s(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(
35*cos(d*x + c)^3 - 20*cos(d*x + c)^2 - (35*cos(d*x + c)^2 + 55*cos(d*x + c
) + 4)*sin(d*x + c) - 51*cos(d*x + c) + 4)*sqrt(a*sin(d*x + c) + a))/(a^3*d
*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^3 - 5*a^3*d*cos(d*x + c)^2 + 2*a^3*d
*cos(d*x + c) + 4*a^3*d - (a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 -
2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

giac [B] time = 3.14, size = 671, normalized size = 3.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*(115*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(
a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/
2*d*x + 1/2*c) + 1)) - 80*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)
 + 1)) + 40*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a)))/(a^(5/2)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 8*sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a)/(a^3*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 16/(((sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)*a^(3/2)*sgn(ta
n(1/2*d*x + 1/2*c) + 1)) + 2*(77*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan
(1/2*d*x + 1/2*c)^2 + a))^7 + 283*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(a) + 199*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^5*a - 299*(sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(3/2) + 15*(sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3*a^2 + 177*(sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(5/2) - 107*(sqrt(a
)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^3 + 23*a^(7/
2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2
 + 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*sq
rt(a) - a)^4*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [B] time = 1.12, size = 356, normalized size = 2.05

$$-\frac{\left(115\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}\ \sqrt{2}}{2\sqrt{a}}\right)\left(\sin^3(dx+c)\right)a^2+32\sqrt{-a(\sin(dx+c)-1)}\ a^{\frac{3}{2}}\left(\sin^2(dx+c)\right)+230\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/32*(115*2^(1/2)*arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))*s
in(d*x+c)^3*a^2+32*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2)*sin(d*x+c)^2+230*2^(1/
2)*arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(d*x+c)^2*a^2-
160*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^3*a^2+148*(-a*(si
n(d*x+c)-1))^(1/2)*sin(d*x+c)*a^(3/2)-38*(-a*(sin(d*x+c)-1))^(3/2)*a^(1/2)*
sin(d*x+c)+115*2^(1/2)*arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2
))*a^2*sin(d*x+c)-320*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)
^2*a^2+32*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2)-160*arctanh((-a*(sin(d*x+c)-1))
^(1/2)/a^(1/2))*a^2*sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(9/2)/(1+sin(d*
x+c))/sin(d*x+c)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(dx+c)^2}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)^2/(a*sin(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin (c+d\,x)^2\,(a+a\,\sin (c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2 (c+dx)}{(a (\sin (c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(csc(c + d*x)**2/(a*(sin(c + d*x) + 1))**(5/2), x)
```

3.84 $\int \frac{\csc^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=224

$$-\frac{39\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4a^{5/2}d}+\frac{219\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{63\cot(c+dx)}{16a^2d\sqrt{a\sin(c+dx)+a}}-\frac{31\cot(c+dx)\csc(c+dx)}{16a^2d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -39/4*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/d+1/4*cot(
d*x+c)*csc(d*x+c)/d/(a+a*sin(d*x+c))^(5/2)+19/16*cot(d*x+c)*csc(d*x+c)/a/d/
(a+a*sin(d*x+c))^(3/2)+219/32*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*s
in(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+63/16*cot(d*x+c)/a^2/d/(a+a*sin(d*x+c))
^(1/2)-31/16*cot(d*x+c)*csc(d*x+c)/a^2/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2766, 2978, 2984, 2985, 2649, 206, 2773}

$$\frac{63\cot(c+dx)}{16a^2d\sqrt{a\sin(c+dx)+a}}-\frac{39\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4a^{5/2}d}+\frac{219\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{31\cot(c+dx)\csc(c+dx)}{16a^2d\sqrt{a\sin(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-39*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*a^(5/2)*d
) + (219*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])]
)/(16*Sqrt[2]*a^(5/2)*d) + (Cot[c + d*x]*Csc[c + d*x])/(4*d*(a + a*Sin[c +
d*x])^(5/2)) + (19*Cot[c + d*x]*Csc[c + d*x])/(16*a*d*(a + a*Sin[c + d*x])^
(3/2)) + (63*Cot[c + d*x])/(16*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (31*Cot[c
+ d*x]*Csc[c + d*x])/(16*a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
```

```
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\cot(c+dx)\csc(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{\int \frac{\csc^3(c+dx)\left(6a-\frac{7}{2}a\sin(c+dx)\right)}{(a+a\sin(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{19\cot(c+dx)\csc(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{\int \frac{\csc^3(c+dx)\left(31a^2-\frac{95}{4}a^2\sin(c+dx\right.}{\sqrt{a+a\sin(c+dx)}}}{8a^4}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{19\cot(c+dx)\csc(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{31\cot(c+dx)\csc(c+dx)}{16a^2d\sqrt{a+a\sin(c+dx)}}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{19\cot(c+dx)\csc(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{63\cot(c+dx)}{16a^2d\sqrt{a+a\sin(c+dx)}}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{19\cot(c+dx)\csc(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{63\cot(c+dx)}{16a^2d\sqrt{a+a\sin(c+dx)}}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{4d(a+a\sin(c+dx))^{5/2}} + \frac{19\cot(c+dx)\csc(c+dx)}{16ad(a+a\sin(c+dx))^{3/2}} + \frac{63\cot(c+dx)}{16a^2d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{39\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4a^{5/2}d} + \frac{219\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} + \frac{\cot(c+dx)\csc(c+}{4d(a+a\sin(c+dx)}$$

Mathematica [C] time = 1.13, size = 680, normalized size = 3.04

$$\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(-16\sin\left(\frac{1}{2}(c+dx)\right) - \frac{40\sin\left(\frac{1}{4}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4}{\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)} + \frac{40\sin\left(\frac{1}{4}(c+dx)\right)\left(\mathrm{s}\right.}{\sin\left(\frac{1}{4}(c\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-16*Sin[(c + d*x)/2] + 8*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]) - 108*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^2 + 54*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - 40*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])^4 - (438 + 438*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/
2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
)^4 + 20*Cot[(c + d*x)/4]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 - Csc[(c
+ d*x)/4]^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 - 156*Log[1 + Cos[(c +
d*x)/2] - Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + 156*L
og[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^4 + Sec[(c + d*x)/4]^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + (2*
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)/(Cos[(c + d*x)/4] - Sin[(c + d*x)/
4])^2 - (40*Sin[(c + d*x)/4]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)/(Cos[
(c + d*x)/4] - Sin[(c + d*x)/4]) - (2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^4)/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^2 + (40*Sin[(c + d*x)/4]*(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])^4)/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4]) + 20
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*Tan[(c + d*x)/4]))/(32*d*(a*(1 + S
in[c + d*x]))^(5/2))
```

fricas [B] time = 0.60, size = 715, normalized size = 3.19

$$\frac{219\sqrt{2}\left(\cos(dx+c)^5 + 3\cos(dx+c)^4 - 3\cos(dx+c)^3 - 7\cos(dx+c)^2 + \left(\cos(dx+c)^4 - 2\cos(dx+c)^3 - \right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(219*sqrt(2)*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 - 3*cos(d*x + c)^3 - 7
*cos(d*x + c)^2 + (cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 5*cos(d*x + c)^2 + 2
*cos(d*x + c) + 4)*sin(d*x + c) + 2*cos(d*x + c) + 4)*sqrt(a)*log(-(a*cos(d
*x + c)^2 + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(
d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*
a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) +
 156*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 - 3*cos(d*x + c)^3 - 7*cos(d*x + c)
^2 + (cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 5*cos(d*x + c)^2 + 2*cos(d*x + c)
 + 4)*sin(d*x + c) + 2*cos(d*x + c) + 4)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*
a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*
cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*
cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 +
cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4
*(63*cos(d*x + c)^4 + 95*cos(d*x + c)^3 - 51*cos(d*x + c)^2 + (63*cos(d*x +
 c)^3 - 32*cos(d*x + c)^2 - 83*cos(d*x + c) + 4)*sin(d*x + c) - 87*cos(d*x
+ c) - 4)*sqrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x
```

```
 + c)^4 - 3*a^3*d*cos(d*x + c)^3 - 7*a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x
 + c) + 4*a^3*d + (a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^3 - 5*a^3*d*
cos(d*x + c)^2 + 2*a^3*d*cos(d*x + c) + 4*a^3*d)*sin(d*x + c))
```

giac [B] time = 1.36, size = 817, normalized size = 3.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(tan(1/2*d*x + 1/2*c)/(a^3*sgn(t
an(1/2*d*x + 1/2*c) + 1)) - 10/(a^3*sgn(tan(1/2*d*x + 1/2*c) + 1))) - 219*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2
*c) + 1)) + 156*arctan(-(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
+ 1/2*c)^2 + a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 7
8*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a
)))/(a^(5/2)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 4*((sqrt(a)*tan(1/2*d*x + 1/2
*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3 - 10*(sqrt(a)*tan(1/2*d*x + 1/2
*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(a) + (sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a + 10*a^(3/2))/(((sqrt(a)*t
an(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^2*a^2*sgn(
tan(1/2*d*x + 1/2*c) + 1)) - 2*(101*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*
tan(1/2*d*x + 1/2*c)^2 + a))^7 + 387*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a
*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(a) + 271*(sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^5*a - 403*(sqrt(a)*tan(1/2*d*x + 1/2
*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(3/2) + 23*(sqrt(a)*tan(1/2*d
*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3*a^2 + 233*(sqrt(a)*tan(
1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(5/2) - 147*(sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^3 + 31*a^
(7/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)
)^2 + 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))
*sqrt(a) - a)^4*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

maple [B] time = 1.28, size = 404, normalized size = 1.80

$$-\frac{\left(-219\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}\ \sqrt{2}}{2\sqrt{a}}\right)\left(\sin^4(dx+c)\right)a^2+312\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)\left(\sin^4(dx+c)\right)a^2-4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/32/a^(9/2)*(-219*2^(1/2)*arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a
^(1/2))*sin(d*x+c)^4*a^2+312*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin
(d*x+c)^4*a^2-438*2^(1/2)*arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(
1/2))*sin(d*x+c)^3*a^2+126*(-a*(sin(d*x+c)-1))^(3/2)*a^(1/2)*sin(d*x+c)^2+6
24*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^3*a^2-219*2^(1/2)*
arctanh(1/2*(-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(d*x+c)^2*a^2+144
*(-a*(sin(d*x+c)-1))^(3/2)*a^(1/2)*sin(d*x+c)-172*(-a*(sin(d*x+c)-1))^(1/2)
*a^(3/2)*sin(d*x+c)^2+312*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*
x+c)^2*a^2+72*(-a*(sin(d*x+c)-1))^(3/2)*a^(1/2)-112*(-a*(sin(d*x+c)-1))^(1/
2)*sin(d*x+c)*a^(3/2)-56*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2))*(-a*(sin(d*x+c)
-1))^(1/2)/(1+sin(d*x+c))/sin(d*x+c)^2/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin\left(c+d\,x\right)^3\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(5/2)),x)
[Out] int(1/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(5/2)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{3}{\left(c + dx \right)}}{\left(a \left(\sin{\left(c + dx \right)} + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+a*sin(d*x+c))**(5/2),x)
[Out] Integral(csc(c + d*x)**3/(a*(sin(c + d*x) + 1))**(5/2), x)
```

3.85 $\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{\sin(e+fx)}}\,dx$

Optimal. Leaf size=37

$$-\frac{2\sqrt{a}\,\sin^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}$$

```
[Out] -2*arcsin(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))*a^(1/2)/f
```

Rubi [A] time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.080, Rules used = {2774, 216}

$$-\frac{2\sqrt{a}\,\sin^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/Sqrt[Sin[e + f*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcSin[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/f
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{\sin(e+fx)}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f}$$
$$= -\frac{2\sqrt{a}\,\sin^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f}$$

Mathematica [C] time = 0.51, size = 164, normalized size = 4.43

$$\frac{(1+i)e^{\frac{1}{2}i(e+fx)}\sqrt{-ie^{-i(e+fx)}\left(-1+e^{2i(e+fx)}\right)}\sqrt{a(\sin(e+fx)+1)}\left(\tan^{-1}\left(\sqrt{-1+e^{2i(e+fx)}}\right)-i\tanh^{-1}\left(\frac{e^{i(e+fx)}}{\sqrt{-1+e^{2i(e+fx)}}}\right.\right.}{\sqrt{2}f\sqrt{-1+e^{2i(e+fx)}}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/Sqrt[Sin[e + f*x]],x]
```

```
[Out] ((1 + I)*E^((I/2)*(e + f*x))*Sqrt[((-I)*(-1 + E^((2*I)*(e + f*x))))/E^(I*(e
 + f*x))]*(ArcTan[Sqrt[-1 + E^((2*I)*(e + f*x))]] - I*ArcTanh[E^(I*(e + f*x
))/Sqrt[-1 + E^((2*I)*(e + f*x))]])*Sqrt[a*(1 + Sin[e + f*x])])/(Sqrt[2]*Sq
rt[-1 + E^((2*I)*(e + f*x))]*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

fricas [B] time = 0.67, size = 330, normalized size = 8.92

$$\left[\frac{\sqrt{-a}\log\left(\frac{128\,a\cos\left(fx+e\right)^5-128\,a\cos\left(fx+e\right)^4-416\,a\cos\left(fx+e\right)^3+128\,a\cos\left(fx+e\right)^2-8\left(16\cos\left(fx+e\right)^4-24\cos\left(fx+e\right)^3-66\cos\left(fx+e\right)^2+\left(16\,\mathrm{c}\right.\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(-a)*log((128*a*cos(f*x + e)^5 - 128*a*cos(f*x + e)^4 - 416*a*cos(
f*x + e)^3 + 128*a*cos(f*x + e)^2 - 8*(16*cos(f*x + e)^4 - 24*cos(f*x + e)^
3 - 66*cos(f*x + e)^2 + (16*cos(f*x + e)^3 + 40*cos(f*x + e)^2 - 26*cos(f*x
 + e) - 51)*sin(f*x + e) + 25*cos(f*x + e) + 51)*sqrt(a*sin(f*x + e) + a)*s
qrt(-a)*sqrt(sin(f*x + e)) + 289*a*cos(f*x + e) + (128*a*cos(f*x + e)^4 + 2
56*a*cos(f*x + e)^3 - 160*a*cos(f*x + e)^2 - 288*a*cos(f*x + e) + a)*sin(f*
```

```
x + e) + a)/(cos(f*x + e) + sin(f*x + e) + 1))/f, 1/2*sqrt(a)*arctan(1/4*(8
*cos(f*x + e)^2 + 8*sin(f*x + e) - 9)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*sqrt
(sin(f*x + e))/(2*a*cos(f*x + e)^3 + a*cos(f*x + e)*sin(f*x + e) - 2*a*cos(
f*x + e)))/f]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a \sin\left(fx+e\right)+a}}{\sqrt{\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/sqrt(sin(f*x + e)), x)
```

maple [B] time = 0.32, size = 320, normalized size = 8.65

$$\frac{\sqrt{-\frac{-1+\cos(fx+e)}{\sin(fx+e)}}\,\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,\left(\sqrt{\sin}\left(fx+e\right)\right)\left(\ln\left(-\frac{\sqrt{2}\,\sqrt{-\frac{-1+\cos(fx+e)}{\sin(fx+e)}}\,\sin(fx+e)+\sin(fx+e)-\cos(fx+e)+1}{\sqrt{2}\,\sqrt{-\frac{-1+\cos(fx+e)}{\sin(fx+e)}}\,\sin(fx+e)-\sin(fx+e)+\cos(fx+e)-1}\right)+4\right.}{2f\left(1-\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)^(1/2),x)
```

```
[Out] 1/2/f*(-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(a*(1+sin(f*x+e)))^(1/2)*sin(f*x+
e)^(1/2)*(ln(-(2^(1/2)*(-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)+sin(f
*x+e)-cos(f*x+e)+1)/(2^(1/2)*(-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)
-sin(f*x+e)+cos(f*x+e)-1))+4*arctan((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(
1/2)+1)+4*arctan((-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*2^(1/2)-1)+ln(-(2^(1/2
)*(-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)-sin(f*x+e)+cos(f*x+e)-1)/(
2^(1/2)*(-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)+sin(f*x+e)-cos(f*x+e
)+1)))*2^(1/2)/(1-cos(f*x+e)+sin(f*x+e))
```

maxima [B] time = 0.93, size = 210, normalized size = 5.68

$$-\frac{2\sqrt{2}\sqrt{a}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)^{\frac{3}{2}}-3\sqrt{2}\left(\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}}\right)\right)+\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\frac{\sin}{\cos(}}\right.\right.\right.}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(2*sqrt(2)*sqrt(a)*(sin(f*x + e)/(cos(f*x + e) + 1))^(3/2) - 3*sqrt(2)
*(sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(sin(f*x + e)/(cos(f*x + e) +
 1)))) + sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(sin(f*x + e)/(cos(f*
x + e) + 1)))))*sqrt(a) + 6*sqrt(2)*sqrt(a)*sqrt(sin(f*x + e)/(cos(f*x + e)
 + 1)) - 2*(3*sqrt(2)*sqrt(a)*sin(f*x + e)/(cos(f*x + e) + 1) + sqrt(2)*sqr
t(a)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)/sqrt(sin(f*x + e)/(cos(f*x + e) +
 1)))/f
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{a+a\,\sin\left(e+f\,x\right)}}{\sqrt{\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)/sin(e + f*x)^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(1/2)/sin(e + f*x)^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}{\sqrt{\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/sin(f*x+e)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))/sqrt(sin(e + f*x)), x)
```

3.86 $\int \frac{\sqrt{a-a\sin(e+fx)}}{\sqrt{-\sin(e+fx)}}\,dx$

Optimal. Leaf size=38

$$\frac{2\sqrt{a}\,\sin^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{a-a\sin(e+fx)}}\right)}{f}$$

```
[Out] 2*arcsin(cos(f*x+e)*a^(1/2)/(a-a*sin(f*x+e))^(1/2))*a^(1/2)/f
```

Rubi [A] time = 0.07, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2774, 216}

$$\frac{2\sqrt{a}\,\sin^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{a-a\sin(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Sin[e + f*x]]/Sqrt[-Sin[e + f*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcSin[(Sqrt[a]*Cos[e + f*x])/Sqrt[a - a*Sin[e + f*x]]])/f
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rubi steps

$$\int \frac{\sqrt{a-a\sin(e+fx)}}{\sqrt{-\sin(e+fx)}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, -\frac{a\cos(e+fx)}{\sqrt{a-a\sin(e+fx)}}\right)}{f}$$
$$= \frac{2\sqrt{a}\,\sin^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{a-a\sin(e+fx)}}\right)}{f}$$

Mathematica [C] time = 0.47, size = 119, normalized size = 3.13

$$-\frac{\sqrt{-1+e^{2i(e+fx)}}\,\sqrt{a-a\sin(e+fx)}\left(\tan^{-1}\left(\sqrt{-1+e^{2i(e+fx)}}\right)+i\tanh^{-1}\left(\frac{e^{i(e+fx)}}{\sqrt{-1+e^{2i(e+fx)}}}\right)\right)}{f\left(e^{i(e+fx)}-i\right)\sqrt{-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sin[e + f*x]]/Sqrt[-Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[-1 + E^((2*I)*(e + f*x))]*(ArcTan[Sqrt[-1 + E^((2*I)*(e + f*x))]] +
 I*ArcTanh[E^(I*(e + f*x))/Sqrt[-1 + E^((2*I)*(e + f*x))]])*Sqrt[a - a*Sin[
e + f*x]])/((-I + E^(I*(e + f*x)))*f*Sqrt[-Sin[e + f*x]]))
```

fricas [B] time = 0.62, size = 341, normalized size = 8.97

$$\left[\frac{\sqrt{-a}\,\log\left(\frac{128\,a\cos\left(fx+e\right)^5-128\,a\cos\left(fx+e\right)^4-416\,a\cos\left(fx+e\right)^3+128\,a\cos\left(fx+e\right)^2+8\left(16\,\cos\left(fx+e\right)^4-24\,\cos\left(fx+e\right)^3-66\,\cos\left(fx+e\right)^2-\left(16\,\text{c}\right.\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)/(-sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(-a)*log((128*a*cos(f*x + e)^5 - 128*a*cos(f*x + e)^4 - 416*a*cos(
f*x + e)^3 + 128*a*cos(f*x + e)^2 + 8*(16*cos(f*x + e)^4 - 24*cos(f*x + e)^
3 - 66*cos(f*x + e)^2 - (16*cos(f*x + e)^3 + 40*cos(f*x + e)^2 - 26*cos(f*x
 + e) - 51)*sin(f*x + e) + 25*cos(f*x + e) + 51)*sqrt(-a*sin(f*x + e) + a)*
sqrt(-a)*sqrt(-sin(f*x + e)) + 289*a*cos(f*x + e) - (128*a*cos(f*x + e)^4 +
 256*a*cos(f*x + e)^3 - 160*a*cos(f*x + e)^2 - 288*a*cos(f*x + e) + a)*sin(
f*x + e) + a)/(cos(f*x + e) - sin(f*x + e) + 1))/f, -1/2*sqrt(a)*arctan(1/4
```

```
*(8*cos(f*x + e)^2 - 8*sin(f*x + e) - 9)*sqrt(-a*sin(f*x + e) + a)*sqrt(a)*
sqrt(-sin(f*x + e))/(2*a*cos(f*x + e)^3 - a*cos(f*x + e)*sin(f*x + e) - 2*a
*cos(f*x + e)))/f]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)/(-sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)-4*sqrt(2*a)*sign(sin(1/2*(f*x+exp(1
))-1/4*pi))*atan((-sqrt(2)+2*(4*sqrt(2)-2*sqrt(-tan(1/2*(1/2*f*x+1/4*(2*exp
(1)-pi)))^4+6*tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^2-1))/(-2*tan(1/2*(1/2*f
*x+1/4*(2*exp(1)-pi)))^2+6))/sqrt(2))/sqrt(2)/f
```

maple [B] time = 0.22, size = 271, normalized size = 7.13

$$\frac{\sqrt{-\frac{-1+\cos(fx+e)}{\sin(fx+e)}}\,\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,\sin\left(fx+e\right)\left(\ln\left(-\frac{\sqrt{2}\,\sqrt{-\frac{-1+\cos(fx+e)}{\sin(fx+e)}}\,\sin(fx+e)+\sin(fx+e)-\cos(fx+e)+1}{\sqrt{2}\,\sqrt{-\frac{-1+\cos(fx+e)}{\sin(fx+e)}}\,\sin(fx+e)-\sin(fx+e)+\cos(fx+e)-1}\right)-\ln\left(-\right.\right.}{2f\sqrt{-\sin\left(fx+e\right)}\,\left(-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))^(1/2)/(-sin(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*(-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-a*(sin(f*x+e)-1))^(1/2)*sin(f*x
+e)*(ln(-(2^(1/2)*(-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)+sin(f*x+e)
-cos(f*x+e)+1)/(2^(1/2)*(-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)-sin(
f*x+e)+cos(f*x+e)-1))-ln(-(2^(1/2)*(-(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(
f*x+e)-sin(f*x+e)+cos(f*x+e)-1)/(2^(1/2)*(-(-1+cos(f*x+e))/sin(f*x+e))^(1/2
)*sin(f*x+e)+sin(f*x+e)-cos(f*x+e)+1)))/(-sin(f*x+e))^(1/2)/(-1+cos(f*x+e)+
sin(f*x+e))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a\sin\left(fx+e\right)+a}}{\sqrt{-\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)/(-sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a*sin(f*x + e) + a)/sqrt(-sin(f*x + e)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{a-a\,\sin\left(e+f\,x\right)}}{\sqrt{-\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*sin(e + f*x))^(1/2)/(-sin(e + f*x))^(1/2),x)
[Out] int((a - a*sin(e + f*x))^(1/2)/(-sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}}{\sqrt{-\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))**(1/2)/(-sin(f*x+e))**(1/2),x)
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1))/sqrt(-sin(e + f*x)), x)
```

3.87 $\int \frac{1}{\sqrt{\sin(x)}\sqrt{1+\sin(x)}}\,dx$

Optimal. Leaf size=17

$$-\sqrt{2}\sin^{-1}\left(\frac{\cos(x)}{\sin(x)+1}\right)$$

```
[Out] -arcsin(cos(x)/(1+sin(x)))*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2781, 216}

$$-\sqrt{2}\sin^{-1}\left(\frac{\cos(x)}{\sin(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sin[x]]*Sqrt[1 + Sin[x]]),x]
```

```
[Out] -(Sqrt[2]*ArcSin[Cos[x]/(1 + Sin[x])])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2781

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
^2], x], x, (b*Cos[e + f*x])/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\sin(x)}\sqrt{1+\sin(x)}}\,dx &= -\left(\sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \frac{\cos(x)}{1+\sin(x)}\right)\right)\\ &= -\sqrt{2}\sin^{-1}\left(\frac{\cos(x)}{1+\sin(x)}\right)\end{aligned}$$

Mathematica [C] time = 2.57, size = 123, normalized size = 7.24

$$\frac{2\sqrt{\sin(x)}\sec^2\left(\frac{x}{4}\right)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(F\left(\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{4}\right)}}\right)\middle|-1\right)-\Pi\left(1-\sqrt{2};\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{4}\right)}}\right)\middle|-1\right)-\Pi\left(1+\sqrt{\right.\right.}{\sqrt{\sin(x)+1}\tan^{\frac{3}{2}}\left(\frac{x}{4}\right)\sqrt{1-\cot^2\left(\frac{x}{4}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sin[x]]*Sqrt[1 + Sin[x]]),x]
```

```
[Out] (2*(EllipticF[ArcSin[1/Sqrt[Tan[x/4]]], -1] - EllipticPi[1 - Sqrt[2], ArcSi
n[1/Sqrt[Tan[x/4]]], -1] - EllipticPi[1 + Sqrt[2], ArcSin[1/Sqrt[Tan[x/4]]]
, -1])*Sec[x/4]^2*(Cos[x/2] + Sin[x/2])*Sqrt[Sin[x]])/(Sqrt[1 - Cot[x/4]^2]
*Sqrt[1 + Sin[x]]*Tan[x/4]^(3/2))
```

fricas [A] time = 0.50, size = 28, normalized size = 1.65

$$2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sin(x)+1}\sqrt{\sin(x)}}{\cos(x)+\sin(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^(1/2)/(1+sin(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*arctan(sqrt(2)*sqrt(sin(x) + 1)*sqrt(sin(x))/(cos(x) + sin(x) + 1
))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sin(x)+1}\sqrt{\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^(1/2)/(1+sin(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(sin(x) + 1)*sqrt(sin(x))), x)
```

maple [B] time = 0.13, size = 52, normalized size = 3.06

$$-\frac{2\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,(1-\cos(x)+\sin(x))\left(\sqrt{\sin}\,(x)\right)\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\right)}{\sqrt{1+\sin(x)}\,(-1+\cos(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x)^(1/2)/(1+sin(x))^(1/2),x)
```

```
[Out] -2*(-(-1+cos(x))/sin(x))^(1/2)*(1-cos(x)+sin(x))*sin(x)^(1/2)*arctan((-(-1+
cos(x))/sin(x))^(1/2))/(1+sin(x))^(1/2)/(-1+cos(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sin(x)+1}\,\sqrt{\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^(1/2)/(1+sin(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(sin(x) + 1)*sqrt(sin(x))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sqrt{\sin(x)}\,\sqrt{\sin(x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^(1/2)*(sin(x) + 1)^(1/2)),x)
```

```
[Out] int(1/(sin(x)^(1/2)*(sin(x) + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sin{\left(x \right)}+1}\,\sqrt{\sin{\left(x \right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)**(1/2)/(1+sin(x))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(sin(x) + 1)*sqrt(sin(x))), x)
```

3.88 $\int \frac{1}{\sqrt{\sin(x)}\sqrt{a+a\sin(x)}}\,dx$

Optimal. Leaf size=42

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\cos(x)}{\sqrt{2}\sqrt{\sin(x)}\sqrt{a\sin(x)+a}}\right)}{\sqrt{a}}$$

```
[Out] -arctan(1/2*cos(x)*a^(1/2)*2^(1/2)/sin(x)^(1/2)/(a+a*sin(x))^(1/2))*2^(1/2)
/a^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2782, 205}

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\cos(x)}{\sqrt{2}\sqrt{\sin(x)}\sqrt{a\sin(x)+a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sin[x]]*Sqrt[a + a*Sin[x]]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Cos[x])/(Sqrt[2]*Sqrt[Sin[x]]*Sqrt[a + a*Sin[x]]
)]])/Sqrt[a])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sin(x)}\sqrt{a+a\sin(x)}}\,dx = -\left((2a)\,\mathrm{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x, \frac{a\cos(x)}{\sqrt{\sin(x)}\sqrt{a+a\sin(x)}}\right)\right)$$
$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\cos(x)}{\sqrt{2}\sqrt{\sin(x)}\sqrt{a+a\sin(x)}}\right)}{\sqrt{a}}$$

Mathematica [C] time = 0.09, size = 125, normalized size = 2.98

$$\frac{2\sqrt{\sin(x)}\sec^2\left(\frac{x}{4}\right)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\left(F\left(\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{4}\right)}}\right)\middle|-1\right)-\Pi\left(1-\sqrt{2};\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{4}\right)}}\right)\middle|-1\right)-\Pi\left(1+\sqrt{2}\right.\right.}{\tan^{\frac{3}{2}}\left(\frac{x}{4}\right)\sqrt{1-\cot^2\left(\frac{x}{4}\right)}\sqrt{a(\sin(x)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sin[x]]*Sqrt[a + a*Sin[x]]),x]
```

```
[Out] (2*(EllipticF[ArcSin[1/Sqrt[Tan[x/4]]], -1] - EllipticPi[1 - Sqrt[2], ArcSi
n[1/Sqrt[Tan[x/4]]], -1] - EllipticPi[1 + Sqrt[2], ArcSin[1/Sqrt[Tan[x/4]]]
, -1])*Sec[x/4]^2*(Cos[x/2] + Sin[x/2])*Sqrt[Sin[x]])/(Sqrt[1 - Cot[x/4]^2]
*Sqrt[a*(1 + Sin[x])]*Tan[x/4]^(3/2))
```

fricas [A] time = 0.60, size = 163, normalized size = 3.88

$$\left[\frac{1}{4}\sqrt{2}\sqrt{-\frac{1}{a}}\log\left(\frac{17\cos(x)^3-4\sqrt{2}\left(3\cos(x)^2+(3\cos(x)+4)\sin(x)-\cos(x)-4\right)\sqrt{a\sin(x)+a}\sqrt{-\frac{1}{a}}\sqrt{\sin(x)}}{\cos(x)^3+3\cos(x)^2+\left(\cos(x)^2-2\cos(x)-4\right)s}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^(1/2)/(a+a*sin(x))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(2)*sqrt(-1/a)*log((17*cos(x)^3 - 4*sqrt(2)*(3*cos(x)^2 + (3*cos(x
) + 4)*sin(x) - cos(x) - 4)*sqrt(a*sin(x) + a)*sqrt(-1/a)*sqrt(sin(x)) + 3*
cos(x)^2 + (17*cos(x)^2 + 14*cos(x) - 4)*sin(x) - 18*cos(x) - 4)/(cos(x)^3
+ 3*cos(x)^2 + (cos(x)^2 - 2*cos(x) - 4)*sin(x) - 2*cos(x) - 4)), 1/2*sqrt(
2)*arctan(1/4*sqrt(2)*sqrt(a*sin(x) + a)*(3*sin(x) - 1)/(sqrt(a)*cos(x)*sqr
t(sin(x))))/sqrt(a)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sin(x)+a}\sqrt{\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^(1/2)/(a+a*sin(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*sin(x) + a)*sqrt(sin(x))), x)
```

maple [A] time = 0.16, size = 54, normalized size = 1.29

$$\frac{2\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,(-1+\cos(x)-\sin(x))\left(\sqrt{\sin}\,(x)\right)\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\right)}{\sqrt{a\,(1+\sin(x))}\,\,(-1+\cos(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x)^(1/2)/(a+a*sin(x))^(1/2),x)
```

```
[Out] 2*(-(-1+cos(x))/sin(x))^(1/2)*(-1+cos(x)-sin(x))*sin(x)^(1/2)*arctan((-(-1+
cos(x))/sin(x))^(1/2))/(a*(1+sin(x)))^(1/2)/(-1+cos(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sin(x)+a}\,\sqrt{\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^(1/2)/(a+a*sin(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*sin(x) + a)*sqrt(sin(x))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\sin(x)}\,\sqrt{a+a\,\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^(1/2)*(a + a*sin(x))^(1/2)),x)
```

```
[Out] int(1/(sin(x)^(1/2)*(a + a*sin(x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\,(\sin(x)+1)}\,\sqrt{\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)**(1/2)/(a+a*sin(x))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(x) + 1))*sqrt(sin(x))), x)
```

3.89 $\int \frac{1}{\sqrt{1-\sin(x)}\,\sqrt{\sin(x)}}\,dx$

Optimal. Leaf size=31

$$\sqrt{2}\,\tanh^{-1}\left(\frac{\cos(x)}{\sqrt{2}\,\sqrt{1-\sin(x)}\,\sqrt{\sin(x)}}\right)$$

```
[Out] arctanh(1/2*cos(x)*2^(1/2)/(1-sin(x))^(1/2)/sin(x)^(1/2))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2782, 206}

$$\sqrt{2}\,\tanh^{-1}\left(\frac{\cos(x)}{\sqrt{2}\,\sqrt{1-\sin(x)}\,\sqrt{\sin(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - Sin[x]]*Sqrt[Sin[x]]),x]
```

```
[Out] Sqrt[2]*ArcTanh[Cos[x]/(Sqrt[2]*Sqrt[1 - Sin[x]]*Sqrt[Sin[x]])]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-\sin(x)}\,\sqrt{\sin(x)}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, -\frac{\cos(x)}{\sqrt{1-\sin(x)}\,\sqrt{\sin(x)}}\right)\right)\\ &= \sqrt{2}\,\tanh^{-1}\left(\frac{\cos(x)}{\sqrt{2}\,\sqrt{1-\sin(x)}\,\sqrt{\sin(x)}}\right)\end{aligned}$$

Mathematica [C] time = 2.52, size = 125, normalized size = 4.03

$$\frac{2\sin(x)\sec^2\left(\frac{x}{4}\right)\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)\left(F\left(\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{4}\right)}}\right)\middle|-1\right)-\Pi\left(-1-\sqrt{2};\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{4}\right)}}\right)\middle|-1\right)-\Pi\left(-1+\cdots\right.\right.}{\sqrt{-((\sin(x)-1)\sin(x))}\tan^{\frac{3}{2}}\left(\frac{x}{4}\right)\sqrt{1-\cot^2\left(\frac{x}{4}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - Sin[x]]*Sqrt[Sin[x]]),x]
```

```
[Out] (2*(EllipticF[ArcSin[1/Sqrt[Tan[x/4]]], -1] - EllipticPi[-1 - Sqrt[2], ArcS
in[1/Sqrt[Tan[x/4]]], -1] - EllipticPi[-1 + Sqrt[2], ArcSin[1/Sqrt[Tan[x/4]
]], -1])*Sec[x/4]^2*(Cos[x/2] - Sin[x/2])*Sin[x])/(Sqrt[1 - Cot[x/4]^2]*Sqr
t[-((-1 + Sin[x])*Sin[x])]*Tan[x/4]^(3/2))
```

fricas [A] time = 0.49, size = 31, normalized size = 1.00

$$\sqrt{2}\log\left(\frac{\sqrt{2}\sqrt{-\sin(x)+1}\sqrt{\sin(x)}+\cos(x)}{\sin(x)-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x))^(1/2)/sin(x)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*log((sqrt(2)*sqrt(-sin(x) + 1)*sqrt(sin(x)) + cos(x))/(sin(x) - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-\sin(x)+1}\sqrt{\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x))^(1/2)/sin(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-sin(x) + 1)*sqrt(sin(x))), x)
```

maple [B] time = 0.16, size = 52, normalized size = 1.68

$$-\frac{2\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,(-1+\cos(x)+\sin(x))\left(\sqrt{\sin}\,(x)\right)\operatorname{arctanh}\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\right)}{\sqrt{1-\sin(x)}\,(-1+\cos(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sin(x))^(1/2)/sin(x)^(1/2),x)
```

```
[Out] -2*(-(-1+cos(x))/sin(x))^(1/2)*(-1+cos(x)+sin(x))*sin(x)^(1/2)*arctanh((-(-
1+cos(x))/sin(x))^(1/2))/(1-sin(x))^(1/2)/(-1+cos(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\sin(x)+1}\,\sqrt{\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x))^(1/2)/sin(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-sin(x) + 1)*sqrt(sin(x))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\sin(x)}\,\sqrt{1-\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^(1/2)*(1 - sin(x))^(1/2)),x)
```

```
[Out] int(1/(sin(x)^(1/2)*(1 - sin(x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{1-\sin{\left(x \right)}}\,\sqrt{\sin{\left(x \right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x))**(1/2)/sin(x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(1 - sin(x))*sqrt(sin(x))), x)
```

3.90 $\int \frac{1}{\sqrt{\sin(x)}\sqrt{a-a\sin(x)}}\,dx$

Optimal. Leaf size=42

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(x)}{\sqrt{2}\sqrt{\sin(x)}\sqrt{a-a\sin(x)}}\right)}{\sqrt{a}}$$

```
[Out] arctanh(1/2*cos(x)*a^(1/2)*2^(1/2)/sin(x)^(1/2)/(a-a*sin(x))^(1/2))*2^(1/2)
/a^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2782, 208}

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(x)}{\sqrt{2}\sqrt{\sin(x)}\sqrt{a-a\sin(x)}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sin[x]]*Sqrt[a - a*Sin[x]]),x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[x])/(Sqrt[2]*Sqrt[Sin[x]]*Sqrt[a - a*Sin[x]])
])/Sqrt[a]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sin(x)}\sqrt{a-a\sin(x)}}\,dx = -\left((2a)\,\text{Subst}\left(\int \frac{1}{2a^2-ax^2}\,dx, x, -\frac{a\cos(x)}{\sqrt{\sin(x)}\sqrt{a-a\sin(x)}}\right)\right)$$
$$= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(x)}{\sqrt{2}\sqrt{\sin(x)}\sqrt{a-a\sin(x)}}\right)}{\sqrt{a}}$$

Mathematica [C] time = 0.10, size = 128, normalized size = 3.05

$$\frac{2\sqrt{\sin(x)}\sec^2\left(\frac{x}{4}\right)\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)\left(F\left(\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{4}\right)}}\right)\middle|-1\right)-\Pi\left(-1-\sqrt{2};\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{4}\right)}}\right)\middle|-1\right)-\Pi\left(-1+\right.\right.}{\tan^{\frac{3}{2}}\left(\frac{x}{4}\right)\sqrt{1-\cot^2\left(\frac{x}{4}\right)}\sqrt{a-a\sin(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sin[x]]*Sqrt[a - a*Sin[x]]),x]
```

```
[Out] (2*(EllipticF[ArcSin[1/Sqrt[Tan[x/4]]], -1] - EllipticPi[-1 - Sqrt[2], ArcS
in[1/Sqrt[Tan[x/4]]], -1] - EllipticPi[-1 + Sqrt[2], ArcSin[1/Sqrt[Tan[x/4]
]], -1])*Sec[x/4]^2*(Cos[x/2] - Sin[x/2])*Sqrt[Sin[x]])/(Sqrt[1 - Cot[x/4]^
2]*Sqrt[a - a*Sin[x]]*Tan[x/4]^(3/2))
```

fricas [A] time = 0.62, size = 168, normalized size = 4.00

$$\left[\frac{\sqrt{2}\log\left(\frac{17\cos(x)^3+3\cos(x)^2+\frac{4\sqrt{2}\left(3\cos(x)^2-(3\cos(x)+4)\sin(x)-\cos(x)-4\right)\sqrt{-a\sin(x)+a}\sqrt{\sin(x)}}{\sqrt{a}}-\left(17\cos(x)^2+14\cos(x)-4\right)\sin(x)-18\cos(x)-4}{\cos(x)^3+3\cos(x)^2-\left(\cos(x)^2-2\cos(x)-4\right)\sin(x)-2\cos(x)-4}\right)}{4\sqrt{a}}, -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^(1/2)/(a-a*sin(x))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(2)*log((17*cos(x)^3 + 3*cos(x)^2 + 4*sqrt(2)*(3*cos(x)^2 - (3*cos
(x) + 4)*sin(x) - cos(x) - 4)*sqrt(-a*sin(x) + a)*sqrt(sin(x))/sqrt(a) - (1
7*cos(x)^2 + 14*cos(x) - 4)*sin(x) - 18*cos(x) - 4)/(cos(x)^3 + 3*cos(x)^2
- (cos(x)^2 - 2*cos(x) - 4)*sin(x) - 2*cos(x) - 4))/sqrt(a), -1/2*sqrt(2)*s
qrt(-1/a)*arctan(1/4*sqrt(2)*sqrt(-a*sin(x) + a)*sqrt(-1/a)*(3*sin(x) + 1)/
(cos(x)*sqrt(sin(x))))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a\sin(x) + a}\,\sqrt{\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^(1/2)/(a-a*sin(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-a*sin(x) + a)*sqrt(sin(x))), x)
```

maple [A] time = 0.15, size = 53, normalized size = 1.26

$$-\frac{2\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,(-1+\cos(x)+\sin(x))\left(\sqrt{\sin}\,(x)\right)\operatorname{arctanh}\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\right)}{\sqrt{-a\,(-1+\sin(x))}\,(-1+\cos(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x)^(1/2)/(a-a*sin(x))^(1/2),x)
```

```
[Out] -2*(-(-1+cos(x))/sin(x))^(1/2)*(-1+cos(x)+sin(x))*sin(x)^(1/2)*arctanh((-(-
1+cos(x))/sin(x))^(1/2))/(-a*(-1+sin(x)))^(1/2)/(-1+cos(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a\sin(x) + a}\,\sqrt{\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^(1/2)/(a-a*sin(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-a*sin(x) + a)*sqrt(sin(x))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\sin(x)}\,\sqrt{a - a\,\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^(1/2)*(a - a*sin(x))^(1/2)),x)
```

```
[Out] int(1/(sin(x)^(1/2)*(a - a*sin(x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a\left(\sin\left(x\right)-1\right)}\,\sqrt{\sin\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)**(1/2)/(a-a*sin(x))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-a*(sin(x) - 1))*sqrt(sin(x))), x)
```

3.91 $\int \frac{\sqrt[3]{\sin(c+dx)}}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=184

$$\frac{4\sqrt[3]{\sin(c+dx)}\,\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2(c+dx)\right)}{9a^2d\sqrt{\cos^2(c+dx)}}-\frac{\sin^{\frac{4}{3}}(c+dx)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sin^2(c+dx)\right)}{36a^2d\sqrt{\cos^2(c+dx)}}-\frac{\sqrt[3]{\text{si}}}{9a}$$

```
[Out] -1/9*cos(d*x+c)*sin(d*x+c)^(1/3)/a^2/d/(1+sin(d*x+c))-1/3*cos(d*x+c)*sin(d*
x+c)^(1/3)/d/(a+a*sin(d*x+c))^2+4/9*cos(d*x+c)*hypergeom([1/6, 1/2],[7/6],s
in(d*x+c)^2)*sin(d*x+c)^(1/3)/a^2/d/(cos(d*x+c)^2)^(1/2)-1/36*cos(d*x+c)*hy
pergeom([1/2, 2/3],[5/3],sin(d*x+c)^2)*sin(d*x+c)^(4/3)/a^2/d/(cos(d*x+c)^2
)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2764, 2978, 2748, 2643}

$$-\frac{\sin^{\frac{4}{3}}(c+dx)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sin^2(c+dx)\right)}{36a^2d\sqrt{\cos^2(c+dx)}}+\frac{4\sqrt[3]{\sin(c+dx)}\,\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2(c+dx)\right)}{9a^2d\sqrt{\cos^2(c+dx)}}-\frac{\sqrt[3]{\text{s}}}{9}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^(1/3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (4*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[c + d*x]^2]*Sin[c + d*
x]^(1/3))/(9*a^2*d*Sqrt[Cos[c + d*x]^2]) - (Cos[c + d*x]*Hypergeometric2F1[
1/2, 2/3, 5/3, Sin[c + d*x]^2]*Sin[c + d*x]^(4/3))/(36*a^2*d*Sqrt[Cos[c + d
*x]^2]) - (Cos[c + d*x]*Sin[c + d*x]^(1/3))/(9*a^2*d*(1 + Sin[c + d*x])) -
(Cos[c + d*x]*Sin[c + d*x]^(1/3))/(3*d*(a + a*Sin[c + d*x])^2)
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2764

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m
*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*
(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && L
tQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c,
 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{\sin(c+dx)}}{(a+a\sin(c+dx))^2}\,dx = -\frac{\cos(c+dx)\sqrt[3]{\sin(c+dx)}}{3d(a+a\sin(c+dx))^2} + \frac{\int \frac{\frac{a}{3}+\frac{2}{3}a\sin(c+dx)}{\sin^{\frac{2}{3}}(c+dx)(a+a\sin(c+dx))}\,dx}{3a^2}$$

$$= -\frac{\cos(c+dx)\sqrt[3]{\sin(c+dx)}}{9a^2d(1+\sin(c+dx))} - \frac{\cos(c+dx)\sqrt[3]{\sin(c+dx)}}{3d(a+a\sin(c+dx))^2} + \frac{\int \frac{\frac{4a^2}{9}-\frac{1}{9}a^2\sin(c+dx)}{\sin^{\frac{2}{3}}(c+dx)}\,dx}{3a^4}$$

$$= -\frac{\cos(c+dx)\sqrt[3]{\sin(c+dx)}}{9a^2d(1+\sin(c+dx))} - \frac{\cos(c+dx)\sqrt[3]{\sin(c+dx)}}{3d(a+a\sin(c+dx))^2} - \frac{\int \sqrt[3]{\sin(c+dx)}\,dx}{27a^2} + \frac{4\int \cdots}{\cdots}$$

$$= \frac{4\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2(c+dx)\right)\sqrt[3]{\sin(c+dx)}}{9a^2d\sqrt{\cos^2(c+dx)}} - \frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sin^2\cdots\right)\cdots}{36a^2d\sqrt{\cos^2(\cdots}}$$

Mathematica [A] time = 0.40, size = 121, normalized size = 0.66

$$\frac{\sqrt[3]{\sin(c+dx)}\sec^3(c+dx)\left(80\cos^2(c+dx)^{3/2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2(c+dx)\right)+27\sin(c+dx)\cos^2(c+dx)^{3/2}\,{}_2F_1\left(\frac{2}{3},\right.\right.}{180a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^(1/3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^3*Sin[c + d*x]^(1/3)*(80*(Cos[c + d*x]^2)^(3/2)*Hypergeometri
c2F1[1/6, 1/2, 7/6, Sin[c + d*x]^2] + 27*(Cos[c + d*x]^2)^(3/2)*Hypergeomet
ric2F1[2/3, 5/2, 5/3, Sin[c + d*x]^2]*Sin[c + d*x] + 4*(-25 + 5*Cos[2*(c +
d*x)] + 27*Sin[c + d*x])))/(180*a^2*d)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sin(dx+c)^{\frac{1}{3}}}{a^2\cos(dx+c)^2-2a^2\sin(dx+c)-2a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(1/3)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-sin(d*x + c)^(1/3)/(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x + c) - 2*a
^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin(dx+c)^{\frac{1}{3}}}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(1/3)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^(1/3)/(a*sin(d*x + c) + a)^2, x)
```

maple [F] time = 0.78, size = 0, normalized size = 0.00

$$\int\frac{\sin^{\frac{1}{3}}(dx+c)}{(a+a\sin(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^(1/3)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] int(sin(d*x+c)^(1/3)/(a+a*sin(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^{\frac{1}{3}}}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(1/3)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^(1/3)/(a*sin(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(c+d\,x)^{1/3}}{(a+a\,\sin(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^(1/3)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] int(sin(c + d*x)^(1/3)/(a + a*sin(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt[3]{\sin(c+dx)}}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**(1/3)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)**(1/3)/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**
2
```

3.92 $\int \sin^3(c+dx)(a+a\sin(c+dx))^{2/3}\,dx$

Optimal. Leaf size=161

$$-\frac{67\cos(c+dx)(a\sin(c+dx)+a)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{55\;2^{5/6}d(\sin(c+dx)+1)^{7/6}}-\frac{3\sin^2(c+dx)\cos(c+dx)(a\sin(c+dx)}{11d}$$

```
[Out] -63/220*cos(d*x+c)*(a+a*sin(d*x+c))^(2/3)/d-3/11*cos(d*x+c)*sin(d*x+c)^2*(a
+a*sin(d*x+c))^(2/3)/d-67/110*cos(d*x+c)*hypergeom([-1/6, 1/2],[3/2],1/2-1/
2*sin(d*x+c))*(a+a*sin(d*x+c))^(2/3)*2^(1/6)/d/(1+sin(d*x+c))^(7/6)-3/44*co
s(d*x+c)*(a+a*sin(d*x+c))^(5/3)/a/d
```

Rubi [A] time = 0.28, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2783, 2968, 3023, 2751, 2652, 2651}

$$-\frac{67\cos(c+dx)(a\sin(c+dx)+a)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{55\;2^{5/6}d(\sin(c+dx)+1)^{7/6}}-\frac{3\sin^2(c+dx)\cos(c+dx)(a\sin(c+dx)}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] (-63*Cos[c + d*x]*(a + a*Sin[c + d*x])^(2/3))/(220*d) - (3*Cos[c + d*x]*Sin
[c + d*x]^2*(a + a*Sin[c + d*x])^(2/3))/(11*d) - (67*Cos[c + d*x]*Hypergeom
etric2F1[-1/6, 1/2, 3/2, (1 - Sin[c + d*x])/2]*(a + a*Sin[c + d*x])^(2/3))/
(55*2^(5/6)*d*(1 + Sin[c + d*x])^(7/6)) - (3*Cos[c + d*x]*(a + a*Sin[c + d*
x])^(5/3))/(44*a*d)
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2783

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + n)), x] + Dist[1/(b*(m + n)), Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n - 2)*Simp[d*(a*c*m + b*d*(n -
 1)) + b*c^2*(m + n) + d*(a*d*m + b*c*(m + 2*n - 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^{2/3}\,dx = -\frac{3\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^{2/3}}{11d} + \frac{3\int \sin(c+dx)\Big(2 \cdots}{\cdots}$$

$$= -\frac{3\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^{2/3}}{11d} + \frac{3\int (a+a\sin(c+ \cdots}{\cdots}$$

$$= -\frac{3\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^{2/3}}{11d} - \frac{3\cos(c+dx)(a+ \cdots}{44a \cdots}$$

$$= -\frac{63\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{220d} - \frac{3\cos(c+dx)\sin^2(c+dx)(a+ \cdots}{11d}$$

$$= -\frac{63\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{220d} - \frac{3\cos(c+dx)\sin^2(c+dx)(a+ \cdots}{11d}$$

$$= -\frac{63\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{220d} - \frac{3\cos(c+dx)\sin^2(c+dx)(a+ \cdots}{11d}$$

Mathematica [A] time = 0.69, size = 160, normalized size = 0.99

$$\frac{3(a(\sin(c+dx)+1))^{2/3}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(67\sqrt{2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+2dx+\pi)\right)\right)+\sqrt{1-\cdots}\right.}{440d\sqrt{1-\sin(c+dx)}\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\cdots\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] (3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a*(1 + Sin[c + d*x]))^(2/3)*(67*S
qrt[2]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*c + Pi + 2*d*x)/4]^2] + Sqrt
[1 - Sin[c + d*x]]*(-144 + 25*Cos[2*(c + d*x)] - 92*Sin[c + d*x] + 10*Sin[3
*(c + d*x)])))/(440*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*Sqrt[1 - Sin[c
+ d*x]])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos\left(dx+c\right)^2-1\right)\left(a\sin\left(dx+c\right)+a\right)^{\frac{2}{3}}\sin\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*(a*sin(d*x + c) + a)^(2/3)*sin(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{2}{3}}\sin(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^(2/3)*sin(d*x + c)^3, x)
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \left(\sin^3(dx+c)\right)(a+a\sin(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+a*sin(d*x+c))^(2/3),x)
[Out] int(sin(d*x+c)^3*(a+a*sin(d*x+c))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{2}{3}}\sin(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^(2/3)*sin(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(c+d\,x)^3\,(a+a\,\sin(c+d\,x))^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + a*sin(c + d*x))^(2/3),x)
[Out] int(sin(c + d*x)^3*(a + a*sin(c + d*x))^(2/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+a*sin(d*x+c))**(2/3),x)
[Out] Timed out
```

3.93 $\int \sin^2(c+dx)(a+a\sin(c+dx))^{2/3}\,dx$

Optimal. Leaf size=126

$$-\frac{19\cos(c+dx)(a\sin(c+dx)+a)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{10\,2^{5/6}d(\sin(c+dx)+1)^{7/6}}-\frac{3\cos(c+dx)(a\sin(c+dx)+a)^{5/3}}{8ad}+\frac{9\,\mathrm{c}}{}$$

```
[Out] 9/40*cos(d*x+c)*(a+a*sin(d*x+c))^(2/3)/d-19/20*cos(d*x+c)*hypergeom([-1/6,
1/2],[3/2],1/2-1/2*sin(d*x+c))*(a+a*sin(d*x+c))^(2/3)*2^(1/6)/d/(1+sin(d*x+
c))^(7/6)-3/8*cos(d*x+c)*(a+a*sin(d*x+c))^(5/3)/a/d
```

Rubi [A] time = 0.15, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2759, 2751, 2652, 2651}

$$-\frac{19\cos(c+dx)(a\sin(c+dx)+a)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{10\,2^{5/6}d(\sin(c+dx)+1)^{7/6}}-\frac{3\cos(c+dx)(a\sin(c+dx)+a)^{5/3}}{8ad}+\frac{9\,\mathrm{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] (9*Cos[c + d*x]*(a + a*Sin[c + d*x])^(2/3))/(40*d) - (19*Cos[c + d*x]*Hyper
geometric2F1[-1/6, 1/2, 3/2, (1 - Sin[c + d*x])/2]*(a + a*Sin[c + d*x])^(2/
3))/(10*2^(5/6)*d*(1 + Sin[c + d*x])^(7/6)) - (3*Cos[c + d*x]*(a + a*Sin[c
+ d*x])^(5/3))/(8*a*d)
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \sin^2(c+dx)(a+a\sin(c+dx))^{2/3}\,dx &= -\frac{3\cos(c+dx)(a+a\sin(c+dx))^{5/3}}{8ad}+\frac{3\int\left(\frac{5a}{3}-a\sin(c+dx)\right)(a+a\,\mathrm{si}}{8a}\\ &= \frac{9\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{40d}-\frac{3\cos(c+dx)(a+a\sin(c+dx))^{5/3}}{8ad}\\ &= \frac{9\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{40d}-\frac{3\cos(c+dx)(a+a\sin(c+dx))^{5/3}}{8ad}\\ &= \frac{9\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{40d}-\frac{19\cos(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\right.}{10\,2^{5/6}d(1+}\end{aligned}$$

Mathematica [A] time = 0.44, size = 151, normalized size = 1.20

$$\frac{3(a(\sin(c+dx)+1))^{2/3}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(19\sqrt{2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+2dx+\pi)\right)\right)+\sqrt{1-\mathrm{s}}\right.}{80d\sqrt{1-\sin(c+dx)}\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] (3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a*(1 + Sin[c + d*x]))^(2/3)*(19*S
qrt[2]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*c + Pi + 2*d*x)/4]^2] + Sqrt
[1 - Sin[c + d*x]]*(5*Cos[2*(c + d*x)] - 14*(2 + Sin[c + d*x]))))/(80*d*(Co
s[(c + d*x)/2] + Sin[(c + d*x)/2])*Sqrt[1 - Sin[c + d*x]])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(\cos(dx+c)^2-1\right)(a\sin(dx+c)+a)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*(a*sin(d*x + c) + a)^(2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sin (dx + c) + a)^{\frac{2}{3}} \sin (dx + c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(2/3)*sin(d*x + c)^2, x)
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \left(\sin^2 (dx + c)\right) (a + a \sin (dx + c))^{\frac{2}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^(2/3),x)
```

```
[Out] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sin (dx + c) + a)^{\frac{2}{3}} \sin (dx + c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(2/3)*sin(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin (c + d\,x)^2\,(a + a\,\sin (c + d\,x))^{2/3} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + a*sin(c + d*x))^(2/3),x)
```

```
[Out] int(sin(c + d*x)^2*(a + a*sin(c + d*x))^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^{\frac{2}{3}}\sin^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+a*sin(d*x+c))**(2/3),x)
[Out] Integral((a*(sin(c + d*x) + 1))**(2/3)*sin(c + d*x)**2, x)
```

3.94 $\int \sin(c+dx)(a+a\sin(c+dx))^{2/3}\,dx$

Optimal. Leaf size=96

$$-\frac{4\sqrt[6]{2}\cos(c+dx)(a\sin(c+dx)+a)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5d(\sin(c+dx)+1)^{7/6}}-\frac{3\cos(c+dx)(a\sin(c+dx)+a)^{2/3}}{5d}$$

```
[Out] -3/5*cos(d*x+c)*(a+a*sin(d*x+c))^(2/3)/d-4/5*cos(d*x+c)*hypergeom([-1/6, 1/
2],[3/2],1/2-1/2*sin(d*x+c))*(a+a*sin(d*x+c))^(2/3)*2^(1/6)/d/(1+sin(d*x+c)
)^(7/6)
```

Rubi [A] time = 0.07, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2751, 2652, 2651}

$$-\frac{4\sqrt[6]{2}\cos(c+dx)(a\sin(c+dx)+a)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5d(\sin(c+dx)+1)^{7/6}}-\frac{3\cos(c+dx)(a\sin(c+dx)+a)^{2/3}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] (-3*Cos[c + d*x]*(a + a*Sin[c + d*x])^(2/3))/(5*d) - (4*2^(1/6)*Cos[c + d*x
]*Hypergeometric2F1[-1/6, 1/2, 3/2, (1 - Sin[c + d*x])/2]*(a + a*Sin[c + d*
x])^(2/3))/(5*d*(1 + Sin[c + d*x])^(7/6))
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \sin(c+dx)(a+a\sin(c+dx))^{2/3}\,dx = -\frac{3\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{5d} + \frac{2}{5}\int (a+a\sin(c+dx))^{2/3}\,dx$$
$$= -\frac{3\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{5d} + \frac{\left(2(a+a\sin(c+dx))^{2/3}\right)\int (1+\text{si}}{5(1+\sin(c+dx))^{2}}$$
$$= -\frac{3\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{5d} - \frac{4\sqrt[6]{2}\cos(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(\right.}{5d(1+\text{s}}$$

Mathematica [A] time = 0.23, size = 138, normalized size = 1.44

$$-\frac{3(a(\sin(c+dx)+1))^{2/3}\left(\sqrt{1-\sin(c+dx)}(\sin(c+dx)+2)-\sqrt{2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+2dx+\pi)\right)\right)\right)\left(\cos\left(\frac{1}{2}(\right.\right.}{5d\sqrt{1-\sin(c+dx)}\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] (-3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a*(1 + Sin[c + d*x]))^(2/3)*(-(S
qrt[2]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*c + Pi + 2*d*x)/4]^2]) + Sqr
t[1 - Sin[c + d*x]]*(2 + Sin[c + d*x])))/(5*d*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])*Sqrt[1 - Sin[c + d*x]])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sin(dx+c)+a)^{\frac{2}{3}}\sin(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^(2/3)*sin(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{2}{3}}\sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^(2/3)*sin(d*x + c), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \sin \left(dx+c\right) \left(a+a\sin \left(dx+c\right)\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+a*sin(d*x+c))^(2/3),x)
[Out] int(sin(d*x+c)*(a+a*sin(d*x+c))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin \left(dx+c\right)+a\right)^{\frac{2}{3}} \sin \left(dx+c\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^(2/3)*sin(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(c+d\,x\right)\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{2/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + a*sin(c + d*x))^(2/3),x)
[Out] int(sin(c + d*x)*(a + a*sin(c + d*x))^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\sin{\left(c + dx \right)} + 1\right)\right)^{\frac{2}{3}} \sin{\left(c + dx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))**(2/3),x)
[Out] Integral((a*(sin(c + d*x) + 1))**(2/3)*sin(c + d*x), x)
```

3.95 $\int (a+a\sin(c+dx))^{2/3}\,dx$

Optimal. Leaf size=66

$$-\frac{2\sqrt[6]{2}\cos(c+dx)(a\sin(c+dx)+a)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{7/6}}$$

```
[Out] -2*cos(d*x+c)*hypergeom([-1/6, 1/2],[3/2],1/2-1/2*sin(d*x+c))*(a+a*sin(d*x+
c))^(2/3)*2^(1/6)/d/(1+sin(d*x+c))^(7/6)
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2652, 2651}

$$-\frac{2\sqrt[6]{2}\cos(c+dx)(a\sin(c+dx)+a)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] (-2*2^(1/6)*Cos[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 3/2, (1 - Sin[c + d*x
])/2]*(a + a*Sin[c + d*x])^(2/3))/(d*(1 + Sin[c + d*x])^(7/6))
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+a\sin(c+dx))^{2/3}\,dx = \frac{(a+a\sin(c+dx))^{2/3}\int (1+\sin(c+dx))^{2/3}\,dx}{(1+\sin(c+dx))^{2/3}}$$
$$= -\frac{2\sqrt[6]{2}\cos(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)(a+a\sin(c+dx))^{2/3}}{d(1+\sin(c+dx))^{7/6}}$$

Mathematica [A] time = 0.20, size = 124, normalized size = 1.88

$$-\frac{3(a(\sin(c+dx)+1))^{2/3}\left(\sqrt{2-2\sin(c+dx)}-2\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+2dx+\pi)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\right.\right.}{2d\sqrt{2-2\sin(c+dx)}\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] (-3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(-2*Hypergeometric2F1[1/6, 1/2, 7
/6, Sin[(2*c + Pi + 2*d*x)/4]^2] + Sqrt[2 - 2*Sin[c + d*x]])*(a*(1 + Sin[c
+ d*x]))^(2/3))/(2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*Sqrt[2 - 2*Sin[c
 + d*x]])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sin(dx+c)+a)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^(2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(2/3), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int (a + a \sin(dx + c))^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(2/3),x)
[Out] int((a+a*sin(d*x+c))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sin(dx + c) + a)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (a + a\,\sin(c + d\,x))^{2/3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(2/3),x)
[Out] int((a + a*sin(c + d*x))^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sin(c + dx) + a)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(2/3),x)
[Out] Integral((a*sin(c + d*x) + a)**(2/3), x)
```

3.96 $\int \csc(c+dx)(a+a\sin(c+dx))^{2/3}\,dx$

Optimal. Leaf size=77

$$-\frac{2\sqrt[6]{2}\cos(c+dx)(a\sin(c+dx)+a)^{2/3}F_1\left(\frac{1}{2};1,-\frac{1}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{7/6}}$$

```
[Out] -2*2^(1/6)*AppellF1(1/2,1,-1/6,3/2,1-sin(d*x+c),1/2-1/2*sin(d*x+c))*cos(d*x
+c)*(a+a*sin(d*x+c))^(2/3)/d/(1+sin(d*x+c))^(7/6)
```

Rubi [A] time = 0.11, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2787, 2785, 130, 429}

$$-\frac{2\sqrt[6]{2}\cos(c+dx)(a\sin(c+dx)+a)^{2/3}F_1\left(\frac{1}{2};1,-\frac{1}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] (-2*2^(1/6)*AppellF1[1/2, 1, -1/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*x]
)/2]*Cos[c + d*x]*(a + a*Sin[c + d*x])^(2/3))/(d*(1 + Sin[c + d*x])^(7/6))
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
```

```
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \csc(c+dx)(a+a\sin(c+dx))^{2/3}\,dx = \frac{(a+a\sin(c+dx))^{2/3}\int \csc(c+dx)(1+\sin(c+dx))^{2/3}\,dx}{(1+\sin(c+dx))^{2/3}}$$

$$= -\frac{\left(\cos(c+dx)(a+a\sin(c+dx))^{2/3}\right)\operatorname{Subst}\left(\int \frac{\sqrt[6]{2-x}}{(1-x)\sqrt{x}}\,dx, x, 1-\sin(c+a$$

$$\frac{}{d\sqrt{1-\sin(c+dx)}\,(1+\sin(c+dx))^{7/6}}$$

$$= -\frac{\left(2\cos(c+dx)(a+a\sin(c+dx))^{2/3}\right)\operatorname{Subst}\left(\int \frac{\sqrt[6]{2-x^2}}{1-x^2}\,dx, x, \sqrt{1-\sin(c}$$

$$\frac{}{d\sqrt{1-\sin(c+dx)}\,(1+\sin(c+dx))^{7/6}}$$

$$= -\frac{2\sqrt[6]{2}F_1\left(\frac{1}{2};1,-\frac{1}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)(a+}{d(1+\sin(c+dx))^{7/6}}$$

Mathematica [F] time = 2.79, size = 0, normalized size = 0.00

$$\int \csc(c+dx)(a+a\sin(c+dx))^{2/3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[c + d*x]*(a + a*Sin[c + d*x])^(2/3),x]
[Out] Integrate[Csc[c + d*x]*(a + a*Sin[c + d*x])^(2/3), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{2}{3}}\csc(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(2/3)*csc(d*x + c), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \csc(dx+c)\,(a+a\sin(dx+c))^{\frac{2}{3}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sin(d*x+c))^(2/3),x)
```

```
[Out] int(csc(d*x+c)*(a+a*sin(d*x+c))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{2}{3}}\csc(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(2/3)*csc(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\ \sin(c+d\,x))^{2/3}}{\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(2/3)/sin(c + d*x),x)
```

```
[Out] int((a + a*sin(c + d*x))^(2/3)/sin(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^{\frac{2}{3}}\csc(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))**(2/3),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**(2/3)*csc(c + d*x), x)
```

3.97 $\int \csc^2(c+dx)(a+a\sin(c+dx))^{2/3}\,dx$

Optimal. Leaf size=77

$$-\frac{2\sqrt[6]{2}\cos(c+dx)(a\sin(c+dx)+a)^{2/3}F_1\left(\frac{1}{2};2,-\frac{1}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{7/6}}$$

```
[Out] -2*2^(1/6)*AppellF1(1/2,2,-1/6,3/2,1-sin(d*x+c),1/2-1/2*sin(d*x+c))*cos(d*x
+c)*(a+a*sin(d*x+c))^(2/3)/d/(1+sin(d*x+c))^(7/6)
```

Rubi [A] time = 0.13, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2787, 2785, 130, 429}

$$-\frac{2\sqrt[6]{2}\cos(c+dx)(a\sin(c+dx)+a)^{2/3}F_1\left(\frac{1}{2};2,-\frac{1}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] (-2*2^(1/6)*AppellF1[1/2, 2, -1/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*x]
)/2]*Cos[c + d*x]*(a + a*Sin[c + d*x])^(2/3))/(d*(1 + Sin[c + d*x])^(7/6))
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
```

```
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^{2/3}\,dx = \frac{(a+a\sin(c+dx))^{2/3}\int \csc^2(c+dx)(1+\sin(c+dx))^{2/3}\,dx}{(1+\sin(c+dx))^{2/3}}$$

$$= -\frac{\left(\cos(c+dx)(a+a\sin(c+dx))^{2/3}\right)\text{Subst}\left(\int \frac{\sqrt[6]{2-x}}{(1-x)^2\sqrt{x}}\,dx, x, 1-\sin(c+$$

$$\overline{d\sqrt{1-\sin(c+dx)}\,(1+\sin(c+dx))^{7/6}}$$

$$= -\frac{\left(2\cos(c+dx)(a+a\sin(c+dx))^{2/3}\right)\text{Subst}\left(\int \frac{\sqrt[6]{2-x^2}}{\left(1-x^2\right)^2}\,dx, x, \sqrt{1-\sin(c}{d\sqrt{1-\sin(c+dx)}\,(1+\sin(c+dx))^{7/6}}$$

$$= -\frac{2\sqrt[6]{2}F_1\left(\frac{1}{2};2,-\frac{1}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)(a}{d(1+\sin(c+dx))^{7/6}}$$

Mathematica [C] time = 14.75, size = 143, normalized size = 1.86

$$-\frac{2e^{i(c+dx)}\left(\left(1+ie^{-i(c+dx)}\right)^{2/3}\left(e^{i(c+dx)}-i\right){}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-ie^{-i(c+dx)}\right)-e^{i(c+dx)}-i\right)(a(\sin(c+dx)+1))^{2/3}}{d\left(e^{i(c+dx)}-i\right)\left(e^{i(c+dx)}+i\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(2/3),x]
```

```
[Out] (-2*E^(I*(c + d*x))*(-I - E^(I*(c + d*x)) + (1 + I/E^(I*(c + d*x)))^(2/3)*(
-I + E^(I*(c + d*x)))*Hypergeometric2F1[1/3, 2/3, 4/3, (-I)/E^(I*(c + d*x))
])*(a*(1 + Sin[c + d*x]))^(2/3))/(d*(-I + E^(I*(c + d*x)))*(I + E^(I*(c + d
*x)))^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(2/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{2}{3}}\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^(2/3)*csc(d*x + c)^2, x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \left(\csc^2(dx+c)\right)(a+a\sin(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^(2/3),x)
[Out] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{2}{3}}\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^(2/3)*csc(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^{2/3}}{\sin(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(2/3)/sin(c + d*x)^2,x)
[Out] int((a + a*sin(c + d*x))^(2/3)/sin(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^{\frac{2}{3}}\csc^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sin(d*x+c))**(2/3),x)
[Out] Integral((a*(sin(c + d*x) + 1))**(2/3)*csc(c + d*x)**2, x)
```

3.98 $\int \sin^3(c+dx)(a+a\sin(c+dx))^{4/3}\,dx$

Optimal. Leaf size=162

$$-\frac{388\ 2^{5/6} a\cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}\ {}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{455d(\sin(c+dx)+1)^{5/6}}-\frac{3\sin^2(c+dx)\cos(c+dx)(a\sin(c+\ldots}{13d}$$

```
[Out] -388/455*2^(5/6)*a*cos(d*x+c)*hypergeom([-5/6, 1/2],[3/2],1/2-1/2*sin(d*x+c
))*(a+a*sin(d*x+c))^(1/3)/d/(1+sin(d*x+c))^(5/6)-72/455*cos(d*x+c)*(a+a*sin
(d*x+c))^(4/3)/d-3/13*cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^(4/3)/d-6/65
*cos(d*x+c)*(a+a*sin(d*x+c))^(7/3)/a/d
```

Rubi [A] time = 0.28, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2783, 2968, 3023, 2751, 2652, 2651}

$$-\frac{388\ 2^{5/6} a\cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}\ {}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{455d(\sin(c+dx)+1)^{5/6}}-\frac{3\sin^2(c+dx)\cos(c+dx)(a\sin(c+\ldots}{13d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (-388*2^(5/6)*a*Cos[c + d*x]*Hypergeometric2F1[-5/6, 1/2, 3/2, (1 - Sin[c +
 d*x])/2]*(a + a*Sin[c + d*x])^(1/3))/(455*d*(1 + Sin[c + d*x])^(5/6)) - (7
2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(4/3))/(455*d) - (3*Cos[c + d*x]*Sin[c
+ d*x]^2*(a + a*Sin[c + d*x])^(4/3))/(13*d) - (6*Cos[c + d*x]*(a + a*Sin[c
+ d*x])^(7/3))/(65*a*d)
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2783

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + n)), x] + Dist[1/(b*(m + n)), Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n - 2)*Simp[d*(a*c*m + b*d*(n -
 1)) + b*c^2*(m + n) + d*(a*d*m + b*c*(m + 2*n - 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^{4/3}\,dx = -\frac{3\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^{4/3}}{13d} + \frac{3\int \sin(c+dx)(a\
$$

$$= -\frac{3\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^{4/3}}{13d} + \frac{3\int(a+a\sin(c+
$$

$$= -\frac{3\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^{4/3}}{13d} - \frac{6\cos(c+dx)(a+}{65a
$$

$$= -\frac{72\cos(c+dx)(a+a\sin(c+dx))^{4/3}}{455d} - \frac{3\cos(c+dx)\sin^2(c+dx)(a+}{13d}
$$

$$= -\frac{72\cos(c+dx)(a+a\sin(c+dx))^{4/3}}{455d} - \frac{3\cos(c+dx)\sin^2(c+dx)(a+}{13d}
$$

$$= -\frac{388\,2^{5/6}a\cos(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[3]{a+a\sin(c}{455d(1+\sin(c+dx))^{5/6}}
$$

Mathematica [C] time = 2.64, size = 373, normalized size = 2.30

$$\frac{(a(\sin(c+dx)+1))^{4/3}\left(-\frac{3}{40}\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)(278\sin(2(c+dx))-35\sin(4(c+dx))+790\,\mathrm{co}}{}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(4/3)*((291*(-1)^(3/4)*(I + E^(I*(c + d*x)))*(20*E^
(I*(c + d*x))*Sqrt[Cos[(2*c + Pi + 2*d*x)/4]^2]*Hypergeometric2F1[-1/3, 1/3
, 2/3, (-I)/E^(I*(c + d*x))] - 2*(1 + I/E^(I*(c + d*x)))^(2/3)*(1 + E^((2*I
)*(c + d*x)))*Hypergeometric2F1[1/2, 5/6, 11/6, Sin[(2*c + Pi + 2*d*x)/4]^2
] + (5*I)*Hypergeometric2F1[1/3, 2/3, 5/3, (-I)/E^(I*(c + d*x))]*Sqrt[2 - 2
*Sin[c + d*x]]))/(20*Sqrt[2]*E^(((3*I)/2)*(c + d*x))*(1 + I/E^(I*(c + d*x))
)^(2/3)*Sqrt[(I*(-I + E^(I*(c + d*x)))^2)/E^(I*(c + d*x))]) - (3*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])*(-1940 + 790*Cos[c + d*x] - 98*Cos[3*(c + d*x)]
 + 278*Sin[2*(c + d*x)] - 35*Sin[4*(c + d*x)]))/40))/(91*d*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^3)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a\cos(dx+c)^4-2\,a\cos(dx+c)^2-\left(a\cos(dx+c)^2-a\right)\sin(dx+c)+a\right)(a\sin(dx+c)+a)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 - (a*cos(d*x + c)^2 - a)*si
n(d*x + c) + a)*(a*sin(d*x + c) + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{4}{3}}\sin(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(4/3)*sin(d*x + c)^3, x)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \left(\sin^3(dx+c)\right)(a+a\sin(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+a*sin(d*x+c))^(4/3),x)
```

```
[Out] int(sin(d*x+c)^3*(a+a*sin(d*x+c))^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{4}{3}}\sin(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(4/3)*sin(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(c+d\,x)^3\,(a+a\,\sin(c+d\,x))^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + a*sin(c + d*x))^(4/3),x)
```

```
[Out] int(sin(c + d*x)^3*(a + a*sin(c + d*x))^(4/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+a*sin(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

3.99 $\int \sin^2(c+dx)(a+a\sin(c+dx))^{4/3}\,dx$

Optimal. Leaf size=127

$$-\frac{37\,2^{5/6}a\cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}\;{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{35d(\sin(c+dx)+1)^{5/6}}-\frac{3\cos(c+dx)(a\sin(c+dx)+a)^{7/3}}{10ad}+$$

```
[Out] -37/35*2^(5/6)*a*cos(d*x+c)*hypergeom([-5/6, 1/2],[3/2],1/2-1/2*sin(d*x+c))
*(a+a*sin(d*x+c))^(1/3)/d/(1+sin(d*x+c))^(5/6)+9/70*cos(d*x+c)*(a+a*sin(d*x
+c))^(4/3)/d-3/10*cos(d*x+c)*(a+a*sin(d*x+c))^(7/3)/a/d
```

Rubi [A] time = 0.14, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2759, 2751, 2652, 2651}

$$-\frac{37\,2^{5/6}a\cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}\;{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{35d(\sin(c+dx)+1)^{5/6}}-\frac{3\cos(c+dx)(a\sin(c+dx)+a)^{7/3}}{10ad}+$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (-37*2^(5/6)*a*Cos[c + d*x]*Hypergeometric2F1[-5/6, 1/2, 3/2, (1 - Sin[c +
d*x])/2]*(a + a*Sin[c + d*x])^(1/3))/(35*d*(1 + Sin[c + d*x])^(5/6)) + (9*C
os[c + d*x]*(a + a*Sin[c + d*x])^(4/3))/(70*d) - (3*Cos[c + d*x]*(a + a*Sin
[c + d*x])^(7/3))/(10*a*d)
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rubi steps

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^{4/3}\,dx = -\frac{3\cos(c+dx)(a+a\sin(c+dx))^{7/3}}{10ad} + \frac{3\int\left(\frac{7a}{3}-a\sin(c+dx)\right)(a+a}{10a}$$

$$= \frac{9\cos(c+dx)(a+a\sin(c+dx))^{4/3}}{70d} - \frac{3\cos(c+dx)(a+a\sin(c+dx))^{7}}{10ad}$$

$$= \frac{9\cos(c+dx)(a+a\sin(c+dx))^{4/3}}{70d} - \frac{3\cos(c+dx)(a+a\sin(c+dx))^{7}}{10ad}$$

$$= -\frac{37\,2^{5/6}a\cos(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[3]{a+a\sin(c+}}{35d(1+\sin(c+dx))^{5/6}}$$

Mathematica [C] time = 2.62, size = 363, normalized size = 2.86

$$\frac{(a(\sin(c+dx)+1))^{4/3}\left(-\frac{3}{10}\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)(22\sin(2(c+dx))+60\cos(c+dx)-7\cos(3(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(4/3)*((111*(-1)^(3/4)*(I + E^(I*(c + d*x)))*(20*E^
(I*(c + d*x))*Sqrt[Cos[(2*c + Pi + 2*d*x)/4]^2]*Hypergeometric2F1[-1/3, 1/3
, 2/3, (-I)/E^(I*(c + d*x))] - 2*(1 + I/E^(I*(c + d*x)))^(2/3)*(1 + E^((2*I
)*(c + d*x)))*Hypergeometric2F1[1/2, 5/6, 11/6, Sin[(2*c + Pi + 2*d*x)/4]^2
```

```
] + (5*I)*Hypergeometric2F1[1/3, 2/3, 5/3, (-I)/E^(I*(c + d*x))]*Sqrt[2 - 2
*Sin[c + d*x]]))/(20*Sqrt[2]*E^(((3*I)/2)*(c + d*x))*(1 + I/E^(I*(c + d*x))
)^(2/3)*Sqrt[(I*(-I + E^(I*(c + d*x)))^2)/E^(I*(c + d*x))]) - (3*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])*(-185 + 60*Cos[c + d*x] - 7*Cos[3*(c + d*x)] +
22*Sin[2*(c + d*x)]))/10))/(28*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a\cos\left(dx+c\right)^2+\left(a\cos\left(dx+c\right)^2-a\right)\sin\left(dx+c\right)-a\right)\left(a\sin\left(dx+c\right)+a\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(-(a*cos(d*x + c)^2 + (a*cos(d*x + c)^2 - a)*sin(d*x + c) - a)*(a*s
in(d*x + c) + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(dx+c\right)+a\right)^{\frac{4}{3}}\sin\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(4/3)*sin(d*x + c)^2, x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int\left(\sin^2\left(dx+c\right)\right)\left(a+a\sin\left(dx+c\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^(4/3),x)
```

```
[Out] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(dx+c\right)+a\right)^{\frac{4}{3}}\sin\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(4/3)*sin(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(c+dx)^2 (a+a\sin(c+dx))^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + a*sin(c + d*x))^(4/3),x)
[Out] int(sin(c + d*x)^2*(a + a*sin(c + d*x))^(4/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^{\frac{4}{3}}\sin^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+a*sin(d*x+c))**(4/3),x)
[Out] Integral((a*(sin(c + d*x) + 1))**(4/3)*sin(c + d*x)**2, x)
```

3.100 $\int \sin(c+dx)(a+a\sin(c+dx))^{4/3}\,dx$

Optimal. Leaf size=97

$$-\frac{8\,2^{5/6}a\cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}\;{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{7d(\sin(c+dx)+1)^{5/6}}-\frac{3\cos(c+dx)(a\sin(c+dx)+a)^{4/3}}{7d}$$

```
[Out] -8/7*2^(5/6)*a*cos(d*x+c)*hypergeom([-5/6, 1/2],[3/2],1/2-1/2*sin(d*x+c))*(
a+a*sin(d*x+c))^(1/3)/d/(1+sin(d*x+c))^(5/6)-3/7*cos(d*x+c)*(a+a*sin(d*x+c)
)^(4/3)/d
```

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2751, 2652, 2651}

$$-\frac{8\,2^{5/6}a\cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}\;{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{7d(\sin(c+dx)+1)^{5/6}}-\frac{3\cos(c+dx)(a\sin(c+dx)+a)^{4/3}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (-8*2^(5/6)*a*Cos[c + d*x]*Hypergeometric2F1[-5/6, 1/2, 3/2, (1 - Sin[c + d
*x])/2]*(a + a*Sin[c + d*x])^(1/3))/(7*d*(1 + Sin[c + d*x])^(5/6)) - (3*Cos
[c + d*x]*(a + a*Sin[c + d*x])^(4/3))/(7*d)
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \sin(c+dx)(a+a\sin(c+dx))^{4/3}\,dx &= -\frac{3\cos(c+dx)(a+a\sin(c+dx))^{4/3}}{7d}+\frac{4}{7}\int (a+a\sin(c+dx))^{4/3}\,dx\\ &= -\frac{3\cos(c+dx)(a+a\sin(c+dx))^{4/3}}{7d}+\frac{\left(4a\sqrt[3]{a+a\sin(c+dx)}\right)\int(1+\mathrm{s}}{7\sqrt[3]{1+\sin(c+dx}}\\ &= -\frac{8\,2^{5/6}a\cos(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[3]{a+a\sin(c+d}}{7d(1+\sin(c+dx))^{5/6}}\end{aligned}$$

Mathematica [C] time = 2.00, size = 351, normalized size = 3.62

$$\frac{(a(\sin(c+dx)+1))^{4/3}\left(-\frac{3}{2}\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)(\sin(2(c+dx))+4\cos(c+dx)-10)+\frac{3(-1)^{3/4}e^{-\frac{3}{2}}}{}\right.}{7d\,(}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(4/3)*((3*(-1)^(3/4)*(I + E^(I*(c + d*x)))*(20*E^(I
*(c + d*x))*Sqrt[Cos[(2*c + Pi + 2*d*x)/4]^2]*Hypergeometric2F1[-1/3, 1/3,
2/3, (-I)/E^(I*(c + d*x))] - 2*(1 + I/E^(I*(c + d*x)))^(2/3)*(1 + E^((2*I)*
(c + d*x)))*Hypergeometric2F1[1/2, 5/6, 11/6, Sin[(2*c + Pi + 2*d*x)/4]^2]
+ (5*I)*Hypergeometric2F1[1/3, 2/3, 5/3, (-I)/E^(I*(c + d*x))]*Sqrt[2 - 2*S
in[c + d*x]]))/(2*Sqrt[2]*E^(((3*I)/2)*(c + d*x))*(1 + I/E^(I*(c + d*x)))^(
2/3)*Sqrt[(I*(-I + E^(I*(c + d*x)))^2)/E^(I*(c + d*x))]) - (3*(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2])*(-10 + 4*Cos[c + d*x] + Sin[2*(c + d*x)]))/2))/(7*
d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(a\cos\left(dx+c\right)^{2}-a\sin\left(dx+c\right)-a\right)\left(a\sin\left(dx+c\right)+a\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(-(a*cos(d*x + c)^2 - a*sin(d*x + c) - a)*(a*sin(d*x + c) + a)^(1/3
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{4}{3}}\sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(4/3)*sin(d*x + c), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \sin(dx+c)\,(a+a\sin(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+a*sin(d*x+c))^(4/3),x)
```

```
[Out] int(sin(d*x+c)*(a+a*sin(d*x+c))^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{4}{3}}\sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(4/3)*sin(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(c+d\,x)\,\left(a+a\,\sin(c+d\,x)\right)^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + a*sin(c + d*x))^(4/3),x)
```

```
[Out] int(sin(c + d*x)*(a + a*sin(c + d*x))^(4/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^{\frac{4}{3}} \sin(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))**(4/3),x)
[Out] Integral((a*(sin(c + d*x) + 1))**(4/3)*sin(c + d*x), x)
```

3.101 $\int (a+a\sin(c+dx))^{4/3}\,dx$

Optimal. Leaf size=67

$$-\frac{2\,2^{5/6}a\cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}\;{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{5/6}}$$

```
[Out] -2*2^(5/6)*a*cos(d*x+c)*hypergeom([-5/6, 1/2],[3/2],1/2-1/2*sin(d*x+c))*(a+
a*sin(d*x+c))^(1/3)/d/(1+sin(d*x+c))^(5/6)
```

Rubi [A] time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2652, 2651}

$$-\frac{2\,2^{5/6}a\cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}\;{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (-2*2^(5/6)*a*Cos[c + d*x]*Hypergeometric2F1[-5/6, 1/2, 3/2, (1 - Sin[c + d
*x])/2]*(a + a*Sin[c + d*x])^(1/3))/(d*(1 + Sin[c + d*x])^(5/6))
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+a\sin(c+dx))^{4/3}\,dx = \frac{\left(a\sqrt[3]{a+a\sin(c+dx)}\right)\int (1+\sin(c+dx))^{4/3}\,dx}{\sqrt[3]{1+\sin(c+dx)}}$$
$$= -\frac{2\,2^{5/6}a\cos(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[3]{a+a\sin(c+dx)}}{d(1+\sin(c+dx))^{5/6}}$$

Mathematica [C] time = 1.72, size = 341, normalized size = 5.09

$$\frac{(a(\sin(c+dx)+1))^{4/3}\left(-\frac{3}{2}(\cos(c+dx)-5)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)+\frac{3(-1)^{3/4}e^{-\frac{3}{2}i(c+dx)}\left(e^{i(c+dx)}+i\right)\left(-2\left(1+ie\right.\right.}{}\right.}{2d\left(\sin\left(\frac{1}{2}(c+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (((-3*(-5 + Cos[c + d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/2 + (3*(-1
)^(3/4)*(I + E^(I*(c + d*x)))*(20*E^(I*(c + d*x))*Sqrt[Cos[(2*c + Pi + 2*d*
x)/4]^2]*Hypergeometric2F1[-1/3, 1/3, 2/3, (-I)/E^(I*(c + d*x))] - 2*(1 + I
/E^(I*(c + d*x)))^(2/3)*(1 + E^((2*I)*(c + d*x)))*Hypergeometric2F1[1/2, 5/
6, 11/6, Sin[(2*c + Pi + 2*d*x)/4]^2] + (5*I)*Hypergeometric2F1[1/3, 2/3, 5
/3, (-I)/E^(I*(c + d*x))]*Sqrt[2 - 2*Sin[c + d*x]]))/(4*Sqrt[2]*E^(((3*I)/2
)*(c + d*x))*(1 + I/E^(I*(c + d*x)))^(2/3)*Sqrt[(I*(-I + E^(I*(c + d*x)))^2
)/E^(I*(c + d*x))]))*(a*(1 + Sin[c + d*x]))^(4/3))/(2*d*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])^3)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sin(dx+c)+a)^{\frac{4}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^(4/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(4/3), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int (a + a\sin(dx + c))^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(4/3),x)
```

```
[Out] int((a+a*sin(d*x+c))^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx + c) + a)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a\,\sin(c + d\,x))^{4/3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(4/3),x)
```

```
[Out] int((a + a*sin(c + d*x))^(4/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(c + dx) + a)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(4/3),x)
```

```
[Out] Integral((a*sin(c + d*x) + a)**(4/3), x)
```

3.102 $\int \csc(c+dx)(a+a\sin(c+dx))^{4/3}\,dx$

Optimal. Leaf size=78

$$-\frac{2\,2^{5/6} a \cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}\,F_1\left(\frac{1}{2};1,-\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{5/6}}$$

```
[Out] -2*2^(5/6)*a*AppellF1(1/2,1,-5/6,3/2,1-sin(d*x+c),1/2-1/2*sin(d*x+c))*cos(d
*x+c)*(a+a*sin(d*x+c))^(1/3)/d/(1+sin(d*x+c))^(5/6)
```

Rubi [A] time = 0.11, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2787, 2785, 130, 429}

$$-\frac{2\,2^{5/6} a \cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}\,F_1\left(\frac{1}{2};1,-\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (-2*2^(5/6)*a*AppellF1[1/2, 1, -5/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*
x])/2]*Cos[c + d*x]*(a + a*Sin[c + d*x])^(1/3))/(d*(1 + Sin[c + d*x])^(5/6)
)
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
```

```
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \csc(c+dx)(a+a\sin(c+dx))^{4/3}\,dx = \frac{\left(a\sqrt[3]{a+a\sin(c+dx)}\right)\int \csc(c+dx)(1+\sin(c+dx))^{4/3}\,dx}{\sqrt[3]{1+\sin(c+dx)}}$$

$$= -\frac{\left(a\cos(c+dx)\sqrt[3]{a+a\sin(c+dx)}\right)\text{Subst}\left(\int \frac{(2-x)^{5/6}}{(1-x)\sqrt{x}}\,dx, x, 1-\sin(c+\right.}{d\sqrt{1-\sin(c+dx)}\,(1+\sin(c+dx))^{5/6}}$$

$$= -\frac{\left(2a\cos(c+dx)\sqrt[3]{a+a\sin(c+dx)}\right)\text{Subst}\left(\int \frac{\left(2-x^2\right)^{5/6}}{1-x^2}\,dx, x, \sqrt{1-\sin(c\right.}{d\sqrt{1-\sin(c+dx)}\,(1+\sin(c+dx))^{5/6}}$$

$$= -\frac{2\,2^{5/6}aF_1\left(\frac{1}{2};1,-\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)\sqrt[3]{}}{d(1+\sin(c+dx))^{5/6}}$$

Mathematica [C] time = 9.61, size = 2791, normalized size = 35.78

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]*(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (3*(a*(1 + Sin[c + d*x]))^(4/3))/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2
) - ((15 + 15*I)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c + d*x
)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*(a*(1 + Sin[c + d*x]))^(4/3)*(1
+ Tan[(c + d*x)/2]))/(d*((5 + 5*I)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)
*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*Sec[(c + d*x)/
2] + AppellF1[5/3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2
- I/2)*(1 + Cot[(c + d*x)/2])]*(Csc[(c + d*x)/2] + Sec[(c + d*x)/2]) + I*Ap
pellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*
```

(1 + Cot[(c + d*x)/2])]*(Csc[(c + d*x)/2] + Sec[(c + d*x)/2]))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + ((15/2 + (15*I)/2)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Tan[(c + d*x)/2]), (1/2 - I/2)*(1 + Tan[(c + d*x)/2])]*(a*(1 + Sin[c + d*x]))^(4/3))/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*((5 + 5*I)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Tan[(c + d*x)/2]), (1/2 - I/2)*(1 + Tan[(c + d*x)/2])] + (AppellF1[5/3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1 + Tan[(c + d*x)/2]), (1/2 - I/2)*(1 + Tan[(c + d*x)/2])] + I*AppellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Tan[(c + d*x)/2]), (1/2 - I/2)*(1 + Tan[(c + d*x)/2])])*(1 + Tan[(c + d*x)/2]))) - (3*Cos[(3*(c + d*x))/2]*Csc[c + d*x]*(a*(1 + Sin[c + d*x]))^(4/3)*((1 + Tan[(c + d*x)/2])/Sqrt[Sec[(c + d*x)/2]^2])^(2/3)*(8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot[(c + d*x)/2]))/(1 + Cot[(c + d*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(c + d*x)/2])/(2 + 2*Tan[(c + d*x)/2])]*(I + Tan[(c + d*x)/2]) - AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(c + d*x)/2])^(1/3)*((-1 - I)*(I + Cot[(c + d*x)/2]))^(1/3)*(1 + Tan[(c + d*x)/2])))/(4*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(1 + Tan[(c + d*x)/2])*((-3*Sec[(c + d*x)/2]^2*((1 + Tan[(c + d*x)/2])/Sqrt[Sec[(c + d*x)/2]^2])^(2/3)*(8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot[(c + d*x)/2]))/(1 + Cot[(c + d*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(c + d*x)/2])/(2 + 2*Tan[(c + d*x)/2])]*(I + Tan[(c + d*x)/2]) - AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(c + d*x)/2])^(1/3)*((-1 - I)*(I + Cot[(c + d*x)/2]))^(1/3)*(1 + Tan[(c + d*x)/2])))/(8*(1 + Tan[(c + d*x)/2])^2) + ((8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot[(c + d*x)/2]))/(1 + Cot[(c + d*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(c + d*x)/2])/(2 + 2*Tan[(c + d*x)/2])]*(I + Tan[(c + d*x)/2]) - AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(c + d*x)/2])^(1/3)*((-1 - I)*(I + Cot[(c + d*x)/2]))^(1/3)*(1 + Tan[(c + d*x)/2]))*(Sqrt[Sec[(c + d*x)/2]^2]/2 - (Tan[(c + d*x)/2]*(1 + Tan[(c + d*x)/2]))/(2*Sqrt[Sec[(c + d*x)/2]^2])))/(2*(1 + Tan[(c + d*x)/2])*((1 + Tan[(c + d*x)/2])/Sqrt[Sec[(c + d*x)/2]^2])^(1/3)) + (3*((1 + Tan[(c + d*x)/2])/Sqrt[Sec[(c + d*x)/2]^2])^(2/3)*(-1/2*(AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(c + d*x)/2])^(1/3)*((-1 - I)*(I + Cot[(c + d*x)/2]))^(1/3)*Sec[(c + d*x)/2]^2) + ((1 + I)*(((1 - I)*(I + Cot[(c + d*x)/2]))/(1 + Cot[(c + d*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(c + d*x)/2])/(2 + 2*Tan[(c + d*x)/2])]*Sec[(c + d*x)/2]^2)/2^(1/3) + ((1/3 + I/3)*2^(2/3)*(((1/2 - I/2)*(I + Cot[(c + d*x)/2])*Csc[(c + d*x)/2]^2)/(1 + Cot[(c + d*x)/2])^2 - ((1/2 - I/2)*Csc[(c + d*x)/2]^2)/(1 + Cot[(c + d*x)/2]))*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(c + d*x)/2])/(2 + 2*Tan[(c + d*x)/2])]*(I + Tan[(c + d*x)/2]))/(((1 - I)*(I + Cot[(c + d*x)/2]))/(1 + Cot[(c + d*x)/2]))^(2/3) - ((1/6 + I/6)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(c + d*x)/2])^(1/

```
3)*Csc[(c + d*x)/2]^2*(1 + Tan[(c + d*x)/2]))/((-1 - I)*(I + Cot[(c + d*x)/
2]))^(2/3) - ((1/3 - I/3)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot
[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*((-1 - I)*(I + Cot[(c +
 d*x)/2]))^(1/3)*Csc[(c + d*x)/2]^2*(1 + Tan[(c + d*x)/2]))/((2 + 2*I) - (2
 - 2*I)*Cot[(c + d*x)/2])^(2/3) - ((2 + 2*I) - (2 - 2*I)*Cot[(c + d*x)/2])^
(1/3)*((-1 - I)*(I + Cot[(c + d*x)/2]))^(1/3)*((-1/30 + I/30)*AppellF1[5/3,
 1/3, 4/3, 8/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c
 + d*x)/2])]*Csc[(c + d*x)/2]^2 - (1/30 + I/30)*AppellF1[5/3, 4/3, 1/3, 8/3
, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*C
sc[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]) + ((2/3 + (2*I)/3)*2^(2/3)*(((1 -
 I)*(I + Cot[(c + d*x)/2]))/(1 + Cot[(c + d*x)/2]))^(1/3)*(I + Tan[(c + d*x
)/2])*(2 + 2*Tan[(c + d*x)/2])*(-((Sec[(c + d*x)/2]^2*((1 + I) + (1 - I)*Ta
n[(c + d*x)/2]))/(2 + 2*Tan[(c + d*x)/2])^2) + ((1/2 - I/2)*Sec[(c + d*x)/2
]^2)/(2 + 2*Tan[(c + d*x)/2]))*(-Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I)
+ (1 - I)*Tan[(c + d*x)/2])/(2 + 2*Tan[(c + d*x)/2])] + (1 - ((1 + I) + (1
- I)*Tan[(c + d*x)/2])/(2 + 2*Tan[(c + d*x)/2]))^(-1/3)))/((1 + I) + (1 - I
)*Tan[(c + d*x)/2])))/(4*(1 + Tan[(c + d*x)/2])))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{4}{3}}\csc(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^(4/3)*csc(d*x + c), x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \csc(dx+c)(a+a\sin(dx+c))^{\frac{4}{3}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sin(d*x+c))^(4/3),x)
```

```
[Out] int(csc(d*x+c)*(a+a*sin(d*x+c))^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin (dx+c)+a)^{\frac{4}{3}}\csc (dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^(4/3)*csc(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin \left(c+d\,x\right)\right)}^{4/3}}{\sin \left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(4/3)/sin(c + d*x),x)
[Out] int((a + a*sin(c + d*x))^(4/3)/sin(c + d*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))**(4/3),x)
[Out] Timed out
```

3.103 $\int \csc^2(c+dx)(a+a\sin(c+dx))^{4/3}\,dx$

Optimal. Leaf size=78

$$-\frac{2\,2^{5/6}a\cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}\,F_1\left(\frac{1}{2};2,-\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{5/6}}$$

```
[Out] -2*2^(5/6)*a*AppellF1(1/2,2,-5/6,3/2,1-sin(d*x+c),1/2-1/2*sin(d*x+c))*cos(d
*x+c)*(a+a*sin(d*x+c))^(1/3)/d/(1+sin(d*x+c))^(5/6)
```

Rubi [A] time = 0.13, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2787, 2785, 130, 429}

$$-\frac{2\,2^{5/6}a\cos(c+dx)\sqrt[3]{a\sin(c+dx)+a}\,F_1\left(\frac{1}{2};2,-\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d(\sin(c+dx)+1)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (-2*2^(5/6)*a*AppellF1[1/2, 2, -5/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*
x])/2]*Cos[c + d*x]*(a + a*Sin[c + d*x])^(1/3))/(d*(1 + Sin[c + d*x])^(5/6)
)
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
```

```
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^{4/3}\,dx = \frac{\left(a\sqrt[3]{a+a\sin(c+dx)}\right)\int \csc^2(c+dx)(1+\sin(c+dx))^{4/3}\,dx}{\sqrt[3]{1+\sin(c+dx)}}$$
$$= -\frac{\left(a\cos(c+dx)\sqrt[3]{a+a\sin(c+dx)}\right)\text{Subst}\left(\int \frac{(2-x)^{5/6}}{(1-x)^2\sqrt{x}}\,dx, x, 1-\sin(c$$
$$d\sqrt{1-\sin(c+dx)}\,(1+\sin(c+dx))^{5/6}$$
$$= -\frac{\left(2a\cos(c+dx)\sqrt[3]{a+a\sin(c+dx)}\right)\text{Subst}\left(\int \frac{\left(2-x^2\right)^{5/6}}{\left(1-x^2\right)^2}\,dx, x, \sqrt{1-\sin$$
$$d\sqrt{1-\sin(c+dx)}\,(1+\sin(c+dx))^{5/6}$$
$$= -\frac{2\,2^{5/6}aF_1\left(\frac{1}{2};2,-\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx$$
$$d(1+\sin(c+dx))^{5/6}$$

Mathematica [C] time = 10.57, size = 2800, normalized size = 35.90

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] ((-1 - Cot[c + d*x])*(a*(1 + Sin[c + d*x]))^(4/3))/(d*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^2) - ((15/2 + (15*I)/2)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2
+ I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*(a*(1 +
Sin[c + d*x]))^(4/3)*(1 + Tan[(c + d*x)/2]))/(d*((5 + 5*I)*AppellF1[2/3, 1/
3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c +
d*x)/2])]*Sec[(c + d*x)/2] + AppellF1[5/3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1 +
Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*(Csc[(c + d*x)/2] +
```

```
Sec[(c + d*x)/2]) + I*AppellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Cot[(c
+ d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*(Csc[(c + d*x)/2] + Sec[(c
+ d*x)/2]))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + ((10 + 10*I)*AppellF
1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Tan[(c + d*x)/2]), (1/2 - I/2)*(1 +
Tan[(c + d*x)/2])]*(a*(1 + Sin[c + d*x]))^(4/3))/(d*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])^2*((5 + 5*I)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + T
an[(c + d*x)/2]), (1/2 - I/2)*(1 + Tan[(c + d*x)/2])] + (AppellF1[5/3, 1/3,
 4/3, 8/3, (1/2 + I/2)*(1 + Tan[(c + d*x)/2]), (1/2 - I/2)*(1 + Tan[(c + d*
x)/2])] + I*AppellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Tan[(c + d*x)/2])
, (1/2 - I/2)*(1 + Tan[(c + d*x)/2])])*(1 + Tan[(c + d*x)/2]))) + (Cos[(3*(
c + d*x))/2]*Csc[c + d*x]*(a*(1 + Sin[c + d*x]))^(4/3)*((1 + Tan[(c + d*x)/
2])/Sqrt[Sec[(c + d*x)/2]^2])^(2/3)*(8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot
[(c + d*x)/2]))/(1 + Cot[(c + d*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5
/3, ((1 + I) + (1 - I)*Tan[(c + d*x)/2])/(2 + 2*Tan[(c + d*x)/2])]*(I + Tan
[(c + d*x)/2]) - AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c + d*x
)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(c +
d*x)/2])^(1/3)*((-1 - I)*(I + Cot[(c + d*x)/2]))^(1/3)*(1 + Tan[(c + d*x)/2
])))/(4*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(1 + Tan[(c + d*x)/2])*((
-3*Sec[(c + d*x)/2]^2*((1 + Tan[(c + d*x)/2])/Sqrt[Sec[(c + d*x)/2]^2])^(2/
3)*(8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot[(c + d*x)/2]))/(1 + Cot[(c + d*x
)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(c + d
*x)/2])/(2 + 2*Tan[(c + d*x)/2])]*(I + Tan[(c + d*x)/2]) - AppellF1[2/3, 1/
3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c +
d*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(c + d*x)/2])^(1/3)*((-1 - I)*(I + Cot
[(c + d*x)/2]))^(1/3)*(1 + Tan[(c + d*x)/2])))/(8*(1 + Tan[(c + d*x)/2])^2)
 + ((8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot[(c + d*x)/2]))/(1 + Cot[(c + d*
x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(c +
d*x)/2])/(2 + 2*Tan[(c + d*x)/2])]*(I + Tan[(c + d*x)/2]) - AppellF1[2/3, 1
/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c +
 d*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(c + d*x)/2])^(1/3)*((-1 - I)*(I + Co
t[(c + d*x)/2]))^(1/3)*(1 + Tan[(c + d*x)/2]))*(Sqrt[Sec[(c + d*x)/2]^2]/2
- (Tan[(c + d*x)/2]*(1 + Tan[(c + d*x)/2]))/(2*Sqrt[Sec[(c + d*x)/2]^2])))/
(2*(1 + Tan[(c + d*x)/2])*((1 + Tan[(c + d*x)/2])/Sqrt[Sec[(c + d*x)/2]^2])
^(1/3)) + (3*((1 + Tan[(c + d*x)/2])/Sqrt[Sec[(c + d*x)/2]^2])^(2/3)*(-1/2*
(AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/
2)*(1 + Cot[(c + d*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(c + d*x)/2])^(1/3)*(
(-1 - I)*(I + Cot[(c + d*x)/2]))^(1/3)*Sec[(c + d*x)/2]^2) + ((1 + I)*(((1
- I)*(I + Cot[(c + d*x)/2]))/(1 + Cot[(c + d*x)/2]))^(1/3)*Hypergeometric2F
1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(c + d*x)/2])/(2 + 2*Tan[(c + d*x)/
2])]*Sec[(c + d*x)/2]^2)/2^(1/3) + ((1/3 + I/3)*2^(2/3)*(((1/2 - I/2)*(I +
Cot[(c + d*x)/2])*Csc[(c + d*x)/2]^2)/(1 + Cot[(c + d*x)/2])^2 - ((1/2 - I/
2)*Csc[(c + d*x)/2]^2)/(1 + Cot[(c + d*x)/2]))*Hypergeometric2F1[1/3, 2/3,
5/3, ((1 + I) + (1 - I)*Tan[(c + d*x)/2])/(2 + 2*Tan[(c + d*x)/2])]*(I + Ta
n[(c + d*x)/2]))/(((1 - I)*(I + Cot[(c + d*x)/2]))/(1 + Cot[(c + d*x)/2]))^
(2/3) - ((1/6 + I/6)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(c +
```

```
 d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(
c + d*x)/2])^(1/3)*Csc[(c + d*x)/2]^2*(1 + Tan[(c + d*x)/2]))/((-1 - I)*(I
+ Cot[(c + d*x)/2]))^(2/3) - ((1/3 - I/3)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2
 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[(c + d*x)/2])]*((-1 -
I)*(I + Cot[(c + d*x)/2]))^(1/3)*Csc[(c + d*x)/2]^2*(1 + Tan[(c + d*x)/2]))
/((2 + 2*I) - (2 - 2*I)*Cot[(c + d*x)/2])^(2/3) - ((2 + 2*I) - (2 - 2*I)*Co
t[(c + d*x)/2])^(1/3)*((-1 - I)*(I + Cot[(c + d*x)/2]))^(1/3)*((-1/30 + I/3
0)*AppellF1[5/3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 -
I/2)*(1 + Cot[(c + d*x)/2])]*Csc[(c + d*x)/2]^2 - (1/30 + I/30)*AppellF1[5/
3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Cot[(c + d*x)/2]), (1/2 - I/2)*(1 + Cot[
(c + d*x)/2])]*Csc[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]) + ((2/3 + (2*I)/3
)*2^(2/3)*(((1 - I)*(I + Cot[(c + d*x)/2]))/(1 + Cot[(c + d*x)/2]))^(1/3)*(
I + Tan[(c + d*x)/2])*(2 + 2*Tan[(c + d*x)/2])*(-((Sec[(c + d*x)/2]^2*((1 +
 I) + (1 - I)*Tan[(c + d*x)/2]))/(2 + 2*Tan[(c + d*x)/2])^2) + ((1/2 - I/2)
*Sec[(c + d*x)/2]^2)/(2 + 2*Tan[(c + d*x)/2]))*(-Hypergeometric2F1[1/3, 2/3
, 5/3, ((1 + I) + (1 - I)*Tan[(c + d*x)/2])/(2 + 2*Tan[(c + d*x)/2])] + (1
- ((1 + I) + (1 - I)*Tan[(c + d*x)/2])/(2 + 2*Tan[(c + d*x)/2]))^(-1/3)))/(
(1 + I) + (1 - I)*Tan[(c + d*x)/2])))/(4*(1 + Tan[(c + d*x)/2]))))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{4}{3}}\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^(4/3)*csc(d*x + c)^2, x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \left(\csc^2(dx+c)\right)(a+a\sin(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^(4/3),x)
```

```
[Out] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^{\frac{4}{3}}\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(4/3)*csc(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^{4/3}}{\sin(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(4/3)/sin(c + d*x)^2,x)
```

```
[Out] int((a + a*sin(c + d*x))^(4/3)/sin(c + d*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sin(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

3.104 $\int \frac{\sin^3(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=161

$$\frac{37\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{40\,2^{5/6}d\sqrt[6]{\sin(c+dx)+1}\,\sqrt[3]{a\sin(c+dx)+a}}-\frac{3\sin^2(c+dx)\cos(c+dx)}{8d\sqrt[3]{a\sin(c+dx)+a}}+\frac{3\cos(c+dx)(a\sin(c+dx)+a)^{2/3}}{40ad}-$$

```
[Out] -99/80*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/3)-3/8*cos(d*x+c)*sin(d*x+c)^2/d/(a
+a*sin(d*x+c))^(1/3)+37/80*cos(d*x+c)*hypergeom([1/2, 5/6],[3/2],1/2-1/2*si
n(d*x+c))*2^(1/6)/d/(1+sin(d*x+c))^(1/6)/(a+a*sin(d*x+c))^(1/3)+3/40*cos(d*
x+c)*(a+a*sin(d*x+c))^(2/3)/a/d
```

Rubi [A] time = 0.25, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2783, 2968, 3023, 2751, 2652, 2651}

$$\frac{37\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{40\,2^{5/6}d\sqrt[6]{\sin(c+dx)+1}\,\sqrt[3]{a\sin(c+dx)+a}}-\frac{3\sin^2(c+dx)\cos(c+dx)}{8d\sqrt[3]{a\sin(c+dx)+a}}+\frac{3\cos(c+dx)(a\sin(c+dx)+a)^{2/3}}{40ad}-$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + a*Sin[c + d*x])^(1/3),x]
```

```
[Out] (-99*Cos[c + d*x])/(80*d*(a + a*Sin[c + d*x])^(1/3)) - (3*Cos[c + d*x]*Sin[
c + d*x]^2)/(8*d*(a + a*Sin[c + d*x])^(1/3)) + (37*Cos[c + d*x]*Hypergeomet
ric2F1[1/2, 5/6, 3/2, (1 - Sin[c + d*x])/2])/(40*2^(5/6)*d*(1 + Sin[c + d*x
])^(1/6)*(a + a*Sin[c + d*x])^(1/3)) + (3*Cos[c + d*x]*(a + a*Sin[c + d*x])
^(2/3))/(40*a*d)
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2783

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + n)), x] + Dist[1/(b*(m + n)), Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n - 2)*Simp[d*(a*c*m + b*d*(n -
 1)) + b*c^2*(m + n) + d*(a*d*m + b*c*(m + 2*n - 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx = -\frac{3\cos(c+dx)\sin^2(c+dx)}{8d\sqrt[3]{a+a\sin(c+dx)}} + \frac{3\int \frac{\sin(c+dx)\left(2a-\frac{1}{3}a\sin(c+dx)\right)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx}{8a}$$

$$= -\frac{3\cos(c+dx)\sin^2(c+dx)}{8d\sqrt[3]{a+a\sin(c+dx)}} + \frac{3\int \frac{2a\sin(c+dx)-\frac{1}{3}a\sin^2(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx}{8a}$$

$$= -\frac{3\cos(c+dx)\sin^2(c+dx)}{8d\sqrt[3]{a+a\sin(c+dx)}} + \frac{3\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{40ad} + \frac{9\int \frac{-\frac{2a^2}{9}+\frac{11}{3}a^2\sin}{\sqrt[3]{a+a\sin(c}}}{40a^2}$$

$$= -\frac{99\cos(c+dx)}{80d\sqrt[3]{a+a\sin(c+dx)}} - \frac{3\cos(c+dx)\sin^2(c+dx)}{8d\sqrt[3]{a+a\sin(c+dx)}} + \frac{3\cos(c+dx)(a+a\sin(c+}{40ad}$$

$$= -\frac{99\cos(c+dx)}{80d\sqrt[3]{a+a\sin(c+dx)}} - \frac{3\cos(c+dx)\sin^2(c+dx)}{8d\sqrt[3]{a+a\sin(c+dx)}} + \frac{3\cos(c+dx)(a+a\sin(c+}{40ad}$$

$$= -\frac{99\cos(c+dx)}{80d\sqrt[3]{a+a\sin(c+dx)}} - \frac{3\cos(c+dx)\sin^2(c+dx)}{8d\sqrt[3]{a+a\sin(c+dx)}} + \frac{37\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2}\right.}{40\,2^{5/6}d\sqrt[6]{1+\sin(c+dx)}}$$

Mathematica [A] time = 0.46, size = 110, normalized size = 0.68

$$\frac{3\cos(c+dx)\left(\sqrt{1-\sin(c+dx)}\,(2\sin(c+dx)+5\cos(2(c+dx))-36)-37\sqrt{2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+2dx+\right.\right.\right.}{80d\sqrt{1-\sin(c+dx)}\,\sqrt[3]{a(\sin(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^3/(a + a*Sin[c + d*x])^(1/3),x]
```

```
[Out] (3*Cos[c + d*x]*(-37*Sqrt[2]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*c + Pi
 + 2*d*x)/4]^2] + Sqrt[1 - Sin[c + d*x]]*(-36 + 5*Cos[2*(c + d*x)] + 2*Sin[
c + d*x])))/(80*d*Sqrt[1 - Sin[c + d*x]]*(a*(1 + Sin[c + d*x]))^(1/3))
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(\cos(dx+c)^2-1\right)\sin(dx+c)}{(a\sin(dx+c)+a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*sin(d*x + c)/(a*sin(d*x + c) + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^3}{(a\sin(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate(sin(d*x + c)^3/(a*sin(d*x + c) + a)^(1/3), x)
```

maple [F] time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(dx+c)}{(a+a\sin(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/3),x)
[Out] int(sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^3}{(a\sin(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate(sin(d*x + c)^3/(a*sin(d*x + c) + a)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(c+d\,x)^3}{(a+a\,\sin(c+d\,x))^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + a*sin(c + d*x))^(1/3),x)
[Out] int(sin(c + d*x)^3/(a + a*sin(c + d*x))^(1/3), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+a*sin(d*x+c))**(1/3),x)
[Out] Timed out
```

3.105 $\int \frac{\sin^2(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=126

$$-\frac{7\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5\,2^{5/6}d\sqrt[6]{\sin(c+dx)+1}\,\sqrt[3]{a\sin(c+dx)+a}}-\frac{3\cos(c+dx)(a\sin(c+dx)+a)^{2/3}}{5ad}+\frac{9\cos(c+dx)}{10d\sqrt[3]{a\sin(c+dx)+a}}$$

```
[Out] 9/10*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/3)-7/10*cos(d*x+c)*hypergeom([1/2, 5/
6],[3/2],1/2-1/2*sin(d*x+c))*2^(1/6)/d/(1+sin(d*x+c))^(1/6)/(a+a*sin(d*x+c)
)^(1/3)-3/5*cos(d*x+c)*(a+a*sin(d*x+c))^(2/3)/a/d
```

Rubi [A] time = 0.13, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2759, 2751, 2652, 2651}

$$-\frac{7\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5\,2^{5/6}d\sqrt[6]{\sin(c+dx)+1}\,\sqrt[3]{a\sin(c+dx)+a}}-\frac{3\cos(c+dx)(a\sin(c+dx)+a)^{2/3}}{5ad}+\frac{9\cos(c+dx)}{10d\sqrt[3]{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + a*Sin[c + d*x])^(1/3),x]
```

```
[Out] (9*Cos[c + d*x])/(10*d*(a + a*Sin[c + d*x])^(1/3)) - (7*Cos[c + d*x]*Hyperg
eometric2F1[1/2, 5/6, 3/2, (1 - Sin[c + d*x])/2])/(5*2^(5/6)*d*(1 + Sin[c +
 d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3)) - (3*Cos[c + d*x]*(a + a*Sin[c + d
*x])^(2/3))/(5*a*d)
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx = -\frac{3\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{5ad}+\frac{3\int \frac{\frac{2a}{3}-a\sin(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx}{5a}$$
$$=\frac{9\cos(c+dx)}{10d\sqrt[3]{a+a\sin(c+dx)}}-\frac{3\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{5ad}+\frac{7}{10}\int \frac{1}{\sqrt[3]{a+a\sin(c}$$
$$=\frac{9\cos(c+dx)}{10d\sqrt[3]{a+a\sin(c+dx)}}-\frac{3\cos(c+dx)(a+a\sin(c+dx))^{2/3}}{5ad}+\frac{\left(7\sqrt[3]{1+\sin(c+dx)}\right.}{10\sqrt[3]{a+a\,\mathrm{si}}}$$
$$=\frac{9\cos(c+dx)}{10d\sqrt[3]{a+a\sin(c+dx)}}-\frac{7\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5\,2^{5/6}d\sqrt[6]{1+\sin(c+dx)}\,\sqrt[3]{a+a\sin(c+dx)}}-\frac{3\cos(c+}{}$$

Mathematica [A] time = 0.29, size = 95, normalized size = 0.75

$$-\frac{3\cos(c+dx)\left(\sqrt{2-2\sin(c+dx)}\,(2\sin(c+dx)-1)-14\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+2dx+\pi)\right)\right)\right)}{10d\sqrt{2-2\sin(c+dx)}\,\sqrt[3]{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + a*Sin[c + d*x])^(1/3),x]
```

```
[Out] (-3*Cos[c + d*x]*(-14*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*c + Pi + 2*d*
x)/4]^2] + Sqrt[2 - 2*Sin[c + d*x]]*(-1 + 2*Sin[c + d*x])))/(10*d*Sqrt[2 -
2*Sin[c + d*x]]*(a*(1 + Sin[c + d*x]))^(1/3))
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\cos(dx+c)^2-1}{(a\sin(dx+c)+a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral(-(cos(d*x + c)^2 - 1)/(a*sin(d*x + c) + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^2}{(a\sin(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate(sin(d*x + c)^2/(a*sin(d*x + c) + a)^(1/3), x)
```

maple [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(dx+c)}{(a+a\sin(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/3),x)
[Out] int(sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^2}{(a\sin(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate(sin(d*x + c)^2/(a*sin(d*x + c) + a)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(c+d\,x)^2}{\left(a+a\,\sin(c+d\,x)\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + a*sin(c + d*x))^(1/3),x)
[Out] int(sin(c + d*x)^2/(a + a*sin(c + d*x))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(c+dx)}{\sqrt[3]{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+a*sin(d*x+c))**(1/3),x)
[Out] Integral(sin(c + d*x)**2/(a*(sin(c + d*x) + 1))**(1/3), x)
```

3.106 $\int \frac{\sin(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=93

$$\frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{2^{5/6}d\sqrt[6]{\sin(c+dx)+1}\,\sqrt[3]{a\sin(c+dx)+a}}-\frac{3\cos(c+dx)}{2d\sqrt[3]{a\sin(c+dx)+a}}$$

```
[Out] -3/2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/3)+1/2*cos(d*x+c)*hypergeom([1/2, 5/6
],[3/2],1/2-1/2*sin(d*x+c))*2^(1/6)/d/(1+sin(d*x+c))^(1/6)/(a+a*sin(d*x+c))
^(1/3)
```

Rubi [A] time = 0.06, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2751, 2652, 2651}

$$\frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{2^{5/6}d\sqrt[6]{\sin(c+dx)+1}\,\sqrt[3]{a\sin(c+dx)+a}}-\frac{3\cos(c+dx)}{2d\sqrt[3]{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + a*Sin[c + d*x])^(1/3),x]
```

```
[Out] (-3*Cos[c + d*x])/(2*d*(a + a*Sin[c + d*x])^(1/3)) + (Cos[c + d*x]*Hypergeo
metric2F1[1/2, 5/6, 3/2, (1 - Sin[c + d*x])/2])/(2^(5/6)*d*(1 + Sin[c + d*x
])^(1/6)*(a + a*Sin[c + d*x])^(1/3))
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx &= -\frac{3\cos(c+dx)}{2d\sqrt[3]{a+a\sin(c+dx)}}-\frac{1}{2}\int \frac{1}{\sqrt[3]{a+a\sin(c+dx)}}\,dx\\ &= -\frac{3\cos(c+dx)}{2d\sqrt[3]{a+a\sin(c+dx)}}-\frac{\sqrt[3]{1+\sin(c+dx)}\int \frac{1}{\sqrt[3]{1+\sin(c+dx)}}\,dx}{2\sqrt[3]{a+a\sin(c+dx)}}\\ &= -\frac{3\cos(c+dx)}{2d\sqrt[3]{a+a\sin(c+dx)}}+\frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{2^{5/6}d\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.15, size = 84, normalized size = 0.90

$$-\frac{3\cos(c+dx)\left(2\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+2dx+\pi)\right)\right)+\sqrt{2-2\sin(c+dx)}\right)}{2d\sqrt{2-2\sin(c+dx)}\sqrt[3]{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + a*Sin[c + d*x])^(1/3),x]
```

```
[Out] (-3*Cos[c + d*x]*(2*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*c + Pi + 2*d*x)
/4]^2] + Sqrt[2 - 2*Sin[c + d*x]]))/(2*d*Sqrt[2 - 2*Sin[c + d*x]]*(a*(1 + S
in[c + d*x]))^(1/3))
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sin(dx+c)}{(a\sin(dx+c)+a)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sin(d*x + c)/(a*sin(d*x + c) + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)}{(a\sin(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate(sin(d*x + c)/(a*sin(d*x + c) + a)^(1/3), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\sin \left(dx + c\right)}{\left(a + a \sin \left(dx + c\right)\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+a*sin(d*x+c))^(1/3),x)
[Out] int(sin(d*x+c)/(a+a*sin(d*x+c))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin \left(dx + c\right)}{\left(a \sin \left(dx + c\right) + a\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate(sin(d*x + c)/(a*sin(d*x + c) + a)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(c+d\,x\right)}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + a*sin(c + d*x))^(1/3),x)
[Out] int(sin(c + d*x)/(a + a*sin(c + d*x))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin{\left(c + dx \right)}}{\sqrt[3]{a \left(\sin{\left(c + dx \right)} + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))**(1/3),x)
[Out] Integral(sin(c + d*x)/(a*(sin(c + d*x) + 1))**(1/3), x)
```

3.107 $\int \frac{1}{\sqrt[3]{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt[6]{2}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt[6]{\sin(c+dx)+1}\,\sqrt[3]{a\sin(c+dx)+a}}$$

```
[Out] -cos(d*x+c)*hypergeom([1/2, 5/6],[3/2],1/2-1/2*sin(d*x+c))*2^(1/6)/d/(1+sin
(d*x+c))^(1/6)/(a+a*sin(d*x+c))^(1/3)
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2652, 2651}

$$-\frac{\sqrt[6]{2}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt[6]{\sin(c+dx)+1}\,\sqrt[3]{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(-1/3),x]
```

```
[Out] -((2^(1/6)*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sin[c + d*x])
/2])/(d*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3)))
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+a\sin(c+dx)}}\,dx = \frac{\sqrt[3]{1+\sin(c+dx)}\int \frac{1}{\sqrt[3]{1+\sin(c+dx)}}\,dx}{\sqrt[3]{a+a\sin(c+dx)}}$$
$$= -\frac{\sqrt[6]{2}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt[6]{1+\sin(c+dx)}\,\sqrt[3]{a+a\sin(c+dx)}}$$

Mathematica [A] time = 0.10, size = 70, normalized size = 1.06

$$\frac{3\sqrt{2}\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+2dx+\pi)\right)\right)}{d\sqrt{1-\sin(c+dx)}\,\sqrt[3]{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(-1/3),x]
```

```
[Out] (3*Sqrt[2]*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*c + Pi + 2*
d*x)/4]^2])/(d*Sqrt[1 - Sin[c + d*x]]*(a*(1 + Sin[c + d*x]))^(1/3))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{(a\sin(dx+c)+a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^(-1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sin(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(-1/3), x)
```

maple [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + a\sin(dx + c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))^(1/3),x)
[Out] int(1/(a+a*sin(d*x+c))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sin(dx + c) + a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^(-1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(a + a\,\sin(c + d\,x))^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sin(c + d*x))^(1/3),x)
[Out] int(1/(a + a*sin(c + d*x))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a\sin(c + dx) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))**(1/3),x)
[Out] Integral((a*sin(c + d*x) + a)**(-1/3), x)
```

3.108 $\int \frac{\csc(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=77

$$-\frac{\sqrt[6]{2}\cos(c+dx)F_1\left(\frac{1}{2};1,\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}$$

```
[Out] -2^(1/6)*AppellF1(1/2,1,5/6,3/2,1-sin(d*x+c),1/2-1/2*sin(d*x+c))*cos(d*x+c)
/d/(1+sin(d*x+c))^(1/6)/(a+a*sin(d*x+c))^(1/3)
```

Rubi [A] time = 0.11, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2787, 2785, 130, 429}

$$-\frac{\sqrt[6]{2}\cos(c+dx)F_1\left(\frac{1}{2};1,\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + a*Sin[c + d*x])^(1/3),x]
```

```
[Out] -((2^(1/6)*AppellF1[1/2, 1, 5/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*x])/
2]*Cos[c + d*x])/(d*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3)))
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
```

```
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx &= \frac{\sqrt[3]{1+\sin(c+dx)}\int \frac{\csc(c+dx)}{\sqrt[3]{1+\sin(c+dx)}}\,dx}{\sqrt[3]{a+a\sin(c+dx)}}\\
&= -\frac{\cos(c+dx)\,\text{Subst}\left(\int \frac{1}{(1-x)(2-x)^{5/6}\sqrt{x}}\,dx, x, 1-\sin(c+dx)\right)}{d\sqrt{1-\sin(c+dx)}\,\sqrt[6]{1+\sin(c+dx)}\,\sqrt[3]{a+a\sin(c+dx)}}\\
&= -\frac{(2\cos(c+dx))\,\text{Subst}\left(\int \frac{1}{(1-x^2)(2-x^2)^{5/6}}\,dx, x, \sqrt{1-\sin(c+dx)}\right)}{d\sqrt{1-\sin(c+dx)}\,\sqrt[6]{1+\sin(c+dx)}\,\sqrt[3]{a+a\sin(c+dx)}}\\
&= -\frac{\sqrt[6]{2}\,F_1\left(\frac{1}{2};1,\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)}{d\sqrt[6]{1+\sin(c+dx)}\,\sqrt[3]{a+a\sin(c+dx)}}
\end{aligned}
$$

Mathematica [F] time = 3.47, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[c + d*x]/(a + a*Sin[c + d*x])^(1/3),x]
[Out] Integrate[Csc[c + d*x]/(a + a*Sin[c + d*x])^(1/3), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(1/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(dx+c)}{(a\sin(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate(csc(d*x + c)/(a*sin(d*x + c) + a)^(1/3), x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\csc(dx+c)}{(a+a\sin(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+a*sin(d*x+c))^(1/3),x)
[Out] int(csc(d*x+c)/(a+a*sin(d*x+c))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(dx+c)}{(a\sin(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate(csc(d*x + c)/(a*sin(d*x + c) + a)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(c+d\,x)\,\left(a+a\,\sin(c+d\,x)\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + a*sin(c + d*x))^(1/3)),x)
[Out] int(1/(sin(c + d*x)*(a + a*sin(c + d*x))^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc (c + dx)}{\sqrt[3]{a (\sin (c + dx) + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))**(1/3),x)
```

```
[Out] Integral(csc(c + d*x)/(a*(sin(c + d*x) + 1))**(1/3), x)
```

3.109 $\int \frac{\csc^2(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=77

$$-\frac{\sqrt[6]{2}\cos(c+dx)F_1\left(\frac{1}{2};2,\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}$$

```
[Out] -2^(1/6)*AppellF1(1/2,2,5/6,3/2,1-sin(d*x+c),1/2-1/2*sin(d*x+c))*cos(d*x+c)
/d/(1+sin(d*x+c))^(1/6)/(a+a*sin(d*x+c))^(1/3)
```

Rubi [A] time = 0.12, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2787, 2785, 130, 429}

$$-\frac{\sqrt[6]{2}\cos(c+dx)F_1\left(\frac{1}{2};2,\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + a*Sin[c + d*x])^(1/3),x]
```

```
[Out] -((2^(1/6)*AppellF1[1/2, 2, 5/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*x])/
2]*Cos[c + d*x])/(d*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3)))
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
```

```
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx &= \frac{\sqrt[3]{1+\sin(c+dx)}\int \frac{\csc^2(c+dx)}{\sqrt[3]{1+\sin(c+dx)}}\,dx}{\sqrt[3]{a+a\sin(c+dx)}}\\
&= -\frac{\cos(c+dx)\,\text{Subst}\left(\int \frac{1}{(1-x)^2(2-x)^{5/6}\sqrt{x}}\,dx, x, 1-\sin(c+dx)\right)}{d\sqrt{1-\sin(c+dx)}\,\sqrt[6]{1+\sin(c+dx)}\,\sqrt[3]{a+a\sin(c+dx)}}\\
&= -\frac{(2\cos(c+dx))\,\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(2-x^2\right)^{5/6}}\,dx, x, \sqrt{1-\sin(c+dx)}\right)}{d\sqrt{1-\sin(c+dx)}\,\sqrt[6]{1+\sin(c+dx)}\,\sqrt[3]{a+a\sin(c+dx)}}\\
&= -\frac{\sqrt[6]{2}\,F_1\left(\frac{1}{2};2,\frac{5}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)}{d\sqrt[6]{1+\sin(c+dx)}\,\sqrt[3]{a+a\sin(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 8.84, size = 184, normalized size = 2.39

$$\frac{2\,2^{2/3}\cos^{\frac{2}{3}}\left(\frac{1}{4}(2c+2dx-\pi)\right)(\cos(c+dx)+i\sin(c+dx))\left(4i\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-ie^{-i(c+dx)}\right)\cos(c+dx)(\sin(c+dx)+\right.}{5d\left(-(-1)^{3/4}e^{-\frac{1}{2}i(c+dx)}\left(e^{i(c+dx)}+i\right)\right)^{2/3}\left(1+e^{2i(c+dx)}\right)\sqrt[3]{a(\sin(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^2/(a + a*Sin[c + d*x])^(1/3),x]
```

```
[Out] (2*2^(2/3)*Cos[(2*c - Pi + 2*d*x)/4]^(2/3)*(Cos[c + d*x] + I*Sin[c + d*x])*
(1 + 4*Sin[c + d*x] + (4*I)*Cos[c + d*x]*Hypergeometric2F1[1/3, 2/3, 4/3, (
-I)/E^(I*(c + d*x))]*(1 + I*Cos[c + d*x] + Sin[c + d*x])^(2/3)))/(5*d*(-(((
```

```
-1)^(3/4)*(I + E^(I*(c + d*x))))/E^((I/2)*(c + d*x))))^(2/3)*(1 + E^((2*I)*
(c + d*x)))*(a*(1 + Sin[c + d*x]))^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(dx+c)^2}{(a\sin(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate(csc(d*x + c)^2/(a*sin(d*x + c) + a)^(1/3), x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(dx+c)}{(a+a\sin(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/3),x)
[Out] int(csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(dx+c)^2}{(a\sin(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate(csc(d*x + c)^2/(a*sin(d*x + c) + a)^(1/3), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin{(c+d\,x)}^2\,{\left(a+a\,\sin{(c+d\,x)}\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(1/3)),x)
[Out] int(1/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(1/3)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2{(c+dx)}}{\sqrt[3]{a\,(\sin{(c+dx)}+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+a*sin(d*x+c))**(1/3),x)
[Out] Integral(csc(c + d*x)**2/(a*(sin(c + d*x) + 1))**(1/3), x)
```

3.110 $\int \frac{\sin^3(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=162

$$-\frac{2\sqrt[6]{2}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{ad\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}-\frac{3\sin^2(c+dx)\cos(c+dx)}{5d(a\sin(c+dx)+a)^{4/3}}+\frac{6\cos(c+dx)}{5ad\sqrt[3]{a\sin(c+dx)+a}}+\frac{6\,c}{5d(a\,\mathrm{sin}}$$

```
[Out] 6/5*cos(d*x+c)/d/(a+a*sin(d*x+c))^(4/3)-3/5*cos(d*x+c)*sin(d*x+c)^2/d/(a+a*
sin(d*x+c))^(4/3)+6/5*cos(d*x+c)/a/d/(a+a*sin(d*x+c))^(1/3)-2*2^(1/6)*cos(d
*x+c)*hypergeom([1/2, 5/6],[3/2],1/2-1/2*sin(d*x+c))/a/d/(1+sin(d*x+c))^(1/
6)/(a+a*sin(d*x+c))^(1/3)
```

Rubi [A] time = 0.27, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2783, 2968, 3019, 2751, 2652, 2651}

$$-\frac{2\sqrt[6]{2}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{ad\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}-\frac{3\sin^2(c+dx)\cos(c+dx)}{5d(a\sin(c+dx)+a)^{4/3}}+\frac{6\cos(c+dx)}{5ad\sqrt[3]{a\sin(c+dx)+a}}+\frac{6\,c}{5d(a\,\mathrm{sin}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (6*Cos[c + d*x])/(5*d*(a + a*Sin[c + d*x])^(4/3)) - (3*Cos[c + d*x]*Sin[c +
 d*x]^2)/(5*d*(a + a*Sin[c + d*x])^(4/3)) + (6*Cos[c + d*x])/(5*a*d*(a + a*
Sin[c + d*x])^(1/3)) - (2*2^(1/6)*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/6,
3/2, (1 - Sin[c + d*x])/2])/(a*d*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*
x])^(1/3))
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2783

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + n)), x] + Dist[1/(b*(m + n)), Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n - 2)*Simp[d*(a*c*m + b*d*(n -
 1)) + b*c^2*(m + n) + d*(a*d*m + b*c*(m + 2*n - 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx = -\frac{3\cos(c+dx)\sin^2(c+dx)}{5d(a+a\sin(c+dx))^{4/3}} + \frac{3\int \frac{\sin(c+dx)\left(2a-\frac{4}{3}a\sin(c+dx)\right)}{(a+a\sin(c+dx))^{4/3}}dx}{5a}$$

$$= -\frac{3\cos(c+dx)\sin^2(c+dx)}{5d(a+a\sin(c+dx))^{4/3}} + \frac{3\int \frac{2a\sin(c+dx)-\frac{4}{3}a\sin^2(c+dx)}{(a+a\sin(c+dx))^{4/3}}dx}{5a}$$

$$= \frac{6\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}} - \frac{3\cos(c+dx)\sin^2(c+dx)}{5d(a+a\sin(c+dx))^{4/3}} - \frac{9\int \frac{-\frac{40a^2}{9}+\frac{20}{9}a^2\sin(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}dx}{25a^3}$$

$$= \frac{6\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}} - \frac{3\cos(c+dx)\sin^2(c+dx)}{5d(a+a\sin(c+dx))^{4/3}} + \frac{6\cos(c+dx)}{5ad\sqrt[3]{a+a\sin(c+dx)}} + -$$

$$= \frac{6\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}} - \frac{3\cos(c+dx)\sin^2(c+dx)}{5d(a+a\sin(c+dx))^{4/3}} + \frac{6\cos(c+dx)}{5ad\sqrt[3]{a+a\sin(c+dx)}} + -$$

$$= \frac{6\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}} - \frac{3\cos(c+dx)\sin^2(c+dx)}{5d(a+a\sin(c+dx))^{4/3}} + \frac{6\cos(c+dx)}{5ad\sqrt[3]{a+a\sin(c+dx)}} - -$$

Mathematica [A] time = 0.50, size = 116, normalized size = 0.72

$$\frac{3\cos(c+dx)\left(20\sqrt{2}\,(\sin(c+dx)+1)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+2dx+\pi)\right)\right)+\sqrt{1-\sin(c+dx)}\,(4\sin(c+dx)+\text{co}\right.}{10d\sqrt{1-\sin(c+dx)}\,(a(\sin(c+dx)+1))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (3*Cos[c + d*x]*(20*Sqrt[2]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*c + Pi
+ 2*d*x)/4]^2]*(1 + Sin[c + d*x]) + Sqrt[1 - Sin[c + d*x]]*(7 + Cos[2*(c +
d*x)] + 4*Sin[c + d*x])))/(10*d*Sqrt[1 - Sin[c + d*x]]*(a*(1 + Sin[c + d*x]
))^(4/3))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\cos(dx+c)^2-1\right)(a\sin(dx+c)+a)^{\frac{2}{3}}\sin(dx+c)}{a^2\cos(dx+c)^2-2a^2\sin(dx+c)-2a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((cos(d*x + c)^2 - 1)*(a*sin(d*x + c) + a)^(2/3)*sin(d*x + c)/(a^2*
cos(d*x + c)^2 - 2*a^2*sin(d*x + c) - 2*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^3}{(a\sin(dx+c)+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^3/(a*sin(d*x + c) + a)^(4/3), x)
```

maple [F] time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(dx+c)}{(a+a\sin(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+a*sin(d*x+c))^(4/3),x)
```

```
[Out] int(sin(d*x+c)^3/(a+a*sin(d*x+c))^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^3}{(a\sin(dx+c)+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^3/(a*sin(d*x + c) + a)^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(c+d\,x)^3}{(a+a\,\sin(c+d\,x))^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + a*sin(c + d*x))^(4/3),x)
```

```
[Out] int(sin(c + d*x)^3/(a + a*sin(c + d*x))^(4/3), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+a*sin(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

3.111 $\int \frac{\sin^2(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=129

$$\frac{13\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5\,2^{5/6}ad\sqrt[6]{\sin(c+dx)+1}\,\sqrt[3]{a\sin(c+dx)+a}}-\frac{3\cos(c+dx)}{2ad\sqrt[3]{a\sin(c+dx)+a}}-\frac{3\cos(c+dx)}{5d(a\sin(c+dx)+a)^{4/3}}$$

```
[Out] -3/5*cos(d*x+c)/d/(a+a*sin(d*x+c))^(4/3)-3/2*cos(d*x+c)/a/d/(a+a*sin(d*x+c)
)^(1/3)+13/10*2^(1/6)*cos(d*x+c)*hypergeom([1/2, 5/6],[3/2],1/2-1/2*sin(d*x
+c))/a/d/(1+sin(d*x+c))^(1/6)/(a+a*sin(d*x+c))^(1/3)
```

Rubi [A] time = 0.14, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2758, 2751, 2652, 2651}

$$\frac{13\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5\,2^{5/6}ad\sqrt[6]{\sin(c+dx)+1}\,\sqrt[3]{a\sin(c+dx)+a}}-\frac{3\cos(c+dx)}{2ad\sqrt[3]{a\sin(c+dx)+a}}-\frac{3\cos(c+dx)}{5d(a\sin(c+dx)+a)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (-3*Cos[c + d*x])/(5*d*(a + a*Sin[c + d*x])^(4/3)) - (3*Cos[c + d*x])/(2*a*
d*(a + a*Sin[c + d*x])^(1/3)) + (13*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/6
, 3/2, (1 - Sin[c + d*x])/2])/(5*2^(5/6)*a*d*(1 + Sin[c + d*x])^(1/6)*(a +
a*Sin[c + d*x])^(1/3))
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2758

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx &= -\frac{3\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}}+\frac{3\int \frac{-\frac{4a}{3}+\frac{5}{3}a\sin(c+dx)}{\sqrt[3]{a+a\sin(c+dx)}}\,dx}{5a^2}\\ &= -\frac{3\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}}-\frac{3\cos(c+dx)}{2ad\sqrt[3]{a+a\sin(c+dx)}}-\frac{13\int \frac{1}{\sqrt[3]{a+a\sin(c+dx)}}\,dx}{10a}\\ &= -\frac{3\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}}-\frac{3\cos(c+dx)}{2ad\sqrt[3]{a+a\sin(c+dx)}}-\frac{\left(13\sqrt[3]{1+\sin(c+dx)}\right)\int \frac{}{\sqrt[3]{1+}}}{10a\sqrt[3]{a+a\sin(c+d}}\\ &= -\frac{3\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}}-\frac{3\cos(c+dx)}{2ad\sqrt[3]{a+a\sin(c+dx)}}+\frac{13\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}\right.}{5\,2^{5/6}ad\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{}}\end{aligned}$$

Mathematica [A] time = 0.33, size = 108, normalized size = 0.84

$$-\frac{3\cos(c+dx)\left(13\sqrt{2}\,(\sin(c+dx)+1)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+2dx+\pi)\right)\right)+\sqrt{1-\sin(c+dx)}\,(5\sin(c+dx)+7\right.}{10d\sqrt{1-\sin(c+dx)}\,(a(\sin(c+dx)+1))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (-3*Cos[c + d*x]*(13*Sqrt[2]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*c + Pi
 + 2*d*x)/4]^2]*(1 + Sin[c + d*x]) + Sqrt[1 - Sin[c + d*x]]*(7 + 5*Sin[c +
d*x])))/(10*d*Sqrt[1 - Sin[c + d*x]]*(a*(1 + Sin[c + d*x]))^(4/3))
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\cos(dx+c)^2-1\right)(a\sin(dx+c)+a)^{\frac{2}{3}}}{a^2\cos(dx+c)^2-2\,a^2\sin(dx+c)-2\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((cos(d*x + c)^2 - 1)*(a*sin(d*x + c) + a)^(2/3)/(a^2*cos(d*x + c)^
2 - 2*a^2*sin(d*x + c) - 2*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin(dx+c)^2}{(a\sin(dx+c)+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^2/(a*sin(d*x + c) + a)^(4/3), x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int\frac{\sin^2(dx+c)}{(a+a\sin(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+a*sin(d*x+c))^(4/3),x)
```

```
[Out] int(sin(d*x+c)^2/(a+a*sin(d*x+c))^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin(dx+c)^2}{(a\sin(dx+c)+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^2/(a*sin(d*x + c) + a)^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(c+d\,x)^2}{(a+a\,\sin(c+d\,x))^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + a*sin(c + d*x))^(4/3),x)
[Out] int(sin(c + d*x)^2/(a + a*sin(c + d*x))^(4/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(c+dx)}{(a(\sin(c+dx)+1))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+a*sin(d*x+c))**(4/3),x)
[Out] Integral(sin(c + d*x)**2/(a*(sin(c + d*x) + 1))**(4/3), x)
```

3.112 $\int \frac{\sin(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=99

$$\frac{3\cos(c+dx)}{5d(a\sin(c+dx)+a)^{4/3}}-\frac{4\sqrt[6]{2}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5ad\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}$$

```
[Out] 3/5*cos(d*x+c)/d/(a+a*sin(d*x+c))^(4/3)-4/5*2^(1/6)*cos(d*x+c)*hypergeom([1
/2, 5/6],[3/2],1/2-1/2*sin(d*x+c))/a/d/(1+sin(d*x+c))^(1/6)/(a+a*sin(d*x+c)
)^(1/3)
```

Rubi [A] time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2750, 2652, 2651}

$$\frac{3\cos(c+dx)}{5d(a\sin(c+dx)+a)^{4/3}}-\frac{4\sqrt[6]{2}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5ad\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (3*Cos[c + d*x])/(5*d*(a + a*Sin[c + d*x])^(4/3)) - (4*2^(1/6)*Cos[c + d*x]
*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sin[c + d*x])/2])/(5*a*d*(1 + Sin[c
+ d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3))
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
```

```
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx &= \frac{3\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}}+\frac{4\int \frac{1}{\sqrt[3]{a+a\sin(c+dx)}}\,dx}{5a}\\ &= \frac{3\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}}+\frac{\left(4\sqrt[3]{1+\sin(c+dx)}\right)\int \frac{1}{\sqrt[3]{1+\sin(c+dx)}}\,dx}{5a\sqrt[3]{a+a\sin(c+dx)}}\\ &= \frac{3\cos(c+dx)}{5d(a+a\sin(c+dx))^{4/3}}-\frac{4\sqrt[6]{2}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5ad\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.29, size = 130, normalized size = 1.31

$$\frac{3\left(8(\sin(c+dx)+1)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+2dx+\pi)\right)\right)+\sqrt{2-2\sin(c+dx)}\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx\right.\right.}{5d\sqrt{2-2\sin(c+dx)}\,(a(\sin(c+dx)+1))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])*(Sqrt[2 - 2*Sin[c + d*x]] + 8*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*c
 + Pi + 2*d*x)/4]^2]*(1 + Sin[c + d*x])))/(5*d*Sqrt[2 - 2*Sin[c + d*x]]*(a*
(1 + Sin[c + d*x]))^(4/3))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{(a\sin(dx+c)+a)^{\frac{2}{3}}\sin(dx+c)}{a^2\cos(dx+c)^2-2a^2\sin(dx+c)-2a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(-(a*sin(d*x + c) + a)^(2/3)*sin(d*x + c)/(a^2*cos(d*x + c)^2 - 2*a
^2*sin(d*x + c) - 2*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)}{(a\sin(dx+c)+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/(a*sin(d*x + c) + a)^(4/3), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)}{(a+a\sin(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+a*sin(d*x+c))^(4/3),x)
```

```
[Out] int(sin(d*x+c)/(a+a*sin(d*x+c))^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)}{(a\sin(dx+c)+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/(a*sin(d*x + c) + a)^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(c+d\,x)}{(a+a\,\sin(c+d\,x))^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + a*sin(c + d*x))^(4/3),x)
```

```
[Out] int(sin(c + d*x)/(a + a*sin(c + d*x))^(4/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)}{(a(\sin(c+dx)+1))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sin(d*x+c))**(4/3),x)
```

```
[Out] Integral(sin(c + d*x)/(a*(sin(c + d*x) + 1))**(4/3), x)
```

3.113 $\int \frac{1}{(a+a\sin(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=69

$$-\frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{11}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{2^{5/6}ad\sqrt[6]{\sin(c+dx)+1}\,\sqrt[3]{a\sin(c+dx)+a}}$$

```
[Out] -1/2*cos(d*x+c)*hypergeom([1/2, 11/6],[3/2],1/2-1/2*sin(d*x+c))*2^(1/6)/a/d
/(1+sin(d*x+c))^(1/6)/(a+a*sin(d*x+c))^(1/3)
```

Rubi [A] time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2652, 2651}

$$-\frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{11}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{2^{5/6}ad\sqrt[6]{\sin(c+dx)+1}\,\sqrt[3]{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(-4/3),x]
```

```
[Out] -((Cos[c + d*x]*Hypergeometric2F1[1/2, 11/6, 3/2, (1 - Sin[c + d*x])/2])/(2
^(5/6)*a*d*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3)))
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(c+dx))^{4/3}}\,dx = \frac{\sqrt[3]{1+\sin(c+dx)}\int \frac{1}{(1+\sin(c+dx))^{4/3}}\,dx}{a\sqrt[3]{a+a\sin(c+dx)}}$$
$$= -\frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{11}{6};\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{2^{5/6}ad\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}$$

Mathematica [A] time = 0.21, size = 130, normalized size = 1.88

$$-\frac{3\left(\sqrt{2-2\sin(c+dx)}-2(\sin(c+dx)+1)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2c+2dx+\pi)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+\right.\right.}{5d\sqrt{2-2\sin(c+dx)}\,(a(\sin(c+dx)+1))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(-4/3),x]
```

```
[Out] (-3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])*(Sqrt[2 - 2*Sin[c + d*x]] - 2*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*
c + Pi + 2*d*x)/4]^2]*(1 + Sin[c + d*x])))/(5*d*Sqrt[2 - 2*Sin[c + d*x]]*(a
*(1 + Sin[c + d*x]))^(4/3))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(a\sin(dx+c)+a)^{\frac{2}{3}}}{a^2\cos(dx+c)^2-2a^2\sin(dx+c)-2a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(-(a*sin(d*x + c) + a)^(2/3)/(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x +
c) - 2*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sin(dx+c)+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(-4/3), x)
```

maple [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+a\sin(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))^(4/3),x)
[Out] int(1/(a+a*sin(d*x+c))^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sin(dx+c)+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^(-4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+a\,\sin(c+d\,x))^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sin(c + d*x))^(4/3),x)
[Out] int(1/(a + a*sin(c + d*x))^(4/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sin(c+dx)+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))**(4/3),x)
[Out] Integral((a*sin(c + d*x) + a)**(-4/3), x)
```

3.114 $\int \frac{\csc(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=80

$$-\frac{\cos(c+dx)F_1\left(\frac{1}{2};1,\frac{11}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{2^{5/6}ad\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}$$

```
[Out] -1/2*AppellF1(1/2,1,11/6,3/2,1-sin(d*x+c),1/2-1/2*sin(d*x+c))*cos(d*x+c)*2^
(1/6)/a/d/(1+sin(d*x+c))^(1/6)/(a+a*sin(d*x+c))^(1/3)
```

Rubi [A] time = 0.12, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2787, 2785, 130, 429}

$$-\frac{\cos(c+dx)F_1\left(\frac{1}{2};1,\frac{11}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{2^{5/6}ad\sqrt[6]{\sin(c+dx)+1}\sqrt[3]{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] -((AppellF1[1/2, 1, 11/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*x])/2]*Cos[
c + d*x])/(2^(5/6)*a*d*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3))
)
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
```

```
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx &= \frac{\sqrt[3]{1+\sin(c+dx)}\int \frac{\csc(c+dx)}{(1+\sin(c+dx))^{4/3}}\,dx}{a\sqrt[3]{a+a\sin(c+dx)}}\\
&= -\frac{\cos(c+dx)\,\text{Subst}\left(\int \frac{1}{(1-x)(2-x)^{11/6}\sqrt{x}}\,dx, x, 1-\sin(c+dx)\right)}{ad\sqrt{1-\sin(c+dx)}\,\sqrt[6]{1+\sin(c+dx)}\,\sqrt[3]{a+a\sin(c+dx)}}\\
&= -\frac{(2\cos(c+dx))\,\text{Subst}\left(\int \frac{1}{(1-x^2)(2-x^2)^{11/6}}\,dx, x, \sqrt{1-\sin(c+dx)}\right)}{ad\sqrt{1-\sin(c+dx)}\,\sqrt[6]{1+\sin(c+dx)}\,\sqrt[3]{a+a\sin(c+dx)}}\\
&= -\frac{F_1\left(\frac{1}{2};1,\frac{11}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)}{2^{5/6}ad\sqrt[6]{1+\sin(c+dx)}\,\sqrt[3]{a+a\sin(c+dx)}}
\end{aligned}$$

Mathematica [F] time = 10.32, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[c + d*x]/(a + a*Sin[c + d*x])^(4/3),x]
[Out] Integrate[Csc[c + d*x]/(a + a*Sin[c + d*x])^(4/3), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(dx+c)}{(a\sin(dx+c)+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(csc(d*x + c)/(a*sin(d*x + c) + a)^(4/3), x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\csc(dx+c)}{(a+a\sin(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+a*sin(d*x+c))^(4/3),x)
```

```
[Out] int(csc(d*x+c)/(a+a*sin(d*x+c))^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(dx+c)}{(a\sin(dx+c)+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)/(a*sin(d*x + c) + a)^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(c+d\,x)\,\left(a+a\,\sin(c+d\,x)\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + a*sin(c + d*x))^(4/3)),x)
```

```
[Out] int(1/(sin(c + d*x)*(a + a*sin(c + d*x))^(4/3)), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)}{(a(\sin(c+dx)+1))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sin(d*x+c))**(4/3),x)
[Out] Integral(csc(c + d*x)/(a*(sin(c + d*x) + 1))**(4/3), x)
```

3.115 $\int \frac{\csc^2(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=80

$$-\frac{\cos(c+dx)F_1\left(\frac{1}{2};2,\frac{11}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{2^{5/6}ad\sqrt[6]{\sin(c+dx)+1}\,\sqrt[3]{a\sin(c+dx)+a}}$$

```
[Out] -1/2*AppellF1(1/2,2,11/6,3/2,1-sin(d*x+c),1/2-1/2*sin(d*x+c))*cos(d*x+c)*2^
(1/6)/a/d/(1+sin(d*x+c))^(1/6)/(a+a*sin(d*x+c))^(1/3)
```

Rubi [A] time = 0.14, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2787, 2785, 130, 429}

$$-\frac{\cos(c+dx)F_1\left(\frac{1}{2};2,\frac{11}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{2^{5/6}ad\sqrt[6]{\sin(c+dx)+1}\,\sqrt[3]{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] -((AppellF1[1/2, 2, 11/6, 3/2, 1 - Sin[c + d*x], (1 - Sin[c + d*x])/2]*Cos[
c + d*x])/(2^(5/6)*a*d*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*x])^(1/3))
)
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
```

```
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(c+dx)}{(a+a\sin(c+dx))^{4/3}}\,dx &= \frac{\sqrt[3]{1+\sin(c+dx)}\int \frac{\csc^2(c+dx)}{(1+\sin(c+dx))^{4/3}}\,dx}{a\sqrt[3]{a+a\sin(c+dx)}}\\
&= -\frac{\cos(c+dx)\,\mathrm{Subst}\left(\int \frac{1}{(1-x)^2(2-x)^{11/6}\sqrt{x}}\,dx, x, 1-\sin(c+dx)\right)}{ad\sqrt{1-\sin(c+dx)}\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}\\
&= -\frac{(2\cos(c+dx))\,\mathrm{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(2-x^2\right)^{11/6}}\,dx, x, \sqrt{1-\sin(c+dx)}\right)}{ad\sqrt{1-\sin(c+dx)}\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}\\
&= -\frac{F_1\left(\frac{1}{2};2,\frac{11}{6};\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)}{2^{5/6}ad\sqrt[6]{1+\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 14.18, size = 230, normalized size = 2.88

$$\frac{8\,2^{2/3}\cos^{\frac{8}{3}}\left(\frac{1}{4}(2c+2dx-\pi)\right)(\cos(2(c+dx))+i\sin(2(c+dx)))\left(14i\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-ie^{-i(c+dx)}\right)(\sin(c+dx)+i\cos\cdots\right.}{55d\left(-1+ie^{i(c+dx)}\right)^3\left(e^{i(c+dx)}-i\right)\left(-(-1)^{3/4}e^{-\frac{1}{2}i(c+dx)}\left(e^{i(c\cdots}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^2/(a + a*Sin[c + d*x])^(4/3),x]
```

```
[Out] (8*2^(2/3)*Cos[(2*c - Pi + 2*d*x)/4]^(8/3)*(Cos[2*(c + d*x)] + I*Sin[2*(c +
 d*x)])*(6 - 14*Cos[2*(c + d*x)] + 35*Sin[c + d*x] + (14*I)*Hypergeometric2
F1[1/3, 2/3, 4/3, (-I)/E^(I*(c + d*x))]*(1 + I*Cos[c + d*x] + Sin[c + d*x])
```

```
^(2/3)*(2*Cos[c + d*x] + Sin[2*(c + d*x)])))/(55*d*(-1 + I*E^(I*(c + d*x)))
^3*(-I + E^(I*(c + d*x)))*(-(((-1)^(3/4)*(I + E^(I*(c + d*x))))/E^((I/2)*(c
 + d*x))))^(2/3)*(a*(1 + Sin[c + d*x]))^(4/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(4/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(dx+c)^2}{(a\sin(dx+c)+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate(csc(d*x + c)^2/(a*sin(d*x + c) + a)^(4/3), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(dx+c)}{(a+a\sin(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+a*sin(d*x+c))^(4/3),x)
[Out] int(csc(d*x+c)^2/(a+a*sin(d*x+c))^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(dx+c)^2}{(a\sin(dx+c)+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sin(d*x+c))^(4/3),x, algorithm="maxima")
[Out] integrate(csc(d*x + c)^2/(a*sin(d*x + c) + a)^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(c+d\,x\right)^2\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(4/3)),x)
[Out] int(1/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(4/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2\left(c+dx\right)}{\left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+a*sin(d*x+c))**(4/3),x)
[Out] Integral(csc(c + d*x)**2/(a*(sin(c + d*x) + 1))**(4/3), x)
```

3.116 $\int \sin^n(e+fx)(1+\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=96

$$-\frac{2(4n+5)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(2n+3)\sqrt{\sin(e+fx)+1}}-\frac{2\cos(e+fx)\sin^{n+1}(e+fx)}{f(2n+3)\sqrt{\sin(e+fx)+1}}$$

```
[Out] -2*(5+4*n)*cos(f*x+e)*hypergeom([1/2, -n],[3/2],1-sin(f*x+e))/f/(3+2*n)/(1+
sin(f*x+e))^(1/2)-2*cos(f*x+e)*sin(f*x+e)^(1+n)/f/(3+2*n)/(1+sin(f*x+e))^(1
/2)
```

Rubi [A] time = 0.11, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2763, 21, 2776, 65}

$$-\frac{2(4n+5)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(2n+3)\sqrt{\sin(e+fx)+1}}-\frac{2\cos(e+fx)\sin^{n+1}(e+fx)}{f(2n+3)\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^n*(1 + Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(5 + 4*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]
])/(f*(3 + 2*n)*Sqrt[1 + Sin[e + f*x]]) - (2*Cos[e + f*x]*Sin[e + f*x]^(1 +
 n))/(f*(3 + 2*n)*Sqrt[1 + Sin[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
```

```
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \sin^n(e+fx)(1+\sin(e+fx))^{3/2}\,dx &= -\frac{2\cos(e+fx)\sin^{1+n}(e+fx)}{f(3+2n)\sqrt{1+\sin(e+fx)}}+\frac{2\int \frac{\sin^n(e+fx)\left(\frac{1}{2}(5+4n)+\frac{1}{2}(5+4n)\sin(e+fx)\right)}{\sqrt{1+\sin(e+fx)}}\,d}{3+2n}\\ &= -\frac{2\cos(e+fx)\sin^{1+n}(e+fx)}{f(3+2n)\sqrt{1+\sin(e+fx)}}+\frac{(5+4n)\int \sin^n(e+fx)\sqrt{1+\sin(e+}}{3+2n}\\ &= -\frac{2\cos(e+fx)\sin^{1+n}(e+fx)}{f(3+2n)\sqrt{1+\sin(e+fx)}}+\frac{((5+4n)\cos(e+fx))\,\mathrm{Subst}\left(\int \frac{x^n}{\sqrt{1-x}}\right.}{f(3+2n)\sqrt{1-\sin(e+fx)}\,\sqrt{1+}}\\ &= -\frac{2(5+4n)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(3+2n)\sqrt{1+\sin(e+fx)}}-\frac{2\cos(e+fx)}{f(3+2n)\sqrt{1}}\end{aligned}$$

Mathematica [C] time = 22.54, size = 5109, normalized size = 53.22

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^n*(1 + Sin[e + f*x])^(3/2),x]
[Out] Result too large to show
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\sin\left(fx+e\right)^{n}\left(\sin\left(fx+e\right)+1\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(1+sin(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral(sin(f*x + e)^n*(sin(f*x + e) + 1)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(fx+e\right)^{n}\left(\sin\left(fx+e\right)+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(1+sin(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(sin(f*x + e)^n*(sin(f*x + e) + 1)^(3/2), x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \left(\sin^{n}\left(fx+e\right)\right)\left(1+\sin\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^n*(1+sin(f*x+e))^(3/2),x)
[Out] int(sin(f*x+e)^n*(1+sin(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(fx+e\right)^{n}\left(\sin\left(fx+e\right)+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(1+sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)^n*(sin(f*x + e) + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\sin\left(e+f\,x\right)}^n\,{\left(\sin\left(e+f\,x\right)+1\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^n*(sin(e + f*x) + 1)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^n*(sin(e + f*x) + 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sin\left(e+fx\right)+1\right)^{\frac{3}{2}}\sin^{n}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**n*(1+sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((sin(e + f*x) + 1)**(3/2)*sin(e + f*x)**n, x)
```

3.117 $\int \sin^n(e+fx)\sqrt{1+\sin(e+fx)}\,dx$

Optimal. Leaf size=43

$$-\frac{2\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -2*cos(f*x+e)*hypergeom([1/2, -n],[3/2],1-sin(f*x+e))/f/(1+sin(f*x+e))^(1/2
)
```

Rubi [A] time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2776, 65}

$$-\frac{2\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^n*Sqrt[1 + Sin[e + f*x]],x]
```

```
[Out] (-2*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]])/(f*Sqrt
[1 + Sin[e + f*x]])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sin^n(e+fx)\sqrt{1+\sin(e+fx)}\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{x^n}{\sqrt{1-x}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$
$$= -\frac{2\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f\sqrt{1+\sin(e+fx)}}$$

Mathematica [C] time = 0.44, size = 186, normalized size = 4.33

$$\frac{2^{1-n}e^{i(e+fx)}\left(-ie^{-i(e+fx)}\left(-1+e^{2i(e+fx)}\right)\right)^{n+1}\sqrt{\sin(e+fx)+1}\left(i(2n-1)\,{}_2F_1\left(1,\frac{1}{4}(2n+3);\frac{1}{4}(3-2n);e^{2i(e+fx)}\right)+(\right.}{f(2n-1)(2n+1)\left(e^{i(e+fx)}+i\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^n*Sqrt[1 + Sin[e + f*x]],x]
```

```
[Out] (2^(1 - n)*E^(I*(e + f*x))*(((-I)*(-1 + E^((2*I)*(e + f*x))))/E^(I*(e + f*x
)))^(1 + n)*(I*(-1 + 2*n)*Hypergeometric2F1[1, (3 + 2*n)/4, (3 - 2*n)/4, E^
((2*I)*(e + f*x))] + E^(I*(e + f*x))*(1 + 2*n)*Hypergeometric2F1[1, (5 + 2*
n)/4, (5 - 2*n)/4, E^((2*I)*(e + f*x))])*Sqrt[1 + Sin[e + f*x]])/((I + E^(I
*(e + f*x)))*f*(-1 + 2*n)*(1 + 2*n))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\sin\left(fx+e\right)^n\sqrt{\sin\left(fx+e\right)+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(1+sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sin(f*x + e)^n*sqrt(sin(f*x + e) + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(fx+e\right)^n\sqrt{\sin\left(fx+e\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(1+sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^n*sqrt(sin(f*x + e) + 1), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \left(\sin^{n}\left(fx+e\right)\right)\sqrt{1+\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^n*(1+sin(f*x+e))^(1/2),x)
[Out] int(sin(f*x+e)^n*(1+sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(fx+e\right)^{n}\sqrt{\sin\left(fx+e\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(1+sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)^n*sqrt(sin(f*x + e) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int {\sin\left(e+f\,x\right)}^n\,\sqrt{\sin\left(e+f\,x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^n*(sin(e + f*x) + 1)^(1/2),x)
[Out] int(sin(e + f*x)^n*(sin(e + f*x) + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sin\left(e+fx\right)+1}\,\sin^{n}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**n*(1+sin(f*x+e))**(1/2),x)
[Out] Integral(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**n, x)
```

3.118 $\int \frac{\sin^n(e+fx)}{\sqrt{1+\sin(e+fx)}}\,dx$

Optimal. Leaf size=58

$$-\frac{\cos(e+fx)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -AppellF1(1/2,-n,1,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos(f*x+e)/f/(1+sin
(f*x+e))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2785, 130, 429}

$$-\frac{\cos(e+fx)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^n/Sqrt[1 + Sin[e + f*x]],x]
```

```
[Out] -((AppellF1[1/2, -n, 1, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e
+ f*x])/(f*Sqrt[1 + Sin[e + f*x]]))
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
```

```
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rubi steps

$$\int \frac{\sin^n(e+fx)}{\sqrt{1+\sin(e+fx)}}\,dx = -\frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^n}{(2-x)\sqrt{x}}\,dx, x, 1-\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{(2\cos(e+fx))\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^n}{2-x^2}\,dx, x, \sqrt{1-\sin(e+fx)}\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)}{f\sqrt{1+\sin(e+fx)}}$$

Mathematica [B] time = 1.58, size = 225, normalized size = 3.88

$$\frac{\sqrt{\sin(e+fx)+1}\,\cos(e+fx)(-\sin(e+fx))^{-n}\sin^n(e+fx)\left(1-\frac{1}{\sin(e+fx)+1}\right)^{-n}\left(4\sqrt{\frac{\sin(e+fx)-1}{\sin(e+fx)+1}}\,(-\sin(e+fx))^nF_1\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^n/Sqrt[1 + Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]^n*Sqrt[1 + Sin[e + f*x]]*(4*AppellF1[-1/2 - n, -
1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)]*(-Sin[e +
f*x])^n*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])] - (1 + 2*n)*AppellF1[1
, 1/2, -n, 2, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e + f*
x]]*(1 - (1 + Sin[e + f*x])^(-1))^n))/(4*f*(1 + 2*n)*(-1 + Sin[e + f*x])*(-
Sin[e + f*x])^n*(1 - (1 + Sin[e + f*x])^(-1))^n)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sin\left(fx+e\right)^n}{\sqrt{\sin\left(fx+e\right)+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(1+sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sin(f*x + e)^n/sqrt(sin(f*x + e) + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{n}}{\sqrt{\sin\left(fx+e\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(1+sin(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sin(f*x + e)^n/sqrt(sin(f*x + e) + 1), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\sin^{n}\left(fx+e\right)}{\sqrt{1+\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^n/(1+sin(f*x+e))^(1/2),x)
[Out] int(sin(f*x+e)^n/(1+sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{n}}{\sqrt{\sin\left(fx+e\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(1+sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)^n/sqrt(sin(f*x + e) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin\left(e+f\,x\right)^{n}}{\sqrt{\sin\left(e+f\,x\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^n/(sin(e + f*x) + 1)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^n/(sin(e + f*x) + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{n}\left(e+fx\right)}{\sqrt{\sin\left(e+fx\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**n/(1+sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)**n/sqrt(sin(e + f*x) + 1), x)
```

3.119 $\int \frac{\sin^n(e+fx)}{(1+\sin(e+fx))^{3/2}}\, dx$

Optimal. Leaf size=60

$$-\frac{\cos(e+fx)F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{2f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -1/2*AppellF1(1/2,-n,2,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos(f*x+e)/f/(1
+sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2785, 130, 429}

$$-\frac{\cos(e+fx)F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{2f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^n/(1 + Sin[e + f*x])^(3/2),x]
```

```
[Out] -(AppellF1[1/2, -n, 2, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e +
 f*x])/(2*f*Sqrt[1 + Sin[e + f*x]])
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
```

```
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rubi steps

$$\int \frac{\sin^n(e+fx)}{(1+\sin(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^n}{(2-x)^2\sqrt{x}}\,dx, x, 1-\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{(2\cos(e+fx))\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^n}{\left(2-x^2\right)^2}\,dx, x, \sqrt{1-\sin(e+fx)}\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)}{2f\sqrt{1+\sin(e+fx)}}$$

Mathematica [B] time = 3.79, size = 263, normalized size = 4.38

$$\frac{\sec(e+fx)\sin^n(e+fx)\left(\sqrt{2-2\sin(e+fx)}\,(\sin(e+fx)+1)^2(-\sin(e+fx))^{-n}F_1\left(1;\frac{1}{2},-n;2;\frac{1}{2}(\sin(e+fx)+1)\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^n/(1 + Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]*Sin[e + f*x]^n*((AppellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2
, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e + f*x]]*(1 + Sin[e + f*x])^2)/(-Sin[e
+ f*x])^n - (4*(1 + Sin[e + f*x])*Sqrt[1 - 2/(1 + Sin[e + f*x])]*(2*(1 + 2*
n)*AppellF1[1/2 - n, -1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e +
f*x])^(-1)] + (-1 + 2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e
 + f*x]), (1 + Sin[e + f*x])^(-1)]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)*(1 -
(1 + Sin[e + f*x])^(-1))^n)))/(8*f*Sqrt[1 + Sin[e + f*x]])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sin\left(fx+e\right)^n\sqrt{\sin\left(fx+e\right)+1}}{\cos\left(fx+e\right)^2-2\,\sin\left(fx+e\right)-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(1+sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sin(f*x + e)^n*sqrt(sin(f*x + e) + 1)/(cos(f*x + e)^2 - 2*sin(f*x
 + e) - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{n}}{\left(\sin\left(fx+e\right)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(1+sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^n/(sin(f*x + e) + 1)^(3/2), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\sin^{n}\left(fx+e\right)}{\left(1+\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^n/(1+sin(f*x+e))^(3/2),x)
```

```
[Out] int(sin(f*x+e)^n/(1+sin(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{n}}{\left(\sin\left(fx+e\right)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(1+sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^n/(sin(f*x + e) + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\sin\left(e+f\,x\right)}^n}{{\left(\sin\left(e+f\,x\right)+1\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^n/(sin(e + f*x) + 1)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^n/(sin(e + f*x) + 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{n}\left(e+fx\right)}{\left(\sin\left(e+fx\right)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**n/(1+sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sin(e + f*x)**n/(sin(e + f*x) + 1)**(3/2), x)
```

3.120 $\int \sin^n(e+fx)(a+a\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=106

$$-\frac{2a^2(4n+5)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(2n+3)\sqrt{a\sin(e+fx)+a}}-\frac{2a^2\cos(e+fx)\sin^{n+1}(e+fx)}{f(2n+3)\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -2*a^2*(5+4*n)*cos(f*x+e)*hypergeom([1/2, -n],[3/2],1-sin(f*x+e))/f/(3+2*n)
/(a+a*sin(f*x+e))^(1/2)-2*a^2*cos(f*x+e)*sin(f*x+e)^(1+n)/f/(3+2*n)/(a+a*si
n(f*x+e))^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2763, 21, 2776, 65}

$$-\frac{2a^2(4n+5)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(2n+3)\sqrt{a\sin(e+fx)+a}}-\frac{2a^2\cos(e+fx)\sin^{n+1}(e+fx)}{f(2n+3)\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^n*(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a^2*(5 + 4*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e +
f*x]])/(f*(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]]) - (2*a^2*Cos[e + f*x]*Sin[e +
 f*x]^(1 + n))/(f*(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
```

```
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sin^n(e+fx)(a+a\sin(e+fx))^{3/2}\,dx = -\frac{2a^2\cos(e+fx)\sin^{1+n}(e+fx)}{f(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{\sin^n(e+fx)\left(\frac{1}{2}a^2(5+4n)+\frac{1}{2}a^2(5+4n)\sin(e\cdots\right.}{\sqrt{a+a\sin(e+fx)}}}{3+2n}$$

$$= -\frac{2a^2\cos(e+fx)\sin^{1+n}(e+fx)}{f(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{(a(5+4n))\int \sin^n(e+fx)\sqrt{a+a\,\mathrm{si}\cdots}}{3+2n}$$

$$= -\frac{2a^2\cos(e+fx)\sin^{1+n}(e+fx)}{f(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{\left(a^3(5+4n)\cos(e+fx)\right)\mathrm{Subst}\left(\int \cdots\right.}{f(3+2n)\sqrt{a-a\sin(e+fx)}\cdots}$$

$$= -\frac{2a^2(5+4n)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(3+2n)\sqrt{a+a\sin(e+fx)}} - \frac{2a^2\cos(e+\cdots}{f(3+2n)\sqrt{\cdots}}$$

Mathematica [C] time = 6.33, size = 5111, normalized size = 48.22

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^n*(a + a*Sin[e + f*x])^(3/2),x]

[Out] Result too large to show
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^(3/2)*sin(f*x + e)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*sin(f*x + e)^n, x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int\left(\sin^{n}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^n*(a+a*sin(f*x+e))^(3/2),x)
[Out] int(sin(f*x+e)^n*(a+a*sin(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*sin(f*x + e)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\sin\left(e+f\,x\right)}^n\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^n*(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int(sin(e + f*x)^n*(a + a*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\sin^{n}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**n*(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*sin(e + f*x)**n, x)
```

3.121 $\int \sin^n(e+fx)\sqrt{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=46

$$-\frac{2a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -2*a*cos(f*x+e)*hypergeom([1/2, -n],[3/2],1-sin(f*x+e))/f/(a+a*sin(f*x+e))^
(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2776, 65}

$$-\frac{2a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (-2*a*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]])/(f*Sq
rt[a + a*Sin[e + f*x]])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sin^n(e+fx)\sqrt{a+a\sin(e+fx)}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{x^n}{\sqrt{a-ax}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$
$$= -\frac{2a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [C] time = 4.29, size = 264, normalized size = 5.74

$$\frac{(1+i)e^{-\frac{1}{2}ifx}\sqrt{a(\sin(e+fx)+1)}\sin^n(e+fx)\left(\sin^2(e)e^{2ifx}-i\sin(2e)e^{2ifx}+\cos^2(e)\left(-e^{2ifx}\right)+1\right)^{-n}\left((2n+1)e^{ifx}\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] ((1 + I)*(E^(I*f*x)*(1 + 2*n)*Hypergeometric2F1[(1 - 2*n)/4, -n, (5 - 2*n)/
4, E^((2*I)*f*x)*(Cos[e] + I*Sin[e])^2]*(Cos[e/2] + I*Sin[e/2]) + (-1 + 2*n
)*Hypergeometric2F1[(-1 - 2*n)/4, -n, (3 - 2*n)/4, E^((2*I)*f*x)*(Cos[e] +
I*Sin[e])^2]*(I*Cos[e/2] + Sin[e/2]))*Sin[e + f*x]^n*Sqrt[a*(1 + Sin[e + f*
x])])/(E^((I/2)*f*x)*f*(-1 + 2*n)*(1 + 2*n)*(1 - E^((2*I)*f*x)*Cos[e]^2 + E
^((2*I)*f*x)*Sin[e]^2 - I*E^((2*I)*f*x)*Sin[2*e])^n*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2]))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a\sin\left(fx+e\right)+a}\,\sin\left(fx+e\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sin(f*x + e) + a)*sin(f*x + e)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin\left(fx+e\right)+a}\,\sin\left(fx+e\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sin(f*x + e) + a)*sin(f*x + e)^n, x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \left(\sin^{n}\left(fx+e\right)\right)\sqrt{a+a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^n*(a+a*sin(f*x+e))^(1/2),x)
[Out] int(sin(f*x+e)^n*(a+a*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin\left(fx+e\right)+a}\,\sin\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sin(f*x + e) + a)*sin(f*x + e)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sin\left(e+f\,x\right)^{n}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^n*(a + a*sin(e + f*x))^(1/2),x)
[Out] int(sin(e + f*x)^n*(a + a*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\sin^{n}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**n*(a+a*sin(f*x+e))**(1/2),x)
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*sin(e + f*x)**n, x)
```

3.122 $\int \frac{\sin^n(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=60

$$-\frac{\cos(e+fx)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -AppellF1(1/2,-n,1,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos(f*x+e)/f/(a+a*s
in(f*x+e))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2787, 2785, 130, 429}

$$-\frac{\cos(e+fx)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^n/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((AppellF1[1/2, -n, 1, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e
+ f*x])/(f*Sqrt[a + a*Sin[e + f*x]]))
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
```

```
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^n(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx &= \frac{\sqrt{1+\sin(e+fx)}\int \frac{\sin^n(e+fx)}{\sqrt{1+\sin(e+fx)}}\,dx}{\sqrt{a+a\sin(e+fx)}}\\
&= -\frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^n}{(2-x)\sqrt{x}}\,dx, x, 1-\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\\
&= -\frac{(2\cos(e+fx))\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^n}{2-x^2}\,dx, x, \sqrt{1-\sin(e+fx)}\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}\\
&= -\frac{F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}
\end{aligned}$$

Mathematica [B] time = 1.34, size = 234, normalized size = 3.90

$$\cos(e+fx)\sqrt{a(\sin(e+fx)+1)}\,\sin^{2n}(e+fx)\left(-\sin^2(e+fx)\right)^{-n}\left(1-\frac{1}{\sin(e+fx)+1}\right)^{-n}\left(4\sqrt{\frac{\sin(e+fx)-1}{\sin(e+fx)+1}}\,(-\sin(e+f$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^n/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]^(2*n)*Sqrt[a*(1 + Sin[e + f*x])]*(4*AppellF1[-1/
2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)]*(-
Sin[e + f*x])^n*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])] - (1 + 2*n)*Ap
pellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Si
```

```
n[e + f*x]]*(1 - (1 + Sin[e + f*x])^(-1))^n))/(4*a*f*(1 + 2*n)*(-1 + Sin[e
+ f*x])*(-Sin[e + f*x]^2)^n*(1 - (1 + Sin[e + f*x])^(-1))^n)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sin\left(fx+e\right)^{n}}{\sqrt{a\sin\left(fx+e\right)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sin(f*x + e)^n/sqrt(a*sin(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{n}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sin(f*x + e)^n/sqrt(a*sin(f*x + e) + a), x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\sin^{n}\left(fx+e\right)}{\sqrt{a+a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^n/(a+a*sin(f*x+e))^(1/2),x)
[Out] int(sin(f*x+e)^n/(a+a*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{n}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)^n/sqrt(a*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin\left(e+f\,x\right)^n}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^n/(a + a*sin(e + f*x))^(1/2),x)
[Out] int(sin(e + f*x)^n/(a + a*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{n}\left(e+fx\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**n/(a+a*sin(f*x+e))**(1/2),x)
[Out] Integral(sin(e + f*x)**n/sqrt(a*(sin(e + f*x) + 1)), x)
```

3.123 $\int \frac{\sin^n(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{\cos(e+fx)F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{2af\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -1/2*AppellF1(1/2,-n,2,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos(f*x+e)/a/f/
(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2787, 2785, 130, 429}

$$-\frac{\cos(e+fx)F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{2af\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^n/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(AppellF1[1/2, -n, 2, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e +
 f*x])/(2*a*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
```

```
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^n(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx &= \frac{\sqrt{1+\sin(e+fx)}\int \frac{\sin^n(e+fx)}{(1+\sin(e+fx))^{3/2}}\,dx}{a\sqrt{a+a\sin(e+fx)}}\\
&= -\frac{\cos(e+fx)\,\mathrm{Subst}\left(\int \frac{(1-x)^n}{(2-x)^2\sqrt{x}}\,dx,x,1-\sin(e+fx)\right)}{af\sqrt{1-\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}\\
&= -\frac{(2\cos(e+fx))\,\mathrm{Subst}\left(\int \frac{\left(1-x^2\right)^n}{\left(2-x^2\right)^2}\,dx,x,\sqrt{1-\sin(e+fx)}\right)}{af\sqrt{1-\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}\\
&= -\frac{F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)}{2af\sqrt{a+a\sin(e+fx)}}
\end{aligned}$$

Mathematica [B] time = 2.27, size = 274, normalized size = 4.22

$$\frac{\sec(e+fx)\sin^n(e+fx)\left(a^2\sqrt{2-2\sin(e+fx)}\,(\sin(e+fx)+1)^2(-\sin(e+fx))^{-n}F_1\left(1;\frac{1}{2},-n;2;\frac{1}{2}(\sin(e+fx)\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^n/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]*Sin[e + f*x]^n*((a^2*AppellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x
])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e + f*x]]*(1 + Sin[e + f*x])^2)/(-Si
n[e + f*x])^n - (4*a*(-1 + Sin[e + f*x])*(2*a*(1 + 2*n)*AppellF1[1/2 - n, -
```

```
1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)] + a*(-1 +
2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e
 + f*x])^(-1)]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)*Sqrt[1 - 2/(1 + Sin[e + f
*x])]*(1 - (1 + Sin[e + f*x])^(-1))^n)))/(8*a^3*f*Sqrt[a*(1 + Sin[e + f*x])
])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{a\sin\left(fx+e\right)+a}\,\sin\left(fx+e\right)^{n}}{a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*sin(f*x + e)^n/(a^2*cos(f*x + e)^2 - 2*a
^2*sin(f*x + e) - 2*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^n/(a*sin(f*x + e) + a)^(3/2), x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\sin^{n}\left(fx+e\right)}{\left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^n/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int(sin(f*x+e)^n/(a+a*sin(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)^n/(a*sin(f*x + e) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin\left(e+f\,x\right)^n}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^n/(a + a*sin(e + f*x))^(3/2),x)
[Out] int(sin(e + f*x)^n/(a + a*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^n\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**n/(a+a*sin(f*x+e))**(3/2),x)
[Out] Integral(sin(e + f*x)**n/(a*(sin(e + f*x) + 1))**(3/2), x)
```

3.124 $\int (d\sin(e+fx))^n (1+\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=130

$$\frac{(4n+5)\cos(e+fx)(d\sin(e+fx))^{n+1}\,{}_2F_1\left(\frac{1}{2},n+1;n+2;\sin(e+fx)\right)}{df(n+1)(2n+3)\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}-\frac{2\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{\sin(e+fx)+1}}$$

```
[Out] -2*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(3+2*n)/(1+sin(f*x+e))^(1/2)+(5+4*n)
*cos(f*x+e)*hypergeom([1/2, 1+n],[2+n],sin(f*x+e))*(d*sin(f*x+e))^(1+n)/d/f
/(1+n)/(3+2*n)/(1-sin(f*x+e))^(1/2)/(1+sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2763, 21, 2776, 64}

$$\frac{(4n+5)\cos(e+fx)(d\sin(e+fx))^{n+1}\,{}_2F_1\left(\frac{1}{2},n+1;n+2;\sin(e+fx)\right)}{df(n+1)(2n+3)\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}-\frac{2\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*Sqrt[1 + Sin[e +
f*x]]) + ((5 + 4*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1 + n, 2 + n, Sin[e
 + f*x]]*(d*Sin[e + f*x])^(1 + n))/(d*f*(1 + n)*(3 + 2*n)*Sqrt[1 - Sin[e +
f*x]]*Sqrt[1 + Sin[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (d\sin(e+fx))^n(1+\sin(e+fx))^{3/2}\,dx = -\frac{2\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{1+\sin(e+fx)}} + \frac{2\int \frac{(d\sin(e+fx))^n\left(\frac{1}{2}d(5+4n)+\frac{1}{2}d(5+4n)\right.}{\sqrt{1+\sin(e+fx)}}}{d(3+2n)}$$

$$= -\frac{2\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{1+\sin(e+fx)}} + \frac{(5+4n)\int (d\sin(e+fx))^n\sqrt{1+}}{3+2n}$$

$$= -\frac{2\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{1+\sin(e+fx)}} + \frac{((5+4n)\cos(e+fx))\,\mathrm{Subst}\left(\int\right.}{f(3+2n)\sqrt{1-\sin(e+fx}}$$

$$= -\frac{2\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{1+\sin(e+fx)}} + \frac{(5+4n)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},1+\right.}{df(1+n)(3+2n)\sqrt{1-}}$$

Mathematica [C] time = 6.31, size = 5129, normalized size = 39.45

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])^(3/2),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d\sin\left(fx+e\right)\right)^{n}\left(\sin\left(fx+e\right)+1\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(1+sin(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral((d*sin(f*x + e))^n*(sin(f*x + e) + 1)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d\sin\left(fx+e\right)\right)^{n}\left(\sin\left(fx+e\right)+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(1+sin(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((d*sin(f*x + e))^n*(sin(f*x + e) + 1)^(3/2), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int\left(d\sin\left(fx+e\right)\right)^{n}\left(1+\sin\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(1+sin(f*x+e))^(3/2),x)
[Out] int((d*sin(f*x+e))^n*(1+sin(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d\sin\left(fx+e\right)\right)^{n}\left(\sin\left(fx+e\right)+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(1+sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e))^n*(sin(f*x + e) + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(d\,\sin\left(e+f\,x\right)\right)}^n\,{\left(\sin\left(e+f\,x\right)+1\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(sin(e + f*x) + 1)^(3/2),x)
```

```
[Out] int((d*sin(e + f*x))^n*(sin(e + f*x) + 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(e+fx\right)\right)^{n}\left(\sin\left(e+fx\right)+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(1+sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*sin(e + f*x))**n*(sin(e + f*x) + 1)**(3/2), x)
```

3.125 $\int (d\sin(e+fx))^n\sqrt{1+\sin(e+fx)}\,dx$

Optimal. Leaf size=72

$$\frac{\cos(e+fx)(d\sin(e+fx))^{n+1}\,{}_2F_1\left(\frac{1}{2},n+1;n+2;\sin(e+fx)\right)}{df(n+1)\sqrt{1-\sin(e+fx)}\,\sqrt{\sin(e+fx)+1}}$$

```
[Out] cos(f*x+e)*hypergeom([1/2, 1+n],[2+n],sin(f*x+e))*(d*sin(f*x+e))^(1+n)/d/f/
(1+n)/(1-sin(f*x+e))^(1/2)/(1+sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2776, 64}

$$\frac{\cos(e+fx)(d\sin(e+fx))^{n+1}\,{}_2F_1\left(\frac{1}{2},n+1;n+2;\sin(e+fx)\right)}{df(n+1)\sqrt{1-\sin(e+fx)}\,\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*Sqrt[1 + Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, 1 + n, 2 + n, Sin[e + f*x]]*(d*Sin[e +
 f*x])^(1 + n))/(d*f*(1 + n)*Sqrt[1 - Sin[e + f*x]]*Sqrt[1 + Sin[e + f*x]])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !IntegerQ[2*n]
```

Rubi steps

$$\int (d\sin(e+fx))^n\sqrt{1+\sin(e+fx)}\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(dx)^n}{\sqrt{1-x}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$
$$= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},1+n;2+n;\sin(e+fx)\right)(d\sin(e+fx))^{1+n}}{df(1+n)\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

Mathematica [C] time = 0.35, size = 215, normalized size = 2.99

$$\frac{(1-i)2^{-n}e^{\frac{1}{2}i(e+fx)}\left(-ie^{-i(e+fx)}\left(-1+e^{2i(e+fx)}\right)\right)^{n+1}\sqrt{\sin(e+fx)+1}\left(i(2n-1)\,{}_2F_1\left(1,\frac{1}{4}(2n+3);\frac{1}{4}(3-2n);e^{2i(e+f}\right.\right.}{f(2n-1)(2n+1)\left(\sin\left(\frac{1}{2}(e+fx)\right)+c\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n*Sqrt[1 + Sin[e + f*x]],x]
```

```
[Out] ((1 - I)*E^((I/2)*(e + f*x))*(((-I)*(-1 + E^((2*I)*(e + f*x))))/E^(I*(e + f
*x)))^(1 + n)*(I*(-1 + 2*n)*Hypergeometric2F1[1, (3 + 2*n)/4, (3 - 2*n)/4,
E^((2*I)*(e + f*x))] + E^(I*(e + f*x))*(1 + 2*n)*Hypergeometric2F1[1, (5 +
2*n)/4, (5 - 2*n)/4, E^((2*I)*(e + f*x))])*(d*Sin[e + f*x])^n*Sqrt[1 + Sin[
e + f*x]])/(2^n*f*(-1 + 2*n)*(1 + 2*n)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*Sin[e + f*x]^n)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d\sin\left(fx+e\right)\right)^n\sqrt{\sin\left(fx+e\right)+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(1+sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e))^n*sqrt(sin(f*x + e) + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)\right)^n\sqrt{\sin\left(fx+e\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(1+sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^n*sqrt(sin(f*x + e) + 1), x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \left(d \sin \left(f x+e\right)\right)^{n} \sqrt{1+\sin \left(f x+e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(1+sin(f*x+e))^(1/2),x)
[Out] int((d*sin(f*x+e))^n*(1+sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \sin \left(f x+e\right)\right)^{n} \sqrt{\sin \left(f x+e\right)+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(1+sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e))^n*sqrt(sin(f*x + e) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\sin\left(e+f\,x\right)\right)}^n\,\sqrt{\sin\left(e+f\,x\right)+1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(sin(e + f*x) + 1)^(1/2),x)
[Out] int((d*sin(e + f*x))^n*(sin(e + f*x) + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \sin{\left(e + f x \right)}\right)^{n} \sqrt{\sin{\left(e + f x \right)} + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(1+sin(f*x+e))**(1/2),x)
[Out] Integral((d*sin(e + f*x))**n*sqrt(sin(e + f*x) + 1), x)
```

3.126 $\int \frac{(d\sin(e+fx))^n}{\sqrt{1+\sin(e+fx)}}\,dx$

Optimal. Leaf size=78

$$-\frac{\cos(e+fx)\sin^{-n}(e+fx)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+fx))^n}{f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -AppellF1(1/2,-n,1,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(d*sin(f
*x+e))^n/f/(sin(f*x+e)^n)/(1+sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2786, 2785, 130, 429}

$$-\frac{\cos(e+fx)\sin^{-n}(e+fx)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+fx))^n}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n/Sqrt[1 + Sin[e + f*x]],x]
```

```
[Out] -((AppellF1[1/2, -n, 1, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e
+ f*x]*(d*Sin[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[1 + Sin[e + f*x]]))
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
```

```
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2786

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[((d/b)^IntPart[n]*(d*Sin[e + f*x])^FracPart[n
])/(b*Sin[e + f*x])^FracPart[n], Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^n}{\sqrt{1+\sin(e+fx)}}\,dx = \left(\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\int \frac{\sin^n(e+fx)}{\sqrt{1+\sin(e+fx)}}\,dx$$

$$= -\frac{\left(\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\text{Subst}\left(\int \frac{(1-x)^n}{(2-x)\sqrt{x}}\,dx, x, 1-\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{\left(2\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^n}{2-x^2}\,dx, x, \sqrt{1-\sin(e+fx}\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e}{f\sqrt{1+\sin(e+fx)}}$$

Mathematica [B] time = 0.41, size = 227, normalized size = 2.91

$$\frac{\sqrt{\sin(e+fx)+1}\,\cos(e+fx)(-\sin(e+fx))^{-n}\left(1-\frac{1}{\sin(e+fx)+1}\right)^{-n}\left(4\sqrt{\frac{\sin(e+fx)-1}{\sin(e+fx)+1}}(-\sin(e+fx))^nF_1\left(-n-\frac{1}{2};-\frac{1}{2},\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n/Sqrt[1 + Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*(d*Sin[e + f*x])^n*Sqrt[1 + Sin[e + f*x]]*(4*AppellF1[-1/2 -
n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)]*(-Sin[
e + f*x])^n*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])] - (1 + 2*n)*Appell
F1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e
+ f*x]]*(1 - (1 + Sin[e + f*x])^(-1))^n))/(4*f*(1 + 2*n)*(-1 + Sin[e + f*x]
)*(-Sin[e + f*x])^n*(1 - (1 + Sin[e + f*x])^(-1))^n)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d\sin\left(fx+e\right)\right)^{n}}{\sqrt{\sin\left(fx+e\right)+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(1+sin(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral((d*sin(f*x + e))^n/sqrt(sin(f*x + e) + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(d\sin\left(fx+e\right)\right)^{n}}{\sqrt{\sin\left(fx+e\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(1+sin(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((d*sin(f*x + e))^n/sqrt(sin(f*x + e) + 1), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int\frac{\left(d\sin\left(fx+e\right)\right)^{n}}{\sqrt{1+\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n/(1+sin(f*x+e))^(1/2),x)
[Out] int((d*sin(f*x+e))^n/(1+sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(d\sin\left(fx+e\right)\right)^{n}}{\sqrt{\sin\left(fx+e\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(1+sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e))^n/sqrt(sin(f*x + e) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,\sin\left(e+f\,x\right)\right)^n}{\sqrt{\sin\left(e+f\,x\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n/(sin(e + f*x) + 1)^(1/2),x)
[Out] int((d*sin(e + f*x))^n/(sin(e + f*x) + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(e+fx\right)\right)^n}{\sqrt{\sin\left(e+fx\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n/(1+sin(f*x+e))**(1/2),x)
[Out] Integral((d*sin(e + f*x))**n/sqrt(sin(e + f*x) + 1), x)
```

3.127 $\int \frac{(d\sin(e+fx))^n}{(1+\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=80

$$-\frac{\cos(e+fx)\sin^{-n}(e+fx)F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+fx))^n}{2f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -1/2*AppellF1(1/2,-n,2,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(d*s
in(f*x+e))^n/f/(sin(f*x+e)^n)/(1+sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2786, 2785, 130, 429}

$$-\frac{\cos(e+fx)\sin^{-n}(e+fx)F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+fx))^n}{2f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n/(1 + Sin[e + f*x])^(3/2),x]
```

```
[Out] -(AppellF1[1/2, -n, 2, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e +
 f*x]*(d*Sin[e + f*x])^n)/(2*f*Sin[e + f*x]^n*Sqrt[1 + Sin[e + f*x]])
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
```

```
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2786

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[((d/b)^IntPart[n]*(d*Sin[e + f*x])^FracPart[n
])/(b*Sin[e + f*x])^FracPart[n], Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^n}{(1+\sin(e+fx))^{3/2}}\,dx = \left(\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\int \frac{\sin^n(e+fx)}{(1+\sin(e+fx))^{3/2}}\,dx$$
$$= -\frac{\left(\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\text{Subst}\left(\int \frac{(1-x)^n}{(2-x)^2\sqrt{x}}\,dx, x, 1-\sin(e+fx\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$
$$= -\frac{\left(2\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^n}{\left(2-x^2\right)^2}\,dx, x, \sqrt{1-\sin(e+f\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$
$$= -\frac{F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(}{2f\sqrt{1+\sin(e+fx)}}$$

Mathematica [B] time = 0.93, size = 265, normalized size = 3.31

$$\frac{\sec(e+fx)\left(\sqrt{2-2\sin(e+fx)}(\sin(e+fx)+1)^2(-\sin(e+fx))^{-n}F_1\left(1;\frac{1}{2},-n;2;\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n/(1 + Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]*(d*Sin[e + f*x])^n*((AppellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x
])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e + f*x]]*(1 + Sin[e + f*x])^2)/(-Si
n[e + f*x])^n - (4*(1 + Sin[e + f*x])*Sqrt[1 - 2/(1 + Sin[e + f*x])]*(2*(1
+ 2*n)*AppellF1[1/2 - n, -1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[
```

```
e + f*x])^(-1)] + (-1 + 2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + S
in[e + f*x]), (1 + Sin[e + f*x])^(-1)]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)*(
1 - (1 + Sin[e + f*x])^(-1))^n)))/(8*f*Sqrt[1 + Sin[e + f*x]])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(d\sin\left(fx+e\right)\right)^{n}\sqrt{\sin\left(fx+e\right)+1}}{\cos\left(fx+e\right)^{2}-2\,\sin\left(fx+e\right)-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(1+sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(d*sin(f*x + e))^n*sqrt(sin(f*x + e) + 1)/(cos(f*x + e)^2 - 2*sin
(f*x + e) - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^{n}}{\left(\sin\left(fx+e\right)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(1+sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^n/(sin(f*x + e) + 1)^(3/2), x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^{n}}{\left(1+\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n/(1+sin(f*x+e))^(3/2),x)
```

```
[Out] int((d*sin(f*x+e))^n/(1+sin(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^{n}}{\left(\sin\left(fx+e\right)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(1+sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e))^n/(sin(f*x + e) + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,\sin\left(e+f\,x\right)\right)^n}{\left(\sin\left(e+f\,x\right)+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n/(sin(e + f*x) + 1)^(3/2),x)
[Out] int((d*sin(e + f*x))^n/(sin(e + f*x) + 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(e+fx\right)\right)^n}{\left(\sin\left(e+fx\right)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n/(1+sin(f*x+e))**(3/2),x)
[Out] Integral((d*sin(e + f*x))**n/(sin(e + f*x) + 1)**(3/2), x)
```

3.128 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=131

$$-\frac{2a^2(4n+5)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(2n+3)\sqrt{a\sin(e+fx)+a}}-\frac{2a^2\cos(e+fx)(d\sin(e}{df(2n+3)\sqrt{a\sin(e+}}$$

```
[Out] -2*a^2*(5+4*n)*cos(f*x+e)*hypergeom([1/2, -n],[3/2],1-sin(f*x+e))*(d*sin(f*
x+e))^n/f/(3+2*n)/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)-2*a^2*cos(f*x+e)*(d
*sin(f*x+e))^(1+n)/d/f/(3+2*n)/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2763, 21, 2776, 67, 65}

$$-\frac{2a^2(4n+5)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(2n+3)\sqrt{a\sin(e+fx)+a}}-\frac{2a^2\cos(e+fx)(d\sin(e}{df(2n+3)\sqrt{a\sin(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a^2*(5 + 4*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e +
f*x]]*(d*Sin[e + f*x])^n)/(f*(3 + 2*n)*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*
x]]) - (2*a^2*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*Sqrt[a
+ a*Sin[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}\,dx = -\frac{2a^2\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{(d\sin(e+fx))^n\left(\frac{1}{2}a^2d(5+4n)+\frac{1}{2}a\ldots\right)}{\sqrt{a+a\sin(e+\ldots}}}{d(3+2n\ldots}$$

$$= -\frac{2a^2\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{(a(5+4n))\int(d\sin(e+fx))\ldots}{3+2\ldots}$$

$$= -\frac{2a^2\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{\left(a^3(5+4n)\cos(e+fx)\right)\mathrm{Su}\ldots}{f(3+2n)\sqrt{a-a\sin(e\ldots}}$$

$$= -\frac{2a^2\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{\left(a^3(5+4n)\cos(e+fx)\sin\ldots\right.}{f(3+2\ldots}$$

$$= -\frac{2a^2(5+4n)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)\sin^{-n}(e+\ldots}{f(3+2n)\sqrt{a+a\sin(e+fx)}}$$

Mathematica [C] time = 6.34, size = 5131, normalized size = 39.17

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(3/2),x]
[Out] Result too large to show
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e))^n, x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)\right)^{n}\left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2),x)
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\sin\left(e+f\,x\right)\right)}^n\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(a + a*sin(e + f*x))^(3/2),x)
[Out] int((d*sin(e + f*x))^n*(a + a*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(d\sin\left(e+fx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**(3/2),x)
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(d*sin(e + f*x))**n, x)
```

3.129 $\int (d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=66

$$-\frac{2a\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -2*a*cos(f*x+e)*hypergeom([1/2, -n],[3/2],1-sin(f*x+e))*(d*sin(f*x+e))^n/f/
(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2776, 67, 65}

$$-\frac{2a\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (-2*a*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*(d*Sin
[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
```

```
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int\frac{(dx)^n}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\text{Subst}\left(\int\frac{x^n}{\sqrt{a-ax}}\,dx,x,s\right.}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)\sin^{-n}(e+fx)(d\sin(e+}{f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [C] time = 0.43, size = 266, normalized size = 4.03

$$\frac{(1+i)e^{-\frac{1}{2}ifx}\sqrt{a(\sin(e+fx)+1)}\,(d\sin(e+fx))^n\left(\sin^2(e)e^{2ifx}-i\sin(2e)e^{2ifx}+\cos^2(e)\left(-e^{2ifx}\right)+1\right)^{-n}\left((2n+1)e\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sin[e + f*x])^n*Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] ((1 + I)*(E^(I*f*x)*(1 + 2*n)*Hypergeometric2F1[(1 - 2*n)/4, -n, (5 - 2*n)/
4, E^((2*I)*f*x)*(Cos[e] + I*Sin[e])^2]*(Cos[e/2] + I*Sin[e/2]) + (-1 + 2*n
)*Hypergeometric2F1[(-1 - 2*n)/4, -n, (3 - 2*n)/4, E^((2*I)*f*x)*(Cos[e] +
I*Sin[e])^2]*(I*Cos[e/2] + Sin[e/2]))*(d*Sin[e + f*x])^n*Sqrt[a*(1 + Sin[e
+ f*x])])/(E^((I/2)*f*x)*f*(-1 + 2*n)*(1 + 2*n)*(1 - E^((2*I)*f*x)*Cos[e]^2
 + E^((2*I)*f*x)*Sin[e]^2 - I*E^((2*I)*f*x)*Sin[2*e])^n*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2]))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a\sin\left(fx+e\right)+a}\,\left(d\sin\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin\left(fx+e\right)+a}\,\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)\right)^{n}\sqrt{a+a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2),x)
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin\left(fx+e\right)+a}\,\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(d\,\sin\left(e+f\,x\right)\right)}^n\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(a + a*sin(e + f*x))^(1/2),x)
[Out] int((d*sin(e + f*x))^n*(a + a*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\left(d\sin\left(e+fx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**(1/2),x)
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(d*sin(e + f*x))**n, x)
```

3.130 $\int \frac{(d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=80

$$-\frac{\cos(e+fx)\sin^{-n}(e+fx)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+fx))^n}{f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -AppellF1(1/2,-n,1,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(d*sin(f
*x+e))^n/f/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2787, 2786, 2785, 130, 429}

$$-\frac{\cos(e+fx)\sin^{-n}(e+fx)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+fx))^n}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((AppellF1[1/2, -n, 1, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e
+ f*x]*(d*Sin[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]]))
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
```

```
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2786

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[((d/b)^IntPart[n]*(d*Sin[e + f*x])^FracPart[n
])/(b*Sin[e + f*x])^FracPart[n], Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\sqrt{1+\sin(e+fx)}\int \frac{(d\sin(e+fx))^n}{\sqrt{1+\sin(e+fx)}}\,dx}{\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(\sin^{-n}(e+fx)(d\sin(e+fx))^n\sqrt{1+\sin(e+fx)}\right)\int \frac{\sin^n(e+fx)}{\sqrt{1+\sin(e+fx)}}\,dx}{\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{\left(\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\mathrm{Subst}\left(\int \frac{(1-x)^n}{(2-x)\sqrt{x}}\,dx, x, 1-\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{\left(2\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\mathrm{Subst}\left(\int \frac{\left(1-x^2\right)^n}{2-x^2}\,dx, x, \sqrt{1-\sin(e+f}\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(}{f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [B] time = 0.71, size = 242, normalized size = 3.02

$$\frac{\cos(e+fx)\sqrt{a(\sin(e+fx)+1)}\,\sin^n(e+fx)\left(-\sin^2(e+fx)\right)^{-n}\left(1-\frac{1}{\sin(e+fx)+1}\right)^{-n}\left(4\sqrt{\frac{\sin(e+fx)-1}{\sin(e+fx)+1}}(-\sin(e+fx$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]^n*(d*Sin[e + f*x])^n*Sqrt[a*(1 + Sin[e + f*x])]*
(4*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e +
 f*x])^(-1)]*(-Sin[e + f*x])^n*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])]
 - (1 + 2*n)*AppellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]
]*Sqrt[2 - 2*Sin[e + f*x]]*(1 - (1 + Sin[e + f*x])^(-1))^n))/(4*a*f*(1 + 2*
n)*(-1 + Sin[e + f*x])*(-Sin[e + f*x]^2)^n*(1 - (1 + Sin[e + f*x])^(-1))^n)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(d\sin\left(fx+e\right)\right)^{n}}{\sqrt{a\sin\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e))^n/sqrt(a*sin(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(d\sin\left(fx+e\right)\right)^{n}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^n/sqrt(a*sin(f*x + e) + a), x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int\frac{\left(d\sin\left(fx+e\right)\right)^{n}}{\sqrt{a+a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] int((d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin\left(fx+e\right)\right)^{n}}{\sqrt{a \sin\left(fx+e\right)+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^n/sqrt(a*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,\sin\left(e+f\,x\right)\right)^{n}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int((d*sin(e + f*x))^n/(a + a*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin\left(e+fx\right)\right)^{n}}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((d*sin(e + f*x))**n/sqrt(a*(sin(e + f*x) + 1)), x)
```

3.131 $\int \frac{(d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=85

$$-\frac{\cos(e+fx)\sin^{-n}(e+fx)F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+fx))^n}{2af\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -1/2*AppellF1(1/2,-n,2,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(d*s
in(f*x+e))^n/a/f/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2787, 2786, 2785, 130, 429}

$$-\frac{\cos(e+fx)\sin^{-n}(e+fx)F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+fx))^n}{2af\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(AppellF1[1/2, -n, 2, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e +
 f*x]*(d*Sin[e + f*x])^n)/(2*a*f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]])
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
```

```
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2786

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[((d/b)^IntPart[n]*(d*Sin[e + f*x])^FracPart[n
])/(b*Sin[e + f*x])^FracPart[n], Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{1+\sin(e+fx)}\int \frac{(d\sin(e+fx))^n}{(1+\sin(e+fx))^{3/2}}\,dx}{a\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(\sin^{-n}(e+fx)(d\sin(e+fx))^n\sqrt{1+\sin(e+fx)}\right)\int \frac{\sin^n(e+fx)}{(1+\sin(e+fx))^{3/2}}\,dx}{a\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{\left(\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\text{Subst}\left(\int \frac{(1-x)^n}{(2-x)^2\sqrt{x}}\,dx, x, 1-\sin(e+f}{af\sqrt{1-\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{\left(2\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^n}{\left(2-x^2\right)^2}\,dx, x, \sqrt{1-\sin(e+}{af\sqrt{1-\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{F_1\left(\frac{1}{2};-n,2;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+fx)(d\,\text{si}}{2af\sqrt{a+a\sin(e+fx)}}$$

Mathematica [B] time = 1.26, size = 276, normalized size = 3.25

$$\sec(e+fx)(d\sin(e+fx))^n \left(a^2\sqrt{2-2\sin(e+fx)}\,(\sin(e+fx)+1)^2(-\sin(e+fx))^{-n}F_1\left(1;\tfrac{1}{2},-n;2;\tfrac{1}{2}(\sin(e+f\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]*(d*Sin[e + f*x])^n*((a^2*AppellF1[1, 1/2, -n, 2, (1 + Sin[e +
 f*x])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e + f*x]]*(1 + Sin[e + f*x])^2)/
(-Sin[e + f*x])^n - (4*a*(-1 + Sin[e + f*x])*(2*a*(1 + 2*n)*AppellF1[1/2 -
n, -1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)] + a*(-
1 + 2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 + S
in[e + f*x])^(-1)]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)*Sqrt[1 - 2/(1 + Sin[e
 + f*x])]*(1 - (1 + Sin[e + f*x])^(-1))^n)))/(8*a^3*f*Sqrt[a*(1 + Sin[e + f
*x])])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{a\sin\left(fx+e\right)+a}\,\left(d\sin\left(fx+e\right)\right)^n}{a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n/(a^2*cos(f*x + e)^2 -
 2*a^2*sin(f*x + e) - 2*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^n}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^(3/2), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin \left(f x+e\right)\right)^{n}}{\left(a+a \sin \left(f x+e\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x)
[Out] int((d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin \left(f x+e\right)\right)^{n}}{\left(a \sin \left(f x+e\right)+a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,\sin\left(e+f\,x\right)\right)}^n}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n/(a + a*sin(e + f*x))^(3/2),x)
[Out] int((d*sin(e + f*x))^n/(a + a*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin{\left(e + f x \right)}\right)^{n}}{\left(a \left(\sin{\left(e + f x \right)} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n/(a+a*sin(f*x+e))**(3/2),x)
[Out] Integral((d*sin(e + f*x))**n/(a*(sin(e + f*x) + 1))**(3/2), x)
```

3.132 $\int \sin^n(e+fx)(1+\sin(e+fx))^m\,dx$

Optimal. Leaf size=71

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -2^(1/2+m)*AppellF1(1/2,-n,1/2-m,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos(f
*x+e)/f/(1+sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2785, 133}

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^n*(1 + Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - Sin[e
 + f*x])/2]*Cos[e + f*x])/(f*Sqrt[1 + Sin[e + f*x]]))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rubi steps

$$\int \sin^n(e+fx)(1+\sin(e+fx))^m\,dx = -\frac{\cos(e+fx)\,\mathrm{Subst}\left(\int \frac{(1-x)^n(2-x)^{-\frac{1}{2}+m}}{\sqrt{x}}\,dx, x, 1-\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$
$$= -\frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2}; -n, \frac{1}{2}-m; \frac{3}{2}; 1-\sin(e+fx), \frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx}{f\sqrt{1+\sin(e+fx)}}$$

Mathematica [B] time = 14.92, size = 2805, normalized size = 39.51

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^n*(1 + Sin[e + f*x])^m,x]
```

```
[Out] (-3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e
+ Pi/2 - f*x)/2]^2]*Cos[e + f*x]*Sin[e + f*x]^(2*n)*(1 + Sin[e + f*x])^m)/(
f*(Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(
-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3/2, 1
- n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^
2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]
^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)*((-3*n*Appell
F1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f
*x)/2]^2]*Cos[e + f*x]^2*Sin[e + f*x]^(-1 + n))/((Sec[(-e + Pi/2 - f*x)/2]^
2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[
(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e
 + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/
2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2
]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) + (3*AppellF1[1/2, -n, 1 + m + n, 3/2, T
an[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sin[e + f*x]^(1 + n
))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan
[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3/2,
1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2
]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/
2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) - (3*m*App
ellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2
- f*x)/2]^2]*Cos[e + f*x]*Sin[e + f*x]^n*Tan[(-e + Pi/2 - f*x)/2])/((Sec[(-
e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2
 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3/2, 1 - n, 1 +
m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + (1 +
 m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[
```

```
(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) + (3*Cos[e + f*x]*Sin
[e + f*x]^n*(-1/3*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 -
f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e
+ Pi/2 - f*x)/2]) - ((1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e
+ Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2
*Tan[(-e + Pi/2 - f*x)/2])/3))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[
1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)
/2]^2] - 2*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]
^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n,
 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + P
i/2 - f*x)/2]^2)) - (3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f
*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Cos[e + f*x]*Sin[e + f*x]^n*(-2*(n*A
ppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e +
Pi/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e +
 Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Sec[(-e + Pi/2 - f*x)/2]^2
*Tan[(-e + Pi/2 - f*x)/2] + 3*(-1/3*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2,
 Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 -
f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2]) - ((1 + m + n)*AppellF1[3/2, -n, 2 + m
+ n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e
+ Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/3) - 2*Tan[(-e + Pi/2 - f*x)/2
]^2*(n*((-3*(1 + m + n)*AppellF1[5/2, 1 - n, 2 + m + n, 7/2, Tan[(-e + Pi/2
 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(
-e + Pi/2 - f*x)/2])/5 + (3*(1 - n)*AppellF1[5/2, 2 - n, 1 + m + n, 7/2, Ta
n[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x
)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/5) + (1 + m + n)*((-3*n*AppellF1[5/2, 1 -
n, 2 + m + n, 7/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]
*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/5 - (3*(2 + m + n)*Ap
pellF1[5/2, -n, 3 + m + n, 7/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2
 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/5))))/((
Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e
+ Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3/2, 1 - n
, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]
+ (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2,
 -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)^2)))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\sin\left(fx+e\right)+1\right)^{m}\sin\left(fx+e\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(1+sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((sin(f*x + e) + 1)^m*sin(f*x + e)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sin\left(fx+e\right)+1\right)^{m}\sin\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(1+sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((sin(f*x + e) + 1)^m*sin(f*x + e)^n, x)
```

maple [F] time = 0.86, size = 0, normalized size = 0.00

$$\int \left(\sin^{n}\left(fx+e\right)\right)\left(1+\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^n*(1+sin(f*x+e))^m,x)
[Out] int(sin(f*x+e)^n*(1+sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sin\left(fx+e\right)+1\right)^{m}\sin\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(1+sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((sin(f*x + e) + 1)^m*sin(f*x + e)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\sin\left(e+f\,x\right)}^n\,{\left(\sin\left(e+f\,x\right)+1\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^n*(sin(e + f*x) + 1)^m,x)
[Out] int(sin(e + f*x)^n*(sin(e + f*x) + 1)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sin{\left(e+fx\right)}+1\right)^{m}\sin^{n}{\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**n*(1+sin(f*x+e))**m,x)
[Out] Integral((sin(e + f*x) + 1)**m*sin(e + f*x)**n, x)
```

3.133 $\int (1-\sin(e+fx))^m(-\sin(e+fx))^n\,dx$

Optimal. Leaf size=68

$$\frac{2^{m+\frac{1}{2}}\cos(e+fx)F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};\sin(e+fx)+1,\frac{1}{2}(\sin(e+fx)+1)\right)}{f\sqrt{1-\sin(e+fx)}}$$

```
[Out] 2^(1/2+m)*AppellF1(1/2,-n,1/2-m,3/2,1+sin(f*x+e),1/2+1/2*sin(f*x+e))*cos(f*
x+e)/f/(1-sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2785, 133}

$$\frac{2^{m+\frac{1}{2}}\cos(e+fx)F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};\sin(e+fx)+1,\frac{1}{2}(\sin(e+fx)+1)\right)}{f\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[e + f*x])^m*(-Sin[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 + Sin[e + f*x], (1 + Sin[e +
 f*x])/2]*Cos[e + f*x])/(f*Sqrt[1 - Sin[e + f*x]])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rubi steps

$$\int (1-\sin(e+fx))^m(-\sin(e+fx))^n\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^n(2-x)^{-\frac{1}{2}+m}}{\sqrt{x}}\,dx, x, 1+\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$
$$= \frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2}; -n, \frac{1}{2}-m; \frac{3}{2}; 1+\sin(e+fx), \frac{1}{2}(1+\sin(e+fx))\right)\cos(e+f}{f\sqrt{1-\sin(e+fx)}}$$

Mathematica [B] time = 2.32, size = 300, normalized size = 4.41

$$-\frac{(2m+3)\cos(e+fx}{f(2m+1)\left((2m+3)F_1\left(m+\frac{1}{2}; -n, m+n+1; m+\frac{3}{2}; \cot^2\left(\frac{1}{4}(2e+2fx+\pi)\right), -\tan^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\right) - 2\,\text{ta}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sin[e + f*x])^m*(-Sin[e + f*x])^n,x]
```

```
[Out] -(((3 + 2*m)*AppellF1[1/2 + m, -n, 1 + m + n, 3/2 + m, Cot[(2*e + Pi + 2*f*
x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Cos[e + f*x]*(1 - Sin[e + f*x])^m*(-
Sin[e + f*x])^n)/(f*(1 + 2*m)*((3 + 2*m)*AppellF1[1/2 + m, -n, 1 + m + n, 3
/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2] - 2*(n*A
ppellF1[3/2 + m, 1 - n, 1 + m + n, 5/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2, -T
an[(2*e - Pi + 2*f*x)/4]^2] + (1 + m + n)*AppellF1[3/2 + m, -n, 2 + m + n,
5/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2])*Tan[(2
*e - Pi + 2*f*x)/4]^2)))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-\sin\left(fx+e\right)\right)^n\left(-\sin\left(fx+e\right)+1\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(f*x+e))^m*(-sin(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((-sin(f*x + e))^n*(-sin(f*x + e) + 1)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\sin\left(fx+e\right)\right)^n\left(-\sin\left(fx+e\right)+1\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(f*x+e))^m*(-sin(f*x+e))^n,x, algorithm="giac")
[Out] integrate((-sin(f*x + e))^n*(-sin(f*x + e) + 1)^m, x)
```

maple [F] time = 1.06, size = 0, normalized size = 0.00

$$\int \left(1-\sin \left(f x +e \right)\right)^{m} \left(-\sin \left(f x +e \right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sin(f*x+e))^m*(-sin(f*x+e))^n,x)
[Out] int((1-sin(f*x+e))^m*(-sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\sin \left(f x + e\right)\right)^{n} \left(-\sin \left(f x + e\right) + 1\right)^{m}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(f*x+e))^m*(-sin(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((-sin(f*x + e))^n*(-sin(f*x + e) + 1)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(-\sin\left(e+f\,x\right)\right)}^n\,{\left(1-\sin\left(e+f\,x\right)\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sin(e + f*x))^n*(1 - sin(e + f*x))^m,x)
[Out] int((-sin(e + f*x))^n*(1 - sin(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(- \sin{\left(e + f x \right)}\right)^{n} \left(1 - \sin{\left(e + f x \right)}\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(f*x+e))**m*(-sin(f*x+e))**n,x)
[Out] Integral((-sin(e + f*x))**n*(1 - sin(e + f*x))**m, x)
```

3.134 $\int (d\sin(e+fx))^n(1+\sin(e+fx))^m\,dx$

Optimal. Leaf size=91

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -2^(1/2+m)*AppellF1(1/2,-n,1/2-m,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos(f
*x+e)*(d*sin(f*x+e))^n/f/(sin(f*x+e)^n)/(1+sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2786, 2785, 133}

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - Sin[e
 + f*x])/2]*Cos[e + f*x]*(d*Sin[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[1 + Sin
[e + f*x]]))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2786

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[((d/b)^IntPart[n]*(d*Sin[e + f*x])^FracPart[n
```

```
])/(b*Sin[e + f*x])^FracPart[n], Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rubi steps

$$\int (d\sin(e+fx))^n(1+\sin(e+fx))^m\,dx = \left(\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\int \sin^n(e+fx)(1+\sin(e+fx))^m\,dx$$

$$= -\frac{\left(\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\right)\text{Subst}\left(\int \frac{(1-x)^n(2-x)^{-\frac{1}{2}+m}}{\sqrt{x}}\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e}{f\sqrt{1+\sin(e+fx)}}$$

Mathematica [B] time = 6.22, size = 2813, normalized size = 30.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])^m,x]

[Out] (-3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e
+ Pi/2 - f*x)/2]^2]*Cos[e + f*x]*Sin[e + f*x]^n*(d*Sin[e + f*x])^n*(1 + Sin
[e + f*x])^m)/(f*(Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m
+ n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*A
ppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e +
Pi/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e +
 Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2
)*((-3*n*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan
[(-e + Pi/2 - f*x)/2]^2]*Cos[e + f*x]^2*Sin[e + f*x]^(-1 + n))/((Sec[(-e +
Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f
*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3/2, 1 - n, 1 + m +
n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + (1 + m +
 n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e
+ Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) + (3*AppellF1[1/2, -n, 1
+ m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sin[
e + f*x]^(1 + n))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 +
m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n
*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e
```

```
+ Pi/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e
 + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]
^2)) - (3*m*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -
Tan[(-e + Pi/2 - f*x)/2]^2]*Cos[e + f*x]*Sin[e + f*x]^n*Tan[(-e + Pi/2 - f*
x)/2])/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2,
 Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3
/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*
x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f
*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) + (3*C
os[e + f*x]*Sin[e + f*x]^n*(-1/3*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Ta
n[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x
)/2]^2*Tan[(-e + Pi/2 - f*x)/2]) - ((1 + m + n)*AppellF1[3/2, -n, 2 + m + n
, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + P
i/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/3))/((Sec[(-e + Pi/2 - f*x)/2]^2)
^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-
e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e +
 Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/2,
 -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^
2])*Tan[(-e + Pi/2 - f*x)/2]^2)) - (3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan
[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Cos[e + f*x]*Sin[e +
f*x]^n*(-2*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]
^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n,
 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Sec[(-e + P
i/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2] + 3*(-1/3*(n*AppellF1[3/2, 1 - n,
1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Se
c[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2]) - ((1 + m + n)*AppellF1[
3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)
/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/3) - 2*Tan[(-e
+ Pi/2 - f*x)/2]^2*(n*((-3*(1 + m + n)*AppellF1[5/2, 1 - n, 2 + m + n, 7/2,
 Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 -
f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/5 + (3*(1 - n)*AppellF1[5/2, 2 - n, 1 +
 m + n, 7/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(
-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/5) + (1 + m + n)*((-3*n*App
ellF1[5/2, 1 - n, 2 + m + n, 7/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi
/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/5 - (3
*(2 + m + n)*AppellF1[5/2, -n, 3 + m + n, 7/2, Tan[(-e + Pi/2 - f*x)/2]^2,
-Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*
x)/2])/5))))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n
, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*Appe
llF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/
2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi
/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)^2
)))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d\sin\left(fx+e\right)\right)^{n}\left(\sin\left(fx+e\right)+1\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(1+sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((d*sin(f*x + e))^n*(sin(f*x + e) + 1)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d\sin\left(fx+e\right)\right)^{n}\left(\sin\left(fx+e\right)+1\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(1+sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((d*sin(f*x + e))^n*(sin(f*x + e) + 1)^m, x)
```

maple [F] time = 0.97, size = 0, normalized size = 0.00

$$\int\left(d\sin\left(fx+e\right)\right)^{n}\left(1+\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(1+sin(f*x+e))^m,x)
[Out] int((d*sin(f*x+e))^n*(1+sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d\sin\left(fx+e\right)\right)^{n}\left(\sin\left(fx+e\right)+1\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(1+sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e))^n*(sin(f*x + e) + 1)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(d\,\sin\left(e+f\,x\right)\right)}^n\,{\left(\sin\left(e+f\,x\right)+1\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(sin(e + f*x) + 1)^m,x)
```

```
[Out] int((d*sin(e + f*x))^n*(sin(e + f*x) + 1)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(e+fx\right)\right)^{n}\left(\sin\left(e+fx\right)+1\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(1+sin(f*x+e))**m,x)
```

```
[Out] Integral((d*sin(e + f*x))**n*(sin(e + f*x) + 1)**m, x)
```

3.135 $\int (1-\sin(e+fx))^m (d\sin(e+fx))^n\,dx$

Optimal. Leaf size=90

$$\frac{2^{m+\frac{1}{2}}\cos(e+fx)(-\sin(e+fx))^{-n}(d\sin(e+fx))^n F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};\sin(e+fx)+1,\frac{1}{2}(\sin(e+fx)+1)\right)}{f\sqrt{1-\sin(e+fx)}}$$

```
[Out] 2^(1/2+m)*AppellF1(1/2,-n,1/2-m,3/2,1+sin(f*x+e),1/2+1/2*sin(f*x+e))*cos(f*
x+e)*(d*sin(f*x+e))^n/f/((-sin(f*x+e))^n)/(1-sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2786, 2785, 133}

$$\frac{2^{m+\frac{1}{2}}\cos(e+fx)(-\sin(e+fx))^{-n}(d\sin(e+fx))^n F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};\sin(e+fx)+1,\frac{1}{2}(\sin(e+fx)+1)\right)}{f\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[e + f*x])^m*(d*Sin[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 + Sin[e + f*x], (1 + Sin[e +
 f*x])/2]*Cos[e + f*x]*(d*Sin[e + f*x])^n)/(f*Sqrt[1 - Sin[e + f*x]]*(-Sin[
e + f*x])^n)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2786

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[((d/b)^IntPart[n]*(d*Sin[e + f*x])^FracPart[n
```

```
])/(b*Sin[e + f*x])^FracPart[n], Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rubi steps

$$\int (1-\sin(e+fx))^m (d\sin(e+fx))^n\,dx = \left((-\sin(e+fx))^{-n}(d\sin(e+fx))^n\right)\int (1-\sin(e+fx))^m(-\sin(e+fx)$$

$$= \frac{\left(\cos(e+fx)(-\sin(e+fx))^{-n}(d\sin(e+fx))^n\right)\text{Subst}\left(\int \frac{(1-x)^n(2-x)^{-\frac{1}{2}+m}}{\sqrt{x}}\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1+\sin(e+fx),\frac{1}{2}(1+\sin(e+fx))\right)\cos(e+f}{f\sqrt{1-\sin(e+fx)}}$$

Mathematica [B] time = 0.72, size = 300, normalized size = 3.33

$$-\frac{(2m+3)\cos(e+f}{f(2m+1)\left((2m+3)F_1\left(m+\frac{1}{2};-n,m+n+1;m+\frac{3}{2};\cot^2\left(\frac{1}{4}(2e+2fx+\pi)\right),-\tan^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)-2\text{ta}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sin[e + f*x])^m*(d*Sin[e + f*x])^n,x]
```

```
[Out] -(((3 + 2*m)*AppellF1[1/2 + m, -n, 1 + m + n, 3/2 + m, Cot[(2*e + Pi + 2*f*
x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Cos[e + f*x]*(1 - Sin[e + f*x])^m*(d
*Sin[e + f*x])^n)/(f*(1 + 2*m)*((3 + 2*m)*AppellF1[1/2 + m, -n, 1 + m + n,
3/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2] - 2*(n*
AppellF1[3/2 + m, 1 - n, 1 + m + n, 5/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2, -
Tan[(2*e - Pi + 2*f*x)/4]^2] + (1 + m + n)*AppellF1[3/2 + m, -n, 2 + m + n,
 5/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2])*Tan[(
2*e - Pi + 2*f*x)/4]^2)))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d\sin\left(fx+e\right)\right)^n\left(-\sin\left(fx+e\right)+1\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(f*x+e))^m*(d*sin(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e))^n*(-sin(f*x + e) + 1)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)\right)^{n}\left(-\sin\left(fx+e\right)+1\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(f*x+e))^m*(d*sin(f*x+e))^n,x, algorithm="giac")
[Out] integrate((d*sin(f*x + e))^n*(-sin(f*x + e) + 1)^m, x)
```

maple [F] time = 0.98, size = 0, normalized size = 0.00

$$\int \left(1-\sin\left(fx+e\right)\right)^{m}\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sin(f*x+e))^m*(d*sin(f*x+e))^n,x)
[Out] int((1-sin(f*x+e))^m*(d*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)\right)^{n}\left(-\sin\left(fx+e\right)+1\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(f*x+e))^m*(d*sin(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e))^n*(-sin(f*x + e) + 1)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\sin\left(e+f\,x\right)\right)}^n\,{\left(1-\sin\left(e+f\,x\right)\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(1 - sin(e + f*x))^m,x)
[Out] int((d*sin(e + f*x))^n*(1 - sin(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(e+fx\right)\right)^{n}\left(1-\sin\left(e+fx\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(f*x+e))**m*(d*sin(f*x+e))**n,x)
```

```
[Out] Integral((d*sin(e + f*x))**n*(1 - sin(e + f*x))**m, x)
```

3.136 $\int \sin^n(e+fx)(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=87

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+f\right.}{f}$$

```
[Out] -2^(1/2+m)*AppellF1(1/2,-n,1/2-m,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos(f
*x+e)*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/f
```

Rubi [A] time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2787, 2785, 133}

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+f\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^n*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - Sin[e
 + f*x])/2]*Cos[e + f*x]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])
^m)/f)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
```

```
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$\int \sin^n(e+fx)(a+a\sin(e+fx))^m\,dx = \left((1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\right)\int \sin^n(e+fx)(1+\sin(e+f$$

$$= -\frac{\left(\cos(e+fx)(1+\sin(e+fx))^{-\frac{1}{2}-m}(a+a\sin(e+fx))^m\right)\text{Subst}\left(\int \frac{(1-x)}{}\right.}{f\sqrt{1-\sin(e+fx)}}$$

$$= -\frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+f}{f}$$

Mathematica [B] time = 6.31, size = 2807, normalized size = 32.26

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^n*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (-3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e
+ Pi/2 - f*x)/2]^2]*Cos[e + f*x]*Sin[e + f*x]^(2*n)*(a + a*Sin[e + f*x])^m)
/(f*(Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan
[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3/2,
1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2
]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/
2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)*((-3*n*Appe
llF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 -
 f*x)/2]^2]*Cos[e + f*x]^2*Sin[e + f*x]^(-1 + n))/((Sec[(-e + Pi/2 - f*x)/2
]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Ta
n[(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(
-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[
3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)
/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) + (3*AppellF1[1/2, -n, 1 + m + n, 3/2,
 Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sin[e + f*x]^(1 +
 n))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2, T
an[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3/2
, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)
```

/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) - (3*m*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Cos[e + f*x]*Sin[e + f*x]^n*Tan[(-e + Pi/2 - f*x)/2])/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) + (3*Cos[e + f*x]*Sin[e + f*x]^n*(-1/3*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2]) - ((1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/3))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) - (3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Cos[e + f*x]*Sin[e + f*x]^n*(-2*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2] + 3*(-1/3*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2]) - ((1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/3) - 2*Tan[(-e + Pi/2 - f*x)/2]^2*(n*((-3*(1 + m + n)*AppellF1[5/2, 1 - n, 2 + m + n, 7/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/5 + (3*(1 - n)*AppellF1[5/2, 2 - n, 1 + m + n, 7/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/5) + (1 + m + n)*((-3*n*AppellF1[5/2, 1 - n, 2 + m + n, 7/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/5 - (3*(2 + m + n)*AppellF1[5/2, -n, 3 + m + n, 7/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/5))))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)^2)))

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\sin\left(fx+e\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*sin(f*x + e)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\sin\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*sin(f*x + e)^n, x)
```

maple [F] time = 1.01, size = 0, normalized size = 0.00

$$\int\left(\sin^{n}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^n*(a+a*sin(f*x+e))^m,x)
[Out] int(sin(f*x+e)^n*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\sin\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^n*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*sin(f*x + e)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\sin\left(e+f\,x\right)^{n}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^n*(a + a*sin(e + f*x))^m,x)
```

```
[Out] int(sin(e + f*x)^n*(a + a*sin(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\sin \left(e + fx\right) + 1\right)\right)^m \sin^n \left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**n*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*sin(e + f*x)**n, x)
```

3.137 $\int (-\sin(e+fx))^n(a-a\sin(e+fx))^m\,dx$

Optimal. Leaf size=85

$$\frac{2^{m+\frac{1}{2}}\cos(e+fx)(1-\sin(e+fx))^{-m-\frac{1}{2}}(a-a\sin(e+fx))^m F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};\sin(e+fx)+1,\frac{1}{2}(\sin(e+fx)+1\right.}{f}$$

```
[Out] 2^(1/2+m)*AppellF1(1/2,-n,1/2-m,3/2,1+sin(f*x+e),1/2+1/2*sin(f*x+e))*cos(f*
x+e)*(1-sin(f*x+e))^(-1/2-m)*(a-a*sin(f*x+e))^m/f
```

Rubi [A] time = 0.11, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2787, 2785, 133}

$$\frac{2^{m+\frac{1}{2}}\cos(e+fx)(1-\sin(e+fx))^{-m-\frac{1}{2}}(a-a\sin(e+fx))^m F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};\sin(e+fx)+1,\frac{1}{2}(\sin(e+fx)+1\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sin[e + f*x])^n*(a - a*Sin[e + f*x])^m,x]
```

```
[Out] (2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 + Sin[e + f*x], (1 + Sin[e +
 f*x])/2]*Cos[e + f*x]*(1 - Sin[e + f*x])^(-1/2 - m)*(a - a*Sin[e + f*x])^m
)/f
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
```

```
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (-\sin(e+fx))^n(a-a\sin(e+fx))^m\,dx = \left((1-\sin(e+fx))^{-m}(a-a\sin(e+fx))^m\right)\int (1-\sin(e+fx))^m(-\sin$$

$$= \frac{\left(\cos(e+fx)(1-\sin(e+fx))^{-\frac{1}{2}-m}(a-a\sin(e+fx))^m\right)\text{Subst}\left(\int \frac{(1}{}\right.}{f\sqrt{1+\sin(e+fx)}}$$

$$= \frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1+\sin(e+fx),\frac{1}{2}(1+\sin(e+fx))\right)\cos(e}{f}$$

Mathematica [B] time = 0.60, size = 301, normalized size = 3.54

$$-\frac{(2m+3)\cos(e+f}{f(2m+1)\left((2m+3)F_1\left(m+\frac{1}{2};-n,m+n+1;m+\frac{3}{2};\cot^2\left(\frac{1}{4}(2e+2fx+\pi)\right),-\tan^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)-2\,t}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sin[e + f*x])^n*(a - a*Sin[e + f*x])^m,x]
```

```
[Out] -((((3 + 2*m)*AppellF1[1/2 + m, -n, 1 + m + n, 3/2 + m, Cot[(2*e + Pi + 2*f*
x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Cos[e + f*x]*(-Sin[e + f*x])^n*(a -
a*Sin[e + f*x])^m)/(f*(1 + 2*m)*((3 + 2*m)*AppellF1[1/2 + m, -n, 1 + m + n,
 3/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2] - 2*(n
*AppellF1[3/2 + m, 1 - n, 1 + m + n, 5/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2,
-Tan[(2*e - Pi + 2*f*x)/4]^2] + (1 + m + n)*AppellF1[3/2 + m, -n, 2 + m + n
, 5/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2])*Tan[
(2*e - Pi + 2*f*x)/4]^2)))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-a\sin\left(fx+e\right)+a\right)^m\left(-\sin\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sin(f*x+e))^n*(a-a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((-a*sin(f*x + e) + a)^m*(-sin(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-a\sin\left(fx+e\right)+a\right)^{m}\left(-\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sin(f*x+e))^n*(a-a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((-a*sin(f*x + e) + a)^m*(-sin(f*x + e))^n, x)
```

maple [F] time = 1.17, size = 0, normalized size = 0.00

$$\int\left(-\sin\left(fx+e\right)\right)^{n}\left(a-a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sin(f*x+e))^n*(a-a*sin(f*x+e))^m,x)
[Out] int((-sin(f*x+e))^n*(a-a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-a\sin\left(fx+e\right)+a\right)^{m}\left(-\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sin(f*x+e))^n*(a-a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((-a*sin(f*x + e) + a)^m*(-sin(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(-\sin\left(e+f\,x\right)\right)}^n\,{\left(a-a\,\sin\left(e+f\,x\right)\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sin(e + f*x))^n*(a - a*sin(e + f*x))^m,x)
[Out] int((-sin(e + f*x))^n*(a - a*sin(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-\sin\left(e+fx\right)\right)^{n}\left(-a\left(\sin\left(e+fx\right)-1\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sin(f*x+e))**n*(a-a*sin(f*x+e))**m,x)
[Out] Integral((-sin(e + f*x))**n*(-a*(sin(e + f*x) - 1))**m, x)
```

3.138 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=107

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}\sin^{-n}(e+fx)(a\sin(e+fx)+a)^m(d\sin(e+fx))^nF_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\right.}{f}$$

```
[Out] -2^(1/2+m)*AppellF1(1/2,-n,1/2-m,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos(f
*x+e)*(d*sin(f*x+e))^n*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/f/(sin(f*
x+e)^n)
```

Rubi [A] time = 0.15, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2787, 2786, 2785, 133}

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}\sin^{-n}(e+fx)(a\sin(e+fx)+a)^m(d\sin(e+fx))^nF_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - Sin[e
 + f*x])/2]*Cos[e + f*x]*(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])^(-1/2 - m)*(
a + a*Sin[e + f*x])^m)/(f*Sin[e + f*x]^n))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2786

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[((d/b)^IntPart[n]*(d*Sin[e + f*x])^FracPart[n
```

```
])/(b*Sin[e + f*x])^FracPart[n], Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m\,dx = \left((1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\right)\int (d\sin(e+fx))^n(1+\sin$$

$$= \left(\sin^{-n}(e+fx)(d\sin(e+fx))^n(1+\sin(e+fx))^{-m}(a+a\sin(e+fx)\right.$$

$$= -\frac{\left(\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n(1+\sin(e+fx))^{-\frac{1}{2}-m}(\right.}{f\sqrt{1-\sin}}$$

$$= -\frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(}{}$$

Mathematica [B] time = 6.24, size = 2815, normalized size = 26.31

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (-3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e
+ Pi/2 - f*x)/2]^2]*Cos[e + f*x]*Sin[e + f*x]^n*(d*Sin[e + f*x])^n*(a + a*S
in[e + f*x])^m)/(f*(Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 +
m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n
*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e
+ Pi/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e
 + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]
^2)*((-3*n*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -T
```

an[(-e + Pi/2 - f*x)/2]^2]*Cos[e + f*x]^2*Sin[e + f*x]^(-1 + n))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) + (3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sin[e + f*x]^(1 + n))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) - (3*m*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Cos[e + f*x]*Sin[e + f*x]^n*Tan[(-e + Pi/2 - f*x)/2])/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) + (3*Cos[e + f*x]*Sin[e + f*x]^n*(-1/3*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2]) - ((1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/3))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) - (3*AppellF1[1/2, -n, 1 + m + n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Cos[e + f*x]*Sin[e + f*x]^n*(-2*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2] + 3*(-1/3*(n*AppellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2]) - ((1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/3) - 2*Tan[(-e + Pi/2 - f*x)/2]^2*(n*((-3*(1 + m + n)*AppellF1[5/2, 1 - n, 2 + m + n, 7/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/5 + (3*(1 - n)*AppellF1[5/2, 2 - n, 1 + m + n, 7/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/5) + (1 + m + n)*((-3*n*AppellF1[5/2, 1 - n, 2 + m + n, 7/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/5 -

```
(3*(2 + m + n)*AppellF1[5/2, -n, 3 + m + n, 7/2, Tan[(-e + Pi/2 - f*x)/2]^2
, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 -
f*x)/2])/5))))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -n, 1 + m +
 n, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] - 2*(n*Ap
pellF1[3/2, 1 - n, 1 + m + n, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + P
i/2 - f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, -n, 2 + m + n, 5/2, Tan[(-e +
Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)
^2)))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)
```

maple [F] time = 1.06, size = 0, normalized size = 0.00

$$\int\left(d\sin\left(fx+e\right)\right)^{n}\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m,x)
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d\ \sin\left(e+f\,x\right)\right)^n\,\left(a+a\ \sin\left(e+f\,x\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(a + a*sin(e + f*x))^m,x)
[Out] int((d*sin(e + f*x))^n*(a + a*sin(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(d\sin\left(e+fx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**m,x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*(d*sin(e + f*x))**n, x)
```

3.139 $\int (d\sin(e+fx))^n(a-a\sin(e+fx))^m\,dx$

Optimal. Leaf size=107

$$\frac{2^{m+\frac{1}{2}}\cos(e+fx)(1-\sin(e+fx))^{-m-\frac{1}{2}}(-\sin(e+fx))^{-n}(a-a\sin(e+fx))^m(d\sin(e+fx))^n F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};\right.}{f}$$

```
[Out] 2^(1/2+m)*AppellF1(1/2,-n,1/2-m,3/2,1+sin(f*x+e),1/2+1/2*sin(f*x+e))*cos(f*
x+e)*(1-sin(f*x+e))^(-1/2-m)*(d*sin(f*x+e))^n*(a-a*sin(f*x+e))^m/f/((-sin(f
*x+e))^n)
```

Rubi [A] time = 0.16, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2787, 2786, 2785, 133}

$$\frac{2^{m+\frac{1}{2}}\cos(e+fx)(1-\sin(e+fx))^{-m-\frac{1}{2}}(-\sin(e+fx))^{-n}(a-a\sin(e+fx))^m(d\sin(e+fx))^n F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(a - a*Sin[e + f*x])^m,x]
```

```
[Out] (2^(1/2 + m)*AppellF1[1/2, -n, 1/2 - m, 3/2, 1 + Sin[e + f*x], (1 + Sin[e +
 f*x])/2]*Cos[e + f*x]*(1 - Sin[e + f*x])^(-1/2 - m)*(d*Sin[e + f*x])^n*(a
- a*Sin[e + f*x])^m)/(f*(-Sin[e + f*x])^n)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2786

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[((d/b)^IntPart[n]*(d*Sin[e + f*x])^FracPart[n
```

```
])/(b*Sin[e + f*x])^FracPart[n], Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))^m\,dx = \left((1-\sin(e+fx))^{-m}(a-a\sin(e+fx))^m\right)\int (1-\sin(e+fx))^m(d\sin($$

$$= \left((1-\sin(e+fx))^{-m}(-\sin(e+fx))^{-n}(d\sin(e+fx))^n(a-a\sin(e+f\right.$$

$$= \frac{\left(\cos(e+fx)(1-\sin(e+fx))^{-\frac{1}{2}-m}(-\sin(e+fx))^{-n}(d\sin(e+fx))^n(\right.}{f\sqrt{1+\sin}}$$

$$= \frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1+\sin(e+fx),\frac{1}{2}(1+\sin(e+fx))\right)\cos(e+}{}$$

Mathematica [B] time = 0.53, size = 301, normalized size = 2.81

$$-\frac{(2m+3)\cos(e+fx}{f(2m+1)\left((2m+3)F_1\left(m+\frac{1}{2};-n,m+n+1;m+\frac{3}{2};\cot^2\left(\frac{1}{4}(2e+2fx+\pi)\right),-\tan^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)-2\,\mathrm{ta}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a - a*Sin[e + f*x])^m,x]
```

```
[Out] -(((3 + 2*m)*AppellF1[1/2 + m, -n, 1 + m + n, 3/2 + m, Cot[(2*e + Pi + 2*f*
x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Cos[e + f*x]*(d*Sin[e + f*x])^n*(a -
 a*Sin[e + f*x])^m)/(f*(1 + 2*m)*((3 + 2*m)*AppellF1[1/2 + m, -n, 1 + m + n
, 3/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2] - 2*(
n*AppellF1[3/2 + m, 1 - n, 1 + m + n, 5/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2,
 -Tan[(2*e - Pi + 2*f*x)/4]^2] + (1 + m + n)*AppellF1[3/2 + m, -n, 2 + m +
```

```
n, 5/2 + m, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2])*Tan
[(2*e - Pi + 2*f*x)/4]^2)))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a-a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((-a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a-a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((-a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)
```

maple [F] time = 1.05, size = 0, normalized size = 0.00

$$\int\left(d\sin\left(fx+e\right)\right)^{n}\left(a-a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a-a*sin(f*x+e))^m,x)
[Out] int((d*sin(f*x+e))^n*(a-a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a-a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((-a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(d\,\sin\left(e+f\,x\right)\right)}^n\,{\left(a-a\,\sin\left(e+f\,x\right)\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(a - a*sin(e + f*x))^m,x)
```

```
[Out] int((d*sin(e + f*x))^n*(a - a*sin(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(e+fx\right)\right)^{n}\left(-a\left(\sin\left(e+fx\right)-1\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a-a*sin(f*x+e))**m,x)
```

```
[Out] Integral((d*sin(e + f*x))**n*(-a*(sin(e + f*x) - 1))**m, x)
```

3.140 $\int \sin^4(c+dx)(a+a\sin(c+dx))^n\,dx$

Optimal. Leaf size=294

$$-\frac{2^{n+\frac{1}{2}}\left(n^4+6n^3+17n^2+12n+9\right)\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\right.}{d(n+1)(n+2)(n+3)(n+4)}$$

```
[Out] (-n^2-n+9)*cos(d*x+c)*(a+a*sin(d*x+c))^n/d/(n^4+10*n^3+35*n^2+50*n+24)-n*co
s(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^n/d/(3+n)/(4+n)-cos(d*x+c)*sin(d*x+c
)^3*(a+a*sin(d*x+c))^n/d/(4+n)-2^(1/2+n)*(n^4+6*n^3+17*n^2+12*n+9)*cos(d*x+
c)*hypergeom([1/2, 1/2-n],[3/2],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(-1/2-n)
*(a+a*sin(d*x+c))^n/d/(4+n)/(n^3+6*n^2+11*n+6)-(n^2+3*n+9)*cos(d*x+c)*(a+a*
sin(d*x+c))^(1+n)/a/d/(4+n)/(n^2+5*n+6)
```

Rubi [A] time = 0.51, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2783, 2983, 2968, 3023, 2751, 2652, 2651}

$$-\frac{2^{n+\frac{1}{2}}\left(n^4+6n^3+17n^2+12n+9\right)\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\right.}{d(n+1)(n+2)(n+3)(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4*(a + a*Sin[c + d*x])^n,x]
```

```
[Out] ((9 - n - n^2)*Cos[c + d*x]*(a + a*Sin[c + d*x])^n)/(d*(1 + n)*(2 + n)*(3 +
 n)*(4 + n)) - (n*Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^n)/(d*(3
 + n)*(4 + n)) - (Cos[c + d*x]*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^n)/(d*(4
 + n)) - (2^(1/2 + n)*(9 + 12*n + 17*n^2 + 6*n^3 + n^4)*Cos[c + d*x]*Hyperg
eometric2F1[1/2, 1/2 - n, 3/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-1
/2 - n)*(a + a*Sin[c + d*x])^n)/(d*(1 + n)*(2 + n)*(3 + n)*(4 + n)) - ((9 +
 3*n + n^2)*Cos[c + d*x]*(a + a*Sin[c + d*x])^(1 + n))/(a*d*(2 + n)*(3 + n)
*(4 + n))
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
```

```
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2783

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + n)), x] + Dist[1/(b*(m + n)), Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n - 2)*Simp[d*(a*c*m + b*d*(n -
 1)) + b*c^2*(m + n) + d*(a*d*m + b*c*(m + 2*n - 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
```

```
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \sin^4(c+dx)(a+a\sin(c+dx))^n\,dx = -\frac{\cos(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^n}{d(4+n)} + \frac{\int \sin^2(c+dx)(a+a\,\mathrm{si}}{}$$

$$= -\frac{n\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^n}{d(3+n)(4+n)} - \frac{\cos(c+dx)\sin^3(c+}{d(4}$$

$$= -\frac{n\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^n}{d(3+n)(4+n)} - \frac{\cos(c+dx)\sin^3(c+}{d(4}$$

$$= -\frac{n\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^n}{d(3+n)(4+n)} - \frac{\cos(c+dx)\sin^3(c+}{d(4}$$

$$= \frac{\left(9-n-n^2\right)\cos(c+dx)(a+a\sin(c+dx))^n}{d(1+n)(2+n)(3+n)(4+n)} - \frac{n\cos(c+dx)\sin^2(c+d}{d(3+n)(}$$

$$= \frac{\left(9-n-n^2\right)\cos(c+dx)(a+a\sin(c+dx))^n}{d(1+n)(2+n)(3+n)(4+n)} - \frac{n\cos(c+dx)\sin^2(c+d}{d(3+n)(}$$

$$= \frac{\left(9-n-n^2\right)\cos(c+dx)(a+a\sin(c+dx))^n}{d(1+n)(2+n)(3+n)(4+n)} - \frac{n\cos(c+dx)\sin^2(c+d}{d(3+n)(}$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[c + d*x]^4*(a + a*Sin[c + d*x])^n,x]
[Out] $Aborted
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(\cos\left(dx+c\right)^4 - 2\,\cos\left(dx+c\right)^2 + 1\right)\left(a\sin\left(dx+c\right)+a\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+a*sin(d*x+c))^n,x, algorithm="fricas")
[Out] integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*(a*sin(d*x + c) + a)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^n \sin(dx+c)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+a*sin(d*x+c))^n,x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

maple [F] time = 3.04, size = 0, normalized size = 0.00

$$\int \left(\sin^4(dx+c)\right)(a+a\sin(dx+c))^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4*(a+a*sin(d*x+c))^n,x)
[Out] int(sin(d*x+c)^4*(a+a*sin(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^n \sin(dx+c)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+a*sin(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sin(c+d\,x)^4\,(a+a\,\sin(c+d\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4*(a + a*sin(c + d*x))^n,x)
[Out] int(sin(c + d*x)^4*(a + a*sin(c + d*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^n \sin^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4*(a+a*sin(d*x+c))**n,x)
[Out] Integral((a*(sin(c + d*x) + 1))**n*sin(c + d*x)**4, x)
```

3.141 $\int \sin^3(c+dx)(a+a\sin(c+dx))^n\,dx$

Optimal. Leaf size=215

$$-\frac{2^{n+\frac{1}{2}} n \left(n^2+3n+5\right)\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(n+1)(n+2)(n+3)}$$

```
[Out] -(4+n)*cos(d*x+c)*(a+a*sin(d*x+c))^n/d/(n^3+6*n^2+11*n+6)-cos(d*x+c)*sin(d*
x+c)^2*(a+a*sin(d*x+c))^n/d/(3+n)-2^(1/2+n)*n*(n^2+3*n+5)*cos(d*x+c)*hyperg
eom([1/2, 1/2-n],[3/2],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(-1/2-n)*(a+a*sin
(d*x+c))^n/d/(n^3+6*n^2+11*n+6)-n*cos(d*x+c)*(a+a*sin(d*x+c))^(1+n)/a/d/(n^
2+5*n+6)
```

Rubi [A] time = 0.29, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2783, 2968, 3023, 2751, 2652, 2651}

$$-\frac{2^{n+\frac{1}{2}} n \left(n^2+3n+5\right)\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(n+1)(n+2)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^n,x]
```

```
[Out] -(((4 + n)*Cos[c + d*x]*(a + a*Sin[c + d*x])^n)/(d*(1 + n)*(2 + n)*(3 + n))
) - (Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^n)/(d*(3 + n)) - (2^(
1/2 + n)*n*(5 + 3*n + n^2)*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2
, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-1/2 - n)*(a + a*Sin[c + d*x])^
n)/(d*(1 + n)*(2 + n)*(3 + n)) - (n*Cos[c + d*x]*(a + a*Sin[c + d*x])^(1 +
n))/(a*d*(6 + 5*n + n^2))
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
```

```
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2783

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + n)), x] + Dist[1/(b*(m + n)), Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n - 2)*Simp[d*(a*c*m + b*d*(n -
 1)) + b*c^2*(m + n) + d*(a*d*m + b*c*(m + 2*n - 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^n\,dx = -\frac{\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^n}{d(3+n)} + \frac{\int \sin(c+dx)(a+a\sin}{}$$

$$= -\frac{\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^n}{d(3+n)} + \frac{\int (a+a\sin(c+dx))^n (}{}$$

$$= -\frac{\cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^n}{d(3+n)} - \frac{n\cos(c+dx)(a+a\sin}{ad\left(6+5n+\right.}$$

$$= -\frac{(4+n)\cos(c+dx)(a+a\sin(c+dx))^n}{d(1+n)(2+n)(3+n)} - \frac{\cos(c+dx)\sin^2(c+dx)(a}{d(3+n)}$$

$$= -\frac{(4+n)\cos(c+dx)(a+a\sin(c+dx))^n}{d(1+n)(2+n)(3+n)} - \frac{\cos(c+dx)\sin^2(c+dx)(a}{d(3+n)}$$

$$= -\frac{(4+n)\cos(c+dx)(a+a\sin(c+dx))^n}{d(1+n)(2+n)(3+n)} - \frac{\cos(c+dx)\sin^2(c+dx)(a}{d(3+n)}$$

Mathematica [C] time = 128.81, size = 60244, normalized size = 280.20

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^n,x]
[Out] Result too large to show
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos\left(dx+c\right)^2-1\right)\left(a\sin\left(dx+c\right)+a\right)^n\sin\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^n,x, algorithm="fricas")
[Out] integral(-(cos(d*x + c)^2 - 1)*(a*sin(d*x + c) + a)^n*sin(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(dx+c\right)+a\right)^n\sin\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^n,x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

maple [F] time = 2.21, size = 0, normalized size = 0.00

$$\int \left(\sin^3(dx+c)\right)(a+a\sin(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+a*sin(d*x+c))^n,x)
[Out] int(sin(d*x+c)^3*(a+a*sin(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^n\sin(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sin(c+d\,x)^3\,(a+a\,\sin(c+d\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + a*sin(c + d*x))^n,x)
[Out] int(sin(c + d*x)^3*(a + a*sin(c + d*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+a*sin(d*x+c))**n,x)
[Out] Timed out
```

3.142 $\int \sin^2(c+dx)(a+a\sin(c+dx))^n\,dx$

Optimal. Leaf size=156

$$-\frac{2^{n+\frac{1}{2}}\left(n^2+n+1\right)\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(n+1)(n+2)}+$$

```
[Out] cos(d*x+c)*(a+a*sin(d*x+c))^n/d/(n^2+3*n+2)-2^(1/2+n)*(n^2+n+1)*cos(d*x+c)*
hypergeom([1/2, 1/2-n],[3/2],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(-1/2-n)*(a
+a*sin(d*x+c))^n/d/(n^2+3*n+2)-cos(d*x+c)*(a+a*sin(d*x+c))^(1+n)/a/d/(2+n)
```

Rubi [A] time = 0.14, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2759, 2751, 2652, 2651}

$$-\frac{2^{n+\frac{1}{2}}\left(n^2+n+1\right)\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(n+1)(n+2)}+$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^n,x]
```

```
[Out] (Cos[c + d*x]*(a + a*Sin[c + d*x])^n)/(d*(2 + 3*n + n^2)) - (2^(1/2 + n)*(1
 + n + n^2)*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 - Sin[c +
d*x])/2]*(1 + Sin[c + d*x])^(-1/2 - n)*(a + a*Sin[c + d*x])^n)/(d*(1 + n)*(
2 + n)) - (Cos[c + d*x]*(a + a*Sin[c + d*x])^(1 + n))/(a*d*(2 + n))
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rubi steps

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^n\,dx = -\frac{\cos(c+dx)(a+a\sin(c+dx))^{1+n}}{ad(2+n)}+\frac{\int (a(1+n)-a\sin(c+dx))(a+a\,\mathrm{s}}{a(2+n)}$$
$$=\frac{\cos(c+dx)(a+a\sin(c+dx))^n}{d\left(2+3n+n^2\right)}-\frac{\cos(c+dx)(a+a\sin(c+dx))^{1+n}}{ad(2+n)}+\frac{(1}{}$$
$$=\frac{\cos(c+dx)(a+a\sin(c+dx))^n}{d\left(2+3n+n^2\right)}-\frac{\cos(c+dx)(a+a\sin(c+dx))^{1+n}}{ad(2+n)}+\frac{((}{}$$
$$=\frac{\cos(c+dx)(a+a\sin(c+dx))^n}{d\left(2+3n+n^2\right)}-\frac{2^{\frac{1}{2}+n}\left(1+n+n^2\right)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\right.}{}$$

Mathematica [C] time = 54.58, size = 28439, normalized size = 182.30

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^n,x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos(dx+c)^2-1\right)(a\sin(dx+c)+a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^n,x, algorithm="fricas")
[Out] integral(-(cos(d*x + c)^2 - 1)*(a*sin(d*x + c) + a)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^n\sin(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^n,x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

maple [F] time = 1.82, size = 0, normalized size = 0.00

$$\int \left(\sin^2(dx+c)\right)(a+a\sin(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^n,x)
[Out] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^n\sin(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(c+d\,x)^2\,(a+a\,\sin(c+d\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + a*sin(c + d*x))^n,x)
[Out] int(sin(c + d*x)^2*(a + a*sin(c + d*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^n\sin^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+a*sin(d*x+c))**n,x)
[Out] Integral((a*(sin(c + d*x) + 1))**n*sin(c + d*x)**2, x)
```

3.143 $\int \sin(c+dx)(a+a\sin(c+dx))^n\,dx$

Optimal. Leaf size=109

$$-\frac{2^{n+\frac{1}{2}} n \cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(n+1)}-\frac{\cos(c+dx)(a}{d(}$$

```
[Out] -cos(d*x+c)*(a+a*sin(d*x+c))^n/d/(1+n)-2^(1/2+n)*n*cos(d*x+c)*hypergeom([1/
2, 1/2-n],[3/2],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(-1/2-n)*(a+a*sin(d*x+c)
)^n/d/(1+n)
```

Rubi [A] time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2751, 2652, 2651}

$$-\frac{2^{n+\frac{1}{2}} n \cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(n+1)}-\frac{\cos(c+dx)(a}{d(}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])^n,x]
```

```
[Out] -((Cos[c + d*x]*(a + a*Sin[c + d*x])^n)/(d*(1 + n))) - (2^(1/2 + n)*n*Cos[c
 + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 - Sin[c + d*x])/2]*(1 + Sin
[c + d*x])^(-1/2 - n)*(a + a*Sin[c + d*x])^n)/(d*(1 + n))
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \sin(c+dx)(a+a\sin(c+dx))^n\,dx = -\frac{\cos(c+dx)(a+a\sin(c+dx))^n}{d(1+n)} + \frac{n\int (a+a\sin(c+dx))^n\,dx}{1+n}$$
$$= -\frac{\cos(c+dx)(a+a\sin(c+dx))^n}{d(1+n)} + \frac{(n(1+\sin(c+dx))^{-n}(a+a\sin(c+d$$
$$1+n$$
$$= -\frac{\cos(c+dx)(a+a\sin(c+dx))^n}{d(1+n)} - \frac{2^{\frac{1}{2}+n}n\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(\right.$$

Mathematica [C] time = 0.44, size = 178, normalized size = 1.63

$$-\frac{\sqrt[4]{-1}\,2^{-2n-1}e^{-\frac{3}{2}i(c+dx)}\left(-(-1)^{3/4}e^{-\frac{1}{2}i(c+dx)}\left(e^{i(c+dx)}+i\right)\right)^{2n+1}\left((n-1)e^{2i(c+dx)}\,{}_2F_1\left(1,n;-n;-ie^{-i(c+dx)}\right)-(n+1)\,{}_2F_1\left(\right.\right.}{d(n-1)(n+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])^n,x]
```

```
[Out] -(((-1)^(1/4)*2^(-1 - 2*n)*(-(((-1)^(3/4)*(I + E^(I*(c + d*x))))/E^((I/2)*(
c + d*x))))^(1 + 2*n)*(E^((2*I)*(c + d*x))*(-1 + n)*Hypergeometric2F1[1, n,
 -n, (-I)/E^(I*(c + d*x))] - (1 + n)*Hypergeometric2F1[1, 2 + n, 2 - n, (-I
)/E^(I*(c + d*x))])*(a*(1 + Sin[c + d*x]))^n)/(d*E^(((3*I)/2)*(c + d*x))*(-
1 + n)*(1 + n)*Sin[(2*c + Pi + 2*d*x)/4]^(2*n)))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sin(dx+c)+a)^n\sin(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^n*sin(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^n\sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^n*sin(d*x + c), x)
```

maple [F] time = 1.53, size = 0, normalized size = 0.00

$$\int \sin\left(dx+c\right)\left(a+a\sin\left(dx+c\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+a*sin(d*x+c))^n,x)
```

```
[Out] int(sin(d*x+c)*(a+a*sin(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(dx+c\right)+a\right)^{n}\sin\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^n*sin(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(c+d\,x\right)\,\left(a+a\,\sin\left(c+d\,x\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + a*sin(c + d*x))^n,x)
```

```
[Out] int(sin(c + d*x)*(a + a*sin(c + d*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(c+dx\right)+1\right)\right)^{n}\sin\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))**n,x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**n*sin(c + d*x), x)
```

3.144 $\int (a + a\sin(c + dx))^n \, dx$

Optimal. Leaf size=74

$$-\frac{2^{n+\frac{1}{2}}\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n \, {}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d}$$

```
[Out] -2^(1/2+n)*cos(d*x+c)*hypergeom([1/2, 1/2-n],[3/2],1/2-1/2*sin(d*x+c))*(1+s
in(d*x+c))^(-1/2-n)*(a+a*sin(d*x+c))^n/d
```

Rubi [A] time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2652, 2651}

$$-\frac{2^{n+\frac{1}{2}}\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n \, {}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^n,x]
```

```
[Out] -((2^(1/2 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 - Sin[c
 + d*x])/2]*(1 + Sin[c + d*x])^(-1/2 - n)*(a + a*Sin[c + d*x])^n)/d)
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+a\sin(c+dx))^n\,dx = \left((1+\sin(c+dx))^{-n}(a+a\sin(c+dx))^n\right)\int (1+\sin(c+dx))^n\,dx$$
$$= -\frac{2^{\frac{1}{2}+n}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{-\frac{1}{2}-n}(a+a\,\text{s}}{d}$$

Mathematica [A] time = 0.17, size = 90, normalized size = 1.22

$$\frac{\sqrt{2}\cos(c+dx)(a(\sin(c+dx)+1))^n\,{}_2F_1\left(\frac{1}{2},n+\frac{1}{2};n+\frac{3}{2};\frac{1}{4}\cos^2(c+dx)\csc^2\left(\frac{1}{4}(2c+2dx-\pi)\right)\right)}{(2dn+d)\sqrt{1-\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^n,x]
```

```
[Out] (Sqrt[2]*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 + n, 3/2 + n, (Cos[c + d*x
]^2*Csc[(2*c - Pi + 2*d*x)/4]^2)/4]*(a*(1 + Sin[c + d*x]))^n)/((d + 2*d*n)*
Sqrt[1 - Sin[c + d*x]])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sin(dx+c)+a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^n, x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int (a+a\sin(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^n,x)
```

```
[Out] int((a+a*sin(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin (dx+c)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\,\sin (c+d\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^n,x)
```

```
[Out] int((a + a*sin(c + d*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin (c+dx)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**n,x)
```

```
[Out] Integral((a*sin(c + d*x) + a)**n, x)
```

3.145 $\int \csc(c+dx)(a+a\sin(c+dx))^n\,dx$

Optimal. Leaf size=85

$$-\frac{2^{n+\frac{1}{2}}\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n F_1\left(\frac{1}{2};1,\frac{1}{2}-n;\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d}$$

```
[Out] -2^(1/2+n)*AppellF1(1/2,1,1/2-n,3/2,1-sin(d*x+c),1/2-1/2*sin(d*x+c))*cos(d*
x+c)*(1+sin(d*x+c))^(-1/2-n)*(a+a*sin(d*x+c))^n/d
```

Rubi [A] time = 0.11, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2787, 2785, 130, 429}

$$-\frac{2^{n+\frac{1}{2}}\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n F_1\left(\frac{1}{2};1,\frac{1}{2}-n;\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + a*Sin[c + d*x])^n,x]
```

```
[Out] -((2^(1/2 + n)*AppellF1[1/2, 1, 1/2 - n, 3/2, 1 - Sin[c + d*x], (1 - Sin[c
+ d*x])/2]*Cos[c + d*x]*(1 + Sin[c + d*x])^(-1/2 - n)*(a + a*Sin[c + d*x])^
n)/d)
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
```

```
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \csc(c+dx)(a+a\sin(c+dx))^n\,dx = \left((1+\sin(c+dx))^{-n}(a+a\sin(c+dx))^n\right)\int \csc(c+dx)(1+\sin(c+dx))^n$$

$$= -\frac{\left(\cos(c+dx)(1+\sin(c+dx))^{-\frac{1}{2}-n}(a+a\sin(c+dx))^n\right)\text{Subst}\left(\int \frac{(2-x)^{-\frac{1}{2}}}{(1-x)\sqrt{}}\right.}{d\sqrt{1-\sin(c+dx)}}$$

$$= -\frac{\left(2\cos(c+dx)(1+\sin(c+dx))^{-\frac{1}{2}-n}(a+a\sin(c+dx))^n\right)\text{Subst}\left(\int \frac{(2-x^2)}{1-}\right.}{d\sqrt{1-\sin(c+dx)}}$$

$$= -\frac{2^{\frac{1}{2}+n}F_1\left(\frac{1}{2};1,\frac{1}{2}-n;\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)(1}{d}$$

Mathematica [C] time = 16.11, size = 2560, normalized size = 30.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]*(a + a*Sin[c + d*x])^n,x]
```

```
[Out] -1/2*(Csc[c + d*x]*(a + a*Sin[c + d*x])^n*(AppellF1[2*n, n, n, 1 + 2*n, (-1
 - I)/(-1 + Tan[(-c + Pi/2 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)
/2])]*((-I + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*(
(I + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n - AppellF
1[2*n, n, n, 1 + 2*n, (1 - I)/(1 + Tan[(-c + Pi/2 - d*x)/2]), (1 + I)/(1 +
Tan[(-c + Pi/2 - d*x)/2])]*((-I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c +
Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*
x)/2]))^n))/(d*n*(Sec[(-c + Pi/2 - d*x)/2]^2)^n*(-1/2*(Tan[(-c + Pi/2 - d*x
```

```
)/2]*(AppellF1[2*n, n, n, 1 + 2*n, (-1 - I)/(-1 + Tan[(-c + Pi/2 - d*x)/2])
, (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)/2])]*((-I + Tan[(-c + Pi/2 - d*x)/2]
)/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(-1 +
Tan[(-c + Pi/2 - d*x)/2]))^n - AppellF1[2*n, n, n, 1 + 2*n, (1 - I)/(1 + Ta
n[(-c + Pi/2 - d*x)/2]), (1 + I)/(1 + Tan[(-c + Pi/2 - d*x)/2])]*((-I + Tan
[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi
/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^n))/(Sec[(-c + Pi/2 - d*x)/2]
^2)^n + ((((1 - I)*n^2*AppellF1[1 + 2*n, n, 1 + n, 2 + 2*n, (-1 - I)/(-1 +
Tan[(-c + Pi/2 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)/2])]*Sec[(-
c + Pi/2 - d*x)/2]^2)/((1 + 2*n)*(-1 + Tan[(-c + Pi/2 - d*x)/2])^2) + ((1 +
 I)*n^2*AppellF1[1 + 2*n, 1 + n, n, 2 + 2*n, (-1 - I)/(-1 + Tan[(-c + Pi/2
- d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)/2])]*Sec[(-c + Pi/2 - d*x)
/2]^2)/((1 + 2*n)*(-1 + Tan[(-c + Pi/2 - d*x)/2])^2))*((-I + Tan[(-c + Pi/2
 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/
2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n + n*AppellF1[2*n, n, n, 1 + 2*n, (-1
 - I)/(-1 + Tan[(-c + Pi/2 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)
/2])]*((-I + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^(-1
 + n)*((I + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*(S
ec[(-c + Pi/2 - d*x)/2]^2/(2*(-1 + Tan[(-c + Pi/2 - d*x)/2])) - (Sec[(-c +
Pi/2 - d*x)/2]^2*(-I + Tan[(-c + Pi/2 - d*x)/2]))/(2*(-1 + Tan[(-c + Pi/2 -
 d*x)/2])^2)) + n*AppellF1[2*n, n, n, 1 + 2*n, (-1 - I)/(-1 + Tan[(-c + Pi/
2 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)/2])]*((-I + Tan[(-c + Pi
/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x
)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^(-1 + n)*(Sec[(-c + Pi/2 - d*x)/2]^2
/(2*(-1 + Tan[(-c + Pi/2 - d*x)/2])) - (Sec[(-c + Pi/2 - d*x)/2]^2*(I + Tan
[(-c + Pi/2 - d*x)/2]))/(2*(-1 + Tan[(-c + Pi/2 - d*x)/2])^2)) - ((-I + Tan
[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi
/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^n*(((-1 - I)*n^2*AppellF1[1 +
 2*n, n, 1 + n, 2 + 2*n, (1 - I)/(1 + Tan[(-c + Pi/2 - d*x)/2]), (1 + I)/(1
 + Tan[(-c + Pi/2 - d*x)/2])]*Sec[(-c + Pi/2 - d*x)/2]^2)/((1 + 2*n)*(1 + T
an[(-c + Pi/2 - d*x)/2])^2) - ((1 - I)*n^2*AppellF1[1 + 2*n, 1 + n, n, 2 +
2*n, (1 - I)/(1 + Tan[(-c + Pi/2 - d*x)/2]), (1 + I)/(1 + Tan[(-c + Pi/2 -
d*x)/2])]*Sec[(-c + Pi/2 - d*x)/2]^2)/((1 + 2*n)*(1 + Tan[(-c + Pi/2 - d*x)
/2])^2)) - n*AppellF1[2*n, n, n, 1 + 2*n, (1 - I)/(1 + Tan[(-c + Pi/2 - d*x
)/2]), (1 + I)/(1 + Tan[(-c + Pi/2 - d*x)/2])]*((-I + Tan[(-c + Pi/2 - d*x)
/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^(-1 + n)*((I + Tan[(-c + Pi/2 - d*x)/2
])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^n*(-1/2*(Sec[(-c + Pi/2 - d*x)/2]^2*(-I
+ Tan[(-c + Pi/2 - d*x)/2]))/(1 + Tan[(-c + Pi/2 - d*x)/2])^2 + Sec[(-c + P
i/2 - d*x)/2]^2/(2*(1 + Tan[(-c + Pi/2 - d*x)/2]))) - n*AppellF1[2*n, n, n,
 1 + 2*n, (1 - I)/(1 + Tan[(-c + Pi/2 - d*x)/2]), (1 + I)/(1 + Tan[(-c + Pi
/2 - d*x)/2])]*((-I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/
2]))^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^(-1
+ n)*(-1/2*(Sec[(-c + Pi/2 - d*x)/2]^2*(I + Tan[(-c + Pi/2 - d*x)/2]))/(1 +
 Tan[(-c + Pi/2 - d*x)/2])^2 + Sec[(-c + Pi/2 - d*x)/2]^2/(2*(1 + Tan[(-c +
 Pi/2 - d*x)/2]))))/(2*n*(Sec[(-c + Pi/2 - d*x)/2]^2)^n)))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sin(dx+c)+a)^n\csc(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^n,x, algorithm="fricas")
[Out] integral((a*sin(d*x + c) + a)^n*csc(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a\sin(dx+c)+a)^n\csc(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^n,x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^n*csc(d*x + c), x)
```

maple [F] time = 1.28, size = 0, normalized size = 0.00

$$\int\csc(dx+c)(a+a\sin(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sin(d*x+c))^n,x)
[Out] int(csc(d*x+c)*(a+a*sin(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a\sin(dx+c)+a)^n\csc(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^n*csc(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(a+a\sin(c+dx))^n}{\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^n/sin(c + d*x),x)
```

```
[Out] int((a + a*sin(c + d*x))^n/sin(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a (\sin (c+dx)+1))^n \csc (c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))**n,x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**n*csc(c + d*x), x)
```

3.146 $\int \csc^2(c+dx)(a+a\sin(c+dx))^n\,dx$

Optimal. Leaf size=85

$$-\frac{2^{n+\frac{1}{2}}\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n F_1\left(\frac{1}{2};2,\frac{1}{2}-n;\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d}$$

```
[Out] -2^(1/2+n)*AppellF1(1/2,2,1/2-n,3/2,1-sin(d*x+c),1/2-1/2*sin(d*x+c))*cos(d*
x+c)*(1+sin(d*x+c))^(-1/2-n)*(a+a*sin(d*x+c))^n/d
```

Rubi [A] time = 0.12, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2787, 2785, 130, 429}

$$-\frac{2^{n+\frac{1}{2}}\cos(c+dx)(\sin(c+dx)+1)^{-n-\frac{1}{2}}(a\sin(c+dx)+a)^n F_1\left(\frac{1}{2};2,\frac{1}{2}-n;\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^n,x]
```

```
[Out] -((2^(1/2 + n)*AppellF1[1/2, 2, 1/2 - n, 3/2, 1 - Sin[c + d*x], (1 - Sin[c
+ d*x])/2]*Cos[c + d*x]*(1 + Sin[c + d*x])^(-1/2 - n)*(a + a*Sin[c + d*x])^
n)/d)
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
```

```
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^n\,dx = \left((1+\sin(c+dx))^{-n}(a+a\sin(c+dx))^n\right)\int \csc^2(c+dx)(1+\sin(c+dx$$

$$= -\frac{\left(\cos(c+dx)(1+\sin(c+dx))^{-\frac{1}{2}-n}(a+a\sin(c+dx))^n\right)\text{Subst}\left(\int \frac{(2-x)}{(1-x)}\right.}{d\sqrt{1-\sin(c+dx)}}$$

$$= -\frac{\left(2\cos(c+dx)(1+\sin(c+dx))^{-\frac{1}{2}-n}(a+a\sin(c+dx))^n\right)\text{Subst}\left(\int \frac{(2-}{(}\right.}{d\sqrt{1-\sin(c+dx)}}$$

$$= -\frac{2^{\frac{1}{2}+n}F_1\left(\frac{1}{2};2,\frac{1}{2}-n;\frac{3}{2};1-\sin(c+dx),\frac{1}{2}(1-\sin(c+dx))\right)\cos(c+dx)}{d}$$

Mathematica [C] time = 26.56, size = 4206, normalized size = 49.48

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^n,x]
```

```
[Out] -((Csc[c + d*x]^2*(a + a*Sin[c + d*x])^n*(-(AppellF1[1 + 2*n, n, n, 2 + 2*n
, (1 - I)/(1 + Tan[(-c + Pi/2 - d*x)/2]), (1 + I)/(1 + Tan[(-c + Pi/2 - d*x
)/2])]*(-1 + Tan[(-c + Pi/2 - d*x)/2])*((-I + Tan[(-c + Pi/2 - d*x)/2])/(1
+ Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c
 + Pi/2 - d*x)/2]))^n) - AppellF1[1 + 2*n, n, n, 2*(1 + n), (-1 - I)/(-1 +
Tan[(-c + Pi/2 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)/2])]*((-I +
 Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c
 + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*(1 + Tan[(-c + Pi/2 -
```

```
 d*x)/2])))/(d*(1 + 2*n)*(Sec[(-c + Pi/2 - d*x)/2]^2)^n*(-1 + Tan[(-c + Pi/
2 - d*x)/2])*(1 + Tan[(-c + Pi/2 - d*x)/2])*(-1/2*((Sec[(-c + Pi/2 - d*x)/2
]^2)^(1 - n)*(-(AppellF1[1 + 2*n, n, n, 2 + 2*n, (1 - I)/(1 + Tan[(-c + Pi/
2 - d*x)/2]), (1 + I)/(1 + Tan[(-c + Pi/2 - d*x)/2])]*(-1 + Tan[(-c + Pi/2
 - d*x)/2])*((-I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]))
^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^n) - App
ellF1[1 + 2*n, n, n, 2*(1 + n), (-1 - I)/(-1 + Tan[(-c + Pi/2 - d*x)/2]), (
-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)/2])]*((-I + Tan[(-c + Pi/2 - d*x)/2])/(
-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan
[(-c + Pi/2 - d*x)/2]))^n*(1 + Tan[(-c + Pi/2 - d*x)/2])))/((1 + 2*n)*(-1 +
 Tan[(-c + Pi/2 - d*x)/2])*(1 + Tan[(-c + Pi/2 - d*x)/2])^2) - ((Sec[(-c +
Pi/2 - d*x)/2]^2)^(1 - n)*(-(AppellF1[1 + 2*n, n, n, 2 + 2*n, (1 - I)/(1 +
Tan[(-c + Pi/2 - d*x)/2]), (1 + I)/(1 + Tan[(-c + Pi/2 - d*x)/2])]*(-1 + Ta
n[(-c + Pi/2 - d*x)/2])*((-I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/
2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/
2]))^n) - AppellF1[1 + 2*n, n, n, 2*(1 + n), (-1 - I)/(-1 + Tan[(-c + Pi/2
 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)/2])]*((-I + Tan[(-c + Pi/2
 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/
2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*(1 + Tan[(-c + Pi/2 - d*x)/2])))/(2*
(1 + 2*n)*(-1 + Tan[(-c + Pi/2 - d*x)/2])^2*(1 + Tan[(-c + Pi/2 - d*x)/2]))
 - (n*Tan[(-c + Pi/2 - d*x)/2]*(-(AppellF1[1 + 2*n, n, n, 2 + 2*n, (1 - I)/
(1 + Tan[(-c + Pi/2 - d*x)/2]), (1 + I)/(1 + Tan[(-c + Pi/2 - d*x)/2])]*(-1
 + Tan[(-c + Pi/2 - d*x)/2])*((-I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c
+ Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 -
d*x)/2]))^n) - AppellF1[1 + 2*n, n, n, 2*(1 + n), (-1 - I)/(-1 + Tan[(-c +
Pi/2 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)/2])]*((-I + Tan[(-c +
 Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 -
d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*(1 + Tan[(-c + Pi/2 - d*x)/2]))
)/((1 + 2*n)*(Sec[(-c + Pi/2 - d*x)/2]^2)^n*(-1 + Tan[(-c + Pi/2 - d*x)/2])
*(1 + Tan[(-c + Pi/2 - d*x)/2])) + (-1/2*(AppellF1[1 + 2*n, n, n, 2*(1 + n)
, (-1 - I)/(-1 + Tan[(-c + Pi/2 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 -
 d*x)/2])]*Sec[(-c + Pi/2 - d*x)/2]^2*((-I + Tan[(-c + Pi/2 - d*x)/2])/(-1
+ Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-
c + Pi/2 - d*x)/2]))^n) - (AppellF1[1 + 2*n, n, n, 2 + 2*n, (1 - I)/(1 + Ta
n[(-c + Pi/2 - d*x)/2]), (1 + I)/(1 + Tan[(-c + Pi/2 - d*x)/2])]*Sec[(-c +
Pi/2 - d*x)/2]^2*((-I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x
)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^n)
/2 - (((1/4 - I/4)*n*(1 + 2*n)*AppellF1[2 + 2*n, n, 1 + n, 1 + 2*(1 + n), (
-1 - I)/(-1 + Tan[(-c + Pi/2 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*
x)/2])]*Sec[(-c + Pi/2 - d*x)/2]^2)/((1 + n)*(-1 + Tan[(-c + Pi/2 - d*x)/2]
)^2) + ((1/4 + I/4)*n*(1 + 2*n)*AppellF1[2 + 2*n, 1 + n, n, 1 + 2*(1 + n),
(-1 - I)/(-1 + Tan[(-c + Pi/2 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d
*x)/2])]*Sec[(-c + Pi/2 - d*x)/2]^2)/((1 + n)*(-1 + Tan[(-c + Pi/2 - d*x)/2
])^2))*((-I + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*
((I + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*(1 + Tan
```

```
[(-c + Pi/2 - d*x)/2]) - n*AppellF1[1 + 2*n, n, n, 2*(1 + n), (-1 - I)/(-1
+ Tan[(-c + Pi/2 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*x)/2])]*((-I
 + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^(-1 + n)*((I
+ Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n*(1 + Tan[(-c
 + Pi/2 - d*x)/2])*(Sec[(-c + Pi/2 - d*x)/2]^2/(2*(-1 + Tan[(-c + Pi/2 - d*
x)/2])) - (Sec[(-c + Pi/2 - d*x)/2]^2*(-I + Tan[(-c + Pi/2 - d*x)/2]))/(2*(
-1 + Tan[(-c + Pi/2 - d*x)/2])^2)) - n*AppellF1[1 + 2*n, n, n, 2*(1 + n), (
-1 - I)/(-1 + Tan[(-c + Pi/2 - d*x)/2]), (-1 + I)/(-1 + Tan[(-c + Pi/2 - d*
x)/2])]*((-I + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^n
*((I + Tan[(-c + Pi/2 - d*x)/2])/(-1 + Tan[(-c + Pi/2 - d*x)/2]))^(-1 + n)*
(1 + Tan[(-c + Pi/2 - d*x)/2])*(Sec[(-c + Pi/2 - d*x)/2]^2/(2*(-1 + Tan[(-c
 + Pi/2 - d*x)/2])) - (Sec[(-c + Pi/2 - d*x)/2]^2*(I + Tan[(-c + Pi/2 - d*x
)/2]))/(2*(-1 + Tan[(-c + Pi/2 - d*x)/2])^2)) - (-1 + Tan[(-c + Pi/2 - d*x)
/2])*((-I + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^n*((I
 + Tan[(-c + Pi/2 - d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^n*(((-1/2 - I/
2)*n*(1 + 2*n)*AppellF1[2 + 2*n, n, 1 + n, 3 + 2*n, (1 - I)/(1 + Tan[(-c +
Pi/2 - d*x)/2]), (1 + I)/(1 + Tan[(-c + Pi/2 - d*x)/2])]*Sec[(-c + Pi/2 - d
*x)/2]^2)/((2 + 2*n)*(1 + Tan[(-c + Pi/2 - d*x)/2])^2) - ((1/2 - I/2)*n*(1
+ 2*n)*AppellF1[2 + 2*n, 1 + n, n, 3 + 2*n, (1 - I)/(1 + Tan[(-c + Pi/2 - d
*x)/2]), (1 + I)/(1 + Tan[(-c + Pi/2 - d*x)/2])]*Sec[(-c + Pi/2 - d*x)/2]^2
)/((2 + 2*n)*(1 + Tan[(-c + Pi/2 - d*x)/2])^2)) - n*AppellF1[1 + 2*n, n, n,
 2 + 2*n, (1 - I)/(1 + Tan[(-c + Pi/2 - d*x)/2]), (1 + I)/(1 + Tan[(-c + Pi
/2 - d*x)/2])]*(-1 + Tan[(-c + Pi/2 - d*x)/2])*((-I + Tan[(-c + Pi/2 - d*x)
/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^(-1 + n)*((I + Tan[(-c + Pi/2 - d*x)/2
])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^n*(-1/2*(Sec[(-c + Pi/2 - d*x)/2]^2*(-I
+ Tan[(-c + Pi/2 - d*x)/2]))/(1 + Tan[(-c + Pi/2 - d*x)/2])^2 + Sec[(-c + P
i/2 - d*x)/2]^2/(2*(1 + Tan[(-c + Pi/2 - d*x)/2]))) - n*AppellF1[1 + 2*n, n
, n, 2 + 2*n, (1 - I)/(1 + Tan[(-c + Pi/2 - d*x)/2]), (1 + I)/(1 + Tan[(-c
+ Pi/2 - d*x)/2])]*(-1 + Tan[(-c + Pi/2 - d*x)/2])*((-I + Tan[(-c + Pi/2 -
d*x)/2])/(1 + Tan[(-c + Pi/2 - d*x)/2]))^n*((I + Tan[(-c + Pi/2 - d*x)/2])/
(1 + Tan[(-c + Pi/2 - d*x)/2]))^(-1 + n)*(-1/2*(Sec[(-c + Pi/2 - d*x)/2]^2*
(I + Tan[(-c + Pi/2 - d*x)/2]))/(1 + Tan[(-c + Pi/2 - d*x)/2])^2 + Sec[(-c
+ Pi/2 - d*x)/2]^2/(2*(1 + Tan[(-c + Pi/2 - d*x)/2])))))/((1 + 2*n)*(Sec[(-c
 + Pi/2 - d*x)/2]^2)^n*(-1 + Tan[(-c + Pi/2 - d*x)/2])*(1 + Tan[(-c + Pi/2
- d*x)/2])))))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sin(dx+c)+a)^n\csc(dx+c)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^n \csc(dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^n,x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \left(\csc^2(dx+c)\right)(a+a\sin(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^n,x)
[Out] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(dx+c)+a)^n \csc(dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^n}{\sin(c+d\,x)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^n/sin(c + d*x)^2,x)
[Out] int((a + a*sin(c + d*x))^n/sin(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^n \csc^2(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sin(d*x+c))**n,x)
[Out] Integral((a*(sin(c + d*x) + 1))**n*csc(c + d*x)**2, x)
```

3.147 $\int (1+\sin(c+dx))^n\,dx$

Optimal. Leaf size=58

$$-\frac{2^{n+\frac{1}{2}}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt{\sin(c+dx)+1}}$$

```
[Out] -2^(1/2+n)*cos(d*x+c)*hypergeom([1/2, 1/2-n],[3/2],1/2-1/2*sin(d*x+c))/d/(1
+sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2651}

$$-\frac{2^{n+\frac{1}{2}}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt{\sin(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[c + d*x])^n,x]
```

```
[Out] -((2^(1/2 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 - Sin[c
 + d*x])/2])/(d*Sqrt[1 + Sin[c + d*x]]))
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int (1+\sin(c+dx))^n\,dx=-\frac{2^{\frac{1}{2}+n}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d\sqrt{1+\sin(c+dx)}}$$

Mathematica [A] time = 0.15, size = 88, normalized size = 1.52

$$\frac{\sqrt{2}\cos(c+dx)(\sin(c+dx)+1)^n\,{}_2F_1\left(\frac{1}{2},n+\frac{1}{2};n+\frac{3}{2};\frac{1}{4}\cos^2(c+dx)\csc^2\left(\frac{1}{4}(2c+2dx-\pi)\right)\right)}{(2dn+d)\sqrt{1-\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[c + d*x])^n,x]
```

```
[Out] (Sqrt[2]*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 + n, 3/2 + n, (Cos[c + d*x
]^2*Csc[(2*c - Pi + 2*d*x)/4]^2)/4]*(1 + Sin[c + d*x])^n)/((d + 2*d*n)*Sqrt
[1 - Sin[c + d*x]])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left((\sin(dx+c)+1)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((sin(d*x + c) + 1)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\sin(dx+c)+1)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((sin(d*x + c) + 1)^n, x)
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int (1+\sin(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sin(d*x+c))^n,x)
```

```
[Out] int((1+sin(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\sin(dx+c)+1)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((sin(d*x + c) + 1)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (\sin (c + d\,x) + 1)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x) + 1)^n,x)
[Out] int((sin(c + d*x) + 1)^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\sin (c + dx) + 1)^n\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(d*x+c))**n,x)
[Out] Integral((sin(c + d*x) + 1)**n, x)
```

3.148 $\int (1-\sin(c+dx))^n\,dx$

Optimal. Leaf size=57

$$\frac{2^{n+\frac{1}{2}}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{d\sqrt{1-\sin(c+dx)}}$$

```
[Out] 2^(1/2+n)*cos(d*x+c)*hypergeom([1/2, 1/2-n],[3/2],1/2+1/2*sin(d*x+c))/d/(1-
sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2651}

$$\frac{2^{n+\frac{1}{2}}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{d\sqrt{1-\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[c + d*x])^n,x]
```

```
[Out] (2^(1/2 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 + Sin[c +
 d*x])/2])/(d*Sqrt[1 - Sin[c + d*x]])
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int (1-\sin(c+dx))^n\,dx=\frac{2^{\frac{1}{2}+n}\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1+\sin(c+dx))\right)}{d\sqrt{1-\sin(c+dx)}}$$

Mathematica [A] time = 0.11, size = 90, normalized size = 1.58

$$\frac{\cos(c+dx)(1-\sin(c+dx))^n\cos^2\left(\frac{1}{4}(2c+2dx+\pi)\right)^{-n-\frac{1}{2}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{4}\cos^2(c+dx)\csc^2\left(\frac{1}{4}(2c+2dx-\pi)\right)\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sin[c + d*x])^n,x]
```

```
[Out] (Cos[c + d*x]*(Cos[(2*c + Pi + 2*d*x)/4]^2)^(-1/2 - n)*Hypergeometric2F1[1/
2, 1/2 - n, 3/2, (Cos[c + d*x]^2*Csc[(2*c - Pi + 2*d*x)/4]^2)/4]*(1 - Sin[c
 + d*x])^n)/d
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left((-\sin(dx+c)+1)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((-sin(d*x + c) + 1)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-\sin(dx+c)+1)^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((-sin(d*x + c) + 1)^n, x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int (1-\sin(dx+c))^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sin(d*x+c))^n,x)
```

```
[Out] int((1-sin(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-\sin(dx+c)+1)^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((-sin(d*x + c) + 1)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (1 - \sin(c + d\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - sin(c + d*x))^n,x)
```

```
[Out] int((1 - sin(c + d*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (1 - \sin(c + dx))^n\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(d*x+c))**n,x)
```

```
[Out] Integral((1 - sin(c + d*x))**n, x)
```

3.149 $\int \sin^3(e+fx)(a+b\sin(e+fx))\,dx$

Optimal. Leaf size=77

$$\frac{a\cos^3(e+fx)}{3f}-\frac{a\cos(e+fx)}{f}-\frac{b\sin^3(e+fx)\cos(e+fx)}{4f}-\frac{3b\sin(e+fx)\cos(e+fx)}{8f}+\frac{3bx}{8}$$

```
[Out] 3/8*b*x-a*cos(f*x+e)/f+1/3*a*cos(f*x+e)^3/f-3/8*b*cos(f*x+e)*sin(f*x+e)/f-1
/4*b*cos(f*x+e)*sin(f*x+e)^3/f
```

Rubi **[A]** time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2748, 2633, 2635, 8}

$$\frac{a\cos^3(e+fx)}{3f}-\frac{a\cos(e+fx)}{f}-\frac{b\sin^3(e+fx)\cos(e+fx)}{4f}-\frac{3b\sin(e+fx)\cos(e+fx)}{8f}+\frac{3bx}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + b*Sin[e + f*x]),x]
```

```
[Out] (3*b*x)/8 - (a*Cos[e + f*x])/f + (a*Cos[e + f*x]^3)/(3*f) - (3*b*Cos[e + f*
x]*Sin[e + f*x])/(8*f) - (b*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int \sin^3(e+fx)(a+b\sin(e+fx))\,dx &= a\int \sin^3(e+fx)\,dx + b\int \sin^4(e+fx)\,dx\\ &= -\frac{b\cos(e+fx)\sin^3(e+fx)}{4f} + \frac{1}{4}(3b)\int \sin^2(e+fx)\,dx - \frac{a\,\mathrm{Subst}\left(\int \left(1-\right.\right.}{}\\ &= -\frac{a\cos(e+fx)}{f} + \frac{a\cos^3(e+fx)}{3f} - \frac{3b\cos(e+fx)\sin(e+fx)}{8f} - \frac{b\cos(e-}{}\\ &= \frac{3bx}{8} - \frac{a\cos(e+fx)}{f} + \frac{a\cos^3(e+fx)}{3f} - \frac{3b\cos(e+fx)\sin(e+fx)}{8f} - \frac{b\,\mathrm{c}}{}\end{aligned}$$

Mathematica [A] time = 0.17, size = 76, normalized size = 0.99

$$-\frac{3a\cos(e+fx)}{4f} + \frac{a\cos(3(e+fx))}{12f} + \frac{3b(e+fx)}{8f} - \frac{b\sin(2(e+fx))}{4f} + \frac{b\sin(4(e+fx))}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*(a + b*Sin[e + f*x]),x]
```

```
[Out] (3*b*(e + f*x))/(8*f) - (3*a*Cos[e + f*x])/(4*f) + (a*Cos[3*(e + f*x)])/(12
*f) - (b*Sin[2*(e + f*x)])/(4*f) + (b*Sin[4*(e + f*x)])/(32*f)
```

fricas [A] time = 0.47, size = 60, normalized size = 0.78

$$\frac{8\,a\cos\left(fx+e\right)^3 + 9\,bfx - 24\,a\cos\left(fx+e\right) + 3\left(2\,b\cos\left(fx+e\right)^3 - 5\,b\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/24*(8*a*cos(f*x + e)^3 + 9*b*f*x - 24*a*cos(f*x + e) + 3*(2*b*cos(f*x + e
)^3 - 5*b*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.14, size = 66, normalized size = 0.86

$$\frac{3}{8}\,bx + \frac{a\cos\left(3\,fx+3\,e\right)}{12\,f} - \frac{3\,a\cos\left(fx+e\right)}{4\,f} + \frac{b\sin\left(4\,fx+4\,e\right)}{32\,f} - \frac{b\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 3/8*b*x + 1/12*a*cos(3*f*x + 3*e)/f - 3/4*a*cos(f*x + e)/f + 1/32*b*sin(4*f
*x + 4*e)/f - 1/4*b*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.22, size = 60, normalized size = 0.78

$$\frac{b\left(-\frac{\left(\sin^{3}\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{a\left(2+\sin^{2}\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*sin(f*x+e)),x)
```

```
[Out] 1/f*(b*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/3*a*
(2+sin(f*x+e)^2)*cos(f*x+e))
```

maxima [A] time = 1.32, size = 57, normalized size = 0.74

$$\frac{32\left(\cos\left(fx+e\right)^{3}-3\cos\left(fx+e\right)\right)a+3\left(12fx+12e+\sin\left(4fx+4e\right)-8\sin\left(2fx+2e\right)\right)b}{96f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/96*(32*(cos(f*x + e)^3 - 3*cos(f*x + e))*a + 3*(12*f*x + 12*e + sin(4*f*x
 + 4*e) - 8*sin(2*f*x + 2*e))*b)/f
```

mupad [B] time = 10.29, size = 111, normalized size = 1.44

$$\frac{3bx}{8}-\frac{-\frac{3b\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{7}}{4}-\frac{11b\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{5}}{4}+4a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}+\frac{11b\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{3}}{4}+\frac{16a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}}{3}+\frac{3b\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{4}+\frac{4a}{3}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}+1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b*sin(e + f*x)),x)
```

```
[Out] (3*b*x)/8 - ((4*a)/3 + (3*b*tan(e/2 + (f*x)/2))/4 + (16*a*tan(e/2 + (f*x)/2
)^2)/3 + 4*a*tan(e/2 + (f*x)/2)^4 + (11*b*tan(e/2 + (f*x)/2)^3)/4 - (11*b*t
```

an(e/2 + (f*x)/2)^5)/4 - (3*b*tan(e/2 + (f*x)/2)^7)/4)/(f*(tan(e/2 + (f*x)/2)^2 + 1)^4)

sympy [A] time = 1.34, size = 144, normalized size = 1.87

$$\begin{cases} -\frac{a\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f} - \frac{2a\cos^3\left(e+fx\right)}{3f} + \frac{3bx\sin^4\left(e+fx\right)}{8} + \frac{3bx\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4} + \frac{3bx\cos^4\left(e+fx\right)}{8} - \frac{5b\sin^3\left(e+fx\right)\cos\left(e+fx\right)}{8f} \\ x\left(a+b\sin\left(e\right)\right)\sin^3\left(e\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*sin(f*x+e)),x)
```

```
[Out] Piecewise((-a*sin(e + f*x)**2*cos(e + f*x)/f - 2*a*cos(e + f*x)**3/(3*f) +
3*b*x*sin(e + f*x)**4/8 + 3*b*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*b*x*c
os(e + f*x)**4/8 - 5*b*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*b*sin(e + f*x
)*cos(e + f*x)**3/(8*f), Ne(f, 0)), (x*(a + b*sin(e))*sin(e)**3, True))
```

3.150 $\int \sin^2(e+fx)(a+b\sin(e+fx))\,dx$

Optimal. Leaf size=55

$$-\frac{a\sin(e+fx)\cos(e+fx)}{2f}+\frac{ax}{2}+\frac{b\cos^3(e+fx)}{3f}-\frac{b\cos(e+fx)}{f}$$

```
[Out] 1/2*a*x-b*cos(f*x+e)/f+1/3*b*cos(f*x+e)^3/f-1/2*a*cos(f*x+e)*sin(f*x+e)/f
```

Rubi [A] time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2748, 2635, 8, 2633}

$$-\frac{a\sin(e+fx)\cos(e+fx)}{2f}+\frac{ax}{2}+\frac{b\cos^3(e+fx)}{3f}-\frac{b\cos(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*x)/2 - (b*Cos[e + f*x])/f + (b*Cos[e + f*x]^3)/(3*f) - (a*Cos[e + f*x]*S
in[e + f*x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \sin^2(e+fx)(a+b\sin(e+fx))\,dx = a\int \sin^2(e+fx)\,dx + b\int \sin^3(e+fx)\,dx$$
$$= -\frac{a\cos(e+fx)\sin(e+fx)}{2f} + \frac{1}{2}a\int 1\,dx - \frac{b\,\mathrm{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos(\right.}{f}$$
$$= \frac{ax}{2} - \frac{b\cos(e+fx)}{f} + \frac{b\cos^3(e+fx)}{3f} - \frac{a\cos(e+fx)\sin(e+fx)}{2f}$$

Mathematica [A] time = 0.06, size = 60, normalized size = 1.09

$$\frac{a(e+fx)}{2f} - \frac{a\sin(2(e+fx))}{4f} - \frac{3b\cos(e+fx)}{4f} + \frac{b\cos(3(e+fx))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2*(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*(e + f*x))/(2*f) - (3*b*Cos[e + f*x])/(4*f) + (b*Cos[3*(e + f*x)])/(12*f
) - (a*Sin[2*(e + f*x)])/(4*f)
```

fricas [A] time = 0.45, size = 46, normalized size = 0.84

$$\frac{2\,b\cos\left(fx+e\right)^3 + 3\,afx - 3\,a\cos\left(fx+e\right)\sin\left(fx+e\right) - 6\,b\cos\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*cos(f*x + e)^3 + 3*a*f*x - 3*a*cos(f*x + e)*sin(f*x + e) - 6*b*cos
(f*x + e))/f
```

giac [A] time = 0.15, size = 50, normalized size = 0.91

$$\frac{1}{2}\,ax + \frac{b\cos\left(3\,fx+3\,e\right)}{12\,f} - \frac{3\,b\cos\left(fx+e\right)}{4\,f} - \frac{a\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*a*x + 1/12*b*cos(3*f*x + 3*e)/f - 3/4*b*cos(f*x + e)/f - 1/4*a*sin(2*f*
x + 2*e)/f
```

maple [A] time = 0.16, size = 49, normalized size = 0.89

$$\frac{-\frac{b\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+a\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*sin(f*x+e)),x)
```

```
[Out] 1/f*(-1/3*b*(2+sin(f*x+e)^2)*cos(f*x+e)+a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f
*x+1/2*e))
```

maxima [A] time = 0.65, size = 48, normalized size = 0.87

$$\frac{3\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)a+4\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)b}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*f*x + 2*e - sin(2*f*x + 2*e))*a + 4*(cos(f*x + e)^3 - 3*cos(f*x
+ e))*b)/f
```

mupad [B] time = 8.57, size = 68, normalized size = 1.24

$$\frac{a\,x}{2}-\frac{-a\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+4\,b\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\frac{4\,b}{3}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b*sin(e + f*x)),x)
```

```
[Out] (a*x)/2 - ((4*b)/3 + a*tan(e/2 + (f*x)/2) - a*tan(e/2 + (f*x)/2)^5 + 4*b*ta
n(e/2 + (f*x)/2)^2)/(f*(tan(e/2 + (f*x)/2)^2 + 1)^3)
```

sympy [A] time = 0.74, size = 92, normalized size = 1.67

$$\begin{cases} \frac{ax\sin^2\left(e+fx\right)}{2}+\frac{ax\cos^2\left(e+fx\right)}{2}-\frac{a\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{b\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{2b\cos^3\left(e+fx\right)}{3f} & \text{for } f\neq 0 \\ x\left(a+b\sin\left(e\right)\right)\sin^2\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*sin(f*x+e)),x)
```

```
[Out] Piecewise((a*x*sin(e + f*x)**2/2 + a*x*cos(e + f*x)**2/2 - a*sin(e + f*x)*c
os(e + f*x)/(2*f) - b*sin(e + f*x)**2*cos(e + f*x)/f - 2*b*cos(e + f*x)**3/
(3*f), Ne(f, 0)), (x*(a + b*sin(e))*sin(e)**2, True))
```

3.151 $\int \sin(e+fx)(a+b\sin(e+fx))\,dx$

Optimal. Leaf size=39

$$-\frac{a\cos(e+fx)}{f}-\frac{b\sin(e+fx)\cos(e+fx)}{2f}+\frac{bx}{2}$$

```
[Out] 1/2*b*x-a*cos(f*x+e)/f-1/2*b*cos(f*x+e)*sin(f*x+e)/f
```

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2734}

$$-\frac{a\cos(e+fx)}{f}-\frac{b\sin(e+fx)\cos(e+fx)}{2f}+\frac{bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*(a + b*Sin[e + f*x]),x]
```

```
[Out] (b*x)/2 - (a*Cos[e + f*x])/f - (b*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \sin(e+fx)(a+b\sin(e+fx))\,dx=\frac{bx}{2}-\frac{a\cos(e+fx)}{f}-\frac{b\cos(e+fx)\sin(e+fx)}{2f}$$

Mathematica [A] time = 0.10, size = 35, normalized size = 0.90

$$-\frac{4a\cos(e+fx)+b(\sin(2(e+fx))-2(e+fx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*(a + b*Sin[e + f*x]),x]
```

```
[Out] -1/4*(4*a*Cos[e + f*x] + b*(-2*(e + f*x) + Sin[2*(e + f*x)]))/f
```

fricas [A] time = 0.48, size = 34, normalized size = 0.87

$$\frac{bfx - b\cos\left(fx + e\right)\sin\left(fx + e\right) - 2\,a\cos\left(fx + e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)),x, algorithm="fricas")
[Out] 1/2*(b*f*x - b*cos(f*x + e)*sin(f*x + e) - 2*a*cos(f*x + e))/f
```

giac [A] time = 0.14, size = 34, normalized size = 0.87

$$\frac{1}{2}\,bx - \frac{a\cos\left(fx + e\right)}{f} - \frac{b\sin\left(2\,fx + 2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)),x, algorithm="giac")
[Out] 1/2*b*x - a*cos(f*x + e)/f - 1/4*b*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.05, size = 39, normalized size = 1.00

$$\frac{b\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2} + \frac{fx}{2} + \frac{e}{2}\right) - \cos\left(fx + e\right)a}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*sin(f*x+e)),x)
[Out] 1/f*(b*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-cos(f*x+e)*a)
```

maxima [A] time = 0.38, size = 36, normalized size = 0.92

$$\frac{\left(2\,fx + 2\,e - \sin\left(2\,fx + 2\,e\right)\right)b - 4\,a\cos\left(fx + e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)),x, algorithm="maxima")
[Out] 1/4*((2*f*x + 2*e - sin(2*f*x + 2*e))*b - 4*a*cos(f*x + e))/f
```

mupad [B] time = 7.19, size = 68, normalized size = 1.74

$$\frac{b\,x}{2}-\frac{-b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+2\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+2\,a}{f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b*sin(e + f*x)),x)
```

```
[Out] (b*x)/2 - (2*a + b*tan(e/2 + (f*x)/2) + 2*a*tan(e/2 + (f*x)/2)^2 - b*tan(e/
2 + (f*x)/2)^3)/(f*(tan(e/2 + (f*x)/2)^2 + 1)^2)
```

sympy [A] time = 0.31, size = 66, normalized size = 1.69

$$\begin{cases} -\frac{a\cos\left(e+fx\right)}{f}+\frac{bx\sin^2\left(e+fx\right)}{2}+\frac{bx\cos^2\left(e+fx\right)}{2}-\frac{b\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f} & \text{for}\ f\neq 0 \\ x\left(a+b\sin\left(e\right)\right)\sin\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)),x)
```

```
[Out] Piecewise((-a*cos(e + f*x)/f + b*x*sin(e + f*x)**2/2 + b*x*cos(e + f*x)**2/
2 - b*sin(e + f*x)*cos(e + f*x)/(2*f), Ne(f, 0)), (x*(a + b*sin(e))*sin(e),
 True))
```

3.152 $\int (a+b\sin(e+fx))\,dx$

Optimal. Leaf size=16

$$ax-\frac{b\cos(e+fx)}{f}$$

[Out] a*x-b*cos(f*x+e)/f

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2638}

$$ax-\frac{b\cos(e+fx)}{f}$$

Antiderivative was successfully verified.

[In] Int[a + b*Sin[e + f*x],x]

[Out] a*x - (b*Cos[e + f*x])/f

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\sin(e+fx))\,dx &= ax+b\int \sin(e+fx)\,dx\\ &= ax-\frac{b\cos(e+fx)}{f}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.69

$$ax+\frac{b\sin(e)\sin(fx)}{f}-\frac{b\cos(e)\cos(fx)}{f}$$

Antiderivative was successfully verified.

[In] Integrate[a + b*Sin[e + f*x],x]

[Out] a*x - (b*Cos[e]*Cos[f*x])/f + (b*Sin[e]*Sin[f*x])/f

fricas [A] time = 0.49, size = 18, normalized size = 1.12

$$\frac{afx - b\cos\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e),x, algorithm="fricas")
[Out] (a*f*x - b*cos(f*x + e))/f
```

giac [A] time = 0.15, size = 17, normalized size = 1.06

$$ax - \frac{b\cos\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e),x, algorithm="giac")
[Out] a*x - b*cos(f*x + e)/f
```

maple [A] time = 0.01, size = 17, normalized size = 1.06

$$ax - \frac{b\cos\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sin(f*x+e),x)
[Out] a*x-b*cos(f*x+e)/f
```

maxima [A] time = 0.88, size = 16, normalized size = 1.00

$$ax - \frac{b\cos\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e),x, algorithm="maxima")
[Out] a*x - b*cos(f*x + e)/f
```

mupad [B] time = 6.51, size = 25, normalized size = 1.56

$$a\,x - \frac{2\,b}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*sin(e + f*x),x)
```

```
[Out] a*x - (2*b)/(f*(tan(e/2 + (f*x)/2)^2 + 1))
```

sympy **[A]** time = 0.16, size = 19, normalized size = 1.19

$$ax + b\left(\begin{cases} -\frac{\cos\left(e+fx\right)}{f} & \text{for } f \neq 0 \\ x\sin\left(e\right) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e),x)
```

```
[Out] a*x + b*Piecewise((-cos(e + f*x)/f, Ne(f, 0)), (x*sin(e), True))
```

3.153 $\int \csc(e+fx)(a+b\sin(e+fx))\,dx$

Optimal. Leaf size=17

$$bx-\frac{a\tanh^{-1}(\cos(e+fx))}{f}$$

```
[Out] b*x-a*arctanh(cos(f*x+e))/f
```

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2735, 3770}

$$bx-\frac{a\tanh^{-1}(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Sin[e + f*x]),x]
```

```
[Out] b*x - (a*ArcTanh[Cos[e + f*x]])/f
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc(e+fx)(a+b\sin(e+fx))\,dx &= bx+a\int \csc(e+fx)\,dx\\ &= bx-\frac{a\tanh^{-1}(\cos(e+fx))}{f}\end{aligned}$$

Mathematica [B] time = 0.02, size = 43, normalized size = 2.53

$$\frac{a\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}-\frac{a\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}+bx$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(a + b*Sin[e + f*x]),x]
```

```
[Out] b*x - (a*Log[Cos[e/2 + (f*x)/2]])/f + (a*Log[Sin[e/2 + (f*x)/2]])/f
```

fricas [B] time = 0.45, size = 38, normalized size = 2.24

$$\frac{2\,bfx - a\log\left(\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right) + a\log\left(-\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*f*x - a*log(1/2*cos(f*x + e) + 1/2) + a*log(-1/2*cos(f*x + e) + 1/
2))/f
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(a/2*ln(abs(tan((f*x+exp(1))/2))
)+2*b/2*(f*x+exp(1))/2)
```

maple [A] time = 0.11, size = 32, normalized size = 1.88

$$bx + \frac{a\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{f} + \frac{be}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sin(f*x+e)),x)
```

```
[Out] b*x+1/f*a*ln(csc(f*x+e)-cot(f*x+e))+1/f*b*e
```

maxima [A] time = 0.64, size = 29, normalized size = 1.71

$$\frac{\left(fx+e\right)b - a\log\left(\cot\left(fx+e\right)+\csc\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] ((f*x + e)*b - a*log(cot(f*x + e) + csc(f*x + e)))/f
```

mupad [B] time = 6.80, size = 85, normalized size = 5.00

$$\frac{a\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f}+\frac{2\,b\,\mathrm{atan}\left(\frac{b\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)-b\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))/sin(e + f*x),x)
```

```
[Out] (a*log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/f + (2*b*atan((b*cos(e/2 + (
f*x)/2) + a*sin(e/2 + (f*x)/2))/(a*cos(e/2 + (f*x)/2) - b*sin(e/2 + (f*x)/2
))))/f
```

sympy [B] time = 6.76, size = 51, normalized size = 3.00

$$a\left(\begin{cases}\frac{x\cot\left(e\right)\csc\left(e\right)}{\cot\left(e\right)+\csc\left(e\right)}+\frac{x\csc^{2}\left(e\right)}{\cot\left(e\right)+\csc\left(e\right)} & \text{for}\ f=0\\ -\frac{\log\left(\cot\left(e+fx\right)+\csc\left(e+fx\right)\right)}{f} & \text{otherwise}\end{cases}\right)+bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)),x)
```

```
[Out] a*Piecewise((x*cot(e)*csc(e)/(cot(e) + csc(e)) + x*csc(e)**2/(cot(e) + csc(
e)), Eq(f, 0)), (-log(cot(e + f*x) + csc(e + f*x))/f, True)) + b*x
```

3.154 $\int \csc^2(e+fx)(a+b\sin(e+fx))\,dx$

Optimal. Leaf size=26

$$-\frac{a\cot(e+fx)}{f}-\frac{b\tanh^{-1}(\cos(e+fx))}{f}$$

[Out] -b*arctanh(cos(f*x+e))/f-a*cot(f*x+e)/f

Rubi [A] time = 0.04, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2748, 3767, 8, 3770}

$$-\frac{a\cot(e+fx)}{f}-\frac{b\tanh^{-1}(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

[In] Int[Csc[e + f*x]^2*(a + b*Sin[e + f*x]),x]

[Out] -((b*ArcTanh[Cos[e + f*x]])/f) - (a*Cot[e + f*x])/f

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^2(e+fx)(a+b\sin(e+fx))\,dx = a\int \csc^2(e+fx)\,dx + b\int \csc(e+fx)\,dx$$
$$= -\frac{b\tanh^{-1}(\cos(e+fx))}{f} - \frac{a\,\text{Subst}(\int 1\,dx, x, \cot(e+fx))}{f}$$
$$= -\frac{b\tanh^{-1}(\cos(e+fx))}{f} - \frac{a\cot(e+fx)}{f}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 2.00

$$-\frac{a\cot(e+fx)}{f} + \frac{b\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f} - \frac{b\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sin[e + f*x]),x]
```

```
[Out] -((a*Cot[e + f*x])/f) - (b*Log[Cos[e/2 + (f*x)/2]])/f + (b*Log[Sin[e/2 + (f
*x)/2]])/f
```

fricas [B] time = 0.49, size = 62, normalized size = 2.38

$$-\frac{b\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)\sin\left(fx+e\right) - b\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)\sin\left(fx+e\right) + 2\,a\cos\left(fx+e\right)}{2\,f\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/2*(b*log(1/2*cos(f*x + e) + 1/2)*sin(f*x + e) - b*log(-1/2*cos(f*x + e)
+ 1/2)*sin(f*x + e) + 2*a*cos(f*x + e))/(f*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(tan((f*x+exp(1))/2)*a/4+(-2*tan
```

```
((f*x+exp(1))/2)*b-a)*1/4/tan((f*x+exp(1))/2)+b/2*ln(abs(tan((f*x+exp(1))/2
))))
```

maple [A] time = 0.17, size = 35, normalized size = 1.35

$$-\frac{a\cot\left(fx+e\right)}{f}+\frac{b\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sin(f*x+e)),x)
```

```
[Out] -a*cot(f*x+e)/f+1/f*b*ln(csc(f*x+e)-cot(f*x+e))
```

maxima [A] time = 0.61, size = 40, normalized size = 1.54

$$-\frac{b\left(\log\left(\cos\left(fx+e\right)+1\right)-\log\left(\cos\left(fx+e\right)-1\right)\right)+\frac{2a}{\tan\left(fx+e\right)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/2*(b*(log(cos(f*x + e) + 1) - log(cos(f*x + e) - 1)) + 2*a/tan(f*x + e))
/f
```

mupad [B] time = 6.76, size = 28, normalized size = 1.08

$$\frac{b\,\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f}-\frac{a\cot\left(e+f\,x\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))/sin(e + f*x)^2,x)
```

```
[Out] (b*log(tan(e/2 + (f*x)/2)))/f - (a*cot(e + f*x))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(e+fx\right)\right)\csc^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sin(f*x+e)),x)
```

```
[Out] Integral((a + b*sin(e + f*x))*csc(e + f*x)**2, x)
```

3.155 $\int \csc^3(e+fx)(a+b\sin(e+fx))\,dx$

Optimal. Leaf size=48

$$-\frac{a\tanh^{-1}(\cos(e+fx))}{2f}-\frac{a\cot(e+fx)\csc(e+fx)}{2f}-\frac{b\cot(e+fx)}{f}$$

```
[Out] -1/2*a*arctanh(cos(f*x+e))/f-b*cot(f*x+e)/f-1/2*a*cot(f*x+e)*csc(f*x+e)/f
```

Rubi [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2748, 3768, 3770, 3767, 8}

$$-\frac{a\tanh^{-1}(\cos(e+fx))}{2f}-\frac{a\cot(e+fx)\csc(e+fx)}{2f}-\frac{b\cot(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Sin[e + f*x]),x]
```

```
[Out] -(a*ArcTanh[Cos[e + f*x]])/(2*f) - (b*Cot[e + f*x])/f - (a*Cot[e + f*x]*Csc
[e + f*x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^3(e+fx)(a+b\sin(e+fx))\,dx &= a\int \csc^3(e+fx)\,dx + b\int \csc^2(e+fx)\,dx\\ &= -\frac{a\cot(e+fx)\csc(e+fx)}{2f} + \frac{1}{2}a\int \csc(e+fx)\,dx - \frac{b\,\text{Subst}(\int 1\,dx, x, \text{co}}{f}\\ &= -\frac{a\tanh^{-1}(\cos(e+fx))}{2f} - \frac{b\cot(e+fx)}{f} - \frac{a\cot(e+fx)\csc(e+fx)}{2f}\end{aligned}$$

Mathematica [A] time = 0.03, size = 91, normalized size = 1.90

$$-\frac{a\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f}+\frac{a\sec^2\left(\frac{1}{2}(e+fx)\right)}{8f}+\frac{a\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2f}-\frac{a\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{2f}-\frac{b\cot(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Sin[e + f*x]),x]
```

```
[Out] -((b*Cot[e + f*x])/f) - (a*Csc[(e + f*x)/2]^2)/(8*f) - (a*Log[Cos[(e + f*x)
/2]])/(2*f) + (a*Log[Sin[(e + f*x)/2]])/(2*f) + (a*Sec[(e + f*x)/2]^2)/(8*f
)
```

fricas [B] time = 0.51, size = 96, normalized size = 2.00

$$\frac{4\,b\cos\left(fx+e\right)\sin\left(fx+e\right)+2\,a\cos\left(fx+e\right)-\left(a\cos\left(fx+e\right)^2-a\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)+\left(a\cos\left(fx+e\right)\right.}{4\left(f\cos\left(fx+e\right)^2-f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*b*cos(f*x + e)*sin(f*x + e) + 2*a*cos(f*x + e) - (a*cos(f*x + e)^2 -
 a)*log(1/2*cos(f*x + e) + 1/2) + (a*cos(f*x + e)^2 - a)*log(-1/2*cos(f*x +
 e) + 1/2))/(f*cos(f*x + e)^2 - f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*((4*tan((f*x+exp(1))/2)^2*a+16*t
an((f*x+exp(1))/2)*b)/64+(-6*tan((f*x+exp(1))/2)^2*a-4*tan((f*x+exp(1))/2)*
b-a)*1/16/tan((f*x+exp(1))/2)^2+a/4*ln(abs(tan((f*x+exp(1))/2))))
```

maple [A] time = 0.29, size = 54, normalized size = 1.12

$$-\frac{a\cot\left(fx+e\right)\csc\left(fx+e\right)}{2f}+\frac{a\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{2f}-\frac{b\cot\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sin(f*x+e)),x)
```

```
[Out] -1/2*a*cot(f*x+e)*csc(f*x+e)/f+1/2/f*a*ln(csc(f*x+e)-cot(f*x+e))-b*cot(f*x+
e)/f
```

maxima [A] time = 0.82, size = 60, normalized size = 1.25

$$\frac{a\left(\frac{2\cos\left(fx+e\right)}{\cos\left(fx+e\right)^2-1}-\log\left(\cos\left(fx+e\right)+1\right)+\log\left(\cos\left(fx+e\right)-1\right)\right)-\frac{4b}{\tan\left(fx+e\right)}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*(a*(2*cos(f*x + e)/(cos(f*x + e)^2 - 1) - log(cos(f*x + e) + 1) + log(c
os(f*x + e) - 1)) - 4*b/tan(f*x + e))/f
```

mupad [B] time = 6.72, size = 81, normalized size = 1.69

$$\frac{b\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{2f}-\frac{\cot\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(\frac{a}{2}+2b\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{4f}+\frac{a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2}{8f}+\frac{a\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))/sin(e + f*x)^3,x)
```

```
[Out] (b*tan(e/2 + (f*x)/2))/(2*f) - (cot(e/2 + (f*x)/2)^2*(a/2 + 2*b*tan(e/2 + (
f*x)/2)))/(4*f) + (a*tan(e/2 + (f*x)/2)^2)/(8*f) + (a*log(tan(e/2 + (f*x)/2
)))/(2*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \sin \left(e + fx\right)\right) \csc^3 \left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sin(f*x+e)),x)
```

```
[Out] Integral((a + b*sin(e + f*x))*csc(e + f*x)**3, x)
```

3.156 $\int \csc^4(e+fx)(a+b\sin(e+fx))\,dx$

Optimal. Leaf size=64

$$-\frac{a\cot^3(e+fx)}{3f}-\frac{a\cot(e+fx)}{f}-\frac{b\tanh^{-1}(\cos(e+fx))}{2f}-\frac{b\cot(e+fx)\csc(e+fx)}{2f}$$

```
[Out] -1/2*b*arctanh(cos(f*x+e))/f-a*cot(f*x+e)/f-1/3*a*cot(f*x+e)^3/f-1/2*b*cot(
f*x+e)*csc(f*x+e)/f
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2748, 3767, 3768, 3770}

$$-\frac{a\cot^3(e+fx)}{3f}-\frac{a\cot(e+fx)}{f}-\frac{b\tanh^{-1}(\cos(e+fx))}{2f}-\frac{b\cot(e+fx)\csc(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Sin[e + f*x]),x]
```

```
[Out] -(b*ArcTanh[Cos[e + f*x]])/(2*f) - (a*Cot[e + f*x])/f - (a*Cot[e + f*x]^3)/
(3*f) - (b*Cot[e + f*x]*Csc[e + f*x])/(2*f)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^4(e+fx)(a+b\sin(e+fx))\,dx = a\int \csc^4(e+fx)\,dx + b\int \csc^3(e+fx)\,dx$$
$$= -\frac{b\cot(e+fx)\csc(e+fx)}{2f} + \frac{1}{2}b\int \csc(e+fx)\,dx - \frac{a\,\text{Subst}\left(\int \left(1+x^2\right)\right.}{f}$$
$$= -\frac{b\tanh^{-1}(\cos(e+fx))}{2f} - \frac{a\cot(e+fx)}{f} - \frac{a\cot^3(e+fx)}{3f} - \frac{b\cot(e+fx)}{2f}$$

Mathematica [A] time = 0.03, size = 115, normalized size = 1.80

$$-\frac{2a\cot(e+fx)}{3f} - \frac{a\cot(e+fx)\csc^2(e+fx)}{3f} - \frac{b\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f} + \frac{b\sec^2\left(\frac{1}{2}(e+fx)\right)}{8f} + \frac{b\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2f} - \frac{b\,\text{l}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Sin[e + f*x]),x]
```

```
[Out] (-2*a*Cot[e + f*x])/(3*f) - (b*Csc[(e + f*x)/2]^2)/(8*f) - (a*Cot[e + f*x]*
Csc[e + f*x]^2)/(3*f) - (b*Log[Cos[(e + f*x)/2]])/(2*f) + (b*Log[Sin[(e + f
*x)/2]])/(2*f) + (b*Sec[(e + f*x)/2]^2)/(8*f)
```

fricas [B] time = 0.51, size = 128, normalized size = 2.00

$$-\frac{8a\cos\left(fx+e\right)^3 - 6b\cos\left(fx+e\right)\sin\left(fx+e\right) + 3\left(b\cos\left(fx+e\right)^2 - b\right)\log\left(\frac{1}{2}\cos\left(fx+e\right) + \frac{1}{2}\right)\sin\left(fx+e\right)}{12\left(f\cos\left(fx+e\right)^2 - f\right)\sin\left(fx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/12*(8*a*cos(f*x + e)^3 - 6*b*cos(f*x + e)*sin(f*x + e) + 3*(b*cos(f*x +
e)^2 - b)*log(1/2*cos(f*x + e) + 1/2)*sin(f*x + e) - 3*(b*cos(f*x + e)^2 -
b)*log(-1/2*cos(f*x + e) + 1/2)*sin(f*x + e) - 12*a*cos(f*x + e))/((f*cos(f
*x + e)^2 - f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*((256/3*tan((f*x+exp(1))/2)^3*a+
256*tan((f*x+exp(1))/2)^2*b+768*tan((f*x+exp(1))/2)*a)/4096+(-22*tan((f*x+e
xp(1))/2)^3*b-9*tan((f*x+exp(1))/2)^2*a-3*tan((f*x+exp(1))/2)*b-a)*1/48/tan
((f*x+exp(1))/2)^3+b/4*ln(abs(tan((f*x+exp(1))/2))))
```

maple [A] time = 0.34, size = 74, normalized size = 1.16

$$-\frac{2a\cot\left(fx+e\right)}{3f}-\frac{a\cot\left(fx+e\right)\left(\csc^{2}\left(fx+e\right)\right)}{3f}-\frac{b\cot\left(fx+e\right)\csc\left(fx+e\right)}{2f}+\frac{b\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sin(f*x+e)),x)
```

```
[Out] -2/3*a*cot(f*x+e)/f-1/3/f*a*cot(f*x+e)*csc(f*x+e)^2-1/2*b*cot(f*x+e)*csc(f*
x+e)/f+1/2/f*b*ln(csc(f*x+e)-cot(f*x+e))
```

maxima [A] time = 0.30, size = 73, normalized size = 1.14

$$\frac{3\,b\left(\frac{2\,\cos\left(fx+e\right)}{\cos\left(fx+e\right)^{2}-1}-\log\left(\cos\left(fx+e\right)+1\right)+\log\left(\cos\left(fx+e\right)-1\right)\right)-\frac{4\left(3\,\tan\left(fx+e\right)^{2}+1\right)a}{\tan\left(fx+e\right)^{3}}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*b*(2*cos(f*x + e)/(cos(f*x + e)^2 - 1) - log(cos(f*x + e) + 1) + lo
g(cos(f*x + e) - 1)) - 4*(3*tan(f*x + e)^2 + 1)*a/tan(f*x + e)^3)/f
```

mupad [B] time = 6.74, size = 111, normalized size = 1.73

$$\frac{3\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{8\,f}+\frac{a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}}{24\,f}+\frac{b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}}{8\,f}+\frac{b\,\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{2\,f}-\frac{\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}\,\left(3\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}+\right.}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))/sin(e + f*x)^4,x)
```

```
[Out] (3*a*tan(e/2 + (f*x)/2))/(8*f) + (a*tan(e/2 + (f*x)/2)^3)/(24*f) + (b*tan(e
/2 + (f*x)/2)^2)/(8*f) + (b*log(tan(e/2 + (f*x)/2)))/(2*f) - (cot(e/2 + (f*
x)/2)^3*(a/3 + b*tan(e/2 + (f*x)/2) + 3*a*tan(e/2 + (f*x)/2)^2))/(8*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sin\left(e + fx\right)\right)\csc^4\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sin(f*x+e)),x)
```

```
[Out] Integral((a + b*sin(e + f*x))*csc(e + f*x)**4, x)
```

3.157 $\int \sin^3(e+fx)(a+b\sin(e+fx))^2\,dx$

Optimal. Leaf size=112

$$\frac{\left(a^2+2b^2\right)\cos^3(e+fx)}{3f}-\frac{\left(a^2+b^2\right)\cos(e+fx)}{f}-\frac{ab\sin^3(e+fx)\cos(e+fx)}{2f}-\frac{3ab\sin(e+fx)\cos(e+fx)}{4f}+\frac{3ab}{4}$$

```
[Out] 3/4*a*b*x-(a^2+b^2)*cos(f*x+e)/f+1/3*(a^2+2*b^2)*cos(f*x+e)^3/f-1/5*b^2*cos
(f*x+e)^5/f-3/4*a*b*cos(f*x+e)*sin(f*x+e)/f-1/2*a*b*cos(f*x+e)*sin(f*x+e)^3
/f
```

Rubi [A] time = 0.10, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2789, 2635, 8, 3013, 373}

$$\frac{\left(a^2+2b^2\right)\cos^3(e+fx)}{3f}-\frac{\left(a^2+b^2\right)\cos(e+fx)}{f}-\frac{ab\sin^3(e+fx)\cos(e+fx)}{2f}-\frac{3ab\sin(e+fx)\cos(e+fx)}{4f}+\frac{3ab}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (3*a*b*x)/4 - ((a^2 + b^2)*Cos[e + f*x])/f + ((a^2 + 2*b^2)*Cos[e + f*x]^3)
/(3*f) - (b^2*Cos[e + f*x]^5)/(5*f) - (3*a*b*Cos[e + f*x]*Sin[e + f*x])/(4*
f) - (a*b*Cos[e + f*x]*Sin[e + f*x]^3)/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \sin^3(e+fx)(a+b\sin(e+fx))^2\,dx = (2ab)\int \sin^4(e+fx)\,dx + \int \sin^3(e+fx)\left(a^2+b^2\sin^2(e+fx)\right)\,dx$$
$$= -\frac{ab\cos(e+fx)\sin^3(e+fx)}{2f} + \frac{1}{2}(3ab)\int \sin^2(e+fx)\,dx - \frac{\mathrm{Subst}\left(\int \left(1\right.\right.}{}$$
$$= -\frac{3ab\cos(e+fx)\sin(e+fx)}{4f} - \frac{ab\cos(e+fx)\sin^3(e+fx)}{2f} + \frac{1}{4}(3ab)\int$$
$$= \frac{3abx}{4} - \frac{\left(a^2+b^2\right)\cos(e+fx)}{f} + \frac{\left(a^2+2b^2\right)\cos^3(e+fx)}{3f} - \frac{b^2\cos^5(e+f}{5f}$$

Mathematica [A] time = 0.35, size = 91, normalized size = 0.81

$$\frac{-30\left(6a^2+5b^2\right)\cos(e+fx)+5\left(4a^2+5b^2\right)\cos(3(e+fx))-3b(b\cos(5(e+fx))-5a(12(e+fx)-8\sin(2(e+f}{240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (-30*(6*a^2 + 5*b^2)*Cos[e + f*x] + 5*(4*a^2 + 5*b^2)*Cos[3*(e + f*x)] - 3*
b*(b*Cos[5*(e + f*x)] - 5*a*(12*(e + f*x) - 8*Sin[2*(e + f*x)] + Sin[4*(e +
 f*x)])))/(240*f)
```

fricas [A] time = 0.49, size = 90, normalized size = 0.80

$$-\frac{12\,b^2\cos\left(fx+e\right)^5 - 45\,abfx - 20\left(a^2+2\,b^2\right)\cos\left(fx+e\right)^3 + 60\left(a^2+b^2\right)\cos\left(fx+e\right) - 15\left(2\,ab\cos\left(fx+e\right)^3\right.}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/60*(12*b^2*cos(f*x + e)^5 - 45*a*b*f*x - 20*(a^2 + 2*b^2)*cos(f*x + e)^3
 + 60*(a^2 + b^2)*cos(f*x + e) - 15*(2*a*b*cos(f*x + e)^3 - 5*a*b*cos(f*x +
 e))*sin(f*x + e))/f
```

giac [A] time = 0.22, size = 128, normalized size = 1.14

$$\frac{3}{4}\,abx - \frac{b^2\cos\left(5\,fx + 5\,e\right)}{80\,f} + \frac{ab\sin\left(4\,fx + 4\,e\right)}{16\,f} - \frac{ab\sin\left(2\,fx + 2\,e\right)}{2\,f} + \frac{\left(4\,a^2 + 5\,b^2\right)\cos\left(3\,fx + 3\,e\right)}{48\,f} - \frac{\left(2\,a^2 + 3\,b^2\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 3/4*a*b*x - 1/80*b^2*cos(5*f*x + 5*e)/f + 1/16*a*b*sin(4*f*x + 4*e)/f - 1/2
*a*b*sin(2*f*x + 2*e)/f + 1/48*(4*a^2 + 5*b^2)*cos(3*f*x + 3*e)/f - 1/8*(2*
a^2 + 3*b^2)*cos(f*x + e)/f - 1/4*(2*a^2 + b^2)*cos(f*x + e)/f
```

maple [A] time = 0.28, size = 95, normalized size = 0.85

$$\frac{-\frac{b^2\left(\frac{8}{3}+\sin^4\left(fx+e\right)+\frac{4\left(\sin^2\left(fx+e\right)\right)}{3}\right)\cos\left(fx+e\right)}{5} + 2ab\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4} + \frac{3fx}{8} + \frac{3e}{8}\right) - \frac{a^2\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(-1/5*b^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+2*a*b*(-1/4*(s
in(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/3*a^2*(2+sin(f*x+e)
^2)*cos(f*x+e))
```

maxima [A] time = 0.72, size = 94, normalized size = 0.84

$$\frac{80\left(\cos\left(fx+e\right)^3 - 3\,\cos\left(fx+e\right)\right)a^2 + 15\left(12\,fx + 12\,e + \sin\left(4\,fx + 4\,e\right) - 8\,\sin\left(2\,fx + 2\,e\right)\right)ab - 16\left(3\,\cos\left(\right.\right.}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/240*(80*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2 + 15*(12*f*x + 12*e + sin(4
*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a*b - 16*(3*cos(f*x + e)^5 - 10*cos(f*x +
 e)^3 + 15*cos(f*x + e))*b^2)/f
```

mupad [B] time = 10.37, size = 157, normalized size = 1.40

$$\frac{3\,a\,b\,x}{4}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{20\,a^2}{3}+\frac{16\,b^2}{3}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(\frac{28\,a^2}{3}+\frac{32\,b^2}{3}\right)+4\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+\frac{4\,a^2}{3}+\frac{16\,b^2}{15}+7\,a\,b\tan}{f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b*sin(e + f*x))^2,x)
```

```
[Out] (3*a*b*x)/4 - (tan(e/2 + (f*x)/2)^2*((20*a^2)/3 + (16*b^2)/3) + tan(e/2 + (
f*x)/2)^4*((28*a^2)/3 + (32*b^2)/3) + 4*a^2*tan(e/2 + (f*x)/2)^6 + (4*a^2)/
3 + (16*b^2)/15 + 7*a*b*tan(e/2 + (f*x)/2)^3 - 7*a*b*tan(e/2 + (f*x)/2)^7 -
 (3*a*b*tan(e/2 + (f*x)/2)^9)/2 + (3*a*b*tan(e/2 + (f*x)/2))/2)/(f*(tan(e/2
 + (f*x)/2)^2 + 1)^5)
```

sympy [A] time = 3.10, size = 221, normalized size = 1.97

$$\begin{cases} -\frac{a^2\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{2a^2\cos^3\left(e+fx\right)}{3f}+\frac{3abx\sin^4\left(e+fx\right)}{4}+\frac{3abx\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{2}+\frac{3abx\cos^4\left(e+fx\right)}{4}-\frac{5ab\sin^3\left(e+fx\right)\cos}{4f} \\ x\left(a+b\sin\left(e\right)\right)^2\sin^3\left(e\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-a**2*sin(e + f*x)**2*cos(e + f*x)/f - 2*a**2*cos(e + f*x)**3/(3
*f) + 3*a*b*x*sin(e + f*x)**4/4 + 3*a*b*x*sin(e + f*x)**2*cos(e + f*x)**2/2
 + 3*a*b*x*cos(e + f*x)**4/4 - 5*a*b*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 3
*a*b*sin(e + f*x)*cos(e + f*x)**3/(4*f) - b**2*sin(e + f*x)**4*cos(e + f*x)
/f - 4*b**2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 8*b**2*cos(e + f*x)**5/
(15*f), Ne(f, 0)), (x*(a + b*sin(e))**2*sin(e)**3, True))
```

3.158 $\int \sin^2(e+fx)(a+b\sin(e+fx))^2\,dx$

Optimal. Leaf size=101

$$-\frac{(4a^2+3b^2)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}x\left(4a^2+3b^2\right)+\frac{2ab\cos^3(e+fx)}{3f}-\frac{2ab\cos(e+fx)}{f}-\frac{b^2\sin^3(e+fx)\cos}{4f}$$

```
[Out] 1/8*(4*a^2+3*b^2)*x-2*a*b*cos(f*x+e)/f+2/3*a*b*cos(f*x+e)^3/f-1/8*(4*a^2+3*
b^2)*cos(f*x+e)*sin(f*x+e)/f-1/4*b^2*cos(f*x+e)*sin(f*x+e)^3/f
```

Rubi [A] time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2789, 2633, 3014, 2635, 8}

$$-\frac{(4a^2+3b^2)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}x\left(4a^2+3b^2\right)+\frac{2ab\cos^3(e+fx)}{3f}-\frac{2ab\cos(e+fx)}{f}-\frac{b^2\sin^3(e+fx)\cos}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((4*a^2 + 3*b^2)*x)/8 - (2*a*b*Cos[e + f*x])/f + (2*a*b*Cos[e + f*x]^3)/(3*
f) - ((4*a^2 + 3*b^2)*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (b^2*Cos[e + f*x]*
Sin[e + f*x]^3)/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
```

```
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int \sin^2(e+fx)(a+b\sin(e+fx))^2\,dx = (2ab)\int \sin^3(e+fx)\,dx + \int \sin^2(e+fx)\left(a^2+b^2\sin^2(e+fx)\right)\,dx$$
$$= -\frac{b^2\cos(e+fx)\sin^3(e+fx)}{4f} + \frac{1}{4}\left(4a^2+3b^2\right)\int \sin^2(e+fx)\,dx - \frac{(2ab)}{}$$
$$= -\frac{2ab\cos(e+fx)}{f} + \frac{2ab\cos^3(e+fx)}{3f} - \frac{\left(4a^2+3b^2\right)\cos(e+fx)\sin(e+}{8f}$$
$$= \frac{1}{8}\left(4a^2+3b^2\right)x - \frac{2ab\cos(e+fx)}{f} + \frac{2ab\cos^3(e+fx)}{3f} - \frac{\left(4a^2+3b^2\right)\cos}{}$$

Mathematica [A] time = 0.16, size = 117, normalized size = 1.16

$$\frac{a^2(e+fx)}{2f} - \frac{a^2\sin(2(e+fx))}{4f} - \frac{3ab\cos(e+fx)}{2f} + \frac{ab\cos(3(e+fx))}{6f} + \frac{3b^2(e+fx)}{8f} - \frac{b^2\sin(2(e+fx))}{4f} + \frac{b^2\sin(4(e+}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*(e + f*x))/(2*f) + (3*b^2*(e + f*x))/(8*f) - (3*a*b*Cos[e + f*x])/(2*f
) + (a*b*Cos[3*(e + f*x)])/(6*f) - (a^2*Sin[2*(e + f*x)])/(4*f) - (b^2*Sin[
2*(e + f*x)])/(4*f) + (b^2*Sin[4*(e + f*x)])/(32*f)
```

fricas [A] time = 0.53, size = 84, normalized size = 0.83

$$\frac{16\,ab\cos\left(fx+e\right)^3 + 3\left(4\,a^2+3\,b^2\right)fx - 48\,ab\cos\left(fx+e\right) + 3\left(2\,b^2\cos\left(fx+e\right)^3 - \left(4\,a^2+5\,b^2\right)\cos\left(fx+e\right)\right)\mathrm{s}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(16*a*b*cos(f*x + e)^3 + 3*(4*a^2 + 3*b^2)*f*x - 48*a*b*cos(f*x + e) +
 3*(2*b^2*cos(f*x + e)^3 - (4*a^2 + 5*b^2)*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.88, size = 86, normalized size = 0.85

$$\frac{1}{8}\left(4\,a^2+3\,b^2\right)x+\frac{ab\cos\left(3\,fx+3\,e\right)}{6\,f}-\frac{3\,ab\cos\left(fx+e\right)}{2\,f}+\frac{b^2\sin\left(4\,fx+4\,e\right)}{32\,f}-\frac{\left(a^2+b^2\right)\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/8*(4*a^2 + 3*b^2)*x + 1/6*a*b*cos(3*f*x + 3*e)/f - 3/2*a*b*cos(f*x + e)/f
 + 1/32*b^2*sin(4*f*x + 4*e)/f - 1/4*(a^2 + b^2)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.21, size = 89, normalized size = 0.88

$$\frac{b^2\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{2ab\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}+a^2\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(b^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2/3*
a*b*(2+sin(f*x+e)^2)*cos(f*x+e)+a^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2
*e))
```

maxima [A] time = 1.00, size = 84, normalized size = 0.83

$$\frac{24\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)a^2+64\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)ab+3\left(12\,fx+12\,e+\sin\left(4\,fx+4\,e\right)\right.}{96\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/96*(24*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2 + 64*(cos(f*x + e)^3 - 3*cos(
f*x + e))*a*b + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*b
^2)/f
```

mupad [B] time = 6.93, size = 85, normalized size = 0.84

$$\frac{\frac{3\,b^2\,\sin\left(4\,e+4\,f\,x\right)}{4}-6\,b^2\,\sin\left(2\,e+2\,f\,x\right)-6\,a^2\,\sin\left(2\,e+2\,f\,x\right)-36\,a\,b\,\cos\left(e+f\,x\right)+4\,a\,b\,\cos\left(3\,e+3\,f\,x\right)+12\,a}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b*sin(e + f*x))^2,x)
```

```
[Out] ((3*b^2*sin(4*e + 4*f*x))/4 - 6*b^2*sin(2*e + 2*f*x) - 6*a^2*sin(2*e + 2*f*
x) - 36*a*b*cos(e + f*x) + 4*a*b*cos(3*e + 3*f*x) + 12*a^2*f*x + 9*b^2*f*x)
/(24*f)
```

sympy [A] time = 1.73, size = 211, normalized size = 2.09

$$\begin{cases} \frac{a^2x\sin^2\left(e+fx\right)}{2}+\frac{a^2x\cos^2\left(e+fx\right)}{2}-\frac{a^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{2ab\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{4ab\cos^3\left(e+fx\right)}{3f}+\frac{3b^2x\sin^4\left(e+fx\right)}{8}+\frac{3b^2x\sin}{} \\ x\left(a+b\sin\left(e\right)\right)^2\sin^2\left(e\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(e + f*x)**2/2 + a**2*x*cos(e + f*x)**2/2 - a**2*sin(e
 + f*x)*cos(e + f*x)/(2*f) - 2*a*b*sin(e + f*x)**2*cos(e + f*x)/f - 4*a*b*c
os(e + f*x)**3/(3*f) + 3*b**2*x*sin(e + f*x)**4/8 + 3*b**2*x*sin(e + f*x)**
2*cos(e + f*x)**2/4 + 3*b**2*x*cos(e + f*x)**4/8 - 5*b**2*sin(e + f*x)**3*c
os(e + f*x)/(8*f) - 3*b**2*sin(e + f*x)*cos(e + f*x)**3/(8*f), Ne(f, 0)), (
x*(a + b*sin(e))**2*sin(e)**2, True))
```

3.159 $\int \sin(e+fx)(a+b\sin(e+fx))^2\,dx$

Optimal. Leaf size=71

$$-\frac{2\left(a^2+b^2\right)\cos(e+fx)}{3f}-\frac{\cos(e+fx)(a+b\sin(e+fx))^2}{3f}-\frac{ab\sin(e+fx)\cos(e+fx)}{3f}+abx$$

```
[Out] a*b*x-2/3*(a^2+b^2)*cos(f*x+e)/f-1/3*a*b*cos(f*x+e)*sin(f*x+e)/f-1/3*cos(f*
x+e)*(a+b*sin(f*x+e))^2/f
```

Rubi [A] time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2753, 2734}

$$-\frac{2\left(a^2+b^2\right)\cos(e+fx)}{3f}-\frac{\cos(e+fx)(a+b\sin(e+fx))^2}{3f}-\frac{ab\sin(e+fx)\cos(e+fx)}{3f}+abx$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] a*b*x - (2*(a^2 + b^2)*Cos[e + f*x])/(3*f) - (a*b*Cos[e + f*x]*Sin[e + f*x]
)/(3*f) - (Cos[e + f*x]*(a + b*Sin[e + f*x])^2)/(3*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int \sin(e+fx)(a+b\sin(e+fx))^2\,dx = -\frac{\cos(e+fx)(a+b\sin(e+fx))^2}{3f} + \frac{1}{3}\int (2b+2a\sin(e+fx))(a+b\sin(e$$

$$= abx - \frac{2\left(a^2+b^2\right)\cos(e+fx)}{3f} - \frac{ab\cos(e+fx)\sin(e+fx)}{3f} - \frac{\cos(e+fx)}{}$$

Mathematica [A] time = 0.22, size = 59, normalized size = 0.83

$$\frac{b(12a(e+fx)-6a\sin(2(e+fx))+b\cos(3(e+fx)))-3\left(4a^2+3b^2\right)\cos(e+fx)}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (-3*(4*a^2 + 3*b^2)*Cos[e + f*x] + b*(12*a*(e + f*x) + b*Cos[3*(e + f*x)] -
 6*a*Sin[2*(e + f*x)]))/(12*f)
```

fricas [A] time = 0.49, size = 55, normalized size = 0.77

$$\frac{b^2\cos\left(fx+e\right)^3+3\,abfx-3\,ab\cos\left(fx+e\right)\sin\left(fx+e\right)-3\left(a^2+b^2\right)\cos\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*cos(f*x + e)^3 + 3*a*b*f*x - 3*a*b*cos(f*x + e)*sin(f*x + e) - 3*(
a^2 + b^2)*cos(f*x + e))/f
```

giac [A] time = 0.28, size = 76, normalized size = 1.07

$$abx + \frac{b^2\cos\left(3\,fx+3\,e\right)}{12\,f} - \frac{b^2\cos\left(fx+e\right)}{4\,f} - \frac{ab\sin\left(2\,fx+2\,e\right)}{2\,f} - \frac{\left(2\,a^2+b^2\right)\cos\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] a*b*x + 1/12*b^2*cos(3*f*x + 3*e)/f - 1/4*b^2*cos(f*x + e)/f - 1/2*a*b*sin(
2*f*x + 2*e)/f - 1/2*(2*a^2 + b^2)*cos(f*x + e)/f
```

maple [A] time = 0.16, size = 64, normalized size = 0.90

$$\frac{-\frac{b^2\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}+2ab\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\cos\left(fx+e\right)a^2}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(-1/3*b^2*(2+sin(f*x+e)^2)*cos(f*x+e)+2*a*b*(-1/2*sin(f*x+e)*cos(f*x+e)
+1/2*f*x+1/2*e)-cos(f*x+e)*a^2)
```

maxima [A] time = 0.64, size = 62, normalized size = 0.87

$$\frac{3\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)ab+2\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)b^2-6\,a^2\cos\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*b + 2*(cos(f*x + e)^3 - 3*cos(f*x
 + e))*b^2 - 6*a^2*cos(f*x + e))/f
```

mupad [B] time = 8.99, size = 103, normalized size = 1.45

$$a\,b\,x-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(4\,a^2+4\,b^2\right)+2\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+2\,a^2+\frac{4\,b^2}{3}-2\,a\,b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+2\,a\,b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b*sin(e + f*x))^2,x)
```

```
[Out] a*b*x - (tan(e/2 + (f*x)/2)^2*(4*a^2 + 4*b^2) + 2*a^2*tan(e/2 + (f*x)/2)^4
+ 2*a^2 + (4*b^2)/3 - 2*a*b*tan(e/2 + (f*x)/2)^5 + 2*a*b*tan(e/2 + (f*x)/2)
)/(f*(tan(e/2 + (f*x)/2)^2 + 1)^3)
```

sympy [A] time = 0.72, size = 107, normalized size = 1.51

$$\begin{cases} -\frac{a^2\cos\left(e+fx\right)}{f}+abx\sin^2\left(e+fx\right)+abx\cos^2\left(e+fx\right)-\frac{ab\sin\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{b^2\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{2b^2\cos^3\left(e+f\right.}{3f} \\ x\left(a+b\sin\left(e\right)\right)^2\sin\left(e\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-a**2*cos(e + f*x)/f + a*b*x*sin(e + f*x)**2 + a*b*x*cos(e + f*x
)**2 - a*b*sin(e + f*x)*cos(e + f*x)/f - b**2*sin(e + f*x)**2*cos(e + f*x)/
f - 2*b**2*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(a + b*sin(e))**2*sin(e), T
rue))
```

3.160 $\int (a+b\sin(e+fx))^2\,dx$

Optimal. Leaf size=50

$$\frac{1}{2}x\left(2a^2+b^2\right)-\frac{2ab\cos(e+fx)}{f}-\frac{b^2\sin(e+fx)\cos(e+fx)}{2f}$$

```
[Out] 1/2*(2*a^2+b^2)*x-2*a*b*cos(f*x+e)/f-1/2*b^2*cos(f*x+e)*sin(f*x+e)/f
```

Rubi [A] time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2644}

$$\frac{1}{2}x\left(2a^2+b^2\right)-\frac{2ab\cos(e+fx)}{f}-\frac{b^2\sin(e+fx)\cos(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x)/2 - (2*a*b*Cos[e + f*x])/f - (b^2*Cos[e + f*x]*Sin[e + f*
x])/(2*f)
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a+b\sin(e+fx))^2\,dx=\frac{1}{2}\left(2a^2+b^2\right)x-\frac{2ab\cos(e+fx)}{f}-\frac{b^2\cos(e+fx)\sin(e+fx)}{2f}$$

Mathematica [A] time = 0.10, size = 46, normalized size = 0.92

$$-\frac{-2\left(2a^2+b^2\right)(e+fx)+8ab\cos(e+fx)+b^2\sin(2(e+fx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -1/4*(-2*(2*a^2 + b^2)*(e + f*x) + 8*a*b*Cos[e + f*x] + b^2*Sin[2*(e + f*x)
])/f
```

fricas [A] time = 0.48, size = 45, normalized size = 0.90

$$-\frac{b^2 \cos\left(fx+e\right)\sin\left(fx+e\right) - \left(2\,a^2+b^2\right)fx + 4\,ab\cos\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*cos(f*x + e)*sin(f*x + e) - (2*a^2 + b^2)*f*x + 4*a*b*cos(f*x + e
))/f
```

giac [A] time = 0.16, size = 45, normalized size = 0.90

$$\frac{1}{2}\left(2\,a^2+b^2\right)x - \frac{2\,ab\cos\left(fx+e\right)}{f} - \frac{b^2\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2 + b^2)*x - 2*a*b*cos(f*x + e)/f - 1/4*b^2*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.08, size = 51, normalized size = 1.02

$$\frac{b^2\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2} + \frac{fx}{2} + \frac{e}{2}\right) - 2ab\cos\left(fx+e\right) + a^2\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(b^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2*a*b*cos(f*x+e)+a^2*(f
*x+e))
```

maxima [A] time = 0.30, size = 46, normalized size = 0.92

$$a^2x + \frac{\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)b^2}{4\,f} - \frac{2\,ab\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/4*(2*f*x + 2*e - sin(2*f*x + 2*e))*b^2/f - 2*a*b*cos(f*x + e)/f
```

mupad [B] time = 6.79, size = 44, normalized size = 0.88

$$-\frac{\frac{b^2\,\sin\left(2\,e+2\,f\,x\right)}{2}+4\,a\,b\,\cos\left(e+f\,x\right)-2\,a^2\,f\,x-b^2\,f\,x}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2,x)
```

```
[Out] -((b^2*sin(2*e + 2*f*x))/2 + 4*a*b*cos(e + f*x) - 2*a^2*f*x - b^2*f*x)/(2*f
)
```

sympy [A] time = 0.40, size = 78, normalized size = 1.56

$$\begin{cases} a^2x - \frac{2ab\cos\left(e+fx\right)}{f} + \frac{b^2x\sin^2\left(e+fx\right)}{2} + \frac{b^2x\cos^2\left(e+fx\right)}{2} - \frac{b^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f} & \text{for}\ f \neq 0 \\ x\left(a+b\sin\left(e\right)\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*x - 2*a*b*cos(e + f*x)/f + b**2*x*sin(e + f*x)**2/2 + b**2*
x*cos(e + f*x)**2/2 - b**2*sin(e + f*x)*cos(e + f*x)/(2*f), Ne(f, 0)), (x*(
a + b*sin(e))**2, True))
```

3.161 $\int \csc(e+fx)(a+b\sin(e+fx))^2\,dx$

Optimal. Leaf size=35

$$-\frac{a^2\tanh^{-1}(\cos(e+fx))}{f}+2abx-\frac{b^2\cos(e+fx)}{f}$$

```
[Out] 2*a*b*x-a^2*arctanh(cos(f*x+e))/f-b^2*cos(f*x+e)/f
```

Rubi [A] time = 0.06, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2746, 2735, 3770}

$$-\frac{a^2\tanh^{-1}(\cos(e+fx))}{f}+2abx-\frac{b^2\cos(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] 2*a*b*x - (a^2*ArcTanh[Cos[e + f*x]])/f - (b^2*Cos[e + f*x])/f
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc(e+fx)(a+b\sin(e+fx))^2\,dx = -\frac{b^2\cos(e+fx)}{f} + \int \csc(e+fx)\left(a^2+2ab\sin(e+fx)\right)dx$$
$$= 2abx - \frac{b^2\cos(e+fx)}{f} + a^2\int \csc(e+fx)\,dx$$
$$= 2abx - \frac{a^2\tanh^{-1}(\cos(e+fx))}{f} - \frac{b^2\cos(e+fx)}{f}$$

Mathematica [B] time = 0.02, size = 76, normalized size = 2.17

$$\frac{a^2\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f} - \frac{a^2\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f} + 2abx + \frac{b^2\sin(e)\sin(fx)}{f} - \frac{b^2\cos(e)\cos(fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] 2*a*b*x - (b^2*Cos[e]*Cos[f*x])/f - (a^2*Log[Cos[e/2 + (f*x)/2]])/f + (a^2*
Log[Sin[e/2 + (f*x)/2]])/f + (b^2*Sin[e]*Sin[f*x])/f
```

fricas [A] time = 0.49, size = 54, normalized size = 1.54

$$\frac{4\,abfx - 2\,b^2\cos\left(fx+e\right) - a^2\log\left(\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right) + a^2\log\left(-\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*b*f*x - 2*b^2*cos(f*x + e) - a^2*log(1/2*cos(f*x + e) + 1/2) + a^2
*log(-1/2*cos(f*x + e) + 1/2))/f
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(a^2/2*ln(abs(tan((f*x+exp(1))/2
)))+4*a*b/2*(f*x+exp(1))/2-b^2/(tan((f*x+exp(1))/2)^2+1))
```

maple [A] time = 0.20, size = 52, normalized size = 1.49

$$2abx + \frac{a^2 \ln\left(\csc\left(fx+e\right) - \cot\left(fx+e\right)\right)}{f} - \frac{b^2 \cos\left(fx+e\right)}{f} + \frac{2abe}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sin(f*x+e))^2,x)
```

```
[Out] 2*a*b*x+1/f*a^2*ln(csc(f*x+e)-cot(f*x+e))-b^2*cos(f*x+e)/f+2/f*a*b*e
```

maxima [A] time = 0.64, size = 44, normalized size = 1.26

$$\frac{2\left(fx+e\right)ab - b^2\cos\left(fx+e\right) - a^2\log\left(\cot\left(fx+e\right) + \csc\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] (2*(f*x + e)*a*b - b^2*cos(f*x + e) - a^2*log(cot(f*x + e) + csc(f*x + e)))
/f
```

mupad [B] time = 6.48, size = 125, normalized size = 3.57

$$\frac{a^2\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f} - \frac{2b^2}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1\right)} + \frac{4ab\,\mathrm{atan}\left(\frac{16a^2b^2}{8a^3b-16a^2b^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)} + \frac{8a^3b\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{8a^3b-16a^2b^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/sin(e + f*x),x)
```

```
[Out] (a^2*log(tan(e/2 + (f*x)/2)))/f - (2*b^2)/(f*(tan(e/2 + (f*x)/2)^2 + 1)) +
(4*a*b*atan((16*a^2*b^2)/(8*a^3*b - 16*a^2*b^2*tan(e/2 + (f*x)/2)) + (8*a^3
*b*tan(e/2 + (f*x)/2))/(8*a^3*b - 16*a^2*b^2*tan(e/2 + (f*x)/2))))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sin\left(e+fx\right)\right)^2 \csc\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Integral((a + b*sin(e + f*x))**2*csc(e + f*x), x)
```

3.162 $\int \csc^2(e+fx)(a+b\sin(e+fx))^2\,dx$

Optimal. Leaf size=34

$$-\frac{a^2\cot(e+fx)}{f}-\frac{2ab\tanh^{-1}(\cos(e+fx))}{f}+b^2x$$

```
[Out] b^2*x-2*a*b*arctanh(cos(f*x+e))/f-a^2*cot(f*x+e)/f
```

Rubi [A] time = 0.07, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2789, 3770, 3012, 8}

$$-\frac{a^2\cot(e+fx)}{f}-\frac{2ab\tanh^{-1}(\cos(e+fx))}{f}+b^2x$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] b^2*x - (2*a*b*ArcTanh[Cos[e + f*x]])/f - (a^2*Cot[e + f*x])/f
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^2(e+fx)(a+b\sin(e+fx))^2\,dx=(2ab)\int \csc(e+fx)\,dx+\int \csc^2(e+fx)\left(a^2+b^2\sin^2(e+fx)\right)\,dx$$
$$=-\frac{2ab\tanh^{-1}(\cos(e+fx))}{f}-\frac{a^2\cot(e+fx)}{f}+b^2\int 1\,dx$$
$$=b^2x-\frac{2ab\tanh^{-1}(\cos(e+fx))}{f}-\frac{a^2\cot(e+fx)}{f}$$

Mathematica [B] time = 0.24, size = 76, normalized size = 2.24

$$\frac{a^2\tan\left(\frac{1}{2}(e+fx)\right)+a^2\left(-\cot\left(\frac{1}{2}(e+fx)\right)\right)+2b\left(2a\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)-2a\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)+be+bfx\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (-(a^2*Cot[(e + f*x)/2]) + 2*b*(b*e + b*f*x - 2*a*Log[Cos[(e + f*x)/2]] + 2
*a*Log[Sin[(e + f*x)/2]]) + a^2*Tan[(e + f*x)/2])/(2*f)
```

fricas [B] time = 0.52, size = 77, normalized size = 2.26

$$\frac{b^2fx\sin\left(fx+e\right)-ab\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)\sin\left(fx+e\right)+ab\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)\sin\left(fx+e\right)-a^2\cos\left(fx+e\right)}{f\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] (b^2*f*x*sin(f*x + e) - a*b*log(1/2*cos(f*x + e) + 1/2)*sin(f*x + e) + a*b*
log(-1/2*cos(f*x + e) + 1/2)*sin(f*x + e) - a^2*cos(f*x + e))/(f*sin(f*x +
e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(tan((f*x+exp(1))/2)*a^2/4+(-4*t
an((f*x+exp(1))/2)*b*a-a^2)*1/4/tan((f*x+exp(1))/2)+2*b^2/2*(f*x+exp(1))/2+
b*a*ln(abs(tan((f*x+exp(1))/2))))
```

maple [A] time = 0.25, size = 52, normalized size = 1.53

$$b^2x - \frac{a^2\cot\left(fx+e\right)}{f} + \frac{2ab\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{f} + \frac{b^2e}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sin(f*x+e))^2,x)
```

```
[Out] b^2*x-a^2*cot(f*x+e)/f+2/f*a*b*ln(csc(f*x+e)-cot(f*x+e))+1/f*b^2*e
```

maxima [A] time = 0.63, size = 52, normalized size = 1.53

$$\frac{\left(fx+e\right)b^2 - ab\left(\log\left(\cos\left(fx+e\right)+1\right) - \log\left(\cos\left(fx+e\right)-1\right)\right) - \frac{a^2}{\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] ((f*x + e)*b^2 - a*b*(log(cos(f*x + e) + 1) - log(cos(f*x + e) - 1)) - a^2/
tan(f*x + e))/f
```

mupad [B] time = 6.83, size = 105, normalized size = 3.09

$$\frac{2\,b^2\,\mathrm{atan}\left(\frac{b\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+2\,a\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{2\,a\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)-b\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f} - \frac{a^2\cot\left(e+f\,x\right)}{f} + \frac{2\,a\,b\,\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/sin(e + f*x)^2,x)
```

```
[Out] (2*b^2*atan((b*cos(e/2 + (f*x)/2) + 2*a*sin(e/2 + (f*x)/2))/(2*a*cos(e/2 +
(f*x)/2) - b*sin(e/2 + (f*x)/2))))/f - (a^2*cot(e + f*x))/f + (2*a*b*log(si
n(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(e+fx\right)\right)^2\csc^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Integral((a + b*sin(e + f*x))**2*csc(e + f*x)**2, x)
```

3.163 $\int \csc^3(e+fx)(a+b\sin(e+fx))^2\,dx$

Optimal. Leaf size=59

$$-\frac{(a^2+2b^2)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{a^2\cot(e+fx)\csc(e+fx)}{2f}-\frac{2ab\cot(e+fx)}{f}$$

```
[Out] -1/2*(a^2+2*b^2)*arctanh(cos(f*x+e))/f-2*a*b*cot(f*x+e)/f-1/2*a^2*cot(f*x+e
)*csc(f*x+e)/f
```

Rubi [A] time = 0.08, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2789, 3767, 8, 3012, 3770}

$$-\frac{(a^2+2b^2)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{a^2\cot(e+fx)\csc(e+fx)}{2f}-\frac{2ab\cot(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -((a^2 + 2*b^2)*ArcTanh[Cos[e + f*x]])/(2*f) - (2*a*b*Cot[e + f*x])/f - (a^
2*Cot[e + f*x]*Csc[e + f*x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^3(e+fx)(a+b\sin(e+fx))^2\,dx = (2ab)\int \csc^2(e+fx)\,dx + \int \csc^3(e+fx)\left(a^2+b^2\sin^2(e+fx)\right)\,dx$$
$$= -\frac{a^2\cot(e+fx)\csc(e+fx)}{2f} + \frac{1}{2}\left(a^2+2b^2\right)\int \csc(e+fx)\,dx - \frac{(2ab)\,\mathrm{Su}}{}$$
$$= -\frac{\left(a^2+2b^2\right)\tanh^{-1}(\cos(e+fx))}{2f} - \frac{2ab\cot(e+fx)}{f} - \frac{a^2\cot(e+fx)\csc}{2f}$$

Mathematica [B] time = 0.47, size = 133, normalized size = 2.25

$$\frac{a^2\left(-\csc^2\left(\frac{1}{2}(e+fx)\right)\right)+a^2\sec^2\left(\frac{1}{2}(e+fx)\right)+4a^2\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)-4a^2\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)+8ab\tan\left(\right.}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (-8*a*b*Cot[(e + f*x)/2] - a^2*Csc[(e + f*x)/2]^2 - 4*a^2*Log[Cos[(e + f*x)
/2]] - 8*b^2*Log[Cos[(e + f*x)/2]] + 4*a^2*Log[Sin[(e + f*x)/2]] + 8*b^2*Lo
g[Sin[(e + f*x)/2]] + a^2*Sec[(e + f*x)/2]^2 + 8*a*b*Tan[(e + f*x)/2])/(8*f
)
```

fricas [B] time = 0.47, size = 129, normalized size = 2.19

$$\frac{8\,ab\cos\left(fx+e\right)\sin\left(fx+e\right)+2\,a^2\cos\left(fx+e\right)-\left(\left(a^2+2\,b^2\right)\cos\left(fx+e\right)^2-a^2-2\,b^2\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\right.}{4\left(f\cos\left(fx+e\right)^2-f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(8*a*b*cos(f*x + e)*sin(f*x + e) + 2*a^2*cos(f*x + e) - ((a^2 + 2*b^2)*
cos(f*x + e)^2 - a^2 - 2*b^2)*log(1/2*cos(f*x + e) + 1/2) + ((a^2 + 2*b^2)*
cos(f*x + e)^2 - a^2 - 2*b^2)*log(-1/2*cos(f*x + e) + 1/2))/(f*cos(f*x + e)
^2 - f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*((4*tan((f*x+exp(1))/2)^2*a^2+32
*tan((f*x+exp(1))/2)*b*a)/64+(-12*tan((f*x+exp(1))/2)^2*b^2-6*tan((f*x+exp(
1))/2)^2*a^2-8*tan((f*x+exp(1))/2)*b*a-a^2)*1/16/tan((f*x+exp(1))/2)^2+(2*b
^2+a^2)/4*ln(abs(tan((f*x+exp(1))/2))))
```

maple [A] time = 0.37, size = 82, normalized size = 1.39

$$-\frac{a^{2} \cot \left(f x+e\right) \csc \left(f x+e\right)}{2 f}+\frac{a^{2} \ln \left(\csc \left(f x+e\right)-\cot \left(f x+e\right)\right)}{2 f}-\frac{2 a b \cot \left(f x+e\right)}{f}+\frac{b^{2} \ln \left(\csc \left(f x+e\right)-\cot \left(\right.\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sin(f*x+e))^2,x)
```

```
[Out] -1/2*a^2*cot(f*x+e)*csc(f*x+e)/f+1/2/f*a^2*ln(csc(f*x+e)-cot(f*x+e))-2*a*b*
cot(f*x+e)/f+1/f*b^2*ln(csc(f*x+e)-cot(f*x+e))
```

maxima [A] time = 0.31, size = 89, normalized size = 1.51

$$\frac{a^{2}\left(\frac{2 \cos\left(f x+e\right)}{\cos\left(f x+e\right)^{2}-1}-\log\left(\cos\left(f x+e\right)+1\right)+\log\left(\cos\left(f x+e\right)-1\right)\right)-2 b^{2}\left(\log\left(\cos\left(f x+e\right)+1\right)-\log\left(\cos\left(f x -\right.\right.\right.}{4 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(a^2*(2*cos(f*x + e)/(cos(f*x + e)^2 - 1) - log(cos(f*x + e) + 1) + log
(cos(f*x + e) - 1)) - 2*b^2*(log(cos(f*x + e) + 1) - log(cos(f*x + e) - 1))
 - 8*a*b/tan(f*x + e))/f
```

mupad [B] time = 6.49, size = 92, normalized size = 1.56

$$\frac{a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2}{8f}+\frac{\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)\left(\frac{a^2}{2}+b^2\right)}{f}-\frac{\cot\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(\frac{a^2}{8}+b\tan\left(\frac{e}{2}+\frac{fx}{2}\right)a\right)}{f}+\frac{a\,b\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/sin(e + f*x)^3,x)
```

```
[Out] (a^2*tan(e/2 + (f*x)/2)^2)/(8*f) + (log(tan(e/2 + (f*x)/2))*(a^2/2 + b^2))/
f - (cot(e/2 + (f*x)/2)^2*(a^2/8 + a*b*tan(e/2 + (f*x)/2)))/f + (a*b*tan(e/
2 + (f*x)/2))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(e+fx\right)\right)^2\csc^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Integral((a + b*sin(e + f*x))**2*csc(e + f*x)**3, x)
```

3.164 $\int \csc^4(e+fx)(a+b\sin(e+fx))^2\,dx$

Optimal. Leaf size=82

$$-\frac{(2a^2+3b^2)\cot(e+fx)}{3f}-\frac{a^2\cot(e+fx)\csc^2(e+fx)}{3f}-\frac{ab\tanh^{-1}(\cos(e+fx))}{f}-\frac{ab\cot(e+fx)\csc(e+fx)}{f}$$

```
[Out] -a*b*arctanh(cos(f*x+e))/f-1/3*(2*a^2+3*b^2)*cot(f*x+e)/f-a*b*cot(f*x+e)*cs
c(f*x+e)/f-1/3*a^2*cot(f*x+e)*csc(f*x+e)^2/f
```

Rubi [A] time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2789, 3768, 3770, 3012, 3767, 8}

$$-\frac{(2a^2+3b^2)\cot(e+fx)}{3f}-\frac{a^2\cot(e+fx)\csc^2(e+fx)}{3f}-\frac{ab\tanh^{-1}(\cos(e+fx))}{f}-\frac{ab\cot(e+fx)\csc(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -((a*b*ArcTanh[Cos[e + f*x]])/f) - ((2*a^2 + 3*b^2)*Cot[e + f*x])/(3*f) - (
a*b*Cot[e + f*x]*Csc[e + f*x])/f - (a^2*Cot[e + f*x]*Csc[e + f*x]^2)/(3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^4(e+fx)(a+b\sin(e+fx))^2\,dx &= (2ab)\int \csc^3(e+fx)\,dx+\int \csc^4(e+fx)\left(a^2+b^2\sin^2(e+fx)\right)\,dx\\ &= -\frac{ab\cot(e+fx)\csc(e+fx)}{f}-\frac{a^2\cot(e+fx)\csc^2(e+fx)}{3f}+(ab)\int \csc(\\ &= -\frac{ab\tanh^{-1}(\cos(e+fx))}{f}-\frac{ab\cot(e+fx)\csc(e+fx)}{f}-\frac{a^2\cot(e+fx)\,c}{3f}\\ &= -\frac{ab\tanh^{-1}(\cos(e+fx))}{f}-\frac{\left(2a^2+3b^2\right)\cot(e+fx)}{3f}-\frac{ab\cot(e+fx)\csc}{f}\end{aligned}$$

Mathematica [A] time = 0.04, size = 132, normalized size = 1.61

$$-\frac{2a^2\cot(e+fx)}{3f}-\frac{a^2\cot(e+fx)\csc^2(e+fx)}{3f}-\frac{ab\csc^2\left(\frac{1}{2}(e+fx)\right)}{4f}+\frac{ab\sec^2\left(\frac{1}{2}(e+fx)\right)}{4f}+\frac{ab\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (-2*a^2*Cot[e + f*x])/(3*f) - (b^2*Cot[e + f*x])/f - (a*b*Csc[(e + f*x)/2]^
2)/(4*f) - (a^2*Cot[e + f*x]*Csc[e + f*x]^2)/(3*f) - (a*b*Log[Cos[(e + f*x)
/2]])/f + (a*b*Log[Sin[(e + f*x)/2]])/f + (a*b*Sec[(e + f*x)/2]^2)/(4*f)
```

fricas [A] time = 0.52, size = 149, normalized size = 1.82

$$-\frac{2\left(2\,a^{2}+3\,b^{2}\right)\cos\left(fx+e\right)^{3}-6\,ab\cos\left(fx+e\right)\sin\left(fx+e\right)+3\left(ab\cos\left(fx+e\right)^{2}-ab\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\right.}{6\left(f\cos\left(fx+e\right)^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*(2*a^2 + 3*b^2)*cos(f*x + e)^3 - 6*a*b*cos(f*x + e)*sin(f*x + e) +
3*(a*b*cos(f*x + e)^2 - a*b)*log(1/2*cos(f*x + e) + 1/2)*sin(f*x + e) - 3*(
a*b*cos(f*x + e)^2 - a*b)*log(-1/2*cos(f*x + e) + 1/2)*sin(f*x + e) - 6*(a^
2 + b^2)*cos(f*x + e))/((f*cos(f*x + e)^2 - f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*((256/3*tan((f*x+exp(1))/2)^3*a^
2+512*tan((f*x+exp(1))/2)^2*b*a+1024*tan((f*x+exp(1))/2)*b^2+768*tan((f*x+e
xp(1))/2)*a^2)/4096+(-44*tan((f*x+exp(1))/2)^3*b*a-12*tan((f*x+exp(1))/2)^2
*b^2-9*tan((f*x+exp(1))/2)^2*a^2-6*tan((f*x+exp(1))/2)*b*a-a^2)*1/48/tan((f
*x+exp(1))/2)^3+b*a/2*ln(abs(tan((f*x+exp(1))/2))))
```

maple [A] time = 0.40, size = 93, normalized size = 1.13

$$-\frac{2a^{2}\cot\left(fx+e\right)}{3f}-\frac{a^{2}\cot\left(fx+e\right)\left(\csc^{2}\left(fx+e\right)\right)}{3f}-\frac{ab\cot\left(fx+e\right)\csc\left(fx+e\right)}{f}+\frac{ab\ln\left(\csc\left(fx+e\right)-\cot\left(fx+\right.\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sin(f*x+e))^2,x)
```

```
[Out] -2/3*a^2*cot(f*x+e)/f-1/3*a^2*cot(f*x+e)*csc(f*x+e)^2/f-a*b*cot(f*x+e)*csc(
f*x+e)/f+1/f*a*b*ln(csc(f*x+e)-cot(f*x+e))-1/f*b^2*cot(f*x+e)
```

maxima [A] time = 0.69, size = 89, normalized size = 1.09

$$\frac{3\,ab\left(\frac{2\,\cos\left(fx+e\right)}{\cos\left(fx+e\right)^{2}-1}-\log\left(\cos\left(fx+e\right)+1\right)+\log\left(\cos\left(fx+e\right)-1\right)\right)-\frac{6\,b^{2}}{\tan\left(fx+e\right)}-\frac{2\left(3\,\tan\left(fx+e\right)^{2}+1\right)a^{2}}{\tan\left(fx+e\right)^{3}}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*a*b*(2*cos(f*x + e)/(cos(f*x + e)^2 - 1) - log(cos(f*x + e) + 1) + l
og(cos(f*x + e) - 1)) - 6*b^2/tan(f*x + e) - 2*(3*tan(f*x + e)^2 + 1)*a^2/t
an(f*x + e)^3)/f
```

mupad [B] time = 6.78, size = 136, normalized size = 1.66

$$\frac{a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3}{24f}+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{3a^2}{8}+\frac{b^2}{2}\right)}{f}-\frac{\cot\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(3a^2+4b^2\right)+\frac{a^2}{3}+2ab\tan\left(\frac{e}{2}+\frac{fx}{2}\right.\right.}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/sin(e + f*x)^4,x)
```

```
[Out] (a^2*tan(e/2 + (f*x)/2)^3)/(24*f) + (tan(e/2 + (f*x)/2)*((3*a^2)/8 + b^2/2)
)/f - (cot(e/2 + (f*x)/2)^3*(tan(e/2 + (f*x)/2)^2*(3*a^2 + 4*b^2) + a^2/3 +
 2*a*b*tan(e/2 + (f*x)/2)))/(8*f) + (a*b*tan(e/2 + (f*x)/2)^2)/(4*f) + (a*b
*log(tan(e/2 + (f*x)/2)))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(e+fx\right)\right)^2\csc^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Integral((a + b*sin(e + f*x))**2*csc(e + f*x)**4, x)
```

3.165 $\int \csc^5(e+fx)(a+b\sin(e+fx))^2\,dx$

Optimal. Leaf size=110

$$-\frac{(3a^2+4b^2)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{(3a^2+4b^2)\cot(e+fx)\csc(e+fx)}{8f}-\frac{a^2\cot(e+fx)\csc^3(e+fx)}{4f}-\frac{2ab\cot^3(\cdot}{3f}$$

```
[Out] -1/8*(3*a^2+4*b^2)*arctanh(cos(f*x+e))/f-2*a*b*cot(f*x+e)/f-2/3*a*b*cot(f*x
+e)^3/f-1/8*(3*a^2+4*b^2)*cot(f*x+e)*csc(f*x+e)/f-1/4*a^2*cot(f*x+e)*csc(f*
x+e)^3/f
```

Rubi [A] time = 0.09, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2789, 3767, 3012, 3768, 3770}

$$-\frac{(3a^2+4b^2)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{(3a^2+4b^2)\cot(e+fx)\csc(e+fx)}{8f}-\frac{a^2\cot(e+fx)\csc^3(e+fx)}{4f}-\frac{2ab\cot^3(\cdot}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -((3*a^2 + 4*b^2)*ArcTanh[Cos[e + f*x]])/(8*f) - (2*a*b*Cot[e + f*x])/f - (
2*a*b*Cot[e + f*x]^3)/(3*f) - ((3*a^2 + 4*b^2)*Cot[e + f*x]*Csc[e + f*x])/(
8*f) - (a^2*Cot[e + f*x]*Csc[e + f*x]^3)/(4*f)
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^5(e+fx)(a+b\sin(e+fx))^2\,dx = (2ab)\int \csc^4(e+fx)\,dx + \int \csc^5(e+fx)\left(a^2+b^2\sin^2(e+fx)\right)\,dx$$
$$= -\frac{a^2\cot(e+fx)\csc^3(e+fx)}{4f} + \frac{1}{4}\left(3a^2+4b^2\right)\int \csc^3(e+fx)\,dx - \frac{(2ab)}{}$$
$$= -\frac{2ab\cot(e+fx)}{f} - \frac{2ab\cot^3(e+fx)}{3f} - \frac{\left(3a^2+4b^2\right)\cot(e+fx)\csc(e+f}{8f}$$
$$= -\frac{\left(3a^2+4b^2\right)\tanh^{-1}(\cos(e+fx))}{8f} - \frac{2ab\cot(e+fx)}{f} - \frac{2ab\cot^3(e+fx)}{3f}$$

Mathematica [B] time = 0.04, size = 255, normalized size = 2.32

$$-\frac{a^2\csc^4\left(\frac{1}{2}(e+fx)\right)}{64f} - \frac{3a^2\csc^2\left(\frac{1}{2}(e+fx)\right)}{32f} + \frac{a^2\sec^4\left(\frac{1}{2}(e+fx)\right)}{64f} + \frac{3a^2\sec^2\left(\frac{1}{2}(e+fx)\right)}{32f} + \frac{3a^2\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{8f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (-4*a*b*Cot[e + f*x])/(3*f) - (3*a^2*Csc[(e + f*x)/2]^2)/(32*f) - (b^2*Csc[
(e + f*x)/2]^2)/(8*f) - (a^2*Csc[(e + f*x)/2]^4)/(64*f) - (2*a*b*Cot[e + f*
x]*Csc[e + f*x]^2)/(3*f) - (3*a^2*Log[Cos[(e + f*x)/2]])/(8*f) - (b^2*Log[C
os[(e + f*x)/2]])/(2*f) + (3*a^2*Log[Sin[(e + f*x)/2]])/(8*f) + (b^2*Log[Si
n[(e + f*x)/2]])/(2*f) + (3*a^2*Sec[(e + f*x)/2]^2)/(32*f) + (b^2*Sec[(e +
f*x)/2]^2)/(8*f) + (a^2*Sec[(e + f*x)/2]^4)/(64*f)
```

fricas [B] time = 0.47, size = 229, normalized size = 2.08

$$\frac{6\left(3a^2+4b^2\right)\cos\left(fx+e\right)^3-6\left(5a^2+4b^2\right)\cos\left(fx+e\right)-3\left(\left(3a^2+4b^2\right)\cos\left(fx+e\right)^4-2\left(3a^2+4b^2\right)\cos\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(6*(3*a^2 + 4*b^2)*cos(f*x + e)^3 - 6*(5*a^2 + 4*b^2)*cos(f*x + e) - 3
*((3*a^2 + 4*b^2)*cos(f*x + e)^4 - 2*(3*a^2 + 4*b^2)*cos(f*x + e)^2 + 3*a^2
 + 4*b^2)*log(1/2*cos(f*x + e) + 1/2) + 3*((3*a^2 + 4*b^2)*cos(f*x + e)^4 -
 2*(3*a^2 + 4*b^2)*cos(f*x + e)^2 + 3*a^2 + 4*b^2)*log(-1/2*cos(f*x + e) +
1/2) + 32*(2*a*b*cos(f*x + e)^3 - 3*a*b*cos(f*x + e))*sin(f*x + e))/(f*cos(
f*x + e)^4 - 2*f*cos(f*x + e)^2 + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*((8192*tan((f*x+exp(1))/2)^4*a^2
+131072/3*tan((f*x+exp(1))/2)^3*b*a+65536*tan((f*x+exp(1))/2)^2*b^2+65536*t
an((f*x+exp(1))/2)^2*a^2+393216*tan((f*x+exp(1))/2)*b*a)/1048576+(-200*tan(
(f*x+exp(1))/2)^4*b^2-150*tan((f*x+exp(1))/2)^4*a^2-144*tan((f*x+exp(1))/2)
^3*b*a-24*tan((f*x+exp(1))/2)^2*b^2-24*tan((f*x+exp(1))/2)^2*a^2-16*tan((f*
x+exp(1))/2)*b*a-3*a^2)*1/384/tan((f*x+exp(1))/2)^4+(4*b^2+3*a^2)/16*ln(abs
(tan((f*x+exp(1))/2))))
```

maple [A] time = 0.43, size = 146, normalized size = 1.33

$$-\frac{a^2\cot\left(fx+e\right)\left(\csc^3\left(fx+e\right)\right)}{4f}-\frac{3a^2\cot\left(fx+e\right)\csc\left(fx+e\right)}{8f}+\frac{3a^2\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{8f}-\frac{4ab\cot\left(\right.}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*sin(f*x+e))^2,x)
```

```
[Out] -1/4*a^2*cot(f*x+e)*csc(f*x+e)^3/f-3/8*a^2*cot(f*x+e)*csc(f*x+e)/f+3/8/f*a^
2*ln(csc(f*x+e)-cot(f*x+e))-4/3*a*b*cot(f*x+e)/f-2/3/f*a*b*cot(f*x+e)*csc(f
*x+e)^2-1/2/f*b^2*cot(f*x+e)*csc(f*x+e)+1/2/f*b^2*ln(csc(f*x+e)-cot(f*x+e))
```

maxima [A] time = 1.01, size = 147, normalized size = 1.34

$$\frac{3\,a^2\left(\frac{2\left(3\,\cos\left(fx+e\right)^3-5\,\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)^4-2\,\cos\left(fx+e\right)^2+1}-3\,\log\left(\cos\left(fx+e\right)+1\right)+3\,\log\left(\cos\left(fx+e\right)-1\right)\right)+12\,b^2\left(\frac{2\,\cos\left(fx+e\right)}{\cos\left(fx+e\right)^2-1}-\log\left(c\right.\right.}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(3*a^2*(2*(3*cos(f*x + e)^3 - 5*cos(f*x + e))/(cos(f*x + e)^4 - 2*cos(
f*x + e)^2 + 1) - 3*log(cos(f*x + e) + 1) + 3*log(cos(f*x + e) - 1)) + 12*b
^2*(2*cos(f*x + e)/(cos(f*x + e)^2 - 1) - log(cos(f*x + e) + 1) + log(cos(f
*x + e) - 1)) - 32*(3*tan(f*x + e)^2 + 1)*a*b/tan(f*x + e)^3)/f
```

mupad [B] time = 6.88, size = 178, normalized size = 1.62

$$\frac{\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\left(\frac{3\,a^2}{8}+\frac{b^2}{2}\right)}{f}+\frac{a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{64\,f}-\frac{\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\left(2\,a^2+2\,b^2\right)+\frac{a^2}{4}+12\,a\,b\tan\left(\frac{e}{2}\right.\right.}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/sin(e + f*x)^5,x)
```

```
[Out] (log(tan(e/2 + (f*x)/2))*((3*a^2)/8 + b^2/2))/f + (a^2*tan(e/2 + (f*x)/2)^4
)/(64*f) - (cot(e/2 + (f*x)/2)^4*(tan(e/2 + (f*x)/2)^2*(2*a^2 + 2*b^2) + a^
2/4 + 12*a*b*tan(e/2 + (f*x)/2)^3 + (4*a*b*tan(e/2 + (f*x)/2))/3))/(16*f) +
 (tan(e/2 + (f*x)/2)^2*(a^2/8 + b^2/8))/f + (a*b*tan(e/2 + (f*x)/2)^3)/(12*
f) + (3*a*b*tan(e/2 + (f*x)/2))/(4*f)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.166 $\int \sin^3(e+fx)(a+b\sin(e+fx))^3\,dx$

Optimal. Leaf size=171

$$\frac{a\left(a^2+6b^2\right)\cos^3(e+fx)}{3f}-\frac{a\left(a^2+3b^2\right)\cos(e+fx)}{f}-\frac{b\left(18a^2+5b^2\right)\sin^3(e+fx)\cos(e+fx)}{24f}-\frac{b\left(18a^2+5b^2\right)\sin$$

```
[Out] 1/16*b*(18*a^2+5*b^2)*x-a*(a^2+3*b^2)*cos(f*x+e)/f+1/3*a*(a^2+6*b^2)*cos(f*
x+e)^3/f-3/5*a*b^2*cos(f*x+e)^5/f-1/16*b*(18*a^2+5*b^2)*cos(f*x+e)*sin(f*x+
e)/f-1/24*b*(18*a^2+5*b^2)*cos(f*x+e)*sin(f*x+e)^3/f-1/6*b^3*cos(f*x+e)*sin
(f*x+e)^5/f
```

Rubi [A] time = 0.21, antiderivative size = 193, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2793, 3023, 2748, 2633, 2635, 8}

$$\frac{a\left(5a^2+12b^2\right)\cos^3(e+fx)}{15f}-\frac{a\left(5a^2+12b^2\right)\cos(e+fx)}{5f}-\frac{b\left(18a^2+5b^2\right)\sin^3(e+fx)\cos(e+fx)}{24f}-\frac{b\left(18a^2+5b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (b*(18*a^2 + 5*b^2)*x)/16 - (a*(5*a^2 + 12*b^2)*Cos[e + f*x])/(5*f) + (a*(5
*a^2 + 12*b^2)*Cos[e + f*x]^3)/(15*f) - (b*(18*a^2 + 5*b^2)*Cos[e + f*x]*Si
n[e + f*x])/(16*f) - (b*(18*a^2 + 5*b^2)*Cos[e + f*x]*Sin[e + f*x]^3)/(24*f
) - (13*a*b^2*Cos[e + f*x]*Sin[e + f*x]^4)/(30*f) - (b^2*Cos[e + f*x]*Sin[e
 + f*x]^4*(a + b*Sin[e + f*x]))/(6*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \sin^3(e+fx)(a+b\sin(e+fx))^3\,dx = -\frac{b^2\cos(e+fx)\sin^4(e+fx)(a+b\sin(e+fx))}{6f} + \frac{1}{6}\int \sin^3(e+fx)\left(2a\right.$$
$$= -\frac{13ab^2\cos(e+fx)\sin^4(e+fx)}{30f} - \frac{b^2\cos(e+fx)\sin^4(e+fx)(a+b\sin$$
$$= -\frac{13ab^2\cos(e+fx)\sin^4(e+fx)}{30f} - \frac{b^2\cos(e+fx)\sin^4(e+fx)(a+b\sin$$
$$= -\frac{b\left(18a^2+5b^2\right)\cos(e+fx)\sin^3(e+fx)}{24f} - \frac{13ab^2\cos(e+fx)\sin^4(e+$$
$$= -\frac{a\left(5a^2+12b^2\right)\cos(e+fx)}{5f} + \frac{a\left(5a^2+12b^2\right)\cos^3(e+fx)}{15f} - \frac{b\left(18a^2+\right.}{}$$
$$= \frac{1}{16}b\left(18a^2+5b^2\right)x - \frac{a\left(5a^2+12b^2\right)\cos(e+fx)}{5f} + \frac{a\left(5a^2+12b^2\right)\cos^3}{15f}$$

Mathematica [A] time = 0.74, size = 147, normalized size = 0.86

$$\frac{20\left(4a^3+15ab^2\right)\cos(3(e+fx)) - 360a\left(2a^2+5b^2\right)\cos(e+fx) + b\left(5\left(-9\left(16a^2+5b^2\right)\sin(2(e+fx)) + 9\left(2a^2\right.\right.\right.}{960f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (-360*a*(2*a^2 + 5*b^2)*Cos[e + f*x] + 20*(4*a^3 + 15*a*b^2)*Cos[3*(e + f*x
)] + b*(-36*a*b*Cos[5*(e + f*x)] + 5*(216*a^2*e + 60*b^2*e + 216*a^2*f*x +
60*b^2*f*x - 9*(16*a^2 + 5*b^2)*Sin[2*(e + f*x)] + 9*(2*a^2 + b^2)*Sin[4*(e
 + f*x)] - b^2*Sin[6*(e + f*x)])))/(960*f)
```

fricas [A] time = 0.52, size = 138, normalized size = 0.81

$$-\frac{144\,ab^2\cos\left(fx+e\right)^5 - 80\left(a^3+6\,ab^2\right)\cos\left(fx+e\right)^3 - 15\left(18\,a^2b+5\,b^3\right)fx + 240\left(a^3+3\,ab^2\right)\cos\left(fx+e\right) + }{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/240*(144*a*b^2*cos(f*x + e)^5 - 80*(a^3 + 6*a*b^2)*cos(f*x + e)^3 - 15*(
18*a^2*b + 5*b^3)*f*x + 240*(a^3 + 3*a*b^2)*cos(f*x + e) + 5*(8*b^3*cos(f*x
```

```
 + e)^5 - 2*(18*a^2*b + 13*b^3)*cos(f*x + e)^3 + 3*(30*a^2*b + 11*b^3)*cos(
f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.20, size = 180, normalized size = 1.05

$$-\frac{3\,ab^2\cos\left(5\,fx+5\,e\right)}{80\,f}-\frac{b^3\sin\left(6\,fx+6\,e\right)}{192\,f}+\frac{1}{16}\left(18\,a^2b+5\,b^3\right)x+\frac{\left(4\,a^3+15\,ab^2\right)\cos\left(3\,fx+3\,e\right)}{48\,f}-\frac{\left(2\,a^3+9\,ab^2\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -3/80*a*b^2*cos(5*f*x + 5*e)/f - 1/192*b^3*sin(6*f*x + 6*e)/f + 1/16*(18*a^
2*b + 5*b^3)*x + 1/48*(4*a^3 + 15*a*b^2)*cos(3*f*x + 3*e)/f - 1/8*(2*a^3 +
9*a*b^2)*cos(f*x + e)/f - 1/4*(2*a^3 + 3*a*b^2)*cos(f*x + e)/f + 3/64*(2*a^
2*b + b^3)*sin(4*f*x + 4*e)/f - 3/64*(16*a^2*b + 5*b^3)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.35, size = 145, normalized size = 0.85

$$\frac{b^3\left(-\frac{\left(\sin^5\left(fx+e\right)+\frac{5\left(\sin^3\left(fx+e\right)\right)}{4}+\frac{15\sin\left(fx+e\right)}{8}\right)\cos\left(fx+e\right)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)-\frac{3ab^2\left(\frac{8}{3}+\sin^4\left(fx+e\right)+\frac{4\left(\sin^2\left(fx+e\right)\right)}{3}\right)\cos\left(fx+e\right)}{5}+3a^2b\left(-\frac{\left(\sin^3\left(fx\right.\right.}{}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(b^3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5
/16*f*x+5/16*e)-3/5*a*b^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+3*
a^2*b*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/3*a^3
*(2+sin(f*x+e)^2)*cos(f*x+e))
```

maxima [A] time = 0.78, size = 145, normalized size = 0.85

$$\frac{320\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)a^3+90\left(12\,fx+12\,e+\sin\left(4\,fx+4\,e\right)-8\,\sin\left(2\,fx+2\,e\right)\right)a^2b-192\left(3\,\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/960*(320*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3 + 90*(12*f*x + 12*e + sin(
4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^2*b - 192*(3*cos(f*x + e)^5 - 10*cos(f
```

```
*x + e)^3 + 15*cos(f*x + e))*a*b^2 + 5*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*
e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*b^3)/f
```

mupad [B] time = 8.40, size = 417, normalized size = 2.44

$$\frac{b\,\mathrm{atan}\left(\frac{b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(18\,a^2+5\,b^2\right)}{8\left(\frac{9\,a^2\,b}{4}+\frac{5\,b^3}{8}\right)}\right)\left(18\,a^2+5\,b^2\right)}{8\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(\frac{9\,a^2\,b}{4}+\frac{5\,b^3}{8}\right)+4\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+\frac{16\,a\,b^2}{5}+\tan\left(\frac{e}{2}\right.\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b*sin(e + f*x))^3,x)
```

```
[Out] (b*atan((b*tan(e/2 + (f*x)/2)*(18*a^2 + 5*b^2))/(8*((9*a^2*b)/4 + (5*b^3)/8
)))*(18*a^2 + 5*b^2))/(8*f) - (tan(e/2 + (f*x)/2)*((9*a^2*b)/4 + (5*b^3)/8)
 + 4*a^3*tan(e/2 + (f*x)/2)^8 + (16*a*b^2)/5 + tan(e/2 + (f*x)/2)^4*(48*a*b
^2 + 16*a^3) + tan(e/2 + (f*x)/2)^6*(32*a*b^2 + (40*a^3)/3) + tan(e/2 + (f*
x)/2)^2*((96*a*b^2)/5 + 8*a^3) - tan(e/2 + (f*x)/2)^11*((9*a^2*b)/4 + (5*b^
3)/8) + tan(e/2 + (f*x)/2)^5*((21*a^2*b)/2 + (33*b^3)/4) - tan(e/2 + (f*x)/
2)^7*((21*a^2*b)/2 + (33*b^3)/4) + tan(e/2 + (f*x)/2)^3*((51*a^2*b)/4 + (85
*b^3)/24) - tan(e/2 + (f*x)/2)^9*((51*a^2*b)/4 + (85*b^3)/24) + (4*a^3)/3)/
(f*(6*tan(e/2 + (f*x)/2)^2 + 15*tan(e/2 + (f*x)/2)^4 + 20*tan(e/2 + (f*x)/2
)^6 + 15*tan(e/2 + (f*x)/2)^8 + 6*tan(e/2 + (f*x)/2)^10 + tan(e/2 + (f*x)/2
)^12 + 1)) - (b*(18*a^2 + 5*b^2)*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2))/(8*f
)
```

sympy [A] time = 5.55, size = 393, normalized size = 2.30

$$\begin{cases} -\frac{a^3\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{2a^3\cos^3\left(e+fx\right)}{3f}+\frac{9a^2bx\sin^4\left(e+fx\right)}{8}+\frac{9a^2bx\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4}+\frac{9a^2bx\cos^4\left(e+fx\right)}{8}-\frac{15a^2b\sin^3\left(e+\cdots\right.}{8\cdots} \\ x\left(a+b\sin\left(e\right)\right)^3\sin^3\left(e\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-a**3*sin(e + f*x)**2*cos(e + f*x)/f - 2*a**3*cos(e + f*x)**3/(3
*f) + 9*a**2*b*x*sin(e + f*x)**4/8 + 9*a**2*b*x*sin(e + f*x)**2*cos(e + f*x
)**2/4 + 9*a**2*b*x*cos(e + f*x)**4/8 - 15*a**2*b*sin(e + f*x)**3*cos(e + f
*x)/(8*f) - 9*a**2*b*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 3*a*b**2*sin(e +
f*x)**4*cos(e + f*x)/f - 4*a*b**2*sin(e + f*x)**2*cos(e + f*x)**3/f - 8*a*b
**2*cos(e + f*x)**5/(5*f) + 5*b**3*x*sin(e + f*x)**6/16 + 15*b**3*x*sin(e +
```

```
f*x)**4*cos(e + f*x)**2/16 + 15*b**3*x*sin(e + f*x)**2*cos(e + f*x)**4/16
+ 5*b**3*x*cos(e + f*x)**6/16 - 11*b**3*sin(e + f*x)**5*cos(e + f*x)/(16*f)
 - 5*b**3*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 5*b**3*sin(e + f*x)*cos(e
 + f*x)**5/(16*f), Ne(f, 0)), (x*(a + b*sin(e))**3*sin(e)**3, True))
```

3.167 $\int \sin^2(e+fx)(a+b\sin(e+fx))^3\,dx$

Optimal. Leaf size=160

$$\frac{b\left(15a^2+4b^2\right)\cos^3(e+fx)}{15f}-\frac{b\left(15a^2+4b^2\right)\cos(e+fx)}{5f}-\frac{a\left(4a^2+9b^2\right)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}ax\left(4a^2+9b^2\right)$$

```
[Out] 1/8*a*(4*a^2+9*b^2)*x-1/5*b*(15*a^2+4*b^2)*cos(f*x+e)/f+1/15*b*(15*a^2+4*b^
2)*cos(f*x+e)^3/f-1/8*a*(4*a^2+9*b^2)*cos(f*x+e)*sin(f*x+e)/f-11/20*a*b^2*c
os(f*x+e)*sin(f*x+e)^3/f-1/5*b^2*cos(f*x+e)*sin(f*x+e)^3*(a+b*sin(f*x+e))/f
```

Rubi [A] time = 0.22, antiderivative size = 180, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2791, 2753, 2734}

$$\frac{\left(-52a^2b^2+3a^4-16b^4\right)\cos(e+fx)}{30bf}+\frac{\left(3a^2-16b^2\right)\cos(e+fx)(a+b\sin(e+fx))^2}{60bf}+\frac{a\left(6a^2-71b^2\right)\sin(e+fx)\cos(e+fx)}{120f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (a*(4*a^2 + 9*b^2)*x)/8 + ((3*a^4 - 52*a^2*b^2 - 16*b^4)*Cos[e + f*x])/(30*
b*f) + (a*(6*a^2 - 71*b^2)*Cos[e + f*x]*Sin[e + f*x])/(120*f) + ((3*a^2 - 1
6*b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^2)/(60*b*f) + (a*Cos[e + f*x]*(a +
 b*Sin[e + f*x])^3)/(20*b*f) - (Cos[e + f*x]*(a + b*Sin[e + f*x])^4)/(5*b*f
)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] && !LtQ[m, -1]
```

Rubi steps

$$\int \sin^2(e+fx)(a+b\sin(e+fx))^3\,dx = -\frac{\cos(e+fx)(a+b\sin(e+fx))^4}{5bf} + \frac{\int(4b-a\sin(e+fx))(a+b\sin(e+\ldots}{5b}$$

$$= \frac{a\cos(e+fx)(a+b\sin(e+fx))^3}{20bf} - \frac{\cos(e+fx)(a+b\sin(e+fx))^4}{5bf} + \ldots$$

$$= \frac{\left(3a^2-16b^2\right)\cos(e+fx)(a+b\sin(e+fx))^2}{60bf} + \frac{a\cos(e+fx)(a+b\sin(e\ldots}{20bf}$$

$$= \frac{1}{8}a\left(4a^2+9b^2\right)x + \frac{\left(3a^4-52a^2b^2-16b^4\right)\cos(e+fx)}{30bf} + \frac{a\left(6a^2-71b^2\right)\ldots}{\ldots}$$

Mathematica [A] time = 0.65, size = 117, normalized size = 0.73

$$\frac{10\left(12a^2b+5b^3\right)\cos(3(e+fx))+15a\left(4\left(4a^2+9b^2\right)(e+fx)-8\left(a^2+3b^2\right)\sin(2(e+fx))+3b^2\sin(4(e+fx))\right)\ldots}{480f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (-60*b*(18*a^2 + 5*b^2)*Cos[e + f*x] + 10*(12*a^2*b + 5*b^3)*Cos[3*(e + f*x
)] - 6*b^3*Cos[5*(e + f*x)] + 15*a*(4*(4*a^2 + 9*b^2)*(e + f*x) - 8*(a^2 +
3*b^2)*Sin[2*(e + f*x)] + 3*b^2*Sin[4*(e + f*x)]))/(480*f)
```

fricas [A] time = 0.49, size = 118, normalized size = 0.74

$$-\frac{24\,b^3\cos\left(fx+e\right)^5-40\left(3\,a^2b+2\,b^3\right)\cos\left(fx+e\right)^3-15\left(4\,a^3+9\,ab^2\right)fx+120\left(3\,a^2b+b^3\right)\cos\left(fx+e\right)-15\left(\ldots\right.}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/120*(24*b^3*cos(f*x + e)^5 - 40*(3*a^2*b + 2*b^3)*cos(f*x + e)^3 - 15*(4
*a^3 + 9*a*b^2)*f*x + 120*(3*a^2*b + b^3)*cos(f*x + e) - 15*(6*a*b^2*cos(f*
x + e)^3 - (4*a^3 + 15*a*b^2)*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.18, size = 129, normalized size = 0.81

$$-\frac{b^3\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{3\,ab^2\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{1}{8}\left(4\,a^3+9\,ab^2\right)x+\frac{\left(12\,a^2b+5\,b^3\right)\cos\left(3\,fx+3\,e\right)}{48\,f}-\frac{\left(18\,a^2b+5\,b^3\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/80*b^3*cos(5*f*x + 5*e)/f + 3/32*a*b^2*sin(4*f*x + 4*e)/f + 1/8*(4*a^3 +
 9*a*b^2)*x + 1/48*(12*a^2*b + 5*b^3)*cos(3*f*x + 3*e)/f - 1/8*(18*a^2*b +
5*b^3)*cos(f*x + e)/f - 1/4*(a^3 + 3*a*b^2)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.27, size = 124, normalized size = 0.78

$$\frac{-\frac{b^3\left(\frac{8}{3}+\sin^4\left(fx+e\right)+\frac{4\left(\sin^2\left(fx+e\right)\right)}{3}\right)\cos\left(fx+e\right)}{5}+3a\,b^2\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-a^2b\left(2+\sin^2\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(-1/5*b^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+3*a*b^2*(-1/4*
(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-a^2*b*(2+sin(f*x+e)
^2)*cos(f*x+e)+a^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e))
```

maxima [A] time = 1.62, size = 121, normalized size = 0.76

$$\frac{120\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)a^3+480\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)a^2b+45\left(12\,fx+12\,e+\sin\left(4\,fx+\right.\right.}{480\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/480*(120*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^3 + 480*(cos(f*x + e)^3 - 3*c
os(f*x + e))*a^2*b + 45*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2
*e))*a*b^2 - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*b^
3)/f
```

mupad [B] time = 8.17, size = 328, normalized size = 2.05

$$\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(4\,a^2+9\,b^2\right)}{4\left(a^3+\frac{9\,a\,b^2}{4}\right)}\right)\left(4\,a^2+9\,b^2\right)}{4\,f}-\frac{4\,a^2\,b-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9\left(a^3+\frac{9\,a\,b^2}{4}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\left(2\,a^3+\frac{21\,a\,b^2}{2}\right)-}{f\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b*sin(e + f*x))^3,x)
```

```
[Out] (a*atan((a*tan(e/2 + (f*x)/2)*(4*a^2 + 9*b^2))/(4*((9*a*b^2)/4 + a^3)))*(4*
a^2 + 9*b^2))/(4*f) - (4*a^2*b - tan(e/2 + (f*x)/2)^9*((9*a*b^2)/4 + a^3) +
 tan(e/2 + (f*x)/2)^3*((21*a*b^2)/2 + 2*a^3) - tan(e/2 + (f*x)/2)^7*((21*a*
b^2)/2 + 2*a^3) + tan(e/2 + (f*x)/2)^2*(20*a^2*b + (16*b^3)/3) + tan(e/2 +
(f*x)/2)^4*(28*a^2*b + (32*b^3)/3) + (16*b^3)/15 + tan(e/2 + (f*x)/2)*((9*a
*b^2)/4 + a^3) + 12*a^2*b*tan(e/2 + (f*x)/2)^6)/(f*(5*tan(e/2 + (f*x)/2)^2
+ 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 + 5*tan(e/2 + (f*x)/2)^
8 + tan(e/2 + (f*x)/2)^10 + 1)) - (a*(4*a^2 + 9*b^2)*(atan(tan(e/2 + (f*x)/
2)) - (f*x)/2))/(4*f)
```

sympy [A] time = 2.85, size = 284, normalized size = 1.78

$$\begin{cases}\frac{a^3x\sin^2\left(e+fx\right)}{2}+\frac{a^3x\cos^2\left(e+fx\right)}{2}-\frac{a^3\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{3a^2b\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{2a^2b\cos^3\left(e+fx\right)}{f}+\frac{9ab^2x\sin^4\left(e+fx\right)}{8}+\frac{9ab}{} \\ x\left(a+b\sin\left(e\right)\right)^3\sin^2\left(e\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((a**3*x*sin(e + f*x)**2/2 + a**3*x*cos(e + f*x)**2/2 - a**3*sin(e
 + f*x)*cos(e + f*x)/(2*f) - 3*a**2*b*sin(e + f*x)**2*cos(e + f*x)/f - 2*a*
*2*b*cos(e + f*x)**3/f + 9*a*b**2*x*sin(e + f*x)**4/8 + 9*a*b**2*x*sin(e +
f*x)**2*cos(e + f*x)**2/4 + 9*a*b**2*x*cos(e + f*x)**4/8 - 15*a*b**2*sin(e
+ f*x)**3*cos(e + f*x)/(8*f) - 9*a*b**2*sin(e + f*x)*cos(e + f*x)**3/(8*f)
- b**3*sin(e + f*x)**4*cos(e + f*x)/f - 4*b**3*sin(e + f*x)**2*cos(e + f*x)
**3/(3*f) - 8*b**3*cos(e + f*x)**5/(15*f), Ne(f, 0)), (x*(a + b*sin(e))**3*
sin(e)**2, True))
```

3.168 $\int \sin(e+fx)(a+b\sin(e+fx))^3\,dx$

Optimal. Leaf size=121

$$-\frac{a\left(a^2+4b^2\right)\cos(e+fx)}{2f}-\frac{b\left(2a^2+3b^2\right)\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}bx\left(4a^2+b^2\right)-\frac{\cos(e+fx)(a+b\sin(e+fx}{4f}$$

```
[Out] 3/8*b*(4*a^2+b^2)*x-1/2*a*(a^2+4*b^2)*cos(f*x+e)/f-1/8*b*(2*a^2+3*b^2)*cos(
f*x+e)*sin(f*x+e)/f-1/4*a*cos(f*x+e)*(a+b*sin(f*x+e))^2/f-1/4*cos(f*x+e)*(a
+b*sin(f*x+e))^3/f
```

Rubi [A] time = 0.11, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2753, 2734}

$$-\frac{a\left(a^2+4b^2\right)\cos(e+fx)}{2f}-\frac{b\left(2a^2+3b^2\right)\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}bx\left(4a^2+b^2\right)-\frac{\cos(e+fx)(a+b\sin(e+fx}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (3*b*(4*a^2 + b^2)*x)/8 - (a*(a^2 + 4*b^2)*Cos[e + f*x])/(2*f) - (b*(2*a^2
+ 3*b^2)*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (a*Cos[e + f*x]*(a + b*Sin[e +
f*x])^2)/(4*f) - (Cos[e + f*x]*(a + b*Sin[e + f*x])^3)/(4*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int \sin(e+fx)(a+b\sin(e+fx))^3\,dx = -\frac{\cos(e+fx)(a+b\sin(e+fx))^3}{4f} + \frac{1}{4}\int (3b+3a\sin(e+fx))(a+b\sin(e$$

$$= -\frac{a\cos(e+fx)(a+b\sin(e+fx))^2}{4f} - \frac{\cos(e+fx)(a+b\sin(e+fx))^3}{4f} + \frac{1}{1}$$

$$= \frac{3}{8}b\left(4a^2+b^2\right)x - \frac{a\left(a^2+4b^2\right)\cos(e+fx)}{2f} - \frac{b\left(2a^2+3b^2\right)\cos(e+fx)\sin}{8f}$$

Mathematica [A] time = 0.35, size = 100, normalized size = 0.83

$$\frac{b\left(-8\left(3a^2+b^2\right)\sin(2(e+fx))+48a^2e+48a^2fx+8ab\cos(3(e+fx))+b^2\sin(4(e+fx))+12b^2e+12b^2fx\right)-8}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (-8*a*(4*a^2 + 9*b^2)*Cos[e + f*x] + b*(48*a^2*e + 12*b^2*e + 48*a^2*f*x +
12*b^2*f*x + 8*a*b*Cos[3*(e + f*x)] - 8*(3*a^2 + b^2)*Sin[2*(e + f*x)] + b^
2*Sin[4*(e + f*x)]))/(32*f)
```

fricas [A] time = 0.52, size = 93, normalized size = 0.77

$$\frac{8\,ab^2\cos\left(fx+e\right)^3+3\left(4\,a^2b+b^3\right)fx-8\left(a^3+3\,ab^2\right)\cos\left(fx+e\right)+\left(2\,b^3\cos\left(fx+e\right)^3-\left(12\,a^2b+5\,b^3\right)\cos\left(f\right.\right.}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(8*a*b^2*cos(f*x + e)^3 + 3*(4*a^2*b + b^3)*f*x - 8*(a^3 + 3*a*b^2)*cos
(f*x + e) + (2*b^3*cos(f*x + e)^3 - (12*a^2*b + 5*b^3)*cos(f*x + e))*sin(f*
x + e))/f
```

giac [A] time = 0.39, size = 116, normalized size = 0.96

$$\frac{ab^2\cos\left(3\,fx+3\,e\right)}{4\,f}-\frac{3\,ab^2\cos\left(fx+e\right)}{4\,f}+\frac{b^3\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{3}{8}\left(4\,a^2b+b^3\right)x-\frac{\left(2\,a^3+3\,ab^2\right)\cos\left(fx+e\right)}{2\,f}-\frac{\left(3\,a^2b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/4*a*b^2*cos(3*f*x + 3*e)/f - 3/4*a*b^2*cos(f*x + e)/f + 1/32*b^3*sin(4*f*
x + 4*e)/f + 3/8*(4*a^2*b + b^3)*x - 1/2*(2*a^3 + 3*a*b^2)*cos(f*x + e)/f -
 1/4*(3*a^2*b + b^3)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.23, size = 104, normalized size = 0.86

$$\frac{b^{3}\left(-\frac{\left(\sin^{3}\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-a\,b^{2}\left(2+\sin^{2}\left(fx+e\right)\right)\cos\left(fx+e\right)+3a^{2}b\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(b^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-a*b^
2*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a^2*b*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1
/2*e)-a^3*cos(f*x+e))
```

maxima [A] time = 0.51, size = 97, normalized size = 0.80

$$\frac{24\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)a^{2}b+32\left(\cos\left(fx+e\right)^{3}-3\,\cos\left(fx+e\right)\right)ab^{2}+\left(12\,fx+12\,e+\sin\left(4\,fx+4\,e\right)\right.}{32\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/32*(24*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*b + 32*(cos(f*x + e)^3 - 3*co
s(f*x + e))*a*b^2 + (12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))
*b^3 - 32*a^3*cos(f*x + e))/f
```

mupad [B] time = 8.08, size = 313, normalized size = 2.59

$$\frac{3\,b\,\mathrm{atan}\left(\frac{3\,b\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(4\,a^{2}+b^{2}\right)}{4\left(3\,a^{2}\,b+\frac{3\,b^{3}}{4}\right)}\right)\left(4\,a^{2}+b^{2}\right)}{4\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(3\,a^{2}\,b+\frac{3\,b^{3}}{4}\right)+2\,a^{3}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}+4\,a\,b^{2}+\tan\left(\frac{e}{2}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b*sin(e + f*x))^3,x)
```

```
[Out] (3*b*atan((3*b*tan(e/2 + (f*x)/2)*(4*a^2 + b^2))/(4*(3*a^2*b + (3*b^3)/4)))
*(4*a^2 + b^2))/(4*f) - (tan(e/2 + (f*x)/2)*(3*a^2*b + (3*b^3)/4) + 2*a^3*t
```

```
an(e/2 + (f*x)/2)^6 + 4*a*b^2 + tan(e/2 + (f*x)/2)^4*(12*a*b^2 + 6*a^3) + t
an(e/2 + (f*x)/2)^2*(16*a*b^2 + 6*a^3) - tan(e/2 + (f*x)/2)^7*(3*a^2*b + (3
*b^3)/4) + tan(e/2 + (f*x)/2)^3*(3*a^2*b + (11*b^3)/4) - tan(e/2 + (f*x)/2)
^5*(3*a^2*b + (11*b^3)/4) + 2*a^3)/(f*(4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 +
 (f*x)/2)^4 + 4*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1)) - (3*b*(4
*a^2 + b^2)*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2))/(4*f)
```

sympy [A] time = 1.66, size = 233, normalized size = 1.93

$$\begin{cases} -\frac{a^3\cos(e+fx)}{f} + \frac{3a^2bx\sin^2(e+fx)}{2} + \frac{3a^2bx\cos^2(e+fx)}{2} - \frac{3a^2b\sin(e+fx)\cos(e+fx)}{2f} - \frac{3ab^2\sin^2(e+fx)\cos(e+fx)}{f} - \frac{2ab^2\cos^3(e+fx)}{f} + \\ x(a+b\sin(e))^3\sin(e) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-a**3*cos(e + f*x)/f + 3*a**2*b*x*sin(e + f*x)**2/2 + 3*a**2*b*x
*cos(e + f*x)**2/2 - 3*a**2*b*sin(e + f*x)*cos(e + f*x)/(2*f) - 3*a*b**2*si
n(e + f*x)**2*cos(e + f*x)/f - 2*a*b**2*cos(e + f*x)**3/f + 3*b**3*x*sin(e
+ f*x)**4/8 + 3*b**3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*b**3*x*cos(e +
 f*x)**4/8 - 5*b**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*b**3*sin(e + f*x
)*cos(e + f*x)**3/(8*f), Ne(f, 0)), (x*(a + b*sin(e))**3*sin(e), True))
```

3.169 $\int (a+b\sin(e+fx))^3\,dx$

Optimal. Leaf size=90

$$-\frac{2b\left(4a^2+b^2\right)\cos(e+fx)}{3f}+\frac{1}{2}ax\left(2a^2+3b^2\right)-\frac{5ab^2\sin(e+fx)\cos(e+fx)}{6f}-\frac{b\cos(e+fx)(a+b\sin(e+fx))^2}{3f}$$

```
[Out] 1/2*a*(2*a^2+3*b^2)*x-2/3*b*(4*a^2+b^2)*cos(f*x+e)/f-5/6*a*b^2*cos(f*x+e)*s
in(f*x+e)/f-1/3*b*cos(f*x+e)*(a+b*sin(f*x+e))^2/f
```

Rubi [A] time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2656, 2734}

$$-\frac{2b\left(4a^2+b^2\right)\cos(e+fx)}{3f}+\frac{1}{2}ax\left(2a^2+3b^2\right)-\frac{5ab^2\sin(e+fx)\cos(e+fx)}{6f}-\frac{b\cos(e+fx)(a+b\sin(e+fx))^2}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*x)/2 - (2*b*(4*a^2 + b^2)*Cos[e + f*x])/(3*f) - (5*a*b^2
*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (b*Cos[e + f*x]*(a + b*Sin[e + f*x])^2)
/(3*f)
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))^3\,dx = -\frac{b\cos(e+fx)(a+b\sin(e+fx))^2}{3f}+\frac{1}{3}\int (a+b\sin(e+fx))\left(3a^2+2b^2+5ab\sin(e\right.$$
$$=\frac{1}{2}a\left(2a^2+3b^2\right)x-\frac{2b\left(4a^2+b^2\right)\cos(e+fx)}{3f}-\frac{5ab^2\cos(e+fx)\sin(e+fx)}{6f}-\frac{b\cos(}{}$$

Mathematica [A] time = 0.17, size = 71, normalized size = 0.79

$$\frac{6a\left(2a^2+3b^2\right)(e+fx)-9b\left(4a^2+b^2\right)\cos(e+fx)-9ab^2\sin(2(e+fx))+b^3\cos(3(e+fx))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (6*a*(2*a^2 + 3*b^2)*(e + f*x) - 9*b*(4*a^2 + b^2)*Cos[e + f*x] + b^3*Cos[3
*(e + f*x)] - 9*a*b^2*Sin[2*(e + f*x)])/(12*f)
```

fricas [A] time = 0.47, size = 71, normalized size = 0.79

$$\frac{2\,b^3\cos\left(fx+e\right)^3-9\,ab^2\cos\left(fx+e\right)\sin\left(fx+e\right)+3\left(2\,a^3+3\,ab^2\right)fx-6\left(3\,a^2b+b^3\right)\cos\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*cos(f*x + e)^3 - 9*a*b^2*cos(f*x + e)*sin(f*x + e) + 3*(2*a^3 +
3*a*b^2)*f*x - 6*(3*a^2*b + b^3)*cos(f*x + e))/f
```

giac [A] time = 0.17, size = 75, normalized size = 0.83

$$\frac{b^3\cos\left(3\,fx+3\,e\right)}{12\,f}-\frac{3\,ab^2\sin\left(2\,fx+2\,e\right)}{4\,f}+\frac{1}{2}\left(2\,a^3+3\,ab^2\right)x-\frac{3\left(4\,a^2b+b^3\right)\cos\left(fx+e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/12*b^3*cos(3*f*x + 3*e)/f - 3/4*a*b^2*sin(2*f*x + 2*e)/f + 1/2*(2*a^3 + 3
*a*b^2)*x - 3/4*(4*a^2*b + b^3)*cos(f*x + e)/f
```

maple [A] time = 0.16, size = 76, normalized size = 0.84

$$\frac{-\frac{b^3\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}+3a\,b^2\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-3a^2b\cos\left(fx+e\right)+\left(fx+e\right)a^3}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(-1/3*b^3*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a*b^2*(-1/2*sin(f*x+e)*cos(f*x+
e)+1/2*f*x+1/2*e)-3*a^2*b*cos(f*x+e)+(f*x+e)*a^3)
```

maxima [A] time = 0.65, size = 74, normalized size = 0.82

$$a^3x+\frac{3\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)ab^2}{4\,f}+\frac{\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)b^3}{3\,f}-\frac{3\,a^2b\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 3/4*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*b^2/f + 1/3*(cos(f*x + e)^3
- 3*cos(f*x + e))*b^3/f - 3*a^2*b*cos(f*x + e)/f
```

mupad [B] time = 6.74, size = 127, normalized size = 1.41

$$a^3\,x-\frac{4\,b^3\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{f}+\frac{8\,b^3\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6}{3\,f}+\frac{3\,a\,b^2\,x}{2}-\frac{6\,a^2\,b\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{f}-\frac{6\,a\,b^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3,x)
```

```
[Out] a^3*x - (4*b^3*cos(e/2 + (f*x)/2)^4)/f + (8*b^3*cos(e/2 + (f*x)/2)^6)/(3*f)
 + (3*a*b^2*x)/2 - (6*a^2*b*cos(e/2 + (f*x)/2)^2)/f - (6*a*b^2*cos(e/2 + (f
*x)/2)^3*sin(e/2 + (f*x)/2))/f + (3*a*b^2*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x
)/2))/f
```

sympy [A] time = 0.74, size = 128, normalized size = 1.42

$$\begin{cases} a^3x-\frac{3a^2b\cos\left(e+fx\right)}{f}+\frac{3ab^2x\sin^2\left(e+fx\right)}{2}+\frac{3ab^2x\cos^2\left(e+fx\right)}{2}-\frac{3ab^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{b^3\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{2b^3\cos^3\left(e+\right.}{3f} \\ x\left(a+b\sin\left(e\right)\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((a**3*x - 3*a**2*b*cos(e + f*x)/f + 3*a*b**2*x*sin(e + f*x)**2/2
+ 3*a*b**2*x*cos(e + f*x)**2/2 - 3*a*b**2*sin(e + f*x)*cos(e + f*x)/(2*f) -
 b**3*sin(e + f*x)**2*cos(e + f*x)/f - 2*b**3*cos(e + f*x)**3/(3*f), Ne(f,
0)), (x*(a + b*sin(e))**3, True))
```

3.170 $\int \csc(e+fx)(a+b\sin(e+fx))^3\,dx$

Optimal. Leaf size=74

$$-\frac{a^3\tanh^{-1}(\cos(e+fx))}{f}+\frac{1}{2}bx\left(6a^2+b^2\right)-\frac{5ab^2\cos(e+fx)}{2f}-\frac{b^2\cos(e+fx)(a+b\sin(e+fx))}{2f}$$

```
[Out] 1/2*b*(6*a^2+b^2)*x-a^3*arctanh(cos(f*x+e))/f-5/2*a*b^2*cos(f*x+e)/f-1/2*b^
2*cos(f*x+e)*(a+b*sin(f*x+e))/f
```

Rubi [A] time = 0.11, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2793, 3023, 2735, 3770}

$$\frac{1}{2}bx\left(6a^2+b^2\right)-\frac{a^3\tanh^{-1}(\cos(e+fx))}{f}-\frac{5ab^2\cos(e+fx)}{2f}-\frac{b^2\cos(e+fx)(a+b\sin(e+fx))}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (b*(6*a^2 + b^2)*x)/2 - (a^3*ArcTanh[Cos[e + f*x]])/f - (5*a*b^2*Cos[e + f*
x])/(2*f) - (b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]))/(2*f)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc(e+fx)(a+b\sin(e+fx))^3\,dx = -\frac{b^2\cos(e+fx)(a+b\sin(e+fx))}{2f}+\frac{1}{2}\int \csc(e+fx)\left(2a^3+b\left(6a^2+b^2\right)\right.$$

$$= -\frac{5ab^2\cos(e+fx)}{2f}-\frac{b^2\cos(e+fx)(a+b\sin(e+fx))}{2f}+\frac{1}{2}\int \csc(e+fx$$

$$= \frac{1}{2}b\left(6a^2+b^2\right)x-\frac{5ab^2\cos(e+fx)}{2f}-\frac{b^2\cos(e+fx)(a+b\sin(e+fx))}{2f}+$$

$$= \frac{1}{2}b\left(6a^2+b^2\right)x-\frac{a^3\tanh^{-1}(\cos(e+fx))}{f}-\frac{5ab^2\cos(e+fx)}{2f}-\frac{b^2\cos(e$$

Mathematica [A] time = 0.17, size = 81, normalized size = 1.09

$$-\frac{-4a^3\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)+4a^3\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)-2b\left(6a^2+b^2\right)(e+fx)+12ab^2\cos(e+fx)+b^3\sin(2(}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] -1/4*(-2*b*(6*a^2 + b^2)*(e + f*x) + 12*a*b^2*Cos[e + f*x] + 4*a^3*Log[Cos[
(e + f*x)/2]] - 4*a^3*Log[Sin[(e + f*x)/2]] + b^3*Sin[2*(e + f*x)])/f
```

fricas [A] time = 0.49, size = 79, normalized size = 1.07

$$-\frac{b^3\cos\left(fx+e\right)\sin\left(fx+e\right)+6\,ab^2\cos\left(fx+e\right)+a^3\log\left(\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)-a^3\log\left(-\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)-}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b^3*cos(f*x + e)*sin(f*x + e) + 6*a*b^2*cos(f*x + e) + a^3*log(1/2*co
s(f*x + e) + 1/2) - a^3*log(-1/2*cos(f*x + e) + 1/2) - (6*a^2*b + b^3)*f*x)
/f
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(a^3/2*ln(abs(tan((f*x+exp(1))/2
)))+(6*a^2*b+b^3)/2*(f*x+exp(1))/2+(tan((f*x+exp(1))/2)^3*b^3-6*tan((f*x+ex
p(1))/2)^2*a*b^2-tan((f*x+exp(1))/2)*b^3-6*a*b^2)*1/2/(tan((f*x+exp(1))/2)^
2+1)^2)
```

maple [A] time = 0.21, size = 92, normalized size = 1.24

$$\frac{a^{3} \ln \left(\csc \left(f x+e\right)-\cot \left(f x+e\right)\right)}{f}+3 a^{2} b x+\frac{3 a^{2} b e}{f}-\frac{3 a \,b^{2} \cos \left(f x+e\right)}{f}-\frac{b^{3} \sin \left(f x+e\right) \cos \left(f x+e\right)}{2 f}+\frac{b^{3} x}{2}+\frac{b^{3} e}{2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sin(f*x+e))^3,x)
```

```
[Out] 1/f*a^3*ln(csc(f*x+e)-cot(f*x+e))+3*a^2*b*x+3/f*a^2*b*e-3*a*b^2*cos(f*x+e)/
f-1/2/f*b^3*sin(f*x+e)*cos(f*x+e)+1/2*b^3*x+1/2/f*b^3*e
```

maxima [A] time = 0.67, size = 71, normalized size = 0.96

$$\frac{12 \left(f x+e\right) a^{2} b+\left(2 \, f x+2 \, e-\sin \left(2 \, f x+2 \, e\right)\right) b^{3}-12 \, a b^{2} \cos \left(f x+e\right)-4 \, a^{3} \log \left(\cot \left(f x+e\right)+\csc \left(f x+e\right)\right)}{4 \, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(12*(f*x + e)*a^2*b + (2*f*x + 2*e - sin(2*f*x + 2*e))*b^3 - 12*a*b^2*c
os(f*x + e) - 4*a^3*log(cot(f*x + e) + csc(f*x + e)))/f
```

mupad [B] time = 6.79, size = 259, normalized size = 3.50

$$\frac{a^3\ \ln\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f}-\frac{b^3\,\mathrm{atan}\left(\frac{2\ \sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)a^3+6\ \cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)a^2\,b+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)b^3}{-2\ \cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)a^3+6\ \sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)a^2\,b+\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)b^3}\right)}{f}-\frac{b^3\ \sin\left(2\,e+2\,f\,x\right)}{4\,f}-\frac{6\,a^2\,b\,\mathrm{atan}\left(\frac{2\ \sin\left(\frac{e}{2}+\right.}{-2\ \cos\left(\frac{e}{2}+\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3/sin(e + f*x),x)
```

```
[Out] (a^3*log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/f - (b^3*atan((b^3*cos(e/2
 + (f*x)/2) + 2*a^3*sin(e/2 + (f*x)/2) + 6*a^2*b*cos(e/2 + (f*x)/2))/(b^3*s
in(e/2 + (f*x)/2) - 2*a^3*cos(e/2 + (f*x)/2) + 6*a^2*b*sin(e/2 + (f*x)/2)))
)/f - (b^3*sin(2*e + 2*f*x))/(4*f) - (6*a^2*b*atan((b^3*cos(e/2 + (f*x)/2)
+ 2*a^3*sin(e/2 + (f*x)/2) + 6*a^2*b*cos(e/2 + (f*x)/2))/(b^3*sin(e/2 + (f*
x)/2) - 2*a^3*cos(e/2 + (f*x)/2) + 6*a^2*b*sin(e/2 + (f*x)/2))))/f - (3*a*b
^2*cos(e + f*x))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(e+fx\right)\right)^3\csc\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))**3,x)
```

```
[Out] Integral((a + b*sin(e + f*x))**3*csc(e + f*x), x)
```

3.171 $\int \csc^2(e+fx)(a+b\sin(e+fx))^3\,dx$

Optimal. Leaf size=68

$$\frac{b\left(a^2-b^2\right)\cos(e+fx)}{f}-\frac{3a^2b\tanh^{-1}(\cos(e+fx))}{f}-\frac{a^2\cot(e+fx)(a+b\sin(e+fx))}{f}+3ab^2x$$

```
[Out] 3*a*b^2*x-3*a^2*b*arctanh(cos(f*x+e))/f+b*(a^2-b^2)*cos(f*x+e)/f-a^2*cot(f*
x+e)*(a+b*sin(f*x+e))/f
```

Rubi [A] time = 0.12, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2792, 3023, 2735, 3770}

$$\frac{b\left(a^2-b^2\right)\cos(e+fx)}{f}-\frac{3a^2b\tanh^{-1}(\cos(e+fx))}{f}-\frac{a^2\cot(e+fx)(a+b\sin(e+fx))}{f}+3ab^2x$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] 3*a*b^2*x - (3*a^2*b*ArcTanh[Cos[e + f*x]])/f + (b*(a^2 - b^2)*Cos[e + f*x]
)/f - (a^2*Cot[e + f*x]*(a + b*Sin[e + f*x]))/f
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^2(e+fx)(a+b\sin(e+fx))^3\,dx = -\frac{a^2\cot(e+fx)(a+b\sin(e+fx))}{f} + \int \csc(e+fx)\left(3a^2b+3ab^2\sin(e+\right.$$

$$= \frac{b\left(a^2-b^2\right)\cos(e+fx)}{f} - \frac{a^2\cot(e+fx)(a+b\sin(e+fx))}{f} + \int \csc(e+$$

$$= 3ab^2x + \frac{b\left(a^2-b^2\right)\cos(e+fx)}{f} - \frac{a^2\cot(e+fx)(a+b\sin(e+fx))}{f} + \left(3\right.$$

$$= 3ab^2x - \frac{3a^2b\tanh^{-1}(\cos(e+fx))}{f} + \frac{b\left(a^2-b^2\right)\cos(e+fx)}{f} - \frac{a^2\cot(e}{}$$

Mathematica [A] time = 0.54, size = 87, normalized size = 1.28

$$\frac{a^3\tan\left(\frac{1}{2}(e+fx)\right)+a^3\left(-\cot\left(\frac{1}{2}(e+fx)\right)\right)+6ab\left(a\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)-a\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)+b(e+fx)\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (-2*b^3*Cos[e + f*x] - a^3*Cot[(e + f*x)/2] + 6*a*b*(b*(e + f*x) - a*Log[Co
s[(e + f*x)/2]] + a*Log[Sin[(e + f*x)/2]]) + a^3*Tan[(e + f*x)/2])/(2*f)
```

fricas [A] time = 0.50, size = 99, normalized size = 1.46

$$-\frac{3\,a^2b\log\left(\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)\sin\left(fx+e\right)-3\,a^2b\log\left(-\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)\sin\left(fx+e\right)+2\,a^3\cos\left(fx+e\right)-}{2\,f\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(3*a^2*b*log(1/2*cos(f*x + e) + 1/2)*sin(f*x + e) - 3*a^2*b*log(-1/2*c
os(f*x + e) + 1/2)*sin(f*x + e) + 2*a^3*cos(f*x + e) - 2*(3*a*b^2*f*x - b^3
*cos(f*x + e))*sin(f*x + e))/(f*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(tan((f*x+exp(1))/2)*a^3/4+(-2*t
an((f*x+exp(1))/2)^3*b*a^2-tan((f*x+exp(1))/2)^2*a^3-4*tan((f*x+exp(1))/2)*
b^3-2*tan((f*x+exp(1))/2)*b*a^2-a^3)*1/4/(tan((f*x+exp(1))/2)^3+tan((f*x+ex
p(1))/2))+3*b*a^2/2*ln(abs(tan((f*x+exp(1))/2)))+6*b^2*a/2*(f*x+exp(1))/2)
```

maple [A] time = 0.27, size = 72, normalized size = 1.06

$$3ab^2x-\frac{a^3\cot\left(fx+e\right)}{f}-\frac{b^3\cos\left(fx+e\right)}{f}+\frac{3a^2b\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{f}+\frac{3ab^2e}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sin(f*x+e))^3,x)
```

```
[Out] 3*a*b^2*x-1/f*a^3*cot(f*x+e)-1/f*b^3*cos(f*x+e)+3/f*a^2*b*ln(csc(f*x+e)-cot
(f*x+e))+3/f*a*b^2*e
```

maxima [A] time = 0.31, size = 68, normalized size = 1.00

$$\frac{6\left(fx+e\right)ab^2-3\,a^2b\left(\log\left(\cos\left(fx+e\right)+1\right)-\log\left(\cos\left(fx+e\right)-1\right)\right)-2\,b^3\cos\left(fx+e\right)-\frac{2\,a^3}{\tan\left(fx+e\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(6*(f*x + e)*a*b^2 - 3*a^2*b*(log(cos(f*x + e) + 1) - log(cos(f*x + e)
- 1)) - 2*b^3*cos(f*x + e) - 2*a^3/tan(f*x + e))/f
```

mupad [B] time = 6.72, size = 194, normalized size = 2.85

$$\frac{a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{2f}-\frac{a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+a^3+4b^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\left(2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3+2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}+\frac{6ab^2\,\mathrm{atan}\left(\frac{36a^2b^4}{36a^3b^3-36a^2b^4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}+\frac{36a^3b^3\tan\left(\frac{e}{2}\right.}{36a^3b^3-36a^2b^4\,\mathrm{t}}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3/sin(e + f*x)^2,x)
```

```
[Out] (a^3*tan(e/2 + (f*x)/2))/(2*f) - (a^3*tan(e/2 + (f*x)/2)^2 + a^3 + 4*b^3*ta
n(e/2 + (f*x)/2))/(f*(2*tan(e/2 + (f*x)/2) + 2*tan(e/2 + (f*x)/2)^3)) + (6*
a*b^2*atan((36*a^2*b^4)/(36*a^3*b^3 - 36*a^2*b^4*tan(e/2 + (f*x)/2)) + (36*
a^3*b^3*tan(e/2 + (f*x)/2))/(36*a^3*b^3 - 36*a^2*b^4*tan(e/2 + (f*x)/2))))/
f + (3*a^2*b*log(tan(e/2 + (f*x)/2)))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(e+fx\right)\right)^3\csc^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sin(f*x+e))**3,x)
```

```
[Out] Integral((a + b*sin(e + f*x))**3*csc(e + f*x)**2, x)
```

3.172 $\int \csc^3(e+fx)(a+b\sin(e+fx))^3\,dx$

Optimal. Leaf size=79

$$-\frac{a(a^2+6b^2)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{5a^2b\cot(e+fx)}{2f}-\frac{a^2\cot(e+fx)\csc(e+fx)(a+b\sin(e+fx))}{2f}+b^3x$$

```
[Out] b^3*x-1/2*a*(a^2+6*b^2)*arctanh(cos(f*x+e))/f-5/2*a^2*b*cot(f*x+e)/f-1/2*a^
2*cot(f*x+e)*csc(f*x+e)*(a+b*sin(f*x+e))/f
```

Rubi [A] time = 0.13, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2792, 3021, 2735, 3770}

$$-\frac{a(a^2+6b^2)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{5a^2b\cot(e+fx)}{2f}-\frac{a^2\cot(e+fx)\csc(e+fx)(a+b\sin(e+fx))}{2f}+b^3x$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] b^3*x - (a*(a^2 + 6*b^2)*ArcTanh[Cos[e + f*x]])/(2*f) - (5*a^2*b*Cot[e + f*
x])/(2*f) - (a^2*Cot[e + f*x]*Csc[e + f*x]*(a + b*Sin[e + f*x]))/(2*f)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^3(e+fx)(a+b\sin(e+fx))^3\,dx = -\frac{a^2\cot(e+fx)\csc(e+fx)(a+b\sin(e+fx))}{2f} + \frac{1}{2}\int \csc^2(e+fx)\left(5a^2b\right.$$
$$= -\frac{5a^2b\cot(e+fx)}{2f} - \frac{a^2\cot(e+fx)\csc(e+fx)(a+b\sin(e+fx))}{2f} + \frac{1}{2}\int$$
$$= b^3x - \frac{5a^2b\cot(e+fx)}{2f} - \frac{a^2\cot(e+fx)\csc(e+fx)(a+b\sin(e+fx))}{2f} +$$
$$= b^3x - \frac{a\left(a^2+6b^2\right)\tanh^{-1}(\cos(e+fx))}{2f} - \frac{5a^2b\cot(e+fx)}{2f} - \frac{a^2\cot(e+}{}$$

Mathematica [A] time = 0.68, size = 152, normalized size = 1.92

$$\frac{a^3\left(-\csc^2\left(\frac{1}{2}(e+fx)\right)\right) + a^3\sec^2\left(\frac{1}{2}(e+fx)\right) + 4a^3\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right) - 4a^3\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right) + 12a^2b\tan}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (8*b^3*e + 8*b^3*f*x - 12*a^2*b*Cot[(e + f*x)/2] - a^3*Csc[(e + f*x)/2]^2 -
 4*a^3*Log[Cos[(e + f*x)/2]] - 24*a*b^2*Log[Cos[(e + f*x)/2]] + 4*a^3*Log[S
in[(e + f*x)/2]] + 24*a*b^2*Log[Sin[(e + f*x)/2]] + a^3*Sec[(e + f*x)/2]^2
+ 12*a^2*b*Tan[(e + f*x)/2])/(8*f)
```

fricas [B] time = 0.50, size = 155, normalized size = 1.96

$$\frac{4\,b^3 f x \cos\left(f x + e\right)^2 - 4\,b^3 f x + 12\,a^2 b \cos\left(f x + e\right) \sin\left(f x + e\right) + 2\,a^3 \cos\left(f x + e\right) + \left(a^3 + 6\,ab^2 - \left(a^3 + 6\,ab^2\right) \cdots}{4\left(f \cos\left(f x \cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^3*f*x*cos(f*x + e)^2 - 4*b^3*f*x + 12*a^2*b*cos(f*x + e)*sin(f*x +
 e) + 2*a^3*cos(f*x + e) + (a^3 + 6*a*b^2 - (a^3 + 6*a*b^2)*cos(f*x + e)^2)
*log(1/2*cos(f*x + e) + 1/2) - (a^3 + 6*a*b^2 - (a^3 + 6*a*b^2)*cos(f*x + e
)^2)*log(-1/2*cos(f*x + e) + 1/2))/(f*cos(f*x + e)^2 - f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*((4*tan((f*x+exp(1))/2)^2*a^3+48
*tan((f*x+exp(1))/2)*b*a^2)/64+(-36*tan((f*x+exp(1))/2)^2*b^2*a-6*tan((f*x+
exp(1))/2)^2*a^3-12*tan((f*x+exp(1))/2)*b*a^2-a^3)*1/16/tan((f*x+exp(1))/2)
^2+2*b^3/2*(f*x+exp(1))/2+(6*b^2*a+a^3)/4*ln(abs(tan((f*x+exp(1))/2))))
```

maple [A] time = 0.36, size = 99, normalized size = 1.25

$$-\frac{a^3 \csc\left(f x + e\right) \cot\left(f x + e\right)}{2f} + \frac{a^3 \ln\left(\csc\left(f x + e\right) - \cot\left(f x + e\right)\right)}{2f} - \frac{3a^2 b \cot\left(f x + e\right)}{f} + \frac{3a\,b^2 \ln\left(\csc\left(f x + e\right) - \cdots\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sin(f*x+e))^3,x)
```

```
[Out] -1/2/f*a^3*csc(f*x+e)*cot(f*x+e)+1/2/f*a^3*ln(csc(f*x+e)-cot(f*x+e))-3*a^2*
b*cot(f*x+e)/f+3/f*a*b^2*ln(csc(f*x+e)-cot(f*x+e))+b^3*x+1/f*b^3*e
```

maxima [A] time = 1.28, size = 102, normalized size = 1.29

$$\frac{4\left(f x + e\right)b^3 + a^3\left(\frac{2\cos\left(f x + e\right)}{\cos\left(f x + e\right)^2 - 1} - \log\left(\cos\left(f x + e\right) + 1\right) + \log\left(\cos\left(f x + e\right) - 1\right)\right) - 6\,ab^2\left(\log\left(\cos\left(f x + e\right) + 1\right) \cdots\right.}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*(f*x + e)*b^3 + a^3*(2*cos(f*x + e)/(cos(f*x + e)^2 - 1) - log(cos(f
*x + e) + 1) + log(cos(f*x + e) - 1)) - 6*a*b^2*(log(cos(f*x + e) + 1) - lo
g(cos(f*x + e) - 1)) - 12*a^2*b/tan(f*x + e))/f
```

mupad [B] time = 6.96, size = 234, normalized size = 2.96

$$\frac{2\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,a^3+6\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,a\,b^2+2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,b^3}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,a^3+6\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,a\,b^2-2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,b^3}\right)}{f}-\frac{a^3\,\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{8\,f}+\frac{a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{8\,f}+\frac{a^3\,\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{2\,f}-\frac{3\,a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3/sin(e + f*x)^3,x)
```

```
[Out] (2*b^3*atan((2*b^3*cos(e/2 + (f*x)/2) + a^3*sin(e/2 + (f*x)/2) + 6*a*b^2*si
n(e/2 + (f*x)/2))/(a^3*cos(e/2 + (f*x)/2) - 2*b^3*sin(e/2 + (f*x)/2) + 6*a*
b^2*cos(e/2 + (f*x)/2))))/f - (a^3*cot(e/2 + (f*x)/2)^2)/(8*f) + (a^3*tan(e
/2 + (f*x)/2)^2)/(8*f) + (a^3*log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(
2*f) - (3*a^2*b*cot(e/2 + (f*x)/2))/(2*f) + (3*a*b^2*log(sin(e/2 + (f*x)/2)
/cos(e/2 + (f*x)/2)))/f + (3*a^2*b*tan(e/2 + (f*x)/2))/(2*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sin\left(e + fx\right)\right)^3 \csc^3\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sin(f*x+e))**3,x)
```

```
[Out] Integral((a + b*sin(e + f*x))**3*csc(e + f*x)**3, x)
```

3.173 $\int \csc^4(e+fx)(a+b\sin(e+fx))^3\,dx$

Optimal. Leaf size=109

$$-\frac{a\left(2a^2+9b^2\right)\cot(e+fx)}{3f}-\frac{b\left(3a^2+2b^2\right)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{7a^2b\cot(e+fx)\csc(e+fx)}{6f}-\frac{a^2\cot(e+fx)\text{cs}}{}$$

```
[Out] -1/2*b*(3*a^2+2*b^2)*arctanh(cos(f*x+e))/f-1/3*a*(2*a^2+9*b^2)*cot(f*x+e)/f
-7/6*a^2*b*cot(f*x+e)*csc(f*x+e)/f-1/3*a^2*cot(f*x+e)*csc(f*x+e)^2*(a+b*sin
(f*x+e))/f
```

Rubi [A] time = 0.18, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2792, 3021, 2748, 3767, 8, 3770}

$$-\frac{a\left(2a^2+9b^2\right)\cot(e+fx)}{3f}-\frac{b\left(3a^2+2b^2\right)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{7a^2b\cot(e+fx)\csc(e+fx)}{6f}-\frac{a^2\cot(e+fx)\text{cs}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] -(b*(3*a^2 + 2*b^2)*ArcTanh[Cos[e + f*x]])/(2*f) - (a*(2*a^2 + 9*b^2)*Cot[e
 + f*x])/(3*f) - (7*a^2*b*Cot[e + f*x]*Csc[e + f*x])/(6*f) - (a^2*Cot[e + f
*x]*Csc[e + f*x]^2*(a + b*Sin[e + f*x]))/(3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
```

```
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^4(e+fx)(a+b\sin(e+fx))^3\,dx = -\frac{a^2\cot(e+fx)\csc^2(e+fx)(a+b\sin(e+fx))}{3f}+\frac{1}{3}\int \csc^3(e+fx)\left(7a^2\right.$$

$$= -\frac{7a^2b\cot(e+fx)\csc(e+fx)}{6f}-\frac{a^2\cot(e+fx)\csc^2(e+fx)(a+b\sin(e$$

$$= -\frac{7a^2b\cot(e+fx)\csc(e+fx)}{6f}-\frac{a^2\cot(e+fx)\csc^2(e+fx)(a+b\sin(e$$

$$= -\frac{b\left(3a^2+2b^2\right)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{7a^2b\cot(e+fx)\csc(e+fx)}{6f}-\frac{a^2$$

$$= -\frac{b\left(3a^2+2b^2\right)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{a\left(2a^2+9b^2\right)\cot(e+fx)}{3f}-\frac{7a^2b$$

Mathematica [B] time = 6.20, size = 525, normalized size = 4.82

$$\frac{\sin^3(e+fx)\csc\left(\frac{1}{2}(e+fx)\right)\left(-2a^3\cos\left(\frac{1}{2}(e+fx)\right)-9ab^2\cos\left(\frac{1}{2}(e+fx)\right)\right)(a\csc(e+fx)+b)^3}{6f(a+b\sin(e+fx))^3}+\frac{\sin^3(e+fx)\,\mathrm{se}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((-2*a^3*Cos[(e + f*x)/2] - 9*a*b^2*Cos[(e + f*x)/2])*Csc[(e + f*x)/2]*(b +
 a*Csc[e + f*x])^3*Sin[e + f*x]^3)/(6*f*(a + b*Sin[e + f*x])^3) - (3*a^2*b*
Csc[(e + f*x)/2]^2*(b + a*Csc[e + f*x])^3*Sin[e + f*x]^3)/(8*f*(a + b*Sin[e
 + f*x])^3) - (a^3*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^2*(b + a*Csc[e + f*x])
^3*Sin[e + f*x]^3)/(24*f*(a + b*Sin[e + f*x])^3) + ((-3*a^2*b - 2*b^3)*(b +
 a*Csc[e + f*x])^3*Log[Cos[(e + f*x)/2]]*Sin[e + f*x]^3)/(2*f*(a + b*Sin[e
+ f*x])^3) + ((3*a^2*b + 2*b^3)*(b + a*Csc[e + f*x])^3*Log[Sin[(e + f*x)/2]
]*Sin[e + f*x]^3)/(2*f*(a + b*Sin[e + f*x])^3) + (3*a^2*b*(b + a*Csc[e + f*
x])^3*Sec[(e + f*x)/2]^2*Sin[e + f*x]^3)/(8*f*(a + b*Sin[e + f*x])^3) + ((b
 + a*Csc[e + f*x])^3*Sec[(e + f*x)/2]*(2*a^3*Sin[(e + f*x)/2] + 9*a*b^2*Sin
[(e + f*x)/2])*Sin[e + f*x]^3)/(6*f*(a + b*Sin[e + f*x])^3) + (a^3*(b + a*C
sc[e + f*x])^3*Sec[(e + f*x)/2]^2*Sin[e + f*x]^3*Tan[(e + f*x)/2])/(24*f*(a
 + b*Sin[e + f*x])^3)
```

fricas [A] time = 0.49, size = 191, normalized size = 1.75

$$\frac{18\,a^2b\cos\left(fx+e\right)\sin\left(fx+e\right)-4\left(2\,a^3+9\,ab^2\right)\cos\left(fx+e\right)^3+3\left(3\,a^2b+2\,b^3-\left(3\,a^2b+2\,b^3\right)\cos\left(fx+e\right)^2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(18*a^2*b*cos(f*x + e)*sin(f*x + e) - 4*(2*a^3 + 9*a*b^2)*cos(f*x + e)
^3 + 3*(3*a^2*b + 2*b^3 - (3*a^2*b + 2*b^3)*cos(f*x + e)^2)*log(1/2*cos(f*x
 + e) + 1/2)*sin(f*x + e) - 3*(3*a^2*b + 2*b^3 - (3*a^2*b + 2*b^3)*cos(f*x
+ e)^2)*log(-1/2*cos(f*x + e) + 1/2)*sin(f*x + e) + 12*(a^3 + 3*a*b^2)*cos(
f*x + e))/((f*cos(f*x + e)^2 - f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*((256/3*tan((f*x+exp(1))/2)^3*a^
3+768*tan((f*x+exp(1))/2)^2*b*a^2+3072*tan((f*x+exp(1))/2)*b^2*a+768*tan((f
*x+exp(1))/2)*a^3)/4096+(-44*tan((f*x+exp(1))/2)^3*b^3-66*tan((f*x+exp(1))/
2)^3*b*a^2-36*tan((f*x+exp(1))/2)^2*b^2*a-9*tan((f*x+exp(1))/2)^2*a^3-9*tan
((f*x+exp(1))/2)*b*a^2-a^3)*1/48/tan((f*x+exp(1))/2)^3+(2*b^3+3*b*a^2)/4*ln
(abs(tan((f*x+exp(1))/2))))
```

maple [A] time = 0.40, size = 122, normalized size = 1.12

$$-\frac{2a^3\cot\left(fx+e\right)}{3f}-\frac{a^3\cot\left(fx+e\right)\left(\csc^2\left(fx+e\right)\right)}{3f}-\frac{3a^2b\cot\left(fx+e\right)\csc\left(fx+e\right)}{2f}+\frac{3a^2b\ln\left(\csc\left(fx+e\right)-\cot\left(f\right.\right.}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sin(f*x+e))^3,x)
```

```
[Out] -2/3/f*a^3*cot(f*x+e)-1/3/f*a^3*cot(f*x+e)*csc(f*x+e)^2-3/2*a^2*b*cot(f*x+e
)*csc(f*x+e)/f+3/2/f*a^2*b*ln(csc(f*x+e)-cot(f*x+e))-3/f*a*b^2*cot(f*x+e)+1
/f*b^3*ln(csc(f*x+e)-cot(f*x+e))
```

maxima [A] time = 0.68, size = 118, normalized size = 1.08

$$\frac{9\,a^2b\left(\frac{2\,\cos\left(fx+e\right)}{\cos\left(fx+e\right)^2-1}-\log\left(\cos\left(fx+e\right)+1\right)+\log\left(\cos\left(fx+e\right)-1\right)\right)-6\,b^3\left(\log\left(\cos\left(fx+e\right)+1\right)-\log\left(\cos\left(fx\right.\right.\right.}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/12*(9*a^2*b*(2*cos(f*x + e)/(cos(f*x + e)^2 - 1) - log(cos(f*x + e) + 1)
+ log(cos(f*x + e) - 1)) - 6*b^3*(log(cos(f*x + e) + 1) - log(cos(f*x + e)
- 1)) - 36*a*b^2/tan(f*x + e) - 4*(3*tan(f*x + e)^2 + 1)*a^3/tan(f*x + e)^3
)/f
```

mupad [B] time = 6.78, size = 150, normalized size = 1.38

$$\frac{\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(\frac{3\,a^2\,b}{2}+b^3\right)}{f}+\frac{a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{24\,f}-\frac{\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(3\,a^3+12\,a\,b^2\right)+\frac{a^3}{3}+3\,a^2\,b\,\tan\right.}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3/sin(e + f*x)^4,x)
```

```
[Out] (log(tan(e/2 + (f*x)/2))*((3*a^2*b)/2 + b^3))/f + (a^3*tan(e/2 + (f*x)/2)^3
)/(24*f) - (cot(e/2 + (f*x)/2)^3*(tan(e/2 + (f*x)/2)^2*(12*a*b^2 + 3*a^3) +
 a^3/3 + 3*a^2*b*tan(e/2 + (f*x)/2)))/(8*f) + (tan(e/2 + (f*x)/2)*((3*a*b^2
)/2 + (3*a^3)/8))/f + (3*a^2*b*tan(e/2 + (f*x)/2)^2)/(8*f)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.174 $\int \csc^5(e+fx)(a+b\sin(e+fx))^3\,dx$

Optimal. Leaf size=134

$$-\frac{b\left(2a^2+b^2\right)\cot(e+fx)}{f}-\frac{3a\left(a^2+4b^2\right)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{3a\left(a^2+4b^2\right)\cot(e+fx)\csc(e+fx)}{8f}-\frac{3a^2b\cot(e$$

```
[Out] -3/8*a*(a^2+4*b^2)*arctanh(cos(f*x+e))/f-b*(2*a^2+b^2)*cot(f*x+e)/f-3/8*a*(
a^2+4*b^2)*cot(f*x+e)*csc(f*x+e)/f-3/4*a^2*b*cot(f*x+e)*csc(f*x+e)^2/f-1/4*
a^2*cot(f*x+e)*csc(f*x+e)^3*(a+b*sin(f*x+e))/f
```

Rubi [A] time = 0.21, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2792, 3021, 2748, 3768, 3770, 3767, 8}

$$-\frac{b\left(2a^2+b^2\right)\cot(e+fx)}{f}-\frac{3a\left(a^2+4b^2\right)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{3a\left(a^2+4b^2\right)\cot(e+fx)\csc(e+fx)}{8f}-\frac{3a^2b\cot(e$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (-3*a*(a^2 + 4*b^2)*ArcTanh[Cos[e + f*x]])/(8*f) - (b*(2*a^2 + b^2)*Cot[e +
 f*x])/f - (3*a*(a^2 + 4*b^2)*Cot[e + f*x]*Csc[e + f*x])/(8*f) - (3*a^2*b*C
ot[e + f*x]*Csc[e + f*x]^2)/(4*f) - (a^2*Cot[e + f*x]*Csc[e + f*x]^3*(a + b
*Sin[e + f*x]))/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
```

```
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^5(e+fx)(a+b\sin(e+fx))^3\,dx = -\frac{a^2\cot(e+fx)\csc^3(e+fx)(a+b\sin(e+fx))}{4f} + \frac{1}{4}\int \csc^4(e+fx)\left(9a^2\right.$$

$$= -\frac{3a^2b\cot(e+fx)\csc^2(e+fx)}{4f} - \frac{a^2\cot(e+fx)\csc^3(e+fx)(a+b\sin(e$$

$$= -\frac{3a^2b\cot(e+fx)\csc^2(e+fx)}{4f} - \frac{a^2\cot(e+fx)\csc^3(e+fx)(a+b\sin(e$$

$$= -\frac{3a\left(a^2+4b^2\right)\cot(e+fx)\csc(e+fx)}{8f} - \frac{3a^2b\cot(e+fx)\csc^2(e+fx)}{4f}$$

$$= -\frac{3a\left(a^2+4b^2\right)\tanh^{-1}(\cos(e+fx))}{8f} - \frac{b\left(2a^2+b^2\right)\cot(e+fx)}{f} - \frac{3a\left(a^2\right.}{}$$

Mathematica [B] time = 6.18, size = 322, normalized size = 2.40

$$-\frac{3\left(a^3+4ab^2\right)\csc^2\left(\frac{1}{2}(e+fx)\right)}{32f}+\frac{3\left(a^3+4ab^2\right)\sec^2\left(\frac{1}{2}(e+fx)\right)}{32f}+\frac{3\left(a^3+4ab^2\right)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{8f}-\frac{3\left(a^3+4a\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((-2*a^2*b*Cos[(e + f*x)/2] - b^3*Cos[(e + f*x)/2])*Csc[(e + f*x)/2])/(2*f)
 - (3*(a^3 + 4*a*b^2)*Csc[(e + f*x)/2]^2)/(32*f) - (a^2*b*Cot[(e + f*x)/2]*
Csc[(e + f*x)/2]^2)/(8*f) - (a^3*Csc[(e + f*x)/2]^4)/(64*f) - (3*(a^3 + 4*a
*b^2)*Log[Cos[(e + f*x)/2]])/(8*f) + (3*(a^3 + 4*a*b^2)*Log[Sin[(e + f*x)/2
]])/(8*f) + (3*(a^3 + 4*a*b^2)*Sec[(e + f*x)/2]^2)/(32*f) + (a^3*Sec[(e + f
*x)/2]^4)/(64*f) + (Sec[(e + f*x)/2]*(2*a^2*b*Sin[(e + f*x)/2] + b^3*Sin[(e
 + f*x)/2]))/(2*f) + (a^2*b*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(8*f)
```

fricas [A] time = 0.51, size = 238, normalized size = 1.78

$$\frac{6\left(a^3+4\,ab^2\right)\cos\left(fx+e\right)^3-2\left(5\,a^3+12\,ab^2\right)\cos\left(fx+e\right)-3\left(\left(a^3+4\,ab^2\right)\cos\left(fx+e\right)^4+a^3+4\,ab^2-2\left(a^3+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(6*(a^3 + 4*a*b^2)*cos(f*x + e)^3 - 2*(5*a^3 + 12*a*b^2)*cos(f*x + e)
- 3*((a^3 + 4*a*b^2)*cos(f*x + e)^4 + a^3 + 4*a*b^2 - 2*(a^3 + 4*a*b^2)*cos
```

```
(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 3*((a^3 + 4*a*b^2)*cos(f*x + e)^
4 + a^3 + 4*a*b^2 - 2*(a^3 + 4*a*b^2)*cos(f*x + e)^2)*log(-1/2*cos(f*x + e)
 + 1/2) + 16*((2*a^2*b + b^3)*cos(f*x + e)^3 - (3*a^2*b + b^3)*cos(f*x + e)
)*sin(f*x + e))/(f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*((8192*tan((f*x+exp(1))/2)^4*a^3
+65536*tan((f*x+exp(1))/2)^3*b*a^2+196608*tan((f*x+exp(1))/2)^2*b^2*a+65536
*tan((f*x+exp(1))/2)^2*a^3+262144*tan((f*x+exp(1))/2)*b^3+589824*tan((f*x+e
xp(1))/2)*b*a^2)/1048576+(-200*tan((f*x+exp(1))/2)^4*b^2*a-50*tan((f*x+exp(
1))/2)^4*a^3-32*tan((f*x+exp(1))/2)^3*b^3-72*tan((f*x+exp(1))/2)^3*b*a^2-24
*tan((f*x+exp(1))/2)^2*b^2*a-8*tan((f*x+exp(1))/2)^2*a^3-8*tan((f*x+exp(1))
/2)*b*a^2-a^3)*1/128/tan((f*x+exp(1))/2)^4+(12*b^2*a+3*a^3)/16*ln(abs(tan((
f*x+exp(1))/2))))
```

maple [A] time = 0.48, size = 166, normalized size = 1.24

$$-\frac{a^{3}\cot\left(fx+e\right)\left(\csc^{3}\left(fx+e\right)\right)}{4f}-\frac{3a^{3}\csc\left(fx+e\right)\cot\left(fx+e\right)}{8f}+\frac{3a^{3}\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{8f}-\frac{2a^{2}b\cot\left(\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*sin(f*x+e))^3,x)
```

```
[Out] -1/4/f*a^3*cot(f*x+e)*csc(f*x+e)^3-3/8/f*a^3*csc(f*x+e)*cot(f*x+e)+3/8/f*a^
3*ln(csc(f*x+e)-cot(f*x+e))-2*a^2*b*cot(f*x+e)/f-a^2*b*cot(f*x+e)*csc(f*x+e
)^2/f-3/2/f*a*b^2*cot(f*x+e)*csc(f*x+e)+3/2/f*a*b^2*ln(csc(f*x+e)-cot(f*x+e
))-1/f*b^3*cot(f*x+e)
```

maxima [A] time = 1.17, size = 162, normalized size = 1.21

$$\frac{a^{3}\left(\frac{2\left(3\cos\left(fx+e\right)^{3}-5\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)^{4}-2\cos\left(fx+e\right)^{2}+1}-3\log\left(\cos\left(fx+e\right)+1\right)+3\log\left(\cos\left(fx+e\right)-1\right)\right)+12\,ab^{2}\left(\frac{2\cos\left(fx+e\right)}{\cos\left(fx+e\right)^{2}-1}-\log\left(\right.\right.}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/16*(a^3*(2*(3*cos(f*x + e)^3 - 5*cos(f*x + e))/(cos(f*x + e)^4 - 2*cos(f*
x + e)^2 + 1) - 3*log(cos(f*x + e) + 1) + 3*log(cos(f*x + e) - 1)) + 12*a*b
^2*(2*cos(f*x + e)/(cos(f*x + e)^2 - 1) - log(cos(f*x + e) + 1) + log(cos(f
*x + e) - 1)) - 16*b^3/tan(f*x + e) - 16*(3*tan(f*x + e)^2 + 1)*a^2*b/tan(f
*x + e)^3)/f
```

mupad [B] time = 6.86, size = 203, normalized size = 1.51

$$\frac{a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{64\,f}-\frac{\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(2\,a^3+6\,a\,b^2\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(18\,a^2\,b+8\,b^3\right)+\frac{a^3}{4}+2\,a^2\,b\,\mathrm{tar}}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3/sin(e + f*x)^5,x)
```

```
[Out] (a^3*tan(e/2 + (f*x)/2)^4)/(64*f) - (cot(e/2 + (f*x)/2)^4*(tan(e/2 + (f*x)/
2)^2*(6*a*b^2 + 2*a^3) + tan(e/2 + (f*x)/2)^3*(18*a^2*b + 8*b^3) + a^3/4 +
2*a^2*b*tan(e/2 + (f*x)/2)))/(16*f) + (tan(e/2 + (f*x)/2)^2*((3*a*b^2)/8 +
a^3/8))/f + (log(tan(e/2 + (f*x)/2))*((3*a*b^2)/2 + (3*a^3)/8))/f + (tan(e/
2 + (f*x)/2)*((9*a^2*b)/8 + b^3/2))/f + (a^2*b*tan(e/2 + (f*x)/2)^3)/(8*f)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.175 $\int (a + b\sin(e + fx))^4\,dx$

Optimal. Leaf size=137

$$-\frac{ab\left(19a^2+16b^2\right)\cos(e+fx)}{6f}-\frac{b^2\left(26a^2+9b^2\right)\sin(e+fx)\cos(e+fx)}{24f}+\frac{1}{8}x\left(8a^4+24a^2b^2+3b^4\right)-\frac{b\cos(e+f}{}$$

```
[Out] 1/8*(8*a^4+24*a^2*b^2+3*b^4)*x-1/6*a*b*(19*a^2+16*b^2)*cos(f*x+e)/f-1/24*b^
2*(26*a^2+9*b^2)*cos(f*x+e)*sin(f*x+e)/f-7/12*a*b*cos(f*x+e)*(a+b*sin(f*x+e
))^2/f-1/4*b*cos(f*x+e)*(a+b*sin(f*x+e))^3/f
```

Rubi [A] time = 0.15, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2656, 2753, 2734}

$$-\frac{ab\left(19a^2+16b^2\right)\cos(e+fx)}{6f}-\frac{b^2\left(26a^2+9b^2\right)\sin(e+fx)\cos(e+fx)}{24f}+\frac{1}{8}x\left(24a^2b^2+8a^4+3b^4\right)-\frac{b\cos(e+f}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^4,x]
```

```
[Out] ((8*a^4 + 24*a^2*b^2 + 3*b^4)*x)/8 - (a*b*(19*a^2 + 16*b^2)*Cos[e + f*x])/(
6*f) - (b^2*(26*a^2 + 9*b^2)*Cos[e + f*x]*Sin[e + f*x])/(24*f) - (7*a*b*Cos
[e + f*x]*(a + b*Sin[e + f*x])^2)/(12*f) - (b*Cos[e + f*x]*(a + b*Sin[e + f
*x])^3)/(4*f)
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\sin(e+fx))^4\,dx = -\frac{b\cos(e+fx)(a+b\sin(e+fx))^3}{4f}+\frac{1}{4}\int (a+b\sin(e+fx))^2\left(4a^2+3b^2+7ab\sin(e\right.$$

$$= -\frac{7ab\cos(e+fx)(a+b\sin(e+fx))^2}{12f}-\frac{b\cos(e+fx)(a+b\sin(e+fx))^3}{4f}+\frac{1}{12}\int (a-$$

$$= \frac{1}{8}\left(8a^4+24a^2b^2+3b^4\right)x-\frac{ab\left(19a^2+16b^2\right)\cos(e+fx)}{6f}-\frac{b^2\left(26a^2+9b^2\right)\cos(e+f}{24f}$$

Mathematica [A] time = 0.37, size = 106, normalized size = 0.77

$$\frac{-96ab\left(4a^2+3b^2\right)\cos(e+fx)+3\left(-8\left(6a^2b^2+b^4\right)\sin(2(e+fx))+4\left(8a^4+24a^2b^2+3b^4\right)(e+fx)+b^4\sin(4(e\right.}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^4,x]
```

```
[Out] (-96*a*b*(4*a^2 + 3*b^2)*Cos[e + f*x] + 32*a*b^3*Cos[3*(e + f*x)] + 3*(4*(8
*a^4 + 24*a^2*b^2 + 3*b^4)*(e + f*x) - 8*(6*a^2*b^2 + b^4)*Sin[2*(e + f*x)]
 + b^4*Sin[4*(e + f*x)]))/(96*f)
```

fricas [A] time = 0.48, size = 106, normalized size = 0.77

$$\frac{32\,ab^3\cos\left(fx+e\right)^3+3\left(8\,a^4+24\,a^2b^2+3\,b^4\right)fx-96\left(a^3b+ab^3\right)\cos\left(fx+e\right)+3\left(2\,b^4\cos\left(fx+e\right)^3-\left(24\,a^2b\right.\right.}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(32*a*b^3*cos(f*x + e)^3 + 3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*f*x - 96*(a^
3*b + a*b^3)*cos(f*x + e) + 3*(2*b^4*cos(f*x + e)^3 - (24*a^2*b^2 + 5*b^4)*
cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.17, size = 112, normalized size = 0.82

$$\frac{ab^3\cos\left(3\,fx+3\,e\right)}{3\,f}+\frac{b^4\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{1}{8}\left(8\,a^4+24\,a^2b^2+3\,b^4\right)x-\frac{\left(4\,a^3b+3\,ab^3\right)\cos\left(fx+e\right)}{f}-\frac{\left(6\,a^2b^2+b^4\right)\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/3*a*b^3*cos(3*f*x + 3*e)/f + 1/32*b^4*sin(4*f*x + 4*e)/f + 1/8*(8*a^4 + 2
4*a^2*b^2 + 3*b^4)*x - (4*a^3*b + 3*a*b^3)*cos(f*x + e)/f - 1/4*(6*a^2*b^2
+ b^4)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.22, size = 116, normalized size = 0.85

$$\frac{b^4\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{4ab^3\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}+6a^2b^2\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)\ldots}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^4,x)
```

```
[Out] 1/f*(b^4*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-4/3*
a*b^3*(2+sin(f*x+e)^2)*cos(f*x+e)+6*a^2*b^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2
*f*x+1/2*e)-4*a^3*b*cos(f*x+e)+a^4*(f*x+e))
```

maxima [A] time = 1.07, size = 113, normalized size = 0.82

$$a^4x+\frac{3\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)a^2b^2}{2\,f}+\frac{4\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)ab^3}{3\,f}+\frac{\left(12\,fx+12\,e+\sin\left(4\,fx+4\ldots\right.\right.}{32\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 3/2*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*b^2/f + 4/3*(cos(f*x + e)^
3 - 3*cos(f*x + e))*a*b^3/f + 1/32*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*si
n(2*f*x + 2*e))*b^4/f - 4*a^3*b*cos(f*x + e)/f
```

mupad [B] time = 6.95, size = 114, normalized size = 0.83

$$\frac{\frac{3\,b^4\,\sin\left(4\,e+4\,f\,x\right)}{4}-6\,b^4\,\sin\left(2\,e+2\,f\,x\right)+8\,a\,b^3\,\cos\left(3\,e+3\,f\,x\right)-36\,a^2\,b^2\,\sin\left(2\,e+2\,f\,x\right)-72\,a\,b^3\,\cos\left(e+f\,x\ldots\right.}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^4,x)
```

```
[Out] ((3*b^4*sin(4*e + 4*f*x))/4 - 6*b^4*sin(2*e + 2*f*x) + 8*a*b^3*cos(3*e + 3*
f*x) - 36*a^2*b^2*sin(2*e + 2*f*x) - 72*a*b^3*cos(e + f*x) - 96*a^3*b*cos(e
 + f*x) + 24*a^4*f*x + 9*b^4*f*x + 72*a^2*b^2*f*x)/(24*f)
```

sympy [A] time = 1.67, size = 240, normalized size = 1.75

$$\begin{cases} a^4x - \frac{4a^3b\cos\left(e+fx\right)}{f} + 3a^2b^2x\sin^2\left(e+fx\right) + 3a^2b^2x\cos^2\left(e+fx\right) - \frac{3a^2b^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{f} - \frac{4ab^3\sin^2\left(e+fx\right)\cos\left(e+f\right.}{f} \\ x\left(a+b\sin\left(e\right)\right)^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((a**4*x - 4*a**3*b*cos(e + f*x)/f + 3*a**2*b**2*x*sin(e + f*x)**2
 + 3*a**2*b**2*x*cos(e + f*x)**2 - 3*a**2*b**2*sin(e + f*x)*cos(e + f*x)/f
- 4*a*b**3*sin(e + f*x)**2*cos(e + f*x)/f - 8*a*b**3*cos(e + f*x)**3/(3*f)
+ 3*b**4*x*sin(e + f*x)**4/8 + 3*b**4*x*sin(e + f*x)**2*cos(e + f*x)**2/4 +
 3*b**4*x*cos(e + f*x)**4/8 - 5*b**4*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3
*b**4*sin(e + f*x)*cos(e + f*x)**3/(8*f), Ne(f, 0)), (x*(a + b*sin(e))**4,
True))
```

3.176 $\int \frac{\sin^4(x)}{a+b\sin(x)}\,dx$

Optimal. Leaf size=110

$$-\frac{ax\left(2a^2+b^2\right)}{2b^4}-\frac{\left(3a^2+2b^2\right)\cos(x)}{3b^3}+\frac{2a^4\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4\sqrt{a^2-b^2}}+\frac{a\sin(x)\cos(x)}{2b^2}-\frac{\sin^2(x)\cos(x)}{3b}$$

```
[Out] -1/2*a*(2*a^2+b^2)*x/b^4-1/3*(3*a^2+2*b^2)*cos(x)/b^3+1/2*a*cos(x)*sin(x)/b
^2-1/3*cos(x)*sin(x)^2/b+2*a^4*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/b^4
/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2793, 3049, 3023, 2735, 2660, 618, 204}

$$-\frac{ax\left(2a^2+b^2\right)}{2b^4}-\frac{\left(3a^2+2b^2\right)\cos(x)}{3b^3}+\frac{2a^4\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4\sqrt{a^2-b^2}}+\frac{a\sin(x)\cos(x)}{2b^2}-\frac{\sin^2(x)\cos(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4/(a + b*Sin[x]),x]
```

```
[Out] -(a*(2*a^2 + b^2)*x)/(2*b^4) + (2*a^4*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^
2]])/(b^4*Sqrt[a^2 - b^2]) - ((3*a^2 + 2*b^2)*Cos[x])/(3*b^3) + (a*Cos[x]*S
in[x])/(2*b^2) - (Cos[x]*Sin[x]^2)/(3*b)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\sin^4(x)}{a+b\sin(x)}\,dx = -\frac{\cos(x)\sin^2(x)}{3b} + \frac{\int \frac{\sin(x)\left(2a+2b\sin(x)-3a\sin^2(x)\right)}{a+b\sin(x)}\,dx}{3b}$$

$$= \frac{a\cos(x)\sin(x)}{2b^2} - \frac{\cos(x)\sin^2(x)}{3b} + \frac{\int \frac{-3a^2+ab\sin(x)+2\left(3a^2+2b^2\right)\sin^2(x)}{a+b\sin(x)}\,dx}{6b^2}$$

$$= -\frac{\left(3a^2+2b^2\right)\cos(x)}{3b^3} + \frac{a\cos(x)\sin(x)}{2b^2} - \frac{\cos(x)\sin^2(x)}{3b} + \frac{\int \frac{-3a^2b-3a\left(2a^2+b^2\right)\sin(x)}{a+b\sin(x)}\,dx}{6b^3}$$

$$= -\frac{a\left(2a^2+b^2\right)x}{2b^4} - \frac{\left(3a^2+2b^2\right)\cos(x)}{3b^3} + \frac{a\cos(x)\sin(x)}{2b^2} - \frac{\cos(x)\sin^2(x)}{3b} + \frac{a^4\int \frac{1}{a+b\sin(x)}\,d}{b^4}$$

$$= -\frac{a\left(2a^2+b^2\right)x}{2b^4} - \frac{\left(3a^2+2b^2\right)\cos(x)}{3b^3} + \frac{a\cos(x)\sin(x)}{2b^2} - \frac{\cos(x)\sin^2(x)}{3b} + \frac{\left(2a^4\right)\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{a\left(2a^2+b^2\right)x}{2b^4} - \frac{\left(3a^2+2b^2\right)\cos(x)}{3b^3} + \frac{a\cos(x)\sin(x)}{2b^2} - \frac{\cos(x)\sin^2(x)}{3b} - \frac{\left(4a^4\right)\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{a\left(2a^2+b^2\right)x}{2b^4} + \frac{2a^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^4\sqrt{a^2-b^2}} - \frac{\left(3a^2+2b^2\right)\cos(x)}{3b^3} + \frac{a\cos(x)\sin(x)}{2b^2} - \frac{\cos(x)\,\mathrm{s}}{3b}$$

Mathematica [A] time = 0.26, size = 98, normalized size = 0.89

$$\frac{-6ax\left(2a^2+b^2\right) - 3b\left(4a^2+3b^2\right)\cos(x) + \frac{24a^4\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + 3ab^2\sin(2x) + b^3\cos(3x)}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4/(a + b*Sin[x]),x]
```

```
[Out] (-6*a*(2*a^2 + b^2)*x + (24*a^4*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/S
qrt[a^2 - b^2] - 3*b*(4*a^2 + 3*b^2)*Cos[x] + b^3*Cos[3*x] + 3*a*b^2*Sin[2*
x])/(12*b^4)
```

fricas [A] time = 0.54, size = 333, normalized size = 3.03

$$\left[-\frac{3\sqrt{-a^2+b^2}\,a^4\log\left(\frac{(2a^2-b^2)\cos(x)^2-2ab\sin(x)-a^2-b^2+2(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2ab\sin(x)-a^2-b^2}\right)-2\left(a^2b^3-b^5\right)\cos(x)^3-3\left(a^3b\right.}{6\left(a^2b^4-b^6\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(-a^2 + b^2)*a^4*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) -
a^2 - b^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2
- 2*a*b*sin(x) - a^2 - b^2)) - 2*(a^2*b^3 - b^5)*cos(x)^3 - 3*(a^3*b^2 - a*
b^4)*cos(x)*sin(x) + 3*(2*a^5 - a^3*b^2 - a*b^4)*x + 6*(a^4*b - b^5)*cos(x)
)/(a^2*b^4 - b^6), -1/6*(6*sqrt(a^2 - b^2)*a^4*arctan(-(a*sin(x) + b)/(sqrt
(a^2 - b^2)*cos(x))) - 2*(a^2*b^3 - b^5)*cos(x)^3 - 3*(a^3*b^2 - a*b^4)*cos
(x)*sin(x) + 3*(2*a^5 - a^3*b^2 - a*b^4)*x + 6*(a^4*b - b^5)*cos(x))/(a^2*b
^4 - b^6)]
```

giac [A] time = 0.20, size = 149, normalized size = 1.35

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^4}{\sqrt{a^2-b^2}\,b^4}-\frac{(2a^3+ab^2)x}{2b^4}-\frac{3ab\tan\left(\frac{1}{2}x\right)^5+6a^2\tan\left(\frac{1}{2}x\right)^4+12a^2\tan\left(\frac{1}{2}x\right.}{3\left(\tan\left(\frac{1}{2}x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b
^2)))*a^4/(sqrt(a^2 - b^2)*b^4) - 1/2*(2*a^3 + a*b^2)*x/b^4 - 1/3*(3*a*b*ta
n(1/2*x)^5 + 6*a^2*tan(1/2*x)^4 + 12*a^2*tan(1/2*x)^2 + 12*b^2*tan(1/2*x)^2
 - 3*a*b*tan(1/2*x) + 6*a^2 + 4*b^2)/((tan(1/2*x)^2 + 1)^3*b^3)
```

maple [B] time = 0.08, size = 213, normalized size = 1.94

$$-\frac{a\left(\tan^5\left(\frac{x}{2}\right)\right)}{b^2\left(\tan^2\left(\frac{x}{2}\right)+1\right)^3}-\frac{2a^2\left(\tan^4\left(\frac{x}{2}\right)\right)}{b^3\left(\tan^2\left(\frac{x}{2}\right)+1\right)^3}-\frac{4\left(\tan^2\left(\frac{x}{2}\right)\right)a^2}{b^3\left(\tan^2\left(\frac{x}{2}\right)+1\right)^3}-\frac{4\left(\tan^2\left(\frac{x}{2}\right)\right)}{b\left(\tan^2\left(\frac{x}{2}\right)+1\right)^3}+\frac{a\tan\left(\frac{x}{2}\right)}{b^2\left(\tan^2\left(\frac{x}{2}\right)+1\right)^3}-\frac{2a^2}{b^3\left(\tan^2\left(\frac{x}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a+b*sin(x)),x)
```

```
[Out] -1/b^2/(tan(1/2*x)^2+1)^3*a*tan(1/2*x)^5-2/b^3/(tan(1/2*x)^2+1)^3*a^2*tan(1
/2*x)^4-4/b^3/(tan(1/2*x)^2+1)^3*tan(1/2*x)^2*a^2-4/b/(tan(1/2*x)^2+1)^3*ta
n(1/2*x)^2+1/b^2/(tan(1/2*x)^2+1)^3*a*tan(1/2*x)-2/b^3/(tan(1/2*x)^2+1)^3*a
^2-4/3/b/(tan(1/2*x)^2+1)^3-2/b^4*arctan(tan(1/2*x))*a^3-1/b^2*arctan(tan(1
/2*x))*a+2*a^4/b^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2
)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.22, size = 1075, normalized size = 9.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a + b*sin(x)),x)
```

```
[Out] - ((2*(3*a^2 + 2*b^2))/(3*b^3) + (a*tan(x/2)^5)/b^2 + (2*a^2*tan(x/2)^4)/b^
3 + (4*tan(x/2)^2*(a^2 + b^2))/b^3 - (a*tan(x/2))/b^2)/(3*tan(x/2)^2 + 3*ta
n(x/2)^4 + tan(x/2)^6 + 1) - (a*atan((8*a^4*tan(x/2))/(8*a^4 + (40*a^6)/b^2
 + (48*a^8)/b^4) + (40*a^6*tan(x/2))/(40*a^6 + 8*a^4*b^2 + (48*a^8)/b^2) +
(48*a^8*tan(x/2))/(48*a^8 + 8*a^4*b^4 + 40*a^6*b^2))*(2*a^2 + b^2))/b^4 - (
a^4*atan(((a^4*((8*(a^4*b^7 + 4*a^6*b^5 + 4*a^8*b^3))/b^8 + (8*tan(x/2)*(2*
a^3*b^9 + 7*a^5*b^7 + 4*a^7*b^5 - 8*a^9*b^3))/b^9 + (a^4*((8*(2*a^2*b^10 +
2*a^4*b^8))/b^8 + (64*a^5*tan(x/2))/b + (a^4*(32*a^2*b^3 + (8*tan(x/2)*(12*
a*b^13 - 8*a^3*b^11))/b^9))/(b^4*(b^2 - a^2)^(1/2))))/(b^4*(b^2 - a^2)^(1/2
)))*1i)/(b^4*(b^2 - a^2)^(1/2)) + (a^4*((8*(a^4*b^7 + 4*a^6*b^5 + 4*a^8*b^3
))/b^8 + (8*tan(x/2)*(2*a^3*b^9 + 7*a^5*b^7 + 4*a^7*b^5 - 8*a^9*b^3))/b^9 -
 (a^4*((8*(2*a^2*b^10 + 2*a^4*b^8))/b^8 + (64*a^5*tan(x/2))/b - (a^4*(32*a^
2*b^3 + (8*tan(x/2)*(12*a*b^13 - 8*a^3*b^11))/b^9))/(b^4*(b^2 - a^2)^(1/2))
))/(b^4*(b^2 - a^2)^(1/2)))*1i)/(b^4*(b^2 - a^2)^(1/2)))/((16*(2*a^10 + a^8
*b^2))/b^8 + (16*tan(x/2)*(8*a^11 + 2*a^7*b^4 + 8*a^9*b^2))/b^9 + (a^4*((8*
(a^4*b^7 + 4*a^6*b^5 + 4*a^8*b^3))/b^8 + (8*tan(x/2)*(2*a^3*b^9 + 7*a^5*b^7
 + 4*a^7*b^5 - 8*a^9*b^3))/b^9 + (a^4*((8*(2*a^2*b^10 + 2*a^4*b^8))/b^8 + (
```

```
64*a^5*tan(x/2))/b + (a^4*(32*a^2*b^3 + (8*tan(x/2)*(12*a*b^13 - 8*a^3*b^11
))/b^9))/(b^4*(b^2 - a^2)^(1/2))))/(b^4*(b^2 - a^2)^(1/2))))/(b^4*(b^2 - a^
2)^(1/2)) - (a^4*((8*(a^4*b^7 + 4*a^6*b^5 + 4*a^8*b^3))/b^8 + (8*tan(x/2)*(
2*a^3*b^9 + 7*a^5*b^7 + 4*a^7*b^5 - 8*a^9*b^3))/b^9 - (a^4*((8*(2*a^2*b^10
+ 2*a^4*b^8))/b^8 + (64*a^5*tan(x/2))/b - (a^4*(32*a^2*b^3 + (8*tan(x/2)*(1
2*a*b^13 - 8*a^3*b^11))/b^9))/(b^4*(b^2 - a^2)^(1/2))))/(b^4*(b^2 - a^2)^(1
/2))))/(b^4*(b^2 - a^2)^(1/2))))*2i)/(b^4*(b^2 - a^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4/(a+b*sin(x)),x)
```

```
[Out] Timed out
```

3.177 $\int \frac{\sin^3(x)}{a+b\sin(x)}\,dx$

Optimal. Leaf size=82

$$\frac{x(2a^2+b^2)}{2b^3}-\frac{2a^3\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}}+\frac{a\cos(x)}{b^2}-\frac{\sin(x)\cos(x)}{2b}$$

```
[Out] 1/2*(2*a^2+b^2)*x/b^3+a*cos(x)/b^2-1/2*cos(x)*sin(x)/b-2*a^3*arctan((b+a*ta
n(1/2*x))/(a^2-b^2)^(1/2))/b^3/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2793, 3023, 2735, 2660, 618, 204}

$$\frac{x(2a^2+b^2)}{2b^3}-\frac{2a^3\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}}+\frac{a\cos(x)}{b^2}-\frac{\sin(x)\cos(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a + b*Sin[x]),x]
```

```
[Out] ((2*a^2 + b^2)*x)/(2*b^3) - (2*a^3*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]]
)/(b^3*Sqrt[a^2 - b^2]) + (a*Cos[x])/b^2 - (Cos[x]*Sin[x])/(2*b)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^3(x)}{a+b\sin(x)}\,dx = -\frac{\cos(x)\sin(x)}{2b} + \frac{\int \frac{a+b\sin(x)-2a\sin^2(x)}{a+b\sin(x)}\,dx}{2b}$$
$$= \frac{a\cos(x)}{b^2} - \frac{\cos(x)\sin(x)}{2b} + \frac{\int \frac{ab+(2a^2+b^2)\sin(x)}{a+b\sin(x)}\,dx}{2b^2}$$
$$= \frac{(2a^2+b^2)\,x}{2b^3} + \frac{a\cos(x)}{b^2} - \frac{\cos(x)\sin(x)}{2b} - \frac{a^3\int \frac{1}{a+b\sin(x)}\,dx}{b^3}$$
$$= \frac{(2a^2+b^2)\,x}{2b^3} + \frac{a\cos(x)}{b^2} - \frac{\cos(x)\sin(x)}{2b} - \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{b^3}$$
$$= \frac{(2a^2+b^2)\,x}{2b^3} + \frac{a\cos(x)}{b^2} - \frac{\cos(x)\sin(x)}{2b} + \frac{(4a^3)\,\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b+2a\tan\left(\frac{x}{2}\right.\right.}{b^3}$$
$$= \frac{(2a^2+b^2)\,x}{2b^3} - \frac{2a^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}} + \frac{a\cos(x)}{b^2} - \frac{\cos(x)\sin(x)}{2b}$$

Mathematica [A] time = 0.11, size = 78, normalized size = 0.95

$$\frac{4a^2x - \frac{8a^3\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + 4ab\cos(x) + 2b^2x - b^2\sin(2x)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a + b*Sin[x]),x]
```

```
[Out] (4*a^2*x + 2*b^2*x - (8*a^3*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[
a^2 - b^2] + 4*a*b*Cos[x] - b^2*Sin[2*x])/(4*b^3)
```

fricas [A] time = 0.53, size = 291, normalized size = 3.55

$$\left[-\frac{\sqrt{-a^2+b^2}\,a^3\log\left(-\frac{(2\,a^2-b^2)\cos(x)^2-2\,ab\sin(x)-a^2-b^2-2\,(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2\,ab\sin(x)-a^2-b^2}\right) + (a^2b^2-b^4)\cos(x)\sin(x) - (\cdots}{2\,(a^2b^3-b^5)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*a^3*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a
^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 -
 2*a*b*sin(x) - a^2 - b^2)) + (a^2*b^2 - b^4)*cos(x)*sin(x) - (2*a^4 - a^2*
b^2 - b^4)*x - 2*(a^3*b - a*b^3)*cos(x))/(a^2*b^3 - b^5), 1/2*(2*sqrt(a^2 -
 b^2)*a^3*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) - (a^2*b^2 - b^4
)*cos(x)*sin(x) + (2*a^4 - a^2*b^2 - b^4)*x + 2*(a^3*b - a*b^3)*cos(x))/(a^
2*b^3 - b^5)]
```

giac [A] time = 0.23, size = 112, normalized size = 1.37

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^3}{\sqrt{a^2-b^2}\,b^3}+\frac{\left(2a^2+b^2\right)x}{2b^3}+\frac{b\tan\left(\frac{1}{2}x\right)^3+2a\tan\left(\frac{1}{2}x\right)^2-b\tan\left(\frac{1}{2}x\right)+2a}{\left(\tan\left(\frac{1}{2}x\right)^2+1\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 -
b^2)))*a^3/(sqrt(a^2 - b^2)*b^3) + 1/2*(2*a^2 + b^2)*x/b^3 + (b*tan(1/2*x)^
3 + 2*a*tan(1/2*x)^2 - b*tan(1/2*x) + 2*a)/((tan(1/2*x)^2 + 1)^2*b^2)
```

maple [A] time = 0.07, size = 142, normalized size = 1.73

$$\frac{\tan^3\left(\frac{x}{2}\right)}{b\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}+\frac{2\left(\tan^2\left(\frac{x}{2}\right)\right)a}{b^2\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}-\frac{\tan\left(\frac{x}{2}\right)}{b\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}+\frac{2a}{b^2\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}+\frac{2\arctan\left(\tan\left(\frac{x}{2}\right)\right)a^2}{b^3}-\frac{2a^3\arctan(}{b^3\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a+b*sin(x)),x)
```

```
[Out] 1/b/(tan(1/2*x)^2+1)^2*tan(1/2*x)^3+2/b^2/(tan(1/2*x)^2+1)^2*tan(1/2*x)^2*a
-1/b/(tan(1/2*x)^2+1)^2*tan(1/2*x)+2/b^2/(tan(1/2*x)^2+1)^2*a+2/b^3*arctan(
tan(1/2*x))*a^2-2*a^3/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(
a^2-b^2)^(1/2))+1/2*x/b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.08, size = 1004, normalized size = 12.24

$$\frac{\frac{2a}{b^2}-\frac{\tan\left(\frac{x}{2}\right)}{b}+\frac{\tan\left(\frac{x}{2}\right)^3}{b}+\frac{2a\tan\left(\frac{x}{2}\right)^2}{b^2}}{\tan\left(\frac{x}{2}\right)^4+2\tan\left(\frac{x}{2}\right)^2+1}-\frac{\mathrm{atan}\left(\frac{40a^3\tan\left(\frac{x}{2}\right)}{8ab^2+40a^3+\frac{48a^5}{b^2}}+\frac{48a^5\tan\left(\frac{x}{2}\right)}{48a^5+40a^3b^2+8ab^4}+\frac{8ab\tan\left(\frac{x}{2}\right)}{8ab+\frac{40a^3}{b}+\frac{48a^5}{b^3}}\right)\left(a^2\,2\mathrm{i}+b^2\,1\mathrm{i}\right)1\mathrm{i}}{b^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a + b*sin(x)),x)
```

```
[Out] ((2*a)/b^2 - tan(x/2)/b + tan(x/2)^3/b + (2*a*tan(x/2)^2)/b^2)/(2*tan(x/2)^
2 + tan(x/2)^4 + 1) - (atan((40*a^3*tan(x/2))/(8*a*b^2 + 40*a^3 + (48*a^5)/
b^2) + (48*a^5*tan(x/2))/(8*a*b^4 + 48*a^5 + 40*a^3*b^2) + (8*a*b*tan(x/2))
/(8*a*b + (40*a^3)/b + (48*a^5)/b^3))*(a^2*2i + b^2*1i)*1i)/b^3 + (a^3*atan
(((a^3*((8*(a^2*b^6 + 4*a^4*b^4 + 4*a^6*b^2))/b^5 + (8*tan(x/2)*(2*a*b^8 +
7*a^3*b^6 + 4*a^5*b^4 - 8*a^7*b^2))/b^6 + (a^3*(64*a^4*tan(x/2) + (8*(2*a*b
^8 + 2*a^3*b^6))/b^5 + (a^3*(32*a^2*b^3 + (8*tan(x/2)*(12*a*b^10 - 8*a^3*b^
8))/b^6))/(b^3*(b^2 - a^2)^(1/2))))/(b^3*(b^2 - a^2)^(1/2)))*1i)/(b^3*(b^2
- a^2)^(1/2)) + (a^3*((8*(a^2*b^6 + 4*a^4*b^4 + 4*a^6*b^2))/b^5 + (8*tan(x/
2)*(2*a*b^8 + 7*a^3*b^6 + 4*a^5*b^4 - 8*a^7*b^2))/b^6 - (a^3*(64*a^4*tan(x/
2) + (8*(2*a*b^8 + 2*a^3*b^6))/b^5 - (a^3*(32*a^2*b^3 + (8*tan(x/2)*(12*a*b
^10 - 8*a^3*b^8))/b^6))/(b^3*(b^2 - a^2)^(1/2))))/(b^3*(b^2 - a^2)^(1/2)))*
```

```
1i)/(b^3*(b^2 - a^2)^(1/2)))/((16*(2*a^7 + a^5*b^2))/b^5 + (16*tan(x/2)*(8*
a^8 + 2*a^4*b^4 + 8*a^6*b^2))/b^6 + (a^3*((8*(a^2*b^6 + 4*a^4*b^4 + 4*a^6*b
^2))/b^5 + (8*tan(x/2)*(2*a*b^8 + 7*a^3*b^6 + 4*a^5*b^4 - 8*a^7*b^2))/b^6 +
 (a^3*(64*a^4*tan(x/2) + (8*(2*a*b^8 + 2*a^3*b^6))/b^5 + (a^3*(32*a^2*b^3 +
 (8*tan(x/2)*(12*a*b^10 - 8*a^3*b^8))/b^6))/(b^3*(b^2 - a^2)^(1/2))))/(b^3*
(b^2 - a^2)^(1/2))))/(b^3*(b^2 - a^2)^(1/2)) - (a^3*((8*(a^2*b^6 + 4*a^4*b^
4 + 4*a^6*b^2))/b^5 + (8*tan(x/2)*(2*a*b^8 + 7*a^3*b^6 + 4*a^5*b^4 - 8*a^7*
b^2))/b^6 - (a^3*(64*a^4*tan(x/2) + (8*(2*a*b^8 + 2*a^3*b^6))/b^5 - (a^3*(3
2*a^2*b^3 + (8*tan(x/2)*(12*a*b^10 - 8*a^3*b^8))/b^6))/(b^3*(b^2 - a^2)^(1/
2))))/(b^3*(b^2 - a^2)^(1/2))))/(b^3*(b^2 - a^2)^(1/2))))*2i)/(b^3*(b^2 - a
^2)^(1/2))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a+b*sin(x)),x)
```

```
[Out] Timed out
```

3.178 $\int \frac{\sin^2(x)}{a+b\sin(x)}\,dx$

Optimal. Leaf size=61

$$\frac{2a^2\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}-\frac{ax}{b^2}-\frac{\cos(x)}{b}$$

```
[Out] -a*x/b^2-cos(x)/b+2*a^2*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/b^2/(a^2-b
^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2746, 12, 2735, 2660, 618, 204}

$$\frac{2a^2\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}-\frac{ax}{b^2}-\frac{\cos(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a + b*Sin[x]),x]
```

```
[Out] -((a*x)/b^2) + (2*a^2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b^2*Sqrt[a
^2 - b^2]) - Cos[x]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^2(x)}{a+b\sin(x)}\,dx &= -\frac{\cos(x)}{b}-\frac{\int \frac{a\sin(x)}{a+b\sin(x)}\,dx}{b}\\
&= -\frac{\cos(x)}{b}-\frac{a\int \frac{\sin(x)}{a+b\sin(x)}\,dx}{b}\\
&= -\frac{ax}{b^2}-\frac{\cos(x)}{b}+\frac{a^2\int \frac{1}{a+b\sin(x)}\,dx}{b^2}\\
&= -\frac{ax}{b^2}-\frac{\cos(x)}{b}+\frac{\left(2a^2\right)\operatorname{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{b^2}\\
&= -\frac{ax}{b^2}-\frac{\cos(x)}{b}-\frac{\left(4a^2\right)\operatorname{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan\left(\frac{x}{2}\right)\right)}{b^2}\\
&= -\frac{ax}{b^2}+\frac{2a^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}-\frac{\cos(x)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 56, normalized size = 0.92

$$-\frac{-\frac{2a^2\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+ax+b\cos(x)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a + b*Sin[x]),x]
```

```
[Out] -((a*x - (2*a^2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] +
 b*Cos[x])/b^2)
```

fricas [A] time = 0.50, size = 231, normalized size = 3.79

$$\left[-\frac{\sqrt{-a^2+b^2}\,a^2\log\left(\frac{\left(2\,a^2-b^2\right)\cos(x)^2-2\,ab\sin(x)-a^2-b^2+2\,(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2\,ab\sin(x)-a^2-b^2}\right)+2\left(a^3-ab^2\right)x+2\left(a^2b-b^3\right)\mathrm{c}}{2\left(a^2b^2-b^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*a^2*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^
2 - b^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 -
2*a*b*sin(x) - a^2 - b^2)) + 2*(a^3 - a*b^2)*x + 2*(a^2*b - b^3)*cos(x))/(a
^2*b^2 - b^4), -(sqrt(a^2 - b^2)*a^2*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2
)*cos(x))) + (a^3 - a*b^2)*x + (a^2*b - b^3)*cos(x))/(a^2*b^2 - b^4)]
```

giac [A] time = 0.36, size = 77, normalized size = 1.26

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^2}{\sqrt{a^2-b^2}\,b^2}-\frac{ax}{b^2}-\frac{2}{\left(\tan\left(\frac{1}{2}x\right)^2+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b
^2)))*a^2/(sqrt(a^2 - b^2)*b^2) - a*x/b^2 - 2/((tan(1/2*x)^2 + 1)*b)
```

maple [A] time = 0.07, size = 72, normalized size = 1.18

$$-\frac{2}{b\left(\tan^2\left(\frac{x}{2}\right)+1\right)}-\frac{2\arctan\left(\tan\left(\frac{x}{2}\right)\right)a}{b^2}+\frac{2a^2\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a+b*sin(x)),x)
```

```
[Out] -2/b/(tan(1/2*x)^2+1)-2/b^2*arctan(tan(1/2*x))*a+2*a^2/b^2/(a^2-b^2)^(1/2)*
arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 6.91, size = 623, normalized size = 10.21

$$-\frac{2}{b\left(\tan\left(\frac{x}{2}\right)^2+1\right)}-\frac{a\,x}{b^2}-\frac{a^2\,\mathrm{atan}\left(\frac{\frac{a^2\left(\frac{32\,a^4}{b}-\frac{32\tan\left(\frac{x}{2}\right)\left(2\,a^5\,b-2\,a^3\,b^3\right)}{b^3}+\frac{a^2\left(32\,a^2\,b^2+64\,a^3\,b\tan\left(\frac{x}{2}\right)+\frac{a^2\left(32\,a^2\,b^3+\frac{32\tan\left(\frac{x}{2}\right)\left(3\,a\,b^7-2\,a^3\,b^5\right)}{b^3}\right)}{b^2\sqrt{b^2-a^2}}\right)}{b^2\sqrt{b^2-a^2}}\right)1\mathrm{i}\cdots}{b^2\sqrt{b^2-a^2}}}{\frac{128\,a^5\tan\left(\frac{x}{2}\right)}{b^3}+\frac{a^2\left(\frac{32\,a^4}{b}-\frac{32\tan\left(\frac{x}{2}\right)\left(2\,a^5\,b-2\,a^3\,b^3\right)}{b^3}+\frac{a^2\left(32\,a^2\,b^2+64\,a^3\,b\tan\left(\frac{x}{2}\right)+\frac{a^2\left(32\,a^2\,b^3+\frac{32\tan\left(\frac{x}{2}\right)\left(3\,a\,b^7-2\,a^3\cdots\right)}{b^3}\cdots\right)}{b^2\sqrt{b^2-a^2}}\cdots\right)}{b^2\sqrt{b^2-a^2}}\cdots\right)}{b^2\sqrt{b^2-a^2}}\cdots}\right)}{b^2\sqrt{b^2\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a + b*sin(x)),x)
```

```
[Out] - 2/(b*(tan(x/2)^2 + 1)) - (a*x)/b^2 - (a^2*atan(((a^2*((32*a^4)/b - (32*ta
n(x/2)*(2*a^5*b - 2*a^3*b^3))/b^3 + (a^2*(32*a^2*b^2 + 64*a^3*b*tan(x/2) +
(a^2*(32*a^2*b^3 + (32*tan(x/2)*(3*a*b^7 - 2*a^3*b^5))/b^3))/(b^2*(b^2 - a^
2)^(1/2))))/(b^2*(b^2 - a^2)^(1/2)))*1i)/(b^2*(b^2 - a^2)^(1/2)) - (a^2*((3
2*tan(x/2)*(2*a^5*b - 2*a^3*b^3))/b^3 - (32*a^4)/b + (a^2*(32*a^2*b^2 + 64*
a^3*b*tan(x/2) - (a^2*(32*a^2*b^3 + (32*tan(x/2)*(3*a*b^7 - 2*a^3*b^5))/b^3
))/(b^2*(b^2 - a^2)^(1/2))))/(b^2*(b^2 - a^2)^(1/2)))*1i)/(b^2*(b^2 - a^2)^
(1/2)))/((128*a^5*tan(x/2))/b^3 + (a^2*((32*a^4)/b - (32*tan(x/2)*(2*a^5*b
- 2*a^3*b^3))/b^3 + (a^2*(32*a^2*b^2 + 64*a^3*b*tan(x/2) + (a^2*(32*a^2*b^3
 + (32*tan(x/2)*(3*a*b^7 - 2*a^3*b^5))/b^3))/(b^2*(b^2 - a^2)^(1/2))))/(b^2
*(b^2 - a^2)^(1/2))))/(b^2*(b^2 - a^2)^(1/2)) + (a^2*((32*tan(x/2)*(2*a^5*b
 - 2*a^3*b^3))/b^3 - (32*a^4)/b + (a^2*(32*a^2*b^2 + 64*a^3*b*tan(x/2) - (a
^2*(32*a^2*b^3 + (32*tan(x/2)*(3*a*b^7 - 2*a^3*b^5))/b^3))/(b^2*(b^2 - a^2)
^(1/2))))/(b^2*(b^2 - a^2)^(1/2))))/(b^2*(b^2 - a^2)^(1/2))))*2i)/(b^2*(b^2
 - a^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+b*sin(x)),x)
```

```
[Out] Timed out
```

3.179 $\int \frac{\sin(x)}{a+b\sin(x)}\,dx$

Optimal. Leaf size=50

$$\frac{x}{b}-\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}$$

```
[Out] x/b-2*a*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/b/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2735, 2660, 618, 204}

$$\frac{x}{b}-\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a + b*Sin[x]),x]
```

```
[Out] x/b - (2*a*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b*Sqrt[a^2 - b^2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin(x)}{a+b\sin(x)}\,dx &= \frac{x}{b}-\frac{a\int \frac{1}{a+b\sin(x)}\,dx}{b}\\
&= \frac{x}{b}-\frac{(2a)\operatorname{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{b}\\
&= \frac{x}{b}+\frac{(4a)\operatorname{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan\left(\frac{x}{2}\right)\right)}{b}\\
&= \frac{x}{b}-\frac{2a\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 47, normalized size = 0.94

$$
\frac{x-\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a + b*Sin[x]),x]
```

```
[Out] (x - (2*a*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2])/b
```

fricas [A] time = 0.78, size = 192, normalized size = 3.84

$$
\left[-\frac{\sqrt{-a^2+b^2}\,a\log\left(-\frac{\left(2a^2-b^2\right)\cos(x)^2-2ab\sin(x)-a^2-b^2-2\left(a\cos(x)\sin(x)+b\cos(x)\right)\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2ab\sin(x)-a^2-b^2}\right)-2\left(a^2-b^2\right)x}{2\left(a^2b-b^3\right)},\ \sqrt{a^2-b^2}\,a\,\mathrm{arctar}
\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*a*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2
 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2
*a*b*sin(x) - a^2 - b^2)) - 2*(a^2 - b^2)*x)/(a^2*b - b^3), (sqrt(a^2 - b^2
)*a*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) + (a^2 - b^2)*x)/(a^2*
b - b^3)]
```

giac [A] time = 0.32, size = 58, normalized size = 1.16

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)a}{\sqrt{a^2-b^2}\,b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 -
b^2)))*a/(sqrt(a^2 - b^2)*b) + x/b
```

maple [A] time = 0.06, size = 54, normalized size = 1.08

$$\frac{2\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{b}-\frac{2a\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a+b*sin(x)),x)
```

```
[Out] 2/b*arctan(tan(1/2*x))-2*a/b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b
)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 6.70, size = 101, normalized size = 2.02

$$\frac{x}{b}-\frac{2\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)\,a^4-\cos\left(\frac{x}{2}\right)\,a^3\,b-3\,\sin\left(\frac{x}{2}\right)\,a^2\,b^2+\cos\left(\frac{x}{2}\right)\,a\,b^3+2\,\sin\left(\frac{x}{2}\right)\,b^4}{{\left(b^2-a^2\right)}^{3/2}\,\left(2\,b\,\sin\left(\frac{x}{2}\right)+a\,\cos\left(\frac{x}{2}\right)\right)}\right)}{b\,\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a + b*sin(x)),x)
```

```
[Out] x/b - (2*a*atanh((a^4*sin(x/2) + 2*b^4*sin(x/2) - 3*a^2*b^2*sin(x/2) + a*b^
3*cos(x/2) - a^3*b*cos(x/2))/((b^2 - a^2)^(3/2)*(2*b*sin(x/2) + a*cos(x/2))
)))/(b*(b^2 - a^2)^(1/2))
```

sympy [A] time = 80.94, size = 236, normalized size = 4.72

$$\begin{cases} \tilde{\infty} x & \text{for}\: a = 0 \wedge b = 0 \\ \frac{b x \tan{\left(\frac{x}{2} \right)}}{b^{2} \tan{\left(\frac{x}{2} \right)} - b \sqrt{b^{2}}} + \frac{2 b}{b^{2} \tan{\left(\frac{x}{2} \right)} - b \sqrt{b^{2}}} - \frac{x \sqrt{b^{2}}}{b^{2} \tan{\left(\frac{x}{2} \right)} - b \sqrt{b^{2}}} & \text{for}\: a = - \sqrt{b^{2}} \\ \frac{b x \tan{\left(\frac{x}{2} \right)}}{b^{2} \tan{\left(\frac{x}{2} \right)} + b \sqrt{b^{2}}} + \frac{2 b}{b^{2} \tan{\left(\frac{x}{2} \right)} + b \sqrt{b^{2}}} + \frac{x \sqrt{b^{2}}}{b^{2} \tan{\left(\frac{x}{2} \right)} + b \sqrt{b^{2}}} & \text{for}\: a = \sqrt{b^{2}} \\ - \frac{\cos{\left(x \right)}}{a} & \text{for}\: b = 0 \\ \frac{x}{b} & \text{for}\: a = 0 \\ - \frac{a \log{\left(\tan{\left(\frac{x}{2} \right)} + \frac{b}{a} - \frac{\sqrt{- a^{2} + b^{2}}}{a} \right)}}{b \sqrt{- a^{2} + b^{2}}} + \frac{a \log{\left(\tan{\left(\frac{x}{2} \right)} + \frac{b}{a} + \frac{\sqrt{- a^{2} + b^{2}}}{a} \right)}}{b \sqrt{- a^{2} + b^{2}}} + \frac{x}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x)),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (b*x*tan(x/2)/(b**2*tan(x/2) - b*sq
rt(b**2)) + 2*b/(b**2*tan(x/2) - b*sqrt(b**2)) - x*sqrt(b**2)/(b**2*tan(x/2
) - b*sqrt(b**2)), Eq(a, -sqrt(b**2))), (b*x*tan(x/2)/(b**2*tan(x/2) + b*sq
rt(b**2)) + 2*b/(b**2*tan(x/2) + b*sqrt(b**2)) + x*sqrt(b**2)/(b**2*tan(x/2
) + b*sqrt(b**2)), Eq(a, sqrt(b**2))), (-cos(x)/a, Eq(b, 0)), (x/b, Eq(a, 0
)), (-a*log(tan(x/2) + b/a - sqrt(-a**2 + b**2)/a)/(b*sqrt(-a**2 + b**2)) +
 a*log(tan(x/2) + b/a + sqrt(-a**2 + b**2)/a)/(b*sqrt(-a**2 + b**2)) + x/b,
 True))
```

3.180 $\int \frac{1}{a+b\sin(x)}\,dx$

Optimal. Leaf size=40

$$\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

```
[Out] 2*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2660, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x])^(-1),x]
```

```
[Out] (2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{a+b\sin(x)}\,dx = 2\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$
$$= -\left(4\,\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx, x, 2b+2a\tan\left(\frac{x}{2}\right)\right)\right)$$
$$= \frac{2\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Mathematica [**A**] time = 0.02, size = 40, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x])^(-1),x]
[Out] (2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
```

fricas [**A**] time = 0.47, size = 148, normalized size = 3.70

$$\left[-\frac{\sqrt{-a^2+b^2}\,\log\left(\frac{\left(2\,a^2-b^2\right)\cos(x)^2-2\,ab\sin(x)-a^2-b^2+2\,(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2\,ab\sin(x)-a^2-b^2}\right)}{2\left(a^2-b^2\right)}, -\frac{\arctan\left(-\frac{a\sin(x)+b}{\sqrt{a^2-b^2}\cos(x)}\right)}{\sqrt{a^2-b^2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)),x, algorithm="fricas")
[Out] [-1/2*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b
^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b
*sin(x) - a^2 - b^2))/(a^2 - b^2), -arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)
*cos(x)))/sqrt(a^2 - b^2)]
```

giac [**A**] time = 0.17, size = 48, normalized size = 1.20

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\text{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b
^2)))/sqrt(a^2 - b^2)
```

maple [A] time = 0.05, size = 39, normalized size = 0.98

$$\frac{2\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(x)),x)
```

```
[Out] 2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 6.88, size = 45, normalized size = 1.12

$$\frac{2\,\mathrm{atan}\left(\frac{b}{\sqrt{a^2-b^2}}+\frac{a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(x)),x)
```

```
[Out] (2*atan(b/(a^2 - b^2)^(1/2) + (a*tan(x/2))/(a^2 - b^2)^(1/2)))/(a^2 - b^2)^
(1/2)
```

sympy [A] time = 9.66, size = 133, normalized size = 3.32

$$\begin{cases} \frac{2\sqrt{b^2}}{b^2\tan\left(\frac{x}{2}\right)-b\sqrt{b^2}} & \text{for } a=-\sqrt{b^2} \\ -\frac{2\sqrt{b^2}}{b^2\tan\left(\frac{x}{2}\right)+b\sqrt{b^2}} & \text{for } a=\sqrt{b^2} \\ \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b} & \text{for } a=0 \\ \frac{\log\left(\tan\left(\frac{x}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{\sqrt{-a^2+b^2}}-\frac{\log\left(\tan\left(\frac{x}{2}\right)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{\sqrt{-a^2+b^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)),x)
```

```
[Out] Piecewise((2*sqrt(b**2)/(b**2*tan(x/2) - b*sqrt(b**2)), Eq(a, -sqrt(b**2)))
, (-2*sqrt(b**2)/(b**2*tan(x/2) + b*sqrt(b**2)), Eq(a, sqrt(b**2))), (log(t
an(x/2))/b, Eq(a, 0)), (log(tan(x/2) + b/a - sqrt(-a**2 + b**2)/a)/sqrt(-a*
*2 + b**2) - log(tan(x/2) + b/a + sqrt(-a**2 + b**2)/a)/sqrt(-a**2 + b**2),
 True))
```

3.181 $\int \frac{\csc(x)}{a+b\sin(x)}\,dx$

Optimal. Leaf size=53

$$-\frac{2b\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}-\frac{\tanh^{-1}(\cos(x))}{a}$$

```
[Out] -arctanh(cos(x))/a-2*b*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/a/(a^2-b^2)
^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2747, 3770, 2660, 618, 204}

$$-\frac{2b\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}-\frac{\tanh^{-1}(\cos(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a + b*Sin[x]),x]
```

```
[Out] (-2*b*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a*Sqrt[a^2 - b^2]) - ArcTa
nh[Cos[x]]/a
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(x)}{a+b\sin(x)}\,dx &= \frac{\int \csc(x)\,dx}{a}-\frac{b\int \frac{1}{a+b\sin(x)}\,dx}{a}\\ &= -\frac{\tanh^{-1}(\cos(x))}{a}-\frac{(2b)\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{a}\\ &= -\frac{\tanh^{-1}(\cos(x))}{a}+\frac{(4b)\,\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan\left(\frac{x}{2}\right)\right)}{a}\\ &= -\frac{2b\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}-\frac{\tanh^{-1}(\cos(x))}{a}\end{aligned}$$

Mathematica [A] time = 0.06, size = 62, normalized size = 1.17

$$\frac{-\frac{2b\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a + b*Sin[x]),x]
```

```
[Out] ((-2*b*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - Log[Cos[
x/2]] + Log[Sin[x/2]])/a
```

fricas [A] time = 0.58, size = 239, normalized size = 4.51

$$\left[-\frac{\sqrt{-a^2+b^2}\,b\log\left(-\frac{\left(2\,a^2-b^2\right)\cos(x)^2-2\,ab\sin(x)-a^2-b^2-2\,(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2\,ab\sin(x)-a^2-b^2}\right)+\left(a^2-b^2\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)\cdots}{2\left(a^3-ab^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*b*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2
 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2
*a*b*sin(x) - a^2 - b^2)) + (a^2 - b^2)*log(1/2*cos(x) + 1/2) - (a^2 - b^2)
*log(-1/2*cos(x) + 1/2))/(a^3 - a*b^2), 1/2*(2*sqrt(a^2 - b^2)*b*arctan(-(a
*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) - (a^2 - b^2)*log(1/2*cos(x) + 1/2)
+ (a^2 - b^2)*log(-1/2*cos(x) + 1/2))/(a^3 - a*b^2)]
```

giac [A] time = 0.18, size = 63, normalized size = 1.19

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)b}{\sqrt{a^2-b^2}\,a}+\frac{\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 -
b^2)))*b/(sqrt(a^2 - b^2)*a) + log(abs(tan(1/2*x)))/a
```

maple [A] time = 0.10, size = 53, normalized size = 1.00

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a}-\frac{2b\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a+b*sin(x)),x)
```

```
[Out] 1/a*ln(tan(1/2*x))-2*b/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a
^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 6.86, size = 122, normalized size = 2.30

$$\frac{\ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{a}+\frac{2\,b\,\mathrm{atanh}\left(\frac{\sqrt{b^2-a^2}\,\left(-1\mathrm{i}\,\sin\left(\frac{x}{2}\right)\,a^2+2\mathrm{i}\,\cos\left(\frac{x}{2}\right)\,a\,b+4\mathrm{i}\,\sin\left(\frac{x}{2}\right)\,b^2\right)}{1\mathrm{i}\,\cos\left(\frac{x}{2}\right)\,a^3+3\mathrm{i}\,\sin\left(\frac{x}{2}\right)\,a^2\,b-2\mathrm{i}\,\cos\left(\frac{x}{2}\right)\,a\,b^2-4\mathrm{i}\,\sin\left(\frac{x}{2}\right)\,b^3}\right)}{a\,\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(a + b*sin(x))),x)
```

```
[Out] log(sin(x/2)/cos(x/2))/a + (2*b*atanh(((b^2 - a^2)^(1/2)*(b^2*sin(x/2)*4i -
 a^2*sin(x/2)*1i + a*b*cos(x/2)*2i))/(a^3*cos(x/2)*1i - b^3*sin(x/2)*4i - a
*b^2*cos(x/2)*2i + a^2*b*sin(x/2)*3i)))/(a*(b^2 - a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(x)}{a+b\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x)),x)
```

```
[Out] Integral(csc(x)/(a + b*sin(x)), x)
```

3.182 $\int \frac{\csc^2(x)}{a+b\sin(x)}\,dx$

Optimal. Leaf size=62

$$\frac{2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}}+\frac{b\tanh^{-1}(\cos(x))}{a^2}-\frac{\cot(x)}{a}$$

```
[Out] b*arctanh(cos(x))/a^2-cot(x)/a+2*b^2*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2
))/a^2/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2802, 12, 2747, 3770, 2660, 618, 204}

$$\frac{2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}}+\frac{b\tanh^{-1}(\cos(x))}{a^2}-\frac{\cot(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a + b*Sin[x]),x]
```

```
[Out] (2*b^2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2*Sqrt[a^2 - b^2]) + (b
*ArcTanh[Cos[x]])/a^2 - Cot[x]/a
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^2(x)}{a+b\sin(x)}\,dx = -\frac{\cot(x)}{a} - \frac{\int \frac{b\csc(x)}{a+b\sin(x)}\,dx}{a}$$
$$= -\frac{\cot(x)}{a} - \frac{b\int \frac{\csc(x)}{a+b\sin(x)}\,dx}{a}$$
$$= -\frac{\cot(x)}{a} - \frac{b\int \csc(x)\,dx}{a^2} + \frac{b^2\int \frac{1}{a+b\sin(x)}\,dx}{a^2}$$
$$= \frac{b\tanh^{-1}(\cos(x))}{a^2} - \frac{\cot(x)}{a} + \frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{a^2}$$
$$= \frac{b\tanh^{-1}(\cos(x))}{a^2} - \frac{\cot(x)}{a} - \frac{(4b^2)\,\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b+2a\tan\left(\frac{x}{2}\right)\right)}{a^2}$$
$$= \frac{2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}} + \frac{b\tanh^{-1}(\cos(x))}{a^2} - \frac{\cot(x)}{a}$$

Mathematica [A] time = 0.25, size = 91, normalized size = 1.47

$$\frac{\csc\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\left(\frac{2b^2\sin(x)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - a\cos(x) + b\sin(x)\left(\log\left(\cos\left(\frac{x}{2}\right)\right) - \log\left(\sin\left(\frac{x}{2}\right)\right)\right)\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a + b*Sin[x]),x]
```

```
[Out] (Csc[x/2]*Sec[x/2]*(-(a*Cos[x]) + (2*b^2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 -
 b^2]]*Sin[x])/Sqrt[a^2 - b^2] + b*(Log[Cos[x/2]] - Log[Sin[x/2]])*Sin[x]))
/(2*a^2)
```

fricas [B] time = 0.56, size = 302, normalized size = 4.87

$$\left[-\frac{\sqrt{-a^2+b^2}\,b^2\log\left(\frac{(2a^2-b^2)\cos(x)^2-2ab\sin(x)-a^2-b^2+2(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2ab\sin(x)-a^2-b^2}\right)\sin(x) - (a^2b-b^3)\log\left(\frac{1}{2}\cos(x\right.}{2(a^4-a^2b^2)\sin(x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*b^2*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^
2 - b^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 -
2*a*b*sin(x) - a^2 - b^2))*sin(x) - (a^2*b - b^3)*log(1/2*cos(x) + 1/2)*sin
(x) + (a^2*b - b^3)*log(-1/2*cos(x) + 1/2)*sin(x) + 2*(a^3 - a*b^2)*cos(x))
/((a^4 - a^2*b^2)*sin(x)), -1/2*(2*sqrt(a^2 - b^2)*b^2*arctan(-(a*sin(x) +
b)/(sqrt(a^2 - b^2)*cos(x)))*sin(x) - (a^2*b - b^3)*log(1/2*cos(x) + 1/2)*s
in(x) + (a^2*b - b^3)*log(-1/2*cos(x) + 1/2)*sin(x) + 2*(a^3 - a*b^2)*cos(x
))/((a^4 - a^2*b^2)*sin(x))]
```

giac [A] time = 0.16, size = 98, normalized size = 1.58

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^2}{\sqrt{a^2-b^2}\,a^2}-\frac{b\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{a^2}+\frac{\tan\left(\frac{1}{2}x\right)}{2a}+\frac{2b\tan\left(\frac{1}{2}x\right)-a}{2a^2\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b
^2)))*b^2/(sqrt(a^2 - b^2)*a^2) - b*log(abs(tan(1/2*x)))/a^2 + 1/2*tan(1/2*
x)/a + 1/2*(2*b*tan(1/2*x) - a)/(a^2*tan(1/2*x))
```

maple [A] time = 0.10, size = 77, normalized size = 1.24

$$\frac{\tan\left(\frac{x}{2}\right)}{2a}-\frac{1}{2a\tan\left(\frac{x}{2}\right)}-\frac{b\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}+\frac{2b^2\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a+b*sin(x)),x)
```

```
[Out] 1/2/a*tan(1/2*x)-1/2/a/tan(1/2*x)-1/a^2*b*ln(tan(1/2*x))+2*b^2/a^2/(a^2-b^2
)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.01, size = 179, normalized size = 2.89

$$\frac{b^3 \ln\left(\tan\left(\frac{x}{2}\right)\right) - a^2 b \ln\left(\tan\left(\frac{x}{2}\right)\right) + b^2 \operatorname{atan}\left(\frac{-a^2 \tan\left(\frac{x}{2}\right) \sqrt{b^2-a^2}\, 1\mathrm{i} + b^2 \tan\left(\frac{x}{2}\right) \sqrt{b^2-a^2}\, 4\mathrm{i} + a\, b \sqrt{b^2-a^2}\, 2\mathrm{i}}{-a^3 - 3 \tan\left(\frac{x}{2}\right) a^2 b + 2\, a\, b^2 + 4 \tan\left(\frac{x}{2}\right) b^3}\right) \sqrt{b^2-a^2}\, 2\mathrm{i}}{a^4 - a^2 b^2} + \frac{\quad}{a^4 \mathrm{ta}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(a + b*sin(x))),x)
```

```
[Out] (b^3*log(tan(x/2)) - a^2*b*log(tan(x/2)) + b^2*atan((b^2*tan(x/2)*(b^2 - a^
2)^(1/2)*4i - a^2*tan(x/2)*(b^2 - a^2)^(1/2)*1i + a*b*(b^2 - a^2)^(1/2)*2i)
/(4*b^3*tan(x/2) + 2*a*b^2 - a^3 - 3*a^2*b*tan(x/2)))*(b^2 - a^2)^(1/2)*2i)
/(a^4 - a^2*b^2) + (a*b^2 - a^3)/(a^4*tan(x) - a^2*b^2*tan(x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(x)}{a + b\sin(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a+b*sin(x)),x)
```

```
[Out] Integral(csc(x)**2/(a + b*sin(x)), x)
```

3.183 $\int \frac{\csc^3(x)}{a+b\sin(x)}\,dx$

Optimal. Leaf size=84

$$\frac{b\cot(x)}{a^2}-\frac{(a^2+2b^2)\tanh^{-1}(\cos(x))}{2a^3}-\frac{2b^3\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3\sqrt{a^2-b^2}}-\frac{\cot(x)\csc(x)}{2a}$$

```
[Out] -1/2*(a^2+2*b^2)*arctanh(cos(x))/a^3+b*cot(x)/a^2-1/2*cot(x)*csc(x)/a-2*b^3
*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/a^3/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2802, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{2b^3\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3\sqrt{a^2-b^2}}-\frac{(a^2+2b^2)\tanh^{-1}(\cos(x))}{2a^3}+\frac{b\cot(x)}{a^2}-\frac{\cot(x)\csc(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a + b*Sin[x]),x]
```

```
[Out] (-2*b^3*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^3*Sqrt[a^2 - b^2]) - (
(a^2 + 2*b^2)*ArcTanh[Cos[x]])/(2*a^3) + (b*Cot[x])/a^2 - (Cot[x]*Csc[x])/(
2*a)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^3(x)}{a+b\sin(x)}\,dx = -\frac{\cot(x)\csc(x)}{2a} + \frac{\int \frac{\csc^2(x)\left(-2b+a\sin(x)+b\sin^2(x)\right)}{a+b\sin(x)}\,dx}{2a}$$
$$= \frac{b\cot(x)}{a^2} - \frac{\cot(x)\csc(x)}{2a} + \frac{\int \frac{\csc(x)\left(a^2+2b^2+ab\sin(x)\right)}{a+b\sin(x)}\,dx}{2a^2}$$
$$= \frac{b\cot(x)}{a^2} - \frac{\cot(x)\csc(x)}{2a} - \frac{b^3\int \frac{1}{a+b\sin(x)}\,dx}{a^3} + \frac{\left(a^2+2b^2\right)\int \csc(x)\,dx}{2a^3}$$
$$= -\frac{\left(a^2+2b^2\right)\tanh^{-1}(\cos(x))}{2a^3} + \frac{b\cot(x)}{a^2} - \frac{\cot(x)\csc(x)}{2a} - \frac{\left(2b^3\right)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan$$
$$= -\frac{\left(a^2+2b^2\right)\tanh^{-1}(\cos(x))}{2a^3} + \frac{b\cot(x)}{a^2} - \frac{\cot(x)\csc(x)}{2a} + \frac{\left(4b^3\right)\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx, x, 2$$
$$= -\frac{2b^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^3\sqrt{a^2-b^2}} - \frac{\left(a^2+2b^2\right)\tanh^{-1}(\cos(x))}{2a^3} + \frac{b\cot(x)}{a^2} - \frac{\cot(x)\csc(x)}{2a}$$

Mathematica [A] time = 0.52, size = 144, normalized size = 1.71

$$\frac{-\frac{16b^3\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - a^2\csc^2\left(\frac{x}{2}\right) + a^2\sec^2\left(\frac{x}{2}\right) + 4a^2\log\left(\sin\left(\frac{x}{2}\right)\right) - 4a^2\log\left(\cos\left(\frac{x}{2}\right)\right) - 4ab\tan\left(\frac{x}{2}\right) + 4ab\cot\left(\frac{x}{2}$$
$$8a^3$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a + b*Sin[x]),x]
```

```
[Out] ((-16*b^3*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 4*a*b
*Cot[x/2] - a^2*Csc[x/2]^2 - 4*a^2*Log[Cos[x/2]] - 8*b^2*Log[Cos[x/2]] + 4*
a^2*Log[Sin[x/2]] + 8*b^2*Log[Sin[x/2]] + a^2*Sec[x/2]^2 - 4*a*b*Tan[x/2])/
(8*a^3)
```

fricas [B] time = 0.69, size = 490, normalized size = 5.83

$$\left[\frac{4\left(a^3b-ab^3\right)\cos(x)\sin(x) + 2\left(b^3\cos(x)^2-b^3\right)\sqrt{-a^2+b^2}\,\log\left(-\frac{\left(2a^2-b^2\right)\cos(x)^2-2ab\sin(x)-a^2-b^2-2\left(a\cos(x)\sin(x)+b\cos\right.}{b^2\cos(x)^2-2ab\sin(x)-a^2-b^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^3*b - a*b^3)*cos(x)*sin(x) + 2*(b^3*cos(x)^2 - b^3)*sqrt(-a^2 +
b^2)*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 - 2*(a*cos(x)*
sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b
^2)) - 2*(a^4 - a^2*b^2)*cos(x) - (a^4 + a^2*b^2 - 2*b^4 - (a^4 + a^2*b^2 -
 2*b^4)*cos(x)^2)*log(1/2*cos(x) + 1/2) + (a^4 + a^2*b^2 - 2*b^4 - (a^4 + a
^2*b^2 - 2*b^4)*cos(x)^2)*log(-1/2*cos(x) + 1/2))/(a^5 - a^3*b^2 - (a^5 - a
^3*b^2)*cos(x)^2), 1/4*(4*(a^3*b - a*b^3)*cos(x)*sin(x) - 4*(b^3*cos(x)^2 -
 b^3)*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) - 2*
(a^4 - a^2*b^2)*cos(x) - (a^4 + a^2*b^2 - 2*b^4 - (a^4 + a^2*b^2 - 2*b^4)*c
os(x)^2)*log(1/2*cos(x) + 1/2) + (a^4 + a^2*b^2 - 2*b^4 - (a^4 + a^2*b^2 -
2*b^4)*cos(x)^2)*log(-1/2*cos(x) + 1/2))/(a^5 - a^3*b^2 - (a^5 - a^3*b^2)*c
os(x)^2)]
```

giac [A] time = 0.26, size = 141, normalized size = 1.68

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^3}{\sqrt{a^2-b^2}\,a^3}+\frac{a\tan\left(\frac{1}{2}x\right)^2-4\,b\tan\left(\frac{1}{2}x\right)}{8\,a^2}+\frac{\left(a^2+2\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{2\,a^3}-\frac{6\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 -
b^2)))*b^3/(sqrt(a^2 - b^2)*a^3) + 1/8*(a*tan(1/2*x)^2 - 4*b*tan(1/2*x))/a^
2 + 1/2*(a^2 + 2*b^2)*log(abs(tan(1/2*x)))/a^3 - 1/8*(6*a^2*tan(1/2*x)^2 +
12*b^2*tan(1/2*x)^2 - 4*a*b*tan(1/2*x) + a^2)/(a^3*tan(1/2*x)^2)
```

maple [A] time = 0.12, size = 112, normalized size = 1.33

$$\frac{\tan^2\left(\frac{x}{2}\right)}{8a}-\frac{\tan\left(\frac{x}{2}\right)b}{2a^2}-\frac{1}{8a\tan\left(\frac{x}{2}\right)^2}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)b^2}{a^3}+\frac{b}{2a^2\tan\left(\frac{x}{2}\right)}-\frac{2b^3\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{a^3\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a+b*sin(x)),x)
```

```
[Out] 1/8/a*tan(1/2*x)^2-1/2/a^2*tan(1/2*x)*b-1/8/a/tan(1/2*x)^2+1/2/a*ln(tan(1/2
*x))+1/a^3*ln(tan(1/2*x))*b^2+1/2*b/a^2/tan(1/2*x)-2*b^3/a^3/(a^2-b^2)^(1/2
)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.39, size = 531, normalized size = 6.32

$$\frac{a^4\left(\frac{\cos(x)}{2}-\frac{\ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{4}+\frac{\cos(2x)\ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{4}\right)-a^2\left(\frac{b^2\ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{4}+\frac{b^2\cos(x)}{2}-\frac{b^2\cos(2x)\ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{4}\right)+\frac{b^4\ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{2}-b^3\,\mathrm{atan}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^3*(a + b*sin(x))),x)
```

```
[Out] (a^4*(cos(x)/2 - log(sin(x/2)/cos(x/2))/4 + (cos(2*x)*log(sin(x/2)/cos(x/2)
))/4) - a^2*((b^2*log(sin(x/2)/cos(x/2)))/4 + (b^2*cos(x))/2 - (b^2*cos(2*x
)*log(sin(x/2)/cos(x/2)))/4) + (b^4*log(sin(x/2)/cos(x/2)))/2 - b^3*atan((b
^4*sin(x/2)*(b^2 - a^2)^(1/2)*8i - a^4*sin(x/2)*(b^2 - a^2)^(1/2)*1i + a*b^
3*cos(x/2)*(b^2 - a^2)^(1/2)*4i + a^3*b*cos(x/2)*(b^2 - a^2)^(1/2)*1i)/(a^5
*cos(x/2) - 8*b^5*sin(x/2) + a^3*b^2*cos(x/2) + 4*a^2*b^3*sin(x/2) - 4*a*b^
4*cos(x/2) + 2*a^4*b*sin(x/2)))*(b^2 - a^2)^(1/2)*1i - (b^4*cos(2*x)*log(si
n(x/2)/cos(x/2)))/2 + (a*b^3*sin(2*x))/2 - (a^3*b*sin(2*x))/2 + b^3*cos(2*x
)*atan((b^4*sin(x/2)*(b^2 - a^2)^(1/2)*8i - a^4*sin(x/2)*(b^2 - a^2)^(1/2)*
1i + a*b^3*cos(x/2)*(b^2 - a^2)^(1/2)*4i + a^3*b*cos(x/2)*(b^2 - a^2)^(1/2)
*1i)/(a^5*cos(x/2) - 8*b^5*sin(x/2) + a^3*b^2*cos(x/2) + 4*a^2*b^3*sin(x/2)
 - 4*a*b^4*cos(x/2) + 2*a^4*b*sin(x/2)))*(b^2 - a^2)^(1/2)*1i)/((a^5*cos(2*
x))/2 - a^5/2 + (a^3*b^2)/2 - (a^3*b^2*cos(2*x))/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(x)}{a+b\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a+b*sin(x)),x)
```

```
[Out] Integral(csc(x)**3/(a + b*sin(x)), x)
```

3.184 $\int \frac{\csc^4(x)}{a+b\sin(x)}\,dx$

Optimal. Leaf size=112

$$\frac{b\cot(x)\csc(x)}{2a^2}+\frac{b\left(a^2+2b^2\right)\tanh^{-1}(\cos(x))}{2a^4}+\frac{2b^4\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4\sqrt{a^2-b^2}}-\frac{\left(2a^2+3b^2\right)\cot(x)}{3a^3}-\frac{\cot(x)\csc^2(x)}{3a}$$

```
[Out] 1/2*b*(a^2+2*b^2)*arctanh(cos(x))/a^4-1/3*(2*a^2+3*b^2)*cot(x)/a^3+1/2*b*co
t(x)*csc(x)/a^2-1/3*cot(x)*csc(x)^2/a+2*b^4*arctan((b+a*tan(1/2*x))/(a^2-b^
2)^(1/2))/a^4/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2802, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2b^4\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4\sqrt{a^2-b^2}}-\frac{\left(2a^2+3b^2\right)\cot(x)}{3a^3}+\frac{b\left(a^2+2b^2\right)\tanh^{-1}(\cos(x))}{2a^4}+\frac{b\cot(x)\csc(x)}{2a^2}-\frac{\cot(x)\csc^2(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^4/(a + b*Sin[x]),x]
```

```
[Out] (2*b^4*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^4*Sqrt[a^2 - b^2]) + (b
*(a^2 + 2*b^2)*ArcTanh[Cos[x]])/(2*a^4) - ((2*a^2 + 3*b^2)*Cot[x])/(3*a^3)
+ (b*Cot[x]*Csc[x])/(2*a^2) - (Cot[x]*Csc[x]^2)/(3*a)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^4(x)}{a+b\sin(x)}\,dx = -\frac{\cot(x)\csc^2(x)}{3a} + \frac{\int \frac{\csc^3(x)\left(-3b+2a\sin(x)+2b\sin^2(x)\right)}{a+b\sin(x)}\,dx}{3a}$$

$$= \frac{b\cot(x)\csc(x)}{2a^2} - \frac{\cot(x)\csc^2(x)}{3a} + \frac{\int \frac{\csc^2(x)\left(2\left(2a^2+3b^2\right)+ab\sin(x)-3b^2\sin^2(x)\right)}{a+b\sin(x)}\,dx}{6a^2}$$

$$= -\frac{\left(2a^2+3b^2\right)\cot(x)}{3a^3} + \frac{b\cot(x)\csc(x)}{2a^2} - \frac{\cot(x)\csc^2(x)}{3a} + \frac{\int \frac{\csc(x)\left(-3b\left(a^2+2b^2\right)-3ab^2\sin(x)\right)}{a+b\sin(x)}\,dx}{6a^3}$$

$$= -\frac{\left(2a^2+3b^2\right)\cot(x)}{3a^3} + \frac{b\cot(x)\csc(x)}{2a^2} - \frac{\cot(x)\csc^2(x)}{3a} + \frac{b^4\int \frac{1}{a+b\sin(x)}\,dx}{a^4} - \frac{\left(b\left(a^2+2b^2\right)\right)}{2a^4}$$

$$= \frac{b\left(a^2+2b^2\right)\tanh^{-1}(\cos(x))}{2a^4} - \frac{\left(2a^2+3b^2\right)\cot(x)}{3a^3} + \frac{b\cot(x)\csc(x)}{2a^2} - \frac{\cot(x)\csc^2(x)}{3a} + \frac{\left(2b^4\right.}{}$$

$$= \frac{b\left(a^2+2b^2\right)\tanh^{-1}(\cos(x))}{2a^4} - \frac{\left(2a^2+3b^2\right)\cot(x)}{3a^3} + \frac{b\cot(x)\csc(x)}{2a^2} - \frac{\cot(x)\csc^2(x)}{3a} - \frac{\left(4b^4\right.}{}$$

$$= \frac{2b^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^4\sqrt{a^2-b^2}} + \frac{b\left(a^2+2b^2\right)\tanh^{-1}(\cos(x))}{2a^4} - \frac{\left(2a^2+3b^2\right)\cot(x)}{3a^3} + \frac{b\cot(x)\csc(x)}{2a^2}$$

Mathematica [A] time = 1.70, size = 125, normalized size = 1.12

$$\frac{a\left(2a^2+3b^2\right)\cos(3x)\csc^3(x) - 3a\cot(x)\csc(x)\left(\left(2a^2+b^2\right)\csc(x)-2ab\right) + 6b\left(a^2+2b^2\right)\left(\log\left(\cos\left(\frac{x}{2}\right)\right) - \log\left(\mathrm{si}\right.\right.}{12a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^4/(a + b*Sin[x]),x]
```

```
[Out] ((24*b^4*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + a*(2*a
^2 + 3*b^2)*Cos[3*x]*Csc[x]^3 - 3*a*Cot[x]*Csc[x]*(-2*a*b + (2*a^2 + b^2)*C
sc[x]) + 6*b*(a^2 + 2*b^2)*(Log[Cos[x/2]] - Log[Sin[x/2]]))/(12*a^4)
```

fricas [B] time = 0.71, size = 577, normalized size = 5.15

$$\left[\frac{4\left(2a^5+a^3b^2-3ab^4\right)\cos(x)^3+6\left(b^4\cos(x)^2-b^4\right)\sqrt{-a^2+b^2}\log\left(\frac{\left(2a^2-b^2\right)\cos(x)^2-2ab\sin(x)-a^2-b^2+2(a\cos(x)\sin(x)+}{b^2\cos(x)^2-2ab\sin(x)-a^2-b^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [1/12*(4*(2*a^5 + a^3*b^2 - 3*a*b^4)*cos(x)^3 + 6*(b^4*cos(x)^2 - b^4)*sqrt
(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 + 2*(a*
cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) -
a^2 - b^2))*sin(x) + 6*(a^4*b - a^2*b^3)*cos(x)*sin(x) + 3*(a^4*b + a^2*b^3
 - 2*b^5 - (a^4*b + a^2*b^3 - 2*b^5)*cos(x)^2)*log(1/2*cos(x) + 1/2)*sin(x)
 - 3*(a^4*b + a^2*b^3 - 2*b^5 - (a^4*b + a^2*b^3 - 2*b^5)*cos(x)^2)*log(-1/
2*cos(x) + 1/2)*sin(x) - 12*(a^5 - a*b^4)*cos(x))/((a^6 - a^4*b^2 - (a^6 -
a^4*b^2)*cos(x)^2)*sin(x)), 1/12*(4*(2*a^5 + a^3*b^2 - 3*a*b^4)*cos(x)^3 +
12*(b^4*cos(x)^2 - b^4)*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 -
b^2)*cos(x)))*sin(x) + 6*(a^4*b - a^2*b^3)*cos(x)*sin(x) + 3*(a^4*b + a^2*b
^3 - 2*b^5 - (a^4*b + a^2*b^3 - 2*b^5)*cos(x)^2)*log(1/2*cos(x) + 1/2)*sin(
x) - 3*(a^4*b + a^2*b^3 - 2*b^5 - (a^4*b + a^2*b^3 - 2*b^5)*cos(x)^2)*log(-
1/2*cos(x) + 1/2)*sin(x) - 12*(a^5 - a*b^4)*cos(x))/((a^6 - a^4*b^2 - (a^6
- a^4*b^2)*cos(x)^2)*sin(x))]
```

giac [A] time = 0.24, size = 194, normalized size = 1.73

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^4}{\sqrt{a^2-b^2}\,a^4}+\frac{a^2\tan\left(\frac{1}{2}x\right)^3-3ab\tan\left(\frac{1}{2}x\right)^2+9a^2\tan\left(\frac{1}{2}x\right)+12b^2\tan\left(\frac{1}{2}x\right.}{24a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b
^2)))*b^4/(sqrt(a^2 - b^2)*a^4) + 1/24*(a^2*tan(1/2*x)^3 - 3*a*b*tan(1/2*x)
^2 + 9*a^2*tan(1/2*x) + 12*b^2*tan(1/2*x))/a^3 - 1/2*(a^2*b + 2*b^3)*log(ab
s(tan(1/2*x)))/a^4 + 1/24*(22*a^2*b*tan(1/2*x)^3 + 44*b^3*tan(1/2*x)^3 - 9*
a^3*tan(1/2*x)^2 - 12*a*b^2*tan(1/2*x)^2 + 3*a^2*b*tan(1/2*x) - a^3)/(a^4*t
an(1/2*x)^3)
```

maple [A] time = 0.12, size = 162, normalized size = 1.45

$$\frac{\tan^3\left(\frac{x}{2}\right)}{24a}-\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)b}{8a^2}+\frac{3\tan\left(\frac{x}{2}\right)}{8a}+\frac{b^2\tan\left(\frac{x}{2}\right)}{2a^3}-\frac{1}{24a\tan\left(\frac{x}{2}\right)^3}-\frac{3}{8a\tan\left(\frac{x}{2}\right)}-\frac{b^2}{2a^3\tan\left(\frac{x}{2}\right)}+\frac{b}{8a^2\tan\left(\frac{x}{2}\right)^2}-\frac{b\ln\left(\tan\left(\frac{x}{2}\right.\right.}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^4/(a+b*sin(x)),x)
```

```
[Out] 1/24/a*tan(1/2*x)^3-1/8/a^2*tan(1/2*x)^2*b+3/8/a*tan(1/2*x)+1/2/a^3*b^2*tan
(1/2*x)-1/24/a/tan(1/2*x)^3-3/8/a/tan(1/2*x)-1/2/a^3/tan(1/2*x)*b^2+1/8/a^2
*b/tan(1/2*x)^2-1/2/a^2*b*ln(tan(1/2*x))-1/a^4*b^3*ln(tan(1/2*x))+2/a^4*b^4
/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.52, size = 586, normalized size = 5.23

$$\frac{a^5\left(\frac{\cos(3x)}{12}-\frac{\cos(x)}{4}\right)-a\left(\frac{b^4\cos(3x)}{8}-\frac{b^4\cos(x)}{8}\right)+a^4\left(\frac{b\sin(2x)}{8}-\frac{3b\ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)\sin(x)}{16}+\frac{b\sin(3x)\ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{16}\right)-a^2\left(\frac{b^3\sin(2\right.}{8}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^4*(a + b*sin(x))),x)
```

```
[Out] (a^5*(cos(3*x)/12 - cos(x)/4) - a*((b^4*cos(3*x))/8 - (b^4*cos(x))/8) + a^4
*((b*sin(2*x))/8 - (3*b*log(sin(x/2)/cos(x/2))*sin(x))/16 + (b*sin(3*x)*log
(sin(x/2)/cos(x/2)))/16) - a^2*((b^3*sin(2*x))/8 - (b^3*sin(3*x)*log(sin(x/
2)/cos(x/2)))/16 + (3*b^3*log(sin(x/2)/cos(x/2))*sin(x))/16) + a^3*((b^2*co
s(3*x))/24 + (b^2*cos(x))/8) - (b^5*sin(3*x)*log(sin(x/2)/cos(x/2)))/8 + (3
```

```
*b^5*log(sin(x/2)/cos(x/2))*sin(x))/8 + (b^4*atan((b^4*sin(x/2)*(b^2 - a^2)
^(1/2)*8i - a^4*sin(x/2)*(b^2 - a^2)^(1/2)*1i + a*b^3*cos(x/2)*(b^2 - a^2)^
(1/2)*4i + a^3*b*cos(x/2)*(b^2 - a^2)^(1/2)*1i)/(a^5*cos(x/2) - 8*b^5*sin(x
/2) + a^3*b^2*cos(x/2) + 4*a^2*b^3*sin(x/2) - 4*a*b^4*cos(x/2) + 2*a^4*b*si
n(x/2)))*sin(3*x)*(b^2 - a^2)^(1/2)*1i)/4 - (b^4*atan((b^4*sin(x/2)*(b^2 -
a^2)^(1/2)*8i - a^4*sin(x/2)*(b^2 - a^2)^(1/2)*1i + a*b^3*cos(x/2)*(b^2 - a
^2)^(1/2)*4i + a^3*b*cos(x/2)*(b^2 - a^2)^(1/2)*1i)/(a^5*cos(x/2) - 8*b^5*s
in(x/2) + a^3*b^2*cos(x/2) + 4*a^2*b^3*sin(x/2) - 4*a*b^4*cos(x/2) + 2*a^4*
b*sin(x/2)))*sin(x)*(b^2 - a^2)^(1/2)*3i)/4)/((3*a^6*sin(x))/8 - (a^6*sin(3
*x))/8 + (a^4*b^2*sin(3*x))/8 - (3*a^4*b^2*sin(x))/8)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4(x)}{a + b\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**4/(a+b*sin(x)),x)
```

```
[Out] Integral(csc(x)**4/(a + b*sin(x)), x)
```

3.185 $\int \frac{\sin^4(x)}{(a+b\sin(x))^2}\,dx$

Optimal. Leaf size=169

$$\frac{a^2\sin^2(x)\cos(x)}{b\left(a^2-b^2\right)(a+b\sin(x))}-\frac{\left(3a^2-b^2\right)\sin(x)\cos(x)}{2b^2\left(a^2-b^2\right)}+\frac{x\left(6a^2+b^2\right)}{2b^4}+\frac{a\left(3a^2-2b^2\right)\cos(x)}{b^3\left(a^2-b^2\right)}-\frac{2a^3\left(3a^2-4b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4\left(a^2-b^2\right)^{3/2}}$$

```
[Out] 1/2*(6*a^2+b^2)*x/b^4-2*a^3*(3*a^2-4*b^2)*arctan((b+a*tan(1/2*x))/(a^2-b^2)
^(1/2))/b^4/(a^2-b^2)^(3/2)+a*(3*a^2-2*b^2)*cos(x)/b^3/(a^2-b^2)-1/2*(3*a^2
-b^2)*cos(x)*sin(x)/b^2/(a^2-b^2)+a^2*cos(x)*sin(x)^2/b/(a^2-b^2)/(a+b*sin(
x))
```

Rubi [A] time = 0.38, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2792, 3049, 3023, 2735, 2660, 618, 204}

$$\frac{x\left(6a^2+b^2\right)}{2b^4}+\frac{a\left(3a^2-2b^2\right)\cos(x)}{b^3\left(a^2-b^2\right)}-\frac{2a^3\left(3a^2-4b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4\left(a^2-b^2\right)^{3/2}}+\frac{a^2\sin^2(x)\cos(x)}{b\left(a^2-b^2\right)(a+b\sin(x))}-\frac{\left(3a^2-b^2\right)\sin(x)\cos(x)}{2b^2\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4/(a + b*Sin[x])^2,x]
```

```
[Out] ((6*a^2 + b^2)*x)/(2*b^4) - (2*a^3*(3*a^2 - 4*b^2)*ArcTan[(b + a*Tan[x/2])/
Sqrt[a^2 - b^2]])/(b^4*(a^2 - b^2)^(3/2)) + (a*(3*a^2 - 2*b^2)*Cos[x])/(b^3
*(a^2 - b^2)) - ((3*a^2 - b^2)*Cos[x]*Sin[x])/(2*b^2*(a^2 - b^2)) + (a^2*Co
s[x]*Sin[x]^2)/(b*(a^2 - b^2)*(a + b*Sin[x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
```

```
0] && !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\sin^4(x)}{(a+b\sin(x))^2}\,dx = \frac{a^2\cos(x)\sin^2(x)}{b\left(a^2-b^2\right)(a+b\sin(x))} - \frac{\int \frac{\sin(x)\left(2a^2-ab\sin(x)-\left(3a^2-b^2\right)\sin^2(x)\right)}{a+b\sin(x)}\,dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{\left(3a^2-b^2\right)\cos(x)\sin(x)}{2b^2\left(a^2-b^2\right)} + \frac{a^2\cos(x)\sin^2(x)}{b\left(a^2-b^2\right)(a+b\sin(x))} - \frac{\int \frac{-a\left(3a^2-b^2\right)+b\left(a^2+b^2\right)\sin(x)+2a\left(3a^2-2b\right.}{a+b\sin(x)}}{2b^2\left(a^2-b^2\right)}$$

$$= \frac{a\left(3a^2-2b^2\right)\cos(x)}{b^3\left(a^2-b^2\right)} - \frac{\left(3a^2-b^2\right)\cos(x)\sin(x)}{2b^2\left(a^2-b^2\right)} + \frac{a^2\cos(x)\sin^2(x)}{b\left(a^2-b^2\right)(a+b\sin(x))} - \frac{\int \frac{-ab\left(3a^2-b^2\right.}{}}{2}$$

$$= \frac{\left(6a^2+b^2\right)x}{2b^4} + \frac{a\left(3a^2-2b^2\right)\cos(x)}{b^3\left(a^2-b^2\right)} - \frac{\left(3a^2-b^2\right)\cos(x)\sin(x)}{2b^2\left(a^2-b^2\right)} + \frac{a^2\cos(x)\sin^2(x)}{b\left(a^2-b^2\right)(a+b\sin(x)}$$

$$= \frac{\left(6a^2+b^2\right)x}{2b^4} + \frac{a\left(3a^2-2b^2\right)\cos(x)}{b^3\left(a^2-b^2\right)} - \frac{\left(3a^2-b^2\right)\cos(x)\sin(x)}{2b^2\left(a^2-b^2\right)} + \frac{a^2\cos(x)\sin^2(x)}{b\left(a^2-b^2\right)(a+b\sin(x)}$$

$$= \frac{\left(6a^2+b^2\right)x}{2b^4} + \frac{a\left(3a^2-2b^2\right)\cos(x)}{b^3\left(a^2-b^2\right)} - \frac{\left(3a^2-b^2\right)\cos(x)\sin(x)}{2b^2\left(a^2-b^2\right)} + \frac{a^2\cos(x)\sin^2(x)}{b\left(a^2-b^2\right)(a+b\sin(x)}$$

$$= \frac{\left(6a^2+b^2\right)x}{2b^4} - \frac{2a^3\left(3a^2-4b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^4\left(a^2-b^2\right)^{3/2}} + \frac{a\left(3a^2-2b^2\right)\cos(x)}{b^3\left(a^2-b^2\right)} - \frac{\left(3a^2-b^2\right)\cos}{2b^2\left(a^2-\right.}$$

Mathematica [A] time = 0.60, size = 115, normalized size = 0.68

$$\frac{4ab\cos(x)\left(\frac{a^3}{(a-b)(a+b)(a+b\sin(x))}+2\right)+12a^2x-\frac{8a^3\left(3a^2-4b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+2b^2x-b^2\sin(2x)}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4/(a + b*Sin[x])^2,x]
```

```
[Out] (12*a^2*x + 2*b^2*x - (8*a^3*(3*a^2 - 4*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a
^2 - b^2]])/(a^2 - b^2)^(3/2) + 4*a*b*Cos[x]*(2 + a^3/((a - b)*(a + b)*(a +
 b*Sin[x]))) - b^2*Sin[2*x])/(4*b^4)
```

fricas [A] time = 0.59, size = 580, normalized size = 3.43

$$\left[\frac{\left(a^4b^3-2\,a^2b^5+b^7\right)\cos(x)^3-\left(3\,a^6-4\,a^4b^2+\left(3\,a^5b-4\,a^3b^3\right)\sin(x)\right)\sqrt{-a^2+b^2}\,\log\left(-\frac{\left(2\,a^2-b^2\right)\cos(x)^2-2\,ab\sin(x)\cdots}{b^2\cos(\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a^4*b^3 - 2*a^2*b^5 + b^7)*cos(x)^3 - (3*a^6 - 4*a^4*b^2 + (3*a^5*b
- 4*a^3*b^3)*sin(x))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*
sin(x) - a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*
cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) + (6*a^7 - 11*a^5*b^2 + 4*a^3*b^4 + a
*b^6)*x + (6*a^6*b - 11*a^4*b^3 + 6*a^2*b^5 - b^7)*cos(x) + ((6*a^6*b - 11*
a^4*b^3 + 4*a^2*b^5 + b^7)*x + 3*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(x))*sin(
x))/(a^5*b^4 - 2*a^3*b^6 + a*b^8 + (a^4*b^5 - 2*a^2*b^7 + b^9)*sin(x)), 1/2
*((a^4*b^3 - 2*a^2*b^5 + b^7)*cos(x)^3 + 2*(3*a^6 - 4*a^4*b^2 + (3*a^5*b -
4*a^3*b^3)*sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*
cos(x))) + (6*a^7 - 11*a^5*b^2 + 4*a^3*b^4 + a*b^6)*x + (6*a^6*b - 11*a^4*b
^3 + 6*a^2*b^5 - b^7)*cos(x) + ((6*a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + b^7)*x
+ 3*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(x))*sin(x))/(a^5*b^4 - 2*a^3*b^6 + a*
b^8 + (a^4*b^5 - 2*a^2*b^7 + b^9)*sin(x))]
```

giac [A] time = 0.17, size = 184, normalized size = 1.09

$$-\frac{2\left(3\,a^5-4\,a^3b^2\right)\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^4-b^6\right)\sqrt{a^2-b^2}}+\frac{2\left(a^3b\tan\left(\frac{1}{2}\,x\right)+a^4\right)}{\left(a^2b^3-b^5\right)\left(a\tan\left(\frac{1}{2}\,x\right)^2+2\,b\tan\left(\frac{1}{2}\,x\right)+a\right)}+\frac{(6\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*(3*a^5 - 4*a^3*b^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2
*x) + b)/sqrt(a^2 - b^2)))/((a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + 2*(a^3*b*tan
(1/2*x) + a^4)/((a^2*b^3 - b^5)*(a*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)) + 1/
2*(6*a^2 + b^2)*x/b^4 + (b*tan(1/2*x)^3 + 4*a*tan(1/2*x)^2 - b*tan(1/2*x) +
 4*a)/((tan(1/2*x)^2 + 1)^2*b^3)
```

maple [A] time = 0.11, size = 266, normalized size = 1.57

$$\frac{\tan^3\left(\frac{x}{2}\right)}{b^2\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}+\frac{4\left(\tan^2\left(\frac{x}{2}\right)\right)a}{b^3\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}-\frac{\tan\left(\frac{x}{2}\right)}{b^2\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}+\frac{4a}{b^3\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}+\frac{6\arctan\left(\tan\left(\frac{x}{2}\right)\right)a^2}{b^4}+\frac{}{b^2\left(\left(\tan^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a+b*sin(x))^2,x)
```

```
[Out] 1/b^2/(tan(1/2*x)^2+1)^2*tan(1/2*x)^3+4/b^3/(tan(1/2*x)^2+1)^2*tan(1/2*x)^2
*a-1/b^2/(tan(1/2*x)^2+1)^2*tan(1/2*x)+4/b^3/(tan(1/2*x)^2+1)^2*a+6/b^4*arc
tan(tan(1/2*x))*a^2+2*a^3/b^2/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)/(a^2-b^2)*t
an(1/2*x)+2*a^4/b^3/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)/(a^2-b^2)-6*a^5/b^4/(
a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))+8*a^3/b^2/(
a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))+1/2*x/b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 11.88, size = 4368, normalized size = 25.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a + b*sin(x))^2,x)
```

```
[Out] (atan((((a^2*6i + b^2*1i)*((8*(a^2*b^11 + 10*a^4*b^9 + 13*a^6*b^7 - 60*a^8*
b^5 + 36*a^10*b^3))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(2*a*b^13 +
 19*a^3*b^11 + 16*a^5*b^9 - 197*a^7*b^7 + 228*a^9*b^5 - 72*a^11*b^3))/(b^13
 - 2*a^2*b^11 + a^4*b^9) - ((a^2*6i + b^2*1i)*((8*(2*a*b^14 + 6*a^3*b^12 -
14*a^5*b^10 + 6*a^7*b^8))/(b^12 - 2*a^2*b^10 + a^4*b^8) - (((8*(4*a^2*b^15
- 8*a^4*b^13 + 4*a^6*b^11))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(12
*a*b^17 - 32*a^3*b^15 + 28*a^5*b^13 - 8*a^7*b^11))/(b^13 - 2*a^2*b^11 + a^4
*b^9))*(a^2*6i + b^2*1i))/(2*b^4) + (8*tan(x/2)*(32*a^4*b^12 - 56*a^6*b^10
```

```
+ 24*a^8*b^8))/(b^13 - 2*a^2*b^11 + a^4*b^9)))/(2*b^4))*1i)/(2*b^4) + ((a^2
*6i + b^2*1i)*((8*(a^2*b^11 + 10*a^4*b^9 + 13*a^6*b^7 - 60*a^8*b^5 + 36*a^1
0*b^3))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(2*a*b^13 + 19*a^3*b^11
 + 16*a^5*b^9 - 197*a^7*b^7 + 228*a^9*b^5 - 72*a^11*b^3))/(b^13 - 2*a^2*b^1
1 + a^4*b^9) + ((a^2*6i + b^2*1i)*((8*(2*a*b^14 + 6*a^3*b^12 - 14*a^5*b^10
+ 6*a^7*b^8))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (((8*(4*a^2*b^15 - 8*a^4*b^13
 + 4*a^6*b^11))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(12*a*b^17 - 32
*a^3*b^15 + 28*a^5*b^13 - 8*a^7*b^11))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(a^2*
6i + b^2*1i))/(2*b^4) + (8*tan(x/2)*(32*a^4*b^12 - 56*a^6*b^10 + 24*a^8*b^8
))/(b^13 - 2*a^2*b^11 + a^4*b^9)))/(2*b^4))*1i)/(2*b^4))/((16*(54*a^11 + 4*
a^5*b^6 + 9*a^7*b^4 - 81*a^9*b^2))/(b^12 - 2*a^2*b^10 + a^4*b^8) - ((a^2*6i
 + b^2*1i)*((8*(a^2*b^11 + 10*a^4*b^9 + 13*a^6*b^7 - 60*a^8*b^5 + 36*a^10*b
^3))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(2*a*b^13 + 19*a^3*b^11 +
16*a^5*b^9 - 197*a^7*b^7 + 228*a^9*b^5 - 72*a^11*b^3))/(b^13 - 2*a^2*b^11 +
 a^4*b^9) - ((a^2*6i + b^2*1i)*((8*(2*a*b^14 + 6*a^3*b^12 - 14*a^5*b^10 + 6
*a^7*b^8))/(b^12 - 2*a^2*b^10 + a^4*b^8) - (((8*(4*a^2*b^15 - 8*a^4*b^13 +
4*a^6*b^11))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(12*a*b^17 - 32*a^
3*b^15 + 28*a^5*b^13 - 8*a^7*b^11))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(a^2*6i
+ b^2*1i))/(2*b^4) + (8*tan(x/2)*(32*a^4*b^12 - 56*a^6*b^10 + 24*a^8*b^8))/
(b^13 - 2*a^2*b^11 + a^4*b^9)))/(2*b^4)))/(2*b^4) + ((a^2*6i + b^2*1i)*((8*
(a^2*b^11 + 10*a^4*b^9 + 13*a^6*b^7 - 60*a^8*b^5 + 36*a^10*b^3))/(b^12 - 2*
a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(2*a*b^13 + 19*a^3*b^11 + 16*a^5*b^9 - 19
7*a^7*b^7 + 228*a^9*b^5 - 72*a^11*b^3))/(b^13 - 2*a^2*b^11 + a^4*b^9) + ((a
^2*6i + b^2*1i)*((8*(2*a*b^14 + 6*a^3*b^12 - 14*a^5*b^10 + 6*a^7*b^8))/(b^1
2 - 2*a^2*b^10 + a^4*b^8) + (((8*(4*a^2*b^15 - 8*a^4*b^13 + 4*a^6*b^11))/(b
^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(12*a*b^17 - 32*a^3*b^15 + 28*a^5
*b^13 - 8*a^7*b^11))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(a^2*6i + b^2*1i))/(2*b
^4) + (8*tan(x/2)*(32*a^4*b^12 - 56*a^6*b^10 + 24*a^8*b^8))/(b^13 - 2*a^2*b
^11 + a^4*b^9)))/(2*b^4)))/(2*b^4) + (16*tan(x/2)*(216*a^12 + 8*a^4*b^8 + 8
2*a^6*b^6 + 126*a^8*b^4 - 432*a^10*b^2))/(b^13 - 2*a^2*b^11 + a^4*b^9)))*(a
^2*6i + b^2*1i)*1i)/b^4 - ((tan(x/2)*(7*a*b^2 - 9*a^3))/(b^2*(a^2 - b^2)) -
 (2*(3*a^4 - 2*a^2*b^2))/(b^3*(a^2 - b^2)) + (4*tan(x/2)^3*(2*a*b^2 - 3*a^3
))/(b^2*(a^2 - b^2)) + (tan(x/2)^5*(a*b^2 - 3*a^3))/(b^2*(a^2 - b^2)) + (2*
tan(x/2)^4*(b^4 - 3*a^4 + a^2*b^2))/(b^3*(a^2 - b^2)) - (2*tan(x/2)^2*(6*a^
4 + b^4 - 5*a^2*b^2))/(b^3*(a^2 - b^2)))/(a + 2*b*tan(x/2) + 3*a*tan(x/2)^2
 + 3*a*tan(x/2)^4 + a*tan(x/2)^6 + 4*b*tan(x/2)^3 + 2*b*tan(x/2)^5) + (a^3*
atan(((a^3*(3*a^2 - 4*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(a^2*b^11 + 10*
a^4*b^9 + 13*a^6*b^7 - 60*a^8*b^5 + 36*a^10*b^3))/(b^12 - 2*a^2*b^10 + a^4*
b^8) + (8*tan(x/2)*(2*a*b^13 + 19*a^3*b^11 + 16*a^5*b^9 - 197*a^7*b^7 + 228
*a^9*b^5 - 72*a^11*b^3))/(b^13 - 2*a^2*b^11 + a^4*b^9) + (a^3*(3*a^2 - 4*b^
2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(2*a*b^14 + 6*a^3*b^12 - 14*a^5*b^10 +
6*a^7*b^8))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(32*a^4*b^12 - 56*a
^6*b^10 + 24*a^8*b^8))/(b^13 - 2*a^2*b^11 + a^4*b^9) + (a^3*((8*(4*a^2*b^15
 - 8*a^4*b^13 + 4*a^6*b^11))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(1
2*a*b^17 - 32*a^3*b^15 + 28*a^5*b^13 - 8*a^7*b^11))/(b^13 - 2*a^2*b^11 + a^
```

```
4*b^9))*(3*a^2 - 4*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(b^10 - 3*a^2*b^8 + 3
*a^4*b^6 - a^6*b^4)))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))*1i)/(b^10 -
 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4) + (a^3*(3*a^2 - 4*b^2)*(-(a + b)^3*(a - b
)^3)^(1/2)*((8*(a^2*b^11 + 10*a^4*b^9 + 13*a^6*b^7 - 60*a^8*b^5 + 36*a^10*b
^3))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(2*a*b^13 + 19*a^3*b^11 +
16*a^5*b^9 - 197*a^7*b^7 + 228*a^9*b^5 - 72*a^11*b^3))/(b^13 - 2*a^2*b^11 +
 a^4*b^9) - (a^3*(3*a^2 - 4*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(2*a*b^14
 + 6*a^3*b^12 - 14*a^5*b^10 + 6*a^7*b^8))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (
8*tan(x/2)*(32*a^4*b^12 - 56*a^6*b^10 + 24*a^8*b^8))/(b^13 - 2*a^2*b^11 + a
^4*b^9) - (a^3*((8*(4*a^2*b^15 - 8*a^4*b^13 + 4*a^6*b^11))/(b^12 - 2*a^2*b^
10 + a^4*b^8) + (8*tan(x/2)*(12*a*b^17 - 32*a^3*b^15 + 28*a^5*b^13 - 8*a^7*
b^11))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(3*a^2 - 4*b^2)*(-(a + b)^3*(a - b)^3
)^(1/2))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))/(b^10 - 3*a^2*b^8 + 3*a
^4*b^6 - a^6*b^4))*1i)/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))/((16*(54*a
^11 + 4*a^5*b^6 + 9*a^7*b^4 - 81*a^9*b^2))/(b^12 - 2*a^2*b^10 + a^4*b^8) +
(16*tan(x/2)*(216*a^12 + 8*a^4*b^8 + 82*a^6*b^6 + 126*a^8*b^4 - 432*a^10*b^
2))/(b^13 - 2*a^2*b^11 + a^4*b^9) + (a^3*(3*a^2 - 4*b^2)*(-(a + b)^3*(a - b
)^3)^(1/2)*((8*(a^2*b^11 + 10*a^4*b^9 + 13*a^6*b^7 - 60*a^8*b^5 + 36*a^10*b
^3))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(2*a*b^13 + 19*a^3*b^11 +
16*a^5*b^9 - 197*a^7*b^7 + 228*a^9*b^5 - 72*a^11*b^3))/(b^13 - 2*a^2*b^11 +
 a^4*b^9) + (a^3*(3*a^2 - 4*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(2*a*b^14
 + 6*a^3*b^12 - 14*a^5*b^10 + 6*a^7*b^8))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (
8*tan(x/2)*(32*a^4*b^12 - 56*a^6*b^10 + 24*a^8*b^8))/(b^13 - 2*a^2*b^11 + a
^4*b^9) + (a^3*((8*(4*a^2*b^15 - 8*a^4*b^13 + 4*a^6*b^11))/(b^12 - 2*a^2*b^
10 + a^4*b^8) + (8*tan(x/2)*(12*a*b^17 - 32*a^3*b^15 + 28*a^5*b^13 - 8*a^7*
b^11))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(3*a^2 - 4*b^2)*(-(a + b)^3*(a - b)^3
)^(1/2))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))/(b^10 - 3*a^2*b^8 + 3*a
^4*b^6 - a^6*b^4)))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4) - (a^3*(3*a^2
- 4*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(a^2*b^11 + 10*a^4*b^9 + 13*a^6*b
^7 - 60*a^8*b^5 + 36*a^10*b^3))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)
*(2*a*b^13 + 19*a^3*b^11 + 16*a^5*b^9 - 197*a^7*b^7 + 228*a^9*b^5 - 72*a^11
*b^3))/(b^13 - 2*a^2*b^11 + a^4*b^9) - (a^3*(3*a^2 - 4*b^2)*(-(a + b)^3*(a
- b)^3)^(1/2)*((8*(2*a*b^14 + 6*a^3*b^12 - 14*a^5*b^10 + 6*a^7*b^8))/(b^12
- 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(32*a^4*b^12 - 56*a^6*b^10 + 24*a^8*b
^8))/(b^13 - 2*a^2*b^11 + a^4*b^9) - (a^3*((8*(4*a^2*b^15 - 8*a^4*b^13 + 4*
a^6*b^11))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(12*a*b^17 - 32*a^3*
b^15 + 28*a^5*b^13 - 8*a^7*b^11))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(3*a^2 - 4
*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4
)))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))/(b^10 - 3*a^2*b^8 + 3*a^4*b^
6 - a^6*b^4)))*(3*a^2 - 4*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*2i)/(b^10 - 3*a
^2*b^8 + 3*a^4*b^6 - a^6*b^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4/(a+b*sin(x))**2,x)
```

```
[Out] Timed out
```

3.186 $\int \frac{\sin^3(x)}{(a+b\sin(x))^2}\,dx$

Optimal. Leaf size=124

$$-\frac{(2a^2-b^2)\cos(x)}{b^2(a^2-b^2)}+\frac{a^2\sin(x)\cos(x)}{b(a^2-b^2)(a+b\sin(x))}+\frac{2a^2(2a^2-3b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3(a^2-b^2)^{3/2}}-\frac{2ax}{b^3}$$

```
[Out] -2*a*x/b^3+2*a^2*(2*a^2-3*b^2)*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/b^3
/(a^2-b^2)^(3/2)-(2*a^2-b^2)*cos(x)/b^2/(a^2-b^2)+a^2*cos(x)*sin(x)/b/(a^2-
b^2)/(a+b*sin(x))
```

Rubi [A] time = 0.22, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2792, 3023, 2735, 2660, 618, 204}

$$-\frac{(2a^2-b^2)\cos(x)}{b^2(a^2-b^2)}+\frac{2a^2(2a^2-3b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3(a^2-b^2)^{3/2}}+\frac{a^2\sin(x)\cos(x)}{b(a^2-b^2)(a+b\sin(x))}-\frac{2ax}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a + b*Sin[x])^2,x]
```

```
[Out] (-2*a*x)/b^3 + (2*a^2*(2*a^2 - 3*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^
2]])/(b^3*(a^2 - b^2)^(3/2)) - ((2*a^2 - b^2)*Cos[x])/(b^2*(a^2 - b^2)) + (
a^2*Cos[x]*Sin[x])/(b*(a^2 - b^2)*(a + b*Sin[x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^3(x)}{(a+b\sin(x))^2}\,dx = \frac{a^2\cos(x)\sin(x)}{b\left(a^2-b^2\right)(a+b\sin(x))} - \frac{\int \frac{a^2-ab\sin(x)-\left(2a^2-b^2\right)\sin^2(x)}{a+b\sin(x)}\,dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{\left(2a^2-b^2\right)\cos(x)}{b^2\left(a^2-b^2\right)} + \frac{a^2\cos(x)\sin(x)}{b\left(a^2-b^2\right)(a+b\sin(x))} - \frac{\int \frac{a^2b+2a\left(a^2-b^2\right)\sin(x)}{a+b\sin(x)}\,dx}{b^2\left(a^2-b^2\right)}$$

$$= -\frac{2ax}{b^3} - \frac{\left(2a^2-b^2\right)\cos(x)}{b^2\left(a^2-b^2\right)} + \frac{a^2\cos(x)\sin(x)}{b\left(a^2-b^2\right)(a+b\sin(x))} + \frac{\left(a^2\left(2a^2-3b^2\right)\right)\int \frac{1}{a+b\sin(x)}\,dx}{b^3\left(a^2-b^2\right)}$$

$$= -\frac{2ax}{b^3} - \frac{\left(2a^2-b^2\right)\cos(x)}{b^2\left(a^2-b^2\right)} + \frac{a^2\cos(x)\sin(x)}{b\left(a^2-b^2\right)(a+b\sin(x))} + \frac{\left(2a^2\left(2a^2-3b^2\right)\right)\text{Subst}\left(\int \frac{1}{a+2bx\cdots}\right.}{b^3\left(a^2-b^2\right)}$$

$$= -\frac{2ax}{b^3} - \frac{\left(2a^2-b^2\right)\cos(x)}{b^2\left(a^2-b^2\right)} + \frac{a^2\cos(x)\sin(x)}{b\left(a^2-b^2\right)(a+b\sin(x))} - \frac{\left(4a^2\left(2a^2-3b^2\right)\right)\text{Subst}\left(\int \frac{\cdots}{-4\left(a^2-\cdots\right.}\right.}{b^3\left(a^2-\cdots\right.}$$

$$= -\frac{2ax}{b^3} + \frac{2a^2\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^3\left(a^2-b^2\right)^{3/2}} - \frac{\left(2a^2-b^2\right)\cos(x)}{b^2\left(a^2-b^2\right)} + \frac{a^2\cos(x)\sin(x)}{b\left(a^2-b^2\right)(a+b\sin(x)\cdots}$$

Mathematica [A] time = 0.43, size = 94, normalized size = 0.76

$$\frac{b\cos(x)\left(-\frac{a^3}{(a-b)(a+b)(a+b\sin(x))}-1\right)+\frac{2a^2\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}-2ax}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a + b*Sin[x])^2,x]
```

```
[Out] (-2*a*x + (2*a^2*(2*a^2 - 3*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/
(a^2 - b^2)^(3/2) + b*Cos[x]*(-1 - a^3/((a - b)*(a + b)*(a + b*Sin[x]))))/b
^3
```

fricas [A] time = 0.56, size = 483, normalized size = 3.90

$$\left[-\frac{\left(2a^5-3a^3b^2+\left(2a^4b-3a^2b^3\right)\sin(x)\right)\sqrt{-a^2+b^2}\,\log\left(\frac{\left(2a^2-b^2\right)\cos(x)^2-2ab\sin(x)-a^2-b^2+2(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+\cdots}}{b^2\cos(x)^2-2ab\sin(x)-a^2-b^2}\right.}{2\left(a^5b^3-2a^3b^5+\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((2*a^5 - 3*a^3*b^2 + (2*a^4*b - 3*a^2*b^3)*sin(x))*sqrt(-a^2 + b^2)*
log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 + 2*(a*cos(x)*sin(x)
 + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) +
 4*(a^6 - 2*a^4*b^2 + a^2*b^4)*x + 2*(2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(x) +
 2*(2*(a^5*b - 2*a^3*b^3 + a*b^5)*x + (a^4*b^2 - 2*a^2*b^4 + b^6)*cos(x))*s
in(x))/(a^5*b^3 - 2*a^3*b^5 + a*b^7 + (a^4*b^4 - 2*a^2*b^6 + b^8)*sin(x)),
-((2*a^5 - 3*a^3*b^2 + (2*a^4*b - 3*a^2*b^3)*sin(x))*sqrt(a^2 - b^2)*arctan
(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) + 2*(a^6 - 2*a^4*b^2 + a^2*b^4)*
x + (2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(x) + (2*(a^5*b - 2*a^3*b^3 + a*b^5)*x
 + (a^4*b^2 - 2*a^2*b^4 + b^6)*cos(x))*sin(x))/(a^5*b^3 - 2*a^3*b^5 + a*b^7
 + (a^4*b^4 - 2*a^2*b^6 + b^8)*sin(x))]
```

giac [A] time = 0.17, size = 204, normalized size = 1.65

$$\frac{2\left(2\,a^4-3\,a^2b^2\right)\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^3-b^5\right)\sqrt{a^2-b^2}}-\frac{2\left(a^2b\tan\left(\frac{1}{2}\,x\right)^3+2\,a^3\tan\left(\frac{1}{2}\,x\right)^2-ab^2\tan\left(\frac{1}{2}\,x\right)^2\right.}{\left(a\tan\left(\frac{1}{2}\,x\right)^4+2\,b\tan\left(\frac{1}{2}\,x\right)^3+2\,a\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] 2*(2*a^4 - 3*a^2*b^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*
x) + b)/sqrt(a^2 - b^2)))/((a^2*b^3 - b^5)*sqrt(a^2 - b^2)) - 2*(a^2*b*tan(
1/2*x)^3 + 2*a^3*tan(1/2*x)^2 - a*b^2*tan(1/2*x)^2 + 3*a^2*b*tan(1/2*x) - 2
*b^3*tan(1/2*x) + 2*a^3 - a*b^2)/((a*tan(1/2*x)^4 + 2*b*tan(1/2*x)^3 + 2*a*
tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)*(a^2*b^2 - b^4)) - 2*a*x/b^3
```

maple [A] time = 0.11, size = 196, normalized size = 1.58

$$-\frac{2}{b^2\left(\tan^2\left(\frac{x}{2}\right)+1\right)}-\frac{4a\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{b^3}-\frac{2a^2\tan\left(\frac{x}{2}\right)}{b\left(\left(\tan^2\left(\frac{x}{2}\right)\right)a+2\tan\left(\frac{x}{2}\right)b+a\right)\left(a^2-b^2\right)}-\frac{2a^3}{b^2\left(\left(\tan^2\left(\frac{x}{2}\right)\right)a+2\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a+b*sin(x))^2,x)
```

```
[Out] -2/b^2/(tan(1/2*x)^2+1)-4/b^3*a*arctan(tan(1/2*x))-2*a^2/b/(tan(1/2*x)^2*a+
2*tan(1/2*x)*b+a)/(a^2-b^2)*tan(1/2*x)-2*a^3/b^2/(tan(1/2*x)^2*a+2*tan(1/2*
```

```
x)*b+a)/(a^2-b^2)+4*a^4/b^3/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)
/(a^2-b^2)^(1/2))-6*a^2/b/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(
a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 9.98, size = 2578, normalized size = 20.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a + b*sin(x))^2,x)
```

```
[Out] ((2*(a*b^2 - 2*a^3))/(b^2*(a^2 - b^2)) - (2*a^2*tan(x/2)^3)/(b*(a^2 - b^2))
 + (2*tan(x/2)^2*(a*b^2 - 2*a^3))/(b^2*(a^2 - b^2)) - (2*tan(x/2)*(3*a^2 -
2*b^2))/(b*(a^2 - b^2)))/(a + 2*b*tan(x/2) + 2*a*tan(x/2)^2 + a*tan(x/2)^4
+ 2*b*tan(x/2)^3) - (4*a*atan((512*a^4*b^5*tan(x/2))/((512*a^4*b^14)/(b^9 -
 2*a^2*b^7 + a^4*b^5) - (1408*a^6*b^12)/(b^9 - 2*a^2*b^7 + a^4*b^5) + (1280
*a^8*b^10)/(b^9 - 2*a^2*b^7 + a^4*b^5) - (384*a^10*b^8)/(b^9 - 2*a^2*b^7 +
a^4*b^5)) - (384*a^6*b^3*tan(x/2))/((512*a^4*b^14)/(b^9 - 2*a^2*b^7 + a^4*b
^5) - (1408*a^6*b^12)/(b^9 - 2*a^2*b^7 + a^4*b^5) + (1280*a^8*b^10)/(b^9 -
2*a^2*b^7 + a^4*b^5) - (384*a^10*b^8)/(b^9 - 2*a^2*b^7 + a^4*b^5))))/b^3 -
(a^2*atan(((a^2*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(4*a^4*b^
6 - 8*a^6*b^4 + 4*a^8*b^2))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(x/2)*(8*a
^3*b^8 - 29*a^5*b^6 + 28*a^7*b^4 - 8*a^9*b^2))/(b^10 - 2*a^2*b^8 + a^4*b^6)
 + (a^2*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(2*a^2*b^10 - 3*a
^4*b^8 + a^6*b^6))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(x/2)*(6*a^3*b^10 -
 10*a^5*b^8 + 4*a^7*b^6))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (a^2*((32*(a^2*b^1
2 - 2*a^4*b^10 + a^6*b^8))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(x/2)*(3*a*
b^14 - 8*a^3*b^12 + 7*a^5*b^10 - 2*a^7*b^8))/(b^10 - 2*a^2*b^8 + a^4*b^6))*
(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5
- a^6*b^3)))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*1i)/(b^9 - 3*a^2*b^7
+ 3*a^4*b^5 - a^6*b^3) + (a^2*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*
((32*(4*a^4*b^6 - 8*a^6*b^4 + 4*a^8*b^2))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32
*tan(x/2)*(8*a^3*b^8 - 29*a^5*b^6 + 28*a^7*b^4 - 8*a^9*b^2))/(b^10 - 2*a^2*
```

```
b^8 + a^4*b^6) - (a^2*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(2*
a^2*b^10 - 3*a^4*b^8 + a^6*b^6))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(x/2)
*(6*a^3*b^10 - 10*a^5*b^8 + 4*a^7*b^6))/(b^10 - 2*a^2*b^8 + a^4*b^6) - (a^2
*((32*(a^2*b^12 - 2*a^4*b^10 + a^6*b^8))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*
tan(x/2)*(3*a*b^14 - 8*a^3*b^12 + 7*a^5*b^10 - 2*a^7*b^8))/(b^10 - 2*a^2*b^
8 + a^4*b^6))*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(b^9 - 3*a^2*b^
7 + 3*a^4*b^5 - a^6*b^3)))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*1i)/(b^
9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))/((64*(4*a^8 - 6*a^6*b^2))/(b^9 - 2*a^
2*b^7 + a^4*b^5) + (64*tan(x/2)*(16*a^9 + 24*a^5*b^4 - 40*a^7*b^2))/(b^10 -
 2*a^2*b^8 + a^4*b^6) + (a^2*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(
(32*(4*a^4*b^6 - 8*a^6*b^4 + 4*a^8*b^2))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*
tan(x/2)*(8*a^3*b^8 - 29*a^5*b^6 + 28*a^7*b^4 - 8*a^9*b^2))/(b^10 - 2*a^2*b
^8 + a^4*b^6) + (a^2*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(2*a
^2*b^10 - 3*a^4*b^8 + a^6*b^6))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(x/2)*
(6*a^3*b^10 - 10*a^5*b^8 + 4*a^7*b^6))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (a^2*
((32*(a^2*b^12 - 2*a^4*b^10 + a^6*b^8))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*t
an(x/2)*(3*a*b^14 - 8*a^3*b^12 + 7*a^5*b^10 - 2*a^7*b^8))/(b^10 - 2*a^2*b^8
 + a^4*b^6))*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(b^9 - 3*a^2*b^7
 + 3*a^4*b^5 - a^6*b^3)))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))/(b^9 -
3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3) - (a^2*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)
^3)^(1/2)*((32*(4*a^4*b^6 - 8*a^6*b^4 + 4*a^8*b^2))/(b^9 - 2*a^2*b^7 + a^4*
b^5) + (32*tan(x/2)*(8*a^3*b^8 - 29*a^5*b^6 + 28*a^7*b^4 - 8*a^9*b^2))/(b^1
0 - 2*a^2*b^8 + a^4*b^6) - (a^2*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2
)*((32*(2*a^2*b^10 - 3*a^4*b^8 + a^6*b^6))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (3
2*tan(x/2)*(6*a^3*b^10 - 10*a^5*b^8 + 4*a^7*b^6))/(b^10 - 2*a^2*b^8 + a^4*b
^6) - (a^2*((32*(a^2*b^12 - 2*a^4*b^10 + a^6*b^8))/(b^9 - 2*a^2*b^7 + a^4*b
^5) + (32*tan(x/2)*(3*a*b^14 - 8*a^3*b^12 + 7*a^5*b^10 - 2*a^7*b^8))/(b^10
- 2*a^2*b^8 + a^4*b^6))*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(b^9
- 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3
)))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))*(2*a^2 - 3*b^2)*(-(a + b)^3*(
a - b)^3)^(1/2)*2i)/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a+b*sin(x))**2,x)
```

```
[Out] Timed out
```

3.187 $\int \frac{\sin^2(x)}{(a+b\sin(x))^2}\,dx$

Optimal. Leaf size=87

$$-\frac{2a\left(a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{3/2}}+\frac{a^2\cos(x)}{b\left(a^2-b^2\right)(a+b\sin(x))}+\frac{x}{b^2}$$

```
[Out] x/b^2-2*a*(a^2-2*b^2)*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/b^2/(a^2-b^2
)^(3/2)+a^2*cos(x)/b/(a^2-b^2)/(a+b*sin(x))
```

Rubi [A] time = 0.13, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2790, 2735, 2660, 618, 204}

$$-\frac{2a\left(a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{3/2}}+\frac{a^2\cos(x)}{b\left(a^2-b^2\right)(a+b\sin(x))}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a + b*Sin[x])^2,x]
```

```
[Out] x/b^2 - (2*a*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b^2*(
a^2 - b^2)^(3/2)) + (a^2*Cos[x])/(b*(a^2 - b^2)*(a + b*Sin[x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(x)}{(a+b\sin(x))^2}\,dx &= \frac{a^2\cos(x)}{b\left(a^2-b^2\right)(a+b\sin(x))}+\frac{\int \frac{ab+\left(a^2-b^2\right)\sin(x)}{a+b\sin(x)}\,dx}{b\left(a^2-b^2\right)}\\
&= \frac{x}{b^2}+\frac{a^2\cos(x)}{b\left(a^2-b^2\right)(a+b\sin(x))}-\frac{\left(a\left(a^2-2b^2\right)\right)\int \frac{1}{a+b\sin(x)}\,dx}{b^2\left(a^2-b^2\right)}\\
&= \frac{x}{b^2}+\frac{a^2\cos(x)}{b\left(a^2-b^2\right)(a+b\sin(x))}-\frac{\left(2a\left(a^2-2b^2\right)\right)\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{b^2\left(a^2-b^2\right)}\\
&= \frac{x}{b^2}+\frac{a^2\cos(x)}{b\left(a^2-b^2\right)(a+b\sin(x))}+\frac{\left(4a\left(a^2-2b^2\right)\right)\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan\right.}{b^2\left(a^2-b^2\right)}\\
&= \frac{x}{b^2}-\frac{2a\left(a^2-2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{3/2}}+\frac{a^2\cos(x)}{b\left(a^2-b^2\right)(a+b\sin(x))}
\end{aligned}$$

Mathematica [A] time = 0.23, size = 83, normalized size = 0.95

$$\frac{-\frac{2a(a^2-2b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{a^2b\cos(x)}{(a-b)(a+b)(a+b\sin(x))}+x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a + b*Sin[x])^2,x]
```

```
[Out] (x - (2*a*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^
2)^(3/2) + (a^2*b*Cos[x])/((a - b)*(a + b)*(a + b*Sin[x])))/b^2
```

fricas [B] time = 0.55, size = 403, normalized size = 4.63

$$\left[\frac{2\left(a^4b-2a^2b^3+b^5\right)x\sin(x)-\left(a^4-2a^2b^2+\left(a^3b-2ab^3\right)\sin(x)\right)\sqrt{-a^2+b^2}\log\left(-\frac{\left(2a^2-b^2\right)\cos(x)^2-2ab\sin(x)-a^2-b^2}{b^2\cos(x)^2-2a}\right.}{2\left(a^5b^2-2a^3b^4+ab^6+\left(a^4b^3-2a^2b^5+b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^4*b - 2*a^2*b^3 + b^5)*x*sin(x) - (a^4 - 2*a^2*b^2 + (a^3*b - 2*
a*b^3)*sin(x))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x)
 - a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)
^2 - 2*a*b*sin(x) - a^2 - b^2)) + 2*(a^5 - 2*a^3*b^2 + a*b^4)*x + 2*(a^4*b
- a^2*b^3)*cos(x))/(a^5*b^2 - 2*a^3*b^4 + a*b^6 + (a^4*b^3 - 2*a^2*b^5 + b^
7)*sin(x)), ((a^4*b - 2*a^2*b^3 + b^5)*x*sin(x) + (a^4 - 2*a^2*b^2 + (a^3*b
 - 2*a*b^3)*sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)
*cos(x))) + (a^5 - 2*a^3*b^2 + a*b^4)*x + (a^4*b - a^2*b^3)*cos(x))/(a^5*b^
2 - 2*a^3*b^4 + a*b^6 + (a^4*b^3 - 2*a^2*b^5 + b^7)*sin(x))]
```

giac [A] time = 0.16, size = 124, normalized size = 1.43

$$-\frac{2\left(a^3-2ab^2\right)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^2-b^4\right)\sqrt{a^2-b^2}}+\frac{2\left(ab\tan\left(\frac{1}{2}x\right)+a^2\right)}{\left(a^2b-b^3\right)\left(a\tan\left(\frac{1}{2}x\right)^2+2b\tan\left(\frac{1}{2}x\right)+a\right)}+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*(a^3 - 2*a*b^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x)
+ b)/sqrt(a^2 - b^2)))/((a^2*b^2 - b^4)*sqrt(a^2 - b^2)) + 2*(a*b*tan(1/2*x
) + a^2)/((a^2*b - b^3)*(a*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)) + x/b^2
```

maple [B] time = 0.10, size = 170, normalized size = 1.95

$$\frac{2\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{b^2}+\frac{2a\tan\left(\frac{x}{2}\right)}{\left(\left(\tan^2\left(\frac{x}{2}\right)\right)a+2\tan\left(\frac{x}{2}\right)b+a\right)\left(a^2-b^2\right)}+\frac{2a^2}{b\left(\left(\tan^2\left(\frac{x}{2}\right)\right)a+2\tan\left(\frac{x}{2}\right)b+a\right)\left(a^2-b^2\right)}-\frac{2a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a+b*sin(x))^2,x)
```

```
[Out] 2/b^2*arctan(tan(1/2*x))+2*a/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)/(a^2-b^2)*ta
n(1/2*x)+2*a^2/b/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)/(a^2-b^2)-2*a^3/b^2/(a^2
-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))+4*a/(a^2-b^2)^
(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 9.95, size = 2562, normalized size = 29.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a + b*sin(x))^2,x)
```

```
[Out] ((2*a^2)/(b*(a^2 - b^2)) + (2*a*tan(x/2))/(a^2 - b^2))/(a + 2*b*tan(x/2) +
a*tan(x/2)^2) - (2*atan((64*a^3*b^3*tan(x/2))/((64*a^3*b^9)/(b^6 - 2*a^2*b^
4 + a^4*b^2) + (128*a^5*b^7)/(b^6 - 2*a^2*b^4 + a^4*b^2) - (192*a^7*b^5)/(b
^6 - 2*a^2*b^4 + a^4*b^2) + (64*a^9*b^3)/(b^6 - 2*a^2*b^4 + a^4*b^2) - (64*
a*b^11)/(b^6 - 2*a^2*b^4 + a^4*b^2)) + (64*a*b^5*tan(x/2))/((64*a^3*b^9)/(b
^6 - 2*a^2*b^4 + a^4*b^2) + (128*a^5*b^7)/(b^6 - 2*a^2*b^4 + a^4*b^2) - (19
```

2*a^7*b^5)/(b^6 - 2*a^2*b^4 + a^4*b^2) + (64*a^9*b^3)/(b^6 - 2*a^2*b^4 + a^
4*b^2) - (64*a*b^11)/(b^6 - 2*a^2*b^4 + a^4*b^2)) - (64*a^5*b*tan(x/2))/((6
4*a^3*b^9)/(b^6 - 2*a^2*b^4 + a^4*b^2) + (128*a^5*b^7)/(b^6 - 2*a^2*b^4 + a
^4*b^2) - (192*a^7*b^5)/(b^6 - 2*a^2*b^4 + a^4*b^2) + (64*a^9*b^3)/(b^6 - 2
*a^2*b^4 + a^4*b^2) - (64*a*b^11)/(b^6 - 2*a^2*b^4 + a^4*b^2))))/b^2 + (a*a
tan(((a*(a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(a^6*b + a^2*b^5 -
2*a^4*b^3))/(b^6 - 2*a^2*b^4 + a^4*b^2) + (32*tan(x/2)*(2*a*b^7 - 2*a^7*b -
9*a^3*b^5 + 8*a^5*b^3))/(b^7 - 2*a^2*b^5 + a^4*b^3) + (a*(a^2 - 2*b^2)*(-(
a + b)^3*(a - b)^3)^(1/2)*((32*(a*b^8 - a^3*b^6))/(b^6 - 2*a^2*b^4 + a^4*b^
2) + (32*tan(x/2)*(4*a^2*b^8 - 6*a^4*b^6 + 2*a^6*b^4))/(b^7 - 2*a^2*b^5 + a
^4*b^3) + (a*(a^2 - 2*b^2)*((32*(a^2*b^9 - 2*a^4*b^7 + a^6*b^5))/(b^6 - 2*a
^2*b^4 + a^4*b^2) + (32*tan(x/2)*(3*a*b^11 - 8*a^3*b^9 + 7*a^5*b^7 - 2*a^7*
b^5))/(b^7 - 2*a^2*b^5 + a^4*b^3))*(-(a + b)^3*(a - b)^3)^(1/2))/(b^8 - 3*a
^2*b^6 + 3*a^4*b^4 - a^6*b^2)))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*1i
)/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2) + (a*(a^2 - 2*b^2)*(-(a + b)^3*(a
- b)^3)^(1/2)*((32*(a^6*b + a^2*b^5 - 2*a^4*b^3))/(b^6 - 2*a^2*b^4 + a^4*b
^2) + (32*tan(x/2)*(2*a*b^7 - 2*a^7*b - 9*a^3*b^5 + 8*a^5*b^3))/(b^7 - 2*a^
2*b^5 + a^4*b^3) - (a*(a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(a*b^
8 - a^3*b^6))/(b^6 - 2*a^2*b^4 + a^4*b^2) + (32*tan(x/2)*(4*a^2*b^8 - 6*a^4
*b^6 + 2*a^6*b^4))/(b^7 - 2*a^2*b^5 + a^4*b^3) - (a*(a^2 - 2*b^2)*((32*(a^2
*b^9 - 2*a^4*b^7 + a^6*b^5))/(b^6 - 2*a^2*b^4 + a^4*b^2) + (32*tan(x/2)*(3*
a*b^11 - 8*a^3*b^9 + 7*a^5*b^7 - 2*a^7*b^5))/(b^7 - 2*a^2*b^5 + a^4*b^3))*(
-(a + b)^3*(a - b)^3)^(1/2))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))/(b^8
- 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*1i)/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6
b^2))/((64(a^5 - 2*a^3*b^2))/(b^6 - 2*a^2*b^4 + a^4*b^2) + (64*tan(x/2)*(
2*a^6 + 4*a^2*b^4 - 6*a^4*b^2))/(b^7 - 2*a^2*b^5 + a^4*b^3) + (a*(a^2 - 2*b
^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(a^6*b + a^2*b^5 - 2*a^4*b^3))/(b^6 -
2*a^2*b^4 + a^4*b^2) + (32*tan(x/2)*(2*a*b^7 - 2*a^7*b - 9*a^3*b^5 + 8*a^5
*b^3))/(b^7 - 2*a^2*b^5 + a^4*b^3) + (a*(a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3
)^(1/2)*((32*(a*b^8 - a^3*b^6))/(b^6 - 2*a^2*b^4 + a^4*b^2) + (32*tan(x/2)*
(4*a^2*b^8 - 6*a^4*b^6 + 2*a^6*b^4))/(b^7 - 2*a^2*b^5 + a^4*b^3) + (a*(a^2
- 2*b^2)*((32*(a^2*b^9 - 2*a^4*b^7 + a^6*b^5))/(b^6 - 2*a^2*b^4 + a^4*b^2)
+ (32*tan(x/2)*(3*a*b^11 - 8*a^3*b^9 + 7*a^5*b^7 - 2*a^7*b^5))/(b^7 - 2*a^2
*b^5 + a^4*b^3))*(-(a + b)^3*(a - b)^3)^(1/2))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4
- a^6*b^2)))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))/(b^8 - 3*a^2*b^6 +
3*a^4*b^4 - a^6*b^2) - (a*(a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(
a^6*b + a^2*b^5 - 2*a^4*b^3))/(b^6 - 2*a^2*b^4 + a^4*b^2) + (32*tan(x/2)*(2
*a*b^7 - 2*a^7*b - 9*a^3*b^5 + 8*a^5*b^3))/(b^7 - 2*a^2*b^5 + a^4*b^3) - (a
*(a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(a*b^8 - a^3*b^6))/(b^6 -
2*a^2*b^4 + a^4*b^2) + (32*tan(x/2)*(4*a^2*b^8 - 6*a^4*b^6 + 2*a^6*b^4))/(b
^7 - 2*a^2*b^5 + a^4*b^3) - (a*(a^2 - 2*b^2)*((32*(a^2*b^9 - 2*a^4*b^7 + a^
6*b^5))/(b^6 - 2*a^2*b^4 + a^4*b^2) + (32*tan(x/2)*(3*a*b^11 - 8*a^3*b^9 +
7*a^5*b^7 - 2*a^7*b^5))/(b^7 - 2*a^2*b^5 + a^4*b^3))*(-(a + b)^3*(a - b)^3)
^(1/2))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))/(b^8 - 3*a^2*b^6 + 3*a^4*
b^4 - a^6*b^2)))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))*(a^2 - 2*b^2)*(-

```
(a + b)^3*(a - b)^3)^(1/2)*2i)/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+b*sin(x))**2,x)
```

```
[Out] Timed out
```

3.188 $\int \frac{\sin(x)}{(a+b\sin(x))^2}\,dx$

Optimal. Leaf size=66

$$-\frac{2b\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}-\frac{a\cos(x)}{\left(a^2-b^2\right)(a+b\sin(x))}$$

```
[Out] -2*b*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)-a*cos(x)/(a^2
-b^2)/(a+b*sin(x))
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2754, 12, 2660, 618, 204}

$$-\frac{2b\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}-\frac{a\cos(x)}{\left(a^2-b^2\right)(a+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a + b*Sin[x])^2,x]
```

```
[Out] (-2*b*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - (a*Cos[
x])/((a^2 - b^2)*(a + b*Sin[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(x)}{(a+b\sin(x))^2}\,dx &= -\frac{a\cos(x)}{\left(a^2-b^2\right)(a+b\sin(x))}+\frac{\int \frac{b}{a+b\sin(x)}\,dx}{-a^2+b^2}\\
&= -\frac{a\cos(x)}{\left(a^2-b^2\right)(a+b\sin(x))}-\frac{b\int \frac{1}{a+b\sin(x)}\,dx}{a^2-b^2}\\
&= -\frac{a\cos(x)}{\left(a^2-b^2\right)(a+b\sin(x))}-\frac{(2b)\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{a^2-b^2}\\
&= -\frac{a\cos(x)}{\left(a^2-b^2\right)(a+b\sin(x))}+\frac{(4b)\,\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan\left(\frac{x}{2}\right)\right)}{a^2-b^2}\\
&= -\frac{2b\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}-\frac{a\cos(x)}{\left(a^2-b^2\right)(a+b\sin(x))}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 67, normalized size = 1.02

$$-\frac{2b\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}-\frac{a\cos(x)}{(a-b)(a+b)(a+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a + b*Sin[x])^2,x]
```

```
[Out] (-2*b*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - (a*Cos[
x])/((a - b)*(a + b)*(a + b*Sin[x]))
```

fricas [A] time = 0.52, size = 266, normalized size = 4.03

$$\left[\frac{\left(b^2\sin(x)+ab\right)\sqrt{-a^2+b^2}\,\log\left(\frac{\left(2\,a^2-b^2\right)\cos(x)^2-2\,ab\sin(x)-a^2-b^2+2\,(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2\,ab\sin(x)-a^2-b^2}\right)-2\left(a^3-ab^2\right)\cos(x)}{2\left(a^5-2\,a^3b^2+ab^4+\left(a^4b-2\,a^2b^3+b^5\right)\sin(x)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*sin(x) + a*b)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(x)^2 - 2*a
*b*sin(x) - a^2 - b^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b
^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) - 2*(a^3 - a*b^2)*cos(x))/(a^5 - 2
*a^3*b^2 + a*b^4 + (a^4*b - 2*a^2*b^3 + b^5)*sin(x)), ((b^2*sin(x) + a*b)*s
qrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) - (a^3 - a*
b^2)*cos(x))/(a^5 - 2*a^3*b^2 + a*b^4 + (a^4*b - 2*a^2*b^3 + b^5)*sin(x))]
```

giac [A] time = 0.17, size = 90, normalized size = 1.36

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)b}{\left(a^2-b^2\right)^{\frac{3}{2}}}-\frac{2\left(b\tan\left(\frac{1}{2}x\right)+a\right)}{\left(a\tan\left(\frac{1}{2}x\right)^2+2\,b\tan\left(\frac{1}{2}x\right)+a\right)\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 -
b^2)))*b/(a^2 - b^2)^(3/2) - 2*(b*tan(1/2*x) + a)/((a*tan(1/2*x)^2 + 2*b*ta
n(1/2*x) + a)*(a^2 - b^2))
```

maple [A] time = 0.09, size = 99, normalized size = 1.50

$$\frac{-8\tan\left(\frac{x}{2}\right)b-8a}{\left(4a^2-4b^2\right)\left(\left(\tan^2\left(\frac{x}{2}\right)\right)a+2\tan\left(\frac{x}{2}\right)b+a\right)}-\frac{8b\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(4a^2-4b^2\right)\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a+b*sin(x))^2,x)
```

```
[Out] 4*(-2*tan(1/2*x)*b-2*a)/(4*a^2-4*b^2)/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)-8*b
/(4*a^2-4*b^2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1
/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 6.49, size = 123, normalized size = 1.86

$$-\frac{\frac{2\,a}{a^2-b^2}+\frac{2\,b\,\tan\left(\frac{x}{2}\right)}{a^2-b^2}}{a\,\tan\left(\frac{x}{2}\right)^2+2\,b\,\tan\left(\frac{x}{2}\right)+a}-\frac{2\,b\,\mathrm{atan}\left(\frac{\left(a^2-b^2\right)\left(\frac{2\,b^2}{{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}+\frac{2\,a\,b\,\tan\left(\frac{x}{2}\right)}{{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}\right)}{2\,b}\right)}{{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a + b*sin(x))^2,x)
```

```
[Out] - ((2*a)/(a^2 - b^2) + (2*b*tan(x/2))/(a^2 - b^2))/(a + 2*b*tan(x/2) + a*ta
n(x/2)^2) - (2*b*atan(((a^2 - b^2)*((2*b^2)/((a + b)^(3/2)*(a - b)^(3/2)) +
 (2*a*b*tan(x/2))/((a + b)^(3/2)*(a - b)^(3/2))))/(2*b)))/((a + b)^(3/2)*(a
 - b)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x))**2,x)
```

```
[Out] Timed out
```

3.189 $\int \frac{1}{(a+b\sin(x))^2}\,dx$

Optimal. Leaf size=65

$$\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+\frac{b\cos(x)}{\left(a^2-b^2\right)(a+b\sin(x))}$$

```
[Out] 2*a*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)+b*cos(x)/(a^2-
b^2)/(a+b*sin(x))
```

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {2664, 12, 2660, 618, 204}

$$\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+\frac{b\cos(x)}{\left(a^2-b^2\right)(a+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x])^(-2),x]
```

```
[Out] (2*a*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + (b*Cos[x
])/((a^2 - b^2)*(a + b*Sin[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+b\sin(x))^2}\,dx &= \frac{b\cos(x)}{\left(a^2-b^2\right)(a+b\sin(x))} - \frac{\int \frac{a}{a+b\sin(x)}\,dx}{-a^2+b^2} \\
&= \frac{b\cos(x)}{\left(a^2-b^2\right)(a+b\sin(x))} + \frac{a\int \frac{1}{a+b\sin(x)}\,dx}{a^2-b^2} \\
&= \frac{b\cos(x)}{\left(a^2-b^2\right)(a+b\sin(x))} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{a^2-b^2} \\
&= \frac{b\cos(x)}{\left(a^2-b^2\right)(a+b\sin(x))} - \frac{(4a)\,\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx, x, 2b+2a\tan\left(\frac{x}{2}\right)\right)}{a^2-b^2} \\
&= \frac{2a\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{b\cos(x)}{\left(a^2-b^2\right)(a+b\sin(x))}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 66, normalized size = 1.02

$$\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{b\cos(x)}{(a-b)(a+b)(a+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x])^(-2),x]
```

```
[Out] (2*a*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + (b*Cos[x
])/((a - b)*(a + b)*(a + b*Sin[x]))
```

fricas [A] time = 0.56, size = 268, normalized size = 4.12

$$\left[\frac{\left(ab\sin(x)+a^2\right)\sqrt{-a^2+b^2}\,\log\left(-\frac{\left(2a^2-b^2\right)\cos(x)^2-2ab\sin(x)-a^2-b^2-2\left(a\cos(x)\sin(x)+b\cos(x)\right)\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2ab\sin(x)-a^2-b^2}\right)+2\left(a^2b-b^3\right)\cos(x}{2\left(a^5-2a^3b^2+ab^4+\left(a^4b-2a^2b^3+b^5\right)\sin(x)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a*b*sin(x) + a^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(x)^2 - 2*
a*b*sin(x) - a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(
b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) + 2*(a^2*b - b^3)*cos(x))/(a^5 -
2*a^3*b^2 + a*b^4 + (a^4*b - 2*a^2*b^3 + b^5)*sin(x)), -((a*b*sin(x) + a^2)
*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) - (a^2*b
- b^3)*cos(x))/(a^5 - 2*a^3*b^2 + a*b^4 + (a^4*b - 2*a^2*b^3 + b^5)*sin(x))
]
```

giac [A] time = 0.17, size = 95, normalized size = 1.46

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)a}{\left(a^2-b^2\right)^{\frac{3}{2}}}+\frac{2\left(b^2\tan\left(\frac{1}{2}x\right)+ab\right)}{\left(a^3-ab^2\right)\left(a\tan\left(\frac{1}{2}x\right)^2+2b\tan\left(\frac{1}{2}x\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b
^2)))*a/(a^2 - b^2)^(3/2) + 2*(b^2*tan(1/2*x) + a*b)/((a^3 - a*b^2)*(a*tan(
1/2*x)^2 + 2*b*tan(1/2*x) + a))
```

maple [A] time = 0.08, size = 98, normalized size = 1.51

$$\frac{\frac{2b^2\tan\left(\frac{x}{2}\right)}{a\left(a^2-b^2\right)}+\frac{2b}{a^2-b^2}}{\left(\tan^2\left(\frac{x}{2}\right)\right)a+2\tan\left(\frac{x}{2}\right)b+a}+\frac{2a\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(x))^2,x)
```

```
[Out] 2*(b^2/a/(a^2-b^2)*tan(1/2*x)+b/(a^2-b^2))/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a
)+2*a/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 6.82, size = 148, normalized size = 2.28

$$\frac{\frac{2b}{a^2-b^2}+\frac{2b^2\tan\left(\frac{x}{2}\right)}{a\left(a^2-b^2\right)}}{a\tan\left(\frac{x}{2}\right)^2+2b\tan\left(\frac{x}{2}\right)+a}+\frac{2a\,\mathrm{atan}\left(\frac{\left(a^2-b^2\right)\left(\frac{2a^2\tan\left(\frac{x}{2}\right)}{(a+b)^{3/2}\,(a-b)^{3/2}}+\frac{2a\left(a^2b-b^3\right)}{(a+b)^{3/2}\left(a^2-b^2\right)(a-b)^{3/2}}\right)}{2a}\right)}{(a+b)^{3/2}\,(a-b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(x))^2,x)
```

```
[Out] ((2*b)/(a^2 - b^2) + (2*b^2*tan(x/2))/(a*(a^2 - b^2)))/(a + 2*b*tan(x/2) +
a*tan(x/2)^2) + (2*a*atan(((a^2 - b^2)*((2*a^2*tan(x/2))/((a + b)^(3/2)*(a
- b)^(3/2)) + (2*a*(a^2*b - b^3))/((a + b)^(3/2)*(a^2 - b^2)*(a - b)^(3/2))
))/(2*a)))/((a + b)^(3/2)*(a - b)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sin(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x))**2,x)
```

```
[Out] Integral((a + b*sin(x))**(-2), x)
```

3.190 $\int \frac{\csc(x)}{(a+b\sin(x))^2}\,dx$

Optimal. Leaf size=93

$$-\frac{2b\left(2a^2-b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{3/2}}-\frac{b^2\cos(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}-\frac{\tanh^{-1}(\cos(x))}{a^2}$$

```
[Out] -2*b*(2*a^2-b^2)*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/a^2/(a^2-b^2)^(3/
2)-arctanh(cos(x))/a^2-b^2*cos(x)/a/(a^2-b^2)/(a+b*sin(x))
```

Rubi [A] time = 0.18, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2802, 3001, 3770, 2660, 618, 204}

$$-\frac{2b\left(2a^2-b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{3/2}}-\frac{b^2\cos(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}-\frac{\tanh^{-1}(\cos(x))}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a + b*Sin[x])^2,x]
```

```
[Out] (-2*b*(2*a^2 - b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2*(a^2 - b
^2)^(3/2)) - ArcTanh[Cos[x]]/a^2 - (b^2*Cos[x])/(a*(a^2 - b^2)*(a + b*Sin[x
]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(x)}{(a+b\sin(x))^2}\,dx &= -\frac{b^2\cos(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}+\frac{\int \frac{\csc(x)\left(a^2-b^2-ab\sin(x)\right)}{a+b\sin(x)}\,dx}{a\left(a^2-b^2\right)} \\
&= -\frac{b^2\cos(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}+\frac{\int \csc(x)\,dx}{a^2}-\frac{\left(b\left(2a^2-b^2\right)\right)\int \frac{1}{a+b\sin(x)}\,dx}{a^2\left(a^2-b^2\right)} \\
&= -\frac{\tanh^{-1}(\cos(x))}{a^2}-\frac{b^2\cos(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}-\frac{\left(2b\left(2a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\text{ta}\right.}{a^2\left(a^2-b^2\right)} \\
&= -\frac{\tanh^{-1}(\cos(x))}{a^2}-\frac{b^2\cos(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}+\frac{\left(4b\left(2a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x\right.}{a^2\left(a^2-b^2\right)} \\
&= -\frac{2b\left(2a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{3/2}}-\frac{\tanh^{-1}(\cos(x))}{a^2}-\frac{b^2\cos(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}
\end{aligned}
$$

<u>Mathematica</u> [A] time = 0.25, size = 99, normalized size = 1.06

$$
\frac{\frac{2b\left(b^2-2a^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}-\frac{ab^2\cos(x)}{(a-b)(a+b)(a+b\sin(x))}+\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)}{a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a + b*Sin[x])^2,x]
```

```
[Out] ((2*b*(-2*a^2 + b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^
(3/2) - Log[Cos[x/2]] + Log[Sin[x/2]] - (a*b^2*Cos[x])/((a - b)*(a + b)*(a
+ b*Sin[x])))/a^2
```

<u>fricas</u> [B] time = 0.86, size = 511, normalized size = 5.49

$$
\left[-\frac{\left(2\,a^3b-ab^3+\left(2\,a^2b^2-b^4\right)\sin(x)\right)\sqrt{-a^2+b^2}\,\log\left(-\frac{\left(2\,a^2-b^2\right)\cos(x)^2-2\,ab\sin(x)-a^2-b^2-2\left(a\cos(x)\sin(x)+b\cos(x)\right)\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2\,ab\sin(x)-a^2-b^2}\right.}{}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((2*a^3*b - a*b^3 + (2*a^2*b^2 - b^4)*sin(x))*sqrt(-a^2 + b^2)*log(-(
(2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*
cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) + 2*(a
^3*b^2 - a*b^4)*cos(x) + (a^5 - 2*a^3*b^2 + a*b^4 + (a^4*b - 2*a^2*b^3 + b^
5)*sin(x))*log(1/2*cos(x) + 1/2) - (a^5 - 2*a^3*b^2 + a*b^4 + (a^4*b - 2*a^
2*b^3 + b^5)*sin(x))*log(-1/2*cos(x) + 1/2))/(a^7 - 2*a^5*b^2 + a^3*b^4 + (
a^6*b - 2*a^4*b^3 + a^2*b^5)*sin(x)), 1/2*(2*(2*a^3*b - a*b^3 + (2*a^2*b^2
- b^4)*sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(
x))) - 2*(a^3*b^2 - a*b^4)*cos(x) - (a^5 - 2*a^3*b^2 + a*b^4 + (a^4*b - 2*a
^2*b^3 + b^5)*sin(x))*log(1/2*cos(x) + 1/2) + (a^5 - 2*a^3*b^2 + a*b^4 + (a
^4*b - 2*a^2*b^3 + b^5)*sin(x))*log(-1/2*cos(x) + 1/2))/(a^7 - 2*a^5*b^2 +
a^3*b^4 + (a^6*b - 2*a^4*b^3 + a^2*b^5)*sin(x))]
```

giac [A] time = 0.25, size = 134, normalized size = 1.44

$$-\frac{2\left(2\,a^2b-b^3\right)\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4-a^2b^2\right)\sqrt{a^2-b^2}}-\frac{2\left(b^3\tan\left(\frac{1}{2}\,x\right)+ab^2\right)}{\left(a^4-a^2b^2\right)\left(a\tan\left(\frac{1}{2}\,x\right)^2+2\,b\tan\left(\frac{1}{2}\,x\right)+a\right)}+\frac{\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*(2*a^2*b - b^3)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x)
+ b)/sqrt(a^2 - b^2)))/((a^4 - a^2*b^2)*sqrt(a^2 - b^2)) - 2*(b^3*tan(1/2*x
) + a*b^2)/((a^4 - a^2*b^2)*(a*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)) + log(ab
s(tan(1/2*x)))/a^2
```

maple [A] time = 0.13, size = 174, normalized size = 1.87

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}-\frac{2b^3\tan\left(\frac{x}{2}\right)}{a^2\left(\left(\tan^2\left(\frac{x}{2}\right)\right)a+2\tan\left(\frac{x}{2}\right)b+a\right)\left(a^2-b^2\right)}-\frac{2b^2}{a\left(\left(\tan^2\left(\frac{x}{2}\right)\right)a+2\tan\left(\frac{x}{2}\right)b+a\right)\left(a^2-b^2\right)}-\frac{4b\,\mathrm{arcta}}{(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a+b*sin(x))^2,x)
```

```
[Out] 1/a^2*ln(tan(1/2*x))-2/a^2*b^3/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)/(a^2-b^2)*
tan(1/2*x)-2/a*b^2/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)/(a^2-b^2)-4*b/(a^2-b^2
)^(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))+2/a^2*b^3/(a^2-b^2
)^(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.82, size = 1356, normalized size = 14.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(a + b*sin(x))^2),x)
```

```
[Out] log(tan(x/2))/a^2 - ((2*b^2)/(a*(a^2 - b^2)) + (2*b^3*tan(x/2))/(a^2*(a^2 -
 b^2)))/(a + 2*b*tan(x/2) + a*tan(x/2)^2) - (b*atan(((b*(2*a^2 - b^2)*(-(a
+ b)^3*(a - b)^3)^(1/2)*((2*tan(x/2)*(a^6 - 4*b^6 + 11*a^2*b^4 - 8*a^4*b^2)
)/(a*b^4 + a^5 - 2*a^3*b^2) - (2*(3*a^4*b - 2*a^2*b^3))/(a^4 - a^2*b^2) + (
b*((2*(a^6*b - a^4*b^3))/(a^4 - a^2*b^2) - (2*tan(x/2)*(3*a^8 - 4*a^2*b^6 +
 11*a^4*b^4 - 10*a^6*b^2))/(a*b^4 + a^5 - 2*a^3*b^2))*(2*a^2 - b^2)*(-(a +
b)^3*(a - b)^3)^(1/2))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))*1i)/(a^8 -
a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2) - (b*(2*a^2 - b^2)*(-(a + b)^3*(a - b)^3)^
(1/2)*((2*(3*a^4*b - 2*a^2*b^3))/(a^4 - a^2*b^2) - (2*tan(x/2)*(a^6 - 4*b^6
 + 11*a^2*b^4 - 8*a^4*b^2))/(a*b^4 + a^5 - 2*a^3*b^2) + (b*((2*(a^6*b - a^4
*b^3))/(a^4 - a^2*b^2) - (2*tan(x/2)*(3*a^8 - 4*a^2*b^6 + 11*a^4*b^4 - 10*a
^6*b^2))/(a*b^4 + a^5 - 2*a^3*b^2))*(2*a^2 - b^2)*(-(a + b)^3*(a - b)^3)^(1
/2))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))*1i)/(a^8 - a^2*b^6 + 3*a^4*b^
4 - 3*a^6*b^2))/((4*(2*a^2*b - b^3))/(a^4 - a^2*b^2) + (4*tan(x/2)*(2*b^4 -
 4*a^2*b^2))/(a*b^4 + a^5 - 2*a^3*b^2) + (b*(2*a^2 - b^2)*(-(a + b)^3*(a -
b)^3)^(1/2)*((2*tan(x/2)*(a^6 - 4*b^6 + 11*a^2*b^4 - 8*a^4*b^2))/(a*b^4 + a
^5 - 2*a^3*b^2) - (2*(3*a^4*b - 2*a^2*b^3))/(a^4 - a^2*b^2) + (b*((2*(a^6*b
 - a^4*b^3))/(a^4 - a^2*b^2) - (2*tan(x/2)*(3*a^8 - 4*a^2*b^6 + 11*a^4*b^4
- 10*a^6*b^2))/(a*b^4 + a^5 - 2*a^3*b^2))*(2*a^2 - b^2)*(-(a + b)^3*(a - b)
^3)^(1/2))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))/(a^8 - a^2*b^6 + 3*a^4
*b^4 - 3*a^6*b^2) + (b*(2*a^2 - b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((2*(3*a^
4*b - 2*a^2*b^3))/(a^4 - a^2*b^2) - (2*tan(x/2)*(a^6 - 4*b^6 + 11*a^2*b^4 -
 8*a^4*b^2))/(a*b^4 + a^5 - 2*a^3*b^2) + (b*((2*(a^6*b - a^4*b^3))/(a^4 - a
^2*b^2) - (2*tan(x/2)*(3*a^8 - 4*a^2*b^6 + 11*a^4*b^4 - 10*a^6*b^2))/(a*b^4
 + a^5 - 2*a^3*b^2))*(2*a^2 - b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(a^8 - a^2
```

```
*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))*(
2*a^2 - b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*2i)/(a^8 - a^2*b^6 + 3*a^4*b^4 -
3*a^6*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(x)}{(a + b\sin(x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x))**2,x)
```

```
[Out] Integral(csc(x)/(a + b*sin(x))**2, x)
```

3.191 $\int \frac{\csc^2(x)}{(a+b\sin(x))^2}\,dx$

Optimal. Leaf size=123

$$\frac{2b\tanh^{-1}(\cos(x))}{a^3}-\frac{\left(a^2-2b^2\right)\cot(x)}{a^2\left(a^2-b^2\right)}-\frac{b^2\cot(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}+\frac{2b^2\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{3/2}}$$

```
[Out] 2*b^2*(3*a^2-2*b^2)*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/a^3/(a^2-b^2)^
(3/2)+2*b*arctanh(cos(x))/a^3-(a^2-2*b^2)*cot(x)/a^2/(a^2-b^2)-b^2*cot(x)/a
/(a^2-b^2)/(a+b*sin(x))
```

Rubi [A] time = 0.33, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2802, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2b^2\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{3/2}}-\frac{\left(a^2-2b^2\right)\cot(x)}{a^2\left(a^2-b^2\right)}-\frac{b^2\cot(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}+\frac{2b\tanh^{-1}(\cos(x))}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a + b*Sin[x])^2,x]
```

```
[Out] (2*b^2*(3*a^2 - 2*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^3*(a^2
- b^2)^(3/2)) + (2*b*ArcTanh[Cos[x]])/a^3 - ((a^2 - 2*b^2)*Cot[x])/(a^2*(a^
2 - b^2)) - (b^2*Cot[x])/(a*(a^2 - b^2)*(a + b*Sin[x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^2(x)}{(a+b\sin(x))^2}\,dx = -\frac{b^2\cot(x)}{a\left(a^2-b^2\right)(a+b\sin(x))} + \frac{\int \frac{\csc^2(x)\left(a^2-2b^2-ab\sin(x)+b^2\sin^2(x)\right)}{a+b\sin(x)}\,dx}{a\left(a^2-b^2\right)}$$

$$= -\frac{\left(a^2-2b^2\right)\cot(x)}{a^2\left(a^2-b^2\right)} - \frac{b^2\cot(x)}{a\left(a^2-b^2\right)(a+b\sin(x))} + \frac{\int \frac{\csc(x)\left(-2b\left(a^2-b^2\right)+ab^2\sin(x)\right)}{a+b\sin(x)}\,dx}{a^2\left(a^2-b^2\right)}$$

$$= -\frac{\left(a^2-2b^2\right)\cot(x)}{a^2\left(a^2-b^2\right)} - \frac{b^2\cot(x)}{a\left(a^2-b^2\right)(a+b\sin(x))} - \frac{(2b)\int \csc(x)\,dx}{a^3} + \frac{\left(b^2\left(3a^2-2b^2\right)\right)\int \frac{}{a+}}{a^3\left(a^2-b^2\right)}$$

$$= \frac{2b\tanh^{-1}(\cos(x))}{a^3} - \frac{\left(a^2-2b^2\right)\cot(x)}{a^2\left(a^2-b^2\right)} - \frac{b^2\cot(x)}{a\left(a^2-b^2\right)(a+b\sin(x))} + \frac{\left(2b^2\left(3a^2-2b^2\right)\right)\mathrm{Su}}{a}$$

$$= \frac{2b\tanh^{-1}(\cos(x))}{a^3} - \frac{\left(a^2-2b^2\right)\cot(x)}{a^2\left(a^2-b^2\right)} - \frac{b^2\cot(x)}{a\left(a^2-b^2\right)(a+b\sin(x))} - \frac{\left(4b^2\left(3a^2-2b^2\right)\right)\mathrm{Su}}{}$$

$$= \frac{2b^2\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{3/2}} + \frac{2b\tanh^{-1}(\cos(x))}{a^3} - \frac{\left(a^2-2b^2\right)\cot(x)}{a^2\left(a^2-b^2\right)} - \frac{b^2\,\mathrm{c}}{a\left(a^2-b^2\right)(}$$

Mathematica [A] time = 0.72, size = 127, normalized size = 1.03

$$\frac{\frac{4b^2\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{2ab^3\cos(x)}{(a-b)(a+b)(a+b\sin(x))} + a\tan\left(\frac{x}{2}\right) - a\cot\left(\frac{x}{2}\right) - 4b\log\left(\sin\left(\frac{x}{2}\right)\right) + 4b\log\left(\cos\left(\frac{x}{2}\right)\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a + b*Sin[x])^2,x]
```

```
[Out] ((4*b^2*(3*a^2 - 2*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^
2)^(3/2) - a*Cot[x/2] + 4*b*Log[Cos[x/2]] - 4*b*Log[Sin[x/2]] + (2*a*b^3*Co
s[x])/((a - b)*(a + b)*(a + b*Sin[x])) + a*Tan[x/2])/(2*a^3)
```

fricas [B] time = 0.84, size = 784, normalized size = 6.37

$$\left[-\frac{2\left(a^5b-3\,a^3b^3+2\,ab^5\right)\cos(x)\sin(x)+\left(3\,a^2b^3-2\,b^5-\left(3\,a^2b^3-2\,b^5\right)\cos(x)^2+\left(3\,a^3b^2-2\,ab^4\right)\sin(x)\right)\sqrt{-a}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(x)*sin(x) + (3*a^2*b^3 - 2*b^5 -
 (3*a^2*b^3 - 2*b^5)*cos(x)^2 + (3*a^3*b^2 - 2*a*b^4)*sin(x))*sqrt(-a^2 + b
^2)*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 + 2*(a*cos(x)*si
n(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2
)) + 2*(a^6 - 2*a^4*b^2 + a^2*b^4)*cos(x) - 2*(a^4*b^2 - 2*a^2*b^4 + b^6 -
(a^4*b^2 - 2*a^2*b^4 + b^6)*cos(x)^2 + (a^5*b - 2*a^3*b^3 + a*b^5)*sin(x))*
log(1/2*cos(x) + 1/2) + 2*(a^4*b^2 - 2*a^2*b^4 + b^6 - (a^4*b^2 - 2*a^2*b^4
 + b^6)*cos(x)^2 + (a^5*b - 2*a^3*b^3 + a*b^5)*sin(x))*log(-1/2*cos(x) + 1/
2))/(a^7*b - 2*a^5*b^3 + a^3*b^5 - (a^7*b - 2*a^5*b^3 + a^3*b^5)*cos(x)^2 +
 (a^8 - 2*a^6*b^2 + a^4*b^4)*sin(x)), -((a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(x
)*sin(x) + (3*a^2*b^3 - 2*b^5 - (3*a^2*b^3 - 2*b^5)*cos(x)^2 + (3*a^3*b^2 -
 2*a*b^4)*sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*c
os(x))) + (a^6 - 2*a^4*b^2 + a^2*b^4)*cos(x) - (a^4*b^2 - 2*a^2*b^4 + b^6 -
 (a^4*b^2 - 2*a^2*b^4 + b^6)*cos(x)^2 + (a^5*b - 2*a^3*b^3 + a*b^5)*sin(x))
*log(1/2*cos(x) + 1/2) + (a^4*b^2 - 2*a^2*b^4 + b^6 - (a^4*b^2 - 2*a^2*b^4
+ b^6)*cos(x)^2 + (a^5*b - 2*a^3*b^3 + a*b^5)*sin(x))*log(-1/2*cos(x) + 1/2
))/(a^7*b - 2*a^5*b^3 + a^3*b^5 - (a^7*b - 2*a^5*b^3 + a^3*b^5)*cos(x)^2 +
(a^8 - 2*a^6*b^2 + a^4*b^4)*sin(x))]
```

giac [A] time = 0.15, size = 234, normalized size = 1.90

$$\frac{2\left(3\,a^2b^2-2\,b^4\right)\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^5-a^3b^2\right)\sqrt{a^2-b^2}}+\frac{4\,a^3b\tan\left(\frac{1}{2}\,x\right)^3-4\,ab^3\tan\left(\frac{1}{2}\,x\right)^3-3\,a^4\tan\left(\frac{1}{2}\,x\right)}{6\left(a^5-a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] 2*(3*a^2*b^2 - 2*b^4)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*
x) + b)/sqrt(a^2 - b^2)))/((a^5 - a^3*b^2)*sqrt(a^2 - b^2)) + 1/6*(4*a^3*b*
tan(1/2*x)^3 - 4*a*b^3*tan(1/2*x)^3 - 3*a^4*tan(1/2*x)^2 + 11*a^2*b^2*tan(1
/2*x)^2 + 4*b^4*tan(1/2*x)^2 - 2*a^3*b*tan(1/2*x) + 14*a*b^3*tan(1/2*x) - 3
```

```
*a^4 + 3*a^2*b^2)/((a^5 - a^3*b^2)*(a*tan(1/2*x)^3 + 2*b*tan(1/2*x)^2 + a*t
an(1/2*x))) - 2*b*log(abs(tan(1/2*x)))/a^3 + 1/2*tan(1/2*x)/a^2
```

maple [A] time = 0.14, size = 201, normalized size = 1.63

$$\frac{\tan\left(\frac{x}{2}\right)}{2a^2}-\frac{1}{2a^2\tan\left(\frac{x}{2}\right)}-\frac{2b\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^3}+\frac{2b^4\tan\left(\frac{x}{2}\right)}{a^3\left(\left(\tan^2\left(\frac{x}{2}\right)\right)a+2\tan\left(\frac{x}{2}\right)b+a\right)\left(a^2-b^2\right)}+\frac{2b^3}{a^2\left(\left(\tan^2\left(\frac{x}{2}\right)\right)a+2\tan\left(\frac{x}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a+b*sin(x))^2,x)
```

```
[Out] 1/2/a^2*tan(1/2*x)-1/2/a^2/tan(1/2*x)-2/a^3*b*ln(tan(1/2*x))+2/a^3*b^4/(tan
(1/2*x)^2*a+2*tan(1/2*x)*b+a)/(a^2-b^2)*tan(1/2*x)+2/a^2*b^3/(tan(1/2*x)^2*
a+2*tan(1/2*x)*b+a)/(a^2-b^2)+6/a*b^2/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1
/2*x)+2*b)/(a^2-b^2)^(1/2))-4/a^3*b^4/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1
/2*x)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.63, size = 1471, normalized size = 11.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(a + b*sin(x))^2),x)
```

```
[Out] tan(x/2)/(2*a^2) - (a - (tan(x/2)^2*(4*b^4 - a^4 + a^2*b^2))/(a*(a^2 - b^2)
) + (2*b*tan(x/2)*(a^2 - 3*b^2))/(a^2 - b^2))/(2*a^3*tan(x/2) + 2*a^3*tan(x
/2)^3 + 4*a^2*b*tan(x/2)^2) - (2*b*log(tan(x/2)))/a^3 - (b^2*atan(((b^2*(3*
a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((2*tan(x/2)*(8*a*b^7 - 2*a^7*b -
 20*a^3*b^5 + 14*a^5*b^3))/(a^7 + a^3*b^4 - 2*a^5*b^2) - (2*(4*a^3*b^4 - 5*
a^5*b^2))/(a^6 - a^4*b^2) + (b^2*(3*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/
```

```
2)*((2*(a^8*b - a^6*b^3))/(a^6 - a^4*b^2) - (2*tan(x/2)*(3*a^10 - 4*a^4*b^6
 + 11*a^6*b^4 - 10*a^8*b^2))/(a^7 + a^3*b^4 - 2*a^5*b^2)))/(a^9 - a^3*b^6 +
 3*a^5*b^4 - 3*a^7*b^2))*1i)/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2) - (b^2
*(3*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((2*(4*a^3*b^4 - 5*a^5*b^2))/
(a^6 - a^4*b^2) - (2*tan(x/2)*(8*a*b^7 - 2*a^7*b - 20*a^3*b^5 + 14*a^5*b^3)
)/(a^7 + a^3*b^4 - 2*a^5*b^2) + (b^2*(3*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)
^(1/2)*((2*(a^8*b - a^6*b^3))/(a^6 - a^4*b^2) - (2*tan(x/2)*(3*a^10 - 4*a^4
*b^6 + 11*a^6*b^4 - 10*a^8*b^2))/(a^7 + a^3*b^4 - 2*a^5*b^2)))/(a^9 - a^3*b
^6 + 3*a^5*b^4 - 3*a^7*b^2))*1i)/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))/(
(4*tan(x/2)*(4*b^6 - 6*a^2*b^4))/(a^7 + a^3*b^4 - 2*a^5*b^2) - (4*(4*b^5 -
6*a^2*b^3))/(a^6 - a^4*b^2) + (b^2*(3*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(
1/2)*((2*tan(x/2)*(8*a*b^7 - 2*a^7*b - 20*a^3*b^5 + 14*a^5*b^3))/(a^7 + a^3
*b^4 - 2*a^5*b^2) - (2*(4*a^3*b^4 - 5*a^5*b^2))/(a^6 - a^4*b^2) + (b^2*(3*a
^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((2*(a^8*b - a^6*b^3))/(a^6 - a^4*
b^2) - (2*tan(x/2)*(3*a^10 - 4*a^4*b^6 + 11*a^6*b^4 - 10*a^8*b^2))/(a^7 + a
^3*b^4 - 2*a^5*b^2)))/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))/(a^9 - a^3*
b^6 + 3*a^5*b^4 - 3*a^7*b^2) + (b^2*(3*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^
(1/2)*((2*(4*a^3*b^4 - 5*a^5*b^2))/(a^6 - a^4*b^2) - (2*tan(x/2)*(8*a*b^7 -
 2*a^7*b - 20*a^3*b^5 + 14*a^5*b^3))/(a^7 + a^3*b^4 - 2*a^5*b^2) + (b^2*(3*
a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((2*(a^8*b - a^6*b^3))/(a^6 - a^4
*b^2) - (2*tan(x/2)*(3*a^10 - 4*a^4*b^6 + 11*a^6*b^4 - 10*a^8*b^2))/(a^7 +
a^3*b^4 - 2*a^5*b^2)))/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))/(a^9 - a^3
*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))*(3*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2
)*2i)/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(x)}{(a+b\sin(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a+b*sin(x))**2,x)
```

```
[Out] Integral(csc(x)**2/(a + b*sin(x))**2, x)
```

3.192 $\int \frac{\csc^3(x)}{(a+b\sin(x))^2}\,dx$

Optimal. Leaf size=168

$$-\frac{\left(a^2-3b^2\right)\cot(x)\csc(x)}{2a^2\left(a^2-b^2\right)}-\frac{b^2\cot(x)\csc(x)}{a\left(a^2-b^2\right)(a+b\sin(x))}-\frac{\left(a^2+6b^2\right)\tanh^{-1}(\cos(x))}{2a^4}-\frac{2b^3\left(4a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{3/2}}$$

```
[Out] -2*b^3*(4*a^2-3*b^2)*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/a^4/(a^2-b^2)
^(3/2)-1/2*(a^2+6*b^2)*arctanh(cos(x))/a^4+b*(2*a^2-3*b^2)*cot(x)/a^3/(a^2-
b^2)-1/2*(a^2-3*b^2)*cot(x)*csc(x)/a^2/(a^2-b^2)-b^2*cot(x)*csc(x)/a/(a^2-b
^2)/(a+b*sin(x))
```

Rubi [A] time = 0.57, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2802, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{2b^3\left(4a^2-3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{3/2}}+\frac{b\left(2a^2-3b^2\right)\cot(x)}{a^3\left(a^2-b^2\right)}-\frac{\left(a^2+6b^2\right)\tanh^{-1}(\cos(x))}{2a^4}-\frac{\left(a^2-3b^2\right)\cot(x)\csc(x)}{2a^2\left(a^2-b^2\right)}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a + b*Sin[x])^2,x]
```

```
[Out] (-2*b^3*(4*a^2 - 3*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^4*(a^2
 - b^2)^(3/2)) - ((a^2 + 6*b^2)*ArcTanh[Cos[x]])/(2*a^4) + (b*(2*a^2 - 3*b^
2)*Cot[x])/(a^3*(a^2 - b^2)) - ((a^2 - 3*b^2)*Cot[x]*Csc[x])/(2*a^2*(a^2 -
b^2)) - (b^2*Cot[x]*Csc[x])/(a*(a^2 - b^2)*(a + b*Sin[x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
```

```
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^3(x)}{(a+b\sin(x))^2}\,dx = -\frac{b^2\cot(x)\csc(x)}{a\left(a^2-b^2\right)(a+b\sin(x))} + \frac{\int \frac{\csc^3(x)\left(a^2-3b^2-ab\sin(x)+2b^2\sin^2(x)\right)}{a+b\sin(x)}\,dx}{a\left(a^2-b^2\right)}$$

$$= -\frac{\left(a^2-3b^2\right)\cot(x)\csc(x)}{2a^2\left(a^2-b^2\right)} - \frac{b^2\cot(x)\csc(x)}{a\left(a^2-b^2\right)(a+b\sin(x))} + \frac{\int \frac{\csc^2(x)\left(-2b\left(2a^2-3b^2\right)+a\left(a^2+b^2\right)\sin(x)+\right.}{a+b\sin(x)}}{2a^2\left(a^2-b^2\right)}$$

$$= \frac{b\left(2a^2-3b^2\right)\cot(x)}{a^3\left(a^2-b^2\right)} - \frac{\left(a^2-3b^2\right)\cot(x)\csc(x)}{2a^2\left(a^2-b^2\right)} - \frac{b^2\cot(x)\csc(x)}{a\left(a^2-b^2\right)(a+b\sin(x))} + \frac{\int \frac{\csc(x)\left(a^4+5a\right.}{}}{}$$

$$= \frac{b\left(2a^2-3b^2\right)\cot(x)}{a^3\left(a^2-b^2\right)} - \frac{\left(a^2-3b^2\right)\cot(x)\csc(x)}{2a^2\left(a^2-b^2\right)} - \frac{b^2\cot(x)\csc(x)}{a\left(a^2-b^2\right)(a+b\sin(x))} - \frac{\left(b^3\left(4a^2-3\right.\right.}{a^4\left(\right.}$$

$$= -\frac{\left(a^2+6b^2\right)\tanh^{-1}(\cos(x))}{2a^4} + \frac{b\left(2a^2-3b^2\right)\cot(x)}{a^3\left(a^2-b^2\right)} - \frac{\left(a^2-3b^2\right)\cot(x)\csc(x)}{2a^2\left(a^2-b^2\right)} - \frac{b^2\,\mathrm{co}}{a\left(a^2-b^2\right.}$$

$$= -\frac{\left(a^2+6b^2\right)\tanh^{-1}(\cos(x))}{2a^4} + \frac{b\left(2a^2-3b^2\right)\cot(x)}{a^3\left(a^2-b^2\right)} - \frac{\left(a^2-3b^2\right)\cot(x)\csc(x)}{2a^2\left(a^2-b^2\right)} - \frac{b^2\,\mathrm{co}}{a\left(a^2-b^2\right.}$$

$$= -\frac{2b^3\left(4a^2-3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{3/2}} - \frac{\left(a^2+6b^2\right)\tanh^{-1}(\cos(x))}{2a^4} + \frac{b\left(2a^2-3b^2\right)\cot(x)}{a^3\left(a^2-b^2\right)} -$$

Mathematica [A] time = 1.01, size = 171, normalized size = 1.02

$$\frac{4\left(a^2+6b^2\right)\log\left(\sin\left(\frac{x}{2}\right)\right) - 4\left(a^2+6b^2\right)\log\left(\cos\left(\frac{x}{2}\right)\right) + \frac{16b^3\left(3b^2-4a^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} - a^2\csc^2\left(\frac{x}{2}\right) + a^2\sec^2\left(\frac{x}{2}\right) - \cdots}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a + b*Sin[x])^2,x]
```

```
[Out] ((16*b^3*(-4*a^2 + 3*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 -
b^2)^(3/2) + 8*a*b*Cot[x/2] - a^2*Csc[x/2]^2 - 4*(a^2 + 6*b^2)*Log[Cos[x/2]
```

```
] + 4*(a^2 + 6*b^2)*Log[Sin[x/2]] + a^2*Sec[x/2]^2 - (8*a*b^4*Cos[x])/((a -
 b)*(a + b)*(a + b*Sin[x])) - 8*a*b*Tan[x/2])/(8*a^4)
```

fricas [B] time = 1.27, size = 1174, normalized size = 6.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(2*a^5*b^2 - 5*a^3*b^4 + 3*a*b^6)*cos(x)^3 - 6*(a^6*b - 2*a^4*b^3
+ a^2*b^5)*cos(x)*sin(x) + 2*(4*a^3*b^3 - 3*a*b^5 - (4*a^3*b^3 - 3*a*b^5)*c
os(x)^2 + (4*a^2*b^4 - 3*b^6 - (4*a^2*b^4 - 3*b^6)*cos(x)^2)*sin(x))*sqrt(-
a^2 + b^2)*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 - 2*(a*c
os(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a
^2 - b^2)) + 2*(a^7 - 6*a^5*b^2 + 11*a^3*b^4 - 6*a*b^6)*cos(x) + (a^7 + 4*a
^5*b^2 - 11*a^3*b^4 + 6*a*b^6 - (a^7 + 4*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*co
s(x)^2 + (a^6*b + 4*a^4*b^3 - 11*a^2*b^5 + 6*b^7 - (a^6*b + 4*a^4*b^3 - 11*
a^2*b^5 + 6*b^7)*cos(x)^2)*sin(x))*log(1/2*cos(x) + 1/2) - (a^7 + 4*a^5*b^2
 - 11*a^3*b^4 + 6*a*b^6 - (a^7 + 4*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*cos(x)^2
 + (a^6*b + 4*a^4*b^3 - 11*a^2*b^5 + 6*b^7 - (a^6*b + 4*a^4*b^3 - 11*a^2*b^
5 + 6*b^7)*cos(x)^2)*sin(x))*log(-1/2*cos(x) + 1/2))/(a^9 - 2*a^7*b^2 + a^5
*b^4 - (a^9 - 2*a^7*b^2 + a^5*b^4)*cos(x)^2 + (a^8*b - 2*a^6*b^3 + a^4*b^5
- (a^8*b - 2*a^6*b^3 + a^4*b^5)*cos(x)^2)*sin(x)), -1/4*(4*(2*a^5*b^2 - 5*a
^3*b^4 + 3*a*b^6)*cos(x)^3 - 6*(a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(x)*sin(x)
- 4*(4*a^3*b^3 - 3*a*b^5 - (4*a^3*b^3 - 3*a*b^5)*cos(x)^2 + (4*a^2*b^4 - 3*
b^6 - (4*a^2*b^4 - 3*b^6)*cos(x)^2)*sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(
x) + b)/(sqrt(a^2 - b^2)*cos(x))) + 2*(a^7 - 6*a^5*b^2 + 11*a^3*b^4 - 6*a*b
^6)*cos(x) + (a^7 + 4*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6 - (a^7 + 4*a^5*b^2 - 1
1*a^3*b^4 + 6*a*b^6)*cos(x)^2 + (a^6*b + 4*a^4*b^3 - 11*a^2*b^5 + 6*b^7 - (
a^6*b + 4*a^4*b^3 - 11*a^2*b^5 + 6*b^7)*cos(x)^2)*sin(x))*log(1/2*cos(x) +
1/2) - (a^7 + 4*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6 - (a^7 + 4*a^5*b^2 - 11*a^3*
b^4 + 6*a*b^6)*cos(x)^2 + (a^6*b + 4*a^4*b^3 - 11*a^2*b^5 + 6*b^7 - (a^6*b
+ 4*a^4*b^3 - 11*a^2*b^5 + 6*b^7)*cos(x)^2)*sin(x))*log(-1/2*cos(x) + 1/2))
/(a^9 - 2*a^7*b^2 + a^5*b^4 - (a^9 - 2*a^7*b^2 + a^5*b^4)*cos(x)^2 + (a^8*b
 - 2*a^6*b^3 + a^4*b^5 - (a^8*b - 2*a^6*b^3 + a^4*b^5)*cos(x)^2)*sin(x))]
```

giac [A] time = 0.18, size = 215, normalized size = 1.28

$$-\frac{2\left(4\,a^{2}b^{3}-3\,b^{5}\right)\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{6}-a^{4}b^{2}\right)\sqrt{a^{2}-b^{2}}}-\frac{2\left(b^{5}\tan\left(\frac{1}{2}\,x\right)+ab^{4}\right)}{\left(a^{6}-a^{4}b^{2}\right)\left(a\tan\left(\frac{1}{2}\,x\right)^{2}+2\,b\tan\left(\frac{1}{2}\,x\right)+a\right)}+\frac{(a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*(4*a^2*b^3 - 3*b^5)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2
*x) + b)/sqrt(a^2 - b^2)))/((a^6 - a^4*b^2)*sqrt(a^2 - b^2)) - 2*(b^5*tan(1
/2*x) + a*b^4)/((a^6 - a^4*b^2)*(a*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)) + 1/
2*(a^2 + 6*b^2)*log(abs(tan(1/2*x)))/a^4 + 1/8*(a^2*tan(1/2*x)^2 - 8*a*b*ta
n(1/2*x))/a^4 - 1/8*(6*a^2*tan(1/2*x)^2 + 36*b^2*tan(1/2*x)^2 - 8*a*b*tan(1
/2*x) + a^2)/(a^4*tan(1/2*x)^2)
```

maple [A] time = 0.15, size = 236, normalized size = 1.40

$$\frac{\tan^2\left(\frac{x}{2}\right)}{8a^2}-\frac{\tan\left(\frac{x}{2}\right)b}{a^3}-\frac{1}{8a^2\tan\left(\frac{x}{2}\right)^2}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a^2}+\frac{3\ln\left(\tan\left(\frac{x}{2}\right)\right)b^2}{a^4}+\frac{b}{a^3\tan\left(\frac{x}{2}\right)}-\frac{2b^5\tan\left(\frac{x}{2}\right)}{a^4\left(\left(\tan^2\left(\frac{x}{2}\right)\right)a+2\tan\left(\frac{x}{2}\right)b+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a+b*sin(x))^2,x)
```

```
[Out] 1/8/a^2*tan(1/2*x)^2-1/a^3*tan(1/2*x)*b-1/8/a^2/tan(1/2*x)^2+1/2/a^2*ln(tan
(1/2*x))+3/a^4*ln(tan(1/2*x))*b^2+b/a^3/tan(1/2*x)-2/a^4*b^5/(tan(1/2*x)^2*
a+2*tan(1/2*x)*b+a)/(a^2-b^2)*tan(1/2*x)-2/a^3*b^4/(tan(1/2*x)^2*a+2*tan(1/
2*x)*b+a)/(a^2-b^2)-8/a^2*b^3/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*
b)/(a^2-b^2)^(1/2))+6/a^4*b^5/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*
b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.67, size = 1576, normalized size = 9.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^3*(a + b*sin(x))^2),x)
```

```
[Out] tan(x/2)^2/(8*a^2) - (a^2/2 - 3*a*b*tan(x/2) + (tan(x/2)^2*(a^4 + 32*b^4 -
17*a^2*b^2))/(2*(a^2 - b^2)) + (4*b*tan(x/2)^3*(2*b^4 - a^4 + a^2*b^2))/(a*
(a^2 - b^2)))/(4*a^4*tan(x/2)^2 + 4*a^4*tan(x/2)^4 + 8*a^3*b*tan(x/2)^3) +
(log(tan(x/2))*(a^2 + 6*b^2))/(2*a^4) - (b*tan(x/2))/a^3 + (b^3*atan(((b^3*
(4*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((a^8*b - 12*a^4*b^5 + 13*a^6*
b^3)/(a^8 - a^6*b^2) - (tan(x/2)*(a^10 - 24*a^2*b^8 + 56*a^4*b^6 - 35*a^6*b
^4 + 2*a^8*b^2))/(a^9 + a^5*b^4 - 2*a^7*b^2) + (b^3*(4*a^2 - 3*b^2)*(-(a +
b)^3*(a - b)^3)^(1/2)*((2*a^10*b - 2*a^8*b^3)/(a^8 - a^6*b^2) - (tan(x/2)*(
6*a^12 - 8*a^6*b^6 + 22*a^8*b^4 - 20*a^10*b^2))/(a^9 + a^5*b^4 - 2*a^7*b^2)
))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))*1i)/(a^10 - a^4*b^6 + 3*a^6*b^
4 - 3*a^8*b^2) - (b^3*(4*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((tan(x/
2)*(a^10 - 24*a^2*b^8 + 56*a^4*b^6 - 35*a^6*b^4 + 2*a^8*b^2))/(a^9 + a^5*b^
4 - 2*a^7*b^2) - (a^8*b - 12*a^4*b^5 + 13*a^6*b^3)/(a^8 - a^6*b^2) + (b^3*(
4*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((2*a^10*b - 2*a^8*b^3)/(a^8 -
a^6*b^2) - (tan(x/2)*(6*a^12 - 8*a^6*b^6 + 22*a^8*b^4 - 20*a^10*b^2))/(a^9
+ a^5*b^4 - 2*a^7*b^2)))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))*1i)/(a^1
0 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))/((2*(21*a^2*b^5 - 18*b^7 + 4*a^4*b^3)
)/(a^8 - a^6*b^2) + (2*tan(x/2)*(18*b^8 - 30*a^2*b^6 + 8*a^4*b^4))/(a^9 + a
^5*b^4 - 2*a^7*b^2) + (b^3*(4*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((a
^8*b - 12*a^4*b^5 + 13*a^6*b^3)/(a^8 - a^6*b^2) - (tan(x/2)*(a^10 - 24*a^2*
b^8 + 56*a^4*b^6 - 35*a^6*b^4 + 2*a^8*b^2))/(a^9 + a^5*b^4 - 2*a^7*b^2) + (
b^3*(4*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((2*a^10*b - 2*a^8*b^3)/(a
^8 - a^6*b^2) - (tan(x/2)*(6*a^12 - 8*a^6*b^6 + 22*a^8*b^4 - 20*a^10*b^2))/
(a^9 + a^5*b^4 - 2*a^7*b^2)))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))/(a
^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2) + (b^3*(4*a^2 - 3*b^2)*(-(a + b)^3*(
a - b)^3)^(1/2)*((tan(x/2)*(a^10 - 24*a^2*b^8 + 56*a^4*b^6 - 35*a^6*b^4 + 2
*a^8*b^2))/(a^9 + a^5*b^4 - 2*a^7*b^2) - (a^8*b - 12*a^4*b^5 + 13*a^6*b^3)/
(a^8 - a^6*b^2) + (b^3*(4*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((2*a^1
0*b - 2*a^8*b^3)/(a^8 - a^6*b^2) - (tan(x/2)*(6*a^12 - 8*a^6*b^6 + 22*a^8*b
^4 - 20*a^10*b^2))/(a^9 + a^5*b^4 - 2*a^7*b^2)))/(a^10 - a^4*b^6 + 3*a^6*b^
4 - 3*a^8*b^2)))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))*(4*a^2 - 3*b^2)
*(-(a + b)^3*(a - b)^3)^(1/2)*2i)/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(x)}{(a + b\sin(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a+b*sin(x))**2,x)
```

```
[Out] Integral(csc(x)**3/(a + b*sin(x))**2, x)
```

3.193 $\int \frac{\sin^5(x)}{(a+b\sin(x))^3}\,dx$

Optimal. Leaf size=243

$$\frac{a^2\sin^3(x)\cos(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2}+\frac{a^2\left(4a^2-7b^2\right)\sin^2(x)\cos(x)}{2b^2\left(a^2-b^2\right)^2(a+b\sin(x))}+\frac{x\left(12a^2+b^2\right)}{2b^5}+\frac{3a\left(4a^4-7a^2b^2+2b^4\right)\cos(x)}{2b^4\left(a^2-b^2\right)^2}-\frac{\left(6a^4-\right.}{}$$

```
[Out] 1/2*(12*a^2+b^2)*x/b^5-a^3*(12*a^4-29*a^2*b^2+20*b^4)*arctan((b+a*tan(1/2*x
))/(a^2-b^2)^(1/2))/b^5/(a^2-b^2)^(5/2)+3/2*a*(4*a^4-7*a^2*b^2+2*b^4)*cos(x
)/b^4/(a^2-b^2)^2-1/2*(6*a^4-10*a^2*b^2+b^4)*cos(x)*sin(x)/b^3/(a^2-b^2)^2+
1/2*a^2*cos(x)*sin(x)^3/b/(a^2-b^2)/(a+b*sin(x))^2+1/2*a^2*(4*a^2-7*b^2)*co
s(x)*sin(x)^2/b^2/(a^2-b^2)^2/(a+b*sin(x))
```

Rubi [A] time = 0.66, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {2792, 3047, 3049, 3023, 2735, 2660, 618, 204}

$$\frac{x\left(12a^2+b^2\right)}{2b^5}+\frac{3a\left(-7a^2b^2+4a^4+2b^4\right)\cos(x)}{2b^4\left(a^2-b^2\right)^2}-\frac{a^3\left(-29a^2b^2+12a^4+20b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5\left(a^2-b^2\right)^{5/2}}+\frac{a^2\left(4a^2-7b^2\right)\sin}{2b^2\left(a^2-b^2\right)^2(a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^5/(a + b*Sin[x])^3,x]
```

```
[Out] ((12*a^2 + b^2)*x)/(2*b^5) - (a^3*(12*a^4 - 29*a^2*b^2 + 20*b^4)*ArcTan[(b
+ a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b^5*(a^2 - b^2)^(5/2)) + (3*a*(4*a^4 - 7*a
^2*b^2 + 2*b^4)*Cos[x])/(2*b^4*(a^2 - b^2)^2) - ((6*a^4 - 10*a^2*b^2 + b^4)
*Cos[x]*Sin[x])/(2*b^3*(a^2 - b^2)^2) + (a^2*Cos[x]*Sin[x]^3)/(2*b*(a^2 - b
^2)*(a + b*Sin[x])^2) + (a^2*(4*a^2 - 7*b^2)*Cos[x]*Sin[x]^2)/(2*b^2*(a^2 -
 b^2)^2*(a + b*Sin[x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
```

```
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\sin^5(x)}{(a+b\sin(x))^3}\,dx = \frac{a^2\cos(x)\sin^3(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{\int \frac{\sin^2(x)\left(3a^2-2ab\sin(x)-2\left(2a^2-b^2\right)\sin^2(x)\right)}{(a+b\sin(x))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= \frac{a^2\cos(x)\sin^3(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{a^2\left(4a^2-7b^2\right)\cos(x)\sin^2(x)}{2b^2\left(a^2-b^2\right)^2(a+b\sin(x))} + \frac{\int \frac{\sin(x)\left(-2a^2\left(4a^2-7b^2\right)+ab\left(a^2-\right.\right.}{a}}{2b^2}$$

$$= -\frac{\left(6a^4-10a^2b^2+b^4\right)\cos(x)\sin(x)}{2b^3\left(a^2-b^2\right)^2} + \frac{a^2\cos(x)\sin^3(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{a^2\left(4a^2-7b^2\right)\cos(x}{2b^2\left(a^2-b^2\right)^2(a+}$$

$$= \frac{3a\left(4a^4-7a^2b^2+2b^4\right)\cos(x)}{2b^4\left(a^2-b^2\right)^2} - \frac{\left(6a^4-10a^2b^2+b^4\right)\cos(x)\sin(x)}{2b^3\left(a^2-b^2\right)^2} + \frac{a^2\cos(x)\sin^3}{2b\left(a^2-b^2\right)(a+b}$$

$$= \frac{\left(12a^2+b^2\right)x}{2b^5} + \frac{3a\left(4a^4-7a^2b^2+2b^4\right)\cos(x)}{2b^4\left(a^2-b^2\right)^2} - \frac{\left(6a^4-10a^2b^2+b^4\right)\cos(x)\sin(x)}{2b^3\left(a^2-b^2\right)^2} + \frac{}{2b}$$

$$= \frac{\left(12a^2+b^2\right)x}{2b^5} + \frac{3a\left(4a^4-7a^2b^2+2b^4\right)\cos(x)}{2b^4\left(a^2-b^2\right)^2} - \frac{\left(6a^4-10a^2b^2+b^4\right)\cos(x)\sin(x)}{2b^3\left(a^2-b^2\right)^2} + \frac{}{2b}$$

$$= \frac{\left(12a^2+b^2\right)x}{2b^5} + \frac{3a\left(4a^4-7a^2b^2+2b^4\right)\cos(x)}{2b^4\left(a^2-b^2\right)^2} - \frac{\left(6a^4-10a^2b^2+b^4\right)\cos(x)\sin(x)}{2b^3\left(a^2-b^2\right)^2} + \frac{}{2b}$$

$$= \frac{\left(12a^2+b^2\right)x}{2b^5} - \frac{a^3\left(12a^4-29a^2b^2+20b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^5\left(a^2-b^2\right)^{5/2}} + \frac{3a\left(4a^4-7a^2b^2+2b^4\right)c}{2b^4\left(a^2-b^2\right)^2}$$

Mathematica [A] time = 1.04, size = 164, normalized size = 0.67

$$\frac{-\frac{2a^5b\cos(x)}{(a-b)(a+b)(a+b\sin(x))^2} + 2x\left(12a^2+b^2\right) + \frac{2a^4b\left(7a^2-10b^2\right)\cos(x)}{(a-b)^2(a+b)^2(a+b\sin(x))} - \frac{4a^3\left(12a^4-29a^2b^2+20b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + 12ab\cos(x) -}{4b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^5/(a + b*Sin[x])^3,x]
```

```
[Out] (2*(12*a^2 + b^2)*x - (4*a^3*(12*a^4 - 29*a^2*b^2 + 20*b^4)*ArcTan[(b + a*T
an[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + 12*a*b*Cos[x] - (2*a^5*b*Cos
[x])/((a - b)*(a + b)*(a + b*Sin[x])^2) + (2*a^4*b*(7*a^2 - 10*b^2)*Cos[x])
/((a - b)^2*(a + b)^2*(a + b*Sin[x])) - b^2*Sin[2*x])/(4*b^5)
```

fricas [B] time = 0.64, size = 1090, normalized size = 4.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(a+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(12*a^8*b^2 - 35*a^6*b^4 + 33*a^4*b^6 - 9*a^2*b^8 - b^10)*x*cos(x)
^2 + 8*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*cos(x)^3 + (12*a^9 - 17*a^
7*b^2 - 9*a^5*b^4 + 20*a^3*b^6 - (12*a^7*b^2 - 29*a^5*b^4 + 20*a^3*b^6)*cos
(x)^2 + 2*(12*a^8*b - 29*a^6*b^3 + 20*a^4*b^5)*sin(x))*sqrt(-a^2 + b^2)*log
(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 - 2*(a*cos(x)*sin(x) +
 b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) - 2
*(12*a^10 - 23*a^8*b^2 - 2*a^6*b^4 + 24*a^4*b^6 - 10*a^2*b^8 - b^10)*x - 2*
(12*a^9*b - 29*a^7*b^3 + 15*a^5*b^5 + 6*a^3*b^7 - 4*a*b^9)*cos(x) - 2*((a^6
*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*cos(x)^3 + 2*(12*a^9*b - 35*a^7*b^3 +
33*a^5*b^5 - 9*a^3*b^7 - a*b^9)*x + (18*a^8*b^2 - 51*a^6*b^4 + 46*a^4*b^6 -
 14*a^2*b^8 + b^10)*cos(x))*sin(x))/(a^8*b^5 - 2*a^6*b^7 + 2*a^2*b^11 - b^1
3 - (a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*cos(x)^2 + 2*(a^7*b^6 - 3*a^5
*b^8 + 3*a^3*b^10 - a*b^12)*sin(x)), -1/2*((12*a^8*b^2 - 35*a^6*b^4 + 33*a^
4*b^6 - 9*a^2*b^8 - b^10)*x*cos(x)^2 + 4*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 -
 a*b^9)*cos(x)^3 - (12*a^9 - 17*a^7*b^2 - 9*a^5*b^4 + 20*a^3*b^6 - (12*a^7*
b^2 - 29*a^5*b^4 + 20*a^3*b^6)*cos(x)^2 + 2*(12*a^8*b - 29*a^6*b^3 + 20*a^4
*b^5)*sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x
))) - (12*a^10 - 23*a^8*b^2 - 2*a^6*b^4 + 24*a^4*b^6 - 10*a^2*b^8 - b^10)*x
 - (12*a^9*b - 29*a^7*b^3 + 15*a^5*b^5 + 6*a^3*b^7 - 4*a*b^9)*cos(x) - ((a^
6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*cos(x)^3 + 2*(12*a^9*b - 35*a^7*b^3 +
 33*a^5*b^5 - 9*a^3*b^7 - a*b^9)*x + (18*a^8*b^2 - 51*a^6*b^4 + 46*a^4*b^6
- 14*a^2*b^8 + b^10)*cos(x))*sin(x))/(a^8*b^5 - 2*a^6*b^7 + 2*a^2*b^11 - b^
13 - (a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*cos(x)^2 + 2*(a^7*b^6 - 3*a^
5*b^8 + 3*a^3*b^10 - a*b^12)*sin(x))]
```

giac [B] time = 0.42, size = 516, normalized size = 2.12

$$-\frac{\left(12\,a^{7}-29\,a^{5}b^{2}+20\,a^{3}b^{4}\right)\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{4}b^{5}-2\,a^{2}b^{7}+b^{9}\right)\sqrt{a^{2}-b^{2}}}+\frac{6\,a^{6}b\tan\left(\frac{1}{2}\,x\right)^{7}-10\,a^{4}b^{3}\tan\left(\frac{1}{2}\,x\right)^{7}+a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(a+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] -(12*a^7 - 29*a^5*b^2 + 20*a^3*b^4)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arct
an((a*tan(1/2*x) + b)/sqrt(a^2 - b^2)))/((a^4*b^5 - 2*a^2*b^7 + b^9)*sqrt(a
^2 - b^2)) + (6*a^6*b*tan(1/2*x)^7 - 10*a^4*b^3*tan(1/2*x)^7 + a^2*b^5*tan(
1/2*x)^7 + 12*a^7*tan(1/2*x)^6 - 5*a^5*b^2*tan(1/2*x)^6 - 20*a^3*b^4*tan(1/
2*x)^6 + 4*a*b^6*tan(1/2*x)^6 + 54*a^6*b*tan(1/2*x)^5 - 90*a^4*b^3*tan(1/2*
x)^5 + 17*a^2*b^5*tan(1/2*x)^5 + 4*b^7*tan(1/2*x)^5 + 36*a^7*tan(1/2*x)^4 -
 15*a^5*b^2*tan(1/2*x)^4 - 66*a^3*b^4*tan(1/2*x)^4 + 24*a*b^6*tan(1/2*x)^4
+ 90*a^6*b*tan(1/2*x)^3 - 162*a^4*b^3*tan(1/2*x)^3 + 55*a^2*b^5*tan(1/2*x)^
3 - 4*b^7*tan(1/2*x)^3 + 36*a^7*tan(1/2*x)^2 - 31*a^5*b^2*tan(1/2*x)^2 - 40
*a^3*b^4*tan(1/2*x)^2 + 20*a*b^6*tan(1/2*x)^2 + 42*a^6*b*tan(1/2*x) - 74*a^
4*b^3*tan(1/2*x) + 23*a^2*b^5*tan(1/2*x) + 12*a^7 - 21*a^5*b^2 + 6*a^3*b^4)
/((a^4*b^4 - 2*a^2*b^6 + b^8)*(a*tan(1/2*x)^4 + 2*b*tan(1/2*x)^3 + 2*a*tan(
1/2*x)^2 + 2*b*tan(1/2*x) + a)^2) + 1/2*(12*a^2 + b^2)*x/b^5
```

maple [B] time = 0.13, size = 712, normalized size = 2.93

$$\frac{\tan^3\left(\frac{x}{2}\right)}{b^3\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}+\frac{6\left(\tan^2\left(\frac{x}{2}\right)\right)a}{b^4\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}-\frac{\tan\left(\frac{x}{2}\right)}{b^3\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}+\frac{6a}{b^4\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}+\frac{12\arctan\left(\tan\left(\frac{x}{2}\right)\right)a^2}{b^5}+\frac{}{b^3\left(\left(\tan\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^5/(a+b*sin(x))^3,x)
```

```
[Out] 1/b^3/(tan(1/2*x)^2+1)^2*tan(1/2*x)^3+6/b^4/(tan(1/2*x)^2+1)^2*tan(1/2*x)^2
*a-1/b^3/(tan(1/2*x)^2+1)^2*tan(1/2*x)+6/b^4/(tan(1/2*x)^2+1)^2*a+12/b^5*ar
ctan(tan(1/2*x))*a^2+5*a^6/b^3/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a
^2*b^2+b^4)*tan(1/2*x)^3-8*a^4/b/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2
*a^2*b^2+b^4)*tan(1/2*x)^3+6*a^7/b^4/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a
^4-2*a^2*b^2+b^4)*tan(1/2*x)^2+3*a^5/b^2/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^
2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^2-18*a^3/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)
^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^2+19*a^6/b^3/(tan(1/2*x)^2*a+2*tan(1/2*x)
*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)-28*a^4/b/(tan(1/2*x)^2*a+2*tan(1/2*x
)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)+6*a^7/b^4/(tan(1/2*x)^2*a+2*tan(1/2
*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)-9*a^5/b^2/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^
2/(a^4-2*a^2*b^2+b^4)-12*a^7/b^5/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan
(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))+29*a^5/b^3/(a^4-2*a^2*b^2+b^4)/(
a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))-20*a^3/b/(a
^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)
^(1/2))+1/2*x/b^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(a+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 15.81, size = 6640, normalized size = 27.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^5/(a + b*sin(x))^3,x)
```

```
[Out] ((3*(4*a^7 + 2*a^3*b^4 - 7*a^5*b^2))/(b^4*(a^4 + b^4 - 2*a^2*b^2)) + (tan(x
/2)^7*(6*a^6 + a^2*b^4 - 10*a^4*b^2))/(b^3*(a^4 + b^4 - 2*a^2*b^2)) + (tan(
x/2)^5*(54*a^6 + 4*b^6 + 17*a^2*b^4 - 90*a^4*b^2))/(b^3*(a^4 + b^4 - 2*a^2*
b^2)) + (tan(x/2)^3*(90*a^6 - 4*b^6 + 55*a^2*b^4 - 162*a^4*b^2))/(b^3*(a^4
+ b^4 - 2*a^2*b^2)) + (tan(x/2)^6*(4*a*b^6 + 12*a^7 - 20*a^3*b^4 - 5*a^5*b^
2))/(b^4*(a^4 + b^4 - 2*a^2*b^2)) + (tan(x/2)^2*(20*a*b^6 + 36*a^7 - 40*a^3
*b^4 - 31*a^5*b^2))/(b^4*(a^4 + b^4 - 2*a^2*b^2)) + (tan(x/2)*(42*a^6 + 23*
a^2*b^4 - 74*a^4*b^2))/(b^3*(a^4 + b^4 - 2*a^2*b^2)) + (3*tan(x/2)^4*(3*a^2
 + 4*b^2)*(2*a*b^4 + 4*a^5 - 7*a^3*b^2))/(b^4*(a^4 + b^4 - 2*a^2*b^2)))/(ta
n(x/2)^2*(4*a^2 + 4*b^2) + tan(x/2)^6*(4*a^2 + 4*b^2) + tan(x/2)^4*(6*a^2 +
 8*b^2) + a^2 + a^2*tan(x/2)^8 + 4*a*b*tan(x/2) + 12*a*b*tan(x/2)^3 + 12*a*
b*tan(x/2)^5 + 4*a*b*tan(x/2)^7) + (atan((((a^2*12i + b^2*1i)*((4*(2*a^2*b^
16 + 40*a^4*b^14 + 108*a^6*b^12 - 872*a^8*b^10 + 1538*a^10*b^8 - 1104*a^12*
b^6 + 288*a^14*b^4))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^1
1) - ((a^2*12i + b^2*1i)*((4*(4*a*b^20 + 28*a^3*b^18 - 120*a^5*b^16 + 164*a
^7*b^14 - 100*a^9*b^12 + 24*a^11*b^10))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4
*a^6*b^13 + a^8*b^11) - (((4*(8*a^2*b^22 - 32*a^4*b^20 + 48*a^6*b^18 - 32*a
^8*b^16 + 8*a^10*b^14))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*
b^11) + (8*tan(x/2)*(12*a*b^24 - 56*a^3*b^22 + 104*a^5*b^20 - 96*a^7*b^18 +
 44*a^9*b^16 - 8*a^11*b^14))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 +
 a^8*b^12))*(a^2*12i + b^2*1i))/(2*b^5) + (8*tan(x/2)*(80*a^4*b^18 - 276*a^
6*b^16 + 360*a^8*b^14 - 212*a^10*b^12 + 48*a^12*b^10))/(b^20 - 4*a^2*b^18 +
 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12)))/(2*b^5) + (8*tan(x/2)*(2*a*b^18 + 39
*a^3*b^16 + 88*a^5*b^14 - 1326*a^7*b^12 + 3134*a^9*b^10 - 3194*a^11*b^8 + 1
536*a^13*b^6 - 288*a^15*b^4))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14
+ a^8*b^12))*1i)/(2*b^5) + ((a^2*12i + b^2*1i)*((4*(2*a^2*b^16 + 40*a^4*b^1
4 + 108*a^6*b^12 - 872*a^8*b^10 + 1538*a^10*b^8 - 1104*a^12*b^6 + 288*a^14*
b^4))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) + ((a^2*12i
+ b^2*1i)*((4*(4*a*b^20 + 28*a^3*b^18 - 120*a^5*b^16 + 164*a^7*b^14 - 100*a
```

^9*b^12 + 24*a^11*b^10))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8
b^11) + (((4(8*a^2*b^22 - 32*a^4*b^20 + 48*a^6*b^18 - 32*a^8*b^16 + 8*a^1
0*b^14))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) + (8*tan(
x/2)*(12*a*b^24 - 56*a^3*b^22 + 104*a^5*b^20 - 96*a^7*b^18 + 44*a^9*b^16 -
8*a^11*b^14))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12))*(a^
2*12i + b^2*1i))/(2*b^5) + (8*tan(x/2)*(80*a^4*b^18 - 276*a^6*b^16 + 360*a^
8*b^14 - 212*a^10*b^12 + 48*a^12*b^10))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4
*a^6*b^14 + a^8*b^12)))/(2*b^5) + (8*tan(x/2)*(2*a*b^18 + 39*a^3*b^16 + 88*
a^5*b^14 - 1326*a^7*b^12 + 3134*a^9*b^10 - 3194*a^11*b^8 + 1536*a^13*b^6 -
288*a^15*b^4))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12))*1i
)/(2*b^5))/((8*(864*a^15 + 20*a^5*b^10 + 11*a^7*b^8 - 2326*a^9*b^6 + 4770*a
^11*b^4 - 3456*a^13*b^2))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^
8*b^11) - ((a^2*12i + b^2*1i)*((4*(2*a^2*b^16 + 40*a^4*b^14 + 108*a^6*b^12
- 872*a^8*b^10 + 1538*a^10*b^8 - 1104*a^12*b^6 + 288*a^14*b^4))/(b^19 - 4*a
^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) - ((a^2*12i + b^2*1i)*((4*(4*
a*b^20 + 28*a^3*b^18 - 120*a^5*b^16 + 164*a^7*b^14 - 100*a^9*b^12 + 24*a^11
*b^10))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) - (((4*(8*
a^2*b^22 - 32*a^4*b^20 + 48*a^6*b^18 - 32*a^8*b^16 + 8*a^10*b^14))/(b^19 -
4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) + (8*tan(x/2)*(12*a*b^24 -
56*a^3*b^22 + 104*a^5*b^20 - 96*a^7*b^18 + 44*a^9*b^16 - 8*a^11*b^14))/(b^
20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12))*(a^2*12i + b^2*1i))/
(2*b^5) + (8*tan(x/2)*(80*a^4*b^18 - 276*a^6*b^16 + 360*a^8*b^14 - 212*a^10
*b^12 + 48*a^12*b^10))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b
^12)))/(2*b^5) + (8*tan(x/2)*(2*a*b^18 + 39*a^3*b^16 + 88*a^5*b^14 - 1326*a
^7*b^12 + 3134*a^9*b^10 - 3194*a^11*b^8 + 1536*a^13*b^6 - 288*a^15*b^4))/(b
^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12)))/(2*b^5) + ((a^2*12
i + b^2*1i)*((4*(2*a^2*b^16 + 40*a^4*b^14 + 108*a^6*b^12 - 872*a^8*b^10 + 1
538*a^10*b^8 - 1104*a^12*b^6 + 288*a^14*b^4))/(b^19 - 4*a^2*b^17 + 6*a^4*b^
15 - 4*a^6*b^13 + a^8*b^11) + ((a^2*12i + b^2*1i)*((4*(4*a*b^20 + 28*a^3*b^
18 - 120*a^5*b^16 + 164*a^7*b^14 - 100*a^9*b^12 + 24*a^11*b^10))/(b^19 - 4*
a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) + (((4*(8*a^2*b^22 - 32*a^4*
b^20 + 48*a^6*b^18 - 32*a^8*b^16 + 8*a^10*b^14))/(b^19 - 4*a^2*b^17 + 6*a^4
*b^15 - 4*a^6*b^13 + a^8*b^11) + (8*tan(x/2)*(12*a*b^24 - 56*a^3*b^22 + 104
*a^5*b^20 - 96*a^7*b^18 + 44*a^9*b^16 - 8*a^11*b^14))/(b^20 - 4*a^2*b^18 +
6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12))*(a^2*12i + b^2*1i))/(2*b^5) + (8*tan(x
/2)*(80*a^4*b^18 - 276*a^6*b^16 + 360*a^8*b^14 - 212*a^10*b^12 + 48*a^12*b^
10))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12)))/(2*b^5) + (
8*tan(x/2)*(2*a*b^18 + 39*a^3*b^16 + 88*a^5*b^14 - 1326*a^7*b^12 + 3134*a^9
*b^10 - 3194*a^11*b^8 + 1536*a^13*b^6 - 288*a^15*b^4))/(b^20 - 4*a^2*b^18 +
6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12)))/(2*b^5) + (16*tan(x/2)*(1728*a^16 +
20*a^4*b^12 + 411*a^6*b^10 + 1314*a^8*b^8 - 7829*a^10*b^6 + 11700*a^12*b^4
- 7344*a^14*b^2))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12))
)*(a^2*12i + b^2*1i)*1i)/b^5 + (a^3*atan(((a^3*(-(a + b)^5*(a - b)^5)^(1/2)
((4(2*a^2*b^16 + 40*a^4*b^14 + 108*a^6*b^12 - 872*a^8*b^10 + 1538*a^10*b^
8 - 1104*a^12*b^6 + 288*a^14*b^4))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*

```
b^13 + a^8*b^11) + (8*tan(x/2)*(2*a*b^18 + 39*a^3*b^16 + 88*a^5*b^14 - 1326
*a^7*b^12 + 3134*a^9*b^10 - 3194*a^11*b^8 + 1536*a^13*b^6 - 288*a^15*b^4))/
(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12) - (a^3*(-(a + b)^5
*(a - b)^5)^(1/2)*(12*a^4 + 20*b^4 - 29*a^2*b^2)*((4*(4*a*b^20 + 28*a^3*b^1
8 - 120*a^5*b^16 + 164*a^7*b^14 - 100*a^9*b^12 + 24*a^11*b^10))/(b^19 - 4*a
^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) + (8*tan(x/2)*(80*a^4*b^18 -
276*a^6*b^16 + 360*a^8*b^14 - 212*a^10*b^12 + 48*a^12*b^10))/(b^20 - 4*a^2*
b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12) - (a^3*((4*(8*a^2*b^22 - 32*a^4*
b^20 + 48*a^6*b^18 - 32*a^8*b^16 + 8*a^10*b^14))/(b^19 - 4*a^2*b^17 + 6*a^4
*b^15 - 4*a^6*b^13 + a^8*b^11) + (8*tan(x/2)*(12*a*b^24 - 56*a^3*b^22 + 104
*a^5*b^20 - 96*a^7*b^18 + 44*a^9*b^16 - 8*a^11*b^14))/(b^20 - 4*a^2*b^18 +
6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12))*(-(a + b)^5*(a - b)^5)^(1/2)*(12*a^4 +
 20*b^4 - 29*a^2*b^2))/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5
*a^8*b^7 - a^10*b^5))))/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 +
5*a^8*b^7 - a^10*b^5)))*(12*a^4 + 20*b^4 - 29*a^2*b^2)*1i)/(2*(b^15 - 5*a^2
*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)) + (a^3*(-(a + b)^
5*(a - b)^5)^(1/2)*((4*(2*a^2*b^16 + 40*a^4*b^14 + 108*a^6*b^12 - 872*a^8*b
^10 + 1538*a^10*b^8 - 1104*a^12*b^6 + 288*a^14*b^4))/(b^19 - 4*a^2*b^17 + 6
*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) + (8*tan(x/2)*(2*a*b^18 + 39*a^3*b^16 +
88*a^5*b^14 - 1326*a^7*b^12 + 3134*a^9*b^10 - 3194*a^11*b^8 + 1536*a^13*b^6
 - 288*a^15*b^4))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12)
+ (a^3*(-(a + b)^5*(a - b)^5)^(1/2)*(12*a^4 + 20*b^4 - 29*a^2*b^2)*((4*(4*a
*b^20 + 28*a^3*b^18 - 120*a^5*b^16 + 164*a^7*b^14 - 100*a^9*b^12 + 24*a^11*
b^10))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) + (8*tan(x/
2)*(80*a^4*b^18 - 276*a^6*b^16 + 360*a^8*b^14 - 212*a^10*b^12 + 48*a^12*b^1
0))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12) + (a^3*((4*(8*
a^2*b^22 - 32*a^4*b^20 + 48*a^6*b^18 - 32*a^8*b^16 + 8*a^10*b^14))/(b^19 -
4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) + (8*tan(x/2)*(12*a*b^24 -
 56*a^3*b^22 + 104*a^5*b^20 - 96*a^7*b^18 + 44*a^9*b^16 - 8*a^11*b^14))/(b^
20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12))*(-(a + b)^5*(a - b)^
5)^(1/2)*(12*a^4 + 20*b^4 - 29*a^2*b^2))/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^1
1 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5))))/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^
11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)))*(12*a^4 + 20*b^4 - 29*a^2*b^2)*1i
)/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5))
)/((8*(864*a^15 + 20*a^5*b^10 + 11*a^7*b^8 - 2326*a^9*b^6 + 4770*a^11*b^4 -
 3456*a^13*b^2))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) +
 (16*tan(x/2)*(1728*a^16 + 20*a^4*b^12 + 411*a^6*b^10 + 1314*a^8*b^8 - 7829
*a^10*b^6 + 11700*a^12*b^4 - 7344*a^14*b^2))/(b^20 - 4*a^2*b^18 + 6*a^4*b^1
6 - 4*a^6*b^14 + a^8*b^12) - (a^3*(-(a + b)^5*(a - b)^5)^(1/2)*((4*(2*a^2*b
^16 + 40*a^4*b^14 + 108*a^6*b^12 - 872*a^8*b^10 + 1538*a^10*b^8 - 1104*a^12
*b^6 + 288*a^14*b^4))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^
11) + (8*tan(x/2)*(2*a*b^18 + 39*a^3*b^16 + 88*a^5*b^14 - 1326*a^7*b^12 + 3
134*a^9*b^10 - 3194*a^11*b^8 + 1536*a^13*b^6 - 288*a^15*b^4))/(b^20 - 4*a^2
*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12) - (a^3*(-(a + b)^5*(a - b)^5)^(
1/2)*(12*a^4 + 20*b^4 - 29*a^2*b^2)*((4*(4*a*b^20 + 28*a^3*b^18 - 120*a^5*b
```

```
^16 + 164*a^7*b^14 - 100*a^9*b^12 + 24*a^11*b^10))/(b^19 - 4*a^2*b^17 + 6*a
^4*b^15 - 4*a^6*b^13 + a^8*b^11) + (8*tan(x/2)*(80*a^4*b^18 - 276*a^6*b^16
+ 360*a^8*b^14 - 212*a^10*b^12 + 48*a^12*b^10))/(b^20 - 4*a^2*b^18 + 6*a^4*
b^16 - 4*a^6*b^14 + a^8*b^12) - (a^3*((4*(8*a^2*b^22 - 32*a^4*b^20 + 48*a^6
*b^18 - 32*a^8*b^16 + 8*a^10*b^14))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6
*b^13 + a^8*b^11) + (8*tan(x/2)*(12*a*b^24 - 56*a^3*b^22 + 104*a^5*b^20 - 9
6*a^7*b^18 + 44*a^9*b^16 - 8*a^11*b^14))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 -
4*a^6*b^14 + a^8*b^12))*(-(a + b)^5*(a - b)^5)^(1/2)*(12*a^4 + 20*b^4 - 29*
a^2*b^2))/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^
10*b^5))))/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a
^10*b^5)))*(12*a^4 + 20*b^4 - 29*a^2*b^2))/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b
^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)) + (a^3*(-(a + b)^5*(a - b)^5)^(1/
2)*((4*(2*a^2*b^16 + 40*a^4*b^14 + 108*a^6*b^12 - 872*a^8*b^10 + 1538*a^10*
b^8 - 1104*a^12*b^6 + 288*a^14*b^4))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^
6*b^13 + a^8*b^11) + (8*tan(x/2)*(2*a*b^18 + 39*a^3*b^16 + 88*a^5*b^14 - 13
26*a^7*b^12 + 3134*a^9*b^10 - 3194*a^11*b^8 + 1536*a^13*b^6 - 288*a^15*b^4)
)/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12) + (a^3*(-(a + b)
^5*(a - b)^5)^(1/2)*(12*a^4 + 20*b^4 - 29*a^2*b^2)*((4*(4*a*b^20 + 28*a^3*b
^18 - 120*a^5*b^16 + 164*a^7*b^14 - 100*a^9*b^12 + 24*a^11*b^10))/(b^19 - 4
*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) + (8*tan(x/2)*(80*a^4*b^18
- 276*a^6*b^16 + 360*a^8*b^14 - 212*a^10*b^12 + 48*a^12*b^10))/(b^20 - 4*a^
2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12) + (a^3*((4*(8*a^2*b^22 - 32*a^
4*b^20 + 48*a^6*b^18 - 32*a^8*b^16 + 8*a^10*b^14))/(b^19 - 4*a^2*b^17 + 6*a
^4*b^15 - 4*a^6*b^13 + a^8*b^11) + (8*tan(x/2)*(12*a*b^24 - 56*a^3*b^22 + 1
04*a^5*b^20 - 96*a^7*b^18 + 44*a^9*b^16 - 8*a^11*b^14))/(b^20 - 4*a^2*b^18
+ 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12))*(-(a + b)^5*(a - b)^5)^(1/2)*(12*a^4
 + 20*b^4 - 29*a^2*b^2))/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 +
 5*a^8*b^7 - a^10*b^5))))/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9
+ 5*a^8*b^7 - a^10*b^5)))*(12*a^4 + 20*b^4 - 29*a^2*b^2))/(2*(b^15 - 5*a^2*
b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5))))*(-(a + b)^5*(a -
 b)^5)^(1/2)*(12*a^4 + 20*b^4 - 29*a^2*b^2)*1i)/(b^15 - 5*a^2*b^13 + 10*a^4
*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**5/(a+b*sin(x))**3,x)
```

```
[Out] Timed out
```

3.194 $\int \frac{\sin^4(x)}{(a+b\sin(x))^3}\,dx$

Optimal. Leaf size=179

$$\frac{a^2\sin^2(x)\cos(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{\left(3a^2-2b^2\right)\cos(x)}{2b^3\left(a^2-b^2\right)}+\frac{3a^2\left(2a^4-5a^2b^2+4b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4\left(a^2-b^2\right)^{5/2}}-\frac{3a^3\left(a^2-2b^2\right)\cos}{2b^3\left(a^2-b^2\right)^2(a+b}$$

```
[Out] -3*a*x/b^4+3*a^2*(2*a^4-5*a^2*b^2+4*b^4)*arctan((b+a*tan(1/2*x))/(a^2-b^2)^
(1/2))/b^4/(a^2-b^2)^(5/2)-1/2*(3*a^2-2*b^2)*cos(x)/b^3/(a^2-b^2)+1/2*a^2*c
os(x)*sin(x)^2/b/(a^2-b^2)/(a+b*sin(x))^2-3/2*a^3*(a^2-2*b^2)*cos(x)/b^3/(a
^2-b^2)^2/(a+b*sin(x))
```

Rubi [A] time = 0.41, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2792, 3031, 3023, 2735, 2660, 618, 204}

$$-\frac{\left(3a^2-2b^2\right)\cos(x)}{2b^3\left(a^2-b^2\right)}+\frac{3a^2\left(-5a^2b^2+2a^4+4b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4\left(a^2-b^2\right)^{5/2}}+\frac{a^2\sin^2(x)\cos(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{3a^3\left(a^2-2b^2\right.}{2b^3\left(a^2-b^2\right)^2(a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4/(a + b*Sin[x])^3,x]
```

```
[Out] (-3*a*x)/b^4 + (3*a^2*(2*a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(b + a*Tan[x/2])/S
qrt[a^2 - b^2]])/(b^4*(a^2 - b^2)^(5/2)) - ((3*a^2 - 2*b^2)*Cos[x])/(2*b^3*
(a^2 - b^2)) + (a^2*Cos[x]*Sin[x]^2)/(2*b*(a^2 - b^2)*(a + b*Sin[x])^2) - (
3*a^3*(a^2 - 2*b^2)*Cos[x])/(2*b^3*(a^2 - b^2)^2*(a + b*Sin[x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^4(x)}{(a+b\sin(x))^3}\,dx = \frac{a^2\cos(x)\sin^2(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{\int \frac{\sin(x)\left(2a^2-2ab\sin(x)-\left(3a^2-2b^2\right)\sin^2(x)\right)}{(a+b\sin(x))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= \frac{a^2\cos(x)\sin^2(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{3a^3\left(a^2-2b^2\right)\cos(x)}{2b^3\left(a^2-b^2\right)^2(a+b\sin(x))} - \frac{\int \frac{3a^2b\left(a^2-2b^2\right)+a\left(3a^2-4b^2\right)\left(a^2-b^2\right)}{a+b\,\mathrm{s}}}{2b^3\left(a^2\right.}$$

$$= -\frac{\left(3a^2-2b^2\right)\cos(x)}{2b^3\left(a^2-b^2\right)} + \frac{a^2\cos(x)\sin^2(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{3a^3\left(a^2-2b^2\right)\cos(x)}{2b^3\left(a^2-b^2\right)^2(a+b\sin(x))} - \frac{\int \frac{3a^2}{}}{}$$

$$= -\frac{3ax}{b^4} - \frac{\left(3a^2-2b^2\right)\cos(x)}{2b^3\left(a^2-b^2\right)} + \frac{a^2\cos(x)\sin^2(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{3a^3\left(a^2-2b^2\right)\cos(x)}{2b^3\left(a^2-b^2\right)^2(a+b\sin(x))} +$$

$$= -\frac{3ax}{b^4} - \frac{\left(3a^2-2b^2\right)\cos(x)}{2b^3\left(a^2-b^2\right)} + \frac{a^2\cos(x)\sin^2(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{3a^3\left(a^2-2b^2\right)\cos(x)}{2b^3\left(a^2-b^2\right)^2(a+b\sin(x))} +$$

$$= -\frac{3ax}{b^4} - \frac{\left(3a^2-2b^2\right)\cos(x)}{2b^3\left(a^2-b^2\right)} + \frac{a^2\cos(x)\sin^2(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{3a^3\left(a^2-2b^2\right)\cos(x)}{2b^3\left(a^2-b^2\right)^2(a+b\sin(x))} -$$

$$= -\frac{3ax}{b^4} + \frac{3a^2\left(2a^4-5a^2b^2+4b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^4\left(a^2-b^2\right)^{5/2}} - \frac{\left(3a^2-2b^2\right)\cos(x)}{2b^3\left(a^2-b^2\right)} + \frac{a^2\cos(x)}{2b\left(a^2-b^2\right)(a}$$

Mathematica [A] time = 0.84, size = 144, normalized size = 0.80

$$\frac{\frac{a^4b\cos(x)}{(a-b)(a+b)(a+b\sin(x))^2} + \frac{6a^2\left(2a^4-5a^2b^2+4b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{a^3b\left(8b^2-5a^2\right)\cos(x)}{(a-b)^2(a+b)^2(a+b\sin(x))} - 6ax - 2b\cos(x)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4/(a + b*Sin[x])^3,x]
```

```
[Out] (-6*a*x + (6*a^2*(2*a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(b + a*Tan[x/2])/Sqrt[a
^2 - b^2]])/(a^2 - b^2)^(5/2) - 2*b*Cos[x] + (a^4*b*Cos[x])/((a - b)*(a + b
)*(a + b*Sin[x])^2) + (a^3*b*(-5*a^2 + 8*b^2)*Cos[x])/((a - b)^2*(a + b)^2*
(a + b*Sin[x])))/(2*b^4)
```

fricas [B] time = 0.63, size = 945, normalized size = 5.28

$$\left[\frac{12\left(a^7b^2-3\,a^5b^4+3\,a^3b^6-ab^8\right)x\cos(x)^2+4\left(a^6b^3-3\,a^4b^5+3\,a^2b^7-b^9\right)\cos(x)^3-3\left(2\,a^8-3\,a^6b^2-a^4b^4+\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(12*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*x*cos(x)^2 + 4*(a^6*b^3
 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cos(x)^3 - 3*(2*a^8 - 3*a^6*b^2 - a^4*b^4 +
4*a^2*b^6 - (2*a^6*b^2 - 5*a^4*b^4 + 4*a^2*b^6)*cos(x)^2 + 2*(2*a^7*b - 5*a
^5*b^3 + 4*a^3*b^5)*sin(x))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(x)^2 -
2*a*b*sin(x) - a^2 - b^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))
/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) - 12*(a^9 - 2*a^7*b^2 + 2*a^3*b
^6 - a*b^8)*x - 2*(6*a^8*b - 15*a^6*b^3 + 7*a^4*b^5 + 4*a^2*b^7 - 2*b^9)*co
s(x) - 2*(12*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*x + (9*a^7*b^2 - 25*
a^5*b^4 + 20*a^3*b^6 - 4*a*b^8)*cos(x))*sin(x))/(a^8*b^4 - 2*a^6*b^6 + 2*a^
2*b^10 - b^12 - (a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*cos(x)^2 + 2*(a^7
*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*sin(x)), 1/2*(6*(a^7*b^2 - 3*a^5*b^4
 + 3*a^3*b^6 - a*b^8)*x*cos(x)^2 + 2*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9
)*cos(x)^3 - 3*(2*a^8 - 3*a^6*b^2 - a^4*b^4 + 4*a^2*b^6 - (2*a^6*b^2 - 5*a^
4*b^4 + 4*a^2*b^6)*cos(x)^2 + 2*(2*a^7*b - 5*a^5*b^3 + 4*a^3*b^5)*sin(x))*s
qrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) - 6*(a^9 -
2*a^7*b^2 + 2*a^3*b^6 - a*b^8)*x - (6*a^8*b - 15*a^6*b^3 + 7*a^4*b^5 + 4*a^
2*b^7 - 2*b^9)*cos(x) - (12*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*x + (
9*a^7*b^2 - 25*a^5*b^4 + 20*a^3*b^6 - 4*a*b^8)*cos(x))*sin(x))/(a^8*b^4 - 2
*a^6*b^6 + 2*a^2*b^10 - b^12 - (a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*co
s(x)^2 + 2*(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*sin(x))]
```

giac [A] time = 0.26, size = 256, normalized size = 1.43

$$\frac{3\left(2\,a^6-5\,a^4b^2+4\,a^2b^4\right)\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4b^4-2\,a^2b^6+b^8\right)\sqrt{a^2-b^2}}-\frac{3\,a^5b\tan\left(\frac{1}{2}\,x\right)^3-6\,a^3b^3\tan\left(\frac{1}{2}\,x\right)^3+4\,a^6\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^4/(a+b*sin(x))^3,x, algorithm="giac")

[Out] 3*(2*a^6 - 5*a^4*b^2 + 4*a^2*b^4)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan ((a*tan(1/2*x) + b)/sqrt(a^2 - b^2)))/((a^4*b^4 - 2*a^2*b^6 + b^8)*sqrt(a^2 - b^2)) - (3*a^5*b*tan(1/2*x)^3 - 6*a^3*b^3*tan(1/2*x)^3 + 4*a^6*tan(1/2*x)^2 + a^4*b^2*tan(1/2*x)^2 - 14*a^2*b^4*tan(1/2*x)^2 + 13*a^5*b*tan(1/2*x) - 22*a^3*b^3*tan(1/2*x) + 4*a^6 - 7*a^4*b^2)/((a^4*b^3 - 2*a^2*b^5 + b^7)*(a*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)^2) - 3*a*x/b^4 - 2/((tan(1/2*x)^2 + 1) *b^3)

maple [B] time = 0.12, size = 634, normalized size = 3.54

$$-\frac{2}{b^3\left(\tan^2\left(\frac{x}{2}\right)+1\right)}-\frac{6a\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{b^4}-\frac{3a^5\left(\tan^3\left(\frac{x}{2}\right)\right)}{b^2\left(\left(\tan^2\left(\frac{x}{2}\right)\right)a+2\tan\left(\frac{x}{2}\right)b+a\right)^2\left(a^4-2a^2b^2+b^4\right)}+\frac{}{\left(\left(\tan^2\left(\frac{x}{2}\right)\right)a+}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^4/(a+b*sin(x))^3,x)

[Out] -2/b^3/(tan(1/2*x)^2+1)-6/b^4*a*arctan(tan(1/2*x))-3*a^5/b^2/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^3+6*a^3/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^3-4*a^6/b^3/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^2-a^4/b/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^2+14*a^2*b/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^2-13*a^5/b^2/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)+22*a^3/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)-4*a^6/b^3/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)+7*a^4/b/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)+6*a^6/b^4/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))-15*a^4/b^2/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))+12*a^2/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^4/(a+b*sin(x))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* h elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for more details)Is 4*b^2-4*a^2 positive or negative?

mupad [B] time = 14.87, size = 5945, normalized size = 33.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a + b*sin(x))^3,x)
```

```
[Out] - ((a^2*(6*a^4 + 2*b^4 - 11*a^2*b^2))/(b^3*(a^2 - b^2)^2) + (3*tan(x/2)^5*(
a^5 - 2*a^3*b^2))/(b^2*(a^2 - b^2)^2) - (3*tan(x/2)^4*(4*a^2*b^4 - 2*a^6 +
a^4*b^2))/(b^3*(a^2 - b^2)^2) + (2*tan(x/2)^2*(6*a^6 + 4*b^6 - 13*a^2*b^4 -
 3*a^4*b^2))/(b^3*(a^2 - b^2)^2) + (4*a*tan(x/2)^3*(6*a^4 + 2*b^4 - 11*a^2*
b^2))/(b^2*(a^2 - b^2)^2) + (a*tan(x/2)*(21*a^4 + 8*b^4 - 38*a^2*b^2))/(b^2
*(a^2 - b^2)^2))/(tan(x/2)^2*(3*a^2 + 4*b^2) + tan(x/2)^4*(3*a^2 + 4*b^2) +
 a^2 + a^2*tan(x/2)^6 + 4*a*b*tan(x/2) + 8*a*b*tan(x/2)^3 + 4*a*b*tan(x/2)^
5) - (6*a*atan(((3*a*((8*(36*a^4*b^11 - 144*a^6*b^9 + 216*a^8*b^7 - 144*a^1
0*b^5 + 36*a^12*b^3))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^
8) + (8*tan(x/2)*(72*a^3*b^13 - 468*a^5*b^11 + 936*a^7*b^9 - 873*a^9*b^7 +
396*a^11*b^5 - 72*a^13*b^3))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 +
 a^8*b^9) - (a*((8*(12*a^2*b^16 - 36*a^4*b^14 + 42*a^6*b^12 - 24*a^8*b^10 +
 6*a^10*b^8))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) - (a*
((8*(4*a^2*b^19 - 16*a^4*b^17 + 24*a^6*b^15 - 16*a^8*b^13 + 4*a^10*b^11))/(
b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(x/2)*(12*a*
b^21 - 56*a^3*b^19 + 104*a^5*b^17 - 96*a^7*b^15 + 44*a^9*b^13 - 8*a^11*b^11
))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9))*3i)/b^4 + (8*ta
n(x/2)*(48*a^3*b^16 - 156*a^5*b^14 + 192*a^7*b^12 - 108*a^9*b^10 + 24*a^11*
b^8))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9))*3i)/b^4))/b^
4 + (3*a*((8*(36*a^4*b^11 - 144*a^6*b^9 + 216*a^8*b^7 - 144*a^10*b^5 + 36*a
^12*b^3))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(
x/2)*(72*a^3*b^13 - 468*a^5*b^11 + 936*a^7*b^9 - 873*a^9*b^7 + 396*a^11*b^5
 - 72*a^13*b^3))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9) +
(a*((8*(12*a^2*b^16 - 36*a^4*b^14 + 42*a^6*b^12 - 24*a^8*b^10 + 6*a^10*b^8)
)/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (a*((8*(4*a^2*b
^19 - 16*a^4*b^17 + 24*a^6*b^15 - 16*a^8*b^13 + 4*a^10*b^11))/(b^16 - 4*a^2
*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(x/2)*(12*a*b^21 - 56*a^
3*b^19 + 104*a^5*b^17 - 96*a^7*b^15 + 44*a^9*b^13 - 8*a^11*b^11))/(b^17 - 4
*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9))*3i)/b^4 + (8*tan(x/2)*(48*a
^3*b^16 - 156*a^5*b^14 + 192*a^7*b^12 - 108*a^9*b^10 + 24*a^11*b^8))/(b^17
- 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9))*3i)/b^4))/b^4)/((16*(54*
a^12 - 216*a^6*b^6 + 378*a^8*b^4 - 243*a^10*b^2))/(b^16 - 4*a^2*b^14 + 6*a^
4*b^12 - 4*a^6*b^10 + a^8*b^8) + (16*tan(x/2)*(216*a^13 + 432*a^5*b^8 - 140
4*a^7*b^6 + 1728*a^9*b^4 - 972*a^11*b^2))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 -
 4*a^6*b^11 + a^8*b^9) - (a*((8*(36*a^4*b^11 - 144*a^6*b^9 + 216*a^8*b^7 -
144*a^10*b^5 + 36*a^12*b^3))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 +
 a^8*b^8) + (8*tan(x/2)*(72*a^3*b^13 - 468*a^5*b^11 + 936*a^7*b^9 - 873*a^9
```

*b^7 + 396*a^11*b^5 - 72*a^13*b^3))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6
*b^11 + a^8*b^9) - (a*((8*(12*a^2*b^16 - 36*a^4*b^14 + 42*a^6*b^12 - 24*a^8
*b^10 + 6*a^10*b^8))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8
) - (a*((8*(4*a^2*b^19 - 16*a^4*b^17 + 24*a^6*b^15 - 16*a^8*b^13 + 4*a^10*b
^11))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(x/2)
*(12*a*b^21 - 56*a^3*b^19 + 104*a^5*b^17 - 96*a^7*b^15 + 44*a^9*b^13 - 8*a^
11*b^11))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9))*3i)/b^4
+ (8*tan(x/2)*(48*a^3*b^16 - 156*a^5*b^14 + 192*a^7*b^12 - 108*a^9*b^10 + 2
4*a^11*b^8))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9))*3i)/b
^4)*3i)/b^4 + (a*((8*(36*a^4*b^11 - 144*a^6*b^9 + 216*a^8*b^7 - 144*a^10*b^
5 + 36*a^12*b^3))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) +
 (8*tan(x/2)*(72*a^3*b^13 - 468*a^5*b^11 + 936*a^7*b^9 - 873*a^9*b^7 + 396*
a^11*b^5 - 72*a^13*b^3))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8
b^9) + (a((8*(12*a^2*b^16 - 36*a^4*b^14 + 42*a^6*b^12 - 24*a^8*b^10 + 6*a
^10*b^8))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (a*((8*
(4*a^2*b^19 - 16*a^4*b^17 + 24*a^6*b^15 - 16*a^8*b^13 + 4*a^10*b^11))/(b^16
 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(x/2)*(12*a*b^21
 - 56*a^3*b^19 + 104*a^5*b^17 - 96*a^7*b^15 + 44*a^9*b^13 - 8*a^11*b^11))/(
b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9))*3i)/b^4 + (8*tan(x/
2)*(48*a^3*b^16 - 156*a^5*b^14 + 192*a^7*b^12 - 108*a^9*b^10 + 24*a^11*b^8)
)/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9))*3i)/b^4)*3i)/b^4
)))/b^4 - (a^2*atan(((a^2*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 4*b^4 - 5*a
^2*b^2)*((8*(36*a^4*b^11 - 144*a^6*b^9 + 216*a^8*b^7 - 144*a^10*b^5 + 36*a^
12*b^3))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(x
/2)*(72*a^3*b^13 - 468*a^5*b^11 + 936*a^7*b^9 - 873*a^9*b^7 + 396*a^11*b^5
 - 72*a^13*b^3))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9) - (
3*a^2*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(12*a^2*b^16 - 36*a^4*b^14 + 42*a^6*
b^12 - 24*a^8*b^10 + 6*a^10*b^8))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b
^10 + a^8*b^8) + (8*tan(x/2)*(48*a^3*b^16 - 156*a^5*b^14 + 192*a^7*b^12 - 1
08*a^9*b^10 + 24*a^11*b^8))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 +
a^8*b^9) - (3*a^2*((8*(4*a^2*b^19 - 16*a^4*b^17 + 24*a^6*b^15 - 16*a^8*b^13
 + 4*a^10*b^11))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) +
(8*tan(x/2)*(12*a*b^21 - 56*a^3*b^19 + 104*a^5*b^17 - 96*a^7*b^15 + 44*a^9*
b^13 - 8*a^11*b^11))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9
))*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 4*b^4 - 5*a^2*b^2))/(2*(b^14 - 5*a
^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)))*(2*a^4 + 4*b^4
 - 5*a^2*b^2))/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6
 - a^10*b^4)))*3i)/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8
*b^6 - a^10*b^4)) + (a^2*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 4*b^4 - 5*a^
2*b^2)*((8*(36*a^4*b^11 - 144*a^6*b^9 + 216*a^8*b^7 - 144*a^10*b^5 + 36*a^1
2*b^3))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(x/
2)*(72*a^3*b^13 - 468*a^5*b^11 + 936*a^7*b^9 - 873*a^9*b^7 + 396*a^11*b^5 -
 72*a^13*b^3))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9) + (3
a^2(-(a + b)^5*(a - b)^5)^(1/2)*((8*(12*a^2*b^16 - 36*a^4*b^14 + 42*a^6*b
^12 - 24*a^8*b^10 + 6*a^10*b^8))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^

10 + a^8*b^8) + (8*tan(x/2)*(48*a^3*b^16 - 156*a^5*b^14 + 192*a^7*b^12 - 10
8*a^9*b^10 + 24*a^11*b^8))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a
^8*b^9) + (3*a^2*((8*(4*a^2*b^19 - 16*a^4*b^17 + 24*a^6*b^15 - 16*a^8*b^13
+ 4*a^10*b^11))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (
8*tan(x/2)*(12*a*b^21 - 56*a^3*b^19 + 104*a^5*b^17 - 96*a^7*b^15 + 44*a^9*b
^13 - 8*a^11*b^11))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9)
)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 4*b^4 - 5*a^2*b^2))/(2*(b^14 - 5*a^
2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)))*(2*a^4 + 4*b^4
- 5*a^2*b^2))/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6
- a^10*b^4)))*3i)/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*
b^6 - a^10*b^4)))/((16*(54*a^12 - 216*a^6*b^6 + 378*a^8*b^4 - 243*a^10*b^2)
)/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (16*tan(x/2)*(2
16*a^13 + 432*a^5*b^8 - 1404*a^7*b^6 + 1728*a^9*b^4 - 972*a^11*b^2))/(b^17
- 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9) - (3*a^2*(-(a + b)^5*(a -
 b)^5)^(1/2)*(2*a^4 + 4*b^4 - 5*a^2*b^2)*((8*(36*a^4*b^11 - 144*a^6*b^9 + 2
16*a^8*b^7 - 144*a^10*b^5 + 36*a^12*b^3))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 -
 4*a^6*b^10 + a^8*b^8) + (8*tan(x/2)*(72*a^3*b^13 - 468*a^5*b^11 + 936*a^7*
b^9 - 873*a^9*b^7 + 396*a^11*b^5 - 72*a^13*b^3))/(b^17 - 4*a^2*b^15 + 6*a^4
*b^13 - 4*a^6*b^11 + a^8*b^9) - (3*a^2*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(12
*a^2*b^16 - 36*a^4*b^14 + 42*a^6*b^12 - 24*a^8*b^10 + 6*a^10*b^8))/(b^16 -
4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(x/2)*(48*a^3*b^16
- 156*a^5*b^14 + 192*a^7*b^12 - 108*a^9*b^10 + 24*a^11*b^8))/(b^17 - 4*a^2*
b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9) - (3*a^2*((8*(4*a^2*b^19 - 16*a^4
*b^17 + 24*a^6*b^15 - 16*a^8*b^13 + 4*a^10*b^11))/(b^16 - 4*a^2*b^14 + 6*a^
4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(x/2)*(12*a*b^21 - 56*a^3*b^19 + 104
*a^5*b^17 - 96*a^7*b^15 + 44*a^9*b^13 - 8*a^11*b^11))/(b^17 - 4*a^2*b^15 +
6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9))*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 4
*b^4 - 5*a^2*b^2))/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8
*b^6 - a^10*b^4)))*(2*a^4 + 4*b^4 - 5*a^2*b^2))/(2*(b^14 - 5*a^2*b^12 + 10*
a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4))))/(2*(b^14 - 5*a^2*b^12 + 10
*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)) + (3*a^2*(-(a + b)^5*(a - b
)^5)^(1/2)*(2*a^4 + 4*b^4 - 5*a^2*b^2)*((8*(36*a^4*b^11 - 144*a^6*b^9 + 216
*a^8*b^7 - 144*a^10*b^5 + 36*a^12*b^3))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4
*a^6*b^10 + a^8*b^8) + (8*tan(x/2)*(72*a^3*b^13 - 468*a^5*b^11 + 936*a^7*b^
9 - 873*a^9*b^7 + 396*a^11*b^5 - 72*a^13*b^3))/(b^17 - 4*a^2*b^15 + 6*a^4*b
^13 - 4*a^6*b^11 + a^8*b^9) + (3*a^2*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(12*a
^2*b^16 - 36*a^4*b^14 + 42*a^6*b^12 - 24*a^8*b^10 + 6*a^10*b^8))/(b^16 - 4*
a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(x/2)*(48*a^3*b^16 -
156*a^5*b^14 + 192*a^7*b^12 - 108*a^9*b^10 + 24*a^11*b^8))/(b^17 - 4*a^2*b^
15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9) + (3*a^2*((8*(4*a^2*b^19 - 16*a^4*b
^17 + 24*a^6*b^15 - 16*a^8*b^13 + 4*a^10*b^11))/(b^16 - 4*a^2*b^14 + 6*a^4*
b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(x/2)*(12*a*b^21 - 56*a^3*b^19 + 104*a
^5*b^17 - 96*a^7*b^15 + 44*a^9*b^13 - 8*a^11*b^11))/(b^17 - 4*a^2*b^15 + 6*
a^4*b^13 - 4*a^6*b^11 + a^8*b^9))*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 4*b
^4 - 5*a^2*b^2))/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b

```
^6 - a^10*b^4)))*(2*a^4 + 4*b^4 - 5*a^2*b^2))/(2*(b^14 - 5*a^2*b^12 + 10*a^
4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4))))/(2*(b^14 - 5*a^2*b^12 + 10*a
^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4))))*(-(a + b)^5*(a - b)^5)^(1/2
)*(2*a^4 + 4*b^4 - 5*a^2*b^2)*3i)/(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6
*b^8 + 5*a^8*b^6 - a^10*b^4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4/(a+b*sin(x))**3,x)
```

```
[Out] Timed out
```

3.195 $\int \frac{\sin^3(x)}{(a+b\sin(x))^3}\,dx$

Optimal. Leaf size=144

$$\frac{a^2\left(2a^2-5b^2\right)\cos(x)}{2b^2\left(a^2-b^2\right)^2(a+b\sin(x))}+\frac{a^2\sin(x)\cos(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{a\left(2a^4-5a^2b^2+6b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3\left(a^2-b^2\right)^{5/2}}+\frac{x}{b^3}$$

```
[Out] x/b^3-a*(2*a^4-5*a^2*b^2+6*b^4)*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/b^
3/(a^2-b^2)^(5/2)+1/2*a^2*cos(x)*sin(x)/b/(a^2-b^2)/(a+b*sin(x))^2+1/2*a^2*
(2*a^2-5*b^2)*cos(x)/b^2/(a^2-b^2)^2/(a+b*sin(x))
```

Rubi [A] time = 0.24, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2792, 3021, 2735, 2660, 618, 204}

$$-\frac{a\left(-5a^2b^2+2a^4+6b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3\left(a^2-b^2\right)^{5/2}}+\frac{a^2\left(2a^2-5b^2\right)\cos(x)}{2b^2\left(a^2-b^2\right)^2(a+b\sin(x))}+\frac{a^2\sin(x)\cos(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2}+\frac{x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a + b*Sin[x])^3,x]
```

```
[Out] x/b^3 - (a*(2*a^4 - 5*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b
^2]])/(b^3*(a^2 - b^2)^(5/2)) + (a^2*Cos[x]*Sin[x])/(2*b*(a^2 - b^2)*(a + b
*Sin[x])^2) + (a^2*(2*a^2 - 5*b^2)*Cos[x])/(2*b^2*(a^2 - b^2)^2*(a + b*Sin[
x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin^3(x)}{(a+b\sin(x))^3}\,dx = \frac{a^2\cos(x)\sin(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{\int \frac{a^2-2ab\sin(x)-2\left(a^2-b^2\right)\sin^2(x)}{(a+b\sin(x))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= \frac{a^2\cos(x)\sin(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{a^2\left(2a^2-5b^2\right)\cos(x)}{2b^2\left(a^2-b^2\right)^2(a+b\sin(x))} + \frac{\int \frac{ab\left(a^2-4b^2\right)+2\left(a^2-b^2\right)^2\sin(x)}{a+b\sin(x)}\,d\ldots}{2b^2\left(a^2-b^2\right)^2 \ldots}$$

$$= \frac{x}{b^3} + \frac{a^2\cos(x)\sin(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{a^2\left(2a^2-5b^2\right)\cos(x)}{2b^2\left(a^2-b^2\right)^2(a+b\sin(x))} - \frac{\left(a\left(2a^4-5a^2b^2+6b^4\ldots\right.\right.}{2b^3\left(a^2-\ldots\right.}$$

$$= \frac{x}{b^3} + \frac{a^2\cos(x)\sin(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{a^2\left(2a^2-5b^2\right)\cos(x)}{2b^2\left(a^2-b^2\right)^2(a+b\sin(x))} - \frac{\left(a\left(2a^4-5a^2b^2+6b^4\ldots\right.\right.}{\ldots}$$

$$= \frac{x}{b^3} + \frac{a^2\cos(x)\sin(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{a^2\left(2a^2-5b^2\right)\cos(x)}{2b^2\left(a^2-b^2\right)^2(a+b\sin(x))} + \frac{\left(2a\left(2a^4-5a^2b^2+6b\ldots\right.\right.}{\ldots}$$

$$= \frac{x}{b^3} - \frac{a\left(2a^4-5a^2b^2+6b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^3\left(a^2-b^2\right)^{5/2}} + \frac{a^2\cos(x)\sin(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{a^2\left(2a\ldots\right.}{2b^2\left(a^2-\ldots\right.}$$

Mathematica [A] time = 0.57, size = 136, normalized size = 0.94

$$\frac{-\frac{a^3b\cos(x)}{(a-b)(a+b)(a+b\sin(x))^2} + \frac{3a^2b\left(a^2-2b^2\right)\cos(x)}{(a-b)^2(a+b)^2(a+b\sin(x))} - \frac{2a\left(2a^4-5a^2b^2+6b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + 2x}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a + b*Sin[x])^3,x]
```

```
[Out] (2*x - (2*a*(2*a^4 - 5*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 -
b^2]])/(a^2 - b^2)^(5/2) - (a^3*b*Cos[x])/((a - b)*(a + b)*(a + b*Sin[x])^2
) + (3*a^2*b*(a^2 - 2*b^2)*Cos[x])/((a - b)^2*(a + b)^2*(a + b*Sin[x])))/(2
*b^3)
```

fricas [B] time = 0.55, size = 819, normalized size = 5.69

$$\left[-\frac{4\left(a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8\right)x\cos(x)^2+\left(2\,a^7-3\,a^5b^2+a^3b^4+6\,ab^6-\left(2\,a^5b^2-5\,a^3b^4+6\,ab^6\right)\cos(x)^2+2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*x*cos(x)^2 + (2*a^7 - 3*a^
5*b^2 + a^3*b^4 + 6*a*b^6 - (2*a^5*b^2 - 5*a^3*b^4 + 6*a*b^6)*cos(x)^2 + 2*
(2*a^6*b - 5*a^4*b^3 + 6*a^2*b^5)*sin(x))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b
^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sq
rt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) - 4*(a^8 - 2*a^6
*b^2 + 2*a^2*b^6 - b^8)*x - 2*(2*a^7*b - 7*a^5*b^3 + 5*a^3*b^5)*cos(x) - 2*
(4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*x + 3*(a^6*b^2 - 3*a^4*b^4 + 2*a
^2*b^6)*cos(x))*sin(x))/(a^8*b^3 - 2*a^6*b^5 + 2*a^2*b^9 - b^11 - (a^6*b^5
- 3*a^4*b^7 + 3*a^2*b^9 - b^11)*cos(x)^2 + 2*(a^7*b^4 - 3*a^5*b^6 + 3*a^3*b
^8 - a*b^10)*sin(x)), -1/2*(2*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*x*cos
(x)^2 - (2*a^7 - 3*a^5*b^2 + a^3*b^4 + 6*a*b^6 - (2*a^5*b^2 - 5*a^3*b^4 + 6
*a*b^6)*cos(x)^2 + 2*(2*a^6*b - 5*a^4*b^3 + 6*a^2*b^5)*sin(x))*sqrt(a^2 - b
^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) - 2*(a^8 - 2*a^6*b^2 +
 2*a^2*b^6 - b^8)*x - (2*a^7*b - 7*a^5*b^3 + 5*a^3*b^5)*cos(x) - (4*(a^7*b
- 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*x + 3*(a^6*b^2 - 3*a^4*b^4 + 2*a^2*b^6)*co
s(x))*sin(x))/(a^8*b^3 - 2*a^6*b^5 + 2*a^2*b^9 - b^11 - (a^6*b^5 - 3*a^4*b^
7 + 3*a^2*b^9 - b^11)*cos(x)^2 + 2*(a^7*b^4 - 3*a^5*b^6 + 3*a^3*b^8 - a*b^1
0)*sin(x))]
```

giac [A] time = 0.22, size = 234, normalized size = 1.62

$$-\frac{\left(2\,a^5-5\,a^3b^2+6\,ab^4\right)\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4b^3-2\,a^2b^5+b^7\right)\sqrt{a^2-b^2}}+\frac{a^4b\tan\left(\frac{1}{2}\,x\right)^3-4\,a^2b^3\tan\left(\frac{1}{2}\,x\right)^3+2\,a^5\tan}{(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] -(2*a^5 - 5*a^3*b^2 + 6*a*b^4)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a
*tan(1/2*x) + b)/sqrt(a^2 - b^2)))/((a^4*b^3 - 2*a^2*b^5 + b^7)*sqrt(a^2 -
b^2)) + (a^4*b*tan(1/2*x)^3 - 4*a^2*b^3*tan(1/2*x)^3 + 2*a^5*tan(1/2*x)^2 -
```

```
 a^3*b^2*tan(1/2*x)^2 - 10*a*b^4*tan(1/2*x)^2 + 7*a^4*b*tan(1/2*x) - 16*a^2
*b^3*tan(1/2*x) + 2*a^5 - 5*a^3*b^2)/((a^4*b^2 - 2*a^2*b^4 + b^6)*(a*tan(1/
2*x)^2 + 2*b*tan(1/2*x) + a)^2) + x/b^3
```

maple [B] time = 0.12, size = 612, normalized size = 4.25

$$\frac{2\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{b^3}+\frac{a^4\left(\tan^3\left(\frac{x}{2}\right)\right)}{b\left(\left(\tan^2\left(\frac{x}{2}\right)\right)a+2\tan\left(\frac{x}{2}\right)b+a\right)^2\left(a^4-2a^2b^2+b^4\right)}-\frac{4b\,a^2\left(\tan^3\left(\frac{x}{2}\right)\right)}{\left(\left(\tan^2\left(\frac{x}{2}\right)\right)a+2\tan\left(\frac{x}{2}\right)b+a\right)^2\left(a^4-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a+b*sin(x))^3,x)
```

```
[Out] 2/b^3*arctan(tan(1/2*x))+a^4/b/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a
^2*b^2+b^4)*tan(1/2*x)^3-4*b*a^2/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2
*a^2*b^2+b^4)*tan(1/2*x)^3+2*a^5/b^2/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a
^4-2*a^2*b^2+b^4)*tan(1/2*x)^2-a^3/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4
-2*a^2*b^2+b^4)*tan(1/2*x)^2-10*b^2*a/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(
a^4-2*a^2*b^2+b^4)*tan(1/2*x)^2+7*a^4/b/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2
/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)-16*b*a^2/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^
2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)+2*a^5/b^2/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a
)^2/(a^4-2*a^2*b^2+b^4)-5*a^3/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^
2*b^2+b^4)-2*a^5/b^3/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*ta
n(1/2*x)+2*b)/(a^2-b^2)^(1/2))+5*a^3/b/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*
arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))-6*b*a/(a^4-2*a^2*b^2+b^4)/
(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 14.90, size = 5756, normalized size = 39.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a + b*sin(x))^3,x)

[Out] (2*atan((((8*(4*a^2*b^10 - 16*a^4*b^8 + 24*a^6*b^6 - 16*a^8*b^4 + 4*a^10*b^
2))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) - (((8*(4*a*b^14
- 8*a^3*b^12 + 6*a^5*b^10 - 4*a^7*b^8 + 2*a^9*b^6))/(b^13 - 4*a^2*b^11 + 6*
a^4*b^9 - 4*a^6*b^7 + a^8*b^5) - (((8*(4*a^2*b^16 - 16*a^4*b^14 + 24*a^6*b^
12 - 16*a^8*b^10 + 4*a^10*b^8))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7
+ a^8*b^5) + (8*tan(x/2)*(12*a*b^18 - 56*a^3*b^16 + 104*a^5*b^14 - 96*a^7*b
^12 + 44*a^9*b^10 - 8*a^11*b^8))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^
8 + a^8*b^6))*1i)/b^3 + (8*tan(x/2)*(24*a^2*b^14 - 68*a^4*b^12 + 72*a^6*b^1
0 - 36*a^8*b^8 + 8*a^10*b^6))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 +
 a^8*b^6))*1i)/b^3 + (8*tan(x/2)*(8*a*b^12 - 72*a^3*b^10 + 124*a^5*b^8 - 10
5*a^7*b^6 + 44*a^9*b^4 - 8*a^11*b^2))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a
^6*b^8 + a^8*b^6))/b^3 + ((8*(4*a^2*b^10 - 16*a^4*b^8 + 24*a^6*b^6 - 16*a^8
*b^4 + 4*a^10*b^2))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) +
 (((((8*(4*a^2*b^16 - 16*a^4*b^14 + 24*a^6*b^12 - 16*a^8*b^10 + 4*a^10*b^8)
)/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (8*tan(x/2)*(12*a
*b^18 - 56*a^3*b^16 + 104*a^5*b^14 - 96*a^7*b^12 + 44*a^9*b^10 - 8*a^11*b^8
))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))*1i)/b^3 + (8*(4*
a*b^14 - 8*a^3*b^12 + 6*a^5*b^10 - 4*a^7*b^8 + 2*a^9*b^6))/(b^13 - 4*a^2*b^
11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (8*tan(x/2)*(24*a^2*b^14 - 68*a^4*b
^12 + 72*a^6*b^10 - 36*a^8*b^8 + 8*a^10*b^6))/(b^14 - 4*a^2*b^12 + 6*a^4*b^
10 - 4*a^6*b^8 + a^8*b^6))*1i)/b^3 + (8*tan(x/2)*(8*a*b^12 - 72*a^3*b^10 +
124*a^5*b^8 - 105*a^7*b^6 + 44*a^9*b^4 - 8*a^11*b^2))/(b^14 - 4*a^2*b^12 +
6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))/b^3)/((((8*(4*a^2*b^10 - 16*a^4*b^8 + 24
*a^6*b^6 - 16*a^8*b^4 + 4*a^10*b^2))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6
*b^7 + a^8*b^5) + (((((8*(4*a^2*b^16 - 16*a^4*b^14 + 24*a^6*b^12 - 16*a^8*b
^10 + 4*a^10*b^8))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) +
(8*tan(x/2)*(12*a*b^18 - 56*a^3*b^16 + 104*a^5*b^14 - 96*a^7*b^12 + 44*a^9*
b^10 - 8*a^11*b^8))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))
*1i)/b^3 + (8*(4*a*b^14 - 8*a^3*b^12 + 6*a^5*b^10 - 4*a^7*b^8 + 2*a^9*b^6))
/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (8*tan(x/2)*(24*a^
2*b^14 - 68*a^4*b^12 + 72*a^6*b^10 - 36*a^8*b^8 + 8*a^10*b^6))/(b^14 - 4*a^
2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))*1i)/b^3 + (8*tan(x/2)*(8*a*b^12
 - 72*a^3*b^10 + 124*a^5*b^8 - 105*a^7*b^6 + 44*a^9*b^4 - 8*a^11*b^2))/(b^1
4 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))*1i)/b^3 - (((8*(4*a^2*b
^10 - 16*a^4*b^8 + 24*a^6*b^6 - 16*a^8*b^4 + 4*a^10*b^2))/(b^13 - 4*a^2*b^1
1 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) - (((8*(4*a*b^14 - 8*a^3*b^12 + 6*a^5*
b^10 - 4*a^7*b^8 + 2*a^9*b^6))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 +
 a^8*b^5) - (((8*(4*a^2*b^16 - 16*a^4*b^14 + 24*a^6*b^12 - 16*a^8*b^10 + 4*
a^10*b^8))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (8*tan(x
/2)*(12*a*b^18 - 56*a^3*b^16 + 104*a^5*b^14 - 96*a^7*b^12 + 44*a^9*b^10 - 8
*a^11*b^8))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))*1i)/b^3
 + (8*tan(x/2)*(24*a^2*b^14 - 68*a^4*b^12 + 72*a^6*b^10 - 36*a^8*b^8 + 8*a^
10*b^6))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))*1i)/b^3 +
```

(8*tan(x/2)*(8*a*b^12 - 72*a^3*b^10 + 124*a^5*b^8 - 105*a^7*b^6 + 44*a^9*b^4 - 8*a^11*b^2))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))*1i)/b^3 + (16*(2*a^9 - 24*a^3*b^6 + 26*a^5*b^4 - 13*a^7*b^2))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (16*tan(x/2)*(8*a^10 + 24*a^2*b^8 - 68*a^4*b^6 + 72*a^6*b^4 - 36*a^8*b^2))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))))/b^3 + ((2*a^5 - 5*a^3*b^2)/(b^2*(a^4 + b^4 - 2*a^2*b^2)) + (tan(x/2)*(7*a^4 - 16*a^2*b^2))/(b*(a^4 + b^4 - 2*a^2*b^2)) + (tan(x/2)^3*(a^4 - 4*a^2*b^2))/(b*(a^4 + b^4 - 2*a^2*b^2)) - (tan(x/2)^2*(5*a*b^2 - 2*a^3)*(a^2 + 2*b^2))/(b^2*(a^4 + b^4 - 2*a^2*b^2)))/(tan(x/2)^2*(2*a^2 + 4*b^2) + a^2 + a^2*tan(x/2)^4 + 4*a*b*tan(x/2) + 4*a*b*tan(x/2)^3) + (a*atan(((a*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(4*a^2*b^10 - 16*a^4*b^8 + 24*a^6*b^6 - 16*a^8*b^4 + 4*a^10*b^2))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (8*tan(x/2)*(8*a*b^12 - 72*a^3*b^10 + 124*a^5*b^8 - 105*a^7*b^6 + 44*a^9*b^4 - 8*a^11*b^2))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6) - (a*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(4*a*b^14 - 8*a^3*b^12 + 6*a^5*b^10 - 4*a^7*b^8 + 2*a^9*b^6))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (8*tan(x/2)*(24*a^2*b^14 - 68*a^4*b^12 + 72*a^6*b^10 - 36*a^8*b^8 + 8*a^10*b^6))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6) - (a*((8*(4*a^2*b^16 - 16*a^4*b^14 + 24*a^6*b^12 - 16*a^8*b^10 + 4*a^10*b^8))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (8*tan(x/2)*(12*a*b^18 - 56*a^3*b^16 + 104*a^5*b^14 - 96*a^7*b^12 + 44*a^9*b^10 - 8*a^11*b^8))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 6*b^4 - 5*a^2*b^2))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)))*(2*a^4 + 6*b^4 - 5*a^2*b^2))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)))*(2*a^4 + 6*b^4 - 5*a^2*b^2)*1i)/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)) + (a*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(4*a^2*b^10 - 16*a^4*b^8 + 24*a^6*b^6 - 16*a^8*b^4 + 4*a^10*b^2))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (8*tan(x/2)*(8*a*b^12 - 72*a^3*b^10 + 124*a^5*b^8 - 105*a^7*b^6 + 44*a^9*b^4 - 8*a^11*b^2))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6) + (a*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(4*a*b^14 - 8*a^3*b^12 + 6*a^5*b^10 - 4*a^7*b^8 + 2*a^9*b^6))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (8*tan(x/2)*(24*a^2*b^14 - 68*a^4*b^12 + 72*a^6*b^10 - 36*a^8*b^8 + 8*a^10*b^6))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6) + (a*((8*(4*a^2*b^16 - 16*a^4*b^14 + 24*a^6*b^12 - 16*a^8*b^10 + 4*a^10*b^8))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (8*tan(x/2)*(12*a*b^18 - 56*a^3*b^16 + 104*a^5*b^14 - 96*a^7*b^12 + 44*a^9*b^10 - 8*a^11*b^8))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 6*b^4 - 5*a^2*b^2))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)))*(2*a^4 + 6*b^4 - 5*a^2*b^2))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)))*(2*a^4 + 6*b^4 - 5*a^2*b^2)*1i)/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)))/((16*(2*a^9 - 24*a^3*b^6 + 26*a^5*b^4 - 13*a^7*b^2))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (16*tan(x/2)*(8*a^10 + 24*a^2*b^8 - 68*a^4*b^6 + 72*a^6*b^4 - 3

```
6*a^8*b^2))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6) - (a*(-(
a + b)^5*(a - b)^5)^(1/2)*((8*(4*a^2*b^10 - 16*a^4*b^8 + 24*a^6*b^6 - 16*a^
8*b^4 + 4*a^10*b^2))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5)
+ (8*tan(x/2)*(8*a*b^12 - 72*a^3*b^10 + 124*a^5*b^8 - 105*a^7*b^6 + 44*a^9*
b^4 - 8*a^11*b^2))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6) -
 (a*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(4*a*b^14 - 8*a^3*b^12 + 6*a^5*b^10 -
4*a^7*b^8 + 2*a^9*b^6))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^
5) + (8*tan(x/2)*(24*a^2*b^14 - 68*a^4*b^12 + 72*a^6*b^10 - 36*a^8*b^8 + 8*
a^10*b^6))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6) - (a*((8*
(4*a^2*b^16 - 16*a^4*b^14 + 24*a^6*b^12 - 16*a^8*b^10 + 4*a^10*b^8))/(b^13
- 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (8*tan(x/2)*(12*a*b^18 -
56*a^3*b^16 + 104*a^5*b^14 - 96*a^7*b^12 + 44*a^9*b^10 - 8*a^11*b^8))/(b^14
 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))*(-(a + b)^5*(a - b)^5)^(
1/2)*(2*a^4 + 6*b^4 - 5*a^2*b^2))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a
^6*b^7 + 5*a^8*b^5 - a^10*b^3)))*(2*a^4 + 6*b^4 - 5*a^2*b^2))/(2*(b^13 - 5*
a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)))*(2*a^4 + 6*b^4
 - 5*a^2*b^2))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5
- a^10*b^3)) + (a*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(4*a^2*b^10 - 16*a^4*b^8
 + 24*a^6*b^6 - 16*a^8*b^4 + 4*a^10*b^2))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 -
4*a^6*b^7 + a^8*b^5) + (8*tan(x/2)*(8*a*b^12 - 72*a^3*b^10 + 124*a^5*b^8 -
105*a^7*b^6 + 44*a^9*b^4 - 8*a^11*b^2))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4
*a^6*b^8 + a^8*b^6) + (a*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(4*a*b^14 - 8*a^3
*b^12 + 6*a^5*b^10 - 4*a^7*b^8 + 2*a^9*b^6))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9
 - 4*a^6*b^7 + a^8*b^5) + (8*tan(x/2)*(24*a^2*b^14 - 68*a^4*b^12 + 72*a^6*b
^10 - 36*a^8*b^8 + 8*a^10*b^6))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8
 + a^8*b^6) + (a*((8*(4*a^2*b^16 - 16*a^4*b^14 + 24*a^6*b^12 - 16*a^8*b^10
+ 4*a^10*b^8))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (8*t
an(x/2)*(12*a*b^18 - 56*a^3*b^16 + 104*a^5*b^14 - 96*a^7*b^12 + 44*a^9*b^10
 - 8*a^11*b^8))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))*(-(
a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 6*b^4 - 5*a^2*b^2))/(2*(b^13 - 5*a^2*b^1
1 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)))*(2*a^4 + 6*b^4 - 5*a^
2*b^2))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*
b^3)))*(2*a^4 + 6*b^4 - 5*a^2*b^2))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10
*a^6*b^7 + 5*a^8*b^5 - a^10*b^3))))*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 6
*b^4 - 5*a^2*b^2)*1i)/(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*
b^5 - a^10*b^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a+b*sin(x))**3,x)
```

```
[Out] Timed out
```

3.196 $\int \frac{\sin^2(x)}{(a+b\sin(x))^3}\,dx$

Optimal. Leaf size=118

$$\frac{\left(a^2+2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}+\frac{a^2\cos(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{a\left(a^2-4b^2\right)\cos(x)}{2b\left(a^2-b^2\right)^2(a+b\sin(x))}$$

```
[Out] (a^2+2*b^2)*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)+1/2*a^
2*cos(x)/b/(a^2-b^2)/(a+b*sin(x))^2-1/2*a*(a^2-4*b^2)*cos(x)/b/(a^2-b^2)^2/
(a+b*sin(x))
```

Rubi [A] time = 0.14, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2790, 2754, 12, 2660, 618, 204}

$$\frac{\left(a^2+2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}+\frac{a^2\cos(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{a\left(a^2-4b^2\right)\cos(x)}{2b\left(a^2-b^2\right)^2(a+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a + b*Sin[x])^3,x]
```

```
[Out] ((a^2 + 2*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
+ (a^2*Cos[x])/(2*b*(a^2 - b^2)*(a + b*Sin[x])^2) - (a*(a^2 - 4*b^2)*Cos[x]
)/(2*b*(a^2 - b^2)^2*(a + b*Sin[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^2(x)}{(a+b\sin(x))^3}\,dx = \frac{a^2\cos(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{\int \frac{2ab+\left(a^2-2b^2\right)\sin(x)}{(a+b\sin(x))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= \frac{a^2\cos(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{a\left(a^2-4b^2\right)\cos(x)}{2b\left(a^2-b^2\right)^2(a+b\sin(x))} + \frac{\int \frac{b\left(a^2+2b^2\right)}{a+b\sin(x)}\,dx}{2b\left(a^2-b^2\right)^2}$$

$$= \frac{a^2\cos(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{a\left(a^2-4b^2\right)\cos(x)}{2b\left(a^2-b^2\right)^2(a+b\sin(x))} + \frac{\left(a^2+2b^2\right)\int \frac{1}{a+b\sin(x)}\,dx}{2\left(a^2-b^2\right)^2}$$

$$= \frac{a^2\cos(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{a\left(a^2-4b^2\right)\cos(x)}{2b\left(a^2-b^2\right)^2(a+b\sin(x))} + \frac{\left(a^2+2b^2\right)\text{Subst}\left(\int \frac{1}{a+2bx+a\cdots}\right.}{\left(a^2-b^2\right)^2}$$

$$= \frac{a^2\cos(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{a\left(a^2-4b^2\right)\cos(x)}{2b\left(a^2-b^2\right)^2(a+b\sin(x))} - \frac{\left(2\left(a^2+2b^2\right)\right)\text{Subst}\left(\int \frac{}{-4(a\cdots}\right.}{\left(a^2\cdots\right.}$$

$$= \frac{\left(a^2+2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{a^2\cos(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{a\left(a^2-4b^2\right)\cos(x)}{2b\left(a^2-b^2\right)^2(a+b\sin(x))}$$

Mathematica [A] time = 0.38, size = 94, normalized size = 0.80

$$\frac{\left(a^2+2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{a\cos(x)\left(3ab-\left(a^2-4b^2\right)\sin(x)\right)}{2(a-b)^2(a+b)^2(a+b\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a + b*Sin[x])^3,x]
```

```
[Out] ((a^2 + 2*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
+ (a*Cos[x]*(3*a*b - (a^2 - 4*b^2)*Sin[x]))/(2*(a - b)^2*(a + b)^2*(a + b*S
in[x])^2)
```

fricas [B] time = 0.52, size = 516, normalized size = 4.37

$$\left[-\frac{2\left(a^5-5a^3b^2+4ab^4\right)\cos(x)\sin(x)+\left(a^4+3a^2b^2+2b^4-\left(a^2b^2+2b^4\right)\cos(x)^2+2\left(a^3b+2ab^3\right)\sin(x)\right)\sqrt{-\cdots}}{4\left(a^8-2a^6b^2+2a^2b^6-b^8-\left(a^6b^2-3a^4b^4+3a^2b^6-b^8\right)\cos\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(a^5 - 5*a^3*b^2 + 4*a*b^4)*cos(x)*sin(x) + (a^4 + 3*a^2*b^2 + 2*b
^4 - (a^2*b^2 + 2*b^4)*cos(x)^2 + 2*(a^3*b + 2*a*b^3)*sin(x))*sqrt(-a^2 + b
^2)*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 + 2*(a*cos(x)*si
n(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2
)) - 6*(a^4*b - a^2*b^3)*cos(x))/(a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8 - (a^6*
b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*
b^5 - a*b^7)*sin(x)), -1/2*((a^5 - 5*a^3*b^2 + 4*a*b^4)*cos(x)*sin(x) + (a^
4 + 3*a^2*b^2 + 2*b^4 - (a^2*b^2 + 2*b^4)*cos(x)^2 + 2*(a^3*b + 2*a*b^3)*si
n(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) - 3*
(a^4*b - a^2*b^3)*cos(x))/(a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8 - (a^6*b^2 - 3
*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a
*b^7)*sin(x))]
```

giac [A] time = 0.15, size = 182, normalized size = 1.54

$$\frac{\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)\left(a^2+2b^2\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{a^3\tan\left(\frac{1}{2}x\right)^3+2ab^2\tan\left(\frac{1}{2}x\right)^3+3a^2b\tan\left(\frac{1}{2}x\right)^2+6b^3\cdots}{\left(a^4-2a^2b^2+b^4\right)\left(a\tan\left(\frac{1}{2}x\right)\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] (pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b^2
)))*(a^2 + 2*b^2)/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) + (a^3*tan(1/2*
x)^3 + 2*a*b^2*tan(1/2*x)^3 + 3*a^2*b*tan(1/2*x)^2 + 6*b^3*tan(1/2*x)^2 - a
^3*tan(1/2*x) + 10*a*b^2*tan(1/2*x) + 3*a^2*b)/((a^4 - 2*a^2*b^2 + b^4)*(a*
tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)^2)
```

maple [B] time = 0.11, size = 265, normalized size = 2.25

$$\frac{\frac{8\left(a^2+2b^2\right)a\left(\tan^3\left(\frac{x}{2}\right)\right)}{8a^4-16a^2b^2+8b^4}+\frac{3b\left(a^2+2b^2\right)\left(\tan^2\left(\frac{x}{2}\right)\right)}{a^4-2a^2b^2+b^4}-\frac{a\left(a^2-10b^2\right)\tan\left(\frac{x}{2}\right)}{a^4-2a^2b^2+b^4}+\frac{3a^2b}{a^4-2a^2b^2+b^4}}{\left(\left(\tan^2\left(\frac{x}{2}\right)\right)a+2\tan\left(\frac{x}{2}\right)b+a\right)^2}+\frac{a^2\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{2\arctan\left(\frac{2a\,\mathrm{t}\cdots}{2\cdots}\right.}{\left(a^4-2a^2b^2+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a+b*sin(x))^3,x)
```

```
[Out] 8*(1/8*(a^2+2*b^2)*a/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^3+3/8*b*(a^2+2*b^2)/(a^
4-2*a^2*b^2+b^4)*tan(1/2*x)^2-1/8*a*(a^2-10*b^2)/(a^4-2*a^2*b^2+b^4)*tan(1/
2*x)+3/8*a^2*b/(a^4-2*a^2*b^2+b^4))/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2+a^2
/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b
^2)^(1/2))+2/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)
+2*b)/(a^2-b^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.23, size = 318, normalized size = 2.69

$$\frac{\frac{3\,a^2\,b}{a^4-2\,a^2\,b^2+b^4}-\frac{a\,\tan\left(\frac{x}{2}\right)\,\left(a^2-10\,b^2\right)}{a^4-2\,a^2\,b^2+b^4}+\frac{a\,{\tan\left(\frac{x}{2}\right)}^3\,\left(a^2+2\,b^2\right)}{a^4-2\,a^2\,b^2+b^4}+\frac{3\,b\,{\tan\left(\frac{x}{2}\right)}^2\,\left(a^2+2\,b^2\right)}{a^4-2\,a^2\,b^2+b^4}}{{\tan\left(\frac{x}{2}\right)}^2\,\left(2\,a^2+4\,b^2\right)+a^2+a^2\,{\tan\left(\frac{x}{2}\right)}^4+4\,a\,b\,\tan\left(\frac{x}{2}\right)+4\,a\,b\,{\tan\left(\frac{x}{2}\right)}^3}+\frac{\mathrm{atan}\left(\frac{\left(\left(\frac{\left(a^2+2\,b^2\right)\,\left(2\,a^4\,b-4\,a^2\,b^3+2\,b^5\right)}{2\,{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{a\,\tan\left(\frac{x}{2}\right)}{{\left(a+b\right)}^{5/2}}\right.\right.}{a^2+2\,b^2}\right.}{{\left(a+b\right)}^{5/2}\,\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a + b*sin(x))^3,x)
```

```
[Out] ((3*a^2*b)/(a^4 + b^4 - 2*a^2*b^2) - (a*tan(x/2)*(a^2 - 10*b^2))/(a^4 + b^4
 - 2*a^2*b^2) + (a*tan(x/2)^3*(a^2 + 2*b^2))/(a^4 + b^4 - 2*a^2*b^2) + (3*b
*tan(x/2)^2*(a^2 + 2*b^2))/(a^4 + b^4 - 2*a^2*b^2))/(tan(x/2)^2*(2*a^2 + 4*
b^2) + a^2 + a^2*tan(x/2)^4 + 4*a*b*tan(x/2) + 4*a*b*tan(x/2)^3) + (atan(((
((a^2 + 2*b^2)*(2*a^4*b + 2*b^5 - 4*a^2*b^3))/(2*(a + b)^(5/2)*(a - b)^(5/2
)*(a^4 + b^4 - 2*a^2*b^2)) + (a*tan(x/2)*(a^2 + 2*b^2))/((a + b)^(5/2)*(a -
 b)^(5/2)))*(a^4 + b^4 - 2*a^2*b^2))/(a^2 + 2*b^2))*(a^2 + 2*b^2))/((a + b)
^(5/2)*(a - b)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+b*sin(x))**3,x)
```

```
[Out] Timed out
```

3.197 $\int \frac{\sin(x)}{(a+b\sin(x))^3}\,dx$

Optimal. Leaf size=103

$$-\frac{3ab\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}-\frac{\left(a^2+2b^2\right)\cos(x)}{2\left(a^2-b^2\right)^2(a+b\sin(x))}-\frac{a\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))^2}$$

```
[Out] -3*a*b*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)-1/2*a*cos(x
)/(a^2-b^2)/(a+b*sin(x))^2-1/2*(a^2+2*b^2)*cos(x)/(a^2-b^2)^2/(a+b*sin(x))
```

Rubi [A] time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2754, 12, 2660, 618, 204}

$$-\frac{3ab\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}-\frac{\left(a^2+2b^2\right)\cos(x)}{2\left(a^2-b^2\right)^2(a+b\sin(x))}-\frac{a\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a + b*Sin[x])^3,x]
```

```
[Out] (-3*a*b*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) - (a*Co
s[x])/(2*(a^2 - b^2)*(a + b*Sin[x])^2) - ((a^2 + 2*b^2)*Cos[x])/(2*(a^2 - b
^2)^2*(a + b*Sin[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sin(x)}{(a+b\sin(x))^3}\,dx = -\frac{a\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{\int \frac{2b-a\sin(x)}{(a+b\sin(x))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{a\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{\left(a^2+2b^2\right)\cos(x)}{2\left(a^2-b^2\right)^2(a+b\sin(x))} + \frac{\int -\frac{3ab}{a+b\sin(x)}\,dx}{2\left(a^2-b^2\right)^2}$$

$$= -\frac{a\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{\left(a^2+2b^2\right)\cos(x)}{2\left(a^2-b^2\right)^2(a+b\sin(x))} - \frac{(3ab)\int \frac{1}{a+b\sin(x)}\,dx}{2\left(a^2-b^2\right)^2}$$

$$= -\frac{a\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{\left(a^2+2b^2\right)\cos(x)}{2\left(a^2-b^2\right)^2(a+b\sin(x))} - \frac{(3ab)\,\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x,\right.}{\left(a^2-b^2\right)^2}$$

$$= -\frac{a\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{\left(a^2+2b^2\right)\cos(x)}{2\left(a^2-b^2\right)^2(a+b\sin(x))} + \frac{(6ab)\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,\right.}{\left(a^2-b^2\right)^2}$$

$$= -\frac{3ab\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} - \frac{a\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{\left(a^2+2b^2\right)\cos(x)}{2\left(a^2-b^2\right)^2(a+b\sin(x))}$$

Mathematica [A]   time = 0.30, size = 94, normalized size = 0.91

$$-\frac{3ab\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}-\frac{\cos(x)\left(b\left(a^2+2b^2\right)\sin(x)+a\left(2a^2+b^2\right)\right)}{2(a-b)^2(a+b)^2(a+b\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a + b*Sin[x])^3,x]
```

```
[Out] (-3*a*b*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) - (Cos[
x]*(a*(2*a^2 + b^2) + b*(a^2 + 2*b^2)*Sin[x]))/(2*(a - b)^2*(a + b)^2*(a +
b*Sin[x])^2)
```

fricas [B]   time = 0.52, size = 490, normalized size = 4.76

$$\left[-\frac{2\left(a^4b+a^2b^3-2\,b^5\right)\cos(x)\sin(x)-3\left(ab^3\cos(x)^2-2\,a^2b^2\sin(x)-a^3b-ab^3\right)\sqrt{-a^2+b^2}\,\log\left(-\frac{\left(2\,a^2-b^2\right)\cos(x}{}\right.}{4\left(a^8-2\,a^6b^2+2\,a^2b^6-b^8-\left(a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8\right)\cos(x)^2+2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(a^4*b + a^2*b^3 - 2*b^5)*cos(x)*sin(x) - 3*(a*b^3*cos(x)^2 - 2*a^
2*b^2*sin(x) - a^3*b - a*b^3)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(x)^2
 - 2*a*b*sin(x) - a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^
2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) + 2*(2*a^5 - a^3*b^2 - a*b^4
)*cos(x))/(a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8 - (a^6*b^2 - 3*a^4*b^4 + 3*a^2
*b^6 - b^8)*cos(x)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*sin(x)), -
1/2*((a^4*b + a^2*b^3 - 2*b^5)*cos(x)*sin(x) + 3*(a*b^3*cos(x)^2 - 2*a^2*b^
2*sin(x) - a^3*b - a*b^3)*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2
- b^2)*cos(x))) + (2*a^5 - a^3*b^2 - a*b^4)*cos(x))/(a^8 - 2*a^6*b^2 + 2*a^
2*b^6 - b^8 - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b -
 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*sin(x))]
```

giac [B]   time = 0.42, size = 189, normalized size = 1.83

$$-\frac{3\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)+b}{\sqrt{a^2-b^2}}\right)\right)ab}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}-\frac{3\,a^3b\tan\left(\frac{1}{2}\,x\right)^3+2\,a^4\tan\left(\frac{1}{2}\,x\right)^2+5\,a^2b^2\tan\left(\frac{1}{2}\,x\right)^2+2\,b^4\,\mathrm{ta}}{\left(a^5-2\,a^3b^2+ab^4\right)\left(a\tan\left(\frac{1}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] -3*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 -
b^2)))*a*b/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) - (3*a^3*b*tan(1/2*x)^
3 + 2*a^4*tan(1/2*x)^2 + 5*a^2*b^2*tan(1/2*x)^2 + 2*b^4*tan(1/2*x)^2 + 5*a^
3*b*tan(1/2*x) + 4*a*b^3*tan(1/2*x) + 2*a^4 + a^2*b^2)/((a^5 - 2*a^3*b^2 +
a*b^4)*(a*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)^2)
```

maple [B] time = 0.11, size = 221, normalized size = 2.15

$$\frac{-\frac{3a^2b\left(\tan^3\left(\frac{x}{2}\right)\right)}{a^4-2a^2b^2+b^4}-\frac{\left(2a^4+5a^2b^2+2b^4\right)\left(\tan^2\left(\frac{x}{2}\right)\right)}{\left(a^4-2a^2b^2+b^4\right)a}-\frac{\left(5a^2+4b^2\right)b\tan\left(\frac{x}{2}\right)}{a^4-2a^2b^2+b^4}-\frac{\left(2a^2+b^2\right)a}{a^4-2a^2b^2+b^4}}{\left(\left(\tan^2\left(\frac{x}{2}\right)\right)a+2\tan\left(\frac{x}{2}\right)b+a\right)^2}-\frac{3ba\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a+b*sin(x))^3,x)
```

```
[Out] 4*(-3/4*a^2*b/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^3-1/4*(2*a^4+5*a^2*b^2+2*b^4)/
(a^4-2*a^2*b^2+b^4)/a*tan(1/2*x)^2-1/4*(5*a^2+4*b^2)*b/(a^4-2*a^2*b^2+b^4)*
tan(1/2*x)-1/4*(2*a^2+b^2)*a/(a^4-2*a^2*b^2+b^4))/(tan(1/2*x)^2*a+2*tan(1/2
*x)*b+a)^2-3*b*a/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/
2*x)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.51, size = 310, normalized size = 3.01

$$-\frac{\frac{2\,a^3+a\,b^2}{a^4-2\,a^2\,b^2+b^4}+\frac{3\,a^2\,b\,{\tan\left(\frac{x}{2}\right)}^3}{a^4-2\,a^2\,b^2+b^4}+\frac{b\,\tan\left(\frac{x}{2}\right)\,\left(5\,a^2+4\,b^2\right)}{a^4-2\,a^2\,b^2+b^4}+\frac{{\tan\left(\frac{x}{2}\right)}^2\,\left(2\,a^2+b^2\right)\,\left(a^2+2\,b^2\right)}{a\,\left(a^4-2\,a^2\,b^2+b^4\right)}}{{\tan\left(\frac{x}{2}\right)}^2\,\left(2\,a^2+4\,b^2\right)+a^2+a^2\,{\tan\left(\frac{x}{2}\right)}^4+4\,a\,b\,\tan\left(\frac{x}{2}\right)+4\,a\,b\,{\tan\left(\frac{x}{2}\right)}^3}-\frac{3\,a\,b\,\mathrm{atan}\left(\frac{\left(\frac{3\,a^2\,b\,\tan\left(\frac{x}{2}\right)}{{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}+\frac{3\,a\,b^2\,\left(2\,a^4-4\,a \cdots\right.}{2\,{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}\,\left(\cdots\right.}\cdots\right.}{3\,a\,b\cdots}\cdots\right.}{{\left(a+b\right)}^{5/2}\,\left(a-b\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a + b*sin(x))^3,x)
```

```
[Out] - ((a*b^2 + 2*a^3)/(a^4 + b^4 - 2*a^2*b^2) + (3*a^2*b*tan(x/2)^3)/(a^4 + b^
4 - 2*a^2*b^2) + (b*tan(x/2)*(5*a^2 + 4*b^2))/(a^4 + b^4 - 2*a^2*b^2) + (ta
n(x/2)^2*(2*a^2 + b^2)*(a^2 + 2*b^2))/(a*(a^4 + b^4 - 2*a^2*b^2)))/(tan(x/2
)^2*(2*a^2 + 4*b^2) + a^2 + a^2*tan(x/2)^4 + 4*a*b*tan(x/2) + 4*a*b*tan(x/2
)^3) - (3*a*b*atan((((3*a^2*b*tan(x/2))/((a + b)^(5/2)*(a - b)^(5/2)) + (3*
a*b^2*(2*a^4 + 2*b^4 - 4*a^2*b^2))/(2*(a + b)^(5/2)*(a - b)^(5/2)*(a^4 + b^
4 - 2*a^2*b^2)))*(a^4 + b^4 - 2*a^2*b^2))/(3*a*b)))/((a + b)^(5/2)*(a - b)^
(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*sin(x))**3,x)
```

```
[Out] Timed out
```

3.198 $\int \frac{1}{(a+b\sin(x))^3}\,dx$

Optimal. Leaf size=102

$$\frac{(2a^2+b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+\frac{3ab\cos(x)}{2(a^2-b^2)^2(a+b\sin(x))}+\frac{b\cos(x)}{2(a^2-b^2)(a+b\sin(x))^2}$$

```
[Out] (2*a^2+b^2)*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)+1/2*b*
cos(x)/(a^2-b^2)/(a+b*sin(x))^2+3/2*a*b*cos(x)/(a^2-b^2)^2/(a+b*sin(x))
```

Rubi [A] time = 0.09, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {2664, 2754, 12, 2660, 618, 204}

$$\frac{(2a^2+b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+\frac{3ab\cos(x)}{2(a^2-b^2)^2(a+b\sin(x))}+\frac{b\cos(x)}{2(a^2-b^2)(a+b\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x])^(-3),x]
```

```
[Out] ((2*a^2 + b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
+ (b*Cos[x])/(2*(a^2 - b^2)*(a + b*Sin[x])^2) + (3*a*b*Cos[x])/(2*(a^2 - b^
2)^2*(a + b*Sin[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(x))^3}\,dx = \frac{b\cos(x)}{2(a^2-b^2)(a+b\sin(x))^2} - \frac{\int \frac{-2a+b\sin(x)}{(a+b\sin(x))^2}\,dx}{2(a^2-b^2)}$$

$$= \frac{b\cos(x)}{2(a^2-b^2)(a+b\sin(x))^2} + \frac{3ab\cos(x)}{2(a^2-b^2)^2(a+b\sin(x))} + \frac{\int \frac{2a^2+b^2}{a+b\sin(x)}\,dx}{2(a^2-b^2)^2}$$

$$= \frac{b\cos(x)}{2(a^2-b^2)(a+b\sin(x))^2} + \frac{3ab\cos(x)}{2(a^2-b^2)^2(a+b\sin(x))} + \frac{(2a^2+b^2)\int \frac{1}{a+b\sin(x)}\,dx}{2(a^2-b^2)^2}$$

$$= \frac{b\cos(x)}{2(a^2-b^2)(a+b\sin(x))^2} + \frac{3ab\cos(x)}{2(a^2-b^2)^2(a+b\sin(x))} + \frac{(2a^2+b^2)\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,d\right.}{(a^2-b^2)^2}$$

$$= \frac{b\cos(x)}{2(a^2-b^2)(a+b\sin(x))^2} + \frac{3ab\cos(x)}{2(a^2-b^2)^2(a+b\sin(x))} - \frac{\left(2(2a^2+b^2)\right)\text{Subst}\left(\int \frac{1}{-4(a^2-b^2}\right.}{(a^2-b^2}$$

$$= \frac{(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{b\cos(x)}{2(a^2-b^2)(a+b\sin(x))^2} + \frac{3ab\cos(x)}{2(a^2-b^2)^2(a+b\sin(x))}$$

Mathematica [A] time = 0.20, size = 93, normalized size = 0.91

$$\frac{(2a^2+b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{b\cos(x)\left(4a^2+3ab\sin(x)-b^2\right)}{2(a-b)^2(a+b)^2(a+b\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x])^(-3),x]
```

```
[Out] ((2*a^2 + b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
+ (b*Cos[x]*(4*a^2 - b^2 + 3*a*b*Sin[x]))/(2*(a - b)^2*(a + b)^2*(a + b*Sin
[x])^2)
```

fricas [B] time = 0.52, size = 516, normalized size = 5.06

$$\left[\frac{6(a^3b^2-ab^4)\cos(x)\sin(x) - \left(2a^4+3a^2b^2+b^4-(2a^2b^2+b^4)\cos(x)^2+2(2a^3b+ab^3)\sin(x)\right)\sqrt{-a^2+b^2}\,\log}{4\left(a^8-2a^6b^2+2a^2b^6-b^8-(a^6b^2-3a^4b^4+3a^2b^6-b^8)\cos(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(6*(a^3*b^2 - a*b^4)*cos(x)*sin(x) - (2*a^4 + 3*a^2*b^2 + b^4 - (2*a^2
*b^2 + b^4)*cos(x)^2 + 2*(2*a^3*b + a*b^3)*sin(x))*sqrt(-a^2 + b^2)*log(((2
*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 + 2*(a*cos(x)*sin(x) + b*co
s(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) + 2*(4*a
^4*b - 5*a^2*b^3 + b^5)*cos(x))/(a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8 - (a^6*b
^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b
^5 - a*b^7)*sin(x)), 1/2*(3*(a^3*b^2 - a*b^4)*cos(x)*sin(x) - (2*a^4 + 3*a^
2*b^2 + b^4 - (2*a^2*b^2 + b^4)*cos(x)^2 + 2*(2*a^3*b + a*b^3)*sin(x))*sqrt
(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) + (4*a^4*b - 5
*a^2*b^3 + b^5)*cos(x))/(a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8 - (a^6*b^2 - 3*a
^4*b^4 + 3*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b
^7)*sin(x))]
```

giac [B] time = 0.17, size = 215, normalized size = 2.11

$$\frac{\left(\pi \left\lfloor \frac{x}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(a) + \arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)\left(2a^2+b^2\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{a^2-b^2}} + \frac{5a^3b^2\tan\left(\frac{1}{2}x\right)^3 - 2ab^4\tan\left(\frac{1}{2}x\right)^3 + 4a^4b\tan\left(\frac{1}{2}x\right)^2 +}{\left(a^6-2a^4b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] (pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b^2
)))*(2*a^2 + b^2)/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) + (5*a^3*b^2*ta
n(1/2*x)^3 - 2*a*b^4*tan(1/2*x)^3 + 4*a^4*b*tan(1/2*x)^2 + 7*a^2*b^3*tan(1/
2*x)^2 - 2*b^5*tan(1/2*x)^2 + 11*a^3*b^2*tan(1/2*x) - 2*a*b^4*tan(1/2*x) +
4*a^4*b - a^2*b^3)/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a*tan(1/2*x)^2 + 2*b*tan(1
/2*x) + a)^2)
```

maple [B] time = 0.10, size = 300, normalized size = 2.94

$$\frac{\frac{b^2\left(5a^2-2b^2\right)\left(\tan^3\left(\frac{x}{2}\right)\right)}{\left(a^4-2a^2b^2+b^4\right)a} + \frac{b\left(4a^4+7a^2b^2-2b^4\right)\left(\tan^2\left(\frac{x}{2}\right)\right)}{\left(a^4-2a^2b^2+b^4\right)a^2} + \frac{b^2\left(11a^2-2b^2\right)\tan\left(\frac{x}{2}\right)}{a\left(a^4-2a^2b^2+b^4\right)} + \frac{2b\left(4a^2-b^2\right)}{2a^4-4a^2b^2+2b^4}}{\left(\left(\tan^2\left(\frac{x}{2}\right)\right)a+2\tan\left(\frac{x}{2}\right)b+a\right)^2} + \frac{2a^2\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{a^2-b^2}} + \frac{a}{\left(a^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(x))^3,x)
```

```
[Out] 2*(1/2*b^2*(5*a^2-2*b^2)/(a^4-2*a^2*b^2+b^4)/a*tan(1/2*x)^3+1/2*b*(4*a^4+7*
a^2*b^2-2*b^4)/(a^4-2*a^2*b^2+b^4)/a^2*tan(1/2*x)^2+1/2*b^2*(11*a^2-2*b^2)/
a/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)+1/2*b*(4*a^2-b^2)/(a^4-2*a^2*b^2+b^4))/(ta
n(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2+2*a^2/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*
arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))+1/(a^4-2*a^2*b^2+b^4)/(a^2
-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.32, size = 349, normalized size = 3.42

$$\frac{\frac{4\,a^2\,b-b^3}{a^4-2\,a^2\,b^2+b^4}+\frac{{\tan\left(\frac{x}{2}\right)}^2\,\left(4\,a^2\,b-b^3\right)\,\left(a^2+2\,b^2\right)}{a^2\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{b\,{\tan\left(\frac{x}{2}\right)}^3\,\left(5\,a^2\,b-2\,b^3\right)}{a\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{b\,\tan\left(\frac{x}{2}\right)\,\left(11\,a^2\,b-2\,b^3\right)}{a\,\left(a^4-2\,a^2\,b^2+b^4\right)}}{{\tan\left(\frac{x}{2}\right)}^2\,\left(2\,a^2+4\,b^2\right)+a^2+a^2\,{\tan\left(\frac{x}{2}\right)}^4+4\,a\,b\,\tan\left(\frac{x}{2}\right)+4\,a\,b\,{\tan\left(\frac{x}{2}\right)}^3}+\frac{\mathrm{atan}\left(\frac{\left(\left(\frac{\left(2\,a^2+b^2\right)\,\left(2\,a^4\,b-4\,a^2\,b^3+2\,b^5\right)}{2\,{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\dots}{2\,a^2+\dots}\right)}{(a+b)^{5\dots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(x))^3,x)
```

```
[Out] ((4*a^2*b - b^3)/(a^4 + b^4 - 2*a^2*b^2) + (tan(x/2)^2*(4*a^2*b - b^3)*(a^2
 + 2*b^2))/(a^2*(a^4 + b^4 - 2*a^2*b^2)) + (b*tan(x/2)^3*(5*a^2*b - 2*b^3))
/(a*(a^4 + b^4 - 2*a^2*b^2)) + (b*tan(x/2)*(11*a^2*b - 2*b^3))/(a*(a^4 + b^
4 - 2*a^2*b^2)))/(tan(x/2)^2*(2*a^2 + 4*b^2) + a^2 + a^2*tan(x/2)^4 + 4*a*b
*tan(x/2) + 4*a*b*tan(x/2)^3) + (atan(((((2*a^2 + b^2)*(2*a^4*b + 2*b^5 - 4
*a^2*b^3))/(2*(a + b)^(5/2)*(a - b)^(5/2)*(a^4 + b^4 - 2*a^2*b^2)) + (a*tan
(x/2)*(2*a^2 + b^2))/((a + b)^(5/2)*(a - b)^(5/2)))*(a^4 + b^4 - 2*a^2*b^2)
)/(2*a^2 + b^2))*(2*a^2 + b^2))/((a + b)^(5/2)*(a - b)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x))**3,x)

[Out] Timed out
```

3.199 $\int \frac{\csc(x)}{(a+b\sin(x))^3}\,dx$

Optimal. Leaf size=145

$$-\frac{\tanh^{-1}(\cos(x))}{a^3}-\frac{b^2\left(5a^2-2b^2\right)\cos(x)}{2a^2\left(a^2-b^2\right)^2(a+b\sin(x))}-\frac{b^2\cos(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{b\left(6a^4-5a^2b^2+2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{5/2}}$$

```
[Out] -b*(6*a^4-5*a^2*b^2+2*b^4)*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/a^3/(a^
2-b^2)^(5/2)-arctanh(cos(x))/a^3-1/2*b^2*cos(x)/a/(a^2-b^2)/(a+b*sin(x))^2-
1/2*b^2*(5*a^2-2*b^2)*cos(x)/a^2/(a^2-b^2)^2/(a+b*sin(x))
```

Rubi [A] time = 0.37, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {2802, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{b\left(-5a^2b^2+6a^4+2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{5/2}}-\frac{b^2\left(5a^2-2b^2\right)\cos(x)}{2a^2\left(a^2-b^2\right)^2(a+b\sin(x))}-\frac{b^2\cos(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{\tanh^{-1}(\cos(x))}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a + b*Sin[x])^3,x]
```

```
[Out] -((b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/
(a^3*(a^2 - b^2)^(5/2))) - ArcTanh[Cos[x]]/a^3 - (b^2*Cos[x])/(2*a*(a^2 - b
^2)*(a + b*Sin[x])^2) - (b^2*(5*a^2 - 2*b^2)*Cos[x])/(2*a^2*(a^2 - b^2)^2*(
a + b*Sin[x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
```

```
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc(x)}{(a+b\sin(x))^3}\,dx = -\frac{b^2\cos(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{\int \frac{\csc(x)\left(2\left(a^2-b^2\right)-2ab\sin(x)+b^2\sin^2(x)\right)}{(a+b\sin(x))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= -\frac{b^2\cos(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{b^2\left(5a^2-2b^2\right)\cos(x)}{2a^2\left(a^2-b^2\right)^2(a+b\sin(x))} + \frac{\int \frac{\csc(x)\left(2\left(a^2-b^2\right)^2-ab\left(4a^2-b^2\right)\text{si}\right.}{a+b\sin(x)}}{2a^2\left(a^2-b^2\right)^2}$$

$$= -\frac{b^2\cos(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{b^2\left(5a^2-2b^2\right)\cos(x)}{2a^2\left(a^2-b^2\right)^2(a+b\sin(x))} + \frac{\int \csc(x)\,dx}{a^3} - \frac{\left(b\left(6a^4-5a\right.\right.}{2}$$

$$= -\frac{\tanh^{-1}(\cos(x))}{a^3} - \frac{b^2\cos(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{b^2\left(5a^2-2b^2\right)\cos(x)}{2a^2\left(a^2-b^2\right)^2(a+b\sin(x))} - \frac{\left(b\left(6a^4\right.\right.}{}$$

$$= -\frac{\tanh^{-1}(\cos(x))}{a^3} - \frac{b^2\cos(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{b^2\left(5a^2-2b^2\right)\cos(x)}{2a^2\left(a^2-b^2\right)^2(a+b\sin(x))} + \frac{\left(2b\left(6a^4\right.\right.}{}$$

$$= -\frac{b\left(6a^4-5a^2b^2+2b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{5/2}} - \frac{\tanh^{-1}(\cos(x))}{a^3} - \frac{b^2\cos(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2}$$

Mathematica [A] time = 0.91, size = 140, normalized size = 0.97

$$-\frac{\frac{2b\left(6a^4-5a^2b^2+2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{ab^2\cos(x)\left(6a^3+b\left(5a^2-2b^2\right)\sin(x)-3ab^2\right)}{(a-b)^2(a+b)^2(a+b\sin(x))^2} - 2\log\left(\sin\left(\frac{x}{2}\right)\right) + 2\log\left(\cos\left(\frac{x}{2}\right)\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a + b*Sin[x])^3,x]
```

```
[Out] -1/2*((2*b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b
^2]])/(a^2 - b^2)^(5/2) + 2*Log[Cos[x/2]] - 2*Log[Sin[x/2]] + (a*b^2*Cos[x]
*(6*a^3 - 3*a*b^2 + b*(5*a^2 - 2*b^2)*Sin[x]))/((a - b)^2*(a + b)^2*(a + b*
Sin[x])^2))/a^3
```

fricas [B] time = 1.39, size = 1027, normalized size = 7.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(5*a^5*b^3 - 7*a^3*b^5 + 2*a*b^7)*cos(x)*sin(x) + (6*a^6*b + a^4*b
^3 - 3*a^2*b^5 + 2*b^7 - (6*a^4*b^3 - 5*a^2*b^5 + 2*b^7)*cos(x)^2 + 2*(6*a^
5*b^2 - 5*a^3*b^4 + 2*a*b^6)*sin(x))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*c
os(x)^2 - 2*a*b*sin(x) - a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a
^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) + 6*(2*a^6*b^2 - 3*a^
4*b^4 + a^2*b^6)*cos(x) + 2*(a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8 - (a^6*b^2 -
 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 -
 a*b^7)*sin(x))*log(1/2*cos(x) + 1/2) - 2*(a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^
8 - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b - 3*a^5*b^3
 + 3*a^3*b^5 - a*b^7)*sin(x))*log(-1/2*cos(x) + 1/2))/(a^11 - 2*a^9*b^2 + 2
*a^5*b^6 - a^3*b^8 - (a^9*b^2 - 3*a^7*b^4 + 3*a^5*b^6 - a^3*b^8)*cos(x)^2 +
 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*sin(x)), -1/2*((5*a^5*b^3 - 7
*a^3*b^5 + 2*a*b^7)*cos(x)*sin(x) - (6*a^6*b + a^4*b^3 - 3*a^2*b^5 + 2*b^7
- (6*a^4*b^3 - 5*a^2*b^5 + 2*b^7)*cos(x)^2 + 2*(6*a^5*b^2 - 5*a^3*b^4 + 2*a
*b^6)*sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x
))) + 3*(2*a^6*b^2 - 3*a^4*b^4 + a^2*b^6)*cos(x) + (a^8 - 2*a^6*b^2 + 2*a^2
*b^6 - b^8 - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b -
3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*sin(x))*log(1/2*cos(x) + 1/2) - (a^8 - 2*a^6
*b^2 + 2*a^2*b^6 - b^8 - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(x)^2 +
 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*sin(x))*log(-1/2*cos(x) + 1/2))/
(a^11 - 2*a^9*b^2 + 2*a^5*b^6 - a^3*b^8 - (a^9*b^2 - 3*a^7*b^4 + 3*a^5*b^6
- a^3*b^8)*cos(x)^2 + 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*sin(x))]
```

giac [A] time = 0.18, size = 246, normalized size = 1.70

$$-\frac{\left(6\,a^4b-5\,a^2b^3+2\,b^5\right)\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^7-2\,a^5b^2+a^3b^4\right)\sqrt{a^2-b^2}}-\frac{7\,a^3b^3\tan\left(\frac{1}{2}\,x\right)^3-4\,ab^5\tan\left(\frac{1}{2}\,x\right)^3+6\,a^4b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] -(6*a^4*b - 5*a^2*b^3 + 2*b^5)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a
*tan(1/2*x) + b)/sqrt(a^2 - b^2)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(a^2 -
b^2)) - (7*a^3*b^3*tan(1/2*x)^3 - 4*a*b^5*tan(1/2*x)^3 + 6*a^4*b^2*tan(1/2*
```

```
x)^2 + 9*a^2*b^4*tan(1/2*x)^2 - 6*b^6*tan(1/2*x)^2 + 17*a^3*b^3*tan(1/2*x)
- 8*a*b^5*tan(1/2*x) + 6*a^4*b^2 - 3*a^2*b^4)/((a^7 - 2*a^5*b^2 + a^3*b^4)*
(a*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)^2) + log(abs(tan(1/2*x)))/a^3
```

maple [B] time = 0.14, size = 614, normalized size = 4.23

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^3}-\frac{7b^3\left(\tan^3\left(\frac{x}{2}\right)\right)}{\left(\left(\tan^2\left(\frac{x}{2}\right)\right)a+2\tan\left(\frac{x}{2}\right)b+a\right)^2\left(a^4-2a^2b^2+b^4\right)}+\frac{4b^5\left(\tan^3\left(\frac{x}{2}\right)\right)}{a^2\left(\left(\tan^2\left(\frac{x}{2}\right)\right)a+2\tan\left(\frac{x}{2}\right)b+a\right)^2\left(a^4-2a^2b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a+b*sin(x))^3,x)
```

```
[Out] 1/a^3*ln(tan(1/2*x))-7*b^3/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b
^2+b^4)*tan(1/2*x)^3+4/a^2*b^5/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a
^2*b^2+b^4)*tan(1/2*x)^3-6*b^2*a/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2
*a^2*b^2+b^4)*tan(1/2*x)^2-9/a*b^4/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4
-2*a^2*b^2+b^4)*tan(1/2*x)^2+6/a^3*b^6/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/
(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^2-17*b^3/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2
/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)+8/a^2*b^5/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)
^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)-6*a*b^2/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)
^2/(a^4-2*a^2*b^2+b^4)+3/a*b^4/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a
^2*b^2+b^4)-6*b*a/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1
/2*x)+2*b)/(a^2-b^2)^(1/2))+5/a*b^3/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arc
tan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))-2/a^3*b^5/(a^4-2*a^2*b^2+b^4)
/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 11.39, size = 2191, normalized size = 15.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(a + b*sin(x))^3),x)

[Out] ((3*(b^4 - 2*a^2*b^2))/(a*(a^4 + b^4 - 2*a^2*b^2)) - (3*tan(x/2)^2*(3*a^2*b
^4 - 2*b^6 + 2*a^4*b^2))/(a^3*(a^4 + b^4 - 2*a^2*b^2)) + (tan(x/2)*(8*b^5 -
 17*a^2*b^3))/(a^2*(a^4 + b^4 - 2*a^2*b^2)) + (b*tan(x/2)^3*(4*b^4 - 7*a^2*
b^2))/(a^2*(a^4 + b^4 - 2*a^2*b^2)))/(tan(x/2)^2*(2*a^2 + 4*b^2) + a^2 + a^
2*tan(x/2)^4 + 4*a*b*tan(x/2) + 4*a*b*tan(x/2)^3) + log(tan(x/2))/a^3 + (b*
atan(((b*(-(a + b)^5*(a - b)^5)^(1/2)*((8*a^7*b + 4*a^3*b^5 - 9*a^5*b^3)/(a
^8 + a^4*b^4 - 2*a^6*b^2) + (tan(x/2)*(8*a*b^10 - 2*a^11 - 36*a^3*b^8 + 68*
a^5*b^6 - 62*a^7*b^4 + 24*a^9*b^2))/(a^11 + a^3*b^8 - 4*a^5*b^6 + 6*a^7*b^4
 - 4*a^9*b^2) - (b*((2*a^10*b + 2*a^6*b^5 - 4*a^8*b^3)/(a^8 + a^4*b^4 - 2*a
^6*b^2) - (tan(x/2)*(6*a^14 - 8*a^4*b^10 + 38*a^6*b^8 - 72*a^8*b^6 + 68*a^1
0*b^4 - 32*a^12*b^2))/(a^11 + a^3*b^8 - 4*a^5*b^6 + 6*a^7*b^4 - 4*a^9*b^2))
*(-(a + b)^5*(a - b)^5)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^13 - a^3*b
^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)))*(6*a^4 + 2*b^4 -
5*a^2*b^2)*1i)/(2*(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 -
5*a^11*b^2)) + (b*(-(a + b)^5*(a - b)^5)^(1/2)*((8*a^7*b + 4*a^3*b^5 - 9*a^
5*b^3)/(a^8 + a^4*b^4 - 2*a^6*b^2) + (tan(x/2)*(8*a*b^10 - 2*a^11 - 36*a^3*
b^8 + 68*a^5*b^6 - 62*a^7*b^4 + 24*a^9*b^2))/(a^11 + a^3*b^8 - 4*a^5*b^6 +
6*a^7*b^4 - 4*a^9*b^2) + (b*((2*a^10*b + 2*a^6*b^5 - 4*a^8*b^3)/(a^8 + a^4*
b^4 - 2*a^6*b^2) - (tan(x/2)*(6*a^14 - 8*a^4*b^10 + 38*a^6*b^8 - 72*a^8*b^6
 + 68*a^10*b^4 - 32*a^12*b^2))/(a^11 + a^3*b^8 - 4*a^5*b^6 + 6*a^7*b^4 - 4*
a^9*b^2))*(-(a + b)^5*(a - b)^5)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^1
3 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)))*(6*a^4 +
 2*b^4 - 5*a^2*b^2)*1i)/(2*(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a
^9*b^4 - 5*a^11*b^2)))/((2*(6*a^4*b + 2*b^5 - 5*a^2*b^3))/(a^8 + a^4*b^4 -
2*a^6*b^2) + (2*tan(x/2)*(2*b^8 - 13*a^2*b^6 + 26*a^4*b^4 - 24*a^6*b^2))/(a
^11 + a^3*b^8 - 4*a^5*b^6 + 6*a^7*b^4 - 4*a^9*b^2) - (b*(-(a + b)^5*(a - b)
^5)^(1/2)*((8*a^7*b + 4*a^3*b^5 - 9*a^5*b^3)/(a^8 + a^4*b^4 - 2*a^6*b^2) +
(tan(x/2)*(8*a*b^10 - 2*a^11 - 36*a^3*b^8 + 68*a^5*b^6 - 62*a^7*b^4 + 24*a^
9*b^2))/(a^11 + a^3*b^8 - 4*a^5*b^6 + 6*a^7*b^4 - 4*a^9*b^2) - (b*((2*a^10*
b + 2*a^6*b^5 - 4*a^8*b^3)/(a^8 + a^4*b^4 - 2*a^6*b^2) - (tan(x/2)*(6*a^14
- 8*a^4*b^10 + 38*a^6*b^8 - 72*a^8*b^6 + 68*a^10*b^4 - 32*a^12*b^2))/(a^11
+ a^3*b^8 - 4*a^5*b^6 + 6*a^7*b^4 - 4*a^9*b^2))*(-(a + b)^5*(a - b)^5)^(1/2
)*(6*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6
 + 10*a^9*b^4 - 5*a^11*b^2)))*(6*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^13 - a^3*b
^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)) + (b*(-(a + b)^5*(
a - b)^5)^(1/2)*((8*a^7*b + 4*a^3*b^5 - 9*a^5*b^3)/(a^8 + a^4*b^4 - 2*a^6*b
^2) + (tan(x/2)*(8*a*b^10 - 2*a^11 - 36*a^3*b^8 + 68*a^5*b^6 - 62*a^7*b^4 +
 24*a^9*b^2))/(a^11 + a^3*b^8 - 4*a^5*b^6 + 6*a^7*b^4 - 4*a^9*b^2) + (b*((2
*a^10*b + 2*a^6*b^5 - 4*a^8*b^3)/(a^8 + a^4*b^4 - 2*a^6*b^2) - (tan(x/2)*(6
*a^14 - 8*a^4*b^10 + 38*a^6*b^8 - 72*a^8*b^6 + 68*a^10*b^4 - 32*a^12*b^2))/
(a^11 + a^3*b^8 - 4*a^5*b^6 + 6*a^7*b^4 - 4*a^9*b^2))*(-(a + b)^5*(a - b)^5
)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a
^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)))*(6*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^13 -
```

```
a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2))))*(-(a + b)^
5*(a - b)^5)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2)*1i)/(a^13 - a^3*b^10 + 5*a^5
*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(x)}{(a+b\sin(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*sin(x))**3,x)
```

```
[Out] Integral(csc(x)/(a + b*sin(x))**3, x)
```

3.200 $\int \frac{\csc^2(x)}{(a+b\sin(x))^3}\,dx$

Optimal. Leaf size=187

$$\frac{3b\tanh^{-1}(\cos(x))}{a^4}-\frac{3b^2\left(2a^2-b^2\right)\cot(x)}{2a^2\left(a^2-b^2\right)^2(a+b\sin(x))}-\frac{b^2\cot(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2}+\frac{3b^2\left(4a^4-5a^2b^2+2b^4\right)\tan^{-1}\left(\cdots\right)}{a^4\left(a^2-b^2\right)^{5/2}}\cdots$$

```
[Out] 3*b^2*(4*a^4-5*a^2*b^2+2*b^4)*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/a^4/
(a^2-b^2)^(5/2)+3*b*arctanh(cos(x))/a^4-1/2*(2*a^4-11*a^2*b^2+6*b^4)*cot(x)
/a^3/(a^2-b^2)^2-1/2*b^2*cot(x)/a/(a^2-b^2)/(a+b*sin(x))^2-3/2*b^2*(2*a^2-b
^2)*cot(x)/a^2/(a^2-b^2)^2/(a+b*sin(x))
```

Rubi [A] time = 0.64, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2802, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{3b^2\left(-5a^2b^2+4a^4+2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{5/2}}-\frac{\left(-11a^2b^2+2a^4+6b^4\right)\cot(x)}{2a^3\left(a^2-b^2\right)^2}-\frac{3b^2\left(2a^2-b^2\right)\cot(x)}{2a^2\left(a^2-b^2\right)^2(a+b\sin(x))}-\frac{\cdots}{2a\left(a^2\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a + b*Sin[x])^3,x]
```

```
[Out] (3*b^2*(4*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]]
)/(a^4*(a^2 - b^2)^(5/2)) + (3*b*ArcTanh[Cos[x]])/a^4 - ((2*a^4 - 11*a^2*b^
2 + 6*b^4)*Cot[x])/(2*a^3*(a^2 - b^2)^2) - (b^2*Cot[x])/(2*a*(a^2 - b^2)*(a
 + b*Sin[x])^2) - (3*b^2*(2*a^2 - b^2)*Cot[x])/(2*a^2*(a^2 - b^2)^2*(a + b*
Sin[x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^2(x)}{(a+b\sin(x))^3}\,dx = -\frac{b^2\cot(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2}+\frac{\int \frac{\csc^2(x)\left(2a^2-3b^2-2ab\sin(x)+2b^2\sin^2(x)\right)}{(a+b\sin(x))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= -\frac{b^2\cot(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{3b^2\left(2a^2-b^2\right)\cot(x)}{2a^2\left(a^2-b^2\right)^2(a+b\sin(x))}+\frac{\int \frac{\csc^2(x)\left(2a^4-11a^2b^2+6b^4-ab(4\right.}{a+}}{2a^2(}$$

$$= -\frac{\left(2a^4-11a^2b^2+6b^4\right)\cot(x)}{2a^3\left(a^2-b^2\right)^2}-\frac{b^2\cot(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{3b^2\left(2a^2-b^2\right)\cot(x)}{2a^2\left(a^2-b^2\right)^2(a+b\sin(x}$$

$$= -\frac{\left(2a^4-11a^2b^2+6b^4\right)\cot(x)}{2a^3\left(a^2-b^2\right)^2}-\frac{b^2\cot(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{3b^2\left(2a^2-b^2\right)\cot(x)}{2a^2\left(a^2-b^2\right)^2(a+b\sin(x}$$

$$= \frac{3b\tanh^{-1}(\cos(x))}{a^4}-\frac{\left(2a^4-11a^2b^2+6b^4\right)\cot(x)}{2a^3\left(a^2-b^2\right)^2}-\frac{b^2\cot(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{3b}{2a^2(a}$$

$$= \frac{3b\tanh^{-1}(\cos(x))}{a^4}-\frac{\left(2a^4-11a^2b^2+6b^4\right)\cot(x)}{2a^3\left(a^2-b^2\right)^2}-\frac{b^2\cot(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{3b}{2a^2(a}$$

$$= \frac{3b^2\left(4a^4-5a^2b^2+2b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{5/2}}+\frac{3b\tanh^{-1}(\cos(x))}{a^4}-\frac{\left(2a^4-11a^2b^2+6b^4\right)c}{2a^3\left(a^2-b^2\right)^2}$$

Mathematica [A] time = 1.38, size = 174, normalized size = 0.93

$$\frac{\frac{a^2b^3\cos(x)}{(a-b)(a+b)(a+b\sin(x))^2}+\frac{ab^3\left(7a^2-4b^2\right)\cos(x)}{(a-b)^2(a+b)^2(a+b\sin(x))}+\frac{6b^2\left(4a^4-5a^2b^2+2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}+a\tan\left(\frac{x}{2}\right)-a\cot\left(\frac{x}{2}\right)-6b\log\left(\sin\left(\right.\right.}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a + b*Sin[x])^3,x]
```

```
[Out] ((6*b^2*(4*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]
])/(a^2 - b^2)^(5/2) - a*Cot[x/2] + 6*b*Log[Cos[x/2]] - 6*b*Log[Sin[x/2]] +
 (a^2*b^3*Cos[x])/((a - b)*(a + b)*(a + b*Sin[x])^2) + (a*b^3*(7*a^2 - 4*b^
2)*Cos[x])/((a - b)^2*(a + b)^2*(a + b*Sin[x])) + a*Tan[x/2])/(2*a^4)
```

fricas [B] time = 1.42, size = 1436, normalized size = 7.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(2*a^7*b^2 - 13*a^5*b^4 + 17*a^3*b^6 - 6*a*b^8)*cos(x)^3 - 2*(4*a^8
*b - 20*a^6*b^3 + 25*a^4*b^5 - 9*a^2*b^7)*cos(x)*sin(x) - 3*(8*a^5*b^3 - 10
*a^3*b^5 + 4*a*b^7 - 2*(4*a^5*b^3 - 5*a^3*b^5 + 2*a*b^7)*cos(x)^2 + (4*a^6*
b^2 - a^4*b^4 - 3*a^2*b^6 + 2*b^8 - (4*a^4*b^4 - 5*a^2*b^6 + 2*b^8)*cos(x)^
2)*sin(x))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^
2 - b^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 -
2*a*b*sin(x) - a^2 - b^2)) - 2*(2*a^9 - 4*a^7*b^2 - 7*a^5*b^4 + 15*a^3*b^6
- 6*a*b^8)*cos(x) + 6*(2*a^7*b^2 - 6*a^5*b^4 + 6*a^3*b^6 - 2*a*b^8 - 2*(a^7
*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cos(x)^2 + (a^8*b - 2*a^6*b^3 + 2*a^2
*b^7 - b^9 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cos(x)^2)*sin(x))*log(
1/2*cos(x) + 1/2) - 6*(2*a^7*b^2 - 6*a^5*b^4 + 6*a^3*b^6 - 2*a*b^8 - 2*(a^7
*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cos(x)^2 + (a^8*b - 2*a^6*b^3 + 2*a^2
*b^7 - b^9 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cos(x)^2)*sin(x))*log(
-1/2*cos(x) + 1/2))/(2*a^11*b - 6*a^9*b^3 + 6*a^7*b^5 - 2*a^5*b^7 - 2*(a^11
*b - 3*a^9*b^3 + 3*a^7*b^5 - a^5*b^7)*cos(x)^2 + (a^12 - 2*a^10*b^2 + 2*a^6
*b^6 - a^4*b^8 - (a^10*b^2 - 3*a^8*b^4 + 3*a^6*b^6 - a^4*b^8)*cos(x)^2)*sin
(x)), 1/2*((2*a^7*b^2 - 13*a^5*b^4 + 17*a^3*b^6 - 6*a*b^8)*cos(x)^3 - (4*a^
8*b - 20*a^6*b^3 + 25*a^4*b^5 - 9*a^2*b^7)*cos(x)*sin(x) - 3*(8*a^5*b^3 - 1
0*a^3*b^5 + 4*a*b^7 - 2*(4*a^5*b^3 - 5*a^3*b^5 + 2*a*b^7)*cos(x)^2 + (4*a^6
*b^2 - a^4*b^4 - 3*a^2*b^6 + 2*b^8 - (4*a^4*b^4 - 5*a^2*b^6 + 2*b^8)*cos(x)
^2)*sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))
) - (2*a^9 - 4*a^7*b^2 - 7*a^5*b^4 + 15*a^3*b^6 - 6*a*b^8)*cos(x) + 3*(2*a^
7*b^2 - 6*a^5*b^4 + 6*a^3*b^6 - 2*a*b^8 - 2*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^
6 - a*b^8)*cos(x)^2 + (a^8*b - 2*a^6*b^3 + 2*a^2*b^7 - b^9 - (a^6*b^3 - 3*a
^4*b^5 + 3*a^2*b^7 - b^9)*cos(x)^2)*sin(x))*log(1/2*cos(x) + 1/2) - 3*(2*a^
7*b^2 - 6*a^5*b^4 + 6*a^3*b^6 - 2*a*b^8 - 2*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^
6 - a*b^8)*cos(x)^2 + (a^8*b - 2*a^6*b^3 + 2*a^2*b^7 - b^9 - (a^6*b^3 - 3*a
^4*b^5 + 3*a^2*b^7 - b^9)*cos(x)^2)*sin(x))*log(-1/2*cos(x) + 1/2))/(2*a^11
*b - 6*a^9*b^3 + 6*a^7*b^5 - 2*a^5*b^7 - 2*(a^11*b - 3*a^9*b^3 + 3*a^7*b^5
- a^5*b^7)*cos(x)^2 + (a^12 - 2*a^10*b^2 + 2*a^6*b^6 - a^4*b^8 - (a^10*b^2
- 3*a^8*b^4 + 3*a^6*b^6 - a^4*b^8)*cos(x)^2)*sin(x))]
```

giac [A] time = 0.23, size = 280, normalized size = 1.50

$$\frac{3\left(4\,a^4b^2-5\,a^2b^4+2\,b^6\right)\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^8-2\,a^6b^2+a^4b^4\right)\sqrt{a^2-b^2}}+\frac{9\,a^3b^4\tan\left(\frac{1}{2}\,x\right)^3-6\,ab^6\tan\left(\frac{1}{2}\,x\right)^3+8\,a^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] 3*(4*a^4*b^2 - 5*a^2*b^4 + 2*b^6)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan
((a*tan(1/2*x) + b)/sqrt(a^2 - b^2)))/((a^8 - 2*a^6*b^2 + a^4*b^4)*sqrt(a^2
 - b^2)) + (9*a^3*b^4*tan(1/2*x)^3 - 6*a*b^6*tan(1/2*x)^3 + 8*a^4*b^3*tan(1
/2*x)^2 + 11*a^2*b^5*tan(1/2*x)^2 - 10*b^7*tan(1/2*x)^2 + 23*a^3*b^4*tan(1/
2*x) - 14*a*b^6*tan(1/2*x) + 8*a^4*b^3 - 5*a^2*b^5)/((a^8 - 2*a^6*b^2 + a^4
*b^4)*(a*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)^2) - 3*b*log(abs(tan(1/2*x)))/a
^4 + 1/2*tan(1/2*x)/a^3 + 1/2*(6*b*tan(1/2*x) - a)/(a^4*tan(1/2*x))
```

maple [B] time = 0.16, size = 641, normalized size = 3.43

$$\frac{\tan\left(\frac{x}{2}\right)}{2a^3}-\frac{1}{2a^3\tan\left(\frac{x}{2}\right)}-\frac{3b\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^4}+\frac{9b^4\left(\tan^3\left(\frac{x}{2}\right)\right)}{a\left(\left(\tan^2\left(\frac{x}{2}\right)\right)a+2\tan\left(\frac{x}{2}\right)b+a\right)^2\left(a^4-2a^2b^2+b^4\right)}-\frac{}{a^3\left(\left(\tan^2\left(\frac{x}{2}\right)\right)a+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a+b*sin(x))^3,x)
```

```
[Out] 1/2/a^3*tan(1/2*x)-1/2/a^3/tan(1/2*x)-3/a^4*b*ln(tan(1/2*x))+9/a*b^4/(tan(1
/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^3-6/a^3*b^6/(t
an(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^3+8*b^3/(t
an(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^2+11/a^2*b
^5/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^2-10/
a^4*b^7/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^
2+23/a*b^4/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*
x)-14/a^3*b^6/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1
/2*x)+8*b^3/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)-5/a^2*b
^5/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)+12/(a^4-2*a^2*b^
2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))*b^2
-15/a^2*b^4/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+
2*b)/(a^2-b^2)^(1/2))+6/a^4*b^6/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(
1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 9.02, size = 2295, normalized size = 12.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(a + b*sin(x))^3),x)
```

```
[Out] tan(x/2)/(2*a^3) - (a^2 + (2*tan(x/2)*(7*a*b^5 + 2*a^5*b - 12*a^3*b^3))/(a^
4 + b^4 - 2*a^2*b^2) + (tan(x/2)^4*(a^6 + 12*b^6 - 17*a^2*b^4 - 2*a^4*b^2))
/(a^4 + b^4 - 2*a^2*b^2) + (2*tan(x/2)^2*(a^6 + 16*b^6 - 26*a^2*b^4))/(a^4
+ b^4 - 2*a^2*b^2) + (2*tan(x/2)^3*(2*a^6*b + 10*b^7 - 9*a^2*b^5 - 12*a^4*b
^3))/(a*(a^4 + b^4 - 2*a^2*b^2)))/(tan(x/2)^3*(4*a^5 + 8*a^3*b^2) + 2*a^5*t
an(x/2) + 2*a^5*tan(x/2)^5 + 8*a^4*b*tan(x/2)^2 + 8*a^4*b*tan(x/2)^4) - (3*
b*log(tan(x/2)))/a^4 - (b^2*atan(((b^2*(-(a + b)^5*(a - b)^5)^(1/2)*(4*a^4
+ 2*b^4 - 5*a^2*b^2)*((12*a^4*b^6 - 27*a^6*b^4 + 18*a^8*b^2)/(a^10 + a^6*b^
4 - 2*a^8*b^2) - (tan(x/2)*(6*a^12*b - 24*a^2*b^11 + 108*a^4*b^9 - 192*a^6*
b^7 + 162*a^8*b^5 - 60*a^10*b^3))/(a^13 + a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 -
 4*a^11*b^2) + (3*b^2*((2*a^12*b + 2*a^8*b^5 - 4*a^10*b^3)/(a^10 + a^6*b^4
- 2*a^8*b^2) - (tan(x/2)*(6*a^16 - 8*a^6*b^10 + 38*a^8*b^8 - 72*a^10*b^6 +
68*a^12*b^4 - 32*a^14*b^2))/(a^13 + a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^1
1*b^2))*(-(a + b)^5*(a - b)^5)^(1/2)*(4*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^14
- a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)))*3i)/(2*(a
^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)) - (b^2
*(-(a + b)^5*(a - b)^5)^(1/2)*(4*a^4 + 2*b^4 - 5*a^2*b^2)*((tan(x/2)*(6*a^1
2*b - 24*a^2*b^11 + 108*a^4*b^9 - 192*a^6*b^7 + 162*a^8*b^5 - 60*a^10*b^3))
/(a^13 + a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^11*b^2) - (12*a^4*b^6 - 27*a
^6*b^4 + 18*a^8*b^2)/(a^10 + a^6*b^4 - 2*a^8*b^2) + (3*b^2*((2*a^12*b + 2*a
^8*b^5 - 4*a^10*b^3)/(a^10 + a^6*b^4 - 2*a^8*b^2) - (tan(x/2)*(6*a^16 - 8*a
^6*b^10 + 38*a^8*b^8 - 72*a^10*b^6 + 68*a^12*b^4 - 32*a^14*b^2))/(a^13 + a^
5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^11*b^2))*(-(a + b)^5*(a - b)^5)^(1/2)*(
4*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 +
10*a^10*b^4 - 5*a^12*b^2)))*3i)/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^
```

```
6 + 10*a^10*b^4 - 5*a^12*b^2)))/((2*(18*b^7 - 45*a^2*b^5 + 36*a^4*b^3))/(a^
10 + a^6*b^4 - 2*a^8*b^2) + (2*tan(x/2)*(18*b^10 - 81*a^2*b^8 + 126*a^4*b^6
 - 72*a^6*b^4))/(a^13 + a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^11*b^2) + (3*
b^2*(-(a + b)^5*(a - b)^5)^(1/2)*(4*a^4 + 2*b^4 - 5*a^2*b^2)*((12*a^4*b^6 -
 27*a^6*b^4 + 18*a^8*b^2)/(a^10 + a^6*b^4 - 2*a^8*b^2) - (tan(x/2)*(6*a^12*
b - 24*a^2*b^11 + 108*a^4*b^9 - 192*a^6*b^7 + 162*a^8*b^5 - 60*a^10*b^3))/(
a^13 + a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^11*b^2) + (3*b^2*((2*a^12*b +
2*a^8*b^5 - 4*a^10*b^3)/(a^10 + a^6*b^4 - 2*a^8*b^2) - (tan(x/2)*(6*a^16 -
8*a^6*b^10 + 38*a^8*b^8 - 72*a^10*b^6 + 68*a^12*b^4 - 32*a^14*b^2))/(a^13 +
 a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^11*b^2))*(-(a + b)^5*(a - b)^5)^(1/2
)*(4*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6
 + 10*a^10*b^4 - 5*a^12*b^2))))/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^
6 + 10*a^10*b^4 - 5*a^12*b^2)) + (3*b^2*(-(a + b)^5*(a - b)^5)^(1/2)*(4*a^4
 + 2*b^4 - 5*a^2*b^2)*((tan(x/2)*(6*a^12*b - 24*a^2*b^11 + 108*a^4*b^9 - 19
2*a^6*b^7 + 162*a^8*b^5 - 60*a^10*b^3))/(a^13 + a^5*b^8 - 4*a^7*b^6 + 6*a^9
*b^4 - 4*a^11*b^2) - (12*a^4*b^6 - 27*a^6*b^4 + 18*a^8*b^2)/(a^10 + a^6*b^4
 - 2*a^8*b^2) + (3*b^2*((2*a^12*b + 2*a^8*b^5 - 4*a^10*b^3)/(a^10 + a^6*b^4
 - 2*a^8*b^2) - (tan(x/2)*(6*a^16 - 8*a^6*b^10 + 38*a^8*b^8 - 72*a^10*b^6 +
 68*a^12*b^4 - 32*a^14*b^2))/(a^13 + a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^
11*b^2))*(-(a + b)^5*(a - b)^5)^(1/2)*(4*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^14
 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2))))/(2*(a^1
4 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2))))*(-(a +
 b)^5*(a - b)^5)^(1/2)*(4*a^4 + 2*b^4 - 5*a^2*b^2)*3i)/(a^14 - a^4*b^10 + 5
*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(x)}{(a + b\sin(x))^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a+b*sin(x))**3,x)
```

```
[Out] Integral(csc(x)**2/(a + b*sin(x))**3, x)
```

3.201 $\int \frac{\csc^3(x)}{(a+b\sin(x))^3}\,dx$

Optimal. Leaf size=241

$$-\frac{b^2\left(7a^2-4b^2\right)\cot(x)\csc(x)}{2a^2\left(a^2-b^2\right)^2(a+b\sin(x))}-\frac{b^2\cot(x)\csc(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2}-\frac{\left(a^2+12b^2\right)\tanh^{-1}(\cos(x))}{2a^5}+\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\cdots}{2a^4\left(a^2-b^2\right)^2}$$

```
[Out] -b^3*(20*a^4-29*a^2*b^2+12*b^4)*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/a^
5/(a^2-b^2)^(5/2)-1/2*(a^2+12*b^2)*arctanh(cos(x))/a^5+3/2*b*(2*a^4-7*a^2*b
^2+4*b^4)*cot(x)/a^4/(a^2-b^2)^2-1/2*(a^4-10*a^2*b^2+6*b^4)*cot(x)*csc(x)/a
^3/(a^2-b^2)^2-1/2*b^2*cot(x)*csc(x)/a/(a^2-b^2)/(a+b*sin(x))^2-1/2*b^2*(7*
a^2-4*b^2)*cot(x)*csc(x)/a^2/(a^2-b^2)^2/(a+b*sin(x))
```

Rubi [A] time = 0.87, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2802, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{b^3\left(-29a^2b^2+20a^4+12b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5\left(a^2-b^2\right)^{5/2}}+\frac{3b\left(-7a^2b^2+2a^4+4b^4\right)\cot(x)}{2a^4\left(a^2-b^2\right)^2}-\frac{\left(a^2+12b^2\right)\tanh^{-1}(\cos(x))}{2a^5}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a + b*Sin[x])^3,x]
```

```
[Out] -((b^3*(20*a^4 - 29*a^2*b^2 + 12*b^4)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^
2]])/(a^5*(a^2 - b^2)^(5/2))) - ((a^2 + 12*b^2)*ArcTanh[Cos[x]])/(2*a^5) +
(3*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Cot[x])/(2*a^4*(a^2 - b^2)^2) - ((a^4 - 10
*a^2*b^2 + 6*b^4)*Cot[x]*Csc[x])/(2*a^3*(a^2 - b^2)^2) - (b^2*Cot[x]*Csc[x]
)/(2*a*(a^2 - b^2)*(a + b*Sin[x])^2) - (b^2*(7*a^2 - 4*b^2)*Cot[x]*Csc[x])/
(2*a^2*(a^2 - b^2)^2*(a + b*Sin[x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
```

```
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^3(x)}{(a+b\sin(x))^3}\,dx = -\frac{b^2\cot(x)\csc(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2} + \frac{\int \frac{\csc^3(x)\left(2\left(a^2-2b^2\right)-2ab\sin(x)+3b^2\sin^2(x)\right)}{(a+b\sin(x))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= -\frac{b^2\cot(x)\csc(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{b^2\left(7a^2-4b^2\right)\cot(x)\csc(x)}{2a^2\left(a^2-b^2\right)^2(a+b\sin(x))} + \frac{\int \frac{\csc^3(x)\left(2\left(a^4-10a^2b^2+6b^4\right)-ab\left(4\right.\right.}{a+}}{2a^2\left(\right.}$$

$$= -\frac{\left(a^4-10a^2b^2+6b^4\right)\cot(x)\csc(x)}{2a^3\left(a^2-b^2\right)^2} - \frac{b^2\cot(x)\csc(x)}{2a\left(a^2-b^2\right)(a+b\sin(x))^2} - \frac{b^2\left(7a^2-4b^2\right)\cot(x)\,\mathrm{c}}{2a^2\left(a^2-b^2\right)^2(a+b\,\mathrm{s}}$$

$$= \frac{3b\left(2a^4-7a^2b^2+4b^4\right)\cot(x)}{2a^4\left(a^2-b^2\right)^2} - \frac{\left(a^4-10a^2b^2+6b^4\right)\cot(x)\csc(x)}{2a^3\left(a^2-b^2\right)^2} - \frac{b^2\cot(x)\csc(x)}{2a\left(a^2-b^2\right)(a+b\,\mathrm{si}}$$

$$= \frac{3b\left(2a^4-7a^2b^2+4b^4\right)\cot(x)}{2a^4\left(a^2-b^2\right)^2} - \frac{\left(a^4-10a^2b^2+6b^4\right)\cot(x)\csc(x)}{2a^3\left(a^2-b^2\right)^2} - \frac{b^2\cot(x)\csc(x)}{2a\left(a^2-b^2\right)(a+b\,\mathrm{si}}$$

$$= -\frac{\left(a^2+12b^2\right)\tanh^{-1}(\cos(x))}{2a^5} + \frac{3b\left(2a^4-7a^2b^2+4b^4\right)\cot(x)}{2a^4\left(a^2-b^2\right)^2} - \frac{\left(a^4-10a^2b^2+6b^4\right)\cot(}{2a^3\left(a^2-b^2\right)^2}$$

$$= -\frac{\left(a^2+12b^2\right)\tanh^{-1}(\cos(x))}{2a^5} + \frac{3b\left(2a^4-7a^2b^2+4b^4\right)\cot(x)}{2a^4\left(a^2-b^2\right)^2} - \frac{\left(a^4-10a^2b^2+6b^4\right)\cot(}{2a^3\left(a^2-b^2\right)^2}$$

$$= -\frac{b^3\left(20a^4-29a^2b^2+12b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^5\left(a^2-b^2\right)^{5/2}} - \frac{\left(a^2+12b^2\right)\tanh^{-1}(\cos(x))}{2a^5} + \frac{3b\left(2a^4-\right.}{2a}$$

Mathematica [A] time = 2.08, size = 220, normalized size = 0.91

$$\frac{-\frac{4a^2b^4\cos(x)}{(a-b)(a+b)(a+b\sin(x))^2}+4\left(a^2+12b^2\right)\log\left(\sin\left(\frac{x}{2}\right)\right)-4\left(a^2+12b^2\right)\log\left(\cos\left(\frac{x}{2}\right)\right)+\frac{12ab^4\left(2b^2-3a^2\right)\cos(x)}{(a-b)^2(a+b)^2(a+b\sin(x))}-a^2\csc^2(}{8a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a + b*Sin[x])^3,x]
```

```
[Out] ((-8*b^3*(20*a^4 - 29*a^2*b^2 + 12*b^4)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 -
b^2]])/(a^2 - b^2)^(5/2) + 12*a*b*Cot[x/2] - a^2*Csc[x/2]^2 - 4*(a^2 + 12*b
^2)*Log[Cos[x/2]] + 4*(a^2 + 12*b^2)*Log[Sin[x/2]] + a^2*Sec[x/2]^2 - (4*a^
2*b^4*Cos[x])/((a - b)*(a + b)*(a + b*Sin[x])^2) + (12*a*b^4*(-3*a^2 + 2*b^
2)*Cos[x])/((a - b)^2*(a + b)^2*(a + b*Sin[x])) - 12*a*b*Tan[x/2])/(8*a^5)
```

fricas [B] time = 2.55, size = 2005, normalized size = 8.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(11*a^8*b^2 - 43*a^6*b^4 + 50*a^4*b^6 - 18*a^2*b^8)*cos(x)^3 + (20
*a^6*b^3 - 9*a^4*b^5 - 17*a^2*b^7 + 12*b^9 + (20*a^4*b^5 - 29*a^2*b^7 + 12*
b^9)*cos(x)^4 - (20*a^6*b^3 + 11*a^4*b^5 - 46*a^2*b^7 + 24*b^9)*cos(x)^2 +
2*(20*a^5*b^4 - 29*a^3*b^6 + 12*a*b^8 - (20*a^5*b^4 - 29*a^3*b^6 + 12*a*b^8
)*cos(x)^2)*sin(x))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*s
in(x) - a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*c
os(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) + 2*(a^10 - 14*a^8*b^2 + 46*a^6*b^4 -
51*a^4*b^6 + 18*a^2*b^8)*cos(x) + (a^10 + 10*a^8*b^2 - 24*a^6*b^4 + 2*a^4*b
^6 + 23*a^2*b^8 - 12*b^10 + (a^8*b^2 + 9*a^6*b^4 - 33*a^4*b^6 + 35*a^2*b^8
- 12*b^10)*cos(x)^4 - (a^10 + 11*a^8*b^2 - 15*a^6*b^4 - 31*a^4*b^6 + 58*a^2
*b^8 - 24*b^10)*cos(x)^2 + 2*(a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 -
 12*a*b^9 - (a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9)*cos(x)
^2)*sin(x))*log(1/2*cos(x) + 1/2) - (a^10 + 10*a^8*b^2 - 24*a^6*b^4 + 2*a^4
*b^6 + 23*a^2*b^8 - 12*b^10 + (a^8*b^2 + 9*a^6*b^4 - 33*a^4*b^6 + 35*a^2*b^
8 - 12*b^10)*cos(x)^4 - (a^10 + 11*a^8*b^2 - 15*a^6*b^4 - 31*a^4*b^6 + 58*a
^2*b^8 - 24*b^10)*cos(x)^2 + 2*(a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7
 - 12*a*b^9 - (a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9)*cos(
x)^2)*sin(x))*log(-1/2*cos(x) + 1/2) + 2*(3*(2*a^7*b^3 - 9*a^5*b^5 + 11*a^3
*b^7 - 4*a*b^9)*cos(x)^3 - (4*a^9*b - 6*a^7*b^3 - 15*a^5*b^5 + 29*a^3*b^7 -
 12*a*b^9)*cos(x))*sin(x))/(a^13 - 2*a^11*b^2 + 2*a^7*b^6 - a^5*b^8 + (a^11
*b^2 - 3*a^9*b^4 + 3*a^7*b^6 - a^5*b^8)*cos(x)^4 - (a^13 - a^11*b^2 - 3*a^9
```

```
*b^4 + 5*a^7*b^6 - 2*a^5*b^8)*cos(x)^2 + 2*(a^12*b - 3*a^10*b^3 + 3*a^8*b^5
 - a^6*b^7 - (a^12*b - 3*a^10*b^3 + 3*a^8*b^5 - a^6*b^7)*cos(x)^2)*sin(x)),
 -1/4*(2*(11*a^8*b^2 - 43*a^6*b^4 + 50*a^4*b^6 - 18*a^2*b^8)*cos(x)^3 - 2*(
20*a^6*b^3 - 9*a^4*b^5 - 17*a^2*b^7 + 12*b^9 + (20*a^4*b^5 - 29*a^2*b^7 + 1
2*b^9)*cos(x)^4 - (20*a^6*b^3 + 11*a^4*b^5 - 46*a^2*b^7 + 24*b^9)*cos(x)^2
+ 2*(20*a^5*b^4 - 29*a^3*b^6 + 12*a*b^8 - (20*a^5*b^4 - 29*a^3*b^6 + 12*a*b
^8)*cos(x)^2)*sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^
2)*cos(x))) + 2*(a^10 - 14*a^8*b^2 + 46*a^6*b^4 - 51*a^4*b^6 + 18*a^2*b^8)*
cos(x) + (a^10 + 10*a^8*b^2 - 24*a^6*b^4 + 2*a^4*b^6 + 23*a^2*b^8 - 12*b^10
 + (a^8*b^2 + 9*a^6*b^4 - 33*a^4*b^6 + 35*a^2*b^8 - 12*b^10)*cos(x)^4 - (a^
10 + 11*a^8*b^2 - 15*a^6*b^4 - 31*a^4*b^6 + 58*a^2*b^8 - 24*b^10)*cos(x)^2
+ 2*(a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9 - (a^9*b + 9*a^
7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9)*cos(x)^2)*sin(x))*log(1/2*cos(x
) + 1/2) - (a^10 + 10*a^8*b^2 - 24*a^6*b^4 + 2*a^4*b^6 + 23*a^2*b^8 - 12*b^
10 + (a^8*b^2 + 9*a^6*b^4 - 33*a^4*b^6 + 35*a^2*b^8 - 12*b^10)*cos(x)^4 - (
a^10 + 11*a^8*b^2 - 15*a^6*b^4 - 31*a^4*b^6 + 58*a^2*b^8 - 24*b^10)*cos(x)^
2 + 2*(a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9 - (a^9*b + 9*
a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9)*cos(x)^2)*sin(x))*log(-1/2*co
s(x) + 1/2) + 2*(3*(2*a^7*b^3 - 9*a^5*b^5 + 11*a^3*b^7 - 4*a*b^9)*cos(x)^3
- (4*a^9*b - 6*a^7*b^3 - 15*a^5*b^5 + 29*a^3*b^7 - 12*a*b^9)*cos(x))*sin(x)
)/(a^13 - 2*a^11*b^2 + 2*a^7*b^6 - a^5*b^8 + (a^11*b^2 - 3*a^9*b^4 + 3*a^7*
b^6 - a^5*b^8)*cos(x)^4 - (a^13 - a^11*b^2 - 3*a^9*b^4 + 5*a^7*b^6 - 2*a^5*
b^8)*cos(x)^2 + 2*(a^12*b - 3*a^10*b^3 + 3*a^8*b^5 - a^6*b^7 - (a^12*b - 3*
a^10*b^3 + 3*a^8*b^5 - a^6*b^7)*cos(x)^2)*sin(x))]
```

giac [B] time = 0.46, size = 514, normalized size = 2.13

$$-\frac{\left(20\,a^4b^3-29\,a^2b^5+12\,b^7\right)\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^9-2\,a^7b^2+a^5b^4\right)\sqrt{a^2-b^2}}-\frac{2\,a^8\tan\left(\frac{1}{2}\,x\right)^6+20\,a^6b^2\tan\left(\frac{1}{2}\,x\right)^6-4\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] -(20*a^4*b^3 - 29*a^2*b^5 + 12*b^7)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arct
an((a*tan(1/2*x) + b)/sqrt(a^2 - b^2)))/((a^9 - 2*a^7*b^2 + a^5*b^4)*sqrt(a
^2 - b^2)) - 1/8*(2*a^8*tan(1/2*x)^6 + 20*a^6*b^2*tan(1/2*x)^6 - 46*a^4*b^4
*tan(1/2*x)^6 + 24*a^2*b^6*tan(1/2*x)^6 - 4*a^7*b*tan(1/2*x)^5 + 104*a^5*b^
3*tan(1/2*x)^5 - 108*a^3*b^5*tan(1/2*x)^5 + 32*a*b^7*tan(1/2*x)^5 + 5*a^8*t
an(1/2*x)^4 - 2*a^6*b^2*tan(1/2*x)^4 + 165*a^4*b^4*tan(1/2*x)^4 - 80*a^2*b^
6*tan(1/2*x)^4 - 16*b^8*tan(1/2*x)^4 - 12*a^7*b*tan(1/2*x)^3 + 72*a^5*b^3*t
an(1/2*x)^3 + 124*a^3*b^5*tan(1/2*x)^3 - 112*a*b^7*tan(1/2*x)^3 + 4*a^8*tan
```

```
(1/2*x)^2 - 28*a^6*b^2*tan(1/2*x)^2 + 124*a^4*b^4*tan(1/2*x)^2 - 76*a^2*b^6
*tan(1/2*x)^2 - 8*a^7*b*tan(1/2*x) + 16*a^5*b^3*tan(1/2*x) - 8*a^3*b^5*tan(
1/2*x) + a^8 - 2*a^6*b^2 + a^4*b^4)/((a^9 - 2*a^7*b^2 + a^5*b^4)*(a*tan(1/2
*x)^3 + 2*b*tan(1/2*x)^2 + a*tan(1/2*x))^2) + 1/2*(a^2 + 12*b^2)*log(abs(ta
n(1/2*x)))/a^5 + 1/8*(a^3*tan(1/2*x)^2 - 12*a^2*b*tan(1/2*x))/a^6
```

maple [B] time = 0.17, size = 686, normalized size = 2.85

$$\frac{\tan^2\left(\frac{x}{2}\right)}{8a^3}-\frac{3\tan\left(\frac{x}{2}\right)b}{2a^4}-\frac{1}{8a^3\tan\left(\frac{x}{2}\right)^2}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a^3}+\frac{6\ln\left(\tan\left(\frac{x}{2}\right)\right)b^2}{a^5}+\frac{3b}{2a^4\tan\left(\frac{x}{2}\right)}-\frac{11b^5\left(\tan\right.}{a^2\left(\left(\tan^2\left(\frac{x}{2}\right)\right)a+2\tan\left(\frac{x}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a+b*sin(x))^3,x)
```

```
[Out] 1/8/a^3*tan(1/2*x)^2-3/2/a^4*tan(1/2*x)*b-1/8/a^3/tan(1/2*x)^2+1/2/a^3*ln(t
an(1/2*x))+6/a^5*ln(tan(1/2*x))*b^2+3/2*b/a^4/tan(1/2*x)-11/a^2*b^5/(tan(1/
2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^3+8/a^4*b^7/(ta
n(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^3-10/a*b^4/
(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^2-13/a^3
*b^6/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x)^2+1
4/a^5*b^8/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*x
)^2-29/a^2*b^5/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(
1/2*x)+20/a^4*b^7/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)*t
an(1/2*x)-10/a*b^4/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)+
7/a^3*b^6/(tan(1/2*x)^2*a+2*tan(1/2*x)*b+a)^2/(a^4-2*a^2*b^2+b^4)-20/a*b^3/
(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^
2)^(1/2))+29/a^3*b^5/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*ta
n(1/2*x)+2*b)/(a^2-b^2)^(1/2))-12/a^5*b^7/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/
2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 9.28, size = 2405, normalized size = 9.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^3*(a + b*sin(x))^3),x)
```

```
[Out] (4*a^2*b*tan(x/2) - a^3/2 + (tan(x/2)^2*(50*a*b^6 - a^7 - 85*a^3*b^4 + 24*a
^5*b^2))/(a^4 + b^4 - 2*a^2*b^2) + (2*tan(x/2)^5*(3*a^6*b + 16*b^7 - 19*a^2
*b^5 - 6*a^4*b^3))/(a^4 + b^4 - 2*a^2*b^2) + (2*tan(x/2)^3*(5*a^6*b + 52*b^
7 - 77*a^2*b^5 + 2*a^4*b^3))/(a^4 + b^4 - 2*a^2*b^2) - (tan(x/2)^4*(a^8 - 1
12*b^8 + 56*a^2*b^6 + 177*a^4*b^4 - 50*a^6*b^2))/(2*a*(a^4 + b^4 - 2*a^2*b^
2)))/(tan(x/2)^4*(8*a^6 + 16*a^4*b^2) + 4*a^6*tan(x/2)^2 + 4*a^6*tan(x/2)^6
 + 16*a^5*b*tan(x/2)^3 + 16*a^5*b*tan(x/2)^5) + tan(x/2)^2/(8*a^3) + (log(t
an(x/2))*(a^2 + 12*b^2))/(2*a^5) - (3*b*tan(x/2))/(2*a^4) + (b^3*atan(((b^3
*(-(a + b)^5*(a - b)^5)^(1/2)*(20*a^4 + 12*b^4 - 29*a^2*b^2)*((a^11*b + 24*
a^5*b^7 - 52*a^7*b^5 + 30*a^9*b^3)/(a^12 + a^8*b^4 - 2*a^10*b^2) - (tan(x/2
)*(a^15 - 48*a^3*b^12 + 212*a^5*b^10 - 363*a^7*b^8 + 290*a^9*b^6 - 98*a^11*
b^4 + 6*a^13*b^2))/(a^15 + a^7*b^8 - 4*a^9*b^6 + 6*a^11*b^4 - 4*a^13*b^2) +
 (b^3*((2*a^14*b + 2*a^10*b^5 - 4*a^12*b^3)/(a^12 + a^8*b^4 - 2*a^10*b^2) -
 (tan(x/2)*(6*a^18 - 8*a^8*b^10 + 38*a^10*b^8 - 72*a^12*b^6 + 68*a^14*b^4 -
 32*a^16*b^2))/(a^15 + a^7*b^8 - 4*a^9*b^6 + 6*a^11*b^4 - 4*a^13*b^2))*(-(a
 + b)^5*(a - b)^5)^(1/2)*(20*a^4 + 12*b^4 - 29*a^2*b^2))/(2*(a^15 - a^5*b^1
0 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)))*1i)/(2*(a^15 - a^5
*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)) - (b^3*(-(a + b
)^5*(a - b)^5)^(1/2)*(20*a^4 + 12*b^4 - 29*a^2*b^2)*((tan(x/2)*(a^15 - 48*a
^3*b^12 + 212*a^5*b^10 - 363*a^7*b^8 + 290*a^9*b^6 - 98*a^11*b^4 + 6*a^13*b
^2))/(a^15 + a^7*b^8 - 4*a^9*b^6 + 6*a^11*b^4 - 4*a^13*b^2) - (a^11*b + 24*
a^5*b^7 - 52*a^7*b^5 + 30*a^9*b^3)/(a^12 + a^8*b^4 - 2*a^10*b^2) + (b^3*((2
*a^14*b + 2*a^10*b^5 - 4*a^12*b^3)/(a^12 + a^8*b^4 - 2*a^10*b^2) - (tan(x/2
)*(6*a^18 - 8*a^8*b^10 + 38*a^10*b^8 - 72*a^12*b^6 + 68*a^14*b^4 - 32*a^16*
b^2))/(a^15 + a^7*b^8 - 4*a^9*b^6 + 6*a^11*b^4 - 4*a^13*b^2))*(-(a + b)^5*(
a - b)^5)^(1/2)*(20*a^4 + 12*b^4 - 29*a^2*b^2))/(2*(a^15 - a^5*b^10 + 5*a^7
*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)))*1i)/(2*(a^15 - a^5*b^10 + 5
*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)))/((144*b^9 - 336*a^2*b^7
 + 211*a^4*b^5 + 20*a^6*b^3)/(a^12 + a^8*b^4 - 2*a^10*b^2) + (2*tan(x/2)*(7
2*b^12 - 294*a^2*b^10 + 422*a^4*b^8 - 229*a^6*b^6 + 20*a^8*b^4))/(a^15 + a^
7*b^8 - 4*a^9*b^6 + 6*a^11*b^4 - 4*a^13*b^2) + (b^3*(-(a + b)^5*(a - b)^5)^
(1/2)*(20*a^4 + 12*b^4 - 29*a^2*b^2)*((a^11*b + 24*a^5*b^7 - 52*a^7*b^5 + 3
0*a^9*b^3)/(a^12 + a^8*b^4 - 2*a^10*b^2) - (tan(x/2)*(a^15 - 48*a^3*b^12 +
212*a^5*b^10 - 363*a^7*b^8 + 290*a^9*b^6 - 98*a^11*b^4 + 6*a^13*b^2))/(a^15
 + a^7*b^8 - 4*a^9*b^6 + 6*a^11*b^4 - 4*a^13*b^2) + (b^3*((2*a^14*b + 2*a^1
0*b^5 - 4*a^12*b^3)/(a^12 + a^8*b^4 - 2*a^10*b^2) - (tan(x/2)*(6*a^18 - 8*a
^8*b^10 + 38*a^10*b^8 - 72*a^12*b^6 + 68*a^14*b^4 - 32*a^16*b^2))/(a^15 + a
^7*b^8 - 4*a^9*b^6 + 6*a^11*b^4 - 4*a^13*b^2))*(-(a + b)^5*(a - b)^5)^(1/2)
*(20*a^4 + 12*b^4 - 29*a^2*b^2))/(2*(a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b
^6 + 10*a^11*b^4 - 5*a^13*b^2))))/(2*(a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*
b^6 + 10*a^11*b^4 - 5*a^13*b^2)) + (b^3*(-(a + b)^5*(a - b)^5)^(1/2)*(20*a^
```

```
4 + 12*b^4 - 29*a^2*b^2)*((tan(x/2)*(a^15 - 48*a^3*b^12 + 212*a^5*b^10 - 36
3*a^7*b^8 + 290*a^9*b^6 - 98*a^11*b^4 + 6*a^13*b^2))/(a^15 + a^7*b^8 - 4*a^
9*b^6 + 6*a^11*b^4 - 4*a^13*b^2) - (a^11*b + 24*a^5*b^7 - 52*a^7*b^5 + 30*a
^9*b^3)/(a^12 + a^8*b^4 - 2*a^10*b^2) + (b^3*((2*a^14*b + 2*a^10*b^5 - 4*a^
12*b^3)/(a^12 + a^8*b^4 - 2*a^10*b^2) - (tan(x/2)*(6*a^18 - 8*a^8*b^10 + 38
*a^10*b^8 - 72*a^12*b^6 + 68*a^14*b^4 - 32*a^16*b^2))/(a^15 + a^7*b^8 - 4*a
^9*b^6 + 6*a^11*b^4 - 4*a^13*b^2))*(-(a + b)^5*(a - b)^5)^(1/2)*(20*a^4 + 1
2*b^4 - 29*a^2*b^2))/(2*(a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11
*b^4 - 5*a^13*b^2))))/(2*(a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^1
1*b^4 - 5*a^13*b^2))))*(-(a + b)^5*(a - b)^5)^(1/2)*(20*a^4 + 12*b^4 - 29*a
^2*b^2)*1i)/(a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^1
3*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(x)}{(a+b\sin(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a+b*sin(x))**3,x)
```

```
[Out] Integral(csc(x)**3/(a + b*sin(x))**3, x)
```

3.202 $\int \frac{1}{(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=182

$$\frac{a\left(2a^2+3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}+\frac{b\left(11a^2+4b^2\right)\cos(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{5ab\cos(c+dx)}{6d\left(a^2-b^2\right)^2(a+b\sin(c+dx))^2}+\frac{\quad}{3d\left(a^2 \cdots\right.}$$

```
[Out] a*(2*a^2+3*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^
(7/2)/d+1/3*b*cos(d*x+c)/(a^2-b^2)/d/(a+b*sin(d*x+c))^3+5/6*a*b*cos(d*x+c)/
(a^2-b^2)^2/d/(a+b*sin(d*x+c))^2+1/6*b*(11*a^2+4*b^2)*cos(d*x+c)/(a^2-b^2)^
3/d/(a+b*sin(d*x+c))
```

Rubi [A] time = 0.22, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2664, 2754, 12, 2660, 618, 204}

$$\frac{a\left(2a^2+3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}+\frac{b\left(11a^2+4b^2\right)\cos(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{5ab\cos(c+dx)}{6d\left(a^2-b^2\right)^2(a+b\sin(c+dx))^2}+\frac{\quad}{3d\left(a^2 \cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^(-4),x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2
- b^2)^(7/2)*d) + (b*Cos[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^3)
 + (5*a*b*Cos[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^2) + (b*(11
*a^2 + 4*b^2)*Cos[c + d*x])/(6*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(c+dx))^4}\,dx = \frac{b\cos(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^3} - \frac{\int \frac{-3a+2b\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx}{3\left(a^2-b^2\right)}$$

$$= \frac{b\cos(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^3} + \frac{5ab\cos(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^2} + \frac{\int \frac{2\left(3a^2+2b^2\right)-5}{(a+b\sin(}}{6\left(a^2\right.}$$

$$= \frac{b\cos(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^3} + \frac{5ab\cos(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^2} + \frac{b\left(11a^2+\right.}{6\left(a^2-b^2\right)^3 a}$$

$$= \frac{b\cos(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^3} + \frac{5ab\cos(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^2} + \frac{b\left(11a^2+\right.}{6\left(a^2-b^2\right)^3 a}$$

$$= \frac{b\cos(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^3} + \frac{5ab\cos(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^2} + \frac{b\left(11a^2+\right.}{6\left(a^2-b^2\right)^3 a}$$

$$= \frac{b\cos(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^3} + \frac{5ab\cos(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^2} + \frac{b\left(11a^2+\right.}{6\left(a^2-b^2\right)^3 a}$$

$$= \frac{a\left(2a^2+3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2} d} + \frac{b\cos(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^3} + \frac{5}{6\left(a^2-b^2\right.}$$

Mathematica [A] time = 1.02, size = 157, normalized size = 0.86

$$\frac{\frac{6a\left(2a^2+3b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}} + \frac{b\cos(c+dx)\left(18a^4+b^2\left(11a^2+4b^2\right)\sin^2(c+dx)+3ab\left(9a^2+b^2\right)\sin(c+dx)-5a^2b^2+2b^4\right)}{(a-b)^3(a+b)^3(a+b\sin(c+dx))^3}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^(-4),x]
```

```
[Out] ((6*a*(2*a^2 + 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^
2 - b^2)^(7/2) + (b*Cos[c + d*x]*(18*a^4 - 5*a^2*b^2 + 2*b^4 + 3*a*b*(9*a^2
 + b^2)*Sin[c + d*x] + b^2*(11*a^2 + 4*b^2)*Sin[c + d*x]^2))/((a - b)^3*(a
+ b)^3*(a + b*Sin[c + d*x])^3))/(6*d)
```

fricas [B] time = 0.59, size = 965, normalized size = 5.30

$$\left[\frac{2\left(11\,a^4b^3-7\,a^2b^5-4\,b^7\right)\cos\left(dx+c\right)^3-6\left(9\,a^5b^2-8\,a^3b^4-ab^6\right)\cos\left(dx+c\right)\sin\left(dx+c\right)-3\left(2\,a^6+9\,a^4b^2+\right.}{12\left(3\left(a^9b^2-4\,a^7b^4+6\,a^5b^6-4\,a^3b^8+ab^{10}\right)d\cos\left(a\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(2*(11*a^4*b^3 - 7*a^2*b^5 - 4*b^7)*cos(d*x + c)^3 - 6*(9*a^5*b^2 - 8
*a^3*b^4 - a*b^6)*cos(d*x + c)*sin(d*x + c) - 3*(2*a^6 + 9*a^4*b^2 + 9*a^2*
b^4 - 3*(2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^2 + (6*a^5*b + 11*a^3*b^3 + 3*
a*b^5 - (2*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2
)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(
a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*
x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 12*(3*a^6*b - 2*a^4*b^3 - b^7
)*cos(d*x + c))/(3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d
*cos(d*x + c)^2 - (a^11 - a^9*b^2 - 6*a^7*b^4 + 14*a^5*b^6 - 11*a^3*b^8 + 3
*a*b^10)*d + ((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*
x + c)^2 - (3*a^10*b - 11*a^8*b^3 + 14*a^6*b^5 - 6*a^4*b^7 - a^2*b^9 + b^11
)*d)*sin(d*x + c)), 1/6*((11*a^4*b^3 - 7*a^2*b^5 - 4*b^7)*cos(d*x + c)^3 -
3*(9*a^5*b^2 - 8*a^3*b^4 - a*b^6)*cos(d*x + c)*sin(d*x + c) + 3*(2*a^6 + 9*
a^4*b^2 + 9*a^2*b^4 - 3*(2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^2 + (6*a^5*b +
 11*a^3*b^3 + 3*a*b^5 - (2*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))
*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)
)) - 6*(3*a^6*b - 2*a^4*b^3 - b^7)*cos(d*x + c))/(3*(a^9*b^2 - 4*a^7*b^4 +
6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 - (a^11 - a^9*b^2 - 6*a^7*
b^4 + 14*a^5*b^6 - 11*a^3*b^8 + 3*a*b^10)*d + ((a^8*b^3 - 4*a^6*b^5 + 6*a^4
*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^2 - (3*a^10*b - 11*a^8*b^3 + 14*a^6
*b^5 - 6*a^4*b^7 - a^2*b^9 + b^11)*d)*sin(d*x + c))]
```

giac [B] time = 0.36, size = 510, normalized size = 2.80

$$\frac{\frac{3\left(2\,a^3+3\,ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\sqrt{a^2-b^2}}+\frac{27\,a^6b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-18\,a^4b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,a^2b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+18\,a^7b\tan\ldots}{\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(2*a^3 + 3*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan(
(a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6)*sqrt(a^2 - b^2)) + (27*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 - 18*a^4*b^4
*tan(1/2*d*x + 1/2*c)^5 + 6*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 + 18*a^7*b*tan(1
/2*d*x + 1/2*c)^4 + 81*a^5*b^3*tan(1/2*d*x + 1/2*c)^4 - 36*a^3*b^5*tan(1/2*
d*x + 1/2*c)^4 + 12*a*b^7*tan(1/2*d*x + 1/2*c)^4 + 108*a^6*b^2*tan(1/2*d*x
+ 1/2*c)^3 + 42*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 - 8*a^2*b^6*tan(1/2*d*x + 1/
2*c)^3 + 8*b^8*tan(1/2*d*x + 1/2*c)^3 + 36*a^7*b*tan(1/2*d*x + 1/2*c)^2 + 1
20*a^5*b^3*tan(1/2*d*x + 1/2*c)^2 - 18*a^3*b^5*tan(1/2*d*x + 1/2*c)^2 + 12*
a*b^7*tan(1/2*d*x + 1/2*c)^2 + 81*a^6*b^2*tan(1/2*d*x + 1/2*c) - 12*a^4*b^4
*tan(1/2*d*x + 1/2*c) + 6*a^2*b^6*tan(1/2*d*x + 1/2*c) + 18*a^7*b - 5*a^5*b
^3 + 2*a^3*b^5)/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*(a*tan(1/2*d*x + 1
/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^3))/d
```

maple [B] time = 0.21, size = 1733, normalized size = 9.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c))^4,x)
```

```
[Out] 9/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^3*b^2*a^3/(a^6-3*a^4*
b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)^5-6/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1
/2*d*x+1/2*c)*b+a)^3*b^4*a/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)
^5+2/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^3*b^6/a/(a^6-3*a^4
*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)^5+6/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(
1/2*d*x+1/2*c)*b+a)^3*b*a^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c
)^4+27/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^3*b^3*a^2/(a^6-3
*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)^4-12/d/(tan(1/2*d*x+1/2*c)^2*a+2
*tan(1/2*d*x+1/2*c)*b+a)^3*b^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/
2*c)^4+4/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^3*b^7/a^2/(a^6
-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)^4+36/d/(tan(1/2*d*x+1/2*c)^2*a
+2*tan(1/2*d*x+1/2*c)*b+a)^3*a^3*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*
d*x+1/2*c)^3+14/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^3*a*b^4
/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)^3-8/3/d/(tan(1/2*d*x+1/2*
c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^3/a*b^6/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(
1/2*d*x+1/2*c)^3+8/3/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^3/
a^3*b^8/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)^3+12/d/(tan(1/2*d*
x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^3*a^4*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6
)*tan(1/2*d*x+1/2*c)^2+40/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+
a)^3*a^2*b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)^2-6/d/(tan(1/
2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^3*b^5/(a^6-3*a^4*b^2+3*a^2*b^4-b
^6)*tan(1/2*d*x+1/2*c)^2+4/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b
+a)^3/a^2*b^7/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)^2+27/d/(tan(
1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^3*b^2*a^3/(a^6-3*a^4*b^2+3*a^2
```

```
*b^4-b^6)*tan(1/2*d*x+1/2*c)-4/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*
c)*b+a)^3*b^4*a/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)+2/d/(tan(1
/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^3*b^6/a/(a^6-3*a^4*b^2+3*a^2*b^
4-b^6)*tan(1/2*d*x+1/2*c)+6/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*
b+a)^3*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*a^4-5/3/d/(tan(1/2*d*x+1/2*c)^2*a+2*
tan(1/2*d*x+1/2*c)*b+a)^3*b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*a^2+2/3/d/(tan(
1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^3*b^5/(a^6-3*a^4*b^2+3*a^2*b^4
-b^6)+2/d*a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a
*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+3/d*a/(a^6-3*a^4*b^2+3*a^2*b^4-b^
6)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
*b^2
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [**B**] time = 10.38, size = 708, normalized size = 3.89

$$\frac{\frac{18\,a^4\,b-5\,a^2\,b^3+2\,b^5}{3\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(6\,a^6\,b+20\,a^4\,b^3-3\,a^2\,b^5+2\,b^7\right)}{a^2\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(6\,a^6\,b+27\,a^4\,b^3-12\,a^2\,b^5+4\,b^7\right)}{a^2\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(27\,a^4 \dots\right.}{a\,\left(a^6-3\,a^4\,b^2+\dots\right.}}{d\,\left(a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(3\,a^3+12\,a\,b^2\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(3\,a^3+12\,a\,b^2\right)+\tan\left(\frac{c}{2}+\dots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(c + d*x))^4,x)
```

```
[Out] ((18*a^4*b + 2*b^5 - 5*a^2*b^3)/(3*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (
2*tan(c/2 + (d*x)/2)^2*(6*a^6*b + 2*b^7 - 3*a^2*b^5 + 20*a^4*b^3))/(a^2*(a^
6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (tan(c/2 + (d*x)/2)^4*(6*a^6*b + 4*b^7
- 12*a^2*b^5 + 27*a^4*b^3))/(a^2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (b*
tan(c/2 + (d*x)/2)*(27*a^4*b + 2*b^5 - 4*a^2*b^3))/(a*(a^6 - b^6 + 3*a^2*b^
4 - 3*a^4*b^2)) + (b*tan(c/2 + (d*x)/2)^5*(9*a^4*b + 2*b^5 - 6*a^2*b^3))/(a
*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (2*b*tan(c/2 + (d*x)/2)^3*(3*a^2 +
2*b^2)*(18*a^4*b + 2*b^5 - 5*a^2*b^3))/(3*a^3*(a^6 - b^6 + 3*a^2*b^4 - 3*a^
```

```
4*b^2)))/(d*(a^3*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^2*(12*a*b^2 + 3*
a^3) + tan(c/2 + (d*x)/2)^4*(12*a*b^2 + 3*a^3) + tan(c/2 + (d*x)/2)^3*(12*a
^2*b + 8*b^3) + a^3 + 6*a^2*b*tan(c/2 + (d*x)/2) + 6*a^2*b*tan(c/2 + (d*x)/
2)^5)) + (a*atan((((a^2*tan(c/2 + (d*x)/2)*(2*a^2 + 3*b^2))/((a + b)^(7/2)*
(a - b)^(7/2)) + (a*(2*a^2 + 3*b^2)*(2*a^6*b - 2*b^7 + 6*a^2*b^5 - 6*a^4*b^
3))/(2*(a + b)^(7/2)*(a - b)^(7/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)))*(a
^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/(3*a*b^2 + 2*a^3))*(2*a^2 + 3*b^2))/(d*(
a + b)^(7/2)*(a - b)^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.203 $\int \sin(e+fx)\sqrt{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=172

$$-\frac{2\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3bf\sqrt{a+b\sin(e+fx)}}-\frac{2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f}+\frac{2a\sqrt{a+b\sin(e+fx)}\,E\Big(\ldots}{3bf\sqrt{\frac{a+b\sin\ldots}{a+\ldots}}}$$

```
[Out] -2/3*cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)/f-2/3*a*(sin(1/2*e+1/4*Pi+1/2*f*x)^2
)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/
2)*(b/(a+b))^(1/2))*(a+b*sin(f*x+e))^(1/2)/b/f/((a+b*sin(f*x+e))/(a+b))^(1/
2)+2/3*(a^2-b^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f
*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(
f*x+e))/(a+b))^(1/2)/b/f/(a+b*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3bf\sqrt{a+b\sin(e+fx)}}-\frac{2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f}+\frac{2a\sqrt{a+b\sin(e+fx)}\,E\Big(\ldots}{3bf\sqrt{\frac{a+b\sin\ldots}{a+\ldots}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (-2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(3*f) + (2*a*EllipticE[(e - Pi/2
 + f*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[e + f*x]])/(3*b*f*Sqrt[(a + b*Sin[
e + f*x])/(a + b)]) - (2*(a^2 - b^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*b)/(a
 + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/(3*b*f*Sqrt[a + b*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int \sin(e+fx)\sqrt{a+b\sin(e+fx)}\,dx = -\frac{2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f} + \frac{2}{3}\int \frac{\frac{b}{2}+\frac{1}{2}a\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,dx$$

$$= -\frac{2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f} + \frac{a\int\sqrt{a+b\sin(e+fx)}\,dx}{3b} - \frac{(a^2 -}{}$$

$$= -\frac{2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f} + \frac{\left(a\sqrt{a+b\sin(e+fx)}\right)\int\sqrt{\frac{a}{a+b}+\frac{b}{}}}{3b\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$

$$= -\frac{2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f} + \frac{2aE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\,\mathrm{s}}}{3bf\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$

Mathematica [A] time = 3.14, size = 143, normalized size = 0.83

$$-\frac{2\left(-\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2b}{a+b}\right)+b\cos(e+fx)(a+b\sin(e+fx))+a(a+b)\sqrt{\frac{a+b\sin(e+}{a+b}}\right.}{3bf\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (-2*(b*Cos[e + f*x]*(a + b*Sin[e + f*x]) + a*(a + b)*EllipticE[(-2*e + Pi -
 2*f*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)] - (a^2 - b^2)*
EllipticF[(-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(
a + b)]))/(3*b*f*Sqrt[a + b*Sin[e + f*x]])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sin\left(fx+e\right)+a}\,\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sin(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sin\left(fx+e\right)+a}\,\sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*sin(f*x + e), x)
```

maple [B] time = 1.17, size = 460, normalized size = 2.67

$$\frac{\frac{2\sqrt{\frac{a+b\sin(fx+e)}{a-b}}\sqrt{-\frac{(\sin(fx+e)-1)b}{a+b}}\sqrt{-\frac{(1+\sin(fx+e))b}{a-b}}\,\mathrm{EllipticF}\left(\sqrt{\frac{a+b\sin(fx+e)}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)a^{2}b}{3}-\frac{2\sqrt{\frac{a+b\sin(fx+e)}{a-b}}\sqrt{-\frac{(\sin(fx+e)-1)b}{a+b}}\sqrt{-\frac{(1+\sin(fx+e))b}{a-b}}}{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/3*(((a+b*sin(f*x+e))/(a-b))^(1/2)*(-(sin(f*x+e)-1)*b/(a+b))^(1/2)*(-(1+si
n(f*x+e))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a
+b))^(1/2))*a^2*b-((a+b*sin(f*x+e))/(a-b))^(1/2)*(-(sin(f*x+e)-1)*b/(a+b))^
(1/2)*(-(1+sin(f*x+e))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(f*x+e))/(a-b))^(1
/2),((a-b)/(a+b))^(1/2))*b^3-((a+b*sin(f*x+e))/(a-b))^(1/2)*(-(sin(f*x+e)-1
)*b/(a+b))^(1/2)*(-(1+sin(f*x+e))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(f*x+e)
)/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3+((a+b*sin(f*x+e))/(a-b))^(1/2)*(-(s
in(f*x+e)-1)*b/(a+b))^(1/2)*(-(1+sin(f*x+e))*b/(a-b))^(1/2)*EllipticE(((a+b
*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^2+sin(f*x+e)^3*b^3+sin(f
*x+e)^2*a*b^2-sin(f*x+e)*b^3-a*b^2)/b^2/cos(f*x+e)/(a+b*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(fx+e\right)+a}\,\sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*sin(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)\,\sqrt{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b*sin(e + f*x))^(1/2),x)
```

```
[Out] int(sin(e + f*x)*(a + b*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sin\left(e + fx\right)} \sin\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e))**(1/2),x)
[Out] Integral(sqrt(a + b*sin(e + f*x))*sin(e + f*x), x)
```

3.204 $\int \sqrt{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=62

$$\frac{2\sqrt{a+b\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$

```
[Out] -2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(
cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(f*x+e))^(1/2)/f
/((a+b*sin(f*x+e))/(a+b))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2655, 2653}

$$\frac{2\sqrt{a+b\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (2*EllipticE[(e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[e + f*x]])/(
f*Sqrt[(a + b*Sin[e + f*x])/(a + b)])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] && !GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{a+b\sin(e+fx)}\,dx = \frac{\sqrt{a+b\sin(e+fx)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin(e+fx)}{a+b}}\,dx}{\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$
$$= \frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(e+fx)}}{f\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$

Mathematica [A] time = 0.07, size = 61, normalized size = 0.98

$$-\frac{2\sqrt{a+b\sin(e+fx)}\,E\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2b}{a+b}\right)}{f\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]],x]
[Out] (-2*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)]*Sqrt[a + b*Sin[e + f*x]
])/(f*Sqrt[(a + b*Sin[e + f*x])/(a + b)])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sin\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sin(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sin(f*x + e) + a), x)
```

maple [B] time = 1.42, size = 239, normalized size = 3.85

$$-\frac{2\left(a-b\right)\sqrt{\frac{a+b\sin\left(fx+e\right)}{a-b}}\,\sqrt{-\frac{\left(\sin\left(fx+e\right)-1\right)b}{a+b}}\,\sqrt{-\frac{\left(1+\sin\left(fx+e\right)\right)b}{a-b}}\,\left(\mathrm{EllipticE}\left(\sqrt{\frac{a+b\sin\left(fx+e\right)}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)a+\mathrm{EllipticE}\left(\sqrt{\right.\right.}{b\cos\left(fx+e\right)\sqrt{a+b\sin}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/b*(a-b)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(-(sin(f*x+e)-1)*b/(a+b))^(1/2)*(
-(1+sin(f*x+e))*b/(a-b))^(1/2)*(EllipticE(((a+b*sin(f*x+e))/(a-b))^(1/2),((
a-b)/(a+b))^(1/2))*a+EllipticE(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a+b))
^(1/2))*b-a*EllipticF(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))-E
llipticF(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b)/cos(f*x+e)/
(a+b*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a), x)
```

mupad [B] time = 6.87, size = 55, normalized size = 0.89

$$\frac{2\,\mathrm{E}\left(\frac{e}{2}-\frac{\pi}{4}+\frac{f\,x}{2}\middle|\frac{2\,b}{a+b}\right)\,\sqrt{a+b\,\sin\left(e+f\,x\right)}}{f\,\sqrt{\frac{a+b\,\sin\left(e+f\,x\right)}{a+b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(1/2),x)
```

```
[Out] (2*ellipticE(e/2 - pi/4 + (f*x)/2, (2*b)/(a + b))*(a + b*sin(e + f*x))^(1/2
))/(f*((a + b*sin(e + f*x))/(a + b))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)), x)
```

3.205 $\int \csc(e+fx)\sqrt{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=128

$$\frac{2b\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}+\frac{2a\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}$$

```
[Out] -2*b*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elliptic
F(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(f*x+e))/(a+b
))^(1/2)/f/(a+b*sin(f*x+e))^(1/2)-2*a*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/s
in(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2,2^(1/2)*(b/
(a+b))^(1/2))*((a+b*sin(f*x+e))/(a+b))^(1/2)/f/(a+b*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2803, 2663, 2661, 2807, 2805}

$$\frac{2b\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}+\frac{2a\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (2*b*EllipticF[(e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])
/(a + b)])/(f*Sqrt[a + b*Sin[e + f*x]]) + (2*a*EllipticPi[2, (e - Pi/2 + f*
x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/(f*Sqrt[a + b*Sin[
e + f*x]])
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rubi steps

$$\int \csc(e+fx)\sqrt{a+b\sin(e+fx)}\,dx = a\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,dx + b\int \frac{1}{\sqrt{a+b\sin(e+fx)}}\,dx$$

$$= \frac{\left(a\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\right)\int \frac{\csc(e+fx)}{\sqrt{\frac{a}{a+b}+\frac{b\sin(e+fx)}{a+b}}}\,dx}{\sqrt{a+b\sin(e+fx)}} + \frac{\left(b\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin(e+f}{a+b}}}}{\sqrt{a+b\sin(e+fx)}}$$

$$= \frac{2bF\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{f\sqrt{a+b\sin(e+fx)}} + \frac{2a\Pi\left(2;\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+f}}$$

Mathematica [A] time = 16.18, size = 89, normalized size = 0.70

$$-\frac{2\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\left(bF\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2b}{a+b}\right)+a\Pi\left(2;\frac{1}{4}(-2e-2fx+\pi)|\frac{2b}{a+b}\right)\right)}{f\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (-2*(b*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)] + a*EllipticPi[2, (-
2*e + Pi - 2*f*x)/4, (2*b)/(a + b)])*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/(f
*Sqrt[a + b*Sin[e + f*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sin \left(fx + e\right) + a} \csc \left(fx + e\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*csc(f*x + e), x)
```

maple [A] time = 1.21, size = 169, normalized size = 1.32

$$\frac{2 \left(a-b\right) \sqrt{\frac{a+b \sin \left(fx+e\right)}{a-b}}\, \sqrt{-\frac{\left(\sin \left(fx+e\right)-1\right) b}{a+b}}\, \sqrt{-\frac{\left(1+\sin \left(fx+e\right)\right) b}{a-b}}\, \left(\operatorname{EllipticF}\left(\sqrt{\frac{a+b \sin \left(fx+e\right)}{a-b}}, \sqrt{\frac{a-b}{a+b}}\right)-\operatorname{EllipticPi}\left(\sqrt{\frac{a+b}{}}\right.\right.}{\cos \left(fx+e\right) \sqrt{a+b \sin \left(fx+e\right)}\, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sin(f*x+e))^(1/2),x)
```

```
[Out] 2*(a-b)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(-(sin(f*x+e)-1)*b/(a+b))^(1/2)*(-(1
+sin(f*x+e))*b/(a-b))^(1/2)*(EllipticF(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b
)/(a+b))^(1/2))-EllipticPi(((a+b*sin(f*x+e))/(a-b))^(1/2),(a-b)/a,((a-b)/(a
+b))^(1/2)))/cos(f*x+e)/(a+b*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sin \left(fx + e\right) + a} \csc \left(fx + e\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sin(f*x + e) + a)*csc(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\,\sin\left(e+f\,x\right)}}{\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(1/2)/sin(e + f*x),x)
[Out] int((a + b*sin(e + f*x))^(1/2)/sin(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin\left(e+fx\right)}\,\csc\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e))**(1/2),x)
[Out] Integral(sqrt(a + b*sin(e + f*x))*csc(e + f*x), x)
```

3.206 $\int \csc^2(e+fx)\sqrt{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=213

$$-\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{f}+\frac{a\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}-\frac{\sqrt{a+b\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\right.}{f\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$

```
[Out] -cot(f*x+e)*(a+b*sin(f*x+e))^(1/2)/f+(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/si
n(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(b/(a+b
))^(1/2))*(a+b*sin(f*x+e))^(1/2)/f/((a+b*sin(f*x+e))/(a+b))^(1/2)-a*(sin(1/
2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+
1/4*Pi+1/2*f*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(f*x+e))/(a+b))^(1/2)/f/(
a+b*sin(f*x+e))^(1/2)-b*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*P
i+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2,2^(1/2)*(b/(a+b))^(1/2))*
((a+b*sin(f*x+e))/(a+b))^(1/2)/f/(a+b*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2796, 3060, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{f}+\frac{a\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}-\frac{\sqrt{a+b\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\right.}{f\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] -((Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/f) - (EllipticE[(e - Pi/2 + f*x)/
2, (2*b)/(a + b)]*Sqrt[a + b*Sin[e + f*x]])/(f*Sqrt[(a + b*Sin[e + f*x])/(a
 + b)]) + (a*EllipticF[(e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e
 + f*x])/(a + b)])/(f*Sqrt[a + b*Sin[e + f*x]]) + (b*EllipticPi[2, (e - Pi/
2 + f*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/(f*Sqrt[a +
b*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3060

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist
[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c
*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c
+ d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \csc^2(e+fx)\sqrt{a+b\sin(e+fx)}\,dx = -\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{f} + \int \frac{\csc(e+fx)\left(\frac{b}{2}-\frac{1}{2}b\sin^2(e+fx)\right)}{\sqrt{a+b\sin(e+fx)}}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{f} - \frac{1}{2}\int \sqrt{a+b\sin(e+fx)}\,dx - \frac{\int \frac{\csc(e+}{}}{}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{f} + \frac{1}{2}a\int \frac{1}{\sqrt{a+b\sin(e+fx)}}\,dx + \frac{1}{2}b\int$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{f} - \frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(e}}{f\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{f} - \frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(e}}{f\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$

Mathematica [C] time = 8.87, size = 312, normalized size = 1.46

$$\frac{-4\cot(e+fx)\sqrt{a+b\sin(e+fx)}-\frac{2b\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\Pi\left(2;\frac{1}{4}(-2e-2fx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(e+fx)}}+\frac{2i\sec(e+fx)\sqrt{-\frac{b(\sin(e+fx)-1)}{a+b}}\sqrt{-\frac{b(\sin(e+fx)+1)}{a-b}}\left(b\left(b\right.\right.}{}}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (((2*I)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Si
n[e + f*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^
(-1)]*Sqrt[a + b*Sin[e + f*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a,
 I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[e + f*x]]], (a + b)/(a - b)])
)*Sec[e + f*x]*Sqrt[-((b*(-1 + Sin[e + f*x]))/(a + b))]*Sqrt[-((b*(1 + Sin[
e + f*x]))/(a - b))])/(a*b*Sqrt[-(a + b)^(-1)]) - 4*Cot[e + f*x]*Sqrt[a + b
*Sin[e + f*x]] - (2*b*EllipticPi[2, (-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)]*S
qrt[(a + b*Sin[e + f*x])/(a + b)])/Sqrt[a + b*Sin[e + f*x]])/(4*f)
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sin\left(fx+e\right)+a}\,\csc\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*csc(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sin\left(fx+e\right)+a}\,\csc\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*csc(f*x + e)^2, x)
```

maple [A] time = 1.35, size = 457, normalized size = 2.15

$$-\frac{a b^{2}\sin\left(fx+e\right)\left(\cos^{2}\left(fx+e\right)\right)-\sqrt{\frac{b\sin\left(fx+e\right)}{a-b}+\frac{a}{a-b}}\sqrt{-\frac{b\sin\left(fx+e\right)}{a-b}-\frac{b}{a-b}}\sqrt{-\frac{b\sin\left(fx+e\right)}{a+b}+\frac{b}{a+b}}\left(\text{EllipticE}\left(\sqrt{\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sin(f*x+e))^(1/2),x)
```

```
[Out] -(a*b^2*sin(f*x+e)*cos(f*x+e)^2-(b/(a-b)*sin(f*x+e)+1/(a-b)*a)^(1/2)*(-b/(a
-b)*sin(f*x+e)-b/(a-b))^(1/2)*(-b/(a+b)*sin(f*x+e)+b/(a+b))^(1/2)*(Elliptic
E((b/(a-b)*sin(f*x+e)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3-EllipticE((
b/(a-b)*sin(f*x+e)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^2-EllipticF((b
/(a-b)*sin(f*x+e)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b+EllipticF((b/
(a-b)*sin(f*x+e)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^2-EllipticPi((b/
(a-b)*sin(f*x+e)+1/(a-b)*a)^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^2+Ellipt
icPi((b/(a-b)*sin(f*x+e)+1/(a-b)*a)^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^3)
*sin(f*x+e)+a^2*b*cos(f*x+e)^2)/a/b/sin(f*x+e)/cos(f*x+e)/(a+b*sin(f*x+e))^
(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(fx+e\right)+a}\,\csc\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*csc(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,\sin\left(e+f\,x\right)}}{\sin\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(1/2)/sin(e + f*x)^2,x)
```

```
[Out] int((a + b*sin(e + f*x))^(1/2)/sin(e + f*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin\left(e+fx\right)}\,\csc^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x))*csc(e + f*x)**2, x)
```

3.207 $\int \frac{\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,dx$

Optimal. Leaf size=132

$$\frac{2\sqrt{a+b\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{bf\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}-\frac{2a\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{bf\sqrt{a+b\sin(e+fx)}}$$

```
[Out] -2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(
cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(f*x+e))^(1/2)/b
/f/((a+b*sin(f*x+e))/(a+b))^(1/2)+2*a*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/s
in(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(b/(a+
b))^(1/2))*((a+b*sin(f*x+e))/(a+b))^(1/2)/b/f/(a+b*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2752, 2663, 2661, 2655, 2653}

$$\frac{2\sqrt{a+b\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{bf\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}-\frac{2a\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{bf\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (2*EllipticE[(e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[e + f*x]])/(
b*f*Sqrt[(a + b*Sin[e + f*x])/(a + b)]) - (2*a*EllipticF[(e - Pi/2 + f*x)/2
, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/(b*f*Sqrt[a + b*Sin[e
+ f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,dx = \frac{\int \sqrt{a+b\sin(e+fx)}\,dx}{b} - \frac{a\int \frac{1}{\sqrt{a+b\sin(e+fx)}}\,dx}{b}$$

$$= \frac{\sqrt{a+b\sin(e+fx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin(e+fx)}{a+b}}\,dx}{b\sqrt{\frac{a+b\sin(e+fx)}{a+b}}} - \frac{\left(a\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin(e+fx)}{a+b}}}\,dx}{b\sqrt{a+b\sin(e+fx)}}$$

$$= \frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(e+fx)}}{bf\sqrt{\frac{a+b\sin(e+fx)}{a+b}}} - \frac{2aF\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+f}{a+b}}}{bf\sqrt{a+b\sin(e+fx)}}$$

Mathematica [A] time = 2.40, size = 94, normalized size = 0.71

$$-\frac{2\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\left((a+b)E\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2b}{a+b}\right)\right)}{bf\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (-2*((a + b)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)] - a*EllipticF[
(-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)])*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/
(b*f*Sqrt[a + b*Sin[e + f*x]])
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sin\left(fx+e\right)}{\sqrt{b\sin\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sin(f*x + e)/sqrt(b*sin(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)}{\sqrt{b\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)/sqrt(b*sin(f*x + e) + a), x)
```

maple [A] time = 1.06, size = 202, normalized size = 1.53

$$-\frac{2\left(a-b\right)\sqrt{\frac{a+b\sin\left(fx+e\right)}{a-b}}\,\sqrt{-\frac{\left(\sin\left(fx+e\right)-1\right)b}{a+b}}\,\sqrt{-\frac{\left(1+\sin\left(fx+e\right)\right)b}{a-b}}\,\left(\text{EllipticE}\left(\sqrt{\frac{a+b\sin\left(fx+e\right)}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)a+\text{EllipticE}\left(\right.\right.}{b^{2}\cos\left(fx+e\right)\sqrt{a+b\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sin(f*x+e))^(1/2),x)
```

```
[Out] -2*(a-b)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(-(sin(f*x+e)-1)*b/(a+b))^(1/2)*(-(
1+sin(f*x+e))*b/(a-b))^(1/2)*(EllipticE(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-
b)/(a+b))^(1/2))*a+EllipticE(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a+b))^(
1/2))*b-EllipticF(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b)/b^
2/cos(f*x+e)/(a+b*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)}{\sqrt{b\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)/sqrt(b*sin(f*x + e) + a), x)
```

mupad [B] time = 7.21, size = 118, normalized size = 0.89

$$\frac{\left(2\,a\,\mathrm{F}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin\left(e+f\,x\right)}}{2}\right)\middle|\frac{2\,b}{a+b}\right)-2\,\left(a+b\right)\,\mathrm{E}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin\left(e+f\,x\right)}}{2}\right)\middle|\frac{2\,b}{a+b}\right)\right)\,\sqrt{{\cos\left(e+f\,x\right)}^2}\,\sqrt{\frac{a+b\,\sin\left(e+f\,x\right)}{a+b}}}{b\,f\,\cos\left(e+f\,x\right)\,\sqrt{a+b\,\sin\left(e+f\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b*sin(e + f*x))^(1/2),x)
```

```
[Out] ((2*a*ellipticF(asin((2^(1/2)*(1 - sin(e + f*x))^(1/2))/2), (2*b)/(a + b))
- 2*(a + b)*ellipticE(asin((2^(1/2)*(1 - sin(e + f*x))^(1/2))/2), (2*b)/(a
+ b)))*(cos(e + f*x)^2)^(1/2)*((a + b*sin(e + f*x))/(a + b))^(1/2))/(b*f*co
s(e + f*x)*(a + b*sin(e + f*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(e+fx\right)}{\sqrt{a+b\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)/sqrt(a + b*sin(e + f*x)), x)
```

3.208 $\int \frac{1}{\sqrt{a+b\sin(e+fx)}}\,dx$

Optimal. Leaf size=62

$$\frac{2\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}$$

```
[Out] -2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(
cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(f*x+e))/(a+b))
^(1/2)/f/(a+b*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2663, 2661}

$$\frac{2\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (2*EllipticF[(e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(
a + b)])/(f*Sqrt[a + b*Sin[e + f*x]])
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sin(e+fx)}}\,dx = \frac{\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin(e+fx)}{a+b}}}\,dx}{\sqrt{a+b\sin(e+fx)}}$$
$$= \frac{2F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{f\sqrt{a+b\sin(e+fx)}}$$

Mathematica [A] time = 0.05, size = 61, normalized size = 0.98

$$-\frac{2\sqrt{\frac{a+b\sin(e+fx)}{a+b}}F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (-2*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x
])/(a + b)])/(f*Sqrt[a + b*Sin[e + f*x]])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{b\sin\left(fx+e\right)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*sin(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sin(f*x + e) + a), x)
```

maple [A] time = 0.71, size = 126, normalized size = 2.03

$$\frac{2\left(a-b\right)\sqrt{\frac{a+b\sin\left(fx+e\right)}{a-b}}\,\sqrt{-\frac{\left(\sin\left(fx+e\right)-1\right)b}{a+b}}\,\sqrt{-\frac{\left(1+\sin\left(fx+e\right)\right)b}{a-b}}\,\operatorname{EllipticF}\left(\sqrt{\frac{a+b\sin\left(fx+e\right)}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)}{b\cos\left(fx+e\right)\sqrt{a+b\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^(1/2),x)
```

```
[Out] 2*(a-b)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(-(sin(f*x+e)-1)*b/(a+b))^(1/2)*(-(1
+sin(f*x+e))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))/b/cos(f*x+e)/(a+b*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sin(f*x + e) + a), x)
```

mupad [B] time = 6.91, size = 55, normalized size = 0.89

$$-\frac{2\,\mathrm{F}\left(\frac{\pi}{4}-\frac{e}{2}-\frac{f\,x}{2}\middle|\frac{2\,b}{a+b}\right)\,\sqrt{\frac{a+b\,\sin\left(e+f\,x\right)}{a+b}}}{f\,\sqrt{a+b\,\sin\left(e+f\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(e + f*x))^(1/2),x)
```

```
[Out] -(2*ellipticF(pi/4 - e/2 - (f*x)/2, (2*b)/(a + b))*((a + b*sin(e + f*x))/(a
 + b))^(1/2))/(f*(a + b*sin(e + f*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**(1/2),x)
[Out] Integral(1/sqrt(a + b*sin(e + f*x)), x)
```

3.209 $\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,dx$

Optimal. Leaf size=63

$$\frac{2\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}$$

```
[Out] -2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi
(cos(1/2*e+1/4*Pi+1/2*f*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(f*x+e))/(a+
b))^(1/2)/f/(a+b*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2807, 2805}

$$\frac{2\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (2*EllipticPi[2, (e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x
])/(a + b)])/(f*Sqrt[a + b*Sin[e + f*x]])
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,dx = \frac{\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\int \frac{\csc(e+fx)}{\sqrt{\frac{a}{a+b}+\frac{b\sin(e+fx)}{a+b}}}\,dx}{\sqrt{a+b\sin(e+fx)}}$$
$$= \frac{2\Pi\left(2;\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}{f\sqrt{a+b\sin(e+fx)}}$$

Mathematica [**A**] time = 0.08, size = 62, normalized size = 0.98

$$-\frac{2\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,\Pi\left(2;\frac{1}{4}(-2e-2fx+\pi)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (-2*EllipticPi[2, (-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e +
 f*x])/(a + b)])/(f*Sqrt[a + b*Sin[e + f*x]])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   faile
d of mode Union(SparseUnivariatePolynomial(SimpleAlgebraicExtension(InnerPr
imeField(7),SparseUnivariatePolynomial(InnerPrimeField(7)),?^2+2)),failed)
cannot be coerced to mode SparseUnivariatePolynomial(SimpleAlgebraicExtensi
on(InnerPrimeField(7),SparseUnivariatePolynomial(InnerPrimeField(7)),?^2+2)
)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)}{\sqrt{b\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)/sqrt(b*sin(f*x + e) + a), x)
```

maple [A] time = 0.99, size = 135, normalized size = 2.14

$$-\frac{2 (a-b) \sqrt{\frac{a+b \sin\left(f x+e\right)}{a-b}}\, \sqrt{-\frac{\left(\sin\left(f x+e\right)-1\right) b}{a+b}}\, \sqrt{-\frac{\left(1+\sin\left(f x+e\right)\right) b}{a-b}}\, \operatorname{EllipticPi}\left(\sqrt{\frac{a+b \sin\left(f x+e\right)}{a-b}}, \frac{a-b}{a}, \sqrt{\frac{a-b}{a+b}}\right)}{a \cos\left(f x+e\right) \sqrt{a+b \sin\left(f x+e\right)}\, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sin(f*x+e))^(1/2),x)
```

```
[Out] -2*(a-b)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(-(sin(f*x+e)-1)*b/(a+b))^(1/2)*(-(
1+sin(f*x+e))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(f*x+e))/(a-b))^(1/2),(a-b
)/a,((a-b)/(a+b))^(1/2))/a/cos(f*x+e)/(a+b*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(f x+e\right)}{\sqrt{b \sin\left(f x+e\right)+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)/sqrt(b*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sin\left(e+f\,x\right)\,\sqrt{a+b\,\sin\left(e+f\,x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/(sin(e + f*x)*(a + b*sin(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(e+f x\right)}{\sqrt{a+b \sin\left(e+f x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)/sqrt(a + b*sin(e + f*x)), x)
```

3.210 $\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,dx$

Optimal. Leaf size=222

$$-\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{af}+\frac{\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}-\frac{\sqrt{a+b\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\right.}{af\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$

```
[Out] -cot(f*x+e)*(a+b*sin(f*x+e))^(1/2)/a/f+(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/
sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(b/(a
+b))^(1/2))*(a+b*sin(f*x+e))^(1/2)/a/f/((a+b*sin(f*x+e))/(a+b))^(1/2)-(sin(
1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*
e+1/4*Pi+1/2*f*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(f*x+e))/(a+b))^(1/2)/f
/(a+b*sin(f*x+e))^(1/2)+b*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4
*Pi+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2,2^(1/2)*(b/(a+b))^(1/2)
)*((a+b*sin(f*x+e))/(a+b))^(1/2)/a/f/(a+b*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2802, 3060, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{af}+\frac{\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{f\sqrt{a+b\sin(e+fx)}}-\frac{\sqrt{a+b\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\right.}{af\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] -((Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(a*f)) - (EllipticE[(e - Pi/2 + f
*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[e + f*x]])/(a*f*Sqrt[(a + b*Sin[e + f*
x])/(a + b)]) + (EllipticF[(e - Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*S
in[e + f*x])/(a + b)])/(f*Sqrt[a + b*Sin[e + f*x]]) - (b*EllipticPi[2, (e -
 Pi/2 + f*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[e + f*x])/(a + b)])/(a*f*Sqr
t[a + b*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3060

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist
[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c
*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c
+ d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sin(e+fx)}}\,dx = -\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{af} + \frac{\int \frac{\csc(e+fx)\left(-\frac{b}{2}-\frac{1}{2}b\sin^2(e+fx)\right)}{\sqrt{a+b\sin(e+fx)}}\,dx}{a}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{af} - \frac{\int \sqrt{a+b\sin(e+fx)}\,dx}{2a} - \frac{\int \frac{\csc(e+fx)\left(\frac{b^2}{2}-\frac{1}{2}ab\sin(\right.}{\sqrt{a+b\sin(e+fx)}}}{ab}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{af} + \frac{1}{2}\int \frac{1}{\sqrt{a+b\sin(e+fx)}}\,dx - \frac{b\int \frac{\csc(e+fx)}{\sqrt{a+b\sin(e+fx)}}}{2a}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{af} - \frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(e+fx)}}{af\sqrt{\frac{a+b\sin(e+fx)}{a+b}}} + -$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin(e+fx)}}{af} - \frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(e+fx)}}{af\sqrt{\frac{a+b\sin(e+fx)}{a+b}}} + \frac{F}{}$$

Mathematica [C] time = 10.05, size = 315, normalized size = 1.42

$$\frac{-4\cot(e+fx)\sqrt{a+b\sin(e+fx)}+\frac{6b\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\,\Pi\left(2;\frac{1}{4}(-2e-2fx+\pi)|\frac{2b}{a+b}\right)}{\sqrt{a+b\sin(e+fx)}}+\frac{2i\sec(e+fx)\sqrt{-\frac{b(\sin(e+fx)-1)}{a+b}}\sqrt{-\frac{b(\sin(e+fx)+1)}{a-b}}\left(b\left(b\Pi\right.\right.}{}}{4af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (((2*I)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Si
n[e + f*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^
(-1)]*Sqrt[a + b*Sin[e + f*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a,
 I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[e + f*x]]], (a + b)/(a - b)])
)*Sec[e + f*x]*Sqrt[-((b*(-1 + Sin[e + f*x]))/(a + b))]*Sqrt[-((b*(1 + Sin[
e + f*x]))/(a - b))])/(a*b*Sqrt[-(a + b)^(-1)]) - 4*Cot[e + f*x]*Sqrt[a + b
*Sin[e + f*x]] + (6*b*EllipticPi[2, (-2*e + Pi - 2*f*x)/4, (2*b)/(a + b)]*S
qrt[(a + b*Sin[e + f*x])/(a + b)])/Sqrt[a + b*Sin[e + f*x]])/(4*a*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^2}{\sqrt{b\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/sqrt(b*sin(f*x + e) + a), x)
```

maple [A] time = 2.92, size = 412, normalized size = 1.86

$$\frac{\sqrt{-\left(-b\sin\left(fx+e\right)-a\right)\left(\cos^2\left(fx+e\right)\right)}\left(-\frac{\sqrt{-\left(-b\sin\left(fx+e\right)-a\right)\left(\cos^2\left(fx+e\right)\right)}}{a\sin\left(fx+e\right)}-\frac{b\left(\frac{a}{b}-1\right)\sqrt{\frac{a+b\sin\left(fx+e\right)}{a-b}}\sqrt{\frac{b\left(1-\sin\left(fx+e\right)\right)}{a+b}}\sqrt{\frac{\left(-\sin\left(f\right.\right.}{a}}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sin(f*x+e))^(1/2),x)
```

```
[Out] (-(-b*sin(f*x+e)-a)*cos(f*x+e)^2)^(1/2)*(-1/a*(-(-b*sin(f*x+e)-a)*cos(f*x+e
)^2)^(1/2)/sin(f*x+e)-1/a*b*(a/b-1)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(b*(1-si
n(f*x+e))/(a+b))^(1/2)*((-sin(f*x+e)-1)*b/(a-b))^(1/2)/(-(-b*sin(f*x+e)-a)*
cos(f*x+e)^2)^(1/2)*((-a/b-1)*EllipticE(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-
b)/(a+b))^(1/2))+EllipticF(((a+b*sin(f*x+e))/(a-b))^(1/2),((a-b)/(a+b))^(1/
2)))+1/a^2*b^2*(a/b-1)*((a+b*sin(f*x+e))/(a-b))^(1/2)*(b*(1-sin(f*x+e))/(a+
b))^(1/2)*((-sin(f*x+e)-1)*b/(a-b))^(1/2)/(-(-b*sin(f*x+e)-a)*cos(f*x+e)^2)
^(1/2)*EllipticPi(((a+b*sin(f*x+e))/(a-b))^(1/2),-(-a/b+1)/a*b,((a-b)/(a+b)
)^(1/2)))/cos(f*x+e)/(a+b*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^2}{\sqrt{b\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^2/sqrt(b*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\sin\left(e+f\,x\right)}^2\,\sqrt{a+b\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/(sin(e + f*x)^2*(a + b*sin(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{2}\left(e+fx\right)}{\sqrt{a+b\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/sqrt(a + b*sin(e + f*x)), x)
```

3.211 $\int \sqrt{\sin(c+dx)}\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=371

$$-\frac{\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-\frac{\sqrt{a+b}\,\tan(c+dx)\sqrt{\frac{a(1-\csc(c+dx))}{a+b}}\sqrt{\frac{a(\csc(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}\sqrt{\sin(c+dx)}}\right)|-\frac{a}{a}\right.}{d}$$

```
[Out] -cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/d/sin(d*x+c)^(1/2)+(a-b)*EllipticE((a+b*
sin(d*x+c))^(1/2)/(a+b)^(1/2)/sin(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^
(1/2)*(a*(1-csc(d*x+c))/(a+b))^(1/2)*(a*(1+csc(d*x+c))/(a-b))^(1/2)*tan(d*x
+c)/a/d-EllipticF((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2)/sin(d*x+c)^(1/2),((-a-
b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-csc(d*x+c))/(a+b))^(1/2)*(a*(1+csc(d*x+c
))/(a-b))^(1/2)*tan(d*x+c)/d+a*EllipticPi((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2
)/sin(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-csc(d*x+
c))/(a+b))^(1/2)*(a*(1+csc(d*x+c))/(a-b))^(1/2)*tan(d*x+c)/b/d
```

Rubi [A] time = 0.56, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2821, 3054, 2809, 12, 2801, 2816, 2994}

$$-\frac{\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-\frac{\sqrt{a+b}\,\tan(c+dx)\sqrt{\frac{a(1-\csc(c+dx))}{a+b}}\sqrt{\frac{a(\csc(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}\sqrt{\sin(c+dx)}}\right)|-\frac{a}{a}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sin[c + d*x]]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -((Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x]])) + ((a - b
)*Sqrt[a + b]*Sqrt[(a*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Csc[c + d*x
]))/(a - b)]*EllipticE[ArcSin[Sqrt[a + b*Sin[c + d*x]]/(Sqrt[a + b]*Sqrt[Si
n[c + d*x]])], -((a + b)/(a - b))]*Tan[c + d*x])/(a*d) - (Sqrt[a + b]*Sqrt[
(a*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Csc[c + d*x]))/(a - b)]*Ellipt
icF[ArcSin[Sqrt[a + b*Sin[c + d*x]]/(Sqrt[a + b]*Sqrt[Sin[c + d*x]])], -((a
 + b)/(a - b))]*Tan[c + d*x])/d + (a*Sqrt[a + b]*Sqrt[(a*(1 - Csc[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Csc[c + d*x]))/(a - b)]*EllipticPi[(a + b)/b, ArcSi
n[Sqrt[a + b*Sin[c + d*x]]/(Sqrt[a + b]*Sqrt[Sin[c + d*x]])], -((a + b)/(a
- b))]*Tan[c + d*x])/(b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2801

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
```

```
 && PosQ[(c + d)/b]
```

Rule 3054

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :
> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[1/b^2, Int[(A*b^2 - a^2*C - 2*a*b*C*Sin[e + f*x])/((a + b*Sin[e + f
*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{\sin(c+dx)}\sqrt{a+b\sin(c+dx)}\,dx = -\frac{\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{\int \frac{-\frac{ab}{2}+\frac{1}{2}ab\sin^2(c+dx)}{\sin^{\frac{3}{2}}(c+dx)\sqrt{a+b\sin(c+dx)}}\,dx}{b}$$

$$= -\frac{\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{1}{2}a\int \frac{\sqrt{\sin(c+dx)}}{\sqrt{a+b\sin(c+dx)}}\,dx + \frac{\int -\cdots}{2\cdots}$$

$$= -\frac{\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{a\sqrt{a+b}\sqrt{\frac{a(1-\csc(c+dx))}{a+b}}\sqrt{\frac{a(1+\csc(c+dx\cdots}{a-b}}\cdots}{\cdots}$$

$$= -\frac{\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{a\sqrt{a+b}\sqrt{\frac{a(1-\csc(c+dx))}{a+b}}\sqrt{\frac{a(1+\csc(c+dx\cdots}{a-b}}\cdots}{\cdots}$$

$$= -\frac{\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{(a-b)\sqrt{a+b}\sqrt{\frac{a(1-\csc(c+dx))}{a+b}}\sqrt{\frac{a(1+\text{cs}\cdots}{\cdots}}\cdots}{\cdots}$$

Mathematica [C] time = 26.83, size = 10847, normalized size = 29.24

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sin[c + d*x]]*Sqrt[a + b*Sin[c + d*x]],x]
[Out] Result too large to show
```

fricas [F] time = 2.70, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{b\sin\left(dx+c\right)+a}\,\sqrt{\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(1/2)*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(d*x + c) + a)*sqrt(sin(d*x + c)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(1/2)*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming c near 0Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Simplification assuming c near 0Unable to check sign: (2*pi/x
/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)ext_reduce Error: Bad Argument TypeSimplificati
on assuming c near 0Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Simplification assuming c nea
r 0Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)ext_reduce Erro
r: Bad Argument TypeDone
```

maple [C] time = 0.76, size = 9823, normalized size = 26.48

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^(1/2)*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sin\left(dx+c\right)+a}\,\sqrt{\sin\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(1/2)*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sin(d*x + c) + a)*sqrt(sin(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\sin\left(c+d\,x\right)}\,\sqrt{a+b\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^(1/2)*(a + b*sin(c + d*x))^(1/2),x)
[Out] int(sin(c + d*x)^(1/2)*(a + b*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin\left(c+dx\right)}\sqrt{\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**(1/2)*(a+b*sin(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sin(c + d*x))*sqrt(sin(c + d*x)), x)
```

3.212 $\int \frac{1}{\sqrt{\sin(c+dx)}\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=109

$$-\frac{2\sqrt{a+b}\tan(c+dx)\sqrt{\frac{a(1-\csc(c+dx))}{a+b}}\sqrt{\frac{a(\csc(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}\sqrt{\sin(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}$$

```
[Out] -2*EllipticF((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2)/sin(d*x+c)^(1/2),((-a-b)/(a
-b))^(1/2))*(a+b)^(1/2)*(a*(1-csc(d*x+c))/(a+b))^(1/2)*(a*(1+csc(d*x+c))/(a
-b))^(1/2)*tan(d*x+c)/a/d
```

Rubi [A] time = 0.07, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2816}

$$-\frac{2\sqrt{a+b}\tan(c+dx)\sqrt{\frac{a(1-\csc(c+dx))}{a+b}}\sqrt{\frac{a(\csc(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}\sqrt{\sin(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sin[c + d*x]]*Sqrt[a + b*Sin[c + d*x]]),x]
```

```
[Out] (-2*Sqrt[a + b]*Sqrt[(a*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Csc[c + d
*x]))/(a - b)]*EllipticF[ArcSin[Sqrt[a + b*Sin[c + d*x]]/(Sqrt[a + b]*Sqrt[
Sin[c + d*x]])], -((a + b)/(a - b))]*Tan[c + d*x])/(a*d)
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sin(c+dx)}\sqrt{a+b\sin(c+dx)}}\,dx = -\frac{2\sqrt{a+b}\sqrt{\frac{a(1-\csc(c+dx))}{a+b}}\sqrt{\frac{a(1+\csc(c+dx))}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}\sqrt{\sin(c+dx)}}\right)|-\frac{a}{a}\right.}{ad}$$

Mathematica [A] time = 3.34, size = 172, normalized size = 1.58

$$\frac{8a\sin^4\left(\frac{1}{4}(2c+2dx-\pi)\right)\sec(c+dx)\sqrt{-\frac{(a+b)\sin(c+dx)(a+b\sin(c+dx))}{a^2(\sin(c+dx)-1)^2}}\sqrt{-\frac{(a+b)\cot^2\left(\frac{1}{4}(2c+2dx-\pi)\right)}{a-b}}F\left(\sin^{-1}\left(\sqrt{-\frac{a+b\sin(c+}{a(\sin(c+dx}}\right.\right.}{d(a+b)\sqrt{\sin(c+dx)}\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sin[c + d*x]]*Sqrt[a + b*Sin[c + d*x]]),x]
```

```
[Out] (8*a*Sqrt[-(((a + b)*Cot[(2*c - Pi + 2*d*x)/4]^2)/(a - b))]*EllipticF[ArcSi
n[Sqrt[-((a + b*Sin[c + d*x])/(a*(-1 + Sin[c + d*x])))]], (2*a)/(a - b)]*Se
c[c + d*x]*Sqrt[-(((a + b)*Sin[c + d*x]*(a + b*Sin[c + d*x]))/(a^2*(-1 + Si
n[c + d*x])^2))]*Sin[(2*c - Pi + 2*d*x)/4]^4)/((a + b)*d*Sqrt[Sin[c + d*x]]
*Sqrt[a + b*Sin[c + d*x]])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{b\sin(dx+c)+a}\sqrt{\sin(dx+c)}}{b\cos(dx+c)^2-a\sin(dx+c)-b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(d*x+c)^(1/2)/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*sqrt(sin(d*x + c))/(b*cos(d*x + c)^2 - a
*sin(d*x + c) - b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{b\sin(dx+c)+a}\sqrt{\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(d*x+c)^(1/2)/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sin(d*x + c) + a)*sqrt(sin(d*x + c))), x)
```

maple [B] time = 0.30, size = 318, normalized size = 2.92

$$-\frac{\sqrt{-\frac{-\sqrt{-a^2+b^2}\,\sin(dx+c)-b\sin(dx+c)+a\cos(dx+c)-a}{\left(b+\sqrt{-a^2+b^2}\right)\sin(dx+c)}}\sqrt{\frac{\sqrt{-a^2+b^2}\,\sin(dx+c)-b\sin(dx+c)+a\cos(dx+c)-a}{\sqrt{-a^2+b^2}\,\sin(dx+c)}}\sqrt{\frac{a(\cos(dx+c)-1)}{\left(b+\sqrt{-a^2+b^2}\right)\sin(dx+c)}}\,\text{Elli}}{d\sqrt{a+b\sin(dx+c)}\,(\text{co}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sin(d*x+c)^(1/2)/(a+b*sin(d*x+c))^(1/2),x)

[Out] -1/d/(a+b*sin(d*x+c))^(1/2)*(-(-(-a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)+a*cos(d*x+c)-a)/(b+(-a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*(((-a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)+a*cos(d*x+c)-a)/(-a^2+b^2)^(1/2)/sin(d*x+c))^(1/2)*(a*(cos(d*x+c)-1)/(b+(-a^2+b^2)^(1/2))/sin(d*x+c))^(1/2)*EllipticF((-(-(-a^2+b^2)^(1/2)*sin(d*x+c)-b*sin(d*x+c)+a*cos(d*x+c)-a)/(b+(-a^2+b^2)^(1/2))/sin(d*x+c))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*sin(d*x+c)^(3/2)*2^(1/2)/(cos(d*x+c)-1)*(b+(-a^2+b^2)^(1/2))/a

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sin\left(dx+c\right)+a}\,\sqrt{\sin\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(d*x+c)^(1/2)/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*sin(d*x + c) + a)*sqrt(sin(d*x + c))), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\sin\left(c+d\,x\right)}\,\sqrt{a+b\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(c + d*x)^(1/2)*(a + b*sin(c + d*x))^(1/2)),x)

[Out] int(1/(sin(c + d*x)^(1/2)*(a + b*sin(c + d*x))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sin\left(c+dx\right)}\,\sqrt{\sin\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(d*x+c)**(1/2)/(a+b*sin(d*x+c))**(1/2),x)

[Out] Integral(1/(sqrt(a + b*sin(c + d*x))*sqrt(sin(c + d*x))), x)

3.213 $\int (d \sin(e + fx))^m (a + b \sin(e + fx))^3 \, dx$

Optimal. Leaf size=270

$$\frac{b\left(3a^2(m+3)+b^2(m+2)\right)\cos(e+fx)(d\sin(e+fx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\sin^2(e+fx)\right)}{d^2f(m+2)(m+3)\sqrt{\cos^2(e+fx)}}+\frac{a\left(a^2(m+2)+3b^2(m\right.}{}$$

```
[Out] -a*b^2*(7+2*m)*cos(f*x+e)*(d*sin(f*x+e))^(1+m)/d/f/(2+m)/(3+m)-b^2*cos(f*x+
e)*(d*sin(f*x+e))^(1+m)*(a+b*sin(f*x+e))/d/f/(3+m)+a*(3*b^2*(1+m)+a^2*(2+m)
)*cos(f*x+e)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(f*x+e)^2)*(d*sin(f*
x+e))^(1+m)/d/f/(1+m)/(2+m)/(cos(f*x+e)^2)^(1/2)+b*(b^2*(2+m)+3*a^2*(3+m))*
cos(f*x+e)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],sin(f*x+e)^2)*(d*sin(f*x+e))^
(2+m)/d^2/f/(2+m)/(3+m)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2793, 3023, 2748, 2643}

$$\frac{b\left(3a^2(m+3)+b^2(m+2)\right)\cos(e+fx)(d\sin(e+fx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\sin^2(e+fx)\right)}{d^2f(m+2)(m+3)\sqrt{\cos^2(e+fx)}}+\frac{a\left(a^2(m+2)+3b^2(m\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^m*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] -((a*b^2*(7 + 2*m)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + m))/(d*f*(2 + m)*(3 +
 m))) + (a*(3*b^2*(1 + m) + a^2*(2 + m))*Cos[e + f*x]*Hypergeometric2F1[1/2
, (1 + m)/2, (3 + m)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + m))/(d*f*(1 +
 m)*(2 + m)*Sqrt[Cos[e + f*x]^2]) + (b*(b^2*(2 + m) + 3*a^2*(3 + m))*Cos[e
+ f*x]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Sin[e + f*x]^2]*(d*Sin[
e + f*x])^(2 + m))/(d^2*f*(2 + m)*(3 + m)*Sqrt[Cos[e + f*x]^2]) - (b^2*Cos[
e + f*x]*(d*Sin[e + f*x])^(1 + m)*(a + b*Sin[e + f*x]))/(d*f*(3 + m))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int (d\sin(e+fx))^m(a+b\sin(e+fx))^3\,dx = -\frac{b^2\cos(e+fx)(d\sin(e+fx))^{1+m}(a+b\sin(e+fx))}{df(3+m)} + \frac{\int (d\sin(e+ \cdots}{\cdots}$$

$$= -\frac{ab^2(7+2m)\cos(e+fx)(d\sin(e+fx))^{1+m}}{df(2+m)(3+m)} - \frac{b^2\cos(e+fx)(d\sin \cdots}{\cdots}$$

$$= -\frac{ab^2(7+2m)\cos(e+fx)(d\sin(e+fx))^{1+m}}{df(2+m)(3+m)} - \frac{b^2\cos(e+fx)(d\sin \cdots}{\cdots}$$

$$= -\frac{ab^2(7+2m)\cos(e+fx)(d\sin(e+fx))^{1+m}}{df(2+m)(3+m)} + \frac{a\left(a^2+\frac{3b^2(1+m)}{2+m}\right)\cos \cdots}{\cdots}$$

Mathematica [A] time = 0.80, size = 199, normalized size = 0.74

$$\frac{\sin(e+fx)\cos(e+fx)(d\sin(e+fx))^m\left(\frac{b\left(3a^2(m+3)+b^2(m+2)\right)\sin(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\sin^2(e+fx)\right)}{(m+2)\sqrt{\cos^2(e+fx)}}+\frac{a(m+3)\left(a^2(m+2)+3b^2(m+1\right.}{(m+1)(m+2}\right.}{f(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sin[e + f*x])^m*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^m*(-((a*b^2*(7 + 2*m))/(2 + m))
 + (a*(3 + m)*(3*b^2*(1 + m) + a^2*(2 + m))*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, Sin[e + f*x]^2])/((1 + m)*(2 + m)*Sqrt[Cos[e + f*x]^2]) + (b*
(b^2*(2 + m) + 3*a^2*(3 + m))*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2,
Sin[e + f*x]^2]*Sin[e + f*x])/((2 + m)*Sqrt[Cos[e + f*x]^2]) - b^2*(a + b*S
in[e + f*x])))/(f*(3 + m))
```

fricas [F] time = 2.59, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(3\,ab^2\cos\left(fx+e\right)^2-a^3-3\,ab^2+\left(b^3\cos\left(fx+e\right)^2-3\,a^2b-b^3\right)\sin\left(fx+e\right)\right)\left(d\sin\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 - 3
*a^2*b - b^3)*sin(f*x + e))*(d*sin(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)+a\right)^3\left(d\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3*(d*sin(f*x + e))^m, x)
```

maple [F] time = 5.11, size = 0, normalized size = 0.00

$$\int\left(d\sin\left(fx+e\right)\right)^m\left(a+b\sin\left(fx+e\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m*(a+b*sin(f*x+e))^3,x)
```

```
[Out] int((d*sin(f*x+e))^m*(a+b*sin(f*x+e))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)^{3}\left(d\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)^3*(d*sin(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(d\,\sin\left(e+f\,x\right)\right)}^m\,{\left(a+b\,\sin\left(e+f\,x\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^m*(a + b*sin(e + f*x))^3,x)
[Out] int((d*sin(e + f*x))^m*(a + b*sin(e + f*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m*(a+b*sin(f*x+e))**3,x)
[Out] Timed out
```

3.214 $\int (d\sin(e+fx))^m(a+b\sin(e+fx))^2\,dx$

Optimal. Leaf size=194

$$\frac{\left(a^2(m+2)+b^2(m+1)\right)\cos(e+fx)(d\sin(e+fx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(e+fx)\right)}{df(m+1)(m+2)\sqrt{\cos^2(e+fx)}}+\frac{2ab\cos(e+fx)(d\sin(e+}{d^2f(}$$

```
[Out] -b^2*cos(f*x+e)*(d*sin(f*x+e))^(1+m)/d/f/(2+m)+(b^2*(1+m)+a^2*(2+m))*cos(f*
x+e)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(f*x+e)^2)*(d*sin(f*x+e))^(1
+m)/d/f/(1+m)/(2+m)/(cos(f*x+e)^2)^(1/2)+2*a*b*cos(f*x+e)*hypergeom([1/2, 1
+1/2*m],[2+1/2*m],sin(f*x+e)^2)*(d*sin(f*x+e))^(2+m)/d^2/f/(2+m)/(cos(f*x+e
)^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2789, 2643, 3014}

$$\frac{\left(a^2(m+2)+b^2(m+1)\right)\cos(e+fx)(d\sin(e+fx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(e+fx)\right)}{df(m+1)(m+2)\sqrt{\cos^2(e+fx)}}+\frac{2ab\cos(e+fx)(d\sin(e+}{d^2f(}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^m*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -((b^2*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + m))/(d*f*(2 + m))) + ((b^2*(1 + m
) + a^2*(2 + m))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2,
Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + m))/(d*f*(1 + m)*(2 + m)*Sqrt[Cos[e +
 f*x]^2]) + (2*a*b*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2
, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(2 + m))/(d^2*f*(2 + m)*Sqrt[Cos[e + f*x
]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
```

```
, f, m}, x]
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (d\sin(e+fx))^m(a+b\sin(e+fx))^2\,dx = \frac{(2ab)\int (d\sin(e+fx))^{1+m}\,dx}{d} + \int (d\sin(e+fx))^m\left(a^2+b^2\sin^2(e+\right.$$

$$= -\frac{b^2\cos(e+fx)(d\sin(e+fx))^{1+m}}{df(2+m)} + \frac{2ab\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4}{}\right.}{d^2f(2+m}$$

$$= -\frac{b^2\cos(e+fx)(d\sin(e+fx))^{1+m}}{df(2+m)} + \frac{\left(a^2+\frac{b^2(1+m)}{2+m}\right)\cos(e+fx)\,{}_2F_1}{df(1}$$

Mathematica [A] time = 0.33, size = 144, normalized size = 0.74

$$-\frac{\cos(e+fx)\sin^2(e+fx)^{\frac{1}{2}(-m-1)}(d\sin(e+fx))^m\left(a\left(a\sin(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3}{2};\cos^2(e+fx)\right)+2b\sqrt{\sin^2(e+\right.\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sin[e + f*x])^m*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -((Cos[e + f*x]*(d*Sin[e + f*x])^m*(Sin[e + f*x]^2)^((-1 - m)/2)*(b^2*Hyper
geometric2F1[1/2, (-1 - m)/2, 3/2, Cos[e + f*x]^2]*Sin[e + f*x] + a*(a*Hype
rgeometric2F1[1/2, (1 - m)/2, 3/2, Cos[e + f*x]^2]*Sin[e + f*x] + 2*b*Hyper
geometric2F1[1/2, -1/2*m, 3/2, Cos[e + f*x]^2]*Sqrt[Sin[e + f*x]^2])))/f)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2\right)\left(d\sin\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*(d*sin(f*x
+ e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)^{2}\left(d\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(d*sin(f*x + e))^m, x)
```

maple [F] time = 7.29, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)\right)^{m}\left(a+b\sin\left(fx+e\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m*(a+b*sin(f*x+e))^2,x)
```

```
[Out] int((d*sin(f*x+e))^m*(a+b*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)^{2}\left(d\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(d*sin(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\sin\left(e+f\,x\right)\right)}^m\,{\left(a+b\,\sin\left(e+f\,x\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^m*(a + b*sin(e + f*x))^2,x)
```

```
[Out] int((d*sin(e + f*x))^m*(a + b*sin(e + f*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m*(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.215 $\int (d\sin(e+fx))^m(a+b\sin(e+fx))\,dx$

Optimal. Leaf size=139

$$\frac{a\cos(e+fx)(d\sin(e+fx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(e+fx)\right)}{df(m+1)\sqrt{\cos^2(e+fx)}}+\frac{b\cos(e+fx)(d\sin(e+fx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2}\right.}{d^2f(m+2)\sqrt{\cos^2(e+fx)}}$$

```
[Out] a*cos(f*x+e)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(f*x+e)^2)*(d*sin(f*
x+e))^(1+m)/d/f/(1+m)/(cos(f*x+e)^2)^(1/2)+b*cos(f*x+e)*hypergeom([1/2, 1+1
/2*m],[2+1/2*m],sin(f*x+e)^2)*(d*sin(f*x+e))^(2+m)/d^2/f/(2+m)/(cos(f*x+e)^
2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2748, 2643}

$$\frac{a\cos(e+fx)(d\sin(e+fx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(e+fx)\right)}{df(m+1)\sqrt{\cos^2(e+fx)}}+\frac{b\cos(e+fx)(d\sin(e+fx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2}\right.}{d^2f(m+2)\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^m*(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[e + f*x]^2
]*(d*Sin[e + f*x])^(1 + m))/(d*f*(1 + m)*Sqrt[Cos[e + f*x]^2]) + (b*Cos[e +
 f*x]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Sin[e + f*x]^2]*(d*Sin[e
 + f*x])^(2 + m))/(d^2*f*(2 + m)*Sqrt[Cos[e + f*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int (d\sin(e+fx))^m(a+b\sin(e+fx))\,dx = a\int (d\sin(e+fx))^m\,dx + \frac{b\int (d\sin(e+fx))^{1+m}\,dx}{d}$$
$$= \frac{a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(e+fx)\right)(d\sin(e+fx))^{1+m}}{df(1+m)\sqrt{\cos^2(e+fx)}} + \frac{b}{}$$

Mathematica [A] time = 0.16, size = 111, normalized size = 0.80

$$\frac{\sqrt{\cos^2(e+fx)}\tan(e+fx)(d\sin(e+fx))^m\left(a(m+2)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(e+fx)\right)+b(m+1)\sin(e+fx)\,{}_2F_1\right.}{f(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sin[e + f*x])^m*(a + b*Sin[e + f*x]),x]
```

```
[Out] (Sqrt[Cos[e + f*x]^2]*(d*Sin[e + f*x])^m*(a*(2 + m)*Hypergeometric2F1[1/2,
(1 + m)/2, (3 + m)/2, Sin[e + f*x]^2] + b*(1 + m)*Hypergeometric2F1[1/2, (2
 + m)/2, (4 + m)/2, Sin[e + f*x]^2]*Sin[e + f*x])*Tan[e + f*x])/(f*(1 + m)*
(2 + m))
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e) + a)*(d*sin(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(d*sin(f*x + e))^m, x)
```

maple [F] time = 1.87, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)\right)^m\left(a+b\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m*(a+b*sin(f*x+e)),x)
```

```
[Out] int((d*sin(f*x+e))^m*(a+b*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(d*sin(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\sin\left(e+f\,x\right)\right)}^m\,\left(a+b\,\sin\left(e+f\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^m*(a + b*sin(e + f*x)),x)
```

```
[Out] int((d*sin(e + f*x))^m*(a + b*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m*(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.216 $\int \frac{(d\sin(e+fx))^m}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=195

$$\frac{b\cos(e+fx)\sin^2(e+fx)^{-m/2}(d\sin(e+fx))^m F_1\left(\frac{1}{2};-\frac{m}{2},1;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)}-\frac{ad\cos(e+fx)\sin^2(e}{}$$

```
[Out] -a*d*AppellF1(1/2,1/2-1/2*m,1,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b^2))
*cos(f*x+e)*(d*sin(f*x+e))^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)/(a^2-b^2)/f+b*
AppellF1(1/2,-1/2*m,1,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b^2))*cos(f*x
+e)*(d*sin(f*x+e))^m/(a^2-b^2)/f/((sin(f*x+e)^2)^(1/2*m))
```

Rubi [A] time = 0.25, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2823, 3189, 429}

$$\frac{b\cos(e+fx)\sin^2(e+fx)^{-m/2}(d\sin(e+fx))^m F_1\left(\frac{1}{2};-\frac{m}{2},1;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)}-\frac{ad\cos(e+fx)\sin^2(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^m/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((a*d*AppellF1[1/2, (1 - m)/2, 1, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]
^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Sin[e + f*x])^(-1 + m)*(Sin[e + f*x]^2)^(
(1 - m)/2))/((a^2 - b^2)*f)) + (b*AppellF1[1/2, -m/2, 1, 3/2, Cos[e + f*x]^
2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Sin[e + f*x])^m)/((
a^2 - b^2)*f*(Sin[e + f*x]^2)^(m/2))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^m}{a+b\sin(e+fx)}\,dx = a\int \frac{(d\sin(e+fx))^m}{a^2-b^2\sin^2(e+fx)}\,dx - \frac{b\int \frac{(d\sin(e+fx))^{1+m}}{a^2-b^2\sin^2(e+fx)}\,dx}{d}$$

$$= -\frac{\left(ad(d\sin(e+fx))^{2\left(-\frac{1}{2}+\frac{m}{2}\right)}\sin^2(e+fx)^{\frac{1}{2}-\frac{m}{2}}\right)\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^{\frac{1}{2}(-1+m)}}{a^2-b^2+b^2x^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{adF_1\left(\frac{1}{2};\frac{1-m}{2},1;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\sin(e+fx))^{-1+m}\sin^2(e+}{\left(a^2-b^2\right)f}$$

Mathematica [B] time = 17.69, size = 1590, normalized size = 8.15

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^m/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((Sec[e + f*x]^2)^(m/2)*(d*Sin[e + f*x])^m*Tan[e + f*x]*(Tan[e + f*x]/Sqrt[
Sec[e + f*x]^2])^m*(a*b*(2 + m)*AppellF1[(1 + m)/2, m/2, 1, (3 + m)/2, -Tan
[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] + (1 + m)*((a^2 - b^2)*Appe
llF1[(2 + m)/2, (-1 + m)/2, 1, (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*T
an[e + f*x]^2] - a^2*Hypergeometric2F1[(1 + m)/2, (2 + m)/2, (4 + m)/2, -Ta
n[e + f*x]^2])*Tan[e + f*x]))/(a^2*b*f*(1 + m)*(2 + m)*(a + b*Sin[e + f*x])
*(((Sec[e + f*x]^2)^(1 + m/2)*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^m*(a*b*(2
 + m)*AppellF1[(1 + m)/2, m/2, 1, (3 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)
*Tan[e + f*x]^2)/a^2] + (1 + m)*((a^2 - b^2)*AppellF1[(2 + m)/2, (-1 + m)/2
, 1, (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - a^2*Hyper
geometric2F1[(1 + m)/2, (2 + m)/2, (4 + m)/2, -Tan[e + f*x]^2])*Tan[e + f*x
]))/(a^2*b*(1 + m)*(2 + m)) + (m*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]^2*(Tan
```

```
[e + f*x]/Sqrt[Sec[e + f*x]^2])^m*(a*b*(2 + m)*AppellF1[(1 + m)/2, m/2, 1,
(3 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] + (1 + m)*((
a^2 - b^2)*AppellF1[(2 + m)/2, (-1 + m)/2, 1, (4 + m)/2, -Tan[e + f*x]^2, (
-1 + b^2/a^2)*Tan[e + f*x]^2] - a^2*Hypergeometric2F1[(1 + m)/2, (2 + m)/2,
 (4 + m)/2, -Tan[e + f*x]^2])*Tan[e + f*x]))/(a^2*b*(1 + m)*(2 + m)) + (m*(
Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^(-1
+ m)*(a*b*(2 + m)*AppellF1[(1 + m)/2, m/2, 1, (3 + m)/2, -Tan[e + f*x]^2, (
(-a^2 + b^2)*Tan[e + f*x]^2)/a^2] + (1 + m)*((a^2 - b^2)*AppellF1[(2 + m)/2
, (-1 + m)/2, 1, (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]
 - a^2*Hypergeometric2F1[(1 + m)/2, (2 + m)/2, (4 + m)/2, -Tan[e + f*x]^2])
*Tan[e + f*x])*(Sqrt[Sec[e + f*x]^2] - Tan[e + f*x]^2/Sqrt[Sec[e + f*x]^2])
)/(a^2*b*(1 + m)*(2 + m)) + ((Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]*(Tan[e + f
*x]/Sqrt[Sec[e + f*x]^2])^m*((1 + m)*((a^2 - b^2)*AppellF1[(2 + m)/2, (-1 +
 m)/2, 1, (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - a^2*
Hypergeometric2F1[(1 + m)/2, (2 + m)/2, (4 + m)/2, -Tan[e + f*x]^2])*Sec[e
+ f*x]^2 + a*b*(2 + m)*(-((m*(1 + m)*AppellF1[1 + (1 + m)/2, 1 + m/2, 1, 1
+ (3 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*
x]^2*Tan[e + f*x])/(3 + m)) + (2*(-a^2 + b^2)*(1 + m)*AppellF1[1 + (1 + m)/
2, m/2, 2, 1 + (3 + m)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^
2]*Sec[e + f*x]^2*Tan[e + f*x])/(a^2*(3 + m))) + (1 + m)*Tan[e + f*x]*((a^2
 - b^2)*(-(((-1 + m)*(2 + m)*AppellF1[1 + (2 + m)/2, 1 + (-1 + m)/2, 1, 1 +
 (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*
Tan[e + f*x])/(4 + m)) + (2*(-1 + b^2/a^2)*(2 + m)*AppellF1[1 + (2 + m)/2,
(-1 + m)/2, 2, 1 + (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^
2]*Sec[e + f*x]^2*Tan[e + f*x])/(4 + m)) - a^2*(2 + m)*Csc[e + f*x]*Sec[e +
 f*x]*(-Hypergeometric2F1[(1 + m)/2, (2 + m)/2, (4 + m)/2, -Tan[e + f*x]^2]
 + (1 + Tan[e + f*x]^2)^((-1 - m)/2)))))/(a^2*b*(1 + m)*(2 + m))))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d \sin\left(fx+e\right)\right)^{m}}{b \sin\left(fx+e\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e))^m/(b*sin(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin\left(fx+e\right)\right)^{m}}{b \sin\left(fx+e\right)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^m/(b*sin(f*x + e) + a), x)
```

maple [F] time = 0.75, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^{m}}{a+b\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m/(a+b*sin(f*x+e)),x)
```

```
[Out] int((d*sin(f*x+e))^m/(a+b*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^{m}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^m/(b*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,\sin\left(e+f\,x\right)\right)^{m}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^m/(a + b*sin(e + f*x)),x)
```

```
[Out] int((d*sin(e + f*x))^m/(a + b*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.217 $\int \frac{(d\sin(e+fx))^m}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=306

$$-\frac{b^2\cos(e+fx)\sin^2(e+fx)^{\frac{1}{2}(-m-1)}(d\sin(e+fx))^{m+1}F_1\left(\frac{1}{2};\frac{1}{2}(-m-1),2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{df\left(a^2-b^2\right)^2}-\frac{a^2d\,\mathrm{co}}{}$$

```
[Out] -b^2*AppellF1(1/2,-1/2-1/2*m,2,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b^2)
)*cos(f*x+e)*(d*sin(f*x+e))^(1+m)*(sin(f*x+e)^2)^(-1/2-1/2*m)/(a^2-b^2)^2/d
/f-a^2*d*AppellF1(1/2,1/2-1/2*m,2,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b
^2))*cos(f*x+e)*(d*sin(f*x+e))^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)/(a^2-b^2)^
2/f+2*a*b*AppellF1(1/2,-1/2*m,2,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b^2
))*cos(f*x+e)*(d*sin(f*x+e))^m/(a^2-b^2)^2/f/((sin(f*x+e)^2)^(1/2*m))
```

Rubi [A] time = 0.42, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2824, 3189, 429, 16}

$$-\frac{b^2\cos(e+fx)\sin^2(e+fx)^{\frac{1}{2}(-m-1)}(d\sin(e+fx))^{m+1}F_1\left(\frac{1}{2};\frac{1}{2}(-m-1),2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{df\left(a^2-b^2\right)^2}-\frac{a^2d\,\mathrm{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^m/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -((b^2*AppellF1[1/2, (-1 - m)/2, 2, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x
]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + m)*(Sin[e + f*x]^2)^(
(-1 - m)/2))/((a^2 - b^2)^2*d*f)) - (a^2*d*AppellF1[1/2, (1 - m)/2, 2, 3/2,
 Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Sin[e
 + f*x])^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/((a^2 - b^2)^2*f) + (2*a*b*
AppellF1[1/2, -m/2, 2, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b
^2))]*Cos[e + f*x]*(d*Sin[e + f*x])^m)/((a^2 - b^2)^2*f*(Sin[e + f*x]^2)^(m
/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
```

```
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2824

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n/((a - b*sin[e +
 f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] && !IntegerQ[m]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^m}{(a+b\sin(e+fx))^2}\,dx = \int\left(\frac{a^2(d\sin(e+fx))^m}{\left(a^2-b^2\sin^2(e+fx)\right)^2} - \frac{2ab\sin(e+fx)(d\sin(e+fx))^m}{\left(a^2-b^2\sin^2(e+fx)\right)^2} + \frac{b^2\sin^2(e+fx)(d\sin}{\left(-a^2+b^2\sin^2(e\right.}\right.$$

$$= a^2\int \frac{(d\sin(e+fx))^m}{\left(a^2-b^2\sin^2(e+fx)\right)^2}\,dx - (2ab)\int \frac{\sin(e+fx)(d\sin(e+fx))^m}{\left(a^2-b^2\sin^2(e+fx)\right)^2}\,dx + b^2\int \frac{\sin}{}$$

$$= \frac{b^2\int \frac{(d\sin(e+fx))^{2+m}}{\left(-a^2+b^2\sin^2(e+fx)\right)^2}\,dx}{d^2} - \frac{(2ab)\int \frac{(d\sin(e+fx))^{1+m}}{\left(a^2-b^2\sin^2(e+fx)\right)^2}\,dx}{d} - \frac{\left(a^2d(d\sin(e+fx))^{2\left(-\frac{1}{2}+\frac{m}{2}\right.}\right.}{}$$

$$= -\frac{a^2dF_1\left(\frac{1}{2};\frac{1-m}{2},2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\sin(e+fx))^{-1+m}\sin^2}{\left(a^2-b^2\right)^2f}$$

$$= -\frac{b^2F_1\left(\frac{1}{2};\frac{1}{2}(-1-m),2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\sin(e+fx))^{1+m}}{\left(a^2-b^2\right)^2df}$$

Mathematica [B] time = 18.88, size = 1790, normalized size = 5.85

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^m/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -(((Sec[e + f*x]^2)^(m/2)*(d*Sin[e + f*x])^m*Tan[e + f*x]*(Tan[e + f*x]/Sqr
t[Sec[e + f*x]^2])^m*(-(a*(a^2 + b^2)*(2 + m)*AppellF1[(1 + m)/2, m/2, 1, (
3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]) + 2*b*(a*b*(2 +
m)*AppellF1[(1 + m)/2, m/2, 2, (3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*T
an[e + f*x]^2] + (a^2 - b^2)*(1 + m)*AppellF1[(2 + m)/2, (-1 + m)/2, 2, (4
+ m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x])))/(a^
3*(a^2 - b^2)*f*(1 + m)*(2 + m)*(a + b*Sin[e + f*x])^2*(-(((Sec[e + f*x]^2)
^(1 + m/2)*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^m*(-(a*(a^2 + b^2)*(2 + m)*A
ppellF1[(1 + m)/2, m/2, 1, (3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e
 + f*x]^2]) + 2*b*(a*b*(2 + m)*AppellF1[(1 + m)/2, m/2, 2, (3 + m)/2, -Tan[
e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + (a^2 - b^2)*(1 + m)*AppellF1[(
2 + m)/2, (-1 + m)/2, 2, (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e +
 f*x]^2]*Tan[e + f*x])))/(a^3*(a^2 - b^2)*(1 + m)*(2 + m))) - (m*(Sec[e + f
*x]^2)^(m/2)*Tan[e + f*x]^2*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^m*(-(a*(a^2
 + b^2)*(2 + m)*AppellF1[(1 + m)/2, m/2, 1, (3 + m)/2, -Tan[e + f*x]^2, (-1
 + b^2/a^2)*Tan[e + f*x]^2]) + 2*b*(a*b*(2 + m)*AppellF1[(1 + m)/2, m/2, 2,
 (3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + (a^2 - b^2)*(
1 + m)*AppellF1[(2 + m)/2, (-1 + m)/2, 2, (4 + m)/2, -Tan[e + f*x]^2, (-1 +
 b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x])))/(a^3*(a^2 - b^2)*(1 + m)*(2 + m))
 - (m*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2
])^(-1 + m)*(Sqrt[Sec[e + f*x]^2] - Tan[e + f*x]^2/Sqrt[Sec[e + f*x]^2])*(-
(a*(a^2 + b^2)*(2 + m)*AppellF1[(1 + m)/2, m/2, 1, (3 + m)/2, -Tan[e + f*x]
^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]) + 2*b*(a*b*(2 + m)*AppellF1[(1 + m)/2,
m/2, 2, (3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + (a^2 -
 b^2)*(1 + m)*AppellF1[(2 + m)/2, (-1 + m)/2, 2, (4 + m)/2, -Tan[e + f*x]^2
, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x])))/(a^3*(a^2 - b^2)*(1 + m)*(
2 + m)) - ((Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]*(Tan[e + f*x]/Sqrt[Sec[e + f
*x]^2])^m*(-(a*(a^2 + b^2)*(2 + m)*(-((m*(1 + m)*AppellF1[1 + (1 + m)/2, 1
+ m/2, 1, 1 + (3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Se
c[e + f*x]^2*Tan[e + f*x])/(3 + m)) + (2*(-1 + b^2/a^2)*(1 + m)*AppellF1[1
+ (1 + m)/2, m/2, 2, 1 + (3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e +
 f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 + m))) + 2*b*((a^2 - b^2)*(1 + m)*
AppellF1[(2 + m)/2, (-1 + m)/2, 2, (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^
2)*Tan[e + f*x]^2]*Sec[e + f*x]^2 + a*b*(2 + m)*(-((m*(1 + m)*AppellF1[1 +
(1 + m)/2, 1 + m/2, 2, 1 + (3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e
 + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 + m)) + (4*(-1 + b^2/a^2)*(1 + m
)*AppellF1[1 + (1 + m)/2, m/2, 3, 1 + (3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2
```

```
/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 + m)) + (a^2 - b^2)*(
1 + m)*Tan[e + f*x]*(-(((-1 + m)*(2 + m)*AppellF1[1 + (2 + m)/2, 1 + (-1 +
m)/2, 2, 1 + (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec
[e + f*x]^2*Tan[e + f*x])/(4 + m)) + (4*(-1 + b^2/a^2)*(2 + m)*AppellF1[1 +
 (2 + m)/2, (-1 + m)/2, 3, 1 + (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*T
an[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(4 + m)))))/(a^3*(a^2 - b^2)*(1
 + m)*(2 + m)))))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(d\sin\left(fx+e\right)\right)^m}{b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(d*sin(f*x + e))^m/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2
 - b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(d\sin\left(fx+e\right)\right)^m}{\left(b\sin\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^m/(b*sin(f*x + e) + a)^2, x)
```

maple [F] time = 1.46, size = 0, normalized size = 0.00

$$\int\frac{\left(d\sin\left(fx+e\right)\right)^m}{\left(a+b\sin\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m/(a+b*sin(f*x+e))^2,x)
```

```
[Out] int((d*sin(f*x+e))^m/(a+b*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(d\sin\left(fx+e\right)\right)^m}{\left(b\sin\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e))^m/(b*sin(f*x + e) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d\,\sin\left(e+f\,x\right)\right)^m}{\left(a+b\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^m/(a + b*sin(e + f*x))^2,x)
[Out] int((d*sin(e + f*x))^m/(a + b*sin(e + f*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m/(a+b*sin(f*x+e))**2,x)
[Out] Timed out
```

3.218 $\int \frac{(d\sin(e+fx))^m}{(a+b\sin(e+fx))^3}\,dx$

Optimal. Leaf size=406

$$-\frac{3ab^2\cos(e+fx)\sin^2(e+fx)^{\frac{1}{2}(-m-1)}(d\sin(e+fx))^{m+1}F_1\left(\frac{1}{2};\frac{1}{2}(-m-1),3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{df\left(a^2-b^2\right)^3}+\frac{3a^2b}{}$$

```
[Out] -3*a*b^2*AppellF1(1/2,-1/2-1/2*m,3,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-
b^2))*cos(f*x+e)*(d*sin(f*x+e))^(1+m)*(sin(f*x+e)^2)^(-1/2-1/2*m)/(a^2-b^2)
^3/d/f-a^3*d*AppellF1(1/2,1/2-1/2*m,3,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a
^2-b^2))*cos(f*x+e)*(d*sin(f*x+e))^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)/(a^2-b
^2)^3/f+b^3*AppellF1(1/2,-1-1/2*m,3,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2
-b^2))*cos(f*x+e)*(d*sin(f*x+e))^m/(a^2-b^2)^3/f/((sin(f*x+e)^2)^(1/2*m))+3
*a^2*b*AppellF1(1/2,-1/2*m,3,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b^2))*
cos(f*x+e)*(d*sin(f*x+e))^m/(a^2-b^2)^3/f/((sin(f*x+e)^2)^(1/2*m))
```

Rubi [A] time = 0.55, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2824, 3189, 429, 16}

$$-\frac{3ab^2\cos(e+fx)\sin^2(e+fx)^{\frac{1}{2}(-m-1)}(d\sin(e+fx))^{m+1}F_1\left(\frac{1}{2};\frac{1}{2}(-m-1),3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{df\left(a^2-b^2\right)^3}-\frac{a^3d\,c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^m/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (-3*a*b^2*AppellF1[1/2, (-1 - m)/2, 3, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e +
f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + m)*(Sin[e + f*x]^2
)^((-1 - m)/2))/((a^2 - b^2)^3*d*f) - (a^3*d*AppellF1[1/2, (1 - m)/2, 3, 3/
2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Sin
[e + f*x])^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/((a^2 - b^2)^3*f) + (b^3*
AppellF1[1/2, (-2 - m)/2, 3, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a
^2 - b^2))]*Cos[e + f*x]*(d*Sin[e + f*x])^m)/((a^2 - b^2)^3*f*(Sin[e + f*x]
^2)^(m/2)) + (3*a^2*b*AppellF1[1/2, -m/2, 3, 3/2, Cos[e + f*x]^2, -((b^2*Co
s[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Sin[e + f*x])^m)/((a^2 - b^2)^3
*f*(Sin[e + f*x]^2)^(m/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2824

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n/((a - b*sin[e +
 f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^m}{(a+b\sin(e+fx))^3}\,dx = \int\left(\frac{a^3(d\sin(e+fx))^m}{\left(a^2-b^2\sin^2(e+fx)\right)^3} - \frac{3a^2b\sin(e+fx)(d\sin(e+fx))^m}{\left(a^2-b^2\sin^2(e+fx)\right)^3} + \frac{3ab^2\sin^2(e+fx)(}{\left(a^2-b^2\sin^2\right.}\right.$$

$$= a^3\int \frac{(d\sin(e+fx))^m}{\left(a^2-b^2\sin^2(e+fx)\right)^3}\,dx - \left(3a^2b\right)\int \frac{\sin(e+fx)(d\sin(e+fx))^m}{\left(a^2-b^2\sin^2(e+fx)\right)^3}\,dx + \left(3ab^2\right)$$

$$= \frac{b^3\int \frac{(d\sin(e+fx))^{3+m}}{\left(-a^2+b^2\sin^2(e+fx)\right)^3}\,dx}{d^3} + \frac{\left(3ab^2\right)\int \frac{(d\sin(e+fx))^{2+m}}{\left(a^2-b^2\sin^2(e+fx)\right)^3}\,dx}{d^2} - \frac{\left(3a^2b\right)\int \frac{(d\sin(e+fx))^{1+m}}{\left(a^2-b^2\sin^2(e+fx)\right)^3}}{d}$$

$$= -\frac{a^3dF_1\left(\frac{1}{2};\frac{1-m}{2},3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\sin(e+fx))^{-1+m}\sin^2}{\left(a^2-b^2\right)^3 f}$$

$$= -\frac{3ab^2F_1\left(\frac{1}{2};\frac{1}{2}(-1-m),3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\sin(e+fx))^{1-}}{\left(a^2-b^2\right)^3 df}$$

Mathematica [B] time = 18.69, size = 2298, normalized size = 5.66

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^m/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] -(((Sec[e + f*x]^2)^(m/2)*(d*Sin[e + f*x])^m*Tan[e + f*x]*(Tan[e + f*x]/Sqr
t[Sec[e + f*x]^2])^m*(-(a*(a^2 + 3*b^2)*(2 + m)*AppellF1[(1 + m)/2, (-2 + m
)/2, 2, (3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]) + b*(4*
a*b*(2 + m)*AppellF1[(1 + m)/2, (-2 + m)/2, 3, (3 + m)/2, -Tan[e + f*x]^2,
(-1 + b^2/a^2)*Tan[e + f*x]^2] + (1 + m)*((3*a^2 + b^2)*AppellF1[(2 + m)/2,
 (-1 + m)/2, 2, (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]
- 4*b^2*AppellF1[(2 + m)/2, (-1 + m)/2, 3, (4 + m)/2, -Tan[e + f*x]^2, (-1
+ b^2/a^2)*Tan[e + f*x]^2])*Tan[e + f*x])))/(a^4*(a^2 - b^2)*f*(1 + m)*(2 +
 m)*(a + b*Sin[e + f*x])^3*(-(((Sec[e + f*x]^2)^(1 + m/2)*(Tan[e + f*x]/Sqr
t[Sec[e + f*x]^2])^m*(-(a*(a^2 + 3*b^2)*(2 + m)*AppellF1[(1 + m)/2, (-2 + m
)/2, 2, (3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]) + b*(4*
a*b*(2 + m)*AppellF1[(1 + m)/2, (-2 + m)/2, 3, (3 + m)/2, -Tan[e + f*x]^2,
(-1 + b^2/a^2)*Tan[e + f*x]^2] + (1 + m)*((3*a^2 + b^2)*AppellF1[(2 + m)/2,
 (-1 + m)/2, 2, (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]
```

```
- 4*b^2*AppellF1[(2 + m)/2, (-1 + m)/2, 3, (4 + m)/2, -Tan[e + f*x]^2, (-1
+ b^2/a^2)*Tan[e + f*x]^2])*Tan[e + f*x]))))/(a^4*(a^2 - b^2)*(1 + m)*(2 + m
))) - (m*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]^2*(Tan[e + f*x]/Sqrt[Sec[e + f
*x]^2])^m*(-(a*(a^2 + 3*b^2)*(2 + m)*AppellF1[(1 + m)/2, (-2 + m)/2, 2, (3
+ m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]) + b*(4*a*b*(2 + m)
*AppellF1[(1 + m)/2, (-2 + m)/2, 3, (3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a
^2)*Tan[e + f*x]^2] + (1 + m)*((3*a^2 + b^2)*AppellF1[(2 + m)/2, (-1 + m)/2
, 2, (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - 4*b^2*App
ellF1[(2 + m)/2, (-1 + m)/2, 3, (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*
Tan[e + f*x]^2])*Tan[e + f*x])))/(a^4*(a^2 - b^2)*(1 + m)*(2 + m)) - (m*(Se
c[e + f*x]^2)^(m/2)*Tan[e + f*x]*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^(-1 +
m)*(Sqrt[Sec[e + f*x]^2] - Tan[e + f*x]^2/Sqrt[Sec[e + f*x]^2])*(-(a*(a^2 +
 3*b^2)*(2 + m)*AppellF1[(1 + m)/2, (-2 + m)/2, 2, (3 + m)/2, -Tan[e + f*x]
^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]) + b*(4*a*b*(2 + m)*AppellF1[(1 + m)/2,
(-2 + m)/2, 3, (3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] +
 (1 + m)*((3*a^2 + b^2)*AppellF1[(2 + m)/2, (-1 + m)/2, 2, (4 + m)/2, -Tan[
e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - 4*b^2*AppellF1[(2 + m)/2, (-1
+ m)/2, 3, (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2])*Tan[
e + f*x])))/(a^4*(a^2 - b^2)*(1 + m)*(2 + m)) - ((Sec[e + f*x]^2)^(m/2)*Tan
[e + f*x]*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^m*(-(a*(a^2 + 3*b^2)*(2 + m)*
(-(((-2 + m)*(1 + m)*AppellF1[1 + (1 + m)/2, 1 + (-2 + m)/2, 2, 1 + (3 + m)
/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e +
f*x])/(3 + m)) + (4*(-1 + b^2/a^2)*(1 + m)*AppellF1[1 + (1 + m)/2, (-2 + m)
/2, 3, 1 + (3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e
 + f*x]^2*Tan[e + f*x])/(3 + m))) + b*((1 + m)*((3*a^2 + b^2)*AppellF1[(2 +
 m)/2, (-1 + m)/2, 2, (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*
x]^2] - 4*b^2*AppellF1[(2 + m)/2, (-1 + m)/2, 3, (4 + m)/2, -Tan[e + f*x]^2
, (-1 + b^2/a^2)*Tan[e + f*x]^2])*Sec[e + f*x]^2 + 4*a*b*(2 + m)*(-(((-2 +
m)*(1 + m)*AppellF1[1 + (1 + m)/2, 1 + (-2 + m)/2, 3, 1 + (3 + m)/2, -Tan[e
 + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 +
 m)) + (6*(-1 + b^2/a^2)*(1 + m)*AppellF1[1 + (1 + m)/2, (-2 + m)/2, 4, 1 +
 (3 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*
Tan[e + f*x])/(3 + m)) + (1 + m)*Tan[e + f*x]*((3*a^2 + b^2)*(-(((-1 + m)*(
2 + m)*AppellF1[1 + (2 + m)/2, 1 + (-1 + m)/2, 2, 1 + (4 + m)/2, -Tan[e + f
*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(4 + m))
 + (4*(-1 + b^2/a^2)*(2 + m)*AppellF1[1 + (2 + m)/2, (-1 + m)/2, 3, 1 + (4
+ m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[
e + f*x])/(4 + m)) - 4*b^2*(-(((-1 + m)*(2 + m)*AppellF1[1 + (2 + m)/2, 1 +
 (-1 + m)/2, 3, 1 + (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]
^2]*Sec[e + f*x]^2*Tan[e + f*x])/(4 + m)) + (6*(-1 + b^2/a^2)*(2 + m)*Appel
lF1[1 + (2 + m)/2, (-1 + m)/2, 4, 1 + (4 + m)/2, -Tan[e + f*x]^2, (-1 + b^2
/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(4 + m))))))/(a^4*(a^2 -
 b^2)*(1 + m)*(2 + m)))))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(d\sin\left(fx+e\right)\right)^{m}}{3\,ab^{2}\cos\left(fx+e\right)^{2}-a^{3}-3\,ab^{2}+\left(b^{3}\cos\left(fx+e\right)^{2}-3\,a^{2}b-b^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(d*sin(f*x + e))^m/(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3
*cos(f*x + e)^2 - 3*a^2*b - b^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^{m}}{\left(b\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^m/(b*sin(f*x + e) + a)^3, x)
```

maple [F] time = 1.77, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^{m}}{\left(a+b\sin\left(fx+e\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m/(a+b*sin(f*x+e))^3,x)
```

```
[Out] int((d*sin(f*x+e))^m/(a+b*sin(f*x+e))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^{m}}{\left(b\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^m/(b*sin(f*x + e) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d\,\sin\left(e+f\,x\right)\right)^m}{\left(a+b\,\sin\left(e+f\,x\right)\right)^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^m/(a + b*sin(e + f*x))^3,x)
[Out] int((d*sin(e + f*x))^m/(a + b*sin(e + f*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m/(a+b*sin(f*x+e))**3,x)
[Out] Timed out
```

3.219 $\int \sin^{-1-\frac{a^2}{a^2+b^2}}(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=142

$$\frac{2a\left(a^2+b^2\right)\cos(c+dx)\sin^{\frac{b^2}{a^2+b^2}}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{b^2}{2\left(a^2+b^2\right)};\frac{1}{2}\left(3-\frac{a^2}{a^2+b^2}\right);\sin^2(c+dx)\right)}{bd\sqrt{\cos^2(c+dx)}}-\frac{\left(a^2+b^2\right)\cos(c+dx)\sin^{-\frac{a}{}}}{d}$$

```
[Out] -(a^2+b^2)*cos(d*x+c)/d/(sin(d*x+c)^(a^2/(a^2+b^2)))+2*a*(a^2+b^2)*cos(d*x+
c)*hypergeom([1/2, 1/2*b^2/(a^2+b^2)],[3/2-1/2*a^2/(a^2+b^2)],sin(d*x+c)^2)
*sin(d*x+c)^(b^2/(a^2+b^2))/b/d/(cos(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2789, 2643, 3011}

$$\frac{2a\left(a^2+b^2\right)\cos(c+dx)\sin^{\frac{b^2}{a^2+b^2}}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{b^2}{2\left(a^2+b^2\right)};\frac{1}{2}\left(3-\frac{a^2}{a^2+b^2}\right);\sin^2(c+dx)\right)}{bd\sqrt{\cos^2(c+dx)}}-\frac{\left(a^2+b^2\right)\cos(c+dx)\sin^{-\frac{a}{}}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^(-1 - a^2/(a^2 + b^2))*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(((a^2 + b^2)*Cos[c + d*x])/(d*Sin[c + d*x]^(a^2/(a^2 + b^2)))) + (2*a*(a^
2 + b^2)*Cos[c + d*x]*Hypergeometric2F1[1/2, b^2/(2*(a^2 + b^2)), (3 - a^2/
(a^2 + b^2))/2, Sin[c + d*x]^2]*Sin[c + d*x]^(b^2/(a^2 + b^2)))/(b*d*Sqrt[C
os[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3011

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(
m + 1)), x] /; FreeQ[{b, e, f, A, C, m}, x] && EqQ[A*(m + 2) + C*(m + 1), 0
]
```

Rubi steps

$$\int \sin^{-1-\frac{a^2}{a^2+b^2}}(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \sin^{-\frac{a^2}{a^2+b^2}}(c+dx)\,dx + \int \sin^{-1-\frac{a^2}{a^2+b^2}}(c+dx)\left(a^2+b^2\sin\right.$$

$$= -\frac{\left(a^2+b^2\right)\cos(c+dx)\sin^{-\frac{a^2}{a^2+b^2}}(c+dx)}{d} + \frac{2a\left(a^2+b^2\right)\cos(c+}{}$$

Mathematica [A] time = 0.30, size = 188, normalized size = 1.32

$$-\frac{\cos(c+dx)\sin^{-\frac{a^2}{a^2+b^2}}(c+dx)\sin^2(c+dx)^{-\frac{b^2}{2\left(a^2+b^2\right)}}\left(\sqrt{\sin^2(c+dx)}\left(a^2\,{}_2F_1\left(\frac{1}{2},\frac{a^2}{2\left(a^2+b^2\right)}+1;\frac{3}{2};\cos^2(c+dx)\right)+b^2\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^(-1 - a^2/(a^2 + b^2))*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((Cos[c + d*x]*(2*a*b*Hypergeometric2F1[1/2, (1 + a^2/(a^2 + b^2))/2, 3/2,
 Cos[c + d*x]^2]*Sin[c + d*x] + (b^2*Hypergeometric2F1[1/2, a^2/(2*(a^2 + b
^2)), 3/2, Cos[c + d*x]^2] + a^2*Hypergeometric2F1[1/2, 1 + a^2/(2*(a^2 + b
^2)), 3/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2]))/(d*Sin[c + d*x]^(a^2/(a^
2 + b^2))*(Sin[c + d*x]^2)^(b^2/(2*(a^2 + b^2)))))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b^2\cos\left(dx+c\right)^2-2\,ab\sin\left(dx+c\right)-a^2-b^2\right)\sin\left(dx+c\right)^{-\frac{2\,a^2+b^2}{a^2+b^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(-1-a^2/(a^2+b^2))*(a+b*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] integral(-(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)*sin(d*x + c
)^(-(2*a^2 + b^2)/(a^2 + b^2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^2\sin(dx+c)^{-\frac{a^2}{a^2+b^2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(-1-a^2/(a^2+b^2))*(a+b*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2*sin(d*x + c)^(-a^2/(a^2 + b^2) - 1), x)
```

maple [F] time = 10.98, size = 0, normalized size = 0.00

$$\int \left(\sin^{-1-\frac{a^2}{a^2+b^2}}(dx+c)\right)(a+b\sin(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^(-1-a^2/(a^2+b^2))*(a+b*sin(d*x+c))^2,x)
```

```
[Out] int(sin(d*x+c)^(-1-a^2/(a^2+b^2))*(a+b*sin(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^2\sin(dx+c)^{-\frac{a^2}{a^2+b^2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(-1-a^2/(a^2+b^2))*(a+b*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2*sin(d*x + c)^(-a^2/(a^2 + b^2) - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\sin(c+d\,x))^2}{\sin(c+d\,x)^{\frac{a^2}{a^2+b^2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/sin(c + d*x)^(a^2/(a^2 + b^2) + 1),x)
```

```
[Out] int((a + b*sin(c + d*x))^2/sin(c + d*x)^(a^2/(a^2 + b^2) + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(c + dx))^2 \sin^{-\frac{a^2}{a^2+b^2}-1}(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**(-1-a**2/(a**2+b**2))*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*sin(c + d*x)**(-a**2/(a**2 + b**2) - 1), x
)
```

3.220 $\int \frac{(1+2\sin(c+dx))^2}{\sin^{\frac{6}{5}}(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{5\sin^{\frac{4}{5}}(c+dx)\cos(c+dx)\,{}_2F_1\left(\frac{2}{5},\frac{1}{2};\frac{7}{5};\sin^2(c+dx)\right)}{d\sqrt{\cos^2(c+dx)}}-\frac{5\cos(c+dx)}{d\sqrt[5]{\sin(c+dx)}}$$

```
[Out] -5*cos(d*x+c)/d/sin(d*x+c)^(1/5)+5*cos(d*x+c)*hypergeom([2/5, 1/2],[7/5],si
n(d*x+c)^2)*sin(d*x+c)^(4/5)/d/(cos(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2789, 2643, 3011}

$$\frac{5\sin^{\frac{4}{5}}(c+dx)\cos(c+dx)\,{}_2F_1\left(\frac{2}{5},\frac{1}{2};\frac{7}{5};\sin^2(c+dx)\right)}{d\sqrt{\cos^2(c+dx)}}-\frac{5\cos(c+dx)}{d\sqrt[5]{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*Sin[c + d*x])^2/Sin[c + d*x]^(6/5),x]
```

```
[Out] (-5*Cos[c + d*x])/(d*Sin[c + d*x]^(1/5)) + (5*Cos[c + d*x]*Hypergeometric2F
1[2/5, 1/2, 7/5, Sin[c + d*x]^2]*Sin[c + d*x]^(4/5))/(d*Sqrt[Cos[c + d*x]^2
])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3011

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(
```

```
m + 1)), x] /; FreeQ[{b, e, f, A, C, m}, x] && EqQ[A*(m + 2) + C*(m + 1), 0
]
```

Rubi steps

$$\int \frac{(1+2\sin(c+dx))^2}{\sin^{\frac{6}{5}}(c+dx)}\,dx = 4\int \frac{1}{\sqrt[5]{\sin(c+dx)}}\,dx + \int \frac{1+4\sin^2(c+dx)}{\sin^{\frac{6}{5}}(c+dx)}\,dx$$
$$= -\frac{5\cos(c+dx)}{d\sqrt[5]{\sin(c+dx)}} + \frac{5\cos(c+dx)\,{}_2F_1\left(\frac{2}{5},\frac{1}{2};\frac{7}{5};\sin^2(c+dx)\right)\sin^{\frac{4}{5}}(c+dx)}{d\sqrt{\cos^2(c+dx)}}$$

Mathematica [A] time = 0.10, size = 73, normalized size = 1.00

$$-\frac{4\sin^{\frac{4}{5}}(c+dx)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{3}{5};\frac{3}{2};\cos^2(c+dx)\right)}{d\sin^2(c+dx)^{2/5}} - \frac{5\cos(c+dx)}{d\sqrt[5]{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*Sin[c + d*x])^2/Sin[c + d*x]^(6/5),x]
```

```
[Out] (-5*Cos[c + d*x])/(d*Sin[c + d*x]^(1/5)) - (4*Cos[c + d*x]*Hypergeometric2F
1[1/2, 3/5, 3/2, Cos[c + d*x]^2]*Sin[c + d*x]^(4/5))/(d*(Sin[c + d*x]^2)^(2
/5))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(4\cos\left(dx+c\right)^2-4\sin\left(dx+c\right)-5\right)\sin\left(dx+c\right)^{\frac{4}{5}}}{\cos\left(dx+c\right)^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*sin(d*x+c))^2/sin(d*x+c)^(6/5),x, algorithm="fricas")
```

```
[Out] integral((4*cos(d*x + c)^2 - 4*sin(d*x + c) - 5)*sin(d*x + c)^(4/5)/(cos(d*
x + c)^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\sin\left(dx+c\right)+1\right)^2}{\sin\left(dx+c\right)^{\frac{6}{5}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*sin(d*x+c))^2/sin(d*x+c)^(6/5),x, algorithm="giac")
```

```
[Out] integrate((2*sin(d*x + c) + 1)^2/sin(d*x + c)^(6/5), x)
```

maple [F] time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{(1+2\sin(dx+c))^2}{\sin(dx+c)^{\frac{6}{5}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*sin(d*x+c))^2/sin(d*x+c)^(6/5),x)
```

```
[Out] int((1+2*sin(d*x+c))^2/sin(d*x+c)^(6/5),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2\sin(dx+c)+1)^2}{\sin(dx+c)^{\frac{6}{5}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*sin(d*x+c))^2/sin(d*x+c)^(6/5),x, algorithm="maxima")
```

```
[Out] integrate((2*sin(d*x + c) + 1)^2/sin(d*x + c)^(6/5), x)
```

mupad [B] time = 7.47, size = 127, normalized size = 1.74

$$-\frac{4\cos(c+dx)\sin(c+dx)^{4/5}\,{}_2F_1\left(\frac{1}{2},\frac{3}{5};\ \frac{3}{2};\ \cos(c+dx)^2\right)}{d\left(\sin(c+dx)^2\right)^{2/5}}-\frac{\cos(c+dx)\left(\sin(c+dx)^2\right)^{1/10}\,{}_2F_1\left(\frac{1}{2},\frac{11}{10};\ \frac{3}{2};\ \cos(\right.}{d\sin(c+dx)^{1/5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*sin(c + d*x) + 1)^2/sin(c + d*x)^(6/5),x)
```

```
[Out] - (4*cos(c + d*x)*sin(c + d*x)^(4/5)*hypergeom([1/2, 3/5], 3/2, cos(c + d*x
)^2))/(d*(sin(c + d*x)^2)^(2/5)) - (cos(c + d*x)*(sin(c + d*x)^2)^(1/10)*hy
pergeom([1/2, 11/10], 3/2, cos(c + d*x)^2))/(d*sin(c + d*x)^(1/5)) - (4*cos
(c + d*x)*sin(c + d*x)^(9/5)*hypergeom([1/10, 1/2], 3/2, cos(c + d*x)^2))/(
d*(sin(c + d*x)^2)^(9/10))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*sin(d*x+c))**2/sin(d*x+c)**(6/5),x)
```

```
[Out] Timed out
```

3.221 $\int \sin^m(c+dx)(a+b\sin(c+dx))^n\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\sin^m(c+dx)(a+b\sin(c+dx))^n,x\right)$$

```
[Out] Unintegrable(sin(d*x+c)^m*(a+b*sin(d*x+c))^n,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sin^m(c+dx)(a+b\sin(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[c + d*x]^m*(a + b*Sin[c + d*x])^n,x]
```

```
[Out] Defer[Int][Sin[c + d*x]^m*(a + b*Sin[c + d*x])^n, x]
```

Rubi steps

$$\int \sin^m(c+dx)(a+b\sin(c+dx))^n\,dx = \int \sin^m(c+dx)(a+b\sin(c+dx))^n\,dx$$

Mathematica [A] time = 2.37, size = 0, normalized size = 0.00

$$\int \sin^m(c+dx)(a+b\sin(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[c + d*x]^m*(a + b*Sin[c + d*x])^n,x]
```

```
[Out] Integrate[Sin[c + d*x]^m*(a + b*Sin[c + d*x])^n, x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sin(dx+c)+a)^n\sin(dx+c)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c) + a)^n*sin(d*x + c)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^n \sin(dx+c)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*sin(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^n*sin(d*x + c)^m, x)
```

maple [A] time = 1.19, size = 0, normalized size = 0.00

$$\int (\sin^m(dx+c))(a+b\sin(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^m*(a+b*sin(d*x+c))^n,x)
[Out] int(sin(d*x+c)^m*(a+b*sin(d*x+c))^n,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^n \sin(dx+c)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^m*(a+b*sin(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^n*sin(d*x + c)^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \sin(c+d\,x)^m\,(a+b\,\sin(c+d\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^m*(a + b*sin(c + d*x))^n,x)
[Out] int(sin(c + d*x)^m*(a + b*sin(c + d*x))^n, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))^n \sin^m(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**m*(a+b*sin(d*x+c))**n,x)
[Out] Integral((a + b*sin(c + d*x))**n*sin(c + d*x)**m, x)
```

3.222 $\int \sin^3(c+dx)(a+b\sin(c+dx))^n\,dx$

Optimal. Leaf size=351

$$\frac{\sqrt{2}\,a\left(2a^2+b^2\left(n^2+5n+4\right)\right)\cos(c+dx)(a+b\sin(c+dx))^n\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\right.}{b^3d(n+2)(n+3)\sqrt{\sin(c+dx)+1}}$$

```
[Out] 2*a*cos(d*x+c)*(a+b*sin(d*x+c))^(1+n)/b^2/d/(2+n)/(3+n)-cos(d*x+c)*sin(d*x+
c)*(a+b*sin(d*x+c))^(1+n)/b/d/(3+n)-(a+b)*(2*a^2+b^2*(2+n)^2)*AppellF1(1/2,
-1-n,1/2,3/2,b*(1-sin(d*x+c))/(a+b),1/2-1/2*sin(d*x+c))*cos(d*x+c)*(a+b*sin
(d*x+c))^n*2^(1/2)/b^3/d/(2+n)/(3+n)/(((a+b*sin(d*x+c))/(a+b))^n)/(1+sin(d*
x+c))^(1/2)+a*(2*a^2+b^2*(n^2+5*n+4))*AppellF1(1/2,-n,1/2,3/2,b*(1-sin(d*x+
c))/(a+b),1/2-1/2*sin(d*x+c))*cos(d*x+c)*(a+b*sin(d*x+c))^n*2^(1/2)/b^3/d/(
2+n)/(3+n)/(((a+b*sin(d*x+c))/(a+b))^n)/(1+sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2793, 3023, 2756, 2665, 139, 138}

$$\frac{\sqrt{2}\,a\left(2a^2+b^2\left(n^2+5n+4\right)\right)\cos(c+dx)(a+b\sin(c+dx))^n\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\right.}{b^3d(n+2)(n+3)\sqrt{\sin(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + b*Sin[c + d*x])^n,x]
```

```
[Out] (2*a*Cos[c + d*x]*(a + b*Sin[c + d*x])^(1 + n))/(b^2*d*(2 + n)*(3 + n)) - (
Cos[c + d*x]*Sin[c + d*x]*(a + b*Sin[c + d*x])^(1 + n))/(b*d*(3 + n)) - (Sq
rt[2]*(a + b)*(2*a^2 + b^2*(2 + n)^2)*AppellF1[1/2, 1/2, -1 - n, 3/2, (1 -
Sin[c + d*x])/2, (b*(1 - Sin[c + d*x]))/(a + b)]*Cos[c + d*x]*(a + b*Sin[c
+ d*x])^n)/(b^3*d*(2 + n)*(3 + n)*Sqrt[1 + Sin[c + d*x]]*((a + b*Sin[c + d*
x])/(a + b))^n) + (Sqrt[2]*a*(2*a^2 + b^2*(4 + 5*n + n^2))*AppellF1[1/2, 1/
2, -n, 3/2, (1 - Sin[c + d*x])/2, (b*(1 - Sin[c + d*x]))/(a + b)]*Cos[c + d
*x]*(a + b*Sin[c + d*x])^n)/(b^3*d*(2 + n)*(3 + n)*Sqrt[1 + Sin[c + d*x]]*(
(a + b*Sin[c + d*x])/(a + b))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
```

```
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 2756

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
```

```
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \sin^3(c+dx)(a+b\sin(c+dx))^n\,dx = -\frac{\cos(c+dx)\sin(c+dx)(a+b\sin(c+dx))^{1+n}}{bd(3+n)} + \frac{\int (a+b\sin(c+dx))^n}{}$$

$$= \frac{2a\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{b^2d(2+n)(3+n)} - \frac{\cos(c+dx)\sin(c+dx)(a+b\,\mathrm{si}}{bd(3+n)}$$

$$= \frac{2a\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{b^2d(2+n)(3+n)} - \frac{\cos(c+dx)\sin(c+dx)(a+b\,\mathrm{si}}{bd(3+n)}$$

$$= \frac{2a\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{b^2d(2+n)(3+n)} - \frac{\cos(c+dx)\sin(c+dx)(a+b\,\mathrm{si}}{bd(3+n)}$$

$$= \frac{2a\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{b^2d(2+n)(3+n)} - \frac{\cos(c+dx)\sin(c+dx)(a+b\,\mathrm{si}}{bd(3+n)}$$

$$= \frac{2a\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{b^2d(2+n)(3+n)} - \frac{\cos(c+dx)\sin(c+dx)(a+b\,\mathrm{si}}{bd(3+n)}$$

Mathematica [F] time = 4.14, size = 0, normalized size = 0.00

$$\int \sin^3(c+dx)(a+b\sin(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[c + d*x]^3*(a + b*Sin[c + d*x])^n,x]
[Out] Integrate[Sin[c + d*x]^3*(a + b*Sin[c + d*x])^n, x]
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(\cos\left(dx+c\right)^2-1\right)\left(b\sin\left(dx+c\right)+a\right)^n\sin\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*(b*sin(d*x + c) + a)^n*sin(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^n \sin(dx+c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

maple [F] time = 0.78, size = 0, normalized size = 0.00

$$\int \left(\sin^3(dx+c)\right)(a+b\sin(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+b*sin(d*x+c))^n,x)
```

```
[Out] int(sin(d*x+c)^3*(a+b*sin(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^n \sin(dx+c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sin(c+d\,x)^3\,(a+b\,\sin(c+d\,x))^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + b*sin(c + d*x))^n,x)
```

```
[Out] int(sin(c + d*x)^3*(a + b*sin(c + d*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+b*sin(d*x+c))**n,x)
```

```
[Out] Timed out
```

3.223 $\int \sin^2(c+dx)(a+b\sin(c+dx))^n\,dx$

Optimal. Leaf size=274

$$-\frac{\sqrt{2}\left(a^2+b^2(n+1)\right)\cos(c+dx)(a+b\sin(c+dx))^n\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+a}{a+b}\right.}{b^2d(n+2)\sqrt{\sin(c+dx)+1}}$$

```
[Out] -cos(d*x+c)*(a+b*sin(d*x+c))^(1+n)/b/d/(2+n)+a*(a+b)*AppellF1(1/2,-1-n,1/2,
3/2,b*(1-sin(d*x+c))/(a+b),1/2-1/2*sin(d*x+c))*cos(d*x+c)*(a+b*sin(d*x+c))^
n*2^(1/2)/b^2/d/(2+n)/(((a+b*sin(d*x+c))/(a+b))^n)/(1+sin(d*x+c))^(1/2)-(a^
2+b^2*(1+n))*AppellF1(1/2,-n,1/2,3/2,b*(1-sin(d*x+c))/(a+b),1/2-1/2*sin(d*x
+c))*cos(d*x+c)*(a+b*sin(d*x+c))^n*2^(1/2)/b^2/d/(2+n)/(((a+b*sin(d*x+c))/(
a+b))^n)/(1+sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2791, 2756, 2665, 139, 138}

$$-\frac{\sqrt{2}\left(a^2+b^2(n+1)\right)\cos(c+dx)(a+b\sin(c+dx))^n\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+a}{a+b}\right.}{b^2d(n+2)\sqrt{\sin(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + b*Sin[c + d*x])^n,x]
```

```
[Out] -((Cos[c + d*x]*(a + b*Sin[c + d*x])^(1 + n))/(b*d*(2 + n))) + (Sqrt[2]*a*(
a + b)*AppellF1[1/2, 1/2, -1 - n, 3/2, (1 - Sin[c + d*x])/2, (b*(1 - Sin[c
+ d*x]))/(a + b)]*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(b^2*d*(2 + n)*Sqrt[
1 + Sin[c + d*x]]*((a + b*Sin[c + d*x])/(a + b))^n) - (Sqrt[2]*(a^2 + b^2*(
1 + n))*AppellF1[1/2, 1/2, -n, 3/2, (1 - Sin[c + d*x])/2, (b*(1 - Sin[c + d
*x]))/(a + b)]*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(b^2*d*(2 + n)*Sqrt[1 +
 Sin[c + d*x]]*((a + b*Sin[c + d*x])/(a + b))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 2756

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] && !LtQ[m, -1]
```

Rubi steps

$$\int \sin^2(c+dx)(a+b\sin(c+dx))^n\,dx = -\frac{\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{bd(2+n)} + \frac{\int (b(1+n)-a\sin(c+dx))(a+b\,\mathrm{s}}{b(2+n)}$$

$$= -\frac{\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{bd(2+n)} - \frac{a\int (a+b\sin(c+dx))^{1+n}\,dx}{b^2(2+n)} + \frac{(a^2}{}$$

$$= -\frac{\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{bd(2+n)} - \frac{(a\cos(c+dx))\,\mathrm{Subst}\left(\int \frac{(a+bx)^{1+n}}{\sqrt{1-x}\,\sqrt{1+x}}\right.}{b^2d(2+n)\sqrt{1-\sin(c+dx)}\,\sqrt{1}}$$

$$= -\frac{\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{bd(2+n)} + \frac{\left(a(-a-b)\cos(c+dx)(a+b\sin(c\right.}{b^2d(2}$$

$$= -\frac{\cos(c+dx)(a+b\sin(c+dx))^{1+n}}{bd(2+n)} + \frac{\sqrt{2}\,a(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-1-n;\frac{3}{2};\frac{1}{2}(1\right.}{}$$

Mathematica [F] time = 6.81, size = 0, normalized size = 0.00

$$\int \sin^2(c+dx)(a+b\sin(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Sin[c + d*x])^n,x]
[Out] Integrate[Sin[c + d*x]^2*(a + b*Sin[c + d*x])^n, x]
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(\cos\left(dx+c\right)^2-1\right)\left(b\sin\left(dx+c\right)+a\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*sin(d*x+c))^n,x, algorithm="fricas")
[Out] integral(-(cos(d*x + c)^2 - 1)*(b*sin(d*x + c) + a)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)+a\right)^n \sin\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*sin(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

maple [F] time = 0.73, size = 0, normalized size = 0.00

$$\int \left(\sin^{2}\left(dx+c\right)\right)\left(a+b\sin\left(dx+c\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+b*sin(d*x+c))^n,x)
[Out] int(sin(d*x+c)^2*(a+b*sin(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)+a\right)^{n}\sin\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*sin(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(c+d\,x\right)^{2}\,\left(a+b\,\sin\left(c+d\,x\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + b*sin(c + d*x))^n,x)
[Out] int(sin(c + d*x)^2*(a + b*sin(c + d*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+b*sin(d*x+c))**n,x)
[Out] Timed out
```

3.224 $\int \sin(c+dx)(a+b\sin(c+dx))^n\,dx$

Optimal. Leaf size=220

$$\frac{\sqrt{2}\,a\cos(c+dx)(a+b\sin(c+dx))^n\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)}{bd\sqrt{\sin(c+dx)+1}}-\frac{\sqrt{2}\,(a+b)\,\cdots}{\cdots}$$

```
[Out] -(a+b)*AppellF1(1/2,-1-n,1/2,3/2,b*(1-sin(d*x+c))/(a+b),1/2-1/2*sin(d*x+c))
*cos(d*x+c)*(a+b*sin(d*x+c))^n*2^(1/2)/b/d/(((a+b*sin(d*x+c))/(a+b))^n)/(1+
sin(d*x+c))^(1/2)+a*AppellF1(1/2,-n,1/2,3/2,b*(1-sin(d*x+c))/(a+b),1/2-1/2*
sin(d*x+c))*cos(d*x+c)*(a+b*sin(d*x+c))^n*2^(1/2)/b/d/(((a+b*sin(d*x+c))/(a
+b))^n)/(1+sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2756, 2665, 139, 138}

$$\frac{\sqrt{2}\,a\cos(c+dx)(a+b\sin(c+dx))^n\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)}{bd\sqrt{\sin(c+dx)+1}}-\frac{\sqrt{2}\,(a+b)\,\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Sin[c + d*x])^n,x]
```

```
[Out] -((Sqrt[2]*(a + b)*AppellF1[1/2, 1/2, -1 - n, 3/2, (1 - Sin[c + d*x])/2, (b
*(1 - Sin[c + d*x]))/(a + b)]*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(b*d*Sqr
t[1 + Sin[c + d*x]]*((a + b*Sin[c + d*x])/(a + b))^n)) + (Sqrt[2]*a*AppellF
1[1/2, 1/2, -n, 3/2, (1 - Sin[c + d*x])/2, (b*(1 - Sin[c + d*x]))/(a + b)]*
Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(b*d*Sqrt[1 + Sin[c + d*x]]*((a + b*Si
n[c + d*x])/(a + b))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 2756

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sin(c+dx)(a+b\sin(c+dx))^n\,dx = \frac{\int (a+b\sin(c+dx))^{1+n}\,dx}{b} - \frac{a\int (a+b\sin(c+dx))^n\,dx}{b}$$

$$= \frac{\cos(c+dx)\,\text{Subst}\left(\int \frac{(a+bx)^{1+n}}{\sqrt{1-x}\,\sqrt{1+x}}\,dx, x, \sin(c+dx)\right)}{bd\sqrt{1-\sin(c+dx)}\,\sqrt{1+\sin(c+dx)}} - \frac{(a\cos(c+dx))\,\text{Sul}}{bd\sqrt{1-\text{si}}}$$

$$= -\frac{\left(a\cos(c+dx)(a+b\sin(c+dx))^n\left(-\frac{a+b\sin(c+dx)}{-a-b}\right)^{-n}\right)\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\right.}{\sqrt{1-x}\,\sqrt{}}\right.}{bd\sqrt{1-\sin(c+dx)}\,\sqrt{1+\sin(c+dx)}}$$

$$= -\frac{\sqrt{2}\,(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-1-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)\cos(c+}{bd\sqrt{1+\sin(c+dx)}}$$

Mathematica [A] time = 0.49, size = 193, normalized size = 0.88

$$\frac{\sec(c+dx)\sqrt{-\frac{b(\sin(c+dx)-1)}{a+b}}\,\sqrt{\frac{b(\sin(c+dx)+1)}{b-a}}\,(a+b\sin(c+dx))^{n+1}\left((n+1)(a+b\sin(c+dx))F_1\left(n+2;\frac{1}{2},\frac{1}{2};n+3\right.\right.}{b^2d(n+1)(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]*(a + b*Sin[c + d*x])^n,x]
```

```
[Out] (Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Sin[c +
 d*x]))/(-a + b)]*(a + b*Sin[c + d*x])^(1 + n)*(-(a*(2 + n)*AppellF1[1 + n,
 1/2, 1/2, 2 + n, (a + b*Sin[c + d*x])/(a - b), (a + b*Sin[c + d*x])/(a + b
)]) + (1 + n)*AppellF1[2 + n, 1/2, 1/2, 3 + n, (a + b*Sin[c + d*x])/(a - b)
, (a + b*Sin[c + d*x])/(a + b)]*(a + b*Sin[c + d*x])))/(b^2*d*(1 + n)*(2 +
n))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sin(dx+c)+a)^n\sin(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c) + a)^n*sin(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^n\sin(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^n*sin(d*x + c), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \sin(dx+c)\,(a+b\sin(dx+c))^n\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+b*sin(d*x+c))^n,x)
```

```
[Out] int(sin(d*x+c)*(a+b*sin(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^n\sin(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^n*sin(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sin(c+dx)\,(a+b\sin(c+dx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + b*sin(c + d*x))^n,x)
```

```
[Out] int(sin(c + d*x)*(a + b*sin(c + d*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c))**n,x)
```

```
[Out] Timed out
```

3.225 $\int (a+b\sin(c+dx))^n\,dx$

Optimal. Leaf size=104

$$-\frac{\sqrt{2}\cos(c+dx)(a+b\sin(c+dx))^n\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)}{d\sqrt{\sin(c+dx)+1}}$$

```
[Out] -AppellF1(1/2,-n,1/2,3/2,b*(1-sin(d*x+c))/(a+b),1/2-1/2*sin(d*x+c))*cos(d*x
+c)*(a+b*sin(d*x+c))^n*2^(1/2)/d/(((a+b*sin(d*x+c))/(a+b))^n)/(1+sin(d*x+c)
)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2665, 139, 138}

$$-\frac{\sqrt{2}\cos(c+dx)(a+b\sin(c+dx))^n\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)}{d\sqrt{\sin(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^n,x]
```

```
[Out] -((Sqrt[2]*AppellF1[1/2, 1/2, -n, 3/2, (1 - Sin[c + d*x])/2, (b*(1 - Sin[c
+ d*x]))/(a + b)]*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(d*Sqrt[1 + Sin[c +
d*x]]*((a + b*Sin[c + d*x])/(a + b))^n))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (a+b\sin(c+dx))^n\,dx = \frac{\cos(c+dx)\,\text{Subst}\left(\int \frac{(a+bx)^n}{\sqrt{1-x}\,\sqrt{1+x}}\,dx, x, \sin(c+dx)\right)}{d\sqrt{1-\sin(c+dx)}\,\sqrt{1+\sin(c+dx)}}$$

$$= \frac{\left(\cos(c+dx)(a+b\sin(c+dx))^n\left(-\frac{a+b\sin(c+dx)}{-a-b}\right)^{-n}\right)\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^n}{\sqrt{1-x}\,\sqrt{1+x}}\,dx, x, \sin(\right.}{d\sqrt{1-\sin(c+dx)}\,\sqrt{1+\sin(c+dx)}}$$

$$= -\frac{\sqrt{2}\,F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(c+dx)),\frac{b(1-\sin(c+dx))}{a+b}\right)\cos(c+dx)(a+b\sin(c+dx))^n}{d\sqrt{1+\sin(c+dx)}}$$

Mathematica [A] time = 0.23, size = 120, normalized size = 1.15

$$\frac{\sec(c+dx)\sqrt{-\frac{b(\sin(c+dx)-1)}{a+b}}\sqrt{\frac{b(\sin(c+dx)+1)}{b-a}}\,(a+b\sin(c+dx))^{n+1}F_1\left(n+1;\frac{1}{2},\frac{1}{2};n+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(n+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[c + d*x])^n,x]
```

```
[Out] (AppellF1[1 + n, 1/2, 1/2, 2 + n, (a + b*Sin[c + d*x])/(a - b), (a + b*Sin[
c + d*x])/(a + b)]*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sq
rt[(b*(1 + Sin[c + d*x]))/(-a + b)]*(a + b*Sin[c + d*x])^(1 + n))/(b*d*(1 +
 n))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sin(dx+c)+a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c) + a)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^n, x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int (a+b\sin(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^n,x)
```

```
[Out] int((a+b*sin(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin(dx+c)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\sin(c+d\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^n,x)
```

```
[Out] int((a + b*sin(c + d*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**n,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**n, x)
```

3.226 $\int \csc(c+dx)(a+b\sin(c+dx))^n\,dx$

Optimal. Leaf size=22

$$\text{Int}\left(\csc(c+dx)(a+b\sin(c+dx))^n,x\right)$$

```
[Out] Unintegrable(csc(d*x+c)*(a+b*sin(d*x+c))^n,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \csc(c+dx)(a+b\sin(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csc[c + d*x]*(a + b*Sin[c + d*x])^n,x]
[Out] Defer[Int][Csc[c + d*x]*(a + b*Sin[c + d*x])^n, x]
```

Rubi steps

$$\int \csc(c+dx)(a+b\sin(c+dx))^n\,dx=\int \csc(c+dx)(a+b\sin(c+dx))^n\,dx$$

Mathematica [A] time = 2.44, size = 0, normalized size = 0.00

$$\int \csc(c+dx)(a+b\sin(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[c + d*x]*(a + b*Sin[c + d*x])^n,x]
[Out] Integrate[Csc[c + d*x]*(a + b*Sin[c + d*x])^n, x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sin(dx+c)+a)^n\csc(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c) + a)^n*csc(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sin(dx+c)+a)^n \csc(dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^n*csc(d*x + c), x)
```

maple [A] time = 1.37, size = 0, normalized size = 0.00

$$\int \csc(dx+c)(a+b\sin(dx+c))^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*sin(d*x+c))^n,x)
[Out] int(csc(d*x+c)*(a+b*sin(d*x+c))^n,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sin(dx+c)+a)^n \csc(dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^n*csc(d*x + c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(a+b\,\sin(c+d\,x))^n}{\sin(c+d\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^n/sin(c + d*x),x)
[Out] int((a + b*sin(c + d*x))^n/sin(c + d*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))^n \csc(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c))**n,x)
[Out] Integral((a + b*sin(c + d*x))**n*csc(c + d*x), x)
```

3.227 $\int (a + a\sin(e + fx))(c - c\sin(e + fx))^4\,dx$

Optimal. Leaf size=116

$$\frac{7ac^4\cos^3(e+fx)}{12f}+\frac{7a\cos^3(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{20f}+\frac{7ac^4\sin(e+fx)\cos(e+fx)}{8f}+\frac{7}{8}ac^4x+\frac{a\cos^3(e+fx)\left(c\right.}{}$$

```
[Out] 7/8*a*c^4*x+7/12*a*c^4*cos(f*x+e)^3/f+7/8*a*c^4*cos(f*x+e)*sin(f*x+e)/f+1/5
*a*cos(f*x+e)^3*(c^2-c^2*sin(f*x+e))^2/f+7/20*a*cos(f*x+e)^3*(c^4-c^4*sin(f
*x+e))/f
```

Rubi [A] time = 0.16, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2736, 2678, 2669, 2635, 8}

$$\frac{7ac^4\cos^3(e+fx)}{12f}+\frac{a\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{5f}+\frac{7a\cos^3(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{20f}+\frac{7ac^4\sin(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (7*a*c^4*x)/8 + (7*a*c^4*Cos[e + f*x]^3)/(12*f) + (7*a*c^4*Cos[e + f*x]*Sin
[e + f*x])/(8*f) + (a*Cos[e + f*x]^3*(c^2 - c^2*Sin[e + f*x])^2)/(5*f) + (7
*a*Cos[e + f*x]^3*(c^4 - c^4*Sin[e + f*x]))/(20*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))(c-c\sin(e+fx))^4\,dx = (ac)\int \cos^2(e+fx)(c-c\sin(e+fx))^3\,dx$$
$$= \frac{a\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{5f}+\frac{1}{5}\left(7ac^2\right)\int \cos^2(e+fx)(c$$
$$= \frac{a\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{5f}+\frac{7a\cos^3(e+fx)\left(c^4-c^4\,\mathrm{si}\right.}{20f}$$
$$= \frac{7ac^4\cos^3(e+fx)}{12f}+\frac{a\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{5f}+\frac{7a\,\mathrm{co}}{}$$
$$= \frac{7ac^4\cos^3(e+fx)}{12f}+\frac{7ac^4\cos(e+fx)\sin(e+fx)}{8f}+\frac{a\cos^3(e+f}{}$$
$$= \frac{7}{8}ac^4x+\frac{7ac^4\cos^3(e+fx)}{12f}+\frac{7ac^4\cos(e+fx)\sin(e+fx)}{8f}+\frac{a\,\mathrm{co}}{}$$

Mathematica [A] time = 0.56, size = 64, normalized size = 0.55

$$\frac{ac^4(120\sin(2(e+fx))-45\sin(4(e+fx))+420\cos(e+fx)+130\cos(3(e+fx))-6\cos(5(e+fx))+420fx)}{480f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (a*c^4*(420*f*x + 420*Cos[e + f*x] + 130*Cos[3*(e + f*x)] - 6*Cos[5*(e + f*
x)] + 120*Sin[2*(e + f*x)] - 45*Sin[4*(e + f*x)]))/(480*f)
```

fricas [A] time = 0.47, size = 77, normalized size = 0.66

$$-\frac{24\,ac^4\cos\left(fx+e\right)^5-160\,ac^4\cos\left(fx+e\right)^3-105\,ac^4fx+15\left(6\,ac^4\cos\left(fx+e\right)^3-7\,ac^4\cos\left(fx+e\right)\right)\sin\left(fx\right.}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] -1/120*(24*a*c^4*cos(f*x + e)^5 - 160*a*c^4*cos(f*x + e)^3 - 105*a*c^4*f*x
+ 15*(6*a*c^4*cos(f*x + e)^3 - 7*a*c^4*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.43, size = 100, normalized size = 0.86

$$\frac{7}{8}\,ac^4x-\frac{ac^4\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{13\,ac^4\cos\left(3\,fx+3\,e\right)}{48\,f}+\frac{7\,ac^4\cos\left(fx+e\right)}{8\,f}-\frac{3\,ac^4\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{ac^4\sin\left(2\,fx+2\right.}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 7/8*a*c^4*x - 1/80*a*c^4*cos(5*f*x + 5*e)/f + 13/48*a*c^4*cos(3*f*x + 3*e)/
f + 7/8*a*c^4*cos(f*x + e)/f - 3/32*a*c^4*sin(4*f*x + 4*e)/f + 1/4*a*c^4*si
n(2*f*x + 2*e)/f
```

maple [A] time = 0.28, size = 149, normalized size = 1.28

$$\frac{-\frac{a\,c^4\left(\frac{8}{3}+\sin^4\left(fx+e\right)+\frac{4\left(\sin^2\left(fx+e\right)\right)}{3}\right)\cos\left(fx+e\right)}{5}-3a\,c^4\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{2a\,c^4\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right.}{3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^4,x)
```

```
[Out] 1/f*(-1/5*a*c^4*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-3*a*c^4*(-1/
4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2/3*a*c^4*(2+sin(
f*x+e)^2)*cos(f*x+e)+2*a*c^4*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+3*a
*c^4*cos(f*x+e)+a*c^4*(f*x+e))
```

maxima [A] time = 0.87, size = 146, normalized size = 1.26

$$-\frac{32\left(3\cos\left(fx+e\right)^{5}-10\cos\left(fx+e\right)^{3}+15\cos\left(fx+e\right)\right)ac^{4}-320\left(\cos\left(fx+e\right)^{3}-3\cos\left(fx+e\right)\right)ac^{4}+45}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] -1/480*(32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a*c^4 -
 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*c^4 + 45*(12*f*x + 12*e + sin(4*f*
x + 4*e) - 8*sin(2*f*x + 2*e))*a*c^4 - 240*(2*f*x + 2*e - sin(2*f*x + 2*e))
*a*c^4 - 480*(f*x + e)*a*c^4 - 1440*a*c^4*cos(f*x + e))/f
```

mupad [B] time = 8.86, size = 292, normalized size = 2.52

$$\frac{7\,a\,c^4\,x}{8}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{a\,c^4\,\left(105\,e+105\,f\,x\right)}{24}-\frac{a\,c^4\,\left(525\,e+525\,f\,x+640\right)}{120}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8\,\left(\frac{a\,c^4\,\left(105\,e+105\,f\,x\right)}{24}-\frac{a\,c^4\,\left(525\,e+525\right.}{120}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^4,x)
```

```
[Out] (7*a*c^4*x)/8 - (tan(e/2 + (f*x)/2)^2*((a*c^4*(105*e + 105*f*x))/24 - (a*c^
4*(525*e + 525*f*x + 640))/120) + tan(e/2 + (f*x)/2)^8*((a*c^4*(105*e + 105
*f*x))/24 - (a*c^4*(525*e + 525*f*x + 720))/120) + tan(e/2 + (f*x)/2)^4*((a
*c^4*(105*e + 105*f*x))/12 - (a*c^4*(1050*e + 1050*f*x + 800))/120) + tan(e
/2 + (f*x)/2)^6*((a*c^4*(105*e + 105*f*x))/12 - (a*c^4*(1050*e + 1050*f*x +
 1920))/120) - (a*c^4*tan(e/2 + (f*x)/2))/4 - (13*a*c^4*tan(e/2 + (f*x)/2)^
3)/2 + (13*a*c^4*tan(e/2 + (f*x)/2)^7)/2 + (a*c^4*tan(e/2 + (f*x)/2)^9)/4 +
 (a*c^4*(105*e + 105*f*x))/120 - (a*c^4*(105*e + 105*f*x + 272))/120)/(f*(t
an(e/2 + (f*x)/2)^2 + 1)^5)
```

sympy [A] time = 3.64, size = 314, normalized size = 2.71

$$\begin{cases} -\frac{9ac^4x\sin^4\left(e+fx\right)}{8}-\frac{9ac^4x\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4}+ac^4x\sin^2\left(e+fx\right)-\frac{9ac^4x\cos^4\left(e+fx\right)}{8}+ac^4x\cos^2\left(e+fx\right)+ac^4x- \\ x\left(a\sin\left(e\right)+a\right)\left(-c\sin\left(e\right)+c\right)^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-9*a*c**4*x*sin(e + f*x)**4/8 - 9*a*c**4*x*sin(e + f*x)**2*cos(e
 + f*x)**2/4 + a*c**4*x*sin(e + f*x)**2 - 9*a*c**4*x*cos(e + f*x)**4/8 + a*
c**4*x*cos(e + f*x)**2 + a*c**4*x - a*c**4*sin(e + f*x)**4*cos(e + f*x)/f +
 15*a*c**4*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*a*c**4*sin(e + f*x)**2*co
s(e + f*x)**3/(3*f) - 2*a*c**4*sin(e + f*x)**2*cos(e + f*x)/f + 9*a*c**4*si
n(e + f*x)*cos(e + f*x)**3/(8*f) - a*c**4*sin(e + f*x)*cos(e + f*x)/f - 8*a
*c**4*cos(e + f*x)**5/(15*f) - 4*a*c**4*cos(e + f*x)**3/(3*f) + 3*a*c**4*co
s(e + f*x)/f, Ne(f, 0)), (x*(a*sin(e) + a)*(-c*sin(e) + c)**4, True))
```

3.228 $\int (a+a\sin(e+fx))(c-c\sin(e+fx))^3\,dx$

Optimal. Leaf size=83

$$\frac{5ac^3\cos^3(e+fx)}{12f}+\frac{a\cos^3(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{4f}+\frac{5ac^3\sin(e+fx)\cos(e+fx)}{8f}+\frac{5}{8}ac^3x$$

```
[Out] 5/8*a*c^3*x+5/12*a*c^3*cos(f*x+e)^3/f+5/8*a*c^3*cos(f*x+e)*sin(f*x+e)/f+1/4
*a*cos(f*x+e)^3*(c^3-c^3*sin(f*x+e))/f
```

Rubi [A] time = 0.11, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2736, 2678, 2669, 2635, 8}

$$\frac{5ac^3\cos^3(e+fx)}{12f}+\frac{a\cos^3(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{4f}+\frac{5ac^3\sin(e+fx)\cos(e+fx)}{8f}+\frac{5}{8}ac^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (5*a*c^3*x)/8 + (5*a*c^3*Cos[e + f*x]^3)/(12*f) + (5*a*c^3*Cos[e + f*x]*Sin
[e + f*x])/(8*f) + (a*Cos[e + f*x]^3*(c^3 - c^3*Sin[e + f*x]))/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
```

```
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))(c-c\sin(e+fx))^3\,dx = (ac)\int \cos^2(e+fx)(c-c\sin(e+fx))^2\,dx$$
$$= \frac{a\cos^3(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{4f}+\frac{1}{4}\left(5ac^2\right)\int \cos^2(e+fx)(c-$$
$$= \frac{5ac^3\cos^3(e+fx)}{12f}+\frac{a\cos^3(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{4f}+\frac{1}{4}\left(5ac^3\right.$$
$$= \frac{5ac^3\cos^3(e+fx)}{12f}+\frac{5ac^3\cos(e+fx)\sin(e+fx)}{8f}+\frac{a\cos^3(e+fx)}{}$$
$$= \frac{5}{8}ac^3x+\frac{5ac^3\cos^3(e+fx)}{12f}+\frac{5ac^3\cos(e+fx)\sin(e+fx)}{8f}+\frac{a\cos}{}$$

Mathematica [A] time = 0.37, size = 54, normalized size = 0.65

$$\frac{ac^3(24\sin(2(e+fx))-3\sin(4(e+fx))+48\cos(e+fx)+16\cos(3(e+fx))+60fx)}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (a*c^3*(60*f*x + 48*Cos[e + f*x] + 16*Cos[3*(e + f*x)] + 24*Sin[2*(e + f*x)
] - 3*Sin[4*(e + f*x)]))/(96*f)
```

fricas [A] time = 0.46, size = 63, normalized size = 0.76

$$\frac{16\,ac^3\cos\left(fx+e\right)^3+15\,ac^3fx-3\left(2\,ac^3\cos\left(fx+e\right)^3-5\,ac^3\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/24*(16*a*c^3*cos(f*x + e)^3 + 15*a*c^3*f*x - 3*(2*a*c^3*cos(f*x + e)^3 -
5*a*c^3*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.19, size = 81, normalized size = 0.98

$$\frac{5}{8}\,ac^{3}x+\frac{ac^{3}\cos\left(3\,fx+3\,e\right)}{6\,f}+\frac{ac^{3}\cos\left(fx+e\right)}{2\,f}-\frac{ac^{3}\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{ac^{3}\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 5/8*a*c^3*x + 1/6*a*c^3*cos(3*f*x + 3*e)/f + 1/2*a*c^3*cos(f*x + e)/f - 1/3
2*a*c^3*sin(4*f*x + 4*e)/f + 1/4*a*c^3*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.23, size = 89, normalized size = 1.07

$$\frac{-a\,c^{3}\left(-\frac{\left(\sin^{3}\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{2a\,c^{3}\left(2+\sin^{2}\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}+2a\,c^{3}\cos\left(fx+e\right)+a\,c^{3}\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(-a*c^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2
/3*a*c^3*(2+sin(f*x+e)^2)*cos(f*x+e)+2*a*c^3*cos(f*x+e)+a*c^3*(f*x+e))
```

maxima [A] time = 1.71, size = 86, normalized size = 1.04

$$\frac{64\left(\cos\left(fx+e\right)^{3}-3\,\cos\left(fx+e\right)\right)ac^{3}-3\left(12\,fx+12\,e+\sin\left(4\,fx+4\,e\right)-8\,\sin\left(2\,fx+2\,e\right)\right)ac^{3}+96\left(fx+e\right.}{96\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/96*(64*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*c^3 - 3*(12*f*x + 12*e + sin(4
*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a*c^3 + 96*(f*x + e)*a*c^3 + 192*a*c^3*co
s(f*x + e))/f
```

mupad [B] time = 9.00, size = 250, normalized size = 3.01

$$\frac{5\,a\,c^3\,x}{8}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{a\,c^3\,\left(15\,e+15\,f\,x\right)}{6}-\frac{a\,c^3\,\left(60\,e+60\,f\,x+32\right)}{24}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,\left(\frac{a\,c^3\,\left(15\,e+15\,f\,x\right)}{6}-\frac{a\,c^3\,\left(60\,e+60\,f\,x+96\right)}{24}\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^3,x)
```

```
[Out] (5*a*c^3*x)/8 - (tan(e/2 + (f*x)/2)^2*((a*c^3*(15*e + 15*f*x))/6 - (a*c^3*(
60*e + 60*f*x + 32))/24) + tan(e/2 + (f*x)/2)^6*((a*c^3*(15*e + 15*f*x))/6
- (a*c^3*(60*e + 60*f*x + 96))/24) + tan(e/2 + (f*x)/2)^4*((a*c^3*(15*e + 1
5*f*x))/4 - (a*c^3*(90*e + 90*f*x + 96))/24) - (3*a*c^3*tan(e/2 + (f*x)/2))
/4 - (11*a*c^3*tan(e/2 + (f*x)/2)^3)/4 + (11*a*c^3*tan(e/2 + (f*x)/2)^5)/4
+ (3*a*c^3*tan(e/2 + (f*x)/2)^7)/4 + (a*c^3*(15*e + 15*f*x))/24 - (a*c^3*(1
5*e + 15*f*x + 32))/24)/(f*(tan(e/2 + (f*x)/2)^2 + 1)^4)
```

sympy [A] time = 1.37, size = 196, normalized size = 2.36

$$\begin{cases} -\frac{3ac^3x\sin^4\left(e+fx\right)}{8}-\frac{3ac^3x\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4}-\frac{3ac^3x\cos^4\left(e+fx\right)}{8}+ac^3x+\frac{5ac^3\sin^3\left(e+fx\right)\cos\left(e+fx\right)}{8f}-\frac{2ac^3\sin^2\left(e+fx\right)\cos\left(e+f\right.}{f} \\ x\left(a\sin\left(e\right)+a\right)\left(-c\sin\left(e\right)+c\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-3*a*c**3*x*sin(e + f*x)**4/8 - 3*a*c**3*x*sin(e + f*x)**2*cos(e
 + f*x)**2/4 - 3*a*c**3*x*cos(e + f*x)**4/8 + a*c**3*x + 5*a*c**3*sin(e + f
*x)**3*cos(e + f*x)/(8*f) - 2*a*c**3*sin(e + f*x)**2*cos(e + f*x)/f + 3*a*c
**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 4*a*c**3*cos(e + f*x)**3/(3*f) + 2
*a*c**3*cos(e + f*x)/f, Ne(f, 0)), (x*(a*sin(e) + a)*(-c*sin(e) + c)**3, Tr
ue))
```

3.229 $\int (a+a\sin(e+fx))(c-c\sin(e+fx))^2\,dx$

Optimal. Leaf size=52

$$\frac{ac^2\cos^3(e+fx)}{3f}+\frac{ac^2\sin(e+fx)\cos(e+fx)}{2f}+\frac{1}{2}ac^2x$$

```
[Out] 1/2*a*c^2*x+1/3*a*c^2*cos(f*x+e)^3/f+1/2*a*c^2*cos(f*x+e)*sin(f*x+e)/f
```

Rubi [A] time = 0.06, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2736, 2669, 2635, 8}

$$\frac{ac^2\cos^3(e+fx)}{3f}+\frac{ac^2\sin(e+fx)\cos(e+fx)}{2f}+\frac{1}{2}ac^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a*c^2*x)/2 + (a*c^2*Cos[e + f*x]^3)/(3*f) + (a*c^2*Cos[e + f*x]*Sin[e + f*
x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
```

```
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int (a+a\sin(e+fx))(c-c\sin(e+fx))^2\,dx &= (ac)\int \cos^2(e+fx)(c-c\sin(e+fx))\,dx \\
&= \frac{ac^2\cos^3(e+fx)}{3f}+\left(ac^2\right)\int \cos^2(e+fx)\,dx \\
&= \frac{ac^2\cos^3(e+fx)}{3f}+\frac{ac^2\cos(e+fx)\sin(e+fx)}{2f}+\frac{1}{2}\left(ac^2\right)\int 1\,dx \\
&= \frac{1}{2}ac^2x+\frac{ac^2\cos^3(e+fx)}{3f}+\frac{ac^2\cos(e+fx)\sin(e+fx)}{2f}
\end{aligned}$$

Mathematica [A] time = 0.28, size = 42, normalized size = 0.81

$$\frac{ac^2(3\sin(2(e+fx))+3\cos(e+fx)+\cos(3(e+fx))+6fx)}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a*c^2*(6*f*x + 3*Cos[e + f*x] + Cos[3*(e + f*x)] + 3*Sin[2*(e + f*x)]))/(1
2*f)
```

fricas [A] time = 0.44, size = 46, normalized size = 0.88

$$\frac{2\,ac^2\cos\left(fx+e\right)^3+3\,ac^2fx+3\,ac^2\cos\left(fx+e\right)\sin\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^2*cos(f*x + e)^3 + 3*a*c^2*f*x + 3*a*c^2*cos(f*x + e)*sin(f*x +
e))/f
```

giac [A] time = 0.19, size = 62, normalized size = 1.19

$$\frac{1}{2}\,ac^2x+\frac{ac^2\cos\left(3\,fx+3\,e\right)}{12\,f}+\frac{ac^2\cos\left(fx+e\right)}{4\,f}+\frac{ac^2\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*a*c^2*x + 1/12*a*c^2*cos(3*f*x + 3*e)/f + 1/4*a*c^2*cos(f*x + e)/f + 1/
4*a*c^2*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.16, size = 77, normalized size = 1.48

$$\frac{-\frac{a c^{2}\left(2+\sin^{2}\left(f x+e\right)\right) \cos\left(f x+e\right)}{3}-a c^{2} \left(-\frac{\sin\left(f x+e\right) \cos\left(f x+e\right)}{2}+\frac{f x}{2}+\frac{e}{2}\right)+a c^{2} \cos \left(f x+e\right)+a c^{2} \left(f x+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(-1/3*a*c^2*(2+sin(f*x+e)^2)*cos(f*x+e)-a*c^2*(-1/2*sin(f*x+e)*cos(f*x+
e)+1/2*f*x+1/2*e)+a*c^2*cos(f*x+e)+a*c^2*(f*x+e))
```

maxima [A] time = 0.59, size = 77, normalized size = 1.48

$$\frac{4\left(\cos\left(fx+e\right)^{3}-3\,\cos\left(fx+e\right)\right)ac^{2}-3\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)ac^{2}+12\left(fx+e\right)ac^{2}+12\,ac^{2}\cos\left(fx+\right.}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*c^2 - 3*(2*f*x + 2*e - sin(2*f*
x + 2*e))*a*c^2 + 12*(f*x + e)*a*c^2 + 12*a*c^2*cos(f*x + e))/f
```

mupad [B] time = 8.96, size = 125, normalized size = 2.40

$$\frac{a\,c^2\,x}{2}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(\frac{3\,a\,c^2\,\left(e+f\,x\right)}{2}-\frac{a\,c^2\,\left(9\,e+9\,f\,x+12\right)}{6}\right)-a\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\frac{a\,c^2\,\left(e+f\,x\right)}{2}+a\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5-\frac{a\,c^2\,(\ldots}{\ldots}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^2,x)
```

```
[Out] (a*c^2*x)/2 - (tan(e/2 + (f*x)/2)^4*((3*a*c^2*(e + f*x))/2 - (a*c^2*(9*e +
9*f*x + 12))/6) - a*c^2*tan(e/2 + (f*x)/2) + (a*c^2*(e + f*x))/2 + a*c^2*ta
```

```
n(e/2 + (f*x)/2)^5 - (a*c^2*(3*e + 3*f*x + 4))/6)/(f*(tan(e/2 + (f*x)/2)^2
+ 1)^3)
```

sympy [A] time = 0.90, size = 133, normalized size = 2.56

$$\begin{cases} -\frac{ac^2x\sin^2(e+fx)}{2} - \frac{ac^2x\cos^2(e+fx)}{2} + ac^2x - \frac{ac^2\sin^2(e+fx)\cos(e+fx)}{f} + \frac{ac^2\sin(e+fx)\cos(e+fx)}{2f} - \frac{2ac^2\cos^3(e+fx)}{3f} + \frac{ac^2\cos(e+f}{f} \\ x(a\sin(e)+a)(-c\sin(e)+c)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-a*c**2*x*sin(e + f*x)**2/2 - a*c**2*x*cos(e + f*x)**2/2 + a*c**
2*x - a*c**2*sin(e + f*x)**2*cos(e + f*x)/f + a*c**2*sin(e + f*x)*cos(e + f
*x)/(2*f) - 2*a*c**2*cos(e + f*x)**3/(3*f) + a*c**2*cos(e + f*x)/f, Ne(f, 0
)), (x*(a*sin(e) + a)*(-c*sin(e) + c)**2, True))
```

3.230 $\int (a+a\sin(e+fx))(c-c\sin(e+fx))\,dx$

Optimal. Leaf size=29

$$\frac{ac\sin(e+fx)\cos(e+fx)}{2f}+\frac{acx}{2}$$

```
[Out] 1/2*a*c*x+1/2*a*c*cos(f*x+e)*sin(f*x+e)/f
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2734}

$$\frac{ac\sin(e+fx)\cos(e+fx)}{2f}+\frac{acx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a*c*x)/2 + (a*c*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))(c-c\sin(e+fx))\,dx=\frac{acx}{2}+\frac{ac\cos(e+fx)\sin(e+fx)}{2f}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 0.86

$$\frac{ac(2(e+fx)+\sin(2(e+fx)))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a*c*(2*(e + f*x) + Sin[2*(e + f*x)]))/(4*f)
```

fricas [A] time = 0.43, size = 26, normalized size = 0.90

$$\frac{acfx + ac\cos\left(fx+e\right)\sin\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*c*f*x + a*c*cos(f*x + e)*sin(f*x + e))/f
```

giac [A] time = 0.17, size = 23, normalized size = 0.79

$$\frac{1}{2}\,acx + \frac{ac\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*a*c*x + 1/4*a*c*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.05, size = 40, normalized size = 1.38

$$\frac{-ca\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+ac\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] 1/f*(-c*a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+a*c*(f*x+e))
```

maxima [A] time = 0.51, size = 37, normalized size = 1.28

$$-\frac{\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)ac-4\left(fx+e\right)ac}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/4*((2*f*x + 2*e - sin(2*f*x + 2*e))*a*c - 4*(f*x + e)*a*c)/f
```

mupad [B] time = 7.15, size = 54, normalized size = 1.86

$$\frac{a\,c\,x}{2}-\frac{a\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3-a\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c - c*sin(e + f*x)),x)
```

```
[Out] (a*c*x)/2 - (a*c*tan(e/2 + (f*x)/2)^3 - a*c*tan(e/2 + (f*x)/2))/(f*(tan(e/2
 + (f*x)/2)^2 + 1)^2)
```

sympy [A] time = 0.30, size = 70, normalized size = 2.41

$$\begin{cases} -\frac{acx\sin^2\left(e+fx\right)}{2}-\frac{acx\cos^2\left(e+fx\right)}{2}+acx+\frac{ac\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f} & \text{for}\ f\neq 0 \\ x\left(a\sin\left(e\right)+a\right)\left(-c\sin\left(e\right)+c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-a*c*x*sin(e + f*x)**2/2 - a*c*x*cos(e + f*x)**2/2 + a*c*x + a*c
*sin(e + f*x)*cos(e + f*x)/(2*f), Ne(f, 0)), (x*(a*sin(e) + a)*(-c*sin(e) +
 c), True))
```

3.231 $\int \frac{a+a\sin(e+fx)}{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=33

$$\frac{2a\cos(e+fx)}{f(c-c\sin(e+fx))}-\frac{ax}{c}$$

```
[Out] -a*x/c+2*a*cos(f*x+e)/f/(c-c*sin(f*x+e))
```

Rubi [A] time = 0.05, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2735, 2648}

$$\frac{2a\cos(e+fx)}{f(c-c\sin(e+fx))}-\frac{ax}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x]),x]
```

```
[Out] -((a*x)/c) + (2*a*Cos[e + f*x])/(f*(c - c*Sin[e + f*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\sin(e+fx)}{c-c\sin(e+fx)}\,dx &= -\frac{ax}{c}+(2a)\int \frac{1}{c-c\sin(e+fx)}\,dx\\ &= -\frac{ax}{c}+\frac{2a\cos(e+fx)}{f(c-c\sin(e+fx))}\end{aligned}$$

Mathematica [B] time = 0.19, size = 83, normalized size = 2.52

$$\frac{a\left(fx\sin\left(e+\frac{fx}{2}\right)+4\sin\left(\frac{fx}{2}\right)-fx\cos\left(\frac{fx}{2}\right)\right)}{cf\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x]),x]
```

```
[Out] (a*(-(f*x*Cos[(f*x)/2]) + 4*Sin[(f*x)/2] + f*x*Sin[e + (f*x)/2]))/(c*f*(Cos
[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

fricas [A] time = 0.44, size = 66, normalized size = 2.00

$$-\frac{afx+\left(afx-2\,a\right)\cos\left(fx+e\right)-\left(afx+2\,a\right)\sin\left(fx+e\right)-2\,a}{cf\cos\left(fx+e\right)-cf\sin\left(fx+e\right)+cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(a*f*x + (a*f*x - 2*a)*cos(f*x + e) - (a*f*x + 2*a)*sin(f*x + e) - 2*a)/(c
*f*cos(f*x + e) - c*f*sin(f*x + e) + c*f)
```

giac [A] time = 0.16, size = 37, normalized size = 1.12

$$-\frac{\frac{\left(fx+e\right)a}{c}+\frac{4\,a}{c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -((f*x + e)*a/c + 4*a/(c*(tan(1/2*f*x + 1/2*e) - 1)))/f
```

maple [A] time = 0.18, size = 43, normalized size = 1.30

$$-\frac{4a}{fc\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{2a\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x)
[Out] -4/f*a/c/(tan(1/2*f*x+1/2*e)-1)-2/f*a/c*arctan(tan(1/2*f*x+1/2*e))
```

maxima [B] time = 0.92, size = 82, normalized size = 2.48

$$-\frac{2\left(a\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}-\frac{1}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}}\right)-\frac{a}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="maxima")
[Out] -2*(a*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c - 1/(c - c*sin(f*x + e)/(c
os(f*x + e) + 1))) - a/(c - c*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

mupad [B] time = 6.81, size = 46, normalized size = 1.39

$$-\frac{a\,x}{c}-\frac{a\,\left(e+f\,x\right)-a\,\left(e+f\,x-4\right)}{c\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c - c*sin(e + f*x)),x)
[Out] - (a*x)/c - (a*(e + f*x) - a*(e + f*x - 4))/(c*f*(tan(e/2 + (f*x)/2) - 1))
```

sympy [A] time = 1.61, size = 88, normalized size = 2.67

$$\begin{cases} -\frac{afx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-cf}+\frac{afx}{cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-cf}-\frac{4a}{cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-cf} & \text{for}\ f\neq 0 \\ \frac{x(a\sin(e)+a)}{-c\sin(e)+c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x)
[Out] Piecewise((-a*f*x*tan(e/2 + f*x/2)/(c*f*tan(e/2 + f*x/2) - c*f) + a*f*x/(c*
f*tan(e/2 + f*x/2) - c*f) - 4*a/(c*f*tan(e/2 + f*x/2) - c*f), Ne(f, 0)), (x
*(a*sin(e) + a)/(-c*sin(e) + c), True))
```

3.232 $\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=30

$$\frac{ac\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}$$

```
[Out] 1/3*a*c*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^3
```

Rubi [A] time = 0.07, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2736, 2671}

$$\frac{ac\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a*c*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^3)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^2}\,dx &= (ac)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^3}\,dx\\ &= \frac{ac\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}\end{aligned}$$

Mathematica [**B**] time = 0.28, size = 74, normalized size = 2.47

$$-\frac{a\left(\cos\left(e+\frac{3fx}{2}\right)-3\cos\left(e+\frac{fx}{2}\right)\right)}{3c^2f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] -1/3*(a*(-3*Cos[e + (f*x)/2] + Cos[e + (3*f*x)/2]))/(c^2*f*(Cos[e/2] - Sin[
e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3)
```

fricas [**B**] time = 0.42, size = 104, normalized size = 3.47

$$\frac{a\cos\left(fx+e\right)^2-a\cos\left(fx+e\right)-\left(a\cos\left(fx+e\right)+2\,a\right)\sin\left(fx+e\right)-2\,a}{3\left(c^2f\cos\left(fx+e\right)^2-c^2f\cos\left(fx+e\right)-2\,c^2f+\left(c^2f\cos\left(fx+e\right)+2\,c^2f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(a*cos(f*x + e)^2 - a*cos(f*x + e) - (a*cos(f*x + e) + 2*a)*sin(f*x + e
) - 2*a)/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) - 2*c^2*f + (c^2*f*cos(
f*x + e) + 2*c^2*f)*sin(f*x + e))
```

giac [**A**] time = 0.20, size = 39, normalized size = 1.30

$$-\frac{2\left(3\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a\right)}{3\,c^2f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*a*tan(1/2*f*x + 1/2*e)^2 + a)/(c^2*f*(tan(1/2*f*x + 1/2*e) - 1)^3)
```

maple [**A**] time = 0.22, size = 56, normalized size = 1.87

$$\frac{2a\left(-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{4}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}\right)}{f\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

```
[Out] 2/f*a/c^2*(-1/(tan(1/2*f*x+1/2*e)-1)-4/3/(tan(1/2*f*x+1/2*e)-1)^3-2/(tan(1/
2*f*x+1/2*e)-1)^2)
```

maxima [B] time = 0.74, size = 217, normalized size = 7.23

$$-\frac{2\left(\frac{a\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-2\right)}{c^2-\frac{3c^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}-\frac{a\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^2-\frac{3c^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(a*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 - 2)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - a*(
3*sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x +
 e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3))/f
```

mupad [B] time = 6.73, size = 56, normalized size = 1.87

$$-\frac{2a\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2-3\right)}{3c^2f\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)-\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c - c*sin(e + f*x))^2,x)
```

```
[Out] -(2*a*cos(e/2 + (f*x)/2)*(2*cos(e/2 + (f*x)/2)^2 - 3))/(3*c^2*f*(cos(e/2 +
(f*x)/2) - sin(e/2 + (f*x)/2))^3)
```

sympy [A] time = 3.71, size = 158, normalized size = 5.27

$$\begin{cases} -\frac{6a\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3c^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-9c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9c^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3c^2f}-\frac{2a}{3c^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-9c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9c^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3c^2f} & \text{for } f \neq 0 \\ \frac{x(a\sin(e)+a)}{(-c\sin(e)+c)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*a*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*
f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 2*a/(3*c**2
*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 +
f*x/2) - 3*c**2*f), Ne(f, 0)), (x*(a*sin(e) + a)/(-c*sin(e) + c)**2, True))
```

3.233 $\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=60

$$\frac{a\cos^3(e+fx)}{15f(c-c\sin(e+fx))^3}+\frac{ac\cos^3(e+fx)}{5f(c-c\sin(e+fx))^4}$$

```
[Out] 1/5*a*c*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^4+1/15*a*cos(f*x+e)^3/f/(c-c*sin(f*
x+e))^3
```

Rubi [A] time = 0.12, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2736, 2672, 2671}

$$\frac{a\cos^3(e+fx)}{15f(c-c\sin(e+fx))^3}+\frac{ac\cos^3(e+fx)}{5f(c-c\sin(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (a*c*Cos[e + f*x]^3)/(5*f*(c - c*Sin[e + f*x])^4) + (a*Cos[e + f*x]^3)/(15*
f*(c - c*Sin[e + f*x])^3)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
```

```
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^3}\,dx &= (ac)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^4}\,dx\\ &= \frac{ac\cos^3(e+fx)}{5f(c-c\sin(e+fx))^4}+\frac{1}{5}a\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^3}\,dx\\ &= \frac{ac\cos^3(e+fx)}{5f(c-c\sin(e+fx))^4}+\frac{a\cos^3(e+fx)}{15f(c-c\sin(e+fx))^3}\end{aligned}$$

Mathematica [A] time = 0.34, size = 96, normalized size = 1.60

$$\frac{a\left(\sin\left(2e+\frac{5fx}{2}\right)+15\cos\left(e+\frac{fx}{2}\right)-5\cos\left(e+\frac{3fx}{2}\right)+5\sin\left(\frac{fx}{2}\right)\right)}{30c^3f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (a*(15*Cos[e + (f*x)/2] - 5*Cos[e + (3*f*x)/2] + 5*Sin[(f*x)/2] + Sin[2*e +
 (5*f*x)/2]))/(30*c^3*f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])^5)
```

fricas [B] time = 0.47, size = 154, normalized size = 2.57

$$-\frac{a\cos\left(fx+e\right)^3-2\,a\cos\left(fx+e\right)^2+3\,a\cos\left(fx+e\right)+\left(a\cos\left(fx+e\right)^2+3\,a\cos\left(fx+e\right)+6\,a\right)\sin\left(\right.}{15\left(c^3f\cos\left(fx+e\right)^3+3\,c^3f\cos\left(fx+e\right)^2-2\,c^3f\cos\left(fx+e\right)-4\,c^3f-\left(c^3f\cos\left(fx+e\right)^2-2\,c^3f\cos\left(fx+e\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(a*cos(f*x + e)^3 - 2*a*cos(f*x + e)^2 + 3*a*cos(f*x + e) + (a*cos(f*
x + e)^2 + 3*a*cos(f*x + e) + 6*a)*sin(f*x + e) + 6*a)/(c^3*f*cos(f*x + e)^
3 + 3*c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*
x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e))
```

giac [A] time = 0.24, size = 84, normalized size = 1.40

$$-\frac{2\left(15\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-15\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+25\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-5\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+4\,a\right)}{15\,c^{3}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*a*tan(1/2*f*x + 1/2*e)^4 - 15*a*tan(1/2*f*x + 1/2*e)^3 + 25*a*tan
(1/2*f*x + 1/2*e)^2 - 5*a*tan(1/2*f*x + 1/2*e) + 4*a)/(c^3*f*(tan(1/2*f*x +
 1/2*e) - 1)^5)
```

maple [A] time = 0.25, size = 86, normalized size = 1.43

$$\frac{2a\left(-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{14}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{8}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{3}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}\right)}{f\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)
```

```
[Out] 2/f*a/c^3*(-1/(tan(1/2*f*x+1/2*e)-1)-14/3/(tan(1/2*f*x+1/2*e)-1)^3-8/5/(tan
(1/2*f*x+1/2*e)-1)^5-4/(tan(1/2*f*x+1/2*e)-1)^4-3/(tan(1/2*f*x+1/2*e)-1)^2)
```

maxima [B] time = 1.05, size = 389, normalized size = 6.48

$$-\frac{2\left(\frac{a\left(\frac{20\,\sin(fx+e)}{\cos(fx+e)+1}-\frac{40\,\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{30\,\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}-\frac{15\,\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}-7\right)}{c^{3}-\frac{5\,c^{3}\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\,c^{3}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{10\,c^{3}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{5\,c^{3}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}-\frac{c^{3}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}}-\frac{3\,a\left(\frac{5\,\sin(fx+e)}{\cos(fx+e)+1}-\frac{5\,\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{5\,\sin(fx+\cdots}{(\cos(fx+e)\cdots}\right.}{c^{3}-\frac{5\,c^{3}\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\,c^{3}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{10\,c^{3}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{5\,c^{3}\sin\cdots}{(\cos(fx\cdots}}\right.}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(a*(20*sin(f*x + e)/(cos(f*x + e) + 1) - 40*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 - 7)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(c
```

```
os(f*x + e) + 1)^5) - 3*a*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^3
 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

mupad [B] time = 7.10, size = 136, normalized size = 2.27

$$\frac{2\,a\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(4\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-5\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+25\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-15\,\cos\left(\frac{e}{2}\right.\right.}{15\,c^3\,f\,\left(\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)-\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c - c*sin(e + f*x))^3,x)
```

```
[Out] (2*a*cos(e/2 + (f*x)/2)*(4*cos(e/2 + (f*x)/2)^4 + 15*sin(e/2 + (f*x)/2)^4 -
 15*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x)/2)^3 - 5*cos(e/2 + (f*x)/2)^3*sin(e/
2 + (f*x)/2) + 25*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^2))/(15*c^3*f*(co
s(e/2 + (f*x)/2) - sin(e/2 + (f*x)/2))^5)
```

sympy [A] time = 7.49, size = 571, normalized size = 9.52

$$\begin{cases} -\frac{30a\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{15c^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)-75c^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+150c^3f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-150c^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+75c^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-15c^3f}+\frac{\cdots}{15c^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)-75c^3f\tan\cdots} \\ \frac{x(a\sin(e)+a)}{(-c\sin(e)+c)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*a*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c*
*3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(
e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 30*a*tan(e/2 +
f*x/2)**3/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 +
150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f
*tan(e/2 + f*x/2) - 15*c**3*f) - 50*a*tan(e/2 + f*x/2)**2/(15*c**3*f*tan(e/
2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)
**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3
*f) + 10*a*tan(e/2 + f*x/2)/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(
e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x
/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 8*a/(15*c**3*f*tan(e/2 +
```

```
f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3
 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f)
, Ne(f, 0)), (x*(a*sin(e) + a)/(-c*sin(e) + c)**3, True))
```

3.234 $\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^4}\,dx$

Optimal. Leaf size=92

$$\frac{2a\cos^3(e+fx)}{105cf(c-c\sin(e+fx))^3}+\frac{2a\cos^3(e+fx)}{35f(c-c\sin(e+fx))^4}+\frac{ac\cos^3(e+fx)}{7f(c-c\sin(e+fx))^5}$$

```
[Out] 1/7*a*c*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^5+2/35*a*cos(f*x+e)^3/f/(c-c*sin(f*
x+e))^4+2/105*a*cos(f*x+e)^3/c/f/(c-c*sin(f*x+e))^3
```

Rubi [A] time = 0.17, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2736, 2672, 2671}

$$\frac{2a\cos^3(e+fx)}{105cf(c-c\sin(e+fx))^3}+\frac{2a\cos^3(e+fx)}{35f(c-c\sin(e+fx))^4}+\frac{ac\cos^3(e+fx)}{7f(c-c\sin(e+fx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (a*c*Cos[e + f*x]^3)/(7*f*(c - c*Sin[e + f*x])^5) + (2*a*Cos[e + f*x]^3)/(3
5*f*(c - c*Sin[e + f*x])^4) + (2*a*Cos[e + f*x]^3)/(105*c*f*(c - c*Sin[e +
f*x])^3)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
```

```
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^4}\,dx &= (ac)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^5}\,dx\\ &= \frac{ac\cos^3(e+fx)}{7f(c-c\sin(e+fx))^5}+\frac{1}{7}(2a)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^4}\,dx\\ &= \frac{ac\cos^3(e+fx)}{7f(c-c\sin(e+fx))^5}+\frac{2a\cos^3(e+fx)}{35f(c-c\sin(e+fx))^4}+\frac{(2a)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^3}\,dx}{35c}\\ &= \frac{ac\cos^3(e+fx)}{7f(c-c\sin(e+fx))^5}+\frac{2a\cos^3(e+fx)}{35f(c-c\sin(e+fx))^4}+\frac{2a\cos^3(e+fx)}{105cf(c-c\sin(e+fx))^3}\end{aligned}$$

Mathematica [A] time = 0.48, size = 109, normalized size = 1.18

$$\frac{a\left(7\sin\left(2e+\frac{5fx}{2}\right)+70\cos\left(e+\frac{fx}{2}\right)-21\cos\left(e+\frac{3fx}{2}\right)+\cos\left(3e+\frac{7fx}{2}\right)+35\sin\left(\frac{fx}{2}\right)\right)}{210c^4f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (a*(70*Cos[e + (f*x)/2] - 21*Cos[e + (3*f*x)/2] + Cos[3*e + (7*f*x)/2] + 35
*Sin[(f*x)/2] + 7*Sin[2*e + (5*f*x)/2]))/(210*c^4*f*(Cos[e/2] - Sin[e/2])*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7)
```

fricas [B] time = 0.43, size = 206, normalized size = 2.24

$$\frac{2\,a\cos\left(fx+e\right)^4+8\,a\cos\left(fx+e\right)^3-9\,a\cos\left(fx+e\right)^2+15\,a\cos\left(fx+e\right)-\left(2\,a\cos\left(fx+e\right)^3-6\,a\,c\right.}{105\left(c^4f\cos\left(fx+e\right)^4-3\,c^4f\cos\left(fx+e\right)^3-8\,c^4f\cos\left(fx+e\right)^2+4\,c^4f\cos\left(fx+e\right)+8\,c^4f+\left(c^4f\cos\left(fx+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(2*a*cos(f*x + e)^4 + 8*a*cos(f*x + e)^3 - 9*a*cos(f*x + e)^2 + 15*a*
cos(f*x + e) - (2*a*cos(f*x + e)^3 - 6*a*cos(f*x + e)^2 - 15*a*cos(f*x + e)
 - 30*a)*sin(f*x + e) + 30*a)/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f*x + e)^
3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*f*cos(f*
x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)*sin(f*x
 + e))
```

giac [A] time = 0.21, size = 114, normalized size = 1.24

$$-\frac{2\left(105\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-210\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+455\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-350\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+273\,a\,t\right.}{105\,c^{4}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -2/105*(105*a*tan(1/2*f*x + 1/2*e)^6 - 210*a*tan(1/2*f*x + 1/2*e)^5 + 455*a
*tan(1/2*f*x + 1/2*e)^4 - 350*a*tan(1/2*f*x + 1/2*e)^3 + 273*a*tan(1/2*f*x
+ 1/2*e)^2 - 56*a*tan(1/2*f*x + 1/2*e) + 23*a)/(c^4*f*(tan(1/2*f*x + 1/2*e)
 - 1)^7)
```

maple [A] time = 0.22, size = 116, normalized size = 1.26

$$\frac{2a\left(-\frac{14}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{28}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{68}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{16}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{8}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}\right.}{f\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x)
```

```
[Out] 2/f*a/c^4*(-14/(tan(1/2*f*x+1/2*e)-1)^4-1/(tan(1/2*f*x+1/2*e)-1)-28/3/(tan(
1/2*f*x+1/2*e)-1)^3-68/5/(tan(1/2*f*x+1/2*e)-1)^5-16/7/(tan(1/2*f*x+1/2*e)-
1)^7-8/(tan(1/2*f*x+1/2*e)-1)^6-4/(tan(1/2*f*x+1/2*e)-1)^2)
```

maxima [B] time = 0.55, size = 561, normalized size = 6.10

$$\frac{2\left(\frac{a\left(\frac{91\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{168\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{280\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}-\frac{175\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{105\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}-13\right)}{c^{4}-\frac{7c^{4}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{21c^{4}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{35c^{4}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{35c^{4}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}-\frac{21c^{4}\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}+\frac{7c^{4}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}-\frac{c^{4}\sin\left(fx+e\right)^{7}}{\left(\cos\left(fx+e\right)+1\right)^{7}}}-\frac{3a\left(\frac{49\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{147\,s}{(\cos(}\right.}{c^{4}-\frac{7c^{4}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{21c^{4}\sin\left(fx+\right.}{\left(\cos\left(fx+e\right)+\right.}}\right.}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 2/105*(a*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 13)/(c^4 -
 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^
4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) +
1)^7) - 3*a*(49*sin(f*x + e)/(cos(f*x + e) + 1) - 147*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 210*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 210*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 35*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6 - 12)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x +
e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^
4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e)
+ 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7))/f
```

mupad [B] time = 7.35, size = 97, normalized size = 1.05

$$\frac{\sqrt{2}\,a\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(\frac{25\cos\left(3\,e+3\,f\,x\right)}{8}-\frac{595\sin\left(e+f\,x\right)}{8}-\frac{43\cos\left(2\,e+2\,f\,x\right)}{2}-\frac{353\cos\left(e+f\,x\right)}{8}+\frac{77\sin\left(2\,e+2\,f\,x\right)}{4}+\frac{21\sin\left(3\,e+3\,f\,x\right)}{8}\right.}{840\,c^4\,f\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{f\,x}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c - c*sin(e + f*x))^4,x)
```

```
[Out] (2^(1/2)*a*cos(e/2 + (f*x)/2)*((25*cos(3*e + 3*f*x))/8 - (595*sin(e + f*x))
/8 - (43*cos(2*e + 2*f*x))/2 - (353*cos(e + f*x))/8 + (77*sin(2*e + 2*f*x))
/4 + (21*sin(3*e + 3*f*x))/8 + 171/2))/(840*c^4*f*cos(e/2 + pi/4 + (f*x)/2)
^7)
```

sympy [A] time = 16.67, size = 1061, normalized size = 11.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-210*a*tan(e/2 + f*x/2)**6/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735
*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f
*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/
```

```
2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 420*a*tan(e/2 +
 f*x/2)**5/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6
 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675
*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*
tan(e/2 + f*x/2) - 105*c**4*f) - 910*a*tan(e/2 + f*x/2)**4/(105*c**4*f*tan(
e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*
x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3
 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4
*f) + 700*a*tan(e/2 + f*x/2)**3/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*
f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e
/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*
x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 546*a*tan(e/2 + f*x/2
)**2/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 220
5*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*
f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/
2 + f*x/2) - 105*c**4*f) + 112*a*tan(e/2 + f*x/2)/(105*c**4*f*tan(e/2 + f*x
/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 -
 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c
**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 46*
a/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c
**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*t
an(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 +
 f*x/2) - 105*c**4*f), Ne(f, 0)), (x*(a*sin(e) + a)/(-c*sin(e) + c)**4, Tru
e))
```

3.235 $\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^5}\,dx$

Optimal. Leaf size=126

$$\frac{2ac\cos^3(e+fx)}{315f\left(c^2-c^2\sin(e+fx)\right)^3}+\frac{2a\cos^3(e+fx)}{105cf(c-c\sin(e+fx))^4}+\frac{a\cos^3(e+fx)}{21f(c-c\sin(e+fx))^5}+\frac{ac\cos^3(e+fx)}{9f(c-c\sin(e+fx))^6}$$

```
[Out] 1/9*a*c*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^6+1/21*a*cos(f*x+e)^3/f/(c-c*sin(f*
x+e))^5+2/105*a*cos(f*x+e)^3/c/f/(c-c*sin(f*x+e))^4+2/315*a*c*cos(f*x+e)^3/
f/(c^2-c^2*sin(f*x+e))^3
```

Rubi [A] time = 0.22, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2736, 2672, 2671}

$$\frac{2ac\cos^3(e+fx)}{315f\left(c^2-c^2\sin(e+fx)\right)^3}+\frac{2a\cos^3(e+fx)}{105cf(c-c\sin(e+fx))^4}+\frac{a\cos^3(e+fx)}{21f(c-c\sin(e+fx))^5}+\frac{ac\cos^3(e+fx)}{9f(c-c\sin(e+fx))^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (a*c*Cos[e + f*x]^3)/(9*f*(c - c*Sin[e + f*x])^6) + (a*Cos[e + f*x]^3)/(21*
f*(c - c*Sin[e + f*x])^5) + (2*a*Cos[e + f*x]^3)/(105*c*f*(c - c*Sin[e + f*
x])^4) + (2*a*c*Cos[e + f*x]^3)/(315*f*(c^2 - c^2*Sin[e + f*x])^3)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^5}\,dx = (ac)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^6}\,dx$$
$$= \frac{ac\cos^3(e+fx)}{9f(c-c\sin(e+fx))^6} + \frac{1}{3}a\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^5}\,dx$$
$$= \frac{ac\cos^3(e+fx)}{9f(c-c\sin(e+fx))^6} + \frac{a\cos^3(e+fx)}{21f(c-c\sin(e+fx))^5} + \frac{(2a)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^4}\,dx}{21c}$$
$$= \frac{ac\cos^3(e+fx)}{9f(c-c\sin(e+fx))^6} + \frac{a\cos^3(e+fx)}{21f(c-c\sin(e+fx))^5} + \frac{2a\cos^3(e+fx)}{105cf(c-c\sin(e+fx))^4} + \frac{(2a)}{}$$
$$= \frac{ac\cos^3(e+fx)}{9f(c-c\sin(e+fx))^6} + \frac{a\cos^3(e+fx)}{21f(c-c\sin(e+fx))^5} + \frac{2a\cos^3(e+fx)}{105cf(c-c\sin(e+fx))^4} + \frac{}{315c}$$

Mathematica [A] time = 0.62, size = 124, normalized size = 0.98

$$\frac{a\left(36\sin\left(2e+\frac{5fx}{2}\right)-\sin\left(4e+\frac{9fx}{2}\right)+315\cos\left(e+\frac{fx}{2}\right)-84\cos\left(e+\frac{3fx}{2}\right)+9\cos\left(3e+\frac{7fx}{2}\right)+189\sin\left(\frac{fx}{2}\right)\right)}{1260c^5f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^5,x]
[Out] (a*(315*Cos[e + (f*x)/2] - 84*Cos[e + (3*f*x)/2] + 9*Cos[3*e + (7*f*x)/2] +
 189*Sin[(f*x)/2] + 36*Sin[2*e + (5*f*x)/2] - Sin[4*e + (9*f*x)/2]))/(1260*
c^5*f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9)
```

fricas [B] time = 0.47, size = 256, normalized size = 2.03

$$-\frac{2a\cos\left(fx+e\right)^5-8a\cos\left(fx+e\right)^4-25a\cos\left(fx+e\right)^3+20a\cos\left(fx+e\right)^2-35a\cos\left(fx+e\right)+}{315\left(c^5f\cos\left(fx+e\right)^5+5c^5f\cos\left(fx+e\right)^4-8c^5f\cos\left(fx+e\right)^3-20c^5f\cos\left(fx+e\right)^2+8c^5f\cos\left(fx+e\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] -1/315*(2*a*cos(f*x + e)^5 - 8*a*cos(f*x + e)^4 - 25*a*cos(f*x + e)^3 + 20*
a*cos(f*x + e)^2 - 35*a*cos(f*x + e) + (2*a*cos(f*x + e)^4 + 10*a*cos(f*x +
 e)^3 - 15*a*cos(f*x + e)^2 - 35*a*cos(f*x + e) - 70*a)*sin(f*x + e) - 70*a
)/(c^5*f*cos(f*x + e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^3 -
 20*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(f*x
 + e)^4 - 4*c^5*f*cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*
x + e) + 16*c^5*f)*sin(f*x + e))
```

giac [A] time = 0.60, size = 144, normalized size = 1.14

$$-\frac{2\left(315\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-945\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+2625\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-3465\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+38\right.}{315\,c^{5}f\left(\tan\left(\frac{1}{2}\,f\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorithm="giac")
```

```
[Out] -2/315*(315*a*tan(1/2*f*x + 1/2*e)^8 - 945*a*tan(1/2*f*x + 1/2*e)^7 + 2625*
a*tan(1/2*f*x + 1/2*e)^6 - 3465*a*tan(1/2*f*x + 1/2*e)^5 + 3843*a*tan(1/2*f
*x + 1/2*e)^4 - 2247*a*tan(1/2*f*x + 1/2*e)^3 + 1143*a*tan(1/2*f*x + 1/2*e)
^2 - 207*a*tan(1/2*f*x + 1/2*e) + 58*a)/(c^5*f*(tan(1/2*f*x + 1/2*e) - 1)^9
)
```

maple [A] time = 0.27, size = 146, normalized size = 1.16

$$\frac{2a\left(-\frac{5}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{32}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{46}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{32}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{9}}-\frac{16}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{8}}-\frac{236}{5\left(\tan\left(\frac{fx}{2}+\right.\right.}\right.}{f\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^5,x)
```

```
[Out] 2/f*a/c^5*(-5/(tan(1/2*f*x+1/2*e)-1)^2-32/(tan(1/2*f*x+1/2*e)-1)^4-1/(tan(1
/2*f*x+1/2*e)-1)-46/3/(tan(1/2*f*x+1/2*e)-1)^3-32/9/(tan(1/2*f*x+1/2*e)-1)^
9-16/(tan(1/2*f*x+1/2*e)-1)^8-236/5/(tan(1/2*f*x+1/2*e)-1)^5-248/7/(tan(1/2
*f*x+1/2*e)-1)^7-148/3/(tan(1/2*f*x+1/2*e)-1)^6)
```

maxima [B] time = 0.61, size = 733, normalized size = 5.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] -2/315*(a*(432*sin(f*x + e)/(cos(f*x + e) + 1) - 1728*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 3612*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5418*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 + 5040*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 3360
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1260*sin(f*x + e)^7/(cos(f*x + e) +
1)^7 - 315*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 83)/(c^5 - 9*c^5*sin(f*x +
 e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^
5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e
)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c
^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) +
 1)^9) - 5*a*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 315*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 147*si
n(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^7/(cos(f*x + e) + 1)^7
- 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*
sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7
/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin
(f*x + e)^9/(cos(f*x + e) + 1)^9))/f
```

mupad [B] time = 8.77, size = 119, normalized size = 0.94

$$\frac{\sqrt{2}\,a\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(\frac{121\,\cos\left(3\,e+3\,f\,x\right)}{4}-\frac{1575\,\sin\left(e+f\,x\right)}{4}-\frac{625\,\cos\left(2\,e+2\,f\,x\right)}{4}-\frac{635\,\cos\left(e+f\,x\right)}{4}+\frac{7\,\cos\left(4\,e+4\,f\,x\right)}{2}+\frac{399\,\sin\left(2\,e+2\,f\right.}{4}\right.}{5040\,c^5\,f\,\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{f\,x}{2}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c - c*sin(e + f*x))^5,x)
```

```
[Out] (2^(1/2)*a*cos(e/2 + (f*x)/2)*((121*cos(3*e + 3*f*x))/4 - (1575*sin(e + f*x
))/4 - (625*cos(2*e + 2*f*x))/4 - (635*cos(e + f*x))/4 + (7*cos(4*e + 4*f*x
))/2 + (399*sin(2*e + 2*f*x))/4 + (141*sin(3*e + 3*f*x))/4 - (15*sin(4*e +
4*f*x))/4 + 1357/4))/(5040*c^5*f*cos(e/2 + pi/4 + (f*x)/2)^9)
```

sympy [A] time = 33.03, size = 1700, normalized size = 13.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((-630*a*tan(e/2 + f*x/2)**8/(315*c**5*f*tan(e/2 + f*x/2)**9 - 283
5*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**
5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*t
an(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/
2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 1890*a*tan(e/2
 + f*x/2)**7/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)
**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 +
 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 2646
0*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5
*f*tan(e/2 + f*x/2) - 315*c**5*f) - 5250*a*tan(e/2 + f*x/2)**6/(315*c**5*f*
tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/
2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f
*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)
**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315
*c**5*f) + 6930*a*tan(e/2 + f*x/2)**5/(315*c**5*f*tan(e/2 + f*x/2)**9 - 283
5*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**
5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*t
an(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/
2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 7686*a*tan(e/2
 + f*x/2)**4/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)
**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 +
 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 2646
0*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5
*f*tan(e/2 + f*x/2) - 315*c**5*f) + 4494*a*tan(e/2 + f*x/2)**3/(315*c**5*f*
tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/
2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f
*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)
**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315
*c**5*f) - 2286*a*tan(e/2 + f*x/2)**2/(315*c**5*f*tan(e/2 + f*x/2)**9 - 283
5*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**
5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*t
an(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/
2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 414*a*tan(e/2
+ f*x/2)/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8
+ 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 396
90*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c*
*5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*t
an(e/2 + f*x/2) - 315*c**5*f) - 116*a/(315*c**5*f*tan(e/2 + f*x/2)**9 - 283
5*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**
5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*t
an(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/
2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f), Ne(f, 0)), (x*(
a*sin(e) + a)/(-c*sin(e) + c)**5, True))
```

3.236 $\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^5\,dx$

Optimal. Leaf size=152

$$\frac{3a^2c^5\cos^5(e+fx)}{10f}+\frac{3a^2\cos^5(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{14f}+\frac{3a^2c^5\sin(e+fx)\cos^3(e+fx)}{8f}+\frac{9a^2c^5\sin(e+fx)\mathrm{co}}{16f}$$

```
[Out] 9/16*a^2*c^5*x+3/10*a^2*c^5*cos(f*x+e)^5/f+9/16*a^2*c^5*cos(f*x+e)*sin(f*x+
e)/f+3/8*a^2*c^5*cos(f*x+e)^3*sin(f*x+e)/f+1/7*a^2*c^3*cos(f*x+e)^5*(c-c*si
n(f*x+e))^2/f+3/14*a^2*cos(f*x+e)^5*(c^5-c^5*sin(f*x+e))/f
```

Rubi [A] time = 0.20, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2736, 2678, 2669, 2635, 8}

$$\frac{3a^2c^5\cos^5(e+fx)}{10f}+\frac{3a^2c^5\sin(e+fx)\cos^3(e+fx)}{8f}+\frac{a^2c^3\cos^5(e+fx)(c-c\sin(e+fx))^2}{7f}+\frac{3a^2\cos^5(e+fx)\left(c^5\right.}{14f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (9*a^2*c^5*x)/16 + (3*a^2*c^5*Cos[e + f*x]^5)/(10*f) + (9*a^2*c^5*Cos[e + f
*x]*Sin[e + f*x])/(16*f) + (3*a^2*c^5*Cos[e + f*x]^3*Sin[e + f*x])/(8*f) +
(a^2*c^3*Cos[e + f*x]^5*(c - c*Sin[e + f*x])^2)/(7*f) + (3*a^2*Cos[e + f*x]
^5*(c^5 - c^5*Sin[e + f*x]))/(14*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^5\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(c-c\sin(e+fx))^3\,dx$$

$$= \frac{a^2c^3\cos^5(e+fx)(c-c\sin(e+fx))^2}{7f} + \frac{1}{7}\left(9a^2c^3\right)\int \cos^4(e+fx$$

$$= \frac{a^2c^3\cos^5(e+fx)(c-c\sin(e+fx))^2}{7f} + \frac{3a^2\cos^5(e+fx)\left(c^5-c^5\right.}{14f}$$

$$= \frac{3a^2c^5\cos^5(e+fx)}{10f} + \frac{a^2c^3\cos^5(e+fx)(c-c\sin(e+fx))^2}{7f} + \frac{3a}{}$$

$$= \frac{3a^2c^5\cos^5(e+fx)}{10f} + \frac{3a^2c^5\cos^3(e+fx)\sin(e+fx)}{8f} + \frac{a^2c^3\cos^5}{}$$

$$= \frac{3a^2c^5\cos^5(e+fx)}{10f} + \frac{9a^2c^5\cos(e+fx)\sin(e+fx)}{16f} + \frac{3a^2c^5\cos^3}{}$$

$$= \frac{9}{16}a^2c^5x + \frac{3a^2c^5\cos^5(e+fx)}{10f} + \frac{9a^2c^5\cos(e+fx)\sin(e+fx)}{16f} +$$

Mathematica [A] time = 1.11, size = 89, normalized size = 0.59

$$\frac{a^2c^5(665\sin(2(e+fx))-35\sin(4(e+fx))-35\sin(6(e+fx))+945\cos(e+fx)+455\cos(3(e+fx))+77\,\mathrm{co}}{2240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (a^2*c^5*(1260*e + 1260*f*x + 945*Cos[e + f*x] + 455*Cos[3*(e + f*x)] + 77*
Cos[5*(e + f*x)] - 5*Cos[7*(e + f*x)] + 665*Sin[2*(e + f*x)] - 35*Sin[4*(e
+ f*x)] - 35*Sin[6*(e + f*x)]))/(2240*f)
```

fricas [A] time = 0.48, size = 103, normalized size = 0.68

$$-\frac{80\,a^2c^5\cos\left(fx+e\right)^7-448\,a^2c^5\cos\left(fx+e\right)^5-315\,a^2c^5fx+35\left(8\,a^2c^5\cos\left(fx+e\right)^5-6\,a^2c^5\cos\left(fx+e\right)^3-9\right.}{560\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] -1/560*(80*a^2*c^5*cos(f*x + e)^7 - 448*a^2*c^5*cos(f*x + e)^5 - 315*a^2*c^
5*f*x + 35*(8*a^2*c^5*cos(f*x + e)^5 - 6*a^2*c^5*cos(f*x + e)^3 - 9*a^2*c^5
*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.23, size = 154, normalized size = 1.01

$$\frac{9}{16}\,a^2c^5x-\frac{a^2c^5\cos\left(7\,fx+7\,e\right)}{448\,f}+\frac{11\,a^2c^5\cos\left(5\,fx+5\,e\right)}{320\,f}+\frac{13\,a^2c^5\cos\left(3\,fx+3\,e\right)}{64\,f}+\frac{27\,a^2c^5\cos\left(fx+e\right)}{64\,f}-\frac{a^2c^5\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^5,x, algorithm="giac")
```

```
[Out] 9/16*a^2*c^5*x - 1/448*a^2*c^5*cos(7*f*x + 7*e)/f + 11/320*a^2*c^5*cos(5*f*
x + 5*e)/f + 13/64*a^2*c^5*cos(3*f*x + 3*e)/f + 27/64*a^2*c^5*cos(f*x + e)/
f - 1/64*a^2*c^5*sin(6*f*x + 6*e)/f - 1/64*a^2*c^5*sin(4*f*x + 4*e)/f + 19/
64*a^2*c^5*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.40, size = 255, normalized size = 1.68

$$\frac{\frac{c^5a^2\left(\frac{16}{5}+\sin^6\left(fx+e\right)+\frac{6\left(\sin^4\left(fx+e\right)\right)}{5}+\frac{8\left(\sin^2\left(fx+e\right)\right)}{5}\right)\cos\left(fx+e\right)}{7}+3c^5a^2\left(-\frac{\left(\sin^5\left(fx+e\right)+\frac{5\left(\sin^3\left(fx+e\right)\right)}{4}+\frac{15\sin\left(fx+e\right)}{8}\right)\cos\left(fx+e\right)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^5,x)
```

```
[Out] 1/f*(1/7*c^5*a^2*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(
f*x+e)+3*c^5*a^2*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(
```

```
f*x+e)+5/16*f*x+5/16*e)+1/5*c^5*a^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos
(f*x+e)-5*c^5*a^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/
8*e)-5/3*c^5*a^2*(2+sin(f*x+e)^2)*cos(f*x+e)+c^5*a^2*(-1/2*sin(f*x+e)*cos(f
*x+e)+1/2*f*x+1/2*e)+3*c^5*a^2*cos(f*x+e)+c^5*a^2*(f*x+e))
```

maxima [A] time = 0.82, size = 256, normalized size = 1.68

$$-\frac{192\left(5\cos\left(fx+e\right)^{7}-21\cos\left(fx+e\right)^{5}+35\cos\left(fx+e\right)^{3}-35\cos\left(fx+e\right)\right)a^{2}c^{5}-448\left(3\cos\left(fx+e\right)^{5}-1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] -1/6720*(192*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 - 35
*cos(f*x + e))*a^2*c^5 - 448*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos
(f*x + e))*a^2*c^5 - 11200*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*c^5 - 105*
(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x +
 2*e))*a^2*c^5 + 1050*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e
))*a^2*c^5 - 1680*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c^5 - 6720*(f*x + e)
*a^2*c^5 - 20160*a^2*c^5*cos(f*x + e))/f
```

mupad [B] time = 9.23, size = 452, normalized size = 2.97

$$\frac{9\,a^2\,c^5\,x}{16}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{a^2\,c^5\,\left(315\,e+315\,f\,x\right)}{80}-\frac{a^2\,c^5\,\left(2205\,e+2205\,f\,x+1792\right)}{560}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}\,\left(\frac{a^2\,c^5\,\left(315\,e+315\,f\,x\right)}{80}-\frac{a^2\,c^5\,(\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^5,x)
```

```
[Out] (9*a^2*c^5*x)/16 - (tan(e/2 + (f*x)/2)^2*((a^2*c^5*(315*e + 315*f*x))/80 -
(a^2*c^5*(2205*e + 2205*f*x + 1792))/560) + tan(e/2 + (f*x)/2)^12*((a^2*c^5
*(315*e + 315*f*x))/80 - (a^2*c^5*(2205*e + 2205*f*x + 3360))/560) + tan(e/
2 + (f*x)/2)^4*((3*a^2*c^5*(315*e + 315*f*x))/80 - (a^2*c^5*(6615*e + 6615*
f*x + 6496))/560) + tan(e/2 + (f*x)/2)^10*((3*a^2*c^5*(315*e + 315*f*x))/80
 - (a^2*c^5*(6615*e + 6615*f*x + 8960))/560) + tan(e/2 + (f*x)/2)^8*((a^2*c
^5*(315*e + 315*f*x))/16 - (a^2*c^5*(11025*e + 11025*f*x + 7840))/560) + ta
n(e/2 + (f*x)/2)^6*((a^2*c^5*(315*e + 315*f*x))/16 - (a^2*c^5*(11025*e + 11
025*f*x + 17920))/560) - (17*a^2*c^5*tan(e/2 + (f*x)/2)^3)/2 + (13*a^2*c^5*
tan(e/2 + (f*x)/2)^5)/8 - (13*a^2*c^5*tan(e/2 + (f*x)/2)^9)/8 + (17*a^2*c^5
*tan(e/2 + (f*x)/2)^11)/2 + (7*a^2*c^5*tan(e/2 + (f*x)/2)^13)/8 + (a^2*c^5*
```

```
(315*e + 315*f*x))/560 - (a^2*c^5*(315*e + 315*f*x + 736))/560 - (7*a^2*c^5
*tan(e/2 + (f*x)/2))/8)/(f*(tan(e/2 + (f*x)/2)^2 + 1)^7)
```

sympy [A] time = 10.58, size = 629, normalized size = 4.14

$$\begin{cases} \frac{15a^2c^5x\sin^6\left(e+fx\right)}{16} + \frac{45a^2c^5x\sin^4\left(e+fx\right)\cos^2\left(e+fx\right)}{16} - \frac{15a^2c^5x\sin^4\left(e+fx\right)}{8} + \frac{45a^2c^5x\sin^2\left(e+fx\right)\cos^4\left(e+fx\right)}{16} - \frac{15a^2c^5x\sin^2\left(e+fx\right)\cos^2}{4} \\ x\left(a\sin\left(e\right)+a\right)^2\left(-c\sin\left(e\right)+c\right)^5 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((15*a**2*c**5*x*sin(e + f*x)**6/16 + 45*a**2*c**5*x*sin(e + f*x)*
*4*cos(e + f*x)**2/16 - 15*a**2*c**5*x*sin(e + f*x)**4/8 + 45*a**2*c**5*x*s
in(e + f*x)**2*cos(e + f*x)**4/16 - 15*a**2*c**5*x*sin(e + f*x)**2*cos(e +
f*x)**2/4 + a**2*c**5*x*sin(e + f*x)**2/2 + 15*a**2*c**5*x*cos(e + f*x)**6/
16 - 15*a**2*c**5*x*cos(e + f*x)**4/8 + a**2*c**5*x*cos(e + f*x)**2/2 + a**
2*c**5*x + a**2*c**5*sin(e + f*x)**6*cos(e + f*x)/f - 33*a**2*c**5*sin(e +
f*x)**5*cos(e + f*x)/(16*f) + 2*a**2*c**5*sin(e + f*x)**4*cos(e + f*x)**3/f
 + a**2*c**5*sin(e + f*x)**4*cos(e + f*x)/f - 5*a**2*c**5*sin(e + f*x)**3*c
os(e + f*x)**3/(2*f) + 25*a**2*c**5*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 8*
a**2*c**5*sin(e + f*x)**2*cos(e + f*x)**5/(5*f) + 4*a**2*c**5*sin(e + f*x)*
*2*cos(e + f*x)**3/(3*f) - 5*a**2*c**5*sin(e + f*x)**2*cos(e + f*x)/f - 15*
a**2*c**5*sin(e + f*x)*cos(e + f*x)**5/(16*f) + 15*a**2*c**5*sin(e + f*x)*c
os(e + f*x)**3/(8*f) - a**2*c**5*sin(e + f*x)*cos(e + f*x)/(2*f) + 16*a**2*
c**5*cos(e + f*x)**7/(35*f) + 8*a**2*c**5*cos(e + f*x)**5/(15*f) - 10*a**2*
c**5*cos(e + f*x)**3/(3*f) + 3*a**2*c**5*cos(e + f*x)/f, Ne(f, 0)), (x*(a*s
in(e) + a)**2*(-c*sin(e) + c)**5, True))
```

3.237 $\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^4\,dx$

Optimal. Leaf size=118

$$\frac{7a^2c^4\cos^5(e+fx)}{30f}+\frac{a^2\cos^5(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{6f}+\frac{7a^2c^4\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{7a^2c^4\sin(e+fx)\cos(e+fx)}{16f}$$

```
[Out] 7/16*a^2*c^4*x+7/30*a^2*c^4*cos(f*x+e)^5/f+7/16*a^2*c^4*cos(f*x+e)*sin(f*x+
e)/f+7/24*a^2*c^4*cos(f*x+e)^3*sin(f*x+e)/f+1/6*a^2*cos(f*x+e)^5*(c^4-c^4*s
in(f*x+e))/f
```

Rubi [A] time = 0.14, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2736, 2678, 2669, 2635, 8}

$$\frac{7a^2c^4\cos^5(e+fx)}{30f}+\frac{a^2\cos^5(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{6f}+\frac{7a^2c^4\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{7a^2c^4\sin(e+fx)\cos(e+fx)}{16f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (7*a^2*c^4*x)/16 + (7*a^2*c^4*Cos[e + f*x]^5)/(30*f) + (7*a^2*c^4*Cos[e + f
*x]*Sin[e + f*x])/(16*f) + (7*a^2*c^4*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) +
 (a^2*Cos[e + f*x]^5*(c^4 - c^4*Sin[e + f*x]))/(6*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^4\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(c-c\sin(e+fx))^2\,dx$$
$$= \frac{a^2\cos^5(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{6f}+\frac{1}{6}\left(7a^2c^3\right)\int \cos^4(e+fx)($$
$$= \frac{7a^2c^4\cos^5(e+fx)}{30f}+\frac{a^2\cos^5(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{6f}+\frac{1}{6}\left(7\right.$$
$$= \frac{7a^2c^4\cos^5(e+fx)}{30f}+\frac{7a^2c^4\cos^3(e+fx)\sin(e+fx)}{24f}+\frac{a^2\cos^5(e-}{}$$
$$= \frac{7a^2c^4\cos^5(e+fx)}{30f}+\frac{7a^2c^4\cos(e+fx)\sin(e+fx)}{16f}+\frac{7a^2c^4\cos^3(}{}$$
$$= \frac{7}{16}a^2c^4x+\frac{7a^2c^4\cos^5(e+fx)}{30f}+\frac{7a^2c^4\cos(e+fx)\sin(e+fx)}{16f}+$$

Mathematica [A] time = 0.73, size = 79, normalized size = 0.67

$$\frac{a^2c^4(255\sin(2(e+fx))+15\sin(4(e+fx))-5\sin(6(e+fx))+240\cos(e+fx)+120\cos(3(e+fx))+24\cos(5}{960f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (a^2*c^4*(420*e + 420*f*x + 240*Cos[e + f*x] + 120*Cos[3*(e + f*x)] + 24*Co
s[5*(e + f*x)] + 255*Sin[2*(e + f*x)] + 15*Sin[4*(e + f*x)] - 5*Sin[6*(e +
f*x)]))/(960*f)
```

fricas [A] time = 0.46, size = 87, normalized size = 0.74

$$\frac{96\,a^2c^4\cos\left(fx+e\right)^5+105\,a^2c^4fx-5\left(8\,a^2c^4\cos\left(fx+e\right)^5-14\,a^2c^4\cos\left(fx+e\right)^3-21\,a^2c^4\cos\left(fx+e\right)\right)\sin\left(\right.}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/240*(96*a^2*c^4*cos(f*x + e)^5 + 105*a^2*c^4*f*x - 5*(8*a^2*c^4*cos(f*x +
 e)^5 - 14*a^2*c^4*cos(f*x + e)^3 - 21*a^2*c^4*cos(f*x + e))*sin(f*x + e))/
f
```

giac [A] time = 0.22, size = 133, normalized size = 1.13

$$\frac{7}{16}\,a^2c^4x+\frac{a^2c^4\cos\left(5\,fx+5\,e\right)}{40\,f}+\frac{a^2c^4\cos\left(3\,fx+3\,e\right)}{8\,f}+\frac{a^2c^4\cos\left(fx+e\right)}{4\,f}-\frac{a^2c^4\sin\left(6\,fx+6\,e\right)}{192\,f}+\frac{a^2c^4\sin\left(4\,fx\right.}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 7/16*a^2*c^4*x + 1/40*a^2*c^4*cos(5*f*x + 5*e)/f + 1/8*a^2*c^4*cos(3*f*x +
3*e)/f + 1/4*a^2*c^4*cos(f*x + e)/f - 1/192*a^2*c^4*sin(6*f*x + 6*e)/f + 1/
64*a^2*c^4*sin(4*f*x + 4*e)/f + 17/64*a^2*c^4*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.34, size = 211, normalized size = 1.79

$$c^4a^2\left(-\frac{\left(\sin^5\left(fx+e\right)+\frac{5\left(\sin^3\left(fx+e\right)\right)}{4}+\frac{15\sin\left(fx+e\right)}{8}\right)\cos\left(fx+e\right)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+\frac{2c^4a^2\left(\frac{8}{3}+\sin^4\left(fx+e\right)+\frac{4\left(\sin^2\left(fx+e\right)\right)}{3}\right)\cos\left(fx+e\right)}{5}-c^4a^2\left(-\frac{\left(\sin\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^4,x)
```

```
[Out] 1/f*(c^4*a^2*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+
e)+5/16*f*x+5/16*e)+2/5*c^4*a^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x
+e)-c^4*a^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-4
```

```
/3*c^4*a^2*(2+sin(f*x+e)^2)*cos(f*x+e)-c^4*a^2*(-1/2*sin(f*x+e)*cos(f*x+e)+
1/2*f*x+1/2*e)+2*c^4*a^2*cos(f*x+e)+c^4*a^2*(f*x+e))
```

maxima [A] time = 0.68, size = 209, normalized size = 1.77

$$\frac{128\left(3\cos\left(fx+e\right)^{5}-10\cos\left(fx+e\right)^{3}+15\cos\left(fx+e\right)\right)a^{2}c^{4}+1280\left(\cos\left(fx+e\right)^{3}-3\cos\left(fx+e\right)\right)a^{2}c^{4}+5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 1/960*(128*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a^2*c^4
 + 1280*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*c^4 + 5*(4*sin(2*f*x + 2*e)^3
 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*a^2*c^4 - 30*(
12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^2*c^4 - 240*(2*f*x
 + 2*e - sin(2*f*x + 2*e))*a^2*c^4 + 960*(f*x + e)*a^2*c^4 + 1920*a^2*c^4*c
os(f*x + e))/f
```

mupad [B] time = 8.90, size = 284, normalized size = 2.41

$$\frac{a^{2}c^{4}\left(105e+270\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+192\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}+890\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{3}+1920\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}-660\tan\left(\frac{e}{2}+\frac{f}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^4,x)
```

```
[Out] (a^2*c^4*(105*e + 270*tan(e/2 + (f*x)/2) + 192*tan(e/2 + (f*x)/2)^2 + 890*t
an(e/2 + (f*x)/2)^3 + 1920*tan(e/2 + (f*x)/2)^4 - 660*tan(e/2 + (f*x)/2)^5
+ 1920*tan(e/2 + (f*x)/2)^6 + 660*tan(e/2 + (f*x)/2)^7 + 960*tan(e/2 + (f*x
)/2)^8 - 890*tan(e/2 + (f*x)/2)^9 + 960*tan(e/2 + (f*x)/2)^10 - 270*tan(e/2
 + (f*x)/2)^11 + 105*f*x + 630*tan(e/2 + (f*x)/2)^2*(e + f*x) + 1575*tan(e/
2 + (f*x)/2)^4*(e + f*x) + 2100*tan(e/2 + (f*x)/2)^6*(e + f*x) + 1575*tan(e
/2 + (f*x)/2)^8*(e + f*x) + 630*tan(e/2 + (f*x)/2)^10*(e + f*x) + 105*tan(e
/2 + (f*x)/2)^12*(e + f*x) + 192))/(240*f*(tan(e/2 + (f*x)/2)^2 + 1)^6)
```

sympy [A] time = 6.93, size = 530, normalized size = 4.49

$$\begin{cases}\frac{5a^{2}c^{4}x\sin^{6}\left(e+fx\right)}{16}+\frac{15a^{2}c^{4}x\sin^{4}\left(e+fx\right)\cos^{2}\left(e+fx\right)}{16}-\frac{3a^{2}c^{4}x\sin^{4}\left(e+fx\right)}{8}+\frac{15a^{2}c^{4}x\sin^{2}\left(e+fx\right)\cos^{4}\left(e+fx\right)}{16}-\frac{3a^{2}c^{4}x\sin^{2}\left(e+fx\right)\cos^{2}\left(e+\right.}{4}\\ x\left(a\sin\left(e\right)+a\right)^{2}\left(-c\sin\left(e\right)+c\right)^{4}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((5*a**2*c**4*x*sin(e + f*x)**6/16 + 15*a**2*c**4*x*sin(e + f*x)**
4*cos(e + f*x)**2/16 - 3*a**2*c**4*x*sin(e + f*x)**4/8 + 15*a**2*c**4*x*sin
(e + f*x)**2*cos(e + f*x)**4/16 - 3*a**2*c**4*x*sin(e + f*x)**2*cos(e + f*x
)**2/4 - a**2*c**4*x*sin(e + f*x)**2/2 + 5*a**2*c**4*x*cos(e + f*x)**6/16 -
 3*a**2*c**4*x*cos(e + f*x)**4/8 - a**2*c**4*x*cos(e + f*x)**2/2 + a**2*c**
4*x - 11*a**2*c**4*sin(e + f*x)**5*cos(e + f*x)/(16*f) + 2*a**2*c**4*sin(e
+ f*x)**4*cos(e + f*x)/f - 5*a**2*c**4*sin(e + f*x)**3*cos(e + f*x)**3/(6*f
) + 5*a**2*c**4*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 8*a**2*c**4*sin(e + f*
x)**2*cos(e + f*x)**3/(3*f) - 4*a**2*c**4*sin(e + f*x)**2*cos(e + f*x)/f -
5*a**2*c**4*sin(e + f*x)*cos(e + f*x)**5/(16*f) + 3*a**2*c**4*sin(e + f*x)*
cos(e + f*x)**3/(8*f) + a**2*c**4*sin(e + f*x)*cos(e + f*x)/(2*f) + 16*a**2
*c**4*cos(e + f*x)**5/(15*f) - 8*a**2*c**4*cos(e + f*x)**3/(3*f) + 2*a**2*c
**4*cos(e + f*x)/f, Ne(f, 0)), (x*(a*sin(e) + a)**2*(-c*sin(e) + c)**4, Tru
e))
```

3.238 $\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^3\,dx$

Optimal. Leaf size=85

$$\frac{a^2c^3\cos^5(e+fx)}{5f}+\frac{a^2c^3\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3a^2c^3\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}a^2c^3x$$

```
[Out] 3/8*a^2*c^3*x+1/5*a^2*c^3*cos(f*x+e)^5/f+3/8*a^2*c^3*cos(f*x+e)*sin(f*x+e)/
f+1/4*a^2*c^3*cos(f*x+e)^3*sin(f*x+e)/f
```

Rubi [A] time = 0.10, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2669, 2635, 8}

$$\frac{a^2c^3\cos^5(e+fx)}{5f}+\frac{a^2c^3\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3a^2c^3\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}a^2c^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (3*a^2*c^3*x)/8 + (a^2*c^3*Cos[e + f*x]^5)/(5*f) + (3*a^2*c^3*Cos[e + f*x]*
Sin[e + f*x])/(8*f) + (a^2*c^3*Cos[e + f*x]^3*Sin[e + f*x])/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
```

```
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^3\,dx &= \left(a^2c^2\right)\int \cos^4(e+fx)(c-c\sin(e+fx))\,dx\\ &= \frac{a^2c^3\cos^5(e+fx)}{5f}+\left(a^2c^3\right)\int \cos^4(e+fx)\,dx\\ &= \frac{a^2c^3\cos^5(e+fx)}{5f}+\frac{a^2c^3\cos^3(e+fx)\sin(e+fx)}{4f}+\frac{1}{4}\left(3a^2c^3\right)\int\\ &= \frac{a^2c^3\cos^5(e+fx)}{5f}+\frac{3a^2c^3\cos(e+fx)\sin(e+fx)}{8f}+\frac{a^2c^3\cos^3(e\\ &= \frac{3}{8}a^2c^3x+\frac{a^2c^3\cos^5(e+fx)}{5f}+\frac{3a^2c^3\cos(e+fx)\sin(e+fx)}{8f}+\frac{a\end{aligned}$$

Mathematica [A] time = 1.64, size = 69, normalized size = 0.81

$$\frac{a^2c^3(40\sin(2(e+fx))+5\sin(4(e+fx))+20\cos(e+fx)+10\cos(3(e+fx))+2\cos(5(e+fx))+60e+60fx}{160f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (a^2*c^3*(60*e + 60*f*x + 20*Cos[e + f*x] + 10*Cos[3*(e + f*x)] + 2*Cos[5*(
e + f*x)] + 40*Sin[2*(e + f*x)] + 5*Sin[4*(e + f*x)]))/(160*f)
```

fricas [A] time = 0.47, size = 71, normalized size = 0.84

$$\frac{8\,a^2c^3\cos\left(fx+e\right)^5+15\,a^2c^3fx+5\left(2\,a^2c^3\cos\left(fx+e\right)^3+3\,a^2c^3\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{40\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/40*(8*a^2*c^3*cos(f*x + e)^5 + 15*a^2*c^3*f*x + 5*(2*a^2*c^3*cos(f*x + e)
^3 + 3*a^2*c^3*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.20, size = 112, normalized size = 1.32

$$\frac{3}{8}\,a^2c^3x+\frac{a^2c^3\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{a^2c^3\cos\left(3\,fx+3\,e\right)}{16\,f}+\frac{a^2c^3\cos\left(fx+e\right)}{8\,f}+\frac{a^2c^3\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{a^2c^3\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 3/8*a^2*c^3*x + 1/80*a^2*c^3*cos(5*f*x + 5*e)/f + 1/16*a^2*c^3*cos(3*f*x +
3*e)/f + 1/8*a^2*c^3*cos(f*x + e)/f + 1/32*a^2*c^3*sin(4*f*x + 4*e)/f + 1/4
*a^2*c^3*sin(2*f*x + 2*e)/f
```

maple [B] time = 0.29, size = 159, normalized size = 1.87

$$\frac{\frac{c^3a^2\left(\frac{8}{3}+\sin^4\left(fx+e\right)+\frac{4\left(\sin^2\left(fx+e\right)\right)}{3}\right)\cos\left(fx+e\right)}{5}+c^3a^2\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{2c^3a^2\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}\cdots}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(1/5*c^3*a^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+c^3*a^2*(-1
/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2/3*c^3*a^2*(2+s
in(f*x+e)^2)*cos(f*x+e)-2*c^3*a^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e
)+c^3*a^2*cos(f*x+e)+a^2*c^3*(f*x+e))
```

maxima [B] time = 1.00, size = 158, normalized size = 1.86

$$\frac{32\left(3\cos\left(fx+e\right)^5-10\cos\left(fx+e\right)^3+15\cos\left(fx+e\right)\right)a^2c^3+320\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)a^2c^3+15\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a^2*c^3
+ 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*c^3 + 15*(12*f*x + 12*e + sin(4
*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^2*c^3 - 240*(2*f*x + 2*e - sin(2*f*x +
2*e))*a^2*c^3 + 480*(f*x + e)*a^2*c^3 + 480*a^2*c^3*cos(f*x + e))/f
```

mupad [B] time = 10.00, size = 220, normalized size = 2.59

$$\frac{3\,a^2\,c^3\,x}{8}+\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8\,\left(\frac{a^2\,c^3\,\left(75\,e+75\,f\,x+80\right)}{40}-\frac{15\,a^2\,c^3\,\left(e+f\,x\right)}{8}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(\frac{a^2\,c^3\,\left(150\,e+150\,f\,x+160\right)}{40}-\frac{15\,a^2\,c^3\,\left(e+f\,x\right)}{4}\right.\cdots}{f\,\left(\tan\left(\frac{e}{2}\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^3,x)
```

```
[Out] (3*a^2*c^3*x)/8 + (tan(e/2 + (f*x)/2)^8*((a^2*c^3*(75*e + 75*f*x + 80))/40
- (15*a^2*c^3*(e + f*x))/8) + tan(e/2 + (f*x)/2)^4*((a^2*c^3*(150*e + 150*f
*x + 160))/40 - (15*a^2*c^3*(e + f*x))/4) + (a^2*c^3*tan(e/2 + (f*x)/2)^3)/
2 - (a^2*c^3*tan(e/2 + (f*x)/2)^7)/2 - (5*a^2*c^3*tan(e/2 + (f*x)/2)^9)/4 +
 (a^2*c^3*(15*e + 15*f*x + 16))/40 + (5*a^2*c^3*tan(e/2 + (f*x)/2))/4 - (3*
a^2*c^3*(e + f*x))/8)/(f*(tan(e/2 + (f*x)/2)^2 + 1)^5)
```

sympy [A] time = 3.04, size = 340, normalized size = 4.00

$$\begin{cases} \frac{3a^2c^3x\sin^4\left(e+fx\right)}{8}+\frac{3a^2c^3x\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4}-a^2c^3x\sin^2\left(e+fx\right)+\frac{3a^2c^3x\cos^4\left(e+fx\right)}{8}-a^2c^3x\cos^2\left(e+fx\right)+a^2c^3\cdots \\ x\left(a\sin\left(e\right)+a\right)^2\left(-c\sin\left(e\right)+c\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((3*a**2*c**3*x*sin(e + f*x)**4/8 + 3*a**2*c**3*x*sin(e + f*x)**2*
cos(e + f*x)**2/4 - a**2*c**3*x*sin(e + f*x)**2 + 3*a**2*c**3*x*cos(e + f*x
)**4/8 - a**2*c**3*x*cos(e + f*x)**2 + a**2*c**3*x + a**2*c**3*sin(e + f*x)
**4*cos(e + f*x)/f - 5*a**2*c**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 4*a**
2*c**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 2*a**2*c**3*sin(e + f*x)**2*
cos(e + f*x)/f - 3*a**2*c**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) + a**2*c**3
*sin(e + f*x)*cos(e + f*x)/f + 8*a**2*c**3*cos(e + f*x)**5/(15*f) - 4*a**2*
c**3*cos(e + f*x)**3/(3*f) + a**2*c**3*cos(e + f*x)/f, Ne(f, 0)), (x*(a*sin
(e) + a)**2*(-c*sin(e) + c)**3, True))
```

3.239 $\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^2\,dx$

Optimal. Leaf size=64

$$\frac{a^2c^2\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3a^2c^2\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}a^2c^2x$$

```
[Out] 3/8*a^2*c^2*x+3/8*a^2*c^2*cos(f*x+e)*sin(f*x+e)/f+1/4*a^2*c^2*cos(f*x+e)^3*
sin(f*x+e)/f
```

Rubi [A] time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2635, 8}

$$\frac{a^2c^2\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3a^2c^2\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}a^2c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (3*a^2*c^2*x)/8 + (3*a^2*c^2*Cos[e + f*x]*Sin[e + f*x])/(8*f) + (a^2*c^2*Co
s[e + f*x]^3*Sin[e + f*x])/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^2\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)\,dx$$
$$= \frac{a^2c^2\cos^3(e+fx)\sin(e+fx)}{4f} + \frac{1}{4}\left(3a^2c^2\right)\int \cos^2(e+fx)\,dx$$
$$= \frac{3a^2c^2\cos(e+fx)\sin(e+fx)}{8f} + \frac{a^2c^2\cos^3(e+fx)\sin(e+fx)}{4f} +$$
$$= \frac{3}{8}a^2c^2x + \frac{3a^2c^2\cos(e+fx)\sin(e+fx)}{8f} + \frac{a^2c^2\cos^3(e+fx)\sin(}{4f}$$

Mathematica [A] time = 0.04, size = 39, normalized size = 0.61

$$\frac{a^2c^2(12(e+fx)+8\sin(2(e+fx))+\sin(4(e+fx)))}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*c^2*(12*(e + f*x) + 8*Sin[2*(e + f*x)] + Sin[4*(e + f*x)]))/(32*f)
```

fricas [A] time = 0.48, size = 54, normalized size = 0.84

$$\frac{3\,a^2c^2fx + \left(2\,a^2c^2\cos\left(fx+e\right)^3 + 3\,a^2c^2\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(3*a^2*c^2*f*x + (2*a^2*c^2*cos(f*x + e)^3 + 3*a^2*c^2*cos(f*x + e))*si
n(f*x + e))/f
```

giac [A] time = 1.25, size = 52, normalized size = 0.81

$$\frac{3}{8}\,a^2c^2x + \frac{a^2c^2\sin\left(4\,fx+4\,e\right)}{32\,f} + \frac{a^2c^2\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

[Out] 3/8*a^2*c^2*x + 1/32*a^2*c^2*sin(4*f*x + 4*e)/f + 1/4*a^2*c^2*sin(2*f*x + 2
*e)/f

maple [A] time = 0.17, size = 88, normalized size = 1.38

$$\frac{c^2a^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-2a^2c^2\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+a^2c^2\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^2,x)

[Out] 1/f*(c^2*a^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-
2*a^2*c^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+a^2*c^2*(f*x+e))

maxima [A] time = 0.31, size = 81, normalized size = 1.27

$$\frac{\left(12\,fx+12\,e+\sin\left(4\,fx+4\,e\right)-8\,\sin\left(2\,fx+2\,e\right)\right)a^2c^2-16\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)a^2c^2+32\left(fx+e\right)a^2c}{32\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^2,x, algorithm="maxima")

[Out] 1/32*((12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^2*c^2 - 16*
(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c^2 + 32*(f*x + e)*a^2*c^2)/f

mupad [B] time = 6.74, size = 36, normalized size = 0.56

$$\frac{a^2\,c^2\,\left(8\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(4\,e+4\,f\,x\right)+12\,f\,x\right)}{32\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^2,x)

[Out] (a^2*c^2*(8*sin(2*e + 2*f*x) + sin(4*e + 4*f*x) + 12*f*x))/(32*f)

sympy [A] time = 1.47, size = 206, normalized size = 3.22

$$\begin{cases} \frac{3a^2c^2x\sin^4\left(e+fx\right)}{8}+\frac{3a^2c^2x\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4}-a^2c^2x\sin^2\left(e+fx\right)+\frac{3a^2c^2x\cos^4\left(e+fx\right)}{8}-a^2c^2x\cos^2\left(e+fx\right)+a^2c^2x \\ x\left(a\sin\left(e\right)+a\right)^2\left(-c\sin\left(e\right)+c\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*a**2*c**2*x*sin(e + f*x)**4/8 + 3*a**2*c**2*x*sin(e + f*x)**2*
cos(e + f*x)**2/4 - a**2*c**2*x*sin(e + f*x)**2 + 3*a**2*c**2*x*cos(e + f*x
)**4/8 - a**2*c**2*x*cos(e + f*x)**2 + a**2*c**2*x - 5*a**2*c**2*sin(e + f*
x)**3*cos(e + f*x)/(8*f) - 3*a**2*c**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) +
 a**2*c**2*sin(e + f*x)*cos(e + f*x)/f, Ne(f, 0)), (x*(a*sin(e) + a)**2*(-c
*sin(e) + c)**2, True))
```

3.240 $\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx$

Optimal. Leaf size=52

$$-\frac{a^2c\cos^3(e+fx)}{3f}+\frac{a^2c\sin(e+fx)\cos(e+fx)}{2f}+\frac{1}{2}a^2cx$$

```
[Out] 1/2*a^2*c*x-1/3*a^2*c*cos(f*x+e)^3/f+1/2*a^2*c*cos(f*x+e)*sin(f*x+e)/f
```

Rubi [A] time = 0.08, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2736, 2669, 2635, 8}

$$-\frac{a^2c\cos^3(e+fx)}{3f}+\frac{a^2c\sin(e+fx)\cos(e+fx)}{2f}+\frac{1}{2}a^2cx$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^2*c*x)/2 - (a^2*c*Cos[e + f*x]^3)/(3*f) + (a^2*c*Cos[e + f*x]*Sin[e + f*
x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
```

```
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))\,dx &= (ac)\int \cos^2(e+fx)(a+a\sin(e+fx))\,dx\\ &= -\frac{a^2c\cos^3(e+fx)}{3f}+\left(a^2c\right)\int \cos^2(e+fx)\,dx\\ &= -\frac{a^2c\cos^3(e+fx)}{3f}+\frac{a^2c\cos(e+fx)\sin(e+fx)}{2f}+\frac{1}{2}\left(a^2c\right)\int 1\,dx\\ &= \frac{1}{2}a^2cx-\frac{a^2c\cos^3(e+fx)}{3f}+\frac{a^2c\cos(e+fx)\sin(e+fx)}{2f}\end{aligned}$$

Mathematica [A] time = 0.33, size = 43, normalized size = 0.83

$$-\frac{a^2c(-3(\sin(2(e+fx))+2fx)+3\cos(e+fx)+\cos(3(e+fx)))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]),x]
```

```
[Out] -1/12*(a^2*c*(3*Cos[e + f*x] + Cos[3*(e + f*x)] - 3*(2*f*x + Sin[2*(e + f*x
)])))/f
```

fricas [A] time = 0.45, size = 46, normalized size = 0.88

$$-\frac{2\,a^2c\cos\left(fx+e\right)^3-3\,a^2cfx-3\,a^2c\cos\left(fx+e\right)\sin\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*a^2*c*cos(f*x + e)^3 - 3*a^2*c*f*x - 3*a^2*c*cos(f*x + e)*sin(f*x +
 e))/f
```

giac [A] time = 0.22, size = 62, normalized size = 1.19

$$\frac{1}{2}\,a^2cx-\frac{a^2c\cos\left(3\,fx+3\,e\right)}{12\,f}-\frac{a^2c\cos\left(fx+e\right)}{4\,f}+\frac{a^2c\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*a^2*c*x - 1/12*a^2*c*cos(3*f*x + 3*e)/f - 1/4*a^2*c*cos(f*x + e)/f + 1/
4*a^2*c*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.16, size = 78, normalized size = 1.50

$$\frac{\frac{a^2c\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}-a^2c\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-a^2c\cos\left(fx+e\right)+a^2c\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x)
```

```
[Out] 1/f*(1/3*a^2*c*(2+sin(f*x+e)^2)*cos(f*x+e)-a^2*c*(-1/2*sin(f*x+e)*cos(f*x+e
)+1/2*f*x+1/2*e)-a^2*c*cos(f*x+e)+a^2*c*(f*x+e))
```

maxima [A] time = 0.67, size = 77, normalized size = 1.48

$$-\frac{4\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)a^2c+3\left(2fx+2e-\sin\left(2fx+2e\right)\right)a^2c-12\left(fx+e\right)a^2c+12a^2c\cos\left(fx+\right.}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/12*(4*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*c + 3*(2*f*x + 2*e - sin(2*f
*x + 2*e))*a^2*c - 12*(f*x + e)*a^2*c + 12*a^2*c*cos(f*x + e))/f
```

mupad [B] time = 9.02, size = 125, normalized size = 2.40

$$\frac{a^2cx}{2}-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(\frac{3a^2c\left(e+fx\right)}{2}-\frac{a^2c\left(9e+9fx-12\right)}{6}\right)-a^2c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{a^2c\left(e+fx\right)}{2}+a^2c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5-\frac{a^2c\left(3e\right.}{}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x)),x)
```

```
[Out] (a^2*c*x)/2 - (tan(e/2 + (f*x)/2)^4*((3*a^2*c*(e + f*x))/2 - (a^2*c*(9*e +
9*f*x - 12))/6) - a^2*c*tan(e/2 + (f*x)/2) + (a^2*c*(e + f*x))/2 + a^2*c*ta
```

```
n(e/2 + (f*x)/2)^5 - (a^2*c*(3*e + 3*f*x - 4))/6)/(f*(tan(e/2 + (f*x)/2)^2
+ 1)^3)
```

sympy [A] time = 0.89, size = 133, normalized size = 2.56

$$\begin{cases} -\frac{a^2cx\sin^2(e+fx)}{2} - \frac{a^2cx\cos^2(e+fx)}{2} + a^2cx + \frac{a^2c\sin^2(e+fx)\cos(e+fx)}{f} + \frac{a^2c\sin(e+fx)\cos(e+fx)}{2f} + \frac{2a^2c\cos^3(e+fx)}{3f} - \frac{a^2c\cos(e+}{f} \\ x(a\sin(e)+a)^2(-c\sin(e)+c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-a**2*c*x*sin(e + f*x)**2/2 - a**2*c*x*cos(e + f*x)**2/2 + a**2*
c*x + a**2*c*sin(e + f*x)**2*cos(e + f*x)/f + a**2*c*sin(e + f*x)*cos(e + f
*x)/(2*f) + 2*a**2*c*cos(e + f*x)**3/(3*f) - a**2*c*cos(e + f*x)/f, Ne(f, 0
)), (x*(a*sin(e) + a)**2*(-c*sin(e) + c), True))
```

3.241 $\int \frac{(a+a\sin(e+fx))^2}{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=57

$$\frac{3a^2\cos(e+fx)}{cf}+\frac{2a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^2}-\frac{3a^2x}{c}$$

```
[Out] -3*a^2*x/c+3*a^2*cos(f*x+e)/c/f+2*a^2*c*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^2
```

Rubi [A] time = 0.14, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2680, 2682, 8}

$$\frac{3a^2\cos(e+fx)}{cf}+\frac{2a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^2}-\frac{3a^2x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x]),x]
```

```
[Out] (-3*a^2*x)/c + (3*a^2*Cos[e + f*x])/(c*f) + (2*a^2*c*Cos[e + f*x]^3)/(f*(c
- c*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(e+fx))^2}{c-c\sin(e+fx)}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^3}\,dx\\
&= \frac{2a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^2}-\left(3a^2\right)\int \frac{\cos^2(e+fx)}{c-c\sin(e+fx)}\,dx\\
&= \frac{3a^2\cos(e+fx)}{cf}+\frac{2a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^2}-\frac{\left(3a^2\right)\int 1\,dx}{c}\\
&= -\frac{3a^2x}{c}+\frac{3a^2\cos(e+fx)}{cf}+\frac{2a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^2}
\end{aligned}
$$

Mathematica [B] time = 0.38, size = 130, normalized size = 2.28

$$
\frac{a^2(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)(3(e+fx)-\cos(e+fx))+\sin\left(\frac{1}{2}(e+\cdots\right.\right.}{cf(\sin(e+fx)-1)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2]*(3*(e + f*x) -
 Cos[e + f*x]) + (-8 - 3*e - 3*f*x + Cos[e + f*x])*Sin[(e + f*x)/2])*(1 + S
in[e + f*x])^2)/(c*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-1 + Sin[e +
f*x]))
```

fricas [A] time = 0.44, size = 105, normalized size = 1.84

$$
-\frac{3\,a^2fx-a^2\cos\left(fx+e\right)^2-4\,a^2+\left(3\,a^2fx-5\,a^2\right)\cos\left(fx+e\right)-\left(3\,a^2fx-a^2\cos\left(fx+e\right)+4\,a^2\right)\sin\left(fx+e\cdots\right.}{cf\cos\left(fx+e\right)-cf\sin\left(fx+e\right)+cf}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(3*a^2*f*x - a^2*cos(f*x + e)^2 - 4*a^2 + (3*a^2*f*x - 5*a^2)*cos(f*x + e)
 - (3*a^2*f*x - a^2*cos(f*x + e) + 4*a^2)*sin(f*x + e))/(c*f*cos(f*x + e) -
 c*f*sin(f*x + e) + c*f)
```

giac [A] time = 0.24, size = 103, normalized size = 1.81

$$-\frac{\frac{3\left(fx+e\right)a^2}{c}+\frac{2\left(4\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+5\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)c}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -(3*(f*x + e)*a^2/c + 2*(4*a^2*tan(1/2*f*x + 1/2*e)^2 - a^2*tan(1/2*f*x + 1
/2*e) + 5*a^2)/((tan(1/2*f*x + 1/2*e)^3 - tan(1/2*f*x + 1/2*e)^2 + tan(1/2*
f*x + 1/2*e) - 1)*c))/f
```

maple [A] time = 0.23, size = 73, normalized size = 1.28

$$-\frac{8a^2}{fc\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{2a^2}{fc\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}-\frac{6a^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x)
```

```
[Out] -8/f*a^2/c/(tan(1/2*f*x+1/2*e)-1)+2/f*a^2/c/(1+tan(1/2*f*x+1/2*e)^2)-6/f*a^
2/c*arctan(tan(1/2*f*x+1/2*e))
```

maxima [B] time = 1.92, size = 218, normalized size = 3.82

$$-\frac{2\left(a^2\left(\frac{\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-2}{c-\frac{c\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{c\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{c\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}}+\frac{\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{c}\right)+2\,a^2\left(\frac{\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{c}-\frac{1}{c-\frac{c\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\right)-\frac{a^2}{c-\frac{c\sin(fx+e}{\cos(fx+}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -2*(a^2*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^2/(cos(f*x + e) +
1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f*x + e)^2/(cos(f*
```

```
x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/c) + 2*a^2*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c
 - 1/(c - c*sin(f*x + e)/(cos(f*x + e) + 1))) - a^2/(c - c*sin(f*x + e)/(co
s(f*x + e) + 1)))/f
```

mupad [B] time = 6.91, size = 118, normalized size = 2.07

$$\frac{3\sqrt{2}\,a^2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(e+f\,x\right)-\frac{\sqrt{2}\,a^2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(6\,e+6\,f\,x-16\right)}{2}}{c\,f\,\left(\sqrt{2}\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)-\sqrt{2}\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}-\frac{3\,a^2\,x}{c}+\frac{2\,a^2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{c\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x)),x)
```

```
[Out] (3*2^(1/2)*a^2*sin(e/2 + (f*x)/2)*(e + f*x) - (2^(1/2)*a^2*sin(e/2 + (f*x)/
2)*(6*e + 6*f*x - 16))/2)/(c*f*(2^(1/2)*cos(e/2 + (f*x)/2) - 2^(1/2)*sin(e/
2 + (f*x)/2))) - (3*a^2*x)/c + (2*a^2*cos(e/2 + (f*x)/2)^2)/(c*f)
```

sympy [A] time = 4.20, size = 454, normalized size = 7.96

$$\begin{cases} -\frac{3a^2fx\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-cf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-cf}+\frac{3a^2fx\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-cf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-cf}-\frac{3a^2fx\tan\left(\right.}{cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-cf\tan^2\left(\frac{e}{2}+\right.} \\ \frac{x(a\sin(e)+a)^2}{-c\sin(e)+c} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-3*a**2*f*x*tan(e/2 + f*x/2)**3/(c*f*tan(e/2 + f*x/2)**3 - c*f*t
an(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + 3*a**2*f*x*tan(e/2 + f*x
/2)**2/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f
*x/2) - c*f) - 3*a**2*f*x*tan(e/2 + f*x/2)/(c*f*tan(e/2 + f*x/2)**3 - c*f*t
an(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + 3*a**2*f*x/(c*f*tan(e/2
+ f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) - 8*a**
2*tan(e/2 + f*x/2)**2/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 +
c*f*tan(e/2 + f*x/2) - c*f) + 2*a**2*tan(e/2 + f*x/2)/(c*f*tan(e/2 + f*x/2)
**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) - 10*a**2/(c*f*
tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f)
, Ne(f, 0)), (x*(a*sin(e) + a)**2/(-c*sin(e) + c), True))
```

3.242 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=72

$$-\frac{2a^2\cos(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)}+\frac{a^2x}{c^2}+\frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}$$

```
[Out] a^2*x/c^2+2/3*a^2*c*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^3-2*a^2*cos(f*x+e)/f/(c
^2-c^2*sin(f*x+e))
```

Rubi [A] time = 0.14, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2680, 8}

$$-\frac{2a^2\cos(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)}+\frac{a^2x}{c^2}+\frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*x)/c^2 + (2*a^2*c*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^3) - (2*a^
2*Cos[e + f*x])/(f*(c^2 - c^2*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^2}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^4}\,dx \\
&= \frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3} - a^2\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^2}\,dx \\
&= \frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3} - \frac{2a^2\cos(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)} + \frac{a^2\int 1\,dx}{c^2} \\
&= \frac{a^2x}{c^2} + \frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3} - \frac{2a^2\cos(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)}
\end{aligned}
$$

Mathematica [A] time = 0.62, size = 121, normalized size = 1.68

$$
-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-3(3e+3fx+8)\cos\left(\frac{1}{2}(e+fx)\right)+(3e+3fx+16)\cos\left(\frac{3}{2}(e+fx)\right)+\cdots\right.}{6c^2f(\sin(e+fx)-1)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] -1/6*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(-3*(8 + 3*e + 3*f*x)*Cos[(
e + f*x)/2] + (16 + 3*e + 3*f*x)*Cos[(3*(e + f*x))/2] + 6*(2*(2 + e + f*x)
+ (e + f*x)*Cos[e + f*x])*Sin[(e + f*x)/2]))/(c^2*f*(-1 + Sin[e + f*x])^2)
```

fricas [B] time = 0.44, size = 158, normalized size = 2.19

$$
-\frac{6\,a^2fx-\left(3\,a^2fx+8\,a^2\right)\cos\left(fx+e\right)^2+4\,a^2+\left(3\,a^2fx-4\,a^2\right)\cos\left(fx+e\right)-\left(6\,a^2fx-4\,a^2+\left(3\,a^2fx-8\,a^2\right)\cdots\right.}{3\left(c^2f\cos\left(fx+e\right)^2-c^2f\cos\left(fx+e\right)-2\,c^2f+\left(c^2f\cos\left(fx+e\right)+2\,c^2f\right)\sin\left(fx+\cdots\right.\right.}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(6*a^2*f*x - (3*a^2*f*x + 8*a^2)*cos(f*x + e)^2 + 4*a^2 + (3*a^2*f*x -
 4*a^2)*cos(f*x + e) - (6*a^2*f*x - 4*a^2 + (3*a^2*f*x - 8*a^2)*cos(f*x + e
))*sin(f*x + e))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) - 2*c^2*f + (c^
2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

giac [A] time = 0.24, size = 60, normalized size = 0.83

$$\frac{\frac{3\,(fx+e)a^2}{c^2} - \frac{8\left(3\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-a^2\right)}{c^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(f*x + e)*a^2/c^2 - 8*(3*a^2*tan(1/2*f*x + 1/2*e) - a^2)/(c^2*(tan(1
/2*f*x + 1/2*e) - 1)^3))/f
```

maple [A] time = 0.24, size = 71, normalized size = 0.99

$$-\frac{16a^2}{3c^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3} - \frac{8a^2}{c^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2} + \frac{2a^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x)
```

```
[Out] -16/3*a^2/c^2/f/(tan(1/2*f*x+1/2*e)-1)^3-8*a^2/c^2/f/(tan(1/2*f*x+1/2*e)-1)
^2+2*a^2/c^2/f*arctan(tan(1/2*f*x+1/2*e))
```

maxima [B] time = 1.18, size = 364, normalized size = 5.06

$$\frac{2\left(a^2\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-4}{c^2-\frac{3c^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^2}\right)-\frac{a^2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-2\right)}{c^2-\frac{3c^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\ldots}{c^2-\frac{3c^2\sin\ldots}{\cos(f\ldots}}\right.}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(a^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 - 4)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3
*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2) - a^2*(3*sin(f*x + e)/(cos(f*
x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2)/(c^2 - 3*c^2*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^
```

```
2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 2*a^2*(3*sin(f*x + e)/(cos(f*x + e
) + 1) - 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

mupad [B] time = 6.87, size = 90, normalized size = 1.25

$$\frac{a^2\,x}{c^2}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(3\,a^2\,\left(e+f\,x\right)-\frac{a^2\,\left(9\,e+9\,f\,x-24\right)}{3}\right)-a^2\,\left(e+f\,x\right)+\frac{a^2\,\left(3\,e+3\,f\,x-8\right)}{3}}{c^2\,f\,{\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x))^2,x)
```

```
[Out] (a^2*x)/c^2 - (tan(e/2 + (f*x)/2)*(3*a^2*(e + f*x) - (a^2*(9*e + 9*f*x - 24
))/3) - a^2*(e + f*x) + (a^2*(3*e + 3*f*x - 8))/3)/(c^2*f*(tan(e/2 + (f*x)/
2) - 1)^3)
```

sympy [A] time = 8.24, size = 473, normalized size = 6.57

$$\begin{cases} \frac{3a^2fx\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3c^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-9c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9c^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3c^2f}-\frac{9a^2fx\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3c^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-9c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9c^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3c^2f}+\frac{}{3c^2f\tan^3\left(\frac{e}{2}\right.} \\ \frac{x(a\sin(e)+a)^2}{(-c\sin(e)+c)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*a**2*f*x*tan(e/2 + f*x/2)**3/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9
*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 9*a**
2*f*x*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2
+ f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 9*a**2*f*x*tan(e/2 +
f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**
2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 3*a**2*f*x/(3*c**2*f*tan(e/2 + f*x/2)**3
 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 2
4*a**2*tan(e/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 +
f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 8*a**2/(3*c**2*f*tan(e/
2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) -
3*c**2*f), Ne(f, 0)), (x*(a*sin(e) + a)**2/(-c*sin(e) + c)**2, True))
```

3.243 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=34

$$\frac{a^2c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}$$

```
[Out] 1/5*a^2*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^5
```

Rubi [A] time = 0.09, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2736, 2671}

$$\frac{a^2c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (a^2*c^2*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*x])^5)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^3}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^5}\,dx\\
&= \frac{a^2c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}
\end{aligned}$$

Mathematica [B] time = 0.40, size = 81, normalized size = 2.38

$$\frac{a^2\left(-10\sin\left(\frac{1}{2}(e+fx)\right)-5\sin\left(\frac{3}{2}(e+fx)\right)+\sin\left(\frac{5}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{10c^3f(\sin(e+fx)-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(-10*Sin[(e + f*x)/2] - 5*Sin[(3
*(e + f*x))/2] + Sin[(5*(e + f*x))/2]))/(10*c^3*f*(-1 + Sin[e + f*x])^3)
```

fricas [B] time = 0.46, size = 168, normalized size = 4.94

$$\frac{a^2\cos\left(fx+e\right)^3+3\,a^2\cos\left(fx+e\right)^2-2\,a^2\cos\left(fx+e\right)-4\,a^2+\left(a^2\cos\left(fx+e\right)^2-2\,a^2\cos\left(fx+e\right)-4\right.}{5\left(c^3f\cos\left(fx+e\right)^3+3\,c^3f\cos\left(fx+e\right)^2-2\,c^3f\cos\left(fx+e\right)-4\,c^3f-\left(c^3f\cos\left(fx+e\right)^2-2\,c^3f\cos\left(fx+e\right)\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/5*(a^2*cos(f*x + e)^3 + 3*a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) - 4*a^2
 + (a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) - 4*a^2)*sin(f*x + e))/(c^3*f*c
os(f*x + e)^3 + 3*c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (
c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e))
```

giac [A] time = 0.27, size = 60, normalized size = 1.76

$$-\frac{2\left(5\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+10\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a^2\right)}{5\,c^3f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/5*(5*a^2*tan(1/2*f*x + 1/2*e)^4 + 10*a^2*tan(1/2*f*x + 1/2*e)^2 + a^2)/(
c^3*f*(tan(1/2*f*x + 1/2*e) - 1)^5)
```

maple [B] time = 0.27, size = 88, normalized size = 2.59

$$\frac{2a^2\left(-\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{8}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{8}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{16}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}\right)}{f\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x)
```

```
[Out] 2/f*a^2/c^3*(-4/(tan(1/2*f*x+1/2*e)-1)^2-1/(tan(1/2*f*x+1/2*e)-1)-8/(tan(1/
2*f*x+1/2*e)-1)^4-8/(tan(1/2*f*x+1/2*e)-1)^3-16/5/(tan(1/2*f*x+1/2*e)-1)^5)
```

maxima [B] time = 2.03, size = 557, normalized size = 16.38

$$-\frac{2\left(\frac{a^2\left(\frac{20\sin(fx+e)}{\cos(fx+e)+1}-\frac{40\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{30\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-7\right)}{c^3-\frac{5c^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10c^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{10c^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5c^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{c^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}-\frac{6a^2\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{5\sin(fx+e}{(\cos(fx+e)+}\right.}{c^3-\frac{5c^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10c^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{10c^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5c^3\sin(f}{(\cos(fx+e}}\right.}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(a^2*(20*sin(f*x + e)/(cos(f*x + e) + 1) - 40*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 7)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*
c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) - 6*a^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/
(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 2
*a^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 - 1)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*
c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5))/f
```

mupad [B] time = 6.99, size = 92, normalized size = 2.71

$$\frac{2\,a^2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left({\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+10\,{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+5\,{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\right)}{5\,c^3\,f\,{\left(\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)-\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x))^3,x)
```

```
[Out] (2*a^2*cos(e/2 + (f*x)/2)*(cos(e/2 + (f*x)/2)^4 + 5*sin(e/2 + (f*x)/2)^4 +
10*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^2))/(5*c^3*f*(cos(e/2 + (f*x)/2)
 - sin(e/2 + (f*x)/2))^5)
```

sympy [A] time = 14.88, size = 354, normalized size = 10.41

$$\begin{cases} -\frac{10a^2\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{5c^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)-25c^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+50c^3f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-50c^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+25c^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-5c^3f} - \frac{}{5c^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)-25c^3f\tan^4\left(\frac{e}{2} \cdots\right.} \\ \frac{x(a\sin(e)+a)^2}{(-c\sin(e)+c)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-10*a**2*tan(e/2 + f*x/2)**4/(5*c**3*f*tan(e/2 + f*x/2)**5 - 25*
c**3*f*tan(e/2 + f*x/2)**4 + 50*c**3*f*tan(e/2 + f*x/2)**3 - 50*c**3*f*tan(
e/2 + f*x/2)**2 + 25*c**3*f*tan(e/2 + f*x/2) - 5*c**3*f) - 20*a**2*tan(e/2
+ f*x/2)**2/(5*c**3*f*tan(e/2 + f*x/2)**5 - 25*c**3*f*tan(e/2 + f*x/2)**4 +
 50*c**3*f*tan(e/2 + f*x/2)**3 - 50*c**3*f*tan(e/2 + f*x/2)**2 + 25*c**3*f*
tan(e/2 + f*x/2) - 5*c**3*f) - 2*a**2/(5*c**3*f*tan(e/2 + f*x/2)**5 - 25*c*
*3*f*tan(e/2 + f*x/2)**4 + 50*c**3*f*tan(e/2 + f*x/2)**3 - 50*c**3*f*tan(e/
2 + f*x/2)**2 + 25*c**3*f*tan(e/2 + f*x/2) - 5*c**3*f), Ne(f, 0)), (x*(a*si
n(e) + a)**2/(-c*sin(e) + c)**3, True))
```

3.244 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^4}\,dx$

Optimal. Leaf size=67

$$\frac{a^2c^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}+\frac{a^2c\cos^5(e+fx)}{35f(c-c\sin(e+fx))^5}$$

```
[Out] 1/7*a^2*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^6+1/35*a^2*c*cos(f*x+e)^5/f/(c-
c*sin(f*x+e))^5
```

Rubi [A] time = 0.14, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2672, 2671}

$$\frac{a^2c^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}+\frac{a^2c\cos^5(e+fx)}{35f(c-c\sin(e+fx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (a^2*c^2*Cos[e + f*x]^5)/(7*f*(c - c*Sin[e + f*x])^6) + (a^2*c*Cos[e + f*x]
^5)/(35*f*(c - c*Sin[e + f*x])^5)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
```

```
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^4}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^6}\,dx\\ &= \frac{a^2c^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}+\frac{1}{7}\left(a^2c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^5}\,dx\\ &= \frac{a^2c^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}+\frac{a^2c\cos^5(e+fx)}{35f(c-c\sin(e+fx))^5}\end{aligned}$$

Mathematica [A] time = 0.63, size = 117, normalized size = 1.75

$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-70\sin\left(\frac{1}{2}(e+fx)\right)-35\sin\left(\frac{3}{2}(e+fx)\right)+7\sin\left(\frac{5}{2}(e+fx)\right)-35\cos\right.}{140c^4f(\sin(e+fx)-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^4,x]
```

```
[Out] -1/140*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(-35*Cos[(e + f*x)/2] + 1
4*Cos[(3*(e + f*x))/2] + Cos[(7*(e + f*x))/2] - 70*Sin[(e + f*x)/2] - 35*Si
n[(3*(e + f*x))/2] + 7*Sin[(5*(e + f*x))/2]))/(c^4*f*(-1 + Sin[e + f*x])^4)
```

fricas [B] time = 0.47, size = 222, normalized size = 3.31

$$-\frac{a^2\cos\left(fx+e\right)^4+4\,a^2\cos\left(fx+e\right)^3+13\,a^2\cos\left(fx+e\right)^2-10\,a^2\cos\left(fx+e\right)-20\,a^2-\left(a^2\cos\left(fx+e\right)\right.}{35\left(c^4f\cos\left(fx+e\right)^4-3\,c^4f\cos\left(fx+e\right)^3-8\,c^4f\cos\left(fx+e\right)^2+4\,c^4f\cos\left(fx+e\right)+8\,c^4f+\left(c^4f\cos\left(fx+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] -1/35*(a^2*cos(f*x + e)^4 + 4*a^2*cos(f*x + e)^3 + 13*a^2*cos(f*x + e)^2 -
10*a^2*cos(f*x + e) - 20*a^2 - (a^2*cos(f*x + e)^3 - 3*a^2*cos(f*x + e)^2 +
 10*a^2*cos(f*x + e) + 20*a^2)*sin(f*x + e))/(c^4*f*cos(f*x + e)^4 - 3*c^4*
f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f
+ (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8
*c^4*f)*sin(f*x + e))
```

giac [A] time = 0.22, size = 128, normalized size = 1.91

$$-\frac{2\left(35\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-35\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+140\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-70\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+91\,a^{2}\tan\right.}{35\,c^{4}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -2/35*(35*a^2*tan(1/2*f*x + 1/2*e)^6 - 35*a^2*tan(1/2*f*x + 1/2*e)^5 + 140*
a^2*tan(1/2*f*x + 1/2*e)^4 - 70*a^2*tan(1/2*f*x + 1/2*e)^3 + 91*a^2*tan(1/2
*f*x + 1/2*e)^2 - 7*a^2*tan(1/2*f*x + 1/2*e) + 6*a^2)/(c^4*f*(tan(1/2*f*x +
 1/2*e) - 1)^7)
```

maple [A] time = 0.27, size = 118, normalized size = 1.76

$$\frac{2a^{2}\left(-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{5}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{14}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{32}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{16}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{128}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{24}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}\right.}{f\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x)
```

```
[Out] 2/f*a^2/c^4*(-1/(tan(1/2*f*x+1/2*e)-1)-5/(tan(1/2*f*x+1/2*e)-1)^2-14/(tan(1
/2*f*x+1/2*e)-1)^3-32/7/(tan(1/2*f*x+1/2*e)-1)^7-16/(tan(1/2*f*x+1/2*e)-1)^
6-128/5/(tan(1/2*f*x+1/2*e)-1)^5-24/(tan(1/2*f*x+1/2*e)-1)^4)
```

maxima [B] time = 0.57, size = 816, normalized size = 12.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 2/105*(2*a^2*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 13)/(c
^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e
```

```
) + 1)^7) - 3*a^2*(49*sin(f*x + e)/(cos(f*x + e) + 1) - 147*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 210*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 210*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 3
5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 12)/(c^4 - 7*c^4*sin(f*x + e)/(cos(
f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x
 + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) - 4*a^2*(14*sin(f*x
 + e)/(cos(f*x + e) + 1) - 42*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(
f*x + e)^3/(cos(f*x + e) + 1)^3 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
2)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*
x + e) + 1)^7))/f
```

mupad [B] time = 7.30, size = 99, normalized size = 1.48

$$\frac{\sqrt{2}\,a^2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{5\,\cos\left(3\,e+3\,f\,x\right)}{8}-\frac{105\,\sin\left(e+f\,x\right)}{8}-\frac{27\,\cos\left(2\,e+2\,f\,x\right)}{4}-\frac{121\,\cos\left(e+f\,x\right)}{8}+\frac{7\,\sin\left(2\,e+2\,f\,x\right)}{2}+\frac{7\,\sin\left(3\,e+3\,f\,x\right)}{8}\right.}{280\,c^4\,f\,\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{f\,x}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x))^4,x)
```

```
[Out] (2^(1/2)*a^2*cos(e/2 + (f*x)/2)*((5*cos(3*e + 3*f*x))/8 - (105*sin(e + f*x)
)/8 - (27*cos(2*e + 2*f*x))/4 - (121*cos(e + f*x))/8 + (7*sin(2*e + 2*f*x))
/2 + (7*sin(3*e + 3*f*x))/8 + 109/4))/(280*c^4*f*cos(e/2 + pi/4 + (f*x)/2)^
7)
```

sympy [A] time = 25.12, size = 1074, normalized size = 16.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-70*a**2*tan(e/2 + f*x/2)**6/(35*c**4*f*tan(e/2 + f*x/2)**7 - 24
5*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f
*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2
 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) + 70*a**2*tan(e/2 +
 f*x/2)**5/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6
+ 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c
```

```
**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan
(e/2 + f*x/2) - 35*c**4*f) - 280*a**2*tan(e/2 + f*x/2)**4/(35*c**4*f*tan(e/
2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2
)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 -
735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) +
 140*a**2*tan(e/2 + f*x/2)**3/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*t
an(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 +
 f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)*
*2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) - 182*a**2*tan(e/2 + f*x/2)**
2/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**
4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan
(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*
x/2) - 35*c**4*f) + 14*a**2*tan(e/2 + f*x/2)/(35*c**4*f*tan(e/2 + f*x/2)**7
 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c
**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*ta
n(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) - 12*a**2/(35*
c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*ta
n(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 +
 f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) -
 35*c**4*f), Ne(f, 0)), (x*(a*sin(e) + a)**2/(-c*sin(e) + c)**4, True))
```

3.245 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^5}\,dx$

Optimal. Leaf size=98

$$\frac{a^2c^2\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{2a^2\cos^5(e+fx)}{315f(c-c\sin(e+fx))^5}+\frac{2a^2c\cos^5(e+fx)}{63f(c-c\sin(e+fx))^6}$$

```
[Out] 1/9*a^2*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^7+2/63*a^2*c*cos(f*x+e)^5/f/(c-
c*sin(f*x+e))^6+2/315*a^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^5
```

Rubi [A] time = 0.18, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2672, 2671}

$$\frac{a^2c^2\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{2a^2\cos^5(e+fx)}{315f(c-c\sin(e+fx))^5}+\frac{2a^2c\cos^5(e+fx)}{63f(c-c\sin(e+fx))^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (a^2*c^2*Cos[e + f*x]^5)/(9*f*(c - c*Sin[e + f*x])^7) + (2*a^2*c*Cos[e + f*
x]^5)/(63*f*(c - c*Sin[e + f*x])^6) + (2*a^2*Cos[e + f*x]^5)/(315*f*(c - c*
Sin[e + f*x])^5)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
```

```
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^5}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^7}\,dx\\
&= \frac{a^2c^2\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{1}{9}\left(2a^2c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^6}\,dx\\
&= \frac{a^2c^2\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{2a^2c\cos^5(e+fx)}{63f(c-c\sin(e+fx))^6}+\frac{1}{63}\left(2a^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^5}\,d\\
&= \frac{a^2c^2\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{2a^2c\cos^5(e+fx)}{63f(c-c\sin(e+fx))^6}+\frac{2a^2\cos^5(e+fx)}{315f(c-c\sin(e+fx))^5}\end{aligned}$$

Mathematica [A] time = 0.58, size = 121, normalized size = 1.23

$$\frac{a^2\left(441\sin\left(\frac{1}{2}(e+fx)\right)+210\sin\left(\frac{3}{2}(e+fx)\right)-36\sin\left(\frac{5}{2}(e+fx)\right)+\sin\left(\frac{9}{2}(e+fx)\right)+315\cos\left(\frac{1}{2}(e+fx)\right)-126}{1260c^5f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (a^2*(315*Cos[(e + f*x)/2] - 126*Cos[(3*(e + f*x))/2] - 9*Cos[(7*(e + f*x))
/2] + 441*Sin[(e + f*x)/2] + 210*Sin[(3*(e + f*x))/2] - 36*Sin[(5*(e + f*x)
)/2] + Sin[(9*(e + f*x))/2]))/(1260*c^5*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)
/2])^9)
```

fricas [B] time = 0.44, size = 278, normalized size = 2.84

$$\frac{2\,a^2\cos\left(fx+e\right)^5-8\,a^2\cos\left(fx+e\right)^4-25\,a^2\cos\left(fx+e\right)^3-85\,a^2\cos\left(fx+e\right)^2+70\,a^2\cos\left(fx+e\right)+140}{315\left(c^5f\cos\left(fx+e\right)^5+5\,c^5f\cos\left(fx+e\right)^4-8\,c^5f\cos\left(fx+e\right)^3-20\,c^5f\cos\left(fx+e\right)^2+8\,c^5f\cos\left(fx+e\right)+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorithm="fricas")
```

[Out] 1/315*(2*a^2*cos(f*x + e)^5 - 8*a^2*cos(f*x + e)^4 - 25*a^2*cos(f*x + e)^3 - 85*a^2*cos(f*x + e)^2 + 70*a^2*cos(f*x + e) + 140*a^2 + (2*a^2*cos(f*x + e)^4 + 10*a^2*cos(f*x + e)^3 - 15*a^2*cos(f*x + e)^2 + 70*a^2*cos(f*x + e) + 140*a^2)*sin(f*x + e))/(c^5*f*cos(f*x + e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f)*sin(f*x + e))

giac [A] time = 0.27, size = 162, normalized size = 1.65

$$-\frac{2\left(315\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8-630\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+2310\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-2520\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 \cdots\right.}{315\,c^5 f\left(\tan \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorithm="giac")

[Out] -2/315*(315*a^2*tan(1/2*f*x + 1/2*e)^8 - 630*a^2*tan(1/2*f*x + 1/2*e)^7 + 2310*a^2*tan(1/2*f*x + 1/2*e)^6 - 2520*a^2*tan(1/2*f*x + 1/2*e)^5 + 3402*a^2*tan(1/2*f*x + 1/2*e)^4 - 1638*a^2*tan(1/2*f*x + 1/2*e)^3 + 1062*a^2*tan(1/2*f*x + 1/2*e)^2 - 108*a^2*tan(1/2*f*x + 1/2*e) + 47*a^2)/(c^5*f*(tan(1/2*f*x + 1/2*e) - 1)^9)

maple [A] time = 0.30, size = 148, normalized size = 1.51

$$\frac{2a^2\left(-\frac{50}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{64}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{272}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{6}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{64}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{48\cdots}{7\left(\tan\left(\frac{fx}{2}\cdots\right.\right.}\cdots\right.}{f\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x)

[Out] 2/f*a^2/c^5*(-50/(tan(1/2*f*x+1/2*e)-1)^4-64/9/(tan(1/2*f*x+1/2*e)-1)^9-272/3/(tan(1/2*f*x+1/2*e)-1)^6-6/(tan(1/2*f*x+1/2*e)-1)^2-1/(tan(1/2*f*x+1/2*e)-1)-64/3/(tan(1/2*f*x+1/2*e)-1)^3-480/7/(tan(1/2*f*x+1/2*e)-1)^7-404/5/(tan(1/2*f*x+1/2*e)-1)^5-32/(tan(1/2*f*x+1/2*e)-1)^8)

maxima [B] time = 1.19, size = 1073, normalized size = 10.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] -2/315*(a^2*(432*sin(f*x + e)/(cos(f*x + e) + 1) - 1728*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 3612*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5418*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 + 5040*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 33
60*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1260*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 - 315*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 83)/(c^5 - 9*c^5*sin(f*x
 + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*
c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9
*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e)
 + 1)^9) - 10*a^2*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 315*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 1
47*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^7/(cos(f*x + e) +
1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126
*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x +
 e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^
5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 14*a^2*(9*sin(f*x + e)/(cos(f*x +
e) + 1) - 36*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 54*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 - 81*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 45*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5 - 30*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1)/(c^5 -
9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*
c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x +
e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/
(cos(f*x + e) + 1)^9))/f
```

mupad [B] time = 8.81, size = 121, normalized size = 1.23

$$\frac{\sqrt{2}\,a^2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{89\,\cos\left(3\,e+3\,f\,x\right)}{4}-\frac{2205\,\sin\left(e+f\,x\right)}{8}-\frac{265\,\cos\left(2\,e+2\,f\,x\right)}{2}-\frac{625\,\cos\left(e+f\,x\right)}{4}+\frac{49\,\cos\left(4\,e+4\,f\,x\right)}{16}+\frac{567\,\sin\left(2\,e+2\right.}{8}\right.}{5040\,c^5\,f\,\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{f\,x}{2}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x))^5,x)
```

```
[Out] (2^(1/2)*a^2*cos(e/2 + (f*x)/2)*((89*cos(3*e + 3*f*x))/4 - (2205*sin(e + f*
x))/8 - (265*cos(2*e + 2*f*x))/2 - (625*cos(e + f*x))/4 + (49*cos(4*e + 4*f
```

```
*x))/16 + (567*sin(2*e + 2*f*x))/8 + (243*sin(3*e + 3*f*x))/8 - (45*sin(4*e
 + 4*f*x))/16 + 4967/16))/(5040*c^5*f*cos(e/2 + pi/4 + (f*x)/2)^9)
```

sympy [A] time = 50.69, size = 1717, normalized size = 17.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**5,x)

[Out] Piecewise((-630*a**2*tan(e/2 + f*x/2)**8/(315*c**5*f*tan(e/2 + f*x/2)**9 -
2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*
c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*
f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan
(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 1260*a**2*t
an(e/2 + f*x/2)**7/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 +
f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2
)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4
+ 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 283
5*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 4620*a**2*tan(e/2 + f*x/2)**6/(31
5*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5
*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*ta
n(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2
 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x
/2) - 315*c**5*f) + 5040*a**2*tan(e/2 + f*x/2)**5/(315*c**5*f*tan(e/2 + f*x
/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7
 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39
690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c
**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 68
04*a**2*tan(e/2 + f*x/2)**4/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*t
an(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/
2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f
*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)
**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 3276*a**2*tan(e/2 + f*x/
2)**3/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 1
1340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*
c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*
f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(
e/2 + f*x/2) - 315*c**5*f) - 2124*a**2*tan(e/2 + f*x/2)**2/(315*c**5*f*tan(
e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 +
f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2
)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3
- 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**
5*f) + 216*a**2*tan(e/2 + f*x/2)/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**
5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*t
```

```
an(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/
2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f
*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 94*a**2/(315*c**5*f
*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e
/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 +
f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2
)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 31
5*c**5*f), Ne(f, 0)), (x*(a*sin(e) + a)**2/(-c*sin(e) + c)**5, True))
```

3.246 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^6}\,dx$

Optimal. Leaf size=132

$$\frac{a^2c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}+\frac{2a^2\cos^5(e+fx)}{1155cf(c-c\sin(e+fx))^5}+\frac{2a^2\cos^5(e+fx)}{231f(c-c\sin(e+fx))^6}+\frac{a^2c\cos^5(e+fx)}{33f(c-c\sin(e+fx))^7}$$

```
[Out] 1/11*a^2*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^8+1/33*a^2*c*cos(f*x+e)^5/f/(c
-c*sin(f*x+e))^7+2/231*a^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^6+2/1155*a^2*cos
(f*x+e)^5/c/f/(c-c*sin(f*x+e))^5
```

Rubi [A] time = 0.23, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2672, 2671}

$$\frac{a^2c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}+\frac{2a^2\cos^5(e+fx)}{1155cf(c-c\sin(e+fx))^5}+\frac{2a^2\cos^5(e+fx)}{231f(c-c\sin(e+fx))^6}+\frac{a^2c\cos^5(e+fx)}{33f(c-c\sin(e+fx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^6,x]
```

```
[Out] (a^2*c^2*Cos[e + f*x]^5)/(11*f*(c - c*Sin[e + f*x])^8) + (a^2*c*Cos[e + f*x
]^5)/(33*f*(c - c*Sin[e + f*x])^7) + (2*a^2*Cos[e + f*x]^5)/(231*f*(c - c*S
in[e + f*x])^6) + (2*a^2*Cos[e + f*x]^5)/(1155*c*f*(c - c*Sin[e + f*x])^5)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^6}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^8}\,dx$$
$$= \frac{a^2c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}+\frac{1}{11}\left(3a^2c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^7}\,dx$$
$$= \frac{a^2c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}+\frac{a^2c\cos^5(e+fx)}{33f(c-c\sin(e+fx))^7}+\frac{1}{33}\left(2a^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^6}$$
$$= \frac{a^2c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}+\frac{a^2c\cos^5(e+fx)}{33f(c-c\sin(e+fx))^7}+\frac{2a^2\cos^5(e+fx)}{231f(c-c\sin(e+fx))^6}+$$
$$= \frac{a^2c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}+\frac{a^2c\cos^5(e+fx)}{33f(c-c\sin(e+fx))^7}+\frac{2a^2\cos^5(e+fx)}{231f(c-c\sin(e+fx))^6}+$$

Mathematica [A] time = 0.74, size = 133, normalized size = 1.01

$$\frac{a^2\left(2541\sin\left(\frac{1}{2}(e+fx)\right)+1155\sin\left(\frac{3}{2}(e+fx)\right)-165\sin\left(\frac{5}{2}(e+fx)\right)+11\sin\left(\frac{9}{2}(e+fx)\right)+2079\cos\left(\frac{1}{2}(e+fx)\right)\right.}{9240c^6f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^6,x]
```

```
[Out] (a^2*(2079*Cos[(e + f*x)/2] - 825*Cos[(3*(e + f*x))/2] - 55*Cos[(7*(e + f*x
))/2] + Cos[(11*(e + f*x))/2] + 2541*Sin[(e + f*x)/2] + 1155*Sin[(3*(e + f*
x))/2] - 165*Sin[(5*(e + f*x))/2] + 11*Sin[(9*(e + f*x))/2]))/(9240*c^6*f*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^11)
```

fricas [B] time = 0.44, size = 332, normalized size = 2.52

$$-\frac{2\,a^2\cos\left(fx+e\right)^6+12\,a^2\cos\left(fx+e\right)^5-25\,a^2\cos\left(fx+e\right)^4-70\,a^2\cos\left(fx+e\right)^3-245\,a^2\cos\left(fx+e\right)^2}{1155\left(c^6f\cos\left(fx+e\right)^6-5\,c^6f\cos\left(fx+e\right)^5-18\,c^6f\cos\left(fx+e\right)^4+20\,c^6f\cos\left(fx+e\right)^3+48\,c^6f\cos\left(fx+e\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^6,x, algorithm="fricas")
```

```
[Out] -1/1155*(2*a^2*cos(f*x + e)^6 + 12*a^2*cos(f*x + e)^5 - 25*a^2*cos(f*x + e)
^4 - 70*a^2*cos(f*x + e)^3 - 245*a^2*cos(f*x + e)^2 + 210*a^2*cos(f*x + e)
+ 420*a^2 - (2*a^2*cos(f*x + e)^5 - 10*a^2*cos(f*x + e)^4 - 35*a^2*cos(f*x
+ e)^3 + 35*a^2*cos(f*x + e)^2 - 210*a^2*cos(f*x + e) - 420*a^2)*sin(f*x +
e))/(c^6*f*cos(f*x + e)^6 - 5*c^6*f*cos(f*x + e)^5 - 18*c^6*f*cos(f*x + e)^
4 + 20*c^6*f*cos(f*x + e)^3 + 48*c^6*f*cos(f*x + e)^2 - 16*c^6*f*cos(f*x +
e) - 32*c^6*f + (c^6*f*cos(f*x + e)^5 + 6*c^6*f*cos(f*x + e)^4 - 12*c^6*f*c
os(f*x + e)^3 - 32*c^6*f*cos(f*x + e)^2 + 16*c^6*f*cos(f*x + e) + 32*c^6*f)
*sin(f*x + e))
```

giac [A] time = 0.26, size = 196, normalized size = 1.48

$$-\frac{2\left(1155\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}-3465\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}+13860\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-23100\,a^2\tan\left(\frac{1}{2}\,fx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^6,x, algorithm="giac")
```

```
[Out] -2/1155*(1155*a^2*tan(1/2*f*x + 1/2*e)^10 - 3465*a^2*tan(1/2*f*x + 1/2*e)^9
 + 13860*a^2*tan(1/2*f*x + 1/2*e)^8 - 23100*a^2*tan(1/2*f*x + 1/2*e)^7 + 37
422*a^2*tan(1/2*f*x + 1/2*e)^6 - 32802*a^2*tan(1/2*f*x + 1/2*e)^5 + 27060*a
^2*tan(1/2*f*x + 1/2*e)^4 - 11220*a^2*tan(1/2*f*x + 1/2*e)^3 + 4895*a^2*tan
(1/2*f*x + 1/2*e)^2 - 517*a^2*tan(1/2*f*x + 1/2*e) + 152*a^2)/(c^6*f*(tan(1
/2*f*x + 1/2*e) - 1)^11)
```

maple [A] time = 0.30, size = 178, normalized size = 1.35

$$\frac{2a^2\left(-\frac{512}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{7}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{30}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{128}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{11}}-\frac{2376}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{}{\left(\tan\left(\frac{f}{2}\right.\right.}\right.}{f\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^6,x)
```

```
[Out] 2/f*a^2/c^6*(-512/3/(tan(1/2*f*x+1/2*e)-1)^9-7/(tan(1/2*f*x+1/2*e)-1)^2-1/(
tan(1/2*f*x+1/2*e)-1)-30/(tan(1/2*f*x+1/2*e)-1)^3-128/11/(tan(1/2*f*x+1/2*e
)-1)^11-2376/7/(tan(1/2*f*x+1/2*e)-1)^7-88/(tan(1/2*f*x+1/2*e)-1)^4-932/5/(
```

```
tan(1/2*f*x+1/2*e)-1)^5-292/(tan(1/2*f*x+1/2*e)-1)^6-288/(tan(1/2*f*x+1/2*e
)-1)^8-64/(tan(1/2*f*x+1/2*e)-1)^10)
```

maxima [B] time = 2.36, size = 1332, normalized size = 10.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^6,x, algorithm="maxima")
```

```
[Out] -2/3465*(5*a^2*(913*sin(f*x + e)/(cos(f*x + e) + 1) - 4565*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 12540*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 25080*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33726*sin(f*x + e)^5/(cos(f*x + e) + 1)^
5 - 33726*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 23100*sin(f*x + e)^7/(cos(f
*x + e) + 1)^7 - 11550*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 3465*sin(f*x +
 e)^9/(cos(f*x + e) + 1)^9 - 693*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 14
6)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*
sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)
^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*
c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x +
 e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 6*a^2*(671*sin(f
*x + e)/(cos(f*x + e) + 1) - 2200*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 660
0*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10890*sin(f*x + e)^4/(cos(f*x + e)
+ 1)^4 + 15246*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 12936*sin(f*x + e)^6/(
cos(f*x + e) + 1)^6 + 9240*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3465*sin(f
*x + e)^8/(cos(f*x + e) + 1)^8 + 1155*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 -
 61)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^
6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x +
e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5
5*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x
 + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 4*a^2*(253*sin
(f*x + e)/(cos(f*x + e) + 1) - 1265*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2
640*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5280*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 + 5313*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 5313*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 + 2310*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 1155*sin(f*
x + e)^8/(cos(f*x + e) + 1)^8 - 23)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e
) + 1) + 55*c^6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*
c^6*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x +
 e) + 1)^6 - 330*c^6*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x
+ e)^8/(cos(f*x + e) + 1)^8 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 +
```

```
11*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x
 + e) + 1)^11))/f
```

mupad [B] time = 9.36, size = 143, normalized size = 1.08

$$-\frac{\sqrt{2}\,a^2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(697\,\cos\left(e+f\,x\right)+\frac{7623\,\sin\left(e+f\,x\right)}{4}+\frac{3977\,\cos\left(2\,e+2\,f\,x\right)}{4}-\frac{3203\,\cos\left(3\,e+3\,f\,x\right)}{16}-\frac{461\,\cos\left(4\,e+4\,f\,x\right)}{8}+\right.}{36960\,c^6\,f\,\cos\left(\frac{e}{2}+\frac{\pi}{4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x))^6,x)
```

```
[Out] -(2^(1/2)*a^2*cos(e/2 + (f*x)/2)*(697*cos(e + f*x) + (7623*sin(e + f*x))/4
+ (3977*cos(2*e + 2*f*x))/4 - (3203*cos(3*e + 3*f*x))/16 - (461*cos(4*e + 4
*f*x))/8 + (75*cos(5*e + 5*f*x))/16 - 462*sin(2*e + 2*f*x) - (4983*sin(3*e
+ 3*f*x))/16 + (187*sin(4*e + 4*f*x))/4 + (77*sin(5*e + 5*f*x))/16 - 12721/
8))/(36960*c^6*f*cos(e/2 + pi/4 + (f*x)/2)^11)
```

sympy [A] time = 84.94, size = 2509, normalized size = 19.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**6,x)
```

```
[Out] Piecewise((-2310*a**2*tan(e/2 + f*x/2)**10/(1155*c**6*f*tan(e/2 + f*x/2)**1
1 - 12705*c**6*f*tan(e/2 + f*x/2)**10 + 63525*c**6*f*tan(e/2 + f*x/2)**9 -
190575*c**6*f*tan(e/2 + f*x/2)**8 + 381150*c**6*f*tan(e/2 + f*x/2)**7 - 533
610*c**6*f*tan(e/2 + f*x/2)**6 + 533610*c**6*f*tan(e/2 + f*x/2)**5 - 381150
*c**6*f*tan(e/2 + f*x/2)**4 + 190575*c**6*f*tan(e/2 + f*x/2)**3 - 63525*c**
6*f*tan(e/2 + f*x/2)**2 + 12705*c**6*f*tan(e/2 + f*x/2) - 1155*c**6*f) + 69
30*a**2*tan(e/2 + f*x/2)**9/(1155*c**6*f*tan(e/2 + f*x/2)**11 - 12705*c**6*
f*tan(e/2 + f*x/2)**10 + 63525*c**6*f*tan(e/2 + f*x/2)**9 - 190575*c**6*f*t
an(e/2 + f*x/2)**8 + 381150*c**6*f*tan(e/2 + f*x/2)**7 - 533610*c**6*f*tan(
e/2 + f*x/2)**6 + 533610*c**6*f*tan(e/2 + f*x/2)**5 - 381150*c**6*f*tan(e/2
 + f*x/2)**4 + 190575*c**6*f*tan(e/2 + f*x/2)**3 - 63525*c**6*f*tan(e/2 + f
*x/2)**2 + 12705*c**6*f*tan(e/2 + f*x/2) - 1155*c**6*f) - 27720*a**2*tan(e/
2 + f*x/2)**8/(1155*c**6*f*tan(e/2 + f*x/2)**11 - 12705*c**6*f*tan(e/2 + f*
x/2)**10 + 63525*c**6*f*tan(e/2 + f*x/2)**9 - 190575*c**6*f*tan(e/2 + f*x/2
)**8 + 381150*c**6*f*tan(e/2 + f*x/2)**7 - 533610*c**6*f*tan(e/2 + f*x/2)**
6 + 533610*c**6*f*tan(e/2 + f*x/2)**5 - 381150*c**6*f*tan(e/2 + f*x/2)**4 +
 190575*c**6*f*tan(e/2 + f*x/2)**3 - 63525*c**6*f*tan(e/2 + f*x/2)**2 + 127
05*c**6*f*tan(e/2 + f*x/2) - 1155*c**6*f) + 46200*a**2*tan(e/2 + f*x/2)**7/
```

(1155*c**6*f*tan(e/2 + f*x/2)**11 - 12705*c**6*f*tan(e/2 + f*x/2)**10 + 635
25*c**6*f*tan(e/2 + f*x/2)**9 - 190575*c**6*f*tan(e/2 + f*x/2)**8 + 381150*
c**6*f*tan(e/2 + f*x/2)**7 - 533610*c**6*f*tan(e/2 + f*x/2)**6 + 533610*c**
6*f*tan(e/2 + f*x/2)**5 - 381150*c**6*f*tan(e/2 + f*x/2)**4 + 190575*c**6*f
*tan(e/2 + f*x/2)**3 - 63525*c**6*f*tan(e/2 + f*x/2)**2 + 12705*c**6*f*tan(
e/2 + f*x/2) - 1155*c**6*f) - 74844*a**2*tan(e/2 + f*x/2)**6/(1155*c**6*f*t
an(e/2 + f*x/2)**11 - 12705*c**6*f*tan(e/2 + f*x/2)**10 + 63525*c**6*f*tan(
e/2 + f*x/2)**9 - 190575*c**6*f*tan(e/2 + f*x/2)**8 + 381150*c**6*f*tan(e/2
 + f*x/2)**7 - 533610*c**6*f*tan(e/2 + f*x/2)**6 + 533610*c**6*f*tan(e/2 +
f*x/2)**5 - 381150*c**6*f*tan(e/2 + f*x/2)**4 + 190575*c**6*f*tan(e/2 + f*x
/2)**3 - 63525*c**6*f*tan(e/2 + f*x/2)**2 + 12705*c**6*f*tan(e/2 + f*x/2) -
 1155*c**6*f) + 65604*a**2*tan(e/2 + f*x/2)**5/(1155*c**6*f*tan(e/2 + f*x/2
)**11 - 12705*c**6*f*tan(e/2 + f*x/2)**10 + 63525*c**6*f*tan(e/2 + f*x/2)**
9 - 190575*c**6*f*tan(e/2 + f*x/2)**8 + 381150*c**6*f*tan(e/2 + f*x/2)**7 -
 533610*c**6*f*tan(e/2 + f*x/2)**6 + 533610*c**6*f*tan(e/2 + f*x/2)**5 - 38
1150*c**6*f*tan(e/2 + f*x/2)**4 + 190575*c**6*f*tan(e/2 + f*x/2)**3 - 63525
*c**6*f*tan(e/2 + f*x/2)**2 + 12705*c**6*f*tan(e/2 + f*x/2) - 1155*c**6*f)
 - 54120*a**2*tan(e/2 + f*x/2)**4/(1155*c**6*f*tan(e/2 + f*x/2)**11 - 12705*
c**6*f*tan(e/2 + f*x/2)**10 + 63525*c**6*f*tan(e/2 + f*x/2)**9 - 190575*c**
6*f*tan(e/2 + f*x/2)**8 + 381150*c**6*f*tan(e/2 + f*x/2)**7 - 533610*c**6*f
*tan(e/2 + f*x/2)**6 + 533610*c**6*f*tan(e/2 + f*x/2)**5 - 381150*c**6*f*ta
n(e/2 + f*x/2)**4 + 190575*c**6*f*tan(e/2 + f*x/2)**3 - 63525*c**6*f*tan(e/
2 + f*x/2)**2 + 12705*c**6*f*tan(e/2 + f*x/2) - 1155*c**6*f) + 22440*a**2*t
an(e/2 + f*x/2)**3/(1155*c**6*f*tan(e/2 + f*x/2)**11 - 12705*c**6*f*tan(e/2
 + f*x/2)**10 + 63525*c**6*f*tan(e/2 + f*x/2)**9 - 190575*c**6*f*tan(e/2 +
f*x/2)**8 + 381150*c**6*f*tan(e/2 + f*x/2)**7 - 533610*c**6*f*tan(e/2 + f*x
/2)**6 + 533610*c**6*f*tan(e/2 + f*x/2)**5 - 381150*c**6*f*tan(e/2 + f*x/2)
4 + 190575*c6*f*tan(e/2 + f*x/2)**3 - 63525*c**6*f*tan(e/2 + f*x/2)**2
 + 12705*c**6*f*tan(e/2 + f*x/2) - 1155*c**6*f) - 9790*a**2*tan(e/2 + f*x/2)
2/(1155*c6*f*tan(e/2 + f*x/2)**11 - 12705*c**6*f*tan(e/2 + f*x/2)**10 +
 63525*c**6*f*tan(e/2 + f*x/2)**9 - 190575*c**6*f*tan(e/2 + f*x/2)**8 + 381
150*c**6*f*tan(e/2 + f*x/2)**7 - 533610*c**6*f*tan(e/2 + f*x/2)**6 + 533610
*c**6*f*tan(e/2 + f*x/2)**5 - 381150*c**6*f*tan(e/2 + f*x/2)**4 + 190575*c*
*6*f*tan(e/2 + f*x/2)**3 - 63525*c**6*f*tan(e/2 + f*x/2)**2 + 12705*c**6*f*
tan(e/2 + f*x/2) - 1155*c**6*f) + 1034*a**2*tan(e/2 + f*x/2)/(1155*c**6*f*t
an(e/2 + f*x/2)**11 - 12705*c**6*f*tan(e/2 + f*x/2)**10 + 63525*c**6*f*tan(
e/2 + f*x/2)**9 - 190575*c**6*f*tan(e/2 + f*x/2)**8 + 381150*c**6*f*tan(e/2
 + f*x/2)**7 - 533610*c**6*f*tan(e/2 + f*x/2)**6 + 533610*c**6*f*tan(e/2 +
f*x/2)**5 - 381150*c**6*f*tan(e/2 + f*x/2)**4 + 190575*c**6*f*tan(e/2 + f*x
/2)**3 - 63525*c**6*f*tan(e/2 + f*x/2)**2 + 12705*c**6*f*tan(e/2 + f*x/2) -
 1155*c**6*f) - 304*a**2/(1155*c**6*f*tan(e/2 + f*x/2)**11 - 12705*c**6*f*t
an(e/2 + f*x/2)**10 + 63525*c**6*f*tan(e/2 + f*x/2)**9 - 190575*c**6*f*tan(
e/2 + f*x/2)**8 + 381150*c**6*f*tan(e/2 + f*x/2)**7 - 533610*c**6*f*tan(e/2
 + f*x/2)**6 + 533610*c**6*f*tan(e/2 + f*x/2)**5 - 381150*c**6*f*tan(e/2 +
f*x/2)**4 + 190575*c**6*f*tan(e/2 + f*x/2)**3 - 63525*c**6*f*tan(e/2 + f*x/

```
2)**2 + 12705*c**6*f*tan(e/2 + f*x/2) - 1155*c**6*f), Ne(f, 0)), (x*(a*sin(
e) + a)**2/(-c*sin(e) + c)**6, True))
```

3.247 $\int (a + a\sin(e + fx))^3(c - c\sin(e + fx))^6 \, dx$

Optimal. Leaf size=180

$$\frac{11a^3c^6\cos^7(e+fx)}{56f}+\frac{11a^3\cos^7(e+fx)\left(c^6-c^6\sin(e+fx)\right)}{72f}+\frac{11a^3c^6\sin(e+fx)\cos^5(e+fx)}{48f}+\frac{55a^3c^6\sin(e+fx}{192}$$

```
[Out] 55/128*a^3*c^6*x+11/56*a^3*c^6*cos(f*x+e)^7/f+55/128*a^3*c^6*cos(f*x+e)*sin
(f*x+e)/f+55/192*a^3*c^6*cos(f*x+e)^3*sin(f*x+e)/f+11/48*a^3*c^6*cos(f*x+e)
^5*sin(f*x+e)/f+1/9*a^3*cos(f*x+e)^7*(c^3-c^3*sin(f*x+e))^2/f+11/72*a^3*cos
(f*x+e)^7*(c^6-c^6*sin(f*x+e))/f
```

Rubi [A] time = 0.21, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2736, 2678, 2669, 2635, 8}

$$\frac{11a^3c^6\cos^7(e+fx)}{56f}+\frac{a^3\cos^7(e+fx)\left(c^3-c^3\sin(e+fx)\right)^2}{9f}+\frac{11a^3\cos^7(e+fx)\left(c^6-c^6\sin(e+fx)\right)}{72f}+\frac{11a^3c^6\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^6,x]
```

```
[Out] (55*a^3*c^6*x)/128 + (11*a^3*c^6*Cos[e + f*x]^7)/(56*f) + (55*a^3*c^6*Cos[e
 + f*x]*Sin[e + f*x])/(128*f) + (55*a^3*c^6*Cos[e + f*x]^3*Sin[e + f*x])/(1
92*f) + (11*a^3*c^6*Cos[e + f*x]^5*Sin[e + f*x])/(48*f) + (a^3*Cos[e + f*x]
^7*(c^3 - c^3*Sin[e + f*x])^2)/(9*f) + (11*a^3*Cos[e + f*x]^7*(c^6 - c^6*Si
n[e + f*x]))/(72*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
```

```
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^6\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)(c-c\sin(e+fx))^3\,dx$$

$$= \frac{a^3\cos^7(e+fx)\left(c^3-c^3\sin(e+fx)\right)^2}{9f} + \frac{1}{9}\left(11a^3c^4\right)\int \cos^6(e+fx$$

$$= \frac{a^3\cos^7(e+fx)\left(c^3-c^3\sin(e+fx)\right)^2}{9f} + \frac{11a^3\cos^7(e+fx)\left(c^6-c^6\right.}{72f}$$

$$= \frac{11a^3c^6\cos^7(e+fx)}{56f} + \frac{a^3\cos^7(e+fx)\left(c^3-c^3\sin(e+fx)\right)^2}{9f} + \frac{11}{}$$

$$= \frac{11a^3c^6\cos^7(e+fx)}{56f} + \frac{11a^3c^6\cos^5(e+fx)\sin(e+fx)}{48f} + \frac{a^3\cos^7(}{}$$

$$= \frac{11a^3c^6\cos^7(e+fx)}{56f} + \frac{55a^3c^6\cos^3(e+fx)\sin(e+fx)}{192f} + \frac{11a^3c^6\,c}{}$$

$$= \frac{11a^3c^6\cos^7(e+fx)}{56f} + \frac{55a^3c^6\cos(e+fx)\sin(e+fx)}{128f} + \frac{55a^3c^6\,c}{}$$

$$= \frac{55}{128}a^3c^6x + \frac{11a^3c^6\cos^7(e+fx)}{56f} + \frac{55a^3c^6\cos(e+fx)\sin(e+fx)}{128f}$$

Mathematica [A] time = 2.14, size = 109, normalized size = 0.61

$$\frac{a^3c^6(18144\sin(2(e+fx))+1512\sin(4(e+fx))-672\sin(6(e+fx))-189\sin(8(e+fx))+16632\cos(e+fx)+}{64512}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^6,x]
```

```
[Out] (a^3*c^6*(27720*e + 27720*f*x + 16632*Cos[e + f*x] + 9744*Cos[3*(e + f*x)]
+ 3024*Cos[5*(e + f*x)] + 324*Cos[7*(e + f*x)] - 28*Cos[9*(e + f*x)] + 1814
4*Sin[2*(e + f*x)] + 1512*Sin[4*(e + f*x)] - 672*Sin[6*(e + f*x)] - 189*Sin
[8*(e + f*x)]))/(64512*f)
```

fricas [A] time = 0.49, size = 119, normalized size = 0.66

$$-\frac{896\,a^3c^6\cos\left(fx+e\right)^9-4608\,a^3c^6\cos\left(fx+e\right)^7-3465\,a^3c^6fx+21\left(144\,a^3c^6\cos\left(fx+e\right)^7-88\,a^3c^6\cos\left(fx+\right.\right.}{8064\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^6,x, algorithm="fricas")
```

```
[Out] -1/8064*(896*a^3*c^6*cos(f*x + e)^9 - 4608*a^3*c^6*cos(f*x + e)^7 - 3465*a^
3*c^6*f*x + 21*(144*a^3*c^6*cos(f*x + e)^7 - 88*a^3*c^6*cos(f*x + e)^5 - 11
0*a^3*c^6*cos(f*x + e)^3 - 165*a^3*c^6*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.29, size = 196, normalized size = 1.09

$$\frac{55}{128}a^3c^6x-\frac{a^3c^6\cos\left(9fx+9e\right)}{2304f}+\frac{9a^3c^6\cos\left(7fx+7e\right)}{1792f}+\frac{3a^3c^6\cos\left(5fx+5e\right)}{64f}+\frac{29a^3c^6\cos\left(3fx+3e\right)}{192f}+\frac{33}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^6,x, algorithm="giac")
```

```
[Out] 55/128*a^3*c^6*x - 1/2304*a^3*c^6*cos(9*f*x + 9*e)/f + 9/1792*a^3*c^6*cos(7
*f*x + 7*e)/f + 3/64*a^3*c^6*cos(5*f*x + 5*e)/f + 29/192*a^3*c^6*cos(3*f*x
+ 3*e)/f + 33/128*a^3*c^6*cos(f*x + e)/f - 3/1024*a^3*c^6*sin(8*f*x + 8*e)/
f - 1/96*a^3*c^6*sin(6*f*x + 6*e)/f + 3/128*a^3*c^6*sin(4*f*x + 4*e)/f + 9/
32*a^3*c^6*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.41, size = 297, normalized size = 1.65

$$-\frac{c^6a^3\left(\frac{128}{35}+\sin^8\left(fx+e\right)+\frac{8\left(\sin^6\left(fx+e\right)\right)}{7}+\frac{48\left(\sin^4\left(fx+e\right)\right)}{35}+\frac{64\left(\sin^2\left(fx+e\right)\right)}{35}\right)\cos\left(fx+e\right)}{9}-3c^6a^3\left(-\frac{\left(\sin^7\left(fx+e\right)+\frac{7\left(\sin^5\left(fx+e\right)\right)}{6}+\frac{35\left(\sin^3\left(fx+e\right)\right)}{24}+\frac{35\sin(f}{16}\right.}{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^6,x)
```

```
[Out] 1/f*(-1/9*c^6*a^3*(128/35+sin(f*x+e)^8+8/7*sin(f*x+e)^6+48/35*sin(f*x+e)^4+
64/35*sin(f*x+e)^2)*cos(f*x+e)-3*c^6*a^3*(-1/8*(sin(f*x+e)^7+7/6*sin(f*x+e)
^5+35/24*sin(f*x+e)^3+35/16*sin(f*x+e))*cos(f*x+e)+35/128*f*x+35/128*e)+8*c
^6*a^3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/1
6*f*x+5/16*e)+6/5*c^6*a^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-6*
c^6*a^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-8/3*c
^6*a^3*(2+sin(f*x+e)^2)*cos(f*x+e)+3*c^6*a^3*cos(f*x+e)+c^6*a^3*(f*x+e))
```

maxima [A] time = 0.33, size = 301, normalized size = 1.67

$$-\frac{1024\left(35\cos\left(fx+e\right)^9-180\cos\left(fx+e\right)^7+378\cos\left(fx+e\right)^5-420\cos\left(fx+e\right)^3+315\cos\left(fx+e\right)\right)a^3c^6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^6,x, algorithm="maxima")
```

```
[Out] -1/322560*(1024*(35*cos(f*x + e)^9 - 180*cos(f*x + e)^7 + 378*cos(f*x + e)^
5 - 420*cos(f*x + e)^3 + 315*cos(f*x + e))*a^3*c^6 - 129024*(3*cos(f*x + e)
^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a^3*c^6 - 860160*(cos(f*x + e)^3
- 3*cos(f*x + e))*a^3*c^6 + 315*(128*sin(2*f*x + 2*e)^3 + 840*f*x + 840*e +
 3*sin(8*f*x + 8*e) + 168*sin(4*f*x + 4*e) - 768*sin(2*f*x + 2*e))*a^3*c^6
- 13440*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin
(2*f*x + 2*e))*a^3*c^6 + 60480*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*
f*x + 2*e))*a^3*c^6 - 322560*(f*x + e)*a^3*c^6 - 967680*a^3*c^6*cos(f*x + e
))/f
```

mupad [B] time = 9.31, size = 403, normalized size = 2.24

$$a^3\,c^6\,\left(3465\,e+9198\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+18432\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+79716\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+138240\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-4284\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^6,x)
```

```
[Out] (a^3*c^6*(3465*e + 9198*tan(e/2 + (f*x)/2) + 18432*tan(e/2 + (f*x)/2)^2 + 7
9716*tan(e/2 + (f*x)/2)^3 + 138240*tan(e/2 + (f*x)/2)^4 - 4284*tan(e/2 + (f
*x)/2)^5 + 387072*tan(e/2 + (f*x)/2)^6 + 176148*tan(e/2 + (f*x)/2)^7 + 2903
04*tan(e/2 + (f*x)/2)^8 + 645120*tan(e/2 + (f*x)/2)^10 - 176148*tan(e/2 + (
f*x)/2)^11 + 236544*tan(e/2 + (f*x)/2)^12 + 4284*tan(e/2 + (f*x)/2)^13 + 12
9024*tan(e/2 + (f*x)/2)^14 - 79716*tan(e/2 + (f*x)/2)^15 + 48384*tan(e/2 +
(f*x)/2)^16 - 9198*tan(e/2 + (f*x)/2)^17 + 3465*f*x + 31185*tan(e/2 + (f*x)
/2)^2*(e + f*x) + 124740*tan(e/2 + (f*x)/2)^4*(e + f*x) + 291060*tan(e/2 +
(f*x)/2)^6*(e + f*x) + 436590*tan(e/2 + (f*x)/2)^8*(e + f*x) + 436590*tan(e
/2 + (f*x)/2)^10*(e + f*x) + 291060*tan(e/2 + (f*x)/2)^12*(e + f*x) + 12474
0*tan(e/2 + (f*x)/2)^14*(e + f*x) + 31185*tan(e/2 + (f*x)/2)^16*(e + f*x) +
 3465*tan(e/2 + (f*x)/2)^18*(e + f*x) + 7424))/(8064*f*(tan(e/2 + (f*x)/2)^
2 + 1)^9)
```

sympy [A] time = 26.39, size = 838, normalized size = 4.66

$$\begin{cases} -\frac{105a^3c^6x\sin^8\left(e+fx\right)}{128}-\frac{105a^3c^6x\sin^6\left(e+fx\right)\cos^2\left(e+fx\right)}{32}+\frac{5a^3c^6x\sin^6\left(e+fx\right)}{2}-\frac{315a^3c^6x\sin^4\left(e+fx\right)\cos^4\left(e+fx\right)}{64}+\frac{15a^3c^6x\sin^4\left(e+fx\right)c}{2} \\ x\left(a\sin\left(e\right)+a\right)^3\left(-c\sin\left(e\right)+c\right)^6 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**6,x)
```

```
[Out] Piecewise((-105*a**3*c**6*x*sin(e + f*x)**8/128 - 105*a**3*c**6*x*sin(e + f
*x)**6*cos(e + f*x)**2/32 + 5*a**3*c**6*x*sin(e + f*x)**6/2 - 315*a**3*c**6
*x*sin(e + f*x)**4*cos(e + f*x)**4/64 + 15*a**3*c**6*x*sin(e + f*x)**4*cos(
e + f*x)**2/2 - 9*a**3*c**6*x*sin(e + f*x)**4/4 - 105*a**3*c**6*x*sin(e + f
*x)**2*cos(e + f*x)**6/32 + 15*a**3*c**6*x*sin(e + f*x)**2*cos(e + f*x)**4/
2 - 9*a**3*c**6*x*sin(e + f*x)**2*cos(e + f*x)**2/2 - 105*a**3*c**6*x*cos(e
 + f*x)**8/128 + 5*a**3*c**6*x*cos(e + f*x)**6/2 - 9*a**3*c**6*x*cos(e + f*
x)**4/4 + a**3*c**6*x - a**3*c**6*sin(e + f*x)**8*cos(e + f*x)/f + 279*a**3
*c**6*sin(e + f*x)**7*cos(e + f*x)/(128*f) - 8*a**3*c**6*sin(e + f*x)**6*co
s(e + f*x)**3/(3*f) + 511*a**3*c**6*sin(e + f*x)**5*cos(e + f*x)**3/(128*f)
 - 11*a**3*c**6*sin(e + f*x)**5*cos(e + f*x)/(2*f) - 16*a**3*c**6*sin(e + f
*x)**4*cos(e + f*x)**5/(5*f) + 6*a**3*c**6*sin(e + f*x)**4*cos(e + f*x)/f +
 385*a**3*c**6*sin(e + f*x)**3*cos(e + f*x)**5/(128*f) - 20*a**3*c**6*sin(e
 + f*x)**3*cos(e + f*x)**3/(3*f) + 15*a**3*c**6*sin(e + f*x)**3*cos(e + f*x
)/(4*f) - 64*a**3*c**6*sin(e + f*x)**2*cos(e + f*x)**7/(35*f) + 8*a**3*c**6
*sin(e + f*x)**2*cos(e + f*x)**3/f - 8*a**3*c**6*sin(e + f*x)**2*cos(e + f*
x)/f + 105*a**3*c**6*sin(e + f*x)*cos(e + f*x)**7/(128*f) - 5*a**3*c**6*sin
(e + f*x)*cos(e + f*x)**5/(2*f) + 9*a**3*c**6*sin(e + f*x)*cos(e + f*x)**3/
(4*f) - 128*a**3*c**6*cos(e + f*x)**9/(315*f) + 16*a**3*c**6*cos(e + f*x)**
5/(5*f) - 16*a**3*c**6*cos(e + f*x)**3/(3*f) + 3*a**3*c**6*cos(e + f*x)/f,
Ne(f, 0)), (x*(a*sin(e) + a)**3*(-c*sin(e) + c)**6, True))
```

3.248 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^5\,dx$

Optimal. Leaf size=145

$$\frac{9a^3c^5\cos^7(e+fx)}{56f}+\frac{a^3\cos^7(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{8f}+\frac{3a^3c^5\sin(e+fx)\cos^5(e+fx)}{16f}+\frac{15a^3c^5\sin(e+fx)\text{co}}{64f}$$

```
[Out] 45/128*a^3*c^5*x+9/56*a^3*c^5*cos(f*x+e)^7/f+45/128*a^3*c^5*cos(f*x+e)*sin(
f*x+e)/f+15/64*a^3*c^5*cos(f*x+e)^3*sin(f*x+e)/f+3/16*a^3*c^5*cos(f*x+e)^5*
sin(f*x+e)/f+1/8*a^3*cos(f*x+e)^7*(c^5-c^5*sin(f*x+e))/f
```

Rubi [A] time = 0.16, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2736, 2678, 2669, 2635, 8}

$$\frac{9a^3c^5\cos^7(e+fx)}{56f}+\frac{a^3\cos^7(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{8f}+\frac{3a^3c^5\sin(e+fx)\cos^5(e+fx)}{16f}+\frac{15a^3c^5\sin(e+fx)\text{co}}{64f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (45*a^3*c^5*x)/128 + (9*a^3*c^5*Cos[e + f*x]^7)/(56*f) + (45*a^3*c^5*Cos[e
+ f*x]*Sin[e + f*x])/(128*f) + (15*a^3*c^5*Cos[e + f*x]^3*Sin[e + f*x])/(64
*f) + (3*a^3*c^5*Cos[e + f*x]^5*Sin[e + f*x])/(16*f) + (a^3*Cos[e + f*x]^7*
(c^5 - c^5*Sin[e + f*x]))/(8*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^5\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)(c-c\sin(e+fx))^2\,dx$$

$$= \frac{a^3\cos^7(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{8f}+\frac{1}{8}\left(9a^3c^4\right)\int \cos^6(e+fx$$

$$= \frac{9a^3c^5\cos^7(e+fx)}{56f}+\frac{a^3\cos^7(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{8f}+\frac{1}{8}\Big($$

$$= \frac{9a^3c^5\cos^7(e+fx)}{56f}+\frac{3a^3c^5\cos^5(e+fx)\sin(e+fx)}{16f}+\frac{a^3\cos^7(e$$

$$= \frac{9a^3c^5\cos^7(e+fx)}{56f}+\frac{15a^3c^5\cos^3(e+fx)\sin(e+fx)}{64f}+\frac{3a^3c^5\cos$$

$$= \frac{9a^3c^5\cos^7(e+fx)}{56f}+\frac{45a^3c^5\cos(e+fx)\sin(e+fx)}{128f}+\frac{15a^3c^5\cos$$

$$= \frac{45}{128}a^3c^5x+\frac{9a^3c^5\cos^7(e+fx)}{56f}+\frac{45a^3c^5\cos(e+fx)\sin(e+fx$$

Mathematica [A] time = 1.21, size = 89, normalized size = 0.61

$$\frac{a^3c^5(1792\sin(2(e+fx))+280\sin(4(e+fx))-7\sin(8(e+fx))+1120\cos(e+fx)+672\cos(3(e+fx))+224}{7168f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (a^3*c^5*(2520*e + 2520*f*x + 1120*Cos[e + f*x] + 672*Cos[3*(e + f*x)] + 22
4*Cos[5*(e + f*x)] + 32*Cos[7*(e + f*x)] + 1792*Sin[2*(e + f*x)] + 280*Sin[
4*(e + f*x)] - 7*Sin[8*(e + f*x)]))/(7168*f)
```

fricas [A] time = 0.47, size = 103, normalized size = 0.71

$$\frac{256\,a^3c^5\cos\left(fx+e\right)^7+315\,a^3c^5fx-7\left(16\,a^3c^5\cos\left(fx+e\right)^7-24\,a^3c^5\cos\left(fx+e\right)^5-30\,a^3c^5\cos\left(fx+e\right)^3-4}{896\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] 1/896*(256*a^3*c^5*cos(f*x + e)^7 + 315*a^3*c^5*f*x - 7*(16*a^3*c^5*cos(f*x
 + e)^7 - 24*a^3*c^5*cos(f*x + e)^5 - 30*a^3*c^5*cos(f*x + e)^3 - 45*a^3*c^
5*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.26, size = 154, normalized size = 1.06

$$\frac{45}{128}\,a^3c^5x+\frac{a^3c^5\cos\left(7\,fx+7\,e\right)}{224\,f}+\frac{a^3c^5\cos\left(5\,fx+5\,e\right)}{32\,f}+\frac{3\,a^3c^5\cos\left(3\,fx+3\,e\right)}{32\,f}+\frac{5\,a^3c^5\cos\left(fx+e\right)}{32\,f}-\frac{a^3c^5\sin\left(8}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^5,x, algorithm="giac")
```

```
[Out] 45/128*a^3*c^5*x + 1/224*a^3*c^5*cos(7*f*x + 7*e)/f + 1/32*a^3*c^5*cos(5*f*
x + 5*e)/f + 3/32*a^3*c^5*cos(3*f*x + 3*e)/f + 5/32*a^3*c^5*cos(f*x + e)/f
- 1/1024*a^3*c^5*sin(8*f*x + 8*e)/f + 5/128*a^3*c^5*sin(4*f*x + 4*e)/f + 1/
4*a^3*c^5*sin(2*f*x + 2*e)/f
```

maple [B] time = 0.43, size = 276, normalized size = 1.90

$$\frac{-c^5a^3\left(-\frac{\left(\sin^7\left(fx+e\right)+\frac{7\left(\sin^5\left(fx+e\right)\right)}{6}+\frac{35\left(\sin^3\left(fx+e\right)\right)}{24}+\frac{35\sin\left(fx+e\right)}{16}\right)\cos\left(fx+e\right)}{8}+\frac{35fx}{128}+\frac{35e}{128}\right)-\frac{2c^5a^3\left(\frac{16}{5}+\sin^6\left(fx+e\right)+\frac{6\left(\sin^4\left(fx+e\right)\right)}{5}+\frac{8\left(\sin^2\left(fx+\right.\right.}{5}}{7}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^5,x)
```

```
[Out] 1/f*(-c^5*a^3*(-1/8*(sin(f*x+e)^7+7/6*sin(f*x+e)^5+35/24*sin(f*x+e)^3+35/16
*sin(f*x+e))*cos(f*x+e)+35/128*f*x+35/128*e)-2/7*c^5*a^3*(16/5+sin(f*x+e)^6
+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)+2*c^5*a^3*(-1/6*(sin(f*x+e)^
5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+6/5*c^5*a^3
*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-2*c^5*a^3*(2+sin(f*x+e)^2)*
cos(f*x+e)-2*c^5*a^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+2*c^5*a^3*c
os(f*x+e)+c^5*a^3*(f*x+e))
```

maxima [B] time = 0.77, size = 282, normalized size = 1.94

$$\frac{6144\left(5\cos\left(fx+e\right)^{7}-21\cos\left(fx+e\right)^{5}+35\cos\left(fx+e\right)^{3}-35\cos\left(fx+e\right)\right)a^{3}c^{5}+43008\left(3\cos\left(fx+e\right)^{5}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] 1/107520*(6144*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 -
35*cos(f*x + e))*a^3*c^5 + 43008*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15
*cos(f*x + e))*a^3*c^5 + 215040*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3*c^5 -
 35*(128*sin(2*f*x + 2*e)^3 + 840*f*x + 840*e + 3*sin(8*f*x + 8*e) + 168*si
n(4*f*x + 4*e) - 768*sin(2*f*x + 2*e))*a^3*c^5 + 1120*(4*sin(2*f*x + 2*e)^3
 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*a^3*c^5 - 5376
0*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^3*c^5 + 107520*(f*x + e)*a^3*c^5 + 215
040*a^3*c^5*cos(f*x + e))/f
```

mupad [B] time = 9.12, size = 372, normalized size = 2.57

$$\frac{a^{3}c^{5}\left(\frac{315e}{2}+581\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+256\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}+2065\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{3}+5376\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}+21\tan\left(\frac{e}{2}+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^5,x)
```

```
[Out] (a^3*c^5*((315*e)/2 + 581*tan(e/2 + (f*x)/2) + 256*tan(e/2 + (f*x)/2)^2 + 2
065*tan(e/2 + (f*x)/2)^3 + 5376*tan(e/2 + (f*x)/2)^4 + 21*tan(e/2 + (f*x)/2
)^5 + 5376*tan(e/2 + (f*x)/2)^6 + 5705*tan(e/2 + (f*x)/2)^7 + 8960*tan(e/2
+ (f*x)/2)^8 - 5705*tan(e/2 + (f*x)/2)^9 + 8960*tan(e/2 + (f*x)/2)^10 - 21*
tan(e/2 + (f*x)/2)^11 + 1792*tan(e/2 + (f*x)/2)^12 - 2065*tan(e/2 + (f*x)/2
)^13 + 1792*tan(e/2 + (f*x)/2)^14 - 581*tan(e/2 + (f*x)/2)^15 + (315*f*x)/2
 + 1260*tan(e/2 + (f*x)/2)^2*(e + f*x) + 4410*tan(e/2 + (f*x)/2)^4*(e + f*x
```

```
) + 8820*tan(e/2 + (f*x)/2)^6*(e + f*x) + 11025*tan(e/2 + (f*x)/2)^8*(e + f
*x) + 8820*tan(e/2 + (f*x)/2)^10*(e + f*x) + 4410*tan(e/2 + (f*x)/2)^12*(e
+ f*x) + 1260*tan(e/2 + (f*x)/2)^14*(e + f*x) + (315*tan(e/2 + (f*x)/2)^16*
(e + f*x))/2 + 256))/(448*f*(tan(e/2 + (f*x)/2)^2 + 1)^8)
```

sympy [A] time = 15.63, size = 740, normalized size = 5.10

$$\begin{cases} -\frac{35a^3c^5x\sin^8\left(e+fx\right)}{128} - \frac{35a^3c^5x\sin^6\left(e+fx\right)\cos^2\left(e+fx\right)}{32} + \frac{5a^3c^5x\sin^6\left(e+fx\right)}{8} - \frac{105a^3c^5x\sin^4\left(e+fx\right)\cos^4\left(e+fx\right)}{64} + \frac{15a^3c^5x\sin^4\left(e+fx\right)\cos}{8} \\ x\left(a\sin\left(e\right)+a\right)^3\left(-c\sin\left(e\right)+c\right)^5 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((-35*a**3*c**5*x*sin(e + f*x)**8/128 - 35*a**3*c**5*x*sin(e + f*x
)**6*cos(e + f*x)**2/32 + 5*a**3*c**5*x*sin(e + f*x)**6/8 - 105*a**3*c**5*x
*sin(e + f*x)**4*cos(e + f*x)**4/64 + 15*a**3*c**5*x*sin(e + f*x)**4*cos(e
+ f*x)**2/8 - 35*a**3*c**5*x*sin(e + f*x)**2*cos(e + f*x)**6/32 + 15*a**3*c
**5*x*sin(e + f*x)**2*cos(e + f*x)**4/8 - a**3*c**5*x*sin(e + f*x)**2 - 35*
a**3*c**5*x*cos(e + f*x)**8/128 + 5*a**3*c**5*x*cos(e + f*x)**6/8 - a**3*c*
*5*x*cos(e + f*x)**2 + a**3*c**5*x + 93*a**3*c**5*sin(e + f*x)**7*cos(e + f
*x)/(128*f) - 2*a**3*c**5*sin(e + f*x)**6*cos(e + f*x)/f + 511*a**3*c**5*si
n(e + f*x)**5*cos(e + f*x)**3/(384*f) - 11*a**3*c**5*sin(e + f*x)**5*cos(e
+ f*x)/(8*f) - 4*a**3*c**5*sin(e + f*x)**4*cos(e + f*x)**3/f + 6*a**3*c**5*
sin(e + f*x)**4*cos(e + f*x)/f + 385*a**3*c**5*sin(e + f*x)**3*cos(e + f*x)
**5/(384*f) - 5*a**3*c**5*sin(e + f*x)**3*cos(e + f*x)**3/(3*f) - 16*a**3*c
**5*sin(e + f*x)**2*cos(e + f*x)**5/(5*f) + 8*a**3*c**5*sin(e + f*x)**2*cos
(e + f*x)**3/f - 6*a**3*c**5*sin(e + f*x)**2*cos(e + f*x)/f + 35*a**3*c**5*
sin(e + f*x)*cos(e + f*x)**7/(128*f) - 5*a**3*c**5*sin(e + f*x)*cos(e + f*x
)**5/(8*f) + a**3*c**5*sin(e + f*x)*cos(e + f*x)/f - 32*a**3*c**5*cos(e + f
*x)**7/(35*f) + 16*a**3*c**5*cos(e + f*x)**5/(5*f) - 4*a**3*c**5*cos(e + f*
x)**3/f + 2*a**3*c**5*cos(e + f*x)/f, Ne(f, 0)), (x*(a*sin(e) + a)**3*(-c*s
in(e) + c)**5, True))
```

3.249 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^4\,dx$

Optimal. Leaf size=112

$$\frac{a^3c^4\cos^7(e+fx)}{7f}+\frac{a^3c^4\sin(e+fx)\cos^5(e+fx)}{6f}+\frac{5a^3c^4\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{5a^3c^4\sin(e+fx)\cos(e+fx)}{16f}$$

```
[Out] 5/16*a^3*c^4*x+1/7*a^3*c^4*cos(f*x+e)^7/f+5/16*a^3*c^4*cos(f*x+e)*sin(f*x+e
)/f+5/24*a^3*c^4*cos(f*x+e)^3*sin(f*x+e)/f+1/6*a^3*c^4*cos(f*x+e)^5*sin(f*x
+e)/f
```

Rubi [A] time = 0.11, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2669, 2635, 8}

$$\frac{a^3c^4\cos^7(e+fx)}{7f}+\frac{a^3c^4\sin(e+fx)\cos^5(e+fx)}{6f}+\frac{5a^3c^4\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{5a^3c^4\sin(e+fx)\cos(e+fx)}{16f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (5*a^3*c^4*x)/16 + (a^3*c^4*Cos[e + f*x]^7)/(7*f) + (5*a^3*c^4*Cos[e + f*x]
*Sin[e + f*x])/(16*f) + (5*a^3*c^4*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) + (a
^3*c^4*Cos[e + f*x]^5*Sin[e + f*x])/(6*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^4\,dx &= \left(a^3c^3\right)\int \cos^6(e+fx)(c-c\sin(e+fx))\,dx\\ &= \frac{a^3c^4\cos^7(e+fx)}{7f}+\left(a^3c^4\right)\int \cos^6(e+fx)\,dx\\ &= \frac{a^3c^4\cos^7(e+fx)}{7f}+\frac{a^3c^4\cos^5(e+fx)\sin(e+fx)}{6f}+\frac{1}{6}\left(5a^3c^4\right)\int\\ &= \frac{a^3c^4\cos^7(e+fx)}{7f}+\frac{5a^3c^4\cos^3(e+fx)\sin(e+fx)}{24f}+\frac{a^3c^4\cos^5(e}{}\\ &= \frac{a^3c^4\cos^7(e+fx)}{7f}+\frac{5a^3c^4\cos(e+fx)\sin(e+fx)}{16f}+\frac{5a^3c^4\cos^3(e}{}\\ &= \frac{5}{16}a^3c^4x+\frac{a^3c^4\cos^7(e+fx)}{7f}+\frac{5a^3c^4\cos(e+fx)\sin(e+fx)}{16f}+\frac{5}{}\end{aligned}$$

Mathematica [A] time = 1.07, size = 89, normalized size = 0.79

$$\frac{a^3c^4(315\sin(2(e+fx))+63\sin(4(e+fx))+7\sin(6(e+fx))+105\cos(e+fx)+63\cos(3(e+fx))+21\cos(5(}{1344f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (a^3*c^4*(420*e + 420*f*x + 105*Cos[e + f*x] + 63*Cos[3*(e + f*x)] + 21*Cos
[5*(e + f*x)] + 3*Cos[7*(e + f*x)] + 315*Sin[2*(e + f*x)] + 63*Sin[4*(e + f
*x)] + 7*Sin[6*(e + f*x)]))/(1344*f)
```

fricas [A] time = 0.46, size = 87, normalized size = 0.78

$$\frac{48\,a^3c^4\cos\left(fx+e\right)^7+105\,a^3c^4fx+7\left(8\,a^3c^4\cos\left(fx+e\right)^5+10\,a^3c^4\cos\left(fx+e\right)^3+15\,a^3c^4\cos\left(fx+e\right)\right)\sin\left(\right.}{336\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/336*(48*a^3*c^4*cos(f*x + e)^7 + 105*a^3*c^4*f*x + 7*(8*a^3*c^4*cos(f*x +
 e)^5 + 10*a^3*c^4*cos(f*x + e)^3 + 15*a^3*c^4*cos(f*x + e))*sin(f*x + e))/
f
```

giac [A] time = 0.23, size = 154, normalized size = 1.38

$$\frac{5}{16}\,a^3c^4x+\frac{a^3c^4\cos\left(7\,fx+7\,e\right)}{448\,f}+\frac{a^3c^4\cos\left(5\,fx+5\,e\right)}{64\,f}+\frac{3\,a^3c^4\cos\left(3\,fx+3\,e\right)}{64\,f}+\frac{5\,a^3c^4\cos\left(fx+e\right)}{64\,f}+\frac{a^3c^4\sin\left(6\right.}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 5/16*a^3*c^4*x + 1/448*a^3*c^4*cos(7*f*x + 7*e)/f + 1/64*a^3*c^4*cos(5*f*x
+ 5*e)/f + 3/64*a^3*c^4*cos(3*f*x + 3*e)/f + 5/64*a^3*c^4*cos(f*x + e)/f +
1/192*a^3*c^4*sin(6*f*x + 6*e)/f + 3/64*a^3*c^4*sin(4*f*x + 4*e)/f + 15/64*
a^3*c^4*sin(2*f*x + 2*e)/f
```

maple [B] time = 0.42, size = 255, normalized size = 2.28

$$\frac{-\frac{c^4a^3\left(\frac{16}{5}+\sin^6\left(fx+e\right)+\frac{6\left(\sin^4\left(fx+e\right)\right)}{5}+\frac{8\left(\sin^2\left(fx+e\right)\right)}{5}\right)\cos\left(fx+e\right)}{7}-c^4a^3\left(-\frac{\left(\sin^5\left(fx+e\right)+\frac{5\left(\sin^3\left(fx+e\right)\right)}{4}+\frac{15\sin\left(fx+e\right)}{8}\right)\cos\left(fx+e\right)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^4,x)
```

```
[Out] 1/f*(-1/7*c^4*a^3*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos
(f*x+e)-c^4*a^3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f
*x+e)+5/16*f*x+5/16*e)+3/5*c^4*a^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(
f*x+e)+3*c^4*a^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8
*e)-c^4*a^3*(2+sin(f*x+e)^2)*cos(f*x+e)-3*c^4*a^3*(-1/2*sin(f*x+e)*cos(f*x+
e)+1/2*f*x+1/2*e)+c^4*a^3*cos(f*x+e)+c^4*a^3*(f*x+e))
```

maxima [B] time = 0.53, size = 256, normalized size = 2.29

$$\frac{192\left(5\cos\left(fx+e\right)^7-21\cos\left(fx+e\right)^5+35\cos\left(fx+e\right)^3-35\cos\left(fx+e\right)\right)a^3c^4+1344\left(3\cos\left(fx+e\right)^5-1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 1/6720*(192*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 - 35*
cos(f*x + e))*a^3*c^4 + 1344*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos
(f*x + e))*a^3*c^4 + 6720*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3*c^4 - 35*(4
*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2
*e))*a^3*c^4 + 630*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*
a^3*c^4 - 5040*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^3*c^4 + 6720*(f*x + e)*a^
3*c^4 + 6720*a^3*c^4*cos(f*x + e))/f
```

mupad [B] time = 10.45, size = 301, normalized size = 2.69

$$\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}\,\left(\frac{a^3\,c^4\,\left(735\,e+735\,f\,x+672\right)}{336}-\frac{35\,a^3\,c^4\,\left(e+f\,x\right)}{16}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}\,\left(\frac{a^3\,c^4\,\left(2205\,e+2205\,f\,x+2016\right)}{336}-\frac{105\,a^3\,c^4\,\left(e+f\,x\right)}{16}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^4,x)
```

```
[Out] (tan(e/2 + (f*x)/2)^12*((a^3*c^4*(735*e + 735*f*x + 672))/336 - (35*a^3*c^4
*(e + f*x))/16) + tan(e/2 + (f*x)/2)^4*((a^3*c^4*(2205*e + 2205*f*x + 2016)
)/336 - (105*a^3*c^4*(e + f*x))/16) + tan(e/2 + (f*x)/2)^8*((a^3*c^4*(3675*
e + 3675*f*x + 3360))/336 - (175*a^3*c^4*(e + f*x))/16) + (7*a^3*c^4*tan(e/
2 + (f*x)/2)^3)/6 + (85*a^3*c^4*tan(e/2 + (f*x)/2)^5)/24 - (85*a^3*c^4*tan(
e/2 + (f*x)/2)^9)/24 - (7*a^3*c^4*tan(e/2 + (f*x)/2)^11)/6 - (11*a^3*c^4*ta
n(e/2 + (f*x)/2)^13)/8 + (a^3*c^4*(105*e + 105*f*x + 96))/336 + (11*a^3*c^4
*tan(e/2 + (f*x)/2))/8 - (5*a^3*c^4*(e + f*x))/16)/(f*(tan(e/2 + (f*x)/2)^2
 + 1)^7) + (5*a^3*c^4*x)/16
```

sympy [A] time = 9.27, size = 631, normalized size = 5.63

$$\begin{cases} -\frac{5a^3c^4x\sin^6\left(e+fx\right)}{16}-\frac{15a^3c^4x\sin^4\left(e+fx\right)\cos^2\left(e+fx\right)}{16}+\frac{9a^3c^4x\sin^4\left(e+fx\right)}{8}-\frac{15a^3c^4x\sin^2\left(e+fx\right)\cos^4\left(e+fx\right)}{16}+\frac{9a^3c^4x\sin^2\left(e+fx\right)\cos^2\left(e\right.}{4} \\ x\left(a\sin\left(e\right)+a\right)^3\left(-c\sin\left(e\right)+c\right)^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-5*a**3*c**4*x*sin(e + f*x)**6/16 - 15*a**3*c**4*x*sin(e + f*x)*
*4*cos(e + f*x)**2/16 + 9*a**3*c**4*x*sin(e + f*x)**4/8 - 15*a**3*c**4*x*si
n(e + f*x)**2*cos(e + f*x)**4/16 + 9*a**3*c**4*x*sin(e + f*x)**2*cos(e + f*
```

```
x)**2/4 - 3*a**3*c**4*x*sin(e + f*x)**2/2 - 5*a**3*c**4*x*cos(e + f*x)**6/1
6 + 9*a**3*c**4*x*cos(e + f*x)**4/8 - 3*a**3*c**4*x*cos(e + f*x)**2/2 + a**
3*c**4*x - a**3*c**4*sin(e + f*x)**6*cos(e + f*x)/f + 11*a**3*c**4*sin(e +
f*x)**5*cos(e + f*x)/(16*f) - 2*a**3*c**4*sin(e + f*x)**4*cos(e + f*x)**3/f
 + 3*a**3*c**4*sin(e + f*x)**4*cos(e + f*x)/f + 5*a**3*c**4*sin(e + f*x)**3
*cos(e + f*x)**3/(6*f) - 15*a**3*c**4*sin(e + f*x)**3*cos(e + f*x)/(8*f) -
8*a**3*c**4*sin(e + f*x)**2*cos(e + f*x)**5/(5*f) + 4*a**3*c**4*sin(e + f*x
)**2*cos(e + f*x)**3/f - 3*a**3*c**4*sin(e + f*x)**2*cos(e + f*x)/f + 5*a**
3*c**4*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 9*a**3*c**4*sin(e + f*x)*cos(e
 + f*x)**3/(8*f) + 3*a**3*c**4*sin(e + f*x)*cos(e + f*x)/(2*f) - 16*a**3*c*
*4*cos(e + f*x)**7/(35*f) + 8*a**3*c**4*cos(e + f*x)**5/(5*f) - 2*a**3*c**4
*cos(e + f*x)**3/f + a**3*c**4*cos(e + f*x)/f, Ne(f, 0)), (x*(a*sin(e) + a)
**3*(-c*sin(e) + c)**4, True))
```

3.250 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^3\,dx$

Optimal. Leaf size=91

$$\frac{a^3c^3\sin(e+fx)\cos^5(e+fx)}{6f}+\frac{5a^3c^3\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{5a^3c^3\sin(e+fx)\cos(e+fx)}{16f}+\frac{5}{16}a^3c^3x$$

```
[Out] 5/16*a^3*c^3*x+5/16*a^3*c^3*cos(f*x+e)*sin(f*x+e)/f+5/24*a^3*c^3*cos(f*x+e)
^3*sin(f*x+e)/f+1/6*a^3*c^3*cos(f*x+e)^5*sin(f*x+e)/f
```

Rubi [A] time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2635, 8}

$$\frac{a^3c^3\sin(e+fx)\cos^5(e+fx)}{6f}+\frac{5a^3c^3\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{5a^3c^3\sin(e+fx)\cos(e+fx)}{16f}+\frac{5}{16}a^3c^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (5*a^3*c^3*x)/16 + (5*a^3*c^3*Cos[e + f*x]*Sin[e + f*x])/(16*f) + (5*a^3*c^
3*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) + (a^3*c^3*Cos[e + f*x]^5*Sin[e + f*x
])/(6*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^3\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)\,dx$$
$$= \frac{a^3c^3\cos^5(e+fx)\sin(e+fx)}{6f} + \frac{1}{6}\left(5a^3c^3\right)\int \cos^4(e+fx)\,dx$$
$$= \frac{5a^3c^3\cos^3(e+fx)\sin(e+fx)}{24f} + \frac{a^3c^3\cos^5(e+fx)\sin(e+fx)}{6f}$$
$$= \frac{5a^3c^3\cos(e+fx)\sin(e+fx)}{16f} + \frac{5a^3c^3\cos^3(e+fx)\sin(e+fx)}{24f}$$
$$= \frac{5}{16}a^3c^3x + \frac{5a^3c^3\cos(e+fx)\sin(e+fx)}{16f} + \frac{5a^3c^3\cos^3(e+fx)\sin}{24f}$$

Mathematica [A] time = 0.05, size = 49, normalized size = 0.54

$$\frac{a^3c^3(45\sin(2(e+fx))+9\sin(4(e+fx))+\sin(6(e+fx))+60e+60fx)}{192f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (a^3*c^3*(60*e + 60*f*x + 45*Sin[2*(e + f*x)] + 9*Sin[4*(e + f*x)] + Sin[6*
(e + f*x)]))/(192*f)
```

fricas [A] time = 0.46, size = 70, normalized size = 0.77

$$\frac{15\,a^3c^3fx + \left(8\,a^3c^3\cos\left(fx+e\right)^5 + 10\,a^3c^3\cos\left(fx+e\right)^3 + 15\,a^3c^3\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/48*(15*a^3*c^3*f*x + (8*a^3*c^3*cos(f*x + e)^5 + 10*a^3*c^3*cos(f*x + e)^
3 + 15*a^3*c^3*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.24, size = 73, normalized size = 0.80

$$\frac{5}{16}\,a^3c^3x + \frac{a^3c^3\sin\left(6\,fx+6\,e\right)}{192\,f} + \frac{3\,a^3c^3\sin\left(4\,fx+4\,e\right)}{64\,f} + \frac{15\,a^3c^3\sin\left(2\,fx+2\,e\right)}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 5/16*a^3*c^3*x + 1/192*a^3*c^3*sin(6*f*x + 6*e)/f + 3/64*a^3*c^3*sin(4*f*x
+ 4*e)/f + 15/64*a^3*c^3*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.24, size = 140, normalized size = 1.54

$$\frac{-c^{3}a^{3}\left(-\frac{\left(\sin^{5}\left(fx+e\right)+\frac{5\left(\sin^{3}\left(fx+e\right)\right)}{4}+\frac{15\sin\left(fx+e\right)}{8}\right)\cos\left(fx+e\right)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+3c^{3}a^{3}\left(-\frac{\left(\sin^{3}\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)\cdots}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(-c^3*a^3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x
+e)+5/16*f*x+5/16*e)+3*c^3*a^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+
e)+3/8*f*x+3/8*e)-3*c^3*a^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+c^3*
a^3*(f*x+e))
```

maxima [A] time = 0.78, size = 132, normalized size = 1.45

$$-\frac{\left(4\,\sin\left(2\,fx+2\,e\right)^{3}+60\,fx+60\,e+9\,\sin\left(4\,fx+4\,e\right)-48\,\sin\left(2\,fx+2\,e\right)\right)a^{3}c^{3}-18\left(12\,fx+12\,e+\sin\left(4\,fx\cdots\right.\right.}{192\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/192*((4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin
(2*f*x + 2*e))*a^3*c^3 - 18*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x
 + 2*e))*a^3*c^3 + 144*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^3*c^3 - 192*(f*x
+ e)*a^3*c^3)/f
```

mupad [B] time = 10.14, size = 143, normalized size = 1.57

$$\frac{5\,a^{3}\,c^{3}\,x}{16}-\frac{\frac{11\,a^{3}\,c^{3}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{8}-\frac{5\,a^{3}\,c^{3}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}{24}+\frac{15\,a^{3}\,c^{3}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}}{4}-\frac{15\,a^{3}\,c^{3}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}}{4}+\frac{5\,a^{3}\,c^{3}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}}{24}-\frac{11\,a^{3}\,c^{3}\,\tan\left(\cdots\right)}{8}}{f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}+1\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^3,x)
```

```
[Out] (5*a^3*c^3*x)/16 - ((5*a^3*c^3*tan(e/2 + (f*x)/2)^3)/24 - (15*a^3*c^3*tan(e
/2 + (f*x)/2)^5)/4 + (15*a^3*c^3*tan(e/2 + (f*x)/2)^7)/4 - (5*a^3*c^3*tan(e
/2 + (f*x)/2)^9)/24 + (11*a^3*c^3*tan(e/2 + (f*x)/2)^11)/8 - (11*a^3*c^3*ta
n(e/2 + (f*x)/2))/8)/(f*(tan(e/2 + (f*x)/2)^2 + 1)^6)
```

sympy [A] time = 5.42, size = 398, normalized size = 4.37

$$\begin{cases} -\frac{5a^3c^3x\sin^6\left(e+fx\right)}{16} - \frac{15a^3c^3x\sin^4\left(e+fx\right)\cos^2\left(e+fx\right)}{16} + \frac{9a^3c^3x\sin^4\left(e+fx\right)}{8} - \frac{15a^3c^3x\sin^2\left(e+fx\right)\cos^4\left(e+fx\right)}{16} + \frac{9a^3c^3x\sin^2\left(e+fx\right)\cos^2}{4} \\ x\left(a\sin\left(e\right)+a\right)^3\left(-c\sin\left(e\right)+c\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-5*a**3*c**3*x*sin(e + f*x)**6/16 - 15*a**3*c**3*x*sin(e + f*x)*
*4*cos(e + f*x)**2/16 + 9*a**3*c**3*x*sin(e + f*x)**4/8 - 15*a**3*c**3*x*si
n(e + f*x)**2*cos(e + f*x)**4/16 + 9*a**3*c**3*x*sin(e + f*x)**2*cos(e + f*
x)**2/4 - 3*a**3*c**3*x*sin(e + f*x)**2/2 - 5*a**3*c**3*x*cos(e + f*x)**6/1
6 + 9*a**3*c**3*x*cos(e + f*x)**4/8 - 3*a**3*c**3*x*cos(e + f*x)**2/2 + a**
3*c**3*x + 11*a**3*c**3*sin(e + f*x)**5*cos(e + f*x)/(16*f) + 5*a**3*c**3*s
in(e + f*x)**3*cos(e + f*x)**3/(6*f) - 15*a**3*c**3*sin(e + f*x)**3*cos(e +
 f*x)/(8*f) + 5*a**3*c**3*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 9*a**3*c**3
*sin(e + f*x)*cos(e + f*x)**3/(8*f) + 3*a**3*c**3*sin(e + f*x)*cos(e + f*x)
/(2*f), Ne(f, 0)), (x*(a*sin(e) + a)**3*(-c*sin(e) + c)**3, True))
```

3.251 $\int (a + a\sin(e + fx))^3 (c - c\sin(e + fx))^2\,dx$

Optimal. Leaf size=85

$$-\frac{a^3c^2\cos^5(e+fx)}{5f}+\frac{a^3c^2\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3a^3c^2\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}a^3c^2x$$

```
[Out] 3/8*a^3*c^2*x-1/5*a^3*c^2*cos(f*x+e)^5/f+3/8*a^3*c^2*cos(f*x+e)*sin(f*x+e)/
f+1/4*a^3*c^2*cos(f*x+e)^3*sin(f*x+e)/f
```

Rubi [A] time = 0.09, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2669, 2635, 8}

$$-\frac{a^3c^2\cos^5(e+fx)}{5f}+\frac{a^3c^2\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3a^3c^2\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}a^3c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (3*a^3*c^2*x)/8 - (a^3*c^2*Cos[e + f*x]^5)/(5*f) + (3*a^3*c^2*Cos[e + f*x]*
Sin[e + f*x])/(8*f) + (a^3*c^2*Cos[e + f*x]^3*Sin[e + f*x])/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
```

```
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^2\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(a+a\sin(e+fx))\,dx$$
$$= -\frac{a^3c^2\cos^5(e+fx)}{5f} + \left(a^3c^2\right)\int \cos^4(e+fx)\,dx$$
$$= -\frac{a^3c^2\cos^5(e+fx)}{5f} + \frac{a^3c^2\cos^3(e+fx)\sin(e+fx)}{4f} + \frac{1}{4}\left(3a^3c^2\right)$$
$$= -\frac{a^3c^2\cos^5(e+fx)}{5f} + \frac{3a^3c^2\cos(e+fx)\sin(e+fx)}{8f} + \frac{a^3c^2\cos^3}{}$$
$$= \frac{3}{8}a^3c^2x - \frac{a^3c^2\cos^5(e+fx)}{5f} + \frac{3a^3c^2\cos(e+fx)\sin(e+fx)}{8f} + \frac{a}{}$$

Mathematica [A] time = 1.65, size = 69, normalized size = 0.81

$$\frac{a^3c^2(40\sin(2(e+fx))+5\sin(4(e+fx))-20\cos(e+fx)-10\cos(3(e+fx))-2\cos(5(e+fx))+60e+60fx}{160f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a^3*c^2*(60*e + 60*f*x - 20*Cos[e + f*x] - 10*Cos[3*(e + f*x)] - 2*Cos[5*(
e + f*x)] + 40*Sin[2*(e + f*x)] + 5*Sin[4*(e + f*x)]))/(160*f)
```

fricas [A] time = 0.47, size = 71, normalized size = 0.84

$$-\frac{8\,a^3c^2\cos\left(fx+e\right)^5-15\,a^3c^2fx-5\left(2\,a^3c^2\cos\left(fx+e\right)^3+3\,a^3c^2\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{40\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/40*(8*a^3*c^2*cos(f*x + e)^5 - 15*a^3*c^2*f*x - 5*(2*a^3*c^2*cos(f*x + e
)^3 + 3*a^3*c^2*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.17, size = 112, normalized size = 1.32

$$\frac{3}{8}a^3c^2x - \frac{a^3c^2\cos\left(5fx+5e\right)}{80f} - \frac{a^3c^2\cos\left(3fx+3e\right)}{16f} - \frac{a^3c^2\cos\left(fx+e\right)}{8f} + \frac{a^3c^2\sin\left(4fx+4e\right)}{32f} + \frac{a^3c^2\sin\left(2fx+2\right.}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 3/8*a^3*c^2*x - 1/80*a^3*c^2*cos(5*f*x + 5*e)/f - 1/16*a^3*c^2*cos(3*f*x +
3*e)/f - 1/8*a^3*c^2*cos(f*x + e)/f + 1/32*a^3*c^2*sin(4*f*x + 4*e)/f + 1/4
*a^3*c^2*sin(2*f*x + 2*e)/f
```

maple [B] time = 0.29, size = 160, normalized size = 1.88

$$\frac{-\frac{c^2a^3\left(\frac{8}{3}+\sin^4\left(fx+e\right)+\frac{4\left(\sin^2\left(fx+e\right)\right)}{3}\right)\cos\left(fx+e\right)}{5} + c^2a^3\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4} + \frac{3fx}{8} + \frac{3e}{8}\right) + \frac{2c^2a^3\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+\right.}{3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(-1/5*c^2*a^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+c^2*a^3*(-
1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+2/3*c^2*a^3*(2+
sin(f*x+e)^2)*cos(f*x+e)-2*c^2*a^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*
e)-c^2*a^3*cos(f*x+e)+c^2*a^3*(f*x+e))
```

maxima [B] time = 1.10, size = 158, normalized size = 1.86

$$-\frac{32\left(3\cos\left(fx+e\right)^5 - 10\cos\left(fx+e\right)^3 + 15\cos\left(fx+e\right)\right)a^3c^2 + 320\left(\cos\left(fx+e\right)^3 - 3\cos\left(fx+e\right)\right)a^3c^2 - 15}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/480*(32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a^3*c^2
 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3*c^2 - 15*(12*f*x + 12*e + sin(
4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^3*c^2 + 240*(2*f*x + 2*e - sin(2*f*x +
 2*e))*a^3*c^2 - 480*(f*x + e)*a^3*c^2 + 480*a^3*c^2*cos(f*x + e))/f
```

mupad [B] time = 10.32, size = 220, normalized size = 2.59

$$\frac{3\,a^3\,c^2\,x}{8}+\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8\,\left(\frac{a^3\,c^2\,\left(75\,e+75\,f\,x-80\right)}{40}-\frac{15\,a^3\,c^2\,\left(e+f\,x\right)}{8}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(\frac{a^3\,c^2\,\left(150\,e+150\,f\,x-160\right)}{40}-\frac{15\,a^3\,c^2\,\left(e+f\,x\right)}{4}\cdots\right)}{f\,\left(\tan\left(\frac{e}{2}\cdots\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^2,x)
```

```
[Out] (3*a^3*c^2*x)/8 + (tan(e/2 + (f*x)/2)^8*((a^3*c^2*(75*e + 75*f*x - 80))/40
- (15*a^3*c^2*(e + f*x))/8) + tan(e/2 + (f*x)/2)^4*((a^3*c^2*(150*e + 150*f
*x - 160))/40 - (15*a^3*c^2*(e + f*x))/4) + (a^3*c^2*tan(e/2 + (f*x)/2)^3)/
2 - (a^3*c^2*tan(e/2 + (f*x)/2)^7)/2 - (5*a^3*c^2*tan(e/2 + (f*x)/2)^9)/4 +
 (a^3*c^2*(15*e + 15*f*x - 16))/40 + (5*a^3*c^2*tan(e/2 + (f*x)/2))/4 - (3*
a^3*c^2*(e + f*x))/8)/(f*(tan(e/2 + (f*x)/2)^2 + 1)^5)
```

sympy [A] time = 3.32, size = 340, normalized size = 4.00

$$\begin{cases} \frac{3a^3c^2x\sin^4\left(e+fx\right)}{8}+\frac{3a^3c^2x\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4}-a^3c^2x\sin^2\left(e+fx\right)+\frac{3a^3c^2x\cos^4\left(e+fx\right)}{8}-a^3c^2x\cos^2\left(e+fx\right)+a^3c^2\cdots \\ x\left(a\sin\left(e\right)+a\right)^3\left(-c\sin\left(e\right)+c\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*a**3*c**2*x*sin(e + f*x)**4/8 + 3*a**3*c**2*x*sin(e + f*x)**2*
cos(e + f*x)**2/4 - a**3*c**2*x*sin(e + f*x)**2 + 3*a**3*c**2*x*cos(e + f*x
)**4/8 - a**3*c**2*x*cos(e + f*x)**2 + a**3*c**2*x - a**3*c**2*sin(e + f*x)
**4*cos(e + f*x)/f - 5*a**3*c**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*a**
3*c**2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) + 2*a**3*c**2*sin(e + f*x)**2*
cos(e + f*x)/f - 3*a**3*c**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) + a**3*c**2
*sin(e + f*x)*cos(e + f*x)/f - 8*a**3*c**2*cos(e + f*x)**5/(15*f) + 4*a**3*
c**2*cos(e + f*x)**3/(3*f) - a**3*c**2*cos(e + f*x)/f, Ne(f, 0)), (x*(a*sin
(e) + a)**3*(-c*sin(e) + c)**2, True))
```

3.252 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))\,dx$

Optimal. Leaf size=82

$$-\frac{5a^3c\cos^3(e+fx)}{12f}-\frac{c\cos^3(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{4f}+\frac{5a^3c\sin(e+fx)\cos(e+fx)}{8f}+\frac{5}{8}a^3cx$$

```
[Out] 5/8*a^3*c*x-5/12*a^3*c*cos(f*x+e)^3/f+5/8*a^3*c*cos(f*x+e)*sin(f*x+e)/f-1/4
*c*cos(f*x+e)^3*(a^3+a^3*sin(f*x+e))/f
```

Rubi [A] time = 0.10, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2736, 2678, 2669, 2635, 8}

$$-\frac{5a^3c\cos^3(e+fx)}{12f}-\frac{c\cos^3(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{4f}+\frac{5a^3c\sin(e+fx)\cos(e+fx)}{8f}+\frac{5}{8}a^3cx$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x]),x]
```

```
[Out] (5*a^3*c*x)/8 - (5*a^3*c*Cos[e + f*x]^3)/(12*f) + (5*a^3*c*Cos[e + f*x]*Sin
[e + f*x])/(8*f) - (c*Cos[e + f*x]^3*(a^3 + a^3*Sin[e + f*x]))/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
```

```
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))\,dx = (ac)\int \cos^2(e+fx)(a+a\sin(e+fx))^2\,dx$$
$$= -\frac{c\cos^3(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4f}+\frac{1}{4}\left(5a^2c\right)\int \cos^2(e+fx)($$
$$= -\frac{5a^3c\cos^3(e+fx)}{12f}-\frac{c\cos^3(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4f}+\frac{1}{4}(5$$
$$= -\frac{5a^3c\cos^3(e+fx)}{12f}+\frac{5a^3c\cos(e+fx)\sin(e+fx)}{8f}-\frac{c\cos^3(e+f}{}$$
$$= \frac{5}{8}a^3cx-\frac{5a^3c\cos^3(e+fx)}{12f}+\frac{5a^3c\cos(e+fx)\sin(e+fx)}{8f}-\frac{c\cos}{}$$

Mathematica [A] time = 0.38, size = 54, normalized size = 0.66

$$\frac{a^3c(24\sin(2(e+fx))-3\sin(4(e+fx))-48\cos(e+fx)-16\cos(3(e+fx))+60fx)}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^3*c*(60*f*x - 48*Cos[e + f*x] - 16*Cos[3*(e + f*x)] + 24*Sin[2*(e + f*x)
] - 3*Sin[4*(e + f*x)]))/(96*f)
```

fricas [A] time = 0.48, size = 63, normalized size = 0.77

$$-\frac{16\,a^3c\cos\left(fx+e\right)^3-15\,a^3cfx+3\left(2\,a^3c\cos\left(fx+e\right)^3-5\,a^3c\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/24*(16*a^3*c*cos(f*x + e)^3 - 15*a^3*c*f*x + 3*(2*a^3*c*cos(f*x + e)^3 -
 5*a^3*c*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.42, size = 81, normalized size = 0.99

$$\frac{5}{8}a^3cx - \frac{a^3c\cos\left(3fx+3e\right)}{6f} - \frac{a^3c\cos\left(fx+e\right)}{2f} - \frac{a^3c\sin\left(4fx+4e\right)}{32f} + \frac{a^3c\sin\left(2fx+2e\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 5/8*a^3*c*x - 1/6*a^3*c*cos(3*f*x + 3*e)/f - 1/2*a^3*c*cos(f*x + e)/f - 1/3
2*a^3*c*sin(4*f*x + 4*e)/f + 1/4*a^3*c*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.24, size = 89, normalized size = 1.09

$$\frac{-a^3c\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+\frac{2a^3c\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}-2a^3c\cos\left(fx+e\right)+a^3c\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e)),x)
```

```
[Out] 1/f*(-a^3*c*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+2
/3*a^3*c*(2+sin(f*x+e)^2)*cos(f*x+e)-2*a^3*c*cos(f*x+e)+a^3*c*(f*x+e))
```

maxima [A] time = 0.83, size = 86, normalized size = 1.05

$$-\frac{64\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)a^3c+3\left(12fx+12e+\sin\left(4fx+4e\right)-8\sin\left(2fx+2e\right)\right)a^3c-96\left(fx+e\right.}{96f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/96*(64*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3*c + 3*(12*f*x + 12*e + sin(
4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^3*c - 96*(f*x + e)*a^3*c + 192*a^3*c*c
os(f*x + e))/f
```

mupad [B] time = 8.81, size = 250, normalized size = 3.05

$$\frac{5a^3cx}{8} - \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(\frac{a^3c\left(15e+15fx\right)}{6}-\frac{a^3c\left(60e+60fx-32\right)}{24}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6\left(\frac{a^3c\left(15e+15fx\right)}{6}-\frac{a^3c\left(60e+60fx-96\right)}{24}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x)),x)
```

```
[Out] (5*a^3*c*x)/8 - (tan(e/2 + (f*x)/2)^2*((a^3*c*(15*e + 15*f*x))/6 - (a^3*c*(
60*e + 60*f*x - 32))/24) + tan(e/2 + (f*x)/2)^6*((a^3*c*(15*e + 15*f*x))/6
- (a^3*c*(60*e + 60*f*x - 96))/24) + tan(e/2 + (f*x)/2)^4*((a^3*c*(15*e + 1
5*f*x))/4 - (a^3*c*(90*e + 90*f*x - 96))/24) - (3*a^3*c*tan(e/2 + (f*x)/2))
/4 - (11*a^3*c*tan(e/2 + (f*x)/2)^3)/4 + (11*a^3*c*tan(e/2 + (f*x)/2)^5)/4
+ (3*a^3*c*tan(e/2 + (f*x)/2)^7)/4 + (a^3*c*(15*e + 15*f*x))/24 - (a^3*c*(1
5*e + 15*f*x - 32))/24)/(f*(tan(e/2 + (f*x)/2)^2 + 1)^4)
```

sympy [A] time = 1.49, size = 196, normalized size = 2.39

$$\begin{cases} -\frac{3a^3cx\sin^4\left(e+fx\right)}{8} - \frac{3a^3cx\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4} - \frac{3a^3cx\cos^4\left(e+fx\right)}{8} + a^3cx + \frac{5a^3c\sin^3\left(e+fx\right)\cos\left(e+fx\right)}{8f} + \frac{2a^3c\sin^2\left(e+fx\right)\cos\left(e\right.}{f} \\ x\left(a\sin\left(e\right)+a\right)^3\left(-c\sin\left(e\right)+c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-3*a**3*c*x*sin(e + f*x)**4/8 - 3*a**3*c*x*sin(e + f*x)**2*cos(e
 + f*x)**2/4 - 3*a**3*c*x*cos(e + f*x)**4/8 + a**3*c*x + 5*a**3*c*sin(e + f
*x)**3*cos(e + f*x)/(8*f) + 2*a**3*c*sin(e + f*x)**2*cos(e + f*x)/f + 3*a**
3*c*sin(e + f*x)*cos(e + f*x)**3/(8*f) + 4*a**3*c*cos(e + f*x)**3/(3*f) - 2
*a**3*c*cos(e + f*x)/f, Ne(f, 0)), (x*(a*sin(e) + a)**3*(-c*sin(e) + c), Tr
ue))
```

3.253 $\int \frac{(a+a\sin(e+fx))^3}{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=94

$$\frac{2a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}+\frac{15a^3\cos(e+fx)}{2cf}+\frac{5a^3\cos^3(e+fx)}{2f(c-c\sin(e+fx))}-\frac{15a^3x}{2c}$$

```
[Out] -15/2*a^3*x/c+15/2*a^3*cos(f*x+e)/c/f+2*a^3*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x
+e))^3+5/2*a^3*cos(f*x+e)^3/f/(c-c*sin(f*x+e))
```

Rubi [A] time = 0.18, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2736, 2680, 2679, 2682, 8}

$$\frac{2a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}+\frac{15a^3\cos(e+fx)}{2cf}+\frac{5a^3\cos^3(e+fx)}{2f(c-c\sin(e+fx))}-\frac{15a^3x}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x]),x]
```

```
[Out] (-15*a^3*x)/(2*c) + (15*a^3*Cos[e + f*x])/(2*c*f) + (2*a^3*c^2*Cos[e + f*x]
^5)/(f*(c - c*Sin[e + f*x])^3) + (5*a^3*Cos[e + f*x]^3)/(2*f*(c - c*Sin[e +
 f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
```

```
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))^3}{c-c\sin(e+fx)}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^4}\,dx\\
&= \frac{2a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}-\left(5a^3c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^2}\,dx\\
&= \frac{2a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}+\frac{5a^3\cos^3(e+fx)}{2f(c-c\sin(e+fx))}-\frac{1}{2}\left(15a^3\right)\int \frac{\cos^2(e+fx)}{c-c\sin(e+fx)}\,dx\\
&= \frac{15a^3\cos(e+fx)}{2cf}+\frac{2a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}+\frac{5a^3\cos^3(e+fx)}{2f(c-c\sin(e+fx))}-\frac{\left(15a^3\right)\int 1\,dx}{2c}\\
&= -\frac{15a^3x}{2c}+\frac{15a^3\cos(e+fx)}{2cf}+\frac{2a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}+\frac{5a^3\cos^3(e+fx)}{2f(c-c\sin(e+fx))}
\end{aligned}$$

Mathematica [A] time = 0.53, size = 153, normalized size = 1.63

$$\frac{a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)(30(e+fx)-\sin(2(e+fx))-16\cos(\right.}{4cf(\sin(e+fx)-1)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(Cos[(e + f
*x)/2]*(30*(e + f*x) - 16*Cos[e + f*x] - Sin[2*(e + f*x)]) + Sin[(e + f*x)/
2]*(-64 - 30*e - 30*f*x + 16*Cos[e + f*x] + Sin[2*(e + f*x)])))/(4*c*f*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(-1 + Sin[e + f*x]))
```

fricas [A] time = 0.44, size = 129, normalized size = 1.37

$$\frac{a^3\cos\left(fx+e\right)^3 - 15\,a^3fx + 8\,a^3\cos\left(fx+e\right)^2 + 16\,a^3 - \left(15\,a^3fx - 23\,a^3\right)\cos\left(fx+e\right) + \left(15\,a^3fx + a^3\cos\left(fx\right.\right.}{2\left(cf\cos\left(fx+e\right) - cf\sin\left(fx+e\right) + cf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(a^3*cos(f*x + e)^3 - 15*a^3*f*x + 8*a^3*cos(f*x + e)^2 + 16*a^3 - (15*
a^3*f*x - 23*a^3)*cos(f*x + e) + (15*a^3*f*x + a^3*cos(f*x + e)^2 - 7*a^3*c
os(f*x + e) + 16*a^3)*sin(f*x + e))/(c*f*cos(f*x + e) - c*f*sin(f*x + e) +
c*f)
```

giac [A] time = 0.29, size = 117, normalized size = 1.24

$$-\frac{\frac{15\left(fx+e\right)a^3}{c} + \frac{32\,a^3}{c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)} + \frac{2\left(a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 - 8\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 - a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) - 8\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2c}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/2*(15*(f*x + e)*a^3/c + 32*a^3/(c*(tan(1/2*f*x + 1/2*e) - 1)) + 2*(a^3*t
an(1/2*f*x + 1/2*e)^3 - 8*a^3*tan(1/2*f*x + 1/2*e)^2 - a^3*tan(1/2*f*x + 1/
2*e) - 8*a^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*c))/f
```

maple [B] time = 0.27, size = 181, normalized size = 1.93

$$-\frac{16a^3}{cf\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)} - \frac{a^3\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2} + \frac{8a^3\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2} + \frac{a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{cf\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2} + \frac{8}{cf\left(1+\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x)
```

```
[Out] -16*a^3/c/f/(tan(1/2*f*x+1/2*e)-1)-a^3/c/f/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1
/2*f*x+1/2*e)^3+8*a^3/c/f/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2+a
^3/c/f/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)+8*a^3/c/f/(1+tan(1/2*f
*x+1/2*e)^2)^2-15*a^3/c/f*arctan(tan(1/2*f*x+1/2*e))
```

maxima [B] time = 1.78, size = 433, normalized size = 4.61

$$-\frac{6\,a^3\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-2}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}+\frac{c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}\right)+a^3\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{3\sin}{(\cos}}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}+\frac{2c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{2c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{c\sin(fx+e}{(\cos(fx+e)+}}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -(6*a^3*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^2/(cos(f*x + e) +
1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/c) + a^3*((sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 3*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 - 4)/(c - c*sin(f*x + e)/(cos(f*x + e) +
 1) + 2*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*c*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c*sin(f*x + e)^5/(co
s(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c) + 6*a^3*(
arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c - 1/(c - c*sin(f*x + e)/(cos(f*x
+ e) + 1))) - 2*a^3/(c - c*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

mupad [B] time = 9.12, size = 219, normalized size = 2.33

$$-\frac{15\,a^3\,x}{2\,c}-\frac{\frac{15\,a^3\,(e+f\,x)}{2}-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(\frac{15\,a^3\,(e+f\,x)}{2}-\frac{a^3\,(15\,e+15\,f\,x-14)}{2}\right)-\frac{a^3\,(15\,e+15\,f\,x-48)}{2}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\left(\frac{15\,a^3\,(e+}{2}\right.}{c\,f\,\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x)),x)
```

```
[Out] - (15*a^3*x)/(2*c) - ((15*a^3*(e + f*x))/2 - tan(e/2 + (f*x)/2)*((15*a^3*(e
 + f*x))/2 - (a^3*(15*e + 15*f*x - 14))/2) - (a^3*(15*e + 15*f*x - 48))/2 +
 tan(e/2 + (f*x)/2)^4*((15*a^3*(e + f*x))/2 - (a^3*(15*e + 15*f*x - 34))/2)
 - tan(e/2 + (f*x)/2)^3*(15*a^3*(e + f*x) - (a^3*(30*e + 30*f*x - 18))/2) +
 tan(e/2 + (f*x)/2)^2*(15*a^3*(e + f*x) - (a^3*(30*e + 30*f*x - 78))/2))/(c
*f*(tan(e/2 + (f*x)/2) - 1)*(tan(e/2 + (f*x)/2)^2 + 1)^2)
```

sympy [A] time = 7.43, size = 1168, normalized size = 12.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-15*a**3*f*x*tan(e/2 + f*x/2)**5/(2*c*f*tan(e/2 + f*x/2)**5 - 2*
c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2
)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 15*a**3*f*x*tan(e/2 + f*x/2)**4/(2
*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/
2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 30*a*
*3*f*x*tan(e/2 + f*x/2)**3/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x
/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(
e/2 + f*x/2) - 2*c*f) + 30*a**3*f*x*tan(e/2 + f*x/2)**2/(2*c*f*tan(e/2 + f*
x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan
(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 15*a**3*f*x*tan(e/2 +
f*x/2)/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e
/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f
) + 15*a**3*f*x/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*
c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2
) - 2*c*f) - 34*a**3*tan(e/2 + f*x/2)**4/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f
*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**
2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 18*a**3*tan(e/2 + f*x/2)**3/(2*c*f*ta
n(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 -
 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 78*a**3*tan(
e/2 + f*x/2)**2/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*
c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2
) - 2*c*f) + 14*a**3*tan(e/2 + f*x/2)/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*ta
n(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 +
 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 48*a**3/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c
*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)
**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f), Ne(f, 0)), (x*(a*sin(e) + a)**3/(-c*
sin(e) + c), True))
```

3.254 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=92

$$-\frac{5a^3\cos(e+fx)}{c^2f}+\frac{2a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}+\frac{5a^3x}{c^2}-\frac{10a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^2}$$

```
[Out] 5*a^3*x/c^2-5*a^3*cos(f*x+e)/c^2/f+2/3*a^3*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+
e))^4-10/3*a^3*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^2
```

Rubi [A] time = 0.19, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2680, 2682, 8}

$$-\frac{5a^3\cos(e+fx)}{c^2f}+\frac{2a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}+\frac{5a^3x}{c^2}-\frac{10a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (5*a^3*x)/c^2 - (5*a^3*Cos[e + f*x])/(c^2*f) + (2*a^3*c^2*Cos[e + f*x]^5)/(
3*f*(c - c*Sin[e + f*x])^4) - (10*a^3*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f
*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^2}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^5}\,dx\\
&= \frac{2a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}-\frac{1}{3}\left(5a^3c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^3}\,dx\\
&= \frac{2a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}-\frac{10a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^2}+\frac{\left(5a^3\right)\int \frac{\cos^2(e+fx)}{c-c\sin(e+fx)}\,dx}{c}\\
&= -\frac{5a^3\cos(e+fx)}{c^2f}+\frac{2a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}-\frac{10a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^2}+\frac{\left(5a^3\right)\int 1\,dx}{c^2}\\
&= \frac{5a^3x}{c^2}-\frac{5a^3\cos(e+fx)}{c^2f}+\frac{2a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}-\frac{10a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^2}\end{aligned}$$

Mathematica [A] time = 1.00, size = 149, normalized size = 1.62

$$\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(6(15e+15fx+23)\cos\left(\frac{1}{2}(e+fx)\right)-(30e+30fx+121)\cos\left(\frac{3}{2}(e+fx)\right)\right.}{12c^2f(\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(6*(23 + 15*e + 15*f*x)*Cos[(e +
 f*x)/2] - (121 + 30*e + 30*f*x)*Cos[(3*(e + f*x))/2] + 3*Cos[(5*(e + f*x))
/2] - 6*(31 + 20*e + 20*f*x + 2*(-2 + 5*e + 5*f*x)*Cos[e + f*x] - Cos[2*(e
+ f*x)])*Sin[(e + f*x)/2]))/(12*c^2*f*(-1 + Sin[e + f*x])^2)
```

fricas [B] time = 0.45, size = 184, normalized size = 2.00

$$-\frac{3\,a^3\cos\left(fx+e\right)^3+30\,a^3fx+8\,a^3-\left(15\,a^3fx+31\,a^3\right)\cos\left(fx+e\right)^2+\left(15\,a^3fx-26\,a^3\right)\cos\left(fx+e\right)-\left(30\,a^3\right.}{3\left(c^2f\cos\left(fx+e\right)^2-c^2f\cos\left(fx+e\right)-2\,c^2f+\left(c^2f\cos\left(fx\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^3*cos(f*x + e)^3 + 30*a^3*f*x + 8*a^3 - (15*a^3*f*x + 31*a^3)*cos
(f*x + e)^2 + (15*a^3*f*x - 26*a^3)*cos(f*x + e) - (30*a^3*f*x - 3*a^3*cos(
f*x + e)^2 - 8*a^3 + (15*a^3*f*x - 34*a^3)*cos(f*x + e))*sin(f*x + e))/(c^2
*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) - 2*c^2*f + (c^2*f*cos(f*x + e) + 2*
c^2*f)*sin(f*x + e))
```

giac [A] time = 0.38, size = 101, normalized size = 1.10

$$\frac{\frac{15\left(fx+e\right)a^{3}}{c^{2}}-\frac{6\,a^{3}}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)c^{2}}+\frac{8\left(3\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-12\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+5\,a^{3}\right)}{c^{2}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{3}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(15*(f*x + e)*a^3/c^2 - 6*a^3/((tan(1/2*f*x + 1/2*e)^2 + 1)*c^2) + 8*(3
*a^3*tan(1/2*f*x + 1/2*e)^2 - 12*a^3*tan(1/2*f*x + 1/2*e) + 5*a^3)/(c^2*(ta
n(1/2*f*x + 1/2*e) - 1)^3))/f
```

maple [A] time = 0.28, size = 121, normalized size = 1.32

$$-\frac{32a^{3}}{3c^{2}f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{16a^{3}}{c^{2}f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}+\frac{8a^{3}}{c^{2}f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{2a^{3}}{c^{2}f\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}+\frac{10a^{3}\,\mathrm{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x)
```

```
[Out] -32/3*a^3/c^2/f/(tan(1/2*f*x+1/2*e)-1)^3-16*a^3/c^2/f/(tan(1/2*f*x+1/2*e)-1
)^2+8*a^3/c^2/f/(tan(1/2*f*x+1/2*e)-1)-2*a^3/c^2/f/(1+tan(1/2*f*x+1/2*e)^2)
+10*a^3/c^2/f*arctan(tan(1/2*f*x+1/2*e))
```

maxima [B] time = 1.50, size = 594, normalized size = 6.46

$$2\left(2\,a^{3}\left(\frac{\frac{12\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{11\,\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{9\,\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}-\frac{3\,\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}-5}{c^{2}-\frac{3\,c^{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{4\,c^{2}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{4\,c^{2}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{3\,c^{2}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}-\frac{c^{2}\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}}+\frac{3\,\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{c^{2}}\right)+3\,a^{3}\left(\frac{\frac{9\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+\cdots}}{c^{2}-\frac{3\,c^{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(2*a^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) - 11*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 3*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 - 5)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*c^
2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^2*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + 3*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^2*sin(f*x + e)^5/(co
s(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2) + 3*a^3
*((9*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 - 4)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(
sin(f*x + e)/(cos(f*x + e) + 1))/c^2) - a^3*(3*sin(f*x + e)/(cos(f*x + e) +
 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2)/(c^2 - 3*c^2*sin(f*x + e)/
(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3) + 3*a^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) -
 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

mupad [B] time = 9.75, size = 218, normalized size = 2.37

$$\frac{5\,a^3\,x}{c^2}+\frac{5\,a^3\,\left(e+f\,x\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(15\,a^3\,\left(e+f\,x\right)-\frac{a^3\,\left(45\,e+45\,f\,x-114\right)}{3}\right)-\frac{a^3\,\left(15\,e+15\,f\,x-46\right)}{3}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(15 \cdots\right.}{c^2\,f\,\left(\mathrm{ta}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^2,x)
```

```
[Out] (5*a^3*x)/c^2 + (5*a^3*(e + f*x) - tan(e/2 + (f*x)/2)*(15*a^3*(e + f*x) - (
a^3*(45*e + 45*f*x - 114))/3) - (a^3*(15*e + 15*f*x - 46))/3 + tan(e/2 + (f
*x)/2)^4*(15*a^3*(e + f*x) - (a^3*(45*e + 45*f*x - 24))/3) + tan(e/2 + (f*x
)/2)^2*(20*a^3*(e + f*x) - (a^3*(60*e + 60*f*x - 82))/3) - tan(e/2 + (f*x)/
2)^3*(20*a^3*(e + f*x) - (a^3*(60*e + 60*f*x - 102))/3))/(c^2*f*(tan(e/2 +
(f*x)/2) - 1)^3*(tan(e/2 + (f*x)/2)^2 + 1))
```

sympy [A] time = 13.65, size = 1282, normalized size = 13.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((15*a**3*f*x*tan(e/2 + f*x/2)**5/(3*c**2*f*tan(e/2 + f*x/2)**5 -
9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*ta
n(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 45*a**3*f*x*tan
(e/2 + f*x/2)**4/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)*
*4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2
*f*tan(e/2 + f*x/2) - 3*c**2*f) + 60*a**3*f*x*tan(e/2 + f*x/2)**3/(3*c**2*f
*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f
*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c*
*2*f) - 60*a**3*f*x*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c
**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e
/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 45*a**3*f*x*tan(e/
2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 1
2*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan
(e/2 + f*x/2) - 3*c**2*f) - 15*a**3*f*x/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c
**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e
/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 24*a**3*tan(e/2 +
f*x/2)**4/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12
*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(
e/2 + f*x/2) - 3*c**2*f) - 102*a**3*tan(e/2 + f*x/2)**3/(3*c**2*f*tan(e/2 +
 f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 -
 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 82
*a**3*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2
+ f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2
 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 114*a**3*tan(e/2 + f*x/2)/(3*c**
2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2
+ f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3
*c**2*f) + 46*a**3/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2
)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c*
*2*f*tan(e/2 + f*x/2) - 3*c**2*f), Ne(f, 0)), (x*(a*sin(e) + a)**3/(-c*sin(
e) + c)**2, True))
```

3.255 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=106

$$\frac{2a^3\cos(e+fx)}{f\left(c^3-c^3\sin(e+fx)\right)}-\frac{a^3x}{c^3}+\frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}-\frac{2a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}$$

```
[Out] -a^3*x/c^3+2/5*a^3*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^5-2/3*a^3*cos(f*x+e)
^3/f/(c-c*sin(f*x+e))^3+2*a^3*cos(f*x+e)/f/(c^3-c^3*sin(f*x+e))
```

Rubi [A] time = 0.19, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2680, 8}

$$\frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}+\frac{2a^3\cos(e+fx)}{f\left(c^3-c^3\sin(e+fx)\right)}-\frac{a^3x}{c^3}-\frac{2a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] -((a^3*x)/c^3) + (2*a^3*c^2*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*x])^5) -
(2*a^3*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^3) + (2*a^3*Cos[e + f*x])/
(f*(c^3 - c^3*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
```

```
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^3}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^6}\,dx$$
$$= \frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} - \left(a^3c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^4}\,dx$$
$$= \frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} - \frac{2a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3} + \frac{a^3\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{c}$$
$$= \frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} - \frac{2a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3} + \frac{2a^3\cos(e+fx)}{f\left(c^3-c^3\sin(e+fx)\right)} - \frac{a^3\int 1\,c}{c^3}$$
$$= -\frac{a^3x}{c^3} + \frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} - \frac{2a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3} + \frac{2a^3\cos(e+fx)}{f\left(c^3-c^3\sin(e+fx)\right)}$$

Mathematica [B] time = 0.46, size = 249, normalized size = 2.35

$$\frac{(a\sin(e+fx)+a)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(48\sin\left(\frac{1}{2}(e+fx)\right)-15(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(24*(Cos[(e + f*x)/2] - Sin[(e + f*x
)/2]) - 44*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 - 15*(e + f*x)*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])^5 + 48*Sin[(e + f*x)/2] - 88*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + 92*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])^4*Sin[(e + f*x)/2])*(a + a*Sin[e + f*x])^3)/(15*f*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^3)
```

fricas [B] time = 0.44, size = 235, normalized size = 2.22

$$\frac{60\,a^3fx-\left(15\,a^3fx-46\,a^3\right)\cos\left(fx+e\right)^3-24\,a^3-\left(45\,a^3fx+2\,a^3\right)\cos\left(fx+e\right)^2+6\left(5\,a^3fx-12\,a^3\right)\cos\left(fx\right.}{15\left(c^3f\cos\left(fx+e\right)^3+3\,c^3f\cos\left(fx+e\right)^2-2\,c^3f\cos\left(fx+e\right)-4\,c^3f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(60*a^3*f*x - (15*a^3*f*x - 46*a^3)*cos(f*x + e)^3 - 24*a^3 - (45*a^3*
f*x + 2*a^3)*cos(f*x + e)^2 + 6*(5*a^3*f*x - 12*a^3)*cos(f*x + e) - (60*a^3
*f*x + 24*a^3 - (15*a^3*f*x + 46*a^3)*cos(f*x + e)^2 + 6*(5*a^3*f*x - 8*a^3
)*cos(f*x + e))*sin(f*x + e))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e)^
2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*
x + e) - 4*c^3*f)*sin(f*x + e))
```

giac [A] time = 0.20, size = 111, normalized size = 1.05

$$-\frac{\frac{15\left(fx+e\right)a^3}{c^3}+\frac{4\left(15\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-30\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+100\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-50\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+13\,a^3\right)}{c^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/15*(15*(f*x + e)*a^3/c^3 + 4*(15*a^3*tan(1/2*f*x + 1/2*e)^4 - 30*a^3*tan
(1/2*f*x + 1/2*e)^3 + 100*a^3*tan(1/2*f*x + 1/2*e)^2 - 50*a^3*tan(1/2*f*x +
 1/2*e) + 13*a^3)/(c^3*(tan(1/2*f*x + 1/2*e) - 1)^5))/f
```

maple [A] time = 0.31, size = 143, normalized size = 1.35

$$-\frac{64a^3}{5c^3f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{32a^3}{c^3f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{80a^3}{3c^3f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{8a^3}{c^3f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{}{c^3f\left(\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x)
```

```
[Out] -64/5*a^3/c^3/f/(tan(1/2*f*x+1/2*e)-1)^5-32*a^3/c^3/f/(tan(1/2*f*x+1/2*e)-1
)^4-80/3*a^3/c^3/f/(tan(1/2*f*x+1/2*e)-1)^3-8*a^3/c^3/f/(tan(1/2*f*x+1/2*e)
-1)^2-4*a^3/c^3/f/(tan(1/2*f*x+1/2*e)-1)-2*a^3/c^3/f*arctan(tan(1/2*f*x+1/2
*e))
```

maxima [B] time = 1.76, size = 785, normalized size = 7.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(a^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) - 145*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 - 22)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) +
10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)
^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^3)
+ a^3*(20*sin(f*x + e)/(cos(f*x + e) + 1) - 40*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 - 7)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5) - 9*a^3*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^3
- 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 6*a^3*
(5*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 - 1)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^
5))/f
```

mupad [B] time = 8.48, size = 203, normalized size = 1.92

$$-\frac{a^3\,x}{c^3}-\frac{a^3\,\left(e+f\,x\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(5\,a^3\,\left(e+f\,x\right)-\frac{a^3\,\left(75\,e+75\,f\,x-200\right)}{15}\right)-\frac{a^3\,\left(15\,e+15\,f\,x-52\right)}{15}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(5\,a^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^3,x)
```

```
[Out] - (a^3*x)/c^3 - (a^3*(e + f*x) - tan(e/2 + (f*x)/2)*(5*a^3*(e + f*x) - (a^3
*(75*e + 75*f*x - 200))/15) - (a^3*(15*e + 15*f*x - 52))/15 + tan(e/2 + (f*
x)/2)^4*(5*a^3*(e + f*x) - (a^3*(75*e + 75*f*x - 60))/15) - tan(e/2 + (f*x)
/2)^3*(10*a^3*(e + f*x) - (a^3*(150*e + 150*f*x - 120))/15) + tan(e/2 + (f*
x)/2)^2*(10*a^3*(e + f*x) - (a^3*(150*e + 150*f*x - 400))/15))/(c^3*f*(tan(
e/2 + (f*x)/2) - 1)^5)
```

sympy [A] time = 26.03, size = 1282, normalized size = 12.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-15*a**3*f*x*tan(e/2 + f*x/2)**5/(15*c**3*f*tan(e/2 + f*x/2)**5
- 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3
*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 75*a**3*
f*x*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2
+ f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)*
*2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 150*a**3*f*x*tan(e/2 + f*x/2
)**3/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c
**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(
e/2 + f*x/2) - 15*c**3*f) + 150*a**3*f*x*tan(e/2 + f*x/2)**2/(15*c**3*f*tan
(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x
/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c
**3*f) - 75*a**3*f*x*tan(e/2 + f*x/2)/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c
**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan
(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 15*a**3*f*x/(1
5*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*t
an(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f
*x/2) - 15*c**3*f) - 60*a**3*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2
)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150
*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 120
*a**3*tan(e/2 + f*x/2)**3/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/
2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2
)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 400*a**3*tan(e/2 + f*x/2)*
*2/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**
3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/
2 + f*x/2) - 15*c**3*f) + 200*a**3*tan(e/2 + f*x/2)/(15*c**3*f*tan(e/2 + f*
x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 -
150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) -
52*a**3/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 15
0*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*t
an(e/2 + f*x/2) - 15*c**3*f), Ne(f, 0)), (x*(a*sin(e) + a)**3/(-c*sin(e) +
c)**3, True))
```

3.256 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^4}\,dx$

Optimal. Leaf size=34

$$\frac{a^3c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}$$

[Out] 1/7*a^3*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^7

Rubi [A] time = 0.09, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2736, 2671}

$$\frac{a^3c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^4,x]

[Out] (a^3*c^3*Cos[e + f*x]^7)/(7*f*(c - c*Sin[e + f*x])^7)

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^4}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^7}\,dx\\ &= \frac{a^3c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}\end{aligned}$$

Mathematica [B] time = 0.80, size = 93, normalized size = 2.74

$$\frac{a^3\left(35\cos\left(\frac{1}{2}(e+fx)\right)-21\cos\left(\frac{3}{2}(e+fx)\right)-7\cos\left(\frac{5}{2}(e+fx)\right)+\cos\left(\frac{7}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+\right.\right.}{28c^4f(\sin(e+fx)-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (a^3*(35*Cos[(e + f*x)/2] - 21*Cos[(3*(e + f*x))/2] - 7*Cos[(5*(e + f*x))/2
] + Cos[(7*(e + f*x))/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/(28*c^4*f*
(-1 + Sin[e + f*x])^4)
```

fricas [B] time = 0.45, size = 222, normalized size = 6.53

$$\frac{a^3\cos\left(fx+e\right)^4-3\,a^3\cos\left(fx+e\right)^3-8\,a^3\cos\left(fx+e\right)^2+4\,a^3\cos\left(fx+e\right)+8\,a^3-\left(a^3\cos\left(fx+e\right)^3+\right.}{7\left(c^4f\cos\left(fx+e\right)^4-3\,c^4f\cos\left(fx+e\right)^3-8\,c^4f\cos\left(fx+e\right)^2+4\,c^4f\cos\left(fx+e\right)+8\,c^4f+\left(c^4f\cos\left(fx+e\right)^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/7*(a^3*cos(f*x + e)^4 - 3*a^3*cos(f*x + e)^3 - 8*a^3*cos(f*x + e)^2 + 4*a
^3*cos(f*x + e) + 8*a^3 - (a^3*cos(f*x + e)^3 + 4*a^3*cos(f*x + e)^2 - 4*a^
3*cos(f*x + e) - 8*a^3)*sin(f*x + e))/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f
*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*
f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)
*sin(f*x + e))
```

giac [B] time = 0.22, size = 77, normalized size = 2.26

$$-\frac{2\left(7\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+35\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+21\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a^3\right)}{7\,c^4f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -2/7*(7*a^3*tan(1/2*f*x + 1/2*e)^6 + 35*a^3*tan(1/2*f*x + 1/2*e)^4 + 21*a^3
*tan(1/2*f*x + 1/2*e)^2 + a^3)/(c^4*f*(tan(1/2*f*x + 1/2*e) - 1)^7)
```

maple [B] time = 0.28, size = 118, normalized size = 3.47

$$\frac{2a^3\left(-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{20}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{48}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{40}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{6}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{32}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{64}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}\right.}{f\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x)
```

```
[Out] 2/f*a^3/c^4*(-1/(tan(1/2*f*x+1/2*e)-1)-20/(tan(1/2*f*x+1/2*e)-1)^3-48/(tan(
1/2*f*x+1/2*e)-1)^5-40/(tan(1/2*f*x+1/2*e)-1)^4-6/(tan(1/2*f*x+1/2*e)-1)^2-
32/(tan(1/2*f*x+1/2*e)-1)^6-64/7/(tan(1/2*f*x+1/2*e)-1)^7)
```

maxima [B] time = 0.89, size = 1045, normalized size = 30.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 2/35*(a^3*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 13)/(c^4
- 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c
^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) +
 1)^7) - a^3*(49*sin(f*x + e)/(cos(f*x + e) + 1) - 147*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 210*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 210*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 35*sin
(f*x + e)^6/(cos(f*x + e) + 1)^6 - 12)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x +
 e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c
^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e)
 + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) - 4*a^3*(14*sin(f*x + e)
/(cos(f*x + e) + 1) - 42*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2)/(c
^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7) + 2*a^3*(7*sin(f*x + e)/(cos(f*x + e) + 1) - 21*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 35*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^4 - 7*c
```

```
^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*si
n(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7
))/f
```

mupad [B] time = 7.01, size = 116, normalized size = 3.41

$$\frac{2\,a^3\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left({\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+21\,{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+35\,{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+7\,\sin\left(\frac{e}{2}+\cdots\right.}{7\,c^4\,f\,{\left(\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)-\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^4,x)
```

```
[Out] (2*a^3*cos(e/2 + (f*x)/2)*(cos(e/2 + (f*x)/2)^6 + 7*sin(e/2 + (f*x)/2)^6 +
35*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^4 + 21*cos(e/2 + (f*x)/2)^4*sin(
e/2 + (f*x)/2)^2))/(7*c^4*f*(cos(e/2 + (f*x)/2) - sin(e/2 + (f*x)/2))^7)
```

sympy [A] time = 41.61, size = 619, normalized size = 18.21

$$\begin{cases} -\frac{14a^3\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)}{7c^4f\tan^7\left(\frac{e}{2}+\frac{fx}{2}\right)-49c^4f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+147c^4f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)-245c^4f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+245c^4f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-147c^4f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+49c^4f\tan\left(\frac{e}{2}+\frac{fx}{2}\right.\cdots} \\ \frac{x(a\sin(e)+a)^3}{(-c\sin(e)+c)^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-14*a**3*tan(e/2 + f*x/2)**6/(7*c**4*f*tan(e/2 + f*x/2)**7 - 49*
c**4*f*tan(e/2 + f*x/2)**6 + 147*c**4*f*tan(e/2 + f*x/2)**5 - 245*c**4*f*ta
n(e/2 + f*x/2)**4 + 245*c**4*f*tan(e/2 + f*x/2)**3 - 147*c**4*f*tan(e/2 + f
*x/2)**2 + 49*c**4*f*tan(e/2 + f*x/2) - 7*c**4*f) - 70*a**3*tan(e/2 + f*x/2
)**4/(7*c**4*f*tan(e/2 + f*x/2)**7 - 49*c**4*f*tan(e/2 + f*x/2)**6 + 147*c*
*4*f*tan(e/2 + f*x/2)**5 - 245*c**4*f*tan(e/2 + f*x/2)**4 + 245*c**4*f*tan(
e/2 + f*x/2)**3 - 147*c**4*f*tan(e/2 + f*x/2)**2 + 49*c**4*f*tan(e/2 + f*x/
2) - 7*c**4*f) - 42*a**3*tan(e/2 + f*x/2)**2/(7*c**4*f*tan(e/2 + f*x/2)**7
- 49*c**4*f*tan(e/2 + f*x/2)**6 + 147*c**4*f*tan(e/2 + f*x/2)**5 - 245*c**4
*f*tan(e/2 + f*x/2)**4 + 245*c**4*f*tan(e/2 + f*x/2)**3 - 147*c**4*f*tan(e/
2 + f*x/2)**2 + 49*c**4*f*tan(e/2 + f*x/2) - 7*c**4*f) - 2*a**3/(7*c**4*f*t
an(e/2 + f*x/2)**7 - 49*c**4*f*tan(e/2 + f*x/2)**6 + 147*c**4*f*tan(e/2 + f
```

```
*x/2)**5 - 245*c**4*f*tan(e/2 + f*x/2)**4 + 245*c**4*f*tan(e/2 + f*x/2)**3
- 147*c**4*f*tan(e/2 + f*x/2)**2 + 49*c**4*f*tan(e/2 + f*x/2) - 7*c**4*f),
Ne(f, 0)), (x*(a*sin(e) + a)**3/(-c*sin(e) + c)**4, True))
```

3.257 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^5}\,dx$

Optimal. Leaf size=69

$$\frac{a^3c^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}+\frac{a^3c^2\cos^7(e+fx)}{63f(c-c\sin(e+fx))^7}$$

```
[Out] 1/9*a^3*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^8+1/63*a^3*c^2*cos(f*x+e)^7/f/(
c-c*sin(f*x+e))^7
```

Rubi [A] time = 0.14, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2672, 2671}

$$\frac{a^3c^2\cos^7(e+fx)}{63f(c-c\sin(e+fx))^7}+\frac{a^3c^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (a^3*c^3*Cos[e + f*x]^7)/(9*f*(c - c*Sin[e + f*x])^8) + (a^3*c^2*Cos[e + f*
x]^7)/(63*f*(c - c*Sin[e + f*x])^7)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
```

```
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^5}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^8}\,dx\\ &= \frac{a^3c^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}+\frac{1}{9}\left(a^3c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^7}\,dx\\ &= \frac{a^3c^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}+\frac{a^3c^2\cos^7(e+fx)}{63f(c-c\sin(e+fx))^7}\end{aligned}$$

Mathematica [A] time = 0.74, size = 135, normalized size = 1.96

$$\frac{a^3\left(189\sin\left(\frac{1}{2}(e+fx)\right)+105\sin\left(\frac{3}{2}(e+fx)\right)-27\sin\left(\frac{5}{2}(e+fx)\right)-\sin\left(\frac{9}{2}(e+fx)\right)+315\cos\left(\frac{1}{2}(e+fx)\right)-18\right.}{504c^5f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (a^3*(315*Cos[(e + f*x)/2] - 189*Cos[(3*(e + f*x))/2] - 63*Cos[(5*(e + f*x)
)/2] + 9*Cos[(7*(e + f*x))/2] + 189*Sin[(e + f*x)/2] + 105*Sin[(3*(e + f*x)
)/2] - 27*Sin[(5*(e + f*x))/2] - Sin[(9*(e + f*x))/2]))/(504*c^5*f*(Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2])^9)
```

fricas [B] time = 0.45, size = 276, normalized size = 4.00

$$-\frac{a^3\cos\left(fx+e\right)^5-4\,a^3\cos\left(fx+e\right)^4+19\,a^3\cos\left(fx+e\right)^3+52\,a^3\cos\left(fx+e\right)^2-28\,a^3\cos\left(fx+e\right)-}{63\left(c^5f\cos\left(fx+e\right)^5+5\,c^5f\cos\left(fx+e\right)^4-8\,c^5f\cos\left(fx+e\right)^3-20\,c^5f\cos\left(fx+e\right)^2+8\,c^5f\cos\left(fx+e\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] -1/63*(a^3*cos(f*x + e)^5 - 4*a^3*cos(f*x + e)^4 + 19*a^3*cos(f*x + e)^3 +
52*a^3*cos(f*x + e)^2 - 28*a^3*cos(f*x + e) - 56*a^3 + (a^3*cos(f*x + e)^4
+ 5*a^3*cos(f*x + e)^3 + 24*a^3*cos(f*x + e)^2 - 28*a^3*cos(f*x + e) - 56*a
^3)*sin(f*x + e))/(c^5*f*cos(f*x + e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*
```

```
cos(f*x + e)^3 - 20*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f
- (c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)^2
+ 8*c^5*f*cos(f*x + e) + 16*c^5*f)*sin(f*x + e))
```

giac [B] time = 0.26, size = 162, normalized size = 2.35

$$-\frac{2\left(63\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-63\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+483\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-315\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+693\,a\right.}{63\,c^{5}f\left(\tan\left(\frac{1}{2}\,fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algorithm="giac")
```

```
[Out] -2/63*(63*a^3*tan(1/2*f*x + 1/2*e)^8 - 63*a^3*tan(1/2*f*x + 1/2*e)^7 + 483*
a^3*tan(1/2*f*x + 1/2*e)^6 - 315*a^3*tan(1/2*f*x + 1/2*e)^5 + 693*a^3*tan(1
/2*f*x + 1/2*e)^4 - 189*a^3*tan(1/2*f*x + 1/2*e)^3 + 225*a^3*tan(1/2*f*x +
1/2*e)^2 - 9*a^3*tan(1/2*f*x + 1/2*e) + 8*a^3)/(c^5*f*(tan(1/2*f*x + 1/2*e)
 - 1)^9)
```

maple [B] time = 0.34, size = 148, normalized size = 2.14

$$\frac{2a^{3}\left(-\frac{7}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{76}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{128}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{9}}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{64}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{8}}-\frac{496}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{86}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}\right.}{f\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x)
```

```
[Out] 2/f*a^3/c^5*(-7/(tan(1/2*f*x+1/2*e)-1)^2-76/(tan(1/2*f*x+1/2*e)-1)^4-128/9/
(tan(1/2*f*x+1/2*e)-1)^9-1/(tan(1/2*f*x+1/2*e)-1)-64/(tan(1/2*f*x+1/2*e)-1)
^8-496/3/(tan(1/2*f*x+1/2*e)-1)^6-86/3/(tan(1/2*f*x+1/2*e)-1)^3-136/(tan(1/
2*f*x+1/2*e)-1)^5-928/7/(tan(1/2*f*x+1/2*e)-1)^7)
```

maxima [B] time = 0.81, size = 1389, normalized size = 20.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] -2/315*(a^3*(432*sin(f*x + e)/(cos(f*x + e) + 1) - 1728*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 3612*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5418*sin(f*x
```

```
+ e)^4/(cos(f*x + e) + 1)^4 + 5040*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 33
60*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1260*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 - 315*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 83)/(c^5 - 9*c^5*sin(f*x
 + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*
c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9
*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e)
 + 1)^9) - 15*a^3*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 315*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 1
47*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^7/(cos(f*x + e) +
1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126
*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x +
 e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^
5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 10*a^3*(9*sin(f*x + e)/(cos(f*x +
e) + 1) - 36*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 84*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 - 63*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 3
6*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 -
 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x
 + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 42*a^3*(9*sin(f*x
 + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 54*sin(
f*x + e)^3/(cos(f*x + e) + 1)^3 - 81*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
45*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 30*sin(f*x + e)^6/(cos(f*x + e) +
1)^6 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126
*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x +
 e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^
5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9))/f
```

mupad [B] time = 8.58, size = 121, normalized size = 1.75

$$\frac{\sqrt{2}\, a^3 \cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(\frac{37\,\cos\left(3\,e+3\,f\,x\right)}{8}-\frac{63\,\sin\left(e+f\,x\right)}{2}-\frac{113\,\cos\left(2\,e+2\,f\,x\right)}{4}-\frac{257\,\cos\left(e+f\,x\right)}{8}+\frac{7\,\cos\left(4\,e+4\,f\,x\right)}{16}+\frac{63\,\sin\left(2\,e+2\,f\,x\right)}{8}\right.}{1008\,c^5\,f\,\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{f\,x}{2}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^5,x)
```

```
[Out] (2^(1/2)*a^3*cos(e/2 + (f*x)/2)*((37*cos(3*e + 3*f*x))/8 - (63*sin(e + f*x)
)/2 - (113*cos(2*e + 2*f*x))/4 - (257*cos(e + f*x))/8 + (7*cos(4*e + 4*f*x)
)/16 + (63*sin(2*e + 2*f*x))/8 + (9*sin(3*e + 3*f*x))/2 - (9*sin(4*e + 4*f*
x))/16 + 1013/16))/(1008*c^5*f*cos(e/2 + pi/4 + (f*x)/2)^9)
```

sympy [A] time = 72.34, size = 1717, normalized size = 24.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((-126*a**3*tan(e/2 + f*x/2)**8/(63*c**5*f*tan(e/2 + f*x/2)**9 - 5
67*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5
*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(
e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f
*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) + 126*a**3*tan(e/2 + f*
x/2)**7/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2
268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**
5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan
(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f
*x/2) - 63*c**5*f) - 966*a**3*tan(e/2 + f*x/2)**6/(63*c**5*f*tan(e/2 + f*x/
2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 -
5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c*
*5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*ta
n(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) + 630*a**3*tan
(e/2 + f*x/2)**5/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/
2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 +
 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c
**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*ta
n(e/2 + f*x/2) - 63*c**5*f) - 1386*a**3*tan(e/2 + f*x/2)**4/(63*c**5*f*tan(
e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*
x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5
 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268
*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) + 37
8*a**3*tan(e/2 + f*x/2)**3/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(
e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f
*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**
4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567
*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) - 450*a**3*tan(e/2 + f*x/2)**2/(63*c*
*5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan
(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 +
f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)*
```

```
*3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**
5*f) + 18*a**3*tan(e/2 + f*x/2)/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f
*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/
2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x
/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2
+ 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) - 16*a**3/(63*c**5*f*tan(e/2 + f
*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7
 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938
*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f
*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f), Ne(f, 0)),
 (x*(a*sin(e) + a)**3/(-c*sin(e) + c)**5, True))
```

3.258 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^6}\,dx$

Optimal. Leaf size=101

$$\frac{a^3c^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}+\frac{2a^3c^2\cos^7(e+fx)}{99f(c-c\sin(e+fx))^8}+\frac{2a^3c\cos^7(e+fx)}{693f(c-c\sin(e+fx))^7}$$

```
[Out] 1/11*a^3*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^9+2/99*a^3*c^2*cos(f*x+e)^7/f/
(c-c*sin(f*x+e))^8+2/693*a^3*c*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^7
```

Rubi [A] time = 0.18, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2672, 2671}

$$\frac{2a^3c^2\cos^7(e+fx)}{99f(c-c\sin(e+fx))^8}+\frac{a^3c^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}+\frac{2a^3c\cos^7(e+fx)}{693f(c-c\sin(e+fx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^6,x]
```

```
[Out] (a^3*c^3*Cos[e + f*x]^7)/(11*f*(c - c*Sin[e + f*x])^9) + (2*a^3*c^2*Cos[e +
 f*x]^7)/(99*f*(c - c*Sin[e + f*x])^8) + (2*a^3*c*Cos[e + f*x]^7)/(693*f*(c
 - c*Sin[e + f*x])^7)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
```

```
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^6}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^9}\,dx \\
&= \frac{a^3c^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}+\frac{1}{11}\left(2a^3c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^8}\,dx \\
&= \frac{a^3c^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}+\frac{2a^3c^2\cos^7(e+fx)}{99f(c-c\sin(e+fx))^8}+\frac{1}{99}\left(2a^3c\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx)} \\
&= \frac{a^3c^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}+\frac{2a^3c^2\cos^7(e+fx)}{99f(c-c\sin(e+fx))^8}+\frac{2a^3c\cos^7(e+fx)}{693f(c-c\sin(e+fx))^7}
\end{aligned}
$$

Mathematica [A] time = 0.90, size = 145, normalized size = 1.44

$$
-\frac{a^3\left(-2079\sin\left(\frac{1}{2}(e+fx)\right)-1155\sin\left(\frac{3}{2}(e+fx)\right)+297\sin\left(\frac{5}{2}(e+fx)\right)+11\sin\left(\frac{9}{2}(e+fx)\right)-2541\cos\left(\frac{1}{2}(e+\right.\right.}{5544c^6f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\right.}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^6,x]
```

```
[Out] -1/5544*(a^3*(-2541*Cos[(e + f*x)/2] + 1485*Cos[(3*(e + f*x))/2] + 462*Cos[
(5*(e + f*x))/2] - 55*Cos[(7*(e + f*x))/2] + Cos[(11*(e + f*x))/2] - 2079*S
in[(e + f*x)/2] - 1155*Sin[(3*(e + f*x))/2] + 297*Sin[(5*(e + f*x))/2] + 11
*Sin[(9*(e + f*x))/2]))/(c^6*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^11)
```

fricas [B] time = 0.50, size = 332, normalized size = 3.29

$$
\frac{2\,a^3\cos\left(fx+e\right)^6+12\,a^3\cos\left(fx+e\right)^5-25\,a^3\cos\left(fx+e\right)^4+161\,a^3\cos\left(fx+e\right)^3+448\,a^3\cos\left(fx+e\right)^2}{693\left(c^6f\cos\left(fx+e\right)^6-5\,c^6f\cos\left(fx+e\right)^5-18\,c^6f\cos\left(fx+e\right)^4+20\,c^6f\cos\left(fx+e\right)^3+48\,c^6f\cos\left(fx+e\right)\right.}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorithm="fricas")
```

```
[Out] 1/693*(2*a^3*cos(f*x + e)^6 + 12*a^3*cos(f*x + e)^5 - 25*a^3*cos(f*x + e)^4
 + 161*a^3*cos(f*x + e)^3 + 448*a^3*cos(f*x + e)^2 - 252*a^3*cos(f*x + e) -
 504*a^3 - (2*a^3*cos(f*x + e)^5 - 10*a^3*cos(f*x + e)^4 - 35*a^3*cos(f*x +
 e)^3 - 196*a^3*cos(f*x + e)^2 + 252*a^3*cos(f*x + e) + 504*a^3)*sin(f*x +
e))/(c^6*f*cos(f*x + e)^6 - 5*c^6*f*cos(f*x + e)^5 - 18*c^6*f*cos(f*x + e)^
4 + 20*c^6*f*cos(f*x + e)^3 + 48*c^6*f*cos(f*x + e)^2 - 16*c^6*f*cos(f*x +
e) - 32*c^6*f + (c^6*f*cos(f*x + e)^5 + 6*c^6*f*cos(f*x + e)^4 - 12*c^6*f*c
os(f*x + e)^3 - 32*c^6*f*cos(f*x + e)^2 + 16*c^6*f*cos(f*x + e) + 32*c^6*f)
*sin(f*x + e))
```

giac [A] time = 0.37, size = 196, normalized size = 1.94

$$-\frac{2\left(693\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}-1386\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}+8085\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-10626\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorithm="giac")
```

```
[Out] -2/693*(693*a^3*tan(1/2*f*x + 1/2*e)^10 - 1386*a^3*tan(1/2*f*x + 1/2*e)^9 +
 8085*a^3*tan(1/2*f*x + 1/2*e)^8 - 10626*a^3*tan(1/2*f*x + 1/2*e)^7 + 21252
*a^3*tan(1/2*f*x + 1/2*e)^6 - 15246*a^3*tan(1/2*f*x + 1/2*e)^5 + 15444*a^3*
tan(1/2*f*x + 1/2*e)^4 - 4950*a^3*tan(1/2*f*x + 1/2*e)^3 + 2959*a^3*tan(1/2
*f*x + 1/2*e)^2 - 176*a^3*tan(1/2*f*x + 1/2*e) + 79*a^3)/(c^6*f*(tan(1/2*f*
x + 1/2*e) - 1)^11)
```

maple [A] time = 0.32, size = 178, normalized size = 1.76

$$\frac{2a^3\left(-\frac{256}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{11}}-\frac{3008}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{9}}-\frac{128}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{10}}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{116}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{292}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{5}{\left(\tan\left(\frac{fx}{2}\right.\right.}\right.}{f\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x)
```

```
[Out] 2/f*a^3/c^6*(-256/11/(tan(1/2*f*x+1/2*e)-1)^11-3008/9/(tan(1/2*f*x+1/2*e)-1
)^9-128/(tan(1/2*f*x+1/2*e)-1)^10-1/(tan(1/2*f*x+1/2*e)-1)-116/3/(tan(1/2*f
*x+1/2*e)-1)^3-292/(tan(1/2*f*x+1/2*e)-1)^5-544/(tan(1/2*f*x+1/2*e)-1)^8-12
6/(tan(1/2*f*x+1/2*e)-1)^4-1480/3/(tan(1/2*f*x+1/2*e)-1)^6-4272/7/(tan(1/2*
f*x+1/2*e)-1)^7-8/(tan(1/2*f*x+1/2*e)-1)^2)
```

maxima [B] time = 1.25, size = 1734, normalized size = 17.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorithm="maxima")

[Out] -2/3465*(5*a^3*(913*sin(f*x + e)/(cos(f*x + e) + 1) - 4565*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 12540*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 25080*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33726*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 33726*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 23100*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 11550*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 3465*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 693*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 146)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 9*a^3*(671*sin(f*x + e)/(cos(f*x + e) + 1) - 2200*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 6600*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10890*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 15246*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 12936*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 9240*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3465*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1155*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 61)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 2*a^3*(341*sin(f*x + e)/(cos(f*x + e) + 1) - 1705*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5115*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6765*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 9471*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 4851*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 3465*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 31)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 12*a^3*(253*sin(f*x + e)/(cos(f*x + e) + 1) - 1265*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2640*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5280*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5313*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 5313*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 2310*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 1155*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 23)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55

```
*c^6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
- 330*c^6*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos
(f*x + e) + 1)^8 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(
f*x + e)^10/(cos(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^
11))/f
```

mupad [B] time = 9.34, size = 143, normalized size = 1.42

$$-\frac{\sqrt{2}\, a^3 \cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(\frac{6635\,\cos\left(e+f\,x\right)}{16}+\frac{13629\,\sin\left(e+f\,x\right)}{16}+565\,\cos\left(2\,e+2\,f\,x\right)-\frac{3527\,\cos\left(3\,e+3\,f\,x\right)}{32}-29\,\cos\left(4\,e+4\,\right.\right.}{22176\,c^6\,f\cos\left(\frac{e}{2}+\frac{\pi}{4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^6,x)
```

```
[Out] -(2^(1/2)*a^3*cos(e/2 + (f*x)/2)*((6635*cos(e + f*x))/16 + (13629*sin(e + f
*x))/16 + 565*cos(2*e + 2*f*x) - (3527*cos(3*e + 3*f*x))/32 - 29*cos(4*e +
4*f*x) + (81*cos(5*e + 5*f*x))/32 - (1617*sin(2*e + 2*f*x))/8 - (5049*sin(3
*e + 3*f*x))/32 + (407*sin(4*e + 4*f*x))/16 + (77*sin(5*e + 5*f*x))/32 - 92
2))/(22176*c^6*f*cos(e/2 + pi/4 + (f*x)/2)^11)
```

sympy [A] time = 118.54, size = 2509, normalized size = 24.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**6,x)
```

```
[Out] Piecewise((-1386*a**3*tan(e/2 + f*x/2)**10/(693*c**6*f*tan(e/2 + f*x/2)**11
 - 7623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 - 11
4345*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 32016
6*c**6*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c
**6*f*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*
f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f) + 2772*a
**3*tan(e/2 + f*x/2)**9/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(
e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2
 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 320166*c**6*f*tan(e/2 +
f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c**6*f*tan(e/2 + f*x
/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*f*tan(e/2 + f*x/2)*
*2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f) - 16170*a**3*tan(e/2 + f*x/
2)**8/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 +
```

38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2 + f*x/2)**8 + 228
690*c**6*f*tan(e/2 + f*x/2)**7 - 320166*c**6*f*tan(e/2 + f*x/2)**6 + 320166
*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c**6*f*tan(e/2 + f*x/2)**4 + 114345*c*
*6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*t
an(e/2 + f*x/2) - 693*c**6*f) + 21252*a**3*tan(e/2 + f*x/2)**7/(693*c**6*f*
tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan(
e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2
 + f*x/2)**7 - 320166*c**6*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 +
f*x/2)**5 - 228690*c**6*f*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x
/2)**3 - 38115*c**6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) -
693*c**6*f) - 42504*a**3*tan(e/2 + f*x/2)**6/(693*c**6*f*tan(e/2 + f*x/2)**
11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 -
114345*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 320
166*c**6*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 228690
*c**6*f*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**
6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f) + 3049
2*a**3*tan(e/2 + f*x/2)**5/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*t
an(e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345*c**6*f*tan(
e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 320166*c**6*f*tan(e/2
 + f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c**6*f*tan(e/2 +
f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*f*tan(e/2 + f*x/
2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f) - 30888*a**3*tan(e/2 + f
*x/2)**4/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(e/2 + f*x/2)**1
0 + 38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2 + f*x/2)**8 +
228690*c**6*f*tan(e/2 + f*x/2)**7 - 320166*c**6*f*tan(e/2 + f*x/2)**6 + 320
166*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c**6*f*tan(e/2 + f*x/2)**4 + 114345
*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*
f*tan(e/2 + f*x/2) - 693*c**6*f) + 9900*a**3*tan(e/2 + f*x/2)**3/(693*c**6*
f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*ta
n(e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e
/2 + f*x/2)**7 - 320166*c**6*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2
+ f*x/2)**5 - 228690*c**6*f*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e/2 + f
*x/2)**3 - 38115*c**6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2)
- 693*c**6*f) - 5918*a**3*tan(e/2 + f*x/2)**2/(693*c**6*f*tan(e/2 + f*x/2)*
*11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 -
 114345*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 32
0166*c**6*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 22869
0*c**6*f*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c*
*6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f) + 352
*a**3*tan(e/2 + f*x/2)/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(e
/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2
+ f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 320166*c**6*f*tan(e/2 + f
*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c**6*f*tan(e/2 + f*x/
2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*f*tan(e/2 + f*x/2)**
2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f) - 158*a**3/(693*c**6*f*tan(e

```
/2 + f*x/2)**11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 +
 f*x/2)**9 - 114345*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*
x/2)**7 - 320166*c**6*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2
)**5 - 228690*c**6*f*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**
3 - 38115*c**6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c
**6*f), Ne(f, 0)), (x*(a*sin(e) + a)**3/(-c*sin(e) + c)**6, True))
```

3.259 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^7}\,dx$

Optimal. Leaf size=132

$$\frac{a^3c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}+\frac{3a^3c^2\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9}+\frac{2a^3\cos^7(e+fx)}{3003f(c-c\sin(e+fx))^7}+\frac{2a^3c\cos^7(e+fx)}{429f(c-c\sin(e+fx))^8}$$

```
[Out] 1/13*a^3*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^10+3/143*a^3*c^2*cos(f*x+e)^7/
f/(c-c*sin(f*x+e))^9+2/429*a^3*c*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^8+2/3003*a
^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^7
```

Rubi [A] time = 0.23, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2672, 2671}

$$\frac{3a^3c^2\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9}+\frac{a^3c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}+\frac{2a^3\cos^7(e+fx)}{3003f(c-c\sin(e+fx))^7}+\frac{2a^3c\cos^7(e+fx)}{429f(c-c\sin(e+fx))^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^7,x]
```

```
[Out] (a^3*c^3*Cos[e + f*x]^7)/(13*f*(c - c*Sin[e + f*x])^10) + (3*a^3*c^2*Cos[e
 + f*x]^7)/(143*f*(c - c*Sin[e + f*x])^9) + (2*a^3*c*Cos[e + f*x]^7)/(429*f*
(c - c*Sin[e + f*x])^8) + (2*a^3*Cos[e + f*x]^7)/(3003*f*(c - c*Sin[e + f*x
])^7)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^7}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{10}}\,dx$$
$$= \frac{a^3c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}} + \frac{1}{13}\left(3a^3c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^9}\,dx$$
$$= \frac{a^3c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}} + \frac{3a^3c^2\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9} + \frac{1}{143}\left(6a^3c\right)\int \frac{\cos^6(e+fx}{(c-c\sin(e+f}$$
$$= \frac{a^3c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}} + \frac{3a^3c^2\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9} + \frac{2a^3c\cos^7(e+fx)}{429f(c-c\sin(e+fx))^8} + \frac{1}{42}$$
$$= \frac{a^3c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}} + \frac{3a^3c^2\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9} + \frac{2a^3c\cos^7(e+fx)}{429f(c-c\sin(e+fx))^8} + \frac{}{30}$$

Mathematica [A] time = 1.98, size = 157, normalized size = 1.19

$$\frac{a^3\left(16302\sin\left(\frac{1}{2}(e+fx)\right)+9009\sin\left(\frac{3}{2}(e+fx)\right)-2288\sin\left(\frac{5}{2}(e+fx)\right)-78\sin\left(\frac{9}{2}(e+fx)\right)+\sin\left(\frac{13}{2}(e+fx)\right)\right.}{48048c^7f\left(\cos\left(\frac{1}{2}(e+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^7,x]
```

```
[Out] (a^3*(18018*Cos[(e + f*x)/2] - 10296*Cos[(3*(e + f*x))/2] - 3003*Cos[(5*(e
+ f*x))/2] + 286*Cos[(7*(e + f*x))/2] - 13*Cos[(11*(e + f*x))/2] + 16302*Si
n[(e + f*x)/2] + 9009*Sin[(3*(e + f*x))/2] - 2288*Sin[(5*(e + f*x))/2] - 78
*Sin[(9*(e + f*x))/2] + Sin[(13*(e + f*x))/2]))/(48048*c^7*f*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^13)
```

fricas [B] time = 0.49, size = 386, normalized size = 2.92

$$-\frac{2\,a^3\cos\left(fx+e\right)^7-12\,a^3\cos\left(fx+e\right)^6-49\,a^3\cos\left(fx+e\right)^5+70\,a^3\cos\left(fx+e\right)^4-567\,a^3\cos\left(fx+e\right)^3-}{3003\left(c^7f\cos\left(fx+e\right)^7+7\,c^7f\cos\left(fx+e\right)^6-18\,c^7f\cos\left(fx+e\right)^5-56\,c^7f\cos\left(fx+e\right)^4+48\,c^7f\cos\left(fx+e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^7,x, algorithm="fricas")
```

```
[Out] -1/3003*(2*a^3*cos(f*x + e)^7 - 12*a^3*cos(f*x + e)^6 - 49*a^3*cos(f*x + e)
^5 + 70*a^3*cos(f*x + e)^4 - 567*a^3*cos(f*x + e)^3 - 1596*a^3*cos(f*x + e)
^2 + 924*a^3*cos(f*x + e) + 1848*a^3 + (2*a^3*cos(f*x + e)^6 + 14*a^3*cos(f
*x + e)^5 - 35*a^3*cos(f*x + e)^4 - 105*a^3*cos(f*x + e)^3 - 672*a^3*cos(f*
x + e)^2 + 924*a^3*cos(f*x + e) + 1848*a^3)*sin(f*x + e))/(c^7*f*cos(f*x +
e)^7 + 7*c^7*f*cos(f*x + e)^6 - 18*c^7*f*cos(f*x + e)^5 - 56*c^7*f*cos(f*x
+ e)^4 + 48*c^7*f*cos(f*x + e)^3 + 112*c^7*f*cos(f*x + e)^2 - 32*c^7*f*cos(
f*x + e) - 64*c^7*f - (c^7*f*cos(f*x + e)^6 - 6*c^7*f*cos(f*x + e)^5 - 24*c
^7*f*cos(f*x + e)^4 + 32*c^7*f*cos(f*x + e)^3 + 80*c^7*f*cos(f*x + e)^2 - 3
2*c^7*f*cos(f*x + e) - 64*c^7*f)*sin(f*x + e))
```

giac [A] time = 0.33, size = 230, normalized size = 1.74

$$-\frac{2\left(3003\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{12}-9009\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}+51051\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}-99099\,a^3\tan\left(\frac{1}{2}\,fx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^7,x, algorithm="giac")
```

```
[Out] -2/3003*(3003*a^3*tan(1/2*f*x + 1/2*e)^12 - 9009*a^3*tan(1/2*f*x + 1/2*e)^1
1 + 51051*a^3*tan(1/2*f*x + 1/2*e)^10 - 99099*a^3*tan(1/2*f*x + 1/2*e)^9 +
216216*a^3*tan(1/2*f*x + 1/2*e)^8 - 246246*a^3*tan(1/2*f*x + 1/2*e)^7 + 285
714*a^3*tan(1/2*f*x + 1/2*e)^6 - 182754*a^3*tan(1/2*f*x + 1/2*e)^5 + 122551
*a^3*tan(1/2*f*x + 1/2*e)^4 - 37609*a^3*tan(1/2*f*x + 1/2*e)^3 + 15171*a^3*
tan(1/2*f*x + 1/2*e)^2 - 1027*a^3*tan(1/2*f*x + 1/2*e) + 310*a^3)/(c^7*f*(t
an(1/2*f*x + 1/2*e) - 1)^13)
```

maple [A] time = 0.37, size = 208, normalized size = 1.58

$$\frac{2a^3\left(-\frac{8832}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{11}}-\frac{50}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{2352}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{8}}-\frac{9}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{512}{13\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{13}}-\frac{6752}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{9}}-\right.}{f\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^7,x)
```

```
[Out] 2/f*a^3/c^7*(-8832/11/(tan(1/2*f*x+1/2*e)-1)^11-50/(tan(1/2*f*x+1/2*e)-1)^3
-2352/(tan(1/2*f*x+1/2*e)-1)^8-9/(tan(1/2*f*x+1/2*e)-1)^2-512/13/(tan(1/2*f
```

```
*x+1/2*e)-1)^13-6752/3/(tan(1/2*f*x+1/2*e)-1)^9-1/(tan(1/2*f*x+1/2*e)-1)-19
2/(tan(1/2*f*x+1/2*e)-1)^4-1148/(tan(1/2*f*x+1/2*e)-1)^6-1600/(tan(1/2*f*x+
1/2*e)-1)^10-540/(tan(1/2*f*x+1/2*e)-1)^5-256/(tan(1/2*f*x+1/2*e)-1)^12-131
12/7/(tan(1/2*f*x+1/2*e)-1)^7)
```

maxima [B] time = 0.88, size = 2078, normalized size = 15.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^7,x, algorithm="maxima")
```

```
[Out] -2/15015*(2*a^3*(4771*sin(f*x + e)/(cos(f*x + e) + 1) - 28626*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + 74932*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 187330
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 265122*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5 - 353496*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 276276*sin(f*x + e)^7
/(cos(f*x + e) + 1)^7 - 207207*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 75075*
sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 30030*sin(f*x + e)^10/(cos(f*x + e) +
 1)^10 - 367)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^
7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x +
e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x +
 e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11
 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(
f*x + e) + 1)^13) + 5*a^3*(3796*sin(f*x + e)/(cos(f*x + e) + 1) - 22776*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 77506*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3 - 193765*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 339768*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 - 453024*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 444444*sin(
f*x + e)^7/(cos(f*x + e) + 1)^7 - 333333*sin(f*x + e)^8/(cos(f*x + e) + 1)^
8 + 180180*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 72072*sin(f*x + e)^10/(cos
(f*x + e) + 1)^10 + 18018*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 3003*sin(
f*x + e)^12/(cos(f*x + e) + 1)^12 - 523)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*
x + e) + 1) + 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
 1287*c^7*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(co
s(f*x + e) + 1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7
*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e)
 + 1)^9 + 286*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x +
e)^11/(cos(f*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12
- c^7*sin(f*x + e)^13/(cos(f*x + e) + 1)^13) - 35*a^3*(611*sin(f*x + e)/(co
s(f*x + e) + 1) - 2379*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 8723*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 - 18590*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33
462*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 40326*sin(f*x + e)^6/(cos(f*x + e
```

```
) + 1)^6 + 40326*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 27027*sin(f*x + e)^8
/(cos(f*x + e) + 1)^8 + 15015*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 4719*si
n(f*x + e)^10/(cos(f*x + e) + 1)^10 + 1287*sin(f*x + e)^11/(cos(f*x + e) +
1)^11 - 47)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^7*
sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x + e)
 + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x + e
)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 +
 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(f*
x + e) + 1)^13) - 154*a^3*(13*sin(f*x + e)/(cos(f*x + e) + 1) - 78*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + 286*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 520
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 936*sin(f*x + e)^5/(cos(f*x + e) + 1
)^5 - 858*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 858*sin(f*x + e)^7/(cos(f*x
 + e) + 1)^7 - 351*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 195*sin(f*x + e)^9
/(cos(f*x + e) + 1)^9 - 1)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) +
78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1
)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^
8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*
c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x
 + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x
+ e)^13/(cos(f*x + e) + 1)^13))/f
```

mupad [B] time = 10.17, size = 165, normalized size = 1.25

$$-\frac{\sqrt{2}\,a^3\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{8993\,\cos\left(e+f\,x\right)}{4}+\frac{57915\,\sin\left(e+f\,x\right)}{8}+\frac{73423\,\cos\left(2\,e+2\,f\,x\right)}{16}-\frac{15365\,\cos\left(3\,e+3\,f\,x\right)}{16}-\frac{6943\,\cos\left(4\,e+4\,f\,x\right)}{16}+\right.}{192192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^7,x)
```

```
[Out] -(2^(1/2)*a^3*cos(e/2 + (f*x)/2)*((8993*cos(e + f*x))/4 + (57915*sin(e + f*
x))/8 + (73423*cos(2*e + 2*f*x))/16 - (15365*cos(3*e + 3*f*x))/16 - (6943*c
os(4*e + 4*f*x))/16 + (937*cos(5*e + 5*f*x))/16 + (77*cos(6*e + 6*f*x))/16
- (6435*sin(2*e + 2*f*x))/4 - (27027*sin(3*e + 3*f*x))/16 + (5005*sin(4*e +
 4*f*x))/16 + (1079*sin(5*e + 5*f*x))/16 - (39*sin(6*e + 6*f*x))/8 - 93061/
16))/(192192*c^7*f*cos(e/2 + pi/4 + (f*x)/2)^13)
```

sympy [A] time = 178.92, size = 3451, normalized size = 26.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**7,x)
```

```
[Out] Piecewise((-6006*a**3*tan(e/2 + f*x/2)**12/(3003*c**7*f*tan(e/2 + f*x/2)**1
3 - 39039*c**7*f*tan(e/2 + f*x/2)**12 + 234234*c**7*f*tan(e/2 + f*x/2)**11
- 858858*c**7*f*tan(e/2 + f*x/2)**10 + 2147145*c**7*f*tan(e/2 + f*x/2)**9 -
 3864861*c**7*f*tan(e/2 + f*x/2)**8 + 5153148*c**7*f*tan(e/2 + f*x/2)**7 -
5153148*c**7*f*tan(e/2 + f*x/2)**6 + 3864861*c**7*f*tan(e/2 + f*x/2)**5 - 2
147145*c**7*f*tan(e/2 + f*x/2)**4 + 858858*c**7*f*tan(e/2 + f*x/2)**3 - 234
234*c**7*f*tan(e/2 + f*x/2)**2 + 39039*c**7*f*tan(e/2 + f*x/2) - 3003*c**7*
f) + 18018*a**3*tan(e/2 + f*x/2)**11/(3003*c**7*f*tan(e/2 + f*x/2)**13 - 39
039*c**7*f*tan(e/2 + f*x/2)**12 + 234234*c**7*f*tan(e/2 + f*x/2)**11 - 8588
58*c**7*f*tan(e/2 + f*x/2)**10 + 2147145*c**7*f*tan(e/2 + f*x/2)**9 - 38648
61*c**7*f*tan(e/2 + f*x/2)**8 + 5153148*c**7*f*tan(e/2 + f*x/2)**7 - 515314
8*c**7*f*tan(e/2 + f*x/2)**6 + 3864861*c**7*f*tan(e/2 + f*x/2)**5 - 2147145
*c**7*f*tan(e/2 + f*x/2)**4 + 858858*c**7*f*tan(e/2 + f*x/2)**3 - 234234*c*
*7*f*tan(e/2 + f*x/2)**2 + 39039*c**7*f*tan(e/2 + f*x/2) - 3003*c**7*f) - 1
02102*a**3*tan(e/2 + f*x/2)**10/(3003*c**7*f*tan(e/2 + f*x/2)**13 - 39039*c
**7*f*tan(e/2 + f*x/2)**12 + 234234*c**7*f*tan(e/2 + f*x/2)**11 - 858858*c*
*7*f*tan(e/2 + f*x/2)**10 + 2147145*c**7*f*tan(e/2 + f*x/2)**9 - 3864861*c*
*7*f*tan(e/2 + f*x/2)**8 + 5153148*c**7*f*tan(e/2 + f*x/2)**7 - 5153148*c**
7*f*tan(e/2 + f*x/2)**6 + 3864861*c**7*f*tan(e/2 + f*x/2)**5 - 2147145*c**7
*f*tan(e/2 + f*x/2)**4 + 858858*c**7*f*tan(e/2 + f*x/2)**3 - 234234*c**7*f*
tan(e/2 + f*x/2)**2 + 39039*c**7*f*tan(e/2 + f*x/2) - 3003*c**7*f) + 198198
*a**3*tan(e/2 + f*x/2)**9/(3003*c**7*f*tan(e/2 + f*x/2)**13 - 39039*c**7*f*
tan(e/2 + f*x/2)**12 + 234234*c**7*f*tan(e/2 + f*x/2)**11 - 858858*c**7*f*t
an(e/2 + f*x/2)**10 + 2147145*c**7*f*tan(e/2 + f*x/2)**9 - 3864861*c**7*f*t
an(e/2 + f*x/2)**8 + 5153148*c**7*f*tan(e/2 + f*x/2)**7 - 5153148*c**7*f*ta
n(e/2 + f*x/2)**6 + 3864861*c**7*f*tan(e/2 + f*x/2)**5 - 2147145*c**7*f*tan
(e/2 + f*x/2)**4 + 858858*c**7*f*tan(e/2 + f*x/2)**3 - 234234*c**7*f*tan(e/
2 + f*x/2)**2 + 39039*c**7*f*tan(e/2 + f*x/2) - 3003*c**7*f) - 432432*a**3*
tan(e/2 + f*x/2)**8/(3003*c**7*f*tan(e/2 + f*x/2)**13 - 39039*c**7*f*tan(e/
2 + f*x/2)**12 + 234234*c**7*f*tan(e/2 + f*x/2)**11 - 858858*c**7*f*tan(e/2
 + f*x/2)**10 + 2147145*c**7*f*tan(e/2 + f*x/2)**9 - 3864861*c**7*f*tan(e/2
 + f*x/2)**8 + 5153148*c**7*f*tan(e/2 + f*x/2)**7 - 5153148*c**7*f*tan(e/2
+ f*x/2)**6 + 3864861*c**7*f*tan(e/2 + f*x/2)**5 - 2147145*c**7*f*tan(e/2 +
 f*x/2)**4 + 858858*c**7*f*tan(e/2 + f*x/2)**3 - 234234*c**7*f*tan(e/2 + f*
x/2)**2 + 39039*c**7*f*tan(e/2 + f*x/2) - 3003*c**7*f) + 492492*a**3*tan(e/
2 + f*x/2)**7/(3003*c**7*f*tan(e/2 + f*x/2)**13 - 39039*c**7*f*tan(e/2 + f*
x/2)**12 + 234234*c**7*f*tan(e/2 + f*x/2)**11 - 858858*c**7*f*tan(e/2 + f*x
/2)**10 + 2147145*c**7*f*tan(e/2 + f*x/2)**9 - 3864861*c**7*f*tan(e/2 + f*x
/2)**8 + 5153148*c**7*f*tan(e/2 + f*x/2)**7 - 5153148*c**7*f*tan(e/2 + f*x/
2)**6 + 3864861*c**7*f*tan(e/2 + f*x/2)**5 - 2147145*c**7*f*tan(e/2 + f*x/2
)**4 + 858858*c**7*f*tan(e/2 + f*x/2)**3 - 234234*c**7*f*tan(e/2 + f*x/2)**
```

```
2 + 39039*c**7*f*tan(e/2 + f*x/2) - 3003*c**7*f) - 571428*a**3*tan(e/2 + f*
x/2)**6/(3003*c**7*f*tan(e/2 + f*x/2)**13 - 39039*c**7*f*tan(e/2 + f*x/2)**
12 + 234234*c**7*f*tan(e/2 + f*x/2)**11 - 858858*c**7*f*tan(e/2 + f*x/2)**1
0 + 2147145*c**7*f*tan(e/2 + f*x/2)**9 - 3864861*c**7*f*tan(e/2 + f*x/2)**8
 + 5153148*c**7*f*tan(e/2 + f*x/2)**7 - 5153148*c**7*f*tan(e/2 + f*x/2)**6
+ 3864861*c**7*f*tan(e/2 + f*x/2)**5 - 2147145*c**7*f*tan(e/2 + f*x/2)**4 +
 858858*c**7*f*tan(e/2 + f*x/2)**3 - 234234*c**7*f*tan(e/2 + f*x/2)**2 + 39
039*c**7*f*tan(e/2 + f*x/2) - 3003*c**7*f) + 365508*a**3*tan(e/2 + f*x/2)**
5/(3003*c**7*f*tan(e/2 + f*x/2)**13 - 39039*c**7*f*tan(e/2 + f*x/2)**12 + 2
34234*c**7*f*tan(e/2 + f*x/2)**11 - 858858*c**7*f*tan(e/2 + f*x/2)**10 + 21
47145*c**7*f*tan(e/2 + f*x/2)**9 - 3864861*c**7*f*tan(e/2 + f*x/2)**8 + 515
3148*c**7*f*tan(e/2 + f*x/2)**7 - 5153148*c**7*f*tan(e/2 + f*x/2)**6 + 3864
861*c**7*f*tan(e/2 + f*x/2)**5 - 2147145*c**7*f*tan(e/2 + f*x/2)**4 + 85885
8*c**7*f*tan(e/2 + f*x/2)**3 - 234234*c**7*f*tan(e/2 + f*x/2)**2 + 39039*c*
*7*f*tan(e/2 + f*x/2) - 3003*c**7*f) - 245102*a**3*tan(e/2 + f*x/2)**4/(300
3*c**7*f*tan(e/2 + f*x/2)**13 - 39039*c**7*f*tan(e/2 + f*x/2)**12 + 234234*
c**7*f*tan(e/2 + f*x/2)**11 - 858858*c**7*f*tan(e/2 + f*x/2)**10 + 2147145*
c**7*f*tan(e/2 + f*x/2)**9 - 3864861*c**7*f*tan(e/2 + f*x/2)**8 + 5153148*c
**7*f*tan(e/2 + f*x/2)**7 - 5153148*c**7*f*tan(e/2 + f*x/2)**6 + 3864861*c*
*7*f*tan(e/2 + f*x/2)**5 - 2147145*c**7*f*tan(e/2 + f*x/2)**4 + 858858*c**7
*f*tan(e/2 + f*x/2)**3 - 234234*c**7*f*tan(e/2 + f*x/2)**2 + 39039*c**7*f*t
an(e/2 + f*x/2) - 3003*c**7*f) + 75218*a**3*tan(e/2 + f*x/2)**3/(3003*c**7*
f*tan(e/2 + f*x/2)**13 - 39039*c**7*f*tan(e/2 + f*x/2)**12 + 234234*c**7*f*
tan(e/2 + f*x/2)**11 - 858858*c**7*f*tan(e/2 + f*x/2)**10 + 2147145*c**7*f*
tan(e/2 + f*x/2)**9 - 3864861*c**7*f*tan(e/2 + f*x/2)**8 + 5153148*c**7*f*t
an(e/2 + f*x/2)**7 - 5153148*c**7*f*tan(e/2 + f*x/2)**6 + 3864861*c**7*f*ta
n(e/2 + f*x/2)**5 - 2147145*c**7*f*tan(e/2 + f*x/2)**4 + 858858*c**7*f*tan(
e/2 + f*x/2)**3 - 234234*c**7*f*tan(e/2 + f*x/2)**2 + 39039*c**7*f*tan(e/2
+ f*x/2) - 3003*c**7*f) - 30342*a**3*tan(e/2 + f*x/2)**2/(3003*c**7*f*tan(e
/2 + f*x/2)**13 - 39039*c**7*f*tan(e/2 + f*x/2)**12 + 234234*c**7*f*tan(e/2
 + f*x/2)**11 - 858858*c**7*f*tan(e/2 + f*x/2)**10 + 2147145*c**7*f*tan(e/2
 + f*x/2)**9 - 3864861*c**7*f*tan(e/2 + f*x/2)**8 + 5153148*c**7*f*tan(e/2
+ f*x/2)**7 - 5153148*c**7*f*tan(e/2 + f*x/2)**6 + 3864861*c**7*f*tan(e/2 +
 f*x/2)**5 - 2147145*c**7*f*tan(e/2 + f*x/2)**4 + 858858*c**7*f*tan(e/2 + f
*x/2)**3 - 234234*c**7*f*tan(e/2 + f*x/2)**2 + 39039*c**7*f*tan(e/2 + f*x/2
) - 3003*c**7*f) + 2054*a**3*tan(e/2 + f*x/2)/(3003*c**7*f*tan(e/2 + f*x/2)
**13 - 39039*c**7*f*tan(e/2 + f*x/2)**12 + 234234*c**7*f*tan(e/2 + f*x/2)**
11 - 858858*c**7*f*tan(e/2 + f*x/2)**10 + 2147145*c**7*f*tan(e/2 + f*x/2)**
9 - 3864861*c**7*f*tan(e/2 + f*x/2)**8 + 5153148*c**7*f*tan(e/2 + f*x/2)**7
 - 5153148*c**7*f*tan(e/2 + f*x/2)**6 + 3864861*c**7*f*tan(e/2 + f*x/2)**5
- 2147145*c**7*f*tan(e/2 + f*x/2)**4 + 858858*c**7*f*tan(e/2 + f*x/2)**3 -
234234*c**7*f*tan(e/2 + f*x/2)**2 + 39039*c**7*f*tan(e/2 + f*x/2) - 3003*c*
*7*f) - 620*a**3/(3003*c**7*f*tan(e/2 + f*x/2)**13 - 39039*c**7*f*tan(e/2 +
 f*x/2)**12 + 234234*c**7*f*tan(e/2 + f*x/2)**11 - 858858*c**7*f*tan(e/2 +
f*x/2)**10 + 2147145*c**7*f*tan(e/2 + f*x/2)**9 - 3864861*c**7*f*tan(e/2 +
```

```
f*x/2)**8 + 5153148*c**7*f*tan(e/2 + f*x/2)**7 - 5153148*c**7*f*tan(e/2 + f
*x/2)**6 + 3864861*c**7*f*tan(e/2 + f*x/2)**5 - 2147145*c**7*f*tan(e/2 + f*
x/2)**4 + 858858*c**7*f*tan(e/2 + f*x/2)**3 - 234234*c**7*f*tan(e/2 + f*x/2
)**2 + 39039*c**7*f*tan(e/2 + f*x/2) - 3003*c**7*f), Ne(f, 0)), (x*(a*sin(e
) + a)**3/(-c*sin(e) + c)**7, True))
```

3.260 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^8}\,dx$

Optimal. Leaf size=166

$$\frac{a^3c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}+\frac{4a^3c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}}+\frac{8a^3\cos^7(e+fx)}{45045cf(c-c\sin(e+fx))^7}+\frac{8a^3\cos^7(e+fx)}{6435f(c-c\sin(e+fx))^8}+$$

```
[Out] 1/15*a^3*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^11+4/195*a^3*c^2*cos(f*x+e)^7/
f/(c-c*sin(f*x+e))^10+4/715*a^3*c*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^9+8/6435*
a^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^8+8/45045*a^3*cos(f*x+e)^7/c/f/(c-c*sin
(f*x+e))^7
```

Rubi [A] time = 0.29, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2672, 2671}

$$\frac{4a^3c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}}+\frac{a^3c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}+\frac{8a^3\cos^7(e+fx)}{45045cf(c-c\sin(e+fx))^7}+\frac{8a^3\cos^7(e+fx)}{6435f(c-c\sin(e+fx))^8}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^8,x]
```

```
[Out] (a^3*c^3*Cos[e + f*x]^7)/(15*f*(c - c*Sin[e + f*x])^11) + (4*a^3*c^2*Cos[e
+ f*x]^7)/(195*f*(c - c*Sin[e + f*x])^10) + (4*a^3*c*Cos[e + f*x]^7)/(715*f
*(c - c*Sin[e + f*x])^9) + (8*a^3*Cos[e + f*x]^7)/(6435*f*(c - c*Sin[e + f*
x])^8) + (8*a^3*Cos[e + f*x]^7)/(45045*c*f*(c - c*Sin[e + f*x])^7)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^8}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{11}}\,dx$$
$$= \frac{a^3c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{1}{15}\left(4a^3c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{10}}\,dx$$
$$= \frac{a^3c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{4a^3c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}} + \frac{1}{65}\left(4a^3c\right)\int \frac{\cos^6(e+fx}{(c-c\sin(e+f}$$
$$= \frac{a^3c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{4a^3c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}} + \frac{4a^3c\cos^7(e+fx)}{715f(c-c\sin(e+fx))^9} + \frac{}{7}$$
$$= \frac{a^3c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{4a^3c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}} + \frac{4a^3c\cos^7(e+fx)}{715f(c-c\sin(e+fx))^9} + \frac{}{6}$$
$$= \frac{a^3c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{4a^3c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}} + \frac{4a^3c\cos^7(e+fx)}{715f(c-c\sin(e+fx))^9} + \frac{}{6}$$

Mathematica [A] time = 1.92, size = 209, normalized size = 1.26

$$\frac{(a\sin(e+fx)+a)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(109395\sin\left(\frac{1}{2}(e+fx)\right)+60060\sin\left(\frac{3}{2}(e+fx)\right)-15015\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^8,x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a + a*Sin[e + f*x])^3*(115830*Cos[(
e + f*x)/2] - 65065*Cos[(3*(e + f*x))/2] - 18018*Cos[(5*(e + f*x))/2] + 136
5*Cos[(7*(e + f*x))/2] - 105*Cos[(11*(e + f*x))/2] + Cos[(15*(e + f*x))/2]
+ 109395*Sin[(e + f*x)/2] + 60060*Sin[(3*(e + f*x))/2] - 15015*Sin[(5*(e +
f*x))/2] - 455*Sin[(9*(e + f*x))/2] + 15*Sin[(13*(e + f*x))/2]))/(360360*f*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^8)
```

fricas [B] time = 0.49, size = 440, normalized size = 2.65

$$\frac{8\,a^{3}\cos\left(fx+e\right)^{8}+64\,a^{3}\cos\left(fx+e\right)^{7}-196\,a^{3}\cos\left(fx+e\right)^{6}-672\,a^{3}\cos\left(fx+e\right)^{5}+735\,a^{3}\cos\left(fx+e\right)^{4}-7}{45045\left(c^{8}f\cos\left(fx+e\right)^{8}-7\,c^{8}f\cos\left(fx+e\right)^{7}-32\,c^{8}f\cos\left(fx+e\right)^{6}+56\,c^{8}f\cos\left(fx+e\right)^{5}+160\,c^{8}f\cos\left(fx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^8,x, algorithm="fricas")
```

```
[Out] 1/45045*(8*a^3*cos(f*x + e)^8 + 64*a^3*cos(f*x + e)^7 - 196*a^3*cos(f*x + e
)^6 - 672*a^3*cos(f*x + e)^5 + 735*a^3*cos(f*x + e)^4 - 7161*a^3*cos(f*x +
e)^3 - 20328*a^3*cos(f*x + e)^2 + 12012*a^3*cos(f*x + e) + 24024*a^3 - (8*a
^3*cos(f*x + e)^7 - 56*a^3*cos(f*x + e)^6 - 252*a^3*cos(f*x + e)^5 + 420*a^
3*cos(f*x + e)^4 + 1155*a^3*cos(f*x + e)^3 + 8316*a^3*cos(f*x + e)^2 - 1201
2*a^3*cos(f*x + e) - 24024*a^3)*sin(f*x + e))/(c^8*f*cos(f*x + e)^8 - 7*c^8
*f*cos(f*x + e)^7 - 32*c^8*f*cos(f*x + e)^6 + 56*c^8*f*cos(f*x + e)^5 + 160
*c^8*f*cos(f*x + e)^4 - 112*c^8*f*cos(f*x + e)^3 - 256*c^8*f*cos(f*x + e)^2
 + 64*c^8*f*cos(f*x + e) + 128*c^8*f + (c^8*f*cos(f*x + e)^7 + 8*c^8*f*cos(
f*x + e)^6 - 24*c^8*f*cos(f*x + e)^5 - 80*c^8*f*cos(f*x + e)^4 + 80*c^8*f*c
os(f*x + e)^3 + 192*c^8*f*cos(f*x + e)^2 - 64*c^8*f*cos(f*x + e) - 128*c^8*
f)*sin(f*x + e))
```

giac [A] time = 0.33, size = 264, normalized size = 1.59

$$-\frac{2\left(45045\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{14}-180180\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{13}+1066065\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{12}-2702700\,a^{3}\,\mathrm{ta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^8,x, algorithm="giac")
```

```
[Out] -2/45045*(45045*a^3*tan(1/2*f*x + 1/2*e)^14 - 180180*a^3*tan(1/2*f*x + 1/2*
e)^13 + 1066065*a^3*tan(1/2*f*x + 1/2*e)^12 - 2702700*a^3*tan(1/2*f*x + 1/2
*e)^11 + 6675669*a^3*tan(1/2*f*x + 1/2*e)^10 - 10210200*a^3*tan(1/2*f*x + 1
/2*e)^9 + 14124825*a^3*tan(1/2*f*x + 1/2*e)^8 - 13178880*a^3*tan(1/2*f*x +
1/2*e)^7 + 11026015*a^3*tan(1/2*f*x + 1/2*e)^6 - 6066060*a^3*tan(1/2*f*x +
1/2*e)^5 + 3088995*a^3*tan(1/2*f*x + 1/2*e)^4 - 864500*a^3*tan(1/2*f*x + 1/
2*e)^3 + 265335*a^3*tan(1/2*f*x + 1/2*e)^2 - 18600*a^3*tan(1/2*f*x + 1/2*e)
 + 4243*a^3)/(c^8*f*(tan(1/2*f*x + 1/2*e) - 1)^15)
```

maple [A] time = 0.45, size = 238, normalized size = 1.43

$$2a^3\left(-\frac{81344}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{11}}-\frac{32288}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{2304}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{13184}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{12}}-\frac{24320}{13\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{13}}-\frac{84112}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{9}}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^8,x)
```

```
[Out] 2/f*a^3/c^8*(-81344/11/(tan(1/2*f*x+1/2*e)-1)^11-32288/7/(tan(1/2*f*x+1/2*e
)-1)^7-2304/(tan(1/2*f*x+1/2*e)-1)^6-13184/3/(tan(1/2*f*x+1/2*e)-1)^12-2432
0/13/(tan(1/2*f*x+1/2*e)-1)^13-84112/9/(tan(1/2*f*x+1/2*e)-1)^9-10/(tan(1/2
*f*x+1/2*e)-1)^2-47072/5/(tan(1/2*f*x+1/2*e)-1)^10-1/(tan(1/2*f*x+1/2*e)-1)
-512/(tan(1/2*f*x+1/2*e)-1)^14-4536/5/(tan(1/2*f*x+1/2*e)-1)^5-188/3/(tan(1
/2*f*x+1/2*e)-1)^3-1024/15/(tan(1/2*f*x+1/2*e)-1)^15-276/(tan(1/2*f*x+1/2*e
)-1)^4-7352/(tan(1/2*f*x+1/2*e)-1)^8)
```

maxima [B] time = 0.94, size = 2422, normalized size = 14.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^8,x, algorithm="maxima")
```

```
[Out] 2/45045*(3*a^3*(17715*sin(f*x + e)/(cos(f*x + e) + 1) - 78960*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + 342160*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 89134
5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1960959*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 - 3043040*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 3912480*sin(f*x +
e)^7/(cos(f*x + e) + 1)^7 - 3687255*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 2
867865*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 1585584*sin(f*x + e)^10/(cos(f
*x + e) + 1)^10 + 720720*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 195195*sin
(f*x + e)^12/(cos(f*x + e) + 1)^12 + 45045*sin(f*x + e)^13/(cos(f*x + e) +
1)^13 - 1181)/(c^8 - 15*c^8*sin(f*x + e)/(cos(f*x + e) + 1) + 105*c^8*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 - 455*c^8*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3 + 1365*c^8*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3003*c^8*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5 + 5005*c^8*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 6435*
c^8*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 6435*c^8*sin(f*x + e)^8/(cos(f*x
+ e) + 1)^8 - 5005*c^8*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 3003*c^8*sin(f
*x + e)^10/(cos(f*x + e) + 1)^10 - 1365*c^8*sin(f*x + e)^11/(cos(f*x + e) +
 1)^11 + 455*c^8*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 105*c^8*sin(f*x +
e)^13/(cos(f*x + e) + 1)^13 + 15*c^8*sin(f*x + e)^14/(cos(f*x + e) + 1)^14
- c^8*sin(f*x + e)^15/(cos(f*x + e) + 1)^15) - 7*a^3*(7845*sin(f*x + e)/(co
s(f*x + e) + 1) - 54915*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 222950*sin(f*
```

x + e)^3/(cos(f*x + e) + 1)^3 - 668850*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1444443*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 2407405*sin(f*x + e)^6/(cos (f*x + e) + 1)^6 + 3063060*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3063060*si n(f*x + e)^8/(cos(f*x + e) + 1)^8 + 2357355*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 1414413*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 630630*sin(f*x + e)^ 11/(cos(f*x + e) + 1)^11 - 210210*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 + 4 5045*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 - 6435*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 - 952)/(c^8 - 15*c^8*sin(f*x + e)/(cos(f*x + e) + 1) + 105*c^8 *sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 455*c^8*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1365*c^8*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3003*c^8*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5005*c^8*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 6435*c^8*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 6435*c^8*sin(f*x + e)^8/(co s(f*x + e) + 1)^8 - 5005*c^8*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 3003*c^8 *sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1365*c^8*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 455*c^8*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 105*c^8*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 15*c^8*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 - c^8*sin(f*x + e)^15/(cos(f*x + e) + 1)^15) - 12*a^3*(1740*sin(f*x + e)/(cos(f*x + e) + 1) - 12180*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 37765* sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 113295*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 204204*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 340340*sin(f*x + e)^6/ (cos(f*x + e) + 1)^6 + 373230*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 373230* sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 240240*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 144144*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 45045*sin(f*x + e)^1 1/(cos(f*x + e) + 1)^11 - 15015*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 116)/(c^8 - 15*c^8*sin(f*x + e)/(cos(f*x + e) + 1) + 105*c^8*sin(f*x + e)^2/(c os(f*x + e) + 1)^2 - 455*c^8*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1365*c^8 *sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3003*c^8*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5005*c^8*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 6435*c^8*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 6435*c^8*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5005*c^8*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 3003*c^8*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1365*c^8*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 455 *c^8*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 105*c^8*sin(f*x + e)^13/(cos(f *x + e) + 1)^13 + 15*c^8*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 - c^8*sin(f* x + e)^15/(cos(f*x + e) + 1)^15) + 6*a^3*(675*sin(f*x + e)/(cos(f*x + e) + 1) - 4725*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 20475*sin(f*x + e)^3/(cos(f *x + e) + 1)^3 - 46410*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 102102*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 130130*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 167310*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 122265*sin(f*x + e)^8/(cos(f* x + e) + 1)^8 + 95095*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 33033*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 15015*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 45)/(c^8 - 15*c^8*sin(f*x + e)/(cos(f*x + e) + 1) + 105*c^8*sin(f*x + e)^ 2/(cos(f*x + e) + 1)^2 - 455*c^8*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1365 *c^8*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3003*c^8*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5005*c^8*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 6435*c^8*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 6435*c^8*sin(f*x + e)^8/(cos(f*x + e) + 1

```
)^8 - 5005*c^8*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 3003*c^8*sin(f*x + e)^
10/(cos(f*x + e) + 1)^10 - 1365*c^8*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 +
 455*c^8*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 105*c^8*sin(f*x + e)^13/(c
os(f*x + e) + 1)^13 + 15*c^8*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 - c^8*si
n(f*x + e)^15/(cos(f*x + e) + 1)^15))/f
```

mupad [B] time = 11.20, size = 187, normalized size = 1.13

$$\frac{\sqrt{2}\,a^3\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{3497111\,\cos\left(3\,e+3\,f\,x\right)}{128}-\frac{25501905\,\sin\left(e+f\,x\right)}{128}-\frac{257861\,\cos\left(2\,e+2\,f\,x\right)}{2}-\frac{5734111\,\cos\left(e+f\,x\right)}{128}+\frac{72047\,\cos\left(4\,e+4\right.}{4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^8,x)
```

```
[Out] (2^(1/2)*a^3*cos(e/2 + (f*x)/2)*((3497111*cos(3*e + 3*f*x))/128 - (25501905
*sin(e + f*x))/128 - (257861*cos(2*e + 2*f*x))/2 - (5734111*cos(e + f*x))/1
28 + (72047*cos(4*e + 4*f*x))/4 - (378579*cos(5*e + 5*f*x))/128 - (1059*cos
(6*e + 6*f*x))/2 + (4251*cos(7*e + 7*f*x))/128 + (2633345*sin(2*e + 2*f*x))
/64 + (7210775*sin(3*e + 3*f*x))/128 - (89375*sin(4*e + 4*f*x))/8 - (504205
*sin(5*e + 5*f*x))/128 + (29765*sin(6*e + 6*f*x))/64 + (4235*sin(7*e + 7*f*
x))/128 + 544369/4))/(5765760*c^8*f*cos(e/2 + pi/4 + (f*x)/2)^15)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**8,x)
```

```
[Out] Timed out
```

3.261 $\int \frac{(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=118

$$-\frac{2a^3c^4\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}-\frac{35c^4\cos^3(e+fx)}{3af}-\frac{14ac^4\cos^5(e+fx)}{f(a\sin(e+fx)+a)^2}-\frac{35c^4\sin(e+fx)\cos(e+fx)}{2af}-\frac{35c^4x}{2a}$$

```
[Out] -35/2*c^4*x/a-35/3*c^4*cos(f*x+e)^3/a/f-35/2*c^4*cos(f*x+e)*sin(f*x+e)/a/f-
2*a^3*c^4*cos(f*x+e)^7/f/(a+a*sin(f*x+e))^4-14*a*c^4*cos(f*x+e)^5/f/(a+a*si
n(f*x+e))^2
```

Rubi [A] time = 0.20, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2736, 2680, 2682, 2635, 8}

$$-\frac{2a^3c^4\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}-\frac{35c^4\cos^3(e+fx)}{3af}-\frac{14ac^4\cos^5(e+fx)}{f(a\sin(e+fx)+a)^2}-\frac{35c^4\sin(e+fx)\cos(e+fx)}{2af}-\frac{35c^4x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^4/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-35*c^4*x)/(2*a) - (35*c^4*Cos[e + f*x]^3)/(3*a*f) - (35*c^4*Cos[e + f*x]*
Sin[e + f*x])/(2*a*f) - (2*a^3*c^4*Cos[e + f*x]^7)/(f*(a + a*Sin[e + f*x])^
4) - (14*a*c^4*Cos[e + f*x]^5)/(f*(a + a*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx = \left(a^4c^4\right)\int \frac{\cos^8(e+fx)}{(a+a\sin(e+fx))^5}\,dx$$
$$= -\frac{2a^3c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} - \left(7a^2c^4\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^3}\,dx$$
$$= -\frac{2a^3c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} - \frac{14ac^4\cos^5(e+fx)}{f(a+a\sin(e+fx))^2} - \left(35c^4\right)\int \frac{\cos^4(e+fx)}{a+a\sin(e+fx)}\,dx$$
$$= -\frac{35c^4\cos^3(e+fx)}{3af} - \frac{2a^3c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} - \frac{14ac^4\cos^5(e+fx)}{f(a+a\sin(e+fx))^2} - \frac{\left(35c^4\right)\int \cos^2}{a}$$
$$= -\frac{35c^4\cos^3(e+fx)}{3af} - \frac{35c^4\cos(e+fx)\sin(e+fx)}{2af} - \frac{2a^3c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} - \frac{14ac^4}{f(a+}$$
$$= -\frac{35c^4x}{2a} - \frac{35c^4\cos^3(e+fx)}{3af} - \frac{35c^4\cos(e+fx)\sin(e+fx)}{2af} - \frac{2a^3c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^4}$$

Mathematica [A] time = 1.44, size = 175, normalized size = 1.48

$$-\frac{c^4(\sin(e+fx)-1)^4\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)(-15\sin(2(e+fx))+141\cos(e+fx)}{12af(\sin(e+fx)+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^4/(a + a*Sin[e + f*x]),x]
```

```
[Out] -1/12*(c^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^4*(Sin
[(e + f*x)/2]*(-384 + 210*e + 210*f*x + 141*Cos[e + f*x] - Cos[3*(e + f*x)]
 - 15*Sin[2*(e + f*x)]) + Cos[(e + f*x)/2]*(210*e + 210*f*x + 141*Cos[e + f
*x] - Cos[3*(e + f*x)] - 15*Sin[2*(e + f*x)])))/(a*f*(Cos[(e + f*x)/2] - Si
n[(e + f*x)/2])^8*(1 + Sin[e + f*x]))
```

fricas [A] time = 0.44, size = 156, normalized size = 1.32

$$\frac{2\,c^4\cos\left(fx+e\right)^4-13\,c^4\cos\left(fx+e\right)^3-105\,c^4fx-72\,c^4\cos\left(fx+e\right)^2-96\,c^4-3\left(35\,c^4fx+51\,c^4\right)\cos\left(fx+\right.}{6\left(af\cos\left(fx+e\right)+af\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*c^4*cos(f*x + e)^4 - 13*c^4*cos(f*x + e)^3 - 105*c^4*f*x - 72*c^4*co
s(f*x + e)^2 - 96*c^4 - 3*(35*c^4*f*x + 51*c^4)*cos(f*x + e) + (2*c^4*cos(f
*x + e)^3 - 105*c^4*f*x + 15*c^4*cos(f*x + e)^2 - 57*c^4*cos(f*x + e) + 96*
c^4)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

giac [A] time = 0.22, size = 135, normalized size = 1.14

$$-\frac{\frac{105\left(fx+e\right)c^4}{a}+\frac{192\,c^4}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{2\left(15\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+66\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+144\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-15\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+70\,c^4\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^3a}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/6*(105*(f*x + e)*c^4/a + 192*c^4/(a*(tan(1/2*f*x + 1/2*e) + 1)) + 2*(15*
c^4*tan(1/2*f*x + 1/2*e)^5 + 66*c^4*tan(1/2*f*x + 1/2*e)^4 + 144*c^4*tan(1/
2*f*x + 1/2*e)^2 - 15*c^4*tan(1/2*f*x + 1/2*e) + 70*c^4)/((tan(1/2*f*x + 1/
2*e)^2 + 1)^3*a))/f
```

maple [A] time = 0.32, size = 219, normalized size = 1.86

$$-\frac{5c^4\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fa\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}-\frac{22c^4\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fa\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}-\frac{48c^4\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fa\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}+\frac{5c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{fa\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}-\frac{}{3fa\left(1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x)
```

```
[Out] -5/f*c^4/a/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5-22/f*c^4/a/(1+ta
n(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^4-48/f*c^4/a/(1+tan(1/2*f*x+1/2*e)
^2)^3*tan(1/2*f*x+1/2*e)^2+5/f*c^4/a/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x
+1/2*e)-70/3/f*c^4/a/(1+tan(1/2*f*x+1/2*e)^2)^3-35/f*c^4/a*arctan(tan(1/2*f
*x+1/2*e))-32/f*c^4/a/(tan(1/2*f*x+1/2*e)+1)
```

maxima [B] time = 0.91, size = 720, normalized size = 6.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/3*(c^4*((7*sin(f*x + e)/(cos(f*x + e) + 1) + 39*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 24*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 9*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6 + 16)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3*a*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 + a*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a*sin(f*x + e)^7/(cos
(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 12*c^4*(
(sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(si
n(f*x + e)/(cos(f*x + e) + 1))/a) + 36*c^4*((sin(f*x + e)/(cos(f*x + e) + 1
) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x +
 e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 24*c^4*(arcta
n(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e)
+ 1))) + 6*c^4/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

mupad [B] time = 10.53, size = 290, normalized size = 2.46

$$\frac{\frac{35\,c^4\,\left(e+f\,x\right)}{2}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{35\,c^4\,\left(e+f\,x\right)}{2}-\frac{c^4\,\left(105\,e+105\,f\,x+110\right)}{6}\right)-\frac{c^4\,\left(105\,e+105\,f\,x+332\right)}{6}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,\left(\frac{35\,c^4\,\left(e+f\,x\right)}{2}-\frac{c}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^4/(a + a*sin(e + f*x)),x)
```

```
[Out] ((35*c^4*(e + f*x))/2 + tan(e/2 + (f*x)/2)*((35*c^4*(e + f*x))/2 - (c^4*(10
5*e + 105*f*x + 110))/6) - (c^4*(105*e + 105*f*x + 332))/6 + tan(e/2 + (f*x
)/2)^6*((35*c^4*(e + f*x))/2 - (c^4*(105*e + 105*f*x + 222))/6) + tan(e/2 +
 (f*x)/2)^5*((105*c^4*(e + f*x))/2 - (c^4*(315*e + 315*f*x + 162))/6) + tan
(e/2 + (f*x)/2)^3*((105*c^4*(e + f*x))/2 - (c^4*(315*e + 315*f*x + 288))/6)
 + tan(e/2 + (f*x)/2)^4*((105*c^4*(e + f*x))/2 - (c^4*(315*e + 315*f*x + 70
8))/6) + tan(e/2 + (f*x)/2)^2*((105*c^4*(e + f*x))/2 - (c^4*(315*e + 315*f*
x + 834))/6))/(a*f*(tan(e/2 + (f*x)/2) + 1)*(tan(e/2 + (f*x)/2)^2 + 1)^3) -
 (35*c^4*x)/(2*a)
```

sympy [A] time = 14.00, size = 2108, normalized size = 17.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**4/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-105*c**4*f*x*tan(e/2 + f*x/2)**7/(6*a*f*tan(e/2 + f*x/2)**7 + 6
*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*
x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*t
an(e/2 + f*x/2) + 6*a*f) - 105*c**4*f*x*tan(e/2 + f*x/2)**6/(6*a*f*tan(e/2
+ f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a
*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/
2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 315*c**4*f*x*tan(e/2 + f*x/2)**5/
(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f
*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f
*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 315*c**4*f*x*tan(e
/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*
a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x
/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 315
*c**4*f*x*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 +
f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*
f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2)
 + 6*a*f) - 315*c**4*f*x*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x/2)**7 + 6
*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*
x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*t
an(e/2 + f*x/2) + 6*a*f) - 105*c**4*f*x*tan(e/2 + f*x/2)/(6*a*f*tan(e/2 + f
*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*
tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)*
*2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 105*c**4*f*x/(6*a*f*tan(e/2 + f*x/2)
**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e
/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 +
6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 222*c**4*tan(e/2 + f*x/2)**6/(6*a*f*tan(e
/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 1
8*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f
```

```
*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 162*c**4*tan(e/2 + f*x/2)**5/(
6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*
x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*
tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 708*c**4*tan(e/2 +
f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*t
an(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**
3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 288*c**4
*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6
 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2
 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f)
 - 834*c**4*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2
+ f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*
a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/
2) + 6*a*f) - 110*c**4*tan(e/2 + f*x/2)/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*
tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)*
*4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/
2 + f*x/2) + 6*a*f) - 332*c**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 +
 f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a
*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2
) + 6*a*f), Ne(f, 0)), (x*(-c*sin(e) + c)**4/(a*sin(e) + a), True))
```

3.262 $\int \frac{(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=92

$$-\frac{2a^2c^3\cos^5(e+fx)}{f(a\sin(e+fx)+a)^3}-\frac{15c^3\cos(e+fx)}{2af}-\frac{5c^3\cos^3(e+fx)}{2f(a\sin(e+fx)+a)}-\frac{15c^3x}{2a}$$

```
[Out] -15/2*c^3*x/a-15/2*c^3*cos(f*x+e)/a/f-2*a^2*c^3*cos(f*x+e)^5/f/(a+a*sin(f*x
+e))^3-5/2*c^3*cos(f*x+e)^3/f/(a+a*sin(f*x+e))
```

Rubi [A] time = 0.18, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2736, 2680, 2679, 2682, 8}

$$-\frac{2a^2c^3\cos^5(e+fx)}{f(a\sin(e+fx)+a)^3}-\frac{15c^3\cos(e+fx)}{2af}-\frac{5c^3\cos^3(e+fx)}{2f(a\sin(e+fx)+a)}-\frac{15c^3x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^3/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-15*c^3*x)/(2*a) - (15*c^3*Cos[e + f*x])/(2*a*f) - (2*a^2*c^3*Cos[e + f*x]
^5)/(f*(a + a*Sin[e + f*x])^3) - (5*c^3*Cos[e + f*x]^3)/(2*f*(a + a*Sin[e +
 f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
```

```
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^4}\,dx\\
&= -\frac{2a^2c^3\cos^5(e+fx)}{f(a+a\sin(e+fx))^3}-\left(5ac^3\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^2}\,dx\\
&= -\frac{2a^2c^3\cos^5(e+fx)}{f(a+a\sin(e+fx))^3}-\frac{5c^3\cos^3(e+fx)}{2f(a+a\sin(e+fx))}-\frac{1}{2}\left(15c^3\right)\int \frac{\cos^2(e+fx)}{a+a\sin(e+fx)}\,dx\\
&= -\frac{15c^3\cos(e+fx)}{2af}-\frac{2a^2c^3\cos^5(e+fx)}{f(a+a\sin(e+fx))^3}-\frac{5c^3\cos^3(e+fx)}{2f(a+a\sin(e+fx))}-\frac{\left(15c^3\right)\int 1\,dx}{2a}\\
&= -\frac{15c^3x}{2a}-\frac{15c^3\cos(e+fx)}{2af}-\frac{2a^2c^3\cos^5(e+fx)}{f(a+a\sin(e+fx))^3}-\frac{5c^3\cos^3(e+fx)}{2f(a+a\sin(e+fx))}
\end{aligned}
$$

Mathematica [A] time = 0.51, size = 155, normalized size = 1.68

$$
\frac{c^3(\sin(e+fx)-1)^3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)(-\sin(2(e+fx))+16\cos(e+fx)+30\right.}{4af(\sin(e+fx)+1)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\right.\right.}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^3/(a + a*Sin[e + f*x]),x]
```

```
[Out] (c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^3*(Sin[(e +
f*x)/2]*(-64 + 30*e + 30*f*x + 16*Cos[e + f*x] - Sin[2*(e + f*x)]) + Cos[(e
 + f*x)/2]*(30*(e + f*x) + 16*Cos[e + f*x] - Sin[2*(e + f*x)])))/(4*a*f*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^6*(1 + Sin[e + f*x]))
```

fricas [A] time = 0.46, size = 128, normalized size = 1.39

$$-\frac{c^3\cos\left(fx+e\right)^3+15\,c^3fx+8\,c^3\cos\left(fx+e\right)^2+16\,c^3+\left(15\,c^3fx+23\,c^3\right)\cos\left(fx+e\right)+\left(15\,c^3fx-c^3\cos\left(f\right.\right.}{2\left(af\cos\left(fx+e\right)+af\sin\left(fx+e\right)+af\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/2*(c^3*cos(f*x + e)^3 + 15*c^3*f*x + 8*c^3*cos(f*x + e)^2 + 16*c^3 + (15
*c^3*f*x + 23*c^3)*cos(f*x + e) + (15*c^3*f*x - c^3*cos(f*x + e)^2 + 7*c^3*
cos(f*x + e) - 16*c^3)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) +
 a*f)
```

giac [A] time = 0.20, size = 117, normalized size = 1.27

$$-\frac{\frac{15\left(fx+e\right)c^3}{a}+\frac{32\,c^3}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{2\left(c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+8\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+8\,c^3\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2a}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/2*(15*(f*x + e)*c^3/a + 32*c^3/(a*(tan(1/2*f*x + 1/2*e) + 1)) + 2*(c^3*t
an(1/2*f*x + 1/2*e)^3 + 8*c^3*tan(1/2*f*x + 1/2*e)^2 - c^3*tan(1/2*f*x + 1/
2*e) + 8*c^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*a))/f
```

maple [B] time = 0.25, size = 181, normalized size = 1.97

$$-\frac{c^3\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{8c^3\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{8c^3}{af\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{15c^3\,\mathrm{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x)
```

```
[Out] -c^3/a/f/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^3-8*c^3/a/f/(1+tan(1
/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2+c^3/a/f/(1+tan(1/2*f*x+1/2*e)^2)^2*
tan(1/2*f*x+1/2*e)-8*c^3/a/f/(1+tan(1/2*f*x+1/2*e)^2)^2-15*c^3/a/f*arctan(t
an(1/2*f*x+1/2*e))-16*c^3/a/f/(tan(1/2*f*x+1/2*e)+1)
```

maxima [B] time = 1.13, size = 424, normalized size = 4.61

$$-\frac{c^3\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+4}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{2a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{2a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{a\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)+6c^3\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin}{(\cos(}}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{a\sin(fx+}{(\cos(fx+e)}}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -(c^3*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x +
e) + 1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*ar
ctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 6*c^3*((sin(f*x + e)/(cos(f*x +
e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos
(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 6*c^3*(
arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x
+ e) + 1))) + 2*c^3/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

mupad [B] time = 8.71, size = 216, normalized size = 2.35

$$\frac{\frac{15c^3(e+fx)}{2}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{15c^3(e+fx)}{2}-\frac{c^3(15e+15fx+14)}{2}\right)-\frac{c^3(15e+15fx+48)}{2}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(\frac{15c^3(e+fx)}{2}-\frac{c^3(15e+}{}\right.}{af\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^3/(a + a*sin(e + f*x)),x)
```

```
[Out] ((15*c^3*(e + f*x))/2 + tan(e/2 + (f*x)/2)*((15*c^3*(e + f*x))/2 - (c^3*(15
*e + 15*f*x + 14))/2) - (c^3*(15*e + 15*f*x + 48))/2 + tan(e/2 + (f*x)/2)^4
*((15*c^3*(e + f*x))/2 - (c^3*(15*e + 15*f*x + 34))/2) + tan(e/2 + (f*x)/2)
^3*(15*c^3*(e + f*x) - (c^3*(30*e + 30*f*x + 18))/2) + tan(e/2 + (f*x)/2)^2
*(15*c^3*(e + f*x) - (c^3*(30*e + 30*f*x + 78))/2))/(a*f*(tan(e/2 + (f*x)/2
) + 1)*(tan(e/2 + (f*x)/2)^2 + 1)^2) - (15*c^3*x)/(2*a)
```

sympy [A] time = 7.44, size = 1170, normalized size = 12.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**3/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-15*c**3*f*x*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*
a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2
)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 15*c**3*f*x*tan(e/2 + f*x/2)**4/(2
*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/
2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 30*c*
*3*f*x*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x
/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(
e/2 + f*x/2) + 2*a*f) - 30*c**3*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*
x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan
(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 15*c**3*f*x*tan(e/2 +
f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e
/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f
) - 15*c**3*f*x/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*
a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2
) + 2*a*f) - 34*c**3*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f
*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**
2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 18*c**3*tan(e/2 + f*x/2)**3/(2*a*f*ta
n(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 +
 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 78*c**3*tan(
e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*
a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2
) + 2*a*f) - 14*c**3*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*ta
n(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 +
 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 48*c**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a
*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)
**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f), Ne(f, 0)), (x*(-c*sin(e) + c)**3/(a*
sin(e) + a), True))
```

3.263 $\int \frac{(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=56

$$-\frac{3c^2\cos(e+fx)}{af}-\frac{2ac^2\cos^3(e+fx)}{f(a\sin(e+fx)+a)^2}-\frac{3c^2x}{a}$$

```
[Out] -3*c^2*x/a-3*c^2*cos(f*x+e)/a/f-2*a*c^2*cos(f*x+e)^3/f/(a+a*sin(f*x+e))^2
```

Rubi [A] time = 0.14, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2680, 2682, 8}

$$-\frac{3c^2\cos(e+fx)}{af}-\frac{2ac^2\cos^3(e+fx)}{f(a\sin(e+fx)+a)^2}-\frac{3c^2x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^2/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-3*c^2*x)/a - (3*c^2*Cos[e + f*x])/(a*f) - (2*a*c^2*Cos[e + f*x]^3)/(f*(a
+ a*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^3}\,dx\\ &= -\frac{2ac^2\cos^3(e+fx)}{f(a+a\sin(e+fx))^2}-\left(3c^2\right)\int \frac{\cos^2(e+fx)}{a+a\sin(e+fx)}\,dx\\ &= -\frac{3c^2\cos(e+fx)}{af}-\frac{2ac^2\cos^3(e+fx)}{f(a+a\sin(e+fx))^2}-\frac{\left(3c^2\right)\int 1\,dx}{a}\\ &= -\frac{3c^2x}{a}-\frac{3c^2\cos(e+fx)}{af}-\frac{2ac^2\cos^3(e+fx)}{f(a+a\sin(e+fx))^2}\end{aligned}$$

Mathematica [B] time = 0.37, size = 129, normalized size = 2.30

$$-\frac{c^2(\sin(e+fx)-1)^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)(3(e+fx)+\cos(e+fx))+\sin\left(\frac{1}{2}(e\right.\right.}{af(\sin(e+fx)+1)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^2/(a + a*Sin[e + f*x]),x]
```

```
[Out] -((c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(Cos[(e + f*x)/2]*(3*(e + f*x)
 + Cos[e + f*x]) + (-8 + 3*e + 3*f*x + Cos[e + f*x])*Sin[(e + f*x)/2])*(-1
+ Sin[e + f*x])^2)/(a*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(1 + Sin[e
+ f*x])))
```

fricas [A] time = 0.46, size = 101, normalized size = 1.80

$$-\frac{3\,c^2fx+c^2\cos\left(fx+e\right)^2+4\,c^2+\left(3\,c^2fx+5\,c^2\right)\cos\left(fx+e\right)+\left(3\,c^2fx+c^2\cos\left(fx+e\right)-4\,c^2\right)\sin\left(fx+e\right)}{af\cos\left(fx+e\right)+af\sin\left(fx+e\right)+af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(3*c^2*f*x + c^2*cos(f*x + e)^2 + 4*c^2 + (3*c^2*f*x + 5*c^2)*cos(f*x + e)
 + (3*c^2*f*x + c^2*cos(f*x + e) - 4*c^2)*sin(f*x + e))/(a*f*cos(f*x + e) +
 a*f*sin(f*x + e) + a*f)
```

giac [A] time = 0.20, size = 100, normalized size = 1.79

$$-\frac{\frac{3(fx+e)c^2}{a}+\frac{2\left(4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+5c^2\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -(3*(f*x + e)*c^2/a + 2*(4*c^2*tan(1/2*f*x + 1/2*e)^2 + c^2*tan(1/2*f*x + 1
/2*e) + 5*c^2)/((tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)^2 + tan(1/2*
f*x + 1/2*e) + 1)*a))/f
```

maple [A] time = 0.21, size = 73, normalized size = 1.30

$$-\frac{2c^2}{fa\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}-\frac{6c^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fa}-\frac{8c^2}{fa\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x)
```

```
[Out] -2/f*c^2/a/(1+tan(1/2*f*x+1/2*e)^2)-6/f*c^2/a*arctan(tan(1/2*f*x+1/2*e))-8/
f*c^2/a/(tan(1/2*f*x+1/2*e)+1)
```

maxima [B] time = 0.87, size = 210, normalized size = 3.75

$$-\frac{2\left(c^2\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)+2c^2\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)+\frac{c^2}{a+\frac{a\sin(fx+}{\cos(fx+e)}}\cdots}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -2*(c^2*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*
```

```
x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/a) + 2*c^2*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a
 + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + c^2/(a + a*sin(f*x + e)/(co
s(f*x + e) + 1)))/f
```

mupad [B] time = 6.99, size = 118, normalized size = 2.11

$$-\frac{3\,c^2\,x}{a}-\frac{3\,\sqrt{2}\,c^2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(e+f\,x\right)-\frac{\sqrt{2}\,c^2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(6\,e+6\,f\,x+16\right)}{2}}{a\,f\,\left(\sqrt{2}\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\sqrt{2}\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}-\frac{2\,c^2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^2/(a + a*sin(e + f*x)),x)
```

```
[Out] - (3*c^2*x)/a - (3*2^(1/2)*c^2*sin(e/2 + (f*x)/2)*(e + f*x) - (2^(1/2)*c^2*
sin(e/2 + (f*x)/2)*(6*e + 6*f*x + 16))/2)/(a*f*(2^(1/2)*cos(e/2 + (f*x)/2)
+ 2^(1/2)*sin(e/2 + (f*x)/2))) - (2*c^2*cos(e/2 + (f*x)/2)^2)/(a*f)
```

sympy [A] time = 3.89, size = 456, normalized size = 8.14

$$\begin{cases}-\frac{3c^2fx\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}-\frac{3c^2fx\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}-\frac{3c^2fx\tan}{af\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+af\tan^2\left(\frac{e}{2}\right.} \\ \frac{x(-c\sin(e)+c)^2}{a\sin(e)+a}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**2/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-3*c**2*f*x*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*t
an(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 3*c**2*f*x*tan(e/2 + f*x
/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f
*x/2) + a*f) - 3*c**2*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*t
an(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 3*c**2*f*x/(a*f*tan(e/2
+ f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 8*c**
2*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 +
a*f*tan(e/2 + f*x/2) + a*f) - 2*c**2*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)
**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 10*c**2/(a*f*
tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f)
, Ne(f, 0)), (x*(-c*sin(e) + c)**2/(a*sin(e) + a), True))
```

3.264 $\int \frac{c-c\sin(e+fx)}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=32

$$-\frac{2c\cos(e+fx)}{f(a\sin(e+fx)+a)}-\frac{cx}{a}$$

```
[Out] -c*x/a-2*c*cos(f*x+e)/f/(a+a*sin(f*x+e))
```

Rubi [A] time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2735, 2648}

$$-\frac{2c\cos(e+fx)}{f(a\sin(e+fx)+a)}-\frac{cx}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])/(a + a*Sin[e + f*x]),x]
```

```
[Out] -((c*x)/a) - (2*c*Cos[e + f*x])/(f*(a + a*Sin[e + f*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{c-c\sin(e+fx)}{a+a\sin(e+fx)}\,dx &= -\frac{cx}{a}+(2c)\int \frac{1}{a+a\sin(e+fx)}\,dx\\ &= -\frac{cx}{a}-\frac{2c\cos(e+fx)}{f(a+a\sin(e+fx))}\end{aligned}$$

Mathematica [B] time = 0.19, size = 79, normalized size = 2.47

$$-\frac{c\left(fx\sin\left(e+\frac{fx}{2}\right)-4\sin\left(\frac{fx}{2}\right)+fx\cos\left(\frac{fx}{2}\right)\right)}{af\left(\sin\left(\frac{e}{2}\right)+\cos\left(\frac{e}{2}\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])/(a + a*Sin[e + f*x]),x]
```

```
[Out] -((c*(f*x*Cos[(f*x)/2] - 4*Sin[(f*x)/2] + f*x*Sin[e + (f*x)/2]))/(a*f*(Cos[
e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))
```

fricas [A] time = 0.46, size = 64, normalized size = 2.00

$$-\frac{cfx+\left(cfx+2\,c\right)\cos\left(fx+e\right)+\left(cfx-2\,c\right)\sin\left(fx+e\right)+2\,c}{af\cos\left(fx+e\right)+af\sin\left(fx+e\right)+af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(c*f*x + (c*f*x + 2*c)*cos(f*x + e) + (c*f*x - 2*c)*sin(f*x + e) + 2*c)/(a
*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

giac [A] time = 0.23, size = 37, normalized size = 1.16

$$-\frac{\frac{\left(fx+e\right)c}{a}+\frac{4\,c}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -((f*x + e)*c/a + 4*c/(a*(tan(1/2*f*x + 1/2*e) + 1)))/f
```

maple [A] time = 0.18, size = 43, normalized size = 1.34

$$-\frac{2c\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fa}-\frac{4c}{fa\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x)
[Out] -2/f*c/a*arctan(tan(1/2*f*x+1/2*e))-4/f*c/a/(tan(1/2*f*x+1/2*e)+1)
```

maxima [B] time = 0.86, size = 77, normalized size = 2.41

$$-\frac{2\left(c\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)+\frac{c}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="maxima")
[Out] -2*(c*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(c
os(f*x + e) + 1))) + c/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

mupad [B] time = 6.64, size = 45, normalized size = 1.41

$$\frac{c\left(e+fx\right)-c\left(e+fx+4\right)}{af\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}-\frac{cx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))/(a + a*sin(e + f*x)),x)
[Out] (c*(e + f*x) - c*(e + f*x + 4))/(a*f*(tan(e/2 + (f*x)/2) + 1)) - (c*x)/a
```

sympy [A] time = 1.82, size = 90, normalized size = 2.81

$$\begin{cases} -\frac{cfx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}-\frac{cfx}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}-\frac{4c}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} & \text{for } f\neq 0 \\ \frac{x(-c\sin(e)+c)}{a\sin(e)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x)
[Out] Piecewise((-c*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2) + a*f) - c*f*x/(a*
f*tan(e/2 + f*x/2) + a*f) - 4*c/(a*f*tan(e/2 + f*x/2) + a*f), Ne(f, 0)), (x
*(-c*sin(e) + c)/(a*sin(e) + a), True))
```

3.265 $\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx$

Optimal. Leaf size=16

$$\frac{\tan(e+fx)}{acf}$$

```
[Out] tan(f*x+e)/a/c/f
```

Rubi [A] time = 0.07, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 3767, 8}

$$\frac{\tan(e+fx)}{acf}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])),x]
```

```
[Out] Tan[e + f*x]/(a*c*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx = \frac{\int \sec^2(e+fx)\,dx}{ac}$$
$$= -\frac{\text{Subst}(\int 1\,dx, x, -\tan(e+fx))}{acf}$$
$$= \frac{\tan(e+fx)}{acf}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{\tan(e+fx)}{acf}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])),x]
[Out] Tan[e + f*x]/(a*c*f)
```

fricas [A] time = 0.43, size = 24, normalized size = 1.50

$$\frac{\sin\left(fx+e\right)}{acf\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="fricas")
[Out] sin(f*x + e)/(a*c*f*cos(f*x + e))
```

giac [A] time = 0.16, size = 17, normalized size = 1.06

$$\frac{\tan\left(fx+e\right)}{acf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="giac")
[Out] tan(f*x + e)/(a*c*f)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+a\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] int(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

maxima [A] time = 0.31, size = 16, normalized size = 1.00

$$\frac{\tan\left(fx+e\right)}{acf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] tan(f*x + e)/(a*c*f)
```

mupad [B] time = 6.85, size = 35, normalized size = 2.19

$$-\frac{2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{acf\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))),x)
```

```
[Out] -(2*tan(e/2 + (f*x)/2))/(a*c*f*(tan(e/2 + (f*x)/2)^2 - 1))
```

sympy [A] time = 1.65, size = 49, normalized size = 3.06

$$\begin{cases} -\frac{2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{acf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-acf} & \text{for } f\neq 0 \\ \frac{x}{(a\sin(e)+a)(-c\sin(e)+c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-2*tan(e/2 + f*x/2)/(a*c*f*tan(e/2 + f*x/2)**2 - a*c*f), Ne(f, 0
)), (x/((a*sin(e) + a)*(-c*sin(e) + c)), True))
```

3.266 $\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2} dx$

Optimal. Leaf size=53

$$\frac{2\tan(e+fx)}{3ac^2f} + \frac{\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)}$$

```
[Out] 1/3*sec(f*x+e)/a/f/(c^2-c^2*sin(f*x+e))+2/3*tan(f*x+e)/a/c^2/f
```

Rubi [A] time = 0.11, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2672, 3767, 8}

$$\frac{2\tan(e+fx)}{3ac^2f} + \frac{\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^2),x]
```

```
[Out] Sec[e + f*x]/(3*a*f*(c^2 - c^2*Sin[e + f*x])) + (2*Tan[e + f*x])/(3*a*c^2*f
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx &= \frac{\int \frac{\sec^2(e+fx)}{c-c\sin(e+fx)}\,dx}{ac}\\
&= \frac{\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)}+\frac{2\int \sec^2(e+fx)\,dx}{3ac^2}\\
&= \frac{\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)}-\frac{2\,\mathrm{Subst}\left(\int 1\,dx,x,-\tan(e+fx)\right)}{3ac^2f}\\
&= \frac{\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)}+\frac{2\tan(e+fx)}{3ac^2f}
\end{aligned}$$

Mathematica [A] time = 0.42, size = 87, normalized size = 1.64

$$\frac{\sin(e+fx)+8\sin(2(e+fx))+\sin(3(e+fx))+4\cos(e+fx)-2\cos(2(e+fx))+4\cos(3(e+fx))-2}{24ac^2f(\sin(e+fx)-1)^2(\sin(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^2),x]
```

```
[Out] (-2 + 4*Cos[e + f*x] - 2*Cos[2*(e + f*x)] + 4*Cos[3*(e + f*x)] + Sin[e + f*
x] + 8*Sin[2*(e + f*x)] + Sin[3*(e + f*x)])/(24*a*c^2*f*(-1 + Sin[e + f*x])
^2*(1 + Sin[e + f*x]))
```

fricas [A] time = 0.44, size = 56, normalized size = 1.06

$$-\frac{2\,\cos\left(fx+e\right)^2+2\,\sin\left(fx+e\right)-1}{3\left(ac^2f\cos\left(fx+e\right)\sin\left(fx+e\right)-ac^2f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*cos(f*x + e)^2 + 2*sin(f*x + e) - 1)/(a*c^2*f*cos(f*x + e)*sin(f*x
+ e) - a*c^2*f*cos(f*x + e))
```

giac [A] time = 0.19, size = 77, normalized size = 1.45

$$-\frac{\frac{3}{ac^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{9\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-12\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+7}{ac^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/6*(3/(a*c^2*(tan(1/2*f*x + 1/2*e) + 1)) + (9*tan(1/2*f*x + 1/2*e)^2 - 12
*tan(1/2*f*x + 1/2*e) + 7)/(a*c^2*(tan(1/2*f*x + 1/2*e) - 1)^3))/f
```

maple [A] time = 0.18, size = 73, normalized size = 1.38

$$\frac{-\frac{2}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{1}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{3}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{1}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}}{f\,a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

```
[Out] 2/f/a/c^2*(-1/3/(tan(1/2*f*x+1/2*e)-1)^3-1/2/(tan(1/2*f*x+1/2*e)-1)^2-3/4/(
tan(1/2*f*x+1/2*e)-1)-1/4/(tan(1/2*f*x+1/2*e)+1))
```

maxima [B] time = 0.71, size = 142, normalized size = 2.68

$$\frac{2\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{3\,\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{3\,\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+1\right)}{3\left(ac^2-\frac{2\,ac^2\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{2\,ac^2\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}-\frac{ac^2\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/((a*c^2 - 2*a*c^2*sin(f*x +
e)/(cos(f*x + e) + 1) + 2*a*c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - a*c^2
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*f)
```

mupad [B] time = 7.01, size = 74, normalized size = 1.40

$$-\frac{2\,\left(3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3-3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)}{3\,a\,c^2\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-1\right)^3\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^2),x)
```

```
[Out] -(2*(tan(e/2 + (f*x)/2) - 3*tan(e/2 + (f*x)/2)^2 + 3*tan(e/2 + (f*x)/2)^3 +
 1))/(3*a*c^2*f*(tan(e/2 + (f*x)/2) - 1)^3*(tan(e/2 + (f*x)/2) + 1))
```

sympy [A] time = 4.02, size = 328, normalized size = 6.19

$$\begin{cases} -\frac{6\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3ac^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)-6ac^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+6ac^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3ac^2f}+\frac{6\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3ac^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)-6ac^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+6ac^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3ac^2f}-\frac{\quad}{3\ldots} \\ \frac{x}{(a\sin(e)+a)(-c\sin(e)+c)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*tan(e/2 + f*x/2)**3/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**
2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) + 6*tan
(e/2 + f*x/2)**2/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*tan(e/2 + f*x
/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) - 2*tan(e/2 + f*x/2)/(3*
a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*
tan(e/2 + f*x/2) - 3*a*c**2*f) - 2/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c*
*2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f), Ne(f,
 0)), (x/((a*sin(e) + a)*(-c*sin(e) + c)**2), True))
```

3.267 $\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=85

$$\frac{2\tan(e+fx)}{5ac^3f}+\frac{\sec(e+fx)}{5af\left(c^3-c^3\sin(e+fx)\right)}+\frac{\sec(e+fx)}{5acf(c-c\sin(e+fx))^2}$$

```
[Out] 1/5*sec(f*x+e)/a/c/f/(c-c*sin(f*x+e))^2+1/5*sec(f*x+e)/a/f/(c^3-c^3*sin(f*x
+e))+2/5*tan(f*x+e)/a/c^3/f
```

Rubi [A] time = 0.16, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2672, 3767, 8}

$$\frac{2\tan(e+fx)}{5ac^3f}+\frac{\sec(e+fx)}{5af\left(c^3-c^3\sin(e+fx)\right)}+\frac{\sec(e+fx)}{5acf(c-c\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^3),x]
```

```
[Out] Sec[e + f*x]/(5*a*c*f*(c - c*Sin[e + f*x])^2) + Sec[e + f*x]/(5*a*f*(c^3 -
c^3*Sin[e + f*x])) + (2*Tan[e + f*x])/(5*a*c^3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx = \frac{\int \frac{\sec^2(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{ac}$$
$$= \frac{\sec(e+fx)}{5acf(c-c\sin(e+fx))^2} + \frac{3\int \frac{\sec^2(e+fx)}{c-c\sin(e+fx)}\,dx}{5ac^2}$$
$$= \frac{\sec(e+fx)}{5acf(c-c\sin(e+fx))^2} + \frac{\sec(e+fx)}{5af\left(c^3-c^3\sin(e+fx)\right)} + \frac{2\int \sec^2(}{5a}$$
$$= \frac{\sec(e+fx)}{5acf(c-c\sin(e+fx))^2} + \frac{\sec(e+fx)}{5af\left(c^3-c^3\sin(e+fx)\right)} - \frac{2\,\mathrm{Subst}(}{}$$
$$= \frac{\sec(e+fx)}{5acf(c-c\sin(e+fx))^2} + \frac{\sec(e+fx)}{5af\left(c^3-c^3\sin(e+fx)\right)} + \frac{2\tan(e+}{5ac^3f}$$

Mathematica [A] time = 0.67, size = 111, normalized size = 1.31

$$-\frac{12\sin(e+fx)+32\sin(2(e+fx))+12\sin(3(e+fx))-8\sin(4(e+fx))+32\cos(e+fx)-12\cos(2(e+fx))}{160ac^3f(\sin(e+fx)-1)^3(\sin(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^3),x]
```

```
[Out] -1/160*(-15 + 32*Cos[e + f*x] - 12*Cos[2*(e + f*x)] + 32*Cos[3*(e + f*x)] +
 3*Cos[4*(e + f*x)] + 12*Sin[e + f*x] + 32*Sin[2*(e + f*x)] + 12*Sin[3*(e +
 f*x)] - 8*Sin[4*(e + f*x)])/(a*c^3*f*(-1 + Sin[e + f*x])^3*(1 + Sin[e + f*
x]))
```

fricas [A] time = 0.44, size = 83, normalized size = 0.98

$$-\frac{4\cos\left(fx+e\right)^2-\left(2\cos\left(fx+e\right)^2-3\right)\sin\left(fx+e\right)-2}{5\left(ac^3f\cos\left(fx+e\right)^3+2\,ac^3f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,ac^3f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/5*(4*cos(f*x + e)^2 - (2*cos(f*x + e)^2 - 3)*sin(f*x + e) - 2)/(a*c^3*f*
cos(f*x + e)^3 + 2*a*c^3*f*cos(f*x + e)*sin(f*x + e) - 2*a*c^3*f*cos(f*x +
e))
```

giac [A] time = 0.30, size = 105, normalized size = 1.24

$$-\frac{\frac{5}{ac^3\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}+\frac{35\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-90\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+120\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-70\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+21}{ac^3\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^5}}{20f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/20*(5/(a*c^3*(tan(1/2*f*x + 1/2*e) + 1)) + (35*tan(1/2*f*x + 1/2*e)^4 -
90*tan(1/2*f*x + 1/2*e)^3 + 120*tan(1/2*f*x + 1/2*e)^2 - 70*tan(1/2*f*x + 1
/2*e) + 21)/(a*c^3*(tan(1/2*f*x + 1/2*e) - 1)^5))/f
```

maple [A] time = 0.25, size = 103, normalized size = 1.21

$$\frac{-\frac{4}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{3}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{5}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{7}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}}{f\,a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)
```

```
[Out] 2/f/a/c^3*(-2/5/(tan(1/2*f*x+1/2*e)-1)^5-1/(tan(1/2*f*x+1/2*e)-1)^4-3/2/(ta
n(1/2*f*x+1/2*e)-1)^3-5/4/(tan(1/2*f*x+1/2*e)-1)^2-7/8/(tan(1/2*f*x+1/2*e)-
1)-1/8/(tan(1/2*f*x+1/2*e)+1))
```

maxima [B] time = 1.58, size = 211, normalized size = 2.48

$$-\frac{2\left(\frac{3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{10\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{10\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}-\frac{5\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}-2\right)}{5\left(ac^3-\frac{4ac^3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{5ac^3\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{5ac^3\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+\frac{4ac^3\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}-\frac{ac^3\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/5*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 10*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 5*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 - 2)/((a*c^3 - 4*a*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 5*a*c^
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*a*c^3*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 + 4*a*c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - a*c^3*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6)*f)
```

mupad [B] time = 7.16, size = 89, normalized size = 1.05

$$-\frac{2\left(5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5-10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3-3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+2\right)}{5\,a\,c^3\,f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-1\right)^5\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^3),x)
```

```
[Out] -(2*(10*tan(e/2 + (f*x)/2)^3 - 3*tan(e/2 + (f*x)/2) - 10*tan(e/2 + (f*x)/2)
^4 + 5*tan(e/2 + (f*x)/2)^5 + 2))/(5*a*c^3*f*(tan(e/2 + (f*x)/2) - 1)^5*(ta
n(e/2 + (f*x)/2) + 1))
```

sympy [A] time = 8.43, size = 614, normalized size = 7.22

$$\begin{cases} -\frac{10\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)}{5ac^3f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)-20ac^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)+25ac^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)-25ac^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+20ac^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-5ac^3f}+\frac{}{5ac^3f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)-20ac^3} \\ \frac{x}{(a\sin(e)+a)(-c\sin(e)+c)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-10*tan(e/2 + f*x/2)**5/(5*a*c**3*f*tan(e/2 + f*x/2)**6 - 20*a*c
**3*f*tan(e/2 + f*x/2)**5 + 25*a*c**3*f*tan(e/2 + f*x/2)**4 - 25*a*c**3*f*t
an(e/2 + f*x/2)**2 + 20*a*c**3*f*tan(e/2 + f*x/2) - 5*a*c**3*f) + 20*tan(e/
2 + f*x/2)**4/(5*a*c**3*f*tan(e/2 + f*x/2)**6 - 20*a*c**3*f*tan(e/2 + f*x/2
)**5 + 25*a*c**3*f*tan(e/2 + f*x/2)**4 - 25*a*c**3*f*tan(e/2 + f*x/2)**2 +
20*a*c**3*f*tan(e/2 + f*x/2) - 5*a*c**3*f) - 20*tan(e/2 + f*x/2)**3/(5*a*c*
*3*f*tan(e/2 + f*x/2)**6 - 20*a*c**3*f*tan(e/2 + f*x/2)**5 + 25*a*c**3*f*ta
n(e/2 + f*x/2)**4 - 25*a*c**3*f*tan(e/2 + f*x/2)**2 + 20*a*c**3*f*tan(e/2 +
 f*x/2) - 5*a*c**3*f) + 6*tan(e/2 + f*x/2)/(5*a*c**3*f*tan(e/2 + f*x/2)**6
- 20*a*c**3*f*tan(e/2 + f*x/2)**5 + 25*a*c**3*f*tan(e/2 + f*x/2)**4 - 25*a*
```

```
c**3*f*tan(e/2 + f*x/2)**2 + 20*a*c**3*f*tan(e/2 + f*x/2) - 5*a*c**3*f) - 4
/(5*a*c**3*f*tan(e/2 + f*x/2)**6 - 20*a*c**3*f*tan(e/2 + f*x/2)**5 + 25*a*c
**3*f*tan(e/2 + f*x/2)**4 - 25*a*c**3*f*tan(e/2 + f*x/2)**2 + 20*a*c**3*f*t
an(e/2 + f*x/2) - 5*a*c**3*f), Ne(f, 0)), (x/((a*sin(e) + a)*(-c*sin(e) + c
)**3), True))
```

3.268 $\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx$

Optimal. Leaf size=118

$$\frac{8\tan(e+fx)}{35ac^4f}+\frac{4\sec(e+fx)}{35af\left(c^4-c^4\sin(e+fx)\right)}+\frac{4\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{\sec(e+fx)}{7acf(c-c\sin(e+fx))^3}$$

```
[Out] 1/7*sec(f*x+e)/a/c/f/(c-c*sin(f*x+e))^3+4/35*sec(f*x+e)/a/f/(c^2-c^2*sin(f*
x+e))^2+4/35*sec(f*x+e)/a/f/(c^4-c^4*sin(f*x+e))+8/35*tan(f*x+e)/a/c^4/f
```

Rubi [A] time = 0.21, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2672, 3767, 8}

$$\frac{8\tan(e+fx)}{35ac^4f}+\frac{4\sec(e+fx)}{35af\left(c^4-c^4\sin(e+fx)\right)}+\frac{4\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{\sec(e+fx)}{7acf(c-c\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^4),x]
```

```
[Out] Sec[e + f*x]/(7*a*c*f*(c - c*Sin[e + f*x])^3) + (4*Sec[e + f*x])/(35*a*f*(c
^2 - c^2*Sin[e + f*x])^2) + (4*Sec[e + f*x])/(35*a*f*(c^4 - c^4*Sin[e + f*x
])) + (8*Tan[e + f*x])/(35*a*c^4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
```

```
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx = \frac{\int \frac{\sec^2(e+fx)}{(c-c\sin(e+fx))^3}\,dx}{ac}$$

$$= \frac{\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{4\int \frac{\sec^2(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{7ac^2}$$

$$= \frac{\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{4\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{12\int \frac{\mathrm{sec}}{c-c}}{35}$$

$$= \frac{\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{4\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{4}{35af\left(c^4\right.}$$

$$= \frac{\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{4\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{4}{35af\left(c^4\right.}$$

$$= \frac{\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{4\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{4}{35af\left(c^4\right.}$$

Mathematica [A] time = 0.74, size = 131, normalized size = 1.11

$$\frac{406\sin(e+fx)+512\sin(2(e+fx))+377\sin(3(e+fx))-384\sin(4(e+fx))-29\sin(5(e+fx))+896\cos(e+}{4480ac^4f(\sin(e+fx)-1)^4(\sin}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^4),x]
```

```
[Out] (-406 + 896*Cos[e + f*x] - 232*Cos[2*(e + f*x)] + 832*Cos[3*(e + f*x)] + 17
4*Cos[4*(e + f*x)] - 64*Cos[5*(e + f*x)] + 406*Sin[e + f*x] + 512*Sin[2*(e
+ f*x)] + 377*Sin[3*(e + f*x)] - 384*Sin[4*(e + f*x)] - 29*Sin[5*(e + f*x)]
)/(4480*a*c^4*f*(-1 + Sin[e + f*x])^4*(1 + Sin[e + f*x]))
```

fricas [A] time = 0.45, size = 111, normalized size = 0.94

$$\frac{8 \cos\left(fx+e\right)^{4}-36 \cos\left(fx+e\right)^{2}+4 \left(6 \cos\left(fx+e\right)^{2}-5\right) \sin\left(fx+e\right)+15}{35 \left(3 ac^{4} f \cos\left(fx+e\right)^{3}-4 ac^{4} f \cos\left(fx+e\right)-\left(ac^{4} f \cos\left(fx+e\right)^{3}-4 ac^{4} f \cos\left(fx+e\right)\right) \sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/35*(8*cos(f*x + e)^4 - 36*cos(f*x + e)^2 + 4*(6*cos(f*x + e)^2 - 5)*sin(f
*x + e) + 15)/(3*a*c^4*f*cos(f*x + e)^3 - 4*a*c^4*f*cos(f*x + e) - (a*c^4*f
*cos(f*x + e)^3 - 4*a*c^4*f*cos(f*x + e))*sin(f*x + e))
```

giac [A] time = 0.23, size = 133, normalized size = 1.13

$$-\frac{\frac{35}{ac^{4}\left(\tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)+1\right)}+\frac{525 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{6}-1960 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{5}+4025 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{4}-4480 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{3}+3143 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{2}-1176}{ac^{4}\left(\tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)-1\right)^{7}}}{280 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -1/280*(35/(a*c^4*(tan(1/2*f*x + 1/2*e) + 1)) + (525*tan(1/2*f*x + 1/2*e)^6
 - 1960*tan(1/2*f*x + 1/2*e)^5 + 4025*tan(1/2*f*x + 1/2*e)^4 - 4480*tan(1/2
*f*x + 1/2*e)^3 + 3143*tan(1/2*f*x + 1/2*e)^2 - 1176*tan(1/2*f*x + 1/2*e) +
 243)/(a*c^4*(tan(1/2*f*x + 1/2*e) - 1)^7))/f
```

maple [A] time = 0.25, size = 133, normalized size = 1.13

$$\frac{-\frac{8}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{38}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{9}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{15}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{17}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{15}{8\left(\tan\left(\frac{fx}{2}+\right.\right.}}{f a c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x)
```

```
[Out] 2/f/a/c^4*(-4/7/(tan(1/2*f*x+1/2*e)-1)^7-2/(tan(1/2*f*x+1/2*e)-1)^6-19/5/(t
an(1/2*f*x+1/2*e)-1)^5-9/2/(tan(1/2*f*x+1/2*e)-1)^4-15/4/(tan(1/2*f*x+1/2*e
)-1)^3-17/8/(tan(1/2*f*x+1/2*e)-1)^2-15/16/(tan(1/2*f*x+1/2*e)-1)-1/16/(tan
(1/2*f*x+1/2*e)+1))
```

maxima [B] time = 0.68, size = 319, normalized size = 2.70

$$-\frac{2\left(\frac{43\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{77\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{7\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{105\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}-\frac{175\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}+\frac{105\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}-\frac{35\sin\left(fx+e\right)^{7}}{\left(\cos\left(fx+e\right)+1\right)^{7}}\right.}{35\left(ac^{4}-\frac{6\,ac^{4}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{14\,ac^{4}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{14\,ac^{4}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{14\,ac^{4}\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}-\frac{14\,ac^{4}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}+\frac{6\,ac^{4}\sin\left(fx+e\right)^{7}}{\left(\cos\left(fx+e\right)+1\right)^{7}}-\frac{ac^{4}\,\mathrm{s}}{(\cos(}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] -2/35*(43*sin(f*x + e)/(cos(f*x + e) + 1) - 77*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 105*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 - 175*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 105*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6 - 35*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 13)/((a*
c^4 - 6*a*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 14*a*c^4*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 - 14*a*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 14*a*c^4
*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 14*a*c^4*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 + 6*a*c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a*c^4*sin(f*x + e)
^8/(cos(f*x + e) + 1)^8)*f)
```

mupad [B] time = 7.27, size = 96, normalized size = 0.81

$$\frac{\frac{2\sin\left(e+f\,x\right)}{5}+\frac{2\cos\left(2\,e+2\,f\,x\right)}{5}-\frac{\cos\left(4\,e+4\,f\,x\right)}{35}-\frac{6\sin\left(3\,e+3\,f\,x\right)}{35}}{a\,c^4\,f\,\left(\frac{7\cos\left(e+f\,x\right)}{4}-\frac{3\cos\left(3\,e+3\,f\,x\right)}{4}-\frac{7\sin\left(2\,e+2\,f\,x\right)}{4}+\frac{\sin\left(4\,e+4\,f\,x\right)}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^4),x)
```

```
[Out] ((2*sin(e + f*x))/5 + (2*cos(2*e + 2*f*x))/5 - cos(4*e + 4*f*x)/35 - (6*sin
(3*e + 3*f*x))/35)/(a*c^4*f*((7*cos(e + f*x))/4 - (3*cos(3*e + 3*f*x))/4 -
(7*sin(2*e + 2*f*x))/4 + sin(4*e + 4*f*x)/8))
```

sympy [A] time = 13.73, size = 1307, normalized size = 11.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-70*tan(e/2 + f*x/2)**7/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a
*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4
```

```
*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*ta
n(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) + 210*tan(
e/2 + f*x/2)**6/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f
*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)
**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 +
 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) - 350*tan(e/2 + f*x/2)**5/(35
*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c*
*4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*
tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e
/2 + f*x/2) - 35*a*c**4*f) + 210*tan(e/2 + f*x/2)**4/(35*a*c**4*f*tan(e/2 +
 f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/
2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3
 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*
c**4*f) + 14*tan(e/2 + f*x/2)**3/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c
**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f
*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(
e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) - 154*tan(e/
2 + f*x/2)**2/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x
/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**
5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 2
10*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) + 86*tan(e/2 + f*x/2)/(35*a*c**
4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*t
an(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/
2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f
*x/2) - 35*a*c**4*f) - 26/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*t
an(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/
2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f
*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f), Ne(f, 0)), (x/((a*
sin(e) + a)*(-c*sin(e) + c)**4), True))
```

3.269 $\int \frac{(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=148

$$-\frac{2a^4c^5\cos^9(e+fx)}{3f(a\sin(e+fx)+a)^6}+\frac{35c^5\cos^3(e+fx)}{a^2f}+\frac{6a^2c^5\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}+\frac{105c^5\sin(e+fx)\cos(e+fx)}{2a^2f}+\frac{105c^5x}{2a^2}+\frac{4}{f(a}$$

```
[Out] 105/2*c^5*x/a^2+35*c^5*cos(f*x+e)^3/a^2/f+105/2*c^5*cos(f*x+e)*sin(f*x+e)/a
^2/f-2/3*a^4*c^5*cos(f*x+e)^9/f/(a+a*sin(f*x+e))^6+6*a^2*c^5*cos(f*x+e)^7/f
/(a+a*sin(f*x+e))^4+42*c^5*cos(f*x+e)^5/f/(a+a*sin(f*x+e))^2
```

Rubi [A] time = 0.24, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2736, 2680, 2682, 2635, 8}

$$\frac{35c^5\cos^3(e+fx)}{a^2f}-\frac{2a^4c^5\cos^9(e+fx)}{3f(a\sin(e+fx)+a)^6}+\frac{6a^2c^5\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}+\frac{105c^5\sin(e+fx)\cos(e+fx)}{2a^2f}+\frac{105c^5x}{2a^2}+\frac{42c}{f(a\,\mathrm{s}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^5/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (105*c^5*x)/(2*a^2) + (35*c^5*Cos[e + f*x]^3)/(a^2*f) + (105*c^5*Cos[e + f*
x]*Sin[e + f*x])/(2*a^2*f) - (2*a^4*c^5*Cos[e + f*x]^9)/(3*f*(a + a*Sin[e +
 f*x])^6) + (6*a^2*c^5*Cos[e + f*x]^7)/(f*(a + a*Sin[e + f*x])^4) + (42*c^5
*Cos[e + f*x]^5)/(f*(a + a*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx &= \left(a^5c^5\right)\int \frac{\cos^{10}(e+fx)}{(a+a\sin(e+fx))^7}\,dx\\
&= -\frac{2a^4c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6} - \left(3a^3c^5\right)\int \frac{\cos^8(e+fx)}{(a+a\sin(e+fx))^5}\,dx\\
&= -\frac{2a^4c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6} + \frac{6a^2c^5\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} + \left(21ac^5\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^3}\\
&= -\frac{2a^4c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6} + \frac{6a^2c^5\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} + \frac{42c^5\cos^5(e+fx)}{f(a+a\sin(e+fx))^2} + \frac{\left(105c^5\right.}{}\\
&= \frac{35c^5\cos^3(e+fx)}{a^2f} - \frac{2a^4c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6} + \frac{6a^2c^5\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} + \frac{42c^5\cos^5(}{f(a+a\sin(}\\
&= \frac{35c^5\cos^3(e+fx)}{a^2f} + \frac{105c^5\cos(e+fx)\sin(e+fx)}{2a^2f} - \frac{2a^4c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6} + \frac{6a}{f(a}\\
&= \frac{105c^5x}{2a^2} + \frac{35c^5\cos^3(e+fx)}{a^2f} + \frac{105c^5\cos(e+fx)\sin(e+fx)}{2a^2f} - \frac{2a^4c^5\cos^9(e+f}{3f(a+a\sin(e+f}\end{aligned}$$

Mathematica [A] time = 0.72, size = 276, normalized size = 1.86

$$\frac{(c - c\sin(e + fx))^5 \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right) \left(256 \sin\left(\frac{1}{2}(e + fx)\right) + 630(e + fx)\left(\sin\left(\frac{1}{2}(e + fx)\right) + \text{co}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^5/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^5*(256*Sin[(e +
 f*x)/2] - 128*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 1664*Sin[(e + f*x)/2
]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 630*(e + f*x)*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^3 + 285*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2])^3 - Cos[3*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 21*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)]))/(12*f*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^10*(a + a*Sin[e + f*x])^2)
```

fricas [A] time = 0.56, size = 238, normalized size = 1.61

$$-\frac{2\,c^5\cos\left(fx+e\right)^5+19\,c^5\cos\left(fx+e\right)^4-106\,c^5\cos\left(fx+e\right)^3+630\,c^5fx-64\,c^5-7\left(45\,c^5fx-77\,c^5\right)\cos\left(fx+\right.}{6\left(a^2f\cos\left(fx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*c^5*cos(f*x + e)^5 + 19*c^5*cos(f*x + e)^4 - 106*c^5*cos(f*x + e)^3
 + 630*c^5*f*x - 64*c^5 - 7*(45*c^5*f*x - 77*c^5)*cos(f*x + e)^2 + (315*c^5
*f*x + 598*c^5)*cos(f*x + e) - (2*c^5*cos(f*x + e)^4 - 17*c^5*cos(f*x + e)^
3 - 630*c^5*f*x - 123*c^5*cos(f*x + e)^2 - 64*c^5 - (315*c^5*f*x + 662*c^5)
*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2
*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

giac [A] time = 0.25, size = 203, normalized size = 1.37

$$\frac{\frac{315\left(fx+e\right)c^5}{a^2}+\frac{2\left(309\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8+969\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+1693\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+3027\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+2901\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+3247\,c\right.}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3a}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/6*(315*(f*x + e)*c^5/a^2 + 2*(309*c^5*tan(1/2*f*x + 1/2*e)^8 + 969*c^5*ta
n(1/2*f*x + 1/2*e)^7 + 1693*c^5*tan(1/2*f*x + 1/2*e)^6 + 3027*c^5*tan(1/2*f
*x + 1/2*e)^5 + 2901*c^5*tan(1/2*f*x + 1/2*e)^4 + 3247*c^5*tan(1/2*f*x + 1/
2*e)^3 + 1995*c^5*tan(1/2*f*x + 1/2*e)^2 + 1173*c^5*tan(1/2*f*x + 1/2*e) +
494*c^5)/((tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x +
1/2*e) + 1)^3*a^2))/f
```

maple [A] time = 0.31, size = 267, normalized size = 1.80

$$\frac{7c^5\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a^2f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}+\frac{46c^5\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a^2f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}+\frac{96c^5\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a^2f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}-\frac{7c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a^2f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}+\frac{}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 7*c^5/a^2/f/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5+46*c^5/a^2/f/(1
+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^4+96*c^5/a^2/f/(1+tan(1/2*f*x+1
/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2-7*c^5/a^2/f/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(
1/2*f*x+1/2*e)+142/3*c^5/a^2/f/(1+tan(1/2*f*x+1/2*e)^2)^3+105*c^5/a^2/f*arc
tan(tan(1/2*f*x+1/2*e))-128/3*c^5/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3+64*c^5/a^2
/f/(tan(1/2*f*x+1/2*e)+1)^2+96*c^5/a^2/f/(tan(1/2*f*x+1/2*e)+1)
```

maxima [B] time = 1.12, size = 1304, normalized size = 8.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(5*c^5*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e)
+ 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^2*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
+ a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)/(cos(f*
x + e) + 1))/a^2) + 2*c^5*((57*sin(f*x + e)/(cos(f*x + e) + 1) + 99*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + 155*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15
3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 135*sin(f*x + e)^5/(cos(f*x + e) +
1)^5 + 85*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 45*sin(f*x + e)^7/(cos(f*x
```

```
+ e) + 1)^7 + 15*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 24)/(a^2 + 3*a^2*sin
(f*x + e)/(cos(f*x + e) + 1) + 6*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
10*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 12*a^2*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 + 12*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 10*a^2*sin(f*x
+ e)^6/(cos(f*x + e) + 1)^6 + 6*a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 3
*a^2*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^2*sin(f*x + e)^9/(cos(f*x + e)
 + 1)^9) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 40*c^5*((12*si
n(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 9*
sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*a^2*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*a^2*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)
 + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 20*c^5*((9*sin(f*x + e)
/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 + 3*a
^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(cos
(f*x + e) + 1))/a^2) - 2*c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) +
 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3) + 10*c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a
^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

mupad [B] time = 10.86, size = 372, normalized size = 2.51

$$\frac{105\,c^5\,x}{2\,a^2}-\frac{\frac{105\,c^5\,\left(e+f\,x\right)}{2}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{315\,c^5\,\left(e+f\,x\right)}{2}-\frac{c^5\,\left(945\,e+945\,f\,x+2346\right)}{6}\right)-\frac{c^5\,\left(315\,e+315\,f\,x+988\right)}{6}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8\,\left(\frac{3\cdots}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^5/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (105*c^5*x)/(2*a^2) - ((105*c^5*(e + f*x))/2 + tan(e/2 + (f*x)/2)*((315*c^5
*(e + f*x))/2 - (c^5*(945*e + 945*f*x + 2346))/6) - (c^5*(315*e + 315*f*x +
 988))/6 + tan(e/2 + (f*x)/2)^8*((315*c^5*(e + f*x))/2 - (c^5*(945*e + 945*
f*x + 618))/6) + tan(e/2 + (f*x)/2)^7*(315*c^5*(e + f*x) - (c^5*(1890*e + 1
890*f*x + 1938))/6) + tan(e/2 + (f*x)/2)^2*(315*c^5*(e + f*x) - (c^5*(1890*
e + 1890*f*x + 3990))/6) + tan(e/2 + (f*x)/2)^6*(525*c^5*(e + f*x) - (c^5*(
3150*e + 3150*f*x + 3386))/6) + tan(e/2 + (f*x)/2)^3*(525*c^5*(e + f*x) - (
c^5*(3150*e + 3150*f*x + 6494))/6) + tan(e/2 + (f*x)/2)^4*(630*c^5*(e + f*x
) - (c^5*(3780*e + 3780*f*x + 5802))/6) + tan(e/2 + (f*x)/2)^5*(630*c^5*(e
+ f*x) - (c^5*(3780*e + 3780*f*x + 6054))/6))/(a^2*f*(tan(e/2 + (f*x)/2) +
tan(e/2 + (f*x)/2)^2 + tan(e/2 + (f*x)/2)^3 + 1)^3)
```

sympy [A] time = 41.19, size = 3641, normalized size = 24.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**5/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((315*c**5*f*x*tan(e/2 + f*x/2)**9/(6*a**2*f*tan(e/2 + f*x/2)**9 +
 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*
tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f
*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 +
18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 945*c**5*f*x*tan(e/2 + f*x/2)**8/(
6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*ta
n(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x
/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36
*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 1890
*c**5*f*x*tan(e/2 + f*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan
(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/
2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*
a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(
e/2 + f*x/2) + 6*a**2*f) + 3150*c**5*f*x*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(
e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2
)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a
**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e
/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 3780*c**5*f*x*tan
(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)
**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a*
*2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/
2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2)
+ 6*a**2*f) + 3780*c**5*f*x*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)*
*9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**
2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2
 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**
2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 3150*c**5*f*x*tan(e/2 + f*x/2)
**3/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2
*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2
+ f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3
 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) +
 1890*c**5*f*x*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*
f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 +
 f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4
+ 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f
*tan(e/2 + f*x/2) + 6*a**2*f) + 945*c**5*f*x*tan(e/2 + f*x/2)/(6*a**2*f*tan
(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/
```

2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*
a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(
e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 315*c**5*f*x/(6*
a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(
e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2
)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a
2*f*tan(e/2 + f*x/2)2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 618*c*
*5*tan(e/2 + f*x/2)**8/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 +
f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 +
72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*
tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f
*x/2) + 6*a**2*f) + 1938*c**5*tan(e/2 + f*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2
)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a
2*f*tan(e/2 + f*x/2)6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e
/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)
2 + 18*a2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 3386*c**5*tan(e/2 + f*x/2)**
6/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f
*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 +
f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 +
36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 6
054*c**5*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(
e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2
)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a
2*f*tan(e/2 + f*x/2)3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e
/2 + f*x/2) + 6*a**2*f) + 5802*c**5*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 +
f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7
+ 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f
*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 +
f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 6494*c**5*tan(e/2 + f*
x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*
a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(
e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2
)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*
f) + 3990*c**5*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*
f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 +
f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4
+ 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f
*tan(e/2 + f*x/2) + 6*a**2*f) + 2346*c**5*tan(e/2 + f*x/2)/(6*a**2*f*tan(e/
2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)*
*7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**
2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2
+ f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 988*c**5/(6*a**2*f*
tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f
*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 +
72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*t

```
an(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f), Ne(f, 0)), (x*
(-c*sin(e) + c)**5/(a*sin(e) + a)**2, True))
```

3.270 $\int \frac{(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=135

$$-\frac{2a^3c^4\cos^7(e+fx)}{3f(a\sin(e+fx)+a)^5}+\frac{35c^4\cos(e+fx)}{2a^2f}+\frac{35c^4\cos^3(e+fx)}{6f\left(a^2\sin(e+fx)+a^2\right)}+\frac{35c^4x}{2a^2}+\frac{14a^4c^4\cos^5(e+fx)}{3f\left(a^2\sin(e+fx)+a^2\right)^3}$$

```
[Out] 35/2*c^4*x/a^2+35/2*c^4*cos(f*x+e)/a^2/f-2/3*a^3*c^4*cos(f*x+e)^7/f/(a+a*si
n(f*x+e))^5+14/3*a^4*c^4*cos(f*x+e)^5/f/(a^2+a^2*sin(f*x+e))^3+35/6*c^4*cos
(f*x+e)^3/f/(a^2+a^2*sin(f*x+e))
```

Rubi [A] time = 0.22, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2736, 2680, 2679, 2682, 8}

$$\frac{35c^4\cos(e+fx)}{2a^2f}-\frac{2a^3c^4\cos^7(e+fx)}{3f(a\sin(e+fx)+a)^5}+\frac{14a^4c^4\cos^5(e+fx)}{3f\left(a^2\sin(e+fx)+a^2\right)^3}+\frac{35c^4\cos^3(e+fx)}{6f\left(a^2\sin(e+fx)+a^2\right)}+\frac{35c^4x}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^4/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (35*c^4*x)/(2*a^2) + (35*c^4*Cos[e + f*x])/(2*a^2*f) - (2*a^3*c^4*Cos[e + f
*x]^7)/(3*f*(a + a*Sin[e + f*x])^5) + (14*a^4*c^4*Cos[e + f*x]^5)/(3*f*(a^2
 + a^2*Sin[e + f*x])^3) + (35*c^4*Cos[e + f*x]^3)/(6*f*(a^2 + a^2*Sin[e + f
*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx &= \left(a^4c^4\right)\int \frac{\cos^8(e+fx)}{(a+a\sin(e+fx))^6}\,dx \\
&= -\frac{2a^3c^4\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5} - \frac{1}{3}\left(7a^2c^4\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^4}\,dx \\
&= -\frac{2a^3c^4\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5} + \frac{14ac^4\cos^5(e+fx)}{3f(a+a\sin(e+fx))^3} + \frac{1}{3}\left(35c^4\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx)} \\
&= -\frac{2a^3c^4\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5} + \frac{14ac^4\cos^5(e+fx)}{3f(a+a\sin(e+fx))^3} + \frac{35c^4\cos^3(e+fx)}{6f\left(a^2+a^2\sin(e+fx)\right)} + \frac{(3}{} \\
&= \frac{35c^4\cos(e+fx)}{2a^2f} - \frac{2a^3c^4\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5} + \frac{14ac^4\cos^5(e+fx)}{3f(a+a\sin(e+fx))^3} + \frac{35c^4\cos}{6f\left(a^2+a^2\right.} \\
&= \frac{35c^4x}{2a^2} + \frac{35c^4\cos(e+fx)}{2a^2f} - \frac{2a^3c^4\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5} + \frac{14ac^4\cos^5(e+fx)}{3f(a+a\sin(e+fx))^3} + \frac{}{6f}
\end{aligned}$$

Mathematica [A] time = 0.49, size = 243, normalized size = 1.80

$$\frac{(c - c\sin(e+fx))^4 \left(\sin\left(\frac{1}{2}(e+fx)\right) + \cos\left(\frac{1}{2}(e+fx)\right)\right)\left(128\sin\left(\frac{1}{2}(e+fx)\right) + 210(e+fx)\left(\sin\left(\frac{1}{2}(e+fx)\right) + \cos\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^4/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^4*(128*Sin[(e +
 f*x)/2] - 64*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 640*Sin[(e + f*x)/2]*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 210*(e + f*x)*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^3 + 72*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^3 - 3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)]))/(12*f*(C
os[(e + f*x)/2] - Sin[(e + f*x)/2])^8*(a + a*Sin[e + f*x])^2)
```

fricas [A] time = 0.51, size = 210, normalized size = 1.56

$$-\frac{3\,c^4\cos\left(fx+e\right)^4 - 30\,c^4\cos\left(fx+e\right)^3 + 210\,c^4 fx - 32\,c^4 - \left(105\,c^4 fx - 193\,c^4\right)\cos\left(fx+e\right)^2 + \left(105\,c^4 fx + 19\right.}{6\left(a^2 f\cos\left(fx+e\right)^2 - a^2 f\cos\left(fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*c^4*cos(f*x + e)^4 - 30*c^4*cos(f*x + e)^3 + 210*c^4*f*x - 32*c^4 -
 (105*c^4*f*x - 193*c^4)*cos(f*x + e)^2 + (105*c^4*f*x + 194*c^4)*cos(f*x +
 e) + (3*c^4*cos(f*x + e)^3 + 210*c^4*f*x + 33*c^4*cos(f*x + e)^2 + 32*c^4
+ (105*c^4*f*x + 226*c^4)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2
 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x +
e))
```

giac [A] time = 0.19, size = 152, normalized size = 1.13

$$\frac{\frac{105\left(fx+e\right)c^4}{a^2} + \frac{6\left(c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3 + 12\,c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 - c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right) + 12\,c^4\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)^2 a^2} + \frac{64\left(3\,c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 + 9\,c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right) + 4\,c^4\right)}{a^2\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/6*(105*(f*x + e)*c^4/a^2 + 6*(c^4*tan(1/2*f*x + 1/2*e)^3 + 12*c^4*tan(1/2
*f*x + 1/2*e)^2 - c^4*tan(1/2*f*x + 1/2*e) + 12*c^4)/((tan(1/2*f*x + 1/2*e)
^2 + 1)^2*a^2) + 64*(3*c^4*tan(1/2*f*x + 1/2*e)^2 + 9*c^4*tan(1/2*f*x + 1/2
*e) + 4*c^4)/(a^2*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

maple [A] time = 0.29, size = 229, normalized size = 1.70

$$\frac{c^4\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a^2 f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{12c^4\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a^2 f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a^2 f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{12c^4}{a^2 f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{35c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x)
```

```
[Out] c^4/a^2/f/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^3+12*c^4/a^2/f/(1+t
an(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2-c^4/a^2/f/(1+tan(1/2*f*x+1/2*e)
^2)^2*tan(1/2*f*x+1/2*e)+12*c^4/a^2/f/(1+tan(1/2*f*x+1/2*e)^2)^2+35*c^4/a^2
/f*arctan(tan(1/2*f*x+1/2*e))-64/3*c^4/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3+32*c^
4/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2+32*c^4/a^2/f/(tan(1/2*f*x+1/2*e)+1)
```

maxima [B] time = 0.87, size = 903, normalized size = 6.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(c^4*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) +
1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^2*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 +
a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)/(cos(f*x
+ e) + 1))/a^2) + 16*c^4*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x +
e) + 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1)
)/a^2) + 12*c^4*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)
```

```
^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 2*c^4*(3*sin(f*x + e
)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*
a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 8*c^4*(3*sin(f*x + e)/(co
s(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3))/f
```

mupad [B] time = 10.09, size = 291, normalized size = 2.16

$$\frac{35\,c^4\,x}{2\,a^2}-\frac{\frac{35\,c^4\,\left(e+f\,x\right)}{2}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{105\,c^4\,\left(e+f\,x\right)}{2}-\frac{c^4\,\left(315\,e+315\,f\,x+786\right)}{6}\right)-\frac{c^4\,\left(105\,e+105\,f\,x+328\right)}{6}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,\left(\frac{105\,c^4 \cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^4/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (35*c^4*x)/(2*a^2) - ((35*c^4*(e + f*x))/2 + tan(e/2 + (f*x)/2)*((105*c^4*(
e + f*x))/2 - (c^4*(315*e + 315*f*x + 786))/6) - (c^4*(105*e + 105*f*x + 32
8))/6 + tan(e/2 + (f*x)/2)^6*((105*c^4*(e + f*x))/2 - (c^4*(315*e + 315*f*x
 + 198))/6) + tan(e/2 + (f*x)/2)^5*((175*c^4*(e + f*x))/2 - (c^4*(525*e + 5
25*f*x + 666))/6) + tan(e/2 + (f*x)/2)^2*((175*c^4*(e + f*x))/2 - (c^4*(525
*e + 525*f*x + 974))/6) + tan(e/2 + (f*x)/2)^4*((245*c^4*(e + f*x))/2 - (c^
4*(735*e + 735*f*x + 868))/6) + tan(e/2 + (f*x)/2)^3*((245*c^4*(e + f*x))/2
 - (c^4*(735*e + 735*f*x + 1428))/6))/(a^2*f*(tan(e/2 + (f*x)/2) + 1)^3*(ta
n(e/2 + (f*x)/2)^2 + 1)^2)
```

sympy [A] time = 23.38, size = 2312, normalized size = 17.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**4/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((105*c**4*f*x*tan(e/2 + f*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2)**7 +
 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*
tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f
*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 315*c**4*f*x*tan(e/2 +
f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 3
0*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*ta
n(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x
/2) + 6*a**2*f) + 525*c**4*f*x*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/
2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*
```

```
a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(
e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 735*c**4*f*x*tan
(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)
**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a*
*2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/
2 + f*x/2) + 6*a**2*f) + 735*c**4*f*x*tan(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2
 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**
5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2
*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 525*c**4*
f*x*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 +
 f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4
+ 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f
*tan(e/2 + f*x/2) + 6*a**2*f) + 315*c**4*f*x*tan(e/2 + f*x/2)/(6*a**2*f*tan
(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/
2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*
a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 105*c
**4*f*x/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*
a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(
e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2
) + 6*a**2*f) + 198*c**4*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**7
+ 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f
*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 +
f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 666*c**4*tan(e/2 + f*x
/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a
**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e
/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2)
 + 6*a**2*f) + 868*c**4*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 +
 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*
tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f
*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 1428*c**4*tan(e/2 + f*x
/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a
**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e
/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2)
 + 6*a**2*f) + 974*c**4*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**7 +
 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*
tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f
*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 786*c**4*tan(e/2 + f*x/
2)/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*
f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 +
 f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6
*a**2*f) + 328*c**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x
/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42
*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan
(e/2 + f*x/2) + 6*a**2*f), Ne(f, 0)), (x*(-c*sin(e) + c)**4/(a*sin(e) + a)*
*2, True))
```

3.271 $\int \frac{(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=90

$$\frac{5c^3\cos(e+fx)}{a^2f}-\frac{2a^2c^3\cos^5(e+fx)}{3f(a\sin(e+fx)+a)^4}+\frac{5c^3x}{a^2}+\frac{10c^3\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

```
[Out] 5*c^3*x/a^2+5*c^3*cos(f*x+e)/a^2/f-2/3*a^2*c^3*cos(f*x+e)^5/f/(a+a*sin(f*x+
e))^4+10/3*c^3*cos(f*x+e)^3/f/(a+a*sin(f*x+e))^2
```

Rubi [A] time = 0.18, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2680, 2682, 8}

$$\frac{5c^3\cos(e+fx)}{a^2f}-\frac{2a^2c^3\cos^5(e+fx)}{3f(a\sin(e+fx)+a)^4}+\frac{5c^3x}{a^2}+\frac{10c^3\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (5*c^3*x)/a^2 + (5*c^3*Cos[e + f*x])/(a^2*f) - (2*a^2*c^3*Cos[e + f*x]^5)/(
3*f*(a + a*Sin[e + f*x])^4) + (10*c^3*Cos[e + f*x]^3)/(3*f*(a + a*Sin[e + f
*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^5}\,dx\\ &= -\frac{2a^2c^3\cos^5(e+fx)}{3f(a+a\sin(e+fx))^4}-\frac{1}{3}\left(5ac^3\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^3}\,dx\\ &= -\frac{2a^2c^3\cos^5(e+fx)}{3f(a+a\sin(e+fx))^4}+\frac{10c^3\cos^3(e+fx)}{3f(a+a\sin(e+fx))^2}+\frac{\left(5c^3\right)\int \frac{\cos^2(e+fx)}{a+a\sin(e+fx)}\,dx}{a}\\ &= \frac{5c^3\cos(e+fx)}{a^2f}-\frac{2a^2c^3\cos^5(e+fx)}{3f(a+a\sin(e+fx))^4}+\frac{10c^3\cos^3(e+fx)}{3f(a+a\sin(e+fx))^2}+\frac{\left(5c^3\right)\int 1\,dx}{a^2}\\ &= \frac{5c^3x}{a^2}+\frac{5c^3\cos(e+fx)}{a^2f}-\frac{2a^2c^3\cos^5(e+fx)}{3f(a+a\sin(e+fx))^4}+\frac{10c^3\cos^3(e+fx)}{3f(a+a\sin(e+fx))^2}\end{aligned}$$

Mathematica [B] time = 0.38, size = 210, normalized size = 2.33

$$\frac{(c-c\sin(e+fx))^3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(16\sin\left(\frac{1}{2}(e+fx)\right)+15(e+fx)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\right.\right.}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(16*Sin[(e + f*x)/2] - 8*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2]) - 56*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])^2 + 15*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 3*Co
s[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)*(c - c*Sin[e + f*x])^3)
/(3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6*(a + a*Sin[e + f*x])^2)
```

fricas [B] time = 0.47, size = 185, normalized size = 2.06

$$\frac{3\,c^3\cos\left(fx+e\right)^3-30\,c^3fx+8\,c^3+\left(15\,c^3fx-31\,c^3\right)\cos\left(fx+e\right)^2-\left(15\,c^3fx+26\,c^3\right)\cos\left(fx+e\right)-\left(30\,c^3fx+\right.}{3\left(a^2f\cos\left(fx+e\right)^2-a^2f\cos\left(fx+e\right)-2\,a^2f-\left(a^2f\cos\left(fx+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*c^3*cos(f*x + e)^3 - 30*c^3*f*x + 8*c^3 + (15*c^3*f*x - 31*c^3)*cos(
f*x + e)^2 - (15*c^3*f*x + 26*c^3)*cos(f*x + e) - (30*c^3*f*x + 3*c^3*cos(f
*x + e)^2 + 8*c^3 + (15*c^3*f*x + 34*c^3)*cos(f*x + e))*sin(f*x + e))/(a^2*
f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a
^2*f)*sin(f*x + e))
```

giac [A] time = 0.20, size = 101, normalized size = 1.12

$$\frac{\frac{15\left(fx+e\right)c^3}{a^2}+\frac{6\,c^3}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)a^2}+\frac{8\left(3\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+12\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+5\,c^3\right)}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(15*(f*x + e)*c^3/a^2 + 6*c^3/((tan(1/2*f*x + 1/2*e)^2 + 1)*a^2) + 8*(3
*c^3*tan(1/2*f*x + 1/2*e)^2 + 12*c^3*tan(1/2*f*x + 1/2*e) + 5*c^3)/(a^2*(ta
n(1/2*f*x + 1/2*e) + 1)^3))/f
```

maple [A] time = 0.26, size = 121, normalized size = 1.34

$$\frac{2c^3}{a^2f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}+\frac{10c^3\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a^2f}-\frac{32c^3}{3a^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{16c^3}{a^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{}{a^2f\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 2*c^3/a^2/f/(1+tan(1/2*f*x+1/2*e)^2)+10*c^3/a^2/f*arctan(tan(1/2*f*x+1/2*e)
)-32/3*c^3/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3+16*c^3/a^2/f/(tan(1/2*f*x+1/2*e)+
1)^2+8*c^3/a^2/f/(tan(1/2*f*x+1/2*e)+1)
```

maxima [B] time = 0.92, size = 590, normalized size = 6.56

$$2\left(2\,c^{3}\left(\frac{\frac{12\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{11\,\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{9\,\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{3\,\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+5}{a^{2}+\frac{3\,a^{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{4\,a^{2}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{4\,a^{2}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{3\,a^{2}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{a^{2}\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}}+\frac{3\,\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{a^{2}}\right)+3\,c^{3}\left(\frac{\frac{9\,\sin\left(fx+\right.}{\cos\left(fx+e\right)}}{a^{2}+\frac{3\,a^{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(2*c^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*a^
2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/(co
s(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 3*c^3
*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(
sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - c^3*(3*sin(f*x + e)/(cos(f*x + e) +
 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/
(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3) + 3*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) +
 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

mupad [B] time = 9.62, size = 217, normalized size = 2.41

$$\frac{5\,c^{3}\,x}{a^{2}}-\frac{5\,c^{3}\,\left(e+f\,x\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(15\,c^{3}\,\left(e+f\,x\right)-\frac{c^{3}\,\left(45\,e+45\,f\,x+114\right)}{3}\right)-\frac{c^{3}\,\left(15\,e+15\,f\,x+46\right)}{3}+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{4}\,\left(15\right.}{a^{2}\,f\,\left(\mathrm{t}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^3/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (5*c^3*x)/a^2 - (5*c^3*(e + f*x) + tan(e/2 + (f*x)/2)*(15*c^3*(e + f*x) - (
c^3*(45*e + 45*f*x + 114))/3) - (c^3*(15*e + 15*f*x + 46))/3 + tan(e/2 + (f
*x)/2)^4*(15*c^3*(e + f*x) - (c^3*(45*e + 45*f*x + 24))/3) + tan(e/2 + (f*x
)/2)^2*(20*c^3*(e + f*x) - (c^3*(60*e + 60*f*x + 82))/3) + tan(e/2 + (f*x)/
2)^3*(20*c^3*(e + f*x) - (c^3*(60*e + 60*f*x + 102))/3))/(a^2*f*(tan(e/2 +
(f*x)/2) + 1)^3*(tan(e/2 + (f*x)/2)^2 + 1))
```

sympy [A] time = 12.76, size = 1282, normalized size = 14.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**3/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((15*c**3*f*x*tan(e/2 + f*x/2)**5/(3*a**2*f*tan(e/2 + f*x/2)**5 +
9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*ta
n(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 45*c**3*f*x*tan
(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)*
*4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2
*f*tan(e/2 + f*x/2) + 3*a**2*f) + 60*c**3*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f
*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f
*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a*
*2*f) + 60*c**3*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a
**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e
/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 45*c**3*f*x*tan(e/
2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 1
2*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan
(e/2 + f*x/2) + 3*a**2*f) + 15*c**3*f*x/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a
**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e
/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 24*c**3*tan(e/2 +
f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12
*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(
e/2 + f*x/2) + 3*a**2*f) + 102*c**3*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 +
 f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 +
 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 82
*c**3*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2
+ f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2
 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 114*c**3*tan(e/2 + f*x/2)/(3*a**
2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2
+ f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3
*a**2*f) + 46*c**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2
)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a*
*2*f*tan(e/2 + f*x/2) + 3*a**2*f), Ne(f, 0)), (x*(-c*sin(e) + c)**3/(a*sin(
e) + a)**2, True))
```

3.272 $\int \frac{(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=70

$$\frac{2c^2\cos(e+fx)}{f\left(a^2\sin(e+fx)+a^2\right)}+\frac{c^2x}{a^2}-\frac{2ac^2\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^3}$$

```
[Out] c^2*x/a^2-2/3*a*c^2*cos(f*x+e)^3/f/(a+a*sin(f*x+e))^3+2*c^2*cos(f*x+e)/f/(a
^2+a^2*sin(f*x+e))
```

Rubi [A] time = 0.13, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2680, 8}

$$\frac{2c^2\cos(e+fx)}{f\left(a^2\sin(e+fx)+a^2\right)}+\frac{c^2x}{a^2}-\frac{2ac^2\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (c^2*x)/a^2 - (2*a*c^2*Cos[e + f*x]^3)/(3*f*(a + a*Sin[e + f*x])^3) + (2*c^
2*Cos[e + f*x])/(f*(a^2 + a^2*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^4}\,dx\\ &= -\frac{2ac^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3}-c^2\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^2}\,dx\\ &= -\frac{2ac^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3}+\frac{2c^2\cos(e+fx)}{f\left(a^2+a^2\sin(e+fx)\right)}+\frac{c^2\int 1\,dx}{a^2}\\ &= \frac{c^2x}{a^2}-\frac{2ac^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3}+\frac{2c^2\cos(e+fx)}{f\left(a^2+a^2\sin(e+fx)\right)}\end{aligned}$$

Mathematica [A] time = 0.62, size = 119, normalized size = 1.70

$$\frac{c^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(3(3e+3fx-8)\cos\left(\frac{1}{2}(e+fx)\right)+(-3e-3fx+16)\cos\left(\frac{3}{2}(e+fx)\right)+6\,\mathrm{s}\right.}{6a^2f(\sin(e+fx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3*(-8 + 3*e + 3*f*x)*Cos[(e + f
*x)/2] + (16 - 3*e - 3*f*x)*Cos[(3*(e + f*x))/2] + 6*(2*(-2 + e + f*x) + (e
 + f*x)*Cos[e + f*x])*Sin[(e + f*x)/2]))/(6*a^2*f*(1 + Sin[e + f*x])^2)
```

fricas [B] time = 0.52, size = 158, normalized size = 2.26

$$-\frac{6\,c^2fx-\left(3\,c^2fx-8\,c^2\right)\cos\left(fx+e\right)^2-4\,c^2+\left(3\,c^2fx+4\,c^2\right)\cos\left(fx+e\right)+\left(6\,c^2fx+4\,c^2+\left(3\,c^2fx+8\,c^2\right)\mathrm{co}\right.}{3\left(a^2f\cos\left(fx+e\right)^2-a^2f\cos\left(fx+e\right)-2\,a^2f-\left(a^2f\cos\left(fx+e\right)+2\,a^2f\right)\sin\left(fx+e\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(6*c^2*f*x - (3*c^2*f*x - 8*c^2)*cos(f*x + e)^2 - 4*c^2 + (3*c^2*f*x +
 4*c^2)*cos(f*x + e) + (6*c^2*f*x + 4*c^2 + (3*c^2*f*x + 8*c^2)*cos(f*x + e
))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^
2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

giac [A] time = 0.17, size = 58, normalized size = 0.83

$$\frac{\frac{3\left(fx+e\right)c^2}{a^2}+\frac{8\left(3\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+c^2\right)}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(f*x + e)*c^2/a^2 + 8*(3*c^2*tan(1/2*f*x + 1/2*e) + c^2)/(a^2*(tan(1
/2*f*x + 1/2*e) + 1)^3))/f
```

maple [A] time = 0.26, size = 71, normalized size = 1.01

$$\frac{2c^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a^2f}-\frac{16c^2}{3a^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{8c^2}{a^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 2*c^2/a^2/f*arctan(tan(1/2*f*x+1/2*e))-16/3*c^2/a^2/f/(tan(1/2*f*x+1/2*e)+1
)^3+8*c^2/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2
```

maxima [B] time = 0.75, size = 361, normalized size = 5.16

$$\frac{2\left(c^2\left(\frac{\frac{9\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\,\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+4}{a^2+\frac{3\,a^2\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\,a^2\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{a^2\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}}+\frac{3\,\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{a^2}\right)-\frac{c^2\left(\frac{3\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\,\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+2\right)}{a^2+\frac{3\,a^2\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\,a^2\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{a^2\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}}+\cdots}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(c^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3
*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - c^2*(3*sin(f*x + e)/(cos(f*
x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^
```

```
2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 2*c^2*(3*sin(f*x + e)/(cos(f*x + e
) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

mupad [B] time = 7.07, size = 89, normalized size = 1.27

$$\frac{c^2\,x}{a^2}-\frac{c^2\,\left(e+f\,x\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(3\,c^2\,\left(e+f\,x\right)-\frac{c^2\,\left(9\,e+9\,f\,x+24\right)}{3}\right)-\frac{c^2\,\left(3\,e+3\,f\,x+8\right)}{3}}{a^2\,f\,{\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^2/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (c^2*x)/a^2 - (c^2*(e + f*x) + tan(e/2 + (f*x)/2)*(3*c^2*(e + f*x) - (c^2*(
9*e + 9*f*x + 24))/3) - (c^2*(3*e + 3*f*x + 8))/3)/(a^2*f*(tan(e/2 + (f*x)/
2) + 1)^3)
```

sympy [A] time = 7.69, size = 473, normalized size = 6.76

$$\begin{cases} \frac{3c^2fx\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f}+\frac{9c^2fx\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f}+\frac{}{3a^2f\tan^3\left(\frac{e}{2}\right.} \\ \frac{x(-c\sin(e)+c)^2}{(a\sin(e)+a)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**2/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*c**2*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9
*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*c**
2*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2
+ f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*c**2*f*x*tan(e/2 +
f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**
2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 3*c**2*f*x/(3*a**2*f*tan(e/2 + f*x/2)**3
 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 2
4*c**2*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 +
f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 8*c**2/(3*a**2*f*tan(e/
2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) +
3*a**2*f), Ne(f, 0)), (x*(-c*sin(e) + c)**2/(a*sin(e) + a)**2, True))
```

3.273 $\int \frac{c-c\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=29

$$-\frac{ac\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^3}$$

```
[Out] -1/3*a*c*cos(f*x+e)^3/f/(a+a*sin(f*x+e))^3
```

Rubi [A] time = 0.07, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2736, 2671}

$$-\frac{ac\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -(a*c*Cos[e + f*x]^3)/(3*f*(a + a*Sin[e + f*x])^3)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{c-c\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx &= (ac)\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^3}\,dx\\ &= -\frac{ac\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3}\end{aligned}$$

Mathematica [B] time = 0.27, size = 70, normalized size = 2.41

$$\frac{c\left(\cos\left(e+\frac{3fx}{2}\right)-3\cos\left(e+\frac{fx}{2}\right)\right)}{3a^2f\left(\sin\left(\frac{e}{2}\right)+\cos\left(\frac{e}{2}\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])/(a + a*Sin[e + f*x])^2,x]
[Out] (c*(-3*Cos[e + (f*x)/2] + Cos[e + (3*f*x)/2]))/(3*a^2*f*(Cos[e/2] + Sin[e/2
])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

fricas [B] time = 0.52, size = 104, normalized size = 3.59

$$-\frac{c\cos\left(fx+e\right)^2-c\cos\left(fx+e\right)+\left(c\cos\left(fx+e\right)+2c\right)\sin\left(fx+e\right)-2c}{3\left(a^2f\cos\left(fx+e\right)^2-a^2f\cos\left(fx+e\right)-2a^2f-\left(a^2f\cos\left(fx+e\right)+2a^2f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
[Out] -1/3*(c*cos(f*x + e)^2 - c*cos(f*x + e) + (c*cos(f*x + e) + 2*c)*sin(f*x +
e) - 2*c)/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos
(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

giac [A] time = 0.18, size = 39, normalized size = 1.34

$$-\frac{2\left(3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right)}{3a^2f\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="giac")
[Out] -2/3*(3*c*tan(1/2*f*x + 1/2*e)^2 + c)/(a^2*f*(tan(1/2*f*x + 1/2*e) + 1)^3)
```

maple [B] time = 0.22, size = 56, normalized size = 1.93

$$\frac{2c\left(\frac{2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{4}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}\right)}{f\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 2/f*c/a^2*(2/(tan(1/2*f*x+1/2*e)+1)^2-1/(tan(1/2*f*x+1/2*e)+1)-4/3/(tan(1/2
*f*x+1/2*e)+1)^3)
```

maxima [B] time = 0.70, size = 215, normalized size = 7.41

$$-\frac{2\left(\frac{c\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2\right)}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}-\frac{c\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(c*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - c*(
3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x +
 e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3))/f
```

mupad [B] time = 7.03, size = 54, normalized size = 1.86

$$\frac{2\,c\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-3\right)}{3\,a^2\,f\,\left(\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (2*c*cos(e/2 + (f*x)/2)*(2*cos(e/2 + (f*x)/2)^2 - 3))/(3*a^2*f*(cos(e/2 + (
f*x)/2) + sin(e/2 + (f*x)/2))^3)
```

sympy [A] time = 3.86, size = 158, normalized size = 5.45

$$\begin{cases} -\frac{6c\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f}-\frac{2c}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} & \text{for } f \neq \\ \frac{x(-c\sin(e)+c)}{(a\sin(e)+a)^2} & \text{otherw} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*c*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*
f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*c/(3*a**2
*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 +
f*x/2) + 3*a**2*f), Ne(f, 0)), (x*(-c*sin(e) + c)/(a*sin(e) + a)**2, True))
```

3.274 $\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx$

Optimal. Leaf size=52

$$\frac{2\tan(e+fx)}{3a^2cf}-\frac{\sec(e+fx)}{3cf\left(a^2\sin(e+fx)+a^2\right)}$$

```
[Out] -1/3*sec(f*x+e)/c/f/(a^2+a^2*sin(f*x+e))+2/3*tan(f*x+e)/a^2/c/f
```

Rubi [A] time = 0.11, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2672, 3767, 8}

$$\frac{2\tan(e+fx)}{3a^2cf}-\frac{\sec(e+fx)}{3cf\left(a^2\sin(e+fx)+a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])),x]
```

```
[Out] -Sec[e + f*x]/(3*c*f*(a^2 + a^2*Sin[e + f*x])) + (2*Tan[e + f*x])/(3*a^2*c*
f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx = \frac{\int \frac{\sec^2(e+fx)}{a+a\sin(e+fx)}\,dx}{ac}$$
$$= -\frac{\sec(e+fx)}{3cf\left(a^2+a^2\sin(e+fx)\right)} + \frac{2\int \sec^2(e+fx)\,dx}{3a^2c}$$
$$= -\frac{\sec(e+fx)}{3cf\left(a^2+a^2\sin(e+fx)\right)} - \frac{2\,\text{Subst}(\int 1\,dx, x, -\tan(e+fx))}{3a^2cf}$$
$$= -\frac{\sec(e+fx)}{3cf\left(a^2+a^2\sin(e+fx)\right)} + \frac{2\tan(e+fx)}{3a^2cf}$$

Mathematica [A] time = 0.48, size = 87, normalized size = 1.67

$$-\frac{\sin(e+fx)+8\sin(2(e+fx))+\sin(3(e+fx))-4\cos(e+fx)+2\cos(2(e+fx))-4\cos(3(e+fx))+2}{24a^2cf(\sin(e+fx)-1)(\sin(e+fx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])),x]
```

```
[Out] -1/24*(2 - 4*Cos[e + f*x] + 2*Cos[2*(e + f*x)] - 4*Cos[3*(e + f*x)] + Sin[e
 + f*x] + 8*Sin[2*(e + f*x)] + Sin[3*(e + f*x)])/(a^2*c*f*(-1 + Sin[e + f*x
])*(1 + Sin[e + f*x])^2)
```

fricas [A] time = 0.43, size = 55, normalized size = 1.06

$$-\frac{2\cos\left(fx+e\right)^2-2\sin\left(fx+e\right)-1}{3\left(a^2cf\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2cf\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/3*(2*cos(f*x + e)^2 - 2*sin(f*x + e) - 1)/(a^2*c*f*cos(f*x + e)*sin(f*x
+ e) + a^2*c*f*cos(f*x + e))
```

giac [A] time = 0.21, size = 77, normalized size = 1.48

$$-\frac{\frac{3}{a^2c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}+\frac{9\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+12\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+7}{a^2c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/6*(3/(a^2*c*(tan(1/2*f*x + 1/2*e) - 1)) + (9*tan(1/2*f*x + 1/2*e)^2 + 12
*tan(1/2*f*x + 1/2*e) + 7)/(a^2*c*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

maple [A] time = 0.26, size = 73, normalized size = 1.40

$$\frac{-\frac{1}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{2}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{1}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{3}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}}{a^2cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x)
```

```
[Out] 2/f/a^2/c*(-1/4/(tan(1/2*f*x+1/2*e)-1)-1/3/(tan(1/2*f*x+1/2*e)+1)^3+1/2/(ta
n(1/2*f*x+1/2*e)+1)^2-3/4/(tan(1/2*f*x+1/2*e)+1))
```

maxima [B] time = 0.72, size = 142, normalized size = 2.73

$$\frac{2\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\,\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{3\,\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}-1\right)}{3\left(a^2c+\frac{2\,a^2c\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{2\,a^2c\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}-\frac{a^2c\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2/3*(sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/((a^2*c + 2*a^2*c*sin(f*x +
e)/(cos(f*x + e) + 1) - 2*a^2*c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - a^2*c
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*f)
```

mupad [B] time = 6.99, size = 74, normalized size = 1.42

$$-\frac{2\left(3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-1\right)}{3\,a^2\,c\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-1\right)\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))),x)
```

```
[Out] -(2*(tan(e/2 + (f*x)/2) + 3*tan(e/2 + (f*x)/2)^2 + 3*tan(e/2 + (f*x)/2)^3 -
 1))/(3*a^2*c*f*(tan(e/2 + (f*x)/2) - 1)*(tan(e/2 + (f*x)/2) + 1)^3)
```

sympy [A] time = 3.92, size = 328, normalized size = 6.31

$$\begin{cases} -\frac{6\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2cf\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+6a^2cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-6a^2cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^2cf} - \frac{6\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2cf\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+6a^2cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-6a^2cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^2cf} - \frac{}{3a^2} \\ \frac{x}{(a\sin(e)+a)^2(-c\sin(e)+c)} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-6*tan(e/2 + f*x/2)**3/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*
c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 6*tan
(e/2 + f*x/2)**2/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*tan(e/2 + f*x
/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 2*tan(e/2 + f*x/2)/(3*
a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*
tan(e/2 + f*x/2) - 3*a**2*c*f) + 2/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2
*c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f), Ne(f,
 0)), (x/((a*sin(e) + a)**2*(-c*sin(e) + c)), True))
```

3.275 $\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=38

$$\frac{\tan^3(e+fx)}{3a^2c^2f}+\frac{\tan(e+fx)}{a^2c^2f}$$

```
[Out] tan(f*x+e)/a^2/c^2/f+1/3*tan(f*x+e)^3/a^2/c^2/f
```

Rubi [A] time = 0.06, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2736, 3767}

$$\frac{\tan^3(e+fx)}{3a^2c^2f}+\frac{\tan(e+fx)}{a^2c^2f}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^2),x]
```

```
[Out] Tan[e + f*x]/(a^2*c^2*f) + Tan[e + f*x]^3/(3*a^2*c^2*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx = \frac{\int \sec^4(e+fx)\,dx}{a^2c^2}$$
$$= -\frac{\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -\tan(e+fx)\right)}{a^2c^2f}$$
$$= \frac{\tan(e+fx)}{a^2c^2f} + \frac{\tan^3(e+fx)}{3a^2c^2f}$$

Mathematica [A] time = 0.05, size = 29, normalized size = 0.76

$$\frac{\frac{1}{3}\tan^3(e+fx)+\tan(e+fx)}{a^2c^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^2),x]
[Out] (Tan[e + f*x] + Tan[e + f*x]^3/3)/(a^2*c^2*f)
```

fricas [A] time = 0.42, size = 37, normalized size = 0.97

$$\frac{\left(2\,\cos\left(fx+e\right)^2+1\right)\sin\left(fx+e\right)}{3\,a^2c^2f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorithm="fricas")
[Out] 1/3*(2*cos(f*x + e)^2 + 1)*sin(f*x + e)/(a^2*c^2*f*cos(f*x + e)^3)
```

giac [A] time = 0.44, size = 30, normalized size = 0.79

$$\frac{\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)}{3\,a^2c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorithm="giac")
[Out] 1/3*(tan(f*x + e)^3 + 3*tan(f*x + e))/(a^2*c^2*f)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + a\sin\left(fx + e\right)\right)^2 \left(c - c\sin\left(fx + e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x)
```

```
[Out] int(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x)
```

maxima [A] time = 0.67, size = 28, normalized size = 0.74

$$\frac{\tan\left(fx + e\right)^3 + 3\,\tan\left(fx + e\right)}{3\,a^2c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(tan(f*x + e)^3 + 3*tan(f*x + e))/(a^2*c^2*f)
```

mupad [B] time = 6.84, size = 63, normalized size = 1.66

$$-\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+3\right)}{3\,a^2\,c^2\,f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^2),x)
```

```
[Out] -(2*tan(e/2 + (f*x)/2)*(3*tan(e/2 + (f*x)/2)^4 - 2*tan(e/2 + (f*x)/2)^2 + 3
))/(3*a^2*c^2*f*(tan(e/2 + (f*x)/2)^2 - 1)^3)
```

sympy [A] time = 5.07, size = 286, normalized size = 7.53

$$\begin{cases} -\frac{6\tan^{5}\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2c^2f\tan^{6}\left(\frac{e}{2}+\frac{fx}{2}\right)-9a^2c^2f\tan^{4}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2c^2f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^2c^2f}+\frac{4\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2c^2f\tan^{6}\left(\frac{e}{2}+\frac{fx}{2}\right)-9a^2c^2f\tan^{4}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2c^2f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)-3 \cdots} \\ \frac{x}{(a\sin(e)+a)^2(-c\sin(e)+c)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*tan(e/2 + f*x/2)**5/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*a*
*2*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**2*
c**2*f) + 4*tan(e/2 + f*x/2)**3/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*a**2
*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**2*c*
*2*f) - 6*tan(e/2 + f*x/2)/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*a**2*c**2
*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**2*c**2*f)
, Ne(f, 0)), (x/((a*sin(e) + a)**2*(-c*sin(e) + c)**2), True))
```

3.276 $\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=76

$$\frac{4\tan^3(e+fx)}{15a^2c^3f}+\frac{4\tan(e+fx)}{5a^2c^3f}+\frac{\sec^3(e+fx)}{5a^2f\left(c^3-c^3\sin(e+fx)\right)}$$

```
[Out] 1/5*sec(f*x+e)^3/a^2/f/(c^3-c^3*sin(f*x+e))+4/5*tan(f*x+e)/a^2/c^3/f+4/15*t
an(f*x+e)^3/a^2/c^3/f
```

Rubi [A] time = 0.11, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2672, 3767}

$$\frac{4\tan^3(e+fx)}{15a^2c^3f}+\frac{4\tan(e+fx)}{5a^2c^3f}+\frac{\sec^3(e+fx)}{5a^2f\left(c^3-c^3\sin(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^3),x]
```

```
[Out] Sec[e + f*x]^3/(5*a^2*f*(c^3 - c^3*Sin[e + f*x])) + (4*Tan[e + f*x])/(5*a^2
*c^3*f) + (4*Tan[e + f*x]^3)/(15*a^2*c^3*f)
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx = \frac{\int \frac{\sec^4(e+fx)}{c-c\sin(e+fx)}\,dx}{a^2c^2}$$
$$= \frac{\sec^3(e+fx)}{5a^2f\left(c^3-c^3\sin(e+fx)\right)}+\frac{4\int \sec^4(e+fx)\,dx}{5a^2c^3}$$
$$= \frac{\sec^3(e+fx)}{5a^2f\left(c^3-c^3\sin(e+fx)\right)}-\frac{4\,\mathrm{Subst}\left(\int \left(1+x^2\right)dx,x,-\tan(e+$$
$$= \frac{\sec^3(e+fx)}{5a^2f\left(c^3-c^3\sin(e+fx)\right)}+\frac{4\tan(e+fx)}{5a^2c^3f}+\frac{4\tan^3(e+fx)}{15a^2c^3f}$$

Mathematica [A] time = 0.89, size = 131, normalized size = 1.72

$$-\frac{18\sin(e+fx)+512\sin(2(e+fx))+27\sin(3(e+fx))+128\sin(4(e+fx))+9\sin(5(e+fx))+128\cos(e+f}{1920a^2c^3f(\sin(e+fx)-1)^3(\sin(e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^3),x]
```

```
[Out] -1/1920*(-54 + 128*Cos[e + f*x] - 72*Cos[2*(e + f*x)] + 192*Cos[3*(e + f*x)
] - 18*Cos[4*(e + f*x)] + 64*Cos[5*(e + f*x)] + 18*Sin[e + f*x] + 512*Sin[2
*(e + f*x)] + 27*Sin[3*(e + f*x)] + 128*Sin[4*(e + f*x)] + 9*Sin[5*(e + f*x
)])/(a^2*c^3*f*(-1 + Sin[e + f*x])^3*(1 + Sin[e + f*x])^2)
```

fricas [A] time = 0.44, size = 86, normalized size = 1.13

$$-\frac{8\cos\left(fx+e\right)^4-4\cos\left(fx+e\right)^2+4\left(2\cos\left(fx+e\right)^2+1\right)\sin\left(fx+e\right)-1}{15\left(a^2c^3f\cos\left(fx+e\right)^3\sin\left(fx+e\right)-a^2c^3f\cos\left(fx+e\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(8*cos(f*x + e)^4 - 4*cos(f*x + e)^2 + 4*(2*cos(f*x + e)^2 + 1)*sin(f
*x + e) - 1)/(a^2*c^3*f*cos(f*x + e)^3*sin(f*x + e) - a^2*c^3*f*cos(f*x + e
)^3)
```

giac [A] time = 0.23, size = 133, normalized size = 1.75

$$-\frac{\frac{5\left(15\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+24\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+13\right)}{a^2c^3\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}+\frac{165\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-480\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+650\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-400\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+113}{a^2c^3\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^5}}{120f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/120*(5*(15*tan(1/2*f*x + 1/2*e)^2 + 24*tan(1/2*f*x + 1/2*e) + 13)/(a^2*c
^3*(tan(1/2*f*x + 1/2*e) + 1)^3) + (165*tan(1/2*f*x + 1/2*e)^4 - 480*tan(1/
2*f*x + 1/2*e)^3 + 650*tan(1/2*f*x + 1/2*e)^2 - 400*tan(1/2*f*x + 1/2*e) +
113)/(a^2*c^3*(tan(1/2*f*x + 1/2*e) - 1)^5))/f
```

maple [A] time = 0.26, size = 133, normalized size = 1.75

$$\frac{-\frac{2}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{1}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{5}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{3}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{11}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{1}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{1}{4\left(\tan\left(\frac{fx}{2}+\right.\right.}}{f\,c^3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x)
```

```
[Out] 2/f/c^3/a^2*(-1/5/(tan(1/2*f*x+1/2*e)-1)^5-1/2/(tan(1/2*f*x+1/2*e)-1)^4-5/6
/(tan(1/2*f*x+1/2*e)-1)^3-3/4/(tan(1/2*f*x+1/2*e)-1)^2-11/16/(tan(1/2*f*x+1
/2*e)-1)-1/12/(tan(1/2*f*x+1/2*e)+1)^3+1/8/(tan(1/2*f*x+1/2*e)+1)^2-5/16/(t
an(1/2*f*x+1/2*e)+1))
```

maxima [B] time = 0.97, size = 335, normalized size = 4.41

$$\frac{2\left(\frac{9\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{21\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{13\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{25\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}-\frac{5\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}-\frac{15\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}+\frac{15\sin\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)}\right.}{15\left(a^2c^3-\frac{2a^2c^3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{2a^2c^3\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{6a^2c^3\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}-\frac{6a^2c^3\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}+\frac{2a^2c^3\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}+\frac{2a^2c^3\sin\left(fx+e\right)^7}{\left(\cos\left(fx+e\right)+1\right)^7}-\frac{a^2c}{(\mathrm{co}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 2/15*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 21*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 13*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 25*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 - 5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 15*sin(f*x + e)^6/(c
```

```
os(f*x + e) + 1)^6 + 15*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 3)/((a^2*c^3
- 2*a^2*c^3*sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^2*c^3*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 6*a^2*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6*a^2*c^
3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2*a^2*c^3*sin(f*x + e)^6/(cos(f*x +
 e) + 1)^6 + 2*a^2*c^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a^2*c^3*sin(f*
x + e)^8/(cos(f*x + e) + 1)^8)*f)
```

mupad [B] time = 7.83, size = 128, normalized size = 1.68

$$-\frac{2\left(15\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7-15\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6-5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5+25\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+13\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3-21\tan\left(\frac{e}{2}+\right.\right.}{15\,a^2\,c^3\,f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)^5\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^3),x)
```

```
[Out] -(2*(9*tan(e/2 + (f*x)/2) - 21*tan(e/2 + (f*x)/2)^2 + 13*tan(e/2 + (f*x)/2)
^3 + 25*tan(e/2 + (f*x)/2)^4 - 5*tan(e/2 + (f*x)/2)^5 - 15*tan(e/2 + (f*x)/
2)^6 + 15*tan(e/2 + (f*x)/2)^7 + 3))/(15*a^2*c^3*f*(tan(e/2 + (f*x)/2) - 1)
^5*(tan(e/2 + (f*x)/2) + 1)^3)
```

sympy [A] time = 15.88, size = 1418, normalized size = 18.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*tan(e/2 + f*x/2)**7/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30
*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*
a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a
**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*
c**3*f) + 30*tan(e/2 + f*x/2)**6/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a
**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a*
*2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**
2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c*
*3*f) + 10*tan(e/2 + f*x/2)**5/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**
2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2
*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*
c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3
*f) - 50*tan(e/2 + f*x/2)**4/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*
c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c
**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c*
```

```
*3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f
) - 26*tan(e/2 + f*x/2)**3/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c*
*3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**
3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3
*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f)
+ 42*tan(e/2 + f*x/2)**2/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3
*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*
f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f
*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) -
18*tan(e/2 + f*x/2)/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*ta
n(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan
(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(
e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) - 6/(15
*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*
a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a
**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a*
*2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f), Ne(f, 0)), (x/((a*sin(e) + a)
**2*(-c*sin(e) + c)**3), True))
```

3.277 $\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx$

Optimal. Leaf size=111

$$\frac{4\tan^3(e+fx)}{21a^2c^4f}+\frac{4\tan(e+fx)}{7a^2c^4f}+\frac{\sec^3(e+fx)}{7a^2f\left(c^4-c^4\sin(e+fx)\right)}+\frac{\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2}$$

```
[Out] 1/7*sec(f*x+e)^3/a^2/f/(c^2-c^2*sin(f*x+e))^2+1/7*sec(f*x+e)^3/a^2/f/(c^4-c
^4*sin(f*x+e))+4/7*tan(f*x+e)/a^2/c^4/f+4/21*tan(f*x+e)^3/a^2/c^4/f
```

Rubi [A] time = 0.16, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2672, 3767}

$$\frac{4\tan^3(e+fx)}{21a^2c^4f}+\frac{4\tan(e+fx)}{7a^2c^4f}+\frac{\sec^3(e+fx)}{7a^2f\left(c^4-c^4\sin(e+fx)\right)}+\frac{\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^4),x]
```

```
[Out] Sec[e + f*x]^3/(7*a^2*f*(c^2 - c^2*Sin[e + f*x])^2) + Sec[e + f*x]^3/(7*a^2
*f*(c^4 - c^4*Sin[e + f*x])) + (4*Tan[e + f*x])/(7*a^2*c^4*f) + (4*Tan[e +
f*x]^3)/(21*a^2*c^4*f)
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx = \frac{\int \frac{\sec^4(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{a^2c^2}$$

$$= \frac{\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{5\int \frac{\sec^4(e+fx)}{c-c\sin(e+fx)}\,dx}{7a^2c^3}$$

$$= \frac{\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{\sec^3(e+fx)}{7a^2f\left(c^4-c^4\sin(e+fx)\right)} + \frac{4\int s$$

$$= \frac{\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{\sec^3(e+fx)}{7a^2f\left(c^4-c^4\sin(e+fx)\right)} - \frac{4\,\mathrm{Su}$$

$$= \frac{\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{\sec^3(e+fx)}{7a^2f\left(c^4-c^4\sin(e+fx)\right)} + \frac{4\tan}{7}$$

Mathematica [A] time = 0.95, size = 151, normalized size = 1.36

$$\frac{120\sin(e+fx)+1088\sin(2(e+fx))+180\sin(3(e+fx))+128\sin(4(e+fx))+60\sin(5(e+fx))-64\sin(6(}{5376a^2c^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^4),x]
```

```
[Out] (-210 + 512*Cos[e + f*x] - 255*Cos[2*(e + f*x)] + 768*Cos[3*(e + f*x)] - 30
*Cos[4*(e + f*x)] + 256*Cos[5*(e + f*x)] + 15*Cos[6*(e + f*x)] + 120*Sin[e
+ f*x] + 1088*Sin[2*(e + f*x)] + 180*Sin[3*(e + f*x)] + 128*Sin[4*(e + f*x)
] + 60*Sin[5*(e + f*x)] - 64*Sin[6*(e + f*x)])/(5376*a^2*c^4*f*(-1 + Sin[e
+ f*x])^4*(1 + Sin[e + f*x])^2)
```

fricas [A] time = 0.43, size = 113, normalized size = 1.02

$$-\frac{16\cos\left(fx+e\right)^4-8\cos\left(fx+e\right)^2-\left(8\cos\left(fx+e\right)^4-12\cos\left(fx+e\right)^2-5\right)\sin\left(fx+e\right)-2}{21\left(a^2c^4f\cos\left(fx+e\right)^5+2\,a^2c^4f\cos\left(fx+e\right)^3\sin\left(fx+e\right)-2\,a^2c^4f\cos\left(fx+e\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] -1/21*(16*cos(f*x + e)^4 - 8*cos(f*x + e)^2 - (8*cos(f*x + e)^4 - 12*cos(f*
x + e)^2 - 5)*sin(f*x + e) - 2)/(a^2*c^4*f*cos(f*x + e)^5 + 2*a^2*c^4*f*cos
(f*x + e)^3*sin(f*x + e) - 2*a^2*c^4*f*cos(f*x + e)^3)
```

giac [A] time = 0.32, size = 161, normalized size = 1.45

$$-\frac{\frac{7\left(9\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+15\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+8\right)}{a^{2}c^{4}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{3}}+\frac{273\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6}-1155\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}+2450\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2870\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+2037\tan}{a^{2}c^{4}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{7}}}{168f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -1/168*(7*(9*tan(1/2*f*x + 1/2*e)^2 + 15*tan(1/2*f*x + 1/2*e) + 8)/(a^2*c^4
*(tan(1/2*f*x + 1/2*e) + 1)^3) + (273*tan(1/2*f*x + 1/2*e)^6 - 1155*tan(1/2
*f*x + 1/2*e)^5 + 2450*tan(1/2*f*x + 1/2*e)^4 - 2870*tan(1/2*f*x + 1/2*e)^3
 + 2037*tan(1/2*f*x + 1/2*e)^2 - 791*tan(1/2*f*x + 1/2*e) + 152)/(a^2*c^4*(
tan(1/2*f*x + 1/2*e) - 1)^7))/f
```

maple [A] time = 0.27, size = 163, normalized size = 1.47

$$\frac{-\frac{4}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{5}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{55}{12\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{23}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{13}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}}{f\,c^{4}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x)
```

```
[Out] 2/f/c^4/a^2*(-2/7/(tan(1/2*f*x+1/2*e)-1)^7-1/(tan(1/2*f*x+1/2*e)-1)^6-2/(ta
n(1/2*f*x+1/2*e)-1)^5-5/2/(tan(1/2*f*x+1/2*e)-1)^4-55/24/(tan(1/2*f*x+1/2*e
)-1)^3-23/16/(tan(1/2*f*x+1/2*e)-1)^2-13/16/(tan(1/2*f*x+1/2*e)-1)-1/24/(ta
n(1/2*f*x+1/2*e)+1)^3+1/16/(tan(1/2*f*x+1/2*e)+1)^2-3/16/(tan(1/2*f*x+1/2*e
)+1))
```

maxima [B] time = 0.94, size = 427, normalized size = 3.85

$$-\frac{2\left(\frac{3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{24\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{76\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{28\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{42\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}-\frac{56\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}-\frac{28\sin\left(fx-\right.}{\left(\cos\left(fx+e\right)\right.}\right.}{21\left(a^{2}c^{4}-\frac{4a^{2}c^{4}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3a^{2}c^{4}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{8a^{2}c^{4}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}-\frac{14a^{2}c^{4}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{14a^{2}c^{4}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}-\frac{8a^{2}c^{4}\sin\left(fx+e\right)^{7}}{\left(\cos\left(fx+e\right)+1\right)^{7}}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] -2/21*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 24*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 - 76*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 28*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 + 42*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 56*sin(f*x + e)^6/
(cos(f*x + e) + 1)^6 - 28*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 42*sin(f*x
+ e)^8/(cos(f*x + e) + 1)^8 - 21*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 6)/(
(a^2*c^4 - 4*a^2*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*c^4*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 8*a^2*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 -
 14*a^2*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 14*a^2*c^4*sin(f*x + e)^6
/(cos(f*x + e) + 1)^6 - 8*a^2*c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3*a
^2*c^4*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 4*a^2*c^4*sin(f*x + e)^9/(cos(
f*x + e) + 1)^9 - a^2*c^4*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)*f)
```

mupad [B] time = 6.98, size = 119, normalized size = 1.07

$$-\frac{\frac{\sin\left(e+f\,x\right)}{3}+\frac{4\,\cos\left(2\,e+2\,f\,x\right)}{21}+\frac{2\,\cos\left(4\,e+4\,f\,x\right)}{21}+\frac{\sin\left(3\,e+3\,f\,x\right)}{14}-\frac{\sin\left(5\,e+5\,f\,x\right)}{42}}{a^2\,c^4\,f\,\left(\frac{\cos\left(5\,e+5\,f\,x\right)}{16}-\frac{3\,\cos\left(3\,e+3\,f\,x\right)}{16}-\frac{7\,\cos\left(e+f\,x\right)}{8}+\frac{\sin\left(2\,e+2\,f\,x\right)}{2}+\frac{\sin\left(4\,e+4\,f\,x\right)}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^4),x)
```

```
[Out] -(sin(e + f*x)/3 + (4*cos(2*e + 2*f*x))/21 + (2*cos(4*e + 4*f*x))/21 + sin(
3*e + 3*f*x)/14 - sin(5*e + 5*f*x)/42)/(a^2*c^4*f*(cos(5*e + 5*f*x)/16 - (3
*cos(3*e + 3*f*x))/16 - (7*cos(e + f*x))/8 + sin(2*e + 2*f*x)/2 + sin(4*e +
 4*f*x)/4))
```

sympy [A] time = 30.15, size = 2213, normalized size = 19.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-42*tan(e/2 + f*x/2)**9/(21*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 8
4*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 63*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 16
8*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 294*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 2
94*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 168*a**2*c**4*f*tan(e/2 + f*x/2)**3 -
63*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 84*a**2*c**4*f*tan(e/2 + f*x/2) - 21*a
**2*c**4*f) + 84*tan(e/2 + f*x/2)**8/(21*a**2*c**4*f*tan(e/2 + f*x/2)**10 -
 84*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 63*a**2*c**4*f*tan(e/2 + f*x/2)**8 +
```

```
168*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 294*a**2*c**4*f*tan(e/2 + f*x/2)**6 +
 294*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 168*a**2*c**4*f*tan(e/2 + f*x/2)**3
- 63*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 84*a**2*c**4*f*tan(e/2 + f*x/2) - 21
*a**2*c**4*f) - 56*tan(e/2 + f*x/2)**7/(21*a**2*c**4*f*tan(e/2 + f*x/2)**10
 - 84*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 63*a**2*c**4*f*tan(e/2 + f*x/2)**8
+ 168*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 294*a**2*c**4*f*tan(e/2 + f*x/2)**6
 + 294*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 168*a**2*c**4*f*tan(e/2 + f*x/2)**
3 - 63*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 84*a**2*c**4*f*tan(e/2 + f*x/2) -
21*a**2*c**4*f) - 112*tan(e/2 + f*x/2)**6/(21*a**2*c**4*f*tan(e/2 + f*x/2)*
*10 - 84*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 63*a**2*c**4*f*tan(e/2 + f*x/2)*
*8 + 168*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 294*a**2*c**4*f*tan(e/2 + f*x/2)
**6 + 294*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 168*a**2*c**4*f*tan(e/2 + f*x/2
)**3 - 63*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 84*a**2*c**4*f*tan(e/2 + f*x/2)
 - 21*a**2*c**4*f) + 84*tan(e/2 + f*x/2)**5/(21*a**2*c**4*f*tan(e/2 + f*x/2
)**10 - 84*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 63*a**2*c**4*f*tan(e/2 + f*x/2
)**8 + 168*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 294*a**2*c**4*f*tan(e/2 + f*x/
2)**6 + 294*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 168*a**2*c**4*f*tan(e/2 + f*x
/2)**3 - 63*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 84*a**2*c**4*f*tan(e/2 + f*x/
2) - 21*a**2*c**4*f) + 56*tan(e/2 + f*x/2)**4/(21*a**2*c**4*f*tan(e/2 + f*x
/2)**10 - 84*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 63*a**2*c**4*f*tan(e/2 + f*x
/2)**8 + 168*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 294*a**2*c**4*f*tan(e/2 + f*
x/2)**6 + 294*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 168*a**2*c**4*f*tan(e/2 + f
*x/2)**3 - 63*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 84*a**2*c**4*f*tan(e/2 + f*
x/2) - 21*a**2*c**4*f) - 152*tan(e/2 + f*x/2)**3/(21*a**2*c**4*f*tan(e/2 +
f*x/2)**10 - 84*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 63*a**2*c**4*f*tan(e/2 +
f*x/2)**8 + 168*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 294*a**2*c**4*f*tan(e/2 +
 f*x/2)**6 + 294*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 168*a**2*c**4*f*tan(e/2
+ f*x/2)**3 - 63*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 84*a**2*c**4*f*tan(e/2 +
 f*x/2) - 21*a**2*c**4*f) + 48*tan(e/2 + f*x/2)**2/(21*a**2*c**4*f*tan(e/2
+ f*x/2)**10 - 84*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 63*a**2*c**4*f*tan(e/2
+ f*x/2)**8 + 168*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 294*a**2*c**4*f*tan(e/2
 + f*x/2)**6 + 294*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 168*a**2*c**4*f*tan(e/
2 + f*x/2)**3 - 63*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 84*a**2*c**4*f*tan(e/2
 + f*x/2) - 21*a**2*c**4*f) + 6*tan(e/2 + f*x/2)/(21*a**2*c**4*f*tan(e/2 +
f*x/2)**10 - 84*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 63*a**2*c**4*f*tan(e/2 +
f*x/2)**8 + 168*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 294*a**2*c**4*f*tan(e/2 +
 f*x/2)**6 + 294*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 168*a**2*c**4*f*tan(e/2
+ f*x/2)**3 - 63*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 84*a**2*c**4*f*tan(e/2 +
 f*x/2) - 21*a**2*c**4*f) - 12/(21*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 84*a*
*2*c**4*f*tan(e/2 + f*x/2)**9 + 63*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 168*a*
*2*c**4*f*tan(e/2 + f*x/2)**7 - 294*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 294*a
**2*c**4*f*tan(e/2 + f*x/2)**4 - 168*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 63*a
**2*c**4*f*tan(e/2 + f*x/2)**2 + 84*a**2*c**4*f*tan(e/2 + f*x/2) - 21*a**2*
c**4*f), Ne(f, 0)), (x/((a*sin(e) + a)**2*(-c*sin(e) + c)**4), True))
```

3.278 $\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx$

Optimal. Leaf size=144

$$\frac{8\tan^3(e+fx)}{63a^2c^5f}+\frac{8\tan(e+fx)}{21a^2c^5f}+\frac{2\sec^3(e+fx)}{21a^2f\left(c^5-c^5\sin(e+fx)\right)}+\frac{2\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2}+\frac{\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3}$$

```
[Out] 1/9*sec(f*x+e)^3/a^2/c^2/f/(c-c*sin(f*x+e))^3+2/21*sec(f*x+e)^3/a^2/c^3/f/(
c-c*sin(f*x+e))^2+2/21*sec(f*x+e)^3/a^2/f/(c^5-c^5*sin(f*x+e))+8/21*tan(f*x
+e)/a^2/c^5/f+8/63*tan(f*x+e)^3/a^2/c^5/f
```

Rubi [**A**] time = 0.22, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2672, 3767}

$$\frac{8\tan^3(e+fx)}{63a^2c^5f}+\frac{8\tan(e+fx)}{21a^2c^5f}+\frac{2\sec^3(e+fx)}{21a^2f\left(c^5-c^5\sin(e+fx)\right)}+\frac{2\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2}+\frac{\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^5),x]
```

```
[Out] Sec[e + f*x]^3/(9*a^2*c^2*f*(c - c*Sin[e + f*x])^3) + (2*Sec[e + f*x]^3)/(2
1*a^2*c^3*f*(c - c*Sin[e + f*x])^2) + (2*Sec[e + f*x]^3)/(21*a^2*f*(c^5 - c
^5*Sin[e + f*x])) + (8*Tan[e + f*x])/(21*a^2*c^5*f) + (8*Tan[e + f*x]^3)/(6
3*a^2*c^5*f)
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx = \frac{\int \frac{\sec^4(e+fx)}{(c-c\sin(e+fx))^3}\,dx}{a^2c^2}$$

$$= \frac{\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3} + \frac{2\int \frac{\sec^4(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{3a^2c^3}$$

$$= \frac{\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3} + \frac{2\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2} + \frac{10\int}{}$$

$$= \frac{\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3} + \frac{2\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2} + \frac{}{21a^2}$$

$$= \frac{\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3} + \frac{2\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2} + \frac{}{21a^2}$$

$$= \frac{\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3} + \frac{2\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2} + \frac{}{21a^2}$$

Mathematica [A] time = 1.16, size = 193, normalized size = 1.34

$$\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(18432\sin(e+fx)+4185\sin(2(e+fx))$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^5),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-5580*Cos[e + f*x] + 13824*Cos[2*(e + f*x)] - 310*Cos[3*(e + f*x)] + 614
4*Cos[4*(e + f*x)] + 930*Cos[5*(e + f*x)] - 512*Cos[6*(e + f*x)] + 18432*Si
n[e + f*x] + 4185*Sin[2*(e + f*x)] + 1024*Sin[3*(e + f*x)] + 1860*Sin[4*(e
+ f*x)] - 3072*Sin[5*(e + f*x)] - 155*Sin[6*(e + f*x)]))/(64512*f*(a + a*Si
n[e + f*x])^2*(c - c*Sin[e + f*x])^5)
```

fricas [A] time = 0.46, size = 143, normalized size = 0.99

$$\frac{16 \cos\left(fx+e\right)^{6} - 72 \cos\left(fx+e\right)^{4} + 30 \cos\left(fx+e\right)^{2} + 2\left(24 \cos\left(fx+e\right)^{4} - 20 \cos\left(fx+e\right)^{2} - 7\right)\sin\left(fx+\right.}{63\left(3\,a^{2}c^{5}f\cos\left(fx+e\right)^{5} - 4\,a^{2}c^{5}f\cos\left(fx+e\right)^{3} - \left(a^{2}c^{5}f\cos\left(fx+e\right)^{5} - 4\,a^{2}c^{5}f\cos\left(fx+e\right)^{3}\right)\sin\left(fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] 1/63*(16*cos(f*x + e)^6 - 72*cos(f*x + e)^4 + 30*cos(f*x + e)^2 + 2*(24*cos
(f*x + e)^4 - 20*cos(f*x + e)^2 - 7)*sin(f*x + e) + 7)/(3*a^2*c^5*f*cos(f*x
 + e)^5 - 4*a^2*c^5*f*cos(f*x + e)^3 - (a^2*c^5*f*cos(f*x + e)^5 - 4*a^2*c^
5*f*cos(f*x + e)^3)*sin(f*x + e))
```

giac [A] time = 0.24, size = 189, normalized size = 1.31

$$-\frac{\frac{21\left(21 \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+36 \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+19\right)}{a^{2}c^{5}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{3}} + \frac{3591 \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{8}-19656 \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{7}+56196 \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6}-95760 \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^{2}c^{5}\left(\mathrm{ta}\right.}}{2016\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorithm="giac")
```

```
[Out] -1/2016*(21*(21*tan(1/2*f*x + 1/2*e)^2 + 36*tan(1/2*f*x + 1/2*e) + 19)/(a^2
*c^5*(tan(1/2*f*x + 1/2*e) + 1)^3) + (3591*tan(1/2*f*x + 1/2*e)^8 - 19656*t
an(1/2*f*x + 1/2*e)^7 + 56196*tan(1/2*f*x + 1/2*e)^6 - 95760*tan(1/2*f*x +
1/2*e)^5 + 107730*tan(1/2*f*x + 1/2*e)^4 - 79464*tan(1/2*f*x + 1/2*e)^3 + 3
8484*tan(1/2*f*x + 1/2*e)^2 - 10944*tan(1/2*f*x + 1/2*e) + 1615)/(a^2*c^5*(
tan(1/2*f*x + 1/2*e) - 1)^9))/f
```

maple [A] time = 0.28, size = 193, normalized size = 1.34

$$\frac{-\frac{8}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{9}} - \frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{8}} - \frac{68}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}} - \frac{46}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}} - \frac{35}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}} - \frac{59}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}} - \frac{19}{2\left(\tan\left(\frac{fx}{2}\right.\right.}}{f\,a^{2}c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x)
```

```
[Out] 2/f/a^2/c^5*(-4/9/(tan(1/2*f*x+1/2*e)-1)^9-2/(tan(1/2*f*x+1/2*e)-1)^8-34/7/
(tan(1/2*f*x+1/2*e)-1)^7-23/3/(tan(1/2*f*x+1/2*e)-1)^6-35/4/(tan(1/2*f*x+1/
```

```
2*e)-1)^5-59/8/(tan(1/2*f*x+1/2*e)-1)^4-19/4/(tan(1/2*f*x+1/2*e)-1)^3-9/4/(
tan(1/2*f*x+1/2*e)-1)^2-57/64/(tan(1/2*f*x+1/2*e)-1)-1/48/(tan(1/2*f*x+1/2*
e)+1)^3+1/32/(tan(1/2*f*x+1/2*e)+1)^2-7/64/(tan(1/2*f*x+1/2*e)+1))
```

maxima [B] time = 0.81, size = 519, normalized size = 3.60

$$-\frac{2\left(\frac{51\sin(fx+e)}{\cos(fx+e)+1}-\frac{39\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{235\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{450\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{306\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{294\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{378\sin(\cdots}{(\cos(fx\cdots}\right.}{63\left(a^2c^5-\frac{6a^2c^5\sin(fx+e)}{\cos(fx+e)+1}+\frac{12a^2c^5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{2a^2c^5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{27a^2c^5\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{36a^2c^5\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{36a^2c^5\sin(fx+e)^7}{(\cos(fx+e)+1)^7}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] -2/63*(51*sin(f*x + e)/(cos(f*x + e) + 1) - 39*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 - 235*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 450*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 - 306*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 294*sin(f*x +
e)^6/(cos(f*x + e) + 1)^6 + 378*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 63*si
n(f*x + e)^8/(cos(f*x + e) + 1)^8 - 273*sin(f*x + e)^9/(cos(f*x + e) + 1)^9
 + 189*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 63*sin(f*x + e)^11/(cos(f*x
+ e) + 1)^11 - 19)/((a^2*c^5 - 6*a^2*c^5*sin(f*x + e)/(cos(f*x + e) + 1) +
12*a^2*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*a^2*c^5*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3 - 27*a^2*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 36*a
^2*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 36*a^2*c^5*sin(f*x + e)^7/(cos
(f*x + e) + 1)^7 + 27*a^2*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 2*a^2*c
^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 12*a^2*c^5*sin(f*x + e)^10/(cos(f*
x + e) + 1)^10 + 6*a^2*c^5*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - a^2*c^5*
sin(f*x + e)^12/(cos(f*x + e) + 1)^12)*f)
```

mupad [B] time = 9.29, size = 180, normalized size = 1.25

$$-\frac{2\left(63\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{11}-189\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}+273\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{9}+63\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{8}-378\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{7}+294\cdots\right.}{63a^2c^5f\left(\tan\left(\frac{e}{2}+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^5),x)
```

```
[Out] -(2*(39*tan(e/2 + (f*x)/2)^2 - 51*tan(e/2 + (f*x)/2) + 235*tan(e/2 + (f*x)/
2)^3 - 450*tan(e/2 + (f*x)/2)^4 + 306*tan(e/2 + (f*x)/2)^5 + 294*tan(e/2 +
(f*x)/2)^6 - 378*tan(e/2 + (f*x)/2)^7 + 63*tan(e/2 + (f*x)/2)^8 + 273*tan(e
/2 + (f*x)/2)^9 - 189*tan(e/2 + (f*x)/2)^10 + 63*tan(e/2 + (f*x)/2)^11 + 19
))/(63*a^2*c^5*f*(tan(e/2 + (f*x)/2) - 1)^9*(tan(e/2 + (f*x)/2) + 1)^3)
```

sympy [A] time = 56.79, size = 3186, normalized size = 22.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((-126*tan(e/2 + f*x/2)**11/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 -
 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**1
0 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)
**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x
/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f
*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 +
f*x/2) - 63*a**2*c**5*f) + 378*tan(e/2 + f*x/2)**10/(63*a**2*c**5*f*tan(e/2
 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(
e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*t
an(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*
f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**
5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c*
*5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) - 546*tan(e/2 + f*x/2)**9/(63*a**2*
c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a*
*2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701
*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2
268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4
+ 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2
 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) - 126*tan(e/2 + f*x/2
)**8/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2
)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*
x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 +
 f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/
2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e
/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) + 756*t
an(e/2 + f*x/2)**7/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*t
an(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*
f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c*
*5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2
*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**
2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c
**5*f) - 588*tan(e/2 + f*x/2)**6/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378
*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 -
126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8
+ 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)*
*5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2
)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/
```

```
2) - 63*a**2*c**5*f) - 612*tan(e/2 + f*x/2)**5/(63*a**2*c**5*f*tan(e/2 + f*
x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 +
 f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/
2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan
(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*t
an(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*
tan(e/2 + f*x/2) - 63*a**2*c**5*f) + 900*tan(e/2 + f*x/2)**4/(63*a**2*c**5*
f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c*
*5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2
*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a
**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126
*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 37
8*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) - 470*tan(e/2 + f*x/2)**3/
(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11
 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)*
*9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/
2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f
*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 +
f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) - 78*tan(e/2
 + f*x/2)**2/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2
 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(
e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*t
an(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*
f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5
*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f)
 + 102*tan(e/2 + f*x/2)/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**
5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*
c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a*
*2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701
*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 75
6*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a
**2*c**5*f) - 38/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan
(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*
tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5
*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c
**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*
c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**
5*f), Ne(f, 0)), (x/((a*sin(e) + a)**2*(-c*sin(e) + c)**5), True))
```

3.279 $\int \frac{(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=161

$$-\frac{2a^4c^5\cos^9(e+fx)}{5f(a\sin(e+fx)+a)^7}-\frac{63c^5\cos(e+fx)}{2a^3f}-\frac{21c^5\cos^3(e+fx)}{2f\left(a^3\sin(e+fx)+a^3\right)}-\frac{63c^5x}{2a^3}+\frac{6a^2c^5\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^5}-\frac{42c^5 \cdots}{5f(a\sin \cdots}$$

```
[Out] -63/2*c^5*x/a^3-63/2*c^5*cos(f*x+e)/a^3/f-2/5*a^4*c^5*cos(f*x+e)^9/f/(a+a*s
in(f*x+e))^7+6/5*a^2*c^5*cos(f*x+e)^7/f/(a+a*sin(f*x+e))^5-42/5*c^5*cos(f*x
+e)^5/f/(a+a*sin(f*x+e))^3-21/2*c^5*cos(f*x+e)^3/f/(a^3+a^3*sin(f*x+e))
```

Rubi [A] time = 0.28, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2736, 2680, 2679, 2682, 8}

$$-\frac{63c^5\cos(e+fx)}{2a^3f}-\frac{2a^4c^5\cos^9(e+fx)}{5f(a\sin(e+fx)+a)^7}+\frac{6a^2c^5\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^5}-\frac{21c^5\cos^3(e+fx)}{2f\left(a^3\sin(e+fx)+a^3\right)}-\frac{63c^5x}{2a^3}-\frac{42c^5 \cdots}{5f(a\sin \cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^5/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-63*c^5*x)/(2*a^3) - (63*c^5*Cos[e + f*x])/(2*a^3*f) - (2*a^4*c^5*Cos[e +
f*x]^9)/(5*f*(a + a*Sin[e + f*x])^7) + (6*a^2*c^5*Cos[e + f*x]^7)/(5*f*(a +
 a*Sin[e + f*x])^5) - (42*c^5*Cos[e + f*x]^5)/(5*f*(a + a*Sin[e + f*x])^3)
- (21*c^5*Cos[e + f*x]^3)/(2*f*(a^3 + a^3*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
```

```
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx = \left(a^5c^5\right)\int \frac{\cos^{10}(e+fx)}{(a+a\sin(e+fx))^8}\,dx$$

$$= -\frac{2a^4c^5\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} - \frac{1}{5}\left(9a^3c^5\right)\int \frac{\cos^8(e+fx)}{(a+a\sin(e+fx))^6}\,dx$$

$$= -\frac{2a^4c^5\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} + \frac{6a^2c^5\cos^7(e+fx)}{5f(a+a\sin(e+fx))^5} + \frac{1}{5}\left(21ac^5\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))}$$

$$= -\frac{2a^4c^5\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} + \frac{6a^2c^5\cos^7(e+fx)}{5f(a+a\sin(e+fx))^5} - \frac{42c^5\cos^5(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{\left(21c^5\right)}{}$$

$$= -\frac{2a^4c^5\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} + \frac{6a^2c^5\cos^7(e+fx)}{5f(a+a\sin(e+fx))^5} - \frac{42c^5\cos^5(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{21}{2f\left(a^3\right.}$$

$$= -\frac{63c^5\cos(e+fx)}{2a^3f} - \frac{2a^4c^5\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} + \frac{6a^2c^5\cos^7(e+fx)}{5f(a+a\sin(e+fx))^5} - \frac{42c^5\cos^5}{5f(a+a\sin}$$

$$= -\frac{63c^5x}{2a^3} - \frac{63c^5\cos(e+fx)}{2a^3f} - \frac{2a^4c^5\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} + \frac{6a^2c^5\cos^7(e+fx)}{5f(a+a\sin(e+fx))^5} - \frac{4}{5f}$$

Mathematica [A] time = 0.86, size = 303, normalized size = 1.88

$$\frac{(c-c\sin(e+fx))^5\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(256\sin\left(\frac{1}{2}(e+fx)\right)-630(e+fx)\left(\sin\left(\frac{1}{2}(e+fx)\right)+c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^5/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^5*(256*Sin[(e +
 f*x)/2] - 128*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 896*Sin[(e + f*x)/2]
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 448*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^3 + 2304*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4
 - 630*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - 160*Cos[e + f*x]
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 + 5*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^5*Sin[2*(e + f*x)]))/(20*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^10
*(a + a*Sin[e + f*x])^3)
```

fricas [A] time = 0.48, size = 285, normalized size = 1.77

$$-\frac{5\,c^5\cos\left(fx+e\right)^5+70\,c^5\cos\left(fx+e\right)^4-1260\,c^5fx-64\,c^5+7\left(45\,c^5fx+113\,c^5\right)\cos\left(fx+e\right)^3+\left(945\,c^5fx-\right.}{10\left(a^3f\cos\left(fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/10*(5*c^5*cos(f*x + e)^5 + 70*c^5*cos(f*x + e)^4 - 1260*c^5*f*x - 64*c^5
 + 7*(45*c^5*f*x + 113*c^5)*cos(f*x + e)^3 + (945*c^5*f*x - 502*c^5)*cos(f*
x + e)^2 - 2*(315*c^5*f*x + 646*c^5)*cos(f*x + e) - (5*c^5*cos(f*x + e)^4 -
 65*c^5*cos(f*x + e)^3 + 1260*c^5*f*x - 64*c^5 - 3*(105*c^5*f*x - 242*c^5)*
cos(f*x + e)^2 + 2*(315*c^5*f*x + 614*c^5)*cos(f*x + e))*sin(f*x + e))/(a^3
*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f
 + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

giac [A] time = 0.24, size = 186, normalized size = 1.16

$$-\frac{\frac{315\left(fx+e\right)c^5}{a^3}+\frac{10\left(c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+16\,c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+16\,c^5\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)^2a^3}+\frac{64\left(10\,c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+45\,c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+85\right.}{a^3\left(\tan\left(\frac{1}{2}fx+\right.\right.}}{10\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/10*(315*(f*x + e)*c^5/a^3 + 10*(c^5*tan(1/2*f*x + 1/2*e)^3 + 16*c^5*tan(
1/2*f*x + 1/2*e)^2 - c^5*tan(1/2*f*x + 1/2*e) + 16*c^5)/((tan(1/2*f*x + 1/2
*e)^2 + 1)^2*a^3) + 64*(10*c^5*tan(1/2*f*x + 1/2*e)^4 + 45*c^5*tan(1/2*f*x
+ 1/2*e)^3 + 85*c^5*tan(1/2*f*x + 1/2*e)^2 + 55*c^5*tan(1/2*f*x + 1/2*e) +
13*c^5)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

maple [A] time = 0.34, size = 277, normalized size = 1.72

$$-\frac{c^5\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a^3 f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{16c^5\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a^3 f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a^3 f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{16c^5}{a^3 f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{63c^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x)
```

```
[Out] -c^5/a^3/f/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^3-16*c^5/a^3/f/(1+
tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2+c^5/a^3/f/(1+tan(1/2*f*x+1/2*e
)^2)^2*tan(1/2*f*x+1/2*e)-16*c^5/a^3/f/(1+tan(1/2*f*x+1/2*e)^2)^2-63*c^5/a^
3/f*arctan(tan(1/2*f*x+1/2*e))-256/5*c^5/a^3/f/(tan(1/2*f*x+1/2*e)+1)^5+128
*c^5/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4-64*c^5/a^3/f/(tan(1/2*f*x+1/2*e)+1)^3-3
2*c^5/a^3/f/(tan(1/2*f*x+1/2*e)+1)^2-64*c^5/a^3/f/(tan(1/2*f*x+1/2*e)+1)
```

maxima [B] time = 1.49, size = 1496, normalized size = 9.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/15*(c^5*((1325*sin(f*x + e)/(cos(f*x + e) + 1) + 2673*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 3805*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 4329*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + 3575*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2
275*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 975*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 + 195*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 304)/(a^3 + 5*a^3*sin(f*
x + e)/(cos(f*x + e) + 1) + 12*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 20
*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 26*a^3*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + 26*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20*a^3*sin(f*x +
e)^6/(cos(f*x + e) + 1)^6 + 12*a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 5*
a^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^3*sin(f*x + e)^9/(cos(f*x + e)
+ 1)^9) + 195*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) + 30*c^5*((105*s
```

```
in(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x + e)^4/(cos(f*x + e)
+ 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*sin(f*x + e)^6/(cos(f*
x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 11*a^3*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 11*a^3*sin(f*x + e)^5/(
cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a^3*sin(f
*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1)
)/a^3) + 20*c^5*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 145*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) +
1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/
a^3) + 2*c^5*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 1
0*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^
5/(cos(f*x + e) + 1)^5) + 40*c^5*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x +
 e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*s
in(f*x + e)^5/(cos(f*x + e) + 1)^5) - 30*c^5*(5*sin(f*x + e)/(cos(f*x + e)
+ 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
+ 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x +
 e) + 1)^5))/f
```

mupad [B] time = 11.14, size = 364, normalized size = 2.26

$$\frac{\frac{63\,c^5\,\left(e+f\,x\right)}{2}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{315\,c^5\,\left(e+f\,x\right)}{2}-\frac{c^5\,\left(1575\,e+1575\,f\,x+4310\right)}{10}\right)-\frac{c^5\,\left(315\,e+315\,f\,x+992\right)}{10}+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8\,\left(\frac{315\,c^5\,\left(e+f\right.}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^5/(a + a*sin(e + f*x))^3,x)
```

```
[Out] ((63*c^5*(e + f*x))/2 + tan(e/2 + (f*x)/2)*((315*c^5*(e + f*x))/2 - (c^5*(1
575*e + 1575*f*x + 4310))/10) - (c^5*(315*e + 315*f*x + 992))/10 + tan(e/2
+ (f*x)/2)^8*((315*c^5*(e + f*x))/2 - (c^5*(1575*e + 1575*f*x + 650))/10) +
 tan(e/2 + (f*x)/2)^7*(378*c^5*(e + f*x) - (c^5*(3780*e + 3780*f*x + 3090))
```

```
/10) + tan(e/2 + (f*x)/2)^2*(378*c^5*(e + f*x) - (c^5*(3780*e + 3780*f*x +
8814))/10) + tan(e/2 + (f*x)/2)^6*(630*c^5*(e + f*x) - (c^5*(6300*e + 6300*
f*x + 7610))/10) + tan(e/2 + (f*x)/2)^3*(630*c^5*(e + f*x) - (c^5*(6300*e +
 6300*f*x + 12230))/10) + tan(e/2 + (f*x)/2)^5*(819*c^5*(e + f*x) - (c^5*(8
190*e + 8190*f*x + 11090))/10) + tan(e/2 + (f*x)/2)^4*(819*c^5*(e + f*x) -
(c^5*(8190*e + 8190*f*x + 14702))/10))/(a^3*f*(tan(e/2 + (f*x)/2) + 1)^5*(t
an(e/2 + (f*x)/2)^2 + 1)^2) - (63*c^5*x)/(2*a^3)
```

sympy [A] time = 72.50, size = 3643, normalized size = 22.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**5/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-315*c**5*f*x*tan(e/2 + f*x/2)**9/(10*a**3*f*tan(e/2 + f*x/2)**9
 + 50*a**3*f*tan(e/2 + f*x/2)**8 + 120*a**3*f*tan(e/2 + f*x/2)**7 + 200*a**
3*f*tan(e/2 + f*x/2)**6 + 260*a**3*f*tan(e/2 + f*x/2)**5 + 260*a**3*f*tan(e
/2 + f*x/2)**4 + 200*a**3*f*tan(e/2 + f*x/2)**3 + 120*a**3*f*tan(e/2 + f*x/
2)**2 + 50*a**3*f*tan(e/2 + f*x/2) + 10*a**3*f) - 1575*c**5*f*x*tan(e/2 + f
*x/2)**8/(10*a**3*f*tan(e/2 + f*x/2)**9 + 50*a**3*f*tan(e/2 + f*x/2)**8 + 1
20*a**3*f*tan(e/2 + f*x/2)**7 + 200*a**3*f*tan(e/2 + f*x/2)**6 + 260*a**3*f
*tan(e/2 + f*x/2)**5 + 260*a**3*f*tan(e/2 + f*x/2)**4 + 200*a**3*f*tan(e/2
+ f*x/2)**3 + 120*a**3*f*tan(e/2 + f*x/2)**2 + 50*a**3*f*tan(e/2 + f*x/2) +
 10*a**3*f) - 3780*c**5*f*x*tan(e/2 + f*x/2)**7/(10*a**3*f*tan(e/2 + f*x/2)
**9 + 50*a**3*f*tan(e/2 + f*x/2)**8 + 120*a**3*f*tan(e/2 + f*x/2)**7 + 200*
a**3*f*tan(e/2 + f*x/2)**6 + 260*a**3*f*tan(e/2 + f*x/2)**5 + 260*a**3*f*ta
n(e/2 + f*x/2)**4 + 200*a**3*f*tan(e/2 + f*x/2)**3 + 120*a**3*f*tan(e/2 + f
*x/2)**2 + 50*a**3*f*tan(e/2 + f*x/2) + 10*a**3*f) - 6300*c**5*f*x*tan(e/2
+ f*x/2)**6/(10*a**3*f*tan(e/2 + f*x/2)**9 + 50*a**3*f*tan(e/2 + f*x/2)**8
+ 120*a**3*f*tan(e/2 + f*x/2)**7 + 200*a**3*f*tan(e/2 + f*x/2)**6 + 260*a**
3*f*tan(e/2 + f*x/2)**5 + 260*a**3*f*tan(e/2 + f*x/2)**4 + 200*a**3*f*tan(e
/2 + f*x/2)**3 + 120*a**3*f*tan(e/2 + f*x/2)**2 + 50*a**3*f*tan(e/2 + f*x/2
) + 10*a**3*f) - 8190*c**5*f*x*tan(e/2 + f*x/2)**5/(10*a**3*f*tan(e/2 + f*x
/2)**9 + 50*a**3*f*tan(e/2 + f*x/2)**8 + 120*a**3*f*tan(e/2 + f*x/2)**7 + 2
00*a**3*f*tan(e/2 + f*x/2)**6 + 260*a**3*f*tan(e/2 + f*x/2)**5 + 260*a**3*f
*tan(e/2 + f*x/2)**4 + 200*a**3*f*tan(e/2 + f*x/2)**3 + 120*a**3*f*tan(e/2
+ f*x/2)**2 + 50*a**3*f*tan(e/2 + f*x/2) + 10*a**3*f) - 8190*c**5*f*x*tan(e
/2 + f*x/2)**4/(10*a**3*f*tan(e/2 + f*x/2)**9 + 50*a**3*f*tan(e/2 + f*x/2)*
*8 + 120*a**3*f*tan(e/2 + f*x/2)**7 + 200*a**3*f*tan(e/2 + f*x/2)**6 + 260*
a**3*f*tan(e/2 + f*x/2)**5 + 260*a**3*f*tan(e/2 + f*x/2)**4 + 200*a**3*f*ta
n(e/2 + f*x/2)**3 + 120*a**3*f*tan(e/2 + f*x/2)**2 + 50*a**3*f*tan(e/2 + f*
x/2) + 10*a**3*f) - 6300*c**5*f*x*tan(e/2 + f*x/2)**3/(10*a**3*f*tan(e/2 +
f*x/2)**9 + 50*a**3*f*tan(e/2 + f*x/2)**8 + 120*a**3*f*tan(e/2 + f*x/2)**7
+ 200*a**3*f*tan(e/2 + f*x/2)**6 + 260*a**3*f*tan(e/2 + f*x/2)**5 + 260*a**
```

3*f*tan(e/2 + f*x/2)**4 + 200*a**3*f*tan(e/2 + f*x/2)**3 + 120*a**3*f*tan(e
/2 + f*x/2)**2 + 50*a**3*f*tan(e/2 + f*x/2) + 10*a**3*f) - 3780*c**5*f*x*ta
n(e/2 + f*x/2)**2/(10*a**3*f*tan(e/2 + f*x/2)**9 + 50*a**3*f*tan(e/2 + f*x/
2)**8 + 120*a**3*f*tan(e/2 + f*x/2)**7 + 200*a**3*f*tan(e/2 + f*x/2)**6 + 2
60*a**3*f*tan(e/2 + f*x/2)**5 + 260*a**3*f*tan(e/2 + f*x/2)**4 + 200*a**3*f
*tan(e/2 + f*x/2)**3 + 120*a**3*f*tan(e/2 + f*x/2)**2 + 50*a**3*f*tan(e/2 +
f*x/2) + 10*a**3*f) - 1575*c**5*f*x*tan(e/2 + f*x/2)/(10*a**3*f*tan(e/2 +
f*x/2)**9 + 50*a**3*f*tan(e/2 + f*x/2)**8 + 120*a**3*f*tan(e/2 + f*x/2)**7
+ 200*a**3*f*tan(e/2 + f*x/2)**6 + 260*a**3*f*tan(e/2 + f*x/2)**5 + 260*a**
3*f*tan(e/2 + f*x/2)**4 + 200*a**3*f*tan(e/2 + f*x/2)**3 + 120*a**3*f*tan(e
/2 + f*x/2)**2 + 50*a**3*f*tan(e/2 + f*x/2) + 10*a**3*f) - 315*c**5*f*x/(10
*a**3*f*tan(e/2 + f*x/2)**9 + 50*a**3*f*tan(e/2 + f*x/2)**8 + 120*a**3*f*ta
n(e/2 + f*x/2)**7 + 200*a**3*f*tan(e/2 + f*x/2)**6 + 260*a**3*f*tan(e/2 + f
*x/2)**5 + 260*a**3*f*tan(e/2 + f*x/2)**4 + 200*a**3*f*tan(e/2 + f*x/2)**3
+ 120*a**3*f*tan(e/2 + f*x/2)**2 + 50*a**3*f*tan(e/2 + f*x/2) + 10*a**3*f)
- 650*c**5*tan(e/2 + f*x/2)**8/(10*a**3*f*tan(e/2 + f*x/2)**9 + 50*a**3*f*t
an(e/2 + f*x/2)**8 + 120*a**3*f*tan(e/2 + f*x/2)**7 + 200*a**3*f*tan(e/2 +
f*x/2)**6 + 260*a**3*f*tan(e/2 + f*x/2)**5 + 260*a**3*f*tan(e/2 + f*x/2)**4
+ 200*a**3*f*tan(e/2 + f*x/2)**3 + 120*a**3*f*tan(e/2 + f*x/2)**2 + 50*a**
3*f*tan(e/2 + f*x/2) + 10*a**3*f) - 3090*c**5*tan(e/2 + f*x/2)**7/(10*a**3*
f*tan(e/2 + f*x/2)**9 + 50*a**3*f*tan(e/2 + f*x/2)**8 + 120*a**3*f*tan(e/2
+ f*x/2)**7 + 200*a**3*f*tan(e/2 + f*x/2)**6 + 260*a**3*f*tan(e/2 + f*x/2)*
*5 + 260*a**3*f*tan(e/2 + f*x/2)**4 + 200*a**3*f*tan(e/2 + f*x/2)**3 + 120*
a**3*f*tan(e/2 + f*x/2)**2 + 50*a**3*f*tan(e/2 + f*x/2) + 10*a**3*f) - 7610
*c**5*tan(e/2 + f*x/2)**6/(10*a**3*f*tan(e/2 + f*x/2)**9 + 50*a**3*f*tan(e/
2 + f*x/2)**8 + 120*a**3*f*tan(e/2 + f*x/2)**7 + 200*a**3*f*tan(e/2 + f*x/2
)**6 + 260*a**3*f*tan(e/2 + f*x/2)**5 + 260*a**3*f*tan(e/2 + f*x/2)**4 + 20
0*a**3*f*tan(e/2 + f*x/2)**3 + 120*a**3*f*tan(e/2 + f*x/2)**2 + 50*a**3*f*t
an(e/2 + f*x/2) + 10*a**3*f) - 11090*c**5*tan(e/2 + f*x/2)**5/(10*a**3*f*ta
n(e/2 + f*x/2)**9 + 50*a**3*f*tan(e/2 + f*x/2)**8 + 120*a**3*f*tan(e/2 + f*
x/2)**7 + 200*a**3*f*tan(e/2 + f*x/2)**6 + 260*a**3*f*tan(e/2 + f*x/2)**5 +
260*a**3*f*tan(e/2 + f*x/2)**4 + 200*a**3*f*tan(e/2 + f*x/2)**3 + 120*a**3
*f*tan(e/2 + f*x/2)**2 + 50*a**3*f*tan(e/2 + f*x/2) + 10*a**3*f) - 14702*c*
*5*tan(e/2 + f*x/2)**4/(10*a**3*f*tan(e/2 + f*x/2)**9 + 50*a**3*f*tan(e/2 +
f*x/2)**8 + 120*a**3*f*tan(e/2 + f*x/2)**7 + 200*a**3*f*tan(e/2 + f*x/2)**
6 + 260*a**3*f*tan(e/2 + f*x/2)**5 + 260*a**3*f*tan(e/2 + f*x/2)**4 + 200*a
3*f*tan(e/2 + f*x/2)3 + 120*a**3*f*tan(e/2 + f*x/2)**2 + 50*a**3*f*tan(
e/2 + f*x/2) + 10*a**3*f) - 12230*c**5*tan(e/2 + f*x/2)**3/(10*a**3*f*tan(e
/2 + f*x/2)**9 + 50*a**3*f*tan(e/2 + f*x/2)**8 + 120*a**3*f*tan(e/2 + f*x/2
)**7 + 200*a**3*f*tan(e/2 + f*x/2)**6 + 260*a**3*f*tan(e/2 + f*x/2)**5 + 26
0*a**3*f*tan(e/2 + f*x/2)**4 + 200*a**3*f*tan(e/2 + f*x/2)**3 + 120*a**3*f*
tan(e/2 + f*x/2)**2 + 50*a**3*f*tan(e/2 + f*x/2) + 10*a**3*f) - 8814*c**5*t
an(e/2 + f*x/2)**2/(10*a**3*f*tan(e/2 + f*x/2)**9 + 50*a**3*f*tan(e/2 + f*x
/2)**8 + 120*a**3*f*tan(e/2 + f*x/2)**7 + 200*a**3*f*tan(e/2 + f*x/2)**6 +
260*a**3*f*tan(e/2 + f*x/2)**5 + 260*a**3*f*tan(e/2 + f*x/2)**4 + 200*a**3*

```
f*tan(e/2 + f*x/2)**3 + 120*a**3*f*tan(e/2 + f*x/2)**2 + 50*a**3*f*tan(e/2
+ f*x/2) + 10*a**3*f) - 4310*c**5*tan(e/2 + f*x/2)/(10*a**3*f*tan(e/2 + f*x
/2)**9 + 50*a**3*f*tan(e/2 + f*x/2)**8 + 120*a**3*f*tan(e/2 + f*x/2)**7 + 2
00*a**3*f*tan(e/2 + f*x/2)**6 + 260*a**3*f*tan(e/2 + f*x/2)**5 + 260*a**3*f
*tan(e/2 + f*x/2)**4 + 200*a**3*f*tan(e/2 + f*x/2)**3 + 120*a**3*f*tan(e/2
+ f*x/2)**2 + 50*a**3*f*tan(e/2 + f*x/2) + 10*a**3*f) - 992*c**5/(10*a**3*f
*tan(e/2 + f*x/2)**9 + 50*a**3*f*tan(e/2 + f*x/2)**8 + 120*a**3*f*tan(e/2 +
 f*x/2)**7 + 200*a**3*f*tan(e/2 + f*x/2)**6 + 260*a**3*f*tan(e/2 + f*x/2)**
5 + 260*a**3*f*tan(e/2 + f*x/2)**4 + 200*a**3*f*tan(e/2 + f*x/2)**3 + 120*a
**3*f*tan(e/2 + f*x/2)**2 + 50*a**3*f*tan(e/2 + f*x/2) + 10*a**3*f), Ne(f,
0)), (x*(-c*sin(e) + c)**5/(a*sin(e) + a)**3, True))
```

3.280 $\int \frac{(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=124

$$-\frac{7c^4\cos(e+fx)}{a^3f}-\frac{2a^3c^4\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^6}-\frac{7c^4x}{a^3}+\frac{14ac^4\cos^5(e+fx)}{15f(a\sin(e+fx)+a)^4}-\frac{14c^4\cos^3(e+fx)}{3af(a\sin(e+fx)+a)^2}$$

```
[Out] -7*c^4*x/a^3-7*c^4*cos(f*x+e)/a^3/f-2/5*a^3*c^4*cos(f*x+e)^7/f/(a+a*sin(f*x
+e))^6+14/15*a*c^4*cos(f*x+e)^5/f/(a+a*sin(f*x+e))^4-14/3*c^4*cos(f*x+e)^3/
a/f/(a+a*sin(f*x+e))^2
```

Rubi [A] time = 0.22, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2680, 2682, 8}

$$-\frac{7c^4\cos(e+fx)}{a^3f}-\frac{2a^3c^4\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^6}-\frac{7c^4x}{a^3}+\frac{14ac^4\cos^5(e+fx)}{15f(a\sin(e+fx)+a)^4}-\frac{14c^4\cos^3(e+fx)}{3af(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^4/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-7*c^4*x)/a^3 - (7*c^4*Cos[e + f*x])/(a^3*f) - (2*a^3*c^4*Cos[e + f*x]^7)/
(5*f*(a + a*Sin[e + f*x])^6) + (14*a*c^4*Cos[e + f*x]^5)/(15*f*(a + a*Sin[e
 + f*x])^4) - (14*c^4*Cos[e + f*x]^3)/(3*a*f*(a + a*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
```

```
&& EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx = \left(a^4c^4\right)\int \frac{\cos^8(e+fx)}{(a+a\sin(e+fx))^7}\,dx$$
$$= -\frac{2a^3c^4\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6} - \frac{1}{5}\left(7a^2c^4\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^5}\,dx$$
$$= -\frac{2a^3c^4\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6} + \frac{14ac^4\cos^5(e+fx)}{15f(a+a\sin(e+fx))^4} + \frac{1}{3}\left(7c^4\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^3}$$
$$= -\frac{2a^3c^4\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6} + \frac{14ac^4\cos^5(e+fx)}{15f(a+a\sin(e+fx))^4} - \frac{14c^4\cos^3(e+fx)}{3af(a+a\sin(e+fx))^2} - \frac{(7c}{}$$
$$= -\frac{7c^4\cos(e+fx)}{a^3f} - \frac{2a^3c^4\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6} + \frac{14ac^4\cos^5(e+fx)}{15f(a+a\sin(e+fx))^4} - \frac{14c^4\cos}{3af(a+a\sin}$$
$$= -\frac{7c^4x}{a^3} - \frac{7c^4\cos(e+fx)}{a^3f} - \frac{2a^3c^4\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6} + \frac{14ac^4\cos^5(e+fx)}{15f(a+a\sin(e+fx))^4} - \frac{1}{3af}$$

Mathematica [B] time = 0.62, size = 270, normalized size = 2.18

$$\frac{(c-c\sin(e+fx))^4\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(96\sin\left(\frac{1}{2}(e+fx)\right)-105(e+fx)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^4/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(96*Sin[(e + f*x)/2] - 48*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2]) - 256*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^2 + 128*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 464*Sin[(e
```

```
+ f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 105*(e + f*x)*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^5 - 15*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])^5)*(c - c*Sin[e + f*x])^4)/(15*f*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])^8*(a + a*Sin[e + f*x])^3)
```

fricas [B] time = 0.44, size = 256, normalized size = 2.06

$$-\frac{15\,c^4\cos\left(fx+e\right)^4-420\,c^4fx-48\,c^4+\left(105\,c^4fx+277\,c^4\right)\cos\left(fx+e\right)^3+\left(315\,c^4fx-134\,c^4\right)\cos\left(fx+e\right)^2}{15\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(15*c^4*cos(f*x + e)^4 - 420*c^4*f*x - 48*c^4 + (105*c^4*f*x + 277*c^
4)*cos(f*x + e)^3 + (315*c^4*f*x - 134*c^4)*cos(f*x + e)^2 - 6*(35*c^4*f*x
+ 74*c^4)*cos(f*x + e) + (15*c^4*cos(f*x + e)^3 - 420*c^4*f*x + 48*c^4 + (1
05*c^4*f*x - 262*c^4)*cos(f*x + e)^2 - 6*(35*c^4*f*x + 66*c^4)*cos(f*x + e)
)*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*co
s(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3
*f)*sin(f*x + e))
```

giac [A] time = 0.21, size = 135, normalized size = 1.09

$$-\frac{\frac{105\left(fx+e\right)c^4}{a^3}+\frac{30\,c^4}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)a^3}+\frac{16\left(15\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+60\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+130\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+80\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+19\,c^4\right)}{a^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/15*(105*(f*x + e)*c^4/a^3 + 30*c^4/((tan(1/2*f*x + 1/2*e)^2 + 1)*a^3) +
16*(15*c^4*tan(1/2*f*x + 1/2*e)^4 + 60*c^4*tan(1/2*f*x + 1/2*e)^3 + 130*c^4
*tan(1/2*f*x + 1/2*e)^2 + 80*c^4*tan(1/2*f*x + 1/2*e) + 19*c^4)/(a^3*(tan(1
/2*f*x + 1/2*e) + 1)^5))/f
```

maple [A] time = 0.31, size = 145, normalized size = 1.17

$$-\frac{2c^4}{a^3f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}-\frac{14c^4\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a^3f}-\frac{128c^4}{5a^3f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}+\frac{64c^4}{a^3f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x)
```

```
[Out] -2*c^4/a^3/f/(1+tan(1/2*f*x+1/2*e)^2)-14*c^4/a^3/f*arctan(tan(1/2*f*x+1/2*e
))-128/5*c^4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^5+64*c^4/a^3/f/(tan(1/2*f*x+1/2*e
)+1)^4-128/3*c^4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^3-16*c^4/a^3/f/(tan(1/2*f*x+1
/2*e)+1)
```

maxima [B] time = 0.97, size = 1096, normalized size = 8.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(3*c^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*si
n(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x
+ e) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 11*
a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/a^3) + 4*c^4*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 1
45*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x +
e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) +
 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(
cos(f*x + e) + 1))/a^3) + c^4*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(
f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 12*c^4*(5*sin(f*x + e)/(cos(f*x
+ e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*
x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10
*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x +
e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 12*c^4*(5*sin(f*x +
e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1
) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5))/f
```

mupad [B] time = 10.90, size = 290, normalized size = 2.34

$$7\,c^4\,\left(e+f\,x\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(35\,c^4\,\left(e+f\,x\right)-\frac{c^4\,\left(525\,e+525\,f\,x+1430\right)}{15}\right)-\frac{c^4\,\left(105\,e+105\,f\,x+334\right)}{15}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,\left(35\,c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^4/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (7*c^4*(e + f*x) + tan(e/2 + (f*x)/2)*(35*c^4*(e + f*x) - (c^4*(525*e + 525
*f*x + 1430))/15) - (c^4*(105*e + 105*f*x + 334))/15 + tan(e/2 + (f*x)/2)^6
*(35*c^4*(e + f*x) - (c^4*(525*e + 525*f*x + 240))/15) + tan(e/2 + (f*x)/2)
^5*(77*c^4*(e + f*x) - (c^4*(1155*e + 1155*f*x + 990))/15) + tan(e/2 + (f*x
)/2)^2*(77*c^4*(e + f*x) - (c^4*(1155*e + 1155*f*x + 2684))/15) + tan(e/2 +
 (f*x)/2)^4*(105*c^4*(e + f*x) - (c^4*(1575*e + 1575*f*x + 2470))/15) + tan
(e/2 + (f*x)/2)^3*(105*c^4*(e + f*x) - (c^4*(1575*e + 1575*f*x + 2540))/15)
)/(a^3*f*(tan(e/2 + (f*x)/2) + 1)^5*(tan(e/2 + (f*x)/2)^2 + 1)) - (7*c^4*x)
/a^3
```

sympy [A] time = 44.74, size = 2314, normalized size = 18.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**4/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-105*c**4*f*x*tan(e/2 + f*x/2)**7/(15*a**3*f*tan(e/2 + f*x/2)**7
 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**
3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e
/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 525*c**4*f*x*tan
(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2
)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 22
5*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*t
an(e/2 + f*x/2) + 15*a**3*f) - 1155*c**4*f*x*tan(e/2 + f*x/2)**5/(15*a**3*f
*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 +
 f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**
3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f
) - 1575*c**4*f*x*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a
**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan
(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*
x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 1575*c**4*f*x*tan(e/2 +
 f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 +
```

```
165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3
*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2
 + f*x/2) + 15*a**3*f) - 1155*c**4*f*x*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e
/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2
)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 16
5*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 52
5*c**4*f*x*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(
e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x
/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 +
75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 105*c**4*f*x/(15*a**3*f*tan(e/2 +
 f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5
 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a*
*3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 240*c*
*4*tan(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 +
 f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**
4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a*
*3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 990*c**4*tan(e/2 + f*x/2)**5/(15*a**3*
f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2
+ f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)*
*3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*
f) - 2470*c**4*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3
*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/
2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2
)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 2540*c**4*tan(e/2 + f*x/2)
**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a*
*3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(
e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/
2) + 15*a**3*f) - 2684*c**4*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)
**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*
a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*ta
n(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 1430*c**4*tan
(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**
6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a
**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(
e/2 + f*x/2) + 15*a**3*f) - 334*c**4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a*
*3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(
e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x
/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(-c*sin(e)
+ c)**4/(a*sin(e) + a)**3, True))
```

3.281 $\int \frac{(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=103

$$-\frac{2c^3\cos(e+fx)}{f\left(a^3\sin(e+fx)+a^3\right)}-\frac{c^3x}{a^3}-\frac{2a^2c^3\cos^5(e+fx)}{5f(a\sin(e+fx)+a)^5}+\frac{2c^3\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^3}$$

```
[Out] -c^3*x/a^3-2/5*a^2*c^3*cos(f*x+e)^5/f/(a+a*sin(f*x+e))^5+2/3*c^3*cos(f*x+e)
^3/f/(a+a*sin(f*x+e))^3-2*c^3*cos(f*x+e)/f/(a^3+a^3*sin(f*x+e))
```

Rubi [A] time = 0.18, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2680, 8}

$$-\frac{2a^2c^3\cos^5(e+fx)}{5f(a\sin(e+fx)+a)^5}-\frac{2c^3\cos(e+fx)}{f\left(a^3\sin(e+fx)+a^3\right)}-\frac{c^3x}{a^3}+\frac{2c^3\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -((c^3*x)/a^3) - (2*a^2*c^3*Cos[e + f*x]^5)/(5*f*(a + a*Sin[e + f*x])^5) +
(2*c^3*Cos[e + f*x]^3)/(3*f*(a + a*Sin[e + f*x])^3) - (2*c^3*Cos[e + f*x])/
(f*(a^3 + a^3*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
```

```
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^6}\,dx$$
$$= -\frac{2a^2c^3\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5} - \left(ac^3\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^4}\,dx$$
$$= -\frac{2a^2c^3\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5} + \frac{2c^3\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3} + \frac{c^3\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^2}\,dx}{a}$$
$$= -\frac{2a^2c^3\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5} + \frac{2c^3\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3} - \frac{2c^3\cos(e+fx)}{f\left(a^3+a^3\sin(e+fx)\right)} - \frac{c^3\int 1}{a^3}$$
$$= -\frac{c^3x}{a^3} - \frac{2a^2c^3\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5} + \frac{2c^3\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3} - \frac{2c^3\cos(e+fx)}{f\left(a^3+a^3\sin(e+fx)\right)}$$

Mathematica [B] time = 0.44, size = 239, normalized size = 2.32

$$\frac{(c-c\sin(e+fx))^3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(48\sin\left(\frac{1}{2}(e+fx)\right)-15(e+fx)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(48*Sin[(e + f*x)/2] - 24*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2]) - 88*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^2 + 44*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 92*Sin[(e + f
*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 15*(e + f*x)*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^5)*(c - c*Sin[e + f*x])^3)/(15*f*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])^6*(a + a*Sin[e + f*x])^3)
```

fricas [B] time = 0.43, size = 233, normalized size = 2.26

$$\frac{60\,c^3fx-\left(15\,c^3fx+46\,c^3\right)\cos\left(fx+e\right)^3+24\,c^3-\left(45\,c^3fx-2\,c^3\right)\cos\left(fx+e\right)^2+6\left(5\,c^3fx+12\,c^3\right)\cos\left(fx+\right.}{15\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2-2\,a^3f\cos\left(fx+e\right)-4\,a^3f+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(60*c^3*f*x - (15*c^3*f*x + 46*c^3)*cos(f*x + e)^3 + 24*c^3 - (45*c^3*
f*x - 2*c^3)*cos(f*x + e)^2 + 6*(5*c^3*f*x + 12*c^3)*cos(f*x + e) + (60*c^3
*f*x - 24*c^3 - (15*c^3*f*x - 46*c^3)*cos(f*x + e)^2 + 6*(5*c^3*f*x + 8*c^3
)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^
2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*
x + e) - 4*a^3*f)*sin(f*x + e))
```

giac [A] time = 0.23, size = 111, normalized size = 1.08

$$-\frac{\frac{15\,(fx+e)c^3}{a^3}+\frac{4\left(15\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+30\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+100\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+50\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+13\,c^3\right)}{a^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/15*(15*(f*x + e)*c^3/a^3 + 4*(15*c^3*tan(1/2*f*x + 1/2*e)^4 + 30*c^3*tan
(1/2*f*x + 1/2*e)^3 + 100*c^3*tan(1/2*f*x + 1/2*e)^2 + 50*c^3*tan(1/2*f*x +
 1/2*e) + 13*c^3)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

maple [A] time = 0.30, size = 143, normalized size = 1.39

$$-\frac{2c^3\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a^3f}-\frac{64c^3}{5a^3f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}+\frac{32c^3}{a^3f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{80c^3}{3a^3f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+-\frac{}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x)
```

```
[Out] -2*c^3/a^3/f*arctan(tan(1/2*f*x+1/2*e))-64/5*c^3/a^3/f/(tan(1/2*f*x+1/2*e)+
1)^5+32*c^3/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4-80/3*c^3/a^3/f/(tan(1/2*f*x+1/2*
e)+1)^3+8*c^3/a^3/f/(tan(1/2*f*x+1/2*e)+1)^2-4*c^3/a^3/f/(tan(1/2*f*x+1/2*e
)+1)
```

maxima [B] time = 1.46, size = 781, normalized size = 7.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(c^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 145*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) +
10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)
^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3)
+ c^3*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5) + 6*c^3*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1)
 + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x +
 e)^5/(cos(f*x + e) + 1)^5) - 9*c^3*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^
5))/f
```

mupad [B] time = 8.94, size = 200, normalized size = 1.94

$$\frac{c^3\,\left(e+f\,x\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(5\,c^3\,\left(e+f\,x\right)-\frac{c^3\,\left(75\,e+75\,f\,x+200\right)}{15}\right)-\frac{c^3\,\left(15\,e+15\,f\,x+52\right)}{15}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(5\,c^3\,\left(e+f\,x\right)\right.}{a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^3/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (c^3*(e + f*x) + tan(e/2 + (f*x)/2)*(5*c^3*(e + f*x) - (c^3*(75*e + 75*f*x
+ 200))/15) - (c^3*(15*e + 15*f*x + 52))/15 + tan(e/2 + (f*x)/2)^4*(5*c^3*(
e + f*x) - (c^3*(75*e + 75*f*x + 60))/15) + tan(e/2 + (f*x)/2)^3*(10*c^3*(e
 + f*x) - (c^3*(150*e + 150*f*x + 120))/15) + tan(e/2 + (f*x)/2)^2*(10*c^3*
(e + f*x) - (c^3*(150*e + 150*f*x + 400))/15))/(a^3*f*(tan(e/2 + (f*x)/2) +
 1)^5) - (c^3*x)/a^3
```

sympy [A] time = 26.24, size = 1284, normalized size = 12.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**3/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-15*c**3*f*x*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**5
+ 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3
*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 75*c**3*
f*x*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2
+ f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)*
*2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 150*c**3*f*x*tan(e/2 + f*x/2
)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a
**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(
e/2 + f*x/2) + 15*a**3*f) - 150*c**3*f*x*tan(e/2 + f*x/2)**2/(15*a**3*f*tan
(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x
/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a
**3*f) - 75*c**3*f*x*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a
**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan
(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 15*c**3*f*x/(1
5*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*t
an(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f
*x/2) + 15*a**3*f) - 60*c**3*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2
)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150
*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 120
*c**3*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/
2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2
)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 400*c**3*tan(e/2 + f*x/2)*
*2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**
3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/
2 + f*x/2) + 15*a**3*f) - 200*c**3*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*
x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 +
150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) -
52*c**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 15
0*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*t
an(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(-c*sin(e) + c)**3/(a*sin(e) +
a)**3, True))
```

3.282 $\int \frac{(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=33

$$-\frac{a^2c^2\cos^5(e+fx)}{5f(a\sin(e+fx)+a)^5}$$

```
[Out] -1/5*a^2*c^2*cos(f*x+e)^5/f/(a+a*sin(f*x+e))^5
```

Rubi [A] time = 0.09, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2736, 2671}

$$-\frac{a^2c^2\cos^5(e+fx)}{5f(a\sin(e+fx)+a)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -(a^2*c^2*Cos[e + f*x]^5)/(5*f*(a + a*Sin[e + f*x])^5)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^5}\,dx\\ &= -\frac{a^2c^2\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5}\end{aligned}$$

Mathematica [B] time = 0.41, size = 81, normalized size = 2.45

$$\frac{c^2\left(10\sin\left(\frac{1}{2}(e+fx)\right)+5\sin\left(\frac{3}{2}(e+fx)\right)-\sin\left(\frac{5}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{10a^3f(\sin(e+fx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(10*Sin[(e + f*x)/2] + 5*Sin[(3*
(e + f*x))/2] - Sin[(5*(e + f*x))/2]))/(10*a^3*f*(1 + Sin[e + f*x])^3)
```

fricas [B] time = 0.42, size = 168, normalized size = 5.09

$$-\frac{c^2\cos\left(fx+e\right)^3+3\,c^2\cos\left(fx+e\right)^2-2\,c^2\cos\left(fx+e\right)-4\,c^2-\left(c^2\cos\left(fx+e\right)^2-2\,c^2\cos\left(fx+e\right)-\right.}{5\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2-2\,a^3f\cos\left(fx+e\right)-4\,a^3f+\left(a^3f\cos\left(fx+e\right)^2-2\,a^3f\cos\left(fx+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/5*(c^2*cos(f*x + e)^3 + 3*c^2*cos(f*x + e)^2 - 2*c^2*cos(f*x + e) - 4*c^
2 - (c^2*cos(f*x + e)^2 - 2*c^2*cos(f*x + e) - 4*c^2)*sin(f*x + e))/(a^3*f*
cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f +
(a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

giac [A] time = 0.26, size = 60, normalized size = 1.82

$$-\frac{2\left(5\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+10\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+c^2\right)}{5\,a^3f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/5*(5*c^2*tan(1/2*f*x + 1/2*e)^4 + 10*c^2*tan(1/2*f*x + 1/2*e)^2 + c^2)/(
a^3*f*(tan(1/2*f*x + 1/2*e) + 1)^5)
```

maple [B] time = 0.27, size = 88, normalized size = 2.67

$$\frac{2c^2\left(\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{8}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{8}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{16}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}\right)}{f\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/f*c^2/a^3*(4/(tan(1/2*f*x+1/2*e)+1)^2+8/(tan(1/2*f*x+1/2*e)+1)^4-1/(tan(1
/2*f*x+1/2*e)+1)-8/(tan(1/2*f*x+1/2*e)+1)^3-16/5/(tan(1/2*f*x+1/2*e)+1)^5)
```

maxima [B] time = 0.71, size = 554, normalized size = 16.79

$$-\frac{2\left(\frac{c^2\left(\frac{20\sin(fx+e)}{\cos(fx+e)+1}+\frac{40\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{30\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+7\right)}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{2c^2\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\cdots\right.}{a^3+\frac{5a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^3\sin\cdots}{(\cos(fx\cdots}}\cdots\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(c^2*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*
a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) + 2*c^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5) - 6*c^2*(5*sin(f*x + e)/(cos(f*x + e) + 1)
 + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*
a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5))/f
```

mupad [B] time = 7.21, size = 90, normalized size = 2.73

$$-\frac{2c^2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4+10\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2+5\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^4\right)}{5a^3f\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^2/(a + a*sin(e + f*x))^3,x)
```

```
[Out] -(2*c^2*cos(e/2 + (f*x)/2)*(cos(e/2 + (f*x)/2)^4 + 5*sin(e/2 + (f*x)/2)^4 +
 10*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^2))/(5*a^3*f*(cos(e/2 + (f*x)/2
) + sin(e/2 + (f*x)/2))^5)
```

sympy [A] time = 14.47, size = 354, normalized size = 10.73

$$\begin{cases} -\frac{10c^2\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{5a^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)+25a^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+50a^3f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+50a^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+25a^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+5a^3f} - \frac{}{5a^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)+25a^3f\tan^4\left(\right.} \\ \frac{x(-c\sin(e)+c)^2}{(a\sin(e)+a)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**2/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-10*c**2*tan(e/2 + f*x/2)**4/(5*a**3*f*tan(e/2 + f*x/2)**5 + 25*
a**3*f*tan(e/2 + f*x/2)**4 + 50*a**3*f*tan(e/2 + f*x/2)**3 + 50*a**3*f*tan(
e/2 + f*x/2)**2 + 25*a**3*f*tan(e/2 + f*x/2) + 5*a**3*f) - 20*c**2*tan(e/2
+ f*x/2)**2/(5*a**3*f*tan(e/2 + f*x/2)**5 + 25*a**3*f*tan(e/2 + f*x/2)**4 +
 50*a**3*f*tan(e/2 + f*x/2)**3 + 50*a**3*f*tan(e/2 + f*x/2)**2 + 25*a**3*f*
tan(e/2 + f*x/2) + 5*a**3*f) - 2*c**2/(5*a**3*f*tan(e/2 + f*x/2)**5 + 25*a*
*3*f*tan(e/2 + f*x/2)**4 + 50*a**3*f*tan(e/2 + f*x/2)**3 + 50*a**3*f*tan(e/
2 + f*x/2)**2 + 25*a**3*f*tan(e/2 + f*x/2) + 5*a**3*f), Ne(f, 0)), (x*(-c*s
in(e) + c)**2/(a*sin(e) + a)**3, True))
```

3.283 $\int \frac{c-c\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=58

$$-\frac{c\cos^3(e+fx)}{15f(a\sin(e+fx)+a)^3}-\frac{ac\cos^3(e+fx)}{5f(a\sin(e+fx)+a)^4}$$

```
[Out] -1/5*a*c*cos(f*x+e)^3/f/(a+a*sin(f*x+e))^4-1/15*c*cos(f*x+e)^3/f/(a+a*sin(f
*x+e))^3
```

Rubi [A] time = 0.11, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2736, 2672, 2671}

$$-\frac{c\cos^3(e+fx)}{15f(a\sin(e+fx)+a)^3}-\frac{ac\cos^3(e+fx)}{5f(a\sin(e+fx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -(a*c*Cos[e + f*x]^3)/(5*f*(a + a*Sin[e + f*x])^4) - (c*Cos[e + f*x]^3)/(15
*f*(a + a*Sin[e + f*x])^3)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
```

```
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{c-c\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx &= (ac)\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^4}\,dx\\ &= -\frac{ac\cos^3(e+fx)}{5f(a+a\sin(e+fx))^4}+\frac{1}{5}c\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^3}\,dx\\ &= -\frac{ac\cos^3(e+fx)}{5f(a+a\sin(e+fx))^4}-\frac{c\cos^3(e+fx)}{15f(a+a\sin(e+fx))^3}\end{aligned}$$

Mathematica [A] time = 0.34, size = 92, normalized size = 1.59

$$\frac{c\left(\sin\left(2e+\frac{5fx}{2}\right)-15\cos\left(e+\frac{fx}{2}\right)+5\cos\left(e+\frac{3fx}{2}\right)+5\sin\left(\frac{fx}{2}\right)\right)}{30a^3f\left(\sin\left(\frac{e}{2}\right)+\cos\left(\frac{e}{2}\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (c*(-15*Cos[e + (f*x)/2] + 5*Cos[e + (3*f*x)/2] + 5*Sin[(f*x)/2] + Sin[2*e
+ (5*f*x)/2]))/(30*a^3*f*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^5)
```

fricas [B] time = 0.49, size = 154, normalized size = 2.66

$$\frac{c\cos\left(fx+e\right)^3-2\,c\cos\left(fx+e\right)^2+3\,c\cos\left(fx+e\right)-\left(c\cos\left(fx+e\right)^2+3\,c\cos\left(fx+e\right)+6\,c\right)\sin}{15\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2-2\,a^3f\cos\left(fx+e\right)-4\,a^3f+\left(a^3f\cos\left(fx+e\right)^2-2\,a^3f\cos\left(fx+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(c*cos(f*x + e)^3 - 2*c*cos(f*x + e)^2 + 3*c*cos(f*x + e) - (c*cos(f*x
 + e)^2 + 3*c*cos(f*x + e) + 6*c)*sin(f*x + e) + 6*c)/(a^3*f*cos(f*x + e)^3
 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x
 + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

giac [A] time = 0.70, size = 84, normalized size = 1.45

$$-\frac{2 \left(15 \, c \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{4} + 15 \, c \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{3} + 25 \, c \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{2} + 5 \, c \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right) + 4 \, c\right)}{15 \, a^{3} f \left(\tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right) + 1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*c*tan(1/2*f*x + 1/2*e)^4 + 15*c*tan(1/2*f*x + 1/2*e)^3 + 25*c*tan
(1/2*f*x + 1/2*e)^2 + 5*c*tan(1/2*f*x + 1/2*e) + 4*c)/(a^3*f*(tan(1/2*f*x +
 1/2*e) + 1)^5)
```

maple [A] time = 0.26, size = 86, normalized size = 1.48

$$\frac{2 c \left(\frac{3}{\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{2}}+\frac{4}{\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{1}{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1}-\frac{14}{3 \left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{8}{5 \left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{5}}\right)}{f \, a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/f*c/a^3*(3/(tan(1/2*f*x+1/2*e)+1)^2+4/(tan(1/2*f*x+1/2*e)+1)^4-1/(tan(1/2
*f*x+1/2*e)+1)-14/3/(tan(1/2*f*x+1/2*e)+1)^3-8/5/(tan(1/2*f*x+1/2*e)+1)^5)
```

maxima [B] time = 0.98, size = 387, normalized size = 6.67

$$-\frac{2 \left(\frac{c \left(\frac{20 \, \sin\left(f x+e\right)}{\cos\left(f x+e\right)+1}+\frac{40 \, \sin\left(f x+e\right)^{2}}{\left(\cos\left(f x+e\right)+1\right)^{2}}+\frac{30 \, \sin\left(f x+e\right)^{3}}{\left(\cos\left(f x+e\right)+1\right)^{3}}+\frac{15 \, \sin\left(f x+e\right)^{4}}{\left(\cos\left(f x+e\right)+1\right)^{4}}+7\right)}{a^{3}+\frac{5 \, a^{3} \sin\left(f x+e\right)}{\cos\left(f x+e\right)+1}+\frac{10 \, a^{3} \sin\left(f x+e\right)^{2}}{\left(\cos\left(f x+e\right)+1\right)^{2}}+\frac{10 \, a^{3} \sin\left(f x+e\right)^{3}}{\left(\cos\left(f x+e\right)+1\right)^{3}}+\frac{5 \, a^{3} \sin\left(f x+e\right)^{4}}{\left(\cos\left(f x+e\right)+1\right)^{4}}+\frac{a^{3} \sin\left(f x+e\right)^{5}}{\left(\cos\left(f x+e\right)+1\right)^{5}}}-\frac{3 \, c \left(\frac{5 \, \sin\left(f x+e\right)}{\cos\left(f x+e\right)+1}+\frac{5 \, \sin\left(f x+e\right)^{2}}{\left(\cos\left(f x+e\right)+1\right)^{2}}+\frac{5 \, \sin(f x+\cdots}{(\cos(f x+e)\cdots}\right.}{a^{3}+\frac{5 \, a^{3} \sin\left(f x+e\right)}{\cos\left(f x+e\right)+1}+\frac{10 \, a^{3} \sin\left(f x+e\right)^{2}}{\left(\cos\left(f x+e\right)+1\right)^{2}}+\frac{10 \, a^{3} \sin\left(f x+e\right)^{3}}{\left(\cos\left(f x+e\right)+1\right)^{3}}+\frac{5 \, a^{3} \sin\cdots}{(\cos(f x\cdots}}\right.}{15 \, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(c*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(c
```

```
os(f*x + e) + 1)^5) - 3*c*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3
 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

mupad [B] time = 7.24, size = 134, normalized size = 2.31

$$-\frac{2\,c\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(4\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+5\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+25\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+15\,\cos}{15\,a^3\,f\,\left(\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))/(a + a*sin(e + f*x))^3,x)
```

```
[Out] -(2*c*cos(e/2 + (f*x)/2)*(4*cos(e/2 + (f*x)/2)^4 + 15*sin(e/2 + (f*x)/2)^4
+ 15*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x)/2)^3 + 5*cos(e/2 + (f*x)/2)^3*sin(e
/2 + (f*x)/2) + 25*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^2))/(15*a^3*f*(c
os(e/2 + (f*x)/2) + sin(e/2 + (f*x)/2))^5)
```

sympy [A] time = 8.43, size = 573, normalized size = 9.88

$$\begin{cases} -\frac{30c\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{15a^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+150a^3f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+150a^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+15a^3f} - \frac{}{15a^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^3f} \\ \frac{x(-c\sin(e)+c)}{(a\sin(e)+a)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*c*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a*
*3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(
e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*c*tan(e/2 +
f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 +
150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f
*tan(e/2 + f*x/2) + 15*a**3*f) - 50*c*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/
2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)
**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3
*f) - 10*c*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(
e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x
/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 8*c/(15*a**3*f*tan(e/2 +
```

```
f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3
 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f)
, Ne(f, 0)), (x*(-c*sin(e) + c)/(a*sin(e) + a)**3, True))
```

3.284 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx$

Optimal. Leaf size=83

$$\frac{2\tan(e+fx)}{5a^3cf}-\frac{\sec(e+fx)}{5cf\left(a^3\sin(e+fx)+a^3\right)}-\frac{\sec(e+fx)}{5acf(a\sin(e+fx)+a)^2}$$

```
[Out] -1/5*sec(f*x+e)/a/c/f/(a+a*sin(f*x+e))^2-1/5*sec(f*x+e)/c/f/(a^3+a^3*sin(f*
x+e))+2/5*tan(f*x+e)/a^3/c/f
```

Rubi [A] time = 0.15, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2672, 3767, 8}

$$\frac{2\tan(e+fx)}{5a^3cf}-\frac{\sec(e+fx)}{5cf\left(a^3\sin(e+fx)+a^3\right)}-\frac{\sec(e+fx)}{5acf(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])),x]
```

```
[Out] -Sec[e + f*x]/(5*a*c*f*(a + a*Sin[e + f*x])^2) - Sec[e + f*x]/(5*c*f*(a^3 +
 a^3*Sin[e + f*x])) + (2*Tan[e + f*x])/(5*a^3*c*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx = \frac{\int \frac{\sec^2(e+fx)}{(a+a\sin(e+fx))^2}\,dx}{ac}$$

$$= -\frac{\sec(e+fx)}{5acf(a+a\sin(e+fx))^2} + \frac{3\int \frac{\sec^2(e+fx)}{a+a\sin(e+fx)}\,dx}{5a^2c}$$

$$= -\frac{\sec(e+fx)}{5acf(a+a\sin(e+fx))^2} - \frac{\sec(e+fx)}{5cf\left(a^3+a^3\sin(e+fx)\right)} + \frac{2\int \sec^2 \cdots}{5 \cdots}$$

$$= -\frac{\sec(e+fx)}{5acf(a+a\sin(e+fx))^2} - \frac{\sec(e+fx)}{5cf\left(a^3+a^3\sin(e+fx)\right)} - \frac{2\,\mathrm{Subst}(\cdots}{\cdots}$$

$$= -\frac{\sec(e+fx)}{5acf(a+a\sin(e+fx))^2} - \frac{\sec(e+fx)}{5cf\left(a^3+a^3\sin(e+fx)\right)} + \frac{2\tan(e \cdots}{5a^3c \cdots}$$

Mathematica [A] time = 0.67, size = 111, normalized size = 1.34

$$\frac{-12\sin(e+fx)-32\sin(2(e+fx))-12\sin(3(e+fx))+8\sin(4(e+fx))+32\cos(e+fx)-12\cos(2(e+fx))+ \cdots}{160a^3cf(\sin(e+fx)-1)(\sin(e+fx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])),x]
```

```
[Out] (-15 + 32*Cos[e + f*x] - 12*Cos[2*(e + f*x)] + 32*Cos[3*(e + f*x)] + 3*Cos[
4*(e + f*x)] - 12*Sin[e + f*x] - 32*Sin[2*(e + f*x)] - 12*Sin[3*(e + f*x)]
+ 8*Sin[4*(e + f*x)])/(160*a^3*c*f*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])^3
)
```

fricas [A] time = 0.44, size = 82, normalized size = 0.99

$$\frac{4\cos\left(fx+e\right)^2+\left(2\cos\left(fx+e\right)^2-3\right)\sin\left(fx+e\right)-2}{5\left(a^3cf\cos\left(fx+e\right)^3-2\,a^3cf\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^3cf\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/5*(4*cos(f*x + e)^2 + (2*cos(f*x + e)^2 - 3)*sin(f*x + e) - 2)/(a^3*c*f*c
os(f*x + e)^3 - 2*a^3*c*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*c*f*cos(f*x + e
))
```

giac [A] time = 0.20, size = 105, normalized size = 1.27

$$-\frac{\frac{5}{a^3c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}+\frac{35\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+90\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+120\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+70\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+21}{a^3c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}}{20\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/20*(5/(a^3*c*(tan(1/2*f*x + 1/2*e) - 1)) + (35*tan(1/2*f*x + 1/2*e)^4 +
90*tan(1/2*f*x + 1/2*e)^3 + 120*tan(1/2*f*x + 1/2*e)^2 + 70*tan(1/2*f*x + 1
/2*e) + 21)/(a^3*c*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

maple [A] time = 0.26, size = 101, normalized size = 1.22

$$\frac{-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{4}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}+\frac{2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{3}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{5}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{7}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}}{a^3cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x)
```

```
[Out] 2/f/a^3/c*(-1/8/(tan(1/2*f*x+1/2*e)-1)-2/5/(tan(1/2*f*x+1/2*e)+1)^5+1/(tan(
1/2*f*x+1/2*e)+1)^4-3/2/(tan(1/2*f*x+1/2*e)+1)^3+5/4/(tan(1/2*f*x+1/2*e)+1)
^2-7/8/(tan(1/2*f*x+1/2*e)+1))
```

maxima [B] time = 0.65, size = 211, normalized size = 2.54

$$-\frac{2\left(\frac{3\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{10\,\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}-\frac{10\,\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}-\frac{5\,\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}+2\right)}{5\left(a^3c+\frac{4\,a^3c\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{5\,a^3c\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{5\,a^3c\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}-\frac{4\,a^3c\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}-\frac{a^3c\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -2/5*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 - 10*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 5*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 + 2)/((a^3*c + 4*a^3*c*sin(f*x + e)/(cos(f*x + e) + 1) + 5*a^3*
c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*a^3*c*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 - 4*a^3*c*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - a^3*c*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6)*f)
```

mupad [B] time = 7.38, size = 89, normalized size = 1.07

$$-\frac{2\,\left(5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+10\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+10\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3-3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-2\right)}{5\,a^3\,c\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-1\right)\,{\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))),x)
```

```
[Out] -(2*(10*tan(e/2 + (f*x)/2)^3 - 3*tan(e/2 + (f*x)/2) + 10*tan(e/2 + (f*x)/2)
^4 + 5*tan(e/2 + (f*x)/2)^5 - 2))/(5*a^3*c*f*(tan(e/2 + (f*x)/2) - 1)*(tan(
e/2 + (f*x)/2) + 1)^5)
```

sympy [A] time = 8.24, size = 614, normalized size = 7.40

$$\begin{cases} -\frac{10\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)}{5a^3cf\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+20a^3cf\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)+25a^3cf\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)-25a^3cf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-20a^3cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-5a^3cf} - \frac{}{5a^3cf\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)+20a^3cf} \\ \frac{x}{(a\sin(e)+a)^3(-c\sin(e)+c)} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-10*tan(e/2 + f*x/2)**5/(5*a**3*c*f*tan(e/2 + f*x/2)**6 + 20*a**
3*c*f*tan(e/2 + f*x/2)**5 + 25*a**3*c*f*tan(e/2 + f*x/2)**4 - 25*a**3*c*f*t
an(e/2 + f*x/2)**2 - 20*a**3*c*f*tan(e/2 + f*x/2) - 5*a**3*c*f) - 20*tan(e/
2 + f*x/2)**4/(5*a**3*c*f*tan(e/2 + f*x/2)**6 + 20*a**3*c*f*tan(e/2 + f*x/2
)**5 + 25*a**3*c*f*tan(e/2 + f*x/2)**4 - 25*a**3*c*f*tan(e/2 + f*x/2)**2 -
20*a**3*c*f*tan(e/2 + f*x/2) - 5*a**3*c*f) - 20*tan(e/2 + f*x/2)**3/(5*a**3
*c*f*tan(e/2 + f*x/2)**6 + 20*a**3*c*f*tan(e/2 + f*x/2)**5 + 25*a**3*c*f*ta
n(e/2 + f*x/2)**4 - 25*a**3*c*f*tan(e/2 + f*x/2)**2 - 20*a**3*c*f*tan(e/2 +
 f*x/2) - 5*a**3*c*f) + 6*tan(e/2 + f*x/2)/(5*a**3*c*f*tan(e/2 + f*x/2)**6
+ 20*a**3*c*f*tan(e/2 + f*x/2)**5 + 25*a**3*c*f*tan(e/2 + f*x/2)**4 - 25*a*
```

```
*3*c*f*tan(e/2 + f*x/2)**2 - 20*a**3*c*f*tan(e/2 + f*x/2) - 5*a**3*c*f) + 4
/(5*a**3*c*f*tan(e/2 + f*x/2)**6 + 20*a**3*c*f*tan(e/2 + f*x/2)**5 + 25*a**
3*c*f*tan(e/2 + f*x/2)**4 - 25*a**3*c*f*tan(e/2 + f*x/2)**2 - 20*a**3*c*f*t
an(e/2 + f*x/2) - 5*a**3*c*f), Ne(f, 0)), (x/((a*sin(e) + a)**3*(-c*sin(e)
+ c)), True))
```

3.285 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=75

$$\frac{4\tan^3(e+fx)}{15a^3c^2f}+\frac{4\tan(e+fx)}{5a^3c^2f}-\frac{\sec^3(e+fx)}{5c^2f\left(a^3\sin(e+fx)+a^3\right)}$$

```
[Out] -1/5*sec(f*x+e)^3/c^2/f/(a^3+a^3*sin(f*x+e))+4/5*tan(f*x+e)/a^3/c^2/f+4/15*
tan(f*x+e)^3/a^3/c^2/f
```

Rubi [A] time = 0.11, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2672, 3767}

$$\frac{4\tan^3(e+fx)}{15a^3c^2f}+\frac{4\tan(e+fx)}{5a^3c^2f}-\frac{\sec^3(e+fx)}{5c^2f\left(a^3\sin(e+fx)+a^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^2),x]
```

```
[Out] -Sec[e + f*x]^3/(5*c^2*f*(a^3 + a^3*Sin[e + f*x])) + (4*Tan[e + f*x])/(5*a^
3*c^2*f) + (4*Tan[e + f*x]^3)/(15*a^3*c^2*f)
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx = \frac{\int \frac{\sec^4(e+fx)}{a+a\sin(e+fx)}\,dx}{a^2c^2}$$
$$= -\frac{\sec^3(e+fx)}{5c^2f\left(a^3+a^3\sin(e+fx)\right)} + \frac{4\int \sec^4(e+fx)\,dx}{5a^3c^2}$$
$$= -\frac{\sec^3(e+fx)}{5c^2f\left(a^3+a^3\sin(e+fx)\right)} - \frac{4\,\mathrm{Subst}\left(\int\left(1+x^2\right)dx,x,-\tan(\ldots}{5a^3c^2f}$$
$$= -\frac{\sec^3(e+fx)}{5c^2f\left(a^3+a^3\sin(e+fx)\right)} + \frac{4\tan(e+fx)}{5a^3c^2f} + \frac{4\tan^3(e+fx)}{15a^3c^2f}$$

Mathematica [A] time = 0.79, size = 131, normalized size = 1.75

$$\frac{18\sin(e+fx)+512\sin(2(e+fx))+27\sin(3(e+fx))+128\sin(4(e+fx))+9\sin(5(e+fx))-128\cos(e+f\ldots}{1920a^3c^2f(\sin(e+fx)-1)^2(\sin(e\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^2),x]
```

```
[Out] (54 - 128*Cos[e + f*x] + 72*Cos[2*(e + f*x)] - 192*Cos[3*(e + f*x)] + 18*Co
s[4*(e + f*x)] - 64*Cos[5*(e + f*x)] + 18*Sin[e + f*x] + 512*Sin[2*(e + f*x
)] + 27*Sin[3*(e + f*x)] + 128*Sin[4*(e + f*x)] + 9*Sin[5*(e + f*x)])/(1920
*a^3*c^2*f*(-1 + Sin[e + f*x])^2*(1 + Sin[e + f*x])^3)
```

fricas [A] time = 0.43, size = 85, normalized size = 1.13

$$-\frac{8\cos\left(fx+e\right)^4-4\cos\left(fx+e\right)^2-4\left(2\cos\left(fx+e\right)^2+1\right)\sin\left(fx+e\right)-1}{15\left(a^3c^2f\cos\left(fx+e\right)^3\sin\left(fx+e\right)+a^3c^2f\cos\left(fx+e\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(8*cos(f*x + e)^4 - 4*cos(f*x + e)^2 - 4*(2*cos(f*x + e)^2 + 1)*sin(f
*x + e) - 1)/(a^3*c^2*f*cos(f*x + e)^3*sin(f*x + e) + a^3*c^2*f*cos(f*x + e
)^3)
```

giac [A] time = 0.29, size = 133, normalized size = 1.77

$$-\frac{\frac{5\left(15\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-24\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+13\right)}{a^{3}c^{2}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^{3}}+\frac{165\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+480\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+650\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+400\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+113}{a^{3}c^{2}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{5}}}{120f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/120*(5*(15*tan(1/2*f*x + 1/2*e)^2 - 24*tan(1/2*f*x + 1/2*e) + 13)/(a^3*c
^2*(tan(1/2*f*x + 1/2*e) - 1)^3) + (165*tan(1/2*f*x + 1/2*e)^4 + 480*tan(1/
2*f*x + 1/2*e)^3 + 650*tan(1/2*f*x + 1/2*e)^2 + 400*tan(1/2*f*x + 1/2*e) +
113)/(a^3*c^2*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

maple [A] time = 0.23, size = 133, normalized size = 1.77

$$\frac{-\frac{1}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{5}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{2}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}+\frac{1}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{5}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}+\frac{3}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}}{f\,c^{2}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x)
```

```
[Out] 2/f/c^2/a^3*(-1/12/(tan(1/2*f*x+1/2*e)-1)^3-1/8/(tan(1/2*f*x+1/2*e)-1)^2-5/
16/(tan(1/2*f*x+1/2*e)-1)-1/5/(tan(1/2*f*x+1/2*e)+1)^5+1/2/(tan(1/2*f*x+1/2
*e)+1)^4-5/6/(tan(1/2*f*x+1/2*e)+1)^3+3/4/(tan(1/2*f*x+1/2*e)+1)^2-11/16/(t
an(1/2*f*x+1/2*e)+1))
```

maxima [B] time = 1.12, size = 335, normalized size = 4.47

$$\frac{2\left(\frac{9\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{21\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{13\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}-\frac{25\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}-\frac{5\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}+\frac{15\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}+\frac{15\sin\left(fx+e\right)^{7}}{\left(\cos\left(fx+e\right)+1\right)^{7}}\right.}{15\left(a^{3}c^{2}+\frac{2a^{3}c^{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{2a^{3}c^{2}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{6a^{3}c^{2}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{6a^{3}c^{2}\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}+\frac{2a^{3}c^{2}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}-\frac{2a^{3}c^{2}\sin\left(fx+e\right)^{7}}{\left(\cos\left(fx+e\right)+1\right)^{7}}-\frac{a^{3}c^{2}}{\left(\cos\right.}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2/15*(9*sin(f*x + e)/(cos(f*x + e) + 1) + 21*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 13*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 25*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 - 5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*sin(f*x + e)^6/(c
```

```
os(f*x + e) + 1)^6 + 15*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3)/((a^3*c^2
+ 2*a^3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^3*c^2*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 - 6*a^3*c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 6*a^3*c^
2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2*a^3*c^2*sin(f*x + e)^6/(cos(f*x +
 e) + 1)^6 - 2*a^3*c^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a^3*c^2*sin(f*
x + e)^8/(cos(f*x + e) + 1)^8)*f)
```

mupad [B] time = 8.43, size = 128, normalized size = 1.71

$$-\frac{2\left(15\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7+15\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6-5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5-25\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+13\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3+21\tan\left(\frac{e}{2}+\right.\right.}{15\,a^3\,c^2\,f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)^3\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^2),x)
```

```
[Out] -(2*(9*tan(e/2 + (f*x)/2) + 21*tan(e/2 + (f*x)/2)^2 + 13*tan(e/2 + (f*x)/2)
^3 - 25*tan(e/2 + (f*x)/2)^4 - 5*tan(e/2 + (f*x)/2)^5 + 15*tan(e/2 + (f*x)/
2)^6 + 15*tan(e/2 + (f*x)/2)^7 - 3))/(15*a^3*c^2*f*(tan(e/2 + (f*x)/2) - 1)
^3*(tan(e/2 + (f*x)/2) + 1)^5)
```

sympy [A] time = 15.76, size = 1418, normalized size = 18.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-30*tan(e/2 + f*x/2)**7/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30
*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*
a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a
**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*
c**2*f) - 30*tan(e/2 + f*x/2)**6/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a
**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a*
*3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**
3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c*
*2*f) + 10*tan(e/2 + f*x/2)**5/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**
3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3
*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*
c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2
*f) + 50*tan(e/2 + f*x/2)**4/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*
c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c
**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c*
```

```
*2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f
) - 26*tan(e/2 + f*x/2)**3/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c*
*2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**
2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2
*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f)
- 42*tan(e/2 + f*x/2)**2/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2
*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*
f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f
*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) -
18*tan(e/2 + f*x/2)/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*ta
n(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan
(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(
e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) + 6/(15
*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*
a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a
**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a*
*3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f), Ne(f, 0)), (x/((a*sin(e) + a)
**3*(-c*sin(e) + c)**2), True))
```

3.286 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=59

$$\frac{\tan^5(e+fx)}{5a^3c^3f}+\frac{2\tan^3(e+fx)}{3a^3c^3f}+\frac{\tan(e+fx)}{a^3c^3f}$$

```
[Out] tan(f*x+e)/a^3/c^3/f+2/3*tan(f*x+e)^3/a^3/c^3/f+1/5*tan(f*x+e)^5/a^3/c^3/f
```

Rubi [A] time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2736, 3767}

$$\frac{\tan^5(e+fx)}{5a^3c^3f}+\frac{2\tan^3(e+fx)}{3a^3c^3f}+\frac{\tan(e+fx)}{a^3c^3f}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^3),x]
```

```
[Out] Tan[e + f*x]/(a^3*c^3*f) + (2*Tan[e + f*x]^3)/(3*a^3*c^3*f) + Tan[e + f*x]^
5/(5*a^3*c^3*f)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx = \frac{\int \sec^6(e+fx)\,dx}{a^3c^3}$$
$$= -\frac{\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -\tan(e+fx)\right)}{a^3c^3f}$$
$$= \frac{\tan(e+fx)}{a^3c^3f} + \frac{2\tan^3(e+fx)}{3a^3c^3f} + \frac{\tan^5(e+fx)}{5a^3c^3f}$$

Mathematica [A] time = 0.13, size = 41, normalized size = 0.69

$$\frac{\frac{1}{5}\tan^5(e+fx) + \frac{2}{3}\tan^3(e+fx) + \tan(e+fx)}{a^3c^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^3),x]
```

```
[Out] (Tan[e + f*x] + (2*Tan[e + f*x]^3)/3 + Tan[e + f*x]^5/5)/(a^3*c^3*f)
```

fricas [A] time = 0.42, size = 47, normalized size = 0.80

$$\frac{\left(8\cos\left(fx+e\right)^4 + 4\cos\left(fx+e\right)^2 + 3\right)\sin\left(fx+e\right)}{15\,a^3c^3f\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(8*cos(f*x + e)^4 + 4*cos(f*x + e)^2 + 3)*sin(f*x + e)/(a^3*c^3*f*cos(
f*x + e)^5)
```

giac [A] time = 0.22, size = 43, normalized size = 0.73

$$\frac{3\tan\left(fx+e\right)^5 + 10\tan\left(fx+e\right)^3 + 15\tan\left(fx+e\right)}{15\,a^3c^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/15*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))/(a^3*c^3*f)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + a\sin\left(fx + e\right)\right)^{3}\left(c - c\sin\left(fx + e\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x)
```

```
[Out] int(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x)
```

maxima [A] time = 0.67, size = 40, normalized size = 0.68

$$\frac{3\,\tan\left(fx + e\right)^{5} + 10\,\tan\left(fx + e\right)^{3} + 15\,\tan\left(fx + e\right)}{15\,a^{3}c^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/15*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))/(a^3*c^3*f)
```

mupad [B] time = 8.34, size = 89, normalized size = 1.51

$$-\frac{2\tan\left(\frac{e}{2} + \frac{fx}{2}\right)\left(15\tan\left(\frac{e}{2} + \frac{fx}{2}\right)^{8} - 20\tan\left(\frac{e}{2} + \frac{fx}{2}\right)^{6} + 58\tan\left(\frac{e}{2} + \frac{fx}{2}\right)^{4} - 20\tan\left(\frac{e}{2} + \frac{fx}{2}\right)^{2} + 15\right)}{15\,a^{3}\,c^{3}\,f\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)^{2} - 1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^3),x)
```

```
[Out] -(2*tan(e/2 + (f*x)/2)*(58*tan(e/2 + (f*x)/2)^4 - 20*tan(e/2 + (f*x)/2)^2 -
 20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 + 15))/(15*a^3*c^3*f*(ta
n(e/2 + (f*x)/2)^2 - 1)^5)
```

sympy [A] time = 14.79, size = 687, normalized size = 11.64

$$\begin{cases} -\frac{30\tan^{9}\left(\frac{e}{2}+\frac{fx}{2}\right)}{15a^{3}c^{3}f\tan^{10}\left(\frac{e}{2}+\frac{fx}{2}\right)-75a^{3}c^{3}f\tan^{8}\left(\frac{e}{2}+\frac{fx}{2}\right)+150a^{3}c^{3}f\tan^{6}\left(\frac{e}{2}+\frac{fx}{2}\right)-150a^{3}c^{3}f\tan^{4}\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^{3}c^{3}f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)-15a^{3}c^{3}f} + \frac{}{15a^{3}c^{3}f\tan^{1}} \\ \frac{x}{\left(a\sin\left(e\right)+a\right)^{3}\left(-c\sin\left(e\right)+c\right)^{3}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*tan(e/2 + f*x/2)**9/(15*a**3*c**3*f*tan(e/2 + f*x/2)**10 - 7
5*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*f*tan(e/2 + f*x/2)**6 - 1
50*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3*f*tan(e/2 + f*x/2)**2 - 1
5*a**3*c**3*f) + 40*tan(e/2 + f*x/2)**7/(15*a**3*c**3*f*tan(e/2 + f*x/2)**1
0 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*f*tan(e/2 + f*x/2)**
6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3*f*tan(e/2 + f*x/2)**
2 - 15*a**3*c**3*f) - 116*tan(e/2 + f*x/2)**5/(15*a**3*c**3*f*tan(e/2 + f*x
/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*f*tan(e/2 + f*
x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3*f*tan(e/2 + f*
x/2)**2 - 15*a**3*c**3*f) + 40*tan(e/2 + f*x/2)**3/(15*a**3*c**3*f*tan(e/2
+ f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*f*tan(e/2
 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3*f*tan(e/2
 + f*x/2)**2 - 15*a**3*c**3*f) - 30*tan(e/2 + f*x/2)/(15*a**3*c**3*f*tan(e/
2 + f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*f*tan(e
/2 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3*f*tan(e
/2 + f*x/2)**2 - 15*a**3*c**3*f), Ne(f, 0)), (x/((a*sin(e) + a)**3*(-c*sin(
e) + c)**3), True))
```

3.287 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx$

Optimal. Leaf size=97

$$\frac{6\tan^5(e+fx)}{35a^3c^4f}+\frac{4\tan^3(e+fx)}{7a^3c^4f}+\frac{6\tan(e+fx)}{7a^3c^4f}+\frac{\sec^5(e+fx)}{7a^3f\left(c^4-c^4\sin(e+fx)\right)}$$

```
[Out] 1/7*sec(f*x+e)^5/a^3/f/(c^4-c^4*sin(f*x+e))+6/7*tan(f*x+e)/a^3/c^4/f+4/7*ta
n(f*x+e)^3/a^3/c^4/f+6/35*tan(f*x+e)^5/a^3/c^4/f
```

Rubi [A] time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2672, 3767}

$$\frac{6\tan^5(e+fx)}{35a^3c^4f}+\frac{4\tan^3(e+fx)}{7a^3c^4f}+\frac{6\tan(e+fx)}{7a^3c^4f}+\frac{\sec^5(e+fx)}{7a^3f\left(c^4-c^4\sin(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^4),x]
```

```
[Out] Sec[e + f*x]^5/(7*a^3*f*(c^4 - c^4*Sin[e + f*x])) + (6*Tan[e + f*x])/(7*a^3
*c^4*f) + (4*Tan[e + f*x]^3)/(7*a^3*c^4*f) + (6*Tan[e + f*x]^5)/(35*a^3*c^4
*f)
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx = \frac{\int \frac{\sec^6(e+fx)}{c-c\sin(e+fx)}\,dx}{a^3c^3}$$

$$= \frac{\sec^5(e+fx)}{7a^3f\left(c^4-c^4\sin(e+fx)\right)} + \frac{6\int \sec^6(e+fx)\,dx}{7a^3c^4}$$

$$= \frac{\sec^5(e+fx)}{7a^3f\left(c^4-c^4\sin(e+fx)\right)} - \frac{6\,\text{Subst}\left(\int\left(1+2x^2+x^4\right)dx, x, -\text{ta}\right.}{7a^3c^4f}$$

$$= \frac{\sec^5(e+fx)}{7a^3f\left(c^4-c^4\sin(e+fx)\right)} + \frac{6\tan(e+fx)}{7a^3c^4f} + \frac{4\tan^3(e+fx)}{7a^3c^4f} + \frac{6}{}$$

Mathematica [A] time = 1.19, size = 193, normalized size = 1.99

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(5120\sin(e+fx)+125\sin(2(e+fx))+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^4),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-500*Cos[e + f*x] + 1280*Cos[2*(e + f*x)] - 250*Cos[3*(e + f*x)] + 1024*
Cos[4*(e + f*x)] - 50*Cos[5*(e + f*x)] + 256*Cos[6*(e + f*x)] + 5120*Sin[e
+ f*x] + 125*Sin[2*(e + f*x)] + 2560*Sin[3*(e + f*x)] + 100*Sin[4*(e + f*x)
] + 512*Sin[5*(e + f*x)] + 25*Sin[6*(e + f*x)]))/(17920*f*(a + a*Sin[e + f*
x])^3*(c - c*Sin[e + f*x])^4)
```

fricas [A] time = 0.47, size = 106, normalized size = 1.09

$$-\frac{16\cos\left(fx+e\right)^6-8\cos\left(fx+e\right)^4-2\cos\left(fx+e\right)^2+2\left(8\cos\left(fx+e\right)^4+4\cos\left(fx+e\right)^2+3\right)\sin\left(fx+e\right)-}{35\left(a^3c^4f\cos\left(fx+e\right)^5\sin\left(fx+e\right)-a^3c^4f\cos\left(fx+e\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] -1/35*(16*cos(f*x + e)^6 - 8*cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 2*(8*cos(f
*x + e)^4 + 4*cos(f*x + e)^2 + 3)*sin(f*x + e) - 1)/(a^3*c^4*f*cos(f*x + e)
^5*sin(f*x + e) - a^3*c^4*f*cos(f*x + e)^5)
```

giac [B] time = 1.43, size = 189, normalized size = 1.95

$$-\frac{\frac{7\left(55\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+180\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+250\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+160\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+43\right)}{a^{3}c^{4}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{5}}+\frac{735\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6}-3360\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}+7315}{}}{560f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -1/560*(7*(55*tan(1/2*f*x + 1/2*e)^4 + 180*tan(1/2*f*x + 1/2*e)^3 + 250*tan
(1/2*f*x + 1/2*e)^2 + 160*tan(1/2*f*x + 1/2*e) + 43)/(a^3*c^4*(tan(1/2*f*x
+ 1/2*e) + 1)^5) + (735*tan(1/2*f*x + 1/2*e)^6 - 3360*tan(1/2*f*x + 1/2*e)^
5 + 7315*tan(1/2*f*x + 1/2*e)^4 - 8820*tan(1/2*f*x + 1/2*e)^3 + 6321*tan(1/
2*f*x + 1/2*e)^2 - 2492*tan(1/2*f*x + 1/2*e) + 461)/(a^3*c^4*(tan(1/2*f*x +
 1/2*e) - 1)^7))/f
```

maple [B] time = 0.29, size = 193, normalized size = 1.99

$$\frac{-\frac{2}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{1}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{21}{10\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{11}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{11}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{15}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{2}{16\left(\tan\right.}}{f\,c^{4}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x)
```

```
[Out] 2/f/c^4/a^3*(-1/7/(tan(1/2*f*x+1/2*e)-1)^7-1/2/(tan(1/2*f*x+1/2*e)-1)^6-21/
20/(tan(1/2*f*x+1/2*e)-1)^5-11/8/(tan(1/2*f*x+1/2*e)-1)^4-11/8/(tan(1/2*f*x
+1/2*e)-1)^3-15/16/(tan(1/2*f*x+1/2*e)-1)^2-21/32/(tan(1/2*f*x+1/2*e)-1)-1/
20/(tan(1/2*f*x+1/2*e)+1)^5+1/8/(tan(1/2*f*x+1/2*e)+1)^4-1/4/(tan(1/2*f*x+1
/2*e)+1)^3+1/4/(tan(1/2*f*x+1/2*e)+1)^2-11/32/(tan(1/2*f*x+1/2*e)+1))
```

maxima [B] time = 1.02, size = 519, normalized size = 5.35

$$\frac{2\left(\frac{25\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{55\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{15\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{130\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{26\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}-\frac{182\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}+\frac{126\sin\left(\right.}{\left(\cos\left(fx\right.\right.}\right.}{35\left(a^{3}c^{4}-\frac{2a^{3}c^{4}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{4a^{3}c^{4}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{10a^{3}c^{4}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{5a^{3}c^{4}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}-\frac{20a^{3}c^{4}\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}+\frac{20a^{3}c^{4}\sin\left(fx+e\right)^{7}}{\left(\cos\left(fx+e\right)+1\right)^{7}}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorithm="maxima")

[Out] 2/35*(25*sin(f*x + e)/(cos(f*x + e) + 1) - 55*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 130*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 26*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 182*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 126*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 105*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 35*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 35*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 35*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 5)/((a^3*c^4 - 2*a^3*c^4*sin(f*x + e)/(cos(f*x + e) + 1) - 4*a^3*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 20*a^3*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20*a^3*c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 5*a^3*c^4*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 10*a^3*c^4*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 4*a^3*c^4*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 2*a^3*c^4*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - a^3*c^4*sin(f*x + e)^12/(cos(f*x + e) + 1)^12)*f)

mupad [B] time = 9.42, size = 180, normalized size = 1.86

$$-\frac{2\left(35\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}-35\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-35\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}+105\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}+126\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}-182\,\mathrm{ta}\right.}{35\,a^3\,c^4\,f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^4),x)

[Out] -(2*(25*tan(e/2 + (f*x)/2) - 55*tan(e/2 + (f*x)/2)^2 + 15*tan(e/2 + (f*x)/2)^3 + 130*tan(e/2 + (f*x)/2)^4 + 26*tan(e/2 + (f*x)/2)^5 - 182*tan(e/2 + (f*x)/2)^6 + 126*tan(e/2 + (f*x)/2)^7 + 105*tan(e/2 + (f*x)/2)^8 - 35*tan(e/2 + (f*x)/2)^9 - 35*tan(e/2 + (f*x)/2)^10 + 35*tan(e/2 + (f*x)/2)^11 + 5))/(35*a^3*c^4*f*(tan(e/2 + (f*x)/2) - 1)^7*(tan(e/2 + (f*x)/2) + 1)^5)

sympy [A] time = 55.96, size = 3186, normalized size = 32.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**4,x)

[Out] Piecewise((-70*tan(e/2 + f*x/2)**11/(35*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 70*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 140*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 350*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 175*a**3*c**4*f*tan(e/2 + f*x/2)**8

- 700*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 700*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 175*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 350*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 140*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 70*a**3*c**4*f*tan(e/2 + f*x/2) - 35*a**3*c**4*f) + 70*tan(e/2 + f*x/2)**10/(35*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 70*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 140*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 350*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 175*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 700*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 700*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 175*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 350*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 140*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 70*a**3*c**4*f*tan(e/2 + f*x/2) - 35*a**3*c**4*f) + 70*tan(e/2 + f*x/2)**9/(35*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 70*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 140*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 350*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 175*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 700*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 700*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 175*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 350*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 140*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 70*a**3*c**4*f*tan(e/2 + f*x/2) - 35*a**3*c**4*f) - 210*tan(e/2 + f*x/2)**8/(35*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 70*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 140*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 350*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 175*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 700*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 700*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 175*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 350*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 140*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 70*a**3*c**4*f*tan(e/2 + f*x/2) - 35*a**3*c**4*f) - 252*tan(e/2 + f*x/2)**7/(35*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 70*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 140*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 350*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 175*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 700*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 700*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 175*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 350*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 140*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 70*a**3*c**4*f*tan(e/2 + f*x/2) - 35*a**3*c**4*f) + 364*tan(e/2 + f*x/2)**6/(35*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 70*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 140*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 350*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 175*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 700*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 700*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 175*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 350*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 140*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 70*a**3*c**4*f*tan(e/2 + f*x/2) - 35*a**3*c**4*f) - 52*tan(e/2 + f*x/2)**5/(35*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 70*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 140*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 350*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 175*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 700*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 700*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 175*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 350*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 140*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 70*a**3*c**4*f*tan(e/2 + f*x/2) - 35*a**3*c**4*f) - 260*tan(e/2 + f*x/2)**4/(35*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 70*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 140*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 350*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 175*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 700*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 700*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 175*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 350*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 140*a**3*c**4*f*

```
tan(e/2 + f*x/2)**2 + 70*a**3*c**4*f*tan(e/2 + f*x/2) - 35*a**3*c**4*f) - 3
0*tan(e/2 + f*x/2)**3/(35*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 70*a**3*c**4*f
*tan(e/2 + f*x/2)**11 - 140*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 350*a**3*c**
4*f*tan(e/2 + f*x/2)**9 + 175*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 700*a**3*c*
*4*f*tan(e/2 + f*x/2)**7 + 700*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 175*a**3*c
**4*f*tan(e/2 + f*x/2)**4 - 350*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 140*a**3*
c**4*f*tan(e/2 + f*x/2)**2 + 70*a**3*c**4*f*tan(e/2 + f*x/2) - 35*a**3*c**4
*f) + 110*tan(e/2 + f*x/2)**2/(35*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 70*a**
3*c**4*f*tan(e/2 + f*x/2)**11 - 140*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 350*
a**3*c**4*f*tan(e/2 + f*x/2)**9 + 175*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 700
*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 700*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 17
5*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 350*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 1
40*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 70*a**3*c**4*f*tan(e/2 + f*x/2) - 35*a
**3*c**4*f) - 50*tan(e/2 + f*x/2)/(35*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 70
*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 140*a**3*c**4*f*tan(e/2 + f*x/2)**10 +
350*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 175*a**3*c**4*f*tan(e/2 + f*x/2)**8 -
 700*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 700*a**3*c**4*f*tan(e/2 + f*x/2)**5
- 175*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 350*a**3*c**4*f*tan(e/2 + f*x/2)**3
 + 140*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 70*a**3*c**4*f*tan(e/2 + f*x/2) -
35*a**3*c**4*f) - 10/(35*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 70*a**3*c**4*f*
tan(e/2 + f*x/2)**11 - 140*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 350*a**3*c**4
*f*tan(e/2 + f*x/2)**9 + 175*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 700*a**3*c**
4*f*tan(e/2 + f*x/2)**7 + 700*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 175*a**3*c*
*4*f*tan(e/2 + f*x/2)**4 - 350*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 140*a**3*c
**4*f*tan(e/2 + f*x/2)**2 + 70*a**3*c**4*f*tan(e/2 + f*x/2) - 35*a**3*c**4*
f), Ne(f, 0)), (x/((a*sin(e) + a)**3*(-c*sin(e) + c)**4), True))
```

3.288 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx$

Optimal. Leaf size=131

$$\frac{2\tan^5(e+fx)}{15a^3c^5f}+\frac{4\tan^3(e+fx)}{9a^3c^5f}+\frac{2\tan(e+fx)}{3a^3c^5f}+\frac{\sec^5(e+fx)}{9a^3f\left(c^5-c^5\sin(e+fx)\right)}+\frac{\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2}$$

```
[Out] 1/9*sec(f*x+e)^5/a^3/c^3/f/(c-c*sin(f*x+e))^2+1/9*sec(f*x+e)^5/a^3/f/(c^5-c
^5*sin(f*x+e))+2/3*tan(f*x+e)/a^3/c^5/f+4/9*tan(f*x+e)^3/a^3/c^5/f+2/15*tan
(f*x+e)^5/a^3/c^5/f
```

Rubi [A] time = 0.17, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2672, 3767}

$$\frac{2\tan^5(e+fx)}{15a^3c^5f}+\frac{4\tan^3(e+fx)}{9a^3c^5f}+\frac{2\tan(e+fx)}{3a^3c^5f}+\frac{\sec^5(e+fx)}{9a^3f\left(c^5-c^5\sin(e+fx)\right)}+\frac{\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^5),x]
```

```
[Out] Sec[e + f*x]^5/(9*a^3*c^3*f*(c - c*Sin[e + f*x])^2) + Sec[e + f*x]^5/(9*a^3
*f*(c^5 - c^5*Sin[e + f*x])) + (2*Tan[e + f*x])/(3*a^3*c^5*f) + (4*Tan[e +
f*x]^3)/(9*a^3*c^5*f) + (2*Tan[e + f*x]^5)/(15*a^3*c^5*f)
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx = \frac{\int \frac{\sec^6(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{a^3c^3}$$
$$= \frac{\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2} + \frac{7\int \frac{\sec^6(e+fx)}{c-c\sin(e+fx)}\,dx}{9a^3c^4}$$
$$= \frac{\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2} + \frac{\sec^5(e+fx)}{9a^3f\left(c^5-c^5\sin(e+fx)\right)} + \frac{2\int \sec$$
$$= \frac{\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2} + \frac{\sec^5(e+fx)}{9a^3f\left(c^5-c^5\sin(e+fx)\right)} - \frac{2\,\mathrm{Subs}}{}$$
$$= \frac{\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2} + \frac{\sec^5(e+fx)}{9a^3f\left(c^5-c^5\sin(e+fx)\right)} + \frac{2\tan(e}{3a^3}$$

Mathematica [A] time = 1.54, size = 213, normalized size = 1.63

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(46080\sin(e+fx)+3500\sin(2(e+fx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^5),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-7875*Cos[e + f*x] + 20480*Cos[2*(e + f*x)] - 3325*Cos[3*(e + f*x)] + 16
384*Cos[4*(e + f*x)] - 175*Cos[5*(e + f*x)] + 4096*Cos[6*(e + f*x)] + 175*C
os[7*(e + f*x)] + 46080*Sin[e + f*x] + 3500*Sin[2*(e + f*x)] + 19456*Sin[3*
(e + f*x)] + 2800*Sin[4*(e + f*x)] + 1024*Sin[5*(e + f*x)] + 700*Sin[6*(e +
 f*x)] - 1024*Sin[7*(e + f*x)]))/(184320*f*(a + a*Sin[e + f*x])^3*(c - c*Si
n[e + f*x])^5)
```

fricas [A] time = 0.46, size = 133, normalized size = 1.02

$$-\frac{32\cos\left(fx+e\right)^6-16\cos\left(fx+e\right)^4-4\cos\left(fx+e\right)^2-\left(16\cos\left(fx+e\right)^6-24\cos\left(fx+e\right)^4-10\cos\left(fx+e\right.}{45\left(a^3c^5f\cos\left(fx+e\right)^7+2\,a^3c^5f\cos\left(fx+e\right)^5\sin\left(fx+e\right)-2\,a^3c^5f\cos\left(fx+e\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] -1/45*(32*cos(f*x + e)^6 - 16*cos(f*x + e)^4 - 4*cos(f*x + e)^2 - (16*cos(f
*x + e)^6 - 24*cos(f*x + e)^4 - 10*cos(f*x + e)^2 - 7)*sin(f*x + e) - 2)/(a
^3*c^5*f*cos(f*x + e)^7 + 2*a^3*c^5*f*cos(f*x + e)^5*sin(f*x + e) - 2*a^3*c
^5*f*cos(f*x + e)^5)
```

giac [A] time = 0.25, size = 217, normalized size = 1.66

$$-\frac{\frac{3\left(435\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+1470\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+2060\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+1330\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+353\right)}{a^{3}c^{5}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{5}}+\frac{4455\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{8}-26460\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algorithm="giac")
```

```
[Out] -1/2880*(3*(435*tan(1/2*f*x + 1/2*e)^4 + 1470*tan(1/2*f*x + 1/2*e)^3 + 2060
*tan(1/2*f*x + 1/2*e)^2 + 1330*tan(1/2*f*x + 1/2*e) + 353)/(a^3*c^5*(tan(1/
2*f*x + 1/2*e) + 1)^5) + (4455*tan(1/2*f*x + 1/2*e)^8 - 26460*tan(1/2*f*x +
 1/2*e)^7 + 78120*tan(1/2*f*x + 1/2*e)^6 - 137340*tan(1/2*f*x + 1/2*e)^5 +
157374*tan(1/2*f*x + 1/2*e)^4 - 118356*tan(1/2*f*x + 1/2*e)^3 + 57744*tan(1
/2*f*x + 1/2*e)^2 - 16596*tan(1/2*f*x + 1/2*e) + 2339)/(a^3*c^5*(tan(1/2*f*
x + 1/2*e) - 1)^9))/f
```

maple [A] time = 0.29, size = 223, normalized size = 1.70

$$\frac{-\frac{4}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{9}}-\frac{2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{8}}-\frac{5}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{49}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{49}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{35}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{49}{8\left(\tan\left(\frac{fx}{2}+\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x)
```

```
[Out] 2/f/a^3/c^5*(-2/9/(tan(1/2*f*x+1/2*e)-1)^9-1/(tan(1/2*f*x+1/2*e)-1)^8-5/2/(
tan(1/2*f*x+1/2*e)-1)^7-49/12/(tan(1/2*f*x+1/2*e)-1)^6-49/10/(tan(1/2*f*x+1
/2*e)-1)^5-35/8/(tan(1/2*f*x+1/2*e)-1)^4-49/16/(tan(1/2*f*x+1/2*e)-1)^3-51/
32/(tan(1/2*f*x+1/2*e)-1)^2-99/128/(tan(1/2*f*x+1/2*e)-1)-1/40/(tan(1/2*f*x
+1/2*e)+1)^5+1/16/(tan(1/2*f*x+1/2*e)+1)^4-13/96/(tan(1/2*f*x+1/2*e)+1)^3+9
/64/(tan(1/2*f*x+1/2*e)+1)^2-29/128/(tan(1/2*f*x+1/2*e)+1))
```

maxima [B] time = 0.44, size = 610, normalized size = 4.66

$$\frac{2\left(\frac{5\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{80\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{190\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{50\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}-\frac{269\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}+\frac{96\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}+\frac{516\sin(\ldots}{(\cos(fx+\ldots}\right.}{45\left(a^{3}c^{5}-\frac{4a^{3}c^{5}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{a^{3}c^{5}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{16a^{3}c^{5}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}-\frac{19a^{3}c^{5}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}-\frac{20a^{3}c^{5}\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}+\frac{45a^{3}c^{5}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}-\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] 2/45*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 80*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 190*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 50*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 - 269*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 96*sin(f*x + e)^6
/(cos(f*x + e) + 1)^6 + 516*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 354*sin(f
*x + e)^8/(cos(f*x + e) + 1)^8 - 69*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 2
40*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 30*sin(f*x + e)^11/(cos(f*x + e)
 + 1)^11 - 90*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 + 45*sin(f*x + e)^13/(c
os(f*x + e) + 1)^13 + 10)/((a^3*c^5 - 4*a^3*c^5*sin(f*x + e)/(cos(f*x + e)
+ 1) + a^3*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 16*a^3*c^5*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 - 19*a^3*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
 20*a^3*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 45*a^3*c^5*sin(f*x + e)^6
/(cos(f*x + e) + 1)^6 - 45*a^3*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 20
*a^3*c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 19*a^3*c^5*sin(f*x + e)^10/(
cos(f*x + e) + 1)^10 - 16*a^3*c^5*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - a
^3*c^5*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 + 4*a^3*c^5*sin(f*x + e)^13/(c
os(f*x + e) + 1)^13 - a^3*c^5*sin(f*x + e)^14/(cos(f*x + e) + 1)^14)*f)
```

mupad [B] time = 8.20, size = 190, normalized size = 1.45

$$-\frac{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{65\cos\left(\frac{5e}{2}+\frac{5fx}{2}\right)}{32}-\frac{225\cos\left(\frac{3e}{2}+\frac{3fx}{2}\right)}{32}-5\cos\left(\frac{7e}{2}+\frac{7fx}{2}\right)+\cos\left(\frac{9e}{2}+\frac{9fx}{2}\right)-\frac{37\cos\left(\frac{11e}{2}+\frac{11fx}{2}\right)}{32}+\frac{5\cos\left(\frac{13\ldots}{2}\right.}{3\ldots}\right.}{2880\,a^{3}\,c^{5}\,f\cos\left(\frac{e}{2}\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^5),x)
```

```
[Out] -(cos(e/2 + (f*x)/2)*((65*cos((5*e)/2 + (5*f*x)/2))/32 - (225*cos((3*e)/2 +
 (3*f*x)/2))/32 - 5*cos((7*e)/2 + (7*f*x)/2) + cos((9*e)/2 + (9*f*x)/2) - (
37*cos((11*e)/2 + (11*f*x)/2))/32 + (5*cos((13*e)/2 + (13*f*x)/2))/32 - (89
*sin(e/2 + (f*x)/2))/4 + 11*sin((3*e)/2 + (3*f*x)/2) - (63*sin((5*e)/2 + (5
*f*x)/2))/8 + (25*sin((7*e)/2 + (7*f*x)/2))/8 - (5*sin((9*e)/2 + (9*f*x)/2)
```

```
)/8 + (3*sin((11*e)/2 + (11*f*x)/2))/8 + sin((13*e)/2 + (13*f*x)/2)/4))/(28
80*a^3*c^5*f*cos(e/2 - pi/4 + (f*x)/2)^5*cos(e/2 + pi/4 + (f*x)/2)^9)
```

sympy [A] time = 98.60, size = 4335, normalized size = 33.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((-90*tan(e/2 + f*x/2)**13/(45*a**3*c**5*f*tan(e/2 + f*x/2)**14 -
180*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 45*a**3*c**5*f*tan(e/2 + f*x/2)**12
+ 720*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 855*a**3*c**5*f*tan(e/2 + f*x/2)**
10 - 900*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 2025*a**3*c**5*f*tan(e/2 + f*x/2
)**8 - 2025*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 900*a**3*c**5*f*tan(e/2 + f*x
/2)**5 + 855*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 720*a**3*c**5*f*tan(e/2 + f*
x/2)**3 - 45*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 180*a**3*c**5*f*tan(e/2 + f*
x/2) - 45*a**3*c**5*f) + 180*tan(e/2 + f*x/2)**12/(45*a**3*c**5*f*tan(e/2 +
 f*x/2)**14 - 180*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 45*a**3*c**5*f*tan(e/2
 + f*x/2)**12 + 720*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 855*a**3*c**5*f*tan(
e/2 + f*x/2)**10 - 900*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 2025*a**3*c**5*f*t
an(e/2 + f*x/2)**8 - 2025*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 900*a**3*c**5*f
*tan(e/2 + f*x/2)**5 + 855*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 720*a**3*c**5*
f*tan(e/2 + f*x/2)**3 - 45*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 180*a**3*c**5*
f*tan(e/2 + f*x/2) - 45*a**3*c**5*f) - 60*tan(e/2 + f*x/2)**11/(45*a**3*c**
5*f*tan(e/2 + f*x/2)**14 - 180*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 45*a**3*c
**5*f*tan(e/2 + f*x/2)**12 + 720*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 855*a**
3*c**5*f*tan(e/2 + f*x/2)**10 - 900*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 2025*
a**3*c**5*f*tan(e/2 + f*x/2)**8 - 2025*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 90
0*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 855*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 7
20*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 45*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1
80*a**3*c**5*f*tan(e/2 + f*x/2) - 45*a**3*c**5*f) - 480*tan(e/2 + f*x/2)**1
0/(45*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 180*a**3*c**5*f*tan(e/2 + f*x/2)**
13 + 45*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 720*a**3*c**5*f*tan(e/2 + f*x/2)
**11 - 855*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 900*a**3*c**5*f*tan(e/2 + f*x
/2)**9 + 2025*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 2025*a**3*c**5*f*tan(e/2 +
f*x/2)**6 + 900*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 855*a**3*c**5*f*tan(e/2 +
 f*x/2)**4 - 720*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 45*a**3*c**5*f*tan(e/2 +
 f*x/2)**2 + 180*a**3*c**5*f*tan(e/2 + f*x/2) - 45*a**3*c**5*f) + 138*tan(e
/2 + f*x/2)**9/(45*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 180*a**3*c**5*f*tan(e
/2 + f*x/2)**13 + 45*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 720*a**3*c**5*f*tan
(e/2 + f*x/2)**11 - 855*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 900*a**3*c**5*f*
tan(e/2 + f*x/2)**9 + 2025*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 2025*a**3*c**5
*f*tan(e/2 + f*x/2)**6 + 900*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 855*a**3*c**
5*f*tan(e/2 + f*x/2)**4 - 720*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 45*a**3*c**
```

5*f*tan(e/2 + f*x/2)**2 + 180*a**3*c**5*f*tan(e/2 + f*x/2) - 45*a**3*c**5*f
) + 708*tan(e/2 + f*x/2)**8/(45*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 180*a**3
*c**5*f*tan(e/2 + f*x/2)**13 + 45*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 720*a*
*3*c**5*f*tan(e/2 + f*x/2)**11 - 855*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 900
*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 2025*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 2
025*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 900*a**3*c**5*f*tan(e/2 + f*x/2)**5 +
855*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 720*a**3*c**5*f*tan(e/2 + f*x/2)**3
- 45*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 180*a**3*c**5*f*tan(e/2 + f*x/2) - 4
5*a**3*c**5*f) - 1032*tan(e/2 + f*x/2)**7/(45*a**3*c**5*f*tan(e/2 + f*x/2)*
*14 - 180*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 45*a**3*c**5*f*tan(e/2 + f*x/2
)**12 + 720*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 855*a**3*c**5*f*tan(e/2 + f*
x/2)**10 - 900*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 2025*a**3*c**5*f*tan(e/2 +
f*x/2)**8 - 2025*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 900*a**3*c**5*f*tan(e/2
+ f*x/2)**5 + 855*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 720*a**3*c**5*f*tan(e/
2 + f*x/2)**3 - 45*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 180*a**3*c**5*f*tan(e/
2 + f*x/2) - 45*a**3*c**5*f) - 192*tan(e/2 + f*x/2)**6/(45*a**3*c**5*f*tan(
e/2 + f*x/2)**14 - 180*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 45*a**3*c**5*f*ta
n(e/2 + f*x/2)**12 + 720*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 855*a**3*c**5*f
*tan(e/2 + f*x/2)**10 - 900*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 2025*a**3*c**
5*f*tan(e/2 + f*x/2)**8 - 2025*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 900*a**3*c
5*f*tan(e/2 + f*x/2)5 + 855*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 720*a**3*
c**5*f*tan(e/2 + f*x/2)**3 - 45*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 180*a**3*
c**5*f*tan(e/2 + f*x/2) - 45*a**3*c**5*f) + 538*tan(e/2 + f*x/2)**5/(45*a**
3*c**5*f*tan(e/2 + f*x/2)**14 - 180*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 45*a
3*c5*f*tan(e/2 + f*x/2)**12 + 720*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 85
5*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 900*a**3*c**5*f*tan(e/2 + f*x/2)**9 +
2025*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 2025*a**3*c**5*f*tan(e/2 + f*x/2)**6
+ 900*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 855*a**3*c**5*f*tan(e/2 + f*x/2)**
4 - 720*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 45*a**3*c**5*f*tan(e/2 + f*x/2)**
2 + 180*a**3*c**5*f*tan(e/2 + f*x/2) - 45*a**3*c**5*f) - 100*tan(e/2 + f*x/
2)**4/(45*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 180*a**3*c**5*f*tan(e/2 + f*x/
2)**13 + 45*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 720*a**3*c**5*f*tan(e/2 + f*
x/2)**11 - 855*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 900*a**3*c**5*f*tan(e/2 +
f*x/2)**9 + 2025*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 2025*a**3*c**5*f*tan(e/
2 + f*x/2)**6 + 900*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 855*a**3*c**5*f*tan(e
/2 + f*x/2)**4 - 720*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 45*a**3*c**5*f*tan(e
/2 + f*x/2)**2 + 180*a**3*c**5*f*tan(e/2 + f*x/2) - 45*a**3*c**5*f) - 380*t
an(e/2 + f*x/2)**3/(45*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 180*a**3*c**5*f*t
an(e/2 + f*x/2)**13 + 45*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 720*a**3*c**5*f
*tan(e/2 + f*x/2)**11 - 855*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 900*a**3*c**
5*f*tan(e/2 + f*x/2)**9 + 2025*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 2025*a**3*
c**5*f*tan(e/2 + f*x/2)**6 + 900*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 855*a**3
*c**5*f*tan(e/2 + f*x/2)**4 - 720*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 45*a**3
*c**5*f*tan(e/2 + f*x/2)**2 + 180*a**3*c**5*f*tan(e/2 + f*x/2) - 45*a**3*c*
*5*f) + 160*tan(e/2 + f*x/2)**2/(45*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 180*

```
a**3*c**5*f*tan(e/2 + f*x/2)**13 + 45*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 72
0*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 855*a**3*c**5*f*tan(e/2 + f*x/2)**10 -
 900*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 2025*a**3*c**5*f*tan(e/2 + f*x/2)**8
 - 2025*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 900*a**3*c**5*f*tan(e/2 + f*x/2)*
*5 + 855*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 720*a**3*c**5*f*tan(e/2 + f*x/2)
**3 - 45*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 180*a**3*c**5*f*tan(e/2 + f*x/2)
 - 45*a**3*c**5*f) - 10*tan(e/2 + f*x/2)/(45*a**3*c**5*f*tan(e/2 + f*x/2)**
14 - 180*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 45*a**3*c**5*f*tan(e/2 + f*x/2)
**12 + 720*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 855*a**3*c**5*f*tan(e/2 + f*x
/2)**10 - 900*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 2025*a**3*c**5*f*tan(e/2 +
f*x/2)**8 - 2025*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 900*a**3*c**5*f*tan(e/2
+ f*x/2)**5 + 855*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 720*a**3*c**5*f*tan(e/2
 + f*x/2)**3 - 45*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 180*a**3*c**5*f*tan(e/2
 + f*x/2) - 45*a**3*c**5*f) - 20/(45*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 180
*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 45*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 7
20*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 855*a**3*c**5*f*tan(e/2 + f*x/2)**10
- 900*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 2025*a**3*c**5*f*tan(e/2 + f*x/2)**
8 - 2025*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 900*a**3*c**5*f*tan(e/2 + f*x/2)
**5 + 855*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 720*a**3*c**5*f*tan(e/2 + f*x/2
)**3 - 45*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 180*a**3*c**5*f*tan(e/2 + f*x/2
) - 45*a**3*c**5*f), Ne(f, 0)), (x/((a*sin(e) + a)**3*(-c*sin(e) + c)**5),
True))
```

3.289 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx$

Optimal. Leaf size=167

$$\frac{16\tan^5(e+fx)}{165a^3c^6f}+\frac{32\tan^3(e+fx)}{99a^3c^6f}+\frac{16\tan(e+fx)}{33a^3c^6f}+\frac{8\sec^5(e+fx)}{99a^3f\left(c^6-c^6\sin(e+fx)\right)}+\frac{8\sec^5(e+fx)}{99a^3f\left(c^3-c^3\sin(e+fx)\right)^2}+\frac{}{11}$$

```
[Out] 1/11*sec(f*x+e)^5/a^3/f/(c^2-c^2*sin(f*x+e))^3+8/99*sec(f*x+e)^5/a^3/f/(c^3
-c^3*sin(f*x+e))^2+8/99*sec(f*x+e)^5/a^3/f/(c^6-c^6*sin(f*x+e))+16/33*tan(f
*x+e)/a^3/c^6/f+32/99*tan(f*x+e)^3/a^3/c^6/f+16/165*tan(f*x+e)^5/a^3/c^6/f
```

Rubi [A] time = 0.22, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2672, 3767}

$$\frac{16\tan^5(e+fx)}{165a^3c^6f}+\frac{32\tan^3(e+fx)}{99a^3c^6f}+\frac{16\tan(e+fx)}{33a^3c^6f}+\frac{8\sec^5(e+fx)}{99a^3f\left(c^6-c^6\sin(e+fx)\right)}+\frac{8\sec^5(e+fx)}{99a^3f\left(c^3-c^3\sin(e+fx)\right)^2}+\frac{}{11}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^6),x]
```

```
[Out] Sec[e + f*x]^5/(11*a^3*f*(c^2 - c^2*Sin[e + f*x])^3) + (8*Sec[e + f*x]^5)/(
99*a^3*f*(c^3 - c^3*Sin[e + f*x])^2) + (8*Sec[e + f*x]^5)/(99*a^3*f*(c^6 -
c^6*Sin[e + f*x])) + (16*Tan[e + f*x])/(33*a^3*c^6*f) + (32*Tan[e + f*x]^3)
/(99*a^3*c^6*f) + (16*Tan[e + f*x]^5)/(165*a^3*c^6*f)
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx = \frac{\int \frac{\sec^6(e+fx)}{(c-c\sin(e+fx))^3}\,dx}{a^3c^3}$$

$$= \frac{\sec^5(e+fx)}{11a^3f\left(c^2-c^2\sin(e+fx)\right)^3} + \frac{8\int \frac{\sec^6(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{11a^3c^4}$$

$$= \frac{\sec^5(e+fx)}{11a^3f\left(c^2-c^2\sin(e+fx)\right)^3} + \frac{8\sec^5(e+fx)}{99a^3f\left(c^3-c^3\sin(e+fx)\right)^2} + \frac{5}{}$$

$$= \frac{\sec^5(e+fx)}{11a^3f\left(c^2-c^2\sin(e+fx)\right)^3} + \frac{8\sec^5(e+fx)}{99a^3f\left(c^3-c^3\sin(e+fx)\right)^2} + \frac{}{9}$$

$$= \frac{\sec^5(e+fx)}{11a^3f\left(c^2-c^2\sin(e+fx)\right)^3} + \frac{8\sec^5(e+fx)}{99a^3f\left(c^3-c^3\sin(e+fx)\right)^2} + \frac{}{9}$$

$$= \frac{\sec^5(e+fx)}{11a^3f\left(c^2-c^2\sin(e+fx)\right)^3} + \frac{8\sec^5(e+fx)}{99a^3f\left(c^3-c^3\sin(e+fx)\right)^2} + \frac{}{9}$$

Mathematica [A] time = 1.63, size = 233, normalized size = 1.40

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(1802240\sin(e+fx)+247170\sin(2(e}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^6),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-411950*Cos[e + f*x] + 1081344*Cos[2*(e + f*x)] - 127330*Cos[3*(e + f*x)
] + 819200*Cos[4*(e + f*x)] + 37450*Cos[5*(e + f*x)] + 163840*Cos[6*(e + f*
x)] + 22470*Cos[7*(e + f*x)] - 16384*Cos[8*(e + f*x)] + 1802240*Sin[e + f*x
] + 247170*Sin[2*(e + f*x)] + 557056*Sin[3*(e + f*x)] + 187250*Sin[4*(e + f
```

```
*x)] - 163840*Sin[5*(e + f*x)] + 37450*Sin[6*(e + f*x)] - 98304*Sin[7*(e +
f*x)] - 3745*Sin[8*(e + f*x)]))/(8110080*f*(a + a*Sin[e + f*x])^3*(c - c*Si
n[e + f*x])^6)
```

fricas [A] time = 0.46, size = 163, normalized size = 0.98

$$\frac{128 \cos\left(fx+e\right)^{8} - 576 \cos\left(fx+e\right)^{6} + 240 \cos\left(fx+e\right)^{4} + 56 \cos\left(fx+e\right)^{2} + 8\left(48 \cos\left(fx+e\right)^{6} - 40 \cos\left(f\right.\right.}{495\left(3\,a^{3}c^{6}f\cos\left(fx+e\right)^{7} - 4\,a^{3}c^{6}f\cos\left(fx+e\right)^{5} - \left(a^{3}c^{6}f\cos\left(fx+e\right)^{7} - 4\,a^{3}c^{6}f\cos\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorithm="fricas")
```

```
[Out] 1/495*(128*cos(f*x + e)^8 - 576*cos(f*x + e)^6 + 240*cos(f*x + e)^4 + 56*co
s(f*x + e)^2 + 8*(48*cos(f*x + e)^6 - 40*cos(f*x + e)^4 - 14*cos(f*x + e)^2
 - 9)*sin(f*x + e) + 27)/(3*a^3*c^6*f*cos(f*x + e)^7 - 4*a^3*c^6*f*cos(f*x
+ e)^5 - (a^3*c^6*f*cos(f*x + e)^7 - 4*a^3*c^6*f*cos(f*x + e)^5)*sin(f*x +
e))
```

giac [A] time = 0.26, size = 245, normalized size = 1.47

$$-\frac{\frac{33\left(555 \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+1920 \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+2710 \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+1760 \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+463\right)}{a^{3}c^{6}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{5}} + \frac{108405 \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{10} - 784080 \tan\left(\frac{1}{2}fx\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorithm="giac")
```

```
[Out] -1/63360*(33*(555*tan(1/2*f*x + 1/2*e)^4 + 1920*tan(1/2*f*x + 1/2*e)^3 + 27
10*tan(1/2*f*x + 1/2*e)^2 + 1760*tan(1/2*f*x + 1/2*e) + 463)/(a^3*c^6*(tan(
1/2*f*x + 1/2*e) + 1)^5) + (108405*tan(1/2*f*x + 1/2*e)^10 - 784080*tan(1/2
*f*x + 1/2*e)^9 + 2901195*tan(1/2*f*x + 1/2*e)^8 - 6652800*tan(1/2*f*x + 1/
2*e)^7 + 10407474*tan(1/2*f*x + 1/2*e)^6 - 11435424*tan(1/2*f*x + 1/2*e)^5
+ 8949270*tan(1/2*f*x + 1/2*e)^4 - 4899840*tan(1/2*f*x + 1/2*e)^3 + 1816265
*tan(1/2*f*x + 1/2*e)^2 - 411664*tan(1/2*f*x + 1/2*e) + 47279)/(a^3*c^6*(ta
n(1/2*f*x + 1/2*e) - 1)^11))/f
```

maple [A] time = 0.32, size = 253, normalized size = 1.51

$$\frac{-\frac{8}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{11}} - \frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{10}} - \frac{106}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{9}} - \frac{23}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{8}} - \frac{33}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}} - \frac{217}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}} - \frac{623}{20\left(\tan\left(\frac{fx}{2}\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x)
```

```
[Out] 2/f/a^3/c^6*(-4/11/(tan(1/2*f*x+1/2*e)-1)^11-2/(tan(1/2*f*x+1/2*e)-1)^10-53
/9/(tan(1/2*f*x+1/2*e)-1)^9-23/2/(tan(1/2*f*x+1/2*e)-1)^8-33/2/(tan(1/2*f*x
+1/2*e)-1)^7-217/12/(tan(1/2*f*x+1/2*e)-1)^6-623/40/(tan(1/2*f*x+1/2*e)-1)^
5-169/16/(tan(1/2*f*x+1/2*e)-1)^4-365/64/(tan(1/2*f*x+1/2*e)-1)^3-303/128/(
tan(1/2*f*x+1/2*e)-1)^2-219/256/(tan(1/2*f*x+1/2*e)-1)-1/80/(tan(1/2*f*x+1/
2*e)+1)^5+1/32/(tan(1/2*f*x+1/2*e)+1)^4-7/96/(tan(1/2*f*x+1/2*e)+1)^3+5/64/
(tan(1/2*f*x+1/2*e)+1)^2-37/256/(tan(1/2*f*x+1/2*e)+1))
```

maxima [B] time = 0.73, size = 703, normalized size = 4.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorithm="maxima")
```

```
[Out] -2/495*(255*sin(f*x + e)/(cos(f*x + e) + 1) + 235*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 3065*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3775*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 667*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 8217*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6 + 2035*sin(f*x + e)^7/(cos(f*x + e) + 1)^7
+ 8745*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 11715*sin(f*x + e)^9/(cos(f*x
+ e) + 1)^9 + 33*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 4917*sin(f*x + e)^
11/(cos(f*x + e) + 1)^11 - 2475*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 181
5*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 1485*sin(f*x + e)^14/(cos(f*x + e
) + 1)^14 - 495*sin(f*x + e)^15/(cos(f*x + e) + 1)^15 - 125)/((a^3*c^6 - 6*
a^3*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*c^6*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 10*a^3*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 50*a^3*c^6
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 34*a^3*c^6*sin(f*x + e)^5/(cos(f*x +
 e) + 1)^5 + 66*a^3*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 110*a^3*c^6*s
in(f*x + e)^7/(cos(f*x + e) + 1)^7 + 110*a^3*c^6*sin(f*x + e)^9/(cos(f*x +
e) + 1)^9 - 66*a^3*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 34*a^3*c^6*s
in(f*x + e)^11/(cos(f*x + e) + 1)^11 + 50*a^3*c^6*sin(f*x + e)^12/(cos(f*x
+ e) + 1)^12 - 10*a^3*c^6*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 - 10*a^3*c^
6*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 + 6*a^3*c^6*sin(f*x + e)^15/(cos(f*
x + e) + 1)^15 - a^3*c^6*sin(f*x + e)^16/(cos(f*x + e) + 1)^16)*f)
```

mupad [B] time = 8.62, size = 185, normalized size = 1.11

$$-\frac{\frac{2\sin\left(e+f\,x\right)}{9}+\frac{2\cos\left(2\,e+2\,f\,x\right)}{15}+\frac{10\cos\left(4\,e+4\,f\,x\right)}{99}+\frac{2\cos\left(6\,e+6\,f\,x\right)}{99}-\frac{\cos\left(8\,e+8\,f\,x\right)}{495}+\frac{34\sin\left(3\,e+3\,f\,x\right)}{495}-\frac{2\sin\left(5\,e+5\,f\,x\right)}{99}-}{a^3\,c^6\,f\,\left(\frac{5\cos\left(5\,e+5\,f\,x\right)}{64}-\frac{17\cos\left(3\,e+3\,f\,x\right)}{64}-\frac{55\cos\left(e+f\,x\right)}{64}+\frac{3\cos\left(7\,e+7\,f\,x\right)}{64}+\frac{33\sin\left(2\,e+2\,f\,x\right)}{64}+\frac{25\sin\left(4\,e+4\,f\,x\right)}{64}+\frac{5\sin\left(6\,e+6\right.}{64}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^6),x)
```

```
[Out] -((2*sin(e + f*x))/9 + (2*cos(2*e + 2*f*x))/15 + (10*cos(4*e + 4*f*x))/99 +
 (2*cos(6*e + 6*f*x))/99 - cos(8*e + 8*f*x)/495 + (34*sin(3*e + 3*f*x))/495
 - (2*sin(5*e + 5*f*x))/99 - (2*sin(7*e + 7*f*x))/165)/(a^3*c^6*f*((5*cos(5
*e + 5*f*x))/64 - (17*cos(3*e + 3*f*x))/64 - (55*cos(e + f*x))/64 + (3*cos(
7*e + 7*f*x))/64 + (33*sin(2*e + 2*f*x))/64 + (25*sin(4*e + 4*f*x))/64 + (5
*sin(6*e + 6*f*x))/64 - sin(8*e + 8*f*x)/128))
```

sympy [A] time = 160.15, size = 5661, normalized size = 33.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**6,x)
```

```
[Out] Piecewise((-990*tan(e/2 + f*x/2)**15/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16
- 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)
**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 +
f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan
(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6
*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3
*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*
a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 29
70*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) + 2970*tan(e/2 + f*x/2)*
*14/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/
2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 +
 f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*ta
n(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c*
*6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a*
*3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 247
50*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 -
 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) -
 495*a**3*c**6*f) - 3630*tan(e/2 + f*x/2)**13/(495*a**3*c**6*f*tan(e/2 + f*
x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2
 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*t
an(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c
**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*
a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 1
6830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**
4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2
)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) - 4950*tan(e/2
+ f*x/2)**12/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e
```

/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*
tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*
c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 5445
0*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 -
 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)
5 + 24750*a3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*
x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 +
 f*x/2) - 495*a**3*c**6*f) + 9834*tan(e/2 + f*x/2)**11/(495*a**3*c**6*f*tan
(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*
f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3
*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 326
70*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9
 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/
2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 +
 f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/
2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) + 66*t
an(e/2 + f*x/2)**10/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*
f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*
c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 1683
0*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10
 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/
2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 +
 f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e
/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*ta
n(e/2 + f*x/2) - 495*a**3*c**6*f) - 23430*tan(e/2 + f*x/2)**9/(495*a**3*c**
6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**
3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 247
50*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**1
1 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*
x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2
 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*ta
n(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f
*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f)
 + 17490*tan(e/2 + f*x/2)**8/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a
3*c6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4
950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**
12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f
*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e
/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*
tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**
6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*
c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) + 4070*tan(e/2 + f*x/2)**7/(495*
a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 +
4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**
13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f

```
*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(
e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f
*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c
**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**
3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3
*c**6*f) - 16434*tan(e/2 + f*x/2)**6/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16
- 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)
**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 +
f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan
(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6
*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3
*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*
a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 29
70*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) + 1334*tan(e/2 + f*x/2)*
*5/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2
)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 +
f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan
(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**
6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**
3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 2475
0*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 -
4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) -
495*a**3*c**6*f) + 7550*tan(e/2 + f*x/2)**4/(495*a**3*c**6*f*tan(e/2 + f*x/
2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 +
 f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan
(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**
6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a*
*3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 168
30*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4
- 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)*
*2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) - 6130*tan(e/2 +
f*x/2)**3/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2
+ f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan
(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**
6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a
**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32
670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5
 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2
)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*
x/2) - 495*a**3*c**6*f) + 470*tan(e/2 + f*x/2)**2/(495*a**3*c**6*f*tan(e/2
+ f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan
(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6
*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a*
*3*c**6*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54
450*a**3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6
```

```
 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/
2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f
*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) + 510*tan(e
/2 + f*x/2)/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/
2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*t
an(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 16830*a**3*c
**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**10 - 54450
*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*x/2)**7 -
32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/2 + f*x/2)*
*5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*tan(e/2 + f*x
/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6*f*tan(e/2 +
f*x/2) - 495*a**3*c**6*f) - 250/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 297
0*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14
+ 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2
)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2
+ f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*ta
n(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6
*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*
c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a*
*3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f), Ne(f, 0)), (x/((a*sin(e) + a
)**3*(-c*sin(e) + c)**6), True))
```

3.290 $\int (a+a\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=137

$$\frac{256ac^5\cos^3(e+fx)}{315f(c-c\sin(e+fx))^{3/2}}+\frac{64ac^4\cos^3(e+fx)}{105f\sqrt{c-c\sin(e+fx)}}+\frac{8ac^3\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{21f}+\frac{2ac^2\cos^3(e+fx)(c-}{9f}$$

```
[Out] 256/315*a*c^5*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)+2/9*a*c^2*cos(f*x+e)^3*
(c-c*sin(f*x+e))^(3/2)/f+64/105*a*c^4*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(1/2)
+8/21*a*c^3*cos(f*x+e)^3*(c-c*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.29, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2674, 2673}

$$\frac{256ac^5\cos^3(e+fx)}{315f(c-c\sin(e+fx))^{3/2}}+\frac{64ac^4\cos^3(e+fx)}{105f\sqrt{c-c\sin(e+fx)}}+\frac{8ac^3\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{21f}+\frac{2ac^2\cos^3(e+fx)(c-}{9f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (256*a*c^5*Cos[e + f*x]^3)/(315*f*(c - c*Sin[e + f*x])^(3/2)) + (64*a*c^4*C
os[e + f*x]^3)/(105*f*Sqrt[c - c*Sin[e + f*x]]) + (8*a*c^3*Cos[e + f*x]^3*S
qrt[c - c*Sin[e + f*x]])/(21*f) + (2*a*c^2*Cos[e + f*x]^3*(c - c*Sin[e + f*
x])^(3/2))/(9*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a + a\sin(e+fx))(c - c\sin(e+fx))^{7/2}\,dx = (ac)\int \cos^2(e+fx)(c - c\sin(e+fx))^{5/2}\,dx$$
$$= \frac{2ac^2\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{9f} + \frac{1}{3}\left(4ac^2\right)\int \cos^2(e+f$$
$$= \frac{8ac^3\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{21f} + \frac{2ac^2\cos^3(e+fx)(c-c\,\mathrm{s}}{9f}$$
$$= \frac{64ac^4\cos^3(e+fx)}{105f\sqrt{c-c\sin(e+fx)}} + \frac{8ac^3\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{21f}$$
$$= \frac{256ac^5\cos^3(e+fx)}{315f(c-c\sin(e+fx))^{3/2}} + \frac{64ac^4\cos^3(e+fx)}{105f\sqrt{c-c\sin(e+fx)}} + \frac{8ac^3\cos}{}$$

Mathematica [A] time = 0.86, size = 104, normalized size = 0.76

$$\frac{ac^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3(-1389\sin(e+fx)+35\sin(3(e+fx))-330\cos(2}{630f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a*c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x]]*(16
06 - 330*Cos[2*(e + f*x)] - 1389*Sin[e + f*x] + 35*Sin[3*(e + f*x)]))/(630*
f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

fricas [A] time = 0.46, size = 179, normalized size = 1.31

$$-\frac{2\left(35\,ac^3\cos\left(fx+e\right)^5-95\,ac^3\cos\left(fx+e\right)^4-226\,ac^3\cos\left(fx+e\right)^3+32\,ac^3\cos\left(fx+e\right)^2-128\,ac^3\cos\left(fx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] -2/315*(35*a*c^3*cos(f*x + e)^5 - 95*a*c^3*cos(f*x + e)^4 - 226*a*c^3*cos(f
*x + e)^3 + 32*a*c^3*cos(f*x + e)^2 - 128*a*c^3*cos(f*x + e) - 256*a*c^3 +
(35*a*c^3*cos(f*x + e)^4 + 130*a*c^3*cos(f*x + e)^3 - 96*a*c^3*cos(f*x + e)
^2 - 128*a*c^3*cos(f*x + e) - 256*a*c^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e)
 + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*c)*(-2*a
*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(2*f*x-pi)+1/2*exp(1))/f-8*
a*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(2*f*x+2*exp(1)+pi))/(4*
f)^2+24*a*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(6*f*x+6*exp(1)-
pi))/(12*f)^2-40*a*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(10*f*x
+10*exp(1)+pi))/(20*f)^2+56*a*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(
1/4*(14*f*x+14*exp(1)-pi))/(28*f)^2+24*a*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/
4*pi))*cos(1/4*(6*f*x+6*exp(1)+pi))/(12*f)^2-40*a*c^3*f*sign(sin(1/2*(f*x+e
xp(1))-1/4*pi))*cos(1/4*(10*f*x+10*exp(1)-pi))/(20*f)^2-224*a*c^3*f*sign(si
n(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(14*f*x+14*exp(1)+pi))/(112*f)^2+288*a*
c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(18*f*x+18*exp(1)-pi))/(14
4*f)^2+48*a*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(2*f*x-pi)+1/2
*exp(1))/(8*f)^2)
```

maple [A] time = 0.71, size = 79, normalized size = 0.58

$$\frac{2\left(\sin\left(fx+e\right)-1\right)c^{4}\left(1+\sin\left(fx+e\right)\right)^{2}a\left(35\left(\sin^{3}\left(fx+e\right)\right)-165\left(\sin^{2}\left(fx+e\right)\right)+321\sin\left(fx+e\right)-319\right)}{315\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 2/315*(sin(f*x+e)-1)*c^4*(1+sin(f*x+e))^2*a*(35*sin(f*x+e)^3-165*sin(f*x+e)
^2+321*sin(f*x+e)-319)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(fx+e)+a)(-c\sin(fx+e)+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\sin(e+fx))\,(c-c\sin(e+fx))^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2),x)
[Out] int((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),x)
[Out] Timed out
```

3.291 $\int (a + a\sin(e + fx))(c - c\sin(e + fx))^{5/2}\,dx$

Optimal. Leaf size=103

$$\frac{64ac^4\cos^3(e+fx)}{105f(c-c\sin(e+fx))^{3/2}}+\frac{16ac^3\cos^3(e+fx)}{35f\sqrt{c-c\sin(e+fx)}}+\frac{2ac^2\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}$$

```
[Out] 64/105*a*c^4*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)+16/35*a*c^3*cos(f*x+e)^3
/f/(c-c*sin(f*x+e))^(1/2)+2/7*a*c^2*cos(f*x+e)^3*(c-c*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.22, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2674, 2673}

$$\frac{64ac^4\cos^3(e+fx)}{105f(c-c\sin(e+fx))^{3/2}}+\frac{16ac^3\cos^3(e+fx)}{35f\sqrt{c-c\sin(e+fx)}}+\frac{2ac^2\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (64*a*c^4*Cos[e + f*x]^3)/(105*f*(c - c*Sin[e + f*x])^(3/2)) + (16*a*c^3*Co
s[e + f*x]^3)/(35*f*Sqrt[c - c*Sin[e + f*x]]) + (2*a*c^2*Cos[e + f*x]^3*Sqr
t[c - c*Sin[e + f*x]])/(7*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
```

```
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a + a\sin(e + fx))(c - c\sin(e + fx))^{5/2}\,dx = (ac)\int \cos^2(e + fx)(c - c\sin(e + fx))^{3/2}\,dx$$
$$= \frac{2ac^2\cos^3(e + fx)\sqrt{c - c\sin(e + fx)}}{7f} + \frac{1}{7}\left(8ac^2\right)\int \cos^2(e + fx)$$
$$= \frac{16ac^3\cos^3(e + fx)}{35f\sqrt{c - c\sin(e + fx)}} + \frac{2ac^2\cos^3(e + fx)\sqrt{c - c\sin(e + fx)}}{7f}$$
$$= \frac{64ac^4\cos^3(e + fx)}{105f(c - c\sin(e + fx))^{3/2}} + \frac{16ac^3\cos^3(e + fx)}{35f\sqrt{c - c\sin(e + fx)}} + \frac{2ac^2\cos^3}{}$$

Mathematica [A] time = 0.57, size = 94, normalized size = 0.91

$$-\frac{ac^2\sqrt{c - c\sin(e + fx)}\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)^3(108\sin(e + fx) + 15\cos(2(e + fx)) - 157)}{105f\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/105*(a*c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(-157 + 15*Cos[2*(e +
 f*x)] + 108*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(f*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]))
```

fricas [A] time = 0.43, size = 152, normalized size = 1.48

$$\frac{2\left(15\,ac^2\cos\left(fx+e\right)^4 + 39\,ac^2\cos\left(fx+e\right)^3 - 8\,ac^2\cos\left(fx+e\right)^2 + 32\,ac^2\cos\left(fx+e\right) + 64\,ac^2 - \left(15\,ac^2\cos\right.\right.}{105\left(f\cos\left(fx+e\right) - f\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*a*c^2*cos(f*x + e)^4 + 39*a*c^2*cos(f*x + e)^3 - 8*a*c^2*cos(f*x
+ e)^2 + 32*a*c^2*cos(f*x + e) + 64*a*c^2 - (15*a*c^2*cos(f*x + e)^3 - 24*a
```

```
*c^2*cos(f*x + e)^2 - 32*a*c^2*cos(f*x + e) - 64*a*c^2)*sin(f*x + e))*sqrt(
-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*c)*(-2*a*c^2*sign(sin
(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(2*f*x-pi)+1/2*exp(1))/f-16*a*c^2*f*sign
(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(2*f*x+2*exp(1)+pi))/(8*f)^2+48*a*c^
2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(6*f*x+6*exp(1)-pi))/(24*f)^
2-80*a*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(10*f*x+10*exp(1)+p
i))/(40*f)^2+112*a*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(14*f*x
+14*exp(1)-pi))/(56*f)^2+24*a*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(
1/4*(6*f*x+6*exp(1)+pi))/(12*f)^2-40*a*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*
pi))*cos(1/4*(10*f*x+10*exp(1)-pi))/(20*f)^2+4*a*c^2*f*sign(sin(1/2*(f*x+ex
p(1))-1/4*pi))*cos(1/4*(2*f*x-pi)+1/2*exp(1))/(2*f)^2)
```

maple [A] time = 0.88, size = 69, normalized size = 0.67

$$-\frac{2\left(\sin\left(fx+e\right)-1\right)c^{3}\left(1+\sin\left(fx+e\right)\right)^{2}a\left(15\left(\sin^{2}\left(fx+e\right)\right)-54\sin\left(fx+e\right)+71\right)}{105\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -2/105*(sin(f*x+e)-1)*c^3*(1+sin(f*x+e))^2*a*(15*sin(f*x+e)^2-54*sin(f*x+e)
+71)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + a\,\sin\left(e + f\,x\right)\right)\,\left(c - c\,\sin\left(e + f\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int c^{2}\sqrt{-c\sin\left(e + fx\right) + c}\,dx + \int\left(-c^{2}\sqrt{-c\sin\left(e + fx\right) + c}\,\sin\left(e + fx\right)\right)dx + \int\left(-c^{2}\sqrt{-c\sin\left(e + fx\right) + c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] a*(Integral(c**2*sqrt(-c*sin(e + f*x) + c), x) + Integral(-c**2*sqrt(-c*sin
(e + f*x) + c)*sin(e + f*x), x) + Integral(-c**2*sqrt(-c*sin(e + f*x) + c)*
sin(e + f*x)**2, x) + Integral(c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)*
*3, x))
```

3.292 $\int (a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=69

$$\frac{8ac^3\cos^3(e+fx)}{15f(c-c\sin(e+fx))^{3/2}}+\frac{2ac^2\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 8/15*a*c^3*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)+2/5*a*c^2*cos(f*x+e)^3/f/(
c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2736, 2674, 2673}

$$\frac{2ac^2\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}+\frac{8ac^3\cos^3(e+fx)}{15f(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (8*a*c^3*Cos[e + f*x]^3)/(15*f*(c - c*Sin[e + f*x])^(3/2)) + (2*a*c^2*Cos[e
 + f*x]^3)/(5*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
```

```
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx &= (ac)\int \cos^2(e+fx)\sqrt{c-c\sin(e+fx)}\,dx\\ &= \frac{2ac^2\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}+\frac{1}{5}\left(4ac^2\right)\int \frac{\cos^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx\\ &= \frac{8ac^3\cos^3(e+fx)}{15f(c-c\sin(e+fx))^{3/2}}+\frac{2ac^2\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.37, size = 82, normalized size = 1.19

$$-\frac{2ac(3\sin(e+fx)-7)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}{15f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(-7 + 3*Sin[e + f*x])*Sqrt[
c - c*Sin[e + f*x]])/(15*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

fricas [A] time = 0.46, size = 109, normalized size = 1.58

$$\frac{2\left(3\,ac\cos\left(fx+e\right)^3-ac\cos\left(fx+e\right)^2+4\,ac\cos\left(fx+e\right)+8\,ac+\left(3\,ac\cos\left(fx+e\right)^2+4\,ac\cos\left(fx+e\right)+8\,a\right.}{15\left(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*a*c*cos(f*x + e)^3 - a*c*cos(f*x + e)^2 + 4*a*c*cos(f*x + e) + 8*a*
c + (3*a*c*cos(f*x + e)^2 + 4*a*c*cos(f*x + e) + 8*a*c)*sin(f*x + e))*sqrt(
-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)sqrt(2*c)*(-2*a*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*c
os(1/4*(2*f*x-pi)+1/2*exp(1))/f+24*a*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))
*cos(1/4*(6*f*x+6*exp(1)+pi))/(12*f)^2-40*a*c*f*sign(sin(1/2*(f*x+exp(1))-1
/4*pi))*cos(1/4*(10*f*x+10*exp(1)-pi))/(20*f)^2+4*a*c*f*sign(sin(1/2*(f*x+e
xp(1))-1/4*pi))*cos(1/4*(2*f*x-pi)+1/2*exp(1))/(2*f)^2)
```

maple [A] time = 0.76, size = 59, normalized size = 0.86

$$\frac{2\left(\sin\left(fx+e\right)-1\right)c^{2}\left(1+\sin\left(fx+e\right)\right)^{2}a\left(3\sin\left(fx+e\right)-7\right)}{15\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 2/15*(sin(f*x+e)-1)*c^2*(1+sin(f*x+e))^2*a*(3*sin(f*x+e)-7)/cos(f*x+e)/(c-c
*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(a+a\,\sin\left(e+f\,x\right)\right)\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int c\sqrt{-c\sin\left(e+fx\right)+c}\,dx+\int\left(-c\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^{2}\left(e+fx\right)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] a*(Integral(c*sqrt(-c*sin(e + f*x) + c), x) + Integral(-c*sqrt(-c*sin(e + f
*x) + c)*sin(e + f*x)**2, x))
```

3.293 $\int (a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=34

$$\frac{2ac^2\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 2/3*a*c^2*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)
```

Rubi [A] time = 0.09, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2736, 2673}

$$\frac{2ac^2\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*a*c^2*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx &= (ac)\int \frac{\cos^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx\\ &= \frac{2ac^2\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}\end{aligned}$$

Mathematica [B] time = 0.13, size = 71, normalized size = 2.09

$$\frac{2a\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*a*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x]])/(3*f
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

fricas [B] time = 0.44, size = 79, normalized size = 2.32

$$-\frac{2\left(a\cos\left(fx+e\right)^{2}-a\cos\left(fx+e\right)-\left(a\cos\left(fx+e\right)+2\,a\right)\sin\left(fx+e\right)-2\,a\right)\sqrt{-c\sin\left(fx+e\right)+c}}{3\left(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(a*cos(f*x + e)^2 - a*cos(f*x + e) - (a*cos(f*x + e) + 2*a)*sin(f*x +
e) - 2*a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*c)*(-2*a*sign(si
n(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(2*f*x-pi)+1/2*exp(1))/f+4*a*f*sign(sin
(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(2*f*x+2*exp(1)+pi))/(2*f)^2-12*a*f*sign
(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(6*f*x+6*exp(1)-pi))/(6*f)^2)
```

maple [A] time = 0.54, size = 47, normalized size = 1.38

$$-\frac{2\left(\sin\left(fx+e\right)-1\right)c\left(1+\sin\left(fx+e\right)\right)^{2}a}{3\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/3*(sin(f*x+e)-1)*c*(1+sin(f*x+e))^2*a/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/
f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \left(a+a\,\sin\left(e+f\,x\right)\right)\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)dx+\int \sqrt{-c\sin\left(e+fx\right)+c}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a*(Integral(sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Integral(sqrt(-c*s
in(e + f*x) + c), x))
```

3.294 $\int \frac{a+a\sin(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=77

$$\frac{2\sqrt{2}\,a\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f}-\frac{2a\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 2*a*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))*2^(1/2)/
f/c^(1/2)-2*a*cos(f*x+e)/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2679, 2649, 206}

$$\frac{2\sqrt{2}\,a\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f}-\frac{2a\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*Sqrt[2]*a*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x
]])])/(Sqrt[c]*f) - (2*a*Cos[e + f*x])/(f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
```

```
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\sin(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx &= (ac)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx\\
&= -\frac{2a\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}+(2a)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx\\
&= -\frac{2a\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}-\frac{(4a)\,\mathrm{Subst}\left(\int \frac{1}{2c-x^2}\,dx,x,-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{f}\\
&= \frac{2\sqrt{2}\,a\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f}-\frac{2a\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.65, size = 135, normalized size = 1.75

$$-\frac{2a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sqrt{c}\,(\sin(e+fx)+1)+\sqrt{2}\,\sqrt{-c(\sin(e+fx)+1)}\,\tan^{-1}\left(\frac{\sqrt{-c(\sin(e+fx)+1)}}{\sqrt{2}\,\sqrt{c}}\right)\right)}{\sqrt{c}\,f\sqrt{c-c\sin(e+fx)}\,\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (-2*a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Sqrt[c]*(1 + Sin[e + f*x]) + S
qrt[2]*ArcTan[Sqrt[-(c*(1 + Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])]*Sqrt[-(c*(1
+ Sin[e + f*x]))]))/(Sqrt[c]*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c
 - c*Sin[e + f*x]])
```

fricas [B] time = 0.45, size = 196, normalized size = 2.55

$$\frac{\frac{\sqrt{2}\left(ac\cos\left(fx+e\right)-ac\sin\left(fx+e\right)+ac\right)\log\left(-\frac{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)+\frac{2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)}{\sqrt{c}}+3\cos\left(fx+e\right)+2}{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right)}{\sqrt{c}}-2\left(a\cos\right.}{cf\cos\left(fx+e\right)-cf\sin\left(fx+e\right)+cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*(a*c*cos(f*x + e) - a*c*sin(f*x + e) + a*c)*log(-(cos(f*x + e)^2 +
 (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos
(f*x + e) + sin(f*x + e) + 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2
 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(c) - 2*(a*cos(
f*x + e) + a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(c*f*cos(f*x + e)
 - c*f*sin(f*x + e) + c*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)2/f*(4*a*atan((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sq
rt(c*tan((f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-c))/sqrt(2)/sqrt(-c)/sign(tan(
(f*x+exp(1))/2)-1)+2*sqrt(c*tan((f*x+exp(1))/2)^2+c)*(1/2*a/sign(tan((f*x+e
xp(1))/2)-1)+1/2*a*tan((f*x+exp(1))/2)/sign(tan((f*x+exp(1))/2)-1))/(c*tan(
(f*x+exp(1))/2)^2+c)+(-4*a*c*atan(sqrt(c)/sqrt(-c))-2*a*sqrt(-c)*sqrt(c))/c
/sqrt(-c)/sqrt(2)*sign(tan((f*x+exp(1))/2)-1))
```

maple [A] time = 0.72, size = 94, normalized size = 1.22

$$-\frac{2\left(\sin\left(fx+e\right)-1\right)\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\, a\left(\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{2}}{2\sqrt{c}}\right)-\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\right)}{c\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2*(sin(f*x+e)-1)*(c*(1+sin(f*x+e)))^(1/2)*a*(c^(1/2)*2^(1/2)*arctanh(1/2*(
c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))-(c*(1+sin(f*x+e)))^(1/2))/c/cos(f*
x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sin\left(fx+e\right)+a}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)/sqrt(-c*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+a\,\sin\left(e+f\,x\right)}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))/(c - c*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{\sin\left(e+fx\right)}{\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int \frac{1}{\sqrt{-c\sin\left(e+fx\right)+c}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a*(Integral(sin(e + f*x)/sqrt(-c*sin(e + f*x) + c), x) + Integral(1/sqrt(-c
*sin(e + f*x) + c), x))
```

3.295 $\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{a\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}\,c^{3/2}f}$$

```
[Out] a*cos(f*x+e)/f/(c-c*sin(f*x+e))^(3/2)-1/2*a*arctanh(1/2*cos(f*x+e)*c^(1/2)*
2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(3/2)/f*2^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2680, 2649, 206}

$$\frac{a\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}\,c^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((a*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(S
qrt[2]*c^(3/2)*f)) + (a*Cos[e + f*x])/(f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
```

```
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx &= (ac)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx\\
&= \frac{a\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}}-\frac{a\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx}{2c}\\
&= \frac{a\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}}+\frac{a\,\text{Subst}\left(\int \frac{1}{2c-x^2}\,dx,x,-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{cf}\\
&= -\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}\,c^{3/2}f}+\frac{a\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.71, size = 107, normalized size = 1.41

$$
\frac{a\sec(e+fx)\left(2\sqrt{c}(\sin(e+fx)+1)-\sqrt{2}(\sin(e+fx)-1)\sqrt{-c(\sin(e+fx)+1)}\tan^{-1}\left(\frac{\sqrt{-c(\sin(e+fx)+1)}}{\sqrt{2}\sqrt{c}}\right)\right)}{2c^{3/2}f\sqrt{c-c\sin(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (a*Sec[e + f*x]*(2*Sqrt[c]*(1 + Sin[e + f*x]) - Sqrt[2]*ArcTan[Sqrt[-(c*(1
+ Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])]*(-1 + Sin[e + f*x])*Sqrt[-(c*(1 + Sin[
e + f*x]))]))/(2*c^(3/2)*f*Sqrt[c - c*Sin[e + f*x]])
```

fricas [B] time = 0.46, size = 255, normalized size = 3.36

$$\frac{\frac{\sqrt{2}\left(ac\cos\left(fx+e\right)^{2}-ac\cos\left(fx+e\right)-2\,ac+\left(ac\cos\left(fx+e\right)+2\,ac\right)\sin\left(fx+e\right)\right)\log\left(-\frac{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)-\frac{2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\left(\cos\left(fx+e\right)+\sin\left(f\right.\right.}{\sqrt{c}}}{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right.}{\sqrt{c}}}{4\left(c^{2}f\cos\left(fx+e\right)^{2}-c^{2}f\cos\left(fx+e\right)-2\,c^{2}f+\left(c^{2}f\cos\left(f\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(a*c*cos(f*x + e)^2 - a*c*cos(f*x + e) - 2*a*c + (a*c*cos(f*x
+ e) + 2*a*c)*sin(f*x + e))*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f
*x + e) - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e)
+ 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin
(f*x + e) - cos(f*x + e) - 2))/sqrt(c) - 4*(a*cos(f*x + e) + a*sin(f*x + e)
 + a)*sqrt(-c*sin(f*x + e) + c))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e)
 - 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*2*(1/2*(-3*a*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(
1))/2)^2+c))^3+a*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)
^2+c))-a*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2
+c))^2-a*sqrt(c)*c)/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))
/2)^2+c))^2-2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))
/2)^2+c))+c)^2/c/sign(tan((f*x+exp(1))/2)-1)-1/2*a*atan((-sqrt(c)*tan((f*x+
exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-c))/sqrt(
2)/sqrt(-c)/c/sign(tan((f*x+exp(1))/2)-1))
```

maple [A] time = 0.63, size = 120, normalized size = 1.58

$$\frac{a\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{2}}{2\sqrt{c}}\right)c\sin\left(fx+e\right)-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{2}}{2\sqrt{c}}\right)c+2\sqrt{c\left(1+\sin\left(fx+e\right)\right.}\right.}{2c^{\frac{5}{2}}\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/2/c^(5/2)*a*(2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2)
)*c*sin(f*x+e)-2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2)
)*c+2*(c*(1+sin(f*x+e)))^(1/2)*c^(1/2))*(c*(1+sin(f*x+e)))^(1/2)/cos(f*x+e)
/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sin\left(fx+e\right)+a}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+a\,\sin\left(e+f\,x\right)}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))/(c - c*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{\sin\left(e+fx\right)}{-c\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)+c\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int \frac{1}{-c\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] a*(Integral(sin(e + f*x)/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqr
t(-c*sin(e + f*x) + c)), x) + Integral(1/(-c*sqrt(-c*sin(e + f*x) + c)*sin(
e + f*x) + c*sqrt(-c*sin(e + f*x) + c)), x))
```

3.296 $\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=113

$$-\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{5/2}f}-\frac{a\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}}+\frac{a\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 1/2*a*cos(f*x+e)/f/(c-c*sin(f*x+e))^(5/2)-1/8*a*cos(f*x+e)/c/f/(c-c*sin(f*x
+e))^(3/2)-1/16*a*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(
1/2))/c^(5/2)/f*2^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2736, 2680, 2650, 2649, 206}

$$-\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{5/2}f}-\frac{a\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}}+\frac{a\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] -(a*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(8*
Sqrt[2]*c^(5/2)*f) + (a*Cos[e + f*x])/(2*f*(c - c*Sin[e + f*x])^(5/2)) - (a
*Cos[e + f*x])/(8*c*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
```

```
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx &= (ac)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,dx \\
&= \frac{a\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a\int \frac{1}{(c-c\sin(e+fx))^{3/2}}\,dx}{4c} \\
&= \frac{a\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}} - \frac{a\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx}{16c^2} \\
&= \frac{a\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}} + \frac{a\,\mathrm{Subst}\left(\int \frac{1}{2c-x^2}\,dx, x, -\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{8c^2 f} \\
&= -\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{5/2}f} + \frac{a\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.97, size = 176, normalized size = 1.56

$$\frac{a\left(2\sqrt{2}\,\sqrt{-c(\sin(e+fx)+1)}\,\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\tan^{-1}\left(\frac{\sqrt{-c(\sin(e+fx)+1)}}{\sqrt{2}\,\sqrt{c}}\right)-2\sqrt{c}\,(-8\sin(e+fx}{32c^{5/2}f\sqrt{c-c\sin(e+fx)}\,\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a*(-2*Sqrt[c]*(-7 + Cos[2*(e + f*x)] - 8*Sin[e + f*x]) + 2*Sqrt[2]*ArcTan[
Sqrt[-(c*(1 + Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])]*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])^4*Sqrt[-(c*(1 + Sin[e + f*x]))]))/(32*c^(5/2)*f*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*S
in[e + f*x]])
```

fricas [B] time = 0.46, size = 336, normalized size = 2.97

$$\frac{\sqrt{2}\left(a\cos\left(fx+e\right)^3+3\,a\cos\left(fx+e\right)^2-2\,a\cos\left(fx+e\right)-\left(a\cos\left(fx+e\right)^2-2\,a\cos\left(fx+e\right)-4\,a\right)\sin\left(fx+\right.\right.}{32\left(c^3f\cos\left(fx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/32*(sqrt(2)*(a*cos(f*x + e)^3 + 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) - (
a*cos(f*x + e)^2 - 2*a*cos(f*x + e) - 4*a)*sin(f*x + e) - 4*a)*sqrt(c)*log(
-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x +
 e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x
 + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x +
e) - 2)) + 4*(a*cos(f*x + e)^2 - 3*a*cos(f*x + e) - (a*cos(f*x + e) + 4*a)*
sin(f*x + e) - 4*a)*sqrt(-c*sin(f*x + e) + c))/(c^3*f*cos(f*x + e)^3 + 3*c^
3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2
 - 2*c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*2*(1/16*(-17*a*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+ex
p(1))/2)^2+c))^7-23*a*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x
+exp(1))/2)^2+c))^6-19*a*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+ex
p(1))/2)^2+c))^5+39*a*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f
*x+exp(1))/2)^2+c))^4+5*a*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x
+exp(1))/2)^2+c))^3+7*a*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+e
xp(1))/2)^2+c))-37*a*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((
f*x+exp(1))/2)^2+c))^2-3*a*sqrt(c)*c^3)/c^2/(-(-sqrt(c)*tan((f*x+exp(1))/2)
+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)
+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)^4/sign(tan((f*x+exp(1))/2)-1)-1/16*a*a
tan((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/
sqrt(2)/sqrt(-c))/sqrt(2)/c^2/sqrt(-c)/sign(tan((f*x+exp(1))/2)-1))
```

maple [A] time = 0.82, size = 189, normalized size = 1.67

$$-\frac{a\left(-\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right)\left(\sin^2\left(fx+e\right)\right)c^3+2\left(c\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}c^{\frac{3}{2}}+2\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\right.}}{2}\right.\right.}{16c^{\frac{11}{2}}\left(\sin\left(fx+e\right)-1\right)\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/16/c^(11/2)*a*(-2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(
1/2))*sin(f*x+e)^2*c^3+2*(c*(1+sin(f*x+e)))^(3/2)*c^(3/2)+2*2^(1/2)*arctanh
(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c^3+4*(c*(1+sin(f
*x+e)))^(1/2)*c^(5/2)-2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/
c^(1/2))*c^3)*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*sin(f
*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sin\left(fx+e\right)+a}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+a\,\sin\left(e+f\,x\right)}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*sin(e + f*x))/(c - c*sin(e + f*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{\sin\left(e+fx\right)}{c^2\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^2\left(e+fx\right)-2c^2\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)+c^2\sqrt{-c\sin\left(e+fx\right)+c}}\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] a*(Integral(sin(e + f*x)/(c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2 -
2*c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c**2*sqrt(-c*sin(e + f*x) +
 c)), x) + Integral(1/(c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2 - 2*c
**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c**2*sqrt(-c*sin(e + f*x) + c)
), x))
```

3.297 $\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}c^{7/2}f}-\frac{a\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{3/2}}-\frac{a\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}}+\frac{a\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}}$$

```
[Out] 1/3*a*cos(f*x+e)/f/(c-c*sin(f*x+e))^(7/2)-1/24*a*cos(f*x+e)/c/f/(c-c*sin(f*
x+e))^(5/2)-1/32*a*cos(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(3/2)-1/64*a*arctanh(1
/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(7/2)/f*2^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2736, 2680, 2650, 2649, 206}

$$-\frac{a\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{3/2}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}c^{7/2}f}-\frac{a\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}}+\frac{a\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] -(a*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(32
*Sqrt[2]*c^(7/2)*f) + (a*Cos[e + f*x])/(3*f*(c - c*Sin[e + f*x])^(7/2)) - (
a*Cos[e + f*x])/(24*c*f*(c - c*Sin[e + f*x])^(5/2)) - (a*Cos[e + f*x])/(32*
c^2*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
```

```
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{a+a\sin(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,dx=(ac)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{9/2}}\,dx$$

$$=\frac{a\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}}-\frac{a\int \frac{1}{(c-c\sin(e+fx))^{5/2}}\,dx}{6c}$$

$$=\frac{a\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}}-\frac{a\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}}-\frac{a\int \frac{1}{(c-c\sin(e+fx))^{3/2}}\,dx}{16c^2}$$

$$=\frac{a\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}}-\frac{a\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}}-\frac{a\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{3/2}}$$

$$=\frac{a\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}}-\frac{a\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}}-\frac{a\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{3/2}}$$

$$=-\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}\,c^{7/2}f}+\frac{a\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}}-\frac{a\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5}}$$

Mathematica [A] time = 1.16, size = 189, normalized size = 1.30

$$\frac{a\left(2\sqrt{c}\left(131\sin(e+fx)+3(\sin(3(e+fx))+38)-14\cos(2(e+fx))\right)+12\sqrt{2}\sqrt{-c(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+\right.\right.}{768c^{7/2}f\sqrt{c-c\sin(e+fx)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\sin\left(\frac{1}{2}(e+fx)\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a*(12*Sqrt[2]*ArcTan[Sqrt[-(c*(1 + Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])]*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^6*Sqrt[-(c*(1 + Sin[e + f*x]))] + 2*Sqrt[
c]*(-14*Cos[2*(e + f*x)] + 131*Sin[e + f*x] + 3*(38 + Sin[3*(e + f*x)]))))/
(768*c^(7/2)*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]])
```

fricas [B] time = 0.46, size = 408, normalized size = 2.81

$$3\sqrt{2}\left(a\cos\left(fx+e\right)^4-3\,a\cos\left(fx+e\right)^3-8\,a\cos\left(fx+e\right)^2+4\,a\cos\left(fx+e\right)+\left(a\cos\left(fx+e\right)^3+4\,a\cos\left(fx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/384*(3*sqrt(2)*(a*cos(f*x + e)^4 - 3*a*cos(f*x + e)^3 - 8*a*cos(f*x + e)^
2 + 4*a*cos(f*x + e) + (a*cos(f*x + e)^3 + 4*a*cos(f*x + e)^2 - 4*a*cos(f*x
 + e) - 8*a)*sin(f*x + e) + 8*a)*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)
*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*
cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 +
 (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(3*a*cos(f*x + e)
^3 - 7*a*cos(f*x + e)^2 + 22*a*cos(f*x + e) + (3*a*cos(f*x + e)^2 + 10*a*co
s(f*x + e) + 32*a)*sin(f*x + e) + 32*a)*sqrt(-c*sin(f*x + e) + c))/(c^4*f*c
os(f*x + e)^4 - 3*c^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*c
os(f*x + e) + 8*c^4*f + (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*
c^4*f*cos(f*x + e) - 8*c^4*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integrat
ion of abs or sign assumes constant sign by intervals (correct if the argum
ent is real):Check [abs(sin((f*t_nostep+exp(1))/2-pi/4))]Unable to check si
gn: (8*pi/t_nostep/2)>(-8*pi/t_nostep/2)Discontinuities at zeroes of sin((f
*t_nostep+exp(1))/2-pi/4) were not checkedUnable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes cons
tant sign by intervals (correct if the argument is real):Check [abs(t_noste
p-1)]Evaluation time: 0.69Not invertible Error: Bad Argument Value
```

maple [A] time = 1.06, size = 243, normalized size = 1.68

$$-\frac{a\left(6\left(c\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{5}{2}} c^{\frac{5}{2}}-3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{2}}{2\sqrt{c}}\right)\left(\sin^{3}\left(fx+e\right)\right)c^{5}-32\left(c\left(1+\sin\left(fx+e\right)\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] -1/192/c^(17/2)*a*(6*(c*(1+sin(f*x+e)))^(5/2)*c^(5/2)-3*2^(1/2)*arctanh(1/2
*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^3*c^5-32*(c*(1+sin(f*
x+e)))^(3/2)*c^(7/2)+9*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)
/c^(1/2))*sin(f*x+e)^2*c^5-24*(c*(1+sin(f*x+e)))^(1/2)*c^(9/2)-9*2^(1/2)*ar
ctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c^5+3*2^(1/2
)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^5)*(c*(1+sin(f*x+
e)))^(1/2)/(sin(f*x+e)-1)^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sin\left(fx+e\right)+a}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+a\,\sin\left(e+f\,x\right)}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c - c*sin(e + f*x))^(7/2),x)
[Out] int((a + a*sin(e + f*x))/(c - c*sin(e + f*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),x)
[Out] Timed out
```

3.298 $\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=145

$$\frac{256a^2c^6\cos^5(e+fx)}{1155f(c-c\sin(e+fx))^{5/2}}+\frac{64a^2c^5\cos^5(e+fx)}{231f(c-c\sin(e+fx))^{3/2}}+\frac{8a^2c^4\cos^5(e+fx)}{33f\sqrt{c-c\sin(e+fx)}}+\frac{2a^2c^3\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{11f}$$

```
[Out] 256/1155*a^2*c^6*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)+64/231*a^2*c^5*cos(f
*x+e)^5/f/(c-c*sin(f*x+e))^(3/2)+8/33*a^2*c^4*cos(f*x+e)^5/f/(c-c*sin(f*x+e
))^(1/2)+2/11*a^2*c^3*cos(f*x+e)^5*(c-c*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.33, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$\frac{256a^2c^6\cos^5(e+fx)}{1155f(c-c\sin(e+fx))^{5/2}}+\frac{64a^2c^5\cos^5(e+fx)}{231f(c-c\sin(e+fx))^{3/2}}+\frac{8a^2c^4\cos^5(e+fx)}{33f\sqrt{c-c\sin(e+fx)}}+\frac{2a^2c^3\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{11f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (256*a^2*c^6*Cos[e + f*x]^5)/(1155*f*(c - c*Sin[e + f*x])^(5/2)) + (64*a^2*
c^5*Cos[e + f*x]^5)/(231*f*(c - c*Sin[e + f*x])^(3/2)) + (8*a^2*c^4*Cos[e +
 f*x]^5)/(33*f*Sqrt[c - c*Sin[e + f*x]]) + (2*a^2*c^3*Cos[e + f*x]^5*Sqrt[c
 - c*Sin[e + f*x]])/(11*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^{7/2}\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(c-c\sin(e+fx))^{3/2}\,dx$$

$$= \frac{2a^2c^3\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{11f} + \frac{1}{11}\left(12a^2c^3\right)\int \cos^4(e+$$

$$= \frac{8a^2c^4\cos^5(e+fx)}{33f\sqrt{c-c\sin(e+fx)}} + \frac{2a^2c^3\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{11f}$$

$$= \frac{64a^2c^5\cos^5(e+fx)}{231f(c-c\sin(e+fx))^{3/2}} + \frac{8a^2c^4\cos^5(e+fx)}{33f\sqrt{c-c\sin(e+fx)}} + \frac{2a^2c^3\cos}{}$$

$$= \frac{256a^2c^6\cos^5(e+fx)}{1155f(c-c\sin(e+fx))^{5/2}} + \frac{64a^2c^5\cos^5(e+fx)}{231f(c-c\sin(e+fx))^{3/2}} + \frac{8a^2}{33f\sqrt{}}$$

Mathematica [B] time = 6.44, size = 1105, normalized size = 7.62

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (7*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(8*f
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^4) - (Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^
(7/2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^4) + (11*Cos[(5*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*
Sin[e + f*x])^(7/2))/(80*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^4) + (Cos[(7*(e + f*x))/2]*(a + a*Sin[e + f*x
])^2*(c - c*Sin[e + f*x])^(7/2))/(112*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + (Cos[(9*(e + f*x))/2]*(a +
a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(48*f*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + (Cos[(11*(e + f
*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(176*f*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + (
```

```
7*Sin[(e + f*x)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(8*f*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^4) + ((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2)*Sin[(3*(e + f*x)
)/2])/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^4) + (11*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2)*Si
n[(5*(e + f*x))/2])/(80*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^4) - ((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*
x])^(7/2)*Sin[(7*(e + f*x))/2])/(112*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((a + a*Sin[e + f*x])^2*(c -
 c*Sin[e + f*x])^(7/2)*Sin[(9*(e + f*x))/2])/(48*f*(Cos[(e + f*x)/2] - Sin[
(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) - ((a + a*Sin[e +
f*x])^2*(c - c*Sin[e + f*x])^(7/2)*Sin[(11*(e + f*x))/2])/(176*f*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)
```

fricas [A] time = 0.48, size = 234, normalized size = 1.61

$$\frac{2\left(105\,a^2c^3\cos\left(fx+e\right)^6+245\,a^2c^3\cos\left(fx+e\right)^5-20\,a^2c^3\cos\left(fx+e\right)^4+32\,a^2c^3\cos\left(fx+e\right)^3-64\,a^2c^3\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*a^2*c^3*cos(f*x + e)^6 + 245*a^2*c^3*cos(f*x + e)^5 - 20*a^2*c^
3*cos(f*x + e)^4 + 32*a^2*c^3*cos(f*x + e)^3 - 64*a^2*c^3*cos(f*x + e)^2 +
256*a^2*c^3*cos(f*x + e) + 512*a^2*c^3 - (105*a^2*c^3*cos(f*x + e)^5 - 140*
a^2*c^3*cos(f*x + e)^4 - 160*a^2*c^3*cos(f*x + e)^3 - 192*a^2*c^3*cos(f*x +
 e)^2 - 256*a^2*c^3*cos(f*x + e) - 512*a^2*c^3)*sin(f*x + e))*sqrt(-c*sin(f
*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
```

```
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sq
rt(2*c)*(-2*a^2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(2*f*x-pi)+1
/2*exp(1))/f-32*a^2*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(2*f*x
+2*exp(1)+pi))/(16*f)^2+96*a^2*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin
(1/4*(6*f*x+6*exp(1)-pi))/(48*f)^2-960*a^2*c^3*f*sign(sin(1/2*(f*x+exp(1))-
1/4*pi))*sin(1/4*(10*f*x+10*exp(1)+pi))/(160*f)^2+1344*a^2*c^3*f*sign(sin(1
/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(14*f*x+14*exp(1)-pi))/(224*f)^2-576*a^2*c
^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(18*f*x+18*exp(1)+pi))/(288
*f)^2+704*a^2*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(22*f*x+22*e
xp(1)-pi))/(352*f)^2+24*a^2*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/
4*(6*f*x+6*exp(1)+pi))/(12*f)^2-40*a^2*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*
pi))*cos(1/4*(10*f*x+10*exp(1)-pi))/(20*f)^2+224*a^2*c^3*f*sign(sin(1/2*(f*
x+exp(1))-1/4*pi))*cos(1/4*(14*f*x+14*exp(1)+pi))/(112*f)^2-288*a^2*c^3*f*s
ign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(18*f*x+18*exp(1)-pi))/(144*f)^2+
80*a^2*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(2*f*x-pi)+1/2*exp(
1))/(8*f)^2)
```

maple [A] time = 0.87, size = 81, normalized size = 0.56

$$\frac{2\left(\sin\left(fx+e\right)-1\right)c^{4}\left(1+\sin\left(fx+e\right)\right)^{3}a^{2}\left(105\left(\sin^{3}\left(fx+e\right)\right)-455\left(\sin^{2}\left(fx+e\right)\right)+755\sin\left(fx+e\right)-533\right)}{1155\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 2/1155*(sin(f*x+e)-1)*c^4*(1+sin(f*x+e))^3*a^2*(105*sin(f*x+e)^3-455*sin(f*
x+e)^2+755*sin(f*x+e)-533)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{2}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(7/2),x)
[Out] int((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c-c*sin(f*x+e))**(7/2),x)
[Out] Timed out
```

3.299 $\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=109

$$\frac{64a^2c^5\cos^5(e+fx)}{315f(c-c\sin(e+fx))^{5/2}}+\frac{16a^2c^4\cos^5(e+fx)}{63f(c-c\sin(e+fx))^{3/2}}+\frac{2a^2c^3\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 64/315*a^2*c^5*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)+16/63*a^2*c^4*cos(f*x+
e)^5/f/(c-c*sin(f*x+e))^(3/2)+2/9*a^2*c^3*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(
1/2)
```

Rubi [A] time = 0.26, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$\frac{2a^2c^3\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}+\frac{16a^2c^4\cos^5(e+fx)}{63f(c-c\sin(e+fx))^{3/2}}+\frac{64a^2c^5\cos^5(e+fx)}{315f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (64*a^2*c^5*Cos[e + f*x]^5)/(315*f*(c - c*Sin[e + f*x])^(5/2)) + (16*a^2*c^
4*Cos[e + f*x]^5)/(63*f*(c - c*Sin[e + f*x])^(3/2)) + (2*a^2*c^3*Cos[e + f*
x]^5)/(9*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
```

```
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}\,dx &= \left(a^2c^2\right)\int \cos^4(e+fx)\sqrt{c-c\sin(e+fx)}\,dx\\ &= \frac{2a^2c^3\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}+\frac{1}{9}\left(8a^2c^3\right)\int \frac{\cos^4(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx\\ &= \frac{16a^2c^4\cos^5(e+fx)}{63f(c-c\sin(e+fx))^{3/2}}+\frac{2a^2c^3\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}+\frac{1}{63}\left(32a^2c^4\right.\\ &= \frac{64a^2c^5\cos^5(e+fx)}{315f(c-c\sin(e+fx))^{5/2}}+\frac{16a^2c^4\cos^5(e+fx)}{63f(c-c\sin(e+fx))^{3/2}}+\frac{2a^2c^3}{9f\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 5.69, size = 96, normalized size = 0.88

$$-\frac{a^2c^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5(220\sin(e+fx)+35\cos(2(e+fx))-249)}{315f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/315*(a^2*c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(-249 + 35*Cos[2*(e
 + f*x)] + 220*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(f*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2]))
```

fricas [B] time = 0.46, size = 201, normalized size = 1.84

$$2\left(35\,a^2c^2\cos\left(fx+e\right)^5-5\,a^2c^2\cos\left(fx+e\right)^4+8\,a^2c^2\cos\left(fx+e\right)^3-16\,a^2c^2\cos\left(fx+e\right)^2+64\,a^2c^2\cos\left(fx-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*a^2*c^2*cos(f*x + e)^5 - 5*a^2*c^2*cos(f*x + e)^4 + 8*a^2*c^2*cos
(f*x + e)^3 - 16*a^2*c^2*cos(f*x + e)^2 + 64*a^2*c^2*cos(f*x + e) + 128*a^2
```

```
*c^2 + (35*a^2*c^2*cos(f*x + e)^4 + 40*a^2*c^2*cos(f*x + e)^3 + 48*a^2*c^2*
cos(f*x + e)^2 + 64*a^2*c^2*cos(f*x + e) + 128*a^2*c^2)*sin(f*x + e))*sqrt(
-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*c)*(-2*a^2*c^2*sign(sin(1/2*(f*x+e
xp(1))-1/4*pi))*cos(1/4*(2*f*x-pi)+1/2*exp(1))/f+24*a^2*c^2*f*sign(sin(1/2*
(f*x+exp(1))-1/4*pi))*cos(1/4*(6*f*x+6*exp(1)+pi))/(12*f)^2-40*a^2*c^2*f*si
gn(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(10*f*x+10*exp(1)-pi))/(20*f)^2+22
4*a^2*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(14*f*x+14*exp(1)+pi
))/(112*f)^2-288*a^2*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(18*f
*x+18*exp(1)-pi))/(144*f)^2+80*a^2*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))
*cos(1/4*(2*f*x-pi)+1/2*exp(1))/(8*f)^2)
```

maple [A] time = 0.84, size = 71, normalized size = 0.65

$$-\frac{2\left(\sin\left(fx+e\right)-1\right)c^{3}\left(1+\sin\left(fx+e\right)\right)^{3}a^{2}\left(35\left(\sin^{2}\left(fx+e\right)\right)-110\sin\left(fx+e\right)+107\right)}{315\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -2/315*(sin(f*x+e)-1)*c^3*(1+sin(f*x+e))^3*a^2*(35*sin(f*x+e)^2-110*sin(f*x
+e)+107)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{2}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + a\,\sin\left(e + f\,x\right)\right)^2\,\left(c - c\,\sin\left(e + f\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int c^2\sqrt{-c\sin\left(e + fx\right) + c}\,dx + \int \left(-2c^2\sqrt{-c\sin\left(e + fx\right) + c}\,\sin^2\left(e + fx\right)\right)dx + \int c^2\sqrt{-c\sin\left(e + fx\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] a**2*(Integral(c**2*sqrt(-c*sin(e + f*x) + c), x) + Integral(-2*c**2*sqrt(-
c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(c**2*sqrt(-c*sin(e + f*x
) + c)*sin(e + f*x)**4, x))
```

3.300 $\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=73

$$\frac{8a^2c^4\cos^5(e+fx)}{35f(c-c\sin(e+fx))^{5/2}}+\frac{2a^2c^3\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 8/35*a^2*c^4*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)+2/7*a^2*c^3*cos(f*x+e)^5
/f/(c-c*sin(f*x+e))^(3/2)
```

Rubi [A] time = 0.20, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$\frac{2a^2c^3\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}+\frac{8a^2c^4\cos^5(e+fx)}{35f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (8*a^2*c^4*Cos[e + f*x]^5)/(35*f*(c - c*Sin[e + f*x])^(5/2)) + (2*a^2*c^3*C
os[e + f*x]^5)/(7*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
```

```
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \frac{2a^2c^3\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}+\frac{1}{7}\left(4a^2c^3\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx$$
$$= \frac{8a^2c^4\cos^5(e+fx)}{35f(c-c\sin(e+fx))^{5/2}}+\frac{2a^2c^3\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}$$

Mathematica [A] time = 1.39, size = 84, normalized size = 1.15

$$-\frac{2a^2c(5\sin(e+fx)-9)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}{35f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a^2*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(-9 + 5*Sin[e + f*x])*Sqr
t[c - c*Sin[e + f*x]])/(35*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

fricas [B] time = 0.44, size = 152, normalized size = 2.08

$$-\frac{2\left(5\,a^2c\cos\left(fx+e\right)^4-a^2c\cos\left(fx+e\right)^3+2\,a^2c\cos\left(fx+e\right)^2-8\,a^2c\cos\left(fx+e\right)-16\,a^2c-\left(5\,a^2c\cos\left(fx+\right.\right.\right.}{35\left(f\cos\left(fx+e\right)-f\sin\left(f\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/35*(5*a^2*c*cos(f*x + e)^4 - a^2*c*cos(f*x + e)^3 + 2*a^2*c*cos(f*x + e)
^2 - 8*a^2*c*cos(f*x + e) - 16*a^2*c - (5*a^2*c*cos(f*x + e)^3 + 6*a^2*c*co
s(f*x + e)^2 + 8*a^2*c*cos(f*x + e) + 16*a^2*c)*sin(f*x + e))*sqrt(-c*sin(f
*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*c)*(-2*a^2*c*sign(sin
(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(2*f*x-pi)+1/2*exp(1))/f+16*a^2*c*f*sign
(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(2*f*x+2*exp(1)+pi))/(8*f)^2-48*a^2*
c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(6*f*x+6*exp(1)-pi))/(24*f)^
2+80*a^2*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(10*f*x+10*exp(1)+p
i))/(40*f)^2-112*a^2*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(14*f*x
+14*exp(1)-pi))/(56*f)^2+24*a^2*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(
1/4*(6*f*x+6*exp(1)+pi))/(12*f)^2-40*a^2*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*
pi))*cos(1/4*(10*f*x+10*exp(1)-pi))/(20*f)^2+4*a^2*c*f*sign(sin(1/2*(f*x+ex
p(1))-1/4*pi))*cos(1/4*(2*f*x-pi)+1/2*exp(1))/(2*f)^2)
```

maple [A] time = 0.84, size = 61, normalized size = 0.84

$$\frac{2 \left(\sin \left(f x +e \right)-1\right) c^{2} \left(1+\sin \left(f x +e \right)\right)^{3} a^{2} \left(5 \sin \left(f x +e \right)-9\right)}{35 \cos \left(f x +e \right) \sqrt{c -c \sin \left(f x +e \right)}\, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 2/35*(sin(f*x+e)-1)*c^2*(1+sin(f*x+e))^3*a^2*(5*sin(f*x+e)-9)/cos(f*x+e)/(c
-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin \left(f x + e\right) + a\right)^{2} \left(-c \sin \left(f x + e\right) + c\right)^{\frac{3}{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int c\sqrt{-c\sin\left(e+fx\right)+c}\,dx + \int c\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)dx + \int \left(-c\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] a**2*(Integral(c*sqrt(-c*sin(e + f*x) + c), x) + Integral(c*sqrt(-c*sin(e +
 f*x) + c)*sin(e + f*x), x) + Integral(-c*sqrt(-c*sin(e + f*x) + c)*sin(e +
 f*x)**2, x) + Integral(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**3, x))
```

3.301 $\int (a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=36

$$\frac{2a^2c^3\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 2/5*a^2*c^3*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)
```

Rubi [A] time = 0.13, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2736, 2673}

$$\frac{2a^2c^3\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*a^2*c^3*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*x])^(5/2))
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx\\ &= \frac{2a^2c^3\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}\end{aligned}$$

Mathematica [B] time = 0.23, size = 73, normalized size = 2.03

$$\frac{2a^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}{5f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*a^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sqrt[c - c*Sin[e + f*x]])/(5
*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

fricas [B] time = 0.43, size = 114, normalized size = 3.17

$$-\frac{2\left(a^2\cos\left(fx+e\right)^3+3\,a^2\cos\left(fx+e\right)^2-2\,a^2\cos\left(fx+e\right)-4\,a^2+\left(a^2\cos\left(fx+e\right)^2-2\,a^2\cos\left(fx+e\right)-4\,a\right.\right.}{5\left(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*(a^2*cos(f*x + e)^3 + 3*a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) - 4*a^
2 + (a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) - 4*a^2)*sin(f*x + e))*sqrt(-c
*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*c)*(2*a^2*sign(sin(1/2*(f*x+exp(1)
)-1/4*pi))*sin(1/4*(2*f*x+2*exp(1)+pi))/f-2*a^2*sign(sin(1/2*(f*x+exp(1))-1
/4*pi))*cos(1/4*(2*f*x-pi)+1/2*exp(1))/f-6*a^2*f*sign(sin(1/2*(f*x+exp(1))-
1/4*pi))*sin(1/4*(6*f*x+6*exp(1)-pi))/(3*f)^2-24*a^2*f*sign(sin(1/2*(f*x+ex
p(1))-1/4*pi))*cos(1/4*(6*f*x+6*exp(1)+pi))/(12*f)^2+40*a^2*f*sign(sin(1/2*
(f*x+exp(1))-1/4*pi))*cos(1/4*(10*f*x+10*exp(1)-pi))/(20*f)^2-4*a^2*f*sign(
sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(2*f*x-pi)+1/2*exp(1))/(2*f)^2)
```

maple [A] time = 0.64, size = 49, normalized size = 1.36

$$-\frac{2\left(\sin\left(fx+e\right)-1\right)c\left(1+\sin\left(fx+e\right)\right)^{3}a^{2}}{5\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/5*(sin(f*x+e)-1)*c*(1+sin(f*x+e))^3*a^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2
)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{2}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*sqrt(-c*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int 2\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)dx+\int\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^{2}\left(e+fx\right)dx+\int\sqrt{-c\sin\left(e+fx\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a**2*(Integral(2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Integral(sqrt
(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(sqrt(-c*sin(e + f*x) +
 c), x))
```

3.302 $\int \frac{(a+a\sin(e+fx))^2}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=115

$$-\frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}-\frac{4a^2\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}+\frac{4\sqrt{2}\,a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f}$$

```
[Out] -2/3*a^2*c*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)+4*a^2*arctanh(1/2*cos(f*x+
e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))*2^(1/2)/f/c^(1/2)-4*a^2*cos(f*x+
e)/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2736, 2679, 2649, 206}

$$-\frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}-\frac{4a^2\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}+\frac{4\sqrt{2}\,a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (4*Sqrt[2]*a^2*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f
*x]])])/(Sqrt[c]*f) - (2*a^2*c*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^(3
/2)) - (4*a^2*Cos[e + f*x])/(f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
```

```
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))^2}{\sqrt{c-c\sin(e+fx)}}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx\\
&= -\frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}+\left(2a^2c\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx\\
&= -\frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}-\frac{4a^2\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}+\left(4a^2\right)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx\\
&= -\frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}-\frac{4a^2\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}-\frac{\left(8a^2\right)\text{Subst}\left(\int \frac{1}{2c-x^2}\,dx,x,-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{f}\\
&= \frac{4\sqrt{2}\,a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f}-\frac{2a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}-\frac{4a^2\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}
\end{aligned}$$

Mathematica [C] time = 0.47, size = 130, normalized size = 1.13

$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(15\sin\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{3}{2}(e+fx)\right)+15\cos\left(\frac{1}{2}(e+fx)\right)-\cos\left(\frac{3}{2}(e+\ldots\right.\right.}{3f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/Sqrt[c - c*Sin[e + f*x]],x]
```

[Out] -1/3*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*((24 + 24*I)*(-1)^(1/4)*Arc
Tan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])] + 15*Cos[(e + f*x)/2] -
Cos[(3*(e + f*x))/2] + 15*Sin[(e + f*x)/2] + Sin[(3*(e + f*x))/2]))/(f*Sqrt
[c - c*Sin[e + f*x]])

fricas [B] time = 0.45, size = 238, normalized size = 2.07

$$\frac{2\left(\frac{3\sqrt{2}\left(a^2c\cos\left(fx+e\right)-a^2c\sin\left(fx+e\right)+a^2c\right)\log\left(-\frac{\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)+\frac{2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)}{\sqrt{c}}+3\cos\left(fx+e\right)+2}{\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right)}{\sqrt{c}}+\right.}{3\left(cf\cos\left(fx+e\right)-cf\,s\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] 2/3*(3*sqrt(2)*(a^2*c*cos(f*x + e) - a^2*c*sin(f*x + e) + a^2*c)*log(-(cos(
f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(-c*sin(f*x +
e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(co
s(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(c)
 + (a^2*cos(f*x + e)^2 - 7*a^2*cos(f*x + e) - 8*a^2 - (a^2*cos(f*x + e) + 8
*a^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c*f*cos(f*x + e) - c*f*sin(
f*x + e) + c*f)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(2/sqrt(c*tan((f*x+exp(1))/2)^2+c)/(c
*tan((f*x+exp(1))/2)^2+c)*(tan((f*x+exp(1))/2)*(tan((f*x+exp(1))/2)*(3/2*a^
2*c/sign(tan((f*x+exp(1))/2)-1)+7/6*a^2*c*tan((f*x+exp(1))/2)/sign(tan((f*x
+exp(1))/2)-1))+3/2*a^2*c/sign(tan((f*x+exp(1))/2)-1))+7/6*a^2*c/sign(tan((
f*x+exp(1))/2)-1))+8*a^2*atan((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*
tan((f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-c))/sqrt(2)/sqrt(-c)/sign(tan((f*x+

```
exp(1))/2)-1)+(-24*a^2*c*atan(sqrt(c)/sqrt(-c))-16*a^2*sqrt(-c)*sqrt(c))/3/
c/sqrt(-c)/sqrt(2)*sign(tan((f*x+exp(1))/2)-1))
```

maple [A] time = 0.92, size = 112, normalized size = 0.97

$$-\frac{2\left(\sin\left(fx+e\right)-1\right)\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,a^{2}\left(6c^{\frac{3}{2}}\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right)-\left(c\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}-6\right.}{3c^{2}\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\ f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/3*(sin(f*x+e)-1)*(c*(1+sin(f*x+e)))^(1/2)*a^2*(6*c^(3/2)*2^(1/2)*arctanh
(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))-(c*(1+sin(f*x+e)))^(3/2)-6*(
c*(1+sin(f*x+e)))^(1/2)*c)/c^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{2}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/sqrt(-c*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\ \sin\left(e+f\,x\right)\right)^{2}}{\sqrt{c-c\ \sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int \frac{2\sin\left(e+fx\right)}{\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int \frac{\sin^{2}\left(e+fx\right)}{\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int \frac{1}{\sqrt{-c\sin\left(e+fx\right)+c}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a**2*(Integral(2*sin(e + f*x)/sqrt(-c*sin(e + f*x) + c), x) + Integral(sin(
e + f*x)**2/sqrt(-c*sin(e + f*x) + c), x) + Integral(1/sqrt(-c*sin(e + f*x)
 + c), x))
```

3.303 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{3\sqrt{2}a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}+\frac{a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^{5/2}}+\frac{3a^2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

```
[Out] a^2*c*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(5/2)-3*a^2*arctanh(1/2*cos(f*x+e)*c^
(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))*2^(1/2)/c^(3/2)/f+3*a^2*cos(f*x+e)/c/
f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2736, 2680, 2679, 2649, 206}

$$-\frac{3\sqrt{2}a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}+\frac{a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^{5/2}}+\frac{3a^2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-3*Sqrt[2]*a^2*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e +
f*x]])])/(c^(3/2)*f) + (a^2*c*Cos[e + f*x]^3)/(f*(c - c*Sin[e + f*x])^(5/2)
) + (3*a^2*Cos[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
```

```
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^{3/2}}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,dx\\
&= \frac{a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^{5/2}}-\frac{1}{2}\left(3a^2\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx\\
&= \frac{a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^{5/2}}+\frac{3a^2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}-\frac{\left(3a^2\right)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx}{c}\\
&= \frac{a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^{5/2}}+\frac{3a^2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}+\frac{\left(6a^2\right)\text{Subst}\left(\int \frac{1}{2c-x^2}\,dx,x,-\frac{c\cdots}{\sqrt{c-\cdots}}\right)}{cf}\\
&= -\frac{3\sqrt{2}\,a^2\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}+\frac{a^2c\cos^3(e+fx)}{f(c-c\sin(e+fx))^{5/2}}+\frac{3a^2\cos(e+fx)}{cf\sqrt{c-c\sin(e+f\cdots}}
\end{aligned}
$$

Mathematica [C] time = 0.69, size = 149, normalized size = 1.30

$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(3\sin\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{3}{2}(e+fx)\right)+3\cos\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{3}{2}(e+fx\right.\right.}{cf(\sin(e+fx)-1)\sqrt{c-c\sin(e+f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(3*Cos[(e + f*x)/2] + Cos[(3*(
e + f*x))/2] + 3*Sin[(e + f*x)/2] - (6 + 6*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)
*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(-1 + Sin[e + f*x]) - Sin[(3*(e + f*x))
/2]))/(c*f*(-1 + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]]))
```

fricas [B] time = 0.46, size = 299, normalized size = 2.60

$$\frac{\frac{3\sqrt{2}\left(a^2c\cos\left(fx+e\right)^2-a^2c\cos\left(fx+e\right)-2a^2c+\left(a^2c\cos\left(fx+e\right)+2a^2c\right)\sin\left(fx+e\right)\right)\log\left(-\frac{\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)-\frac{2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\left(\cos\left(fx+e\right)\right.}{\sqrt{c}}}{\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+\right.}\right.}{\sqrt{c}}}{2\left(c^2f\cos\left(fx+e\right)^2-c^2f\cos\left(fx+e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(3*sqrt(2)*(a^2*c*cos(f*x + e)^2 - a^2*c*cos(f*x + e) - 2*a^2*c + (a^2*
c*cos(f*x + e) + 2*a^2*c)*sin(f*x + e))*log(-(cos(f*x + e)^2 + (cos(f*x + e
) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + s
in(f*x + e) + 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x +
 e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(c) - 4*(a^2*cos(f*x + e)^2
+ 2*a^2*cos(f*x + e) + a^2 - (a^2*cos(f*x + e) - a^2)*sin(f*x + e))*sqrt(-c
*sin(f*x + e) + c))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) - 2*c^2*f +
(c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
```

```
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)2/f*(2*sqrt(c*tan((f*x+exp(1))/2)^2+c)*(-1/2*a^2/c/sig
n(tan((f*x+exp(1))/2)-1)-1/2*a^2*tan((f*x+exp(1))/2)/c/sign(tan((f*x+exp(1)
)/2)-1))/(c*tan((f*x+exp(1))/2)^2+c)+2*(-(3*a^2*(-sqrt(c)*tan((f*x+exp(1))/
2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3-a^2*c*(-sqrt(c)*tan((f*x+exp(1))/2)+s
qrt(c*tan((f*x+exp(1))/2)^2+c))+a^2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+s
qrt(c*tan((f*x+exp(1))/2)^2+c))^2+a^2*sqrt(c)*c)/(-(-sqrt(c)*tan((f*x+exp(1
))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1
))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)^2/c/sign(tan((f*x+exp(1))/2)-1)-3
*a^2*atan((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^
2+c))/sqrt(2)/sqrt(-c))/sqrt(2)/sqrt(-c)/c/sign(tan((f*x+exp(1))/2)-1)))
```

maple [A] time = 0.78, size = 145, normalized size = 1.26

$$\frac{a^2\left(3\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right)c\sin\left(fx+e\right)-3\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right)c-2\sqrt{c\left(1+\sin\left(fx+\right.\right.}}{c^{\frac{5}{2}}\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\ f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] a^2*(3*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c*sin(
f*x+e)-3*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c-2*
(c*(1+sin(f*x+e)))^(1/2)*c^(1/2)*sin(f*x+e)+4*(c*(1+sin(f*x+e)))^(1/2)*c^(1
/2))*(c*(1+sin(f*x+e)))^(1/2)/c^(5/2)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^2}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x))^(3/2),x)

[Out] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{2\sin\left(e+fx\right)}{-c\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)+c\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int \frac{\sin^2\left(e+fx\right)}{-c\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(3/2),x)

[Out] a**2*(Integral(2*sin(e + f*x)/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin(e + f*x) + c)), x) + Integral(sin(e + f*x)**2/(-c*sqrt(-c*sin (e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin(e + f*x) + c)), x) + Integral(1 /(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin(e + f*x) + c)), x))

3.304 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=122

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}\,c^{5/2}f}+\frac{a^2c\cos^3(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{3a^2\cos(e+fx)}{4cf(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 1/2*a^2*c*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(7/2)-3/4*a^2*cos(f*x+e)/c/f/(c-c
*sin(f*x+e))^(3/2)+3/8*a^2*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(
f*x+e))^(1/2))/c^(5/2)/f*2^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2736, 2680, 2649, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}\,c^{5/2}f}+\frac{a^2c\cos^3(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{3a^2\cos(e+fx)}{4cf(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (3*a^2*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/
(4*Sqrt[2]*c^(5/2)*f) + (a^2*c*Cos[e + f*x]^3)/(2*f*(c - c*Sin[e + f*x])^(7
/2)) - (3*a^2*Cos[e + f*x])/(4*c*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
```

```
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^{5/2}}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{9/2}}\,dx\\ &= \frac{a^2c\cos^3(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{1}{4}\left(3a^2\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx\\ &= \frac{a^2c\cos^3(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{3a^2\cos(e+fx)}{4cf(c-c\sin(e+fx))^{3/2}}+\frac{\left(3a^2\right)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx}{8c^2}\\ &= \frac{a^2c\cos^3(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{3a^2\cos(e+fx)}{4cf(c-c\sin(e+fx))^{3/2}}-\frac{\left(3a^2\right)\operatorname{Subst}\left(\int \frac{1}{2c-x^2}\,dx,x,-\frac{}{}\right.}{4c^2f}\\ &= \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}c^{5/2}f}+\frac{a^2c\cos^3(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{3a^2\cos(e+fx)}{4cf(c-c\sin(e+fx))^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.97, size = 163, normalized size = 1.34

$$\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(3\sin\left(\frac{1}{2}(e+fx)\right)+5\sin\left(\frac{3}{2}(e+fx)\right)+3\cos\left(\frac{1}{2}(e+fx)\right)-5\cos\left(\frac{3}{2}(e+f\right.\right.}{8c^2f(\sin(e+fx)-1)^2\sqrt{c-}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(3*Cos[(e + f*x)/2] - 5*Cos[(3*(
e + f*x))/2] + 3*Sin[(e + f*x)/2] + (3 + 3*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)
```

```
*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(-3 + Cos[2*(e + f*x)] + 4*Sin[e + f*x]
) + 5*Sin[(3*(e + f*x))/2]))/(8*c^2*f*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[
e + f*x]])
```

fricas [B] time = 0.46, size = 362, normalized size = 2.97

$$\frac{3\sqrt{2}\left(a^2\cos\left(fx+e\right)^3+3\,a^2\cos\left(fx+e\right)^2-2\,a^2\cos\left(fx+e\right)-4\,a^2-\left(a^2\cos\left(fx+e\right)^2-2\,a^2\cos\left(fx+e\right)-4\right.}{16\left(c^3f\,\mathrm{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*sqrt(2)*(a^2*cos(f*x + e)^3 + 3*a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x
+ e) - 4*a^2 - (a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) - 4*a^2)*sin(f*x +
e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sq
rt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e
) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x +
 e) - cos(f*x + e) - 2)) + 4*(5*a^2*cos(f*x + e)^2 + a^2*cos(f*x + e) - 4*a
^2 - (5*a^2*cos(f*x + e) + 4*a^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/
(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c
^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e)
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*2*(1/8*(-5*a^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+ex
p(1))/2)^2+c))^7+29*a^2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f
*x+exp(1))/2)^2+c))^6+17*a^2*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*
x+exp(1))/2)^2+c))^5-13*a^2*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*
tan((f*x+exp(1))/2)^2+c))^4+9*a^2*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*
tan((f*x+exp(1))/2)^2+c))^3-13*a^2*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c
*tan((f*x+exp(1))/2)^2+c))-9*a^2*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+
sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+a^2*sqrt(c)*c^3)/c^2/(-(-sqrt(c)*tan((f*
x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-2*sqrt(c)*(-sqrt(c)*tan((f*
x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)^4/sign(tan((f*x+exp(1))/2)
```

```
-1)+3/8*a^2*atan((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(
1))/2)^2+c))/sqrt(2)/sqrt(-c))/sqrt(2)/c^2/sqrt(-c)/sign(tan((f*x+exp(1))/2
)-1))
```

maple [A] time = 0.90, size = 191, normalized size = 1.57

$$-\frac{a^{2}\left(3\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right)\left(\sin^{2}\left(fx+e\right)\right)c^{2}-6\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right)\sin\left(fx+e\right)c^{2}+1\right.}{8c^{\frac{9}{2}}\left(\sin\left(fx+e\right)-1\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/8/c^(9/2)*a^2*(3*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^
(1/2))*sin(f*x+e)^2*c^2-6*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1
/2)/c^(1/2))*sin(f*x+e)*c^2+10*(c*(1+sin(f*x+e)))^(3/2)*c^(1/2)+3*2^(1/2)*a
rctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^2-12*(c*(1+sin(f*x+e
)))^(1/2)*c^(3/2))*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*
sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\sin\left(fx+e\right)+a\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^{2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.305 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=156

$$\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}\,c^{7/2}f}+\frac{a^2\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{3/2}}+\frac{a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{9/2}}-\frac{a^2\cos(e+fx)}{4cf(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 1/3*a^2*c*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(9/2)-1/4*a^2*cos(f*x+e)/c/f/(c-c
*sin(f*x+e))^(5/2)+1/16*a^2*cos(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(3/2)+1/32*a^
2*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(7/2)/f*
2^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2736, 2680, 2650, 2649, 206}

$$\frac{a^2\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{3/2}}+\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}\,c^{7/2}f}+\frac{a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{9/2}}-\frac{a^2\cos(e+fx)}{4cf(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a^2*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(1
6*Sqrt[2]*c^(7/2)*f) + (a^2*c*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^(9/
2)) - (a^2*Cos[e + f*x])/(4*c*f*(c - c*Sin[e + f*x])^(5/2)) + (a^2*Cos[e +
f*x])/(16*c^2*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^{7/2}}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{11/2}}\,dx\\
&= \frac{a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{9/2}}-\frac{1}{2}a^2\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,dx\\
&= \frac{a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{9/2}}-\frac{a^2\cos(e+fx)}{4cf(c-c\sin(e+fx))^{5/2}}+\frac{a^2\int \frac{1}{(c-c\sin(e+fx))^{3/2}}\,dx}{8c^2}\\
&= \frac{a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{9/2}}-\frac{a^2\cos(e+fx)}{4cf(c-c\sin(e+fx))^{5/2}}+\frac{a^2\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{3/2}}\\
&= \frac{a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{9/2}}-\frac{a^2\cos(e+fx)}{4cf(c-c\sin(e+fx))^{5/2}}+\frac{a^2\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{3/2}}\\
&= \frac{a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}c^{7/2}f}+\frac{a^2c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{9/2}}-\frac{a^2\cos(e+fx)}{4cf(c-c\sin(e+fx))^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.98, size = 307, normalized size = 1.97

$$a^2(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(64\sin\left(\frac{1}{2}(e+fx)\right)+3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(32*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2]) - 28*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 + 3*(Cos[(e + f*x)/2
] - Sin[(e + f*x)/2])^5 - (3 + 3*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4
)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6 + 64*Sin[
(e + f*x)/2] - 56*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2]
+ 6*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/2])*(1 + Sin[e +
f*x])^2)/(48*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])
^(7/2))
```

fricas [B] time = 0.49, size = 440, normalized size = 2.82

$$3\sqrt{2}\left(a^2\cos\left(fx+e\right)^4-3\,a^2\cos\left(fx+e\right)^3-8\,a^2\cos\left(fx+e\right)^2+4\,a^2\cos\left(fx+e\right)+8\,a^2+\left(a^2\cos\left(fx+e\right)^3+4\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/192*(3*sqrt(2)*(a^2*cos(f*x + e)^4 - 3*a^2*cos(f*x + e)^3 - 8*a^2*cos(f*x
 + e)^2 + 4*a^2*cos(f*x + e) + 8*a^2 + (a^2*cos(f*x + e)^3 + 4*a^2*cos(f*x
+ e)^2 - 4*a^2*cos(f*x + e) - 8*a^2)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x
+ e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*
x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)
/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4
*(3*a^2*cos(f*x + e)^3 + 25*a^2*cos(f*x + e)^2 - 10*a^2*cos(f*x + e) - 32*a
^2 + (3*a^2*cos(f*x + e)^2 - 22*a^2*cos(f*x + e) - 32*a^2)*sin(f*x + e))*sq
rt(-c*sin(f*x + e) + c))/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f*x + e)^3 - 8
*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*f*cos(f*x + e
)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)*sin(f*x + e)
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integrat
ion of abs or sign assumes constant sign by intervals (correct if the argum
ent is real):Check [abs(sin((f*t_nostep+exp(1))/2-pi/4))]Unable to check si
gn: (8*pi/t_nostep/2)>(-8*pi/t_nostep/2)Discontinuities at zeroes of sin((f
*t_nostep+exp(1))/2-pi/4) were not checkedUnable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes cons
tant sign by intervals (correct if the argument is real):Check [abs(t_noste
p-1)]Evaluation time: 0.88Not invertible Error: Bad Argument Value
```

maple [A] time = 1.02, size = 245, normalized size = 1.57

$$-\frac{a^2\left(3\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right)\left(\sin^3\left(fx+e\right)\right)c^4+24\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ c^{\frac{7}{2}}-32\left(c\left(1+\sin\left(fx+e\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] -1/96*a^2*(3*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*
sin(f*x+e)^3*c^4+24*(c*(1+sin(f*x+e)))^(1/2)*c^(7/2)-32*(c*(1+sin(f*x+e)))^
(3/2)*c^(5/2)-6*(c*(1+sin(f*x+e)))^(5/2)*c^(3/2)-9*2^(1/2)*arctanh(1/2*(c*(
1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^4+9*2^(1/2)*arctanh(1/
2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c^4-3*2^(1/2)*arctan
```

```
h(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^4)*(c*(1+sin(f*x+e)))^(1/
2)/c^(15/2)/(sin(f*x+e)-1)^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^2}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(7/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^2}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x))^(7/2),x)
[Out] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(7/2),x)
[Out] Timed out
```

3.306 $\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=190

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{256\sqrt{2}\,c^{9/2}f}+\frac{3a^2\cos(e+fx)}{256c^3f(c-c\sin(e+fx))^{3/2}}+\frac{a^2\cos(e+fx)}{64c^2f(c-c\sin(e+fx))^{5/2}}+\frac{a^2c\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{1}}$$

```
[Out] 1/4*a^2*c*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(11/2)-1/8*a^2*cos(f*x+e)/c/f/(c-
c*sin(f*x+e))^(7/2)+1/64*a^2*cos(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(5/2)+3/256*
a^2*cos(f*x+e)/c^3/f/(c-c*sin(f*x+e))^(3/2)+3/512*a^2*arctanh(1/2*cos(f*x+e
)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(9/2)/f*2^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2736, 2680, 2650, 2649, 206}

$$\frac{3a^2\cos(e+fx)}{256c^3f(c-c\sin(e+fx))^{3/2}}+\frac{a^2\cos(e+fx)}{64c^2f(c-c\sin(e+fx))^{5/2}}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{256\sqrt{2}\,c^{9/2}f}+\frac{a^2c\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (3*a^2*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/
(256*Sqrt[2]*c^(9/2)*f) + (a^2*c*Cos[e + f*x]^3)/(4*f*(c - c*Sin[e + f*x])^
(11/2)) - (a^2*Cos[e + f*x])/(8*c*f*(c - c*Sin[e + f*x])^(7/2)) + (a^2*Cos[
e + f*x])/(64*c^2*f*(c - c*Sin[e + f*x])^(5/2)) + (3*a^2*Cos[e + f*x])/(256
*c^3*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^{9/2}}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{13/2}}\,dx$$

$$= \frac{a^2c\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{1}{8}\left(3a^2\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{9/2}}\,dx$$

$$= \frac{a^2c\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^2\cos(e+fx)}{8cf(c-c\sin(e+fx))^{7/2}} + \frac{a^2\int \frac{1}{(c-c\sin(e+fx))^{5/2}}\,dx}{16c^2}$$

$$= \frac{a^2c\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^2\cos(e+fx)}{8cf(c-c\sin(e+fx))^{7/2}} + \frac{a^2\cos(e+fx)}{64c^2f(c-c\sin(e+fx))^{5/2}}$$

$$= \frac{a^2c\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^2\cos(e+fx)}{8cf(c-c\sin(e+fx))^{7/2}} + \frac{a^2\cos(e+fx)}{64c^2f(c-c\sin(e+fx))^{5/2}}$$

$$= \frac{a^2c\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^2\cos(e+fx)}{8cf(c-c\sin(e+fx))^{7/2}} + \frac{a^2\cos(e+fx)}{64c^2f(c-c\sin(e+fx))^{5/2}}$$

$$= \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{256\sqrt{2}c^{9/2}f} + \frac{a^2c\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^2\cos(e+fx)}{8cf(c-c\sin(e+fx))}$$

Mathematica [C] time = 1.46, size = 371, normalized size = 1.95

$$\frac{a^2(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(256\sin\left(\frac{1}{2}(e+fx)\right)+3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(128*(Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2]) - 96*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 + 4*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^5 + 3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7 - (3 +
 3*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^8 + 256*Sin[(e + f*x)/2] - 192*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + 8*(Cos[(e + f*x)/2] - Sin[
(e + f*x)/2])^4*Sin[(e + f*x)/2] + 6*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
6*Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2)/(256*f*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])^4*(c - c*Sin[e + f*x])^(9/2))
```

fricas [B] time = 0.47, size = 523, normalized size = 2.75

$$\frac{3\sqrt{2}\left(a^2\cos\left(fx+e\right)^5+5\,a^2\cos\left(fx+e\right)^4-8\,a^2\cos\left(fx+e\right)^3-20\,a^2\cos\left(fx+e\right)^2+8\,a^2\cos\left(fx+e\right)+16\,a^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/1024*(3*sqrt(2)*(a^2*cos(f*x + e)^5 + 5*a^2*cos(f*x + e)^4 - 8*a^2*cos(f*
x + e)^3 - 20*a^2*cos(f*x + e)^2 + 8*a^2*cos(f*x + e) + 16*a^2 - (a^2*cos(f
*x + e)^4 - 4*a^2*cos(f*x + e)^3 - 12*a^2*cos(f*x + e)^2 + 8*a^2*cos(f*x +
e) + 16*a^2)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(
-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*
x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(
f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(3*a^2*cos(f*x + e)^4 +
 13*a^2*cos(f*x + e)^3 + 86*a^2*cos(f*x + e)^2 - 52*a^2*cos(f*x + e) - 128*
a^2 - (3*a^2*cos(f*x + e)^3 - 10*a^2*cos(f*x + e)^2 + 76*a^2*cos(f*x + e) +
 128*a^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^5*f*cos(f*x + e)^5 +
5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f*x + e)^2 +
 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x
+ e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f)*sin(f*x
 + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(9/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*2*(1/512*(-509*a^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*
x+exp(1))/2)^2+c))^15-1491*a^2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c
*tan((f*x+exp(1))/2)^2+c))^14-7411*a^2*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt
(c*tan((f*x+exp(1))/2)^2+c))^13-7937*a^2*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1
))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^12+711*a^2*c^2*(-sqrt(c)*tan((f*x+ex
p(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^11+22673*a^2*sqrt(c)*c^2*(-sqrt(c
)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^10+8457*a^2*c^3*(-sq
rt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^9-23829*a^2*sqrt
(c)*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^8-83
91*a^2*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7
```

```
+18823*a^2*sqrt(c)*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1)
)/2)^2+c))^6-3657*a^2*c^5*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp
(1))/2)^2+c))^5-10627*a^2*sqrt(c)*c^5*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*
tan((f*x+exp(1))/2)^2+c))^4+5693*a^2*c^6*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt
(c*tan((f*x+exp(1))/2)^2+c))^3+115*a^2*c^7*(-sqrt(c)*tan((f*x+exp(1))/2)+sq
rt(c*tan((f*x+exp(1))/2)^2+c))-2565*a^2*sqrt(c)*c^6*(-sqrt(c)*tan((f*x+exp(
1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-39*a^2*sqrt(c)*c^7)/c^4/(-(-sqrt(
c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-2*sqrt(c)*(-sqrt(
c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)^8/sign(tan((f*x+
exp(1))/2)-1)+3/512*a^2*atan((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*t
an((f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-c))/sqrt(2)/c^4/sqrt(-c)/sign(tan((f
*x+exp(1))/2)-1))
```

maple [A] time = 1.30, size = 299, normalized size = 1.57

$$\frac{a^2\left(6\left(c\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{7}{2}}c^{\frac{5}{2}}-3\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right)\left(\sin^4\left(fx+e\right)\right)c^6-44\left(c\left(1+\sin\left(fx+e\right)\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] 1/512/c^(21/2)*a^2*(6*(c*(1+sin(f*x+e)))^(7/2)*c^(5/2)-3*2^(1/2)*arctanh(1/
2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^4*c^6-44*(c*(1+sin(f
*x+e)))^(5/2)*c^(7/2)+12*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/
2)/c^(1/2))*sin(f*x+e)^3*c^6-88*(c*(1+sin(f*x+e)))^(3/2)*c^(9/2)-18*2^(1/2)
*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^6+48*
(c*(1+sin(f*x+e)))^(1/2)*c^(11/2)+12*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))
^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c^6-3*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+
e)))^(1/2)*2^(1/2)/c^(1/2))*c^6)*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)^3/
cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^2}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^2}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x))^(9/2),x)
[Out] int((a + a*sin(e + f*x))^2/(c - c*sin(e + f*x))^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(9/2),x)
[Out] Timed out
```

3.307 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=145

$$\frac{256a^3c^7\cos^7(e+fx)}{3003f(c-c\sin(e+fx))^{7/2}}+\frac{64a^3c^6\cos^7(e+fx)}{429f(c-c\sin(e+fx))^{5/2}}+\frac{24a^3c^5\cos^7(e+fx)}{143f(c-c\sin(e+fx))^{3/2}}+\frac{2a^3c^4\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 256/3003*a^3*c^7*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)+64/429*a^3*c^6*cos(f
*x+e)^7/f/(c-c*sin(f*x+e))^(5/2)+24/143*a^3*c^5*cos(f*x+e)^7/f/(c-c*sin(f*x
+e))^(3/2)+2/13*a^3*c^4*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$\frac{2a^3c^4\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}+\frac{24a^3c^5\cos^7(e+fx)}{143f(c-c\sin(e+fx))^{3/2}}+\frac{64a^3c^6\cos^7(e+fx)}{429f(c-c\sin(e+fx))^{5/2}}+\frac{256a^3c^7\cos^7(e+fx)}{3003f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (256*a^3*c^7*Cos[e + f*x]^7)/(3003*f*(c - c*Sin[e + f*x])^(7/2)) + (64*a^3*
c^6*Cos[e + f*x]^7)/(429*f*(c - c*Sin[e + f*x])^(5/2)) + (24*a^3*c^5*Cos[e
+ f*x]^7)/(143*f*(c - c*Sin[e + f*x])^(3/2)) + (2*a^3*c^4*Cos[e + f*x]^7)/(
13*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^{7/2}\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)\sqrt{c-c\sin(e+fx)}\,dx$$
$$= \frac{2a^3c^4\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}} + \frac{1}{13}\left(12a^3c^4\right)\int \frac{\cos^6(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \frac{24a^3c^5\cos^7(e+fx)}{143f(c-c\sin(e+fx))^{3/2}} + \frac{2a^3c^4\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}} + \frac{1}{143}\left(96a^3\right.$$
$$= \frac{64a^3c^6\cos^7(e+fx)}{429f(c-c\sin(e+fx))^{5/2}} + \frac{24a^3c^5\cos^7(e+fx)}{143f(c-c\sin(e+fx))^{3/2}} + \frac{2a^3c}{13f\sqrt{c}}$$
$$= \frac{256a^3c^7\cos^7(e+fx)}{3003f(c-c\sin(e+fx))^{7/2}} + \frac{64a^3c^6\cos^7(e+fx)}{429f(c-c\sin(e+fx))^{5/2}} + \frac{24}{143f}$$

Mathematica [A] time = 6.27, size = 112, normalized size = 0.77

$$\frac{a^3c^3\cos^6(e+fx)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(-6377\sin(e+fx)+231\sin(3(e+fx))}{6006f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a^3*c^3*Cos[e + f*x]^6*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Si
n[e + f*x]]*(5230 - 1890*Cos[2*(e + f*x)] - 6377*Sin[e + f*x] + 231*Sin[3*(
e + f*x)]))/(6006*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7)
```

fricas [B] time = 0.45, size = 265, normalized size = 1.83

$$\frac{2\left(231\,a^3c^3\cos\left(fx+e\right)^7-21\,a^3c^3\cos\left(fx+e\right)^6+28\,a^3c^3\cos\left(fx+e\right)^5-40\,a^3c^3\cos\left(fx+e\right)^4+64\,a^3c^3\cos\left(f\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(231*a^3*c^3*cos(f*x + e)^7 - 21*a^3*c^3*cos(f*x + e)^6 + 28*a^3*c^3
*cos(f*x + e)^5 - 40*a^3*c^3*cos(f*x + e)^4 + 64*a^3*c^3*cos(f*x + e)^3 - 1
28*a^3*c^3*cos(f*x + e)^2 + 512*a^3*c^3*cos(f*x + e) + 1024*a^3*c^3 + (231*
a^3*c^3*cos(f*x + e)^6 + 252*a^3*c^3*cos(f*x + e)^5 + 280*a^3*c^3*cos(f*x +
 e)^4 + 320*a^3*c^3*cos(f*x + e)^3 + 384*a^3*c^3*cos(f*x + e)^2 + 512*a^3*c
^3*cos(f*x + e) + 1024*a^3*c^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*
cos(f*x + e) - f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*c)*(-2*a^3*c^3*sign(s
in(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(2*f*x-pi)+1/2*exp(1))/f+5760*a^3*c^3*
f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(6*f*x+6*exp(1)+pi))/(192*f)^2
-9600*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(10*f*x+10*exp(1
)-pi))/(320*f)^2+1344*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*
(14*f*x+14*exp(1)+pi))/(224*f)^2-1728*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1
/4*pi))*cos(1/4*(18*f*x+18*exp(1)-pi))/(288*f)^2+1408*a^3*c^3*f*sign(sin(1/
2*(f*x+exp(1))-1/4*pi))*cos(1/4*(22*f*x+22*exp(1)+pi))/(704*f)^2-1664*a^3*c
^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(26*f*x+26*exp(1)-pi))/(832
*f)^2+352*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(2*f*x-pi)+1
/2*exp(1))/(16*f)^2)
```

maple [A] time = 0.89, size = 81, normalized size = 0.56

$$\frac{2\left(\sin\left(fx+e\right)-1\right)c^{4}\left(1+\sin\left(fx+e\right)\right)^{4}a^{3}\left(231\left(\sin^{3}\left(fx+e\right)\right)-945\left(\sin^{2}\left(fx+e\right)\right)+1421\sin\left(fx+e\right)-83\right.}{3003\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 2/3003*(sin(f*x+e)-1)*c^4*(1+sin(f*x+e))^4*a^3*(231*sin(f*x+e)^3-945*sin(f*
x+e)^2+1421*sin(f*x+e)-835)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{3}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(7/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^3\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(7/2),x)
[Out] int((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**(7/2),x)
[Out] Timed out
```

3.308 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=109

$$\frac{64a^3c^6\cos^7(e+fx)}{693f(c-c\sin(e+fx))^{7/2}}+\frac{16a^3c^5\cos^7(e+fx)}{99f(c-c\sin(e+fx))^{5/2}}+\frac{2a^3c^4\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 64/693*a^3*c^6*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)+16/99*a^3*c^5*cos(f*x+
e)^7/f/(c-c*sin(f*x+e))^(5/2)+2/11*a^3*c^4*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^
(3/2)
```

Rubi [A] time = 0.27, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$\frac{2a^3c^4\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}+\frac{16a^3c^5\cos^7(e+fx)}{99f(c-c\sin(e+fx))^{5/2}}+\frac{64a^3c^6\cos^7(e+fx)}{693f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (64*a^3*c^6*Cos[e + f*x]^7)/(693*f*(c - c*Sin[e + f*x])^(7/2)) + (16*a^3*c^
5*Cos[e + f*x]^7)/(99*f*(c - c*Sin[e + f*x])^(5/2)) + (2*a^3*c^4*Cos[e + f*
x]^7)/(11*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
```

```
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}\,dx=\left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$=\frac{2a^3c^4\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}+\frac{1}{11}\left(8a^3c^4\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx$$
$$=\frac{16a^3c^5\cos^7(e+fx)}{99f(c-c\sin(e+fx))^{5/2}}+\frac{2a^3c^4\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}+\frac{1}{99}\left(32a^3\right.$$
$$=\frac{64a^3c^6\cos^7(e+fx)}{693f(c-c\sin(e+fx))^{7/2}}+\frac{16a^3c^5\cos^7(e+fx)}{99f(c-c\sin(e+fx))^{5/2}}+\frac{2a^3c}{11f(c-}$$

Mathematica [B] time = 6.48, size = 1105, normalized size = 10.14

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (5*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2))/(8*f
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^6) - (5*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x]
)^(5/2))/(24*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^6) + (Cos[(5*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*
Sin[e + f*x])^(5/2))/(16*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^6) - (5*Cos[(7*(e + f*x))/2]*(a + a*Sin[e + f
*x])^3*(c - c*Sin[e + f*x])^(5/2))/(112*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)
/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (Cos[(9*(e + f*x))/2]*(a
+ a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2))/(144*f*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - (Cos[(11*(e
+ f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2))/(176*f*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
+ (5*Sin[(e + f*x)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2))/(8
*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])^6) + (5*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2)*Sin[(3*(e +
 f*x))/2])/(24*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^6) + ((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2)
```

```
*Sin[(5*(e + f*x))/2])/(16*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(
e + f*x)/2] + Sin[(e + f*x)/2])^6) + (5*(a + a*Sin[e + f*x])^3*(c - c*Sin[e
 + f*x])^(5/2)*Sin[(7*(e + f*x))/2])/(112*f*(Cos[(e + f*x)/2] - Sin[(e + f*
x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + ((a + a*Sin[e + f*x])^3
*(c - c*Sin[e + f*x])^(5/2)*Sin[(9*(e + f*x))/2])/(144*f*(Cos[(e + f*x)/2]
- Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + ((a + a*Si
n[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2)*Sin[(11*(e + f*x))/2])/(176*f*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6
)
```

fricas [B] time = 0.45, size = 234, normalized size = 2.15

$$-\frac{2\left(63\,a^3c^2\cos\left(fx+e\right)^6-7\,a^3c^2\cos\left(fx+e\right)^5+10\,a^3c^2\cos\left(fx+e\right)^4-16\,a^3c^2\cos\left(fx+e\right)^3+32\,a^3c^2\cos\left(f\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/693*(63*a^3*c^2*cos(f*x + e)^6 - 7*a^3*c^2*cos(f*x + e)^5 + 10*a^3*c^2*c
os(f*x + e)^4 - 16*a^3*c^2*cos(f*x + e)^3 + 32*a^3*c^2*cos(f*x + e)^2 - 128
*a^3*c^2*cos(f*x + e) - 256*a^3*c^2 - (63*a^3*c^2*cos(f*x + e)^5 + 70*a^3*c
^2*cos(f*x + e)^4 + 80*a^3*c^2*cos(f*x + e)^3 + 96*a^3*c^2*cos(f*x + e)^2 +
 128*a^3*c^2*cos(f*x + e) + 256*a^3*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e)
 + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sq
rt(2*c)*(-2*a^3*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(2*f*x-pi)+1
/2*exp(1))/f+32*a^3*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(2*f*x
+2*exp(1)+pi))/(16*f)^2-96*a^3*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin
```

```
(1/4*(6*f*x+6*exp(1)-pi))/(48*f)^2+960*a^3*c^2*f*sign(sin(1/2*(f*x+exp(1))-
1/4*pi))*sin(1/4*(10*f*x+10*exp(1)+pi))/(160*f)^2-1344*a^3*c^2*f*sign(sin(1
/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(14*f*x+14*exp(1)-pi))/(224*f)^2+576*a^3*c
^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(18*f*x+18*exp(1)+pi))/(288
*f)^2-704*a^3*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(22*f*x+22*e
xp(1)-pi))/(352*f)^2+24*a^3*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/
4*(6*f*x+6*exp(1)+pi))/(12*f)^2-40*a^3*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*
pi))*cos(1/4*(10*f*x+10*exp(1)-pi))/(20*f)^2+224*a^3*c^2*f*sign(sin(1/2*(f*
x+exp(1))-1/4*pi))*cos(1/4*(14*f*x+14*exp(1)+pi))/(112*f)^2-288*a^3*c^2*f*s
ign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(18*f*x+18*exp(1)-pi))/(144*f)^2+
80*a^3*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(2*f*x-pi)+1/2*exp(
1))/(8*f)^2)
```

maple [A] time = 0.76, size = 71, normalized size = 0.65

$$-\frac{2\left(\sin\left(fx+e\right)-1\right)c^{3}\left(1+\sin\left(fx+e\right)\right)^{4}a^{3}\left(63\left(\sin^{2}\left(fx+e\right)\right)-182\sin\left(fx+e\right)+151\right)}{693\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -2/693*(sin(f*x+e)-1)*c^3*(1+sin(f*x+e))^4*a^3*(63*sin(f*x+e)^2-182*sin(f*x
+e)+151)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{3}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(a+a\,\sin\left(e+f\,x\right)\right)^{3}\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.309 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=73

$$\frac{8a^3c^5\cos^7(e+fx)}{63f(c-c\sin(e+fx))^{7/2}}+\frac{2a^3c^4\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 8/63*a^3*c^5*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)+2/9*a^3*c^4*cos(f*x+e)^7
/f/(c-c*sin(f*x+e))^(5/2)
```

Rubi [A] time = 0.20, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$\frac{2a^3c^4\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}+\frac{8a^3c^5\cos^7(e+fx)}{63f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (8*a^3*c^5*Cos[e + f*x]^7)/(63*f*(c - c*Sin[e + f*x])^(7/2)) + (2*a^3*c^4*C
os[e + f*x]^7)/(9*f*(c - c*Sin[e + f*x])^(5/2))
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
```

```
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx$$
$$= \frac{2a^3c^4\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}+\frac{1}{9}\left(4a^3c^4\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx$$
$$= \frac{8a^3c^5\cos^7(e+fx)}{63f(c-c\sin(e+fx))^{7/2}}+\frac{2a^3c^4\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}$$

Mathematica [A] time = 3.09, size = 84, normalized size = 1.15

$$-\frac{2a^3c(7\sin(e+fx)-11)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7}{63f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a^3*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(-11 + 7*Sin[e + f*x])*Sq
rt[c - c*Sin[e + f*x]])/(63*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

fricas [B] time = 0.46, size = 179, normalized size = 2.45

$$-\frac{2\left(7\,a^3c\cos\left(fx+e\right)^5+17\,a^3c\cos\left(fx+e\right)^4-2\,a^3c\cos\left(fx+e\right)^3+4\,a^3c\cos\left(fx+e\right)^2-16\,a^3c\cos\left(fx+e\right)-\right.}{63\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/63*(7*a^3*c*cos(f*x + e)^5 + 17*a^3*c*cos(f*x + e)^4 - 2*a^3*c*cos(f*x +
 e)^3 + 4*a^3*c*cos(f*x + e)^2 - 16*a^3*c*cos(f*x + e) - 32*a^3*c + (7*a^3*
c*cos(f*x + e)^4 - 10*a^3*c*cos(f*x + e)^3 - 12*a^3*c*cos(f*x + e)^2 - 16*a
^3*c*cos(f*x + e) - 32*a^3*c)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*co
s(f*x + e) - f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*c)*(-2*a
^3*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(2*f*x-pi)+1/2*exp(1))/f+8*
a^3*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(2*f*x+2*exp(1)+pi))/(4*
f)^2-24*a^3*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(6*f*x+6*exp(1)-
pi))/(12*f)^2+40*a^3*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(10*f*x
+10*exp(1)+pi))/(20*f)^2-56*a^3*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(
1/4*(14*f*x+14*exp(1)-pi))/(28*f)^2+24*a^3*c*f*sign(sin(1/2*(f*x+exp(1))-1/
4*pi))*cos(1/4*(6*f*x+6*exp(1)+pi))/(12*f)^2-40*a^3*c*f*sign(sin(1/2*(f*x+e
xp(1))-1/4*pi))*cos(1/4*(10*f*x+10*exp(1)-pi))/(20*f)^2-224*a^3*c*f*sign(si
n(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(14*f*x+14*exp(1)+pi))/(112*f)^2+288*a^
3*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(18*f*x+18*exp(1)-pi))/(14
4*f)^2+48*a^3*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(2*f*x-pi)+1/2
*exp(1))/(8*f)^2)
```

maple [A] time = 0.83, size = 61, normalized size = 0.84

$$\frac{2 \left(\sin \left(f x +e \right)-1\right) c^{2} \left(1+\sin \left(f x +e \right)\right)^{4} a^{3} \left(7 \sin \left(f x +e \right)-11\right)}{63 \cos \left(f x +e \right) \sqrt{c -c \sin \left(f x +e \right)}\, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 2/63*(sin(f*x+e)-1)*c^2*(1+sin(f*x+e))^4*a^3*(7*sin(f*x+e)-11)/cos(f*x+e)/(
c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin \left(f x +e \right)+a \right)^{3} \left(-c \sin \left(f x +e \right)+c \right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+a\,\sin\left(e+f\,x\right)\right)^3\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int c\sqrt{-c\sin\left(e+fx\right)+c}\,dx+\int 2c\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)dx+\int\left(-2c\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] a**3*(Integral(c*sqrt(-c*sin(e + f*x) + c), x) + Integral(2*c*sqrt(-c*sin(e
 + f*x) + c)*sin(e + f*x), x) + Integral(-2*c*sqrt(-c*sin(e + f*x) + c)*sin
(e + f*x)**3, x) + Integral(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**4, x
))
```

3.310 $\int (a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=36

$$\frac{2a^3c^4\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}}$$

```
[Out] 2/7*a^3*c^4*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)
```

Rubi [A] time = 0.13, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2736, 2673}

$$\frac{2a^3c^4\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*a^3*c^4*Cos[e + f*x]^7)/(7*f*(c - c*Sin[e + f*x])^(7/2))
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx\\ &= \frac{2a^3c^4\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}}\end{aligned}$$

Mathematica [B] time = 0.38, size = 73, normalized size = 2.03

$$\frac{2a^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7}{7f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*a^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*Sqrt[c - c*Sin[e + f*x]])/(7
*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

fricas [B] time = 0.43, size = 141, normalized size = 3.92

$$\frac{2\left(a^3\cos\left(fx+e\right)^4-3\,a^3\cos\left(fx+e\right)^3-8\,a^3\cos\left(fx+e\right)^2+4\,a^3\cos\left(fx+e\right)+8\,a^3-\left(a^3\cos\left(fx+e\right)^3+4\,a^3\right.\right.}{7\left(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)+f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/7*(a^3*cos(f*x + e)^4 - 3*a^3*cos(f*x + e)^3 - 8*a^3*cos(f*x + e)^2 + 4*a
^3*cos(f*x + e) + 8*a^3 - (a^3*cos(f*x + e)^3 + 4*a^3*cos(f*x + e)^2 - 4*a^
3*cos(f*x + e) - 8*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x
+ e) - f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*c)*(-2*a^3*sign(sin(1
/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(2*f*x-pi)+1/2*exp(1))/f+240*a^3*f*sign(si
n(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(2*f*x+2*exp(1)+pi))/(8*f)^2-720*a^3*f*
sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(6*f*x+6*exp(1)-pi))/(24*f)^2-80
*a^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(10*f*x+10*exp(1)+pi))/(4
```

```
0*f)^2+112*a^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(14*f*x+14*exp(
1)-pi))/(56*f)^2-72*a^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(6*f*x
+6*exp(1)+pi))/(12*f)^2+120*a^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/
4*(10*f*x+10*exp(1)-pi))/(20*f)^2-12*a^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi
))*cos(1/4*(2*f*x-pi)+1/2*exp(1))/(2*f)^2)
```

maple [A] time = 0.73, size = 49, normalized size = 1.36

$$-\frac{2\left(\sin\left(fx+e\right)-1\right)c\left(1+\sin\left(fx+e\right)\right)^{4}a^{3}}{7\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(1/2),x)
[Out] -2/7*(sin(f*x+e)-1)*c*(1+sin(f*x+e))^4*a^3/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2
)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{3}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^3*sqrt(-c*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2),x)
[Out] int((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int 3\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)dx+\int 3\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^{2}\left(e+fx\right)dx+\int\sqrt{-c\sin\left(e+fx\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a**3*(Integral(3*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Integral(3*sq
rt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(sqrt(-c*sin(e + f*x)
 + c)*sin(e + f*x)**3, x) + Integral(sqrt(-c*sin(e + f*x) + c), x))
```

3.311 $\int \frac{(a+a\sin(e+fx))^3}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=151

$$-\frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}-\frac{4a^3c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}-\frac{8a^3\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}+\frac{8\sqrt{2}\,a^3\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f}$$

```
[Out] -2/5*a^3*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)-4/3*a^3*c*cos(f*x+e)^3/f
/(c-c*sin(f*x+e))^(3/2)+8*a^3*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*s
in(f*x+e))^(1/2))*2^(1/2)/f/c^(1/2)-8*a^3*cos(f*x+e)/f/(c-c*sin(f*x+e))^(1/
2)
```

Rubi [A] time = 0.32, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2736, 2679, 2649, 206}

$$-\frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}-\frac{4a^3c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}-\frac{8a^3\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}+\frac{8\sqrt{2}\,a^3\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (8*Sqrt[2]*a^3*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f
*x]])])/(Sqrt[c]*f) - (2*a^3*c^2*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*x])^
(5/2)) - (4*a^3*c*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^(3/2)) - (8*a^3
*Cos[e + f*x])/(f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{\sqrt{c-c\sin(e+fx)}}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,dx$$

$$= -\frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} + \left(2a^3c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx$$

$$= -\frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{4a^3c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} + \left(4a^3c\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))}$$

$$= -\frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{4a^3c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} - \frac{8a^3\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}} + \left(8a^3\right)$$

$$= -\frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{4a^3c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} - \frac{8a^3\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}} - \frac{\left(16a\right.}{}$$

$$= \frac{8\sqrt{2}\,a^3\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f} - \frac{2a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{4a^3c\cos^3(e+fx)}{3f(c-c\sin(e+fx))}$$

Mathematica [C] time = 0.75, size = 156, normalized size = 1.03

$$-\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(330\sin\left(\frac{1}{2}(e+fx)\right)+35\sin\left(\frac{3}{2}(e+fx)\right)-3\sin\left(\frac{5}{2}(e+fx)\right)+330\cos\right.}{30f\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] -1/30*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*((480 + 480*I)*(-1)^(1/4)*
ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])] + 330*Cos[(e + f*x)/2
] - 35*Cos[(3*(e + f*x))/2] - 3*Cos[(5*(e + f*x))/2] + 330*Sin[(e + f*x)/2]
 + 35*Sin[(3*(e + f*x))/2] - 3*Sin[(5*(e + f*x))/2]))/(f*Sqrt[c - c*Sin[e +
 f*x]])
```

fricas [B] time = 0.45, size = 265, normalized size = 1.75

$$2\left(\frac{30\,\sqrt{2}\left(a^{3}c\cos\left(fx+e\right)-a^{3}c\sin\left(fx+e\right)+a^{3}c\right)\log\left(-\frac{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)+\frac{2\,\sqrt{2}\,\sqrt{-c\sin\left(fx+e\right)+c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)}{\sqrt{c}}+3\,\cos\left(fx+e\right)+2}{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right)}{\sqrt{c}}+\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(30*sqrt(2)*(a^3*c*cos(f*x + e) - a^3*c*sin(f*x + e) + a^3*c)*log(-(co
s(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(-c*sin(f*x
+ e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(
cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(
c) + (3*a^3*cos(f*x + e)^3 + 19*a^3*cos(f*x + e)^2 - 76*a^3*cos(f*x + e) -
92*a^3 + (3*a^3*cos(f*x + e)^2 - 16*a^3*cos(f*x + e) - 92*a^3)*sin(f*x + e)
)*sqrt(-c*sin(f*x + e) + c))/(c*f*cos(f*x + e) - c*f*sin(f*x + e) + c*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(2/sqrt(c*tan((f*x+exp(1
))/2)^2+c)/(c*tan((f*x+exp(1))/2)^2+c)^2*(tan((f*x+exp(1))/2)*(tan((f*x+exp
(1))/2)*(tan((f*x+exp(1))/2)*(tan((f*x+exp(1))/2)*(7/2*a^3*c^2/sign(tan((f*
```

```
x+exp(1))/2)-1)+73/30*a^3*c^2*tan((f*x+exp(1))/2)/sign(tan((f*x+exp(1))/2)-
1))+19/3*a^3*c^2/sign(tan((f*x+exp(1))/2)-1))+19/3*a^3*c^2/sign(tan((f*x+ex
p(1))/2)-1))+7/2*a^3*c^2/sign(tan((f*x+exp(1))/2)-1))+73/30*a^3*c^2/sign(ta
n((f*x+exp(1))/2)-1))+16*a^3*atan((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqr
t(c*tan((f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-c))/sqrt(2)/sqrt(-c)/sign(tan((
f*x+exp(1))/2)-1)+(-240*a^3*c*atan(sqrt(c)/sqrt(-c))-184*a^3*sqrt(-c)*sqrt(
c))/15/c/sqrt(-c)/sqrt(2)*sign(tan((f*x+exp(1))/2)-1))
```

maple [A] time = 1.13, size = 129, normalized size = 0.85

$$-\frac{2\left(\sin\left(fx+e\right)-1\right)\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\, a^{3}\left(60c^{\frac{5}{2}}\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right)-3\left(c\left(1+\sin\left(fx+e\right)\right)\right)\right.}{15c^{3}\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\ f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/15*(sin(f*x+e)-1)*(c*(1+sin(f*x+e)))^(1/2)*a^3*(60*c^(5/2)*2^(1/2)*arcta
nh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))-3*(c*(1+sin(f*x+e)))^(5/2)
-10*(c*(1+sin(f*x+e)))^(3/2)*c-60*c^2*(c*(1+sin(f*x+e)))^(1/2))/c^3/cos(f*x
+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{3}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/sqrt(-c*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\ \sin\left(e+f\,x\right)\right)^{3}}{\sqrt{c-c\ \sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int \frac{3\sin\left(e+fx\right)}{\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int \frac{3\sin^2\left(e+fx\right)}{\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int \frac{\sin^3\left(e+fx\right)}{\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int \frac{1}{\sqrt{-c\sin\left(e+f\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a**3*(Integral(3*sin(e + f*x)/sqrt(-c*sin(e + f*x) + c), x) + Integral(3*si
n(e + f*x)**2/sqrt(-c*sin(e + f*x) + c), x) + Integral(sin(e + f*x)**3/sqrt
(-c*sin(e + f*x) + c), x) + Integral(1/sqrt(-c*sin(e + f*x) + c), x))
```

3.312 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=150

$$-\frac{10\sqrt{2}\,a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}+\frac{a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^{7/2}}+\frac{5a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}+\frac{10a^3\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

```
[Out] a^3*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(7/2)+5/3*a^3*cos(f*x+e)^3/f/(c-c*s
in(f*x+e))^(3/2)-10*a^3*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x
+e))^(1/2))*2^(1/2)/c^(3/2)/f+10*a^3*cos(f*x+e)/c/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2736, 2680, 2679, 2649, 206}

$$\frac{a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^{7/2}}-\frac{10\sqrt{2}\,a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}+\frac{5a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}+\frac{10a^3\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-10*Sqrt[2]*a^3*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e +
 f*x]])])/(c^(3/2)*f) + (a^3*c^2*Cos[e + f*x]^5)/(f*(c - c*Sin[e + f*x])^(7
/2)) + (5*a^3*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^(3/2)) + (10*a^3*Co
s[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
```

```
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{3/2}}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{9/2}}\,dx\\
&= \frac{a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^{7/2}} - \frac{1}{2}\left(5a^3c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx\\
&= \frac{a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^{7/2}} + \frac{5a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} - \left(5a^3\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,d\\
&= \frac{a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^{7/2}} + \frac{5a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} + \frac{10a^3\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}} - \frac{\left(10a^3\right)}{}\\
&= \frac{a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^{7/2}} + \frac{5a^3\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} + \frac{10a^3\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}} + \frac{\left(20a^3\right)}{}\\
&= -\frac{10\sqrt{2}\,a^3\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f} + \frac{a^3c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^{7/2}} + \frac{5a^3\cos^3(e+fx}{3f(c-c\sin(e+fx}
\end{aligned}$$

Mathematica [C] time = 0.84, size = 173, normalized size = 1.15

$$-\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(50\sin\left(\frac{1}{2}(e+fx)\right)-25\sin\left(\frac{3}{2}(e+fx)\right)+\sin\left(\frac{5}{2}(e+fx)\right)+50\cos\left(\frac{1}{2}(\right.\right.}{6cf(\sin(e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/6*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(50*Cos[(e + f*x)/2] + 25*C
os[(3*(e + f*x))/2] + Cos[(5*(e + f*x))/2] + 50*Sin[(e + f*x)/2] - (120 + 1
20*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(-1
+ Sin[e + f*x]) - 25*Sin[(3*(e + f*x))/2] + Sin[(5*(e + f*x))/2]))/(c*f*(-1
 + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])
```

fricas [B] time = 0.46, size = 326, normalized size = 2.17

$$\frac{\frac{15\sqrt{2}\left(a^3c\cos(fx+e)^2-a^3c\cos(fx+e)-2a^3c+\left(a^3c\cos(fx+e)+2a^3c\right)\sin(fx+e)\right)\log\left(-\frac{\cos(fx+e)^2+(\cos(fx+e)-2)\sin(fx+e)-\frac{2\sqrt{2}\sqrt{-c\sin(fx+e)+c}\,(\cos(fx}{\sqrt{c}}}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(}\right.}{\sqrt{c}}}{3\left(c^2f\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(15*sqrt(2)*(a^3*c*cos(f*x + e)^2 - a^3*c*cos(f*x + e) - 2*a^3*c + (a^3
*c*cos(f*x + e) + 2*a^3*c)*sin(f*x + e))*log(-(cos(f*x + e)^2 + (cos(f*x +
e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) +
sin(f*x + e) + 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x
+ e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(c) - 2*(a^3*cos(f*x + e)^3
 + 13*a^3*cos(f*x + e)^2 + 18*a^3*cos(f*x + e) + 6*a^3 + (a^3*cos(f*x + e)^
2 - 12*a^3*cos(f*x + e) + 6*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(
c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) - 2*c^2*f + (c^2*f*cos(f*x + e) +
 2*c^2*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(2/sqrt(c*tan((f*x+exp(1))/2)^2+c)/(c
*tan((f*x+exp(1))/2)^2+c)*(tan((f*x+exp(1))/2)*(tan((f*x+exp(1))/2)*(-5/2*a
^3/sign(tan((f*x+exp(1))/2)-1)-13/6*a^3*tan((f*x+exp(1))/2)/sign(tan((f*x+e
xp(1))/2)-1))-5/2*a^3/sign(tan((f*x+exp(1))/2)-1))-13/6*a^3/sign(tan((f*x+e
xp(1))/2)-1))+2*((-6*a^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(
1))/2)^2+c))^3+2*a^3*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1)
)/2)^2+c))-2*a^3*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(
1))/2)^2+c))^2-2*a^3*sqrt(c)*c)/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan(
(f*x+exp(1))/2)^2+c))^2-2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan(
(f*x+exp(1))/2)^2+c))+c)^2/c/sign(tan((f*x+exp(1))/2)-1)-10*a^3*atan((-sqrt
(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/sqrt(2)/sq
rt(-c))/sqrt(2)/sqrt(-c)/c/sign(tan((f*x+exp(1))/2)-1)))
```

maple [A] time = 1.02, size = 189, normalized size = 1.26

$$\frac{2a^3\left(15\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{2}}{2\sqrt{c}}\right)\sin\left(fx+e\right)c^2-12\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,c^{\frac{3}{2}}\sin\left(fx+e\right)-\left(c\left(1+\sin\left(f\right.\right.\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 2/3*a^3*(15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*s
in(f*x+e)*c^2-12*(c*(1+sin(f*x+e)))^(1/2)*c^(3/2)*sin(f*x+e)-(c*(1+sin(f*x+
e)))^(3/2)*c^(1/2)*sin(f*x+e)-15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/
2)*2^(1/2)/c^(1/2))*c^2+18*(c*(1+sin(f*x+e)))^(1/2)*c^(3/2)+(c*(1+sin(f*x+e
)))^(3/2)*c^(1/2))*(c*(1+sin(f*x+e)))^(1/2)/c^(7/2)/cos(f*x+e)/(c-c*sin(f*x
+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^3}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^3}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^(3/2),x)
[Out] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int \frac{3\sin\left(e+fx\right)}{-c\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)+c\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int \frac{3\sin^2\left(e+\right.}{-c\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(3/2),x)
[Out] a**3*(Integral(3*sin(e + f*x)/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) +
c*sqrt(-c*sin(e + f*x) + c)), x) + Integral(3*sin(e + f*x)**2/(-c*sqrt(-c*s
in(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin(e + f*x) + c)), x) + Integral
(sin(e + f*x)**3/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin
(e + f*x) + c)), x) + Integral(1/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)
 + c*sqrt(-c*sin(e + f*x) + c)), x))
```

3.313 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=157

$$\frac{15a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{5/2}f}+\frac{a^3c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}-\frac{15a^3\cos(e+fx)}{4c^2f\sqrt{c-c\sin(e+fx)}}-\frac{5a^3\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 1/2*a^3*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(9/2)-5/4*a^3*cos(f*x+e)^3/f/(c
-c*sin(f*x+e))^(5/2)+15/4*a^3*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*s
in(f*x+e))^(1/2))/c^(5/2)/f*2^(1/2)-15/4*a^3*cos(f*x+e)/c^2/f/(c-c*sin(f*x+
e))^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2736, 2680, 2679, 2649, 206}

$$\frac{a^3c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}-\frac{15a^3\cos(e+fx)}{4c^2f\sqrt{c-c\sin(e+fx)}}+\frac{15a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{5/2}f}-\frac{5a^3\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (15*a^3*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])
/(2*Sqrt[2]*c^(5/2)*f) + (a^3*c^2*Cos[e + f*x]^5)/(2*f*(c - c*Sin[e + f*x])
^(9/2)) - (5*a^3*Cos[e + f*x]^3)/(4*f*(c - c*Sin[e + f*x])^(5/2)) - (15*a^3
*Cos[e + f*x])/(4*c^2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{5/2}}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{11/2}}\,dx$$
$$= \frac{a^3c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} - \frac{1}{4}\left(5a^3c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,dx$$
$$= \frac{a^3c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} - \frac{5a^3\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{5/2}} + \frac{\left(15a^3\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx}{8c}$$
$$= \frac{a^3c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} - \frac{5a^3\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{5/2}} - \frac{15a^3\cos(e+fx)}{4c^2f\sqrt{c-c\sin(e+fx)}} + \frac{(15 \ldots}{\ldots}$$
$$= \frac{a^3c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} - \frac{5a^3\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{5/2}} - \frac{15a^3\cos(e+fx)}{4c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(15 \ldots}{\ldots}$$
$$= \frac{15a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{5/2}f} + \frac{a^3c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} - \frac{5a^3\cos^3(e+fx)}{4f(c-c\sin(e+fx))^{5/2}}$$

Mathematica [C] time = 1.05, size = 187, normalized size = 1.19

$$\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-5\sin\left(\frac{1}{2}(e+fx)\right)+15\sin\left(\frac{3}{2}(e+fx)\right)+2\sin\left(\frac{5}{2}(e+fx)\right)-5\cos\left(\frac{1}{2}(e+ \ldots\right.\right.}{4c^2f \ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(-5*Cos[(e + f*x)/2] - 15*Cos[(3
*(e + f*x))/2] + 2*Cos[(5*(e + f*x))/2] - 5*Sin[(e + f*x)/2] + (15 + 15*I)*
(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(-3 + Cos[
2*(e + f*x)] + 4*Sin[e + f*x]) + 15*Sin[(3*(e + f*x))/2] + 2*Sin[(5*(e + f*
x))/2]))/(4*c^2*f*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

fricas [B] time = 0.47, size = 388, normalized size = 2.47

$$\frac{15\sqrt{2}\left(a^3\cos\left(fx+e\right)^3+3a^3\cos\left(fx+e\right)^2-2a^3\cos\left(fx+e\right)-4a^3-\left(a^3\cos\left(fx+e\right)^2-2a^3\cos\left(fx+e\right)-4 \ldots\right.\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/8*(15*sqrt(2)*(a^3*cos(f*x + e)^3 + 3*a^3*cos(f*x + e)^2 - 2*a^3*cos(f*x
+ e) - 4*a^3 - (a^3*cos(f*x + e)^2 - 2*a^3*cos(f*x + e) - 4*a^3)*sin(f*x +
e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sq
rt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e
) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x +
 e) - cos(f*x + e) - 2)) - 4*(4*a^3*cos(f*x + e)^3 - 13*a^3*cos(f*x + e)^2
- 13*a^3*cos(f*x + e) + 4*a^3 + (4*a^3*cos(f*x + e)^2 + 17*a^3*cos(f*x + e)
 + 4*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^3*f*cos(f*x + e)^3 +
3*c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x +
e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)2/f*(2*sqrt(c*tan((f*x+exp(1))/2)^2+c)*(1/2*a^3/c^2/si
gn(tan((f*x+exp(1))/2)-1)+1/2*a^3*tan((f*x+exp(1))/2)/c^2/sign(tan((f*x+exp
(1))/2)-1))/(c*tan((f*x+exp(1))/2)^2+c)+2*(1/4*(7*a^3*(-sqrt(c)*tan((f*x+ex
p(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7+81*a^3*sqrt(c)*(-sqrt(c)*tan((f
*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6+53*a^3*c*(-sqrt(c)*tan((f*
x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5-65*a^3*sqrt(c)*c*(-sqrt(c)*
tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4+13*a^3*c^2*(-sqrt(c)
*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3-33*a^3*c^3*(-sqrt(c
)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+19*a^3*sqrt(c)*c^2*(
-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+5*a^3*sqrt(
c)*c^3)/c^2/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c)
)^2-2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c)
)+c)^4/sign(tan((f*x+exp(1))/2)-1)+15/4*a^3*atan((-sqrt(c)*tan((f*x+exp(1))
/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-c))/sqrt(2)/c^2/
sqrt(-c)/sign(tan((f*x+exp(1))/2)-1)))
```

maple [A] time = 1.17, size = 239, normalized size = 1.52

$$-\frac{a^{3}\left(15\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right)\left(\sin^{2}\left(fx+e\right)\right)c^{2}-8\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ c^{\frac{3}{2}}\left(\sin^{2}\left(fx+e\right)\right)-30\sqrt{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/4/c^(9/2)*a^3*(15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c
^(1/2))*sin(f*x+e)^2*c^2-8*(c*(1+sin(f*x+e)))^(1/2)*c^(3/2)*sin(f*x+e)^2-30
*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c
^2+18*(c*(1+sin(f*x+e)))^(3/2)*c^(1/2)+16*(c*(1+sin(f*x+e)))^(1/2)*c^(3/2)*
sin(f*x+e)+15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))
*c^2-36*(c*(1+sin(f*x+e)))^(1/2)*c^(3/2))*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x
+e)-1)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{3}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.314 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=157

$$-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{7/2}f}+\frac{a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^{11/2}}+\frac{5a^3\cos(e+fx)}{8c^2f(c-c\sin(e+fx))^{3/2}}-\frac{5a^3\cos^3(e+fx)}{12f(c-c\sin(e+fx))^{7/2}}$$

```
[Out] 1/3*a^3*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(11/2)-5/12*a^3*cos(f*x+e)^3/f/
(c-c*sin(f*x+e))^(7/2)+5/8*a^3*cos(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(3/2)-5/16
*a^3*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(7/2)
/f*2^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2736, 2680, 2649, 206}

$$\frac{a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^{11/2}}+\frac{5a^3\cos(e+fx)}{8c^2f(c-c\sin(e+fx))^{3/2}}-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{7/2}f}-\frac{5a^3\cos^3(e+fx)}{12f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-5*a^3*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])
/(8*Sqrt[2]*c^(7/2)*f) + (a^3*c^2*Cos[e + f*x]^5)/(3*f*(c - c*Sin[e + f*x])
^(11/2)) - (5*a^3*Cos[e + f*x]^3)/(12*f*(c - c*Sin[e + f*x])^(7/2)) + (5*a^
3*Cos[e + f*x])/(8*c^2*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{7/2}}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{13/2}}\,dx \\
&= \frac{a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^{11/2}} - \frac{1}{6}\left(5a^3c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{9/2}}\,dx \\
&= \frac{a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^{11/2}} - \frac{5a^3\cos^3(e+fx)}{12f(c-c\sin(e+fx))^{7/2}} + \frac{\left(5a^3\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx}{8c} \\
&= \frac{a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^{11/2}} - \frac{5a^3\cos^3(e+fx)}{12f(c-c\sin(e+fx))^{7/2}} + \frac{5a^3\cos(e+fx)}{8c^2f(c-c\sin(e+fx))^{3/2}} - \\
&= \frac{a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^{11/2}} - \frac{5a^3\cos^3(e+fx)}{12f(c-c\sin(e+fx))^{7/2}} + \frac{5a^3\cos(e+fx)}{8c^2f(c-c\sin(e+fx))^{3/2}} + \\
&= -\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{7/2}f} + \frac{a^3c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^{11/2}} - \frac{5a^3\cos^3(e+fx)}{12f(c-c\sin(e+fx))
\end{aligned}$$

Mathematica [C] time = 1.57, size = 307, normalized size = 1.96

$$a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(64\sin\left(\frac{1}{2}(e+fx)\right)+33\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+f\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(32*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2]) - 52*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 + 33*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^5 + (15 + 15*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(
1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6 + 64*S
in[(e + f*x)/2] - 104*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)
/2] + 66*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/2])*(1 + Sin
[e + f*x])^3)/(24*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e +
f*x])^(7/2))
```

fricas [B] time = 0.46, size = 440, normalized size = 2.80

$$\frac{15\sqrt{2}\left(a^3\cos\left(fx+e\right)^4-3\,a^3\cos\left(fx+e\right)^3-8\,a^3\cos\left(fx+e\right)^2+4\,a^3\cos\left(fx+e\right)+8\,a^3+\left(a^3\cos\left(fx+e\right)^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/96*(15*sqrt(2)*(a^3*cos(f*x + e)^4 - 3*a^3*cos(f*x + e)^3 - 8*a^3*cos(f*x
 + e)^2 + 4*a^3*cos(f*x + e) + 8*a^3 + (a^3*cos(f*x + e)^3 + 4*a^3*cos(f*x
+ e)^2 - 4*a^3*cos(f*x + e) - 8*a^3)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x
+ e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*
x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)
/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4
*(33*a^3*cos(f*x + e)^3 + 19*a^3*cos(f*x + e)^2 - 46*a^3*cos(f*x + e) - 32*
a^3 + (33*a^3*cos(f*x + e)^2 + 14*a^3*cos(f*x + e) - 32*a^3)*sin(f*x + e))*
sqrt(-c*sin(f*x + e) + c))/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f*x + e)^3 -
 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*f*cos(f*x +
 e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)*sin(f*x +
e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
```

```
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integrat
ion of abs or sign assumes constant sign by intervals (correct if the argum
ent is real):Check [abs(sin((f*t_nostep+exp(1))/2-pi/4))]Unable to check si
gn: (8*pi/t_nostep/2)>(-8*pi/t_nostep/2)Discontinuities at zeroes of sin((f
*t_nostep+exp(1))/2-pi/4) were not checkedUnable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes cons
tant sign by intervals (correct if the argument is real):Check [abs(t_noste
p-1)]Evaluation time: 0.99Not invertible Error: Bad Argument Value
```

maple [A] time = 1.00, size = 245, normalized size = 1.56

$$\frac{a^3\left(15\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right)\left(\sin^3\left(fx+e\right)\right)c^3-45\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right)\left(\sin^2\left(fx+e\right)\right)c^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/48/c^(13/2)*a^3*(15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/
c^(1/2))*sin(f*x+e)^3*c^3-45*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2
^(1/2)/c^(1/2))*sin(f*x+e)^2*c^3+66*(c*(1+sin(f*x+e)))^(5/2)*c^(1/2)+45*2^(
1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c^3-1
60*(c*(1+sin(f*x+e)))^(3/2)*c^(3/2)-15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)
))^(1/2)*2^(1/2)/c^(1/2))*c^3+120*(c*(1+sin(f*x+e)))^(1/2)*c^(5/2))*(c*(1+s
in(f*x+e)))^(1/2)/(sin(f*x+e)-1)^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{3}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(7/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^3}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^(7/2),x)
[Out] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(7/2),x)
[Out] Timed out
```

3.315 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=191

$$-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}c^{9/2}f}-\frac{5a^3\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{3/2}}+\frac{a^3c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{13/2}}+\frac{5a^3\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 1/4*a^3*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(13/2)-5/24*a^3*cos(f*x+e)^3/f/
(c-c*sin(f*x+e))^(9/2)+5/32*a^3*cos(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(5/2)-5/1
28*a^3*cos(f*x+e)/c^3/f/(c-c*sin(f*x+e))^(3/2)-5/256*a^3*arctanh(1/2*cos(f*
x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(9/2)/f*2^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2736, 2680, 2650, 2649, 206}

$$\frac{a^3c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{13/2}}-\frac{5a^3\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{3/2}}+\frac{5a^3\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{5/2}}-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}c^{9/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (-5*a^3*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])
/(128*Sqrt[2]*c^(9/2)*f) + (a^3*c^2*Cos[e + f*x]^5)/(4*f*(c - c*Sin[e + f*x
])^(13/2)) - (5*a^3*Cos[e + f*x]^3)/(24*f*(c - c*Sin[e + f*x])^(9/2)) + (5*
a^3*Cos[e + f*x])/(32*c^2*f*(c - c*Sin[e + f*x])^(5/2)) - (5*a^3*Cos[e + f*
x])/(128*c^3*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{9/2}}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{15/2}}\,dx$$

$$= \frac{a^3c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{13/2}} - \frac{1}{8}\left(5a^3c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{11/2}}\,dx$$

$$= \frac{a^3c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{13/2}} - \frac{5a^3\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} + \frac{\left(5a^3\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,dx}{16c}$$

$$= \frac{a^3c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{13/2}} - \frac{5a^3\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} + \frac{5a^3\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{5/2}} -$$

$$= \frac{a^3c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{13/2}} - \frac{5a^3\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} + \frac{5a^3\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{5/2}} -$$

$$= \frac{a^3c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{13/2}} - \frac{5a^3\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} + \frac{5a^3\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{5/2}} -$$

$$= -\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}c^{9/2}f} + \frac{a^3c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{13/2}} - \frac{5a^3\cos^3(e+fx)}{24f(c-c\sin(e+fx))}$$

Mathematica [C] time = 2.65, size = 371, normalized size = 1.94

$$\frac{a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(768\sin\left(\frac{1}{2}(e+fx)\right)-15\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(384*(Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2]) - 544*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 + 236*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])^5 - 15*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7 +
(15 + 15*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])
]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8 + 768*Sin[(e + f*x)/2] - 1088*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + 472*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/2] - 30*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^6*Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3)/(384*f*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^(9/2))
```

fricas [B] time = 0.47, size = 523, normalized size = 2.74

$$\frac{15\sqrt{2}\left(a^3\cos\left(fx+e\right)^5+5a^3\cos\left(fx+e\right)^4-8a^3\cos\left(fx+e\right)^3-20a^3\cos\left(fx+e\right)^2+8a^3\cos\left(fx+e\right)+16a\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/1536*(15*sqrt(2)*(a^3*cos(f*x + e)^5 + 5*a^3*cos(f*x + e)^4 - 8*a^3*cos(f
*x + e)^3 - 20*a^3*cos(f*x + e)^2 + 8*a^3*cos(f*x + e) + 16*a^3 - (a^3*cos(
f*x + e)^4 - 4*a^3*cos(f*x + e)^3 - 12*a^3*cos(f*x + e)^2 + 8*a^3*cos(f*x +
 e) + 16*a^3)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt
(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f
*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos
(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(15*a^3*cos(f*x + e)^4
 - 191*a^3*cos(f*x + e)^3 - 338*a^3*cos(f*x + e)^2 + 252*a^3*cos(f*x + e) +
 384*a^3 - (15*a^3*cos(f*x + e)^3 + 206*a^3*cos(f*x + e)^2 - 132*a^3*cos(f*
x + e) - 384*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^5*f*cos(f*x +
 e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f*x
+ e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5*f*
cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f)
*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(9/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*2*(1/768*(-783*a^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*
x+exp(1))/2)^2+c))^15-993*a^3*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*
tan((f*x+exp(1))/2)^2+c))^14-14913*a^3*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt
(c*tan((f*x+exp(1))/2)^2+c))^13-11259*a^3*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(
1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^12+285*a^3*c^2*(-sqrt(c)*tan((f*x+e
xp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^11+28715*a^3*sqrt(c)*c^2*(-sqrt(
c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^10+17363*a^3*c^3*(-
sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^9-37271*a^3*sq
rt(c)*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^8-
8989*a^3*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))
```

```
^7+36189*a^3*sqrt(c)*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(
1))/2)^2+c))^6-6547*a^3*c^5*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+e
xp(1))/2)^2+c))^5-17777*a^3*sqrt(c)*c^5*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(
c*tan((f*x+exp(1))/2)^2+c))^4+5583*a^3*c^6*(-sqrt(c)*tan((f*x+exp(1))/2)+sq
rt(c*tan((f*x+exp(1))/2)^2+c))^3+193*a^3*c^7*(-sqrt(c)*tan((f*x+exp(1))/2)+
sqrt(c*tan((f*x+exp(1))/2)^2+c))-5351*a^3*sqrt(c)*c^6*(-sqrt(c)*tan((f*x+ex
p(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-61*a^3*sqrt(c)*c^7)/c^4/(-(-sqr
t(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-2*sqrt(c)*(-sqr
t(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)^8/sign(tan((f*
x+exp(1))/2)-1)-5/256*a^3*atan((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c
*tan((f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-c))/sqrt(2)/c^4/sqrt(-c)/sign(tan(
(f*x+exp(1))/2)-1))
```

maple [A] time = 1.12, size = 299, normalized size = 1.57

$$-\frac{a^3\left(-15\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right)\left(\sin^4\left(fx+e\right)\right)c^5+30c^{\frac{3}{2}}\left(c\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{7}{2}}+60\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c}}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] -1/768/c^(19/2)*a^3*(-15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/
2)/c^(1/2))*sin(f*x+e)^4*c^5+30*c^(3/2)*(c*(1+sin(f*x+e)))^(7/2)+60*2^(1/2)
*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^3*c^5+292
*(c*(1+sin(f*x+e)))^(5/2)*c^(5/2)-90*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))
^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^5-440*(c*(1+sin(f*x+e)))^(3/2)*c^(7/
2)+60*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x
+e)*c^5+240*(c*(1+sin(f*x+e)))^(1/2)*c^(9/2)-15*2^(1/2)*arctanh(1/2*(c*(1+s
in(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^5)*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e
)-1)^3/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^3}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^3}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^(9/2),x)
[Out] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(9/2),x)
[Out] Timed out
```

3.316 $\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=225

$$-\frac{3a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{512\sqrt{2}c^{11/2}f}-\frac{3a^3\cos(e+fx)}{512c^4f(c-c\sin(e+fx))^{3/2}}-\frac{a^3\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{5/2}}+\frac{a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))}$$

```
[Out] 1/5*a^3*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(15/2)-1/8*a^3*cos(f*x+e)^3/f/(
c-c*sin(f*x+e))^(11/2)+1/16*a^3*cos(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(7/2)-1/1
28*a^3*cos(f*x+e)/c^3/f/(c-c*sin(f*x+e))^(5/2)-3/512*a^3*cos(f*x+e)/c^4/f/(
c-c*sin(f*x+e))^(3/2)-3/1024*a^3*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-
c*sin(f*x+e))^(1/2))/c^(11/2)/f*2^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2736, 2680, 2650, 2649, 206}

$$\frac{a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{15/2}}-\frac{3a^3\cos(e+fx)}{512c^4f(c-c\sin(e+fx))^{3/2}}-\frac{a^3\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{5/2}}+\frac{a^3\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^(11/2),x]
```

```
[Out] (-3*a^3*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])
/(512*Sqrt[2]*c^(11/2)*f) + (a^3*c^2*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*
x])^(15/2)) - (a^3*Cos[e + f*x]^3)/(8*f*(c - c*Sin[e + f*x])^(11/2)) + (a^3
*Cos[e + f*x])/(16*c^2*f*(c - c*Sin[e + f*x])^(7/2)) - (a^3*Cos[e + f*x])/(
128*c^3*f*(c - c*Sin[e + f*x])^(5/2)) - (3*a^3*Cos[e + f*x])/(512*c^4*f*(c
- c*Sin[e + f*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{11/2}}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{17/2}}\,dx$$

$$= \frac{a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{15/2}} - \frac{1}{2}\left(a^3c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{13/2}}\,dx$$

$$= \frac{a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{15/2}} - \frac{a^3\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{11/2}} + \frac{\left(3a^3\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{9/2}}\,dx}{16c}$$

$$= \frac{a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{15/2}} - \frac{a^3\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{11/2}} + \frac{a^3\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{7/2}}$$

$$= \frac{a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{15/2}} - \frac{a^3\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{11/2}} + \frac{a^3\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{7/2}}$$

$$= \frac{a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{15/2}} - \frac{a^3\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{11/2}} + \frac{a^3\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{7/2}}$$

$$= \frac{a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{15/2}} - \frac{a^3\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{11/2}} + \frac{a^3\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{7/2}}$$

$$= -\frac{3a^3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{512\sqrt{2}c^{11/2}f} + \frac{a^3c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{15/2}} - \frac{a^3\cos^3(e+fx)}{8f(c-c\sin(e+fx))}$$

Mathematica [C] time = 4.11, size = 435, normalized size = 1.93

$$\frac{a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(4096\sin\left(\frac{1}{2}(e+fx)\right)-15\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c - c*Sin[e + f*x])^(11/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(2048*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2]) - 2688*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 + 992*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])^5 - 20*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7
- 15*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9 + (15 + 15*I)*(-1)^(1/4)*ArcTa
n[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])^10 + 4096*Sin[(e + f*x)/2] - 5376*(Cos[(e + f*x)/2] - Sin[(e +
```

```
f*x)/2])^2*Sin[(e + f*x)/2] + 1984*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*
Sin[(e + f*x)/2] - 40*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6*Sin[(e + f*x)
/2] - 30*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8*Sin[(e + f*x)/2])*(1 + Sin
[e + f*x])^3)/(2560*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e
+ f*x])^(11/2))
```

fricas [B] time = 0.51, size = 600, normalized size = 2.67

$$15\sqrt{2}\left(a^3\cos\left(fx+e\right)^6 - 5a^3\cos\left(fx+e\right)^5 - 18a^3\cos\left(fx+e\right)^4 + 20a^3\cos\left(fx+e\right)^3 + 48a^3\cos\left(fx+e\right)^2 - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(11/2),x, algorithm="fricas")
```

```
[Out] 1/10240*(15*sqrt(2)*(a^3*cos(f*x + e)^6 - 5*a^3*cos(f*x + e)^5 - 18*a^3*cos
(f*x + e)^4 + 20*a^3*cos(f*x + e)^3 + 48*a^3*cos(f*x + e)^2 - 16*a^3*cos(f*
x + e) - 32*a^3 + (a^3*cos(f*x + e)^5 + 6*a^3*cos(f*x + e)^4 - 12*a^3*cos(f
*x + e)^3 - 32*a^3*cos(f*x + e)^2 + 16*a^3*cos(f*x + e) + 32*a^3)*sin(f*x +
 e))*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*s
qrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x +
e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x
+ e) - cos(f*x + e) - 2)) + 4*(15*a^3*cos(f*x + e)^5 - 65*a^3*cos(f*x + e)^
4 + 812*a^3*cos(f*x + e)^3 + 1796*a^3*cos(f*x + e)^2 - 1144*a^3*cos(f*x + e
) - 2048*a^3 + (15*a^3*cos(f*x + e)^4 + 80*a^3*cos(f*x + e)^3 + 892*a^3*cos
(f*x + e)^2 - 904*a^3*cos(f*x + e) - 2048*a^3)*sin(f*x + e))*sqrt(-c*sin(f*
x + e) + c))/(c^6*f*cos(f*x + e)^6 - 5*c^6*f*cos(f*x + e)^5 - 18*c^6*f*cos(
f*x + e)^4 + 20*c^6*f*cos(f*x + e)^3 + 48*c^6*f*cos(f*x + e)^2 - 16*c^6*f*c
os(f*x + e) - 32*c^6*f + (c^6*f*cos(f*x + e)^5 + 6*c^6*f*cos(f*x + e)^4 - 1
2*c^6*f*cos(f*x + e)^3 - 32*c^6*f*cos(f*x + e)^2 + 16*c^6*f*cos(f*x + e) +
32*c^6*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(11/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*2*(1/5120*(-5135*a^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((
f*x+exp(1))/2)^2+c))^19-15645*a^3*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqr
```

```
t(c*tan((f*x+exp(1))/2)^2+c))^18-150755*a^3*c*(-sqrt(c)*tan((f*x+exp(1))/2)
+sqrt(c*tan((f*x+exp(1))/2)^2+c))^17-270725*a^3*sqrt(c)*c*(-sqrt(c)*tan((f*
x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^16-385756*a^3*c^2*(-sqrt(c)*t
an((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^15+333260*a^3*sqrt(c)*c
^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^14+983140
*a^3*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^13+
258460*a^3*sqrt(c)*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1)
)/2)^2+c))^12-1312610*a^3*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x
+exp(1))/2)^2+c))^11-597862*a^3*sqrt(c)*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+s
qrt(c*tan((f*x+exp(1))/2)^2+c))^10+1413590*a^3*c^5*(-sqrt(c)*tan((f*x+exp(1
))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^9+283610*a^3*sqrt(c)*c^5*(-sqrt(c)*t
an((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^8-1071020*a^3*c^6*(-sqr
t(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7+237340*a^3*sqrt
(c)*c^6*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6+26
8532*a^3*c^7*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))
^5-352180*a^3*sqrt(c)*c^7*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp
(1))/2)^2+c))^4+96505*a^3*c^8*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x
+exp(1))/2)^2+c))^3+1205*a^3*c^9*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((
f*x+exp(1))/2)^2+c))-38245*a^3*sqrt(c)*c^8*(-sqrt(c)*tan((f*x+exp(1))/2)+sq
rt(c*tan((f*x+exp(1))/2)^2+c))^2-317*a^3*sqrt(c)*c^9)/c^5/(-(-sqrt(c)*tan((
f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-2*sqrt(c)*(-sqrt(c)*tan((
f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)^10/sign(tan((f*x+exp(1))
/2)-1)-3/1024*a^3*atan((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*
x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-c))/sqrt(2)/c^5/sqrt(-c)/sign(tan((f*x+exp
(1))/2)-1))
```

maple [A] time = 1.12, size = 353, normalized size = 1.57

$$\frac{a^3\left(15\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right)\left(\sin^5\left(fx+e\right)\right)c^7-30\left(c\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{9}{2}}c^{\frac{5}{2}}+280\left(c\left(1+\sin\left(fx+e\right.\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(11/2),x)
```

```
[Out] 1/5120*a^3*(15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2)
)*sin(f*x+e)^5*c^7-30*(c*(1+sin(f*x+e)))^(9/2)*c^(5/2)+280*(c*(1+sin(f*x+e)
))^(7/2)*c^(7/2)+1024*(c*(1+sin(f*x+e)))^(5/2)*c^(9/2)-1120*(c*(1+sin(f*x+e
)))^(3/2)*c^(11/2)+480*(c*(1+sin(f*x+e)))^(1/2)*c^(13/2)-75*2^(1/2)*arctanh
(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^4*c^7+150*2^(1/2)
*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^3*c^7-150
*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2
*c^7+75*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f
```

```
*x+e)*c^7-15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*
c^7)*(c*(1+sin(f*x+e)))^(1/2)/c^(25/2)/(sin(f*x+e)-1)^4/cos(f*x+e)/(c-c*sin
(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{3}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(11/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)^(11/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^3}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^(11/2),x)
[Out] int((a + a*sin(e + f*x))^3/(c - c*sin(e + f*x))^(11/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(11/2),x)
[Out] Timed out
```

3.317 $\int \frac{(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=132

$$-\frac{256c^3\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{5af}+\frac{64c^2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{5af}+\frac{2\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{5af}$$

```
[Out] 64/5*c^2*sec(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f+8/5*c*sec(f*x+e)*(c-c*sin(f*
x+e))^(5/2)/a/f+2/5*sec(f*x+e)*(c-c*sin(f*x+e))^(7/2)/a/f-256/5*c^3*sec(f*x
+e)*(c-c*sin(f*x+e))^(1/2)/a/f
```

Rubi [A] time = 0.34, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$\frac{64c^2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{5af}-\frac{256c^3\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{5af}+\frac{2\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{5af}+$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(7/2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-256*c^3*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(5*a*f) + (64*c^2*Sec[e +
f*x]*(c - c*Sin[e + f*x])^(3/2))/(5*a*f) + (8*c*Sec[e + f*x]*(c - c*Sin[e +
 f*x])^(5/2))/(5*a*f) + (2*Sec[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(5*a*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx = \frac{\int \sec^2(e+fx)(c-c\sin(e+fx))^{9/2}\,dx}{ac}$$
$$= \frac{2\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{5af} + \frac{12\int \sec^2(e+fx)(c-c\sin(e+fx))^{7/2}\,dx}{5a}$$
$$= \frac{8c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{5af} + \frac{2\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{5af} + \frac{(32c)\int}{}$$
$$= \frac{64c^2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{5af} + \frac{8c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{5af} + \frac{2\,\mathrm{se}}{}$$
$$= -\frac{256c^3\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{5af} + \frac{64c^2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{5af} +$$

Mathematica [A] time = 2.19, size = 112, normalized size = 0.85

$$\frac{c^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(-175\sin(e+fx)+\sin(3(e+fx))-14\cos(2(e+fx)}{10af(\sin(e+fx)+1)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(7/2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] (c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(-350 -
 14*Cos[2*(e + f*x)] - 175*Sin[e + f*x] + Sin[3*(e + f*x)]))/(10*a*f*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x]))
```

fricas [A] time = 0.46, size = 74, normalized size = 0.56

$$-\frac{2\left(7\,c^3\cos\left(fx+e\right)^2+84\,c^3-\left(c^3\cos\left(fx+e\right)^2-44\,c^3\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{5\,af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -2/5*(7*c^3*cos(f*x + e)^2 + 84*c^3 - (c^3*cos(f*x + e)^2 - 44*c^3)*sin(f*x
 + e))*sqrt(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.64, size = 69, normalized size = 0.52

$$\frac{2c^4\left(\sin\left(fx+e\right)-1\right)\left(\sin^3\left(fx+e\right)-7\left(\sin^2\left(fx+e\right)\right)+43\sin\left(fx+e\right)+91\right)}{5a\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] 2/5*c^4/a*(sin(f*x+e)-1)*(sin(f*x+e)^3-7*sin(f*x+e)^2+43*sin(f*x+e)+91)/cos
(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [B] time = 1.80, size = 238, normalized size = 1.80

$$\frac{2\left(91\,c^{\frac{7}{2}}+\frac{86\,c^{\frac{7}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{336\,c^{\frac{7}{2}}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{266\,c^{\frac{7}{2}}\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{490\,c^{\frac{7}{2}}\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+\frac{266\,c^{\frac{7}{2}}\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}+\frac{336\,c^{\frac{7}{2}}\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}+\cdots\right.}{5\left(a+\frac{a\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)f\left(\frac{\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2/5*(91*c^(7/2) + 86*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 336*c^(7/2)*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 266*c^(7/2)*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 490*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 266*c^(7/2)*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 336*c^(7/2)*sin(f*x + e)^6/(cos(f*x +
 e) + 1)^6 + 86*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 91*c^(7/2)*si
n(f*x + e)^8/(cos(f*x + e) + 1)^8)/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))
*f*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c - c\,\sin\left(e + f\,x\right)\right)^{7/2}}{a + a\,\sin\left(e + f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(7/2)/(a + a*sin(e + f*x)),x)
[Out] int((c - c*sin(e + f*x))^(7/2)/(a + a*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e)),x)
[Out] Timed out
```

3.318 $\int \frac{(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=98

$$-\frac{64c^2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3af}+\frac{2\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3af}+\frac{16c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3af}$$

```
[Out] 16/3*c*sec(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f+2/3*sec(f*x+e)*(c-c*sin(f*x+e)
)^(5/2)/a/f-64/3*c^2*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f
```

Rubi [A] time = 0.27, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$-\frac{64c^2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3af}+\frac{2\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3af}+\frac{16c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-64*c^2*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*a*f) + (16*c*Sec[e + f*x
]*(c - c*Sin[e + f*x])^(3/2))/(3*a*f) + (2*Sec[e + f*x]*(c - c*Sin[e + f*x]
)^(5/2))/(3*a*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
```

```
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx = \frac{\int \sec^2(e+fx)(c-c\sin(e+fx))^{7/2}\,dx}{ac}$$
$$= \frac{2\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3af} + \frac{8\int \sec^2(e+fx)(c-c\sin(e+fx))^{5/2}\,dx}{3a}$$
$$= \frac{16c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3af} + \frac{2\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3af} + \frac{(32c)}{}$$
$$= -\frac{64c^2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3af} + \frac{16c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3af} + \frac{2\,\mathrm{s}}{}$$

Mathematica [A] time = 0.75, size = 102, normalized size = 1.04

$$-\frac{c^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(20\sin(e+fx)+\cos(2(e+fx))+45)}{3af(\sin(e+fx)+1)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] -1/3*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(45 + Cos[2*(e + f*x)] + 20
*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a*f*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(1 + Sin[e + f*x]))
```

fricas [A] time = 0.44, size = 58, normalized size = 0.59

$$-\frac{2\left(c^2\cos\left(fx+e\right)^2+10\,c^2\sin\left(fx+e\right)+22\,c^2\right)\sqrt{-c\sin\left(fx+e\right)+c}}{3\,af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -2/3*(c^2*cos(f*x + e)^2 + 10*c^2*sin(f*x + e) + 22*c^2)*sqrt(-c*sin(f*x +
e) + c)/(a*f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.70, size = 59, normalized size = 0.60

$$-\frac{2c^{3}\left(\sin\left(fx+e\right)-1\right)\left(\sin^{2}\left(fx+e\right)-10\sin\left(fx+e\right)-23\right)}{3a\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] -2/3*c^3/a*(sin(f*x+e)-1)*(sin(f*x+e)^2-10*sin(f*x+e)-23)/cos(f*x+e)/(c-c*s
in(f*x+e))^(1/2)/f
```

maxima [B] time = 0.66, size = 192, normalized size = 1.96

$$\frac{2\left(23\,c^{\frac{5}{2}}+\frac{20\,c^{\frac{5}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{65\,c^{\frac{5}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{40\,c^{\frac{5}{2}}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{65\,c^{\frac{5}{2}}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{20\,c^{\frac{5}{2}}\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}+\frac{23\,c^{\frac{5}{2}}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}\right)}{3\left(a+\frac{a\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)f\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2/3*(23*c^(5/2) + 20*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 65*c^(5/2)*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 40*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 65*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 20*c^(5/2)*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5 + 23*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) +
 1)^6)/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*f*(sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 1)^(5/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x)),x)
```

```
[Out] int((c - c*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.319 $\int \frac{(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=60

$$\frac{2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{af}-\frac{8c\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{af}$$

```
[Out] 2*sec(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f-8*c*sec(f*x+e)*(c-c*sin(f*x+e))^(1/
2)/a/f
```

Rubi [A] time = 0.20, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$\frac{2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{af}-\frac{8c\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-8*c*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*f) + (2*Sec[e + f*x]*(c - c
*Sin[e + f*x])^(3/2))/(a*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
```

```
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx &= \frac{\int \sec^2(e+fx)(c-c\sin(e+fx))^{5/2}\,dx}{ac}\\ &= \frac{2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{af}+\frac{4\int \sec^2(e+fx)(c-c\sin(e+fx))^{3/2}\,dx}{a}\\ &= -\frac{8c\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{af}+\frac{2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{af}\end{aligned}$$

Mathematica [A] time = 0.31, size = 88, normalized size = 1.47

$$-\frac{2c(\sin(e+fx)+3)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{af(\sin(e+fx)+1)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-2*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3 + Sin[e + f*x])*Sqrt[c - c*S
in[e + f*x]])/(a*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])
)
```

fricas [A] time = 0.44, size = 41, normalized size = 0.68

$$-\frac{2\left(c\sin\left(fx+e\right)+3\,c\right)\sqrt{-c\sin\left(fx+e\right)+c}}{af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -2*(c*sin(f*x + e) + 3*c)*sqrt(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
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((211106232532992*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))+211106232532
992*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1))
)^2+70368744177664*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/
2*f*x+2*exp(1)))^3-633318697598976*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*
pi))*tan(1/2*exp(1))^2+2814749767106560*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))
-1/4*pi))*tan(1/2*exp(1))^3+1477743627730944*sqrt(2)*c*sign(sin(1/2*(f*x+ex
p(1))-1/4*pi))*tan(1/2*exp(1))^4-844424930131968*sqrt(2)*c*sign(sin(1/2*(f*
x+exp(1))-1/4*pi))*tan(1/2*exp(1))^5+70368744177664*sqrt(2)*c*sign(sin(1/2*
(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^6-3166593487994880*sqrt(2)*c*sign(sin
(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^2+1407374883553280*sqrt(2)*c*sig
n(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^3+3166593487994880*sqrt(2)*
c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^4-422212465065984*sqrt
(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^5-211106232532992*
sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^6+351843720888
320*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1))
)-844424930131968*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(
1))-422212465065984*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*ex
p(1))-3166593487994880*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2
*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^2+3166593487994880*sqrt(2)*c*sign(si
n(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^4
-211106232532992*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*
f*x+2*exp(1)))^2*tan(1/2*exp(1))^6-3166593487994880*sqrt(2)*c*sign(sin(1/2*
(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/4*exp(1))^2-42221
24650659840*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2
*exp(1)))^2*tan(1/4*exp(1))^3+3166593487994880*sqrt(2)*c*sign(sin(1/2*(f*x+
exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/4*exp(1))^4+1266637395
197952*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(
1)))^2*tan(1/4*exp(1))^5-211106232532992*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1)
)-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/4*exp(1))^6+1266637395197952
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*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2
*tan(1/4*exp(1))-1055531162664960*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*p
i))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^2-2814749767106560*sqrt(2
)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/
2*exp(1))^3+1055531162664960*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*t
an(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^4+844424930131968*sqrt(2)*c*si
gn(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(
1))^5-70368744177664*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(
1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^6-1055531162664960*sqrt(2)*c*sign(sin(
1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/4*exp(1))^2+1
407374883553280*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f
*x+2*exp(1)))^3*tan(1/4*exp(1))^3+1055531162664960*sqrt(2)*c*sign(sin(1/2*(
f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/4*exp(1))^4-422212
465065984*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*e
xp(1)))^3*tan(1/4*exp(1))^5-70368744177664*sqrt(2)*c*sign(sin(1/2*(f*x+exp(
1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/4*exp(1))^6+84442493013196
8*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^
3*tan(1/2*exp(1))-422212465065984*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*p
i))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/4*exp(1))+9499780463984640*sqrt(2)*
c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(1/4*exp(1))^2+29
554872554618880*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1)
)^2*tan(1/4*exp(1))^3-9499780463984640*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-
1/4*pi))*tan(1/2*exp(1))^2*tan(1/4*exp(1))^4-8866461766385664*sqrt(2)*c*sig
n(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(1/4*exp(1))^5+6333186
97598976*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan
(1/4*exp(1))^6-8866461766385664*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi)
)*tan(1/2*exp(1))^2*tan(1/4*exp(1))-42221246506598400*sqrt(2)*c*sign(sin(1/
2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^3*tan(1/4*exp(1))^2+562949953421312
00*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^3*tan(1/4*e
xp(1))^3+42221246506598400*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan
(1/2*exp(1))^3*tan(1/4*exp(1))^4-16888498602639360*sqrt(2)*c*sign(sin(1/2*(
f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^3*tan(1/4*exp(1))^5-2814749767106560*s
qrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^3*tan(1/4*exp(1
))^6-16888498602639360*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2
*exp(1))^3*tan(1/4*exp(1))-22166154415964160*sqrt(2)*c*sign(sin(1/2*(f*x+ex
p(1))-1/4*pi))*tan(1/2*exp(1))^4*tan(1/4*exp(1))^2-12666373951979520*sqrt(2
)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^4*tan(1/4*exp(1))^3+
22166154415964160*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(
1))^4*tan(1/4*exp(1))^4+3799912185593856*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1)
)-1/4*pi))*tan(1/2*exp(1))^4*tan(1/4*exp(1))^5-1477743627730944*sqrt(2)*c*s
ign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^4*tan(1/4*exp(1))^6+37999
12185593856*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^4*
tan(1/4*exp(1))+12666373951979520*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*p
i))*tan(1/2*exp(1))^5*tan(1/4*exp(1))^2-16888498602639360*sqrt(2)*c*sign(si
n(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^5*tan(1/4*exp(1))^3-12666373951
```

979520*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^5*tan(1/4*exp(1))^4+5066549580791808*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^5*tan(1/4*exp(1))^5+844424930131968*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^5*tan(1/4*exp(1))^6+5066549580791808*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^5*tan(1/4*exp(1))-1055531162664960*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^6*tan(1/4*exp(1))^2+4222124650659840*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^6*tan(1/4*exp(1))^3+1055531162664960*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^6*tan(1/4*exp(1))^4-1266637395197952*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^6*tan(1/4*exp(1))^5-70368744177664*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^6*tan(1/4*exp(1))^6-1266637395197952*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^6*tan(1/4*exp(1))-2744381022928896*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^2+3588805953060864*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^4-70368744177664*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^6-5277655813324800*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/4*exp(1))^2+1407374883553280*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/4*exp(1))^3+5277655813324800*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/4*exp(1))^4-422212465065984*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/4*exp(1))^5-351843720888320*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/4*exp(1))^6-422212465065984*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/4*exp(1))+12666373951979520*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/4*exp(1))^2-16888498602639360*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/4*exp(1))^3-12666373951979520*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/4*exp(1))^4+5066549580791808*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/4*exp(1))^5+844424930131968*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/4*exp(1))^6+5066549580791808*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/4*exp(1))+47498902319923200*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^2*tan(1/4*exp(1))^2+63331869759897600*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^2*tan(1/4*exp(1))^3-47498902319923200*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^2*tan(1/4*exp(1))^4-18999560927969280*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^2*tan(1/4*exp(1))^5+3166593487994880*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^2*tan(1/4*exp(1))^6-18999560927969280*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^2*tan(1/4*exp(1))-47498902319923200*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2 *exp(1))^4*tan(1/4*exp(1))^2-6333186975989760O*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^4*tan(1/4*exp(1))^3+47498902319923200*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^4*tan(1/4*exp(1))^4+18999560927969280*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^4*tan(1/4*exp(1))^5-3166593487994880*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^4*tan(1/4*exp(1))^6+18999560927969280*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^4*tan(1/4*exp(1))+3166593487994880*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^6*tan(1/4*exp(1))^2+4222124650659840*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^6*tan(1/4*exp(1))^3-3166593487994880*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^6*tan(1/4*exp(1))^4-1266637395197952*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^6*tan(1/4*exp(1))^5+211106232532992*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^6*tan(1/4*exp(1))^6-1266637395197952*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^6*tan(1/4*exp(1))+15832967439974400*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^2*tan(1/4*exp(1))^2-21110623253299200*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^2*tan(1/4*exp(1))^3-15832967439974400*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^2*tan(1/4*exp(1))^4+6333186975989760*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^2*tan(1/4*exp(1))^5+1055531162664960*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^2*tan(1/4*exp(1))^6+6333186975989760*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^2*tan(1/4*exp(1))+42221246506598400*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^3*tan(1/4*exp(1))^2+56294995342131200*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^3*tan(1/4*exp(1))^3-42221246506598400*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^3*tan(1/4*exp(1))^4-16888498602639360*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^3*tan(1/4*exp(1))^5+2814749767106560*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^3*tan(1/4*exp(1))^6-16888498602639360*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^3*tan(1/4*exp(1))-15832967439974400*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^4*tan(1/4*exp(1))^2+21110623253299200*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^4*tan(1/4*exp(1))^3+15832967439974400*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^4*tan(1/4*exp(1))^4-633318697598

9760*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)
))^3*tan(1/2*exp(1))^4*tan(1/4*exp(1))^5-1055531162664960*sqrt(2)*c*sign(si
n(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^4
*tan(1/4*exp(1))^6-6333186975989760*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4
*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^4*tan(1/4*exp(1))-12666
373951979520*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+
2*exp(1)))^3*tan(1/2*exp(1))^5*tan(1/4*exp(1))^2-16888498602639360*sqrt(2)*
c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*
exp(1))^5*tan(1/4*exp(1))^3+12666373951979520*sqrt(2)*c*sign(sin(1/2*(f*x+e
xp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^5*tan(1/4*exp
(1))^4+5066549580791808*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/
2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^5*tan(1/4*exp(1))^5-844424930131968
*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3
*tan(1/2*exp(1))^5*tan(1/4*exp(1))^6+5066549580791808*sqrt(2)*c*sign(sin(1/
2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^5*tan
(1/4*exp(1))+1055531162664960*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*
tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^6*tan(1/4*exp(1))^2-140737488
3553280*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp
(1)))^3*tan(1/2*exp(1))^6*tan(1/4*exp(1))^3-1055531162664960*sqrt(2)*c*sign
(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1)
)^6*tan(1/4*exp(1))^4+422212465065984*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1
/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^6*tan(1/4*exp(1))^5+7
0368744177664*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x
+2*exp(1)))^3*tan(1/2*exp(1))^6*tan(1/4*exp(1))^6+422212465065984*sqrt(2)*c
sign(sin(1/2(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*e
xp(1))^6*tan(1/4*exp(1))-12666373951979520*sqrt(2)*c*sign(sin(1/2*(f*x+exp(
1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))*tan(1/4*exp(1))^
2-16888498602639360*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1
/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))*tan(1/4*exp(1))^3+12666373951979520*sqr
t(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan
(1/2*exp(1))*tan(1/4*exp(1))^4+5066549580791808*sqrt(2)*c*sign(sin(1/2*(f*x
+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))*tan(1/4*exp
(1))^5-844424930131968*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2
*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))*tan(1/4*exp(1))^6+5066549580791808*s
qrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*t
an(1/2*exp(1))*tan(1/4*exp(1))+41165715343933440*sqrt(2)*c*sign(sin(1/2*(f*
x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^2*tan(1/4*ex
p(1))^2-71776119061217280*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(
1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^2*tan(1/4*exp(1))^3-411657153439334
40*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))
*tan(1/2*exp(1))^2*tan(1/4*exp(1))^4+21532835718365184*sqrt(2)*c*sign(sin(1
/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^2*tan(
1/4*exp(1))^5+2744381022928896*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))
tan(1/2(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^2*tan(1/4*exp(1))^6+2153283571
8365184*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp

(1)))*tan(1/2*exp(1))^2*tan(1/4*exp(1))-53832089295912960*sqrt(2)*c*sign(si
n(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^4*t
an(1/4*exp(1))^2+54887620458577920*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*
pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^4*tan(1/4*exp(1))^3+538320
89295912960*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2
*exp(1)))*tan(1/2*exp(1))^4*tan(1/4*exp(1))^4-16466286137573376*sqrt(2)*c*s
ign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1
))^4*tan(1/4*exp(1))^5-3588805953060864*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))
-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^4*tan(1/4*exp(1))^6-1
6466286137573376*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*
f*x+2*exp(1)))*tan(1/2*exp(1))^4*tan(1/4*exp(1))+1055531162664960*sqrt(2)*c
sign(sin(1/2(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp
(1))^6*tan(1/4*exp(1))^2-7036874417766400*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1
))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^6*tan(1/4*exp(1))^3
-1055531162664960*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2
*f*x+2*exp(1)))*tan(1/2*exp(1))^6*tan(1/4*exp(1))^4+2111062325329920*sqrt(2
)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*
exp(1))^6*tan(1/4*exp(1))^5+70368744177664*sqrt(2)*c*sign(sin(1/2*(f*x+exp(
1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^6*tan(1/4*exp(1))^
6+2111062325329920*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/
2*f*x+2*exp(1)))*tan(1/2*exp(1))^6*tan(1/4*exp(1)))/(422212465065984*a*tan(
1/2*exp(1))-140737488355328*a*tan(1/2*exp(1))^3*tan(1/4*exp(1))^6+422212465
065984*a*tan(1/2*exp(1))^2*tan(1/4*exp(1))^6-422212465065984*a*tan(1/2*exp(
1))^3*tan(1/4*exp(1))^4-140737488355328*a*tan(1/4*exp(1))^6+126663739519795
2*a*tan(1/2*exp(1))^2*tan(1/4*exp(1))^4-422212465065984*a*tan(1/2*exp(1))^3
*tan(1/4*exp(1))^2-422212465065984*a*tan(1/4*exp(1))^4+1266637395197952*a*t
an(1/2*exp(1))^2*tan(1/4*exp(1))^2-140737488355328*a*tan(1/2*exp(1))^3+4222
12465065984*a*tan(1/2*exp(1))^2-422212465065984*a*tan(1/4*exp(1))^2+1266637
395197952*a*tan(1/2*exp(1))*tan(1/4*exp(1))^2+1266637395197952*a*tan(1/2*ex
p(1))*tan(1/4*exp(1))^4+422212465065984*a*tan(1/2*exp(1))*tan(1/4*exp(1))^6
-140737488355328*a)/(-2*tan(1/2*(1/2*f*x+2*exp(1)))^3-6*tan(1/2*(1/2*f*x+2*
exp(1)))^3*tan(1/2*exp(1))+tan(1/2*(1/2*f*x+2*exp(1)))^4*tan(1/2*exp(1))^3+
2*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^3-3*tan(1/2*(1/2*f*x+2*exp(
1)))^4*tan(1/2*exp(1))^2+6*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^2+
tan(1/2*(1/2*f*x+2*exp(1)))^4-tan(1/2*exp(1))^3+3*tan(1/2*exp(1))^2-3*tan(1
/2*(1/2*f*x+2*exp(1)))^4*tan(1/2*exp(1))+6*tan(1/2*(1/2*f*x+2*exp(1)))*tan(
1/2*exp(1))^2+2*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^3-6*tan(1/2*(1/
2*f*x+2*exp(1)))*tan(1/2*exp(1))-2*tan(1/2*(1/2*f*x+2*exp(1)))+3*tan(1/2*ex
p(1))-1)+1/2*(158456325028528675187087900672*sqrt(2)*c*sign(sin(1/2*(f*x+ex
p(1))-1/4*pi))*tan(1/2*exp(1))^3-316912650057057350374175801344O*sqrt(2)*c*
sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^3+950737950171172051122527404032*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^5-475
368975085586025561263702016*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*ta
n(1/2*exp(1))+950737950171172051122527404032*sqrt(2)*c*sign(sin(1/2*(f*x+ex
p(1))-1/4*pi))*tan(1/4*exp(1))+9507379501711720511225274040320*sqrt(2)*c*si

```
gn(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(1/4*exp(1))^3-285221
38505135161533675822120096*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(
1/2*exp(1))^2*tan(1/4*exp(1))^5-28522138505135161533675822120096*sqrt(2)*c*s
ign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(1/4*exp(1))-2376844
8754279301278063185100800*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1
/2*exp(1))^3*tan(1/4*exp(1))^2+23768448754279301278063185100800*sqrt(2)*c*si
gn(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^3*tan(1/4*exp(1))^4-158456
3250285286751870879006720*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1
/2*exp(1))^3*tan(1/4*exp(1))^6+7130534626283790383418955530240*sqrt(2)*c*si
gn(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/4*exp(1))^2-71305346
26283790383418955530240*sqrt(2)*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/
2*exp(1))*tan(1/4*exp(1))^4+4753689750855860255612637020016*sqrt(2)*c*sign(s
in(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/4*exp(1))^6)*ln(abs(2*ta
n(1/2*exp(1))^3-2*sqrt(2)*sqrt(tan(1/2*exp(1))^2+1)*(tan(1/2*exp(1))^2+1)+6
*tan(1/2*exp(1))^2-6*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^2+2*tan(1/
2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^3-6*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1
/2*exp(1))+2*tan(1/2*(1/2*f*x+2*exp(1)))-6*tan(1/2*exp(1))-2)/abs(2*tan(1/2
*exp(1))^3+2*sqrt(2)*sqrt(tan(1/2*exp(1))^2+1)*(tan(1/2*exp(1))^2+1)+6*tan(
1/2*exp(1))^2-6*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^2+2*tan(1/2*(1/
2*f*x+2*exp(1)))*tan(1/2*exp(1))^3-6*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*ex
p(1))+2*tan(1/2*(1/2*f*x+2*exp(1)))-6*tan(1/2*exp(1))-2))/sqrt(2)/sqrt(tan(
1/2*exp(1))^2+1)/(tan(1/2*exp(1))^2+1)/(1584563250285286751870879006720*a*ta
n(1/4*exp(1))^6+4753689750855860255612637020016*a*tan(1/4*exp(1))^4+47536897
50855860255612637020016*a*tan(1/4*exp(1))^2+1584563250285286751870879006720*a
))/f
```

maple [A] time = 0.63, size = 49, normalized size = 0.82

$$\frac{2c^2\left(\sin\left(fx+e\right)-1\right)\left(3+\sin\left(fx+e\right)\right)}{a\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\;f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x)
[Out] 2*c^2/a*(sin(f*x+e)-1)*(3+sin(f*x+e))/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [B] time = 0.92, size = 146, normalized size = 2.43

$$\frac{2\left(3\,c^{\frac{3}{2}}+\frac{2\,c^{\frac{3}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{6\,c^{\frac{3}{2}}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{2\,c^{\frac{3}{2}}\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{3\,c^{\frac{3}{2}}\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}\right)}{\left(a+\frac{a\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)f\left(\frac{\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2*(3*c^(3/2) + 2*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 6*c^(3/2)*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 2*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 3*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/((a + a*sin(f*x + e)/(c
os(f*x + e) + 1))*f*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2))
```

mupad [B] time = 7.29, size = 90, normalized size = 1.50

$$-\frac{2\,c\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(22\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+2\sin\left(\frac{3\,e}{2}+\frac{3\,f\,x}{2}\right)^2+4\,\sin\left(2\,e+2\,f\,x\right)-12\right)}{a\,f\,\left(4\sin\left(e+f\,x\right)^2+\sin\left(e+f\,x\right)+\sin\left(3\,e+3\,f\,x\right)-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x)),x)
```

```
[Out] -(2*c*(-c*(sin(e + f*x) - 1))^(1/2)*(4*sin(2*e + 2*f*x) + 22*sin(e/2 + (f*x
)/2)^2 + 2*sin((3*e)/2 + (3*f*x)/2)^2 - 12))/(a*f*(sin(e + f*x) + sin(3*e +
 3*f*x) + 4*sin(e + f*x)^2 - 4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c\sqrt{-c\sin\left(e+fx\right)+c}}{\sin\left(e+fx\right)+1}\,dx+\int\left(-\frac{c\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)}{\sin\left(e+fx\right)+1}\right)dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] (Integral(c*sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x) + 1), x) + Integral(-c*
sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)/(sin(e + f*x) + 1), x))/a
```

3.320 $\int \frac{\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=29

$$-\frac{2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{af}$$

```
[Out] -2*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f
```

Rubi [A] time = 0.13, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2736, 2673}

$$-\frac{2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c*Sin[e + f*x]]/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-2*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx &= \frac{\int \sec^2(e+fx)(c-c\sin(e+fx))^{3/2}\,dx}{ac}\\ &= -\frac{2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{af}\end{aligned}$$

Mathematica [A] time = 0.10, size = 29, normalized size = 1.00

$$-\frac{2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c*Sin[e + f*x]]/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-2*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*f)
```

fricas [A] time = 0.44, size = 29, normalized size = 1.00

$$-\frac{2\sqrt{-c\sin\left(fx+e\right)+c}}{af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)-2*sqrt(2*c)*sign(sin(1/2*(f*x+exp(1
))-1/4*pi))/a/f/((-cos(1/4*(2*f*x+2*exp(1)-pi))+1)/(cos(1/4*(2*f*x+2*exp(1)
-pi))+1)-1)
```

maple [A] time = 0.61, size = 39, normalized size = 1.34

$$\frac{2c\left(\sin\left(fx+e\right)-1\right)}{a\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] 2*c/a*(sin(f*x+e)-1)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [B] time = 0.78, size = 76, normalized size = 2.62

$$\frac{2\left(\sqrt{c}+\frac{\sqrt{c}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)}{\left(a+\frac{a\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)f\sqrt{\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(c) + sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)/((a + a*sin(f*x +
 e)/(cos(f*x + e) + 1))*f*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))
```

mupad [B] time = 0.20, size = 40, normalized size = 1.38

$$-\frac{4\cos\left(e+fx\right)\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}{af\left(\cos\left(2e+2fx\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x)),x)
```

```
[Out] -(4*cos(e + f*x)*(-c*(sin(e + f*x) - 1))^(1/2))/(a*f*(cos(2*e + 2*f*x) + 1)
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sqrt{-c\sin\left(e+fx\right)+c}}{\sin\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x) + 1), x)/a
```

3.321 $\int \frac{1}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=83

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}\,a\sqrt{c}\,f}-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf}$$

```
[Out] 1/2*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a/f*2^(1
/2)/c^(1/2)-sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/c/f
```

Rubi [A] time = 0.16, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2736, 2675, 2649, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}\,a\sqrt{c}\,f}-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(Sqrt[2]
*a*Sqrt[c]*f) - (Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*c*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
```

```
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx &= \frac{\int \sec^2(e+fx)\sqrt{c-c\sin(e+fx)}\,dx}{ac}\\
&= -\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf}+\frac{\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx}{2a}\\
&= -\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf}-\frac{\text{Subst}\left(\int \frac{1}{2c-x^2}\,dx,x,-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{af}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}\,a\sqrt{c}\,f}-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf}
\end{aligned}$$

Mathematica [C] time = 0.33, size = 97, normalized size = 1.17

$$-\frac{\cos(e+fx)\left(1+(1+i)\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(\tan\left(\frac{1}{4}(e+fx)\right)+1\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\right)}{af(\sin(e+fx)+1)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] -((Cos[e + f*x]*(1 + (1 + I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 +
Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))/(a*f*(1 + Sin[e
+ f*x])*Sqrt[c - c*Sin[e + f*x]]))
```

fricas [B] time = 0.48, size = 154, normalized size = 1.86

$$\frac{\sqrt{2}\sqrt{c}\cos\left(fx+e\right)\log\left(-\frac{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)+\frac{2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)}{\sqrt{c}}+3\cos\left(fx+e\right)+2}{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right)-4\sqrt{-}}{4acf\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*sqrt(c)*cos(f*x + e)*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)
*sin(f*x + e) + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x
 + e) + 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) +
2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*sqrt(-c*sin(f*x + e) + c))/(a*c*f*
cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*((2*c*sqrt(2)*atan(
sqrt(c)/sqrt(-c))-2*c*atan(sqrt(c)/sqrt(-c))+sqrt(-c)*sqrt(c))/(2*a*c*sqrt(
-c)*sqrt(2)-4*a*c*sqrt(-c))*sign(tan((f*x+exp(1))/2)-1)+2*(1/2*(-sqrt(c)*ta
n((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/a/(-(-sqrt(c)*ta
n((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+2*sqrt(c)*(-sqrt(c)*ta
n((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)/sign(tan((f*x+exp(1))
/2)-1)+1/2*atan((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1
))/2)^2+c))/sqrt(2)/sqrt(-c))/sqrt(2)/sqrt(-c)/a/sign(tan((f*x+exp(1))/2)-1
)))
```

maple [A] time = 0.96, size = 86, normalized size = 1.04

$$\frac{\left(\sin\left(fx+e\right)-1\right)\left(-\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right)c\sqrt{c\left(1+\sin\left(fx+e\right)\right)}+2c^{\frac{3}{2}}\right)}{2a\,c^{\frac{3}{2}}\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\ f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/2/a*(sin(f*x+e)-1)*(-2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)
/c^(1/2))*c*(c*(1+sin(f*x+e)))^(1/2)+2*c^(3/2))/c^(3/2)/cos(f*x+e)/(c-c*sin
(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+a\,\sin\left(e+f\,x\right)\right)\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)+\sqrt{-c\sin\left(e+fx\right)+c}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + sqrt(-c*sin(e + f*x) +
 c)), x)/a
```

3.322 $\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=117

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}\,ac^{3/2}f}+\frac{3\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}}-\frac{\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 3/4*cos(f*x+e)/a/f/(c-c*sin(f*x+e))^(3/2)+3/8*arctanh(1/2*cos(f*x+e)*c^(1/2
)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a/c^(3/2)/f*2^(1/2)-sec(f*x+e)/a/c/f/(c-c
*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2736, 2687, 2650, 2649, 206}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}\,ac^{3/2}f}+\frac{3\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}}-\frac{\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (3*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(4*S
qrt[2]*a*c^(3/2)*f) + (3*Cos[e + f*x])/(4*a*f*(c - c*Sin[e + f*x])^(3/2)) -
 Sec[e + f*x]/(a*c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
```

```
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{\sec^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx}{ac}$$

$$= -\frac{\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}} + \frac{3\int \frac{1}{(c-c\sin(e+fx))^{3/2}}\,dx}{2a}$$

$$= \frac{3\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}} - \frac{\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}} + \frac{3\int \frac{1}{\sqrt{c-c\sin(}}}{8ac}$$

$$= \frac{3\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}} - \frac{\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}} - \frac{3\,\mathrm{Subst}\Big(\int}{}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}\,ac^{3/2}f} + \frac{3\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}} - \frac{\mathrm{s}}{acf\sqrt{c}}$$

Mathematica [C] time = 0.62, size = 125, normalized size = 1.07

$$-\frac{\sec(e+fx)\left(-3\sin(e+fx)+(3+3i)\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(\tan\left(\frac{1}{4}(e+fx)\right)+1\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\right.\right.}{4acf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -1/4*(Sec[e + f*x]*(1 + (3 + 3*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*
(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2]) - 3*Sin[e + f*x]))/(a*c*f*Sqrt[c - c*Sin[e + f*x
]])
```

fricas [B] time = 0.46, size = 207, normalized size = 1.77

$$\frac{3\sqrt{2}\left(\cos\left(fx+e\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)\right)\sqrt{c}\log\left(-\frac{c\cos\left(fx+e\right)^{2}+2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\sqrt{c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)+3}{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)}\right.}{16\left(ac^{2}f\cos\left(fx+e\right)\sin\left(fx+e\right)-ac^{2}f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*sqrt(2)*(cos(f*x + e)*sin(f*x + e) - cos(f*x + e))*sqrt(c)*log(-(c*
cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e)
+ sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e
) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) -
 2)) - 4*sqrt(-c*sin(f*x + e) + c)*(3*sin(f*x + e) - 1))/(a*c^2*f*cos(f*x +
 e)*sin(f*x + e) - a*c^2*f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*(-1/4*(sqrt(c)*ta
n((f*x+exp(1))/2)-sqrt(c)-sqrt(c*tan((f*x+exp(1))/2)^2+c))/a/c/(-(-sqrt(c)*
tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+2*sqrt(c)*(-sqrt(c)*
```

```
tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)/sign(tan((f*x+exp(1
))/2)-1)+1/8*(-3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2
+c))^3+c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-sqr
t(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-sqrt(
c)*c)/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-2*
sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)^2
/a/c/sign(tan((f*x+exp(1))/2)-1)+3/8*atan((-sqrt(c)*tan((f*x+exp(1))/2)+sqr
t(c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-c))/sqrt(2)/sqrt(-c)/a/
c/sign(tan((f*x+exp(1))/2)-1))
```

maple [A] time = 0.82, size = 134, normalized size = 1.15

$$-\frac{3\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{2}}{2\sqrt{c}}\right)\sin\left(fx+e\right)c-3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{2}}{2\sqrt{c}}\right)c\sqrt{c}}{8c^{\frac{5}{2}}a\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/8/c^(5/2)/a*(3*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*
x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c-3*2^(1/2)*arctanh(1/2*(c*(1+sin(
f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c*(c*(1+sin(f*x+e)))^(1/2)-6*c^(3/2)*sin(f*
x+e)+2*c^(3/2))/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+a\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{-c\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^{2}\left(e+fx\right)+c\sqrt{-c\sin\left(e+fx\right)+c}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2 + c*sqrt(-c*sin(e
+ f*x) + c)), x)/a
```

3.323 $\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=156

$$\frac{15\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}\,ac^{5/2}f}-\frac{5\sec(e+fx)}{8ac^2f\sqrt{c-c\sin(e+fx)}}+\frac{15\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}}+\frac{\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 15/32*cos(f*x+e)/a/c/f/(c-c*sin(f*x+e))^(3/2)+1/4*sec(f*x+e)/a/c/f/(c-c*sin
(f*x+e))^(3/2)+15/64*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e)
)^(1/2))/a/c^(5/2)/f*2^(1/2)-5/8*sec(f*x+e)/a/c^2/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2736, 2681, 2687, 2650, 2649, 206}

$$-\frac{5\sec(e+fx)}{8ac^2f\sqrt{c-c\sin(e+fx)}}+\frac{15\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}\,ac^{5/2}f}+\frac{15\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}}+\frac{\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (15*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(32
*Sqrt[2]*a*c^(5/2)*f) + (15*Cos[e + f*x])/(32*a*c*f*(c - c*Sin[e + f*x])^(3
/2)) + Sec[e + f*x]/(4*a*c*f*(c - c*Sin[e + f*x])^(3/2)) - (5*Sec[e + f*x])
/(8*a*c^2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
```

```
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{\sec^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= \frac{\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}} + \frac{5\int \frac{\sec^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx}{8ac^2}$$

$$= \frac{\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}} - \frac{5\sec(e+fx)}{8ac^2 f\sqrt{c-c\sin(e+fx)}} + \frac{15\int \frac{}{(c-}}{}$$

$$= \frac{15\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}} + \frac{\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}} - \frac{}{8ac^2 f}$$

$$= \frac{15\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}} + \frac{\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}} - \frac{}{8ac^2 f}$$

$$= \frac{15\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}\,ac^{5/2}f} + \frac{15\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}} + \frac{}{4ac}$$

Mathematica [C] time = 0.84, size = 162, normalized size = 1.04

$$\frac{\left(\frac{1}{128}+\frac{i}{128}\right)\cos(e+fx)\left((1-i)(40\sin(e+fx)+15\cos(2(e+fx))-9)-60\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(\tan\left(\frac{1}{4}(e\right.\right.\right.\right.}{ac^2 f(\sin(e+fx)-1)^2(\sin(e+fx)+1)\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] ((1/128 + I/128)*Cos[e + f*x]*(-60*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)
*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2]) + (1 - I)*(-9 + 15*Cos[2*(e + f*x)] + 40*Sin[e
+ f*x])))/(a*c^2*f*(-1 + Sin[e + f*x])^2*(1 + Sin[e + f*x])*Sqrt[c - c*Sin[
e + f*x]])
```

fricas [A] time = 0.47, size = 241, normalized size = 1.54

$$\frac{15\sqrt{2}\left(\cos\left(fx+e\right)^3+2\cos\left(fx+e\right)\sin\left(fx+e\right)-2\cos\left(fx+e\right)\right)\sqrt{c}\log\left(-\frac{c\cos\left(fx+e\right)^2+2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\sqrt{c}\left(c\right.}{\cos\left(fx+e\right)^2}\right.}{128\left(ac^3 f\cos\left(fx+e\right)^3+2ac^3 f\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/128*(15*sqrt(2)*(cos(f*x + e)^3 + 2*cos(f*x + e)*sin(f*x + e) - 2*cos(f*x
 + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)
*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x
+ e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*
x + e) - cos(f*x + e) - 2)) - 4*(15*cos(f*x + e)^2 + 20*sin(f*x + e) - 12)*
sqrt(-c*sin(f*x + e) + c))/(a*c^3*f*cos(f*x + e)^3 + 2*a*c^3*f*cos(f*x + e)
*sin(f*x + e) - 2*a*c^3*f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*(-1/8*(sqrt(c)*ta
n((f*x+exp(1))/2)-sqrt(c)-sqrt(c*tan((f*x+exp(1))/2)^2+c))/c^2/a/(-(-sqrt(c
)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+2*sqrt(c)*(-sqrt(c
)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)/sign(tan((f*x+exp
(1))/2)-1)+1/64*(-41*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/
2)^2+c))^7-127*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1)
)/2)^2+c))^6-91*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^
2+c))^5+143*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))
/2)^2+c))^4-3*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^
2+c))^3+47*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c
))-93*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^
2+c))^2-11*sqrt(c)*c^3)/c^2/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x
+exp(1))/2)^2+c))^2-2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x
+exp(1))/2)^2+c))+c)^4/a/sign(tan((f*x+exp(1))/2)-1)+15/64*atan((-sqrt(c)*t
an((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-c
))/sqrt(2)/c^2/sqrt(-c)/a/sign(tan((f*x+exp(1))/2)-1))
```

maple [A] time = 1.52, size = 210, normalized size = 1.35

$$-\frac{15\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{2}}{2\sqrt{c}}\right)\left(\sin^{2}\left(fx+e\right)\right)c^{2}-30c^{\frac{5}{2}}\left(\sin^{2}\left(fx+e\right)\right)-30\sqrt{c\left(1\right.}\cdots}{64c^{\frac{9}{2}}a\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/64/c^(9/2)/a*(15*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(
f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^2-30*c^(5/2)*sin(f*x+e)^2-30
*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1
/2)/c^(1/2))*sin(f*x+e)*c^2+40*c^(5/2)*sin(f*x+e)+15*(c*(1+sin(f*x+e)))^(1/
2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^2+6*c^(5
/2))/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+a\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c^{2}\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^{3}\left(e+fx\right)-c^{2}\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^{2}\left(e+fx\right)-c^{2}\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)+c^{2}\sqrt{-c\sin\left(e+fx\right)+c}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(1/(c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**3 - c**2*sqrt(-c*s
in(e + f*x) + c)*sin(e + f*x)**2 - c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f
*x) + c**2*sqrt(-c*sin(e + f*x) + c)), x)/a
```

3.324 $\int \frac{(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=176

$$\frac{4096c^3\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^2f}-\frac{1024c^2\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^2f}+\frac{2\sec^3(e+fx)(c-c\sin(e-}{5a^2cf}$$

```
[Out] 4096/15*c^3*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^2/f-1024/5*c^2*sec(f*x+e)
^3*(c-c*sin(f*x+e))^(5/2)/a^2/f+128/5*c*sec(f*x+e)^3*(c-c*sin(f*x+e))^(7/2)
/a^2/f+32/15*sec(f*x+e)^3*(c-c*sin(f*x+e))^(9/2)/a^2/f+2/5*sec(f*x+e)^3*(c-
c*sin(f*x+e))^(11/2)/a^2/c/f
```

Rubi [A] time = 0.42, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$-\frac{1024c^2\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^2f}+\frac{4096c^3\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^2f}+\frac{2\sec^3(e+fx)(c-c\sin(e}{5a^2cf}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(9/2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (4096*c^3*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(15*a^2*f) - (1024*c^2
*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(5*a^2*f) + (128*c*Sec[e + f*x]
^3*(c - c*Sin[e + f*x])^(7/2))/(5*a^2*f) + (32*Sec[e + f*x]^3*(c - c*Sin[e
+ f*x])^(9/2))/(15*a^2*f) + (2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(11/2))/
(5*a^2*c*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \sec^4(e+fx)(c-c\sin(e+fx))^{13/2}\,dx}{a^2c^2}$$
$$= \frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^2cf} + \frac{16\int \sec^4(e+fx)(c-c\sin(e+fx))^{11/2}\,dx}{5a^2c}$$
$$= \frac{32\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{15a^2f} + \frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^2cf} + \frac{64\int$$
$$= \frac{128c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^2f} + \frac{32\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{15a^2f} + \frac{2\,\mathrm{s}$$
$$= -\frac{1024c^2\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^2f} + \frac{128c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^2f}$$
$$= \frac{4096c^3\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^2f} - \frac{1024c^2\sec^3(e+fx)(c-c\sin(e+fx))^{5/}}{5a^2f}$$

Mathematica [A] time = 3.12, size = 124, normalized size = 0.70

$$\frac{c^4\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(8568\sin(e+fx)+56\sin(3(e+fx))-1044\cos(2(e+}{60a^2f(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(9/2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (c^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(6825 -
 1044*Cos[2*(e + f*x)] + 3*Cos[4*(e + f*x)] + 8568*Sin[e + f*x] + 56*Sin[3*
(e + f*x)]))/(60*a^2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f
*x])^2)
```

fricas [A] time = 0.47, size = 104, normalized size = 0.59

$$\frac{2\left(3\,c^4\cos\left(fx+e\right)^4-264\,c^4\cos\left(fx+e\right)^2+984\,c^4+28\left(c^4\cos\left(fx+e\right)^2+38\,c^4\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+\right.}}{15\left(a^2f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 2/15*(3*c^4*cos(f*x + e)^4 - 264*c^4*cos(f*x + e)^2 + 984*c^4 + 28*(c^4*cos
(f*x + e)^2 + 38*c^4)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*
x + e)*sin(f*x + e) + a^2*f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.67, size = 91, normalized size = 0.52

$$-\frac{2c^5\left(\sin\left(fx+e\right)-1\right)\left(3\left(\sin^4\left(fx+e\right)\right)-28\left(\sin^3\left(fx+e\right)\right)+258\left(\sin^2\left(fx+e\right)\right)+1092\sin\left(fx+e\right)+723\right)}{15a^2\left(1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\;f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x)
```

```
[Out] -2/15*c^5/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(3*sin(f*x+e)^4-28*sin(f*x+e)^3
+258*sin(f*x+e)^2+1092*sin(f*x+e)+723)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [B] time = 1.02, size = 380, normalized size = 2.16

$$-\frac{2\left(723\,c^{\frac{9}{2}}+\frac{2184\,c^{\frac{9}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{5370\,c^{\frac{9}{2}}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{10696\,c^{\frac{9}{2}}\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{15021\,c^{\frac{9}{2}}\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+\frac{21168\,c^{\frac{9}{2}}\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}+\frac{20748}{(\cos}\right.}{15\left(a^2+\frac{3\,a^2\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\,a^2\sin(}{(\cos(fx}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/15*(723*c^(9/2) + 2184*c^(9/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 5370*c^
(9/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10696*c^(9/2)*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 15021*c^(9/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 21
168*c^(9/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20748*c^(9/2)*sin(f*x + e
)^6/(cos(f*x + e) + 1)^6 + 21168*c^(9/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^
7 + 15021*c^(9/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 10696*c^(9/2)*sin(f
*x + e)^9/(cos(f*x + e) + 1)^9 + 5370*c^(9/2)*sin(f*x + e)^10/(cos(f*x + e)
 + 1)^10 + 2184*c^(9/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 723*c^(9/2)
*sin(f*x + e)^12/(cos(f*x + e) + 1)^12)/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x
 + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3)*f*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(9/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-c\,\sin\left(e+f\,x\right)\right)^{9/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(9/2)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] int((c - c*sin(e + f*x))^(9/2)/(a + a*sin(e + f*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.325 $\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=136

$$\frac{256c^2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f}+\frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3a^2cf}+\frac{8\sec^3(e+fx)(c-c\sin(e+fx))^{7}}{a^2f}$$

```
[Out] 256/3*c^2*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^2/f-64*c*sec(f*x+e)^3*(c-c*
sin(f*x+e))^(5/2)/a^2/f+8*sec(f*x+e)^3*(c-c*sin(f*x+e))^(7/2)/a^2/f+2/3*sec
(f*x+e)^3*(c-c*sin(f*x+e))^(9/2)/a^2/c/f
```

Rubi [A] time = 0.33, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$\frac{256c^2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f}+\frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3a^2cf}+\frac{8\sec^3(e+fx)(c-c\sin(e+fx))^{7}}{a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(7/2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (256*c^2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(3*a^2*f) - (64*c*Sec[e
 + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(a^2*f) + (8*Sec[e + f*x]^3*(c - c*Si
n[e + f*x])^(7/2))/(a^2*f) + (2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(9/2))/
(3*a^2*c*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \sec^4(e+fx)(c-c\sin(e+fx))^{11/2}\,dx}{a^2c^2}$$
$$= \frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3a^2cf} + \frac{4\int \sec^4(e+fx)(c-c\sin(e+fx))^{9/2}\,dx}{a^2c}$$
$$= \frac{8\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{a^2f} + \frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3a^2cf} + \frac{32\int \mathrm{se}}{}$$
$$= -\frac{64c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{a^2f} + \frac{8\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{a^2f} + \frac{2\,\mathrm{se}}{}$$
$$= \frac{256c^2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f} - \frac{64c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{a^2f} + \frac{8}{}$$

Mathematica [A] time = 1.25, size = 112, normalized size = 0.82

$$\frac{c^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(273\sin(e+fx)+\sin(3(e+fx))-30\cos(2(e+fx))+}{6a^2f(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(7/2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(210 -
30*Cos[2*(e + f*x)] + 273*Sin[e + f*x] + Sin[3*(e + f*x)]))/(6*a^2*f*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2)
```

fricas [A] time = 0.44, size = 91, normalized size = 0.67

$$-\frac{2\left(15\,c^3\cos\left(fx+e\right)^2-60\,c^3-\left(c^3\cos\left(fx+e\right)^2+68\,c^3\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{3\left(a^2f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(15*c^3*cos(f*x + e)^2 - 60*c^3 - (c^3*cos(f*x + e)^2 + 68*c^3)*sin(f*
x + e))*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a^2*f*
cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.84, size = 79, normalized size = 0.58

$$\frac{2c^{4}\left(\sin\left(fx+e\right)-1\right)\left(\sin^{3}\left(fx+e\right)-15\left(\sin^{2}\left(fx+e\right)\right)-69\sin\left(fx+e\right)-45\right)}{3a^{2}\left(1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 2/3*c^4/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(sin(f*x+e)^3-15*sin(f*x+e)^2-69*
sin(f*x+e)-45)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [B] time = 0.82, size = 334, normalized size = 2.46

$$-\frac{2\left(45\,c^{\frac{7}{2}}+\frac{138\,c^{\frac{7}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{285\,c^{\frac{7}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{544\,c^{\frac{7}{2}}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{630\,c^{\frac{7}{2}}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{812\,c^{\frac{7}{2}}\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}+\frac{630\,c^{\frac{7}{2}}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}\cdots}{3\left(a^{2}+\frac{3\,a^{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\,a^{2}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{a^{2}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}\right)f\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(45*c^(7/2) + 138*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 285*c^(7/2
)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 544*c^(7/2)*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 630*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 812*c^(7/2
)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 630*c^(7/2)*sin(f*x + e)^6/(cos(f*x
 + e) + 1)^6 + 544*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 285*c^(7/2
)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 138*c^(7/2)*sin(f*x + e)^9/(cos(f*x
```

```
 + e) + 1)^9 + 45*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)/((a^2 + 3*
a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*f*(sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 1)^(7/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-c\,\sin\left(e+f\,x\right)\right)^{7/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(7/2)/(a + a*sin(e + f*x))^2,x)
[Out] int((c - c*sin(e + f*x))^(7/2)/(a + a*sin(e + f*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**2,x)
[Out] Timed out
```

3.326 $\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=100

$$\frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{a^2cf}-\frac{16\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{a^2f}+\frac{64c\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f}$$

```
[Out] 64/3*c*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^2/f-16*sec(f*x+e)^3*(c-c*sin(f
*x+e))^(5/2)/a^2/f+2*sec(f*x+e)^3*(c-c*sin(f*x+e))^(7/2)/a^2/c/f
```

Rubi [A] time = 0.26, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$\frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{a^2cf}-\frac{16\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{a^2f}+\frac{64c\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (64*c*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(3*a^2*f) - (16*Sec[e + f*
x]^3*(c - c*Sin[e + f*x])^(5/2))/(a^2*f) + (2*Sec[e + f*x]^3*(c - c*Sin[e +
 f*x])^(7/2))/(a^2*c*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
```

```
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx &= \frac{\int \sec^4(e+fx)(c-c\sin(e+fx))^{9/2}\,dx}{a^2c^2}\\ &= \frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{a^2cf}+\frac{8\int \sec^4(e+fx)(c-c\sin(e+fx))^{7/2}\,dx}{a^2c}\\ &= -\frac{16\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{a^2f}+\frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{a^2cf}-\frac{32\int\\ &= \frac{64c\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f}-\frac{16\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{a^2f}+\frac{2\,\mathrm{se}\end{aligned}$$

Mathematica [A] time = 0.80, size = 104, normalized size = 1.04

$$\frac{c^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(36\sin(e+fx)-3\cos(2(e+fx))+25)}{3a^2f(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(25 - 3*Cos[2*(e + f*x)] + 36*Si
n[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])*(1 + Sin[e + f*x])^2)
```

fricas [A] time = 0.43, size = 76, normalized size = 0.76

$$-\frac{2\left(3\,c^2\cos\left(fx+e\right)^2-18\,c^2\sin\left(fx+e\right)-14\,c^2\right)\sqrt{-c\sin\left(fx+e\right)+c}}{3\left(a^2f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(3*c^2*cos(f*x + e)^2 - 18*c^2*sin(f*x + e) - 14*c^2)*sqrt(-c*sin(f*x
+ e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a^2*f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.82, size = 71, normalized size = 0.71

$$-\frac{2c^{3}\left(\sin\left(fx+e\right)-1\right)\left(3\left(\sin^{2}\left(fx+e\right)\right)+18\sin\left(fx+e\right)+11\right)}{3a^{2}\left(1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x)
```

```
[Out] -2/3*c^3/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(3*sin(f*x+e)^2+18*sin(f*x+e)+11
)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [B] time = 1.18, size = 288, normalized size = 2.88

$$-\frac{2\left(11\,c^{\frac{5}{2}}+\frac{36\,c^{\frac{5}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{56\,c^{\frac{5}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{108\,c^{\frac{5}{2}}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{90\,c^{\frac{5}{2}}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{108\,c^{\frac{5}{2}}\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}+\frac{56\,c^{\frac{5}{2}}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}+\cdots}{3\left(a^{2}+\frac{3\,a^{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\,a^{2}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{a^{2}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}\right)f\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(11*c^(5/2) + 36*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 56*c^(5/2)*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 108*c^(5/2)*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 90*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 108*c^(5/2)*s
in(f*x + e)^5/(cos(f*x + e) + 1)^5 + 56*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 + 36*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 11*c^(5/2)*sin(
f*x + e)^8/(cos(f*x + e) + 1)^8)/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) +
 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3)*f*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2))
```

mupad [B] time = 11.68, size = 360, normalized size = 3.60

$$\frac{\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\left(\frac{2\,c^2}{a^2\,f}-\frac{c^2\,e^{e\,1i+f\,x\,1i}\,2i}{a^2\,f}\right)}{e^{e\,1i+f\,x\,1i}-i}+\frac{16\,c^2\,e^{e\,1i+f\,x\,1i}\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}}{a^2\,f\left(e^{e\,1i+f\,x\,1i}-i\right)\left(e^{e\,1i+f\,x\,1i}+1i\right)}-\frac{c^2\,e^{e\,1i+f}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] ((c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((
2*c^2)/(a^2*f) - (c^2*exp(e*1i + f*x*1i)*2i)/(a^2*f)))/(exp(e*1i + f*x*1i)
- 1i) + (16*c^2*exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (e
xp(e*1i + f*x*1i)*1i)/2))^(1/2))/(a^2*f*(exp(e*1i + f*x*1i) - 1i)*(exp(e*1i
 + f*x*1i) + 1i)) - (c^2*exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1
i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*32i)/(3*a^2*f*(exp(e*1i + f*x*1i)
- 1i)*(exp(e*1i + f*x*1i) + 1i)^2) - (32*c^2*exp(e*1i + f*x*1i)*(c - c*((ex
p(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(3*a^2*f*(exp
(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.327 $\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=68

$$\frac{8\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f}-\frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{a^2cf}$$

```
[Out] 8/3*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^2/f-2*sec(f*x+e)^3*(c-c*sin(f*x+e
))^(5/2)/a^2/c/f
```

Rubi [A] time = 0.20, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$\frac{8\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f}-\frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{a^2cf}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (8*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(3*a^2*f) - (2*Sec[e + f*x]^3
*(c - c*Sin[e + f*x])^(5/2))/(a^2*c*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
```

```
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx &= \frac{\int \sec^4(e+fx)(c-c\sin(e+fx))^{7/2}\,dx}{a^2c^2}\\ &= -\frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{a^2cf}-\frac{4\int \sec^4(e+fx)(c-c\sin(e+fx))^{5/2}\,dx}{a^2c}\\ &= \frac{8\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f}-\frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{a^2cf}\end{aligned}$$

Mathematica [A] time = 0.37, size = 92, normalized size = 1.35

$$\frac{2c(3\sin(e+fx)+1)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{3a^2f(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (2*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(1 + 3*Sin[e + f*x])*Sqrt[c - c*
Sin[e + f*x]])/(3*a^2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e +
f*x])^2)
```

fricas [A] time = 0.42, size = 57, normalized size = 0.84

$$\frac{2\left(3c\sin\left(fx+e\right)+c\right)\sqrt{-c\sin\left(fx+e\right)+c}}{3\left(a^2f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 2/3*(3*c*sin(f*x + e) + c)*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x + e)*si
n(f*x + e) + a^2*f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.91, size = 61, normalized size = 0.90

$$-\frac{2c^{2}\left(\sin\left(fx+e\right)-1\right)\left(3\sin\left(fx+e\right)+1\right)}{3a^{2}\left(1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x)
```

```
[Out] -2/3*c^2/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(3*sin(f*x+e)+1)/cos(f*x+e)/(c-c
*sin(f*x+e))^(1/2)/f
```

maxima [B] time = 0.43, size = 239, normalized size = 3.51

$$-\frac{2\left(c^{\frac{3}{2}}+\frac{6\,c^{\frac{3}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\,c^{\frac{3}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{12\,c^{\frac{3}{2}}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{3\,c^{\frac{3}{2}}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{6\,c^{\frac{3}{2}}\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}+\frac{c^{\frac{3}{2}}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}\right)}{3\left(a^{2}+\frac{3\,a^{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\,a^{2}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{a^{2}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}\right)f\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(c^(3/2) + 6*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^(3/2)*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 12*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 3*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 6*c^(3/2)*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5 + c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)/((
a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*f*(sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 1)^(3/2))
```

mupad [B] time = 10.32, size = 120, normalized size = 1.76

$$-\frac{4\,c\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\sqrt{c-c\,\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\,\left(2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,3\mathrm{i}+3\mathrm{i}\right)}{3\,a^{2}\,f\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)}^{3}\,\left(1+\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] -(4*c*exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i +
f*x*1i)*1i)/2))^(1/2)*(2*exp(e*1i + f*x*1i) - exp(e*2i + f*x*2i)*3i + 3i))/
(3*a^2*f*(exp(e*1i + f*x*1i) + 1i)^3*(exp(e*1i + f*x*1i)*1i + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c\sqrt{-c\sin\left(e+fx\right)+c}}{\sin^2\left(e+fx\right)+2\sin\left(e+fx\right)+1}\,dx+\int\left(-\frac{c\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)}{\sin^2\left(e+fx\right)+2\sin\left(e+fx\right)+1}\right)dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] (Integral(c*sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1
), x) + Integral(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)/(sin(e + f*x)**2
 + 2*sin(e + f*x) + 1), x))/a**2
```

3.328 $\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=36

$$-\frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2cf}$$

```
[Out] -2/3*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^2/c/f
```

Rubi [A] time = 0.14, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2736, 2673}

$$-\frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2cf}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c*Sin[e + f*x]]/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (-2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(3*a^2*c*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx &= \frac{\int \sec^4(e+fx)(c-c\sin(e+fx))^{5/2}\,dx}{a^2c^2}\\ &= -\frac{2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2cf}\end{aligned}$$

Mathematica [B] time = 0.13, size = 73, normalized size = 2.03

$$-\frac{2\sqrt{c-c\sin(e+fx)}}{3a^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c*Sin[e + f*x]]/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (-2*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

fricas [A] time = 0.42, size = 46, normalized size = 1.28

$$-\frac{2\sqrt{-c\sin\left(fx+e\right)+c}}{3\left(a^2f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a^2*f*cos
(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)sqrt(2*c)/3*(-3*((-cos(1/4*(2*f*x+2*exp(1)-pi))+1)/(cos(1/4*(2*f*x+
2*exp(1)-pi))+1))^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))-sign(sin(1/2*(f*x+ex
p(1))-1/4*pi)))/a^2/((-cos(1/4*(2*f*x+2*exp(1)-pi))+1)/(cos(1/4*(2*f*x+2*ex
p(1)-pi))+1)-1)^3/f
```

maple [A] time = 0.75, size = 49, normalized size = 1.36

$$\frac{2c\left(\sin\left(fx+e\right)-1\right)}{3a^2\left(1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 2/3*c/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [B] time = 0.98, size = 149, normalized size = 4.14

$$\frac{2\left(\sqrt{c}+\frac{2\sqrt{c}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{\sqrt{c}\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}\right)}{3\left(a^2+\frac{3a^2\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3a^2\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{a^2\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}\right)f\sqrt{\frac{\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(sqrt(c) + 2*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + sqrt(c)*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4)/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) +
 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3)*f*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))
```

mupad [B] time = 9.35, size = 227, normalized size = 6.31

$$-\frac{4\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\left(\sin\left(2e+2fx\right)-4\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2-\sin\left(e+fx\right)^2 2\mathrm{i}+2+2\mathrm{i}\right)}{3a^2f\left(-4\sin\left(e+fx\right)^2+\sin\left(e+fx\right)+\sin\left(3e+3fx\right)+4\right)}+\frac{4\sqrt{-c\left(\sin\left(e+f\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (4*(-c*(sin(e + f*x) - 1))^(1/2)*(sin(e + f*x)*1i + 2*sin(2*e + 2*f*x) + si
n(3*e + 3*f*x)*1i - 2*sin(e/2 + (f*x)/2)^2 + 2*sin((3*e)/2 + (3*f*x)/2)^2 -
 sin(e + f*x)^2*4i + 4i))/(3*a^2*f*(4*sin(e + f*x) + 4*sin(3*e + 3*f*x) + 2
*sin(2*e + 2*f*x)^2 - 8*sin(e + f*x)^2 + 8)) - (4*(-c*(sin(e + f*x) - 1))^(
1/2)*(sin(2*e + 2*f*x) - 4*sin(e/2 + (f*x)/2)^2 - sin(e + f*x)^2*2i + (2 +
2i)))/(3*a^2*f*(sin(e + f*x) + sin(3*e + 3*f*x) - 4*sin(e + f*x)^2 + 4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{-c\sin\left(e+fx\right)+c}}{\sin^2\left(e+fx\right)+2\sin\left(e+fx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Integral(sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),
x)/a**2
```

3.329 $\int \frac{1}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=124

$$-\frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2c^2f}-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f}$$

```
[Out] -1/3*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^2/c^2/f+1/4*arctanh(1/2*cos(f*x+
e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a^2/f*2^(1/2)/c^(1/2)-1/2*sec(f*
x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/c/f
```

Rubi [A] time = 0.22, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2736, 2675, 2649, 206}

$$-\frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2c^2f}-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(2*Sqrt[
2]*a^2*Sqrt[c]*f) - (Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(2*a^2*c*f) - (
Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(3*a^2*c^2*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
```

```
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\int \sec^4(e+fx)(c-c\sin(e+fx))^{3/2}\,dx}{a^2c^2}$$
$$= -\frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2c^2f} + \frac{\int \sec^2(e+fx)\sqrt{c-c\sin(e}{2a^2c}$$
$$= -\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf} - \frac{\sec^3(e+fx)(c-c\sin(e+fx))}{3a^2c^2f}$$
$$= -\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf} - \frac{\sec^3(e+fx)(c-c\sin(e+fx))}{3a^2c^2f}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f} - \frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf} - \sec$$

Mathematica [C] time = 0.51, size = 109, normalized size = 0.88

$$\frac{\cos(e+fx)\left(-3\sin(e+fx)+(-3-3i)\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(\tan\left(\frac{1}{4}(e+fx)\right)+1\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\right.\right.\right.}{6a^2f(\sin(e+fx)+1)^2\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] (Cos[e + f*x]*(-5 - (3 + 3*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 +
 Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 3*Sin[e + f*x
]))/(6*a^2*f*(1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

fricas [A] time = 0.45, size = 204, normalized size = 1.65

$$\frac{3\sqrt{2}\left(\cos\left(fx+e\right)\sin\left(fx+e\right)+\cos\left(fx+e\right)\right)\sqrt{c}\,\log\left(-\frac{c\cos\left(fx+e\right)^{2}+2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\,\sqrt{c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)+3}{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)}}{24\left(a^{2}cf\cos\left(fx+e\right)\sin\left(fx+e\right)+a^{2}cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/24*(3*sqrt(2)*(cos(f*x + e)*sin(f*x + e) + cos(f*x + e))*sqrt(c)*log(-(c*
cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e)
+ sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e
) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) -
 2)) - 4*sqrt(-c*sin(f*x + e) + c)*(3*sin(f*x + e) + 5))/(a^2*c*f*cos(f*x +
 e)*sin(f*x + e) + a^2*c*f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*((30*c*sqrt(2)*atan
(sqrt(c)/sqrt(-c))-42*c*atan(sqrt(c)/sqrt(-c))-15*sqrt(-c)*sqrt(2)*sqrt(c)+
22*sqrt(-c)*sqrt(c))/(84*a^2*c*sqrt(-c)*sqrt(2)-120*a^2*c*sqrt(-c))*sign(ta
n((f*x+exp(1))/2)-1)+2*(1/12*(9*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f
*x+exp(1))/2)^2+c))^5-15*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((
f*x+exp(1))/2)^2+c))^4-10*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+e
xp(1))/2)^2+c))^3+21*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(
1))/2)^2+c))+30*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp
(1))/2)^2+c))^2+5*sqrt(c)*c^2)/a^2/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*t
an((f*x+exp(1))/2)^2+c))^2+2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*t
an((f*x+exp(1))/2)^2+c))+c)^3/sign(tan((f*x+exp(1))/2)-1)+1/4*atan((-sqrt(c
)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt
(-c))/sqrt(2)/a^2/sqrt(-c)/sign(tan((f*x+exp(1))/2)-1)))
```

maple [A] time = 1.25, size = 109, normalized size = 0.88

$$\frac{\left(\sin\left(fx+e\right)-1\right)\left(10c^{\frac{7}{2}}+6c^{\frac{7}{2}}\sin\left(fx+e\right)-3\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right)c^{2}\left(c\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\right)}{12a^{2}c^{\frac{7}{2}}\left(1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\ f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/12/a^2*(sin(f*x+e)-1)/c^(7/2)/(1+sin(f*x+e))*(10*c^(7/2)+6*c^(7/2)*sin(f*
x+e)-3*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^2*(c
*(1+sin(f*x+e)))^(3/2))/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{2}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^2*sqrt(-c*sin(f*x + e) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+a\ \sin\left(e+f\,x\right)\right)^{2}\,\sqrt{c-c\ \sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{-c\sin\left(e+fx\right)+c}\ \sin^{2}\left(e+fx\right)+2\sqrt{-c\sin\left(e+fx\right)+c}\ \sin\left(e+fx\right)+\sqrt{-c\sin\left(e+fx\right)+c}}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2 + 2*sqrt(-c*sin(e + f
*x) + c)*sin(e + f*x) + sqrt(-c*sin(e + f*x) + c)), x)/a**2
```

3.330 $\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=155

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}a^2c^{3/2}f}-\frac{\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2c^2f}+\frac{5\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}}-\frac{5\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 5/8*cos(f*x+e)/a^2/f/(c-c*sin(f*x+e))^(3/2)+5/16*arctanh(1/2*cos(f*x+e)*c^(
1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a^2/c^(3/2)/f*2^(1/2)-5/6*sec(f*x+e)/a
^2/c/f/(c-c*sin(f*x+e))^(1/2)-1/3*sec(f*x+e)^3*(c-c*sin(f*x+e))^(1/2)/a^2/c
^2/f
```

Rubi [A] time = 0.25, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2736, 2675, 2687, 2650, 2649, 206}

$$-\frac{\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2c^2f}+\frac{5\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}a^2c^{3/2}f}+\frac{5\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}}-\frac{5\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (5*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(8*S
qrt[2]*a^2*c^(3/2)*f) + (5*Cos[e + f*x])/(8*a^2*f*(c - c*Sin[e + f*x])^(3/2
)) - (5*Sec[e + f*x])/(6*a^2*c*f*Sqrt[c - c*Sin[e + f*x]]) - (Sec[e + f*x]^
3*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*c^2*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \sec^4(e+fx)\sqrt{c-c\sin(e+fx)}\,dx}{a^2c^2}$$

$$= -\frac{\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2c^2f} + \frac{5\int \frac{\sec^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}dx}{6a^2c}$$

$$= -\frac{5\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}} - \frac{\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2c^2f} +$$

$$= \frac{5\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}} - \frac{5\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}} - \frac{\sec^3(e$$

$$= \frac{5\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}} - \frac{5\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}} - \frac{\sec^3(e$$

$$= \frac{5\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}a^2c^{3/2}f} + \frac{5\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}} - \frac{}{6a^2c}$$

Mathematica [C] time = 0.79, size = 164, normalized size = 1.06

$$\frac{\left(\frac{1}{96}+\frac{i}{96}\right)\cos(e+fx)\left((1-i)(-20\sin(e+fx)+15\cos(2(e+fx))+11)+60\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(\tan\left(\frac{1}{4}(e\right.\right.\right.\right.}{a^2cf(\sin(e+fx)-1)(\sin(e+fx)+1)^2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] ((1/96 + I/96)*Cos[e + f*x]*(60*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1
 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^3 + (1 - I)*(11 + 15*Cos[2*(e + f*x)] - 20*Sin[e +
 f*x]))))/(a^2*c*f*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e
 + f*x]])
```

fricas [A] time = 0.45, size = 186, normalized size = 1.20

$$\frac{15\sqrt{2}\sqrt{c}\cos\left(fx+e\right)^3\log\left(-\frac{c\cos\left(fx+e\right)^2+2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\sqrt{c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)+3c\cos\left(fx+e\right)+\left(c\cos\left(fx+e\right)-2c\right)\sin\left(f\right.}{\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right.}{96a^2c^2f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/96*(15*sqrt(2)*sqrt(c)*cos(f*x + e)^3*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*
sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*c
os(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 +
(cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(15*cos(f*x + e)^2
 - 10*sin(f*x + e) - 2)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^2*f*cos(f*x + e)^
3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*(1/16*(-3*(-sqrt(
c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3+c*(-sqrt(c)*tan((
f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-sqrt(c)*(-sqrt(c)*tan((f*x+
exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-sqrt(c)*c)/a^2/(-(-sqrt(c)*ta
n((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-2*sqrt(c)*(-sqrt(c)*ta
n((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)^2/c/sign(tan((f*x+exp
(1))/2)-1)-1/6*(-3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)
^2+c))^5+6*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)
^2+c))^4+4*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))
^3-9*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-12*
sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-
2*sqrt(c)*c^2)/a^2/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/
2)^2+c))^2+2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/
2)^2+c))+c)^3/c/sign(tan((f*x+exp(1))/2)-1)+5/16*atan((-sqrt(c)*tan((f*x+ex
p(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-c))/sqrt(2)
/a^2/sqrt(-c)/c/sign(tan((f*x+exp(1))/2)-1))
```

maple [A] time = 1.25, size = 157, normalized size = 1.01

$$-\frac{15\left(c\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right)\sin\left(fx+e\right)c-15\left(c\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\sqrt{2}\ \operatorname{arct}}{48c^{\frac{7}{2}}a^{2}\left(1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)\sqrt{c-c\,\mathrm{s}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/48/c^(7/2)/a^2*(15*(c*(1+sin(f*x+e)))^(3/2)*2^(1/2)*arctanh(1/2*(c*(1+si
n(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c-15*(c*(1+sin(f*x+e)))^(3/2)*
2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c-20*c^(5/2)*
sin(f*x+e)-30*c^(5/2)*sin(f*x+e)^2+26*c^(5/2))/(1+sin(f*x+e))/cos(f*x+e)/(c
-c*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.331 $\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=192

$$\frac{35\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{64\sqrt{2}\,a^2c^{5/2}f}-\frac{\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+fx)}}-\frac{35\sec(e+fx)}{48a^2c^2f\sqrt{c-c\sin(e+fx)}}+\frac{35\cos(e+fx)}{64a^2cf(c-c\sin(e+fx)}$$

```
[Out] 35/64*cos(f*x+e)/a^2/c/f/(c-c*sin(f*x+e))^(3/2)+7/24*sec(f*x+e)/a^2/c/f/(c-
c*sin(f*x+e))^(3/2)+35/128*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(
f*x+e))^(1/2))/a^2/c^(5/2)/f*2^(1/2)-35/48*sec(f*x+e)/a^2/c^2/f/(c-c*sin(f*
x+e))^(1/2)-1/3*sec(f*x+e)^3/a^2/c^2/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2736, 2687, 2681, 2650, 2649, 206}

$$-\frac{\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+fx)}}-\frac{35\sec(e+fx)}{48a^2c^2f\sqrt{c-c\sin(e+fx)}}+\frac{35\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{64\sqrt{2}\,a^2c^{5/2}f}+\frac{35\cos(e+fx)}{64a^2cf(c-c\sin(e+f}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (35*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(64
*Sqrt[2]*a^2*c^(5/2)*f) + (35*Cos[e + f*x])/(64*a^2*c*f*(c - c*Sin[e + f*x]
)^(3/2)) + (7*Sec[e + f*x])/(24*a^2*c*f*(c - c*Sin[e + f*x])^(3/2)) - (35*S
ec[e + f*x])/(48*a^2*c^2*f*Sqrt[c - c*Sin[e + f*x]]) - Sec[e + f*x]^3/(3*a^
2*c^2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{\sec^4(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx}{a^2c^2}$$

$$= -\frac{\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+fx)}} + \frac{7\int \frac{\sec^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx}{6a^2c}$$

$$= \frac{7\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}} - \frac{\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+fx)}} + \frac{35}{}$$

$$= \frac{7\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}} - \frac{35\sec(e+fx)}{48a^2c^2f\sqrt{c-c\sin(e+fx)}} - \frac{}{3}$$

$$= \frac{35\cos(e+fx)}{64a^2cf(c-c\sin(e+fx))^{3/2}} + \frac{7\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}} -$$

$$= \frac{35\cos(e+fx)}{64a^2cf(c-c\sin(e+fx))^{3/2}} + \frac{7\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}} -$$

$$= \frac{35\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{64\sqrt{2}\,a^2c^{5/2}f} + \frac{35\cos(e+fx)}{64a^2cf(c-c\sin(e+fx))^{3/2}} +$$

Mathematica [C] time = 1.17, size = 156, normalized size = 0.81

$$-\frac{\left(\frac{1}{1536}+\frac{i}{1536}\right)\sec^3(e+fx)\Big((1-i)(-329\sin(e+fx)-105\sin(3(e+fx))+70\cos(2(e+fx))+102)+840\sqrt[4]{-}}{a^2c^2f\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] ((-1/1536 - I/1536)*Sec[e + f*x]^3*(840*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^
(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])^3 + (1 - I)*(102 + 70*Cos[2*(e + f*x)] - 3
29*Sin[e + f*x] - 105*Sin[3*(e + f*x)])))/(a^2*c^2*f*Sqrt[c - c*Sin[e + f*x
]])
```

fricas [A] time = 0.47, size = 241, normalized size = 1.26

$$\frac{105\sqrt{2}\left(\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)-\cos\left(fx+e\right)^{3}\right)\sqrt{c}\log\left(-\frac{c\cos\left(fx+e\right)^{2}+2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\sqrt{c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right.}{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx\right.}\right.}{768\left(a^{2}c^{3}f\cos\left(fx+e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/768*(105*sqrt(2)*(cos(f*x + e)^3*sin(f*x + e) - cos(f*x + e)^3)*sqrt(c)*l
og(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*
x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(
f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x
 + e) - 2)) + 4*(35*cos(f*x + e)^2 - 7*(15*cos(f*x + e)^2 + 8)*sin(f*x + e)
 + 8)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^3*f*cos(f*x + e)^3*sin(f*x + e) - a
^2*c^3*f*cos(f*x + e)^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*(-1/48*(-15*(-sqr
t(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5+33*sqrt(c)*(-sq
rt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4+22*c*(-sqrt(c)
*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3-51*c^2*(-sqrt(c)*ta
n((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-66*sqrt(c)*c*(-sqrt(c)*t
an((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-11*sqrt(c)*c^2)/a^2/c
^2/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+2*sqr
t(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)^3/si
gn(tan((f*x+exp(1))/2)-1)+1/128*(-53*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*t
an((f*x+exp(1))/2)^2+c))^7-179*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c
*tan((f*x+exp(1))/2)^2+c))^6-127*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan
((f*x+exp(1))/2)^2+c))^5+195*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c
*tan((f*x+exp(1))/2)^2+c))^4-7*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan
((f*x+exp(1))/2)^2+c))^3+67*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f
*x+exp(1))/2)^2+c))-121*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*ta
n((f*x+exp(1))/2)^2+c))^2-15*sqrt(c)*c^3)/a^2/c^2/(-(-sqrt(c)*tan((f*x+exp(
```

```
1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(
1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)^4/sign(tan((f*x+exp(1))/2)-1)+35
/128*atan((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^
2+c))/sqrt(2)/sqrt(-c))/sqrt(2)/a^2/c^2/sqrt(-c)/sign(tan((f*x+exp(1))/2)-1
))
```

maple [A] time = 1.19, size = 233, normalized size = 1.21

$$-\frac{105\left(c\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right)\left(\sin^2\left(fx+e\right)\right)c^2+70c^{\frac{7}{2}}\left(\sin^2\left(fx+e\right)\right)-210c^{\frac{7}{2}}}{38}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/384/c^(11/2)/a^2*(105*(c*(1+sin(f*x+e)))^(3/2)*2^(1/2)*arctanh(1/2*(c*(1
+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^2+70*c^(7/2)*sin(f*x+e)
^2-210*c^(7/2)*sin(f*x+e)^3-210*(c*(1+sin(f*x+e)))^(3/2)*2^(1/2)*arctanh(1/
2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c^2+322*c^(7/2)*sin(
f*x+e)+105*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^
2*(c*(1+sin(f*x+e)))^(3/2)-86*c^(7/2))/(1+sin(f*x+e))/(sin(f*x+e)-1)/cos(f*
x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+a\,\sin\left(e+f\,x\right)\right)^2\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c^2\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^4\left(e+fx\right)-2c^2\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^2\left(e+fx\right)+c^2\sqrt{-c\sin\left(e+fx\right)+c}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(1/(c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**4 - 2*c**2*sqrt(-c
*sin(e + f*x) + c)*sin(e + f*x)**2 + c**2*sqrt(-c*sin(e + f*x) + c)), x)/a*
*2
```

3.332 $\int \frac{(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=174

$$\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{3a^3c^2f}-\frac{4096c^2\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^3f}+\frac{32\sec^5(e+fx)(c-c\sin(e+fx}{3a^3cf}$$

```
[Out] -4096/15*c^2*sec(f*x+e)^5*(c-c*sin(f*x+e))^(5/2)/a^3/f+1024/3*c*sec(f*x+e)^
5*(c-c*sin(f*x+e))^(7/2)/a^3/f-128*sec(f*x+e)^5*(c-c*sin(f*x+e))^(9/2)/a^3/
f+32/3*sec(f*x+e)^5*(c-c*sin(f*x+e))^(11/2)/a^3/c/f+2/3*sec(f*x+e)^5*(c-c*s
in(f*x+e))^(13/2)/a^3/c^2/f
```

Rubi [A] time = 0.40, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{3a^3c^2f}-\frac{4096c^2\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^3f}+\frac{32\sec^5(e+fx)(c-c\sin(e+fx}{3a^3cf}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(9/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-4096*c^2*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(5/2))/(15*a^3*f) + (1024*c*
Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(7/2))/(3*a^3*f) - (128*Sec[e + f*x]^5*
(c - c*Sin[e + f*x])^(9/2))/(a^3*f) + (32*Sec[e + f*x]^5*(c - c*Sin[e + f*x
])^(11/2))/(3*a^3*c*f) + (2*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(13/2))/(3*
a^3*c^2*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{\int \sec^6(e+fx)(c-c\sin(e+fx))^{15/2}\,dx}{a^3c^3}$$
$$= \frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{3a^3c^2f} + \frac{16\int \sec^6(e+fx)(c-c\sin(e+fx))^{13/2}\,dx}{3a^3c^2}$$
$$= \frac{32\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{3a^3cf} + \frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{3a^3c^2f} + \frac{64\int$$
$$= -\frac{128\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{a^3f} + \frac{32\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{3a^3cf} + \frac{2$$
$$= \frac{1024c\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^3f} - \frac{128\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{a^3f} +$$
$$= -\frac{4096c^2\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^3f} + \frac{1024c\sec^5(e+fx)(c-c\sin(e+fx))^{7}}{3a^3f}$$

Mathematica [A] time = 3.15, size = 124, normalized size = 0.71

$$\frac{c^4\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(-7800\sin(e+fx)+200\sin(3(e+fx))+2740\cos(2(e}{60a^3f(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(9/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (c^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(-5649
+ 2740*Cos[2*(e + f*x)] + 5*Cos[4*(e + f*x)] - 7800*Sin[e + f*x] + 200*Sin[
3*(e + f*x)]))/(60*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e +
 f*x])^3)
```

fricas [A] time = 0.43, size = 119, normalized size = 0.68

$$-\frac{2\left(5\,c^4\cos\left(fx+e\right)^4+680\,c^4\cos\left(fx+e\right)^2-1048\,c^4+100\left(c^4\cos\left(fx+e\right)^2-10\,c^4\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(f\right.}}{15\left(a^3f\cos\left(fx+e\right)^3-2\,a^3f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^3f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -2/15*(5*c^4*cos(f*x + e)^4 + 680*c^4*cos(f*x + e)^2 - 1048*c^4 + 100*(c^4*
cos(f*x + e)^2 - 10*c^4)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos
(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.79, size = 91, normalized size = 0.52

$$-\frac{2c^5\left(\sin\left(fx+e\right)-1\right)\left(5\left(\sin^4\left(fx+e\right)\right)-100\left(\sin^3\left(fx+e\right)\right)-690\left(\sin^2\left(fx+e\right)\right)-900\sin\left(fx+e\right)-363\right)}{15a^3\left(1+\sin\left(fx+e\right)\right)^2\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\;f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x)
```

```
[Out] -2/15*c^5/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(5*sin(f*x+e)^4-100*sin(f*x+e
)^3-690*sin(f*x+e)^2-900*sin(f*x+e)-363)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/
f
```

maxima [B] time = 0.84, size = 472, normalized size = 2.71

$$\frac{2\left(363\,c^{\frac{9}{2}}+\frac{1800\,c^{\frac{9}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{5301\,c^{\frac{9}{2}}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{11600\,c^{\frac{9}{2}}\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{21343\,c^{\frac{9}{2}}\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+\frac{30200\,c^{\frac{9}{2}}\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}+\frac{40065\,c^{\frac{9}{2}}\cdots}{\left(\cos\left(f\right.\right.\cdots}\right.}{15\left(a^3+\frac{5\,a^3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{10\,a^3\cdots}{\left(\cos\cdots\right.}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 2/15*(363*c^(9/2) + 1800*c^(9/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 5301*c^(
9/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 11600*c^(9/2)*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 21343*c^(9/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 302
00*c^(9/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 40065*c^(9/2)*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6 + 40800*c^(9/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7
 + 40065*c^(9/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 30200*c^(9/2)*sin(f*
x + e)^9/(cos(f*x + e) + 1)^9 + 21343*c^(9/2)*sin(f*x + e)^10/(cos(f*x + e)
 + 1)^10 + 11600*c^(9/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 5301*c^(9/
2)*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 + 1800*c^(9/2)*sin(f*x + e)^13/(co
s(f*x + e) + 1)^13 + 363*c^(9/2)*sin(f*x + e)^14/(cos(f*x + e) + 1)^14)/((a
^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*f*(si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(9/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-c\,\sin\left(e+f\,x\right)\right)^{9/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(9/2)/(a + a*sin(e + f*x))^3,x)
```

```
[Out] int((c - c*sin(e + f*x))^(9/2)/(a + a*sin(e + f*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.333 $\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=134

$$\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{a^3c^2f}-\frac{24\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{a^3cf}+\frac{64\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{a^3f}$$

```
[Out] -256/5*c*sec(f*x+e)^5*(c-c*sin(f*x+e))^(5/2)/a^3/f+64*sec(f*x+e)^5*(c-c*sin
(f*x+e))^(7/2)/a^3/f-24*sec(f*x+e)^5*(c-c*sin(f*x+e))^(9/2)/a^3/c/f+2*sec(f
*x+e)^5*(c-c*sin(f*x+e))^(11/2)/a^3/c^2/f
```

Rubi [A] time = 0.33, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{a^3c^2f}-\frac{24\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{a^3cf}+\frac{64\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(7/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-256*c*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*f) + (64*Sec[e +
f*x]^5*(c - c*Sin[e + f*x])^(7/2))/(a^3*f) - (24*Sec[e + f*x]^5*(c - c*Sin[
e + f*x])^(9/2))/(a^3*c*f) + (2*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(11/2))
/(a^3*c^2*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{\int \sec^6(e+fx)(c-c\sin(e+fx))^{13/2}\,dx}{a^3c^3}$$
$$= \frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{a^3c^2f} + \frac{12\int \sec^6(e+fx)(c-c\sin(e+fx))^{11/2}\,dx}{a^3c^2}$$
$$= -\frac{24\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{a^3cf} + \frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{a^3c^2f} - \frac{96 \cdots}{}$$
$$= \frac{64\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{a^3f} - \frac{24\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{a^3cf} + \frac{2\sec \cdots}{}$$
$$= -\frac{256c\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f} + \frac{64\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{a^3f} - \frac{2 \cdots}{}$$

Mathematica [A] time = 1.28, size = 114, normalized size = 0.85

$$\frac{c^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(-235\sin(e+fx)+5\sin(3(e+fx))+90\cos(2(e+fx \cdots}{10a^3f(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(7/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(-182 +
 90*Cos[2*(e + f*x)] - 235*Sin[e + f*x] + 5*Sin[3*(e + f*x)]))/(10*a^3*f*(C
os[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3)
```

fricas [A] time = 0.43, size = 106, normalized size = 0.79

$$-\frac{2\left(45\,c^3\cos\left(fx+e\right)^2-68\,c^3+5\left(c^3\cos\left(fx+e\right)^2-12\,c^3\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{5\left(a^3f\cos\left(fx+e\right)^3-2\,a^3f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^3f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -2/5*(45*c^3*cos(f*x + e)^2 - 68*c^3 + 5*(c^3*cos(f*x + e)^2 - 12*c^3)*sin(
f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x
 + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.36, size = 81, normalized size = 0.60

$$\frac{2c^4\left(\sin\left(fx+e\right)-1\right)\left(5\left(\sin^3\left(fx+e\right)\right)+45\left(\sin^2\left(fx+e\right)\right)+55\sin\left(fx+e\right)+23\right)}{5a^3\left(1+\sin\left(fx+e\right)\right)^2\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/5*c^4/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(5*sin(f*x+e)^3+45*sin(f*x+e)^2
+55*sin(f*x+e)+23)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [B] time = 1.00, size = 426, normalized size = 3.18

$$\frac{2\left(23\,c^{\frac{7}{2}}+\frac{110\,c^{\frac{7}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{318\,c^{\frac{7}{2}}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{590\,c^{\frac{7}{2}}\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{1065\,c^{\frac{7}{2}}\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+\frac{1220\,c^{\frac{7}{2}}\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}+\frac{1540\,c^{\frac{7}{2}}\sin\left(fx+e\cdots}{\left(\cos\left(fx+e\right)+1\right)^6}+\cdots\right.}{5\left(a^3+\frac{5\,a^3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{10\,a^3\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{10\,a^3\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 2/5*(23*c^(7/2) + 110*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 318*c^(7/2)
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 590*c^(7/2)*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 1065*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1220*c^(7/
2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1540*c^(7/2)*sin(f*x + e)^6/(cos(f
*x + e) + 1)^6 + 1220*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1065*c^
(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 590*c^(7/2)*sin(f*x + e)^9/(cos
```

```
(f*x + e) + 1)^9 + 318*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 110*
c^(7/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 23*c^(7/2)*sin(f*x + e)^12/
(cos(f*x + e) + 1)^12)/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a
^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(
cos(f*x + e) + 1)^5)*f*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2))
```

mupad [B] time = 13.87, size = 542, normalized size = 4.04

$$-\frac{\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\left(\frac{2\,c^3}{a^3\,f}-\frac{c^3\,e^{e\,1i+f\,x\,1i}\,2i}{a^3\,f}\right)}{e^{e\,1i+f\,x\,1i}-i}-\frac{24\,c^3\,e^{e\,1i+f\,x\,1i}\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}}{a^3\,f\left(e^{e\,1i+f\,x\,1i}-i\right)\left(e^{e\,1i+f\,x\,1i}+1i\right)}+\frac{c^3\,e^{e\,1i}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(7/2)/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (c^3*exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f
*x*1i)*1i)/2))^(1/2)*32i)/(a^3*f*(exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*
1i) + 1i)^2) - (24*c^3*exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)
/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(a^3*f*(exp(e*1i + f*x*1i) - 1i)*(e
xp(e*1i + f*x*1i) + 1i)) - ((c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i
 + f*x*1i)*1i)/2))^(1/2)*((2*c^3)/(a^3*f) - (c^3*exp(e*1i + f*x*1i)*2i)/(a^
3*f)))/(exp(e*1i + f*x*1i) - 1i) + (288*c^3*exp(e*1i + f*x*1i)*(c - c*((exp
(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(5*a^3*f*(exp(
e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^3) - (c^3*exp(e*1i + f*x*1i)
*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*25
6i)/(5*a^3*f*(exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^4) - (128*
c^3*exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*
x*1i)*1i)/2))^(1/2))/(5*a^3*f*(exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i)
 + 1i)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.334 $\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=104

$$-\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{a^3c^2f}+\frac{16\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^3cf}-\frac{64\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^3f}$$

```
[Out] -64/15*sec(f*x+e)^5*(c-c*sin(f*x+e))^(5/2)/a^3/f+16/3*sec(f*x+e)^5*(c-c*sin
(f*x+e))^(7/2)/a^3/c/f-2*sec(f*x+e)^5*(c-c*sin(f*x+e))^(9/2)/a^3/c^2/f
```

Rubi [A] time = 0.27, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$-\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{a^3c^2f}+\frac{16\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^3cf}-\frac{64\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-64*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(5/2))/(15*a^3*f) + (16*Sec[e + f*
x]^5*(c - c*Sin[e + f*x])^(7/2))/(3*a^3*c*f) - (2*Sec[e + f*x]^5*(c - c*Sin
[e + f*x])^(9/2))/(a^3*c^2*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
```

```
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{\int \sec^6(e+fx)(c-c\sin(e+fx))^{11/2}\,dx}{a^3c^3}$$
$$= -\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{a^3c^2f} - \frac{8\int \sec^6(e+fx)(c-c\sin(e+fx))^{9/2}\,dx}{a^3c^2}$$
$$= \frac{16\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^3cf} - \frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{a^3c^2f} + \frac{32\int s}{}$$
$$= -\frac{64\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^3f} + \frac{16\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^3cf} - \frac{2\,se}{}$$

Mathematica [A] time = 0.87, size = 104, normalized size = 1.00

$$\frac{c^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(-20\sin(e+fx)+15\cos(2(e+fx))-29)}{15a^3f(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-29 + 15*Cos[2*(e + f*x)] - 20*
Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(15*a^3*f*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2])*(1 + Sin[e + f*x])^3)
```

fricas [A] time = 0.42, size = 91, normalized size = 0.88

$$-\frac{2\left(15\,c^2\cos\left(fx+e\right)^2-10\,c^2\sin\left(fx+e\right)-22\,c^2\right)\sqrt{-c\sin\left(fx+e\right)+c}}{15\left(a^3f\cos\left(fx+e\right)^3-2\,a^3f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^3f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -2/15*(15*c^2*cos(f*x + e)^2 - 10*c^2*sin(f*x + e) - 22*c^2)*sqrt(-c*sin(f*
x + e) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a
^3*f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.84, size = 71, normalized size = 0.68

$$\frac{2c^{3}\left(\sin\left(fx+e\right)-1\right)\left(15\left(\sin^{2}\left(fx+e\right)\right)+10\sin\left(fx+e\right)+7\right)}{15a^{3}\left(1+\sin\left(fx+e\right)\right)^{2}\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/15*c^3/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(15*sin(f*x+e)^2+10*sin(f*x+e)
+7)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [B] time = 1.09, size = 380, normalized size = 3.65

$$\frac{2\left(7c^{\frac{5}{2}}+\frac{20c^{\frac{5}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{95c^{\frac{5}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{80c^{\frac{5}{2}}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{250c^{\frac{5}{2}}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{120c^{\frac{5}{2}}\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}+\frac{250c^{\frac{5}{2}}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}+\frac{80}{(c}\right.}{15\left(a^{3}+\frac{5a^{3}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{10a^{3}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{10a^{3}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{5a^{3}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{a^{3}\sin(f}{(\cos(fx+}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 2/15*(7*c^(5/2) + 20*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 95*c^(5/2)*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 80*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 250*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 120*c^(5/2)*si
n(f*x + e)^5/(cos(f*x + e) + 1)^5 + 250*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 + 80*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 95*c^(5/2)*sin(
f*x + e)^8/(cos(f*x + e) + 1)^8 + 20*c^(5/2)*sin(f*x + e)^9/(cos(f*x + e) +
 1)^9 + 7*c^(5/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)/((a^3 + 5*a^3*sin(
```

```
f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*f*(sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 1)^(5/2))
```

mupad [B] time = 11.95, size = 453, normalized size = 4.36

$$-\frac{4\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\sqrt{c-c\,\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}}{a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\mathrm{i}\right)\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)}+\frac{c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\sqrt{c-c\,\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\,32\mathrm{i}}{3\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\mathrm{i}\right)\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)}^2}+\frac{352\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f}}{15\,a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (c^2*exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f
*x*1i)*1i)/2))^(1/2)*32i)/(3*a^3*f*(exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*
x*1i) + 1i)^2) - (4*c^2*exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i
)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(a^3*f*(exp(e*1i + f*x*1i) - 1i)*(
exp(e*1i + f*x*1i) + 1i)) + (352*c^2*exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i
 - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(15*a^3*f*(exp(e*1i +
 f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^3) - (c^2*exp(e*1i + f*x*1i)*(c -
c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*128i)/(5
*a^3*f*(exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^4) - (64*c^2*exp
(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1
i)/2))^(1/2))/(5*a^3*f*(exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^
5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.335 $\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=73

$$\frac{8\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^3cf}-\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^3c^2f}$$

```
[Out] 8/15*sec(f*x+e)^5*(c-c*sin(f*x+e))^(5/2)/a^3/c/f-2/3*sec(f*x+e)^5*(c-c*sin(
f*x+e))^(7/2)/a^3/c^2/f
```

Rubi [A] time = 0.20, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2736, 2674, 2673}

$$\frac{8\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^3cf}-\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^3c^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (8*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(5/2))/(15*a^3*c*f) - (2*Sec[e + f*x
]^5*(c - c*Sin[e + f*x])^(7/2))/(3*a^3*c^2*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
```

```
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx &= \frac{\int \sec^6(e+fx)(c-c\sin(e+fx))^{9/2}\,dx}{a^3c^3}\\ &= -\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^3c^2f}-\frac{4\int \sec^6(e+fx)(c-c\sin(e+fx))^{7/2}\,dx}{3a^3c^2}\\ &= \frac{8\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^3cf}-\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^3c^2f}\end{aligned}$$

Mathematica [A] time = 0.39, size = 92, normalized size = 1.26

$$\frac{2c(5\sin(e+fx)-1)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{15a^3f(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (2*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + 5*Sin[e + f*x])*Sqrt[c - c
*Sin[e + f*x]])/(15*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e
+ f*x])^3)
```

fricas [A] time = 0.46, size = 74, normalized size = 1.01

$$-\frac{2\left(5\,c\sin\left(fx+e\right)-c\right)\sqrt{-c\sin\left(fx+e\right)+c}}{15\left(a^3f\cos\left(fx+e\right)^3-2\,a^3f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^3f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -2/15*(5*c*sin(f*x + e) - c)*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*x + e)^
3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.82, size = 61, normalized size = 0.84

$$-\frac{2c^{2}\left(\sin\left(fx+e\right)-1\right)\left(5\sin\left(fx+e\right)-1\right)}{15a^{3}\left(1+\sin\left(fx+e\right)\right)^{2}\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x)
```

```
[Out] -2/15*c^2/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(5*sin(f*x+e)-1)/cos(f*x+e)/(
c-c*sin(f*x+e))^(1/2)/f
```

maxima [B] time = 0.85, size = 331, normalized size = 4.53

$$\frac{2\left(c^{\frac{3}{2}}-\frac{10\,c^{\frac{3}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{4\,c^{\frac{3}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{30\,c^{\frac{3}{2}}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{6\,c^{\frac{3}{2}}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}-\frac{30\,c^{\frac{3}{2}}\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}+\frac{4\,c^{\frac{3}{2}}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}-\frac{10\,c^{\frac{3}{2}}\sin\left(f\cdots\right.}{\left(\cos\left(fx+e\right)\cdots\right.}}{15\left(a^{3}+\frac{5\,a^{3}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{10\,a^{3}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{10\,a^{3}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{5\,a^{3}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{a^{3}\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}\right)f\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\cdots\right.}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 2/15*(c^(3/2) - 10*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 4*c^(3/2)*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 - 30*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 6*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 30*c^(3/2)*sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5 + 4*c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
 - 10*c^(3/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + c^(3/2)*sin(f*x + e)^8/
(cos(f*x + e) + 1)^8)/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5)*f*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2))
```

mupad [B] time = 10.91, size = 355, normalized size = 4.86

$$\frac{c\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\sqrt{c-c\,\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\,8\mathrm{i}}{3\,a^{3}\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\mathrm{i}\right)\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)^{2}}+\frac{136\,c\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\sqrt{c-c\,\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}}{15\,a^{3}\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\mathrm{i}\right)\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)^{3}}-\frac{c\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\sqrt{\cdots}}{5\,a^{3}\,f\,\left(\mathrm{e}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (c*exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x
*1i)*1i)/2))^(1/2)*8i)/(3*a^3*f*(exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1
i) + 1i)^2) + (136*c*exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2
 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(15*a^3*f*(exp(e*1i + f*x*1i) - 1i)*(
exp(e*1i + f*x*1i) + 1i)^3) - (c*exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f
*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*64i)/(5*a^3*f*(exp(e*1i +
f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^4) - (32*c*exp(e*1i + f*x*1i)*(c -
c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(5*a^3*
f*(exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.336 $\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=36

$$-\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^2f}$$

```
[Out] -2/5*sec(f*x+e)^5*(c-c*sin(f*x+e))^(5/2)/a^3/c^2/f
```

Rubi [A] time = 0.12, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2736, 2673}

$$-\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c*Sin[e + f*x]]/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-2*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*c^2*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx &= \frac{\int \sec^6(e+fx)(c-c\sin(e+fx))^{7/2}\,dx}{a^3c^3}\\ &= -\frac{2\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^2f}\end{aligned}$$

Mathematica [B] time = 0.15, size = 73, normalized size = 2.03

$$-\frac{2\sqrt{c-c\sin(e+fx)}}{5a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c*Sin[e + f*x]]/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-2*Sqrt[c - c*Sin[e + f*x]])/(5*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

fricas [A] time = 0.43, size = 61, normalized size = 1.69

$$\frac{2\sqrt{-c\sin\left(fx+e\right)+c}}{5\left(a^3f\cos\left(fx+e\right)^3-2\,a^3f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^3f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*
sin(f*x + e) - 2*a^3*f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*c)/10*(-10*((-co
s(1/4*(2*f*x+2*exp(1)-pi))+1)/(cos(1/4*(2*f*x+2*exp(1)-pi))+1))^2*sign(sin(
1/2*(f*x+exp(1))-1/4*pi))-5*((-cos(1/4*(2*f*x+2*exp(1)-pi))+1)/(cos(1/4*(2*
f*x+2*exp(1)-pi))+1))^4*sign(sin(1/2*(f*x+exp(1))-1/4*pi))-sign(sin(1/2*(f*
x+exp(1))-1/4*pi)))/a^3/((-cos(1/4*(2*f*x+2*exp(1)-pi))+1)/(cos(1/4*(2*f*x+
2*exp(1)-pi))+1)-1)^5/f
```

maple [A] time = 0.66, size = 49, normalized size = 1.36

$$\frac{2c\left(\sin\left(fx+e\right)-1\right)}{5a^3\left(1+\sin\left(fx+e\right)\right)^2\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/5*c/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)
/f
```

maxima [B] time = 1.13, size = 218, normalized size = 6.06

$$\frac{2\left(\sqrt{c}+\frac{3\sqrt{c}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{3\sqrt{c}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{\sqrt{c}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}\right)}{5\left(a^{3}+\frac{5a^{3}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{10a^{3}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{10a^{3}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{5a^{3}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{a^{3}\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}\right)f\sqrt{\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 2/5*(sqrt(c) + 3*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sqrt(c)*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 + sqrt(c)*sin(f*x + e)^6/(cos(f*x + e) +
1)^6)/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*
sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5)*f*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))
```

mupad [B] time = 9.88, size = 90, normalized size = 2.50

$$\frac{\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{c-c\,\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\,16\mathrm{i}}{5\,a^3\,f\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)}^5\,\left(1+\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (exp(e*3i + f*x*3i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1
i)*1i)/2))^(1/2)*16i)/(5*a^3*f*(exp(e*1i + f*x*1i) + 1i)^5*(exp(e*1i + f*x*
1i)*1i + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{-c\sin\left(e+fx\right)+c}}{\sin^{3}\left(e+fx\right)+3\sin^{2}\left(e+fx\right)+3\sin\left(e+fx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Integral(sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3
*sin(e + f*x) + 1), x)/a**3
```

3.337 $\int \frac{1}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=160

$$-\frac{\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^3f}-\frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{6a^3c^2f}-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{4a^3cf}+\frac{\tanh^{-1}}{4}$$

```
[Out] -1/6*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/c^2/f-1/5*sec(f*x+e)^5*(c-c*si
n(f*x+e))^(5/2)/a^3/c^3/f+1/8*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*s
in(f*x+e))^(1/2))/a^3/f*2^(1/2)/c^(1/2)-1/4*sec(f*x+e)*(c-c*sin(f*x+e))^(1/
2)/a^3/c/f
```

Rubi [A] time = 0.29, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2736, 2675, 2649, 206}

$$-\frac{\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^3f}-\frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{6a^3c^2f}-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{4a^3cf}+\frac{\tanh^{-1}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(4*Sqrt[
2]*a^3*Sqrt[c]*f) - (Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(4*a^3*c*f) - (
Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(6*a^3*c^2*f) - (Sec[e + f*x]^5*
(c - c*Sin[e + f*x])^(5/2))/(5*a^3*c^3*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\int \sec^6(e+fx)(c-c\sin(e+fx))^{5/2}\,dx}{a^3c^3}$$

$$= -\frac{\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^3f} + \frac{\int \sec^4(e+fx)(c-c\sin(e+$$

$$= -\frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{6a^3c^2f} - \frac{\sec^5(e+fx)(c-c\sin(e+f}{5a^3c^3f}$$

$$= -\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{4a^3cf} - \frac{\sec^3(e+fx)(c-c\sin(e+fx))}{6a^3c^2f}$$

$$= -\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{4a^3cf} - \frac{\sec^3(e+fx)(c-c\sin(e+fx))}{6a^3c^2f}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}a^3\sqrt{c}f} - \frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{4a^3cf} - \text{sec}$$

Mathematica [C] time = 0.61, size = 189, normalized size = 1.18

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-15\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{60}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-12 - 10*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 15*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^4 - (15 + 15*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4
)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5))/(60*a^3
*f*(1 + Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]])
```

fricas [A] time = 0.48, size = 241, normalized size = 1.51

$$\frac{15\sqrt{2}\left(\cos\left(fx+e\right)^3-2\cos\left(fx+e\right)\sin\left(fx+e\right)-2\cos\left(fx+e\right)\right)\sqrt{c}\log\left(-\frac{c\cos\left(fx+e\right)^2+2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\sqrt{c}\left(\cdots\right.}{\cos\left(fx+e\right)\cdots}\right.}{240\left(a^3cf\cos\left(fx+e\right)^3-2a^3cf\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/240*(15*sqrt(2)*(cos(f*x + e)^3 - 2*cos(f*x + e)*sin(f*x + e) - 2*cos(f*x
 + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)
*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x
+ e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*
x + e) - cos(f*x + e) - 2)) - 4*(15*cos(f*x + e)^2 - 40*sin(f*x + e) - 52)*
sqrt(-c*sin(f*x + e) + c))/(a^3*c*f*cos(f*x + e)^3 - 2*a^3*c*f*cos(f*x + e)
*sin(f*x + e) - 2*a^3*c*f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*((870*c*sqrt(2)*ata
n(sqrt(c)/sqrt(-c))-1230*c*atan(sqrt(c)/sqrt(-c))-850*sqrt(-c)*sqrt(2)*sqrt
(c)+1203*sqrt(-c)*sqrt(c))/(4920*a^3*c*sqrt(-c)*sqrt(2)-6960*a^3*c*sqrt(-c)
)*sign(tan((f*x+exp(1))/2)-1)+2*(1/120*(105*(-sqrt(c)*tan((f*x+exp(1))/2)+s
qrt(c*tan((f*x+exp(1))/2)^2+c))^9-435*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)
+sqrt(c*tan((f*x+exp(1))/2)^2+c))^8+580*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqr
t(c*tan((f*x+exp(1))/2)^2+c))^7+620*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)
```

```
+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6-1258*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+
sqrt(c*tan((f*x+exp(1))/2)^2+c))^5-490*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1
))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4+900*c^3*(-sqrt(c)*tan((f*x+exp(1))
/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3+265*c^4*(-sqrt(c)*tan((f*x+exp(1))/2
)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+860*sqrt(c)*c^3*(-sqrt(c)*tan((f*x+exp(1
))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+37*sqrt(c)*c^4)/a^3/(-(-sqrt(c)*ta
n((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+2*sqrt(c)*(-sqrt(c)*ta
n((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)^5/sign(tan((f*x+exp(1
))/2)-1)+1/8*atan((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp
(1))/2)^2+c))/sqrt(2)/sqrt(-c))/sqrt(2)/a^3/sqrt(-c)/sign(tan((f*x+exp(1))/
2)-1)))
```

maple [A] time = 1.18, size = 122, normalized size = 0.76

$$-\frac{\left(\sin\left(fx+e\right)-1\right)\left(-30c^{\frac{11}{2}}\left(\sin^{2}\left(fx+e\right)\right)-80c^{\frac{11}{2}}\sin\left(fx+e\right)-74c^{\frac{11}{2}}+15\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right)c^{3}}{120a^{3}c^{\frac{11}{2}}\left(1+\sin\left(fx+e\right)\right)^{2}\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\ f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/120*(sin(f*x+e)-1)*(-30*c^(11/2)*sin(f*x+e)^2-80*c^(11/2)*sin(f*x+e)-74*
c^(11/2)+15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c
^3*(c*(1+sin(f*x+e)))^(5/2))/a^3/c^(11/2)/(1+sin(f*x+e))^2/cos(f*x+e)/(c-c*
sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+a\,\sin\left(e+f\,x\right)\right)^3\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^{3}\left(e+fx\right)+3\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^{2}\left(e+fx\right)+3\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)+\sqrt{-c\sin\left(e+fx\right)+c}}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**3 + 3*sqrt(-c*sin(e + f
*x) + c)*sin(e + f*x)**2 + 3*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + sqrt(
-c*sin(e + f*x) + c)), x)/a**3
```

3.338 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=191

$$\frac{7\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}\,a^3c^{3/2}f}-\frac{\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5a^3c^3f}-\frac{7\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{30a^3c^2f}+\frac{7\cos(e}{16a^3f(c-c\sin}$$

```
[Out] 7/16*cos(f*x+e)/a^3/f/(c-c*sin(f*x+e))^(3/2)-1/5*sec(f*x+e)^5*(c-c*sin(f*x+
e))^(3/2)/a^3/c^3/f+7/32*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*
x+e))^(1/2))/a^3/c^(3/2)/f*2^(1/2)-7/12*sec(f*x+e)/a^3/c/f/(c-c*sin(f*x+e))
^(1/2)-7/30*sec(f*x+e)^3*(c-c*sin(f*x+e))^(1/2)/a^3/c^2/f
```

Rubi [A] time = 0.33, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2736, 2675, 2687, 2650, 2649, 206}

$$-\frac{\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5a^3c^3f}-\frac{7\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{30a^3c^2f}+\frac{7\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}\,a^3c^{3/2}f}+\frac{7\cos(}{16a^3f(c-c\,s}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (7*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(16*
Sqrt[2]*a^3*c^(3/2)*f) + (7*Cos[e + f*x])/(16*a^3*f*(c - c*Sin[e + f*x])^(3
/2)) - (7*Sec[e + f*x])/(12*a^3*c*f*Sqrt[c - c*Sin[e + f*x]]) - (7*Sec[e +
f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(30*a^3*c^2*f) - (Sec[e + f*x]^5*(c - c*Si
n[e + f*x])^(3/2))/(5*a^3*c^3*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \sec^6(e+fx)(c-c\sin(e+fx))^{3/2}\,dx}{a^3c^3}$$

$$= -\frac{\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5a^3c^3f} + \frac{7\int \sec^4(e+fx)\sqrt{c-c\sin}}{10a^3c^2}$$

$$= -\frac{7\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{30a^3c^2f} - \frac{\sec^5(e+fx)(c-c\sin(e+}{5a^3c^3f}$$

$$= -\frac{7\sec(e+fx)}{12a^3cf\sqrt{c-c\sin(e+fx)}} - \frac{7\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{30a^3c^2f}$$

$$= \frac{7\cos(e+fx)}{16a^3f(c-c\sin(e+fx))^{3/2}} - \frac{7\sec(e+fx)}{12a^3cf\sqrt{c-c\sin(e+fx)}} - \frac{7\sec}{}$$

$$= \frac{7\cos(e+fx)}{16a^3f(c-c\sin(e+fx))^{3/2}} - \frac{7\sec(e+fx)}{12a^3cf\sqrt{c-c\sin(e+fx)}} - \frac{7\sec}{}$$

$$= \frac{7\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}\,a^3c^{3/2}f} + \frac{7\cos(e+fx)}{16a^3f(c-c\sin(e+fx))^{3/2}} - \frac{}{12a}$$

Mathematica [C] time = 1.19, size = 174, normalized size = 0.91

$$\frac{\left(\frac{1}{1920}+\frac{i}{1920}\right)\cos(e+fx)\left((1-i)(-231\sin(e+fx)+105\sin(3(e+fx))+350\cos(2(e+fx))+206)+840\sqrt[4]{-1}\right.}{a^3cf(\sin(e+fx)-1)(\sin(}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] ((1/1920 + I/1920)*Cos[e + f*x]*(840*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/
4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^5 + (1 - I)*(206 + 350*Cos[2*(e + f*x)] - 231
*Sin[e + f*x] + 105*Sin[3*(e + f*x)])))/(a^3*c*f*(-1 + Sin[e + f*x])*(1 + S
in[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]])
```

fricas [A] time = 0.47, size = 238, normalized size = 1.25

$$\frac{105\sqrt{2}\left(\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)+\cos\left(fx+e\right)^{3}\right)\sqrt{c}\,\log\left(-\frac{c\cos\left(fx+e\right)^{2}+2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\,\sqrt{c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)\right.}{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+2\right)\sin\left(\right.}\right.}{960\left(a^{3}c^{2}f\cos\left(fx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/960*(105*sqrt(2)*(cos(f*x + e)^3*sin(f*x + e) + cos(f*x + e)^3)*sqrt(c)*l
og(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*
x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(
f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x
 + e) - 2)) - 4*(175*cos(f*x + e)^2 + 21*(5*cos(f*x + e)^2 - 4)*sin(f*x + e
) - 36)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^2*f*cos(f*x + e)^3*sin(f*x + e) +
 a^3*c^2*f*cos(f*x + e)^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes constan
t sign by intervals (correct if the argument is real):Check [abs(sin((f*t_n
ostep+exp(1))/2-pi/4))]Unable to check sign: (8*pi/t_nostep/2)>(-8*pi/t_nos
tep/2)Discontinuities at zeroes of sin((f*t_nostep+exp(1))/2-pi/4) were not
 checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
```

```
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Warning, integration of abs or sign assumes constant sign by inte
rvals (correct if the argument is real):Check [abs(t_nostep-1)]Not invertib
le Error: Bad Argument Value
```

maple [A] time = 1.25, size = 170, normalized size = 0.89

$$-\frac{105\left(c\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{5}{2}}\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right)\sin\left(fx+e\right)c-105\left(c\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{5}{2}}\sqrt{2}\ \text{arc}}{480c^{\frac{9}{2}}a^{3}\left(1+\sin\left(fx+e\right)\right)^{2}\cos\left(f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/480/c^(9/2)/a^3*(105*(c*(1+sin(f*x+e)))^(5/2)*2^(1/2)*arctanh(1/2*(c*(1+
sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c-105*(c*(1+sin(f*x+e)))^(5/
2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c+42*c^(7/
2)*sin(f*x+e)-350*c^(7/2)*sin(f*x+e)^2-210*c^(7/2)*sin(f*x+e)^3+278*c^(7/2)
)/(1+sin(f*x+e))^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(3/2),x)
[Out] Timed out
```

3.339 $\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=228

$$\frac{63\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}\,a^3c^{5/2}f}-\frac{\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5a^3c^3f}-\frac{3\sec^3(e+fx)}{10a^3c^2f\sqrt{c-c\sin(e+fx)}}-\frac{21\sec(e+fx)}{32a^3c^2f\sqrt{c-c\sin(e}$$

```
[Out] 63/128*cos(f*x+e)/a^3/c/f/(c-c*sin(f*x+e))^(3/2)+21/80*sec(f*x+e)/a^3/c/f/(
c-c*sin(f*x+e))^(3/2)+63/256*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*si
n(f*x+e))^(1/2))/a^3/c^(5/2)/f*2^(1/2)-21/32*sec(f*x+e)/a^3/c^2/f/(c-c*sin(
f*x+e))^(1/2)-3/10*sec(f*x+e)^3/a^3/c^2/f/(c-c*sin(f*x+e))^(1/2)-1/5*sec(f*
x+e)^5*(c-c*sin(f*x+e))^(1/2)/a^3/c^3/f
```

Rubi [A] time = 0.41, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2736, 2675, 2687, 2681, 2650, 2649, 206}

$$-\frac{\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5a^3c^3f}-\frac{3\sec^3(e+fx)}{10a^3c^2f\sqrt{c-c\sin(e+fx)}}-\frac{21\sec(e+fx)}{32a^3c^2f\sqrt{c-c\sin(e+fx)}}+\frac{63\tanh^{-1}\left(\frac{\sqrt{c}\cos(}{\sqrt{2}\sqrt{c-c\sin}}\right.}{128\sqrt{2}\,a^3c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (63*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(12
8*Sqrt[2]*a^3*c^(5/2)*f) + (63*Cos[e + f*x])/(128*a^3*c*f*(c - c*Sin[e + f*
x])^(3/2)) + (21*Sec[e + f*x])/(80*a^3*c*f*(c - c*Sin[e + f*x])^(3/2)) - (2
1*Sec[e + f*x])/(32*a^3*c^2*f*Sqrt[c - c*Sin[e + f*x]]) - (3*Sec[e + f*x]^3
)/(10*a^3*c^2*f*Sqrt[c - c*Sin[e + f*x]]) - (Sec[e + f*x]^5*Sqrt[c - c*Sin[
e + f*x]])/(5*a^3*c^3*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \sec^6(e+fx)\sqrt{c-c\sin(e+fx)}\,dx}{a^3c^3}$$

$$= -\frac{\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5a^3c^3f} + \frac{9\int \frac{\sec^4(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx}{10a^3c^2}$$

$$= -\frac{3\sec^3(e+fx)}{10a^3c^2f\sqrt{c-c\sin(e+fx)}} - \frac{\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5a^3c^3f}$$

$$= \frac{21\sec(e+fx)}{80a^3cf(c-c\sin(e+fx))^{3/2}} - \frac{3\sec^3(e+fx)}{10a^3c^2f\sqrt{c-c\sin(e+fx)}} - \text{sec}$$

$$= \frac{21\sec(e+fx)}{80a^3cf(c-c\sin(e+fx))^{3/2}} - \frac{21\sec(e+fx)}{32a^3c^2f\sqrt{c-c\sin(e+fx)}} - \frac{}{10}$$

$$= \frac{63\cos(e+fx)}{128a^3cf(c-c\sin(e+fx))^{3/2}} + \frac{21\sec(e+fx)}{80a^3cf(c-c\sin(e+fx))^{3/2}} - \frac{}{3}$$

$$= \frac{63\cos(e+fx)}{128a^3cf(c-c\sin(e+fx))^{3/2}} + \frac{21\sec(e+fx)}{80a^3cf(c-c\sin(e+fx))^{3/2}} - \frac{}{3}$$

$$= \frac{63\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}\,a^3c^{5/2}f} + \frac{63\cos(e+fx)}{128a^3cf(c-c\sin(e+fx))^{3/2}} +$$

Mathematica [C] time = 1.54, size = 443, normalized size = 1.94

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-240\cos^4(e+fx)+75\left(\cos\left(\frac{1}{2}(e+fx\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-240*Cos[e + f*x]^4 - 32*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 80*(C
os[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
^2 + 20*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e +
f*x)/2])^5 + 75*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] +
```

```
 Sin[(e + f*x)/2])^5 - (315 + 315*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/
4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^5 + 40*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^5 + 150*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f
*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5))/(640*a^3*f*(1 + Sin[e + f*
x])^3*(c - c*Sin[e + f*x])^(5/2))
```

fricas [A] time = 0.47, size = 208, normalized size = 0.91

$$\frac{315\sqrt{2}\sqrt{c}\cos\left(fx+e\right)^{5}\log\left(-\frac{c\cos\left(fx+e\right)^{2}+2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\sqrt{c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)+3c\cos\left(fx+e\right)+\left(c\cos\left(fx+e\right)-2c\right)\mathrm{si}}{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right.}{2560\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/2560*(315*sqrt(2)*sqrt(c)*cos(f*x + e)^5*log(-(c*cos(f*x + e)^2 + 2*sqrt(
2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*
c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2
 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(315*cos(f*x +
e)^4 - 42*cos(f*x + e)^2 - 6*(35*cos(f*x + e)^2 + 24)*sin(f*x + e) - 16)*sq
rt(-c*sin(f*x + e) + c))/(a^3*c^3*f*cos(f*x + e)^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
```

```
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of a
bs or sign assumes constant sign by intervals (correct if the argument is r
eal):Check [abs(sin((f*t_nostep+exp(1))/2-pi/4))]Unable to check sign: (8*p
i/t_nostep/2)>(-8*pi/t_nostep/2)Discontinuities at zeroes of sin((f*t_noste
p+exp(1))/2-pi/4) were not checkedUnable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable t
o check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign ass
umes constant sign by intervals (correct if the argument is real):Check [ab
s(t_nostep-1)]Evaluation time: 0.44Not invertible Error: Bad Argument Value
```

maple [A] time = 1.30, size = 246, normalized size = 1.08

$$-\frac{315\left(c\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{5}{2}}\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right)\left(\sin^{2}\left(fx+e\right)\right)c^{2}+1176c^{\frac{9}{2}}\left(\sin^{2}\left(fx+e\right)\right)-420c^{\frac{9}{2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/1280/c^(13/2)/a^3*(315*(c*(1+sin(f*x+e)))^(5/2)*2^(1/2)*arctanh(1/2*(c*(
1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^2+1176*c^(9/2)*sin(f*x
+e)^2-420*c^(9/2)*sin(f*x+e)^3-630*c^(9/2)*sin(f*x+e)^4-630*(c*(1+sin(f*x+e
)))^(5/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin
(f*x+e)*c^2+708*c^(9/2)*sin(f*x+e)+315*(c*(1+sin(f*x+e)))^(5/2)*2^(1/2)*arc
tanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^2-514*c^(9/2))/(1+sin(
f*x+e))^2/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+a\,\sin\left(e+f\,x\right)\right)^3\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2)),x)
[Out] int(1/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(5/2),x)
[Out] Timed out
```

3.340 $\int \sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=43

$$-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -1/4*a*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2738}

$$-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] -(a*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(4*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}\,dx = -\frac{a\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 0.40, size = 83, normalized size = 1.93

$$-\frac{c^3\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\,\sqrt{c-c\sin(e+fx)}\,(8(\sin(3(e+fx))-7\sin(e+fx))-28\cos(2(e+fx))+\mathrm{co}}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] -1/32*(c^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]
*(-28*Cos[2*(e + f*x)] + Cos[4*(e + f*x)] + 8*(-7*Sin[e + f*x] + Sin[3*(e +
 f*x)])))/f
```

fricas [B] time = 0.45, size = 95, normalized size = 2.21

$$-\frac{\left(c^3\cos\left(fx+e\right)^4-8\,c^3\cos\left(fx+e\right)^2+7\,c^3+4\left(c^3\cos\left(fx+e\right)^2-2\,c^3\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-}}{4\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] -1/4*(c^3*cos(f*x + e)^4 - 8*c^3*cos(f*x + e)^2 + 7*c^3 + 4*(c^3*cos(f*x +
e)^2 - 2*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
 c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(
2*c)*(-56*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1)
)-1/4*pi))*sin(f*x+exp(1))/(8*f)^2+72*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*p
i))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(3*f*x+3*exp(1))/(24*f)^2-480*c^3
*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*co
s(2*f*x+2*exp(1))/(32*f)^2+64*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign
(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(4*f*x+4*exp(1))/(64*f)^2+32*c^3*f*sign(s
in(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-2*f*x-
2*exp(1))/(-32*f)^2)
```

maple [B] time = 0.44, size = 103, normalized size = 2.40

$$\frac{\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{7}{2}}\sin\left(fx+e\right)\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,\left(\cos^{6}\left(fx+e\right)+\sin\left(fx+e\right)\left(\cos^{4}\left(fx+e\right)\right)+\left(\cos^{2}\right.\right.}{4f\cos\left(fx+e\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/4/f*(-c*(sin(f*x+e)-1))^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)*(cos(f*
x+e)^6+sin(f*x+e)*cos(f*x+e)^4+cos(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^2+4*sin(f
*x+e)+4)/cos(f*x+e)^7
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(7/2), x)
```

mupad [B] time = 8.25, size = 99, normalized size = 2.30

$$\frac{c^{3}\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\,\left(28\cos\left(e+fx\right)+27\cos\left(3e+3fx\right)-\cos\left(5e+5fx\right)+4\right.}{32f\left(\cos\left(2e+2fx\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] (c^3*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(28*cos(e +
 f*x) + 27*cos(3*e + 3*f*x) - cos(5*e + 5*f*x) + 48*sin(2*e + 2*f*x) - 8*si
n(4*e + 4*f*x)))/(32*f*(cos(2*e + 2*f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

3.341 $\int \sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=43

$$-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -1/3*a*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2738}

$$-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] -(a*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}\,dx=-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 0.30, size = 74, normalized size = 1.72

$$\frac{c^2\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\,\sqrt{c-c\sin(e+fx)}\,(15\sin(e+fx)-\sin(3(e+fx))+6\cos(2(e+fx)))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2),x]
```

[Out] (c^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(6*Co
s[2*(e + f*x)] + 15*Sin[e + f*x] - Sin[3*(e + f*x)]))/(12*f)

fricas [B] time = 0.44, size = 83, normalized size = 1.93

$$\frac{\left(3\,c^{2}\cos\left(fx+e\right)^{2}-3\,c^{2}-\left(c^{2}\cos\left(fx+e\right)^{2}-4\,c^{2}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{3\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="frica
s")

[Out] 1/3*(3*c^2*cos(f*x + e)^2 - 3*c^2 - (c^2*cos(f*x + e)^2 - 4*c^2)*sin(f*x +
e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac"
)

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-40*c^2*f*sign(sin(1
/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(f*x+exp(1))
/(8*f)^2+24*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(
1))-1/4*pi))*sin(3*f*x+3*exp(1))/(24*f)^2-4*c^2*f*sign(sin(1/2*(f*x+exp(1))
-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(2*f*x+2*exp(1))/(4*f)^2)

maple [B] time = 0.34, size = 78, normalized size = 1.81

$$\frac{\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{5}{2}}\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\;\sin\left(fx+e\right)\left(\cos^{4}\left(fx+e\right)+\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)+2\sin\right.}{3f\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(5/2),x)

```
[Out] 1/3/f*(-c*(sin(f*x+e)-1))^(5/2)*(a*(1+sin(f*x+e)))^(1/2)*sin(f*x+e)*(cos(f*
x+e)^4+cos(f*x+e)^2*sin(f*x+e)+2*sin(f*x+e)+2)/cos(f*x+e)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sin\left(fx+e\right)+a}\left(-c \sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(5/2), x)
```

mupad [B] time = 7.71, size = 88, normalized size = 2.05

$$\frac{c^2\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(6\,\cos\left(e+f\,x\right)+6\,\cos\left(3\,e+3\,f\,x\right)+14\,\sin\left(2\,e+2\,f\,x\right)-\right.}{12\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] (c^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(6*cos(e +
f*x) + 6*cos(3*e + 3*f*x) + 14*sin(2*e + 2*f*x) - sin(4*e + 4*f*x)))/(12*f*
(cos(2*e + 2*f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.342 $\int \sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=43

$$-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

[Out] -1/2*a*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(1/2)

Rubi [A] time = 0.08, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2738}

$$-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2),x]

[Out] -(a*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*f*Sqrt[a + a*Sin[e + f*x]])

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}\,dx=-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 0.22, size = 60, normalized size = 1.40

$$\frac{c\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\,\sqrt{c-c\sin(e+fx)}\,(4\sin(e+fx)+\cos(2(e+fx)))}{4f}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2),x]

```
[Out] (c*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(Cos[2*(e + f*x)] + 4*Sin[e + f*
x])*Sqrt[c - c*Sin[e + f*x]])/(4*f)
```

fricas [A] time = 0.44, size = 61, normalized size = 1.42

$$\frac{\left(c\cos\left(fx+e\right)^2+2\,c\sin\left(fx+e\right)-c\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{2\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(c*cos(f*x + e)^2 + 2*c*sin(f*x + e) - c)*sqrt(a*sin(f*x + e) + a)*sqrt
(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-2*c*f*sign(sin(1/2*(f*x+exp(1))-
1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(f*x+exp(1))/(2*f)^2-8*c*f*s
ign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(2*
f*x+2*exp(1))/(8*f)^2)
```

maple [A] time = 0.31, size = 61, normalized size = 1.42

$$\frac{\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{3}{2}}\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,\sin\left(fx+e\right)\left(\cos^2\left(fx+e\right)+\sin\left(fx+e\right)+1\right)}{2f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/2/f*(-c*(sin(f*x+e)-1))^(3/2)*(a*(1+sin(f*x+e)))^(1/2)*sin(f*x+e)*(cos(f*
x+e)^2+sin(f*x+e)+1)/cos(f*x+e)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(3/2), x)
```

mupad [B] time = 0.84, size = 71, normalized size = 1.65

$$\frac{c\sqrt{a\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\left(\sin\left(e+f\,x\right)-1\right)}\,\left(\cos\left(e+f\,x\right)+\cos\left(3\,e+3\,f\,x\right)+4\,\sin\left(2\,e+2\,f\,x\right)\right)}{4\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] (c*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(cos(e + f*x)
 + cos(3*e + 3*f*x) + 4*sin(2*e + 2*f*x)))/(4*f*(cos(2*e + 2*f*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(-c*(sin(e + f*x) - 1))**(3/2), x)
```

3.343 $\int \sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=41

$$-\frac{a\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -a*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2738}

$$-\frac{a\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] -((a*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]))
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}\,dx=-\frac{a\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 0.09, size = 39, normalized size = 0.95

$$\frac{\tan(e+fx)\sqrt{a(\sin(e+fx)+1)}\,\sqrt{c-c\sin(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*Tan[e + f*x])/f
```

fricas [A] time = 0.43, size = 43, normalized size = 1.05

$$\frac{\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin\left(fx+e\right)}{f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e)/(f*cos(f*x
+ e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)sqrt(2*a)*sqrt(2*c)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2
*(f*x+exp(1))-1/4*pi))*sin(1/2*(f*x+exp(1))-1/4*pi)^2/f
```

maple [A] time = 0.31, size = 44, normalized size = 1.07

$$\frac{\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\,\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,\sin\left(fx+e\right)}{f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/f*(-c*(sin(f*x+e)-1))^(1/2)*(a*(1+sin(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c), x)
```

mupad [B] time = 7.01, size = 47, normalized size = 1.15

$$\frac{\sin\left(2\,e+2\,f\,x\right)\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}}{2\,f\,\cos\left(e+f\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] (sin(2*e + 2*f*x)*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2
))/(2*f*cos(e + f*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*sqrt(-c*(sin(e + f*x) - 1)), x)
```

3.344 $\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=52

$$-\frac{a\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -a*cos(f*x+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1
/2)
```

Rubi [A] time = 0.10, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2737, 2667, 31}

$$-\frac{a\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] -((a*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{(ac\cos(e+fx))\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$
$$= -\frac{(a\cos(e+fx))\,\text{Subst}\left(\int \frac{1}{c+x}\,dx, x, -c\sin(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$
$$= -\frac{a\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

Mathematica [C] time = 0.93, size = 119, normalized size = 2.29

$$-\frac{\sqrt{2}\left(e^{i(e+fx)}-i\right)\left(fx+2i\log\left(i-e^{i(e+fx)}\right)\right)\sqrt{a(\sin(e+fx)+1)}}{f\left(e^{i(e+fx)}+i\right)\sqrt{ice^{-i(e+fx)}\left(e^{i(e+fx)}-i\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[2]*(-I + E^(I*(e + f*x)))*(f*x + (2*I)*Log[I - E^(I*(e + f*x))])*Sq
rt[a*(1 + Sin[e + f*x])])/(Sqrt[(I*c*(-I + E^(I*(e + f*x)))^2)/E^(I*(e + f*
x))]*(I + E^(I*(e + f*x)))*f))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{c\sin\left(fx+e\right)-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*sin(f*x + e
) - c), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8
*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2)*(1/2*ln(tan(1/2*(1/2*f*x+1/4*(2*exp(
1)-pi)))^2)/sign(tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^3+tan(1/2*(1/2*f*x+1/
4*(2*exp(1)-pi))))-ln(tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^2+1)/sign(tan(1/
2*(1/2*f*x+1/4*(2*exp(1)-pi)))^3+tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))))*sig
n(cos(1/2*(f*x+exp(1))-1/4*pi))/sqrt(c)/f
```

maple [B] time = 0.26, size = 106, normalized size = 2.04

$$-\frac{\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,\left(-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)\left(2\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)\right)}{f\left(1-\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(a*(1+sin(f*x+e)))^(1/2)*(-1+cos(f*x+e)+sin(f*x+e))*(2*ln(-(-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))-ln(2/(cos(f*x+e)+1)))/(1-cos(f*x+e)+sin(f*x+e))
/(-c*(sin(f*x+e)-1))^(1/2)
```

maxima [A] time = 0.92, size = 63, normalized size = 1.21

$$\frac{\frac{2\,\sqrt{a}\,\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{\sqrt{c}}-\frac{\sqrt{a}\,\log\left(\frac{\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+1\right)}{\sqrt{c}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] (2*sqrt(a)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/sqrt(c) - sqrt(a)*log(s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(c))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+a\,\sin\left(e+f\,x\right)}}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)/(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(1/2)/(c - c*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))/sqrt(-c*(sin(e + f*x) - 1)), x)
```

3.345 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=40

$$\frac{a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}$$

```
[Out] a*cos(f*x+e)/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2738}

$$\frac{a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (a*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2))
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}$$

Mathematica [B] time = 0.21, size = 84, normalized size = 2.10

$$\frac{\sqrt{a(\sin(e+fx)+1)}\,\sqrt{c-c\sin(e+fx)}}{c^2 f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])/(c^2*f*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

fricas [A] time = 0.44, size = 59, normalized size = 1.48

$$-\frac{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{c^2f\cos\left(fx+e\right)\sin\left(fx+e\right)-c^2f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] -sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*f*cos(f*x + e)*sin
(f*x + e) - c^2*f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8
*pi/x/2)-sqrt(2*a)/8*(tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^2+1/tan(1/2*(1/2
*f*x+1/4*(2*exp(1)-pi)))^2)*sign(cos(1/2*(f*x+exp(1))-1/4*pi))/sqrt(2)/sqrt
(c)/c/f/sign(tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^3+tan(1/2*(1/2*f*x+1/4*(2
*exp(1)-pi))))
```

maple [A] time = 0.27, size = 68, normalized size = 1.70

$$\frac{\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,\sin\left(fx+e\right)\left(-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)}{f\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{3}{2}}\left(1-\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/f*(a*(1+sin(f*x+e)))^(1/2)*sin(f*x+e)*(-1+cos(f*x+e)+sin(f*x+e))/(-c*(sin
(f*x+e)-1))^(3/2)/(1-cos(f*x+e)+sin(f*x+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\sin\left(fx+e\right)+a}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+a\,\sin\left(e+f\,x\right)}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)/(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(1/2)/(c - c*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}{\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))/(-c*(sin(e + f*x) - 1))**(3/2), x)
```

3.346 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=43

$$\frac{a\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 1/2*a*cos(f*x+e)/f/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2738}

$$\frac{a\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a*Cos[e + f*x])/(2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2))
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{a\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}$$

Mathematica [B] time = 0.22, size = 87, normalized size = 2.02

$$\frac{\sqrt{a(\sin(e+fx)+1)}\,\sqrt{c-c\sin(e+fx)}}{2c^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])/(2*c^3*f*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

fricas [A] time = 0.44, size = 73, normalized size = 1.70

$$-\frac{\sqrt{a \sin\left(fx+e\right)+a}\,\sqrt{-c \sin\left(fx+e\right)+c}}{2\left(c^{3} f \cos\left(fx+e\right)^{3}+2\,c^{3} f \cos\left(fx+e\right) \sin\left(fx+e\right)-2\,c^{3} f \cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] -1/2*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^3*f*cos(f*x + e)
^3 + 2*c^3*f*cos(f*x + e)*sin(f*x + e) - 2*c^3*f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [B] time = 0.28, size = 96, normalized size = 2.23

$$-\frac{\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\ \sin\left(fx+e\right)\left(\sin\left(fx+e\right)\cos\left(fx+e\right)-\left(\cos^{2}\left(fx+e\right)\right)-3\sin\left(fx+e\right)-2\cos\left(fx+e\right.\right.}{2f\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{5}{2}}\left(1-\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/2/f*(a*(1+sin(f*x+e)))^(1/2)*sin(f*x+e)*(sin(f*x+e)*cos(f*x+e)-cos(f*x+e
)^2-3*sin(f*x+e)-2*cos(f*x+e)+3)/(-c*(sin(f*x+e)-1))^(5/2)/(1-cos(f*x+e)+si
n(f*x+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a \sin\left(fx+e\right)+a}}{\left(-c \sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(5/2), x)
```

mupad [B] time = 8.89, size = 142, normalized size = 3.30

$$\frac{4\sqrt{a\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\left(\sin\left(e+f\,x\right)-1\right)}\,\left(10\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-2\sin\left(\frac{3\,e}{2}+\frac{3\,f\,x}{2}\right)^2+4\,\sin\left(2\,e+\right.\right.}{c^3\,f\,\left(30\sin\left(e+f\,x\right)^2+48\,\sin\left(e+f\,x\right)-52\sin\left(2\,e+2\,f\,x\right)^2+2\sin\left(3\,e+3\,f\,x\right)^2+40\,\sin\left(3\,e+3\,f\,x\right)-8\,\mathrm{s}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)/(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] (4*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(4*sin(2*e +
2*f*x) + 10*sin(e/2 + (f*x)/2)^2 - 2*sin((3*e)/2 + (3*f*x)/2)^2 - 4))/(c^3*
f*(48*sin(e + f*x) + 40*sin(3*e + 3*f*x) - 8*sin(5*e + 5*f*x) - 52*sin(2*e
+ 2*f*x)^2 + 2*sin(3*e + 3*f*x)^2 + 30*sin(e + f*x)^2 - 32))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}{\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))/(-c*(sin(e + f*x) - 1))**(5/2), x)
```

3.347 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=43

$$\frac{a\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}$$

```
[Out] 1/3*a*cos(f*x+e)/f/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2738}

$$\frac{a\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2))
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{7/2}}\,dx=\frac{a\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}}$$

Mathematica [B] time = 0.28, size = 87, normalized size = 2.02

$$\frac{\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}}{3c^4f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])/(3*c^4*f*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

fricas [B] time = 0.43, size = 90, normalized size = 2.09

$$-\frac{\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{3\left(3\,c^4f\cos\left(fx+e\right)^3-4\,c^4f\cos\left(fx+e\right)-\left(c^4f\cos\left(fx+e\right)^3-4\,c^4f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] -1/3*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^4*f*cos(f*x +
e)^3 - 4*c^4*f*cos(f*x + e) - (c^4*f*cos(f*x + e)^3 - 4*c^4*f*cos(f*x + e))
*sin(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [B] time = 0.30, size = 120, normalized size = 2.79

$$-\frac{\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\ \sin\left(fx+e\right)\left(\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)+\cos^3\left(fx+e\right)+3\sin\left(fx+e\right)\cos\left(fx+e\right)}{3f\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{7}{2}}\left(1-\cos\left(fx+e\right)+\sin\left(fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] -1/3/f*(a*(1+sin(f*x+e)))^(1/2)*sin(f*x+e)*(cos(f*x+e)^2*sin(f*x+e)+cos(f*x
+e)^3+3*sin(f*x+e)*cos(f*x+e)-4*cos(f*x+e)^2-7*sin(f*x+e)-4*cos(f*x+e)+7)/(
-c*(sin(f*x+e)-1))^(7/2)/(1-cos(f*x+e)+sin(f*x+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\sin\left(fx+e\right)+a}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(7/2), x)
```

mupad [B] time = 10.98, size = 190, normalized size = 4.42

$$-\frac{e^{e\,4i+f\,x\,4i}\,\sqrt{a+a\,\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\,\sqrt{c-c\,\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\,16i}{3\,c^4\,f\,\left(1+14\,e^{e\,6i+f\,x\,6i}-e^{e\,8i+f\,x\,8i}-14\,e^{e\,2i+f\,x\,2i}+e^{e\,1i+f\,x\,1i}\,6i-e^{e\,3i+f\,x\,3i}\,14i-e^{e\,5i+f\,x\,5i}\,14i+e^{e\,7i+f\,x\,7i}\,6i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)/(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] -(exp(e*4i + f*x*4i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*
1i)*1i)/2))^(1/2)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)
*1i)/2))^(1/2)*16i)/(3*c^4*f*(exp(e*1i + f*x*1i)*6i - 14*exp(e*2i + f*x*2i)
 - exp(e*3i + f*x*3i)*14i - exp(e*5i + f*x*5i)*14i + 14*exp(e*6i + f*x*6i)
+ exp(e*7i + f*x*7i)*6i - exp(e*8i + f*x*8i) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

3.348 $\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=89

$$-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{10f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}{5f}$$

```
[Out] -1/10*a^2*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(1/2)-1/5*a*
cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.19, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2740, 2738}

$$-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{10f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] -(a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(10*f*Sqrt[a + a*Sin[e + f*x
]]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2))/
(5*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}\,dx = -\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}}{5f} + \frac{1}{5}(2a$$

$$= -\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{10f\sqrt{a+a\sin(e+fx)}} - \frac{a\cos(e+fx)\sqrt{a+a\sin}}{}$$

Mathematica [**A**] time = 1.04, size = 146, normalized size = 1.64

$$-\frac{c^3(\sin(e+fx)-1)^3(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin(e+fx)}\,(70\sin(e+fx)+5\sin(3(e+fx))-\sin(5(e+fx))}{80f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] -1/80*(c^3*(-1 + Sin[e + f*x])^3*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Si
n[e + f*x]]*(20*Cos[2*(e + f*x)] + 5*Cos[4*(e + f*x)] + 70*Sin[e + f*x] + 5
*Sin[3*(e + f*x)] - Sin[5*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)
/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

fricas [**A**] time = 0.45, size = 101, normalized size = 1.13

$$\frac{\left(5\,ac^3\cos\left(fx+e\right)^4-5\,ac^3-2\left(ac^3\cos\left(fx+e\right)^4-2\,ac^3\cos\left(fx+e\right)^2-4\,ac^3\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}}{10\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/10*(5*a*c^3*cos(f*x + e)^4 - 5*a*c^3 - 2*(a*c^3*cos(f*x + e)^4 - 2*a*c^3*
cos(f*x + e)^2 - 4*a*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*si
n(f*x + e) + c)/(f*cos(f*x + e))
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sq
rt(2*a)*sqrt(2*c)*(-112*a*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos
(1/2*(f*x+exp(1))-1/4*pi))*sin(f*x+exp(1))/(16*f)^2-288*a*c^3*f*sign(sin(1/
2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(3*f*x+3*exp(
1))/(96*f)^2+160*a*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f
*x+exp(1))-1/4*pi))*sin(5*f*x+5*exp(1))/(160*f)^2-16*a*c^3*f*sign(sin(1/2*(
f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(2*f*x+2*exp(1))
/(16*f)^2-32*a*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+e
xp(1))-1/4*pi))*cos(4*f*x+4*exp(1))/(32*f)^2-16*a*c^3*f*sign(sin(1/2*(f*x+e
xp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-2*f*x-2*exp(1))/(-1
6*f)^2)
```

maple [A] time = 0.33, size = 106, normalized size = 1.19

$$\frac{\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{7}{2}}\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)\left(2\left(\cos^{6}\left(fx+e\right)\right)+\sin\left(fx+e\right)\left(\cos^{4}\left(fx+e\right)\right)+}{10f\cos\left(fx+e\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/10/f*(-c*(sin(f*x+e)-1))^(7/2)*(a*(1+sin(f*x+e)))^(3/2)*sin(f*x+e)*(2*cos
(f*x+e)^6+sin(f*x+e)*cos(f*x+e)^4+2*cos(f*x+e)^4+3*cos(f*x+e)^2*sin(f*x+e)+
6*sin(f*x+e)+6)/cos(f*x+e)^7
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(7/2), x)
```

mupad [B] time = 8.92, size = 111, normalized size = 1.25

$$\frac{a\,c^3\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(20\,\cos\left(e+f\,x\right)+25\,\cos\left(3\,e+3\,f\,x\right)+5\,\cos\left(5\,e+5\,f\,x\right)\right.}{80\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] (a*c^3*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(20*cos(e
 + f*x) + 25*cos(3*e + 3*f*x) + 5*cos(5*e + 5*f*x) + 75*sin(2*e + 2*f*x) +
4*sin(4*e + 4*f*x) - sin(6*e + 6*f*x)))/(80*f*(cos(2*e + 2*f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

3.349 $\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=89

$$-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}{4f}$$

```
[Out] -1/6*a^2*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(1/2)-1/4*a*c
os(f*x+e)*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.18, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2740, 2738}

$$-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] -(a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(6*f*Sqrt[a + a*Sin[e + f*x]
]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2))/(
4*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}\,dx = -\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}{4f} + \frac{1}{2}a\int$$
$$= -\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6f\sqrt{a+a\sin(e+fx)}} - \frac{a\cos(e+fx)\sqrt{a+a\sin}}{}$$

Mathematica [A] time = 0.65, size = 137, normalized size = 1.54

$$\frac{c^2(\sin(e+fx)-1)^2(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin(e+fx)}\,(8(9\sin(e+fx)+\sin(3(e+fx)))+12\cos(2(e+fx}{96f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (c^2*(-1 + Sin[e + f*x])^2*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e +
f*x]]*(12*Cos[2*(e + f*x)] + 3*Cos[4*(e + f*x)] + 8*(9*Sin[e + f*x] + Sin[3
*(e + f*x)])))/(96*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^3)
```

fricas [A] time = 0.45, size = 87, normalized size = 0.98

$$\frac{\left(3\,ac^2\cos\left(fx+e\right)^4-3\,ac^2+4\left(ac^2\cos\left(fx+e\right)^2+2\,ac^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{12\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/12*(3*a*c^2*cos(f*x + e)^4 - 3*a*c^2 + 4*(a*c^2*cos(f*x + e)^2 + 2*a*c^2)
*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*
x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(
2*c)*(-24*a*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(
1))-1/4*pi))*sin(f*x+exp(1))/(8*f)^2-24*a*c^2*f*sign(sin(1/2*(f*x+exp(1))-1
/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(3*f*x+3*exp(1))/(24*f)^2-32*
a*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi
))*cos(2*f*x+2*exp(1))/(32*f)^2-64*a*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi
))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(4*f*x+4*exp(1))/(64*f)^2-32*a*c^2
*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*co
s(-2*f*x-2*exp(1))/(-32*f)^2)
```

maple [A] time = 0.29, size = 90, normalized size = 1.01

$$\frac{\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\left(3\left(\cos^{4}\left(fx+e\right)\right)+\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)+\right.}{12f\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/12/f*(-c*(sin(f*x+e)-1))^(5/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)*(3*cos
(f*x+e)^4+cos(f*x+e)^2*sin(f*x+e)+4*cos(f*x+e)^2+5*sin(f*x+e)+5)/cos(f*x+e)
^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(5/2), x)
```

mupad [B] time = 1.75, size = 100, normalized size = 1.12

$$\frac{a\,c^2\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(12\,\cos\left(e+f\,x\right)+15\,\cos\left(3\,e+3\,f\,x\right)+3\,\cos\left(5\,e+5\,f\,x\right.\right.}{96\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] (a*c^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(12*cos(e
 + f*x) + 15*cos(3*e + 3*f*x) + 3*cos(5*e + 5*f*x) + 80*sin(2*e + 2*f*x) +
8*sin(4*e + 4*f*x)))/(96*f*(cos(2*e + 2*f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.350 $\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=89

$$-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}{3f}$$

```
[Out] -1/3*a^2*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(1/2)-1/3*a*c
os(f*x+e)*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.18, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2740, 2738}

$$-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(3*f*Sqrt[a + a*Sin[e + f*x]
]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2))/(
3*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}\,dx = -\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}{3f}+\frac{1}{3}(2a$$
$$= -\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3f\sqrt{a+a\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a+a\sin}}{}$$

Mathematica [A] time = 0.43, size = 70, normalized size = 0.79

$$-\frac{c(\sin(e+fx)-1)(9\sin(e+fx)+\sin(3(e+fx)))\sec^3(e+fx)(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin(e+fx)}}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/12*(c*Sec[e + f*x]^3*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(3/2)*Sq
rt[c - c*Sin[e + f*x]]*(9*Sin[e + f*x] + Sin[3*(e + f*x)]))/f
```

fricas [A] time = 0.44, size = 60, normalized size = 0.67

$$\frac{\left(ac\cos\left(fx+e\right)^2+2\,ac\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/3*(a*c*cos(f*x + e)^2 + 2*a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
 e) + c)*sin(f*x + e)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
```

```
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-24*a*c*f*sign(sin(1/2*(f*x+exp(1
))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(f*x+exp(1))/(8*f)^2-24*a
*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*
sin(3*f*x+3*exp(1))/(24*f)^2)
```

maple [A] time = 0.26, size = 55, normalized size = 0.62

$$\frac{\left(2+\cos^2\left(fx+e\right)\right)\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}}{3f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/3/f*(2+cos(f*x+e)^2)*(-c*(sin(f*x+e)-1))^(3/2)*sin(f*x+e)*(a*(1+sin(f*x+e
)))^(3/2)/cos(f*x+e)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(3/2), x)
```

mupad [B] time = 0.89, size = 66, normalized size = 0.74

$$\frac{a\,c\,\left(10\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(4\,e+4\,f\,x\right)\right)\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}}{12\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] (a*c*(10*sin(2*e + 2*f*x) + sin(4*e + 4*f*x))*(a*(sin(e + f*x) + 1))^(1/2)*
(-c*(sin(e + f*x) - 1))^(1/2))/(12*f*(cos(2*e + 2*f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.351 $\int (a + a\sin(e + fx))^{3/2}\sqrt{c - c\sin(e + fx)}\, dx$

Optimal. Leaf size=43

$$\frac{c\cos(e + fx)(a\sin(e + fx) + a)^{3/2}}{2f\sqrt{c - c\sin(e + fx)}}$$

```
[Out] 1/2*c*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2738}

$$\frac{c\cos(e + fx)(a\sin(e + fx) + a)^{3/2}}{2f\sqrt{c - c\sin(e + fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a + a\sin(e + fx))^{3/2}\sqrt{c - c\sin(e + fx)}\, dx = \frac{c\cos(e + fx)(a + a\sin(e + fx))^{3/2}}{2f\sqrt{c - c\sin(e + fx)}}$$

Mathematica [A] time = 0.21, size = 60, normalized size = 1.40

$$-\frac{a\sec(e + fx)\sqrt{a(\sin(e + fx) + 1)}\sqrt{c - c\sin(e + fx)}\,(\cos(2(e + fx)) - 4\sin(e + fx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] -1/4*(a*Sec[e + f*x]*(Cos[2*(e + f*x)] - 4*Sin[e + f*x])*Sqrt[a*(1 + Sin[e
+ f*x])]*Sqrt[c - c*Sin[e + f*x]])/f
```

fricas [A] time = 0.42, size = 61, normalized size = 1.42

$$-\frac{\left(a\cos\left(fx+e\right)^{2}-2\,a\sin\left(fx+e\right)-a\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{2\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] -1/2*(a*cos(f*x + e)^2 - 2*a*sin(f*x + e) - a)*sqrt(a*sin(f*x + e) + a)*sqr
t(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-2*a*f*sign(sin(1/2*(f*x+exp(1))-
1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(f*x+exp(1))/(2*f)^2+8*a*f*s
ign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(2*
f*x+2*exp(1))/(8*f)^2)
```

maple [A] time = 0.30, size = 63, normalized size = 1.47

$$-\frac{\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\;\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)\left(-1-\left(\cos^{2}\left(fx+e\right)\right)+\sin\left(fx+e\right)\right)}{2f\cos\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*(-c*(sin(f*x+e)-1))^(1/2)*(a*(1+sin(f*x+e)))^(3/2)*sin(f*x+e)*(-1-co
s(f*x+e)^2+sin(f*x+e))/cos(f*x+e)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x + e) + c), x)
```

mupad [B] time = 7.35, size = 71, normalized size = 1.65

$$-\frac{a\sqrt{a\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\left(\sin\left(e+f\,x\right)-1\right)}\,\left(\cos\left(e+f\,x\right)+\cos\left(3\,e+3\,f\,x\right)-4\,\sin\left(2\,e+2\,f\,x\right)\right)}{4\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] -(a*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(cos(e + f*x
) + cos(3*e + 3*f*x) - 4*sin(2*e + 2*f*x)))/(4*f*(cos(2*e + 2*f*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*sqrt(-c*(sin(e + f*x) - 1)), x)
```

3.352 $\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=96

$$-\frac{2a^2\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -2*a^2*cos(f*x+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e)
)^(1/2)-a*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2740, 2737, 2667, 31}

$$-\frac{2a^2\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (-2*a^2*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c - c*Sin[e + f*x]]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(f*Sqrt[
c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx &= -\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}}+(2a)\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx\\
&= -\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}}+\frac{\left(2a^2c\cos(e+fx)\right)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}\\
&= -\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}}-\frac{\left(2a^2\cos(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{c+x}\,dx,x,-c\sin(\ldots\right)}{f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+f\ldots}}\\
&= -\frac{2a^2\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}}
\end{aligned}
$$

Mathematica [A] time = 0.34, size = 113, normalized size = 1.18

$$
-\frac{(a(\sin(e+fx)+1))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin(e+fx)+4\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+\ldots\right.\right.\right.}{f\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] -(((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2)*(4*Lo
g[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Sin[e + f*x]))/(f*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x]]))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}{c\sin\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x + e) + c)/(c*sin(f*x +
 e) - c), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (8*pi/x/2)>(-8*pi/x/2)4*sqrt(2*a)*(a*sqrt(c*tan(1/2*exp(1)
)^2+c)*(-25165824*tan(1/2*exp(1))^5-75497472*tan(1/2*exp(1))^4+83886080*tan
(1/2*exp(1))^3+50331648*tan(1/2*exp(1))^2-25165824*tan(1/2*exp(1))-8388608)
+a*sqrt(c*tan(1/2*exp(1))^2+c)*(25165824*tan(1/2*exp(1))^5+75497472*tan(1/2
*exp(1))^4-83886080*tan(1/2*exp(1))^3-50331648*tan(1/2*exp(1))^2+25165824*t
an(1/2*exp(1))+8388608)*tan(1/4*exp(1))^6+a*sqrt(c*tan(1/2*exp(1))^2+c)*(16
7772160*tan(1/2*exp(1))^6+503316480*tan(1/2*exp(1))^5-1006632960*tan(1/2*ex
p(1))^4-1677721600*tan(1/2*exp(1))^3+1509949440*tan(1/2*exp(1))^2+503316480
*tan(1/2*exp(1)))*tan(1/4*exp(1))^3+a*sqrt(c*tan(1/2*exp(1))^2+c)*(37748736
0*tan(1/2*exp(1))^5+1132462080*tan(1/2*exp(1))^4-1258291200*tan(1/2*exp(1))
^3-754974720*tan(1/2*exp(1))^2+377487360*tan(1/2*exp(1))+125829120)*tan(1/4
*exp(1))^2+a*sqrt(c*tan(1/2*exp(1))^2+c)*(-50331648*tan(1/2*exp(1))^6-15099
4944*tan(1/2*exp(1))^5+301989888*tan(1/2*exp(1))^4+503316480*tan(1/2*exp(1)
)^3-452984832*tan(1/2*exp(1))^2-150994944*tan(1/2*exp(1)))*tan(1/4*exp(1))^
5+a*sqrt(c*tan(1/2*exp(1))^2+c)*(-50331648*tan(1/2*exp(1))^6-150994944*tan(
1/2*exp(1))^5+301989888*tan(1/2*exp(1))^4+503316480*tan(1/2*exp(1))^3-45298
4832*tan(1/2*exp(1))^2-150994944*tan(1/2*exp(1)))*tan(1/4*exp(1))+a*sqrt(c*
tan(1/2*exp(1))^2+c)*(-377487360*tan(1/2*exp(1))^5-1132462080*tan(1/2*exp(1
))^4+1258291200*tan(1/2*exp(1))^3+754974720*tan(1/2*exp(1))^2-377487360*tan
(1/2*exp(1))-125829120)*tan(1/4*exp(1))^4)*ln(abs(-2*tan(1/2*exp(1))^3+6*ta
n(1/2*exp(1))^2+(tan(1/2*(1/2*f*x+2*exp(1)))-1/tan(1/2*(1/2*f*x+2*exp(1)))
```

```
*(tan(1/2*exp(1))^3+3*tan(1/2*exp(1))^2-3*tan(1/2*exp(1))-1)+6*tan(1/2*exp(
1))-2))/f/(-8388608*sqrt(2)*c*tan(1/2*exp(1))^7+8388608*sqrt(2)*c+(-8388608
*sqrt(2)*c*tan(1/2*exp(1))^7-25165824*sqrt(2)*c*tan(1/2*exp(1))^6+8388608*s
qrt(2)*c*tan(1/2*exp(1))^5-41943040*sqrt(2)*c*tan(1/2*exp(1))^4+41943040*sq
rt(2)*c*tan(1/2*exp(1))^3-8388608*sqrt(2)*c*tan(1/2*exp(1))^2+8388608*sqrt(
2)*c+25165824*sqrt(2)*c*tan(1/2*exp(1)))*tan(1/4*exp(1))^6+(-25165824*sqrt(
2)*c*tan(1/2*exp(1))^7-75497472*sqrt(2)*c*tan(1/2*exp(1))^6+25165824*sqrt(2
)*c*tan(1/2*exp(1))^5-125829120*sqrt(2)*c*tan(1/2*exp(1))^4+125829120*sqrt(
2)*c*tan(1/2*exp(1))^3-25165824*sqrt(2)*c*tan(1/2*exp(1))^2+25165824*sqrt(2
)*c+75497472*sqrt(2)*c*tan(1/2*exp(1)))*tan(1/4*exp(1))^2+(-25165824*sqrt(2
)*c*tan(1/2*exp(1))^7-75497472*sqrt(2)*c*tan(1/2*exp(1))^6+25165824*sqrt(2)
*c*tan(1/2*exp(1))^5-125829120*sqrt(2)*c*tan(1/2*exp(1))^4+125829120*sqrt(2
)*c*tan(1/2*exp(1))^3-25165824*sqrt(2)*c*tan(1/2*exp(1))^2+25165824*sqrt(2)
*c+75497472*sqrt(2)*c*tan(1/2*exp(1)))*tan(1/4*exp(1))^4-25165824*sqrt(2)*c
*tan(1/2*exp(1))^6+8388608*sqrt(2)*c*tan(1/2*exp(1))^5-41943040*sqrt(2)*c*t
an(1/2*exp(1))^4+41943040*sqrt(2)*c*tan(1/2*exp(1))^3-8388608*sqrt(2)*c*tan
(1/2*exp(1))^2+25165824*sqrt(2)*c*tan(1/2*exp(1)))
```

maple [B] time = 0.27, size = 252, normalized size = 2.62

$$\frac{\left(\sin\left(fx+e\right)\cos\left(fx+e\right)-2\sin\left(fx+e\right)\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)+4\sin\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-\left(\cos^2\left(\cdots\right.\right.\right.}{f\left(\sin\left(fx+e\right)\cos\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/f*(sin(f*x+e)*cos(f*x+e)-2*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+4*sin(f*x+e)*l
n(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-cos(f*x+e)^2-2*cos(f*x+e)*ln(2/(c
os(f*x+e)+1))+4*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-sin(f
*x+e)+2*ln(2/(cos(f*x+e)+1))-4*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+1
)*(a*(1+sin(f*x+e)))^(3/2)/(sin(f*x+e)*cos(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)
+cos(f*x+e)-2)/(-c*(sin(f*x+e)-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/sqrt(-c*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)/(c - c*sin(e + f*x))^(1/2),x)
[Out] int((a + a*sin(e + f*x))^(3/2)/(c - c*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(1/2),x)
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)/sqrt(-c*(sin(e + f*x) - 1)), x)
```

3.353 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=97

$$\frac{a^2\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] a*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f/(c-c*sin(f*x+e))^(3/2)+a^2*cos(f*x+e)
*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2739, 2737, 2667, 31}

$$\frac{a^2\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(f*(c - c*Sin[e + f*x])^(3/2)) +
(a^2*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt
[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx &= \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f(c-c\sin(e+fx))^{3/2}} - \frac{a\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c}\\
&= \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f(c-c\sin(e+fx))^{3/2}} - \frac{\left(a^2\cos(e+fx)\right)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}\\
&= \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f(c-c\sin(e+fx))^{3/2}} + \frac{\left(a^2\cos(e+fx)\right)\mathrm{Subst}\left(\int \frac{1}{c+x}\,dx, x, -c\sin(e+\right.}{cf\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}\\
&= \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f(c-c\sin(e+fx))^{3/2}} + \frac{a^2\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.47, size = 153, normalized size = 1.58

$$\frac{2a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin(e+fx)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right.}{cf(\sin(e+fx)-1)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+f\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(-1 -
 Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2]]*Sin[e + f*x]))/(c*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + S
in[e + f*x])*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}{c^{2}\cos\left(fx+e\right)^{2}+2\,c^{2}\sin\left(fx+e\right)-2\,c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f*x
 + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [B] time = 0.24, size = 375, normalized size = 3.87

$$\left(2\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)\cos\left(fx+e\right)-\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)\sin\left(fx+e\right)\cos\left(fx+e\right)-2\left(\cos^{2}\left(f\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/f*(2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)-ln(
2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)-2*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))+cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-2*sin(f*x+e)*cos(f
*x+e)-4*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*sin(f*x+e)*
ln(2/(cos(f*x+e)+1))+2*cos(f*x+e)^2-2*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))+cos(f*x+e)*ln(2/(cos(f*x+e)+1))+2*sin(f*x+e)+4*ln(-(-1+cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*ln(2/(cos(f*x+e)+1))-2)*(a*(1+sin(f*x+e))
)^(3/2)/(sin(f*x+e)*cos(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-c*
(sin(f*x+e)-1))^(3/2)
```

maxima [A] time = 0.90, size = 137, normalized size = 1.41

$$-\frac{\frac{2\,a^{\frac{3}{2}}\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{c^{\frac{3}{2}}}-\frac{a^{\frac{3}{2}}\log\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)}{c^{\frac{3}{2}}}+\frac{4\,a^{\frac{3}{2}}\sqrt{c}\sin\left(fx+e\right)}{\left(c^{2}-\frac{2\,c^{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{c^{2}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)\left(\cos\left(fx+e\right)+1\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -(2*a^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(3/2) - a^(3/2)*log(
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(3/2) + 4*a^(3/2)*sqrt(c)*sin(f*
x + e)/((c^2 - 2*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + c^2*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)/(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(3/2)/(c - c*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}{\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)/(-c*(sin(e + f*x) - 1))**(3/2), x)
```

3.354 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=42

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 1/4*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(5/2)
```

Rubi [A] time = 0.09, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(4*f*(c - c*Sin[e + f*x])^(5/2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4f(c-c\sin(e+fx))^{5/2}}$$

Mathematica [B] time = 0.48, size = 99, normalized size = 2.36

$$\frac{a\sin(e+fx)\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{c^2f(\sin(e+fx)-1)^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sin[e + f*x]*Sqrt[a*(1 + Sin[e + f
*x])])/(c^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*S
qrt[c - c*Sin[e + f*x]])
```

fricas [B] time = 0.44, size = 80, normalized size = 1.90

$$-\frac{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\,a\sin\left(fx+e\right)}{c^3f\cos\left(fx+e\right)^3+2\,c^3f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,c^3f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] -sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*a*sin(f*x + e)/(c^3*f*c
os(f*x + e)^3 + 2*c^3*f*cos(f*x + e)*sin(f*x + e) - 2*c^3*f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [B] time = 0.23, size = 90, normalized size = 2.14

$$-\frac{\left(-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}}{f\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{5}{2}}\left(\sin\left(fx+e\right)\cos\left(fx+e\right)+\cos^2\left(fx+e\right)-2\sin\left(fx+e\right)+\cos\left(fx+e\right)-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/f*(-1+cos(f*x+e)+sin(f*x+e))*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)/(-c*(si
n(f*x+e)-1))^(5/2)/(sin(f*x+e)*cos(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x
+e)-2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)/(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(3/2)/(c - c*sin(e + f*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}{\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)/(-c*(sin(e + f*x) - 1))**(5/2), x)
```

3.355 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=88

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{24cf(c-c\sin(e+fx))^{5/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{6f(c-c\sin(e+fx))^{7/2}}$$

```
[Out] 1/6*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(7/2)+1/24*cos(f*x
+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(5/2)
```

Rubi [A] time = 0.18, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2743, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{24cf(c-c\sin(e+fx))^{5/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{6f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(6*f*(c - c*Sin[e + f*x])^(7/2))
 + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(24*c*f*(c - c*Sin[e + f*x])^(5
/2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{6f(c-c\sin(e+fx))^{7/2}} + \frac{\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{6c}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{6f(c-c\sin(e+fx))^{7/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{24cf(c-c\sin(e+fx))^{5/2}}$$

Mathematica [A] time = 0.58, size = 106, normalized size = 1.20

$$-\frac{a(3\sin(e+fx)+1)\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{6c^3f(\sin(e+fx)-1)^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] -1/6*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(1
 + 3*Sin[e + f*x]))/(c^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[
e + f*x])^3*Sqrt[c - c*Sin[e + f*x]])
```

fricas [A] time = 0.44, size = 101, normalized size = 1.15

$$-\frac{\left(3\,a\sin\left(fx+e\right)+a\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{6\left(3\,c^4f\cos\left(fx+e\right)^3-4\,c^4f\cos\left(fx+e\right)-\left(c^4f\cos\left(fx+e\right)^3-4\,c^4f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] -1/6*(3*a*sin(f*x + e) + a)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
 c)/(3*c^4*f*cos(f*x + e)^3 - 4*c^4*f*cos(f*x + e) - (c^4*f*cos(f*x + e)^3
- 4*c^4*f*cos(f*x + e))*sin(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [A] time = 0.26, size = 141, normalized size = 1.60

$$\frac{\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)\left(\cos^{3}\left(fx+e\right)+\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)-4\left(\cos^{2}\left(fx+e\right)\right)+3\sin\left(fx\right.\right.}{6f\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{7}{2}}\left(\sin\left(fx+e\right)\cos\left(fx+e\right)+\cos^{2}\left(fx+e\right)-2\sin\left(fx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/6/f*(a*(1+sin(f*x+e)))^(3/2)*sin(f*x+e)*(cos(f*x+e)^3+cos(f*x+e)^2*sin(f*
x+e)-4*cos(f*x+e)^2+3*sin(f*x+e)*cos(f*x+e)-7*cos(f*x+e)-10*sin(f*x+e)+10)/
(-c*(sin(f*x+e)-1))^(7/2)/(sin(f*x+e)*cos(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+
cos(f*x+e)-2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e) + c)^(7/2), x)
```

mupad [B] time = 10.65, size = 124, normalized size = 1.41

$$-\frac{a\sqrt{a+a\sin\left(e+fx\right)}\sqrt{c-c\sin\left(e+fx\right)}+3a\sin\left(e+fx\right)\sqrt{a+a\sin\left(e+fx\right)}\sqrt{c-c\sin\left(e+fx\right)}}{\frac{9c^4f\cos\left(3e+3fx\right)}{2}+\frac{21c^4f\sin\left(2e+2fx\right)}{2}-\frac{3c^4f\sin\left(4e+4fx\right)}{4}-\frac{21c^4f\cos\left(e+fx\right)}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)/(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] -(a*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2) + 3*a*sin(e + f*x
)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2))/((9*c^4*f*cos(3*e
+ 3*f*x))/2 + (21*c^4*f*sin(2*e + 2*f*x))/2 - (3*c^4*f*sin(4*e + 4*f*x))/4
- (21*c^4*f*cos(e + f*x))/2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

3.356 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{9/2}} \, dx$

Optimal. Leaf size=92

$$\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{4f(c-c\sin(e+fx))^{9/2}} - \frac{a^2\cos(e+fx)}{12cf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}$$

```
[Out] -1/12*a^2*cos(f*x+e)/c/f/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(1/2)+1/4*
a*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f/(c-c*sin(f*x+e))^(9/2)
```

Rubi [A] time = 0.18, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2739, 2738}

$$\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{4f(c-c\sin(e+fx))^{9/2}} - \frac{a^2\cos(e+fx)}{12cf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(4*f*(c - c*Sin[e + f*x])^(9/2))
- (a^2*Cos[e + f*x])/(12*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^
(7/2))
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{4f(c-c\sin(e+fx))^{9/2}} - \frac{a\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{7/2}}\,dx}{4c}$$
$$= \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{4f(c-c\sin(e+fx))^{9/2}} - \frac{a^2\cos(e+fx)}{12cf\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}}$$

Mathematica [A] time = 1.07, size = 106, normalized size = 1.15

$$\frac{a(2\sin(e+fx)+1)\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{6c^4f(\sin(e+fx)-1)^4\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(1 + 2*
Sin[e + f*x]))/(6*c^4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e +
 f*x])^4*Sqrt[c - c*Sin[e + f*x]])
```

fricas [A] time = 0.45, size = 114, normalized size = 1.24

$$\frac{\left(2\,a\sin\left(fx+e\right)+a\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{6\left(c^5f\cos\left(fx+e\right)^5-8\,c^5f\cos\left(fx+e\right)^3+8\,c^5f\cos\left(fx+e\right)+4\left(c^5f\cos\left(fx+e\right)^3-2\,c^5f\cos\left(fx+e\right)\right)\sin\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="frica
s")
```

```
[Out] 1/6*(2*a*sin(f*x + e) + a)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
c)/(c^5*f*cos(f*x + e)^5 - 8*c^5*f*cos(f*x + e)^3 + 8*c^5*f*cos(f*x + e) +
4*(c^5*f*cos(f*x + e)^3 - 2*c^5*f*cos(f*x + e))*sin(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="giac"
)
```

[Out] Timed out

maple [B] time = 0.28, size = 169, normalized size = 1.84

$$-\frac{\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)\left(\sin\left(fx+e\right)\left(\cos^{3}\left(fx+e\right)\right)-\left(\cos^{4}\left(fx+e\right)\right)-5\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx\right.\right.}{6f\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{9}{2}}\left(\sin\left(fx+e\right)\cos\left(fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] -1/6/f*(a*(1+sin(f*x+e)))^(3/2)*sin(f*x+e)*(sin(f*x+e)*cos(f*x+e)^3-cos(f*x
+e)^4-5*cos(f*x+e)^2*sin(f*x+e)-4*cos(f*x+e)^3-7*sin(f*x+e)*cos(f*x+e)+12*c
os(f*x+e)^2+17*sin(f*x+e)+10*cos(f*x+e)-17)/(-c*(sin(f*x+e)-1))^(9/2)/(sin(
f*x+e)*cos(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e) + c)^(9/2), x)
```

mupad [B] time = 11.81, size = 195, normalized size = 2.12

$$\frac{\left(\frac{16\,a\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{3\,c^5\,f}+\frac{32\,a\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{3\,c^5\,f}\right)\,\sqrt{c-c\,\sin\left(e+\right.}}{84\,\cos\left(e+f\,x\right)\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}-54\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)+2\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)-96\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(2\,e+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)/(c - c*sin(e + f*x))^(9/2),x)
```

```
[Out] (((16*a*exp(e*5i + f*x*5i)*(a + a*sin(e + f*x))^(1/2))/(3*c^5*f) + (32*a*ex
p(e*5i + f*x*5i)*sin(e + f*x)*(a + a*sin(e + f*x))^(1/2))/(3*c^5*f))*(c - c
*sin(e + f*x))^(1/2))/(84*cos(e + f*x)*exp(e*5i + f*x*5i) - 54*exp(e*5i + f
*x*5i)*cos(3*e + 3*f*x) + 2*exp(e*5i + f*x*5i)*cos(5*e + 5*f*x) - 96*exp(e*
5i + f*x*5i)*sin(2*e + 2*f*x) + 16*exp(e*5i + f*x*5i)*sin(4*e + 4*f*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(9/2),x)
[Out] Timed out
```

3.357 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=92

$$\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{5f(c-c\sin(e+fx))^{11/2}}-\frac{a^2\cos(e+fx)}{20cf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{9/2}}$$

```
[Out] -1/20*a^2*cos(f*x+e)/c/f/(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(1/2)+1/5*
a*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f/(c-c*sin(f*x+e))^(11/2)
```

Rubi [A] time = 0.18, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2739, 2738}

$$\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{5f(c-c\sin(e+fx))^{11/2}}-\frac{a^2\cos(e+fx)}{20cf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c - c*Sin[e + f*x])^(11/2),x]
```

```
[Out] (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(5*f*(c - c*Sin[e + f*x])^(11/2))
 - (a^2*Cos[e + f*x])/(20*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])
^(9/2))
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{5f(c-c\sin(e+fx))^{11/2}} - \frac{a\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{9/2}}\,dx}{5c}$$
$$= \frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{5f(c-c\sin(e+fx))^{11/2}} - \frac{a^2\cos(e+fx)}{20cf\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{9/2}}$$

Mathematica [A] time = 1.49, size = 106, normalized size = 1.15

$$-\frac{a(5\sin(e+fx)+3)\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{20c^5f(\sin(e+fx)-1)^5\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c - c*Sin[e + f*x])^(11/2),x]
```

```
[Out] -1/20*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(
3 + 5*Sin[e + f*x]))/(c^5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin
[e + f*x])^5*Sqrt[c - c*Sin[e + f*x]])
```

fricas [A] time = 0.48, size = 131, normalized size = 1.42

$$\frac{\left(5\,a\sin\left(fx+e\right)+3\,a\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{20\left(5\,c^6f\cos\left(fx+e\right)^5-20\,c^6f\cos\left(fx+e\right)^3+16\,c^6f\cos\left(fx+e\right)-\left(c^6f\cos\left(fx+e\right)^5-12\,c^6f\cos\left(fx+e\right)^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="fric
as")
```

```
[Out] 1/20*(5*a*sin(f*x + e) + 3*a)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)/(5*c^6*f*cos(f*x + e)^5 - 20*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x
+ e) - (c^6*f*cos(f*x + e)^5 - 12*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x +
 e))*sin(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

maple [B] time = 0.28, size = 196, normalized size = 2.13

$$-\frac{\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)\left(3\sin\left(fx+e\right)\left(\cos^{4}\left(fx+e\right)\right)+3\left(\cos^{5}\left(fx+e\right)\right)+15\sin\left(fx+e\right)\left(\cos^{3}\left(\right.\right.\right.}{20f\left(-c\left(\sin\left(fx+e\right)\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/2),x)
```

```
[Out] -1/20/f*(a*(1+sin(f*x+e)))^(3/2)*sin(f*x+e)*(3*sin(f*x+e)*cos(f*x+e)^4+3*co
s(f*x+e)^5+15*sin(f*x+e)*cos(f*x+e)^3-18*cos(f*x+e)^4-51*cos(f*x+e)^2*sin(f
*x+e)-36*cos(f*x+e)^3-45*sin(f*x+e)*cos(f*x+e)+96*cos(f*x+e)^2+98*sin(f*x+e
)+53*cos(f*x+e)-98)/(-c*(sin(f*x+e)-1))^(11/2)/(sin(f*x+e)*cos(f*x+e)+cos(f
*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e) + c)^(11/2), x)
```

mupad [B] time = 12.59, size = 225, normalized size = 2.45

$$\frac{\left(\frac{a\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,48\mathrm{i}}{5\,c^6\,f}+\frac{a\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sin\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{c^6\,f}\right.}{\cos\left(e+f\,x\right)\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,264\mathrm{i}-\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)\,220\mathrm{i}+\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)\,20\mathrm{i}-\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sin\left(2\,e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)/(c - c*sin(e + f*x))^(11/2),x)
```

```
[Out] (((a*exp(e*6i + f*x*6i)*(a + a*sin(e + f*x))^(1/2)*48i)/(5*c^6*f) + (a*exp(
e*6i + f*x*6i)*sin(e + f*x)*(a + a*sin(e + f*x))^(1/2)*16i)/(c^6*f))*(c - c
```

```
*sin(e + f*x))^(1/2))/(cos(e + f*x)*exp(e*6i + f*x*6i)*264i - exp(e*6i + f*
x*6i)*cos(3*e + 3*f*x)*220i + exp(e*6i + f*x*6i)*cos(5*e + 5*f*x)*20i - exp
(e*6i + f*x*6i)*sin(2*e + 2*f*x)*330i + exp(e*6i + f*x*6i)*sin(4*e + 4*f*x)
*88i - exp(e*6i + f*x*6i)*sin(6*e + 6*f*x)*2i)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

3.358 $\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=134

$$-\frac{a^3\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{15f\sqrt{a\sin(e+fx)+a}}-\frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}{15f}-\frac{a\cos(e+fx)(a\,\mathrm{si}}{}$$

```
[Out] -1/6*a*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/f-1/15*a^3*
cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(1/2)-2/15*a^2*cos(f*x
+e)*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.27, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2740, 2738}

$$-\frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}{15f}-\frac{a^3\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{15f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)(a\,\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] -(a^3*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(15*f*Sqrt[a + a*Sin[e + f*x
]]) - (2*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/
2))/(15*f) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x]
)^(7/2))/(6*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}\,dx = -\frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{6f}+\frac{1}{3}$$
$$= -\frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}}{15f} - a$$
$$= -\frac{a^3\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{15f\sqrt{a+a\sin(e+fx)}} - 2a^2\cos(e+fx)\sqrt{a+a}$$

Mathematica [A] time = 1.38, size = 156, normalized size = 1.16

$$-\frac{c^3(\sin(e+fx)-1)^3(a(\sin(e+fx)+1))^{5/2}\sqrt{c-c\sin(e+fx)}\,(600\sin(e+fx)+100\sin(3(e+fx))+12\sin(5(}{960f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\sin\left(\frac{1}{2}(e+fx)\right)+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] -1/960*(c^3*(-1 + Sin[e + f*x])^3*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*S
in[e + f*x]]*(75*Cos[2*(e + f*x)] + 30*Cos[4*(e + f*x)] + 5*Cos[6*(e + f*x)
] + 600*Sin[e + f*x] + 100*Sin[3*(e + f*x)] + 12*Sin[5*(e + f*x)]))/(f*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5
)
```

fricas [A] time = 0.45, size = 112, normalized size = 0.84

$$\frac{\left(5\,a^2c^3\cos\left(fx+e\right)^6-5\,a^2c^3+2\left(3\,a^2c^3\cos\left(fx+e\right)^4+4\,a^2c^3\cos\left(fx+e\right)^2+8\,a^2c^3\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx\right.}}{30\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/30*(5*a^2*c^3*cos(f*x + e)^6 - 5*a^2*c^3 + 2*(3*a^2*c^3*cos(f*x + e)^4 +
4*a^2*c^3*cos(f*x + e)^2 + 8*a^2*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a
)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-80*a^2*c^3*f*sign(sin(1/2*(f*
x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(f*x+exp(1))/(16*f
)^2-480*a^2*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(
1))-1/4*pi))*sin(3*f*x+3*exp(1))/(96*f)^2-160*a^2*c^3*f*sign(sin(1/2*(f*x+e
xp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(5*f*x+5*exp(1))/(160
*f)^2-64*a^2*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp
(1))-1/4*pi))*cos(2*f*x+2*exp(1))/(64*f)^2-768*a^2*c^3*f*sign(sin(1/2*(f*x+
exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(4*f*x+4*exp(1))/(25
6*f)^2-384*a^2*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+e
xp(1))-1/4*pi))*cos(6*f*x+6*exp(1))/(384*f)^2-384*a^2*c^3*f*sign(sin(1/2*(f
*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-2*f*x-2*exp(1))
/(-128*f)^2-256*a^2*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(
f*x+exp(1))-1/4*pi))*cos(-4*f*x-4*exp(1))/(-256*f)^2)
```

maple [A] time = 0.32, size = 116, normalized size = 0.87

$$\frac{\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{7}{2}}\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)\left(5\left(\cos^{6}\left(fx+e\right)\right)+\sin\left(fx+e\right)\left(\cos^{4}\left(fx+e\right)\right)+6}{30f\cos\left(fx+e\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/30/f*(-c*(sin(f*x+e)-1))^(7/2)*(a*(1+sin(f*x+e)))^(5/2)*sin(f*x+e)*(5*cos
(f*x+e)^6+sin(f*x+e)*cos(f*x+e)^4+6*cos(f*x+e)^4+3*cos(f*x+e)^2*sin(f*x+e)+
8*cos(f*x+e)^2+11*sin(f*x+e)+11)/cos(f*x+e)^7
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(7/2), x)
```

mupad [**B**] time = 9.94, size = 124, normalized size = 0.93

$$\frac{a^2\,c^3\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(75\,\cos\left(e+f\,x\right)+105\,\cos\left(3\,e+3\,f\,x\right)+35\,\cos\left(5\,e+5\right.\right.}{960\,f\,\left(\cos\left(2\,e+2\,f\,x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] (a^2*c^3*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(75*cos
(e + f*x) + 105*cos(3*e + 3*f*x) + 35*cos(5*e + 5*f*x) + 5*cos(7*e + 7*f*x)
 + 700*sin(2*e + 2*f*x) + 112*sin(4*e + 4*f*x) + 12*sin(6*e + 6*f*x)))/(960
*f*(cos(2*e + 2*f*x) + 1))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

3.359 $\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=134

$$-\frac{2a^3\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{15f\sqrt{a\sin(e+fx)+a}}-\frac{a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}{5f}-\frac{a\cos(e+fx)(a\sin$$

```
[Out] -1/5*a*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/f-2/15*a^3*
cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(1/2)-1/5*a^2*cos(f*x+
e)*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.27, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2740, 2738}

$$-\frac{2a^3\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{15f\sqrt{a\sin(e+fx)+a}}-\frac{a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}{5f}-\frac{a\cos(e+fx)(a\sin$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*a^3*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(15*f*Sqrt[a + a*Sin[e + f
*x]]) - (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/
2))/(5*f) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])
^(5/2))/(5*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}\,dx = -\frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{5f} + \frac{1}{5}$$
$$= -\frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}{5f} - \frac{a\,c}{}$$
$$= -\frac{2a^3\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{15f\sqrt{a+a\sin(e+fx)}} - \frac{a^2\cos(e+fx)\sqrt{a+a}}{}$$

Mathematica [A] time = 0.54, size = 77, normalized size = 0.57

$$\frac{a^2c^2(150\sin(e+fx)+25\sin(3(e+fx))+3\sin(5(e+fx)))\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\,\sqrt{c-c\sin(e+fx)}}{240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a^2*c^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(
150*Sin[e + f*x] + 25*Sin[3*(e + f*x)] + 3*Sin[5*(e + f*x)]))/(240*f)
```

fricas [A] time = 0.46, size = 85, normalized size = 0.63

$$\frac{\left(3\,a^2c^2\cos\left(fx+e\right)^4+4\,a^2c^2\cos\left(fx+e\right)^2+8\,a^2c^2\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/15*(3*a^2*c^2*cos(f*x + e)^4 + 4*a^2*c^2*cos(f*x + e)^2 + 8*a^2*c^2)*sqrt
(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e)/(f*cos(f*x + e)
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-80*a^2*c^2*f*sign(s
in(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(f*x+exp
(1))/(16*f)^2-480*a^2*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2
*(f*x+exp(1))-1/4*pi))*sin(3*f*x+3*exp(1))/(96*f)^2-160*a^2*c^2*f*sign(sin(
1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(5*f*x+5*ex
p(1))/(160*f)^2)
```

maple [A] time = 0.28, size = 67, normalized size = 0.50

$$\frac{\left(3\left(\cos^4\left(fx+e\right)\right)+4\left(\cos^2\left(fx+e\right)\right)+8\right)\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{5}{2}}}{15f\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/15/f*(3*cos(f*x+e)^4+4*cos(f*x+e)^2+8)*(-c*(sin(f*x+e)-1))^(5/2)*sin(f*x+
e)*(a*(1+sin(f*x+e)))^(5/2)/cos(f*x+e)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(5/2), x)
```

mupad [B] time = 1.50, size = 83, normalized size = 0.62

$$\frac{a^2\,c^2\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(175\,\sin\left(2\,e+2\,f\,x\right)+28\,\sin\left(4\,e+4\,f\,x\right)+3\,\sin\left(6\,e+6\right.\right.}{240\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] (a^2*c^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(175*si
n(2*e + 2*f*x) + 28*sin(4*e + 4*f*x) + 3*sin(6*e + 6*f*x)))/(240*f*(cos(2*e
 + 2*f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.360 $\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=89

$$\frac{c^2\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}{4f}$$

```
[Out] 1/6*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(1/2)+1/4*c*co
s(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.17, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2740, 2738}

$$\frac{c^2\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*f*Sqrt[c - c*Sin[e + f*x]]
) + (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(4
*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}\,dx = \frac{c\cos(e+fx)(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{4f} + \frac{1}{2}c\int$$

$$= \frac{c^2\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f\sqrt{c-c\sin(e+fx)}} + \frac{c\cos(e+fx)(a+a\sin}{}$$

Mathematica [A] time = 0.69, size = 133, normalized size = 1.49

$$-\frac{c(\sin(e+fx)-1)(a(\sin(e+fx)+1))^{5/2}\sqrt{c-c\sin(e+fx)}\,(8(9\sin(e+fx)+\sin(3(e+fx)))-12\cos(2(e+f}{96f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/96*(c*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e
+ f*x]]*(-12*Cos[2*(e + f*x)] - 3*Cos[4*(e + f*x)] + 8*(9*Sin[e + f*x] + Si
n[3*(e + f*x)])))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^5)
```

fricas [A] time = 0.46, size = 87, normalized size = 0.98

$$-\frac{\left(3\,a^2c\cos\left(fx+e\right)^4-3\,a^2c-4\left(a^2c\cos\left(fx+e\right)^2+2\,a^2c\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)}}{12\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] -1/12*(3*a^2*c*cos(f*x + e)^4 - 3*a^2*c - 4*(a^2*c*cos(f*x + e)^2 + 2*a^2*c
)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f
*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(
2*c)*(-24*a^2*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(
1))-1/4*pi))*sin(f*x+exp(1))/(8*f)^2-24*a^2*c*f*sign(sin(1/2*(f*x+exp(1))-1
/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(3*f*x+3*exp(1))/(24*f)^2+32*
a^2*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi
))*cos(2*f*x+2*exp(1))/(32*f)^2+64*a^2*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi
))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(4*f*x+4*exp(1))/(64*f)^2+32*a^2*c
*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*co
s(-2*f*x-2*exp(1))/(-32*f)^2)
```

maple [A] time = 0.28, size = 91, normalized size = 1.02

$$\frac{\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{5}{2}}\left(3\left(\cos^{4}\left(fx+e\right)\right)-\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)+4\right.}{12f\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/12/f*(-c*(sin(f*x+e)-1))^(3/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)*(3*cos
(f*x+e)^4-cos(f*x+e)^2*sin(f*x+e)+4*cos(f*x+e)^2-5*sin(f*x+e)+5)/cos(f*x+e)
^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(3/2), x)
```

mupad [B] time = 8.27, size = 100, normalized size = 1.12

$$-\frac{a^2\,c\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(12\,\cos\left(e+f\,x\right)+15\,\cos\left(3\,e+3\,f\,x\right)+3\,\cos\left(5\,e+5\,f\,x\right.\right.}{96\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] -(a^2*c*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(12*cos(
e + f*x) + 15*cos(3*e + 3*f*x) + 3*cos(5*e + 5*f*x) - 80*sin(2*e + 2*f*x) -
 8*sin(4*e + 4*f*x)))/(96*f*(cos(2*e + 2*f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.361 $\int (a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=43

$$\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/3*c*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2738}

$$\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}\,dx=\frac{c\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 0.27, size = 72, normalized size = 1.67

$$-\frac{a^2\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}\,(-15\sin(e+fx)+\sin(3(e+fx))+6\cos(2(e+fx)))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] -1/12*(a^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]
*(6*Cos[2*(e + f*x)] - 15*Sin[e + f*x] + Sin[3*(e + f*x)]))/f
```

fricas [B] time = 0.44, size = 82, normalized size = 1.91

$$-\frac{\left(3\,a^{2}\cos\left(fx+e\right)^{2}-3\,a^{2}+\left(a^{2}\cos\left(fx+e\right)^{2}-4\,a^{2}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{3\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] -1/3*(3*a^2*cos(f*x + e)^2 - 3*a^2 + (a^2*cos(f*x + e)^2 - 4*a^2)*sin(f*x +
 e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-40*a^2*f*sign(sin(1
/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(f*x+exp(1))
/(8*f)^2+24*a^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(
1))-1/4*pi))*sin(3*f*x+3*exp(1))/(24*f)^2+4*a^2*f*sign(sin(1/2*(f*x+exp(1))
-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(2*f*x+2*exp(1))/(4*f)^2)
```

maple [B] time = 0.32, size = 80, normalized size = 1.86

$$-\frac{\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\ \sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{5}{2}}\left(-\left(\cos^{4}\left(fx+e\right)\right)+\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)\right.}{3f\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/3/f*(-c*(sin(f*x+e)-1))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)*(-cos(
f*x+e)^4+cos(f*x+e)^2*sin(f*x+e)-2+2*sin(f*x+e))/cos(f*x+e)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x + e) + c), x)
```

mupad [B] time = 7.75, size = 86, normalized size = 2.00

$$-\frac{a^2\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(6\,\cos\left(e+f\,x\right)+6\,\cos\left(3\,e+3\,f\,x\right)-14\,\sin\left(2\,e+2\,f\,x\right)+\right.}{12\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] -(a^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(6*cos(e +
 f*x) + 6*cos(3*e + 3*f*x) - 14*sin(2*e + 2*f*x) + sin(4*e + 4*f*x)))/(12*f
*(cos(2*e + 2*f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.362 $\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=141

$$-\frac{4a^3\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)(a\sin(e+fx)+a)}{2f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -1/2*a*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(1/2)-4*a^3*cos
(f*x+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-2*
a^2*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2740, 2737, 2667, 31}

$$-\frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}-\frac{4a^3\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)(a\sin(e+fx)+a)}{2f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (-4*a^3*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c - c*Sin[e + f*x]]) - (2*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(f*S
qrt[c - c*Sin[e + f*x]]) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f
*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
```

```
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}} + (2a)\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$$

$$= -\frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}} + \left(4a^2\right)$$

$$= -\frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}} + \frac{\left(4a^3\right.}{\sqrt{a+}}$$

$$= -\frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}} - \frac{\left(4a^3\right.}{}$$

$$= -\frac{4a^3\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} -$$

Mathematica [A] time = 0.59, size = 127, normalized size = 0.90

$$-\frac{(a(\sin(e+fx)+1))^{5/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(12\sin(e+fx)-\cos(2(e+fx))+32\log\left(\cos\left(\frac{1}{2}(e\right.\right.\right.}{4f\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] -1/4*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2)*(-
Cos[2*(e + f*x)] + 32*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 12*Sin[e +
```

```
f*x]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sqrt[c - c*Sin[e + f*x]]
)
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{c\sin\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*sqrt(a*sin(f*x +
 e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*sin(f*x + e) - c), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [B] time = 0.30, size = 315, normalized size = 2.23

$$-\frac{\left(\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)+\cos^3\left(fx+e\right)-6\sin\left(fx+e\right)\cos\left(fx+e\right)-16\sin\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+}{\sin\left(fx+\right.}\right.\right.}{2f\left(\left(\cos^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*(cos(f*x+e)^2*sin(f*x+e)+cos(f*x+e)^3-6*sin(f*x+e)*cos(f*x+e)-16*sin
(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+8*sin(f*x+e)*ln(2/(cos(f
*x+e)+1))+5*cos(f*x+e)^2-16*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f
*x+e))+8*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+5*sin(f*x+e)-cos(f*x+e)+16*ln(-(-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-8*ln(2/(cos(f*x+e)+1))-5)*(a*(1+sin(f*x
+e)))^(5/2)/(cos(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^3+2*sin(f*x+e)*cos(f*x+e)+3
*cos(f*x+e)^2-4*sin(f*x+e)+2*cos(f*x+e)-4)/(-c*(sin(f*x+e)-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/sqrt(-c*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(5/2)/(c - c*sin(e + f*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.363 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=144

$$\frac{4a^3\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}+\frac{a\cos(e+fx)(a\sin(e+fx)+a)}{f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] a*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(3/2)+4*a^3*cos(f*x+
e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+2*a^2
*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2739, 2740, 2737, 2667, 31}

$$\frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}+\frac{4a^3\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{a\cos(e+fx)(a\sin(e+fx)+a)}{f(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(f*(c - c*Sin[e + f*x])^(3/2))
+ (4*a^3*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*
Sqrt[c - c*Sin[e + f*x]]) + (2*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(
c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
```

```
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f(c-c\sin(e+fx))^{3/2}} - \frac{(2a)\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f(c-c\sin(e+fx))^{3/2}} + \frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{\left(4a^2\right)\int\cdots}{\cdots}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f(c-c\sin(e+fx))^{3/2}} + \frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{\left(4a^3\cdots\right.}{\sqrt{a+a\cdots}}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f(c-c\sin(e+fx))^{3/2}} + \frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} + \frac{\left(4a^3\cos\cdots\right.}{c\cdots}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f(c-c\sin(e+fx))^{3/2}} + \frac{4a^3\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2\cdots}{\cdots}$$

Mathematica [A] time = 0.81, size = 169, normalized size = 1.17

$$-\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos(2(e+fx))+16\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}\right.\right.\right.}{2cf(\sin(e+fx)-1)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/2*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*
(7 + Cos[2*(e + f*x)] + 16*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + (2 -
16*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])*Sin[e + f*x]))/(c*f*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{c^2\cos\left(fx+e\right)^2+2\,c^2\sin\left(fx+e\right)-2\,c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*sqrt(a*sin(f*x +
 e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e)
 - 2*c^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [B] time = 0.27, size = 439, normalized size = 3.05

$$-\frac{\left(\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)-8\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)\cos\left(fx+e\right)+4\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)\sin\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/f*(cos(f*x+e)^2*sin(f*x+e)-8*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*
sin(f*x+e)*cos(f*x+e)+4*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)+cos(f*x+
e)^3+8*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-4*cos(f*x+e)
^2*ln(2/(cos(f*x+e)+1))+5*sin(f*x+e)*cos(f*x+e)+16*sin(f*x+e)*ln(-(-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))-8*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-6*cos(f*x+e
)^2+8*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-4*cos(f*x+e)*ln
(2/(cos(f*x+e)+1))-6*sin(f*x+e)-cos(f*x+e)-16*ln(-(-1+cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))+8*ln(2/(cos(f*x+e)+1))+6)*(a*(1+sin(f*x+e)))^(5/2)/(cos(f*x+e
)^2*sin(f*x+e)-cos(f*x+e)^3+2*sin(f*x+e)*cos(f*x+e)+3*cos(f*x+e)^2-4*sin(f*
x+e)+2*cos(f*x+e)-4)/(-c*(sin(f*x+e)-1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a \sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c \sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(5/2)/(c - c*sin(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.364 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=147

$$-\frac{a^3\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf(c-c\sin(e+fx))^{3/2}}+\frac{a\cos(e+fx)(a\sin(e+fx)+a}{2f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 1/2*a*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(5/2)-a^2*cos(f*
x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))^(3/2)-a^3*cos(f*x+e)*ln(1-
sin(f*x+e))/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2739, 2737, 2667, 31}

$$-\frac{a^3\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf(c-c\sin(e+fx))^{3/2}}+\frac{a\cos(e+fx)(a\sin(e+fx)+a}{2f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f*(c - c*Sin[e + f*x])^(5/2)
) - (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c*f*(c - c*Sin[e + f*x])^(
3/2)) - (a^3*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^2*f*Sqrt[a + a*Sin[e +
f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
```

```
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{c}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{a^2\int \frac{\sqrt{a+\cdots}}{\sqrt{c-\cdots}}}{\cdots}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{\left(a^3 c\cdots\right.}{c\sqrt{a+a\cdots}}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}} - \frac{\left(a^3\cos(\cdots\right.}{c^2 f\cdots}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}} - \frac{a^3\cos\cdots}{c^2 f\sqrt{a+\cdots}}$$

Mathematica [A] time = 1.16, size = 190, normalized size = 1.29

$$\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos(2(e+fx))\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\cdots\right.\right.}{c^2 f(\sin(e+fx)-1)^2\sqrt{c-c\sin(e+f\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(-2 -
 3*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Cos[2*(e + f*x)]*Log[Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2]] + 4*(1 + Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/
2]])*Sin[e + f*x]))/(c^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[
e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 1.27, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2\cos\left(fx+e\right)^2 - 2\,a^2\sin\left(fx+e\right) - 2\,a^2\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{3\,c^3\cos\left(fx+e\right)^2 - 4\,c^3 - \left(c^3\cos\left(fx+e\right)^2 - 4\,c^3\right)\sin\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral((a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*sqrt(a*sin(f*x +
 e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos
(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [B] time = 0.26, size = 553, normalized size = 3.76

$$\frac{\left(\sin\left(fx+e\right)\left(\cos^2\left(fx+e\right)\right)\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right) - 2\sin\left(fx+e\right)\left(\cos^2\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right) + \left(\cos\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/f*(sin(f*x+e)*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-2*sin(f*x+e)*cos(f*x+e)^2
*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+cos(f*x+e)^3*ln(2/(cos(f*x+e)+1
))-2*cos(f*x+e)^3*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*cos(f*x+e)^2
```

```
*sin(f*x+e)+2*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)-4*ln(-(-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)+2*cos(f*x+e)^3-3*cos(f*x+e
)^2*ln(2/(cos(f*x+e)+1))+6*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))-4*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+8*sin(f*x+e)*ln(-(-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))-2*cos(f*x+e)^2-2*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+4*c
os(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*sin(f*x+e)-2*cos(f*x
+e)+4*ln(2/(cos(f*x+e)+1))-8*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2)*
(a*(1+sin(f*x+e)))^(5/2)/(cos(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^3+2*sin(f*x+e)
*cos(f*x+e)+3*cos(f*x+e)^2-4*sin(f*x+e)+2*cos(f*x+e)-4)/(-c*(sin(f*x+e)-1))
^(5/2)
```

maxima [A] time = 1.92, size = 184, normalized size = 1.25

$$-\frac{\frac{8\,a^{\frac{5}{2}}\sqrt{c}\,\sin\left(fx+e\right)^{2}}{\left(c^{3}-\frac{4\,c^{3}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{6\,c^{3}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{4\,c^{3}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{c^{3}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}\right)\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{2\,a^{\frac{5}{2}}\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{c^{\frac{5}{2}}}+\frac{a^{\frac{5}{2}}\log\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)}{c^{\frac{5}{2}}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -(8*a^(5/2)*sqrt(c)*sin(f*x + e)^2/((c^3 - 4*c^3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 6*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^3*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*(cos(f*x + e)
 + 1)^2) - 2*a^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(5/2) + a^(
5/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(5/2))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(5/2)/(c - c*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.365 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=42

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f(c-c\sin(e+fx))^{7/2}}$$

```
[Out] 1/6*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(7/2)
```

Rubi [A] time = 0.09, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*f*(c - c*Sin[e + f*x])^(7/2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}}$$

Mathematica [B] time = 0.98, size = 110, normalized size = 2.62

$$\frac{a^2(3\cos(2(e+fx))-5)\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{6c^3f(\sin(e+fx)-1)^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a^2*(-5 + 3*Cos[2*(e + f*x)])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a
*(1 + Sin[e + f*x])])/(6*c^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 +
Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]])
```

fricas [B] time = 0.43, size = 109, normalized size = 2.60

$$\frac{\left(3 a^2 \cos\left(fx+e\right)^2 - 4 a^2\right)\sqrt{a \sin\left(fx+e\right)+a}\,\sqrt{-c \sin\left(fx+e\right)+c}}{3\left(3 c^4 f \cos\left(fx+e\right)^3 - 4 c^4 f \cos\left(fx+e\right) - \left(c^4 f \cos\left(fx+e\right)^3 - 4 c^4 f \cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/3*(3*a^2*cos(f*x + e)^2 - 4*a^2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
 + e) + c)/(3*c^4*f*cos(f*x + e)^3 - 4*c^4*f*cos(f*x + e) - (c^4*f*cos(f*x
+ e)^3 - 4*c^4*f*cos(f*x + e))*sin(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [B] time = 0.28, size = 130, normalized size = 3.10

$$\frac{\left(-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)\left(\cos^2\left(fx+e\right)-4\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right.}{3f\left(\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)-\left(\cos^3\left(fx+e\right)\right)+2\sin\left(fx+e\right)\cos\left(fx+e\right)+3\left(\cos^2\left(fx+e\right)\right)-4\sin\left(fx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/3/f*(-1+cos(f*x+e)+sin(f*x+e))*(cos(f*x+e)^2-4)*(a*(1+sin(f*x+e)))^(5/2)*
sin(f*x+e)/(cos(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^3+2*sin(f*x+e)*cos(f*x+e)+3*
cos(f*x+e)^2-4*sin(f*x+e)+2*cos(f*x+e)-4)/(-c*(sin(f*x+e)-1))^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e) + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(5/2)/(c - c*sin(e + f*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

3.366 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=88

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{48cf(c-c\sin(e+fx))^{7/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

```
[Out] 1/8*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(9/2)+1/48*cos(f*x
+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(7/2)
```

Rubi [A] time = 0.19, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2743, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{48cf(c-c\sin(e+fx))^{7/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(8*f*(c - c*Sin[e + f*x])^(9/2))
+ (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(48*c*f*(c - c*Sin[e + f*x])^(7
/2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{9/2}} + \frac{\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx}{8c}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{9/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{48cf(c-c\sin(e+fx))^{7/2}}$$

Mathematica [**A**] time = 2.25, size = 118, normalized size = 1.34

$$\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(4\sin(e+fx)-3\cos(2(e+fx))+5)}{12c^4f(\sin(e+fx)-1)^4\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(5 -
3*Cos[2*(e + f*x)] + 4*Sin[e + f*x]))/(12*c^4*f*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])*(-1 + Sin[e + f*x])^4*Sqrt[c - c*Sin[e + f*x]])
```

fricas [**A**] time = 0.45, size = 133, normalized size = 1.51

$$-\frac{\left(3\,a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-4\,a^2\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{6\left(c^5f\cos\left(fx+e\right)^5-8\,c^5f\cos\left(fx+e\right)^3+8\,c^5f\cos\left(fx+e\right)+4\left(c^5f\cos\left(fx+e\right)^3-2\,c^5f\cos\left(fx+e\right)\right)\sin\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="frica
s")
```

```
[Out] -1/6*(3*a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 4*a^2)*sqrt(a*sin(f*x + e
) + a)*sqrt(-c*sin(f*x + e) + c)/(c^5*f*cos(f*x + e)^5 - 8*c^5*f*cos(f*x +
e)^3 + 8*c^5*f*cos(f*x + e) + 4*(c^5*f*cos(f*x + e)^3 - 2*c^5*f*cos(f*x + e
))*sin(f*x + e))
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [B] time = 0.28, size = 199, normalized size = 2.26

$$-\frac{\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)\left(\sin\left(fx+e\right)\left(\cos^{3}\left(fx+e\right)\right)-\left(\cos^{4}\left(fx+e\right)\right)-5\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx\right.\right.}{6f\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{9}{2}}\left(\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)-\left(\cos^{3}\left(fx+e\right)\right)+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] -1/6/f*(a*(1+sin(f*x+e)))^(5/2)*sin(f*x+e)*(sin(f*x+e)*cos(f*x+e)^3-cos(f*x
+e)^4-5*cos(f*x+e)^2*sin(f*x+e)-4*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)+9*co
s(f*x+e)^2+14*sin(f*x+e)+10*cos(f*x+e)-14)/(-c*(sin(f*x+e)-1))^(9/2)/(cos(f
*x+e)^2*sin(f*x+e)-cos(f*x+e)^3+2*sin(f*x+e)*cos(f*x+e)+3*cos(f*x+e)^2-4*si
n(f*x+e)+2*cos(f*x+e)-4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e) + c)^(9/2), x)
```

mupad [B] time = 11.57, size = 242, normalized size = 2.75

$$\frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{40\,a^2\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{3\,c^5\,f}+\frac{32\,a^2\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sin\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{3\,c^5\,f}-\frac{8\,a^2\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}}{\ }\right.}{84\,\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}-54\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)+2\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)-96\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sin\left(2\,e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c - c*sin(e + f*x))^(9/2),x)
```

```
[Out] ((c - c*sin(e + f*x))^(1/2)*((40*a^2*exp(e*5i + f*x*5i)*(a + a*sin(e + f*x)
)^(1/2))/(3*c^5*f) + (32*a^2*exp(e*5i + f*x*5i)*sin(e + f*x)*(a + a*sin(e +
```

```
 f*x))^(1/2))/(3*c^5*f) - (8*a^2*exp(e*5i + f*x*5i)*cos(2*e + 2*f*x)*(a + a
*sin(e + f*x))^(1/2))/(c^5*f)))/(84*cos(e + f*x)*exp(e*5i + f*x*5i) - 54*ex
p(e*5i + f*x*5i)*cos(3*e + 3*f*x) + 2*exp(e*5i + f*x*5i)*cos(5*e + 5*f*x) -
 96*exp(e*5i + f*x*5i)*sin(2*e + 2*f*x) + 16*exp(e*5i + f*x*5i)*sin(4*e + 4
*f*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

3.367 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=133

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{240c^2f(c-c\sin(e+fx))^{7/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{40cf(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

```
[Out] 1/10*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(11/2)+1/40*cos(f
*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(9/2)+1/240*cos(f*x+e)*(a
+a*sin(f*x+e))^(5/2)/c^2/f/(c-c*sin(f*x+e))^(7/2)
```

Rubi [A] time = 0.28, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2743, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{240c^2f(c-c\sin(e+fx))^{7/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{40cf(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(11/2),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(10*f*(c - c*Sin[e + f*x])^(11/2)
) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(40*c*f*(c - c*Sin[e + f*x])^
(9/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(240*c^2*f*(c - c*Sin[e
+ f*x])^(7/2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{9/2}}\,dx}{5c}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{40cf(c-c\sin(e+fx))^{9/2}} + \frac{\int \frac{(a+a\sin(e}{(c-c\sin(e}}{40}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{40cf(c-c\sin(e+fx))^{9/2}} + \frac{\cos(e+f}{240c^2f(}$$

Mathematica [A] time = 3.42, size = 118, normalized size = 0.89

$$-\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(10\sin(e+fx)-5\cos(2(e+fx))+9)}{30c^5f(\sin(e+fx)-1)^5\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(11/2),x]
```

```
[Out] -1/30*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]
*(9 - 5*Cos[2*(e + f*x)] + 10*Sin[e + f*x]))/(c^5*f*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])*(-1 + Sin[e + f*x])^5*Sqrt[c - c*Sin[e + f*x]])
```

fricas [A] time = 0.47, size = 148, normalized size = 1.11

$$-\frac{\left(5\,a^2\cos\left(fx+e\right)^2-5\,a^2\sin\left(fx+e\right)-7\,a^2\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right.}}{15\left(5\,c^6f\cos\left(fx+e\right)^5-20\,c^6f\cos\left(fx+e\right)^3+16\,c^6f\cos\left(fx+e\right)-\left(c^6f\cos\left(fx+e\right)^5-12\,c^6f\cos\left(fx+e\right)^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="fric
as")
```

```
[Out] -1/15*(5*a^2*cos(f*x + e)^2 - 5*a^2*sin(f*x + e) - 7*a^2)*sqrt(a*sin(f*x +
e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^6*f*cos(f*x + e)^5 - 20*c^6*f*cos(f*
x + e)^3 + 16*c^6*f*cos(f*x + e) - (c^6*f*cos(f*x + e)^5 - 12*c^6*f*cos(f*x
 + e)^3 + 16*c^6*f*cos(f*x + e))*sin(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

maple [A] time = 0.28, size = 226, normalized size = 1.70

$$-\frac{\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{5}{2}}\left(2\sin\left(fx+e\right)\left(\cos^{4}\left(fx+e\right)\right)+2\left(\cos^{5}\left(fx+e\right)\right)+10\sin\left(fx+e\right)\left(\cos^{3}\right.\right.}{15f\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{11}{2}}\left(\left(\cos^{2}\left(fx+e\right)\right)\mathrm{s}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/2),x)
```

```
[Out] -1/15/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)*(2*sin(f*x+e)*cos(f*x+e)^4+2*co
s(f*x+e)^5+10*sin(f*x+e)*cos(f*x+e)^3-12*cos(f*x+e)^4-34*cos(f*x+e)^2*sin(f
*x+e)-24*cos(f*x+e)^3-25*sin(f*x+e)*cos(f*x+e)+59*cos(f*x+e)^2+62*sin(f*x+e
)+37*cos(f*x+e)-62)/(-c*(sin(f*x+e)-1))^(11/2)/(cos(f*x+e)^2*sin(f*x+e)-cos
(f*x+e)^3+2*sin(f*x+e)*cos(f*x+e)+3*cos(f*x+e)^2-4*sin(f*x+e)+2*cos(f*x+e)-
4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e) + c)^(11/2), x)
```

mupad [B] time = 12.08, size = 273, normalized size = 2.05

$$\frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{a^2\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,96\mathrm{i}}{5\,c^6\,f}+\frac{a^2\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sin\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+\right.}}{3\,c^6\,f}\right.}{\cos\left(e+f\,x\right)\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,264\mathrm{i}-\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)\,220\mathrm{i}+\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)\,20\mathrm{i}-\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sin\left(2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c - c*sin(e + f*x))^(11/2),x)
```

```
[Out] ((c - c*sin(e + f*x))^(1/2)*((a^2*exp(e*6i + f*x*6i)*(a + a*sin(e + f*x))^(
1/2)*96i)/(5*c^6*f) + (a^2*exp(e*6i + f*x*6i)*sin(e + f*x)*(a + a*sin(e + f
*x))^(1/2)*64i)/(3*c^6*f) - (a^2*exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(a + a
*sin(e + f*x))^(1/2)*32i)/(3*c^6*f)))/(cos(e + f*x)*exp(e*6i + f*x*6i)*264i
 - exp(e*6i + f*x*6i)*cos(3*e + 3*f*x)*220i + exp(e*6i + f*x*6i)*cos(5*e +
5*f*x)*20i - exp(e*6i + f*x*6i)*sin(2*e + 2*f*x)*330i + exp(e*6i + f*x*6i)*
sin(4*e + 4*f*x)*88i - exp(e*6i + f*x*6i)*sin(6*e + 6*f*x)*2i)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

3.368 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{13/2}}\,dx$

Optimal. Leaf size=140

$$\frac{a^3\cos(e+fx)}{60c^2f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{9/2}}-\frac{a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15cf(c-c\sin(e+fx))^{11/2}}+\frac{a\cos(e+fx)(a\sin(e+fx)\cdots}{6f(c-c\sin(e+fx))^{13\cdots}}$$

```
[Out] 1/6*a*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(13/2)+1/60*a^3*
cos(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(1/2)-1/15*a^2*cos
(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))^(11/2)
```

Rubi [A] time = 0.27, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2739, 2738}

$$\frac{a^3\cos(e+fx)}{60c^2f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{9/2}}-\frac{a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15cf(c-c\sin(e+fx))^{11/2}}+\frac{a\cos(e+fx)(a\sin(e+fx)\cdots}{6f(c-c\sin(e+fx))^{13\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(13/2),x]
```

```
[Out] (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(6*f*(c - c*Sin[e + f*x])^(13/2
)) - (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(15*c*f*(c - c*Sin[e + f*x
])^(11/2)) + (a^3*Cos[e + f*x])/(60*c^2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*S
in[e + f*x])^(9/2))
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{6f(c-c\sin(e+fx))^{13/2}} - \frac{a\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{11/2}}\,dx}{3c}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{6f(c-c\sin(e+fx))^{13/2}} - \frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15cf(c-c\sin(e+fx))^{11/2}} + \frac{a^2\int \frac{\sqrt{a}}{(c-}}{}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{6f(c-c\sin(e+fx))^{13/2}} - \frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15cf(c-c\sin(e+fx))^{11/2}} + \frac{}{60c^2f\sqrt{}}$$

Mathematica [A] time = 4.91, size = 118, normalized size = 0.84

$$\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(36\sin(e+fx)-15\cos(2(e+fx))+29)}{120c^6f(\sin(e+fx)-1)^6\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c - c*Sin[e + f*x])^(13/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(29 -
 15*Cos[2*(e + f*x)] + 36*Sin[e + f*x]))/(120*c^6*f*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])*(-1 + Sin[e + f*x])^6*Sqrt[c - c*Sin[e + f*x]])
```

fricas [A] time = 0.48, size = 162, normalized size = 1.16

$$\frac{\left(15\,a^2\cos\left(fx+e\right)^2-18\,a^2\sin\left(fx+e\right)-22\,a^2\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-}}{60\left(c^7f\cos\left(fx+e\right)^7-18\,c^7f\cos\left(fx+e\right)^5+48\,c^7f\cos\left(fx+e\right)^3-32\,c^7f\cos\left(fx+e\right)+2\left(3\,c^7f\cos\left(fx+e\right)^5\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/2),x, algorithm="fric
as")
```

```
[Out] 1/60*(15*a^2*cos(f*x + e)^2 - 18*a^2*sin(f*x + e) - 22*a^2)*sqrt(a*sin(f*x
+ e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^7*f*cos(f*x + e)^7 - 18*c^7*f*cos(f*
x + e)^5 + 48*c^7*f*cos(f*x + e)^3 - 32*c^7*f*cos(f*x + e) + 2*(3*c^7*f*cos
(f*x + e)^5 - 16*c^7*f*cos(f*x + e)^3 + 16*c^7*f*cos(f*x + e))*sin(f*x + e)
)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

maple [B] time = 0.32, size = 252, normalized size = 1.80

$$\frac{\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)\left(7\sin\left(fx+e\right)\left(\cos^{5}\left(fx+e\right)\right)-7\left(\cos^{6}\left(fx+e\right)\right)-49\sin\left(fx+e\right)\left(\cos^{4}\left(\right.\right.\right.}{60f\left(-c\left(\sin\left(fx\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/2),x)
```

```
[Out] 1/60/f*(a*(1+sin(f*x+e)))^(5/2)*sin(f*x+e)*(7*sin(f*x+e)*cos(f*x+e)^5-7*cos
(f*x+e)^6-49*sin(f*x+e)*cos(f*x+e)^4-42*cos(f*x+e)^5-119*sin(f*x+e)*cos(f*x
+e)^3+168*cos(f*x+e)^4+343*cos(f*x+e)^2*sin(f*x+e)+224*cos(f*x+e)^3+202*sin
(f*x+e)*cos(f*x+e)-545*cos(f*x+e)^2-444*sin(f*x+e)-242*cos(f*x+e)+444)/(-c*
(sin(f*x+e)-1))^(13/2)/(cos(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^3+2*sin(f*x+e)*c
os(f*x+e)+3*cos(f*x+e)^2-4*sin(f*x+e)+2*cos(f*x+e)-4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e) + c)^(13/2), x)
```

mupad [B] time = 12.24, size = 287, normalized size = 2.05

$$-\frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{464\,a^2\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{15\,c^7\,f}+\frac{192\,a^2\,\mathrm{e}^{e\,7\mathrm{i}+f\,x}}{}\right.}{-858\,\cos\left(e+f\,x\right)\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}+858\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)-130\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)+2\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\mathrm{co}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c - c*sin(e + f*x))^(13/2),x)
```

```
[Out] -((c - c*sin(e + f*x))^(1/2)*((464*a^2*exp(e*7i + f*x*7i)*(a + a*sin(e + f*
x))^(1/2))/(15*c^7*f) + (192*a^2*exp(e*7i + f*x*7i)*sin(e + f*x)*(a + a*sin
(e + f*x))^(1/2))/(5*c^7*f) - (16*a^2*exp(e*7i + f*x*7i)*cos(2*e + 2*f*x)*(
a + a*sin(e + f*x))^(1/2))/(c^7*f)))/(858*exp(e*7i + f*x*7i)*cos(3*e + 3*f*
x) - 858*cos(e + f*x)*exp(e*7i + f*x*7i) - 130*exp(e*7i + f*x*7i)*cos(5*e +
 5*f*x) + 2*exp(e*7i + f*x*7i)*cos(7*e + 7*f*x) + 1144*exp(e*7i + f*x*7i)*s
in(2*e + 2*f*x) - 416*exp(e*7i + f*x*7i)*sin(4*e + 4*f*x) + 24*exp(e*7i + f
*x*7i)*sin(6*e + 6*f*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(13/2),x)
```

```
[Out] Timed out
```

3.369 $\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{9/2}\,dx$

Optimal. Leaf size=179

$$-\frac{a^4\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{35f\sqrt{a\sin(e+fx)+a}}-\frac{a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{9/2}}{14f}-\frac{3a^2\cos(e+fx)(a$$

```
[Out] -3/28*a^2*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(9/2)/f-1/8*a*
cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(9/2)/f-1/35*a^4*cos(f*x
+e)*(c-c*sin(f*x+e))^(9/2)/f/(a+a*sin(f*x+e))^(1/2)-1/14*a^3*cos(f*x+e)*(c-
c*sin(f*x+e))^(9/2)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.37, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2740, 2738}

$$-\frac{3a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{9/2}}{28f}-\frac{a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx)}{14f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] -(a^4*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(35*f*Sqrt[a + a*Sin[e + f*x
]]) - (a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(9/2)
)/(14*f) - (3*a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*
x])^(9/2))/(28*f) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e
 + f*x])^(9/2))/(8*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
```

```
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{9/2}\,dx = -\frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{9/2}}{8f}+\frac{1}{4}(3$$
$$= -\frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{9/2}}{28f}-a$$
$$= -\frac{a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{9/2}}{14f}-\frac{3a^2}{}$$
$$= -\frac{a^4\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{35f\sqrt{a+a\sin(e+fx)}}-\frac{a^3\cos(e+fx)\sqrt{a+a\sin}}{}$$

Mathematica [A] time = 5.29, size = 127, normalized size = 0.71

$$\frac{a^3c^4\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\,\sqrt{c-c\sin(e+fx)}\,(19600\sin(e+fx)+3920\sin(3(e+fx))+784\sin(5(e+}{3584}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (a^3*c^4*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(
1960*Cos[2*(e + f*x)] + 980*Cos[4*(e + f*x)] + 280*Cos[6*(e + f*x)] + 35*Co
s[8*(e + f*x)] + 19600*Sin[e + f*x] + 3920*Sin[3*(e + f*x)] + 784*Sin[5*(e
+ f*x)] + 80*Sin[7*(e + f*x)]))/(35840*f)
```

fricas [A] time = 0.48, size = 128, normalized size = 0.72

$$\frac{\left(35\,a^3c^4\cos\left(fx+e\right)^8-35\,a^3c^4+8\left(5\,a^3c^4\cos\left(fx+e\right)^6+6\,a^3c^4\cos\left(fx+e\right)^4+8\,a^3c^4\cos\left(fx+e\right)^2+16\,a^3c^4\right)}{280\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2),x, algorithm="frica
s")
```

```
[Out] 1/280*(35*a^3*c^4*cos(f*x + e)^8 - 35*a^3*c^4 + 8*(5*a^3*c^4*cos(f*x + e)^6
 + 6*a^3*c^4*cos(f*x + e)^4 + 8*a^3*c^4*cos(f*x + e)^2 + 16*a^3*c^4)*sin(f*
x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sq
rt(2*c)*(-4480*a^3*c^4*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f
*x+exp(1))-1/4*pi))*sin(f*x+exp(1))/(128*f)^2-8064*a^3*c^4*f*sign(sin(1/2*(
f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(3*f*x+3*exp(1))
/(384*f)^2-4480*a^3*c^4*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(
f*x+exp(1))-1/4*pi))*sin(5*f*x+5*exp(1))/(640*f)^2-896*a^3*c^4*f*sign(sin(1
/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(7*f*x+7*exp
(1))/(896*f)^2-2560*a^3*c^4*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1
/2*(f*x+exp(1))-1/4*pi))*cos(2*f*x+2*exp(1))/(512*f)^2-9216*a^3*c^4*f*sign(
sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(4*f*x+
4*exp(1))/(1024*f)^2-7680*a^3*c^4*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign
(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(6*f*x+6*exp(1))/(1536*f)^2-2048*a^3*c^4*
f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos
(8*f*x+8*exp(1))/(2048*f)^2-4608*a^3*c^4*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi
))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-2*f*x-2*exp(1))/(-512*f)^2-5120*
a^3*c^4*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*
pi))*cos(-4*f*x-4*exp(1))/(-1024*f)^2-1536*a^3*c^4*f*sign(sin(1/2*(f*x+exp(
1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-6*f*x-6*exp(1))/(-1536
*f)^2)
```

maple [A] time = 0.37, size = 143, normalized size = 0.80

$$\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{9}{2}}\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{7}{2}}\sin\left(fx+e\right)\left(35\left(\cos^{8}\left(fx+e\right)\right)+5\left(\cos^{6}\left(fx+e\right)\right)\sin\left(fx+e\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] 1/280/f*(-c*(sin(f*x+e)-1))^(9/2)*(a*(1+sin(f*x+e)))^(7/2)*sin(f*x+e)*(35*c
os(f*x+e)^8+5*cos(f*x+e)^6*sin(f*x+e)+40*cos(f*x+e)^6+13*sin(f*x+e)*cos(f*x
+e)^4+48*cos(f*x+e)^4+29*cos(f*x+e)^2*sin(f*x+e)+64*cos(f*x+e)^2+93*sin(f*x
+e)+93)/cos(f*x+e)^9
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e) + c)^(9/2), x)
```

mupad [B] time = 11.30, size = 376, normalized size = 2.10

$$\mathrm{e}^{-e\,8\mathrm{i}-f\,x\,8\mathrm{i}}\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{35\,a^3\,c^4\,\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,\sin\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{32\,f}+\frac{7\,a^3\,c^4\,\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{64\,f}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(9/2),x)
```

```
[Out] (exp(- e*8i - f*x*8i)*(c - c*sin(e + f*x))^(1/2)*((35*a^3*c^4*exp(e*8i + f*
x*8i)*sin(e + f*x)*(a + a*sin(e + f*x))^(1/2))/(32*f) + (7*a^3*c^4*exp(e*8i
 + f*x*8i)*cos(2*e + 2*f*x)*(a + a*sin(e + f*x))^(1/2))/(64*f) + (7*a^3*c^4
*exp(e*8i + f*x*8i)*cos(4*e + 4*f*x)*(a + a*sin(e + f*x))^(1/2))/(128*f) +
(a^3*c^4*exp(e*8i + f*x*8i)*cos(6*e + 6*f*x)*(a + a*sin(e + f*x))^(1/2))/(6
4*f) + (a^3*c^4*exp(e*8i + f*x*8i)*cos(8*e + 8*f*x)*(a + a*sin(e + f*x))^(1
/2))/(512*f) + (7*a^3*c^4*exp(e*8i + f*x*8i)*sin(3*e + 3*f*x)*(a + a*sin(e
+ f*x))^(1/2))/(32*f) + (7*a^3*c^4*exp(e*8i + f*x*8i)*sin(5*e + 5*f*x)*(a +
```

```
 a*sin(e + f*x))^(1/2))/(160*f) + (a^3*c^4*exp(e*8i + f*x*8i)*sin(7*e + 7*f
*x)*(a + a*sin(e + f*x))^(1/2))/(224*f)))/(2*cos(e + f*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(9/2),x)
[Out] Timed out
```

3.370 $\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=179

$$-\frac{2a^4\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{35f\sqrt{a\sin(e+fx)+a}}-\frac{4a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}{35f}-\frac{a^2\cos(e+fx)(a$$

```
[Out] -1/7*a^2*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/f-1/7*a*c
os(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2)/f-2/35*a^4*cos(f*x+
e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(1/2)-4/35*a^3*cos(f*x+e)*(c-c
*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.37, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2740, 2738}

$$-\frac{2a^4\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{35f\sqrt{a\sin(e+fx)+a}}-\frac{4a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}{35f}-\frac{a^2\cos(e+fx)(a$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-2*a^4*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(35*f*Sqrt[a + a*Sin[e + f
*x]]) - (4*a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(
7/2))/(35*f) - (a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
f*x])^(7/2))/(7*f) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[
e + f*x])^(7/2))/(7*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
```

```
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{7/2}\,dx = -\frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}}{7f} + \frac{1}{7}$$
$$= -\frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{7f} - $$
$$= -\frac{4a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}}{35f} - $$
$$= -\frac{2a^4\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{35f\sqrt{a+a\sin(e+fx)}} - \frac{4a^3\cos(e+fx)\sqrt{a+}}{}$$

Mathematica [A] time = 0.99, size = 87, normalized size = 0.49

$$\frac{a^3c^3(1225\sin(e+fx)+245\sin(3(e+fx))+49\sin(5(e+fx))+5\sin(7(e+fx)))\sec(e+fx)\sqrt{a(\sin(e+fx)+}}{2240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a^3*c^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(
1225*Sin[e + f*x] + 245*Sin[3*(e + f*x)] + 49*Sin[5*(e + f*x)] + 5*Sin[7*(e
 + f*x)]))/(2240*f)
```

fricas [A] time = 0.44, size = 101, normalized size = 0.56

$$\frac{\left(5\,a^3c^3\cos\left(fx+e\right)^6+6\,a^3c^3\cos\left(fx+e\right)^4+8\,a^3c^3\cos\left(fx+e\right)^2+16\,a^3c^3\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+\right.}}{35\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/35*(5*a^3*c^3*cos(f*x + e)^6 + 6*a^3*c^3*cos(f*x + e)^4 + 8*a^3*c^3*cos(f
*x + e)^2 + 16*a^3*c^3)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*
sin(f*x + e)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-4480*a
^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*p
i))*sin(f*x+exp(1))/(128*f)^2-8064*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*
pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(3*f*x+3*exp(1))/(384*f)^2-4480*
a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*
pi))*sin(5*f*x+5*exp(1))/(640*f)^2-896*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-
1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(7*f*x+7*exp(1))/(896*f)^2)
```

maple [A] time = 0.31, size = 77, normalized size = 0.43

$$\frac{\left(5\left(\cos^{6}\left(fx+e\right)\right)+6\left(\cos^{4}\left(fx+e\right)\right)+8\left(\cos^{2}\left(fx+e\right)\right)+16\right)\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{7}{2}}\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{7}{2}}}{35f\cos\left(fx+e\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/35/f*(5*cos(f*x+e)^6+6*cos(f*x+e)^4+8*cos(f*x+e)^2+16)*(-c*(sin(f*x+e)-1)
)^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)/cos(f*x+e)^7
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e) + c)^(7/2), x)
```

mupad [B] time = 10.65, size = 179, normalized size = 1.00

$$\frac{\frac{1225\,a^3\,c^3\,\sin\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}{32}+\frac{245\,a^3\,c^3\,\sin\left(3\,e+3\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}{32}+\frac{49\,a^3\,c^3\,\sin\left(5\,e+5\,f\,x\right)\cdots}{\cdots}}{70\,f\,\cos\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] ((1225*a^3*c^3*sin(e + f*x)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))
^(1/2))/32 + (245*a^3*c^3*sin(3*e + 3*f*x)*(a + a*sin(e + f*x))^(1/2)*(c -
c*sin(e + f*x))^(1/2))/32 + (49*a^3*c^3*sin(5*e + 5*f*x)*(a + a*sin(e + f*x
))^(1/2)*(c - c*sin(e + f*x))^(1/2))/32 + (5*a^3*c^3*sin(7*e + 7*f*x)*(a +
a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2))/32)/(70*f*cos(e + f*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

3.371 $\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=134

$$\frac{c^3\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{15f\sqrt{c-c\sin(e+fx)}}+\frac{2c^2\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{15f}+\frac{c\cos(e+fx)(a\sin$$

```
[Out] 1/6*c*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2)/f+1/15*c^3*c
os(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(1/2)+2/15*c^2*cos(f*x+
e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.26, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2740, 2738}

$$\frac{2c^2\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{15f}+\frac{c^3\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{15f\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)(a\sin$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (c^3*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(15*f*Sqrt[c - c*Sin[e + f*x]
]) + (2*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]
])/(15*f) + (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])
^(3/2))/(6*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{5/2}\,dx = \frac{c\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}}{6f} + \frac{1}{3}(2$$

$$= \frac{2c^2\cos(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}}{15f} + \frac{c\,c$$

$$= \frac{c^3\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{15f\sqrt{c-c\sin(e+fx)}} + \frac{2c^2\cos(e+fx)(a+a\,s$$

Mathematica [A] time = 1.26, size = 107, normalized size = 0.80

$$\frac{a^3c^2\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}\,(600\sin(e+fx)+100\sin(3(e+fx))+12\sin(5(e+fx)}{960f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a^3*c^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(
-75*Cos[2*(e + f*x)] - 30*Cos[4*(e + f*x)] - 5*Cos[6*(e + f*x)] + 600*Sin[e
 + f*x] + 100*Sin[3*(e + f*x)] + 12*Sin[5*(e + f*x)]))/(960*f)
```

fricas [A] time = 0.46, size = 112, normalized size = 0.84

$$-\frac{\left(5\,a^3c^2\cos\left(fx+e\right)^6 - 5\,a^3c^2 - 2\left(3\,a^3c^2\cos\left(fx+e\right)^4 + 4\,a^3c^2\cos\left(fx+e\right)^2 + 8\,a^3c^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(f\right.}}{30\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] -1/30*(5*a^3*c^2*cos(f*x + e)^6 - 5*a^3*c^2 - 2*(3*a^3*c^2*cos(f*x + e)^4 +
 4*a^3*c^2*cos(f*x + e)^2 + 8*a^3*c^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) +
a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-80*a^3*c^2*f*sign(sin(1/2*(f*
x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(f*x+exp(1))/(16*f
)^2-480*a^3*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(
1))-1/4*pi))*sin(3*f*x+3*exp(1))/(96*f)^2-160*a^3*c^2*f*sign(sin(1/2*(f*x+e
xp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(5*f*x+5*exp(1))/(160
*f)^2+64*a^3*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp
(1))-1/4*pi))*cos(2*f*x+2*exp(1))/(64*f)^2+768*a^3*c^2*f*sign(sin(1/2*(f*x+
exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(4*f*x+4*exp(1))/(25
6*f)^2+384*a^3*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+e
xp(1))-1/4*pi))*cos(6*f*x+6*exp(1))/(384*f)^2+384*a^3*c^2*f*sign(sin(1/2*(f
*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-2*f*x-2*exp(1))
/(-128*f)^2+256*a^3*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(
f*x+exp(1))-1/4*pi))*cos(-4*f*x-4*exp(1))/(-256*f)^2)
```

maple [A] time = 0.31, size = 116, normalized size = 0.87

$$-\frac{\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{7}{2}}\left(-5\left(\cos^{6}\left(fx+e\right)\right)+\sin\left(fx+e\right)\left(\cos^{4}\left(fx+e\right)\right)\right.}{30f\cos\left(fx+e\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/30/f*(-c*(sin(f*x+e)-1))^(5/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(-5*c
os(f*x+e)^6+sin(f*x+e)*cos(f*x+e)^4-6*cos(f*x+e)^4+3*cos(f*x+e)^2*sin(f*x+e
)-8*cos(f*x+e)^2+11*sin(f*x+e)-11)/cos(f*x+e)^7
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e) + c)^(5/2), x)
```

mupad [B] time = 10.29, size = 124, normalized size = 0.93

$$-\frac{a^3\,c^2\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(75\,\cos\left(e+f\,x\right)+105\,\cos\left(3\,e+3\,f\,x\right)+35\,\cos\left(5\,e+\right.\right.}{960\,f\,\left(\cos\left(2\,e+2\,f\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] -(a^3*c^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(75*co
s(e + f*x) + 105*cos(3*e + 3*f*x) + 35*cos(5*e + 5*f*x) + 5*cos(7*e + 7*f*x
) - 700*sin(2*e + 2*f*x) - 112*sin(4*e + 4*f*x) - 12*sin(6*e + 6*f*x)))/(96
0*f*(cos(2*e + 2*f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.372 $\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=89

$$\frac{c^2\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{5f}$$

```
[Out] 1/10*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(1/2)+1/5*c*c
os(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.17, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2740, 2738}

$$\frac{c^2\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(10*f*Sqrt[c - c*Sin[e + f*x]
]) + (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]])/(
5*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}\,dx = \frac{c\cos(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}}{5f} + \frac{1}{5}(2c$$

$$= \frac{c^2\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10f\sqrt{c-c\sin(e+fx)}} + \frac{c\cos(e+fx)(a+a\sin$$

Mathematica [A] time = 0.96, size = 93, normalized size = 1.04

$$-\frac{a^3c\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\,\sqrt{c-c\sin(e+fx)}\,(-70\sin(e+fx)-5\sin(3(e+fx))+\sin(5(e+fx))+2}{80f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/80*(a^3*c*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x
]]*(20*Cos[2*(e + f*x)] + 5*Cos[4*(e + f*x)] - 70*Sin[e + f*x] - 5*Sin[3*(e
 + f*x)] + Sin[5*(e + f*x)]))/f
```

fricas [A] time = 0.43, size = 101, normalized size = 1.13

$$-\frac{\left(5\,a^3c\cos\left(fx+e\right)^4-5\,a^3c+2\left(a^3c\cos\left(fx+e\right)^4-2\,a^3c\cos\left(fx+e\right)^2-4\,a^3c\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)}}{10\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] -1/10*(5*a^3*c*cos(f*x + e)^4 - 5*a^3*c + 2*(a^3*c*cos(f*x + e)^4 - 2*a^3*c
*cos(f*x + e)^2 - 4*a^3*c)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*s
in(f*x + e) + c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sq
rt(2*a)*sqrt(2*c)*(-112*a^3*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos
(1/2*(f*x+exp(1))-1/4*pi))*sin(f*x+exp(1))/(16*f)^2-288*a^3*c*f*sign(sin(1/
2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(3*f*x+3*exp(
1))/(96*f)^2+160*a^3*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f
*x+exp(1))-1/4*pi))*sin(5*f*x+5*exp(1))/(160*f)^2+16*a^3*c*f*sign(sin(1/2*(
f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(2*f*x+2*exp(1))
/(16*f)^2+32*a^3*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+e
xp(1))-1/4*pi))*cos(4*f*x+4*exp(1))/(32*f)^2+16*a^3*c*f*sign(sin(1/2*(f*x+e
xp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-2*f*x-2*exp(1))/(-1
6*f)^2)
```

maple [A] time = 0.31, size = 106, normalized size = 1.19

$$-\frac{\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{3}{2}}\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{7}{2}}\sin\left(fx+e\right)\left(-2\left(\cos^{6}\left(fx+e\right)\right)+\sin\left(fx+e\right)\left(\cos^{4}\left(fx+e\right)\right)}{10f\cos\left(fx+e\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/10/f*(-c*(sin(f*x+e)-1))^(3/2)*(a*(1+sin(f*x+e)))^(7/2)*sin(f*x+e)*(-2*c
os(f*x+e)^6+sin(f*x+e)*cos(f*x+e)^4-2*cos(f*x+e)^4+3*cos(f*x+e)^2*sin(f*x+e
)+6*sin(f*x+e)-6)/cos(f*x+e)^7
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e) + c)^(3/2), x)
```

mupad [B] time = 9.19, size = 109, normalized size = 1.22

$$-\frac{a^3\,c\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(20\,\cos\left(e+f\,x\right)+25\,\cos\left(3\,e+3\,f\,x\right)+5\,\cos\left(5\,e+5\,f\right.}{80\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] -(a^3*c*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(20*cos(
e + f*x) + 25*cos(3*e + 3*f*x) + 5*cos(5*e + 5*f*x) - 75*sin(2*e + 2*f*x) -
 4*sin(4*e + 4*f*x) + sin(6*e + 6*f*x)))/(80*f*(cos(2*e + 2*f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.373 $\int (a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=43

$$\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/4*c*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2738}

$$\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}\,dx=\frac{c\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 0.34, size = 82, normalized size = 1.91

$$\frac{a^3\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}\,(56\sin(e+fx)-8\sin(3(e+fx))-28\cos(2(e+fx))+\cos}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (a^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(-28*
Cos[2*(e + f*x)] + Cos[4*(e + f*x)] + 56*Sin[e + f*x] - 8*Sin[3*(e + f*x)])
)/(32*f)
```

fricas [B] time = 0.44, size = 95, normalized size = 2.21

$$\frac{\left(a^{3} \cos \left(f x+e\right)^{4}-8 a^{3} \cos \left(f x+e\right)^{2}+7 a^{3}-4 \left(a^{3} \cos \left(f x+e\right)^{2}-2 a^{3}\right) \sin \left(f x+e\right)\right) \sqrt{a \sin \left(f x+e\right)+a} \sqrt{-c}}{4 f \cos \left(f x+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/4*(a^3*cos(f*x + e)^4 - 8*a^3*cos(f*x + e)^2 + 7*a^3 - 4*(a^3*cos(f*x + e
)^2 - 2*a^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(
2*c)*(-56*a^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1)
)-1/4*pi))*sin(f*x+exp(1))/(8*f)^2+72*a^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*p
i))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(3*f*x+3*exp(1))/(24*f)^2+480*a^3
*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*co
s(2*f*x+2*exp(1))/(32*f)^2-64*a^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign
(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(4*f*x+4*exp(1))/(64*f)^2-32*a^3*f*sign(s
in(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-2*f*x-
2*exp(1))/(-32*f)^2)
```

maple [B] time = 0.34, size = 103, normalized size = 2.40

$$-\frac{\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\,\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{7}{2}}\sin\left(fx+e\right)\left(-\left(\cos^{6}\left(fx+e\right)\right)+\sin\left(fx+e\right)\left(\cos^{4}\left(fx+e\right)\right)+\right.}{4f\cos\left(fx+e\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/4/f*(-c*(sin(f*x+e)-1))^(1/2)*(a*(1+sin(f*x+e)))^(7/2)*sin(f*x+e)*(-cos(
f*x+e)^6+sin(f*x+e)*cos(f*x+e)^4+cos(f*x+e)^2*sin(f*x+e)+cos(f*x+e)^2+4*sin
(f*x+e)-4)/cos(f*x+e)^7
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*sqrt(-c*sin(f*x + e) + c), x)
```

mupad [B] time = 8.29, size = 99, normalized size = 2.30

$$-\frac{a^{3}\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(28\,\cos\left(e+f\,x\right)+27\,\cos\left(3\,e+3\,f\,x\right)-\cos\left(5\,e+5\,f\,x\right)-\right.}{32\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] -(a^3*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(28*cos(e
+ f*x) + 27*cos(3*e + 3*f*x) - cos(5*e + 5*f*x) - 48*sin(2*e + 2*f*x) + 8*s
in(4*e + 4*f*x)))/(32*f*(cos(2*e + 2*f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.374 $\int \frac{(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=184

$$-\frac{8a^4\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{4a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}-\frac{a^2\cos(e+fx)(a\sin(e+fx)+a}{f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -a^2*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(1/2)-1/3*a*cos(f
*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(1/2)-8*a^4*cos(f*x+e)*ln(1
-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-4*a^3*cos(f*x+
e)*(a+a*sin(f*x+e))^(1/2)/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2740, 2737, 2667, 31}

$$-\frac{4a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}-\frac{a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f\sqrt{c-c\sin(e+fx)}}-\frac{8a^4\cos(e+fx)\log(1-\sin(e+fx}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (-8*a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c - c*Sin[e + f*x]]) - (4*a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(f*S
qrt[c - c*Sin[e + f*x]]) - (a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(f
*Sqrt[c - c*Sin[e + f*x]]) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3
*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}} + (2a)\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= -\frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}} + \left(4a^2\right)$$
$$= -\frac{4a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a\cos}{}$$
$$= -\frac{4a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a\cos}{}$$
$$= -\frac{4a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a\cos}{}$$
$$= -\frac{8a^4\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{4a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - $$

Mathematica [A] time = 1.04, size = 150, normalized size = 0.82

$$-\frac{a^3(\sin(e+fx)+1)^3\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(87\sin(e+fx)-\sin(3(e+fx))-\right.}{12f\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] -1/12*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*Sqrt[
a*(1 + Sin[e + f*x])]*(-12*Cos[2*(e + f*x)] + 192*Log[Cos[(e + f*x)/2] - Si
n[(e + f*x)/2]] + 87*Sin[e + f*x] - Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^7*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+\right.}}{c\sin\left(fx+e\right)-c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f
*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*sin(f*x + e)
 - c), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [B] time = 0.32, size = 367, normalized size = 1.99

$$\frac{\left(\sin\left(fx+e\right)\left(\cos^3\left(fx+e\right)\right)-\left(\cos^4\left(fx+e\right)\right)+5\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)+6\left(\cos^3\left(fx+e\right)\right)+24\sin\left(f\right.\right.}{3f\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/3/f*(sin(f*x+e)*cos(f*x+e)^3-cos(f*x+e)^4+5*cos(f*x+e)^2*sin(f*x+e)+6*cos
(f*x+e)^3+24*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+24*cos(f*x+e)*ln(2/(cos(f*x+e)
+1))-22*sin(f*x+e)*cos(f*x+e)-48*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/
```

```
sin(f*x+e))+17*cos(f*x+e)^2-48*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))-24*ln(2/(cos(f*x+e)+1))+16*sin(f*x+e)-6*cos(f*x+e)+48*ln(-(-1+cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))-16)*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*co
s(f*x+e)^3+cos(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+
e)*cos(f*x+e)-8*cos(f*x+e)^2+8*sin(f*x+e)-4*cos(f*x+e)+8)/(-c*(sin(f*x+e)-1
))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a \sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\sqrt{-c \sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)/sqrt(-c*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(7/2)/(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(7/2)/(c - c*sin(e + f*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.375 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=192

$$\frac{12a^4\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{6a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}+\frac{3a^2\cos(e+fx)(a\sin(e+fx)+}{2cf\sqrt{c-c\sin(e+fx)}}$$

```
[Out] a*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(3/2)+3/2*a^2*cos(f*
x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(1/2)+12*a^4*cos(f*x+e)*ln
(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+6*a^3*cos(
f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2739, 2740, 2737, 2667, 31}

$$\frac{6a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}+\frac{3a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}}+\frac{12a^4\cos(e+fx)\log(1-\sin(e+f}{cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(f*(c - c*Sin[e + f*x])^(3/2))
+ (12*a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c - c*Sin[e + f*x]]) + (6*a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/
(c*f*Sqrt[c - c*Sin[e + f*x]]) + (3*a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(
3/2))/(2*c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{f(c-c\sin(e+fx))^{3/2}} - \frac{(3a)\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{f(c-c\sin(e+fx))^{3/2}} + \frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}} - \frac{(6a}{}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{f(c-c\sin(e+fx))^{3/2}} + \frac{6a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} + \frac{3a^2 c}{}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{f(c-c\sin(e+fx))^{3/2}} + \frac{6a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} + \frac{3a^2 c}{}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{f(c-c\sin(e+fx))^{3/2}} + \frac{6a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} + \frac{3a^2 c}{}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{f(c-c\sin(e+fx))^{3/2}} + \frac{12a^4\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} +$$

Mathematica [A] time = 1.63, size = 179, normalized size = 0.93

$$-\frac{a^3\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin(3(e+fx))+18\cos(2(e+fx))+192\log\left(\cos\right.\right.}{8cf(\sin(e+fx)-1)\sqrt{c-c\sin(e+fx)}\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/8*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*
(44 + 18*Cos[2*(e + f*x)] + 192*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] +
(39 - 192*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])*Sin[e + f*x] + Sin[3*(e
 + f*x)]))/(c*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])*S
qrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 1.18, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+\right.}}{c^2\cos\left(fx+e\right)^2+2\,c^2\sin\left(fx+e\right)-2\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f
*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f*x +
e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [B] time = 0.29, size = 491, normalized size = 2.56

$$\frac{\left(\sin\left(fx+e\right)\left(\cos^3\left(fx+e\right)\right)-\left(\cos^4\left(fx+e\right)\right)+8\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)-48\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/2/f*(sin(f*x+e)*cos(f*x+e)^3-cos(f*x+e)^4+8*cos(f*x+e)^2*sin(f*x+e)-48*ln
(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)+24*ln(2/(cos
(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)+9*cos(f*x+e)^3+48*cos(f*x+e)^2*ln(-(-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))-24*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+25*si
n(f*x+e)*cos(f*x+e)+96*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)-48*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-33*cos(f*x+e)^2+48*cos(f*x+e)*ln(-(-1+
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-24*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-34*si
n(f*x+e)-9*cos(f*x+e)-96*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+48*ln(2
/(cos(f*x+e)+1))+34)*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(
f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8
*cos(f*x+e)^2+8*sin(f*x+e)-4*cos(f*x+e)+8)/(-c*(sin(f*x+e)-1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(7/2)/(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(7/2)/(c - c*sin(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.376 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=195

$$-\frac{6a^4\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{3a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f\sqrt{c-c\sin(e+fx)}}-\frac{3a^2\cos(e+fx)(a\sin(e+fx)+}{2cf(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 1/2*a*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(5/2)-3/2*a^2*co
s(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(3/2)-6*a^4*cos(f*x+e)
*ln(1-sin(f*x+e))/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-3*a^3
*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c^2/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2739, 2740, 2737, 2667, 31}

$$-\frac{3a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f\sqrt{c-c\sin(e+fx)}}-\frac{6a^4\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{3a^2\cos(e+fx)(a\sin(e+fx)+}{2cf(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(2*f*(c - c*Sin[e + f*x])^(5/2)
) - (3*a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c*f*(c - c*Sin[e + f
*x])^(3/2)) - (6*a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^2*f*Sqrt[a + a*
Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (3*a^3*Cos[e + f*x]*Sqrt[a + a*Si
n[e + f*x]])/(c^2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{(3a)\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{2c}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{3/2}} + \frac{(3a^2)\cdots}{\cdots}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{3/2}} - \frac{3a^3\,\mathrm{c}\cdots}{\cdots}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{3/2}} - \frac{3a^3\,\mathrm{c}\cdots}{\cdots}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{3/2}} - \frac{3a^3\,\mathrm{c}\cdots}{\cdots}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{5/2}} - \frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{3/2}} - \frac{6a\cdots}{c^2f\sqrt{\cdots}}$$

Mathematica [A] time = 1.94, size = 207, normalized size = 1.06

$$\frac{a^3\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin(3(e+fx))-72\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+\cdots\right.\right.\right.}{4c^2f(\sin(e+fx)-1)^2\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(-28
- 72*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 4*Cos[2*(e + f*x)]*(-1 + 6*
Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]) + (41 + 96*Log[Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]])*Sin[e + f*x] + Sin[3*(e + f*x)]))/(4*c^2*f*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 1.54, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\cdots\right.}}{3\,c^3\cos\left(fx+e\right)^2-4\,c^3-\left(c^3\cos\left(fx+e\right)^2-4\,c^3\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral((3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f
*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^3*cos(f*x
+ e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [B] time = 0.30, size = 618, normalized size = 3.17

$$\frac{\left(\sin\left(fx+e\right)\left(\cos^3\left(fx+e\right)\right)-6\sin\left(fx+e\right)\left(\cos^2\left(fx+e\right)\right)\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)+12\sin\left(fx+e\right)\left(\cos^2\left(fx+e\right)\right)\mathrm{l}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/f*(sin(f*x+e)*cos(f*x+e)^3-6*sin(f*x+e)*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))
+12*sin(f*x+e)*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-cos(
f*x+e)^4-6*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))+12*cos(f*x+e)^3*ln(-(-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))-11*cos(f*x+e)^2*sin(f*x+e)-12*ln(2/(cos(f*x+e)
+1))*sin(f*x+e)*cos(f*x+e)+24*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*si
n(f*x+e)*cos(f*x+e)-10*cos(f*x+e)^3+18*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-36
*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-6*sin(f*x+e)*cos(f
*x+e)+24*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-48*sin(f*x+e)*ln(-(-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))+17*cos(f*x+e)^2+12*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-2
4*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+16*sin(f*x+e)+10*co
s(f*x+e)-24*ln(2/(cos(f*x+e)+1))+48*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))-16)*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4*co
s(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^2
+8*sin(f*x+e)-4*cos(f*x+e)+8)/(-c*(sin(f*x+e)-1))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\ \sin\left(e+f\,x\right)\right)^{7/2}}{\left(c-c\ \sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(7/2)/(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(7/2)/(c - c*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.377 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=193

$$\frac{a^4\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f(c-c\sin(e+fx))^{3/2}}-\frac{a^2\cos(e+fx)(a\sin(e+fx)+a}{2cf(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 1/3*a*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(7/2)-1/2*a^2*co
s(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(5/2)+a^3*cos(f*x+e)*(
a+a*sin(f*x+e))^(1/2)/c^2/f/(c-c*sin(f*x+e))^(3/2)+a^4*cos(f*x+e)*ln(1-sin(
f*x+e))/c^3/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2739, 2737, 2667, 31}

$$\frac{a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f(c-c\sin(e+fx))^{3/2}}+\frac{a^4\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{a^2\cos(e+fx)(a\sin(e+fx)+a}{2cf(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*f*(c - c*Sin[e + f*x])^(7/2)
) - (a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c*f*(c - c*Sin[e + f*x
])^(5/2)) + (a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c^2*f*(c - c*Sin[e
 + f*x])^(3/2)) + (a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^3*f*Sqrt[a +
a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{c}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} + \frac{a^2\int \frac{(a}{(c}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} + \frac{a^3\cos}{c^2}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} + \frac{a^3\cos}{c^2}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} + \frac{a^3\cos}{c^2}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} + \frac{a^3\cos}{c^2}$$

Mathematica [A] time = 2.32, size = 232, normalized size = 1.20

$$\frac{a^3\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-3\sin(3(e+fx))\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+f\right.\right.\right.}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(-34
- 30*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 18*Cos[2*(e + f*x)]*(1 + Lo
g[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]) + 9*(4 + 5*Log[Cos[(e + f*x)/2] - S
in[(e + f*x)/2]])*Sin[e + f*x] - 3*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]
*Sin[3*(e + f*x)]))/(6*c^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Si
n[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 2.25, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx\right.}}{c^4\cos\left(fx+e\right)^4-8\,c^4\cos\left(fx+e\right)^2+8\,c^4+4\left(c^4\cos\left(fx+e\right)^2-2\,c^4\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] integral(-(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(
f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^4*cos(f*x +
 e)^4 - 8*c^4*cos(f*x + e)^2 + 8*c^4 + 4*(c^4*cos(f*x + e)^2 - 2*c^4)*sin(f
*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [B] time = 0.29, size = 748, normalized size = 3.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] -1/3/f*(-20-12*sin(f*x+e)*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+20*sin(f*x+e)+6
*cos(f*x+e)+28*cos(f*x+e)^2-14*sin(f*x+e)*cos(f*x+e)+24*sin(f*x+e)*cos(f*x+
```

```
e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+24*ln(-(-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)-12*ln(2/(cos(f*x+e)+1))*sin(f*x+e)
*cos(f*x+e)-48*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+24*c
os(f*x+e)^2*ln(2/(cos(f*x+e)+1))-6*cos(f*x+e)^3-8*cos(f*x+e)^4+48*ln(-(-1+c
os(f*x+e)+sin(f*x+e))/sin(f*x+e))-24*ln(2/(cos(f*x+e)+1))+24*sin(f*x+e)*ln(
2/(cos(f*x+e)+1))-48*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+
12*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-24*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))-6*sin(f*x+e)*cos(f*x+e)^3*ln(-(-1+cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))+3*sin(f*x+e)*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))-9*cos(f*x+e)^3*ln
(2/(cos(f*x+e)+1))+18*cos(f*x+e)^3*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))-14*cos(f*x+e)^2*sin(f*x+e)+8*sin(f*x+e)*cos(f*x+e)^3+6*cos(f*x+e)^4*ln(-
(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-3*cos(f*x+e)^4*ln(2/(cos(f*x+e)+1)))
*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4*cos(f*x+e
)^2*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^2+8*sin(
f*x+e)-4*cos(f*x+e)+8)/(-c*(sin(f*x+e)-1))^(7/2)
```

maxima [A] time = 1.01, size = 336, normalized size = 1.74

$$-\frac{\frac{6a^{\frac{7}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^{\frac{7}{2}}}-\frac{3a^{\frac{7}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{c^{\frac{7}{2}}}+\frac{4\left(\frac{3a^{\frac{7}{2}}\sqrt{c}\sin(fx+e)}{\cos(fx+e)+1}-\frac{6a^{\frac{7}{2}}\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{22a^{\frac{7}{2}}\sqrt{c}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{6a^{\frac{7}{2}}\sqrt{c}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{3a^{\frac{7}{2}}\sqrt{c}\,\mathrm{s}\cdots}{(\cos(fx\cdots}\right.}{c^4-\frac{6c^4\sin(fx+e)}{\cos(fx+e)+1}+\frac{15c^4\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{20c^4\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15c^4\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{6c^4\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{c^4\,\mathrm{s}\cdots}{(\cos\cdots}}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] -1/3*(6*a^(7/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(7/2) - 3*a^(7/2
)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(7/2) + 4*(3*a^(7/2)*sqrt(
c)*sin(f*x + e)/(cos(f*x + e) + 1) - 6*a^(7/2)*sqrt(c)*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 22*a^(7/2)*sqrt(c)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 -
6*a^(7/2)*sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3*a^(7/2)*sqrt(c)*s
in(f*x + e)^5/(cos(f*x + e) + 1)^5)/(c^4 - 6*c^4*sin(f*x + e)/(cos(f*x + e)
 + 1) + 15*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 20*c^4*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 15*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 6*c^4*s
in(f*x + e)^5/(cos(f*x + e) + 1)^5 + c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^
6))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^{7/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(7/2)/(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(7/2)/(c - c*sin(e + f*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

3.378 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=42

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

```
[Out] 1/8*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(9/2)
```

Rubi [A] time = 0.09, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(8*f*(c - c*Sin[e + f*x])^(9/2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

Mathematica [B] time = 4.50, size = 115, normalized size = 2.74

$$-\frac{a^3(\sin(3(e+fx))-7\sin(e+fx))\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{4c^4f(\sin(e+fx)-1)^4\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] -1/4*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*
(-7*Sin[e + f*x] + Sin[3*(e + f*x)]))/(c^4*f*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])*(-1 + Sin[e + f*x])^4*Sqrt[c - c*Sin[e + f*x]])
```

fricas [B] time = 0.48, size = 127, normalized size = 3.02

$$-\frac{\left(a^3\cos\left(fx+e\right)^2-2\,a^3\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin\left(fx+e\right)}{c^5f\cos\left(fx+e\right)^5-8\,c^5f\cos\left(fx+e\right)^3+8\,c^5f\cos\left(fx+e\right)+4\left(c^5f\cos\left(fx+e\right)^3-2\,c^5f\cos\left(fx+e\right)\right)\sin\left(f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="frica
s")
```

```
[Out] -(a^3*cos(f*x + e)^2 - 2*a^3)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)*sin(f*x + e)/(c^5*f*cos(f*x + e)^5 - 8*c^5*f*cos(f*x + e)^3 + 8*c^5*f
*cos(f*x + e) + 4*(c^5*f*cos(f*x + e)^3 - 2*c^5*f*cos(f*x + e))*sin(f*x + e
))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [B] time = 0.26, size = 154, normalized size = 3.67

$$-\frac{\left(-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)\left(\cos^2\left(fx+e\right)\right.}{f\left(\sin\left(fx+e\right)\left(\cos^3\left(fx+e\right)\right)+\cos^4\left(fx+e\right)-4\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)+3\left(\cos^3\left(fx+e\right)\right)-4\sin\left(f\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] -1/f*(-1+cos(f*x+e)+sin(f*x+e))*(cos(f*x+e)^2-2)*(a*(1+sin(f*x+e)))^(7/2)*s
in(f*x+e)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)+3
*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^2+8*sin(f*x+e)-4*cos(f*x
+e)+8)/(-c*(sin(f*x+e)-1))^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e) + c)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^{7/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(7/2)/(c - c*sin(e + f*x))^(9/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(7/2)/(c - c*sin(e + f*x))^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

3.379 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=88

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{80cf(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

```
[Out] 1/10*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(11/2)+1/80*cos(f
*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(9/2)
```

Rubi [A] time = 0.19, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2743, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{80cf(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(11/2),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(10*f*(c - c*Sin[e + f*x])^(11/2)
) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(80*c*f*(c - c*Sin[e + f*x])^
(9/2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx}{10c}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{80cf(c-c\sin(e+fx))^{9/2}}$$

Mathematica [B] time = 6.59, size = 331, normalized size = 3.76

$$-\frac{(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}{2f(c-c\sin(e+fx))^{11/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7}+\frac{2(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\right.\right.}{f(c-c\sin(e+fx))^{11/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(11/2),x]
```

```
[Out] (8*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2))/(5*f
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2)) - (3*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2)) + (2*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2)) - ((Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(2*f*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2))
```

fricas [B] time = 0.44, size = 163, normalized size = 1.85

$$-\frac{\left(5\,a^3\cos\left(fx+e\right)^2-6\,a^3+5\left(a^3\cos\left(fx+e\right)^2-2\,a^3\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c}}{10\left(5\,c^6f\cos\left(fx+e\right)^5-20\,c^6f\cos\left(fx+e\right)^3+16\,c^6f\cos\left(fx+e\right)-\left(c^6f\cos\left(fx+e\right)^5-12\,c^6f\cos\left(fx+e\right)^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="fric
as")
```

```
[Out] -1/10*(5*a^3*cos(f*x + e)^2 - 6*a^3 + 5*(a^3*cos(f*x + e)^2 - 2*a^3)*sin(f*
x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^6*f*cos(f*x
 + e)^5 - 20*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e) - (c^6*f*cos(f*x
+ e)^5 - 12*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e))*sin(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

maple [B] time = 0.30, size = 247, normalized size = 2.81

$$\frac{\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{7}{2}}\left(\sin\left(fx+e\right)\left(\cos^{4}\left(fx+e\right)\right)+\cos^{5}\left(fx+e\right)+5\sin\left(fx+e\right)\left(\cos^{3}\left(fx+e\right.\right.\right.}{10f\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{11}{2}}\left(\sin\left(fx+e\right)\left(\cos^{3}\left(fx+e\right)\right)+\cos^{4}\left(fx+e\right)-4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/2),x)
```

```
[Out] 1/10/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(sin(f*x+e)*cos(f*x+e)^4+cos(f*x
+e)^5+5*sin(f*x+e)*cos(f*x+e)^3-6*cos(f*x+e)^4-22*cos(f*x+e)^2*sin(f*x+e)-1
7*cos(f*x+e)^3-10*sin(f*x+e)*cos(f*x+e)+32*cos(f*x+e)^2+36*sin(f*x+e)+26*co
s(f*x+e)-36)/(-c*(sin(f*x+e)-1))^(11/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)
^4-4*cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f
*x+e)^2+8*sin(f*x+e)-4*cos(f*x+e)+8)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e) + c)^(11/2), x)
```

mupad [B] time = 12.45, size = 317, normalized size = 3.60

$$\frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{a^3\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,112\mathrm{i}}{5\,c^6\,f}+\frac{a^3\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sin\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,56\mathrm{i}}{c^6\,f}-\frac{a^3\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}}{}\right.}{\cos\left(e+f\,x\right)\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,264\mathrm{i}-\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)\,220\mathrm{i}+\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)\,20\mathrm{i}-\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sin\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(7/2)/(c - c*sin(e + f*x))^(11/2),x)
```

```
[Out] ((c - c*sin(e + f*x))^(1/2)*((a^3*exp(e*6i + f*x*6i)*(a + a*sin(e + f*x))^(
1/2)*112i)/(5*c^6*f) + (a^3*exp(e*6i + f*x*6i)*sin(e + f*x)*(a + a*sin(e +
f*x))^(1/2)*56i)/(c^6*f) - (a^3*exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(a + a*
sin(e + f*x))^(1/2)*16i)/(c^6*f) - (a^3*exp(e*6i + f*x*6i)*sin(3*e + 3*f*x)
*(a + a*sin(e + f*x))^(1/2)*8i)/(c^6*f)))/(cos(e + f*x)*exp(e*6i + f*x*6i)*
264i - exp(e*6i + f*x*6i)*cos(3*e + 3*f*x)*220i + exp(e*6i + f*x*6i)*cos(5*
e + 5*f*x)*20i - exp(e*6i + f*x*6i)*sin(2*e + 2*f*x)*330i + exp(e*6i + f*x*
6i)*sin(4*e + 4*f*x)*88i - exp(e*6i + f*x*6i)*sin(6*e + 6*f*x)*2i)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

3.380 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{13/2}}\,dx$

Optimal. Leaf size=133

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{480c^2f(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{60cf(c-c\sin(e+fx))^{11/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}$$

```
[Out] 1/12*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(13/2)+1/60*cos(f
*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(11/2)+1/480*cos(f*x+e)*(
a+a*sin(f*x+e))^(7/2)/c^2/f/(c-c*sin(f*x+e))^(9/2)
```

Rubi [A] time = 0.29, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2743, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{480c^2f(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{60cf(c-c\sin(e+fx))^{11/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(13/2),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(12*f*(c - c*Sin[e + f*x])^(13/2)
) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(60*c*f*(c - c*Sin[e + f*x])^
(11/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(480*c^2*f*(c - c*Sin[e
 + f*x])^(9/2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{11/2}}\,dx}{6c}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{60cf(c-c\sin(e+fx))^{11/2}} + \frac{\int \frac{(a+a\sin(e}{(c-c\sin(e}}{60}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{60cf(c-c\sin(e+fx))^{11/2}} + \frac{\cos(e+f}{480c^2f(}$$

Mathematica [B] time = 6.66, size = 335, normalized size = 2.52

$$-\frac{(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}{3f(c-c\sin(e+fx))^{13/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7}+\frac{3(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\right.\right.}{2f(c-c\sin(e+fx))^{13/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(13/2),x]
```

```
[Out] (4*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2)) - (12
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2))/(5*f
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2)) + (3*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(7/2))/(2*f*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2)) - ((Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2))
```

fricas [A] time = 0.46, size = 178, normalized size = 1.34

$$\frac{\left(15\,a^3\cos\left(fx+e\right)^2-18\,a^3+2\left(5\,a^3\cos\left(fx+e\right)^2-11\,a^3\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx\right.}}{30\left(c^7f\cos\left(fx+e\right)^7-18\,c^7f\cos\left(fx+e\right)^5+48\,c^7f\cos\left(fx+e\right)^3-32\,c^7f\cos\left(fx+e\right)+2\left(3\,c^7f\cos\left(fx+e\right)^5\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/2),x, algorithm="fric
as")
```

```
[Out] 1/30*(15*a^3*cos(f*x + e)^2 - 18*a^3 + 2*(5*a^3*cos(f*x + e)^2 - 11*a^3)*si
n(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^7*f*cos(f
*x + e)^7 - 18*c^7*f*cos(f*x + e)^5 + 48*c^7*f*cos(f*x + e)^3 - 32*c^7*f*co
s(f*x + e) + 2*(3*c^7*f*cos(f*x + e)^5 - 16*c^7*f*cos(f*x + e)^3 + 16*c^7*f
*cos(f*x + e))*sin(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

maple [B] time = 0.31, size = 276, normalized size = 2.08

$$-\frac{\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{7}{2}}\left(3\sin\left(fx+e\right)\left(\cos^{5}\left(fx+e\right)\right)-3\left(\cos^{6}\left(fx+e\right)\right)-21\sin\left(fx+e\right)\left(\cos^{4}\right.\right.}{30f\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{13}{2}}\left(\sin\left(fx+e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/2),x)
```

```
[Out] -1/30/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(3*sin(f*x+e)*cos(f*x+e)^5-3*co
s(f*x+e)^6-21*sin(f*x+e)*cos(f*x+e)^4-18*cos(f*x+e)^5-51*sin(f*x+e)*cos(f*x
+e)^3+72*cos(f*x+e)^4+157*cos(f*x+e)^2*sin(f*x+e)+106*cos(f*x+e)^3+78*sin(f
*x+e)*cos(f*x+e)-235*cos(f*x+e)^2-196*sin(f*x+e)-118*cos(f*x+e)+196)/(-c*(s
in(f*x+e)-1))^(13/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4*cos(f*x+e)^2*s
in(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^2+8*sin(f*x+e
)-4*cos(f*x+e)+8)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e) + c)^(13/2), x)
```

mupad [B] time = 12.06, size = 330, normalized size = 2.48

$$-\frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{224\,a^3\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{5\,c^7\,f}+\frac{416\,a^3\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\sin\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\right.}}{5\,c^7\,f}\right.}{-858\,\cos\left(e+f\,x\right)\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}+858\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)-130\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)+2\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(7/2)/(c - c*sin(e + f*x))^(13/2),x)
```

```
[Out] -((c - c*sin(e + f*x))^(1/2)*((224*a^3*exp(e*7i + f*x*7i)*(a + a*sin(e + f*
x))^(1/2))/(5*c^7*f) + (416*a^3*exp(e*7i + f*x*7i)*sin(e + f*x)*(a + a*sin(
e + f*x))^(1/2))/(5*c^7*f) - (32*a^3*exp(e*7i + f*x*7i)*cos(2*e + 2*f*x)*(a
 + a*sin(e + f*x))^(1/2))/(c^7*f) - (32*a^3*exp(e*7i + f*x*7i)*sin(3*e + 3*
f*x)*(a + a*sin(e + f*x))^(1/2))/(3*c^7*f)))/(858*exp(e*7i + f*x*7i)*cos(3*
e + 3*f*x) - 858*cos(e + f*x)*exp(e*7i + f*x*7i) - 130*exp(e*7i + f*x*7i)*c
os(5*e + 5*f*x) + 2*exp(e*7i + f*x*7i)*cos(7*e + 7*f*x) + 1144*exp(e*7i + f
*x*7i)*sin(2*e + 2*f*x) - 416*exp(e*7i + f*x*7i)*sin(4*e + 4*f*x) + 24*exp(
e*7i + f*x*7i)*sin(6*e + 6*f*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(13/2),x)
```

```
[Out] Timed out
```

3.381 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{15/2}}\,dx$

Optimal. Leaf size=178

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2240c^3f(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{280c^2f(c-c\sin(e+fx))^{11/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{56cf(c-c\sin(e+fx))^{13/2}}+\frac{\cos(e+}{14f}$$

```
[Out] 1/14*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(15/2)+1/56*cos(f
*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(13/2)+1/280*cos(f*x+e)*(
a+a*sin(f*x+e))^(7/2)/c^2/f/(c-c*sin(f*x+e))^(11/2)+1/2240*cos(f*x+e)*(a+a*
sin(f*x+e))^(7/2)/c^3/f/(c-c*sin(f*x+e))^(9/2)
```

Rubi [A] time = 0.38, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2743, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2240c^3f(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{280c^2f(c-c\sin(e+fx))^{11/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{56cf(c-c\sin(e+fx))^{13/2}}+\frac{\cos(e+}{14f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(15/2),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(14*f*(c - c*Sin[e + f*x])^(15/2)
) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(56*c*f*(c - c*Sin[e + f*x])^
(13/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(280*c^2*f*(c - c*Sin[e
 + f*x])^(11/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(2240*c^3*f*(c
 - c*Sin[e + f*x])^(9/2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
```

```
SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{15/2}}\,dx = \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}} + \frac{3\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{13/2}}\,dx}{14c}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{56cf(c-c\sin(e+fx))^{13/2}} + \frac{\int \frac{(a+a\sin(e}{(c-c\sin(e}}{28}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{56cf(c-c\sin(e+fx))^{13/2}} + \frac{\cos(e+f}{280c^2f(}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}} + \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{56cf(c-c\sin(e+fx))^{13/2}} + \frac{\cos(e+f}{280c^2f(}$$

Mathematica [A] time = 6.69, size = 333, normalized size = 1.87

$$-\frac{(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}{4f(c-c\sin(e+fx))^{15/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7}+\frac{6(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\right.\right.}{5f(c-c\sin(e+fx))^{15/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(15/2),x]
```

```
[Out] (8*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2))/(7*f
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2)) - (2*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2)) + (6*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(7/2))/(5*f*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2)) - ((Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(4*f*(Cos[(
e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2))
```

fricas [A] time = 0.49, size = 192, normalized size = 1.08

$$\frac{\left(63\,a^3\cos\left(fx+e\right)^2-76\,a^3+7\left(5\,a^3\cos\left(fx+e\right)^2-12\,a^3\right)\sin\left(fx+e\right)\right.}{140\left(7\,c^8f\cos\left(fx+e\right)^7-56\,c^8f\cos\left(fx+e\right)^5+112\,c^8f\cos\left(fx+e\right)^3-64\,c^8f\cos\left(fx+e\right)-\left(c^8f\cos\left(fx+e\right)^7\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/2),x, algorithm="fric
as")
```

```
[Out] 1/140*(63*a^3*cos(f*x + e)^2 - 76*a^3 + 7*(5*a^3*cos(f*x + e)^2 - 12*a^3)*s
in(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(7*c^8*f*co
s(f*x + e)^7 - 56*c^8*f*cos(f*x + e)^5 + 112*c^8*f*cos(f*x + e)^3 - 64*c^8*
f*cos(f*x + e) - (c^8*f*cos(f*x + e)^7 - 24*c^8*f*cos(f*x + e)^5 + 80*c^8*f
*cos(f*x + e)^3 - 64*c^8*f*cos(f*x + e))*sin(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

maple [A] time = 0.35, size = 302, normalized size = 1.70

$$-\frac{\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{7}{2}}\left(13\left(\cos^{6}\left(fx+e\right)\right)\sin\left(fx+e\right)+13\left(\cos^{7}\left(fx+e\right)\right)+91\sin\left(fx+e\right)\left(\mathrm{co}\right.\right.}{140f\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/2),x)
```

```
[Out] -1/140/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(13*cos(f*x+e)^6*sin(f*x+e)+13
*cos(f*x+e)^7+91*sin(f*x+e)*cos(f*x+e)^5-104*cos(f*x+e)^6-403*sin(f*x+e)*co
s(f*x+e)^4-312*cos(f*x+e)^5-637*sin(f*x+e)*cos(f*x+e)^3+1040*cos(f*x+e)^4+1
712*cos(f*x+e)^2*sin(f*x+e)+1075*cos(f*x+e)^3+756*sin(f*x+e)*cos(f*x+e)-246
8*cos(f*x+e)^2-1672*sin(f*x+e)-916*cos(f*x+e)+1672)/(-c*(sin(f*x+e)-1))^(15
/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)+3*cos(f
*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^2+8*sin(f*x+e)-4*cos(f*x+e)+8)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e) + c)^(15/2), x)
```

mupad [B] time = 13.32, size = 647, normalized size = 3.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(7/2)/(c - c*sin(e + f*x))^(15/2),x)
```

```
[Out] ((c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((
a^3*exp(e*6i + f*x*6i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*
x*1i)*1i)/2))^(1/2)*144i)/(5*c^8*f) - (8*a^3*exp(e*5i + f*x*5i)*(a + a*((ex
p(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(c^8*f) + (34
4*a^3*exp(e*7i + f*x*7i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i +
f*x*1i)*1i)/2))^(1/2))/(5*c^8*f) - (a^3*exp(e*8i + f*x*8i)*(a + a*((exp(- e
*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*2848i)/(35*c^8*f) -
 (344*a^3*exp(e*9i + f*x*9i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1
i + f*x*1i)*1i)/2))^(1/2))/(5*c^8*f) + (a^3*exp(e*10i + f*x*10i)*(a + a*((e
xp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*144i)/(5*c^8*
f) + (8*a^3*exp(e*11i + f*x*11i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp
(e*1i + f*x*1i)*1i)/2))^(1/2))/(c^8*f)))/(exp(e*1i + f*x*1i)*14i - 90*exp(e
*2i + f*x*2i) - exp(e*3i + f*x*3i)*350i + 910*exp(e*4i + f*x*4i) + exp(e*5i
 + f*x*5i)*1638i - 2002*exp(e*6i + f*x*6i) - exp(e*7i + f*x*7i)*1430i - exp
(e*9i + f*x*9i)*1430i + 2002*exp(e*10i + f*x*10i) + exp(e*11i + f*x*11i)*16
38i - 910*exp(e*12i + f*x*12i) - exp(e*13i + f*x*13i)*350i + 90*exp(e*14i +
 f*x*14i) + exp(e*15i + f*x*15i)*14i - exp(e*16i + f*x*16i) + 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(15/2),x)
```

```
[Out] Timed out
```

3.382 $\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{17/2}}\,dx$

Optimal. Leaf size=188

$$-\frac{a^4\cos(e+fx)}{280c^3f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{11/2}}+\frac{a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{56c^2f(c-c\sin(e+fx))^{13/2}}-\frac{3a^2\cos(e+fx)(a\sin(e}{56cf(c-c\sin(e+}$$

```
[Out] 1/8*a*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(17/2)-3/56*a^2*
cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(15/2)-1/280*a^4*cos
(f*x+e)/c^3/f/(c-c*sin(f*x+e))^(11/2)/(a+a*sin(f*x+e))^(1/2)+1/56*a^3*cos(f
*x+e)*(a+a*sin(f*x+e))^(1/2)/c^2/f/(c-c*sin(f*x+e))^(13/2)
```

Rubi [A] time = 0.38, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2739, 2738}

$$-\frac{a^4\cos(e+fx)}{280c^3f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{11/2}}+\frac{a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{56c^2f(c-c\sin(e+fx))^{13/2}}-\frac{3a^2\cos(e+fx)(a\sin(e}{56cf(c-c\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(17/2),x]
```

```
[Out] (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(8*f*(c - c*Sin[e + f*x])^(17/2
)) - (3*a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(56*c*f*(c - c*Sin[e +
 f*x])^(15/2)) + (a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(56*c^2*f*(c -
 c*Sin[e + f*x])^(13/2)) - (a^4*Cos[e + f*x])/(280*c^3*f*Sqrt[a + a*Sin[e +
 f*x]]*(c - c*Sin[e + f*x])^(11/2))
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
```

```
+ 1, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{17/2}}\,dx = \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{17/2}} - \frac{(3a)\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{15/2}}\,dx}{8c}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{17/2}} - \frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{56cf(c-c\sin(e+fx))^{15/2}} + \frac{(3a^2)}{}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{17/2}} - \frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{56cf(c-c\sin(e+fx))^{15/2}} + \frac{a^3\cos}{56}$$

$$= \frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{17/2}} - \frac{3a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{56cf(c-c\sin(e+fx))^{15/2}} + \frac{a^3\cos}{56}$$

Mathematica [A] time = 6.76, size = 329, normalized size = 1.75

$$-\frac{(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}{5f(c-c\sin(e+fx))^{17/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7}+\frac{(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}\right.\right.}{f(c-c\sin(e+fx))^{17/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)/(c - c*Sin[e + f*x])^(17/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2)) - (12*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2))/(7*f*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2)) + ((Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2)) - ((Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(5*f*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2))
```

fricas [A] time = 0.50, size = 204, normalized size = 1.09

$$-\frac{\left(14\,a^3\cos\left(fx+e\right)^2-17\,a^3+\left(7\,a^3\cos\left(fx+e\right)^2-18\,a^3\right)}{35\left(c^9f\cos\left(fx+e\right)^9-32\,c^9f\cos\left(fx+e\right)^7+160\,c^9f\cos\left(fx+e\right)^5-256\,c^9f\cos\left(fx+e\right)^3+128\,c^9f\cos\left(fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(17/2),x, algorithm="fric
as")
```

```
[Out] -1/35*(14*a^3*cos(f*x + e)^2 - 17*a^3 + (7*a^3*cos(f*x + e)^2 - 18*a^3)*sin
(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^9*f*cos(f*
x + e)^9 - 32*c^9*f*cos(f*x + e)^7 + 160*c^9*f*cos(f*x + e)^5 - 256*c^9*f*c
os(f*x + e)^3 + 128*c^9*f*cos(f*x + e) + 8*(c^9*f*cos(f*x + e)^7 - 10*c^9*f
*cos(f*x + e)^5 + 24*c^9*f*cos(f*x + e)^3 - 16*c^9*f*cos(f*x + e))*sin(f*x
+ e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(17/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

maple [A] time = 0.38, size = 328, normalized size = 1.74

$$\frac{\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{7}{2}}\sin\left(fx+e\right)\left(3\sin\left(fx+e\right)\left(\cos^{7}\left(fx+e\right)\right)-3\left(\cos^{8}\left(fx+e\right)\right)-27\left(\cos^{6}\left(fx+e\right)\right)\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(17/2),x)
```

```
[Out] 1/35/f*(a*(1+sin(f*x+e)))^(7/2)*sin(f*x+e)*(3*sin(f*x+e)*cos(f*x+e)^7-3*cos
(f*x+e)^8-27*cos(f*x+e)^6*sin(f*x+e)-24*cos(f*x+e)^7-93*sin(f*x+e)*cos(f*x+
e)^5+120*cos(f*x+e)^6+333*sin(f*x+e)*cos(f*x+e)^4+240*cos(f*x+e)^5+387*sin(
f*x+e)*cos(f*x+e)^3-720*cos(f*x+e)^4-970*cos(f*x+e)^2*sin(f*x+e)-583*cos(f*
x+e)^3-367*sin(f*x+e)*cos(f*x+e)+1337*cos(f*x+e)^2+769*sin(f*x+e)+402*cos(f
*x+e)-769)/(-c*(sin(f*x+e)-1))^(17/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4
-4*cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f*x
+e)^2+8*sin(f*x+e)-4*cos(f*x+e)+8)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(17/2),x, algorithm="maxi
ma")
```

```
[Out] Timed out
```

mupad [B] time = 13.98, size = 673, normalized size = 3.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(7/2)/(c - c*sin(e + f*x))^(17/2),x)
```

```
[Out] -((c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(
(a^3*exp(e*6i + f*x*6i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f
*x*1i)*1i)/2))^(1/2)*64i)/(5*c^9*f) + (256*a^3*exp(e*7i + f*x*7i)*(a + a*((
exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(5*c^9*f) -
 (a^3*exp(e*8i + f*x*8i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i +
f*x*1i)*1i)/2))^(1/2)*832i)/(7*c^9*f) - (1024*a^3*exp(e*9i + f*x*9i)*(a + a
*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(7*c^9*f
) + (a^3*exp(e*10i + f*x*10i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*
1i + f*x*1i)*1i)/2))^(1/2)*832i)/(7*c^9*f) + (256*a^3*exp(e*11i + f*x*11i)*
(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(5
*c^9*f) - (a^3*exp(e*12i + f*x*12i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (
exp(e*1i + f*x*1i)*1i)/2))^(1/2)*64i)/(5*c^9*f)))/(exp(e*1i + f*x*1i)*16i -
 119*exp(e*2i + f*x*2i) - exp(e*3i + f*x*3i)*544i + 1700*exp(e*4i + f*x*4i)
 + exp(e*5i + f*x*5i)*3808i - 6188*exp(e*6i + f*x*6i) - exp(e*7i + f*x*7i)*
7072i + 4862*exp(e*8i + f*x*8i) + 4862*exp(e*10i + f*x*10i) + exp(e*11i + f
*x*11i)*7072i - 6188*exp(e*12i + f*x*12i) - exp(e*13i + f*x*13i)*3808i + 17
00*exp(e*14i + f*x*14i) + exp(e*15i + f*x*15i)*544i - 119*exp(e*16i + f*x*1
6i) - exp(e*17i + f*x*17i)*16i + exp(e*18i + f*x*18i) + 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(17/2),x)
```

```
[Out] Timed out
```

3.383 $\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=139

$$\frac{4c^3\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] 1/2*c*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(1/2)+4*c^3*cos(
f*x+e)*ln(1+sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+2*c
^2*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2740, 2737, 2667, 31}

$$\frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}+\frac{4c^3\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(5/2)/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (4*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt
[c - c*Sin[e + f*x]]) + (2*c^2*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sq
rt[a + a*Sin[e + f*x]]) + (c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*f*
Sqrt[a + a*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
```

```
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} + (2c)\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$$

$$= \frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} + \left(4c^2\right)\int$$

$$= \frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} + \frac{\left(4ac^3 c\right.}{\sqrt{a+a s}}$$

$$= \frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} + \frac{\left(4c^3\cos(\right.}{f\sqrt{}}$$

$$= \frac{4c^3\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + \frac{c\,c}{}$$

Mathematica [A] time = 0.55, size = 136, normalized size = 0.98

$$-\frac{c^2(\sin(e+fx)-1)^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(12\sin(e+fx)+\cos(2(e+fx))-3\right.}{4f\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(5/2)/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -1/4*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*(Cos[
2*(e + f*x)] - 32*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 12*Sin[e + f*x
```

```
])*Sqrt[c - c*Sin[e + f*x]])/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*Sqr
t[a*(1 + Sin[e + f*x])])
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(c^2\cos\left(fx+e\right)^2+2\,c^2\sin\left(fx+e\right)-2\,c^2\right)\sqrt{-c\sin\left(fx+e\right)+c}}{\sqrt{a\sin\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2)*sqrt(-c*sin(f*x
 + e) + c)/sqrt(a*sin(f*x + e) + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (8*pi/x/2)>(-8*pi/x/2)4*sqrt(2*c)*(c^2*sqrt(a*tan(1/2*exp(
1))^2+a)*(54043195528445952*tan(1/2*exp(1))^5-162129586585337856*tan(1/2*ex
p(1))^4-180143985094819840*tan(1/2*exp(1))^3+108086391056891904*tan(1/2*exp
(1))^2+54043195528445952*tan(1/2*exp(1))-18014398509481984)+c^2*sqrt(a*tan(
1/2*exp(1))^2+a)*(108086391056891904*tan(1/2*exp(1))^6-324259173170675712*t
an(1/2*exp(1))^5-648518346341351424*tan(1/2*exp(1))^4+1080863910568919040*t
an(1/2*exp(1))^3+972777519512027136*tan(1/2*exp(1))^2-324259173170675712*ta
n(1/2*exp(1)))*tan(1/4*exp(1))^5+c^2*sqrt(a*tan(1/2*exp(1))^2+a)*(108086391
056891904*tan(1/2*exp(1))^6-324259173170675712*tan(1/2*exp(1))^5-6485183463
41351424*tan(1/2*exp(1))^4+1080863910568919040*tan(1/2*exp(1))^3+9727775195
12027136*tan(1/2*exp(1))^2-324259173170675712*tan(1/2*exp(1)))*tan(1/4*exp(
1))+c^2*sqrt(a*tan(1/2*exp(1))^2+a)*(810647932926689280*tan(1/2*exp(1))^5-2
431943798780067840*tan(1/2*exp(1))^4-2702159776422297600*tan(1/2*exp(1))^3+
1621295865853378560*tan(1/2*exp(1))^2+810647932926689280*tan(1/2*exp(1))-27
0215977642229760)*tan(1/4*exp(1))^4+c^2*sqrt(a*tan(1/2*exp(1))^2+a)*(-54043
195528445952*tan(1/2*exp(1))^5+162129586585337856*tan(1/2*exp(1))^4+1801439
85094819840*tan(1/2*exp(1))^3-108086391056891904*tan(1/2*exp(1))^2-54043195
528445952*tan(1/2*exp(1))+18014398509481984)*tan(1/4*exp(1))^6+c^2*sqrt(a*t
an(1/2*exp(1))^2+a)*(-360287970189639680*tan(1/2*exp(1))^6+108086391056891
```

```
040*tan(1/2*exp(1))^5+2161727821137838080*tan(1/2*exp(1))^4-360287970189639
6800*tan(1/2*exp(1))^3-3242591731706757120*tan(1/2*exp(1))^2+10808639105689
19040*tan(1/2*exp(1)))*tan(1/4*exp(1))^3+c^2*sqrt(a*tan(1/2*exp(1))^2+a)*(-
810647932926689280*tan(1/2*exp(1))^5+2431943798780067840*tan(1/2*exp(1))^4+
2702159776422297600*tan(1/2*exp(1))^3-1621295865853378560*tan(1/2*exp(1))^2
-810647932926689280*tan(1/2*exp(1))+270215977642229760)*tan(1/4*exp(1))^2)*
ln(abs(2*tan(1/2*exp(1))^3+6*tan(1/2*exp(1))^2+(tan(1/2*(1/2*f*x+2*exp(1)))
-1/tan(1/2*(1/2*f*x+2*exp(1))))*(tan(1/2*exp(1))^3-3*tan(1/2*exp(1))^2-3*ta
n(1/2*exp(1))+1)-6*tan(1/2*exp(1))-2))/f/(-9007199254740992*sqrt(2)*a*tan(1
/2*exp(1))^7-9007199254740992*sqrt(2)*a+(-9007199254740992*sqrt(2)*a*tan(1/
2*exp(1))^7+27021597764222976*sqrt(2)*a*tan(1/2*exp(1))^6+9007199254740992*
sqrt(2)*a*tan(1/2*exp(1))^5+45035996273704960*sqrt(2)*a*tan(1/2*exp(1))^4+4
5035996273704960*sqrt(2)*a*tan(1/2*exp(1))^3+9007199254740992*sqrt(2)*a*tan
(1/2*exp(1))^2-9007199254740992*sqrt(2)*a+27021597764222976*sqrt(2)*a*tan(1
/2*exp(1)))*tan(1/4*exp(1))^6+(-27021597764222976*sqrt(2)*a*tan(1/2*exp(1))
^7+81064793292668928*sqrt(2)*a*tan(1/2*exp(1))^6+27021597764222976*sqrt(2)*
a*tan(1/2*exp(1))^5+135107988821114880*sqrt(2)*a*tan(1/2*exp(1))^4+13510798
8821114880*sqrt(2)*a*tan(1/2*exp(1))^3+27021597764222976*sqrt(2)*a*tan(1/2*
exp(1))^2-27021597764222976*sqrt(2)*a+81064793292668928*sqrt(2)*a*tan(1/2*e
xp(1)))*tan(1/4*exp(1))^2+(-27021597764222976*sqrt(2)*a*tan(1/2*exp(1))^7+8
1064793292668928*sqrt(2)*a*tan(1/2*exp(1))^6+27021597764222976*sqrt(2)*a*ta
n(1/2*exp(1))^5+135107988821114880*sqrt(2)*a*tan(1/2*exp(1))^4+135107988821
114880*sqrt(2)*a*tan(1/2*exp(1))^3+27021597764222976*sqrt(2)*a*tan(1/2*exp(
1))^2-27021597764222976*sqrt(2)*a+81064793292668928*sqrt(2)*a*tan(1/2*exp(1
)))*tan(1/4*exp(1))^4+27021597764222976*sqrt(2)*a*tan(1/2*exp(1))^6+9007199
254740992*sqrt(2)*a*tan(1/2*exp(1))^5+45035996273704960*sqrt(2)*a*tan(1/2*e
xp(1))^4+45035996273704960*sqrt(2)*a*tan(1/2*exp(1))^3+9007199254740992*sqr
t(2)*a*tan(1/2*exp(1))^2+27021597764222976*sqrt(2)*a*tan(1/2*exp(1)))
```

maple [B] time = 0.29, size = 320, normalized size = 2.30

$$-\frac{\left(\cos^3\left(fx+e\right)-\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)+5\left(\cos^2\left(fx+e\right)\right)+6\sin\left(fx+e\right)\cos\left(fx+e\right)-16\cos\left(fx+e\right.\right.}{2f\left(\cos^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*(cos(f*x+e)^3-cos(f*x+e)^2*sin(f*x+e)+5*cos(f*x+e)^2+6*sin(f*x+e)*co
s(f*x+e)-16*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+8*cos(f*x
+e)*ln(2/(cos(f*x+e)+1))+16*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
*x+e))-8*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-cos(f*x+e)-5*sin(f*x+e)+16*ln(-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-8*ln(2/(cos(f*x+e)+1))-5)*(-c*(sin(f*x+
```

```
e)-1))^(5/2)/(cos(f*x+e)^3+cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2+2*sin(f*x
+e)*cos(f*x+e)-2*cos(f*x+e)-4*sin(f*x+e)+4)/(a*(1+sin(f*x+e)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(5/2)/sqrt(a*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int((c - c*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.384 $\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=93

$$\frac{2c^2\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] 2*c^2*cos(f*x+e)*ln(1+sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))
^(1/2)+c*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2740, 2737, 2667, 31}

$$\frac{2c^2\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(3/2)/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (2*c^2*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt
[c - c*Sin[e + f*x]]) + (c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sqrt[a
 + a*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx &= \frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}+(2c)\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx \\
&= \frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}+\frac{\left(2ac^2\cos(e+fx)\right)\int \frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} \\
&= \frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}+\frac{\left(2c^2\cos(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{a+x}\,dx,x,a\sin(e+\right.}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} \\
&= \frac{2c^2\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}
\end{aligned}
$$

Mathematica [A] time = 0.31, size = 119, normalized size = 1.28

$$
\frac{c(\sin(e+fx)-1)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin(e+fx)-4\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right.\right.}{f\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(3/2)/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])*(-4*Log[Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2]] + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(f
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((-c*sin(f*x + e) + c)^(3/2)/sqrt(a*sin(f*x + e) + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (8*pi/x/2)>(-8*pi/x/2)4*sqrt(2*c)*(c*sqrt(a*tan(1/2*exp(1)
)^2+a)*(25165824*tan(1/2*exp(1))^5-75497472*tan(1/2*exp(1))^4-83886080*tan(
1/2*exp(1))^3+50331648*tan(1/2*exp(1))^2+25165824*tan(1/2*exp(1))-8388608)+
c*sqrt(a*tan(1/2*exp(1))^2+a)*(50331648*tan(1/2*exp(1))^6-150994944*tan(1/2
*exp(1))^5-301989888*tan(1/2*exp(1))^4+503316480*tan(1/2*exp(1))^3+45298483
2*tan(1/2*exp(1))^2-150994944*tan(1/2*exp(1)))*tan(1/4*exp(1))^5+c*sqrt(a*t
an(1/2*exp(1))^2+a)*(50331648*tan(1/2*exp(1))^6-150994944*tan(1/2*exp(1))^5
-301989888*tan(1/2*exp(1))^4+503316480*tan(1/2*exp(1))^3+452984832*tan(1/2*
exp(1))^2-150994944*tan(1/2*exp(1)))*tan(1/4*exp(1))+c*sqrt(a*tan(1/2*exp(1
))^2+a)*(377487360*tan(1/2*exp(1))^5-1132462080*tan(1/2*exp(1))^4-125829120
0*tan(1/2*exp(1))^3+754974720*tan(1/2*exp(1))^2+377487360*tan(1/2*exp(1))-1
25829120)*tan(1/4*exp(1))^4+c*sqrt(a*tan(1/2*exp(1))^2+a)*(-25165824*tan(1/
2*exp(1))^5+75497472*tan(1/2*exp(1))^4+83886080*tan(1/2*exp(1))^3-50331648*
tan(1/2*exp(1))^2-25165824*tan(1/2*exp(1))+8388608)*tan(1/4*exp(1))^6+c*sqr
t(a*tan(1/2*exp(1))^2+a)*(-167772160*tan(1/2*exp(1))^6+503316480*tan(1/2*ex
p(1))^5+1006632960*tan(1/2*exp(1))^4-1677721600*tan(1/2*exp(1))^3-150994944
0*tan(1/2*exp(1))^2+503316480*tan(1/2*exp(1)))*tan(1/4*exp(1))^3+c*sqrt(a*t
an(1/2*exp(1))^2+a)*(-377487360*tan(1/2*exp(1))^5+1132462080*tan(1/2*exp(1)
)^4+1258291200*tan(1/2*exp(1))^3-754974720*tan(1/2*exp(1))^2-377487360*tan(
1/2*exp(1))+125829120)*tan(1/4*exp(1))^2)*ln(abs(2*tan(1/2*exp(1))^3+6*tan(
1/2*exp(1))^2+(tan(1/2*(1/2*f*x+2*exp(1)))-1/tan(1/2*(1/2*f*x+2*exp(1))))*(
tan(1/2*exp(1))^3-3*tan(1/2*exp(1))^2-3*tan(1/2*exp(1))+1)-6*tan(1/2*exp(1)
)-2))/f/(-8388608*sqrt(2)*a*tan(1/2*exp(1))^7-8388608*sqrt(2)*a+(-8388608*s
```

```
qrt(2)*a*tan(1/2*exp(1))^7+25165824*sqrt(2)*a*tan(1/2*exp(1))^6+8388608*sqr
t(2)*a*tan(1/2*exp(1))^5+41943040*sqrt(2)*a*tan(1/2*exp(1))^4+41943040*sqrt
(2)*a*tan(1/2*exp(1))^3+8388608*sqrt(2)*a*tan(1/2*exp(1))^2-8388608*sqrt(2)
*a+25165824*sqrt(2)*a*tan(1/2*exp(1)))*tan(1/4*exp(1))^6+(-25165824*sqrt(2)
*a*tan(1/2*exp(1))^7+75497472*sqrt(2)*a*tan(1/2*exp(1))^6+25165824*sqrt(2)*
a*tan(1/2*exp(1))^5+125829120*sqrt(2)*a*tan(1/2*exp(1))^4+125829120*sqrt(2)
*a*tan(1/2*exp(1))^3+25165824*sqrt(2)*a*tan(1/2*exp(1))^2-25165824*sqrt(2)*
a+75497472*sqrt(2)*a*tan(1/2*exp(1)))*tan(1/4*exp(1))^2+(-25165824*sqrt(2)*
a*tan(1/2*exp(1))^7+75497472*sqrt(2)*a*tan(1/2*exp(1))^6+25165824*sqrt(2)*a
*tan(1/2*exp(1))^5+125829120*sqrt(2)*a*tan(1/2*exp(1))^4+125829120*sqrt(2)*
a*tan(1/2*exp(1))^3+25165824*sqrt(2)*a*tan(1/2*exp(1))^2-25165824*sqrt(2)*a
+75497472*sqrt(2)*a*tan(1/2*exp(1)))*tan(1/4*exp(1))^4+25165824*sqrt(2)*a*t
an(1/2*exp(1))^6+8388608*sqrt(2)*a*tan(1/2*exp(1))^5+41943040*sqrt(2)*a*tan
(1/2*exp(1))^4+41943040*sqrt(2)*a*tan(1/2*exp(1))^3+8388608*sqrt(2)*a*tan(1
/2*exp(1))^2+25165824*sqrt(2)*a*tan(1/2*exp(1)))
```

maple [B] time = 0.27, size = 261, normalized size = 2.81

$$-\frac{\left(\sin\left(fx+e\right)\cos\left(fx+e\right)+4\sin\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-2\sin\left(fx+e\right)\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)+\cos^{2}}{f\left(\sin\left(fx+e\right)\mathrm{co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(sin(f*x+e)*cos(f*x+e)+4*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))-2*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+cos(f*x+e)^2-4*cos(f*x+e)*ln(-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+2*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-sin(
f*x+e)+4*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-2*ln(2/(cos(f*x+e)+1))-
1)*(-c*(sin(f*x+e)-1))^(3/2)/(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f*x+
e)-cos(f*x+e)+2)/(a*(1+sin(f*x+e)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(3/2)/sqrt(a*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^(1/2),x)
[Out] int((c - c*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(1/2),x)
[Out] Integral((-c*(sin(e + f*x) - 1))**(3/2)/sqrt(a*(sin(e + f*x) + 1)), x)
```

3.385 $\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=49

$$\frac{c\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] c*cos(f*x+e)*ln(1+sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/
2)
```

Rubi [A] time = 0.10, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2737, 2667, 31}

$$\frac{c\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c*Sin[e + f*x]]/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (c*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
 c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{(ac\cos(e+fx))\int \frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$= \frac{(c\cos(e+fx))\,\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\sin(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$= \frac{c\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

Mathematica [C] time = 0.98, size = 118, normalized size = 2.41

$$-\frac{\sqrt{2}\left(e^{i(e+fx)}+i\right)\left(fx+2i\log\left(e^{i(e+fx)}+i\right)\right)\sqrt{c-c\sin(e+fx)}}{f\left(e^{i(e+fx)}-i\right)\sqrt{-iae^{-i(e+fx)}\left(e^{i(e+fx)}+i\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c*Sin[e + f*x]]/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[2]*(I + E^(I*(e + f*x)))*(f*x + (2*I)*Log[I + E^(I*(e + f*x))])*Sqr
t[c - c*Sin[e + f*x]])/((-I + E^(I*(e + f*x)))*Sqrt[((-I)*a*(I + E^(I*(e +
f*x)))^2)/E^(I*(e + f*x))]*f))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c\sin\left(fx+e\right)+c}}{\sqrt{a\sin\left(fx+e\right)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c*sin(f*x + e) + c)/sqrt(a*sin(f*x + e) + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)4*sqrt(2*c)*(-1/2*ln(ta
n(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^2+1)+1/2*ln(abs(tan(1/2*(1/2*f*x+1/4*(2*
exp(1)-pi)))^2-1)))*sign(sin(1/2*(f*x+exp(1))-1/4*pi))/sqrt(2)/sqrt(a)/f/si
gn(tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^4-1)
```

maple [B] time = 0.23, size = 106, normalized size = 2.16

$$-\frac{\left(1-\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\,\left(\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)-2\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\right)}{f\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,\left(-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(1-cos(f*x+e)+sin(f*x+e))*(-c*(sin(f*x+e)-1))^(1/2)*(ln(2/(cos(f*x+e)+
1))-2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)))/(a*(1+sin(f*x+e)))^(1/2)/
(-1+cos(f*x+e)+sin(f*x+e))
```

maxima [A] time = 0.60, size = 64, normalized size = 1.31

$$-\frac{\frac{2\,\sqrt{c}\,\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{\sqrt{a}}-\frac{\sqrt{c}\,\log\left(\frac{\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+1\right)}{\sqrt{a}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -(2*sqrt(c)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/sqrt(a) - sqrt(c)*log(
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(a))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int((c - c*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(sin(e + f*x) - 1))/sqrt(a*(sin(e + f*x) + 1)), x)
```

3.386 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=46

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] arctanh(sin(f*x+e))*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1
/2)
```

Rubi [A] time = 0.09, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2741, 3770}

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx &= \frac{\cos(e+fx)\int \sec(e+fx)\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\\ &= \frac{\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

Mathematica [A]   time = 0.23, size = 89, normalized size = 1.93

$$-\frac{\cos(e+fx)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\right)}{f\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] -((Cos[e + f*x]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*
x)/2] + Sin[(e + f*x)/2]]))/(f*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e
+ f*x]]))
```

fricas [A]   time = 0.51, size = 160, normalized size = 3.48

$$\left[\frac{\sqrt{ac}\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)^{3}}\right)}{2\,acf}, -\frac{\sqrt{-ac}\,\arctan\left(\frac{\sqrt{-ac}\,\sqrt{a\sin\left(fx+e\right)+a}}{ac\cos\left(fx+e\right)}\right.}{acf}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] [1/2*sqrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*
sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x +
e)^3)/(a*c*f), -sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(
-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e)))/(a*c*f)]
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)), x)
```

maple [B] time = 0.25, size = 92, normalized size = 2.00

$$-\frac{\left(\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)-\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)\right)\cos\left(fx+e\right)}{f\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-ln(-(-1+cos(f*x+e)-sin(f*x
+e))/sin(f*x+e)))*cos(f*x+e)/(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(
1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(e + f*x) + 1))*sqrt(-c*(sin(e + f*x) - 1))), x)
```

3.387 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=95

$$\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/2*cos(f*x+e)/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/2*arctanh(
sin(f*x+e))*cos(f*x+e)/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2743, 2741, 3770}

$$\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] Cos[e + f*x]/(2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) + (A
rcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
 c*Sin[e + f*x]])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} + \frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,\sqrt{}}}{2c}$$

$$= \frac{\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} + \frac{\cos(e+fx}{2c\sqrt{a+a\sin(e+}}$$

$$= \frac{\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} + \frac{\tanh^{-1}(\sin(}{2cf\sqrt{a+a\sin(e}}$$

Mathematica [A] time = 0.40, size = 161, normalized size = 1.69

$$-\frac{\cos(e+fx)\left(-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)+\sin(e+f}{2cf(\sin(e+fx)-1)\sqrt{a(\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -1/2*(Cos[e + f*x]*(1 - Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[
(e + f*x)/2] + Sin[(e + f*x)/2]] + (Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x]))/(c*f*(-1 + Sin
[e + f*x])*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

fricas [A] time = 0.53, size = 311, normalized size = 3.27

$$\left[\frac{\sqrt{ac}\left(\cos\left(fx+e\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)\right)\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\mathrm{si}}{\cos\left(fx+e\right)^{3}}\right.}{4\left(ac^{2}f\cos\left(fx+e\right)\sin\left(fx+e\right)-ac^{2}f\cos\left(fx+e\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] [1/4*(sqrt(a*c)*(cos(f*x + e)*sin(f*x + e) - cos(f*x + e))*log(-(a*c*cos(f*
x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-
c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) - 2*sqrt(a*sin(f*x + e) +
 a)*sqrt(-c*sin(f*x + e) + c))/(a*c^2*f*cos(f*x + e)*sin(f*x + e) - a*c^2*f
*cos(f*x + e)), -1/2*(sqrt(-a*c)*(cos(f*x + e)*sin(f*x + e) - cos(f*x + e))
*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*
cos(f*x + e)*sin(f*x + e))) + sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c))/(a*c^2*f*cos(f*x + e)*sin(f*x + e) - a*c^2*f*cos(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(3/2)), x)
```

maple [A] time = 0.27, size = 165, normalized size = 1.74

$$\frac{\left(\sin\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-\sin\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right) \cdots}{2f\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/2/f*(sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-sin(f*x+e)*ln(
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))+ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+sin(f*x+e))*cos(f*x+e)/(
a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(3/2)),x)
[Out] int(1/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(3/2),x)
[Out] Integral(1/(sqrt(a*(sin(e + f*x) + 1))*(-c*(sin(e + f*x) - 1))**(3/2)), x)
```

3.388 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=140

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{4c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{4cf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{\cos(e+}{4f\sqrt{a\sin(e+fx)+a}\,(}$$

```
[Out] 1/4*cos(f*x+e)/f/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)+1/4*cos(f*x+
e)/c/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/4*arctanh(sin(f*x+e)
)*cos(f*x+e)/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2743, 2741, 3770}

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{4c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{4cf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{\cos(e+}{4f\sqrt{a\sin(e+fx)+a}\,(}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] Cos[e + f*x]/(4*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) + Co
s[e + f*x]/(4*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) + (A
rcTanh[Sin[e + f*x]]*Cos[e + f*x])/(4*c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} + \frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c}}{2c}$$

$$= \frac{\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} + \frac{c}{4cf\sqrt{a+a\sin(e}}$$

$$= \frac{\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} + \frac{c}{4cf\sqrt{a+a\sin(e}}$$

$$= \frac{\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} + \frac{c}{4cf\sqrt{a+a\sin(e}}$$

Mathematica [A] time = 0.63, size = 224, normalized size = 1.60

$$\frac{\cos(e+fx)\left(-3\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\cos(2(e+fx))\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (Cos[e + f*x]*(4 - 3*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Cos[2*(e +
f*x)]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Si
n[(e + f*x)/2]]) + 3*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (-2 + 4*Log
[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 4*Log[Cos[(e + f*x)/2] + Sin[(e + f
*x)/2]])*Sin[e + f*x]))/(8*c^2*f*(-1 + Sin[e + f*x])^2*Sqrt[a*(1 + Sin[e +
f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

fricas [A] time = 0.52, size = 376, normalized size = 2.69

$$\left[\frac{\left(\cos\left(fx+e\right)^{3}+2\cos\left(fx+e\right)\sin\left(fx+e\right)-2\cos\left(fx+e\right)\right)\sqrt{ac}\,\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\,\sqrt{a\sin\left(fx\right.}}{\cos\left(fx+e\right)^{3}}\right.}{8\left(ac^{3}f\cos\left(fx+e\right)^{3}+2\,ac^{3}f\cos\left(fx+e\right)\sin\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] [1/8*((cos(f*x + e)^3 + 2*cos(f*x + e)*sin(f*x + e) - 2*cos(f*x + e))*sqrt(
a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin
(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) + 2*
sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(sin(f*x + e) - 2))/(a*c
^3*f*cos(f*x + e)^3 + 2*a*c^3*f*cos(f*x + e)*sin(f*x + e) - 2*a*c^3*f*cos(f
*x + e)), -1/4*((cos(f*x + e)^3 + 2*cos(f*x + e)*sin(f*x + e) - 2*cos(f*x +
 e))*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
+ e) + c)/(a*c*cos(f*x + e)*sin(f*x + e))) - sqrt(a*sin(f*x + e) + a)*sqrt(
-c*sin(f*x + e) + c)*(sin(f*x + e) - 2))/(a*c^3*f*cos(f*x + e)^3 + 2*a*c^3*
f*cos(f*x + e)*sin(f*x + e) - 2*a*c^3*f*cos(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(5/2)), x)
```

maple [B] time = 0.27, size = 252, normalized size = 1.80

$$\frac{\left(\left(\cos^{2}\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-\left(\cos^{2}\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+2\sin\left(fx+e\right)\ln\left(-\frac{-}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/4/f*(cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-cos(f*x+e)^2
*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+2*sin(f*x+e)*ln(-(-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))-2*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*
x+e))+2*cos(f*x+e)^2+3*sin(f*x+e)-2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))+2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-2)*cos(f*x+e)/(a*(1+sin(f*
x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(e + f*x) + 1))*(-c*(sin(e + f*x) - 1))**(5/2)), x)
```

3.389 $\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=191

$$-\frac{12c^4\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{6c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}-\frac{3c^2\cos(e+fx)(c-c\sin(e+fx)}{2af\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -c*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(3/2)-3/2*c^2*cos(f
*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f/(a+a*sin(f*x+e))^(1/2)-12*c^4*cos(f*x+e)*l
n(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-6*c^3*cos
(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2739, 2740, 2737, 2667, 31}

$$-\frac{6c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}-\frac{3c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a\sin(e+fx)+a}}-\frac{12c^4\cos(e+fx)\log(\sin(e+fx)+}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(7/2)/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-12*c^4*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*
Sqrt[c - c*Sin[e + f*x]]) - (6*c^3*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(
a*f*Sqrt[a + a*Sin[e + f*x]]) - (3*c^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3
/2))/(2*a*f*Sqrt[a + a*Sin[e + f*x]]) - (c*Cos[e + f*x]*(c - c*Sin[e + f*x]
)^(5/2))/(f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{f(a+a\sin(e+fx))^{3/2}} - \frac{(3c)\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}$$

$$= -\frac{3c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{f(a+a\sin(e+fx))^{3/2}} - \frac{(6c^2)}{}$$

$$= -\frac{6c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{3c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{c\cos}{}$$

$$= -\frac{6c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{3c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{c\cos}{}$$

$$= -\frac{6c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{3c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{c\cos}{}$$

$$= -\frac{12c^4\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{6c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} -$$

Mathematica [A] time = 1.61, size = 162, normalized size = 0.85

$$\frac{c^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin(3(e+fx))-18\cos(2(e+fx))-192\log\left(\sin\left(\frac{1}{2}(e\right.\right.\right.}{8f(a(\sin(e+fx)+1))^{3/2}\left(\cos\left(\frac{1}{2}(e+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(7/2)/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(-44 -
18*Cos[2*(e + f*x)] - 192*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (39 -
192*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x] + Sin[3*(e + f*x
)]))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2
))
```

fricas [F] time = 1.11, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(3\,c^3\cos\left(fx+e\right)^2-4\,c^3-\left(c^3\cos\left(fx+e\right)^2-4\,c^3\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)}}{a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f
*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*cos(f*x +
e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [B] time = 0.30, size = 501, normalized size = 2.62

$$-\frac{\left(\sin\left(fx+e\right)\left(\cos^{3}\left(fx+e\right)\right)+\cos^{4}\left(fx+e\right)+8\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)-48\sin\left(fx+e\right)\cos\left(fx+e\right)\ln\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/2/f*(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4+8*cos(f*x+e)^2*sin(f*x+e)-48*s
in(f*x+e)*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+24*ln(2/(co
s(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)-9*cos(f*x+e)^3-48*cos(f*x+e)^2*ln(-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))+24*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+25*s
in(f*x+e)*cos(f*x+e)+96*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))-48*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+33*cos(f*x+e)^2-48*cos(f*x+e)*ln(-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+24*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-34*s
in(f*x+e)+9*cos(f*x+e)+96*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-48*ln(
2/(cos(f*x+e)+1))-34)*(-c*(sin(f*x+e)-1))^(7/2)/(sin(f*x+e)*cos(f*x+e)^3-co
s(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)
+8*cos(f*x+e)^2+8*sin(f*x+e)+4*cos(f*x+e)-8)/(a*(1+sin(f*x+e)))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-c\,\sin\left(e+f\,x\right)\right)^{7/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(7/2)/(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int((c - c*sin(e + f*x))^(7/2)/(a + a*sin(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.390 $\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=143

$$-\frac{4c^3\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}-\frac{c\cos(e+fx)(c-c\sin(e+fx))}{f(a\sin(e+fx)+a)^{3/2}}$$

```
[Out] -c*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(3/2)-4*c^3*cos(f*x
+e)*ln(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-2*c^
2*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2739, 2740, 2737, 2667, 31}

$$-\frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}-\frac{4c^3\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{c\cos(e+fx)(c-c\sin(e+fx))}{f(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-4*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*S
qrt[c - c*Sin[e + f*x]]) - (2*c^2*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a
*f*Sqrt[a + a*Sin[e + f*x]]) - (c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/
(f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
```

```
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{f(a+a\sin(e+fx))^{3/2}} - \frac{(2c)\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}$$

$$= -\frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{f(a+a\sin(e+fx))^{3/2}} - \frac{\left(4c^2\right)\int$$

$$= -\frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{f(a+a\sin(e+fx))^{3/2}} - \frac{\left(4c^3\right.}{\sqrt{a+a}}$$

$$= -\frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{f(a+a\sin(e+fx))^{3/2}} - \frac{\left(4c^3\cos\right.}{a}$$

$$= -\frac{4c^3\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} -$$

Mathematica [A] time = 0.77, size = 153, normalized size = 1.07

$$-\frac{c^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos(2(e+fx))+16\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e\right.\right.\right.}{2f(a(\sin(e+fx)+1))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/2*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(7
 + Cos[2*(e + f*x)] + 16*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 2*(-1 +
 8*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x]))/(f*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2))
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^2\cos\left(fx+e\right)^2+2\,c^2\sin\left(fx+e\right)-2\,c^2\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2)*sqrt(a*sin(f*x +
 e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e)
 - 2*a^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [B] time = 0.27, size = 446, normalized size = 3.12

$$\frac{\left(\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)+4\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)\sin\left(fx+e\right)\cos\left(fx+e\right)-8\sin\left(fx+e\right)\cos\left(fx+e\right)\ln\left(-\frac{-1}{\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/f*(cos(f*x+e)^2*sin(f*x+e)+4*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)-8
*sin(f*x+e)*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-cos(f*x+e
)^3+4*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-8*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)-s
in(f*x+e))/sin(f*x+e))+5*sin(f*x+e)*cos(f*x+e)-8*sin(f*x+e)*ln(2/(cos(f*x+e
)+1))+16*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+6*cos(f*x+e)
^2+4*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-8*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))-6*sin(f*x+e)+cos(f*x+e)-8*ln(2/(cos(f*x+e)+1))+16*ln(-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-6)*(-c*(sin(f*x+e)-1))^(5/2)/(cos(f*x+e
)^3+cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2+2*sin(f*x+e)*cos(f*x+e)-2*cos(f*
x+e)-4*sin(f*x+e)+4)/(a*(1+sin(f*x+e)))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int((c - c*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.391 $\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{c^2\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f(a\sin(e+fx)+a)^{3/2}}$$

```
[Out] -c^2*cos(f*x+e)*ln(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e)
)^(1/2)-c*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(3/2)
```

Rubi [A] time = 0.20, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2739, 2737, 2667, 31}

$$-\frac{c^2\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((c^2*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sq
rt[c - c*Sin[e + f*x]])) - (c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*(a
+ a*Sin[e + f*x])^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx &= -\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f(a+a\sin(e+fx))^{3/2}}-\frac{c\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}\\
&= -\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f(a+a\sin(e+fx))^{3/2}}-\frac{\left(c^2\cos(e+fx)\right)\int \frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}\\
&= -\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f(a+a\sin(e+fx))^{3/2}}-\frac{\left(c^2\cos(e+fx)\right)\mathrm{Subst}\left(\int \frac{1}{a+x}\,dx,x,a\sin(e+
}{af\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}\\
&= -\frac{c^2\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}-\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f(a+a\sin(e+fx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.44, size = 134, normalized size = 1.38

$$-\frac{2c\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin(e+fx)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)+
}{f(a(\sin(e+fx)+1))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(1 + L
og[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + Log[Cos[(e + f*x)/2] + Sin[(e + f
*x)/2]]*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin
[e + f*x]))^(3/2))
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(3/2)/(a^2*cos(f*x
 + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [B] time = 0.28, size = 388, normalized size = 4.00

$$\frac{\left(\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)\sin\left(fx+e\right)\cos\left(fx+e\right)-2\sin\left(fx+e\right)\cos\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+\left(\cos^{2}\left(fx\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/f*(ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)-2*sin(f*x+e)*cos(f*x+e)*ln(
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-2
*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+2*sin(f*x+e)*cos(f
*x+e)-2*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+4*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))+2*cos(f*x+e)^2+cos(f*x+e)*ln(2/(cos(f*x+e)+1))-2*cos(f
*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-2*sin(f*x+e)-2*ln(2/(cos(f
*x+e)+1))+4*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-2)*(-c*(sin(f*x+e)-1
))^(3/2)/(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)/(a*
(1+sin(f*x+e)))^(3/2)
```

maxima [A] time = 1.00, size = 136, normalized size = 1.40

$$\frac{\frac{2\,c^{\frac{3}{2}}\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{a^{\frac{3}{2}}}-\frac{c^{\frac{3}{2}}\log\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)}{a^{\frac{3}{2}}}-\frac{4\,\sqrt{a}\,c^{\frac{3}{2}}\sin\left(fx+e\right)}{\left(a^{2}+\frac{2\,a^{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{a^{2}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)\left(\cos\left(fx+e\right)+1\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] (2*c^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(3/2) - c^(3/2)*log(s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(3/2) - 4*sqrt(a)*c^(3/2)*sin(f*x
 + e)/((a^2 + 2*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + a^2*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2)*(cos(f*x + e) + 1)))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int((c - c*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((-c*(sin(e + f*x) - 1))**(3/2)/(a*(sin(e + f*x) + 1))**(3/2), x)
```

3.392 $\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{c\cos(e+fx)}{f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -c*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2738}

$$-\frac{c\cos(e+fx)}{f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c*Sin[e + f*x]]/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((c*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]))
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx=-\frac{c\cos(e+fx)}{f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Mathematica [B] time = 0.19, size = 85, normalized size = 2.07

$$-\frac{\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}}{a^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c*Sin[e + f*x]]/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])/(a^2*f*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3))
```

fricas [A] time = 0.43, size = 58, normalized size = 1.41

$$-\frac{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{a^{2}f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^{2}f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] -sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x + e)*sin
(f*x + e) + a^2*f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8
*pi/x/2)sqrt(2*c)*sqrt(2)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f
*x+1/4*(2*exp(1)-pi)))^2/sqrt(a)/(tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^2-1)
^2/a/f/sign(-tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^2+1)
```

maple [A] time = 0.27, size = 68, normalized size = 1.66

$$\frac{\left(1-\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)\sin\left(fx+e\right)\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}}{f\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\left(-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/f*(1-cos(f*x+e)+sin(f*x+e))*sin(f*x+e)*(-c*(sin(f*x+e)-1))^(1/2)/(a*(1+si
n(f*x+e)))^(3/2)/(-1+cos(f*x+e)+sin(f*x+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\sin\left(fx+e\right)+c}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(3/2), x)
```

mupad [B] time = 7.52, size = 52, normalized size = 1.27

$$-\frac{2\cos\left(e+fx\right)\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}{af\left(\cos\left(2e+2fx\right)+1\right)\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] -(2*cos(e + f*x)*(-c*(sin(e + f*x) - 1))^(1/2))/(a*f*(cos(2*e + 2*f*x) + 1)
*(a*(sin(e + f*x) + 1))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(-c*(sin(e + f*x) - 1))/(a*(sin(e + f*x) + 1))**(3/2), x)
```

3.393 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=95

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -1/2*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)+1/2*arctanh
(sin(f*x+e))*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2743, 2741, 3770}

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] -Cos[e + f*x]/(2*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]) + (
ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
- c*Sin[e + f*x]])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}\ldots}}{2\ldots}$$

$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{\cos(e+\ldots}{2a\sqrt{a+a\sin(e\ldots}}$$

$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{\tanh^{-1}(\sin\ldots}{2af\sqrt{a+a\sin\ldots}}$$

Mathematica [A] time = 0.41, size = 148, normalized size = 1.56

$$-\frac{\cos(e+fx)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)+\sin(e+fx)\ldots}{2f(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] -1/2*(Cos[e + f*x]*(1 + Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[
(e + f*x)/2] + Sin[(e + f*x)/2]] + (Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x]))/(f*(a*(1 + Sin
[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e + f*x]])
```

fricas [A] time = 0.52, size = 305, normalized size = 3.21

$$\left[\frac{\sqrt{ac}\left(\cos\left(fx+e\right)\sin\left(fx+e\right)+\cos\left(fx+e\right)\right)\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\ldots}{\cos\left(fx+e\right)^{3}}\ldots\right.}{4\left(a^{2}cf\cos\left(fx+e\right)\sin\left(fx+e\right)+a^{2}cf\cos\left(fx+e\right)\right)}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] [1/4*(sqrt(a*c)*(cos(f*x + e)*sin(f*x + e) + cos(f*x + e))*log(-(a*c*cos(f*
x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-
c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) - 2*sqrt(a*sin(f*x + e) +
 a)*sqrt(-c*sin(f*x + e) + c))/(a^2*c*f*cos(f*x + e)*sin(f*x + e) + a^2*c*f
*cos(f*x + e)), -1/2*(sqrt(-a*c)*(cos(f*x + e)*sin(f*x + e) + cos(f*x + e))
*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*
cos(f*x + e)*sin(f*x + e))) + sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c))/(a^2*c*f*cos(f*x + e)*sin(f*x + e) + a^2*c*f*cos(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x + e) + c)), x)
```

maple [A] time = 0.27, size = 167, normalized size = 1.76

$$-\frac{\left(\sin\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-\sin\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-\sin\left(fx+e\right)+\ln\left(-\frac{-1+\cos}{}\right.\right.}{2f\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*(sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-sin(f*x+e)*ln
(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-sin(f*x+e)+ln(-(-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))-ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)))*cos(f*x+e)/
(a*(1+sin(f*x+e)))^(3/2)/(-c*(sin(f*x+e)-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x + e) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(1/2)),x)
[Out] int(1/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(1/2),x)
[Out] Integral(1/((a*(sin(e + f*x) + 1))**(3/2)*sqrt(-c*(sin(e + f*x) - 1))), x)
```

3.394 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=143

$$\frac{\cos(e+fx)}{2af\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}+\frac{\cos(e+fx)\tan}{2acf\sqrt{a\sin(e+fx)}}$$

```
[Out] -1/2*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2)+1/2*cos(f*x
+e)/a/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/2*arctanh(sin(f*x+e
))*cos(f*x+e)/a/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2743, 2741, 3770}

$$\frac{\cos(e+fx)}{2af\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}+\frac{\cos(e+fx)\tan}{2acf\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -Cos[e + f*x]/(2*f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2)) +
 Cos[e + f*x]/(2*a*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) +
 (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a*c*f*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c - c*Sin[e + f*x]])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{\int \frac{}{\sqrt{a+a\sin(e}}}{}$$

$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{}{2af\sqrt{a+a}}$$

$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{}{2af\sqrt{a+a}}$$

$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{}{2af\sqrt{a+a}}$$

Mathematica [A] time = 0.62, size = 170, normalized size = 1.19

$$\frac{\cos(e+fx)\left(-2\sin(e+fx)+\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\cos(2(e+fx))\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\right.\right.\right.}{4cf(\sin(e+fx)-1)(a(\sin(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (Cos[e + f*x]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Cos[2*(e + f*x)]*
(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2]]) - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - 2*Sin[e + f*x]))/(4
*c*f*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e + f*
x]])
```

fricas [A] time = 0.48, size = 262, normalized size = 1.83

$$\left[\frac{\sqrt{ac}\cos\left(fx+e\right)^{3}\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)-2\sqrt{ac}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\sin\left(fx+e\right)}{\cos\left(fx+e\right)^{3}}\right)+2\sqrt{a\sin\left(fx+e\right)}}{4\,a^{2}c^{2}f\cos\left(fx+e\right)^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] [1/4*(sqrt(a*c)*cos(f*x + e)^3*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e
) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x
+ e))/cos(f*x + e)^3) + 2*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c
)*sin(f*x + e))/(a^2*c^2*f*cos(f*x + e)^3), -1/2*(sqrt(-a*c)*arctan(sqrt(-a
*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*si
n(f*x + e)))*cos(f*x + e)^3 - sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)*sin(f*x + e))/(a^2*c^2*f*cos(f*x + e)^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(3/2)), x)
```

maple [A] time = 0.24, size = 115, normalized size = 0.80

$$\frac{\left(-\left(\cos^{2}\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+\left(\cos^{2}\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+\sin\left(fx+e\right)\right)\cos\left(\right.}{2f\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/2/f*(-cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+cos(f*x+e)^
2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+sin(f*x+e))*cos(f*x+e)/(a*(1+s
in(f*x+e)))^(3/2)/(-c*(sin(f*x+e)-1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/((a*(sin(e + f*x) + 1))**(3/2)*(-c*(sin(e + f*x) - 1))**(3/2)),
x)
```

3.395 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=191

$$\frac{3\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8ac^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{3\cos(e+fx)}{8acf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{3\cos}{8af\sqrt{a\sin(e+fx)}}$$

```
[Out] -1/2*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2)+3/8*cos(f*x
+e)/a/f/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)+3/8*cos(f*x+e)/a/c/f/
(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+3/8*arctanh(sin(f*x+e))*cos(f
*x+e)/a/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2743, 2741, 3770}

$$\frac{3\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8ac^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{3\cos(e+fx)}{8acf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{3\cos}{8af\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] -Cos[e + f*x]/(2*f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2)) +
 (3*Cos[e + f*x])/(8*a*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2
)) + (3*Cos[e + f*x])/(8*a*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x]
)^(3/2)) + (3*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(8*a*c^2*f*Sqrt[a + a*Sin
[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
```

```
SumSimplerQ[n, 1])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{3\int \frac{}{\sqrt{a+a\sin}}}{}$$

$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{}{8af\sqrt{a+a}}$$

$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{}{8af\sqrt{a+a}}$$

$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{}{8af\sqrt{a+a}}$$

$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{}{8af\sqrt{a+a}}$$

Mathematica [A] time = 0.77, size = 287, normalized size = 1.50

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2\cos^2(e+fx)-\left(\cos\left(\frac{1}{2}(e+fx)\right)-\mathrm{si}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(2*Cos[e + f*x]^2 - (Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + (Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])^2 - 3*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^2 + 3*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2))/(8*f*(a*(1 + Sin[e
+ f*x]))^(3/2)*(c - c*Sin[e + f*x])^(5/2))
```

fricas [A] time = 0.53, size = 377, normalized size = 1.97

$$\left[\frac{3\left(\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)-\cos\left(fx+e\right)^{3}\right)\sqrt{ac}\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\cdots}{\cos\left(fx+e\right)^{3}}\cdots\right)}{16\left(a^{2}c^{3}f\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)-\cdots\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] [1/16*(3*(cos(f*x + e)^3*sin(f*x + e) - cos(f*x + e)^3)*sqrt(a*c)*log(-(a*c
*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)
*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) - 2*(3*cos(f*x + e
)^2 + 3*sin(f*x + e) - 1)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c
))/(a^2*c^3*f*cos(f*x + e)^3*sin(f*x + e) - a^2*c^3*f*cos(f*x + e)^3), -1/8
*(3*(cos(f*x + e)^3*sin(f*x + e) - cos(f*x + e)^3)*sqrt(-a*c)*arctan(sqrt(-
a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*s
in(f*x + e))) + (3*cos(f*x + e)^2 + 3*sin(f*x + e) - 1)*sqrt(a*sin(f*x + e)
 + a)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^3*f*cos(f*x + e)^3*sin(f*x + e) - a
^2*c^3*f*cos(f*x + e)^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(5/2)), x)
```

maple [A] time = 0.27, size = 229, normalized size = 1.20

$$-\frac{\left(3\sin\left(fx+e\right)\left(\cos^{2}\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-3\sin\left(fx+e\right)\left(\cos^{2}\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(\cdots\right.}{\sin\left(fx+e\right)}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/8/f*(3*sin(f*x+e)*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)-3*sin(f*x+e)*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*co
s(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))+3*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-cos(f*x+e)^2
-3*sin(f*x+e)+1)*cos(f*x+e)/(a*(1+sin(f*x+e)))^(3/2)/(-c*(sin(f*x+e)-1))^(5
/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.396 $\int \frac{(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=237

$$\frac{24c^5\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{12c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}+\frac{3c^3\cos(e+fx)(c-c\sin(e+fx}{a^2f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] 2*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a/f/(a+a*sin(f*x+e))^(3/2)-1/2*c*co
s(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(5/2)+3*c^3*cos(f*x+e)*(
c-c*sin(f*x+e))^(3/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)+24*c^5*cos(f*x+e)*ln(1+s
in(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+12*c^4*cos(f
*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2739, 2740, 2737, 2667, 31}

$$\frac{12c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}+\frac{3c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a\sin(e+fx)+a}}+\frac{24c^5\cos(e+fx)\log(\sin(e+fx)+}{a^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(9/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (24*c^5*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c - c*Sin[e + f*x]]) + (12*c^4*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])
/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) + (3*c^3*Cos[e + f*x]*(c - c*Sin[e + f*x]
)^(3/2))/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) + (2*c^2*Cos[e + f*x]*(c - c*Sin[
e + f*x])^(5/2))/(a*f*(a + a*Sin[e + f*x])^(3/2)) - (c*Cos[e + f*x]*(c - c*
Sin[e + f*x])^(7/2))/(2*f*(a + a*Sin[e + f*x])^(5/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a+a\sin(e+fx))^{5/2}} - \frac{(2c)\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx}{a}$$

$$= \frac{2c^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{af(a+a\sin(e+fx))^{3/2}} - \frac{c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a+a\sin(e+fx))^{5/2}} + \frac{(6c^2)\int}{}$$

$$= \frac{3c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{2c^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{af(a+a\sin(e+fx))^{3/2}} - \frac{c\cos}{}$$

$$= \frac{12c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{2c^2 c}{}$$

$$= \frac{12c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{2c^2 c}{}$$

$$= \frac{12c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{2c^2 c}{}$$

$$= \frac{24c^5\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{12c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} +$$

Mathematica [A] time = 5.12, size = 202, normalized size = 0.85

$$\frac{c^4\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(320\sin(e+fx)+24\sin(3(e+fx))+\cos(4(e+fx))+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(9/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (c^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(273 +
Cos[4*(e + f*x)] + Cos[2*(e + f*x)]*(106 - 384*Log[Cos[(e + f*x)/2] + Sin[(
e + f*x)/2]]) + 1152*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 320*Sin[e +
 f*x] + 1536*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] + 24*Sin
[3*(e + f*x)]))/(16*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e +
 f*x]))^(5/2))
```

fricas [F] time = 1.93, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(c^4\cos\left(fx+e\right)^4-8\,c^4\cos\left(fx+e\right)^2+8\,c^4+4\left(c^4\cos\left(fx+e\right)^2-2\,c^4\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+\right.}}{3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-(c^4*cos(f*x + e)^4 - 8*c^4*cos(f*x + e)^2 + 8*c^4 + 4*(c^4*cos(f
*x + e)^2 - 2*c^4)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
 e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f
*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [B] time = 0.32, size = 685, normalized size = 2.89

$$\frac{\left(-132-192\sin\left(fx+e\right)\cos\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-132\sin\left(fx+e\right)+74\cos\left(fx+e\right)+58\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/2/f*(-132-96*sin(f*x+e)*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
*x+e))-192*sin(f*x+e)*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
+48*sin(f*x+e)*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-132*sin(f*x+e)+74*cos(f*x+
e)+143*cos(f*x+e)^2+58*sin(f*x+e)*cos(f*x+e)+96*ln(2/(cos(f*x+e)+1))*sin(f*
x+e)*cos(f*x+e)+144*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-cos(f*x+e)^5-73*cos(f
*x+e)^3-288*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+384*ln(
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-11*cos(f*x+e)^4-192*ln(2/(cos(f*x+e
)+1))-192*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+96*cos(f*x+e)*ln(2/(cos(f*x+e)+1)
```

```
)-48*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))+85*cos(f*x+e)^2*sin(f*x+e)-12*sin(f*
x+e)*cos(f*x+e)^3+sin(f*x+e)*cos(f*x+e)^4+96*cos(f*x+e)^3*ln(-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))-192*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))+384*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)))*(-c*(si
n(f*x+e)-1))^(9/2)/(sin(f*x+e)*cos(f*x+e)^4+cos(f*x+e)^5+4*sin(f*x+e)*cos(f
*x+e)^3-5*cos(f*x+e)^4-12*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^3-8*sin(f*x+
e)*cos(f*x+e)+20*cos(f*x+e)^2+16*sin(f*x+e)+8*cos(f*x+e)-16)/(a*(1+sin(f*x+
e)))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c \sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}{\left(a \sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(9/2)/(a*sin(f*x + e) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-c\,\sin\left(e+f\,x\right)\right)^{9/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(9/2)/(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int((c - c*sin(e + f*x))^(9/2)/(a + a*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.397 $\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=193

$$\frac{6c^4\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{3c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}+\frac{3c^2\cos(e+fx)(c-c\sin(e+fx}{2af(a\sin(e+fx)+a)^{3/2}}$$

```
[Out] 3/2*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f/(a+a*sin(f*x+e))^(3/2)-1/2*c*
cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(5/2)+6*c^4*cos(f*x+e)
*ln(1+sin(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+3*c^3
*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2739, 2740, 2737, 2667, 31}

$$\frac{3c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}+\frac{6c^4\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{3c^2\cos(e+fx)(c-c\sin(e+fx}{2af(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(7/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (6*c^4*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]]*
Sqrt[c - c*Sin[e + f*x]]) + (3*c^3*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(
a^2*f*Sqrt[a + a*Sin[e + f*x]]) + (3*c^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^
(3/2))/(2*a*f*(a + a*Sin[e + f*x])^(3/2)) - (c*Cos[e + f*x]*(c - c*Sin[e +
f*x])^(5/2))/(2*f*(a + a*Sin[e + f*x])^(5/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a+a\sin(e+fx))^{5/2}} - \frac{(3c)\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx}{2a}$$

$$= \frac{3c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af(a+a\sin(e+fx))^{3/2}} - \frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a+a\sin(e+fx))^{5/2}} + \frac{(3c^2)}{}$$

$$= \frac{3c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af(a+a\sin(e+fx))^{3/2}} - \frac{c\cos}{}$$

$$= \frac{3c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af(a+a\sin(e+fx))^{3/2}} - \frac{c\cos}{}$$

$$= \frac{3c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af(a+a\sin(e+fx))^{3/2}} - \frac{c\cos}{}$$

$$= \frac{6c^4\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{3c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} +$$

Mathematica [A] time = 2.00, size = 187, normalized size = 0.97

$$\frac{c^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin(3(e+fx))+\cos(2(e+fx))\left(4-24\log\left(\sin\left(\frac{1}{2}(e\right.\right.\right.\right.}{4f(a(\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(7/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(28 + C
os[2*(e + f*x)]*(4 - 24*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + 72*Log[
Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (41 + 96*Log[Cos[(e + f*x)/2] + Sin[
(e + f*x)/2]])*Sin[e + f*x] + Sin[3*(e + f*x)]))/(4*f*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2))
```

fricas [F] time = 1.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(3\,c^3\cos\left(fx+e\right)^2-4\,c^3-\left(c^3\cos\left(fx+e\right)^2-4\,c^3\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right.}}{3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")

[Out] integral((3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.31, size = 633, normalized size = 3.28

$$-\frac{\left(\sin\left(fx+e\right)\left(\cos^{3}\left(fx+e\right)\right)+12\sin\left(fx+e\right)\left(\cos^{2}\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-6\sin\left(fx+e\right)\left(\cos^{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x)

[Out] -1/f*(sin(f*x+e)*cos(f*x+e)^3+12*sin(f*x+e)*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-6*sin(f*x+e)*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+cos(f*x+e)^4-12*cos(f*x+e)^3*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+6*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))-11*cos(f*x+e)^2*sin(f*x+e)+24*sin(f*x+e)*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-12*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)+10*cos(f*x+e)^3+36*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-18*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-6*sin(f*x+e)*cos(f*x+e)-48*sin(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+24*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-17*cos(f*x+e)^2+24*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-12*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+16*sin(f*x+e)-10*cos(f*x+e)-48*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+24*ln(2/(cos(f*x+e)+1))+16)*(-c*(sin(f*x+e)-1))^(7/2)/(sin(f*x+e)*cos(f*x+e)^3-cos(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)+8*cos(f*x+e)^2+8*sin(f*x+e)+4*cos(f*x+e)-8)/(a*(1+sin(f*x+e)))^(5/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(7/2)/(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int((c - c*sin(e + f*x))^(7/2)/(a + a*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.398 $\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=143

$$\frac{c^3\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a\sin(e+fx)+a)^{3/2}}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a\sin(e+fx)+a)^{5/2}}$$

```
[Out] -1/2*c*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(5/2)+c^3*cos(f
*x+e)*ln(1+sin(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+
c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+e))^(3/2)
```

Rubi [A] time = 0.30, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2739, 2737, 2667, 31}

$$\frac{c^3\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a\sin(e+fx)+a)^{3/2}}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sq
rt[c - c*Sin[e + f*x]]) + (c^2*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*f*
(a + a*Sin[e + f*x])^(3/2)) - (c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(
2*f*(a + a*Sin[e + f*x])^(5/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
```

```
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{5/2}} - \frac{c\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx}{a}$$

$$= \frac{c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{5/2}} + \frac{c^2\int \frac{\sqrt{c-}}{\sqrt{a+}}}{}$$

$$= \frac{c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{5/2}} + \frac{\left(c^3\,\mathrm{c}\right.}{a\sqrt{a+a}}$$

$$= \frac{c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{5/2}} + \frac{\left(c^3\cos(\right.}{a^2 f}$$

$$= \frac{c^3\cos(e+fx)\log(1+\sin(e+fx))}{a^2 f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} -$$

Mathematica [A] time = 1.19, size = 172, normalized size = 1.20

$$\frac{c^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-\cos(2(e+fx))\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+f\right.\right.\right.}{f(a(\sin(e+fx)+1))^{5/2}\left(\mathrm{cos}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

[Out] (c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(2 + 3*
Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - Cos[2*(e + f*x)]*Log[Cos[(e + f*
x)/2] + Sin[(e + f*x)/2]] + 4*(1 + Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]
)*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f
*x]))^(5/2))

fricas [F] time = 1.17, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^2\cos\left(fx+e\right)^2+2\,c^2\sin\left(fx+e\right)-2\,c^2\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="frica
s")

[Out] integral((c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2)*sqrt(a*sin(f*x +
 e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos
(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac"
)

[Out] Timed out

maple [B] time = 0.26, size = 567, normalized size = 3.97

$$-\frac{\left(\sin\left(fx+e\right)\left(\cos^2\left(fx+e\right)\right)\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)-2\sin\left(fx+e\right)\left(\cos^2\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-\left(\cos\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x)

[Out] -1/f*(sin(f*x+e)*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-2*sin(f*x+e)*cos(f*x+e)^
2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-cos(f*x+e)^3*ln(2/(cos(f*x+e)+
1))+2*cos(f*x+e)^3*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+2*cos(f*x+e)^

```
2*sin(f*x+e)+2*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)-4*sin(f*x+e)*cos(
f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-2*cos(f*x+e)^3+3*cos(f*x+
e)^2*ln(2/(cos(f*x+e)+1))-6*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))-4*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+8*sin(f*x+e)*ln(-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))+2*cos(f*x+e)^2+2*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-4*
cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-2*sin(f*x+e)+2*cos(f*
x+e)-4*ln(2/(cos(f*x+e)+1))+8*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-2)
*(-c*(sin(f*x+e)-1))^(5/2)/(cos(f*x+e)^3+cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+
e)^2+2*sin(f*x+e)*cos(f*x+e)-2*cos(f*x+e)-4*sin(f*x+e)+4)/(a*(1+sin(f*x+e))
)^(5/2)
```

maxima [A] time = 0.44, size = 183, normalized size = 1.28

$$\frac{\frac{8\sqrt{a}c^{\frac{5}{2}}\sin\left(fx+e\right)^{2}}{\left(a^{3}+\frac{4a^{3}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{6a^{3}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{4a^{3}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{a^{3}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}\right)\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{2c^{\frac{5}{2}}\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{a^{\frac{5}{2}}}+\frac{c^{\frac{5}{2}}\log\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)}{a^{\frac{5}{2}}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] (8*sqrt(a)*c^(5/2)*sin(f*x + e)^2/((a^3 + 4*a^3*sin(f*x + e)/(cos(f*x + e)
+ 1) + 6*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^3*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*(cos(f*x + e)
+ 1)^2) - 2*c^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(5/2) + c^(5
/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(5/2))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int((c - c*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.399 $\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=42

$$-\frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

```
[Out] -1/4*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(5/2)
```

Rubi [A] time = 0.09, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2742}

$$-\frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -(Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(4*f*(a + a*Sin[e + f*x])^(5/2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a+a\sin(e+fx))^{5/2}}$$

Mathematica [B] time = 0.46, size = 86, normalized size = 2.05

$$\frac{c\sin(e+fx)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f(a(\sin(e+fx)+1))^{5/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sin[e + f*x]*Sqrt[c - c*Sin[e + f*
x]])/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2))
```

fricas [B] time = 0.44, size = 80, normalized size = 1.90

$$-\frac{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\,c\sin\left(fx+e\right)}{a^{3}f\cos\left(fx+e\right)^{3}-2\,a^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^{3}f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] -sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*c*sin(f*x + e)/(a^3*f*c
os(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [B] time = 0.29, size = 93, normalized size = 2.21

$$-\frac{\sin\left(fx+e\right)\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{3}{2}}\left(-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)\right)}{f\left(\cos^{2}\left(fx+e\right)-\sin\left(fx+e\right)\cos\left(fx+e\right)+\cos\left(fx+e\right)+2\sin\left(fx+e\right)-2\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/f*sin(f*x+e)*(-c*(sin(f*x+e)-1))^(3/2)*(-1+cos(f*x+e)-sin(f*x+e))/(cos(f
*x+e)^2-sin(f*x+e)*cos(f*x+e)+cos(f*x+e)+2*sin(f*x+e)-2)/(a*(1+sin(f*x+e)))
^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a)^(5/2), x)
```

mupad [B] time = 8.21, size = 118, normalized size = 2.81

$$\frac{2c\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\left(-2\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2+2\sin\left(\frac{3e}{2}+\frac{3fx}{2}\right)^2+2\sin\left(2e+2fx\right)\right)}{a^2f\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(-8\sin\left(e+fx\right)^2+4\sin\left(e+fx\right)+2\sin\left(2e+2fx\right)^2+4\sin\left(3e+3fx\right)+8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] (2*c*(-c*(sin(e + f*x) - 1))^(1/2)*(2*sin(2*e + 2*f*x) - 2*sin(e/2 + (f*x)/
2)^2 + 2*sin((3*e)/2 + (3*f*x)/2)^2))/(a^2*f*(a*(sin(e + f*x) + 1))^(1/2)*(
4*sin(e + f*x) + 4*sin(3*e + 3*f*x) + 2*sin(2*e + 2*f*x)^2 - 8*sin(e + f*x)
^2 + 8))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((-c*(sin(e + f*x) - 1))**(3/2)/(a*(sin(e + f*x) + 1))**(5/2), x)
```

3.400 $\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=43

$$-\frac{c\cos(e+fx)}{2f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -1/2*c*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2738}

$$-\frac{c\cos(e+fx)}{2f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c*Sin[e + f*x]]/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -(c*Cos[e + f*x])/(2*f*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{c\cos(e+fx)}{2f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}$$

Mathematica [B] time = 0.22, size = 87, normalized size = 2.02

$$-\frac{\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}}{2a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c*Sin[e + f*x]]/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/2*(Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])/(a^3*f*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

fricas [A] time = 0.42, size = 73, normalized size = 1.70

$$\frac{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{2\left(a^3f\cos\left(fx+e\right)^3-2\,a^3f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^3f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/2*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*x + e)^
3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [B] time = 0.28, size = 92, normalized size = 2.14

$$-\frac{\sin\left(fx+e\right)\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\,\left(\cos^2\left(fx+e\right)+\sin\left(fx+e\right)\cos\left(fx+e\right)+2\cos\left(fx+e\right)-3\sin\left(fx+\right.\right.}{2f\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{5}{2}}\left(-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/2/f*sin(f*x+e)*(-c*(sin(f*x+e)-1))^(1/2)*(cos(f*x+e)^2+sin(f*x+e)*cos(f*
x+e)+2*cos(f*x+e)-3*sin(f*x+e)-3)/(a*(1+sin(f*x+e)))^(5/2)/(-1+cos(f*x+e)+s
in(f*x+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\sin\left(fx+e\right)+c}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(5/2), x)
```

mupad [B] time = 7.70, size = 103, normalized size = 2.40

$$-\frac{2\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\left(-4\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2+\sin\left(2e+2fx\right)+2\right)}{a^2 f\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(-8\sin\left(e+fx\right)^2+4\sin\left(e+fx\right)+2\sin\left(2e+2fx\right)^2+4\sin\left(3e+3fx\right)+8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] -(2*(-c*(sin(e + f*x) - 1))^(1/2)*(sin(2*e + 2*f*x) - 4*sin(e/2 + (f*x)/2)^
2 + 2))/(a^2*f*(a*(sin(e + f*x) + 1))^(1/2)*(4*sin(e + f*x) + 4*sin(3*e + 3
*f*x) + 2*sin(2*e + 2*f*x)^2 - 8*sin(e + f*x)^2 + 8))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(-c*(sin(e + f*x) - 1))/(a*(sin(e + f*x) + 1))**(5/2), x)
```

3.401 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=140

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{4a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)}{4af(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e}{4f(a\sin(e+fx)+a}$$

```
[Out] -1/4*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2)-1/4*cos(f*x
+e)/a/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)+1/4*arctanh(sin(f*x+e
))*cos(f*x+e)/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2743, 2741, 3770}

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{4a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)}{4af(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e}{4f(a\sin(e+fx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] -Cos[e + f*x]/(4*f*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]]) - C
os[e + f*x]/(4*a*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]) + (
ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(4*a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[
c - c*Sin[e + f*x]])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} + \frac{\int \frac{1}{(a+a\sin(e+fx))^{3/2}}}{2a}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{c}{4af(a+a\sin(e}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{c}{4af(a+a\sin(e}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{c}{4af(a+a\sin(e}$$

Mathematica [A] time = 0.65, size = 211, normalized size = 1.51

$$\cos(e+fx)\left(-3\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\cos(2(e+fx))\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] (Cos[e + f*x]*(-4 - 3*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Cos[2*(e +
 f*x)]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + S
in[(e + f*x)/2]]) + 3*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (-2 - 4*Lo
g[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 4*Log[Cos[(e + f*x)/2] + Sin[(e +
f*x)/2]])*Sin[e + f*x]))/(8*f*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e
 + f*x]])
```

fricas [A] time = 0.53, size = 376, normalized size = 2.69

$$\left[\frac{\left(\cos\left(fx+e\right)^{3}-2\cos\left(fx+e\right)\sin\left(fx+e\right)-2\cos\left(fx+e\right)\right)\sqrt{ac}\,\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)-2\sqrt{ac}\sqrt{a\sin\left(fx+e\right.}}{\cos\left(fx+e\right)^{3}}\right.}{8\left(a^{3}cf\cos\left(fx+e\right)^{3}-2\,a^{3}cf\cos\left(fx+e\right)\sin\left(f\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] [1/8*((cos(f*x + e)^3 - 2*cos(f*x + e)*sin(f*x + e) - 2*cos(f*x + e))*sqrt(
a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin
(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) + 2*
sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(sin(f*x + e) + 2))/(a^3
*c*f*cos(f*x + e)^3 - 2*a^3*c*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*c*f*cos(f
*x + e)), -1/4*((cos(f*x + e)^3 - 2*cos(f*x + e)*sin(f*x + e) - 2*cos(f*x +
 e))*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
+ e) + c)/(a*c*cos(f*x + e)*sin(f*x + e))) - sqrt(a*sin(f*x + e) + a)*sqrt(
-c*sin(f*x + e) + c)*(sin(f*x + e) + 2))/(a^3*c*f*cos(f*x + e)^3 - 2*a^3*c*
f*cos(f*x + e)*sin(f*x + e) - 2*a^3*c*f*cos(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x + e) + c)), x)
```

maple [B] time = 0.27, size = 252, normalized size = 1.80

$$\left(-\left(\cos^{2}\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+\left(\cos^{2}\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+2\sin\left(fx+e\right)\ln\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/4/f*(-cos(f*x+e)^2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+cos(f*x+e)^
2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*sin(f*x+e)*ln(-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))-2*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f
*x+e))-2*cos(f*x+e)^2+2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-2*ln(-(-
1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+3*sin(f*x+e)+2)*cos(f*x+e)/(a*(1+sin(f
*x+e)))^(5/2)/(-c*(sin(f*x+e)-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x + e) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/((a*(sin(e + f*x) + 1))**(5/2)*sqrt(-c*(sin(e + f*x) - 1))), x)
```

3.402 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=188

$$\frac{3\cos(e+fx)}{8a^2f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{3\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8a^2cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{3\,c}{8af(a\sin(e+fx)+}$$

```
[Out] -1/4*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2)-3/8*cos(f*x
+e)/a/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2)+3/8*cos(f*x+e)/a^2/f/
(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+3/8*arctanh(sin(f*x+e))*cos(f
*x+e)/a^2/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2743, 2741, 3770}

$$\frac{3\cos(e+fx)}{8a^2f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{3\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8a^2cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{3\,c}{8af(a\sin(e+fx)+}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -Cos[e + f*x]/(4*f*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2)) -
 (3*Cos[e + f*x])/(8*a*f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3
/2)) + (3*Cos[e + f*x])/(8*a^2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*
x])^(3/2)) + (3*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(8*a^2*c*f*Sqrt[a + a*S
in[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
```

```
SumSimplerQ[n, 1])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} + \frac{3\int \frac{}{(a+a\sin(e+}}{}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{}{8af(a+a\sin}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{}{8af(a+a\sin}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{}{8af(a+a\sin}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{}{8af(a+a\sin}$$

Mathematica [A] time = 0.75, size = 287, normalized size = 1.53

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-2\cos^2(e+fx)+\left(\sin\left(\frac{1}{2}(e+fx)\right)+\text{co}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-2*Cos[e + f*x]^2 - (Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + (Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^4 - 3*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^4 + 3*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2])^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4))/(8*f*(a*(1 + Sin[e
 + f*x]))^(5/2)*(c - c*Sin[e + f*x])^(3/2))
```

fricas [A] time = 0.50, size = 371, normalized size = 1.97

$$\left[\frac{3\left(\cos\left(fx+e\right)^3\sin\left(fx+e\right)+\cos\left(fx+e\right)^3\right)\sqrt{ac}\log\left(-\frac{ac\cos\left(fx+e\right)^3-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)}}{\cos\left(fx+e\right)^3}\right.}{16\left(a^3c^2f\cos\left(fx+e\right)^3\sin\left(fx+e\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] [1/16*(3*(cos(f*x + e)^3*sin(f*x + e) + cos(f*x + e)^3)*sqrt(a*c)*log(-(a*c
*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)
*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) - 2*(3*cos(f*x + e
)^2 - 3*sin(f*x + e) - 1)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c
))/(a^3*c^2*f*cos(f*x + e)^3*sin(f*x + e) + a^3*c^2*f*cos(f*x + e)^3), -1/8
*(3*(cos(f*x + e)^3*sin(f*x + e) + cos(f*x + e)^3)*sqrt(-a*c)*arctan(sqrt(-
a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*s
in(f*x + e))) + (3*cos(f*x + e)^2 - 3*sin(f*x + e) - 1)*sqrt(a*sin(f*x + e)
 + a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^2*f*cos(f*x + e)^3*sin(f*x + e) + a
^3*c^2*f*cos(f*x + e)^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(3/2)), x)
```

maple [A] time = 0.27, size = 229, normalized size = 1.22

$$\frac{\left(3\sin\left(fx+e\right)\left(\cos^2\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-3\sin\left(fx+e\right)\left(\cos^2\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(\right.}{\sin\left(fx+e\right)}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/8/f*(3*sin(f*x+e)*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
-3*sin(f*x+e)*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*cos
(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))-3*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-cos(f*x+e)^2+
3*sin(f*x+e)+1)*cos(f*x+e)/(a*(1+sin(f*x+e)))^(5/2)/(-c*(sin(f*x+e)-1))^(3/
2)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a \sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c \sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(3/2)), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(3/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.403 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=236

$$\frac{3\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8a^2c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{3\cos(e+fx)}{8a^2cf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{3}{8a^2f\sqrt{a\sin(e+}}$$

```
[Out] -1/4*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2)-1/2*cos(f*x
+e)/a/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2)+3/8*cos(f*x+e)/a^2/f/
(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)+3/8*cos(f*x+e)/a^2/c/f/(c-c*s
in(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+3/8*arctanh(sin(f*x+e))*cos(f*x+e)/
a^2/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2743, 2741, 3770}

$$\frac{3\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8a^2c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{3\cos(e+fx)}{8a^2cf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{3}{8a^2f\sqrt{a\sin(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] -Cos[e + f*x]/(4*f*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2)) -
 Cos[e + f*x]/(2*a*f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2))
 + (3*Cos[e + f*x])/(8*a^2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^
(5/2)) + (3*Cos[e + f*x])/(8*a^2*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e
+ f*x])^(3/2)) + (3*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(8*a^2*c^2*f*Sqrt[a
 + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} + \frac{\int \frac{}{(a+a\sin(e+fx}}{}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{}{2af(a+a\sin}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{}{2af(a+a\sin}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{}{2af(a+a\sin}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{}{2af(a+a\sin}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{}{2af(a+a\sin}$$

Mathematica [A] time = 0.97, size = 237, normalized size = 1.00

$$\frac{\sec^3(e+fx)\left(22\sin(e+fx)+6\sin(3(e+fx))-9\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-12\cos(2(e+fx))\left(1\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (Sec[e + f*x]^3*(-9*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 12*Cos[2*(e
+ f*x)]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] +
```

```
Sin[(e + f*x)/2]]) - 3*Cos[4*(e + f*x)]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*
x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + 9*Log[Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2]] + 22*Sin[e + f*x] + 6*Sin[3*(e + f*x)]))/(64*a^2*c^2*f
*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

fricas [A] time = 0.53, size = 288, normalized size = 1.22

$$\left[\frac{3\sqrt{ac}\cos\left(fx+e\right)^{5}\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)^{3}}\right)+2\left(3\,\cos\left(fx+e\right.\right.}{16\,a^{3}c^{3}f\cos\left(fx+e\right)^{5}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] [1/16*(3*sqrt(a*c)*cos(f*x + e)^5*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x
+ e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f
*x + e))/cos(f*x + e)^3) + 2*(3*cos(f*x + e)^2 + 2)*sqrt(a*sin(f*x + e) + a
)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/(a^3*c^3*f*cos(f*x + e)^5), -1/8*
(3*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
e) + c)/(a*c*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e)^5 - (3*cos(f*x + e)^2
 + 2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/(a^3
*c^3*f*cos(f*x + e)^5)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(5/2)), x)
```

maple [A] time = 0.28, size = 134, normalized size = 0.57

$$\frac{\left(3\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\left(\cos^{4}\left(fx+e\right)\right)-3\left(\cos^{4}\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+3\left(\cos^{2}\left(fx+e\right)\right.\right.}{8f\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{5}{2}}\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/8/f*(3*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^4-3*cos(f*x+
e)^4*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+3*cos(f*x+e)^2*sin(f*x+e)+2
*sin(f*x+e))*cos(f*x+e)/(a*(1+sin(f*x+e)))^(5/2)/(-c*(sin(f*x+e)-1))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.404 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=110

$$\frac{c2^{n+\frac{1}{2}}\cos(e+fx)(1-\sin(e+fx))^{\frac{1}{2}-n}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2}(2m+1),\frac{1}{2}(1-2n);\frac{1}{2}(2m\right.}{f(2m+1)}$$

```
[Out] 2^(1/2+n)*c*cos(f*x+e)*hypergeom([1/2-n, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))
*(1-sin(f*x+e))^(1/2-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+n)/f/(1+2*m
)
```

Rubi [A] time = 0.15, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2745, 2689, 70, 69}

$$\frac{c2^{n+\frac{1}{2}}\cos(e+fx)(1-\sin(e+fx))^{\frac{1}{2}-n}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2}(2m+1),\frac{1}{2}(1-2n);\frac{1}{2}(2m\right.}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + n)*c*Cos[e + f*x]*Hypergeometric2F1[(1 + 2*m)/2, (1 - 2*n)/2, (3
+ 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 - n)*(a + a*Sin[e +
 f*x])^m*(c - c*Sin[e + f*x])^(-1 + n))/(f*(1 + 2*m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = \left(\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^m\right)\int \cos^{2n}$$

$$= \frac{\left(c^2\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{\frac{1}{2}(-1-2m)+m}(\right.}{}$$

$$= \frac{\left(2^{-\frac{1}{2}+n}c^2\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-\frac{1}{2}+\frac{1}{2}(}\right.}{}$$

$$= \frac{2^{\frac{1}{2}+n}c\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}(1+2m),\frac{1}{2}(1-2n);\frac{1}{2}(3+2m);\frac{1}{2}(1+\sin\right.}{f}$$

Mathematica [C] time = 2.96, size = 365, normalized size = 3.32

$$\frac{4(2n+3)\sin\left(\frac{1}{8}(2e+2\right.}{f(2n+1)\left((2n+3)\cos^2\left(\frac{1}{8}(2e+2fx-\pi)\right)F_1\left(n+\frac{1}{2};-2m,2(m+n)+1;n+\frac{3}{2};\tan^2\left(\frac{1}{8}(-2e-2fx+\pi)\right),-\tan^2\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n,x]
```

```
[Out] (4*(3 + 2*n)*AppellF1[1/2 + n, -2*m, 1 + 2*(m + n), 3/2 + n, Tan[(-2*e + Pi
 - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2]*Cos[(2*e - Pi + 2*f*x)/8]^3*(
```

```
a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^n*Sin[(2*e - Pi + 2*f*x)/8])/(
f*(1 + 2*n)*((3 + 2*n)*AppellF1[1/2 + n, -2*m, 1 + 2*(m + n), 3/2 + n, Tan[
(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2]*Cos[(2*e - Pi + 2*f
*x)/8]^2 - 2*(2*m*AppellF1[3/2 + n, 1 - 2*m, 1 + 2*(m + n), 5/2 + n, Tan[(-
2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2] + (1 + 2*m + 2*n)*App
ellF1[3/2 + n, -2*m, 2*(1 + m + n), 5/2 + n, Tan[(-2*e + Pi - 2*f*x)/8]^2,
-Tan[(2*e - Pi + 2*f*x)/8]^2])*Sin[(2*e - Pi + 2*f*x)/8]^2))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n, x)
```

maple [F] time = 1.72, size = 0, normalized size = 0.00

$$\int\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\left(c-c\,\sin\left(e+f\,x\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n,x)
[Out] int((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^m\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*(-c*(sin(e + f*x) - 1))**n, x)
```

3.405 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3\,dx$

Optimal. Leaf size=86

$$-\frac{a^4c^3 2^{m+\frac{1}{2}}\cos^7(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-4}\,{}_2F_1\left(\frac{7}{2},\frac{1}{2}-m;\frac{9}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{7f}$$

```
[Out] -1/7*2^(1/2+m)*a^4*c^3*cos(f*x+e)^7*hypergeom([7/2, 1/2-m],[9/2],1/2-1/2*si
n(f*x+e))*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(-4+m)/f
```

Rubi [A] time = 0.14, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2689, 70, 69}

$$-\frac{a^4c^3 2^{m+\frac{1}{2}}\cos^7(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-4}\,{}_2F_1\left(\frac{7}{2},\frac{1}{2}-m;\frac{9}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] -(2^(1/2 + m)*a^4*c^3*Cos[e + f*x]^7*Hypergeometric2F1[7/2, 1/2 - m, 9/2, (
1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^(-4
+ m))/(7*f)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
```

```
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)(a+a\sin(e+fx))^{-3+m}\,dx$$

$$= \frac{\left(a^5c^3\cos^7(e+fx)\right)\text{Subst}\left(\int (a-ax)^{5/2}(a+ax)^{-\frac{1}{2}+m}\,dx, x, \sin(e-\right.}{f(a-a\sin(e+fx))^{7/2}(a+a\sin(e+fx))^{7/2}}$$

$$= \frac{\left(2^{-\frac{1}{2}+m}a^5c^3\cos^7(e+fx)(a+a\sin(e+fx))^{-4+m}\left(\frac{a+a\sin(e+fx)}{a}\right)^{\frac{1}{2}-m}\right.}{f(a-a\sin(e+f}$$

$$= -\frac{2^{\frac{1}{2}+m}a^4c^3\cos^7(e+fx)\,{}_2F_1\left(\frac{7}{2},\frac{1}{2}-m;\frac{9}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+}{7f}$$

Mathematica [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] $Aborted
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(3\,c^3\cos\left(fx+e\right)^2-4\,c^3-\left(c^3\cos\left(fx+e\right)^2-4\,c^3\right)\sin\left(fx+e\right)\right)\left(a\sin\left(fx+e\right)+a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(
f*x + e))*(a*sin(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(c\sin\left(fx+e\right)-c\right)^{3}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate(-(c*sin(f*x + e) - c)^3*(a*sin(f*x + e) + a)^m, x)
```

maple [F] time = 5.57, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(c\sin\left(fx+e\right)-c\right)^{3}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -integrate((c*sin(f*x + e) - c)^3*(a*sin(f*x + e) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^3,x)
```

```
[Out] int((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c^3\left(\int 3\left(a\sin\left(e+fx\right)+a\right)^m\sin\left(e+fx\right)dx+\int\left(-3\left(a\sin\left(e+fx\right)+a\right)^m\sin^2\left(e+fx\right)\right)dx+\int\left(a\sin\left(e+f\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**3,x)
```

```
[Out] -c**3*(Integral(3*(a*sin(e + f*x) + a)**m*sin(e + f*x), x) + Integral(-3*(a
*sin(e + f*x) + a)**m*sin(e + f*x)**2, x) + Integral((a*sin(e + f*x) + a)**
m*sin(e + f*x)**3, x) + Integral(-(a*sin(e + f*x) + a)**m, x))
```

3.406 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^2\,dx$

Optimal. Leaf size=86

$$-\frac{a^3c^2 2^{m+\frac{1}{2}}\cos^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-3}\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-m;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{5f}$$

```
[Out] -1/5*2^(1/2+m)*a^3*c^2*cos(f*x+e)^5*hypergeom([5/2, 1/2-m],[7/2],1/2-1/2*si
n(f*x+e))*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(-3+m)/f
```

Rubi [A] time = 0.14, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2689, 70, 69}

$$-\frac{a^3c^2 2^{m+\frac{1}{2}}\cos^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-3}\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-m;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] -(2^(1/2 + m)*a^3*c^2*Cos[e + f*x]^5*Hypergeometric2F1[5/2, 1/2 - m, 7/2, (
1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^(-3
+ m))/(5*f)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
```

```
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^2\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(a+a\sin(e+fx))^{-2+m}\,dx$$
$$= \frac{\left(a^4c^2\cos^5(e+fx)\right)\text{Subst}\left(\int (a-ax)^{3/2}(a+ax)^{-\frac{1}{2}+m}\,dx,x,\sin(e\right.}{f(a-a\sin(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}$$
$$= \frac{\left(2^{-\frac{1}{2}+m}a^4c^2\cos^5(e+fx)(a+a\sin(e+fx))^{-3+m}\left(\frac{a+a\sin(e+fx)}{a}\right)^{\frac{1}{2}-m}\right.}{f(a-a\sin(e+f}$$
$$= -\frac{2^{\frac{1}{2}+m}a^3c^2\cos^5(e+fx)\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-m;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+}{5f}$$

Mathematica [C] time = 153.51, size = 88512, normalized size = 1029.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(c^2\cos\left(fx+e\right)^2+2\,c^2\sin\left(fx+e\right)-2\,c^2\right)\left(a\sin\left(fx+e\right)+a\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2)*(a*sin(f*x + e)
 + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\sin\left(fx+e\right)-c\right)^{2}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((c*sin(f*x + e) - c)^2*(a*sin(f*x + e) + a)^m, x)
```

maple [F] time = 6.57, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\sin\left(fx+e\right)-c\right)^{2}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((c*sin(f*x + e) - c)^2*(a*sin(f*x + e) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^2,x)
```

```
[Out] int((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int\left(-2\left(a\sin\left(e+fx\right)+a\right)^m\sin\left(e+fx\right)\right)dx+\int\left(a\sin\left(e+fx\right)+a\right)^m\sin^2\left(e+fx\right)dx+\int\left(a\sin\left(e+fx\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**2,x)
```

```
[Out] c**2*(Integral(-2*(a*sin(e + f*x) + a)**m*sin(e + f*x), x) + Integral((a*si
n(e + f*x) + a)**m*sin(e + f*x)**2, x) + Integral((a*sin(e + f*x) + a)**m,
x))
```

3.407 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))\,dx$

Optimal. Leaf size=84

$$-\frac{a^2c2^{m+\frac{1}{2}}\cos^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3f}$$

```
[Out] -1/3*2^(1/2+m)*a^2*c*cos(f*x+e)^3*hypergeom([3/2, 1/2-m],[5/2],1/2-1/2*sin(
f*x+e))*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(-2+m)/f
```

Rubi [A] time = 0.11, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2736, 2689, 70, 69}

$$-\frac{a^2c2^{m+\frac{1}{2}}\cos^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]),x]
```

```
[Out] -(2^(1/2 + m)*a^2*c*Cos[e + f*x]^3*Hypergeometric2F1[3/2, 1/2 - m, 5/2, (1
- Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^(-2 +
m))/(3*f)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
```

```
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))\,dx = (ac)\int \cos^2(e+fx)(a+a\sin(e+fx))^{-1+m}\,dx$$

$$= \frac{\left(a^3c\cos^3(e+fx)\right)\mathrm{Subst}\left(\int \sqrt{a-ax}\,(a+ax)^{-\frac{1}{2}+m}\,dx, x, \sin(e+f\right.}{f(a-a\sin(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}$$

$$= \frac{\left(2^{-\frac{1}{2}+m}a^3c\cos^3(e+fx)(a+a\sin(e+fx))^{-2+m}\left(\frac{a+a\sin(e+fx)}{a}\right)^{\frac{1}{2}-m}\right)}{f(a-a\sin(e+fx)}$$

$$= -\frac{2^{\frac{1}{2}+m}a^2c\cos^3(e+fx)\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\mathrm{si}}{3f}$$

Mathematica [C] time = 1.66, size = 261, normalized size = 3.11

$$-\frac{(-1)^{3/4}c2^{-2m-1}e^{-\frac{3}{2}i(e+fx)}\left(-(-1)^{3/4}e^{-\frac{1}{2}i(e+fx)}\left(e^{i(e+fx)}+i\right)\right)^{2m+1}(\sin(e+fx)-1)\left((m-1)me^{2i(e+fx)}\,{}_2F_1\left(1,m;-m;\right.\right.}{f(m}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]),x]
```

```
[Out] -((((-1)^(3/4)*2^(-1 - 2*m)*c*(-(((-1)^(3/4)*(I + E^(I*(e + f*x))))/E^((I/2)
*(e + f*x))))^(1 + 2*m)*(E^((2*I)*(e + f*x))*(-1 + m)*m*Hypergeometric2F1[1
, m, -m, (-I)/E^(I*(e + f*x))] - (1 + m)*(2*E^(I*(e + f*x))*(-1 + m)*Hyperg
eometric2F1[1, 1 + m, 1 - m, (-I)/E^(I*(e + f*x))] - m*Hypergeometric2F1[1,
```

```
2 + m, 2 - m, (-I)/E^(I*(e + f*x))]]))*(-1 + Sin[e + f*x])*(a*(1 + Sin[e +
f*x]))^m)/(E^(((3*I)/2)*(e + f*x))*f*(-1 + m)*m*(1 + m)*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])^2*Sin[(2*e + Pi + 2*f*x)/4]^(2*m)))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(c\sin\left(fx+e\right)-c\right)\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x, algorithm="fricas")
[Out] integral(-(c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(c\sin\left(fx+e\right)-c\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x, algorithm="giac")
[Out] integrate(-(c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)^m, x)
```

maple [F] time = 1.80, size = 0, normalized size = 0.00

$$\int\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x)
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(c\sin\left(fx+e\right)-c\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x, algorithm="maxima")
[Out] -integrate((c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(c-c\,\sin\left(e+f\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x)),x)
```

```
[Out] int((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int\left(a\sin\left(e+fx\right)+a\right)^{m}\sin\left(e+fx\right)dx+\int\left(-\left(a\sin\left(e+fx\right)+a\right)^{m}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e)),x)
```

```
[Out] -c*(Integral((a*sin(e + f*x) + a)**m*sin(e + f*x), x) + Integral(-(a*sin(e
+ f*x) + a)**m, x))
```

3.408 $\int \frac{(a+a\sin(e+fx))^m}{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=76

$$\frac{2^{m+\frac{1}{2}}\sec(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{cf}$$

```
[Out] 2^(1/2+m)*hypergeom([-1/2, 1/2-m],[1/2],1/2-1/2*sin(f*x+e))*sec(f*x+e)*(1+s
in(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^m/c/f
```

Rubi [A] time = 0.13, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2689, 70, 69}

$$\frac{2^{m+\frac{1}{2}}\sec(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{cf}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x]),x]
```

```
[Out] (2^(1/2 + m)*Hypergeometric2F1[-1/2, 1/2 - m, 1/2, (1 - Sin[e + f*x])/2]*Se
c[e + f*x]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^m)/(c*f)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
```

```
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{c-c\sin(e+fx)}\,dx = \frac{\int \sec^2(e+fx)(a+a\sin(e+fx))^{1+m}\,dx}{ac}$$

$$= \frac{\left(a\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}\right)\mathrm{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{(a-ax)^{3/2}}\,dx, x, \sin\right.}{cf}$$

$$= \frac{\left(2^{-\frac{1}{2}+m}a\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,(a+a\sin(e+fx))^m\left(\frac{a+a\sin(e+fx)}{a}\right)^{\frac{1}{2}-m}\right)\mathrm{Subs}}{cf}$$

$$= \frac{2^{\frac{1}{2}+m}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec(e+fx)(1+\sin(e+fx))^{\frac{1}{2}-m}(a+a\,\mathrm{s}}{cf}$$

Mathematica [C] time = 6.39, size = 3844, normalized size = 50.58

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x]),x]
```

```
[Out] -1/2*((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Cot[(-e + Pi/2 - f*x)/4]*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])^2*(a + a*Sin[e + f*x])^m*(-(AppellF1[-1/2, -2*
m, 2*m, 1/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[
(-e + Pi/2 - f*x)/4]^2)^(2*m)) + (3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e +
 Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2*
```

(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Ta
n[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2,
 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2
] + AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
+ Pi/2 - f*x)/4]^2])*Tan[(-e + Pi/2 - f*x)/4]^2)))/(f*(c - c*Sin[e + f*x])*
(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^2*(-1/2*(
m*(Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(-(AppellF1[-1/2, -2*m, 2*m, 1/2, Tan[
(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)
/4]^2)^(2*m)) + (3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2
, -Tan[(-e + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2*(1 - Tan[(-e + Pi
/2 - f*x)/4]^2)^(2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*
x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/
2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2,
 -2*m, 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^
2])*Tan[(-e + Pi/2 - f*x)/4]^2))) - ((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Csc
[(-e + Pi/2 - f*x)/4]^2*(-(AppellF1[-1/2, -2*m, 2*m, 1/2, Tan[(-e + Pi/2 -
f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m))
 + (3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +
Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2*(1 - Tan[(-e + Pi/2 - f*x)/4]^
2)^(2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan
[(-e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e +
Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*
m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Tan[(-e +
 Pi/2 - f*x)/4]^2)))/8 + ((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Cot[(-e + Pi/2
 - f*x)/4]*(-(m*AppellF1[-1/2, -2*m, 2*m, 1/2, Tan[(-e + Pi/2 - f*x)/4]^2,
-Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Tan[(-e + P
i/2 - f*x)/4]) - (Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(m*AppellF1[1/2, 1 - 2*
m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(
-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4] + m*AppellF1[1/2, -2*m, 1 +
2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e
+ Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]) + (3*AppellF1[1/2, -2*m, 2*m,
3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/
2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*
m))/(2*(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-
e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/
2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m,
5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Tan[(-e + Pi
/2 - f*x)/4]^2)) + (3*Tan[(-e + Pi/2 - f*x)/4]^2*(-1/3*(m*AppellF1[3/2, 1 -
 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Se
c[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]) - (m*AppellF1[3/2, -2*m,
 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec
[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/3)*(1 - Tan[(-e + Pi/2 -
f*x)/4]^2)^(2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]
^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Ta
n[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m

```
, 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*T
an[(-e + Pi/2 - f*x)/4]^2) - (3*m*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + P
i/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Ta
n[(-e + Pi/2 - f*x)/4]^3*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(-1 + 2*m))/(3*Ap
pellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f
*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^
2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e
 + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Tan[(-e + Pi/2 - f*x)/4]
^2) - (3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
 + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2*(1 - Tan[(-e + Pi/2 - f*x)/
4]^2)^(2*m)*(-2*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4
]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(
-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Sec[(-e + Pi/2 - f*x)/
4]^2*Tan[(-e + Pi/2 - f*x)/4] + 3*(-1/3*(m*AppellF1[3/2, 1 - 2*m, 2*m, 5/2,
 Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 -
f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]) - (m*AppellF1[3/2, -2*m, 1 + 2*m, 5/2,
Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f
*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/3) - 4*m*Tan[(-e + Pi/2 - f*x)/4]^2*((-6
*m*AppellF1[5/2, 1 - 2*m, 1 + 2*m, 7/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-
e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/
5 + (3*(1 - 2*m)*AppellF1[5/2, 2 - 2*m, 2*m, 7/2, Tan[(-e + Pi/2 - f*x)/4]^
2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 -
 f*x)/4])/10 - (3*(1 + 2*m)*AppellF1[5/2, -2*m, 2 + 2*m, 7/2, Tan[(-e + Pi/
2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[
(-e + Pi/2 - f*x)/4])/10)))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2
 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*
m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1
[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x
)/4]^2])*Tan[(-e + Pi/2 - f*x)/4]^2)^2))/2))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{c\sin\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{c\sin\left(fx+e\right)-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x, algorithm="giac")
[Out] integrate(-(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c), x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^{m}}{c-c\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x)
[Out] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{c\sin\left(fx+e\right)-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x, algorithm="maxima")
[Out] -integrate((a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^{m}}{c-c\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x)),x)
[Out] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\left(a\sin\left(e+fx\right)+a\right)^{m}}{\sin\left(e+fx\right)-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c-c*sin(f*x+e)),x)
[Out] -Integral((a*sin(e + f*x) + a)**m/(sin(e + f*x) - 1), x)/c
```

3.409 $\int \frac{(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=86

$$\frac{2^{m+\frac{1}{2}}\sec^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3ac^2f}$$

```
[Out] 1/3*2^(1/2+m)*hypergeom([-3/2, 1/2-m],[-1/2],1/2-1/2*sin(f*x+e))*sec(f*x+e)
^3*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(1+m)/a/c^2/f
```

Rubi [A] time = 0.13, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2689, 70, 69}

$$\frac{2^{m+\frac{1}{2}}\sec^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3ac^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (2^(1/2 + m)*Hypergeometric2F1[-3/2, 1/2 - m, -1/2, (1 - Sin[e + f*x])/2]*S
ec[e + f*x]^3*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^(1 + m))/(3
*a*c^2*f)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^2}\,dx = \frac{\int \sec^4(e+fx)(a+a\sin(e+fx))^{2+m}\,dx}{a^2c^2}$$

$$= \frac{\left(\sec^3(e+fx)(a-a\sin(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{(a-ax)^{5/2}}\,dx,x,s\right.}{c^2f}$$

$$= \frac{\left(2^{-\frac{1}{2}+m}\sec^3(e+fx)(a-a\sin(e+fx))^{3/2}(a+a\sin(e+fx))^{1+m}\left(\frac{a+a\sin(e+fx)}{a}\right)^{\frac{1}{2}-m}\right)S}{c^2f}$$

$$= \frac{2^{\frac{1}{2}+m}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec^3(e+fx)(1+\sin(e+fx))^{\frac{1}{2}-m}(a+}{3ac^2f}$$

Mathematica [C] time = 6.42, size = 5391, normalized size = 62.69

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{c^{2}\cos\left(fx+e\right)^{2}+2\,c^{2}\sin\left(fx+e\right)-2\,c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^m/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) -
 2*c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(c\sin\left(fx+e\right)-c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)^2, x)
```

maple [F] time = 1.13, size = 0, normalized size = 0.00

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^{m}}{\left(c-c\sin\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(c\sin\left(fx+e\right)-c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(c-c\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^2,x)
[Out] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\left(a\sin\left(e+fx\right)+a\right)^m}{\sin^2\left(e+fx\right)-2\sin\left(e+fx\right)+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**2,x)
[Out] Integral((a*sin(e + f*x) + a)**m/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1), x)
/c**2
```

3.410 $\int \frac{(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=86

$$\frac{2^{m+\frac{1}{2}}\sec^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+2}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}-m;-\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{5a^2c^3f}$$

```
[Out] 1/5*2^(1/2+m)*hypergeom([-5/2, 1/2-m],[-3/2],1/2-1/2*sin(f*x+e))*sec(f*x+e)
^5*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(2+m)/a^2/c^3/f
```

Rubi [A] time = 0.13, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2736, 2689, 70, 69}

$$\frac{2^{m+\frac{1}{2}}\sec^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+2}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}-m;-\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{5a^2c^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (2^(1/2 + m)*Hypergeometric2F1[-5/2, 1/2 - m, -3/2, (1 - Sin[e + f*x])/2]*S
ec[e + f*x]^5*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^(2 + m))/(5
*a^2*c^3*f)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^3}\,dx = \frac{\int \sec^6(e+fx)(a+a\sin(e+fx))^{3+m}\,dx}{a^3c^3}$$

$$= \frac{\left(\sec^5(e+fx)(a-a\sin(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{(a-ax)^{7/2}}\,dx,x,s\right.}{ac^3f}$$

$$= \frac{\left(2^{-\frac{1}{2}+m}\sec^5(e+fx)(a-a\sin(e+fx))^{5/2}(a+a\sin(e+fx))^{2+m}\left(\frac{a+a\sin(e+fx)}{a}\right)^{\frac{1}{2}-m}\right)S}{ac^3f}$$

$$= \frac{2^{\frac{1}{2}+m}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}-m;-\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec^5(e+fx)(1+\sin(e+fx))^{\frac{1}{2}-m}(a+}{5a^2c^3f}$$

Mathematica [C] time = 22.78, size = 7184, normalized size = 83.53

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x])^3,x]

[Out] Result too large to show
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{3\,c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}-\left(c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x, algorithm="fricas")
[Out] integral(-(a*sin(f*x + e) + a)^m/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f
*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(c\sin\left(fx+e\right)-c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x, algorithm="giac")
[Out] integrate(-(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)^3, x)
```

maple [F] time = 1.09, size = 0, normalized size = 0.00

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^{m}}{\left(c-c\sin\left(fx+e\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x)
[Out] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(c\sin\left(fx+e\right)-c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -integrate((a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(c-c\,\sin\left(e+f\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^3,x)
[Out] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**3,x)
[Out] Timed out
```

3.411 $\int (a+a\sin(e+fx))^m (c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=160

$$\frac{64c^3\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+5)\left(4m^2+8m+3\right)\sqrt{c-c\sin(e+fx)}}+\frac{16c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}\,(a\sin(e+fx)+a)^m}{f\left(4m^2+16m+15\right)}+\frac{2c\cos$$

```
[Out] 2*c*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2)/f/(5+2*m)+64*c^3*c
os(f*x+e)*(a+a*sin(f*x+e))^m/f/(5+2*m)/(4*m^2+8*m+3)/(c-c*sin(f*x+e))^(1/2)
+16*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2)/f/(4*m^2+16*m+
15)
```

Rubi [A] time = 0.25, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2740, 2738}

$$\frac{16c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}\,(a\sin(e+fx)+a)^m}{f\left(4m^2+16m+15\right)}+\frac{64c^3\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+5)\left(4m^2+8m+3\right)\sqrt{c-c\sin(e+fx)}}+\frac{2c\cos$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (64*c^3*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(5 + 2*m)*(3 + 8*m + 4*m^2)
*Sqrt[c - c*Sin[e + f*x]]) + (16*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sq
rt[c - c*Sin[e + f*x]])/(f*(15 + 16*m + 4*m^2)) + (2*c*Cos[e + f*x]*(a + a*
Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(3/2))/(f*(5 + 2*m))
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int (a+a\sin(e+fx))^m (c-c\sin(e+fx))^{5/2}\,dx = \frac{2c\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3/2}}{f(5+2m)} + \frac{(8c)\int}{}$$

$$= \frac{16c^2\cos(e+fx)(a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}}{f\left(15+16m+4m^2\right)} + \frac{2c\cos}{}$$

$$= \frac{64c^3\cos(e+fx)(a+a\sin(e+fx))^m}{f\left(15+46m+36m^2+8m^3\right)\sqrt{c-c\sin(e+fx)}} + \frac{16c^2\cos(e+}{}$$

Mathematica [A] time = 2.51, size = 149, normalized size = 0.93

$$-\frac{c^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(a(\sin(e+fx)+1))^m\left(4\left(4m^2+16m+7\right)\sin(e+fx)\right.}{f(2m+1)(2m+3)(2m+5)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+f\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] -((c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^m*Sqrt[
c - c*Sin[e + f*x]]*(-89 - 56*m - 12*m^2 + (3 + 8*m + 4*m^2)*Cos[2*(e + f*x
)] + 4*(7 + 16*m + 4*m^2)*Sin[e + f*x]))/(f*(1 + 2*m)*(3 + 2*m)*(5 + 2*m)*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])))
```

fricas [A] time = 0.47, size = 267, normalized size = 1.67

$$-\frac{2\left(\left(4c^2m^2+8c^2m+3c^2\right)\cos\left(fx+e\right)^3-\left(4c^2m^2+24c^2m+11c^2\right)\cos\left(fx+e\right)^2-32c^2-2\left(4c^2m^2+16c^2m\right.\right.}{8fm^3+36fm^2+46fm+\left(8fm^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -2*((4*c^2*m^2 + 8*c^2*m + 3*c^2)*cos(f*x + e)^3 - (4*c^2*m^2 + 24*c^2*m +
11*c^2)*cos(f*x + e)^2 - 32*c^2 - 2*(4*c^2*m^2 + 16*c^2*m + 23*c^2)*cos(f*x
 + e) + ((4*c^2*m^2 + 8*c^2*m + 3*c^2)*cos(f*x + e)^2 - 32*c^2 + 2*(4*c^2*m
^2 + 16*c^2*m + 7*c^2)*cos(f*x + e))*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c
)*(a*sin(f*x + e) + a)^m/(8*f*m^3 + 36*f*m^2 + 46*f*m + (8*f*m^3 + 36*f*m^2
 + 46*f*m + 15*f)*cos(f*x + e) - (8*f*m^3 + 36*f*m^2 + 46*f*m + 15*f)*sin(f
*x + e) + 15*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate((-c*sin(f*x + e) + c)^(5/2)*(a*sin(f*x + e) + a)^m, x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x)
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x)
```

maxima [A] time = 0.47, size = 290, normalized size = 1.81

$$-\frac{2\left(\left(4\,m^{2}+24\,m+43\right)a^{m}c^{\frac{5}{2}}-\frac{\left(12\,m^{2}+40\,m-15\right)a^{m}c^{\frac{5}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{2\left(4\,m^{2}+8\,m+35\right)a^{m}c^{\frac{5}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{2\left(4\,m^{2}+8\,m+35\right)a^{m}c^{\frac{5}{2}}\sin\left(f\right.}{\left(\cos\left(fx+e\right)+1\right)^{3}}\right.}{\left(8\,m^{3}+36\,m^{2}+46\,m+15\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")
[Out] -2*((4*m^2 + 24*m + 43)*a^m*c^(5/2) - (12*m^2 + 40*m - 15)*a^m*c^(5/2)*sin(
f*x + e)/(cos(f*x + e) + 1) + 2*(4*m^2 + 8*m + 35)*a^m*c^(5/2)*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 + 2*(4*m^2 + 8*m + 35)*a^m*c^(5/2)*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3 - (12*m^2 + 40*m - 15)*a^m*c^(5/2)*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 + (4*m^2 + 24*m + 43)*a^m*c^(5/2)*sin(f*x + e)^5/(cos(f*x +
 e) + 1)^5)*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - m*log(sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + 1))/((8*m^3 + 36*m^2 + 46*m + 15)*f*(sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2))
```

mupad [B] time = 10.05, size = 163, normalized size = 1.02

$$\frac{c^{2}\left(a\left(\sin\left(e+f\,x\right)+1\right)\right)^{m}\sqrt{-c\left(\sin\left(e+f\,x\right)-1\right)}\,\left(3\cos\left(3\,e+3\,f\,x\right)-175\cos\left(e+f\,x\right)+28\sin\left(2\,e+2\,f\,x\right.\right.}{2\,f\,\left(\sin\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] (c^2*(a*(sin(e + f*x) + 1))^m*(-c*(sin(e + f*x) - 1))^(1/2)*(3*cos(3*e + 3*
f*x) - 175*cos(e + f*x) + 28*sin(2*e + 2*f*x) + 16*m^2*sin(2*e + 2*f*x) - 1
04*m*cos(e + f*x) + 8*m*cos(3*e + 3*f*x) - 20*m^2*cos(e + f*x) + 64*m*sin(2
*e + 2*f*x) + 4*m^2*cos(3*e + 3*f*x)))/(2*f*(sin(e + f*x) - 1)*(46*m + 36*m
^2 + 8*m^3 + 15))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.412 $\int (a + a\sin(e + fx))^m (c - c\sin(e + fx))^{3/2} dx$

Optimal. Leaf size=100

$$\frac{8c^2\cos(e+fx)(a\sin(e+fx)+a)^m}{f\left(4m^2+8m+3\right)\sqrt{c-c\sin(e+fx)}}+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}\,(a\sin(e+fx)+a)^m}{f(2m+3)}$$

```
[Out] 8*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(4*m^2+8*m+3)/(c-c*sin(f*x+e))^(1/2)+
2*c*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2)/f/(3+2*m)
```

Rubi [A] time = 0.15, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2740, 2738}

$$\frac{8c^2\cos(e+fx)(a\sin(e+fx)+a)^m}{f\left(4m^2+8m+3\right)\sqrt{c-c\sin(e+fx)}}+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}\,(a\sin(e+fx)+a)^m}{f(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (8*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(3 + 8*m + 4*m^2)*Sqrt[c - c
*Sin[e + f*x]]) + (2*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[c - c*Sin[e
 + f*x]])/(f*(3 + 2*m))
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rubi steps

$$\int (a + a\sin(e + fx))^m (c - c\sin(e + fx))^{3/2}\,dx = \frac{2c\cos(e+fx)(a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}}{f(3+2m)} + \frac{(4c)\int(\ldots}{\ldots}$$
$$= \frac{8c^2\cos(e+fx)(a+a\sin(e+fx))^m}{f\left(3+8m+4m^2\right)\sqrt{c-c\sin(e+fx)}} + \frac{2c\cos(e+fx)(a+a\sin\ldots}{f(\ldots}$$

Mathematica [A] time = 0.48, size = 110, normalized size = 1.10

$$-\frac{2c\sqrt{c-c\sin(e+fx)}\,((2m+1)\sin(e+fx)-2m-5)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(a(\sin(e+fx)+1))^m}{f(2m+1)(2m+3)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^m*Sqrt[c
 - c*Sin[e + f*x]]*(-5 - 2*m + (1 + 2*m)*Sin[e + f*x]))/(f*(1 + 2*m)*(3 + 2
*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

fricas [A] time = 0.47, size = 145, normalized size = 1.45

$$\frac{2\left((2\,cm+c)\cos\left(fx+e\right)^2+(2\,cm+5\,c)\cos\left(fx+e\right)-\left((2\,cm+c)\cos\left(fx+e\right)-4\,c\right)\sin\left(fx+e\right)+4\,c\right)\sqrt{-c\sin\ldots}}{4\,fm^2+8\,fm+\left(4\,fm^2+8\,fm+3\,f\right)\cos\left(fx+e\right)-\left(4\,fm^2+8\,fm+3\,f\right)\sin\left(fx+\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 2*((2*c*m + c)*cos(f*x + e)^2 + (2*c*m + 5*c)*cos(f*x + e) - ((2*c*m + c)*c
os(f*x + e) - 4*c)*sin(f*x + e) + 4*c)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x
 + e) + a)^m/(4*f*m^2 + 8*f*m + (4*f*m^2 + 8*f*m + 3*f)*cos(f*x + e) - (4*f
*m^2 + 8*f*m + 3*f)*sin(f*x + e) + 3*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e) + a)^m, x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \left(a + a\sin\left(fx + e\right)\right)^{m} \left(c - c\sin\left(fx + e\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x)
```

maxima [B] time = 0.44, size = 193, normalized size = 1.93

$$-\frac{2\left(a^{m}c^{\frac{3}{2}}(2m+5) - \frac{a^{m}c^{\frac{3}{2}}(2m-3)\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1} - \frac{a^{m}c^{\frac{3}{2}}(2m-3)\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}} + \frac{a^{m}c^{\frac{3}{2}}(2m+5)\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}\right)e^{\left(2m\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)-m\log\left(\right.\right.}}{\left(4m^{2}+8m+3\right)f\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] -2*(a^m*c^(3/2)*(2*m + 5) - a^m*c^(3/2)*(2*m - 3)*sin(f*x + e)/(cos(f*x + e
) + 1) - a^m*c^(3/2)*(2*m - 3)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^m*c^
(3/2)*(2*m + 5)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*e^(2*m*log(sin(f*x + e
)/(cos(f*x + e) + 1) + 1) - m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))
/((4*m^2 + 8*m + 3)*f*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2))
```

mupad [B] time = 1.19, size = 94, normalized size = 0.94

$$-\frac{c\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)^{m}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(10\,\cos\left(e+f\,x\right)-\sin\left(2\,e+2\,f\,x\right)+4\,m\,\cos\left(e+f\,x\right)-2\right.}{f\,\left(\sin\left(e+f\,x\right)-1\right)\,\left(4\,m^{2}+8\,m+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] -(c*(a*(sin(e + f*x) + 1))^m*(-c*(sin(e + f*x) - 1))^(1/2)*(10*cos(e + f*x)
 - sin(2*e + 2*f*x) + 4*m*cos(e + f*x) - 2*m*sin(2*e + 2*f*x)))/(f*(sin(e +
 f*x) - 1)*(8*m + 4*m^2 + 3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.413 $\int (a + a\sin(e + fx))^m \sqrt{c - c\sin(e + fx)}\, dx$

Optimal. Leaf size=46

$$\frac{2c\cos(e + fx)(a\sin(e + fx) + a)^m}{f(2m + 1)\sqrt{c - c\sin(e + fx)}}$$

```
[Out] 2*c*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2738}

$$\frac{2c\cos(e + fx)(a\sin(e + fx) + a)^m}{f(2m + 1)\sqrt{c - c\sin(e + fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e + f
*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a + a\sin(e + fx))^m \sqrt{c - c\sin(e + fx)}\, dx = \frac{2c\cos(e + fx)(a + a\sin(e + fx))^m}{f(1 + 2m)\sqrt{c - c\sin(e + fx)}}$$

Mathematica [A] time = 0.17, size = 85, normalized size = 1.85

$$\frac{2\sqrt{c - c\sin(e + fx)}\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)(a(\sin(e + fx) + 1))^m}{f(2m + 1)\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])^m*Sqrt[c - c*Sin[e + f*x]],x]

[Out] (2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^m*Sqrt[c - c*Sin[e + f*x]])/(f*(1 + 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))

fricas [A] time = 0.49, size = 76, normalized size = 1.65

$$\frac{2\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)}{2\,fm+\left(2\,fm+f\right)\cos\left(fx+e\right)-\left(2\,fm+f\right)\sin\left(fx+e\right)+f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] 2*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m*(cos(f*x + e) + sin(f*x + e) + 1)/(2*f*m + (2*f*m + f)*cos(f*x + e) - (2*f*m + f)*sin(f*x + e) + f)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int\left(a+a\sin\left(fx+e\right)\right)^{m}\sqrt{c-c\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x)

[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x)

maxima [B] time = 0.45, size = 116, normalized size = 2.52

$$-\frac{2\left(a^{m}\sqrt{c}+\frac{a^{m}\sqrt{c}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)e^{\left(2\,m\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)-m\log\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)\right)}}{f(2\,m+1)\sqrt{\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*(a^m*sqrt(c) + a^m*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1))*e^(2*m*log(s
in(f*x + e)/(cos(f*x + e) + 1) + 1) - m*log(sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 1))/(f*(2*m + 1)*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))
```

mupad [B] time = 0.45, size = 53, normalized size = 1.15

$$-\frac{2\cos\left(e+f\,x\right)\,\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)^m\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}}{f\,\left(2\,m+1\right)\,\left(\sin\left(e+f\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] -(2*cos(e + f*x)*(a*(sin(e + f*x) + 1))^m*(-c*(sin(e + f*x) - 1))^(1/2))/(f
*(2*m + 1)*(sin(e + f*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^m\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*sqrt(-c*(sin(e + f*x) - 1)), x)
```

3.414 $\int \frac{(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=68

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] cos(f*x+e)*hypergeom([1, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(a+a*sin(f*x+e)
)^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2745, 2667, 68}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*
(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
```

```
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\cos(e+fx)\int \sec(e+fx)(a+a\sin(e+fx))^{\frac{1}{2}+m}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$= \frac{(a\cos(e+fx))\,\text{Subst}\left(\int \frac{(a+x)^{-\frac{1}{2}+m}}{a-x}\,dx, x, a\sin(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$= \frac{\cos(e+fx)\,{}_2F_1\left(1,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}}$$

Mathematica [B] time = 1.46, size = 157, normalized size = 2.31

$$\frac{2^{-2m-\frac{3}{2}}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(\sin(e+fx)+1))^m\left(4^m\,{}_2F_1\left(1,2m;2m+1;\sin\left(\frac{1}{4}(2e+2fx+\pi)\right)\right.\right.}{fm\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2^(-3/2 - 2*m)*(4^m*Hypergeometric2F1[1, 2*m, 1 + 2*m, Sin[(2*e + Pi + 2*f
*x)/4]] - Hypergeometric2F1[2*m, 2*m, 1 + 2*m, (1 - Tan[(2*e - Pi + 2*f*x)/
8]^2)/2]*(Sec[(2*e - Pi + 2*f*x)/8]^2)^(2*m))*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(a*(1 + Sin[e + f*x]))^m)/(f*m*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^m}{c\sin\left(fx+e\right)-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e)
- c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e) + c), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^{m}}{\sqrt{c-c\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
[Out] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^(1/2),x)
[Out] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}}{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(1/2),x)
[Out] Integral((a*(sin(e + f*x) + 1))**m/sqrt(-c*(sin(e + f*x) - 1)), x)
```

3.415 $\int \frac{(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(2,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{2cf(2m+1)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/2*cos(f*x+e)*hypergeom([2, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(a+a*sin(f*
x+e))^m/c/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2745, 2667, 68}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(2,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{2cf(2m+1)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[2, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*
(a + a*Sin[e + f*x])^m)/(2*c*f*(1 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
```

```
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{3/2}}\,dx &= \frac{\cos(e+fx)\int \sec^3(e+fx)(a+a\sin(e+fx))^{\frac{3}{2}+m}\,dx}{ac\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}\\
&= \frac{\left(a^2\cos(e+fx)\right)\operatorname{Subst}\left(\int \frac{(a+x)^{-\frac{1}{2}+m}}{(a-x)^2}\,dx, x, a\sin(e+fx)\right)}{cf\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}\\
&= \frac{\cos(e+fx)\,{}_2F_1\left(2,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^m}{2cf(1+2m)\sqrt{c-c\sin(e+fx)}}
\end{aligned}$$

Mathematica [C] time = 17.23, size = 3006, normalized size = 40.62

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/8*((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2])^3*(a + a*Sin[e + f*x])^m*(AppellF1[1, -2*m, 2*m, 2, Tan[(-e + Pi/2 - f*
x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Ta
n[(-e + Pi/2 - f*x)/4]^2 - (AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)
/4]^2, -Cot[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^2*(Csc[(-e + P
i/2 - f*x)/4]^2)^(2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(1 - Cot[(-e
 + Pi/2 - f*x)/4]^2)^(2*m) + (2^(1 - 2*m)*AppellF1[1 + 2*m, 2*m, 1, 2 + 2*m
, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)/4]^2]*(-1 +
 Tan[(-e + Pi/2 - f*x)/4]^2)*(1 - Tan[(-e + Pi/2 - f*x)/4]^4)^(2*m))/(1 + 2
*m)))/(Sqrt[2]*f*(c - c*Sin[e + f*x])^(3/2)*(Cos[Pi/4 + (e - Pi/2 + f*x)/2]
 - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^3*(-1/8*(m*Cos[(-e + Pi/2 - f*x)/4]*(Cos
[(-e + Pi/2 - f*x)/4]^2)^(-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/4]*(AppellF1[1, -
2*m, 2*m, 2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[
(-e + Pi/2 - f*x)/4]^2)^(2*m)*Tan[(-e + Pi/2 - f*x)/4]^2 - (AppellF1[1, -2*
m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e
 + Pi/2 - f*x)/4]^2*(Csc[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(1 - Tan[(-e + Pi/2
```

```
- f*x)/4]^2)^(2*m))/(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(2*m) + (2^(1 - 2*m)*A
ppellF1[1 + 2*m, 2*m, 1, 2 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - T
an[(-e + Pi/2 - f*x)/4]^2]*(-1 + Tan[(-e + Pi/2 - f*x)/4]^2)*(1 - Tan[(-e +
 Pi/2 - f*x)/4]^4)^(2*m))/(1 + 2*m)))/Sqrt[2] + ((Cos[(-e + Pi/2 - f*x)/4]^
2)^(2*m)*((AppellF1[1, -2*m, 2*m, 2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +
 Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m)*Tan[(-e + Pi/2 -
f*x)/4])/2 + m*AppellF1[1, -2*m, 2*m, 2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(
-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Tan[(-e + Pi/2 -
f*x)/4]^3 + (Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Tan[(-e + Pi/2 - f*x)/4]^2*(
-1/2*(m*AppellF1[2, 1 - 2*m, 2*m, 3, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +
 Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]) - (
m*AppellF1[2, -2*m, 1 + 2*m, 3, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2
 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/2) + (m*
AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 - f*x
)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^3*(Csc[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(1 -
Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(2*m) +
 m*AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 -
f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^3*(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(-1
- 2*m)*(Csc[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]
^2)^(2*m) + (AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e
 + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]*(Csc[(-e + Pi/2 - f*x)/4]^2)^
(1 + 2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(2*(1 - Cot[(-e + Pi/2 -
f*x)/4]^2)^(2*m)) - (Cot[(-e + Pi/2 - f*x)/4]^2*(Csc[(-e + Pi/2 - f*x)/4]^2
)^(2*m)*((m*AppellF1[2, 1 - 2*m, 2*m, 3, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(
-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]*Csc[(-e + Pi/2 - f*x)/4]^2)
/2 + (m*AppellF1[2, -2*m, 1 + 2*m, 3, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e
+ Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]*Csc[(-e + Pi/2 - f*x)/4]^2)/2)
*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(
2*m) + (m*AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e +
Pi/2 - f*x)/4]^2]*Csc[(-e + Pi/2 - f*x)/4]*(Csc[(-e + Pi/2 - f*x)/4]^2)^(2*
m)*Sec[(-e + Pi/2 - f*x)/4]*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(-1 + 2*m))/(1
 - Cot[(-e + Pi/2 - f*x)/4]^2)^(2*m) + (AppellF1[1 + 2*m, 2*m, 1, 2 + 2*m,
(1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e
 + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]*(1 - Tan[(-e + Pi/2 - f*x)/4]^
4)^(2*m))/(2^(2*m)*(1 + 2*m)) + (2^(1 - 2*m)*(-1/2*((1 + 2*m)*AppellF1[2 +
2*m, 2*m, 2, 3 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/
2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(2 + 2*
m) - (m*(1 + 2*m)*AppellF1[2 + 2*m, 1 + 2*m, 1, 3 + 2*m, (1 - Tan[(-e + Pi/
2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^
2*Tan[(-e + Pi/2 - f*x)/4])/(2*(2 + 2*m)))*(-1 + Tan[(-e + Pi/2 - f*x)/4]^2
)*(1 - Tan[(-e + Pi/2 - f*x)/4]^4)^(2*m))/(1 + 2*m) - (2^(2 - 2*m)*m*Appell
F1[1 + 2*m, 2*m, 1, 2 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-
e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]^3
*(-1 + Tan[(-e + Pi/2 - f*x)/4]^2)*(1 - Tan[(-e + Pi/2 - f*x)/4]^4)^(-1 + 2
*m))/(1 + 2*m)))/(8*Sqrt[2])))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}}{c^{2}\cos\left(fx+e\right)^{2}+2\,c^{2}\sin\left(fx+e\right)-2\,c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(c^2*cos(f*x + e
)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)^(3/2), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^{m}}{\left(c-c\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^(3/2),x)
[Out] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^m}{\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(3/2),x)
[Out] Integral((a*(sin(e + f*x) + 1))**m/(-c*(sin(e + f*x) - 1))**(3/2), x)
```

3.416 $\int \frac{(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=74

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(3,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{4c^2f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/4*cos(f*x+e)*hypergeom([3, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(a+a*sin(f*
x+e))^m/c^2/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2745, 2667, 68}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(3,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{4c^2f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[3, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*
(a + a*Sin[e + f*x])^m)/(4*c^2*f*(1 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
```

```
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\cos(e+fx)\int \sec^5(e+fx)(a+a\sin(e+fx))^{\frac{5}{2}+m}\,dx}{a^2c^2\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\left(a^3\cos(e+fx)\right)\operatorname{Subst}\left(\int \frac{(a+x)^{-\frac{1}{2}+m}}{(a-x)^3}\,dx, x, a\sin(e+fx)\right)}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(3,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^m}{4c^2f(1+2m)\sqrt{c-c\sin(e+fx)}}$$

Mathematica [C] time = 21.98, size = 5136, normalized size = 69.41

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x])^(5/2),x]
[Out] Result too large to show
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(-\frac{\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^m}{3\,c^3\cos\left(fx+e\right)^2-4\,c^3-\left(c^3\cos\left(fx+e\right)^2-4\,c^3\right)\sin\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")
[Out] integral(-sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(3*c^3*cos(f*x +
 e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)^(5/2), x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^{m}}{\left(c-c\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x)
[Out] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^(5/2),x)
[Out] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.417 $\int \frac{(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=68

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] cos(f*x+e)*hypergeom([1, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(a+a*sin(f*x+e)
)^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2745, 2667, 68}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*
(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
```

```
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\cos(e+fx)\int \sec(e+fx)(a+a\sin(e+fx))^{\frac{1}{2}+m}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\,\mathrm{Subst}\left(\int \frac{(a+x)^{-\frac{1}{2}+m}}{a-x}\,dx, x, a\sin(e+fx)\right)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(1,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}}$$

Mathematica [B] time = 0.53, size = 157, normalized size = 2.31

$$\frac{2^{-2m-\frac{3}{2}}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(\sin(e+fx)+1))^m\left(4^m\,{}_2F_1\left(1,2m;2m+1;\sin\left(\frac{1}{4}(2e+2fx+\pi)\right)\right)\right.}{fm\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2^(-3/2 - 2*m)*(4^m*Hypergeometric2F1[1, 2*m, 1 + 2*m, Sin[(2*e + Pi + 2*f
*x)/4]] - Hypergeometric2F1[2*m, 2*m, 1 + 2*m, (1 - Tan[(2*e - Pi + 2*f*x)/
8]^2)/2]*(Sec[(2*e - Pi + 2*f*x)/8]^2)^(2*m))*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(a*(1 + Sin[e + f*x]))^m)/(f*m*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^m}{c\sin\left(fx+e\right)-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e)
- c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e) + c), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^{m}}{\sqrt{c-c\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
[Out] int((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^(1/2),x)
[Out] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}}{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(1/2),x)
[Out] Integral((a*(sin(e + f*x) + 1))**m/sqrt(-c*(sin(e + f*x) - 1)), x)
```

3.418 $\int \frac{(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx$

Optimal. Leaf size=68

$$\frac{\cos(e+fx)(c\sin(e+fx)+c)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{a-a\sin(e+fx)}}$$

```
[Out] cos(f*x+e)*hypergeom([1, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(c+c*sin(f*x+e)
)^m/f/(1+2*m)/(a-a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2745, 2667, 68}

$$\frac{\cos(e+fx)(c\sin(e+fx)+c)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{a-a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + c*Sin[e + f*x])^m/Sqrt[a - a*Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*
(c + c*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[a - a*Sin[e + f*x]])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
```

```
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx &= \frac{\cos(e+fx)\int \sec(e+fx)(c+c\sin(e+fx))^{\frac{1}{2}+m}\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}\\
&= \frac{(c\cos(e+fx))\,\text{Subst}\left(\int \frac{(c+x)^{-\frac{1}{2}+m}}{c-x}\,dx, x, c\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}\\
&= \frac{\cos(e+fx)\,{}_2F_1\left(1,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx))\right)(c+c\sin(e+fx))^m}{f(1+2m)\sqrt{a-a\sin(e+fx)}}
\end{aligned}
$$

Mathematica [B] time = 0.44, size = 157, normalized size = 2.31

$$
\frac{2^{-2m-\frac{3}{2}}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(c(\sin(e+fx)+1))^m\left(4^m\,{}_2F_1\left(1,2m;2m+1;\sin\left(\frac{1}{4}(2e+2fx+\pi)\right)\right)\right.}{fm\sqrt{a-a\sin(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + c*Sin[e + f*x])^m/Sqrt[a - a*Sin[e + f*x]],x]
```

```
[Out] (2^(-3/2 - 2*m)*(4^m*Hypergeometric2F1[1, 2*m, 1 + 2*m, Sin[(2*e + Pi + 2*f
*x)/4]] - Hypergeometric2F1[2*m, 2*m, 1 + 2*m, (1 - Tan[(2*e - Pi + 2*f*x)/
8]^2)/2]*(Sec[(2*e - Pi + 2*f*x)/8]^2)^(2*m))*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(c*(1 + Sin[e + f*x]))^m)/(f*m*Sqrt[a - a*Sin[e + f*x]])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$
\text{integral}\left(-\frac{\sqrt{-a\sin\left(fx+e\right)+a}\left(c\sin\left(fx+e\right)+c\right)^m}{a\sin\left(fx+e\right)-a}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^m/(a*sin(f*x + e)
- a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sin\left(fx+e\right)+c\right)^{m}}{\sqrt{-a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((c*sin(f*x + e) + c)^m/sqrt(-a*sin(f*x + e) + a), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\left(c+c\sin\left(fx+e\right)\right)^{m}}{\sqrt{a-a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)
[Out] int((c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sin\left(fx+e\right)+c\right)^{m}}{\sqrt{-a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((c*sin(f*x + e) + c)^m/sqrt(-a*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+c\,\sin\left(e+f\,x\right)\right)}^m}{\sqrt{a-a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + c*sin(e + f*x))^m/(a - a*sin(e + f*x))^(1/2),x)
[Out] int((c + c*sin(e + f*x))^m/(a - a*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(\sin\left(e+fx\right)+1\right)\right)^{m}}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))**m/(a-a*sin(f*x+e))**(1/2),x)
[Out] Integral((c*(sin(e + f*x) + 1))**m/sqrt(-a*(sin(e + f*x) - 1)), x)
```

3.419 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}\,dx$

Optimal. Leaf size=164

$$\frac{2\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{c^2f(2m+5)\left(4m^2+8m+3\right)}+\frac{2\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m}}{cf\left(4m^2+16m+15\right)}$$

```
[Out] cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m)/f/(5+2*m)+2*cos(f*x+e
)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m)/c/f/(4*m^2+16*m+15)+2*cos(f*x+
e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/c^2/f/(8*m^3+36*m^2+46*m+15)
```

Rubi [A] time = 0.22, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2743, 2742}

$$\frac{2\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{c^2f(2m+5)\left(4m^2+8m+3\right)}+\frac{2\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m}}{cf\left(4m^2+16m+15\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 - m),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 - m))/(f*(5 +
 2*m)) + (2*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 -
m))/(c*f*(15 + 16*m + 4*m^2)) + (2*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c -
 c*Sin[e + f*x])^(-1 - m))/(c^2*f*(5 + 2*m)*(3 + 8*m + 4*m^2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}\,dx = \frac{\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}}{f(5+2m)} + \frac{2\int(}{}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}}{f(5+2m)} + \frac{2\,\mathrm{co}}{}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}}{f(5+2m)} + \frac{2\,\mathrm{co}}{}$$

Mathematica [A] time = 8.53, size = 174, normalized size = 1.06

$$\frac{2^{-m-2}\cos\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)\sin^{-2m-5}\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-3}\left(\cos\left(\frac{1}{2}(e+f\right.\right.}{f(2m+1)(2m+3)(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 - m),x]
```

```
[Out] (2^(-2 - m)*Cos[(-e + Pi/2 - f*x)/2]*Sin[(-e + Pi/2 - f*x)/2]^(-5 - 2*m)*(a
 + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 - m)*(4*(2 + 3*m + m^2) + Cos
[2*(-e + Pi/2 - f*x)] - 2*(3 + 2*m)*Sin[e + f*x]))/(f*(1 + 2*m)*(3 + 2*m)*(
5 + 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(-3 - m)))
```

fricas [A] time = 0.49, size = 101, normalized size = 0.62

$$-\frac{\left(2\,\cos\left(fx+e\right)^3+2\,(2\,m+3)\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(4\,m^2+12\,m+9\right)\cos\left(fx+e\right)\right)\left(a\sin\left(fx+e\right)+a\right)^m}{8\,fm^3+36\,fm^2+46\,fm+15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x, algorithm="fricas")
```

```
[Out] -(2*cos(f*x + e)^3 + 2*(2*m + 3)*cos(f*x + e)*sin(f*x + e) - (4*m^2 + 12*m
+ 9)*cos(f*x + e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 3)/(8
*f*m^3 + 36*f*m^2 + 46*f*m + 15*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^m\left(-c\sin\left(fx+e\right)+c\right)^{-m-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 3), x)
```

maple [F] time = 1.02, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m} \left(c-c\sin\left(fx+e\right)\right)^{-3-m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x)
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 3), x)
```

mupad [B] time = 8.25, size = 149, normalized size = 0.91

$$-\frac{2\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)^{m}\,\left(15\,\cos\left(e+f\,x\right)-\cos\left(3\,e+3\,f\,x\right)-6\,\sin\left(2\,e+2\,f\,x\right)+24\,m\,\cos\left(e+f\,x\right)+8\,m\right.}{c^{3}\,f\,\left(-c\,\left(\sin\left(e+f\,x\right)-1\right)\right)^{m}\,\left(8\,m^{3}+36\,m^{2}+46\,m+15\right)\,\left(15\,\sin\left(e+f\,x\right)+6\,\cos\left(2\,e+2\,f\,x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^(m + 3),x)
[Out] -(2*(a*(sin(e + f*x) + 1))^m*(15*cos(e + f*x) - cos(3*e + 3*f*x) - 6*sin(2*
e + 2*f*x) + 24*m*cos(e + f*x) + 8*m^2*cos(e + f*x) - 4*m*sin(2*e + 2*f*x))
)/(c^3*f*(-c*(sin(e + f*x) - 1))^m*(46*m + 36*m^2 + 8*m^3 + 15)*(15*sin(e +
 f*x) + 6*cos(2*e + 2*f*x) - sin(3*e + 3*f*x) - 10))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-3-m),x)
[Out] Timed out
```

3.420 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}\,dx$

Optimal. Leaf size=101

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{cf\left(4m^2+8m+3\right)}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{f(2m+3)}$$

```
[Out] cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m)/f/(3+2*m)+cos(f*x+e)*
(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/c/f/(4*m^2+8*m+3)
```

Rubi [A] time = 0.13, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2743, 2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{cf\left(4m^2+8m+3\right)}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{f(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))/(f*(3 +
 2*m)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m)
)/(c*f*(3 + 8*m + 4*m^2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a + a\sin(e + fx))^m (c - c\sin(e + fx))^{-2-m}\, dx = \frac{\cos(e + fx)(a + a\sin(e + fx))^m (c - c\sin(e + fx))^{-2-m}}{f(3 + 2m)} + \frac{\int(}{}$$

$$= \frac{\cos(e + fx)(a + a\sin(e + fx))^m (c - c\sin(e + fx))^{-2-m}}{f(3 + 2m)} + \frac{\mathrm{co}}{}$$

Mathematica [A] time = 3.04, size = 136, normalized size = 1.35

$$-\frac{2^{-m}\cos\left(\frac{1}{2}\left(-e - fx + \frac{\pi}{2}\right)\right)\sin^{-2m-3}\left(\frac{1}{2}\left(-e - fx + \frac{\pi}{2}\right)\right)(\sin(e + fx) - 2(m + 1))(a\sin(e + fx) + a)^m (c - c\sin(e}{f\left(8m^2 + 16m + 6\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m),x]
```

```
[Out] -((Cos[(-e + Pi/2 - f*x)/2]*Sin[(-e + Pi/2 - f*x)/2]^(-3 - 2*m)*(-2*(1 + m)
 + Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))/(2^m
*f*(6 + 16*m + 8*m^2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(-2 - m))))
```

fricas [A] time = 0.46, size = 72, normalized size = 0.71

$$\frac{\left(2\,(m + 1)\cos\left(fx + e\right) - \cos\left(fx + e\right)\sin\left(fx + e\right)\right)\left(a\sin\left(fx + e\right) + a\right)^m \left(-c\sin\left(fx + e\right) + c\right)^{-m-2}}{4\,fm^2 + 8\,fm + 3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x, algorithm="fricas")
```

```
[Out] (2*(m + 1)*cos(f*x + e) - cos(f*x + e)*sin(f*x + e))*(a*sin(f*x + e) + a)^m
*(-c*sin(f*x + e) + c)^(-m - 2)/(4*f*m^2 + 8*f*m + 3*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx + e\right) + a\right)^m \left(-c\sin\left(fx + e\right) + c\right)^{-m-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 2), x)
```

maple [F] time = 0.79, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m} \left(c-c\sin\left(fx+e\right)\right)^{-2-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x)
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 2), x)
```

mupad [B] time = 0.88, size = 111, normalized size = 1.10

$$-\frac{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)^m\,\left(\sin\left(2\,e+2\,f\,x\right)+8\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+4\,m\,\left(2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)-4\right)}{c^2\,f\,\left(-c\,\left(\sin\left(e+f\,x\right)-1\right)\right)^m\,\left(4\,m^2+8\,m+3\right)\,\left(2\,\sin\left(e+f\,x\right)^2-4\,\sin\left(e+f\,x\right)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^(m + 2),x)
[Out] -((a*(sin(e + f*x) + 1))^m*(sin(2*e + 2*f*x) + 8*sin(e/2 + (f*x)/2)^2 + 4*m
*(2*sin(e/2 + (f*x)/2)^2 - 1) - 4))/(c^2*f*(-c*(sin(e + f*x) - 1))^m*(8*m +
 4*m^2 + 3)*(2*sin(e + f*x)^2 - 4*sin(e + f*x) + 2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-2-m),x)
[Out] Timed out
```

3.421 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}\,dx$

Optimal. Leaf size=46

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{f(2m+1)}$$

```
[Out] cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/f/(1+2*m)
```

Rubi [A] time = 0.07, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2742}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 +
 2*m))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}\,dx=\frac{\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}}{f(1+2m)}$$

Mathematica [B] time = 1.53, size = 107, normalized size = 2.33

$$\frac{2^{-m}\sin\left(\frac{1}{4}(2e+2fx+\pi)\right)\cos^{-2m-1}\left(\frac{1}{4}(2e+2fx+\pi)\right)(a(\sin(e+fx)+1))^m(c-c\sin(e+fx))^{-m}\left(\cos\left(\frac{1}{2}(e+f\right.\right.}{cf(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] (Cos[(2*e + Pi + 2*f*x)/4]^(-1 - 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^(2*m)*(a*(1 + Sin[e + f*x]))^m*Sin[(2*e + Pi + 2*f*x)/4])/(2^m*c*f*(1 + 2*
m)*(c - c*Sin[e + f*x])^m)
```

fricas [A] time = 0.46, size = 44, normalized size = 0.96

$$\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-1}\cos\left(fx+e\right)}{2\,fm+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x, algorithm="fricas")
```

```
[Out] (a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 1)*cos(f*x + e)/(2*f*m +
 f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 1), x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{-1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 1), x)
```

mupad [B] time = 0.41, size = 58, normalized size = 1.26

$$-\frac{\cos\left(e+f\,x\right)\,{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)}^m}{c\,f\,\left(2\,m+1\right)\,{\left(-c\,\left(\sin\left(e+f\,x\right)-1\right)\right)}^m\,\left(\sin\left(e+f\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^(m + 1),x)
```

```
[Out] -(cos(e + f*x)*(a*(sin(e + f*x) + 1))^m)/(c*f*(2*m + 1)*(-c*(sin(e + f*x) -
 1))^m*(sin(e + f*x) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-1-m),x)
```

```
[Out] Timed out
```

3.422 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-m}\,dx$

Optimal. Leaf size=112

$$\frac{c2^{\frac{1}{2}-m}\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{2}(2m+1),\frac{1}{2}(2m+1);\frac{1}{2}(2\right.}{f(2m+1)}$$

```
[Out] 2^(1/2-m)*c*cos(f*x+e)*hypergeom([1/2+m, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))
*(1-sin(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/f/(1+2*m
)
```

Rubi [A] time = 0.16, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2745, 2689, 70, 69}

$$\frac{c2^{\frac{1}{2}-m}\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{2}(2m+1),\frac{1}{2}(2m+1);\frac{1}{2}(2\right.}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x])^m,x]
```

```
[Out] (2^(1/2 - m)*c*Cos[e + f*x]*Hypergeometric2F1[(1 + 2*m)/2, (1 + 2*m)/2, (3
+ 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a + a*Sin[e +
 f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 + 2*m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-m}\,dx = \left(\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^m\right)\int \mathrm{co}$$

$$= \frac{\left(c^2\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{\frac{1}{2}(-1-2m)+}\right.}{}$$

$$= \frac{\left(2^{-\frac{1}{2}-m}c^2\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-\frac{1}{2}}\right.}{}$$

$$= \frac{2^{\frac{1}{2}-m}c\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}(1+2m),\frac{1}{2}(1+2m);\frac{1}{2}(3+2m);\frac{1}{2}(1+\right.}{}$$

Mathematica [C] time = 2.92, size = 388, normalized size = 3.46

$$\frac{2^{1-m}(2m-3)\sin^2\left(\frac{1}{8}(2e+2fx+3\pi)\right)\cos^{1-2m}\left(\frac{1}{4}(2e+2fx+\pi)\right.}{f(2m-1)\left(2\sin^2\left(\frac{1}{8}(2e+2fx-\pi)\right)\left(2mF_1\left(\frac{3}{2}-m;1-2m,1;\frac{5}{2}-m;\tan^2\left(\frac{1}{8}(-2e-2fx+\pi)\right),-\tan^2\left(\frac{1}{8}(2e+2\right.\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c - c*Sin[e + f*x])^m,x]
```

```
[Out] (2^(1 - m)*(-3 + 2*m)*AppellF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[(-2*e + Pi -
2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2]*Cos[(2*e + Pi + 2*f*x)/4]^(1 - 2
```

```
*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*m)*(a*(1 + Sin[e + f*x]))^m*Si
n[(2*e + 3*Pi + 2*f*x)/8]^2)/(f*(-1 + 2*m)*(c - c*Sin[e + f*x])^m*((-3 + 2*
m)*AppellF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(
2*e - Pi + 2*f*x)/8]^2]*Cos[(2*e - Pi + 2*f*x)/8]^2 + 2*(2*m*AppellF1[3/2 -
 m, 1 - 2*m, 1, 5/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f
*x)/8]^2] + AppellF1[3/2 - m, -2*m, 2, 5/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^
2, -Tan[(2*e - Pi + 2*f*x)/8]^2])*Sin[(2*e - Pi + 2*f*x)/8]^2))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(-c\sin\left(fx+e\right)+c\right)^{m}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(-c\sin\left(fx+e\right)+c\right)^{m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)^m, x)
```

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x)
[Out] int((a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(-c\sin\left(fx+e\right)+c\right)^{m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(c-c\,\sin\left(e+f\,x\right)\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^m,x)
```

```
[Out] int((a + a*sin(e + f*x))^m/(c - c*sin(e + f*x))^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/((c-c*sin(f*x+e))**m),x)
```

```
[Out] Timed out
```

3.423 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1-m}\,dx$

Optimal. Leaf size=114

$$\frac{c^2 2^{\frac{3}{2}-m}\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{2}(2m-1),\frac{1}{2}(2m+1);\frac{1}{2}(\right.}{f(2m+1)}$$

```
[Out] 2^(3/2-m)*c^2*cos(f*x+e)*hypergeom([1/2+m, -1/2+m],[3/2+m],1/2+1/2*sin(f*x+
e))*(1-sin(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/f/(1+
2*m)
```

Rubi [A] time = 0.19, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2745, 2689, 70, 69}

$$\frac{c^2 2^{\frac{3}{2}-m}\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{2}(2m-1),\frac{1}{2}(2m+1);\frac{1}{2}(\right.}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(1 - m),x]
```

```
[Out] (2^(3/2 - m)*c^2*Cos[e + f*x]*Hypergeometric2F1[(-1 + 2*m)/2, (1 + 2*m)/2,
(3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a + a*Sin[
e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 + 2*m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1-m}\,dx = \left(\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^m\right)\int \text{c}$$

$$= \frac{\left(c^2\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{\frac{1}{2}(-1-2m)}\right.}{}$$

$$= \frac{\left(2^{\frac{1}{2}-m}c^3\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-\frac{1}{2}}\right.}{}$$

$$= \frac{2^{\frac{3}{2}-m}c^2\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}(-1+2m),\frac{1}{2}(1+2m);\frac{1}{2}(3+2m);\frac{1}{2}\right.}{}$$

Mathematica [C] time = 7.18, size = 602, normalized size = 5.28

$$-\frac{}{f(2m-1)\left(2\tan^2\left(\frac{1}{8}(2e+2fx-\pi)\right)\left(2mF_1\left(\frac{3}{2}-m;1-2m,2;\frac{5}{2}-m;\tan^2\left(\frac{1}{8}(-2e-2fx+\pi)\right),-\tan^2\left(\frac{1}{8}(2e+\right.\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(1 - m),x]
```

```
[Out] -((2^(2 - m)*c*(-3 + 2*m)*(AppellF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-2*e +
Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2] - AppellF1[1/2 - m, -2*m, 3
```

```
, 3/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2])*Cos
[(2*e + Pi + 2*f*x)/4]^(3 - 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(
-1 + m))*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^m)/(f*(-1 + 2*m)*(c - c
*Sin[e + f*x])^m*((-3 + 2*m)*AppellF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-2*e
+ Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2] + (3 - 2*m)*AppellF1[1/2
- m, -2*m, 3, 3/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x
)/8]^2] + 2*(2*m*AppellF1[3/2 - m, 1 - 2*m, 2, 5/2 - m, Tan[(-2*e + Pi - 2*
f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2] - 2*m*AppellF1[3/2 - m, 1 - 2*m, 3
, 5/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2] + 2*
AppellF1[3/2 - m, -2*m, 3, 5/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e
 - Pi + 2*f*x)/8]^2] - 3*AppellF1[3/2 - m, -2*m, 4, 5/2 - m, Tan[(-2*e + Pi
 - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2])*Tan[(2*e - Pi + 2*f*x)/8]^2)
))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 1), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x)
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1 - m),x)
[Out] int((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1 - m), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(1-m),x)
[Out] Timed out
```

3.424 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2-m}\,dx$

Optimal. Leaf size=114

$$\frac{c^3 2^{\frac{5}{2}-m}\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{2}(2m-3),\frac{1}{2}(2m+1);\frac{1}{2}(\right.}{f(2m+1)}$$

```
[Out] 2^(5/2-m)*c^3*cos(f*x+e)*hypergeom([1/2+m, -3/2+m],[3/2+m],1/2+1/2*sin(f*x+
e))*(1-sin(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/f/(1+
2*m)
```

Rubi [A] time = 0.18, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2745, 2689, 70, 69}

$$\frac{c^3 2^{\frac{5}{2}-m}\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{2}(2m-3),\frac{1}{2}(2m+1);\frac{1}{2}(\right.}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(2 - m),x]
```

```
[Out] (2^(5/2 - m)*c^3*Cos[e + f*x]*Hypergeometric2F1[(-3 + 2*m)/2, (1 + 2*m)/2,
(3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a + a*Sin[
e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 + 2*m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2-m}\,dx = \left(\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^m\right)\int \mathrm{c}$$

$$= \frac{\left(c^2\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{\frac{1}{2}(-1-2m)}\right.}{}$$

$$= \frac{\left(2^{\frac{3}{2}-m}c^4\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-\frac{1}{2}}\right.}{}$$

$$= \frac{2^{\frac{5}{2}-m}c^3\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}(-3+2m),\frac{1}{2}(1+2m);\frac{1}{2}(3+2m);\frac{1}{2}\right.}{}$$

Mathematica [C] time = 12.71, size = 1201, normalized size = 10.54

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(2 - m),x]
```

```
[Out] (2^(4 - m)*(-3 + 2*m)*(AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 -
 f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 2*AppellF1[1/2 - m, -2*m, 4, 3/2
 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[1
/2 - m, -2*m, 5, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x
```

```
)/4]^2])*Cos[(-e + Pi/2 - f*x)/4]*Sin[(-e + Pi/2 - f*x)/4]*Sin[(-e + Pi/2 -
 f*x)/2]^(4 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(2 - m))/(f*
(-1 + 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(2 - m))*((-3 + 2*m)*Ap
pellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi
/2 - f*x)/4]^2] + (6 - 4*m)*AppellF1[1/2 - m, -2*m, 4, 3/2 - m, Tan[(-e + P
i/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 3*AppellF1[1/2 - m, -2*m, 5
, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 2*m*A
ppellF1[1/2 - m, -2*m, 5, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + P
i/2 - f*x)/4]^2] - 8*AppellF1[3/2 - m, -2*m, 5, 5/2 - m, Tan[(-e + Pi/2 - f
*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2 + 8*Appel
lF1[3/2 - m, -2*m, 5, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2
- f*x)/4]^2]*Cos[(-e + Pi/2 - f*x)/2]*Sec[(-e + Pi/2 - f*x)/4]^2 + 4*m*Appe
llF1[3/2 - m, 1 - 2*m, 3, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + P
i/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2 - 8*m*AppellF1[3/2 - m, 1 - 2*m
, 4, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Tan[
(-e + Pi/2 - f*x)/4]^2 + 4*m*AppellF1[3/2 - m, 1 - 2*m, 5, 5/2 - m, Tan[(-e
 + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^
2 + 6*AppellF1[3/2 - m, -2*m, 4, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[
(-e + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2 + 10*AppellF1[3/2 - m, -
2*m, 6, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*T
an[(-e + Pi/2 - f*x)/4]^2))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m),x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 2), x)
```

maple [F] time = 0.81, size = 0, normalized size = 0.00

$$\int\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{2-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{2-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(2 - m),x)
```

```
[Out] int((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(2 - m), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(2-m),x)
```

```
[Out] Timed out
```

3.425 $\int (a+a\sin(e+fx))(c+d\sin(e+fx))^4\,dx$

Optimal. Leaf size=227

$$-\frac{a\left(12c^2+35cd+16d^2\right)\cos(e+fx)(c+d\sin(e+fx))^2}{60f}-\frac{ad\left(24c^3+130c^2d+116cd^2+45d^3\right)\sin(e+fx)\cos(e+$$

```
[Out] 1/8*a*(8*c^4+16*c^3*d+24*c^2*d^2+12*c*d^3+3*d^4)*x-1/30*a*(12*c^4+95*c^3*d+
112*c^2*d^2+80*c*d^3+16*d^4)*cos(f*x+e)/f-1/120*a*d*(24*c^3+130*c^2*d+116*c
*d^2+45*d^3)*cos(f*x+e)*sin(f*x+e)/f-1/60*a*(12*c^2+35*c*d+16*d^2)*cos(f*x+
e)*(c+d*sin(f*x+e))^2/f-1/20*a*(4*c+5*d)*cos(f*x+e)*(c+d*sin(f*x+e))^3/f-1/
5*a*cos(f*x+e)*(c+d*sin(f*x+e))^4/f
```

Rubi [A] time = 0.28, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2753, 2734}

$$-\frac{a\left(112c^2d^2+95c^3d+12c^4+80cd^3+16d^4\right)\cos(e+fx)}{30f}-\frac{a\left(12c^2+35cd+16d^2\right)\cos(e+fx)(c+d\sin(e+fx))^2$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^4,x]
```

```
[Out] (a*(8*c^4 + 16*c^3*d + 24*c^2*d^2 + 12*c*d^3 + 3*d^4)*x)/8 - (a*(12*c^4 + 9
5*c^3*d + 112*c^2*d^2 + 80*c*d^3 + 16*d^4)*Cos[e + f*x])/(30*f) - (a*d*(24*
c^3 + 130*c^2*d + 116*c*d^2 + 45*d^3)*Cos[e + f*x]*Sin[e + f*x])/(120*f) -
(a*(12*c^2 + 35*c*d + 16*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(60*f) -
 (a*(4*c + 5*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(20*f) - (a*Cos[e + f*
x]*(c + d*Sin[e + f*x])^4)/(5*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

```
&& IntegerQ[2*m]
```

Rubi steps

$$\int (a+a\sin(e+fx))(c+d\sin(e+fx))^4\,dx = -\frac{a\cos(e+fx)(c+d\sin(e+fx))^4}{5f}+\frac{1}{5}\int (c+d\sin(e+fx))^3(a(5$$

$$=-\frac{a(4c+5d)\cos(e+fx)(c+d\sin(e+fx))^3}{20f}-\frac{a\cos(e+fx)(c+}{5f}$$

$$=-\frac{a\left(12c^2+35cd+16d^2\right)\cos(e+fx)(c+d\sin(e+fx))^2}{60f}-\frac{a(4c}{}$$

$$=\frac{1}{8}a\left(8c^4+16c^3d+24c^2d^2+12cd^3+3d^4\right)x-\frac{a\left(12c^4+95c^3d+1\right.}{}$$

Mathematica [A] time = 1.36, size = 207, normalized size = 0.91

$$\frac{a(\sin(e+fx)+1)\left(10d^2\left(24c^2+16cd+5d^2\right)\cos(3(e+fx))+15\left(-8d\left(4c^3+6c^2d+4cd^2+d^3\right)\sin(2(e+fx))\right.\right.}{48}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^4,x]
```

```
[Out] (a*(1 + Sin[e + f*x])*(-60*(8*c^4 + 32*c^3*d + 36*c^2*d^2 + 24*c*d^3 + 5*d^
4)*Cos[e + f*x] + 10*d^2*(24*c^2 + 16*c*d + 5*d^2)*Cos[3*(e + f*x)] - 6*d^4
*Cos[5*(e + f*x)] + 15*(4*(8*c^4 + 16*c^3*d + 24*c^2*d^2 + 12*c*d^3 + 3*d^4
)*f*x - 8*d*(4*c^3 + 6*c^2*d + 4*c*d^2 + d^3)*Sin[2*(e + f*x)] + d^3*(4*c +
 d)*Sin[4*(e + f*x)])))/(480*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)
```

fricas [A] time = 0.47, size = 204, normalized size = 0.90

$$-\frac{24\,ad^4\cos\left(fx+e\right)^5-80\left(3\,ac^2d^2+2\,acd^3+ad^4\right)\cos\left(fx+e\right)^3-15\left(8\,ac^4+16\,ac^3d+24\,ac^2d^2+12\,acd^3+3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] -1/120*(24*a*d^4*cos(f*x + e)^5 - 80*(3*a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(
f*x + e)^3 - 15*(8*a*c^4 + 16*a*c^3*d + 24*a*c^2*d^2 + 12*a*c*d^3 + 3*a*d^4
```

```
)*f*x + 120*(a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4)*cos(f*x +
 e) - 15*(2*(4*a*c*d^3 + a*d^4)*cos(f*x + e)^3 - (16*a*c^3*d + 24*a*c^2*d^2
 + 20*a*c*d^3 + 5*a*d^4)*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.18, size = 272, normalized size = 1.20

$$-\frac{ad^4\cos\left(5fx+5e\right)}{80f}+\frac{acd^3\cos\left(3fx+3e\right)}{3f}+\frac{acd^3\sin\left(4fx+4e\right)}{8f}+\frac{ad^4\sin\left(4fx+4e\right)}{32f}+\frac{1}{8}\left(8ac^4+24ac^2d^2+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] -1/80*a*d^4*cos(5*f*x + 5*e)/f + 1/3*a*c*d^3*cos(3*f*x + 3*e)/f + 1/8*a*c*d
^3*sin(4*f*x + 4*e)/f + 1/32*a*d^4*sin(4*f*x + 4*e)/f + 1/8*(8*a*c^4 + 24*a
*c^2*d^2 + 3*a*d^4)*x + 1/2*(4*a*c^3*d + 3*a*c*d^3)*x + 1/48*(24*a*c^2*d^2
+ 5*a*d^4)*cos(3*f*x + 3*e)/f - 1/8*(8*a*c^4 + 36*a*c^2*d^2 + 5*a*d^4)*cos(
f*x + e)/f - (4*a*c^3*d + 3*a*c*d^3)*cos(f*x + e)/f - (a*c^3*d + a*c*d^3)*s
in(2*f*x + 2*e)/f - 1/4*(6*a*c^2*d^2 + a*d^4)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.33, size = 259, normalized size = 1.14

$$\frac{-a\,c^4\cos\left(fx+e\right)+4a\,c^3d\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-2a\,c^2d^2\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)+4ac\,d^3\left(-\frac{\left(\mathrm{s}\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^4,x)
```

```
[Out] 1/f*(-a*c^4*cos(f*x+e)+4*a*c^3*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)
-2*a*c^2*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+4*a*c*d^3*(-1/4*(sin(f*x+e)^3+3/2*
sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/5*a*d^4*(8/3+sin(f*x+e)^4+4/3*sin(f
*x+e)^2)*cos(f*x+e)+a*c^4*(f*x+e)-4*a*c^3*d*cos(f*x+e)+6*a*c^2*d^2*(-1/2*si
n(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-4/3*a*c*d^3*(2+sin(f*x+e)^2)*cos(f*x+e)+
a*d^4*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e))
```

maxima [A] time = 0.39, size = 250, normalized size = 1.10

$$\frac{480\left(fx+e\right)ac^4+480\left(2fx+2e-\sin\left(2fx+2e\right)\right)ac^3d+960\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)ac^2d^2+720\left(2f\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 1/480*(480*(f*x + e)*a*c^4 + 480*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*c^3*d +
 960*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*c^2*d^2 + 720*(2*f*x + 2*e - sin(2
*f*x + 2*e))*a*c^2*d^2 + 640*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*c*d^3 + 60
*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a*c*d^3 - 32*(3*co
s(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a*d^4 + 15*(12*f*x + 12
*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a*d^4 - 480*a*c^4*cos(f*x + e)
- 1920*a*c^3*d*cos(f*x + e))/f
```

mupad [B] time = 9.94, size = 559, normalized size = 2.46

$$\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(8\,c^4+16\,c^3\,d+24\,c^2\,d^2+12\,c\,d^3+3\,d^4\right)}{4\left(2\,a\,c^4+4\,a\,c^3\,d+6\,a\,c^2\,d^2+3\,a\,c\,d^3+\frac{3\,a\,d^4}{4}\right)}\right)\left(8\,c^4+16\,c^3\,d+24\,c^2\,d^2+12\,c\,d^3+3\,d^4\right)}{4\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(8\,a\,c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^4,x)
```

```
[Out] (a*atan((a*tan(e/2 + (f*x)/2)*(12*c*d^3 + 16*c^3*d + 8*c^4 + 3*d^4 + 24*c^2
*d^2))/(4*(2*a*c^4 + (3*a*d^4)/4 + 6*a*c^2*d^2 + 3*a*c*d^3 + 4*a*c^3*d)))*(
12*c*d^3 + 16*c^3*d + 8*c^4 + 3*d^4 + 24*c^2*d^2))/(4*f) - (tan(e/2 + (f*x)
/2)^2*(8*a*c^4 + (16*a*d^4)/3 + 40*a*c^2*d^2 + (80*a*c*d^3)/3 + 32*a*c^3*d)
 + tan(e/2 + (f*x)/2)^4*(12*a*c^4 + (32*a*d^4)/3 + 56*a*c^2*d^2 + (112*a*c*
d^3)/3 + 48*a*c^3*d) + tan(e/2 + (f*x)/2)*((3*a*d^4)/4 + 6*a*c^2*d^2 + 3*a*
c*d^3 + 4*a*c^3*d) + tan(e/2 + (f*x)/2)^8*(2*a*c^4 + 8*a*c^3*d) + 2*a*c^4 +
 (16*a*d^4)/15 + tan(e/2 + (f*x)/2)^6*(8*a*c^4 + 24*a*c^2*d^2 + 16*a*c*d^3
+ 32*a*c^3*d) - tan(e/2 + (f*x)/2)^9*((3*a*d^4)/4 + 6*a*c^2*d^2 + 3*a*c*d^3
 + 4*a*c^3*d) + tan(e/2 + (f*x)/2)^3*((7*a*d^4)/2 + 12*a*c^2*d^2 + 14*a*c*d
^3 + 8*a*c^3*d) - tan(e/2 + (f*x)/2)^7*((7*a*d^4)/2 + 12*a*c^2*d^2 + 14*a*c
*d^3 + 8*a*c^3*d) + 8*a*c^2*d^2 + (16*a*c*d^3)/3 + 8*a*c^3*d)/(f*(5*tan(e/2
 + (f*x)/2)^2 + 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 + 5*tan(e
/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 + 1))
```

sympy [A] time = 6.57, size = 580, normalized size = 2.56

$$\begin{cases} ac^4x-\frac{ac^4\cos\left(e+fx\right)}{f}+2ac^3dx\sin^2\left(e+fx\right)+2ac^3dx\cos^2\left(e+fx\right)-\frac{2ac^3d\sin\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{4ac^3d\cos\left(e+fx\right)}{f}+3a \\ x\left(c+d\sin\left(e\right)\right)^4\left(a\sin\left(e\right)+a\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((a*c**4*x - a*c**4*cos(e + f*x)/f + 2*a*c**3*d*x*sin(e + f*x)**2
+ 2*a*c**3*d*x*cos(e + f*x)**2 - 2*a*c**3*d*sin(e + f*x)*cos(e + f*x)/f - 4
*a*c**3*d*cos(e + f*x)/f + 3*a*c**2*d**2*x*sin(e + f*x)**2 + 3*a*c**2*d**2*
x*cos(e + f*x)**2 - 6*a*c**2*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*a*c**2
*d**2*sin(e + f*x)*cos(e + f*x)/f - 4*a*c**2*d**2*cos(e + f*x)**3/f + 3*a*c
*d**3*x*sin(e + f*x)**4/2 + 3*a*c*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2 +
3*a*c*d**3*x*cos(e + f*x)**4/2 - 5*a*c*d**3*sin(e + f*x)**3*cos(e + f*x)/(2
*f) - 4*a*c*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*a*c*d**3*sin(e + f*x)*c
os(e + f*x)**3/(2*f) - 8*a*c*d**3*cos(e + f*x)**3/(3*f) + 3*a*d**4*x*sin(e
+ f*x)**4/8 + 3*a*d**4*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*a*d**4*x*cos
(e + f*x)**4/8 - a*d**4*sin(e + f*x)**4*cos(e + f*x)/f - 5*a*d**4*sin(e + f
*x)**3*cos(e + f*x)/(8*f) - 4*a*d**4*sin(e + f*x)**2*cos(e + f*x)**3/(3*f)
- 3*a*d**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 8*a*d**4*cos(e + f*x)**5/(1
5*f), Ne(f, 0)), (x*(c + d*sin(e))**4*(a*sin(e) + a), True))
```

3.426 $\int (a+a\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=162

$$-\frac{ad\left(6c^2+20cd+9d^2\right)\sin(e+fx)\cos(e+fx)}{24f}-\frac{a\left(3c^3+16c^2d+12cd^2+4d^3\right)\cos(e+fx)}{6f}+\frac{1}{8}ax\left(8c^3+12c^2d\right.$$

```
[Out] 1/8*a*(8*c^3+12*c^2*d+12*c*d^2+3*d^3)*x-1/6*a*(3*c^3+16*c^2*d+12*c*d^2+4*d^
3)*cos(f*x+e)/f-1/24*a*d*(6*c^2+20*c*d+9*d^2)*cos(f*x+e)*sin(f*x+e)/f-1/12*
a*(3*c+4*d)*cos(f*x+e)*(c+d*sin(f*x+e))^2/f-1/4*a*cos(f*x+e)*(c+d*sin(f*x+e
))^3/f
```

Rubi [A] time = 0.19, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2753, 2734}

$$-\frac{a\left(16c^2d+3c^3+12cd^2+4d^3\right)\cos(e+fx)}{6f}-\frac{ad\left(6c^2+20cd+9d^2\right)\sin(e+fx)\cos(e+fx)}{24f}+\frac{1}{8}ax\left(12c^2d+8c^3\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a*(8*c^3 + 12*c^2*d + 12*c*d^2 + 3*d^3)*x)/8 - (a*(3*c^3 + 16*c^2*d + 12*c
*d^2 + 4*d^3)*Cos[e + f*x])/(6*f) - (a*d*(6*c^2 + 20*c*d + 9*d^2)*Cos[e + f
*x]*Sin[e + f*x])/(24*f) - (a*(3*c + 4*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])
^2)/(12*f) - (a*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(4*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int (a+a\sin(e+fx))(c+d\sin(e+fx))^3\,dx = -\frac{a\cos(e+fx)(c+d\sin(e+fx))^3}{4f} + \frac{1}{4}\int (c+d\sin(e+fx))^2(a(4c$$

$$= -\frac{a(3c+4d)\cos(e+fx)(c+d\sin(e+fx))^2}{12f} - \frac{a\cos(e+fx)(c+d}{4f}$$

$$= \frac{1}{8}a\left(8c^3+12c^2d+12cd^2+3d^3\right)x - \frac{a\left(3c^3+16c^2d+12cd^2+4d^3\right)}{6f}$$

Mathematica [A] time = 0.80, size = 124, normalized size = 0.77

$$\frac{a\left(3\left(-8d\left(3c^2+3cd+d^2\right)\sin(2(e+fx))+4fx\left(8c^3+12c^2d+12cd^2+3d^3\right)+d^3\sin(4(e+fx))\right)-24\left(4c^3+12c\right.\right.}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a*(-24*(4*c^3 + 12*c^2*d + 9*c*d^2 + 3*d^3)*Cos[e + f*x] + 8*d^2*(3*c + d)
*Cos[3*(e + f*x)] + 3*(4*(8*c^3 + 12*c^2*d + 12*c*d^2 + 3*d^3)*f*x - 8*d*(3
*c^2 + 3*c*d + d^2)*Sin[2*(e + f*x)] + d^3*Sin[4*(e + f*x)])))/(96*f)
```

fricas [A] time = 0.46, size = 145, normalized size = 0.90

$$\frac{8\left(3\,acd^2+ad^3\right)\cos\left(fx+e\right)^3+3\left(8\,ac^3+12\,ac^2d+12\,acd^2+3\,ad^3\right)fx-24\left(ac^3+3\,ac^2d+3\,acd^2+ad^3\right)\cos\left(f\right.}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/24*(8*(3*a*c*d^2 + a*d^3)*cos(f*x + e)^3 + 3*(8*a*c^3 + 12*a*c^2*d + 12*a
*c*d^2 + 3*a*d^3)*f*x - 24*(a*c^3 + 3*a*c^2*d + 3*a*c*d^2 + a*d^3)*cos(f*x
+ e) + 3*(2*a*d^3*cos(f*x + e)^3 - (12*a*c^2*d + 12*a*c*d^2 + 5*a*d^3)*cos(
f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.47, size = 191, normalized size = 1.18

$$\frac{acd^2\cos\left(3\,fx+3\,e\right)}{4\,f}+\frac{ad^3\cos\left(3\,fx+3\,e\right)}{12\,f}+\frac{ad^3\sin\left(4\,fx+4\,e\right)}{32\,f}-\frac{3\,acd^2\sin\left(2\,fx+2\,e\right)}{4\,f}+\frac{1}{2}\left(2\,ac^3+3\,acd^2\right)x+\frac{3}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/4*a*c*d^2*cos(3*f*x + 3*e)/f + 1/12*a*d^3*cos(3*f*x + 3*e)/f + 1/32*a*d^3
*sin(4*f*x + 4*e)/f - 3/4*a*c*d^2*sin(2*f*x + 2*e)/f + 1/2*(2*a*c^3 + 3*a*c
*d^2)*x + 3/8*(4*a*c^2*d + a*d^3)*x - 1/4*(4*a*c^3 + 9*a*c*d^2)*cos(f*x + e
)/f - 3/4*(4*a*c^2*d + a*d^3)*cos(f*x + e)/f - 1/4*(3*a*c^2*d + a*d^3)*sin(
2*f*x + 2*e)/f
```

maple [A] time = 0.29, size = 182, normalized size = 1.12

$$\frac{-a\,c^{3}\cos\left(fx+e\right)+3a\,c^{2}d\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-ac\,d^{2}\left(2+\sin^{2}\left(fx+e\right)\right)\cos\left(fx+e\right)+a\,d^{3}\left(-\frac{\left(\sin^{3}(\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(-a*c^3*cos(f*x+e)+3*a*c^2*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)
-a*c*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+a*d^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+
e))*cos(f*x+e)+3/8*f*x+3/8*e)+a*c^3*(f*x+e)-3*a*c^2*d*cos(f*x+e)+3*a*c*d^2*
(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/3*a*d^3*(2+sin(f*x+e)^2)*cos(f
*x+e))
```

maxima [A] time = 0.33, size = 175, normalized size = 1.08

$$\frac{96\left(fx+e\right)ac^{3}+72\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)ac^{2}d+96\left(\cos\left(fx+e\right)^{3}-3\,\cos\left(fx+e\right)\right)acd^{2}+72\left(2\,fx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/96*(96*(f*x + e)*a*c^3 + 72*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*c^2*d + 96
*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*c*d^2 + 72*(2*f*x + 2*e - sin(2*f*x +
2*e))*a*c*d^2 + 32*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*d^3 + 3*(12*f*x + 12
*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a*d^3 - 96*a*c^3*cos(f*x + e) -
 288*a*c^2*d*cos(f*x + e))/f
```

mupad [B] time = 8.17, size = 460, normalized size = 2.84

$$\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(8\,c^3+12\,c^2\,d+12\,c\,d^2+3\,d^3\right)}{4\left(2\,a\,c^3+3\,a\,c^2\,d+3\,a\,c\,d^2+\frac{3\,a\,d^3}{4}\right)}\right)\left(8\,c^3+12\,c^2\,d+12\,c\,d^2+3\,d^3\right)}{4\,f}-\frac{a\,\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{f\,x}{2}\right)\left(8\,c^3+\right.}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^3,x)
```

```
[Out] (a*atan((a*tan(e/2 + (f*x)/2)*(12*c*d^2 + 12*c^2*d + 8*c^3 + 3*d^3))/(4*(2*
a*c^3 + (3*a*d^3)/4 + 3*a*c*d^2 + 3*a*c^2*d)))*(12*c*d^2 + 12*c^2*d + 8*c^3
 + 3*d^3))/(4*f) - (a*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2)*(12*c*d^2 + 12*c
^2*d + 8*c^3 + 3*d^3))/(4*f) - (tan(e/2 + (f*x)/2)^3*((11*a*d^3)/4 + 3*a*c*
d^2 + 3*a*c^2*d) - tan(e/2 + (f*x)/2)^7*((3*a*d^3)/4 + 3*a*c*d^2 + 3*a*c^2*
d) - tan(e/2 + (f*x)/2)^5*((11*a*d^3)/4 + 3*a*c*d^2 + 3*a*c^2*d) + tan(e/2
+ (f*x)/2)^6*(2*a*c^3 + 6*a*c^2*d) + 2*a*c^3 + (4*a*d^3)/3 + tan(e/2 + (f*x
)/2)^4*(6*a*c^3 + 4*a*d^3 + 12*a*c*d^2 + 18*a*c^2*d) + tan(e/2 + (f*x)/2)^2
*(6*a*c^3 + (16*a*d^3)/3 + 16*a*c*d^2 + 18*a*c^2*d) + tan(e/2 + (f*x)/2)*((
3*a*d^3)/4 + 3*a*c*d^2 + 3*a*c^2*d) + 4*a*c*d^2 + 6*a*c^2*d)/(f*(4*tan(e/2
+ (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*tan(e/2 + (f*x)/2)^6 + tan(e/2 +
(f*x)/2)^8 + 1))
```

sympy [A] time = 2.68, size = 386, normalized size = 2.38

$$\begin{cases} ac^3x - \frac{ac^3\cos(e+fx)}{f} + \frac{3ac^2dx\sin^2(e+fx)}{2} + \frac{3ac^2dx\cos^2(e+fx)}{2} - \frac{3ac^2d\sin(e+fx)\cos(e+fx)}{2f} - \frac{3ac^2d\cos(e+fx)}{f} + \frac{3acd^2x\sin^2(e+fx)}{2} \\ x(c+d\sin(e))^3(a\sin(e)+a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((a*c**3*x - a*c**3*cos(e + f*x)/f + 3*a*c**2*d*x*sin(e + f*x)**2/
2 + 3*a*c**2*d*x*cos(e + f*x)**2/2 - 3*a*c**2*d*sin(e + f*x)*cos(e + f*x)/(
2*f) - 3*a*c**2*d*cos(e + f*x)/f + 3*a*c*d**2*x*sin(e + f*x)**2/2 + 3*a*c*d
**2*x*cos(e + f*x)**2/2 - 3*a*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*a*c
*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a*c*d**2*cos(e + f*x)**3/f + 3*a*
d**3*x*sin(e + f*x)**4/8 + 3*a*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3
*a*d**3*x*cos(e + f*x)**4/8 - 5*a*d**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) -
 a*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*a*d**3*sin(e + f*x)*cos(e + f*x)
**3/(8*f) - 2*a*d**3*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(c + d*sin(e))**3
*(a*sin(e) + a), True))
```

3.427 $\int (a + a\sin(e + fx))(c + d\sin(e + fx))^2\,dx$

Optimal. Leaf size=99

$$-\frac{2a\left(c^2+3cd+d^2\right)\cos(e+fx)}{3f}+\frac{1}{2}ax\left(2c^2+2cd+d^2\right)-\frac{a\cos(e+fx)(c+d\sin(e+fx))^2}{3f}-\frac{ad(2c+3d)\sin(e+}{6f}$$

```
[Out] 1/2*a*(2*c^2+2*c*d+d^2)*x-2/3*a*(c^2+3*c*d+d^2)*cos(f*x+e)/f-1/6*a*d*(2*c+3
*d)*cos(f*x+e)*sin(f*x+e)/f-1/3*a*cos(f*x+e)*(c+d*sin(f*x+e))^2/f
```

Rubi [A] time = 0.09, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2753, 2734}

$$-\frac{2a\left(c^2+3cd+d^2\right)\cos(e+fx)}{3f}+\frac{1}{2}ax\left(2c^2+2cd+d^2\right)-\frac{a\cos(e+fx)(c+d\sin(e+fx))^2}{3f}-\frac{ad(2c+3d)\sin(e+}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a*(2*c^2 + 2*c*d + d^2)*x)/2 - (2*a*(c^2 + 3*c*d + d^2)*Cos[e + f*x])/(3*f
) - (a*d*(2*c + 3*d)*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (a*Cos[e + f*x]*(c
+ d*Sin[e + f*x])^2)/(3*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int (a + a\sin(e + fx))(c + d\sin(e + fx))^2\,dx = -\frac{a\cos(e + fx)(c + d\sin(e + fx))^2}{3f} + \frac{1}{3}\int (c + d\sin(e + fx))(a(3c$$

$$= \frac{1}{2}a\left(2c^2 + 2cd + d^2\right)x - \frac{2a\left(c^2 + 3cd + d^2\right)\cos(e + fx)}{3f} - \frac{ad(2c + 3$$

Mathematica [A] time = 0.40, size = 89, normalized size = 0.90

$$\frac{a\left(-3\left(4c^2 + 8cd + 3d^2\right)\cos(e + fx) + 12c^2fx - 6cd\sin(2(e + fx)) + 12cdfx - 3d^2\sin(2(e + fx)) + d^2\cos(3(e +\right.}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a*(12*c^2*f*x + 12*c*d*f*x + 6*d^2*f*x - 3*(4*c^2 + 8*c*d + 3*d^2)*Cos[e +
 f*x] + d^2*Cos[3*(e + f*x)] - 6*c*d*Sin[2*(e + f*x)] - 3*d^2*Sin[2*(e + f*
x)]))/(12*f)
```

fricas [A] time = 0.44, size = 90, normalized size = 0.91

$$\frac{2\,ad^2\cos\left(fx + e\right)^3 + 3\left(2\,ac^2 + 2\,acd + ad^2\right)fx - 3\left(2\,acd + ad^2\right)\cos\left(fx + e\right)\sin\left(fx + e\right) - 6\left(ac^2 + 2\,acd + ad^2\right.}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*d^2*cos(f*x + e)^3 + 3*(2*a*c^2 + 2*a*c*d + a*d^2)*f*x - 3*(2*a*c*
d + a*d^2)*cos(f*x + e)*sin(f*x + e) - 6*(a*c^2 + 2*a*c*d + a*d^2)*cos(f*x
+ e))/f
```

giac [A] time = 0.16, size = 117, normalized size = 1.18

$$acdx + \frac{ad^2\cos\left(3\,fx + 3\,e\right)}{12\,f} - \frac{2\,acd\cos\left(fx + e\right)}{f} - \frac{acd\sin\left(2\,fx + 2\,e\right)}{2\,f} - \frac{ad^2\sin\left(2\,fx + 2\,e\right)}{4\,f} + \frac{1}{2}\left(2\,ac^2 + ad^2\right)x - \frac{(4\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] a*c*d*x + 1/12*a*d^2*cos(3*f*x + 3*e)/f - 2*a*c*d*cos(f*x + e)/f - 1/2*a*c*
d*sin(2*f*x + 2*e)/f - 1/4*a*d^2*sin(2*f*x + 2*e)/f + 1/2*(2*a*c^2 + a*d^2)
*x - 1/4*(4*a*c^2 + 3*a*d^2)*cos(f*x + e)/f
```

maple [A] time = 0.19, size = 115, normalized size = 1.16

$$\frac{-a\,c^2\cos\left(fx+e\right)+2acd\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{a\,d^2\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}+a\,c^2\left(fx+e\right)-2acd\cos\left(f\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(-a*c^2*cos(f*x+e)+2*a*c*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1
/3*a*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+a*c^2*(f*x+e)-2*a*c*d*cos(f*x+e)+a*d^2
*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e))
```

maxima [A] time = 0.31, size = 112, normalized size = 1.13

$$\frac{12\left(fx+e\right)ac^2+6\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)acd+4\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)ad^2+3\left(2\,fx+2\,e-\mathrm{s}\right.}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(12*(f*x + e)*a*c^2 + 6*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*c*d + 4*(co
s(f*x + e)^3 - 3*cos(f*x + e))*a*d^2 + 3*(2*f*x + 2*e - sin(2*f*x + 2*e))*a
*d^2 - 12*a*c^2*cos(f*x + e) - 24*a*c*d*cos(f*x + e))/f
```

mupad [B] time = 6.99, size = 108, normalized size = 1.09

$$-\frac{\frac{3\,a\,d^2\,\sin\left(2\,e+2\,f\,x\right)}{2}-\frac{a\,d^2\,\cos\left(3\,e+3\,f\,x\right)}{2}+6\,a\,c^2\,\cos\left(e+f\,x\right)+\frac{9\,a\,d^2\,\cos\left(e+f\,x\right)}{2}+3\,a\,c\,d\,\sin\left(2\,e+2\,f\,x\right)-6\,a\,c^2\,f\,x-}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^2,x)
```

```
[Out] -((3*a*d^2*sin(2*e + 2*f*x))/2 - (a*d^2*cos(3*e + 3*f*x))/2 + 6*a*c^2*cos(e
 + f*x) + (9*a*d^2*cos(e + f*x))/2 + 3*a*c*d*sin(2*e + 2*f*x) - 6*a*c^2*f*x
 - 3*a*d^2*f*x + 12*a*c*d*cos(e + f*x) - 6*a*c*d*f*x)/(6*f)
```

sympy [A] time = 1.33, size = 199, normalized size = 2.01

$$\begin{cases} ac^2x-\frac{ac^2\cos\left(e+fx\right)}{f}+acdx\sin^2\left(e+fx\right)+acdx\cos^2\left(e+fx\right)-\frac{acd\sin\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{2acd\cos\left(e+fx\right)}{f}+\frac{ad^2x\sin^2\left(e\right.}{2} \\ x\left(c+d\sin\left(e\right)\right)^2\left(a\sin\left(e\right)+a\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a*c**2*x - a*c**2*cos(e + f*x)/f + a*c*d*x*sin(e + f*x)**2 + a*c
*d*x*cos(e + f*x)**2 - a*c*d*sin(e + f*x)*cos(e + f*x)/f - 2*a*c*d*cos(e +
f*x)/f + a*d**2*x*sin(e + f*x)**2/2 + a*d**2*x*cos(e + f*x)**2/2 - a*d**2*s
in(e + f*x)**2*cos(e + f*x)/f - a*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*
a*d**2*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(c + d*sin(e))**2*(a*sin(e) + a
), True))
```

3.428 $\int (a + a\sin(e + fx))(c + d\sin(e + fx))\,dx$

Optimal. Leaf size=48

$$-\frac{a(c+d)\cos(e+fx)}{f}+\frac{1}{2}ax(2c+d)-\frac{ad\sin(e+fx)\cos(e+fx)}{2f}$$

```
[Out] 1/2*a*(2*c+d)*x-a*(c+d)*cos(f*x+e)/f-1/2*a*d*cos(f*x+e)*sin(f*x+e)/f
```

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2734}

$$-\frac{a(c+d)\cos(e+fx)}{f}+\frac{1}{2}ax(2c+d)-\frac{ad\sin(e+fx)\cos(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (a*(2*c + d)*x)/2 - (a*(c + d)*Cos[e + f*x])/f - (a*d*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Cos[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))(c+d\sin(e+fx))\,dx=\frac{1}{2}a(2c+d)x-\frac{a(c+d)\cos(e+fx)}{f}-\frac{ad\cos(e+fx)\sin(e+fx)}{2f}$$

Mathematica [A] time = 0.11, size = 45, normalized size = 0.94

$$\frac{a(-4(c+d)\cos(e+fx)+4cfx-d\sin(2(e+fx))+2de+2dfx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (a*(2*d*e + 4*c*f*x + 2*d*f*x - 4*(c + d)*Cos[e + f*x] - d*Sin[2*(e + f*x)]
))/(4*f)
```

fricas [A] time = 0.43, size = 48, normalized size = 1.00

$$-\frac{ad\cos\left(fx+e\right)\sin\left(fx+e\right)-(2\,ac+ad)fx+2\,(ac+ad)\cos\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*d*cos(f*x + e)*sin(f*x + e) - (2*a*c + a*d)*f*x + 2*(a*c + a*d)*cos
(f*x + e))/f
```

giac [A] time = 0.22, size = 55, normalized size = 1.15

$$acx+\frac{1}{2}\,adx-\frac{ac\cos\left(fx+e\right)}{f}-\frac{ad\cos\left(fx+e\right)}{f}-\frac{ad\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] a*c*x + 1/2*a*d*x - a*c*cos(f*x + e)/f - a*d*cos(f*x + e)/f - 1/4*a*d*sin(2
*f*x + 2*e)/f
```

maple [A] time = 0.10, size = 59, normalized size = 1.23

$$\frac{da\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-ca\cos\left(fx+e\right)-da\cos\left(fx+e\right)+ac\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] 1/f*(d*a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-c*a*cos(f*x+e)-d*a*cos(
f*x+e)+a*c*(f*x+e))
```

maxima [A] time = 0.31, size = 57, normalized size = 1.19

$$\frac{4\left(fx+e\right)ac+\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)ad-4\,ac\cos\left(fx+e\right)-4\,ad\cos\left(fx+e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(f*x + e)*a*c + (2*f*x + 2*e - sin(2*f*x + 2*e))*a*d - 4*a*c*cos(f*x
 + e) - 4*a*d*cos(f*x + e))/f
```

mupad [B] time = 6.99, size = 100, normalized size = 2.08

$$a\,c\,x+\frac{a\,d\,x}{2}-\frac{-a\,d\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+\left(2\,a\,c+2\,a\,d\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a\,d\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+2\,a\,c+2\,a\,d}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c + d*sin(e + f*x)),x)
```

```
[Out] a*c*x + (a*d*x)/2 - (2*a*c + 2*a*d + tan(e/2 + (f*x)/2)^2*(2*a*c + 2*a*d) -
 a*d*tan(e/2 + (f*x)/2)^3 + a*d*tan(e/2 + (f*x)/2))/(f*(2*tan(e/2 + (f*x)/2
)^2 + tan(e/2 + (f*x)/2)^4 + 1))
```

sympy [A] time = 0.72, size = 94, normalized size = 1.96

$$\begin{cases} acx - \frac{ac\cos\left(e+fx\right)}{f} + \frac{adx\sin^2\left(e+fx\right)}{2} + \frac{adx\cos^2\left(e+fx\right)}{2} - \frac{ad\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f} - \frac{ad\cos\left(e+fx\right)}{f} & \text{for}\ f \neq 0 \\ x\left(c+d\sin\left(e\right)\right)\left(a\sin\left(e\right)+a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((a*c*x - a*c*cos(e + f*x)/f + a*d*x*sin(e + f*x)**2/2 + a*d*x*cos
(e + f*x)**2/2 - a*d*sin(e + f*x)*cos(e + f*x)/(2*f) - a*d*cos(e + f*x)/f,
Ne(f, 0)), (x*(c + d*sin(e))*(a*sin(e) + a), True))
```

3.429 $\int (a+a\sin(e+fx))\,dx$

Optimal. Leaf size=16

$$ax-\frac{a\cos(e+fx)}{f}$$

```
[Out] a*x-a*cos(f*x+e)/f
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2638}

$$ax-\frac{a\cos(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[a + a*Sin[e + f*x],x]
```

```
[Out] a*x - (a*Cos[e + f*x])/f
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))\,dx &= ax+a\int \sin(e+fx)\,dx\\ &= ax-\frac{a\cos(e+fx)}{f}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.69

$$\frac{a\sin(e)\sin(fx)}{f}-\frac{a\cos(e)\cos(fx)}{f}+ax$$

Antiderivative was successfully verified.

```
[In] Integrate[a + a*Sin[e + f*x],x]
```

```
[Out] a*x - (a*Cos[e]*Cos[f*x])/f + (a*Sin[e]*Sin[f*x])/f
```

fricas [A] time = 0.44, size = 18, normalized size = 1.12

$$\frac{afx - a\cos\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*sin(f*x+e),x, algorithm="fricas")
[Out] (a*f*x - a*cos(f*x + e))/f
```

giac [A] time = 0.17, size = 17, normalized size = 1.06

$$ax - \frac{a\cos\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*sin(f*x+e),x, algorithm="giac")
[Out] a*x - a*cos(f*x + e)/f
```

maple [A] time = 0.01, size = 17, normalized size = 1.06

$$ax - \frac{a\cos\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+a*sin(f*x+e),x)
[Out] a*x-a*cos(f*x+e)/f
```

maxima [A] time = 0.32, size = 16, normalized size = 1.00

$$ax - \frac{a\cos\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*sin(f*x+e),x, algorithm="maxima")
[Out] a*x - a*cos(f*x + e)/f
```

mupad [B] time = 6.72, size = 25, normalized size = 1.56

$$a\,x - \frac{2\,a}{f\,\left({\tan\left(\frac{e}{2} + \frac{f\,x}{2}\right)}^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + a*sin(e + f*x),x)
```

```
[Out] a*x - (2*a)/(f*(tan(e/2 + (f*x)/2)^2 + 1))
```

sympy [A] time = 0.23, size = 19, normalized size = 1.19

$$ax + a\left(\begin{cases} -\frac{\cos\left(e+fx\right)}{f} & \text{for } f \neq 0 \\ x\sin\left(e\right) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*sin(f*x+e),x)
```

```
[Out] a*x + a*Piecewise((-cos(e + f*x)/f, Ne(f, 0)), (x*sin(e), True))
```

3.430 $\int \frac{a+a\sin(e+fx)}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=63

$$\frac{ax}{d}-\frac{2a(c-d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{df\sqrt{c^2-d^2}}$$

```
[Out] a*x/d-2*a*(c-d)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d/f/(c^2-d
^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2735, 2660, 618, 204}

$$\frac{ax}{d}-\frac{2a(c-d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{df\sqrt{c^2-d^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x]),x]
```

```
[Out] (a*x)/d - (2*a*(c - d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d
*Sqrt[c^2 - d^2]*f)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\sin(e+fx)}{c+d\sin(e+fx)}\,dx &= \frac{ax}{d}-\frac{(a(c-d))\int \frac{1}{c+d\sin(e+fx)}\,dx}{d}\\ &= \frac{ax}{d}-\frac{(2a(c-d))\,\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx,x,\tan\left(\frac{1}{2}(e+fx)\right)\right)}{df}\\ &= \frac{ax}{d}+\frac{(4a(c-d))\,\text{Subst}\left(\int \frac{1}{-4\left(c^2-d^2\right)-x^2}\,dx,x,2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{df}\\ &= \frac{ax}{d}-\frac{2a(c-d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d\sqrt{c^2-d^2}\,f}\end{aligned}$$

Mathematica [C] time = 0.32, size = 182, normalized size = 2.89

$$\frac{a(\sin(e+fx)+1)\left(fx\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}-2(c-d)(\cos(e)-i\sin(e))\tan^{-1}\left(\frac{(\cos(e)-i\sin(e))\sec\left(\frac{fx}{2}\right)\left(c\sin\left(\frac{fx}{2}\right.\right.}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin}}\right.\right.}{df\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x]),x]
```

```
[Out] (a*(-2*(c - d)*ArcTan[(Sec[(f*x)/2]*(Cos[e] - I*Sin[e])*(d*Cos[e + (f*x)/2]
 + c*Sin[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(Cos[e]
- I*Sin[e]) + Sqrt[c^2 - d^2]*f*x*Sqrt[(Cos[e] - I*Sin[e])^2])*(1 + Sin[e +
 f*x]))/(d*Sqrt[c^2 - d^2]*f*Sqrt[(Cos[e] - I*Sin[e])^2]*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^2)
```

fricas [A] time = 0.48, size = 228, normalized size = 3.62

$$\left[\frac{2\,afx + a\sqrt{-\frac{c-d}{c+d}}\,\log\left(\frac{(2\,c^2-d^2)\cos(fx+e)^2-2\,cd\sin(fx+e)-c^2-d^2+2\,((c^2+cd)\cos(fx+e)\sin(fx+e)+(cd+d^2)\cos(fx+e))\sqrt{-\frac{c-d}{c+d}}}{d^2\cos(fx+e)^2-2\,cd\sin(fx+e)-c^2-d^2}\right)}{2\,df}, \; afx \ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*f*x + a*sqrt(-(c - d)/(c + d))*log(((2*c^2 - d^2)*cos(f*x + e)^2
- 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e)
 + (c*d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 -
2*c*d*sin(f*x + e) - c^2 - d^2)))/(d*f), (a*f*x + a*sqrt((c - d)/(c + d))*a
rctan(-(c*sin(f*x + e) + d)*sqrt((c - d)/(c + d))/((c - d)*cos(f*x + e))))/
(d*f)]
```

giac [A] time = 0.22, size = 86, normalized size = 1.37

$$\frac{\frac{(fx+e)a}{d} - \frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)(ac-ad)}{\sqrt{c^2-d^2}\,d}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] ((f*x + e)*a/d - 2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan
(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))*(a*c - a*d)/(sqrt(c^2 - d^2)*d))/f
```

maple [B] time = 0.20, size = 119, normalized size = 1.89

$$-\frac{2a\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)c}{fd\sqrt{c^2-d^2}}+\frac{2a\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{f\sqrt{c^2-d^2}}+\frac{2a\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] -2/f*a/d/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^
(1/2))*c+2/f*a/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2
-d^2)^(1/2))+2/f*a/d*arctan(tan(1/2*f*x+1/2*e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 7.39, size = 449, normalized size = 7.13

$$\frac{2\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,\left(c+d\right)}-\frac{2\,a\,\mathrm{atanh}\left(\frac{3\,d^2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,{\left(d^2-c^2\right)}^{3/2}-2\,c^4\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{d^2-c^2}-2\,c^2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,{\left(d^2-c^2\right)}^{3/2}+d^4\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{d^2-c^2}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c + d*sin(e + f*x)),x)
```

```
[Out] (2*a*atan(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(c + d)) - (2*a*atanh(
(3*d^2*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2) - 2*c^4*sin(e/2 + (f*x)/2)*(d^2
 - c^2)^(1/2) - 2*c^2*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2) + d^4*sin(e/2 +
(f*x)/2)*(d^2 - c^2)^(1/2) + 2*c^2*d^2*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)
 + 3*c^2*d^2*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2) + c*d*cos(e/2 + (f*x)/2)*
(d^2 - c^2)^(3/2) + c*d^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2) + c^3*d*cos(
e/2 + (f*x)/2)*(d^2 - c^2)^(1/2) + 4*c*d^3*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(
1/2) - 2*c^3*d*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2))/(2*(c*d + d^2)*(c^3*co
s(e/2 + (f*x)/2) - 2*d^3*sin(e/2 + (f*x)/2) - c*d^2*cos(e/2 + (f*x)/2) + 2*
c^2*d*sin(e/2 + (f*x)/2))))*(d^2 - c^2)^(1/2))/(d*f*(c + d)) + (2*a*c*atan(
sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(d*f*(c + d))
```

sympy [A] time = 119.91, size = 537, normalized size = 8.52

$$\begin{cases} \frac{\tilde{\infty} x(a\sin{(e)}+a)}{\sin{(e)}} \\ \frac{ad^2fx\tan{\left(\frac{e}{2}+\frac{fx}{2}\right)}}{d^3f\tan{\left(\frac{e}{2}+\frac{fx}{2}\right)}-f\left(d^2\right)^{\frac{3}{2}}}+\frac{2ad^2}{d^3f\tan{\left(\frac{e}{2}+\frac{fx}{2}\right)}-f\left(d^2\right)^{\frac{3}{2}}}-\frac{adfx\sqrt{d^2}}{d^3f\tan{\left(\frac{e}{2}+\frac{fx}{2}\right)}-f\left(d^2\right)^{\frac{3}{2}}}+\frac{2ad\sqrt{d^2}}{d^3f\tan{\left(\frac{e}{2}+\frac{fx}{2}\right)}-f\left(d^2\right)^{\frac{3}{2}}} \\ \frac{ad^2fx\tan{\left(\frac{e}{2}+\frac{fx}{2}\right)}}{d^3f\tan{\left(\frac{e}{2}+\frac{fx}{2}\right)}+f\left(d^2\right)^{\frac{3}{2}}}+\frac{2ad^2}{d^3f\tan{\left(\frac{e}{2}+\frac{fx}{2}\right)}+f\left(d^2\right)^{\frac{3}{2}}}+\frac{adfx\sqrt{d^2}}{d^3f\tan{\left(\frac{e}{2}+\frac{fx}{2}\right)}+f\left(d^2\right)^{\frac{3}{2}}}-\frac{2ad\sqrt{d^2}}{d^3f\tan{\left(\frac{e}{2}+\frac{fx}{2}\right)}+f\left(d^2\right)^{\frac{3}{2}}} \\ \frac{ax+\frac{a\log{\left(\tan{\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)}}{f}}{d} \\ \frac{ax-\frac{a\cos{(e+fx)}}{f}}{c} \\ \frac{x(a\sin{(e)}+a)}{c+d\sin{(e)}} \\ -\frac{ac\log{\left(\tan{\left(\frac{e}{2}+\frac{fx}{2}\right)}+\frac{d}{c}-\frac{\sqrt{-c^2+d^2}}{c}\right)}}{df\sqrt{-c^2+d^2}}+\frac{ac\log{\left(\tan{\left(\frac{e}{2}+\frac{fx}{2}\right)}+\frac{d}{c}+\frac{\sqrt{-c^2+d^2}}{c}\right)}}{df\sqrt{-c^2+d^2}}+\frac{a\log{\left(\tan{\left(\frac{e}{2}+\frac{fx}{2}\right)}+\frac{d}{c}-\frac{\sqrt{-c^2+d^2}}{c}\right)}}{f\sqrt{-c^2+d^2}}-\frac{a\log{\left(\tan{\left(\frac{e}{2}+\frac{fx}{2}\right)}+\frac{d}{c}+\frac{\sqrt{-c^2+d^2}}{c}\right)}}{f\sqrt{-c^2+d^2}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((zoo*x*(a*sin(e) + a)/sin(e), Eq(c, 0) & Eq(d, 0) & Eq(f, 0)), (a
*d**2*f*x*tan(e/2 + f*x/2)/(d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)) + 2*
a*d**2/(d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)) - a*d*f*x*sqrt(d**2)/(d*
*3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)) + 2*a*d*sqrt(d**2)/(d**3*f*tan(e/2
 + f*x/2) - f*(d**2)**(3/2)), Eq(c, -sqrt(d**2))), (a*d**2*f*x*tan(e/2 + f*
x/2)/(d**3*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)) + 2*a*d**2/(d**3*f*tan(e/2
 + f*x/2) + f*(d**2)**(3/2)) + a*d*f*x*sqrt(d**2)/(d**3*f*tan(e/2 + f*x/2)
+ f*(d**2)**(3/2)) - 2*a*d*sqrt(d**2)/(d**3*f*tan(e/2 + f*x/2) + f*(d**2)**
(3/2)), Eq(c, sqrt(d**2))), ((a*x + a*log(tan(e/2 + f*x/2))/f)/d, Eq(c, 0))
, ((a*x - a*cos(e + f*x)/f)/c, Eq(d, 0)), (x*(a*sin(e) + a)/(c + d*sin(e)),
 Eq(f, 0)), (-a*c*log(tan(e/2 + f*x/2) + d/c - sqrt(-c**2 + d**2)/c)/(d*f*s
qrt(-c**2 + d**2)) + a*c*log(tan(e/2 + f*x/2) + d/c + sqrt(-c**2 + d**2)/c)
/(d*f*sqrt(-c**2 + d**2)) + a*log(tan(e/2 + f*x/2) + d/c - sqrt(-c**2 + d**
2)/c)/(f*sqrt(-c**2 + d**2)) - a*log(tan(e/2 + f*x/2) + d/c + sqrt(-c**2 +
d**2)/c)/(f*sqrt(-c**2 + d**2)) + a*x/d, True))
```

3.431 $\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=83

$$\frac{2a\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c+d)\sqrt{c^2-d^2}}-\frac{a\cos(e+fx)}{f(c+d)(c+d\sin(e+fx))}$$

```
[Out] -a*cos(f*x+e)/(c+d)/f/(c+d*sin(f*x+e))+2*a*arctan((d+c*tan(1/2*f*x+1/2*e))/
(c^2-d^2)^(1/2))/(c+d)/f/(c^2-d^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2754, 12, 2660, 618, 204}

$$\frac{2a\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c+d)\sqrt{c^2-d^2}}-\frac{a\cos(e+fx)}{f(c+d)(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (2*a*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)*Sqrt[c^2 -
d^2]*f) - (a*Cos[e + f*x])/((c + d)*f*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^2}\,dx &= -\frac{a\cos(e+fx)}{(c+d)f(c+d\sin(e+fx))}-\frac{\int \frac{a(c-d)}{c+d\sin(e+fx)}\,dx}{-c^2+d^2}\\
&= -\frac{a\cos(e+fx)}{(c+d)f(c+d\sin(e+fx))}+\frac{a\int \frac{1}{c+d\sin(e+fx)}\,dx}{c+d}\\
&= -\frac{a\cos(e+fx)}{(c+d)f(c+d\sin(e+fx))}+\frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx,x,\tan\left(\frac{1}{2}(e+fx)\right)\right)}{(c+d)f}\\
&= -\frac{a\cos(e+fx)}{(c+d)f(c+d\sin(e+fx))}-\frac{(4a)\,\mathrm{Subst}\left(\int \frac{1}{-4\left(c^2-d^2\right)-x^2}\,dx,x,2d+2c\tan\left(\frac{1}{2}(e+
\end{aligned}
$$

$$
= \frac{2a\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}\,f}-\frac{a\cos(e+fx)}{(c+d)f(c+d\sin(e+fx))}
$$

Mathematica [C] time = 0.59, size = 220, normalized size = 2.65

$$
\frac{a(\sin(e+fx)+1)\left(2\sqrt{c^2-d^2}\,\csc(e)\sqrt{(\cos(e)-i\sin(e))^2}\,(c\cos(e)+d\sin(fx))+4d(\cos(e)-i\sin(e))(c+d\sin(\right.}{2df(c+d)\sqrt{c^2-d^2}\,\sqrt{(\cos(e)-i\sin(e))^2}\,\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right.}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a*(1 + Sin[e + f*x])*(2*Sqrt[c^2 - d^2]*Csc[e]*Sqrt[(Cos[e] - I*Sin[e])^2]
*(c*Cos[e] + d*Sin[f*x]) + 4*d*ArcTan[(Sec[(f*x)/2]*(Cos[e] - I*Sin[e])*(d*
Cos[e + (f*x)/2] + c*Sin[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e
])^2])]*(Cos[e] - I*Sin[e])*(c + d*Sin[e + f*x])))/(2*d*(c + d)*Sqrt[c^2 -
d^2]*f*Sqrt[(Cos[e] - I*Sin[e])^2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*
(c + d*Sin[e + f*x]))
```

fricas [A] time = 0.49, size = 362, normalized size = 4.36

$$\left[-\frac{\left(ad\sin\left(fx+e\right)+ac\right)\sqrt{-c^2+d^2}\,\log\left(\frac{\left(2\,c^2-d^2\right)\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2+2\left(c\cos\left(fx+e\right)\sin\left(fx+e\right)+d\cos\left(fx+e\right)\right)\sqrt{-c^2+d^2}}{d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2}\right.}{2\left(\left(c^3d+c^2d^2-cd^3-d^4\right)f\sin\left(fx+e\right)+\left(c^4+c^3d-c^2d^2-cd^3\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((a*d*sin(f*x + e) + a*c)*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x
 + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) +
 d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e)
 - c^2 - d^2)) + 2*(a*c^2 - a*d^2)*cos(f*x + e))/((c^3*d + c^2*d^2 - c*d^3
- d^4)*f*sin(f*x + e) + (c^4 + c^3*d - c^2*d^2 - c*d^3)*f), -((a*d*sin(f*x
+ e) + a*c)*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*c
os(f*x + e))) + (a*c^2 - a*d^2)*cos(f*x + e))/((c^3*d + c^2*d^2 - c*d^3 - d
^4)*f*sin(f*x + e) + (c^4 + c^3*d - c^2*d^2 - c*d^3)*f)]
```

giac [A] time = 0.34, size = 129, normalized size = 1.55

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)a}{\sqrt{c^2-d^2}\,(c+d)}-\frac{ad\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+ac}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\,d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+c\right)\left(c^2+cd\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e
) + d)/sqrt(c^2 - d^2)))*a/(sqrt(c^2 - d^2)*(c + d)) - (a*d*tan(1/2*f*x + 1
```

```
/2*e) + a*c)/((c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)*(c^
2 + c*d)))/f
```

maple [A] time = 0.24, size = 147, normalized size = 1.77

$$-\frac{2ad\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d+c\right)(c+d)c}-\frac{2a}{f\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d+c\right)(c+d)}+\frac{2a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

```
[Out] -2/f*a/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)*d/(c+d)/c*tan(1/2*
f*x+1/2*e)-2/f*a/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)+2/
f*a/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)
^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 6.95, size = 140, normalized size = 1.69

$$\frac{2\,a\,\mathrm{atan}\left(\frac{(c+d)\left(\frac{2\,a\,\left(d^2+c\,d\right)}{(c+d)^{5/2}\,\sqrt{c-d}}+\frac{2\,a\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{(c+d)^{3/2}\,\sqrt{c-d}}\right)}{2\,a}\right)}{f\,(c+d)^{3/2}\,\sqrt{c-d}}-\frac{\frac{2\,a}{c+d}+\frac{2\,a\,d\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{c\,(c+d)}}{f\,\left(c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+2\,d\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c + d*sin(e + f*x))^2,x)
```

```
[Out] (2*a*atan(((c + d)*((2*a*(c*d + d^2))/((c + d)^(5/2)*(c - d)^(1/2)) + (2*a*
c*tan(e/2 + (f*x)/2))/((c + d)^(3/2)*(c - d)^(1/2))))/(2*a)))/(f*(c + d)^(3
/2)*(c - d)^(1/2)) - ((2*a)/(c + d) + (2*a*d*tan(e/2 + (f*x)/2))/(c*(c + d)
))/(f*(c + 2*d*tan(e/2 + (f*x)/2) + c*tan(e/2 + (f*x)/2)^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.432 $\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=134

$$\frac{a(2c-d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c+d)\left(c^2-d^2\right)^{3/2}}-\frac{a(c-2d)\cos(e+fx)}{2f(c-d)(c+d)^2(c+d\sin(e+fx))}-\frac{a\cos(e+fx)}{2f(c+d)(c+d\sin(e+fx))^2}$$

```
[Out] a*(2*c-d)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(c+d)/(c^2-d^2)^
(3/2)/f-1/2*a*cos(f*x+e)/(c+d)/f/(c+d*sin(f*x+e))^2-1/2*a*(c-2*d)*cos(f*x+e
)/(c-d)/(c+d)^2/f/(c+d*sin(f*x+e))
```

Rubi [A] time = 0.18, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2754, 12, 2660, 618, 204}

$$\frac{a(2c-d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c+d)\left(c^2-d^2\right)^{3/2}}-\frac{a(c-2d)\cos(e+fx)}{2f(c-d)(c+d)^2(c+d\sin(e+fx))}-\frac{a\cos(e+fx)}{2f(c+d)(c+d\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a*(2*c - d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)*(c^
2 - d^2)^(3/2)*f) - (a*Cos[e + f*x])/(2*(c + d)*f*(c + d*Sin[e + f*x])^2) -
 (a*(c - 2*d)*Cos[e + f*x])/(2*(c - d)*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^3}\,dx &= -\frac{a\cos(e+fx)}{2(c+d)f(c+d\sin(e+fx))^2} - \frac{\int \frac{-2a(c-d)-a(c-d)\sin(e+fx)}{(c+d\sin(e+fx))^2}\,dx}{2\left(c^2-d^2\right)} \\
&= -\frac{a\cos(e+fx)}{2(c+d)f(c+d\sin(e+fx))^2} - \frac{a(c-2d)\cos(e+fx)}{2(c-d)(c+d)^2f(c+d\sin(e+fx))} + \frac{\int \frac{a(c-d)(2c-d)}{c+d\sin(e+f}}{2\left(c^2-d^2\right.} \\
&= -\frac{a\cos(e+fx)}{2(c+d)f(c+d\sin(e+fx))^2} - \frac{a(c-2d)\cos(e+fx)}{2(c-d)(c+d)^2f(c+d\sin(e+fx))} + \frac{(a(2c-d))}{2(c-} \\
&= -\frac{a\cos(e+fx)}{2(c+d)f(c+d\sin(e+fx))^2} - \frac{a(c-2d)\cos(e+fx)}{2(c-d)(c+d)^2f(c+d\sin(e+fx))} + \frac{(a(2c-d))\,\mathrm{S}}{} \\
&= -\frac{a\cos(e+fx)}{2(c+d)f(c+d\sin(e+fx))^2} - \frac{a(c-2d)\cos(e+fx)}{2(c-d)(c+d)^2f(c+d\sin(e+fx))} - \frac{(2a(2c-d))}{} \\
&= \frac{a(2c-d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c-d)(c+d)^2\sqrt{c^2-d^2}\,f} - \frac{a\cos(e+fx)}{2(c+d)f(c+d\sin(e+fx))^2} - \frac{a(c-2d)\,\mathrm{c}}{2(c-d)(c+d)^2f}
\end{aligned}
$$

Mathematica [C] time = 1.20, size = 242, normalized size = 1.81

$$\frac{a(\sin(e+fx)+1)\left(\frac{4(2c-d)(\cos(e)-i\sin(e))\tan^{-1}\left(\frac{(\cos(e)-i\sin(e))\sec\left(\frac{fx}{2}\right)\left(c\sin\left(\frac{fx}{2}\right)+d\cos\left(e+\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)}{(c-d)\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}+\frac{2(c+d)\csc(e)(c\cos(e)+d\sin(fx))}{d(c+d\sin(e+fx))^2}+\frac{2(c-2}{}\right.}{4f(c+d)^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a*(1 + Sin[e + f*x])*((4*(2*c - d)*ArcTan[(Sec[(f*x)/2]*(Cos[e] - I*Sin[e]
)*(d*Cos[e + (f*x)/2] + c*Sin[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*
Sin[e])^2])]*(Cos[e] - I*Sin[e]))/((c - d)*Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I
*Sin[e])^2]) + (2*(c + d)*Csc[e]*(c*Cos[e] + d*Sin[f*x]))/(d*(c + d*Sin[e +
 f*x])^2) + ((-4*c + 2*d)*Cot[e] + 2*(c - 2*d)*Csc[e]*Sin[f*x])/((c - d)*(c
 + d*Sin[e + f*x]))))/(4*(c + d)^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
2)
```

fricas [B] time = 0.48, size = 803, normalized size = 5.99

$$\left[\frac{2\left(ac^3d-2\,ac^2d^2-acd^3+2\,ad^4\right)\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(2\,ac^3-ac^2d+2\,acd^2-ad^3-\left(2\,acd^2-ad^3\right)\cos}{4\left(\left(c^5d^2+c^4d^3-2\,c^3d^4-2\,c^2d^5+cd^6+d^7\right)f\,c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a*c^3*d - 2*a*c^2*d^2 - a*c*d^3 + 2*a*d^4)*cos(f*x + e)*sin(f*x +
e) + (2*a*c^3 - a*c^2*d + 2*a*c*d^2 - a*d^3 - (2*a*c*d^2 - a*d^3)*cos(f*x +
 e)^2 + 2*(2*a*c^2*d - a*c*d^2)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2
- d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*
sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*
d*sin(f*x + e) - c^2 - d^2)) + 2*(2*a*c^4 - 2*a*c^3*d - 3*a*c^2*d^2 + 2*a*c
*d^3 + a*d^4)*cos(f*x + e))/((c^5*d^2 + c^4*d^3 - 2*c^3*d^4 - 2*c^2*d^5 + c
*d^6 + d^7)*f*cos(f*x + e)^2 - 2*(c^6*d + c^5*d^2 - 2*c^4*d^3 - 2*c^3*d^4 +
 c^2*d^5 + c*d^6)*f*sin(f*x + e) - (c^7 + c^6*d - c^5*d^2 - c^4*d^3 - c^3*d
^4 - c^2*d^5 + c*d^6 + d^7)*f), 1/2*((a*c^3*d - 2*a*c^2*d^2 - a*c*d^3 + 2*a
*d^4)*cos(f*x + e)*sin(f*x + e) + (2*a*c^3 - a*c^2*d + 2*a*c*d^2 - a*d^3 -
(2*a*c*d^2 - a*d^3)*cos(f*x + e)^2 + 2*(2*a*c^2*d - a*c*d^2)*sin(f*x + e))*
```

```
sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))
) + (2*a*c^4 - 2*a*c^3*d - 3*a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e))/(
(c^5*d^2 + c^4*d^3 - 2*c^3*d^4 - 2*c^2*d^5 + c*d^6 + d^7)*f*cos(f*x + e)^2
- 2*(c^6*d + c^5*d^2 - 2*c^4*d^3 - 2*c^3*d^4 + c^2*d^5 + c*d^6)*f*sin(f*x +
 e) - (c^7 + c^6*d - c^5*d^2 - c^4*d^3 - c^3*d^4 - c^2*d^5 + c*d^6 + d^7)*f
)]
```

giac [B] time = 0.36, size = 384, normalized size = 2.87

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)(2ac-ad)}{\left(c^3+c^2d-cd^2-d^3\right)\sqrt{c^2-d^2}}-\frac{3ac^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2ac^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2acd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+2ac^4\tan\left(\frac{1}{2}fx+\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e)
+ d)/sqrt(c^2 - d^2)))*(2*a*c - a*d)/((c^3 + c^2*d - c*d^2 - d^3)*sqrt(c^2
- d^2)) - (3*a*c^3*d*tan(1/2*f*x + 1/2*e)^3 - 2*a*c^2*d^2*tan(1/2*f*x + 1/2
*e)^3 - 2*a*c*d^3*tan(1/2*f*x + 1/2*e)^3 + 2*a*c^4*tan(1/2*f*x + 1/2*e)^2 -
 2*a*c^3*d*tan(1/2*f*x + 1/2*e)^2 + 3*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 - 4*
a*c*d^3*tan(1/2*f*x + 1/2*e)^2 - 2*a*d^4*tan(1/2*f*x + 1/2*e)^2 + 5*a*c^3*d
*tan(1/2*f*x + 1/2*e) - 6*a*c^2*d^2*tan(1/2*f*x + 1/2*e) - 2*a*c*d^3*tan(1/
2*f*x + 1/2*e) + 2*a*c^4 - 2*a*c^3*d - a*c^2*d^2)/((c^5 + c^4*d - c^3*d^2 -
 c^2*d^3)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2))/f
```

maple [B] time = 0.28, size = 1104, normalized size = 8.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

```
[Out] -3/f*a/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*d/(c^3+c^2*d-c*d
^2-d^3)*c*tan(1/2*f*x+1/2*e)^3+2/f*a/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+
1/2*e)*d+c)^2*d^2/(c^3+c^2*d-c*d^2-d^3)*tan(1/2*f*x+1/2*e)^3+2/f*a/(tan(1/2
*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^3/(c^3+c^2*d-c*d^2-d^3)/c*tan
(1/2*f*x+1/2*e)^3-2/f*a/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2
/(c^3+c^2*d-c*d^2-d^3)*c^2*tan(1/2*f*x+1/2*e)^2+2/f*a/(tan(1/2*f*x+1/2*e)^2
*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*c*tan(1/2*f*x+1/2*e)^2
*d-3/f*a/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d
^2-d^3)*tan(1/2*f*x+1/2*e)^2*d^2+4/f*a/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*
```

```
x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)/c*tan(1/2*f*x+1/2*e)^2*d^3+2/f*a/(tan
(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)/c^2*t
an(1/2*f*x+1/2*e)^2*d^4-5/f*a/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*
d+c)^2*d*c/(c^3+c^2*d-c*d^2-d^3)*tan(1/2*f*x+1/2*e)+6/f*a/(tan(1/2*f*x+1/2*
e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^2/(c^3+c^2*d-c*d^2-d^3)*tan(1/2*f*x+1/
2*e)+2/f*a/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^3/c/(c^3+c
^2*d-c*d^2-d^3)*tan(1/2*f*x+1/2*e)-2/f*a/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*
f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*c^2+2/f*a/(tan(1/2*f*x+1/2*e)^2*c+2
*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*c*d+1/f*a/(tan(1/2*f*x+1/2
*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*d^2+2/f*a/(c^3+c^
2*d-c*d^2-d^3)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2
-d^2)^(1/2))*c-1/f*a/(c^3+c^2*d-c*d^2-d^3)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*
tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 8.97, size = 445, normalized size = 3.32

$$-\frac{\frac{-2\,a\,c^2+2\,a\,c\,d+a\,d^2}{-c^3-c^2\,d+c\,d^2+d^3}+\frac{a\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(c^2+2\,d^2\right)\,\left(-2\,c^2+2\,c\,d+d^2\right)}{c^2\,\left(-c^3-c^2\,d+c\,d^2+d^3\right)}+\frac{a\,d\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(-5\,c^2+6\,c\,d+2\,d^2\right)}{c\,\left(-c^3-c^2\,d+c\,d^2+d^3\right)}+\frac{a\,d\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(-3\,c^2+2\,c\,d+2\,d^2\right)}{c\,\left(-c^3-c^2\,d+c\,d^2+d^3\right)}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(2\,c^2+4\,d^2\right)+c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+c^2+4\,c\,d\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+4\,c\,d\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c + d*sin(e + f*x))^3,x)
```

```
[Out] - ((a*d^2 - 2*a*c^2 + 2*a*c*d)/(c*d^2 - c^2*d - c^3 + d^3) + (a*tan(e/2 + (
f*x)/2)^2*(c^2 + 2*d^2)*(2*c*d - 2*c^2 + d^2))/(c^2*(c*d^2 - c^2*d - c^3 +
d^3)) + (a*d*tan(e/2 + (f*x)/2)*(6*c*d - 5*c^2 + 2*d^2))/(c*(c*d^2 - c^2*d
- c^3 + d^3)) + (a*d*tan(e/2 + (f*x)/2)^3*(2*c*d - 3*c^2 + 2*d^2))/(c*(c*d^
2 - c^2*d - c^3 + d^3)))/(f*(tan(e/2 + (f*x)/2)^2*(2*c^2 + 4*d^2) + c^2*tan
```

```
(e/2 + (f*x)/2)^4 + c^2 + 4*c*d*tan(e/2 + (f*x)/2)^3 + 4*c*d*tan(e/2 + (f*x
)/2))) - (a*atan((((a*(2*c - d)*(2*c*d^3 - 2*c^3*d + 2*d^4 - 2*c^2*d^2))/(2
*(c + d)^(5/2)*(c - d)^(3/2)*(c*d^2 - c^2*d - c^3 + d^3)) + (a*c*tan(e/2 +
(f*x)/2)*(2*c - d))/((c + d)^(5/2)*(c - d)^(3/2)))*(c*d^2 - c^2*d - c^3 + d
^3))/(2*a*c - a*d))*(2*c - d))/(f*(c + d)^(5/2)*(c - d)^(3/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.433 $\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^4}\,dx$

Optimal. Leaf size=192

$$\frac{a\left(2c^2-2cd+d^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c+d)\left(c^2-d^2\right)^{5/2}}-\frac{a(c-4d)(2c-d)\cos(e+fx)}{6f(c-d)^2(c+d)^3(c+d\sin(e+fx))}-\frac{a(2c-3d)\cos(e+fx)}{6f(c-d)(c+d)^2(c+d\sin(e+$$

```
[Out] a*(2*c^2-2*c*d+d^2)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(c+d)/
(c^2-d^2)^(5/2)/f-1/3*a*cos(f*x+e)/(c+d)/f/(c+d*sin(f*x+e))^3-1/6*a*(2*c-3*
d)*cos(f*x+e)/(c-d)/(c+d)^2/f/(c+d*sin(f*x+e))^2-1/6*a*(c-4*d)*(2*c-d)*cos(
f*x+e)/(c-d)^2/(c+d)^3/f/(c+d*sin(f*x+e))
```

Rubi [A] time = 0.33, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2754, 12, 2660, 618, 204}

$$\frac{a\left(2c^2-2cd+d^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c+d)\left(c^2-d^2\right)^{5/2}}-\frac{a(c-4d)(2c-d)\cos(e+fx)}{6f(c-d)^2(c+d)^3(c+d\sin(e+fx))}-\frac{a(2c-3d)\cos(e+fx)}{6f(c-d)(c+d)^2(c+d\sin(e+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^4,x]
```

```
[Out] (a*(2*c^2 - 2*c*d + d^2)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/
((c + d)*(c^2 - d^2)^(5/2)*f) - (a*Cos[e + f*x])/(3*(c + d)*f*(c + d*Sin[e
+ f*x])^3) - (a*(2*c - 3*d)*Cos[e + f*x])/(6*(c - d)*(c + d)^2*f*(c + d*Sin
[e + f*x])^2) - (a*(c - 4*d)*(2*c - d)*Cos[e + f*x])/(6*(c - d)^2*(c + d)^3
*f*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^4}\,dx = -\frac{a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^3} - \frac{\int \frac{-3a(c-d)-2a(c-d)\sin(e+fx)}{(c+d\sin(e+fx))^3}\,dx}{3\left(c^2-d^2\right)}$$

$$= -\frac{a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^3} - \frac{a(2c-3d)\cos(e+fx)}{6(c-d)(c+d)^2f(c+d\sin(e+fx))^2} + \frac{\int \frac{2a(3c-2d}{}}{}$$

$$= -\frac{a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^3} - \frac{a(2c-3d)\cos(e+fx)}{6(c-d)(c+d)^2f(c+d\sin(e+fx))^2} - \frac{a(c-}{6(c-d)^2(}$$

$$= -\frac{a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^3} - \frac{a(2c-3d)\cos(e+fx)}{6(c-d)(c+d)^2f(c+d\sin(e+fx))^2} - \frac{a(c-}{6(c-d)^2(}$$

$$= -\frac{a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^3} - \frac{a(2c-3d)\cos(e+fx)}{6(c-d)(c+d)^2f(c+d\sin(e+fx))^2} - \frac{a(c-}{6(c-d)^2(}$$

$$= -\frac{a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^3} - \frac{a(2c-3d)\cos(e+fx)}{6(c-d)(c+d)^2f(c+d\sin(e+fx))^2} - \frac{a(c-}{6(c-d)^2(}$$

$$= \frac{a\left(2c^2-2cd+d^2\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c-d)^2(c+d)^3\sqrt{c^2-d^2}\,f} - \frac{a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^3} - \frac{}{6(c-d}$$

Mathematica [C] time = 2.71, size = 428, normalized size = 2.23

$$\frac{a(\sin(e+fx)+1)\left(\frac{2c\left(4c^4-18c^3d+14c^2d^2-27cd^3+12d^4\right)\cot(e)-d\csc(e)\left(-24c^4\sin(fx)+30c^3d\sin(2e+fx)+78c^3d\sin(fx)-30c^2d^2\sin(2e+fx)+\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^4,x]
```

```
[Out] (a*(1 + Sin[e + f*x])*((24*(2*c^2 - 2*c*d + d^2)*ArcTan[(Sec[(f*x)/2]*(Cos[
e] - I*Sin[e])*(d*Cos[e + (f*x)/2] + c*Sin[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt
```

```
[(Cos[e] - I*Sin[e])^2])]*(Cos[e] - I*Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e
] - I*Sin[e])^2]) + (2*c*(4*c^4 - 18*c^3*d + 14*c^2*d^2 - 27*c*d^3 + 12*d^4
)*Cot[e] - d*Csc[e]*(3*d*(4*c^3 - 16*c^2*d + 6*c*d^2 + d^3)*Cos[e + 2*f*x]
- 3*d^2*(2*c^2 - 2*c*d + d^2)*Cos[3*e + 2*f*x] - 24*c^4*Sin[f*x] + 78*c^3*d
*Sin[f*x] - 24*c^2*d^2*Sin[f*x] + 12*c*d^3*Sin[f*x] - 12*d^4*Sin[f*x] + 30*
c^3*d*Sin[2*e + f*x] - 30*c^2*d^2*Sin[2*e + f*x] + 15*c*d^3*Sin[2*e + f*x]
+ 2*c^2*d^2*Sin[2*e + 3*f*x] - 9*c*d^3*Sin[2*e + 3*f*x] + 4*d^4*Sin[2*e + 3
*f*x]))/(d*(c + d*Sin[e + f*x])^3)))/(24*(c - d)^2*(c + d)^3*f*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])^2)
```

fricas [B] time = 0.56, size = 1344, normalized size = 7.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(2*(2*a*c^4*d^2 - 9*a*c^3*d^3 + 2*a*c^2*d^4 + 9*a*c*d^5 - 4*a*d^6)*c
os(f*x + e)^3 - 6*(2*a*c^5*d - 7*a*c^4*d^2 + 8*a*c^2*d^4 - 2*a*c*d^5 - a*d^
6)*cos(f*x + e)*sin(f*x + e) - 3*(2*a*c^5 - 2*a*c^4*d + 7*a*c^3*d^2 - 6*a*c
^2*d^3 + 3*a*c*d^4 - 3*(2*a*c^3*d^2 - 2*a*c^2*d^3 + a*c*d^4)*cos(f*x + e)^2
 + (6*a*c^4*d - 6*a*c^3*d^2 + 5*a*c^2*d^3 - 2*a*c*d^4 + a*d^5 - (2*a*c^2*d^
3 - 2*a*c*d^4 + a*d^5)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(-c^2 + d^2)*log((
(2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*
x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2
 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 12*(a*c^6 - 2*a*c^5*d - a*c^4*d^2 + a
*c^3*d^3 + a*c^2*d^4 + a*c*d^5 - a*d^6)*cos(f*x + e))/(3*(c^8*d^2 + c^7*d^3
 - 3*c^6*d^4 - 3*c^5*d^5 + 3*c^4*d^6 + 3*c^3*d^7 - c^2*d^8 - c*d^9)*f*cos(f
*x + e)^2 - (c^10 + c^9*d - 6*c^6*d^4 - 6*c^5*d^5 + 8*c^4*d^6 + 8*c^3*d^7 -
 3*c^2*d^8 - 3*c*d^9)*f + ((c^7*d^3 + c^6*d^4 - 3*c^5*d^5 - 3*c^4*d^6 + 3*c
^3*d^7 + 3*c^2*d^8 - c*d^9 - d^10)*f*cos(f*x + e)^2 - (3*c^9*d + 3*c^8*d^2
- 8*c^7*d^3 - 8*c^6*d^4 + 6*c^5*d^5 + 6*c^4*d^6 - c*d^9 - d^10)*f)*sin(f*x
+ e)), -1/6*((2*a*c^4*d^2 - 9*a*c^3*d^3 + 2*a*c^2*d^4 + 9*a*c*d^5 - 4*a*d^6
)*cos(f*x + e)^3 - 3*(2*a*c^5*d - 7*a*c^4*d^2 + 8*a*c^2*d^4 - 2*a*c*d^5 - a
*d^6)*cos(f*x + e)*sin(f*x + e) - 3*(2*a*c^5 - 2*a*c^4*d + 7*a*c^3*d^2 - 6*
a*c^2*d^3 + 3*a*c*d^4 - 3*(2*a*c^3*d^2 - 2*a*c^2*d^3 + a*c*d^4)*cos(f*x + e
)^2 + (6*a*c^4*d - 6*a*c^3*d^2 + 5*a*c^2*d^3 - 2*a*c*d^4 + a*d^5 - (2*a*c^2
*d^3 - 2*a*c*d^4 + a*d^5)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(c^2 - d^2)*arc
tan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) - 6*(a*c^6 - 2*a*
c^5*d - a*c^4*d^2 + a*c^3*d^3 + a*c^2*d^4 + a*c*d^5 - a*d^6)*cos(f*x + e))/
(3*(c^8*d^2 + c^7*d^3 - 3*c^6*d^4 - 3*c^5*d^5 + 3*c^4*d^6 + 3*c^3*d^7 - c^2
*d^8 - c*d^9)*f*cos(f*x + e)^2 - (c^10 + c^9*d - 6*c^6*d^4 - 6*c^5*d^5 + 8*
c^4*d^6 + 8*c^3*d^7 - 3*c^2*d^8 - 3*c*d^9)*f + ((c^7*d^3 + c^6*d^4 - 3*c^5*
d^5 - 3*c^4*d^6 + 3*c^3*d^7 + 3*c^2*d^8 - c*d^9 - d^10)*f*cos(f*x + e)^2 -
```

```
(3*c^9*d + 3*c^8*d^2 - 8*c^7*d^3 - 8*c^6*d^4 + 6*c^5*d^5 + 6*c^4*d^6 - c*d^
9 - d^10)*f)*sin(f*x + e))]
```

giac [B] time = 1.38, size = 808, normalized size = 4.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(2*a*c^2 - 2*a*c*d + a*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c)
 + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^5 + c^4*d - 2*
c^3*d^2 - 2*c^2*d^3 + c*d^4 + d^5)*sqrt(c^2 - d^2)) - (12*a*c^6*d*tan(1/2*f
*x + 1/2*e)^5 - 15*a*c^5*d^2*tan(1/2*f*x + 1/2*e)^5 - 12*a*c^4*d^3*tan(1/2*
f*x + 1/2*e)^5 + 6*a*c^3*d^4*tan(1/2*f*x + 1/2*e)^5 + 6*a*c^2*d^5*tan(1/2*f
*x + 1/2*e)^5 + 6*a*c^7*tan(1/2*f*x + 1/2*e)^4 - 12*a*c^6*d*tan(1/2*f*x + 1
/2*e)^4 + 30*a*c^5*d^2*tan(1/2*f*x + 1/2*e)^4 - 51*a*c^4*d^3*tan(1/2*f*x +
1/2*e)^4 - 18*a*c^3*d^4*tan(1/2*f*x + 1/2*e)^4 + 18*a*c^2*d^5*tan(1/2*f*x +
 1/2*e)^4 + 12*a*c*d^6*tan(1/2*f*x + 1/2*e)^4 + 36*a*c^6*d*tan(1/2*f*x + 1/
2*e)^3 - 72*a*c^5*d^2*tan(1/2*f*x + 1/2*e)^3 + 12*a*c^4*d^3*tan(1/2*f*x + 1
/2*e)^3 - 30*a*c^3*d^4*tan(1/2*f*x + 1/2*e)^3 + 4*a*c^2*d^5*tan(1/2*f*x + 1
/2*e)^3 + 12*a*c*d^6*tan(1/2*f*x + 1/2*e)^3 + 8*a*d^7*tan(1/2*f*x + 1/2*e)^
3 + 12*a*c^7*tan(1/2*f*x + 1/2*e)^2 - 24*a*c^6*d*tan(1/2*f*x + 1/2*e)^2 + 3
6*a*c^5*d^2*tan(1/2*f*x + 1/2*e)^2 - 84*a*c^4*d^3*tan(1/2*f*x + 1/2*e)^2 +
18*a*c^2*d^5*tan(1/2*f*x + 1/2*e)^2 + 12*a*c*d^6*tan(1/2*f*x + 1/2*e)^2 + 2
4*a*c^6*d*tan(1/2*f*x + 1/2*e) - 57*a*c^5*d^2*tan(1/2*f*x + 1/2*e) + 12*a*c
^3*d^4*tan(1/2*f*x + 1/2*e) + 6*a*c^2*d^5*tan(1/2*f*x + 1/2*e) + 6*a*c^7 -
12*a*c^6*d - 2*a*c^5*d^2 + 3*a*c^4*d^3 + 2*a*c^3*d^4)/((c^8 + c^7*d - 2*c^6
*d^2 - 2*c^5*d^3 + c^4*d^4 + c^3*d^5)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1
/2*f*x + 1/2*e) + c)^3))/f
```

maple [B] time = 0.32, size = 3104, normalized size = 16.17

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^4,x)
```

```
[Out] -4/f*a/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*d^4/(c^5+c^4*d-2
*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*tan(1/2*f*x+1/2*e)-2/f*a/(tan(1/2*f*x+1/2*e)^
2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*d^4/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^
5)*tan(1/2*f*x+1/2*e)^5-2/f*a/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*
d+c)^3/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*c^4*tan(1/2*f*x+1/2*e)^4-4
/f*a/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^5+c^4*d-2*c^3*d
```

```
^2-2*c^2*d^3+c*d^4+d^5)*c^4*tan(1/2*f*x+1/2*e)^2+4/f*a/(tan(1/2*f*x+1/2*e)^
2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*c
^3*d+2/3/f*a/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^5+c^4*d
-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*c^2*d^2+2/f*a/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^
3+c*d^4+d^5)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d
^2)^(1/2))*c^2+1/f*a/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)/(c^2-d^2)^(1
/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*d^2-1/f*a/(tan
(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^5+c^4*d-2*c^3*d^2-2*c^2*
d^3+c*d^4+d^5)*c*d^3+6/f*a/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c
)^3/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*tan(1/2*f*x+1/2*e)^4*d^4+10/f
*a/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*d^4/(c^5+c^4*d-2*c^3
*d^2-2*c^2*d^3+c*d^4+d^5)*tan(1/2*f*x+1/2*e)^3-4/3/f*a/(tan(1/2*f*x+1/2*e)^
2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/c*d^5/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+
d^5)*tan(1/2*f*x+1/2*e)^3-4/f*a/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e
)*d+c)^3/c^2*d^6/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*tan(1/2*f*x+1/2*
e)^3-8/3/f*a/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/c^3*d^7/(c
^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*tan(1/2*f*x+1/2*e)^3+8/f*a/(tan(1/2
*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+
c*d^4+d^5)*c^3*tan(1/2*f*x+1/2*e)^2*d-12/f*a/(tan(1/2*f*x+1/2*e)^2*c+2*tan(
1/2*f*x+1/2*e)*d+c)^3/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*c^2*tan(1/2
*f*x+1/2*e)^2*d^2+28/f*a/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^
3/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*c*tan(1/2*f*x+1/2*e)^2*d^3-6/f*
a/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^5+c^4*d-2*c^3*d^2-
2*c^2*d^3+c*d^4+d^5)/c*tan(1/2*f*x+1/2*e)^2*d^5-2/f*a/(tan(1/2*f*x+1/2*e)^2
*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*c^
4-2/3/f*a/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^5+c^4*d-2*
c^3*d^2-2*c^2*d^3+c*d^4+d^5)*d^4-4/f*a/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*
x+1/2*e)*d+c)^3/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)/c^2*tan(1/2*f*x+1
/2*e)^2*d^6+17/f*a/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^5
+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*c*tan(1/2*f*x+1/2*e)^4*d^3-6/f*a/(tan
(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^5+c^4*d-2*c^3*d^2-2*c^2*
d^3+c*d^4+d^5)/c*tan(1/2*f*x+1/2*e)^4*d^5-4/f*a/(tan(1/2*f*x+1/2*e)^2*c+2*t
an(1/2*f*x+1/2*e)*d+c)^3/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)/c^2*tan(
1/2*f*x+1/2*e)^4*d^6-12/f*a/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+
c)^3*c^3*d/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*tan(1/2*f*x+1/2*e)^3+2
4/f*a/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^2*d^2/(c^5+c^4*
d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*tan(1/2*f*x+1/2*e)^3-4/f*a/(tan(1/2*f*x+1/
2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c*d^3/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c
*d^4+d^5)*tan(1/2*f*x+1/2*e)^3-8/f*a/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+
1/2*e)*d+c)^3*d*c^3/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*tan(1/2*f*x+1
/2*e)+19/f*a/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*d^2*c^2/(c
^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*tan(1/2*f*x+1/2*e)-2/f*a/(tan(1/2*f
*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*d^5/c/(c^5+c^4*d-2*c^3*d^2-2*c^2*
d^3+c*d^4+d^5)*tan(1/2*f*x+1/2*e)-2/f*a/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^
4+d^5)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1
```

```
/2))*c*d-4/f*a/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*d*c^3/(c
^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*tan(1/2*f*x+1/2*e)^5+5/f*a/(tan(1/2
*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*d^2*c^2/(c^5+c^4*d-2*c^3*d^2-2*
c^2*d^3+c*d^4+d^5)*tan(1/2*f*x+1/2*e)^5+4/f*a/(tan(1/2*f*x+1/2*e)^2*c+2*tan
(1/2*f*x+1/2*e)*d+c)^3*d^3*c/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*tan(
1/2*f*x+1/2*e)^5-2/f*a/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*
d^5/c/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*tan(1/2*f*x+1/2*e)^5+4/f*a/
(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^5+c^4*d-2*c^3*d^2-2*
c^2*d^3+c*d^4+d^5)*c^3*tan(1/2*f*x+1/2*e)^4*d-10/f*a/(tan(1/2*f*x+1/2*e)^2*
c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^5+c^4*d-2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)*c^2
*tan(1/2*f*x+1/2*e)^4*d^2
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [**B**] time = 9.71, size = 877, normalized size = 4.57

$$\frac{a\,\mathrm{atan}\left(\frac{\left(\frac{a\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(2\,c^2-2\,c\,d+d^2\right)}{(c+d)^{7/2}\,(c-d)^{5/2}}+\frac{a\left(2\,c^2-2\,c\,d+d^2\right)\left(2\,c^5\,d+2\,c^4\,d^2-4\,c^3\,d^3-4\,c^2\,d^4+2\,c\,d^5+2\,d^6\right)}{2\,(c+d)^{7/2}\,(c-d)^{5/2}\left(c^5+c^4\,d-2\,c^3\,d^2-2\,c^2\,d^3+c\,d^4+d^5\right)}\right)\left(c^5+c^4\,d-2\,c^3\,d^2-2\,c^2\,d^3+c\,d^4+d^5\right)}{2\,a\,c^2-2\,a\,c\,d+a\,d^2}\right)\left(2\,c^2-2\,c\,d\right.}{f\,(c+d)^{7/2}\,(c-d)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c + d*sin(e + f*x))^4,x)
```

```
[Out] (a*atan((((a*c*tan(e/2 + (f*x)/2)*(2*c^2 - 2*c*d + d^2))/((c + d)^(7/2)*(c
- d)^(5/2)) + (a*(2*c^2 - 2*c*d + d^2)*(2*c*d^5 + 2*c^5*d + 2*d^6 - 4*c^2*d
^4 - 4*c^3*d^3 + 2*c^4*d^2))/(2*(c + d)^(7/2)*(c - d)^(5/2)*(c*d^4 + c^4*d
+ c^5 + d^5 - 2*c^2*d^3 - 2*c^3*d^2)))*(c*d^4 + c^4*d + c^5 + d^5 - 2*c^2*d
^3 - 2*c^3*d^2))/(2*a*c^2 + a*d^2 - 2*a*c*d))*(2*c^2 - 2*c*d + d^2))/(f*(c
+ d)^(7/2)*(c - d)^(5/2)) - ((6*a*c^4 + 2*a*d^4 - 2*a*c^2*d^2 + 3*a*c*d^3 -
 12*a*c^3*d)/(3*(c*d^4 + c^4*d + c^5 + d^5 - 2*c^2*d^3 - 2*c^3*d^2)) + (a*t
```

```
an(e/2 + (f*x)/2)^4*(6*c*d^5 - 4*c^5*d + 2*c^6 + 4*d^6 - 6*c^2*d^4 - 17*c^3
*d^3 + 10*c^4*d^2))/(c^2*(c*d^4 + c^4*d + c^5 + d^5 - 2*c^2*d^3 - 2*c^3*d^2
)) + (2*a*tan(e/2 + (f*x)/2)^2*(3*c*d^5 - 4*c^5*d + 2*c^6 + 2*d^6 - 14*c^3*
d^3 + 6*c^4*d^2))/(c^2*(c*d^4 + c^4*d + c^5 + d^5 - 2*c^2*d^3 - 2*c^3*d^2))
 + (a*d*tan(e/2 + (f*x)/2)*(4*c*d^3 - 19*c^3*d + 8*c^4 + 2*d^4))/(c*(c*d^4
+ c^4*d + c^5 + d^5 - 2*c^2*d^3 - 2*c^3*d^2)) + (a*d*tan(e/2 + (f*x)/2)^5*(
2*c*d^3 - 5*c^3*d + 4*c^4 + 2*d^4 - 4*c^2*d^2))/(c*(c*d^4 + c^4*d + c^5 + d
^5 - 2*c^2*d^3 - 2*c^3*d^2)) + (2*a*d*tan(e/2 + (f*x)/2)^3*(3*c^2 + 2*d^2)*
(3*c*d^3 - 12*c^3*d + 6*c^4 + 2*d^4 - 2*c^2*d^2))/(3*c^3*(c*d^4 + c^4*d + c
^5 + d^5 - 2*c^2*d^3 - 2*c^3*d^2)))/(f*(c^3*tan(e/2 + (f*x)/2)^6 + tan(e/2
+ (f*x)/2)^2*(12*c*d^2 + 3*c^3) + tan(e/2 + (f*x)/2)^4*(12*c*d^2 + 3*c^3) +
 tan(e/2 + (f*x)/2)^3*(12*c^2*d + 8*d^3) + c^3 + 6*c^2*d*tan(e/2 + (f*x)/2)
 + 6*c^2*d*tan(e/2 + (f*x)/2)^5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))**4,x)
```

```
[Out] Timed out
```

3.434 $\int (a + a\sin(e + fx))^2 (c + d\sin(e + fx))^4 \, dx$

Optimal. Leaf size=318

$$\frac{a^2\left(4c^2-48cd-55d^2\right)\cos(e+fx)(c+d\sin(e+fx))^3}{120df}+\frac{a^2\left(4c^3-48c^2d-123cd^2-64d^3\right)\cos(e+fx)(c+d\sin}{120df}$$

```
[Out] 1/16*a^2*(24*c^4+64*c^3*d+84*c^2*d^2+48*c*d^3+11*d^4)*x+1/60*a^2*(4*c^5-48*
c^4*d-311*c^3*d^2-448*c^2*d^3-288*c*d^4-64*d^5)*cos(f*x+e)/d/f+1/240*a^2*(8
*c^4-96*c^3*d-438*c^2*d^2-464*c*d^3-165*d^4)*cos(f*x+e)*sin(f*x+e)/f+1/120*
a^2*(4*c^3-48*c^2*d-123*c*d^2-64*d^3)*cos(f*x+e)*(c+d*sin(f*x+e))^2/d/f+1/1
20*a^2*(4*c^2-48*c*d-55*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^3/d/f+1/30*a^2*(c-
12*d)*cos(f*x+e)*(c+d*sin(f*x+e))^4/d/f-1/6*a^2*cos(f*x+e)*(c+d*sin(f*x+e))
^5/d/f
```

Rubi [A] time = 0.46, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2763, 2753, 2734}

$$\frac{a^2\left(-311c^3d^2-448c^2d^3-48c^4d+4c^5-288cd^4-64d^5\right)\cos(e+fx)}{60df}+\frac{a^2\left(4c^2-48cd-55d^2\right)\cos(e+fx)(c+d}{120df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^4,x]
```

```
[Out] (a^2*(24*c^4 + 64*c^3*d + 84*c^2*d^2 + 48*c*d^3 + 11*d^4)*x)/16 + (a^2*(4*c
^5 - 48*c^4*d - 311*c^3*d^2 - 448*c^2*d^3 - 288*c*d^4 - 64*d^5)*Cos[e + f*x
])/(60*d*f) + (a^2*(8*c^4 - 96*c^3*d - 438*c^2*d^2 - 464*c*d^3 - 165*d^4)*C
os[e + f*x]*Sin[e + f*x])/(240*f) + (a^2*(4*c^3 - 48*c^2*d - 123*c*d^2 - 64
*d^3)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(120*d*f) + (a^2*(4*c^2 - 48*c*d
 - 55*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(120*d*f) + (a^2*(c - 12*d)
*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(30*d*f) - (a^2*Cos[e + f*x]*(c + d*S
in[e + f*x])^5)/(6*d*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^4\,dx = -\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^5}{6df} + \frac{\int \left(11a^2d - a^2(c-12d)\sin(\right.}{}$$

$$= \frac{a^2(c-12d)\cos(e+fx)(c+d\sin(e+fx))^4}{30df} - \frac{a^2\cos(e+fx)(c+a}{6df}$$

$$= \frac{a^2\left(4c^2-48cd-55d^2\right)\cos(e+fx)(c+d\sin(e+fx))^3}{120df} + \frac{a^2(c-1}{}$$

$$= \frac{a^2\left(4c^3-48c^2d-123cd^2-64d^3\right)\cos(e+fx)(c+d\sin(e+fx))^2}{120df}$$

$$= \frac{1}{16}a^2\left(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4\right)x + \frac{a^2\left(4c^5-48c^4a\right.}{}$$

Mathematica [A] time = 1.40, size = 262, normalized size = 0.82

$$-\frac{a^2\cos(e+fx)\left(30\left(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4\right)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\left(10d^2\left(36c^2+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^4,x]
```

```
[Out] -1/240*(a^2*Cos[e + f*x]*(30*(24*c^4 + 64*c^3*d + 84*c^2*d^2 + 48*c*d^3 + 1
1*d^4)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(32*(1
5*c^4 + 50*c^3*d + 60*c^2*d^2 + 36*c*d^3 + 8*d^4) + 15*(8*c^4 + 64*c^3*d +
84*c^2*d^2 + 48*c*d^3 + 11*d^4)*Sin[e + f*x] + 64*d*(5*c^3 + 15*c^2*d + 9*c
*d^2 + 2*d^3)*Sin[e + f*x]^2 + 10*d^2*(36*c^2 + 48*c*d + 11*d^2)*Sin[e + f*
x]^3 + 96*d^3*(2*c + d)*Sin[e + f*x]^4 + 40*d^4*Sin[e + f*x]^5)))/(f*Sqrt[C
os[e + f*x]^2])
```

fricas [A] time = 0.47, size = 299, normalized size = 0.94

$$-\frac{96\left(2\,a^2cd^3+a^2d^4\right)\cos\left(fx+e\right)^5-320\left(a^2c^3d+3\,a^2c^2d^2+3\,a^2cd^3+a^2d^4\right)\cos\left(fx+e\right)^3-15\left(24\,a^2c^4+64\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] -1/240*(96*(2*a^2*c*d^3 + a^2*d^4)*cos(f*x + e)^5 - 320*(a^2*c^3*d + 3*a^2*
c^2*d^2 + 3*a^2*c*d^3 + a^2*d^4)*cos(f*x + e)^3 - 15*(24*a^2*c^4 + 64*a^2*c
^3*d + 84*a^2*c^2*d^2 + 48*a^2*c*d^3 + 11*a^2*d^4)*f*x + 480*(a^2*c^4 + 4*a
^2*c^3*d + 6*a^2*c^2*d^2 + 4*a^2*c*d^3 + a^2*d^4)*cos(f*x + e) + 5*(8*a^2*d
^4*cos(f*x + e)^5 - 2*(36*a^2*c^2*d^2 + 48*a^2*c*d^3 + 19*a^2*d^4)*cos(f*x
+ e)^3 + 3*(8*a^2*c^4 + 64*a^2*c^3*d + 108*a^2*c^2*d^2 + 80*a^2*c*d^3 + 21*
a^2*d^4)*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.22, size = 458, normalized size = 1.44

$$\frac{a^2cd^3\cos\left(3\,fx+3\,e\right)}{3\,f}-\frac{a^2d^4\sin\left(6\,fx+6\,e\right)}{192\,f}+\frac{a^2d^4\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{1}{16}\left(8\,a^2c^4+64\,a^2c^3d+36\,a^2c^2d^2+48\,a^2c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/3*a^2*c*d^3*cos(3*f*x + 3*e)/f - 1/192*a^2*d^4*sin(6*f*x + 6*e)/f + 1/32*
a^2*d^4*sin(4*f*x + 4*e)/f + 1/16*(8*a^2*c^4 + 64*a^2*c^3*d + 36*a^2*c^2*d^
2 + 48*a^2*c*d^3 + 5*a^2*d^4)*x + 1/8*(8*a^2*c^4 + 24*a^2*c^2*d^2 + 3*a^2*d
^4)*x - 1/40*(2*a^2*c*d^3 + a^2*d^4)*cos(5*f*x + 5*e)/f + 1/24*(8*a^2*c^3*d
 + 24*a^2*c^2*d^2 + 10*a^2*c*d^3 + 5*a^2*d^4)*cos(3*f*x + 3*e)/f - 1/4*(8*a
^2*c^4 + 12*a^2*c^3*d + 36*a^2*c^2*d^2 + 10*a^2*c*d^3 + 5*a^2*d^4)*cos(f*x
+ e)/f - (4*a^2*c^3*d + 3*a^2*c*d^3)*cos(f*x + e)/f + 1/64*(12*a^2*c^2*d^2
+ 16*a^2*c*d^3 + 3*a^2*d^4)*sin(4*f*x + 4*e)/f - 1/64*(16*a^2*c^4 + 128*a^2
*c^3*d + 96*a^2*c^2*d^2 + 128*a^2*c*d^3 + 15*a^2*d^4)*sin(2*f*x + 2*e)/f -
1/4*(6*a^2*c^2*d^2 + a^2*d^4)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.39, size = 462, normalized size = 1.45

$$a^2c^4\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{4a^2c^3d\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}+6a^2c^2d^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^4,x)
```

```
[Out] 1/f*(a^2*c^4*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-4/3*a^2*c^3*d*(2+si
n(f*x+e)^2)*cos(f*x+e)+6*a^2*c^2*d^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*co
s(f*x+e)+3/8*f*x+3/8*e)-4/5*a^2*c*d^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*c
os(f*x+e)+a^2*d^4*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos
(f*x+e)+5/16*f*x+5/16*e)-2*a^2*c^4*cos(f*x+e)+8*a^2*c^3*d*(-1/2*sin(f*x+e)*
cos(f*x+e)+1/2*f*x+1/2*e)-4*a^2*c^2*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+8*a^2*c
*d^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2/5*a^2*
d^4*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+a^2*c^4*(f*x+e)-4*a^2*c^
3*d*cos(f*x+e)+6*a^2*c^2*d^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-4/3
*a^2*c*d^3*(2+sin(f*x+e)^2)*cos(f*x+e)+a^2*d^4*(-1/4*(sin(f*x+e)^3+3/2*sin(
f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e))
```

maxima [A] time = 0.35, size = 451, normalized size = 1.42

$$240\left(2fx+2e-\sin\left(2fx+2e\right)\right)a^2c^4+960\left(fx+e\right)a^2c^4+1280\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)a^2c^3d+1920\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 1/960*(240*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c^4 + 960*(f*x + e)*a^2*c^4
 + 1280*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*c^3*d + 1920*(2*f*x + 2*e - s
in(2*f*x + 2*e))*a^2*c^3*d + 3840*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*c^2
*d^2 + 180*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^2*c^2*
d^2 + 1440*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c^2*d^2 - 256*(3*cos(f*x +
e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a^2*c*d^3 + 1280*(cos(f*x + e)^
3 - 3*cos(f*x + e))*a^2*c*d^3 + 240*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*s
in(2*f*x + 2*e))*a^2*c*d^3 - 128*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15
*cos(f*x + e))*a^2*d^4 + 5*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*
f*x + 4*e) - 48*sin(2*f*x + 2*e))*a^2*d^4 + 30*(12*f*x + 12*e + sin(4*f*x +
 4*e) - 8*sin(2*f*x + 2*e))*a^2*d^4 - 1920*a^2*c^4*cos(f*x + e) - 3840*a^2*
c^3*d*cos(f*x + e))/f
```

mupad [B] time = 9.92, size = 865, normalized size = 2.72

$$\frac{a^2\,\mathrm{atan}\left(\frac{a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(24\,c^4+64\,c^3\,d+84\,c^2\,d^2+48\,c\,d^3+11\,d^4\right)}{8\,\left(3\,a^2\,c^4+8\,a^2\,c^3\,d+\frac{21\,a^2\,c^2\,d^2}{2}+6\,a^2\,c\,d^3+\frac{11\,a^2\,d^4}{8}\right)}\right)\,\left(24\,c^4+64\,c^3\,d+84\,c^2\,d^2+48\,c\,d^3+11\,d^4\right)}{8\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^4,x)
```

```
[Out] (a^2*atan((a^2*tan(e/2 + (f*x)/2)*(48*c*d^3 + 64*c^3*d + 24*c^4 + 11*d^4 +
84*c^2*d^2))/(8*(3*a^2*c^4 + (11*a^2*d^4)/8 + 6*a^2*c*d^3 + 8*a^2*c^3*d + (
21*a^2*c^2*d^2)/2)))*(48*c*d^3 + 64*c^3*d + 24*c^4 + 11*d^4 + 84*c^2*d^2))/
(8*f) - (tan(e/2 + (f*x)/2)^8*(20*a^2*c^4 + 16*a^2*c*d^3 + 56*a^2*c^3*d + 4
8*a^2*c^2*d^2) + tan(e/2 + (f*x)/2)^10*(4*a^2*c^4 + 8*a^2*c^3*d) + tan(e/2
+ (f*x)/2)*(a^2*c^4 + (11*a^2*d^4)/8 + 6*a^2*c*d^3 + 8*a^2*c^3*d + (21*a^2*
c^2*d^2)/2) + 4*a^2*c^4 + (32*a^2*d^4)/15 - tan(e/2 + (f*x)/2)^11*(a^2*c^4
+ (11*a^2*d^4)/8 + 6*a^2*c*d^3 + 8*a^2*c^3*d + (21*a^2*c^2*d^2)/2) + tan(e/
2 + (f*x)/2)^5*(2*a^2*c^4 + (47*a^2*d^4)/4 + 28*a^2*c*d^3 + 16*a^2*c^3*d +
33*a^2*c^2*d^2) - tan(e/2 + (f*x)/2)^7*(2*a^2*c^4 + (47*a^2*d^4)/4 + 28*a^2
*c*d^3 + 16*a^2*c^3*d + 33*a^2*c^2*d^2) + tan(e/2 + (f*x)/2)^3*(3*a^2*c^4 +
 (187*a^2*d^4)/24 + 34*a^2*c*d^3 + 24*a^2*c^3*d + (87*a^2*c^2*d^2)/2) - tan
(e/2 + (f*x)/2)^9*(3*a^2*c^4 + (187*a^2*d^4)/24 + 34*a^2*c*d^3 + 24*a^2*c^3
*d + (87*a^2*c^2*d^2)/2) + tan(e/2 + (f*x)/2)^4*(40*a^2*c^4 + 32*a^2*d^4 +
128*a^2*c*d^3 + 144*a^2*c^3*d + 192*a^2*c^2*d^2) + tan(e/2 + (f*x)/2)^2*(20
*a^2*c^4 + (64*a^2*d^4)/5 + (288*a^2*c*d^3)/5 + 72*a^2*c^3*d + 96*a^2*c^2*d
^2) + tan(e/2 + (f*x)/2)^6*(40*a^2*c^4 + (64*a^2*d^4)/3 + 96*a^2*c*d^3 + (4
00*a^2*c^3*d)/3 + 160*a^2*c^2*d^2) + (48*a^2*c*d^3)/5 + (40*a^2*c^3*d)/3 +
16*a^2*c^2*d^2)/(f*(6*tan(e/2 + (f*x)/2)^2 + 15*tan(e/2 + (f*x)/2)^4 + 20*t
an(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 + 6*tan(e/2 + (f*x)/2)^10 + t
an(e/2 + (f*x)/2)^12 + 1))
```

sympy [A] time = 9.42, size = 1136, normalized size = 3.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((a**2*c**4*x*sin(e + f*x)**2/2 + a**2*c**4*x*cos(e + f*x)**2/2 +
a**2*c**4*x - a**2*c**4*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**2*c**4*cos(e
 + f*x)/f + 4*a**2*c**3*d*x*sin(e + f*x)**2 + 4*a**2*c**3*d*x*cos(e + f*x)*
```

```
*2 - 4*a**2*c**3*d*sin(e + f*x)**2*cos(e + f*x)/f - 4*a**2*c**3*d*sin(e + f
*x)*cos(e + f*x)/f - 8*a**2*c**3*d*cos(e + f*x)**3/(3*f) - 4*a**2*c**3*d*co
s(e + f*x)/f + 9*a**2*c**2*d**2*x*sin(e + f*x)**4/4 + 9*a**2*c**2*d**2*x*si
n(e + f*x)**2*cos(e + f*x)**2/2 + 3*a**2*c**2*d**2*x*sin(e + f*x)**2 + 9*a*
*2*c**2*d**2*x*cos(e + f*x)**4/4 + 3*a**2*c**2*d**2*x*cos(e + f*x)**2 - 15*
a**2*c**2*d**2*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 12*a**2*c**2*d**2*sin(e
 + f*x)**2*cos(e + f*x)/f - 9*a**2*c**2*d**2*sin(e + f*x)*cos(e + f*x)**3/(
4*f) - 3*a**2*c**2*d**2*sin(e + f*x)*cos(e + f*x)/f - 8*a**2*c**2*d**2*cos(
e + f*x)**3/f + 3*a**2*c*d**3*x*sin(e + f*x)**4 + 6*a**2*c*d**3*x*sin(e + f
*x)**2*cos(e + f*x)**2 + 3*a**2*c*d**3*x*cos(e + f*x)**4 - 4*a**2*c*d**3*si
n(e + f*x)**4*cos(e + f*x)/f - 5*a**2*c*d**3*sin(e + f*x)**3*cos(e + f*x)/f
 - 16*a**2*c*d**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 4*a**2*c*d**3*sin
(e + f*x)**2*cos(e + f*x)/f - 3*a**2*c*d**3*sin(e + f*x)*cos(e + f*x)**3/f
- 32*a**2*c*d**3*cos(e + f*x)**5/(15*f) - 8*a**2*c*d**3*cos(e + f*x)**3/(3*
f) + 5*a**2*d**4*x*sin(e + f*x)**6/16 + 15*a**2*d**4*x*sin(e + f*x)**4*cos(
e + f*x)**2/16 + 3*a**2*d**4*x*sin(e + f*x)**4/8 + 15*a**2*d**4*x*sin(e + f
*x)**2*cos(e + f*x)**4/16 + 3*a**2*d**4*x*sin(e + f*x)**2*cos(e + f*x)**2/4
 + 5*a**2*d**4*x*cos(e + f*x)**6/16 + 3*a**2*d**4*x*cos(e + f*x)**4/8 - 11*
a**2*d**4*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 2*a**2*d**4*sin(e + f*x)**4
*cos(e + f*x)/f - 5*a**2*d**4*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 5*a**
2*d**4*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 8*a**2*d**4*sin(e + f*x)**2*cos
(e + f*x)**3/(3*f) - 5*a**2*d**4*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 3*a*
*2*d**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 16*a**2*d**4*cos(e + f*x)**5/(
15*f), Ne(f, 0)), (x*(c + d*sin(e))**4*(a*sin(e) + a)**2, True))
```

3.435 $\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=233

$$\frac{a^2\left(c^2-10cd-12d^2\right)\cos(e+fx)(c+d\sin(e+fx))^2}{20df}+\frac{3}{8}a^2x(2c+d)\left(2c^2+3cd+2d^2\right)+\frac{a^2\left(2c^3-20c^2d-57cd^2\right.}{}$$

```
[Out] 3/8*a^2*(2*c+d)*(2*c^2+3*c*d+2*d^2)*x+1/10*a^2*(c^4-10*c^3*d-44*c^2*d^2-40*
c*d^3-12*d^4)*cos(f*x+e)/d/f+1/40*a^2*(2*c^3-20*c^2*d-57*c*d^2-30*d^3)*cos(
f*x+e)*sin(f*x+e)/f+1/20*a^2*(c^2-10*c*d-12*d^2)*cos(f*x+e)*(c+d*sin(f*x+e)
)^2/d/f+1/20*a^2*(c-10*d)*cos(f*x+e)*(c+d*sin(f*x+e))^3/d/f-1/5*a^2*cos(f*x
+e)*(c+d*sin(f*x+e))^4/d/f
```

Rubi [A] time = 0.31, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2763, 2753, 2734}

$$\frac{a^2\left(-44c^2d^2-10c^3d+c^4-40cd^3-12d^4\right)\cos(e+fx)}{10df}+\frac{a^2\left(c^2-10cd-12d^2\right)\cos(e+fx)(c+d\sin(e+fx))^2}{20df}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (3*a^2*(2*c + d)*(2*c^2 + 3*c*d + 2*d^2)*x)/8 + (a^2*(c^4 - 10*c^3*d - 44*c
^2*d^2 - 40*c*d^3 - 12*d^4)*Cos[e + f*x])/(10*d*f) + (a^2*(2*c^3 - 20*c^2*d
 - 57*c*d^2 - 30*d^3)*Cos[e + f*x]*Sin[e + f*x])/(40*f) + (a^2*(c^2 - 10*c*
d - 12*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(20*d*f) + (a^2*(c - 10*d)
*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(20*d*f) - (a^2*Cos[e + f*x]*(c + d*S
in[e + f*x])^4)/(5*d*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

```
&& IntegerQ[2*m]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^3\,dx = -\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^4}{5df} + \frac{\int \left(9a^2d-a^2(c-10d)\sin(e\right.}{}$$

$$= \frac{a^2(c-10d)\cos(e+fx)(c+d\sin(e+fx))^3}{20df} - \frac{a^2\cos(e+fx)(c+a}{5df}$$

$$= \frac{a^2\left(c^2-10cd-12d^2\right)\cos(e+fx)(c+d\sin(e+fx))^2}{20df} + \frac{a^2(c-10}{}$$

$$= \frac{3}{8}a^2(2c+d)\left(2c^2+3cd+2d^2\right)x + \frac{a^2\left(c^4-10c^3d-44c^2d^2-40cd\right.}{10df}$$

Mathematica [A] time = 0.94, size = 204, normalized size = 0.88

$$-\frac{a^2\cos(e+fx)\left(30\left(4c^3+8c^2d+7cd^2+2d^3\right)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\left(8d\left(5c^2+10cd+3d^2\right)\sin^2(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] -1/40*(a^2*Cos[e + f*x]*(30*(4*c^3 + 8*c^2*d + 7*c*d^2 + 2*d^3)*ArcSin[Sqrt
[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(8*(10*c^3 + 25*c^2*d +
20*c*d^2 + 6*d^3) + 5*(4*c^3 + 24*c^2*d + 21*c*d^2 + 6*d^3)*Sin[e + f*x] +
8*d*(5*c^2 + 10*c*d + 3*d^2)*Sin[e + f*x]^2 + 10*d^2*(3*c + 2*d)*Sin[e + f*
x]^3 + 8*d^3*Sin[e + f*x]^4)))/(f*Sqrt[Cos[e + f*x]^2])
```

fricas [A] time = 0.46, size = 217, normalized size = 0.93

$$-\frac{8\,a^2d^3\cos\left(fx+e\right)^5-40\left(a^2c^2d+2\,a^2cd^2+a^2d^3\right)\cos\left(fx+e\right)^3-15\left(4\,a^2c^3+8\,a^2c^2d+7\,a^2cd^2+2\,a^2d^3\right)fx+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/40*(8*a^2*d^3*cos(f*x + e)^5 - 40*(a^2*c^2*d + 2*a^2*c*d^2 + a^2*d^3)*co
s(f*x + e)^3 - 15*(4*a^2*c^3 + 8*a^2*c^2*d + 7*a^2*c*d^2 + 2*a^2*d^3)*f*x +
 80*(a^2*c^3 + 3*a^2*c^2*d + 3*a^2*c*d^2 + a^2*d^3)*cos(f*x + e) - 5*(2*(3*
a^2*c*d^2 + 2*a^2*d^3)*cos(f*x + e)^3 - (4*a^2*c^3 + 24*a^2*c^2*d + 27*a^2*
c*d^2 + 10*a^2*d^3)*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.21, size = 324, normalized size = 1.39

$$-\frac{a^2d^3\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{a^2d^3\cos\left(3\,fx+3\,e\right)}{12\,f}-\frac{3\,a^2cd^2\sin\left(2\,fx+2\,e\right)}{4\,f}+\frac{1}{8}\left(4\,a^2c^3+24\,a^2c^2d+9\,a^2cd^2+6\,a^2d^3\right)\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/80*a^2*d^3*cos(5*f*x + 5*e)/f + 1/12*a^2*d^3*cos(3*f*x + 3*e)/f - 3/4*a^
2*c*d^2*sin(2*f*x + 2*e)/f + 1/8*(4*a^2*c^3 + 24*a^2*c^2*d + 9*a^2*c*d^2 +
6*a^2*d^3)*x + 1/2*(2*a^2*c^3 + 3*a^2*c*d^2)*x + 1/48*(12*a^2*c^2*d + 24*a^
2*c*d^2 + 5*a^2*d^3)*cos(3*f*x + 3*e)/f - 1/8*(16*a^2*c^3 + 18*a^2*c^2*d +
36*a^2*c*d^2 + 5*a^2*d^3)*cos(f*x + e)/f - 3/4*(4*a^2*c^2*d + a^2*d^3)*cos(
f*x + e)/f + 1/32*(3*a^2*c*d^2 + 2*a^2*d^3)*sin(4*f*x + 4*e)/f - 1/4*(a^2*c
^3 + 6*a^2*c^2*d + 3*a^2*c*d^2 + 2*a^2*d^3)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.33, size = 329, normalized size = 1.41

$$\frac{a^2c^3\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-a^2c^2d\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)+3a^2c\,d^2\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(\cdots\right)}{4}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(a^2*c^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-a^2*c^2*d*(2+sin(f*
x+e)^2)*cos(f*x+e)+3*a^2*c*d^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+
```

```
e)+3/8*f*x+3/8*e)-1/5*a^2*d^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e
)-2*a^2*c^3*cos(f*x+e)+6*a^2*c^2*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*
e)-2*a^2*c*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+2*a^2*d^3*(-1/4*(sin(f*x+e)^3+3/
2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+a^2*c^3*(f*x+e)-3*a^2*c^2*d*cos(f*x
+e)+3*a^2*c*d^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/3*a^2*d^3*(2+s
in(f*x+e)^2)*cos(f*x+e))
```

maxima [A] time = 0.33, size = 318, normalized size = 1.36

$$\frac{120\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)a^2c^3+480\left(fx+e\right)a^2c^3+480\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)a^2c^2d+720\left(2\,f\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/480*(120*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c^3 + 480*(f*x + e)*a^2*c^3
 + 480*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*c^2*d + 720*(2*f*x + 2*e - sin
(2*f*x + 2*e))*a^2*c^2*d + 960*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*c*d^2
+ 45*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^2*c*d^2 + 36
0*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c*d^2 - 32*(3*cos(f*x + e)^5 - 10*co
s(f*x + e)^3 + 15*cos(f*x + e))*a^2*d^3 + 160*(cos(f*x + e)^3 - 3*cos(f*x +
 e))*a^2*d^3 + 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a
^2*d^3 - 960*a^2*c^3*cos(f*x + e) - 1440*a^2*c^2*d*cos(f*x + e))/f
```

mupad [B] time = 8.42, size = 611, normalized size = 2.62

$$\frac{3\,a^2\,\mathrm{atan}\left(\frac{3\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,c+d\right)\,\left(2\,c^2+3\,c\,d+2\,d^2\right)}{4\,\left(3\,a^2\,c^3+6\,a^2\,c^2\,d+\frac{21\,a^2\,c\,d^2}{4}+\frac{3\,a^2\,d^3}{2}\right)}\right)\,\left(2\,c+d\right)\,\left(2\,c^2+3\,c\,d+2\,d^2\right)}{4\,f}-\frac{3\,a^2\,\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{f\,x}{2}\right)\,\left(4\,c^3\right.}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^3,x)
```

```
[Out] (3*a^2*atan((3*a^2*tan(e/2 + (f*x)/2)*(2*c + d)*(3*c*d + 2*c^2 + 2*d^2))/(4
*(3*a^2*c^3 + (3*a^2*d^3)/2 + (21*a^2*c*d^2)/4 + 6*a^2*c^2*d)))*(2*c + d)*(
3*c*d + 2*c^2 + 2*d^2))/(4*f) - (3*a^2*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2)
*(7*c*d^2 + 8*c^2*d + 4*c^3 + 2*d^3))/(4*f) - (tan(e/2 + (f*x)/2)^8*(4*a^2*
c^3 + 6*a^2*c^2*d) - tan(e/2 + (f*x)/2)^9*(a^2*c^3 + (3*a^2*d^3)/2 + (21*a^
2*c*d^2)/4 + 6*a^2*c^2*d) + tan(e/2 + (f*x)/2)^3*(2*a^2*c^3 + 7*a^2*d^3 + (
33*a^2*c*d^2)/2 + 12*a^2*c^2*d) - tan(e/2 + (f*x)/2)^7*(2*a^2*c^3 + 7*a^2*d
^3 + (33*a^2*c*d^2)/2 + 12*a^2*c^2*d) + tan(e/2 + (f*x)/2)^6*(16*a^2*c^3 +
```

```
4*a^2*d^3 + 24*a^2*c*d^2 + 36*a^2*c^2*d) + tan(e/2 + (f*x)/2)^2*(16*a^2*c^3
 + 12*a^2*d^3 + 40*a^2*c*d^2 + 44*a^2*c^2*d) + tan(e/2 + (f*x)/2)^4*(24*a^2
*c^3 + 20*a^2*d^3 + 56*a^2*c*d^2 + 64*a^2*c^2*d) + 4*a^2*c^3 + (12*a^2*d^3)
/5 + tan(e/2 + (f*x)/2)*(a^2*c^3 + (3*a^2*d^3)/2 + (21*a^2*c*d^2)/4 + 6*a^2
*c^2*d) + 8*a^2*c*d^2 + 10*a^2*c^2*d)/(f*(5*tan(e/2 + (f*x)/2)^2 + 10*tan(e
/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 + 5*tan(e/2 + (f*x)/2)^8 + tan(e/
2 + (f*x)/2)^10 + 1))
```

sympy [A] time = 4.89, size = 729, normalized size = 3.13

$$\begin{cases} \frac{a^2c^3x\sin^2\left(e+fx\right)}{2}+\frac{a^2c^3x\cos^2\left(e+fx\right)}{2}+a^2c^3x-\frac{a^2c^3\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{2a^2c^3\cos\left(e+fx\right)}{f}+3a^2c^2dx\sin^2\left(e+fx\right)+3a^2c^2 \\ x\left(c+d\sin\left(e\right)\right)^3\left(a\sin\left(e\right)+a\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((a**2*c**3*x*sin(e + f*x)**2/2 + a**2*c**3*x*cos(e + f*x)**2/2 +
a**2*c**3*x - a**2*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**2*c**3*cos(e
 + f*x)/f + 3*a**2*c**2*d*x*sin(e + f*x)**2 + 3*a**2*c**2*d*x*cos(e + f*x)*
*2 - 3*a**2*c**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 3*a**2*c**2*d*sin(e + f
*x)*cos(e + f*x)/f - 2*a**2*c**2*d*cos(e + f*x)**3/f - 3*a**2*c**2*d*cos(e
+ f*x)/f + 9*a**2*c*d**2*x*sin(e + f*x)**4/8 + 9*a**2*c*d**2*x*sin(e + f*x)
**2*cos(e + f*x)**2/4 + 3*a**2*c*d**2*x*sin(e + f*x)**2/2 + 9*a**2*c*d**2*x
*cos(e + f*x)**4/8 + 3*a**2*c*d**2*x*cos(e + f*x)**2/2 - 15*a**2*c*d**2*sin
(e + f*x)**3*cos(e + f*x)/(8*f) - 6*a**2*c*d**2*sin(e + f*x)**2*cos(e + f*x
)/f - 9*a**2*c*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 3*a**2*c*d**2*sin(
e + f*x)*cos(e + f*x)/(2*f) - 4*a**2*c*d**2*cos(e + f*x)**3/f + 3*a**2*d**3
*x*sin(e + f*x)**4/4 + 3*a**2*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + 3*
a**2*d**3*x*cos(e + f*x)**4/4 - a**2*d**3*sin(e + f*x)**4*cos(e + f*x)/f -
5*a**2*d**3*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 4*a**2*d**3*sin(e + f*x)**
2*cos(e + f*x)**3/(3*f) - a**2*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*a**2
*d**3*sin(e + f*x)*cos(e + f*x)**3/(4*f) - 8*a**2*d**3*cos(e + f*x)**5/(15*
f) - 2*a**2*d**3*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(c + d*sin(e))**3*(a*
sin(e) + a)**2, True))
```

3.436 $\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=156

$$-\frac{a^2\left(12c^2+16cd+7d^2\right)\cos(e+fx)}{6f}-\frac{a^2\left(12c^2+16cd+7d^2\right)\sin(e+fx)\cos(e+fx)}{24f}+\frac{1}{8}a^2x\left(12c^2+16cd+7d^2\right)$$

```
[Out] 1/8*a^2*(12*c^2+16*c*d+7*d^2)*x-1/6*a^2*(12*c^2+16*c*d+7*d^2)*cos(f*x+e)/f-
1/24*a^2*(12*c^2+16*c*d+7*d^2)*cos(f*x+e)*sin(f*x+e)/f-1/12*(8*c-d)*d*cos(f
*x+e)*(a+a*sin(f*x+e))^2/f-1/4*d^2*cos(f*x+e)*(a+a*sin(f*x+e))^3/a/f
```

Rubi [A] time = 0.20, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2761, 2751, 2644}

$$-\frac{a^2\left(12c^2+16cd+7d^2\right)\cos(e+fx)}{6f}-\frac{a^2\left(12c^2+16cd+7d^2\right)\sin(e+fx)\cos(e+fx)}{24f}+\frac{1}{8}a^2x\left(12c^2+16cd+7d^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*(12*c^2 + 16*c*d + 7*d^2)*x)/8 - (a^2*(12*c^2 + 16*c*d + 7*d^2)*Cos[e
+ f*x])/(6*f) - (a^2*(12*c^2 + 16*c*d + 7*d^2)*Cos[e + f*x]*Sin[e + f*x])/(
24*f) - ((8*c - d)*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^2)/(12*f) - (d^2*Cos
[e + f*x]*(a + a*Sin[e + f*x])^3)/(4*a*f)
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
```

```
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^2\,dx = -\frac{d^2\cos(e+fx)(a+a\sin(e+fx))^3}{4af}+\frac{\int (a+a\sin(e+fx))^2\left(a\right.}{}$$

$$= -\frac{(8c-d)d\cos(e+fx)(a+a\sin(e+fx))^2}{12f}-\frac{d^2\cos(e+fx)(a+}{4af}$$

$$= \frac{1}{8}a^2\left(12c^2+16cd+7d^2\right)x-\frac{a^2\left(12c^2+16cd+7d^2\right)\cos(e+fx)}{6f}$$

Mathematica [A] time = 0.54, size = 148, normalized size = 0.95

$$-\frac{a^2\cos(e+fx)\left(6\left(12c^2+16cd+7d^2\right)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\left(3\left(4c^2+16cd+7d^2\right)\sin(e+fx)\right.\right.}{24f\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -1/24*(a^2*Cos[e + f*x]*(6*(12*c^2 + 16*c*d + 7*d^2)*ArcSin[Sqrt[1 - Sin[e
+ f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(16*(3*c^2 + 5*c*d + 2*d^2) + 3*(4*
c^2 + 16*c*d + 7*d^2)*Sin[e + f*x] + 16*d*(c + d)*Sin[e + f*x]^2 + 6*d^2*Si
n[e + f*x]^3)))/(f*Sqrt[Cos[e + f*x]^2])
```

fricas [A] time = 0.45, size = 145, normalized size = 0.93

$$\frac{16\left(a^2cd+a^2d^2\right)\cos\left(fx+e\right)^3+3\left(12\,a^2c^2+16\,a^2cd+7\,a^2d^2\right)fx-48\left(a^2c^2+2\,a^2cd+a^2d^2\right)\cos\left(fx+e\right)+3\left(\right.}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(16*(a^2*c*d + a^2*d^2)*cos(f*x + e)^3 + 3*(12*a^2*c^2 + 16*a^2*c*d +
7*a^2*d^2)*f*x - 48*(a^2*c^2 + 2*a^2*c*d + a^2*d^2)*cos(f*x + e) + 3*(2*a^2
```

```
*d^2*cos(f*x + e)^3 - (4*a^2*c^2 + 16*a^2*c*d + 9*a^2*d^2)*cos(f*x + e))*si
n(f*x + e))/f
```

giac [A] time = 0.35, size = 208, normalized size = 1.33

$$-\frac{2\,a^2cd\cos\left(fx+e\right)}{f}+\frac{a^2d^2\sin\left(4\,fx+4\,e\right)}{32\,f}-\frac{a^2d^2\sin\left(2\,fx+2\,e\right)}{4\,f}+\frac{1}{8}\left(4\,a^2c^2+16\,a^2cd+3\,a^2d^2\right)x+\frac{1}{2}\left(2\,a^2c^2+a^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -2*a^2*c*d*cos(f*x + e)/f + 1/32*a^2*d^2*sin(4*f*x + 4*e)/f - 1/4*a^2*d^2*s
in(2*f*x + 2*e)/f + 1/8*(4*a^2*c^2 + 16*a^2*c*d + 3*a^2*d^2)*x + 1/2*(2*a^2
*c^2 + a^2*d^2)*x + 1/6*(a^2*c*d + a^2*d^2)*cos(3*f*x + 3*e)/f - 1/2*(4*a^2
*c^2 + 3*a^2*c*d + 3*a^2*d^2)*cos(f*x + e)/f - 1/4*(a^2*c^2 + 4*a^2*c*d + a
^2*d^2)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.26, size = 219, normalized size = 1.40

$$\frac{a^2c^2\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{2a^2cd\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}+a^2d^2\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(a^2*c^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2/3*a^2*c*d*(2+sin(
f*x+e)^2)*cos(f*x+e)+a^2*d^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)
+3/8*f*x+3/8*e)-2*a^2*c^2*cos(f*x+e)+4*a^2*c*d*(-1/2*sin(f*x+e)*cos(f*x+e)+
1/2*f*x+1/2*e)-2/3*a^2*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+a^2*c^2*(f*x+e)-2*a^
2*c*d*cos(f*x+e)+a^2*d^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e))
```

maxima [A] time = 0.33, size = 211, normalized size = 1.35

$$\frac{24\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)a^2c^2+96\left(fx+e\right)a^2c^2+64\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)a^2cd+96\left(2\,fx+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/96*(24*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c^2 + 96*(f*x + e)*a^2*c^2 +
64*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*c*d + 96*(2*f*x + 2*e - sin(2*f*x
```

```
+ 2*e))*a^2*c*d + 64*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^2*d^2 + 3*(12*f*x
+ 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^2*d^2 + 24*(2*f*x + 2*e -
 sin(2*f*x + 2*e))*a^2*d^2 - 192*a^2*c^2*cos(f*x + e) - 192*a^2*c*d*cos(f*x
 + e))/f
```

mupad [B] time = 8.34, size = 440, normalized size = 2.82

$$\frac{a^2\,\mathrm{atan}\left(\frac{a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(12\,c^2+16\,c\,d+7\,d^2\right)}{4\,\left(3\,a^2\,c^2+4\,a^2\,c\,d+\frac{7\,a^2\,d^2}{4}\right)}\right)\,\left(12\,c^2+16\,c\,d+7\,d^2\right)}{4\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(a^2\,c^2+4\,a^2\,c\,d+\frac{7\,a^2\,d^2}{4}\right)-\tan\left(\frac{e}{2}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^2,x)
```

```
[Out] (a^2*atan((a^2*tan(e/2 + (f*x)/2)*(16*c*d + 12*c^2 + 7*d^2))/(4*(3*a^2*c^2
+ (7*a^2*d^2)/4 + 4*a^2*c*d)))*(16*c*d + 12*c^2 + 7*d^2))/(4*f) - (tan(e/2
+ (f*x)/2)*(a^2*c^2 + (7*a^2*d^2)/4 + 4*a^2*c*d) - tan(e/2 + (f*x)/2)^7*(a^
2*c^2 + (7*a^2*d^2)/4 + 4*a^2*c*d) + tan(e/2 + (f*x)/2)^3*(a^2*c^2 + (15*a^
2*d^2)/4 + 4*a^2*c*d) - tan(e/2 + (f*x)/2)^5*(a^2*c^2 + (15*a^2*d^2)/4 + 4*
a^2*c*d) + tan(e/2 + (f*x)/2)^4*(12*a^2*c^2 + 8*a^2*d^2 + 20*a^2*c*d) + tan
(e/2 + (f*x)/2)^2*(12*a^2*c^2 + (32*a^2*d^2)/3 + (68*a^2*c*d)/3) + tan(e/2
+ (f*x)/2)^6*(4*a^2*c^2 + 4*a^2*c*d) + 4*a^2*c^2 + (8*a^2*d^2)/3 + (20*a^2*
c*d)/3)/(f*(4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*tan(e/2 + (
f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1)) - (a^2*(atan(tan(e/2 + (f*x)/2)) - (
f*x)/2)*(16*c*d + 12*c^2 + 7*d^2))/(4*f)
```

sympy [A] time = 2.36, size = 459, normalized size = 2.94

$$\begin{cases} \frac{a^2c^2x\sin^2\left(e+fx\right)}{2}+\frac{a^2c^2x\cos^2\left(e+fx\right)}{2}+a^2c^2x-\frac{a^2c^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{2a^2c^2\cos\left(e+fx\right)}{f}+2a^2cdx\sin^2\left(e+fx\right)+2a^2cd \\ x\left(c+d\sin\left(e\right)\right)^2\left(a\sin\left(e\right)+a\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c**2*x*sin(e + f*x)**2/2 + a**2*c**2*x*cos(e + f*x)**2/2 +
a**2*c**2*x - a**2*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**2*c**2*cos(e
 + f*x)/f + 2*a**2*c*d*x*sin(e + f*x)**2 + 2*a**2*c*d*x*cos(e + f*x)**2 - 2
*a**2*c*d*sin(e + f*x)**2*cos(e + f*x)/f - 2*a**2*c*d*sin(e + f*x)*cos(e +
f*x)/f - 4*a**2*c*d*cos(e + f*x)**3/(3*f) - 2*a**2*c*d*cos(e + f*x)/f + 3*a
```

```
**2*d**2*x*sin(e + f*x)**4/8 + 3*a**2*d**2*x*sin(e + f*x)**2*cos(e + f*x)**
2/4 + a**2*d**2*x*sin(e + f*x)**2/2 + 3*a**2*d**2*x*cos(e + f*x)**4/8 + a**
2*d**2*x*cos(e + f*x)**2/2 - 5*a**2*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f)
 - 2*a**2*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*a**2*d**2*sin(e + f*x)*co
s(e + f*x)**3/(8*f) - a**2*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 4*a**2*d*
*2*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(c + d*sin(e))**2*(a*sin(e) + a)**2
, True))
```

3.437 $\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=94

$$-\frac{2a^2(3c+2d)\cos(e+fx)}{3f}-\frac{a^2(3c+2d)\sin(e+fx)\cos(e+fx)}{6f}+\frac{1}{2}a^2x(3c+2d)-\frac{d\cos(e+fx)(a\sin(e+fx)+a}{3f}$$

```
[Out] 1/2*a^2*(3*c+2*d)*x-2/3*a^2*(3*c+2*d)*cos(f*x+e)/f-1/6*a^2*(3*c+2*d)*cos(f*
x+e)*sin(f*x+e)/f-1/3*d*cos(f*x+e)*(a+a*sin(f*x+e))^2/f
```

Rubi [A] time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2751, 2644}

$$-\frac{2a^2(3c+2d)\cos(e+fx)}{3f}-\frac{a^2(3c+2d)\sin(e+fx)\cos(e+fx)}{6f}+\frac{1}{2}a^2x(3c+2d)-\frac{d\cos(e+fx)(a\sin(e+fx)+a}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x]),x]
```

```
[Out] (a^2*(3*c + 2*d)*x)/2 - (2*a^2*(3*c + 2*d)*Cos[e + f*x])/(3*f) - (a^2*(3*c
+ 2*d)*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (d*Cos[e + f*x]*(a + a*Sin[e + f*
x])^2)/(3*f)
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))\,dx = -\frac{d\cos(e+fx)(a+a\sin(e+fx))^2}{3f} + \frac{1}{3}(3c+2d)\int (a+a\sin(e+f$$

$$= \frac{1}{2}a^2(3c+2d)x - \frac{2a^2(3c+2d)\cos(e+fx)}{3f} - \frac{a^2(3c+2d)\cos(e+f}{6f}$$

Mathematica [A] time = 0.33, size = 106, normalized size = 1.13

$$-\frac{a^2\cos(e+fx)\left(6(3c+2d)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\left(3(c+2d)\sin(e+fx)+2(6c+5d)+2d\sin^2(e\right.\right.}{6f\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x]),x]
```

```
[Out] -1/6*(a^2*Cos[e + f*x]*(6*(3*c + 2*d)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]
] + Sqrt[Cos[e + f*x]^2]*(2*(6*c + 5*d) + 3*(c + 2*d)*Sin[e + f*x] + 2*d*Si
n[e + f*x]^2)))/(f*Sqrt[Cos[e + f*x]^2])
```

fricas [A] time = 0.44, size = 82, normalized size = 0.87

$$\frac{2\,a^2d\cos\left(fx+e\right)^3+3\left(3\,a^2c+2\,a^2d\right)fx-3\left(a^2c+2\,a^2d\right)\cos\left(fx+e\right)\sin\left(fx+e\right)-12\left(a^2c+a^2d\right)\cos\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^2*d*cos(f*x + e)^3 + 3*(3*a^2*c + 2*a^2*d)*f*x - 3*(a^2*c + 2*a^2*
d)*cos(f*x + e)*sin(f*x + e) - 12*(a^2*c + a^2*d)*cos(f*x + e))/f
```

giac [A] time = 0.17, size = 109, normalized size = 1.16

$$a^2cx+\frac{a^2d\cos\left(3\,fx+3\,e\right)}{12\,f}-\frac{a^2d\cos\left(fx+e\right)}{f}+\frac{1}{2}\left(a^2c+2\,a^2d\right)x-\frac{\left(8\,a^2c+3\,a^2d\right)\cos\left(fx+e\right)}{4\,f}-\frac{\left(a^2c+2\,a^2d\right)\sin\left(2\right.}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] a^2*c*x + 1/12*a^2*d*cos(3*f*x + 3*e)/f - a^2*d*cos(f*x + e)/f + 1/2*(a^2*c
 + 2*a^2*d)*x - 1/4*(8*a^2*c + 3*a^2*d)*cos(f*x + e)/f - 1/4*(a^2*c + 2*a^2
*d)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.19, size = 117, normalized size = 1.24

$$\frac{a^2c\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{a^2d\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}-2a^2c\cos\left(fx+e\right)+2a^2d\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e)),x)
```

```
[Out] 1/f*(a^2*c*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/3*a^2*d*(2+sin(f*x+
e)^2)*cos(f*x+e)-2*a^2*c*cos(f*x+e)+2*a^2*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2
*f*x+1/2*e)+a^2*c*(f*x+e)-a^2*d*cos(f*x+e))
```

maxima [A] time = 0.31, size = 114, normalized size = 1.21

$$\frac{3\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)a^2c+12\left(fx+e\right)a^2c+4\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)a^2d+6\left(2\,fx+2\,e-\mathrm{s}\right.}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c + 12*(f*x + e)*a^2*c + 4*(co
s(f*x + e)^3 - 3*cos(f*x + e))*a^2*d + 6*(2*f*x + 2*e - sin(2*f*x + 2*e))*a
^2*d - 24*a^2*c*cos(f*x + e) - 12*a^2*d*cos(f*x + e))/f
```

mupad [B] time = 6.92, size = 91, normalized size = 0.97

$$-\frac{\frac{3\,a^2\,c\,\sin\left(2\,e+2\,f\,x\right)}{2}-\frac{a^2\,d\,\cos\left(3\,e+3\,f\,x\right)}{2}+3\,a^2\,d\,\sin\left(2\,e+2\,f\,x\right)+12\,a^2\,c\,\cos\left(e+f\,x\right)+\frac{21\,a^2\,d\,\cos\left(e+f\,x\right)}{2}-9\,a^2\,c\,f\,x}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x)),x)
```

```
[Out] -((3*a^2*c*sin(2*e + 2*f*x))/2 - (a^2*d*cos(3*e + 3*f*x))/2 + 3*a^2*d*sin(2
*e + 2*f*x) + 12*a^2*c*cos(e + f*x) + (21*a^2*d*cos(e + f*x))/2 - 9*a^2*c*f
*x - 6*a^2*d*f*x)/(6*f)
```

sympy [A] time = 0.98, size = 199, normalized size = 2.12

$$\begin{cases}\frac{a^2cx\sin^2\left(e+fx\right)}{2}+\frac{a^2cx\cos^2\left(e+fx\right)}{2}+a^2cx-\frac{a^2c\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{2a^2c\cos\left(e+fx\right)}{f}+a^2dx\sin^2\left(e+fx\right)+a^2dx\cos^2\left(e\right.\\ x\left(c+d\sin\left(e\right)\right)\left(a\sin\left(e\right)+a\right)^2\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((a**2*c*x*sin(e + f*x)**2/2 + a**2*c*x*cos(e + f*x)**2/2 + a**2*c
*x - a**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**2*c*cos(e + f*x)/f + a**
2*d*x*sin(e + f*x)**2 + a**2*d*x*cos(e + f*x)**2 - a**2*d*sin(e + f*x)**2*c
os(e + f*x)/f - a**2*d*sin(e + f*x)*cos(e + f*x)/f - 2*a**2*d*cos(e + f*x)*
*3/(3*f) - a**2*d*cos(e + f*x)/f, Ne(f, 0)), (x*(c + d*sin(e))*(a*sin(e) +
a)**2, True))
```

3.438 $\int (a+a\sin(e+fx))^2\,dx$

Optimal. Leaf size=45

$$-\frac{2a^2\cos(e+fx)}{f}-\frac{a^2\sin(e+fx)\cos(e+fx)}{2f}+\frac{3a^2x}{2}$$

```
[Out] 3/2*a^2*x-2*a^2*cos(f*x+e)/f-1/2*a^2*cos(f*x+e)*sin(f*x+e)/f
```

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2644}

$$-\frac{2a^2\cos(e+fx)}{f}-\frac{a^2\sin(e+fx)\cos(e+fx)}{2f}+\frac{3a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (3*a^2*x)/2 - (2*a^2*Cos[e + f*x])/f - (a^2*Cos[e + f*x]*Sin[e + f*x])/(2*f
)
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2\,dx=\frac{3a^2x}{2}-\frac{2a^2\cos(e+fx)}{f}-\frac{a^2\cos(e+fx)\sin(e+fx)}{2f}$$

Mathematica [A] time = 0.19, size = 34, normalized size = 0.76

$$-\frac{a^2(-6(e+fx)+\sin(2(e+fx))+8\cos(e+fx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -1/4*(a^2*(-6*(e + f*x) + 8*Cos[e + f*x] + Sin[2*(e + f*x)]))/f
```

fricas [A] time = 0.42, size = 41, normalized size = 0.91

$$\frac{3 a^2 f x - a^2 \cos\left(f x + e\right) \sin\left(f x + e\right) - 4 a^2 \cos\left(f x + e\right)}{2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2,x, algorithm="fricas")
[Out] 1/2*(3*a^2*f*x - a^2*cos(f*x + e)*sin(f*x + e) - 4*a^2*cos(f*x + e))/f
```

giac [A] time = 0.16, size = 40, normalized size = 0.89

$$\frac{3}{2} a^2 x - \frac{2 a^2 \cos\left(f x + e\right)}{f} - \frac{a^2 \sin\left(2 f x + 2 e\right)}{4 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2,x, algorithm="giac")
[Out] 3/2*a^2*x - 2*a^2*cos(f*x + e)/f - 1/4*a^2*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.09, size = 52, normalized size = 1.16

$$\frac{a^2 \left(-\frac{\sin\left(f x+e\right) \cos\left(f x+e\right)}{2} + \frac{f x}{2} + \frac{e}{2}\right) - 2 \cos\left(f x + e\right) a^2 + a^2 \left(f x + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2,x)
[Out] 1/f*(a^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2*cos(f*x+e)*a^2+a^2*(f
*x+e))
```

maxima [A] time = 0.30, size = 47, normalized size = 1.04

$$a^2 x + \frac{\left(2 f x + 2 e - \sin\left(2 f x + 2 e\right)\right) a^2}{4 f} - \frac{2 a^2 \cos\left(f x + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2,x, algorithm="maxima")
[Out] a^2*x + 1/4*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2/f - 2*a^2*cos(f*x + e)/f
```

mupad [B] time = 6.87, size = 123, normalized size = 2.73

$$\frac{3\,a^2\,x}{2}-\frac{a^2\,\left(\frac{3\,e}{2}+\frac{3\,f\,x}{2}\right)-a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3-a^2\,\left(\frac{3\,e}{2}+\frac{3\,f\,x}{2}-4\right)+a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(2\,a^2\,\left(\frac{3\,e}{2}+\frac{3}{}\right.\right.}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2,x)
```

```
[Out] (3*a^2*x)/2 - (a^2*((3*e)/2 + (3*f*x)/2) - a^2*tan(e/2 + (f*x)/2)^3 - a^2*(
(3*e)/2 + (3*f*x)/2 - 4) + a^2*tan(e/2 + (f*x)/2) + tan(e/2 + (f*x)/2)^2*(2
*a^2*((3*e)/2 + (3*f*x)/2) - a^2*(3*e + 3*f*x - 4)))/(f*(tan(e/2 + (f*x)/2)
^2 + 1)^2)
```

sympy [A] time = 0.37, size = 78, normalized size = 1.73

$$\begin{cases} \frac{a^2x\sin^2\left(e+fx\right)}{2}+\frac{a^2x\cos^2\left(e+fx\right)}{2}+a^2x-\frac{a^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{2a^2\cos\left(e+fx\right)}{f} & \text{for } f\neq 0 \\ x\left(a\sin\left(e\right)+a\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(e + f*x)**2/2 + a**2*x*cos(e + f*x)**2/2 + a**2*x - a
**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**2*cos(e + f*x)/f, Ne(f, 0)), (x*
(a*sin(e) + a)**2, True))
```

3.439 $\int \frac{(a+a\sin(e+fx))^2}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=92

$$\frac{2a^2(c-d)^2\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f\sqrt{c^2-d^2}}-\frac{a^2x(c-2d)}{d^2}-\frac{a^2\cos(e+fx)}{df}$$

```
[Out] -a^2*(c-2*d)*x/d^2-a^2*cos(f*x+e)/d/f+2*a^2*(c-d)^2*arctan((d+c*tan(1/2*f*x
+1/2*e))/(c^2-d^2)^(1/2))/d^2/f/(c^2-d^2)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2746, 2735, 2660, 618, 204}

$$\frac{2a^2(c-d)^2\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f\sqrt{c^2-d^2}}-\frac{a^2x(c-2d)}{d^2}-\frac{a^2\cos(e+fx)}{df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x]),x]
```

```
[Out] -((a^2*(c - 2*d)*x)/d^2) + (2*a^2*(c - d)^2*ArcTan[(d + c*Tan[(e + f*x)/2])
/Sqrt[c^2 - d^2]])/(d^2*Sqrt[c^2 - d^2]*f) - (a^2*Cos[e + f*x])/(d*f)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{c+d\sin(e+fx)}\,dx = -\frac{a^2\cos(e+fx)}{df}+\frac{\int \frac{a^2d-a^2(c-2d)\sin(e+fx)}{c+d\sin(e+fx)}\,dx}{d}$$

$$= -\frac{a^2(c-2d)x}{d^2}-\frac{a^2\cos(e+fx)}{df}+\frac{\left(a^2(c-d)^2\right)\int \frac{1}{c+d\sin(e+fx)}\,dx}{d^2}$$

$$= -\frac{a^2(c-2d)x}{d^2}-\frac{a^2\cos(e+fx)}{df}+\frac{\left(2a^2(c-d)^2\right)\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx,x,\tan\left(\frac{1}{2}(e+\right.\right.}{d^2f}$$

$$= -\frac{a^2(c-2d)x}{d^2}-\frac{a^2\cos(e+fx)}{df}-\frac{\left(4a^2(c-d)^2\right)\text{Subst}\left(\int \frac{1}{-4\left(c^2-d^2\right)-x^2}\,dx,x,2d+2c\,\mathrm{t}\right.}{d^2f}$$

$$= -\frac{a^2(c-2d)x}{d^2}+\frac{2a^2(c-d)^2\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^2\sqrt{c^2-d^2}\,f}-\frac{a^2\cos(e+fx)}{df}$$

Mathematica [A] time = 0.41, size = 130, normalized size = 1.41

$$-\frac{a^2(\sin(e+fx)+1)^2\left(\sqrt{c^2-d^2}\,((c-2d)(e+fx)+d\cos(e+fx))-2(c-d)^2\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{d^2f\sqrt{c^2-d^2}\,\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x]),x]
```

```
[Out] -((a^2*(-2*(c - d)^2*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]] + Sqr
t[c^2 - d^2]*((c - 2*d)*(e + f*x) + d*Cos[e + f*x]))*(1 + Sin[e + f*x])^2)/
(d^2*Sqrt[c^2 - d^2]*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4))
```

fricas [A] time = 0.48, size = 296, normalized size = 3.22

$$\left[-\frac{2\,a^2d\cos\left(fx+e\right)+2\left(a^2c-2\,a^2d\right)fx+\left(a^2c-a^2d\right)\sqrt{-\frac{c-d}{c+d}}\,\log\left(\frac{\left(2\,c^2-d^2\right)\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2+2\left(\left(c^2+cd\right)\cos\right.}{d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(\right.}\right.}{2\,d^2f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a^2*d*cos(f*x + e) + 2*(a^2*c - 2*a^2*d)*f*x + (a^2*c - a^2*d)*sqr
t(-(c - d)/(c + d))*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e)
- c^2 - d^2 + 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*
x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) -
c^2 - d^2)))/(d^2*f), -(a^2*d*cos(f*x + e) + (a^2*c - 2*a^2*d)*f*x + (a^2*c
 - a^2*d)*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sqrt((c - d)/(
c + d))/((c - d)*cos(f*x + e))))/(d^2*f)]
```

giac [A] time = 0.21, size = 136, normalized size = 1.48

$$-\frac{\frac{\left(a^2c-2\,a^2d\right)\left(fx+e\right)}{d^2}+\frac{2\,a^2}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)d}-\frac{2\left(a^2c^2-2\,a^2cd+a^2d^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(c\right)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\sqrt{c^2-d^2}\,d^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -((a^2*c - 2*a^2*d)*(f*x + e)/d^2 + 2*a^2/((tan(1/2*f*x + 1/2*e)^2 + 1)*d)
- 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c
) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*
d^2))/f
```

maple [B] time = 0.25, size = 228, normalized size = 2.48

$$\frac{2a^2 \arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)c^2}{f\,d^2\sqrt{c^2-d^2}}-\frac{4a^2 \arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)c}{fd\sqrt{c^2-d^2}}+\frac{2a^2 \arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{f\sqrt{c^2-d^2}}-\frac{2a^2}{fd\left(1+\tan^2\left(\frac{fx}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x)
```

```
[Out] 2/f*a^2/d^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^
2)^(1/2))*c^2-4/f*a^2/d/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+
2*d)/(c^2-d^2)^(1/2))*c+2/f*a^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x
+1/2*e)+2*d)/(c^2-d^2)^(1/2))-2/f*a^2/d/(1+tan(1/2*f*x+1/2*e)^2)-2/f*a^2/d^
2*arctan(tan(1/2*f*x+1/2*e))*c+4/f*a^2/d*arctan(tan(1/2*f*x+1/2*e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 7.68, size = 940, normalized size = 10.22

$$\frac{4\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,\left(c+d\right)}-\frac{a^2\,\cos\left(e+f\,x\right)}{f\,\left(c+d\right)}+\frac{2\,a^2\,c\,\mathrm{atan}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{d\,f\,\left(c+d\right)}-\frac{2\,a^2\,c^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{d^2\,f\,\left(c+d\right)}-\frac{a^2\,c\,\cos\left(e+f\,x\right)}{d\,f\,\left(c+d\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c + d*sin(e + f*x)),x)
```

```
[Out] (4*a^2*atan(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(c + d)) - (a^2*cos(
e + f*x))/(f*(c + d)) + (2*a^2*c*atan(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)
))/(d*f*(c + d)) - (2*a^2*c^2*atan(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/
(d^2*f*(c + d)) + (a^2*atan(((3*d^2*sin(e/2 + (f*x)/2)*(2*c^3*d - 2*c*d^3 -
 c^4 + d^4)^(3/2) - 2*c^6*sin(e/2 + (f*x)/2)*(2*c^3*d - 2*c*d^3 - c^4 + d^4
```

```
)^(1/2) - 2*c^2*sin(e/2 + (f*x)/2)*(2*c^3*d - 2*c*d^3 - c^4 + d^4)^(3/2) +
7*d^6*sin(e/2 + (f*x)/2)*(2*c^3*d - 2*c*d^3 - c^4 + d^4)^(1/2) + 10*c*d^5*s
in(e/2 + (f*x)/2)*(2*c^3*d - 2*c*d^3 - c^4 + d^4)^(1/2) + 4*c^5*d*sin(e/2 +
 (f*x)/2)*(2*c^3*d - 2*c*d^3 - c^4 + d^4)^(1/2) + 4*c^2*d^4*cos(e/2 + (f*x)
/2)*(2*c^3*d - 2*c*d^3 - c^4 + d^4)^(1/2) - 3*c^3*d^3*cos(e/2 + (f*x)/2)*(2
*c^3*d - 2*c*d^3 - c^4 + d^4)^(1/2) - 2*c^4*d^2*cos(e/2 + (f*x)/2)*(2*c^3*d
 - 2*c*d^3 - c^4 + d^4)^(1/2) - 9*c^2*d^4*sin(e/2 + (f*x)/2)*(2*c^3*d - 2*c
*d^3 - c^4 + d^4)^(1/2) - 12*c^3*d^3*sin(e/2 + (f*x)/2)*(2*c^3*d - 2*c*d^3
- c^4 + d^4)^(1/2) + 6*c^4*d^2*sin(e/2 + (f*x)/2)*(2*c^3*d - 2*c*d^3 - c^4
+ d^4)^(1/2) + c*d*cos(e/2 + (f*x)/2)*(2*c^3*d - 2*c*d^3 - c^4 + d^4)^(3/2)
 + 4*c*d^5*cos(e/2 + (f*x)/2)*(2*c^3*d - 2*c*d^3 - c^4 + d^4)^(1/2) + c^5*d
*cos(e/2 + (f*x)/2)*(2*c^3*d - 2*c*d^3 - c^4 + d^4)^(1/2))*1i)/(d*(c + d)*(
3*c^5*d*cos(e/2 + (f*x)/2) - 5*c*d^5*cos(e/2 + (f*x)/2) - 10*d^6*sin(e/2 +
(f*x)/2) + 16*c*d^5*sin(e/2 + (f*x)/2) + 8*c^2*d^4*cos(e/2 + (f*x)/2) + 2*c
^3*d^3*cos(e/2 + (f*x)/2) - 8*c^4*d^2*cos(e/2 + (f*x)/2) + 4*c^2*d^4*sin(e/
2 + (f*x)/2) - 16*c^3*d^3*sin(e/2 + (f*x)/2) + 6*c^4*d^2*sin(e/2 + (f*x)/2)
)))*(-(c + d)*(c - d)^3)^(1/2)*2i)/(d^2*f*(c + d)) - (a^2*c*cos(e + f*x))/(
d*f*(c + d))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.440 $\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=112

$$-\frac{2a^2(c-d)^2(c+2d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f\left(c^2-d^2\right)^{3/2}}+\frac{a^2(c-d)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}+\frac{a^2x}{d^2}$$

```
[Out] a^2*x/d^2-2*a^2*(c-d)^2*(c+2*d)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(
1/2))/d^2/(c^2-d^2)^(3/2)/f+a^2*(c-d)*cos(f*x+e)/d/(c+d)/f/(c+d*sin(f*x+e))
```

Rubi [A] time = 0.18, antiderivative size = 115, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2762, 2735, 2660, 618, 204}

$$-\frac{2a^2(c-d)(c+2d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f(c+d)\sqrt{c^2-d^2}}+\frac{a^2(c-d)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}+\frac{a^2x}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*x)/d^2 - (2*a^2*(c - d)*(c + 2*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt
[c^2 - d^2]])/(d^2*(c + d)*Sqrt[c^2 - d^2]*f) + (a^2*(c - d)*Cos[e + f*x])/
(d*(c + d)*f*(c + d*Sin[e + f*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^2}\,dx = \frac{a^2(c-d)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))} - \frac{a\int \frac{-2ad-a(c+d)\sin(e+fx)}{c+d\sin(e+fx)}\,dx}{d(c+d)}$$

$$= \frac{a^2x}{d^2} + \frac{a^2(c-d)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))} - \frac{\left(a^2(c-d)(c+2d)\right)\int \frac{1}{c+d\sin(e+fx)}\,dx}{d^2(c+d)}$$

$$= \frac{a^2x}{d^2} + \frac{a^2(c-d)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))} - \frac{\left(2a^2(c-d)(c+2d)\right)\operatorname{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx, x, \text{t}\right.}{d^2(c+d)f}$$

$$= \frac{a^2x}{d^2} + \frac{a^2(c-d)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))} + \frac{\left(4a^2(c-d)(c+2d)\right)\operatorname{Subst}\left(\int \frac{1}{-4\left(c^2-d^2\right)-x^2}\,dx, x\right.}{d^2(c+d)f}$$

$$= \frac{a^2x}{d^2} - \frac{2a^2(c-d)(c+2d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^2(c+d)\sqrt{c^2-d^2}\,f} + \frac{a^2(c-d)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))}$$

Mathematica [A] time = 0.49, size = 139, normalized size = 1.24

$$\frac{a^2(\sin(e+fx)+1)^2\left(-\frac{2(c^2+cd-2d^2)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}+\frac{d(c-d)\cos(e+fx)}{(c+d)(c+d\sin(e+fx))}+e+fx\right)}{d^2f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*(1 + Sin[e + f*x])^2*(e + f*x - (2*(c^2 + c*d - 2*d^2)*ArcTan[(d + c*T
an[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)*Sqrt[c^2 - d^2]) + ((c - d)*d*C
os[e + f*x])/((c + d)*(c + d*Sin[e + f*x]))))/(d^2*f*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^4)
```

fricas [A] time = 0.48, size = 476, normalized size = 4.25

$$\left[\frac{2\left(a^2cd+a^2d^2\right)fx\sin\left(fx+e\right)+2\left(a^2c^2+a^2cd\right)fx+\left(a^2c^2+2\,a^2cd+\left(a^2cd+2\,a^2d^2\right)\sin\left(fx+e\right)\right)\sqrt{-\frac{c-d}{c+d}}\,\log}{2\left(\left(cd^3+d^4\right)f\sin\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^2*c*d + a^2*d^2)*f*x*sin(f*x + e) + 2*(a^2*c^2 + a^2*c*d)*f*x +
(a^2*c^2 + 2*a^2*c*d + (a^2*c*d + 2*a^2*d^2)*sin(f*x + e))*sqrt(-(c - d)/(c
 + d))*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 +
 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(
-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) +
 2*(a^2*c*d - a^2*d^2)*cos(f*x + e))/((c*d^3 + d^4)*f*sin(f*x + e) + (c^2*d
^2 + c*d^3)*f), ((a^2*c*d + a^2*d^2)*f*x*sin(f*x + e) + (a^2*c^2 + a^2*c*d)
*f*x + (a^2*c^2 + 2*a^2*c*d + (a^2*c*d + 2*a^2*d^2)*sin(f*x + e))*sqrt((c -
 d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sqrt((c - d)/(c + d))/((c - d)*co
s(f*x + e))) + (a^2*c*d - a^2*d^2)*cos(f*x + e))/((c*d^3 + d^4)*f*sin(f*x +
 e) + (c^2*d^2 + c*d^3)*f)]
```

giac [A] time = 0.22, size = 205, normalized size = 1.83

$$\frac{\frac{(fx+e)a^2}{d^2} - \frac{2\left(a^2c^2+a^2cd-2\,a^2d^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(cd^2+d^3\right)\sqrt{c^2-d^2}} + \frac{2\left(a^2cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-a^2d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+a^2c^2-a^2cd\right)}{\left(c^2d+cd^2\right)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+c\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] ((f*x + e)*a^2/d^2 - 2*(a^2*c^2 + a^2*c*d - 2*a^2*d^2)*(pi*floor(1/2*(f*x +
 e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2))
)/((c*d^2 + d^3)*sqrt(c^2 - d^2)) + 2*(a^2*c*d*tan(1/2*f*x + 1/2*e) - a^2*d
^2*tan(1/2*f*x + 1/2*e) + a^2*c^2 - a^2*c*d)/((c^2*d + c*d^2)*(c*tan(1/2*f*
x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)))/f
```

maple [B] time = 0.29, size = 389, normalized size = 3.47

$$\frac{2a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d+c\right)(c+d)} - \frac{2a^2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d+c\right)(c+d)\,c} + \frac{}{fd\left(\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*tan(1/2*f*x
+1/2*e)-2*a^2/f*d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)/c
*tan(1/2*f*x+1/2*e)+2*a^2/f/d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*
d+c)/(c+d)*c-2*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d
)-2*a^2/f/d^2/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)
/(c^2-d^2)^(1/2))*c^2-2*a^2/f/d/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1
/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c+4*a^2/f/(c+d)/(c^2-d^2)^(1/2)*arctan(
1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))+2*a^2/f/d^2*arctan(tan(1/
2*f*x+1/2*e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 11.24, size = 2836, normalized size = 25.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c + d*sin(e + f*x))^2,x)
```

```
[Out] ((2*(a^2*c - a^2*d))/(d*(c + d)) + (2*a^2*tan(e/2 + (f*x)/2)*(c - d))/(c*(c
 + d)))/(f*(c + 2*d*tan(e/2 + (f*x)/2) + c*tan(e/2 + (f*x)/2)^2)) + (2*a^2*
atan((192*a^6*c^3*d*tan(e/2 + (f*x)/2))/((128*a^6*c^3*d^5)/(2*c*d^3 + d^4 +
 c^2*d^2) - (512*a^6*c^2*d^6)/(2*c*d^3 + d^4 + c^2*d^2) - (320*a^6*c*d^7)/(
2*c*d^3 + d^4 + c^2*d^2) + (512*a^6*c^4*d^4)/(2*c*d^3 + d^4 + c^2*d^2) + (1
92*a^6*c^5*d^3)/(2*c*d^3 + d^4 + c^2*d^2)) - (320*a^6*c*d^3*tan(e/2 + (f*x)
/2))/((128*a^6*c^3*d^5)/(2*c*d^3 + d^4 + c^2*d^2) - (512*a^6*c^2*d^6)/(2*c*
d^3 + d^4 + c^2*d^2) - (320*a^6*c*d^7)/(2*c*d^3 + d^4 + c^2*d^2) + (512*a^6
*c^4*d^4)/(2*c*d^3 + d^4 + c^2*d^2) + (192*a^6*c^5*d^3)/(2*c*d^3 + d^4 + c^
2*d^2)) + (128*a^6*c^2*d^2*tan(e/2 + (f*x)/2))/((128*a^6*c^3*d^5)/(2*c*d^3
+ d^4 + c^2*d^2) - (512*a^6*c^2*d^6)/(2*c*d^3 + d^4 + c^2*d^2) - (320*a^6*c
*d^7)/(2*c*d^3 + d^4 + c^2*d^2) + (512*a^6*c^4*d^4)/(2*c*d^3 + d^4 + c^2*d^
2) + (192*a^6*c^5*d^3)/(2*c*d^3 + d^4 + c^2*d^2))))/(d^2*f) + (a^2*atan(((a
^2*(-(c + d)^3*(c - d))^(1/2)*(c + 2*d)*((32*(a^4*c^4*d + a^4*c^2*d^3 + 2*a
^4*c^3*d^2))/(2*c*d^3 + d^4 + c^2*d^2) - (32*tan(e/2 + (f*x)/2)*(2*a^4*c*d^
5 + 2*a^4*c^5*d - 8*a^4*c^2*d^4 - 4*a^4*c^3*d^3 + 4*a^4*c^4*d^2))/(2*c*d^4
+ d^5 + c^2*d^3) + (a^2*(-(c + d)^3*(c - d))^(1/2)*(c + 2*d)*((32*tan(e/2 +
 (f*x)/2)*(4*a^2*c*d^7 + 2*a^2*c^2*d^6 - 4*a^2*c^3*d^5 - 2*a^2*c^4*d^4))/(2
*c*d^4 + d^5 + c^2*d^3) - (32*(a^2*c*d^6 - a^2*c^3*d^4))/(2*c*d^3 + d^4 + c
^2*d^2) + (a^2*((32*(c^2*d^7 + 2*c^3*d^6 + c^4*d^5))/(2*c*d^3 + d^4 + c^2*d
^2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^9 + 6*c^2*d^8 + c^3*d^7 - 4*c^4*d^6 - 2
*c^5*d^5))/(2*c*d^4 + d^5 + c^2*d^3))*(-(c + d)^3*(c - d))^(1/2)*(c + 2*d))
/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2)))/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d
^2))*1i)/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2) + (a^2*(-(c + d)^3*(c - d))^
(1/2)*(c + 2*d)*((32*(a^4*c^4*d + a^4*c^2*d^3 + 2*a^4*c^3*d^2))/(2*c*d^3 +
d^4 + c^2*d^2) - (32*tan(e/2 + (f*x)/2)*(2*a^4*c*d^5 + 2*a^4*c^5*d - 8*a^4*
c^2*d^4 - 4*a^4*c^3*d^3 + 4*a^4*c^4*d^2))/(2*c*d^4 + d^5 + c^2*d^3) + (a^2*
(-(c + d)^3*(c - d))^(1/2)*(c + 2*d)*((32*(a^2*c*d^6 - a^2*c^3*d^4))/(2*c*d
^3 + d^4 + c^2*d^2) - (32*tan(e/2 + (f*x)/2)*(4*a^2*c*d^7 + 2*a^2*c^2*d^6 -
 4*a^2*c^3*d^5 - 2*a^2*c^4*d^4))/(2*c*d^4 + d^5 + c^2*d^3) + (a^2*((32*(c^2
*d^7 + 2*c^3*d^6 + c^4*d^5))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x
)/2)*(3*c*d^9 + 6*c^2*d^8 + c^3*d^7 - 4*c^4*d^6 - 2*c^5*d^5))/(2*c*d^4 + d^
5 + c^2*d^3))*(-(c + d)^3*(c - d))^(1/2)*(c + 2*d))/(3*c*d^4 + d^5 + 3*c^2*
```

```
d^3 + c^3*d^2)))/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2))*1i)/(3*c*d^4 + d^5
+ 3*c^2*d^3 + c^3*d^2))/((64*(2*a^6*c^3 - 4*a^6*c*d^2 + 2*a^6*c^2*d))/(2*c*
d^3 + d^4 + c^2*d^2) + (64*tan(e/2 + (f*x)/2)*(2*a^6*c^4 - 4*a^6*c*d^3 + 4*
a^6*c^3*d - 2*a^6*c^2*d^2))/(2*c*d^4 + d^5 + c^2*d^3) - (a^2*(-(c + d)^3*(c
 - d))^(1/2)*(c + 2*d)*((32*(a^4*c^4*d + a^4*c^2*d^3 + 2*a^4*c^3*d^2))/(2*c
*d^3 + d^4 + c^2*d^2) - (32*tan(e/2 + (f*x)/2)*(2*a^4*c*d^5 + 2*a^4*c^5*d -
 8*a^4*c^2*d^4 - 4*a^4*c^3*d^3 + 4*a^4*c^4*d^2))/(2*c*d^4 + d^5 + c^2*d^3)
+ (a^2*(-(c + d)^3*(c - d))^(1/2)*(c + 2*d)*((32*tan(e/2 + (f*x)/2)*(4*a^2*
c*d^7 + 2*a^2*c^2*d^6 - 4*a^2*c^3*d^5 - 2*a^2*c^4*d^4))/(2*c*d^4 + d^5 + c^
2*d^3) - (32*(a^2*c*d^6 - a^2*c^3*d^4))/(2*c*d^3 + d^4 + c^2*d^2) + (a^2*((
32*(c^2*d^7 + 2*c^3*d^6 + c^4*d^5))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2
 + (f*x)/2)*(3*c*d^9 + 6*c^2*d^8 + c^3*d^7 - 4*c^4*d^6 - 2*c^5*d^5))/(2*c*d
^4 + d^5 + c^2*d^3))*(-(c + d)^3*(c - d))^(1/2)*(c + 2*d))/(3*c*d^4 + d^5 +
 3*c^2*d^3 + c^3*d^2)))/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2)))/(3*c*d^4 +
d^5 + 3*c^2*d^3 + c^3*d^2) + (a^2*(-(c + d)^3*(c - d))^(1/2)*(c + 2*d)*((32
*(a^4*c^4*d + a^4*c^2*d^3 + 2*a^4*c^3*d^2))/(2*c*d^3 + d^4 + c^2*d^2) - (32
*tan(e/2 + (f*x)/2)*(2*a^4*c*d^5 + 2*a^4*c^5*d - 8*a^4*c^2*d^4 - 4*a^4*c^3*
d^3 + 4*a^4*c^4*d^2))/(2*c*d^4 + d^5 + c^2*d^3) + (a^2*(-(c + d)^3*(c - d))
^(1/2)*(c + 2*d)*((32*(a^2*c*d^6 - a^2*c^3*d^4))/(2*c*d^3 + d^4 + c^2*d^2)
- (32*tan(e/2 + (f*x)/2)*(4*a^2*c*d^7 + 2*a^2*c^2*d^6 - 4*a^2*c^3*d^5 - 2*a
^2*c^4*d^4))/(2*c*d^4 + d^5 + c^2*d^3) + (a^2*((32*(c^2*d^7 + 2*c^3*d^6 + c
^4*d^5))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^9 + 6*c^
2*d^8 + c^3*d^7 - 4*c^4*d^6 - 2*c^5*d^5))/(2*c*d^4 + d^5 + c^2*d^3))*(-(c +
 d)^3*(c - d))^(1/2)*(c + 2*d))/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2)))/(3*
c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2)))/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2))
)*(-(c + d)^3*(c - d))^(1/2)*(c + 2*d)*2i)/(f*(3*c*d^4 + d^5 + 3*c^2*d^3 +
c^3*d^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.441 $\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=138

$$\frac{3a^2\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c+d)^2\sqrt{c^2-d^2}}-\frac{a^2(c+4d)\cos(e+fx)}{2df(c+d)^2(c+d\sin(e+fx))}+\frac{a^2(c-d)\cos(e+fx)}{2df(c+d)(c+d\sin(e+fx))^2}$$

```
[Out] 1/2*a^2*(c-d)*cos(f*x+e)/d/(c+d)/f/(c+d*sin(f*x+e))^2-1/2*a^2*(c+4*d)*cos(f
*x+e)/d/(c+d)^2/f/(c+d*sin(f*x+e))+3*a^2*arctan((d+c*tan(1/2*f*x+1/2*e))/(c
^2-d^2)^(1/2))/(c+d)^2/f/(c^2-d^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2762, 2754, 12, 2660, 618, 204}

$$\frac{3a^2\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c+d)^2\sqrt{c^2-d^2}}-\frac{a^2(c+4d)\cos(e+fx)}{2df(c+d)^2(c+d\sin(e+fx))}+\frac{a^2(c-d)\cos(e+fx)}{2df(c+d)(c+d\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (3*a^2*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)^2*Sqrt[c^
2 - d^2]*f) + (a^2*(c - d)*Cos[e + f*x])/(2*d*(c + d)*f*(c + d*Sin[e + f*x]
)^2) - (a^2*(c + 4*d)*Cos[e + f*x])/(2*d*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^3}\,dx = \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a\int \frac{-4ad-a(c+3d)\sin(e+fx)}{(c+d\sin(e+fx))^2}\,dx}{2d(c+d)}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2(c+4d)\cos(e+fx)}{2d(c+d)^2f(c+d\sin(e+fx))} + \frac{a\int \frac{3a(c-d)d}{c+d\sin(e+fx)}}{2(c-d)d(c+d}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2(c+4d)\cos(e+fx)}{2d(c+d)^2f(c+d\sin(e+fx))} + \frac{(3a^2)\int \frac{1}{c+d\sin(e}}{2(c+d)^2}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2(c+4d)\cos(e+fx)}{2d(c+d)^2f(c+d\sin(e+fx))} + \frac{(3a^2)\,\text{Subst}\Big(\int}{}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2(c+4d)\cos(e+fx)}{2d(c+d)^2f(c+d\sin(e+fx))} - \frac{(6a^2)\,\text{Subst}\Big(\int}{}$$

$$= \frac{3a^2\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c+d)^2\sqrt{c^2-d^2}\,f} + \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2(c+4d)\cos(e}{2d(c+d)^2f(c+d\,\text{si}}$$

Mathematica [A] time = 0.62, size = 140, normalized size = 1.01

$$\frac{a^2\cos(e+fx)\left(-\frac{(c+4d)\sin(e+fx)+4c+d}{(c+d)(c+d\sin(e+fx))^2} - \frac{6\tanh^{-1}\left(\frac{\sqrt{c-d}\sqrt{1-\sin(e+fx)}}{\sqrt{-c-d}\sqrt{\sin(e+fx)+1}}\right)}{(-c-d)^{3/2}\sqrt{c-d}\sqrt{\cos^2(e+fx)}}\right)}{2f(c+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a^2*Cos[e + f*x]*((-6*ArcTanh[(Sqrt[c - d]*Sqrt[1 - Sin[e + f*x]])/(Sqrt[-
c - d]*Sqrt[1 + Sin[e + f*x]])])/((-c - d)^(3/2)*Sqrt[c - d]*Sqrt[Cos[e + f
*x]^2]) - (4*c + d + (c + 4*d)*Sin[e + f*x])/((c + d)*(c + d*Sin[e + f*x])^
2)))/(2*(c + d)*f)
```

fricas [B] time = 0.50, size = 679, normalized size = 4.92

$$\left[\frac{2\left(a^2c^3+4\,a^2c^2d-a^2cd^2-4\,a^2d^3\right)\cos\left(fx+e\right)\sin\left(fx+e\right)-3\left(a^2d^2\cos\left(fx+e\right)^2-2\,a^2cd\sin\left(fx+e\right)-a^2c\right.}{4\left(\left(c^4d^2+2\,c^3d^3-2\,cd^5-d^6\right)f\cos\left(fx+e\right)^2-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2*c^3 + 4*a^2*c^2*d - a^2*c*d^2 - 4*a^2*d^3)*cos(f*x + e)*sin(f*
x + e) - 3*(a^2*d^2*cos(f*x + e)^2 - 2*a^2*c*d*sin(f*x + e) - a^2*c^2 - a^2
*d^2)*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x +
e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2
 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(4*a^2*
c^3 + a^2*c^2*d - 4*a^2*c*d^2 - a^2*d^3)*cos(f*x + e))/((c^4*d^2 + 2*c^3*d^
3 - 2*c*d^5 - d^6)*f*cos(f*x + e)^2 - 2*(c^5*d + 2*c^4*d^2 - 2*c^2*d^4 - c*
d^5)*f*sin(f*x + e) - (c^6 + 2*c^5*d + c^4*d^2 - c^2*d^4 - 2*c*d^5 - d^6)*f
), 1/2*((a^2*c^3 + 4*a^2*c^2*d - a^2*c*d^2 - 4*a^2*d^3)*cos(f*x + e)*sin(f*
x + e) - 3*(a^2*d^2*cos(f*x + e)^2 - 2*a^2*c*d*sin(f*x + e) - a^2*c^2 - a^2
*d^2)*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x
 + e))) + (4*a^2*c^3 + a^2*c^2*d - 4*a^2*c*d^2 - a^2*d^3)*cos(f*x + e))/((c
^4*d^2 + 2*c^3*d^3 - 2*c*d^5 - d^6)*f*cos(f*x + e)^2 - 2*(c^5*d + 2*c^4*d^2
 - 2*c^2*d^4 - c*d^5)*f*sin(f*x + e) - (c^6 + 2*c^5*d + c^4*d^2 - c^2*d^4 -
 2*c*d^5 - d^6)*f)]
```

giac [B] time = 0.26, size = 348, normalized size = 2.52

$$\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)a^2}{\left(c^2+2cd+d^2\right)\sqrt{c^2-d^2}}+\frac{a^2c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-4a^2c^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2a^2cd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-4a^2c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] (3*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e
) + d)/sqrt(c^2 - d^2)))*a^2/((c^2 + 2*c*d + d^2)*sqrt(c^2 - d^2)) + (a^2*c
^3*tan(1/2*f*x + 1/2*e)^3 - 4*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^3 - 2*a^2*c*d^
2*tan(1/2*f*x + 1/2*e)^3 - 4*a^2*c^3*tan(1/2*f*x + 1/2*e)^2 - a^2*c^2*d*tan
(1/2*f*x + 1/2*e)^2 - 8*a^2*c*d^2*tan(1/2*f*x + 1/2*e)^2 - 2*a^2*d^3*tan(1/
2*f*x + 1/2*e)^2 - a^2*c^3*tan(1/2*f*x + 1/2*e) - 12*a^2*c^2*d*tan(1/2*f*x
+ 1/2*e) - 2*a^2*c*d^2*tan(1/2*f*x + 1/2*e) - 4*a^2*c^3 - a^2*c^2*d)/((c^4
+ 2*c^3*d + c^2*d^2)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) +
 c)^2))/f
```

maple [B] time = 0.32, size = 799, normalized size = 5.79

$$\frac{a^{2}c\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f\left(\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d+c\right)^{2}\left(c^{2}+2cd+d^{2}\right)}-\frac{4a^{2}\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d}{f\left(\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d+c\right)^{2}\left(c^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x)
```

```
[Out] a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c
*tan(1/2*f*x+1/2*e)^3-4*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*
d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*d-2*a^2/f/(tan(1/2*f*x+1/2*e)^2
*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^3*d^2-4
*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*
c*tan(1/2*f*x+1/2*e)^2-a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d
+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2*d-8*a^2/f/(tan(1/2*f*x+1/2*e)^2*
c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^2*d^2-2*
a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c
^2*tan(1/2*f*x+1/2*e)^2*d^3-a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2
*e)*d+c)^2*c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)-12*a^2/f/(tan(1/2*f*x+1/2*e
)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*d-2*a^
2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^2+2*c*d+d^2)*t
an(1/2*f*x+1/2*e)*d^2-4*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*
d+c)^2/(c^2+2*c*d+d^2)*c-a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)
*d+c)^2/(c^2+2*c*d+d^2)*d+3*a^2/f/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/
2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 9.43, size = 362, normalized size = 2.62

$$\frac{3\,a^2\,\mathrm{atan}\left(\frac{\left(\frac{3\,a^2\,\left(2\,c^2\,d+4\,c\,d^2+2\,d^3\right)}{2\,(c+d)^{5/2}\,\sqrt{c-d}\,\left(c^2+2\,c\,d+d^2\right)}+\frac{3\,a^2\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{(c+d)^{5/2}\,\sqrt{c-d}}\right)\left(c^2+2\,c\,d+d^2\right)}{3\,a^2}\right)}{f\,(c+d)^{5/2}\,\sqrt{c-d}}-\frac{\frac{4\,a^2\,c+a^2\,d}{c^2+2\,c\,d+d^2}+\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(a^2\,c^2+12\,a^2\,c\,d+2\,a^2\,d^2\right)}{c\,\left(c^2+2\,c\,d+d^2\right)}+\frac{a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right.}{c\,\left(c^2\right.}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(2\,c^2+4\,d^2\right)+c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c + d*sin(e + f*x))^3,x)
```

```
[Out] (3*a^2*atan((((3*a^2*(4*c*d^2 + 2*c^2*d + 2*d^3))/(2*(c + d)^(5/2)*(c - d)^
(1/2)*(2*c*d + c^2 + d^2)) + (3*a^2*c*tan(e/2 + (f*x)/2))/((c + d)^(5/2)*(c
 - d)^(1/2)))*(2*c*d + c^2 + d^2))/(3*a^2)))/(f*(c + d)^(5/2)*(c - d)^(1/2)
) - ((4*a^2*c + a^2*d)/(2*c*d + c^2 + d^2) + (tan(e/2 + (f*x)/2)*(a^2*c^2 +
 2*a^2*d^2 + 12*a^2*c*d))/(c*(2*c*d + c^2 + d^2)) + (a^2*tan(e/2 + (f*x)/2)
^3*(4*c*d - c^2 + 2*d^2))/(c*(2*c*d + c^2 + d^2)) + (tan(e/2 + (f*x)/2)^2*(
c^2 + 2*d^2)*(4*a^2*c + a^2*d))/(c^2*(2*c*d + c^2 + d^2)))/(f*(tan(e/2 + (f
*x)/2)^2*(2*c^2 + 4*d^2) + c^2*tan(e/2 + (f*x)/2)^4 + c^2 + 4*c*d*tan(e/2 +
 (f*x)/2)^3 + 4*c*d*tan(e/2 + (f*x)/2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.442 $\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^4}\,dx$

Optimal. Leaf size=207

$$\frac{a^2(3c-2d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c-d)(c+d)^3\sqrt{c^2-d^2}}-\frac{a^2\left(c^2+6cd-10d^2\right)\cos(e+fx)}{6df(c-d)(c+d)^3(c+d\sin(e+fx))}-\frac{a^2(c+6d)\cos(e+fx)}{6df(c+d)^2(c+d\sin(e+fx))^2}+\frac{}{3df}$$

```
[Out] 1/3*a^2*(c-d)*cos(f*x+e)/d/(c+d)/f/(c+d*sin(f*x+e))^3-1/6*a^2*(c+6*d)*cos(f
*x+e)/d/(c+d)^2/f/(c+d*sin(f*x+e))^2-1/6*a^2*(c^2+6*c*d-10*d^2)*cos(f*x+e)/
(c-d)/d/(c+d)^3/f/(c+d*sin(f*x+e))+a^2*(3*c-2*d)*arctan((d+c*tan(1/2*f*x+1/
2*e))/(c^2-d^2)^(1/2))/(c-d)/(c+d)^3/f/(c^2-d^2)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2762, 2754, 12, 2660, 618, 204}

$$\frac{a^2(3c-2d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c-d)(c+d)^3\sqrt{c^2-d^2}}-\frac{a^2\left(c^2+6cd-10d^2\right)\cos(e+fx)}{6df(c-d)(c+d)^3(c+d\sin(e+fx))}-\frac{a^2(c+6d)\cos(e+fx)}{6df(c+d)^2(c+d\sin(e+fx))^2}+\frac{}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^4,x]
```

```
[Out] (a^2*(3*c - 2*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c - d)
*(c + d)^3*Sqrt[c^2 - d^2]*f) + (a^2*(c - d)*Cos[e + f*x])/(3*d*(c + d)*f*(
c + d*Sin[e + f*x])^3) - (a^2*(c + 6*d)*Cos[e + f*x])/(6*d*(c + d)^2*f*(c +
 d*Sin[e + f*x])^2) - (a^2*(c^2 + 6*c*d - 10*d^2)*Cos[e + f*x])/(6*(c - d)*
d*(c + d)^3*f*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^4}\,dx = \frac{a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^3} - \frac{a\int \frac{-6ad-a(c+5d)\sin(e+fx)}{(c+d\sin(e+fx))^3}\,dx}{3d(c+d)}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^3} - \frac{a^2(c+6d)\cos(e+fx)}{6d(c+d)^2f(c+d\sin(e+fx))^2} + \frac{a\int \frac{10a(c-d)d+a(c}{(c+d}}{6(c-}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^3} - \frac{a^2(c+6d)\cos(e+fx)}{6d(c+d)^2f(c+d\sin(e+fx))^2} - \frac{a^2\left(c^2+6cd\right.}{6(c-d)d(c+d}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^3} - \frac{a^2(c+6d)\cos(e+fx)}{6d(c+d)^2f(c+d\sin(e+fx))^2} - \frac{a^2\left(c^2+6cd\right.}{6(c-d)d(c+d}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^3} - \frac{a^2(c+6d)\cos(e+fx)}{6d(c+d)^2f(c+d\sin(e+fx))^2} - \frac{a^2\left(c^2+6cd\right.}{6(c-d)d(c+d}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^3} - \frac{a^2(c+6d)\cos(e+fx)}{6d(c+d)^2f(c+d\sin(e+fx))^2} - \frac{a^2\left(c^2+6cd\right.}{6(c-d)d(c+d}$$

$$= \frac{a^2(3c-2d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c-d)(c+d)^3\sqrt{c^2-d^2}\,f} + \frac{a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^3} - \frac{a^2(c+6}{6d(c+d)^2f}$$

Mathematica [A] time = 2.56, size = 196, normalized size = 0.95

$$\frac{a^2\cos(e+fx)\left(-\frac{d(\sin(e+fx)+1)^2}{(c+d\sin(e+fx))^3}-\frac{(3c-2d)\left(\frac{6\tanh^{-1}\left(\frac{\sqrt{c-d}\sqrt{1-\sin(e+fx)}}{\sqrt{-c-d}\sqrt{\sin(e+fx)+1}}\right)}{\sqrt{-c-d}\sqrt{c-d}}-\frac{\sqrt{\cos^2(e+fx)}((c+4d)\sin(e+fx)+4c+d)}{(c+d\sin(e+fx))^2}\right)}{2(c+d)^2\sqrt{\cos^2(e+fx)}}\right)}{3f(d-c)(c+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^4,x]
```

```
[Out] (a^2*Cos[e + f*x]*(-((d*(1 + Sin[e + f*x])^2)/(c + d*Sin[e + f*x])^3) - ((3
*c - 2*d)*((6*ArcTanh[(Sqrt[c - d]*Sqrt[1 - Sin[e + f*x]])/(Sqrt[-c - d]*Sq
rt[1 + Sin[e + f*x]])])/(Sqrt[-c - d]*Sqrt[c - d]) - (Sqrt[Cos[e + f*x]^2]*
(4*c + d + (c + 4*d)*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2))/(2*(c + d)^2*S
qrt[Cos[e + f*x]^2])))/(3*(-c + d)*(c + d)*f)
```

fricas [B] time = 0.56, size = 1366, normalized size = 6.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(2*(a^2*c^4*d + 6*a^2*c^3*d^2 - 11*a^2*c^2*d^3 - 6*a^2*c*d^4 + 10*a^
2*d^5)*cos(f*x + e)^3 - 6*(a^2*c^5 + 6*a^2*c^4*d - 8*a^2*c^3*d^2 - 8*a^2*c^
2*d^3 + 7*a^2*c*d^4 + 2*a^2*d^5)*cos(f*x + e)*sin(f*x + e) - 3*(3*a^2*c^4 -
 2*a^2*c^3*d + 9*a^2*c^2*d^2 - 6*a^2*c*d^3 - 3*(3*a^2*c^2*d^2 - 2*a^2*c*d^3
)*cos(f*x + e)^2 + (9*a^2*c^3*d - 6*a^2*c^2*d^2 + 3*a^2*c*d^3 - 2*a^2*d^4 -
 (3*a^2*c*d^3 - 2*a^2*d^4)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(-c^2 + d^2)*l
og(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*co
s(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x +
e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 12*(2*a^2*c^5 - a^2*c^4*d - 2*a^2
*c^3*d^2 - a^2*c^2*d^3 + 2*a^2*d^5)*cos(f*x + e))/(3*(c^7*d^2 + 2*c^6*d^3 -
 c^5*d^4 - 4*c^4*d^5 - c^3*d^6 + 2*c^2*d^7 + c*d^8)*f*cos(f*x + e)^2 - (c^9
 + 2*c^8*d + 2*c^7*d^2 + 2*c^6*d^3 - 4*c^5*d^4 - 10*c^4*d^5 - 2*c^3*d^6 + 6
*c^2*d^7 + 3*c*d^8)*f + ((c^6*d^3 + 2*c^5*d^4 - c^4*d^5 - 4*c^3*d^6 - c^2*d
^7 + 2*c*d^8 + d^9)*f*cos(f*x + e)^2 - (3*c^8*d + 6*c^7*d^2 - 2*c^6*d^3 - 1
0*c^5*d^4 - 4*c^4*d^5 + 2*c^3*d^6 + 2*c^2*d^7 + 2*c*d^8 + d^9)*f)*sin(f*x +
 e)), -1/6*((a^2*c^4*d + 6*a^2*c^3*d^2 - 11*a^2*c^2*d^3 - 6*a^2*c*d^4 + 10*
a^2*d^5)*cos(f*x + e)^3 - 3*(a^2*c^5 + 6*a^2*c^4*d - 8*a^2*c^3*d^2 - 8*a^2*
c^2*d^3 + 7*a^2*c*d^4 + 2*a^2*d^5)*cos(f*x + e)*sin(f*x + e) - 3*(3*a^2*c^4
 - 2*a^2*c^3*d + 9*a^2*c^2*d^2 - 6*a^2*c*d^3 - 3*(3*a^2*c^2*d^2 - 2*a^2*c*d
^3)*cos(f*x + e)^2 + (9*a^2*c^3*d - 6*a^2*c^2*d^2 + 3*a^2*c*d^3 - 2*a^2*d^4
 - (3*a^2*c*d^3 - 2*a^2*d^4)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(c^2 - d^2)*
arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) - 6*(2*a^2*c^5
 - a^2*c^4*d - 2*a^2*c^3*d^2 - a^2*c^2*d^3 + 2*a^2*d^5)*cos(f*x + e))/(3*(c
^7*d^2 + 2*c^6*d^3 - c^5*d^4 - 4*c^4*d^5 - c^3*d^6 + 2*c^2*d^7 + c*d^8)*f*c
os(f*x + e)^2 - (c^9 + 2*c^8*d + 2*c^7*d^2 + 2*c^6*d^3 - 4*c^5*d^4 - 10*c^4
*d^5 - 2*c^3*d^6 + 6*c^2*d^7 + 3*c*d^8)*f + ((c^6*d^3 + 2*c^5*d^4 - c^4*d^5
 - 4*c^3*d^6 - c^2*d^7 + 2*c*d^8 + d^9)*f*cos(f*x + e)^2 - (3*c^8*d + 6*c^7
*d^2 - 2*c^6*d^3 - 10*c^5*d^4 - 4*c^4*d^5 + 2*c^3*d^6 + 2*c^2*d^7 + 2*c*d^8
 + d^9)*f)*sin(f*x + e))]
```

giac [B] time = 0.59, size = 781, normalized size = 3.77

$$\frac{3\left(3a^2c-2a^2d\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(c^4+2c^3d-2cd^3-d^4\right)\sqrt{c^2-d^2}}+\frac{3a^2c^6\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-18a^2c^5d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+12a^2c^3d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+6a^2c^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^4,x, algorithm="giac")

[Out] 1/3*(3*(3*a^2*c - 2*a^2*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^4 + 2*c^3*d - 2*c*d^3 - d^4)*sqrt(c^2 - d^2)) + (3*a^2*c^6*tan(1/2*f*x + 1/2*e)^5 - 18*a^2*c^5*d*tan(1/2*f*x + 1/2*e)^5 + 12*a^2*c^3*d^3*tan(1/2*f*x + 1/2*e)^5 + 6*a^2*c^2*d^4*tan(1/2*f*x + 1/2*e)^5 - 12*a^2*c^6*tan(1/2*f*x + 1/2*e)^4 + 9*a^2*c^5*d*tan(1/2*f*x + 1/2*e)^4 - 54*a^2*c^4*d^2*tan(1/2*f*x + 1/2*e)^4 + 24*a^2*c^3*d^3*tan(1/2*f*x + 1/2*e)^4 + 36*a^2*c^2*d^4*tan(1/2*f*x + 1/2*e)^4 + 12*a^2*c*d^5*tan(1/2*f*x + 1/2*e)^4 - 72*a^2*c^5*d*tan(1/2*f*x + 1/2*e)^3 + 42*a^2*c^4*d^2*tan(1/2*f*x + 1/2*e)^3 - 12*a^2*c^3*d^3*tan(1/2*f*x + 1/2*e)^3 + 40*a^2*c^2*d^4*tan(1/2*f*x + 1/2*e)^3 + 24*a^2*c*d^5*tan(1/2*f*x + 1/2*e)^3 + 8*a^2*d^6*tan(1/2*f*x + 1/2*e)^3 - 24*a^2*c^6*tan(1/2*f*x + 1/2*e)^2 + 12*a^2*c^5*d*tan(1/2*f*x + 1/2*e)^2 - 72*a^2*c^4*d^2*tan(1/2*f*x + 1/2*e)^2 + 66*a^2*c^3*d^3*tan(1/2*f*x + 1/2*e)^2 + 36*a^2*c^2*d^4*tan(1/2*f*x + 1/2*e)^2 + 12*a^2*c*d^5*tan(1/2*f*x + 1/2*e)^2 - 3*a^2*c^6*tan(1/2*f*x + 1/2*e) - 54*a^2*c^5*d*tan(1/2*f*x + 1/2*e) + 42*a^2*c^4*d^2*tan(1/2*f*x + 1/2*e) + 24*a^2*c^3*d^3*tan(1/2*f*x + 1/2*e) + 6*a^2*c^2*d^4*tan(1/2*f*x + 1/2*e) - 12*a^2*c^6 + 7*a^2*c^5*d + 6*a^2*c^4*d^2 + 2*a^2*c^3*d^3)/((c^7 + 2*c^6*d - 2*c^4*d^3 - c^3*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^3))/f

maple [B] time = 0.36, size = 2425, normalized size = 11.71

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^4,x)

[Out] -2*a^2/f/(c^4+2*c^3*d-2*c*d^3-d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*d-4*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^3/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^4-8*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^3/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^2+a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^3/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^5+8*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^4*d^3+4*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^5*d^3-a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^3/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)+7/3*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^4+2*c^3*d-2*c*d^3-d^4)*c^2*d+8*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)*d^3+22*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^4+2*c^3*d-2*c

```
*d^3-d^4)*tan(1/2*f*x+1/2*e)^2*d^3-24*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1
/2*f*x+1/2*e)*d+c)^3*c^2*d/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^3+1
2*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/c/(c^4+2*c^3*d-
2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^4*d^4+4*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan
(1/2*f*x+1/2*e)*d+c)^3/c^2/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^4*d
^5-6*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^2/(c^4+2*c
^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^5*d+2*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*
tan(1/2*f*x+1/2*e)*d+c)^3/c/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^5*
d^4+3*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^2/(c^4+2*
c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^4*d-18*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+
2*tan(1/2*f*x+1/2*e)*d+c)^3*c/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^
4*d^2+2*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/c/(c^4+2*
c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)*d^4+14*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+
2*tan(1/2*f*x+1/2*e)*d+c)^3*c/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)*
d^2-18*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^2/(c^4+2
*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)*d-24*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2
*tan(1/2*f*x+1/2*e)*d+c)^3*c/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^2
*d^2+12*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/c/(c^4+2*
c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^2*d^4+4*a^2/f/(tan(1/2*f*x+1/2*e)^2*c
+2*tan(1/2*f*x+1/2*e)*d+c)^3/c^2/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*
e)^2*d^5+14*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c*d^2
/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^3+40/3*a^2/f/(tan(1/2*f*x+1/2
*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/c*d^4/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2
*f*x+1/2*e)^3+8*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/c
^2*d^5/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^3+8/3*a^2/f/(tan(1/2*f*
x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/c^3*d^6/(c^4+2*c^3*d-2*c*d^3-d^4)*
tan(1/2*f*x+1/2*e)^3+4*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d
+c)^3*c^2/(c^4+2*c^3*d-2*c*d^3-d^4)*tan(1/2*f*x+1/2*e)^2*d-4*a^2/f/(tan(1/2
*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*d^3/(c^4+2*c^3*d-2*c*d^3-d^4)*t
an(1/2*f*x+1/2*e)^3+2*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+
c)^3/(c^4+2*c^3*d-2*c*d^3-d^4)*c*d^2+3*a^2/f/(c^4+2*c^3*d-2*c*d^3-d^4)/(c^2
-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c-4*a^
2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^4+2*c^3*d-2*c*d^
3-d^4)*c^3+2/3*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c
^4+2*c^3*d-2*c*d^3-d^4)*d^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 10.32, size = 735, normalized size = 3.55

$$-\frac{\frac{-12\,a^2\,c^3+7\,a^2\,c^2\,d+6\,a^2\,c\,d^2+2\,a^2\,d^3}{3\,\left(-c^4-2\,c^3\,d+2\,c\,d^3+d^4\right)}+\frac{a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\left(c^4-6\,c^3\,d+4\,c\,d^3+2\,d^4\right)}{c\,\left(-c^4-2\,c^3\,d+2\,c\,d^3+d^4\right)}+\frac{2\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(-4\,c^5+2\,c^4\,d-12\,c^3\,d^2+11\,c^2\,d^3+6\,c\,d^4+2\,d^5 \right.}{c^2\,\left(-c^4-2\,c^3\,d+2\,c\,d^3+d^4\right)}}{f\,\left(c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(3\,c^3+12\,c\,d^2\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4 \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c + d*sin(e + f*x))^4,x)
```

```
[Out] - ((2*a^2*d^3 - 12*a^2*c^3 + 6*a^2*c*d^2 + 7*a^2*c^2*d)/(3*(2*c*d^3 - 2*c^3
*d - c^4 + d^4)) + (a^2*tan(e/2 + (f*x)/2)^5*(4*c*d^3 - 6*c^3*d + c^4 + 2*d
^4))/(c*(2*c*d^3 - 2*c^3*d - c^4 + d^4)) + (2*a^2*tan(e/2 + (f*x)/2)^2*(6*c
*d^4 + 2*c^4*d - 4*c^5 + 2*d^5 + 11*c^2*d^3 - 12*c^3*d^2))/(c^2*(2*c*d^3 -
2*c^3*d - c^4 + d^4)) + (a^2*tan(e/2 + (f*x)/2)^4*(12*c*d^4 + 3*c^4*d - 4*c
^5 + 4*d^5 + 8*c^2*d^3 - 18*c^3*d^2))/(c^2*(2*c*d^3 - 2*c^3*d - c^4 + d^4))
 + (a^2*tan(e/2 + (f*x)/2)*(8*c*d^3 - 18*c^3*d - c^4 + 2*d^4 + 14*c^2*d^2))
/(c*(2*c*d^3 - 2*c^3*d - c^4 + d^4)) + (2*a^2*d*tan(e/2 + (f*x)/2)^3*(3*c^2
 + 2*d^2)*(6*c*d^2 + 7*c^2*d - 12*c^3 + 2*d^3))/(3*c^3*(2*c*d^3 - 2*c^3*d -
 c^4 + d^4)))/(f*(c^3*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^2*(12*c*d^2
 + 3*c^3) + tan(e/2 + (f*x)/2)^4*(12*c*d^2 + 3*c^3) + tan(e/2 + (f*x)/2)^3*
(12*c^2*d + 8*d^3) + c^3 + 6*c^2*d*tan(e/2 + (f*x)/2) + 6*c^2*d*tan(e/2 + (
f*x)/2)^5)) - (a^2*atan((((a^2*(3*c - 2*d)*(4*c*d^4 - 2*c^4*d + 2*d^5 - 4*c
^3*d^2))/(2*(c + d)^(7/2)*(c - d)^(3/2)*(2*c*d^3 - 2*c^3*d - c^4 + d^4)) +
(a^2*c*tan(e/2 + (f*x)/2)*(3*c - 2*d))/((c + d)^(7/2)*(c - d)^(3/2)))*(2*c*
d^3 - 2*c^3*d - c^4 + d^4))/(3*a^2*c - 2*a^2*d))*(3*c - 2*d))/(f*(c + d)^(7
/2)*(c - d)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**4,x)
```

```
[Out] Timed out
```

3.443 $\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^5}\,dx$

Optimal. Leaf size=286

$$\frac{a^2\left(12c^2-16cd+7d^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{4f(c-d)^2(c+d)^4\sqrt{c^2-d^2}}-\frac{a^2\left(2c^2+16cd-21d^2\right)\cos(e+fx)}{24df(c-d)(c+d)^3(c+d\sin(e+fx))^2}-\frac{a^2\left(2c^3+16c^2d-59cd^2\right.}{24df(c-d)^2(c+d)^4(c}$$

```
[Out] 1/4*a^2*(c-d)*cos(f*x+e)/d/(c+d)/f/(c+d*sin(f*x+e))^4-1/12*a^2*(c+8*d)*cos(
f*x+e)/d/(c+d)^2/f/(c+d*sin(f*x+e))^3-1/24*a^2*(2*c^2+16*c*d-21*d^2)*cos(f*
x+e)/(c-d)/d/(c+d)^3/f/(c+d*sin(f*x+e))^2-1/24*a^2*(2*c^3+16*c^2*d-59*c*d^2
+32*d^3)*cos(f*x+e)/(c-d)^2/d/(c+d)^4/f/(c+d*sin(f*x+e))+1/4*a^2*(12*c^2-16
*c*d+7*d^2)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(c-d)^2/(c+d)^
4/f/(c^2-d^2)^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2762, 2754, 12, 2660, 618, 204}

$$\frac{a^2\left(12c^2-16cd+7d^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{4f(c-d)^2(c+d)^4\sqrt{c^2-d^2}}-\frac{a^2\left(16c^2d+2c^3-59cd^2+32d^3\right)\cos(e+fx)}{24df(c-d)^2(c+d)^4(c+d\sin(e+fx))}-\frac{a^2\left(2c^2+16cd-\right.}{24df(c-d)(c+d)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^5,x]
```

```
[Out] (a^2*(12*c^2 - 16*c*d + 7*d^2)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d
^2]])/(4*(c - d)^2*(c + d)^4*Sqrt[c^2 - d^2]*f) + (a^2*(c - d)*Cos[e + f*x]
)/(4*d*(c + d)*f*(c + d*Sin[e + f*x])^4) - (a^2*(c + 8*d)*Cos[e + f*x])/(12
*d*(c + d)^2*f*(c + d*Sin[e + f*x])^3) - (a^2*(2*c^2 + 16*c*d - 21*d^2)*Cos
[e + f*x])/(24*(c - d)*d*(c + d)^3*f*(c + d*Sin[e + f*x])^2) - (a^2*(2*c^3
+ 16*c^2*d - 59*c*d^2 + 32*d^3)*Cos[e + f*x])/(24*(c - d)^2*d*(c + d)^4*f*(
c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^5}\,dx = \frac{a^2(c-d)\cos(e+fx)}{4d(c+d)f(c+d\sin(e+fx))^4} - \frac{a\int \frac{-8ad-a(c+7d)\sin(e+fx)}{(c+d\sin(e+fx))^4}\,dx}{4d(c+d)}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{4d(c+d)f(c+d\sin(e+fx))^4} - \frac{a^2(c+8d)\cos(e+fx)}{12d(c+d)^2f(c+d\sin(e+fx))^3} + \frac{a\int \frac{21a(c-d)d+2a(}{(c+d}}{12(c-}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{4d(c+d)f(c+d\sin(e+fx))^4} - \frac{a^2(c+8d)\cos(e+fx)}{12d(c+d)^2f(c+d\sin(e+fx))^3} - \frac{a^2\left(2c^2+16c\right.}{24(c-d)d(c+}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{4d(c+d)f(c+d\sin(e+fx))^4} - \frac{a^2(c+8d)\cos(e+fx)}{12d(c+d)^2f(c+d\sin(e+fx))^3} - \frac{a^2\left(2c^2+16c\right.}{24(c-d)d(c+}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{4d(c+d)f(c+d\sin(e+fx))^4} - \frac{a^2(c+8d)\cos(e+fx)}{12d(c+d)^2f(c+d\sin(e+fx))^3} - \frac{a^2\left(2c^2+16c\right.}{24(c-d)d(c+}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{4d(c+d)f(c+d\sin(e+fx))^4} - \frac{a^2(c+8d)\cos(e+fx)}{12d(c+d)^2f(c+d\sin(e+fx))^3} - \frac{a^2\left(2c^2+16c\right.}{24(c-d)d(c+}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{4d(c+d)f(c+d\sin(e+fx))^4} - \frac{a^2(c+8d)\cos(e+fx)}{12d(c+d)^2f(c+d\sin(e+fx))^3} - \frac{a^2\left(2c^2+16c\right.}{24(c-d)d(c+}$$

$$= \frac{a^2\left(12c^2-16cd+7d^2\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{4(c-d)^2(c+d)^4\sqrt{c^2-d^2}\,f} + \frac{a^2(c-d)\cos(e+fx)}{4d(c+d)f(c+d\sin(e+fx))^4} - \frac{}{12}$$

Mathematica [A] time = 4.89, size = 269, normalized size = 0.94

$$-\frac{a^2\cos(e+fx)\left(\frac{\left(12c^2-16cd+7d^2\right)(c+d\sin(e+fx))^2\left(\sqrt{-c-d}\,\sqrt{c-d}\,\sqrt{\cos^2(e+fx)}\,((c+4d)\sin(e+fx)+4c+d)-6(c+d\sin(e+fx))^2\tanh^{-1}\left(\frac{\sqrt{c-d}\,\sqrt{1-}}{\sqrt{-c-d}\,\sqrt{\sin}}\right.\right.}{(-c-d)^{7/2}(c-d)^{3/2}\sqrt{\cos^2(e+fx)}}\right.}{24f(d-c)(c+d)(c+d\sin(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^5,x]
```

```
[Out] -1/24*(a^2*Cos[e + f*x]*(6*d*(1 + Sin[e + f*x])^2 + (2*(5*c - 2*d)*d*(1 + S
in[e + f*x])^2*(c + d*Sin[e + f*x]))/((c - d)*(c + d)) + ((12*c^2 - 16*c*d
+ 7*d^2)*(c + d*Sin[e + f*x])^2*(-6*ArcTanh[(Sqrt[c - d]*Sqrt[1 - Sin[e + f
*x]])/(Sqrt[-c - d]*Sqrt[1 + Sin[e + f*x]])]*(c + d*Sin[e + f*x])^2 + Sqrt[
```

```
-c - d]*Sqrt[c - d]*Sqrt[Cos[e + f*x]^2]*(4*c + d + (c + 4*d)*Sin[e + f*x])
))/((-c - d)^(7/2)*(c - d)^(3/2)*Sqrt[Cos[e + f*x]^2])))/((-c + d)*(c + d)*
f*(c + d*Sin[e + f*x])^4)
```

fricas [B] time = 0.62, size = 2151, normalized size = 7.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] [1/48*(2*(8*a^2*c^6*d + 64*a^2*c^5*d^2 - 208*a^2*c^4*d^3 + 16*a^2*c^3*d^4 +
 221*a^2*c^2*d^5 - 80*a^2*c*d^6 - 21*a^2*d^7)*cos(f*x + e)^3 - 3*(12*a^2*c^
6 - 16*a^2*c^5*d + 79*a^2*c^4*d^2 - 96*a^2*c^3*d^3 + 54*a^2*c^2*d^4 - 16*a^
2*c*d^5 + 7*a^2*d^6 + (12*a^2*c^2*d^4 - 16*a^2*c*d^5 + 7*a^2*d^6)*cos(f*x +
 e)^4 - 2*(36*a^2*c^4*d^2 - 48*a^2*c^3*d^3 + 33*a^2*c^2*d^4 - 16*a^2*c*d^5
+ 7*a^2*d^6)*cos(f*x + e)^2 + 4*(12*a^2*c^5*d - 16*a^2*c^4*d^2 + 19*a^2*c^3
*d^3 - 16*a^2*c^2*d^4 + 7*a^2*c*d^5 - (12*a^2*c^3*d^3 - 16*a^2*c^2*d^4 + 7*
a^2*c*d^5)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2
)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f
*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin
(f*x + e) - c^2 - d^2)) - 6*(16*a^2*c^7 - 20*a^2*c^6*d - 45*a^2*c^4*d^3 + 1
6*a^2*c^3*d^4 + 74*a^2*c^2*d^5 - 32*a^2*c*d^6 - 9*a^2*d^7)*cos(f*x + e) + 2
*((2*a^2*c^5*d^2 + 16*a^2*c^4*d^3 - 61*a^2*c^3*d^4 + 16*a^2*c^2*d^5 + 59*a^
2*c*d^6 - 32*a^2*d^7)*cos(f*x + e)^3 - 3*(4*a^2*c^7 + 32*a^2*c^6*d - 79*a^2
*c^5*d^2 - 16*a^2*c^4*d^3 + 70*a^2*c^3*d^4 + 5*a^2*c*d^6 - 16*a^2*d^7)*cos(
f*x + e))*sin(f*x + e))/((c^8*d^4 + 2*c^7*d^5 - 2*c^6*d^6 - 6*c^5*d^7 + 6*c
^3*d^9 + 2*c^2*d^10 - 2*c*d^11 - d^12)*f*cos(f*x + e)^4 - 2*(3*c^10*d^2 + 6
*c^9*d^3 - 5*c^8*d^4 - 16*c^7*d^5 - 2*c^6*d^6 + 12*c^5*d^7 + 6*c^4*d^8 - c^
2*d^10 - 2*c*d^11 - d^12)*f*cos(f*x + e)^2 + (c^12 + 2*c^11*d + 4*c^10*d^2
+ 6*c^9*d^3 - 11*c^8*d^4 - 28*c^7*d^5 + 28*c^5*d^7 + 11*c^4*d^8 - 6*c^3*d^9
 - 4*c^2*d^10 - 2*c*d^11 - d^12)*f - 4*((c^9*d^3 + 2*c^8*d^4 - 2*c^7*d^5 -
6*c^6*d^6 + 6*c^4*d^8 + 2*c^3*d^9 - 2*c^2*d^10 - c*d^11)*f*cos(f*x + e)^2 -
 (c^11*d + 2*c^10*d^2 - c^9*d^3 - 4*c^8*d^4 - 2*c^7*d^5 + 2*c^5*d^7 + 4*c^4
*d^8 + c^3*d^9 - 2*c^2*d^10 - c*d^11)*f)*sin(f*x + e)), 1/24*((8*a^2*c^6*d
+ 64*a^2*c^5*d^2 - 208*a^2*c^4*d^3 + 16*a^2*c^3*d^4 + 221*a^2*c^2*d^5 - 80*
a^2*c*d^6 - 21*a^2*d^7)*cos(f*x + e)^3 - 3*(12*a^2*c^6 - 16*a^2*c^5*d + 79*
a^2*c^4*d^2 - 96*a^2*c^3*d^3 + 54*a^2*c^2*d^4 - 16*a^2*c*d^5 + 7*a^2*d^6 +
(12*a^2*c^2*d^4 - 16*a^2*c*d^5 + 7*a^2*d^6)*cos(f*x + e)^4 - 2*(36*a^2*c^4*
d^2 - 48*a^2*c^3*d^3 + 33*a^2*c^2*d^4 - 16*a^2*c*d^5 + 7*a^2*d^6)*cos(f*x +
 e)^2 + 4*(12*a^2*c^5*d - 16*a^2*c^4*d^2 + 19*a^2*c^3*d^3 - 16*a^2*c^2*d^4
+ 7*a^2*c*d^5 - (12*a^2*c^3*d^3 - 16*a^2*c^2*d^4 + 7*a^2*c*d^5)*cos(f*x + e
)^2)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 -
 d^2)*cos(f*x + e))) - 3*(16*a^2*c^7 - 20*a^2*c^6*d - 45*a^2*c^4*d^3 + 16*a
^2*c^3*d^4 + 74*a^2*c^2*d^5 - 32*a^2*c*d^6 - 9*a^2*d^7)*cos(f*x + e) + ((2*
```

```
a^2*c^5*d^2 + 16*a^2*c^4*d^3 - 61*a^2*c^3*d^4 + 16*a^2*c^2*d^5 + 59*a^2*c*d
^6 - 32*a^2*d^7)*cos(f*x + e)^3 - 3*(4*a^2*c^7 + 32*a^2*c^6*d - 79*a^2*c^5*
d^2 - 16*a^2*c^4*d^3 + 70*a^2*c^3*d^4 + 5*a^2*c*d^6 - 16*a^2*d^7)*cos(f*x +
 e))*sin(f*x + e))/((c^8*d^4 + 2*c^7*d^5 - 2*c^6*d^6 - 6*c^5*d^7 + 6*c^3*d^
9 + 2*c^2*d^10 - 2*c*d^11 - d^12)*f*cos(f*x + e)^4 - 2*(3*c^10*d^2 + 6*c^9*
d^3 - 5*c^8*d^4 - 16*c^7*d^5 - 2*c^6*d^6 + 12*c^5*d^7 + 6*c^4*d^8 - c^2*d^1
0 - 2*c*d^11 - d^12)*f*cos(f*x + e)^2 + (c^12 + 2*c^11*d + 4*c^10*d^2 + 6*c
^9*d^3 - 11*c^8*d^4 - 28*c^7*d^5 + 28*c^5*d^7 + 11*c^4*d^8 - 6*c^3*d^9 - 4*
c^2*d^10 - 2*c*d^11 - d^12)*f - 4*((c^9*d^3 + 2*c^8*d^4 - 2*c^7*d^5 - 6*c^6
*d^6 + 6*c^4*d^8 + 2*c^3*d^9 - 2*c^2*d^10 - c*d^11)*f*cos(f*x + e)^2 - (c^1
1*d + 2*c^10*d^2 - c^9*d^3 - 4*c^8*d^4 - 2*c^7*d^5 + 2*c^5*d^7 + 4*c^4*d^8
+ c^3*d^9 - 2*c^2*d^10 - c*d^11)*f)*sin(f*x + e))]
```

giac [B] time = 2.32, size = 1555, normalized size = 5.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^5,x, algorithm="giac")
```

```
[Out] 1/12*(3*(12*a^2*c^2 - 16*a^2*c*d + 7*a^2*d^2)*(pi*floor(1/2*(f*x + e)/pi +
1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^6 +
 2*c^5*d - c^4*d^2 - 4*c^3*d^3 - c^2*d^4 + 2*c*d^5 + d^6)*sqrt(c^2 - d^2))
+ (12*a^2*c^9*tan(1/2*f*x + 1/2*e)^7 - 96*a^2*c^8*d*tan(1/2*f*x + 1/2*e)^7
+ 45*a^2*c^7*d^2*tan(1/2*f*x + 1/2*e)^7 + 96*a^2*c^6*d^3*tan(1/2*f*x + 1/2*
e)^7 + 24*a^2*c^5*d^4*tan(1/2*f*x + 1/2*e)^7 - 48*a^2*c^4*d^5*tan(1/2*f*x +
 1/2*e)^7 - 24*a^2*c^3*d^6*tan(1/2*f*x + 1/2*e)^7 - 48*a^2*c^9*tan(1/2*f*x
+ 1/2*e)^6 + 84*a^2*c^8*d*tan(1/2*f*x + 1/2*e)^6 - 432*a^2*c^7*d^2*tan(1/2*
f*x + 1/2*e)^6 + 411*a^2*c^6*d^3*tan(1/2*f*x + 1/2*e)^6 + 336*a^2*c^5*d^4*t
an(1/2*f*x + 1/2*e)^6 - 24*a^2*c^4*d^5*tan(1/2*f*x + 1/2*e)^6 - 192*a^2*c^3
*d^6*tan(1/2*f*x + 1/2*e)^6 - 72*a^2*c^2*d^7*tan(1/2*f*x + 1/2*e)^6 + 12*a^
2*c^9*tan(1/2*f*x + 1/2*e)^5 - 480*a^2*c^8*d*tan(1/2*f*x + 1/2*e)^5 + 597*a
^2*c^7*d^2*tan(1/2*f*x + 1/2*e)^5 - 480*a^2*c^6*d^3*tan(1/2*f*x + 1/2*e)^5
+ 836*a^2*c^5*d^4*tan(1/2*f*x + 1/2*e)^5 + 208*a^2*c^4*d^5*tan(1/2*f*x + 1/
2*e)^5 - 152*a^2*c^3*d^6*tan(1/2*f*x + 1/2*e)^5 - 256*a^2*c^2*d^7*tan(1/2*f
*x + 1/2*e)^5 - 96*a^2*c*d^8*tan(1/2*f*x + 1/2*e)^5 - 144*a^2*c^9*tan(1/2*f
*x + 1/2*e)^4 + 204*a^2*c^8*d*tan(1/2*f*x + 1/2*e)^4 - 1104*a^2*c^7*d^2*tan
(1/2*f*x + 1/2*e)^4 + 1617*a^2*c^6*d^3*tan(1/2*f*x + 1/2*e)^4 - 48*a^2*c^5*
d^4*tan(1/2*f*x + 1/2*e)^4 + 406*a^2*c^4*d^5*tan(1/2*f*x + 1/2*e)^4 - 256*a
^2*c^3*d^6*tan(1/2*f*x + 1/2*e)^4 - 184*a^2*c^2*d^7*tan(1/2*f*x + 1/2*e)^4
- 128*a^2*c*d^8*tan(1/2*f*x + 1/2*e)^4 - 48*a^2*d^9*tan(1/2*f*x + 1/2*e)^4
- 12*a^2*c^9*tan(1/2*f*x + 1/2*e)^3 - 672*a^2*c^8*d*tan(1/2*f*x + 1/2*e)^3
+ 1035*a^2*c^7*d^2*tan(1/2*f*x + 1/2*e)^3 - 672*a^2*c^6*d^3*tan(1/2*f*x + 1
/2*e)^3 + 1220*a^2*c^5*d^4*tan(1/2*f*x + 1/2*e)^3 - 80*a^2*c^4*d^5*tan(1/2*
f*x + 1/2*e)^3 - 152*a^2*c^3*d^6*tan(1/2*f*x + 1/2*e)^3 - 256*a^2*c^2*d^7*t
```

```
an(1/2*f*x + 1/2*e)^3 - 96*a^2*c*d^8*tan(1/2*f*x + 1/2*e)^3 - 144*a^2*c^9*t
an(1/2*f*x + 1/2*e)^2 + 188*a^2*c^8*d*tan(1/2*f*x + 1/2*e)^2 - 656*a^2*c^7*
d^2*tan(1/2*f*x + 1/2*e)^2 + 1201*a^2*c^6*d^3*tan(1/2*f*x + 1/2*e)^2 - 16*a
^2*c^5*d^4*tan(1/2*f*x + 1/2*e)^2 - 120*a^2*c^4*d^5*tan(1/2*f*x + 1/2*e)^2
- 192*a^2*c^3*d^6*tan(1/2*f*x + 1/2*e)^2 - 72*a^2*c^2*d^7*tan(1/2*f*x + 1/2
*e)^2 - 12*a^2*c^9*tan(1/2*f*x + 1/2*e) - 288*a^2*c^8*d*tan(1/2*f*x + 1/2*e
) + 499*a^2*c^7*d^2*tan(1/2*f*x + 1/2*e) + 32*a^2*c^6*d^3*tan(1/2*f*x + 1/2
*e) - 64*a^2*c^5*d^4*tan(1/2*f*x + 1/2*e) - 80*a^2*c^4*d^5*tan(1/2*f*x + 1/
2*e) - 24*a^2*c^3*d^6*tan(1/2*f*x + 1/2*e) - 48*a^2*c^9 + 68*a^2*c^8*d + 16
*a^2*c^7*d^2 - 5*a^2*c^6*d^3 - 16*a^2*c^5*d^4 - 6*a^2*c^4*d^5)/((c^10 + 2*c
^9*d - c^8*d^2 - 4*c^7*d^3 - c^6*d^4 + 2*c^5*d^5 + c^4*d^6)*(c*tan(1/2*f*x
+ 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^4))/f
```

maple [B] time = 0.41, size = 6466, normalized size = 22.61

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^5,x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 10.35, size = 1411, normalized size = 4.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c + d*sin(e + f*x))^5,x)
```

```
[Out] (a^2*atan((4*((a^2*(12*c^2 - 16*c*d + 7*d^2)*(16*c*d^6 + 8*c^6*d + 8*d^7 -
8*c^2*d^5 - 32*c^3*d^4 - 8*c^4*d^3 + 16*c^5*d^2))/(32*(c + d)^(9/2)*(c - d)
^(5/2)*(2*c*d^5 + 2*c^5*d + c^6 + d^6 - c^2*d^4 - 4*c^3*d^3 - c^4*d^2)) + (
```

```
a^2*c*tan(e/2 + (f*x)/2)*(12*c^2 - 16*c*d + 7*d^2))/(4*(c + d)^(9/2)*(c - d
)^(5/2)))*(2*c*d^5 + 2*c^5*d + c^6 + d^6 - c^2*d^4 - 4*c^3*d^3 - c^4*d^2))/
(12*a^2*c^2 + 7*a^2*d^2 - 16*a^2*c*d))*(12*c^2 - 16*c*d + 7*d^2))/(4*f*(c +
 d)^(9/2)*(c - d)^(5/2)) - ((48*a^2*c^5 + 6*a^2*d^5 + 16*a^2*c*d^4 - 68*a^2
*c^4*d + 5*a^2*c^2*d^3 - 16*a^2*c^3*d^2)/(12*(2*c*d^5 + 2*c^5*d + c^6 + d^6
 - c^2*d^4 - 4*c^3*d^3 - c^4*d^2)) + (a^2*tan(e/2 + (f*x)/2)*(80*c*d^5 + 28
8*c^5*d + 12*c^6 + 24*d^6 + 64*c^2*d^4 - 32*c^3*d^3 - 499*c^4*d^2))/(12*c*(
2*c*d^5 + 2*c^5*d + c^6 + d^6 - c^2*d^4 - 4*c^3*d^3 - c^4*d^2)) + (a^2*tan(
e/2 + (f*x)/2)^5*(256*c*d^7 + 480*c^7*d - 12*c^8 + 96*d^8 + 152*c^2*d^6 - 2
08*c^3*d^5 - 836*c^4*d^4 + 480*c^5*d^3 - 597*c^6*d^2))/(12*c^3*(2*c*d^5 + 2
*c^5*d + c^6 + d^6 - c^2*d^4 - 4*c^3*d^3 - c^4*d^2)) + (a^2*tan(e/2 + (f*x)
/2)^3*(256*c*d^7 + 672*c^7*d + 12*c^8 + 96*d^8 + 152*c^2*d^6 + 80*c^3*d^5 -
 1220*c^4*d^4 + 672*c^5*d^3 - 1035*c^6*d^2))/(12*c^3*(2*c*d^5 + 2*c^5*d + c
^6 + d^6 - c^2*d^4 - 4*c^3*d^3 - c^4*d^2)) + (a^2*tan(e/2 + (f*x)/2)^6*(64*
c*d^6 - 28*c^6*d + 16*c^7 + 24*d^7 + 8*c^2*d^5 - 112*c^3*d^4 - 137*c^4*d^3
+ 144*c^5*d^2))/(4*c^2*(2*c*d^5 + 2*c^5*d + c^6 + d^6 - c^2*d^4 - 4*c^3*d^3
 - c^4*d^2)) + (a^2*tan(e/2 + (f*x)/2)^2*(192*c*d^6 - 188*c^6*d + 144*c^7 +
 72*d^7 + 120*c^2*d^5 + 16*c^3*d^4 - 1201*c^4*d^3 + 656*c^5*d^2))/(12*c^2*(
2*c*d^5 + 2*c^5*d + c^6 + d^6 - c^2*d^4 - 4*c^3*d^3 - c^4*d^2)) - (a^2*tan(
e/2 + (f*x)/2)^7*(4*c^6 - 32*c^5*d - 16*c*d^5 - 8*d^6 + 8*c^2*d^4 + 32*c^3*
d^3 + 15*c^4*d^2))/(4*c*(2*c*d^5 + 2*c^5*d + c^6 + d^6 - c^2*d^4 - 4*c^3*d^
3 - c^4*d^2)) + (a^2*tan(e/2 + (f*x)/2)^4*(3*c^4 + 8*d^4 + 24*c^2*d^2)*(16*
c*d^4 - 68*c^4*d + 48*c^5 + 6*d^5 + 5*c^2*d^3 - 16*c^3*d^2))/(12*c^4*(2*c*d
^5 + 2*c^5*d + c^6 + d^6 - c^2*d^4 - 4*c^3*d^3 - c^4*d^2)))/(f*(tan(e/2 + (
f*x)/2)^4*(6*c^4 + 16*d^4 + 48*c^2*d^2) + c^4*tan(e/2 + (f*x)/2)^8 + c^4 +
tan(e/2 + (f*x)/2)^2*(4*c^4 + 24*c^2*d^2) + tan(e/2 + (f*x)/2)^6*(4*c^4 + 2
4*c^2*d^2) + tan(e/2 + (f*x)/2)^3*(32*c*d^3 + 24*c^3*d) + tan(e/2 + (f*x)/2
)^5*(32*c*d^3 + 24*c^3*d) + 8*c^3*d*tan(e/2 + (f*x)/2) + 8*c^3*d*tan(e/2 +
(f*x)/2)^7))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**5,x)
```

```
[Out] Timed out
```

3.444 $\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=215

$$-\frac{a^3d\left(18c^2+54cd+23d^2\right)\sin^3(e+fx)\cos(e+fx)}{24f}-\frac{a^3\left(24c^3+90c^2d+78cd^2+23d^3\right)\sin(e+fx)\cos(e+fx)}{16f}$$

```
[Out] 1/16*a^3*(40*c^3+90*c^2*d+78*c*d^2+23*d^3)*x-4*a^3*(c+d)^3*cos(f*x+e)/f+1/3
*a^3*(c+d)^2*(c+7*d)*cos(f*x+e)^3/f-3/5*a^3*d^2*(c+d)*cos(f*x+e)^5/f-1/16*a
^3*(24*c^3+90*c^2*d+78*c*d^2+23*d^3)*cos(f*x+e)*sin(f*x+e)/f-1/24*a^3*d*(18
*c^2+54*c*d+23*d^2)*cos(f*x+e)*sin(f*x+e)^3/f-1/6*a^3*d^3*cos(f*x+e)*sin(f*
x+e)^5/f
```

Rubi [A] time = 0.54, antiderivative size = 326, normalized size of antiderivative = 1.52, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2763, 2968, 3023, 2753, 2734}

$$-\frac{a^3\left(107c^3d^2+472c^2d^3-18c^4d+2c^5+456cd^4+136d^5\right)\cos(e+fx)}{60d^2f}-\frac{a^3\left(2c^2-18cd+115d^2\right)\cos(e+fx)(c+}{120d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a^3*(40*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)*x)/16 - (a^3*(2*c^5 - 18*c^4*d
 + 107*c^3*d^2 + 472*c^2*d^3 + 456*c*d^4 + 136*d^5)*Cos[e + f*x])/(60*d^2*f
) - (a^3*(4*c^4 - 36*c^3*d + 216*c^2*d^2 + 626*c*d^3 + 345*d^4)*Cos[e + f*x
]*Sin[e + f*x])/(240*d*f) - (a^3*(2*c^3 - 18*c^2*d + 111*c*d^2 + 136*d^3)*C
os[e + f*x]*(c + d*Sin[e + f*x])^2)/(120*d^2*f) - (a^3*(2*c^2 - 18*c*d + 11
5*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(120*d^2*f) + (a^3*(2*c - 13*d)
*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(30*d^2*f) - (Cos[e + f*x]*(a^3 + a^3
*Sin[e + f*x])*(c + d*Sin[e + f*x])^4)/(6*d*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
```

```
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^3\,dx = -\frac{\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^4}{6df}+\frac{\int(a+$$

$$= -\frac{\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^4}{6df}+\frac{\int(c+$$

$$= \frac{a^3(2c-13d)\cos(e+fx)(c+d\sin(e+fx))^4}{30d^2f}-\frac{\cos(e+fx)\left(a^3+\right.}{}$$

$$= -\frac{a^3\left(2c^2-18cd+115d^2\right)\cos(e+fx)(c+d\sin(e+fx))^3}{120d^2f}+\frac{a^3(2}{}$$

$$= -\frac{a^3\left(2c^3-18c^2d+111cd^2+136d^3\right)\cos(e+fx)(c+d\sin(e+f}{120d^2f}$$

$$= \frac{1}{16}a^3\left(40c^3+90c^2d+78cd^2+23d^3\right)x-\frac{a^3\left(2c^5-18c^4d+107c\right.}{}$$

Mathematica [A] time = 1.39, size = 233, normalized size = 1.08

$$-\frac{a^3\cos(e+fx)\left(30\left(40c^3+90c^2d+78cd^2+23d^3\right)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\left(10d\left(18c^2+54cd+2\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] -1/240*(a^3*Cos[e + f*x]*(30*(40*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)*ArcSin
[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(16*(55*c^3 + 135*c
^2*d + 114*c*d^2 + 34*d^3) + 15*(24*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)*Sin
[e + f*x] + 16*(5*c^3 + 45*c^2*d + 57*c*d^2 + 17*d^3)*Sin[e + f*x]^2 + 10*d
*(18*c^2 + 54*c*d + 23*d^2)*Sin[e + f*x]^3 + 144*d^2*(c + d)*Sin[e + f*x]^4
 + 40*d^3*Sin[e + f*x]^5)))/(f*Sqrt[Cos[e + f*x]^2])
```

fricas [A] time = 0.48, size = 261, normalized size = 1.21

$$-\frac{144\left(a^3cd^2+a^3d^3\right)\cos\left(fx+e\right)^5-80\left(a^3c^3+9\,a^3c^2d+15\,a^3cd^2+7\,a^3d^3\right)\cos\left(fx+e\right)^3-15\left(40\,a^3c^3+90\,a^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/240*(144*(a^3*c*d^2 + a^3*d^3)*cos(f*x + e)^5 - 80*(a^3*c^3 + 9*a^3*c^2*
d + 15*a^3*c*d^2 + 7*a^3*d^3)*cos(f*x + e)^3 - 15*(40*a^3*c^3 + 90*a^3*c^2*
d + 78*a^3*c*d^2 + 23*a^3*d^3)*f*x + 960*(a^3*c^3 + 3*a^3*c^2*d + 3*a^3*c*d
^2 + a^3*d^3)*cos(f*x + e) + 5*(8*a^3*d^3*cos(f*x + e)^5 - 2*(18*a^3*c^2*d
+ 54*a^3*c*d^2 + 31*a^3*d^3)*cos(f*x + e)^3 + 3*(24*a^3*c^3 + 102*a^3*c^2*d
 + 114*a^3*c*d^2 + 41*a^3*d^3)*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.27, size = 373, normalized size = 1.73

$$\frac{a^3d^3\cos\left(3\,fx+3\,e\right)}{12\,f}-\frac{a^3d^3\sin\left(6\,fx+6\,e\right)}{192\,f}-\frac{3\,a^3cd^2\sin\left(2\,fx+2\,e\right)}{4\,f}+\frac{1}{16}\left(24\,a^3c^3+90\,a^3c^2d+54\,a^3cd^2+23\,a^3d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/12*a^3*d^3*cos(3*f*x + 3*e)/f - 1/192*a^3*d^3*sin(6*f*x + 6*e)/f - 3/4*a^
3*c*d^2*sin(2*f*x + 2*e)/f + 1/16*(24*a^3*c^3 + 90*a^3*c^2*d + 54*a^3*c*d^2
 + 23*a^3*d^3)*x + 1/2*(2*a^3*c^3 + 3*a^3*c*d^2)*x - 3/80*(a^3*c*d^2 + a^3*
d^3)*cos(5*f*x + 5*e)/f + 1/48*(4*a^3*c^3 + 36*a^3*c^2*d + 51*a^3*c*d^2 + 1
5*a^3*d^3)*cos(3*f*x + 3*e)/f - 3/8*(10*a^3*c^3 + 18*a^3*c^2*d + 23*a^3*c*d
^2 + 5*a^3*d^3)*cos(f*x + e)/f - 3/4*(4*a^3*c^2*d + a^3*d^3)*cos(f*x + e)/f
 + 3/64*(2*a^3*c^2*d + 6*a^3*c*d^2 + 3*a^3*d^3)*sin(4*f*x + 4*e)/f - 3/64*(
16*a^3*c^3 + 64*a^3*c^2*d + 48*a^3*c*d^2 + 21*a^3*d^3)*sin(2*f*x + 2*e)/f
```

maple [B] time = 0.40, size = 481, normalized size = 2.24

$$\frac{-\frac{a^3c^3\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}+3a^3c^2d\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{3a^3c\,d^2\left(\frac{8}{3}+\sin^4\left(fx+e\right)+\frac{4\left(\sin^2\left(fx+e\right)\right)}{3}\right)\cos}{5}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(-1/3*a^3*c^3*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a^3*c^2*d*(-1/4*(sin(f*x+e)
^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3/5*a^3*c*d^2*(8/3+sin(f*x+e)^
4+4/3*sin(f*x+e)^2)*cos(f*x+e)+a^3*d^3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3
+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+3*a^3*c^3*(-1/2*sin(f*x+e)*co
s(f*x+e)+1/2*f*x+1/2*e)-3*a^3*c^2*d*(2+sin(f*x+e)^2)*cos(f*x+e)+9*a^3*c*d^2
*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3/5*a^3*d^3*
(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-3*a^3*c^3*cos(f*x+e)+9*a^3*c
^2*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-3*a^3*c*d^2*(2+sin(f*x+e)^2
```

```
)*cos(f*x+e)+3*a^3*d^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f
*x+3/8*e)+a^3*c^3*(f*x+e)-3*a^3*c^2*d*cos(f*x+e)+3*a^3*c*d^2*(-1/2*sin(f*x+
e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/3*a^3*d^3*(2+sin(f*x+e)^2)*cos(f*x+e))
```

maxima [B] time = 0.36, size = 469, normalized size = 2.18

$$\frac{320\left(\cos\left(fx+e\right)^{3}-3\cos\left(fx+e\right)\right)a^{3}c^{3}+720\left(2fx+2e-\sin\left(2fx+2e\right)\right)a^{3}c^{3}+960\left(fx+e\right)a^{3}c^{3}+2880\left(\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/960*(320*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3*c^3 + 720*(2*f*x + 2*e - s
in(2*f*x + 2*e))*a^3*c^3 + 960*(f*x + e)*a^3*c^3 + 2880*(cos(f*x + e)^3 - 3
*cos(f*x + e))*a^3*c^2*d + 90*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f
*x + 2*e))*a^3*c^2*d + 2160*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^3*c^2*d - 19
2*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a^3*c*d^2 + 2880
*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3*c*d^2 + 270*(12*f*x + 12*e + sin(4*f
*x + 4*e) - 8*sin(2*f*x + 2*e))*a^3*c*d^2 + 720*(2*f*x + 2*e - sin(2*f*x +
2*e))*a^3*c*d^2 - 192*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x +
e))*a^3*d^3 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3*d^3 + 5*(4*sin(2*f*
x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*a^3*
d^3 + 90*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^3*d^3 -
2880*a^3*c^3*cos(f*x + e) - 2880*a^3*c^2*d*cos(f*x + e))/f
```

mupad [B] time = 8.48, size = 773, normalized size = 3.60

$$\frac{a^{3}\,\mathrm{atan}\left(\frac{a^{3}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(40c^{3}+90c^{2}d+78cd^{2}+23d^{3}\right)}{8\left(5a^{3}c^{3}+\frac{45a^{3}c^{2}d}{4}+\frac{39a^{3}cd^{2}}{4}+\frac{23a^{3}d^{3}}{8}\right)}\right)\left(40c^{3}+90c^{2}d+78cd^{2}+23d^{3}\right)}{8f}-\frac{a^{3}\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)-\frac{fx}{2}\right)\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^3,x)
```

```
[Out] (a^3*atan((a^3*tan(e/2 + (f*x)/2)*(78*c*d^2 + 90*c^2*d + 40*c^3 + 23*d^3))/
(8*(5*a^3*c^3 + (23*a^3*d^3)/8 + (39*a^3*c*d^2)/4 + (45*a^3*c^2*d)/4)))*(78
*c*d^2 + 90*c^2*d + 40*c^3 + 23*d^3))/(8*f) - (a^3*(atan(tan(e/2 + (f*x)/2)
) - (f*x)/2)*(78*c*d^2 + 90*c^2*d + 40*c^3 + 23*d^3))/(8*f) - (tan(e/2 + (f
*x)/2)^10*(6*a^3*c^3 + 6*a^3*c^2*d) - tan(e/2 + (f*x)/2)^11*(3*a^3*c^3 + (2
3*a^3*d^3)/8 + (39*a^3*c*d^2)/4 + (45*a^3*c^2*d)/4) + tan(e/2 + (f*x)/2)^8*
(34*a^3*c^3 + 4*a^3*d^3 + 36*a^3*c*d^2 + 66*a^3*c^2*d) + tan(e/2 + (f*x)/2)
```

```
^5*(6*a^3*c^3 + (75*a^3*d^3)/4 + (75*a^3*c*d^2)/2 + (57*a^3*c^2*d)/2) - tan
(e/2 + (f*x)/2)^7*(6*a^3*c^3 + (75*a^3*d^3)/4 + (75*a^3*c*d^2)/2 + (57*a^3*
c^2*d)/2) + tan(e/2 + (f*x)/2)^4*(76*a^3*c^3 + 64*a^3*d^3 + 192*a^3*c*d^2 +
 204*a^3*c^2*d) + tan(e/2 + (f*x)/2)^6*((220*a^3*c^3)/3 + (136*a^3*d^3)/3 +
 152*a^3*c*d^2 + 180*a^3*c^2*d) + tan(e/2 + (f*x)/2)^2*(38*a^3*c^3 + (136*a
^3*d^3)/5 + (456*a^3*c*d^2)/5 + 102*a^3*c^2*d) + tan(e/2 + (f*x)/2)^3*(9*a^
3*c^3 + (391*a^3*d^3)/24 + (189*a^3*c*d^2)/4 + (159*a^3*c^2*d)/4) - tan(e/2
 + (f*x)/2)^9*(9*a^3*c^3 + (391*a^3*d^3)/24 + (189*a^3*c*d^2)/4 + (159*a^3*
c^2*d)/4) + (22*a^3*c^3)/3 + (68*a^3*d^3)/15 + tan(e/2 + (f*x)/2)*(3*a^3*c^
3 + (23*a^3*d^3)/8 + (39*a^3*c*d^2)/4 + (45*a^3*c^2*d)/4) + (76*a^3*c*d^2)/
5 + 18*a^3*c^2*d)/(f*(6*tan(e/2 + (f*x)/2)^2 + 15*tan(e/2 + (f*x)/2)^4 + 20
*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 + 6*tan(e/2 + (f*x)/2)^10 +
 tan(e/2 + (f*x)/2)^12 + 1))
```

sympy [A] time = 9.29, size = 1176, normalized size = 5.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((3*a**3*c**3*x*sin(e + f*x)**2/2 + 3*a**3*c**3*x*cos(e + f*x)**2/
2 + a**3*c**3*x - a**3*c**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*a**3*c**3*si
n(e + f*x)*cos(e + f*x)/(2*f) - 2*a**3*c**3*cos(e + f*x)**3/(3*f) - 3*a**3*
c**3*cos(e + f*x)/f + 9*a**3*c**2*d*x*sin(e + f*x)**4/8 + 9*a**3*c**2*d*x*s
in(e + f*x)**2*cos(e + f*x)**2/4 + 9*a**3*c**2*d*x*sin(e + f*x)**2/2 + 9*a*
*3*c**2*d*x*cos(e + f*x)**4/8 + 9*a**3*c**2*d*x*cos(e + f*x)**2/2 - 15*a**3
*c**2*d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 9*a**3*c**2*d*sin(e + f*x)**2*
cos(e + f*x)/f - 9*a**3*c**2*d*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 9*a**3*
c**2*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 6*a**3*c**2*d*cos(e + f*x)**3/f -
3*a**3*c**2*d*cos(e + f*x)/f + 27*a**3*c*d**2*x*sin(e + f*x)**4/8 + 27*a**3
*c*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*a**3*c*d**2*x*sin(e + f*x)*
*2/2 + 27*a**3*c*d**2*x*cos(e + f*x)**4/8 + 3*a**3*c*d**2*x*cos(e + f*x)**2
/2 - 3*a**3*c*d**2*sin(e + f*x)**4*cos(e + f*x)/f - 45*a**3*c*d**2*sin(e +
f*x)**3*cos(e + f*x)/(8*f) - 4*a**3*c*d**2*sin(e + f*x)**2*cos(e + f*x)**3/
f - 9*a**3*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 27*a**3*c*d**2*sin(e + f
*x)*cos(e + f*x)**3/(8*f) - 3*a**3*c*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) -
 8*a**3*c*d**2*cos(e + f*x)**5/(5*f) - 6*a**3*c*d**2*cos(e + f*x)**3/f + 5*
a**3*d**3*x*sin(e + f*x)**6/16 + 15*a**3*d**3*x*sin(e + f*x)**4*cos(e + f*x
)**2/16 + 9*a**3*d**3*x*sin(e + f*x)**4/8 + 15*a**3*d**3*x*sin(e + f*x)**2*
cos(e + f*x)**4/16 + 9*a**3*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 5*a*
*3*d**3*x*cos(e + f*x)**6/16 + 9*a**3*d**3*x*cos(e + f*x)**4/8 - 11*a**3*d*
*3*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 3*a**3*d**3*sin(e + f*x)**4*cos(e
+ f*x)/f - 5*a**3*d**3*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 15*a**3*d**3
*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*a**3*d**3*sin(e + f*x)**2*cos(e + f
```

```
*x)**3/f - a**3*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 5*a**3*d**3*sin(e + f
*x)*cos(e + f*x)**5/(16*f) - 9*a**3*d**3*sin(e + f*x)*cos(e + f*x)**3/(8*f)
 - 8*a**3*d**3*cos(e + f*x)**5/(5*f) - 2*a**3*d**3*cos(e + f*x)**3/(3*f), N
e(f, 0)), (x*(c + d*sin(e))**3*(a*sin(e) + a)**3, True))
```

3.445 $\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=164

$$\frac{a^3\left(c^2+6cd+5d^2\right)\cos^3(e+fx)}{3f}-\frac{a^3\left(12c^2+30cd+13d^2\right)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}a^3x\left(20c^2+30cd+13d^2\right)-$$

```
[Out] 1/8*a^3*(20*c^2+30*c*d+13*d^2)*x-4*a^3*(c+d)^2*cos(f*x+e)/f+1/3*a^3*(c^2+6*
c*d+5*d^2)*cos(f*x+e)^3/f-1/5*a^3*d^2*cos(f*x+e)^5/f-1/8*a^3*(12*c^2+30*c*d
+13*d^2)*cos(f*x+e)*sin(f*x+e)/f-1/4*a^3*d*(2*c+3*d)*cos(f*x+e)*sin(f*x+e)^
3/f
```

Rubi [A] time = 0.26, antiderivative size = 189, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2761, 2751, 2645, 2638, 2635, 8, 2633}

$$\frac{a^3\left(20c^2+30cd+13d^2\right)\cos^3(e+fx)}{60f}-\frac{a^3\left(20c^2+30cd+13d^2\right)\cos(e+fx)}{5f}-\frac{3a^3\left(20c^2+30cd+13d^2\right)\sin(e+f$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a^3*(20*c^2 + 30*c*d + 13*d^2)*x)/8 - (a^3*(20*c^2 + 30*c*d + 13*d^2)*Cos[
e + f*x])/(5*f) + (a^3*(20*c^2 + 30*c*d + 13*d^2)*Cos[e + f*x]^3)/(60*f) -
(3*a^3*(20*c^2 + 30*c*d + 13*d^2)*Cos[e + f*x]*Sin[e + f*x])/(40*f) - ((10*
c - d)*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^3)/(20*f) - (d^2*Cos[e + f*x]*(a
 + a*Sin[e + f*x])^4)/(5*a*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^2\,dx = -\frac{d^2\cos(e+fx)(a+a\sin(e+fx))^4}{5af} + \frac{\int (a+a\sin(e+fx))^3\left(a\left(5\right.\right.}{}$$

$$= -\frac{(10c-d)d\cos(e+fx)(a+a\sin(e+fx))^3}{20f} - \frac{d^2\cos(e+fx)(a+}{5af}$$

$$= -\frac{(10c-d)d\cos(e+fx)(a+a\sin(e+fx))^3}{20f} - \frac{d^2\cos(e+fx)(a+}{5af}$$

$$= \frac{1}{20}a^3\left(20c^2+30cd+13d^2\right)x - \frac{(10c-d)d\cos(e+fx)(a+a\sin(e}{20f}$$

$$= \frac{1}{20}a^3\left(20c^2+30cd+13d^2\right)x - \frac{3a^3\left(20c^2+30cd+13d^2\right)\cos(e+}{20f}$$

$$= \frac{1}{8}a^3\left(20c^2+30cd+13d^2\right)x - \frac{a^3\left(20c^2+30cd+13d^2\right)\cos(e+fx}{5f}$$

Mathematica [A] time = 0.74, size = 177, normalized size = 1.08

$$-\frac{a^3\cos(e+fx)\left(30\left(20c^2+30cd+13d^2\right)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\left(8\left(5c^2+30cd+19d^2\right)\sin^2(e+\right.\right.}{120f\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -1/120*(a^3*Cos[e + f*x]*(30*(20*c^2 + 30*c*d + 13*d^2)*ArcSin[Sqrt[1 - Sin
[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(8*(55*c^2 + 90*c*d + 38*d^2) +
15*(12*c^2 + 30*c*d + 13*d^2)*Sin[e + f*x] + 8*(5*c^2 + 30*c*d + 19*d^2)*Si
n[e + f*x]^2 + 30*d*(2*c + 3*d)*Sin[e + f*x]^3 + 24*d^2*Sin[e + f*x]^4)))/(
f*Sqrt[Cos[e + f*x]^2])
```

fricas [A] time = 0.46, size = 180, normalized size = 1.10

$$-\frac{24\,a^3d^2\cos\left(fx+e\right)^5-40\left(a^3c^2+6\,a^3cd+5\,a^3d^2\right)\cos\left(fx+e\right)^3-15\left(20\,a^3c^2+30\,a^3cd+13\,a^3d^2\right)fx+480\left(a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/120*(24*a^3*d^2*cos(f*x + e)^5 - 40*(a^3*c^2 + 6*a^3*c*d + 5*a^3*d^2)*co
s(f*x + e)^3 - 15*(20*a^3*c^2 + 30*a^3*c*d + 13*a^3*d^2)*f*x + 480*(a^3*c^2
```

```
 + 2*a^3*c*d + a^3*d^2)*cos(f*x + e) - 15*(2*(2*a^3*c*d + 3*a^3*d^2)*cos(f*
x + e)^3 - (12*a^3*c^2 + 34*a^3*c*d + 19*a^3*d^2)*cos(f*x + e))*sin(f*x + e
))/f
```

giac [A] time = 0.21, size = 251, normalized size = 1.53

$$-\frac{a^3d^2\cos\left(5\,fx+5\,e\right)}{80\,f}-\frac{2\,a^3cd\cos\left(fx+e\right)}{f}-\frac{a^3d^2\sin\left(2\,fx+2\,e\right)}{4\,f}+\frac{3}{8}\left(4\,a^3c^2+10\,a^3cd+3\,a^3d^2\right)x+\frac{1}{2}\left(2\,a^3c^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/80*a^3*d^2*cos(5*f*x + 5*e)/f - 2*a^3*c*d*cos(f*x + e)/f - 1/4*a^3*d^2*s
in(2*f*x + 2*e)/f + 3/8*(4*a^3*c^2 + 10*a^3*c*d + 3*a^3*d^2)*x + 1/2*(2*a^3
*c^2 + a^3*d^2)*x + 1/48*(4*a^3*c^2 + 24*a^3*c*d + 17*a^3*d^2)*cos(3*f*x +
3*e)/f - 1/8*(30*a^3*c^2 + 36*a^3*c*d + 23*a^3*d^2)*cos(f*x + e)/f + 1/32*(
2*a^3*c*d + 3*a^3*d^2)*sin(4*f*x + 4*e)/f - 1/4*(3*a^3*c^2 + 8*a^3*c*d + 3*
a^3*d^2)*sin(2*f*x + 2*e)/f
```

maple [B] time = 0.32, size = 319, normalized size = 1.95

$$-\frac{c^2a^3\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}+2a^3cd\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{a^3d^2\left(\frac{8}{3}+\sin^4\left(fx+e\right)+\frac{4\left(\sin^2\left(fx+e\right)\right)}{3}\right)\cos\left(f\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(-1/3*c^2*a^3*(2+sin(f*x+e)^2)*cos(f*x+e)+2*a^3*c*d*(-1/4*(sin(f*x+e)^3
+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/5*a^3*d^2*(8/3+sin(f*x+e)^4+4/
3*sin(f*x+e)^2)*cos(f*x+e)+3*c^2*a^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/
2*e)-2*a^3*c*d*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a^3*d^2*(-1/4*(sin(f*x+e)^3+3/
2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3*c^2*a^3*cos(f*x+e)+6*a^3*c*d*(-1/
2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-a^3*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+
c^2*a^3*(f*x+e)-2*a^3*c*d*cos(f*x+e)+a^3*d^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/
2*f*x+1/2*e))
```

maxima [A] time = 0.35, size = 308, normalized size = 1.88

$$160\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)a^3c^2+360\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)a^3c^2+480\left(fx+e\right)a^3c^2+960\left(\mathrm{co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/480*(160*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3*c^2 + 360*(2*f*x + 2*e - s
in(2*f*x + 2*e))*a^3*c^2 + 480*(f*x + e)*a^3*c^2 + 960*(cos(f*x + e)^3 - 3*
cos(f*x + e))*a^3*c*d + 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x
+ 2*e))*a^3*c*d + 720*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^3*c*d - 32*(3*cos(
f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a^3*d^2 + 480*(cos(f*x +
e)^3 - 3*cos(f*x + e))*a^3*d^2 + 45*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*s
in(2*f*x + 2*e))*a^3*d^2 + 120*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^3*d^2 - 1
440*a^3*c^2*cos(f*x + e) - 960*a^3*c*d*cos(f*x + e))/f
```

mupad [B] time = 8.36, size = 493, normalized size = 3.01

$$\frac{a^3\,\mathrm{atan}\left(\frac{a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(20\,c^2+30\,c\,d+13\,d^2\right)}{4\left(5\,a^3\,c^2+\frac{15\,a^3\,c\,d}{2}+\frac{13\,a^3\,d^2}{4}\right)}\right)\left(20\,c^2+30\,c\,d+13\,d^2\right)}{4\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(3\,a^3\,c^2+\frac{15\,a^3\,c\,d}{2}+\frac{13\,a^3\,d^2}{4}\right)-\tan\left(\frac{e}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^2,x)
```

```
[Out] (a^3*atan((a^3*tan(e/2 + (f*x)/2)*(30*c*d + 20*c^2 + 13*d^2))/(4*(5*a^3*c^2
 + (13*a^3*d^2)/4 + (15*a^3*c*d)/2)))*(30*c*d + 20*c^2 + 13*d^2))/(4*f) - (
tan(e/2 + (f*x)/2)*(3*a^3*c^2 + (13*a^3*d^2)/4 + (15*a^3*c*d)/2) - tan(e/2
+ (f*x)/2)^9*(3*a^3*c^2 + (13*a^3*d^2)/4 + (15*a^3*c*d)/2) + tan(e/2 + (f*x
)/2)^3*(6*a^3*c^2 + (25*a^3*d^2)/2 + 19*a^3*c*d) - tan(e/2 + (f*x)/2)^7*(6*
a^3*c^2 + (25*a^3*d^2)/2 + 19*a^3*c*d) + tan(e/2 + (f*x)/2)^6*(28*a^3*c^2 +
 12*a^3*d^2 + 40*a^3*c*d) + tan(e/2 + (f*x)/2)^2*((92*a^3*c^2)/3 + (76*a^3*
d^2)/3 + 56*a^3*c*d) + tan(e/2 + (f*x)/2)^4*((136*a^3*c^2)/3 + (116*a^3*d^2
)/3 + 80*a^3*c*d) + tan(e/2 + (f*x)/2)^8*(6*a^3*c^2 + 4*a^3*c*d) + (22*a^3*
c^2)/3 + (76*a^3*d^2)/15 + 12*a^3*c*d)/(f*(5*tan(e/2 + (f*x)/2)^2 + 10*tan(
e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 + 5*tan(e/2 + (f*x)/2)^8 + tan(e
/2 + (f*x)/2)^10 + 1)) - (a^3*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2)*(30*c*d
+ 20*c^2 + 13*d^2))/(4*f)
```

sympy [A] time = 4.71, size = 702, normalized size = 4.28

$$\begin{cases}\frac{3a^3c^2x\sin^2\left(e+fx\right)}{2}+\frac{3a^3c^2x\cos^2\left(e+fx\right)}{2}+a^3c^2x-\frac{a^3c^2\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{3a^3c^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{2a^3c^2\cos^3\left(e+fx\right)}{3f}-\frac{3a^3c}{} \\ x\left(c+d\sin\left(e\right)\right)^2\left(a\sin\left(e\right)+a\right)^3\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*a**3*c**2*x*sin(e + f*x)**2/2 + 3*a**3*c**2*x*cos(e + f*x)**2/
2 + a**3*c**2*x - a**3*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*a**3*c**2*si
n(e + f*x)*cos(e + f*x)/(2*f) - 2*a**3*c**2*cos(e + f*x)**3/(3*f) - 3*a**3*
c**2*cos(e + f*x)/f + 3*a**3*c*d*x*sin(e + f*x)**4/4 + 3*a**3*c*d*x*sin(e +
 f*x)**2*cos(e + f*x)**2/2 + 3*a**3*c*d*x*sin(e + f*x)**2 + 3*a**3*c*d*x*co
s(e + f*x)**4/4 + 3*a**3*c*d*x*cos(e + f*x)**2 - 5*a**3*c*d*sin(e + f*x)**3
*cos(e + f*x)/(4*f) - 6*a**3*c*d*sin(e + f*x)**2*cos(e + f*x)/f - 3*a**3*c*
d*sin(e + f*x)*cos(e + f*x)**3/(4*f) - 3*a**3*c*d*sin(e + f*x)*cos(e + f*x)
/f - 4*a**3*c*d*cos(e + f*x)**3/f - 2*a**3*c*d*cos(e + f*x)/f + 9*a**3*d**2
*x*sin(e + f*x)**4/8 + 9*a**3*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + a*
*3*d**2*x*sin(e + f*x)**2/2 + 9*a**3*d**2*x*cos(e + f*x)**4/8 + a**3*d**2*x
*cos(e + f*x)**2/2 - a**3*d**2*sin(e + f*x)**4*cos(e + f*x)/f - 15*a**3*d**
2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*a**3*d**2*sin(e + f*x)**2*cos(e +
f*x)**3/(3*f) - 3*a**3*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 9*a**3*d**2*si
n(e + f*x)*cos(e + f*x)**3/(8*f) - a**3*d**2*sin(e + f*x)*cos(e + f*x)/(2*f
) - 8*a**3*d**2*cos(e + f*x)**5/(15*f) - 2*a**3*d**2*cos(e + f*x)**3/f, Ne(
f, 0)), (x*(c + d*sin(e))**2*(a*sin(e) + a)**3, True))
```

3.446 $\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=110

$$\frac{a^3(c+3d)\cos^3(e+fx)}{3f}-\frac{4a^3(c+d)\cos(e+fx)}{f}-\frac{3a^3(4c+5d)\sin(e+fx)\cos(e+fx)}{8f}+\frac{5}{8}a^3x(4c+3d)-\frac{a^3d\sin^3(e$$

```
[Out] 5/8*a^3*(4*c+3*d)*x-4*a^3*(c+d)*cos(f*x+e)/f+1/3*a^3*(c+3*d)*cos(f*x+e)^3/f
-3/8*a^3*(4*c+5*d)*cos(f*x+e)*sin(f*x+e)/f-1/4*a^3*d*cos(f*x+e)*sin(f*x+e)^
3/f
```

Rubi [A] time = 0.10, antiderivative size = 117, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2751, 2645, 2638, 2635, 8, 2633}

$$\frac{a^3(4c+3d)\cos^3(e+fx)}{12f}-\frac{a^3(4c+3d)\cos(e+fx)}{f}-\frac{3a^3(4c+3d)\sin(e+fx)\cos(e+fx)}{8f}+\frac{5}{8}a^3x(4c+3d)-\frac{d\cos(e$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x]),x]
```

```
[Out] (5*a^3*(4*c + 3*d)*x)/8 - (a^3*(4*c + 3*d)*Cos[e + f*x])/f + (a^3*(4*c + 3*
d)*Cos[e + f*x]^3)/(12*f) - (3*a^3*(4*c + 3*d)*Cos[e + f*x]*Sin[e + f*x])/(
8*f) - (d*Cos[e + f*x]*(a + a*Sin[e + f*x])^3)/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))\,dx = -\frac{d\cos(e+fx)(a+a\sin(e+fx))^3}{4f}+\frac{1}{4}(4c+3d)\int (a+a\sin(e+$$

$$= -\frac{d\cos(e+fx)(a+a\sin(e+fx))^3}{4f}+\frac{1}{4}(4c+3d)\int \left(a^3+3a^3\sin\right.$$

$$= \frac{1}{4}a^3(4c+3d)x-\frac{d\cos(e+fx)(a+a\sin(e+fx))^3}{4f}+\frac{1}{4}\left(a^3(4c+3\right.$$

$$= \frac{1}{4}a^3(4c+3d)x-\frac{3a^3(4c+3d)\cos(e+fx)}{4f}-\frac{3a^3(4c+3d)\cos(e+}{8f}$$

$$= \frac{5}{8}a^3(4c+3d)x-\frac{a^3(4c+3d)\cos(e+fx)}{f}+\frac{a^3(4c+3d)\cos^3(e+}{12f}$$

Mathematica [A] time = 0.52, size = 120, normalized size = 1.09

$$-\frac{a^3\cos(e+fx)\left(30(4c+3d)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\,\left(8(c+3d)\sin^2(e+fx)+9(4c+5d)\sin(e+\right.\right.}{24f\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x]),x]
```

```
[Out] -1/24*(a^3*Cos[e + f*x]*(30*(4*c + 3*d)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[
2]] + Sqrt[Cos[e + f*x]^2]*(88*c + 72*d + 9*(4*c + 5*d)*Sin[e + f*x] + 8*(c
 + 3*d)*Sin[e + f*x]^2 + 6*d*Sin[e + f*x]^3)))/(f*Sqrt[Cos[e + f*x]^2])
```

fricas [A] time = 0.45, size = 108, normalized size = 0.98

$$\frac{8\left(a^{3}c+3\,a^{3}d\right)\cos\left(fx+e\right)^{3}+15\left(4\,a^{3}c+3\,a^{3}d\right)fx-96\left(a^{3}c+a^{3}d\right)\cos\left(fx+e\right)+3\left(2\,a^{3}d\cos\left(fx+e\right)^{3}-\left(12\,a\right.\right.}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/24*(8*(a^3*c + 3*a^3*d)*cos(f*x + e)^3 + 15*(4*a^3*c + 3*a^3*d)*f*x - 96*
(a^3*c + a^3*d)*cos(f*x + e) + 3*(2*a^3*d*cos(f*x + e)^3 - (12*a^3*c + 17*a
^3*d)*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.20, size = 138, normalized size = 1.25

$$a^{3}cx-\frac{a^{3}d\cos\left(fx+e\right)}{f}+\frac{a^{3}d\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{3}{8}\left(4\,a^{3}c+5\,a^{3}d\right)x+\frac{\left(a^{3}c+3\,a^{3}d\right)\cos\left(3\,fx+3\,e\right)}{12\,f}-\frac{3\left(5\,a^{3}c+3\,a^{3}d\right.}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] a^3*c*x - a^3*d*cos(f*x + e)/f + 1/32*a^3*d*sin(4*f*x + 4*e)/f + 3/8*(4*a^3
*c + 5*a^3*d)*x + 1/12*(a^3*c + 3*a^3*d)*cos(3*f*x + 3*e)/f - 3/4*(5*a^3*c
+ 3*a^3*d)*cos(f*x + e)/f - 1/4*(3*a^3*c + 4*a^3*d)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.25, size = 178, normalized size = 1.62

$$\frac{-\frac{a^{3}c\left(2+\sin^{2}\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}+a^{3}d\left(-\frac{\left(\sin^{3}\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+3a^{3}c\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e)),x)
```

```
[Out] 1/f*(-1/3*a^3*c*(2+sin(f*x+e)^2)*cos(f*x+e)+a^3*d*(-1/4*(sin(f*x+e)^3+3/2*s
in(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+3*a^3*c*(-1/2*sin(f*x+e)*cos(f*x+e)+1/
2*f*x+1/2*e)-a^3*d*(2+sin(f*x+e)^2)*cos(f*x+e)-3*a^3*c*cos(f*x+e)+3*a^3*d*(
-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+a^3*c*(f*x+e)-a^3*d*cos(f*x+e))
```

maxima [A] time = 0.35, size = 171, normalized size = 1.55

$$\frac{32\left(\cos\left(fx+e\right)^{3}-3\cos\left(fx+e\right)\right)a^{3}c+72\left(2fx+2e-\sin\left(2fx+2e\right)\right)a^{3}c+96\left(fx+e\right)a^{3}c+96\left(\cos\left(fx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/96*(32*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3*c + 72*(2*f*x + 2*e - sin(2*
f*x + 2*e))*a^3*c + 96*(f*x + e)*a^3*c + 96*(cos(f*x + e)^3 - 3*cos(f*x + e
))*a^3*d + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^3*d
+ 72*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^3*d - 288*a^3*c*cos(f*x + e) - 96*a
^3*d*cos(f*x + e))/f
```

mupad [B] time = 8.07, size = 330, normalized size = 3.00

$$\frac{5a^{3}\operatorname{atan}\left(\frac{5a^{3}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(4c+3d)}{4\left(5a^{3}c+\frac{15a^{3}d}{4}\right)}\right)(4c+3d)}{4f}-\frac{5a^{3}\left(\operatorname{atan}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)-\frac{fx}{2}\right)(4c+3d)}{4f}-\frac{\frac{22a^{3}c}{3}+6a^{3}d+\tan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x)),x)
```

```
[Out] (5*a^3*atan((5*a^3*tan(e/2 + (f*x)/2)*(4*c + 3*d))/(4*(5*a^3*c + (15*a^3*d)
/4)))*(4*c + 3*d))/(4*f) - (5*a^3*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2)*(4*c
 + 3*d))/(4*f) - ((22*a^3*c)/3 + 6*a^3*d + tan(e/2 + (f*x)/2)*(3*a^3*c + (1
5*a^3*d)/4) + tan(e/2 + (f*x)/2)^6*(6*a^3*c + 2*a^3*d) - tan(e/2 + (f*x)/2)
^7*(3*a^3*c + (15*a^3*d)/4) + tan(e/2 + (f*x)/2)^3*(3*a^3*c + (23*a^3*d)/4)
 - tan(e/2 + (f*x)/2)^5*(3*a^3*c + (23*a^3*d)/4) + tan(e/2 + (f*x)/2)^4*(22
*a^3*c + 18*a^3*d) + tan(e/2 + (f*x)/2)^2*((70*a^3*c)/3 + 22*a^3*d))/(f*(4*
tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*tan(e/2 + (f*x)/2)^6 + ta
n(e/2 + (f*x)/2)^8 + 1))
```

sympy [A] time = 2.00, size = 371, normalized size = 3.37

$$\begin{cases}\frac{3a^{3}cx\sin^{2}\left(e+fx\right)}{2}+\frac{3a^{3}cx\cos^{2}\left(e+fx\right)}{2}+a^{3}cx-\frac{a^{3}c\sin^{2}\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{3a^{3}c\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{2a^{3}c\cos^{3}\left(e+fx\right)}{3f}-\frac{3a^{3}c\cos\left(\right.}{f} \\ x\left(c+d\sin\left(e\right)\right)\left(a\sin\left(e\right)+a\right)^{3}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((3*a**3*c*x*sin(e + f*x)**2/2 + 3*a**3*c*x*cos(e + f*x)**2/2 + a*
*3*c*x - a**3*c*sin(e + f*x)**2*cos(e + f*x)/f - 3*a**3*c*sin(e + f*x)*cos(
e + f*x)/(2*f) - 2*a**3*c*cos(e + f*x)**3/(3*f) - 3*a**3*c*cos(e + f*x)/f +
 3*a**3*d*x*sin(e + f*x)**4/8 + 3*a**3*d*x*sin(e + f*x)**2*cos(e + f*x)**2/
4 + 3*a**3*d*x*sin(e + f*x)**2/2 + 3*a**3*d*x*cos(e + f*x)**4/8 + 3*a**3*d*
x*cos(e + f*x)**2/2 - 5*a**3*d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*a**3*
d*sin(e + f*x)**2*cos(e + f*x)/f - 3*a**3*d*sin(e + f*x)*cos(e + f*x)**3/(8
*f) - 3*a**3*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*a**3*d*cos(e + f*x)**3/f
 - a**3*d*cos(e + f*x)/f, Ne(f, 0)), (x*(c + d*sin(e))*(a*sin(e) + a)**3, T
rue))
```

3.447 $\int (a + a\sin(e + fx))^3 \, dx$

Optimal. Leaf size=63

$$\frac{a^3\cos^3(e+fx)}{3f} - \frac{4a^3\cos(e+fx)}{f} - \frac{3a^3\sin(e+fx)\cos(e+fx)}{2f} + \frac{5a^3x}{2}$$

```
[Out] 5/2*a^3*x-4*a^3*cos(f*x+e)/f+1/3*a^3*cos(f*x+e)^3/f-3/2*a^3*cos(f*x+e)*sin(
f*x+e)/f
```

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2645, 2638, 2635, 8, 2633}

$$\frac{a^3\cos^3(e+fx)}{3f} - \frac{4a^3\cos(e+fx)}{f} - \frac{3a^3\sin(e+fx)\cos(e+fx)}{2f} + \frac{5a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (5*a^3*x)/2 - (4*a^3*Cos[e + f*x])/f + (a^3*Cos[e + f*x]^3)/(3*f) - (3*a^3*
Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int (a+a\sin(e+fx))^3\,dx &= \int \left(a^3+3a^3\sin(e+fx)+3a^3\sin^2(e+fx)+a^3\sin^3(e+fx)\right)dx\\
&= a^3x+a^3\int \sin^3(e+fx)\,dx+\left(3a^3\right)\int \sin(e+fx)\,dx+\left(3a^3\right)\int \sin^2(e+fx)\,dx\\
&= a^3x-\frac{3a^3\cos(e+fx)}{f}-\frac{3a^3\cos(e+fx)\sin(e+fx)}{2f}+\frac{1}{2}\left(3a^3\right)\int 1\,dx-\frac{a^3\,\mathrm{Subst}\left(\int\right.}{}\\
&= \frac{5a^3x}{2}-\frac{4a^3\cos(e+fx)}{f}+\frac{a^3\cos^3(e+fx)}{3f}-\frac{3a^3\cos(e+fx)\sin(e+fx)}{2f}
\end{aligned}$$

Mathematica [A] time = 0.35, size = 44, normalized size = 0.70

$$\frac{a^3(-9\sin(2(e+fx))-45\cos(e+fx)+\cos(3(e+fx))+30e+30fx)}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (a^3*(30*e + 30*f*x - 45*Cos[e + f*x] + Cos[3*(e + f*x)] - 9*Sin[2*(e + f*x
)]))/(12*f)
```

fricas [A] time = 0.47, size = 54, normalized size = 0.86

$$\frac{2\,a^3\cos\left(fx+e\right)^3+15\,a^3fx-9\,a^3\cos\left(fx+e\right)\sin\left(fx+e\right)-24\,a^3\cos\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^3*cos(f*x + e)^3 + 15*a^3*f*x - 9*a^3*cos(f*x + e)*sin(f*x + e) -
24*a^3*cos(f*x + e))/f
```

giac [A] time = 0.51, size = 58, normalized size = 0.92

$$\frac{5}{2}a^3x + \frac{a^3\cos\left(3fx + 3e\right)}{12f} - \frac{15a^3\cos\left(fx + e\right)}{4f} - \frac{3a^3\sin\left(2fx + 2e\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 5/2*a^3*x + 1/12*a^3*cos(3*f*x + 3*e)/f - 15/4*a^3*cos(f*x + e)/f - 3/4*a^3
*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.18, size = 74, normalized size = 1.17

$$\frac{-\frac{a^3\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3} + 3a^3\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2} + \frac{fx}{2} + \frac{e}{2}\right) - 3a^3\cos\left(fx + e\right) + \left(fx + e\right)a^3}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(-1/3*a^3*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a^3*(-1/2*sin(f*x+e)*cos(f*x+e)
+1/2*f*x+1/2*e)-3*a^3*cos(f*x+e)+(f*x+e)*a^3)
```

maxima [A] time = 0.32, size = 72, normalized size = 1.14

$$a^3x + \frac{\left(\cos\left(fx + e\right)^3 - 3\cos\left(fx + e\right)\right)a^3}{3f} + \frac{3\left(2fx + 2e - \sin\left(2fx + 2e\right)\right)a^3}{4f} - \frac{3a^3\cos\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 1/3*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3/f + 3/4*(2*f*x + 2*e - si
n(2*f*x + 2*e))*a^3/f - 3*a^3*cos(f*x + e)/f
```

mupad [B] time = 9.12, size = 156, normalized size = 2.48

$$\frac{5a^3x}{2} - \frac{\frac{5a^3\left(e+fx\right)}{2} - 3a^3\tan\left(\frac{e}{2} + \frac{fx}{2}\right)^5 - \frac{a^3\left(15e+15fx-44\right)}{6} + \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^4\left(\frac{15a^3\left(e+fx\right)}{2} - \frac{a^3\left(45e+45fx-36\right)}{6}\right) + \tan\left(\right.}{f\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3,x)
```

```
[Out] (5*a^3*x)/2 - ((5*a^3*(e + f*x))/2 - 3*a^3*tan(e/2 + (f*x)/2)^5 - (a^3*(15*
e + 15*f*x - 44))/6 + tan(e/2 + (f*x)/2)^4*((15*a^3*(e + f*x))/2 - (a^3*(45
*e + 45*f*x - 36))/6) + tan(e/2 + (f*x)/2)^2*((15*a^3*(e + f*x))/2 - (a^3*(
45*e + 45*f*x - 96))/6) + 3*a^3*tan(e/2 + (f*x)/2))/(f*(tan(e/2 + (f*x)/2)^
2 + 1)^3)
```

sympy [A] time = 0.74, size = 121, normalized size = 1.92

$$\begin{cases} \frac{3a^3x\sin^2(e+fx)}{2} + \frac{3a^3x\cos^2(e+fx)}{2} + a^3x - \frac{a^3\sin^2(e+fx)\cos(e+fx)}{f} - \frac{3a^3\sin(e+fx)\cos(e+fx)}{2f} - \frac{2a^3\cos^3(e+fx)}{3f} - \frac{3a^3\cos(e+fx)}{f} \\ x(a\sin(e)+a)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(e + f*x)**2/2 + 3*a**3*x*cos(e + f*x)**2/2 + a**3*x
 - a**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*a**3*sin(e + f*x)*cos(e + f*x)/(
2*f) - 2*a**3*cos(e + f*x)**3/(3*f) - 3*a**3*cos(e + f*x)/f, Ne(f, 0)), (x*
(a*sin(e) + a)**3, True))
```

3.448 $\int \frac{(a+a\sin(e+fx))^3}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=143

$$-\frac{2a^3(c-d)^3\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f\sqrt{c^2-d^2}}+\frac{a^3x\left(2c^2-6cd+7d^2\right)}{2d^3}+\frac{a^3(2c-5d)\cos(e+fx)}{2d^2f}-\frac{\cos(e+fx)\left(a^3\sin(e+f\right.}{2df}$$

```
[Out] 1/2*a^3*(2*c^2-6*c*d+7*d^2)*x/d^3+1/2*a^3*(2*c-5*d)*cos(f*x+e)/d^2/f-1/2*co
s(f*x+e)*(a^3+a^3*sin(f*x+e))/d/f-2*a^3*(c-d)^3*arctan((d+c*tan(1/2*f*x+1/2
*e))/(c^2-d^2)^(1/2))/d^3/f/(c^2-d^2)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2763, 2968, 3023, 2735, 2660, 618, 204}

$$-\frac{2a^3(c-d)^3\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f\sqrt{c^2-d^2}}+\frac{a^3x\left(2c^2-6cd+7d^2\right)}{2d^3}+\frac{a^3(2c-5d)\cos(e+fx)}{2d^2f}-\frac{\cos(e+fx)\left(a^3\sin(e+f\right.}{2df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x]),x]
```

```
[Out] (a^3*(2*c^2 - 6*c*d + 7*d^2)*x)/(2*d^3) - (2*a^3*(c - d)^3*ArcTan[(d + c*Ta
n[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^3*Sqrt[c^2 - d^2]*f) + (a^3*(2*c - 5*d
)*Cos[e + f*x])/(2*d^2*f) - (Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(2*d*f)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{c+d\sin(e+fx)}\,dx = -\frac{\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2df}+\frac{\int \frac{(a+a\sin(e+fx))\left(a^2(c+2d)-a^2(2c-5d)\sin(e+fx)\right)}{c+d\sin(e+fx)}dx}{2d}$$

$$= -\frac{\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2df}+\frac{\int \frac{a^3(c+2d)+\left(-a^3(2c-5d)+a^3(c+2d)\right)\sin(e+fx)-a^3(2c-5d)}{c+d\sin(e+fx)}}{2d}$$

$$= \frac{a^3(2c-5d)\cos(e+fx)}{2d^2f}-\frac{\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2df}+\frac{\int \frac{a^3d(c+2d)+a^3\left(2c^2-6cd\right.}{c+d\sin(e+}}{2d^2}$$

$$= \frac{a^3\left(2c^2-6cd+7d^2\right)x}{2d^3}+\frac{a^3(2c-5d)\cos(e+fx)}{2d^2f}-\frac{\cos(e+fx)\left(a^3+a^3\sin(e+fx\right.}{2df}$$

$$= \frac{a^3\left(2c^2-6cd+7d^2\right)x}{2d^3}+\frac{a^3(2c-5d)\cos(e+fx)}{2d^2f}-\frac{\cos(e+fx)\left(a^3+a^3\sin(e+fx\right.}{2df}$$

$$= \frac{a^3\left(2c^2-6cd+7d^2\right)x}{2d^3}+\frac{a^3(2c-5d)\cos(e+fx)}{2d^2f}-\frac{\cos(e+fx)\left(a^3+a^3\sin(e+fx\right.}{2df}$$

$$= \frac{a^3\left(2c^2-6cd+7d^2\right)x}{2d^3}-\frac{2a^3(c-d)^3\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3\sqrt{c^2-d^2}\,f}+\frac{a^3(2c-5d)\cos(e+}{2d^2f}$$

Mathematica [A] time = 0.66, size = 162, normalized size = 1.13

$$\frac{a^3(\sin(e+fx)+1)^3\left(\sqrt{c^2-d^2}\left(2\left(2c^2-6cd+7d^2\right)(e+fx)+4d(c-3d)\cos(e+fx)+d^2(-\sin(2(e+fx)))\right)-\right.}{4d^3f\sqrt{c^2-d^2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x]),x]
```

```
[Out] (a^3*(1 + Sin[e + f*x])^3*(-8*(c - d)^3*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqr
t[c^2 - d^2]] + Sqrt[c^2 - d^2]*(2*(2*c^2 - 6*c*d + 7*d^2)*(e + f*x) + 4*(c
 - 3*d)*d*Cos[e + f*x] - d^2*Sin[2*(e + f*x)])))/(4*d^3*Sqrt[c^2 - d^2]*f*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
```

fricas [A] time = 0.51, size = 404, normalized size = 2.83

$$\left[-\frac{a^3d^2\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(2\,a^3c^2-6\,a^3cd+7\,a^3d^2\right)fx-\left(a^3c^2-2\,a^3cd+a^3d^2\right)\sqrt{-\frac{c-d}{c+d}}\,\log\left(\frac{\left(2\,c^2-d^2\right)\cos\left(f\right.}{}\right.}{2\,d^3f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a^3*d^2*cos(f*x + e)*sin(f*x + e) - (2*a^3*c^2 - 6*a^3*c*d + 7*a^3*d
^2)*f*x - (a^3*c^2 - 2*a^3*c*d + a^3*d^2)*sqrt(-(c - d)/(c + d))*log(((2*c^
2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*((c^2 + c*d)*c
os(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c + d))
)/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 2*(a^3*c*d - 3*a
^3*d^2)*cos(f*x + e))/(d^3*f), -1/2*(a^3*d^2*cos(f*x + e)*sin(f*x + e) - (2
*a^3*c^2 - 6*a^3*c*d + 7*a^3*d^2)*f*x - 2*(a^3*c^2 - 2*a^3*c*d + a^3*d^2)*s
qrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sqrt((c - d)/(c + d))/((c
 - d)*cos(f*x + e))) - 2*(a^3*c*d - 3*a^3*d^2)*cos(f*x + e))/(d^3*f)]
```

giac [A] time = 0.26, size = 239, normalized size = 1.67

$$\frac{\frac{\left(2\,a^3c^2-6\,a^3cd+7\,a^3d^2\right)\left(fx+e\right)}{d^3}-\frac{4\left(a^3c^3-3\,a^3c^2d+3\,a^3cd^2-a^3d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\sqrt{c^2-d^2}\,d^3}+\frac{2\left(a^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+2\,a^3c\tan\cdots\right.}{}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*((2*a^3*c^2 - 6*a^3*c*d + 7*a^3*d^2)*(f*x + e)/d^3 - 4*(a^3*c^3 - 3*a^3
*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) +
arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*d^3)
 + 2*(a^3*d*tan(1/2*f*x + 1/2*e)^3 + 2*a^3*c*tan(1/2*f*x + 1/2*e)^2 - 6*a^3
*d*tan(1/2*f*x + 1/2*e)^2 - a^3*d*tan(1/2*f*x + 1/2*e) + 2*a^3*c - 6*a^3*d)
/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*d^2))/f
```

maple [B] time = 0.26, size = 480, normalized size = 3.36

$$-\frac{2a^3\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)c^3}{f\,d^3\sqrt{c^2-d^2}}+\frac{6a^3\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)c^2}{f\,d^2\sqrt{c^2-d^2}}-\frac{6a^3\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)c}{f\,d\sqrt{c^2-d^2}}+\frac{2a^3\arctan\left(\frac{2c\tan\cdots}{2\cdots}\right.}{f\sqrt{c^2-\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x)
```

```
[Out] -2/f*a^3/d^3/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d
^2)^(1/2))*c^3+6/f*a^3/d^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*
e)+2*d)/(c^2-d^2)^(1/2))*c^2-6/f*a^3/d/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(
1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c+2/f*a^3/(c^2-d^2)^(1/2)*arctan(1/2*(
2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))+1/f*a^3/d/(1+tan(1/2*f*x+1/2*e
)^2)^2*tan(1/2*f*x+1/2*e)^3+2/f*a^3/d^2/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*
f*x+1/2*e)^2*c-6/f*a^3/d/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2-1/
f*a^3/d/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)+2/f*a^3/d^2/(1+tan(1/
2*f*x+1/2*e)^2)^2*c-6/f*a^3/d/(1+tan(1/2*f*x+1/2*e)^2)^2+2/f*a^3/d^3*arctan
(tan(1/2*f*x+1/2*e))*c^2-6/f*a^3/d^2*arctan(tan(1/2*f*x+1/2*e))*c+7/f*a^3/d
*arctan(tan(1/2*f*x+1/2*e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 9.22, size = 3382, normalized size = 23.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c + d*sin(e + f*x)),x)
```

```
[Out] ((2*(a^3*c - 3*a^3*d))/d^2 + (a^3*tan(e/2 + (f*x)/2)^3)/d - (a^3*tan(e/2 +
(f*x)/2))/d + (2*tan(e/2 + (f*x)/2)^2*(a^3*c - 3*a^3*d))/d^2)/(f*(2*tan(e/2
 + (f*x)/2)^2 + tan(e/2 + (f*x)/2)^4 + 1)) + (2*a^3*atan(((a^3*((8*(49*a^6*
c^2*d^6 - 84*a^6*c^3*d^5 + 64*a^6*c^4*d^4 - 24*a^6*c^5*d^3 + 4*a^6*c^6*d^2)
)/d^5 + (8*tan(e/2 + (f*x)/2)*(94*a^6*c*d^8 - 144*a^6*c^2*d^7 + 19*a^6*c^3*
d^6 + 116*a^6*c^4*d^5 - 116*a^6*c^5*d^4 + 48*a^6*c^6*d^3 - 8*a^6*c^7*d^2))/
d^6 + (a^3*((8*tan(e/2 + (f*x)/2)*(8*a^3*c*d^9 - 24*a^3*c^2*d^8 + 24*a^3*c^
3*d^7 - 8*a^3*c^4*d^6))/d^6 - (8*(14*a^3*c*d^8 - 16*a^3*c^2*d^7 + 2*a^3*c^3
*d^6))/d^5 + (a^3*(32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 - 8*c^3*d^
8))/d^6)*(c^2 - 3*c*d + (7*d^2)/2)*1i)/d^3)*(c^2 - 3*c*d + (7*d^2)/2)*1i)/d
```

^3)*(c^2 - 3*c*d + (7*d^2)/2))/d^3 + (a^3*((8*(49*a^6*c^2*d^6 - 84*a^6*c^3*
d^5 + 64*a^6*c^4*d^4 - 24*a^6*c^5*d^3 + 4*a^6*c^6*d^2))/d^5 + (8*tan(e/2 +
(f*x)/2)*(94*a^6*c*d^8 - 144*a^6*c^2*d^7 + 19*a^6*c^3*d^6 + 116*a^6*c^4*d^5
 - 116*a^6*c^5*d^4 + 48*a^6*c^6*d^3 - 8*a^6*c^7*d^2))/d^6 + (a^3*((8*(14*a^
3*c*d^8 - 16*a^3*c^2*d^7 + 2*a^3*c^3*d^6))/d^5 - (8*tan(e/2 + (f*x)/2)*(8*a
^3*c*d^9 - 24*a^3*c^2*d^8 + 24*a^3*c^3*d^7 - 8*a^3*c^4*d^6))/d^6 + (a^3*(32
*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 - 8*c^3*d^8))/d^6)*(c^2 - 3*c*d
 + (7*d^2)/2)*1i)/d^3)*(c^2 - 3*c*d + (7*d^2)/2)*1i)/d^3)*(c^2 - 3*c*d + (7
d^2)/2))/d^3)/((16(2*a^9*c^7 - 14*a^9*c*d^6 - 8*a^9*c^6*d + 47*a^9*c^2*d^
5 - 55*a^9*c^3*d^4 + 21*a^9*c^4*d^3 + 7*a^9*c^5*d^2))/d^5 + (16*tan(e/2 + (
f*x)/2)*(8*a^9*c^8 - 98*a^9*c*d^7 - 72*a^9*c^7*d + 462*a^9*c^2*d^6 - 926*a^
9*c^3*d^5 + 1034*a^9*c^4*d^4 - 704*a^9*c^5*d^3 + 296*a^9*c^6*d^2))/d^6 - (a
^3*((8*(49*a^6*c^2*d^6 - 84*a^6*c^3*d^5 + 64*a^6*c^4*d^4 - 24*a^6*c^5*d^3 +
 4*a^6*c^6*d^2))/d^5 + (8*tan(e/2 + (f*x)/2)*(94*a^6*c*d^8 - 144*a^6*c^2*d^
7 + 19*a^6*c^3*d^6 + 116*a^6*c^4*d^5 - 116*a^6*c^5*d^4 + 48*a^6*c^6*d^3 - 8
*a^6*c^7*d^2))/d^6 + (a^3*((8*tan(e/2 + (f*x)/2)*(8*a^3*c*d^9 - 24*a^3*c^2*
d^8 + 24*a^3*c^3*d^7 - 8*a^3*c^4*d^6))/d^6 - (8*(14*a^3*c*d^8 - 16*a^3*c^2*
d^7 + 2*a^3*c^3*d^6))/d^5 + (a^3*(32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*
d^10 - 8*c^3*d^8))/d^6)*(c^2 - 3*c*d + (7*d^2)/2)*1i)/d^3)*(c^2 - 3*c*d + (
7*d^2)/2)*1i)/d^3)*(c^2 - 3*c*d + (7*d^2)/2)*1i)/d^3 + (a^3*((8*(49*a^6*c^2
*d^6 - 84*a^6*c^3*d^5 + 64*a^6*c^4*d^4 - 24*a^6*c^5*d^3 + 4*a^6*c^6*d^2))/d
^5 + (8*tan(e/2 + (f*x)/2)*(94*a^6*c*d^8 - 144*a^6*c^2*d^7 + 19*a^6*c^3*d^6
 + 116*a^6*c^4*d^5 - 116*a^6*c^5*d^4 + 48*a^6*c^6*d^3 - 8*a^6*c^7*d^2))/d^6
 + (a^3*((8*(14*a^3*c*d^8 - 16*a^3*c^2*d^7 + 2*a^3*c^3*d^6))/d^5 - (8*tan(e
/2 + (f*x)/2)*(8*a^3*c*d^9 - 24*a^3*c^2*d^8 + 24*a^3*c^3*d^7 - 8*a^3*c^4*d^
6))/d^6 + (a^3*(32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 - 8*c^3*d^8))
/d^6)*(c^2 - 3*c*d + (7*d^2)/2)*1i)/d^3)*(c^2 - 3*c*d + (7*d^2)/2)*1i)/d^3)
*(c^2 - 3*c*d + (7*d^2)/2)*1i)/d^3))*(c^2 - 3*c*d + (7*d^2)/2))/(d^3*f) + (
a^3*atan(((a^3*(-(c + d)*(c - d)^5)^(1/2)*((8*(49*a^6*c^2*d^6 - 84*a^6*c^3*
d^5 + 64*a^6*c^4*d^4 - 24*a^6*c^5*d^3 + 4*a^6*c^6*d^2))/d^5 + (8*tan(e/2 +
(f*x)/2)*(94*a^6*c*d^8 - 144*a^6*c^2*d^7 + 19*a^6*c^3*d^6 + 116*a^6*c^4*d^5
 - 116*a^6*c^5*d^4 + 48*a^6*c^6*d^3 - 8*a^6*c^7*d^2))/d^6 + (a^3*(-(c + d)*
(c - d)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(8*a^3*c*d^9 - 24*a^3*c^2*d^8 + 24*
a^3*c^3*d^7 - 8*a^3*c^4*d^6))/d^6 - (8*(14*a^3*c*d^8 - 16*a^3*c^2*d^7 + 2*a
^3*c^3*d^6))/d^5 + (a^3*(32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 - 8*
c^3*d^8))/d^6)*(-(c + d)*(c - d)^5)^(1/2))/(d^3*(c + d))))/(d^3*(c + d)))*1
i)/(d^3*(c + d)) + (a^3*(-(c + d)*(c - d)^5)^(1/2)*((8*(49*a^6*c^2*d^6 - 84
*a^6*c^3*d^5 + 64*a^6*c^4*d^4 - 24*a^6*c^5*d^3 + 4*a^6*c^6*d^2))/d^5 + (8*t
an(e/2 + (f*x)/2)*(94*a^6*c*d^8 - 144*a^6*c^2*d^7 + 19*a^6*c^3*d^6 + 116*a^
6*c^4*d^5 - 116*a^6*c^5*d^4 + 48*a^6*c^6*d^3 - 8*a^6*c^7*d^2))/d^6 + (a^3*(
-(c + d)*(c - d)^5)^(1/2)*((8*(14*a^3*c*d^8 - 16*a^3*c^2*d^7 + 2*a^3*c^3*d^
6))/d^5 - (8*tan(e/2 + (f*x)/2)*(8*a^3*c*d^9 - 24*a^3*c^2*d^8 + 24*a^3*c^3*
d^7 - 8*a^3*c^4*d^6))/d^6 + (a^3*(32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*
d^10 - 8*c^3*d^8))/d^6)*(-(c + d)*(c - d)^5)^(1/2))/(d^3*(c + d))))/(d^3*(c
 + d)))*1i)/(d^3*(c + d)))/((16*(2*a^9*c^7 - 14*a^9*c*d^6 - 8*a^9*c^6*d + 4

```
7*a^9*c^2*d^5 - 55*a^9*c^3*d^4 + 21*a^9*c^4*d^3 + 7*a^9*c^5*d^2))/d^5 + (16
*tan(e/2 + (f*x)/2)*(8*a^9*c^8 - 98*a^9*c*d^7 - 72*a^9*c^7*d + 462*a^9*c^2*
d^6 - 926*a^9*c^3*d^5 + 1034*a^9*c^4*d^4 - 704*a^9*c^5*d^3 + 296*a^9*c^6*d^
2))/d^6 - (a^3*(-(c + d)*(c - d)^5)^(1/2)*((8*(49*a^6*c^2*d^6 - 84*a^6*c^3*
d^5 + 64*a^6*c^4*d^4 - 24*a^6*c^5*d^3 + 4*a^6*c^6*d^2))/d^5 + (8*tan(e/2 +
(f*x)/2)*(94*a^6*c*d^8 - 144*a^6*c^2*d^7 + 19*a^6*c^3*d^6 + 116*a^6*c^4*d^5
 - 116*a^6*c^5*d^4 + 48*a^6*c^6*d^3 - 8*a^6*c^7*d^2))/d^6 + (a^3*(-(c + d)*
(c - d)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(8*a^3*c*d^9 - 24*a^3*c^2*d^8 + 24*
a^3*c^3*d^7 - 8*a^3*c^4*d^6))/d^6 - (8*(14*a^3*c*d^8 - 16*a^3*c^2*d^7 + 2*a
^3*c^3*d^6))/d^5 + (a^3*(32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 - 8*
c^3*d^8))/d^6)*(-(c + d)*(c - d)^5)^(1/2))/(d^3*(c + d))))/(d^3*(c + d))))/
(d^3*(c + d)) + (a^3*(-(c + d)*(c - d)^5)^(1/2)*((8*(49*a^6*c^2*d^6 - 84*a^
6*c^3*d^5 + 64*a^6*c^4*d^4 - 24*a^6*c^5*d^3 + 4*a^6*c^6*d^2))/d^5 + (8*tan(
e/2 + (f*x)/2)*(94*a^6*c*d^8 - 144*a^6*c^2*d^7 + 19*a^6*c^3*d^6 + 116*a^6*c
^4*d^5 - 116*a^6*c^5*d^4 + 48*a^6*c^6*d^3 - 8*a^6*c^7*d^2))/d^6 + (a^3*(-(c
 + d)*(c - d)^5)^(1/2)*((8*(14*a^3*c*d^8 - 16*a^3*c^2*d^7 + 2*a^3*c^3*d^6))
/d^5 - (8*tan(e/2 + (f*x)/2)*(8*a^3*c*d^9 - 24*a^3*c^2*d^8 + 24*a^3*c^3*d^7
 - 8*a^3*c^4*d^6))/d^6 + (a^3*(32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^1
0 - 8*c^3*d^8))/d^6)*(-(c + d)*(c - d)^5)^(1/2))/(d^3*(c + d))))/(d^3*(c +
d))))/(d^3*(c + d))))*(-(c + d)*(c - d)^5)^(1/2)*2i)/(d^3*f*(c + d))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.449 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=161

$$\frac{2a^3(c-d)^2(2c+3d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f(c+d)\sqrt{c^2-d^2}}-\frac{a^3x(2c-3d)}{d^3}-\frac{2a^3c\cos(e+fx)}{d^2f(c+d)}+\frac{(c-d)\cos(e+fx)\left(a^3\sin(e+fx)\right.}{df(c+d)(c+d\sin(e+fx))}$$

```
[Out] -a^3*(2*c-3*d)*x/d^3-2*a^3*c*cos(f*x+e)/d^2/(c+d)/f+(c-d)*cos(f*x+e)*(a^3+a
^3*sin(f*x+e))/d/(c+d)/f/(c+d*sin(f*x+e))+2*a^3*(c-d)^2*(2*c+3*d)*arctan((d
+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d^3/(c+d)/f/(c^2-d^2)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2762, 2968, 3023, 2735, 2660, 618, 204}

$$\frac{2a^3(c-d)^2(2c+3d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f(c+d)\sqrt{c^2-d^2}}-\frac{2a^3c\cos(e+fx)}{d^2f(c+d)}-\frac{a^3x(2c-3d)}{d^3}+\frac{(c-d)\cos(e+fx)\left(a^3\sin(e+fx)\right.}{df(c+d)(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -((a^3*(2*c - 3*d)*x)/d^3) + (2*a^3*(c - d)^2*(2*c + 3*d)*ArcTan[(d + c*Tan
[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^3*(c + d)*Sqrt[c^2 - d^2]*f) - (2*a^3*c
*Cos[e + f*x])/(d^2*(c + d)*f) + ((c - d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f
*x]))/(d*(c + d)*f*(c + d*Sin[e + f*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^2}\,dx = \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))} - \frac{a\int \frac{(a+a\sin(e+fx))(a(c-3d)-2ac\sin(e+fx))}{c+d\sin(e+fx)}\,dx}{d(c+d)}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))} - \frac{a\int \frac{a^2(c-3d)+\left(-2a^2c+a^2(c-3d)\right)\sin(e+fx)-2a^2c\sin^{\cdots}}{c+d\sin(e+fx)}}{d(c+d)}$$

$$= -\frac{2a^3c\cos(e+fx)}{d^2(c+d)f} + \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))} - \frac{a\int \frac{a^2(c-3d)d+a^2(2c-3\cdots}{c+d\sin(\cdots}}{d^2(c+\cdots}$$

$$= -\frac{a^3(2c-3d)x}{d^3} - \frac{2a^3c\cos(e+fx)}{d^2(c+d)f} + \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))} + \frac{\left(a^3\cdots\right.}{\cdots}$$

$$= -\frac{a^3(2c-3d)x}{d^3} - \frac{2a^3c\cos(e+fx)}{d^2(c+d)f} + \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))} + \frac{\left(2a\cdots\right.}{\cdots}$$

$$= -\frac{a^3(2c-3d)x}{d^3} - \frac{2a^3c\cos(e+fx)}{d^2(c+d)f} + \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))} - \frac{\left(4a\cdots\right.}{\cdots}$$

$$= -\frac{a^3(2c-3d)x}{d^3} + \frac{2a^3(c-d)^2(2c+3d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3(c+d)\sqrt{c^2-d^2}\,f} - \frac{2a^3c\cos(e+fx)}{d^2(c+d)f} + \cdots$$

Mathematica [A] time = 0.68, size = 162, normalized size = 1.01

$$\frac{a^3(\sin(e+fx)+1)^3\left(\frac{2(2c+3d)(c-d)^2\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}} + (3d-2c)(e+fx) - \frac{d(c-d)^2\cos(e+fx)}{(c+d)(c+d\sin(e+fx))} - d\cos(e+fx)\right)}{d^3 f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a^3*(1 + Sin[e + f*x])^3*((-2*c + 3*d)*(e + f*x) + (2*(c - d)^2*(2*c + 3*d
)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)*Sqrt[c^2 - d^2
]) - d*Cos[e + f*x] - ((c - d)^2*d*Cos[e + f*x])/((c + d)*(c + d*Sin[e + f*
x]))))/(d^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
```

fricas [A] time = 0.50, size = 645, normalized size = 4.01

$$\left[-\frac{2\left(2\,a^3c^3-a^3c^2d-3\,a^3cd^2\right)fx+\left(2\,a^3c^3+a^3c^2d-3\,a^3cd^2+\left(2\,a^3c^2d+a^3cd^2-3\,a^3d^3\right)\sin\left(fx+e\right)\right)\sqrt{-\frac{c-d}{c+d}}\,1\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(2*a^3*c^3 - a^3*c^2*d - 3*a^3*c*d^2)*f*x + (2*a^3*c^3 + a^3*c^2*d
 - 3*a^3*c*d^2 + (2*a^3*c^2*d + a^3*c*d^2 - 3*a^3*d^3)*sin(f*x + e))*sqrt(-
(c - d)/(c + d))*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c
^2 - d^2 + 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x +
 e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2
 - d^2)) + 2*(2*a^3*c^2*d - a^3*c*d^2 + a^3*d^3)*cos(f*x + e) + 2*((2*a^3*c
^2*d - a^3*c*d^2 - 3*a^3*d^3)*f*x + (a^3*c*d^2 + a^3*d^3)*cos(f*x + e))*sin
(f*x + e))/((c*d^4 + d^5)*f*sin(f*x + e) + (c^2*d^3 + c*d^4)*f), -((2*a^3*c
^3 - a^3*c^2*d - 3*a^3*c*d^2)*f*x + (2*a^3*c^3 + a^3*c^2*d - 3*a^3*c*d^2 +
(2*a^3*c^2*d + a^3*c*d^2 - 3*a^3*d^3)*sin(f*x + e))*sqrt((c - d)/(c + d))*a
rctan(-(c*sin(f*x + e) + d)*sqrt((c - d)/(c + d))/((c - d)*cos(f*x + e))) +
 (2*a^3*c^2*d - a^3*c*d^2 + a^3*d^3)*cos(f*x + e) + ((2*a^3*c^2*d - a^3*c*d
^2 - 3*a^3*d^3)*f*x + (a^3*c*d^2 + a^3*d^3)*cos(f*x + e))*sin(f*x + e))/((c
*d^4 + d^5)*f*sin(f*x + e) + (c^2*d^3 + c*d^4)*f)]
```

giac [B] time = 0.25, size = 395, normalized size = 2.45

$$\frac{\frac{2\left(2\,a^3c^3-a^3c^2d-4\,a^3cd^2+3\,a^3d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(cd^3+d^4\right)\sqrt{c^2-d^2}}-\frac{2\left(a^3c^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2\,a^3cd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+a^3d^3\tan\left(\frac{1}{2}\cdots\right.\right.}{\left(c\tan\cdots\right.}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] (2*(2*a^3*c^3 - a^3*c^2*d - 4*a^3*c*d^2 + 3*a^3*d^3)*(pi*floor(1/2*(f*x + e
)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/
((c*d^3 + d^4)*sqrt(c^2 - d^2)) - 2*(a^3*c^2*d*tan(1/2*f*x + 1/2*e)^3 - 2*a
^3*c*d^2*tan(1/2*f*x + 1/2*e)^3 + a^3*d^3*tan(1/2*f*x + 1/2*e)^3 + 2*a^3*c^
3*tan(1/2*f*x + 1/2*e)^2 - a^3*c^2*d*tan(1/2*f*x + 1/2*e)^2 + a^3*c*d^2*tan
(1/2*f*x + 1/2*e)^2 + 3*a^3*c^2*d*tan(1/2*f*x + 1/2*e) + a^3*d^3*tan(1/2*f*
x + 1/2*e) + 2*a^3*c^3 - a^3*c^2*d + a^3*c*d^2)/((c*tan(1/2*f*x + 1/2*e)^4
```

```
+ 2*d*tan(1/2*f*x + 1/2*e)^3 + 2*c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x
 + 1/2*e) + c)*(c^2*d^2 + c*d^3)) - (2*a^3*c - 3*a^3*d)*(f*x + e)/d^3)/f
```

maple [B] time = 0.31, size = 600, normalized size = 3.73

$$-\frac{2a^3c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{fd\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d+c\right)(c+d)}+\frac{4a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d+c\right)(c+d)}-\frac{}{f\left(\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x)
```

```
[Out] -2*a^3/f/d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*c*tan(1/
2*f*x+1/2*e)+4*a^3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d
)*tan(1/2*f*x+1/2*e)-2*a^3/f*d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)
*d+c)/(c+d)/c*tan(1/2*f*x+1/2*e)-2*a^3/f/d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(
1/2*f*x+1/2*e)*d+c)/(c+d)*c^2+4*a^3/f/d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f
*x+1/2*e)*d+c)/(c+d)*c-2*a^3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)
*d+c)/(c+d)+4*a^3/f/d^3/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1
/2*e)+2*d)/(c^2-d^2)^(1/2))*c^3-2*a^3/f/d^2/(c+d)/(c^2-d^2)^(1/2)*arctan(1/
2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c^2-8*a^3/f/d/(c+d)/(c^2-d^
2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c+6*a^3/f
/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1
/2))-2*a^3/f/d^2/(1+tan(1/2*f*x+1/2*e)^2)-4*a^3/f/d^3*arctan(tan(1/2*f*x+1/
2*e))*c+6*a^3/f/d^2*arctan(tan(1/2*f*x+1/2*e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 12.95, size = 5079, normalized size = 31.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c + d*sin(e + f*x))^2,x)

[Out] - ((2*(2*a^3*c^2 + a^3*d^2 - a^3*c*d))/(d^2*(c + d)) + (2*tan(e/2 + (f*x)/2
)^2*(2*a^3*c^2 + a^3*d^2 - a^3*c*d))/(d^2*(c + d)) + (2*tan(e/2 + (f*x)/2)*
(3*a^3*c^2 + a^3*d^2))/(c*d*(c + d)) + (2*tan(e/2 + (f*x)/2)^3*(a^3*c^2 + a
^3*d^2 - 2*a^3*c*d))/(c*d*(c + d)))/(f*(c + 2*d*tan(e/2 + (f*x)/2) + 2*c*ta
n(e/2 + (f*x)/2)^2 + c*tan(e/2 + (f*x)/2)^4 + 2*d*tan(e/2 + (f*x)/2)^3)) -
(2*a^3*atan(((a^3*(2*c - 3*d)*((32*(9*a^6*c^2*d^6 + 6*a^6*c^3*d^5 - 11*a^6*
c^4*d^4 - 4*a^6*c^5*d^3 + 4*a^6*c^6*d^2))/(2*c*d^6 + d^7 + c^2*d^5) + (32*t
an(e/2 + (f*x)/2)*(9*a^6*c*d^8 + 36*a^6*c^2*d^7 - 41*a^6*c^3*d^6 - 34*a^6*c
^4*d^5 + 34*a^6*c^5*d^4 + 8*a^6*c^6*d^3 - 8*a^6*c^7*d^2))/(2*c*d^7 + d^8 +
c^2*d^6) + (a^3*(2*c - 3*d)*((32*tan(e/2 + (f*x)/2)*(6*a^3*c*d^10 - 2*a^3*c
^2*d^9 - 10*a^3*c^3*d^8 + 2*a^3*c^4*d^7 + 4*a^3*c^5*d^6))/(2*c*d^7 + d^8 +
c^2*d^6) - (32*(3*a^3*c*d^9 + a^3*c^2*d^8 - 3*a^3*c^3*d^7 - a^3*c^4*d^6))/(
2*c*d^6 + d^7 + c^2*d^5) + (a^3*((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^8))/(2*c
*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^12 + 6*c^2*d^11 + c^3
*d^10 - 4*c^4*d^9 - 2*c^5*d^8))/(2*c*d^7 + d^8 + c^2*d^6))*(2*c - 3*d)*1i)/
d^3)*1i)/d^3))/d^3 + (a^3*(2*c - 3*d)*((32*(9*a^6*c^2*d^6 + 6*a^6*c^3*d^5 -
 11*a^6*c^4*d^4 - 4*a^6*c^5*d^3 + 4*a^6*c^6*d^2))/(2*c*d^6 + d^7 + c^2*d^5)
 + (32*tan(e/2 + (f*x)/2)*(9*a^6*c*d^8 + 36*a^6*c^2*d^7 - 41*a^6*c^3*d^6 -
34*a^6*c^4*d^5 + 34*a^6*c^5*d^4 + 8*a^6*c^6*d^3 - 8*a^6*c^7*d^2))/(2*c*d^7
+ d^8 + c^2*d^6) + (a^3*(2*c - 3*d)*((32*(3*a^3*c*d^9 + a^3*c^2*d^8 - 3*a^3
*c^3*d^7 - a^3*c^4*d^6))/(2*c*d^6 + d^7 + c^2*d^5) - (32*tan(e/2 + (f*x)/2)
*(6*a^3*c*d^10 - 2*a^3*c^2*d^9 - 10*a^3*c^3*d^8 + 2*a^3*c^4*d^7 + 4*a^3*c^5
*d^6))/(2*c*d^7 + d^8 + c^2*d^6) + (a^3*((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^
8))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^12 + 6*c^2*d^
11 + c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8))/(2*c*d^7 + d^8 + c^2*d^6))*(2*c - 3
*d)*1i)/d^3)*1i)/d^3))/d^3)/((64*(4*a^9*c^6 + 27*a^9*c*d^5 - 20*a^9*c^5*d -
 63*a^9*c^2*d^4 + 33*a^9*c^3*d^3 + 19*a^9*c^4*d^2))/(2*c*d^6 + d^7 + c^2*d^
5) - (64*tan(e/2 + (f*x)/2)*(40*a^9*c^6*d - 54*a^9*c*d^6 - 16*a^9*c^7 + 90*
a^9*c^2*d^5 + 42*a^9*c^3*d^4 - 130*a^9*c^4*d^3 + 28*a^9*c^5*d^2))/(2*c*d^7
+ d^8 + c^2*d^6) + (a^3*(2*c - 3*d)*((32*(9*a^6*c^2*d^6 + 6*a^6*c^3*d^5 - 1
1*a^6*c^4*d^4 - 4*a^6*c^5*d^3 + 4*a^6*c^6*d^2))/(2*c*d^6 + d^7 + c^2*d^5) +
 (32*tan(e/2 + (f*x)/2)*(9*a^6*c*d^8 + 36*a^6*c^2*d^7 - 41*a^6*c^3*d^6 - 34
*a^6*c^4*d^5 + 34*a^6*c^5*d^4 + 8*a^6*c^6*d^3 - 8*a^6*c^7*d^2))/(2*c*d^7 +
d^8 + c^2*d^6) + (a^3*(2*c - 3*d)*((32*tan(e/2 + (f*x)/2)*(6*a^3*c*d^10 - 2
*a^3*c^2*d^9 - 10*a^3*c^3*d^8 + 2*a^3*c^4*d^7 + 4*a^3*c^5*d^6))/(2*c*d^7 +
d^8 + c^2*d^6) - (32*(3*a^3*c*d^9 + a^3*c^2*d^8 - 3*a^3*c^3*d^7 - a^3*c^4*d
^6))/(2*c*d^6 + d^7 + c^2*d^5) + (a^3*((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^8)
)/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^12 + 6*c^2*d^11
 + c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8))/(2*c*d^7 + d^8 + c^2*d^6))*(2*c - 3*d
)*1i)/d^3)*1i)/d^3)*1i)/d^3 - (a^3*(2*c - 3*d)*((32*(9*a^6*c^2*d^6 + 6*a^6*
c^3*d^5 - 11*a^6*c^4*d^4 - 4*a^6*c^5*d^3 + 4*a^6*c^6*d^2))/(2*c*d^6 + d^7 +
 c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(9*a^6*c*d^8 + 36*a^6*c^2*d^7 - 41*a^6*c
^3*d^6 - 34*a^6*c^4*d^5 + 34*a^6*c^5*d^4 + 8*a^6*c^6*d^3 - 8*a^6*c^7*d^2))/
```

(2*c*d^7 + d^8 + c^2*d^6) + (a^3*(2*c - 3*d)*((32*(3*a^3*c*d^9 + a^3*c^2*d^8 - 3*a^3*c^3*d^7 - a^3*c^4*d^6))/(2*c*d^6 + d^7 + c^2*d^5) - (32*tan(e/2 + (f*x)/2)*(6*a^3*c*d^10 - 2*a^3*c^2*d^9 - 10*a^3*c^3*d^8 + 2*a^3*c^4*d^7 + 4*a^3*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6) + (a^3*((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^8))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^12 + 6*c^2*d^11 + c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8))/(2*c*d^7 + d^8 + c^2*d^6))*(2*c - 3*d)*1i)/d^3)*1i)/d^3)*1i)/d^3))*(2*c - 3*d))/(d^3*f) - (a^3*atan(((a^3*(-(c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d)*((32*(9*a^6*c^2*d^6 + 6*a^6*c^3*d^5 - 11*a^6*c^4*d^4 - 4*a^6*c^5*d^3 + 4*a^6*c^6*d^2))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(9*a^6*c*d^8 + 36*a^6*c^2*d^7 - 41*a^6*c^3*d^6 - 34*a^6*c^4*d^5 + 34*a^6*c^5*d^4 + 8*a^6*c^6*d^3 - 8*a^6*c^7*d^2))/(2*c*d^7 + d^8 + c^2*d^6) + (a^3*(-(c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d)*((32*tan(e/2 + (f*x)/2)*(6*a^3*c*d^10 - 2*a^3*c^2*d^9 - 10*a^3*c^3*d^8 + 2*a^3*c^4*d^7 + 4*a^3*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6) - (32*(3*a^3*c*d^9 + a^3*c^2*d^8 - 3*a^3*c^3*d^7 - a^3*c^4*d^6))/(2*c*d^6 + d^7 + c^2*d^5) + (a^3*((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^8))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^12 + 6*c^2*d^11 + c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8))/(2*c*d^7 + d^8 + c^2*d^6))*(-(c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3)))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))*1i)/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3) + (a^3*(-(c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d)*((32*(9*a^6*c^2*d^6 + 6*a^6*c^3*d^5 - 11*a^6*c^4*d^4 - 4*a^6*c^5*d^3 + 4*a^6*c^6*d^2))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(9*a^6*c*d^8 + 36*a^6*c^2*d^7 - 41*a^6*c^3*d^6 - 34*a^6*c^4*d^5 + 34*a^6*c^5*d^4 + 8*a^6*c^6*d^3 - 8*a^6*c^7*d^2))/(2*c*d^7 + d^8 + c^2*d^6) + (a^3*(-(c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d)*((32*(3*a^3*c*d^9 + a^3*c^2*d^8 - 3*a^3*c^3*d^7 - a^3*c^4*d^6))/(2*c*d^6 + d^7 + c^2*d^5) - (32*tan(e/2 + (f*x)/2)*(6*a^3*c*d^10 - 2*a^3*c^2*d^9 - 10*a^3*c^3*d^8 + 2*a^3*c^4*d^7 + 4*a^3*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6) + (a^3*((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^8))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^12 + 6*c^2*d^11 + c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8))/(2*c*d^7 + d^8 + c^2*d^6))*(-(c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3)))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))*1i)/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))/((64*(4*a^9*c^6 + 27*a^9*c*d^5 - 20*a^9*c^5*d - 63*a^9*c^2*d^4 + 33*a^9*c^3*d^3 + 19*a^9*c^4*d^2))/(2*c*d^6 + d^7 + c^2*d^5) - (64*tan(e/2 + (f*x)/2)*(40*a^9*c^6*d - 54*a^9*c*d^6 - 16*a^9*c^7 + 90*a^9*c^2*d^5 + 42*a^9*c^3*d^4 - 130*a^9*c^4*d^3 + 28*a^9*c^5*d^2))/(2*c*d^7 + d^8 + c^2*d^6) + (a^3*(-(c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d)*((32*(9*a^6*c^2*d^6 + 6*a^6*c^3*d^5 - 11*a^6*c^4*d^4 - 4*a^6*c^5*d^3 + 4*a^6*c^6*d^2))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(9*a^6*c*d^8 + 36*a^6*c^2*d^7 - 41*a^6*c^3*d^6 - 34*a^6*c^4*d^5 + 34*a^6*c^5*d^4 + 8*a^6*c^6*d^3 - 8*a^6*c^7*d^2))/(2*c*d^7 + d^8 + c^2*d^6) + (a^3*(-(c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d)*((32*tan(e/2 + (f*x)/2)*(6*a^3*c*d^10 - 2*a^3*c^2*d^9 - 10*a^3*c^3*d^8 + 2*a^3*c^4*d^7 + 4*a^3*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6) - (32*(3*a^3*c*d^9 + a^3*c^2*d^8 - 3*a^3*c^3*d^7 - a^3*c^4*d^6))/(2*c*d^6 + d^7 + c^2*d^5) + (a^3*((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^8))/(2*c*d^6 + d^7 + c^2*d^5) + (32

```
*tan(e/2 + (f*x)/2)*(3*c*d^12 + 6*c^2*d^11 + c^3*d^10 - 4*c^4*d^9 - 2*c^5*d
^8))/(2*c*d^7 + d^8 + c^2*d^6))*(-(c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d))/(
3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3)))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3
)))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3) - (a^3*(-(c + d)^3*(c - d)^3)^(1/
2)*(2*c + 3*d)*((32*(9*a^6*c^2*d^6 + 6*a^6*c^3*d^5 - 11*a^6*c^4*d^4 - 4*a^6
*c^5*d^3 + 4*a^6*c^6*d^2))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/
2)*(9*a^6*c*d^8 + 36*a^6*c^2*d^7 - 41*a^6*c^3*d^6 - 34*a^6*c^4*d^5 + 34*a^6
*c^5*d^4 + 8*a^6*c^6*d^3 - 8*a^6*c^7*d^2))/(2*c*d^7 + d^8 + c^2*d^6) + (a^3
*(-(c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d)*((32*(3*a^3*c*d^9 + a^3*c^2*d^8 -
 3*a^3*c^3*d^7 - a^3*c^4*d^6))/(2*c*d^6 + d^7 + c^2*d^5) - (32*tan(e/2 + (f
*x)/2)*(6*a^3*c*d^10 - 2*a^3*c^2*d^9 - 10*a^3*c^3*d^8 + 2*a^3*c^4*d^7 + 4*a
^3*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6) + (a^3*((32*(c^2*d^10 + 2*c^3*d^9 +
c^4*d^8))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^12 + 6*
c^2*d^11 + c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8))/(2*c*d^7 + d^8 + c^2*d^6))*(-
(c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^
3)))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3)))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c
^3*d^3)))*(-(c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d)*2i)/(f*(3*c*d^5 + d^6 +
3*c^2*d^4 + c^3*d^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.450 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=187

$$-\frac{a^3(c-d)\left(2c^2+6cd+7d^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f(c+d)^2\sqrt{c^2-d^2}}+\frac{a^3(c-d)(2c+5d)\cos(e+fx)}{2d^2f(c+d)^2(c+d\sin(e+fx))}+\frac{(c-d)\cos(e+fx)\left(a^3\sin\right.}{2df(c+d)(c+d\sin(}$$

```
[Out] a^3*x/d^3+1/2*(c-d)*cos(f*x+e)*(a^3+a^3*sin(f*x+e))/d/(c+d)/f/(c+d*sin(f*x+
e))^2+1/2*a^3*(c-d)*(2*c+5*d)*cos(f*x+e)/d^2/(c+d)^2/f/(c+d*sin(f*x+e))-a^3
*(c-d)*(2*c^2+6*c*d+7*d^2)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))
/d^3/(c+d)^2/f/(c^2-d^2)^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2762, 2968, 3021, 2735, 2660, 618, 204}

$$-\frac{a^3(c-d)\left(2c^2+6cd+7d^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f(c+d)^2\sqrt{c^2-d^2}}+\frac{a^3(c-d)(2c+5d)\cos(e+fx)}{2d^2f(c+d)^2(c+d\sin(e+fx))}+\frac{(c-d)\cos(e+fx)\left(a^3\sin\right.}{2df(c+d)(c+d\sin(}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a^3*x)/d^3 - (a^3*(c - d)*(2*c^2 + 6*c*d + 7*d^2)*ArcTan[(d + c*Tan[(e + f
*x)/2])/Sqrt[c^2 - d^2]])/(d^3*(c + d)^2*Sqrt[c^2 - d^2]*f) + ((c - d)*Cos[
e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(2*d*(c + d)*f*(c + d*Sin[e + f*x])^2) +
 (a^3*(c - d)*(2*c + 5*d)*Cos[e + f*x])/(2*d^2*(c + d)^2*f*(c + d*Sin[e + f
*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^3}\,dx = \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a\int \frac{(a+a\sin(e+fx))(a(c-5d)-2a(c+d)\sin(e+fx))}{(c+d\sin(e+fx))^2}dx}{2d(c+d)}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a\int \frac{a^2(c-5d)+\left(a^2(c-5d)-2a^2(c+d)\right)\sin(e+fx)-2a^2(c}{(c+d\sin(e+fx))^2}}{2d(c+d)}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{a^3(c-d)(2c+5d)\cos(e+fx)}{2d^2(c+d)^2f(c+d\sin(e+fx))} + \frac{a\int \frac{a^2(}{}}{}$$

$$= \frac{a^3x}{d^3} + \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{a^3(c-d)(2c+5d)\cos(e+fx)}{2d^2(c+d)^2f(c+d\sin(e+fx))} -$$

$$= \frac{a^3x}{d^3} + \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{a^3(c-d)(2c+5d)\cos(e+fx)}{2d^2(c+d)^2f(c+d\sin(e+fx))} -$$

$$= \frac{a^3x}{d^3} + \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{a^3(c-d)(2c+5d)\cos(e+fx)}{2d^2(c+d)^2f(c+d\sin(e+fx))} +$$

$$= \frac{a^3x}{d^3} - \frac{a^3(c-d)\left(2c^2+6cd+7d^2\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3(c+d)^2\sqrt{c^2-d^2}\,f} + \frac{(c-d)\cos(e+fx)\left(a^3+\right.}{2d(c+d)f(c+d\sin}$$

Mathematica [A] time = 1.01, size = 196, normalized size = 1.05

$$\frac{a^3(\sin(e+fx)+1)^3\left(\frac{3d\left(c^2+cd-2d^2\right)\cos(e+fx)}{(c+d)^2(c+d\sin(e+fx))} - \frac{2\left(2c^3+4c^2d+cd^2-7d^3\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{(c+d)^2\sqrt{c^2-d^2}} - \frac{d(c-d)^2\cos(e+fx)}{(c+d)(c+d\sin(e+fx))^2} + 2(e+fx)\right)}{2d^3f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[e + f*x])^3*(2*(e + f*x) - (2*(2*c^3 + 4*c^2*d + c*d^2 - 7*d^
3)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)^2*Sqrt[c^2 -
d^2]) - ((c - d)^2*d*Cos[e + f*x])/((c + d)*(c + d*Sin[e + f*x])^2) + (3*d*
(c^2 + c*d - 2*d^2)*Cos[e + f*x])/((c + d)^2*(c + d*Sin[e + f*x]))))/(2*d^3
*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
```

fricas [B] time = 0.54, size = 1064, normalized size = 5.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^3*c^2*d^2 + 2*a^3*c*d^3 + a^3*d^4)*f*x*cos(f*x + e)^2 - 4*(a^3*c
^4 + 2*a^3*c^3*d + 2*a^3*c^2*d^2 + 2*a^3*c*d^3 + a^3*d^4)*f*x - (2*a^3*c^4
+ 6*a^3*c^3*d + 9*a^3*c^2*d^2 + 6*a^3*c*d^3 + 7*a^3*d^4 - (2*a^3*c^2*d^2 +
6*a^3*c*d^3 + 7*a^3*d^4)*cos(f*x + e)^2 + 2*(2*a^3*c^3*d + 6*a^3*c^2*d^2 +
7*a^3*c*d^3)*sin(f*x + e))*sqrt(-(c - d)/(c + d))*log(((2*c^2 - d^2)*cos(f*
x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*((c^2 + c*d)*cos(f*x + e)*sin
(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x
+ e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 2*(2*a^3*c^3*d + 4*a^3*c^2*d^2
- 5*a^3*c*d^3 - a^3*d^4)*cos(f*x + e) - 2*(4*(a^3*c^3*d + 2*a^3*c^2*d^2 + a
^3*c*d^3)*f*x + 3*(a^3*c^2*d^2 + a^3*c*d^3 - 2*a^3*d^4)*cos(f*x + e))*sin(f
*x + e))/((c^2*d^5 + 2*c*d^6 + d^7)*f*cos(f*x + e)^2 - 2*(c^3*d^4 + 2*c^2*d
^5 + c*d^6)*f*sin(f*x + e) - (c^4*d^3 + 2*c^3*d^4 + 2*c^2*d^5 + 2*c*d^6 + d
^7)*f), 1/2*(2*(a^3*c^2*d^2 + 2*a^3*c*d^3 + a^3*d^4)*f*x*cos(f*x + e)^2 - 2
*(a^3*c^4 + 2*a^3*c^3*d + 2*a^3*c^2*d^2 + 2*a^3*c*d^3 + a^3*d^4)*f*x - (2*a
^3*c^4 + 6*a^3*c^3*d + 9*a^3*c^2*d^2 + 6*a^3*c*d^3 + 7*a^3*d^4 - (2*a^3*c^2
*d^2 + 6*a^3*c*d^3 + 7*a^3*d^4)*cos(f*x + e)^2 + 2*(2*a^3*c^3*d + 6*a^3*c^2
*d^2 + 7*a^3*c*d^3)*sin(f*x + e))*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x
+ e) + d)*sqrt((c - d)/(c + d))/((c - d)*cos(f*x + e))) - (2*a^3*c^3*d + 4*
a^3*c^2*d^2 - 5*a^3*c*d^3 - a^3*d^4)*cos(f*x + e) - (4*(a^3*c^3*d + 2*a^3*c
^2*d^2 + a^3*c*d^3)*f*x + 3*(a^3*c^2*d^2 + a^3*c*d^3 - 2*a^3*d^4)*cos(f*x +
 e))*sin(f*x + e))/((c^2*d^5 + 2*c*d^6 + d^7)*f*cos(f*x + e)^2 - 2*(c^3*d^4
 + 2*c^2*d^5 + c*d^6)*f*sin(f*x + e) - (c^4*d^3 + 2*c^3*d^4 + 2*c^2*d^5 + 2
*c*d^6 + d^7)*f)]
```

giac [B] time = 0.41, size = 522, normalized size = 2.79

$$\frac{(fx+e)a^3}{d^3} - \frac{\left(2\,a^3c^3+4\,a^3c^2d+a^3cd^2-7\,a^3d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(c^2d^3+2\,cd^4+d^5\right)\sqrt{c^2-d^2}} + \frac{a^3c^4d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+5\,a^3c^3d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((f*x + e)*a^3/d^3 - (2*a^3*c^3 + 4*a^3*c^2*d + a^3*c*d^2 - 7*a^3*d^3)*(pi*
floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/
```

```
sqrt(c^2 - d^2)))/((c^2*d^3 + 2*c*d^4 + d^5)*sqrt(c^2 - d^2)) + (a^3*c^4*d*
tan(1/2*f*x + 1/2*e)^3 + 5*a^3*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 4*a^3*c^2*d
^3*tan(1/2*f*x + 1/2*e)^3 - 2*a^3*c*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*a^3*c^5*
tan(1/2*f*x + 1/2*e)^2 + 4*a^3*c^4*d*tan(1/2*f*x + 1/2*e)^2 - a^3*c^3*d^2*t
an(1/2*f*x + 1/2*e)^2 + 7*a^3*c^2*d^3*tan(1/2*f*x + 1/2*e)^2 - 10*a^3*c*d^4
*tan(1/2*f*x + 1/2*e)^2 - 2*a^3*d^5*tan(1/2*f*x + 1/2*e)^2 + 7*a^3*c^4*d*ta
n(1/2*f*x + 1/2*e) + 11*a^3*c^3*d^2*tan(1/2*f*x + 1/2*e) - 16*a^3*c^2*d^3*t
an(1/2*f*x + 1/2*e) - 2*a^3*c*d^4*tan(1/2*f*x + 1/2*e) + 2*a^3*c^5 + 4*a^3*
c^4*d - 5*a^3*c^3*d^2 - a^3*c^2*d^3)/((c^4*d^2 + 2*c^3*d^3 + c^2*d^4)*(c*ta
n(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2))/f
```

maple [B] time = 0.34, size = 1400, normalized size = 7.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x)
```

```
[Out] a^3/f/d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)
*c^2*tan(1/2*f*x+1/2*e)^3+5*a^3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2
*e)*d+c)^2/(c^2+2*c*d+d^2)*c*tan(1/2*f*x+1/2*e)^3-4*a^3/f*d/(tan(1/2*f*x+1/
2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3-2
*a^3/f*d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d
^2)/c*tan(1/2*f*x+1/2*e)^3+2*a^3/f/d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*
x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^3*tan(1/2*f*x+1/2*e)^2+4*a^3/f/d/(tan(1/2
*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^2*tan(1/2*f*x
+1/2*e)^2-a^3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*
c*d+d^2)*c*tan(1/2*f*x+1/2*e)^2+7*a^3/f*d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2
*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2-10*a^3/f*d^2/(tan(1
/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c*tan(1/2*f*x
+1/2*e)^2-2*a^3/f*d^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(
c^2+2*c*d+d^2)/c^2*tan(1/2*f*x+1/2*e)^2+7*a^3/f/d/(tan(1/2*f*x+1/2*e)^2*c+2
*tan(1/2*f*x+1/2*e)*d+c)^2*c^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)+11*a^3/f/
(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+d^2)*tan(1
/2*f*x+1/2*e)-16*a^3/f*d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^
2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)-2*a^3/f*d^2/(tan(1/2*f*x+1/2*e)^2*c+2*
tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)+2*a^3/f/d^2/
(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^3+4*a
^3/f/d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*
c^2-5*a^3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+
d^2)*c-a^3/f*d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c
*d+d^2)-2*a^3/f/d^3/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2
*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c^3-4*a^3/f/d^2/(c^2+2*c*d+d^2)/(c^2-d^2)
^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c^2-a^3/f/d
/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c
```

```
^2-d^2)^(1/2))*c+7*a^3/f/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*ta
n(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))+2*a^3/f/d^3*arctan(tan(1/2*f*x+1/2*e
))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 14.27, size = 6246, normalized size = 33.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c + d*sin(e + f*x))^3,x)
```

```
[Out] ((2*a^3*c^3 - a^3*d^3 - 5*a^3*c*d^2 + 4*a^3*c^2*d)/(d^2*(2*c*d + c^2 + d^2)
) + (tan(e/2 + (f*x)/2)^3*(a^3*c^3 - 2*a^3*d^3 - 4*a^3*c*d^2 + 5*a^3*c^2*d)
)/(c*d*(2*c*d + c^2 + d^2)) + (tan(e/2 + (f*x)/2)*(7*a^3*c^3 - 2*a^3*d^3 -
16*a^3*c*d^2 + 11*a^3*c^2*d))/(c*d*(2*c*d + c^2 + d^2)) + (tan(e/2 + (f*x)/
2)^2*(c^2 + 2*d^2)*(2*a^3*c^3 - a^3*d^3 - 5*a^3*c*d^2 + 4*a^3*c^2*d))/(c^2*
d^2*(2*c*d + c^2 + d^2)))/(f*(tan(e/2 + (f*x)/2)^2*(2*c^2 + 4*d^2) + c^2*ta
n(e/2 + (f*x)/2)^4 + c^2 + 4*c*d*tan(e/2 + (f*x)/2)^3 + 4*c*d*tan(e/2 + (f*
x)/2))) - (2*a^3*atan(-(((((((8*(4*c^2*d^12 + 16*c^3*d^11 + 24*c^4*d^10 + 1
6*c^5*d^9 + 4*c^6*d^8))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) +
 (8*tan(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*d^13 + 64*c^3*d^12 + 16*c^4*d^11
 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c
^3*d^7 + c^4*d^6))*1i)/d^3 - (8*(4*c*d^10 + 2*c^2*d^9 - 6*c^3*d^8 - 2*c^4*d
^7 + 2*c^5*d^6))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan
(e/2 + (f*x)/2)*(28*c*d^11 + 52*c^2*d^10 + 4*c^3*d^9 - 44*c^4*d^8 - 32*c^5*
d^7 - 8*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d
^3 + (8*(4*c^2*d^6 + 16*c^3*d^5 + 24*c^4*d^4 + 16*c^5*d^3 + 4*c^6*d^2))/(4*
c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) - (8*tan(e/2 + (f*x)/2)*(41*
c*d^8 - 46*c^2*d^7 - 99*c^3*d^6 - 36*c^4*d^5 + 36*c^5*d^4 + 32*c^6*d^3 + 8*
c^7*d^2))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))/d^3 + ((((8*(
4*c*d^10 + 2*c^2*d^9 - 6*c^3*d^8 - 2*c^4*d^7 + 2*c^5*d^6))/(4*c*d^8 + d^9 +
 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (((8*(4*c^2*d^12 + 16*c^3*d^11 + 24*c^4
*d^10 + 16*c^5*d^9 + 4*c^6*d^8))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c
```

^4*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*d^13 + 64*c^3*d^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3 - (8*tan(e/2 + (f*x)/2)*(28*c*d^11 + 52*c^2*d^10 + 4*c^3*d^9 - 44*c^4*d^8 - 32*c^5*d^7 - 8*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3 + (8*(4*c^2*d^6 + 16*c^3*d^5 + 24*c^4*d^4 + 16*c^5*d^3 + 4*c^6*d^2))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) - (8*tan(e/2 + (f*x)/2)*(41*c*d^8 - 46*c^2*d^7 - 99*c^3*d^6 - 36*c^4*d^5 + 36*c^5*d^4 + 32*c^6*d^3 + 8*c^7*d^2))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))/d^3)/((16*(18*c^4*d - 49*c*d^4 + 2*c^5 + 29*c^3*d^2))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) - (((((((8*(4*c^2*d^12 + 16*c^3*d^11 + 24*c^4*d^10 + 16*c^5*d^9 + 4*c^6*d^8))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*d^13 + 64*c^3*d^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3 - (8*(4*c*d^10 + 2*c^2*d^9 - 6*c^3*d^8 - 2*c^4*d^7 + 2*c^5*d^6))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(28*c*d^11 + 52*c^2*d^10 + 4*c^3*d^9 - 44*c^4*d^8 - 32*c^5*d^7 - 8*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3 + (8*(4*c^2*d^6 + 16*c^3*d^5 + 24*c^4*d^4 + 16*c^5*d^3 + 4*c^6*d^2))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) - (8*tan(e/2 + (f*x)/2)*(41*c*d^8 - 46*c^2*d^7 - 99*c^3*d^6 - 36*c^4*d^5 + 36*c^5*d^4 + 32*c^6*d^3 + 8*c^7*d^2))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3 + (((((8*(4*c*d^10 + 2*c^2*d^9 - 6*c^3*d^8 - 2*c^4*d^7 + 2*c^5*d^6))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (((8*(4*c^2*d^12 + 16*c^3*d^11 + 24*c^4*d^10 + 16*c^5*d^9 + 4*c^6*d^8))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*d^13 + 64*c^3*d^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3 - (8*tan(e/2 + (f*x)/2)*(28*c*d^11 + 52*c^2*d^10 + 4*c^3*d^9 - 44*c^4*d^8 - 32*c^5*d^7 - 8*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3 + (8*(4*c^2*d^6 + 16*c^3*d^5 + 24*c^4*d^4 + 16*c^5*d^3 + 4*c^6*d^2))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) - (8*tan(e/2 + (f*x)/2)*(41*c*d^8 - 46*c^2*d^7 - 99*c^3*d^6 - 36*c^4*d^5 + 36*c^5*d^4 + 32*c^6*d^3 + 8*c^7*d^2))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3 - (16*tan(e/2 + (f*x)/2)*(28*c*d^5 - 32*c^5*d - 8*c^6 + 52*c^2*d^4 + 4*c^3*d^3 - 44*c^4*d^2))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))))/(d^3*f) + (a^3*atan(((a^3*(-(c + d)^5*(c - d))^(1/2)*(3*c*d + c^2 + (7*d^2)/2)*((8*(4*a^6*c^2*d^6 + 16*a^6*c^3*d^5 + 24*a^6*c^4*d^4 + 16*a^6*c^5*d^3 + 4*a^6*c^6*d^2))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) - (8*tan(e/2 + (f*x)/2)*(41*a^6*c*d^8 - 46*a^6*c^2*d^7 - 99*a^6*c^3*d^6 - 36*a^6*c^4*d^5 + 36*a^6*c^5*d^4 + 32*a^6*c^6*d^3 + 8*a^6*c^7*d^2))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (a^3*(-(c + d)^5*(c - d))^(1/2)*(3*c*d + c^2 + (7*d^2)/2)*((8*tan(e/2 + (f*x)/2)*(28*a^3*c*d^11 + 52*a^3*c^2*d^10 + 4*a^3*c^3*d^9 - 44*a^3*c^4*d^8 - 32*a^3*c^5*d^7 - 8*a^3*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) - (8*(4*a^3*c*d^10 + 2*a^3*c^2*d^9 - 6*a^3*c^3*d^8 - 2*a^3*c^4*d^7 + 2*a^3*c^5*d^6))/(4*c*d^8 + d^9

+ 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (a^3*(-(c + d)^5*(c - d))^(1/2)*((8*(4
*c^2*d^12 + 16*c^3*d^11 + 24*c^4*d^10 + 16*c^5*d^9 + 4*c^6*d^8))/(4*c*d^8 +
d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^14
+ 48*c^2*d^13 + 64*c^3*d^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^
7*d^8))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*(3*c*d + c^2 +
(7*d^2)/2))/(5*c*d^7 + d^8 + 10*c^2*d^6 + 10*c^3*d^5 + 5*c^4*d^4 + c^5*d^3)
))/(5*c*d^7 + d^8 + 10*c^2*d^6 + 10*c^3*d^5 + 5*c^4*d^4 + c^5*d^3))*1i)/(5*
c*d^7 + d^8 + 10*c^2*d^6 + 10*c^3*d^5 + 5*c^4*d^4 + c^5*d^3) + (a^3*(-(c +
d)^5*(c - d))^(1/2)*(3*c*d + c^2 + (7*d^2)/2)*((8*(4*a^6*c^2*d^6 + 16*a^6*c
^3*d^5 + 24*a^6*c^4*d^4 + 16*a^6*c^5*d^3 + 4*a^6*c^6*d^2))/(4*c*d^8 + d^9 +
6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) - (8*tan(e/2 + (f*x)/2)*(41*a^6*c*d^8 - 4
6*a^6*c^2*d^7 - 99*a^6*c^3*d^6 - 36*a^6*c^4*d^5 + 36*a^6*c^5*d^4 + 32*a^6*c
^6*d^3 + 8*a^6*c^7*d^2))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6)
+ (a^3*(-(c + d)^5*(c - d))^(1/2)*(3*c*d + c^2 + (7*d^2)/2)*((8*(4*a^3*c*d
^10 + 2*a^3*c^2*d^9 - 6*a^3*c^3*d^8 - 2*a^3*c^4*d^7 + 2*a^3*c^5*d^6))/(4*c*
d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) - (8*tan(e/2 + (f*x)/2)*(28*a^
3*c*d^11 + 52*a^3*c^2*d^10 + 4*a^3*c^3*d^9 - 44*a^3*c^4*d^8 - 32*a^3*c^5*d^
7 - 8*a^3*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (a
^3*(-(c + d)^5*(c - d))^(1/2)*((8*(4*c^2*d^12 + 16*c^3*d^11 + 24*c^4*d^10 +
16*c^5*d^9 + 4*c^6*d^8))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5)
+ (8*tan(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*d^13 + 64*c^3*d^12 + 16*c^4*d^
11 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4
*c^3*d^7 + c^4*d^6))*(3*c*d + c^2 + (7*d^2)/2))/(5*c*d^7 + d^8 + 10*c^2*d^6
+ 10*c^3*d^5 + 5*c^4*d^4 + c^5*d^3)))/(5*c*d^7 + d^8 + 10*c^2*d^6 + 10*c^3
*d^5 + 5*c^4*d^4 + c^5*d^3))*1i)/(5*c*d^7 + d^8 + 10*c^2*d^6 + 10*c^3*d^5 +
5*c^4*d^4 + c^5*d^3))/((16*(2*a^9*c^5 - 49*a^9*c*d^4 + 18*a^9*c^4*d + 29*a
^9*c^3*d^2))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (16*tan(e/
2 + (f*x)/2)*(8*a^9*c^6 - 28*a^9*c*d^5 + 32*a^9*c^5*d - 52*a^9*c^2*d^4 - 4*
a^9*c^3*d^3 + 44*a^9*c^4*d^2))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^
4*d^6) - (a^3*(-(c + d)^5*(c - d))^(1/2)*(3*c*d + c^2 + (7*d^2)/2)*((8*(4*a
^6*c^2*d^6 + 16*a^6*c^3*d^5 + 24*a^6*c^4*d^4 + 16*a^6*c^5*d^3 + 4*a^6*c^6*d
^2))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) - (8*tan(e/2 + (f*x)
/2)*(41*a^6*c*d^8 - 46*a^6*c^2*d^7 - 99*a^6*c^3*d^6 - 36*a^6*c^4*d^5 + 36*a
^6*c^5*d^4 + 32*a^6*c^6*d^3 + 8*a^6*c^7*d^2))/(4*c*d^9 + d^10 + 6*c^2*d^8 +
4*c^3*d^7 + c^4*d^6) + (a^3*(-(c + d)^5*(c - d))^(1/2)*(3*c*d + c^2 + (7*d
^2)/2)*((8*tan(e/2 + (f*x)/2)*(28*a^3*c*d^11 + 52*a^3*c^2*d^10 + 4*a^3*c^3*
d^9 - 44*a^3*c^4*d^8 - 32*a^3*c^5*d^7 - 8*a^3*c^6*d^6))/(4*c*d^9 + d^10 + 6
*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) - (8*(4*a^3*c*d^10 + 2*a^3*c^2*d^9 - 6*a^3*
c^3*d^8 - 2*a^3*c^4*d^7 + 2*a^3*c^5*d^6))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^
3*d^6 + c^4*d^5) + (a^3*(-(c + d)^5*(c - d))^(1/2)*((8*(4*c^2*d^12 + 16*c^3
*d^11 + 24*c^4*d^10 + 16*c^5*d^9 + 4*c^6*d^8))/(4*c*d^8 + d^9 + 6*c^2*d^7 +
4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*d^13 + 64
*c^3*d^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(4*c*d^9 +
d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*(3*c*d + c^2 + (7*d^2)/2))/(5*c*d
^7 + d^8 + 10*c^2*d^6 + 10*c^3*d^5 + 5*c^4*d^4 + c^5*d^3)))/(5*c*d^7 + d^8

```
+ 10*c^2*d^6 + 10*c^3*d^5 + 5*c^4*d^4 + c^5*d^3)))/(5*c*d^7 + d^8 + 10*c^2*
d^6 + 10*c^3*d^5 + 5*c^4*d^4 + c^5*d^3) + (a^3*(-(c + d)^5*(c - d))^(1/2)*(
3*c*d + c^2 + (7*d^2)/2)*((8*(4*a^6*c^2*d^6 + 16*a^6*c^3*d^5 + 24*a^6*c^4*d
^4 + 16*a^6*c^5*d^3 + 4*a^6*c^6*d^2))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^
6 + c^4*d^5) - (8*tan(e/2 + (f*x)/2)*(41*a^6*c*d^8 - 46*a^6*c^2*d^7 - 99*a^
6*c^3*d^6 - 36*a^6*c^4*d^5 + 36*a^6*c^5*d^4 + 32*a^6*c^6*d^3 + 8*a^6*c^7*d^
2))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (a^3*(-(c + d)^5*(
c - d))^(1/2)*(3*c*d + c^2 + (7*d^2)/2)*((8*(4*a^3*c*d^10 + 2*a^3*c^2*d^9 -
 6*a^3*c^3*d^8 - 2*a^3*c^4*d^7 + 2*a^3*c^5*d^6))/(4*c*d^8 + d^9 + 6*c^2*d^7
 + 4*c^3*d^6 + c^4*d^5) - (8*tan(e/2 + (f*x)/2)*(28*a^3*c*d^11 + 52*a^3*c^2
*d^10 + 4*a^3*c^3*d^9 - 44*a^3*c^4*d^8 - 32*a^3*c^5*d^7 - 8*a^3*c^6*d^6))/(
4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (a^3*(-(c + d)^5*(c - d
))^(1/2)*((8*(4*c^2*d^12 + 16*c^3*d^11 + 24*c^4*d^10 + 16*c^5*d^9 + 4*c^6*d
^8))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)
/2)*(12*c*d^14 + 48*c^2*d^13 + 64*c^3*d^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32
*c^6*d^9 - 8*c^7*d^8))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*
(3*c*d + c^2 + (7*d^2)/2))/(5*c*d^7 + d^8 + 10*c^2*d^6 + 10*c^3*d^5 + 5*c^4
*d^4 + c^5*d^3)))/(5*c*d^7 + d^8 + 10*c^2*d^6 + 10*c^3*d^5 + 5*c^4*d^4 + c^
5*d^3)))/(5*c*d^7 + d^8 + 10*c^2*d^6 + 10*c^3*d^5 + 5*c^4*d^4 + c^5*d^3)))*
(-(c + d)^5*(c - d))^(1/2)*(3*c*d + c^2 + (7*d^2)/2)*2i)/(f*(5*c*d^7 + d^8
+ 10*c^2*d^6 + 10*c^3*d^5 + 5*c^4*d^4 + c^5*d^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.451 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^4}\,dx$

Optimal. Leaf size=207

$$\frac{5a^3\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c+d)^3\sqrt{c^2-d^2}}-\frac{a^3\left(2c^2+9cd+22d^2\right)\cos(e+fx)}{6d^2f(c+d)^3(c+d\sin(e+fx))}+\frac{a^3(c-d)(2c+7d)\cos(e+fx)}{6d^2f(c+d)^2(c+d\sin(e+fx))^2}+\frac{(c-d)\cos(e}{3df(c+}$$

```
[Out] 1/3*(c-d)*cos(f*x+e)*(a^3+a^3*sin(f*x+e))/d/(c+d)/f/(c+d*sin(f*x+e))^3+1/6*
a^3*(c-d)*(2*c+7*d)*cos(f*x+e)/d^2/(c+d)^2/f/(c+d*sin(f*x+e))^2-1/6*a^3*(2*
c^2+9*c*d+22*d^2)*cos(f*x+e)/d^2/(c+d)^3/f/(c+d*sin(f*x+e))+5*a^3*arctan((d
+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(c+d)^3/f/(c^2-d^2)^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2762, 2968, 3021, 2754, 12, 2660, 618, 204}

$$\frac{5a^3\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c+d)^3\sqrt{c^2-d^2}}-\frac{a^3\left(2c^2+9cd+22d^2\right)\cos(e+fx)}{6d^2f(c+d)^3(c+d\sin(e+fx))}+\frac{a^3(c-d)(2c+7d)\cos(e+fx)}{6d^2f(c+d)^2(c+d\sin(e+fx))^2}+\frac{(c-d)\cos(e}{3df(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^4,x]
```

```
[Out] (5*a^3*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)^3*Sqrt[c^
2 - d^2]*f) + ((c - d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(3*d*(c + d)*
f*(c + d*Sin[e + f*x])^3) + (a^3*(c - d)*(2*c + 7*d)*Cos[e + f*x])/(6*d^2*(
c + d)^2*f*(c + d*Sin[e + f*x])^2) - (a^3*(2*c^2 + 9*c*d + 22*d^2)*Cos[e +
f*x])/(6*d^2*(c + d)^3*f*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
```

```
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^4}\,dx = \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^3} - \frac{a\int \frac{(a+a\sin(e+fx))(a(c-7d)-2a(c+2d)\sin(e+fx))}{(c+d\sin(e+fx))^3}}{3d(c+d)}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^3} - \frac{a\int \frac{a^2(c-7d)+\left(a^2(c-7d)-2a^2(c+2d)\right)\sin(e+fx)-2a}{(c+d\sin(e+fx))^3}}{3d(c+d)}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^3} + \frac{a^3(c-d)(2c+7d)\cos(e+fx)}{6d^2(c+d)^2f(c+d\sin(e+fx))^2} + \frac{a\int}{}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^3} + \frac{a^3(c-d)(2c+7d)\cos(e+fx)}{6d^2(c+d)^2f(c+d\sin(e+fx))^2} - \frac{a^3\left(\right.}{6d}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^3} + \frac{a^3(c-d)(2c+7d)\cos(e+fx)}{6d^2(c+d)^2f(c+d\sin(e+fx))^2} - \frac{a^3\left(\right.}{6d}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^3} + \frac{a^3(c-d)(2c+7d)\cos(e+fx)}{6d^2(c+d)^2f(c+d\sin(e+fx))^2} - \frac{a^3\left(\right.}{6d}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^3} + \frac{a^3(c-d)(2c+7d)\cos(e+fx)}{6d^2(c+d)^2f(c+d\sin(e+fx))^2} - \frac{a^3\left(\right.}{6d}$$

$$= \frac{5a^3\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c+d)^3\sqrt{c^2-d^2}\,f} + \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^3} + \frac{a^3(c-d)}{6d^2(c+d}$$

Mathematica [A] time = 2.35, size = 178, normalized size = 0.86

$$\frac{a^3\cos(e+fx)\left(-\frac{(\sin(e+fx)+1)^2}{(c+d\sin(e+fx))^3}-\frac{5(\sin(e+fx)+1)}{2(c+d)(c+d\sin(e+fx))^2}-\frac{15}{2(c+d)^2(c+d\sin(e+fx))}+\frac{15\tanh^{-1}\left(\frac{\sqrt{c-d}\sqrt{1-\sin(e+fx)}}{\sqrt{-c-d}\sqrt{\sin(e+fx)+1}}\right)}{(-c-d)^{5/2}\sqrt{c-d}\sqrt{\cos^2(e+fx)}}\right)}{3f(c+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^4,x]
```

```
[Out] (a^3*Cos[e + f*x]*((15*ArcTanh[(Sqrt[c - d]*Sqrt[1 - Sin[e + f*x]])/(Sqrt[-
c - d]*Sqrt[1 + Sin[e + f*x]])])/((-c - d)^(5/2)*Sqrt[c - d]*Sqrt[Cos[e + f
*x]^2]) - (1 + Sin[e + f*x])^2/(c + d*Sin[e + f*x])^3 - (5*(1 + Sin[e + f*x
]))/(2*(c + d)*(c + d*Sin[e + f*x])^2) - 15/(2*(c + d)^2*(c + d*Sin[e + f*x
]))))/(3*(c + d)*f)
```

fricas [B] time = 0.54, size = 1106, normalized size = 5.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(2*(2*a^3*c^4 + 9*a^3*c^3*d + 20*a^3*c^2*d^2 - 9*a^3*c*d^3 - 22*a^3*
d^4)*cos(f*x + e)^3 - 6*(3*a^3*c^4 + 16*a^3*c^3*d - 16*a^3*c*d^3 - 3*a^3*d^
4)*cos(f*x + e)*sin(f*x + e) + 15*(3*a^3*c*d^2*cos(f*x + e)^2 - a^3*c^3 - 3
*a^3*c*d^2 + (a^3*d^3*cos(f*x + e)^2 - 3*a^3*c^2*d - a^3*d^3)*sin(f*x + e))
*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) -
c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^
2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 12*(4*a^3*c^4
+ 3*a^3*c^3*d - 3*a^3*c*d^3 - 4*a^3*d^4)*cos(f*x + e))/(3*(c^6*d^2 + 3*c^5*
d^3 + 2*c^4*d^4 - 2*c^3*d^5 - 3*c^2*d^6 - c*d^7)*f*cos(f*x + e)^2 - (c^8 +
3*c^7*d + 5*c^6*d^2 + 7*c^5*d^3 + 3*c^4*d^4 - 7*c^3*d^5 - 9*c^2*d^6 - 3*c*d
^7)*f + ((c^5*d^3 + 3*c^4*d^4 + 2*c^3*d^5 - 2*c^2*d^6 - 3*c*d^7 - d^8)*f*co
s(f*x + e)^2 - (3*c^7*d + 9*c^6*d^2 + 7*c^5*d^3 - 3*c^4*d^4 - 7*c^3*d^5 - 5
*c^2*d^6 - 3*c*d^7 - d^8)*f)*sin(f*x + e)), -1/6*((2*a^3*c^4 + 9*a^3*c^3*d
+ 20*a^3*c^2*d^2 - 9*a^3*c*d^3 - 22*a^3*d^4)*cos(f*x + e)^3 - 3*(3*a^3*c^4
+ 16*a^3*c^3*d - 16*a^3*c*d^3 - 3*a^3*d^4)*cos(f*x + e)*sin(f*x + e) + 15*(
3*a^3*c*d^2*cos(f*x + e)^2 - a^3*c^3 - 3*a^3*c*d^2 + (a^3*d^3*cos(f*x + e)^
2 - 3*a^3*c^2*d - a^3*d^3)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x
 + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) - 6*(4*a^3*c^4 + 3*a^3*c^3*d - 3
*a^3*c*d^3 - 4*a^3*d^4)*cos(f*x + e))/(3*(c^6*d^2 + 3*c^5*d^3 + 2*c^4*d^4 -
 2*c^3*d^5 - 3*c^2*d^6 - c*d^7)*f*cos(f*x + e)^2 - (c^8 + 3*c^7*d + 5*c^6*d
^2 + 7*c^5*d^3 + 3*c^4*d^4 - 7*c^3*d^5 - 9*c^2*d^6 - 3*c*d^7)*f + ((c^5*d^3
 + 3*c^4*d^4 + 2*c^3*d^5 - 2*c^2*d^6 - 3*c*d^7 - d^8)*f*cos(f*x + e)^2 - (3
*c^7*d + 9*c^6*d^2 + 7*c^5*d^3 - 3*c^4*d^4 - 7*c^3*d^5 - 5*c^2*d^6 - 3*c*d^
7 - d^8)*f)*sin(f*x + e))]
```

giac [B] time = 0.55, size = 667, normalized size = 3.22

$$\frac{15\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)a^3}{\left(c^3+3c^2d+3cd^2+d^3\right)\sqrt{c^2-d^2}}+\frac{9a^3c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-18a^3c^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-18a^3c^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-6a^3c^2d^3\tan\left(\frac{1}{2}f\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^4,x, algorithm="giac")

[Out] 1/3*(15*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))*a^3/((c^3 + 3*c^2*d + 3*c*d^2 + d^3)*sqrt(c^2 - d^2)) + (9*a^3*c^5*tan(1/2*f*x + 1/2*e)^5 - 18*a^3*c^4*d*tan(1/2*f*x + 1/2*e)^5 - 18*a^3*c^3*d^2*tan(1/2*f*x + 1/2*e)^5 - 6*a^3*c^2*d^3*tan(1/2*f*x + 1/2*e)^5 - 18*a^3*c^5*tan(1/2*f*x + 1/2*e)^4 + 9*a^3*c^4*d*tan(1/2*f*x + 1/2*e)^4 - 90*a^3*c^3*d^2*tan(1/2*f*x + 1/2*e)^4 - 54*a^3*c^2*d^3*tan(1/2*f*x + 1/2*e)^4 - 12*a^3*c*d^4*tan(1/2*f*x + 1/2*e)^4 - 132*a^3*c^4*d*tan(1/2*f*x + 1/2*e)^3 - 54*a^3*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 100*a^3*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 - 36*a^3*c*d^4*tan(1/2*f*x + 1/2*e)^3 - 8*a^3*d^5*tan(1/2*f*x + 1/2*e)^3 - 48*a^3*c^5*tan(1/2*f*x + 1/2*e)^2 - 36*a^3*c^4*d*tan(1/2*f*x + 1/2*e)^2 - 180*a^3*c^3*d^2*tan(1/2*f*x + 1/2*e)^2 - 54*a^3*c^2*d^3*tan(1/2*f*x + 1/2*e)^2 - 12*a^3*c*d^4*tan(1/2*f*x + 1/2*e)^2 - 9*a^3*c^5*tan(1/2*f*x + 1/2*e) - 114*a^3*c^4*d*tan(1/2*f*x + 1/2*e) - 36*a^3*c^3*d^2*tan(1/2*f*x + 1/2*e) - 6*a^3*c^2*d^3*tan(1/2*f*x + 1/2*e) - 22*a^3*c^5 - 9*a^3*c^4*d - 2*a^3*c^3*d^2)/((c^6 + 3*c^5*d + 3*c^4*d^2 + c^3*d^3)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^3))/f

maple [B] time = 0.38, size = 1924, normalized size = 9.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^4,x)

[Out] -30*a^3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^4*d^2-18*a^3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*d^2/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^3-60*a^3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^2*d^2-12*a^3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)*d^2+3*a^3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^2/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5-6*a^3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^2/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^4-16*a^3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^2/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^2-3*a^3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^2/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)-3*a^3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^3+3*c^2*d+3*c*d^2+d^3)*c*d-2*a^3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/c/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5*d^3+3*a^3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^4*d-18*a^3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/c/(c^3+3*c

```
^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^4*d^3-4*a^3/f/(tan(1/2*f*x+1/2*e)^2*c+
2*tan(1/2*f*x+1/2*e)*d+c)^3/c^2/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e
)^4*d^4-44*a^3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c*d/(c
^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^3-100/3*a^3/f/(tan(1/2*f*x+1/2*e
)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/c*d^3/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f
*x+1/2*e)^3-12*a^3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/c^
2*d^4/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^3-8/3*a^3/f/(tan(1/2*f*x
+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/c^3*d^5/(c^3+3*c^2*d+3*c*d^2+d^3)*t
an(1/2*f*x+1/2*e)^3-12*a^3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d
+c)^3*c/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^2*d-18*a^3/f/(tan(1/2*
f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/c/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(
1/2*f*x+1/2*e)^2*d^3-4*a^3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d
+c)^3/c^2/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^2*d^4-38*a^3/f/(tan(
1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c/(c^3+3*c^2*d+3*c*d^2+d^3)*
tan(1/2*f*x+1/2*e)*d-2*a^3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d
+c)^3/c/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)*d^3-6*a^3/f/(tan(1/2*f
*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1
/2*f*x+1/2*e)^5*d-6*a^3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)
^3/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5*d^2+5*a^3/f/(c^3+3*c^2*d+
3*c*d^2+d^3)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d
^2)^(1/2))-22/3*a^3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(
c^3+3*c^2*d+3*c*d^2+d^3)*c^2-2/3*a^3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*
x+1/2*e)*d+c)^3/(c^3+3*c^2*d+3*c*d^2+d^3)*d^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 10.36, size = 649, normalized size = 3.14

$$\frac{5\,a^3\,\mathrm{atan}\left(\frac{\left(\frac{5\,a^3\,\left(2\,c^3\,d+6\,c^2\,d^2+6\,c\,d^3+2\,d^4\right)}{2\,{\left(c+d\right)}^{7/2}\,\sqrt{c-d}\,\left(c^3+3\,c^2\,d+3\,c\,d^2+d^3\right)}+\frac{5\,a^3\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{{\left(c+d\right)}^{7/2}\,\sqrt{c-d}}\right)\,\left(c^3+3\,c^2\,d+3\,c\,d^2+d^3\right)}{5\,a^3}\right)}{f\,{\left(c+d\right)}^{7/2}\,\sqrt{c-d}}-\frac{\frac{22\,a^3\,c^2+9\,a^3\,c\,d+2\,a^3\,d^2}{3\,\left(c^3+3\,c^2\,d+3\,c\,d^2+d^3\right)}+\frac{a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\left(-3\,c^3+6\,c\cdots\right.}{c\,\left(c^3+3\,c^2\,d+3\,c\,d\cdots\right.}}{f\,\left(c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c + d*sin(e + f*x))^4,x)
```

```
[Out] (5*a^3*atan((((5*a^3*(6*c*d^3 + 2*c^3*d + 2*d^4 + 6*c^2*d^2))/(2*(c + d)^(7
/2)*(c - d)^(1/2)*(3*c*d^2 + 3*c^2*d + c^3 + d^3)) + (5*a^3*c*tan(e/2 + (f*
x)/2))/((c + d)^(7/2)*(c - d)^(1/2)))*(3*c*d^2 + 3*c^2*d + c^3 + d^3))/(5*a
^3)))/(f*(c + d)^(7/2)*(c - d)^(1/2)) - ((22*a^3*c^2 + 2*a^3*d^2 + 9*a^3*c*
d)/(3*(3*c*d^2 + 3*c^2*d + c^3 + d^3)) + (a^3*tan(e/2 + (f*x)/2)^5*(6*c*d^2
 + 6*c^2*d - 3*c^3 + 2*d^3))/(c*(3*c*d^2 + 3*c^2*d + c^3 + d^3)) + (a^3*tan
(e/2 + (f*x)/2)*(12*c*d^2 + 38*c^2*d + 3*c^3 + 2*d^3))/(c*(3*c*d^2 + 3*c^2*
d + c^3 + d^3)) + (2*a^3*tan(e/2 + (f*x)/2)^2*(9*c*d^3 + 6*c^3*d + 8*c^4 +
2*d^4 + 30*c^2*d^2))/(c^2*(3*c*d^2 + 3*c^2*d + c^3 + d^3)) + (a^3*tan(e/2 +
 (f*x)/2)^4*(18*c*d^3 - 3*c^3*d + 6*c^4 + 4*d^4 + 30*c^2*d^2))/(c^2*(3*c*d^
2 + 3*c^2*d + c^3 + d^3)) + (2*a^3*d*tan(e/2 + (f*x)/2)^3*(3*c^2 + 2*d^2)*(
9*c*d + 22*c^2 + 2*d^2))/(3*c^3*(3*c*d^2 + 3*c^2*d + c^3 + d^3)))/(f*(c^3*t
an(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^2*(12*c*d^2 + 3*c^3) + tan(e/2 + (
f*x)/2)^4*(12*c*d^2 + 3*c^3) + tan(e/2 + (f*x)/2)^3*(12*c^2*d + 8*d^3) + c^
3 + 6*c^2*d*tan(e/2 + (f*x)/2) + 6*c^2*d*tan(e/2 + (f*x)/2)^5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**4,x)
```

```
[Out] Timed out
```

3.452 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^5}\,dx$

Optimal. Leaf size=289

$$\frac{5a^3(4c-3d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{4f(c-d)(c+d)^4\sqrt{c^2-d^2}}-\frac{a^3\left(2c^2+12cd+45d^2\right)\cos(e+fx)}{24d^2f(c+d)^3(c+d\sin(e+fx))^2}-\frac{a^3\left(2c^3+12c^2d+43cd^2-72d^3\right)\cos(e}{24d^2f(c-d)(c+d)^4(c+d\sin(e+f}$$

```
[Out] 1/4*(c-d)*cos(f*x+e)*(a^3+a^3*sin(f*x+e))/d/(c+d)/f/(c+d*sin(f*x+e))^4+1/12
*a^3*(c-d)*(2*c+9*d)*cos(f*x+e)/d^2/(c+d)^2/f/(c+d*sin(f*x+e))^3-1/24*a^3*(
2*c^2+12*c*d+45*d^2)*cos(f*x+e)/d^2/(c+d)^3/f/(c+d*sin(f*x+e))^2-1/24*a^3*(
2*c^3+12*c^2*d+43*c*d^2-72*d^3)*cos(f*x+e)/(c-d)/d^2/(c+d)^4/f/(c+d*sin(f*x
+e))+5/4*a^3*(4*c-3*d)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(c-
d)/(c+d)^4/f/(c^2-d^2)^(1/2)
```

Rubi [A] time = 0.70, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2762, 2968, 3021, 2754, 12, 2660, 618, 204}

$$\frac{5a^3(4c-3d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{4f(c-d)(c+d)^4\sqrt{c^2-d^2}}-\frac{a^3\left(12c^2d+2c^3+43cd^2-72d^3\right)\cos(e+fx)}{24d^2f(c-d)(c+d)^4(c+d\sin(e+fx))}-\frac{a^3\left(2c^2+12cd+45d^2\right)\cos(e}{24d^2f(c+d)^3(c+d\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^5,x]
```

```
[Out] (5*a^3*(4*c - 3*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(4*(c
- d)*(c + d)^4*Sqrt[c^2 - d^2]*f) + ((c - d)*Cos[e + f*x]*(a^3 + a^3*Sin[e
+ f*x]))/(4*d*(c + d)*f*(c + d*Sin[e + f*x])^4) + (a^3*(c - d)*(2*c + 9*d)*
Cos[e + f*x])/(12*d^2*(c + d)^2*f*(c + d*Sin[e + f*x])^3) - (a^3*(2*c^2 + 1
2*c*d + 45*d^2)*Cos[e + f*x])/(24*d^2*(c + d)^3*f*(c + d*Sin[e + f*x])^2) -
 (a^3*(2*c^3 + 12*c^2*d + 43*c*d^2 - 72*d^3)*Cos[e + f*x])/(24*(c - d)*d^2*
(c + d)^4*f*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^5}\,dx = \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4d(c+d)f(c+d\sin(e+fx))^4} - \frac{a\int \frac{(a+a\sin(e+fx))(a(c-9d)-2a(c+3d)\sin(e+fx))}{(c+d\sin(e+fx))^4}\,d}{4d(c+d)}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4d(c+d)f(c+d\sin(e+fx))^4} - \frac{a\int \frac{a^2(c-9d)+\left(a^2(c-9d)-2a^2(c+3d)\right)\sin(e+fx)-2a^2(}{(c+d\sin(e+fx))^4}}{4d(c+d)}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4d(c+d)f(c+d\sin(e+fx))^4} + \frac{a^3(c-d)(2c+9d)\cos(e+fx)}{12d^2(c+d)^2f(c+d\sin(e+fx))^3} + \frac{a\int}{}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4d(c+d)f(c+d\sin(e+fx))^4} + \frac{a^3(c-d)(2c+9d)\cos(e+fx)}{12d^2(c+d)^2f(c+d\sin(e+fx))^3} - \frac{a^3(2}{24d}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4d(c+d)f(c+d\sin(e+fx))^4} + \frac{a^3(c-d)(2c+9d)\cos(e+fx)}{12d^2(c+d)^2f(c+d\sin(e+fx))^3} - \frac{a^3(2}{24d}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4d(c+d)f(c+d\sin(e+fx))^4} + \frac{a^3(c-d)(2c+9d)\cos(e+fx)}{12d^2(c+d)^2f(c+d\sin(e+fx))^3} - \frac{a^3(2}{24d}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4d(c+d)f(c+d\sin(e+fx))^4} + \frac{a^3(c-d)(2c+9d)\cos(e+fx)}{12d^2(c+d)^2f(c+d\sin(e+fx))^3} - \frac{a^3(2}{24d}$$

$$= \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4d(c+d)f(c+d\sin(e+fx))^4} + \frac{a^3(c-d)(2c+9d)\cos(e+fx)}{12d^2(c+d)^2f(c+d\sin(e+fx))^3} - \frac{a^3(2}{24d}$$

$$= \frac{5a^3(4c-3d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{4(c-d)(c+d)^4\sqrt{c^2-d^2}\,f} + \frac{(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{4d(c+d)f(c+d\sin(e+fx))^4} + \frac{a}{12}$$

Mathematica [A] time = 3.21, size = 240, normalized size = 0.83

$$\frac{a^3\cos(e+fx)\left(-\frac{(4c-3d)\left(-\frac{\sqrt{\cos^2(e+fx)}\left((2c^2+9cd+22d^2)\sin^2(e+fx)+(9c^2+48cd+9d^2)\sin(e+fx)+22c^2+9cd+2d^2\right)}{6(c+d)^3(c+d\sin(e+fx))^3}-\frac{5\tanh^{-1}\left(\frac{\sqrt{c-d}\sqrt{1-\sin(e+fx)}}{\sqrt{-c-d}\sqrt{\sin(e+fx)+1}}\right)}{(-c-d)^{7/2}\sqrt{c-d}}\right)}{\sqrt{\cos^2(e+fx)}}-\frac{d(s}{(c+}\right.}{4f(d-c)(c+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^5,x]
```

```
[Out] (a^3*Cos[e + f*x]*(-((d*(1 + Sin[e + f*x])^3)/(c + d*Sin[e + f*x])^4) - ((4
*c - 3*d)*((-5*ArcTanh[(Sqrt[c - d]*Sqrt[1 - Sin[e + f*x]])/(Sqrt[-c - d]*S
qrt[1 + Sin[e + f*x]])])/((-c - d)^(7/2)*Sqrt[c - d]) - (Sqrt[Cos[e + f*x]^
2]*(22*c^2 + 9*c*d + 2*d^2 + (9*c^2 + 48*c*d + 9*d^2)*Sin[e + f*x] + (2*c^2
 + 9*c*d + 22*d^2)*Sin[e + f*x]^2))/(6*(c + d)^3*(c + d*Sin[e + f*x])^3)))/
Sqrt[Cos[e + f*x]^2]))/(4*(-c + d)*(c + d)*f)
```

fricas [B] time = 0.59, size = 2009, normalized size = 6.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] [1/48*(2*(8*a^3*c^6 + 48*a^3*c^5*d + 164*a^3*c^4*d^2 - 276*a^3*c^3*d^3 - 21
7*a^3*c^2*d^4 + 228*a^3*c*d^5 + 45*a^3*d^6)*cos(f*x + e)^3 - 15*(4*a^3*c^5
- 3*a^3*c^4*d + 24*a^3*c^3*d^2 - 18*a^3*c^2*d^3 + 4*a^3*c*d^4 - 3*a^3*d^5 +
 (4*a^3*c*d^4 - 3*a^3*d^5)*cos(f*x + e)^4 - 2*(12*a^3*c^3*d^2 - 9*a^3*c^2*d
^3 + 4*a^3*c*d^4 - 3*a^3*d^5)*cos(f*x + e)^2 + 4*(4*a^3*c^4*d - 3*a^3*c^3*d
^2 + 4*a^3*c^2*d^3 - 3*a^3*c*d^4 - (4*a^3*c^2*d^3 - 3*a^3*c*d^4)*cos(f*x +
e)^2)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*
c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x +
 e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2
)) - 6*(32*a^3*c^6 + 4*a^3*c^5*d + 13*a^3*c^4*d^2 - 88*a^3*c^3*d^3 - 62*a^3
*c^2*d^4 + 84*a^3*c*d^5 + 17*a^3*d^6)*cos(f*x + e) + 2*((2*a^3*c^5*d + 12*a
^3*c^4*d^2 + 41*a^3*c^3*d^3 - 84*a^3*c^2*d^4 - 43*a^3*c*d^5 + 72*a^3*d^6)*c
os(f*x + e)^3 - 3*(12*a^3*c^6 + 79*a^3*c^5*d - 72*a^3*c^4*d^2 - 98*a^3*c^3*
d^3 + 28*a^3*c^2*d^4 + 19*a^3*c*d^5 + 32*a^3*d^6)*cos(f*x + e))*sin(f*x + e
))/((c^7*d^4 + 3*c^6*d^5 + c^5*d^6 - 5*c^4*d^7 - 5*c^3*d^8 + c^2*d^9 + 3*c*
d^10 + d^11)*f*cos(f*x + e)^4 - 2*(3*c^9*d^2 + 9*c^8*d^3 + 4*c^7*d^4 - 12*c
^6*d^5 - 14*c^5*d^6 - 2*c^4*d^7 + 4*c^3*d^8 + 4*c^2*d^9 + 3*c*d^10 + d^11)*
```

```
f*cos(f*x + e)^2 + (c^11 + 3*c^10*d + 7*c^9*d^2 + 13*c^8*d^3 + 2*c^7*d^4 -
26*c^6*d^5 - 26*c^5*d^6 + 2*c^4*d^7 + 13*c^3*d^8 + 7*c^2*d^9 + 3*c*d^10 + d
^11)*f - 4*((c^8*d^3 + 3*c^7*d^4 + c^6*d^5 - 5*c^5*d^6 - 5*c^4*d^7 + c^3*d^
8 + 3*c^2*d^9 + c*d^10)*f*cos(f*x + e)^2 - (c^10*d + 3*c^9*d^2 + 2*c^8*d^3
- 2*c^7*d^4 - 4*c^6*d^5 - 4*c^5*d^6 - 2*c^4*d^7 + 2*c^3*d^8 + 3*c^2*d^9 + c
*d^10)*f)*sin(f*x + e)), 1/24*((8*a^3*c^6 + 48*a^3*c^5*d + 164*a^3*c^4*d^2
- 276*a^3*c^3*d^3 - 217*a^3*c^2*d^4 + 228*a^3*c*d^5 + 45*a^3*d^6)*cos(f*x +
 e)^3 - 15*(4*a^3*c^5 - 3*a^3*c^4*d + 24*a^3*c^3*d^2 - 18*a^3*c^2*d^3 + 4*a
^3*c*d^4 - 3*a^3*d^5 + (4*a^3*c*d^4 - 3*a^3*d^5)*cos(f*x + e)^4 - 2*(12*a^3
*c^3*d^2 - 9*a^3*c^2*d^3 + 4*a^3*c*d^4 - 3*a^3*d^5)*cos(f*x + e)^2 + 4*(4*a
^3*c^4*d - 3*a^3*c^3*d^2 + 4*a^3*c^2*d^3 - 3*a^3*c*d^4 - (4*a^3*c^2*d^3 - 3
*a^3*c*d^4)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*
x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) - 3*(32*a^3*c^6 + 4*a^3*c^5*d +
 13*a^3*c^4*d^2 - 88*a^3*c^3*d^3 - 62*a^3*c^2*d^4 + 84*a^3*c*d^5 + 17*a^3*d
^6)*cos(f*x + e) + ((2*a^3*c^5*d + 12*a^3*c^4*d^2 + 41*a^3*c^3*d^3 - 84*a^3
*c^2*d^4 - 43*a^3*c*d^5 + 72*a^3*d^6)*cos(f*x + e)^3 - 3*(12*a^3*c^6 + 79*a
^3*c^5*d - 72*a^3*c^4*d^2 - 98*a^3*c^3*d^3 + 28*a^3*c^2*d^4 + 19*a^3*c*d^5
+ 32*a^3*d^6)*cos(f*x + e))*sin(f*x + e))/((c^7*d^4 + 3*c^6*d^5 + c^5*d^6 -
 5*c^4*d^7 - 5*c^3*d^8 + c^2*d^9 + 3*c*d^10 + d^11)*f*cos(f*x + e)^4 - 2*(3
*c^9*d^2 + 9*c^8*d^3 + 4*c^7*d^4 - 12*c^6*d^5 - 14*c^5*d^6 - 2*c^4*d^7 + 4*
c^3*d^8 + 4*c^2*d^9 + 3*c*d^10 + d^11)*f*cos(f*x + e)^2 + (c^11 + 3*c^10*d
+ 7*c^9*d^2 + 13*c^8*d^3 + 2*c^7*d^4 - 26*c^6*d^5 - 26*c^5*d^6 + 2*c^4*d^7
+ 13*c^3*d^8 + 7*c^2*d^9 + 3*c*d^10 + d^11)*f - 4*((c^8*d^3 + 3*c^7*d^4 + c
^6*d^5 - 5*c^5*d^6 - 5*c^4*d^7 + c^3*d^8 + 3*c^2*d^9 + c*d^10)*f*cos(f*x +
e)^2 - (c^10*d + 3*c^9*d^2 + 2*c^8*d^3 - 2*c^7*d^4 - 4*c^6*d^5 - 4*c^5*d^6
- 2*c^4*d^7 + 2*c^3*d^8 + 3*c^2*d^9 + c*d^10)*f)*sin(f*x + e))]
```

giac [B] time = 2.22, size = 1338, normalized size = 4.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^5,x, algorithm="giac")
```

```
[Out] 1/12*(15*(4*a^3*c - 3*a^3*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arc
tan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^5 + 3*c^4*d + 2*c^3*
d^2 - 2*c^2*d^3 - 3*c*d^4 - d^5)*sqrt(c^2 - d^2)) + (36*a^3*c^8*tan(1/2*f*x
 + 1/2*e)^7 - 117*a^3*c^7*d*tan(1/2*f*x + 1/2*e)^7 - 48*a^3*c^6*d^2*tan(1/2
*f*x + 1/2*e)^7 + 48*a^3*c^5*d^3*tan(1/2*f*x + 1/2*e)^7 + 72*a^3*c^4*d^4*ta
n(1/2*f*x + 1/2*e)^7 + 24*a^3*c^3*d^5*tan(1/2*f*x + 1/2*e)^7 - 72*a^3*c^8*t
an(1/2*f*x + 1/2*e)^6 + 132*a^3*c^7*d*tan(1/2*f*x + 1/2*e)^6 - 675*a^3*c^6*
d^2*tan(1/2*f*x + 1/2*e)^6 + 360*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^6 + 288*a
^3*c^3*d^5*tan(1/2*f*x + 1/2*e)^6 + 72*a^3*c^2*d^6*tan(1/2*f*x + 1/2*e)^6 +
 36*a^3*c^8*tan(1/2*f*x + 1/2*e)^5 - 813*a^3*c^7*d*tan(1/2*f*x + 1/2*e)^5 +
 288*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 - 892*a^3*c^5*d^3*tan(1/2*f*x + 1/2
```

```
*e)^5 + 552*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 + 664*a^3*c^3*d^5*tan(1/2*f*
x + 1/2*e)^5 + 384*a^3*c^2*d^6*tan(1/2*f*x + 1/2*e)^5 + 96*a^3*c*d^7*tan(1/
2*f*x + 1/2*e)^5 - 264*a^3*c^8*tan(1/2*f*x + 1/2*e)^4 + 108*a^3*c^7*d*tan(1
/2*f*x + 1/2*e)^4 - 2001*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^4 + 936*a^3*c^5*d
^3*tan(1/2*f*x + 1/2*e)^4 + 202*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^4 + 864*a^
3*c^3*d^5*tan(1/2*f*x + 1/2*e)^4 + 440*a^3*c^2*d^6*tan(1/2*f*x + 1/2*e)^4 +
 192*a^3*c*d^7*tan(1/2*f*x + 1/2*e)^4 + 48*a^3*d^8*tan(1/2*f*x + 1/2*e)^4 -
 36*a^3*c^8*tan(1/2*f*x + 1/2*e)^3 - 1299*a^3*c^7*d*tan(1/2*f*x + 1/2*e)^3
+ 576*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^3 - 1036*a^3*c^5*d^3*tan(1/2*f*x + 1
/2*e)^3 + 1176*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 + 664*a^3*c^3*d^5*tan(1/2
*f*x + 1/2*e)^3 + 384*a^3*c^2*d^6*tan(1/2*f*x + 1/2*e)^3 + 96*a^3*c*d^7*tan
(1/2*f*x + 1/2*e)^3 - 280*a^3*c^8*tan(1/2*f*x + 1/2*e)^2 + 12*a^3*c^7*d*tan
(1/2*f*x + 1/2*e)^2 - 1289*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^2 + 960*a^3*c^5
*d^3*tan(1/2*f*x + 1/2*e)^2 + 552*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^2 + 288*
a^3*c^3*d^5*tan(1/2*f*x + 1/2*e)^2 + 72*a^3*c^2*d^6*tan(1/2*f*x + 1/2*e)^2
- 36*a^3*c^8*tan(1/2*f*x + 1/2*e) - 587*a^3*c^7*d*tan(1/2*f*x + 1/2*e) + 33
6*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e) + 248*a^3*c^5*d^3*tan(1/2*f*x + 1/2*e) +
 120*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e) + 24*a^3*c^3*d^5*tan(1/2*f*x + 1/2*e)
 - 88*a^3*c^8 + 36*a^3*c^7*d + 37*a^3*c^6*d^2 + 24*a^3*c^5*d^3 + 6*a^3*c^4*
d^4)/((c^9 + 3*c^8*d + 2*c^7*d^2 - 2*c^6*d^3 - 3*c^5*d^4 - c^4*d^5)*(c*tan(
1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^4))/f
```

maple [B] time = 0.42, size = 5149, normalized size = 17.82

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^5,x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 10.10, size = 1231, normalized size = 4.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c + d*sin(e + f*x))^5,x)
```

```
[Out] - ((6*a^3*d^4 - 88*a^3*c^4 + 24*a^3*c*d^3 + 36*a^3*c^3*d + 37*a^3*c^2*d^2)/
(12*(3*c*d^4 - 3*c^4*d - c^5 + d^5 + 2*c^2*d^3 - 2*c^3*d^2)) + (a^3*tan(e/2
 + (f*x)/2)^7*(24*c*d^4 - 39*c^4*d + 12*c^5 + 8*d^5 + 16*c^2*d^3 - 16*c^3*d
^2))/(4*c*(3*c*d^4 - 3*c^4*d - c^5 + d^5 + 2*c^2*d^3 - 2*c^3*d^2)) + (a^3*t
an(e/2 + (f*x)/2)^6*(96*c*d^5 + 44*c^5*d - 24*c^6 + 24*d^6 + 120*c^2*d^4 -
225*c^4*d^2))/(4*c^2*(3*c*d^4 - 3*c^4*d - c^5 + d^5 + 2*c^2*d^3 - 2*c^3*d^2
)) + (a^3*tan(e/2 + (f*x)/2)*(120*c*d^4 - 587*c^4*d - 36*c^5 + 24*d^5 + 248
*c^2*d^3 + 336*c^3*d^2))/(12*c*(3*c*d^4 - 3*c^4*d - c^5 + d^5 + 2*c^2*d^3 -
 2*c^3*d^2)) + (a^3*tan(e/2 + (f*x)/2)^5*(384*c*d^6 - 813*c^6*d + 36*c^7 +
96*d^7 + 664*c^2*d^5 + 552*c^3*d^4 - 892*c^4*d^3 + 288*c^5*d^2))/(12*c^3*(3
*c*d^4 - 3*c^4*d - c^5 + d^5 + 2*c^2*d^3 - 2*c^3*d^2)) + (a^3*tan(e/2 + (f*
x)/2)^3*(384*c*d^6 - 1299*c^6*d - 36*c^7 + 96*d^7 + 664*c^2*d^5 + 1176*c^3*
d^4 - 1036*c^4*d^3 + 576*c^5*d^2))/(12*c^3*(3*c*d^4 - 3*c^4*d - c^5 + d^5 +
 2*c^2*d^3 - 2*c^3*d^2)) + (a^3*tan(e/2 + (f*x)/2)^2*(288*c*d^5 + 12*c^5*d
- 280*c^6 + 72*d^6 + 552*c^2*d^4 + 960*c^3*d^3 - 1289*c^4*d^2))/(12*c^2*(3*
c*d^4 - 3*c^4*d - c^5 + d^5 + 2*c^2*d^3 - 2*c^3*d^2)) + (a^3*tan(e/2 + (f*x
)/2)^4*(3*c^4 + 8*d^4 + 24*c^2*d^2)*(24*c*d^3 + 36*c^3*d - 88*c^4 + 6*d^4 +
 37*c^2*d^2))/(12*c^4*(3*c*d^4 - 3*c^4*d - c^5 + d^5 + 2*c^2*d^3 - 2*c^3*d^
2)))/(f*(tan(e/2 + (f*x)/2)^4*(6*c^4 + 16*d^4 + 48*c^2*d^2) + c^4*tan(e/2 +
 (f*x)/2)^8 + c^4 + tan(e/2 + (f*x)/2)^2*(4*c^4 + 24*c^2*d^2) + tan(e/2 + (
f*x)/2)^6*(4*c^4 + 24*c^2*d^2) + tan(e/2 + (f*x)/2)^3*(32*c*d^3 + 24*c^3*d)
 + tan(e/2 + (f*x)/2)^5*(32*c*d^3 + 24*c^3*d) + 8*c^3*d*tan(e/2 + (f*x)/2)
+ 8*c^3*d*tan(e/2 + (f*x)/2)^7)) - (5*a^3*atan((4*((5*a^3*(4*c - 3*d)*(24*c
*d^5 - 8*c^5*d + 8*d^6 + 16*c^2*d^4 - 16*c^3*d^3 - 24*c^4*d^2))/(32*(c + d)
^(9/2)*(c - d)^(3/2)*(3*c*d^4 - 3*c^4*d - c^5 + d^5 + 2*c^2*d^3 - 2*c^3*d^2
)) + (5*a^3*c*tan(e/2 + (f*x)/2)*(4*c - 3*d))/(4*(c + d)^(9/2)*(c - d)^(3/2
)))*(3*c*d^4 - 3*c^4*d - c^5 + d^5 + 2*c^2*d^3 - 2*c^3*d^2))/(20*a^3*c - 15
*a^3*d))*(4*c - 3*d))/(4*f*(c + d)^(9/2)*(c - d)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**5,x)
```

```
[Out] Timed out
```

3.453 $\int \frac{(c+d\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=189

$$\frac{d^2\left(6c^2-20cd+9d^2\right)\sin(e+fx)\cos(e+fx)}{6af}+\frac{2d\left(3c^3-16c^2d+12cd^2-4d^3\right)\cos(e+fx)}{3af}+\frac{dx\left(8c^3-12c^2d+\right.}{2a}$$

```
[Out] 1/2*d*(8*c^3-12*c^2*d+12*c*d^2-3*d^3)*x/a+2/3*d*(3*c^3-16*c^2*d+12*c*d^2-4*
d^3)*cos(f*x+e)/a/f+1/6*d^2*(6*c^2-20*c*d+9*d^2)*cos(f*x+e)*sin(f*x+e)/a/f+
1/3*(3*c-4*d)*d*cos(f*x+e)*(c+d*sin(f*x+e))^2/a/f-(c-d)*cos(f*x+e)*(c+d*sin
(f*x+e))^3/f/(a+a*sin(f*x+e))
```

Rubi [A] time = 0.23, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2767, 2753, 2734}

$$\frac{2d\left(-16c^2d+3c^3+12cd^2-4d^3\right)\cos(e+fx)}{3af}+\frac{d^2\left(6c^2-20cd+9d^2\right)\sin(e+fx)\cos(e+fx)}{6af}+\frac{dx\left(-12c^2d+8c^3\right.}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^4/(a + a*Sin[e + f*x]),x]
```

```
[Out] (d*(8*c^3 - 12*c^2*d + 12*c*d^2 - 3*d^3)*x)/(2*a) + (2*d*(3*c^3 - 16*c^2*d
+ 12*c*d^2 - 4*d^3)*Cos[e + f*x])/(3*a*f) + (d^2*(6*c^2 - 20*c*d + 9*d^2)*C
os[e + f*x]*Sin[e + f*x])/(6*a*f) + ((3*c - 4*d)*d*Cos[e + f*x]*(c + d*Sin[
e + f*x])^2)/(3*a*f) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(f*(a
+ a*Sin[e + f*x]))
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^3}{f(a+a\sin(e+fx))} - \frac{d\int(-a(4c-3d)+a(3c-4d)\sin(e+fx)}{a^2}$$

$$= \frac{(3c-4d)d\cos(e+fx)(c+d\sin(e+fx))^2}{3af} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^3}{f(a+a\sin(e+fx))}$$

$$= \frac{d\left(8c^3-12c^2d+12cd^2-3d^3\right)x}{2a} + \frac{2d\left(3c^3-16c^2d+12cd^2-4d^3\right)\cos(e+fx)}{3af} + \frac{d^2}{}$$

Mathematica [A] time = 0.41, size = 234, normalized size = 1.24

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-3d^2\left(24c^2-16cd+7d^2\right)\cos(e+fx)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^4/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(24*(c - d)^4*Sin[(e + f*x)/2] - 6*d
*(-8*c^3 + 12*c^2*d - 12*c*d^2 + 3*d^3)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2]) - 3*d^2*(24*c^2 - 16*c*d + 7*d^2)*Cos[e + f*x]*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2]) + d^4*Cos[3*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e +
f*x)/2]) - 3*(4*c - d)*d^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sin[2*(e +
 f*x)]))/(12*a*f*(1 + Sin[e + f*x]))
```

fricas [A] time = 0.48, size = 349, normalized size = 1.85

$$\frac{2\,d^4\cos\left(fx+e\right)^4-6\,c^4+24\,c^3d-36\,c^2d^2+24\,cd^3-6\,d^4+\left(12\,cd^3-d^4\right)\cos\left(fx+e\right)^3+3\left(8\,c^3d-12\,c^2d^2+12\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*d^4*cos(f*x + e)^4 - 6*c^4 + 24*c^3*d - 36*c^2*d^2 + 24*c*d^3 - 6*d^
4 + (12*c*d^3 - d^4)*cos(f*x + e)^3 + 3*(8*c^3*d - 12*c^2*d^2 + 12*c*d^3 -
3*d^4)*f*x - 12*(3*c^2*d^2 - 2*c*d^3 + d^4)*cos(f*x + e)^2 - 3*(2*c^4 - 8*c
^3*d + 24*c^2*d^2 - 12*c*d^3 + 5*d^4 - (8*c^3*d - 12*c^2*d^2 + 12*c*d^3 - 3
*d^4)*f*x)*cos(f*x + e) + (2*d^4*cos(f*x + e)^3 + 6*c^4 - 24*c^3*d + 36*c^2
*d^2 - 24*c*d^3 + 6*d^4 + 3*(8*c^3*d - 12*c^2*d^2 + 12*c*d^3 - 3*d^4)*f*x -
 3*(4*c*d^3 - d^4)*cos(f*x + e)^2 - 3*(12*c^2*d^2 - 4*c*d^3 + 3*d^4)*cos(f*
x + e))*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

giac [A] time = 0.22, size = 306, normalized size = 1.62

$$\frac{\frac{3\left(8c^3d-12c^2d^2+12cd^3-3d^4\right)(fx+e)}{a}-\frac{12\left(c^4-4c^3d+6c^2d^2-4cd^3+d^4\right)}{a\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}+\frac{2\left(12cd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-3d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-36c^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/6*(3*(8*c^3*d - 12*c^2*d^2 + 12*c*d^3 - 3*d^4)*(f*x + e)/a - 12*(c^4 - 4*
c^3*d + 6*c^2*d^2 - 4*c*d^3 + d^4)/(a*(tan(1/2*f*x + 1/2*e) + 1)) + 2*(12*c
*d^3*tan(1/2*f*x + 1/2*e)^5 - 3*d^4*tan(1/2*f*x + 1/2*e)^5 - 36*c^2*d^2*tan
(1/2*f*x + 1/2*e)^4 + 24*c*d^3*tan(1/2*f*x + 1/2*e)^4 - 6*d^4*tan(1/2*f*x +
 1/2*e)^4 - 72*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 + 48*c*d^3*tan(1/2*f*x + 1/2*
e)^2 - 24*d^4*tan(1/2*f*x + 1/2*e)^2 - 12*c*d^3*tan(1/2*f*x + 1/2*e) + 3*d^
4*tan(1/2*f*x + 1/2*e) - 36*c^2*d^2 + 24*c*d^3 - 10*d^4)/((tan(1/2*f*x + 1/
2*e)^2 + 1)^3*a))/f
```

maple [B] time = 0.23, size = 673, normalized size = 3.56

$$\frac{4\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c\,d^3}{af\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}-\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d^4}{af\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}-\frac{12\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c^2d^2}{af\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}+\frac{8\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c\,d^3}{af\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}-\frac{2\left(\tan\right.}{af\left(1+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e)),x)
```

```
[Out] 4/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5*c*d^3-1/a/f/(1+tan(1/
2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5*d^4-12/a/f/(1+tan(1/2*f*x+1/2*e)^2)^
```

```
3*tan(1/2*f*x+1/2*e)^4*c^2*d^2+8/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x
+1/2*e)^4*c*d^3-2/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^4*d^4-2
4/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2*c^2*d^2+16/a/f/(1+tan
(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2*c*d^3-8/a/f/(1+tan(1/2*f*x+1/2*e)
^2)^3*tan(1/2*f*x+1/2*e)^2*d^4-4/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x
+1/2*e)*c*d^3+1/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)*d^4-12/a/
f/(1+tan(1/2*f*x+1/2*e)^2)^3*c^2*d^2+8/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*c*d^3
-10/3/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*d^4+8/a/f*d*arctan(tan(1/2*f*x+1/2*e))
*c^3-12/a/f*arctan(tan(1/2*f*x+1/2*e))*c^2*d^2+12/a/f*arctan(tan(1/2*f*x+1/
2*e))*c*d^3-3/a/f*arctan(tan(1/2*f*x+1/2*e))*d^4-2/a/f/(tan(1/2*f*x+1/2*e)+
1)*c^4+8/a/f/(tan(1/2*f*x+1/2*e)+1)*c^3*d-12/a/f/(tan(1/2*f*x+1/2*e)+1)*c^2
*d^2+8/a/f/(tan(1/2*f*x+1/2*e)+1)*c*d^3-2/a/f/(tan(1/2*f*x+1/2*e)+1)*d^4
```

maxima [B] time = 0.44, size = 725, normalized size = 3.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/3*(d^4*((7*sin(f*x + e)/(cos(f*x + e) + 1) + 39*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 24*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 9*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6 + 16)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3*a*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 + a*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a*sin(f*x + e)^7/(cos
(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 12*c*d^3
*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
+ 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(
sin(f*x + e)/(cos(f*x + e) + 1))/a) + 36*c^2*d^2*((sin(f*x + e)/(cos(f*x +
e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos
(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 24*c^3*
d*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f
*x + e) + 1))) + 6*c^4/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

mupad [B] time = 9.79, size = 451, normalized size = 2.39

$$\frac{d\,\mathrm{atan}\left(\frac{d\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(8\,c^3-12\,c^2\,d+12\,c\,d^2-3\,d^3\right)}{8\,c^3\,d-12\,c^2\,d^2+12\,c\,d^3-3\,d^4}\right)\left(8\,c^3-12\,c^2\,d+12\,c\,d^2-3\,d^3\right)}{a\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(12\,c^2\,d^2-4\,c\,d^3+\frac{7\,d^4}{3}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^4/(a + a*sin(e + f*x)),x)
```

```
[Out] (d*atan((d*tan(e/2 + (f*x)/2)*(12*c*d^2 - 12*c^2*d + 8*c^3 - 3*d^3))/(12*c*
d^3 + 8*c^3*d - 3*d^4 - 12*c^2*d^2))*(12*c*d^2 - 12*c^2*d + 8*c^3 - 3*d^3))
/(a*f) - (tan(e/2 + (f*x)/2)*((7*d^4)/3 - 4*c*d^3 + 12*c^2*d^2) + tan(e/2 +
 (f*x)/2)^6*(2*c^4 - 8*c^3*d - 12*c*d^3 + 3*d^4 + 12*c^2*d^2) + tan(e/2 + (
f*x)/2)^4*(6*c^4 - 24*c^3*d - 32*c*d^3 + 8*d^4 + 48*c^2*d^2) + tan(e/2 + (f
*x)/2)^2*(6*c^4 - 24*c^3*d - 36*c*d^3 + 13*d^4 + 60*c^2*d^2) + tan(e/2 + (f
*x)/2)^5*(3*d^4 - 12*c*d^3 + 12*c^2*d^2) + tan(e/2 + (f*x)/2)^3*(8*d^4 - 16
*c*d^3 + 24*c^2*d^2) - 16*c*d^3 - 8*c^3*d + 2*c^4 + (16*d^4)/3 + 24*c^2*d^2
)/(f*(a + a*tan(e/2 + (f*x)/2) + 3*a*tan(e/2 + (f*x)/2)^2 + 3*a*tan(e/2 + (
f*x)/2)^3 + 3*a*tan(e/2 + (f*x)/2)^4 + 3*a*tan(e/2 + (f*x)/2)^5 + a*tan(e/2
 + (f*x)/2)^6 + a*tan(e/2 + (f*x)/2)^7))
```

sympy [A] time = 15.64, size = 8605, normalized size = 45.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**4/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-12*c**4*tan(e/2 + f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*
tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)*
*4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/
2 + f*x/2) + 6*a*f) - 36*c**4*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**
7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2
 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*
a*f*tan(e/2 + f*x/2) + 6*a*f) - 36*c**4*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2
+ f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a
*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/
2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 12*c**4/(6*a*f*tan(e/2 + f*x/2)**
7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2
 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*
a*f*tan(e/2 + f*x/2) + 6*a*f) + 24*c**3*d*f*x*tan(e/2 + f*x/2)**7/(6*a*f*ta
n(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5
```

```
+ 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2
+ f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 24*c**3*d*f*x*tan(e/2 + f*x
/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(
e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 +
 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 72*c**3*d*f
*x*tan(e/2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)*
*6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e
/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*
f) + 72*c**3*d*f*x*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*t
an(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**
4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2
 + f*x/2) + 6*a*f) + 72*c**3*d*f*x*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x
/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*ta
n(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2
 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 72*c**3*d*f*x*tan(e/2 + f*x/2)**2/(6*a
*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2
)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan
(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 24*c**3*d*f*x*tan(e/2
+ f*x/2)/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*ta
n(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3
 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 24*c**3*d
*f*x/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/
2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 1
8*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 48*c**3*d*tan
(e/2 + f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 1
8*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f
*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 1
44*c**3*d*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 +
f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*
f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2)
 + 6*a*f) + 144*c**3*d*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a
*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/
2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan
(e/2 + f*x/2) + 6*a*f) + 48*c**3*d/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e
/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 +
18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f
*x/2) + 6*a*f) - 36*c**2*d**2*f*x*tan(e/2 + f*x/2)**7/(6*a*f*tan(e/2 + f*x/
2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan
(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2
+ 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 36*c**2*d**2*f*x*tan(e/2 + f*x/2)**6/(6
*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x
/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*t
an(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 108*c**2*d**2*f*x*ta
n(e/2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 +
18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 +
```

f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) -
108*c**2*d**2*f*x*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*ta
n(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4
 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2
+ f*x/2) + 6*a*f) - 108*c**2*d**2*f*x*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 +
f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f
*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)
2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 108*c2*d**2*f*x*tan(e/2 + f*x/2)*
*2/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2
+ f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*
a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 36*c**2*d**2*f*
x*tan(e/2 + f*x/2)/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 +
 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 +
 f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) -
 36*c**2*d**2*f*x/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 +
18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 +
f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) -
72*c**2*d**2*tan(e/2 + f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2
 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18
*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x
/2) + 6*a*f) - 72*c**2*d**2*tan(e/2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7
+ 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 +
 f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*
f*tan(e/2 + f*x/2) + 6*a*f) - 288*c**2*d**2*tan(e/2 + f*x/2)**4/(6*a*f*tan(
e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 +
18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 +
f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 144*c**2*d**2*tan(e/2 + f*x/2
)**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/
2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 1
8*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 360*c**2*d**2
*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6
 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2
 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f)
 - 72*c**2*d**2*tan(e/2 + f*x/2)/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2
 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18
*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x
/2) + 6*a*f) - 144*c**2*d**2/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f
*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f
*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2)
+ 6*a*f) + 36*c*d**3*f*x*tan(e/2 + f*x/2)**7/(6*a*f*tan(e/2 + f*x/2)**7 + 6
*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*
x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*t
an(e/2 + f*x/2) + 6*a*f) + 36*c*d**3*f*x*tan(e/2 + f*x/2)**6/(6*a*f*tan(e/2
 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*
a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x

/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 108*c*d**3*f*x*tan(e/2 + f*x/2)*
*5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2
+ f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*
a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 108*c*d**3*f*x*
tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6
+ 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2
+ f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f)
+ 108*c*d**3*f*x*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan
(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4
+ 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 +
f*x/2) + 6*a*f) + 108*c*d**3*f*x*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x/
2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan
(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2
+ 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 36*c*d**3*f*x*tan(e/2 + f*x/2)/(6*a*f*t
an(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5
+ 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2
+ f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 36*c*d**3*f*x/(6*a*f*tan(e
/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 1
8*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f
*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 72*c*d**3*tan(e/2 + f*x/2)**6/
(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f
*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f
*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 72*c*d**3*tan(e/2
+ f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f
*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)
3 + 18*a*f*tan(e/2 + f*x/2)2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 192*c*
d**3*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2
)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan
(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*
a*f) + 96*c*d**3*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan
(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4
+ 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 +
f*x/2) + 6*a*f) + 216*c*d**3*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x/2)**
7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2
+ f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*
a*f*tan(e/2 + f*x/2) + 6*a*f) + 24*c*d**3*tan(e/2 + f*x/2)/(6*a*f*tan(e/2 +
f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*
f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2
)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 96*c*d**3/(6*a*f*tan(e/2 + f*x/2)*
*7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/
2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6
*a*f*tan(e/2 + f*x/2) + 6*a*f) - 9*d**4*f*x*tan(e/2 + f*x/2)**7/(6*a*f*tan(
e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 +
18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 +
f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 9*d**4*f*x*tan(e/2 + f*x/2)**

```
6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 +
 f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a
*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 27*d**4*f*x*tan(
e/2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18
*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*
x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 27
*d**4*f*x*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 +
f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*
f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2)
 + 6*a*f) - 27*d**4*f*x*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*
a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x
/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*ta
n(e/2 + f*x/2) + 6*a*f) - 27*d**4*f*x*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 +
f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f
*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)
**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 9*d**4*f*x*tan(e/2 + f*x/2)/(6*a*f*
tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**
5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/
2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 9*d**4*f*x/(6*a*f*tan(e/2
 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*
a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x
/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 18*d**4*tan(e/2 + f*x/2)**6/(6*a
*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2
)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan
(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 18*d**4*tan(e/2 + f*x/
2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e
/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 +
18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 48*d**4*tan(
e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18
*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*
x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 48
*d**4*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/
2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*ta
n(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6
*a*f) - 78*d**4*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(
e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 +
 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 +
f*x/2) + 6*a*f) - 14*d**4*tan(e/2 + f*x/2)/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a
*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/
2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan
(e/2 + f*x/2) + 6*a*f) - 32*d**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2
 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18
*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x
/2) + 6*a*f), Ne(f, 0)), (x*(c + d*sin(e))**4/(a*sin(e) + a), True))
```

3.454 $\int \frac{(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=121

$$\frac{2d\left(c^2-3cd+d^2\right)\cos(e+fx)}{af}+\frac{3dx\left(2c^2-2cd+d^2\right)}{2a}+\frac{d^2(2c-3d)\sin(e+fx)\cos(e+fx)}{2af}-\frac{(c-d)\cos(e+fx)(c}{f(a\sin(e+}$$

```
[Out] 3/2*d*(2*c^2-2*c*d+d^2)*x/a+2*d*(c^2-3*c*d+d^2)*cos(f*x+e)/a/f+1/2*(2*c-3*d
)*d^2*cos(f*x+e)*sin(f*x+e)/a/f-(c-d)*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*
sin(f*x+e))
```

Rubi [A] time = 0.13, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2767, 2734}

$$\frac{2d\left(c^2-3cd+d^2\right)\cos(e+fx)}{af}+\frac{3dx\left(2c^2-2cd+d^2\right)}{2a}+\frac{d^2(2c-3d)\sin(e+fx)\cos(e+fx)}{2af}-\frac{(c-d)\cos(e+fx)(c}{f(a\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^3/(a + a*Sin[e + f*x]),x]
```

```
[Out] (3*d*(2*c^2 - 2*c*d + d^2)*x)/(2*a) + (2*d*(c^2 - 3*c*d + d^2)*Cos[e + f*x]
)/(a*f) + ((2*c - 3*d)*d^2*Cos[e + f*x]*Sin[e + f*x])/(2*a*f) - ((c - d)*Co
s[e + f*x]*(c + d*Sin[e + f*x])^2)/(f*(a + a*Sin[e + f*x]))
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{f(a+a\sin(e+fx))} - \frac{d\int(-a(3c-2d)+a(2c-3d)\sin(e+fx}{a^2}$$
$$= \frac{3d\left(2c^2-2cd+d^2\right)x}{2a} + \frac{2d\left(c^2-3cd+d^2\right)\cos(e+fx)}{af} + \frac{(2c-3d)d^2\cos(e+fx)\mathrm{s}}{2af}$$

Mathematica [A] time = 0.59, size = 192, normalized size = 1.59

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(d\cos\left(\frac{1}{2}(e+fx)\right)\left(6\left(2c^2-2cd+d^2\right)(e+fx)-4d(3c-d)\cos(e+fx)+d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^3/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(d*Cos[(e + f*x)/2]*(6*(2*c^2 - 2*c*
d + d^2)*(e + f*x) - 4*(3*c - d)*d*Cos[e + f*x] - d^2*Sin[2*(e + f*x)]) + S
in[(e + f*x)/2]*(2*(4*c^3 + 6*c^2*d*(-2 + e + f*x) - 6*c*d^2*(-2 + e + f*x)
 + d^3*(-4 + 3*e + 3*f*x)) - 4*(3*c - d)*d^2*Cos[e + f*x] - d^3*Sin[2*(e +
f*x)])))/(4*a*f*(1 + Sin[e + f*x]))
```

fricas [B] time = 0.54, size = 236, normalized size = 1.95

$$\frac{d^3\cos\left(fx+e\right)^3-2c^3+6c^2d-6cd^2+2d^3+3\left(2c^2d-2cd^2+d^3\right)fx-2\left(3cd^2-d^3\right)\cos\left(fx+e\right)^2-\left(2c^3-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(d^3*cos(f*x + e)^3 - 2*c^3 + 6*c^2*d - 6*c*d^2 + 2*d^3 + 3*(2*c^2*d -
2*c*d^2 + d^3)*f*x - 2*(3*c*d^2 - d^3)*cos(f*x + e)^2 - (2*c^3 - 6*c^2*d +
12*c*d^2 - 3*d^3 - 3*(2*c^2*d - 2*c*d^2 + d^3)*f*x)*cos(f*x + e) - (d^3*cos
(f*x + e)^2 - 2*c^3 + 6*c^2*d - 6*c*d^2 + 2*d^3 - 3*(2*c^2*d - 2*c*d^2 + d^
3)*f*x + (6*c*d^2 - d^3)*cos(f*x + e))*sin(f*x + e))/(a*f*cos(f*x + e) + a*
f*sin(f*x + e) + a*f)
```

giac [A] time = 0.19, size = 172, normalized size = 1.42

$$\frac{\frac{3\left(2c^2d-2cd^2+d^3\right)(fx+e)}{a} - \frac{4\left(c^3-3c^2d+3cd^2-d^3\right)}{a\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)} + \frac{2\left(d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-6cd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-6cd^2\right.}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)^2a}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*(3*(2*c^2*d - 2*c*d^2 + d^3)*(f*x + e)/a - 4*(c^3 - 3*c^2*d + 3*c*d^2 -
 d^3)/(a*(tan(1/2*f*x + 1/2*e) + 1)) + 2*(d^3*tan(1/2*f*x + 1/2*e)^3 - 6*c*
d^2*tan(1/2*f*x + 1/2*e)^2 + 2*d^3*tan(1/2*f*x + 1/2*e)^2 - d^3*tan(1/2*f*x
 + 1/2*e) - 6*c*d^2 + 2*d^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*a))/f
```

maple [B] time = 0.26, size = 364, normalized size = 3.01

$$\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d^3}{af\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{6\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c\,d^2}{af\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d^3}{af\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d^3}{af\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{\quad}{af\left(1+\text{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x)
```

```
[Out] 1/a/f/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^3*d^3-6/a/f/(1+tan(1/2*
f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*c*d^2+2/a/f/(1+tan(1/2*f*x+1/2*e)^2)^2
*tan(1/2*f*x+1/2*e)^2*d^3-1/a/f/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*
e)*d^3-6/a/f/(1+tan(1/2*f*x+1/2*e)^2)^2*c*d^2+2/a/f/(1+tan(1/2*f*x+1/2*e)^2
)^2*d^3+6/a/f*d*arctan(tan(1/2*f*x+1/2*e))*c^2-6/a/f*arctan(tan(1/2*f*x+1/2
*e))*c*d^2+3/a/f*arctan(tan(1/2*f*x+1/2*e))*d^3-2/a/f/(tan(1/2*f*x+1/2*e)+1
)*c^3+6/a/f/(tan(1/2*f*x+1/2*e)+1)*c^2*d-6/a/f/(tan(1/2*f*x+1/2*e)+1)*c*d^2
+2/a/f/(tan(1/2*f*x+1/2*e)+1)*d^3
```

maxima [B] time = 0.44, size = 425, normalized size = 3.51

$$\frac{d^3\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+4}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{2a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{2a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{a\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)-6\,cd^2\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(\ldots}{(\cos(\ldots}}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{a\sin(fx+e\ldots}{(\cos(fx+e)+\ldots}}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] (d^3*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arc
tan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 6*c*d^2*((sin(f*x + e)/(cos(f*x +
```

```
 e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(co
s(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 6*c^2*
d*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f
*x + e) + 1))) - 2*c^3/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

mupad [B] time = 9.43, size = 282, normalized size = 2.33

$$\frac{3\,d\,\mathrm{atan}\left(\frac{3\,d\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,c^2-2\,c\,d+d^2\right)}{6\,c^2\,d-6\,c\,d^2+3\,d^3}\right)\,\left(2\,c^2-2\,c\,d+d^2\right)}{a\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(6\,c\,d^2-d^3\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(2\,c^3-6\,c^2\,d\right.}{f\,\left(a\,\tan\left(\frac{e}{2}+\frac{f}{\ }\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^3/(a + a*sin(e + f*x)),x)
```

```
[Out] (3*d*atan((3*d*tan(e/2 + (f*x)/2)*(2*c^2 - 2*c*d + d^2))/(6*c^2*d - 6*c*d^2
 + 3*d^3))*(2*c^2 - 2*c*d + d^2))/(a*f) - (tan(e/2 + (f*x)/2)*(6*c*d^2 - d^
3) + tan(e/2 + (f*x)/2)^4*(6*c*d^2 - 6*c^2*d + 2*c^3 - 3*d^3) + tan(e/2 + (
f*x)/2)^2*(18*c*d^2 - 12*c^2*d + 4*c^3 - 5*d^3) + 12*c*d^2 - 6*c^2*d + tan(
e/2 + (f*x)/2)^3*(6*c*d^2 - 3*d^3) + 2*c^3 - 4*d^3)/(f*(a + a*tan(e/2 + (f*
x)/2) + 2*a*tan(e/2 + (f*x)/2)^2 + 2*a*tan(e/2 + (f*x)/2)^3 + a*tan(e/2 + (
f*x)/2)^4 + a*tan(e/2 + (f*x)/2)^5))
```

sympy [A] time = 8.35, size = 3602, normalized size = 29.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**3/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-4*c**3*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*t
an(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2
+ 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 8*c**3*tan(e/2 + f*x/2)**2/(2*a*f*tan(e
/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*
a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*c**3/(2*a*f*t
an(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3
+ 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 6*c**2*d*f*
x*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**
4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 +
 f*x/2) + 2*a*f) + 6*c**2*d*f*x*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)
**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2
 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 12*c**2*d*f*x*tan(e/2 + f*
```

x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(
e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*
f) + 12*c**2*d*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*t
an(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2
+ 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 6*c**2*d*f*x*tan(e/2 + f*x/2)/(2*a*f*ta
n(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 +
4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 6*c**2*d*f*x
/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f
*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 12
*c**2*d*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*
x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan
(e/2 + f*x/2) + 2*a*f) + 24*c**2*d*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x
/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(
e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 12*c**2*d/(2*a*f*tan(e/
2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a
*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 6*c*d**2*f*x*tan
(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4
*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/
2) + 2*a*f) - 6*c*d**2*f*x*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 +
2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*
x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 12*c*d**2*f*x*tan(e/2 + f*x/2)*
*3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 +
f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) -
12*c*d**2*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/
2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a
*f*tan(e/2 + f*x/2) + 2*a*f) - 6*c*d**2*f*x*tan(e/2 + f*x/2)/(2*a*f*tan(e/2
+ f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*
f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 6*c*d**2*f*x/(2*a
*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)
3 + 4*a*f*tan(e/2 + f*x/2)2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 12*c*d*
*2*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)*
*4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2
+ f*x/2) + 2*a*f) - 12*c*d**2*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**
5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 +
f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 36*c*d**2*tan(e/2 + f*x/2)**
2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 +
f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 1
2*c*d**2*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/
2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e
/2 + f*x/2) + 2*a*f) - 24*c*d**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2
+ f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*
f*tan(e/2 + f*x/2) + 2*a*f) + 3*d**3*f*x*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2
+ f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*
f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 3*d**3*f*x*tan(e/
2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*

```
f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2)
+ 2*a*f) + 6*d**3*f*x*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*
f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)*
*2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 6*d**3*f*x*tan(e/2 + f*x/2)**2/(2*a*
f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)*
*3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 3*d**3*f
*x*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4
+ 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f
*x/2) + 2*a*f) + 3*d**3*f*x/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*
x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan
(e/2 + f*x/2) + 2*a*f) + 6*d**3*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)
**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2
 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 6*d**3*tan(e/2 + f*x/2)**3
/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f
*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 10
*d**3*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/
2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e
/2 + f*x/2) + 2*a*f) + 2*d**3*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 +
 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*
x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 8*d**3/(2*a*f*tan(e/2 + f*x/2)*
*5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2
+ f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f), Ne(f, 0)), (x*(c + d*sin(e))
**3/(a*sin(e) + a), True))
```

3.455 $\int \frac{(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=62

$$-\frac{(c-d)^2\cos(e+fx)}{af(\sin(e+fx)+1)}+\frac{dx(2c-d)}{a}-\frac{d^2\cos(e+fx)}{af}$$

```
[Out] (2*c-d)*d*x/a-d^2*cos(f*x+e)/a/f-(c-d)^2*cos(f*x+e)/a/f/(1+sin(f*x+e))
```

Rubi [A] time = 0.14, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2746, 2735, 2648}

$$-\frac{(c-d)^2\cos(e+fx)}{af(\sin(e+fx)+1)}+\frac{dx(2c-d)}{a}-\frac{d^2\cos(e+fx)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^2/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((2*c - d)*d*x)/a - (d^2*Cos[e + f*x])/(a*f) - ((c - d)^2*Cos[e + f*x])/(a*
f*(1 + Sin[e + f*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx = -\frac{d^2\cos(e+fx)}{af} + \frac{\int \frac{ac^2+a(2c-d)d\sin(e+fx)}{a+a\sin(e+fx)}\,dx}{a}$$
$$= \frac{(2c-d)dx}{a} - \frac{d^2\cos(e+fx)}{af} + (c-d)^2\int \frac{1}{a+a\sin(e+fx)}\,dx$$
$$= \frac{(2c-d)dx}{a} - \frac{d^2\cos(e+fx)}{af} - \frac{(c-d)^2\cos(e+fx)}{f(a+a\sin(e+fx))}$$

Mathematica [**A**] time = 0.46, size = 122, normalized size = 1.97

$$-\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)\left(-2c^2-2cd(e+fx-2)+d^2(e+fx-2)+d^2\cos(e+fx\right.\right.}{af(\sin(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^2/(a + a*Sin[e + f*x]),x]
```

```
[Out] -(((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(d*Cos[(e + f*x)/2]*(-((2*c - d)*(
e + f*x)) + d*Cos[e + f*x]) + (-2*c^2 - 2*c*d*(-2 + e + f*x) + d^2*(-2 + e
+ f*x) + d^2*Cos[e + f*x])*Sin[(e + f*x)/2]))/(a*f*(1 + Sin[e + f*x])))
```

fricas [**B**] time = 0.48, size = 142, normalized size = 2.29

$$-\frac{d^2\cos\left(fx+e\right)^2-\left(2\,cd-d^2\right)fx+c^2-2\,cd+d^2-\left(\left(2\,cd-d^2\right)fx-c^2+2\,cd-2\,d^2\right)\cos\left(fx+e\right)-\left(\left(2\,cd-d\right.\right.}{af\cos\left(fx+e\right)+af\sin\left(fx+e\right)+af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(d^2*cos(f*x + e)^2 - (2*c*d - d^2)*f*x + c^2 - 2*c*d + d^2 - ((2*c*d - d^
2)*f*x - c^2 + 2*c*d - 2*d^2)*cos(f*x + e) - ((2*c*d - d^2)*f*x - d^2*cos(f
*x + e) + c^2 - 2*c*d + d^2)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(f*x
+ e) + a*f)
```

giac [**B**] time = 0.15, size = 143, normalized size = 2.31

$$\frac{\frac{\left(2\,cd-d^2\right)\left(fx+e\right)}{a}-\frac{2\left(c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-2\,cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+c^2-2\,cd+2\,d^2\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] ((2*c*d - d^2)*(f*x + e)/a - 2*(c^2*tan(1/2*f*x + 1/2*e)^2 - 2*c*d*tan(1/2*
f*x + 1/2*e)^2 + d^2*tan(1/2*f*x + 1/2*e)^2 + d^2*tan(1/2*f*x + 1/2*e) + c^
2 - 2*c*d + 2*d^2)/((tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)^2 + tan(
1/2*f*x + 1/2*e) + 1)*a))/f
```

maple [B] time = 0.24, size = 140, normalized size = 2.26

$$-\frac{2d^2}{af\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}+\frac{4d\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c}{af}-\frac{2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d^2}{af}-\frac{2c^2}{af\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{}{af\left(\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x)
```

```
[Out] -2/a/f*d^2/(1+tan(1/2*f*x+1/2*e)^2)+4/a/f*d*arctan(tan(1/2*f*x+1/2*e))*c-2/
a/f*arctan(tan(1/2*f*x+1/2*e))*d^2-2/a/f/(tan(1/2*f*x+1/2*e)+1)*c^2+4/a/f/(
tan(1/2*f*x+1/2*e)+1)*c*d-2/a/f/(tan(1/2*f*x+1/2*e)+1)*d^2
```

maxima [B] time = 0.45, size = 209, normalized size = 3.37

$$-\frac{2\left(d^2\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)-2\,cd\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)+\frac{c^2}{a+\frac{a\sin(fx-}{\cos(fx+e}}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] -2*(d^2*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/a) - 2*c*d*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a
 + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + c^2/(a + a*sin(f*x + e)/(co
s(f*x + e) + 1)))/f
```

mupad [B] time = 7.38, size = 124, normalized size = 2.00

$$-\frac{d^2\,f\,x-2\,c\,d\,f\,x}{a\,f}-\frac{{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(2\,c^2-4\,c\,d+2\,d^2\right)-4\,c\,d+2\,c^2+4\,d^2+2\,d^2\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+a\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+a\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^2/(a + a*sin(e + f*x)),x)
```

```
[Out] - (d^2*f*x - 2*c*d*f*x)/(a*f) - (tan(e/2 + (f*x)/2)^2*(2*c^2 - 4*c*d + 2*d^
2) - 4*c*d + 2*c^2 + 4*d^2 + 2*d^2*tan(e/2 + (f*x)/2))/(f*(a + a*tan(e/2 +
(f*x)/2) + a*tan(e/2 + (f*x)/2)^2 + a*tan(e/2 + (f*x)/2)^3))
```

sympy [A] time = 3.67, size = 940, normalized size = 15.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**2/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-2*c**2*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e
/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 2*c**2/(a*f*tan(e/2 + f*x/2)
**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*c*d*f*x*tan
(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*t
an(e/2 + f*x/2) + a*f) + 2*c*d*f*x*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2
)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*c*d*f*x*ta
n(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan
(e/2 + f*x/2) + a*f) + 2*c*d*f*x/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f
*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 4*c*d*tan(e/2 + f*x/2)**2/(a*f*tan
(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) +
4*c*d/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*
x/2) + a*f) - d**2*f*x*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*t
an(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - d**2*f*x*tan(e/2 + f*x/2
)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x
/2) + a*f) - d**2*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e
/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - d**2*f*x/(a*f*tan(e/2 + f*x/
2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 2*d**2*tan(
e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*ta
n(e/2 + f*x/2) + a*f) - 2*d**2*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 +
a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 4*d**2/(a*f*tan(e/2
 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f), Ne(f,
 0)), (x*(c + d*sin(e))**2/(a*sin(e) + a), True))
```

3.456 $\int \frac{c+d\sin(e+fx)}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=35

$$\frac{dx}{a}-\frac{(c-d)\cos(e+fx)}{f(a\sin(e+fx)+a)}$$

```
[Out] d*x/a-(c-d)*cos(f*x+e)/f/(a+a*sin(f*x+e))
```

Rubi [A] time = 0.05, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2735, 2648}

$$\frac{dx}{a}-\frac{(c-d)\cos(e+fx)}{f(a\sin(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])/(a + a*Sin[e + f*x]),x]
```

```
[Out] (d*x)/a - ((c - d)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{c+d\sin(e+fx)}{a+a\sin(e+fx)}\,dx &= \frac{dx}{a}-(-c+d)\int \frac{1}{a+a\sin(e+fx)}\,dx\\ &= \frac{dx}{a}-\frac{(c-d)\cos(e+fx)}{f(a+a\sin(e+fx))}\end{aligned}$$

Mathematica [B] time = 0.17, size = 79, normalized size = 2.26

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)(2c+d(e+fx-2))+d(e+fx)\cos\left(\frac{1}{2}(e+fx)\right)\right)}{af(\sin(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(d*(e + f*x)*Cos[(e + f*x)/2] + (2*c
 + d*(-2 + e + f*x))*Sin[(e + f*x)/2]))/(a*f*(1 + Sin[e + f*x]))
```

fricas [A] time = 0.49, size = 66, normalized size = 1.89

$$\frac{dfx+\left(dfx-c+d\right)\cos\left(fx+e\right)+\left(dfx+c-d\right)\sin\left(fx+e\right)-c+d}{af\cos\left(fx+e\right)+af\sin\left(fx+e\right)+af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] (d*f*x + (d*f*x - c + d)*cos(f*x + e) + (d*f*x + c - d)*sin(f*x + e) - c +
d)/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

giac [A] time = 0.15, size = 40, normalized size = 1.14

$$\frac{\frac{\left(fx+e\right)d}{a}-\frac{2\,(c-d)}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] ((f*x + e)*d/a - 2*(c - d)/(a*(tan(1/2*f*x + 1/2*e) + 1)))/f
```

maple [A] time = 0.15, size = 65, normalized size = 1.86

$$\frac{2d\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{2c}{af\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{2d}{af\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

```
[Out] 2/a/f*d*arctan(tan(1/2*f*x+1/2*e))-2/a/f/(tan(1/2*f*x+1/2*e)+1)*c+2/a/f/(ta
n(1/2*f*x+1/2*e)+1)*d
```

maxima [B] time = 0.43, size = 78, normalized size = 2.23

$$\frac{2\left(d\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)-\frac{c}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2*(d*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(co
s(f*x + e) + 1))) - c/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

mupad [B] time = 6.83, size = 35, normalized size = 1.00

$$\frac{d\,x}{a}-\frac{2\,c-2\,d}{a\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))/(a + a*sin(e + f*x)),x)
```

```
[Out] (d*x)/a - (2*c - 2*d)/(a*f*(tan(e/2 + (f*x)/2) + 1))
```

sympy [A] time = 1.82, size = 109, normalized size = 3.11

$$\begin{cases} -\frac{2c}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}+\frac{dfx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}+\frac{dfx}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}+\frac{2d}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} & \text{for } f\neq 0 \\ \frac{x(c+d\sin(e))}{a\sin(e)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-2*c/(a*f*tan(e/2 + f*x/2) + a*f) + d*f*x*tan(e/2 + f*x/2)/(a*f*
tan(e/2 + f*x/2) + a*f) + d*f*x/(a*f*tan(e/2 + f*x/2) + a*f) + 2*d/(a*f*tan
(e/2 + f*x/2) + a*f), Ne(f, 0)), (x*(c + d*sin(e))/(a*sin(e) + a), True))
```

3.457 $\int \frac{1}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=23

$$-\frac{\cos(e+fx)}{f(a\sin(e+fx)+a)}$$

```
[Out] -cos(f*x+e)/f/(a+a*sin(f*x+e))
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2648}

$$-\frac{\cos(e+fx)}{f(a\sin(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(-1),x]
```

```
[Out] -(Cos[e + f*x]/(f*(a + a*Sin[e + f*x])))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{a+a\sin(e+fx)}\,dx = -\frac{\cos(e+fx)}{f(a+a\sin(e+fx))}$$

Mathematica [B] time = 0.04, size = 48, normalized size = 2.09

$$\frac{2\sin\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f(a\sin(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(-1),x]
```

```
[Out] (2*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(f*(a + a*Sin[e
+ f*x]))
```

fricas [A] time = 0.43, size = 42, normalized size = 1.83

$$-\frac{\cos\left(fx+e\right)-\sin\left(fx+e\right)+1}{af\cos\left(fx+e\right)+af\sin\left(fx+e\right)+af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e)),x, algorithm="fricas")
[Out] -(cos(f*x + e) - sin(f*x + e) + 1)/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a
*f)
```

giac [A] time = 0.41, size = 22, normalized size = 0.96

$$-\frac{2}{af\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e)),x, algorithm="giac")
[Out] -2/(a*f*(tan(1/2*f*x + 1/2*e) + 1))
```

maple [A] time = 0.12, size = 22, normalized size = 0.96

$$-\frac{2}{fa\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e)),x)
[Out] -2/f/a/(tan(1/2*f*x+1/2*e)+1)
```

maxima [A] time = 0.31, size = 27, normalized size = 1.17

$$-\frac{2}{\left(a+\frac{a\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e)),x, algorithm="maxima")
[Out] -2/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*f)
```

mupad [B] time = 6.97, size = 21, normalized size = 0.91

$$-\frac{2}{a\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sin(e + f*x)),x)
```

```
[Out] -2/(a*f*(tan(e/2 + (f*x)/2) + 1))
```

sympy [A] time = 0.93, size = 27, normalized size = 1.17

$$\begin{cases} -\frac{2}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} & \text{for}\ f\neq 0 \\ \frac{x}{a\sin(e)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-2/(a*f*tan(e/2 + f*x/2) + a*f), Ne(f, 0)), (x/(a*sin(e) + a), T
rue))
```

3.458 $\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=89

$$-\frac{2d\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\sqrt{c^2-d^2}}-\frac{\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}$$

```
[Out] -cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))-2*d*arctan((d+c*tan(1/2*f*x+1/2*e))/(c
^2-d^2)^(1/2))/a/(c-d)/f/(c^2-d^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2747, 2648, 2660, 618, 204}

$$-\frac{2d\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\sqrt{c^2-d^2}}-\frac{\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])),x]
```

```
[Out] (-2*d*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a*(c - d)*Sqrt[c^2
 - d^2]*f) - Cos[e + f*x]/((c - d)*f*(a + a*Sin[e + f*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx = \frac{\int \frac{1}{a+a\sin(e+fx)}\,dx}{c-d} - \frac{d\int \frac{1}{c+d\sin(e+fx)}\,dx}{a(c-d)}$$

$$= -\frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))} - \frac{(2d)\,\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx, x, \tan\right.}{a(c-d)f}$$

$$= -\frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))} + \frac{(4d)\,\text{Subst}\left(\int \frac{1}{-4\left(c^2-d^2\right)-x^2}\,dx, x, 2\right.}{a(c-d)f}$$

$$= -\frac{2d\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)\sqrt{c^2-d^2}\,f} - \frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))}$$

Mathematica [A] time = 0.36, size = 114, normalized size = 1.28

$$\frac{\cos(e+fx)\left(\frac{2d\tanh^{-1}\left(\frac{\sqrt{c-d}\,\sqrt{1-\sin(e+fx)}}{\sqrt{-c-d}\,\sqrt{\sin(e+fx)+1}}\right)}{\sqrt{-c-d}\,\sqrt{c-d}\,\sqrt{\cos^2(e+fx)}} + \frac{1}{\sin(e+fx)+1}\right)}{af(d-c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])),x]
```

```
[Out] (Cos[e + f*x]*((2*d*ArcTanh[(Sqrt[c - d]*Sqrt[1 - Sin[e + f*x]])/(Sqrt[-c -
 d]*Sqrt[1 + Sin[e + f*x]])])/(Sqrt[-c - d]*Sqrt[c - d]*Sqrt[Cos[e + f*x]^2
]) + (1 + Sin[e + f*x])^(-1)))/(a*(-c + d)*f)
```

fricas [B] time = 0.47, size = 489, normalized size = 5.49

$$\left[\frac{\sqrt{-c^2+d^2}\left(d\cos\left(fx+e\right)+d\sin\left(fx+e\right)+d\right)\log\left(\frac{\left(2c^2-d^2\right)\cos\left(fx+e\right)^2-2cd\sin\left(fx+e\right)-c^2-d^2+2\left(c\cos\left(fx+e\right)\sin\left(fx+e\right)+d\cos\right.}{d^2\cos\left(fx+e\right)^2-2cd\sin\left(fx+e\right)-c^2-d^2}\right.}{2\left(\left(ac^3-ac^2d-acd^2+ad^3\right)f\cos\left(fx+e\right)+\left(ac^3-ac^2d-acd^2+ad^3\right)f\,\mathrm{s}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-c^2 + d^2)*(d*cos(f*x + e) + d*sin(f*x + e) + d)*log(((2*c^2 -
d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*si
n(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*
sin(f*x + e) - c^2 - d^2)) - 2*c^2 + 2*d^2 - 2*(c^2 - d^2)*cos(f*x + e) + 2
*(c^2 - d^2)*sin(f*x + e))/((a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*cos(f*x +
 e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*sin(f*x + e) + (a*c^3 - a*c^2*d
 - a*c*d^2 + a*d^3)*f), (sqrt(c^2 - d^2)*(d*cos(f*x + e) + d*sin(f*x + e) +
 d)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) - c^2 + d^
2 - (c^2 - d^2)*cos(f*x + e) + (c^2 - d^2)*sin(f*x + e))/((a*c^3 - a*c^2*d
- a*c*d^2 + a*d^3)*f*cos(f*x + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*s
in(f*x + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f)]
```

giac [A] time = 1.21, size = 100, normalized size = 1.12

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)d}{(ac-ad)\sqrt{c^2-d^2}}+\frac{1}{(ac-ad)\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*
e) + d)/sqrt(c^2 - d^2)))*d/((a*c - a*d)*sqrt(c^2 - d^2)) + 1/((a*c - a*d)*
(tan(1/2*f*x + 1/2*e) + 1)))/f
```

maple [A] time = 0.29, size = 87, normalized size = 0.98

$$-\frac{2d\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{af\,(c-d)\sqrt{c^2-d^2}}-\frac{2}{af\,(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] -2/a/f*d/(c-d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2
-d^2)^(1/2))-2/a/f/(c-d)/(tan(1/2*f*x+1/2*e)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 6.99, size = 121, normalized size = 1.36

$$\frac{2\,d\,\mathrm{atan}\left(\frac{\frac{d\,\left(2\,a\,d^2-2\,a\,c\,d\right)}{a\,\sqrt{c+d}\,{\left(c-d\right)}^{3/2}}-\frac{2\,c\,d\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(a\,c-a\,d\right)}{a\,\sqrt{c+d}\,{\left(c-d\right)}^{3/2}}}{2\,d}\right)}{a\,f\,\sqrt{c+d}\,{\left(c-d\right)}^{3/2}}-\frac{2}{f\,\left(a+a\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(c-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))),x)
```

```
[Out] (2*d*atan(((d*(2*a*d^2 - 2*a*c*d))/(a*(c + d)^(1/2)*(c - d)^(3/2)) - (2*c*d
*tan(e/2 + (f*x)/2)*(a*c - a*d))/(a*(c + d)^(1/2)*(c - d)^(3/2)))/(2*d)))/(
a*f*(c + d)^(1/2)*(c - d)^(3/2)) - 2/(f*(a + a*tan(e/2 + (f*x)/2))*(c - d))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.459 $\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=150

$$-\frac{2d(2c+d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\left(c^2-d^2\right)^{3/2}}-\frac{d(c+2d)\cos(e+fx)}{af(c-d)^2(c+d)(c+d\sin(e+fx))}-\frac{\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(\ldots}$$

```
[Out] -2*d*(2*c+d)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a/(c-d)/(c^2-
d^2)^(3/2)/f-d*(c+2*d)*cos(f*x+e)/a/(c-d)^2/(c+d)/f/(c+d*sin(f*x+e))-cos(f*
x+e)/(c-d)/f/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))
```

Rubi [A] time = 0.18, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2768, 2754, 12, 2660, 618, 204}

$$-\frac{2d(2c+d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\left(c^2-d^2\right)^{3/2}}-\frac{d(c+2d)\cos(e+fx)}{af(c-d)^2(c+d)(c+d\sin(e+fx))}-\frac{\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] (-2*d*(2*c + d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a*(c - d
)*(c^2 - d^2)^(3/2)*f) - (d*(c + 2*d)*Cos[e + f*x])/(a*(c - d)^2*(c + d)*f*
(c + d*Sin[e + f*x])) - Cos[e + f*x]/((c - d)*f*(a + a*Sin[e + f*x])*(c + d
*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx = -\frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))} + \frac{d\int \frac{-2a+a\sin(e+}{(c+d\sin(e+f}}{a^2(c-d)}$$

$$= -\frac{d(c+2d)\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))} - \frac{\cos(e+}{(c-d)f(a+a\sin(e+f}$$

$$= -\frac{d(c+2d)\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))} - \frac{\cos(e+}{(c-d)f(a+a\sin(e+f}$$

$$= -\frac{d(c+2d)\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))} - \frac{\cos(e+}{(c-d)f(a+a\sin(e+f}$$

$$= -\frac{d(c+2d)\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))} - \frac{\cos(e+}{(c-d)f(a+a\sin(e+f}$$

$$= -\frac{2d(2c+d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)^2(c+d)\sqrt{c^2-d^2}\,f} - \frac{d(c+2d)\cos(e+f}{a(c-d)^2(c+d)f(c+d\sin}$$

Mathematica [A] time = 0.65, size = 162, normalized size = 1.08

$$\frac{\cos(e+fx)\left(-\frac{d}{(\sin(e+fx)+1)(c+d\sin(e+fx))} + \frac{c+2d}{(c-d)(\sin(e+fx)+1)} + \frac{2d(2c+d)\tanh^{-1}\left(\frac{\sqrt{c-d}\sqrt{1-\sin(e+fx)}}{\sqrt{-c-d}\sqrt{\sin(e+fx)+1}}\right)}{\sqrt{-c-d}\,(c-d)^{3/2}\sqrt{\cos^2(e+fx)}}\right)}{af(d-c)(c+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] (Cos[e + f*x]*((2*d*(2*c + d)*ArcTanh[(Sqrt[c - d]*Sqrt[1 - Sin[e + f*x]])/
(Sqrt[-c - d]*Sqrt[1 + Sin[e + f*x]])])/(Sqrt[-c - d]*(c - d)^(3/2)*Sqrt[Co
s[e + f*x]^2]) + (c + 2*d)/((c - d)*(1 + Sin[e + f*x])) - d/((1 + Sin[e + f
*x])*(c + d*Sin[e + f*x]))))/(a*(-c + d)*(c + d)*f)
```

fricas [B] time = 0.49, size = 1120, normalized size = 7.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*c^4 - 4*c^2*d^2 + 2*d^4 + 2*(c^3*d + 2*c^2*d^2 - c*d^3 - 2*d^4)*cos
(f*x + e)^2 + (2*c^2*d + 3*c*d^2 + d^3 - (2*c*d^2 + d^3)*cos(f*x + e)^2 + (
2*c^2*d + c*d^2)*cos(f*x + e) + (2*c^2*d + 3*c*d^2 + d^3 + (2*c*d^2 + d^3)*
cos(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(-((2*c^2 - d^2)*cos(f*x +
e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 - 2*(c*cos(f*x + e)*sin(f*x + e) + d*
cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) -
c^2 - d^2)) + 2*(c^4 + c^3*d - c*d^3 - d^4)*cos(f*x + e) - 2*(c^4 - 2*c^2*d
^2 + d^4 - (c^3*d + 2*c^2*d^2 - c*d^3 - 2*d^4)*cos(f*x + e))*sin(f*x + e))/
((a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f*cos(
f*x + e)^2 - (a*c^6 - a*c^5*d - 2*a*c^4*d^2 + 2*a*c^3*d^3 + a*c^2*d^4 - a*c
*d^5)*f*cos(f*x + e) - (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f - ((a*
c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f*cos(f*x
+ e) + (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f)*sin(f*x + e)), (c^4 -
 2*c^2*d^2 + d^4 + (c^3*d + 2*c^2*d^2 - c*d^3 - 2*d^4)*cos(f*x + e)^2 - (2*
c^2*d + 3*c*d^2 + d^3 - (2*c*d^2 + d^3)*cos(f*x + e)^2 + (2*c^2*d + c*d^2)*
cos(f*x + e) + (2*c^2*d + 3*c*d^2 + d^3 + (2*c*d^2 + d^3)*cos(f*x + e))*sin
(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*co
s(f*x + e))) + (c^4 + c^3*d - c*d^3 - d^4)*cos(f*x + e) - (c^4 - 2*c^2*d^2
+ d^4 - (c^3*d + 2*c^2*d^2 - c*d^3 - 2*d^4)*cos(f*x + e))*sin(f*x + e))/((a
*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f*cos(f*x
 + e)^2 - (a*c^6 - a*c^5*d - 2*a*c^4*d^2 + 2*a*c^3*d^3 + a*c^2*d^4 - a*c*d^
5)*f*cos(f*x + e) - (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f - ((a*c^5
*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f*cos(f*x + e
) + (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f)*sin(f*x + e))]
```

giac [B] time = 0.34, size = 311, normalized size = 2.07

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)(2cd+d^2)}{(ac^3-ac^2d-acd^2+ad^3)\sqrt{c^2-d^2}}+\frac{c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2c^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+\cdots}{(ac^4-ac^3d-ac^2d^2+acd^3)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2d\tan\left(\frac{1}{2}fx+\cdots\right.\right.}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*
e) + d)/sqrt(c^2 - d^2)))*(2*c*d + d^2)/((a*c^3 - a*c^2*d - a*c*d^2 + a*d^3
)*sqrt(c^2 - d^2)) + (c^3*tan(1/2*f*x + 1/2*e)^2 + c^2*d*tan(1/2*f*x + 1/2*
e)^2 + d^3*tan(1/2*f*x + 1/2*e)^2 + 2*c^2*d*tan(1/2*f*x + 1/2*e) + 3*c*d^2*
tan(1/2*f*x + 1/2*e) + d^3*tan(1/2*f*x + 1/2*e) + c^3 + c^2*d + c*d^2)/((a*
c^4 - a*c^3*d - a*c^2*d^2 + a*c*d^3)*(c*tan(1/2*f*x + 1/2*e)^3 + c*tan(1/2*
```

```
f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e)^2 + c*tan(1/2*f*x + 1/2*e) + 2*d*
tan(1/2*f*x + 1/2*e) + c)))/f
```

maple [A] time = 0.34, size = 273, normalized size = 1.82

$$-\frac{2d^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af(c-d)^2\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d+c\right)(c+d)c}-\frac{2d^2}{af(c-d)^2\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

```
[Out] -2/a/f*d^3/(c-d)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)/
c*tan(1/2*f*x+1/2*e)-2/a/f*d^2/(c-d)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*
x+1/2*e)*d+c)/(c+d)-4/a/f*d/(c-d)^2/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*t
an(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c-2/a/f*d^2/(c-d)^2/(c+d)/(c^2-d^2)
^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))-2/a/f/(c-d)
^2/(tan(1/2*f*x+1/2*e)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 8.57, size = 309, normalized size = 2.06

$$-\frac{\frac{2\left(c^2+c\,d+d^2\right)}{(c+d)\,(c-d)^2}+\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(d^2+2\,c\,d\right)}{c\,(c-d)^2}+\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\left(c^3+c^2\,d+d^3\right)}{c\,(c+d)\,(c-d)^2}}{f\left(a\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+(a\,c+2\,a\,d)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+(a\,c+2\,a\,d)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\,c\right)}-\frac{2\,d\,\mathrm{atan}\left(\frac{\frac{d\,(2\,c+d)\left(2\,a\,c^3\,d-2\,a\right.}{a\,(c+d)^{3/2}}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^2),x)
```

```
[Out] - ((2*(c*d + c^2 + d^2))/((c + d)*(c - d)^2) + (2*tan(e/2 + (f*x)/2)*(2*c*d
 + d^2))/(c*(c - d)^2) + (2*tan(e/2 + (f*x)/2)^2*(c^2*d + c^3 + d^3))/(c*(c
 + d)*(c - d)^2))/(f*(a*c + tan(e/2 + (f*x)/2)^2*(a*c + 2*a*d) + tan(e/2 +
(f*x)/2)*(a*c + 2*a*d) + a*c*tan(e/2 + (f*x)/2)^3)) - (2*d*atan(((d*(2*c +
d)*(2*a*d^4 - 2*a*c^2*d^2 - 2*a*c*d^3 + 2*a*c^3*d))/(a*(c + d)^(3/2)*(c - d
)^(5/2)) + (2*c*d*tan(e/2 + (f*x)/2)*(2*c + d)*(a*c^3 + a*d^3 - a*c*d^2 - a
*c^2*d))/(a*(c + d)^(3/2)*(c - d)^(5/2)))/(4*c*d + 2*d^2))*(2*c + d))/(a*f*
(c + d)^(3/2)*(c - d)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.460 $\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=213

$$-\frac{3d\left(2c^2+2cd+d^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\left(c^2-d^2\right)^{5/2}}-\frac{d(2c+d)(c+4d)\cos(e+fx)}{2af(c-d)^3(c+d)^2(c+d\sin(e+fx))}-\frac{d(2c+3d)\cos(e+}{2af(c-d)^2(c+d)(c+d\,\text{si}}$$

```
[Out] -3*d*(2*c^2+2*c*d+d^2)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a/(
c-d)/(c^2-d^2)^(5/2)/f-1/2*d*(2*c+3*d)*cos(f*x+e)/a/(c-d)^2/(c+d)/f/(c+d*si
n(f*x+e))^2-cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2-1/2*d*(2
*c+d)*(c+4*d)*cos(f*x+e)/a/(c-d)^3/(c+d)^2/f/(c+d*sin(f*x+e))
```

Rubi [A] time = 0.32, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2768, 2754, 12, 2660, 618, 204}

$$-\frac{3d\left(2c^2+2cd+d^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\left(c^2-d^2\right)^{5/2}}-\frac{d(2c+d)(c+4d)\cos(e+fx)}{2af(c-d)^3(c+d)^2(c+d\sin(e+fx))}-\frac{d(2c+3d)\cos(e+}{2af(c-d)^2(c+d)(c+d\,\text{si}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] (-3*d*(2*c^2 + 2*c*d + d^2)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]
])/(a*(c - d)*(c^2 - d^2)^(5/2)*f) - (d*(2*c + 3*d)*Cos[e + f*x])/(2*a*(c -
 d)^2*(c + d)*f*(c + d*Sin[e + f*x])^2) - Cos[e + f*x]/((c - d)*f*(a + a*Si
n[e + f*x])*(c + d*Sin[e + f*x])^2) - (d*(2*c + d)*(c + 4*d)*Cos[e + f*x])/
(2*a*(c - d)^3*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx = -\frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))^2} + \frac{d\int \frac{-3a+2a\sin(}{(c+d\sin(e+}}{a^2(c-}$$

$$= -\frac{d(2c+3d)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{\cos(}{(c-d)f(a+a\sin(e+}$$

$$= -\frac{d(2c+3d)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{\cos(}{(c-d)f(a+a\sin(e+}$$

$$= -\frac{d(2c+3d)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{\cos(}{(c-d)f(a+a\sin(e+}$$

$$= -\frac{d(2c+3d)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{\cos(}{(c-d)f(a+a\sin(e+}$$

$$= -\frac{d(2c+3d)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{\cos(}{(c-d)f(a+a\sin(e+}$$

$$= -\frac{3d\left(2c^2+2cd+d^2\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)^3(c+d)^2\sqrt{c^2-d^2}\,f} - \frac{d(2c+3d}{2a(c-d)^2(c+d)}$$

Mathematica [A] time = 1.98, size = 230, normalized size = 1.08

$$\frac{\cos(e+fx)\left(\frac{2c^2+9cd+4d^2}{(c-d)^2(c+d)(\sin(e+fx)+1)} - \frac{6d\left(2c^2+2cd+d^2\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\sqrt{1-\sin(e+fx)}}{\sqrt{-c-d}\sqrt{\sin(e+fx)+1}}\right)}{(-c-d)^{3/2}(c-d)^{5/2}\sqrt{\cos^2(e+fx)}} - \frac{d(4c+d)}{(c-d)(c+d)(\sin(e+fx)+1)(c+d\sin(e+fx))} - \frac{}{(\sin(}\right.}{2af(d-c)(c+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] (Cos[e + f*x]*((-6*d*(2*c^2 + 2*c*d + d^2)*ArcTanh[(Sqrt[c - d]*Sqrt[1 - Si
n[e + f*x]])/(Sqrt[-c - d]*Sqrt[1 + Sin[e + f*x]])])/((-c - d)^(3/2)*(c - d
)^(5/2)*Sqrt[Cos[e + f*x]^2]) + (2*c^2 + 9*c*d + 4*d^2)/((c - d)^2*(c + d)*
(1 + Sin[e + f*x])) - d/((1 + Sin[e + f*x])*(c + d*Sin[e + f*x])^2) - (d*(4
*c + d))/((c - d)*(c + d)*(1 + Sin[e + f*x])*(c + d*Sin[e + f*x]))))/(2*a*(
-c + d)*(c + d)*f)
```

fricas [B] time = 0.57, size = 2365, normalized size = 11.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*c^6 - 12*c^4*d^2 + 12*c^2*d^4 - 4*d^6 - 2*(2*c^4*d^2 + 9*c^3*d^3 +
2*c^2*d^4 - 9*c*d^5 - 4*d^6)*cos(f*x + e)^3 + 2*(4*c^5*d + 12*c^4*d^2 - 2*c
^3*d^3 - 15*c^2*d^4 - 2*c*d^5 + 3*d^6)*cos(f*x + e)^2 - 3*(2*c^4*d + 6*c^3*
d^2 + 7*c^2*d^3 + 4*c*d^4 + d^5 - (2*c^2*d^3 + 2*c*d^4 + d^5)*cos(f*x + e)^
3 - (4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*cos(f*x + e)^2 + (2*c^4*d + 2*c
^3*d^2 + 3*c^2*d^3 + 2*c*d^4 + d^5)*cos(f*x + e) + (2*c^4*d + 6*c^3*d^2 + 7
*c^2*d^3 + 4*c*d^4 + d^5 - (2*c^2*d^3 + 2*c*d^4 + d^5)*cos(f*x + e)^2 + 2*(
2*c^3*d^2 + 2*c^2*d^3 + c*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)
*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*
cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x
+ e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(2*c^6 + 4*c^5*d + 8*c^4*d^2
+ 7*c^3*d^3 - 7*c^2*d^4 - 11*c*d^5 - 3*d^6)*cos(f*x + e) - 2*(2*c^6 - 6*c^4
*d^2 + 6*c^2*d^4 - 2*d^6 - (2*c^4*d^2 + 9*c^3*d^3 + 2*c^2*d^4 - 9*c*d^5 - 4
*d^6)*cos(f*x + e)^2 - (4*c^5*d + 14*c^4*d^2 + 7*c^3*d^3 - 13*c^2*d^4 - 11*
c*d^5 - d^6)*cos(f*x + e))*sin(f*x + e))/((a*c^7*d^2 - a*c^6*d^3 - 3*a*c^5*
d^4 + 3*a*c^4*d^5 + 3*a*c^3*d^6 - 3*a*c^2*d^7 - a*c*d^8 + a*d^9)*f*cos(f*x
+ e)^3 + (2*a*c^8*d - a*c^7*d^2 - 7*a*c^6*d^3 + 3*a*c^5*d^4 + 9*a*c^4*d^5 -
 3*a*c^3*d^6 - 5*a*c^2*d^7 + a*c*d^8 + a*d^9)*f*cos(f*x + e)^2 - (a*c^9 - a
*c^8*d - 2*a*c^7*d^2 + 2*a*c^6*d^3 + 2*a*c^3*d^6 - 2*a*c^2*d^7 - a*c*d^8 +
a*d^9)*f*cos(f*x + e) - (a*c^9 + a*c^8*d - 4*a*c^7*d^2 - 4*a*c^6*d^3 + 6*a*
c^5*d^4 + 6*a*c^4*d^5 - 4*a*c^3*d^6 - 4*a*c^2*d^7 + a*c*d^8 + a*d^9)*f + ((
a*c^7*d^2 - a*c^6*d^3 - 3*a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^3*d^6 - 3*a*c^2*d
^7 - a*c*d^8 + a*d^9)*f*cos(f*x + e)^2 - 2*(a*c^8*d - a*c^7*d^2 - 3*a*c^6*d
^3 + 3*a*c^5*d^4 + 3*a*c^4*d^5 - 3*a*c^3*d^6 - a*c^2*d^7 + a*c*d^8)*f*cos(f
*x + e) - (a*c^9 + a*c^8*d - 4*a*c^7*d^2 - 4*a*c^6*d^3 + 6*a*c^5*d^4 + 6*a*
c^4*d^5 - 4*a*c^3*d^6 - 4*a*c^2*d^7 + a*c*d^8 + a*d^9)*f)*sin(f*x + e)), 1/
2*(2*c^6 - 6*c^4*d^2 + 6*c^2*d^4 - 2*d^6 - (2*c^4*d^2 + 9*c^3*d^3 + 2*c^2*d
^4 - 9*c*d^5 - 4*d^6)*cos(f*x + e)^3 + (4*c^5*d + 12*c^4*d^2 - 2*c^3*d^3 -
15*c^2*d^4 - 2*c*d^5 + 3*d^6)*cos(f*x + e)^2 - 3*(2*c^4*d + 6*c^3*d^2 + 7*c
^2*d^3 + 4*c*d^4 + d^5 - (2*c^2*d^3 + 2*c*d^4 + d^5)*cos(f*x + e)^3 - (4*c^
3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*cos(f*x + e)^2 + (2*c^4*d + 2*c^3*d^2 +
3*c^2*d^3 + 2*c*d^4 + d^5)*cos(f*x + e) + (2*c^4*d + 6*c^3*d^2 + 7*c^2*d^3
+ 4*c*d^4 + d^5 - (2*c^2*d^3 + 2*c*d^4 + d^5)*cos(f*x + e)^2 + 2*(2*c^3*d^2
 + 2*c^2*d^3 + c*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(
c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (2*c^6 + 4*c^5*d + 8*
c^4*d^2 + 7*c^3*d^3 - 7*c^2*d^4 - 11*c*d^5 - 3*d^6)*cos(f*x + e) - (2*c^6 -
 6*c^4*d^2 + 6*c^2*d^4 - 2*d^6 - (2*c^4*d^2 + 9*c^3*d^3 + 2*c^2*d^4 - 9*c*d
```

```
^5 - 4*d^6)*cos(f*x + e)^2 - (4*c^5*d + 14*c^4*d^2 + 7*c^3*d^3 - 13*c^2*d^4
 - 11*c*d^5 - d^6)*cos(f*x + e))*sin(f*x + e))/((a*c^7*d^2 - a*c^6*d^3 - 3*
a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^3*d^6 - 3*a*c^2*d^7 - a*c*d^8 + a*d^9)*f*co
s(f*x + e)^3 + (2*a*c^8*d - a*c^7*d^2 - 7*a*c^6*d^3 + 3*a*c^5*d^4 + 9*a*c^4
*d^5 - 3*a*c^3*d^6 - 5*a*c^2*d^7 + a*c*d^8 + a*d^9)*f*cos(f*x + e)^2 - (a*c
^9 - a*c^8*d - 2*a*c^7*d^2 + 2*a*c^6*d^3 + 2*a*c^3*d^6 - 2*a*c^2*d^7 - a*c*
d^8 + a*d^9)*f*cos(f*x + e) - (a*c^9 + a*c^8*d - 4*a*c^7*d^2 - 4*a*c^6*d^3
+ 6*a*c^5*d^4 + 6*a*c^4*d^5 - 4*a*c^3*d^6 - 4*a*c^2*d^7 + a*c*d^8 + a*d^9)*
f + ((a*c^7*d^2 - a*c^6*d^3 - 3*a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^3*d^6 - 3*a
*c^2*d^7 - a*c*d^8 + a*d^9)*f*cos(f*x + e)^2 - 2*(a*c^8*d - a*c^7*d^2 - 3*a
*c^6*d^3 + 3*a*c^5*d^4 + 3*a*c^4*d^5 - 3*a*c^3*d^6 - a*c^2*d^7 + a*c*d^8)*f
*cos(f*x + e) - (a*c^9 + a*c^8*d - 4*a*c^7*d^2 - 4*a*c^6*d^3 + 6*a*c^5*d^4
+ 6*a*c^4*d^5 - 4*a*c^3*d^6 - 4*a*c^2*d^7 + a*c*d^8 + a*d^9)*f)*sin(f*x + e
))]
```

giac [B] time = 1.86, size = 482, normalized size = 2.26

$$-\frac{\frac{3\left(2c^2d+2cd^2+d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(ac^5-ac^4d-2ac^3d^2+2ac^2d^3+acd^4-ad^5\right)\sqrt{c^2-d^2}}+\frac{7c^3d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+2c^2d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2cd^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+6c^4d\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -(3*(2*c^2*d + 2*c*d^2 + d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + ar
ctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a*c^5 - a*c^4*d - 2*a
*c^3*d^2 + 2*a*c^2*d^3 + a*c*d^4 - a*d^5)*sqrt(c^2 - d^2)) + (7*c^3*d^3*tan
(1/2*f*x + 1/2*e)^3 + 2*c^2*d^4*tan(1/2*f*x + 1/2*e)^3 - 2*c*d^5*tan(1/2*f*
x + 1/2*e)^3 + 6*c^4*d^2*tan(1/2*f*x + 1/2*e)^2 + 2*c^3*d^3*tan(1/2*f*x + 1
/2*e)^2 + 11*c^2*d^4*tan(1/2*f*x + 1/2*e)^2 + 4*c*d^5*tan(1/2*f*x + 1/2*e)^
2 - 2*d^6*tan(1/2*f*x + 1/2*e)^2 + 17*c^3*d^3*tan(1/2*f*x + 1/2*e) + 6*c^2*
d^4*tan(1/2*f*x + 1/2*e) - 2*c*d^5*tan(1/2*f*x + 1/2*e) + 6*c^4*d^2 + 2*c^3
*d^3 - c^2*d^4)/((a*c^7 - a*c^6*d - 2*a*c^5*d^2 + 2*a*c^4*d^3 + a*c^3*d^4 -
 a*c^2*d^5)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2) +
2/((a*c^3 - 3*a*c^2*d + 3*a*c*d^2 - a*d^3)*(tan(1/2*f*x + 1/2*e) + 1)))/f
```

maple [B] time = 0.33, size = 1224, normalized size = 5.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

```
[Out] -7/a/f*d^3/(c-d)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c
^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3-2/a/f*d^4/(c-d)^3/(tan(1/2*f*x+1/2*e)^2*
c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+2/a/f*d^
5/(c-d)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^2+2*c*d+
d^2)*tan(1/2*f*x+1/2*e)^3-6/a/f*d^2/(c-d)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1
/2*f*x+1/2*e)*d+c)^2*c^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2-2/a/f*d^3/(c-
d)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+d^2)*
tan(1/2*f*x+1/2*e)^2-11/a/f*d^4/(c-d)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f
*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2-4/a/f*d^5/(c-d)^3/(ta
n(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^2+2*c*d+d^2)*tan(1/2*
f*x+1/2*e)^2+2/a/f*d^6/(c-d)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)
*d+c)^2/c^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2-17/a/f*d^3/(c-d)^3/(tan(1/
2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*tan(1/2*f*x+
1/2*e)-6/a/f*d^4/(c-d)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^
2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)+2/a/f*d^5/(c-d)^3/(tan(1/2*f*x+1/2*e)^
2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)-6/a/f*
d^2/(c-d)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+
d^2)*c^2-2/a/f*d^3/(c-d)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c
)^2/(c^2+2*c*d+d^2)*c+1/a/f*d^4/(c-d)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f
*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)-6/a/f*d/(c-d)^3/(c^2+2*c*d+d^2)/(c^2-d^2)^
(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c^2-6/a/f*d^
2/(c-d)^3/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e
)+2*d)/(c^2-d^2)^(1/2))*c-3/a/f*d^3/(c-d)^3/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)
*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))-2/a/f/(c-d)^3/(ta
n(1/2*f*x+1/2*e)+1)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [**B**] time = 10.20, size = 753, normalized size = 3.54

$$\frac{3\,d\,\mathrm{atan}\left(\frac{\frac{3\,d\,\left(2\,c^2+2\,c\,d+d^2\right)\,\left(-2\,a\,c^5\,d+2\,a\,c^4\,d^2+4\,a\,c^3\,d^3-4\,a\,c^2\,d^4-2\,a\,c\,d^5+2\,a\,d^6\right)}{2\,a\,{\left(c+d\right)}^{5/2}\,{\left(c-d\right)}^{7/2}}-\frac{3\,c\,d\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,c^2+2\,c\,d+d^2\right)\,\left(a\,c^5-a\,c^4\,d-2\,a\,c^3\,d^2+2\,a\,c^2\,d^3+a\,c\,d^4-a\,d^5\right)}{a\,{\left(c+d\right)}^{5/2}\,{\left(c-d\right)}^{7/2}}}{6\,c^2\,d+6\,c\,d^2+3\,d^3}\right)\,\left(2\right.}{a\,f\,{\left(c+d\right)}^{5/2}\,{\left(c-d\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^3),x)
```

```
[Out] (3*d*atan(((3*d*(2*c*d + 2*c^2 + d^2)*(2*a*d^6 - 4*a*c^2*d^4 + 4*a*c^3*d^3
+ 2*a*c^4*d^2 - 2*a*c*d^5 - 2*a*c^5*d))/(2*a*(c + d)^(5/2)*(c - d)^(7/2)) -
 (3*c*d*tan(e/2 + (f*x)/2)*(2*c*d + 2*c^2 + d^2)*(a*c^5 - a*d^5 + 2*a*c^2*d
^3 - 2*a*c^3*d^2 + a*c*d^4 - a*c^4*d))/(a*(c + d)^(5/2)*(c - d)^(7/2)))/(6*
c*d^2 + 6*c^2*d + 3*d^3))*(2*c*d + 2*c^2 + d^2))/(a*f*(c + d)^(5/2)*(c - d)
^(7/2)) - ((2*c*d^3 + 4*c^3*d + 2*c^4 - d^4 + 8*c^2*d^2)/((c + d)*(c^2 - d^
2)*(c^2 - 2*c*d + d^2)) - (tan(e/2 + (f*x)/2)^3*(2*c*d^5 + 8*c^5*d - 2*d^6
+ 13*c^2*d^4 + 17*c^3*d^3 + 22*c^4*d^2))/(c^2*(c^2 - 2*c*d + d^2)*(c*d^2 -
c^2*d - c^3 + d^3)) + (tan(e/2 + (f*x)/2)^2*(4*c*d^4 + 4*c^4*d + 4*c^5 - 2*
d^5 + 21*c^2*d^3 + 14*c^3*d^2))/(c^2*(c^2 - d^2)*(c^2 - 2*c*d + d^2)) - (ta
n(e/2 + (f*x)/2)^4*(2*c*d^4 + 4*c^4*d + 2*c^5 - 2*d^5 + 7*c^2*d^3 + 2*c^3*d
^2))/(c*(c^2 - 2*c*d + d^2)*(c*d^2 - c^2*d - c^3 + d^3)) + (tan(e/2 + (f*x)
/2)*(5*c*d^4 + 8*c^4*d - 2*d^5 + 27*c^2*d^3 + 22*c^3*d^2))/(c*(c + d)*(c^2
- d^2)*(c^2 - 2*c*d + d^2)))/(f*(tan(e/2 + (f*x)/2)^2*(2*a*c^2 + 4*a*d^2 +
4*a*c*d) + tan(e/2 + (f*x)/2)^3*(2*a*c^2 + 4*a*d^2 + 4*a*c*d) + a*c^2 + tan
(e/2 + (f*x)/2)*(a*c^2 + 4*a*c*d) + tan(e/2 + (f*x)/2)^4*(a*c^2 + 4*a*c*d)
+ a*c^2*tan(e/2 + (f*x)/2)^5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.461 $\int \frac{(c+d\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=260

$$\frac{d\left(c^2+10cd-12d^2\right)\cos(e+fx)(c+d\sin(e+fx))^2}{3a^2f}+\frac{5d^2x(2c-d)\left(2c^2-3cd+2d^2\right)}{2a^2}+\frac{d^2\left(2c^3+20c^2d-57cd^2+\right.}{}$$

```
[Out] 5/2*(2*c-d)*d^2*(2*c^2-3*c*d+2*d^2)*x/a^2+2/3*d*(c^4+10*c^3*d-44*c^2*d^2+40
*c*d^3-12*d^4)*cos(f*x+e)/a^2/f+1/6*d^2*(2*c^3+20*c^2*d-57*c*d^2+30*d^3)*co
s(f*x+e)*sin(f*x+e)/a^2/f+1/3*d*(c^2+10*c*d-12*d^2)*cos(f*x+e)*(c+d*sin(f*x
+e))^2/a^2/f-1/3*(c-d)*(c+10*d)*cos(f*x+e)*(c+d*sin(f*x+e))^3/a^2/f/(1+sin(
f*x+e))-1/3*(c-d)*cos(f*x+e)*(c+d*sin(f*x+e))^4/f/(a+a*sin(f*x+e))^2
```

Rubi [A] time = 0.50, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2765, 2977, 2753, 2734}

$$\frac{2d\left(-44c^2d^2+10c^3d+c^4+40cd^3-12d^4\right)\cos(e+fx)}{3a^2f}+\frac{d\left(c^2+10cd-12d^2\right)\cos(e+fx)(c+d\sin(e+fx))^2}{3a^2f}+\frac{d^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^5/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (5*(2*c - d)*d^2*(2*c^2 - 3*c*d + 2*d^2)*x)/(2*a^2) + (2*d*(c^4 + 10*c^3*d
- 44*c^2*d^2 + 40*c*d^3 - 12*d^4)*Cos[e + f*x])/(3*a^2*f) + (d^2*(2*c^3 + 2
0*c^2*d - 57*c*d^2 + 30*d^3)*Cos[e + f*x]*Sin[e + f*x])/(6*a^2*f) + (d*(c^2
 + 10*c*d - 12*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(3*a^2*f) - ((c -
d)*(c + 10*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(3*a^2*f*(1 + Sin[e + f*
x])) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(3*f*(a + a*Sin[e + f*
x])^2)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
```

```
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^4}{3f(a+a\sin(e+fx))^2} - \frac{\int \frac{(c+d\sin(e+fx))^3\left(-a\left(c^2+6cd-4d^2\right)+3a(c-2d)d\sin\right.}{a+a\sin(e+fx)}}{3a^2}$$

$$= -\frac{(c-d)(c+10d)\cos(e+fx)(c+d\sin(e+fx))^3}{3a^2f(1+\sin(e+fx))} - \frac{(c-d)\cos(e+fx)(c+d\sin(e}{3f(a+a\sin(e+fx))^2}$$

$$= \frac{d\left(c^2+10cd-12d^2\right)\cos(e+fx)(c+d\sin(e+fx))^2}{3a^2f} - \frac{(c-d)(c+10d)\cos(e+fx)}{3a^2f(1+\sin(e}$$

$$= \frac{5(2c-d)d^2\left(2c^2-3cd+2d^2\right)x}{2a^2} + \frac{2d\left(c^4+10c^3d-44c^2d^2+40cd^3-12d^4\right)\cos(e+}{3a^2f}$$

Mathematica [B] time = 1.70, size = 837, normalized size = 3.22

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(48\sin\left(\frac{1}{2}(e+fx)\right)c^5+240d\sin\left(\frac{1}{2}(e+fx)\right)c^4-1440d^2\sin\left(\frac{1}{2}(e+fx)\right)c^3 \cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^5/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3*d*(80*c^4 + 80*c^3*d*(-4 + 3*e +
3*f*x) - 80*c^2*d^2*(-5 + 6*e + 6*f*x) + 35*c*d^3*(-7 + 12*e + 12*f*x) - 4*
d^4*(-13 + 30*e + 30*f*x))*Cos[(e + f*x)/2] - (16*c^5 + 160*c^4*d + 80*c^3*
d^2*(-10 + 3*e + 3*f*x) - 40*c^2*d^3*(-41 + 12*e + 12*f*x) - 6*d^5*(-57 + 2
0*e + 20*f*x) + 5*c*d^4*(-239 + 84*e + 84*f*x))*Cos[(3*(e + f*x))/2] + 120*
c^2*d^3*Cos[(5*(e + f*x))/2] - 75*c*d^4*Cos[(5*(e + f*x))/2] + 30*d^5*Cos[(
5*(e + f*x))/2] + 15*c*d^4*Cos[(7*(e + f*x))/2] - 3*d^5*Cos[(7*(e + f*x))/2
] - d^5*Cos[(9*(e + f*x))/2] + 48*c^5*Sin[(e + f*x)/2] + 240*c^4*d*Sin[(e +
 f*x)/2] - 1440*c^3*d^2*Sin[(e + f*x)/2] + 2640*c^2*d^3*Sin[(e + f*x)/2] -
1905*c*d^4*Sin[(e + f*x)/2] + 516*d^5*Sin[(e + f*x)/2] + 720*c^3*d^2*e*Sin[
(e + f*x)/2] - 1440*c^2*d^3*e*Sin[(e + f*x)/2] + 1260*c*d^4*e*Sin[(e + f*x)
/2] - 360*d^5*e*Sin[(e + f*x)/2] + 720*c^3*d^2*f*x*Sin[(e + f*x)/2] - 1440*
c^2*d^3*f*x*Sin[(e + f*x)/2] + 1260*c*d^4*f*x*Sin[(e + f*x)/2] - 360*d^5*f*
x*Sin[(e + f*x)/2] - 360*c^2*d^3*Sin[(3*(e + f*x))/2] + 315*c*d^4*Sin[(3*(e
 + f*x))/2] - 118*d^5*Sin[(3*(e + f*x))/2] + 240*c^3*d^2*e*Sin[(3*(e + f*x)
)/2] - 480*c^2*d^3*e*Sin[(3*(e + f*x))/2] + 420*c*d^4*e*Sin[(3*(e + f*x))/2
] - 120*d^5*e*Sin[(3*(e + f*x))/2] + 240*c^3*d^2*f*x*Sin[(3*(e + f*x))/2] -
 480*c^2*d^3*f*x*Sin[(3*(e + f*x))/2] + 420*c*d^4*f*x*Sin[(3*(e + f*x))/2]
- 120*d^5*f*x*Sin[(3*(e + f*x))/2] - 120*c^2*d^3*Sin[(5*(e + f*x))/2] + 75*
c*d^4*Sin[(5*(e + f*x))/2] - 30*d^5*Sin[(5*(e + f*x))/2] + 15*c*d^4*Sin[(7*
(e + f*x))/2] - 3*d^5*Sin[(7*(e + f*x))/2] + d^5*Sin[(9*(e + f*x))/2]))/(48
*a^2*f*(1 + Sin[e + f*x])^2)
```

fricas [B] time = 0.46, size = 578, normalized size = 2.22

$$\frac{2\,d^5\cos\left(fx+e\right)^5+2\,c^5-10\,c^4d+20\,c^3d^2-20\,c^2d^3+10\,cd^4-2\,d^5-\left(15\,cd^4-4\,d^5\right)\cos\left(fx+e\right)^4-2\left(30\,c^2d^3 \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*d^5*cos(f*x + e)^5 + 2*c^5 - 10*c^4*d + 20*c^3*d^2 - 20*c^2*d^3 + 10
*c*d^4 - 2*d^5 - (15*c*d^4 - 4*d^5)*cos(f*x + e)^4 - 2*(30*c^2*d^3 - 15*c*d
^4 + 8*d^5)*cos(f*x + e)^3 - 30*(4*c^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*f
```

```
*x + (2*c^5 + 20*c^4*d - 100*c^3*d^2 + 220*c^2*d^3 - 155*c*d^4 + 46*d^5 + 1
5*(4*c^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*f*x)*cos(f*x + e)^2 + (4*c^5 +
10*c^4*d - 80*c^3*d^2 + 260*c^2*d^3 - 190*c*d^4 + 62*d^5 - 15*(4*c^3*d^2 -
8*c^2*d^3 + 7*c*d^4 - 2*d^5)*f*x)*cos(f*x + e) - (2*d^5*cos(f*x + e)^4 + 2*
c^5 - 10*c^4*d + 20*c^3*d^2 - 20*c^2*d^3 + 10*c*d^4 - 2*d^5 + (15*c*d^4 - 2
*d^5)*cos(f*x + e)^3 + 30*(4*c^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*f*x - 3
*(20*c^2*d^3 - 15*c*d^4 + 6*d^5)*cos(f*x + e)^2 - (2*c^5 + 20*c^4*d - 100*c
^3*d^2 + 280*c^2*d^3 - 200*c*d^4 + 64*d^5 - 15*(4*c^3*d^2 - 8*c^2*d^3 + 7*c
*d^4 - 2*d^5)*f*x)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*
f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

giac [B] time = 0.24, size = 977, normalized size = 3.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/6*(15*(4*c^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*(f*x + e)/a^2 - 2*(6*c^5*
tan(1/2*f*x + 1/2*e)^8 - 60*c^3*d^2*tan(1/2*f*x + 1/2*e)^8 + 120*c^2*d^3*ta
n(1/2*f*x + 1/2*e)^8 - 105*c*d^4*tan(1/2*f*x + 1/2*e)^8 + 30*d^5*tan(1/2*f*
x + 1/2*e)^8 + 6*c^5*tan(1/2*f*x + 1/2*e)^7 + 30*c^4*d*tan(1/2*f*x + 1/2*e)
^7 - 180*c^3*d^2*tan(1/2*f*x + 1/2*e)^7 + 360*c^2*d^3*tan(1/2*f*x + 1/2*e)^
7 - 315*c*d^4*tan(1/2*f*x + 1/2*e)^7 + 90*d^5*tan(1/2*f*x + 1/2*e)^7 + 22*c
^5*tan(1/2*f*x + 1/2*e)^6 + 10*c^4*d*tan(1/2*f*x + 1/2*e)^6 - 260*c^3*d^2*t
an(1/2*f*x + 1/2*e)^6 + 680*c^2*d^3*tan(1/2*f*x + 1/2*e)^6 - 595*c*d^4*tan(
1/2*f*x + 1/2*e)^6 + 170*d^5*tan(1/2*f*x + 1/2*e)^6 + 18*c^5*tan(1/2*f*x +
1/2*e)^5 + 90*c^4*d*tan(1/2*f*x + 1/2*e)^5 - 540*c^3*d^2*tan(1/2*f*x + 1/2*
e)^5 + 1200*c^2*d^3*tan(1/2*f*x + 1/2*e)^5 - 945*c*d^4*tan(1/2*f*x + 1/2*e)
^5 + 270*d^5*tan(1/2*f*x + 1/2*e)^5 + 30*c^5*tan(1/2*f*x + 1/2*e)^4 + 30*c^
4*d*tan(1/2*f*x + 1/2*e)^4 - 420*c^3*d^2*tan(1/2*f*x + 1/2*e)^4 + 1200*c^2*
d^3*tan(1/2*f*x + 1/2*e)^4 - 975*c*d^4*tan(1/2*f*x + 1/2*e)^4 + 306*d^5*tan
(1/2*f*x + 1/2*e)^4 + 18*c^5*tan(1/2*f*x + 1/2*e)^3 + 90*c^4*d*tan(1/2*f*x
+ 1/2*e)^3 - 540*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 + 1320*c^2*d^3*tan(1/2*f*x
+ 1/2*e)^3 - 1005*c*d^4*tan(1/2*f*x + 1/2*e)^3 + 310*d^5*tan(1/2*f*x + 1/2*
e)^3 + 18*c^5*tan(1/2*f*x + 1/2*e)^2 + 30*c^4*d*tan(1/2*f*x + 1/2*e)^2 - 30
0*c^3*d^2*tan(1/2*f*x + 1/2*e)^2 + 840*c^2*d^3*tan(1/2*f*x + 1/2*e)^2 - 645
*c*d^4*tan(1/2*f*x + 1/2*e)^2 + 198*d^5*tan(1/2*f*x + 1/2*e)^2 + 6*c^5*tan(
1/2*f*x + 1/2*e) + 30*c^4*d*tan(1/2*f*x + 1/2*e) - 180*c^3*d^2*tan(1/2*f*x
+ 1/2*e) + 480*c^2*d^3*tan(1/2*f*x + 1/2*e) - 375*c*d^4*tan(1/2*f*x + 1/2*e
) + 114*d^5*tan(1/2*f*x + 1/2*e) + 4*c^5 + 10*c^4*d - 80*c^3*d^2 + 200*c^2*
d^3 - 160*c*d^4 + 48*d^5)/((tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)^2
 + tan(1/2*f*x + 1/2*e) + 1)^3*a^2))/f
```

maple [B] time = 0.32, size = 982, normalized size = 3.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x)
```

```
[Out] -4/3/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*c^5+4/3/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*
d^5-22/3/a^2/f*d^5/(1+tan(1/2*f*x+1/2*e)^2)^3-10/a^2/f*d^5*arctan(tan(1/2*f
*x+1/2*e))-2/a^2/f/(tan(1/2*f*x+1/2*e)+1)*c^5-8/a^2/f/(tan(1/2*f*x+1/2*e)+1
)*d^5+2/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*c^5-2/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2
*d^5+20/3/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*c^4*d-40/3/a^2/f/(tan(1/2*f*x+1/2*
e)+1)^3*c^3*d^2+40/3/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*c^2*d^3-20/3/a^2/f/(tan
(1/2*f*x+1/2*e)+1)^3*c*d^4+35/a^2/f*d^4*arctan(tan(1/2*f*x+1/2*e))*c+30/a^2
/f/(tan(1/2*f*x+1/2*e)+1)*c*d^4-10/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*c^4*d+20/
a^2/f*d^2*arctan(tan(1/2*f*x+1/2*e))*c^3-40/a^2/f*d^3*arctan(tan(1/2*f*x+1/
2*e))*c^2+20/a^2/f*d^4/(1+tan(1/2*f*x+1/2*e)^2)^3*c-6/a^2/f*d^5/(1+tan(1/2*
f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^4-16/a^2/f*d^5/(1+tan(1/2*f*x+1/2*e)^2)^
3*tan(1/2*f*x+1/2*e)^2+2/a^2/f*d^5/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1
/2*e)-20/a^2/f*d^3/(1+tan(1/2*f*x+1/2*e)^2)^3*c^2+20/a^2/f/(tan(1/2*f*x+1/2
*e)+1)*c^3*d^2-40/a^2/f/(tan(1/2*f*x+1/2*e)+1)*c^2*d^3+20/a^2/f/(tan(1/2*f*
x+1/2*e)+1)^2*c^3*d^2-20/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*c^2*d^3+10/a^2/f/(t
an(1/2*f*x+1/2*e)+1)^2*c*d^4-2/a^2/f*d^5/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2
*f*x+1/2*e)^5-40/a^2/f*d^3/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2*
c^2+40/a^2/f*d^4/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2*c-5/a^2/f*
d^4/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)*c+5/a^2/f*d^4/(1+tan(1/2*
f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5*c-20/a^2/f*d^3/(1+tan(1/2*f*x+1/2*e)^2
)^3*tan(1/2*f*x+1/2*e)^4*c^2+20/a^2/f*d^4/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/
2*f*x+1/2*e)^4*c
```

maxima [B] time = 0.48, size = 1312, normalized size = 5.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(5*c*d^4*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin(f
*x + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e
) + 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^2*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^
```

```
6 + a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)/(cos(
f*x + e) + 1))/a^2) - 2*d^5*((57*sin(f*x + e)/(cos(f*x + e) + 1) + 99*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 155*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
153*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 135*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5 + 85*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 45*sin(f*x + e)^7/(cos(f*
x + e) + 1)^7 + 15*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 24)/(a^2 + 3*a^2*s
in(f*x + e)/(cos(f*x + e) + 1) + 6*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
+ 10*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 12*a^2*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 12*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 10*a^2*sin(f*
x + e)^6/(cos(f*x + e) + 1)^6 + 6*a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 +
 3*a^2*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^2*sin(f*x + e)^9/(cos(f*x +
e) + 1)^9) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 40*c^2*d^3*(
(12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e)
+ 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*a^2*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*a
^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/(cos(f*x + e) +
 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 20*c^3*d^2*((9*si
n(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/
(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x
 + e)/(cos(f*x + e) + 1))/a^2) - 2*c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) +
 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(
f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3) - 10*c^4*d*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1
)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

mupad [B] time = 9.43, size = 692, normalized size = 2.66

$$\frac{5\,d^2\,\mathrm{atan}\left(\frac{5\,d^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,c-d\right)\,\left(2\,c^2-3\,c\,d+2\,d^2\right)}{20\,c^3\,d^2-40\,c^2\,d^3+35\,c\,d^4-10\,d^5}\right)\,\left(2\,c-d\right)\,\left(2\,c^2-3\,c\,d+2\,d^2\right)}{a^2\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7\,\left(2\,c^5+10\,c^4\,d-60\,c^3\,d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^5/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (5*d^2*atan((5*d^2*tan(e/2 + (f*x)/2)*(2*c - d)*(2*c^2 - 3*c*d + 2*d^2))/(3
5*c*d^4 - 10*d^5 - 40*c^2*d^3 + 20*c^3*d^2))*(2*c - d)*(2*c^2 - 3*c*d + 2*d
^2))/(a^2*f) - (tan(e/2 + (f*x)/2)^7*(10*c^4*d - 105*c*d^4 + 2*c^5 + 30*d^5
 + 120*c^2*d^3 - 60*c^3*d^2) + tan(e/2 + (f*x)/2)^2*(10*c^4*d - 215*c*d^4 +
 6*c^5 + 66*d^5 + 280*c^2*d^3 - 100*c^3*d^2) + tan(e/2 + (f*x)/2)^4*(10*c^4
*d - 325*c*d^4 + 10*c^5 + 102*d^5 + 400*c^2*d^3 - 140*c^3*d^2) + tan(e/2 +
```

```
(f*x)/2)^5*(30*c^4*d - 315*c*d^4 + 6*c^5 + 90*d^5 + 400*c^2*d^3 - 180*c^3*d
^2) + tan(e/2 + (f*x)/2)^3*(30*c^4*d - 335*c*d^4 + 6*c^5 + (310*d^5)/3 + 44
0*c^2*d^3 - 180*c^3*d^2) + tan(e/2 + (f*x)/2)^6*((10*c^4*d)/3 - (595*c*d^4)
/3 + (22*c^5)/3 + (170*d^5)/3 + (680*c^2*d^3)/3 - (260*c^3*d^2)/3) - (160*c
*d^4)/3 + (10*c^4*d)/3 + tan(e/2 + (f*x)/2)^8*(2*c^5 - 35*c*d^4 + 10*d^5 +
40*c^2*d^3 - 20*c^3*d^2) + tan(e/2 + (f*x)/2)*(10*c^4*d - 125*c*d^4 + 2*c^5
 + 38*d^5 + 160*c^2*d^3 - 60*c^3*d^2) + (4*c^5)/3 + 16*d^5 + (200*c^2*d^3)/
3 - (80*c^3*d^2)/3)/(f*(6*a^2*tan(e/2 + (f*x)/2)^2 + 10*a^2*tan(e/2 + (f*x)
/2)^3 + 12*a^2*tan(e/2 + (f*x)/2)^4 + 12*a^2*tan(e/2 + (f*x)/2)^5 + 10*a^2*
tan(e/2 + (f*x)/2)^6 + 6*a^2*tan(e/2 + (f*x)/2)^7 + 3*a^2*tan(e/2 + (f*x)/2
)^8 + a^2*tan(e/2 + (f*x)/2)^9 + a^2 + 3*a^2*tan(e/2 + (f*x)/2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**5/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.462 $\int \frac{(c+d\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=195

$$\frac{d^2\left(2c^2+16cd-21d^2\right)\sin(e+fx)\cos(e+fx)}{6a^2f}+\frac{d^2x\left(12c^2-16cd+7d^2\right)}{2a^2}+\frac{2d\left(c^3+8c^2d-20cd^2+8d^3\right)\cos(e+$$

```
[Out] 1/2*d^2*(12*c^2-16*c*d+7*d^2)*x/a^2+2/3*d*(c^3+8*c^2*d-20*c*d^2+8*d^3)*cos(
f*x+e)/a^2/f+1/6*d^2*(2*c^2+16*c*d-21*d^2)*cos(f*x+e)*sin(f*x+e)/a^2/f-1/3*
(c-d)*(c+8*d)*cos(f*x+e)*(c+d*sin(f*x+e))^2/a^2/f/(1+sin(f*x+e))-1/3*(c-d)*
cos(f*x+e)*(c+d*sin(f*x+e))^3/f/(a+a*sin(f*x+e))^2
```

Rubi [A] time = 0.36, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2765, 2977, 2734}

$$\frac{2d\left(8c^2d+c^3-20cd^2+8d^3\right)\cos(e+fx)}{3a^2f}+\frac{d^2\left(2c^2+16cd-21d^2\right)\sin(e+fx)\cos(e+fx)}{6a^2f}+\frac{d^2x\left(12c^2-16cd+\right.}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^4/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (d^2*(12*c^2 - 16*c*d + 7*d^2)*x)/(2*a^2) + (2*d*(c^3 + 8*c^2*d - 20*c*d^2
+ 8*d^3)*Cos[e + f*x])/(3*a^2*f) + (d^2*(2*c^2 + 16*c*d - 21*d^2)*Cos[e + f
*x]*Sin[e + f*x])/(6*a^2*f) - ((c - d)*(c + 8*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^2)/(3*a^2*f*(1 + Sin[e + f*x])) - ((c - d)*Cos[e + f*x]*(c + d*Sin[
e + f*x])^3)/(3*f*(a + a*Sin[e + f*x])^2)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
```

```
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^3}{3f(a+a\sin(e+fx))^2} - \frac{\int \frac{(c+d\sin(e+fx))^2\left(-a\left(c^2+5cd-3d^2\right)+a(2c-5d)d\sin(e}{a+a\sin(e+fx)}}{3a^2}$$

$$= -\frac{(c-d)(c+8d)\cos(e+fx)(c+d\sin(e+fx))^2}{3a^2f(1+\sin(e+fx))} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+}{3f(a+a\sin(e+fx))^2}$$

$$= \frac{d^2\left(12c^2-16cd+7d^2\right)x}{2a^2} + \frac{2d\left(c^3+8c^2d-20cd^2+8d^3\right)\cos(e+fx)}{3a^2f} + \frac{d^2\left(2c^2+16\right.}{}$$

Mathematica [A] time = 1.95, size = 378, normalized size = 1.94

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(3d\cos\left(\frac{1}{2}(e+fx)\right)\left(64c^3+48c^2d(3e+3fx-4)-32cd^2(6e+6fx-5)+7d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^4/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3*d*(64*c^3 + 48*c^2*d*(-4 + 3*e +
3*f*x) - 32*c*d^2*(-5 + 6*e + 6*f*x) + 7*d^3*(-7 + 12*e + 12*f*x))*Cos[(e +
 f*x)/2] - (16*c^4 + 128*c^3*d + 48*c^2*d^2*(-10 + 3*e + 3*f*x) - 16*c*d^3*
(-41 + 12*e + 12*f*x) + d^4*(-239 + 84*e + 84*f*x))*Cos[(3*(e + f*x))/2] +
3*((16*c - 5*d)*d^3*Cos[(5*(e + f*x))/2] + d^4*Cos[(7*(e + f*x))/2] + 2*(8*
c^4 + 32*c^3*d - 144*c^2*d^2 + 144*c*d^3 - 50*d^4 + 96*c^2*d^2*e - 128*c*d^
```

```
3*e + 56*d^4*e + 96*c^2*d^2*f*x - 128*c*d^3*f*x + 56*d^4*f*x + d^2*(48*c^2*
(e + f*x) - 64*c*d*(1 + e + f*x) + d^2*(27 + 28*e + 28*f*x))*Cos[e + f*x] -
 2*(8*c - 3*d)*d^3*Cos[2*(e + f*x)] + d^4*Cos[3*(e + f*x)])*Sin[(e + f*x)/2
])))/(48*a^2*f*(1 + Sin[e + f*x])^2)
```

fricas [B] time = 0.45, size = 440, normalized size = 2.26

$$-\frac{3\,d^4\cos\left(fx+e\right)^4 - 2\,c^4 + 8\,c^3d - 12\,c^2d^2 + 8\,cd^3 - 2\,d^4 + 6\left(4\,cd^3 - d^4\right)\cos\left(fx+e\right)^3 + 6\left(12\,c^2d^2 - 16\,cd^3 + \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*d^4*cos(f*x + e)^4 - 2*c^4 + 8*c^3*d - 12*c^2*d^2 + 8*c*d^3 - 2*d^4
 + 6*(4*c*d^3 - d^4)*cos(f*x + e)^3 + 6*(12*c^2*d^2 - 16*c*d^3 + 7*d^4)*f*x
 - (2*c^4 + 16*c^3*d - 60*c^2*d^2 + 88*c*d^3 - 31*d^4 + 3*(12*c^2*d^2 - 16*
c*d^3 + 7*d^4)*f*x)*cos(f*x + e)^2 - (4*c^4 + 8*c^3*d - 48*c^2*d^2 + 104*c*
d^3 - 38*d^4 - 3*(12*c^2*d^2 - 16*c*d^3 + 7*d^4)*f*x)*cos(f*x + e) + (3*d^4
*cos(f*x + e)^3 + 2*c^4 - 8*c^3*d + 12*c^2*d^2 - 8*c*d^3 + 2*d^4 + 6*(12*c^
2*d^2 - 16*c*d^3 + 7*d^4)*f*x - 3*(8*c*d^3 - 3*d^4)*cos(f*x + e)^2 - (2*c^4
 + 16*c^3*d - 60*c^2*d^2 + 112*c*d^3 - 40*d^4 - 3*(12*c^2*d^2 - 16*c*d^3 +
7*d^4)*f*x)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f
*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

giac [A] time = 0.24, size = 338, normalized size = 1.73

$$\frac{\frac{3\left(12\,c^2d^2-16\,cd^3+7\,d^4\right)\left(fx+e\right)}{a^2} + \frac{6\left(d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-8\,cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+4\,d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-8\,cd^3+4\,d^4\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2a^2} - \frac{4\left(3\,c^4\tan\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*(12*c^2*d^2 - 16*c*d^3 + 7*d^4)*(f*x + e)/a^2 + 6*(d^4*tan(1/2*f*x +
 1/2*e)^3 - 8*c*d^3*tan(1/2*f*x + 1/2*e)^2 + 4*d^4*tan(1/2*f*x + 1/2*e)^2 -
 d^4*tan(1/2*f*x + 1/2*e) - 8*c*d^3 + 4*d^4)/((tan(1/2*f*x + 1/2*e)^2 + 1)^
2*a^2) - 4*(3*c^4*tan(1/2*f*x + 1/2*e)^2 - 18*c^2*d^2*tan(1/2*f*x + 1/2*e)^
2 + 24*c*d^3*tan(1/2*f*x + 1/2*e)^2 - 9*d^4*tan(1/2*f*x + 1/2*e)^2 + 3*c^4*
tan(1/2*f*x + 1/2*e) + 12*c^3*d*tan(1/2*f*x + 1/2*e) - 54*c^2*d^2*tan(1/2*f
*x + 1/2*e) + 60*c*d^3*tan(1/2*f*x + 1/2*e) - 21*d^4*tan(1/2*f*x + 1/2*e) +
 2*c^4 + 4*c^3*d - 24*c^2*d^2 + 28*c*d^3 - 10*d^4)/(a^2*(tan(1/2*f*x + 1/2*
e) + 1)^3))/f
```

maple [B] time = 0.30, size = 618, normalized size = 3.17

$$\frac{d^4\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a^2f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{8d^3\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c}{a^2f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{4d^4\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a^2f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{d^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a^2f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{\quad}{a^2f\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 1/a^2/f*d^4/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^3-8/a^2/f*d^3/(1+
tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*c+4/a^2/f*d^4/(1+tan(1/2*f*x+1
/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2-1/a^2/f*d^4/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(
1/2*f*x+1/2*e)-8/a^2/f*d^3/(1+tan(1/2*f*x+1/2*e)^2)^2*c+4/a^2/f*d^4/(1+tan(
1/2*f*x+1/2*e)^2)^2+12/a^2/f*d^2*arctan(tan(1/2*f*x+1/2*e))*c^2-16/a^2/f*d^
3*arctan(tan(1/2*f*x+1/2*e))*c+7/a^2/f*d^4*arctan(tan(1/2*f*x+1/2*e))-2*c^4
/a^2/f/(tan(1/2*f*x+1/2*e)+1)+12/a^2/f/(tan(1/2*f*x+1/2*e)+1)*c^2*d^2-16/a^
2/f/(tan(1/2*f*x+1/2*e)+1)*c*d^3+6/a^2/f/(tan(1/2*f*x+1/2*e)+1)*d^4+2*c^4/a
^2/f/(tan(1/2*f*x+1/2*e)+1)^2-8/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*c^3*d+12/a^2
/f/(tan(1/2*f*x+1/2*e)+1)^2*c^2*d^2-8/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*c*d^3+
2/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*d^4-4/3*c^4/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3
+16/3/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*c^3*d-8/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3
*c^2*d^2+16/3/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*c*d^3-4/3/a^2/f/(tan(1/2*f*x+1
/2*e)+1)^3*d^4
```

maxima [B] time = 0.45, size = 908, normalized size = 4.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(d^4*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) +
1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^2*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 +
a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)/(cos(f*x
+ e) + 1))/a^2) - 16*c*d^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x
+ e) + 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^
```

```
3/(cos(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*si
n(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) +
1))/a^2) + 12*c^2*d^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) +
3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 2*c^4*(3*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^
2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - 8*c^3*d*(3*sin(f*x
+ e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) +
 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3))/f
```

mupad [B] time = 9.21, size = 478, normalized size = 2.45

$$\frac{d^2 \operatorname{atan}\left(\frac{d^2 \tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(12\,c^2-16\,c\,d+7\,d^2\right)}{12\,c^2\,d^2-16\,c\,d^3+7\,d^4}\right)\left(12\,c^2-16\,c\,d+7\,d^2\right)}{a^2\,f} - \tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\left(2\,c^4+8\,c^3\,d-36\,c^2\,d^2+48\,c\,d^3-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^4/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (d^2*atan((d^2*tan(e/2 + (f*x)/2)*(12*c^2 - 16*c*d + 7*d^2))/(7*d^4 - 16*c*
d^3 + 12*c^2*d^2))*(12*c^2 - 16*c*d + 7*d^2))/(a^2*f) - (tan(e/2 + (f*x)/2)
^5*(48*c*d^3 + 8*c^3*d + 2*c^4 - 21*d^4 - 36*c^2*d^2) + tan(e/2 + (f*x)/2)^
3*(112*c*d^3 + 16*c^3*d + 4*c^4 - 42*d^4 - 72*c^2*d^2) + tan(e/2 + (f*x)/2)
^4*((224*c*d^3)/3 + (8*c^3*d)/3 + (16*c^4)/3 - (98*d^4)/3 - 40*c^2*d^2) + t
an(e/2 + (f*x)/2)^2*((256*c*d^3)/3 + (16*c^3*d)/3 + (14*c^4)/3 - (97*d^4)/3
 - 44*c^2*d^2) + (80*c*d^3)/3 + (8*c^3*d)/3 + tan(e/2 + (f*x)/2)^6*(16*c*d^
3 + 2*c^4 - 7*d^4 - 12*c^2*d^2) + (4*c^4)/3 - (32*d^4)/3 + tan(e/2 + (f*x)/
2)*(64*c*d^3 + 8*c^3*d + 2*c^4 - 25*d^4 - 36*c^2*d^2) - 16*c^2*d^2)/(f*(5*a
^2*tan(e/2 + (f*x)/2)^2 + 7*a^2*tan(e/2 + (f*x)/2)^3 + 7*a^2*tan(e/2 + (f*x
)/2)^4 + 5*a^2*tan(e/2 + (f*x)/2)^5 + 3*a^2*tan(e/2 + (f*x)/2)^6 + a^2*tan(
e/2 + (f*x)/2)^7 + a^2 + 3*a^2*tan(e/2 + (f*x)/2)))
```

sympy [A] time = 28.14, size = 8950, normalized size = 45.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**4/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-12*c**4*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*
a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(
e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2
)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 12*c**4*tan(e/2 + f*x/2)**5
/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*
tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f
*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a
**2*f) - 32*c**4*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**
2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2
 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**
2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 24*c**4*tan(e/2 + f*x/2)**3/(6
*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan
(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/
2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2
*f) - 28*c**4*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f
*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 +
f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 +
 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 12*c**4*tan(e/2 + f*x/2)/(6*a**2*
f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 +
 f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3
+ 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) -
8*c**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a
**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e
/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2)
 + 6*a**2*f) - 48*c**3*d*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**7
+ 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f
*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 +
f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 16*c**3*d*tan(e/2 + f*
x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*
a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(
e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2
) + 6*a**2*f) - 96*c**3*d*tan(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**7
 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*
f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 +
 f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 32*c**3*d*tan(e/2 + f
*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30
*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan
(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/
2) + 6*a**2*f) - 48*c**3*d*tan(e/2 + f*x/2)/(6*a**2*f*tan(e/2 + f*x/2)**7 +
 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*
tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f
*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 16*c**3*d/(6*a**2*f*tan
(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/
2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*
a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 36*c*
```

*2*d**2*f*x*tan(e/2 + f*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*t
an(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*
x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 1
8*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 108*c**2*d**2*f*x*tan(e/2 + f*x/2)*
*6/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*
f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 +
 f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6
*a**2*f) + 180*c**2*d**2*f*x*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)
7 + 18*a2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a*
*2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/
2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 252*c**2*d**2*f*x*
tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x
/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42
*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan
(e/2 + f*x/2) + 6*a**2*f) + 252*c**2*d**2*f*x*tan(e/2 + f*x/2)**3/(6*a**2*f
*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 +
f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 +
 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 1
80*c**2*d**2*f*x*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**
2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2
 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**
2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 108*c**2*d**2*f*x*tan(e/2 + f*
x/2)/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**
2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2
 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) +
 6*a**2*f) + 36*c**2*d**2*f*x/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan
(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/
2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*
a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 72*c**2*d**2*tan(e/2 + f*x/2)**6/(6*a
2*f*tan(e/2 + f*x/2)7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e
/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)
3 + 30*a2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f
) + 216*c**2*d**2*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a*
*2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/
2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)*
*2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 240*c**2*d**2*tan(e/2 + f*x/2
)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**
2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2
 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) +
 6*a**2*f) + 432*c**2*d**2*tan(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**
7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2
*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2
+ f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 264*c**2*d**2*tan(e/
2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6
 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*

f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 216*c**2*d**2*tan(e/2 + f*x/2)/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 96*c**2*d**2/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 48*c*d**3*f*x*tan(e/2 + f*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 144*c*d**3*f*x*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 240*c*d**3*f*x*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 336*c*d**3*f*x*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 336*c*d**3*f*x*tan(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 240*c*d**3*f*x*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 144*c*d**3*f*x*tan(e/2 + f*x/2)/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 48*c*d**3*f*x/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 96*c*d**3*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 288*c*d**3*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 448*c*d**3*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)*

```
*5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**
2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 672*c*d*
*3*tan(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 +
f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 +
 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*
tan(e/2 + f*x/2) + 6*a**2*f) - 512*c*d**3*tan(e/2 + f*x/2)**2/(6*a**2*f*tan
(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/
2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*
a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 384*c
*d**3*tan(e/2 + f*x/2)/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 +
f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 +
 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*
tan(e/2 + f*x/2) + 6*a**2*f) - 160*c*d**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 1
8*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*ta
n(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x
/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 21*d**4*f*x*tan(e/2 + f*x
/2)**7/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a
**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e
/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2)
 + 6*a**2*f) + 63*d**4*f*x*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**
7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2
*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2
+ f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 105*d**4*f*x*tan(e/2
 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6
+ 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f
*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 +
f*x/2) + 6*a**2*f) + 147*d**4*f*x*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f
*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 +
42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*t
an(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 147*d**4*f*x*
tan(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x
/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42
*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan
(e/2 + f*x/2) + 6*a**2*f) + 105*d**4*f*x*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(
e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2
)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a
**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 63*d**
4*f*x*tan(e/2 + f*x/2)/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 +
f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 +
 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*
tan(e/2 + f*x/2) + 6*a**2*f) + 21*d**4*f*x/(6*a**2*f*tan(e/2 + f*x/2)**7 +
18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*t
an(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*
x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 42*d**4*tan(e/2 + f*x/2)
**6/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2
```

```
*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2
+ f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) +
6*a**2*f) + 126*d**4*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18
*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan
(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/
2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 196*d**4*tan(e/2 + f*x/2)*
*4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*
f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 +
 f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6
*a**2*f) + 252*d**4*tan(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*
a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(
e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2
)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 194*d**4*tan(e/2 + f*x/2)**
2/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f
*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 +
f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*
a**2*f) + 150*d**4*tan(e/2 + f*x/2)/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2
*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2
+ f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2
 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 64*d**4/(6*a**2*f*tan(e/2 + f*x
/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42
*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan
(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f), Ne(f, 0)), (x*(c
 + d*sin(e))**4/(a*sin(e) + a)**2, True))
```

3.463 $\int \frac{(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=120

$$\frac{d^2(c-4d)\cos(e+fx)}{3a^2f}+\frac{d^2x(3c-2d)}{a^2}-\frac{(c+6d)(c-d)^2\cos(e+fx)}{3a^2f(\sin(e+fx)+1)}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a\sin(e+fx)+a)^2}$$

```
[Out] (3*c-2*d)*d^2*x/a^2+1/3*(c-4*d)*d^2*cos(f*x+e)/a^2/f-1/3*(c-d)^2*(c+6*d)*co
s(f*x+e)/a^2/f/(1+sin(f*x+e))-1/3*(c-d)*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+
a*sin(f*x+e))^2
```

Rubi [A] time = 0.37, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2765, 2968, 3023, 2735, 2648}

$$\frac{d^2(c-4d)\cos(e+fx)}{3a^2f}+\frac{d^2x(3c-2d)}{a^2}-\frac{(c+6d)(c-d)^2\cos(e+fx)}{3a^2f(\sin(e+fx)+1)}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((3*c - 2*d)*d^2*x)/a^2 + ((c - 4*d)*d^2*Cos[e + f*x])/(3*a^2*f) - ((c - d)
^2*(c + 6*d)*Cos[e + f*x])/(3*a^2*f*(1 + Sin[e + f*x])) - ((c - d)*Cos[e +
f*x]*(c + d*Sin[e + f*x])^2)/(3*f*(a + a*Sin[e + f*x])^2)
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
```

```
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a+a\sin(e+fx))^2} - \frac{\int \frac{(c+d\sin(e+fx))\left(-a\left(c^2+4cd-2d^2\right)+a(c-4d)d\sin(e+}{a+a\sin(e+fx)}}{3a^2}$$

$$= -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a+a\sin(e+fx))^2} - \frac{\int \frac{-ac\left(c^2+4cd-2d^2\right)+\left(ac(c-4d)d-ad\left(c^2+4cd-2d^2\right)\right)\sin}{a+a\sin(e+fx)}}{3a^2}$$

$$= \frac{(c-4d)d^2\cos(e+fx)}{3a^2f} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a+a\sin(e+fx))^2} - \frac{\int \frac{-a^2c\left(c^2+4cd-2d^2\right)-}{a+a\,s}}{}$$

$$= \frac{(3c-2d)d^2x}{a^2} + \frac{(c-4d)d^2\cos(e+fx)}{3a^2f} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a+a\sin(e+fx))^2} + \frac{(}{}$$

$$= \frac{(3c-2d)d^2x}{a^2} + \frac{(c-4d)d^2\cos(e+fx)}{3a^2f} - \frac{(c-d)^2(c+6d)\cos(e+fx)}{3f\left(a^2+a^2\sin(e+fx)\right)} - \frac{(c-d)\cos(e}{3f(a}$$

Mathematica [A] time = 0.35, size = 212, normalized size = 1.77

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(3d^2(3c-2d)(e+fx)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3+2(c-d)^3\sin\left(\frac{1}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^2,x]

[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(c - d)^3*Sin[(e + f*x)/2] - (c - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(c - d)^2*(c + 8*d)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 3*(3*c - 2*d)*d^2*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 3*d^3*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3))/(3*a^2*f*(1 + Sin[e + f*x])^2)

fricas [B] time = 0.45, size = 308, normalized size = 2.57

$$-\frac{3\,d^3\cos\left(fx+e\right)^3 - c^3 + 3\,c^2d - 3\,cd^2 + d^3 + 6\left(3\,cd^2 - 2\,d^3\right)fx - \left(c^3 + 6\,c^2d - 15\,cd^2 + 11\,d^3 + 3\left(3\,cd^2 - 2\,d^3\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorithm="fricas")

[Out] -1/3*(3*d^3*cos(f*x + e)^3 - c^3 + 3*c^2*d - 3*c*d^2 + d^3 + 6*(3*c*d^2 - 2*d^3)*f*x - (c^3 + 6*c^2*d - 15*c*d^2 + 11*d^3 + 3*(3*c*d^2 - 2*d^3)*f*x)*cos(f*x + e)^2 - (2*c^3 + 3*c^2*d - 12*c*d^2 + 13*d^3 - 3*(3*c*d^2 - 2*d^3)*f*x)*cos(f*x + e) - (3*d^3*cos(f*x + e)^2 - c^3 + 3*c^2*d - 3*c*d^2 + d^3 - 6*(3*c*d^2 - 2*d^3)*f*x + (c^3 + 6*c^2*d - 15*c*d^2 + 14*d^3 - 3*(3*c*d^2 - 2*d^3)*f*x)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))

giac [A] time = 0.95, size = 209, normalized size = 1.74

$$\frac{\frac{3\left(3\,cd^2-2\,d^3\right)\left(fx+e\right)}{a^2} - \frac{6\,d^3}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)a^2} - \frac{2\left(3\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 - 9\,cd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 + 6\,d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 + 3\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) + 9\,c^2d\,\mathrm{t}\cdots\right.}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\cdots\right.\right.}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorithm="giac")

[Out] 1/3*(3*(3*c*d^2 - 2*d^3)*(f*x + e)/a^2 - 6*d^3/((tan(1/2*f*x + 1/2*e)^2 + 1)*a^2) - 2*(3*c^3*tan(1/2*f*x + 1/2*e)^2 - 9*c*d^2*tan(1/2*f*x + 1/2*e)^2 + 6*d^3*tan(1/2*f*x + 1/2*e)^2 + 3*c^3*tan(1/2*f*x + 1/2*e) + 9*c^2*d*tan(1/2*f*x + 1/2*e) - 27*c*d^2*tan(1/2*f*x + 1/2*e) + 15*d^3*tan(1/2*f*x + 1/2*e) + 2*c^3 + 3*c^2*d - 12*c*d^2 + 7*d^3)/(a^2*(tan(1/2*f*x + 1/2*e) + 1)^3))/f

maple [B] time = 0.27, size = 340, normalized size = 2.83

$$-\frac{2d^3}{a^2 f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}+\frac{6d^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c}{a^2 f}-\frac{4d^3\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a^2 f}-\frac{2c^3}{a^2 f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x)
```

```
[Out] -2/a^2/f*d^3/(1+tan(1/2*f*x+1/2*e)^2)+6/a^2/f*d^2*arctan(tan(1/2*f*x+1/2*e)
)*c-4/a^2/f*d^3*arctan(tan(1/2*f*x+1/2*e))-2/a^2/f/(tan(1/2*f*x+1/2*e)+1)*c
^3+6/a^2/f/(tan(1/2*f*x+1/2*e)+1)*c*d^2-4/a^2/f/(tan(1/2*f*x+1/2*e)+1)*d^3+
2/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*c^3-6/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*c^2*d
+6/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*c*d^2-2/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*d^
3-4/3/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*c^3+4/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*c
^2*d-4/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*c*d^2+4/3/a^2/f/(tan(1/2*f*x+1/2*e)+1
)^3*d^3
```

maxima [B] time = 0.44, size = 591, normalized size = 4.92

$$-\frac{2\left(2\,d^3\left(\frac{\frac{12\,\sin(fx+e)}{\cos(fx+e)+1}+\frac{11\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{9\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\,\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+5}{a^2+\frac{3\,a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{4\,a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{4\,a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\,a^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^2\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3\,\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)-3\,cd^2\left(\frac{\frac{9\,\sin(fx+e)}{\cos(fx+e)+1}\cdots}{a^2+\frac{3\,a^2\sin(fx+e)}{\cos(fx+e)+1}\cdots}\cdots\right)\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(2*d^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*a
^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 3*c*
d^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arct
an(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + c^3*(3*sin(f*x + e)/(cos(f*x + e
) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x +
e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*c^2*d*(3*sin(f*x + e)/(cos(f*x + e) +
```

```
1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

mupad [**B**] time = 8.39, size = 298, normalized size = 2.48

$$\frac{2\,d^2\,\mathrm{atan}\left(\frac{2\,d^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(3\,c-2\,d)}{6\,c\,d^2-4\,d^3}\right)\,(3\,c-2\,d)}{a^2\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(2\,c^3+6\,c^2\,d-18\,c\,d^2+12\,d^3\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{10\,c}{3}\right.}{f\,\left(a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^3/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (2*d^2*atan((2*d^2*tan(e/2 + (f*x)/2)*(3*c - 2*d))/(6*c*d^2 - 4*d^3))*(3*c
- 2*d))/(a^2*f) - (tan(e/2 + (f*x)/2)^3*(6*c^2*d - 18*c*d^2 + 2*c^3 + 12*d^
3) + tan(e/2 + (f*x)/2)^2*(2*c^2*d - 14*c*d^2 + (10*c^3)/3 + (44*d^3)/3) -
8*c*d^2 + 2*c^2*d + tan(e/2 + (f*x)/2)^4*(2*c^3 - 6*c*d^2 + 4*d^3) + (4*c^3
)/3 + (20*d^3)/3 + tan(e/2 + (f*x)/2)*(6*c^2*d - 18*c*d^2 + 2*c^3 + 16*d^3)
)/(f*(4*a^2*tan(e/2 + (f*x)/2)^2 + 4*a^2*tan(e/2 + (f*x)/2)^3 + 3*a^2*tan(e
/2 + (f*x)/2)^4 + a^2*tan(e/2 + (f*x)/2)^5 + a^2 + 3*a^2*tan(e/2 + (f*x)/2)
))
```

sympy [**A**] time = 14.92, size = 3585, normalized size = 29.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*c**3*tan(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a*
*2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/
2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*c**3*tan(e/2 + f*
x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a
**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/
2 + f*x/2) + 3*a**2*f) - 10*c**3*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*
x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12
*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*c**
3*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2
)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a*
*2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*c**3/(3*a**2*f*tan(e/2 + f*x/2)**5 +
9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*ta
n(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 18*c**2*d*tan(e
/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4
```

```
 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f
*tan(e/2 + f*x/2) + 3*a**2*f) - 6*c**2*d*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(
e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)
**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f)
 - 18*c**2*d*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(
e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2
)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*c**2*d/(3*a**2*f*tan(e/2 +
 f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 +
 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*
c*d**2*f*x*tan(e/2 + f*x/2)**5/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan
(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/
2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 27*c*d**2*f*x*tan(e/2 + f*x
/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a*
*2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2
 + f*x/2) + 3*a**2*f) + 36*c*d**2*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2
 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3
 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) +
36*c*d**2*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*
tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f
*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 27*c*d**2*f*x*tan(e/2 +
f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a*
*2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2
 + f*x/2) + 3*a**2*f) + 9*c*d**2*f*x/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2
*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2
+ f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 18*c*d**2*tan(e/2 + f
*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*
a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e
/2 + f*x/2) + 3*a**2*f) + 54*c*d**2*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 +
 f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 +
 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 42
*c*d**2*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/
2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)*
*2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 54*c*d**2*tan(e/2 + f*x/2)/(3*
a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e
/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2)
+ 3*a**2*f) + 24*c*d**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 +
f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 +
 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*d**3*f*x*tan(e/2 + f*x/2)**5/(3*
a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e
/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2)
+ 3*a**2*f) - 18*d**3*f*x*tan(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5
 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f
*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 24*d**3*f*x*
tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/
2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a
```

```
**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 24*d**3*f*x*tan(e/2 + f*x/2)**2/(3*a**
2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2
+ f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3
*a**2*f) - 18*d**3*f*x*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a
**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e
/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*d**3*f*x/(3*a**2
*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 +
 f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*
a**2*f) - 12*d**3*tan(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**
2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2
 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 36*d**3*tan(e/2 + f*
x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a
**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/
2 + f*x/2) + 3*a**2*f) - 44*d**3*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*
x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12
*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 48*d*
*3*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/
2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a
**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 20*d**3/(3*a**2*f*tan(e/2 + f*x/2)**5
+ 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*
tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f), Ne(f, 0)), (x*
(c + d*sin(e))**3/(a*sin(e) + a)**2, True))
```

3.464 $\int \frac{(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=85

$$-\frac{(c-d)(c+4d)\cos(e+fx)}{3a^2f(\sin(e+fx)+1)}+\frac{d^2x}{a^2}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{3f(a\sin(e+fx)+a)^2}$$

```
[Out] d^2*x/a^2-1/3*(c-d)*(c+4*d)*cos(f*x+e)/a^2/f/(1+sin(f*x+e))-1/3*(c-d)*cos(f
*x+e)*(c+d*sin(f*x+e))/f/(a+a*sin(f*x+e))^2
```

Rubi [A] time = 0.14, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2760, 2735, 2648}

$$-\frac{(c-d)(c+4d)\cos(e+fx)}{3a^2f(\sin(e+fx)+1)}+\frac{d^2x}{a^2}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (d^2*x)/a^2 - ((c - d)*(c + 4*d)*Cos[e + f*x])/(3*a^2*f*(1 + Sin[e + f*x]))
 - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x]))/(3*f*(a + a*Sin[e + f*x])^2)
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2760

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^m*(c + d*Sin[e + f*x]))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*c*d*(m - 1) + b*(d^2 + c^2*(m + 1))
 + d*(a*d*(m - 1) + b*c*(m + 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{3f(a+a\sin(e+fx))^2} - \frac{\int \frac{-a\left(c^2+3cd-d^2\right)-3ad^2\sin(e+fx)}{a+a\sin(e+fx)}\,dx}{3a^2}$$

$$= \frac{d^2x}{a^2} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{3f(a+a\sin(e+fx))^2} + \frac{((c-d)(c+4d))\int \frac{1}{a+a\sin(e+fx)}\,dx}{3a}$$

$$= \frac{d^2x}{a^2} - \frac{(c-d)(c+4d)\cos(e+fx)}{3f\left(a^2+a^2\sin(e+fx)\right)} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{3f(a+a\sin(e+fx))^2}$$

Mathematica [B] time = 0.28, size = 172, normalized size = 2.02

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2\left(c^2+4cd-5d^2\right)\sin\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2+\right.}{3a^2f(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(c - d)^2*Sin[(e + f*x)/2] - (c -
 d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(c^2 + 4*c*d - 5*d^2)*Sin[(
e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 3*d^2*(e + f*x)*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])^3))/(3*a^2*f*(1 + Sin[e + f*x])^2)
```

fricas [B] time = 0.45, size = 197, normalized size = 2.32

$$-\frac{6\,d^2fx-\left(3\,d^2fx+c^2+4\,cd-5\,d^2\right)\cos\left(fx+e\right)^2-c^2+2\,cd-d^2+\left(3\,d^2fx-2\,c^2-2\,cd+4\,d^2\right)\cos\left(fx+e\right)}{3\left(a^2f\cos\left(fx+e\right)^2-a^2f\cos\left(fx+e\right)-2\,a^2f-\left(a^2f\cos\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(6*d^2*f*x - (3*d^2*f*x + c^2 + 4*c*d - 5*d^2)*cos(f*x + e)^2 - c^2 +
2*c*d - d^2 + (3*d^2*f*x - 2*c^2 - 2*c*d + 4*d^2)*cos(f*x + e) + (6*d^2*f*x
 + c^2 - 2*c*d + d^2 + (3*d^2*f*x - c^2 - 4*c*d + 5*d^2)*cos(f*x + e))*sin(
f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos
(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

giac [A] time = 0.17, size = 132, normalized size = 1.55

$$\frac{\frac{3\left(fx+e\right)d^2}{a^2} - \frac{2\left(3\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+6\,cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-9\,d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,c^2+2\,cd-4\,d^2\right)}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(f*x + e)*d^2/a^2 - 2*(3*c^2*tan(1/2*f*x + 1/2*e)^2 - 3*d^2*tan(1/2*
f*x + 1/2*e)^2 + 3*c^2*tan(1/2*f*x + 1/2*e) + 6*c*d*tan(1/2*f*x + 1/2*e) -
9*d^2*tan(1/2*f*x + 1/2*e) + 2*c^2 + 2*c*d - 4*d^2)/(a^2*(tan(1/2*f*x + 1/2
*e) + 1)^3))/f
```

maple [B] time = 0.25, size = 213, normalized size = 2.51

$$\frac{2d^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a^2f} - \frac{2c^2}{a^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)} + \frac{2d^2}{a^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)} + \frac{2c^2}{a^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2} - \frac{\quad}{a^2f\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 2/a^2/f*d^2*arctan(tan(1/2*f*x+1/2*e))-2/a^2/f/(tan(1/2*f*x+1/2*e)+1)*c^2+2
/a^2/f/(tan(1/2*f*x+1/2*e)+1)*d^2+2/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*c^2-4/a^
2/f/(tan(1/2*f*x+1/2*e)+1)^2*c*d+2/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*d^2-4/3/a
^2/f/(tan(1/2*f*x+1/2*e)+1)^3*c^2+8/3/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*c*d-4/
3/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*d^2
```

maxima [B] time = 0.43, size = 360, normalized size = 4.24

$$\frac{2\left(d^2\left(\frac{\frac{9\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\,\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+4}{a^2+\frac{3\,a^2\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\,a^2\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{a^2\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}} + \frac{3\,\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{a^2}\right) - \frac{c^2\left(\frac{3\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\,\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+2\right)}{a^2+\frac{3\,a^2\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\,a^2\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{a^2\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}} - \frac{\quad}{a^2+\frac{3\,a^2}{\cos}}\right.}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(d^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x
```

```
+ e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3
*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - c^2*(3*sin(f*x + e)/(cos(f*
x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^
2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - 2*c*d*(3*sin(f*x + e)/(cos(f*x + e
) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

mupad [B] time = 7.43, size = 93, normalized size = 1.09

$$\frac{d^2 x}{a^2}-\frac{\frac{4\,c\,d}{3}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(2\,c^2-2\,d^2\right)+\frac{4\,c^2}{3}-\frac{8\,d^2}{3}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,c^2+4\,c\,d-6\,d^2\right)}{a^2\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^2/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (d^2*x)/a^2 - ((4*c*d)/3 + tan(e/2 + (f*x)/2)^2*(2*c^2 - 2*d^2) + (4*c^2)/3
 - (8*d^2)/3 + tan(e/2 + (f*x)/2)*(4*c*d + 2*c^2 - 6*d^2))/(a^2*f*(tan(e/2
+ (f*x)/2) + 1)^3)
```

sympy [A] time = 8.06, size = 915, normalized size = 10.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*c**2*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a*
*2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*c**2*t
an(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**
2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*c**2/(3*a**2*f*tan(e/2 + f*x/
2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f
) - 12*c*d*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/
2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*c*d/(3*a**2*f*tan
(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2)
 + 3*a**2*f) + 3*d**2*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**3
 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9
*d**2*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(
e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*d**2*f*x*tan(e/
2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9
*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 3*d**2*f*x/(3*a**2*f*tan(e/2 + f*x/2
)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f)
```

```
 + 6*d**2*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(
e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 18*d**2*tan(e/2 +
 f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a*
*2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 8*d**2/(3*a**2*f*tan(e/2 + f*x/2)**3 +
9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f), Ne(f,
 0)), (x*(c + d*sin(e))**2/(a*sin(e) + a)**2, True))
```

3.465 $\int \frac{c+d \sin(e+fx)}{(a+a\sin(e+fx))^2} \, dx$

Optimal. Leaf size=65

$$-\frac{(c+2d)\cos(e+fx)}{3f\left(a^2\sin(e+fx)+a^2\right)}-\frac{(c-d)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

```
[Out] -1/3*(c-d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^2-1/3*(c+2*d)*cos(f*x+e)/f/(a^2+a^
2*sin(f*x+e))
```

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2750, 2648}

$$-\frac{(c+2d)\cos(e+fx)}{3f\left(a^2\sin(e+fx)+a^2\right)}-\frac{(c-d)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -((c - d)*Cos[e + f*x])/(3*f*(a + a*Sin[e + f*x])^2) - ((c + 2*d)*Cos[e + f
*x])/(3*f*(a^2 + a^2*Sin[e + f*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{c+d\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx = -\frac{(c-d)\cos(e+fx)}{3f(a+a\sin(e+fx))^2} + \frac{(c+2d)\int \frac{1}{a+a\sin(e+fx)}\,dx}{3a}$$
$$= -\frac{(c-d)\cos(e+fx)}{3f(a+a\sin(e+fx))^2} - \frac{(c+2d)\cos(e+fx)}{3f\left(a^2+a^2\sin(e+fx)\right)}$$

Mathematica [**A**] time = 0.06, size = 43, normalized size = 0.66

$$-\frac{\cos(e+fx)((c+2d)\sin(e+fx)+2c+d)}{3a^2f(\sin(e+fx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -1/3*(Cos[e + f*x]*(2*c + d + (c + 2*d)*Sin[e + f*x]))/(a^2*f*(1 + Sin[e +
f*x])^2)
```

fricas [**A**] time = 0.41, size = 117, normalized size = 1.80

$$\frac{(c+2d)\cos\left(fx+e\right)^2+(2c+d)\cos\left(fx+e\right)+\left((c+2d)\cos\left(fx+e\right)-c+d\right)\sin\left(fx+e\right)+c-d}{3\left(a^2f\cos\left(fx+e\right)^2-a^2f\cos\left(fx+e\right)-2a^2f-\left(a^2f\cos\left(fx+e\right)+2a^2f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*((c + 2*d)*cos(f*x + e)^2 + (2*c + d)*cos(f*x + e) + ((c + 2*d)*cos(f*x
 + e) - c + d)*sin(f*x + e) + c - d)/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x
+ e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

giac [**A**] time = 0.22, size = 68, normalized size = 1.05

$$-\frac{2\left(3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2c+d\right)}{3a^2f\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*c*tan(1/2*f*x + 1/2*e)^2 + 3*c*tan(1/2*f*x + 1/2*e) + 3*d*tan(1/2*f
*x + 1/2*e) + 2*c + d)/(a^2*f*(tan(1/2*f*x + 1/2*e) + 1)^3)
```

maple [A] time = 0.22, size = 70, normalized size = 1.08

$$-\frac{\frac{-2c+2d}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2(2c-2d)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2c}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{f\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 2/f/a^2*(-1/2*(-2*c+2*d)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(2*c-2*d)/(tan(1/2*f*
x+1/2*e)+1)^3-c/(tan(1/2*f*x+1/2*e)+1))
```

maxima [B] time = 0.33, size = 214, normalized size = 3.29

$$-\frac{2\left(\frac{c\left(\frac{3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+2\right)}{a^{2}+\frac{3a^{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3a^{2}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{a^{2}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}}+\frac{d\left(\frac{3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{a^{2}+\frac{3a^{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3a^{2}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{a^{2}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(c*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + d*(
3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x +
 e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3))/f
```

mupad [B] time = 7.21, size = 97, normalized size = 1.49

$$-\frac{2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(\frac{5\,c}{2}+\frac{d}{2}-\frac{c\,\cos\left(e+f\,x\right)}{2}+\frac{d\,\cos\left(e+f\,x\right)}{2}+\frac{3\,c\,\sin\left(e+f\,x\right)}{2}+\frac{3\,d\,\sin\left(e+f\,x\right)}{2}\right)}{3\,a^2\,f\,\left(\frac{3\,\sqrt{2}\,\cos\left(\frac{e}{2}-\frac{\pi}{4}+\frac{f\,x}{2}\right)}{2}-\frac{\sqrt{2}\,\cos\left(\frac{3\,e}{2}+\frac{\pi}{4}+\frac{3\,f\,x}{2}\right)}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))/(a + a*sin(e + f*x))^2,x)
```

```
[Out] -(2*cos(e/2 + (f*x)/2)*((5*c)/2 + d/2 - (c*cos(e + f*x))/2 + (d*cos(e + f*x
))/2 + (3*c*sin(e + f*x))/2 + (3*d*sin(e + f*x))/2))/(3*a^2*f*((3*2^(1/2)*c
os(e/2 - pi/4 + (f*x)/2))/2 - (2^(1/2)*cos((3*e)/2 + pi/4 + (3*f*x)/2))/2))
```

sympy [A] time = 3.65, size = 372, normalized size = 5.72

$$\begin{cases} -\frac{6c\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} - \frac{6c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} - \frac{}{3a^2f\tan^3(} \\ \frac{x(c+d\sin(e))}{(a\sin(e)+a)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*c*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*
f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*c*tan(e/2
 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*
a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*c/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9
*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*d*t
an(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**
2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*d/(3*a**2*f*tan(e/2 + f*x/2)*
*3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f),
Ne(f, 0)), (x*(c + d*sin(e))/(a*sin(e) + a)**2, True))
```

3.466 $\int \frac{1}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=55

$$-\frac{\cos(e+fx)}{3f\left(a^2\sin(e+fx)+a^2\right)}-\frac{\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

```
[Out] -1/3*cos(f*x+e)/f/(a+a*sin(f*x+e))^2-1/3*cos(f*x+e)/f/(a^2+a^2*sin(f*x+e))
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2650, 2648}

$$-\frac{\cos(e+fx)}{3f\left(a^2\sin(e+fx)+a^2\right)}-\frac{\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(-2),x]
```

```
[Out] -Cos[e + f*x]/(3*f*(a + a*Sin[e + f*x])^2) - Cos[e + f*x]/(3*f*(a^2 + a^2*S
in[e + f*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\sin(e+fx))^2}\,dx &= -\frac{\cos(e+fx)}{3f(a+a\sin(e+fx))^2}+\frac{\int \frac{1}{a+a\sin(e+fx)}\,dx}{3a}\\ &= -\frac{\cos(e+fx)}{3f(a+a\sin(e+fx))^2}-\frac{\cos(e+fx)}{3f\left(a^2+a^2\sin(e+fx)\right)}\end{aligned}$$

Mathematica [A] time = 0.10, size = 54, normalized size = 0.98

$$-\frac{-4\sin(e+fx)+\sin(2(e+fx))+4\cos(e+fx)+\cos(2(e+fx))-3}{6a^2f(\sin(e+fx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(-2),x]
```

```
[Out] -1/6*(-3 + 4*Cos[e + f*x] + Cos[2*(e + f*x)] - 4*Sin[e + f*x] + Sin[2*(e +
f*x)])/(a^2*f*(1 + Sin[e + f*x])^2)
```

fricas [A] time = 0.44, size = 95, normalized size = 1.73

$$\frac{\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)+2\cos\left(fx+e\right)+1}{3\left(a^2f\cos\left(fx+e\right)^2-a^2f\cos\left(fx+e\right)-2a^2f-\left(a^2f\cos\left(fx+e\right)+2a^2f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(f*x + e)^2 + (cos(f*x + e) - 1)*sin(f*x + e) + 2*cos(f*x + e) + 1)
/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e)
 + 2*a^2*f)*sin(f*x + e))
```

giac [A] time = 0.16, size = 50, normalized size = 0.91

$$-\frac{2\left(3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2\right)}{3a^2f\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*tan(1/2*f*x + 1/2*e)^2 + 3*tan(1/2*f*x + 1/2*e) + 2)/(a^2*f*(tan(1/
2*f*x + 1/2*e) + 1)^3)
```

maple [A] time = 0.14, size = 53, normalized size = 0.96

$$\frac{\frac{2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{4}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{f\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 2/f/a^2*(1/(tan(1/2*f*x+1/2*e)+1)^2-2/3/(tan(1/2*f*x+1/2*e)+1)^3-1/(tan(1/2
*f*x+1/2*e)+1))
```

maxima [B] time = 0.32, size = 117, normalized size = 2.13

$$-\frac{2\left(\frac{3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+2\right)}{3\left(a^{2}+\frac{3a^{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3a^{2}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{a^{2}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 2)/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*f)
```

mupad [B] time = 6.98, size = 76, normalized size = 1.38

$$-\frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)-\frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}-3\right)}{3}}{a^{2}f\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sin(e + f*x))^2,x)
```

```
[Out] -(2*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2) - (2*cos(e/2 + (f*x)/2)*(cos(e/
2 + (f*x)/2)^2 - 3))/3)/(a^2*f*(cos(e/2 + (f*x)/2) + sin(e/2 + (f*x)/2))^3)
```

sympy [A] time = 1.80, size = 221, normalized size = 4.02

$$\begin{cases}-\frac{6\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^{2}f\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^{2}f}-\frac{6\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^{2}f\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^{2}f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^{2}f}-\frac{}{3a^{2}f\tan} \\ \frac{x}{\left(a\sin\left(e\right)+a\right)^{2}}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*
tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*tan(e/2 + f
*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2
*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*
f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f), Ne(f, 0)), (
x/(a*sin(e) + a)**2, True))
```

3.467 $\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=131

$$\frac{2d^2\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^2\sqrt{c^2-d^2}}-\frac{(c-4d)\cos(e+fx)}{3a^2f(c-d)^2(\sin(e+fx)+1)}-\frac{\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}$$

```
[Out] -1/3*(c-4*d)*cos(f*x+e)/a^2/(c-d)^2/f/(1+sin(f*x+e))-1/3*cos(f*x+e)/(c-d)/f
/(a+a*sin(f*x+e))^2+2*d^2*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/
a^2/(c-d)^2/f/(c^2-d^2)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2766, 2978, 12, 2660, 618, 204}

$$\frac{2d^2\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^2\sqrt{c^2-d^2}}-\frac{(c-4d)\cos(e+fx)}{3a^2f(c-d)^2(\sin(e+fx)+1)}-\frac{\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])),x]
```

```
[Out] (2*d^2*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^2*(c - d)^2*Sqr
t[c^2 - d^2]*f) - ((c - 4*d)*Cos[e + f*x])/(3*a^2*(c - d)^2*f*(1 + Sin[e +
f*x])) - Cos[e + f*x]/(3*(c - d)*f*(a + a*Sin[e + f*x])^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx = -\frac{\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2} - \frac{\int \frac{-a(c-3d)-ad\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx}{3a^2(c-d)}$$

$$= -\frac{(c-4d)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2}$$

$$= -\frac{(c-4d)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2}$$

$$= -\frac{(c-4d)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2}$$

$$= -\frac{(c-4d)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2}$$

$$= \frac{2d^2\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^2\sqrt{c^2-d^2}\,f} - \frac{(c-4d)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - 3($$

Mathematica [A] time = 0.35, size = 204, normalized size = 1.56

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\frac{6d^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + 2(c-d)\sin\left(\frac{1}{2}(e+fx)\right) + \right.}{3a^2f(c-d)^2(\sin}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(c - d)*Sin[(e + f*x)/2] - (c - d
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(c - 4*d)*Sin[(e + f*x)/2]*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (6*d^2*ArcTan[(d + c*Tan[(e + f*x)/2]
)/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/Sqrt[c^2 - d^2]
))/(3*a^2*(c - d)^2*f*(1 + Sin[e + f*x])^2)
```

fricas [B] time = 0.48, size = 989, normalized size = 7.55

$$\left[\frac{2\,c^{3}-2\,c^{2}d-2\,cd^{2}+2\,d^{3}+2\left(c^{3}-4\,c^{2}d-cd^{2}+4\,d^{3}\right)\cos\left(fx+e\right)^{2}-3\left(d^{2}\cos\left(fx+e\right)^{2}-d^{2}\cos\left(fx+e\right)-2\,\right.}{6\left(\left(a^{2}c^{4}-2\,a^{2}c^{3}d+2\,a^{2}cd^{3}-a^{2}d^{4}\right)f\cos\left(fx+e\right)^{2}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/6*(2*c^3 - 2*c^2*d - 2*c*d^2 + 2*d^3 + 2*(c^3 - 4*c^2*d - c*d^2 + 4*d^3)
*cos(f*x + e)^2 - 3*(d^2*cos(f*x + e)^2 - d^2*cos(f*x + e) - 2*d^2 - (d^2*c
os(f*x + e) + 2*d^2)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(
f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e
) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x +
 e) - c^2 - d^2)) + 2*(2*c^3 - 5*c^2*d - 2*c*d^2 + 5*d^3)*cos(f*x + e) - 2*
(c^3 - c^2*d - c*d^2 + d^3 - (c^3 - 4*c^2*d - c*d^2 + 4*d^3)*cos(f*x + e))*
sin(f*x + e))/((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x +
e)^2 - (a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e) - 2*(
a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f - ((a^2*c^4 - 2*a^2*c^3*d
+ 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e) + 2*(a^2*c^4 - 2*a^2*c^3*d + 2*a^2*
c*d^3 - a^2*d^4)*f)*sin(f*x + e)), 1/3*(c^3 - c^2*d - c*d^2 + d^3 + (c^3 -
4*c^2*d - c*d^2 + 4*d^3)*cos(f*x + e)^2 - 3*(d^2*cos(f*x + e)^2 - d^2*cos(f
*x + e) - 2*d^2 - (d^2*cos(f*x + e) + 2*d^2)*sin(f*x + e))*sqrt(c^2 - d^2)*
arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (2*c^3 - 5*c
^2*d - 2*c*d^2 + 5*d^3)*cos(f*x + e) - (c^3 - c^2*d - c*d^2 + d^3 - (c^3 -
4*c^2*d - c*d^2 + 4*d^3)*cos(f*x + e))*sin(f*x + e))/((a^2*c^4 - 2*a^2*c^3*
d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e)^2 - (a^2*c^4 - 2*a^2*c^3*d + 2*a^
2*c*d^3 - a^2*d^4)*f*cos(f*x + e) - 2*(a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3
- a^2*d^4)*f - ((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x +
 e) + 2*(a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f)*sin(f*x + e))]
```

giac [A] time = 0.19, size = 195, normalized size = 1.49

$$\frac{2\left(\frac{3\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(c\right)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^{2}-d^{2}}}\right)\right)d^{2}}{\left(a^{2}c^{2}-2\,a^{2}cd+a^{2}d^{2}\right)\sqrt{c^{2}-d^{2}}}-\frac{3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-6\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-9\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,c-5\,d}{\left(a^{2}c^{2}-2\,a^{2}cd+a^{2}d^{2}\right)\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{3}}\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 2/3*(3*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1
/2*e) + d)/sqrt(c^2 - d^2)))*d^2/((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*sqrt(c^2
- d^2)) - (3*c*tan(1/2*f*x + 1/2*e)^2 - 6*d*tan(1/2*f*x + 1/2*e)^2 + 3*c*ta
n(1/2*f*x + 1/2*e) - 9*d*tan(1/2*f*x + 1/2*e) + 2*c - 5*d)/((a^2*c^2 - 2*a^
2*c*d + a^2*d^2)*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

maple [A] time = 0.31, size = 175, normalized size = 1.34

$$\frac{2d^2 \arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{a^2 f (c-d)^2 \sqrt{c^2-d^2}} - \frac{2c}{a^2 f (c-d)^2 \left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)} + \frac{4d}{a^2 f (c-d)^2 \left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)} - \frac{4}{3a^2 f (c-d) \left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x)
```

```
[Out] 2/a^2/f*d^2/(c-d)^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)
/(c^2-d^2)^(1/2))-2/a^2/f/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)*c+4/a^2/f/(c-d)^2/
(tan(1/2*f*x+1/2*e)+1)*d-4/3/a^2/f/(c-d)/(tan(1/2*f*x+1/2*e)+1)^3+2/a^2/f/(
c-d)/(tan(1/2*f*x+1/2*e)+1)^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 8.02, size = 250, normalized size = 1.91

$$\frac{2\,d^2\,\mathrm{atan}\left(\frac{\frac{d^2\,\left(2\,a^2\,c^2\,d-4\,a^2\,c\,d^2+2\,a^2\,d^3\right)}{a^2\,\sqrt{c+d}\,{\left(c-d\right)}^{5/2}}+\frac{2\,c\,d^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(a^2\,c^2-2\,a^2\,c\,d+a^2\,d^2\right)}{a^2\,\sqrt{c+d}\,{\left(c-d\right)}^{5/2}}}{2\,d^2}\right)}{a^2\,f\,\sqrt{c+d}\,{\left(c-d\right)}^{5/2}} - \frac{\frac{2\,\left(2\,c-5\,d\right)}{3\,{\left(c-d\right)}^2}+\frac{2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(c-3\,d\right)}{{\left(c-d\right)}^2}+\frac{2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right.}{\left(c-\right.}}{f\,\left(a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+3\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+3\,a^2\,\mathrm{t}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))),x)
```

```
[Out] (2*d^2*atan(((d^2*(2*a^2*d^3 - 4*a^2*c*d^2 + 2*a^2*c^2*d))/(a^2*(c + d)^(1/
2)*(c - d)^(5/2)) + (2*c*d^2*tan(e/2 + (f*x)/2)*(a^2*c^2 + a^2*d^2 - 2*a^2*
c*d))/(a^2*(c + d)^(1/2)*(c - d)^(5/2)))/(2*d^2)))/(a^2*f*(c + d)^(1/2)*(c
- d)^(5/2)) - ((2*(2*c - 5*d))/(3*(c - d)^2) + (2*tan(e/2 + (f*x)/2)*(c - 3
*d))/(c - d)^2 + (2*tan(e/2 + (f*x)/2)^2*(c - 2*d))/(c - d)^2)/(f*(3*a^2*ta
n(e/2 + (f*x)/2)^2 + a^2*tan(e/2 + (f*x)/2)^3 + a^2 + 3*a^2*tan(e/2 + (f*x)
/2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.468 $\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=221

$$\frac{2d^2(3c+2d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^3(c+d)\sqrt{c^2-d^2}}-\frac{d\left(c^2-6cd-10d^2\right)\cos(e+fx)}{3a^2f(c-d)^3(c+d)(c+d\sin(e+fx))}-\frac{(c-6d)\cos(e+fx)}{3a^2f(c-d)^2(\sin(e+fx)+1)(c+}$$

```
[Out] -1/3*d*(c^2-6*c*d-10*d^2)*cos(f*x+e)/a^2/(c-d)^3/(c+d)/f/(c+d*sin(f*x+e))-1
/3*(c-6*d)*cos(f*x+e)/a^2/(c-d)^2/f/(1+sin(f*x+e))/(c+d*sin(f*x+e))-1/3*cos
(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))+2*d^2*(3*c+2*d)*arctan(
(d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^2/(c-d)^3/(c+d)/f/(c^2-d^2)^(1/
2)
```

Rubi [A] time = 0.42, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2766, 2978, 2754, 12, 2660, 618, 204}

$$\frac{2d^2(3c+2d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^3(c+d)\sqrt{c^2-d^2}}-\frac{d\left(c^2-6cd-10d^2\right)\cos(e+fx)}{3a^2f(c-d)^3(c+d)(c+d\sin(e+fx))}-\frac{(c-6d)\cos(e+fx)}{3a^2f(c-d)^2(\sin(e+fx)+1)(c+}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] (2*d^2*(3*c + 2*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^2*(
c - d)^3*(c + d)*Sqrt[c^2 - d^2]*f) - (d*(c^2 - 6*c*d - 10*d^2)*Cos[e + f*x
])/(3*a^2*(c - d)^3*(c + d)*f*(c + d*Sin[e + f*x])) - ((c - 6*d)*Cos[e + f*
x])/(3*a^2*(c - d)^2*f*(1 + Sin[e + f*x])*(c + d*Sin[e + f*x])) - Cos[e + f
*x]/(3*(c - d)*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx = -\frac{\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))} - \frac{\int \frac{-a(c-4}{(a+a\sin(e+}}{3}$$

$$= -\frac{(c-6d)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))} - \frac{}{3(c-d)f(}$$

$$= -\frac{d\left(c^2-6cd-10d^2\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))} - \frac{(c-6d}{3a^2(c-d)^2f(1+\sin}$$

$$= -\frac{d\left(c^2-6cd-10d^2\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))} - \frac{(c-6d}{3a^2(c-d)^2f(1+\sin}$$

$$= -\frac{d\left(c^2-6cd-10d^2\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))} - \frac{(c-6d}{3a^2(c-d)^2f(1+\sin}$$

$$= -\frac{d\left(c^2-6cd-10d^2\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))} - \frac{(c-6d}{3a^2(c-d)^2f(1+\sin}$$

$$= \frac{2d^2(3c+2d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^3(c+d)\sqrt{c^2-d^2}\,f} - \frac{d\left(c^2-6cd-10d^2\right)\mathrm{c}}{3a^2(c-d)^3(c+d)f(c+d}$$

Mathematica [A] time = 1.37, size = 267, normalized size = 1.21

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\frac{6d^2(3c+2d)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}} + \frac{3d^3\cos(e+fx)\left(\sin\left(\frac{1}{2}(e+f\right.\right.}{(c+d)(c+d\sin}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(c - d)*Sin[(e + f*x)/2] - (c - d
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(c - 7*d)*Sin[(e + f*x)/2]*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (6*d^2*(3*c + 2*d)*ArcTan[(d + c*Tan[
(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/((c
```

```
 + d)*Sqrt[c^2 - d^2]) + (3*d^3*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^3)/((c + d)*(c + d*Sin[e + f*x]))))/(3*a^2*(c - d)^3*f*(1 + Sin[e +
 f*x])^2)
```

fricas [B] time = 0.56, size = 2297, normalized size = 10.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/6*(2*c^5 - 2*c^4*d - 4*c^3*d^2 + 4*c^2*d^3 + 2*c*d^4 - 2*d^5 - 2*(c^4*d
- 6*c^3*d^2 - 11*c^2*d^3 + 6*c*d^4 + 10*d^5)*cos(f*x + e)^3 + 2*(c^5 - 5*c^
4*d - 8*c^3*d^2 + c^2*d^3 + 7*c*d^4 + 4*d^5)*cos(f*x + e)^2 - 3*(6*c^2*d^2
+ 10*c*d^3 + 4*d^4 - (3*c*d^3 + 2*d^4)*cos(f*x + e)^3 - (3*c^2*d^2 + 8*c*d^
3 + 4*d^4)*cos(f*x + e)^2 + (3*c^2*d^2 + 5*c*d^3 + 2*d^4)*cos(f*x + e) + (6
*c^2*d^2 + 10*c*d^3 + 4*d^4 - (3*c*d^3 + 2*d^4)*cos(f*x + e)^2 + (3*c^2*d^2
 + 5*c*d^3 + 2*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(-((2*c
^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 - 2*(c*cos(f*x +
e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2
*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(2*c^5 - 5*c^4*d - 16*c^3*d^2 - 8*c^2*d
^3 + 14*c*d^4 + 13*d^5)*cos(f*x + e) - 2*(c^5 - c^4*d - 2*c^3*d^2 + 2*c^2*d
^3 + c*d^4 - d^5 - (c^4*d - 6*c^3*d^2 - 11*c^2*d^3 + 6*c*d^4 + 10*d^5)*cos(
f*x + e)^2 - (c^5 - 4*c^4*d - 14*c^3*d^2 - 10*c^2*d^3 + 13*c*d^4 + 14*d^5)*
cos(f*x + e))*sin(f*x + e))/((a^2*c^6*d - 2*a^2*c^5*d^2 - a^2*c^4*d^3 + 4*a
^2*c^3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e)^3 + (a^2*c
^7 - 5*a^2*c^5*d^2 + 2*a^2*c^4*d^3 + 7*a^2*c^3*d^4 - 4*a^2*c^2*d^5 - 3*a^2*
c*d^6 + 2*a^2*d^7)*f*cos(f*x + e)^2 - (a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2
+ 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f*co
s(f*x + e) - 2*(a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2
*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f + ((a^2*c^6*d - 2*a^2*c^5
*d^2 - a^2*c^4*d^3 + 4*a^2*c^3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f
*cos(f*x + e)^2 - (a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*
a^2*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e) - 2*(a^2*
c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2
*d^5 - a^2*c*d^6 + a^2*d^7)*f)*sin(f*x + e)), 1/3*(c^5 - c^4*d - 2*c^3*d^2
+ 2*c^2*d^3 + c*d^4 - d^5 - (c^4*d - 6*c^3*d^2 - 11*c^2*d^3 + 6*c*d^4 + 10*
d^5)*cos(f*x + e)^3 + (c^5 - 5*c^4*d - 8*c^3*d^2 + c^2*d^3 + 7*c*d^4 + 4*d^
5)*cos(f*x + e)^2 + 3*(6*c^2*d^2 + 10*c*d^3 + 4*d^4 - (3*c*d^3 + 2*d^4)*cos
(f*x + e)^3 - (3*c^2*d^2 + 8*c*d^3 + 4*d^4)*cos(f*x + e)^2 + (3*c^2*d^2 + 5
*c*d^3 + 2*d^4)*cos(f*x + e) + (6*c^2*d^2 + 10*c*d^3 + 4*d^4 - (3*c*d^3 + 2
*d^4)*cos(f*x + e)^2 + (3*c^2*d^2 + 5*c*d^3 + 2*d^4)*cos(f*x + e))*sin(f*x
+ e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x
 + e))) + (2*c^5 - 5*c^4*d - 16*c^3*d^2 - 8*c^2*d^3 + 14*c*d^4 + 13*d^5)*co
s(f*x + e) - (c^5 - c^4*d - 2*c^3*d^2 + 2*c^2*d^3 + c*d^4 - d^5 - (c^4*d -
```

```
6*c^3*d^2 - 11*c^2*d^3 + 6*c*d^4 + 10*d^5)*cos(f*x + e)^2 - (c^5 - 4*c^4*d
- 14*c^3*d^2 - 10*c^2*d^3 + 13*c*d^4 + 14*d^5)*cos(f*x + e))*sin(f*x + e))/
((a^2*c^6*d - 2*a^2*c^5*d^2 - a^2*c^4*d^3 + 4*a^2*c^3*d^4 - a^2*c^2*d^5 - 2
*a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e)^3 + (a^2*c^7 - 5*a^2*c^5*d^2 + 2*a^2*c
^4*d^3 + 7*a^2*c^3*d^4 - 4*a^2*c^2*d^5 - 3*a^2*c*d^6 + 2*a^2*d^7)*f*cos(f*x
 + e)^2 - (a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*
d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e) - 2*(a^2*c^7 - a^
2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5 - a
^2*c*d^6 + a^2*d^7)*f + ((a^2*c^6*d - 2*a^2*c^5*d^2 - a^2*c^4*d^3 + 4*a^2*c
^3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e)^2 - (a^2*c^7 -
 a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5
- a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e) - 2*(a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*
d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*
f)*sin(f*x + e))]
```

giac [A] time = 0.21, size = 320, normalized size = 1.45

$$\frac{2\left(\frac{3\left(3\,cd^2+2\,d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(a^2c^4-2\,a^2c^3d+2\,a^2cd^3-a^2d^4\right)\sqrt{c^2-d^2}}+\frac{3\left(d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+cd^3\right)}{\left(a^2c^5-2\,a^2c^4d+2\,a^2c^2d^3-a^2cd^4\right)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+c\right)}-\cdots\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 2/3*(3*(3*c*d^2 + 2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan(
(c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^2*c^4 - 2*a^2*c^3*d + 2*
a^2*c*d^3 - a^2*d^4)*sqrt(c^2 - d^2)) + 3*(d^4*tan(1/2*f*x + 1/2*e) + c*d^3
)/((a^2*c^5 - 2*a^2*c^4*d + 2*a^2*c^2*d^3 - a^2*c*d^4)*(c*tan(1/2*f*x + 1/2
*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)) - (3*c*tan(1/2*f*x + 1/2*e)^2 - 9*d*
tan(1/2*f*x + 1/2*e)^2 + 3*c*tan(1/2*f*x + 1/2*e) - 15*d*tan(1/2*f*x + 1/2*
e) + 2*c - 8*d)/((a^2*c^3 - 3*a^2*c^2*d + 3*a^2*c*d^2 - a^2*d^3)*(tan(1/2*f
*x + 1/2*e) + 1)^3))/f
```

maple [A] time = 0.32, size = 361, normalized size = 1.63

$$\frac{2d^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a^2f\,(c-d)^3\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d+c\right)(c+d)\,c}+\frac{2d^3}{a^2f\,(c-d)^3\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2/a^2/f*d^4/(c-d)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)
/c*tan(1/2*f*x+1/2*e)+2/a^2/f*d^3/(c-d)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2
*f*x+1/2*e)*d+c)/(c+d)+6/a^2/f*d^2/(c-d)^3/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2
*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c+4/a^2/f*d^3/(c-d)^3/(c+d)/
(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))-2/
a^2/f/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)*c+6/a^2/f/(c-d)^3/(tan(1/2*f*x+1/2*e)+
1)*d-4/3/a^2/f/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^3+2/a^2/f/(c-d)^2/(tan(1/2*f*
x+1/2*e)+1)^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 10.49, size = 625, normalized size = 2.83

$$\frac{\frac{2\left(-2c^3+6c^2d+8cd^2+3d^3\right)}{3(c+d)(c-d)\left(c^2-2cd+d^2\right)}+\frac{2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(-5c^3+11c^2d+30cd^2+9d^3\right)}{3c(c-d)\left(c^2-2cd+d^2\right)}+\frac{2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(-3c^4+8c^3d+27c^2d^2+25cd^3+3d^4\right)}{3c(c+d)(c-d)\left(c^2-2cd+d^2\right)}+\frac{2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{c(c}}{f\left(a^2c+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(3a^2c+2a^2d\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(3a^2c+2a^2d\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(4a^2c+6a^2d\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^2),x)
```

```
[Out] ((2*(8*c*d^2 + 6*c^2*d - 2*c^3 + 3*d^3))/(3*(c + d)*(c - d)*(c^2 - 2*c*d +
d^2)) + (2*tan(e/2 + (f*x)/2)^2*(30*c*d^2 + 11*c^2*d - 5*c^3 + 9*d^3))/(3*c
*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)*(25*c*d^3 + 8*c^3*d -
 3*c^4 + 3*d^4 + 27*c^2*d^2))/(3*c*(c + d)*(c - d)*(c^2 - 2*c*d + d^2)) + (
2*tan(e/2 + (f*x)/2)^3*(7*c*d^3 + 2*c^3*d - c^4 + 3*d^4 + 9*c^2*d^2))/(c*(c
 + d)*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)^4*(2*c^3*d - c^4
 + d^4 + 3*c^2*d^2))/(c*(c + d)*(c - d)*(c^2 - 2*c*d + d^2)))/(f*(a^2*c + t
an(e/2 + (f*x)/2)*(3*a^2*c + 2*a^2*d) + tan(e/2 + (f*x)/2)^4*(3*a^2*c + 2*a
^2*d) + tan(e/2 + (f*x)/2)^2*(4*a^2*c + 6*a^2*d) + tan(e/2 + (f*x)/2)^3*(4*
```

```
a^2*c + 6*a^2*d) + a^2*c*tan(e/2 + (f*x)/2)^5)) - (2*d^2*atan(((d^2*(3*c +
2*d)*(2*a^2*d^5 - 4*a^2*c*d^4 - 2*a^2*c^4*d + 4*a^2*c^3*d^2))/(a^2*(c + d)^
(3/2)*(c - d)^(7/2)) - (2*c*d^2*tan(e/2 + (f*x)/2)*(3*c + 2*d)*(a^2*c^4 - a
^2*d^4 + 2*a^2*c*d^3 - 2*a^2*c^3*d))/(a^2*(c + d)^(3/2)*(c - d)^(7/2)))/(6*
c*d^2 + 4*d^3))*(3*c + 2*d))/(a^2*f*(c + d)^(3/2)*(c - d)^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.469 $\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=294

$$\frac{d^2\left(12c^2+16cd+7d^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^4(c+d)^2\sqrt{c^2-d^2}}-\frac{d\left(2c^2-16cd-21d^2\right)\cos(e+fx)}{6a^2f(c-d)^3(c+d)(c+d\sin(e+fx))^2}-\frac{d\left(2c^3-16c^2d-59cd^2-\right.}{6a^2f(c-d)^4(c+d)^2(c}$$

```
[Out] -1/6*d*(2*c^2-16*c*d-21*d^2)*cos(f*x+e)/a^2/(c-d)^3/(c+d)/f/(c+d*sin(f*x+e)
)^2-1/3*(c-8*d)*cos(f*x+e)/a^2/(c-d)^2/f/(1+sin(f*x+e))/(c+d*sin(f*x+e))^2-
1/3*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2-1/6*d*(2*c^3-1
6*c^2*d-59*c*d^2-32*d^3)*cos(f*x+e)/a^2/(c-d)^4/(c+d)^2/f/(c+d*sin(f*x+e))+
d^2*(12*c^2+16*c*d+7*d^2)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/
a^2/(c-d)^4/(c+d)^2/f/(c^2-d^2)^(1/2)
```

Rubi [A] time = 0.62, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2766, 2978, 2754, 12, 2660, 618, 204}

$$\frac{d^2\left(12c^2+16cd+7d^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^4(c+d)^2\sqrt{c^2-d^2}}-\frac{d\left(-16c^2d+2c^3-59cd^2-32d^3\right)\cos(e+fx)}{6a^2f(c-d)^4(c+d)^2(c+d\sin(e+fx))}-\frac{d\left(2c^2-16cd-\right.}{6a^2f(c-d)^3(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] (d^2*(12*c^2 + 16*c*d + 7*d^2)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d
^2]])/(a^2*(c - d)^4*(c + d)^2*Sqrt[c^2 - d^2]*f) - (d*(2*c^2 - 16*c*d - 21
*d^2)*Cos[e + f*x])/(6*a^2*(c - d)^3*(c + d)*f*(c + d*Sin[e + f*x])^2) - ((
c - 8*d)*Cos[e + f*x])/(3*a^2*(c - d)^2*f*(1 + Sin[e + f*x])*(c + d*Sin[e +
 f*x])^2) - Cos[e + f*x]/(3*(c - d)*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[e +
 f*x])^2) - (d*(2*c^3 - 16*c^2*d - 59*c*d^2 - 32*d^3)*Cos[e + f*x])/(6*a^2*
(c - d)^4*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
```

```
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx = -\frac{\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2} - \frac{\int \frac{-a(c-5 \ldots}{(a+a\sin(e+\ldots}}{3 \ldots}$$

$$= -\frac{(c-8d)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))^2} - \frac{\ldots}{3(c-d)f(a \ldots}$$

$$= -\frac{d\left(2c^2-16cd-21d^2\right)\cos(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2} - \frac{(c-8d \ldots}{3a^2(c-d)^2f(1+\sin(\ldots}$$

$$= -\frac{d\left(2c^2-16cd-21d^2\right)\cos(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2} - \frac{(c-8d \ldots}{3a^2(c-d)^2f(1+\sin(\ldots}$$

$$= -\frac{d\left(2c^2-16cd-21d^2\right)\cos(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2} - \frac{(c-8d \ldots}{3a^2(c-d)^2f(1+\sin(\ldots}$$

$$= -\frac{d\left(2c^2-16cd-21d^2\right)\cos(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2} - \frac{(c-8d \ldots}{3a^2(c-d)^2f(1+\sin(\ldots}$$

$$= -\frac{d\left(2c^2-16cd-21d^2\right)\cos(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2} - \frac{(c-8d \ldots}{3a^2(c-d)^2f(1+\sin(\ldots}$$

$$= \frac{d^2\left(12c^2+16cd+7d^2\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^4(c+d)^2\sqrt{c^2-d^2}\,f} - \frac{d\left(2c^2-16cd \ldots\right.}{6a^2(c-d)^3(c+ \ldots}$$

Mathematica [A] time = 1.20, size = 338, normalized size = 1.15

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\frac{6d^2\left(12c^2+16cd+7d^2\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{(c+d)^2\sqrt{c^2-d^2}} + \frac{3d^3(7c+4d)\cos(e+ \ldots}{(c \ldots} \ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(4*(c - d)*Sin[(e + f*x)/2] - 2*(c -
 d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 4*(c - 10*d)*Sin[(e + f*x)/2]*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (6*d^2*(12*c^2 + 16*c*d + 7*d^2)*A
rcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^3)/((c + d)^2*Sqrt[c^2 - d^2]) + (3*(c - d)*d^3*Cos[e + f*x]*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/((c + d)*(c + d*Sin[e + f*x])^2) + (3
*d^3*(7*c + 4*d)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/((c
+ d)^2*(c + d*Sin[e + f*x]))))/(6*a^2*(c - d)^4*f*(1 + Sin[e + f*x])^2)
```

fricas [B] time = 0.63, size = 3540, normalized size = 12.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/12*(4*c^7 - 4*c^6*d - 12*c^5*d^2 + 12*c^4*d^3 + 12*c^3*d^4 - 12*c^2*d^5
 - 4*c*d^6 + 4*d^7 - 2*(2*c^5*d^2 - 16*c^4*d^3 - 61*c^3*d^4 - 16*c^2*d^5 +
59*c*d^6 + 32*d^7)*cos(f*x + e)^4 - 2*(4*c^6*d - 28*c^5*d^2 - 118*c^4*d^3 -
 106*c^3*d^4 + 71*c^2*d^5 + 134*c*d^6 + 43*d^7)*cos(f*x + e)^3 + 2*(2*c^7 -
 12*c^6*d - 36*c^5*d^2 - 54*c^4*d^3 - 39*c^3*d^4 + 39*c^2*d^5 + 73*c*d^6 +
27*d^7)*cos(f*x + e)^2 + 3*(24*c^4*d^2 + 80*c^3*d^3 + 102*c^2*d^4 + 60*c*d^
5 + 14*d^6 + (12*c^2*d^4 + 16*c*d^5 + 7*d^6)*cos(f*x + e)^4 - (24*c^3*d^3 +
 44*c^2*d^4 + 30*c*d^5 + 7*d^6)*cos(f*x + e)^3 - (12*c^4*d^2 + 64*c^3*d^3 +
 107*c^2*d^4 + 76*c*d^5 + 21*d^6)*cos(f*x + e)^2 + (12*c^4*d^2 + 40*c^3*d^3
 + 51*c^2*d^4 + 30*c*d^5 + 7*d^6)*cos(f*x + e) + (24*c^4*d^2 + 80*c^3*d^3 +
 102*c^2*d^4 + 60*c*d^5 + 14*d^6 - (12*c^2*d^4 + 16*c*d^5 + 7*d^6)*cos(f*x
+ e)^3 - 2*(12*c^3*d^3 + 28*c^2*d^4 + 23*c*d^5 + 7*d^6)*cos(f*x + e)^2 + (1
2*c^4*d^2 + 40*c^3*d^3 + 51*c^2*d^4 + 30*c*d^5 + 7*d^6)*cos(f*x + e))*sin(f
*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x
 + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-
c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 4*(2*c
^7 - 5*c^6*d - 36*c^5*d^2 - 75*c^4*d^3 - 39*c^3*d^4 + 60*c^2*d^5 + 73*c*d^6
 + 20*d^7)*cos(f*x + e) - 2*(2*c^7 - 2*c^6*d - 6*c^5*d^2 + 6*c^4*d^3 + 6*c^
3*d^4 - 6*c^2*d^5 - 2*c*d^6 + 2*d^7 + (2*c^5*d^2 - 16*c^4*d^3 - 61*c^3*d^4
- 16*c^2*d^5 + 59*c*d^6 + 32*d^7)*cos(f*x + e)^3 - (4*c^6*d - 30*c^5*d^2 -
102*c^4*d^3 - 45*c^3*d^4 + 87*c^2*d^5 + 75*c*d^6 + 11*d^7)*cos(f*x + e)^2 -
 2*(c^7 - 4*c^6*d - 33*c^5*d^2 - 78*c^4*d^3 - 42*c^3*d^4 + 63*c^2*d^5 + 74*
c*d^6 + 19*d^7)*cos(f*x + e))*sin(f*x + e))/((a^2*c^8*d^2 - 2*a^2*c^7*d^3 -
 2*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 6*a^2*c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^
9 - a^2*d^10)*f*cos(f*x + e)^4 - (2*a^2*c^9*d - 3*a^2*c^8*d^2 - 6*a^2*c^7*d
^3 + 10*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 12*a^2*c^4*d^6 - 2*a^2*c^3*d^7 + 6*a^
2*c^2*d^8 - a^2*d^10)*f*cos(f*x + e)^3 - (a^2*c^10 + 2*a^2*c^9*d - 7*a^2*c^
8*d^2 - 8*a^2*c^7*d^3 + 18*a^2*c^6*d^4 + 12*a^2*c^5*d^5 - 22*a^2*c^4*d^6 -
8*a^2*c^3*d^7 + 13*a^2*c^2*d^8 + 2*a^2*c*d^9 - 3*a^2*d^10)*f*cos(f*x + e)^2
```

+ (a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f*cos(f*x + e) + 2*(a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f - ((a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 6*a^2*c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f*cos(f*x + e)^3 + 2*(a^2*c^9*d - a^2*c^8*d^2 - 4*a^2*c^7*d^3 + 4*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 6*a^2*c^4*d^6 - 4*a^2*c^3*d^7 + 4*a^2*c^2*d^8 + a^2*c*d^9 - a^2*d^10)*f*cos(f*x + e)^2 - (a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f*cos(f*x + e) - 2*(a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f)*sin(f*x + e)), -1/6*(2*c^7 - 2*c^6*d - 6*c^5*d^2 + 6*c^4*d^3 + 6*c^3*d^4 - 6*c^2*d^5 - 2*c*d^6 + 2*d^7 - (2*c^5*d^2 - 16*c^4*d^3 - 61*c^3*d^4 - 16*c^2*d^5 + 59*c*d^6 + 32*d^7)*cos(f*x + e)^4 - (4*c^6*d - 28*c^5*d^2 - 118*c^4*d^3 - 106*c^3*d^4 + 71*c^2*d^5 + 134*c*d^6 + 43*d^7)*cos(f*x + e)^3 + (2*c^7 - 12*c^6*d - 36*c^5*d^2 - 54*c^4*d^3 - 39*c^3*d^4 + 39*c^2*d^5 + 73*c*d^6 + 27*d^7)*cos(f*x + e)^2 + 3*(24*c^4*d^2 + 80*c^3*d^3 + 102*c^2*d^4 + 60*c*d^5 + 14*d^6 + (12*c^2*d^4 + 16*c*d^5 + 7*d^6)*cos(f*x + e)^4 - (24*c^3*d^3 + 44*c^2*d^4 + 30*c*d^5 + 7*d^6)*cos(f*x + e)^3 - (12*c^4*d^2 + 64*c^3*d^3 + 107*c^2*d^4 + 76*c*d^5 + 21*d^6)*cos(f*x + e)^2 + (12*c^4*d^2 + 40*c^3*d^3 + 51*c^2*d^4 + 30*c*d^5 + 7*d^6)*cos(f*x + e) + (24*c^4*d^2 + 80*c^3*d^3 + 102*c^2*d^4 + 60*c*d^5 + 14*d^6 - (12*c^2*d^4 + 16*c*d^5 + 7*d^6)*cos(f*x + e)^3 - 2*(12*c^3*d^3 + 28*c^2*d^4 + 23*c*d^5 + 7*d^6)*cos(f*x + e)^2 + (12*c^4*d^2 + 40*c^3*d^3 + 51*c^2*d^4 + 30*c*d^5 + 7*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + 2*(2*c^7 - 5*c^6*d - 36*c^5*d^2 - 75*c^4*d^3 - 39*c^3*d^4 + 60*c^2*d^5 + 73*c*d^6 + 20*d^7)*cos(f*x + e) - (2*c^7 - 2*c^6*d - 6*c^5*d^2 + 6*c^4*d^3 + 6*c^3*d^4 - 6*c^2*d^5 - 2*c*d^6 + 2*d^7 + (2*c^5*d^2 - 16*c^4*d^3 - 61*c^3*d^4 - 16*c^2*d^5 + 59*c*d^6 + 32*d^7)*cos(f*x + e)^3 - (4*c^6*d - 30*c^5*d^2 - 102*c^4*d^3 - 45*c^3*d^4 + 87*c^2*d^5 + 75*c*d^6 + 11*d^7)*cos(f*x + e)^2 - 2*(c^7 - 4*c^6*d - 33*c^5*d^2 - 78*c^4*d^3 - 42*c^3*d^4 + 63*c^2*d^5 + 74*c*d^6 + 19*d^7)*cos(f*x + e))*sin(f*x + e))/((a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 6*a^2*c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f*cos(f*x + e)^4 - (2*a^2*c^9*d - 3*a^2*c^8*d^2 - 6*a^2*c^7*d^3 + 10*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 12*a^2*c^4*d^6 - 2*a^2*c^3*d^7 + 6*a^2*c^2*d^8 - a^2*d^10)*f*cos(f*x + e)^3 - (a^2*c^10 + 2*a^2*c^9*d - 7*a^2*c^8*d^2 - 8*a^2*c^7*d^3 + 18*a^2*c^6*d^4 + 12*a^2*c^5*d^5 - 22*a^2*c^4*d^6 - 8*a^2*c^3*d^7 + 13*a^2*c^2*d^8 + 2*a^2*c*d^9 - 3*a^2*d^10)*f*cos(f*x + e)^2 + (a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f*cos(f*x + e) + 2*(a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f - ((a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 6*a^2*c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f*cos(f*x + e)^3 + 2*(a^2*c^9*d - a^2*c^8*d^2 - 4*a^2*c^7*d^3 + 4*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 6*a^2*c^4*d^6 - 4*a^2*c^3*d^7 + 4*a^2*c^2*d^8 + a^2*c*d^9 - a^2*d^10)*f*cos(f*x + e)^2 - (a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f*cos(f*x + e) - 2*(a^2*c^10 -

```
 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10
)*f)*sin(f*x + e))]
```

giac [B] time = 0.44, size = 614, normalized size = 2.09

$$\frac{3\left(12c^2d^2+16cd^3+7d^4\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(a^2c^6-2a^2c^5d-a^2c^4d^2+4a^2c^3d^3-a^2c^2d^4-2a^2cd^5+a^2d^6\right)\sqrt{c^2-d^2}}+\frac{3\left(9c^3d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+4c^2d^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2cd^6\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/3*(3*(12*c^2*d^2 + 16*c*d^3 + 7*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sg
n(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^2*c^6 - 2*
a^2*c^5*d - a^2*c^4*d^2 + 4*a^2*c^3*d^3 - a^2*c^2*d^4 - 2*a^2*c*d^5 + a^2*d
^6)*sqrt(c^2 - d^2)) + 3*(9*c^3*d^4*tan(1/2*f*x + 1/2*e)^3 + 4*c^2*d^5*tan(
1/2*f*x + 1/2*e)^3 - 2*c*d^6*tan(1/2*f*x + 1/2*e)^3 + 8*c^4*d^3*tan(1/2*f*x
 + 1/2*e)^2 + 4*c^3*d^4*tan(1/2*f*x + 1/2*e)^2 + 15*c^2*d^5*tan(1/2*f*x + 1
/2*e)^2 + 8*c*d^6*tan(1/2*f*x + 1/2*e)^2 - 2*d^7*tan(1/2*f*x + 1/2*e)^2 + 2
3*c^3*d^4*tan(1/2*f*x + 1/2*e) + 12*c^2*d^5*tan(1/2*f*x + 1/2*e) - 2*c*d^6*
tan(1/2*f*x + 1/2*e) + 8*c^4*d^3 + 4*c^3*d^4 - c^2*d^5)/((a^2*c^8 - 2*a^2*c
^7*d - a^2*c^6*d^2 + 4*a^2*c^5*d^3 - a^2*c^4*d^4 - 2*a^2*c^3*d^5 + a^2*c^2*
d^6)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2) - 2*(3*c*
tan(1/2*f*x + 1/2*e)^2 - 12*d*tan(1/2*f*x + 1/2*e)^2 + 3*c*tan(1/2*f*x + 1/
2*e) - 21*d*tan(1/2*f*x + 1/2*e) + 2*c - 11*d)/((a^2*c^4 - 4*a^2*c^3*d + 6*
a^2*c^2*d^2 - 4*a^2*c*d^3 + a^2*d^4)*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

maple [B] time = 0.35, size = 1313, normalized size = 4.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x)
```

```
[Out] 9/a^2/f*d^4/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(
c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+4/a^2/f*d^5/(c-d)^4/(tan(1/2*f*x+1/2*e)
^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3-2/a^2
/f*d^6/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^2+2
*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+8/a^2/f*d^3/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*c+
2*tan(1/2*f*x+1/2*e)*d+c)^2*c^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2+4/a^2/
f*d^4/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*
c*d+d^2)*tan(1/2*f*x+1/2*e)^2+15/a^2/f*d^5/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*c+
```

```
2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2+8/a^2/f*d^
6/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^2+2*c*d+
d^2)*tan(1/2*f*x+1/2*e)^2-2/a^2/f*d^7/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan
(1/2*f*x+1/2*e)*d+c)^2/c^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2+23/a^2/f*d^
4/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^
2)*c*tan(1/2*f*x+1/2*e)+12/a^2/f*d^5/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan(
1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)-2/a^2/f*d^6/(c-d)^
4/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c*tan
(1/2*f*x+1/2*e)+8/a^2/f*d^3/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1
/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^2+4/a^2/f*d^4/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*
c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c-1/a^2/f*d^5/(c-d)^4/(tan(1/
2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)+12/a^2/f*d^2/(
c-d)^4/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2
*d)/(c^2-d^2)^(1/2))*c^2+16/a^2/f*d^3/(c-d)^4/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/
2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c+7/a^2/f*d^4/(
c-d)^4/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2
*d)/(c^2-d^2)^(1/2))-2/a^2/f/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)*c+8/a^2/f/(c-d)
^4/(tan(1/2*f*x+1/2*e)+1)*d-4/3/a^2/f/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^3+2/a^
2/f/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 10.81, size = 1199, normalized size = 4.08

$$\frac{\frac{-4c^5+14c^4d+40c^3d^2+46c^2d^3+12cd^4-3d^5}{3(c+d)\left(c^2-d^2\right)\left(c^3-3c^2d+3cd^2-d^3\right)}+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5\left(-2c^6+4c^5d+38c^4d^2+40c^3d^3+23c^2d^4+4cd^5-2d^6\right)}{c^2\left(c^5-3c^4d+2c^3d^2+2c^2d^3-3cd^4+d^5\right)}+\frac{2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(-6c^6+16c^5\right.}{3c^2\left(c^2\right.}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(3a^2c^2+4da^2c\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(5a^2c^2+1\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^3),x)
```

```
[Out] ((12*c*d^4 + 14*c^4*d - 4*c^5 - 3*d^5 + 46*c^2*d^3 + 40*c^3*d^2)/(3*(c + d)
*(c^2 - d^2)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (tan(e/2 + (f*x)/2)^5*(4*c*
d^5 + 4*c^5*d - 2*c^6 - 2*d^6 + 23*c^2*d^4 + 40*c^3*d^3 + 38*c^4*d^2))/(c^2
*(c^5 - 3*c^4*d - 3*c*d^4 + d^5 + 2*c^2*d^3 + 2*c^3*d^2)) + (2*tan(e/2 + (f
*x)/2)^3*(33*c*d^5 + 16*c^5*d - 6*c^6 - 9*d^6 + 177*c^2*d^4 + 212*c^3*d^3 +
 102*c^4*d^2))/(3*c^2*(c^2 - d^2)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (tan(e
/2 + (f*x)/2)*(33*c*d^4 + 20*c^4*d - 6*c^5 - 6*d^5 + 160*c^2*d^3 + 114*c^3*
d^2))/(3*c*(c^2 - d^2)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (tan(e/2 + (f*x)/
2)^2*(6*c*d^6 + 16*c^6*d - 14*c^7 - 6*d^7 + 232*c^2*d^5 + 583*c^3*d^4 + 532
*c^4*d^3 + 226*c^5*d^2))/(3*c^2*(c + d)*(c^2 - d^2)*(3*c*d^2 - 3*c^2*d + c^
3 - d^3)) + (tan(e/2 + (f*x)/2)^4*(48*c*d^6 + 14*c^6*d - 16*c^7 - 18*d^7 +
303*c^2*d^5 + 522*c^3*d^4 + 502*c^4*d^3 + 220*c^5*d^2))/(3*c^2*(c + d)*(c^2
 - d^2)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (tan(e/2 + (f*x)/2)^6*(4*c*d^5 +
 4*c^5*d - 2*c^6 - 2*d^6 + 9*c^2*d^4 + 8*c^3*d^3 + 14*c^4*d^2))/(c*(c - d)*
(2*c*d + c^2 + d^2)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)))/(f*(tan(e/2 + (f*x)/2
)*(3*a^2*c^2 + 4*a^2*c*d) + tan(e/2 + (f*x)/2)^2*(5*a^2*c^2 + 4*a^2*d^2 + 1
2*a^2*c*d) + tan(e/2 + (f*x)/2)^5*(5*a^2*c^2 + 4*a^2*d^2 + 12*a^2*c*d) + ta
n(e/2 + (f*x)/2)^3*(7*a^2*c^2 + 12*a^2*d^2 + 16*a^2*c*d) + tan(e/2 + (f*x)/
2)^4*(7*a^2*c^2 + 12*a^2*d^2 + 16*a^2*c*d) + tan(e/2 + (f*x)/2)^6*(3*a^2*c^
2 + 4*a^2*c*d) + a^2*c^2 + a^2*c^2*tan(e/2 + (f*x)/2)^7)) - (d^2*atan(((d^2
*(16*c*d + 12*c^2 + 7*d^2)*(4*a^2*c*d^6 - 2*a^2*d^7 - 2*a^2*c^6*d + 2*a^2*c
^2*d^5 - 8*a^2*c^3*d^4 + 2*a^2*c^4*d^3 + 4*a^2*c^5*d^2))/(2*a^2*(c + d)^(5/
2)*(c - d)^(9/2)) + (c*d^2*tan(e/2 + (f*x)/2)*(16*c*d + 12*c^2 + 7*d^2)*(2*
a^2*c*d^5 - a^2*d^6 - a^2*c^6 + 2*a^2*c^5*d + a^2*c^2*d^4 - 4*a^2*c^3*d^3 +
 a^2*c^4*d^2))/(a^2*(c + d)^(5/2)*(c - d)^(9/2)))/(16*c*d^3 + 7*d^4 + 12*c^
2*d^2))*(16*c*d + 12*c^2 + 7*d^2))/(a^2*f*(c + d)^(5/2)*(c - d)^(9/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.470 $\int \frac{(c+d\sin(e+fx))^6}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=354

$$-\frac{(c-d)\left(2c^2+18cd+115d^2\right)\cos(e+fx)(c+d\sin(e+fx))^3}{15f\left(a^3\sin(e+fx)+a^3\right)}+\frac{d\left(2c^3+18c^2d+111cd^2-136d^3\right)\cos(e+fx)(c+}{15a^3f}$$

```
[Out] 1/2*d^3*(40*c^3-90*c^2*d+78*c*d^2-23*d^3)*x/a^3+2/15*d*(2*c^5+18*c^4*d+107*
c^3*d^2-472*c^2*d^3+456*c*d^4-136*d^5)*cos(f*x+e)/a^3/f+1/30*d^2*(4*c^4+36*
c^3*d+216*c^2*d^2-626*c*d^3+345*d^4)*cos(f*x+e)*sin(f*x+e)/a^3/f+1/15*d*(2*
c^3+18*c^2*d+111*c*d^2-136*d^3)*cos(f*x+e)*(c+d*sin(f*x+e))^2/a^3/f-1/15*(c
-d)*(2*c^2+18*c*d+115*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^3/f/(a^3+a^3*sin(f*x
+e))-1/15*(c-d)*(2*c+13*d)*cos(f*x+e)*(c+d*sin(f*x+e))^4/a/f/(a+a*sin(f*x+e
))^2-1/5*(c-d)*cos(f*x+e)*(c+d*sin(f*x+e))^5/f/(a+a*sin(f*x+e))^3
```

Rubi [A] time = 0.79, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2765, 2977, 2753, 2734}

$$\frac{2d\left(107c^3d^2-472c^2d^3+18c^4d+2c^5+456cd^4-136d^5\right)\cos(e+fx)}{15a^3f}-\frac{(c-d)\left(2c^2+18cd+115d^2\right)\cos(e+fx)(c}{15f\left(a^3\sin(e+fx)+a^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^6/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (d^3*(40*c^3 - 90*c^2*d + 78*c*d^2 - 23*d^3)*x)/(2*a^3) + (2*d*(2*c^5 + 18*
c^4*d + 107*c^3*d^2 - 472*c^2*d^3 + 456*c*d^4 - 136*d^5)*Cos[e + f*x])/(15*
a^3*f) + (d^2*(4*c^4 + 36*c^3*d + 216*c^2*d^2 - 626*c*d^3 + 345*d^4)*Cos[e
+ f*x]*Sin[e + f*x])/(30*a^3*f) + (d*(2*c^3 + 18*c^2*d + 111*c*d^2 - 136*d^
3)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(15*a^3*f) - ((c - d)*(2*c^2 + 18*c
*d + 115*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(15*f*(a^3 + a^3*Sin[e +
 f*x])) - ((c - d)*(2*c + 13*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(15*a*
f*(a + a*Sin[e + f*x])^2) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^5)/(
5*f*(a + a*Sin[e + f*x])^3)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^6}{(a+a\sin(e+fx))^3}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^5}{5f(a+a\sin(e+fx))^3} - \frac{\int \frac{(c+d\sin(e+fx))^4\left(-a\left(2c^2+8cd-5d^2\right)+a(3c-8d)d\sin(}{(a+a\sin(e+fx))^2}}{5a^2}$$

$$= -\frac{(c-d)(2c+13d)\cos(e+fx)(c+d\sin(e+fx))^4}{15af(a+a\sin(e+fx))^2} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+}{5f(a+a\sin(e+fx))^3}$$

$$= -\frac{(c-d)\left(2c^2+18cd+115d^2\right)\cos(e+fx)(c+d\sin(e+fx))^3}{15f\left(a^3+a^3\sin(e+fx)\right)} - \frac{(c-d)(2c+13d)\,\mathrm{co}}{15af(a}$$

$$= \frac{d\left(2c^3+18c^2d+111cd^2-136d^3\right)\cos(e+fx)(c+d\sin(e+fx))^2}{15a^3f} - \frac{(c-d)\left(2c^2+18\right.}{}$$

$$= \frac{d^3\left(40c^3-90c^2d+78cd^2-23d^3\right)x}{2a^3} + \frac{2d\left(2c^5+18c^4d+107c^3d^2-472c^2d^3+456c\right.}{15a^3f}$$

Mathematica [C] time = 2.86, size = 560, normalized size = 1.58

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(16\left(2c^2+26cd+197d^2\right)(c-d)^4\sin\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^6/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(48*(c - d)^6*Sin[(e + f*x)/2] - 24*
(c - d)^6*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 32*(c - d)^5*(c + 14*d)*S
in[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 16*(c - d)^5*(c +
 14*d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 16*(c - d)^4*(2*c^2 + 26*c
*d + 197*d^2)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 60
*d^3*(-40*c^3 + 90*c^2*d - 78*c*d^2 + 23*d^3)*(e + f*x)*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^5 + 10*d^6*Cos[3*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^5 - 45*d^4*(20*c^2 - 24*c*d + 9*d^2)*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^5*(Cos[e + f*x] - I*Sin[e + f*x]) - 45*d^4*(20*c^2 - 24*c*d + 9*d
^2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(Cos[e + f*x] + I*Sin[e + f*x])
 - (45*I)*(2*c - d)*d^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(Cos[2*(e +
 f*x)] - I*Sin[2*(e + f*x)]) + (45*I)*(2*c - d)*d^5*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^5*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)])))/(120*a^3*f*(1 +
Sin[e + f*x])^3)
```

fricas [B] time = 0.49, size = 823, normalized size = 2.32

$$\frac{10\,d^6\cos\left(fx+e\right)^6+6\,c^6-36\,c^5d+90\,c^4d^2-120\,c^3d^3+90\,c^2d^4-36\,cd^5+6\,d^6+15\left(6\,cd^5-d^6\right)\cos\left(fx+e\right)^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^6/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/30*(10*d^6*cos(f*x + e)^6 + 6*c^6 - 36*c^5*d + 90*c^4*d^2 - 120*c^3*d^3 +
 90*c^2*d^4 - 36*c*d^5 + 6*d^6 + 15*(6*c*d^5 - d^6)*cos(f*x + e)^5 - 10*(45
*c^2*d^4 - 36*c*d^5 + 14*d^6)*cos(f*x + e)^4 - (4*c^6 + 36*c^5*d + 210*c^4*
d^2 - 1280*c^3*d^3 + 3510*c^2*d^4 - 2694*c*d^5 + 839*d^6 - 15*(40*c^3*d^3 -
 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*f*x)*cos(f*x + e)^3 - 60*(40*c^3*d^3 - 90*
c^2*d^4 + 78*c*d^5 - 23*d^6)*f*x + (8*c^6 + 72*c^5*d - 30*c^4*d^2 - 760*c^3
*d^3 + 2520*c^2*d^4 - 2148*c*d^5 + 668*d^6 + 45*(40*c^3*d^3 - 90*c^2*d^4 +
78*c*d^5 - 23*d^6)*f*x)*cos(f*x + e)^2 + 6*(3*c^6 + 12*c^5*d + 45*c^4*d^2 -
 360*c^3*d^3 + 945*c^2*d^4 - 768*c*d^5 + 233*d^6 - 5*(40*c^3*d^3 - 90*c^2*d
^4 + 78*c*d^5 - 23*d^6)*f*x)*cos(f*x + e) + (10*d^6*cos(f*x + e)^5 - 6*c^6
+ 36*c^5*d - 90*c^4*d^2 + 120*c^3*d^3 - 90*c^2*d^4 + 36*c*d^5 - 6*d^6 - 5*(
18*c*d^5 - 5*d^6)*cos(f*x + e)^4 - 5*(90*c^2*d^4 - 54*c*d^5 + 23*d^6)*cos(f
*x + e)^3 - 60*(40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*f*x + (4*c^6 +
 36*c^5*d + 210*c^4*d^2 - 1280*c^3*d^3 + 3060*c^2*d^4 - 2424*c*d^5 + 724*d^
6 + 15*(40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*f*x)*cos(f*x + e)^2 +
6*(2*c^6 + 18*c^5*d + 30*c^4*d^2 - 340*c^3*d^3 + 930*c^2*d^4 - 762*c*d^5 +
232*d^6 - 5*(40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*f*x)*cos(f*x + e)
)*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*co
s(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3
*f)*sin(f*x + e))
```

giac [B] time = 0.29, size = 776, normalized size = 2.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^6/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/30*(15*(40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*(f*x + e)/a^3 + 10*(
18*c*d^5*tan(1/2*f*x + 1/2*e)^5 - 9*d^6*tan(1/2*f*x + 1/2*e)^5 - 90*c^2*d^4
*tan(1/2*f*x + 1/2*e)^4 + 108*c*d^5*tan(1/2*f*x + 1/2*e)^4 - 36*d^6*tan(1/2
*f*x + 1/2*e)^4 - 180*c^2*d^4*tan(1/2*f*x + 1/2*e)^2 + 216*c*d^5*tan(1/2*f*
x + 1/2*e)^2 - 84*d^6*tan(1/2*f*x + 1/2*e)^2 - 18*c*d^5*tan(1/2*f*x + 1/2*e
) + 9*d^6*tan(1/2*f*x + 1/2*e) - 90*c^2*d^4 + 108*c*d^5 - 40*d^6)/((tan(1/2
```

```
*f*x + 1/2*e)^2 + 1)^3*a^3) - 4*(15*c^6*tan(1/2*f*x + 1/2*e)^4 - 300*c^3*d^
3*tan(1/2*f*x + 1/2*e)^4 + 675*c^2*d^4*tan(1/2*f*x + 1/2*e)^4 - 540*c*d^5*t
an(1/2*f*x + 1/2*e)^4 + 150*d^6*tan(1/2*f*x + 1/2*e)^4 + 30*c^6*tan(1/2*f*x
 + 1/2*e)^3 + 90*c^5*d*tan(1/2*f*x + 1/2*e)^3 - 1500*c^3*d^3*tan(1/2*f*x +
1/2*e)^3 + 3150*c^2*d^4*tan(1/2*f*x + 1/2*e)^3 - 2430*c*d^5*tan(1/2*f*x + 1
/2*e)^3 + 660*d^6*tan(1/2*f*x + 1/2*e)^3 + 40*c^6*tan(1/2*f*x + 1/2*e)^2 +
90*c^5*d*tan(1/2*f*x + 1/2*e)^2 + 300*c^4*d^2*tan(1/2*f*x + 1/2*e)^2 - 2900
*c^3*d^3*tan(1/2*f*x + 1/2*e)^2 + 5400*c^2*d^4*tan(1/2*f*x + 1/2*e)^2 - 399
0*c*d^5*tan(1/2*f*x + 1/2*e)^2 + 1060*d^6*tan(1/2*f*x + 1/2*e)^2 + 20*c^6*t
an(1/2*f*x + 1/2*e) + 90*c^5*d*tan(1/2*f*x + 1/2*e) + 150*c^4*d^2*tan(1/2*f
*x + 1/2*e) - 1900*c^3*d^3*tan(1/2*f*x + 1/2*e) + 3600*c^2*d^4*tan(1/2*f*x
+ 1/2*e) - 2670*c*d^5*tan(1/2*f*x + 1/2*e) + 710*d^6*tan(1/2*f*x + 1/2*e) +
 7*c^6 + 18*c^5*d + 30*c^4*d^2 - 440*c^3*d^3 + 855*c^2*d^4 - 642*c*d^5 + 17
2*d^6)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

maple [B] time = 0.29, size = 1340, normalized size = 3.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^6/(a+a*sin(f*x+e))^3,x)
```

```
[Out] -60/a^3/f*d^4/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2*c^2+72/a^3/f*
d^5/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2*c-6/a^3/f*d^5/(1+tan(1/
2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)*c+6/a^3/f*d^5/(1+tan(1/2*f*x+1/2*e)^2)
^3*tan(1/2*f*x+1/2*e)^5*c-30/a^3/f*d^4/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f
*x+1/2*e)^4*c^2+36/a^3/f*d^5/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^
4*c-8/a^3/f/(tan(1/2*f*x+1/2*e)+1)^2*d^6-16/3/a^3/f/(tan(1/2*f*x+1/2*e)+1)^
3*c^6+8/3/a^3/f/(tan(1/2*f*x+1/2*e)+1)^3*d^6+4/a^3/f/(tan(1/2*f*x+1/2*e)+1)
^4*c^6+4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*d^6-8/5/a^3/f/(tan(1/2*f*x+1/2*e)+1
)^5*c^6-8/5/a^3/f/(tan(1/2*f*x+1/2*e)+1)^5*d^6-40/3/a^3/f*d^6/(1+tan(1/2*f*
x+1/2*e)^2)^3-23/a^3/f*d^6*arctan(tan(1/2*f*x+1/2*e))-2/a^3/f/(tan(1/2*f*x+
1/2*e)+1)*c^6-20/a^3/f/(tan(1/2*f*x+1/2*e)+1)*d^6+4/a^3/f/(tan(1/2*f*x+1/2*
e)+1)^2*c^6-12/a^3/f/(tan(1/2*f*x+1/2*e)+1)^2*c^5*d+40/a^3/f/(tan(1/2*f*x+1
/2*e)+1)^2*c^3*d^3-60/a^3/f/(tan(1/2*f*x+1/2*e)+1)^2*c^2*d^4+36/a^3/f/(tan(
1/2*f*x+1/2*e)+1)^2*c*d^5+24/a^3/f/(tan(1/2*f*x+1/2*e)+1)^3*c^5*d-30/a^3/f*
d^4/(1+tan(1/2*f*x+1/2*e)^2)^3*c^2+3/a^3/f*d^6/(1+tan(1/2*f*x+1/2*e)^2)^3*t
an(1/2*f*x+1/2*e)-3/a^3/f*d^6/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)
^5-12/a^3/f*d^6/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^4-28/a^3/f*d^
6/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2+60/a^3/f/(tan(1/2*f*x+1/2
*e)+1)^4*c^2*d^4-24/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*c*d^5+48/5/a^3/f/(tan(1/
2*f*x+1/2*e)+1)^5*c^5*d-24/a^3/f/(tan(1/2*f*x+1/2*e)+1)^5*c^4*d^2+32/a^3/f/
(tan(1/2*f*x+1/2*e)+1)^5*c^3*d^3-24/a^3/f/(tan(1/2*f*x+1/2*e)+1)^5*c^2*d^4+
48/5/a^3/f/(tan(1/2*f*x+1/2*e)+1)^5*c*d^5-40/a^3/f/(tan(1/2*f*x+1/2*e)+1)^3
*c^4*d^2+80/3/a^3/f/(tan(1/2*f*x+1/2*e)+1)^3*c^3*d^3-8/a^3/f/(tan(1/2*f*x+1
```

```
/2*e)+1)^3*c*d^5-24/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*c^5*d+60/a^3/f/(tan(1/2*
f*x+1/2*e)+1)^4*c^4*d^2-80/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*c^3*d^3-90/a^3/f/
(tan(1/2*f*x+1/2*e)+1)*c^2*d^4+72/a^3/f/(tan(1/2*f*x+1/2*e)+1)*c*d^5+36/a^3
/f*d^5/(1+tan(1/2*f*x+1/2*e)^2)^3*c+40/a^3/f*d^3*arctan(tan(1/2*f*x+1/2*e))
*c^3-90/a^3/f*d^4*arctan(tan(1/2*f*x+1/2*e))*c^2+78/a^3/f*d^5*arctan(tan(1/
2*f*x+1/2*e))*c+40/a^3/f/(tan(1/2*f*x+1/2*e)+1)*c^3*d^3
```

maxima [**B**] time = 0.84, size = 1993, normalized size = 5.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^6/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/15*(d^6*((2375*sin(f*x + e)/(cos(f*x + e) + 1) + 5347*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 9230*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 12622*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 13340*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
 11684*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 8050*sin(f*x + e)^7/(cos(f*x +
 e) + 1)^7 + 4370*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1725*sin(f*x + e)^9
/(cos(f*x + e) + 1)^9 + 345*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 544)/(a
^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 13*a^3*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 25*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 38*a^3*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 + 46*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
46*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 38*a^3*sin(f*x + e)^7/(cos(f*x
 + e) + 1)^7 + 25*a^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 13*a^3*sin(f*x
+ e)^9/(cos(f*x + e) + 1)^9 + 5*a^3*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 +
 a^3*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 345*arctan(sin(f*x + e)/(cos(
f*x + e) + 1))/a^3) - 6*c*d^5*((1325*sin(f*x + e)/(cos(f*x + e) + 1) + 2673
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3805*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 4329*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3575*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5 + 2275*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 975*sin(f*x +
e)^7/(cos(f*x + e) + 1)^7 + 195*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 304)/
(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 12*a^3*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 20*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 26*a^3*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 26*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
+ 20*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 12*a^3*sin(f*x + e)^7/(cos(f
*x + e) + 1)^7 + 5*a^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^3*sin(f*x +
e)^9/(cos(f*x + e) + 1)^9) + 195*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^
3) + 90*c^2*d^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f
*x + e) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
11*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x
```

```
+ e) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x
+ e)/(cos(f*x + e) + 1))/a^3) - 40*c^3*d^3*((95*sin(f*x + e)/(cos(f*x + e)
+ 1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*si
n(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
+ 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*
x + e)/(cos(f*x + e) + 1))/a^3) + 2*c^6*(20*sin(f*x + e)/(cos(f*x + e) + 1)
 + 40*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x
+ e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a
^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 60*c^4*d^2*(5*sin(f*x
+ e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3
+ 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 36*c^5
*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(
cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

mupad [B] time = 9.71, size = 898, normalized size = 2.54

$$\frac{d^3\,\mathrm{atan}\left(\frac{d^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(40\,c^3-90\,c^2\,d+78\,c\,d^2-23\,d^3\right)}{40\,c^3\,d^3-90\,c^2\,d^4+78\,c\,d^5-23\,d^6}\right)\left(40\,c^3-90\,c^2\,d+78\,c\,d^2-23\,d^3\right)}{a^3\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9\,\left(4\,c^6+12\,c^5\,d-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^6/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (d^3*atan((d^3*tan(e/2 + (f*x)/2)*(78*c*d^2 - 90*c^2*d + 40*c^3 - 23*d^3))/
(78*c*d^5 - 23*d^6 - 90*c^2*d^4 + 40*c^3*d^3))*(78*c*d^2 - 90*c^2*d + 40*c^
3 - 23*d^3))/(a^3*f) - (tan(e/2 + (f*x)/2)^9*(12*c^5*d - 390*c*d^5 + 4*c^6
+ 115*d^6 + 450*c^2*d^4 - 200*c^3*d^3) - (608*c*d^5)/5 + (12*c^5*d)/5 + tan
(e/2 + (f*x)/2)^10*(2*c^6 - 78*c*d^5 + 23*d^6 + 90*c^2*d^4 - 40*c^3*d^3) +
tan(e/2 + (f*x)/2)*(12*c^5*d - 530*c*d^5 + (8*c^6)/3 + (475*d^6)/3 + 630*c^
2*d^4 - (760*c^3*d^3)/3 + 20*c^4*d^2) + (14*c^6)/15 + (544*d^6)/15 + 144*c^
2*d^4 - (176*c^3*d^3)/3 + 4*c^4*d^2 + tan(e/2 + (f*x)/2)^8*(12*c^5*d - 988*
c*d^5 + (34*c^6)/3 + (874*d^6)/3 + 1140*c^2*d^4 - (1520*c^3*d^3)/3 + 40*c^4
*d^2) + tan(e/2 + (f*x)/2)^3*(48*c^5*d - 2052*c*d^5 + 12*c^6 + (1846*d^6)/3
 + 2460*c^2*d^4 - 960*c^3*d^3 + 60*c^4*d^2) + tan(e/2 + (f*x)/2)^7*(48*c^5*
```

```
d - 1820*c*d^5 + (44*c^6)/3 + (1610*d^6)/3 + 2100*c^2*d^4 - (2560*c^3*d^3)/
3 + 20*c^4*d^2) + tan(e/2 + (f*x)/2)^5*(72*c^5*d - 2952*c*d^5 + 20*c^6 + (2
668*d^6)/3 + 3480*c^2*d^4 - 1360*c^3*d^3 + 60*c^4*d^2) + tan(e/2 + (f*x)/2)
^2*((96*c^5*d)/5 - (5954*c*d^5)/5 + (122*c^6)/15 + (5347*d^6)/15 + 1422*c^2
*d^4 - (1688*c^3*d^3)/3 + 52*c^4*d^2) + tan(e/2 + (f*x)/2)^4*((216*c^5*d)/5
 - (14004*c*d^5)/5 + (104*c^6)/5 + (12622*d^6)/15 + 3372*c^2*d^4 - 1376*c^3
*d^3 + 132*c^4*d^2) + tan(e/2 + (f*x)/2)^6*((192*c^5*d)/5 - (13208*c*d^5)/5
 + (344*c^6)/15 + (11684*d^6)/15 + 3144*c^2*d^4 - (4016*c^3*d^3)/3 + 124*c^
4*d^2))/(f*(13*a^3*tan(e/2 + (f*x)/2)^2 + 25*a^3*tan(e/2 + (f*x)/2)^3 + 38*
a^3*tan(e/2 + (f*x)/2)^4 + 46*a^3*tan(e/2 + (f*x)/2)^5 + 46*a^3*tan(e/2 + (
f*x)/2)^6 + 38*a^3*tan(e/2 + (f*x)/2)^7 + 25*a^3*tan(e/2 + (f*x)/2)^8 + 13*
a^3*tan(e/2 + (f*x)/2)^9 + 5*a^3*tan(e/2 + (f*x)/2)^10 + a^3*tan(e/2 + (f*x
)/2)^11 + a^3 + 5*a^3*tan(e/2 + (f*x)/2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**6/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.471 $\int \frac{(c+d\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=278

$$-\frac{(c-d)\left(2c^2+15cd+76d^2\right)\cos(e+fx)(c+d\sin(e+fx))^2}{15f\left(a^3\sin(e+fx)+a^3\right)}+\frac{d^3x\left(20c^2-30cd+13d^2\right)}{2a^3}+\frac{d^2\left(4c^3+30c^2d+146c\right.}{}$$

```
[Out] 1/2*d^3*(20*c^2-30*c*d+13*d^2)*x/a^3+2/15*d*(2*c^4+15*c^3*d+72*c^2*d^2-180*
c*d^3+76*d^4)*cos(f*x+e)/a^3/f+1/30*d^2*(4*c^3+30*c^2*d+146*c*d^2-195*d^3)*
cos(f*x+e)*sin(f*x+e)/a^3/f-1/15*(c-d)*(2*c^2+15*c*d+76*d^2)*cos(f*x+e)*(c+
d*sin(f*x+e))^2/f/(a^3+a^3*sin(f*x+e))-1/15*(c-d)*(2*c+11*d)*cos(f*x+e)*(c+
d*sin(f*x+e))^3/a/f/(a+a*sin(f*x+e))^2-1/5*(c-d)*cos(f*x+e)*(c+d*sin(f*x+e)
)^4/f/(a+a*sin(f*x+e))^3
```

Rubi [A] time = 0.61, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2765, 2977, 2734}

$$\frac{2d\left(72c^2d^2+15c^3d+2c^4-180cd^3+76d^4\right)\cos(e+fx)}{15a^3f}-\frac{(c-d)\left(2c^2+15cd+76d^2\right)\cos(e+fx)(c+d\sin(e+f}{15f\left(a^3\sin(e+fx)+a^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^5/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (d^3*(20*c^2 - 30*c*d + 13*d^2)*x)/(2*a^3) + (2*d*(2*c^4 + 15*c^3*d + 72*c^
2*d^2 - 180*c*d^3 + 76*d^4)*Cos[e + f*x])/(15*a^3*f) + (d^2*(4*c^3 + 30*c^2
*d + 146*c*d^2 - 195*d^3)*Cos[e + f*x]*Sin[e + f*x])/(30*a^3*f) - ((c - d)*
(2*c^2 + 15*c*d + 76*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(15*f*(a^3 +
 a^3*Sin[e + f*x])) - ((c - d)*(2*c + 11*d)*Cos[e + f*x]*(c + d*Sin[e + f*x
])^3)/(15*a*f*(a + a*Sin[e + f*x])^2) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^4)/(5*f*(a + a*Sin[e + f*x])^3)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
```

```
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^4}{5f(a+a\sin(e+fx))^3} - \frac{\int \frac{(c+d\sin(e+fx))^3(-a(2c-d)(c+4d)+a(2c-7d)d\sin(}{(a+a\sin(e+fx))^2}}{5a^2}$$

$$= -\frac{(c-d)(2c+11d)\cos(e+fx)(c+d\sin(e+fx))^3}{15af(a+a\sin(e+fx))^2} - \frac{(c-d)\cos(e+fx)(c+d\sin(}{5f(a+a\sin(e+fx))^{}}$$

$$= -\frac{(c-d)\left(2c^2+15cd+76d^2\right)\cos(e+fx)(c+d\sin(e+fx))^2}{15f\left(a^3+a^3\sin(e+fx)\right)} - \frac{(c-d)(2c+11d)\,\mathrm{c}}{15af(a}$$

$$= \frac{d^3\left(20c^2-30cd+13d^2\right)x}{2a^3} + \frac{2d\left(2c^4+15c^3d+72c^2d^2-180cd^3+76d^4\right)\cos(e+f}{15a^3f}$$

Mathematica [B] time = 7.90, size = 992, normalized size = 3.57

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-160\cos\left(\frac{3}{2}(e+fx)\right)c^5+320\sin\left(\frac{1}{2}(e+fx)\right)c^5-32\sin\left(\frac{5}{2}(e+fx)\right)c^5+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^5/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(1200*c^4*d*Cos[(e + f*x)/2] + 4800*
c^3*d^2*Cos[(e + f*x)/2] - 21600*c^2*d^3*Cos[(e + f*x)/2] + 22500*c*d^4*Cos
[(e + f*x)/2] - 7560*d^5*Cos[(e + f*x)/2] + 12000*c^2*d^3*(e + f*x)*Cos[(e
+ f*x)/2] - 18000*c*d^4*(e + f*x)*Cos[(e + f*x)/2] + 7800*d^5*(e + f*x)*Cos
[(e + f*x)/2] - 160*c^5*Cos[(3*(e + f*x))/2] - 1200*c^4*d*Cos[(3*(e + f*x))
/2] - 3200*c^3*d^2*Cos[(3*(e + f*x))/2] + 18400*c^2*d^3*Cos[(3*(e + f*x))/2
] - 24300*c*d^4*Cos[(3*(e + f*x))/2] + 9230*d^5*Cos[(3*(e + f*x))/2] - 6000
*c^2*d^3*(e + f*x)*Cos[(3*(e + f*x))/2] + 9000*c*d^4*(e + f*x)*Cos[(3*(e +
f*x))/2] - 3900*d^5*(e + f*x)*Cos[(3*(e + f*x))/2] + 1500*c*d^4*Cos[(5*(e +
 f*x))/2] - 750*d^5*Cos[(5*(e + f*x))/2] - 1200*c^2*d^3*(e + f*x)*Cos[(5*(e
 + f*x))/2] + 1800*c*d^4*(e + f*x)*Cos[(5*(e + f*x))/2] - 780*d^5*(e + f*x)
*Cos[(5*(e + f*x))/2] + 300*c*d^4*Cos[(7*(e + f*x))/2] - 105*d^5*Cos[(7*(e
+ f*x))/2] - 15*d^5*Cos[(9*(e + f*x))/2] + 320*c^5*Sin[(e + f*x)/2] + 1200*
c^4*d*Sin[(e + f*x)/2] + 6400*c^3*d^2*Sin[(e + f*x)/2] - 29600*c^2*d^3*Sin[
(e + f*x)/2] + 35100*c*d^4*Sin[(e + f*x)/2] - 12760*d^5*Sin[(e + f*x)/2] +
12000*c^2*d^3*(e + f*x)*Sin[(e + f*x)/2] - 18000*c*d^4*(e + f*x)*Sin[(e + f
*x)/2] + 7800*d^5*(e + f*x)*Sin[(e + f*x)/2] + 2400*c^3*d^2*Sin[(3*(e + f*x
))/2] - 7200*c^2*d^3*Sin[(3*(e + f*x))/2] + 4500*c*d^4*Sin[(3*(e + f*x))/2]
 - 930*d^5*Sin[(3*(e + f*x))/2] + 6000*c^2*d^3*(e + f*x)*Sin[(3*(e + f*x))/
2] - 9000*c*d^4*(e + f*x)*Sin[(3*(e + f*x))/2] + 3900*d^5*(e + f*x)*Sin[(3*
(e + f*x))/2] - 32*c^5*Sin[(5*(e + f*x))/2] - 240*c^4*d*Sin[(5*(e + f*x))/2
] - 1120*c^3*d^2*Sin[(5*(e + f*x))/2] + 5120*c^2*d^3*Sin[(5*(e + f*x))/2] -
 7260*c*d^4*Sin[(5*(e + f*x))/2] + 2782*d^5*Sin[(5*(e + f*x))/2] - 1200*c^2
*d^3*(e + f*x)*Sin[(5*(e + f*x))/2] + 1800*c*d^4*(e + f*x)*Sin[(5*(e + f*x)
)/2] - 780*d^5*(e + f*x)*Sin[(5*(e + f*x))/2] + 300*c*d^4*Sin[(7*(e + f*x))
/2] - 105*d^5*Sin[(7*(e + f*x))/2] + 15*d^5*Sin[(9*(e + f*x))/2]))/(480*f*(
a + a*Sin[e + f*x])^3)
```

fricas [B] time = 0.50, size = 653, normalized size = 2.35

$$15\,d^5\cos\left(fx+e\right)^5 + 6\,c^5 - 30\,c^4d + 60\,c^3d^2 - 60\,c^2d^3 + 30\,cd^4 - 6\,d^5 - 30\left(5\,cd^4 - 2\,d^5\right)\cos\left(fx+e\right)^4 - \left(4\,c^5 + 3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/30*(15*d^5*cos(f*x + e)^5 + 6*c^5 - 30*c^4*d + 60*c^3*d^2 - 60*c^2*d^3 +
30*c*d^4 - 6*d^5 - 30*(5*c*d^4 - 2*d^5)*cos(f*x + e)^4 - (4*c^5 + 30*c^4*d
+ 140*c^3*d^2 - 640*c^2*d^3 + 1170*c*d^4 - 449*d^5 - 15*(20*c^2*d^3 - 30*c*
d^4 + 13*d^5)*f*x)*cos(f*x + e)^3 - 60*(20*c^2*d^3 - 30*c*d^4 + 13*d^5)*f*x
 + (8*c^5 + 60*c^4*d - 20*c^3*d^2 - 380*c^2*d^3 + 840*c*d^4 - 358*d^5 + 45*
(20*c^2*d^3 - 30*c*d^4 + 13*d^5)*f*x)*cos(f*x + e)^2 + 6*(3*c^5 + 10*c^4*d
```

```
+ 30*c^3*d^2 - 180*c^2*d^3 + 315*c*d^4 - 128*d^5 - 5*(20*c^2*d^3 - 30*c*d^4
 + 13*d^5)*f*x)*cos(f*x + e) - (15*d^5*cos(f*x + e)^4 + 6*c^5 - 30*c^4*d +
60*c^3*d^2 - 60*c^2*d^3 + 30*c*d^4 - 6*d^5 + 15*(10*c*d^4 - 3*d^5)*cos(f*x
+ e)^3 + 60*(20*c^2*d^3 - 30*c*d^4 + 13*d^5)*f*x - (4*c^5 + 30*c^4*d + 140*
c^3*d^2 - 640*c^2*d^3 + 1020*c*d^4 - 404*d^5 + 15*(20*c^2*d^3 - 30*c*d^4 +
13*d^5)*f*x)*cos(f*x + e)^2 - 6*(2*c^5 + 15*c^4*d + 20*c^3*d^2 - 170*c^2*d^
3 + 310*c*d^4 - 127*d^5 - 5*(20*c^2*d^3 - 30*c*d^4 + 13*d^5)*f*x)*cos(f*x +
 e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f
*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*
a^3*f)*sin(f*x + e))
```

giac [B] time = 0.25, size = 564, normalized size = 2.03

$$\frac{\frac{15\left(20\,c^{2}d^{3}-30\,cd^{4}+13\,d^{5}\right)\left(fx+e\right)}{a^{3}}+\frac{30\left(d^{5}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-10\,cd^{4}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+6\,d^{5}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-d^{5}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-10\,cd^{4}+6\,d^{5}\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)^{2}a^{3}}-\frac{4\left(15\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/30*(15*(20*c^2*d^3 - 30*c*d^4 + 13*d^5)*(f*x + e)/a^3 + 30*(d^5*tan(1/2*f
*x + 1/2*e)^3 - 10*c*d^4*tan(1/2*f*x + 1/2*e)^2 + 6*d^5*tan(1/2*f*x + 1/2*e
)^2 - d^5*tan(1/2*f*x + 1/2*e) - 10*c*d^4 + 6*d^5)/((tan(1/2*f*x + 1/2*e)^2
 + 1)^2*a^3) - 4*(15*c^5*tan(1/2*f*x + 1/2*e)^4 - 150*c^2*d^3*tan(1/2*f*x +
 1/2*e)^4 + 225*c*d^4*tan(1/2*f*x + 1/2*e)^4 - 90*d^5*tan(1/2*f*x + 1/2*e)^
4 + 30*c^5*tan(1/2*f*x + 1/2*e)^3 + 75*c^4*d*tan(1/2*f*x + 1/2*e)^3 - 750*c
^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 1050*c*d^4*tan(1/2*f*x + 1/2*e)^3 - 405*d^5
*tan(1/2*f*x + 1/2*e)^3 + 40*c^5*tan(1/2*f*x + 1/2*e)^2 + 75*c^4*d*tan(1/2*
f*x + 1/2*e)^2 + 200*c^3*d^2*tan(1/2*f*x + 1/2*e)^2 - 1450*c^2*d^3*tan(1/2*
f*x + 1/2*e)^2 + 1800*c*d^4*tan(1/2*f*x + 1/2*e)^2 - 665*d^5*tan(1/2*f*x +
1/2*e)^2 + 20*c^5*tan(1/2*f*x + 1/2*e) + 75*c^4*d*tan(1/2*f*x + 1/2*e) + 10
0*c^3*d^2*tan(1/2*f*x + 1/2*e) - 950*c^2*d^3*tan(1/2*f*x + 1/2*e) + 1200*c*
d^4*tan(1/2*f*x + 1/2*e) - 445*d^5*tan(1/2*f*x + 1/2*e) + 7*c^5 + 15*c^4*d
+ 20*c^3*d^2 - 220*c^2*d^3 + 285*c*d^4 - 107*d^5)/(a^3*(tan(1/2*f*x + 1/2*e
) + 1)^5))/f
```

maple [B] time = 0.29, size = 924, normalized size = 3.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 6/a^3/f/(tan(1/2*f*x+1/2*e)+1)^2*d^5-16/3/a^3/f/(tan(1/2*f*x+1/2*e)+1)^3*c^
5-4/3/a^3/f/(tan(1/2*f*x+1/2*e)+1)^3*d^5+4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*c
^5-4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*d^5-8/5/a^3/f/(tan(1/2*f*x+1/2*e)+1)^5*
c^5+8/5/a^3/f/(tan(1/2*f*x+1/2*e)+1)^5*d^5+6/a^3/f*d^5/(1+tan(1/2*f*x+1/2*e
)^2)^2+13/a^3/f*d^5*arctan(tan(1/2*f*x+1/2*e))-2/a^3/f/(tan(1/2*f*x+1/2*e)+
1)*c^5+12/a^3/f/(tan(1/2*f*x+1/2*e)+1)*d^5+4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^2
*c^5-10/a^3/f*d^4/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*c+40/3/a^
3/f/(tan(1/2*f*x+1/2*e)+1)^3*c^2*d^3-20/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*c^4*
d+40/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*c^3*d^2-40/a^3/f/(tan(1/2*f*x+1/2*e)+1)
^4*c^2*d^3+20/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*c*d^4+8/a^3/f/(tan(1/2*f*x+1/2
*e)+1)^5*c^4*d-16/a^3/f/(tan(1/2*f*x+1/2*e)+1)^5*c^3*d^2+16/a^3/f/(tan(1/2*
f*x+1/2*e)+1)^5*c^2*d^3-1/a^3/f*d^5/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+
1/2*e)-8/a^3/f/(tan(1/2*f*x+1/2*e)+1)^5*c*d^4-10/a^3/f/(tan(1/2*f*x+1/2*e)+
1)^2*c^4*d+20/a^3/f/(tan(1/2*f*x+1/2*e)+1)^2*c^2*d^3-20/a^3/f/(tan(1/2*f*x+
1/2*e)+1)^2*c*d^4+20/a^3/f/(tan(1/2*f*x+1/2*e)+1)^3*c^4*d-80/3/a^3/f/(tan(1
/2*f*x+1/2*e)+1)^3*c^3*d^2+6/a^3/f*d^5/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f
*x+1/2*e)^2+20/a^3/f*d^3*arctan(tan(1/2*f*x+1/2*e))*c^2-30/a^3/f*d^4*arctan
(tan(1/2*f*x+1/2*e))*c+20/a^3/f/(tan(1/2*f*x+1/2*e)+1)*c^2*d^3-30/a^3/f/(ta
n(1/2*f*x+1/2*e)+1)*c*d^4+1/a^3/f*d^5/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*
x+1/2*e)^3-10/a^3/f*d^4/(1+tan(1/2*f*x+1/2*e)^2)^2*c
```

maxima [B] time = 0.49, size = 1504, normalized size = 5.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/15*(d^5*((1325*sin(f*x + e)/(cos(f*x + e) + 1) + 2673*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 3805*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 4329*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 + 3575*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 22
75*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 975*sin(f*x + e)^7/(cos(f*x + e) +
 1)^7 + 195*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 304)/(a^3 + 5*a^3*sin(f*x
 + e)/(cos(f*x + e) + 1) + 12*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 20*
a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 26*a^3*sin(f*x + e)^4/(cos(f*x +
e) + 1)^4 + 26*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20*a^3*sin(f*x + e
)^6/(cos(f*x + e) + 1)^6 + 12*a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 5*a
^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^3*sin(f*x + e)^9/(cos(f*x + e) +
 1)^9) + 195*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) - 30*c*d^4*((105*
sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*sin(f*x + e)^6/(cos(f
*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 11*a^3*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 11*a^3*sin(f*x + e)^5/
```

```
(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a^3*sin(
f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1
))/a^3) + 20*c^2*d^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 145*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x +
e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*si
n(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) +
 1))/a^3) - 2*c^5*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1
) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5) - 40*c^3*d^2*(5*sin(f*x + e)/(cos(f*x + e) + 1
) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(
cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 30*c^4*d*(5*sin(f*x + e)/(co
s(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10
*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5))/f
```

mupad [B] time = 9.54, size = 652, normalized size = 2.35

$$\frac{d^3 \operatorname{atan}\left(\frac{d^3 \tan\left(\frac{e}{2}+\frac{f x}{2}\right)\left(20 c^2-30 c d+13 d^2\right)}{20 c^2 d^3-30 c d^4+13 d^5}\right)\left(20 c^2-30 c d+13 d^2\right)}{a^3 f}-\frac{\tan\left(\frac{e}{2}+\frac{f x}{2}\right)^6\left(\frac{28 c^5}{3}+10 c^4 d+\frac{80 c^3 d^2}{3}-\frac{700 c^2 d^3}{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^5/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (d^3*atan((d^3*tan(e/2 + (f*x)/2)*(20*c^2 - 30*c*d + 13*d^2))/(13*d^5 - 30*
c*d^4 + 20*c^2*d^3))*(20*c^2 - 30*c*d + 13*d^2))/(a^3*f) - (tan(e/2 + (f*x)
/2)^6*(350*c*d^4 + 10*c^4*d + (28*c^5)/3 - (455*d^5)/3 - (700*c^2*d^3)/3 +
(80*c^3*d^2)/3) + tan(e/2 + (f*x)/2)^2*(426*c*d^4 + 14*c^4*d + (36*c^5)/5 -
 (891*d^5)/5 - 252*c^2*d^3 + 32*c^3*d^2) + tan(e/2 + (f*x)/2)^5*(550*c*d^4
+ 30*c^4*d + (32*c^5)/3 - (715*d^5)/3 - (980*c^2*d^3)/3 + (40*c^3*d^2)/3) +
 tan(e/2 + (f*x)/2)^3*(610*c*d^4 + 30*c^4*d + (28*c^5)/3 - (761*d^5)/3 - (1
060*c^2*d^3)/3 + (80*c^3*d^2)/3) + tan(e/2 + (f*x)/2)^4*(698*c*d^4 + 22*c^4
*d + (68*c^5)/5 - (1443*d^5)/5 - 436*c^2*d^3 + 56*c^3*d^2) + tan(e/2 + (f*x
)/2)^7*(150*c*d^4 + 10*c^4*d + 4*c^5 - 65*d^5 - 100*c^2*d^3) + 48*c*d^4 + 2
```

```
*c^4*d + tan(e/2 + (f*x)/2)^8*(30*c*d^4 + 2*c^5 - 13*d^5 - 20*c^2*d^3) + ta
n(e/2 + (f*x)/2)*(210*c*d^4 + 10*c^4*d + (8*c^5)/3 - (265*d^5)/3 - (380*c^2
*d^3)/3 + (40*c^3*d^2)/3) + (14*c^5)/15 - (304*d^5)/15 - (88*c^2*d^3)/3 + (
8*c^3*d^2)/3)/(f*(12*a^3*tan(e/2 + (f*x)/2)^2 + 20*a^3*tan(e/2 + (f*x)/2)^3
 + 26*a^3*tan(e/2 + (f*x)/2)^4 + 26*a^3*tan(e/2 + (f*x)/2)^5 + 20*a^3*tan(e
/2 + (f*x)/2)^6 + 12*a^3*tan(e/2 + (f*x)/2)^7 + 5*a^3*tan(e/2 + (f*x)/2)^8
+ a^3*tan(e/2 + (f*x)/2)^9 + a^3 + 5*a^3*tan(e/2 + (f*x)/2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**5/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.472 $\int \frac{(c+d\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=195

$$\frac{d^2\left(2c^2+10cd-27d^2\right)\cos(e+fx)}{15a^3f}-\frac{(c-d)^2\left(2c^2+12cd+45d^2\right)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}+\frac{d^3x(4c-3d)}{a^3}-\frac{(c-d)\cos(e+fx}{5f(a\sin(e}$$

```
[Out] (4*c-3*d)*d^3*x/a^3+1/15*d^2*(2*c^2+10*c*d-27*d^2)*cos(f*x+e)/a^3/f-1/15*(c
-d)^2*(2*c^2+12*c*d+45*d^2)*cos(f*x+e)/f/(a^3+a^3*sin(f*x+e))-1/15*(c-d)*(2
*c+9*d)*cos(f*x+e)*(c+d*sin(f*x+e))^2/a/f/(a+a*sin(f*x+e))^2-1/5*(c-d)*cos(
f*x+e)*(c+d*sin(f*x+e))^3/f/(a+a*sin(f*x+e))^3
```

Rubi [A] time = 0.61, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2765, 2977, 2968, 3023, 2735, 2648}

$$\frac{d^2\left(2c^2+10cd-27d^2\right)\cos(e+fx)}{15a^3f}-\frac{(c-d)^2\left(2c^2+12cd+45d^2\right)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}+\frac{d^3x(4c-3d)}{a^3}-\frac{(c-d)\cos(e+fx}{5f(a\sin(e}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^4/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((4*c - 3*d)*d^3*x)/a^3 + (d^2*(2*c^2 + 10*c*d - 27*d^2)*Cos[e + f*x])/(15*
a^3*f) - ((c - d)^2*(2*c^2 + 12*c*d + 45*d^2)*Cos[e + f*x])/(15*f*(a^3 + a^
3*Sin[e + f*x])) - ((c - d)*(2*c + 9*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2
)/(15*a*f*(a + a*Sin[e + f*x])^2) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*
x])^3)/(5*f*(a + a*Sin[e + f*x])^3)
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a+a\sin(e+fx))^3} - \frac{\int \frac{(c+d\sin(e+fx))^2\left(-a\left(2c^2+6cd-3d^2\right)+a(c-6d)d\sin}{(a+a\sin(e+fx))^2}}{5a^2}$$

$$= -\frac{(c-d)(2c+9d)\cos(e+fx)(c+d\sin(e+fx))^2}{15af(a+a\sin(e+fx))^2} - \frac{(c-d)\cos(e+fx)(c+d\sin(e}{5f(a+a\sin(e+fx))^3}$$

$$= -\frac{(c-d)(2c+9d)\cos(e+fx)(c+d\sin(e+fx))^2}{15af(a+a\sin(e+fx))^2} - \frac{(c-d)\cos(e+fx)(c+d\sin(e}{5f(a+a\sin(e+fx))^3}$$

$$= \frac{d^2\left(2c^2+10cd-27d^2\right)\cos(e+fx)}{15a^3f} - \frac{(c-d)(2c+9d)\cos(e+fx)(c+d\sin(e+fx}{15af(a+a\sin(e+fx))^2}$$

$$= \frac{(4c-3d)d^3x}{a^3} + \frac{d^2\left(2c^2+10cd-27d^2\right)\cos(e+fx)}{15a^3f} - \frac{(c-d)(2c+9d)\cos(e+fx)(}{15af(a+a\sin(e}$$

$$= \frac{(4c-3d)d^3x}{a^3} + \frac{d^2\left(2c^2+10cd-27d^2\right)\cos(e+fx)}{15a^3f} - \frac{(c-d)^2\left(2c^2+12cd+45d^2\right)}{15f\left(a^3+a^3\sin(e+\right.}$$

Mathematica [B] time = 1.45, size = 683, normalized size = 3.50

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(80c^4\sin\left(\frac{1}{2}(e+fx)\right)-8c^4\sin\left(\frac{5}{2}(e+fx)\right)+240c^3d\sin\left(\frac{1}{2}(e+fx)\right)-48c^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^4/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(15*d*(16*c^3 + 48*c^2*d - 15*d^3*(-
5 + 4*e + 4*f*x) + 16*c*d^2*(-9 + 5*e + 5*f*x))*Cos[(e + f*x)/2] - 5*(8*c^4
 + 48*c^3*d + 96*c^2*d^2 - 9*d^4*(-27 + 10*e + 10*f*x) + 8*c*d^3*(-46 + 15*
e + 15*f*x))*Cos[(3*(e + f*x))/2] + 75*d^4*Cos[(5*(e + f*x))/2] - 120*c*d^3
*e*Cos[(5*(e + f*x))/2] + 90*d^4*e*Cos[(5*(e + f*x))/2] - 120*c*d^3*f*x*Cos
[(5*(e + f*x))/2] + 90*d^4*f*x*Cos[(5*(e + f*x))/2] + 15*d^4*Cos[(7*(e + f*
x))/2] + 80*c^4*Sin[(e + f*x)/2] + 240*c^3*d*Sin[(e + f*x)/2] + 960*c^2*d^2
*Sin[(e + f*x)/2] - 2960*c*d^3*Sin[(e + f*x)/2] + 1755*d^4*Sin[(e + f*x)/2]
 + 1200*c*d^3*e*Sin[(e + f*x)/2] - 900*d^4*e*Sin[(e + f*x)/2] + 1200*c*d^3*
f*x*Sin[(e + f*x)/2] - 900*d^4*f*x*Sin[(e + f*x)/2] + 360*c^2*d^2*Sin[(3*(e
 + f*x))/2] - 720*c*d^3*Sin[(3*(e + f*x))/2] + 225*d^4*Sin[(3*(e + f*x))/2]
 + 600*c*d^3*e*Sin[(3*(e + f*x))/2] - 450*d^4*e*Sin[(3*(e + f*x))/2] + 600*
c*d^3*f*x*Sin[(3*(e + f*x))/2] - 450*d^4*f*x*Sin[(3*(e + f*x))/2] - 8*c^4*S
```

```
in[(5*(e + f*x))/2] - 48*c^3*d*Sin[(5*(e + f*x))/2] - 168*c^2*d^2*Sin[(5*(e
 + f*x))/2] + 512*c*d^3*Sin[(5*(e + f*x))/2] - 363*d^4*Sin[(5*(e + f*x))/2]
 - 120*c*d^3*e*Sin[(5*(e + f*x))/2] + 90*d^4*e*Sin[(5*(e + f*x))/2] - 120*c
*d^3*f*x*Sin[(5*(e + f*x))/2] + 90*d^4*f*x*Sin[(5*(e + f*x))/2] + 15*d^4*Si
n[(7*(e + f*x))/2]))/(120*a^3*f*(1 + Sin[e + f*x])^3)
```

fricas [B] time = 0.47, size = 494, normalized size = 2.53

$$-\frac{15\,d^4\cos\left(fx+e\right)^4-3\,c^4+12\,c^3d-18\,c^2d^2+12\,cd^3-3\,d^4+\left(2\,c^4+12\,c^3d+42\,c^2d^2-128\,cd^3+117\,d^4-15\,(\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(15*d^4*cos(f*x + e)^4 - 3*c^4 + 12*c^3*d - 18*c^2*d^2 + 12*c*d^3 - 3
*d^4 + (2*c^4 + 12*c^3*d + 42*c^2*d^2 - 128*c*d^3 + 117*d^4 - 15*(4*c*d^3 -
 3*d^4)*f*x)*cos(f*x + e)^3 + 60*(4*c*d^3 - 3*d^4)*f*x - (4*c^4 + 24*c^3*d
- 6*c^2*d^2 - 76*c*d^3 + 84*d^4 + 45*(4*c*d^3 - 3*d^4)*f*x)*cos(f*x + e)^2
- 3*(3*c^4 + 8*c^3*d + 18*c^2*d^2 - 72*c*d^3 + 63*d^4 - 10*(4*c*d^3 - 3*d^4
)*f*x)*cos(f*x + e) + (15*d^4*cos(f*x + e)^3 + 3*c^4 - 12*c^3*d + 18*c^2*d^
2 - 12*c*d^3 + 3*d^4 + 60*(4*c*d^3 - 3*d^4)*f*x - (2*c^4 + 12*c^3*d + 42*c^
2*d^2 - 128*c*d^3 + 102*d^4 + 15*(4*c*d^3 - 3*d^4)*f*x)*cos(f*x + e)^2 - 6*
(c^4 + 6*c^3*d + 6*c^2*d^2 - 34*c*d^3 + 31*d^4 - 5*(4*c*d^3 - 3*d^4)*f*x)*c
os(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 -
 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x +
 e) - 4*a^3*f)*sin(f*x + e))
```

giac [B] time = 0.26, size = 395, normalized size = 2.03

$$-\frac{\frac{30\,d^4}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)a^3}-\frac{15\left(4\,cd^3-3\,d^4\right)\left(fx+e\right)}{a^3}+\frac{2\left(15\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-60\,cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+45\,d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+30\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/15*(30*d^4/((tan(1/2*f*x + 1/2*e)^2 + 1)*a^3) - 15*(4*c*d^3 - 3*d^4)*(f*
x + e)/a^3 + 2*(15*c^4*tan(1/2*f*x + 1/2*e)^4 - 60*c*d^3*tan(1/2*f*x + 1/2*
e)^4 + 45*d^4*tan(1/2*f*x + 1/2*e)^4 + 30*c^4*tan(1/2*f*x + 1/2*e)^3 + 60*c
^3*d*tan(1/2*f*x + 1/2*e)^3 - 300*c*d^3*tan(1/2*f*x + 1/2*e)^3 + 210*d^4*ta
n(1/2*f*x + 1/2*e)^3 + 40*c^4*tan(1/2*f*x + 1/2*e)^2 + 60*c^3*d*tan(1/2*f*x
```

```
 + 1/2*e)^2 + 120*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 - 580*c*d^3*tan(1/2*f*x +
1/2*e)^2 + 360*d^4*tan(1/2*f*x + 1/2*e)^2 + 20*c^4*tan(1/2*f*x + 1/2*e) + 6
0*c^3*d*tan(1/2*f*x + 1/2*e) + 60*c^2*d^2*tan(1/2*f*x + 1/2*e) - 380*c*d^3*
tan(1/2*f*x + 1/2*e) + 240*d^4*tan(1/2*f*x + 1/2*e) + 7*c^4 + 12*c^3*d + 12
*c^2*d^2 - 88*c*d^3 + 57*d^4)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

maple [B] time = 0.29, size = 593, normalized size = 3.04

$$-\frac{2d^4}{a^3 f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}+\frac{8d^3 \arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c}{a^3 f}-\frac{6d^4 \arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a^3 f}-\frac{2c^4}{a^3 f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x)
```

```
[Out] -2/a^3/f*d^4/(1+tan(1/2*f*x+1/2*e)^2)+8/a^3/f*d^3*arctan(tan(1/2*f*x+1/2*e)
)*c-6/a^3/f*d^4*arctan(tan(1/2*f*x+1/2*e))-2/a^3/f/(tan(1/2*f*x+1/2*e)+1)*c
^4+8/a^3/f/(tan(1/2*f*x+1/2*e)+1)*c*d^3-6/a^3/f/(tan(1/2*f*x+1/2*e)+1)*d^4+
4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^2*c^4-8/a^3/f/(tan(1/2*f*x+1/2*e)+1)^2*c^3*d
+8/a^3/f/(tan(1/2*f*x+1/2*e)+1)^2*c*d^3-4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^2*d^
4+4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*c^4-16/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*c^
3*d+24/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*c^2*d^2-16/a^3/f/(tan(1/2*f*x+1/2*e)+
1)^4*c*d^3+4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*d^4-8/5/a^3/f/(tan(1/2*f*x+1/2*
e)+1)^5*c^4+32/5/a^3/f/(tan(1/2*f*x+1/2*e)+1)^5*c^3*d-48/5/a^3/f/(tan(1/2*f
*x+1/2*e)+1)^5*c^2*d^2+32/5/a^3/f/(tan(1/2*f*x+1/2*e)+1)^5*c*d^3-8/5/a^3/f/
(tan(1/2*f*x+1/2*e)+1)^5*d^4-16/3/a^3/f*c^4/(tan(1/2*f*x+1/2*e)+1)^3+16/a^3
/f*c^3/(tan(1/2*f*x+1/2*e)+1)^3*d-16/a^3/f*c^2/(tan(1/2*f*x+1/2*e)+1)^3*d^2
+16/3/a^3/f*c/(tan(1/2*f*x+1/2*e)+1)^3*d^3
```

maxima [B] time = 0.45, size = 1101, normalized size = 5.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(3*d^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*si
n(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x
+ e) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 11*
a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x + e
```

```
)/(cos(f*x + e) + 1))/a^3) - 4*c*d^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) +
 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x
+ e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a
^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/a^3) + c^4*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
+ 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(co
s(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
+ a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 12*c^2*d^2*(5*sin(f*x + e)/(co
s(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*
sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 12*c^3*d*(5*si
n(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x
+ e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

mupad [B] time = 9.03, size = 440, normalized size = 2.26

$$\frac{2\,d^3\,\mathrm{atan}\left(\frac{2\,d^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(4\,c-3\,d)}{8\,c\,d^3-6\,d^4}\right)\,(4\,c-3\,d)}{a^3\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(\frac{22\,c^4}{3}+8\,c^3\,d+16\,c^2\,d^2-\frac{256\,c\,d^3}{3}+64\,d^4\right)+\tan\left(\frac{e}{2}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^4/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (2*d^3*atan((2*d^3*tan(e/2 + (f*x)/2)*(4*c - 3*d))/(8*c*d^3 - 6*d^4))*(4*c
- 3*d))/(a^3*f) - (tan(e/2 + (f*x)/2)^4*(8*c^3*d - (256*c*d^3)/3 + (22*c^4)
/3 + 64*d^4 + 16*c^2*d^2) + tan(e/2 + (f*x)/2)^3*(16*c^3*d - (272*c*d^3)/3
+ (20*c^4)/3 + 80*d^4 + 8*c^2*d^2) + tan(e/2 + (f*x)/2)^2*((48*c^3*d)/5 - (
1336*c*d^3)/15 + (94*c^4)/15 + (378*d^4)/5 + (88*c^2*d^2)/5) + tan(e/2 + (f
*x)/2)^5*(8*c^3*d - 40*c*d^3 + 4*c^4 + 30*d^4) - (176*c*d^3)/15 + (8*c^3*d)
/5 + tan(e/2 + (f*x)/2)^6*(2*c^4 - 8*c*d^3 + 6*d^4) + (14*c^4)/15 + (48*d^4
)/5 + tan(e/2 + (f*x)/2)*(8*c^3*d - (152*c*d^3)/3 + (8*c^4)/3 + 42*d^4 + 8*
c^2*d^2) + (8*c^2*d^2)/5)/(f*(11*a^3*tan(e/2 + (f*x)/2)^2 + 15*a^3*tan(e/2
+ (f*x)/2)^3 + 15*a^3*tan(e/2 + (f*x)/2)^4 + 11*a^3*tan(e/2 + (f*x)/2)^5 +
5*a^3*tan(e/2 + (f*x)/2)^6 + a^3*tan(e/2 + (f*x)/2)^7 + a^3 + 5*a^3*tan(e/2
 + (f*x)/2)))
```

sympy [A] time = 48.33, size = 7373, normalized size = 37.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**4/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*c**4*tan(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75
*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*t
an(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 +
f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*c**4*tan(e/2 + f*x
/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165
*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*t
an(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f
*x/2) + 15*a**3*f) - 110*c**4*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/
2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 22
5*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*
tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 100*c**4*ta
n(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/
2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 2
25*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*
tan(e/2 + f*x/2) + 15*a**3*f) - 94*c**4*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(
e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/
2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 1
65*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 4
0*c**4*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2
+ f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)*
*4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a
**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 14*c**4/(15*a**3*f*tan(e/2 + f*x/2)**
7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a*
*3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(
e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 120*c**3*d*tan(
e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)
**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225
*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*ta
n(e/2 + f*x/2) + 15*a**3*f) - 120*c**3*d*tan(e/2 + f*x/2)**4/(15*a**3*f*tan
(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x
/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 +
165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) -
240*c**3*d*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*t
an(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 +
f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2
 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 144*c**3*d*tan(e/2 + f*x/2)**2
/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*
```

f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 120*c**3*d*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 24*c**3*d/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 240*c**2*d**2*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 120*c**2*d**2*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 264*c**2*d**2*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 120*c**2*d**2*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 24*c**2*d**2/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 60*c*d**3*f*x*tan(e/2 + f*x/2)**7/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 300*c*d**3*f*x*tan(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 660*c*d**3*f*x*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 900*c*d**3*f*x*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 900*c*d**3*f*x*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2)

```
 + 15*a**3*f) + 660*c*d**3*f*x*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x
/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 2
25*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f
*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 300*c*d**3
*f*x*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 +
f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4
 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**
3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 60*c*d**3*f*x/(15*a**3*f*tan(e/2 + f*x/
2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 22
5*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*
tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 120*c*d**3*
tan(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*
x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 +
 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*
f*tan(e/2 + f*x/2) + 15*a**3*f) + 600*c*d**3*tan(e/2 + f*x/2)**5/(15*a**3*f
*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 +
 f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**
3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f
) + 1280*c*d**3*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**
3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e
/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/
2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 1360*c*d**3*tan(e/2 + f*x
/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165
*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*t
an(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f
*x/2) + 15*a**3*f) + 1336*c*d**3*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f
*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 +
 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3
*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 760*c*d*
*3*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*
x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 +
 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*
f*tan(e/2 + f*x/2) + 15*a**3*f) + 176*c*d**3/(15*a**3*f*tan(e/2 + f*x/2)**7
 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**
3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e
/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 45*d**4*f*x*tan(
e/2 + f*x/2)**7/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)
**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225
*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*ta
n(e/2 + f*x/2) + 15*a**3*f) - 225*d**4*f*x*tan(e/2 + f*x/2)**6/(15*a**3*f*t
an(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f
*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3
+ 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f)
- 495*d**4*f*x*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3
*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/
```

```
2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2
)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 675*d**4*f*x*tan(e/2 + f*x
/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165
*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*t
an(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f
*x/2) + 15*a**3*f) - 675*d**4*f*x*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 +
f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5
+ 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**
3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 495*d**
4*f*x*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/
2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2
)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75
*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 225*d**4*f*x*tan(e/2 + f*x/2)/(15*a
**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(
e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x
/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a
**3*f) - 45*d**4*f*x/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f
*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4
+ 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3
*f*tan(e/2 + f*x/2) + 15*a**3*f) - 90*d**4*tan(e/2 + f*x/2)**6/(15*a**3*f*t
an(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f
*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3
+ 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f)
- 450*d**4*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*t
an(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 +
f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2
 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 960*d**4*tan(e/2 + f*x/2)**4/(
15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*
tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 +
 f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) +
15*a**3*f) - 1200*d**4*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 +
 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*
f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2
 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 1134*d**4*tan(e/2
+ f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6
+ 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**
3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/
2 + f*x/2) + 15*a**3*f) - 630*d**4*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*
x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 +
225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*
f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 144*d**4/
(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f
*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2
+ f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) +
 15*a**3*f), Ne(f, 0)), (x*(c + d*sin(e))**4/(a*sin(e) + a)**3, True))
```

3.473 $\int \frac{(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=142

$$-\frac{(c-d)\left(2c^2+11cd+29d^2\right)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}+\frac{d^3x}{a^3}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a\sin(e+fx)+a)^3}-\frac{(c-d)^2(2c+7d)\cos(e}{15af(a\sin(e+fx)+}$$

```
[Out] d^3*x/a^3-1/15*(c-d)^2*(2*c+7*d)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^2-1/15*(c-
d)*(2*c^2+11*c*d+29*d^2)*cos(f*x+e)/f/(a^3+a^3*sin(f*x+e))-1/5*(c-d)*cos(f*
x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(f*x+e))^3
```

Rubi [A] time = 0.33, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2765, 2968, 3019, 2735, 2648}

$$-\frac{(c-d)\left(2c^2+11cd+29d^2\right)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}+\frac{d^3x}{a^3}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a\sin(e+fx)+a)^3}-\frac{(c-d)^2(2c+7d)\cos(e}{15af(a\sin(e+fx)+}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (d^3*x)/a^3 - ((c - d)^2*(2*c + 7*d)*Cos[e + f*x])/(15*a*f*(a + a*Sin[e + f
*x])^2) - ((c - d)*(2*c^2 + 11*c*d + 29*d^2)*Cos[e + f*x])/(15*f*(a^3 + a^3
*Sin[e + f*x])) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(5*f*(a + a
*Sin[e + f*x])^3)
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
```

```
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a+a\sin(e+fx))^3} - \frac{\int \frac{(c+d\sin(e+fx))\left(-a\left(2c^2+5cd-2d^2\right)-5ad^2\sin(e+fx)\right.}{(a+a\sin(e+fx))^2}}{5a^2}$$

$$= -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a+a\sin(e+fx))^3} - \frac{\int \frac{-ac\left(2c^2+5cd-2d^2\right)+\left(-5acd^2-ad\left(2c^2+5cd-2d^2\right)\right)\sin}{(a+a\sin(e+fx))^2}}{5a^2}$$

$$= -\frac{(c-d)^2(2c+7d)\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a+a\sin(e+fx))^3} + \frac{\int \frac{a^2\left(2c^3+\right.}{}}{}$$

$$= \frac{d^3x}{a^3} - \frac{(c-d)^2(2c+7d)\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a+a\sin(e+fx))^3} + \frac{((c}{}$$

$$= \frac{d^3x}{a^3} - \frac{(c-d)^2(2c+7d)\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{(c-d)\left(2c^2+11cd+29d^2\right)\cos(e+fx)}{15f\left(a^3+a^3\sin(e+fx)\right)} - \frac{(c}{}$$

Mathematica [B] time = 5.61, size = 408, normalized size = 2.87

$$\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(40c^3\sin\left(\frac{1}{2}(e+fx)\right)-4c^3\sin\left(\frac{5}{2}(e+fx)\right)+30d\cos\left(\frac{1}{2}(e+fx)\right)\left(3c^2+6c\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(30*d*(3*c^2 + 6*c*d + d^2*(-9 + 5*e
 + 5*f*x))*Cos[(e + f*x)/2] - 5*(4*c^3 + 18*c^2*d + 24*c*d^2 + d^3*(-46 + 1
5*e + 15*f*x))*Cos[(3*(e + f*x))/2] - 15*d^3*e*Cos[(5*(e + f*x))/2] - 15*d^
3*f*x*Cos[(5*(e + f*x))/2] + 40*c^3*Sin[(e + f*x)/2] + 90*c^2*d*Sin[(e + f*
x)/2] + 240*c*d^2*Sin[(e + f*x)/2] - 370*d^3*Sin[(e + f*x)/2] + 150*d^3*e*S
in[(e + f*x)/2] + 150*d^3*f*x*Sin[(e + f*x)/2] + 90*c*d^2*Sin[(3*(e + f*x))
/2] - 90*d^3*Sin[(3*(e + f*x))/2] + 75*d^3*e*Sin[(3*(e + f*x))/2] + 75*d^3*
f*x*Sin[(3*(e + f*x))/2] - 4*c^3*Sin[(5*(e + f*x))/2] - 18*c^2*d*Sin[(5*(e
+ f*x))/2] - 42*c*d^2*Sin[(5*(e + f*x))/2] + 64*d^3*Sin[(5*(e + f*x))/2] -
15*d^3*e*Sin[(5*(e + f*x))/2] - 15*d^3*f*x*Sin[(5*(e + f*x))/2]))/(60*a^3*f
*(1 + Sin[e + f*x])^3)
```

fricas [B] time = 0.44, size = 352, normalized size = 2.48

$$-\frac{60\,d^3fx-\left(15\,d^3fx-2\,c^3-9\,c^2d-21\,cd^2+32\,d^3\right)\cos\left(fx+e\right)^3-3\,c^3+9\,c^2d-9\,cd^2+3\,d^3-\left(45\,d^3fx+4\,c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(60*d^3*f*x - (15*d^3*f*x - 2*c^3 - 9*c^2*d - 21*c*d^2 + 32*d^3)*cos(
f*x + e)^3 - 3*c^3 + 9*c^2*d - 9*c*d^2 + 3*d^3 - (45*d^3*f*x + 4*c^3 + 18*c
^2*d - 3*c*d^2 - 19*d^3)*cos(f*x + e)^2 + 3*(10*d^3*f*x - 3*c^3 - 6*c^2*d -
 9*c*d^2 + 18*d^3)*cos(f*x + e) + (60*d^3*f*x + 3*c^3 - 9*c^2*d + 9*c*d^2 -
 3*d^3 - (15*d^3*f*x + 2*c^3 + 9*c^2*d + 21*c*d^2 - 32*d^3)*cos(f*x + e)^2
+ 3*(10*d^3*f*x - 2*c^3 - 9*c^2*d - 6*c*d^2 + 17*d^3)*cos(f*x + e))*sin(f*x
 + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e
) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f
*x + e))
```

giac [B] time = 0.23, size = 280, normalized size = 1.97

$$\frac{15\left(fx+e\right)d^3}{a^3}-\frac{2\left(15\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-15\,d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+30\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+45\,c^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-75\,d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+40\,c^3\tan\left(\frac{1}{2}\,f\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*(f*x + e)*d^3/a^3 - 2*(15*c^3*tan(1/2*f*x + 1/2*e)^4 - 15*d^3*tan(
1/2*f*x + 1/2*e)^4 + 30*c^3*tan(1/2*f*x + 1/2*e)^3 + 45*c^2*d*tan(1/2*f*x +
 1/2*e)^3 - 75*d^3*tan(1/2*f*x + 1/2*e)^3 + 40*c^3*tan(1/2*f*x + 1/2*e)^2 +
 45*c^2*d*tan(1/2*f*x + 1/2*e)^2 + 60*c*d^2*tan(1/2*f*x + 1/2*e)^2 - 145*d^
3*tan(1/2*f*x + 1/2*e)^2 + 20*c^3*tan(1/2*f*x + 1/2*e) + 45*c^2*d*tan(1/2*f
*x + 1/2*e) + 30*c*d^2*tan(1/2*f*x + 1/2*e) - 95*d^3*tan(1/2*f*x + 1/2*e) +
 7*c^3 + 9*c^2*d + 6*c*d^2 - 22*d^3)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

maple [B] time = 0.25, size = 438, normalized size = 3.08

$$\frac{2d^3 \arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a^3 f}-\frac{2c^3}{a^3 f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{2d^3}{a^3 f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{4c^3}{a^3 f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{}{a^3 f\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/a^3/f*d^3*arctan(tan(1/2*f*x+1/2*e))-2*c^3/a^3/f/(tan(1/2*f*x+1/2*e)+1)+2
/a^3/f/(tan(1/2*f*x+1/2*e)+1)*d^3+4*c^3/a^3/f/(tan(1/2*f*x+1/2*e)+1)^2-6/a^
3/f/(tan(1/2*f*x+1/2*e)+1)^2*c^2*d+2/a^3/f/(tan(1/2*f*x+1/2*e)+1)^2*d^3+4*c
^3/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4-12/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*c^2*d+1
2/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*c*d^2-4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*d^3
-8/5*c^3/a^3/f/(tan(1/2*f*x+1/2*e)+1)^5+24/5/a^3/f/(tan(1/2*f*x+1/2*e)+1)^5
*c^2*d-24/5/a^3/f/(tan(1/2*f*x+1/2*e)+1)^5*c*d^2+8/5/a^3/f/(tan(1/2*f*x+1/2
*e)+1)^5*d^3-16/3*c^3/a^3/f/(tan(1/2*f*x+1/2*e)+1)^3+12/a^3/f/(tan(1/2*f*x+
1/2*e)+1)^3*c^2*d-8/a^3/f/(tan(1/2*f*x+1/2*e)+1)^3*c*d^2+4/3/a^3/f/(tan(1/2
*f*x+1/2*e)+1)^3*d^3
```

maxima [B] time = 0.44, size = 784, normalized size = 5.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 2/15*(d^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 145*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 1
0*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^
```

```
5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) -
 c^3*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1
)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f
*x + e) + 1)^5) - 6*c*d^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1
) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5) - 9*c^2*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) +
 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1
)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^
3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) +
1)^5))/f
```

mupad [B] time = 9.90, size = 240, normalized size = 1.69

$$\frac{d^3\,x}{a^3}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(2\,c^3-2\,d^3\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{16\,c^3}{3}+6\,c^2\,d+8\,c\,d^2-\frac{58\,d^3}{3}\right)+\frac{4\,c\,d^2}{5}+\frac{6\,c^2\,d}{5}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\cdots}{f\,\left(a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+5\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+10\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+10\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^3/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (d^3*x)/a^3 - (tan(e/2 + (f*x)/2)^4*(2*c^3 - 2*d^3) + tan(e/2 + (f*x)/2)^2*
(8*c*d^2 + 6*c^2*d + (16*c^3)/3 - (58*d^3)/3) + (4*c*d^2)/5 + (6*c^2*d)/5 +
 tan(e/2 + (f*x)/2)^3*(6*c^2*d + 4*c^3 - 10*d^3) + (14*c^3)/15 - (44*d^3)/1
5 + tan(e/2 + (f*x)/2)*(4*c*d^2 + 6*c^2*d + (8*c^3)/3 - (38*d^3)/3))/(f*(10
*a^3*tan(e/2 + (f*x)/2)^2 + 10*a^3*tan(e/2 + (f*x)/2)^3 + 5*a^3*tan(e/2 + (
f*x)/2)^4 + a^3*tan(e/2 + (f*x)/2)^5 + a^3 + 5*a^3*tan(e/2 + (f*x)/2)))
```

sympy [A] time = 25.97, size = 2640, normalized size = 18.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*c**3*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75
*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*t
an(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*c**3*tan(
```

e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 80*c**3*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 40*c**3*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 14*c**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 90*c**2*d*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 90*c**2*d*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 90*c**2*d*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 18*c**2*d/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 120*c*d**2*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*c*d**2*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 12*c*d**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 15*d**3*f*x*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 75*d**3*f*x*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 150*d**3*f*x*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 150*d**3*f*x*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 75*d**3*f*x*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 15*d**3*f*x/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 30*d**3*tan(e/2 + f*x

```
/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150
*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*ta
n(e/2 + f*x/2) + 15*a**3*f) + 150*d**3*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e
/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2
)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**
3*f) + 290*d**3*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**
3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e
/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 190*d**3*tan(e/2
 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 +
150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f
*tan(e/2 + f*x/2) + 15*a**3*f) + 44*d**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 7
5*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*
tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0)), (
x*(c + d*sin(e))**3/(a*sin(e) + a)**3, True))
```

3.474 $\int \frac{(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=125

$$-\frac{\left(2c^2+6cd+7d^2\right)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{5f(a\sin(e+fx)+a)^3}-\frac{(c-d)(2c+5d)\cos(e+fx)}{15af(a\sin(e+fx)+a)^2}$$

```
[Out] -1/15*(c-d)*(2*c+5*d)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^2-1/15*(2*c^2+6*c*d+7
*d^2)*cos(f*x+e)/f/(a^3+a^3*sin(f*x+e))-1/5*(c-d)*cos(f*x+e)*(c+d*sin(f*x+e
))/f/(a+a*sin(f*x+e))^3
```

Rubi [A] time = 0.18, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2760, 2750, 2648}

$$-\frac{\left(2c^2+6cd+7d^2\right)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{5f(a\sin(e+fx)+a)^3}-\frac{(c-d)(2c+5d)\cos(e+fx)}{15af(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -((c - d)*(2*c + 5*d)*Cos[e + f*x])/(15*a*f*(a + a*Sin[e + f*x])^2) - ((2*c
^2 + 6*c*d + 7*d^2)*Cos[e + f*x])/(15*f*(a^3 + a^3*Sin[e + f*x])) - ((c - d
)*Cos[e + f*x]*(c + d*Sin[e + f*x]))/(5*f*(a + a*Sin[e + f*x])^3)
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2760

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
```

```
*x])^m*(c + d*Sin[e + f*x]))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*c*d*(m - 1) + b*(d^2 + c^2*(m + 1))
 + d*(a*d*(m - 1) + b*c*(m + 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{5f(a+a\sin(e+fx))^3} - \frac{\int \frac{-a\left(2c^2+4cd-d^2\right)-ad(c+4d)\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx}{5a^2}$$

$$= -\frac{(c-d)(2c+5d)\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{5f(a+a\sin(e+fx))^3} + \frac{\left(2c^2+6c\right.}{}$$

$$= -\frac{(c-d)(2c+5d)\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{\left(2c^2+6cd+7d^2\right)\cos(e+fx)}{15f\left(a^3+a^3\sin(e+fx)\right)} - \frac{(c-d)\cos(e+f}{5f(a+a\,s}$$

Mathematica [A] time = 0.13, size = 84, normalized size = 0.67

$$-\frac{\cos(e+fx)\left(\left(2c^2+6cd+7d^2\right)\sin^2(e+fx)+6\left(c^2+3cd+d^2\right)\sin(e+fx)+7c^2+6cd+2d^2\right)}{15a^3f(\sin(e+fx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -1/15*(Cos[e + f*x]*(7*c^2 + 6*c*d + 2*d^2 + 6*(c^2 + 3*c*d + d^2)*Sin[e +
f*x] + (2*c^2 + 6*c*d + 7*d^2)*Sin[e + f*x]^2))/(a^3*f*(1 + Sin[e + f*x])^3
)
```

fricas [B] time = 0.44, size = 242, normalized size = 1.94

$$-\frac{\left(2\,c^2+6\,cd+7\,d^2\right)\cos\left(fx+e\right)^3-\left(4\,c^2+12\,cd-d^2\right)\cos\left(fx+e\right)^2-3\,c^2+6\,cd-3\,d^2-3\left(3\,c^2+4\,cd+3\,d^2\right.}{15\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2-2\,a^3f\cos\left(fx+e\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/15*((2*c^2 + 6*c*d + 7*d^2)*cos(f*x + e)^3 - (4*c^2 + 12*c*d - d^2)*cos(
f*x + e)^2 - 3*c^2 + 6*c*d - 3*d^2 - 3*(3*c^2 + 4*c*d + 3*d^2)*cos(f*x + e)
```

```
 - ((2*c^2 + 6*c*d + 7*d^2)*cos(f*x + e)^2 - 3*c^2 + 6*c*d - 3*d^2 + 6*(c^2
 + 3*c*d + d^2)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f
*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 -
2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

giac [A] time = 2.21, size = 181, normalized size = 1.45

$$-\frac{2\left(15\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+30\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+30\,cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+40\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+30\,cd\,\mathrm{ta}\right.}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*c^2*tan(1/2*f*x + 1/2*e)^4 + 30*c^2*tan(1/2*f*x + 1/2*e)^3 + 30*c
*d*tan(1/2*f*x + 1/2*e)^3 + 40*c^2*tan(1/2*f*x + 1/2*e)^2 + 30*c*d*tan(1/2*
f*x + 1/2*e)^2 + 20*d^2*tan(1/2*f*x + 1/2*e)^2 + 20*c^2*tan(1/2*f*x + 1/2*e
) + 30*c*d*tan(1/2*f*x + 1/2*e) + 10*d^2*tan(1/2*f*x + 1/2*e) + 7*c^2 + 6*c
*d + 2*d^2)/(a^3*f*(tan(1/2*f*x + 1/2*e) + 1)^5)
```

maple [A] time = 0.24, size = 139, normalized size = 1.11

$$\frac{-\frac{-8c^2+16cd-8d^2}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2\left(4c^2-8cd+4d^2\right)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{2\left(8c^2-12cd+4d^2\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2c^2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}+\frac{4c(c-d)}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}}{f\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/f/a^3*(-1/4*(-8*c^2+16*c*d-8*d^2)/(tan(1/2*f*x+1/2*e)+1)^4-1/5*(4*c^2-8*c
*d+4*d^2)/(tan(1/2*f*x+1/2*e)+1)^5-1/3*(8*c^2-12*c*d+4*d^2)/(tan(1/2*f*x+1/
2*e)+1)^3-c^2/(tan(1/2*f*x+1/2*e)+1)+2*c*(c-d)/(tan(1/2*f*x+1/2*e)+1)^2)
```

maxima [B] time = 0.34, size = 553, normalized size = 4.42

$$-\frac{2\left(\frac{c^2\left(\frac{20\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{40\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{30\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{15\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+7\right)}{a^3+\frac{5a^3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{10a^3\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{10a^3\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{5a^3\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+\frac{a^3\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}}+\frac{2d^2\left(\frac{5\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{10\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+1\right.}{a^3+\frac{5a^3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{10a^3\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{10a^3\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{5a^3\sin}{\left(\cos(fx\right.}}\right.}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(c^2*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*
a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) + 2*d^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5) + 6*c*d*(5*sin(f*x + e)/(cos(f*x + e) + 1)
 + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*
a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5))/f
```

mupad [B] time = 7.49, size = 218, normalized size = 1.74

$$\frac{2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(6\,c\,d-4\,c^2\,\cos\left(e+f\,x\right)+d^2\,\cos\left(e+f\,x\right)+\frac{25\,c^2\,\sin\left(e+f\,x\right)}{2}+\frac{5\,d^2\,\sin\left(e+f\,x\right)}{2}+\frac{53\,c^2}{4}+\frac{13\,d^2}{4}-\frac{9\,c^2\,c\cdots}{\cdots}\right.}{15\,a^3\,f\left(\frac{5\,\sqrt{2}\,\cos\left(\frac{3\,e}{2}+\frac{\pi}{4}+\frac{3\,f\cdots}{2}\right)}{4}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^2/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*(6*c*d - 4*c^2*cos(e + f*x) + d^2*cos(e + f*x) + (25*
c^2*sin(e + f*x))/2 + (5*d^2*sin(e + f*x))/2 + (53*c^2)/4 + (13*d^2)/4 - (9
*c^2*cos(2*e + 2*f*x))/4 - (9*d^2*cos(2*e + 2*f*x))/4 - (5*c^2*sin(2*e + 2*
f*x))/4 + (5*d^2*sin(2*e + 2*f*x))/4 + 3*c*d*cos(e + f*x) + 15*c*d*sin(e +
f*x) - 3*c*d*cos(2*e + 2*f*x)))/(15*a^3*f*((5*2^(1/2)*cos((3*e)/2 + pi/4 +
(3*f*x)/2))/4 - (5*2^(1/2)*cos(e/2 - pi/4 + (f*x)/2))/2 + (2^(1/2)*cos((5*e
)/2 - pi/4 + (5*f*x)/2))/4))
```

sympy [A] time = 15.74, size = 1365, normalized size = 10.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*c**2*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75
*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*t
```

```
an(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*c**2*tan(
e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)
**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*
a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 80*c**2*tan(e/2 + f*x/2)**2/(15*a**3
*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2
 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2)
+ 15*a**3*f) - 40*c**2*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75
*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*t
an(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 14*c**2/(15*
a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan
(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x
/2) + 15*a**3*f) - 60*c*d*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**
5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a*
*3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*c*d
*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f
*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2
+ 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*c*d*tan(e/2 + f*x/2)/(15*a**
3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/
2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2)
 + 15*a**3*f) - 12*c*d/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 +
 f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**
2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 40*d**2*tan(e/2 + f*x/2)**2/(
15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*
tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 +
f*x/2) + 15*a**3*f) - 20*d**2*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)*
*5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a
**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 4*d**
2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3
*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2
 + f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(c + d*sin(e))**2/(a*sin(e) + a)**3,
True))
```

3.475 $\int \frac{c+d\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=102

$$-\frac{(2c+3d)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}-\frac{(2c+3d)\cos(e+fx)}{15af(a\sin(e+fx)+a)^2}-\frac{(c-d)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

```
[Out] -1/5*(c-d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^3-1/15*(2*c+3*d)*cos(f*x+e)/a/f/(a
+a*sin(f*x+e))^2-1/15*(2*c+3*d)*cos(f*x+e)/f/(a^3+a^3*sin(f*x+e))
```

Rubi [A] time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2750, 2650, 2648}

$$-\frac{(2c+3d)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}-\frac{(2c+3d)\cos(e+fx)}{15af(a\sin(e+fx)+a)^2}-\frac{(c-d)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -((c - d)*Cos[e + f*x])/(5*f*(a + a*Sin[e + f*x])^3) - ((2*c + 3*d)*Cos[e +
 f*x])/(15*a*f*(a + a*Sin[e + f*x])^2) - ((2*c + 3*d)*Cos[e + f*x])/(15*f*(
a^3 + a^3*Sin[e + f*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
```

```
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{c+d\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx &= -\frac{(c-d)\cos(e+fx)}{5f(a+a\sin(e+fx))^3}+\frac{(2c+3d)\int \frac{1}{(a+a\sin(e+fx))^2}\,dx}{5a}\\ &= -\frac{(c-d)\cos(e+fx)}{5f(a+a\sin(e+fx))^3}-\frac{(2c+3d)\cos(e+fx)}{15af(a+a\sin(e+fx))^2}+\frac{(2c+3d)\int \frac{1}{a+a\sin(e+fx)}\,dx}{15a^2}\\ &= -\frac{(c-d)\cos(e+fx)}{5f(a+a\sin(e+fx))^3}-\frac{(2c+3d)\cos(e+fx)}{15af(a+a\sin(e+fx))^2}-\frac{(2c+3d)\cos(e+fx)}{15f\left(a^3+a^3\sin(e+fx)\right)}\end{aligned}$$

Mathematica [A] time = 0.08, size = 63, normalized size = 0.62

$$-\frac{\cos(e+fx)\left((2c+3d)\sin^2(e+fx)+(6c+9d)\sin(e+fx)+7c+3d\right)}{15a^3f(\sin(e+fx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -1/15*(Cos[e + f*x]*(7*c + 3*d + (6*c + 9*d)*Sin[e + f*x] + (2*c + 3*d)*Sin
[e + f*x]^2))/(a^3*f*(1 + Sin[e + f*x])^3)
```

fricas [A] time = 0.42, size = 190, normalized size = 1.86

$$-\frac{(2\,c+3\,d)\cos\left(fx+e\right)^3-2\,(2\,c+3\,d)\cos\left(fx+e\right)^2-3\,(3\,c+2\,d)\cos\left(fx+e\right)-\left((2\,c+3\,d)\cos\left(fx+e\right)^2+3\right.}{15\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2-2\,a^3f\cos\left(fx+e\right)-4\,a^3f+\left(a^3f\cos\left(fx+e\right)^2-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/15*((2*c + 3*d)*cos(f*x + e)^3 - 2*(2*c + 3*d)*cos(f*x + e)^2 - 3*(3*c +
 2*d)*cos(f*x + e) - ((2*c + 3*d)*cos(f*x + e)^2 + 3*(2*c + 3*d)*cos(f*x +
e) - 3*c + 3*d)*sin(f*x + e) - 3*c + 3*d)/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*c
os(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*
a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

giac [A] time = 0.26, size = 130, normalized size = 1.27

$$-\frac{2\left(15\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+30\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+15\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+40\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+15\,d\tan\left(\right.\right.}{15\,a^{3}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*c*tan(1/2*f*x + 1/2*e)^4 + 30*c*tan(1/2*f*x + 1/2*e)^3 + 15*d*tan
(1/2*f*x + 1/2*e)^3 + 40*c*tan(1/2*f*x + 1/2*e)^2 + 15*d*tan(1/2*f*x + 1/2*
e)^2 + 20*c*tan(1/2*f*x + 1/2*e) + 15*d*tan(1/2*f*x + 1/2*e) + 7*c + 3*d)/(
a^3*f*(tan(1/2*f*x + 1/2*e) + 1)^5)
```

maple [A] time = 0.19, size = 114, normalized size = 1.12

$$\frac{-\frac{-4c+2d}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2(4c-4d)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{2(8c-6d)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{-8c+8d}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{2c}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{f\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/f/a^3*(-1/2*(-4*c+2*d)/(tan(1/2*f*x+1/2*e)+1)^2-1/5*(4*c-4*d)/(tan(1/2*f*
x+1/2*e)+1)^5-1/3*(8*c-6*d)/(tan(1/2*f*x+1/2*e)+1)^3-1/4*(-8*c+8*d)/(tan(1/
2*f*x+1/2*e)+1)^4-c/(tan(1/2*f*x+1/2*e)+1))
```

maxima [B] time = 0.33, size = 387, normalized size = 3.79

$$-\frac{2\left(\frac{c\left(\frac{20\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{40\,\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{30\,\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{15\,\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+7\right)}{a^{3}+\frac{5\,a^{3}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{10\,a^{3}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{10\,a^{3}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{5\,a^{3}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{a^{3}\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}}+\frac{3\,d\left(\frac{5\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{5\,\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{5\,\sin\left(f\right.}{\left(\cos\left(fx+\right.\right.}\right.}{a^{3}+\frac{5\,a^{3}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{10\,a^{3}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{10\,a^{3}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{5\,a^{3}\,s}{\left(\cos\left(\right.\right.}}\right.}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(c*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^
```

```
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5) + 3*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3
 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

mupad [B] time = 7.25, size = 150, normalized size = 1.47

$$\frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{53c}{4}+3d-4c\cos\left(e+fx\right)+\frac{3d\cos\left(e+fx\right)}{2}+\frac{25c\sin\left(e+fx\right)}{2}+\frac{15d\sin\left(e+fx\right)}{2}-\frac{9c\cos\left(2e+2fx\right)}{4}-\frac{3d\cos\left(2e+2fx\right)}{2}-\frac{5c\sin\left(2e+2fx\right)}{4}\right)}{15a^3f\left(\frac{5\sqrt{2}\cos\left(\frac{3e}{2}+\frac{\pi}{4}+\frac{3fx}{2}\right)}{4}-\frac{5\sqrt{2}\cos\left(\frac{e}{2}-\frac{\pi}{4}+\frac{fx}{2}\right)}{2}+\frac{\sqrt{2}\cos\left(\frac{5e}{2}-\frac{\pi}{4}+\frac{5fx}{2}\right)}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((53*c)/4 + 3*d - 4*c*cos(e + f*x) + (3*d*cos(e + f*x
))/2 + (25*c*sin(e + f*x))/2 + (15*d*sin(e + f*x))/2 - (9*c*cos(2*e + 2*f*x
))/4 - (3*d*cos(2*e + 2*f*x))/2 - (5*c*sin(2*e + 2*f*x))/4))/(15*a^3*f*((5*
2^(1/2)*cos((3*e)/2 + pi/4 + (3*f*x)/2))/4 - (5*2^(1/2)*cos(e/2 - pi/4 + (f
*x)/2))/2 + (2^(1/2)*cos((5*e)/2 - pi/4 + (5*f*x)/2))/4))
```

sympy [A] time = 9.06, size = 1015, normalized size = 9.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*c*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a*
*3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(
e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*c*tan(e/2 +
f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 +
150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f
*tan(e/2 + f*x/2) + 15*a**3*f) - 80*c*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/
2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)
**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3
*f) - 40*c*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(
e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x
/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 14*c/(15*a**3*f*tan(e/2
+ f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**
```

```
3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f
) - 30*d*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan
(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*
x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*d*tan(e/2 + f*x/2)**
2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3
*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2
 + f*x/2) + 15*a**3*f) - 30*d*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)*
*5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a
**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 6*d/(
15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*
tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 +
f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(c + d*sin(e))/(a*sin(e) + a)**3, True))
```

3.476 $\int \frac{1}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=83

$$-\frac{2\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}-\frac{2\cos(e+fx)}{15af(a\sin(e+fx)+a)^2}-\frac{\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

```
[Out] -1/5*cos(f*x+e)/f/(a+a*sin(f*x+e))^3-2/15*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^2
-2/15*cos(f*x+e)/f/(a^3+a^3*sin(f*x+e))
```

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2650, 2648}

$$-\frac{2\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}-\frac{2\cos(e+fx)}{15af(a\sin(e+fx)+a)^2}-\frac{\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(-3),x]
```

```
[Out] -Cos[e + f*x]/(5*f*(a + a*Sin[e + f*x])^3) - (2*Cos[e + f*x])/(15*a*f*(a +
a*Sin[e + f*x])^2) - (2*Cos[e + f*x])/(15*f*(a^3 + a^3*Sin[e + f*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3}\,dx = -\frac{\cos(e+fx)}{5f(a+a\sin(e+fx))^3} + \frac{2\int \frac{1}{(a+a\sin(e+fx))^2}\,dx}{5a}$$
$$= -\frac{\cos(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{2\cos(e+fx)}{15af(a+a\sin(e+fx))^2} + \frac{2\int \frac{1}{a+a\sin(e+fx)}\,dx}{15a^2}$$
$$= -\frac{\cos(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{2\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{2\cos(e+fx)}{15f\left(a^3+a^3\sin(e+fx)\right)}$$

Mathematica [A] time = 0.12, size = 76, normalized size = 0.92

$$\frac{15\sin(e+fx)-6\sin(2(e+fx))-\sin(3(e+fx))-15\cos(e+fx)-6\cos(2(e+fx))+\cos(3(e+fx))+10}{30a^3f(\sin(e+fx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(-3),x]
```

```
[Out] (10 - 15*Cos[e + f*x] - 6*Cos[2*(e + f*x)] + Cos[3*(e + f*x)] + 15*Sin[e +
f*x] - 6*Sin[2*(e + f*x)] - Sin[3*(e + f*x)])/(30*a^3*f*(1 + Sin[e + f*x])^
3)
```

fricas [A] time = 0.44, size = 147, normalized size = 1.77

$$-\frac{2\,\cos\left(fx+e\right)^3-4\,\cos\left(fx+e\right)^2-\left(2\,\cos\left(fx+e\right)^2+6\,\cos\left(fx+e\right)-3\right)\sin\left(fx+e\right)-9\,\cos}{15\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2-2\,a^3f\cos\left(fx+e\right)-4\,a^3f+\left(a^3f\cos\left(fx+e\right)^2-2\,a^3f\cos\left(fx\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(2*cos(f*x + e)^3 - 4*cos(f*x + e)^2 - (2*cos(f*x + e)^2 + 6*cos(f*x
+ e) - 3)*sin(f*x + e) - 9*cos(f*x + e) - 3)/(a^3*f*cos(f*x + e)^3 + 3*a^3*
f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 -
 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

giac [A] time = 0.39, size = 78, normalized size = 0.94

$$-\frac{2\left(15\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+30\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+40\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+20\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+7\right)}{15\,a^3f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*tan(1/2*f*x + 1/2*e)^4 + 30*tan(1/2*f*x + 1/2*e)^3 + 40*tan(1/2*f
*x + 1/2*e)^2 + 20*tan(1/2*f*x + 1/2*e) + 7)/(a^3*f*(tan(1/2*f*x + 1/2*e) +
 1)^5)
```

maple [A] time = 0.16, size = 85, normalized size = 1.02

$$\frac{\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{8}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{16}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}+\frac{4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}}{f\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/f/a^3*(2/(tan(1/2*f*x+1/2*e)+1)^2-4/5/(tan(1/2*f*x+1/2*e)+1)^5-8/3/(tan(1
/2*f*x+1/2*e)+1)^3-1/(tan(1/2*f*x+1/2*e)+1)+2/(tan(1/2*f*x+1/2*e)+1)^4)
```

maxima [B] time = 0.34, size = 203, normalized size = 2.45

$$-\frac{2\left(\frac{20\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{40\,\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{30\,\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{15\,\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+7\right)}{15\left(a^{3}+\frac{5\,a^{3}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{10\,a^{3}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{10\,a^{3}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{5\,a^{3}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{a^{3}\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 7)/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5)*f)
```

mupad [B] time = 6.96, size = 133, normalized size = 1.60

$$-\frac{2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(7\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}+20\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+40\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}+30\,\cos\left(\cdots\right.\right.}{15\,a^{3}\,f\left(\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sin(e + f*x))^3,x)
```

```
[Out] -(2*cos(e/2 + (f*x)/2)*(7*cos(e/2 + (f*x)/2)^4 + 15*sin(e/2 + (f*x)/2)^4 +
30*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x)/2)^3 + 20*cos(e/2 + (f*x)/2)^3*sin(e/
2 + (f*x)/2) + 40*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^2))/(15*a^3*f*(co
s(e/2 + (f*x)/2) + sin(e/2 + (f*x)/2))^5)
```

sympy [A] time = 3.54, size = 558, normalized size = 6.72

$$\begin{cases} -\frac{30\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{15a^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^3f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+150a^3f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+150a^3f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+15a^3f} - \frac{}{15a^3f\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)+75a^3f} \\ \frac{x}{(a\sin(e)+a)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3
*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/
2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*tan(e/2 + f*x/
2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*
a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan
(e/2 + f*x/2) + 15*a**3*f) - 80*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*
x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 +
150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) -
40*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*
x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 +
 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 14/(15*a**3*f*tan(e/2 + f*x/2)**
5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a*
*3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0
)), (x/(a*sin(e) + a)**3, True))
```

3.477 $\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=186

$$-\frac{\left(2c^2-9cd+22d^2\right)\cos(e+fx)}{15f(c-d)^3\left(a^3\sin(e+fx)+a^3\right)}-\frac{2d^3\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^3\sqrt{c^2-d^2}}-\frac{(2c-7d)\cos(e+fx)}{15af(c-d)^2(a\sin(e+fx)+a)^2}-\frac{\cos(e}{5f(c-d)(a\sin}$$

```
[Out] -1/5*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^3-1/15*(2*c-7*d)*cos(f*x+e)/a/(c-d
)^2/f/(a+a*sin(f*x+e))^2-1/15*(2*c^2-9*c*d+22*d^2)*cos(f*x+e)/(c-d)^3/f/(a^
3+a^3*sin(f*x+e))-2*d^3*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^
3/(c-d)^3/f/(c^2-d^2)^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2766, 2978, 12, 2660, 618, 204}

$$-\frac{2d^3\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^3\sqrt{c^2-d^2}}-\frac{\left(2c^2-9cd+22d^2\right)\cos(e+fx)}{15f(c-d)^3\left(a^3\sin(e+fx)+a^3\right)}-\frac{(2c-7d)\cos(e+fx)}{15af(c-d)^2(a\sin(e+fx)+a)^2}-\frac{\cos(e}{5f(c-d)(a\sin}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])),x]
```

```
[Out] (-2*d^3*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^3*(c - d)^3*Sq
rt[c^2 - d^2]*f) - Cos[e + f*x]/(5*(c - d)*f*(a + a*Sin[e + f*x])^3) - ((2*
c - 7*d)*Cos[e + f*x])/(15*a*(c - d)^2*f*(a + a*Sin[e + f*x])^2) - ((2*c^2
- 9*c*d + 22*d^2)*Cos[e + f*x])/(15*(c - d)^3*f*(a^3 + a^3*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx = -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{\int \frac{-a(2c-5d)-2ad\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx}{5a^2(c-d)}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2c-7d)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2c-7d)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2c-7d)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2c-7d)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2c-7d)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2}$$

$$= -\frac{2d^3\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^3\sqrt{c^2-d^2}\,f} - \frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - 15$$

Mathematica [A] time = 0.71, size = 301, normalized size = 1.62

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2\left(2c^2-9cd+22d^2\right)\sin\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4 -\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(6*(c - d)^2*Sin[(e + f*x)/2] - 3*(c
 - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(2*c - 7*d)*(c - d)*Sin[(
e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (c - d)*(-2*c + 7*d)*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 2*(2*c^2 - 9*c*d + 22*d^2)*Sin[(e
```

```
 + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - (30*d^3*ArcTan[(d + c*
Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
/Sqrt[c^2 - d^2]))/(15*a^3*(c - d)^3*f*(1 + Sin[e + f*x])^3)
```

fricas [B] time = 0.53, size = 1744, normalized size = 9.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/30*(6*c^4 - 12*c^3*d + 12*c*d^3 - 6*d^4 - 2*(2*c^4 - 9*c^3*d + 20*c^2*d^
2 + 9*c*d^3 - 22*d^4)*cos(f*x + e)^3 + 2*(4*c^4 - 18*c^3*d + 25*c^2*d^2 + 1
8*c*d^3 - 29*d^4)*cos(f*x + e)^2 + 15*(d^3*cos(f*x + e)^3 + 3*d^3*cos(f*x +
 e)^2 - 2*d^3*cos(f*x + e) - 4*d^3 + (d^3*cos(f*x + e)^2 - 2*d^3*cos(f*x +
e) - 4*d^3)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^
2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos
(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2
 - d^2)) + 6*(3*c^4 - 11*c^3*d + 15*c^2*d^2 + 11*c*d^3 - 18*d^4)*cos(f*x +
e) - 2*(3*c^4 - 6*c^3*d + 6*c*d^3 - 3*d^4 - (2*c^4 - 9*c^3*d + 20*c^2*d^2 +
 9*c*d^3 - 22*d^4)*cos(f*x + e)^2 - 3*(2*c^4 - 9*c^3*d + 15*c^2*d^2 + 9*c*d
^3 - 17*d^4)*cos(f*x + e))*sin(f*x + e))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^
3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^3 + 3*(a^3*c^
5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*
cos(f*x + e)^2 - 2*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 -
 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e) - 4*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c
^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f + ((a^3*c^5 - 3*a^3*c^4*d
 + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^2
- 2*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 +
a^3*d^5)*f*cos(f*x + e) - 4*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*
c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f)*sin(f*x + e)), 1/15*(3*c^4 - 6*c^3*d +
6*c*d^3 - 3*d^4 - (2*c^4 - 9*c^3*d + 20*c^2*d^2 + 9*c*d^3 - 22*d^4)*cos(f*x
 + e)^3 + (4*c^4 - 18*c^3*d + 25*c^2*d^2 + 18*c*d^3 - 29*d^4)*cos(f*x + e)^
2 + 15*(d^3*cos(f*x + e)^3 + 3*d^3*cos(f*x + e)^2 - 2*d^3*cos(f*x + e) - 4*
d^3 + (d^3*cos(f*x + e)^2 - 2*d^3*cos(f*x + e) - 4*d^3)*sin(f*x + e))*sqrt(
c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + 3
*(3*c^4 - 11*c^3*d + 15*c^2*d^2 + 11*c*d^3 - 18*d^4)*cos(f*x + e) - (3*c^4
- 6*c^3*d + 6*c*d^3 - 3*d^4 - (2*c^4 - 9*c^3*d + 20*c^2*d^2 + 9*c*d^3 - 22*
d^4)*cos(f*x + e)^2 - 3*(2*c^4 - 9*c^3*d + 15*c^2*d^2 + 9*c*d^3 - 17*d^4)*c
os(f*x + e))*sin(f*x + e))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*
c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^3 + 3*(a^3*c^5 - 3*a^3*c^4*
d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^2
 - 2*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 +
 a^3*d^5)*f*cos(f*x + e) - 4*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3
*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f + ((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d
```

```
^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^2 - 2*(a^3*c^5 -
 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos
(f*x + e) - 4*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^
3*c*d^4 + a^3*d^5)*f)*sin(f*x + e))]
```

giac [B] time = 0.34, size = 364, normalized size = 1.96

$$-\frac{2\left(\frac{15\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)d^3}{\left(a^3c^3-3a^3c^2d+3a^3cd^2-a^3d^3\right)\sqrt{c^2-d^2}}+\frac{15c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-45cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+45d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+30c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}\cdots\right.}{\cdots}\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -2/15*(15*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x
+ 1/2*e) + d)/sqrt(c^2 - d^2)))*d^3/((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 -
 a^3*d^3)*sqrt(c^2 - d^2)) + (15*c^2*tan(1/2*f*x + 1/2*e)^4 - 45*c*d*tan(1/
2*f*x + 1/2*e)^4 + 45*d^2*tan(1/2*f*x + 1/2*e)^4 + 30*c^2*tan(1/2*f*x + 1/2
*e)^3 - 105*c*d*tan(1/2*f*x + 1/2*e)^3 + 135*d^2*tan(1/2*f*x + 1/2*e)^3 + 4
0*c^2*tan(1/2*f*x + 1/2*e)^2 - 135*c*d*tan(1/2*f*x + 1/2*e)^2 + 185*d^2*tan
(1/2*f*x + 1/2*e)^2 + 20*c^2*tan(1/2*f*x + 1/2*e) - 75*c*d*tan(1/2*f*x + 1/
2*e) + 115*d^2*tan(1/2*f*x + 1/2*e) + 7*c^2 - 24*c*d + 32*d^2)/((a^3*c^3 -
3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

maple [A] time = 0.33, size = 325, normalized size = 1.75

$$-\frac{2d^3\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{a^3f\left(c-d\right)^3\sqrt{c^2-d^2}}+\frac{4c}{a^3f\left(c-d\right)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{6d}{a^3f\left(c-d\right)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{\cdots}{3a^3f\left(c-d\right)^2\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x)
```

```
[Out] -2/a^3/f*d^3/(c-d)^3/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d
)/(c^2-d^2)^(1/2))+4/a^3/f/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^2*c-6/a^3/f/(c-d)
^2/(tan(1/2*f*x+1/2*e)+1)^2*d-16/3/a^3/f/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^3*c
+20/3/a^3/f/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^3*d-2/a^3/f/(c-d)^3/(tan(1/2*f*x
+1/2*e)+1)*c^2+6/a^3/f/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)*c*d-6/a^3/f/(c-d)^3/(
tan(1/2*f*x+1/2*e)+1)*d^2-8/5/a^3/f/(c-d)/(tan(1/2*f*x+1/2*e)+1)^5+4/a^3/f/
(c-d)/(tan(1/2*f*x+1/2*e)+1)^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 10.07, size = 466, normalized size = 2.51

$$\frac{2\,d^3\,\operatorname{atan}\left(\frac{\frac{d^3\,\left(-2\,a^3\,c^3\,d+6\,a^3\,c^2\,d^2-6\,a^3\,c\,d^3+2\,a^3\,d^4\right)}{a^3\,\sqrt{c+d}\,{\left(c-d\right)}^{7/2}}-\frac{2\,c\,d^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(a^3\,c^3-3\,a^3\,c^2\,d+3\,a^3\,c\,d^2-a^3\,d^3\right)}{a^3\,\sqrt{c+d}\,{\left(c-d\right)}^{7/2}}}{2\,d^3}\right)}{a^3\,f\,\sqrt{c+d}\,{\left(c-d\right)}^{7/2}}-\frac{\frac{2\,\left(7\,c^2-24\,c\,d+32\,d^2\right)}{15\,\left(c-d\right)\,\left(c^2-2\,c\,d+d^2\right)}+\frac{2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(4\,c^2-\cdots\right.}{3\,\left(c-d\right)\,\left(c^2-2\cdots\right.}}{f\,\left(a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5+5\,a^3\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))),x)
```

```
[Out] (2*d^3*atan(((d^3*(2*a^3*d^4 - 6*a^3*c*d^3 - 2*a^3*c^3*d + 6*a^3*c^2*d^2))/
(a^3*(c + d)^(1/2)*(c - d)^(7/2)) - (2*c*d^3*tan(e/2 + (f*x)/2)*(a^3*c^3 -
a^3*d^3 + 3*a^3*c*d^2 - 3*a^3*c^2*d))/(a^3*(c + d)^(1/2)*(c - d)^(7/2)))/(2
*d^3)))/(a^3*f*(c + d)^(1/2)*(c - d)^(7/2)) - ((2*(7*c^2 - 24*c*d + 32*d^2)
)/(15*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)*(4*c^2 - 15*c*d
+ 23*d^2))/(3*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)^4*(c^2 -
 3*c*d + 3*d^2))/((c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)^3*(2
*c^2 - 7*c*d + 9*d^2))/((c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2
)^2*(8*c^2 - 27*c*d + 37*d^2))/(3*(c - d)*(c^2 - 2*c*d + d^2)))/(f*(10*a^3*
tan(e/2 + (f*x)/2)^2 + 10*a^3*tan(e/2 + (f*x)/2)^3 + 5*a^3*tan(e/2 + (f*x)/
2)^4 + a^3*tan(e/2 + (f*x)/2)^5 + a^3 + 5*a^3*tan(e/2 + (f*x)/2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.478 $\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=298

$$-\frac{\left(2c^2-12cd+45d^2\right)\cos(e+fx)}{15f(c-d)^3\left(a^3\sin(e+fx)+a^3\right)(c+d\sin(e+fx))}-\frac{2d^3(4c+3d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^4(c+d)\sqrt{c^2-d^2}}-\frac{d\left(2c^3-12c^2d+43c\ldots\right.}{15a^3f(c-d)^4(c+\ldots}$$

```
[Out] -1/15*d*(2*c^3-12*c^2*d+43*c*d^2+72*d^3)*cos(f*x+e)/a^3/(c-d)^4/(c+d)/f/(c+
d*sin(f*x+e))-1/5*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))-1/
15*(2*c-9*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))-1/1
5*(2*c^2-12*c*d+45*d^2)*cos(f*x+e)/(c-d)^3/f/(a^3+a^3*sin(f*x+e))/(c+d*sin(
f*x+e))-2*d^3*(4*c+3*d)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^
3/(c-d)^4/(c+d)/f/(c^2-d^2)^(1/2)
```

Rubi [A] time = 0.73, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2766, 2978, 2754, 12, 2660, 618, 204}

$$-\frac{2d^3(4c+3d)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^4(c+d)\sqrt{c^2-d^2}}-\frac{d\left(-12c^2d+2c^3+43cd^2+72d^3\right)\cos(e+fx)}{15a^3f(c-d)^4(c+d)(c+d\sin(e+fx))}-\frac{\left(2c^2-12cd+45\ldots\right.}{15f(c-d)^3\left(a^3\sin(e+fx)\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] (-2*d^3*(4*c + 3*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^3*
(c - d)^4*(c + d)*Sqrt[c^2 - d^2]*f) - (d*(2*c^3 - 12*c^2*d + 43*c*d^2 + 72
*d^3)*Cos[e + f*x])/(15*a^3*(c - d)^4*(c + d)*f*(c + d*Sin[e + f*x])) - Cos
[e + f*x]/(5*(c - d)*f*(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])) - ((2*c
 - 9*d)*Cos[e + f*x])/(15*a*(c - d)^2*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[e
 + f*x])) - ((2*c^2 - 12*c*d + 45*d^2)*Cos[e + f*x])/(15*(c - d)^3*f*(a^3 +
 a^3*Sin[e + f*x])*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
```

```
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx = -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))} - \frac{\int \frac{-2a(c-3d\ldots}{(a+a\sin(e+f\ldots}}{5a\ldots}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))} - \frac{\ldots}{15a(c-d)^2f\ldots}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))} - \frac{\ldots}{15a(c-d)^2f\ldots}$$

$$= -\frac{d\left(2c^3-12c^2d+43cd^2+72d^3\right)\cos(e+fx)}{15a^3(c-d)^4(c+d)f(c+d\sin(e+fx))} - \frac{\ldots}{5(c-d)f(a+a\,\mathrm{s}\ldots}$$

$$= -\frac{d\left(2c^3-12c^2d+43cd^2+72d^3\right)\cos(e+fx)}{15a^3(c-d)^4(c+d)f(c+d\sin(e+fx))} - \frac{\ldots}{5(c-d)f(a+a\,\mathrm{s}\ldots}$$

$$= -\frac{d\left(2c^3-12c^2d+43cd^2+72d^3\right)\cos(e+fx)}{15a^3(c-d)^4(c+d)f(c+d\sin(e+fx))} - \frac{\ldots}{5(c-d)f(a+a\,\mathrm{s}\ldots}$$

$$= -\frac{d\left(2c^3-12c^2d+43cd^2+72d^3\right)\cos(e+fx)}{15a^3(c-d)^4(c+d)f(c+d\sin(e+fx))} - \frac{\ldots}{5(c-d)f(a+a\,\mathrm{s}\ldots}$$

$$= -\frac{2d^3(4c+3d)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^4(c+d)\sqrt{c^2-d^2}\,f} - \frac{d\left(2c^3-12c^2d+43cd^2+\ldots\right.}{15a^3(c-d)^4(c+d)f(c\ldots}$$

Mathematica [A] time = 2.55, size = 361, normalized size = 1.21

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2\left(2c^2-14cd+57d^2\right)\sin\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(6*(c - d)^2*Sin[(e + f*x)/2] - 3*(c
 - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 4*(c - 6*d)*(c - d)*Sin[(e
+ f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 2*(c - 6*d)*(c - d)*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 2*(2*c^2 - 14*c*d + 57*d^2)*Sin[(e +
 f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - (30*d^3*(4*c + 3*d)*ArcT
an[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^5)/((c + d)*Sqrt[c^2 - d^2]) - (15*d^4*Cos[e + f*x]*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^5)/((c + d)*(c + d*Sin[e + f*x]))))/(15*a^3*(c - d)^
4*f*(1 + Sin[e + f*x])^3)
```

fricas [B] time = 0.60, size = 3235, normalized size = 10.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [-1/30*(6*c^6 - 12*c^5*d - 6*c^4*d^2 + 24*c^3*d^3 - 6*c^2*d^4 - 12*c*d^5 +
6*d^6 - 2*(2*c^5*d - 12*c^4*d^2 + 41*c^3*d^3 + 84*c^2*d^4 - 43*c*d^5 - 72*d
^6)*cos(f*x + e)^4 - 2*(2*c^6 - 6*c^5*d + 5*c^4*d^2 + 147*c^3*d^3 + 164*c^2
*d^4 - 141*c*d^5 - 171*d^6)*cos(f*x + e)^3 + 2*(4*c^6 - 19*c^5*d + 22*c^4*d
^2 + 128*c^3*d^3 + 64*c^2*d^4 - 109*c*d^5 - 90*d^6)*cos(f*x + e)^2 + 15*(16
*c^2*d^3 + 28*c*d^4 + 12*d^5 + (4*c*d^4 + 3*d^5)*cos(f*x + e)^4 - (4*c^2*d^
3 + 11*c*d^4 + 6*d^5)*cos(f*x + e)^3 - (12*c^2*d^3 + 29*c*d^4 + 15*d^5)*cos
(f*x + e)^2 + 2*(4*c^2*d^3 + 7*c*d^4 + 3*d^5)*cos(f*x + e) + (16*c^2*d^3 +
28*c*d^4 + 12*d^5 - (4*c*d^4 + 3*d^5)*cos(f*x + e)^3 - (4*c^2*d^3 + 15*c*d^
4 + 9*d^5)*cos(f*x + e)^2 + 2*(4*c^2*d^3 + 7*c*d^4 + 3*d^5)*cos(f*x + e))*s
in(f*x + e))*sqrt(-c^2 + d^2)*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*si
n(f*x + e) - c^2 - d^2 - 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*s
qrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 6
*(3*c^6 - 11*c^5*d + 12*c^4*d^2 + 82*c^3*d^3 + 47*c^2*d^4 - 71*c*d^5 - 62*d
^6)*cos(f*x + e) - 2*(3*c^6 - 6*c^5*d - 3*c^4*d^2 + 12*c^3*d^3 - 3*c^2*d^4
- 6*c*d^5 + 3*d^6 + (2*c^5*d - 12*c^4*d^2 + 41*c^3*d^3 + 84*c^2*d^4 - 43*c*
d^5 - 72*d^6)*cos(f*x + e)^3 - (2*c^6 - 8*c^5*d + 17*c^4*d^2 + 106*c^3*d^3
+ 80*c^2*d^4 - 98*c*d^5 - 99*d^6)*cos(f*x + e)^2 - 3*(2*c^6 - 9*c^5*d + 13*
c^4*d^2 + 78*c^3*d^3 + 48*c^2*d^4 - 69*c*d^5 - 63*d^6)*cos(f*x + e))*sin(f*
x + e))/((a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 5*a^3*c
^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e)^4 - (a^3*c^8 -
 a^3*c^7*d - 5*a^3*c^6*d^2 + 7*a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 11*a^3*c^3*d^5
 + a^3*c^2*d^6 + 5*a^3*c*d^7 - 2*a^3*d^8)*f*cos(f*x + e)^3 - (3*a^3*c^8 - 4
*a^3*c^7*d - 12*a^3*c^6*d^2 + 20*a^3*c^5*d^3 + 10*a^3*c^4*d^4 - 28*a^3*c^3*
d^5 + 4*a^3*c^2*d^6 + 12*a^3*c*d^7 - 5*a^3*d^8)*f*cos(f*x + e)^2 + 2*(a^3*c
^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^
```

2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e) + 4*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f - ((a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 5*a^3*c^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e)^3 + (a^3*c^8 - 8*a^3*c^6*d^2 + 8*a^3*c^5*d^3 + 10*a^3*c^4*d^4 - 16*a^3*c^3*d^5 + 8*a^3*c*d^7 - 3*a^3*d^8)*f*cos(f*x + e)^2 - 2*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e) - 4*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f)*sin(f*x + e)), -1/15*(3*c^6 - 6*c^5*d - 3*c^4*d^2 + 12*c^3*d^3 - 3*c^2*d^4 - 6*c*d^5 + 3*d^6 - (2*c^5*d - 12*c^4*d^2 + 41*c^3*d^3 + 84*c^2*d^4 - 43*c*d^5 - 72*d^6)*cos(f*x + e)^4 - (2*c^6 - 6*c^5*d + 5*c^4*d^2 + 147*c^3*d^3 + 164*c^2*d^4 - 141*c*d^5 - 171*d^6)*cos(f*x + e)^3 + (4*c^6 - 19*c^5*d + 22*c^4*d^2 + 128*c^3*d^3 + 64*c^2*d^4 - 109*c*d^5 - 90*d^6)*cos(f*x + e)^2 - 15*(16*c^2*d^3 + 28*c*d^4 + 12*d^5 + (4*c*d^4 + 3*d^5)*cos(f*x + e)^4 - (4*c^2*d^3 + 11*c*d^4 + 6*d^5)*cos(f*x + e)^3 - (12*c^2*d^3 + 29*c*d^4 + 15*d^5)*cos(f*x + e)^2 + 2*(4*c^2*d^3 + 7*c*d^4 + 3*d^5)*cos(f*x + e) + (16*c^2*d^3 + 28*c*d^4 + 12*d^5 - (4*c*d^4 + 3*d^5)*cos(f*x + e)^3 - (4*c^2*d^3 + 15*c*d^4 + 9*d^5)*cos(f*x + e)^2 + 2*(4*c^2*d^3 + 7*c*d^4 + 3*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + 3*(3*c^6 - 11*c^5*d + 12*c^4*d^2 + 82*c^3*d^3 + 47*c^2*d^4 - 71*c*d^5 - 62*d^6)*cos(f*x + e) - (3*c^6 - 6*c^5*d - 3*c^4*d^2 + 12*c^3*d^3 - 3*c^2*d^4 - 6*c*d^5 + 3*d^6 + (2*c^5*d - 12*c^4*d^2 + 41*c^3*d^3 + 84*c^2*d^4 - 43*c*d^5 - 72*d^6)*cos(f*x + e)^3 - (2*c^6 - 8*c^5*d + 17*c^4*d^2 + 106*c^3*d^3 + 80*c^2*d^4 - 98*c*d^5 - 99*d^6)*cos(f*x + e)^2 - 3*(2*c^6 - 9*c^5*d + 13*c^4*d^2 + 78*c^3*d^3 + 48*c^2*d^4 - 69*c*d^5 - 63*d^6)*cos(f*x + e))*sin(f*x + e))/((a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 5*a^3*c^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e)^4 - (a^3*c^8 - a^3*c^7*d - 5*a^3*c^6*d^2 + 7*a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 11*a^3*c^3*d^5 + a^3*c^2*d^6 + 5*a^3*c*d^7 - 2*a^3*d^8)*f*cos(f*x + e)^3 - (3*a^3*c^8 - 4*a^3*c^7*d - 12*a^3*c^6*d^2 + 20*a^3*c^5*d^3 + 10*a^3*c^4*d^4 - 28*a^3*c^3*d^5 + 4*a^3*c^2*d^6 + 12*a^3*c*d^7 - 5*a^3*d^8)*f*cos(f*x + e)^2 + 2*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e) + 4*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f - ((a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 5*a^3*c^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e)^3 + (a^3*c^8 - 8*a^3*c^6*d^2 + 8*a^3*c^5*d^3 + 10*a^3*c^4*d^4 - 16*a^3*c^3*d^5 + 8*a^3*c*d^7 - 3*a^3*d^8)*f*cos(f*x + e)^2 - 2*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e) - 4*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f)*sin(f*x + e))]

giac [A] time = 1.31, size = 517, normalized size = 1.73

$$-\frac{2\left(\frac{15\left(4\,cd^3+3\,d^4\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(a^3c^5-3\,a^3c^4d+2\,a^3c^3d^2+2\,a^3c^2d^3-3\,a^3cd^4+a^3d^5\right)\sqrt{c^2-d^2}}+\frac{15\left(d^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+cd^4\right)}{\left(a^3c^6-3\,a^3c^5d+2\,a^3c^4d^2+2\,a^3c^3d^3-3\,a^3c^2d^4+a^3cd^5\right)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,d\ldots\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -2/15*(15*(4*c*d^3 + 3*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arct
an((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^3*c^5 - 3*a^3*c^4*d +
 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*sqrt(c^2 - d^2)) +
15*(d^5*tan(1/2*f*x + 1/2*e) + c*d^4)/((a^3*c^6 - 3*a^3*c^5*d + 2*a^3*c^4*d
^2 + 2*a^3*c^3*d^3 - 3*a^3*c^2*d^4 + a^3*c*d^5)*(c*tan(1/2*f*x + 1/2*e)^2 +
 2*d*tan(1/2*f*x + 1/2*e) + c)) + (15*c^2*tan(1/2*f*x + 1/2*e)^4 - 60*c*d*t
an(1/2*f*x + 1/2*e)^4 + 90*d^2*tan(1/2*f*x + 1/2*e)^4 + 30*c^2*tan(1/2*f*x
+ 1/2*e)^3 - 150*c*d*tan(1/2*f*x + 1/2*e)^3 + 300*d^2*tan(1/2*f*x + 1/2*e)^
3 + 40*c^2*tan(1/2*f*x + 1/2*e)^2 - 190*c*d*tan(1/2*f*x + 1/2*e)^2 + 420*d^
2*tan(1/2*f*x + 1/2*e)^2 + 20*c^2*tan(1/2*f*x + 1/2*e) - 110*c*d*tan(1/2*f*
x + 1/2*e) + 270*d^2*tan(1/2*f*x + 1/2*e) + 7*c^2 - 34*c*d + 72*d^2)/((a^3*
c^4 - 4*a^3*c^3*d + 6*a^3*c^2*d^2 - 4*a^3*c*d^3 + a^3*d^4)*(tan(1/2*f*x + 1
/2*e) + 1)^5))/f
```

maple [A] time = 0.35, size = 511, normalized size = 1.71

$$-\frac{2d^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a^3f\left(c-d\right)^4\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d+c\right)\left(c+d\right)c}-\frac{2d^4}{a^3f\left(c-d\right)^4\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2\tan\left(\frac{fx}{2}+\ldots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x)
```

```
[Out] -2/a^3/f*d^5/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d
)/c*tan(1/2*f*x+1/2*e)-2/a^3/f*d^4/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/
2*f*x+1/2*e)*d+c)/(c+d)-8/a^3/f*d^3/(c-d)^4/(c+d)/(c^2-d^2)^(1/2)*arctan(1/
2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c-6/a^3/f*d^4/(c-d)^4/(c+d)
/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))-1
6/3/a^3/f/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^3*c+8/a^3/f/(c-d)^3/(tan(1/2*f*x+1
/2*e)+1)^3*d-8/a^3/f/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^2*d+4/a^3/f/(c-d)^3/(ta
n(1/2*f*x+1/2*e)+1)^2*c-2/a^3/f/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)*c^2+8/a^3/f/
```

```
(c-d)^4/(tan(1/2*f*x+1/2*e)+1)*c*d-12/a^3/f/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)*
d^2-8/5/a^3/f/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^5+4/a^3/f/(c-d)^2/(tan(1/2*f*x
+1/2*e)+1)^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 10.30, size = 987, normalized size = 3.31

$$-\frac{\frac{2\left(7c^4-27c^3d+38c^2d^2+72cd^3+15d^4\right)}{15(c+d)(c-d)\left(c^3-3c^2d+3cd^2-d^3\right)}+\frac{4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(5c^4-18c^3d+19c^2d^2+84cd^3+15d^4\right)}{3c(c-d)\left(c^3-3c^2d+3cd^2-d^3\right)}+\frac{2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(20c^5-76c^4d+106c^3d^2+346c^2 \cdots}{15c(c+d)(c-d)\left(c^3-3c^2d+3cd^2 \cdots}}{f\left(a^3c+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(5a^3c+2a^3d\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6\left(5a^3c+2a^3d \cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^2),x)
```

```
[Out] - ((2*(72*c*d^3 - 27*c^3*d + 7*c^4 + 15*d^4 + 38*c^2*d^2))/(15*(c + d)*(c -
 d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (4*tan(e/2 + (f*x)/2)^3*(84*c*d^3 -
18*c^3*d + 5*c^4 + 15*d^4 + 19*c^2*d^2))/(3*c*(c - d)*(3*c*d^2 - 3*c^2*d +
c^3 - d^3)) + (2*tan(e/2 + (f*x)/2)*(219*c*d^4 - 76*c^4*d + 20*c^5 + 15*d^5
 + 346*c^2*d^3 + 106*c^3*d^2))/(15*c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d + c
^3 - d^3)) + (2*tan(e/2 + (f*x)/2)^6*(c^5 - 3*c^4*d + d^5 + 6*c^2*d^3 + 2*c
^3*d^2))/(c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (2*tan(e/2 +
 (f*x)/2)^5*(13*c*d^4 - 6*c^4*d + 2*c^5 + 5*d^5 + 24*c^2*d^3 + 4*c^3*d^2))/
(c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (2*tan(e/2 + (f*x)/2)
^4*(135*c*d^4 - 27*c^4*d + 11*c^5 + 30*d^5 + 162*c^2*d^3 + 4*c^3*d^2))/(3*c
*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (2*tan(e/2 + (f*x)/2)^2
*(690*c*d^4 - 137*c^4*d + 47*c^5 + 75*d^5 + 812*c^2*d^3 + 88*c^3*d^2))/(15*
c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)))/(f*(a^3*c + tan(e/2 + (
f*x)/2)*(5*a^3*c + 2*a^3*d) + tan(e/2 + (f*x)/2)^6*(5*a^3*c + 2*a^3*d) + ta
n(e/2 + (f*x)/2)^2*(11*a^3*c + 10*a^3*d) + tan(e/2 + (f*x)/2)^5*(11*a^3*c +
 10*a^3*d) + tan(e/2 + (f*x)/2)^3*(15*a^3*c + 20*a^3*d) + tan(e/2 + (f*x)/2
```

```
)^4*(15*a^3*c + 20*a^3*d) + a^3*c*tan(e/2 + (f*x)/2)^7)) - (2*d^3*atan(((d^
3*(4*c + 3*d)*(2*a^3*d^6 - 6*a^3*c*d^5 + 2*a^3*c^5*d + 4*a^3*c^2*d^4 + 4*a^
3*c^3*d^3 - 6*a^3*c^4*d^2))/(a^3*(c + d)^(3/2)*(c - d)^(9/2)) + (2*c*d^3*ta
n(e/2 + (f*x)/2)*(4*c + 3*d)*(a^3*c^5 + a^3*d^5 - 3*a^3*c*d^4 - 3*a^3*c^4*d
 + 2*a^3*c^2*d^3 + 2*a^3*c^3*d^2))/(a^3*(c + d)^(3/2)*(c - d)^(9/2)))/(8*c*
d^3 + 6*d^4))*(4*c + 3*d))/(a^3*f*(c + d)^(3/2)*(c - d)^(9/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.479 $\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=378

$$-\frac{(2c^2-15cd+76d^2)\cos(e+fx)}{15f(c-d)^3\left(a^3\sin(e+fx)+a^3\right)(c+d\sin(e+fx))^2}-\frac{d^3\left(20c^2+30cd+13d^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^5(c+d)^2\sqrt{c^2-d^2}}-\frac{d\left(4c^3-\right.}{30a^3}$$

```
[Out] -1/30*d*(4*c^3-30*c^2*d+146*c*d^2+195*d^3)*cos(f*x+e)/a^3/(c-d)^4/(c+d)/f/(
c+d*sin(f*x+e))^2-1/5*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)
)^2-1/15*(2*c-11*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+
e))^2-1/15*(2*c^2-15*c*d+76*d^2)*cos(f*x+e)/(c-d)^3/f/(a^3+a^3*sin(f*x+e))/
(c+d*sin(f*x+e))^2-1/30*d*(4*c^4-30*c^3*d+142*c^2*d^2+525*c*d^3+304*d^4)*co
s(f*x+e)/a^3/(c-d)^5/(c+d)^2/f/(c+d*sin(f*x+e))-d^3*(20*c^2+30*c*d+13*d^2)*
arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^3/(c-d)^5/(c+d)^2/f/(c^2
-d^2)^(1/2)
```

Rubi [A] time = 0.96, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2766, 2978, 2754, 12, 2660, 618, 204}

$$-\frac{d^3\left(20c^2+30cd+13d^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^5(c+d)^2\sqrt{c^2-d^2}}-\frac{d\left(142c^2d^2-30c^3d+4c^4+525cd^3+304d^4\right)\cos(e+fx)}{30a^3f(c-d)^5(c+d)^2(c+d\sin(e+fx))}-\frac{d\left(-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] -((d^3*(20*c^2 + 30*c*d + 13*d^2)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2
- d^2]])/(a^3*(c - d)^5*(c + d)^2*Sqrt[c^2 - d^2]*f)) - (d*(4*c^3 - 30*c^2*
d + 146*c*d^2 + 195*d^3)*Cos[e + f*x])/(30*a^3*(c - d)^4*(c + d)*f*(c + d*S
in[e + f*x])^2) - Cos[e + f*x]/(5*(c - d)*f*(a + a*Sin[e + f*x])^3*(c + d*S
in[e + f*x])^2) - ((2*c - 11*d)*Cos[e + f*x])/(15*a*(c - d)^2*f*(a + a*Sin[
e + f*x])^2*(c + d*Sin[e + f*x])^2) - ((2*c^2 - 15*c*d + 76*d^2)*Cos[e + f*
x])/(15*(c - d)^3*f*(a^3 + a^3*Sin[e + f*x])*(c + d*Sin[e + f*x])^2) - (d*(
4*c^4 - 30*c^3*d + 142*c^2*d^2 + 525*c*d^3 + 304*d^4)*Cos[e + f*x])/(30*a^3
*(c - d)^5*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
```

```
Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx = -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2} - \frac{\int \frac{-a(2c-7}{(a+a\sin(e+}}{5}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2} - \frac{}{15a(c-d)^2}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2} - \frac{}{15a(c-d)^2}$$

$$= -\frac{d\left(4c^3-30c^2d+146cd^2+195d^3\right)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^2} - \frac{}{5(c-d)f(a+}$$

$$= -\frac{d\left(4c^3-30c^2d+146cd^2+195d^3\right)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^2} - \frac{}{5(c-d)f(a+}$$

$$= -\frac{d\left(4c^3-30c^2d+146cd^2+195d^3\right)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^2} - \frac{}{5(c-d)f(a+}$$

$$= -\frac{d\left(4c^3-30c^2d+146cd^2+195d^3\right)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^2} - \frac{}{5(c-d)f(a+}$$

$$= -\frac{d\left(4c^3-30c^2d+146cd^2+195d^3\right)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^2} - \frac{}{5(c-d)f(a+}$$

$$= -\frac{d^3\left(20c^2+30cd+13d^2\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^5(c+d)^2\sqrt{c^2-d^2}\,f} - \frac{d\left(4c^3-30c^2\right.}{30a^3(c-}$$

Mathematica [B] time = 6.32, size = 1066, normalized size = 2.82

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] -((d^3*(20*c^2 + 30*c*d + 13*d^2)*ArcTan[(Sec[(e + f*x)/2]*(d*Cos[(e + f*x)
/2] + c*Sin[(e + f*x)/2]))/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])^6)/((c - d)^5*(c + d)^2*Sqrt[c^2 - d^2]*f*(a + a*Sin[e + f*x])^3)) +
 ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-400*c^5*d*Cos[(e + f*x)/2] + 3400
*c^4*d^2*Cos[(e + f*x)/2] + 19340*c^3*d^3*Cos[(e + f*x)/2] + 30400*c^2*d^4*
Cos[(e + f*x)/2] + 19940*c*d^5*Cos[(e + f*x)/2] + 4810*d^6*Cos[(e + f*x)/2]
 - 160*c^6*Cos[(3*(e + f*x))/2] + 848*c^5*d*Cos[(3*(e + f*x))/2] - 2400*c^4
*d^2*Cos[(3*(e + f*x))/2] - 19396*c^3*d^3*Cos[(3*(e + f*x))/2] - 35280*c^2*
d^4*Cos[(3*(e + f*x))/2] - 24742*c*d^5*Cos[(3*(e + f*x))/2] - 5810*d^6*Cos[
(3*(e + f*x))/2] - 1260*c^3*d^3*Cos[(5*(e + f*x))/2] - 2640*c^2*d^4*Cos[(5*
(e + f*x))/2] - 2250*c*d^5*Cos[(5*(e + f*x))/2] - 870*d^6*Cos[(5*(e + f*x))
/2] + 32*c^5*d*Cos[(7*(e + f*x))/2] - 200*c^4*d^2*Cos[(7*(e + f*x))/2] + 83
6*c^3*d^3*Cos[(7*(e + f*x))/2] + 4480*c^2*d^4*Cos[(7*(e + f*x))/2] + 5747*c
*d^5*Cos[(7*(e + f*x))/2] + 2200*d^6*Cos[(7*(e + f*x))/2] - 135*c*d^5*Cos[(
9*(e + f*x))/2] - 90*d^6*Cos[(9*(e + f*x))/2] + 320*c^6*Sin[(e + f*x)/2] -
1520*c^5*d*Sin[(e + f*x)/2] + 4568*c^4*d^2*Sin[(e + f*x)/2] + 27340*c^3*d^3
*Sin[(e + f*x)/2] + 40904*c^2*d^4*Sin[(e + f*x)/2] + 26020*c*d^5*Sin[(e + f
*x)/2] + 6318*d^6*Sin[(e + f*x)/2] + 800*c^4*d^2*Sin[(3*(e + f*x))/2] + 750
0*c^3*d^3*Sin[(3*(e + f*x))/2] + 13280*c^2*d^4*Sin[(3*(e + f*x))/2] + 9690*
c*d^5*Sin[(3*(e + f*x))/2] + 2750*d^6*Sin[(3*(e + f*x))/2] - 32*c^6*Sin[(5*
(e + f*x))/2] + 80*c^5*d*Sin[(5*(e + f*x))/2] - 32*c^4*d^2*Sin[(5*(e + f*x)
)/2] - 6820*c^3*d^3*Sin[(5*(e + f*x))/2] - 18080*c^2*d^4*Sin[(5*(e + f*x))/
2] - 15670*c*d^5*Sin[(5*(e + f*x))/2] - 4266*d^6*Sin[(5*(e + f*x))/2] - 60*
c^2*d^4*Sin[(7*(e + f*x))/2] + 135*c*d^5*Sin[(7*(e + f*x))/2] + 60*d^6*Sin[
(7*(e + f*x))/2] + 8*c^4*d^2*Sin[(9*(e + f*x))/2] - 60*c^3*d^3*Sin[(9*(e +
f*x))/2] + 284*c^2*d^4*Sin[(9*(e + f*x))/2] + 915*c*d^5*Sin[(9*(e + f*x))/2
] + 518*d^6*Sin[(9*(e + f*x))/2]))/(480*(c - d)^5*(c + d)^2*f*(a + a*Sin[e
+ f*x])^3*(c + d*Sin[e + f*x])^2)
```

fricas [B] time = 0.74, size = 5226, normalized size = 13.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/60*(12*c^8 - 24*c^7*d - 24*c^6*d^2 + 72*c^5*d^3 - 72*c^3*d^5 + 24*c^2*d
^6 + 24*c*d^7 - 12*d^8 + 2*(4*c^6*d^2 - 30*c^5*d^3 + 138*c^4*d^4 + 555*c^3*
```

d^5 + 162*c^2*d^6 - 525*c*d^7 - 304*d^8)*cos(f*x + e)^5 - 2*(8*c^7*d - 52*c
^6*d^2 + 216*c^5*d^3 + 1086*c^4*d^4 + 984*c^3*d^5 - 621*c^2*d^6 - 1208*c*d^
7 - 413*d^8)*cos(f*x + e)^4 - 2*(4*c^8 - 6*c^7*d - 20*c^6*d^2 + 768*c^5*d^3
 + 2676*c^4*d^4 + 2307*c^3*d^5 - 1573*c^2*d^6 - 3069*c*d^7 - 1087*d^8)*cos(
f*x + e)^3 + 4*(4*c^8 - 20*c^7*d + 19*c^6*d^2 + 330*c^5*d^3 + 699*c^4*d^4 +
 345*c^3*d^5 - 526*c^2*d^6 - 655*c*d^7 - 196*d^8)*cos(f*x + e)^2 - 15*(80*c
^4*d^3 + 280*c^3*d^4 + 372*c^2*d^5 + 224*c*d^6 + 52*d^7 + (20*c^2*d^5 + 30*
c*d^6 + 13*d^7)*cos(f*x + e)^5 + (40*c^3*d^4 + 120*c^2*d^5 + 116*c*d^6 + 39
*d^7)*cos(f*x + e)^4 - (20*c^4*d^3 + 110*c^3*d^4 + 193*c^2*d^5 + 142*c*d^6
+ 39*d^7)*cos(f*x + e)^3 - (60*c^4*d^3 + 290*c^3*d^4 + 479*c^2*d^5 + 340*c*
d^6 + 91*d^7)*cos(f*x + e)^2 + 2*(20*c^4*d^3 + 70*c^3*d^4 + 93*c^2*d^5 + 56
*c*d^6 + 13*d^7)*cos(f*x + e) + (80*c^4*d^3 + 280*c^3*d^4 + 372*c^2*d^5 + 2
24*c*d^6 + 52*d^7 + (20*c^2*d^5 + 30*c*d^6 + 13*d^7)*cos(f*x + e)^4 - 2*(20
*c^3*d^4 + 50*c^2*d^5 + 43*c*d^6 + 13*d^7)*cos(f*x + e)^3 - (20*c^4*d^3 + 1
50*c^3*d^4 + 293*c^2*d^5 + 228*c*d^6 + 65*d^7)*cos(f*x + e)^2 + 2*(20*c^4*d
^3 + 70*c^3*d^4 + 93*c^2*d^5 + 56*c*d^6 + 13*d^7)*cos(f*x + e))*sin(f*x + e
))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e)
- c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 +
d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 12*(3*c^8 -
11*c^7*d + 9*c^6*d^2 + 213*c^5*d^3 + 475*c^4*d^4 + 237*c^3*d^5 - 359*c^2*d^
6 - 439*c*d^7 - 128*d^8)*cos(f*x + e) - 2*(6*c^8 - 12*c^7*d - 12*c^6*d^2 +
36*c^5*d^3 - 36*c^3*d^5 + 12*c^2*d^6 + 12*c*d^7 - 6*d^8 + (4*c^6*d^2 - 30*c
^5*d^3 + 138*c^4*d^4 + 555*c^3*d^5 + 162*c^2*d^6 - 525*c*d^7 - 304*d^8)*cos
(f*x + e)^4 + (8*c^7*d - 48*c^6*d^2 + 186*c^5*d^3 + 1224*c^4*d^4 + 1539*c^3
*d^5 - 459*c^2*d^6 - 1733*c*d^7 - 717*d^8)*cos(f*x + e)^3 - 2*(2*c^8 - 7*c^
7*d + 14*c^6*d^2 + 291*c^5*d^3 + 726*c^4*d^4 + 384*c^3*d^5 - 557*c^2*d^6 -
668*c*d^7 - 185*d^8)*cos(f*x + e)^2 - 6*(2*c^8 - 9*c^7*d + 11*c^6*d^2 + 207
*c^5*d^3 + 475*c^4*d^4 + 243*c^3*d^5 - 361*c^2*d^6 - 441*c*d^7 - 127*d^8)*c
os(f*x + e))*sin(f*x + e))/((a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^6*d^5 -
6*a^3*c^5*d^6 - 6*a^3*c^4*d^7 + 8*a^3*c^3*d^8 - 3*a^3*c*d^10 + a^3*d^11)*f*
cos(f*x + e)^5 + (2*a^3*c^10*d - 3*a^3*c^9*d^2 - 9*a^3*c^8*d^3 + 16*a^3*c^7
*d^4 + 12*a^3*c^6*d^5 - 30*a^3*c^5*d^6 - 2*a^3*c^4*d^7 + 24*a^3*c^3*d^8 - 6
*a^3*c^2*d^9 - 7*a^3*c*d^10 + 3*a^3*d^11)*f*cos(f*x + e)^4 - (a^3*c^11 + a^
3*c^10*d - 9*a^3*c^9*d^2 - a^3*c^8*d^3 + 26*a^3*c^7*d^4 - 6*a^3*c^6*d^5 - 3
4*a^3*c^5*d^6 + 14*a^3*c^4*d^7 + 21*a^3*c^3*d^8 - 11*a^3*c^2*d^9 - 5*a^3*c*
d^10 + 3*a^3*d^11)*f*cos(f*x + e)^3 - (3*a^3*c^11 + a^3*c^10*d - 23*a^3*c^9
*d^2 + 3*a^3*c^8*d^3 + 62*a^3*c^7*d^4 - 22*a^3*c^6*d^5 - 78*a^3*c^5*d^6 + 3
8*a^3*c^4*d^7 + 47*a^3*c^3*d^8 - 27*a^3*c^2*d^9 - 11*a^3*c*d^10 + 7*a^3*d^1
1)*f*cos(f*x + e)^2 + 2*(a^3*c^11 - a^3*c^10*d - 5*a^3*c^9*d^2 + 5*a^3*c^8*
d^3 + 10*a^3*c^7*d^4 - 10*a^3*c^6*d^5 - 10*a^3*c^5*d^6 + 10*a^3*c^4*d^7 + 5
*a^3*c^3*d^8 - 5*a^3*c^2*d^9 - a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e) + 4*(a
^3*c^11 - a^3*c^10*d - 5*a^3*c^9*d^2 + 5*a^3*c^8*d^3 + 10*a^3*c^7*d^4 - 10*
a^3*c^6*d^5 - 10*a^3*c^5*d^6 + 10*a^3*c^4*d^7 + 5*a^3*c^3*d^8 - 5*a^3*c^2*d
^9 - a^3*c*d^10 + a^3*d^11)*f + ((a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^6*d
^5 - 6*a^3*c^5*d^6 - 6*a^3*c^4*d^7 + 8*a^3*c^3*d^8 - 3*a^3*c*d^10 + a^3*d^1

```
1)*f*cos(f*x + e)^4 - 2*(a^3*c^10*d - 2*a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3
*c^7*d^4 + 2*a^3*c^6*d^5 - 12*a^3*c^5*d^6 + 2*a^3*c^4*d^7 + 8*a^3*c^3*d^8 -
 3*a^3*c^2*d^9 - 2*a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e)^3 - (a^3*c^11 + 3*
a^3*c^10*d - 13*a^3*c^9*d^2 - 7*a^3*c^8*d^3 + 42*a^3*c^7*d^4 - 2*a^3*c^6*d^
5 - 58*a^3*c^5*d^6 + 18*a^3*c^4*d^7 + 37*a^3*c^3*d^8 - 17*a^3*c^2*d^9 - 9*a
^3*c*d^10 + 5*a^3*d^11)*f*cos(f*x + e)^2 + 2*(a^3*c^11 - a^3*c^10*d - 5*a^3
*c^9*d^2 + 5*a^3*c^8*d^3 + 10*a^3*c^7*d^4 - 10*a^3*c^6*d^5 - 10*a^3*c^5*d^6
 + 10*a^3*c^4*d^7 + 5*a^3*c^3*d^8 - 5*a^3*c^2*d^9 - a^3*c*d^10 + a^3*d^11)*
f*cos(f*x + e) + 4*(a^3*c^11 - a^3*c^10*d - 5*a^3*c^9*d^2 + 5*a^3*c^8*d^3 +
 10*a^3*c^7*d^4 - 10*a^3*c^6*d^5 - 10*a^3*c^5*d^6 + 10*a^3*c^4*d^7 + 5*a^3*
c^3*d^8 - 5*a^3*c^2*d^9 - a^3*c*d^10 + a^3*d^11)*f)*sin(f*x + e)), -1/30*(6
*c^8 - 12*c^7*d - 12*c^6*d^2 + 36*c^5*d^3 - 36*c^3*d^5 + 12*c^2*d^6 + 12*c*
d^7 - 6*d^8 + (4*c^6*d^2 - 30*c^5*d^3 + 138*c^4*d^4 + 555*c^3*d^5 + 162*c^2
*d^6 - 525*c*d^7 - 304*d^8)*cos(f*x + e)^5 - (8*c^7*d - 52*c^6*d^2 + 216*c^
5*d^3 + 1086*c^4*d^4 + 984*c^3*d^5 - 621*c^2*d^6 - 1208*c*d^7 - 413*d^8)*co
s(f*x + e)^4 - (4*c^8 - 6*c^7*d - 20*c^6*d^2 + 768*c^5*d^3 + 2676*c^4*d^4 +
 2307*c^3*d^5 - 1573*c^2*d^6 - 3069*c*d^7 - 1087*d^8)*cos(f*x + e)^3 + 2*(4
*c^8 - 20*c^7*d + 19*c^6*d^2 + 330*c^5*d^3 + 699*c^4*d^4 + 345*c^3*d^5 - 52
6*c^2*d^6 - 655*c*d^7 - 196*d^8)*cos(f*x + e)^2 - 15*(80*c^4*d^3 + 280*c^3*
d^4 + 372*c^2*d^5 + 224*c*d^6 + 52*d^7 + (20*c^2*d^5 + 30*c*d^6 + 13*d^7)*c
os(f*x + e)^5 + (40*c^3*d^4 + 120*c^2*d^5 + 116*c*d^6 + 39*d^7)*cos(f*x + e
)^4 - (20*c^4*d^3 + 110*c^3*d^4 + 193*c^2*d^5 + 142*c*d^6 + 39*d^7)*cos(f*x
 + e)^3 - (60*c^4*d^3 + 290*c^3*d^4 + 479*c^2*d^5 + 340*c*d^6 + 91*d^7)*cos
(f*x + e)^2 + 2*(20*c^4*d^3 + 70*c^3*d^4 + 93*c^2*d^5 + 56*c*d^6 + 13*d^7)*
cos(f*x + e) + (80*c^4*d^3 + 280*c^3*d^4 + 372*c^2*d^5 + 224*c*d^6 + 52*d^7
 + (20*c^2*d^5 + 30*c*d^6 + 13*d^7)*cos(f*x + e)^4 - 2*(20*c^3*d^4 + 50*c^2
*d^5 + 43*c*d^6 + 13*d^7)*cos(f*x + e)^3 - (20*c^4*d^3 + 150*c^3*d^4 + 293*
c^2*d^5 + 228*c*d^6 + 65*d^7)*cos(f*x + e)^2 + 2*(20*c^4*d^3 + 70*c^3*d^4 +
 93*c^2*d^5 + 56*c*d^6 + 13*d^7)*cos(f*x + e))*sin(f*x + e))*sqrt(c^2 - d^2
)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + 6*(3*c^8 -
 11*c^7*d + 9*c^6*d^2 + 213*c^5*d^3 + 475*c^4*d^4 + 237*c^3*d^5 - 359*c^2*d
^6 - 439*c*d^7 - 128*d^8)*cos(f*x + e) - (6*c^8 - 12*c^7*d - 12*c^6*d^2 + 3
6*c^5*d^3 - 36*c^3*d^5 + 12*c^2*d^6 + 12*c*d^7 - 6*d^8 + (4*c^6*d^2 - 30*c^
5*d^3 + 138*c^4*d^4 + 555*c^3*d^5 + 162*c^2*d^6 - 525*c*d^7 - 304*d^8)*cos(
f*x + e)^4 + (8*c^7*d - 48*c^6*d^2 + 186*c^5*d^3 + 1224*c^4*d^4 + 1539*c^3*
d^5 - 459*c^2*d^6 - 1733*c*d^7 - 717*d^8)*cos(f*x + e)^3 - 2*(2*c^8 - 7*c^7
*d + 14*c^6*d^2 + 291*c^5*d^3 + 726*c^4*d^4 + 384*c^3*d^5 - 557*c^2*d^6 - 6
68*c*d^7 - 185*d^8)*cos(f*x + e)^2 - 6*(2*c^8 - 9*c^7*d + 11*c^6*d^2 + 207*
c^5*d^3 + 475*c^4*d^4 + 243*c^3*d^5 - 361*c^2*d^6 - 441*c*d^7 - 127*d^8)*co
s(f*x + e))*sin(f*x + e))/((a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^6*d^5 - 6
*a^3*c^5*d^6 - 6*a^3*c^4*d^7 + 8*a^3*c^3*d^8 - 3*a^3*c*d^10 + a^3*d^11)*f*c
os(f*x + e)^5 + (2*a^3*c^10*d - 3*a^3*c^9*d^2 - 9*a^3*c^8*d^3 + 16*a^3*c^7*
d^4 + 12*a^3*c^6*d^5 - 30*a^3*c^5*d^6 - 2*a^3*c^4*d^7 + 24*a^3*c^3*d^8 - 6*
a^3*c^2*d^9 - 7*a^3*c*d^10 + 3*a^3*d^11)*f*cos(f*x + e)^4 - (a^3*c^11 + a^3
*c^10*d - 9*a^3*c^9*d^2 - a^3*c^8*d^3 + 26*a^3*c^7*d^4 - 6*a^3*c^6*d^5 - 34
```

```
*a^3*c^5*d^6 + 14*a^3*c^4*d^7 + 21*a^3*c^3*d^8 - 11*a^3*c^2*d^9 - 5*a^3*c*d
^10 + 3*a^3*d^11)*f*cos(f*x + e)^3 - (3*a^3*c^11 + a^3*c^10*d - 23*a^3*c^9*
d^2 + 3*a^3*c^8*d^3 + 62*a^3*c^7*d^4 - 22*a^3*c^6*d^5 - 78*a^3*c^5*d^6 + 38
*a^3*c^4*d^7 + 47*a^3*c^3*d^8 - 27*a^3*c^2*d^9 - 11*a^3*c*d^10 + 7*a^3*d^11
)*f*cos(f*x + e)^2 + 2*(a^3*c^11 - a^3*c^10*d - 5*a^3*c^9*d^2 + 5*a^3*c^8*d
^3 + 10*a^3*c^7*d^4 - 10*a^3*c^6*d^5 - 10*a^3*c^5*d^6 + 10*a^3*c^4*d^7 + 5*
a^3*c^3*d^8 - 5*a^3*c^2*d^9 - a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e) + 4*(a^
3*c^11 - a^3*c^10*d - 5*a^3*c^9*d^2 + 5*a^3*c^8*d^3 + 10*a^3*c^7*d^4 - 10*a
^3*c^6*d^5 - 10*a^3*c^5*d^6 + 10*a^3*c^4*d^7 + 5*a^3*c^3*d^8 - 5*a^3*c^2*d^
9 - a^3*c*d^10 + a^3*d^11)*f + ((a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^6*d^
5 - 6*a^3*c^5*d^6 - 6*a^3*c^4*d^7 + 8*a^3*c^3*d^8 - 3*a^3*c*d^10 + a^3*d^11
)*f*cos(f*x + e)^4 - 2*(a^3*c^10*d - 2*a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*
c^7*d^4 + 2*a^3*c^6*d^5 - 12*a^3*c^5*d^6 + 2*a^3*c^4*d^7 + 8*a^3*c^3*d^8 -
3*a^3*c^2*d^9 - 2*a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e)^3 - (a^3*c^11 + 3*a
^3*c^10*d - 13*a^3*c^9*d^2 - 7*a^3*c^8*d^3 + 42*a^3*c^7*d^4 - 2*a^3*c^6*d^5
 - 58*a^3*c^5*d^6 + 18*a^3*c^4*d^7 + 37*a^3*c^3*d^8 - 17*a^3*c^2*d^9 - 9*a^
3*c*d^10 + 5*a^3*d^11)*f*cos(f*x + e)^2 + 2*(a^3*c^11 - a^3*c^10*d - 5*a^3*
c^9*d^2 + 5*a^3*c^8*d^3 + 10*a^3*c^7*d^4 - 10*a^3*c^6*d^5 - 10*a^3*c^5*d^6
+ 10*a^3*c^4*d^7 + 5*a^3*c^3*d^8 - 5*a^3*c^2*d^9 - a^3*c*d^10 + a^3*d^11)*f
*cos(f*x + e) + 4*(a^3*c^11 - a^3*c^10*d - 5*a^3*c^9*d^2 + 5*a^3*c^8*d^3 +
10*a^3*c^7*d^4 - 10*a^3*c^6*d^5 - 10*a^3*c^5*d^6 + 10*a^3*c^4*d^7 + 5*a^3*c
^3*d^8 - 5*a^3*c^2*d^9 - a^3*c*d^10 + a^3*d^11)*f)*sin(f*x + e))]
```

giac [B] time = 2.04, size = 794, normalized size = 2.10

$$-\frac{\frac{15\left(20c^2d^3+30cd^4+13d^5\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(a^3c^7-3a^3c^6d+a^3c^5d^2+5a^3c^4d^3-5a^3c^3d^4-a^3c^2d^5+3a^3cd^6-a^3d^7\right)\sqrt{c^2-d^2}}+\frac{15\left(11c^3d^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+6c^2d^6\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2cd^7\tan\left(\frac{1}{2}fx+\cdots\right.\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/15*(15*(20*c^2*d^3 + 30*c*d^4 + 13*d^5)*(pi*floor(1/2*(f*x + e)/pi + 1/2
)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^3*c^7
- 3*a^3*c^6*d + a^3*c^5*d^2 + 5*a^3*c^4*d^3 - 5*a^3*c^3*d^4 - a^3*c^2*d^5 +
 3*a^3*c*d^6 - a^3*d^7)*sqrt(c^2 - d^2)) + 15*(11*c^3*d^5*tan(1/2*f*x + 1/2
*e)^3 + 6*c^2*d^6*tan(1/2*f*x + 1/2*e)^3 - 2*c*d^7*tan(1/2*f*x + 1/2*e)^3 +
 10*c^4*d^4*tan(1/2*f*x + 1/2*e)^2 + 6*c^3*d^5*tan(1/2*f*x + 1/2*e)^2 + 19*
c^2*d^6*tan(1/2*f*x + 1/2*e)^2 + 12*c*d^7*tan(1/2*f*x + 1/2*e)^2 - 2*d^8*ta
n(1/2*f*x + 1/2*e)^2 + 29*c^3*d^5*tan(1/2*f*x + 1/2*e) + 18*c^2*d^6*tan(1/2
*f*x + 1/2*e) - 2*c*d^7*tan(1/2*f*x + 1/2*e) + 10*c^4*d^4 + 6*c^3*d^5 - c^2
*d^6)/((a^3*c^9 - 3*a^3*c^8*d + a^3*c^7*d^2 + 5*a^3*c^6*d^3 - 5*a^3*c^5*d^4
```

```
 - a^3*c^4*d^5 + 3*a^3*c^3*d^6 - a^3*c^2*d^7)*(c*tan(1/2*f*x + 1/2*e)^2 + 2
*d*tan(1/2*f*x + 1/2*e) + c)^2) + 2*(15*c^2*tan(1/2*f*x + 1/2*e)^4 - 75*c*d
*tan(1/2*f*x + 1/2*e)^4 + 150*d^2*tan(1/2*f*x + 1/2*e)^4 + 30*c^2*tan(1/2*f
*x + 1/2*e)^3 - 195*c*d*tan(1/2*f*x + 1/2*e)^3 + 525*d^2*tan(1/2*f*x + 1/2*
e)^3 + 40*c^2*tan(1/2*f*x + 1/2*e)^2 - 245*c*d*tan(1/2*f*x + 1/2*e)^2 + 745
*d^2*tan(1/2*f*x + 1/2*e)^2 + 20*c^2*tan(1/2*f*x + 1/2*e) - 145*c*d*tan(1/2
*f*x + 1/2*e) + 485*d^2*tan(1/2*f*x + 1/2*e) + 7*c^2 - 44*c*d + 127*d^2)/((
a^3*c^5 - 5*a^3*c^4*d + 10*a^3*c^3*d^2 - 10*a^3*c^2*d^3 + 5*a^3*c*d^4 - a^3
*d^5)*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

maple [B] time = 0.37, size = 1462, normalized size = 3.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x)
```

```
[Out] -6/a^3/f*d^5/(c-d)^5/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/
(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2-12/a^3/f*d^7/(c-d)^5/(tan(1/2*f*x+1/2*
e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2+2
/a^3/f*d^8/(c-d)^5/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/c^2/
(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2-29/a^3/f*d^5/(c-d)^5/(tan(1/2*f*x+1/2*
e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*tan(1/2*f*x+1/2*e)+2/a
^3/f*d^7/(c-d)^5/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2
*c*d+d^2)/c*tan(1/2*f*x+1/2*e)-20/a^3/f*d^3/(c-d)^5/(c^2+2*c*d+d^2)/(c^2-d^
2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c^2-30/a^
3/f*d^4/(c-d)^5/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x
+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c+2/a^3/f*d^7/(c-d)^5/(tan(1/2*f*x+1/2*e)^2*c
+2*tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3-11/a^3/
f*d^5/(c-d)^5/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*
c*d+d^2)*tan(1/2*f*x+1/2*e)^3-10/a^3/f*d^4/(c-d)^5/(tan(1/2*f*x+1/2*e)^2*c+
2*tan(1/2*f*x+1/2*e)*d+c)^2*c^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2-8/5/a^
3/f/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^5+4/a^3/f/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)
^4-2/a^3/f/(c-d)^5/(tan(1/2*f*x+1/2*e)+1)*c^2-20/a^3/f/(c-d)^5/(tan(1/2*f*x
+1/2*e)+1)*d^2+1/a^3/f*d^6/(c-d)^5/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/
2*e)*d+c)^2/(c^2+2*c*d+d^2)+10/a^3/f/(c-d)^5/(tan(1/2*f*x+1/2*e)+1)*c*d+4/a
^3/f/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)^2*c-10/a^3/f/(c-d)^4/(tan(1/2*f*x+1/2*e
)+1)^2*d-16/3/a^3/f/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)^3*c+28/3/a^3/f/(c-d)^4/(
tan(1/2*f*x+1/2*e)+1)^3*d-18/a^3/f*d^6/(c-d)^5/(tan(1/2*f*x+1/2*e)^2*c+2*ta
n(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)-10/a^3/f*d^4/(c-
d)^5/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^
2-6/a^3/f*d^5/(c-d)^5/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(
c^2+2*c*d+d^2)*c-13/a^3/f*d^5/(c-d)^5/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arcta
n(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))-6/a^3/f*d^6/(c-d)^5/(ta
n(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*
```

```
x+1/2*e)^3-19/a^3/f*d^6/(c-d)^5/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e
)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 11.65, size = 1660, normalized size = 4.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^3),x)
```

```
[Out] (d^3*atan(((d^3*(30*c*d + 20*c^2 + 13*d^2)*(2*a^3*d^8 - 6*a^3*c*d^7 - 2*a^3
*c^7*d + 2*a^3*c^2*d^6 + 10*a^3*c^3*d^5 - 10*a^3*c^4*d^4 - 2*a^3*c^5*d^3 +
6*a^3*c^6*d^2))/(2*a^3*(c + d)^(5/2)*(c - d)^(11/2)) - (c*d^3*tan(e/2 + (f*
x)/2)*(30*c*d + 20*c^2 + 13*d^2)*(a^3*c^7 - a^3*d^7 + 3*a^3*c*d^6 - 3*a^3*c
^6*d - a^3*c^2*d^5 - 5*a^3*c^3*d^4 + 5*a^3*c^4*d^3 + a^3*c^5*d^2))/(a^3*(c
+ d)^(5/2)*(c - d)^(11/2)))/(30*c*d^4 + 13*d^5 + 20*c^2*d^3))*(30*c*d + 20*
c^2 + 13*d^2))/(a^3*f*(c + d)^(5/2)*(c - d)^(11/2)) - ((90*c*d^5 - 60*c^5*d
 + 14*c^6 - 15*d^6 + 404*c^2*d^4 + 420*c^3*d^3 + 92*c^4*d^2)/(15*(c + d)^2*
(c - d)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x)/2)^
7*(2*c*d^7 - 10*c^7*d + 4*c^8 - 2*d^8 + 49*c^2*d^6 + 141*c^3*d^5 + 200*c^4*
d^4 + 122*c^5*d^3 - 2*c^6*d^2))/(c^2*(c - d)*(2*c*d + c^2 + d^2)*(c^4 - 4*c
^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x)/2)^6*(114*c*d^7 - 54*
c^7*d + 28*c^8 - 30*d^8 + 759*c^2*d^6 + 1707*c^3*d^5 + 1960*c^4*d^4 + 870*c
^5*d^3 - 62*c^6*d^2))/(3*c^2*(c - d)*(2*c*d + c^2 + d^2)*(c^4 - 4*c^3*d - 4
*c*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x)/2)^5*(270*c*d^7 - 62*c^7*d +
32*c^8 - 60*d^8 + 1857*c^2*d^6 + 3763*c^3*d^5 + 3560*c^4*d^4 + 1294*c^5*d^3
 - 70*c^6*d^2))/(3*c^2*(c - d)*(2*c*d + c^2 + d^2)*(c^4 - 4*c^3*d - 4*c*d^3
 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x)/2)^8*(6*c*d^6 - 6*c^6*d + 2*c^7 - 2
*d^7 + 11*c^2*d^5 + 20*c^3*d^4 + 30*c^4*d^3 + 2*c^5*d^2))/(c*(c - d)*(2*c*d
 + c^2 + d^2)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*
x)/2)^2*(30*c*d^7 - 290*c^7*d + 108*c^8 - 30*d^8 + 2501*c^2*d^6 + 8725*c^3*
d^5 + 10616*c^4*d^4 + 4810*c^5*d^3 - 10*c^6*d^2))/(15*c^2*(c + d)^2*(c - d)
*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x)/2)^3*(570*
```

```
c*d^7 - 314*c^7*d + 140*c^8 - 150*d^8 + 7945*c^2*d^6 + 19441*c^3*d^5 + 1860
0*c^4*d^4 + 6898*c^5*d^3 - 210*c^6*d^2))/(15*c^2*(c + d)^2*(c - d)*(c^4 - 4
*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x)/2)^4*(1650*c*d^6 -
614*c^6*d + 204*c^7 - 300*d^7 + 10235*c^2*d^5 + 14330*c^3*d^4 + 7254*c^4*d^
3 + 316*c^5*d^2))/(15*c^2*(c + d)*(c - d)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 +
6*c^2*d^2)) + (tan(e/2 + (f*x)/2)*(195*c*d^6 - 154*c^6*d + 40*c^7 - 30*d^7
+ 1901*c^2*d^5 + 3400*c^3*d^4 + 2018*c^4*d^3 + 190*c^5*d^2))/(15*c*(c + d)^
2*(c - d)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)))/(f*(tan(e/2 + (f*x)
/2)*(5*a^3*c^2 + 4*a^3*c*d) + tan(e/2 + (f*x)/2)^2*(12*a^3*c^2 + 4*a^3*d^2
+ 20*a^3*c*d) + tan(e/2 + (f*x)/2)^7*(12*a^3*c^2 + 4*a^3*d^2 + 20*a^3*c*d)
+ tan(e/2 + (f*x)/2)^3*(20*a^3*c^2 + 20*a^3*d^2 + 44*a^3*c*d) + tan(e/2 + (
f*x)/2)^6*(20*a^3*c^2 + 20*a^3*d^2 + 44*a^3*c*d) + tan(e/2 + (f*x)/2)^4*(26
*a^3*c^2 + 40*a^3*d^2 + 60*a^3*c*d) + tan(e/2 + (f*x)/2)^5*(26*a^3*c^2 + 40
*a^3*d^2 + 60*a^3*c*d) + tan(e/2 + (f*x)/2)^8*(5*a^3*c^2 + 4*a^3*c*d) + a^3
*c^2 + a^3*c^2*tan(e/2 + (f*x)/2)^9))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.480 $\int \frac{A+B\sin(x)}{(1+\sin(x))^4}\,dx$

Optimal. Leaf size=75

$$-\frac{2(3A+4B)\cos(x)}{105(\sin(x)+1)}-\frac{2(3A+4B)\cos(x)}{105(\sin(x)+1)^2}-\frac{(3A+4B)\cos(x)}{35(\sin(x)+1)^3}-\frac{(A-B)\cos(x)}{7(\sin(x)+1)^4}$$

```
[Out] -1/7*(A-B)*cos(x)/(1+sin(x))^4-1/35*(3*A+4*B)*cos(x)/(1+sin(x))^3-2/105*(3*
A+4*B)*cos(x)/(1+sin(x))^2-2/105*(3*A+4*B)*cos(x)/(1+sin(x))
```

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2750, 2650, 2648}

$$-\frac{2(3A+4B)\cos(x)}{105(\sin(x)+1)}-\frac{2(3A+4B)\cos(x)}{105(\sin(x)+1)^2}-\frac{(3A+4B)\cos(x)}{35(\sin(x)+1)^3}-\frac{(A-B)\cos(x)}{7(\sin(x)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[x])/(1 + Sin[x])^4,x]
```

```
[Out] -((A - B)*Cos[x])/(7*(1 + Sin[x])^4) - ((3*A + 4*B)*Cos[x])/(35*(1 + Sin[x]
)^3) - (2*(3*A + 4*B)*Cos[x])/(105*(1 + Sin[x])^2) - (2*(3*A + 4*B)*Cos[x])
/(105*(1 + Sin[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\sin(x)}{(1+\sin(x))^4}\,dx = -\frac{(A-B)\cos(x)}{7(1+\sin(x))^4}+\frac{1}{7}(3A+4B)\int \frac{1}{(1+\sin(x))^3}\,dx$$
$$= -\frac{(A-B)\cos(x)}{7(1+\sin(x))^4}-\frac{(3A+4B)\cos(x)}{35(1+\sin(x))^3}+\frac{1}{35}(2(3A+4B))\int \frac{1}{(1+\sin(x))^2}\,dx$$
$$= -\frac{(A-B)\cos(x)}{7(1+\sin(x))^4}-\frac{(3A+4B)\cos(x)}{35(1+\sin(x))^3}-\frac{2(3A+4B)\cos(x)}{105(1+\sin(x))^2}+\frac{1}{105}(2(3A+4B))\int \frac{1}{1+\sin}$$
$$= -\frac{(A-B)\cos(x)}{7(1+\sin(x))^4}-\frac{(3A+4B)\cos(x)}{35(1+\sin(x))^3}-\frac{2(3A+4B)\cos(x)}{105(1+\sin(x))^2}-\frac{2(3A+4B)\cos(x)}{105(1+\sin(x))}$$

Mathematica [A] time = 0.06, size = 55, normalized size = 0.73

$$-\frac{\cos(x)\left((6A+8B)\sin^3(x)+8(3A+4B)\sin^2(x)+13(3A+4B)\sin(x)+36A+13B\right)}{105(\sin(x)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[x])/(1 + Sin[x])^4,x]
```

```
[Out] -1/105*(Cos[x]*(36*A + 13*B + 13*(3*A + 4*B)*Sin[x] + 8*(3*A + 4*B)*Sin[x]^
2 + (6*A + 8*B)*Sin[x]^3))/(1 + Sin[x])^4
```

fricas [B] time = 0.44, size = 150, normalized size = 2.00

$$\frac{2\,(3\,A+4\,B)\cos(x)^4+8\,(3\,A+4\,B)\cos(x)^3-9\,(3\,A+4\,B)\cos(x)^2-15\,(4\,A+3\,B)\cos(x)+\left(2\,(3\,A+4\,B)\cos\right.}{105\left(\cos(x)^4-3\,\cos(x)^3-8\,\cos(x)^2-\left(\cos(x)^3+4\,\cos(x)^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(x))/(1+sin(x))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(2*(3*A + 4*B)*cos(x)^4 + 8*(3*A + 4*B)*cos(x)^3 - 9*(3*A + 4*B)*cos(
x)^2 - 15*(4*A + 3*B)*cos(x) + (2*(3*A + 4*B)*cos(x)^3 - 6*(3*A + 4*B)*cos(
x)^2 - 15*(3*A + 4*B)*cos(x) + 15*A - 15*B)*sin(x) - 15*A + 15*B)/(cos(x)^4
 - 3*cos(x)^3 - 8*cos(x)^2 - (cos(x)^3 + 4*cos(x)^2 - 4*cos(x) - 8)*sin(x)
+ 4*cos(x) + 8)
```

giac [A] time = 0.20, size = 112, normalized size = 1.49

$$-\frac{2\left(105\,A\tan\left(\frac{1}{2}\,x\right)^6+315\,A\tan\left(\frac{1}{2}\,x\right)^5+105\,B\tan\left(\frac{1}{2}\,x\right)^5+630\,A\tan\left(\frac{1}{2}\,x\right)^4+175\,B\tan\left(\frac{1}{2}\,x\right)^4+630\,A\tan\right.}{105\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(x))/(1+sin(x))^4,x, algorithm="giac")
```

```
[Out] -2/105*(105*A*tan(1/2*x)^6 + 315*A*tan(1/2*x)^5 + 105*B*tan(1/2*x)^5 + 630*
A*tan(1/2*x)^4 + 175*B*tan(1/2*x)^4 + 630*A*tan(1/2*x)^3 + 280*B*tan(1/2*x)
^3 + 441*A*tan(1/2*x)^2 + 168*B*tan(1/2*x)^2 + 147*A*tan(1/2*x) + 91*B*tan(
1/2*x) + 36*A + 13*B)/(tan(1/2*x) + 1)^7
```

maple [A] time = 0.10, size = 115, normalized size = 1.53

$$-\frac{2(8A-8B)}{7\left(\tan\left(\frac{x}{2}\right)+1\right)^{7}}-\frac{2(36A-32B)}{5\left(\tan\left(\frac{x}{2}\right)+1\right)^{5}}-\frac{-6A+2B}{\left(\tan\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{2(18A-10B)}{3\left(\tan\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{-32A+24B}{2\left(\tan\left(\frac{x}{2}\right)+1\right)^{4}}-\frac{2A}{\tan\left(\frac{x}{2}\right)+1}-\frac{-24A+}{3\left(\tan\left(\frac{x}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(x))/(1+sin(x))^4,x)
```

```
[Out] -2/7*(8*A-8*B)/(tan(1/2*x)+1)^7-2/5*(36*A-32*B)/(tan(1/2*x)+1)^5-(-6*A+2*B)
/(tan(1/2*x)+1)^2-2/3*(18*A-10*B)/(tan(1/2*x)+1)^3-1/2*(-32*A+24*B)/(tan(1/
2*x)+1)^4-2*A/(tan(1/2*x)+1)-1/3*(-24*A+24*B)/(tan(1/2*x)+1)^6
```

maxima [B] time = 0.33, size = 309, normalized size = 4.12

$$-\frac{2B\left(\frac{91\sin(x)}{\cos(x)+1}+\frac{168\sin(x)^2}{(\cos(x)+1)^2}+\frac{280\sin(x)^3}{(\cos(x)+1)^3}+\frac{175\sin(x)^4}{(\cos(x)+1)^4}+\frac{105\sin(x)^5}{(\cos(x)+1)^5}+13\right)}{105\left(\frac{7\sin(x)}{\cos(x)+1}+\frac{21\sin(x)^2}{(\cos(x)+1)^2}+\frac{35\sin(x)^3}{(\cos(x)+1)^3}+\frac{35\sin(x)^4}{(\cos(x)+1)^4}+\frac{21\sin(x)^5}{(\cos(x)+1)^5}+\frac{7\sin(x)^6}{(\cos(x)+1)^6}+\frac{\sin(x)^7}{(\cos(x)+1)^7}+1\right)}-\frac{2A\left(\frac{49\sin(x)}{\cos(x)+1}+\right.}{35\left(\frac{7\sin(x)}{\cos(x)+1}+\frac{21\sin}{(\cos(x}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(x))/(1+sin(x))^4,x, algorithm="maxima")
```

```
[Out] -2/105*B*(91*sin(x)/(cos(x) + 1) + 168*sin(x)^2/(cos(x) + 1)^2 + 280*sin(x)
^3/(cos(x) + 1)^3 + 175*sin(x)^4/(cos(x) + 1)^4 + 105*sin(x)^5/(cos(x) + 1)
^5 + 13)/(7*sin(x)/(cos(x) + 1) + 21*sin(x)^2/(cos(x) + 1)^2 + 35*sin(x)^3/
(cos(x) + 1)^3 + 35*sin(x)^4/(cos(x) + 1)^4 + 21*sin(x)^5/(cos(x) + 1)^5 +
7*sin(x)^6/(cos(x) + 1)^6 + sin(x)^7/(cos(x) + 1)^7 + 1) - 2/35*A*(49*sin(x
)/(cos(x) + 1) + 147*sin(x)^2/(cos(x) + 1)^2 + 210*sin(x)^3/(cos(x) + 1)^3
+ 210*sin(x)^4/(cos(x) + 1)^4 + 105*sin(x)^5/(cos(x) + 1)^5 + 35*sin(x)^6/(
cos(x) + 1)^6 + 12)/(7*sin(x)/(cos(x) + 1) + 21*sin(x)^2/(cos(x) + 1)^2 + 3
5*sin(x)^3/(cos(x) + 1)^3 + 35*sin(x)^4/(cos(x) + 1)^4 + 21*sin(x)^5/(cos(x
) + 1)^5 + 7*sin(x)^6/(cos(x) + 1)^6 + sin(x)^7/(cos(x) + 1)^7 + 1)
```

mupad [B] time = 7.07, size = 94, normalized size = 1.25

$$-\frac{2A\tan\left(\frac{x}{2}\right)^6+(6A+2B)\tan\left(\frac{x}{2}\right)^5+\left(12A+\frac{10B}{3}\right)\tan\left(\frac{x}{2}\right)^4+\left(12A+\frac{16B}{3}\right)\tan\left(\frac{x}{2}\right)^3+\left(\frac{42A}{5}+\frac{16B}{5}\right)\tan\left(\frac{x}{2}\right)^2}{\left(\tan\left(\frac{x}{2}\right)+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(x))/(sin(x) + 1)^4,x)
```

```
[Out] -((24*A)/35 + (26*B)/105 + 2*A*tan(x/2)^6 + tan(x/2)*((14*A)/5 + (26*B)/15)
 + tan(x/2)^5*(6*A + 2*B) + tan(x/2)^4*(12*A + (10*B)/3) + tan(x/2)^3*(12*A
 + (16*B)/3) + tan(x/2)^2*((42*A)/5 + (16*B)/5))/(tan(x/2) + 1)^7
```

sympy [B] time = 7.70, size = 889, normalized size = 11.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(x))/(1+sin(x))**4,x)
```

```
[Out] -210*A*tan(x/2)**6/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*tan(x/2)**5 +
3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*tan(x/2) + 105
) - 630*A*tan(x/2)**5/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*tan(x/2)**5
 + 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*tan(x/2) +
105) - 1260*A*tan(x/2)**4/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*tan(x/2
)**5 + 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*tan(x/2
) + 105) - 1260*A*tan(x/2)**3/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*tan
(x/2)**5 + 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*tan
(x/2) + 105) - 882*A*tan(x/2)**2/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*
tan(x/2)**5 + 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*
tan(x/2) + 105) - 294*A*tan(x/2)/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*
tan(x/2)**5 + 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*
tan(x/2) + 105) - 72*A/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*tan(x/2)**
5 + 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*tan(x/2) +
 105) - 210*B*tan(x/2)**5/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*tan(x/2
)**5 + 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*tan(x/2
) + 105) - 350*B*tan(x/2)**4/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*tan(
x/2)**5 + 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*tan(
x/2) + 105) - 560*B*tan(x/2)**3/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*t
an(x/2)**5 + 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*t
an(x/2) + 105) - 336*B*tan(x/2)**2/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 220
5*tan(x/2)**5 + 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 73
5*tan(x/2) + 105) - 182*B*tan(x/2)/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 220
5*tan(x/2)**5 + 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 73
5*tan(x/2) + 105) - 26*B/(105*tan(x/2)**7 + 735*tan(x/2)**6 + 2205*tan(x/2)
**5 + 3675*tan(x/2)**4 + 3675*tan(x/2)**3 + 2205*tan(x/2)**2 + 735*tan(x/2)
 + 105)
```

3.481 $\int \frac{A+B\sin(x)}{(1-\sin(x))^4}\,dx$

Optimal. Leaf size=81

$$\frac{2(3A-4B)\cos(x)}{105(1-\sin(x))}+\frac{2(3A-4B)\cos(x)}{105(1-\sin(x))^2}+\frac{(3A-4B)\cos(x)}{35(1-\sin(x))^3}+\frac{(A+B)\cos(x)}{7(1-\sin(x))^4}$$

```
[Out] 1/7*(A+B)*cos(x)/(1-sin(x))^4+1/35*(3*A-4*B)*cos(x)/(1-sin(x))^3+2/105*(3*A
-4*B)*cos(x)/(1-sin(x))^2+2/105*(3*A-4*B)*cos(x)/(1-sin(x))
```

Rubi [A] time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2750, 2650, 2648}

$$\frac{2(3A-4B)\cos(x)}{105(1-\sin(x))}+\frac{2(3A-4B)\cos(x)}{105(1-\sin(x))^2}+\frac{(3A-4B)\cos(x)}{35(1-\sin(x))^3}+\frac{(A+B)\cos(x)}{7(1-\sin(x))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[x])/(1 - Sin[x])^4,x]
```

```
[Out] ((A + B)*Cos[x])/(7*(1 - Sin[x])^4) + ((3*A - 4*B)*Cos[x])/(35*(1 - Sin[x])
^3) + (2*(3*A - 4*B)*Cos[x])/(105*(1 - Sin[x])^2) + (2*(3*A - 4*B)*Cos[x])/
(105*(1 - Sin[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\sin(x)}{(1-\sin(x))^4}\,dx = \frac{(A+B)\cos(x)}{7(1-\sin(x))^4} + \frac{1}{7}(3A-4B)\int \frac{1}{(1-\sin(x))^3}\,dx$$
$$= \frac{(A+B)\cos(x)}{7(1-\sin(x))^4} + \frac{(3A-4B)\cos(x)}{35(1-\sin(x))^3} + \frac{1}{35}(2(3A-4B))\int \frac{1}{(1-\sin(x))^2}\,dx$$
$$= \frac{(A+B)\cos(x)}{7(1-\sin(x))^4} + \frac{(3A-4B)\cos(x)}{35(1-\sin(x))^3} + \frac{2(3A-4B)\cos(x)}{105(1-\sin(x))^2} + \frac{1}{105}(2(3A-4B))\int \frac{1}{1-\sin(x}$$
$$= \frac{(A+B)\cos(x)}{7(1-\sin(x))^4} + \frac{(3A-4B)\cos(x)}{35(1-\sin(x))^3} + \frac{2(3A-4B)\cos(x)}{105(1-\sin(x))^2} + \frac{2(3A-4B)\cos(x)}{105(1-\sin(x))}$$

Mathematica [A] time = 0.07, size = 54, normalized size = 0.67

$$\frac{\cos(x)\left((8B-6A)\sin^3(x)+8(3A-4B)\sin^2(x)+(52B-39A)\sin(x)+36A-13B\right)}{105(\sin(x)-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[x])/(1 - Sin[x])^4,x]
```

```
[Out] (Cos[x]*(36*A - 13*B + (-39*A + 52*B)*Sin[x] + 8*(3*A - 4*B)*Sin[x]^2 + (-6
*A + 8*B)*Sin[x]^3))/(105*(-1 + Sin[x])^4)
```

fricas [B] time = 0.43, size = 150, normalized size = 1.85

$$-\frac{2\,(3\,A-4\,B)\cos(x)^4+8\,(3\,A-4\,B)\cos(x)^3-9\,(3\,A-4\,B)\cos(x)^2-15\,(4\,A-3\,B)\cos(x)-\left(2\,(3\,A-4\,B)\,\mathrm{co}\right.}{105\left(\cos(x)^4-3\,\cos(x)^3-8\,\cos(x)^2+\left(\cos(x)^3+4\,\cos(x)^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(x))/(1-sin(x))^4,x, algorithm="fricas")
```

```
[Out] -1/105*(2*(3*A - 4*B)*cos(x)^4 + 8*(3*A - 4*B)*cos(x)^3 - 9*(3*A - 4*B)*cos
(x)^2 - 15*(4*A - 3*B)*cos(x) - (2*(3*A - 4*B)*cos(x)^3 - 6*(3*A - 4*B)*cos
(x)^2 - 15*(3*A - 4*B)*cos(x) + 15*A + 15*B)*sin(x) - 15*A - 15*B)/(cos(x)^
4 - 3*cos(x)^3 - 8*cos(x)^2 + (cos(x)^3 + 4*cos(x)^2 - 4*cos(x) - 8)*sin(x)
 + 4*cos(x) + 8)
```

giac [A] time = 0.27, size = 112, normalized size = 1.38

$$-\frac{2\left(105\,A\tan\left(\frac{1}{2}\,x\right)^6-315\,A\tan\left(\frac{1}{2}\,x\right)^5+105\,B\tan\left(\frac{1}{2}\,x\right)^5+630\,A\tan\left(\frac{1}{2}\,x\right)^4-175\,B\tan\left(\frac{1}{2}\,x\right)^4-630\,A\tan\left(\right.\right.}{105\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(x))/(1-sin(x))^4,x, algorithm="giac")
```

```
[Out] -2/105*(105*A*tan(1/2*x)^6 - 315*A*tan(1/2*x)^5 + 105*B*tan(1/2*x)^5 + 630*
A*tan(1/2*x)^4 - 175*B*tan(1/2*x)^4 - 630*A*tan(1/2*x)^3 + 280*B*tan(1/2*x)
^3 + 441*A*tan(1/2*x)^2 - 168*B*tan(1/2*x)^2 - 147*A*tan(1/2*x) + 91*B*tan(
1/2*x) + 36*A - 13*B)/(tan(1/2*x) - 1)^7
```

maple [A] time = 0.10, size = 115, normalized size = 1.42

$$-\frac{2(18A+10B)}{3\left(\tan\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{24A+24B}{3\left(\tan\left(\frac{x}{2}\right)-1\right)^{6}}-\frac{2(8A+8B)}{7\left(\tan\left(\frac{x}{2}\right)-1\right)^{7}}-\frac{2A}{\tan\left(\frac{x}{2}\right)-1}-\frac{2(36A+32B)}{5\left(\tan\left(\frac{x}{2}\right)-1\right)^{5}}-\frac{6A+2B}{\left(\tan\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{32A+2\cdots}{2\left(\tan\left(\frac{x}{2}\right)-\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(x))/(1-sin(x))^4,x)
```

```
[Out] -2/3*(18*A+10*B)/(tan(1/2*x)-1)^3-1/3*(24*A+24*B)/(tan(1/2*x)-1)^6-2/7*(8*A
+8*B)/(tan(1/2*x)-1)^7-2*A/(tan(1/2*x)-1)-2/5*(36*A+32*B)/(tan(1/2*x)-1)^5-
(6*A+2*B)/(tan(1/2*x)-1)^2-1/2*(32*A+24*B)/(tan(1/2*x)-1)^4
```

maxima [B] time = 0.33, size = 309, normalized size = 3.81

$$-\frac{2B\left(\frac{91\sin(x)}{\cos(x)+1}-\frac{168\sin(x)^{2}}{(\cos(x)+1)^{2}}+\frac{280\sin(x)^{3}}{(\cos(x)+1)^{3}}-\frac{175\sin(x)^{4}}{(\cos(x)+1)^{4}}+\frac{105\sin(x)^{5}}{(\cos(x)+1)^{5}}-13\right)}{105\left(\frac{7\sin(x)}{\cos(x)+1}-\frac{21\sin(x)^{2}}{(\cos(x)+1)^{2}}+\frac{35\sin(x)^{3}}{(\cos(x)+1)^{3}}-\frac{35\sin(x)^{4}}{(\cos(x)+1)^{4}}+\frac{21\sin(x)^{5}}{(\cos(x)+1)^{5}}-\frac{7\sin(x)^{6}}{(\cos(x)+1)^{6}}+\frac{\sin(x)^{7}}{(\cos(x)+1)^{7}}-1\right)}+\frac{2A\left(\frac{49\sin(x)}{\cos(x)+1}-\cdots\right.}{35\left(\frac{7\sin(x)}{\cos(x)+1}-\frac{21\sin\cdots}{(\cos(x)\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(x))/(1-sin(x))^4,x, algorithm="maxima")
```

```
[Out] -2/105*B*(91*sin(x)/(cos(x) + 1) - 168*sin(x)^2/(cos(x) + 1)^2 + 280*sin(x)
^3/(cos(x) + 1)^3 - 175*sin(x)^4/(cos(x) + 1)^4 + 105*sin(x)^5/(cos(x) + 1)
^5 - 13)/(7*sin(x)/(cos(x) + 1) - 21*sin(x)^2/(cos(x) + 1)^2 + 35*sin(x)^3/
(cos(x) + 1)^3 - 35*sin(x)^4/(cos(x) + 1)^4 + 21*sin(x)^5/(cos(x) + 1)^5 -
7*sin(x)^6/(cos(x) + 1)^6 + sin(x)^7/(cos(x) + 1)^7 - 1) + 2/35*A*(49*sin(x
)/(cos(x) + 1) - 147*sin(x)^2/(cos(x) + 1)^2 + 210*sin(x)^3/(cos(x) + 1)^3
- 210*sin(x)^4/(cos(x) + 1)^4 + 105*sin(x)^5/(cos(x) + 1)^5 - 35*sin(x)^6/(
cos(x) + 1)^6 - 12)/(7*sin(x)/(cos(x) + 1) - 21*sin(x)^2/(cos(x) + 1)^2 + 3
5*sin(x)^3/(cos(x) + 1)^3 - 35*sin(x)^4/(cos(x) + 1)^4 + 21*sin(x)^5/(cos(x
) + 1)^5 - 7*sin(x)^6/(cos(x) + 1)^6 + sin(x)^7/(cos(x) + 1)^7 - 1)
```

mupad [B] time = 7.06, size = 97, normalized size = 1.20

$$-\frac{2A\tan\left(\frac{x}{2}\right)^{6}+(2B-6A)\tan\left(\frac{x}{2}\right)^{5}+\left(12A-\frac{10B}{3}\right)\tan\left(\frac{x}{2}\right)^{4}+\left(\frac{16B}{3}-12A\right)\tan\left(\frac{x}{2}\right)^{3}+\left(\frac{42A}{5}-\frac{16B}{5}\right)\tan\left(\frac{x}{2}\right)^{2}\cdots}{\left(\tan\left(\frac{x}{2}\right)-1\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(x))/(sin(x) - 1)^4,x)
```

```
[Out] -((24*A)/35 - (26*B)/105 + 2*A*tan(x/2)^6 - tan(x/2)*((14*A)/5 - (26*B)/15)
 - tan(x/2)^5*(6*A - 2*B) + tan(x/2)^4*(12*A - (10*B)/3) - tan(x/2)^3*(12*A
 - (16*B)/3) + tan(x/2)^2*((42*A)/5 - (16*B)/5))/(tan(x/2) - 1)^7
```

sympy [B] time = 7.87, size = 887, normalized size = 10.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(x))/(1-sin(x))**4,x)
```

```
[Out] -210*A*tan(x/2)**6/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*tan(x/2)**5 -
3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*tan(x/2) - 105
) + 630*A*tan(x/2)**5/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*tan(x/2)**5
 - 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*tan(x/2) -
105) - 1260*A*tan(x/2)**4/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*tan(x/2
)**5 - 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*tan(x/2
) - 105) + 1260*A*tan(x/2)**3/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*tan
(x/2)**5 - 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*tan
(x/2) - 105) - 882*A*tan(x/2)**2/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*
tan(x/2)**5 - 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*
tan(x/2) - 105) + 294*A*tan(x/2)/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*
tan(x/2)**5 - 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*
tan(x/2) - 105) - 72*A/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*tan(x/2)**
5 - 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*tan(x/2) -
 105) - 210*B*tan(x/2)**5/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*tan(x/2
)**5 - 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*tan(x/2
) - 105) + 350*B*tan(x/2)**4/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*tan(
x/2)**5 - 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*tan(
x/2) - 105) - 560*B*tan(x/2)**3/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*t
an(x/2)**5 - 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*t
an(x/2) - 105) + 336*B*tan(x/2)**2/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 220
5*tan(x/2)**5 - 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 73
5*tan(x/2) - 105) - 182*B*tan(x/2)/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 220
5*tan(x/2)**5 - 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 73
5*tan(x/2) - 105) + 26*B/(105*tan(x/2)**7 - 735*tan(x/2)**6 + 2205*tan(x/2)
**5 - 3675*tan(x/2)**4 + 3675*tan(x/2)**3 - 2205*tan(x/2)**2 + 735*tan(x/2)
 - 105)
```

3.482 $\int (a+a\sin(e+fx))(c+d\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=290

$$-\frac{2a\left(15c^2+56cd+25d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}-\frac{2a\left(c^2-d^2\right)\left(15c^2+56cd+25d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(\right.\right.}{105df\sqrt{c+d\sin(e+fx)}}$$

```
[Out] -2/35*a*(5*c+7*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/f-2/7*a*cos(f*x+e)*(c+d
*sin(f*x+e))^(5/2)/f-2/105*a*(15*c^2+56*c*d+25*d^2)*cos(f*x+e)*(c+d*sin(f*x
+e))^(1/2)/f-2/105*a*(15*c^3+161*c^2*d+145*c*d^2+63*d^3)*(sin(1/2*e+1/4*Pi+
1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*
f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d/f/((c+d*sin(f*x+e))/
(c+d))^(1/2)+2/105*a*(c^2-d^2)*(15*c^2+56*c*d+25*d^2)*(sin(1/2*e+1/4*Pi+1/2
*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x
),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d/f/(c+d*sin(f*x+
e))^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a\left(15c^2+56cd+25d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}-\frac{2a\left(c^2-d^2\right)\left(15c^2+56cd+25d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(\right.\right.}{105df\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*a*(15*c^2 + 56*c*d + 25*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(10
5*f) - (2*a*(5*c + 7*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(35*f) - (
2*a*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(7*f) + (2*a*(15*c^3 + 161*c^2
*d + 145*c*d^2 + 63*d^3)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[
c + d*Sin[e + f*x]])/(105*d*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*a*(c
^2 - d^2)*(15*c^2 + 56*c*d + 25*d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c
 + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(105*d*f*Sqrt[c + d*Sin[e + f*x]
])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int (a+a\sin(e+fx))(c+d\sin(e+fx))^{5/2}\,dx = -\frac{2a\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{7f}+\frac{2}{7}\int (c+d\sin(e+fx))^{3/2}$$

$$= -\frac{2a(5c+7d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{35f}-\frac{2a\cos(e+fx)(}{}$$

$$= -\frac{2a\left(15c^2+56cd+25d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}-\frac{2a(5}{}$$

$$= -\frac{2a\left(15c^2+56cd+25d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}-\frac{2a(5}{}$$

$$= -\frac{2a\left(15c^2+56cd+25d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}-\frac{2a(5}{}$$

$$= -\frac{2a\left(15c^2+56cd+25d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}-\frac{2a(5}{}$$

Mathematica [C] time = 6.76, size = 3531, normalized size = 12.18

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] a*((c^3*Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Cs
c[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1
 + Cot[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Co
s[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(
-1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]]
)/(Sqrt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e
]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 +
 Cot[e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Co
t[e]^2] + c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2
]*Sin[e]])) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]
^2]*Sin[e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e
]]])/Sqrt[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[
e]^2]*Sin[e]]))/(7*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (23*c^2
*d*Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*
```

(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Co
t[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x
 - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 -
(c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sq
rt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*S
qrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[
e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^
2] + c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin
[e]])) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*S
in[e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/
Sqrt[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]
*Sin[e]]))/(15*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (29*c*d^2*S
ec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*(c +
 d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]
^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x - A
rcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*C
sc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[1
 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[
1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^2
] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] +
 c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]]
)) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e
]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqrt
[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin
[e]]))/(21*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (3*d^3*Sec[e]*(
1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*(c + d*Cos[
f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(1
- (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x - ArcTan[C
ot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*Csc[e])/
(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[1 + Cot[
e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot
[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^2] - d*C
os[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] + c*Csc[
e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]])) - ((
2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d^
2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqrt[1 + Co
t[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]]))/
(5*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + ((1 + Sin[e + f*x])*Sqr
t[c + d*Sin[e + f*x]]*(-1/210*((180*c^2 + 308*c*d + 115*d^2)*Cos[e]*Cos[f*x
])/f + (d^2*Cos[3*e]*Cos[3*f*x])/(14*f) - (d*(15*c + 7*d)*Cos[2*f*x]*Sin[2*
e])/(35*f) + ((180*c^2 + 308*c*d + 115*d^2)*Sin[e]*Sin[f*x])/(210*f) - (d*(
15*c + 7*d)*Cos[2*e]*Sin[2*f*x])/(35*f) - (d^2*Sin[3*e]*Sin[3*f*x])/(14*f)
+ (2*(15*c^3 + 161*c^2*d + 145*c*d^2 + 63*d^3)*Tan[e])/(105*d*f)))/(Cos[e/2
 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2 + (18*c^2*AppellF1[1/2, 1/2, 1/2, 3/2,
-((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*S

```
qrt[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c +
 d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]
^2]*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan
[e]]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Ta
n[e]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[
1 + Tan[e]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]
) + d*Sqrt[1 + Tan[e]^2])]*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt
[1 + Tan[e]^2]])/(7*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2*Sqrt[1 +
Tan[e]^2]) + (2*c^3*AppellF1[1/2, 1/2, 1/2, 3/2, -((Sec[e]*(c + d*Cos[e]*Si
n[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 - (c*
Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan
[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d*Sq
rt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]]]*(1 + Sin[e + f*x])*S
qrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])
/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f*x
+ ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2])]
*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]])/(d*f*(Cos
[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2*Sqrt[1 + Tan[e]^2]) + (34*c*d*Appel
lF1[1/2, 1/2, 1/2, 3/2, -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*S
qrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e
]^2])))), -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]
^2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec
[e]*Sec[f*x + ArcTan[Tan[e]]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2]
 - d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + T
an[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1
+ Tan[e]^2])/(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2])]*Sqrt[c + d*Cos[e]*Sin[f*
x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]])/(15*f*(Cos[e/2 + (f*x)/2] + Sin[e/
2 + (f*x)/2])^2*Sqrt[1 + Tan[e]^2]) + (10*d^2*AppellF1[1/2, 1/2, 1/2, 3/2,
-((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*S
qrt[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c +
 d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]
^2]*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan
[e]]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Ta
n[e]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[
1 + Tan[e]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]
) + d*Sqrt[1 + Tan[e]^2])]*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt
[1 + Tan[e]^2]])/(21*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2*Sqrt[1 +
 Tan[e]^2]))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ac^2 + 2\,acd + ad^2 - \left(2\,acd + ad^2\right)\cos\left(fx + e\right)^2 - \left(ad^2\cos\left(fx + e\right)^2 - ac^2 - 2\,acd - ad^2\right)\sin\left(fx + e\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a*c^2 + 2*a*c*d + a*d^2 - (2*a*c*d + a*d^2)*cos(f*x + e)^2 - (a*d
^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2)*sin(f*x + e))*sqrt(d*sin(f*x +
 e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2), x)
```

maple [B] time = 1.72, size = 1315, normalized size = 4.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] 2/105*a*(-161*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2
)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))*c^4*d-25*c*d^4-77*c^2*d^3-45*c^3*d^2-15*((c+d*sin(f*x+
e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^
(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^5+63*
((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f
*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^
(1/2))*d^5-88*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2
)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))*d^5+15*d^5*sin(f*x+e)^5+21*d^5*sin(f*x+e)^4+10*d^5*sin
(f*x+e)^3-21*d^5*sin(f*x+e)^2-25*d^5*sin(f*x+e)+45*c^3*d^2*sin(f*x+e)^2+77*
c^2*d^3*sin(f*x+e)^2-35*c*d^4*sin(f*x+e)^2-90*c^2*d^3*sin(f*x+e)-98*c*d^4*s
in(f*x+e)+60*c*d^4*sin(f*x+e)^4+90*c^2*d^3*sin(f*x+e)^3+98*c*d^4*sin(f*x+e)
^3-130*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(
1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2))*c^3*d^2+120*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d
/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(
c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^4*d+176*((c+d*sin(f*x+e))/(c-d))^(1/2)*(
-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^3*d^2-32*((c+d*sin(f*x+
e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^
(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^3
```

```
-176*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+
sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c
+d))^(1/2))*c*d^4+98*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d
))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))
^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^3+145*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(s
in(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d
*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^4)/d^2/cos(f*x+e)/(c+d*s
in(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(fx+e)+a)(d\sin(fx+e)+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+a\sin(e+fx))\,(c+d\sin(e+fx))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int c^2\sqrt{c+d\sin(e+fx)}\,dx+\int c^2\sqrt{c+d\sin(e+fx)}\sin(e+fx)\,dx+\int d^2\sqrt{c+d\sin(e+fx)}\sin^2(e+f$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] a*(Integral(c**2*sqrt(c + d*sin(e + f*x)), x) + Integral(c**2*sqrt(c + d*si
n(e + f*x))*sin(e + f*x), x) + Integral(d**2*sqrt(c + d*sin(e + f*x))*sin(e
 + f*x)**2, x) + Integral(d**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**3, x)
 + Integral(2*c*d*sqrt(c + d*sin(e + f*x))*sin(e + f*x), x) + Integral(2*c*
d*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2, x))
```

3.483 $\int (a + a\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=231

$$-\frac{2a(3c+5d)\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15df\sqrt{c+d\sin(e+fx)}}+\frac{2a\left(3c^2+20cd+9d^2\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e-\right.\right.}{15df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] -2/5*a*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/f-2/15*a*(3*c+5*d)*cos(f*x+e)*(c+d
*sin(f*x+e))^(1/2)/f-2/15*a*(3*c^2+20*c*d+9*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)
^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(
1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d/f/((c+d*sin(f*x+e))/(c+d))^(
1/2)+2/15*a*(3*c+5*d)*(c^2-d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2
*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1
/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a(3c+5d)\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15df\sqrt{c+d\sin(e+fx)}}+\frac{2a\left(3c^2+20cd+9d^2\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e-\right.\right.}{15df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a*(3*c + 5*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*f) - (2*a*Cos[
e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(5*f) + (2*a*(3*c^2 + 20*c*d + 9*d^2)*
EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(15*
d*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*a*(3*c + 5*d)*(c^2 - d^2)*Elli
pticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)]
)/(15*d*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int (a + a\sin(e + fx))(c + d\sin(e + fx))^{3/2}\,dx = -\frac{2a\cos(e + fx)(c + d\sin(e + fx))^{3/2}}{5f} + \frac{2}{5}\int \sqrt{c + d\sin(e + fx)}$$

$$= -\frac{2a(3c + 5d)\cos(e + fx)\sqrt{c + d\sin(e + fx)}}{15f} - \frac{2a\cos(e + fx)(c}{}$$

$$= -\frac{2a(3c + 5d)\cos(e + fx)\sqrt{c + d\sin(e + fx)}}{15f} - \frac{2a\cos(e + fx)(c}{}$$

$$= -\frac{2a(3c + 5d)\cos(e + fx)\sqrt{c + d\sin(e + fx)}}{15f} - \frac{2a\cos(e + fx)(c}{}$$

$$= -\frac{2a(3c + 5d)\cos(e + fx)\sqrt{c + d\sin(e + fx)}}{15f} - \frac{2a\cos(e + fx)(c}{}$$

Mathematica [C] time = 6.40, size = 2625, normalized size = 11.36

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] a*((c^2*Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Cs
c[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1
 + Cot[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Co
s[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(
-1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]]
)/(Sqrt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e
]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 +
 Cot[e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Co
t[e]^2] + c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2
]*Sin[e]])) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]
^2]*Sin[e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e
]]])/Sqrt[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[
e]^2]*Sin[e]]))/(5*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (4*c*d*
Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*(c
+ d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e
]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x -
ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*
```

```
Csc[e])/(d*Sqrt[1 + Cot[e]^2])))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[
1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt
[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^
2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2]
+ c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]
])) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[
e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqr
t[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Si
n[e]]))/(3*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (3*d^2*Sec[e]*(
1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*(c + d*Cos[
f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(1
- (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x - ArcTan[C
ot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*Csc[e])/
(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[1 + Cot[
e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot
[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^2] - d*C
os[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] + c*Csc[
e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]])) - ((
2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d^
2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqrt[1 + Co
t[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]]))/
(5*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + ((1 + Sin[e + f*x])*Sqr
t[c + d*Sin[e + f*x]]*((-2*(6*c + 5*d)*Cos[e]*Cos[f*x])/(15*f) - (d*Cos[2*f
*x]*Sin[2*e])/(5*f) + (2*(6*c + 5*d)*Sin[e]*Sin[f*x])/(15*f) - (d*Cos[2*e]*
Sin[2*f*x])/(5*f) + (2*(3*c^2 + 20*c*d + 9*d^2)*Tan[e])/(15*d*f)))/(Cos[e/2
 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2 + (8*c*AppellF1[1/2, 1/2, 1/2, 3/2, -((
Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt
[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*
Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]
*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]
]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e
]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 +
 Tan[e]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) +
 d*Sqrt[1 + Tan[e]^2])]*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1
+ Tan[e]^2]])/(5*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2*Sqrt[1 + Tan
[e]^2]) + (2*c^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((Sec[e]*(c + d*Cos[e]*Sin[f
*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 - (c*Sec
[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Ta
n[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d*Sqrt[
1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]]]*(1 + Sin[e + f*x])*Sqrt
[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(c
*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f*x + A
rcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2])]*Sq
rt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]])/(d*f*(Cos[e/
2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2*Sqrt[1 + Tan[e]^2]) + (2*d*AppellF1[1/
```

```
2, 1/2, 1/2, 3/2, -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1
+ Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))
)), -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/
(d*Sqrt[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Se
c[f*x + ArcTan[Tan[e]]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*S
in[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^
2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[
e]^2])/(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2])]*Sqrt[c + d*Cos[e]*Sin[f*x + Ar
cTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]])/(3*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*
x)/2])^2*Sqrt[1 + Tan[e]^2]))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(ad\cos\left(fx+e\right)^{2}-ac-ad-(ac+ad)\sin\left(fx+e\right)\right)\sqrt{d\sin\left(fx+e\right)+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*d*cos(f*x + e)^2 - a*c - a*d - (a*c + a*d)*sin(f*x + e))*sqrt(
d*sin(f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2), x)
```

maple [B] time = 1.46, size = 1034, normalized size = 4.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] 2/15*a*(18*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(
-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c
-d)/(c+d))^(1/2))*c^3*d+14*c^2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)
-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+
e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^2-18*c*((c+d*sin(f*x+e))/(c-d))^(1/
```

```
2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Elliptic
F(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^3-14*((c+d*sin(f*x+
e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^
(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^4-3*(
(c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*
x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(
1/2))*c^4-20*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)
*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),(
(c-d)/(c+d))^(1/2))*c^3*d-6*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)
*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))
/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^2+20*((c+d*sin(f*x+e))/(c-d))^(1/2
)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE
(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^3+9*((c+d*sin(f*x+
e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^
(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^4+3*d
^4*sin(f*x+e)^4+9*c*d^3*sin(f*x+e)^3+5*d^4*sin(f*x+e)^3+6*c^2*d^2*sin(f*x+e
)^2+5*c*d^3*sin(f*x+e)^2-3*d^4*sin(f*x+e)^2-9*c*d^3*sin(f*x+e)-5*d^4*sin(f*
x+e)-6*c^2*d^2-5*c*d^3)/d^2/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin\left(fx + e\right) + a\right)\left(d \sin\left(fx + e\right) + c\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + a\,\sin\left(e + f\,x\right)\right)\,\left(c + d\,\sin\left(e + f\,x\right)\right)^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int c\sqrt{c + d\sin\left(e + fx\right)}\, dx + \int c\sqrt{c + d\sin\left(e + fx\right)}\sin\left(e + fx\right) dx + \int d\sqrt{c + d\sin\left(e + fx\right)}\sin\left(e + fx\right) d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] a*(Integral(c*sqrt(c + d*sin(e + f*x)), x) + Integral(c*sqrt(c + d*sin(e +
f*x))*sin(e + f*x), x) + Integral(d*sqrt(c + d*sin(e + f*x))*sin(e + f*x),
x) + Integral(d*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2, x))
```

3.484 $\int (a+a\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=179

$$-\frac{2a\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3df\sqrt{c+d\sin(e+fx)}}-\frac{2a\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f}+\frac{2a(c+3d)\sqrt{c+d\sin(e+}}{3df\sqrt{c}}$$

```
[Out] -2/3*a*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f-2/3*a*(c+3*d)*(sin(1/2*e+1/4*Pi+
1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*
f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d/f/((c+d*sin(f*x+e))/
(c+d))^(1/2)+2/3*a*(c^2-d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+
1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2)
)*((c+d*sin(f*x+e))/(c+d))^(1/2)/d/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3df\sqrt{c+d\sin(e+fx)}}-\frac{2a\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f}+\frac{2a(c+3d)\sqrt{c+d\sin(e+}}{3df\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*a*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*f) + (2*a*(c + 3*d)*Ellipti
cE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*d*f*Sqrt
[(c + d*Sin[e + f*x])/(c + d)]) - (2*a*(c^2 - d^2)*EllipticF[(e - Pi/2 + f*
x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*d*f*Sqrt[c + d*
Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int (a+a\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx = -\frac{2a\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f} + \frac{2}{3}\int \frac{\frac{1}{2}a(3c+d)+\frac{1}{2}a(c+3d}{\sqrt{c+d\sin(e+}}$$

$$= -\frac{2a\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f} + \frac{(a(c+3d))\int \sqrt{c+d\sin(e+}}{3d}$$

$$= -\frac{2a\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f} + \frac{\left(a(c+3d)\sqrt{c+d\sin(e+fx}\right.}{3d\sqrt{\frac{c+d\,\mathrm{si}}{c}}}$$

$$= -\frac{2a\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f} + \frac{2a(c+3d)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\right.}{3df\sqrt{\frac{c+d\,\mathrm{si}}{c}}}$$

Mathematica [C] time = 6.28, size = 1736, normalized size = 9.70

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] a*((c*Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[
e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 +
 Cot[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[
f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1
 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/
(Sqrt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]
]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + C
ot[e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[
e]^2] + c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*
Sin[e]])) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2
]*Sin[e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]
])/Sqrt[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]
^2]*Sin[e]]))/(3*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (d*Sec[e]
*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*(c + d*Co
s[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(
1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x - ArcTan
[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*Csc[e]
)/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[1 + Co
t[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + C
```

```
ot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^2] - d
*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] + c*Cs
c[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]])) -
((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(
d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqrt[1 +
Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]])
)/(f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + ((1 + Sin[e + f*x])*Sqr
t[c + d*Sin[e + f*x]]*((-2*Cos[e]*Cos[f*x])/(3*f) + (2*Sin[e]*Sin[f*x])/(3*
f) + (2*(c + 3*d)*Tan[e])/(3*d*f)))/(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2
])^2 + (2*AppellF1[1/2, 1/2, 1/2, 3/2, -((Sec[e]*(c + d*Cos[e]*Sin[f*x + Ar
cTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d
*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*
Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan
[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqr
t[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e]
 + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f*x + ArcTan[T
an[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2])]*Sqrt[c +
d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]])/(3*f*(Cos[e/2 + (f*
x)/2] + Sin[e/2 + (f*x)/2])^2*Sqrt[1 + Tan[e]^2]) + (2*c*AppellF1[1/2, 1/2,
 1/2, 3/2, -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e
]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((
Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt
[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x +
 ArcTan[Tan[e]]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x
+ ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sq
rt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/
(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2])]*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Ta
n[e]]]*Sqrt[1 + Tan[e]^2]])/(d*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^
2*Sqrt[1 + Tan[e]^2]))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)+c}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

maple [B] time = 1.27, size = 657, normalized size = 3.67

$$2a\left(4\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}}\sqrt{-\frac{\left(\sin\left(fx+e\right)-1\right)d}{c+d}}\sqrt{-\frac{d\left(1+\sin\left(fx+e\right)\right)}{c-d}}\ \mathrm{EllipticF}\left(\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}},\sqrt{\frac{c-d}{c+d}}\right)c^2d-4\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}}\ \sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/3*a*(4*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d
*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))*c^2*d-4*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(
c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-
d))^(1/2),((c-d)/(c+d))^(1/2))*d^3-((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*
x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(
f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^3-3*((c+d*sin(f*x+e))/(c-d))^(1
/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ellipti
cE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d+((c+d*sin(f*x+
e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^
(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^2+3
*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(
f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))
^(1/2))*d^3+d^3*sin(f*x+e)^3+c*d^2*sin(f*x+e)^2-d^3*sin(f*x+e)-c*d^2)/d^2/c
os(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(a+a\,\sin\left(e+f\,x\right)\right)\,\sqrt{c+d\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sqrt{c + d\sin\left(e + fx\right)}\,\sin\left(e + fx\right)dx + \int \sqrt{c + d\sin\left(e + fx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] a*(Integral(sqrt(c + d*sin(e + f*x))*sin(e + f*x), x) + Integral(sqrt(c + d
*sin(e + f*x)), x))
```

3.485 $\int \frac{a+a\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=138

$$\frac{2a\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2a(c-d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df\sqrt{c+d\sin(e+fx)}}$$

```
[Out] -2*a*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elliptic
E(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)
/d/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+2*a*(c-d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)
^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2
)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d/f/(c+d*sin(f*x+e))^(1/2
)
```

Rubi [A] time = 0.12, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2752, 2663, 2661, 2655, 2653}

$$\frac{2a\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2a(c-d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (2*a*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])
/(d*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*a*(c - d)*EllipticF[(e - Pi/
2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(d*f*Sqrt[c
+ d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{a+a\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx = \frac{a\int \sqrt{c+d\sin(e+fx)}\,dx}{d} + \frac{(-ac+ad)\int \frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{d}$$

$$= \frac{\left(a\sqrt{c+d\sin(e+fx)}\right)\int \sqrt{\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}}\,dx}{d\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} + \frac{\left((-ac+ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)\int \frac{}{\sqrt{\frac{c}{c+d}}}}{d\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{2aE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} - \frac{2a(c-d)F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{}}{df\sqrt{c+d\sin(e+fx)}}$$

Mathematica [C] time = 6.25, size = 880, normalized size = 6.38

$$a\left(\frac{\sec(e)\left(-\frac{F_1\left(-\frac{1}{2};-\frac{1}{2},-\frac{1}{2};\frac{1}{2};-\frac{\csc(e)\left(c+d\cos\left(fx-\tan^{-1}(\cot(e))\right)\sqrt{\cot^2(e)+1}\,\sin(e)\right)}{d\sqrt{\cot^2(e)+1}\left(1-\frac{c\csc(e)}{d\sqrt{\cot^2(e)+1}}\right)},-\frac{\csc(e)\left(c+d\cos\left(fx-\tan^{-1}(\cot(e))\right)\sqrt{\cot^2(e)+1}\,\sin(e)\right)}{d\sqrt{\cot^2(e)+1}\left(-\frac{c\csc(e)}{d\sqrt{\cot^2(e)+1}}-1\right)}\right)\cot(e)\sin\left(fx-\tan\cdots\right.}{\sqrt{\cot^2(e)+1}\sqrt{\frac{\cos\left(fx-\tan^{-1}(\cot(e))\right)\sqrt{\cot^2(e)+1}\,d+\sqrt{\cot^2(e)+1}\,d}{d\sqrt{\cot^2(e)+1}-c\csc(e)}}\sqrt{\frac{d\sqrt{\cot^2(e)+1}-d\cos\left(fx-\tan^{-1}(\cot(e))\right)\sqrt{\cot^2(e)+1}}{\sqrt{\cot^2(e)+1}\,d+c\csc(e)}}\sqrt{c+d\cos\left(fx-\tan^{-1}(\cot(e))\cdots\right.}}\right.\cdots}{f\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+\sin\left(\frac{e}{2}\cdots\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] a*((Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]
*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + C
ot[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*
x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 -
 (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(S
qrt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*
Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot
[e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]
^2] + c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Si
n[e]])) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*
Sin[e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])
/Sqrt[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2
]*Sin[e]]))/(f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (2*(1 + Sin[e
 + f*x])*Sqrt[c + d*Sin[e + f*x]]*Tan[e])/(d*f*(Cos[e/2 + (f*x)/2] + Sin[e/
2 + (f*x)/2])^2) + (2*AppellF1[1/2, 1/2, 1/2, 3/2, -((Sec[e]*(c + d*Cos[e]*
Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 - (
c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcT
an[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d*
Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]]]*(1 + Sin[e + f*x])
*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2
])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f*
```

```
x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2]
)]*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]])/(d*f*(C
os[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2*Sqrt[1 + Tan[e]^2]))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a\sin\left(fx+e\right)+a}{\sqrt{d\sin\left(fx+e\right)+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sin\left(fx+e\right)+a}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

maple [A] time = 1.31, size = 203, normalized size = 1.47

$$-\frac{2a\left(c-d\right)\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}}\,\sqrt{-\frac{\left(\sin\left(fx+e\right)-1\right)d}{c+d}}\,\sqrt{-\frac{d\left(1+\sin\left(fx+e\right)\right)}{c-d}}\,\left(\text{EllipticE}\left(\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}},\sqrt{\frac{c-d}{c+d}}\right)c+\text{EllipticE}\left(\right.\right.}{d^{2}\cos\left(fx+e\right)\sqrt{c+d\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] -2*a*(c-d)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(
-d*(1+sin(f*x+e))/(c-d))^(1/2)*(EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((
c-d)/(c+d))^(1/2))*c+EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))
^(1/2))*d-2*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d
)/d^2/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sin\left(fx+e\right)+a}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

mupad [B] time = 7.62, size = 176, normalized size = 1.28

$$\frac{a\left(2\,c\,\mathrm{F}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin\left(e+f\,x\right)}}{2}\right)\middle|\frac{2\,d}{c+d}\right)-2\,\left(c+d\right)\,\mathrm{E}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin\left(e+f\,x\right)}}{2}\right)\middle|\frac{2\,d}{c+d}\right)\right)\,\sqrt{{\cos\left(e+f\,x\right)}^2}\,\sqrt{\frac{c+d\,\sin\left(e+f\,x\right)}{c+d}}}{d\,f\,\cos\left(e+f\,x\right)\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c + d*sin(e + f*x))^(1/2),x)
```

```
[Out] (a*(2*c*ellipticF(asin((2^(1/2)*(1 - sin(e + f*x))^(1/2))/2), (2*d)/(c + d)
) - 2*(c + d)*ellipticE(asin((2^(1/2)*(1 - sin(e + f*x))^(1/2))/2), (2*d)/(
c + d)))*(cos(e + f*x)^2)^(1/2)*((c + d*sin(e + f*x))/(c + d))^(1/2))/(d*f*
cos(e + f*x)*(c + d*sin(e + f*x))^(1/2)) - (2*a*ellipticF(pi/4 - e/2 - (f*x
)/2, (2*d)/(c + d))*((c + d*sin(e + f*x))/(c + d))^(1/2))/(f*(c + d*sin(e +
 f*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{\sin\left(e+fx\right)}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx+\int \frac{1}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] a*(Integral(sin(e + f*x)/sqrt(c + d*sin(e + f*x)), x) + Integral(1/sqrt(c +
 d*sin(e + f*x)), x))
```

3.486 $\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=169

$$-\frac{2a\cos(e+fx)}{f(c+d)\sqrt{c+d\sin(e+fx)}}+\frac{2a\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df\sqrt{c+d\sin(e+fx)}}-\frac{2a\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\right.}{df(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] -2*a*cos(f*x+e)/(c+d)/f/(c+d*sin(f*x+e))^(1/2)+2*a*(sin(1/2*e+1/4*Pi+1/2*f*
x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2
^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d/(c+d)/f/((c+d*sin(f*x+e))/
(c+d))^(1/2)-2*a*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f
*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(
f*x+e))/(c+d))^(1/2)/d/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a\cos(e+fx)}{f(c+d)\sqrt{c+d\sin(e+fx)}}+\frac{2a\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df\sqrt{c+d\sin(e+fx)}}-\frac{2a\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\right.}{df(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a*Cos[e + f*x])/((c + d)*f*Sqrt[c + d*Sin[e + f*x]]) - (2*a*EllipticE[(
e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(d*(c + d)*f*Sq
rt[(c + d*Sin[e + f*x])/(c + d)]) + (2*a*EllipticF[(e - Pi/2 + f*x)/2, (2*d
)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(d*f*Sqrt[c + d*Sin[e + f*x]
])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^{3/2}}\,dx = -\frac{2a\cos(e+fx)}{(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{2\int \frac{-\frac{1}{2}a(c-d)+\frac{1}{2}a(c-d)\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx}{c^2-d^2}$$

$$= -\frac{2a\cos(e+fx)}{(c+d)f\sqrt{c+d\sin(e+fx)}} + \frac{a\int \frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{d} - \frac{a\int \sqrt{c+d\sin(e+fx)}\,dx}{d(c+d)}$$

$$= -\frac{2a\cos(e+fx)}{(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{\left(a\sqrt{c+d\sin(e+fx)}\right)\int \sqrt{\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}}\,dx}{d(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} + \cdots$$

$$= -\frac{2a\cos(e+fx)}{(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{2aE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{d(c+d)f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} + \cdots$$

Mathematica [C] time = 6.39, size = 938, normalized size = 5.55

$$a\left(\frac{\sqrt{c+d\sin(e+fx)}\left(\frac{2\csc(e)(c\cos(e)+d\sin(fx))}{d(c+d)f(c+d\sin(e+fx))}-\frac{2\csc(e)\sec(e)}{d(c+d)f}\right)(\sin(e+fx)+1)}{\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)^2} - \frac{\sec(e)\left(-\frac{F_1\left(-\frac{1}{2};-\frac{1}{2},-\frac{1}{2};\frac{1}{2};-\frac{\csc(e)\cdots}{\cdots}\right)}{\sqrt{\cot^2(e)+1}\sqrt{\frac{\cos\left(fx-\tan^{-1}(\cot\cdots\right)}{d\sqrt{\cdots}}}}\cdots\right)}{\cdots}\cdots\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] a*(((1 + Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]*((-2*Csc[e]*Sec[e])/(d*(c +
 d)*f) + (2*Csc[e]*(c*Cos[e] + d*Sin[f*x]))/(d*(c + d)*f*(c + d*Sin[e + f*x
```

```
]))))/(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2 - (Sec[e]*(1 + Sin[e + f*
x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*(c + d*Cos[f*x - ArcTan[C
ot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(1 - (c*Csc[e])/(
d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1
 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*Csc[e])/(d*Sqrt[1 + Co
t[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[1 + Cot[e]^2]*Sqrt[(d*
Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqr
t[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^2] - d*Cos[f*x - ArcTa
n[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] + c*Csc[e])]*Sqrt[c +
d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]])) - ((2*d*Sin[e]*(c
+ d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d^2*Cos[e]^2 + d
^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqrt[1 + Cot[e]^2])/Sqrt[
c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]]))/((c + d)*f*(Co
s[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (2*AppellF1[1/2, 1/2, 1/2, 3/2,
 -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*
Sqrt[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c
+ d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e
]^2]*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Ta
n[e]]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[T
an[e]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt
[1 + Tan[e]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e
]) + d*Sqrt[1 + Tan[e]^2])]*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqr
t[1 + Tan[e]^2]])/(d*(c + d)*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2*
Sqrt[1 + Tan[e]^2]))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)+c}}{d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(d^2*cos(f*x + e)^2
 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sin\left(fx+e\right)+a}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(3/2), x)
```

maple [A] time = 1.09, size = 246, normalized size = 1.46

$$\frac{2\left(\sqrt{-\frac{d(1+\sin(fx+e))}{c-d}}\ \mathrm{EllipticE}\left(\sqrt{\frac{c+d\sin(fx+e)}{c-d}}, \sqrt{\frac{c-d}{c+d}}\right)\sqrt{\frac{c+d\sin(fx+e)}{c-d}}\ \sqrt{-\frac{(\sin(fx+e)-1)d}{c+d}}\ c^2 - \sqrt{-\frac{d(1+\sin(fx+e))}{c-d}}\ \mathrm{El}}{d^2\,(c+d)\cos\left(fx+e\right)\sqrt{c+d\,\mathrm{s}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] 2*((-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2)
,((c-d)/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+
d))^(1/2)*c^2-(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(
c-d))^(1/2),((c-d)/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+
e)-1)*d/(c+d))^(1/2)*d^2+d^2*sin(f*x+e)^2-d^2)/d^2*a/(c+d)/cos(f*x+e)/(c+d*
sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sin\left(fx+e\right)+a}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+a\,\sin\left(e+f\,x\right)}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))/(c + d*sin(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.487 $\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=237

$$-\frac{2a(c-3d)\cos(e+fx)}{3f(c-d)(c+d)^2\sqrt{c+d\sin(e+fx)}}-\frac{2a\cos(e+fx)}{3f(c+d)(c+d\sin(e+fx))^{3/2}}+\frac{2a\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3df(c+d)\sqrt{c+d\sin(e+fx)}}$$

```
[Out] -2/3*a*cos(f*x+e)/(c+d)/f/(c+d*sin(f*x+e))^(3/2)-2/3*a*(c-3*d)*cos(f*x+e)/(
c-d)/(c+d)^2/f/(c+d*sin(f*x+e))^(1/2)+2/3*a*(c-3*d)*(sin(1/2*e+1/4*Pi+1/2*f
*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),
2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/(c-d)/d/(c+d)^2/f/((c+d*sin
(f*x+e))/(c+d))^(1/2)-2/3*a*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1
/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))
*((c+d*sin(f*x+e))/(c+d))^(1/2)/d/(c+d)/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a(c-3d)\cos(e+fx)}{3f(c-d)(c+d)^2\sqrt{c+d\sin(e+fx)}}-\frac{2a\cos(e+fx)}{3f(c+d)(c+d\sin(e+fx))^{3/2}}+\frac{2a\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3df(c+d)\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*a*Cos[e + f*x])/(3*(c + d)*f*(c + d*Sin[e + f*x])^(3/2)) - (2*a*(c - 3*
d)*Cos[e + f*x])/(3*(c - d)*(c + d)^2*f*Sqrt[c + d*Sin[e + f*x]]) - (2*a*(c
 - 3*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x
]])/(3*(c - d)*d*(c + d)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*a*Ell
ipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)
])/(3*d*(c + d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{2a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}a(c-d)-\frac{1}{2}a(c-d)\sin(e+fx)}{(c+d\sin(e+fx))^{3/2}}dx}{3\left(c^2-d^2\right)}$$

$$= -\frac{2a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{2a(c-3d)\cos(e+fx)}{3(c-d)(c+d)^2f\sqrt{c+d\sin(e+fx)}} + \frac{4\int \frac{\frac{1}{4}a(}{}}{}$$

$$= -\frac{2a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{2a(c-3d)\cos(e+fx)}{3(c-d)(c+d)^2f\sqrt{c+d\sin(e+fx)}} - \frac{(a(c-}{}$$

$$= -\frac{2a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{2a(c-3d)\cos(e+fx)}{3(c-d)(c+d)^2f\sqrt{c+d\sin(e+fx)}} - \frac{\left(a(c-\right.}{}$$

$$= -\frac{2a\cos(e+fx)}{3(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{2a(c-3d)\cos(e+fx)}{3(c-d)(c+d)^2f\sqrt{c+d\sin(e+fx)}} - \frac{2a(c-}{}$$

Mathematica [C] time = 6.77, size = 1870, normalized size = 7.89

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] a*(((1 + Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]*((-2*(c - 3*d)*Csc[e]*Sec[e
])/(3*(c - d)*d*(c + d)^2*f) + (2*Csc[e]*(c*Cos[e] + d*Sin[f*x]))/(3*d*(c +
 d)*f*(c + d*Sin[e + f*x])^2) - (2*Csc[e]*(3*c*Cos[e] - d*Cos[e] - c*Sin[f*
x] + 3*d*Sin[f*x]))/(3*(c - d)*(c + d)^2*f*(c + d*Sin[e + f*x]))))/(Cos[e/2
 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2 - (c*Sec[e]*(1 + Sin[e + f*x])*(-((Appe
llF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt
[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + C
ot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]
*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]
*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot
[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]
^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*S
qrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] + c*Csc[e])]*Sqrt[c + d*Cos[f*x -
ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]])) - ((2*d*Sin[e]*(c + d*Cos[f*x
```

```
- ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2)
 - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqrt[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*
x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]]))/(3*(c - d)*(c + d)^2*f*(Co
s[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (d*Sec[e]*(1 + Sin[e + f*x])*(-
((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]
]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(1 - (c*Csc[e])/(d*Sqrt
[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot
[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2
]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1
 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 +
Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^2] - d*Cos[f*x - ArcTan[Cot[
e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] + c*Csc[e])]*Sqrt[c + d*Cos[
f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]])) - ((2*d*Sin[e]*(c + d*Co
s[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d^2*Cos[e]^2 + d^2*Sin
[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqrt[1 + Cot[e]^2])/Sqrt[c + d*
Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]]))/((c - d)*(c + d)^2*f
*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) - (2*AppellF1[1/2, 1/2, 1/2,
3/2, -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))
/(d*Sqrt[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]
*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + T
an[e]^2]*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTa
n[Tan[e]]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcT
an[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*
Sqrt[1 + Tan[e]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*S
ec[e]) + d*Sqrt[1 + Tan[e]^2])]*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]
*Sqrt[1 + Tan[e]^2]])/(3*(c - d)*(c + d)^2*f*(Cos[e/2 + (f*x)/2] + Sin[e/2
+ (f*x)/2])^2*Sqrt[1 + Tan[e]^2]) + (2*c*AppellF1[1/2, 1/2, 1/2, 3/2, -((Se
c[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1
 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*Co
s[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(
-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]]]
*(1 + Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e]]
]*Sqrt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 + T
an[e]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) + d
*Sqrt[1 + Tan[e]^2])]*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 +
Tan[e]^2]])/((c - d)*d*(c + d)^2*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2]
)^2*Sqrt[1 + Tan[e]^2]))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)+c}}{3\,cd^2\cos\left(fx+e\right)^2-c^3-3\,cd^2+\left(d^3\cos\left(fx+e\right)^2-3\,c^2d-d^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(3*c*d^2*cos(f*x +
e)^2 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e)),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sin\left(fx+e\right)+a}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(5/2), x)
```

maple [B] time = 4.68, size = 884, normalized size = 3.73

$$\frac{\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^{2}\left(fx+e\right)\right)}\,a\left(\frac{\frac{2d\left(\cos^{2}\left(fx+e\right)\right)}{\left(c^{2}-d^{2}\right)\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^{2}\left(fx+e\right)\right)}}+\frac{2c\left(\frac{c}{d}-1\right)\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}}\,\sqrt{\frac{d\left(1-\sin\left(fx+e\right)\right)}{c+d}}\,\sqrt{\frac{\left(-\sin\left(fx\right.\right.}{c-}}}{\left(c^{2}-d^{2}\right)\sqrt{-\left(-d\sin\left(fx+e\right)-c\right.}}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*a*(1/d*(2*d*cos(f*x+e)^2/(c^2-d^2)/
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x
+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^
(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(
c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e)
)/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))))+(-c+d)/d*(2/3/(c^2-d^2)/d*(-(-d*sin(f*x+e)-c)*cos(
f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/3*d*cos(f*x+e)^2/(c^2-d^2)^2*c/(-(-d*s
in(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*c^2*d^2+3*d^4)*(c/d
```

```
-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*
x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+8/3*c*d/(c^2-d^2)^2*(c/d-
1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x
+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*Ell
ipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*
sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e)
)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sin\left(fx+e\right)+a}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+a\,\sin\left(e+f\,x\right)}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c + d*sin(e + f*x))^(5/2),x)
[Out] int((a + a*sin(e + f*x))/(c + d*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))**(5/2),x)
[Out] Timed out
```

3.488 $\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=318

$$-\frac{2a\left(3c^2-20cd+9d^2\right)\cos(e+fx)}{15f(c-d)^2(c+d)^3\sqrt{c+d\sin(e+fx)}}-\frac{2a\left(3c^2-20cd+9d^2\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15df(c-d)^2(c+d)^3\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{15f}{}$$

```
[Out] -2/5*a*cos(f*x+e)/(c+d)/f/(c+d*sin(f*x+e))^(5/2)-2/15*a*(3*c-5*d)*cos(f*x+e
)/(c-d)/(c+d)^2/f/(c+d*sin(f*x+e))^(3/2)-2/15*a*(3*c^2-20*c*d+9*d^2)*cos(f*
x+e)/(c-d)^2/(c+d)^3/f/(c+d*sin(f*x+e))^(1/2)+2/15*a*(3*c^2-20*c*d+9*d^2)*(
sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(
1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/(c-d)
^2/d/(c+d)^3/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-2/15*a*(3*c-5*d)*(sin(1/2*e+1
/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*P
i+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/(c-d)/d/
(c+d)^2/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a\left(3c^2-20cd+9d^2\right)\cos(e+fx)}{15f(c-d)^2(c+d)^3\sqrt{c+d\sin(e+fx)}}-\frac{2a\left(3c^2-20cd+9d^2\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15df(c-d)^2(c+d)^3\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{15f}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-2*a*Cos[e + f*x])/(5*(c + d)*f*(c + d*Sin[e + f*x])^(5/2)) - (2*a*(3*c -
5*d)*Cos[e + f*x])/(15*(c - d)*(c + d)^2*f*(c + d*Sin[e + f*x])^(3/2)) - (2
*a*(3*c^2 - 20*c*d + 9*d^2)*Cos[e + f*x])/(15*(c - d)^2*(c + d)^3*f*Sqrt[c
+ d*Sin[e + f*x]]) - (2*a*(3*c^2 - 20*c*d + 9*d^2)*EllipticE[(e - Pi/2 + f*
x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(15*(c - d)^2*d*(c + d)^3*f*
Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*a*(3*c - 5*d)*EllipticF[(e - Pi/2
+ f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(15*(c - d)*d*
(c + d)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^{7/2}}\,dx = -\frac{2a\cos(e+fx)}{5(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{2\int \frac{-\frac{5}{2}a(c-d)-\frac{3}{2}a(c-d)\sin(e+fx)}{(c+d\sin(e+fx))^{5/2}}\,dx}{5\left(c^2-d^2\right)}$$

$$= -\frac{2a\cos(e+fx)}{5(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{2a(3c-5d)\cos(e+fx)}{15(c-d)(c+d)^2f(c+d\sin(e+fx))^{3/2}} + \frac{4\int \cdots}{\cdots}$$

$$= -\frac{2a\cos(e+fx)}{5(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{2a(3c-5d)\cos(e+fx)}{15(c-d)(c+d)^2f(c+d\sin(e+fx))^{3/2}} - \frac{2a\cdots}{15(c\cdots}$$

$$= -\frac{2a\cos(e+fx)}{5(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{2a(3c-5d)\cos(e+fx)}{15(c-d)(c+d)^2f(c+d\sin(e+fx))^{3/2}} - \frac{2a\cdots}{15(c\cdots}$$

$$= -\frac{2a\cos(e+fx)}{5(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{2a(3c-5d)\cos(e+fx)}{15(c-d)(c+d)^2f(c+d\sin(e+fx))^{3/2}} - \frac{2a\cdots}{15(c\cdots}$$

$$= -\frac{2a\cos(e+fx)}{5(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{2a(3c-5d)\cos(e+fx)}{15(c-d)(c+d)^2f(c+d\sin(e+fx))^{3/2}} - \frac{2a\cdots}{15(c\cdots}$$

Mathematica [C] time = 7.15, size = 2815, normalized size = 8.85

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] a*(((1 + Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]*((-2*(3*c^2 - 20*c*d + 9*d^
2)*Csc[e]*Sec[e])/(15*(c - d)^2*d*(c + d)^3*f) + (2*Csc[e]*(c*Cos[e] + d*Si
n[f*x]))/(5*d*(c + d)*f*(c + d*Sin[e + f*x])^3) - (2*Csc[e]*(5*c*Cos[e] - 3
*d*Cos[e] - 3*c*Sin[f*x] + 5*d*Sin[f*x]))/(15*(c - d)*(c + d)^2*f*(c + d*Si
n[e + f*x])^2) - (2*Csc[e]*(15*c^2*Cos[e] - 12*c*d*Cos[e] + 5*d^2*Cos[e] -
3*c^2*Sin[f*x] + 20*c*d*Sin[f*x] - 9*d^2*Sin[f*x]))/(15*(c - d)^2*(c + d)^3
*f*(c + d*Sin[e + f*x]))))/(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2 - (c
^2*Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*
(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Co
t[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x
```

- ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] + c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]])) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqrt[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]]))/(5*(c - d)^2*(c + d)^3*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (4*c*d*Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] + c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]])) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqrt[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]]))/(3*(c - d)^2*(c + d)^3*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) - (3*d^2*Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Csc[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] + c*Csc[e])]*Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]])) - ((2*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqrt[1 + Cot[e]^2])/Sqrt[c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]]))/(5*(c - d)^2*(c + d)^3*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) - (8*c*AppellF1[1/2, 1/2, 1/2, 3/2, -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2])]*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[

```
e]^2]])/(5*(c - d)^2*(c + d)^3*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^
2*Sqrt[1 + Tan[e]^2]) + (2*c^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((Sec[e]*(c +
d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^
2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*Cos[e]*Sin[f
*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 - (c*Se
c[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]]]*(1 + Sin[
e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 +
 Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2] +
 d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) + d*Sqrt[1 +
Tan[e]^2])]*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]]
)/((c - d)^2*d*(c + d)^3*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2*Sqrt
[1 + Tan[e]^2]) + (2*d*AppellF1[1/2, 1/2, 1/2, 3/2, -((Sec[e]*(c + d*Cos[e]
*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 -
(c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*Cos[e]*Sin[f*x + Arc
Tan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d
*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]]]*(1 + Sin[e + f*x]
)*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^
2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f
*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2
])]*Sqrt[c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]])/(3*(c
- d)^2*(c + d)^3*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2*Sqrt[1 + Tan
[e]^2]))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)+c}}{d^4\cos\left(fx+e\right)^4+c^4+6\,c^2d^2+d^4-2\left(3\,c^2d^2+d^4\right)\cos\left(fx+e\right)^2-4\left(cd^3\cos\left(fx+e\right)^2-c^3d-cd^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(d^4*cos(f*x + e)^4
+ c^4 + 6*c^2*d^2 + d^4 - 2*(3*c^2*d^2 + d^4)*cos(f*x + e)^2 - 4*(c*d^3*cos
(f*x + e)^2 - c^3*d - c*d^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sin\left(fx+e\right)+a}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(7/2),x, algorithm="giac")
```

[Out] integrate((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(7/2), x)

maple [B] time = 6.40, size = 1046, normalized size = 3.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(7/2),x)

[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*a*(1/d*(2/3/(c^2-d^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/3*d*cos(f*x+e)^2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*c^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+8/3*c*d/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(-c+d)/d*(2/5/(c^2-d^2)/d^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^3+16/15*c/(c^2-d^2)^2/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+2/15*d*cos(f*x+e)^2/(c^2-d^2)^3*(23*c^2+9*d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(15*c^3+17*c*d^2)/(15*c^6-45*c^4*d^2+45*c^2*d^4-15*d^6)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/15*d*(23*c^2+9*d^2)/(c^2-d^2)^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a \sin\left(fx + e\right) + a}{\left(d \sin\left(fx + e\right) + c\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(7/2),x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(7/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + a \sin\left(e + f x\right)}{\left(c + d \sin\left(e + f x\right)\right)^{7/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c + d*sin(e + f*x))^(7/2),x)
[Out] int((a + a*sin(e + f*x))/(c + d*sin(e + f*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))**(7/2),x)
[Out] Timed out
```

3.489 $\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=378

$$\frac{4a^2\left(5c^3-45c^2d-141cd^2-75d^3\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{315df}+\frac{4a^2\left(c^2-d^2\right)\left(5c^3-45c^2d-141cd^2-75d^3\right)\sqrt{\ldots}}{315d^2f\sqrt{c+d\sin\ldots}}$$

```
[Out] 4/315*a^2*(5*c*(c-9*d)-56*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/d/f+4/63*a
^2*(c-9*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(5/2)/d/f-2/9*a^2*cos(f*x+e)*(c+d*si
n(f*x+e))^(7/2)/d/f+4/315*a^2*(5*c^3-45*c^2*d-141*c*d^2-75*d^3)*cos(f*x+e)*
(c+d*sin(f*x+e))^(1/2)/d/f+4/315*a^2*(5*c^4-45*c^3*d-381*c^2*d^2-435*c*d^3-
168*d^4)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elli
pticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(
1/2)/d^2/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-4/315*a^2*(c^2-d^2)*(5*c^3-45*c^2
*d-141*c*d^2-75*d^3)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1
/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*
sin(f*x+e))/(c+d))^(1/2)/d^2/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2763, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{4a^2\left(-45c^2d+5c^3-141cd^2-75d^3\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{315df}+\frac{4a^2\left(c^2-d^2\right)\left(-45c^2d+5c^3-141cd^2-75d\ldots\right)}{315d^2f\sqrt{c+d\,s\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (4*a^2*(5*c^3 - 45*c^2*d - 141*c*d^2 - 75*d^3)*Cos[e + f*x]*Sqrt[c + d*Sin[
e + f*x]])/(315*d*f) + (4*a^2*(5*c*(c - 9*d) - 56*d^2)*Cos[e + f*x]*(c + d*
Sin[e + f*x])^(3/2))/(315*d*f) + (4*a^2*(c - 9*d)*Cos[e + f*x]*(c + d*Sin[e
 + f*x])^(5/2))/(63*d*f) - (2*a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(7/2))/
(9*d*f) - (4*a^2*(5*c^4 - 45*c^3*d - 381*c^2*d^2 - 435*c*d^3 - 168*d^4)*Ell
ipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(315*d^
2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^2*(c^2 - d^2)*(5*c^3 - 45*c^
2*d - 141*c*d^2 - 75*d^3)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt
[(c + d*Sin[e + f*x])/(c + d)])/(315*d^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
```

```
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^{5/2}\,dx = -\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{7/2}}{9df} + \frac{2\int\left(8a^2d-a^2(c-9d)\right.}{}$$

$$= \frac{4a^2(c-9d)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{63df} - \frac{2a^2\cos(e+fx)(}{}$$

$$= \frac{4a^2\left(5c(c-9d)-56d^2\right)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{315df} + \frac{4a^2(}{}$$

$$= \frac{4a^2\left(5c^3-45c^2d-141cd^2-75d^3\right)\cos(e+fx)\sqrt{c+d\sin(e+fx}}{315df}$$

$$= \frac{4a^2\left(5c^3-45c^2d-141cd^2-75d^3\right)\cos(e+fx)\sqrt{c+d\sin(e+fx}}{315df}$$

$$= \frac{4a^2\left(5c^3-45c^2d-141cd^2-75d^3\right)\cos(e+fx)\sqrt{c+d\sin(e+fx}}{315df}$$

$$= \frac{4a^2\left(5c^3-45c^2d-141cd^2-75d^3\right)\cos(e+fx)\sqrt{c+d\sin(e+fx}}{315df}$$

Mathematica [A] time = 1.98, size = 322, normalized size = 0.85

$$\frac{a^2(\sin(e+fx)+1)^2\left(16\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\left(\left(5c^4-45c^3d-381c^2d^2-435cd^3-168d^4\right)\left((c+d)E\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2}{c+}\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a^2*(1 + Sin[e + f*x])^2*(16*(-(d^2*(235*c^3 + 405*c^2*d + 309*c*d^2 + 75*
d^3)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]) + (5*c^4 - 45*c^3*d -
 381*c^2*d^2 - 435*c*d^3 - 168*d^4)*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/
4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]))*Sqr
t[(c + d*Sin[e + f*x])/(c + d)] - d*(c + d*Sin[e + f*x])*(2*(20*c^3 + 1080*
c^2*d + 1671*c*d^2 + 690*d^3)*Cos[e + f*x] + 2*d*(-5*d*(19*c + 18*d)*Cos[3*
(e + f*x)] + (150*c^2 + 540*c*d + 259*d^2 - 35*d^2*Cos[2*(e + f*x)])*Sin[2*
(e + f*x)]))))/(1260*d^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sqrt[c +
 d*Sin[e + f*x]])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2d^2\cos\left(fx+e\right)^4+2\,a^2c^2+4\,a^2cd+2\,a^2d^2-\left(a^2c^2+4\,a^2cd+3\,a^2d^2\right)\cos\left(fx+e\right)^2+2\left(a^2c^2+2\,a\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*d^2*cos(f*x + e)^4 + 2*a^2*c^2 + 4*a^2*c*d + 2*a^2*d^2 - (a^2
*c^2 + 4*a^2*c*d + 3*a^2*d^2)*cos(f*x + e)^2 + 2*(a^2*c^2 + 2*a^2*c*d + a^2
*d^2 - (a^2*c*d + a^2*d^2)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(d*sin(f*x + e
) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^2\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(5/2), x)
```

maple [B] time = 1.59, size = 1614, normalized size = 4.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] -2/315*a^2*(-426*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(
1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/
2),((c-d)/(c+d))^(1/2))*c^2*d^4+5*c^4*d^2+90*((c+d*sin(f*x+e))/(c-d))^(1/2)
*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(
```

```
((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^5*d-870*((c+d*sin(f*x
+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))
^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^5+
10*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+si
n(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d
))^(1/2))*c^5*d-570*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d)
)^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2))*c^4*d^2-1012*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(s
in(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d
*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^3*d^3+84*((c+d*sin(f*x+e))
/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/
2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^4+10
02*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+si
n(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d
))^(1/2))*c*d^5+772*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d)
)^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2))*c^4*d^2+780*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(si
n(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*
sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^3*d^3+270*c^3*d^3+150*c*d^5
-10*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+s
in(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+
d))^(1/2))*c^6-336*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))
^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(
1/2),((c-d)/(c+d))^(1/2))*d^6+486*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x
+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f
*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^6-130*c*d^5*sin(f*x+e)^5-170*c^2
*d^4*sin(f*x+e)^4-360*c*d^5*sin(f*x+e)^4-80*c^3*d^3*sin(f*x+e)^3-540*c^2*d^
4*sin(f*x+e)^3-376*c*d^5*sin(f*x+e)^3-5*c^4*d^2*sin(f*x+e)^2-270*c^3*d^3*si
n(f*x+e)^2-224*c^2*d^4*sin(f*x+e)^2+210*c*d^5*sin(f*x+e)^2+80*c^3*d^3*sin(f
*x+e)+540*c^2*d^4*sin(f*x+e)+506*c*d^5*sin(f*x+e)-90*d^6*sin(f*x+e)^5-77*d^
6*sin(f*x+e)^4-60*d^6*sin(f*x+e)^3+112*d^6*sin(f*x+e)^2+150*d^6*sin(f*x+e)-
35*d^6*sin(f*x+e)^6+394*c^2*d^4)/d^3/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{2}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int c^2\sqrt{c+d\sin\left(e+fx\right)}\,dx+\int 2c^2\sqrt{c+d\sin\left(e+fx\right)}\sin\left(e+fx\right)dx+\int c^2\sqrt{c+d\sin\left(e+fx\right)}\sin^2\left(e\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] a**2*(Integral(c**2*sqrt(c + d*sin(e + f*x)), x) + Integral(2*c**2*sqrt(c +
 d*sin(e + f*x))*sin(e + f*x), x) + Integral(c**2*sqrt(c + d*sin(e + f*x))*
sin(e + f*x)**2, x) + Integral(d**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**
2, x) + Integral(2*d**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**3, x) + Inte
gral(d**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**4, x) + Integral(2*c*d*sqr
t(c + d*sin(e + f*x))*sin(e + f*x), x) + Integral(4*c*d*sqrt(c + d*sin(e +
f*x))*sin(e + f*x)**2, x) + Integral(2*c*d*sqrt(c + d*sin(e + f*x))*sin(e +
 f*x)**3, x))
```

3.490 $\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=298

$$\frac{4a^2\left(c^2-7cd-10d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{35df}+\frac{4a^2\left(c^2-7cd-10d^2\right)\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\right.\right.}{35d^2f\sqrt{c+d\sin(e+fx)}}$$

```
[Out] 4/35*a^2*(c-7*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/d/f-2/7*a^2*cos(f*x+e)*(
c+d*sin(f*x+e))^(5/2)/d/f+4/35*a^2*(c^2-7*c*d-10*d^2)*cos(f*x+e)*(c+d*sin(f
*x+e))^(1/2)/d/f+4/35*a^2*(c+3*d)*(c^2-10*c*d-7*d^2)*(sin(1/2*e+1/4*Pi+1/2*
f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x)
,2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^2/f/((c+d*sin(f*x+e))/(c
+d))^(1/2)-4/35*a^2*(c^2-7*c*d-10*d^2)*(c^2-d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)
^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(
1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^2/f/(c+d*sin(f*x+e))
^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2763, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{4a^2\left(c^2-7cd-10d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{35df}+\frac{4a^2\left(c^2-7cd-10d^2\right)\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\right.\right.}{35d^2f\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (4*a^2*(c^2 - 7*c*d - 10*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(35*d*
f) + (4*a^2*(c - 7*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(35*d*f) - (
2*a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(7*d*f) - (4*a^2*(c + 3*d)*(
c^2 - 10*c*d - 7*d^2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c +
 d*Sin[e + f*x]])/(35*d^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^2*(c
^2 - 7*c*d - 10*d^2)*(c^2 - d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d
)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(35*d^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
```

```
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^{3/2}\,dx = -\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{7df} + \frac{2\int \left(6a^2d-a^2(c-7d)\right.}{}$$

$$= \frac{4a^2(c-7d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{35df} - \frac{2a^2\cos(e+fx)(}{}$$

$$= \frac{4a^2\left(c^2-7cd-10d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{35df} + \frac{4a^2(c-}{}$$

$$= \frac{4a^2\left(c^2-7cd-10d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{35df} + \frac{4a^2(c-}{}$$

$$= \frac{4a^2\left(c^2-7cd-10d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{35df} + \frac{4a^2(c-}{}$$

$$= \frac{4a^2\left(c^2-7cd-10d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{35df} + \frac{4a^2(c-}{}$$

Mathematica [A] time = 2.11, size = 262, normalized size = 0.88

$$\frac{a^2\left(d\cos(e+fx)\left(-4c^3-d\left(36c^2+168cd+95d^2\right)\sin(e+fx)-112c^2d+2d^2(13c+14d)\cos(2(e+fx))-106cd^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (a^2*(8*(c^4 - 6*c^3*d - 44*c^2*d^2 - 58*c*d^3 - 21*d^4)*EllipticE[(-2*e +
Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - 8*(c^4 -
 7*c^3*d - 11*c^2*d^2 + 7*c*d^3 + 10*d^4)*EllipticF[(-2*e + Pi - 2*f*x)/4,
(2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + d*Cos[e + f*x]*(-4*c^3
- 112*c^2*d - 106*c*d^2 - 28*d^3 + 2*d^2*(13*c + 14*d)*Cos[2*(e + f*x)] - d
```

```
*(36*c^2 + 168*c*d + 95*d^2)*Sin[e + f*x] + 5*d^3*Sin[3*(e + f*x)])))/(70*d
^2*f*Sqrt[c + d*Sin[e + f*x]])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(2\,a^2c+2\,a^2d-\left(a^2c+2\,a^2d\right)\cos\left(fx+e\right)^2-\left(a^2d\cos\left(fx+e\right)^2-2\,a^2c-2\,a^2d\right)\sin\left(fx+e\right)\right)\sqrt{d\sin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((2*a^2*c + 2*a^2*d - (a^2*c + 2*a^2*d)*cos(f*x + e)^2 - (a^2*d*cos
(f*x + e)^2 - 2*a^2*c - 2*a^2*d)*sin(f*x + e))*sqrt(d*sin(f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^2\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(3/2), x)
```

maple [B] time = 1.63, size = 1316, normalized size = 4.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] -2/35*a^2*(14*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2
)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))*c^4*d+20*c*d^4+28*c^2*d^3+c^3*d^2-2*((c+d*sin(f*x+e))/
(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2
)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^5-42*((c+
d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e
))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2
))*d^5+62*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-
d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-
d)/(c+d))^(1/2))*d^5-5*d^5*sin(f*x+e)^5-14*d^5*sin(f*x+e)^4-15*d^5*sin(f*x+
e)^3+14*d^5*sin(f*x+e)^2+20*d^5*sin(f*x+e)-c^3*d^2*sin(f*x+e)^2-28*c^2*d^3*
sin(f*x+e)^2-7*c*d^4*sin(f*x+e)^2+9*c^2*d^3*sin(f*x+e)+42*c*d^4*sin(f*x+e)-
13*c*d^4*sin(f*x+e)^4-9*c^2*d^3*sin(f*x+e)^3-42*c*d^4*sin(f*x+e)^3+76*((c+d
```

```
*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e)
)/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)
)*c^3*d^2+2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*
(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((
c-d)/(c+d))^(1/2))*c^4*d-68*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)
*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))
/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^3*d^2-64*((c+d*sin(f*x+e))/(c-d))^(1/2
)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF
(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^3+68*((c+d*sin(f
*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d
))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^
4+28*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+
sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c
+d))^(1/2))*c^2*d^3-74*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c
+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))*c*d^4)/d^3/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/
f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(fx+e)+a)^2(d\sin(fx+e)+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+a\,\sin(e+f\,x))^2\,(c+d\,\sin(e+f\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^(3/2),x)
[Out] int((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int c\sqrt{c+d\sin(e+fx)}\,dx+\int 2c\sqrt{c+d\sin(e+fx)}\sin(e+fx)\,dx+\int c\sqrt{c+d\sin(e+fx)}\sin^2(e+fx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] a**2*(Integral(c*sqrt(c + d*sin(e + f*x)), x) + Integral(2*c*sqrt(c + d*sin
(e + f*x))*sin(e + f*x), x) + Integral(c*sqrt(c + d*sin(e + f*x))*sin(e + f
*x)**2, x) + Integral(d*sqrt(c + d*sin(e + f*x))*sin(e + f*x), x) + Integra
l(2*d*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2, x) + Integral(d*sqrt(c + d*
sin(e + f*x))*sin(e + f*x)**3, x))
```

3.491 $\int (a+a\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=239

$$\frac{4a^2(c-5d)\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^2f\sqrt{c+d\sin(e+fx)}}-\frac{4a^2\left(c^2-5cd-12d^2\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx\right.\right.}{15d^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] -2/5*a^2*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/d/f+4/15*a^2*(c-5*d)*cos(f*x+e)*
(c+d*sin(f*x+e))^(1/2)/d/f+4/15*a^2*(c^2-5*c*d-12*d^2)*(sin(1/2*e+1/4*Pi+1/
2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*
x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^2/f/((c+d*sin(f*x+e))/
(c+d))^(1/2)-4/15*a^2*(c-5*d)*(c^2-d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)
/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(
c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^2/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2763, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{4a^2(c-5d)\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^2f\sqrt{c+d\sin(e+fx)}}-\frac{4a^2\left(c^2-5cd-12d^2\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx\right.\right.}{15d^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (4*a^2*(c - 5*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*d*f) - (2*a^2*C
os[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(5*d*f) - (4*a^2*(c^2 - 5*c*d - 12*
d^2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])
/(15*d^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^2*(c - 5*d)*(c^2 - d^
2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(
c + d)])/(15*d^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}\,dx = -\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{5df} + \frac{2\int \left(4a^2d-a^2(c-5d)\sin\right.}{}$$

$$= \frac{4a^2(c-5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15df} - \frac{2a^2\cos(e+fx)(c+}{5df}$$

$$= \frac{4a^2(c-5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15df} - \frac{2a^2\cos(e+fx)(c+}{5df}$$

$$= \frac{4a^2(c-5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15df} - \frac{2a^2\cos(e+fx)(c+}{5df}$$

$$= \frac{4a^2(c-5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15df} - \frac{2a^2\cos(e+fx)(c+}{5df}$$

Mathematica [A] time = 1.41, size = 244, normalized size = 1.02

$$-\frac{a^2(\sin(e+fx)+1)^2\left(-d\cos(e+fx)\left(-2c^2-4d(2c+5d)\sin(e+fx)-20cd+3d^2\cos(2(e+fx))-3d^2\right)+4\left(c\right.\right.}{15d^2f\left(\sin\left(\frac{1}{2}(e+fx\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] -1/15*(a^2*(1 + Sin[e + f*x])^2*(-4*(c^3 - 4*c^2*d - 17*c*d^2 - 12*d^3)*Ell
ipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c +
 d)] + 4*(c^3 - 5*c^2*d - c*d^2 + 5*d^3)*EllipticF[(-2*e + Pi - 2*f*x)/4, (
2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - d*Cos[e + f*x]*(-2*c^2 -
 20*c*d - 3*d^2 + 3*d^2*Cos[2*(e + f*x)] - 4*d*(2*c + 5*d)*Sin[e + f*x])))/
(d^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sqrt[c + d*Sin[e + f*x]])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2\right)\sqrt{d\sin\left(fx+e\right)+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*sqrt(d*sin(f*x
+ e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^2\sqrt{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*sqrt(d*sin(f*x + e) + c), x)
```

maple [B] time = 1.45, size = 1035, normalized size = 4.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/15*a^2*(2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)
*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),(
(c-d)/(c+d))^(1/2))*c^3*d-34*c^2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+
e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*
x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^2-2*c*((c+d*sin(f*x+e))/(c-d))^(1
/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ellipti
cF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^3+34*((c+d*sin(f*x
+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))
^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^4-2*
((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f
*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^
(1/2))*c^4+10*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2
)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))*c^3*d+26*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-
1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^2-10*((c+d*sin(f*x+e))/(c-d))^(1
/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ellipti
cE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^3-24*((c+d*sin(f
*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d
))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^4-
3*d^4*sin(f*x+e)^4-4*c*d^3*sin(f*x+e)^3-10*d^4*sin(f*x+e)^3-c^2*d^2*sin(f*x
```

```
+e)^2-10*c*d^3*sin(f*x+e)^2+3*d^4*sin(f*x+e)^2+4*c*d^3*sin(f*x+e)+10*d^4*si
n(f*x+e)+c^2*d^2+10*c*d^3)/d^3/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{2}\sqrt{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*sqrt(d*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,\sqrt{c+d\,\sin\left(e+f\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int 2\sqrt{c+d\sin\left(e+fx\right)}\,\sin\left(e+fx\right)dx+\int \sqrt{c+d\sin\left(e+fx\right)}\,\sin^{2}\left(e+fx\right)dx+\int \sqrt{c+d\sin\left(e+fx\right)}\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] a**2*(Integral(2*sqrt(c + d*sin(e + f*x))*sin(e + f*x), x) + Integral(sqrt(
c + d*sin(e + f*x))*sin(e + f*x)**2, x) + Integral(sqrt(c + d*sin(e + f*x))
, x))
```

3.492 $\int \frac{(a+a\sin(e+fx))^2}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=189

$$\frac{4a^2(c-2d)(c-d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^2f\sqrt{c+d\sin(e+fx)}}-\frac{4a^2(c-3d)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-$$

```
[Out] -2/3*a^2*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d/f+4/3*a^2*(c-3*d)*(sin(1/2*e+1
/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*P
i+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^2/f/((c+d*sin(
f*x+e))/(c+d))^(1/2)-4/3*a^2*(c-2*d)*(c-d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1
/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(
d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^2/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2763, 2752, 2663, 2661, 2655, 2653}

$$\frac{4a^2(c-2d)(c-d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^2f\sqrt{c+d\sin(e+fx)}}-\frac{4a^2(c-3d)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*a^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*d*f) - (4*a^2*(c - 3*d)*E
llipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*d^
2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^2*(c - 2*d)*(c - d)*Elliptic
F[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3
*d^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{2a^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3df} + \frac{2\int \frac{2a^2d-a^2(c-3d)\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx}{3d}$$

$$= -\frac{2a^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3df} - \frac{\left(2a^2(c-3d)\right)\int \sqrt{c+d\sin(e+fx)}\,dx}{3d^2} + \frac{(2}{}$$

$$= -\frac{2a^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3df} - \frac{\left(2a^2(c-3d)\sqrt{c+d\sin(e+fx)}\right)\int \sqrt{\frac{c}{c+d}+}}{3d^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

$$= -\frac{2a^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3df} - \frac{4a^2(c-3d)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\,\mathrm{s}}}{3d^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Mathematica [A] time = 1.08, size = 193, normalized size = 1.02

$$-\frac{2a^2(\sin(e+fx)+1)^2\left(2\left(c^2-3cd+2d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)-2\left(c^2-2cd-3d^2\right)\sqrt{\frac{c+d\sin}{c+}}\right.}{3d^2f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4\sqrt{c+d\sin}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*a^2*(1 + Sin[e + f*x])^2*(d*Cos[e + f*x]*(c + d*Sin[e + f*x]) - 2*(c^2
- 2*c*d - 3*d^2)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c +
d*Sin[e + f*x])/(c + d)] + 2*(c^2 - 3*c*d + 2*d^2)*EllipticF[(-2*e + Pi - 2
*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)]))/(3*d^2*f*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sqrt[c + d*Sin[e + f*x]])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2}{\sqrt{d\sin\left(fx+e\right)+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)/sqrt(d*sin(f*x
+ e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^2}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/sqrt(d*sin(f*x + e) + c), x)
```

maple [B] time = 1.46, size = 758, normalized size = 4.01

$$-\frac{2a^2\left(2\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}}\,\sqrt{-\frac{\left(\sin\left(fx+e\right)-1\right)d}{c+d}}\,\sqrt{-\frac{d\left(1+\sin\left(fx+e\right)\right)}{c-d}}\,\operatorname{EllipticF}\left(\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}},\sqrt{\frac{c-d}{c+d}}\right)c^2d-12\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/3*a^2*(2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*
(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((
c-d)/(c+d))^(1/2))*c^2*d-12*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)
*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))
/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^2*c+10*((c+d*sin(f*x+e))/(c-d))^(1/2)*
(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF((
(c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^3-2*((c+d*sin(f*x+e))/
(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2
)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^3+6*((c+d
*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e)
)/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)
)*c^2*d+2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-
d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-
d)/(c+d))^(1/2))*c*d^2-6*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/
(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c
-d))^(1/2),((c-d)/(c+d))^(1/2))*d^3-d^3*sin(f*x+e)^3-c*d^2*sin(f*x+e)^2+d^3
*sin(f*x+e)+c*d^2)/d^3/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^2}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^2/sqrt(d*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^2}{\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c + d*sin(e + f*x))^(1/2),x)
[Out] int((a + a*sin(e + f*x))^2/(c + d*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{2\sin\left(e+fx\right)}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx+\int \frac{\sin^2\left(e+fx\right)}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx+\int \frac{1}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**(1/2),x)
[Out] a**2*(Integral(2*sin(e + f*x)/sqrt(c + d*sin(e + f*x)), x) + Integral(sin(e
 + f*x)**2/sqrt(c + d*sin(e + f*x)), x) + Integral(1/sqrt(c + d*sin(e + f*x
)), x))
```

3.493 $\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=189

$$-\frac{4a^2(c-d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{d^2f\sqrt{c+d\sin(e+fx)}}+\frac{4a^2c\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{d^2f(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{2a^2(c-d)}{df(c+d)\sqrt{c+}}$$

```
[Out] 2*a^2*(c-d)*cos(f*x+e)/d/(c+d)/f/(c+d*sin(f*x+e))^(1/2)-4*a^2*c*(sin(1/2*e+
1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*
Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^2/(c+d)/f/((c
+d*sin(f*x+e))/(c+d))^(1/2)+4*a^2*(c-d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)
/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(
c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^2/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2762, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4a^2(c-d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{d^2f\sqrt{c+d\sin(e+fx)}}+\frac{4a^2c\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{d^2f(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{2a^2(c-d)}{df(c+d)\sqrt{c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*a^2*(c - d)*Cos[e + f*x])/(d*(c + d)*f*Sqrt[c + d*Sin[e + f*x]]) + (4*a^
2*c*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/
(d^2*(c + d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (4*a^2*(c - d)*Ellipti
cF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(
d^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{(2a)\int \frac{-ad-ac\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx}{d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{\left(2a^2(c-d)\right)\int \frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{d^2} + \frac{\left(2a^2c\right)\int \sqrt{c+d\,\mathrm{s}}}{d^2(c+}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{c+d\sin(e+fx)}} + \frac{\left(2a^2c\sqrt{c+d\sin(e+fx)}\right)\int \sqrt{\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}}\,dx}{d^2(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} -$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{c+d\sin(e+fx)}} + \frac{4a^2cE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{d^2(c+d)f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} - \frac{4}{}$$

Mathematica [A] time = 0.91, size = 175, normalized size = 0.93

$$-\frac{2a^2(\sin(e+fx)+1)^2\left(2c(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,E\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)-(c-d)\left(2(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{4}(-2\right.\right.\right.}{d^2f(c+d)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a^2*(1 + Sin[e + f*x])^2*(2*c*(c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4,
(2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - (c - d)*(d*Cos[e + f*x]
 + 2*(c + d)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Si
n[e + f*x])/(c + d)])))/(d^2*(c + d)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^4*Sqrt[c + d*Sin[e + f*x]])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2\right)\sqrt{d\sin\left(fx+e\right)+c}}{d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*sqrt(d*sin(f*x +
 e) + c)/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^2}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(3/2), x)
```

maple [A] time = 1.40, size = 463, normalized size = 2.45

$$-\frac{2\left(2\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}}\,\sqrt{-\frac{\left(\sin\left(fx+e\right)-1\right)d}{c+d}}\,\sqrt{-\frac{d\left(1+\sin\left(fx+e\right)\right)}{c-d}}\;\mathrm{EllipticE}\left(\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}},\sqrt{\frac{c-d}{c+d}}\right)c^3-2\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}}\;\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] -2*(2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1
+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(
c+d))^(1/2))*c^3-2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))
^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(
1/2),((c-d)/(c+d))^(1/2))*c*d^2-2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x
+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f
*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d+2*((c+d*sin(f*x+e))/(c-d))^(
1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ellipt
icF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^3+c*d^2*sin(f*x+e
)^2-d^3*sin(f*x+e)^2-c*d^2+d^3)/d^3*a^2/(c+d)/cos(f*x+e)/(c+d*sin(f*x+e))^(
1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^2}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + a\,\sin\left(e + f\,x\right)\right)^2}{\left(c + d\,\sin\left(e + f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c + d*sin(e + f*x))^(3/2),x)
[Out] int((a + a*sin(e + f*x))^2/(c + d*sin(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**(3/2),x)
[Out] Timed out
```

3.494 $\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=247

$$\frac{4a^2(c+2d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^2f(c+d)\sqrt{c+d\sin(e+fx)}}-\frac{4a^2(c+3d)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^2f(c+d)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{4a\cdots}{3df(c\cdots}$$

```
[Out] 2/3*a^2*(c-d)*cos(f*x+e)/d/(c+d)/f/(c+d*sin(f*x+e))^(3/2)-4/3*a^2*(c+3*d)*c
os(f*x+e)/d/(c+d)^2/f/(c+d*sin(f*x+e))^(1/2)+4/3*a^2*(c+3*d)*(sin(1/2*e+1/4
*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+
1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^2/(c+d)^2/f/((c+
d*sin(f*x+e))/(c+d))^(1/2)-4/3*a^2*(c+2*d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1
/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(
d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^2/(c+d)/f/(c+d*sin(f*x+e))
^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2762, 2754, 2752, 2663, 2661, 2655, 2653}

$$\frac{4a^2(c+2d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^2f(c+d)\sqrt{c+d\sin(e+fx)}}-\frac{4a^2(c+3d)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^2f(c+d)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{4a\cdots}{3df(c\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*a^2*(c - d)*Cos[e + f*x])/(3*d*(c + d)*f*(c + d*Sin[e + f*x])^(3/2)) - (
4*a^2*(c + 3*d)*Cos[e + f*x])/(3*d*(c + d)^2*f*Sqrt[c + d*Sin[e + f*x]]) -
(4*a^2*(c + 3*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Si
n[e + f*x]])/(3*d^2*(c + d)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^
2*(c + 2*d)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e
+ f*x])/(c + d)])/(3*d^2*(c + d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
```

```
ntegerQ[m] && EqQ[c, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{(2a)\int \frac{-3ad-a(c+2d)\sin(e+fx)}{(c+d\sin(e+fx))^{3/2}}\,dx}{3d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{4a^2(c+3d)\cos(e+fx)}{3d(c+d)^2f\sqrt{c+d\sin(e+fx)}} + \frac{(4a)\int \frac{a(c-d)a}{\cdots}}{3(c}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{4a^2(c+3d)\cos(e+fx)}{3d(c+d)^2f\sqrt{c+d\sin(e+fx)}} + \frac{\left(2a^2(c+2d)\right.}{3}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{4a^2(c+3d)\cos(e+fx)}{3d(c+d)^2f\sqrt{c+d\sin(e+fx)}} - \frac{\left(2a^2(c+3d)\right.}{}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{4a^2(c+3d)\cos(e+fx)}{3d(c+d)^2f\sqrt{c+d\sin(e+fx)}} - \frac{4a^2(c+3d)E}{}$$

Mathematica [A] time = 1.77, size = 207, normalized size = 0.84

$$-\frac{2a^2(\sin(e+fx)+1)^2\left(d\cos(e+fx)\left(c^2+2d(c+3d)\sin(e+fx)+6cd+d^2\right)+2(c+2d)(c+d)^2\left(\frac{c+d\sin(e+fx)}{c+d}\right.\right.}{3d^2f(c+d)^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+f\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*a^2*(1 + Sin[e + f*x])^2*(-2*(c + d)^2*(c + 3*d)*EllipticE[(-2*e + Pi -
 2*f*x)/4, (2*d)/(c + d)]*((c + d*Sin[e + f*x])/(c + d))^(3/2) + 2*(c + d)^
2*(c + 2*d)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*((c + d*Sin[e +
 f*x])/(c + d))^(3/2) + d*Cos[e + f*x]*(c^2 + 6*c*d + d^2 + 2*d*(c + 3*d)*S
in[e + f*x])))/(3*d^2*(c + d)^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(
c + d*Sin[e + f*x])^(3/2))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2\right)\sqrt{d\sin\left(fx+e\right)+c}}{3\,cd^2\cos\left(fx+e\right)^2-c^3-3\,cd^2+\left(d^3\cos\left(fx+e\right)^2-3\,c^2d-d^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*sqrt(d*sin(f*x +
 e) + c)/(3*c*d^2*cos(f*x + e)^2 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3*
c^2*d - d^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\sin\left(fx+e\right)+a\right)^2}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(5/2), x)
```

maple [B] time = 1.52, size = 1221, normalized size = 4.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] -2/3*a^2*((2*c*d^3+6*d^4)*sin(f*x+e)*cos(f*x+e)^2-2*(-d/(c-d)*sin(f*x+e)-d/
(c-d))^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(d/(c-d)*sin(f*x+e)+1/(c-d
)*c)^(1/2)*d*(EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^
(1/2))*c^3+3*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(
1/2))*c^2*d-EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1
/2))*c*d^2-3*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(
1/2))*d^3-EllipticF((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2
))*c^2*d+EllipticF((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2)
)*d^3)*sin(f*x+e)+(c^2*d^2+6*c*d^3+d^4)*cos(f*x+e)^2+2*(d/(c-d)*sin(f*x+e)+
1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d
/(c-d))^(1/2)*EllipticF((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^
(1/2))*c^3*d-2*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/
```

```
(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticF((d/(c-d)*sin(f*x
+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c*d^3-2*(d/(c-d)*sin(f*x+e)+1/(c-
d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d
))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2)
)*c^4-6*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^
(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(
c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^3*d+2*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(
1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2
)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^2*d
^2+6*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/
2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d
)*c)^(1/2),((c-d)/(c+d))^(1/2))*c*d^3)/(c+d)^2/(c+d*sin(f*x+e))^(3/2)/d^3/c
os(f*x+e)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(fx+e)+a)^2}{(d\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+a\,\sin(e+f\,x))^2}{(c+d\,\sin(e+f\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c + d*sin(e + f*x))^(5/2),x)
[Out] int((a + a*sin(e + f*x))^2/(c + d*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**(5/2),x)
[Out] Timed out
```

3.495 $\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=320

$$-\frac{4a^2\left(c^2+5cd-12d^2\right)\cos(e+fx)}{15df(c-d)(c+d)^3\sqrt{c+d\sin(e+fx)}}-\frac{4a^2\left(c^2+5cd-12d^2\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^2f(c-d)(c+d)^3\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{4a^2(}{}$$

```
[Out] 2/5*a^2*(c-d)*cos(f*x+e)/d/(c+d)/f/(c+d*sin(f*x+e))^(5/2)-4/15*a^2*(c+5*d)*
cos(f*x+e)/d/(c+d)^2/f/(c+d*sin(f*x+e))^(3/2)-4/15*a^2*(c^2+5*c*d-12*d^2)*c
os(f*x+e)/(c-d)/d/(c+d)^3/f/(c+d*sin(f*x+e))^(1/2)+4/15*a^2*(c^2+5*c*d-12*d
^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE
(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/
(c-d)/d^2/(c+d)^3/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-4/15*a^2*(c+5*d)*(sin(1/
2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+
1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^2
/(c+d)^2/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2762, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4a^2\left(c^2+5cd-12d^2\right)\cos(e+fx)}{15df(c-d)(c+d)^3\sqrt{c+d\sin(e+fx)}}-\frac{4a^2\left(c^2+5cd-12d^2\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^2f(c-d)(c+d)^3\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{4a^2(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (2*a^2*(c - d)*Cos[e + f*x])/(5*d*(c + d)*f*(c + d*Sin[e + f*x])^(5/2)) - (
4*a^2*(c + 5*d)*Cos[e + f*x])/(15*d*(c + d)^2*f*(c + d*Sin[e + f*x])^(3/2))
 - (4*a^2*(c^2 + 5*c*d - 12*d^2)*Cos[e + f*x])/(15*(c - d)*d*(c + d)^3*f*Sq
rt[c + d*Sin[e + f*x]]) - (4*a^2*(c^2 + 5*c*d - 12*d^2)*EllipticE[(e - Pi/2
 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(15*(c - d)*d^2*(c + d)
^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^2*(c + 5*d)*EllipticF[(e -
Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(15*d^2*(
c + d)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
```

```
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2}{(c+d\sin(e+fx))^{7/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{(2a)\int \frac{-5ad-a(c+4d)\sin(e+fx)}{(c+d\sin(e+fx))^{5/2}}\,dx}{5d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{4a^2(c+5d)\cos(e+fx)}{15d(c+d)^2f(c+d\sin(e+fx))^{3/2}} + \frac{(4a)\int \frac{6a(c-}{}}{15}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{4a^2(c+5d)\cos(e+fx)}{15d(c+d)^2f(c+d\sin(e+fx))^{3/2}} - \frac{4a^2\left(c^2+\right.}{15(c-d)d(}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{4a^2(c+5d)\cos(e+fx)}{15d(c+d)^2f(c+d\sin(e+fx))^{3/2}} - \frac{4a^2\left(c^2+\right.}{15(c-d)d(}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{4a^2(c+5d)\cos(e+fx)}{15d(c+d)^2f(c+d\sin(e+fx))^{3/2}} - \frac{4a^2\left(c^2+\right.}{15(c-d)d(}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{4a^2(c+5d)\cos(e+fx)}{15d(c+d)^2f(c+d\sin(e+fx))^{3/2}} - \frac{4a^2\left(c^2+\right.}{15(c-d)d(}$$

Mathematica [A] time = 2.03, size = 283, normalized size = 0.88

$$\frac{2a^2(\sin(e+fx)+1)^2\Big(d\cos(e+fx)\left(-2\left(c^2+5cd-12d^2\right)(c+d\sin(e+fx))^2-2(c-d)(c+5d)(c+d)(c+d\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (2*a^2*(1 + Sin[e + f*x])^2*(-2*((11*c - 5*d)*d^2*EllipticF[(-2*e + Pi - 2*
f*x)/4, (2*d)/(c + d)] - (c^2 + 5*c*d - 12*d^2)*((c + d)*EllipticE[(-2*e +
```

```
Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c
 + d)]))*(c + d*Sin[e + f*x])^2*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + d*Cos[
e + f*x]*(3*(c - d)^2*(c + d)^2 - 2*(c - d)*(c + d)*(c + 5*d)*(c + d*Sin[e
+ f*x]) - 2*(c^2 + 5*c*d - 12*d^2)*(c + d*Sin[e + f*x])^2)))/(15*(c - d)*d^
2*(c + d)^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c + d*Sin[e + f*x])^
(5/2))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2\right)\sqrt{d\sin\left(fx+e\right)+c}}{d^4\cos\left(fx+e\right)^4+c^4+6\,c^2d^2+d^4-2\left(3\,c^2d^2+d^4\right)\cos\left(fx+e\right)^2-4\left(cd^3\cos\left(fx+e\right)^2-c^3d-cd^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*sqrt(d*sin(f*x
+ e) + c)/(d^4*cos(f*x + e)^4 + c^4 + 6*c^2*d^2 + d^4 - 2*(3*c^2*d^2 + d^4)
*cos(f*x + e)^2 - 4*(c*d^3*cos(f*x + e)^2 - c^3*d - c*d^3)*sin(f*x + e)), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^2}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(7/2), x)
```

maple [B] time = 6.39, size = 1436, normalized size = 4.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(7/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*a^2*(1/d^2*(2*d*cos(f*x+e)^2/(c^2-d
^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin
(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-
d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e
```

```
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*
x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2))))+2*(-c+d)/d^2*(2/3/(c^2-d^2)/d*(-(-d*sin(f*x+e)
-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/3*d*cos(f*x+e)^2/(c^2-d^2)^2*c
/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*c^2*d^2+3*d
^4)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*(
(-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Elli
pticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+8/3*c*d/(c^2-d^2)
^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((
-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/
d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+Elliptic
F(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(c^2-2*c*d+d^2)/d^2
*(2/5/(c^2-d^2)/d^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d
)^3+16/15*c/(c^2-d^2)^2/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+
e)+c/d)^2+2/15*d*cos(f*x+e)^2/(c^2-d^2)^3*(23*c^2+9*d^2)/(-(-d*sin(f*x+e)-c
)*cos(f*x+e)^2)^(1/2)+2*(15*c^3+17*c*d^2)/(15*c^6-45*c^4*d^2+45*c^2*d^4-15*
d^6)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*
((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Ell
ipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/15*d*(23*c^2+9
*d^2)/(c^2-d^2)^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/
(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)
^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^
(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos
(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{2}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^2}{\left(c+d\,\sin\left(e+f\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2/(c + d*sin(e + f*x))^(7/2),x)
[Out] int((a + a*sin(e + f*x))^2/(c + d*sin(e + f*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**(7/2),x)
[Out] Timed out
```

3.496 $\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=467

$$-\frac{4a^3\left(4c^2-33cd+189d^2\right)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{693d^2f}-\frac{4a^3\left(4c^3-33c^2d+182cd^2+231d^3\right)\cos(e+fx)(c+}{693d^2f}$$

```
[Out] -4/693*a^3*(4*c^3-33*c^2*d+182*c*d^2+231*d^3)*cos(f*x+e)*(c+d*sin(f*x+e))^(
3/2)/d^2/f-4/693*a^3*(4*c^2-33*c*d+189*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^(5/
2)/d^2/f+8/99*a^3*(c-6*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(7/2)/d^2/f-2/11*cos(
f*x+e)*(a^3+a^3*sin(f*x+e))*(c+d*sin(f*x+e))^(7/2)/d/f-4/693*a^3*(4*c^4-33*
c^3*d+177*c^2*d^2+561*c*d^3+315*d^4)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d^2/
f-4/693*a^3*(c+3*d)*(4*c^4-45*c^3*d+309*c^2*d^2+525*c*d^3+231*d^4)*(sin(1/2
*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1
/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^3/f/((c+d*
sin(f*x+e))/(c+d))^(1/2)+4/693*a^3*(c^2-d^2)*(4*c^4-33*c^3*d+177*c^2*d^2+56
1*c*d^3+315*d^4)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f
*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(
f*x+e))/(c+d))^(1/2)/d^3/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.03, antiderivative size = 467, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2763, 2968, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4a^3\left(4c^2-33cd+189d^2\right)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{693d^2f}-\frac{4a^3\left(-33c^2d+4c^3+182cd^2+231d^3\right)\cos(e+fx)(c}{693d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-4*a^3*(4*c^4 - 33*c^3*d + 177*c^2*d^2 + 561*c*d^3 + 315*d^4)*Cos[e + f*x]
*Sqrt[c + d*Sin[e + f*x]])/(693*d^2*f) - (4*a^3*(4*c^3 - 33*c^2*d + 182*c*d
^2 + 231*d^3)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(693*d^2*f) - (4*a^3
*(4*c^2 - 33*c*d + 189*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(693*d
^2*f) + (8*a^3*(c - 6*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(7/2))/(99*d^2*f
) - (2*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x])*(c + d*Sin[e + f*x])^(7/2))/(1
1*d*f) + (4*a^3*(c + 3*d)*(4*c^4 - 45*c^3*d + 309*c^2*d^2 + 525*c*d^3 + 231
*d^4)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]]
)/(693*d^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (4*a^3*(c^2 - d^2)*(4*c^
4 - 33*c^3*d + 177*c^2*d^2 + 561*c*d^3 + 315*d^4)*EllipticF[(e - Pi/2 + f*x
```

```
)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(693*d^3*f*Sqrt[c +
 d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int (a + a\sin(e + fx))^3(c + d\sin(e + fx))^{5/2}\,dx = -\frac{2\cos(e + fx)\left(a^3 + a^3\sin(e + fx)\right)(c + d\sin(e + fx))^{7/2}}{11df} + \frac{2}{}$$

$$= -\frac{2\cos(e + fx)\left(a^3 + a^3\sin(e + fx)\right)(c + d\sin(e + fx))^{7/2}}{11df} + \frac{2}{}$$

$$= \frac{8a^3(c - 6d)\cos(e + fx)(c + d\sin(e + fx))^{7/2}}{99d^2f} - \frac{2\cos(e + fx)\left(\right.}{}$$

$$= -\frac{4a^3\left(4c^2 - 33cd + 189d^2\right)\cos(e + fx)(c + d\sin(e + fx))^{5/2}}{693d^2f} +$$

$$= -\frac{4a^3\left(4c^3 - 33c^2d + 182cd^2 + 231d^3\right)\cos(e + fx)(c + d\sin(e +}{693d^2f}$$

$$= -\frac{4a^3\left(4c^4 - 33c^3d + 177c^2d^2 + 561cd^3 + 315d^4\right)\cos(e + fx)\sqrt{}}{693d^2f}$$

$$= -\frac{4a^3\left(4c^4 - 33c^3d + 177c^2d^2 + 561cd^3 + 315d^4\right)\cos(e + fx)\sqrt{}}{693d^2f}$$

$$= -\frac{4a^3\left(4c^4 - 33c^3d + 177c^2d^2 + 561cd^3 + 315d^4\right)\cos(e + fx)\sqrt{}}{693d^2f}$$

$$= -\frac{4a^3\left(4c^4 - 33c^3d + 177c^2d^2 + 561cd^3 + 315d^4\right)\cos(e + fx)\sqrt{}}{693d^2f}$$

Mathematica [A] time = 1.90, size = 377, normalized size = 0.81

$$\frac{a^3(\sin(e + fx) + 1)^3\left(d(c + d\sin(e + fx))\left(d^2\left(452c^2 + 2508cd + 1701d^2\right)\cos(3(e + fx)) - 4d\left(6c^3 + 990c^2d + 2\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (a^3*(1 + Sin[e + f*x])^3*(-32*(d^2*(c^4 + 858*c^3*d + 1668*c^2*d^2 + 1254*
c*d^3 + 315*d^4)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] + (4*c^5 -
 33*c^4*d + 174*c^3*d^2 + 1452*c^2*d^3 + 1806*c*d^4 + 693*d^5)*((c + d)*Ell
ipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f
*x)/4, (2*d)/(c + d)]))*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + d*(c + d*Sin[e
 + f*x])*(2*(32*c^4 - 264*c^3*d - 8994*c^2*d^2 - 13926*c*d^3 - 5859*d^4)*Co
s[e + f*x] + d^2*(452*c^2 + 2508*c*d + 1701*d^2)*Cos[3*(e + f*x)] - 63*d^4*
Cos[5*(e + f*x)] - 4*d*(6*c^3 + 990*c^2*d + 2401*c*d^2 + 1155*d^3)*Sin[2*(e
 + f*x)] + 14*d^3*(23*c + 33*d)*Sin[4*(e + f*x)])))/(5544*d^3*f*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])^6*Sqrt[c + d*Sin[e + f*x]])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(4\,a^3c^2 + 8\,a^3cd + 4\,a^3d^2 + \left(2\,a^3cd + 3\,a^3d^2\right)\cos\left(fx + e\right)^4 - \left(3\,a^3c^2 + 10\,a^3cd + 7\,a^3d^2\right)\cos\left(fx + e\right)^2 -\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((4*a^3*c^2 + 8*a^3*c*d + 4*a^3*d^2 + (2*a^3*c*d + 3*a^3*d^2)*cos(f
*x + e)^4 - (3*a^3*c^2 + 10*a^3*c*d + 7*a^3*d^2)*cos(f*x + e)^2 + (a^3*d^2*
cos(f*x + e)^4 + 4*a^3*c^2 + 8*a^3*c*d + 4*a^3*d^2 - (a^3*c^2 + 6*a^3*c*d +
 5*a^3*d^2)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(d*sin(f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx + e\right) + a\right)^3 \left(d\sin\left(fx + e\right) + c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^(5/2), x)
```

maple [B] time = 1.70, size = 1926, normalized size = 4.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] 2/693*a^3*(2128*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1
/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))*c^4*d^3+116*c^3*d^4*sin(f*x+e)^4+1122*c^2*d^5*sin(f*
```

```
x+e)^4+1274*c*d^6*sin(f*x+e)^4-c^4*d^3*sin(f*x+e)^3+528*c^3*d^4*sin(f*x+e)^
3+1942*c^2*d^5*sin(f*x+e)^3+1188*c*d^6*sin(f*x+e)^3-4*c^5*d^2*sin(f*x+e)^2+
33*c^4*d^3*sin(f*x+e)^2+980*c^3*d^4*sin(f*x+e)^2+462*c^2*d^5*sin(f*x+e)^2-8
68*c*d^6*sin(f*x+e)^2+c^4*d^3*sin(f*x+e)-528*c^3*d^4*sin(f*x+e)-2216*c^2*d^
5*sin(f*x+e)-2046*c*d^6*sin(f*x+e)+224*c*d^6*sin(f*x+e)^6+274*c^2*d^5*sin(f
*x+e)^5+858*c*d^6*sin(f*x+e)^5+4*c^5*d^2-1096*c^3*d^4-33*c^4*d^3-630*c*d^6-
1584*c^2*d^5+63*d^7*sin(f*x+e)^7+231*d^7*sin(f*x+e)^6+315*d^7*sin(f*x+e)^5+
231*d^7*sin(f*x+e)^4+252*d^7*sin(f*x+e)^3-462*d^7*sin(f*x+e)^2-630*d^7*sin(
f*x+e)+1386*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*
(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((
c-d)/(c+d))^(1/2))*d^7-2016*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)
*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))
/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^7-8*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(
sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+
d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^7+4176*((c+d*sin(f*x+e))/
(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2
)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^3*d^4-120
*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(
f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))
^(1/2))*c^2*d^5-4104*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d
))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))
^(1/2),((c-d)/(c+d))^(1/2))*c*d^6+66*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(
f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*si
n(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^6*d+8*((c+d*sin(f*x+e))/(c-d)
)^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ell
ipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^6*d-72*((c+d*s
in(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/
(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*
c^5*d^2-340*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*
(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((
c-d)/(c+d))^(1/2))*c^5*d^2-2970*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e
)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x
+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^4*d^3-3264*((c+d*sin(f*x+e))/(c-d)
)^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ell
ipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^3*d^4+1518*((c
+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+
e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/
2))*c^2*d^5+3612*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(
1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/
2),((c-d)/(c+d))^(1/2))*c*d^6)/d^4/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{3}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+a\,\sin\left(e+f\,x\right)\right)^3\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^(5/2),x)
[Out] int((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c+d*sin(f*x+e))**(5/2),x)
[Out] Timed out
```

3.497 $\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=390

$$-\frac{4a^3\left(4c^2-27cd+119d^2\right)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{315d^2f}-\frac{4a^3\left(4c^3-27c^2d+114cd^2+165d^3\right)\cos(e+fx)\sqrt{}}{315d^2f}$$

[Out] -4/315*a^3*(4*c^2-27*c*d+119*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/d^2/f+8/63*a^3*(c-5*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(5/2)/d^2/f-2/9*cos(f*x+e)*(a^3+a^3*sin(f*x+e))*(c+d*sin(f*x+e))^(5/2)/d/f-4/315*a^3*(4*c^3-27*c^2*d+114*c*d^2+165*d^3)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d^2/f-4/315*a^3*(4*c^4-27*c^3*d+111*c^2*d^2+579*c*d^3+357*d^4)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^3/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+4/315*a^3*(c^2-d^2)*(4*c^3-27*c^2*d+114*c*d^2+165*d^3)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^3/f/(c+d*sin(f*x+e))^(1/2)

Rubi [A] time = 0.78, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2763, 2968, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4a^3\left(4c^2-27cd+119d^2\right)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{315d^2f}-\frac{4a^3\left(-27c^2d+4c^3+114cd^2+165d^3\right)\cos(e+fx)}{315d^2f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(3/2),x]

[Out] (-4*a^3*(4*c^3 - 27*c^2*d + 114*c*d^2 + 165*d^3)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(315*d^2*f) - (4*a^3*(4*c^2 - 27*c*d + 119*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(315*d^2*f) + (8*a^3*(c - 5*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(63*d^2*f) - (2*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x])*(c + d*Sin[e + f*x])^(5/2))/(9*d*f) + (4*a^3*(4*c^4 - 27*c^3*d + 111*c^2*d^2 + 579*c*d^3 + 357*d^4)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(315*d^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (4*a^3*(c^2 - d^2)*(4*c^3 - 27*c^2*d + 114*c*d^2 + 165*d^3)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(315*d^3*f*Sqrt[c + d*Sin[e + f*x]])

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
```

```
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int (a + a\sin(e + fx))^3(c + d\sin(e + fx))^{3/2}\,dx = -\frac{2\cos(e + fx)\left(a^3 + a^3\sin(e + fx)\right)(c + d\sin(e + fx))^{5/2}}{9df} + \frac{2\int}{}$$

$$= -\frac{2\cos(e + fx)\left(a^3 + a^3\sin(e + fx)\right)(c + d\sin(e + fx))^{5/2}}{9df} + \frac{2\int}{}$$

$$= \frac{8a^3(c - 5d)\cos(e + fx)(c + d\sin(e + fx))^{5/2}}{63d^2f} - \frac{2\cos(e + fx)\left(a^3\right.}{}$$

$$= -\frac{4a^3\left(4c^2 - 27cd + 119d^2\right)\cos(e + fx)(c + d\sin(e + fx))^{3/2}}{315d^2f} + \frac{8}{}$$

$$= -\frac{4a^3\left(4c^3 - 27c^2d + 114cd^2 + 165d^3\right)\cos(e + fx)\sqrt{c + d\sin(e +}}{315d^2f}$$

$$= -\frac{4a^3\left(4c^3 - 27c^2d + 114cd^2 + 165d^3\right)\cos(e + fx)\sqrt{c + d\sin(e +}}{315d^2f}$$

$$= -\frac{4a^3\left(4c^3 - 27c^2d + 114cd^2 + 165d^3\right)\cos(e + fx)\sqrt{c + d\sin(e +}}{315d^2f}$$

$$= -\frac{4a^3\left(4c^3 - 27c^2d + 114cd^2 + 165d^3\right)\cos(e + fx)\sqrt{c + d\sin(e +}}{315d^2f}$$

Mathematica [A] time = 2.27, size = 318, normalized size = 0.82

$$\frac{a^3(\sin(e + fx) + 1)^3\left(d(c + d\sin(e + fx))\left(2d\left(5d(10c + 27d)\cos(3(e + fx)) - \sin(2(e + fx))\left(6c^2 + 432cd - 35d^2\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(3/2),x]

[Out] (a^3*(1 + Sin[e + f*x])^3*(-16*(d^2*(c^3 + 387*c^2*d + 471*c*d^2 + 165*d^3)
*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] + (4*c^4 - 27*c^3*d + 111*
c^2*d^2 + 579*c*d^3 + 357*d^4)*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2
*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]))*Sqrt[(c
```

```
+ d*Sin[e + f*x])/(c + d)] + d*(c + d*Sin[e + f*x])*((32*c^3 - 216*c^2*d -
3828*c*d^2 - 2910*d^3)*Cos[e + f*x] + 2*d*(5*d*(10*c + 27*d)*Cos[3*(e + f*x
)] - (6*c^2 + 432*c*d + 511*d^2 - 35*d^2*Cos[2*(e + f*x)])*Sin[2*(e + f*x)]
))))/(1260*d^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*Sqrt[c + d*Sin[e +
 f*x]])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(a^3 d\cos\left(fx+e\right)^4+4\,a^3c+4\,a^3d-\left(3\,a^3c+5\,a^3d\right)\cos\left(fx+e\right)^2+\left(4\,a^3c+4\,a^3d-\left(a^3c+3\,a^3d\right)\cos\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*d*cos(f*x + e)^4 + 4*a^3*c + 4*a^3*d - (3*a^3*c + 5*a^3*d)*co
s(f*x + e)^2 + (4*a^3*c + 4*a^3*d - (a^3*c + 3*a^3*d)*cos(f*x + e)^2)*sin(f
*x + e))*sqrt(d*sin(f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^3\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^(3/2), x)
```

maple [B] time = 1.48, size = 1613, normalized size = 4.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] -2/315*a^3*(492*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1
/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))*c^2*d^4-4*c^4*d^2-54*((c+d*sin(f*x+e))/(c-d))^(1/2)*
(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE((
(c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^5*d-1158*((c+d*sin(f*x
+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))
^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^5-
8*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin
(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d)
```

```
)^(1/2))*c^5*d+60*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^
(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1
/2),((c-d)/(c+d))^(1/2))*c^4*d^2-1048*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin
(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*s
in(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^3*d^3-1104*((c+d*sin(f*x+e))
/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/
2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^4+10
56*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+si
n(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d
))^(1/2))*c*d^5+214*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d)
)^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2))*c^4*d^2+1212*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(s
in(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d
*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^3*d^3+27*c^3*d^3+330*c*d^5
+8*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+si
n(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d
))^(1/2))*c^6-714*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^
(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1
/2),((c-d)/(c+d))^(1/2))*d^6+1044*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x
+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f
*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^6-85*c*d^5*sin(f*x+e)^5-53*c^2*d
^4*sin(f*x+e)^4-351*c*d^5*sin(f*x+e)^4+c^3*d^3*sin(f*x+e)^3-243*c^2*d^4*sin
(f*x+e)^3-619*c*d^5*sin(f*x+e)^3+4*c^4*d^2*sin(f*x+e)^2-27*c^3*d^3*sin(f*x+
e)^2-413*c^2*d^4*sin(f*x+e)^2+21*c*d^5*sin(f*x+e)^2-c^3*d^3*sin(f*x+e)+243*
c^2*d^4*sin(f*x+e)+704*c*d^5*sin(f*x+e)-135*d^6*sin(f*x+e)^5-203*d^6*sin(f*
x+e)^4-195*d^6*sin(f*x+e)^3+238*d^6*sin(f*x+e)^2+330*d^6*sin(f*x+e)-35*d^6*
sin(f*x+e)^6+466*c^2*d^4)/d^4/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{3}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^3\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int c\sqrt{c+d\sin\left(e+fx\right)}\,dx+\int 3c\sqrt{c+d\sin\left(e+fx\right)}\sin\left(e+fx\right)dx+\int 3c\sqrt{c+d\sin\left(e+fx\right)}\sin^2\left(e+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] a**3*(Integral(c*sqrt(c + d*sin(e + f*x)), x) + Integral(3*c*sqrt(c + d*sin
(e + f*x))*sin(e + f*x), x) + Integral(3*c*sqrt(c + d*sin(e + f*x))*sin(e +
 f*x)**2, x) + Integral(c*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**3, x) + In
tegral(d*sqrt(c + d*sin(e + f*x))*sin(e + f*x), x) + Integral(3*d*sqrt(c +
d*sin(e + f*x))*sin(e + f*x)**2, x) + Integral(3*d*sqrt(c + d*sin(e + f*x))
*sin(e + f*x)**3, x) + Integral(d*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**4,
 x))
```

3.498 $\int (a+a\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=318

$$-\frac{4a^3\left(4c^2-21cd+65d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105d^2f}-\frac{4a^3\left(c^2-d^2\right)\left(4c^2-21cd+65d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e\right.\right.}{105d^3f\sqrt{c+d\sin(e+fx)}}$$

```
[Out] 8/35*a^3*(c-4*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/d^2/f-2/7*cos(f*x+e)*(a^
3+a^3*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/d/f-4/105*a^3*(4*c^2-21*c*d+65*d^2
)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d^2/f-4/105*a^3*(4*c^3-21*c^2*d+62*c*d^
2+147*d^3)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*El
lipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))
^(1/2)/d^3/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+4/105*a^3*(c^2-d^2)*(4*c^2-21*c
*d+65*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*El
lipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e)
)/(c+d))^(1/2)/d^3/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2763, 2968, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4a^3\left(4c^2-21cd+65d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105d^2f}-\frac{4a^3\left(c^2-d^2\right)\left(4c^2-21cd+65d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e\right.\right.}{105d^3f\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-4*a^3*(4*c^2 - 21*c*d + 65*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(1
05*d^2*f) + (8*a^3*(c - 4*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(35*d
^2*f) - (2*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2)
)/(7*d*f) + (4*a^3*(4*c^3 - 21*c^2*d + 62*c*d^2 + 147*d^3)*EllipticE[(e - P
i/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(105*d^3*f*Sqrt[(c +
 d*Sin[e + f*x])/(c + d)]) - (4*a^3*(c^2 - d^2)*(4*c^2 - 21*c*d + 65*d^2)*E
llipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c +
d)])/(105*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
```

```
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}\,dx = -\frac{2\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^{3/2}}{7df}+\frac{2\int$$

$$= -\frac{2\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^{3/2}}{7df}+\frac{2\int$$

$$= \frac{8a^3(c-4d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{35d^2f}-\frac{2\cos(e+fx)\left(a^3\right.}{}$$

$$= -\frac{4a^3\left(4c^2-21cd+65d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105d^2f}+\frac{8a^3(}{}$$

$$= -\frac{4a^3\left(4c^2-21cd+65d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105d^2f}+\frac{8a^3(}{}$$

$$= -\frac{4a^3\left(4c^2-21cd+65d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105d^2f}+\frac{8a^3(}{}$$

$$= -\frac{4a^3\left(4c^2-21cd+65d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105d^2f}+\frac{8a^3(}{}$$

Mathematica [A] time = 2.72, size = 266, normalized size = 0.84

$$-\frac{a^3\left(-2d\cos(e+fx)\left(16c^3+d\left(4c^2-336cd-565d^2\right)\sin(e+fx)-84c^2d+18d^2(2c+7d)\cos(2(e+fx))-55\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] -1/420*(a^3*(16*(4*c^4 - 17*c^3*d + 41*c^2*d^2 + 209*c*d^3 + 147*d^4)*Ellip
ticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d
)] - 16*(4*c^4 - 21*c^3*d + 61*c^2*d^2 + 21*c*d^3 - 65*d^4)*EllipticF[(-2*e
 + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - 2*d*C
os[e + f*x]*(16*c^3 - 84*c^2*d - 556*c*d^2 - 126*d^3 + 18*d^2*(2*c + 7*d)*C
os[2*(e + f*x)] + d*(4*c^2 - 336*c*d - 565*d^2)*Sin[e + f*x] + 15*d^3*Sin[3
*(e + f*x)])))/(d^3*f*Sqrt[c + d*Sin[e + f*x]])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)\right)\sqrt{d\sin\left(fx+e\right)+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(
f*x + e))*sqrt(d*sin(f*x + e) + c), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.38, size = 1316, normalized size = 4.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/105*a^3*(42*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2
)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))*c^4*d-130*c*d^4-21*c^2*d^3+4*c^3*d^2-8*((c+d*sin(f*x+e
))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(
1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^5+294*
((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f
*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^
(1/2))*d^5-424*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/
2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2)
,((c-d)/(c+d))^(1/2))*d^5+15*d^5*sin(f*x+e)^5+63*d^5*sin(f*x+e)^4+115*d^5*s
in(f*x+e)^3-63*d^5*sin(f*x+e)^2-130*d^5*sin(f*x+e)-4*c^3*d^2*sin(f*x+e)^2+2
1*c^2*d^3*sin(f*x+e)^2+112*c*d^4*sin(f*x+e)^2+c^2*d^3*sin(f*x+e)-84*c*d^4*s
in(f*x+e)+18*c*d^4*sin(f*x+e)^4-c^2*d^3*sin(f*x+e)^3+84*c*d^4*sin(f*x+e)^3-
116*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+s
in(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+
d))^(1/2))*c^3*d^2+8*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d
))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))
```

```
^(1/2),((c-d)/(c+d))^(1/2))*c^4*d-48*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(
f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*si
n(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^3*d^2+416*((c+d*sin(f*x+e))/(
c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)
*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^3+48*(
(c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*
x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(
1/2))*c*d^4-336*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1
/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))*c^2*d^3+124*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*
x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(
f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^4)/d^4/cos(f*x+e)/(c+d*sin(f*
x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin\left(fx+e\right)+a\right)^{3} \sqrt{d \sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^3*sqrt(d*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^3\,\sqrt{c+d\,\sin\left(e+f\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^(1/2),x)
[Out] int((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{3} \left(\int 3 \sqrt{c + d \sin\left(e + f x\right)} \sin\left(e + f x\right)\, dx + \int 3 \sqrt{c + d \sin\left(e + f x\right)} \sin^{2}\left(e + f x\right)\, dx + \int \sqrt{c + d \sin\left(e + f x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c+d*sin(f*x+e))**(1/2),x)
[Out] a**3*(Integral(3*sqrt(c + d*sin(e + f*x))*sin(e + f*x), x) + Integral(3*sqr
t(c + d*sin(e + f*x))*sin(e + f*x)**2, x) + Integral(sqrt(c + d*sin(e + f*x
))*sin(e + f*x)**3, x) + Integral(sqrt(c + d*sin(e + f*x)), x))
```

3.499 $\int \frac{(a+a\sin(e+fx))^3}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=258

$$-\frac{4a^3(c-d)\left(4c^2-11cd+15d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^3f\sqrt{c+d\sin(e+fx)}}+\frac{4a^3\left(4c^2-15cd+27d^2\right)\sqrt{c+d\sin(e+fx}}{15d^3f\sqrt{\frac{c+d\sin(e+fx}{c+d}}}$$

```
[Out] 8/15*a^3*(c-3*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d^2/f-2/5*cos(f*x+e)*(a^
3+a^3*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/d/f-4/15*a^3*(4*c^2-15*c*d+27*d^2)
*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(co
s(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^3
/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+4/15*a^3*(c-d)*(4*c^2-11*c*d+15*d^2)*(sin
(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2
*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/
d^3/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2763, 2968, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4a^3(c-d)\left(4c^2-11cd+15d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^3f\sqrt{c+d\sin(e+fx)}}+\frac{4a^3\left(4c^2-15cd+27d^2\right)\sqrt{c+d\sin(e+fx}}{15d^3f\sqrt{\frac{c+d\sin(e+fx}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (8*a^3*(c - 3*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*d^2*f) - (2*Cos
[e + f*x]*(a^3 + a^3*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/(5*d*f) + (4*a
^3*(4*c^2 - 15*c*d + 27*d^2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*S
qrt[c + d*Sin[e + f*x]])/(15*d^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (4
*a^3*(c - d)*(4*c^2 - 11*c*d + 15*d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/
(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(15*d^3*f*Sqrt[c + d*Sin[e + f
*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{2\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)\sqrt{c+d\sin(e+fx)}}{5df} + \frac{2\int \frac{(a+a\sin(e+fx))\left(a^2(c+3d)-\dots\right.}{\sqrt{c+d\sin(e\dots}}}{5d}$$

$$= -\frac{2\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)\sqrt{c+d\sin(e+fx)}}{5df} + \frac{2\int \frac{a^3(c+3d)+\left(-2a^3(c-3d)+a^3(\dots\right.}{\sqrt{\dots}}}{\dots}$$

$$= \frac{8a^3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15d^2f} - \frac{2\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)\sqrt{c\dots}}{5df}$$

$$= \frac{8a^3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15d^2f} - \frac{2\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)\sqrt{c\dots}}{5df}$$

$$= \frac{8a^3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15d^2f} - \frac{2\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)\sqrt{c\dots}}{5df}$$

$$= \frac{8a^3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15d^2f} - \frac{2\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)\sqrt{c\dots}}{5df}$$

Mathematica [A] time = 1.62, size = 246, normalized size = 0.95

$$-\frac{a^3(\sin(e+fx)+1)^3\left(-d\cos(e+fx)\left(8c^2+2d(c-15d)\sin(e+fx)-30cd+3d^2\cos(2(e+fx))-3d^2\right)-4\left(4c^3\dots\right.\right.}{15d^3f\left(\sin\left(\frac{1}{2}(e+\dots\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] -1/15*(a^3*(1 + Sin[e + f*x])^3*(4*(4*c^3 - 11*c^2*d + 12*c*d^2 + 27*d^3)*E
llipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c
 + d)] - 4*(4*c^3 - 15*c^2*d + 26*c*d^2 - 15*d^3)*EllipticF[(-2*e + Pi - 2*
f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - d*Cos[e + f*x]*
(8*c^2 - 30*c*d - 3*d^2 + 3*d^2*Cos[2*(e + f*x)] + 2*(c - 15*d)*d*Sin[e + f
*x])))/(d^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*Sqrt[c + d*Sin[e + f*
x]])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{3\,a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}\right)\sin\left(fx+e\right)}{\sqrt{d\sin\left(fx+e\right)+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(
f*x + e))/sqrt(d*sin(f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\sin\left(fx+e\right)+a\right)^{3}}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/sqrt(d*sin(f*x + e) + c), x)
```

maple [B] time = 1.57, size = 1035, normalized size = 4.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/15*a^3*(8*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*
(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((
```

```
c-d)/(c+d))^(1/2))*c^3*d-36*c^2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e
)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x
+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^2+112*c*((c+d*sin(f*x+e))/(c-d))^(
1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ellipt
icF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^3-84*((c+d*sin(f*
x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d)
)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^4-8
*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(
f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))
^(1/2))*c^4+30*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/
2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2)
,((c-d)/(c+d))^(1/2))*c^3*d-46*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)
-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+
e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^2-30*((c+d*sin(f*x+e))/(c-d))^(
1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ellipt
icE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^3+54*((c+d*sin(
f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-
d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^4
+3*d^4*sin(f*x+e)^4-c*d^3*sin(f*x+e)^3+15*d^4*sin(f*x+e)^3-4*c^2*d^2*sin(f*
x+e)^2+15*c*d^3*sin(f*x+e)^2-3*d^4*sin(f*x+e)^2+c*d^3*sin(f*x+e)-15*d^4*sin
(f*x+e)+4*c^2*d^2-15*c*d^3)/d^4/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^3}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^3/sqrt(d*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^3}{\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c + d*sin(e + f*x))^(1/2),x)
[Out] int((a + a*sin(e + f*x))^3/(c + d*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int \frac{3\sin\left(e+fx\right)}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx+\int \frac{3\sin^2\left(e+fx\right)}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx+\int \frac{\sin^3\left(e+fx\right)}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx+\int \frac{1}{\sqrt{c+d\sin\left(e+f\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] a**3*(Integral(3*sin(e + f*x)/sqrt(c + d*sin(e + f*x)), x) + Integral(3*sin
(e + f*x)**2/sqrt(c + d*sin(e + f*x)), x) + Integral(sin(e + f*x)**3/sqrt(c
 + d*sin(e + f*x)), x) + Integral(1/sqrt(c + d*sin(e + f*x)), x))
```

3.500 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=270

$$-\frac{4a^3\left(4c^2-5cd-3d^2\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^3f(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{4a^3(4c-5d)(c-d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx\right.\right.}{3d^3f\sqrt{c+d\sin(e+fx)}}$$

```
[Out] 2*(c-d)*cos(f*x+e)*(a^3+a^3*sin(f*x+e))/d/(c+d)/f/(c+d*sin(f*x+e))^(1/2)-4/
3*a^3*(2*c-d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d^2/(c+d)/f+4/3*a^3*(4*c^2-
5*c*d-3*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*
EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e
))^(1/2)/d^3/(c+d)/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-4/3*a^3*(4*c-5*d)*(c-d)
*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(co
s(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(
1/2)/d^3/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2762, 2968, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4a^3\left(4c^2-5cd-3d^2\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^3f(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{4a^3(2c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3d^2f(c+d)}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*(c - d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(d*(c + d)*f*Sqrt[c + d*S
in[e + f*x]]) - (4*a^3*(2*c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*
d^2*(c + d)*f) - (4*a^3*(4*c^2 - 5*c*d - 3*d^2)*EllipticE[(e - Pi/2 + f*x)/
2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*d^3*(c + d)*f*Sqrt[(c + d*Si
n[e + f*x])/(c + d)]) + (4*a^3*(4*c - 5*d)*(c - d)*EllipticF[(e - Pi/2 + f*
x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*d^3*f*Sqrt[c +
d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{(2a)\int \frac{(a+a\sin(e+fx))(a(c-2d)-a(2c-d)\sin(e+f}{\sqrt{c+d\sin(e+fx)}}}{d(c+d)}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{(2a)\int \frac{a^2(c-2d)+\left(a^2(c-2d)-a^2(2c-d)\right)\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}}{d(c+d)}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{4a^3(2c-d)\cos(e+fx)\sqrt{c+d\sin(e+}}{3d^2(c+d)f}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{4a^3(2c-d)\cos(e+fx)\sqrt{c+d\sin(e+}}{3d^2(c+d)f}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{4a^3(2c-d)\cos(e+fx)\sqrt{c+d\sin(e+}}{3d^2(c+d)f}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{4a^3(2c-d)\cos(e+fx)\sqrt{c+d\sin(e+}}{3d^2(c+d)f}$$

Mathematica [A] time = 1.45, size = 234, normalized size = 0.87

$$-\frac{2a^3(\sin(e+fx)+1)^3\left(d\cos(e+fx)\left(4c^2+d(c+d)\sin(e+fx)-5cd+3d^2\right)+2\left(4c^3-5c^2d-4cd^2+5d^3\right)\sqrt{\frac{c-}{}}\right.}{3d^3f(c+d)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a^3*(1 + Sin[e + f*x])^3*(-2*(4*c^3 - c^2*d - 8*c*d^2 - 3*d^3)*Elliptic
E[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)]
+ 2*(4*c^3 - 5*c^2*d - 4*c*d^2 + 5*d^3)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2
*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + d*Cos[e + f*x]*(4*c^2 - 5
*c*d + 3*d^2 + d*(c + d)*Sin[e + f*x])))/(3*d^3*(c + d)*f*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^6*Sqrt[c + d*Sin[e + f*x]])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(3\,a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}\right)\sin\left(fx+e\right)\right)\sqrt{d\sin\left(fx+e\right)+c}}{d^{2}\cos\left(fx+e\right)^{2}-2\,cd\sin\left(fx+e\right)-c^{2}-d^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f
*x + e))*sqrt(d*sin(f*x + e) + c)/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e)
- c^2 - d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{3}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(3/2), x)
```

maple [B] time = 1.41, size = 1031, normalized size = 3.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] -2/3*(8*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*
(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)
/(c+d))^(1/2))*c^3*d-16*c^2*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)
*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))
```

```
/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^2-8*c*((c+d*sin(f*x+e))/(c-d))^(1/2)*(
-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^3+16*((c+d*sin(f*x+e))/
(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2
)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^4-8*((c+d
*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e)
)/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)
)*c^4+10*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d
*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))*c^3*d+14*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/
(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c
-d))^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^2-10*((c+d*sin(f*x+e))/(c-d))^(1/2)*(
-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*c*d^3-6*((c+d*sin(f*x+e))
/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/
2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^4-c*d^3*
sin(f*x+e)^3-d^4*sin(f*x+e)^3-4*c^2*d^2*sin(f*x+e)^2+5*c*d^3*sin(f*x+e)^2-3
*d^4*sin(f*x+e)^2+c*d^3*sin(f*x+e)+d^4*sin(f*x+e)+4*c^2*d^2-5*c*d^3+3*d^4)*
a^3/d^4/(c+d)/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a \sin\left(fx+e\right)+a\right)^3}{\left(d \sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^3}{\left(c+d\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c + d*sin(e + f*x))^(3/2),x)
[Out] int((a + a*sin(e + f*x))^3/(c + d*sin(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.501 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=280

$$\frac{4a^3\left(4c^2+5cd-3d^2\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^3f(c+d)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{4a^3(c-d)(4c+5d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\right.\right.}{3d^3f(c+d)\sqrt{c+d\sin(e+fx)}}$$

```
[Out] 2/3*(c-d)*cos(f*x+e)*(a^3+a^3*sin(f*x+e))/d/(c+d)/f/(c+d*sin(f*x+e))^(3/2)+
8/3*a^3*(c-d)*(c+2*d)*cos(f*x+e)/d^2/(c+d)^2/f/(c+d*sin(f*x+e))^(1/2)-4/3*a
^3*(4*c^2+5*c*d-3*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi
+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d
*sin(f*x+e))^(1/2)/d^3/(c+d)^2/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+4/3*a^3*(c-
d)*(4*c+5*d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*
EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+
e))/(c+d))^(1/2)/d^3/(c+d)/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2762, 2968, 3021, 2752, 2663, 2661, 2655, 2653}

$$\frac{4a^3\left(4c^2+5cd-3d^2\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^3f(c+d)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{8a^3(c-d)(c+2d)\cos(e+fx)}{3d^2f(c+d)^2\sqrt{c+d\sin(e+fx)}}-\frac{4a^3(c-d)(4c}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*(c - d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(3*d*(c + d)*f*(c + d*Sin
[e + f*x])^(3/2)) + (8*a^3*(c - d)*(c + 2*d)*Cos[e + f*x])/(3*d^2*(c + d)^2
*f*Sqrt[c + d*Sin[e + f*x]]) + (4*a^3*(4*c^2 + 5*c*d - 3*d^2)*EllipticE[(e
- Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*d^3*(c + d)^2*
f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (4*a^3*(c - d)*(4*c + 5*d)*Elliptic
F[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3
*d^3*(c + d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{(2a)\int \frac{(a+a\sin(e+fx))(a(c-4d)-a(2c+d)\sin(e+f}{(c+d\sin(e+fx))^{3/2}}}{3d(c+d)}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{(2a)\int \frac{a^2(c-4d)+\left(a^2(c-4d)-a^2(2c+d)\right)\sin(e+fx)}{(c+d\sin(e+fx))^{3/2}}}{3d(c+d)}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} + \frac{8a^3(c-d)(c+2d)\cos(e+fx)}{3d^2(c+d)^2f\sqrt{c+d\sin(e+fx)}} + \frac{(4a}{}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} + \frac{8a^3(c-d)(c+2d)\cos(e+fx)}{3d^2(c+d)^2f\sqrt{c+d\sin(e+fx)}} - \frac{(2a}{}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} + \frac{8a^3(c-d)(c+2d)\cos(e+fx)}{3d^2(c+d)^2f\sqrt{c+d\sin(e+fx)}} + \frac{(2a}{}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} + \frac{8a^3(c-d)(c+2d)\cos(e+fx)}{3d^2(c+d)^2f\sqrt{c+d\sin(e+fx)}} + \frac{4a^3}{}$$

Mathematica [A] time = 1.55, size = 232, normalized size = 0.83

$$-\frac{2a^3(\sin(e+fx)+1)^3\left(d(d-c)\cos(e+fx)\left(4c^2+d(5c+9d)\sin(e+fx)+9cd+d^2\right)+2(c+d)\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{3/2}}{3d^3f(c+d)^2\left(\sin\left(\frac{1}{2}(e+fx)\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*a^3*(1 + Sin[e + f*x])^3*(2*(c + d)*(d^2*(c + 5*d)*EllipticF[(-2*e + Pi
 - 2*f*x)/4, (2*d)/(c + d)] + (4*c^2 + 5*c*d - 3*d^2)*((c + d)*EllipticE[(-
2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2
*d)/(c + d)]))*((c + d*Sin[e + f*x])/(c + d))^(3/2) + d*(-c + d)*Cos[e + f*
x]*(4*c^2 + 9*c*d + d^2 + d*(5*c + 9*d)*Sin[e + f*x])))/(3*d^3*(c + d)^2*f*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c + d*Sin[e + f*x])^(3/2))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)\right)\sqrt{d\sin\left(fx+e\right)+c}}{3\,cd^2\cos\left(fx+e\right)^2-c^3-3\,cd^2+\left(d^3\cos\left(fx+e\right)^2-3\,c^2d-d^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f
*x + e))*sqrt(d*sin(f*x + e) + c)/(3*c*d^2*cos(f*x + e)^2 - c^3 - 3*c*d^2 +
 (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^3}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(5/2), x)
```

maple [B] time = 5.41, size = 1257, normalized size = 4.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*a^3*(-2/d^4/(cos(f*x+e)^2*sin(f*x+e
)*d+c*cos(f*x+e)^2)^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*(-d/(c+d)*sin
(f*x+e)+d/(c+d))^(1/2)*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(EllipticE((d/(
c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^2-EllipticE((d/(c-d
```

```
)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*d^2+2*EllipticF((d/(c-d)
*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^2-6*EllipticF((d/(c-d)*
sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c*d+4*EllipticF((d/(c-d)*s
in(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*d^2)+3/d^3*(c^2-2*c*d+d^2)*
(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^
2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2
)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*E
llipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*
(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-si
n(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1
)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF((
(c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+1/d^3*(-c^3+3*c^2*d-3*
c*d^2+d^3)*(2/3/(c^2-d^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*
x+e)+c/d)^2+8/3*d*cos(f*x+e)^2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)
^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*c^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(
c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2))+8/3*c*d/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e)
)/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^3}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^3}{\left(c+d\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c + d*sin(e + f*x))^(5/2),x)
[Out] int((a + a*sin(e + f*x))^3/(c + d*sin(e + f*x))^(5/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**(5/2),x)
[Out] Timed out
```

3.502 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=336

$$-\frac{4a^3\left(4c^2+15cd+27d^2\right)\cos(e+fx)}{15d^2f(c+d)^3\sqrt{c+d\sin(e+fx)}}+\frac{4a^3\left(4c^2+11cd+15d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^3f(c+d)^2\sqrt{c+d\sin(e+fx)}}-\frac{4a^3\left(4c^2+\right.}{}$$

```
[Out] 2/5*(c-d)*cos(f*x+e)*(a^3+a^3*sin(f*x+e))/d/(c+d)/f/(c+d*sin(f*x+e))^(5/2)+
8/15*a^3*(c-d)*(c+3*d)*cos(f*x+e)/d^2/(c+d)^2/f/(c+d*sin(f*x+e))^(3/2)-4/15
*a^3*(4*c^2+15*c*d+27*d^2)*cos(f*x+e)/d^2/(c+d)^3/f/(c+d*sin(f*x+e))^(1/2)+
4/15*a^3*(4*c^2+15*c*d+27*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*
e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/
2))*(c+d*sin(f*x+e))^(1/2)/d^3/(c+d)^3/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-4/1
5*a^3*(4*c^2+11*c*d+15*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1
/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))
*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^3/(c+d)^2/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.72, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2762, 2968, 3021, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4a^3\left(4c^2+15cd+27d^2\right)\cos(e+fx)}{15d^2f(c+d)^3\sqrt{c+d\sin(e+fx)}}+\frac{4a^3\left(4c^2+11cd+15d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^3f(c+d)^2\sqrt{c+d\sin(e+fx)}}-\frac{4a^3\left(4c^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (2*(c - d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(5*d*(c + d)*f*(c + d*Sin
[e + f*x])^(5/2)) + (8*a^3*(c - d)*(c + 3*d)*Cos[e + f*x])/(15*d^2*(c + d)^
2*f*(c + d*Sin[e + f*x])^(3/2)) - (4*a^3*(4*c^2 + 15*c*d + 27*d^2)*Cos[e +
f*x])/(15*d^2*(c + d)^3*f*Sqrt[c + d*Sin[e + f*x]]) - (4*a^3*(4*c^2 + 15*c*
d + 27*d^2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e +
 f*x]])/(15*d^3*(c + d)^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^3*(4
*c^2 + 11*c*d + 15*d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(
c + d*Sin[e + f*x])/(c + d)])/(15*d^3*(c + d)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
```

```
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^{7/2}}\,dx = \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{(2a)\int \frac{(a+a\sin(e+fx))(a(c-6d)-a(2c+3d)\sin(e}{(c+d\sin(e+fx))^{5/2}}}{5d(c+d)}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{(2a)\int \frac{a^2(c-6d)+\left(a^2(c-6d)-a^2(2c+3d)\right)\sin(e+}{(c+d\sin(e+fx))}}{5d(c+d)}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} + \frac{8a^3(c-d)(c+3d)\cos(e+fx)}{15d^2(c+d)^2f(c+d\sin(e+fx))^{3/2}} +$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} + \frac{8a^3(c-d)(c+3d)\cos(e+fx)}{15d^2(c+d)^2f(c+d\sin(e+fx))^{3/2}} -$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} + \frac{8a^3(c-d)(c+3d)\cos(e+fx)}{15d^2(c+d)^2f(c+d\sin(e+fx))^{3/2}} -$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} + \frac{8a^3(c-d)(c+3d)\cos(e+fx)}{15d^2(c+d)^2f(c+d\sin(e+fx))^{3/2}} -$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} + \frac{8a^3(c-d)(c+3d)\cos(e+fx)}{15d^2(c+d)^2f(c+d\sin(e+fx))^{3/2}} -$$

Mathematica [A] time = 2.12, size = 298, normalized size = 0.89

$$-\frac{2a^3(\sin(e+fx)+1)^3\Big(d\cos(e+fx)\left(4c^4+15c^3d+2d^2\left(4c^2+15cd+27d^2\right)\sin^2(e+fx)+55c^2d^2+d\left(9c^3+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-2*a^3*(1 + Sin[e + f*x])^3*(-2*((c - 15*d)*d^2*EllipticF[(-2*e + Pi - 2*f
*x)/4, (2*d)/(c + d)] + (4*c^2 + 15*c*d + 27*d^2)*((c + d)*EllipticE[(-2*e
+ Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/
(c + d)]))*(c + d*Sin[e + f*x])^2*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + d*Co
s[e + f*x]*(4*c^4 + 15*c^3*d + 55*c^2*d^2 + 15*c*d^3 + 3*d^4 + d*(9*c^3 + 4
```

```
5*c^2*d + 115*c*d^2 + 15*d^3)*Sin[e + f*x] + 2*d^2*(4*c^2 + 15*c*d + 27*d^2
)*Sin[e + f*x]^2)))/(15*d^3*(c + d)^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^6*(c + d*Sin[e + f*x])^(5/2))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(3\,a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}\right)\sin\left(fx+e\right)\right)\sqrt{d\sin\left(fx+e\right)+c}}{d^{4}\cos\left(fx+e\right)^{4}+c^{4}+6\,c^{2}d^{2}+d^{4}-2\left(3\,c^{2}d^{2}+d^{4}\right)\cos\left(fx+e\right)^{2}-4\left(cd^{3}\cos\left(fx+e\right)^{2}-c^{3}d-cd^{3}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(
f*x + e))*sqrt(d*sin(f*x + e) + c)/(d^4*cos(f*x + e)^4 + c^4 + 6*c^2*d^2 +
d^4 - 2*(3*c^2*d^2 + d^4)*cos(f*x + e)^2 - 4*(c*d^3*cos(f*x + e)^2 - c^3*d
- c*d^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{3}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(7/2), x)
```

maple [B] time = 6.60, size = 1589, normalized size = 4.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(7/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*a^3*(2/d^3*(c/d-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))+3*(-c+d)/d^3*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-
d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))
/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2
)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d)
```

```
)^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))
^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c
-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((
c-d)/(c+d))^(1/2))))+3/d^3*(c^2-2*c*d+d^2)*(2/3/(c^2-d^2)/d*(-(-d*sin(f*x+e
)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/3*d*cos(f*x+e)^2/(c^2-d^2)^2*
c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*c^2*d^2+3*
d^4)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*
((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Ell
ipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+8/3*c*d/(c^2-d^2
)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*(
(-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c
/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+Ellipti
cF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+1/d^3*(-c^3+3*c^2*
d-3*c*d^2+d^3)*(2/5/(c^2-d^2)/d^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(
sin(f*x+e)+c/d)^3+16/15*c/(c^2-d^2)^2/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(
1/2)/(sin(f*x+e)+c/d)^2+2/15*d*cos(f*x+e)^2/(c^2-d^2)^3*(23*c^2+9*d^2)/(-(-
d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(15*c^3+17*c*d^2)/(15*c^6-45*c^4*d^2+
45*c^2*d^4-15*d^6)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))
/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e
)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/
15*d*(23*c^2+9*d^2)/(c^2-d^2)^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(
1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)
-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))
^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^3}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^3}{\left(c+d\,\sin\left(e+f\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c + d*sin(e + f*x))^(7/2),x)
[Out] int((a + a*sin(e + f*x))^3/(c + d*sin(e + f*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**(7/2),x)
[Out] Timed out
```

3.503 $\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=419

$$-\frac{4a^3\left(4c^2+21cd+65d^2\right)\cos(e+fx)}{105d^2f(c+d)^3(c+d\sin(e+fx))^{3/2}}+\frac{4a^3\left(4c^2+21cd+65d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{105d^3f(c+d)^3\sqrt{c+d\sin(e+fx)}}-\frac{4a^3\left(4c^3\right.}{105d^2}$$

```
[Out] 2/7*(c-d)*cos(f*x+e)*(a^3+a^3*sin(f*x+e))/d/(c+d)/f/(c+d*sin(f*x+e))^(7/2)+
8/35*a^3*(c-d)*(c+4*d)*cos(f*x+e)/d^2/(c+d)^2/f/(c+d*sin(f*x+e))^(5/2)-4/10
5*a^3*(4*c^2+21*c*d+65*d^2)*cos(f*x+e)/d^2/(c+d)^3/f/(c+d*sin(f*x+e))^(3/2)
-4/105*a^3*(4*c^3+21*c^2*d+62*c*d^2-147*d^3)*cos(f*x+e)/(c-d)/d^2/(c+d)^4/f
/(c+d*sin(f*x+e))^(1/2)+4/105*a^3*(4*c^3+21*c^2*d+62*c*d^2-147*d^3)*(sin(1/
2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+
1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/(c-d)/d^3/(
c+d)^4/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-4/105*a^3*(4*c^2+21*c*d+65*d^2)*(si
n(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/
2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)
/d^3/(c+d)^3/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.93, antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2762, 2968, 3021, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4a^3\left(21c^2d+4c^3+62cd^2-147d^3\right)\cos(e+fx)}{105d^2f(c-d)(c+d)^4\sqrt{c+d\sin(e+fx)}}-\frac{4a^3\left(4c^2+21cd+65d^2\right)\cos(e+fx)}{105d^2f(c+d)^3(c+d\sin(e+fx))^{3/2}}+\frac{4a^3\left(4c^2+21cd+65d^2\right.}{105d^3f(c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(9/2),x]
```

```
[Out] (2*(c - d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(7*d*(c + d)*f*(c + d*Sin
[e + f*x])^(7/2)) + (8*a^3*(c - d)*(c + 4*d)*Cos[e + f*x])/(35*d^2*(c + d)^
2*f*(c + d*Sin[e + f*x])^(5/2)) - (4*a^3*(4*c^2 + 21*c*d + 65*d^2)*Cos[e +
f*x])/(105*d^2*(c + d)^3*f*(c + d*Sin[e + f*x])^(3/2)) - (4*a^3*(4*c^3 + 21
*c^2*d + 62*c*d^2 - 147*d^3)*Cos[e + f*x])/(105*(c - d)*d^2*(c + d)^4*f*Sqr
t[c + d*Sin[e + f*x]]) - (4*a^3*(4*c^3 + 21*c^2*d + 62*c*d^2 - 147*d^3)*Ell
ipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(105*(c
 - d)*d^3*(c + d)^4*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*a^3*(4*c^2 +
 21*c*d + 65*d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*
Sin[e + f*x])/(c + d)])/(105*d^3*(c + d)^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
```

```
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3}{(c+d\sin(e+fx))^{9/2}}\,dx = \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} - \frac{(2a)\int \frac{(a+a\sin(e+fx))(a(c-8d)-a(2c+5d)\sin(e+}{(c+d\sin(e+fx))^{7/2}}}{7d(c+d)}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} - \frac{(2a)\int \frac{a^2(c-8d)+\left(a^2(c-8d)-a^2(2c+5d)\right)\sin(e+f}{(c+d\sin(e+fx))^{7/2}}}{7d(c+d)}$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} + \frac{8a^3(c-d)(c+4d)\cos(e+fx)}{35d^2(c+d)^2f(c+d\sin(e+fx))^{5/2}} +$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} + \frac{8a^3(c-d)(c+4d)\cos(e+fx)}{35d^2(c+d)^2f(c+d\sin(e+fx))^{5/2}} -$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} + \frac{8a^3(c-d)(c+4d)\cos(e+fx)}{35d^2(c+d)^2f(c+d\sin(e+fx))^{5/2}} -$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} + \frac{8a^3(c-d)(c+4d)\cos(e+fx)}{35d^2(c+d)^2f(c+d\sin(e+fx))^{5/2}} -$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} + \frac{8a^3(c-d)(c+4d)\cos(e+fx)}{35d^2(c+d)^2f(c+d\sin(e+fx))^{5/2}} -$$

$$= \frac{2(c-d)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} + \frac{8a^3(c-d)(c+4d)\cos(e+fx)}{35d^2(c+d)^2f(c+d\sin(e+fx))^{5/2}} -$$

Mathematica [A] time = 3.70, size = 351, normalized size = 0.84

$$-\frac{2a^3(\sin(e+fx)+1)^3\Big(d\cos(e+fx)\left(2(c-d)\left(4c^2+21cd+65d^2\right)(c+d)(c+d\sin(e+fx))^2+2\left(4c^3+21c^2d+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(9/2),x]
```

```
[Out] (-2*a^3*(1 + Sin[e + f*x])^3*(-2*(d^2*(c^2 - 126*c*d + 65*d^2)*EllipticF[(-
2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] + (4*c^3 + 21*c^2*d + 62*c*d^2 - 147*d^
```

```
3)*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(
-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]))*(c + d*Sin[e + f*x])^3*Sqrt[(c + d*S
in[e + f*x])/(c + d)] + d*Cos[e + f*x]*(15*(c - d)^3*(c + d)^3 - 9*(c - d)^
2*(c + d)^2*(3*c + 7*d)*(c + d*Sin[e + f*x]) + 2*(c - d)*(c + d)*(4*c^2 + 2
1*c*d + 65*d^2)*(c + d*Sin[e + f*x])^2 + 2*(4*c^3 + 21*c^2*d + 62*c*d^2 - 1
47*d^3)*(c + d*Sin[e + f*x])^3)))/(105*(c - d)*d^3*(c + d)^4*f*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])^6*(c + d*Sin[e + f*x])^(7/2))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+\right.\right.}{5\,cd^4\cos\left(fx+e\right)^4+c^5+10\,c^3d^2+5\,cd^4-10\left(c^3d^2+cd^4\right)\cos\left(fx+e\right)^2+\left(d^5\cos\left(fx+e\right)^4+5\,c^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(
f*x + e))*sqrt(d*sin(f*x + e) + c)/(5*c*d^4*cos(f*x + e)^4 + c^5 + 10*c^3*d
^2 + 5*c*d^4 - 10*(c^3*d^2 + c*d^4)*cos(f*x + e)^2 + (d^5*cos(f*x + e)^4 +
5*c^4*d + 10*c^2*d^3 + d^5 - 2*(5*c^2*d^3 + d^5)*cos(f*x + e)^2)*sin(f*x +
e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^3}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(9/2), x)
```

maple [B] time = 9.86, size = 2079, normalized size = 4.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(9/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*a^3*((-c^3+3*c^2*d-3*c*d^2+d^3)/d^3
*(2/7/(c^2-d^2)/d^3*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d
```

```
)^4+24/35/(c^2-d^2)^2/d^2*c*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*
x+e)+c/d)^3+2/105*(71*c^2+25*d^2)/d/(c^2-d^2)^3*(-(-d*sin(f*x+e)-c)*cos(f*x
+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+32/105*d*cos(f*x+e)^2/(c^2-d^2)^4*c*(11*c^2
+13*d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(105*c^4+254*c^2*d^2+25*
d^4)/(105*c^8-420*c^6*d^2+630*c^4*d^4-420*c^2*d^6+105*d^8)*(c/d-1)*((c+d*si
n(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c
-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+
e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+32/105*c*d*(11*c^2+13*d^2)/(c^2-d^2)^
4*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-
sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d
-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF
(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+1/d^3*(2*d*cos(f*x+e
)^2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)
*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e
)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d
*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*
sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/
(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+3*(-c+d)/d^3*(2/3/(c^2-d^2)/d*(-(-d*
sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/3*d*cos(f*x+e)^2/(c^
2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*c
^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d
))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^
(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+8/3*c*d
/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d)
)^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(
1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)
)+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+3*(c^2-2*
c*d+d^2)/d^3*(2/5/(c^2-d^2)/d^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(si
n(f*x+e)+c/d)^3+16/15*c/(c^2-d^2)^2/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/
2)/(sin(f*x+e)+c/d)^2+2/15*d*cos(f*x+e)^2/(c^2-d^2)^3*(23*c^2+9*d^2)/(-(-d*
sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(15*c^3+17*c*d^2)/(15*c^6-45*c^4*d^2+45
*c^2*d^4-15*d^6)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(
c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^
2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/15
*d*(23*c^2+9*d^2)/(c^2-d^2)^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-
sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c
)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((
c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(
1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{3}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(9/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^3}{\left(c+d\,\sin\left(e+f\,x\right)\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3/(c + d*sin(e + f*x))^(9/2),x)
[Out] int((a + a*sin(e + f*x))^3/(c + d*sin(e + f*x))^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**(9/2),x)
[Out] Timed out
```

3.504 $\int \frac{(c+d\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=246

$$\frac{(3c-5d)\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3af\sqrt{c+d\sin(e+fx)}}-\frac{\left(3c^2-20cd+9d^2\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\right.}{3af\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] -(c-d)*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))+1/3*(3*c-5*d)*d
*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/a/f+1/3*(3*c^2-20*c*d+9*d^2)*(sin(1/2*e+
1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*
Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/a/f/((c+d*sin(f
*x+e))/(c+d))^(1/2)-1/3*(3*c-5*d)*(c^2-d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(
1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*
(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/a/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2767, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{(3c-5d)\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3af\sqrt{c+d\sin(e+fx)}}-\frac{\left(3c^2-20cd+9d^2\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\right.}{3af\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((3*c - 5*d)*d*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*a*f) - ((c - d)*Co
s[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(f*(a + a*Sin[e + f*x])) - ((3*c^2 -
 20*c*d + 9*d^2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Si
n[e + f*x]])/(3*a*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((3*c - 5*d)*(c^2
 - d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*
x])/(c + d)])/(3*a*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{f(a+a\sin(e+fx))} - \frac{d\int\left(-\frac{1}{2}a(5c-3d)+\frac{1}{2}a(3c-5d)\sin(e\right.}{a^2}$$

$$= \frac{(3c-5d)d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3af} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3}}{f(a+a\sin(e+fx))}$$

$$= \frac{(3c-5d)d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3af} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3}}{f(a+a\sin(e+fx))}$$

$$= \frac{(3c-5d)d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3af} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3}}{f(a+a\sin(e+fx))}$$

$$= \frac{(3c-5d)d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3af} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3}}{f(a+a\sin(e+fx))}$$

Mathematica [A] time = 1.38, size = 298, normalized size = 1.21

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(-d\left(15c^2-12cd+5d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)+\left(3c^2-20\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*(-3*(c - d)^2*(c + d*Sin[e + f*x])
 - 2*d^2*Cos[e + f*x]*(c + d*Sin[e + f*x]) + (6*(c - d)^2*Sin[(e + f*x)/2]*
(c + d*Sin[e + f*x]))/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - d*(15*c^2 - 1
2*c*d + 5*d^2)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*
Sin[e + f*x])/(c + d)] + (3*c^2 - 20*c*d + 9*d^2)*((c + d)*EllipticE[(-2*e
+ Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/
(c + d)])*Sqrt[(c + d*Sin[e + f*x])/(c + d)]))/(3*a*f*(1 + Sin[e + f*x])*Sq
rt[c + d*Sin[e + f*x]])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2\right)\sqrt{d\sin\left(fx+e\right)+c}}{a\sin\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(-(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)*sqrt(d*sin(
f*x + e) + c)/(a*sin(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a), x)
```

maple [B] time = 1.65, size = 1372, normalized size = 5.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] 1/3*(cos(f*x+e)^2*sin(f*x+e)*d+c*cos(f*x+e)^2)^(1/2)*(12*(d/(c-d)*sin(f*x+e
)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)
-d/(c-d))^(1/2)*EllipticF((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d)
)^(1/2))*c^3*d-4*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+
d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticF((d/(c-d)*sin(f
*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^2-12*(d/(c-d)*sin(f*x+e)+
1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d
/(c-d))^(1/2)*EllipticF((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^
(1/2))*c*d^3+4*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/
(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticF((d/(c-d)*sin(f*x
+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*d^4+3*(d/(c-d)*sin(f*x+e)+1/(c-d)
*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))
^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*
```

```
c^4-20*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(
1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c
-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^3*d+6*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1
/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)
*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^2*d^
2+20*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/
2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d
)*c)^(1/2),((c-d)/(c+d))^(1/2))*c*d^3-9*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2
)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*E
llipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*d^4-2*d^
4*sin(f*x+e)*cos(f*x+e)^2-3*c^2*cos(f*x+e)^2*d^2+4*cos(f*x+e)^2*c*d^3-3*cos
(f*x+e)^2*d^4+3*c^3*d*sin(f*x+e)-9*c^2*d^2*sin(f*x+e)+9*c*d^3*sin(f*x+e)-3*
d^4*sin(f*x+e)-3*c^3*d+9*c^2*d^2-9*c*d^3+3*d^4)/d/(-(c+d*sin(f*x+e))*(sin(f
*x+e)-1)*(1+sin(f*x+e)))^(1/2)/a/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c+d\,\sin\left(e+f\,x\right)\right)^{5/2}}{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x)),x)
```

```
[Out] int((c + d*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^{2}\sqrt{c+d\sin\left(e+fx\right)}}{\sin\left(e+fx\right)+1}\,dx+\int \frac{d^{2}\sqrt{c+d\sin\left(e+fx\right)}\,\sin^{2}\left(e+fx\right)}{\sin\left(e+fx\right)+1}\,dx+\int \frac{2cd\sqrt{c+d\sin\left(e+fx\right)}\,\sin\left(e+fx\right)}{\sin\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] (Integral(c**2*sqrt(c + d*sin(e + f*x))/(sin(e + f*x) + 1), x) + Integral(d
**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2/(sin(e + f*x) + 1), x) + Integ
ral(2*c*d*sqrt(c + d*sin(e + f*x))*sin(e + f*x)/(sin(e + f*x) + 1), x))/a
```

3.505 $\int \frac{(c+d\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=186

$$\frac{(c^2-d^2)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{af\sqrt{c+d\sin(e+fx)}}-\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a\sin(e+fx)+a)}-\frac{(c-3d)\sqrt{c+d\sin(e+fx}}{af\sqrt{\frac{c+d}{}}}$$

```
[Out] -(c-d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))+(c-3*d)*(sin(1/
2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+
1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/a/f/((c+d*s
in(f*x+e))/(c+d))^(1/2)-(c^2-d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1
/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^
(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/a/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2767, 2752, 2663, 2661, 2655, 2653}

$$\frac{(c^2-d^2)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{af\sqrt{c+d\sin(e+fx)}}-\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a\sin(e+fx)+a)}-\frac{(c-3d)\sqrt{c+d\sin(e+fx}}{af\sqrt{\frac{c+d}{}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] -(((c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(f*(a + a*Sin[e + f*x])))
 - ((c - 3*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e
 + f*x]])/(a*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((c^2 - d^2)*EllipticF
[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(a*
f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx = -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a+a\sin(e+fx))} - \frac{d\int \frac{-\frac{1}{2}a(3c-d)+\frac{1}{2}a(c-3d)\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx}{a^2}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a+a\sin(e+fx))} - \frac{(c-3d)\int \sqrt{c+d\sin(e+fx)}\,dx}{2a} + \frac{(c^2 -}{}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a+a\sin(e+fx))} - \frac{\left((c-3d)\sqrt{c+d\sin(e+fx)}\right)\int \sqrt{\frac{c}{c+d}+}}{2a\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a+a\sin(e+fx))} - \frac{(c-3d)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\,\mathrm{si}}}{af\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Mathematica [A] time = 1.53, size = 223, normalized size = 1.20

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2(c-d)\sin\left(\frac{1}{2}(e+fx)\right)(c+d\sin(e+fx))-\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+\right.\right.\right.}{af(\mathrm{s}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(c - d)*Sin[(e + f*x)/2]*(c + d*S
in[e + f*x]) - (Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-((c^2 - 2*c*d - 3*d^
2)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x]
)/(c + d)]) + (c - d)*(c + d*Sin[e + f*x] + (c + d)*EllipticF[(-2*e + Pi -
2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)]))))/(a*f*(1 + S
in[e + f*x])*Sqrt[c + d*Sin[e + f*x]])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{a\sin\left(fx+e\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algorithm="giac")
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a), x)
```

maple [B] time = 1.60, size = 925, normalized size = 4.97

$$\frac{\sqrt{\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)d+c\left(\cos^2\left(fx+e\right)\right)}\left(2\sqrt{\frac{d\sin\left(fx+e\right)}{c-d}+\frac{c}{c-d}}\sqrt{-\frac{d\sin\left(fx+e\right)}{c+d}+\frac{d}{c+d}}\sqrt{-\frac{d\sin\left(fx+e\right)}{c-d}-\frac{}{c}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x)
[Out] (cos(f*x+e)^2*sin(f*x+e)*d+c*cos(f*x+e)^2)^(1/2)*(2*(d/(c-d)*sin(f*x+e)+1/(
c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c
-d))^(1/2)*EllipticF((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/
2))*c^2*d-2*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+
d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticF((d/(c-d)*sin(f*x+e)
+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*d^3+(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(
1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2
)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^3-3
*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(
-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)
^(1/2),((c-d)/(c+d))^(1/2))*c^2*d-(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/
(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*Ellipti
cE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c*d^2+3*(d/(c-
d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d
)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),
((c-d)/(c+d))^(1/2))*d^3-c*cos(f*x+e)^2*d^2+cos(f*x+e)^2*d^3+c^2*d*sin(f*x+
e)-2*c*d^2*sin(f*x+e)+d^3*sin(f*x+e)-c^2*d+2*c*d^2-d^3)/d/(-(c+d*sin(f*x+e)
)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/a/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/
f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c+d\,\sin\left(e+f\,x\right)\right)^{3/2}}{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x)),x)
[Out] int((c + d*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c\sqrt{c+d\sin\left(e+fx\right)}}{\sin\left(e+fx\right)+1}\,dx + \int \frac{d\sqrt{c+d\sin\left(e+fx\right)}\sin\left(e+fx\right)}{\sin\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e)),x)
[Out] (Integral(c*sqrt(c + d*sin(e + f*x))/(sin(e + f*x) + 1), x) + Integral(d*sq
rt(c + d*sin(e + f*x))*sin(e + f*x)/(sin(e + f*x) + 1), x))/a
```

3.506 $\int \frac{\sqrt{c+d\sin(e+fx)}}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=170

$$-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a\sin(e+fx)+a)}+\frac{(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{af\sqrt{c+d\sin(e+fx)}}-\frac{\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx\right.\right.}{af\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] -cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))+(sin(1/2*e+1/4*Pi+1/2
*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x
),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/a/f/((c+d*sin(f*x+e))/(c+
d))^(1/2)-(c+d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*
x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f
*x+e))/(c+d))^(1/2)/a/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2769, 2752, 2663, 2661, 2655, 2653}

$$-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a\sin(e+fx)+a)}+\frac{(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{af\sqrt{c+d\sin(e+fx)}}-\frac{\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx\right.\right.}{af\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/(a + a*Sin[e + f*x]),x]
```

```
[Out] -((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(f*(a + a*Sin[e + f*x]))) - (Elli
pticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(a*f*Sqr
t[(c + d*Sin[e + f*x])/(c + d)]) + ((c + d)*EllipticF[(e - Pi/2 + f*x)/2, (
2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(a*f*Sqrt[c + d*Sin[e + f
*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2769

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a
*f*(a + b*Sin[e + f*x])), x] + Dist[(d*n)/(a*b), Int[(c + d*Sin[e + f*x])^(
n - 1)*(a - b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (IntegerQ[2*n
] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{a+a\sin(e+fx)}\,dx = -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a+a\sin(e+fx))} + \frac{d\int \frac{a-a\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx}{2a^2}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a+a\sin(e+fx))} - \frac{\int \sqrt{c+d\sin(e+fx)}\,dx}{2a} + \frac{(c+d)\int \frac{1}{\sqrt{c+d\sin(e+f\cdots}}}{2a}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a+a\sin(e+fx))} - \frac{\sqrt{c+d\sin(e+fx)}\int \sqrt{\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}}\,dx}{2a\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} + \cdots$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(a+a\sin(e+fx))} - \frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{af\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} + \frac{(c\cdots}{\cdots}$$

Mathematica [A] time = 1.12, size = 201, normalized size = 1.18

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2\sin\left(\frac{1}{2}(e+fx)\right)(c+d\sin(e+fx))-\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\cdots\right.}{af(\sin(e+fx)\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*Sin[(e + f*x)/2]*(c + d*Sin[e + f
*x]) - (Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c + d*Sin[e + f*x] - (c + d)*
EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(
c + d)] + (c + d)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c +
 d*Sin[e + f*x])/(c + d)])))/(a*f*(1 + Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x
]])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\sin\left(fx+e\right)+c}}{a\sin\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

[Out] integral(sqrt(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d \sin\left(fx+e\right)+c}}{a \sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algorithm="giac")

[Out] integrate(sqrt(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a), x)

maple [A] time = 1.59, size = 382, normalized size = 2.25

$$\frac{\sqrt{\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)d+c\left(\cos^2\left(fx+e\right)\right)}\left(\sqrt{\frac{d\sin\left(fx+e\right)}{c-d}+\frac{c}{c-d}}\sqrt{-\frac{d\sin\left(fx+e\right)}{c+d}+\frac{d}{c+d}}\sqrt{-\frac{d\sin\left(fx+e\right)}{c-d}-\frac{d}{c-d}}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x)

[Out] (cos(f*x+e)^2*sin(f*x+e)*d+c*cos(f*x+e)^2)^(1/2)*((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^2-(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*d^2-cos(f*x+e)^2*d^2+c*d*sin(f*x+e)-d^2*sin(f*x+e)-c*d+d^2)/d/(-(c+d*sin(f*x+e))*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/a/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d \sin\left(fx+e\right)+c}}{a \sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algorithm="maxima")

[Out] integrate(sqrt(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c+d\,\sin\left(e+f\,x\right)}}{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x)),x)
[Out] int((c + d*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{c+d\sin\left(e+fx\right)}}{\sin\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e)),x)
[Out] Integral(sqrt(c + d*sin(e + f*x))/(sin(e + f*x) + 1), x)/a
```

3.507 $\int \frac{1}{(a+a\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=181

$$-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(c-d)(a\sin(e+fx)+a)}+\frac{\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{af\sqrt{c+d\sin(e+fx)}}-\frac{\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+\cdots}\right)}{af(c-d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] -cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(c-d)/f/(a+a*sin(f*x+e))+(sin(1/2*e+1/4*
Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1
/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/a/(c-d)/f/((c+d*sin
(f*x+e))/(c+d))^(1/2)-(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+
1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d
*sin(f*x+e))/(c+d))^(1/2)/a/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2768, 2752, 2663, 2661, 2655, 2653}

$$-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f(c-d)(a\sin(e+fx)+a)}+\frac{\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{af\sqrt{c+d\sin(e+fx)}}-\frac{\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+\cdots}\right)}{af(c-d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] -((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/((c - d)*f*(a + a*Sin[e + f*x])))
 - (EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/
(a*(c - d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (EllipticF[(e - Pi/2 + f
*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(a*f*Sqrt[c + d*S
in[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(c-d)f(a+a\sin(e+fx))} + \frac{d\int \frac{-\frac{a}{2}-\frac{1}{2}a\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx}{a^2(c-d)}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(c-d)f(a+a\sin(e+fx))} + \frac{\int \frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{2a} - \frac{\int \sqrt{c+d}}{2}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(c-d)f(a+a\sin(e+fx))} - \frac{\sqrt{c+d\sin(e+fx)}\int \sqrt{\frac{c}{c+d}+}}{2a(c-d)\sqrt{\frac{c+d\sin(e+f}{c+d}}}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{(c-d)f(a+a\sin(e+fx))} - \frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d}}{a(c-d)f\sqrt{\frac{c+d\sin(e+f}{c+d}}}$$

Mathematica [A] time = 1.15, size = 210, normalized size = 1.16

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2\sin\left(\frac{1}{2}(e+fx)\right)(c+d\sin(e+fx))-\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\right.}{af(c-d)(\sin(e+fx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*Sin[(e + f*x)/2]*(c + d*Sin[e + f
*x]) - (Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c + d*Sin[e + f*x] - (c + d)*
EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(
c + d)] + (c - d)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c +
 d*Sin[e + f*x])/(c + d)])))/(a*(c - d)*f*(1 + Sin[e + f*x])*Sqrt[c + d*Sin
[e + f*x]])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{d\sin\left(fx+e\right)+c}}{ad\cos\left(fx+e\right)^2-ac-ad-(ac+ad)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*sin(f*x + e) + c)/(a*d*cos(f*x + e)^2 - a*c - a*d - (a*c +
 a*d)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)), x)
```

maple [A] time = 3.47, size = 443, normalized size = 2.45

$$\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^2\left(fx+e\right)\right)}\left(-\frac{-\left(\sin^2\left(fx+e\right)\right)d-c\sin\left(fx+e\right)+d\sin\left(fx+e\right)+c}{\left(c-d\right)\sqrt{\left(-d\sin\left(fx+e\right)-c\right)\left(\sin\left(fx+e\right)-1\right)\left(1+\sin\left(fx+e\right)\right)}}-\frac{2d\left(\frac{c}{d}-1\right)\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}}\ \sqrt{\frac{d}{-}}}{(2c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a*(-(-sin(f*x+e)^2*d-c*sin(f*x+e)+d
*sin(f*x+e)+c)/(c-d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2
)-2*d/(2*c-2*d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c
+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2
)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-d/(c-
d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((
-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/
d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+Elliptic
F(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))/cos(f*x+e)/(c+d*sin
(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+a\,\sin\left(e+f\,x\right)\right)\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{c+d\sin\left(e+fx\right)}\,\sin\left(e+fx\right)+\sqrt{c+d\sin\left(e+fx\right)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c + d*sin(e + f*x))*sin(e + f*x) + sqrt(c + d*sin(e + f*x)
)), x)/a
```

3.508 $\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=244

$$-\frac{d(c+3d)\cos(e+fx)}{af(c-d)^2(c+d)\sqrt{c+d\sin(e+fx)}}-\frac{\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)\sqrt{c+d\sin(e+fx)}}+\frac{\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+\right.\right.}{af(c-d)\sqrt{c+d\sin}}$$

```
[Out] -d*(c+3*d)*cos(f*x+e)/a/(c-d)^2/(c+d)/f/(c+d*sin(f*x+e))^(1/2)-cos(f*x+e)/(
c-d)/f/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2)+(c+3*d)*(sin(1/2*e+1/4*Pi+1/
2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*
x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/a/(c-d)^2/(c+d)/f/((c+d*
sin(f*x+e))/(c+d))^(1/2)-(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*
Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((
c+d*sin(f*x+e))/(c+d))^(1/2)/a/(c-d)/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2768, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{d(c+3d)\cos(e+fx)}{af(c-d)^2(c+d)\sqrt{c+d\sin(e+fx)}}-\frac{\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)\sqrt{c+d\sin(e+fx)}}+\frac{\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+\right.\right.}{af(c-d)\sqrt{c+d\sin}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -((d*(c + 3*d)*Cos[e + f*x])/(a*(c - d)^2*(c + d)*f*Sqrt[c + d*Sin[e + f*x]
])) - Cos[e + f*x]/((c - d)*f*(a + a*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
) - ((c + 3*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[
e + f*x]])/(a*(c - d)^2*(c + d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (El
lipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d
)])/(a*(c - d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))\sqrt{c+d\sin(e+fx)}} + \frac{d\int \frac{-\frac{3a}{2}+\frac{1}{2}a\sin}{(c+d\sin(e}}{a^2(c-}$$

$$= -\frac{d(c+3d)\cos(e+fx)}{a(c-d)^2(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{\cos(}{(c-d)f(a+a\sin(e+}$$

$$= -\frac{d(c+3d)\cos(e+fx)}{a(c-d)^2(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{\cos(}{(c-d)f(a+a\sin(e+}$$

$$= -\frac{d(c+3d)\cos(e+fx)}{a(c-d)^2(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{\cos(}{(c-d)f(a+a\sin(e+}$$

$$= -\frac{d(c+3d)\cos(e+fx)}{a(c-d)^2(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{\cos(}{(c-d)f(a+a\sin(e+}$$

Mathematica [A] time = 2.06, size = 264, normalized size = 1.08

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(-\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)+\left(c^2+4cd+3d^2\right)\sqrt{}\right.}{af(c-d)^2(c+d)(\sin}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*((-2*((c + d)^2*Cos[(e + f*x)/2] +
 d*(2*(c + d) + (c + 3*d)*Cos[e + f*x])*Sin[(e + f*x)/2]))/(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2]) + (c + 3*d)*(c + d*Sin[e + f*x]) + (c^2 + 4*c*d + 3*d
^2)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x
])/(c + d)] - (c^2 - d^2)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*S
qrt[(c + d*Sin[e + f*x])/(c + d)]))/(a*(c - d)^2*(c + d)*f*(1 + Sin[e + f*x
])*Sqrt[c + d*Sin[e + f*x]])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\sin\left(fx+e\right)+c}}{ac^2+2\,acd+ad^2-\left(2\,acd+ad^2\right)\cos\left(fx+e\right)^2-\left(ad^2\cos\left(fx+e\right)^2-ac^2-2\,acd-ad^2\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sin(f*x + e) + c)/(a*c^2 + 2*a*c*d + a*d^2 - (2*a*c*d + a*d
^2)*cos(f*x + e)^2 - (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2)*sin(f
*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2)), x)
```

maple [B] time = 1.97, size = 925, normalized size = 3.79

$$\frac{\sqrt{\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)d+c\left(\cos^2\left(fx+e\right)\right)}\,\left(\sqrt{\frac{d\sin\left(fx+e\right)}{c-d}+\frac{c}{c-d}}\,\sqrt{-\frac{d\sin\left(fx+e\right)}{c+d}+\frac{d}{c+d}}\,\sqrt{-\frac{d\sin\left(fx+e\right)}{c-d}-\frac{d}{c-d}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] (cos(f*x+e)^2*sin(f*x+e)*d+c*cos(f*x+e)^2)^(1/2)*((d/(c-d)*sin(f*x+e)+1/(c-
d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d
))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2)
)*c^3+3*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^
(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(
c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^2*d-(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/
2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*
EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c*d^2-3
*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(
-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticE((d/(c-d)*sin(f*x+e)+1/(c-d)*c)
^(1/2),((c-d)/(c+d))^(1/2))*d^3-4*(d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/
(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d)*sin(f*x+e)-d/(c-d))^(1/2)*Ellipti
cF((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),((c-d)/(c+d))^(1/2))*c^2*d+4*(d/(c-
d)*sin(f*x+e)+1/(c-d)*c)^(1/2)*(-d/(c+d)*sin(f*x+e)+d/(c+d))^(1/2)*(-d/(c-d
)*sin(f*x+e)-d/(c-d))^(1/2)*EllipticF((d/(c-d)*sin(f*x+e)+1/(c-d)*c)^(1/2),
((c-d)/(c+d))^(1/2))*d^3-c*cos(f*x+e)^2*d^2-3*cos(f*x+e)^2*d^3+c^2*d*sin(f*
```

```
x+e)-d^3*sin(f*x+e)-c^2*d+d^3)/d/(-(c+d*sin(f*x+e))*(sin(f*x+e)-1)*(1+sin(f
*x+e)))^(1/2)/(c^2-d^2)/(c-d)/a/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+a\,\sin\left(e+f\,x\right)\right)\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^(3/2)),x)
[Out] int(1/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c\sqrt{c+d\sin\left(e+fx\right)}\sin\left(e+fx\right)+c\sqrt{c+d\sin\left(e+fx\right)}+d\sqrt{c+d\sin\left(e+fx\right)}\sin^{2}\left(e+fx\right)+d\sqrt{c+d\sin\left(e+fx\right)}\sin\left(e+fx\right)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))**(3/2),x)
[Out] Integral(1/(c*sqrt(c + d*sin(e + f*x))*sin(e + f*x) + c*sqrt(c + d*sin(e +
f*x)) + d*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2 + d*sqrt(c + d*sin(e + f
*x))*sin(e + f*x)), x)/a
```

3.509 $\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=333

$$-\frac{d\left(3c^2+20cd+9d^2\right)\cos(e+fx)}{3af(c-d)^3(c+d)^2\sqrt{c+d\sin(e+fx)}}-\frac{\left(3c^2+20cd+9d^2\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3af(c-d)^3(c+d)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{}{3af(c-}$$

```
[Out] -1/3*d*(3*c+5*d)*cos(f*x+e)/a/(c-d)^2/(c+d)/f/(c+d*sin(f*x+e))^(3/2)-cos(f*
x+e)/(c-d)/f/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2)-1/3*d*(3*c^2+20*c*d+9*
d^2)*cos(f*x+e)/a/(c-d)^3/(c+d)^2/f/(c+d*sin(f*x+e))^(1/2)+1/3*(3*c^2+20*c*
d+9*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elli
pticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(
1/2)/a/(c-d)^3/(c+d)^2/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-1/3*(3*c+5*d)*(sin(
1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*
e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/a
/(c-d)^2/(c+d)/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2768, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{d\left(3c^2+20cd+9d^2\right)\cos(e+fx)}{3af(c-d)^3(c+d)^2\sqrt{c+d\sin(e+fx)}}-\frac{\left(3c^2+20cd+9d^2\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3af(c-d)^3(c+d)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{}{3af(c-}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] -(d*(3*c + 5*d)*Cos[e + f*x])/(3*a*(c - d)^2*(c + d)*f*(c + d*Sin[e + f*x])
^(3/2)) - Cos[e + f*x]/((c - d)*f*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])
^(3/2)) - (d*(3*c^2 + 20*c*d + 9*d^2)*Cos[e + f*x])/(3*a*(c - d)^3*(c + d)^
2*f*Sqrt[c + d*Sin[e + f*x]]) - ((3*c^2 + 20*c*d + 9*d^2)*EllipticE[(e - Pi
/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*a*(c - d)^3*(c + d
)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((3*c + 5*d)*EllipticF[(e - Pi/
2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*a*(c - d)
^2*(c + d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
```

```
- d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))^{3/2}} + \frac{d\int \frac{-\frac{5a}{2}+\frac{3}{2}a\,\mathrm{s}}{(c+d\sin(e}}{a^2(c-}$$

$$= -\frac{d(3c+5d)\cos(e+fx)}{3a(c-d)^2(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{\mathrm{co}}{(c-d)f(a+a\sin(e}$$

$$= -\frac{d(3c+5d)\cos(e+fx)}{3a(c-d)^2(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{\mathrm{co}}{(c-d)f(a+a\sin(e}$$

$$= -\frac{d(3c+5d)\cos(e+fx)}{3a(c-d)^2(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{\mathrm{co}}{(c-d)f(a+a\sin(e}$$

$$= -\frac{d(3c+5d)\cos(e+fx)}{3a(c-d)^2(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{\mathrm{co}}{(c-d)f(a+a\sin(e}$$

$$= -\frac{d(3c+5d)\cos(e+fx)}{3a(c-d)^2(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{\mathrm{co}}{(c-d)f(a+a\sin(e}$$

Mathematica [A] time = 4.26, size = 367, normalized size = 1.10

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(2(c+d\sin(e+fx))\left(\frac{3\sin\left(\frac{1}{2}(e+fx)\right)}{\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)}-\frac{\frac{d^2\cos(e+fx)\left(8c^2+d(7c+3d)\sin(e+fx)+3cd-\right.}{(c+d\sin(e+fx))^2}}{(c+d)^2}\right.\right.}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*(((3*c^2 + 20*c*d + 9*d^2)*(c + d*
Sin[e + f*x]) + d*(15*c^2 + 12*c*d + 5*d^2)*EllipticF[(-2*e + Pi - 2*f*x)/4
, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + (3*c^2 + 20*c*d + 9*d
^2)*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[
```

```
(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)])*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/
(c + d)^2 + 2*(c + d*Sin[e + f*x])*((3*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]) - (3*c^2 + 13*c*d + 6*d^2 + (d^2*Cos[e + f*x]*(8*c^2 +
3*c*d - d^2 + d*(7*c + 3*d)*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2)/(c + d)^
2)))/(3*a*(c - d)^3*f*(1 + Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\sin\left(fx+e\right)+c}}{ad^3\cos\left(fx+e\right)^4+ac^3+3\,ac^2d+3\,acd^2+ad^3-\left(3\,ac^2d+3\,acd^2+2\,ad^3\right)\cos\left(fx+e\right)^2+\left(ac^3+3\,a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sin(f*x + e) + c)/(a*d^3*cos(f*x + e)^4 + a*c^3 + 3*a*c^2*d
 + 3*a*c*d^2 + a*d^3 - (3*a*c^2*d + 3*a*c*d^2 + 2*a*d^3)*cos(f*x + e)^2 + (
a*c^3 + 3*a*c^2*d + 3*a*c*d^2 + a*d^3 - (3*a*c*d^2 + a*d^3)*cos(f*x + e)^2)
*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2)), x)
```

maple [B] time = 6.55, size = 1291, normalized size = 3.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a*(-d/(c-d)^2*(2*d*cos(f*x+e)^2/(c^
2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*
sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/
(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*
x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x
+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^
```

```
(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin
(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))))+1/(c-d)^2*(-(-sin(f*x+e)^2*d-c*sin(f*x+e)+d
*sin(f*x+e)+c)/(c-d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2
)-2*d/(2*c-2*d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c
+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2
)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-d/(c-
d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((
-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/
d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+Elliptic
F(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))-d/(c-d)*(2/3/(c^2-d
^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/3*d*cos(
f*x+e)^2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2
)/(3*c^4-6*c^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(
f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*co
s(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1
/2))+8/3*c*d/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f
*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos
(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))
)))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate(1/((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^(5/2)),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c^2\sqrt{c+d\sin\left(e+fx\right)}\sin\left(e+fx\right)+c^2\sqrt{c+d\sin\left(e+fx\right)}+2cd\sqrt{c+d\sin\left(e+fx\right)}\sin^2\left(e+fx\right)+2cd\sqrt{c+d\sin\left(e+fx\right)}\sin\left(e+fx\right)+d^2\sqrt{c+d\sin\left(e+fx\right)}\sin^3}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(1/(c**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x) + c**2*sqrt(c + d*si
n(e + f*x)) + 2*c*d*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2 + 2*c*d*sqrt(c
 + d*sin(e + f*x))*sin(e + f*x) + d**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x
)**3 + d**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2), x)/a
```

3.510 $\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2} dx$

Optimal. Leaf size=256

$$\frac{(c+5d)\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f\sqrt{c+d\sin(e+fx)}}-\frac{\left(c^2+5cd-12d^2\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] -1/3*(c-d)*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^2-1/3*(c-d)
*(c+5*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/a^2/f/(1+sin(f*x+e))+1/3*(c^2+5*
c*d-12*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*E
llipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e)
)^(1/2)/a^2/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-1/3*(c+5*d)*(c^2-d^2)*(sin(1/2
*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1
/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/a^2/
f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2765, 2977, 2752, 2663, 2661, 2655, 2653}

$$\frac{(c+5d)\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f\sqrt{c+d\sin(e+fx)}}-\frac{\left(c^2+5cd-12d^2\right)\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -((c - d)*(c + 5*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*a^2*f*(1 + Si
n[e + f*x])) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(3*f*(a +
a*Sin[e + f*x])^2) - ((c^2 + 5*c*d - 12*d^2)*EllipticE[(e - Pi/2 + f*x)/2,
(2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*a^2*f*Sqrt[(c + d*Sin[e + f*x])
/(c + d)]) + ((c + 5*d)*(c^2 - d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c
+ d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*a^2*f*Sqrt[c + d*Sin[e + f*x]]
)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
```

```
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{3f(a+a\sin(e+fx))^2} - \frac{\int \frac{\sqrt{c+d\sin(e+fx)}\left(-\frac{1}{2}a\left(2c^2+7cd-3d^2\right)+\frac{1}{2}a(c-7d)}{a+a\sin(e+fx)}}{3a^2}$$

$$= -\frac{(c-d)(c+5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(1+\sin(e+fx))} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+}{3f(a+a\sin(e+fx))^2}$$

$$= -\frac{(c-d)(c+5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(1+\sin(e+fx))} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+}{3f(a+a\sin(e+fx))^2}$$

$$= -\frac{(c-d)(c+5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(1+\sin(e+fx))} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+}{3f(a+a\sin(e+fx))^2}$$

$$= -\frac{(c-d)(c+5d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(1+\sin(e+fx))} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+}{3f(a+a\sin(e+fx))^2}$$

Mathematica [A] time = 2.61, size = 310, normalized size = 1.21

$$\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4\left(-\left(c^2+5cd-6d^2\right)(c+d\sin(e+fx))+\left(c^2+5cd-12d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\left((c\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-((c^2 + 5*c*d - 6*d^2)*(c + d*Si
n[e + f*x])) + ((c - d)*(7*d*Cos[(e + f*x)/2] - (c + 6*d)*Cos[(3*(e + f*x))
/2] + (3*c + 11*d)*Sin[(e + f*x)/2])*(c + d*Sin[e + f*x]))/(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^3 + d^2*(-11*c + 5*d)*EllipticF[(-2*e + Pi - 2*f*x)/4
```

```
, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + (c^2 + 5*c*d - 12*d^2
)*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-
2*e + Pi - 2*f*x)/4, (2*d)/(c + d)])*Sqrt[(c + d*Sin[e + f*x])/(c + d)]))/(
3*a^2*f*(1 + Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2\right)\sqrt{d\sin\left(fx+e\right)+c}}{a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)*sqrt(d*sin(f
*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a)^2, x)
```

maple [B] time = 5.56, size = 1372, normalized size = 5.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^2*(2*d^3*(c/d-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*
x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2)))+6*c*d^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*
(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*
x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)
/(c+d))^(1/2))-4*d^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e
```

```
))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x
+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+
3*d*(c^2-2*c*d+d^2)*(-(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)/(
(-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)-2*d/(2*c-2*d)*(c/d-1
)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+
e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+
d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-d/(c-d)*(c/d-1)*((c+d*sin(f
*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d)
)^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*s
in(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c
-d))^(1/2),((c-d)/(c+d))^(1/2))))+(c^3-3*c^2*d+3*c*d^2-d^3)*(-1/3/(c-d)*(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/3*(-sin(f*x+e)^2*d-
c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)^2*(c-3*d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)
-1)*(1+sin(f*x+e)))^(1/2)+2*d^2/(3*c^2-6*c*d+3*d^2)*(c/d-1)*((c+d*sin(f*x+e
))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1
/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-
d))^(1/2),((c-d)/(c+d))^(1/2))-1/3*d*(c-3*d)/(c-d)^2*(c/d-1)*((c+d*sin(f*x+
e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(
1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(
f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d)
)^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c+d\,\sin\left(e+f\,x\right)\right)^{5/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^2,x)
[Out] int((c + d*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**2,x)
[Out] Timed out
```

3.511 $\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=237

$$-\frac{(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(\sin(e+fx)+1)}+\frac{(c+d)(c+2d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f\sqrt{c+d\sin(e+fx)}}-\frac{(c+3d)\sqrt{c+d\sin\cdots}}{3\cdots}$$

```
[Out] -1/3*(c+3*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/a^2/f/(1+sin(f*x+e))-1/3*(c-
d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^2+1/3*(c+3*d)*(sin(
1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*
e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/a^2/f/((c
+d*sin(f*x+e))/(c+d))^(1/2)-1/3*(c+d)*(c+2*d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)
^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2
)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/a^2/f/(c+d*sin(f*x+e))^(1
/2)
```

Rubi [A] time = 0.54, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2765, 2978, 2752, 2663, 2661, 2655, 2653}

$$-\frac{(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(\sin(e+fx)+1)}+\frac{(c+d)(c+2d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f\sqrt{c+d\sin(e+fx)}}-\frac{(c+3d)\sqrt{c+d\sin\cdots}}{3\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -((c + 3*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*a^2*f*(1 + Sin[e + f*
x])) - ((c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*f*(a + a*Sin[e +
f*x])^2) - ((c + 3*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c +
 d*Sin[e + f*x]])/(3*a^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((c + d)*(
c + 2*d)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f
*x])/(c + d)])/(3*a^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
```

```
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx = -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f(a+a\sin(e+fx))^2} - \frac{\int \frac{-\frac{1}{2}a\left(2c^2+5cd-d^2\right)-\frac{1}{2}ad(c+5d)\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx}{3a^2}$$

$$= -\frac{(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(1+\sin(e+fx))} - \frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f(a+a\sin(e+fx))^2}$$

$$= -\frac{(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(1+\sin(e+fx))} - \frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f(a+a\sin(e+fx))^2}$$

$$= -\frac{(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(1+\sin(e+fx))} - \frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f(a+a\sin(e+fx))^2}$$

$$= -\frac{(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(1+\sin(e+fx))} - \frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f(a+a\sin(e+fx))^2}$$

Mathematica [A] time = 2.77, size = 283, normalized size = 1.19

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4\left(-2d^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)-(c+3d)(c+d\sin(e+fx))\right.}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-((c + 3*d)*(c + d*Sin[e + f*x]))
 + ((4*d*Cos[(e + f*x)/2] - (c + 3*d)*Cos[(3*(e + f*x))/2] + (3*c + 5*d)*Si
n[(e + f*x)/2])*(c + d*Sin[e + f*x]))/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
^3 - 2*d^2*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[
e + f*x])/(c + d)] + (c + 3*d)*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2
*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)])*Sqrt[(c +
```

```
d*Sin[e + f*x])/(c + d)]))/(3*a^2*f*(1 + Sin[e + f*x])^2*Sqrt[c + d*Sin[e
+ f*x]])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(d \sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{a^2 \cos\left(fx+e\right)^2 - 2\,a^2 \sin\left(fx+e\right) - 2\,a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(d*sin(f*x + e) + c)^(3/2)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x +
e) - 2*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(a \sin\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a)^2, x)
```

maple [B] time = 5.28, size = 1049, normalized size = 4.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^2*(2*d^2*(c/d-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))+2*d*(c-d)*(-(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*s
in(f*x+e)+c)/(c-d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)-
2*d/(2*c-2*d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d
))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^
(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-d/(c-d)
*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-s
in(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-
```

```
1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(
((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(c^2-2*c*d+d^2)*(-1/3
/(c-d)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/3*(-sin(f
*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)^2*(c-3*d)/((-d*sin(f*x+e)-c)*(
sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)+2*d^2/(3*c^2-6*c*d+3*d^2)*(c/d-1)*((c+d
*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d
/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f
*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-1/3*d*(c-3*d)/(c-d)^2*(c/d-1)*((c+
d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*
d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(
((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x
+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)
/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c+d\,\sin\left(e+f\,x\right)\right)^{3/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^2,x)
[Out] int((c + d*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c\sqrt{c+d\sin\left(e+fx\right)}}{\sin^{2}\left(e+fx\right)+2\sin\left(e+fx\right)+1}\,dx+\int \frac{d\sqrt{c+d\sin\left(e+fx\right)}\,\sin\left(e+fx\right)}{\sin^{2}\left(e+fx\right)+2\sin\left(e+fx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] (Integral(c*sqrt(c + d*sin(e + f*x))/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1)
, x) + Integral(d*sqrt(c + d*sin(e + f*x))*sin(e + f*x)/(sin(e + f*x)**2 +
2*sin(e + f*x) + 1), x))/a**2
```

3.512 $\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=233

$$-\frac{c\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(c-d)(\sin(e+fx)+1)}+\frac{(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f\sqrt{c+d\sin(e+fx)}}-\frac{c\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+f\right.\right.}{3a^2f(c-d)\sqrt{\frac{c+d\sin(e+f}{c+d}}}$$

```
[Out] -1/3*c*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/a^2/(c-d)/f/(1+sin(f*x+e))-1/3*cos
(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^2+1/3*c*(sin(1/2*e+1/4*Pi
+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2
*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/a^2/(c-d)/f/((c+d*sin
(f*x+e))/(c+d))^(1/2)-1/3*(c+d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2
*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1
/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/a^2/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2764, 2978, 2752, 2663, 2661, 2655, 2653}

$$-\frac{c\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(c-d)(\sin(e+fx)+1)}+\frac{(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f\sqrt{c+d\sin(e+fx)}}-\frac{c\sqrt{c+d\sin(e+fx)}E\left(\frac{1}{2}\left(e+f\right.\right.}{3a^2f(c-d)\sqrt{\frac{c+d\sin(e+f}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -(c*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*a^2*(c - d)*f*(1 + Sin[e + f*
x])) - (Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*f*(a + a*Sin[e + f*x])^2)
 - (c*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]]
)/(3*a^2*(c - d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((c + d)*EllipticF
[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*
a^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2764

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m
*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*
(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && L
tQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c,
 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
```

```
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx = -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f(a+a\sin(e+fx))^2} + \frac{\int \frac{\frac{1}{2}a(2c+d)+\frac{1}{2}ad\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx}{3a^2}$$

$$= -\frac{c\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2(c-d)f(1+\sin(e+fx))} - \frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f(a+a\sin(e+fx))^2} - \frac{\int \frac{\frac{a^2d^2}{2}+\frac{1}{2}a^2cd\,\mathrm{s}\ldots}{\sqrt{c+d\sin(e\ldots}}}{3a^4(c-\ldots}$$

$$= -\frac{c\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2(c-d)f(1+\sin(e+fx))} - \frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f(a+a\sin(e+fx))^2} - \frac{c\int \sqrt{c+d\sin\ldots}}{6a^2(c\ldots}$$

$$= -\frac{c\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2(c-d)f(1+\sin(e+fx))} - \frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f(a+a\sin(e+fx))^2} - \frac{\left(c\sqrt{c+d\sin\ldots}\right.}{6a\ldots}$$

$$= -\frac{c\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2(c-d)f(1+\sin(e+fx))} - \frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f(a+a\sin(e+fx))^2} - \frac{cE\left(\frac{1}{2}\left(e-\frac{\pi}{2}\ldots\right.\right.}{3a^2(\ldots}$$

Mathematica [A] time = 2.55, size = 256, normalized size = 1.10

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4\left(-\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)-c(c+d\sin(e+fx))+\ldots\right.}{3a^2f(c-d)(\sin(e+fx)+1\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-(c*(c + d*Sin[e + f*x])) + ((d*C
os[(e + f*x)/2] - c*Cos[(3*(e + f*x))/2] + (3*c - d)*Sin[(e + f*x)/2])*(c +
 d*Sin[e + f*x]))/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + c*(c + d)*Ellip
ticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d
)] - (c^2 - d^2)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c +
d*Sin[e + f*x])/(c + d)]))/(3*a^2*(c - d)*f*(1 + Sin[e + f*x])^2*Sqrt[c + d
*Sin[e + f*x]])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{d\sin\left(fx+e\right)+c}}{a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*sin(f*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e)
 - 2*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{d\sin\left(fx+e\right)+c}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^2, x)
```

maple [B] time = 5.18, size = 906, normalized size = 3.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^2*(d*(-(-sin(f*x+e)^2*d-c*sin(f*x
+e)+d*sin(f*x+e)+c)/(c-d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+sin(f*x+e)))
^(1/2)-2*d/(2*c-2*d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e
))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x
+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-
d/(c-d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/
2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*
((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+Ell
ipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(c-d)*(-1/3/(c
-d)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/3*(-sin(f*x+
e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)^2*(c-3*d)/((-d*sin(f*x+e)-c)*(sin
(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)+2*d^2/(3*c^2-6*c*d+3*d^2)*(c/d-1)*((c+d*si
n(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c
```

```
-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+
e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-1/3*d*(c-3*d)/(c-d)^2*(c/d-1)*((c+d*s
in(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(
c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c
+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e)
)/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d \sin\left(fx+e\right)+c}}{\left(a \sin\left(fx+e\right)+a\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c+d\,\sin\left(e+f\,x\right)}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] int((c + d*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{c+d \sin\left(e+fx\right)}}{\sin^{2}\left(e+fx\right)+2 \sin\left(e+fx\right)+1}\, dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Integral(sqrt(c + d*sin(e + f*x))/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1), x
)/a**2
```

3.513 $\int \frac{1}{(a+a\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=257

$$-\frac{(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(c-d)^2(\sin(e+fx)+1)}+\frac{(c-2d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f(c-d)\sqrt{c+d\sin(e+fx)}}-\frac{(c-3d)\sqrt{c+d\sin(e+}}{3a^2f(c-d}$$

```
[Out] -1/3*(c-3*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/a^2/(c-d)^2/f/(1+sin(f*x+e))
-1/3*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(c-d)/f/(a+a*sin(f*x+e))^2+1/3*(c-3*
d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(
cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/a
^2/(c-d)^2/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-1/3*(c-2*d)*(sin(1/2*e+1/4*Pi+1
/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f
*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/a^2/(c-d)/f/(c+
d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2766, 2978, 2752, 2663, 2661, 2655, 2653}

$$-\frac{(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2f(c-d)^2(\sin(e+fx)+1)}+\frac{(c-2d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f(c-d)\sqrt{c+d\sin(e+fx)}}-\frac{(c-3d)\sqrt{c+d\sin(e+}}{3a^2f(c-d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] -((c - 3*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*a^2*(c - d)^2*f*(1 +
Sin[e + f*x])) - (Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*(c - d)*f*(a +
a*Sin[e + f*x])^2) - ((c - 3*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)
]*Sqrt[c + d*Sin[e + f*x]])/(3*a^2*(c - d)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c
 + d)]) + ((c - 2*d)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c +
 d*Sin[e + f*x])/(c + d)])/(3*a^2*(c - d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
```

```
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3(c-d)f(a+a\sin(e+fx))^2} - \frac{\int \frac{-\frac{1}{2}a(2c-5d)-\frac{1}{2}ad\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c+d\sin(e+fx)}}}{3a^2(c-d)}$$

$$= -\frac{(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{\cos(e+fx)\sqrt{c+d\,s}}{3(c-d)f(a+a\sin}$$

$$= -\frac{(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{\cos(e+fx)\sqrt{c+d\,s}}{3(c-d)f(a+a\sin}$$

$$= -\frac{(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{\cos(e+fx)\sqrt{c+d\,s}}{3(c-d)f(a+a\sin}$$

$$= -\frac{(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{\cos(e+fx)\sqrt{c+d\,s}}{3(c-d)f(a+a\sin}$$

Mathematica [A] time = 2.47, size = 290, normalized size = 1.13

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4\left(-2d^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)-(c-3d)(c+d\sin(e+fx))\right.}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-((c - 3*d)*(c + d*Sin[e + f*x]))
 - ((2*d*Cos[(e + f*x)/2] + (c - 3*d)*Cos[(3*(e + f*x))/2] + (-3*c + 7*d)*S
in[(e + f*x)/2])*(c + d*Sin[e + f*x]))/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^3 - 2*d^2*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin
[e + f*x])/(c + d)] + (c - 3*d)*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (
2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)])*Sqrt[(c
```

```
+ d*Sin[e + f*x])/(c + d)]))/(3*a^2*(c - d)^2*f*(1 + Sin[e + f*x])^2*Sqrt[c
 + d*Sin[e + f*x]])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{d\sin\left(fx+e\right)+c}}{2\,a^2c+2\,a^2d-\left(a^2c+2\,a^2d\right)\cos\left(fx+e\right)^2-\left(a^2d\cos\left(fx+e\right)^2-2\,a^2c-2\,a^2d\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d*sin(f*x + e) + c)/(2*a^2*c + 2*a^2*d - (a^2*c + 2*a^2*d)*co
s(f*x + e)^2 - (a^2*d*cos(f*x + e)^2 - 2*a^2*c - 2*a^2*d)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^2\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^2*sqrt(d*sin(f*x + e) + c)), x)
```

maple [A] time = 3.72, size = 507, normalized size = 1.97

$$\frac{\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^2\left(fx+e\right)\right)}\left(-\frac{\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^2\left(fx+e\right)\right)}}{3(c-d)\left(1+\sin\left(fx+e\right)\right)^2}-\frac{\left(-\left(\sin^2\left(fx+e\right)\right)d-c\sin\left(fx+e\right)+d\sin\left(fx+e\right)+c\right)(c}{3(c-d)^2\sqrt{\left(-d\sin\left(fx+e\right)-c\right)\left(\sin\left(fx+e\right)-1\right)\left(1+\sin(\right.}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^2*(-1/3/(c-d)*(-(-d*sin(f*x+e)-c)
*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/3*(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*s
in(f*x+e)+c)/(c-d)^2*(c-3*d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+sin(f*x+e
)))^(1/2)+2*d^2/(3*c^2-6*c*d+3*d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*
(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*
```

```
x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)
/(c+d))^(1/2))-1/3*d*(c-3*d)/(c-d)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)
*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(
1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(
c+d))^(1/2))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^2\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^2*sqrt(d*sin(f*x + e) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{c+d\sin\left(e+fx\right)}\sin^2\left(e+fx\right)+2\sqrt{c+d\sin\left(e+fx\right)}\sin\left(e+fx\right)+\sqrt{c+d\sin\left(e+fx\right)}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2 + 2*sqrt(c + d*sin(e +
 f*x))*sin(e + f*x) + sqrt(c + d*sin(e + f*x))), x)/a**2
```

3.514 $\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=326

$$-\frac{d\left(c^2-5cd-12d^2\right)\cos(e+fx)}{3a^2f(c-d)^3(c+d)\sqrt{c+d\sin(e+fx)}}-\frac{\left(c^2-5cd-12d^2\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f(c-d)^3(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{\quad}{3a^2f(c-}$$

```
[Out] -1/3*d*(c^2-5*c*d-12*d^2)*cos(f*x+e)/a^2/(c-d)^3/(c+d)/f/(c+d*sin(f*x+e))^(
1/2)-1/3*(c-5*d)*cos(f*x+e)/a^2/(c-d)^2/f/(1+sin(f*x+e))/(c+d*sin(f*x+e))^(
1/2)-1/3*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2)+1/3*(
c^2-5*c*d-12*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*
f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(
f*x+e))^(1/2)/a^2/(c-d)^3/(c+d)/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-1/3*(c-5*d
)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(c
os(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^
(1/2)/a^2/(c-d)^2/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.64, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2766, 2978, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{d\left(c^2-5cd-12d^2\right)\cos(e+fx)}{3a^2f(c-d)^3(c+d)\sqrt{c+d\sin(e+fx)}}-\frac{\left(c^2-5cd-12d^2\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3a^2f(c-d)^3(c+d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{\quad}{3a^2f(c-}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -(d*(c^2 - 5*c*d - 12*d^2)*Cos[e + f*x])/(3*a^2*(c - d)^3*(c + d)*f*Sqrt[c
+ d*Sin[e + f*x]]) - ((c - 5*d)*Cos[e + f*x])/(3*a^2*(c - d)^2*f*(1 + Sin[e
 + f*x])*Sqrt[c + d*Sin[e + f*x]]) - Cos[e + f*x]/(3*(c - d)*f*(a + a*Sin[e
 + f*x])^2*Sqrt[c + d*Sin[e + f*x]]) - ((c^2 - 5*c*d - 12*d^2)*EllipticE[(e
 - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*a^2*(c - d)^3
*(c + d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((c - 5*d)*EllipticF[(e -
Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*a^2*(c
 - d)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
```

```
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}} - \frac{\int \frac{-\frac{1}{2}a(2c \cdots}{(a+a\sin(e \cdots}}{\cdots}$$

$$= -\frac{(c-5d)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))\sqrt{c+d\sin(e+fx)}} - \frac{\cdots}{3(c-d)f(\cdots}$$

$$= -\frac{d\left(c^2-5cd-12d^2\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{(c-5 \cdots}{3a^2(c-d)^2f(1+\sin \cdots}$$

$$= -\frac{d\left(c^2-5cd-12d^2\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{(c-5 \cdots}{3a^2(c-d)^2f(1+\sin \cdots}$$

$$= -\frac{d\left(c^2-5cd-12d^2\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{(c-5 \cdots}{3a^2(c-d)^2f(1+\sin \cdots}$$

$$= -\frac{d\left(c^2-5cd-12d^2\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{(c-5 \cdots}{3a^2(c-d)^2f(1+\sin \cdots}$$

Mathematica [A] time = 4.75, size = 405, normalized size = 1.24

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4\left(\frac{(c^2-5cd-12d^2)(c+d\sin(e+fx))+(c^2-5cd-12d^2)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\left((c+d)E\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)-cF\left(\right.\right.}{c+d}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*((c + d*Sin[e + f*x])*((-2*(c^2 -
5*c*d - 9*d^2))/(c + d) + (2*(c - d)*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^3 + (-c + d)/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (2
*(c - 6*d)*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (6*d^3
*Cos[e + f*x])/((c + d)*(c + d*Sin[e + f*x]))) + ((c^2 - 5*c*d - 12*d^2)*(c
 + d*Sin[e + f*x]) - d^2*(11*c + 5*d)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d
)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + (c^2 - 5*c*d - 12*d^2)*((c
+ d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e +
Pi - 2*f*x)/4, (2*d)/(c + d)])*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(c + d))
)/(3*a^2*(c - d)^3*f*(1 + Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\sin\left(fx+e\right)+c}}{a^2d^2\cos\left(fx+e\right)^4+2\,a^2c^2+4\,a^2cd+2\,a^2d^2-\left(a^2c^2+4\,a^2cd+3\,a^2d^2\right)\cos\left(fx+e\right)^2+2\left(a^2c^2+2\,a^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d*sin(f*x + e) + c)/(a^2*d^2*cos(f*x + e)^4 + 2*a^2*c^2 + 4*a
^2*c*d + 2*a^2*d^2 - (a^2*c^2 + 4*a^2*c*d + 3*a^2*d^2)*cos(f*x + e)^2 + 2*(
a^2*c^2 + 2*a^2*c*d + a^2*d^2 - (a^2*c*d + a^2*d^2)*cos(f*x + e)^2)*sin(f*x
 + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^2\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(3/2)), x)
```

maple [B] time = 5.98, size = 1299, normalized size = 3.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^2*(d^2/(c-d)^2*(2*d*cos(f*x+e)^2/
(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c
+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)
*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin
(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(
f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d
))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*
sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(
c-d))^(1/2),((c-d)/(c+d))^(1/2))))-d/(c-d)^2*(-(-sin(f*x+e)^2*d-c*sin(f*x+e
)+d*sin(f*x+e)+c)/(c-d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(
1/2)-2*d/(2*c-2*d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))
/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e
)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-d/
(c-d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)
*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((
-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+Ellip
ticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+1/(c-d)*(-1/3/(c
-d)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/3*(-sin(f*x+
e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)^2*(c-3*d)/((-d*sin(f*x+e)-c)*(sin
(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)+2*d^2/(3*c^2-6*c*d+3*d^2)*(c/d-1)*((c+d*si
n(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c
-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+
e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-1/3*d*(c-3*d)/(c-d)^2*(c/d-1)*((c+d*s
in(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(
c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c
+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e)
)/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+a\,\sin\left(e+f\,x\right)\right)^2\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\frac{1}{c\sqrt{c+d\sin\left(e+fx\right)}\,\sin^2\left(e+fx\right)+2c\sqrt{c+d\sin\left(e+fx\right)}\,\sin\left(e+fx\right)+c\sqrt{c+d\sin\left(e+fx\right)}+d\sqrt{c+d\sin\left(e+fx\right)}\,\sin^3\left(e+fx\right)+2d\sqrt{c+d\sin\left(e+fx\right)}\,\sin^2\left(e\right)}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/(c*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2 + 2*c*sqrt(c + d*sin
(e + f*x))*sin(e + f*x) + c*sqrt(c + d*sin(e + f*x)) + d*sqrt(c + d*sin(e +
 f*x))*sin(e + f*x)**3 + 2*d*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2 + d*s
qrt(c + d*sin(e + f*x))*sin(e + f*x)), x)/a**2
```

3.515 $\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=405

$$-\frac{d(c+3d)\left(c^2-10cd-7d^2\right)\cos(e+fx)}{3a^2f(c-d)^4(c+d)^2\sqrt{c+d\sin(e+fx)}}-\frac{d\left(c^2-7cd-10d^2\right)\cos(e+fx)}{3a^2f(c-d)^3(c+d)(c+d\sin(e+fx))^{3/2}}+\frac{\left(c^2-7cd-10d^2\right)\sqrt{\frac{c+d\sin(e+}{c+d}}}{3a^2f(c-d)^3(c+d)}$$

```
[Out] -1/3*d*(c^2-7*c*d-10*d^2)*cos(f*x+e)/a^2/(c-d)^3/(c+d)/f/(c+d*sin(f*x+e))^(
3/2)-1/3*(c-7*d)*cos(f*x+e)/a^2/(c-d)^2/f/(1+sin(f*x+e))/(c+d*sin(f*x+e))^(
3/2)-1/3*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2)-1/3*d
*(c+3*d)*(c^2-10*c*d-7*d^2)*cos(f*x+e)/a^2/(c-d)^4/(c+d)^2/f/(c+d*sin(f*x+e
))^(1/2)+1/3*(c+3*d)*(c^2-10*c*d-7*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)
/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(
c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/a^2/(c-d)^4/(c+d)^2/f/((c+d*sin(f*x+e))
/(c+d))^(1/2)-1/3*(c^2-7*c*d-10*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/si
n(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d
))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/a^2/(c-d)^3/(c+d)/f/(c+d*sin(f*x+e
))^(1/2)
```

Rubi [A] time = 0.83, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2766, 2978, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{d(c+3d)\left(c^2-10cd-7d^2\right)\cos(e+fx)}{3a^2f(c-d)^4(c+d)^2\sqrt{c+d\sin(e+fx)}}-\frac{d\left(c^2-7cd-10d^2\right)\cos(e+fx)}{3a^2f(c-d)^3(c+d)(c+d\sin(e+fx))^{3/2}}+\frac{\left(c^2-7cd-10d^2\right)\sqrt{\frac{c+d\sin(e+}{c+d}}}{3a^2f(c-d)^3(c+d)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] -(d*(c^2 - 7*c*d - 10*d^2)*Cos[e + f*x])/(3*a^2*(c - d)^3*(c + d)*f*(c + d*
Sin[e + f*x])^(3/2)) - ((c - 7*d)*Cos[e + f*x])/(3*a^2*(c - d)^2*f*(1 + Sin
[e + f*x])*(c + d*Sin[e + f*x])^(3/2)) - Cos[e + f*x]/(3*(c - d)*f*(a + a*S
in[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2)) - (d*(c + 3*d)*(c^2 - 10*c*d - 7
*d^2)*Cos[e + f*x])/(3*a^2*(c - d)^4*(c + d)^2*f*Sqrt[c + d*Sin[e + f*x]])
- ((c + 3*d)*(c^2 - 10*c*d - 7*d^2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c
+ d)]*Sqrt[c + d*Sin[e + f*x]])/(3*a^2*(c - d)^4*(c + d)^2*f*Sqrt[(c + d*Si
n[e + f*x])/(c + d)]) + ((c^2 - 7*c*d - 10*d^2)*EllipticF[(e - Pi/2 + f*x)/
2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*a^2*(c - d)^3*(c +
 d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
```

```
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}\ldots}{(a+a\,\mathrm{s}\ldots}}{\ldots}$$

$$= -\frac{(c-7d)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))^{3/2}} - \frac{\ldots}{3(c-\ldots}$$

$$= -\frac{d\left(c^2-7cd-10d^2\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{(c\ldots}{3a^2(c-d)^2f(1+\ldots}$$

$$= -\frac{d\left(c^2-7cd-10d^2\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{(c\ldots}{3a^2(c-d)^2f(1+\ldots}$$

$$= -\frac{d\left(c^2-7cd-10d^2\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{(c\ldots}{3a^2(c-d)^2f(1+\ldots}$$

$$= -\frac{d\left(c^2-7cd-10d^2\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{(c\ldots}{3a^2(c-d)^2f(1+\ldots}$$

$$= -\frac{d\left(c^2-7cd-10d^2\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{(c\ldots}{3a^2(c-d)^2f(1+\ldots}$$

Mathematica [A] time = 6.68, size = 674, normalized size = 1.66

$$\frac{d\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4\left(-\frac{2\left(26c^2d+28cd^2+10d^3\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{\sqrt{c+d\sin(e+fx)}}+\frac{2\left(c^3-7c^2d-37cd^2-21d^3\right)\cos^2\ldots}{d\left(1-\sin^2(e\ldots\right.}\right.}{6f(c-d)^4(c+d)^2(a\sin\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sqrt[c + d*Sin[e + f*x]]*((-2*(c^3
 - 7*c^2*d - 27*c*d^2 - 15*d^3))/(3*(c - d)^4*(c + d)^2) + (2*Sin[(e + f*x)
```

```
/2])/(3*(c - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3) - 1/(3*(c - d)^3
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2) + (2*(c*Sin[(e + f*x)/2] - 9*d*Si
n[(e + f*x)/2]))/(3*(c - d)^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])) + (2*d
^3*Cos[e + f*x])/(3*(c - d)^3*(c + d)*(c + d*Sin[e + f*x])^2) + (4*(5*c*d^3
*Cos[e + f*x] + 3*d^4*Cos[e + f*x]))/(3*(c - d)^4*(c + d)^2*(c + d*Sin[e +
f*x]))))/(f*(a + a*Sin[e + f*x])^2) + (d*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^4*((-2*(26*c^2*d + 28*c*d^2 + 10*d^3)*EllipticF[(-e + Pi/2 - f*x)/2, (2
*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/Sqrt[c + d*Sin[e + f*x]] +
 (2*(c^3 - 7*c^2*d - 37*c*d^2 - 21*d^3)*Cos[e + f*x]^2*Sqrt[c + d*Sin[e + f
*x]])/(d*(1 - Sin[e + f*x]^2)) - ((-c^3 + 7*c^2*d + 37*c*d^2 + 21*d^3)*((2*
(c + d)*EllipticE[(-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f
*x])/(c + d)])/Sqrt[c + d*Sin[e + f*x]] - (2*c*EllipticF[(-e + Pi/2 - f*x)/
2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/Sqrt[c + d*Sin[e + f*
x]]))/d))/(6*(c - d)^4*(c + d)^2*f*(a + a*Sin[e + f*x])^2)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{}{2\,a^2c^3+6\,a^2c^2d+6\,a^2cd^2+2\,a^2d^3+\left(3\,a^2cd^2+2\,a^2d^3\right)\cos\left(fx+e\right)^4-\left(a^2c^3+6\,a^2c^2d+9\,a^2cd^2+4\,a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d*sin(f*x + e) + c)/(2*a^2*c^3 + 6*a^2*c^2*d + 6*a^2*c*d^2 +
2*a^2*d^3 + (3*a^2*c*d^2 + 2*a^2*d^3)*cos(f*x + e)^4 - (a^2*c^3 + 6*a^2*c^2
*d + 9*a^2*c*d^2 + 4*a^2*d^3)*cos(f*x + e)^2 + (a^2*d^3*cos(f*x + e)^4 + 2*
a^2*c^3 + 6*a^2*c^2*d + 6*a^2*c*d^2 + 2*a^2*d^3 - 3*(a^2*c^2*d + 2*a^2*c*d^
2 + a^2*d^3)*cos(f*x + e)^2)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^2\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(5/2)), x)
```

maple [B] time = 8.66, size = 1758, normalized size = 4.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^2*(2*d^2/(c-d)^3*(2*d*cos(f*x+e)^
2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*(
(c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-
1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*s
in(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*si
n(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c
-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+
d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))
/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))-2/(c-d)^3*d*(-(-sin(f*x+e)^2*d-c*sin(f
*x+e)+d*sin(f*x+e)+c)/(c-d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+sin(f*x+e)
))^(1/2)-2*d/(2*c-2*d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x
+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f
*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)
)-d/(c-d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(
1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2
)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+E
llipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+1/(c-d)^2*(-
1/3/(c-d)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/3*(-si
n(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)^2*(c-3*d)/((-d*sin(f*x+e)-c
)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)+2*d^2/(3*c^2-6*c*d+3*d^2)*(c/d-1)*((
c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1
)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*si
n(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-1/3*d*(c-3*d)/(c-d)^2*(c/d-1)*(
(c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-
1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*Ellipti
cE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(
f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+d^2/(c-d)^2*(2/3/(c^2-d^2)/d*(-
(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/3*d*cos(f*x+e)^2
/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4
-6*c^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/
(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)
^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+8/3
*c*d/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(
c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^
2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(
1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(
f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^(5/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{c^2\sqrt{c+d\sin\left(e+fx\right)}\sin^2\left(e+fx\right)+2c^2\sqrt{c+d\sin\left(e+fx\right)}\sin\left(e+fx\right)+c^2\sqrt{c+d\sin\left(e+fx\right)}+2cd\sqrt{c+d\sin\left(e+fx\right)}\sin^3\left(e+fx\right)+4cd\sqrt{c+d\sin\left(e+fx\right)}\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(1/(c**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2 + 2*c**2*sqrt(c +
 d*sin(e + f*x))*sin(e + f*x) + c**2*sqrt(c + d*sin(e + f*x)) + 2*c*d*sqrt(
c + d*sin(e + f*x))*sin(e + f*x)**3 + 4*c*d*sqrt(c + d*sin(e + f*x))*sin(e
+ f*x)**2 + 2*c*d*sqrt(c + d*sin(e + f*x))*sin(e + f*x) + d**2*sqrt(c + d*s
in(e + f*x))*sin(e + f*x)**4 + 2*d**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x)
**3 + d**2*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**2), x)/a**2
```

3.516 $\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=322

$$-\frac{\left(4c^2+15cd+27d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{30f\left(a^3\sin(e+fx)+a^3\right)}+\frac{(c+d)\left(4c^2+11cd+15d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\right.\right.}{30a^3f\sqrt{c+d\sin(e+fx)}}$$

```
[Out] -1/5*(c-d)*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^3-2/15*(c-d
)*(c+3*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+e))^2-1/30*(4*
c^2+15*c*d+27*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a^3+a^3*sin(f*x+e))
+1/30*(4*c^2+15*c*d+27*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1
/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))
*(c+d*sin(f*x+e))^(1/2)/a^3/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-1/30*(c+d)*(4*
c^2+11*c*d+15*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2
*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*si
n(f*x+e))/(c+d))^(1/2)/a^3/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.83, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2765, 2977, 2978, 2752, 2663, 2661, 2655, 2653}

$$-\frac{\left(4c^2+15cd+27d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{30f\left(a^3\sin(e+fx)+a^3\right)}+\frac{(c+d)\left(4c^2+11cd+15d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\right.\right.}{30a^3f\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-2*(c - d)*(c + 3*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*a*f*(a + a
*Sin[e + f*x])^2) - ((4*c^2 + 15*c*d + 27*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[
e + f*x]])/(30*f*(a^3 + a^3*Sin[e + f*x])) - ((c - d)*Cos[e + f*x]*(c + d*S
in[e + f*x])^(3/2))/(5*f*(a + a*Sin[e + f*x])^3) - ((4*c^2 + 15*c*d + 27*d^
2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(
30*a^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((c + d)*(4*c^2 + 11*c*d + 1
5*d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x
])/(c + d)])/(30*a^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
```

```
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{5f(a+a\sin(e+fx))^3} - \frac{\int \frac{\sqrt{c+d\sin(e+fx)}\left(-\frac{1}{2}a\left(4c^2+9cd-3d^2\right)-\frac{1}{2}ad(c+9d\right.}{(a+a\sin(e+fx))^2}}{5a^2}$$

$$= -\frac{2(c-d)(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15af(a+a\sin(e+fx))^2} - \frac{(c-d)\cos(e+fx)(c+d\sin(e}{5f(a+a\sin(e+fx))^3}$$

$$= -\frac{2(c-d)(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15af(a+a\sin(e+fx))^2} - \frac{\left(4c^2+15cd+27d^2\right)\cos(e+fx}{30f\left(a^3+a^3\sin(e\right.}$$

$$= -\frac{2(c-d)(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15af(a+a\sin(e+fx))^2} - \frac{\left(4c^2+15cd+27d^2\right)\cos(e+fx}{30f\left(a^3+a^3\sin(e\right.}$$

$$= -\frac{2(c-d)(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15af(a+a\sin(e+fx))^2} - \frac{\left(4c^2+15cd+27d^2\right)\cos(e+fx}{30f\left(a^3+a^3\sin(e\right.}$$

$$= -\frac{2(c-d)(c+3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15af(a+a\sin(e+fx))^2} - \frac{\left(4c^2+15cd+27d^2\right)\cos(e+fx}{30f\left(a^3+a^3\sin(e\right.}$$

Mathematica [A] time = 5.81, size = 385, normalized size = 1.20

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^6\left(-\left(4c^2+15cd+27d^2\right)(c+d\sin(e+fx)) - \frac{(c+d\sin(e+fx))\left(\left(20c^2+74cd+90d^2\right)\cos(\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(-((4*c^2 + 15*c*d + 27*d^2)*(c +
d*Sin[e + f*x])) - ((-2*d*(35*c + 57*d)*Cos[(e + f*x)/2] + (20*c^2 + 74*c*d
 + 90*d^2)*Cos[(3*(e + f*x))/2] + 2*(-3*(6*c^2 + 11*c*d + 29*d^2) + 2*(2*c^
2 + 7*c*d - 9*d^2)*Cos[e + f*x] + (4*c^2 + 15*c*d + 27*d^2)*Cos[2*(e + f*x)
])*Sin[(e + f*x)/2])*(c + d*Sin[e + f*x]))/(2*(Cos[(e + f*x)/2] + Sin[(e +
f*x)/2])^5) + (c - 15*d)*d^2*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)
]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + (4*c^2 + 15*c*d + 27*d^2)*((c + d)*E
```

```
llipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2
*f*x)/4, (2*d)/(c + d)])*Sqrt[(c + d*Sin[e + f*x])/(c + d)]))/(30*a^3*f*(1
+ Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d^2\cos\left(fx+e\right)^2 - 2\,cd\sin\left(fx+e\right) - c^2 - d^2\right)\sqrt{d\sin\left(fx+e\right)+c}}{3\,a^3\cos\left(fx+e\right)^2 - 4\,a^3 + \left(a^3\cos\left(fx+e\right)^2 - 4\,a^3\right)\sin\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral((d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)*sqrt(d*sin(f
*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*s
in(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a)^3, x)
```

maple [B] time = 7.63, size = 1615, normalized size = 5.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^3*(2*d^3*(c/d-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))+(c^3-3*c^2*d+3*c*d^2-d^3)*(-1/5/(c-d)*(-(-d*s
in(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^3-2/15*(c-3*d)/(c-d)^2*(-(-
d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/30*(-sin(f*x+e)^2*d-
c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)^3*(4*c^2-15*c*d+27*d^2)/((-d*sin(f*x+e)-
c)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)+2*(-c*d^2-15*d^3)/(60*c^3-180*c^2*d
```

```
+180*c*d^2-60*d^3)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))
/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e
)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-1/
30*d*(4*c^2-15*c*d+27*d^2)/(c-d)^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(
d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x
+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/
2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+
d))^(1/2))))+3*d^2*(c-d)*(-(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c
-d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)-2*d/(2*c-2*d)*(
c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin
(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF
(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-d/(c-d)*(c/d-1)*((c+d*
sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/
(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+3*d*(c^2-2*c*d+d^2)*(-1/3/(c-d)*(-(-
d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/3*(-sin(f*x+e)^2*d-c
*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)^2*(c-3*d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-
1)*(1+sin(f*x+e)))^(1/2)+2*d^2/(3*c^2-6*c*d+3*d^2)*(c/d-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))-1/3*d*(c-3*d)/(c-d)^2*(c/d-1)*((c+d*sin(f*x+e
))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1
/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f
*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))
^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\sin(fx+e)+c)^{\frac{5}{2}}}{(a\sin(fx+e)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,\sin(e+f\,x))^{5/2}}{(a+a\,\sin(e+f\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^3,x)
```

```
[Out] int((c + d*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.517 $\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=323

$$-\frac{(4c^2+5cd-3d^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{30f(c-d)\left(a^3\sin(e+fx)+a^3\right)}-\frac{(4c^2+5cd-3d^2)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{30a^3f(c-d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] -1/5*(c-d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^3-2/15*(c+2
*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+e))^2-1/30*(4*c^2+5*
c*d-3*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(c-d)/f/(a^3+a^3*sin(f*x+e))+1
/30*(4*c^2+5*c*d-3*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*P
i+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+
d*sin(f*x+e))^(1/2)/a^3/(c-d)/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-1/30*(c+d)*(
4*c+5*d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elli
pticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/
(c+d))^(1/2)/a^3/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.88, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2765, 2978, 2752, 2663, 2661, 2655, 2653}

$$-\frac{(4c^2+5cd-3d^2)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{30f(c-d)\left(a^3\sin(e+fx)+a^3\right)}-\frac{(4c^2+5cd-3d^2)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{30a^3f(c-d)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -((c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(5*f*(a + a*Sin[e + f*x])^
3) - (2*(c + 2*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*a*f*(a + a*Sin
[e + f*x])^2) - ((4*c^2 + 5*c*d - 3*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*
x]])/(30*(c - d)*f*(a^3 + a^3*Sin[e + f*x])) - ((4*c^2 + 5*c*d - 3*d^2)*Ell
ipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(30*a^3
*(c - d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((c + d)*(4*c + 5*d)*Ellip
ticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])
/(30*a^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
```

```
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx = -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3} - \frac{\int \frac{-\frac{1}{2}a\left(4c^2+7cd-d^2\right)-\frac{1}{2}ad(3c+7d)\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}\,dx}{5a^2}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3} - \frac{2(c+2d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15af(a+a\sin(e+fx))^2}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3} - \frac{2(c+2d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15af(a+a\sin(e+fx))^2}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3} - \frac{2(c+2d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15af(a+a\sin(e+fx))^2}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3} - \frac{2(c+2d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15af(a+a\sin(e+fx))^2}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3} - \frac{2(c+2d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15af(a+a\sin(e+fx))^2}$$

Mathematica [A] time = 6.21, size = 441, normalized size = 1.37

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^6\left(\frac{2(c+d\sin(e+fx))\left(\frac{\left(4c^2+5cd-3d^2\right)\sin\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4}{c-d}+6(c-d)\sin\left(\frac{1}{2}(e+fx)\right)-2(c+\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*((2*(6*(c - d)*Sin[(e + f*x)/2] +
3*(-c + d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 4*(c + 2*d)*Sin[(e + f*x
)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 2*(c + 2*d)*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^3 + ((4*c^2 + 5*c*d - 3*d^2)*Sin[(e + f*x)/2]*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^4)/(c - d))*(c + d*Sin[e + f*x]))/(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^5 - ((4*c^2 + 5*c*d - 3*d^2)*(c + d*Sin[e + f*x
]) - d^2*(c + 5*d)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c
+ d*Sin[e + f*x])/(c + d)] + (-4*c^2 - 5*c*d + 3*d^2)*((c + d)*EllipticE[(-
2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2
*d)/(c + d)])*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(c - d)))/(30*a^3*f*(1 +
Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{3\,a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(d*sin(f*x + e) + c)^(3/2)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*c
os(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a)^3, x)
```

maple [B] time = 7.64, size = 1462, normalized size = 4.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^3*((c^2-2*c*d+d^2)*(-1/5/(c-d)*(-
(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^3-2/15*(c-3*d)/(c-d)^2
*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/30*(-sin(f*x+e)
^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)^3*(4*c^2-15*c*d+27*d^2)/((-d*sin(f*
x+e)-c)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)+2*(-c*d^2-15*d^3)/(60*c^3-180*
c^2*d+180*c*d^2-60*d^3)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*
x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(
f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2
))-1/30*d*(4*c^2-15*c*d+27*d^2)/(c-d)^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1
/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*si
n(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d)
)^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))))+d^2*(-(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)
/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)-2*d/(2*c-2*d)*(c/d
-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*
x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-d/(c-d)*(c/d-1)*((c+d*sin
(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-
d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d
*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/
(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+2*d*(c-d)*(-1/3/(c-d)*(-(-d*sin(f*x+e)-
c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/3*(-sin(f*x+e)^2*d-c*sin(f*x+e)+d
*sin(f*x+e)+c)/(c-d)^2*(c-3*d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+sin(f*x
+e)))^(1/2)+2*d^2/(3*c^2-6*c*d+3*d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2
)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(
f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-
d)/(c+d))^(1/2))-1/3*d*(c-3*d)/(c-d)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/
2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))
^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)
/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c+d\,\sin\left(e+f\,x\right)\right)^{3/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^3,x)
[Out] int((c + d*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**3,x)
[Out] Timed out
```

3.518 $\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=334

$$-\frac{\left(4c^2-5cd-3d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{30f(c-d)^2\left(a^3\sin(e+fx)+a^3\right)}-\frac{\left(4c^2-5cd-3d^2\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{30a^3f(c-d)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] -1/5*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^3-1/15*(2*c-d)*co
s(f*x+e)*(c+d*sin(f*x+e))^(1/2)/a/(c-d)/f/(a+a*sin(f*x+e))^2-1/30*(4*c^2-5*
c*d-3*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(c-d)^2/f/(a^3+a^3*sin(f*x+e))
+1/30*(4*c^2-5*c*d-3*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4
*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(
c+d*sin(f*x+e))^(1/2)/a^3/(c-d)^2/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-1/30*(4*
c-5*d)*(c+d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*
EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+
e))/(c+d))^(1/2)/a^3/(c-d)/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.77, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2764, 2978, 2752, 2663, 2661, 2655, 2653}

$$-\frac{\left(4c^2-5cd-3d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{30f(c-d)^2\left(a^3\sin(e+fx)+a^3\right)}-\frac{\left(4c^2-5cd-3d^2\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{30a^3f(c-d)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -(Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(5*f*(a + a*Sin[e + f*x])^3) - ((2
*c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*a*(c - d)*f*(a + a*Sin[e
 + f*x])^2) - ((4*c^2 - 5*c*d - 3*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]
])/(30*(c - d)^2*f*(a^3 + a^3*Sin[e + f*x])) - ((4*c^2 - 5*c*d - 3*d^2)*Ell
ipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(30*a^3
*(c - d)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((4*c - 5*d)*(c + d)*Ell
ipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)
])/(30*a^3*(c - d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2764

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m
*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*
(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && L
tQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c,
 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
```

```
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx = -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3} + \frac{\int \frac{\frac{1}{2}a(4c+d)+\frac{3}{2}ad\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}\,dx}{5a^2}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3} - \frac{(2c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15a(c-d)f(a+a\sin(e+fx))^2} - \frac{\int -\frac{1}{2}a^2 \cdots}{\cdots}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3} - \frac{(2c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15a(c-d)f(a+a\sin(e+fx))^2} - \frac{(4c^2 - \cdots}{\cdots}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3} - \frac{(2c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15a(c-d)f(a+a\sin(e+fx))^2} - \frac{(4c^2 - \cdots}{\cdots}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3} - \frac{(2c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15a(c-d)f(a+a\sin(e+fx))^2} - \frac{(4c^2 - \cdots}{\cdots}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5f(a+a\sin(e+fx))^3} - \frac{(2c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15a(c-d)f(a+a\sin(e+fx))^2} - \frac{(4c^2 - \cdots}{\cdots}$$

Mathematica [A] time = 5.89, size = 449, normalized size = 1.34

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^6 \left(-\left(4c^2-5cd-3d^2\right)(c+d\sin(e+fx)) + \frac{2(c+d\sin(e+fx))\left(\left(4c^2-5cd-3d^2\right)\sin\left(\frac{1}{2}(e+f \cdots\right.\right.}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(-((4*c^2 - 5*c*d - 3*d^2)*(c + d*
Sin[e + f*x])) + (2*(6*(c - d)^2*Sin[(e + f*x)/2] - 3*(c - d)^2*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2]) + 2*(c - d)*(2*c - d)*Sin[(e + f*x)/2]*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^2 - (c - d)*(2*c - d)*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^3 + (4*c^2 - 5*c*d - 3*d^2)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^4)*(c + d*Sin[e + f*x]))/(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^5 + (c - 5*d)*d^2*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]
*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + (4*c^2 - 5*c*d - 3*d^2)*((c + d)*Elli
pticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*
x)/4, (2*d)/(c + d)])*Sqrt[(c + d*Sin[e + f*x])/(c + d)]))/(30*a^3*(c - d)^
2*f*(1 + Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{d\sin\left(fx+e\right)+c}}{3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*sin(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos
(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{d\sin\left(fx+e\right)+c}}{\left(a\sin\left(fx+e\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^3, x)
```

maple [B] time = 6.80, size = 1056, normalized size = 3.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^3*((c-d)*(-1/5/(c-d)*(-(-d*sin(f*
x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^3-2/15*(c-3*d)/(c-d)^2*(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/30*(-sin(f*x+e)^2*d-c*sin
(f*x+e)+d*sin(f*x+e)+c)/(c-d)^3*(4*c^2-15*c*d+27*d^2)/((-d*sin(f*x+e)-c)*(s
in(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)+2*(-c*d^2-15*d^3)/(60*c^3-180*c^2*d+180*
c*d^2-60*d^3)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d
))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^
(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-1/30*d*
(4*c^2-15*c*d+27*d^2)/(c-d)^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-
sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c
)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((
c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(
1/2))))+d*(-1/3/(c-d)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e)
)^2-1/3*(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)^2*(c-3*d)/((-d*
sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)+2*d^2/(3*c^2-6*c*d+3*d^2
)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-
sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Ellipt
icF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-1/3*d*(c-3*d)/(c-d)
^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((
-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/
d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+Elliptic
F(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*si
n(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d \sin\left(fx+e\right)+c}}{\left(a \sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c+d\,\sin\left(e+f\,x\right)}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^3,x)
```

```
[Out] int((c + d*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{c+d\sin(e+fx)}}{\sin^3(e+fx)+3\sin^2(e+fx)+3\sin(e+fx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Integral(sqrt(c + d*sin(e + f*x))/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*
sin(e + f*x) + 1), x)/a**3
```

3.519 $\int \frac{1}{(a+a\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=344

$$-\frac{\left(4c^2-15cd+27d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{30f(c-d)^3\left(a^3\sin(e+fx)+a^3\right)}+\frac{\left(4c^2-11cd+15d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{30a^3f(c-d)^2\sqrt{c+d\sin(e+fx)}}-$$

```
[Out] -1/5*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(c-d)/f/(a+a*sin(f*x+e))^3-2/15*(c-3
*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/a/(c-d)^2/f/(a+a*sin(f*x+e))^2-1/30*(
4*c^2-15*c*d+27*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(c-d)^3/f/(a^3+a^3*s
in(f*x+e))+1/30*(4*c^2-15*c*d+27*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/s
in(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+
d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/a^3/(c-d)^3/f/((c+d*sin(f*x+e))/(c+d))^(1
/2)-1/30*(4*c^2-11*c*d+15*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*
e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/
2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/a^3/(c-d)^2/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.77, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2766, 2978, 2752, 2663, 2661, 2655, 2653}

$$-\frac{\left(4c^2-15cd+27d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{30f(c-d)^3\left(a^3\sin(e+fx)+a^3\right)}+\frac{\left(4c^2-11cd+15d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{30a^3f(c-d)^2\sqrt{c+d\sin(e+fx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] -(Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(5*(c - d)*f*(a + a*Sin[e + f*x])^
3) - (2*(c - 3*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*a*(c - d)^2*f*
(a + a*Sin[e + f*x])^2) - ((4*c^2 - 15*c*d + 27*d^2)*Cos[e + f*x]*Sqrt[c +
d*Sin[e + f*x]])/(30*(c - d)^3*f*(a^3 + a^3*Sin[e + f*x])) - ((4*c^2 - 15*c
*d + 27*d^2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e
+ f*x]])/(30*a^3*(c - d)^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((4*c^2
- 11*c*d + 15*d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d
*Sin[e + f*x])/(c + d)])/(30*a^3*(c - d)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
```

```
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{\int \frac{-\frac{1}{2}a(4c-9d)-\frac{3}{2}ad\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}\,d}{5a^2(c-d)}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{2(c-3d)\cos(e+fx)\sqrt{c+d\,\mathrm{s}}}{15a(c-d)^2f(a+a\sin(e-}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{2(c-3d)\cos(e+fx)\sqrt{c+d\,\mathrm{s}}}{15a(c-d)^2f(a+a\sin(e-}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{2(c-3d)\cos(e+fx)\sqrt{c+d\,\mathrm{s}}}{15a(c-d)^2f(a+a\sin(e-}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{2(c-3d)\cos(e+fx)\sqrt{c+d\,\mathrm{s}}}{15a(c-d)^2f(a+a\sin(e-}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{2(c-3d)\cos(e+fx)\sqrt{c+d\,\mathrm{s}}}{15a(c-d)^2f(a+a\sin(e-}$$

Mathematica [A] time = 6.32, size = 445, normalized size = 1.29

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^6\left(-\left(4c^2-15cd+27d^2\right)(c+d\sin(e+fx))+\frac{2(c+d\sin(e+fx))\left(\left(4c^2-15cd+27d^2\right)\sin\left(\right.\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(-((4*c^2 - 15*c*d + 27*d^2)*(c +
d*Sin[e + f*x])) + (2*(6*(c - d)^2*Sin[(e + f*x)/2] - 3*(c - d)^2*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2]) + 4*(c - 3*d)*(c - d)*Sin[(e + f*x)/2]*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^2 - 2*(c - 3*d)*(c - d)*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^3 + (4*c^2 - 15*c*d + 27*d^2)*Sin[(e + f*x)/2]*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^4)*(c + d*Sin[e + f*x]))/(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^5 + d^2*(c + 15*d)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(
c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + (4*c^2 - 15*c*d + 27*d^2)*((c
+ d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e +
Pi - 2*f*x)/4, (2*d)/(c + d)])*Sqrt[(c + d*Sin[e + f*x])/(c + d)]))/(30*a^3
*(c - d)^3*f*(1 + Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\sin\left(fx+e\right)+c}}{a^3d\cos\left(fx+e\right)^4+4\,a^3c+4\,a^3d-\left(3\,a^3c+5\,a^3d\right)\cos\left(fx+e\right)^2+\left(4\,a^3c+4\,a^3d-\left(a^3c+3\,a^3d\right)\cos\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d*sin(f*x + e) + c)/(a^3*d*cos(f*x + e)^4 + 4*a^3*c + 4*a^3*d
 - (3*a^3*c + 5*a^3*d)*cos(f*x + e)^2 + (4*a^3*c + 4*a^3*d - (a^3*c + 3*a^3
*d)*cos(f*x + e)^2)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^3\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^3*sqrt(d*sin(f*x + e) + c)), x)
```

maple [A] time = 4.63, size = 593, normalized size = 1.72

$$\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^2\left(fx+e\right)\right)}\left(-\frac{\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^2\left(fx+e\right)\right)}}{5(c-d)\left(1+\sin\left(fx+e\right)\right)^3}-\frac{2(c-3d)\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^2\left(fx+e\right)\right)}}{15(c-d)^2\left(1+\sin\left(fx+e\right)\right)^2}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^3*(-1/5/(c-d)*(-(-d*sin(f*x+e)-c)
*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^3-2/15*(c-3*d)/(c-d)^2*(-(-d*sin(f*x+e)
-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/30*(-sin(f*x+e)^2*d-c*sin(f*x+e)
+d*sin(f*x+e)+c)/(c-d)^3*(4*c^2-15*c*d+27*d^2)/((-d*sin(f*x+e)-c)*(sin(f*x+
e)-1)*(1+sin(f*x+e)))^(1/2)+2*(-c*d^2-15*d^3)/(60*c^3-180*c^2*d+180*c*d^2-6
0*d^3)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2
)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*E
llipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-1/30*d*(4*c^2-
15*c*d+27*d^2)/(c-d)^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x
+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f
*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c
+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))
/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{3}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^3*sqrt(d*sin(f*x + e) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{c+d\sin\left(e+fx\right)}\sin^{3}\left(e+fx\right)+3\sqrt{c+d\sin\left(e+fx\right)}\sin^{2}\left(e+fx\right)+3\sqrt{c+d\sin\left(e+fx\right)}\sin\left(e+fx\right)+\sqrt{c+d\sin\left(e+fx\right)}}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c + d*sin(e + f*x))*sin(e + f*x)**3 + 3*sqrt(c + d*sin(e +
 f*x))*sin(e + f*x)**2 + 3*sqrt(c + d*sin(e + f*x))*sin(e + f*x) + sqrt(c +
 d*sin(e + f*x))), x)/a**3
```

3.520 $\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=423

$$-\frac{\left(4c^2-21cd+65d^2\right)\cos(e+fx)}{30f(c-d)^3\left(a^3\sin(e+fx)+a^3\right)\sqrt{c+d\sin(e+fx)}}+\frac{\left(4c^2-21cd+65d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{30a^3f(c-d)^3\sqrt{c+d\sin(e+fx)}}$$

```
[Out] -1/30*d*(4*c^3-21*c^2*d+62*c*d^2+147*d^3)*cos(f*x+e)/a^3/(c-d)^4/(c+d)/f/(c
+d*sin(f*x+e))^(1/2)-1/5*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^3/(c+d*sin(f*x
+e))^(1/2)-2/15*(c-4*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^2/(c+d*sin(
f*x+e))^(1/2)-1/30*(4*c^2-21*c*d+65*d^2)*cos(f*x+e)/(c-d)^3/f/(a^3+a^3*sin(
f*x+e))/(c+d*sin(f*x+e))^(1/2)+1/30*(4*c^3-21*c^2*d+62*c*d^2+147*d^3)*(sin(
1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*
e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/a^3/(c-d)
^4/(c+d)/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-1/30*(4*c^2-21*c*d+65*d^2)*(sin(1
/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e
+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/a^
3/(c-d)^3/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.03, antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2766, 2978, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{d\left(-21c^2d+4c^3+62cd^2+147d^3\right)\cos(e+fx)}{30a^3f(c-d)^4(c+d)\sqrt{c+d\sin(e+fx)}}-\frac{\left(4c^2-21cd+65d^2\right)\cos(e+fx)}{30f(c-d)^3\left(a^3\sin(e+fx)+a^3\right)\sqrt{c+d\sin(e+fx)}}+\frac{\left(4c^2-2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -(d*(4*c^3 - 21*c^2*d + 62*c*d^2 + 147*d^3)*Cos[e + f*x])/(30*a^3*(c - d)^4
*(c + d)*f*Sqrt[c + d*Sin[e + f*x]]) - Cos[e + f*x]/(5*(c - d)*f*(a + a*Sin
[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]]) - (2*(c - 4*d)*Cos[e + f*x])/(15*a*(
c - d)^2*f*(a + a*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]]) - ((4*c^2 - 21*
c*d + 65*d^2)*Cos[e + f*x])/(30*(c - d)^3*f*(a^3 + a^3*Sin[e + f*x])*Sqrt[c
 + d*Sin[e + f*x]]) - ((4*c^3 - 21*c^2*d + 62*c*d^2 + 147*d^3)*EllipticE[(e
 - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(30*a^3*(c - d)^
4*(c + d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((4*c^2 - 21*c*d + 65*d^2
)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c
 + d)])/(30*a^3*(c - d)^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
```

```
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}} - \frac{\int \frac{-\frac{1}{2}a(\cdots}{(a+a\sin\cdots}}{\cdots}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}} - \frac{\cdots}{15a(c-\cdots}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}} - \frac{\cdots}{15a(c-\cdots}$$

$$= -\frac{d\left(4c^3-21c^2d+62cd^2+147d^3\right)\cos(e+fx)}{30a^3(c-d)^4(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{\cdots}{5(c-d)f(a\cdots}$$

$$= -\frac{d\left(4c^3-21c^2d+62cd^2+147d^3\right)\cos(e+fx)}{30a^3(c-d)^4(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{\cdots}{5(c-d)f(a\cdots}$$

$$= -\frac{d\left(4c^3-21c^2d+62cd^2+147d^3\right)\cos(e+fx)}{30a^3(c-d)^4(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{\cdots}{5(c-d)f(a\cdots}$$

$$= -\frac{d\left(4c^3-21c^2d+62cd^2+147d^3\right)\cos(e+fx)}{30a^3(c-d)^4(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{\cdots}{5(c-d)f(a\cdots}$$

Mathematica [A] time = 6.54, size = 745, normalized size = 1.76

$$\frac{d\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^6\left(-\frac{2\left(-c^2d-126cd^2-65d^3\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\middle|\frac{2d}{c+d}\right)}{\sqrt{c+d\sin(e+fx)}} + \frac{2\left(4c^3-21c^2d+62cd^2+147d^3\right)\cdots}{d\left(1-\sin^2\cdots\right.}\cdots\right.}{60f(c-d)^4(c+d)(a\,\mathrm{s}\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*Sqrt[c + d*Sin[e + f*x]]*(-1/15*(4
*c^3 - 21*c^2*d + 62*c*d^2 + 117*d^3)/((c - d)^4*(c + d)) + (2*Sin[(e + f*x
```

```
)/2])/(5*(c - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) - 1/(5*(c - d)^
2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + (-2*c + 11*d)/(15*(c - d)^3*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^2) + (2*(2*c*Sin[(e + f*x)/2] - 11*d*Si
n[(e + f*x)/2]))/(15*(c - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3) + (
4*c^2*Sin[(e + f*x)/2] - 25*c*d*Sin[(e + f*x)/2] + 87*d^2*Sin[(e + f*x)/2])
/(15*(c - d)^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])) - (2*d^4*Cos[e + f*x]
)/((c - d)^4*(c + d)*(c + d*Sin[e + f*x]))))/(f*(a + a*Sin[e + f*x])^3) + (
d*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*((-2*(-(c^2*d) - 126*c*d^2 - 65*d
^3)*EllipticF[(-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])
/(c + d)])/Sqrt[c + d*Sin[e + f*x]] + (2*(4*c^3 - 21*c^2*d + 62*c*d^2 + 147
*d^3)*Cos[e + f*x]^2*Sqrt[c + d*Sin[e + f*x]])/(d*(1 - Sin[e + f*x]^2)) - (
(-4*c^3 + 21*c^2*d - 62*c*d^2 - 147*d^3)*((2*(c + d)*EllipticE[(-e + Pi/2 -
 f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/Sqrt[c + d*Sin[
e + f*x]] - (2*c*EllipticF[(-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*
Sin[e + f*x])/(c + d)])/Sqrt[c + d*Sin[e + f*x]]))/d))/(60*(c - d)^4*(c + d
)*f*(a + a*Sin[e + f*x])^3)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\sin\left(fx\right.}}{4\,a^3c^2+8\,a^3cd+4\,a^3d^2+\left(2\,a^3cd+3\,a^3d^2\right)\cos\left(fx+e\right)^4-\left(3\,a^3c^2+10\,a^3cd+7\,a^3d^2\right)\cos\left(fx+e\right)^2+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d*sin(f*x + e) + c)/(4*a^3*c^2 + 8*a^3*c*d + 4*a^3*d^2 + (2*a
^3*c*d + 3*a^3*d^2)*cos(f*x + e)^4 - (3*a^3*c^2 + 10*a^3*c*d + 7*a^3*d^2)*c
os(f*x + e)^2 + (a^3*d^2*cos(f*x + e)^4 + 4*a^3*c^2 + 8*a^3*c*d + 4*a^3*d^2
 - (a^3*c^2 + 6*a^3*c*d + 5*a^3*d^2)*cos(f*x + e)^2)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^3\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^(3/2)), x)
```

maple [B] time = 8.77, size = 1851, normalized size = 4.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^3*(1/(c-d)*(-1/5/(c-d)*(-(-d*sin(
f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^3-2/15*(c-3*d)/(c-d)^2*(-(-d*s
in(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/30*(-sin(f*x+e)^2*d-c*s
in(f*x+e)+d*sin(f*x+e)+c)/(c-d)^3*(4*c^2-15*c*d+27*d^2)/((-d*sin(f*x+e)-c)*
(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)+2*(-c*d^2-15*d^3)/(60*c^3-180*c^2*d+18
0*c*d^2-60*d^3)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c
+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2
)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-1/30*
d*(4*c^2-15*c*d+27*d^2)/(c-d)^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(
1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)
-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))
^(1/2))))-d^3/(c-d)^3*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f
*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-s
in(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)
*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))
^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+
e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*
x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+
d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+
d^2/(c-d)^3*(-(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)/((-d*sin(
f*x+e)-c)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)-2*d/(2*c-2*d)*(c/d-1)*((c+d*
sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/
(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*
x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-d/(c-d)*(c/d-1)*((c+d*sin(f*x+e))/(
c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/
2),((c-d)/(c+d))^(1/2))))-d/(c-d)^2*(-1/3/(c-d)*(-(-d*sin(f*x+e)-c)*cos(f*x
+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/3*(-sin(f*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e
)+c)/(c-d)^2*(c-3*d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2
)+2*d^2/(3*c^2-6*c*d+3*d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-si
n(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*
cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^
(1/2))-1/3*d*(c-3*d)/(c-d)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-s
in(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)
*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c
-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1
/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^(3/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c\sqrt{c+d\sin\left(e+fx\right)}\sin^3\left(e+fx\right)+3c\sqrt{c+d\sin\left(e+fx\right)}\sin^2\left(e+fx\right)+3c\sqrt{c+d\sin\left(e+fx\right)}\sin\left(e+fx\right)+c\sqrt{c+d\sin\left(e+fx\right)}+d\sqrt{c+d\sin\left(e+fx\right)}\sin^4\left(e+f\right.}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/(c*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**3 + 3*c*sqrt(c + d*sin
(e + f*x))*sin(e + f*x)**2 + 3*c*sqrt(c + d*sin(e + f*x))*sin(e + f*x) + c*
sqrt(c + d*sin(e + f*x)) + d*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**4 + 3*d
*sqrt(c + d*sin(e + f*x))*sin(e + f*x)**3 + 3*d*sqrt(c + d*sin(e + f*x))*si
n(e + f*x)**2 + d*sqrt(c + d*sin(e + f*x))*sin(e + f*x)), x)/a**3
```

3.521 $\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=518

$$-\frac{\left(4c^2-27cd+119d^2\right)\cos(e+fx)}{30f(c-d)^3\left(a^3\sin(e+fx)+a^3\right)(c+d\sin(e+fx))^{3/2}}-\frac{d\left(4c^3-27c^2d+114cd^2+165d^3\right)\cos(e+fx)}{30a^3f(c-d)^4(c+d)(c+d\sin(e+fx))^{3/2}}+\frac{\left(4c^3-\right.}{}$$

```
[Out] -1/30*d*(4*c^3-27*c^2*d+114*c*d^2+165*d^3)*cos(f*x+e)/a^3/(c-d)^4/(c+d)/f/(
c+d*sin(f*x+e))^(3/2)-1/5*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^3/(c+d*sin(f*
x+e))^(3/2)-2/15*(c-5*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^2/(c+d*sin
(f*x+e))^(3/2)-1/30*(4*c^2-27*c*d+119*d^2)*cos(f*x+e)/(c-d)^3/f/(a^3+a^3*si
n(f*x+e))/(c+d*sin(f*x+e))^(3/2)-1/30*d*(4*c^4-27*c^3*d+111*c^2*d^2+579*c*d
^3+357*d^4)*cos(f*x+e)/a^3/(c-d)^5/(c+d)^2/f/(c+d*sin(f*x+e))^(1/2)+1/30*(4
*c^4-27*c^3*d+111*c^2*d^2+579*c*d^3+357*d^4)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^
(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)
*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/a^3/(c-d)^5/(c+d)^2/f/((c+d*sin(f*
x+e))/(c+d))^(1/2)-1/30*(4*c^3-27*c^2*d+114*c*d^2+165*d^3)*(sin(1/2*e+1/4*P
i+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/
2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/a^3/(c-d)^4/
(c+d)/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.21, antiderivative size = 518, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2766, 2978, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{d\left(111c^2d^2-27c^3d+4c^4+579cd^3+357d^4\right)\cos(e+fx)}{30a^3f(c-d)^5(c+d)^2\sqrt{c+d\sin(e+fx)}}-\frac{d\left(-27c^2d+4c^3+114cd^2+165d^3\right)\cos(e+fx)}{30a^3f(c-d)^4(c+d)(c+d\sin(e+fx))^{3/2}}-\frac{}{30}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] -(d*(4*c^3 - 27*c^2*d + 114*c*d^2 + 165*d^3)*Cos[e + f*x])/(30*a^3*(c - d)^
4*(c + d)*f*(c + d*Sin[e + f*x])^(3/2)) - Cos[e + f*x]/(5*(c - d)*f*(a + a*
Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(3/2)) - (2*(c - 5*d)*Cos[e + f*x])/(1
5*a*(c - d)^2*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2)) - ((4*c^
2 - 27*c*d + 119*d^2)*Cos[e + f*x])/(30*(c - d)^3*f*(a^3 + a^3*Sin[e + f*x]
)*(c + d*Sin[e + f*x])^(3/2)) - (d*(4*c^4 - 27*c^3*d + 111*c^2*d^2 + 579*c*
d^3 + 357*d^4)*Cos[e + f*x])/(30*a^3*(c - d)^5*(c + d)^2*f*Sqrt[c + d*Sin[e
 + f*x]]) - ((4*c^4 - 27*c^3*d + 111*c^2*d^2 + 579*c*d^3 + 357*d^4)*Ellipti
cE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(30*a^3*(c
```

```
- d)^5*(c + d)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((4*c^3 - 27*c^2*d
 + 114*c*d^2 + 165*d^3)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(
c + d*Sin[e + f*x])/(c + d)])/(30*a^3*(c - d)^4*(c + d)*f*Sqrt[c + d*Sin[e
+ f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
```

```
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}a(}{(a+a\sin}}{}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}} - \frac{}{15a(c-}$$

$$= -\frac{\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}} - \frac{}{15a(c-}$$

$$= -\frac{d\left(4c^3-27c^2d+114cd^2+165d^3\right)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{}{5(c-d)f(a+}$$

$$= -\frac{d\left(4c^3-27c^2d+114cd^2+165d^3\right)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{}{5(c-d)f(a+}$$

$$= -\frac{d\left(4c^3-27c^2d+114cd^2+165d^3\right)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{}{5(c-d)f(a+}$$

$$= -\frac{d\left(4c^3-27c^2d+114cd^2+165d^3\right)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{}{5(c-d)f(a+}$$

$$= -\frac{d\left(4c^3-27c^2d+114cd^2+165d^3\right)\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{}{5(c-d)f(a+}$$

Mathematica [A] time = 6.92, size = 828, normalized size = 1.60

$$\frac{\sqrt{c+d\sin(e+fx)}\left(-\frac{2\cos(e+fx)d^4}{3(c-d)^4(c+d)(c+d\sin(e+fx))^2} - \frac{4c^4-27dc^3+111d^2c^2+449d^3c+267d^4}{15(c-d)^5(c+d)^2} + \frac{4\left(c\sin\left(\frac{1}{2}(e+fx)\right)-8d\sin\left(\frac{1}{2}(e+fx)\right)\right)}{15(c-d)^4\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3} + \right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*Sqrt[c + d*Sin[e + f*x]]*(-1/15*(4
*c^4 - 27*c^3*d + 111*c^2*d^2 + 449*c*d^3 + 267*d^4)/((c - d)^5*(c + d)^2)
+ (2*Sin[(e + f*x)/2])/(5*(c - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5
) - 1/(5*(c - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) - (2*(c - 8*d))
/(15*(c - d)^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2) + (4*(c*Sin[(e + f*
x)/2] - 8*d*Sin[(e + f*x)/2]))/(15*(c - d)^4*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^3) + (4*c^2*Sin[(e + f*x)/2] - 35*c*d*Sin[(e + f*x)/2] + 177*d^2*Si
n[(e + f*x)/2])/(15*(c - d)^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])) - (2*d
^4*Cos[e + f*x])/(3*(c - d)^4*(c + d)*(c + d*Sin[e + f*x])^2) - (2*(13*c*d^
4*Cos[e + f*x] + 9*d^5*Cos[e + f*x]))/(3*(c - d)^5*(c + d)^2*(c + d*Sin[e +
 f*x]))))/(f*(a + a*Sin[e + f*x])^3) + (d*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2])^6*((-2*(-(c^3*d) - 387*c^2*d^2 - 471*c*d^3 - 165*d^4)*EllipticF[(-e +
Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/Sqrt[c +
d*Sin[e + f*x]] + (2*(4*c^4 - 27*c^3*d + 111*c^2*d^2 + 579*c*d^3 + 357*d^4)
*Cos[e + f*x]^2*Sqrt[c + d*Sin[e + f*x]])/(d*(1 - Sin[e + f*x]^2)) - ((-4*c
^4 + 27*c^3*d - 111*c^2*d^2 - 579*c*d^3 - 357*d^4)*((2*(c + d)*EllipticE[(-
e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/Sqrt[
c + d*Sin[e + f*x]] - (2*c*EllipticF[(-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sq
rt[(c + d*Sin[e + f*x])/(c + d)])/Sqrt[c + d*Sin[e + f*x]]))/d))/(60*(c - d
)^5*(c + d)^2*f*(a + a*Sin[e + f*x])^3)
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{}{a^3d^3\cos\left(fx+e\right)^6-4\,a^3c^3-12\,a^3c^2d-12\,a^3cd^2-4\,a^3d^3-3\left(a^3c^2d+3\,a^3cd^2+2\,a^3d^3\right)\cos\left(fx+e\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(d*sin(f*x + e) + c)/(a^3*d^3*cos(f*x + e)^6 - 4*a^3*c^3 - 12
*a^3*c^2*d - 12*a^3*c*d^2 - 4*a^3*d^3 - 3*(a^3*c^2*d + 3*a^3*c*d^2 + 2*a^3*
d^3)*cos(f*x + e)^4 + 3*(a^3*c^3 + 5*a^3*c^2*d + 7*a^3*c*d^2 + 3*a^3*d^3)*c
os(f*x + e)^2 - (4*a^3*c^3 + 12*a^3*c^2*d + 12*a^3*c*d^2 + 4*a^3*d^3 + 3*(a
^3*c*d^2 + a^3*d^3)*cos(f*x + e)^4 - (a^3*c^3 + 9*a^3*c^2*d + 15*a^3*c*d^2
+ 7*a^3*d^3)*cos(f*x + e)^2)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^3\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")

[Out] integrate(1/((a*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^(5/2)), x)

maple [B] time = 11.46, size = 2311, normalized size = 4.46

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x)

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/a^3*(1/(c-d)^2*(-1/5/(c-d)*(-(-d*si
n(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^3-2/15*(c-3*d)/(c-d)^2*(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/30*(-sin(f*x+e)^2*d-c
*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)^3*(4*c^2-15*c*d+27*d^2)/((-d*sin(f*x+e)-c
)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)+2*(-c*d^2-15*d^3)/(60*c^3-180*c^2*d+
180*c*d^2-60*d^3)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/
(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)
^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-1/3
0*d*(4*c^2-15*c*d+27*d^2)/(c-d)^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d
*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+
e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d
))^(1/2))))-d^3/(c-d)^3*(2/3/(c^2-d^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^
(1/2)/(sin(f*x+e)+c/d)^2+8/3*d*cos(f*x+e)^2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-
c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*c^2*d^2+3*d^4)*(c/d-1)*((c+d*
sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/
(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*
x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+8/3*c*d/(c^2-d^2)^2*(c/d-1)*((c+d*s
in(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(
c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c
+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e)
)/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))-3*d^3/(c-d)^4*(2*d*cos(f*x+e)^2/(c^2-
d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*si
n(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c
-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+
e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e
))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1
/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f
*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))
^(1/2),((c-d)/(c+d))^(1/2))))+3/(c-d)^4*d^2*(-(-sin(f*x+e)^2*d-c*sin(f*x+e)
+d*sin(f*x+e)+c)/(c-d)/((-d*sin(f*x+e)-c)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1
/2)-2*d/(2*c-2*d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/
```

```
(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)
^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-d/(
c-d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*
((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-
c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+Ellipt
icF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))-2/(c-d)^3*d*(-1/3
/(c-d)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(1+sin(f*x+e))^2-1/3*(-sin(f
*x+e)^2*d-c*sin(f*x+e)+d*sin(f*x+e)+c)/(c-d)^2*(c-3*d)/((-d*sin(f*x+e)-c)*(
sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)+2*d^2/(3*c^2-6*c*d+3*d^2)*(c/d-1)*((c+d
*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d
/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f
*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-1/3*d*(c-3*d)/(c-d)^2*(c/d-1)*((c+
d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*
d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(
((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x
+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)
/f
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^(5/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.522 $\int \sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=161

$$-\frac{4a(c+d)\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{35f\sqrt{a\sin(e+fx)+a}}-\frac{12d^2(c+d)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{35af}-\frac{2a\cos(e+fx)(c+d\,\mathrm{s}}{7f\sqrt{a\sin(e+f}}$$

```
[Out] -12/35*d^2*(c+d)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/a/f-4/35*a*(c+d)*(15*c^2
+10*c*d+7*d^2)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/7*a*cos(f*x+e)*(c+d*si
n(f*x+e))^3/f/(a+a*sin(f*x+e))^(1/2)-8/35*(5*c-d)*d*(c+d)*cos(f*x+e)*(a+a*s
in(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.28, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2770, 2761, 2751, 2646}

$$-\frac{4a(c+d)\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{35f\sqrt{a\sin(e+fx)+a}}-\frac{12d^2(c+d)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{35af}-\frac{2a\cos(e+fx)(c+d\,\mathrm{s}}{7f\sqrt{a\sin(e+f}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (-4*a*(c + d)*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*x])/(35*f*Sqrt[a + a*Sin[
e + f*x]]) - (8*(5*c - d)*d*(c + d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/
(35*f) - (12*d^2*(c + d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(35*a*f)
- (2*a*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(7*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^3\,dx = -\frac{2a\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a+a\sin(e+fx)}} + \frac{1}{7}(6(c+d)) \int \sqrt{a+a\sin$$

$$= -\frac{12d^2(c+d)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{35af} - \frac{2a\cos(e+fx)}{7f\sqrt{a+}}$$

$$= -\frac{8(5c-d)d(c+d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{35f} - \frac{12d^2(c+d}{}$$

$$= -\frac{4a(c+d)\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{35f\sqrt{a+a\sin(e+fx)}} - \frac{8(5c-d)d(c+d)}{}$$

Mathematica [A] time = 0.52, size = 146, normalized size = 0.91

$$-\frac{\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(140c^3+d\left(140c^2+112cd+47d^2\right)\sin(e+fx)+280\right.}{70f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^3,x]

[Out] -1/70*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(14
0*c^3 + 280*c^2*d + 266*c*d^2 + 76*d^3 - 6*d^2*(7*c + 2*d)*Cos[2*(e + f*x)]
```

```
 + d*(140*c^2 + 112*c*d + 47*d^2)*Sin[e + f*x] - 5*d^3*Sin[3*(e + f*x)]))/(
f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

fricas [A] time = 0.44, size = 240, normalized size = 1.49

$$\frac{2\left(5\,d^{3}\cos\left(fx+e\right)^{4}+3\left(7\,cd^{2}+2\,d^{3}\right)\cos\left(fx+e\right)^{3}-35\,c^{3}-35\,c^{2}d-49\,cd^{2}-9\,d^{3}-\left(35\,c^{2}d+7\,cd^{2}+12\,d^{3}\right)\cos\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 2/35*(5*d^3*cos(f*x + e)^4 + 3*(7*c*d^2 + 2*d^3)*cos(f*x + e)^3 - 35*c^3 -
35*c^2*d - 49*c*d^2 - 9*d^3 - (35*c^2*d + 7*c*d^2 + 12*d^3)*cos(f*x + e)^2
- (35*c^3 + 70*c^2*d + 77*c*d^2 + 22*d^3)*cos(f*x + e) + (5*d^3*cos(f*x + e
)^3 + 35*c^3 + 35*c^2*d + 49*c*d^2 + 9*d^3 - (21*c*d^2 + d^3)*cos(f*x + e)^
2 - (35*c^2*d + 28*c*d^2 + 13*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f
*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*(2*f*
(4*c^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+6*c*d^2*sign(cos(1/2*(f*x+exp(1))
-1/4*pi)))*sin(1/4*(2*f*x-pi)+1/2*exp(1))/(2*f)^2-8*f*(6*d^3*sign(cos(1/2*(
f*x+exp(1))-1/4*pi))+24*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(
2*f*x+2*exp(1)+pi))/(8*f)^2-24*f*(6*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+
24*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(6*f*x+6*exp(1)-pi))/(
24*f)^2+80*d^3*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(10*f*x+10*exp(
1)+pi))/(40*f)^2+112*d^3*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(14*f
*x+14*exp(1)-pi))/(56*f)^2-72*c*d^2*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*si
n(1/4*(6*f*x+6*exp(1)+pi))/(12*f)^2-120*c*d^2*f*sign(cos(1/2*(f*x+exp(1))-1
/4*pi))*sin(1/4*(10*f*x+10*exp(1)-pi))/(20*f)^2)
```

maple [A] time = 0.92, size = 141, normalized size = 0.88

$$\frac{2\left(1+\sin\left(fx+e\right)\right)a\left(\sin\left(fx+e\right)-1\right)\left(5d^3\left(\sin^3\left(fx+e\right)\right)+21c\,d^2\left(\sin^2\left(fx+e\right)\right)+6d^3\left(\sin^2\left(fx+e\right)\right)+35}{35\cos\left(fx+e\right)\sqrt{a+a\sin\left(f\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^3,x)
```

```
[Out] 2/35*(1+sin(f*x+e))*a*(sin(f*x+e)-1)*(5*d^3*sin(f*x+e)^3+21*c*d^2*sin(f*x+e
)^2+6*d^3*sin(f*x+e)^2+35*c^2*d*sin(f*x+e)+28*c*d^2*sin(f*x+e)+8*d^3*sin(f*
x+e)+35*c^3+70*c^2*d+56*c*d^2+16*d^3)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(c+d\,\sin\left(e+f\,x\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^3,x)
```

```
[Out] int((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\left(c+d\sin\left(e+fx\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(c + d*sin(e + f*x))**3, x)
```

3.523 $\int \sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=112

$$-\frac{2a\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{4d(5c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)-}{5af}$$

```
[Out] -2/5*d^2*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/a/f-2/15*a*(15*c^2+10*c*d+7*d^2)
*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-4/15*(5*c-d)*d*cos(f*x+e)*(a+a*sin(f*x
+e))^(1/2)/f
```

Rubi [A] time = 0.17, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2761, 2751, 2646}

$$-\frac{2a\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{4d(5c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)-}{5af}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (-2*a*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]
]) - (4*(5*c - d)*d*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) - (2*d^2*
Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*a*f)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
```

```
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2\,dx = -\frac{2d^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5af} + \frac{2\int \sqrt{a+a\sin(e+fx)}}{}$$

$$= -\frac{4(5c-d)d\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} - \frac{2d^2\cos(e+fx)(a}{5}$$

$$= -\frac{2a\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{4(5c-d)d\cos(e+fx)\sqrt{}}{15f}$$

Mathematica [A] time = 0.29, size = 111, normalized size = 0.99

$$-\frac{\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(30c^2+4d(5c+2d)\sin(e+fx)+40cd-3d^2\cos(2(e\right.}{15f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -1/15*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(30
*c^2 + 40*c*d + 19*d^2 - 3*d^2*Cos[2*(e + f*x)] + 4*d*(5*c + 2*d)*Sin[e + f
*x]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

fricas [A] time = 0.47, size = 157, normalized size = 1.40

$$\frac{2\left(3\,d^2\cos\left(fx+e\right)^3-\left(10\,cd+d^2\right)\cos\left(fx+e\right)^2-15\,c^2-10\,cd-7\,d^2-\left(15\,c^2+20\,cd+11\,d^2\right)\cos\left(fx+e\right)-\right.}{15\left(f\cos\left(fx+e\right)+f\,s\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 2/15*(3*d^2*cos(f*x + e)^3 - (10*c*d + d^2)*cos(f*x + e)^2 - 15*c^2 - 10*c*
d - 7*d^2 - (15*c^2 + 20*c*d + 11*d^2)*cos(f*x + e) - (3*d^2*cos(f*x + e)^2
```

```
 - 15*c^2 - 10*c*d - 7*d^2 + 2*(5*c*d + 2*d^2)*cos(f*x + e))*sin(f*x + e))*
sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*(2*f*(4*c^2*sign(cos(1/2*(f*x+e
xp(1))-1/4*pi))+2*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(2*f*x-pi
)+1/2*exp(1))/(2*f)^2-2*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(2*f
*x+2*exp(1)+pi))/f-24*d^2*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(6*f
*x+6*exp(1)+pi))/(12*f)^2-40*d^2*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(1
/4*(10*f*x+10*exp(1)-pi))/(20*f)^2-6*c*d*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi
))*cos(1/4*(6*f*x+6*exp(1)-pi))/(3*f)^2)
```

maple [A] time = 0.81, size = 92, normalized size = 0.82

$$\frac{2\left(1+\sin\left(fx+e\right)\right)a\left(\sin\left(fx+e\right)-1\right)\left(3d^{2}\left(\sin^{2}\left(fx+e\right)\right)+10cd\sin\left(fx+e\right)+4d^{2}\sin\left(fx+e\right)+15c^{2}+20cd}{15\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2/15*(1+sin(f*x+e))*a*(sin(f*x+e)-1)*(3*d^2*sin(f*x+e)^2+10*c*d*sin(f*x+e)+
4*d^2*sin(f*x+e)+15*c^2+20*c*d+8*d^2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + a\,\sin\left(e + f\,x\right)}\,\left(c + d\,\sin\left(e + f\,x\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^2,x)
[Out] int((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e + fx\right) + 1\right)}\left(c + d\sin\left(e + fx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e))**2,x)
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(c + d*sin(e + f*x))**2, x)
```

3.524 $\int \sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=62

$$-\frac{2a(3c+d)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2d\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

```
[Out] -2/3*a*(3*c+d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/3*d*cos(f*x+e)*(a+a*si
n(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2751, 2646}

$$-\frac{2a(3c+d)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2d\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]),x]
```

```
[Out] (-2*a*(3*c + d)*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*d*Cos[e +
 f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))\,dx = -\frac{2d\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f}+\frac{1}{3}(3c+d)\int \sqrt{a+a\sin(e$$

$$= -\frac{2a(3c+d)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}-\frac{2d\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f}$$

Mathematica [A] time = 0.12, size = 82, normalized size = 1.32

$$-\frac{2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(3c+d\sin(e+fx)+2d)}{3f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]),x]
```

```
[Out] (-2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(3*c +
 2*d + d*Sin[e + f*x]))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

fricas [A] time = 0.46, size = 85, normalized size = 1.37

$$-\frac{2\left(d\cos\left(fx+e\right)^{2}+(3\,c+2\,d)\cos\left(fx+e\right)+\left(d\cos\left(fx+e\right)-3\,c-d\right)\sin\left(fx+e\right)+3\,c+d\right)\sqrt{a\sin\left(fx+e\right.}}{3\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -2/3*(d*cos(f*x + e)^2 + (3*c + 2*d)*cos(f*x + e) + (d*cos(f*x + e) - 3*c -
 d)*sin(f*x + e) + 3*c + d)*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*si
n(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
```

```
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*(2*c*sign(cos
(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(2*f*x-pi)+1/2*exp(1))/f-4*d*f*sign(cos(
1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(2*f*x+2*exp(1)+pi))/(2*f)^2-12*d*f*sign(
cos(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(6*f*x+6*exp(1)-pi))/(6*f)^2)
```

maple [A] time = 0.64, size = 58, normalized size = 0.94

$$\frac{2\left(1+\sin\left(fx+e\right)\right)a\left(\sin\left(fx+e\right)-1\right)\left(d\sin\left(fx+e\right)+3c+2d\right)}{3\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e)),x)
```

```
[Out] 2/3*(1+sin(f*x+e))*a*(sin(f*x+e)-1)*(d*sin(f*x+e)+3*c+2*d)/cos(f*x+e)/(a+a*
sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(c+d\,\sin\left(e+f\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x)),x)
```

```
[Out] int((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(c+d\sin\left(e+fx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(c + d*sin(e + f*x)), x)
```

3.525 $\int \sqrt{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=26

$$-\frac{2a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -2*a*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2646}

$$-\frac{2a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (-2*a*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,dx=-\frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [B] time = 0.04, size = 65, normalized size = 2.50

$$\frac{2\sqrt{a(\sin(e+fx)+1)}\left(\sin\left(\frac{1}{2}(e+fx)\right)-\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (2*(-Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x]])/(f*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

fricas [B] time = 0.43, size = 50, normalized size = 1.92

$$-\frac{2\sqrt{a\sin\left(fx+e\right)+a}\left(\cos\left(fx+e\right)-\sin\left(fx+e\right)+1\right)}{f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/(f*cos(f*x +
e) + f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2*sqrt(2*a)*sign(cos(1/2*(f*x+exp(1)
)-1/4*pi))*sin(1/2*(f*x+exp(1))-1/4*pi)/f
```

maple [A] time = 0.51, size = 43, normalized size = 1.65

$$\frac{2\left(1+\sin\left(fx+e\right)\right)a\left(\sin\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 2*(1+sin(f*x+e))*a*(sin(f*x+e)-1)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a), x)
```

mupad [B] time = 7.42, size = 33, normalized size = 1.27

$$-\frac{2\cos\left(e+f\,x\right)\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}}{f\,\left(\sin\left(e+f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2),x)
[Out] -(2*cos(e + f*x)*(a*(sin(e + f*x) + 1))^(1/2))/(f*(sin(e + f*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin\left(e+fx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2),x)
[Out] Integral(sqrt(a*sin(e + f*x) + a), x)
```

3.526 $\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=61

$$-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{d}\,f\sqrt{c+d}}$$

```
[Out] -2*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))*a
^(1/2)/f/d^(1/2)/(c+d)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2773, 208}

$$-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{d}\,f\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x]),x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*
Sin[e + f*x]])])/(Sqrt[d]*Sqrt[c + d]*f)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx = -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{ac+ad-dx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f}$$

$$= -\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{d}\,\sqrt{c+d}\,f}$$

Mathematica [C] time = 5.49, size = 657, normalized size = 10.77

$$\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\left(\cos\left(\frac{e}{2}\right)+i\sin\left(\frac{e}{2}\right)\right)\sqrt{a(\sin(e+fx)+1)}\left(\text{RootSum}\left[\#1^4de^{2ie}+2i\#1^2ce^{ie}-d\&, \frac{\#1^3\left(-\sqrt{d}\right)e^{ie}fx\sqrt{c+d}-2i\#1^3\sqrt{\ldots}}{\ldots}\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x]),x]
```

```
[Out] ((1/8 + I/8)*(RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((
1 + I)*d*Sqrt[E^((-I)*e)]*f*x - (2 - 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f
*x) - #1] - I*Sqrt[d]*Sqrt[c + d]*f*x*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[E^((I/
2)*f*x) - #1]*#1 + ((1 - I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 + 2*I)*c*Log
[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - Sqrt[d]*Sqrt[c + d]*E^(I*e)*f
*x*#1^3 - (2*I)*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/(
d - I*c*E^(I*e)*#1^2) & ] - I*RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I
)*e)*#1^4 & , ((1 - I)*d*Sqrt[E^((-I)*e)]*f*x + (2 + 2*I)*d*Sqrt[E^((-I)*e)
]*Log[E^((I/2)*f*x) - #1] + Sqrt[d]*Sqrt[c + d]*f*x*#1 + (2*I)*Sqrt[d]*Sqrt
[c + d]*Log[E^((I/2)*f*x) - #1]*#1 - ((1 + I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)]
+ ((2 - 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - I*Sqrt[d]*S
qrt[c + d]*E^(I*e)*f*x*#1^3 + 2*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*
x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) & ])*(Cos[e/2] + I*Sin[e/2])*Sqrt[a*(
1 + Sin[e + f*x])])/(Sqrt[d]*Sqrt[c + d]*f*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2]))
```

fricas [A] time = 0.55, size = 464, normalized size = 7.61

$$\left[\frac{\sqrt{\frac{a}{cd+d^2}}\,\log\left(\frac{ad^2\cos\left(fx+e\right)^3-ac^2-2\,acd-ad^2-\left(6\,acd+7\,ad^2\right)\cos\left(fx+e\right)^2+4\left(c^2d+4\,cd^2+3\,d^3-\left(cd^2+d^3\right)\cos\left(fx+e\right)^2+\left(c^2d+3\,cd^2+2\,d^3\right)\cos\left(fx+e\right)\cdots\right.}{d^2\cos\left(fx+e\right)^3+\left(2\,cd+d^2\right)\cos\left(fx+e\right)^2\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^
2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^
2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d +
 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x
 + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) +
 (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*co
s(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^
2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*
c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e)))/f, -sqrt(-a/(c*d + d^2
))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(
c*d + d^2))/(a*cos(f*x + e)))/f]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)-sqrt(2*a)*sqrt(2)*sign(cos(1/2*(f*x
+exp(1))-1/4*pi))*atan(sqrt(2)*d*sin(1/4*(2*f*x+2*exp(1)-pi))/sqrt(-d^2-c*d
))/sqrt(-d^2-c*d)/f
```

maple [A] time = 0.79, size = 80, normalized size = 1.31

$$-\frac{2a\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\ \operatorname{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\ d}{\sqrt{a(c+d)d}}\right)}{\sqrt{a\left(c+d\right)d}\ \cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}\ f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] -2*a*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)/(a*(c+d)*d)^(1/2)*arctanh((-a
*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/
2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\sin\left(fx+e\right)+a}}{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+a\,\sin\left(e+f\,x\right)}}{c+d\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x)),x)
```

```
[Out] int((a + a*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}{c+d\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))/(c + d*sin(e + f*x)), x)
```

3.527 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=105

$$-\frac{a\cos(e+fx)}{f(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{d}\,f(c+d)^{3/2}}$$

```
[Out] -arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))*a^(
1/2)/(c+d)^(3/2)/f/d^(1/2)-a*cos(f*x+e)/(c+d)/f/(c+d*sin(f*x+e))/(a+a*sin(f
*x+e))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2772, 2773, 208}

$$-\frac{a\cos(e+fx)}{f(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{d}\,f(c+d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*S
in[e + f*x]])]])/(Sqrt[d]*(c + d)^(3/2)*f)) - (a*Cos[e + f*x])/((c + d)*f*Sq
rt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^2}\,dx = -\frac{a\cos(e+fx)}{(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))}+\frac{\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{2(c+d)}$$

$$= -\frac{a\cos(e+fx)}{(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))}-\frac{a\,\text{Subst}\left(\int \frac{1}{ac+ad-dx^2}\,dx,x,\frac{a\cos}{\sqrt{a+a\,s}}\right.}{(c+d)f}$$

$$= -\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{d}\,(c+d)^{3/2}f}-\frac{a\cos(e+fx)}{(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))}$$

Mathematica [C] time = 6.02, size = 871, normalized size = 8.30

$$\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\sqrt{a(\sin(e+fx)+1)}\left(\left(\cos\left(\frac{e}{2}\right)+i\sin\left(\frac{e}{2}\right)\right)\left((-1+i)x\cos(e)+(1+i)x\sin(e)+\frac{\text{RootSum}\left[de^{2ie}\#1^4+2ice^{ie}\#1^2-d\&,\frac{-\sqrt{d}\sqrt{c+d}e^{ie}fx\#1^3-2i\sqrt{d}\sqrt{c+d}e^{ie}\cdots}{\cdots}\right]}{\cdots}\right)\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] ((1/4 + I/4)*Sqrt[a*(1 + Sin[e + f*x])]*(((Cos[e/2] + I*Sin[e/2])*((-1 + I)
*x*Cos[e] + (RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1
 + I)*d*Sqrt[E^((-I)*e)]*f*x - (2 - 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f*
x) - #1] - I*Sqrt[d]*Sqrt[c + d]*f*x*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2
)*f*x) - #1]*#1 + ((1 - I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 + 2*I)*c*Log[
E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*
x*#1^3 - (2*I)*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/(d
 - I*c*E^(I*e)*#1^2) & ]*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]
)/(4*f) + (1 + I)*x*Sin[e]))/(Sqrt[d]*(c + d)^(3/2)*(Cos[e] + I*(-1 + Sin[e
]))*Sqrt[Cos[e] - I*Sin[e]]) + ((Cos[e/2] + I*Sin[e/2])*((1 - I)*x*Cos[e] -
 (1 + I)*x*Sin[e] + (RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4
 & , ((1 - I)*d*Sqrt[E^((-I)*e)]*f*x + (2 + 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^(
(I/2)*f*x) - #1] + Sqrt[d]*Sqrt[c + d]*f*x*#1 + (2*I)*Sqrt[d]*Sqrt[c + d]*L
og[E^((I/2)*f*x) - #1]*#1 - ((1 + I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 - 2
```

```
*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - I*Sqrt[d]*Sqrt[c + d
]*E^(I*e)*f*x*#1^3 + 2*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*
#1^3)/(d - I*c*E^(I*e)*#1^2) & ]*Sqrt[Cos[e] - I*Sin[e]]*(-1 - I*Cos[e] + S
in[e]))/(4*f)))/(Sqrt[d]*(c + d)^(3/2)*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[
e] - I*Sin[e]]) - ((2 - 2*I)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d
)*f*(c + d*Sin[e + f*x]))))/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
```

fricas [B] time = 0.56, size = 786, normalized size = 7.49

$$\left[\frac{\left(d\cos\left(fx+e\right)^{2}-c\cos\left(fx+e\right)-\left(d\cos\left(fx+e\right)+c+d\right)\sin\left(fx+e\right)-c-d\right)\sqrt{\frac{a}{cd+d^{2}}}\log\left(\frac{ad^{2}\cos\left(fx+e\right)^{3}-ac^{2}-2\cdots}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/4*((d*cos(f*x + e)^2 - c*cos(f*x + e) - (d*cos(f*x + e) + c + d)*sin(f*x
 + e) - c - d)*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*
c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d
^3 - (c*d^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e)
- (c^2*d + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt
(a*sin(f*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(
f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4
*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos
(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x +
 e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*sqrt(a*s
in(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1))/((c*d + d^2)*f*cos(f*x
+ e)^2 - (c^2 + c*d)*f*cos(f*x + e) - (c^2 + 2*c*d + d^2)*f - ((c*d + d^2)*
f*cos(f*x + e) + (c^2 + 2*c*d + d^2)*f)*sin(f*x + e)), -1/2*((d*cos(f*x + e
)^2 - c*cos(f*x + e) - (d*cos(f*x + e) + c + d)*sin(f*x + e) - c - d)*sqrt(
-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2
*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*sqrt(a*sin(f*x + e) + a)*(co
s(f*x + e) - sin(f*x + e) + 1))/((c*d + d^2)*f*cos(f*x + e)^2 - (c^2 + c*d)
*f*cos(f*x + e) - (c^2 + 2*c*d + d^2)*f - ((c*d + d^2)*f*cos(f*x + e) + (c^
2 + 2*c*d + d^2)*f)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)4*sqrt(2*a)*(-sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(2*f*x+2*e
xp(1)-pi))/(-4*c-4*d)/(-2*d*sin(1/4*(2*f*x+2*exp(1)-pi))^2+c+d)-1/2*sign(co
s(1/2*(f*x+exp(1))-1/4*pi))*atan(sqrt(2)*d*sin(1/4*(2*f*x+2*exp(1)-pi))/sqr
t(-d^2-c*d))/sqrt(2)/sqrt(-d^2-c*d)/(2*c+2*d))/f
```

maple [A] time = 1.27, size = 155, normalized size = 1.48

$$-\frac{\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\left(\operatorname{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,d}{\sqrt{a(c+d)d}}\right)\sin\left(fx+e\right)ad+\operatorname{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(fx+e\right.\right.}}{\sqrt{a(c+d)d}}\right.\right.}{(c+d)\left(c+d\sin\left(fx+e\right)\right)\sqrt{a\left(c+d\right)d}\,\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x)
```

```
[Out] -(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(arctanh((-a*(sin(f*x+e)-1))^(1/2
)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*a*d+arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(
a*(c+d)*d)^(1/2))*a*c+(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2))/(c+d)/(c
+d*sin(f*x+e))/(a*(c+d)*d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\sin\left(fx+e\right)+a}}{\left(d\sin\left(fx+e\right)+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\,\sin\left(e+f\,x\right)}}{\left(c+d\,\sin\left(e+f\,x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x))^2,x)
```

```
[Out] int((a + a*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**2,x)
[Out] Timed out
```

3.528 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=154

$$-\frac{3a\cos(e+fx)}{4f(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}-\frac{a\cos(e+fx)}{2f(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^2}-\frac{3\sqrt{a}\,\tanh^{}}{4\sqrt{}}$$

```
[Out] -3/4*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))
*a^(1/2)/(c+d)^(5/2)/f/d^(1/2)-1/2*a*cos(f*x+e)/(c+d)/f/(c+d*sin(f*x+e))^2/
(a+a*sin(f*x+e))^(1/2)-3/4*a*cos(f*x+e)/(c+d)^2/f/(c+d*sin(f*x+e))/(a+a*sin
(f*x+e))^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2772, 2773, 208}

$$-\frac{3a\cos(e+fx)}{4f(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}-\frac{a\cos(e+fx)}{2f(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^2}-\frac{3\sqrt{a}\,\tanh^{}}{4\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (-3*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*
Sin[e + f*x]])])/(4*Sqrt[d]*(c + d)^(5/2)*f) - (a*Cos[e + f*x])/(2*(c + d)*
f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2) - (3*a*Cos[e + f*x])/(4*
(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
```

```
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^3}\,dx = -\frac{a\cos(e+fx)}{2(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} + \frac{3\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^2}\,dx}{4(c+d)}$$

$$= -\frac{a\cos(e+fx)}{2(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} - \frac{3a\cos(e+fx)}{4(c+d)^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{a\cos(e+fx)}{2(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} - \frac{3a\cos(e+fx)}{4(c+d)^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{3\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4\sqrt{d}\,(c+d)^{5/2}f} - \frac{a\cos(e+fx)}{2(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))}$$

Mathematica [C] time = 7.55, size = 920, normalized size = 5.97

$$\frac{\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{a(\sin(e+fx)+1)}\left(3\left(\cos\left(\frac{e}{2}\right)+i\sin\left(\frac{e}{2}\right)\right)\left((-1+i)x\cos(e)+(1+i)x\sin(e)+\frac{\text{RootSum}\left[de^{2ie}\#1^4+2ice^{ie}\#1^2-d\&,\frac{-\sqrt{d}\sqrt{c+d}e^{ie}fx\#1^3-2i\sqrt{d}\sqrt{c+\cdots}}{\cdots}\right]}{\cdots}\right)\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((1/16 + I/16)*Sqrt[a*(1 + Sin[e + f*x])]*((3*(Cos[e/2] + I*Sin[e/2])*((-1
+ I)*x*Cos[e] + (RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & ,
 ((1 + I)*d*Sqrt[E^((-I)*e)]*f*x - (2 - 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2
)*f*x) - #1] - I*Sqrt[d]*Sqrt[c + d]*f*x*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[E^(
(I/2)*f*x) - #1]*#1 + ((1 - I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 + 2*I)*c*
Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - Sqrt[d]*Sqrt[c + d]*E^(I*e
)*f*x*#1^3 - (2*I)*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3
)/(d - I*c*E^(I*e)*#1^2) & ]*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin
[e]])/(4*f) + (1 + I)*x*Sin[e]))/(Sqrt[d]*(c + d)^(5/2)*(Cos[e] + I*(-1 + S
in[e]))*Sqrt[Cos[e] - I*Sin[e]]) + (3*(Cos[e/2] + I*Sin[e/2])*((1 - I)*x*Co
s[e] - (1 + I)*x*Sin[e] + (RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e
)*#1^4 & , ((1 - I)*d*Sqrt[E^((-I)*e)]*f*x + (2 + 2*I)*d*Sqrt[E^((-I)*e)]*L
og[E^((I/2)*f*x) - #1] + Sqrt[d]*Sqrt[c + d]*f*x*#1 + (2*I)*Sqrt[d]*Sqrt[c
+ d]*Log[E^((I/2)*f*x) - #1]*#1 - ((1 + I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + (
```

```
(2 - 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - I*Sqrt[d]*Sqrt
[c + d]*E^(I*e)*f*x*#1^3 + 2*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x)
- #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) & ]*Sqrt[Cos[e] - I*Sin[e]]*(-1 - I*Cos[
e] + Sin[e]))/(4*f)))/(Sqrt[d]*(c + d)^(5/2)*(Cos[e] + I*(-1 + Sin[e]))*Sqr
t[Cos[e] - I*Sin[e]]) - ((4 - 4*I)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/(
(c + d)*f*(c + d*Sin[e + f*x])^2) - ((6 - 6*I)*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2]))/((c + d)^2*f*(c + d*Sin[e + f*x]))))/(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])
```

fricas [B] time = 0.63, size = 1250, normalized size = 8.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d -
 d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e)
- c^2 - 2*c*d - d^2)*sin(f*x + e))*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x +
 e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c
^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 +
2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e)
)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a
*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*
d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3
 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x +
 e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x
 + e))) + 4*(3*d*cos(f*x + e)^2 + (5*c + 2*d)*cos(f*x + e) + (3*d*cos(f*x +
 e) - 5*c + d)*sin(f*x + e) + 5*c - d)*sqrt(a*sin(f*x + e) + a))/((c^2*d^2
+ 2*c*d^3 + d^4)*f*cos(f*x + e)^3 + (2*c^3*d + 5*c^2*d^2 + 4*c*d^3 + d^4)*f
*cos(f*x + e)^2 - (c^4 + 2*c^3*d + 2*c^2*d^2 + 2*c*d^3 + d^4)*f*cos(f*x + e
) - (c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4)*f + ((c^2*d^2 + 2*c*d^3 + d
^4)*f*cos(f*x + e)^2 - 2*(c^3*d + 2*c^2*d^2 + c*d^3)*f*cos(f*x + e) - (c^4
+ 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4)*f)*sin(f*x + e)), -1/8*(3*(d^2*cos(f
*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*
cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2
)*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d
*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*(3*d*co
s(f*x + e)^2 + (5*c + 2*d)*cos(f*x + e) + (3*d*cos(f*x + e) - 5*c + d)*sin(
f*x + e) + 5*c - d)*sqrt(a*sin(f*x + e) + a))/((c^2*d^2 + 2*c*d^3 + d^4)*f*
cos(f*x + e)^3 + (2*c^3*d + 5*c^2*d^2 + 4*c*d^3 + d^4)*f*cos(f*x + e)^2 - (
c^4 + 2*c^3*d + 2*c^2*d^2 + 2*c*d^3 + d^4)*f*cos(f*x + e) - (c^4 + 4*c^3*d
+ 6*c^2*d^2 + 4*c*d^3 + d^4)*f + ((c^2*d^2 + 2*c*d^3 + d^4)*f*cos(f*x + e)^
2 - 2*(c^3*d + 2*c^2*d^2 + c*d^3)*f*cos(f*x + e) - (c^4 + 4*c^3*d + 6*c^2*d
^2 + 4*c*d^3 + d^4)*f)*sin(f*x + e))]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)4*sqrt(2*a)*((-5*c*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*
sin(1/4*(2*f*x+2*exp(1)-pi))+6*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4
*(2*f*x+2*exp(1)-pi))^3-5*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(2*f
*x+2*exp(1)-pi)))/(2*d*sin(1/4*(2*f*x+2*exp(1)-pi))^2-c-d)^2/(-16*c^2-16*d^
2-32*c*d)-3/2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*atan(sqrt(2)*d*sin(1/4*(2*
f*x+2*exp(1)-pi))/sqrt(-d^2-c*d))/sqrt(2)/sqrt(-d^2-c*d)/(8*c^2+8*d^2+16*c*
d))/f
```

maple [**A**] time = 1.45, size = 254, normalized size = 1.65

$$-\frac{\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\left(3\,\mathrm{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,d}{\sqrt{a\left(c+d\right)d}}\right)\left(\sin^{2}\left(fx+e\right)\right)a\,d^{2}+6\,\mathrm{arctanh}\left(\frac{\sqrt{-a}}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x)
```

```
[Out] -1/4*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(3*arctanh((-a*(sin(f*x+e)-1)
)^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a*d^2+6*arctanh((-a*(sin(f*x+e)-1
))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*a*c*d+3*(-a*(sin(f*x+e)-1))^(1/2)*
(a*(c+d)*d)^(1/2)*sin(f*x+e)*d+3*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+
d)*d)^(1/2))*a*c^2+5*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*c+2*(-a*(s
in(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*d)/(c+d)^2/(c+d*sin(f*x+e))^2/(a*(c+d
)*d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\sin\left(fx+e\right)+a}}{\left(d\sin\left(fx+e\right)+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
[Out] integrate(sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\,\sin\left(e+f\,x\right)}}{\left(c+d\,\sin\left(e+f\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x))^3,x)
[Out] int((a + a*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**3,x)
[Out] Timed out
```

3.529 $\int (a + a\sin(e + fx))^{3/2}(c + d\sin(e + fx))^3\,dx$

Optimal. Leaf size=231

$$\frac{4a^2(c-17d)(c+d)\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{315df\sqrt{a\sin(e+fx)+a}}-\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}+\frac{2a^2(c-17d)\cos(e+f}{63df\sqrt{a\sin(}$$

```
[Out] 4/105*(c-17*d)*d*(c+d)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f+4/315*a^2*(c-17*
d)*(c+d)*(15*c^2+10*c*d+7*d^2)*cos(f*x+e)/d/f/(a+a*sin(f*x+e))^(1/2)+2/63*a
^2*(c-17*d)*cos(f*x+e)*(c+d*sin(f*x+e))^3/d/f/(a+a*sin(f*x+e))^(1/2)-2/9*a^
2*cos(f*x+e)*(c+d*sin(f*x+e))^4/d/f/(a+a*sin(f*x+e))^(1/2)+8/315*a*(c-17*d)
*(5*c-d)*(c+d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.38, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2763, 21, 2770, 2761, 2751, 2646}

$$\frac{4a^2(c-17d)(c+d)\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{315df\sqrt{a\sin(e+fx)+a}}-\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}+\frac{2a^2(c-17d)\cos(e+f}{63df\sqrt{a\sin(}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (4*a^2*(c - 17*d)*(c + d)*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*x])/(315*d*f*
Sqrt[a + a*Sin[e + f*x]]) + (8*a*(c - 17*d)*(5*c - d)*(c + d)*Cos[e + f*x]*
Sqrt[a + a*Sin[e + f*x]])/(315*f) + (4*(c - 17*d)*d*(c + d)*Cos[e + f*x]*(a
 + a*Sin[e + f*x])^(3/2))/(105*f) + (2*a^2*(c - 17*d)*Cos[e + f*x]*(c + d*S
in[e + f*x])^3)/(63*d*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a^2*Cos[e + f*x]*(c
+ d*Sin[e + f*x])^4)/(9*d*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3\,dx = -\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{\left(-\frac{1}{2}a^2(c-17d)-\frac{1}{2}a^2(c-17d}{\sqrt{a+a\,s}}}{9}$$

$$= -\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a+a\sin(e+fx)}} - \frac{(a(c-17d))\int\sqrt{a+a\sin}}{}$$

$$= \frac{2a^2(c-17d)\cos(e+fx)(c+d\sin(e+fx))^3}{63df\sqrt{a+a\sin(e+fx)}} - \frac{2a^2\cos(e+fx)(}{9df\sqrt{a+a}}$$

$$= \frac{4(c-17d)d(c+d)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{105f} + \frac{2a^2(c-1}{}$$

$$= \frac{8a(c-17d)(5c-d)(c+d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{315f} + \frac{4(c}{}$$

$$= \frac{4a^2(c-17d)(c+d)\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{315df\sqrt{a+a\sin(e+fx)}} + \frac{8a(c-17}{}$$

Mathematica [A] time = 1.69, size = 203, normalized size = 0.88

$$-\frac{a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(840c^3\sin(e+fx)+4200c^3-4d\left(189c^2+351cd+1\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] -1/1260*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]
*(4200*c^3 + 9828*c^2*d + 8892*c*d^2 + 2689*d^3 - 4*d*(189*c^2 + 351*c*d +
137*d^2)*Cos[2*(e + f*x)] + 35*d^3*Cos[4*(e + f*x)] + 840*c^3*Sin[e + f*x]
+ 4536*c^2*d*Sin[e + f*x] + 4554*c*d^2*Sin[e + f*x] + 1598*d^3*Sin[e + f*x]
 - 270*c*d^2*Sin[3*(e + f*x)] - 170*d^3*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2]))
```

fricas [A] time = 0.46, size = 339, normalized size = 1.47

$$-\frac{2\left(35\,ad^3\cos\left(fx+e\right)^5-5\left(27\,acd^2+10\,ad^3\right)\cos\left(fx+e\right)^4+420\,ac^3+756\,ac^2d+684\,acd^2+188\,ad^3-\left(189\,a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -2/315*(35*a*d^3*cos(f*x + e)^5 - 5*(27*a*c*d^2 + 10*a*d^3)*cos(f*x + e)^4
+ 420*a*c^3 + 756*a*c^2*d + 684*a*c*d^2 + 188*a*d^3 - (189*a*c^2*d + 351*a*
c*d^2 + 172*a*d^3)*cos(f*x + e)^3 + (105*a*c^3 + 378*a*c^2*d + 387*a*c*d^2
+ 134*a*d^3)*cos(f*x + e)^2 + (525*a*c^3 + 1323*a*c^2*d + 1287*a*c*d^2 + 40
9*a*d^3)*cos(f*x + e) - (35*a*d^3*cos(f*x + e)^4 + 420*a*c^3 + 756*a*c^2*d
+ 684*a*c*d^2 + 188*a*d^3 + 5*(27*a*c*d^2 + 17*a*d^3)*cos(f*x + e)^3 - 3*(6
3*a*c^2*d + 72*a*c*d^2 + 29*a*d^3)*cos(f*x + e)^2 - (105*a*c^3 + 567*a*c^2*
d + 603*a*c*d^2 + 221*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e
) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)sqrt(2*a)*(-40*f*(-2*a*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-6*a*c*d^2*si
gn(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(10*f*x+10*exp(1)+pi))/(40*f)^2-5
6*f*(-2*a*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-6*a*c*d^2*sign(cos(1/2*(f*
x+exp(1))-1/4*pi)))*cos(1/4*(14*f*x+14*exp(1)-pi))/(56*f)^2+12*f*(-2*a*d^3*
sign(cos(1/2*(f*x+exp(1))-1/4*pi))-6*a*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*
pi))-6*a*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(6*f*x+6*exp(1)+
pi))/(12*f)^2+20*f*(-2*a*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-6*a*c*d^2*s
ign(cos(1/2*(f*x+exp(1))-1/4*pi))-6*a*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*p
i)))*sin(1/4*(10*f*x+10*exp(1)-pi))/(20*f)^2-8*f*(8*a*c^3*sign(cos(1/2*(f*x
```

```
+exp(1))-1/4*pi))+6*a*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+18*a*c*d^2*sig
n(cos(1/2*(f*x+exp(1))-1/4*pi))+24*a*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi
)))*cos(1/4*(2*f*x+2*exp(1)+pi))/(8*f)^2-24*f*(8*a*c^3*sign(cos(1/2*(f*x+ex
p(1))-1/4*pi))+6*a*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+18*a*c*d^2*sign(c
os(1/2*(f*x+exp(1))-1/4*pi))+24*a*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))
*cos(1/4*(6*f*x+6*exp(1)-pi))/(24*f)^2+8*f*(16*a*c^3*sign(cos(1/2*(f*x+exp(
1))-1/4*pi))+6*a*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+24*a*c*d^2*sign(cos
(1/2*(f*x+exp(1))-1/4*pi))+24*a*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*s
in(1/4*(2*f*x-pi)+1/2*exp(1))/(8*f)^2+224*a*d^3*f*sign(cos(1/2*(f*x+exp(1))
-1/4*pi))*sin(1/4*(14*f*x+14*exp(1)+pi))/(112*f)^2+288*a*d^3*f*sign(cos(1/2
*(f*x+exp(1))-1/4*pi))*sin(1/4*(18*f*x+18*exp(1)-pi))/(144*f)^2)
```

maple [A] time = 0.92, size = 195, normalized size = 0.84

$$\frac{2\left(1+\sin\left(fx+e\right)\right)a^{2}\left(\sin\left(fx+e\right)-1\right)\left(35d^{3}\left(\sin^{4}\left(fx+e\right)\right)+135c\,d^{2}\left(\sin^{3}\left(fx+e\right)\right)+85d^{3}\left(\sin^{3}\left(fx+e\right)\right)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^3,x)
```

```
[Out] 2/315*(1+sin(f*x+e))*a^2*(sin(f*x+e)-1)*(35*d^3*sin(f*x+e)^4+135*c*d^2*sin(
f*x+e)^3+85*d^3*sin(f*x+e)^3+189*c^2*d*sin(f*x+e)^2+351*c*d^2*sin(f*x+e)^2+
102*d^3*sin(f*x+e)^2+105*c^3*sin(f*x+e)+567*c^2*d*sin(f*x+e)+468*c*d^2*sin(
f*x+e)+136*d^3*sin(f*x+e)+525*c^3+1134*c^2*d+936*c*d^2+272*d^3)/cos(f*x+e)/
(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^3,x)
```

```
[Out] int((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(c+d\sin\left(e+fx\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c+d*sin(f*x+e))**3,x)
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(c + d*sin(e + f*x))**3, x)
```

3.530 $\int (a + a\sin(e + fx))^{3/2}(c + d\sin(e + fx))^2\, dx$

Optimal. Leaf size=157

$$-\frac{8a^2\left(35c^2+42cd+19d^2\right)\cos(e+fx)}{105f\sqrt{a\sin(e+fx)+a}}-\frac{2a\left(35c^2+42cd+19d^2\right)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{105f}-\frac{4d(7c-d)\cos}{}$$

```
[Out] -4/35*(7*c-d)*d*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-2/7*d^2*cos(f*x+e)*(a+a
*sin(f*x+e))^(5/2)/a/f-8/105*a^2*(35*c^2+42*c*d+19*d^2)*cos(f*x+e)/f/(a+a*s
in(f*x+e))^(1/2)-2/105*a*(35*c^2+42*c*d+19*d^2)*cos(f*x+e)*(a+a*sin(f*x+e))
^(1/2)/f
```

Rubi [A] time = 0.23, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2761, 2751, 2647, 2646}

$$-\frac{8a^2\left(35c^2+42cd+19d^2\right)\cos(e+fx)}{105f\sqrt{a\sin(e+fx)+a}}-\frac{2a\left(35c^2+42cd+19d^2\right)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{105f}-\frac{4d(7c-d)\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (-8*a^2*(35*c^2 + 42*c*d + 19*d^2)*Cos[e + f*x])/(105*f*Sqrt[a + a*Sin[e +
f*x]]) - (2*a*(35*c^2 + 42*c*d + 19*d^2)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*
x]])/(105*f) - (4*(7*c - d)*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(35*
f) - (2*d^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(7*a*f)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2\,dx = -\frac{2d^2\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{7af}+\frac{2\int (a+a\sin(e+fx)}{}$$

$$= -\frac{4(7c-d)d\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{35f}-\frac{2d^2\cos(e+fx}{}$$

$$= -\frac{2a\left(35c^2+42cd+19d^2\right)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105f}-\frac{4(}{}$$

$$= -\frac{8a^2\left(35c^2+42cd+19d^2\right)\cos(e+fx)}{105f\sqrt{a+a\sin(e+fx)}}-\frac{2a\left(35c^2+42cd+19\right.}{}$$

Mathematica [A] time = 0.88, size = 136, normalized size = 0.87

$$-\frac{a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\left(140c^2+504cd+253d^2\right)\sin(e+fx)+700c^2-6d\right.}{210f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -1/210*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*
(700*c^2 + 1092*c*d + 494*d^2 - 6*d*(14*c + 13*d)*Cos[2*(e + f*x)] + (140*c
^2 + 504*c*d + 253*d^2)*Sin[e + f*x] - 15*d^2*Sin[3*(e + f*x)]))/(f*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2]))
```

fricas [A] time = 0.45, size = 229, normalized size = 1.46

$$\frac{2\left(15\,ad^2\cos\left(fx+e\right)^4+3\left(14\,acd+13\,ad^2\right)\cos\left(fx+e\right)^3-140\,ac^2-168\,acd-76\,ad^2-\left(35\,ac^2+84\,acd+43\,a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 2/105*(15*a*d^2*cos(f*x + e)^4 + 3*(14*a*c*d + 13*a*d^2)*cos(f*x + e)^3 - 1
40*a*c^2 - 168*a*c*d - 76*a*d^2 - (35*a*c^2 + 84*a*c*d + 43*a*d^2)*cos(f*x
+ e)^2 - (175*a*c^2 + 294*a*c*d + 143*a*d^2)*cos(f*x + e) + (15*a*d^2*cos(f
*x + e)^3 + 140*a*c^2 + 168*a*c*d + 76*a*d^2 - 6*(7*a*c*d + 4*a*d^2)*cos(f*
x + e)^2 - (35*a*c^2 + 126*a*c*d + 67*a*d^2)*cos(f*x + e))*sin(f*x + e))*sq
rt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*(12*f*(-2*
a*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-4*a*c*d*sign(cos(1/2*(f*x+exp(1))-
1/4*pi)))*sin(1/4*(6*f*x+6*exp(1)+pi))/(12*f)^2+20*f*(-2*a*d^2*sign(cos(1/2
*(f*x+exp(1))-1/4*pi))-4*a*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*
(10*f*x+10*exp(1)-pi))/(20*f)^2+2*f*(4*a*c^2*sign(cos(1/2*(f*x+exp(1))-1/4*
pi))+2*a*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+4*a*c*d*sign(cos(1/2*(f*x+e
xp(1))-1/4*pi)))*sin(1/4*(2*f*x-pi)+1/2*exp(1))/(2*f)^2-8*f*(8*a*c^2*sign(c
os(1/2*(f*x+exp(1))-1/4*pi))+6*a*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+16*
a*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(2*f*x+2*exp(1)+pi))/(8*f
)^2-24*f*(8*a*c^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+6*a*d^2*sign(cos(1/2*(
f*x+exp(1))-1/4*pi))+16*a*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(
6*f*x+6*exp(1)-pi))/(24*f)^2+80*a*d^2*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*
```

```
cos(1/4*(10*f*x+10*exp(1)+pi))/(40*f)^2+112*a*d^2*f*sign(cos(1/2*(f*x+exp(1
))-1/4*pi))*cos(1/4*(14*f*x+14*exp(1)-pi))/(56*f)^2)
```

maple [A] time = 0.89, size = 130, normalized size = 0.83

$$\frac{2\left(1+\sin\left(fx+e\right)\right)a^{2}\left(\sin\left(fx+e\right)-1\right)\left(15d^{2}\left(\sin^{3}\left(fx+e\right)\right)+42cd\left(\sin^{2}\left(fx+e\right)\right)+39d^{2}\left(\sin^{2}\left(fx+e\right)\right)+3\right.}{105\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2/105*(1+sin(f*x+e))*a^2*(sin(f*x+e)-1)*(15*d^2*sin(f*x+e)^3+42*c*d*sin(f*x
+e)^2+39*d^2*sin(f*x+e)^2+35*c^2*sin(f*x+e)+126*c*d*sin(f*x+e)+52*d^2*sin(f
*x+e)+175*c^2+252*c*d+104*d^2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^2,x)
```

```
[Out] int((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(c+d\sin\left(e+fx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(c + d*sin(e + f*x))**2, x)
```

3.531 $\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=101

$$-\frac{8a^2(5c+3d)\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{2a(5c+3d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{2d\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

```
[Out] -2/5*d*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-8/15*a^2*(5*c+3*d)*cos(f*x+e)/f/
(a+a*sin(f*x+e))^(1/2)-2/15*a*(5*c+3*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2751, 2647, 2646}

$$-\frac{8a^2(5c+3d)\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{2a(5c+3d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{2d\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x]),x]
```

```
[Out] (-8*a^2*(5*c + 3*d)*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*(5
*c + 3*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) - (2*d*Cos[e + f*x]
*(a + a*Sin[e + f*x])^(3/2))/(5*f)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))\,dx = -\frac{2d\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f}+\frac{1}{5}(5c+3d)\int (a+a\sin$$

$$= -\frac{2a(5c+3d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f}-\frac{2d\cos(e+fx)(}{}$$

$$= -\frac{8a^2(5c+3d)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}}-\frac{2a(5c+3d)\cos(e+fx)\sqrt{a+a\sin}}{15f}$$

Mathematica [A] time = 0.39, size = 101, normalized size = 1.00

$$-\frac{a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(2(5c+9d)\sin(e+fx)+50c-3d\cos(2(e+fx))+}{15f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x]),x]
```

```
[Out] -1/15*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(
50*c + 39*d - 3*d*Cos[2*(e + f*x)] + 2*(5*c + 9*d)*Sin[e + f*x]))/(f*(Cos[(
e + f*x)/2] + Sin[(e + f*x)/2]))
```

fricas [A] time = 0.45, size = 136, normalized size = 1.35

$$\frac{2\left(3\,ad\cos\left(fx+e\right)^3-(5\,ac+6\,ad)\cos\left(fx+e\right)^2-20\,ac-12\,ad-(25\,ac+21\,ad)\cos\left(fx+e\right)-\left(3\,ad\cos\left(fx\right.\right.\right.}{15\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 2/15*(3*a*d*cos(f*x + e)^3 - (5*a*c + 6*a*d)*cos(f*x + e)^2 - 20*a*c - 12*a
*d - (25*a*c + 21*a*d)*cos(f*x + e) - (3*a*d*cos(f*x + e)^2 - 20*a*c - 12*a
*d + (5*a*c + 9*a*d)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(
f*cos(f*x + e) + f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*(-2*f*(2*a*c*sign(
cos(1/2*(f*x+exp(1))-1/4*pi))+2*a*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos
(1/4*(2*f*x+2*exp(1)+pi))/(2*f)^2-6*f*(2*a*c*sign(cos(1/2*(f*x+exp(1))-1/4*
pi))+2*a*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(6*f*x+6*exp(1)-pi))
/(6*f)^2+2*f*(4*a*c*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+2*a*d*sign(cos(1/2*(
f*x+exp(1))-1/4*pi)))*sin(1/4*(2*f*x-pi)+1/2*exp(1))/(2*f)^2-24*a*d*f*sign(
cos(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(6*f*x+6*exp(1)+pi))/(12*f)^2-40*a*d*
f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(10*f*x+10*exp(1)-pi))/(20*f)^
2)
```

maple [A] time = 0.71, size = 77, normalized size = 0.76

$$\frac{2\left(1+\sin\left(fx+e\right)\right)a^{2}\left(\sin\left(fx+e\right)-1\right)\left(\sin\left(fx+e\right)(5c+9d)-3\left(\cos^{2}\left(fx+e\right)\right)d+25c+21d\right)}{15\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e)),x)
```

```
[Out] 2/15*(1+sin(f*x+e))*a^2*(sin(f*x+e)-1)*(sin(f*x+e)*(5*c+9*d)-3*cos(f*x+e)^2
*d+25*c+21*d)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x)),x)
```

```
[Out] int((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(c+d\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c+d*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(c + d*sin(e + f*x)), x)
```

3.532 $\int (a+a\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=59

$$-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

```
[Out] -8/3*a^2*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/3*a*cos(f*x+e)*(a+a*sin(f*x+
e))^(1/2)/f
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2647, 2646}

$$-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-8*a^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*Cos[e + f*x]*Sq
rt[a + a*Sin[e + f*x]])/(3*f)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^{3/2}\,dx &= -\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f}+\frac{1}{3}(4a)\int \sqrt{a+a\sin(e+fx)}\,dx\\ &= -\frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}-\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f}\end{aligned}$$

Mathematica [A] time = 0.14, size = 89, normalized size = 1.51

$$-\frac{(a(\sin(e+fx)+1))^{3/2}\left(-9\sin\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{3}{2}(e+fx)\right)+9\cos\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{3}{2}(e+fx)\right)\right)}{3f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/3*((a*(1 + Sin[e + f*x]))^(3/2)*(9*Cos[(e + f*x)/2] + Cos[(3*(e + f*x))/
2] - 9*Sin[(e + f*x)/2] + Sin[(3*(e + f*x))/2]))/(f*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^3)
```

fricas [A] time = 0.42, size = 76, normalized size = 1.29

$$-\frac{2\left(a\cos\left(fx+e\right)^2+5\,a\cos\left(fx+e\right)+\left(a\cos\left(fx+e\right)-4\,a\right)\sin\left(fx+e\right)+4\,a\right)\sqrt{a\sin\left(fx+e\right)+a}}{3\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(a*cos(f*x + e)^2 + 5*a*cos(f*x + e) + (a*cos(f*x + e) - 4*a)*sin(f*x
+ e) + 4*a)*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*(2*a*sign(cos
(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(2*f*x-pi)+1/2*exp(1))/f-4*a*f*sign(cos(
1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(2*f*x+2*exp(1)+pi))/(2*f)^2-12*a*f*sign(
cos(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(6*f*x+6*exp(1)-pi))/(6*f)^2)
```

maple [A] time = 0.66, size = 53, normalized size = 0.90

$$\frac{2\left(1+\sin\left(fx+e\right)\right)a^2\left(\sin\left(fx+e\right)-1\right)\left(\sin\left(fx+e\right)+5\right)}{3\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}\;f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] 2/3*(1+sin(f*x+e))*a^2*(sin(f*x+e)-1)*(sin(f*x+e)+5)/cos(f*x+e)/(a+a*sin(f*
x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin\left(e+fx\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*sin(e + f*x) + a)**(3/2), x)
```

3.533 $\int \frac{(a+a\sin(e+fx))^{3/2}}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=98

$$\frac{2a^{3/2}(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f\sqrt{c+d}}-\frac{2a^2\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}$$

```
[Out] 2*a^(3/2)*(c-d)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x
+e))^(1/2))/d^(3/2)/f/(c+d)^(1/2)-2*a^2*cos(f*x+e)/d/f/(a+a*sin(f*x+e))^(1/
2)
```

Rubi [A] time = 0.20, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2763, 21, 2773, 208}

$$\frac{2a^{3/2}(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f\sqrt{c+d}}-\frac{2a^2\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x]),x]
```

```
[Out] (2*a^(3/2)*(c - d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt
[a + a*Sin[e + f*x]])])/(d^(3/2)*Sqrt[c + d]*f) - (2*a^2*Cos[e + f*x])/(d*f
*Sqrt[a + a*Sin[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
```

```
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{c+d\sin(e+fx)}\,dx = -\frac{2a^2\cos(e+fx)}{df\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{-\frac{1}{2}a^2(c-d)-\frac{1}{2}a^2(c-d)\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))}\,dx}{d}$$

$$= -\frac{2a^2\cos(e+fx)}{df\sqrt{a+a\sin(e+fx)}} - \frac{(a(c-d))\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{d}$$

$$= -\frac{2a^2\cos(e+fx)}{df\sqrt{a+a\sin(e+fx)}} + \frac{\left(2a^2(c-d)\right)\text{Subst}\left(\int \frac{1}{ac+ad-dx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{df}$$

$$= \frac{2a^{3/2}(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sin(e+fx)}}\right)}{d^{3/2}\sqrt{c+d}\,f} - \frac{2a^2\cos(e+fx)}{df\sqrt{a+a\sin(e+fx)}}$$

Mathematica [B] time = 2.29, size = 233, normalized size = 2.38

$$(a(\sin(e+fx)+1))^{3/2}\left(2\sqrt{d}\,\sqrt{c+d}\,\sin\left(\frac{1}{2}(e+fx)\right)-2\sqrt{d}\,\sqrt{c+d}\,\cos\left(\frac{1}{2}(e+fx)\right)+(c-d)\left(\log\left(-\sec^2\left(\frac{1}{4}(e+f\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x]),x]
```

```
[Out] ((-2*Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] + (c - d)*(Log[-(Sec[(e + f*x)/4]
^2*(c + d + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] - Sqrt[d]*Sqrt[c + d]*Sin[
(e + f*x)/2]))] - Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c + d]*(-1
+ 2*Tan[(e + f*x)/4] + Tan[(e + f*x)/4]^2)]) + 2*Sqrt[d]*Sqrt[c + d]*Sin[(e
 + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2))/(d^(3/2)*Sqrt[c + d]*f*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^3)
```

fricas [B] time = 0.57, size = 651, normalized size = 6.64

$$\left[-\frac{\left(ac - ad + (ac - ad)\cos\left(fx + e\right) + (ac - ad)\sin\left(fx + e\right)\right)\sqrt{\frac{a}{cd+d^2}}\,\log\left(\frac{ad^2\cos\left(fx+e\right)^3 - ac^2 - 2\,acd - ad^2 - \left(6\,acd + 7\,ad^2\right)\cos}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/2*((a*c - a*d + (a*c - a*d)*cos(f*x + e) + (a*c - a*d)*sin(f*x + e))*sq
rt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*
a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3
)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^
2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) +
 a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2
*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x +
 e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2
 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos
(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(a*cos(f*x + e) - a*sin(f
*x + e) + a)*sqrt(a*sin(f*x + e) + a))/(d*f*cos(f*x + e) + d*f*sin(f*x + e)
 + d*f), ((a*c - a*d + (a*c - a*d)*cos(f*x + e) + (a*c - a*d)*sin(f*x + e))
*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) -
 c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*(a*cos(f*x + e) - a*si
n(f*x + e) + a)*sqrt(a*sin(f*x + e) + a))/(d*f*cos(f*x + e) + d*f*sin(f*x +
 e) + d*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
```

```
Warning, integration of abs or sign assumes constant sign by intervals (cor
rect if the argument is real):Check [abs(cos((f*x+exp(1))/2-pi/4))]Evaluati
on time: 49.25Not invertible Error: Bad Argument Value
```

maple [A] time = 1.12, size = 137, normalized size = 1.40

$$-\frac{2a\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,\left(-\operatorname{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,d}{\sqrt{a(c+d)d}}\right)ac+a\operatorname{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,d}{\sqrt{a(c+d)d}}\right)\right.}{d\sqrt{a\left(c+d\right)d}\,\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] -2*a*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-arctanh((-a*(sin(f*x+e)-1))
^(1/2)*d/(a*(c+d)*d)^(1/2))*a*c+a*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c
+d)*d)^(1/2))*d+(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2))/d/(a*(c+d)*d)^
(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}{c+d\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x)),x)
```

```
[Out] int((a + a*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.534 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=119

$$\frac{a^2(c-d)\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}-\frac{a^{3/2}(c+3d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f(c+d)^{3/2}}$$

```
[Out] -a^(3/2)*(c+3*d)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*
x+e))^(1/2))/d^(3/2)/(c+d)^(3/2)/f+a^2*(c-d)*cos(f*x+e)/d/(c+d)/f/(c+d*sin(
f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2762, 21, 2773, 208}

$$\frac{a^2(c-d)\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}-\frac{a^{3/2}(c+3d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f(c+d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -((a^(3/2)*(c + 3*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sq
rt[a + a*Sin[e + f*x]])])/(d^(3/2)*(c + d)^(3/2)*f)) + (a^2*(c - d)*Cos[e +
 f*x])/(d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
```

```
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^2}\,dx = \frac{a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))} - \frac{a\int \frac{-\frac{1}{2}a(c+3d)-\frac{1}{2}a(c+3d)\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))}\,dx}{d(c+d)}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))} + \frac{(a(c+3d))\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{2d(c+d)}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))} - \frac{\left(a^2(c+3d)\right)\operatorname{Subst}\left(\int \frac{1}{ac+ad-dx^2}\right.}{d(c+d)f}$$

$$= -\frac{a^{3/2}(c+3d)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sin(e+fx)}}\right)}{d^{3/2}(c+d)^{3/2}f} + \frac{a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin}$$

Mathematica [B] time = 2.35, size = 268, normalized size = 2.25

$$-\frac{(a(\sin(e+fx)+1))^{3/2}\left(2\sqrt{d}\,(c-d)\sqrt{c+d}\,\sin\left(\frac{1}{2}(e+fx)\right)-2\sqrt{d}\,(c-d)\sqrt{c+d}\,\cos\left(\frac{1}{2}(e+fx)\right)+(c+3d)(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -1/2*((a*(1 + Sin[e + f*x]))^(3/2)*(-2*(c - d)*Sqrt[d]*Sqrt[c + d]*Cos[(e +
 f*x)/2] + 2*(c - d)*Sqrt[d]*Sqrt[c + d]*Sin[(e + f*x)/2] + (c + 3*d)*(Log[
-(Sec[(e + f*x)/4]^2*(c + d + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] - Sqrt[d
]*Sqrt[c + d]*Sin[(e + f*x)/2]))] - Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d
]*Sqrt[c + d]*(-1 + 2*Tan[(e + f*x)/4] + Tan[(e + f*x)/4]^2)])*(c + d*Sin[e
 + f*x])))/(d^(3/2)*(c + d)^(3/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3
*(c + d*Sin[e + f*x]))
```

fricas [B] time = 0.60, size = 970, normalized size = 8.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((a*c^2 + 4*a*c*d + 3*a*d^2 - (a*c*d + 3*a*d^2)*cos(f*x + e)^2 + (a*c
^2 + 3*a*c*d)*cos(f*x + e) + (a*c^2 + 4*a*c*d + 3*a*d^2 + (a*c*d + 3*a*d^2)
*cos(f*x + e))*sin(f*x + e))*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3
- a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d +
 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)
*cos(f*x + e) - (c^2*d + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(
f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d +
 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 +
2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*
c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) +
(d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))
) + 4*(a*c - a*d + (a*c - a*d)*cos(f*x + e) - (a*c - a*d)*sin(f*x + e))*sqr
t(a*sin(f*x + e) + a))/((c*d^2 + d^3)*f*cos(f*x + e)^2 - (c^2*d + c*d^2)*f*
cos(f*x + e) - (c^2*d + 2*c*d^2 + d^3)*f - ((c*d^2 + d^3)*f*cos(f*x + e) +
(c^2*d + 2*c*d^2 + d^3)*f)*sin(f*x + e)), 1/2*((a*c^2 + 4*a*c*d + 3*a*d^2 -
 (a*c*d + 3*a*d^2)*cos(f*x + e)^2 + (a*c^2 + 3*a*c*d)*cos(f*x + e) + (a*c^2
 + 4*a*c*d + 3*a*d^2 + (a*c*d + 3*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(-
a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*
d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*(a*c - a*d + (a*c - a*d)*cos(
f*x + e) - (a*c - a*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c*d^2 + d^
3)*f*cos(f*x + e)^2 - (c^2*d + c*d^2)*f*cos(f*x + e) - (c^2*d + 2*c*d^2 + d
^3)*f - ((c*d^2 + d^3)*f*cos(f*x + e) + (c^2*d + 2*c*d^2 + d^3)*f)*sin(f*x
+ e))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

[Out] Timed out

maple [B] time = 1.42, size = 233, normalized size = 1.96

$$\frac{a\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,\left(-\sin\left(fx+e\right)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,d}{\sqrt{acd+a\,d^{2}}}\right)ad\left(c+3d\right)-\operatorname{arctanh}\left(\frac{\sqrt{a}\cdots}{\cdots}\right)\cdots\right.}{d\left(c+d\right)\left(c+d\sin\left(fx+e\right)\right)\sqrt{a}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x)
```

```
[Out] a*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-sin(f*x+e)*arctanh((a-a*sin(f*
x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*d*(c+3*d)-arctanh((a-a*sin(f*x+e))^(1/
2)*d/(a*c*d+a*d^2)^(1/2))*a*c^2-3*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a
*d^2)^(1/2))*a*c*d+(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*c-(a-a*sin(f*x+
e))^(1/2)*(a*(c+d)*d)^(1/2)*d)/d/(c+d)/(c+d*sin(f*x+e))/(a*(c+d)*d)^(1/2)/c
os(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}{\left(c+d\,\sin\left(e+f\,x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^2,x)
```

```
[Out] int((a + a*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.535 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=179

$$-\frac{a^{3/2}(c+7d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4d^{3/2}f(c+d)^{5/2}}-\frac{a^2(c+7d)\cos(e+fx)}{4df(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}+\frac{a^2(\ldots}{2df(c+d)\sqrt{a\sin\ldots}}$$

```
[Out] -1/4*a^(3/2)*(c+7*d)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*si
n(f*x+e))^(1/2))/d^(3/2)/(c+d)^(5/2)/f+1/2*a^2*(c-d)*cos(f*x+e)/d/(c+d)/f/(
c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2)-1/4*a^2*(c+7*d)*cos(f*x+e)/d/(c+d)
^2/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2762, 21, 2772, 2773, 208}

$$-\frac{a^{3/2}(c+7d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4d^{3/2}f(c+d)^{5/2}}-\frac{a^2(c+7d)\cos(e+fx)}{4df(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}+\frac{a^2(\ldots}{2df(c+d)\sqrt{a\sin\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] -(a^(3/2)*(c + 7*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqr
t[a + a*Sin[e + f*x]])])/(4*d^(3/2)*(c + d)^(5/2)*f) + (a^2*(c - d)*Cos[e +
 f*x])/(2*d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2) - (a
^2*(c + 7*d)*Cos[e + f*x])/(4*d*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d
*Sin[e + f*x]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^3}\,dx = \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} - \frac{a\int \frac{-\frac{1}{2}a(c+7d)-\frac{1}{2}a(c+7d)\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))}}{2d(c+d)}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} + \frac{(a(c+7d))\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^2}\,d}{4d(c+d)}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} - \frac{a^2(c+7d)\cos(}{4d(c+d)^2f\sqrt{a+a\sin(e+f}}$$

$$= \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} - \frac{a^2(c+7d)\cos(}{4d(c+d)^2f\sqrt{a+a\sin(e+f}}$$

$$= -\frac{a^{3/2}(c+7d)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sin(e+fx)}}\right)}{4d^{3/2}(c+d)^{5/2}f} + \frac{a^2(c-d)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\,s}$$

Mathematica [A] time = 4.06, size = 313, normalized size = 1.75

$$\frac{(a(\sin(e+fx)+1))^{3/2}\left(\frac{4\sqrt{d}\,\sin\left(\frac{1}{2}(e+fx)\right)\left(-c^2+d(c+7d)\sin(e+fx)+7cd+2d^2\right)}{(c+d)^2(c+d\sin(e+fx))^2} - \frac{4\sqrt{d}\,\cos\left(\frac{1}{2}(e+fx)\right)\left(-c^2+d(c+7d)\sin(e+fx)+7cd+2d^2\right)}{(c+d)^2(c+d\sin(e+fx))^2} - \right.}{16d^{3/2}f\left(\sin\left(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*((-2*(c + 7*d)*(Log[-(Sec[(e + f*x)/4]^2*(c +
 d + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] - Sqrt[d]*Sqrt[c + d]*Sin[(e + f*
x)/2]))] - Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c + d]*(-1 + 2*Tan
[(e + f*x)/4] + Tan[(e + f*x)/4]^2)]))/(c + d)^(5/2) - (4*Sqrt[d]*Cos[(e +
f*x)/2]*(-c^2 + 7*c*d + 2*d^2 + d*(c + 7*d)*Sin[e + f*x]))/((c + d)^2*(c +
d*Sin[e + f*x])^2) + (4*Sqrt[d]*Sin[(e + f*x)/2]*(-c^2 + 7*c*d + 2*d^2 + d*
(c + 7*d)*Sin[e + f*x]))/((c + d)^2*(c + d*Sin[e + f*x])^2)))/(16*d^(3/2)*f
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

fricas [B] time = 0.66, size = 1558, normalized size = 8.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/16*((a*c^3 + 9*a*c^2*d + 15*a*c*d^2 + 7*a*d^3 - (a*c*d^2 + 7*a*d^3)*cos
(f*x + e)^3 - (2*a*c^2*d + 15*a*c*d^2 + 7*a*d^3)*cos(f*x + e)^2 + (a*c^3 +
7*a*c^2*d + a*c*d^2 + 7*a*d^3)*cos(f*x + e) + (a*c^3 + 9*a*c^2*d + 15*a*c*d
^2 + 7*a*d^3 - (a*c*d^2 + 7*a*d^3)*cos(f*x + e)^2 + 2*(a*c^2*d + 7*a*c*d^2)
*cos(f*x + e))*sin(f*x + e))*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3
- a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d +
 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)
*cos(f*x + e) - (c^2*d + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(
f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d +
 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 +
2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*
c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) +
(d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))
) + 4*(a*c^2 - 6*a*c*d + 5*a*d^2 - (a*c*d + 7*a*d^2)*cos(f*x + e)^2 + (a*c^
2 - 7*a*c*d - 2*a*d^2)*cos(f*x + e) - (a*c^2 - 6*a*c*d + 5*a*d^2 + (a*c*d +
 7*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c^2*d^3 +
 2*c*d^4 + d^5)*f*cos(f*x + e)^3 + (2*c^3*d^2 + 5*c^2*d^3 + 4*c*d^4 + d^5)*
f*cos(f*x + e)^2 - (c^4*d + 2*c^3*d^2 + 2*c^2*d^3 + 2*c*d^4 + d^5)*f*cos(f*
x + e) - (c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f + ((c^2*d^3 + 2*
c*d^4 + d^5)*f*cos(f*x + e)^2 - 2*(c^3*d^2 + 2*c^2*d^3 + c*d^4)*f*cos(f*x +
 e) - (c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f)*sin(f*x + e)), 1/8
*((a*c^3 + 9*a*c^2*d + 15*a*c*d^2 + 7*a*d^3 - (a*c*d^2 + 7*a*d^3)*cos(f*x +
 e)^3 - (2*a*c^2*d + 15*a*c*d^2 + 7*a*d^3)*cos(f*x + e)^2 + (a*c^3 + 7*a*c^
2*d + a*c*d^2 + 7*a*d^3)*cos(f*x + e) + (a*c^3 + 9*a*c^2*d + 15*a*c*d^2 + 7
*a*d^3 - (a*c*d^2 + 7*a*d^3)*cos(f*x + e)^2 + 2*(a*c^2*d + 7*a*c*d^2)*cos(f
*x + e))*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e)
+ a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*
(a*c^2 - 6*a*c*d + 5*a*d^2 - (a*c*d + 7*a*d^2)*cos(f*x + e)^2 + (a*c^2 - 7*
a*c*d - 2*a*d^2)*cos(f*x + e) - (a*c^2 - 6*a*c*d + 5*a*d^2 + (a*c*d + 7*a*d
^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c^2*d^3 + 2*c*d
^4 + d^5)*f*cos(f*x + e)^3 + (2*c^3*d^2 + 5*c^2*d^3 + 4*c*d^4 + d^5)*f*cos(
f*x + e)^2 - (c^4*d + 2*c^3*d^2 + 2*c^2*d^3 + 2*c*d^4 + d^5)*f*cos(f*x + e)
 - (c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f + ((c^2*d^3 + 2*c*d^4
+ d^5)*f*cos(f*x + e)^2 - 2*(c^3*d^2 + 2*c^2*d^3 + c*d^4)*f*cos(f*x + e) -
(c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f)*sin(f*x + e))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.50, size = 429, normalized size = 2.40

$$\frac{\left(-\operatorname{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\, d}{\sqrt{a(c+d)d}}\right)\left(\sin^2\left(fx+e\right)\right)a^2 c\, d^2-7\operatorname{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\, d}{\sqrt{a(c+d)d}}\right)\left(\sin^2\left(fx+e\right)\right)a^2 d^3-2\,\mathrm{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/4*(-arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a
^2*c*d^2-7*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e
)^2*a^2*d^3-2*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*
x+e)*a^2*c^2*d-14*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*si
n(f*x+e)*a^2*c*d^2+(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*c*d+7*(-a*(s
in(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*d^2-arctanh((-a*(sin(f*x+e)-1))^(1/2)
*d/(a*(c+d)*d)^(1/2))*a^2*c^3-7*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d
)*d)^(1/2))*a^2*c^2*d+(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2-8*(
-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d-9*(-a*(sin(f*x+e)-1))^(1/2
)*(a*(c+d)*d)^(1/2)*a*d^2)*(-a*(sin(f*x+e)-1))^(1/2)*(1+sin(f*x+e))/(a*(c+d
)*d)^(1/2)/(c+d*sin(f*x+e))^2/(c+d)^2/d/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}{\left(c+d\,\sin\left(e+f\,x\right)\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^3,x)
```

```
[Out] int((a + a*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.536 $\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=328

$$-\frac{2a^3\left(3c^2-38cd+355d^2\right)\cos(e+fx)(c+d\sin(e+fx))^3}{693d^2f\sqrt{a\sin(e+fx)+a}}-\frac{4a^3(c+d)\left(15c^2+10cd+7d^2\right)\left(3c^2-38cd+355d^2\right.}{3465d^2f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -4/1155*a*(c+d)*(3*c^2-38*c*d+355*d^2)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-
4/3465*a^3*(c+d)*(15*c^2+10*c*d+7*d^2)*(3*c^2-38*c*d+355*d^2)*cos(f*x+e)/d^
2/f/(a+a*sin(f*x+e))^(1/2)-2/693*a^3*(3*c^2-38*c*d+355*d^2)*cos(f*x+e)*(c+d
*sin(f*x+e))^3/d^2/f/(a+a*sin(f*x+e))^(1/2)+2/99*a^3*(3*c-23*d)*cos(f*x+e)*
(c+d*sin(f*x+e))^4/d^2/f/(a+a*sin(f*x+e))^(1/2)-8/3465*a^2*(5*c-d)*(c+d)*(3
*c^2-38*c*d+355*d^2)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d/f-2/11*a^2*cos(f*x
+e)*(c+d*sin(f*x+e))^4*(a+a*sin(f*x+e))^(1/2)/d/f
```

Rubi [A] time = 0.66, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2763, 2981, 2770, 2761, 2751, 2646}

$$-\frac{2a^3\left(3c^2-38cd+355d^2\right)\cos(e+fx)(c+d\sin(e+fx))^3}{693d^2f\sqrt{a\sin(e+fx)+a}}-\frac{8a^2(5c-d)(c+d)\left(3c^2-38cd+355d^2\right)\cos(e+fx}{3465df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (-4*a^3*(c + d)*(15*c^2 + 10*c*d + 7*d^2)*(3*c^2 - 38*c*d + 355*d^2)*Cos[e
+ f*x])/(3465*d^2*f*Sqrt[a + a*Sin[e + f*x]]) - (8*a^2*(5*c - d)*(c + d)*(3
*c^2 - 38*c*d + 355*d^2)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3465*d*f)
- (4*a*(c + d)*(3*c^2 - 38*c*d + 355*d^2)*Cos[e + f*x]*(a + a*Sin[e + f*x])
^(3/2))/(1155*f) - (2*a^3*(3*c^2 - 38*c*d + 355*d^2)*Cos[e + f*x]*(c + d*Si
n[e + f*x])^3)/(693*d^2*f*Sqrt[a + a*Sin[e + f*x]]) + (2*a^3*(3*c - 23*d)*C
os[e + f*x]*(c + d*Sin[e + f*x])^4)/(99*d^2*f*Sqrt[a + a*Sin[e + f*x]]) - (
2*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^4)/(11*d*f
)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3\,dx = -\frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^4}{11df} + \frac{2\int$$

$$= \frac{2a^3(3c-23d)\cos(e+fx)(c+d\sin(e+fx))^4}{99d^2f\sqrt{a+a\sin(e+fx)}} - \frac{2a^2\cos(e+f$$

$$= -\frac{2a^3\left(3c^2-38cd+355d^2\right)\cos(e+fx)(c+d\sin(e+fx))^3}{693d^2f\sqrt{a+a\sin(e+fx)}} + \frac{2$$

$$= -\frac{4a(c+d)\left(3c^2-38cd+355d^2\right)\cos(e+fx)(a+a\sin(e+fx)}{1155f}$$

$$= -\frac{8a^2(5c-d)(c+d)\left(3c^2-38cd+355d^2\right)\cos(e+fx)\sqrt{a+a\,\mathrm{si}}}{3465df}$$

$$= -\frac{4a^3(c+d)\left(15c^2+10cd+7d^2\right)\left(3c^2-38cd+355d^2\right)\cos(e+}{3465d^2f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 6.27, size = 246, normalized size = 0.75

$$-\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(51744c^3\sin(e+fx)+164472c^3+199980c^2d\sin(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] -1/27720*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x
])]*(164472*c^3 + 411840*c^2*d + 373098*c*d^2 + 114640*d^3 - 8*(693*c^3 + 5
940*c^2*d + 8382*c*d^2 + 3250*d^3)*Cos[2*(e + f*x)] + 70*d^2*(33*c + 32*d)*
Cos[4*(e + f*x)] + 51744*c^3*Sin[e + f*x] + 199980*c^2*d*Sin[e + f*x] + 205
656*c*d^2*Sin[e + f*x] + 69890*d^3*Sin[e + f*x] - 5940*c^2*d*Sin[3*(e + f*x
)] - 17160*c*d^2*Sin[3*(e + f*x)] - 8675*d^3*Sin[3*(e + f*x)] + 315*d^3*Sin
[5*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

fricas [A] time = 0.49, size = 493, normalized size = 1.50

$$-\frac{2\left(315\,a^2d^3\cos\left(fx+e\right)^6+35\left(33\,a^2cd^2+32\,a^2d^3\right)\cos\left(fx+e\right)^5+7392\,a^2c^3+15840\,a^2c^2d+13728\,a^2cd^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -2/3465*(315*a^2*d^3*cos(f*x + e)^6 + 35*(33*a^2*c*d^2 + 32*a^2*d^3)*cos(f*
x + e)^5 + 7392*a^2*c^3 + 15840*a^2*c^2*d + 13728*a^2*c*d^2 + 4000*a^2*d^3
- 5*(297*a^2*c^2*d + 627*a^2*c*d^2 + 320*a^2*d^3)*cos(f*x + e)^4 - (693*a^2
*c^3 + 5940*a^2*c^2*d + 9537*a^2*c*d^2 + 4370*a^2*d^3)*cos(f*x + e)^3 + (25
41*a^2*c^3 + 8415*a^2*c^2*d + 8679*a^2*c*d^2 + 2965*a^2*d^3)*cos(f*x + e)^2
 + 2*(5313*a^2*c^3 + 14355*a^2*c^2*d + 13827*a^2*c*d^2 + 4465*a^2*d^3)*cos(
f*x + e) + (315*a^2*d^3*cos(f*x + e)^5 - 7392*a^2*c^3 - 15840*a^2*c^2*d - 1
3728*a^2*c*d^2 - 4000*a^2*d^3 - 35*(33*a^2*c*d^2 + 23*a^2*d^3)*cos(f*x + e)
^4 - 5*(297*a^2*c^2*d + 858*a^2*c*d^2 + 481*a^2*d^3)*cos(f*x + e)^3 + 3*(23
1*a^2*c^3 + 1485*a^2*c^2*d + 1749*a^2*c*d^2 + 655*a^2*d^3)*cos(f*x + e)^2 +
 2*(1617*a^2*c^3 + 6435*a^2*c^2*d + 6963*a^2*c*d^2 + 2465*a^2*d^3)*cos(f*x
+ e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x +
e) + f)
```

giac [F(-2)]   time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
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able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
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to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*(112*f*(4*a^
```

```
2*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+6*a^2*c*d^2*sign(cos(1/2*(f*x+exp(
1))-1/4*pi)))*sin(1/4*(14*f*x+14*exp(1)+pi))/(112*f)^2+144*f*(4*a^2*d^3*sig
n(cos(1/2*(f*x+exp(1))-1/4*pi))+6*a^2*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*p
i)))*sin(1/4*(18*f*x+18*exp(1)-pi))/(144*f)^2-160*f*(-18*a^2*d^3*sign(cos(1
/2*(f*x+exp(1))-1/4*pi))-48*a^2*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-24
*a^2*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(10*f*x+10*exp(1)+pi
))/(160*f)^2-224*f*(-18*a^2*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-48*a^2*c
*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-24*a^2*c^2*d*sign(cos(1/2*(f*x+exp(
1))-1/4*pi)))*cos(1/4*(14*f*x+14*exp(1)-pi))/(224*f)^2+8*f*(24*a^2*c^3*sign
(cos(1/2*(f*x+exp(1))-1/4*pi))+12*a^2*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi)
)+42*a^2*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+48*a^2*c^2*d*sign(cos(1/2
*(f*x+exp(1))-1/4*pi)))*sin(1/4*(2*f*x-pi)+1/2*exp(1))/(8*f)^2-16*f*(32*a^2
*c^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+22*a^2*d^3*sign(cos(1/2*(f*x+exp(1)
)-1/4*pi))+72*a^2*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+84*a^2*c^2*d*sig
n(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(2*f*x+2*exp(1)+pi))/(16*f)^2-48*f
*(32*a^2*c^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+22*a^2*d^3*sign(cos(1/2*(f*
x+exp(1))-1/4*pi))+72*a^2*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+84*a^2*c
^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(6*f*x+6*exp(1)-pi))/(48*f
)^2+12*f*(-2*a^2*c^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-4*a^2*d^3*sign(cos(
1/2*(f*x+exp(1))-1/4*pi))-12*a^2*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-1
2*a^2*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(6*f*x+6*exp(1)+pi)
)/(12*f)^2+20*f*(-2*a^2*c^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-4*a^2*d^3*si
gn(cos(1/2*(f*x+exp(1))-1/4*pi))-12*a^2*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4
*pi))-12*a^2*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(10*f*x+10*e
xp(1)-pi))/(20*f)^2-576*a^2*d^3*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(1/
4*(18*f*x+18*exp(1)+pi))/(288*f)^2-704*a^2*d^3*f*sign(cos(1/2*(f*x+exp(1))-
1/4*pi))*cos(1/4*(22*f*x+22*exp(1)-pi))/(352*f)^2)
```

maple [A] time = 0.89, size = 249, normalized size = 0.76

$$\frac{2\left(1+\sin\left(fx+e\right)\right)a^3\left(\sin\left(fx+e\right)-1\right)\left(315d^3\left(\sin^5\left(fx+e\right)\right)+1155c\,d^2\left(\sin^4\left(fx+e\right)\right)+1120d^3\left(\sin^4\left(fx+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^3,x)
```

```
[Out] 2/3465*(1+sin(f*x+e))*a^3*(sin(f*x+e)-1)*(315*d^3*sin(f*x+e)^5+1155*c*d^2*s
in(f*x+e)^4+1120*d^3*sin(f*x+e)^4+1485*c^2*d*sin(f*x+e)^3+4290*c*d^2*sin(f*
x+e)^3+1775*d^3*sin(f*x+e)^3+693*c^3*sin(f*x+e)^2+5940*c^2*d*sin(f*x+e)^2+7
227*c*d^2*sin(f*x+e)^2+2130*d^3*sin(f*x+e)^2+3234*c^3*sin(f*x+e)+11385*c^2*
d*sin(f*x+e)+9636*c*d^2*sin(f*x+e)+2840*d^3*sin(f*x+e)+9933*c^3+22770*c^2*d
+19272*c*d^2+5680*d^3)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^3,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^3,x)
[Out] int((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c+d*sin(f*x+e))**3,x)
[Out] Timed out
```

3.537 $\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=202

$$-\frac{64a^3\left(21c^2+30cd+13d^2\right)\cos(e+fx)}{315f\sqrt{a\sin(e+fx)+a}}-\frac{16a^2\left(21c^2+30cd+13d^2\right)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{315f}-\frac{2a\left(21c^2+\cdots\right.}{}$$

```
[Out] -2/105*a*(21*c^2+30*c*d+13*d^2)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-4/63*(9
*c-d)*d*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f-2/9*d^2*cos(f*x+e)*(a+a*sin(f*x
+e))^(7/2)/a/f-64/315*a^3*(21*c^2+30*c*d+13*d^2)*cos(f*x+e)/f/(a+a*sin(f*x+
e))^(1/2)-16/315*a^2*(21*c^2+30*c*d+13*d^2)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/
2)/f
```

Rubi [A] time = 0.27, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2761, 2751, 2647, 2646}

$$-\frac{16a^2\left(21c^2+30cd+13d^2\right)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{315f}-\frac{64a^3\left(21c^2+30cd+13d^2\right)\cos(e+fx)}{315f\sqrt{a\sin(e+fx)+a}}-\frac{2a\left(21c^2+\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (-64*a^3*(21*c^2 + 30*c*d + 13*d^2)*Cos[e + f*x])/(315*f*Sqrt[a + a*Sin[e +
 f*x]]) - (16*a^2*(21*c^2 + 30*c*d + 13*d^2)*Cos[e + f*x]*Sqrt[a + a*Sin[e
+ f*x]])/(315*f) - (2*a*(21*c^2 + 30*c*d + 13*d^2)*Cos[e + f*x]*(a + a*Sin[
e + f*x])^(3/2))/(105*f) - (4*(9*c - d)*d*Cos[e + f*x]*(a + a*Sin[e + f*x])
^(5/2))/(63*f) - (2*d^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(9*a*f)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2\,dx = -\frac{2d^2\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{9af} + \frac{2\int(a+a\sin(e+fx))^{5}\cdots}{\cdots}$$
$$= -\frac{4(9c-d)d\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{63f} - \frac{2d^2\cos(e+fx)\cdots}{\cdots}$$
$$= -\frac{2a\left(21c^2+30cd+13d^2\right)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{105f} - \frac{4(\cdots}{\cdots}$$
$$= -\frac{16a^2\left(21c^2+30cd+13d^2\right)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{315f} - \frac{2\cdots}{\cdots}$$
$$= -\frac{64a^3\left(21c^2+30cd+13d^2\right)\cos(e+fx)}{315f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2\left(21c^2+30cd+\cdots\right.}{\cdots}$$

Mathematica [A] time = 3.30, size = 180, normalized size = 0.89

$$-\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-4\left(63c^2+360cd+254d^2\right)\cos(2(e+fx))+2352c\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -1/1260*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x]
)]*(7476*c^2 + 12480*c*d + 5653*d^2 - 4*(63*c^2 + 360*c*d + 254*d^2)*Cos[2*
(e + f*x)] + 35*d^2*Cos[4*(e + f*x)] + 2352*c^2*Sin[e + f*x] + 6060*c*d*Sin
[e + f*x] + 3116*d^2*Sin[e + f*x] - 180*c*d*Sin[3*(e + f*x)] - 260*d^2*Sin[
3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

fricas [A] time = 0.48, size = 339, normalized size = 1.68

$$-\frac{2\left(35\,a^2d^2\cos\left(fx+e\right)^5-5\left(18\,a^2cd+19\,a^2d^2\right)\cos\left(fx+e\right)^4+672\,a^2c^2+960\,a^2cd+416\,a^2d^2-\left(63\,a^2c^2+3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -2/315*(35*a^2*d^2*cos(f*x + e)^5 - 5*(18*a^2*c*d + 19*a^2*d^2)*cos(f*x + e
)^4 + 672*a^2*c^2 + 960*a^2*c*d + 416*a^2*d^2 - (63*a^2*c^2 + 360*a^2*c*d +
 289*a^2*d^2)*cos(f*x + e)^3 + (231*a^2*c^2 + 510*a^2*c*d + 263*a^2*d^2)*co
s(f*x + e)^2 + 2*(483*a^2*c^2 + 870*a^2*c*d + 419*a^2*d^2)*cos(f*x + e) - (
35*a^2*d^2*cos(f*x + e)^4 + 672*a^2*c^2 + 960*a^2*c*d + 416*a^2*d^2 + 10*(9
*a^2*c*d + 13*a^2*d^2)*cos(f*x + e)^3 - 3*(21*a^2*c^2 + 90*a^2*c*d + 53*a^2
*d^2)*cos(f*x + e)^2 - 2*(147*a^2*c^2 + 390*a^2*c*d + 211*a^2*d^2)*cos(f*x
+ e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x +
e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
```

```
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)sqrt(2*a)*(-20*f*(-2*a^2*d^2*sign(cos(1/2*(f*x+exp(1))-1
/4*pi))-2*a^2*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(10*f*x+10*ex
p(1)+pi))/(20*f)^2-28*f*(-2*a^2*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-2*a^
2*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(14*f*x+14*exp(1)-pi))/(2
8*f)^2-4*f*(8*a^2*c^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+6*a^2*d^2*sign(cos
(1/2*(f*x+exp(1))-1/4*pi))+14*a^2*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*c
os(1/4*(2*f*x+2*exp(1)+pi))/(4*f)^2-12*f*(8*a^2*c^2*sign(cos(1/2*(f*x+exp(1
))-1/4*pi))+6*a^2*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+14*a^2*c*d*sign(co
s(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(6*f*x+6*exp(1)-pi))/(12*f)^2+8*f*(24*
a^2*c^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+14*a^2*d^2*sign(cos(1/2*(f*x+exp
(1))-1/4*pi))+32*a^2*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(2*f*x
-pi)+1/2*exp(1))/(8*f)^2+12*f*(-2*a^2*c^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi)
)-4*a^2*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-8*a^2*c*d*sign(cos(1/2*(f*x+
exp(1))-1/4*pi)))*sin(1/4*(6*f*x+6*exp(1)+pi))/(12*f)^2+20*f*(-2*a^2*c^2*si
gn(cos(1/2*(f*x+exp(1))-1/4*pi))-4*a^2*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi
))-8*a^2*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(10*f*x+10*exp(1)-
pi))/(20*f)^2+224*a^2*d^2*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(14*
f*x+14*exp(1)+pi))/(112*f)^2+288*a^2*d^2*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi
))*sin(1/4*(18*f*x+18*exp(1)-pi))/(144*f)^2)
```

maple [A] time = 0.94, size = 168, normalized size = 0.83

$$\frac{2\left(1+\sin\left(fx+e\right)\right)a^{3}\left(\sin\left(fx+e\right)-1\right)\left(35d^{2}\left(\sin^{4}\left(fx+e\right)\right)+90cd\left(\sin^{3}\left(fx+e\right)\right)+130d^{2}\left(\sin^{3}\left(fx+e\right)\right)+6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2/315*(1+sin(f*x+e))*a^3*(sin(f*x+e)-1)*(35*d^2*sin(f*x+e)^4+90*c*d*sin(f*x
+e)^3+130*d^2*sin(f*x+e)^3+63*c^2*sin(f*x+e)^2+360*c*d*sin(f*x+e)^2+219*d^2
*sin(f*x+e)^2+294*c^2*sin(f*x+e)+690*c*d*sin(f*x+e)+292*d^2*sin(f*x+e)+903*
c^2+1380*c*d+584*d^2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + a\,\sin\left(e + f\,x\right)\right)^{5/2}\,\left(c + d\,\sin\left(e + f\,x\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^2,x)
[Out] int((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e + fx\right) + 1\right)\right)^{\frac{5}{2}}\left(c + d\sin\left(e + fx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c+d*sin(f*x+e))**2,x)
[Out] Integral((a*(sin(e + f*x) + 1))**(5/2)*(c + d*sin(e + f*x))**2, x)
```

3.538 $\int (a + a\sin(e + fx))^{5/2}(c + d\sin(e + fx))\,dx$

Optimal. Leaf size=138

$$-\frac{64a^3(7c+5d)\cos(e+fx)}{105f\sqrt{a\sin(e+fx)+a}}-\frac{16a^2(7c+5d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{105f}-\frac{2a(7c+5d)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{35f}$$

```
[Out] -2/35*a*(7*c+5*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-2/7*d*cos(f*x+e)*(a+a
*sin(f*x+e))^(5/2)/f-64/105*a^3*(7*c+5*d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/
2)-16/105*a^2*(7*c+5*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.11, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2751, 2647, 2646}

$$-\frac{64a^3(7c+5d)\cos(e+fx)}{105f\sqrt{a\sin(e+fx)+a}}-\frac{16a^2(7c+5d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{105f}-\frac{2a(7c+5d)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{35f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x]),x]
```

```
[Out] (-64*a^3*(7*c + 5*d)*Cos[e + f*x])/(105*f*Sqrt[a + a*Sin[e + f*x]]) - (16*a
^2*(7*c + 5*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(105*f) - (2*a*(7*c +
 5*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(35*f) - (2*d*Cos[e + f*x]*(
a + a*Sin[e + f*x])^(5/2))/(7*f)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
```

```
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))\,dx = -\frac{2d\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{7f}+\frac{1}{7}(7c+5d)\int (a+a\sin$$

$$= -\frac{2a(7c+5d)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{35f} - \frac{2d\cos(e+fx}{}$$

$$= -\frac{16a^2(7c+5d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105f} - \frac{2a(7c+5d)\,\mathrm{c}}{}$$

$$= -\frac{64a^3(7c+5d)\cos(e+fx)}{105f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2(7c+5d)\cos(e+fx)\sqrt{a+a}}{105f}$$

Mathematica [A] time = 1.50, size = 119, normalized size = 0.86

$$-\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)((392c+505d)\sin(e+fx)-6(7c+20d)\cos(2(e+}{210f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x]),x]
```

```
[Out] -1/210*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])
]*(1246*c + 1040*d - 6*(7*c + 20*d)*Cos[2*(e + f*x)] + (392*c + 505*d)*Sin[
e + f*x] - 15*d*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
)
```

fricas [A] time = 0.44, size = 206, normalized size = 1.49

$$\frac{2\left(15\,a^2d\cos\left(fx+e\right)^4+3\left(7\,a^2c+20\,a^2d\right)\cos\left(fx+e\right)^3-224\,a^2c-160\,a^2d-\left(77\,a^2c+85\,a^2d\right)\cos\left(fx+e\right)^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 2/105*(15*a^2*d*cos(f*x + e)^4 + 3*(7*a^2*c + 20*a^2*d)*cos(f*x + e)^3 - 22
4*a^2*c - 160*a^2*d - (77*a^2*c + 85*a^2*d)*cos(f*x + e)^2 - 2*(161*a^2*c +
```

```
 145*a^2*d)*cos(f*x + e) + (15*a^2*d*cos(f*x + e)^3 + 224*a^2*c + 160*a^2*d
 - 3*(7*a^2*c + 15*a^2*d)*cos(f*x + e)^2 - 2*(49*a^2*c + 65*a^2*d)*cos(f*x
+ e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x +
e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sq
rt(2*a)*(2*f*(6*a^2*c*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+4*a^2*d*sign(cos(1
/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(2*f*x-pi)+1/2*exp(1))/(2*f)^2-8*f*(16*a^
2*c*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+14*a^2*d*sign(cos(1/2*(f*x+exp(1))-1
/4*pi)))*cos(1/4*(2*f*x+2*exp(1)+pi))/(8*f)^2-24*f*(16*a^2*c*sign(cos(1/2*(
f*x+exp(1))-1/4*pi))+14*a^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(
6*f*x+6*exp(1)-pi))/(24*f)^2+12*f*(-2*a^2*c*sign(cos(1/2*(f*x+exp(1))-1/4*p
i))-4*a^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(6*f*x+6*exp(1)+pi)
)/(12*f)^2+20*f*(-2*a^2*c*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-4*a^2*d*sign(c
os(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(10*f*x+10*exp(1)-pi))/(20*f)^2+80*a^
2*d*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(10*f*x+10*exp(1)+pi))/(40
*f)^2+112*a^2*d*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(14*f*x+14*exp
(1)-pi))/(56*f)^2)
```

maple [A] time = 0.78, size = 99, normalized size = 0.72

$$\frac{2\left(1+\sin\left(fx+e\right)\right)a^{3}\left(\sin\left(fx+e\right)-1\right)\left(-15\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)d+(98c+130d)\sin\left(fx+e\right)+(-21c\right.}{105\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e)),x)
```

```
[Out] 2/105*(1+sin(f*x+e))*a^3*(sin(f*x+e)-1)*(-15*cos(f*x+e)^2*sin(f*x+e)*d+(98*
c+130*d)*sin(f*x+e)+(-21*c-60*d)*cos(f*x+e)^2+322*c+290*d)/cos(f*x+e)/(a+a*
sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x)),x)
[Out] int((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}\left(c+d\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c+d*sin(f*x+e)),x)
[Out] Integral((a*(sin(e + f*x) + 1))**(5/2)*(c + d*sin(e + f*x)), x)
```

3.539 $\int (a + a\sin(e + fx))^{5/2}\,dx$

Optimal. Leaf size=89

$$-\frac{64a^3\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{16a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

```
[Out] -2/5*a*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-64/15*a^3*cos(f*x+e)/f/(a+a*sin(
f*x+e))^(1/2)-16/15*a^2*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2647, 2646}

$$-\frac{64a^3\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{16a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-64*a^3*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) - (16*a^2*Cos[e + f*
x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) - (2*a*Cos[e + f*x]*(a + a*Sin[e + f*x]
)^(3/2))/(5*f)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}\,dx = -\frac{2a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f} + \frac{1}{5}(8a)\int (a+a\sin(e+fx))^{3/2}\,dx$$

$$= -\frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} - \frac{2a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f} + \frac{1}{15}$$

$$= -\frac{64a^3\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} - \frac{2a\cos(e+fx)(}{}$$

Mathematica [A] time = 0.30, size = 117, normalized size = 1.31

$$-\frac{(a(\sin(e+fx)+1))^{5/2}\left(-150\sin\left(\frac{1}{2}(e+fx)\right)+25\sin\left(\frac{3}{2}(e+fx)\right)+3\sin\left(\frac{5}{2}(e+fx)\right)+150\cos\left(\frac{1}{2}(e+fx)\right)+\right.}{30f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/30*((a*(1 + Sin[e + f*x]))^(5/2)*(150*Cos[(e + f*x)/2] + 25*Cos[(3*(e +
f*x))/2] - 3*Cos[(5*(e + f*x))/2] - 150*Sin[(e + f*x)/2] + 25*Sin[(3*(e + f
*x))/2] + 3*Sin[(5*(e + f*x))/2]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
^5)
```

fricas [A] time = 0.44, size = 115, normalized size = 1.29

$$\frac{2\left(3\,a^2\cos\left(fx+e\right)^3-11\,a^2\cos\left(fx+e\right)^2-46\,a^2\cos\left(fx+e\right)-32\,a^2-\left(3\,a^2\cos\left(fx+e\right)^2+14\,a^2\cos\left(fx+e\right.\right.\right.}{15\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*a^2*cos(f*x + e)^3 - 11*a^2*cos(f*x + e)^2 - 46*a^2*cos(f*x + e) -
32*a^2 - (3*a^2*cos(f*x + e)^2 + 14*a^2*cos(f*x + e) - 32*a^2)*sin(f*x + e)
)*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*(2*a^2*sign(cos(1/2*(f*x+exp(1)
)-1/4*pi))*sin(1/4*(2*f*x-pi)+1/2*exp(1))/f-2*a^2*sign(cos(1/2*(f*x+exp(1))
-1/4*pi))*cos(1/4*(2*f*x+2*exp(1)+pi))/f-24*a^2*f*sign(cos(1/2*(f*x+exp(1))
-1/4*pi))*sin(1/4*(6*f*x+6*exp(1)+pi))/(12*f)^2-40*a^2*f*sign(cos(1/2*(f*x+
exp(1))-1/4*pi))*sin(1/4*(10*f*x+10*exp(1)-pi))/(20*f)^2+4*a^2*f*sign(cos(1
/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(2*f*x-pi)+1/2*exp(1))/(2*f)^2-6*a^2*f*sig
n(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(6*f*x+6*exp(1)-pi))/(3*f)^2)
```

maple [A] time = 0.77, size = 65, normalized size = 0.73

$$\frac{2\left(1+\sin\left(fx+e\right)\right)a^{3}\left(\sin\left(fx+e\right)-1\right)\left(3\left(\sin^{2}\left(fx+e\right)\right)+14\sin\left(fx+e\right)+43\right)}{15\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 2/15*(1+sin(f*x+e))*a^3*(sin(f*x+e)-1)*(3*sin(f*x+e)^2+14*sin(f*x+e)+43)/co
s(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(e+fx\right)+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((a*sin(e + f*x) + a)**(5/2), x)
```

3.540 $\int \frac{(a+a\sin(e+fx))^{5/2}}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=142

$$-\frac{2a^{5/2}(c-d)^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{5/2}f\sqrt{c+d}}+\frac{2a^3(3c-7d)\cos(e+fx)}{3d^2f\sqrt{a\sin(e+fx)+a}}-\frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df}$$

```
[Out] -2*a^(5/2)*(c-d)^2*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(
f*x+e))^(1/2))/d^(5/2)/f/(c+d)^(1/2)+2/3*a^3*(3*c-7*d)*cos(f*x+e)/d^2/f/(a+
a*sin(f*x+e))^(1/2)-2/3*a^2*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d/f
```

Rubi [A] time = 0.41, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2763, 2981, 2773, 208}

$$\frac{2a^3(3c-7d)\cos(e+fx)}{3d^2f\sqrt{a\sin(e+fx)+a}}-\frac{2a^{5/2}(c-d)^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{5/2}f\sqrt{c+d}}-\frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x]),x]
```

```
[Out] (-2*a^(5/2)*(c - d)^2*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*S
qrt[a + a*Sin[e + f*x]])])/(d^(5/2)*Sqrt[c + d]*f) + (2*a^3*(3*c - 7*d)*Cos
[e + f*x])/(3*d^2*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a^2*Cos[e + f*x]*Sqrt[a
+ a*Sin[e + f*x]])/(3*d*f)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
```

```
0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{c+d\sin(e+fx)}\,dx = -\frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3df} + \frac{2\int \frac{\sqrt{a+a\sin(e+fx)}\left(\frac{1}{2}a^2(c+3d)-\frac{1}{2}a^2(3c-7d)\sin(e+f}{c+d\sin(e+fx)}}{3d}$$

$$= \frac{2a^3(3c-7d)\cos(e+fx)}{3d^2f\sqrt{a+a\sin(e+fx)}} - \frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3df} + \frac{\left(a^2(c-d)^2\right)\int}{d}$$

$$= \frac{2a^3(3c-7d)\cos(e+fx)}{3d^2f\sqrt{a+a\sin(e+fx)}} - \frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3df} - \frac{\left(2a^3(c-d)^2\right)\mathrm{Su}}{}$$

$$= -\frac{2a^{5/2}(c-d)^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{5/2}\sqrt{c+d}\,f} + \frac{2a^3(3c-7d)\cos(e+fx)}{3d^2f\sqrt{a+a\sin(e+fx)}} - \frac{2a^2\cos(}{}$$

Mathematica [B] time = 3.60, size = 330, normalized size = 2.32

$$\frac{(a(\sin(e+fx)+1))^{5/2}\left(6\sqrt{d}\,(5d-2c)\sin\left(\frac{1}{2}(e+fx)\right)+6\sqrt{d}\,(2c-5d)\cos\left(\frac{1}{2}(e+fx)\right)+\frac{3(c-d)^2\left(2\log\left(\sqrt{d}\sqrt{c+d}\left(\tan^2\right.\right.\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x]),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(6*(2*c - 5*d)*Sqrt[d]*Cos[(e + f*x)/2] - 2*d
^(3/2)*Cos[(3*(e + f*x))/2] - (3*(c - d)^2*(e + f*x - 2*Log[Sec[(e + f*x)/4
]^2] + 2*Log[-(Sec[(e + f*x)/4]^2*(c + d + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x
)/2] - Sqrt[d]*Sqrt[c + d]*Sin[(e + f*x)/2]))]))/Sqrt[c + d] + (3*(c - d)^2
*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[(c + d)*Sec[(e + f*x)/4]^2 +
Sqrt[d]*Sqrt[c + d]*(-1 + 2*Tan[(e + f*x)/4] + Tan[(e + f*x)/4]^2)]))/Sqrt[
c + d] + 6*Sqrt[d]*(-2*c + 5*d)*Sin[(e + f*x)/2] - 2*d^(3/2)*Sin[(3*(e + f*
x))/2]))/(6*d^(5/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

fricas [B] time = 0.59, size = 868, normalized size = 6.11

$$\left[\frac{3\left(a^2c^2 - 2\,a^2cd + a^2d^2 + \left(a^2c^2 - 2\,a^2cd + a^2d^2\right)\cos\left(fx+e\right) + \left(a^2c^2 - 2\,a^2cd + a^2d^2\right)\sin\left(fx+e\right)\right)\sqrt{\frac{a}{cd+d^2}}\,\log\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^2*c^2 - 2*a^2*c*d + a^2*d^2 + (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*co
s(f*x + e) + (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*sin(f*x + e))*sqrt(a/(c*d + d^
2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^
2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(f*x + e)
^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2 + 3*d^3 + (c
*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(a/(c*
d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^
2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x +
 e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2
 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^
2 - 2*c*d - d^2)*sin(f*x + e))) - 4*(a^2*d*cos(f*x + e)^2 - 3*a^2*c + 7*a^2
*d - (3*a^2*c - 8*a^2*d)*cos(f*x + e) + (a^2*d*cos(f*x + e) + 3*a^2*c - 7*a
^2*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(d^2*f*cos(f*x + e) + d^2*f*s
in(f*x + e) + d^2*f), -1/3*(3*(a^2*c^2 - 2*a^2*c*d + a^2*d^2 + (a^2*c^2 - 2
*a^2*c*d + a^2*d^2)*cos(f*x + e) + (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*sin(f*x
+ e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x +
 e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) + 2*(a^2*d*cos(f*x +
e)^2 - 3*a^2*c + 7*a^2*d - (3*a^2*c - 8*a^2*d)*cos(f*x + e) + (a^2*d*cos(f*
x + e) + 3*a^2*c - 7*a^2*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(d^2*f*
cos(f*x + e) + d^2*f*sin(f*x + e) + d^2*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Warning, integration of abs or sign assumes constant sign by intervals (cor
rect if the argument is real):Check [abs(cos((f*x+exp(1))/2-pi/4))]Evaluati
on time: 109.9Not invertible Error: Bad Argument Value
```

maple [A] time = 1.43, size = 229, normalized size = 1.61

$$-\frac{2a\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\left(3\,\mathrm{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,d}{\sqrt{a(c+d)d}}\right)a^2c^2-6\,\mathrm{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}}{\sqrt{a(c+d)d}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] -2/3*a*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(3*arctanh((-a*(sin(f*x+e)-
1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c^2-6*arctanh((-a*(sin(f*x+e)-1))^(1/2)*
d/(a*(c+d)*d)^(1/2))*a^2*c*d+3*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)
*d)^(1/2))*a^2*d^2-(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*d-3*(-a*(sin
(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*c+9*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d
)*d)^(1/2)*a*d)/d^2/(a*(c+d)*d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}{c+d\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x)),x)
```

```
[Out] int((a + a*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.541 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=166

$$\frac{a^{5/2}(c-d)(3c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{5/2}f(c+d)^{3/2}}-\frac{a^3(3c+d)\cos(e+fx)}{d^2f(c+d)\sqrt{a\sin(e+fx)+a}}+\frac{a^2(c-d)\cos(e+fx)\sqrt{a\sin(e}{df(c+d)(c+d\sin(e+}$$

```
[Out] a^(5/2)*(c-d)*(3*c+5*d)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a
*sin(f*x+e))^(1/2))/d^(5/2)/(c+d)^(3/2)/f-a^3*(3*c+d)*cos(f*x+e)/d^2/(c+d)/
f/(a+a*sin(f*x+e))^(1/2)+a^2*(c-d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d/(c+d
)/f/(c+d*sin(f*x+e))
```

Rubi [A] time = 0.39, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2762, 2981, 2773, 208}

$$-\frac{a^3(3c+d)\cos(e+fx)}{d^2f(c+d)\sqrt{a\sin(e+fx)+a}}+\frac{a^{5/2}(c-d)(3c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{5/2}f(c+d)^{3/2}}+\frac{a^2(c-d)\cos(e+fx)\sqrt{a\sin(}{df(c+d)(c+d\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a^(5/2)*(c - d)*(3*c + 5*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c
 + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(5/2)*(c + d)^(3/2)*f) - (a^3*(3*c + d
)*Cos[e + f*x])/(d^2*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]) + (a^2*(c - d)*Cos
[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(d*(c + d)*f*(c + d*Sin[e + f*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

```
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^2}\,dx = \frac{a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{d(c+d)f(c+d\sin(e+fx))} - \frac{a\int \frac{\sqrt{a+a\sin(e+fx)}\left(\frac{1}{2}a(c-5d)-\frac{1}{2}a(3c+d)\sin(e+f\right.}{c+d\sin(e+fx)}}{d(c+d)}$$

$$= -\frac{a^3(3c+d)\cos(e+fx)}{d^2(c+d)f\sqrt{a+a\sin(e+fx)}} + \frac{a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{d(c+d)f(c+d\sin(e+fx))} - \frac{\left(a^2(c\right.}{}$$

$$= -\frac{a^3(3c+d)\cos(e+fx)}{d^2(c+d)f\sqrt{a+a\sin(e+fx)}} + \frac{a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{d(c+d)f(c+d\sin(e+fx))} + \frac{\left(a^3(c\right.}{}$$

$$= \frac{a^{5/2}(c-d)(3c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{5/2}(c+d)^{3/2}f} - \frac{a^3(3c+d)\cos(e+fx)}{d^2(c+d)f\sqrt{a+a\sin(e+fx)}} +$$

Mathematica [B] time = 4.09, size = 350, normalized size = 2.11

$$(a(\sin(e+fx)+1))^{5/2}\left(\frac{(-3c^2-2cd+5d^2)\left(2\log\left(\sqrt{d}\sqrt{c+d}\left(\tan^2\left(\frac{1}{4}(e+fx)\right)+2\tan\left(\frac{1}{4}(e+fx)\right)-1\right)+(c+d)\sec^2\left(\frac{1}{4}(e+fx)\right)\right)-2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right.\right.}{(c+d)^{3/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(-8*Sqrt[d]*Cos[(e + f*x)/2] + ((3*c^2 + 2*c*
d - 5*d^2)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[-(Sec[(e + f*x)/4]^
2*(c + d + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] - Sqrt[d]*Sqrt[c + d]*Sin[(
e + f*x)/2]))]))/(c + d)^(3/2) + ((-3*c^2 - 2*c*d + 5*d^2)*(e + f*x - 2*Log
[Sec[(e + f*x)/4]^2] + 2*Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c +
d]*(-1 + 2*Tan[(e + f*x)/4] + Tan[(e + f*x)/4]^2)]))/(c + d)^(3/2) + 8*Sqrt
[d]*Sin[(e + f*x)/2] - (4*(c - d)^2*Sqrt[d]*(Cos[(e + f*x)/2] - Sin[(e + f*
x)/2]))/((c + d)*(c + d*Sin[e + f*x]))))/(4*d^(5/2)*f*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^5)
```

fricas [B] time = 0.63, size = 1322, normalized size = 7.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/4*((3*a^2*c^3 + 5*a^2*c^2*d - 3*a^2*c*d^2 - 5*a^2*d^3 - (3*a^2*c^2*d + 2
*a^2*c*d^2 - 5*a^2*d^3)*cos(f*x + e)^2 + (3*a^2*c^3 + 2*a^2*c^2*d - 5*a^2*c
*d^2)*cos(f*x + e) + (3*a^2*c^3 + 5*a^2*c^2*d - 3*a^2*c*d^2 - 5*a^2*d^3 + (
3*a^2*c^2*d + 2*a^2*c*d^2 - 5*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a/(
c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d
+ 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(
f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2 + 3*
d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sq
rt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f
*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*s
in(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c
*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x +
 e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(3*a^2*c^2 - 2*a^2*c*d - a^2*d^
2 + 2*(a^2*c*d + a^2*d^2)*cos(f*x + e)^2 + (3*a^2*c^2 + a^2*d^2)*cos(f*x +
e) - (3*a^2*c^2 - 2*a^2*c*d - a^2*d^2 - 2*(a^2*c*d + a^2*d^2)*cos(f*x + e))
*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c*d^3 + d^4)*f*cos(f*x + e)^2 -
(c^2*d^2 + c*d^3)*f*cos(f*x + e) - (c^2*d^2 + 2*c*d^3 + d^4)*f - ((c*d^3 +
```

```
d^4)*f*cos(f*x + e) + (c^2*d^2 + 2*c*d^3 + d^4)*f)*sin(f*x + e)), -1/2*((3*
a^2*c^3 + 5*a^2*c^2*d - 3*a^2*c*d^2 - 5*a^2*d^3 - (3*a^2*c^2*d + 2*a^2*c*d^
2 - 5*a^2*d^3)*cos(f*x + e)^2 + (3*a^2*c^3 + 2*a^2*c^2*d - 5*a^2*c*d^2)*cos
(f*x + e) + (3*a^2*c^3 + 5*a^2*c^2*d - 3*a^2*c*d^2 - 5*a^2*d^3 + (3*a^2*c^2
*d + 2*a^2*c*d^2 - 5*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-a/(c*d + d^
2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/
(c*d + d^2))/(a*cos(f*x + e))) - 2*(3*a^2*c^2 - 2*a^2*c*d - a^2*d^2 + 2*(a^
2*c*d + a^2*d^2)*cos(f*x + e)^2 + (3*a^2*c^2 + a^2*d^2)*cos(f*x + e) - (3*a
^2*c^2 - 2*a^2*c*d - a^2*d^2 - 2*(a^2*c*d + a^2*d^2)*cos(f*x + e))*sin(f*x
+ e))*sqrt(a*sin(f*x + e) + a))/((c*d^3 + d^4)*f*cos(f*x + e)^2 - (c^2*d^2
+ c*d^3)*f*cos(f*x + e) - (c^2*d^2 + 2*c*d^3 + d^4)*f - ((c*d^3 + d^4)*f*co
s(f*x + e) + (c^2*d^2 + 2*c*d^3 + d^4)*f)*sin(f*x + e))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.48, size = 393, normalized size = 2.37

$$-\frac{a^2 \left(1+\sin\left(fx+e\right)\right) \sqrt{-a\left(\sin\left(fx+e\right)-1\right)} \left(-\sin\left(fx+e\right) d \left(3\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\, d}{\sqrt{acd+a\,d^2}}\right) a\,c^2+2\,\mathrm{arctanh}\left(\frac{\sqrt{\ldots}}{\ldots}\right.\right.\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x)
```

```
[Out] -a^2*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-sin(f*x+e)*d*(3*arctanh((a-
a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c^2+2*arctanh((a-a*sin(f*x+e))
^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c*d-5*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c
*d+a*d^2)^(1/2))*a*d^2-2*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*c-2*(a-a*
sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*d)-3*arctanh((a-a*sin(f*x+e))^(1/2)*d/(
a*c*d+a*d^2)^(1/2))*a*c^3-2*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^
(1/2))*a*c^2*d+5*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c*
d^2+3*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*c^2+(a-a*sin(f*x+e))^(1/2)*(
a*(c+d)*d)^(1/2)*d^2)/d^2/(c+d)/(c+d*sin(f*x+e))/(a*(c+d)*d)^(1/2)/cos(f*x+
e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a \sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(d \sin\left(fx+e\right)+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}{\left(c+d\,\sin\left(e+f\,x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^2,x)
[Out] int((a + a*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**2,x)
[Out] Timed out
```

3.542 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=194

$$-\frac{a^{5/2}\left(3c^2+10cd+19d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{4d^{5/2}f(c+d)^{5/2}}+\frac{3a^3(c-d)(c+3d)\cos(e+fx)}{4d^2f(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}+\frac{a^2(c}{}$$

```
[Out] -1/4*a^(5/2)*(3*c^2+10*c*d+19*d^2)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)
^(1/2)/(a+a*sin(f*x+e))^(1/2))/d^(5/2)/(c+d)^(5/2)/f+3/4*a^3*(c-d)*(c+3*d)*
cos(f*x+e)/d^2/(c+d)^2/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)+1/2*a^2*(c
-d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d/(c+d)/f/(c+d*sin(f*x+e))^2
```

Rubi [A] time = 0.44, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2762, 2980, 2773, 208}

$$-\frac{a^{5/2}\left(3c^2+10cd+19d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{4d^{5/2}f(c+d)^{5/2}}+\frac{3a^3(c-d)(c+3d)\cos(e+fx)}{4d^2f(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}+\frac{a^2(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] -(a^(5/2)*(3*c^2 + 10*c*d + 19*d^2)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/
(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(4*d^(5/2)*(c + d)^(5/2)*f) + (a^2
*(c - d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(2*d*(c + d)*f*(c + d*Sin[e
 + f*x])^2) + (3*a^3*(c - d)*(c + 3*d)*Cos[e + f*x])/(4*d^2*(c + d)^2*f*Sqr
t[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^3}\,dx = \frac{a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a\int \frac{\sqrt{a+a\sin(e+fx)}\left(\frac{1}{2}a(c-9d)-\frac{1}{2}a(3c+5d)\sin(e\cdots\right.}{(c+d\sin(e+fx))^2}}{2d(c+d)}$$

$$= \frac{a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{3a^3(c-d)(c+3d)\cos(e+f\cdots}{4d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}\,(c+d\cdots}$$

$$= \frac{a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{3a^3(c-d)(c+3d)\cos(e+f\cdots}{4d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}\,(c+d\cdots}$$

$$= -\frac{a^{5/2}\left(3c^2+10cd+19d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sin(e+fx)}}\right)}{4d^{5/2}(c+d)^{5/2}f} + \frac{a^2(c-d)\cos(e+fx)\sqrt{a\cdots}}{2d(c+d)f(c+d\sin\cdots}$$

Mathematica [A] time = 4.92, size = 379, normalized size = 1.95

$$(a(\sin(e+fx)+1))^{5/2}\left(-\frac{4\sqrt{d}\left(-5c^2-6cd+11d^2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(c+d)^2(c+d\sin(e+fx))}+\frac{\left(3c^2+10cd+19d^2\right)\left(2\log\left(\sqrt{d}\sqrt{c+d}\left(\tan^2\left(\frac{1}{4}(e+fx)\right)+2\tan\right.\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(-(((3*c^2 + 10*c*d + 19*d^2)*(e + f*x - 2*Lo
g[Sec[(e + f*x)/4]^2] + 2*Log[-(Sec[(e + f*x)/4]^2*(c + d + Sqrt[d]*Sqrt[c
+ d]*Cos[(e + f*x)/2] - Sqrt[d]*Sqrt[c + d]*Sin[(e + f*x)/2]))]))/(c + d)^(
5/2)) + ((3*c^2 + 10*c*d + 19*d^2)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2
*Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c + d]*(-1 + 2*Tan[(e + f*x)
/4] + Tan[(e + f*x)/4]^2)]))/(c + d)^(5/2) - (8*(c - d)^2*Sqrt[d]*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2]))/((c + d)*(c + d*Sin[e + f*x])^2) - (4*Sqrt[d]
*(-5*c^2 - 6*c*d + 11*d^2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d)^
2*(c + d*Sin[e + f*x]))))/(16*d^(5/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^5)
```

fricas [B] time = 0.71, size = 1994, normalized size = 10.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/16*((3*a^2*c^4 + 16*a^2*c^3*d + 42*a^2*c^2*d^2 + 48*a^2*c*d^3 + 19*a^2*
d^4 - (3*a^2*c^2*d^2 + 10*a^2*c*d^3 + 19*a^2*d^4)*cos(f*x + e)^3 - (6*a^2*c
^3*d + 23*a^2*c^2*d^2 + 48*a^2*c*d^3 + 19*a^2*d^4)*cos(f*x + e)^2 + (3*a^2*
c^4 + 10*a^2*c^3*d + 22*a^2*c^2*d^2 + 10*a^2*c*d^3 + 19*a^2*d^4)*cos(f*x +
e) + (3*a^2*c^4 + 16*a^2*c^3*d + 42*a^2*c^2*d^2 + 48*a^2*c*d^3 + 19*a^2*d^4
 - (3*a^2*c^2*d^2 + 10*a^2*c*d^3 + 19*a^2*d^4)*cos(f*x + e)^2 + 2*(3*a^2*c^
3*d + 10*a^2*c^2*d^2 + 19*a^2*c*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a/(c*
d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d +
7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(f*
x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2 + 3*d^
3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt
(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x
 + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin
(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d
 - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e
) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(3*a^2*c^3 + 3*a^2*c^2*d - 15*a^2
```

```
*c*d^2 + 9*a^2*d^3 + (5*a^2*c^2*d + 6*a^2*c*d^2 - 11*a^2*d^3)*cos(f*x + e)^
2 + (3*a^2*c^3 + 8*a^2*c^2*d - 9*a^2*c*d^2 - 2*a^2*d^3)*cos(f*x + e) - (3*a
^2*c^3 + 3*a^2*c^2*d - 15*a^2*c*d^2 + 9*a^2*d^3 - (5*a^2*c^2*d + 6*a^2*c*d^
2 - 11*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c^2
*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e)^3 + (2*c^3*d^3 + 5*c^2*d^4 + 4*c*d^5 +
 d^6)*f*cos(f*x + e)^2 - (c^4*d^2 + 2*c^3*d^3 + 2*c^2*d^4 + 2*c*d^5 + d^6)*
f*cos(f*x + e) - (c^4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^6)*f + ((c^
2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e)^2 - 2*(c^3*d^3 + 2*c^2*d^4 + c*d^5)*f
*cos(f*x + e) - (c^4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^6)*f)*sin(f*
x + e)), 1/8*((3*a^2*c^4 + 16*a^2*c^3*d + 42*a^2*c^2*d^2 + 48*a^2*c*d^3 + 1
9*a^2*d^4 - (3*a^2*c^2*d^2 + 10*a^2*c*d^3 + 19*a^2*d^4)*cos(f*x + e)^3 - (6
*a^2*c^3*d + 23*a^2*c^2*d^2 + 48*a^2*c*d^3 + 19*a^2*d^4)*cos(f*x + e)^2 + (
3*a^2*c^4 + 10*a^2*c^3*d + 22*a^2*c^2*d^2 + 10*a^2*c*d^3 + 19*a^2*d^4)*cos(
f*x + e) + (3*a^2*c^4 + 16*a^2*c^3*d + 42*a^2*c^2*d^2 + 48*a^2*c*d^3 + 19*a
^2*d^4 - (3*a^2*c^2*d^2 + 10*a^2*c*d^3 + 19*a^2*d^4)*cos(f*x + e)^2 + 2*(3*
a^2*c^3*d + 10*a^2*c^2*d^2 + 19*a^2*c*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt
(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c -
2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*(3*a^2*c^3 + 3*a^2*c^2*d -
15*a^2*c*d^2 + 9*a^2*d^3 + (5*a^2*c^2*d + 6*a^2*c*d^2 - 11*a^2*d^3)*cos(f*x
 + e)^2 + (3*a^2*c^3 + 8*a^2*c^2*d - 9*a^2*c*d^2 - 2*a^2*d^3)*cos(f*x + e)
- (3*a^2*c^3 + 3*a^2*c^2*d - 15*a^2*c*d^2 + 9*a^2*d^3 - (5*a^2*c^2*d + 6*a^
2*c*d^2 - 11*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))
/((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e)^3 + (2*c^3*d^3 + 5*c^2*d^4 + 4*c
*d^5 + d^6)*f*cos(f*x + e)^2 - (c^4*d^2 + 2*c^3*d^3 + 2*c^2*d^4 + 2*c*d^5 +
 d^6)*f*cos(f*x + e) - (c^4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^6)*f
+ ((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e)^2 - 2*(c^3*d^3 + 2*c^2*d^4 + c*
d^5)*f*cos(f*x + e) - (c^4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^6)*f)*
sin(f*x + e))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.81, size = 567, normalized size = 2.92

$$-\frac{a\left(2\sin\left(fx+e\right)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,d}{\sqrt{acd+a\,d^{2}}}\right)a^{2}cd\left(3c^{2}+10cd+19d^{2}\right)-\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,d}{\sqrt{acd+a\,d^{2}}}\right)a^{2}d^{2}\left(3c^{2}+10\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x)
```

```
[Out] -1/4*a*(2*sin(f*x+e)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*
a^2*c*d*(3*c^2+10*c*d+19*d^2)-arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2
)^(1/2))*a^2*d^2*(3*c^2+10*c*d+19*d^2)*cos(f*x+e)^2+3*arctanh((a-a*sin(f*x+
e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^4+10*arctanh((a-a*sin(f*x+e))^(1/2)*
d/(a*c*d+a*d^2)^(1/2))*a^2*c^3*d+22*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d
+a*d^2)^(1/2))*a^2*c^2*d^2+10*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2
)^(1/2))*a^2*c*d^3+19*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))
*a^2*d^4+5*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*c^2*d+6*(a-a*sin(f*x+e)
)^(3/2)*(a*(c+d)*d)^(1/2)*c*d^2-11*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)
*d^3-3*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^3-13*(a-a*sin(f*x+e))^(
1/2)*(a*(c+d)*d)^(1/2)*a*c^2*d+3*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a
*c*d^2+13*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^3)*(-a*(sin(f*x+e)-1
))^(1/2)*(1+sin(f*x+e))/(a*(c+d)*d)^(1/2)/(c+d*sin(f*x+e))^2/(c+d)^2/d^2/co
s(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}{\left(c+d\,\sin\left(e+f\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^3,x)
```

```
[Out] int((a + a*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^3, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.543 $\int \frac{(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=178

$$-\frac{4d\left(21c^2-12cd+7d^2\right)\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{2d^2(9c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15af}-\frac{2d\cos(e+fx)(c+d\sin(e+}{5f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -(c-d)^3*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(
1/2)/f/a^(1/2)-4/15*d*(21*c^2-12*c*d+7*d^2)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(
1/2)-2/5*d*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(f*x+e))^(1/2)-2/15*(9*c
-d)*d^2*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/a/f
```

Rubi [A] time = 0.44, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2778, 2968, 3023, 2751, 2649, 206}

$$-\frac{4d\left(21c^2-12cd+7d^2\right)\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{2d^2(9c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15af}-\frac{2d\cos(e+fx)(c+d\sin(e+}{5f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^3/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[2]*(c - d)^3*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin
[e + f*x]])])/(Sqrt[a]*f)) - (4*d*(21*c^2 - 12*c*d + 7*d^2)*Cos[e + f*x])/(
15*f*Sqrt[a + a*Sin[e + f*x]]) - (2*(9*c - d)*d^2*Cos[e + f*x]*Sqrt[a + a*S
in[e + f*x]])/(15*a*f) - (2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(5*f*Sqr
t[a + a*Sin[e + f*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{2d\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a+a\sin(e+fx)}} - \frac{\int \frac{(c+d\sin(e+fx))\left(-a\left(5c^2-cd+4d^2\right)-a(9c-d)d\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}}{5a}$$

$$= -\frac{2d\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a+a\sin(e+fx)}} - \frac{\int \frac{-ac\left(5c^2-cd+4d^2\right)+\left(-ac(9c-d)d-ad\left(5c^2-cd+4d^2\right)\right)\sin(}{\sqrt{a+a\sin(e+fx)}}}{5a}$$

$$= -\frac{2(9c-d)d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15af} - \frac{2d\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{4d\left(21c^2-12cd+7d^2\right)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{2(9c-d)d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15af}$$

$$= -\frac{4d\left(21c^2-12cd+7d^2\right)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{2(9c-d)d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15af}$$

$$= -\frac{\sqrt{2}\,(c-d)^3\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,f} - \frac{4d\left(21c^2-12cd+7d^2\right)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - 2(9$$

Mathematica [C] time = 0.59, size = 155, normalized size = 0.87

$$-\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-2d\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-90c^2-2d(15c-d)\sin(e+fx\right.\right.}{30f\sqrt{a(\mathrm{si}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^3/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -1/30*((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((-60 - 60*I)*(-1)^(3/4)*(c -
d)^3*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - 2*d*(Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2])*(-90*c^2 + 30*c*d - 29*d^2 + 3*d^2*Cos[2*(e +
 f*x)] - 2*(15*c - d)*d*Sin[e + f*x])))/(f*Sqrt[a*(1 + Sin[e + f*x])])
```

fricas [B] time = 0.48, size = 387, normalized size = 2.17

$$-\frac{\frac{15\sqrt{2}\left(ac^3-3ac^2d+3acd^2-ad^3+\left(ac^3-3ac^2d+3acd^2-ad^3\right)\cos\left(fx+e\right)+\left(ac^3-3ac^2d+3acd^2-ad^3\right)\sin\left(fx+e\right)\right)\log\left(-\frac{\cos\left(fx+e\right)^2-\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right.}{\cos\left(fx+e\right)^2}\right.}{\sqrt{a}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/30*(15*sqrt(2)*(a*c^3 - 3*a*c^2*d + 3*a*c*d^2 - a*d^3 + (a*c^3 - 3*a*c^2
*d + 3*a*c*d^2 - a*d^3)*cos(f*x + e) + (a*c^3 - 3*a*c^2*d + 3*a*c*d^2 - a*d
^3)*sin(f*x + e))*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) +
2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a
) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) -
 cos(f*x + e) - 2))/sqrt(a) - 4*(3*d^3*cos(f*x + e)^3 - 45*c^2*d + 30*c*d^2
 - 17*d^3 - (15*c*d^2 - 4*d^3)*cos(f*x + e)^2 - (45*c^2*d - 15*c*d^2 + 16*d
^3)*cos(f*x + e) - (3*d^3*cos(f*x + e)^2 - 45*c^2*d + 30*c*d^2 - 17*d^3 + (
15*c*d^2 - d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a*f*
cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(2/sqrt(a*tan((f*x+ex
p(1))/2)^2+a)/(a*tan((f*x+exp(1))/2)^2+a)^2*(tan((f*x+exp(1))/2)*(tan((f*x+
exp(1))/2)*(tan((f*x+exp(1))/2)*(tan((f*x+exp(1))/2)*(1/3600*tan((f*x+exp(1
))/2)*(1560*a^2*d^3*sign(tan((f*x+exp(1))/2)+1)-1800*a^2*c*d^2*sign(tan((f*
x+exp(1))/2)+1)+5400*a^2*c^2*d*sign(tan((f*x+exp(1))/2)+1))+1/3600*(-1800*a
^2*d^3*sign(tan((f*x+exp(1))/2)+1)+5400*a^2*c*d^2*sign(tan((f*x+exp(1))/2)+
1)-5400*a^2*c^2*d*sign(tan((f*x+exp(1))/2)+1)))+1/3600*(4800*a^2*d^3*sign(t
an((f*x+exp(1))/2)+1)-7200*a^2*c*d^2*sign(tan((f*x+exp(1))/2)+1)+10800*a^2*
c^2*d*sign(tan((f*x+exp(1))/2)+1)))+1/3600*(-4800*a^2*d^3*sign(tan((f*x+exp
(1))/2)+1)+7200*a^2*c*d^2*sign(tan((f*x+exp(1))/2)+1)-10800*a^2*c^2*d*sign(
tan((f*x+exp(1))/2)+1)))+1/3600*(1800*a^2*d^3*sign(tan((f*x+exp(1))/2)+1)-5
```

```
400*a^2*c*d^2*sign(tan((f*x+exp(1))/2)+1)+5400*a^2*c^2*d*sign(tan((f*x+exp(
1))/2)+1)))+1/3600*(-1560*a^2*d^3*sign(tan((f*x+exp(1))/2)+1)+1800*a^2*c*d^
2*sign(tan((f*x+exp(1))/2)+1)-5400*a^2*c^2*d*sign(tan((f*x+exp(1))/2)+1)))+
sqrt(2)*(c^3-d^3+3*c*d^2-3*c^2*d)*atan((-sqrt(a)*tan((f*x+exp(1))/2)-sqrt(a
)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/sqrt(-a)/sign(tan((f*x
+exp(1))/2)+1)+(-15*a*c^3*sqrt(2)*atan(sqrt(a)/sqrt(-a))+45*a*c^2*d*sqrt(2)
*atan(sqrt(a)/sqrt(-a))-45*a*c*d^2*sqrt(2)*atan(sqrt(a)/sqrt(-a))+15*a*d^3*
sqrt(2)*atan(sqrt(a)/sqrt(-a))+45*c^2*d*sqrt(-a)*sqrt(2)*sqrt(a)-30*c*d^2*s
qrt(-a)*sqrt(2)*sqrt(a)+17*d^3*sqrt(-a)*sqrt(2)*sqrt(a))/15/a/sqrt(-a)*sign
(tan((f*x+exp(1))/2)+1))
```

maple [A] time = 1.23, size = 285, normalized size = 1.60

$$-\frac{\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,\left(15a^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,\sqrt{2}}{2\sqrt{a}}\right)c^{3}-45a^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a}}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/15*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(15*a^(5/2)*2^(1/2)*arctanh(
1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^3-45*a^(5/2)*2^(1/2)*arctanh(
1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^2*d+45*a^(5/2)*2^(1/2)*arctan
h(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c*d^2-15*a^(5/2)*2^(1/2)*arct
anh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d^3+6*d^3*(a-a*sin(f*x+e))^
(5/2)-30*(a-a*sin(f*x+e))^(3/2)*a*c*d^2-10*(a-a*sin(f*x+e))^(3/2)*a*d^3+90*
c^2*d*a^2*(a-a*sin(f*x+e))^(1/2)+30*a^2*d^3*(a-a*sin(f*x+e))^(1/2))/a^3/cos
(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{3}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^3/sqrt(a*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c+d\,\sin\left(e+f\,x\right)\right)^{3}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^3/(a + a*sin(e + f*x))^(1/2),x)
[Out] int((c + d*sin(e + f*x))^3/(a + a*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+d\sin\left(e+fx\right)\right)^3}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(1/2),x)
[Out] Integral((c + d*sin(e + f*x))**3/sqrt(a*(sin(e + f*x) + 1)), x)
```

3.544 $\int \frac{(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=123

$$-\frac{4d(3c-d)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{\sqrt{2}\,(c-d)^2\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\,f}-\frac{2d^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3af}$$

```
[Out] -(c-d)^2*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(
1/2)/f/a^(1/2)-4/3*(3*c-d)*d*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/3*d^2*co
s(f*x+e)*(a+a*sin(f*x+e))^(1/2)/a/f
```

Rubi [A] time = 0.20, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2761, 2751, 2649, 206}

$$-\frac{4d(3c-d)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{\sqrt{2}\,(c-d)^2\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\,f}-\frac{2d^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^2/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[2]*(c - d)^2*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin
[e + f*x]])])/(Sqrt[a]*f)) - (4*(3*c - d)*d*Cos[e + f*x])/(3*f*Sqrt[a + a*S
in[e + f*x]]) - (2*d^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*a*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{2d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3af}+\frac{2\int \frac{\frac{1}{2}a\left(3c^2+d^2\right)+a(3c-d)d\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx}{3a}$$

$$= -\frac{4(3c-d)d\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}-\frac{2d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3af}+(c-d)^2\int \frac{}{\sqrt{a+a\,\mathrm{s}}}$$

$$= -\frac{4(3c-d)d\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}-\frac{2d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3af}-\frac{\left(2(c-d)^2\right)\mathrm{Subst}\left(\right.}{}$$

$$= -\frac{\sqrt{2}\,(c-d)^2\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,f}-\frac{4(3c-d)d\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}-\frac{2d^2\cos(e+fx)\sqrt{}}{3}$$

Mathematica [C] time = 0.39, size = 125, normalized size = 1.02

$$-\frac{2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(d\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(6c+d\sin(e+fx)-d)-(3+3i)(\right.}{3f\sqrt{a(\sin(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^2/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (-2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((-3 - 3*I)*(-1)^(3/4)*(c - d)^2*
ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] + d*(Cos[(e + f*x)/
```

```
2] - Sin[(e + f*x)/2])*(6*c - d + d*Sin[e + f*x])))/(3*f*Sqrt[a*(1 + Sin[e
+ f*x])])
```

fricas [B] time = 0.46, size = 286, normalized size = 2.33

$$\frac{\frac{3\sqrt{2}\left(ac^2-2\,acd+ad^2+\left(ac^2-2\,acd+ad^2\right)\cos\left(fx+e\right)+\left(ac^2-2\,acd+ad^2\right)\sin\left(fx+e\right)\right)\log\left(-\frac{\cos\left(fx+e\right)^2-\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)-\frac{2\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}\left(\cos\left(fx+e\right.\right.}{\sqrt{a}}}{\cos\left(fx+e\right)^2-\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(f\right.}\right.}{\sqrt{a}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(2)*(a*c^2 - 2*a*c*d + a*d^2 + (a*c^2 - 2*a*c*d + a*d^2)*cos(f*x
 + e) + (a*c^2 - 2*a*c*d + a*d^2)*sin(f*x + e))*log(-(cos(f*x + e)^2 - (cos
(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x +
 e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (co
s(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) - 4*(d^2*cos(f*x
+ e)^2 + 6*c*d - 2*d^2 + (6*c*d - d^2)*cos(f*x + e) + (d^2*cos(f*x + e) - 6
*c*d + 2*d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a*f*cos(f*x + e) + a
*f*sin(f*x + e) + a*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(2/sqrt(a*t
an((f*x+exp(1))/2)^2+a)/(a*tan((f*x+exp(1))/2)^2+a)*(tan((f*x+exp(1))/2)*(t
an((f*x+exp(1))/2)*(-1/144*tan((f*x+exp(1))/2)*(24*a*d^2*sign(tan((f*x+exp(
1))/2)+1)-144*a*c*d*sign(tan((f*x+exp(1))/2)+1))-1/144*(-72*a*d^2*sign(tan(
(f*x+exp(1))/2)+1)+144*a*c*d*sign(tan((f*x+exp(1))/2)+1)))-1/144*(72*a*d^2*
sign(tan((f*x+exp(1))/2)+1)-144*a*c*d*sign(tan((f*x+exp(1))/2)+1)))-1/144*(
-24*a*d^2*sign(tan((f*x+exp(1))/2)+1)+144*a*c*d*sign(tan((f*x+exp(1))/2)+1)
))+sqrt(2)*(c^2+d^2-2*c*d)*atan((-sqrt(a)*tan((f*x+exp(1))/2)-sqrt(a)+sqrt(
```

```
a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/sqrt(-a)/sign(tan((f*x+exp(1)
)/2)+1)+(-3*a*c^2*sqrt(2)*atan(sqrt(a)/sqrt(-a))+6*a*c*d*sqrt(2)*atan(sqrt(
a)/sqrt(-a))-3*a*d^2*sqrt(2)*atan(sqrt(a)/sqrt(-a))+6*c*d*sqrt(-a)*sqrt(2)*
sqrt(a)-2*d^2*sqrt(-a)*sqrt(2)*sqrt(a))/3/a/sqrt(-a)*sign(tan((f*x+exp(1))/
2)+1))
```

maple [A] time = 1.25, size = 185, normalized size = 1.50

$$-\frac{\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,\left(3a^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,\sqrt{2}}{2\sqrt{a}}\right)c^{2}-6a^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(f\right.}}{2\sqrt{}}\right.\right.}{3a^{2}\cos\left(fx+e\right)\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/3*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(3*a^(3/2)*2^(1/2)*arctanh(1/
2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^2-6*a^(3/2)*2^(1/2)*arctanh(1/2
*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c*d+3*a^(3/2)*2^(1/2)*arctanh(1/2*
(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d^2-2*(a-a*sin(f*x+e))^(3/2)*d^2+12
*a*c*d*(a-a*sin(f*x+e))^(1/2))/a^2/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{2}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^2/sqrt(a*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c+d\,\sin\left(e+f\,x\right)\right)^{2}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^2/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^2/(a + a*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + d\sin\left(e + fx\right)\right)^2}{\sqrt{a\left(\sin\left(e + fx\right) + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((c + d*sin(e + f*x))**2/sqrt(a*(sin(e + f*x) + 1)), x)
```

3.545 $\int \frac{c+d\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=79

$$-\frac{\sqrt{2}\,(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\,f}-\frac{2d\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -(c-d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1/
2)/f/a^(1/2)-2*d*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2751, 2649, 206}

$$-\frac{\sqrt{2}\,(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\,f}-\frac{2d\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[2]*(c - d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e
 + f*x]])])/(Sqrt[a]*f)) - (2*d*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{c+d\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{2d\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} + (c-d)\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx$$

$$= -\frac{2d\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{(2(c-d))\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f}$$

$$= -\frac{\sqrt{2}\,(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,f} - \frac{2d\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [C] time = 0.21, size = 106, normalized size = 1.34

$$\frac{2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(d\left(\sin\left(\frac{1}{2}(e+fx)\right)-\cos\left(\frac{1}{2}(e+fx)\right)\right)+(1+i)(-1)^{3/4}(c-d)\tanh^{-1}\left(\left(\frac{1}{2}\right.\right.\right.}{f\sqrt{a(\sin(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((1 + I)*(-1)^(3/4)*(c - d)*ArcTan
h[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] + d*(-Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])))/(f*Sqrt[a*(1 + Sin[e + f*x])])
```

fricas [B] time = 0.47, size = 214, normalized size = 2.71

$$-\frac{\frac{\sqrt{2}\left(ac-ad+(ac-ad)\cos\left(fx+e\right)+(ac-ad)\sin\left(fx+e\right)\right)\log\left(-\frac{\cos\left(fx+e\right)^2-\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)+\frac{2\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}\left(\cos\left(fx+e\right)-\sin\left(fx+e\right)+1\right)}{\sqrt{a}}+3\cos\left(fx+e\right)\cdots}{\cos\left(fx+e\right)^2-\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right.}{\sqrt{a}}}{2\left(af\cos\left(fx+e\right)+af\sin\left(fx+e\right)+af\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*(a*c - a*d + (a*c - a*d)*cos(f*x + e) + (a*c - a*d)*sin(f*x +
 e))*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqr
```

```
t(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x
 + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e)
 - 2))/sqrt(a) + 4*(d*cos(f*x + e) - d*sin(f*x + e) + d)*sqrt(a*sin(f*x + e
) + a))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)2/f*(2*sqrt(a*tan((f*x+exp(1))/2)^2+a)*(-1/2*d/sign(ta
n((f*x+exp(1))/2)+1)+1/2*d*tan((f*x+exp(1))/2)/sign(tan((f*x+exp(1))/2)+1))
/(a*tan((f*x+exp(1))/2)^2+a)+sqrt(2)*(c-d)*atan((-sqrt(a)*tan((f*x+exp(1))/
2)-sqrt(a)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/sqrt(-a)/sign
(tan((f*x+exp(1))/2)+1)+(-a*c*sqrt(2)*atan(sqrt(a)/sqrt(-a))+a*d*sqrt(2)*at
an(sqrt(a)/sqrt(-a))+d*sqrt(-a)*sqrt(2)*sqrt(a))/a/sqrt(-a)*sign(tan((f*x+e
xp(1))/2)+1))
```

maple [A] time = 1.00, size = 128, normalized size = 1.62

$$-\frac{\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,\left(\sqrt{a}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,\sqrt{2}}{2\sqrt{a}}\right)c-\sqrt{a}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx\right.}}{2\sqrt{a}}\right.\right.}{a\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(a^(1/2)*2^(1/2)*arctanh(1/2*(a-a
*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c-a^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(
f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d+2*(a-a*sin(f*x+e))^(1/2)*d)/a/cos(f*x+e)/(
a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d\sin\left(fx+e\right)+c}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)/sqrt(a*sin(f*x + e) + a), x)
```

mupad [B] time = 8.96, size = 151, normalized size = 1.91

$$-\frac{c\,\mathrm{F}\left(\frac{\pi}{4}-\frac{e}{2}-\frac{f\,x}{2}\middle|1\right)\,\sqrt{\frac{2\,\left(a+a\,\sin\left(e+f\,x\right)\right)}{a}}}{f\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}-\frac{d\,\left(4\,\mathrm{E}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin\left(e+f\,x\right)}}{2}\right)\middle|1\right)-2\,\mathrm{F}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin\left(e+f\,x\right)}}{2}\right)\middle|1\right)\right)\,\sqrt{c}}{f\,\cos\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] - (c*ellipticF(pi/4 - e/2 - (f*x)/2, 1)*((2*(a + a*sin(e + f*x)))/a)^(1/2))
/(f*(a + a*sin(e + f*x))^(1/2)) - (d*(4*ellipticE(asin((2^(1/2)*(1 - sin(e
+ f*x))^(1/2))/2), 1) - 2*ellipticF(asin((2^(1/2)*(1 - sin(e + f*x))^(1/2))
/2), 1))*(cos(e + f*x)^2)^(1/2)*((a + a*sin(e + f*x))/(2*a))^(1/2))/(f*cos(
e + f*x)*(a + a*sin(e + f*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+d\sin\left(e+fx\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((c + d*sin(e + f*x))/sqrt(a*(sin(e + f*x) + 1)), x)
```

3.546 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=47

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}$$

```
[Out] -arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/f*2^(1/2)/a
^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2649, 206}

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]
)]])/(Sqrt[a]*f))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f}$$
$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,f}$$

Mathematica [C] time = 0.05, size = 73, normalized size = 1.55

$$\frac{(2+2i)(-1)^{3/4}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(\tan\left(\frac{1}{4}(e+fx)\right)-1\right)\right)}{f\sqrt{a(\sin(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] ((2 + 2*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4]
)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(f*Sqrt[a*(1 + Sin[e + f*x])])
```

fricas [A] time = 0.48, size = 167, normalized size = 3.55

$$\left[\frac{\sqrt{2}\log\left(-\frac{\cos(fx+e)^2-(\cos(fx+e)-2)\sin(fx+e)-\frac{2\sqrt{2}\sqrt{a\sin(fx+e)+a}(\cos(fx+e)-\sin(fx+e)+1)}{\sqrt{a}}+3\cos(fx+e)+2}{\cos(fx+e)^2-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)}{2\sqrt{a}\,f}, \frac{\sqrt{2}\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\cdots}}{\cdots}\right)}{f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqr
t(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3
*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(
f*x + e) - 2))/(sqrt(a)*f), sqrt(2)*sqrt(-1/a)*arctan(sqrt(2)*sqrt(a*sin(f*
x + e) + a)*sqrt(-1/a)/cos(f*x + e))/f]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)4*(1/8*sqrt(a)*ln(abs(sin(1/4*(2*f*x+2*exp(1)-pi))+1/s
in(1/4*(2*f*x+2*exp(1)-pi))+2))/sqrt(2)/a/sign(cos(1/2*(f*x+exp(1))-1/4*pi)
)-1/8*sqrt(a)*ln(abs(sin(1/4*(2*f*x+2*exp(1)-pi))+1/sin(1/4*(2*f*x+2*exp(1)
-pi))-2))/sqrt(2)/a/sign(cos(1/2*(f*x+exp(1))-1/4*pi)))/f
```

maple [A] time = 0.52, size = 75, normalized size = 1.60

$$-\frac{\left(1+\sin \left(f x+e\right)\right) \sqrt{-a \left(\sin \left(f x+e\right)-1\right)}\, \sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{-a \left(\sin \left(f x+e\right)-1\right)}\, \sqrt{2}}{2 \sqrt{a}}\right)}{\sqrt{a}\, \cos \left(f x+e\right) \sqrt{a+a \sin \left(f x+e\right)}\, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2)*arctanh(1/2*(-a*(
sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a \sin \left(f x+e\right)+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*sin(f*x + e) + a), x)
```

mupad [B] time = 7.72, size = 49, normalized size = 1.04

$$-\frac{\mathrm{F}\left(\frac{\pi}{4}-\frac{e}{2}-\frac{f\,x}{2}\middle|1\right)\,\sqrt{\frac{2\,\left(a+a\,\sin\left(e+f\,x\right)\right)}{a}}}{f\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] -(ellipticF(pi/4 - e/2 - (f*x)/2, 1)*((2*(a + a*sin(e + f*x)))/a)^(1/2))/(f
*(a + a*sin(e + f*x))^(1/2))
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sin\left(e+fx\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sin(e + f*x) + a), x)
```

3.547 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=123

$$\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f(c-d)\sqrt{c+d}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f(c-d)}$$

```
[Out] -arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1/2)/(c-
d)/f/a^(1/2)+2*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+
e))^(1/2))*d^(1/2)/(c-d)/f/a^(1/2)/(c+d)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2780, 2649, 206, 2773, 208}

$$\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f(c-d)\sqrt{c+d}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f(c-d)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]
)])/(Sqrt[a]*(c - d)*f)) + (2*Sqrt[d]*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x]
)/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[a]*(c - d)*Sqrt[c + d]*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2780

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e +
f*x]], x], x] - Dist[d/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))}\,dx = \frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{c-d} - \frac{d\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{a(c-d)}$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{(c-d)f} + \frac{(2d)\,\text{Subst}\left(\int \frac{1}{ac+ad-dx^2}\right.}{(c-}$$

$$= -\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,(c-d)f} + \frac{2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sin(e+f}}\right.}{\sqrt{a}\,(c-d)\sqrt{c+d}\,f}$$

Mathematica [C] time = 1.80, size = 215, normalized size = 1.75

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sqrt{d}\left(\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)+\sqrt{d}\cos\left(\frac{1}{2}(e+\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])),x]
```

```
[Out] (((2 + 2*I)*(-1)^(3/4)*Sqrt[c + d]*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan
[(e + f*x)/4])] + Sqrt[d]*(Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Co
s[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])] - Log[Sec[(e + f*x)/4]^2*(Sqrt[
c + d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])]))*(Cos[(e +
```

```
f*x)/2] + Sin[(e + f*x)/2]))/((c - d)*Sqrt[c + d]*f*Sqrt[a*(1 + Sin[e + f*x
])])
```

fricas [B] time = 0.58, size = 685, normalized size = 5.57

$$\left[-\frac{\sqrt{\frac{d}{ac+ad}}\,\log\left(\frac{d^2\cos\left(fx+e\right)^3-\left(6\,cd+7\,d^2\right)\cos\left(fx+e\right)^2-c^2-2\,cd-d^2-4\left(\left(cd+d^2\right)\cos\left(fx+e\right)^2-c^2-4\,cd-3\,d^2-\left(c^2+3\,cd+2\,d^2\right)\cos\left(fx+e\right)+\left(c^2+4\,cd+\cdots\right.\right.}{d^2\cos\left(fx+e\right)^3+\left(2\,cd+d^2\right)\cos\left(fx+e\right)^2-c^2-2\,cd-d^2\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*cos(f*
x + e)^2 - c^2 - 2*c*d - d^2 - 4*((c*d + d^2)*cos(f*x + e)^2 - c^2 - 4*c*d
- 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d + 3*d^2 + (c*d
+ d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d/(a*c + a
*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - c^2 - 2*c
*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^
3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x
+ e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*
x + e))) + sqrt(2)*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) +
 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(
a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e)
- cos(f*x + e) - 2))/sqrt(a))/((c - d)*f), 1/2*(2*sqrt(-d/(a*c + a*d))*arct
an(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a
*d))/(d*cos(f*x + e))) - sqrt(2)*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*
sin(f*x + e) + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x +
 e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)
*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a))/((c - d)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes
 constant sign by intervals (correct if the argument is real):Check [abs(co
s((f*t_nostep+exp(1))/2-pi/4))]Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Dis
continuities at zeroes of cos((f*t_nostep+exp(1))/2-pi/4) were not checkedU
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
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```
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/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign b
y intervals (correct if the argument is real):Check [abs(t_nostep+1)]sym2po
ly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argu
ment Value
```

maple [A] time = 1.24, size = 131, normalized size = 1.07

$$-\frac{\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,\left(-2d\,\mathrm{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,d}{\sqrt{a(c+d)d}}\right)a^{\frac{3}{2}}+\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}}{2\sqrt{a}}\right.\right.}{\left(c-d\right)\sqrt{a\left(c+d\right)d}\,a^{\frac{3}{2}}\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] -(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-2*d*arctanh((-a*(sin(f*x+e)-1))
^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)+2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))
^(1/2)*2^(1/2)/a^(1/2))*a*(a*(c+d)*d)^(1/2))/(c-d)/(a*(c+d)*d)^(1/2)/a^(3/2
)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(c+d\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))),x)
[Out] int(1/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\left(c+d\sin\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e)),x)
[Out] Integral(1/(sqrt(a*(sin(e + f*x) + 1))*(c + d*sin(e + f*x))), x)
```

3.548 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=175

$$\frac{d\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f(c-d)^2}+\frac{\sqrt{d}(3c+d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\,\mathrm{co}}{\sqrt{c+d}\sqrt{a\,\mathrm{sin}}}\right.}{\sqrt{a}f(c-d)^2(c+d)^{3/2}}$$

```
[Out] -arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1/2)/(c-
d)^2/f/a^(1/2)+(3*c+d)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*
sin(f*x+e))^(1/2))*d^(1/2)/(c-d)^2/(c+d)^(3/2)/f/a^(1/2)+d*cos(f*x+e)/(c^2-
d^2)/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2779, 2985, 2649, 206, 2773, 208}

$$\frac{d\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f(c-d)^2}+\frac{\sqrt{d}(3c+d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\,\mathrm{co}}{\sqrt{c+d}\sqrt{a\,\mathrm{sin}}}\right.}{\sqrt{a}f(c-d)^2(c+d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]
)])/(Sqrt[a]*(c - d)^2*f)) + (Sqrt[d]*(3*c + d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Co
s[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[a]*(c - d)^2*(c
+ d)^(3/2)*f) + (d*Cos[e + f*x])/((c^2 - d^2)*f*Sqrt[a + a*Sin[e + f*x]]*(c
 + d*Sin[e + f*x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2}\,dx = \frac{d\cos(e+fx)}{(c^2-d^2)\,f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))} + \frac{\int \frac{a(2c+d)-}{\sqrt{a+a\sin(e+f}}}{2a\,(}$$

$$= \frac{d\cos(e+fx)}{(c^2-d^2)\,f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))} + \frac{\int \frac{1}{\sqrt{a+a\sin(e+f}}}{(c-d)^2}$$

$$= \frac{d\cos(e+fx)}{(c^2-d^2)\,f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))} - \frac{2\,\mathrm{Subst}\left(\int \frac{}{2a}\right.}{}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,(c-d)^2 f} + \frac{\sqrt{d}\,(3c+d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos}{\sqrt{c+d}\sqrt{a+a}}\right.}{\sqrt{a}\,(c-d)^2(c+d)^{3/2}f}$$

Mathematica [C] time = 3.56, size = 324, normalized size = 1.85

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\frac{4d(c-d)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(c+d)(c+d\sin(e+fx))} + \frac{\sqrt{d}\,(3c+d)\left(2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\sin\left(\frac{1}{2}(e+fx\right.\right.\right.\right.}{(c+d)}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((8 + 8*I)*(-1)^(3/4)*ArcTanh[(1/2 +
 I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] + (Sqrt[d]*(3*c + d)*(e + f*x - 2
*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*
Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])]))/(c + d)^(3/2) - (Sqrt[d]*(3
*c + d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sq
rt[c + d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])]))/(c + d)
^(3/2) + (4*(c - d)*d*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d)*(c +
d*Sin[e + f*x]))))/(4*(c - d)^2*f*Sqrt[a*(1 + Sin[e + f*x])])
```

fricas [B] time = 0.84, size = 1494, normalized size = 8.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] [-1/4*((3*a*c^2 + 4*a*c*d + a*d^2 - (3*a*c*d + a*d^2)*cos(f*x + e)^2 + (3*a
*c^2 + a*c*d)*cos(f*x + e) + (3*a*c^2 + 4*a*c*d + a*d^2 + (3*a*c*d + a*d^2)
*cos(f*x + e))*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 -
(6*c*d + 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 4*((c*d + d^2)*cos(f*x
 + e)^2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 +
 4*c*d + 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e
) + a)*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(
f*x + e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x +
e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2
- (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2
 - 2*c*d - d^2)*sin(f*x + e))) + 2*sqrt(2)*(a*c^2 + 2*a*c*d + a*d^2 - (a*c*
d + a*d^2)*cos(f*x + e)^2 + (a*c^2 + a*c*d)*cos(f*x + e) + (a*c^2 + 2*a*c*d
 + a*d^2 + (a*c*d + a*d^2)*cos(f*x + e))*sin(f*x + e))*log(-(cos(f*x + e)^2
 - (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(co
s(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^
2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) + 4*(c*d -
 d^2 + (c*d - d^2)*cos(f*x + e) - (c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x
+ e) + a))/((a*c^3*d - a*c^2*d^2 - a*c*d^3 + a*d^4)*f*cos(f*x + e)^2 - (a*c
^4 - a*c^3*d - a*c^2*d^2 + a*c*d^3)*f*cos(f*x + e) - (a*c^4 - 2*a*c^2*d^2 +
 a*d^4)*f - ((a*c^3*d - a*c^2*d^2 - a*c*d^3 + a*d^4)*f*cos(f*x + e) + (a*c^
4 - 2*a*c^2*d^2 + a*d^4)*f)*sin(f*x + e)), -1/2*((3*a*c^2 + 4*a*c*d + a*d^2
 - (3*a*c*d + a*d^2)*cos(f*x + e)^2 + (3*a*c^2 + a*c*d)*cos(f*x + e) + (3*a
*c^2 + 4*a*c*d + a*d^2 + (3*a*c*d + a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt
(-d/(a*c + a*d))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c -
2*d)*sqrt(-d/(a*c + a*d))/(d*cos(f*x + e))) + sqrt(2)*(a*c^2 + 2*a*c*d + a*
d^2 - (a*c*d + a*d^2)*cos(f*x + e)^2 + (a*c^2 + a*c*d)*cos(f*x + e) + (a*c^
2 + 2*a*c*d + a*d^2 + (a*c*d + a*d^2)*cos(f*x + e))*sin(f*x + e))*log(-(cos
(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x +
e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(co
s(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a)
 + 2*(c*d - d^2 + (c*d - d^2)*cos(f*x + e) - (c*d - d^2)*sin(f*x + e))*sqrt
(a*sin(f*x + e) + a))/((a*c^3*d - a*c^2*d^2 - a*c*d^3 + a*d^4)*f*cos(f*x +
e)^2 - (a*c^4 - a*c^3*d - a*c^2*d^2 + a*c*d^3)*f*cos(f*x + e) - (a*c^4 - 2*
a*c^2*d^2 + a*d^4)*f - ((a*c^3*d - a*c^2*d^2 - a*c*d^3 + a*d^4)*f*cos(f*x +
 e) + (a*c^4 - 2*a*c^2*d^2 + a*d^4)*f)*sin(f*x + e))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.86, size = 449, normalized size = 2.57

$$\frac{\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,\left(\sin\left(fx+e\right)d\left(3\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,d}{\sqrt{acd+a\,d^{2}}}\right)a^{\frac{7}{2}}cd+\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin\left(f\right.}}{\sqrt{acd+a}}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x)
```

```
[Out] (1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)/a^(7/2)*(sin(f*x+e)*d*(3*arctanh((
a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(7/2)*c*d+arctanh((a-a*sin(f
*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(7/2)*d^2-arctanh(1/2*(a-a*sin(f*x+e)
)^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*2^(1/2)*a^3*c-arctanh(1/2*(a-a*s
in(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*2^(1/2)*a^3*d)+3*arctan
h((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(7/2)*c^2*d+arctanh((a-a*
sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(7/2)*c*d^2+(a-a*sin(f*x+e))^(1/
2)*(a*(c+d)*d)^(1/2)*a^(5/2)*c*d-(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a
^(5/2)*d^2-(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(
1/2)/a^(1/2))*a^3*c^2-(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e)
)^(1/2)*2^(1/2)/a^(1/2))*a^3*c*d)/(c-d)^2/(c+d)/(c+d*sin(f*x+e))/(a*(c+d)*d
)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^2),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.549 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=247

$$\frac{d(7c+d)\cos(e+fx)}{4f\left(c^2-d^2\right)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}+\frac{d\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^2}+\frac{\sqrt{d}\,(1\ldots}{\ldots}$$

```
[Out] -arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1/2)/(c-
d)^3/f/a^(1/2)+1/4*(15*c^2+10*c*d+7*d^2)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)
/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))*d^(1/2)/(c-d)^3/(c+d)^(5/2)/f/a^(1/2)+
1/2*d*cos(f*x+e)/(c^2-d^2)/f/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2)+1/4*
d*(7*c+d)*cos(f*x+e)/(c^2-d^2)^2/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.73, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2779, 2984, 2985, 2649, 206, 2773, 208}

$$\frac{d(7c+d)\cos(e+fx)}{4f\left(c^2-d^2\right)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}+\frac{d\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^2}+\frac{\sqrt{d}\,(1\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]
)])/(Sqrt[a]*(c - d)^3*f)) + (Sqrt[d]*(15*c^2 + 10*c*d + 7*d^2)*ArcTanh[(Sq
rt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(4*Sqr
t[a]*(c - d)^3*(c + d)^(5/2)*f) + (d*Cos[e + f*x])/(2*(c^2 - d^2)*f*Sqrt[a
+ a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2) + (d*(7*c + d)*Cos[e + f*x])/(4*(
c^2 - d^2)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^3}\,dx = \frac{d\cos(e+fx)}{2\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} + \frac{\int \frac{a(4c+}{\sqrt{a+a\sin(}}}{4}$$

$$= \frac{d\cos(e+fx)}{2\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} + \frac{}{4\left(c^2-d^2\right)}$$

$$= \frac{d\cos(e+fx)}{2\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} + \frac{}{4\left(c^2-d^2\right)}$$

$$= \frac{d\cos(e+fx)}{2\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} + \frac{}{4\left(c^2-d^2\right)}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,(c-d)^3 f} + \frac{\sqrt{d}\left(15c^2+10cd+7d^2\right)\tanh^{-}}{4\sqrt{a}\,(c-d)^3(c+}$$

Mathematica [C] time = 4.98, size = 414, normalized size = 1.68

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\frac{\sqrt{d}\left(15c^2+10cd+7d^2\right)\left(2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)+\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)\right)\right)-2\log\left(s\right.\right.}{(c-d)^3(c+d)^{5/2}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(((32 + 32*I)*(-1)^(3/4)*ArcTanh[(1/
2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])])/(c - d)^3 + (Sqrt[d]*(15*c^2
+ 10*c*d + 7*d^2)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x
)/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])]
))/((c - d)^3*(c + d)^(5/2)) + (Sqrt[d]*(15*c^2 + 10*c*d + 7*d^2)*(e + f*x
- 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[
d]*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])]))/((-c + d)^3*(c + d)^(5/2
)) + (8*d*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c - d)*(c + d)*(c + d*Si
n[e + f*x])^2) + (4*d*(7*c + d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c
- d)^2*(c + d)^2*(c + d*Sin[e + f*x]))))/(16*f*Sqrt[a*(1 + Sin[e + f*x])])
```

fricas [B] time = 1.20, size = 2903, normalized size = 11.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, algorithm="fricas"
)

[Out] [1/16*((15*a*c^4 + 40*a*c^3*d + 42*a*c^2*d^2 + 24*a*c*d^3 + 7*a*d^4 - (15*a
*c^2*d^2 + 10*a*c*d^3 + 7*a*d^4)*cos(f*x + e)^3 - (30*a*c^3*d + 35*a*c^2*d^
2 + 24*a*c*d^3 + 7*a*d^4)*cos(f*x + e)^2 + (15*a*c^4 + 10*a*c^3*d + 22*a*c^
2*d^2 + 10*a*c*d^3 + 7*a*d^4)*cos(f*x + e) + (15*a*c^4 + 40*a*c^3*d + 42*a*
c^2*d^2 + 24*a*c*d^3 + 7*a*d^4 - (15*a*c^2*d^2 + 10*a*c*d^3 + 7*a*d^4)*cos(
f*x + e)^2 + 2*(15*a*c^3*d + 10*a*c^2*d^2 + 7*a*c*d^3)*cos(f*x + e))*sin(f*
x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*cos(f
*x + e)^2 - c^2 - 2*c*d - d^2 - 4*((c*d + d^2)*cos(f*x + e)^2 - c^2 - 4*c*d
 - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d + 3*d^2 + (c*d
 + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d/(a*c +
a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - c^2 - 2*
c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)
^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x
 + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f
*x + e))) + 8*sqrt(2)*(a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4
- (a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e)^3 - (2*a*c^3*d + 5*a*c^2*d^2
 + 4*a*c*d^3 + a*d^4)*cos(f*x + e)^2 + (a*c^4 + 2*a*c^3*d + 2*a*c^2*d^2 + 2
*a*c*d^3 + a*d^4)*cos(f*x + e) + (a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d
^3 + a*d^4 - (a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e)^2 + 2*(a*c^3*d +
2*a*c^2*d^2 + a*c*d^3)*cos(f*x + e))*sin(f*x + e))*log(-(cos(f*x + e)^2 - (
cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*
x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 -
(cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) - 4*(9*c^3*d -
 15*c^2*d^2 + 3*c*d^3 + 3*d^4 + (7*c^2*d^2 - 6*c*d^3 - d^4)*cos(f*x + e)^2
+ (9*c^3*d - 8*c^2*d^2 - 3*c*d^3 + 2*d^4)*cos(f*x + e) - (9*c^3*d - 15*c^2*
d^2 + 3*c*d^3 + 3*d^4 - (7*c^2*d^2 - 6*c*d^3 - d^4)*cos(f*x + e))*sin(f*x +
 e))*sqrt(a*sin(f*x + e) + a))/((a*c^5*d^2 - a*c^4*d^3 - 2*a*c^3*d^4 + 2*a*
c^2*d^5 + a*c*d^6 - a*d^7)*f*cos(f*x + e)^3 + (2*a*c^6*d - a*c^5*d^2 - 5*a*
c^4*d^3 + 2*a*c^3*d^4 + 4*a*c^2*d^5 - a*c*d^6 - a*d^7)*f*cos(f*x + e)^2 - (
a*c^7 - a*c^6*d - a*c^5*d^2 + a*c^4*d^3 - a*c^3*d^4 + a*c^2*d^5 + a*c*d^6 -
 a*d^7)*f*cos(f*x + e) - (a*c^7 + a*c^6*d - 3*a*c^5*d^2 - 3*a*c^4*d^3 + 3*a
*c^3*d^4 + 3*a*c^2*d^5 - a*c*d^6 - a*d^7)*f + ((a*c^5*d^2 - a*c^4*d^3 - 2*a
*c^3*d^4 + 2*a*c^2*d^5 + a*c*d^6 - a*d^7)*f*cos(f*x + e)^2 - 2*(a*c^6*d - a
*c^5*d^2 - 2*a*c^4*d^3 + 2*a*c^3*d^4 + a*c^2*d^5 - a*c*d^6)*f*cos(f*x + e)
- (a*c^7 + a*c^6*d - 3*a*c^5*d^2 - 3*a*c^4*d^3 + 3*a*c^3*d^4 + 3*a*c^2*d^5
- a*c*d^6 - a*d^7)*f)*sin(f*x + e)), -1/8*((15*a*c^4 + 40*a*c^3*d + 42*a*c^
```

```
2*d^2 + 24*a*c*d^3 + 7*a*d^4 - (15*a*c^2*d^2 + 10*a*c*d^3 + 7*a*d^4)*cos(f*
x + e)^3 - (30*a*c^3*d + 35*a*c^2*d^2 + 24*a*c*d^3 + 7*a*d^4)*cos(f*x + e)^
2 + (15*a*c^4 + 10*a*c^3*d + 22*a*c^2*d^2 + 10*a*c*d^3 + 7*a*d^4)*cos(f*x +
 e) + (15*a*c^4 + 40*a*c^3*d + 42*a*c^2*d^2 + 24*a*c*d^3 + 7*a*d^4 - (15*a*
c^2*d^2 + 10*a*c*d^3 + 7*a*d^4)*cos(f*x + e)^2 + 2*(15*a*c^3*d + 10*a*c^2*d
^2 + 7*a*c*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-d/(a*c + a*d))*arctan(1/2
*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a*d))/(
d*cos(f*x + e))) - 4*sqrt(2)*(a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 +
 a*d^4 - (a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e)^3 - (2*a*c^3*d + 5*a*
c^2*d^2 + 4*a*c*d^3 + a*d^4)*cos(f*x + e)^2 + (a*c^4 + 2*a*c^3*d + 2*a*c^2*
d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e) + (a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 +
4*a*c*d^3 + a*d^4 - (a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e)^2 + 2*(a*c
^3*d + 2*a*c^2*d^2 + a*c*d^3)*cos(f*x + e))*sin(f*x + e))*log(-(cos(f*x + e
)^2 - (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*
(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x +
e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) + 2*(9*
c^3*d - 15*c^2*d^2 + 3*c*d^3 + 3*d^4 + (7*c^2*d^2 - 6*c*d^3 - d^4)*cos(f*x
+ e)^2 + (9*c^3*d - 8*c^2*d^2 - 3*c*d^3 + 2*d^4)*cos(f*x + e) - (9*c^3*d -
15*c^2*d^2 + 3*c*d^3 + 3*d^4 - (7*c^2*d^2 - 6*c*d^3 - d^4)*cos(f*x + e))*si
n(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a*c^5*d^2 - a*c^4*d^3 - 2*a*c^3*d^4
 + 2*a*c^2*d^5 + a*c*d^6 - a*d^7)*f*cos(f*x + e)^3 + (2*a*c^6*d - a*c^5*d^2
 - 5*a*c^4*d^3 + 2*a*c^3*d^4 + 4*a*c^2*d^5 - a*c*d^6 - a*d^7)*f*cos(f*x + e
)^2 - (a*c^7 - a*c^6*d - a*c^5*d^2 + a*c^4*d^3 - a*c^3*d^4 + a*c^2*d^5 + a*
c*d^6 - a*d^7)*f*cos(f*x + e) - (a*c^7 + a*c^6*d - 3*a*c^5*d^2 - 3*a*c^4*d^
3 + 3*a*c^3*d^4 + 3*a*c^2*d^5 - a*c*d^6 - a*d^7)*f + ((a*c^5*d^2 - a*c^4*d^
3 - 2*a*c^3*d^4 + 2*a*c^2*d^5 + a*c*d^6 - a*d^7)*f*cos(f*x + e)^2 - 2*(a*c^
6*d - a*c^5*d^2 - 2*a*c^4*d^3 + 2*a*c^3*d^4 + a*c^2*d^5 - a*c*d^6)*f*cos(f*
x + e) - (a*c^7 + a*c^6*d - 3*a*c^5*d^2 - 3*a*c^4*d^3 + 3*a*c^3*d^4 + 3*a*c
^2*d^5 - a*c*d^6 - a*d^7)*f)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
```

```
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integrat
ion of abs or sign assumes constant sign by intervals (correct if the argum
ent is real):Check [abs(cos((f*t_nostep+exp(1))/2-pi/4))]Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Discontinuities at zeroes of cos((f*t_nostep+exp(1))
/2-pi/4) were not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
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le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
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x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
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pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
```

ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or si
gn assumes constant sign by intervals (correct if the argument is real):Che
ck [abs(t_nostep+1)]Warning, need to choose a branch for the root of a poly
nomial with parameters. This might be wrong.Non regular value [0] was disca
rded and replaced randomly by 0=[-49]Warning, need to choose a branch for t
he root of a polynomial with parameters. This might be wrong.Non regular va
lue [0] was discarded and replaced randomly by 0=[8]Warning, need to choose
 a branch for the root of a polynomial with parameters. This might be wrong
.Non regular value [0] was discarded and replaced randomly by 0=[-6]Warning
, need to choose a branch for the root of a polynomial with parameters. Thi
s might be wrong.Non regular value [0] was discarded and replaced randomly
by 0=[-13]Warning, need to choose a branch for the root of a polynomial wit
h parameters. This might be wrong.Non regular value [0] was discarded and r
eplaced randomly by 0=[-14]Warning, need to choose a branch for the root of
 a polynomial with parameters. This might be wrong.Non regular value [0] wa
s discarded and replaced randomly by 0=[45]Warning, need to choose a branch
 for the root of a polynomial with parameters. This might be wrong.Non regu
lar value [0] was discarded and replaced randomly by 0=[-35]Warning, need t
o choose a branch for the root of a polynomial with parameters. This might
be wrong.Non regular value [0] was discarded and replaced randomly by 0=[-2
9]Warning, need to choose a branch for the root of a polynomial with parame
ters. This might be wrong.Non regular value [0] was discarded and replaced
randomly by 0=[-4]Precision problem choosing root in common_EXT, current pr
ecision 14Warning, need to choose a branch for the root of a polynomial wit
h parameters. This might be wrong.Non regular value [0] was discarded and r
eplaced randomly by 0=[-62]Warning, need to choose a branch for the root of
 a polynomial with parameters. This might be wrong.Non regular value [0] wa
s discarded and replaced randomly by 0=[-43]Warning, need to choose a branc
h for the root of a polynomial with parameters. This might be wrong.Non reg
ular value [0] was discarded and replaced randomly by 0=[96]Warning, need t
o choose a branch for the root of a polynomial with parameters. This might
be wrong.Non regular value [0] was discarded and replaced randomly by 0=[98
]Warning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.Non regular value [0] was discarded and replaced r

```
andomly by 0=[72]Warning, need to choose a branch for the root of a polynom
ial with parameters. This might be wrong.Non regular value [0] was discarde
d and replaced randomly by 0=[-41]Warning, need to choose a branch for the
root of a polynomial with parameters. This might be wrong.Non regular value
 [0] was discarded and replaced randomly by 0=[72]Warning, need to choose a
 branch for the root of a polynomial with parameters. This might be wrong.N
on regular value [0] was discarded and replaced randomly by 0=[-52]Warning,
 need to choose a branch for the root of a polynomial with parameters. This
 might be wrong.Non regular value [0] was discarded and replaced randomly b
y 0=[-42]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.Non regular value [0] was discarded and re
placed randomly by 0=[1]Warning, need to choose a branch for the root of a
polynomial with parameters. This might be wrong.Non regular value [0] was d
iscarded and replaced randomly by 0=[-38]Warning, need to choose a branch f
or the root of a polynomial with parameters. This might be wrong.Non regula
r value [0] was discarded and replaced randomly by 0=[-96]Warning, need to
choose a branch for the root of a polynomial with parameters. This might be
 wrong.Non regular value [0] was discarded and replaced randomly by 0=[-63]
Warning, need to choose a branch for the root of a polynomial with paramete
rs. This might be wrong.Non regular value [0] was discarded and replaced ra
ndomly by 0=[-85]Warning, need to choose a branch for the root of a polynom
ial with parameters. This might be wrong.Non regular value [0] was discarde
d and replaced randomly by 0=[-32]Warning, need to choose a branch for the
root of a polynomial with parameters. This might be wrong.Non regular value
 [0] was discarded and replaced randomly by 0=[64]Warning, need to choose a
 branch for the root of a polynomial with parameters. This might be wrong.N
on regular value [0] was discarded and replaced randomly by 0=[24]Evaluatio
n time: 0.76sym2poly/r2sym(const gen & e,const index_m & i,const vecteur &
l) Error: Bad Argument Value
```

maple [B] time = 2.20, size = 1065, normalized size = 4.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/4*(-4*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(
1/2)/a^(1/2))*sin(f*x+e)^2*a^5*c^2*d^2-4*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(
1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^5*d^4-4*(a*(c
+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))
*a^5*c^2*d^2-8*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1
/2)*2^(1/2)/a^(1/2))*a^5*c^3*d+(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*
a^(7/2)*d^4+7*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(11/
2)*sin(f*x+e)^2*d^5+15*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2
```

```
))*a^(11/2)*c^4*d+10*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))
*a^(11/2)*c^3*d^2+7*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*
a^(11/2)*c^2*d^3+(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(9/2)*d^4-8*
(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(
1/2))*sin(f*x+e)^2*a^5*c*d^3-8*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(s
in(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^5*c^3*d-16*(a*(c+d)*d)^(1
/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+
e)*a^5*c^2*d^2-8*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^
(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^5*c*d^3+30*arctanh((-a*(sin(f*x+e)-1))^
(1/2)*d/(a*(c+d)*d)^(1/2))*a^(11/2)*sin(f*x+e)*c^3*d^2+20*arctanh((-a*(sin(
f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(11/2)*sin(f*x+e)*c^2*d^3+14*arctan
h((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(11/2)*sin(f*x+e)*c*d^4+
9*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(9/2)*c^3*d-(-a*(sin(f*x+e)
-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(9/2)*c^2*d^2-7*(-a*(sin(f*x+e)-1))^(3/2)*(a
*(c+d)*d)^(1/2)*a^(7/2)*c^2*d^2+6*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/
2)*a^(7/2)*c*d^3-4*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1)
)^(1/2)*2^(1/2)/a^(1/2))*a^5*c^4-9*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1
/2)*a^(9/2)*c*d^3+15*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))
*a^(11/2)*sin(f*x+e)^2*c^2*d^3+10*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c
+d)*d)^(1/2))*a^(11/2)*sin(f*x+e)^2*c*d^4)*(-a*(sin(f*x+e)-1))^(1/2)*(1+sin
(f*x+e))/a^(11/2)/(a*(c+d)*d)^(1/2)/(c+d*sin(f*x+e))^2/(c+d)^2/(c-d)^3/cos(
f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^3),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.550 $\int \frac{(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=192

$$-\frac{(c+11d)(c-d)^2\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}\,a^{3/2}f}+\frac{d^2(3c-7d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{6a^2f}+\frac{d\left(3c^2-24cd+13d^2\right)}{3af\sqrt{a\sin(e+f}}$$

```
[Out] -1/2*(c-d)*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(f*x+e))^(3/2)-1/4*(c-d)
^2*(c+11*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/
a^(3/2)/f*2^(1/2)+1/3*d*(3*c^2-24*c*d+13*d^2)*cos(f*x+e)/a/f/(a+a*sin(f*x+e
))^(1/2)+1/6*(3*c-7*d)*d^2*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/a^2/f
```

Rubi [A] time = 0.46, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2765, 2968, 3023, 2751, 2649, 206}

$$\frac{d^2(3c-7d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{6a^2f}-\frac{(c+11d)(c-d)^2\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}\,a^{3/2}f}+\frac{d\left(3c^2-24cd+13d^2\right)c}{3af\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((c - d)^2*(c + 11*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*S
in[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*f) + (d*(3*c^2 - 24*c*d + 13*d^2)*Cos[e
+ f*x])/(3*a*f*Sqrt[a + a*Sin[e + f*x]]) + ((3*c - 7*d)*d^2*Cos[e + f*x]*Sq
rt[a + a*Sin[e + f*x]])/(6*a^2*f) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*
x])^2)/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{(c+d\sin(e+fx))\left(-\frac{1}{2}a\left(c^2+7cd-4d^2\right)+\frac{1}{2}a(3c-7d)d\,\mathrm{s}}{\sqrt{a+a\sin(e+fx)}}}{2a^2}$$

$$= -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}ac\left(c^2+7cd-4d^2\right)+\left(\frac{1}{2}ac(3c-7d)d-\frac{1}{2}ad\left(c^2+7cd-\right.\right.}{\sqrt{a+a\sin(e+}}}{2a^2}$$

$$= \frac{(3c-7d)d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{6a^2f} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{2f(a+a\sin(e+fx))^{3/2}}$$

$$= \frac{d\left(3c^2-24cd+13d^2\right)\cos(e+fx)}{3af\sqrt{a+a\sin(e+fx)}} + \frac{(3c-7d)d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{6a^2f} -$$

$$= \frac{d\left(3c^2-24cd+13d^2\right)\cos(e+fx)}{3af\sqrt{a+a\sin(e+fx)}} + \frac{(3c-7d)d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{6a^2f} -$$

$$= -\frac{(c-d)^2(c+11d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f} + \frac{d\left(3c^2-24cd+13d^2\right)\cos(e+fx)}{3af\sqrt{a+a\sin(e+fx)}}$$

Mathematica [C] time = 0.55, size = 328, normalized size = 1.71

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-18d^2(2c-d)\cos\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2+18d^2(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(6*(c - d)^3*Sin[(e + f*x)/2] - 3*(c
 - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (3 + 3*I)*(-1)^(3/4)*(c - d
)^2*(c + 11*d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 18*(2*c - d)*d^2*Cos[(e + f*x)/2]*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 2*d^3*Cos[(3*(e + f*x))/2]*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^2 + 18*(2*c - d)*d^2*Sin[(e + f*x)/2]*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^2 - 2*d^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^2*Sin[(3*(e + f*x))/2]))/(6*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

fricas [B] time = 0.46, size = 494, normalized size = 2.57

$$-\frac{3\sqrt{2}\left(2c^3+18c^2d-42cd^2+22d^3-\left(c^3+9c^2d-21cd^2+11d^3\right)\cos\left(fx+e\right)^2+\left(c^3+9c^2d-21cd^2+11d^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/24*(3*sqrt(2)*(2*c^3 + 18*c^2*d - 42*c*d^2 + 22*d^3 - (c^3 + 9*c^2*d - 2
1*c*d^2 + 11*d^3)*cos(f*x + e)^2 + (c^3 + 9*c^2*d - 21*c*d^2 + 11*d^3)*cos(
f*x + e) + (2*c^3 + 18*c^2*d - 42*c*d^2 + 22*d^3 + (c^3 + 9*c^2*d - 21*c*d^
2 + 11*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*
sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1)
+ 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x +
e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(4*d^3*cos(
f*x + e)^3 - 3*c^3 + 9*c^2*d - 9*c*d^2 + 3*d^3 - 4*(9*c*d^2 - 4*d^3)*cos(f*
x + e)^2 - 3*(c^3 - 3*c^2*d + 15*c*d^2 - 5*d^3)*cos(f*x + e) - (4*d^3*cos(f
*x + e)^2 - 3*c^3 + 9*c^2*d - 9*c*d^2 + 3*d^3 + 12*(3*c*d^2 - d^3)*cos(f*x
+ e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a^2*f*cos(f*x + e)^2 - a^2*f
*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(2/sqrt(a*t
an((f*x+exp(1))/2)^2+a)/(a*tan((f*x+exp(1))/2)^2+a)*(tan((f*x+exp(1))/2)*(t
an((f*x+exp(1))/2)*(-1/144*tan((f*x+exp(1))/2)*(96*d^3*sign(tan((f*x+exp(1)
)/2)+1)-216*c*d^2*sign(tan((f*x+exp(1))/2)+1))-1/144*(-144*d^3*sign(tan((f*
x+exp(1))/2)+1)+216*c*d^2*sign(tan((f*x+exp(1))/2)+1)))-1/144*(144*d^3*sign
(tan((f*x+exp(1))/2)+1)-216*c*d^2*sign(tan((f*x+exp(1))/2)+1)))-1/144*(-96*
d^3*sign(tan((f*x+exp(1))/2)+1)+216*c*d^2*sign(tan((f*x+exp(1))/2)+1)))+2*(
1/4*(-3*c^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^
```

```
3+3*d^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3-9*
c*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+9*c^
2*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+a*c^3*
(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))-a*d^3*(-sqrt
(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+sqrt(a)*c^3*(-sqrt
(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-sqrt(a)*d^3*(-sq
rt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+3*a*c*d^2*(-sq
rt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))-3*a*c^2*d*(-sqrt
(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+3*sqrt(a)*c*d^2*(-
sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-3*sqrt(a)*c^
2*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+sqrt(a
)*a*c^3-sqrt(a)*a*d^3+3*sqrt(a)*a*c*d^2-3*sqrt(a)*a*c^2*d)/(-(-sqrt(a)*tan(
(f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+2*sqrt(a)*(-sqrt(a)*tan(
(f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+a)^2/a/sign(tan((f*x+exp(1
))/2)+1)+1/4*(c^3+11*d^3-21*c*d^2+9*c^2*d)*atan((-sqrt(a)*tan((f*x+exp(1))/
2)-sqrt(a)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/sqrt(2)/sqrt(
-a)/a/sign(tan((f*x+exp(1))/2)+1)))
```

maple [B] time = 1.09, size = 490, normalized size = 2.55

$$-\frac{\left(\sin\left(fx+e\right)\left(3\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{a}}\right)a^{2}c^{3}+27\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{a}}\right)a^{2}c^{2}d-63\sqrt{2}\ \operatorname{arctan}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/12*(sin(f*x+e)*(3*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*a^2*c^3+27*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2)
)*a^2*c^2*d-63*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*
a^2*c*d^2+33*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^
2*d^3-8*(a-a*sin(f*x+e))^(3/2)*d^3*a^(1/2)+72*c*d^2*a^(3/2)*(a-a*sin(f*x+e)
)^(1/2)-24*d^3*a^(3/2)*(a-a*sin(f*x+e))^(1/2))+3*2^(1/2)*arctanh(1/2*(a-a*s
in(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^3+27*2^(1/2)*arctanh(1/2*(a-a*sin(f
*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d-63*2^(1/2)*arctanh(1/2*(a-a*sin(f*x
+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d^2+33*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e
))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^3-8*(a-a*sin(f*x+e))^(3/2)*d^3*a^(1/2)+6*(a
-a*sin(f*x+e))^(1/2)*a^(3/2)*c^3-18*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^2*d+90
*c*d^2*a^(3/2)*(a-a*sin(f*x+e))^(1/2)-30*d^3*a^(3/2)*(a-a*sin(f*x+e))^(1/2)
)*(-a*(sin(f*x+e)-1))^(1/2)/a^(7/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{3}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^3/(a*sin(f*x + e) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c+d\,\sin\left(e+f\,x\right)\right)^{3}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^3/(a + a*sin(e + f*x))^(3/2),x)
[Out] int((c + d*sin(e + f*x))^3/(a + a*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+d\sin\left(e+fx\right)\right)^{3}}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(3/2),x)
[Out] Integral((c + d*sin(e + f*x))**3/(a*(sin(e + f*x) + 1))**(3/2), x)
```

3.551 $\int \frac{(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=138

$$-\frac{(c-d)(c+7d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}\,a^{3/2}f}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{2f(a\sin(e+fx)+a)^{3/2}}+\frac{d(c-5d)\cos(e+fx)}{2af\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -1/2*(c-d)*cos(f*x+e)*(c+d*sin(f*x+e))/f/(a+a*sin(f*x+e))^(3/2)-1/4*(c-d)*(
c+7*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/
2)/f*2^(1/2)+1/2*(c-5*d)*d*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2760, 2751, 2649, 206}

$$-\frac{(c-d)(c+7d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}\,a^{3/2}f}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{2f(a\sin(e+fx)+a)^{3/2}}+\frac{d(c-5d)\cos(e+fx)}{2af\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((c - d)*(c + 7*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[
e + f*x]])])/(2*Sqrt[2]*a^(3/2)*f) + ((c - 5*d)*d*Cos[e + f*x])/(2*a*f*Sqrt
[a + a*Sin[e + f*x]]) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x]))/(2*f*(a
 + a*Sin[e + f*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2760

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^m*(c + d*Sin[e + f*x]))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*c*d*(m - 1) + b*(d^2 + c^2*(m + 1))
 + d*(a*d*(m - 1) + b*c*(m + 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{2f(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}a\left(c^2+5cd-2d^2\right)+\frac{1}{2}a(c-5d)d\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx}{2a^2}$$

$$= \frac{(c-5d)d\cos(e+fx)}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{2f(a+a\sin(e+fx))^{3/2}} + \frac{((c-d)(c+7d}{}$$

$$= \frac{(c-5d)d\cos(e+fx)}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{2f(a+a\sin(e+fx))^{3/2}} - \frac{((c-d)(c+7d}{}$$

$$= -\frac{(c-d)(c+7d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} + \frac{(c-5d)d\cos(e+fx)}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(c-d)\,\mathrm{co}}{2f}$$

Mathematica [C] time = 0.34, size = 239, normalized size = 1.73

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left((1+i)(-1)^{3/4}\left(c^2+6cd-7d^2\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\tanh^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(c - d)^2*Sin[(e + f*x)/2] - (c -
 d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (1 + I)*(-1)^(3/4)*(c^2 + 6*c
```

```
*d - 7*d^2)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^2 - 4*d^2*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^2 + 4*d^2*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^2))/(2*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

fricas [B] time = 0.45, size = 380, normalized size = 2.75

$$-\frac{\sqrt{2}\left(\left(c^2+6\,cd-7\,d^2\right)\cos\left(fx+e\right)^2-2\,c^2-12\,cd+14\,d^2-\left(c^2+6\,cd-7\,d^2\right)\cos\left(fx+e\right)-\left(2\,c^2+12\,cd-14\,d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(2)*((c^2 + 6*c*d - 7*d^2)*cos(f*x + e)^2 - 2*c^2 - 12*c*d + 14*d
^2 - (c^2 + 6*c*d - 7*d^2)*cos(f*x + e) - (2*c^2 + 12*c*d - 14*d^2 + (c^2 +
 6*c*d - 7*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2
+ 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) +
 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*
x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(4*d^2*
cos(f*x + e)^2 + c^2 - 2*c*d + d^2 + (c^2 - 2*c*d + 5*d^2)*cos(f*x + e) + (
4*d^2*cos(f*x + e) - c^2 + 2*c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) +
 a))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x
+ e) + 2*a^2*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)2/f*(2*sqrt(a*tan((f*x+exp(1))/2)^2+a)*(-1/2*d^2/a/sig
n(tan((f*x+exp(1))/2)+1)+1/2*d^2*tan((f*x+exp(1))/2)/a/sign(tan((f*x+exp(1)
)/2)+1))/(a*tan((f*x+exp(1))/2)^2+a)+2*(1/4*(-3*c^2*(-sqrt(a)*tan((f*x+exp(
1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3-3*d^2*(-sqrt(a)*tan((f*x+exp(1))/
2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+6*c*d*(-sqrt(a)*tan((f*x+exp(1))/2)+s
qrt(a*tan((f*x+exp(1))/2)^2+a))^3+a*c^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(
a*tan((f*x+exp(1))/2)^2+a))+a*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan(
(f*x+exp(1))/2)^2+a))+sqrt(a)*c^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan(
```

```
(f*x+exp(1))/2)^2+a))^2+sqrt(a)*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*ta
n((f*x+exp(1))/2)^2+a))^2-2*a*c*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan(
(f*x+exp(1))/2)^2+a))-2*sqrt(a)*c*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*ta
n((f*x+exp(1))/2)^2+a))^2+sqrt(a)*a*c^2+sqrt(a)*a*d^2-2*sqrt(a)*a*c*d)/(-(-
sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+2*sqrt(a)*(-
sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+a)^2/a/sign(ta
n((f*x+exp(1))/2)+1)+1/4*(c^2-7*d^2+6*c*d)*atan((-sqrt(a)*tan((f*x+exp(1))/
2)-sqrt(a)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/sqrt(2)/sqrt(
-a)/a/sign(tan((f*x+exp(1))/2)+1)))
```

maple [B] time = 1.01, size = 316, normalized size = 2.29

$$-\frac{\left(\sin\left(fx+e\right)\left(\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{a}}\right)a\,c^{2}+6\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{a}}\right)acd-7\sqrt{2}\ \operatorname{arctanh}\left(\sqrt{\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/4/a^(5/2)*(sin(f*x+e)*(2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2
)/a^(1/2))*a*c^2+6*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/
2))*a*c*d-7*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*d
^2+8*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*d^2)+2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e
))^(1/2)*2^(1/2)/a^(1/2))*a*c^2+6*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2
)*2^(1/2)/a^(1/2))*a*c*d-7*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/
2)/a^(1/2))*a*d^2+2*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*c^2-4*(a-a*sin(f*x+e))^(
1/2)*a^(1/2)*c*d+10*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*d^2)*(-a*(sin(f*x+e)-1))
^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^2/(a*sin(f*x + e) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^2/(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^2/(a + a*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + d \sin \left(e + fx\right)\right)^{2}}{\left(a \left(\sin \left(e + fx\right) + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((c + d*sin(e + f*x))**2/(a*(sin(e + f*x) + 1))**(3/2), x)
```

3.552 $\int \frac{c+d\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=87

$$-\frac{(c+3d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}\,a^{3/2}f}-\frac{(c-d)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

```
[Out] -1/2*(c-d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)-1/4*(c+3*d)*arctanh(1/2*cos(
f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2)/f*2^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2750, 2649, 206}

$$-\frac{(c+3d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}\,a^{3/2}f}-\frac{(c-d)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((c + 3*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]
]])])/(2*Sqrt[2]*a^(3/2)*f) - ((c - d)*Cos[e + f*x])/(2*f*(a + a*Sin[e + f*x
])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
```

```
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{c+d\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx &= -\frac{(c-d)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}}+\frac{(c+3d)\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{4a}\\ &= -\frac{(c-d)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}}-\frac{(c+3d)\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,\frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{2af}\\ &= -\frac{(c+3d)\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f}-\frac{(c-d)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.20, size = 150, normalized size = 1.72

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2(c-d)\sin\left(\frac{1}{2}(e+fx)\right)+(d-c)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)+(1+\right.}{2f(a(\sin(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(c - d)*Sin[(e + f*x)/2] + (-c +
d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (1 + I)*(-1)^(3/4)*(c + 3*d)*Arc
Tanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^2))/(2*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

fricas [B] time = 0.44, size = 293, normalized size = 3.37

$$\frac{\sqrt{2}\left((c+3\,d)\cos\left(fx+e\right)^{2}-(c+3\,d)\cos\left(fx+e\right)-\left((c+3\,d)\cos\left(fx+e\right)+2\,c+6\,d\right)\sin\left(fx+e\right)-2\,c-6\,d\right)}{8\left(a^{2}f\cos\left(f\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*((c + 3*d)*cos(f*x + e)^2 - (c + 3*d)*cos(f*x + e) - ((c + 3*d
)*cos(f*x + e) + 2*c + 6*d)*sin(f*x + e) - 2*c - 6*d)*sqrt(a)*log(-(a*cos(f
*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(
f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*
a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) +
 4*((c - d)*cos(f*x + e) - (c - d)*sin(f*x + e) + c - d)*sqrt(a*sin(f*x + e
) + a))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f
*x + e) + 2*a^2*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*2*(1/4*(-3*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(
1))/2)^2+a))^3+3*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)
^2+a))^3+a*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))
-a*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+sqrt(a)
*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-sqrt(a)
*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+sqrt(a)
*a*c-sqrt(a)*a*d)/(-(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2
)^2+a))^2+2*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2
)^2+a))+a)^2/a/sign(tan((f*x+exp(1))/2)+1)+1/4*(c+3*d)*atan((-sqrt(a)*tan((
f*x+exp(1))/2)-sqrt(a)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/s
qrt(2)/sqrt(-a)/a/sign(tan((f*x+exp(1))/2)+1))
```

maple [B] time = 0.86, size = 176, normalized size = 2.02

$$-\frac{\left(\sin \left(f x+e\right) \sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{a-a \sin \left(f x+e\right)}\, \sqrt{2}}{2 \sqrt{a}}\right) a \left(c+3 d\right)+\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{a-a \sin \left(f x+e\right)}\, \sqrt{2}}{2 \sqrt{a}}\right) a c+3 \sqrt{2}\, \operatorname{arctanh}\left(\right.}{4 a^{\frac{5}{2}} \cos \left(f x+e\right) \sqrt{a +}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/4/a^(5/2)*(sin(f*x+e)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)
/a^(1/2))*a*(c+3*d)+2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1
/2))*a*c+3*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*d+
```

```
2*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*c-2*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*d)*(-a*
(sin(f*x+e)-1))^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d\sin\left(fx+e\right)+c}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c+d\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))/(a + a*sin(e + f*x))^(3/2),x)
[Out] int((c + d*sin(e + f*x))/(a + a*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+d\sin\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2),x)
[Out] Integral((c + d*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(3/2), x)
```

3.553 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

```
[Out] -1/2*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)-1/4*arctanh(1/2*cos(f*x+e)*a^(1/2)
*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2)/f*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2650, 2649, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(-3/2),x]
```

```
[Out] -ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])]/(2*Sqrt
[2]*a^(3/2)*f) - Cos[e + f*x]/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+a\sin(e+fx))^{3/2}}\,dx &= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}}+\frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{4a}\\
&= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}}-\frac{\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,\frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{2af}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}}
\end{aligned}
$$

Mathematica [C] time = 0.18, size = 108, normalized size = 1.40

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)-\cos\left(\frac{1}{2}(e+fx)\right)+(1+i)(-1)^{3/4}(\sin(e+fx)+1)\tanh^{-1}\left(\right.\right.}{2f(a(\sin(e+fx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(-3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-Cos[(e + f*x)/2] + Sin[(e + f*x)/2
] + (1 + I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4
])]*(1 + Sin[e + f*x])))/(2*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

fricas [B] time = 0.45, size = 252, normalized size = 3.27

$$\frac{\sqrt{2}\left(\cos\left(fx+e\right)^2-\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2\right)\sqrt{a}\log\left(-\frac{a\cos\left(fx+e\right)^2-2\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{a}}{\cos\left(fx+\right.}\right.}{8\left(a^2f\cos\left(fx+e\right)^2-a^2f\cos\left(fx+e\right)-2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x +
e) - 2)*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)
*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x
+ e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*
x + e) - cos(f*x + e) - 2)) + 4*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - si
```

```
n(f*x + e) + 1))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^
2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*2*(1/4*(-3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1)
)/2)^2+a))^3+a*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a
))+sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2
+sqrt(a)*a)/(-(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a)
)^2+2*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a)
)+a)^2/a/sign(tan((f*x+exp(1))/2)+1)+1/4*atan((-sqrt(a)*tan((f*x+exp(1))/2)
-sqrt(a)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/sqrt(2)/sqrt(-a
)/a/sign(tan((f*x+exp(1))/2)+1))
```

maple [A] time = 0.81, size = 125, normalized size = 1.62

$$-\frac{\left(\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{a}}\right)a^{2}\sin\left(fx+e\right)+2\sqrt{a-a\sin\left(fx+e\right)}\ a^{\frac{3}{2}}+\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{a}}\right)\right.}{4a^{\frac{7}{2}}\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}\ f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/4/a^(7/2)*(2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a
^2*sin(f*x+e)+2*(a-a*sin(f*x+e))^(1/2)*a^(3/2)+2^(1/2)*arctanh(1/2*(a-a*sin
(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2)*(-a*(sin(f*x+e)-1))^(1/2)/cos(f*x+e)/(
a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a + a\,\sin\left(e + f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sin(e + f*x))^(3/2),x)
[Out] int(1/(a + a*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a \sin\left(e + f x\right) + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(3/2),x)
[Out] Integral((a*sin(e + f*x) + a)**(-3/2), x)
```

3.554 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=164

$$-\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f(c-d)^2\sqrt{c+d}}-\frac{(c-5d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^2}-\frac{\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}$$

```
[Out] -1/2*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(3/2)-1/4*(c-5*d)*arctanh(1/2*cos(
f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2)/(c-d)^2/f*2^(1/2)-2*
d^(3/2)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/
2))/a^(3/2)/(c-d)^2/f/(c+d)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2766, 2985, 2649, 206, 2773, 208}

$$-\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f(c-d)^2\sqrt{c+d}}-\frac{(c-5d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^2}-\frac{\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])),x]
```

```
[Out] -((c - 5*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]
])])/(2*Sqrt[2]*a^(3/2)*(c - d)^2*f) - (2*d^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[d]*
Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(a^(3/2)*(c - d)^2*S
qrt[c + d]*f) - Cos[e + f*x]/(2*(c - d)*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx = -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}a(c-4d)-\frac{1}{2}ad\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))}}{2a^2(c-d)}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}} + \frac{(c-5d)\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{4a(c-d)^2}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}} - \frac{(c-5d)\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x\right.}{2a(c-d)^2 f}$$

$$= -\frac{(c-5d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}(c-d)^2 f} - \frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos}{\sqrt{c+d}\sqrt{a+a\,s}}\right.}{a^{3/2}(c-d)^2\sqrt{c+d}}$$

Mathematica [C] time = 1.81, size = 385, normalized size = 2.35

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-\frac{d^{3/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)+\sqrt{d}\cos\left(\frac{1}{2}\right.\right.\right.\right.}{\sqrt{c+d}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(c - d)*Sin[(e + f*x)/2] - (c - d
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (1 + I)*(-1)^(3/4)*(c - 5*d)*ArcT
anh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^2 - (d^(3/2)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Se
c[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e +
 f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/Sqrt[c + d] + (d^(3/2)
*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c +
d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^2)/Sqrt[c + d]))/(2*(c - d)^2*f*(a*(1 + Sin[e + f*x]
))^(3/2))
```

fricas [B] time = 0.80, size = 1299, normalized size = 7.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*((c - 5*d)*cos(f*x + e)^2 - (c - 5*d)*cos(f*x + e) - ((c - 5
*d)*cos(f*x + e) + 2*c - 10*d)*sin(f*x + e) - 2*c + 10*d)*sqrt(a)*log(-(a*c
os(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) -
sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e)
+ 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2
)) - 4*(a*d*cos(f*x + e)^2 - a*d*cos(f*x + e) - 2*a*d - (a*d*cos(f*x + e) +
 2*a*d)*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d
+ 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - 4*((c*d + d^2)*cos(f*x + e)^2
 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d
+ 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*
sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x + e
)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d^
2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2
+ d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*
d - d^2)*sin(f*x + e))) - 4*((c - d)*cos(f*x + e) - (c - d)*sin(f*x + e) +
c - d)*sqrt(a*sin(f*x + e) + a))/((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x
 + e)^2 - (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e) - 2*(a^2*c^2 - 2*a
^2*c*d + a^2*d^2)*f - ((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e) + 2*(
a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f)*sin(f*x + e)), -1/8*(sqrt(2)*((c - 5*d)*c
os(f*x + e)^2 - (c - 5*d)*cos(f*x + e) - ((c - 5*d)*cos(f*x + e) + 2*c - 10
*d)*sin(f*x + e) - 2*c + 10*d)*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*s
qrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos
(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (c
os(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 8*(a*d*cos(f*x + e)^2
- a*d*cos(f*x + e) - 2*a*d - (a*d*cos(f*x + e) + 2*a*d)*sin(f*x + e))*sqrt(
-d/(a*c + a*d))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2
*d)*sqrt(-d/(a*c + a*d))/(d*cos(f*x + e))) - 4*((c - d)*cos(f*x + e) - (c -
 d)*sin(f*x + e) + c - d)*sqrt(a*sin(f*x + e) + a))/((a^2*c^2 - 2*a^2*c*d +
 a^2*d^2)*f*cos(f*x + e)^2 - (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e)
 - 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f - ((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f
*cos(f*x + e) + 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
```

*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(cos((f*t_nostep+exp(1))/2-pi/4))]Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Discontinuities at zeroes of cos((f*t_nostep+exp(1))/2-pi/4) were not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi

```
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
n: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes consta
nt sign by intervals (correct if the argument is real):Check [abs(t_nostep+
1)]Evaluation time: 1.29sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value
```

maple [B] time = 1.32, size = 338, normalized size = 2.06

$$-\frac{\left(\sin\left(fx+e\right)\left(8d^{2}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,d}{\sqrt{acd+a\,d^{2}}}\right)a^{\frac{3}{2}}+\sqrt{a\left(c+d\right)d}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,\sqrt{2}}{2\sqrt{a}}\right)ac-5\sqrt{a\left(c+d\right)}\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] -1/4/a^(5/2)*(sin(f*x+e)*(8*d^2*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d
^2)^(1/2))*a^(3/2)+(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(
1/2)*2^(1/2)/a^(1/2))*a*c-5*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(
f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*d)+8*d^2*arctanh((a-a*sin(f*x+e))^(1/2)*d/
(a*c*d+a*d^2)^(1/2))*a^(3/2)+(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin
(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*c-5*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2
*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*d+2*(a*(c+d)*d)^(1/2)*(a-a*sin(f
*x+e))^(1/2)*a^(1/2)*c-2*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*d
)*(-a*(sin(f*x+e)-1))^(1/2)/(a*(c+d)*d)^(1/2)/(c-d)^2/cos(f*x+e)/(a+a*sin(f
*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))),x)
[Out] int(1/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e)),x)
[Out] Timed out
```

3.555 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=243

$$-\frac{d^{3/2}(5c+3d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f(c-d)^3(c+d)^{3/2}}-\frac{(c-9d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^3}-\frac{d(c+3d)\cos(e}{2af(c-d)^2(c+d)\sqrt{a\sin(e+fx)}}$$

```
[Out] -d^(3/2)*(5*c+3*d)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(
f*x+e))^(1/2))/a^(3/2)/(c-d)^3/(c+d)^(3/2)/f-1/2*cos(f*x+e)/(c-d)/f/(a+a*si
n(f*x+e))^(3/2)/(c+d*sin(f*x+e))-1/4*(c-9*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)
*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2)/(c-d)^3/f*2^(1/2)-1/2*d*(c+3*d)*co
s(f*x+e)/a/(c-d)^2/(c+d)/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.74, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2766, 2984, 2985, 2649, 206, 2773, 208}

$$-\frac{d^{3/2}(5c+3d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f(c-d)^3(c+d)^{3/2}}-\frac{(c-9d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^3}-\frac{d(c+3d)\cos(e}{2af(c-d)^2(c+d)\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] -((c - 9*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]
]])])/(2*Sqrt[2]*a^(3/2)*(c - d)^3*f) - (d^(3/2)*(5*c + 3*d)*ArcTanh[(Sqrt[a
]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(a^(3/2)*(
c - d)^3*(c + d)^(3/2)*f) - Cos[e + f*x]/(2*(c - d)*f*(a + a*Sin[e + f*x])^
(3/2)*(c + d*Sin[e + f*x])) - (d*(c + 3*d)*Cos[e + f*x])/(2*a*(c - d)^2*(c
+ d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx = -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} - \frac{\int \frac{-\frac{1}{2}a(c}{\sqrt{a+a\sin}}}{}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} - \frac{}{2a(c-d)^2}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} - \frac{}{2a(c-d)^2}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} - \frac{}{2a(c-d)^2}$$

$$= -\frac{(c-9d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}(c-d)^3f} - \frac{d^{3/2}(5c+3d)\tanh^{-1}\left(\frac{}{\sqrt{c}}\right)}{a^{3/2}(c-d)^3(c+}$$

Mathematica [C] time = 4.56, size = 491, normalized size = 2.02

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\frac{d^{3/2}(5c+3d)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)+\sqrt{d}\cos\right.\right.\right.}{(d-c)^3(c+d)^{3/2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((4*Sin[(e + f*x)/2])/(c - d)^2 - (2
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(c - d)^2 + ((2 + 2*I)*(-1)^(3/4)*(
c - 9*d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^2)/(c - d)^3 + (d^(3/2)*(5*c + 3*d)*(e + f*x -
2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]
*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^2)/((-c + d)^3*(c + d)^(3/2)) + (d^(3/2)*(5*c + 3*d)*(e + f*x -
2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]
*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^2)/((c - d)^3*(c + d)^(3/2)) - (4*d^2*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/((c - d)^2*(c + d)*(c
+ d*Sin[e + f*x]))))/(4*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

fricas [B] time = 1.23, size = 2515, normalized size = 10.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, algorithm="fricas"
)

[Out] [1/8*(sqrt(2)*((c^2*d - 8*c*d^2 - 9*d^3)*cos(f*x + e)^3 - 2*c^3 + 14*c^2*d
+ 34*c*d^2 + 18*d^3 + (c^3 - 6*c^2*d - 25*c*d^2 - 18*d^3)*cos(f*x + e)^2 -
(c^3 - 7*c^2*d - 17*c*d^2 - 9*d^3)*cos(f*x + e) - (2*c^3 - 14*c^2*d - 34*c*
d^2 - 18*d^3 - (c^2*d - 8*c*d^2 - 9*d^3)*cos(f*x + e)^2 + (c^3 - 7*c^2*d -
17*c*d^2 - 9*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^
2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e)
 + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(
f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 2*(10*a
*c^2*d + 16*a*c*d^2 + 6*a*d^3 - (5*a*c*d^2 + 3*a*d^3)*cos(f*x + e)^3 - (5*a
*c^2*d + 13*a*c*d^2 + 6*a*d^3)*cos(f*x + e)^2 + (5*a*c^2*d + 8*a*c*d^2 + 3*
a*d^3)*cos(f*x + e) + (10*a*c^2*d + 16*a*c*d^2 + 6*a*d^3 - (5*a*c*d^2 + 3*a
*d^3)*cos(f*x + e)^2 + (5*a*c^2*d + 8*a*c*d^2 + 3*a*d^3)*cos(f*x + e))*sin(
f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*cos
(f*x + e)^2 - c^2 - 2*c*d - d^2 + 4*((c*d + d^2)*cos(f*x + e)^2 - c^2 - 4*c
*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d + 3*d^2 + (c
*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d/(a*c
+ a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - c^2 -
2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x +
e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f
*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin
(f*x + e))) + 4*(c^3 - c^2*d - c*d^2 + d^3 + (c^2*d + 2*c*d^2 - 3*d^3)*cos(
f*x + e)^2 + (c^3 + c*d^2 - 2*d^3)*cos(f*x + e) - (c^3 - c^2*d - c*d^2 + d^
3 - (c^2*d + 2*c*d^2 - 3*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x +
e) + a))/((a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e
)^3 + (a^2*c^5 - 4*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + 3*a^2*c*d^4 - 2*a^2*d^5)*f
*cos(f*x + e)^2 - (a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^
2*c*d^4 - a^2*d^5)*f*cos(f*x + e) - 2*(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2
+ 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f + ((a^2*c^4*d - 2*a^2*c^3*d^2 + 2*
a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e)^2 - (a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^
2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e) - 2*(a^2*c^5 - a^2*
c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f)*sin(f*x + e
)), 1/8*(sqrt(2)*((c^2*d - 8*c*d^2 - 9*d^3)*cos(f*x + e)^3 - 2*c^3 + 14*c^2
*d + 34*c*d^2 + 18*d^3 + (c^3 - 6*c^2*d - 25*c*d^2 - 18*d^3)*cos(f*x + e)^2
 - (c^3 - 7*c^2*d - 17*c*d^2 - 9*d^3)*cos(f*x + e) - (2*c^3 - 14*c^2*d - 34
*c*d^2 - 18*d^3 - (c^2*d - 8*c*d^2 - 9*d^3)*cos(f*x + e)^2 + (c^3 - 7*c^2*d
 - 17*c*d^2 - 9*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x +
```

```
e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x +
 e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(c
os(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(1
0*a*c^2*d + 16*a*c*d^2 + 6*a*d^3 - (5*a*c*d^2 + 3*a*d^3)*cos(f*x + e)^3 - (
5*a*c^2*d + 13*a*c*d^2 + 6*a*d^3)*cos(f*x + e)^2 + (5*a*c^2*d + 8*a*c*d^2 +
 3*a*d^3)*cos(f*x + e) + (10*a*c^2*d + 16*a*c*d^2 + 6*a*d^3 - (5*a*c*d^2 +
3*a*d^3)*cos(f*x + e)^2 + (5*a*c^2*d + 8*a*c*d^2 + 3*a*d^3)*cos(f*x + e))*s
in(f*x + e))*sqrt(-d/(a*c + a*d))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*si
n(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a*d))/(d*cos(f*x + e))) + 4*(c^3 - c^2
*d - c*d^2 + d^3 + (c^2*d + 2*c*d^2 - 3*d^3)*cos(f*x + e)^2 + (c^3 + c*d^2
- 2*d^3)*cos(f*x + e) - (c^3 - c^2*d - c*d^2 + d^3 - (c^2*d + 2*c*d^2 - 3*d
^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^2*c^4*d - 2*a
^2*c^3*d^2 + 2*a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e)^3 + (a^2*c^5 - 4*a^2*c^3
*d^2 + 2*a^2*c^2*d^3 + 3*a^2*c*d^4 - 2*a^2*d^5)*f*cos(f*x + e)^2 - (a^2*c^5
 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f*cos(f
*x + e) - 2*(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^
4 - a^2*d^5)*f + ((a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c*d^4 - a^2*d^5)*f*cos
(f*x + e)^2 - (a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*
d^4 - a^2*d^5)*f*cos(f*x + e) - 2*(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*
a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Warning, integration of abs or sign assumes constant sign by inter
vals (correct if the argument is real):Check [abs(cos((f*t_nostep+exp(1))/2
-pi/4))]Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Discontinuities at zeroes
```

of cos((f*t_nostep+exp(1))/2-pi/4) were not checkedUnable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/

```
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of
abs or sign assumes constant sign by intervals (correct if the argument is
real):Check [abs(t_nostep+1)]Evaluation time: 0.57Error: Bad Argument Type
```

maple [B] time = 1.81, size = 978, normalized size = 4.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x)
```

```
[Out] -1/4/a^(5/2)*((a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/
2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a*c^2*d-8*(a*(c+d)*d)^(1/2)*2^(1/2)*arctan
h(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a*c*d^2-9*(a*
(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2
))*sin(f*x+e)^2*a*d^3+20*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1
/2))*a^(3/2)*sin(f*x+e)^2*c*d^3+12*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(
c+d)*d)^(1/2))*a^(3/2)*sin(f*x+e)^2*d^4+(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1
/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a*c^3-7*(a*(c+d)*d
)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(
f*x+e)*a*c^2*d-17*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))
^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a*c*d^2-9*(a*(c+d)*d)^(1/2)*2^(1/2)*arct
anh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a*d^3+20*arct
anh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*sin(f*x+e)*c^2*d
^2+32*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*sin(f*
x+e)*c*d^3+12*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2
)*sin(f*x+e)*d^4+2*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(
f*x+e)*c^2*d+4*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+
e)*c*d^2-6*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*d
^3+(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/
a^(1/2))*a*c^3-8*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^
(1/2)*2^(1/2)/a^(1/2))*a*c^2*d-9*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*
(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a*c*d^2+20*arctanh((-a*(sin(f*x+e)-1
))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*c^2*d^2+12*arctanh((-a*(sin(f*x+e)-1)
)^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*c*d^3+2*(-a*(sin(f*x+e)-1))^(1/2)*(a*(
c+d)*d)^(1/2)*a^(1/2)*c^3+2*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(
1/2)*c*d^2-4*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*d^3)*(-a*(
sin(f*x+e)-1))^(1/2)/(a*(c+d)*d)^(1/2)/(c+d*sin(f*x+e))/(c+d)/(c-d)^3/cos(f
*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^2),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.556 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=318

$$-\frac{d^{3/2}\left(35c^2+42cd+19d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{4a^{3/2}f(c-d)^4(c+d)^{5/2}}-\frac{(c-13d)\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}\,a^{3/2}f(c-d)^4}-\frac{d(2c}{4af(c-d)^3(c+d)^2}$$

```
[Out] -1/4*d^(3/2)*(35*c^2+42*c*d+19*d^2)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d
)^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2)/(c-d)^4/(c+d)^(5/2)/f-1/2*cos(f*x+e
)/(c-d)/f/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2-1/4*(c-13*d)*arctanh(1/
2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2)/(c-d)^4/f*2^(1
/2)-1/2*d*(c+2*d)*cos(f*x+e)/a/(c-d)^2/(c+d)/f/(c+d*sin(f*x+e))^2/(a+a*sin(
f*x+e))^(1/2)-1/4*d*(2*c+d)*(c+7*d)*cos(f*x+e)/a/(c-d)^3/(c+d)^2/f/(c+d*sin
(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.11, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2766, 2984, 2985, 2649, 206, 2773, 208}

$$-\frac{d^{3/2}\left(35c^2+42cd+19d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{4a^{3/2}f(c-d)^4(c+d)^{5/2}}-\frac{(c-13d)\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}\,a^{3/2}f(c-d)^4}-\frac{d(2c}{4af(c-d)^3(c+d)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] -((c - 13*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x
]])])/(2*Sqrt[2]*a^(3/2)*(c - d)^4*f) - (d^(3/2)*(35*c^2 + 42*c*d + 19*d^2)
*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x
]])])/(4*a^(3/2)*(c - d)^4*(c + d)^(5/2)*f) - Cos[e + f*x]/(2*(c - d)*f*(a
+ a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2) - (d*(c + 2*d)*Cos[e + f*x]
)/(2*a*(c - d)^2*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2)
 - (d*(2*c + d)*(c + 7*d)*Cos[e + f*x])/(4*a*(c - d)^3*(c + d)^2*f*Sqrt[a +
 a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx = -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} - \frac{\int \frac{-\frac{1}{2}a(\ldots}{\sqrt{a+a\sin\ldots}}}{\ldots}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} - \frac{\ldots}{2a(c-d)\ldots}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} - \frac{\ldots}{2a(c-d)\ldots}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} - \frac{\ldots}{2a(c-d)\ldots}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} - \frac{\ldots}{2a(c-d)\ldots}$$

$$= -\frac{(c-13d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}(c-d)^4 f} - \frac{d^{3/2}\left(35c^2+42cd+19d\ldots\right.}{4a^{3/2}(c-\ldots}$$

Mathematica [C] time = 6.52, size = 570, normalized size = 1.79

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-\frac{d^{3/2}(35c^2+42cd+19d^2)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\sin\left(\frac{1}{2}(e\ldots\right.\right.\right.}{(c+d)^{5/2}}\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(16*(c - d)*Sin[(e + f*x)/2] - 8*(c
- d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (8 + 8*I)*(-1)^(3/4)*(c - 13*d
)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^2 - (d^(3/2)*(35*c^2 + 42*c*d + 19*d^2)*(e + f*x - 2*L
```

```
og[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Co
s[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^2)/(c + d)^(5/2) + (d^(3/2)*(35*c^2 + 42*c*d + 19*d^2)*(e + f*x - 2
*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*
Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^2)/(c + d)^(5/2) - (8*(c - d)^2*d^2*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/((c + d)*(c + d*Sin[e + f
*x])^2) - (4*(c - d)*d^2*(11*c + 5*d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/((c + d)^2*(c + d*Sin[e + f*x])))
)/(16*(c - d)^4*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

fricas [B] time = 2.09, size = 4133, normalized size = 13.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, algorithm="fricas"
)
```

```
[Out] [-1/16*(2*sqrt(2)*(2*c^5 - 18*c^4*d - 92*c^3*d^2 - 148*c^2*d^3 - 102*c*d^4
- 26*d^5 + (c^3*d^2 - 11*c^2*d^3 - 25*c*d^4 - 13*d^5)*cos(f*x + e)^4 - (2*c
^4*d - 21*c^3*d^2 - 61*c^2*d^3 - 51*c*d^4 - 13*d^5)*cos(f*x + e)^3 - (c^5 -
 7*c^4*d - 66*c^3*d^2 - 146*c^2*d^3 - 127*c*d^4 - 39*d^5)*cos(f*x + e)^2 +
(c^5 - 9*c^4*d - 46*c^3*d^2 - 74*c^2*d^3 - 51*c*d^4 - 13*d^5)*cos(f*x + e)
+ (2*c^5 - 18*c^4*d - 92*c^3*d^2 - 148*c^2*d^3 - 102*c*d^4 - 26*d^5 - (c^3*
d^2 - 11*c^2*d^3 - 25*c*d^4 - 13*d^5)*cos(f*x + e)^3 - 2*(c^4*d - 10*c^3*d^
2 - 36*c^2*d^3 - 38*c*d^4 - 13*d^5)*cos(f*x + e)^2 + (c^5 - 9*c^4*d - 46*c^
3*d^2 - 74*c^2*d^3 - 51*c*d^4 - 13*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(a)
*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f
*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin
(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*
x + e) - 2)) - (70*a*c^4*d + 224*a*c^3*d^2 + 276*a*c^2*d^3 + 160*a*c*d^4 +
38*a*d^5 + (35*a*c^2*d^3 + 42*a*c*d^4 + 19*a*d^5)*cos(f*x + e)^4 - (70*a*c^
3*d^2 + 119*a*c^2*d^3 + 80*a*c*d^4 + 19*a*d^5)*cos(f*x + e)^3 - (35*a*c^4*d
 + 182*a*c^3*d^2 + 292*a*c^2*d^3 + 202*a*c*d^4 + 57*a*d^5)*cos(f*x + e)^2 +
 (35*a*c^4*d + 112*a*c^3*d^2 + 138*a*c^2*d^3 + 80*a*c*d^4 + 19*a*d^5)*cos(f
*x + e) + (70*a*c^4*d + 224*a*c^3*d^2 + 276*a*c^2*d^3 + 160*a*c*d^4 + 38*a*
d^5 - (35*a*c^2*d^3 + 42*a*c*d^4 + 19*a*d^5)*cos(f*x + e)^3 - 2*(35*a*c^3*d
^2 + 77*a*c^2*d^3 + 61*a*c*d^4 + 19*a*d^5)*cos(f*x + e)^2 + (35*a*c^4*d + 1
12*a*c^3*d^2 + 138*a*c^2*d^3 + 80*a*c*d^4 + 19*a*d^5)*cos(f*x + e))*sin(f*x
 + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*cos(f*
x + e)^2 - c^2 - 2*c*d - d^2 - 4*((c*d + d^2)*cos(f*x + e)^2 - c^2 - 4*c*d
- 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d + 3*d^2 + (c*d
+ d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d/(a*c + a
*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - c^2 - 2*c
```

d - d^2 + 2(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^
3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x
+ e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*
x + e))) + 4*(2*c^5 - 2*c^4*d - 4*c^3*d^2 + 4*c^2*d^3 + 2*c*d^4 - 2*d^5 - (
2*c^3*d^2 + 13*c^2*d^3 - 8*c*d^4 - 7*d^5)*cos(f*x + e)^3 + (4*c^4*d + 15*c^
3*d^2 - 14*c^2*d^3 - 9*c*d^4 + 4*d^5)*cos(f*x + e)^2 + (2*c^5 + 2*c^4*d + 1
3*c^3*d^2 + 3*c^2*d^3 - 15*c*d^4 - 5*d^5)*cos(f*x + e) - (2*c^5 - 2*c^4*d -
 4*c^3*d^2 + 4*c^2*d^3 + 2*c*d^4 - 2*d^5 - (2*c^3*d^2 + 13*c^2*d^3 - 8*c*d^
4 - 7*d^5)*cos(f*x + e)^2 - (4*c^4*d + 17*c^3*d^2 - c^2*d^3 - 17*c*d^4 - 3*
d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^2*c^6*d^2 -
2*a^2*c^5*d^3 - a^2*c^4*d^4 + 4*a^2*c^3*d^5 - a^2*c^2*d^6 - 2*a^2*c*d^7 + a
^2*d^8)*f*cos(f*x + e)^4 - (2*a^2*c^7*d - 3*a^2*c^6*d^2 - 4*a^2*c^5*d^3 + 7
*a^2*c^4*d^4 + 2*a^2*c^3*d^5 - 5*a^2*c^2*d^6 + a^2*d^8)*f*cos(f*x + e)^3 -
(a^2*c^8 + 2*a^2*c^7*d - 6*a^2*c^6*d^2 - 6*a^2*c^5*d^3 + 12*a^2*c^4*d^4 + 6
*a^2*c^3*d^5 - 10*a^2*c^2*d^6 - 2*a^2*c*d^7 + 3*a^2*d^8)*f*cos(f*x + e)^2 +
 (a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f*cos(
f*x + e) + 2*(a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2
*d^8)*f - ((a^2*c^6*d^2 - 2*a^2*c^5*d^3 - a^2*c^4*d^4 + 4*a^2*c^3*d^5 - a^2
*c^2*d^6 - 2*a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^3 + 2*(a^2*c^7*d - a^2*c^6
*d^2 - 3*a^2*c^5*d^3 + 3*a^2*c^4*d^4 + 3*a^2*c^3*d^5 - 3*a^2*c^2*d^6 - a^2*
c*d^7 + a^2*d^8)*f*cos(f*x + e)^2 - (a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^
4 - 4*a^2*c^2*d^6 + a^2*d^8)*f*cos(f*x + e) - 2*(a^2*c^8 - 4*a^2*c^6*d^2 +
6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f)*sin(f*x + e)), -1/8*(sqrt(2)*(2
*c^5 - 18*c^4*d - 92*c^3*d^2 - 148*c^2*d^3 - 102*c*d^4 - 26*d^5 + (c^3*d^2
- 11*c^2*d^3 - 25*c*d^4 - 13*d^5)*cos(f*x + e)^4 - (2*c^4*d - 21*c^3*d^2 -
61*c^2*d^3 - 51*c*d^4 - 13*d^5)*cos(f*x + e)^3 - (c^5 - 7*c^4*d - 66*c^3*d^
2 - 146*c^2*d^3 - 127*c*d^4 - 39*d^5)*cos(f*x + e)^2 + (c^5 - 9*c^4*d - 46*
c^3*d^2 - 74*c^2*d^3 - 51*c*d^4 - 13*d^5)*cos(f*x + e) + (2*c^5 - 18*c^4*d
- 92*c^3*d^2 - 148*c^2*d^3 - 102*c*d^4 - 26*d^5 - (c^3*d^2 - 11*c^2*d^3 - 2
5*c*d^4 - 13*d^5)*cos(f*x + e)^3 - 2*(c^4*d - 10*c^3*d^2 - 36*c^2*d^3 - 38*
c*d^4 - 13*d^5)*cos(f*x + e)^2 + (c^5 - 9*c^4*d - 46*c^3*d^2 - 74*c^2*d^3 -
 51*c*d^4 - 13*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e
)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x +
e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(co
s(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + (70*a
*c^4*d + 224*a*c^3*d^2 + 276*a*c^2*d^3 + 160*a*c*d^4 + 38*a*d^5 + (35*a*c^2
*d^3 + 42*a*c*d^4 + 19*a*d^5)*cos(f*x + e)^4 - (70*a*c^3*d^2 + 119*a*c^2*d^
3 + 80*a*c*d^4 + 19*a*d^5)*cos(f*x + e)^3 - (35*a*c^4*d + 182*a*c^3*d^2 + 2
92*a*c^2*d^3 + 202*a*c*d^4 + 57*a*d^5)*cos(f*x + e)^2 + (35*a*c^4*d + 112*a
*c^3*d^2 + 138*a*c^2*d^3 + 80*a*c*d^4 + 19*a*d^5)*cos(f*x + e) + (70*a*c^4*
d + 224*a*c^3*d^2 + 276*a*c^2*d^3 + 160*a*c*d^4 + 38*a*d^5 - (35*a*c^2*d^3
+ 42*a*c*d^4 + 19*a*d^5)*cos(f*x + e)^3 - 2*(35*a*c^3*d^2 + 77*a*c^2*d^3 +
61*a*c*d^4 + 19*a*d^5)*cos(f*x + e)^2 + (35*a*c^4*d + 112*a*c^3*d^2 + 138*a
*c^2*d^3 + 80*a*c*d^4 + 19*a*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(-d/(a*c
+ a*d))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt

```
(-d/(a*c + a*d))/(d*cos(f*x + e))) + 2*(2*c^5 - 2*c^4*d - 4*c^3*d^2 + 4*c^2
*d^3 + 2*c*d^4 - 2*d^5 - (2*c^3*d^2 + 13*c^2*d^3 - 8*c*d^4 - 7*d^5)*cos(f*x
 + e)^3 + (4*c^4*d + 15*c^3*d^2 - 14*c^2*d^3 - 9*c*d^4 + 4*d^5)*cos(f*x + e
)^2 + (2*c^5 + 2*c^4*d + 13*c^3*d^2 + 3*c^2*d^3 - 15*c*d^4 - 5*d^5)*cos(f*x
 + e) - (2*c^5 - 2*c^4*d - 4*c^3*d^2 + 4*c^2*d^3 + 2*c*d^4 - 2*d^5 - (2*c^3
*d^2 + 13*c^2*d^3 - 8*c*d^4 - 7*d^5)*cos(f*x + e)^2 - (4*c^4*d + 17*c^3*d^2
 - c^2*d^3 - 17*c*d^4 - 3*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x +
 e) + a))/((a^2*c^6*d^2 - 2*a^2*c^5*d^3 - a^2*c^4*d^4 + 4*a^2*c^3*d^5 - a^2
*c^2*d^6 - 2*a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^4 - (2*a^2*c^7*d - 3*a^2*c
^6*d^2 - 4*a^2*c^5*d^3 + 7*a^2*c^4*d^4 + 2*a^2*c^3*d^5 - 5*a^2*c^2*d^6 + a^
2*d^8)*f*cos(f*x + e)^3 - (a^2*c^8 + 2*a^2*c^7*d - 6*a^2*c^6*d^2 - 6*a^2*c^
5*d^3 + 12*a^2*c^4*d^4 + 6*a^2*c^3*d^5 - 10*a^2*c^2*d^6 - 2*a^2*c*d^7 + 3*a
^2*d^8)*f*cos(f*x + e)^2 + (a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2
*c^2*d^6 + a^2*d^8)*f*cos(f*x + e) + 2*(a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4
*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f - ((a^2*c^6*d^2 - 2*a^2*c^5*d^3 - a^2*c^4
*d^4 + 4*a^2*c^3*d^5 - a^2*c^2*d^6 - 2*a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^
3 + 2*(a^2*c^7*d - a^2*c^6*d^2 - 3*a^2*c^5*d^3 + 3*a^2*c^4*d^4 + 3*a^2*c^3*
d^5 - 3*a^2*c^2*d^6 - a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^2 - (a^2*c^8 - 4*
a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f*cos(f*x + e) - 2*(
a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f)*sin(f
*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assu
```

mes constant sign by intervals (correct if the argument is real):Check [abs
(cos((f*t_nostep+exp(1))/2-pi/4))]Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Discontinuities at zeroes of cos((f*t_nostep+exp(1))/2-pi/4) were not check
edUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x

```
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Warning, integration of abs or sign assumes constant sign by intervals (c
orrect if the argument is real):Check [abs(t_nostep+1)]Evaluation time: 0.6
2Unable to divide, perhaps due to rounding error%%%{%%%{1,[1]%%%},[18,1,3,0
]%%%}+%%%{%%{[%%%{-6,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[17,1,3,0]%%%}+%%%{
%%{[%%%{-12,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[17,1,2,1]%%%}+%%%{%%%{15,[2
]%%%},[16,1,3,0]%%%}+%%%{%%%{72,[2]%%%},[16,1,2,1]%%%}+%%%{%%%{48,[2]%%%},[
16,1,1,2]%%%}+%%%{%%{[%%%{-32,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[15,1,3,0]
%%%}+%%%{%%{[%%%{-144,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[15,1,2,1]%%%}+%%%
{%%{[%%%{-288,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[15,1,1,2]%%%}+%%%{%%{[%%%
{-64,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[15,1,0,3]%%%}+%%%{%%%{60,[3]%%%},[
14,1,3,0]%%%}+%%%{%%%{168,[3]%%%},[14,1,2,1]%%%}+%%%{%%%{432,[3]%%%},[14,1,
1,2]%%%}+%%%{%%%{384,[3]%%%},[14,1,0,3]%%%}+%%%{%%{[%%%{-72,[3]%%%},0]:[1,0
,%%%{-1,[1]%%%}]%%},[13,1,3,0]%%%}+%%%{%%{[%%%{-240,[3]%%%},0]:[1,0,%%%{-1,
[1]%%%}]%%},[13,1,2,1]%%%}+%%%{%%{[%%%{192,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%
%},[13,1,1,2]%%%}+%%%{%%{[%%%{-384,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[13,1
,0,3]%%%}+%%%{%%%{100,[4]%%%},[12,1,3,0]%%%}+%%%{%%%{72,[4]%%%},[12,1,2,1]%
%%}+%%%{%%%{-528,[4]%%%},[12,1,1,2]%%%}+%%%{%%%{-1408,[4]%%%},[12,1,0,3]%%%
}+%%%{%%{[%%%{-96,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[11,1,3,0]%%%}+%%%{%%{
[%%%{144,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[11,1,2,1]%%%}+%%%{%%{[%%%{288,
[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[11,1,1,2]%%%}+%%%{%%{[%%%{2112,[4]%%%},
0]:[1,0,%%%{-1,[1]%%%}]%%},[11,1,0,3]%%%}+%%%{%%%{54,[5]%%%},[10,1,3,0]%%%}
+%%%{%%%{-24,[5]%%%},[10,1,2,1]%%%}+%%%{%%%{-912,[5]%%%},[10,1,1,2]%%%}+%%%
{%%%{2304,[5]%%%},[10,1,0,3]%%%}+%%%{%%{[%%%{-100,[5]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[9,1,3,0]%%%}+%%%{%%{[%%%{504,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},
[9,1,2,1]%%%}+%%%{%%{[%%%{-384,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[9,1,1,2]
%%%}+%%%{%%{[%%%{-3328,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[9,1,0,3]%%%}+%%%
{%%%{-54,[6]%%%},[8,1,3,0]%%%}+%%%{%%%{24,[6]%%%},[8,1,2,1]%%%}+%%%{%%%{912
,[6]%%%},[8,1,1,2]%%%}+%%%{%%%{-2304,[6]%%%},[8,1,0,3]%%%}+%%%{%%{[%%%{-96,
[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,1,3,0]%%%}+%%%{%%{[%%%{144,[6]%%%},0]
:[1,0,%%%{-1,[1]%%%}]%%},[7,1,2,1]%%%}+%%%{%%{[%%%{288,[6]%%%},0]:[1,0,%%%{
-1,[1]%%%}]%%},[7,1,1,2]%%%}+%%%{%%{[%%%{2112,[6]%%%},0]:[1,0,%%%{-1,[1]%%%
}]%%},[7,1,0,3]%%%}+%%%{%%%{-100,[7]%%%},[6,1,3,0]%%%}+%%%{%%%{-72,[7]%%%},
[6,1,2,1]%%%}+%%%{%%%{528,[7]%%%},[6,1,1,2]%%%}+%%%{%%%{1408,[7]%%%},[6,1,0
,3]%%%}+%%%{%%{[%%%{-72,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,1,3,0]%%%}+%%
%{%%{[%%%{-240,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,1,2,1]%%%}+%%%{%%{[%%%
{192,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,1,1,2]%%%}+%%%{%%{[%%%{-384,[7]%
```

%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,1,0,3]%%%}+%%%{%%%{-60,[8]%%%},[4,1,3,0]%
%%}+%%%{%%%{-168,[8]%%%},[4,1,2,1]%%%}+%%%{%%%{-432,[8]%%%},[4,1,1,2]%%%}+%
%%{%%%{-384,[8]%%%},[4,1,0,3]%%%}+%%%{%%{[%%%{-32,[8]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[3,1,3,0]%%%}+%%%{%%{[%%%{-144,[8]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%}
,[3,1,2,1]%%%}+%%%{%%{[%%%{-288,[8]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,1,1,2
]%%%}+%%%{%%{[%%%{-64,[8]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,1,0,3]%%%}+%%%{
%%%{-15,[9]%%%},[2,1,3,0]%%%}+%%%{%%%{-72,[9]%%%},[2,1,2,1]%%%}+%%%{%%%{-48
,[9]%%%},[2,1,1,2]%%%}+%%%{%%{[%%%{-6,[9]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1
,1,3,0]%%%}+%%%{%%{[%%%{-12,[9]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,1,2,1]%%%
}+%%%{%%%{-1,[10]%%%},[0,1,3,0]%%%} / %%%{%%{poly1[%%%{-1,[1]%%%},0]:[1,0,%
%%{-1,[1]%%%}]%%},[18,0,3,0]%%%}+%%%{%%%{6,[2]%%%},[17,0,3,0]%%%}+%%%{%%%{1
2,[2]%%%},[17,0,2,1]%%%}+%%%{%%{poly1[%%%{-15,[2]%%%},0]:[1,0,%%%{-1,[1]%%%
}]%%},[16,0,3,0]%%%}+%%%{%%{[%%%{-72,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[16
,0,2,1]%%%}+%%%{%%{poly1[%%%{-48,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[16,0,1
,2]%%%}+%%%{%%%{32,[3]%%%},[15,0,3,0]%%%}+%%%{%%%{144,[3]%%%},[15,0,2,1]%%%
}+%%%{%%%{288,[3]%%%},[15,0,1,2]%%%}+%%%{%%%{64,[3]%%%},[15,0,0,3]%%%}+%%%{
%%{poly1[%%%{-60,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[14,0,3,0]%%%}+%%%{%%{[
%%%{-168,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[14,0,2,1]%%%}+%%%{%%{poly1[%%%
{-432,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[14,0,1,2]%%%}+%%%{%%{[%%%{-384,[3
]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[14,0,0,3]%%%}+%%%{%%%{72,[4]%%%},[13,0,3,
0]%%%}+%%%{%%%{240,[4]%%%},[13,0,2,1]%%%}+%%%{%%%{-192,[4]%%%},[13,0,1,2]%%
%}+%%%{%%%{384,[4]%%%},[13,0,0,3]%%%}+%%%{%%{poly1[%%%{-100,[4]%%%},0]:[1,0
,%%%{-1,[1]%%%}]%%},[12,0,3,0]%%%}+%%%{%%{[%%%{-72,[4]%%%},0]:[1,0,%%%{-1,[
1]%%%}]%%},[12,0,2,1]%%%}+%%%{%%{poly1[%%%{528,[4]%%%},0]:[1,0,%%%{-1,[1]%%
%}]%%},[12,0,1,2]%%%}+%%%{%%{[%%%{1408,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[
12,0,0,3]%%%}+%%%{%%%{96,[5]%%%},[11,0,3,0]%%%}+%%%{%%%{-144,[5]%%%},[11,0,
2,1]%%%}+%%%{%%%{-288,[5]%%%},[11,0,1,2]%%%}+%%%{%%%{-2112,[5]%%%},[11,0,0,
3]%%%}+%%%{%%{poly1[%%%{-54,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[10,0,3,0]%%
%}+%%%{%%{[%%%{24,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[10,0,2,1]%%%}+%%%{%%{
poly1[%%%{912,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[10,0,1,2]%%%}+%%%{%%{[%%%
{-2304,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[10,0,0,3]%%%}+%%%{%%%{100,[6]%%%
},[9,0,3,0]%%%}+%%%{%%%{-504,[6]%%%},[9,0,2,1]%%%}+%%%{%%%{384,[6]%%%},[9,0
,1,2]%%%}+%%%{%%%{3328,[6]%%%},[9,0,0,3]%%%}+%%%{%%{poly1[%%%{54,[6]%%%},0]
:[1,0,%%%{-1,[1]%%%}]%%},[8,0,3,0]%%%}+%%%{%%{[%%%{-24,[6]%%%},0]:[1,0,%%%{
-1,[1]%%%}]%%},[8,0,2,1]%%%}+%%%{%%{poly1[%%%{-912,[6]%%%},0]:[1,0,%%%{-1,[
1]%%%}]%%},[8,0,1,2]%%%}+%%%{%%{[%%%{2304,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%
},[8,0,0,3]%%%}+%%%{%%%{96,[7]%%%},[7,0,3,0]%%%}+%%%{%%%{-144,[7]%%%},[7,0,
2,1]%%%}+%%%{%%%{-288,[7]%%%},[7,0,1,2]%%%}+%%%{%%%{-2112,[7]%%%},[7,0,0,3]
%%%}+%%%{%%{poly1[%%%{100,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,0,3,0]%%%}+
%%%{%%{[%%%{72,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,0,2,1]%%%}+%%%{%%{poly
1[%%%{-528,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,0,1,2]%%%}+%%%{%%{[%%%{-14
08,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,0,0,3]%%%}+%%%{%%%{72,[8]%%%},[5,0
,3,0]%%%}+%%%{%%%{240,[8]%%%},[5,0,2,1]%%%}+%%%{%%%{-192,[8]%%%},[5,0,1,2]%
%%}+%%%{%%%{384,[8]%%%},[5,0,0,3]%%%}+%%%{%%{poly1[%%%{60,[8]%%%},0]:[1,0,%
%%{-1,[1]%%%}]%%},[4,0,3,0]%%%}+%%%{%%{[%%%{168,[8]%%%},0]:[1,0,%%%{-1,[1]%

```
%%}]%%},[4,0,2,1]%%%}+%%%{%%{poly1[%%%{432,[8]%%%},0]:[1,0,%%%{-1,[1]%%%}]%
%},[4,0,1,2]%%%}+%%%{%%{[%%%{384,[8]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,0,
3]%%%}+%%%{%%%{32,[9]%%%},[3,0,3,0]%%%}+%%%{%%%{144,[9]%%%},[3,0,2,1]%%%}+%
%%{%%%{288,[9]%%%},[3,0,1,2]%%%}+%%%{%%%{64,[9]%%%},[3,0,0,3]%%%}+%%%{%%{po
ly1[%%%{15,[9]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,3,0]%%%}+%%%{%%{[%%%{72,
[9]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,2,1]%%%}+%%%{%%{poly1[%%%{48,[9]%%%
},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,1,2]%%%}+%%%{%%%{6,[10]%%%},[1,0,3,0]%%%}
+%%%{%%%{12,[10]%%%},[1,0,2,1]%%%}+%%%{%%{poly1[%%%{1,[10]%%%},0]:[1,0,%%%{
-1,[1]%%%}]%%},[0,0,3,0]%%%} Error: Bad Argument Value
```

maple [B] time = 2.43, size = 2219, normalized size = 6.98

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x)
```

```
[Out] -1/4*(-a*(sin(f*x+e)-1))^(1/2)*(-13*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(
-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*d^5+42*arctanh((
-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*c^3*d^3+19*arctanh((-
a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*c^2*d^4+2*(-a*(sin(f*x
+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^5-3*(-a*(sin(f*x+e)-1))^(1/2)*(a*
(c+d)*d)^(1/2)*a^(3/2)*d^5+19*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*
d)^(1/2))*a^(5/2)*sin(f*x+e)^3*d^6+(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-
a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^5-11*(a*(c+d)*d)^
(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^
4*d-25*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1
/2)/a^(1/2))*a^2*c^3*d^2-13*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(
f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d^3+4*(-a*(sin(f*x+e)-1))^(1/2)*(
a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c^4*d+17*(-a*(sin(f*x+e)-1))^(1/2)*(a*(
c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c^3*d^2-(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)
*d)^(1/2)*a^(3/2)*sin(f*x+e)*c^2*d^3-17*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*
d)^(1/2)*a^(3/2)*sin(f*x+e)*c*d^4+2*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(
1/2)*a^(3/2)*sin(f*x+e)^2*c^3*d^2+2*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(
1/2)*a^(3/2)*sin(f*x+e)^2*c^2*d^3+19*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a
*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*d^6+35*arctanh((-a*(sin(f*x+e)-1))^(1
/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*c^4*d^2+5*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+
d)*d)^(1/2)*a^(1/2)*d^5-2*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/
2)*sin(f*x+e)^2*c*d^4-13*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x
+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*d^5-11*(-a*(sin(f*x+e)-1))^
(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c^2*d^3+6*(-a*(sin(f*x+e)-1))^(3
/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c*d^4-51*(a*(c+d)*d)^(1/2)*2^(1/2)
*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c*
d^4-9*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/
2)/a^(1/2))*sin(f*x+e)*a^2*c^4*d-47*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(
```

```
-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^3*d^2-63*(a*(c+d
)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*s
in(f*x+e)*a^2*c^2*d^3-26*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x
+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c*d^4+(a*(c+d)*d)^(1/2)*2^(1/
2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*
c^3*d^2-11*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*
2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^2*d^3-25*(a*(c+d)*d)^(1/2)*2^(1/2)*arct
anh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c*d^4+2
*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^
(1/2))*sin(f*x+e)^2*a^2*c^4*d-21*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*
(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^3*d^2-61*(a*(c+d)
*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*si
n(f*x+e)^2*a^2*c^2*d^3+5*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2
)*sin(f*x+e)*d^5-11*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^2
*d^3+6*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c*d^4+(a*(c+d)*d
)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*
c^5+35*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f
*x+e)^3*c^2*d^4+42*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a
^(5/2)*sin(f*x+e)^3*c*d^5+70*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d
)^(1/2))*a^(5/2)*sin(f*x+e)^2*c^3*d^3+119*arctanh((-a*(sin(f*x+e)-1))^(1/2)
*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c^2*d^4+80*arctanh((-a*(sin(f*x+
e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c*d^5-2*(-a*(sin(f*x
+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*d^5+35*arctanh((-a*(si
n(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c^4*d^2+112*arct
anh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c^3*d
^3+103*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f
*x+e)*c^2*d^4+38*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(
5/2)*sin(f*x+e)*c*d^5-3*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)
*sin(f*x+e)*d^5+2*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^4*d
+11*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^3*d^2+(-a*(sin(f*
x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^2*d^3-13*(-a*(sin(f*x+e)-1))^(1/
2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c*d^4)/a^(7/2)/(a*(c+d)*d)^(1/2)/(c+d*sin(f*x+
e))^2/(c+d)^2/(c-d)^4/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, algorithm="maxima"
)

[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a + a\sin\left(e + f\,x\right)\right)^{3/2}\left(c + d\sin\left(e + f\,x\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^3),x)
[Out] int(1/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**3,x)
[Out] Timed out
```

3.557 $\int \frac{(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=194

$$-\frac{3\left(c^2+6cd+25d^2\right)(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}\,a^{5/2}f}+\frac{d^2(c-9d)\cos(e+fx)}{4a^2f\sqrt{a\sin(e+fx)+a}}-\frac{(3c+13d)(c-d)^2\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}}$$

```
[Out] -1/16*(c-d)^2*(3*c+13*d)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)-1/4*(c-d)*co
s(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(f*x+e))^(5/2)-3/32*(c-d)*(c^2+6*c*d+
25*d^2)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5
/2)/f*2^(1/2)+1/4*(c-9*d)*d^2*cos(f*x+e)/a^2/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2765, 2968, 3019, 2751, 2649, 206}

$$-\frac{3\left(c^2+6cd+25d^2\right)(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}\,a^{5/2}f}+\frac{d^2(c-9d)\cos(e+fx)}{4a^2f\sqrt{a\sin(e+fx)+a}}-\frac{(3c+13d)(c-d)^2\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-3*(c - d)*(c^2 + 6*c*d + 25*d^2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*
Sqrt[a + a*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*f) - ((c - d)^2*(3*c + 13*d
)*Cos[e + f*x])/(16*a*f*(a + a*Sin[e + f*x])^(3/2)) + ((c - 9*d)*d^2*Cos[e
+ f*x])/(4*a^2*f*Sqrt[a + a*Sin[e + f*x]]) - ((c - d)*Cos[e + f*x]*(c + d*S
in[e + f*x])^2)/(4*f*(a + a*Sin[e + f*x])^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\int \frac{(c+d\sin(e+fx))\left(-\frac{1}{2}a\left(3c^2+9cd-4d^2\right)+\frac{1}{2}a(c-9d)d\,s\cdots}{(a+a\sin(e+fx))^{3/2}}}{4a^2}$$

$$= -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\int \frac{-\frac{1}{2}ac\left(3c^2+9cd-4d^2\right)+\left(\frac{1}{2}ac(c-9d)d-\frac{1}{2}ad\left(3c^2+9cd\cdots\right.\right.}{(a+a\sin(e+f\cdots}}{4a^2}$$

$$= -\frac{(c-d)^2(3c+13d)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a+a\sin(e+fx))^{5/2}} + \frac{\int \frac{\frac{1}{4}a^2(\cdots}{\cdots}}{\cdots}$$

$$= -\frac{(c-d)^2(3c+13d)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + \frac{(c-9d)d^2\cos(e+fx)}{4a^2f\sqrt{a+a\sin(e+fx)}} - \frac{(c-d)\cos(e+fx)(c\cdots}{4f(a+a\sin(\cdots}$$

$$= -\frac{(c-d)^2(3c+13d)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + \frac{(c-9d)d^2\cos(e+fx)}{4a^2f\sqrt{a+a\sin(e+fx)}} - \frac{(c-d)\cos(e+fx)(c\cdots}{4f(a+a\sin(\cdots}$$

$$= -\frac{3(c-d)\left(c^2+6cd+25d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}f} - \frac{(c-d)^2(3c+13d)\cos(e+\cdots}{16af(a+a\sin(e+fx))^{\cdots}}$$

Mathematica [C] time = 0.75, size = 400, normalized size = 2.06

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(11c^3\sin\left(\frac{1}{2}(e+fx)\right)-3c^3\sin\left(\frac{3}{2}(e+fx)\right)-11c^3\cos\left(\frac{1}{2}(e+fx)\right)-3c^3\cos\left(\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^3/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-11*c^3*Cos[(e + f*x)/2] + 9*c^2*d*
Cos[(e + f*x)/2] + 15*c*d^2*Cos[(e + f*x)/2] - 45*d^3*Cos[(e + f*x)/2] - 3*
c^3*Cos[(3*(e + f*x))/2] - 15*c^2*d*Cos[(3*(e + f*x))/2] + 39*c*d^2*Cos[(3*
(e + f*x))/2] - 69*d^3*Cos[(3*(e + f*x))/2] + 16*d^3*Cos[(5*(e + f*x))/2] +
 11*c^3*Sin[(e + f*x)/2] - 9*c^2*d*Sin[(e + f*x)/2] - 15*c*d^2*Sin[(e + f*x
)/2] + 45*d^3*Sin[(e + f*x)/2] + (6 + 6*I)*(-1)^(3/4)*(c^3 + 5*c^2*d + 19*c
*d^2 - 25*d^3)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 3*c^3*Sin[(3*(e + f*x))/2] - 15*c^2*d
*Sin[(3*(e + f*x))/2] + 39*c*d^2*Sin[(3*(e + f*x))/2] - 69*d^3*Sin[(3*(e +
f*x))/2] - 16*d^3*Sin[(5*(e + f*x))/2]))/(32*f*(a*(1 + Sin[e + f*x]))^(5/2)
)
```

fricas [B] time = 0.46, size = 608, normalized size = 3.13

$$-\frac{3\sqrt{2}\left(\left(c^3+5c^2d+19cd^2-25d^3\right)\cos\left(fx+e\right)^3-4c^3-20c^2d-76cd^2+100d^3+3\left(c^3+5c^2d+19cd^2-25\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/64*(3*sqrt(2)*((c^3 + 5*c^2*d + 19*c*d^2 - 25*d^3)*cos(f*x + e)^3 - 4*c^
3 - 20*c^2*d - 76*c*d^2 + 100*d^3 + 3*(c^3 + 5*c^2*d + 19*c*d^2 - 25*d^3)*c
os(f*x + e)^2 - 2*(c^3 + 5*c^2*d + 19*c*d^2 - 25*d^3)*cos(f*x + e) - (4*c^3
 + 20*c^2*d + 76*c*d^2 - 100*d^3 - (c^3 + 5*c^2*d + 19*c*d^2 - 25*d^3)*cos(
f*x + e)^2 + 2*(c^3 + 5*c^2*d + 19*c*d^2 - 25*d^3)*cos(f*x + e))*sin(f*x +
e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqr
t(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e)
 - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x +
e) - cos(f*x + e) - 2)) + 4*(32*d^3*cos(f*x + e)^3 - 4*c^3 + 12*c^2*d - 12*
c*d^2 + 4*d^3 - (3*c^3 + 15*c^2*d - 39*c*d^2 + 53*d^3)*cos(f*x + e)^2 - (7*
c^3 + 3*c^2*d - 27*c*d^2 + 81*d^3)*cos(f*x + e) - (32*d^3*cos(f*x + e)^2 -
4*c^3 + 12*c^2*d - 12*c*d^2 + 4*d^3 + (3*c^3 + 15*c^2*d - 39*c*d^2 + 85*d^3
)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a^3*f*cos(f*x + e)
^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f
*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)2/f*(2*sqrt(a*tan((f*x+exp(1))/2)^2+a)*(-1/2*d^3/a^2/s
ign(tan((f*x+exp(1))/2)+1)+1/2*d^3*tan((f*x+exp(1))/2)/a^2/sign(tan((f*x+ex
p(1))/2)+1))/(a*tan((f*x+exp(1))/2)^2+a)+2*(1/32*(-29*c^3*(-sqrt(a)*tan((f*
x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^7-43*d^3*(-sqrt(a)*tan((f*x+e
xp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^7+57*c*d^2*(-sqrt(a)*tan((f*x+ex
p(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^7+15*c^2*d*(-sqrt(a)*tan((f*x+exp
(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^7+75*sqrt(a)*c^3*(-sqrt(a)*tan((f*
x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^6+237*sqrt(a)*d^3*(-sqrt(a)*t
```

```
an((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^6-399*sqrt(a)*c*d^2*(-s
qrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^6+87*sqrt(a)*c^
2*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^6-55*a*c
^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^5-161*a*d
^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^5+267*a*c
*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^5-51*a*
c^2*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^5-91*s
qrt(a)*a*c^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))
^4-221*sqrt(a)*a*d^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/
2)^2+a))^4+351*sqrt(a)*a*c*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*
x+exp(1))/2)^2+a))^4-39*sqrt(a)*a*c^2*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(
a*tan((f*x+exp(1))/2)^2+a))^4+a^2*c^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*
tan((f*x+exp(1))/2)^2+a))^3-25*a^2*d^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a
*tan((f*x+exp(1))/2)^2+a))^3+51*a^2*c*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqr
t(a*tan((f*x+exp(1))/2)^2+a))^3-27*a^2*c^2*d*(-sqrt(a)*tan((f*x+exp(1))/2)+
sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+27*a^3*c^3*(-sqrt(a)*tan((f*x+exp(1))/2)
+sqrt(a*tan((f*x+exp(1))/2)^2+a))+93*a^3*d^3*(-sqrt(a)*tan((f*x+exp(1))/2)+
sqrt(a*tan((f*x+exp(1))/2)^2+a))+65*sqrt(a)*a^2*c^3*(-sqrt(a)*tan((f*x+exp(
1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+103*sqrt(a)*a^2*d^3*(-sqrt(a)*tan
((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-159*a^3*c*d^2*(-sqrt(a)
*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+39*a^3*c^2*d*(-sqrt(a
)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))-141*sqrt(a)*a^2*c*d^
2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-27*sqrt(
a)*a^2*c^2*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))
^2+7*sqrt(a)*a^3*c^3+17*sqrt(a)*a^3*d^3-27*sqrt(a)*a^3*c*d^2+3*sqrt(a)*a^3*
c^2*d)/a^2/(-(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))
^2+2*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))
+a)^4/sign(tan((f*x+exp(1))/2)+1)+1/32*(3*c^3-75*d^3+57*c*d^2+15*c^2*d)*ata
n((-sqrt(a)*tan((f*x+exp(1))/2)-sqrt(a)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sq
rt(2)/sqrt(-a))/sqrt(2)/a^2/sqrt(-a)/sign(tan((f*x+exp(1))/2)+1)))
```

maple [B] time = 1.32, size = 688, normalized size = 3.55

$$-\frac{\left(2\sin\left(fx+e\right)\left(64d^{3}a^{\frac{3}{2}}\sqrt{a-a\sin\left(fx+e\right)}+3\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{a}}\right)a^{2}c^{3}+15\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(}}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/32*(2*sin(f*x+e)*(64*d^3*a^(3/2)*(a-a*sin(f*x+e))^(1/2)+3*2^(1/2)*arctan
h(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^3+15*2^(1/2)*arctanh(1/
2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d+57*2^(1/2)*arctanh(1/2*
```

```
(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d^2-75*2^(1/2)*arctanh(1/2*(a
-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^3)+(-3*2^(1/2)*arctanh(1/2*(a-a
*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^3-15*2^(1/2)*arctanh(1/2*(a-a*sin
(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d-57*2^(1/2)*arctanh(1/2*(a-a*sin(f
*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d^2+75*2^(1/2)*arctanh(1/2*(a-a*sin(f*x
+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^3-64*d^3*a^(3/2)*(a-a*sin(f*x+e))^(1/2))*
cos(f*x+e)^2+6*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*
a^2*c^3+30*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*
c^2*d+114*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c
*d^2-150*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^
3-6*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^3-30*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^
2*d+78*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c*d^2-42*(a-a*sin(f*x+e))^(3/2)*d^3*a
^(1/2)+20*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^3+36*(a-a*sin(f*x+e))^(1/2)*a^(3
/2)*c^2*d-132*c*d^2*a^(3/2)*(a-a*sin(f*x+e))^(1/2)+204*d^3*a^(3/2)*(a-a*sin
(f*x+e))^(1/2))*(-a*(sin(f*x+e)-1))^(1/2)/a^(9/2)/(1+sin(f*x+e))/cos(f*x+e)
/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin \left(f x + e\right) + c\right)^{3}}{\left(a \sin \left(f x + e\right) + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^3/(a*sin(f*x + e) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^3/(a + a*sin(e + f*x))^(5/2),x)
[Out] int((c + d*sin(e + f*x))^3/(a + a*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.558 $\int \frac{(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=147

$$-\frac{\left(3c^2+10cd+19d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{4f(a\sin(e+fx)+a)^{5/2}}-\frac{3(c-d)(c+3d)\cos(e}{16af(a\sin(e+fx)+}$$

```
[Out] -3/16*(c-d)*(c+3*d)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)-1/4*(c-d)*cos(f*x
+e)*(c+d*sin(f*x+e))/f/(a+a*sin(f*x+e))^(5/2)-1/32*(3*c^2+10*c*d+19*d^2)*ar
ctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/f*2^(1
/2)
```

Rubi [A] time = 0.23, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2760, 2750, 2649, 206}

$$-\frac{\left(3c^2+10cd+19d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f}-\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{4f(a\sin(e+fx)+a)^{5/2}}-\frac{3(c-d)(c+3d)\cos(e}{16af(a\sin(e+fx)+}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -((3*c^2 + 10*c*d + 19*d^2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a
+ a*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*f) - (3*(c - d)*(c + 3*d)*Cos[e +
f*x])/(16*a*f*(a + a*Sin[e + f*x])^(3/2)) - ((c - d)*Cos[e + f*x]*(c + d*Si
n[e + f*x]))/(4*f*(a + a*Sin[e + f*x])^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2760

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^m*(c + d*Sin[e + f*x]))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*(2*m + 1)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*c*d*(m - 1) + b*(d^2 + c^2*(m + 1))
 + d*(a*d*(m - 1) + b*c*(m + 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\int \frac{-\frac{1}{2}a\left(3c^2+7cd-2d^2\right)-\frac{1}{2}ad(c+7d)\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{3(c-d)(c+3d)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{4f(a+a\sin(e+fx))^{5/2}} + \frac{\left(3c^2+10\right.}{}$$

$$= -\frac{3(c-d)(c+3d)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\left(3c^2+10\right.}{}$$

$$= -\frac{\left(3c^2+10cd+19d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}f} - \frac{3(c-d)(c+3d)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(}{}$$

Mathematica [C] time = 0.54, size = 252, normalized size = 1.71

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2\left(3c^2+10cd-13d^2\right)\sin\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^2/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(c - d)^2*Sin[(e + f*x)/2] - 4*(c
 - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(3*c^2 + 10*c*d - 13*d^2)
```

```
*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (c - d)*(3*c +
13*d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + (1 + I)*(-1)^(3/4)*(3*c^2 +
 10*c*d + 19*d^2)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4))/(16*f*(a*(1 + Sin[e + f*x]))^(5/2)
)
```

fricas [B] time = 0.47, size = 492, normalized size = 3.35

$$\sqrt{2}\left(\left(3\,c^2+10\,cd+19\,d^2\right)\cos\left(fx+e\right)^3+3\left(3\,c^2+10\,cd+19\,d^2\right)\cos\left(fx+e\right)^2-12\,c^2-40\,cd-76\,d^2-2\left(3\,c^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(sqrt(2)*((3*c^2 + 10*c*d + 19*d^2)*cos(f*x + e)^3 + 3*(3*c^2 + 10*c*d
 + 19*d^2)*cos(f*x + e)^2 - 12*c^2 - 40*c*d - 76*d^2 - 2*(3*c^2 + 10*c*d +
19*d^2)*cos(f*x + e) + ((3*c^2 + 10*c*d + 19*d^2)*cos(f*x + e)^2 - 12*c^2 -
 40*c*d - 76*d^2 - 2*(3*c^2 + 10*c*d + 19*d^2)*cos(f*x + e))*sin(f*x + e))*
sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)
*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2
*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) -
 cos(f*x + e) - 2)) + 4*((3*c^2 + 10*c*d - 13*d^2)*cos(f*x + e)^2 + 4*c^2 -
 8*c*d + 4*d^2 + (7*c^2 + 2*c*d - 9*d^2)*cos(f*x + e) - (4*c^2 - 8*c*d + 4*
d^2 - (3*c^2 + 10*c*d - 13*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x
+ e) + a))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x
 + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*s
in(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*2*(1/32*(-29*c^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+
exp(1))/2)^2+a))^7+19*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp
(1))/2)^2+a))^7+10*c*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1)
)/2)^2+a))^7+75*sqrt(a)*c^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+e
xp(1))/2)^2+a))^6-133*sqrt(a)*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan(
```

```
(f*x+exp(1))/2)^2+a))^6+58*sqrt(a)*c*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a
*tan((f*x+exp(1))/2)^2+a))^6-55*a*c^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*
tan((f*x+exp(1))/2)^2+a))^5+89*a*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*t
an((f*x+exp(1))/2)^2+a))^5-34*a*c*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*ta
n((f*x+exp(1))/2)^2+a))^5-91*sqrt(a)*a*c^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sq
rt(a*tan((f*x+exp(1))/2)^2+a))^4+117*sqrt(a)*a*d^2*(-sqrt(a)*tan((f*x+exp(1
))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^4-26*sqrt(a)*a*c*d*(-sqrt(a)*tan((f*
x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^4+a^2*c^2*(-sqrt(a)*tan((f*x+
exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+17*a^2*d^2*(-sqrt(a)*tan((f*x
+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3-18*a^2*c*d*(-sqrt(a)*tan((f*
x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+27*a^3*c^2*(-sqrt(a)*tan((f
*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))-53*a^3*d^2*(-sqrt(a)*tan((f*
x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+65*sqrt(a)*a^2*c^2*(-sqrt(a)*
tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-47*sqrt(a)*a^2*d^2*(
-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+26*a^3*c*d*
(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))-18*sqrt(a)*a
^2*c*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+7*s
qrt(a)*a^3*c^2-9*sqrt(a)*a^3*d^2+2*sqrt(a)*a^3*c*d)/a^2/(-(-sqrt(a)*tan((f*
x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+2*sqrt(a)*(-sqrt(a)*tan((f*
x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+a)^4/sign(tan((f*x+exp(1))/2)
+1)+1/32*(3*c^2+19*d^2+10*c*d)*atan((-sqrt(a)*tan((f*x+exp(1))/2)-sqrt(a)+s
qrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/sqrt(2)/a^2/sqrt(-a)/sign
(tan((f*x+exp(1))/2)+1))
```

maple [B] time = 1.43, size = 378, normalized size = 2.57

$$\frac{\left(-2\sin\left(fx+e\right)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}\ a^{2}\left(3c^{2}+10cd+19d^{2}\right)+\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}\ a^{2}\left(3c^{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/32/a^(9/2)*(-2*sin(f*x+e)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1
/2))*2^(1/2)*a^2*(3*c^2+10*c*d+19*d^2)+arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2
^(1/2)/a^(1/2))*2^(1/2)*a^2*(3*c^2+10*c*d+19*d^2)*cos(f*x+e)^2-6*2^(1/2)*ar
ctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2-20*2^(1/2)*arctan
h(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d-38*2^(1/2)*arctanh(1/
2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2+6*(a-a*sin(f*x+e))^(3/2)*
a^(1/2)*c^2+20*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c*d-26*(a-a*sin(f*x+e))^(3/2)
*a^(1/2)*d^2-20*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^2-24*(a-a*sin(f*x+e))^(1/2
)*a^(3/2)*c*d+44*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d^2)*(-a*(sin(f*x+e)-1))^(1
/2)/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^2/(a*sin(f*x + e) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c+d\,\sin\left(e+f\,x\right)\right)^2}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^2/(a + a*sin(e + f*x))^(5/2),x)
[Out] int((c + d*sin(e + f*x))^2/(a + a*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(5/2),x)
[Out] Timed out
```

3.559 $\int \frac{c+d\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=126

$$-\frac{(3c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}\,a^{5/2}f}-\frac{(3c+5d)\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}}-\frac{(c-d)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

```
[Out] -1/4*(c-d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)-1/16*(3*c+5*d)*cos(f*x+e)/a/
f/(a+a*sin(f*x+e))^(3/2)-1/32*(3*c+5*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1
/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/f*2^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2750, 2650, 2649, 206}

$$-\frac{(3c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}\,a^{5/2}f}-\frac{(3c+5d)\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}}-\frac{(c-d)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -((3*c + 5*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*
x]])])/(16*Sqrt[2]*a^(5/2)*f) - ((c - d)*Cos[e + f*x])/(4*f*(a + a*Sin[e +
f*x])^(5/2)) - ((3*c + 5*d)*Cos[e + f*x])/(16*a*f*(a + a*Sin[e + f*x])^(3/2
))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
```

```
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{c+d\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(c-d)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} + \frac{(3c+5d)\int \frac{1}{(a+a\sin(e+fx))^{3/2}}\,dx}{8a}$$

$$= -\frac{(c-d)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3c+5d)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + \frac{(3c+5d)\int \frac{1}{\sqrt{a+a\sin(e+fx}}}{32a^2}$$

$$= -\frac{(c-d)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3c+5d)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(3c+5d)\,\text{Subst}\left(\int \frac{1}{2a-x}\right.}{16a^2}$$

$$= -\frac{(3c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}f} - \frac{(c-d)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3c+5d)\cos}{16af(a+a\sin}$$

Mathematica [C] time = 0.35, size = 227, normalized size = 1.80

$$\frac{\left(\sin\left(\tfrac{1}{2}(e+fx)\right)+\cos\left(\tfrac{1}{2}(e+fx)\right)\right)\left(8(c-d)\sin\left(\tfrac{1}{2}(e+fx)\right)-(3c+5d)\left(\sin\left(\tfrac{1}{2}(e+fx)\right)+\cos\left(\tfrac{1}{2}(e+fx)\right)\right)^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(c - d)*Sin[(e + f*x)/2] + 4*(-c
+ d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(3*c + 5*d)*Sin[(e + f*x)/2]
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (3*c + 5*d)*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^3 + (1 + I)*(-1)^(3/4)*(3*c + 5*d)*ArcTanh[(1/2 + I/2)*(-
1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4))
/(16*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

fricas [B] time = 0.47, size = 392, normalized size = 3.11

$$\frac{\sqrt{2}\left((3\,c+5\,d)\cos\left(fx+e\right)^3+3\,(3\,c+5\,d)\cos\left(fx+e\right)^2-2\,(3\,c+5\,d)\cos\left(fx+e\right)+\left((3\,c+5\,d)\cos\left(fx+e\right)^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(sqrt(2)*((3*c + 5*d)*cos(f*x + e)^3 + 3*(3*c + 5*d)*cos(f*x + e)^2 -
2*(3*c + 5*d)*cos(f*x + e) + ((3*c + 5*d)*cos(f*x + e)^2 - 2*(3*c + 5*d)*co
s(f*x + e) - 12*c - 20*d)*sin(f*x + e) - 12*c - 20*d)*sqrt(a)*log(-(a*cos(f
*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(
f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*
a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) +
 4*((3*c + 5*d)*cos(f*x + e)^2 + (7*c + d)*cos(f*x + e) + ((3*c + 5*d)*cos(
f*x + e) - 4*c + 4*d)*sin(f*x + e) + 4*c - 4*d)*sqrt(a*sin(f*x + e) + a))/(
a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^
3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*2*(1/32*(-29*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+ex
p(1))/2)^2+a))^7+5*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/
2)^2+a))^7+75*sqrt(a)*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1
))/2)^2+a))^6+29*sqrt(a)*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+ex
p(1))/2)^2+a))^6-55*a*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1
))/2)^2+a))^5-17*a*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/
2)^2+a))^5-91*sqrt(a)*a*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp
(1))/2)^2+a))^4-13*sqrt(a)*a*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*
x+exp(1))/2)^2+a))^4+a^2*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+ex
p(1))/2)^2+a))^3-9*a^2*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(
1))/2)^2+a))^3+27*a^3*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1
))/2)^2+a))+13*a^3*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/
2)^2+a))+65*sqrt(a)*a^2*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp
(1))/2)^2+a))^2-9*sqrt(a)*a^2*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f
```

```
*x+exp(1))/2)^2+a))^2+7*sqrt(a)*a^3*c+sqrt(a)*a^3*d)/a^2/(-(-sqrt(a)*tan((f
*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+2*sqrt(a)*(-sqrt(a)*tan((f
*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+a)^4/sign(tan((f*x+exp(1))/2
)+1)+1/32*(3*c+5*d)*atan((-sqrt(a)*tan((f*x+exp(1))/2)-sqrt(a)+sqrt(a*tan((
f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/sqrt(2)/a^2/sqrt(-a)/sign(tan((f*x+e
xp(1))/2)+1))
```

maple [B] time = 1.18, size = 279, normalized size = 2.21

$$-\frac{\left(2\sin\left(fx+e\right)\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{a}}\right)a^{3}\left(3c+5d\right)-\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{a}}\right)a^{3}\left(3c+5d\right)\left(\cos\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/32*(2*sin(f*x+e)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1
/2))*a^3*(3*c+5*d)-2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/
2))*a^3*(3*c+5*d)*cos(f*x+e)^2+20*(a-a*sin(f*x+e))^(1/2)*a^(5/2)*c+12*(a-a*
sin(f*x+e))^(1/2)*a^(5/2)*d-6*(a-a*sin(f*x+e))^(3/2)*a^(3/2)*c-10*(a-a*sin(
f*x+e))^(3/2)*a^(3/2)*d+6*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2
)/a^(1/2))*a^3*c+10*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1
/2))*a^3*d)*(-a*(sin(f*x+e)-1))^(1/2)/a^(11/2)/(1+sin(f*x+e))/cos(f*x+e)/(a
+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d \sin\left(fx+e\right)+c}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c+d\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))/(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int((c + d*sin(e + f*x))/(a + a*sin(e + f*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + d \sin\left(e + fx\right)}{\left(a \left(\sin\left(e + fx\right) + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((c + d*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(5/2), x)
```

3.560 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=107

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f}-\frac{3\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}}-\frac{\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

```
[Out] -1/4*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)-3/16*cos(f*x+e)/a/f/(a+a*sin(f*x+e
))^(3/2)-3/32*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2)
)/a^(5/2)/f*2^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2650, 2649, 206}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f}-\frac{3\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}}-\frac{\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(-5/2),x]
```

```
[Out] (-3*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(16
*Sqrt[2]*a^(5/2)*f) - Cos[e + f*x]/(4*f*(a + a*Sin[e + f*x])^(5/2)) - (3*Co
s[e + f*x])/(16*a*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
```

```
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} + \frac{3\int \frac{1}{(a+a\sin(e+fx))^{3/2}}\,dx}{8a}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{3\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{32a^2}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{3\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} - \frac{3\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \ldots\right.}{16a^2f}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}f} - \frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{3\cos(e+fx)}{16af(a+a\sin(e+fx))^{\ldots}}$$

Mathematica [C] time = 0.15, size = 196, normalized size = 1.83

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(8\sin\left(\frac{1}{2}(e+fx)\right)-3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3+6\sin\left(\frac{1}{2}(e+f\ldots\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(-5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*Sin[(e + f*x)/2] - 4*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2]) + 6*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^2 - 3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + (3 + 3*I)*(-1)^(3
/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^4))/(16*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

fricas [B] time = 0.49, size = 320, normalized size = 2.99

$$\frac{3\sqrt{2}\left(\cos\left(fx+e\right)^3+3\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)^2-2\cos\left(fx+e\right)-4\right)\sin\left(fx+e\right)-2\cos\left(fx+e\right)-4\right)\ldots}{64\left(a^3f\cos\left(fx+e\right)^3+3\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(3*sqrt(2)*(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*co
s(f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x + e) - 4)*sqrt(a)*log(-(a*cos(f*x
+ e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x
 + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/
(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*
(3*cos(f*x + e)^2 + (3*cos(f*x + e) - 4)*sin(f*x + e) + 7*cos(f*x + e) + 4)
*sqrt(a*sin(f*x + e) + a))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 -
 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x +
 e) - 4*a^3*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*2*(1/32*(-29*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(
1))/2)^2+a))^7+75*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp
(1))/2)^2+a))^6-55*a*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/
2)^2+a))^5-91*sqrt(a)*a*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1
))/2)^2+a))^4+a^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^
2+a))^3+27*a^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a
))+65*sqrt(a)*a^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^
2+a))^2+7*sqrt(a)*a^3)/a^2/(-(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+
exp(1))/2)^2+a))^2+2*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+
exp(1))/2)^2+a))+a)^4/sign(tan((f*x+exp(1))/2)+1)+3/32*atan((-sqrt(a)*tan((
f*x+exp(1))/2)-sqrt(a)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/s
qrt(2)/a^2/sqrt(-a)/sign(tan((f*x+exp(1))/2)+1))
```

maple [B] time = 0.91, size = 195, normalized size = 1.82

$$-\frac{\left(\sin\left(fx+e\right)\left(6\sqrt{a-a\sin\left(fx+e\right)}\,a^{\frac{3}{2}}+6\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,\sqrt{2}}{2\sqrt{a}}\right)a^{2}\right)-3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}}{2\sqrt{a}}\right.\right.}{32a^{\frac{9}{2}}\left(1+\sin\left(fx+e\right)\right)\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/32/a^(9/2)*(sin(f*x+e)*(6*(a-a*sin(f*x+e))^(1/2)*a^(3/2)+6*2^(1/2)*arcta
nh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2)-3*2^(1/2)*arctanh(1/2*(
a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*cos(f*x+e)^2+14*(a-a*sin(f*x+e))
^(1/2)*a^(3/2)+6*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2)
)*a^2)*(-a*(sin(f*x+e)-1))^(1/2)/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))
^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int(1/(a + a*sin(e + f*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(e+fx\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((a*sin(e + f*x) + a)**(-5/2), x)
```

3.561 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=218

$$-\frac{\left(3c^2-14cd+43d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f(c-d)^3}+\frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{5/2}f(c-d)^3\sqrt{c+d}}-\frac{(3c-11d)\cos(e+fx}{16af(c-d)^2(a\sin(e+fx)}$$

```
[Out] -1/4*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(5/2)-1/16*(3*c-11*d)*cos(f*x+e)/a
/(c-d)^2/f/(a+a*sin(f*x+e))^(3/2)-1/32*(3*c^2-14*c*d+43*d^2)*arctanh(1/2*co
s(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/(c-d)^3/f*2^(1/2)+
2*d^(5/2)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(
1/2))/a^(5/2)/(c-d)^3/f/(c+d)^(1/2)
```

Rubi [A] time = 0.74, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2766, 2978, 2985, 2649, 206, 2773, 208}

$$-\frac{\left(3c^2-14cd+43d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f(c-d)^3}+\frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{5/2}f(c-d)^3\sqrt{c+d}}-\frac{(3c-11d)\cos(e+fx}{16af(c-d)^2(a\sin(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])),x]
```

```
[Out] -((3*c^2 - 14*c*d + 43*d^2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a
+ a*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*(c - d)^3*f) + (2*d^(5/2)*ArcTanh[
(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(a^
(5/2)*(c - d)^3*Sqrt[c + d]*f) - Cos[e + f*x]/(4*(c - d)*f*(a + a*Sin[e + f
*x])^(5/2)) - ((3*c - 11*d)*Cos[e + f*x])/(16*a*(c - d)^2*f*(a + a*Sin[e +
f*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx = -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{\int \frac{-\frac{1}{2}a(3c-8d)-\frac{3}{2}ad\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx)}}{4a^2(c-d)}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{(3c-11d)\cos(e+fx}{16a(c-d)^2f(a+a\sin(e+}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{(3c-11d)\cos(e+fx}{16a(c-d)^2f(a+a\sin(e+}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{(3c-11d)\cos(e+fx}{16a(c-d)^2f(a+a\sin(e+}$$

$$= -\frac{(3c^2-14cd+43d^2)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}(c-d)^3f} + \frac{2d^{5/2}\tanh^{-}}{a^{5/2}(c}$$

Mathematica [C] time = 3.29, size = 501, normalized size = 2.30

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\frac{(1+i)(-1)^{3/4}(3c^2-14cd+43d^2)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(\tan\left(\frac{1}{4}(e+fx\right.\right.\right.}{(c-d)^3}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((8*Sin[(e + f*x)/2])/(c - d) - (4*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(c - d) + (2*(3*c - 11*d)*Sin[(e + f*
x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(c - d)^2 + ((-3*c + 11*d)*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/(c - d)^2 + ((1 + I)*(-1)^(3/4)*(3*
c^2 - 14*c*d + 43*d^2)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4
])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)/(c - d)^3 + (8*d^(5/2)*(e + f*
x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqr
t[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^4)/((c - d)^3*Sqrt[c + d]) + (8*d^(5/2)*(e + f*x - 2*Log[Sec
[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e +
 f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^4)/((-c + d)^3*Sqrt[c + d]))/(16*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

fricas [B] time = 1.06, size = 2015, normalized size = 9.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/64*(sqrt(2)*((3*c^2 - 14*c*d + 43*d^2)*cos(f*x + e)^3 + 3*(3*c^2 - 14*c
*d + 43*d^2)*cos(f*x + e)^2 - 12*c^2 + 56*c*d - 172*d^2 - 2*(3*c^2 - 14*c*d
 + 43*d^2)*cos(f*x + e) + ((3*c^2 - 14*c*d + 43*d^2)*cos(f*x + e)^2 - 12*c^
2 + 56*c*d - 172*d^2 - 2*(3*c^2 - 14*c*d + 43*d^2)*cos(f*x + e))*sin(f*x +
e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqr
t(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e)
 - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x +
e) - cos(f*x + e) - 2)) + 32*(a*d^2*cos(f*x + e)^3 + 3*a*d^2*cos(f*x + e)^2
 - 2*a*d^2*cos(f*x + e) - 4*a*d^2 + (a*d^2*cos(f*x + e)^2 - 2*a*d^2*cos(f*x
 + e) - 4*a*d^2)*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3
- (6*c*d + 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - 4*((c*d + d^2)*cos(f
*x + e)^2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2
 + 4*c*d + 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x +
 e) + a)*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*co
s(f*x + e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x
+ e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^
2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c
^2 - 2*c*d - d^2)*sin(f*x + e))) - 4*((3*c^2 - 14*c*d + 11*d^2)*cos(f*x + e
)^2 + 4*c^2 - 8*c*d + 4*d^2 + (7*c^2 - 22*c*d + 15*d^2)*cos(f*x + e) - (4*c
^2 - 8*c*d + 4*d^2 - (3*c^2 - 14*c*d + 11*d^2)*cos(f*x + e))*sin(f*x + e))*
sqrt(a*sin(f*x + e) + a))/((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*
f*cos(f*x + e)^3 + 3*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(
f*x + e)^2 - 2*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x +
e) - 4*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f + ((a^3*c^3 - 3*a^
3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^2 - 2*(a^3*c^3 - 3*a^3*c^2*
d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e) - 4*(a^3*c^3 - 3*a^3*c^2*d + 3*a^
3*c*d^2 - a^3*d^3)*f)*sin(f*x + e)), -1/64*(sqrt(2)*((3*c^2 - 14*c*d + 43*d
^2)*cos(f*x + e)^3 + 3*(3*c^2 - 14*c*d + 43*d^2)*cos(f*x + e)^2 - 12*c^2 +
56*c*d - 172*d^2 - 2*(3*c^2 - 14*c*d + 43*d^2)*cos(f*x + e) + ((3*c^2 - 14*
c*d + 43*d^2)*cos(f*x + e)^2 - 12*c^2 + 56*c*d - 172*d^2 - 2*(3*c^2 - 14*c*
d + 43*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*
sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1)
+ 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x +
e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 64*(a*d^2*cos
(f*x + e)^3 + 3*a*d^2*cos(f*x + e)^2 - 2*a*d^2*cos(f*x + e) - 4*a*d^2 + (a*
d^2*cos(f*x + e)^2 - 2*a*d^2*cos(f*x + e) - 4*a*d^2)*sin(f*x + e))*sqrt(-d/
(a*c + a*d))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)
```

```
*sqrt(-d/(a*c + a*d))/(d*cos(f*x + e))) - 4*((3*c^2 - 14*c*d + 11*d^2)*cos(
f*x + e)^2 + 4*c^2 - 8*c*d + 4*d^2 + (7*c^2 - 22*c*d + 15*d^2)*cos(f*x + e)
 - (4*c^2 - 8*c*d + 4*d^2 - (3*c^2 - 14*c*d + 11*d^2)*cos(f*x + e))*sin(f*x
 + e))*sqrt(a*sin(f*x + e) + a))/((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^
3*d^3)*f*cos(f*x + e)^3 + 3*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)
*f*cos(f*x + e)^2 - 2*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos
(f*x + e) - 4*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f + ((a^3*c^3
 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^2 - 2*(a^3*c^3 - 3*a
^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e) - 4*(a^3*c^3 - 3*a^3*c^2*d
 + 3*a^3*c*d^2 - a^3*d^3)*f)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Warning, integration of abs or sign assumes constant sign by inter
vals (correct if the argument is real):Check [abs(cos((f*t_nostep+exp(1))/2
-pi/4))]Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Discontinuities at zeroes
of cos((f*t_nostep+exp(1))/2-pi/4) were not checkedUnable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
```

)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
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k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
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)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
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/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x
/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign by

```
intervals (correct if the argument is real):Check [abs(t_nostep+1)]Evaluat
ion time: 1.56sym2poly/r2sym(const gen & e,const index_m & i,const vecteur
& l) Error: Bad Argument Value
```

maple [B] time = 1.67, size = 732, normalized size = 3.36

$$\left(\sin\left(fx+e\right)\left(128d^{3}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,d}{\sqrt{acd+a\,d^{2}}}\right)a^{\frac{5}{2}}-6\sqrt{a\left(c+d\right)d}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,\sqrt{2}}{2\sqrt{a}}\right)a^{2}c^{2}+28\sqrt{a\,(}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] 1/32/a^(9/2)*(sin(f*x+e)*(128*d^3*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a
*d^2)^(1/2))*a^(5/2)-6*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e
))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2+28*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*
(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d-86*(a*(c+d)*d)^(1/2)*2^(1/2
)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2)+(-64*d^3*arc
tanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)+3*(a*(c+d)*d)^(1
/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2-14*
(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2
))*a^2*c*d+43*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*
2^(1/2)/a^(1/2))*a^2*d^2)*cos(f*x+e)^2+128*d^3*arctanh((a-a*sin(f*x+e))^(1/
2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)-6*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(
a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2+28*(a*(c+d)*d)^(1/2)*2^(1/2)
*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d-86*(a*(c+d)*d)
^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2+
6*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^2-28*(a*(c+d)*d)^(1/2)
*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c*d+22*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(
3/2)*a^(1/2)*d^2-20*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^2+72
*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c*d-52*(a*(c+d)*d)^(1/2)*
(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d^2)*(-a*(sin(f*x+e)-1))^(1/2)/(1+sin(f*x+e)
)/(a*(c+d)*d)^(1/2)/(c-d)^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))),x)
[Out] int(1/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e)),x)
[Out] Timed out
```

3.562 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=313

$$-\frac{\left(3c^2-22cd+115d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f(c-d)^4}+\frac{d^{5/2}(7c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{5/2}f(c-d)^4(c+d)^{3/2}}-\frac{d(c}{16a^2f(c-d)^3(c+}$$

```
[Out] d^(5/2)*(7*c+5*d)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f
*x+e))^(1/2))/a^(5/2)/(c-d)^4/(c+d)^(3/2)/f-1/4*cos(f*x+e)/(c-d)/f/(a+a*sin
(f*x+e))^(5/2)/(c+d*sin(f*x+e))-3/16*(c-5*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*si
n(f*x+e))^(3/2)/(c+d*sin(f*x+e))-1/32*(3*c^2-22*c*d+115*d^2)*arctanh(1/2*co
s(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/(c-d)^4/f*2^(1/2)-
1/16*(c-7*d)*d*(3*c+5*d)*cos(f*x+e)/a^2/(c-d)^3/(c+d)/f/(c+d*sin(f*x+e))/(a
+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.10, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2766, 2978, 2984, 2985, 2649, 206, 2773, 208}

$$-\frac{\left(3c^2-22cd+115d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f(c-d)^4}+\frac{d^{5/2}(7c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{5/2}f(c-d)^4(c+d)^{3/2}}-\frac{d(c}{16a^2f(c-d)^3(c+}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] -((3*c^2 - 22*c*d + 115*d^2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a
 + a*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*(c - d)^4*f) + (d^(5/2)*(7*c + 5*
d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f
*x]])])/(a^(5/2)*(c - d)^4*(c + d)^(3/2)*f) - Cos[e + f*x]/(4*(c - d)*f*(a
+ a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])) - (3*(c - 5*d)*Cos[e + f*x])/
(16*a*(c - d)^2*f*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])) - ((c -
7*d)*d*(3*c + 5*d)*Cos[e + f*x])/(16*a^2*(c - d)^3*(c + d)*f*Sqrt[a + a*Sin
[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
```

```
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx = -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} - \frac{\int \frac{-\frac{1}{2}a}{(a+a\sin}}{}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} - \frac{}{16a(c-}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} - \frac{}{16a(c-}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} - \frac{}{16a(c-}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} - \frac{}{16a(c-}$$

$$= -\frac{\left(3c^2-22cd+115d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}(c-d)^4f} + \frac{d^{5/2}(7c+}{}$$

Mathematica [C] time = 5.84, size = 570, normalized size = 1.82

$$\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left((1+i)(-1)^{3/4}\left(3c^2-22cd+115d^2\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4\tan$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(c - d)^2*Sin[(e + f*x)/2] - 4*(c
 - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(3*c - 19*d)*(c - d)*Sin[
(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (c - d)*(-3*c + 19*d
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + (1 + I)*(-1)^(3/4)*(3*c^2 - 22*
c*d + 115*d^2)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (4*d^(5/2)*(7*c + 5*d)*(e + f*x - 2*L
og[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Co
s[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^4)/(c + d)^(3/2) - (4*d^(5/2)*(7*c + 5*d)*(e + f*x - 2*Log[Sec[(e +
 f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e + f*x)
/2] + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)/
(c + d)^(3/2) + (16*(c - d)*d^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])^4)/((c + d)*(c + d*Sin[e + f*x]))))/(16*(c
 - d)^4*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

fricas [B] time = 1.95, size = 3719, normalized size = 11.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] [1/64*(sqrt(2)*((3*c^3*d - 19*c^2*d^2 + 93*c*d^3 + 115*d^4)*cos(f*x + e)^4
+ 12*c^4 - 64*c^3*d + 296*c^2*d^2 + 832*c*d^3 + 460*d^4 - (3*c^4 - 13*c^3*d
 + 55*c^2*d^2 + 301*c*d^3 + 230*d^4)*cos(f*x + e)^3 - (9*c^4 - 42*c^3*d + 1
84*c^2*d^2 + 810*c*d^3 + 575*d^4)*cos(f*x + e)^2 + 2*(3*c^4 - 16*c^3*d + 74
*c^2*d^2 + 208*c*d^3 + 115*d^4)*cos(f*x + e) + (12*c^4 - 64*c^3*d + 296*c^2
*d^2 + 832*c*d^3 + 460*d^4 - (3*c^3*d - 19*c^2*d^2 + 93*c*d^3 + 115*d^4)*co
s(f*x + e)^3 - (3*c^4 - 10*c^3*d + 36*c^2*d^2 + 394*c*d^3 + 345*d^4)*cos(f*
x + e)^2 + 2*(3*c^4 - 16*c^3*d + 74*c^2*d^2 + 208*c*d^3 + 115*d^4)*cos(f*x
+ e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f
*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) -
 (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e)
```

+ 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 16*(28*a*c^2*d^2 + 48*a*c*d^3 + 2
0*a*d^4 + (7*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^4 - (7*a*c^2*d^2 + 19*a*c*d^3
+ 10*a*d^4)*cos(f*x + e)^3 - (21*a*c^2*d^2 + 50*a*c*d^3 + 25*a*d^4)*cos(f*x
+ e)^2 + 2*(7*a*c^2*d^2 + 12*a*c*d^3 + 5*a*d^4)*cos(f*x + e) + (28*a*c^2*d
^2 + 48*a*c*d^3 + 20*a*d^4 - (7*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^3 - (7*a*c^
2*d^2 + 26*a*c*d^3 + 15*a*d^4)*cos(f*x + e)^2 + 2*(7*a*c^2*d^2 + 12*a*c*d^3
+ 5*a*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*
x + e)^3 - (6*c*d + 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 4*((c*d + d
^2)*cos(f*x + e)^2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x +
e) + (c^2 + 4*c*d + 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*
sin(f*x + e) + a)*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e)
+ (d^2*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))
*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2
*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x
+ e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) - 4*(4*c^4 - 8*c^3*d + 8*c*d^3 -
4*d^4 - (3*c^3*d - 19*c^2*d^2 - 19*c*d^3 + 35*d^4)*cos(f*x + e)^3 + (3*c^4
- 15*c^3*d - 7*c^2*d^2 - c*d^3 + 20*d^4)*cos(f*x + e)^2 + (7*c^4 - 20*c^3*d
- 26*c^2*d^2 - 12*c*d^3 + 51*d^4)*cos(f*x + e) - (4*c^4 - 8*c^3*d + 8*c*d^
3 - 4*d^4 - (3*c^3*d - 19*c^2*d^2 - 19*c*d^3 + 35*d^4)*cos(f*x + e)^2 - (3*
c^4 - 12*c^3*d - 26*c^2*d^2 - 20*c*d^3 + 55*d^4)*cos(f*x + e))*sin(f*x + e)
)*sqrt(a*sin(f*x + e) + a))/((a^3*c^5*d - 3*a^3*c^4*d^2 + 2*a^3*c^3*d^3 + 2
*a^3*c^2*d^4 - 3*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e)^4 - (a^3*c^6 - a^3*c^5
*d - 4*a^3*c^4*d^2 + 6*a^3*c^3*d^3 + a^3*c^2*d^4 - 5*a^3*c*d^5 + 2*a^3*d^6)
*f*cos(f*x + e)^3 - (3*a^3*c^6 - 4*a^3*c^5*d - 9*a^3*c^4*d^2 + 16*a^3*c^3*d
^3 + a^3*c^2*d^4 - 12*a^3*c*d^5 + 5*a^3*d^6)*f*cos(f*x + e)^2 + 2*(a^3*c^6
- 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a
^3*d^6)*f*cos(f*x + e) + 4*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3
*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f - ((a^3*c^5*d - 3*a^3*c^4*d^2
+ 2*a^3*c^3*d^3 + 2*a^3*c^2*d^4 - 3*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e)^3
+ (a^3*c^6 - 7*a^3*c^4*d^2 + 8*a^3*c^3*d^3 + 3*a^3*c^2*d^4 - 8*a^3*c*d^5 +
3*a^3*d^6)*f*cos(f*x + e)^2 - 2*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^
3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e) - 4*(a^3*c^
6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 +
a^3*d^6)*f)*sin(f*x + e)), 1/64*(sqrt(2)*((3*c^3*d - 19*c^2*d^2 + 93*c*d^3
+ 115*d^4)*cos(f*x + e)^4 + 12*c^4 - 64*c^3*d + 296*c^2*d^2 + 832*c*d^3 +
460*d^4 - (3*c^4 - 13*c^3*d + 55*c^2*d^2 + 301*c*d^3 + 230*d^4)*cos(f*x + e
)^3 - (9*c^4 - 42*c^3*d + 184*c^2*d^2 + 810*c*d^3 + 575*d^4)*cos(f*x + e)^2
+ 2*(3*c^4 - 16*c^3*d + 74*c^2*d^2 + 208*c*d^3 + 115*d^4)*cos(f*x + e) + (
12*c^4 - 64*c^3*d + 296*c^2*d^2 + 832*c*d^3 + 460*d^4 - (3*c^3*d - 19*c^2*d
^2 + 93*c*d^3 + 115*d^4)*cos(f*x + e)^3 - (3*c^4 - 10*c^3*d + 36*c^2*d^2 +
394*c*d^3 + 345*d^4)*cos(f*x + e)^2 + 2*(3*c^4 - 16*c^3*d + 74*c^2*d^2 + 20
8*c*d^3 + 115*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)
^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e
) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos
(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 32*(28

```
*a*c^2*d^2 + 48*a*c*d^3 + 20*a*d^4 + (7*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^4 -
 (7*a*c^2*d^2 + 19*a*c*d^3 + 10*a*d^4)*cos(f*x + e)^3 - (21*a*c^2*d^2 + 50*
a*c*d^3 + 25*a*d^4)*cos(f*x + e)^2 + 2*(7*a*c^2*d^2 + 12*a*c*d^3 + 5*a*d^4)
*cos(f*x + e) + (28*a*c^2*d^2 + 48*a*c*d^3 + 20*a*d^4 - (7*a*c*d^3 + 5*a*d^
4)*cos(f*x + e)^3 - (7*a*c^2*d^2 + 26*a*c*d^3 + 15*a*d^4)*cos(f*x + e)^2 +
2*(7*a*c^2*d^2 + 12*a*c*d^3 + 5*a*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(-d/
(a*c + a*d))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)
*sqrt(-d/(a*c + a*d))/(d*cos(f*x + e))) - 4*(4*c^4 - 8*c^3*d + 8*c*d^3 - 4*
d^4 - (3*c^3*d - 19*c^2*d^2 - 19*c*d^3 + 35*d^4)*cos(f*x + e)^3 + (3*c^4 -
15*c^3*d - 7*c^2*d^2 - c*d^3 + 20*d^4)*cos(f*x + e)^2 + (7*c^4 - 20*c^3*d -
 26*c^2*d^2 - 12*c*d^3 + 51*d^4)*cos(f*x + e) - (4*c^4 - 8*c^3*d + 8*c*d^3
- 4*d^4 - (3*c^3*d - 19*c^2*d^2 - 19*c*d^3 + 35*d^4)*cos(f*x + e)^2 - (3*c^
4 - 12*c^3*d - 26*c^2*d^2 - 20*c*d^3 + 55*d^4)*cos(f*x + e))*sin(f*x + e))*
sqrt(a*sin(f*x + e) + a))/((a^3*c^5*d - 3*a^3*c^4*d^2 + 2*a^3*c^3*d^3 + 2*a
^3*c^2*d^4 - 3*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e)^4 - (a^3*c^6 - a^3*c^5*d
 - 4*a^3*c^4*d^2 + 6*a^3*c^3*d^3 + a^3*c^2*d^4 - 5*a^3*c*d^5 + 2*a^3*d^6)*f
*cos(f*x + e)^3 - (3*a^3*c^6 - 4*a^3*c^5*d - 9*a^3*c^4*d^2 + 16*a^3*c^3*d^3
 + a^3*c^2*d^4 - 12*a^3*c*d^5 + 5*a^3*d^6)*f*cos(f*x + e)^2 + 2*(a^3*c^6 -
2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3
*d^6)*f*cos(f*x + e) + 4*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d
^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f - ((a^3*c^5*d - 3*a^3*c^4*d^2 +
 2*a^3*c^3*d^3 + 2*a^3*c^2*d^4 - 3*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e)^3 +
(a^3*c^6 - 7*a^3*c^4*d^2 + 8*a^3*c^3*d^3 + 3*a^3*c^2*d^4 - 8*a^3*c*d^5 + 3*
a^3*d^6)*f*cos(f*x + e)^2 - 2*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*
c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e) - 4*(a^3*c^6
- 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a
^3*d^6)*f)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
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check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
```

x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
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(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs
or sign assumes constant sign by intervals (correct if the argument is rea
l):Check [abs(cos((f*t_nostep+exp(1))/2-pi/4))]Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Discontinuities at zeroes of cos((f*t_nostep+exp(1))/2-pi/4) w
ere not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
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)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
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nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
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to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
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*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
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sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
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i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
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Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2

```
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
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to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration
of abs or sign assumes constant sign by intervals (correct if the argument
is real):Check [abs(t_nostep+1)]Evaluation time: 0.78Error: Bad Argument Ty
pe
```

maple [B] time = 2.39, size = 1972, normalized size = 6.30

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x)
```

```
[Out] -1/32*(323*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*
2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c*d^3-35*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(
1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^3*d+167*(a*
(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2
))*sin(f*x+e)*a^2*c^2*d^2-84*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^
(3/2)*sin(f*x+e)*c*d^3-6*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2
)*c^4+32*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*d^4-160*arctan
h((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*d^5-3
20*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e
)^2*d^5-160*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*
sin(f*x+e)*d^5-224*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a
^(5/2)*c^2*d^3-160*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a
^(5/2)*c*d^4+20*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^4+3*(
a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1
```

```
/2))*a^2*c^4-84*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^3*d-2
0*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^2*d^2+52*(-a*(sin(f
*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c*d^3+32*(-a*(sin(f*x+e)-1))^(1/2
)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*d^4-448*arctanh((-a*(sin(f*x+e)-1)
)^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c^2*d^3-38*(-a*(sin(f*x+e)-
1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*d^4+38*(-a*(sin(f*x+e)-1))^(
3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^3*d+6*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d
)^(1/2)*a^(1/2)*c^2*d^2-224*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)
^(1/2))*a^(5/2)*sin(f*x+e)^3*c*d^4-224*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/
(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c^2*d^3-608*arctanh((-a*(sin(f*x+e)
-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c*d^4-544*arctanh((-a*
(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c*d^4+148*(-a
*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*d^4-38*(-a*(sin
(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c*d^3+3*(a*(c+d)*d)^(1/2)*2^(1/
2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*
c^3*d-19*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^
(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^2*d^2+93*(a*(c+d)*d)^(1/2)*2^(1/2)*arctan
h(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c*d^3+301
*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^
(1/2))*sin(f*x+e)^2*a^2*c*d^3+55*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*
(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^2*d^2-13*(a*(c+d)
*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*si
n(f*x+e)^2*a^2*c^3*d-6*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*
sin(f*x+e)*c^3*d+38*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin
(f*x+e)*c^2*d^2+6*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f
*x+e)*c*d^3+6*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/
2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^4+115*(a*(c+d)*d)^(1/2)*2^(1/2)*arctan
h(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*d^4+115*(a*
(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2
))*sin(f*x+e)^3*a^2*d^4+3*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*
x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^4+230*(a*(c+d)*d)^(1/2)*
2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2
*a^2*d^4+20*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*
c^3*d-84*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c^2
*d^2-19*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(
1/2)/a^(1/2))*a^2*c^3*d-32*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3
/2)*sin(f*x+e)^2*c*d^3+93*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*
x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d^2+115*(a*(c+d)*d)^(1/2)*2^(1/2)*a
rctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d^3)*(-a*(sin(f
*x+e)-1))^(1/2)/a^(9/2)/(1+sin(f*x+e))/(a*(c+d)*d)^(1/2)/(c+d*sin(f*x+e))/(
c+d)/(c-d)^4/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a + a\,\sin\left(e + f\,x\right)\right)^{5/2}\,\left(c + d\,\sin\left(e + f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^2),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.563 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=400

$$-\frac{3\left(c^2-10cd+73d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}\,a^{5/2}f(c-d)^5}+\frac{3d^{5/2}\left(21c^2+30cd+13d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4a^{5/2}f(c-d)^5(c+d)^{5/2}}-\frac{}{16a^2}$$

```
[Out] 3/4*d^(5/2)*(21*c^2+30*c*d+13*d^2)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)
^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/(c-d)^5/(c+d)^(5/2)/f-1/4*cos(f*x+e)
/(c-d)/f/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2-1/16*(3*c-19*d)*cos(f*x+
e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2-3/32*(c^2-10*c*d+7
3*d^2)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/
2)/(c-d)^5/f*2^(1/2)-1/16*d*(3*c^2-20*c*d-31*d^2)*cos(f*x+e)/a^2/(c-d)^3/(c
+d)/f/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2)-3/16*d*(c+3*d)*(c^2-10*c*d-
7*d^2)*cos(f*x+e)/a^2/(c-d)^4/(c+d)^2/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(
1/2)
```

Rubi [A] time = 1.52, antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2766, 2978, 2984, 2985, 2649, 206, 2773, 208}

$$-\frac{3d(c+3d)\left(c^2-10cd-7d^2\right)\cos(e+fx)}{16a^2f(c-d)^4(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}-\frac{d\left(3c^2-20cd-31d^2\right)\cos(e+fx)}{16a^2f(c-d)^3(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] (-3*(c^2 - 10*c*d + 73*d^2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a
+ a*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*(c - d)^5*f) + (3*d^(5/2)*(21*c^2
+ 30*c*d + 13*d^2)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt
[a + a*Sin[e + f*x]])])/(4*a^(5/2)*(c - d)^5*(c + d)^(5/2)*f) - Cos[e + f*x
]/(4*(c - d)*f*(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^2) - ((3*c -
 19*d)*Cos[e + f*x])/(16*a*(c - d)^2*f*(a + a*Sin[e + f*x])^(3/2)*(c + d*Si
n[e + f*x])^2) - (d*(3*c^2 - 20*c*d - 31*d^2)*Cos[e + f*x])/(16*a^2*(c - d)
^3*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2) - (3*d*(c + 3
*d)*(c^2 - 10*c*d - 7*d^2)*Cos[e + f*x])/(16*a^2*(c - d)^4*(c + d)^2*f*Sqrt
[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx = -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - \frac{\int \frac{-\frac{3}{2}a \ldots}{(a+a\sin \ldots}}{\ldots}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - \frac{\ldots}{16a(c-d \ldots}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - \frac{\ldots}{16a(c-d \ldots}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - \frac{\ldots}{16a(c-d \ldots}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - \frac{\ldots}{16a(c-d \ldots}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - \frac{\ldots}{16a(c-d \ldots}$$

$$= -\frac{3\left(c^2-10cd+73d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}(c-d)^5 f} + \frac{3d^{5/2}\left(21c^2 \ldots\right.}{\ldots}$$

Mathematica [C] time = 9.36, size = 958, normalized size = 2.40

$$\frac{3\left(3\cos\left(\frac{1}{2}(e+fx)\right)d^4 - 3\sin\left(\frac{1}{2}(e+fx)\right)d^4 + 5c\cos\left(\frac{1}{2}(e+fx)\right)d^3 - 5c\sin\left(\frac{1}{2}(e+fx)\right)d^3\right)\left(\cos\left(\frac{1}{2}(e+fx)\right) + \ldots\right.}{4(c-d)^4(c+d)^2 f(a(\sin(e+fx)+1))^{5/2}(c+d\sin(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] (Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(2*(c - d)^3*f*(a*
(1 + Sin[e + f*x]))^(5/2)) - (Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2/(4*(c
- d)^3*f*(a*(1 + Sin[e + f*x]))^(5/2)) - (3*(c - 9*d)*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^4)/(16*(c - d)^4*f*(a*(1 + Sin[e + f*x]))^(5/2)) + ((3 + 3
*I)*(c^2 - 10*c*d + 73*d^2)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(e + f*x)/4]
*(Cos[(e + f*x)/4] - Sin[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
```

```
])^5)/((16*(-1)^(1/4)*c^5 - 80*(-1)^(1/4)*c^4*d + 160*(-1)^(1/4)*c^3*d^2 -
160*(-1)^(1/4)*c^2*d^3 + 80*(-1)^(1/4)*c*d^4 - 16*(-1)^(1/4)*d^5)*f*(a*(1 +
 Sin[e + f*x]))^(5/2)) + (3*d^(5/2)*(21*c^2 + 30*c*d + 13*d^2)*(e + f*x - 2
*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*
Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^5)/(16*(c - d)^5*(c + d)^(5/2)*f*(a*(1 + Sin[e + f*x]))^(5/2)) +
(3*d^(5/2)*(21*c^2 + 30*c*d + 13*d^2)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2]
+ 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d
]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(16*(-c + d)
^5*(c + d)^(5/2)*f*(a*(1 + Sin[e + f*x]))^(5/2)) + (3*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^3*(c*Sin[(e + f*x)/2] - 9*d*Sin[(e + f*x)/2]))/(8*(c - d)^
4*f*(a*(1 + Sin[e + f*x]))^(5/2)) + ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
5*(d^3*Cos[(e + f*x)/2] - d^3*Sin[(e + f*x)/2]))/(2*(c - d)^3*(c + d)*f*(a*
(1 + Sin[e + f*x]))^(5/2)*(c + d*Sin[e + f*x])^2) + (3*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^5*(5*c*d^3*Cos[(e + f*x)/2] + 3*d^4*Cos[(e + f*x)/2] - 5*
c*d^3*Sin[(e + f*x)/2] - 3*d^4*Sin[(e + f*x)/2]))/(4*(c - d)^4*(c + d)^2*f*
(a*(1 + Sin[e + f*x]))^(5/2)*(c + d*Sin[e + f*x]))
```

fricas [B] time = 3.54, size = 5999, normalized size = 15.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, algorithm="fricas"
)
```

```
[Out] [-1/64*(3*sqrt(2)*(4*c^6 - 24*c^5*d + 156*c^4*d^2 + 944*c^3*d^3 + 1596*c^2*
d^4 + 1128*c*d^5 + 292*d^6 + (c^4*d^2 - 8*c^3*d^3 + 54*c^2*d^4 + 136*c*d^5
+ 73*d^6)*cos(f*x + e)^5 + (2*c^5*d - 13*c^4*d^2 + 84*c^3*d^3 + 434*c^2*d^4
 + 554*c*d^5 + 219*d^6)*cos(f*x + e)^4 - (c^6 - 4*c^5*d + 25*c^4*d^2 + 328*
c^3*d^3 + 779*c^2*d^4 + 700*c*d^5 + 219*d^6)*cos(f*x + e)^3 - (3*c^6 - 14*c
^5*d + 89*c^4*d^2 + 892*c^3*d^3 + 1957*c^2*d^4 + 1682*c*d^5 + 511*d^6)*cos(
f*x + e)^2 + 2*(c^6 - 6*c^5*d + 39*c^4*d^2 + 236*c^3*d^3 + 399*c^2*d^4 + 28
2*c*d^5 + 73*d^6)*cos(f*x + e) + (4*c^6 - 24*c^5*d + 156*c^4*d^2 + 944*c^3*
d^3 + 1596*c^2*d^4 + 1128*c*d^5 + 292*d^6 + (c^4*d^2 - 8*c^3*d^3 + 54*c^2*d
^4 + 136*c*d^5 + 73*d^6)*cos(f*x + e)^4 - 2*(c^5*d - 7*c^4*d^2 + 46*c^3*d^3
 + 190*c^2*d^4 + 209*c*d^5 + 73*d^6)*cos(f*x + e)^3 - (c^6 - 2*c^5*d + 11*c
^4*d^2 + 420*c^3*d^3 + 1159*c^2*d^4 + 1118*c*d^5 + 365*d^6)*cos(f*x + e)^2
+ 2*(c^6 - 6*c^5*d + 39*c^4*d^2 + 236*c^3*d^3 + 399*c^2*d^4 + 282*c*d^5 + 7
3*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(
2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a
*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2
- (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 12*(84*a*c^4*d^2 +
 288*a*c^3*d^3 + 376*a*c^2*d^4 + 224*a*c*d^5 + 52*a*d^6 + (21*a*c^2*d^4 + 3
0*a*c*d^5 + 13*a*d^6)*cos(f*x + e)^5 + (42*a*c^3*d^3 + 123*a*c^2*d^4 + 116*
```

a*c*d^5 + 39*a*d^6)*cos(f*x + e)^4 - (21*a*c^4*d^2 + 114*a*c^3*d^3 + 196*a*c^2*d^4 + 142*a*c*d^5 + 39*a*d^6)*cos(f*x + e)^3 - (63*a*c^4*d^2 + 300*a*c^3*d^3 + 486*a*c^2*d^4 + 340*a*c*d^5 + 91*a*d^6)*cos(f*x + e)^2 + 2*(21*a*c^4*d^2 + 72*a*c^3*d^3 + 94*a*c^2*d^4 + 56*a*c*d^5 + 13*a*d^6)*cos(f*x + e) + (84*a*c^4*d^2 + 288*a*c^3*d^3 + 376*a*c^2*d^4 + 224*a*c*d^5 + 52*a*d^6 + (21*a*c^2*d^4 + 30*a*c*d^5 + 13*a*d^6)*cos(f*x + e)^4 - 2*(21*a*c^3*d^3 + 51*a*c^2*d^4 + 43*a*c*d^5 + 13*a*d^6)*cos(f*x + e)^3 - (21*a*c^4*d^2 + 156*a*c^3*d^3 + 298*a*c^2*d^4 + 228*a*c*d^5 + 65*a*d^6)*cos(f*x + e)^2 + 2*(21*a*c^4*d^2 + 72*a*c^3*d^3 + 94*a*c^2*d^4 + 56*a*c*d^5 + 13*a*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - 4*((c*d + d^2)*cos(f*x + e)^2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d + 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(4*c^6 - 8*c^5*d - 4*c^4*d^2 + 16*c^3*d^3 - 4*c^2*d^4 - 8*c*d^5 + 4*d^6 - 3*(c^4*d^2 - 8*c^3*d^3 - 30*c^2*d^4 + 16*c*d^5 + 21*d^6)*cos(f*x + e)^4 - (6*c^5*d - 41*c^4*d^2 - 152*c^3*d^3 - 78*c^2*d^4 + 170*c*d^5 + 95*d^6)*cos(f*x + e)^3 + (3*c^6 - 16*c^5*d - 31*c^4*d^2 - 84*c^3*d^3 - 23*c^2*d^4 + 100*c*d^5 + 51*d^6)*cos(f*x + e)^2 + (7*c^6 - 18*c^5*d - 79*c^4*d^2 - 196*c^3*d^3 - 15*c^2*d^4 + 214*c*d^5 + 87*d^6)*cos(f*x + e) - (4*c^6 - 8*c^5*d - 4*c^4*d^2 + 16*c^3*d^3 - 4*c^2*d^4 - 8*c*d^5 + 4*d^6 + 3*(c^4*d^2 - 8*c^3*d^3 - 30*c^2*d^4 + 16*c*d^5 + 21*d^6)*cos(f*x + e)^3 - 2*(3*c^5*d - 22*c^4*d^2 - 64*c^3*d^3 + 6*c^2*d^4 + 61*c*d^5 + 16*d^6)*cos(f*x + e)^2 - (3*c^6 - 10*c^5*d - 75*c^4*d^2 - 212*c^3*d^3 - 11*c^2*d^4 + 222*c*d^5 + 83*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^3*c^7*d^2 - 3*a^3*c^6*d^3 + a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 5*a^3*c^3*d^6 - a^3*c^2*d^7 + 3*a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^5 + (2*a^3*c^8*d - 3*a^3*c^7*d^2 - 7*a^3*c^6*d^3 + 13*a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 17*a^3*c^3*d^6 + 3*a^3*c^2*d^7 + 7*a^3*c*d^8 - 3*a^3*d^9)*f*cos(f*x + e)^4 - (a^3*c^9 + a^3*c^8*d - 8*a^3*c^7*d^2 + 18*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 16*a^3*c^3*d^6 + 8*a^3*c^2*d^7 + 5*a^3*c*d^8 - 3*a^3*d^9)*f*cos(f*x + e)^3 - (3*a^3*c^9 + a^3*c^8*d - 20*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 42*a^3*c^5*d^4 - 18*a^3*c^4*d^5 - 36*a^3*c^3*d^6 + 20*a^3*c^2*d^7 + 11*a^3*c*d^8 - 7*a^3*d^9)*f*cos(f*x + e)^2 + 2*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e) + 4*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f + ((a^3*c^7*d^2 - 3*a^3*c^6*d^3 + a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 5*a^3*c^3*d^6 - a^3*c^2*d^7 + 3*a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^4 - 2*(a^3*c^8*d - 2*a^3*c^7*d^2 - 2*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^3*d^6 + 2*a^3*c^2*d^7 + 2*a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^3 - (a^3*c^9 + 3*a^3*c^8*d - 12*a^3*c^7*d^2 - 4*a^3*c^6*d^3 + 30*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 28*a

^3*c^3*d^6 + 12*a^3*c^2*d^7 + 9*a^3*c*d^8 - 5*a^3*d^9)*f*cos(f*x + e)^2 + 2
*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a
^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f*cos(f*x
 + e) + 4*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*
d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*
f)*sin(f*x + e)), -1/64*(3*sqrt(2)*(4*c^6 - 24*c^5*d + 156*c^4*d^2 + 944*c^
3*d^3 + 1596*c^2*d^4 + 1128*c*d^5 + 292*d^6 + (c^4*d^2 - 8*c^3*d^3 + 54*c^2
*d^4 + 136*c*d^5 + 73*d^6)*cos(f*x + e)^5 + (2*c^5*d - 13*c^4*d^2 + 84*c^3*
d^3 + 434*c^2*d^4 + 554*c*d^5 + 219*d^6)*cos(f*x + e)^4 - (c^6 - 4*c^5*d +
25*c^4*d^2 + 328*c^3*d^3 + 779*c^2*d^4 + 700*c*d^5 + 219*d^6)*cos(f*x + e)^
3 - (3*c^6 - 14*c^5*d + 89*c^4*d^2 + 892*c^3*d^3 + 1957*c^2*d^4 + 1682*c*d^
5 + 511*d^6)*cos(f*x + e)^2 + 2*(c^6 - 6*c^5*d + 39*c^4*d^2 + 236*c^3*d^3 +
 399*c^2*d^4 + 282*c*d^5 + 73*d^6)*cos(f*x + e) + (4*c^6 - 24*c^5*d + 156*c
^4*d^2 + 944*c^3*d^3 + 1596*c^2*d^4 + 1128*c*d^5 + 292*d^6 + (c^4*d^2 - 8*c
^3*d^3 + 54*c^2*d^4 + 136*c*d^5 + 73*d^6)*cos(f*x + e)^4 - 2*(c^5*d - 7*c^4
*d^2 + 46*c^3*d^3 + 190*c^2*d^4 + 209*c*d^5 + 73*d^6)*cos(f*x + e)^3 - (c^6
 - 2*c^5*d + 11*c^4*d^2 + 420*c^3*d^3 + 1159*c^2*d^4 + 1118*c*d^5 + 365*d^6
)*cos(f*x + e)^2 + 2*(c^6 - 6*c^5*d + 39*c^4*d^2 + 236*c^3*d^3 + 399*c^2*d^
4 + 282*c*d^5 + 73*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x
 + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*
x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)
/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 2
4*(84*a*c^4*d^2 + 288*a*c^3*d^3 + 376*a*c^2*d^4 + 224*a*c*d^5 + 52*a*d^6 +
(21*a*c^2*d^4 + 30*a*c*d^5 + 13*a*d^6)*cos(f*x + e)^5 + (42*a*c^3*d^3 + 123
*a*c^2*d^4 + 116*a*c*d^5 + 39*a*d^6)*cos(f*x + e)^4 - (21*a*c^4*d^2 + 114*a
*c^3*d^3 + 196*a*c^2*d^4 + 142*a*c*d^5 + 39*a*d^6)*cos(f*x + e)^3 - (63*a*c
^4*d^2 + 300*a*c^3*d^3 + 486*a*c^2*d^4 + 340*a*c*d^5 + 91*a*d^6)*cos(f*x +
e)^2 + 2*(21*a*c^4*d^2 + 72*a*c^3*d^3 + 94*a*c^2*d^4 + 56*a*c*d^5 + 13*a*d^
6)*cos(f*x + e) + (84*a*c^4*d^2 + 288*a*c^3*d^3 + 376*a*c^2*d^4 + 224*a*c*d
^5 + 52*a*d^6 + (21*a*c^2*d^4 + 30*a*c*d^5 + 13*a*d^6)*cos(f*x + e)^4 - 2*(
21*a*c^3*d^3 + 51*a*c^2*d^4 + 43*a*c*d^5 + 13*a*d^6)*cos(f*x + e)^3 - (21*a
*c^4*d^2 + 156*a*c^3*d^3 + 298*a*c^2*d^4 + 228*a*c*d^5 + 65*a*d^6)*cos(f*x
+ e)^2 + 2*(21*a*c^4*d^2 + 72*a*c^3*d^3 + 94*a*c^2*d^4 + 56*a*c*d^5 + 13*a*
d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(-d/(a*c + a*d))*arctan(1/2*sqrt(a*sin
(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a*d))/(d*cos(f*x +
 e))) + 4*(4*c^6 - 8*c^5*d - 4*c^4*d^2 + 16*c^3*d^3 - 4*c^2*d^4 - 8*c*d^5 +
 4*d^6 - 3*(c^4*d^2 - 8*c^3*d^3 - 30*c^2*d^4 + 16*c*d^5 + 21*d^6)*cos(f*x +
 e)^4 - (6*c^5*d - 41*c^4*d^2 - 152*c^3*d^3 - 78*c^2*d^4 + 170*c*d^5 + 95*d
^6)*cos(f*x + e)^3 + (3*c^6 - 16*c^5*d - 31*c^4*d^2 - 84*c^3*d^3 - 23*c^2*d
^4 + 100*c*d^5 + 51*d^6)*cos(f*x + e)^2 + (7*c^6 - 18*c^5*d - 79*c^4*d^2 -
196*c^3*d^3 - 15*c^2*d^4 + 214*c*d^5 + 87*d^6)*cos(f*x + e) - (4*c^6 - 8*c^
5*d - 4*c^4*d^2 + 16*c^3*d^3 - 4*c^2*d^4 - 8*c*d^5 + 4*d^6 + 3*(c^4*d^2 - 8
*c^3*d^3 - 30*c^2*d^4 + 16*c*d^5 + 21*d^6)*cos(f*x + e)^3 - 2*(3*c^5*d - 22
*c^4*d^2 - 64*c^3*d^3 + 6*c^2*d^4 + 61*c*d^5 + 16*d^6)*cos(f*x + e)^2 - (3*
c^6 - 10*c^5*d - 75*c^4*d^2 - 212*c^3*d^3 - 11*c^2*d^4 + 222*c*d^5 + 83*d^6

```
)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^3*c^7*d^2 - 3*a
^3*c^6*d^3 + a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 5*a^3*c^3*d^6 - a^3*c^2*d^7 + 3*
a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^5 + (2*a^3*c^8*d - 3*a^3*c^7*d^2 - 7*a^
3*c^6*d^3 + 13*a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 17*a^3*c^3*d^6 + 3*a^3*c^2*d^7
 + 7*a^3*c*d^8 - 3*a^3*d^9)*f*cos(f*x + e)^4 - (a^3*c^9 + a^3*c^8*d - 8*a^3
*c^7*d^2 + 18*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 16*a^3*c^3*d^6 + 8*a^3*c^2*d^7
+ 5*a^3*c*d^8 - 3*a^3*d^9)*f*cos(f*x + e)^3 - (3*a^3*c^9 + a^3*c^8*d - 20*a
^3*c^7*d^2 + 4*a^3*c^6*d^3 + 42*a^3*c^5*d^4 - 18*a^3*c^4*d^5 - 36*a^3*c^3*d
^6 + 20*a^3*c^2*d^7 + 11*a^3*c*d^8 - 7*a^3*d^9)*f*cos(f*x + e)^2 + 2*(a^3*c
^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*
d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e) +
 4*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6
*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f + ((a
^3*c^7*d^2 - 3*a^3*c^6*d^3 + a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 5*a^3*c^3*d^6 -
a^3*c^2*d^7 + 3*a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^4 - 2*(a^3*c^8*d - 2*a^
3*c^7*d^2 - 2*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^3*d^6 + 2*a^3*c^2*d^7 +
 2*a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^3 - (a^3*c^9 + 3*a^3*c^8*d - 12*a^3*
c^7*d^2 - 4*a^3*c^6*d^3 + 30*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 28*a^3*c^3*d^6 +
 12*a^3*c^2*d^7 + 9*a^3*c*d^8 - 5*a^3*d^9)*f*cos(f*x + e)^2 + 2*(a^3*c^9 -
a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 -
 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e) + 4*(a
^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*
c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f)*sin(f*x +
 e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
```

```
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Warning, integration of abs or sign assumes constant sign by in
tervals (correct if the argument is real):Check [abs(cos((f*t_nostep+exp(1)
)/2-pi/4))]Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Discontinuities at zero
es of cos((f*t_nostep+exp(1))/2-pi/4) were not checkedUnable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
```

```
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*p
i/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(t_nostep+1)]Eval
uation time: 1.17index.cc index_m i_lex_is_greater Error: Bad Argument Valu
e
```

maple [B] time = 3.26, size = 3535, normalized size = 8.84

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/32/a^(9/2)*(-a*(sin(f*x+e)-1))^(1/2)*(-219*(a*(c+d)*d)^(1/2)*2^(1/2)*arct
anh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d^4+504*arctanh(
(-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*c^4*d^3+720*arctanh(
(-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*c^3*d^4+312*arctanh(
(-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*c^2*d^5+72*(-a*(sin(
f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*d^6+6*(-a*(sin(f*x+e)-1))^(3/2)*
(a*(c+d)*d)^(1/2)*a^(1/2)*c^6-20*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2
)*a^(3/2)*c^6-56*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*d^6+31
2*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)
^4*d^7+624*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*s
in(f*x+e)^3*d^7+312*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*
a^(5/2)*sin(f*x+e)^2*d^7+42*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(
f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^4*d^2-276*(a*(c+d)*d)^
```

(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*
x+e)^3*a^2*c^3*d^3-1140*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+
e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^2*d^4-1254*(a*(c+d)*d)^(1/
2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e
)^3*a^2*c*d^5+12*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^
(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^5*d-69*(a*(c+d)*d)^(1/2)*2^(1/2)*
arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^4
d^2+12(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c^5*
d-96*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c^4*d^2
-120*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c^3*d^3
-144*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c^2*d^4
+204*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c*d^5-6
(a(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^
(1/2))*sin(f*x+e)*a^2*c^6-219*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(si
n(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^4*a^2*d^6-438*(a*(c+d)*d)^(1
/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+
e)^3*a^2*d^6+6*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+
e)^2*c^4*d^2-219*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^
(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*d^6-20*(-a*(sin(f*x+e)-1))^(1/2)*(a
*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*c^4*d^2-48*(-a*(sin(f*x+e)-1))^(3/2)*(
a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)^2*c^3*d^3-180*(-a*(sin(f*x+e)-1))^(3/2)
(a(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)^2*c^2*d^4+96*(-a*(sin(f*x+e)-1))^(3/2
)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)^2*c*d^5-3*(a*(c+d)*d)^(1/2)*2^(1/2)*
arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^6
-1032*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/
2)/a^(1/2))*sin(f*x+e)^2*a^2*c^3*d^3-2013*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh
(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^2*d^4-12
84*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/
a^(1/2))*sin(f*x+e)^2*a^2*c*d^5+42*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-
a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^5*d-276*(a*(c+d)*
d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin
(f*x+e)*a^2*c^4*d^2-1140*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x
+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^3*d^3-1254*(a*(c+d)*d)^(1/2
)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)
*a^2*c^2*d^4-438*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^
(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c*d^5-3*(a*(c+d)*d)^(1/2)*2^(1/2)*arc
tanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^4*a^2*c^4*d^
2+24*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2
)/a^(1/2))*sin(f*x+e)^4*a^2*c^3*d^3-162*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1
/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^4*a^2*c^2*d^4-408*
(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(
1/2))*sin(f*x+e)^4*a^2*c*d^5-6*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(s
in(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^5*d+624*arctanh((-a
*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c*d^6-112*(-
a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*d^6+96*(-a*(si

```
n(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^5*d+136*(-a*(sin(f*x+e)-1))^
(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^4*d^2+40*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+
d)*d)^(1/2)*a^(3/2)*c^3*d^3-60*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*
a^(3/2)*c^2*d^4-136*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c*d
^5+2736*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(
f*x+e)^2*c^3*d^4+2448*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2)
)*a^(5/2)*sin(f*x+e)^3*c^2*d^5+2064*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*
(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*c*d^6+504*arctanh((-a*(sin(f*x+e)-1))^
(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c^4*d^3+3696*arctanh((-a*(s
in(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c^2*d^5+1968*
arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2
*c*d^6-172*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2
*d^6+1008*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*si
n(f*x+e)*c^4*d^3+2448*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2)
)*a^(5/2)*sin(f*x+e)*c^3*d^4+2064*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c
+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c^2*d^5+126*(-a*(sin(f*x+e)-1))^(3/2)*(a*(
c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)^2*d^6+144*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+
d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*d^6-48*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)
^(1/2)*a^(1/2)*c^5*d-60*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)
*c^4*d^2+48*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^3*d^3-66*
(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^2*d^4+48*(-a*(sin(f*x
+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c*d^5-3*(a*(c+d)*d)^(1/2)*2^(1/2)*a
rctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^6+504*arctanh((
-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^4*c^2*d^5+
720*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+
e)^4*c*d^6+1008*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5
/2)*sin(f*x+e)^3*c^3*d^4+192*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^
(3/2)*sin(f*x+e)^2*c^2*d^4-232*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*
a^(3/2)*sin(f*x+e)^2*c*d^5-40*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a
^(3/2)*sin(f*x+e)*c^5*d+192*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(
3/2)*sin(f*x+e)*c^4*d^2+544*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(
3/2)*sin(f*x+e)*c^3*d^3-80*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3
/2)*sin(f*x+e)*c^2*d^4-504*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3
/2)*sin(f*x+e)*c*d^5+24*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+
e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^5*d-162*(a*(c+d)*d)^(1/2)*2^(1/2)*arcta
nh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^4*d^2-408*(a*(c+d)*
d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2
*c^3*d^3+232*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)
^2*c^3*d^3)/(1+sin(f*x+e))/(a*(c+d)*d)^(1/2)/(c+d*sin(f*x+e))^2/(c+d)^2/(c-
d)^5/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^3),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.564 $\int \sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=203

$$-\frac{5a(c+d)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{8f\sqrt{a\sin(e+fx)+a}}-\frac{5a(c+d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{12f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)(c+d\sin(}{3f\sqrt{a\sin(e+fx)}}$$

```
[Out] -5/8*(c+d)^3*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*
sin(f*x+e))^(1/2))*a^(1/2)/f/d^(1/2)-5/12*a*(c+d)*cos(f*x+e)*(c+d*sin(f*x+e
))^(3/2)/f/(a+a*sin(f*x+e))^(1/2)-1/3*a*cos(f*x+e)*(c+d*sin(f*x+e))^(5/2)/f
/(a+a*sin(f*x+e))^(1/2)-5/8*a*(c+d)^2*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(
a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2770, 2775, 205}

$$-\frac{5a(c+d)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{8f\sqrt{a\sin(e+fx)+a}}-\frac{5a(c+d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{12f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)(c+d\sin(}{3f\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-5*Sqrt[a]*(c + d)^3*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(8*Sqrt[d]*f) - (5*a*(c + d)^2*Cos[e
 + f*x]*Sqrt[c + d*Sin[e + f*x]])/(8*f*Sqrt[a + a*Sin[e + f*x]]) - (5*a*(c
+ d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(12*f*Sqrt[a + a*Sin[e + f*x]
]) - (a*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(3*f*Sqrt[a + a*Sin[e + f*
x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
```

```
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}\,dx = -\frac{a\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{3f\sqrt{a+a\sin(e+fx)}} + \frac{1}{6}(5(c+d))\int \sqrt{a+a\,\mathrm{si}}$$

$$= -\frac{5a(c+d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{12f\sqrt{a+a\sin(e+fx)}} - \frac{a\cos(e+fx)(c}{3f\sqrt{a+a}}$$

$$= -\frac{5a(c+d)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{8f\sqrt{a+a\sin(e+fx)}} - \frac{5a(c+d)\cos(e}{12f\sqrt{a}}$$

$$= -\frac{5a(c+d)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{8f\sqrt{a+a\sin(e+fx)}} - \frac{5a(c+d)\cos(e}{12f\sqrt{a}}$$

$$= -\frac{5\sqrt{a}\,(c+d)^3\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}\right)}{8\sqrt{d}\,f} - \frac{5a(c+d)^2}{8}$$

Mathematica [C] time = 3.73, size = 391, normalized size = 1.93

$$-\frac{\sqrt{a(\sin(e+fx)+1)}\left(2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))\left(33c^2+2d(13c+5d)\sin(e+f\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/48*(Sqrt[a*(1 + Sin[e + f*x])]*((15*(c + d)^3*(Log[(2*(-1)^(1/4)*c - 2*(
-1)^(3/4)*d*E^(I*(e + f*x)) + 2*Sqrt[d]*Sqrt[2*c*E^(I*(e + f*x)) - I*d*(-1
+ E^((2*I)*(e + f*x)))]])/(Sqrt[d]*E^(I*e))] - Log[(2*E^((I/2)*(e - 2*f*x))*
((-1)^(3/4)*d + (-1)^(1/4)*c*E^(I*(e + f*x)) + I*Sqrt[d]*Sqrt[2*c*E^(I*(e +
 f*x)) - I*d*(-1 + E^((2*I)*(e + f*x)))])*f)/Sqrt[d]])*(I*Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])*Sqrt[(Cos[e + f*x] + I*Sin[e + f*x])*(c + d*Sin[e + f*x
])])/Sqrt[d] + 2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(c + d*Sin[e + f*x])
*(33*c^2 + 40*c*d + 19*d^2 - 4*d^2*Cos[2*(e + f*x)] + 2*d*(13*c + 5*d)*Sin[
e + f*x])))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x
]])
```

fricas [B] time = 1.07, size = 1257, normalized size = 6.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] [1/192*(15*(c^3 + 3*c^2*d + 3*c*d^2 + d^3 + (c^3 + 3*c^2*d + 3*c*d^2 + d^3)
*cos(f*x + e) + (c^3 + 3*c^2*d + 3*c*d^2 + d^3)*sin(f*x + e))*sqrt(-a/d)*lo
g((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 +
 a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*
d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*
a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f*x + e)^4 - c^3*d + 17*c^2*d^2 - 59*
c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f*x + e)^3 - 2*(5*c^2*d^2 - 26*c*d^3
+ 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c^2*d^2 + 31*c*d^3 - 25*d^4)*cos(f*x
+ e) + (16*d^4*cos(f*x + e)^3 + c^3*d - 17*c^2*d^2 + 59*c*d^3 - 51*d^4 - 8*
(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*
x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqr
t(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^3 + 289*a*d^4)*co
s(f*x + e) + (128*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 +
4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2
- 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*
d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1
)) + 8*(8*d^2*cos(f*x + e)^3 - 2*(13*c*d + d^2)*cos(f*x + e)^2 - 33*c^2 - 1
4*c*d - 13*d^2 - (33*c^2 + 40*c*d + 23*d^2)*cos(f*x + e) - (8*d^2*cos(f*x +
 e)^2 - 33*c^2 - 14*c*d - 13*d^2 + 2*(13*c*d + 5*d^2)*cos(f*x + e))*sin(f*x
 + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(f*cos(f*x + e) +
 f*sin(f*x + e) + f), 1/96*(15*(c^3 + 3*c^2*d + 3*c*d^2 + d^3 + (c^3 + 3*c^
2*d + 3*c*d^2 + d^3)*cos(f*x + e) + (c^3 + 3*c^2*d + 3*c*d^2 + d^3)*sin(f*x
 + e))*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8
*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c
)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c*d - a*d^2)*cos(f*x + e)*sin(f*
x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x + e))) + 4*(8*d^2*cos(f*x + e)^3
```

```
 - 2*(13*c*d + d^2)*cos(f*x + e)^2 - 33*c^2 - 14*c*d - 13*d^2 - (33*c^2 + 4
0*c*d + 23*d^2)*cos(f*x + e) - (8*d^2*cos(f*x + e)^2 - 33*c^2 - 14*c*d - 13
*d^2 + 2*(13*c*d + 5*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) +
 a)*sqrt(d*sin(f*x + e) + c))/(f*cos(f*x + e) + f*sin(f*x + e) + f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+a\sin\left(fx+e\right)}\,\left(c+d\sin\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.565 $\int \sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=156

$$-\frac{3a(c+d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}-\frac{3\sqrt{a}\,(c+d)^2\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{a\sin(e+fx)+}}\right.}{4\sqrt{d}\,f}$$

```
[Out] -3/4*(c+d)^2*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*
sin(f*x+e))^(1/2))*a^(1/2)/f/d^(1/2)-1/2*a*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2
)/f/(a+a*sin(f*x+e))^(1/2)-3/4*a*(c+d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/
(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2770, 2775, 205}

$$-\frac{3a(c+d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}-\frac{3\sqrt{a}\,(c+d)^2\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{a\sin(e+fx)+}}\right.}{4\sqrt{d}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-3*Sqrt[a]*(c + d)^2*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(4*Sqrt[d]*f) - (3*a*(c + d)*Cos[e +
 f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*f*Sqrt[a + a*Sin[e + f*x]]) - (a*Cos[e +
 f*x]*(c + d*Sin[e + f*x])^(3/2))/(2*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}\,dx = -\frac{a\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} + \frac{1}{4}(3(c+d))\int \sqrt{a+a\sin}$$

$$= -\frac{3a(c+d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f\sqrt{a+a\sin(e+fx)}} - \frac{a\cos(e+fx)(c+d}{2f\sqrt{a+a\sin}}$$

$$= -\frac{3a(c+d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f\sqrt{a+a\sin(e+fx)}} - \frac{a\cos(e+fx)(c+d}{2f\sqrt{a+a\sin}}$$

$$= -\frac{3\sqrt{a}\,(c+d)^2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{4\sqrt{d}\,f} - \frac{3a(c+d)\cos}{4f}$$

Mathematica [C] time = 2.05, size = 365, normalized size = 2.34

$$\frac{\sqrt{a(\sin(e+fx)+1)}\left(-2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))(5c+2d\sin(e+fx)+3d) - \frac{3i(c+}{}\right.}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sqrt[a*(1 + Sin[e + f*x])]*(((-3*I)*(c + d)^2*(Log[(2*(-1)^(1/4)*c - 2*(-1
)^(3/4)*d*E^(I*(e + f*x)) + 2*Sqrt[d]*Sqrt[2*c*E^(I*(e + f*x)) - I*d*(-1 +
E^((2*I)*(e + f*x)))])/(Sqrt[d]*E^(I*e))] - Log[(2*E^((I/2)*(e - 2*f*x))*((
-1)^(3/4)*d + (-1)^(1/4)*c*E^(I*(e + f*x)) + I*Sqrt[d]*Sqrt[2*c*E^(I*(e + f
*x)) - I*d*(-1 + E^((2*I)*(e + f*x)))])*f)/Sqrt[d]])*(Cos[(e + f*x)/2] - I*
Sin[(e + f*x)/2])*Sqrt[(Cos[e + f*x] + I*Sin[e + f*x])*(c + d*Sin[e + f*x])
```

```
])/Sqrt[d] - 2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(c + d*Sin[e + f*x])*(
5*c + 3*d + 2*d*Sin[e + f*x])))/(8*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*
Sqrt[c + d*Sin[e + f*x]])
```

fricas [B] time = 0.91, size = 1069, normalized size = 6.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/32*(3*(c^2 + 2*c*d + d^2 + (c^2 + 2*c*d + d^2)*cos(f*x + e) + (c^2 + 2*c
*d + d^2)*sin(f*x + e))*sqrt(-a/d)*log((128*a*d^4*cos(f*x + e)^5 + a*c^4 +
4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f
*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a
*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(
f*x + e)^4 - c^3*d + 17*c^2*d^2 - 59*c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(
f*x + e)^3 - 2*(5*c^2*d^2 - 26*c*d^3 + 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*
c^2*d^2 + 31*c*d^3 - 25*d^4)*cos(f*x + e) + (16*d^4*cos(f*x + e)^3 + c^3*d
- 17*c^2*d^2 + 59*c*d^3 - 51*d^4 - 8*(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(
5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x +
 e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*
c^2*d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x + e)^4
 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d
^4)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2
+ 32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x +
 e))/(cos(f*x + e) + sin(f*x + e) + 1)) - 8*(2*d*cos(f*x + e)^2 + (5*c + 3*
d)*cos(f*x + e) + (2*d*cos(f*x + e) - 5*c - d)*sin(f*x + e) + 5*c + d)*sqrt
(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(f*cos(f*x + e) + f*sin(f*x
+ e) + f), 1/16*(3*(c^2 + 2*c*d + d^2 + (c^2 + 2*c*d + d^2)*cos(f*x + e) +
(c^2 + 2*c*d + d^2)*sin(f*x + e))*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x + e)^
2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) +
 a)*sqrt(d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c*d -
 a*d^2)*cos(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x + e))
) - 4*(2*d*cos(f*x + e)^2 + (5*c + 3*d)*cos(f*x + e) + (2*d*cos(f*x + e) -
5*c - d)*sin(f*x + e) + 5*c + d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x +
e) + c))/(f*cos(f*x + e) + f*sin(f*x + e) + f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sin\left(fx+e\right)+a}\left(d \sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+a\sin\left(fx+e\right)}\,\left(c+d\sin\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(c + d*sin(e + f*x))**(3/2), x)
```

3.566 $\int \sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=105

$$-\frac{a\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}-\frac{\sqrt{a}\,(c+d)\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{d}\,f}$$

```
[Out] -(c+d)*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*
x+e))^(1/2))*a^(1/2)/f/d^(1/2)-a*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+a*s
in(f*x+e))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2770, 2775, 205}

$$-\frac{a\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}-\frac{\sqrt{a}\,(c+d)\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{d}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[a]*(c + d)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e
+ f*x]]*Sqrt[c + d*Sin[e + f*x]])]])/(Sqrt[d]*f)) - (a*Cos[e + f*x]*Sqrt[c +
 d*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
```

```
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}\,dx = -\frac{a\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + \frac{1}{2}(c+d)\int \frac{\sqrt{a+a\sin(e+f}}{\sqrt{c+d\sin(e+f}}$$

$$= -\frac{a\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} - \frac{(a(c+d))\,\text{Subst}\left(\int \frac{1}{a+dx^2}\,dx,\right.}{}$$

$$= -\frac{\sqrt{a}\,(c+d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{d}\,f} - \frac{a\cos(e+fx)\sqrt{c}}{f\sqrt{a+a\,\text{si}}}$$

Mathematica [C] time = 1.31, size = 350, normalized size = 3.33

$$\frac{\sqrt{a(\sin(e+fx)+1)}\left(-\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))}{f} - \frac{i(c+d)\left(\cos\left(\frac{1}{2}(e+fx)\right)-i\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\log\left(\frac{e^{-ie}\left(2\sqrt{d}\sqrt{2ce^{i(e+fx)}-id}\right.}{}\right.\right.}{}\right.}{2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[a*(1 + Sin[e + f*x])]*((-2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(c +
 d*Sin[e + f*x]))/f - (I*(c + d)*(Log[(2*(-1)^(1/4)*c - 2*(-1)^(3/4)*d*E^(I
*(e + f*x)) + 2*Sqrt[d]*Sqrt[2*c*E^(I*(e + f*x)) - I*d*(-1 + E^((2*I)*(e +
f*x)))])/(Sqrt[d]*E^(I*e))] - Log[(2*E^((I/2)*(e - 2*f*x))*((-1)^(3/4)*d +
(-1)^(1/4)*c*E^(I*(e + f*x)) + I*Sqrt[d]*Sqrt[2*c*E^(I*(e + f*x)) - I*d*(-1
 + E^((2*I)*(e + f*x)))])*f)/Sqrt[d]])*(Cos[(e + f*x)/2] - I*Sin[(e + f*x)/
2])*Sqrt[(Cos[e + f*x] + I*Sin[e + f*x])*(c + d*Sin[e + f*x])])/(Sqrt[d]*f)
))/(2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]])
```

fricas [B] time = 0.86, size = 945, normalized size = 9.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/8*(((c + d)*cos(f*x + e) + (c + d)*sin(f*x + e) + c + d)*sqrt(-a/d)*log(
(128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a
*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^
3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*
d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f*x + e)^4 - c^3*d + 17*c^2*d^2 - 59*c*
d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f*x + e)^3 - 2*(5*c^2*d^2 - 26*c*d^3 +
33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c^2*d^2 + 31*c*d^3 - 25*d^4)*cos(f*x +
e) + (16*d^4*cos(f*x + e)^3 + c^3*d - 17*c^2*d^2 + 59*c*d^3 - 51*d^4 - 8*(3
*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x
+ e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(
-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(
f*x + e) + (128*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*
a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2 -
6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^
3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1))
 - 8*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*(cos(f*x + e) - sin(
f*x + e) + 1))/(f*cos(f*x + e) + f*sin(f*x + e) + f), 1/4*(((c + d)*cos(f*x
 + e) + (c + d)*sin(f*x + e) + c + d)*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x +
 e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x +
e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c
*d - a*d^2)*cos(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x +
 e))) - 4*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*(cos(f*x + e) -
 sin(f*x + e) + 1))/(f*cos(f*x + e) + f*sin(f*x + e) + f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+a\sin\left(fx+e\right)}\,\sqrt{c+d\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c+d\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\sqrt{c+d\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*sqrt(c + d*sin(e + f*x)), x)
```

3.567 $\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=61

$$-\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{d}f}$$

```
[Out] -2*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)
)^(1/2))*a^(1/2)/f/d^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2775, 205}

$$-\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{d}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[d]*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{a+dx^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}\right)}{f}$$
$$= -\frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{d}\,f}$$

Mathematica [C] time = 1.15, size = 305, normalized size = 5.00

$$-\frac{i\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-i\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\log\left(\frac{e^{-ie}\left(2\sqrt{d}\sqrt{2ce^{i(e+fx)}-id\left(-1+e^{2i(e+fx)}\right)}+2\sqrt[4]{-1}\,c-2(-1)^{3/4}de^{i(\cdots}\right.}{\sqrt{d}}\right.\right.}{\sqrt{d}\,f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] ((-I)*(Log[(2*(-1)^(1/4)*c - 2*(-1)^(3/4)*d*E^(I*(e + f*x)) + 2*Sqrt[d]*Sqr
t[2*c*E^(I*(e + f*x)) - I*d*(-1 + E^((2*I)*(e + f*x)))])/(Sqrt[d]*E^(I*e))]
 - Log[((-1 - I)*E^((I/2)*(e - 2*f*x))*(-((-1)^(1/4)*d) + (-1)^(3/4)*c*E^(I
*(e + f*x)) - Sqrt[d]*Sqrt[2*c*E^(I*(e + f*x)) - I*d*(-1 + E^((2*I)*(e + f*
x)))])*f)/Sqrt[d]])*(Cos[(e + f*x)/2] - I*Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin
[e + f*x])]*Sqrt[(Cos[e + f*x] + I*Sin[e + f*x])*(c + d*Sin[e + f*x])])/(Sq
rt[d]*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]])
```

fricas [B] time = 0.84, size = 777, normalized size = 12.74

$$\left[\frac{\sqrt{-\frac{a}{d}}\,\log\left(\frac{128\,ad^4\cos\left(fx+e\right)^5+ac^4+4\,ac^3d+6\,ac^2d^2+4\,acd^3+ad^4+128\left(2\,acd^3-ad^4\right)\cos\left(fx+e\right)^4-32\left(5\,ac^2d^2-14\,acd^3+13\,ad^4\right)\cos\left(fx+e\right)^3-32\left(ac\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/4*sqrt(-a/d)*log((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2
*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a
```

```
*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^
2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f*x + e)^4 - c^3*d
+ 17*c^2*d^2 - 59*c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f*x + e)^3 - 2*(5*c
^2*d^2 - 26*c*d^3 + 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c^2*d^2 + 31*c*d^3
- 25*d^4)*cos(f*x + e) + (16*d^4*cos(f*x + e)^3 + c^3*d - 17*c^2*d^2 + 59*c
*d^3 - 51*d^4 - 8*(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5*c^2*d^2 - 14*c*d^
3 + 13*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin
(f*x + e) + c)*sqrt(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d
^3 + 289*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*
d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3
- 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a
*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)
+ sin(f*x + e) + 1))/f, 1/2*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^
2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sq
rt(d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c*d - a*d^2
)*cos(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x + e)))/f]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\sin\left(fx+e\right)+a}}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

maple [B] time = 0.54, size = 2707, normalized size = 44.38

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/f*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*(c+d*sin(f
*x+e))^(1/2)*(a*(1+sin(f*x+e)))^(1/2)*((d^2/c^2)^(1/2)*sin(f*x+e)*cos(f*x+e
)*(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2
*d^2-4*d^4)*c)^(1/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)*arctan(((d^2/c^2)^(1/2)*c*s
in(f*x+e)+d*cos(f*x+e)-d)/((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d
)*d)^(1/2)/((d^2/c^2)^(1/2)*c*sin(f*x+e)-d*cos(f*x+e)+d)*((d^2/c^2)^(1/2)*c
^2-d^2)*c*((d^2/c^2)^(1/2)-1)/(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c
^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2))*c*d+cos(f*x+e)^2*arctan((
```

(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2
)*c)^(1/2))*d^5+sin(f*x+e)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(
f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*d^5-arctan(((c+d*sin(f*x+e))
/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*c^3*
d^2+arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d
^2/c^2)^(1/2)*c)^(1/2))*c^2*d^3+arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c
*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*c*d^4-(d^2/c^2)^(1/2)*c
os(f*x+e)*(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/
2)-4*c^2*d^2-4*d^4)*c)^(1/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)*arctan(((d^2/c^2)^(
1/2)*c*sin(f*x+e)+d*cos(f*x+e)-d)/((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(
f*x+e)+d)*d)^(1/2)/((d^2/c^2)^(1/2)*c*sin(f*x+e)-d*cos(f*x+e)+d)*((d^2/c^2)
^(1/2)*c^2-d^2)*c*((d^2/c^2)^(1/2)-1)/(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/
2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2))*c^2+sin(f*x+e)*co
s(f*x+e)*(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2
)-4*c^2*d^2-4*d^4)*c)^(1/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)*arctan(((d^2/c^2)^(1
/2)*c*sin(f*x+e)+d*cos(f*x+e)-d)/((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f
*x+e)+d)*d)^(1/2)/((d^2/c^2)^(1/2)*c*sin(f*x+e)-d*cos(f*x+e)+d)*((d^2/c^2)^
(1/2)*c^2-d^2)*c*((d^2/c^2)^(1/2)-1)/(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2
)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2))*d^2-arctan(((c+d*s
in(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(
1/2))*d^5-(d^2/c^2)^(1/2)*cos(f*x+e)^2*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^
(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*c^3*d^2+2*(d^2/c
^2)^(1/2)*cos(f*x+e)^2*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+
e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*c^2*d^3-(d^2/c^2)^(1/2)*cos(f*x+
e)^2*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(
d^2/c^2)^(1/2)*c)^(1/2))*c*d^4-(d^2/c^2)^(1/2)*sin(f*x+e)*arctan(((c+d*sin(
f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2
))*c^4*d+(d^2/c^2)^(1/2)*sin(f*x+e)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/
2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*c^3*d^2+(d^2/c^2)^(
1/2)*sin(f*x+e)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d
)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*c^2*d^3-(d^2/c^2)^(1/2)*sin(f*x+e)*arct
an(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^
(1/2)*c)^(1/2))*c*d^4-sin(f*x+e)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*
c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*c^2*d^3-sin(f*x+e)*arc
tan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)
^(1/2)*c)^(1/2))*c*d^4+cos(f*x+e)^2*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/
2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*c^2*d^3-2*cos(f*x+e
)^2*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d
^2/c^2)^(1/2)*c)^(1/2))*c*d^4+(d^2/c^2)^(1/2)*arctan(((c+d*sin(f*x+e))/((d^
2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*c^4*d-(d^
2/c^2)^(1/2)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(
1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*c^3*d^2-(d^2/c^2)^(1/2)*arctan(((c+d*sin(f
*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2)
)*c^2*d^3+(d^2/c^2)^(1/2)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f
*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*c*d^4+sin(f*x+e)*arctan(((c+d

```
*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)
^(1/2))*c^3*d^2-cos(f*x+e)*(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+
d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)*ar
ctan(((d^2/c^2)^(1/2)*c*sin(f*x+e)+d*cos(f*x+e)-d)/((c+d*sin(f*x+e))/((d^2/
c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/((d^2/c^2)^(1/2)*c*sin(f*x+e)-d*cos(f*x
+e)+d)*((d^2/c^2)^(1/2)*c^2-d^2)*c*((d^2/c^2)^(1/2)-1)/(((d^2/c^2)^(1/2)*c^
4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2))*
c*d)/d^2/cos(f*x+e)/(cos(f*x+e)^2*d^2+c^2-d^2)/(-(d^2/c^2)^(1/2)*c)^(1/2)/(
c^2-2*c*d+d^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\sin\left(fx+e\right)+a}}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+a\,\sin\left(e+f\,x\right)}}{\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))/sqrt(c + d*sin(e + f*x)), x)
```

3.568 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=45

$$-\frac{2a\cos(e+fx)}{f(c+d)\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}$$

```
[Out] -2*a*cos(f*x+e)/(c+d)/f/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2771}

$$-\frac{2a\cos(e+fx)}{f(c+d)\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a*Cos[e + f*x])/((c + d)*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e +
f*x]])
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^{3/2}}\,dx=-\frac{2a\cos(e+fx)}{(c+d)f\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}$$

Mathematica [A] time = 0.19, size = 84, normalized size = 1.87

$$-\frac{2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f(c+d)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x]])/((c +
 d)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]])
```

fricas [B] time = 0.45, size = 129, normalized size = 2.87

$$\frac{2\sqrt{a\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+c}\left(\cos\left(fx+e\right)-\sin\left(fx+e\right)+1\right)}{\left(cd+d^2\right)f\cos\left(fx+e\right)^2-\left(c^2+cd\right)f\cos\left(fx+e\right)-\left(c^2+2\,cd+d^2\right)f-\left(\left(cd+d^2\right)f\cos\left(fx+e\right)+\left(c^2+2\,cd+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] 2*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*(cos(f*x + e) - sin(f*x
 + e) + 1)/((c*d + d^2)*f*cos(f*x + e)^2 - (c^2 + c*d)*f*cos(f*x + e) - (c^
2 + 2*c*d + d^2)*f - ((c*d + d^2)*f*cos(f*x + e) + (c^2 + 2*c*d + d^2)*f)*s
in(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)-8*sqrt(2*a)*(-1024*d^4+2048*c*d^3-1
024*c^2*d^2)*sqrt(c*tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^4+d*tan(1/2*(1/2*f
*x+1/4*(2*exp(1)-pi)))^4+2*c*tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^2-6*d*tan
(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^2+c+d)*sign(cos(1/2*(f*x+exp(1))-1/4*pi))
*tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))/f/(2048*d^5-2048*c*d^4-2048*c^2*d^3+2
048*c^3*d^2)/(c*tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^4+d*tan(1/2*(1/2*f*x+1
/4*(2*exp(1)-pi)))^4+2*c*tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^2-6*d*tan(1/2
*(1/2*f*x+1/4*(2*exp(1)-pi)))^2+c+d)
```

maple [B] time = 0.31, size = 99, normalized size = 2.20

$$\frac{2\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{c+d\sin\left(fx+e\right)}\,\left(\left(\cos^2\left(fx+e\right)\right)d+c\sin\left(fx+e\right)+d\sin\left(fx+e\right)-c-d\right)}{f\cos\left(fx+e\right)\left(\left(\cos^2\left(fx+e\right)\right)d^2+c^2-d^2\right)\left(c+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] 2/f*(a*(1+sin(f*x+e)))^(1/2)*(c+d*sin(f*x+e))^(1/2)*(cos(f*x+e)^2*d+c*sin(f
*x+e)+d*sin(f*x+e)-c-d)/cos(f*x+e)/(cos(f*x+e)^2*d^2+c^2-d^2)/(c+d)
```

maxima [B] time = 1.03, size = 179, normalized size = 3.98

$$-\frac{2\left(\sqrt{a}\,c-\frac{\sqrt{a}\,(c-2\,d)\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{\sqrt{a}\,(c-2\,d)\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{\sqrt{a}\,c\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}\right)\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)}{\left(c+d+\frac{(c+d)\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)\left(c+\frac{2\,d\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{c\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -2*(sqrt(a)*c - sqrt(a)*(c - 2*d)*sin(f*x + e)/(cos(f*x + e) + 1) + sqrt(a)
*(c - 2*d)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - sqrt(a)*c*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/((c + d + (c
 + d)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(c + 2*d*sin(f*x + e)/(cos(f*x +
 e) + 1) + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)^(3/2)*f)
```

mupad [B] time = 9.00, size = 145, normalized size = 3.22

$$-\frac{4\,\left(2\,c\,\cos\left(e+f\,x\right)+d\,\sin\left(2\,e+2\,f\,x\right)\right)\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{c+d\,\mathrm{si}}}{f\,\left(c+d\right)\,\left(4\,c\,d+4\,c^2\,\sin\left(e+f\,x\right)+3\,d^2\,\sin\left(e+f\,x\right)+4\,c^2+2\,d^2-2\,d^2\,\cos\left(2\,e+2\,f\,x\right)-d^2\,\sin\left(3\,e+3\,f\,x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] -(4*(2*c*cos(e + f*x) + d*sin(2*e + 2*f*x))*(a*(sin(e + f*x) + 1))^(1/2)*(c
 + d*sin(e + f*x))^(1/2))/(f*(c + d)*(4*c*d + 4*c^2*sin(e + f*x) + 3*d^2*si
n(e + f*x) + 4*c^2 + 2*d^2 - 2*d^2*cos(2*e + 2*f*x) - d^2*sin(3*e + 3*f*x)
+ 8*c*d*sin(e + f*x) - 4*c*d*cos(2*e + 2*f*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}{\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))/(c + d*sin(e + f*x))**(3/2), x)
```

3.569 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{4a\cos(e+fx)}{3f(c+d)^2\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}-\frac{2a\cos(e+fx)}{3f(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{3/2}}$$

```
[Out] -2/3*a*cos(f*x+e)/(c+d)/f/(c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)-4/3
*a*cos(f*x+e)/(c+d)^2/f/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2772, 2771}

$$-\frac{4a\cos(e+fx)}{3f(c+d)^2\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}-\frac{2a\cos(e+fx)}{3f(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*a*Cos[e + f*x])/(3*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*
x])^(3/2)) - (4*a*Cos[e + f*x])/(3*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c + d*Sin[e + f*x]])
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{2a\cos(e+fx)}{3(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}} + \frac{2\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^{3/2}}\,dx}{3(c+d)}$$
$$= -\frac{2a\cos(e+fx)}{3(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}} - \frac{4a\cos(e+}{3(c+d)^2f\sqrt{a+a\sin(e+f}}$$

Mathematica [A] time = 0.27, size = 100, normalized size = 1.05

$$-\frac{2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(3c+2d\sin(e+fx)+d)}{3f(c+d)^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(3*c +
 d + 2*d*Sin[e + f*x]))/(3*(c + d)^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(c + d*Sin[e + f*x])^(3/2))
```

fricas [B] time = 0.46, size = 300, normalized size = 3.16

$$\frac{2\left(2\,d\cos\left(fx+e\right)^{2}+(3\,c+d\right.}{3\left(\left(c^{2}d^{2}+2\,cd^{3}+d^{4}\right)f\cos\left(fx+e\right)^{3}+\left(2\,c^{3}d+5\,c^{2}d^{2}+4\,cd^{3}+d^{4}\right)f\cos\left(fx+e\right)^{2}-\left(c^{4}+2\,c^{3}d+2\,c^{2}d^{2}+2\,c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] 2/3*(2*d*cos(f*x + e)^2 + (3*c + d)*cos(f*x + e) + (2*d*cos(f*x + e) - 3*c
+ d)*sin(f*x + e) + 3*c - d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) +
 c)/((c^2*d^2 + 2*c*d^3 + d^4)*f*cos(f*x + e)^3 + (2*c^3*d + 5*c^2*d^2 + 4*
c*d^3 + d^4)*f*cos(f*x + e)^2 - (c^4 + 2*c^3*d + 2*c^2*d^2 + 2*c*d^3 + d^4)
*f*cos(f*x + e) - (c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4)*f + ((c^2*d^2
 + 2*c*d^3 + d^4)*f*cos(f*x + e)^2 - 2*(c^3*d + 2*c^2*d^2 + c*d^3)*f*cos(f*
x + e) - (c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4)*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)8*sqrt(2*a)*((-(21233664*d^10-424673
28*c*d^9-21233664*c^2*d^8+84934656*c^3*d^7-21233664*c^4*d^6-42467328*c^5*d^
5+21233664*c^6*d^4)*tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^2/(-42467328*d^11+
42467328*c*d^10+127401984*c^2*d^9-127401984*c^3*d^8-127401984*c^4*d^7+12740
1984*c^5*d^6+42467328*c^6*d^5-42467328*c^7*d^4)-(-70778880*d^10+254803968*c
*d^9-268959744*c^2*d^8-56623104*c^3*d^7+297271296*c^4*d^6-198180864*c^5*d^5
+42467328*c^6*d^4)/(-42467328*d^11+42467328*c*d^10+127401984*c^2*d^9-127401
984*c^3*d^8-127401984*c^4*d^7+127401984*c^5*d^6+42467328*c^6*d^5-42467328*c
^7*d^4))*tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^2-(21233664*d^10-42467328*c*d
^9-21233664*c^2*d^8+84934656*c^3*d^7-21233664*c^4*d^6-42467328*c^5*d^5+2123
3664*c^6*d^4)/(-42467328*d^11+42467328*c*d^10+127401984*c^2*d^9-127401984*c
^3*d^8-127401984*c^4*d^7+127401984*c^5*d^6+42467328*c^6*d^5-42467328*c^7*d^
4))/sqrt(c*tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^4+d*tan(1/2*(1/2*f*x+1/4*(2
*exp(1)-pi)))^4+2*c*tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^2-6*d*tan(1/2*(1/2
*f*x+1/4*(2*exp(1)-pi)))^2+c+d)/(c*tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^4+d
*tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^4+2*c*tan(1/2*(1/2*f*x+1/4*(2*exp(1)-
pi)))^2-6*d*tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^2+c+d)*sign(cos(1/2*(f*x+e
xp(1))-1/4*pi))*tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))/f
```

maple [B] time = 0.34, size = 222, normalized size = 2.34

$$\frac{2\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{c+d\sin\left(fx+e\right)}\,\left(2\left(\cos^{4}\left(fx+e\right)\right)d^{3}+\sin\left(fx+e\right)\left(\cos^{2}\left(fx+e\right)\right)c\,d^{2}+\sin\left(fx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] 2/3/f*(a*(1+sin(f*x+e)))^(1/2)*(c+d*sin(f*x+e))^(1/2)*(2*cos(f*x+e)^4*d^3+s
in(f*x+e)*cos(f*x+e)^2*c*d^2+sin(f*x+e)*cos(f*x+e)^2*d^3+4*c^2*cos(f*x+e)^2
*d+c*cos(f*x+e)^2*d^2-3*cos(f*x+e)^2*d^3+3*c^3*sin(f*x+e)+5*c^2*d*sin(f*x+e
)+c*d^2*sin(f*x+e)-d^3*sin(f*x+e)-3*c^3-5*c^2*d-c*d^2+d^3)/cos(f*x+e)/(cos(
f*x+e)^2*d^2+c^2-d^2)^2/(c+d)^2
```

maxima [B] time = 1.05, size = 340, normalized size = 3.58

$$-\frac{2\left(\left(3c^2+cd\right)\sqrt{a}-\frac{\left(3c^2-9cd-2d^2\right)\sqrt{a}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{2\left(3c^2-4cd+3d^2\right)\sqrt{a}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{2\left(3c^2-4cd+3d^2\right)\sqrt{a}\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{\left(3c^2-9cd\right.}{\left(c\right.}}{3\left(c^2+2cd+d^2+\frac{2\left(c^2+2cd+d^2\right)\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{\left(c^2+2cd+d^2\right)\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}\right)\left(c+\frac{2d\sin(}{\cos(fx-}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -2/3*((3*c^2 + c*d)*sqrt(a) - (3*c^2 - 9*c*d - 2*d^2)*sqrt(a)*sin(f*x + e)/
(cos(f*x + e) + 1) + 2*(3*c^2 - 4*c*d + 3*d^2)*sqrt(a)*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 - 2*(3*c^2 - 4*c*d + 3*d^2)*sqrt(a)*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + (3*c^2 - 9*c*d - 2*d^2)*sqrt(a)*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 - (3*c^2 + c*d)*sqrt(a)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*(sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^2/((c^2 + 2*c*d + d^2 + 2*(c^2 + 2*c*d
+ d^2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + (c^2 + 2*c*d + d^2)*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4)*(c + 2*d*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2)^(5/2)*f)
```

mupad [B] time = 13.90, size = 353, normalized size = 3.72

$$-\frac{\sqrt{c+d\sin\left(e+fx\right)}\left(\frac{e^{e\,1i+fx\,1i}\sqrt{a+a\sin\left(e+fx\right)}\,8i}{3df\left(c\,1i+d\,1i\right)^2}+\frac{8\,e^{e\,4i+fx\,4i}\sqrt{a+a\sin\left(e+fx\right)}}{3df\left(c\,1i+d\,1i\right)^2}+\frac{8c\,e^{e\,2i+fx\,2i}\sqrt{a+a\sin\left(e+fx\right)}}{d^2f\left(c\,1i+d\,1i\right)^2}+\frac{c\,e^{e\,3i+fx\,3i}}{d^2}\right.}{e^{e\,5i+fx\,5i}-\frac{(c+d)^2\,1i}{\left(c\,1i+d\,1i\right)^2}-\frac{2\,e^{e\,3i+fx\,3i}\left(2c^2+2cd+d^2\right)}{d^2}+\frac{e^{e\,1i+fx\,1i}\left(4c+d\right)}{d}+\frac{e^{e\,2i+fx\,2i}\left(c+d\right)^2\left(2c^2+2cd+d^2\right)2i}{d^2\left(c\,1i+d\,1i\right)^2}-\frac{e^{e\,4i+fx\,4i}\left(c+\right.}{d\left(c\,1i+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] -((c + d*sin(e + f*x))^(1/2)*((exp(e*1i + f*x*1i)*(a + a*sin(e + f*x))^(1/2
)*8i)/(3*d*f*(c*1i + d*1i)^2) + (8*exp(e*4i + f*x*4i)*(a + a*sin(e + f*x))^
(1/2))/(3*d*f*(c*1i + d*1i)^2) + (8*c*exp(e*2i + f*x*2i)*(a + a*sin(e + f*x
))^(1/2))/(d^2*f*(c*1i + d*1i)^2) + (c*exp(e*3i + f*x*3i)*(a + a*sin(e + f*
x))^(1/2)*8i)/(d^2*f*(c*1i + d*1i)^2)))/(exp(e*5i + f*x*5i) - ((c + d)^2*1i
)/(c*1i + d*1i)^2 - (2*exp(e*3i + f*x*3i)*(2*c*d + 2*c^2 + d^2))/d^2 + (exp
(e*1i + f*x*1i)*(4*c + d))/d + (exp(e*2i + f*x*2i)*(c + d)^2*(2*c*d + 2*c^2
 + d^2)*2i)/(d^2*(c*1i + d*1i)^2) - (exp(e*4i + f*x*4i)*(c + d)^2*(4*c + d)
*1i)/(d*(c*1i + d*1i)^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}{\left(c+d\sin\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))/(c + d*sin(e + f*x))**(5/2), x)
```

3.570 $\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=142

$$-\frac{16a\cos(e+fx)}{15f(c+d)^3\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}-\frac{8a\cos(e+fx)}{15f(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{3/2}}-\frac{}{5f(c}$$

```
[Out] -2/5*a*cos(f*x+e)/(c+d)/f/(c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)-8/1
5*a*cos(f*x+e)/(c+d)^2/f/(c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)-16/1
5*a*cos(f*x+e)/(c+d)^3/f/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2772, 2771}

$$-\frac{16a\cos(e+fx)}{15f(c+d)^3\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}-\frac{8a\cos(e+fx)}{15f(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{3/2}}-\frac{}{5f(c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-2*a*Cos[e + f*x])/(5*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*
x])^(5/2)) - (8*a*Cos[e + f*x])/(15*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c
 + d*Sin[e + f*x])^(3/2)) - (16*a*Cos[e + f*x])/(15*(c + d)^3*f*Sqrt[a + a*
Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^{7/2}}\,dx = -\frac{2a\cos(e+fx)}{5(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}} + \frac{4\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^{5/2}}\,dx}{5(c+d)}$$

$$= -\frac{2a\cos(e+fx)}{5(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}} - \frac{8a\cos(e+}{15(c+d)^2f\sqrt{a+a\sin(e+f}$$

$$= -\frac{2a\cos(e+fx)}{5(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}} - \frac{8a\cos(e+}{15(c+d)^2f\sqrt{a+a\sin(e+f}$$

Mathematica [A] time = 0.39, size = 128, normalized size = 0.90

$$-\frac{2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(15c^2+4d(5c+d)\sin(e+fx)+10cd+8d^2\sin^2(e+\right.}{15f(c+d)^3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(15*c^
2 + 10*c*d + 3*d^2 + 4*d*(5*c + d)*Sin[e + f*x] + 8*d^2*Sin[e + f*x]^2))/(1
5*(c + d)^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c + d*Sin[e + f*x])^(5
/2))
```

fricas [B] time = 0.52, size = 556, normalized size = 3.92

$$\frac{}{15\left(\left(c^3d^3+3c^2d^4+3cd^5+d^6\right)f\cos\left(fx+e\right)^4-3\left(c^4d^2+3c^3d^3+3c^2d^4+cd^5\right)f\cos\left(fx+e\right)^3-\left(3c^5d+12c^4d\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] 2/15*(8*d^2*cos(f*x + e)^3 - 4*(5*c*d - d^2)*cos(f*x + e)^2 - 15*c^2 + 10*c
*d - 7*d^2 - (15*c^2 + 10*c*d + 11*d^2)*cos(f*x + e) - (8*d^2*cos(f*x + e)^
2 - 15*c^2 + 10*c*d - 7*d^2 + 4*(5*c*d + d^2)*cos(f*x + e))*sin(f*x + e))*s
qrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/((c^3*d^3 + 3*c^2*d^4 + 3*
```

```
c*d^5 + d^6)*f*cos(f*x + e)^4 - 3*(c^4*d^2 + 3*c^3*d^3 + 3*c^2*d^4 + c*d^5)
*f*cos(f*x + e)^3 - (3*c^5*d + 12*c^4*d^2 + 20*c^3*d^3 + 18*c^2*d^4 + 9*c*d
^5 + 2*d^6)*f*cos(f*x + e)^2 + (c^6 + 3*c^5*d + 6*c^4*d^2 + 10*c^3*d^3 + 9*
c^2*d^4 + 3*c*d^5)*f*cos(f*x + e) + (c^6 + 6*c^5*d + 15*c^4*d^2 + 20*c^3*d^
3 + 15*c^2*d^4 + 6*c*d^5 + d^6)*f - ((c^3*d^3 + 3*c^2*d^4 + 3*c*d^5 + d^6)*
f*cos(f*x + e)^3 + (3*c^4*d^2 + 10*c^3*d^3 + 12*c^2*d^4 + 6*c*d^5 + d^6)*f*
cos(f*x + e)^2 - (3*c^5*d + 9*c^4*d^2 + 10*c^3*d^3 + 6*c^2*d^4 + 3*c*d^5 +
d^6)*f*cos(f*x + e) - (c^6 + 6*c^5*d + 15*c^4*d^2 + 20*c^3*d^3 + 15*c^2*d^4
 + 6*c*d^5 + d^6)*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)8*sqrt(2*a)*((((-(54358179840000*d^1
6-108716359680000*c*d^15-163074539520000*c^2*d^14+434865438720000*c^3*d^13+
108716359680000*c^4*d^12-652298158080000*c^5*d^11+108716359680000*c^6*d^10+
434865438720000*c^7*d^9-163074539520000*c^8*d^8-108716359680000*c^9*d^7+543
58179840000*c^10*d^6)*tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^2/(-108716359680
000*d^17+108716359680000*c*d^16+543581798400000*c^2*d^15-543581798400000*c^
3*d^14-1087163596800000*c^4*d^13+1087163596800000*c^5*d^12+1087163596800000
*c^6*d^11-1087163596800000*c^7*d^10-543581798400000*c^8*d^9+543581798400000
*c^9*d^8+108716359680000*c^10*d^7-108716359680000*c^11*d^6)-(-3623878656000
00*d^16+1304596316160000*c*d^15-652298158080000*c^2*d^14-2899102924800000*c
^3*d^13+3913788948480000*c^4*d^12+869730877440000*c^5*d^11-4203699240960000
*c^6*d^10+1739461754880000*c^7*d^9+1087163596800000*c^8*d^8-101468602368000
0*c^9*d^7+217432719360000*c^10*d^6)/(-108716359680000*d^17+108716359680000*
c*d^16+543581798400000*c^2*d^15-543581798400000*c^3*d^14-1087163596800000*c
^4*d^13+1087163596800000*c^5*d^12+1087163596800000*c^6*d^11-108716359680000
0*c^7*d^10-543581798400000*c^8*d^9+543581798400000*c^9*d^8+108716359680000*
c^10*d^7-108716359680000*c^11*d^6))*tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^2-
(1021933780992000*d^16-4595078135808000*c*d^15+6443256250368000*c^2*d^14+75
3766760448000*c^3*d^13-10944113541120000*c^4*d^12+10465761558528000*c^5*d^1
1-1203127713792000*c^6*d^10-4812510855168000*c^7*d^9+4355902144512000*c^8*d
^8-1811939328000000*c^9*d^7+326149079040000*c^10*d^6)/(-108716359680000*d^1
7+108716359680000*c*d^16+543581798400000*c^2*d^15-543581798400000*c^3*d^14-
1087163596800000*c^4*d^13+1087163596800000*c^5*d^12+1087163596800000*c^6*d^
11-1087163596800000*c^7*d^10-543581798400000*c^8*d^9+543581798400000*c^9*d^
8+108716359680000*c^10*d^7-108716359680000*c^11*d^6))*tan(1/2*(1/2*f*x+1/4*
(2*exp(1)-pi)))^2-(-362387865600000*d^16+1304596316160000*c*d^15-6522981580
```

```
80000*c^2*d^14-2899102924800000*c^3*d^13+3913788948480000*c^4*d^12+86973087
7440000*c^5*d^11-4203699240960000*c^6*d^10+1739461754880000*c^7*d^9+1087163
596800000*c^8*d^8-1014686023680000*c^9*d^7+217432719360000*c^10*d^6)/(-1087
16359680000*d^17+108716359680000*c*d^16+543581798400000*c^2*d^15-5435817984
00000*c^3*d^14-1087163596800000*c^4*d^13+1087163596800000*c^5*d^12+10871635
96800000*c^6*d^11-1087163596800000*c^7*d^10-543581798400000*c^8*d^9+5435817
98400000*c^9*d^8+108716359680000*c^10*d^7-108716359680000*c^11*d^6))*tan(1/
2*(1/2*f*x+1/4*(2*exp(1)-pi)))^2-(54358179840000*d^16-108716359680000*c*d^1
5-163074539520000*c^2*d^14+434865438720000*c^3*d^13+108716359680000*c^4*d^1
2-652298158080000*c^5*d^11+108716359680000*c^6*d^10+434865438720000*c^7*d^9
-163074539520000*c^8*d^8-108716359680000*c^9*d^7+54358179840000*c^10*d^6)/(
-108716359680000*d^17+108716359680000*c*d^16+543581798400000*c^2*d^15-54358
1798400000*c^3*d^14-1087163596800000*c^4*d^13+1087163596800000*c^5*d^12+108
7163596800000*c^6*d^11-1087163596800000*c^7*d^10-543581798400000*c^8*d^9+54
3581798400000*c^9*d^8+108716359680000*c^10*d^7-108716359680000*c^11*d^6))/s
qrt(c*tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^4+d*tan(1/2*(1/2*f*x+1/4*(2*exp(
1)-pi)))^4+2*c*tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^2-6*d*tan(1/2*(1/2*f*x+
1/4*(2*exp(1)-pi)))^2+c+d)/(c*tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^4+d*tan(
1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^4+2*c*tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))
^2-6*d*tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))^2+c+d)^2*sign(cos(1/2*(f*x+exp(
1))-1/4*pi))*tan(1/2*(1/2*f*x+1/4*(2*exp(1)-pi)))/f
```

maple [B] time = 0.36, size = 430, normalized size = 3.03

$$\frac{2\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{c+d\sin\left(fx+e\right)}\,\left(19c^{3}\left(\cos^{2}\left(fx+e\right)\right)d^{2}-11\sin\left(fx+e\right)\left(\cos^{2}\left(fx+e\right)\right)d^{5}+25\left(c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(7/2),x)
```

```
[Out] 2/15/f*(a*(1+sin(f*x+e)))^(1/2)*(c+d*sin(f*x+e))^(1/2)*(4*sin(f*x+e)*cos(f*
x+e)^4*d^5+4*sin(f*x+e)*cos(f*x+e)^4*c*d^4+7*sin(f*x+e)*cos(f*x+e)^2*c^3*d^
2+3*sin(f*x+e)*cos(f*x+e)^2*c^2*d^3-15*sin(f*x+e)*cos(f*x+e)^2*c*d^4-11*c*d
^4-6*c^2*d^3-22*c^3*d^2-35*c^4*d-15*cos(f*x+e)^2*c^2*d^3-23*cos(f*x+e)^4*d^
5+22*cos(f*x+e)^2*d^5+8*cos(f*x+e)^6*d^5+7*d^5*sin(f*x+e)-2*c*cos(f*x+e)^4*
d^4+19*c^3*cos(f*x+e)^2*d^2+13*c*cos(f*x+e)^2*d^4+6*c^2*d^3*sin(f*x+e)+11*c
*d^4*sin(f*x+e)+21*cos(f*x+e)^4*c^2*d^3-11*sin(f*x+e)*cos(f*x+e)^2*d^5+25*c
os(f*x+e)^2*c^4*d+35*sin(f*x+e)*c^4*d+22*sin(f*x+e)*c^3*d^2-7*d^5-15*c^5+15
*c^5*sin(f*x+e))/cos(f*x+e)/(cos(f*x+e)^2*d^2+c^2-d^2)^3/(c+d)^3
```

maxima [B] time = 1.34, size = 544, normalized size = 3.83

$$-\frac{2\left(\left(15\,c^{3}+10\,c^{2}d+3\,cd^{2}\right)\sqrt{a}-\frac{\left(15\,c^{3}-60\,c^{2}d-25\,cd^{2}-6\,d^{3}\right)\sqrt{a}\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{\left(45\,c^{3}-40\,c^{2}d+93\,cd^{2}+10\,d^{3}\right)\sqrt{a}\,\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{5\left(9\,c^{3}-2\right.}{}}{15\left(c^{3}+3\,c^{2}d+3\,cd^{2}+d^{3}+\frac{3\left(c^{3}+3\right.}{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] -2/15*((15*c^3 + 10*c^2*d + 3*c*d^2)*sqrt(a) - (15*c^3 - 60*c^2*d - 25*c*d^
2 - 6*d^3)*sqrt(a)*sin(f*x + e)/(cos(f*x + e) + 1) + (45*c^3 - 40*c^2*d + 9
3*c*d^2 + 10*d^3)*sqrt(a)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*(9*c^3 -
22*c^2*d + 13*c*d^2 - 12*d^3)*sqrt(a)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
 5*(9*c^3 - 22*c^2*d + 13*c*d^2 - 12*d^3)*sqrt(a)*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 - (45*c^3 - 40*c^2*d + 93*c*d^2 + 10*d^3)*sqrt(a)*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5 + (15*c^3 - 60*c^2*d - 25*c*d^2 - 6*d^3)*sqrt(a)*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6 - (15*c^3 + 10*c^2*d + 3*c*d^2)*sqrt(a)*sin
(f*x + e)^7/(cos(f*x + e) + 1)^7)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)
^3/((c^3 + 3*c^2*d + 3*c*d^2 + d^3 + 3*(c^3 + 3*c^2*d + 3*c*d^2 + d^3)*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*(c^3 + 3*c^2*d + 3*c*d^2 + d^3)*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + (c^3 + 3*c^2*d + 3*c*d^2 + d^3)*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6)*(c + 2*d*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2)^(7/2)*f)
```

mupad [B] time = 16.35, size = 501, normalized size = 3.53

$$-\frac{\sqrt{c+d\,\sin\left(e+f\,x\right)}\left(\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,32\mathrm{i}}{15\,d\,f\,{\left(c+d\right)}^{3}}-\frac{32\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{15\,d\,f\,{\left(c+d\right)}^{3}}+\frac{\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\left(240\,c^{2}+80\,d^{2}\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{15\,d^{3}\,f\,{\left(c+d\right)}^{3}}\right.}{\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}+\frac{{\left(c\,1\mathrm{i}+d\,1\mathrm{i}\right)}^{3}}{{\left(c+d\right)}^{3}}-\frac{3\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\left(4\,c^{2}+2\,c\,d+d^{2}\right)}{d^{2}}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\left(6\,c+d\right)}{d}+\frac{\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\left(8\,c^{3}+12\,c^{2}\,d+12\,c\,d^{2}+3\,d^{3}\right)}{d^{3}}+\frac{\mathrm{e}^{e\,6\mathrm{i}+f}}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x))^(7/2),x)
```

```
[Out] -((c + d*sin(e + f*x))^(1/2)*((exp(e*1i + f*x*1i)*(a + a*sin(e + f*x))^(1/2
)*32i)/(15*d*f*(c + d)^3) - (32*exp(e*6i + f*x*6i)*(a + a*sin(e + f*x))^(1/
2))/(15*d*f*(c + d)^3) + (exp(e*4i + f*x*4i)*(240*c^2 + 80*d^2)*(a + a*sin(
e + f*x))^(1/2))/(15*d^3*f*(c + d)^3) - (exp(e*3i + f*x*3i)*(c^2*240i + d^2
*80i)*(a + a*sin(e + f*x))^(1/2))/(15*d^3*f*(c + d)^3) + (32*c*exp(e*2i + f
```

```
*x*2i)*(a + a*sin(e + f*x))^(1/2))/(3*d^2*f*(c + d)^3) - (c*exp(e*5i + f*x*
5i)*(a + a*sin(e + f*x))^(1/2)*32i)/(3*d^2*f*(c + d)^3)))/(exp(e*7i + f*x*7
i) + (c*1i + d*1i)^3/(c + d)^3 - (3*exp(e*5i + f*x*5i)*(2*c*d + 4*c^2 + d^2
))/d^2 - (exp(e*1i + f*x*1i)*(6*c + d))/d + (exp(e*3i + f*x*3i)*(12*c*d^2 +
 12*c^2*d + 8*c^3 + 3*d^3))/d^3 + (exp(e*6i + f*x*6i)*(c*6i + d*1i))/d - (3
*exp(e*2i + f*x*2i)*(c*1i + d*1i)^3*(2*c*d + 4*c^2 + d^2))/(d^2*(c + d)^3)
+ (exp(e*4i + f*x*4i)*(c*1i + d*1i)^3*(12*c*d^2 + 12*c^2*d + 8*c^3 + 3*d^3)
)/(d^3*(c + d)^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

3.571 $\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=285

$$\frac{5a^{3/2}(c-15d)(c+d)^3\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{64d^{3/2}f}-\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{7/2}}{4df\sqrt{a\sin(e+fx)+a}}+\frac{a^2(c-15d)\cos(\cdots}{24df\sqrt{\cdots}}$$

```
[Out] 5/64*a^(3/2)*(c-15*d)*(c+d)^3*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*
x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))/d^(3/2)/f+5/96*a^2*(c-15*d)*(c+d)*cos(f
*x+e)*(c+d*sin(f*x+e))^(3/2)/d/f/(a+a*sin(f*x+e))^(1/2)+1/24*a^2*(c-15*d)*c
os(f*x+e)*(c+d*sin(f*x+e))^(5/2)/d/f/(a+a*sin(f*x+e))^(1/2)-1/4*a^2*cos(f*x
+e)*(c+d*sin(f*x+e))^(7/2)/d/f/(a+a*sin(f*x+e))^(1/2)+5/64*a^2*(c-15*d)*(c+
d)^2*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2763, 21, 2770, 2775, 205}

$$\frac{5a^{3/2}(c-15d)(c+d)^3\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{64d^{3/2}f}-\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{7/2}}{4df\sqrt{a\sin(e+fx)+a}}+\frac{a^2(c-15d)\cos(\cdots}{24df\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (5*a^(3/2)*(c - 15*d)*(c + d)^3*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt
[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(64*d^(3/2)*f) + (5*a^2*(c
 - 15*d)*(c + d)^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(64*d*f*Sqrt[a +
a*Sin[e + f*x]]) + (5*a^2*(c - 15*d)*(c + d)*Cos[e + f*x]*(c + d*Sin[e + f*
x])^(3/2))/(96*d*f*Sqrt[a + a*Sin[e + f*x]]) + (a^2*(c - 15*d)*Cos[e + f*x]
*(c + d*Sin[e + f*x])^(5/2))/(24*d*f*Sqrt[a + a*Sin[e + f*x]]) - (a^2*Cos[e
 + f*x]*(c + d*Sin[e + f*x])^(7/2))/(4*d*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}\,dx = -\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{7/2}}{4df\sqrt{a+a\sin(e+fx)}} + \frac{\int \frac{\left(-\frac{1}{2}a^2(c-15d)-\frac{1}{2}a^2(c-15\right.}{\sqrt{a+}}}{}$$

$$= -\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{7/2}}{4df\sqrt{a+a\sin(e+fx)}} - \frac{(a(c-15d))\int\sqrt{a+a}}{}$$

$$= \frac{a^2(c-15d)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{24df\sqrt{a+a\sin(e+fx)}} - \frac{a^2\cos(e+fx)}{4df\sqrt{a}}$$

$$= \frac{5a^2(c-15d)(c+d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{96df\sqrt{a+a\sin(e+fx)}} + \frac{a^2(c-}{}$$

$$= \frac{5a^2(c-15d)(c+d)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{64df\sqrt{a+a\sin(e+fx)}} + \frac{5a^2(c-}{}$$

$$= \frac{5a^2(c-15d)(c+d)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{64df\sqrt{a+a\sin(e+fx)}} + \frac{5a^2(c-}{}$$

$$= \frac{5a^{3/2}(c-15d)(c+d)^3\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{64d^{3/2}f} + \frac{5a}{}$$

Mathematica [A] time = 1.37, size = 318, normalized size = 1.12

$$\frac{(a(\sin(e+fx)+1))^{3/2}\left(-\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c+d\sin(e+fx)}\left(15c^3+2d\left(59c^2+190cd+93d^2\right)\sin(e+fx)+455c^2d-4d^2(17c+15d)\,c\right.}{3d}\right.}{128f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*((-5*(c - 15*d)*(c + d)^3*(2*ArcTan[(Sqrt[2]*
Sqrt[d]*Sin[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] + ArcTanh[(Sqr
t[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] - Log[Sqr
t[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*x]]]))/d^(3/2
) - (2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]*(15*c
```

```
^3 + 455*c^2*d + 653*c*d^2 + 285*d^3 - 4*d^2*(17*c + 15*d)*Cos[2*(e + f*x)]
 + 2*d*(59*c^2 + 190*c*d + 93*d^2)*Sin[e + f*x] - 12*d^3*Sin[3*(e + f*x)]))
/(3*d)))/(128*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

fricas [B] time = 1.59, size = 1579, normalized size = 5.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] [-1/1536*(15*(a*c^4 - 12*a*c^3*d - 42*a*c^2*d^2 - 44*a*c*d^3 - 15*a*d^4 + (
a*c^4 - 12*a*c^3*d - 42*a*c^2*d^2 - 44*a*c*d^3 - 15*a*d^4)*cos(f*x + e) + (
a*c^4 - 12*a*c^3*d - 42*a*c^2*d^2 - 44*a*c*d^3 - 15*a*d^4)*sin(f*x + e))*sq
rt(-a/d)*log((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 +
4*a*c*d^3 + a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2*d^
2 - 14*a*c*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 + 9*a
*c*d^3 - 4*a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f*x + e)^4 - c^3*d + 17*c^
2*d^2 - 59*c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f*x + e)^3 - 2*(5*c^2*d^2
- 26*c*d^3 + 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c^2*d^2 + 31*c*d^3 - 25*d^
4)*cos(f*x + e) + (16*d^4*cos(f*x + e)^3 + c^3*d - 17*c^2*d^2 + 59*c*d^3 -
51*d^4 - 8*(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5*c^2*d^2 - 14*c*d^3 + 13*
d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x +
e) + c)*sqrt(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^3 + 28
9*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d + 6*a
*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 - 32*(5
*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*c^2*d^
2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f
*x + e) + 1)) - 8*(48*a*d^3*cos(f*x + e)^4 - 15*a*c^3 - 337*a*c^2*d - 341*a
*c*d^2 - 147*a*d^3 + 8*(17*a*c*d^2 + 15*a*d^3)*cos(f*x + e)^3 - 2*(59*a*c^2
*d + 122*a*c*d^2 + 63*a*d^3)*cos(f*x + e)^2 - (15*a*c^3 + 455*a*c^2*d + 721
*a*c*d^2 + 345*a*d^3)*cos(f*x + e) + (48*a*d^3*cos(f*x + e)^3 + 15*a*c^3 +
337*a*c^2*d + 341*a*c*d^2 + 147*a*d^3 - 8*(17*a*c*d^2 + 9*a*d^3)*cos(f*x +
e)^2 - 2*(59*a*c^2*d + 190*a*c*d^2 + 99*a*d^3)*cos(f*x + e))*sin(f*x + e))*
sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d*f*cos(f*x + e) + d*f*
sin(f*x + e) + d*f), -1/768*(15*(a*c^4 - 12*a*c^3*d - 42*a*c^2*d^2 - 44*a*c
*d^3 - 15*a*d^4 + (a*c^4 - 12*a*c^3*d - 42*a*c^2*d^2 - 44*a*c*d^3 - 15*a*d^
4)*cos(f*x + e) + (a*c^4 - 12*a*c^3*d - 42*a*c^2*d^2 - 44*a*c*d^3 - 15*a*d^
4)*sin(f*x + e))*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d -
 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*
x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c*d - a*d^2)*cos(f*x +
 e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x + e))) - 4*(48*a*d^3*c
os(f*x + e)^4 - 15*a*c^3 - 337*a*c^2*d - 341*a*c*d^2 - 147*a*d^3 + 8*(17*a*
c*d^2 + 15*a*d^3)*cos(f*x + e)^3 - 2*(59*a*c^2*d + 122*a*c*d^2 + 63*a*d^3)*
```

```
cos(f*x + e)^2 - (15*a*c^3 + 455*a*c^2*d + 721*a*c*d^2 + 345*a*d^3)*cos(f*x
 + e) + (48*a*d^3*cos(f*x + e)^3 + 15*a*c^3 + 337*a*c^2*d + 341*a*c*d^2 + 1
47*a*d^3 - 8*(17*a*c*d^2 + 9*a*d^3)*cos(f*x + e)^2 - 2*(59*a*c^2*d + 190*a*
c*d^2 + 99*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt
(d*sin(f*x + e) + c))/(d*f*cos(f*x + e) + d*f*sin(f*x + e) + d*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin\left(fx+e\right)+a\right)^{\frac{3}{2}} \left(d \sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(5/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(a+a \sin\left(fx+e\right)\right)^{\frac{3}{2}} \left(c+d \sin\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin\left(fx+e\right)+a\right)^{\frac{3}{2}} \left(d \sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(5/2),x)
[Out] int((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c+d*sin(f*x+e))**(5/2),x)
[Out] Timed out
```

3.572 $\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=228

$$\frac{a^{3/2}(c-11d)(c+d)^2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{8d^{3/2}f}-\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{3df\sqrt{a\sin(e+fx)+a}}+\frac{a^2(c-11d)\cos(e}{12df\sqrt{a}}$$

```
[Out] 1/8*a^(3/2)*(c-11*d)*(c+d)^2*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*x
+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))/d^(3/2)/f+1/12*a^2*(c-11*d)*cos(f*x+e)*(
c+d*sin(f*x+e))^(3/2)/d/f/(a+a*sin(f*x+e))^(1/2)-1/3*a^2*cos(f*x+e)*(c+d*si
n(f*x+e))^(5/2)/d/f/(a+a*sin(f*x+e))^(1/2)+1/8*a^2*(c-11*d)*(c+d)*cos(f*x+e
)*(c+d*sin(f*x+e))^(1/2)/d/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2763, 21, 2770, 2775, 205}

$$\frac{a^{3/2}(c-11d)(c+d)^2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{8d^{3/2}f}-\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{3df\sqrt{a\sin(e+fx)+a}}+\frac{a^2(c-11d)\cos(e}{12df\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (a^(3/2)*(c - 11*d)*(c + d)^2*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a
 + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(8*d^(3/2)*f) + (a^2*(c - 11
*d)*(c + d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(8*d*f*Sqrt[a + a*Sin[e
+ f*x]]) + (a^2*(c - 11*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(12*d*f
*Sqrt[a + a*Sin[e + f*x]]) - (a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/
(3*d*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}\,dx = -\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{3df\sqrt{a+a\sin(e+fx)}} + \frac{\int \frac{\left(-\frac{1}{2}a^2(c-11d)-\frac{1}{2}a^2(c-11}{\sqrt{a+}}}{}$$

$$= -\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{3df\sqrt{a+a\sin(e+fx)}} - \frac{(a(c-11d))\int\sqrt{a+a}}{}$$

$$= \frac{a^2(c-11d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{12df\sqrt{a+a\sin(e+fx)}} - \frac{a^2\cos(e+fx}{3df\sqrt{a}}$$

$$= \frac{a^2(c-11d)(c+d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{8df\sqrt{a+a\sin(e+fx)}} + \frac{a^2(c-11}{}$$

$$= \frac{a^2(c-11d)(c+d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{8df\sqrt{a+a\sin(e+fx)}} + \frac{a^2(c-11}{}$$

$$= \frac{a^{3/2}(c-11d)(c+d)^2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{8d^{3/2}f} + \frac{a^2(}{}$$

Mathematica [A] time = 0.88, size = 281, normalized size = 1.23

$$\frac{(a(\sin(e+fx)+1))^{3/2}\left(\frac{(c-11d)(c+d)^2\left(-2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{4}(2e+2fx-\pi)\right)}{\sqrt{c+d\sin(e+fx)}}\right)+\log\left(\sqrt{c+d\sin(e+fx)}+\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)-\tanh^{-1}\left(\frac{\sqrt{2}\ }{}\right.\right)}{d^{3/2}}\right.}{16f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\mathrm{co}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*(((c - 11*d)*(c + d)^2*(-2*ArcTan[(Sqrt[2]*Sq
rt[d]*Sin[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] - ArcTanh[(Sqrt[
2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] + Log[Sqrt[
2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*x]]]))/d^(3/2)
- (2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]*(3*c^2
+ 52*c*d + 37*d^2 - 4*d^2*Cos[2*(e + f*x)] + 2*d*(7*c + 11*d)*Sin[e + f*x])
)/(3*d)))/(16*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

fricas [B] time = 1.07, size = 1337, normalized size = 5.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [-1/192*(3*(a*c^3 - 9*a*c^2*d - 21*a*c*d^2 - 11*a*d^3 + (a*c^3 - 9*a*c^2*d
- 21*a*c*d^2 - 11*a*d^3)*cos(f*x + e) + (a*c^3 - 9*a*c^2*d - 21*a*c*d^2 - 1
1*a*d^3)*sin(f*x + e))*sqrt(-a/d)*log((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4
*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*
x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*
c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f
*x + e)^4 - c^3*d + 17*c^2*d^2 - 59*c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f
*x + e)^3 - 2*(5*c^2*d^2 - 26*c*d^3 + 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c
^2*d^2 + 31*c*d^3 - 25*d^4)*cos(f*x + e) + (16*d^4*cos(f*x + e)^3 + c^3*d -
 17*c^2*d^2 + 59*c*d^3 - 51*d^4 - 8*(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5
*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x +
e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c
^2*d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x + e)^4
+ a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^
4)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 +
 32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x +
e))/(cos(f*x + e) + sin(f*x + e) + 1)) - 8*(8*a*d^2*cos(f*x + e)^3 - 3*a*c^
2 - 38*a*c*d - 19*a*d^2 - 14*(a*c*d + a*d^2)*cos(f*x + e)^2 - (3*a*c^2 + 52
*a*c*d + 41*a*d^2)*cos(f*x + e) - (8*a*d^2*cos(f*x + e)^2 - 3*a*c^2 - 38*a*
c*d - 19*a*d^2 + 2*(7*a*c*d + 11*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*
sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d*f*cos(f*x + e) + d*f*sin(f*x
 + e) + d*f), -1/96*(3*(a*c^3 - 9*a*c^2*d - 21*a*c*d^2 - 11*a*d^3 + (a*c^3
- 9*a*c^2*d - 21*a*c*d^2 - 11*a*d^3)*cos(f*x + e) + (a*c^3 - 9*a*c^2*d - 21
*a*c*d^2 - 11*a*d^3)*sin(f*x + e))*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x + e)
^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e)
+ a)*sqrt(d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c*d
- a*d^2)*cos(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x + e)
)) - 4*(8*a*d^2*cos(f*x + e)^3 - 3*a*c^2 - 38*a*c*d - 19*a*d^2 - 14*(a*c*d
+ a*d^2)*cos(f*x + e)^2 - (3*a*c^2 + 52*a*c*d + 41*a*d^2)*cos(f*x + e) - (8
*a*d^2*cos(f*x + e)^2 - 3*a*c^2 - 38*a*c*d - 19*a*d^2 + 2*(7*a*c*d + 11*a*d
^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e
) + c))/(d*f*cos(f*x + e) + d*f*sin(f*x + e) + d*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(a + a\sin\left(fx + e\right)\right)^{\frac{3}{2}} \left(c + d\sin\left(fx + e\right)\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx + e\right) + a\right)^{\frac{3}{2}}\left(d\sin\left(fx + e\right) + c\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a + a\,\sin\left(e + f\,x\right)\right)}^{3/2}\,{\left(c + d\,\sin\left(e + f\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.573 $\int (a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=171

$$\frac{a^{3/2}(c-7d)(c+d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{4d^{3/2}f}-\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2df\sqrt{a\sin(e+fx)+a}}+\frac{a^2(c-7d)\cos(e+fx}{4df\sqrt{a\sin(e}}$$

```
[Out] 1/4*a^(3/2)*(c-7*d)*(c+d)*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*x+e)
)^(1/2)/(c+d*sin(f*x+e))^(1/2))/d^(3/2)/f-1/2*a^2*cos(f*x+e)*(c+d*sin(f*x+e
))^(3/2)/d/f/(a+a*sin(f*x+e))^(1/2)+1/4*a^2*(c-7*d)*cos(f*x+e)*(c+d*sin(f*x
+e))^(1/2)/d/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2763, 21, 2770, 2775, 205}

$$\frac{a^{3/2}(c-7d)(c+d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{4d^{3/2}f}-\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2df\sqrt{a\sin(e+fx)+a}}+\frac{a^2(c-7d)\cos(e+fx}{4df\sqrt{a\sin(e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (a^(3/2)*(c - 7*d)*(c + d)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a +
a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(4*d^(3/2)*f) + (a^2*(c - 7*d)*
Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*d*f*Sqrt[a + a*Sin[e + f*x]]) - (
a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(2*d*f*Sqrt[a + a*Sin[e + f*x]
])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}\,dx = -\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2df\sqrt{a+a\sin(e+fx)}} + \frac{\int \frac{\left(-\frac{1}{2}a^2(c-7d)-\frac{1}{2}a^2(c-7d)\sin(\ldots\right.}{\sqrt{a+a\sin(\ldots}}}{2d\ldots}$$

$$= -\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2df\sqrt{a+a\sin(e+fx)}} - \frac{(a(c-7d))\int\sqrt{a+a\sin(\ldots}}{\ldots}$$

$$= \frac{a^2(c-7d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4df\sqrt{a+a\sin(e+fx)}} - \frac{a^2\cos(e+fx)(c+d\ldots}{2df\sqrt{a+a\sin\ldots}}$$

$$= \frac{a^2(c-7d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4df\sqrt{a+a\sin(e+fx)}} - \frac{a^2\cos(e+fx)(c+d\ldots}{2df\sqrt{a+a\sin\ldots}}$$

$$= \frac{a^{3/2}(c-7d)(c+d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{4d^{3/2}f} + \frac{a^2(c-7\ldots}{\ldots}$$

Mathematica [A] time = 0.59, size = 247, normalized size = 1.44

$$\frac{(a(\sin(e+fx)+1))^{3/2}\left(\frac{(c-7d)(c+d)\left(-2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{4}(2e+2fx-\pi)\right)}{\sqrt{c+d\sin(e+fx)}}\right)+\log\left(\sqrt{c+d\sin(e+fx)}+\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)-\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{d}\cos\ldots}{\sqrt{c+\ldots}}\right.\ldots}{d^{3/2}}\ldots\right)}{8f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*(((c - 7*d)*(c + d)*(-2*ArcTan[(Sqrt[2]*Sqrt[
d]*Sin[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] - ArcTanh[(Sqrt[2]*
Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] + Log[Sqrt[2]*
Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*x]]]))/d^(3/2) - (
2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]*(c + 7*d +
 2*d*Sin[e + f*x]))/d))/(8*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

fricas [B] time = 1.00, size = 1127, normalized size = 6.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [-1/32*((a*c^2 - 6*a*c*d - 7*a*d^2 + (a*c^2 - 6*a*c*d - 7*a*d^2)*cos(f*x +
e) + (a*c^2 - 6*a*c*d - 7*a*d^2)*sin(f*x + e))*sqrt(-a/d)*log((128*a*d^4*co
s(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2
*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)
*cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x
+ e)^2 - 8*(16*d^4*cos(f*x + e)^4 - c^3*d + 17*c^2*d^2 - 59*c*d^3 + 51*d^4
+ 24*(c*d^3 - d^4)*cos(f*x + e)^3 - 2*(5*c^2*d^2 - 26*c*d^3 + 33*d^4)*cos(f
*x + e)^2 - (c^3*d - 7*c^2*d^2 + 31*c*d^3 - 25*d^4)*cos(f*x + e) + (16*d^4*
cos(f*x + e)^3 + c^3*d - 17*c^2*d^2 + 59*c*d^3 - 51*d^4 - 8*(3*c*d^3 - 5*d^
4)*cos(f*x + e)^2 - 2*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e))*sin(f*x
 + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-a/d) + (a*c^
4 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (1
28*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d
^4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5
*a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*
cos(f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)) + 8*(2*a*d*c
os(f*x + e)^2 + a*c + 5*a*d + (a*c + 7*a*d)*cos(f*x + e) + (2*a*d*cos(f*x +
 e) - a*c - 5*a*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x +
e) + c))/(d*f*cos(f*x + e) + d*f*sin(f*x + e) + d*f), -1/16*((a*c^2 - 6*a*c
*d - 7*a*d^2 + (a*c^2 - 6*a*c*d - 7*a*d^2)*cos(f*x + e) + (a*c^2 - 6*a*c*d
- 7*a*d^2)*sin(f*x + e))*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 +
 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(
d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c*d - a*d^2)*c
os(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x + e))) + 4*(2*
a*d*cos(f*x + e)^2 + a*c + 5*a*d + (a*c + 7*a*d)*cos(f*x + e) + (2*a*d*cos(
f*x + e) - a*c - 5*a*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f
*x + e) + c))/(d*f*cos(f*x + e) + d*f*sin(f*x + e) + d*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin\left(fx+e\right)+a\right)^{\frac{3}{2}} \sqrt{d \sin\left(fx+e\right)+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(a+a \sin\left(fx+e\right)\right)^{\frac{3}{2}} \sqrt{c+d \sin\left(fx+e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin\left(fx+e\right)+a\right)^{\frac{3}{2}} \sqrt{d \sin\left(fx+e\right)+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,\sqrt{c+d\,\sin\left(e+f\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\sin{\left(e + f x \right)} + 1\right)\right)^{\frac{3}{2}} \sqrt{c + d \sin{\left(e + f x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*sqrt(c + d*sin(e + f*x)), x)
```

3.574 $\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=111

$$\frac{a^{3/2}(c-3d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{d^{3/2}f}-\frac{a^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{df\sqrt{a\sin(e+fx)+a}}$$

```
[Out] a^(3/2)*(c-3*d)*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c
+d*sin(f*x+e))^(1/2))/d^(3/2)/f-a^2*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d/f/(
a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2763, 21, 2775, 205}

$$\frac{a^{3/2}(c-3d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{d^{3/2}f}-\frac{a^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{df\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (a^(3/2)*(c - 3*d)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e
+ f*x]]*Sqrt[c + d*Sin[e + f*x]])]])/(d^(3/2)*f) - (a^2*Cos[e + f*x]*Sqrt[c
+ d*Sin[e + f*x]])/(d*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
```

```
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c+d\sin(e+fx)}}\,dx &= -\frac{a^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{df\sqrt{a+a\sin(e+fx)}}+\frac{\int \frac{-\frac{1}{2}a^2(c-3d)-\frac{1}{2}a^2(c-3d)\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{d}\\
&= -\frac{a^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{df\sqrt{a+a\sin(e+fx)}}-\frac{(a(c-3d))\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx}{2d}\\
&= -\frac{a^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{df\sqrt{a+a\sin(e+fx)}}+\frac{\left(a^2(c-3d)\right)\text{Subst}\left(\int \frac{1}{a+dx^2}\,dx,x,\frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{df}\\
&= \frac{a^{3/2}(c-3d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{d^{3/2}f}-\frac{a^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{df\sqrt{a+a\sin(e+fx)}}
\end{aligned}$$

Mathematica [B] time = 0.58, size = 301, normalized size = 2.71

$$\frac{(a(\sin(e+fx)+1))^{3/2}\left(2\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)\sqrt{c+d\sin(e+fx)}-2(c-3d)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{4}(2e+2fx-\pi)\right)}{\sqrt{c+d\sin(e+fx)}}\right)-2\sqrt{d}\ \cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*(-2*(c - 3*d)*ArcTan[(Sqrt[2]*Sqrt[d]*Sin[(2*
e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] - (c - 3*d)*ArcTanh[(Sqrt[2]*
Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] + c*Log[Sqrt[2
]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*x]]] - 3*d*Log[S
qrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*x]]] - 2*Sq
rt[d]*Cos[(e + f*x)/2]*Sqrt[c + d*Sin[e + f*x]] + 2*Sqrt[d]*Sin[(e + f*x)/2
]*Sqrt[c + d*Sin[e + f*x]]))/(2*d^(3/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2])^3)
```

fricas [B] time = 0.93, size = 989, normalized size = 8.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [-1/8*((a*c - 3*a*d + (a*c - 3*a*d)*cos(f*x + e) + (a*c - 3*a*d)*sin(f*x +
e))*sqrt(-a/d)*log((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*
d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*
c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2
 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f*x + e)^4 - c^3*d +
 17*c^2*d^2 - 59*c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f*x + e)^3 - 2*(5*c^
2*d^2 - 26*c*d^3 + 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c^2*d^2 + 31*c*d^3 -
 25*d^4)*cos(f*x + e) + (16*d^4*cos(f*x + e)^3 + c^3*d - 17*c^2*d^2 + 59*c*
d^3 - 51*d^4 - 8*(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5*c^2*d^2 - 14*c*d^3
 + 13*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(
f*x + e) + c)*sqrt(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^
3 + 289*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d
 + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 -
 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*
c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e) +
 sin(f*x + e) + 1)) + 8*(a*cos(f*x + e) - a*sin(f*x + e) + a)*sqrt(a*sin(f*
x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d*f*cos(f*x + e) + d*f*sin(f*x + e)
+ d*f), -1/4*((a*c - 3*a*d + (a*c - 3*a*d)*cos(f*x + e) + (a*c - 3*a*d)*sin
(f*x + e))*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2
 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
 + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c*d - a*d^2)*cos(f*x + e)*si
n(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x + e))) + 4*(a*cos(f*x + e) -
 a*sin(f*x + e) + a)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d*
f*cos(f*x + e) + d*f*sin(f*x + e) + d*f)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{c+d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/sqrt(d*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}{\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)/sqrt(c + d*sin(e + f*x)), x)
```

3.575 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=117

$$\frac{2a^2(c-d)\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}-\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}\right)}{d^{3/2}f}$$

```
[Out] -2*a^(3/2)*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*si
n(f*x+e))^(1/2))/d^(3/2)/f+2*a^2*(c-d)*cos(f*x+e)/d/(c+d)/f/(a+a*sin(f*x+e)
)^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2762, 21, 2775, 205}

$$\frac{2a^2(c-d)\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}-\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}\right)}{d^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a^(3/2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c + d*Sin[e + f*x]])])/(d^(3/2)*f) + (2*a^2*(c - d)*Cos[e + f*x])/(d*
(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
```

```
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}} - \frac{(2a)\int \frac{-\frac{1}{2}a(c+d)-\frac{1}{2}a(c+d)\sin(e+fx}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+f}}}{d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}} + \frac{a\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx}{d}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}} - \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{a+dx^2}\,dx, x, \frac{}{\sqrt{a}}\right.}{df}$$

$$= -\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{d^{3/2}f} + \frac{2a^2(c-d)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}\sqrt{c+d\,\text{s}}}$$

Mathematica [B] time = 7.41, size = 377, normalized size = 3.22

$$\frac{(a(\sin(e+fx)+1))^{3/2}\left(-2c\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)+2c\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)+2(c+d)\sqrt{c+d\sin(e+fx)}\tan^{-1}\left(\frac{\sqrt{2}}{}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*(2*c*Sqrt[d]*Cos[(e + f*x)/2] - 2*d^(3/2)*Cos
[(e + f*x)/2] - 2*c*Sqrt[d]*Sin[(e + f*x)/2] + 2*d^(3/2)*Sin[(e + f*x)/2] +
 2*(c + d)*ArcTan[(Sqrt[2]*Sqrt[d]*Sin[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Si
n[e + f*x]]]*Sqrt[c + d*Sin[e + f*x]] + (c + d)*ArcTanh[(Sqrt[2]*Sqrt[d]*Co
s[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]]*Sqrt[c + d*Sin[e + f*x]]
 - c*Log[Sqrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*x
]]]*Sqrt[c + d*Sin[e + f*x]] - d*Log[Sqrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)
/4] + Sqrt[c + d*Sin[e + f*x]]]*Sqrt[c + d*Sin[e + f*x]]))/(d^(3/2)*(c + d)
*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c + d*Sin[e + f*x]])
```

fricas [B] time = 0.87, size = 1297, normalized size = 11.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [-1/4*((a*c^2 + 2*a*c*d + a*d^2 - (a*c*d + a*d^2)*cos(f*x + e)^2 + (a*c^2 +
 a*c*d)*cos(f*x + e) + (a*c^2 + 2*a*c*d + a*d^2 + (a*c*d + a*d^2)*cos(f*x +
 e))*sin(f*x + e))*sqrt(-a/d)*log((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*c
^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x +
e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3*
d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f*x +
 e)^4 - c^3*d + 17*c^2*d^2 - 59*c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f*x +
 e)^3 - 2*(5*c^2*d^2 - 26*c*d^3 + 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c^2*d
^2 + 31*c*d^3 - 25*d^4)*cos(f*x + e) + (16*d^4*cos(f*x + e)^3 + c^3*d - 17*
c^2*d^2 + 59*c*d^3 - 51*d^4 - 8*(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5*c^2
*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) +
 a)*sqrt(d*sin(f*x + e) + c)*sqrt(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c^2*d
^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x + e)^4 + a*
c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4)*c
os(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 + 32*
(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e))/
(cos(f*x + e) + sin(f*x + e) + 1)) + 8*(a*c - a*d + (a*c - a*d)*cos(f*x + e
) - (a*c - a*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c))/((c*d^2 + d^3)*f*cos(f*x + e)^2 - (c^2*d + c*d^2)*f*cos(f*x + e) - (c
^2*d + 2*c*d^2 + d^3)*f - ((c*d^2 + d^3)*f*cos(f*x + e) + (c^2*d + 2*c*d^2
+ d^3)*f)*sin(f*x + e)), -1/2*((a*c^2 + 2*a*c*d + a*d^2 - (a*c*d + a*d^2)*c
os(f*x + e)^2 + (a*c^2 + a*c*d)*cos(f*x + e) + (a*c^2 + 2*a*c*d + a*d^2 + (
a*c*d + a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a/d)*arctan(1/4*(8*d^2*cos(
f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f
*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (
3*a*c*d - a*d^2)*cos(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(
f*x + e))) + 4*(a*c - a*d + (a*c - a*d)*cos(f*x + e) - (a*c - a*d)*sin(f*x
```

```
+ e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((c*d^2 + d^3)*f*c
os(f*x + e)^2 - (c^2*d + c*d^2)*f*cos(f*x + e) - (c^2*d + 2*c*d^2 + d^3)*f
- ((c*d^2 + d^3)*f*cos(f*x + e) + (c^2*d + 2*c*d^2 + d^3)*f)*sin(f*x + e))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [B] time = 0.56, size = 6630, normalized size = 56.67

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a \sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(d \sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}{\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)/(c + d*sin(e + f*x))**(3/2), x)
```

3.576 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=115

$$\frac{2a^2(c-d)\cos(e+fx)}{3df(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{3/2}}-\frac{2a^2(c+5d)\cos(e+fx)}{3df(c+d)^2\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}$$

```
[Out] 2/3*a^2*(c-d)*cos(f*x+e)/d/(c+d)/f/(c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^
(1/2)-2/3*a^2*(c+5*d)*cos(f*x+e)/d/(c+d)^2/f/(a+a*sin(f*x+e))^(1/2)/(c+d*si
n(f*x+e))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2762, 21, 2771}

$$\frac{2a^2(c-d)\cos(e+fx)}{3df(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{3/2}}-\frac{2a^2(c+5d)\cos(e+fx)}{3df(c+d)^2\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*a^2*(c - d)*Cos[e + f*x])/(3*d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d
*Sin[e + f*x])^(3/2)) - (2*a^2*(c + 5*d)*Cos[e + f*x])/(3*d*(c + d)^2*f*Sqr
t[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)}{3d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}} - \frac{(2a)\int \frac{-\frac{1}{2}a(c+5d)-\frac{1}{2}a(c+5d)\sin(e}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+}}{3d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{3d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}} + \frac{(a(c+5d))\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^{3/2}}}{3d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{3d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}} - \frac{2a^2(c+5d)\cos}{3d(c+d)^2f\sqrt{a+a\sin(e+f}}$$

Mathematica [A] time = 0.59, size = 104, normalized size = 0.90

$$-\frac{2a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)((c+5d)\sin(e+fx)+5c+d)}{3f(c+d)^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(5*c
 + d + (c + 5*d)*Sin[e + f*x]))/(3*(c + d)^2*f*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])*(c + d*Sin[e + f*x])^(3/2))
```

fricas [B] time = 0.50, size = 323, normalized size = 2.81

$$\frac{2\left((ac+5\,ad)\cos\left(fx+e\right)^2+4\,ac-4\,ad+(\right.}{3\left(\left(c^2d^2+2\,cd^3+d^4\right)f\cos\left(fx+e\right)^3+\left(2\,c^3d+5\,c^2d^2+4\,cd^3+d^4\right)f\cos\left(fx+e\right)^2-\left(c^4+2\,c^3d+2\,c^2d^2+2\,cd^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] 2/3*((a*c + 5*a*d)*cos(f*x + e)^2 + 4*a*c - 4*a*d + (5*a*c + a*d)*cos(f*x +
 e) - (4*a*c - 4*a*d - (a*c + 5*a*d)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin
(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/((c^2*d^2 + 2*c*d^3 + d^4)*f*cos(f*
x + e)^3 + (2*c^3*d + 5*c^2*d^2 + 4*c*d^3 + d^4)*f*cos(f*x + e)^2 - (c^4 +
2*c^3*d + 2*c^2*d^2 + 2*c*d^3 + d^4)*f*cos(f*x + e) - (c^4 + 4*c^3*d + 6*c^
2*d^2 + 4*c*d^3 + d^4)*f + ((c^2*d^2 + 2*c*d^3 + d^4)*f*cos(f*x + e)^2 - 2*
(c^3*d + 2*c^2*d^2 + c*d^3)*f*cos(f*x + e) - (c^4 + 4*c^3*d + 6*c^2*d^2 + 4
*c*d^3 + d^4)*f)*sin(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [B] time = 0.31, size = 345, normalized size = 3.00

$$-\frac{2\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\sqrt{c+d\sin\left(fx+e\right)}\,\left(\sin\left(fx+e\right)\left(\cos^{4}\left(fx+e\right)\right)c\,d^{2}+5\sin\left(fx+e\right)\left(\cos^{4}\left(fx+e\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] -2/3/f*(a*(1+sin(f*x+e)))^(3/2)*(c+d*sin(f*x+e))^(1/2)*(sin(f*x+e)*cos(f*x+
e)^4*c*d^2+5*sin(f*x+e)*cos(f*x+e)^4*d^3-2*c^2*cos(f*x+e)^4*d-7*c*cos(f*x+e
)^4*d^2-9*cos(f*x+e)^4*d^3-sin(f*x+e)*cos(f*x+e)^2*c^3+sin(f*x+e)*cos(f*x+e
)^2*c^2*d-11*sin(f*x+e)*cos(f*x+e)^2*c*d^2-13*sin(f*x+e)*cos(f*x+e)^2*d^3-3
*c^3*cos(f*x+e)^2-5*c^2*cos(f*x+e)^2*d+15*c*cos(f*x+e)^2*d^2+17*cos(f*x+e)^
2*d^3-8*c^3*sin(f*x+e)-8*c^2*d*sin(f*x+e)+8*c*d^2*sin(f*x+e)+8*d^3*sin(f*x+
e)+8*c^3+8*c^2*d-8*c*d^2-8*d^3)/cos(f*x+e)^3/(cos(f*x+e)^2*d^2+c^2-d^2)^2/(
c+d)^2
```

maxima [B] time = 1.27, size = 307, normalized size = 2.67

$$-\frac{2\left(\left(5\,c^{2}+cd\right)a^{\frac{3}{2}}-\frac{\left(3\,c^{2}-19\,cd-2\,d^{2}\right)a^{\frac{3}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{2\left(4\,c^{2}-7\,cd+9\,d^{2}\right)a^{\frac{3}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{2\left(4\,c^{2}-7\,cd+9\,d^{2}\right)a^{\frac{3}{2}}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{\left(3\,c^{2}-19\,cd-\cdots\right.}{\left(\cos\left(\cdots\right.\right.}}{3\left(c^{2}+2\,cd+d^{2}+\frac{\left(c^{2}+2\,cd+d^{2}\right)\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)\left(c+\frac{2\,d\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{c\sin\left(f\cdots\right.}{\left(\cos\left(fx+\cdots\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -2/3*((5*c^2 + c*d)*a^(3/2) - (3*c^2 - 19*c*d - 2*d^2)*a^(3/2)*sin(f*x + e)
/(cos(f*x + e) + 1) + 2*(4*c^2 - 7*c*d + 9*d^2)*a^(3/2)*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 - 2*(4*c^2 - 7*c*d + 9*d^2)*a^(3/2)*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + (3*c^2 - 19*c*d - 2*d^2)*a^(3/2)*sin(f*x + e)^4/(cos(f*x +
e) + 1)^4 - (5*c^2 + c*d)*a^(3/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*(sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/((c^2 + 2*c*d + d^2 + (c^2 + 2*c*d +
d^2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(c + 2*d*sin(f*x + e)/(cos(f*x +
e) + 1) + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)^(5/2)*f)
```

mupad [B] time = 14.40, size = 387, normalized size = 3.37

$$-\frac{\sqrt{c+d\,\sin\left(e+f\,x\right)}\,\left(\frac{a\,{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,\left(c+5\,d\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,4{}\mathrm{i}}{3\,d^2\,f\,{\left(c\,1{}\mathrm{i}+d\,1{}\mathrm{i}\right)}^2}+\frac{a\,{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,\left(3\,c-d\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,4{}\mathrm{i}}{d^2\,f\,{\left(c\,1{}\mathrm{i}+d\,1{}\mathrm{i}\right)}^2}-\frac{a\,{\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}\,\left(c\,3{}\mathrm{i}-d\,1{}\mathrm{i}\right)\,\sqrt{a+a\,\mathrm{si}\cdots}}{d^2\,f\,{\left(c\,1{}\mathrm{i}+d\,1{}\mathrm{i}\right)}^2}\cdots\right.}{{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}-\frac{{\left(c+d\right)}^2\,1{}\mathrm{i}}{{\left(c\,1{}\mathrm{i}+d\,1{}\mathrm{i}\right)}^2}-\frac{2\,{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,\left(2\,c^2+2\,c\,d+d^2\right)}{d^2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,\left(4\,c+d\right)}{d}+\frac{{\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}\,{\left(c+d\right)}^2\,\left(2\,c^2+2\,c\,d+d^2\right)\,2{}\mathrm{i}}{d^2\,{\left(c\,1{}\mathrm{i}+d\,1{}\mathrm{i}\right)}^2}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] -((c + d*sin(e + f*x))^(1/2)*((a*exp(e*1i + f*x*1i)*(c + 5*d)*(a + a*sin(e
+ f*x))^(1/2)*4i)/(3*d^2*f*(c*1i + d*1i)^2) + (a*exp(e*3i + f*x*3i)*(3*c -
d)*(a + a*sin(e + f*x))^(1/2)*4i)/(d^2*f*(c*1i + d*1i)^2) - (a*exp(e*2i + f
*x*2i)*(c*3i - d*1i)*(a + a*sin(e + f*x))^(1/2)*4i)/(d^2*f*(c*1i + d*1i)^2)
 - (a*exp(e*4i + f*x*4i)*(c*1i + d*5i)*(a + a*sin(e + f*x))^(1/2)*4i)/(3*d^
2*f*(c*1i + d*1i)^2)))/(exp(e*5i + f*x*5i) - ((c + d)^2*1i)/(c*1i + d*1i)^2
 - (2*exp(e*3i + f*x*3i)*(2*c*d + 2*c^2 + d^2))/d^2 + (exp(e*1i + f*x*1i)*(
4*c + d))/d + (exp(e*2i + f*x*2i)*(c + d)^2*(2*c*d + 2*c^2 + d^2)*2i)/(d^2*
(c*1i + d*1i)^2) - (exp(e*4i + f*x*4i)*(c + d)^2*(4*c + d)*1i)/(d*(c*1i + d
*1i)^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}{\left(c+d\sin\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)/(c + d*sin(e + f*x))**(5/2), x)
```

3.577 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=172

$$-\frac{4a^2(c+9d)\cos(e+fx)}{15df(c+d)^3\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}-\frac{2a^2(c+9d)\cos(e+fx)}{15df(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{3/2}}+\frac{\cdots}{5d\cdots}$$

```
[Out] 2/5*a^2*(c-d)*cos(f*x+e)/d/(c+d)/f/(c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^
(1/2)-2/15*a^2*(c+9*d)*cos(f*x+e)/d/(c+d)^2/f/(c+d*sin(f*x+e))^(3/2)/(a+a*s
in(f*x+e))^(1/2)-4/15*a^2*(c+9*d)*cos(f*x+e)/d/(c+d)^3/f/(a+a*sin(f*x+e))^(
1/2)/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2762, 21, 2772, 2771}

$$-\frac{4a^2(c+9d)\cos(e+fx)}{15df(c+d)^3\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}-\frac{2a^2(c+9d)\cos(e+fx)}{15df(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{3/2}}+\frac{\cdots}{5d\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (2*a^2*(c - d)*Cos[e + f*x])/(5*d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d
*Sin[e + f*x])^(5/2)) - (2*a^2*(c + 9*d)*Cos[e + f*x])/(15*d*(c + d)^2*f*Sq
rt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2)) - (4*a^2*(c + 9*d)*Cos[e
 + f*x])/(15*d*(c + d)^3*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]
])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{7/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}} - \frac{(2a)\int \frac{-\frac{1}{2}a(c+9d)-\frac{1}{2}a(c+9d)\sin(e}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e}}{5d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}} + \frac{(a(c+9d))\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^{5/}}}{5d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}} - \frac{2a^2(c+9d)\,c}{15d(c+d)^2f\sqrt{a+a\sin(e+}}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{5d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}} - \frac{2a^2(c+9d)\,c}{15d(c+d)^2f\sqrt{a+a\sin(e+}}$$

Mathematica [A] time = 0.90, size = 140, normalized size = 0.81

$$-\frac{2a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\left(5c^2+46cd+9d^2\right)\sin(e+fx)+25c^2-d(c+9d}{15f(c+d)^3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-2*a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(25*
c^2 + 13*c*d + 12*d^2 - d*(c + 9*d)*Cos[2*(e + f*x)] + (5*c^2 + 46*c*d + 9*
d^2)*Sin[e + f*x]))/(15*(c + d)^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(
c + d*Sin[e + f*x])^(5/2))
```

fricas [B] time = 0.49, size = 598, normalized size = 3.48

$$\overline{15\left(\left(c^3d^3+3\,c^2d^4+3\,cd^5+d^6\right)f\cos\left(fx+e\right)^4-3\left(c^4d^2+3\,c^3d^3+3\,c^2d^4+cd^5\right)f\cos\left(fx+e\right)^3-\left(3\,c^5d+12\,c^4d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] 2/15*(2*(a*c*d + 9*a*d^2)*cos(f*x + e)^3 - 20*a*c^2 + 32*a*c*d - 12*a*d^2 -
 (5*a*c^2 + 44*a*c*d - 9*a*d^2)*cos(f*x + e)^2 - (25*a*c^2 + 14*a*c*d + 21*
a*d^2)*cos(f*x + e) + (20*a*c^2 - 32*a*c*d + 12*a*d^2 - 2*(a*c*d + 9*a*d^2)
*cos(f*x + e)^2 - (5*a*c^2 + 46*a*c*d + 9*a*d^2)*cos(f*x + e))*sin(f*x + e)
)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/((c^3*d^3 + 3*c^2*d^4 +
 3*c*d^5 + d^6)*f*cos(f*x + e)^4 - 3*(c^4*d^2 + 3*c^3*d^3 + 3*c^2*d^4 + c*d
^5)*f*cos(f*x + e)^3 - (3*c^5*d + 12*c^4*d^2 + 20*c^3*d^3 + 18*c^2*d^4 + 9*
c*d^5 + 2*d^6)*f*cos(f*x + e)^2 + (c^6 + 3*c^5*d + 6*c^4*d^2 + 10*c^3*d^3 +
 9*c^2*d^4 + 3*c*d^5)*f*cos(f*x + e) + (c^6 + 6*c^5*d + 15*c^4*d^2 + 20*c^3
*d^3 + 15*c^2*d^4 + 6*c*d^5 + d^6)*f - ((c^3*d^3 + 3*c^2*d^4 + 3*c*d^5 + d^
6)*f*cos(f*x + e)^3 + (3*c^4*d^2 + 10*c^3*d^3 + 12*c^2*d^4 + 6*c*d^5 + d^6)
*f*cos(f*x + e)^2 - (3*c^5*d + 9*c^4*d^2 + 10*c^3*d^3 + 6*c^2*d^4 + 3*c*d^5
 + d^6)*f*cos(f*x + e) - (c^6 + 6*c^5*d + 15*c^4*d^2 + 20*c^3*d^3 + 15*c^2*
d^4 + 6*c*d^5 + d^6)*f)*sin(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [B] time = 0.36, size = 625, normalized size = 3.63

$$-\frac{2\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\sqrt{c+d\sin\left(fx+e\right)}\,\left(114c^{3}\left(\cos^{2}\left(fx+e\right)\right)d^{2}+63\sin\left(fx+e\right)\left(\cos^{2}\left(fx+e\right)\right)d^{5}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(7/2),x)
```

```
[Out] -2/15/f*(a*(1+sin(f*x+e)))^(3/2)*(c+d*sin(f*x+e))^(1/2)*(-57*sin(f*x+e)*cos
(f*x+e)^4*d^5-9*sin(f*x+e)*cos(f*x+e)^4*c*d^4-90*sin(f*x+e)*cos(f*x+e)^2*c^
3*d^2-146*sin(f*x+e)*cos(f*x+e)^2*c^2*d^3+15*sin(f*x+e)*cos(f*x+e)^2*c*d^4+
8*c*d^4-80*c^2*d^3-48*c^3*d^2+56*c^4*d+186*cos(f*x+e)^2*c^2*d^3+18*sin(f*x+
e)*cos(f*x+e)^6*d^5-cos(f*x+e)^6*c^2*d^3-12*cos(f*x+e)^6*c*d^4-13*cos(f*x+e
)^4*c^4*d-63*cos(f*x+e)^4*c^3*d^2-5*sin(f*x+e)*cos(f*x+e)^2*c^5+78*cos(f*x+
e)^4*d^5-75*cos(f*x+e)^2*d^5-27*cos(f*x+e)^6*d^5-24*d^5*sin(f*x+e)+57*sin(f
*x+e)*cos(f*x+e)^4*c^2*d^3+3*sin(f*x+e)*cos(f*x+e)^2*c^4*d+2*sin(f*x+e)*cos
(f*x+e)^6*c*d^4+9*sin(f*x+e)*cos(f*x+e)^4*c^3*d^2+23*c*cos(f*x+e)^4*d^4+114
*c^3*cos(f*x+e)^2*d^2-19*c*cos(f*x+e)^2*d^4+80*c^2*d^3*sin(f*x+e)-8*c*d^4*s
in(f*x+e)-105*cos(f*x+e)^4*c^2*d^3+63*sin(f*x+e)*cos(f*x+e)^2*d^5-31*cos(f*
x+e)^2*c^4*d-56*sin(f*x+e)*c^4*d+48*sin(f*x+e)*c^3*d^2+24*d^5-15*cos(f*x+e)
^2*c^5+40*c^5-40*c^5*sin(f*x+e))/cos(f*x+e)^3/(cos(f*x+e)^2*d^2+c^2-d^2)^3/
(c+d)^3
```

maxima [B] time = 1.22, size = 505, normalized size = 2.94

$$-\frac{2\left(\left(25\,c^{3}+12\,c^{2}d+3\,cd^{2}\right)a^{\frac{3}{2}}-\frac{\left(15\,c^{3}-130\,c^{2}d-39\,cd^{2}-6\,d^{3}\right)a^{\frac{3}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{\left(65\,c^{3}-78\,c^{2}d+223\,cd^{2}+30\,d^{3}\right)a^{\frac{3}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{5\left(11\,c^{3}-\right.}{}}{15\left(c^{3}+3\,c^{2}d+3\,cd\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] -2/15*((25*c^3 + 12*c^2*d + 3*c*d^2)*a^(3/2) - (15*c^3 - 130*c^2*d - 39*c*d
^2 - 6*d^3)*a^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + (65*c^3 - 78*c^2*d +
223*c*d^2 + 30*d^3)*a^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*(11*c^3
 - 44*c^2*d + 33*c*d^2 - 24*d^3)*a^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3 + 5*(11*c^3 - 44*c^2*d + 33*c*d^2 - 24*d^3)*a^(3/2)*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 - (65*c^3 - 78*c^2*d + 223*c*d^2 + 30*d^3)*a^(3/2)*sin(f*x +
```

```
 e)^5/(cos(f*x + e) + 1)^5 + (15*c^3 - 130*c^2*d - 39*c*d^2 - 6*d^3)*a^(3/2
)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - (25*c^3 + 12*c^2*d + 3*c*d^2)*a^(3/
2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 1)^2/((c^3 + 3*c^2*d + 3*c*d^2 + d^3 + 2*(c^3 + 3*c^2*d + 3*c*d^2 + d^3
)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + (c^3 + 3*c^2*d + 3*c*d^2 + d^3)*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4)*(c + 2*d*sin(f*x + e)/(cos(f*x + e) + 1)
+ c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)^(7/2)*f)
```

mupad [B] time = 17.48, size = 541, normalized size = 3.15

$$\frac{\sqrt{c+d\,\sin\left(e+f\,x\right)}\,\left(\frac{8\,a\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\left(c+9\,d\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{15\,d^2\,f\,{\left(c+d\right)}^3}-\frac{8\,a\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(9\,c^2-4\,c\,d+3\,d^2\right)}{3\,d^3\,f\,{\left(c+d\right)}^3}+\frac{8\,a\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\cdots}{3\,d^3\cdots}\right.}{\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}+\frac{{\left(c\,1\mathrm{i}+d\,1\mathrm{i}\right)}^3}{{\left(c+d\right)}^3}-\frac{3\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\left(4\,c^2+2\,c\,d+d^2\right)}{d^2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\left(6\,c+d\right)}{d}+\frac{\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\left(8\,c^3+12\,c^2\,d+12\,c\cdots\right)}{d^3}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^(7/2),x)
```

```
[Out] ((c + d*sin(e + f*x))^(1/2)*((8*a*exp(e*6i + f*x*6i)*(c + 9*d)*(a + a*sin(e
 + f*x))^(1/2))/(15*d^2*f*(c + d)^3) - (8*a*exp(e*4i + f*x*4i)*(a + a*sin(e
 + f*x))^(1/2)*(9*c^2 - 4*c*d + 3*d^2))/(3*d^3*f*(c + d)^3) + (8*a*exp(e*3i
 + f*x*3i)*(a + a*sin(e + f*x))^(1/2)*(c^2*9i - c*d*4i + d^2*3i))/(3*d^3*f*
(c + d)^3) - (8*a*exp(e*1i + f*x*1i)*(c*1i + d*9i)*(a + a*sin(e + f*x))^(1/
2))/(15*d^2*f*(c + d)^3) + (8*a*c*exp(e*5i + f*x*5i)*(c*1i + d*9i)*(a + a*s
in(e + f*x))^(1/2))/(3*d^3*f*(c + d)^3) - (8*a*c*exp(e*2i + f*x*2i)*(c + 9*
d)*(a + a*sin(e + f*x))^(1/2))/(3*d^3*f*(c + d)^3)))/(exp(e*7i + f*x*7i) +
(c*1i + d*1i)^3/(c + d)^3 - (3*exp(e*5i + f*x*5i)*(2*c*d + 4*c^2 + d^2))/d^
2 - (exp(e*1i + f*x*1i)*(6*c + d))/d + (exp(e*3i + f*x*3i)*(12*c*d^2 + 12*c
^2*d + 8*c^3 + 3*d^3))/d^3 + (exp(e*6i + f*x*6i)*(c*6i + d*1i))/d - (3*exp(
e*2i + f*x*2i)*(c*1i + d*1i)^3*(2*c*d + 4*c^2 + d^2))/(d^2*(c + d)^3) + (ex
p(e*4i + f*x*4i)*(c*1i + d*1i)^3*(12*c*d^2 + 12*c^2*d + 8*c^3 + 3*d^3))/(d^
3*(c + d)^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

3.578 $\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=229

$$-\frac{16a^2(c+13d)\cos(e+fx)}{105df(c+d)^4\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}-\frac{8a^2(c+13d)\cos(e+fx)}{105df(c+d)^3\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{3/2}}-$$

```
[Out] 2/7*a^2*(c-d)*cos(f*x+e)/d/(c+d)/f/(c+d*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^
(1/2)-2/35*a^2*(c+13*d)*cos(f*x+e)/d/(c+d)^2/f/(c+d*sin(f*x+e))^(5/2)/(a+a*
sin(f*x+e))^(1/2)-8/105*a^2*(c+13*d)*cos(f*x+e)/d/(c+d)^3/f/(c+d*sin(f*x+e)
)^(3/2)/(a+a*sin(f*x+e))^(1/2)-16/105*a^2*(c+13*d)*cos(f*x+e)/d/(c+d)^4/f/(
a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2762, 21, 2772, 2771}

$$-\frac{16a^2(c+13d)\cos(e+fx)}{105df(c+d)^4\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}-\frac{8a^2(c+13d)\cos(e+fx)}{105df(c+d)^3\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(9/2),x]
```

```
[Out] (2*a^2*(c - d)*Cos[e + f*x])/(7*d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d
*Sin[e + f*x])^(7/2)) - (2*a^2*(c + 13*d)*Cos[e + f*x])/(35*d*(c + d)^2*f*S
qrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2)) - (8*a^2*(c + 13*d)*Cos
[e + f*x])/(105*d*(c + d)^3*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])
^(3/2)) - (16*a^2*(c + 13*d)*Cos[e + f*x])/(105*d*(c + d)^4*f*Sqrt[a + a*Si
n[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
```

```
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{9/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)}{7d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{7/2}} - \frac{(2a)\int \frac{-\frac{1}{2}a(c+13d)-\frac{1}{2}a(c+13d)\sin(}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+}}{7d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{7d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{7/2}} + \frac{(a(c+13d))\int \frac{\sqrt{a+a\sin(e+fx)}}{(c+d\sin(e+fx))^{7/2}}}{7d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{7d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{7/2}} - \frac{2a^2(c+13d)\,c}{35d(c+d)^2f\sqrt{a+a\sin(e+}}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{7d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{7/2}} - \frac{2a^2(c+13d)\,c}{35d(c+d)^2f\sqrt{a+a\sin(e+}}$$

$$= \frac{2a^2(c-d)\cos(e+fx)}{7d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{7/2}} - \frac{2a^2(c+13d)\,c}{35d(c+d)^2f\sqrt{a+a\sin(e+}}$$

Mathematica [A] time = 1.53, size = 193, normalized size = 0.84

$$-\frac{2a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(175c^3-2d\left(7c^2+92cd+13d^2\right)\cos(2(e+fx))+\right.}{105f(c+d)^4\left(\sin\left(\frac{1}{2}(e+fx)\right)+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(9/2),x]
```

```
[Out] (-2*a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(175
*c^3 + 147*c^2*d + 253*c*d^2 + 41*d^3 - 2*d*(7*c^2 + 92*c*d + 13*d^2)*Cos[2
*(e + f*x)] + (35*c^3 + 469*c^2*d + 191*c*d^2 + 117*d^3)*Sin[e + f*x] - 2*c
*d^2*Sin[3*(e + f*x)] - 26*d^3*Sin[3*(e + f*x)]))/(105*(c + d)^4*f*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])*(c + d*Sin[e + f*x])^(7/2))
```

fricas [B] time = 0.56, size = 937, normalized size = 4.09

$$\frac{}{105\left(\left(c^4d^4+4c^3d^5+6c^2d^6+4cd^7+d^8\right)f\cos\left(fx+e\right)^5+\left(4c^5d^3+17c^4d^4+28c^3d^5+22c^2d^6+8cd^7+d^8\right)f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(9/2),x, algorithm="frica
s")
```

```
[Out] 2/105*(8*(a*c*d^2 + 13*a*d^3)*cos(f*x + e)^4 - 140*a*c^3 + 308*a*c^2*d - 24
4*a*c*d^2 + 76*a*d^3 + 4*(7*a*c^2*d + 92*a*c*d^2 + 13*a*d^3)*cos(f*x + e)^3
 - (35*a*c^3 + 441*a*c^2*d - 167*a*c*d^2 + 195*a*d^3)*cos(f*x + e)^2 - (175
*a*c^3 + 161*a*c^2*d + 437*a*c*d^2 + 67*a*d^3)*cos(f*x + e) + (140*a*c^3 -
308*a*c^2*d + 244*a*c*d^2 - 76*a*d^3 + 8*(a*c*d^2 + 13*a*d^3)*cos(f*x + e)^
3 - 4*(7*a*c^2*d + 90*a*c*d^2 - 13*a*d^3)*cos(f*x + e)^2 - (35*a*c^3 + 469*
a*c^2*d + 193*a*c*d^2 + 143*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f
*x + e) + a)*sqrt(d*sin(f*x + e) + c)/((c^4*d^4 + 4*c^3*d^5 + 6*c^2*d^6 + 4
*c*d^7 + d^8)*f*cos(f*x + e)^5 + (4*c^5*d^3 + 17*c^4*d^4 + 28*c^3*d^5 + 22*
c^2*d^6 + 8*c*d^7 + d^8)*f*cos(f*x + e)^4 - 2*(3*c^6*d^2 + 12*c^5*d^3 + 19*
c^4*d^4 + 16*c^3*d^5 + 9*c^2*d^6 + 4*c*d^7 + d^8)*f*cos(f*x + e)^3 - 2*(2*c
^7*d + 11*c^6*d^2 + 28*c^5*d^3 + 43*c^4*d^4 + 42*c^3*d^5 + 25*c^2*d^6 + 8*c
*d^7 + d^8)*f*cos(f*x + e)^2 + (c^8 + 4*c^7*d + 12*c^6*d^2 + 28*c^5*d^3 + 3
8*c^4*d^4 + 28*c^3*d^5 + 12*c^2*d^6 + 4*c*d^7 + d^8)*f*cos(f*x + e) + (c^8
+ 8*c^7*d + 28*c^6*d^2 + 56*c^5*d^3 + 70*c^4*d^4 + 56*c^3*d^5 + 28*c^2*d^6
+ 8*c*d^7 + d^8)*f + ((c^4*d^4 + 4*c^3*d^5 + 6*c^2*d^6 + 4*c*d^7 + d^8)*f*c
os(f*x + e)^4 - 4*(c^5*d^3 + 4*c^4*d^4 + 6*c^3*d^5 + 4*c^2*d^6 + c*d^7)*f*c
os(f*x + e)^3 - 2*(3*c^6*d^2 + 14*c^5*d^3 + 27*c^4*d^4 + 28*c^3*d^5 + 17*c^
```

```
2*d^6 + 6*c*d^7 + d^8)*f*cos(f*x + e)^2 + 4*(c^7*d + 4*c^6*d^2 + 7*c^5*d^3
+ 8*c^4*d^4 + 7*c^3*d^5 + 4*c^2*d^6 + c*d^7)*f*cos(f*x + e) + (c^8 + 8*c^7*
d + 28*c^6*d^2 + 56*c^5*d^3 + 70*c^4*d^4 + 56*c^3*d^5 + 28*c^2*d^6 + 8*c*d^
7 + d^8)*f)*sin(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(9/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [B] time = 0.48, size = 979, normalized size = 4.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(9/2),x)
```

```
[Out] -2/105/f*(a*(1+sin(f*x+e)))^(3/2)*(c+d*sin(f*x+e))^(1/2)*(776*c^4*d^3*sin(f
*x+e)-136*c^3*d^4*sin(f*x+e)-424*c^2*d^5*sin(f*x+e)+120*c*d^6*sin(f*x+e)+8*
sin(f*x+e)*cos(f*x+e)^8*c*d^6+29*sin(f*x+e)*cos(f*x+e)^6*c^3*d^4+371*sin(f*
x+e)*cos(f*x+e)^6*c^2*d^5-113*sin(f*x+e)*cos(f*x+e)^6*c*d^6+106*sin(f*x+e)*
cos(f*x+e)^4*c^5*d^2+754*sin(f*x+e)*cos(f*x+e)^4*c^4*d^3+72*sin(f*x+e)*cos(
f*x+e)^4*c^3*d^4-944*sin(f*x+e)*cos(f*x+e)^4*c^2*d^5+382*sin(f*x+e)*cos(f*x
+e)^4*c*d^6+7*sin(f*x+e)*cos(f*x+e)^2*c^6*d-887*sin(f*x+e)*cos(f*x+e)^2*c^5
*d^2-1797*sin(f*x+e)*cos(f*x+e)^2*c^4*d^3-25*sin(f*x+e)*cos(f*x+e)^2*c^3*d^
4+997*sin(f*x+e)*cos(f*x+e)^2*c^2*d^5-397*sin(f*x+e)*cos(f*x+e)^2*c*d^6-280
*sin(f*x+e)*c^7-156*cos(f*x+e)^8*d^7+635*cos(f*x+e)^6*d^7-954*cos(f*x+e)^4*
d^7-105*cos(f*x+e)^2*c^7+627*cos(f*x+e)^2*d^7-296*c^5*d^2+136*c^3*d^4+280*c
^7-776*c^4*d^3-120*c*d^6+424*c^2*d^5+504*c^6*d-152*d^7+152*d^7*sin(f*x+e)-2
59*cos(f*x+e)^2*c^6*d+2185*cos(f*x+e)^2*c^4*d^3-43*cos(f*x+e)^2*c^3*d^4-120
9*cos(f*x+e)^2*c^2*d^5+457*cos(f*x+e)^2*c*d^6-504*sin(f*x+e)*c^6*d+296*sin(
f*x+e)*c^5*d^2+104*sin(f*x+e)*cos(f*x+e)^8*d^7-4*cos(f*x+e)^8*c^2*d^5-64*co
s(f*x+e)^8*c*d^6-455*sin(f*x+e)*cos(f*x+e)^6*d^7+4*cos(f*x+e)^6*c^4*d^3-149
*cos(f*x+e)^6*c^3*d^4-443*cos(f*x+e)^6*c^2*d^5+345*cos(f*x+e)^6*c*d^6+750*s
in(f*x+e)*cos(f*x+e)^4*d^7-112*cos(f*x+e)^4*c^6*d-670*cos(f*x+e)^4*c^5*d^2-
1398*cos(f*x+e)^4*c^4*d^3+56*cos(f*x+e)^4*c^3*d^4+1232*cos(f*x+e)^4*c^2*d^5
-618*cos(f*x+e)^4*c*d^6-35*sin(f*x+e)*cos(f*x+e)^2*c^7-551*sin(f*x+e)*cos(f
*x+e)^2*d^7+1035*cos(f*x+e)^2*c^5*d^2)/cos(f*x+e)^3/(cos(f*x+e)^2*d^2+c^2-d
^2)^4/(c+d)^4
```

maxima [B] time = 1.33, size = 750, normalized size = 3.28

$$-\frac{2\left(\left(175\,c^4+133\,c^3d+69\,c^2d^2+15\,cd^3\right)a^{\frac{3}{2}}-\frac{3\left(35\,c^4-385\,c^3d-189\,c^2d^2-67\,cd^3-10\,d^4\right)a^{\frac{3}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{18\left(35\,c^4-28\,c^3d+166\,c^2d^2\cdots\right.}{\left(\cos\left(f\cdots\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(9/2),x, algorithm="maxim
a")
```

```
[Out] -2/105*((175*c^4 + 133*c^3*d + 69*c^2*d^2 + 15*c*d^3)*a^(3/2) - 3*(35*c^4 -
 385*c^3*d - 189*c^2*d^2 - 67*c*d^3 - 10*d^4)*a^(3/2)*sin(f*x + e)/(cos(f*x
 + e) + 1) + 18*(35*c^4 - 28*c^3*d + 166*c^2*d^2 + 44*c*d^3 + 7*d^4)*a^(3/2
)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 14*(35*c^4 - 220*c^3*d + 102*c^2*d^
2 - 244*c*d^3 - 25*d^4)*a^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 42*(2
0*c^4 - 61*c^3*d + 117*c^2*d^2 - 55*c*d^3 + 35*d^4)*a^(3/2)*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 - 42*(20*c^4 - 61*c^3*d + 117*c^2*d^2 - 55*c*d^3 + 35*
d^4)*a^(3/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 14*(35*c^4 - 220*c^3*d +
 102*c^2*d^2 - 244*c*d^3 - 25*d^4)*a^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1
)^6 - 18*(35*c^4 - 28*c^3*d + 166*c^2*d^2 + 44*c*d^3 + 7*d^4)*a^(3/2)*sin(f
*x + e)^7/(cos(f*x + e) + 1)^7 + 3*(35*c^4 - 385*c^3*d - 189*c^2*d^2 - 67*c
*d^3 - 10*d^4)*a^(3/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - (175*c^4 + 133
*c^3*d + 69*c^2*d^2 + 15*c*d^3)*a^(3/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9
)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^3/((c^4 + 4*c^3*d + 6*c^2*d^2 +
 4*c*d^3 + d^4 + 3*(c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4)*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 + 3*(c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4)*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + (c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d
^4)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)*(c + 2*d*sin(f*x + e)/(cos(f*x + e
) + 1) + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)^(9/2)*f)
```

mupad [B] time = 20.21, size = 807, normalized size = 3.52

$$\frac{\sqrt{c+d\,\sin\left(e+f\,x\right)}\,\left(\frac{32\,a\,{\mathrm{e}}^{e\,8{}\mathrm{i}+f\,x\,8{}\mathrm{i}}\,\left(c+13\,d\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{105\,d^2\,f\,{\left(c+d\right)}^4}-\frac{16\,a\,{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(9\,c^3-5\,c^2\,d+9\,c\,d^2-d^3\right)}{3\,d^4\,f\,{\left(c+d\right)}^4}-\frac{16\,a\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\cdots}}{\cdots}\right.}{{\mathrm{e}}^{e\,9{}\mathrm{i}+f\,x\,9{}\mathrm{i}}+\frac{{\left(c\,1{}\mathrm{i}+d\,1{}\mathrm{i}\right)}^4\,1{}\mathrm{i}}{{\left(c+d\right)}^4}-\frac{4\,{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,\left(8\,c^3+6\,c^2\,d+6\,c\,d^2+d^3\right)}{d^3}-\frac{4\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\left(6\,c^2+2\,c\,d+\cdots\right.}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^(9/2),x)
```

```
[Out] ((c + d*sin(e + f*x))^(1/2)*((32*a*exp(e*8i + f*x*8i)*(c + 13*d)*(a + a*sin
(e + f*x))^(1/2))/(105*d^2*f*(c + d)^4) - (16*a*exp(e*4i + f*x*4i)*(a + a*s
in(e + f*x))^(1/2)*(9*c*d^2 - 5*c^2*d + 9*c^3 - d^3))/(3*d^4*f*(c + d)^4) -
 (16*a*exp(e*5i + f*x*5i)*(a + a*sin(e + f*x))^(1/2)*(c*d^2*9i - c^2*d*5i +
 c^3*9i - d^3*1i))/(3*d^4*f*(c + d)^4) - (16*a*exp(e*6i + f*x*6i)*(a + a*si
n(e + f*x))^(1/2)*(c*d^2 + 65*c^2*d + 5*c^3 + 13*d^3))/(15*d^4*f*(c + d)^4)
 - (16*a*exp(e*3i + f*x*3i)*(a + a*sin(e + f*x))^(1/2)*(c*d^2*1i + c^2*d*65
i + c^3*5i + d^3*13i))/(15*d^4*f*(c + d)^4) + (32*a*exp(e*1i + f*x*1i)*(c*1
i + d*13i)*(a + a*sin(e + f*x))^(1/2))/(105*d^2*f*(c + d)^4) + (32*a*c*exp(
e*7i + f*x*7i)*(c*1i + d*13i)*(a + a*sin(e + f*x))^(1/2))/(15*d^3*f*(c + d)
^4) + (32*a*c*exp(e*2i + f*x*2i)*(c + 13*d)*(a + a*sin(e + f*x))^(1/2))/(15
*d^3*f*(c + d)^4)))/(exp(e*9i + f*x*9i) + ((c*1i + d*1i)^4*1i)/(c + d)^4 -
(4*exp(e*3i + f*x*3i)*(6*c*d^2 + 6*c^2*d + 8*c^3 + d^3))/d^3 - (4*exp(e*7i
+ f*x*7i)*(2*c*d + 6*c^2 + d^2))/d^2 + (exp(e*1i + f*x*1i)*(8*c + d))/d + (
2*exp(e*5i + f*x*5i)*(12*c*d^3 + 16*c^3*d + 8*c^4 + 3*d^4 + 24*c^2*d^2))/d^
4 - (exp(e*6i + f*x*6i)*(c*1i + d*1i)^4*(6*c*d^2 + 6*c^2*d + 8*c^3 + d^3)*4
i)/(d^3*(c + d)^4) - (exp(e*2i + f*x*2i)*(c*1i + d*1i)^4*(2*c*d + 6*c^2 + d
^2)*4i)/(d^2*(c + d)^4) + (exp(e*8i + f*x*8i)*(8*c + d)*(c*1i + d*1i)^4*1i)
/(d*(c + d)^4) + (exp(e*4i + f*x*4i)*(c*1i + d*1i)^4*(12*c*d^3 + 16*c^3*d +
 8*c^4 + 3*d^4 + 24*c^2*d^2)*2i)/(d^4*(c + d)^4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

3.579 $\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=377

$$-\frac{a^{5/2}(c+d)^3\left(3c^2-34cd+283d^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{128d^{5/2}f}-\frac{a^3\left(3c^2-34cd+283d^2\right)\cos(e+fx)(c+}{240d^2f\sqrt{a\sin(e+fx)+}}$$

```
[Out] -1/128*a^(5/2)*(c+d)^3*(3*c^2-34*c*d+283*d^2)*arctan(cos(f*x+e)*a^(1/2)*d^(
1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))/d^(5/2)/f-1/192*a^3*(c+
d)*(3*c^2-34*c*d+283*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/d^2/f/(a+a*sin(
f*x+e))^(1/2)-1/240*a^3*(3*c^2-34*c*d+283*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^
(5/2)/d^2/f/(a+a*sin(f*x+e))^(1/2)+3/40*a^3*(c-7*d)*cos(f*x+e)*(c+d*sin(f*x
+e))^(7/2)/d^2/f/(a+a*sin(f*x+e))^(1/2)-1/5*a^2*cos(f*x+e)*(c+d*sin(f*x+e))
^(7/2)*(a+a*sin(f*x+e))^(1/2)/d/f-1/128*a^3*(c+d)^2*(3*c^2-34*c*d+283*d^2)*
cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d^2/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.85, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2763, 2981, 2770, 2775, 205}

$$-\frac{a^3\left(3c^2-34cd+283d^2\right)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{240d^2f\sqrt{a\sin(e+fx)+a}}-\frac{a^3(c+d)\left(3c^2-34cd+283d^2\right)\cos(e+fx)(c+d\,\text{si}}{192d^2f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] -(a^(5/2)*(c + d)^3*(3*c^2 - 34*c*d + 283*d^2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[
e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(128*d^(5/2
)*f) - (a^3*(c + d)^2*(3*c^2 - 34*c*d + 283*d^2)*Cos[e + f*x]*Sqrt[c + d*Si
n[e + f*x]])/(128*d^2*f*Sqrt[a + a*Sin[e + f*x]]) - (a^3*(c + d)*(3*c^2 - 3
4*c*d + 283*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(192*d^2*f*Sqrt[a
 + a*Sin[e + f*x]]) - (a^3*(3*c^2 - 34*c*d + 283*d^2)*Cos[e + f*x]*(c + d*S
in[e + f*x])^(5/2))/(240*d^2*f*Sqrt[a + a*Sin[e + f*x]]) + (3*a^3*(c - 7*d)
*Cos[e + f*x]*(c + d*Sin[e + f*x])^(7/2))/(40*d^2*f*Sqrt[a + a*Sin[e + f*x]
]) - (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(7/2))
/(5*d*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{5/2}\,dx = -\frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{7/2}}{5df} + \frac{\int}{}$$

$$= \frac{3a^3(c-7d)\cos(e+fx)(c+d\sin(e+fx))^{7/2}}{40d^2f\sqrt{a+a\sin(e+fx)}} - \frac{a^2\cos(e+fx}{}$$

$$= -\frac{a^3\left(3c^2-34cd+283d^2\right)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{240d^2f\sqrt{a+a\sin(e+fx)}} +$$

$$= -\frac{a^3(c+d)\left(3c^2-34cd+283d^2\right)\cos(e+fx)(c+d\sin(e+fx}{192d^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{a^3(c+d)^2\left(3c^2-34cd+283d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+}}{128d^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{a^3(c+d)^2\left(3c^2-34cd+283d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+}}{128d^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{a^{5/2}(c+d)^3\left(3c^2-34cd+283d^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin}}\right.}{128d^{5/2}f}$$

Mathematica [A] time = 2.80, size = 395, normalized size = 1.05

$$\frac{(a(\sin(e+fx)+1))^{5/2}\left(\frac{\left(3c^2-34cd+283d^2\right)(c+d)^3\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{4}(2e+2fx-\pi)\right)}{\sqrt{c+d\sin(e+fx)}}\right)-\log\left(\sqrt{c+d\sin(e+fx)}+\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)+\tan}{d^{5/2}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(((c + d)^3*(3*c^2 - 34*c*d + 283*d^2)*(2*Arc
Tan[(Sqrt[2]*Sqrt[d]*Sin[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] +
 ArcTanh[(Sqrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x
]]] - Log[Sqrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*
x]]]))/d^(5/2) + (2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e
+ f*x]]*(45*c^4 - 390*c^3*d - 8396*c^2*d^2 - 12762*c*d^3 - 5521*d^4 + 4*d^2
```

```
*(93*c^2 + 488*c*d + 331*d^2)*Cos[2*(e + f*x)] - 48*d^4*Cos[4*(e + f*x)] -
30*c^3*d*Sin[e + f*x] - 3322*c^2*d^2*Sin[e + f*x] - 7774*c*d^3*Sin[e + f*x]
 - 3874*d^4*Sin[e + f*x] + 252*c*d^3*Sin[3*(e + f*x)] + 348*d^4*Sin[3*(e +
f*x)]))/(15*d^2)))/(256*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

fricas [B] time = 2.06, size = 2083, normalized size = 5.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] [1/15360*(15*(3*a^2*c^5 - 25*a^2*c^4*d + 190*a^2*c^3*d^2 + 750*a^2*c^2*d^3
+ 815*a^2*c*d^4 + 283*a^2*d^5 + (3*a^2*c^5 - 25*a^2*c^4*d + 190*a^2*c^3*d^2
 + 750*a^2*c^2*d^3 + 815*a^2*c*d^4 + 283*a^2*d^5)*cos(f*x + e) + (3*a^2*c^5
 - 25*a^2*c^4*d + 190*a^2*c^3*d^2 + 750*a^2*c^2*d^3 + 815*a^2*c*d^4 + 283*a
^2*d^5)*sin(f*x + e))*sqrt(-a/d)*log((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*
a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x
 + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c
^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f*
x + e)^4 - c^3*d + 17*c^2*d^2 - 59*c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f*
x + e)^3 - 2*(5*c^2*d^2 - 26*c*d^3 + 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c^
2*d^2 + 31*c*d^3 - 25*d^4)*cos(f*x + e) + (16*d^4*cos(f*x + e)^3 + c^3*d -
17*c^2*d^2 + 59*c*d^3 - 51*d^4 - 8*(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5*
c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e
) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c^
2*d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x + e)^4 +
 a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4
)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 +
32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e
))/(cos(f*x + e) + sin(f*x + e) + 1)) - 8*(384*a^2*d^4*cos(f*x + e)^5 - 45*
a^2*c^4 + 360*a^2*c^3*d + 5446*a^2*c^2*d^2 + 6688*a^2*c*d^3 + 2671*a^2*d^4
- 1008*(a^2*c*d^3 + a^2*d^4)*cos(f*x + e)^4 - 8*(93*a^2*c^2*d^2 + 488*a^2*c
*d^3 + 379*a^2*d^4)*cos(f*x + e)^3 + 2*(15*a^2*c^3*d + 1289*a^2*c^2*d^2 + 2
565*a^2*c*d^3 + 1291*a^2*d^4)*cos(f*x + e)^2 - (45*a^2*c^4 - 390*a^2*c^3*d
- 8768*a^2*c^2*d^2 - 14714*a^2*c*d^3 - 6893*a^2*d^4)*cos(f*x + e) - (384*a^
2*d^4*cos(f*x + e)^4 - 45*a^2*c^4 + 360*a^2*c^3*d + 5446*a^2*c^2*d^2 + 6688
*a^2*c*d^3 + 2671*a^2*d^4 + 48*(21*a^2*c*d^3 + 29*a^2*d^4)*cos(f*x + e)^3 -
 8*(93*a^2*c^2*d^2 + 362*a^2*c*d^3 + 205*a^2*d^4)*cos(f*x + e)^2 - 2*(15*a^
2*c^3*d + 1661*a^2*c^2*d^2 + 4013*a^2*c*d^3 + 2111*a^2*d^4)*cos(f*x + e))*s
in(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d^2*f*cos(
f*x + e) + d^2*f*sin(f*x + e) + d^2*f), 1/7680*(15*(3*a^2*c^5 - 25*a^2*c^4*
d + 190*a^2*c^3*d^2 + 750*a^2*c^2*d^3 + 815*a^2*c*d^4 + 283*a^2*d^5 + (3*a^
2*c^5 - 25*a^2*c^4*d + 190*a^2*c^3*d^2 + 750*a^2*c^2*d^3 + 815*a^2*c*d^4 +
```

```
283*a^2*d^5)*cos(f*x + e) + (3*a^2*c^5 - 25*a^2*c^4*d + 190*a^2*c^3*d^2 + 7
50*a^2*c^2*d^3 + 815*a^2*c*d^4 + 283*a^2*d^5)*sin(f*x + e))*sqrt(a/d)*arcta
n(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x +
 e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*c
os(f*x + e)^3 - (3*a*c*d - a*d^2)*cos(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*
d + 2*a*d^2)*cos(f*x + e))) - 4*(384*a^2*d^4*cos(f*x + e)^5 - 45*a^2*c^4 +
360*a^2*c^3*d + 5446*a^2*c^2*d^2 + 6688*a^2*c*d^3 + 2671*a^2*d^4 - 1008*(a^
2*c*d^3 + a^2*d^4)*cos(f*x + e)^4 - 8*(93*a^2*c^2*d^2 + 488*a^2*c*d^3 + 379
*a^2*d^4)*cos(f*x + e)^3 + 2*(15*a^2*c^3*d + 1289*a^2*c^2*d^2 + 2565*a^2*c*
d^3 + 1291*a^2*d^4)*cos(f*x + e)^2 - (45*a^2*c^4 - 390*a^2*c^3*d - 8768*a^2
*c^2*d^2 - 14714*a^2*c*d^3 - 6893*a^2*d^4)*cos(f*x + e) - (384*a^2*d^4*cos(
f*x + e)^4 - 45*a^2*c^4 + 360*a^2*c^3*d + 5446*a^2*c^2*d^2 + 6688*a^2*c*d^3
 + 2671*a^2*d^4 + 48*(21*a^2*c*d^3 + 29*a^2*d^4)*cos(f*x + e)^3 - 8*(93*a^2
*c^2*d^2 + 362*a^2*c*d^3 + 205*a^2*d^4)*cos(f*x + e)^2 - 2*(15*a^2*c^3*d +
1661*a^2*c^2*d^2 + 4013*a^2*c*d^3 + 2111*a^2*d^4)*cos(f*x + e))*sin(f*x + e
))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d^2*f*cos(f*x + e) +
 d^2*f*sin(f*x + e) + d^2*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(5/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{\frac{5}{2}}\left(c+d\sin\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + a\sin\left(e + f x\right)\right)^{5/2} \left(c + d\sin\left(e + f x\right)\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.580 $\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=312

$$-\frac{a^{5/2}(c+d)^2\left(3c^2-26cd+163d^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{64d^{5/2}f}-\frac{a^3\left(3c^2-26cd+163d^2\right)\cos(e+fx)(c+}{96d^2f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -1/64*a^(5/2)*(c+d)^2*(3*c^2-26*c*d+163*d^2)*arctan(cos(f*x+e)*a^(1/2)*d^(1
/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))/d^(5/2)/f-1/96*a^3*(3*c^
2-26*c*d+163*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/d^2/f/(a+a*sin(f*x+e))^
(1/2)+1/24*a^3*(3*c-17*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(5/2)/d^2/f/(a+a*sin(
f*x+e))^(1/2)-1/4*a^2*cos(f*x+e)*(c+d*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1
/2)/d/f-1/64*a^3*(c+d)*(3*c^2-26*c*d+163*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^(
1/2)/d^2/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.73, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2763, 2981, 2770, 2775, 205}

$$-\frac{a^3\left(3c^2-26cd+163d^2\right)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{96d^2f\sqrt{a\sin(e+fx)+a}}-\frac{a^3(c+d)\left(3c^2-26cd+163d^2\right)\cos(e+fx)\sqrt{c+d\,\mathrm{s}}}{64d^2f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(a^(5/2)*(c + d)^2*(3*c^2 - 26*c*d + 163*d^2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[
e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(64*d^(5/2)
*f) - (a^3*(c + d)*(3*c^2 - 26*c*d + 163*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[e
 + f*x]])/(64*d^2*f*Sqrt[a + a*Sin[e + f*x]]) - (a^3*(3*c^2 - 26*c*d + 163*
d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(96*d^2*f*Sqrt[a + a*Sin[e +
f*x]]) + (a^3*(3*c - 17*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(24*d^2
*f*Sqrt[a + a*Sin[e + f*x]]) - (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(5/2))/(4*d*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
```

```
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a + a\sin(e + fx))^{5/2}(c + d\sin(e + fx))^{3/2}\,dx = -\frac{a^2\cos(e + fx)\sqrt{a + a\sin(e + fx)}\,(c + d\sin(e + fx))^{5/2}}{4df} + -$$

$$= \frac{a^3(3c - 17d)\cos(e + fx)(c + d\sin(e + fx))^{5/2}}{24d^2f\sqrt{a + a\sin(e + fx)}} - \frac{a^2\cos(e + f}{}$$

$$= -\frac{a^3\left(3c^2 - 26cd + 163d^2\right)\cos(e + fx)(c + d\sin(e + fx))^{3/2}}{96d^2f\sqrt{a + a\sin(e + fx)}} +$$

$$= -\frac{a^3(c + d)\left(3c^2 - 26cd + 163d^2\right)\cos(e + fx)\sqrt{c + d\sin(e + f}}{64d^2f\sqrt{a + a\sin(e + fx)}}$$

$$= -\frac{a^3(c + d)\left(3c^2 - 26cd + 163d^2\right)\cos(e + fx)\sqrt{c + d\sin(e + f}}{64d^2f\sqrt{a + a\sin(e + fx)}}$$

$$= -\frac{a^{5/2}(c + d)^2\left(3c^2 - 26cd + 163d^2\right)\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin}}\right.}{64d^{5/2}f}$$

Mathematica [A] time = 1.73, size = 327, normalized size = 1.05

$$\frac{(a(\sin(e + fx) + 1))^{5/2}\left(\frac{\left(3c^2-26cd+163d^2\right)(c+d)^2\left(2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{d}\,\sin\left(\frac{1}{4}(2e+2fx-\pi)\right)}{\sqrt{c+d\sin(e+fx)}}\right)-\log\left(\sqrt{c+d\sin(e+fx)}+\sqrt{2}\,\sqrt{d}\,\cos\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)+\tan}{d^{5/2}}\right.}{128}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(((c + d)^2*(3*c^2 - 26*c*d + 163*d^2)*(2*Arc
Tan[(Sqrt[2]*Sqrt[d]*Sin[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] +
 ArcTanh[(Sqrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x
]]] - Log[Sqrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*
x]]]))/d^(5/2) + (2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e
+ f*x]]*(9*c^3 - 63*c^2*d - 773*c*d^2 - 581*d^3 + 4*d^2*(9*c + 23*d)*Cos[2*
(e + f*x)] - 2*d*(3*c^2 + 158*c*d + 181*d^2)*Sin[e + f*x] + 12*d^3*Sin[3*(e
 + f*x)]))/(3*d^2)))/(128*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

fricas [B] time = 1.69, size = 1751, normalized size = 5.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/1536*(3*(3*a^2*c^4 - 20*a^2*c^3*d + 114*a^2*c^2*d^2 + 300*a^2*c*d^3 + 16
3*a^2*d^4 + (3*a^2*c^4 - 20*a^2*c^3*d + 114*a^2*c^2*d^2 + 300*a^2*c*d^3 + 1
63*a^2*d^4)*cos(f*x + e) + (3*a^2*c^4 - 20*a^2*c^3*d + 114*a^2*c^2*d^2 + 30
0*a^2*c*d^3 + 163*a^2*d^4)*sin(f*x + e))*sqrt(-a/d)*log((128*a*d^4*cos(f*x
+ e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d
^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f
*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2
 - 8*(16*d^4*cos(f*x + e)^4 - c^3*d + 17*c^2*d^2 - 59*c*d^3 + 51*d^4 + 24*(
c*d^3 - d^4)*cos(f*x + e)^3 - 2*(5*c^2*d^2 - 26*c*d^3 + 33*d^4)*cos(f*x + e
)^2 - (c^3*d - 7*c^2*d^2 + 31*c*d^3 - 25*d^4)*cos(f*x + e) + (16*d^4*cos(f*
x + e)^3 + c^3*d - 17*c^2*d^2 + 59*c*d^3 - 51*d^4 - 8*(3*c*d^3 - 5*d^4)*cos
(f*x + e)^2 - 2*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e))*sin(f*x + e))
*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-a/d) + (a*c^4 - 28
*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (128*a*d
^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 2
56*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4
)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*
x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)) + 8*(48*a^2*d^3*co
s(f*x + e)^4 + 9*a^2*c^3 - 57*a^2*c^2*d - 493*a^2*c*d^2 - 299*a^2*d^3 + 8*(
9*a^2*c*d^2 + 23*a^2*d^3)*cos(f*x + e)^3 - 2*(3*a^2*c^2*d + 122*a^2*c*d^2 +
 119*a^2*d^3)*cos(f*x + e)^2 + (9*a^2*c^3 - 63*a^2*c^2*d - 809*a^2*c*d^2 -
673*a^2*d^3)*cos(f*x + e) + (48*a^2*d^3*cos(f*x + e)^3 - 9*a^2*c^3 + 57*a^2
*c^2*d + 493*a^2*c*d^2 + 299*a^2*d^3 - 8*(9*a^2*c*d^2 + 17*a^2*d^3)*cos(f*x
 + e)^2 - 2*(3*a^2*c^2*d + 158*a^2*c*d^2 + 187*a^2*d^3)*cos(f*x + e))*sin(f
*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d^2*f*cos(f*x
+ e) + d^2*f*sin(f*x + e) + d^2*f), 1/768*(3*(3*a^2*c^4 - 20*a^2*c^3*d + 11
4*a^2*c^2*d^2 + 300*a^2*c*d^3 + 163*a^2*d^4 + (3*a^2*c^4 - 20*a^2*c^3*d + 1
14*a^2*c^2*d^2 + 300*a^2*c*d^3 + 163*a^2*d^4)*cos(f*x + e) + (3*a^2*c^4 - 2
0*a^2*c^3*d + 114*a^2*c^2*d^2 + 300*a^2*c*d^3 + 163*a^2*d^4)*sin(f*x + e))*
sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d -
 d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(
a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c*d - a*d^2)*cos(f*x + e)*sin(f*x + e)
- (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x + e))) + 4*(48*a^2*d^3*cos(f*x + e)^4 +
 9*a^2*c^3 - 57*a^2*c^2*d - 493*a^2*c*d^2 - 299*a^2*d^3 + 8*(9*a^2*c*d^2 +
23*a^2*d^3)*cos(f*x + e)^3 - 2*(3*a^2*c^2*d + 122*a^2*c*d^2 + 119*a^2*d^3)*
cos(f*x + e)^2 + (9*a^2*c^3 - 63*a^2*c^2*d - 809*a^2*c*d^2 - 673*a^2*d^3)*c
```

```
os(f*x + e) + (48*a^2*d^3*cos(f*x + e)^3 - 9*a^2*c^3 + 57*a^2*c^2*d + 493*a
^2*c*d^2 + 299*a^2*d^3 - 8*(9*a^2*c*d^2 + 17*a^2*d^3)*cos(f*x + e)^2 - 2*(3
*a^2*c^2*d + 158*a^2*c*d^2 + 187*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(
a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d^2*f*cos(f*x + e) + d^2*f*s
in(f*x + e) + d^2*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(fx+e)+a)^{\frac{5}{2}}(d\sin(fx+e)+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))^{\frac{5}{2}}\,(c+d\sin(fx+e))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(fx+e)+a)^{\frac{5}{2}}(d\sin(fx+e)+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+a\sin(e+fx))^{5/2}\,(c+d\sin(e+fx))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.581 $\int (a+a\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=241

$$-\frac{a^{5/2}(c+d)\left(c^2-6cd+25d^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{8d^{5/2}f}-\frac{a^3\left(c^2-6cd+25d^2\right)\cos(e+fx)\sqrt{c+d\sin(e}{8d^2f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -1/8*a^(5/2)*(c+d)*(c^2-6*c*d+25*d^2)*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+
a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))/d^(5/2)/f+1/12*a^3*(3*c-13*d)*c
os(f*x+e)*(c+d*sin(f*x+e))^(3/2)/d^2/f/(a+a*sin(f*x+e))^(1/2)-1/3*a^2*cos(f
*x+e)*(c+d*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/d/f-1/8*a^3*(c^2-6*c*d+
25*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d^2/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2763, 2981, 2770, 2775, 205}

$$-\frac{a^3\left(c^2-6cd+25d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{8d^2f\sqrt{a\sin(e+fx)+a}}-\frac{a^{5/2}(c+d)\left(c^2-6cd+25d^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(}}\right)}{8d^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] -(a^(5/2)*(c + d)*(c^2 - 6*c*d + 25*d^2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*
x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(8*d^(5/2)*f) - (
a^3*(c^2 - 6*c*d + 25*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(8*d^2*f*
Sqrt[a + a*Sin[e + f*x]]) + (a^3*(3*c - 13*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(3/2))/(12*d^2*f*Sqrt[a + a*Sin[e + f*x]]) - (a^2*Cos[e + f*x]*Sqrt[a
+ a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2))/(3*d*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
```

```
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}\,dx = -\frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}}{3df} + \frac{\int \sqrt{\ldots}}{\ldots}$$

$$= \frac{a^3(3c-13d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{12d^2f\sqrt{a+a\sin(e+fx)}} - \frac{a^2\cos(e+fx)\ldots}{\ldots}$$

$$= -\frac{a^3\left(c^2-6cd+25d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{8d^2f\sqrt{a+a\sin(e+fx)}} + \frac{a^3(3c\ldots}{\ldots}$$

$$= -\frac{a^3\left(c^2-6cd+25d^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{8d^2f\sqrt{a+a\sin(e+fx)}} + \frac{a^3(3c\ldots}{\ldots}$$

$$= -\frac{a^{5/2}(c+d)\left(c^2-6cd+25d^2\right)\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}\ldots\right.}{8d^{5/2}f}$$

Mathematica [A] time = 0.91, size = 285, normalized size = 1.18

$$\frac{(a(\sin(e+fx)+1))^{5/2}\left(\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c+d\sin(e+fx)}\left(3c^2-2d(c+17d)\sin(e+fx)-16cd+4d^2\cos(2(e+fx))-79d^2\right)}{3d^2}+\frac{(c+d\ldots}{\ldots}\right.}{16f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(((c + d)*(c^2 - 6*c*d + 25*d^2)*(2*ArcTan[(S
qrt[2]*Sqrt[d]*Sin[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] + ArcTa
nh[(Sqrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] -
Log[Sqrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*x]]]))
/d^(5/2) + (2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]
]*(3*c^2 - 16*c*d - 79*d^2 + 4*d^2*Cos[2*(e + f*x)] - 2*d*(c + 17*d)*Sin[e
+ f*x]))/(3*d^2)))/(16*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

fricas [B] time = 1.14, size = 1455, normalized size = 6.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/192*(3*(a^2*c^3 - 5*a^2*c^2*d + 19*a^2*c*d^2 + 25*a^2*d^3 + (a^2*c^3 - 5
*a^2*c^2*d + 19*a^2*c*d^2 + 25*a^2*d^3)*cos(f*x + e) + (a^2*c^3 - 5*a^2*c^2
*d + 19*a^2*c*d^2 + 25*a^2*d^3)*sin(f*x + e))*sqrt(-a/d)*log((128*a*d^4*cos
(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*
a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*
cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x +
 e)^2 - 8*(16*d^4*cos(f*x + e)^4 - c^3*d + 17*c^2*d^2 - 59*c*d^3 + 51*d^4 +
 24*(c*d^3 - d^4)*cos(f*x + e)^3 - 2*(5*c^2*d^2 - 26*c*d^3 + 33*d^4)*cos(f*
x + e)^2 - (c^3*d - 7*c^2*d^2 + 31*c*d^3 - 25*d^4)*cos(f*x + e) + (16*d^4*c
os(f*x + e)^3 + c^3*d - 17*c^2*d^2 + 59*c*d^3 - 51*d^4 - 8*(3*c*d^3 - 5*d^4
)*cos(f*x + e)^2 - 2*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e))*sin(f*x
+ e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-a/d) + (a*c^4
 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (12
8*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^
4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*
a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*c
os(f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)) + 8*(8*a^2*d^
2*cos(f*x + e)^3 + 3*a^2*c^2 - 14*a^2*c*d - 49*a^2*d^2 - 2*(a^2*c*d + 13*a^
2*d^2)*cos(f*x + e)^2 + (3*a^2*c^2 - 16*a^2*c*d - 83*a^2*d^2)*cos(f*x + e)
- (8*a^2*d^2*cos(f*x + e)^2 + 3*a^2*c^2 - 14*a^2*c*d - 49*a^2*d^2 + 2*(a^2*
c*d + 17*a^2*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt
(d*sin(f*x + e) + c))/(d^2*f*cos(f*x + e) + d^2*f*sin(f*x + e) + d^2*f), 1/
96*(3*(a^2*c^3 - 5*a^2*c^2*d + 19*a^2*c*d^2 + 25*a^2*d^3 + (a^2*c^3 - 5*a^2
*c^2*d + 19*a^2*c*d^2 + 25*a^2*d^3)*cos(f*x + e) + (a^2*c^3 - 5*a^2*c^2*d +
 19*a^2*c*d^2 + 25*a^2*d^3)*sin(f*x + e))*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f
*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*
x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3
*a*c*d - a*d^2)*cos(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f
*x + e))) + 4*(8*a^2*d^2*cos(f*x + e)^3 + 3*a^2*c^2 - 14*a^2*c*d - 49*a^2*d
^2 - 2*(a^2*c*d + 13*a^2*d^2)*cos(f*x + e)^2 + (3*a^2*c^2 - 16*a^2*c*d - 83
*a^2*d^2)*cos(f*x + e) - (8*a^2*d^2*cos(f*x + e)^2 + 3*a^2*c^2 - 14*a^2*c*d
 - 49*a^2*d^2 + 2*(a^2*c*d + 17*a^2*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a
*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d^2*f*cos(f*x + e) + d^2*f*si
n(f*x + e) + d^2*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
```

)

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*sqrt(d*sin(f*x + e) + c), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(a + a\sin\left(fx + e\right)\right)^{\frac{5}{2}} \sqrt{c + d\sin\left(fx + e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(1/2),x)
[Out] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx + e\right) + a\right)^{\frac{5}{2}} \sqrt{d\sin\left(fx + e\right) + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*sqrt(d*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,\sqrt{c+d\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(1/2),x)
[Out] int((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c+d*sin(f*x+e))**(1/2),x)
[Out] Timed out
```

3.582 $\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=178

$$-\frac{a^{5/2}\left(3c^2-10cd+19d^2\right)\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}\right)}{4d^{5/2}f}+\frac{3a^3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4d^2f\sqrt{a\sin(e+fx)+a}}-\frac{a^2\cos(}{}$$

```
[Out] -1/4*a^(5/2)*(3*c^2-10*c*d+19*d^2)*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*s
in(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))/d^(5/2)/f+3/4*a^3*(c-3*d)*cos(f*x+
e)*(c+d*sin(f*x+e))^(1/2)/d^2/f/(a+a*sin(f*x+e))^(1/2)-1/2*a^2*cos(f*x+e)*(
a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2)/d/f
```

Rubi [A] time = 0.43, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2763, 2981, 2775, 205}

$$-\frac{a^{5/2}\left(3c^2-10cd+19d^2\right)\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}\right)}{4d^{5/2}f}+\frac{3a^3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4d^2f\sqrt{a\sin(e+fx)+a}}-\frac{a^2\cos(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] -(a^(5/2)*(3*c^2 - 10*c*d + 19*d^2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(
Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(4*d^(5/2)*f) + (3*a^3
*(c - 3*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*d^2*f*Sqrt[a + a*Sin[e
 + f*x]]) - (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f
*x]])/(2*d*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
```

```
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}{2df} + \frac{\int \frac{\sqrt{a+a\sin(e+fx)}\left(\frac{1}{2}a^2(c+5d)\right.}{\sqrt{c+d\sin(}}}{2d}$$

$$= \frac{3a^3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4d^2f\sqrt{a+a\sin(e+fx)}} - \frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,\sqrt{}}{2df}$$

$$= \frac{3a^3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4d^2f\sqrt{a+a\sin(e+fx)}} - \frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,\sqrt{}}{2df}$$

$$= -\frac{a^{5/2}\left(3c^2-10cd+19d^2\right)\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}\right)}{4d^{5/2}f} + \frac{3a^3(c-3d)\cos(e}{4d^2f\sqrt{a}}$$

Mathematica [A] time = 0.80, size = 256, normalized size = 1.44

$$\frac{(a(\sin(e+fx)+1))^{5/2}\left(\frac{(3c^2-10cd+19d^2)\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{4}(2e+2fx-\pi)\right)}{\sqrt{c+d\sin(e+fx)}}\right)-\log\left(\sqrt{c+d\sin(e+fx)}+\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)+\tanh^{-1}\left(\frac{\sqrt{2}\ \cdots}{\cdots}\right.\right.}{d^{5/2}}\cdots\right.}{8f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(((3*c^2 - 10*c*d + 19*d^2)*(2*ArcTan[(Sqrt[2
]*Sqrt[d]*Sin[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] + ArcTanh[(S
qrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] - Log[S
qrt[2]*Sqrt[d]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*x]]]))/d^(5
/2) + (2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(3*c - 11*d - 2*d*Sin[e + f*
x])*Sqrt[c + d*Sin[e + f*x]])/d^2))/(8*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^5)
```

fricas [B] time = 1.08, size = 1219, normalized size = 6.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/32*((3*a^2*c^2 - 10*a^2*c*d + 19*a^2*d^2 + (3*a^2*c^2 - 10*a^2*c*d + 19*
a^2*d^2)*cos(f*x + e) + (3*a^2*c^2 - 10*a^2*c*d + 19*a^2*d^2)*sin(f*x + e))
*sqrt(-a/d)*log((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2
 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2
*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 +
9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f*x + e)^4 - c^3*d + 17
*c^2*d^2 - 59*c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f*x + e)^3 - 2*(5*c^2*d
^2 - 26*c*d^3 + 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c^2*d^2 + 31*c*d^3 - 25
*d^4)*cos(f*x + e) + (16*d^4*cos(f*x + e)^3 + c^3*d - 17*c^2*d^2 + 59*c*d^3
 - 51*d^4 - 8*(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5*c^2*d^2 - 14*c*d^3 +
13*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x
 + e) + c)*sqrt(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^3 +
 289*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d +
6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 - 32
*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*c^2
*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e) + si
```

```
n(f*x + e) + 1)) - 8*(2*a^2*d*cos(f*x + e)^2 - 3*a^2*c + 9*a^2*d - (3*a^2*c
 - 11*a^2*d)*cos(f*x + e) + (2*a^2*d*cos(f*x + e) + 3*a^2*c - 9*a^2*d)*sin(
f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d^2*f*cos(f*x
 + e) + d^2*f*sin(f*x + e) + d^2*f), 1/16*((3*a^2*c^2 - 10*a^2*c*d + 19*a^2
*d^2 + (3*a^2*c^2 - 10*a^2*c*d + 19*a^2*d^2)*cos(f*x + e) + (3*a^2*c^2 - 10
*a^2*c*d + 19*a^2*d^2)*sin(f*x + e))*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x +
e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e
) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c*
d - a*d^2)*cos(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x +
e))) - 4*(2*a^2*d*cos(f*x + e)^2 - 3*a^2*c + 9*a^2*d - (3*a^2*c - 11*a^2*d)
*cos(f*x + e) + (2*a^2*d*cos(f*x + e) + 3*a^2*c - 9*a^2*d)*sin(f*x + e))*sq
rt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(d^2*f*cos(f*x + e) + d^2*
f*sin(f*x + e) + d^2*f)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + a \sin\left(f x + e\right)\right)^{\frac{5}{2}}}{\sqrt{c + d \sin\left(f x + e\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a \sin\left(f x + e\right) + a\right)^{\frac{5}{2}}}{\sqrt{d \sin\left(f x + e\right) + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/sqrt(d*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}{\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^(1/2),x)
[Out] int((a + a*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(1/2),x)
[Out] Timed out
```

3.583 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=180

$$\frac{a^{5/2}(3c-5d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{d^{5/2}f}-\frac{a^3(3c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{d^2f(c+d)\sqrt{a\sin(e+fx)+a}}+\frac{2a^2(c-d)\cos(e+}{df(c+d)\sqrt{c}}$$

```
[Out] a^(5/2)*(3*c-5*d)*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*x+e))^(1/2)/
(c+d*sin(f*x+e))^(1/2))/d^(5/2)/f+2*a^2*(c-d)*cos(f*x+e)*(a+a*sin(f*x+e))^(
1/2)/d/(c+d)/f/(c+d*sin(f*x+e))^(1/2)-a^3*(3*c-d)*cos(f*x+e)*(c+d*sin(f*x+e
))^(1/2)/d^2/(c+d)/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2762, 2981, 2775, 205}

$$-\frac{a^3(3c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{d^2f(c+d)\sqrt{a\sin(e+fx)+a}}+\frac{a^{5/2}(3c-5d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{d^{5/2}f}+\frac{2a^2(c-d)\cos(e+}{df(c+d)\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (a^(5/2)*(3*c - 5*d)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(d^(5/2)*f) + (2*a^2*(c - d)*Cos[e +
f*x]*Sqrt[a + a*Sin[e + f*x]])/(d*(c + d)*f*Sqrt[c + d*Sin[e + f*x]]) - (a^
3*(3*c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d^2*(c + d)*f*Sqrt[a +
a*Sin[e + f*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

```
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{(2a)\int \frac{\sqrt{a+a\sin(e+fx)}\left(\frac{1}{2}a(c-3d)-\frac{1}{2}a(3c-d)\sin\cdots\right.}{\sqrt{c+d\sin(e+fx)}}}{d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{a^3(3c-d)\cos(e+fx)\sqrt{c+d\sin(e+\cdots}}{d^2(c+d)f\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{d(c+d)f\sqrt{c+d\sin(e+fx)}} - \frac{a^3(3c-d)\cos(e+fx)\sqrt{c+d\sin(e+\cdots}}{d^2(c+d)f\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{a^{5/2}(3c-5d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{d^{5/2}f} + \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin\cdots}}{d(c+d)f\sqrt{c+d\sin(e+\cdots}}$$

Mathematica [A] time = 1.01, size = 263, normalized size = 1.46

$$\frac{(a(\sin(e+fx)+1))^{5/2}\left(\frac{(5d-3c)\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{4}(2e+2fx-\pi)\right)}{\sqrt{c+d\sin(e+fx)}}\right)-\log\left(\sqrt{c+d\sin(e+fx)}+\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)+\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{d}\cos\left(\frac{1}{4}(\right.}{\sqrt{c+d\sin}}\right.\right.}{d^{5/2}}\right.}{2f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(((-3*c + 5*d)*(2*ArcTan[(Sqrt[2]*Sqrt[d]*Sin
[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] + ArcTanh[(Sqrt[2]*Sqrt[d
]*Cos[(2*e - Pi + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] - Log[Sqrt[2]*Sqrt[d
]*Cos[(2*e - Pi + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*x]]]))/d^(5/2) - (2*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])*(3*c^2 - 3*c*d + 2*d^2 + d*(c + d)*Sin[e
+ f*x]))/(d^2*(c + d)*Sqrt[c + d*Sin[e + f*x]])))/(2*f*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^5)
```

fricas [B] time = 1.00, size = 1665, normalized size = 9.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/8*((3*a^2*c^3 + a^2*c^2*d - 7*a^2*c*d^2 - 5*a^2*d^3 - (3*a^2*c^2*d - 2*a
^2*c*d^2 - 5*a^2*d^3)*cos(f*x + e)^2 + (3*a^2*c^3 - 2*a^2*c^2*d - 5*a^2*c*d
^2)*cos(f*x + e) + (3*a^2*c^3 + a^2*c^2*d - 7*a^2*c*d^2 - 5*a^2*d^3 + (3*a^
2*c^2*d - 2*a^2*c*d^2 - 5*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-a/d)*l
og((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3
+ a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c
*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4
*a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f*x + e)^4 - c^3*d + 17*c^2*d^2 - 59
*c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f*x + e)^3 - 2*(5*c^2*d^2 - 26*c*d^3
 + 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c^2*d^2 + 31*c*d^3 - 25*d^4)*cos(f*x
 + e) + (16*d^4*cos(f*x + e)^3 + c^3*d - 17*c^2*d^2 + 59*c*d^3 - 51*d^4 - 8
*(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f
*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sq
rt(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^3 + 289*a*d^4)*c
os(f*x + e) + (128*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 +
 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2
```

```
 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c
*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) +
1)) + 8*(3*a^2*c^2 - 4*a^2*c*d + a^2*d^2 + (a^2*c*d + a^2*d^2)*cos(f*x + e)
^2 + (3*a^2*c^2 - 3*a^2*c*d + 2*a^2*d^2)*cos(f*x + e) - (3*a^2*c^2 - 4*a^2*
c*d + a^2*d^2 - (a^2*c*d + a^2*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(
f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((c*d^3 + d^4)*f*cos(f*x + e)^2 - (
c^2*d^2 + c*d^3)*f*cos(f*x + e) - (c^2*d^2 + 2*c*d^3 + d^4)*f - ((c*d^3 + d
^4)*f*cos(f*x + e) + (c^2*d^2 + 2*c*d^3 + d^4)*f)*sin(f*x + e)), 1/4*((3*a^
2*c^3 + a^2*c^2*d - 7*a^2*c*d^2 - 5*a^2*d^3 - (3*a^2*c^2*d - 2*a^2*c*d^2 -
5*a^2*d^3)*cos(f*x + e)^2 + (3*a^2*c^3 - 2*a^2*c^2*d - 5*a^2*c*d^2)*cos(f*x
 + e) + (3*a^2*c^3 + a^2*c^2*d - 7*a^2*c*d^2 - 5*a^2*d^3 + (3*a^2*c^2*d - 2
*a^2*c*d^2 - 5*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a/d)*arctan(1/4*(8
*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqr
t(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x +
 e)^3 - (3*a*c*d - a*d^2)*cos(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*
d^2)*cos(f*x + e))) + 4*(3*a^2*c^2 - 4*a^2*c*d + a^2*d^2 + (a^2*c*d + a^2*d
^2)*cos(f*x + e)^2 + (3*a^2*c^2 - 3*a^2*c*d + 2*a^2*d^2)*cos(f*x + e) - (3*
a^2*c^2 - 4*a^2*c*d + a^2*d^2 - (a^2*c*d + a^2*d^2)*cos(f*x + e))*sin(f*x +
 e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((c*d^3 + d^4)*f*co
s(f*x + e)^2 - (c^2*d^2 + c*d^3)*f*cos(f*x + e) - (c^2*d^2 + 2*c*d^3 + d^4)
*f - ((c*d^3 + d^4)*f*cos(f*x + e) + (c^2*d^2 + 2*c*d^3 + d^4)*f)*sin(f*x +
 e))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\left(a + a \sin\left(f x + e\right)\right)^{\frac{5}{2}}}{\left(c + d \sin\left(f x + e\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}{\left(c+d\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.584 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=183

$$-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{d^{5/2}f}+\frac{2a^3(c-d)(3c+7d)\cos(e+fx)}{3d^2f(c+d)^2\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}+\frac{2a^2(c-d)\cos(e\ldots}{3df(c+d)(\ldots}$$

```
[Out] -2*a^(5/2)*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*si
n(f*x+e))^(1/2))/d^(5/2)/f+2/3*a^2*(c-d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/
d/(c+d)/f/(c+d*sin(f*x+e))^(3/2)+2/3*a^3*(c-d)*(3*c+7*d)*cos(f*x+e)/d^2/(c+
d)^2/f/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2762, 2980, 2775, 205}

$$\frac{2a^3(c-d)(3c+7d)\cos(e+fx)}{3d^2f(c+d)^2\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{d^{5/2}f}+\frac{2a^2(c-d)\cos(e+\ldots}{3df(c+d)(c\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*a^(5/2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c + d*Sin[e + f*x]])])/(d^(5/2)*f) + (2*a^2*(c - d)*Cos[e + f*x]*Sqrt
[a + a*Sin[e + f*x]])/(3*d*(c + d)*f*(c + d*Sin[e + f*x])^(3/2)) + (2*a^3*(
c - d)*(3*c + 7*d)*Cos[e + f*x])/(3*d^2*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

```
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} - \frac{(2a)\int \frac{\sqrt{a+a\sin(e+fx)}\left(\frac{1}{2}a(c-7d)-\frac{3}{2}a(c+d)\sin\right.}{(c+d\sin(e+fx))^{3/2}}}{3d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} + \frac{2a^3(c-d)(3c+7d)\cos(e+}{3d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}\sqrt{c+}}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3d(c+d)f(c+d\sin(e+fx))^{3/2}} + \frac{2a^3(c-d)(3c+7d)\cos(e+}{3d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}\sqrt{c+}}$$

$$= -\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{d^{5/2}f} + \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+}}{3d(c+d)f(c+d\sin(e+fx))^{3/2}}$$

Mathematica [A] time = 8.16, size = 261, normalized size = 1.43

$$\frac{(a(\sin(e+fx)+1))^{5/2}\left(\frac{2(c-d)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(3c^2+4d(c+2d)\sin(e+fx)+8cd+d^2\right)}{3d^2(c+d)^2(c+d\sin(e+fx))^{3/2}}+\frac{2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{4}(2e+2fx-\pi)\right)}{\sqrt{c+d\sin(e+fx)}}\right)-\log\left(\sqrt{d}\right.}{}\right.}{f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*((2*ArcTan[(Sqrt[2]*Sqrt[d]*Sin[(2*e - Pi + 2
*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] + ArcTanh[(Sqrt[2]*Sqrt[d]*Cos[(2*e - P
i + 2*f*x)/4])/Sqrt[c + d*Sin[e + f*x]]] - Log[Sqrt[2]*Sqrt[d]*Cos[(2*e - P
i + 2*f*x)/4] + Sqrt[c + d*Sin[e + f*x]]])/d^(5/2) + (2*(c - d)*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])*(3*c^2 + 8*c*d + d^2 + 4*d*(c + 2*d)*Sin[e + f*x
]))/(3*d^2*(c + d)^2*(c + d*Sin[e + f*x])^(3/2))))/(f*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^5)
```

fricas [B] time = 0.99, size = 2297, normalized size = 12.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] [-1/12*(3*(a^2*c^4 + 4*a^2*c^3*d + 6*a^2*c^2*d^2 + 4*a^2*c*d^3 + a^2*d^4 -
(a^2*c^2*d^2 + 2*a^2*c*d^3 + a^2*d^4)*cos(f*x + e)^3 - (2*a^2*c^3*d + 5*a^2
*c^2*d^2 + 4*a^2*c*d^3 + a^2*d^4)*cos(f*x + e)^2 + (a^2*c^4 + 2*a^2*c^3*d +
 2*a^2*c^2*d^2 + 2*a^2*c*d^3 + a^2*d^4)*cos(f*x + e) + (a^2*c^4 + 4*a^2*c^3
*d + 6*a^2*c^2*d^2 + 4*a^2*c*d^3 + a^2*d^4 - (a^2*c^2*d^2 + 2*a^2*c*d^3 + a
^2*d^4)*cos(f*x + e)^2 + 2*(a^2*c^3*d + 2*a^2*c^2*d^2 + a^2*c*d^3)*cos(f*x
+ e))*sin(f*x + e))*sqrt(-a/d)*log((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*
c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x +
 e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3
*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2 - 8*(16*d^4*cos(f*x
+ e)^4 - c^3*d + 17*c^2*d^2 - 59*c*d^3 + 51*d^4 + 24*(c*d^3 - d^4)*cos(f*x
+ e)^3 - 2*(5*c^2*d^2 - 26*c*d^3 + 33*d^4)*cos(f*x + e)^2 - (c^3*d - 7*c^2*
d^2 + 31*c*d^3 - 25*d^4)*cos(f*x + e) + (16*d^4*cos(f*x + e)^3 + c^3*d - 17
*c^2*d^2 + 59*c*d^3 - 51*d^4 - 8*(3*c*d^3 - 5*d^4)*cos(f*x + e)^2 - 2*(5*c^
2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e)
+ a)*sqrt(d*sin(f*x + e) + c)*sqrt(-a/d) + (a*c^4 - 28*a*c^3*d + 230*a*c^2*
```

```
d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x + e)^4 + a
*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4)*
cos(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 + 32
*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e))
/(cos(f*x + e) + sin(f*x + e) + 1)) + 8*(3*a^2*c^3 + a^2*c^2*d - 11*a^2*c*d
^2 + 7*a^2*d^3 + 4*(a^2*c^2*d + a^2*c*d^2 - 2*a^2*d^3)*cos(f*x + e)^2 + (3*
a^2*c^3 + 5*a^2*c^2*d - 7*a^2*c*d^2 - a^2*d^3)*cos(f*x + e) - (3*a^2*c^3 +
a^2*c^2*d - 11*a^2*c*d^2 + 7*a^2*d^3 - 4*(a^2*c^2*d + a^2*c*d^2 - 2*a^2*d^3
)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c))/((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e)^3 + (2*c^3*d^3 + 5*c^2*d^4
+ 4*c*d^5 + d^6)*f*cos(f*x + e)^2 - (c^4*d^2 + 2*c^3*d^3 + 2*c^2*d^4 + 2*c*
d^5 + d^6)*f*cos(f*x + e) - (c^4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^
6)*f + ((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e)^2 - 2*(c^3*d^3 + 2*c^2*d^4
 + c*d^5)*f*cos(f*x + e) - (c^4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^6
)*f)*sin(f*x + e)), -1/6*(3*(a^2*c^4 + 4*a^2*c^3*d + 6*a^2*c^2*d^2 + 4*a^2*
c*d^3 + a^2*d^4 - (a^2*c^2*d^2 + 2*a^2*c*d^3 + a^2*d^4)*cos(f*x + e)^3 - (2
*a^2*c^3*d + 5*a^2*c^2*d^2 + 4*a^2*c*d^3 + a^2*d^4)*cos(f*x + e)^2 + (a^2*c
^4 + 2*a^2*c^3*d + 2*a^2*c^2*d^2 + 2*a^2*c*d^3 + a^2*d^4)*cos(f*x + e) + (a
^2*c^4 + 4*a^2*c^3*d + 6*a^2*c^2*d^2 + 4*a^2*c*d^3 + a^2*d^4 - (a^2*c^2*d^2
 + 2*a^2*c*d^3 + a^2*d^4)*cos(f*x + e)^2 + 2*(a^2*c^3*d + 2*a^2*c^2*d^2 + a
^2*c*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a/d)*arctan(1/4*(8*d^2*cos(f*x +
 e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x +
e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(a/d)/(2*a*d^2*cos(f*x + e)^3 - (3*a*c
*d - a*d^2)*cos(f*x + e)*sin(f*x + e) - (a*c^2 - a*c*d + 2*a*d^2)*cos(f*x +
 e))) + 4*(3*a^2*c^3 + a^2*c^2*d - 11*a^2*c*d^2 + 7*a^2*d^3 + 4*(a^2*c^2*d
+ a^2*c*d^2 - 2*a^2*d^3)*cos(f*x + e)^2 + (3*a^2*c^3 + 5*a^2*c^2*d - 7*a^2*
c*d^2 - a^2*d^3)*cos(f*x + e) - (3*a^2*c^3 + a^2*c^2*d - 11*a^2*c*d^2 + 7*a
^2*d^3 - 4*(a^2*c^2*d + a^2*c*d^2 - 2*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*
sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((c^2*d^4 + 2*c*d^5 + d^
6)*f*cos(f*x + e)^3 + (2*c^3*d^3 + 5*c^2*d^4 + 4*c*d^5 + d^6)*f*cos(f*x + e
)^2 - (c^4*d^2 + 2*c^3*d^3 + 2*c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e) - (c
^4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^6)*f + ((c^2*d^4 + 2*c*d^5 + d
^6)*f*cos(f*x + e)^2 - 2*(c^3*d^3 + 2*c^2*d^4 + c*d^5)*f*cos(f*x + e) - (c^
4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^6)*f)*sin(f*x + e))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [B] time = 0.62, size = 16223, normalized size = 88.65

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a \sin\left(fx + e\right) + a\right)^{\frac{5}{2}}}{\left(d \sin\left(fx + e\right) + c\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + a\,\sin\left(e + f\,x\right)\right)^{5/2}}{\left(c + d\,\sin\left(e + f\,x\right)\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.585 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=189

$$-\frac{2a^3\left(3c^2+14cd+43d^2\right)\cos(e+fx)}{15d^2f(c+d)^3\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}+\frac{2a^3(c-d)(3c+11d)\cos(e+fx)}{15d^2f(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{3/2}}+$$

```
[Out] 2/15*a^3*(c-d)*(3*c+11*d)*cos(f*x+e)/d^2/(c+d)^2/f/(c+d*sin(f*x+e))^(3/2)/(
a+a*sin(f*x+e))^(1/2)+2/5*a^2*(c-d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d/(c+
d)/f/(c+d*sin(f*x+e))^(5/2)-2/15*a^3*(3*c^2+14*c*d+43*d^2)*cos(f*x+e)/d^2/(
c+d)^3/f/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2762, 2980, 2771}

$$-\frac{2a^3\left(3c^2+14cd+43d^2\right)\cos(e+fx)}{15d^2f(c+d)^3\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}+\frac{2a^3(c-d)(3c+11d)\cos(e+fx)}{15d^2f(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (2*a^2*(c - d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(5*d*(c + d)*f*(c + d
*Sin[e + f*x])^(5/2)) + (2*a^3*(c - d)*(3*c + 11*d)*Cos[e + f*x])/(15*d^2*(
c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2)) - (2*a^3*(3
*c^2 + 14*c*d + 43*d^2)*Cos[e + f*x])/(15*d^2*(c + d)^3*f*Sqrt[a + a*Sin[e
+ f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
```

```
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{7/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} - \frac{(2a)\int \frac{\sqrt{a+a\sin(e+fx)}\left(\frac{1}{2}a(c-11d)-\frac{1}{2}a(3c+7d)\right.}{(c+d\sin(e+fx))^{5/2}}}{5d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} + \frac{2a^3(c-d)(3c+11d)\cos(e}{15d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}\,(c+}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{5d(c+d)f(c+d\sin(e+fx))^{5/2}} + \frac{2a^3(c-d)(3c+11d)\cos(e}{15d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}\,(c+}$$

Mathematica [A] time = 2.11, size = 152, normalized size = 0.80

$$-\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(4\left(7c^2+46cd+7d^2\right)\sin(e+fx)-\left(3c^2+14cd+4\right.\right.}{15f(c+d)^3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(c+d\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] -1/15*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]
*(89*c^2 + 42*c*d + 49*d^2 - (3*c^2 + 14*c*d + 43*d^2)*Cos[2*(e + f*x)] + 4
*(7*c^2 + 46*c*d + 7*d^2)*Sin[e + f*x]))/((c + d)^3*f*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])*(c + d*Sin[e + f*x])^(5/2))
```

fricas [B] time = 0.52, size = 654, normalized size = 3.46

$$-\frac{2\left(32\,a^{2}c^{2}-64\,a\right.}{15\left(\left(c^{3}d^{3}+3\,c^{2}d^{4}+3\,cd^{5}+d^{6}\right)f\cos\left(fx+e\right)^{4}-3\left(c^{4}d^{2}+3\,c^{3}d^{3}+3\,c^{2}d^{4}+cd^{5}\right)f\cos\left(fx+e\right)^{3}-\left(3\,c^{5}d+12\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(7/2),x, algorithm="frica
s")
```

```
[Out] -2/15*(32*a^2*c^2 - 64*a^2*c*d + 32*a^2*d^2 - (3*a^2*c^2 + 14*a^2*c*d + 43*
a^2*d^2)*cos(f*x + e)^3 + (11*a^2*c^2 + 78*a^2*c*d - 29*a^2*d^2)*cos(f*x +
e)^2 + 2*(23*a^2*c^2 + 14*a^2*c*d + 23*a^2*d^2)*cos(f*x + e) - (32*a^2*c^2
- 64*a^2*c*d + 32*a^2*d^2 - (3*a^2*c^2 + 14*a^2*c*d + 43*a^2*d^2)*cos(f*x +
 e)^2 - 2*(7*a^2*c^2 + 46*a^2*c*d + 7*a^2*d^2)*cos(f*x + e))*sin(f*x + e))*
sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/((c^3*d^3 + 3*c^2*d^4 + 3
*c*d^5 + d^6)*f*cos(f*x + e)^4 - 3*(c^4*d^2 + 3*c^3*d^3 + 3*c^2*d^4 + c*d^5
)*f*cos(f*x + e)^3 - (3*c^5*d + 12*c^4*d^2 + 20*c^3*d^3 + 18*c^2*d^4 + 9*c*
d^5 + 2*d^6)*f*cos(f*x + e)^2 + (c^6 + 3*c^5*d + 6*c^4*d^2 + 10*c^3*d^3 + 9
*c^2*d^4 + 3*c*d^5)*f*cos(f*x + e) + (c^6 + 6*c^5*d + 15*c^4*d^2 + 20*c^3*d
^3 + 15*c^2*d^4 + 6*c*d^5 + d^6)*f - ((c^3*d^3 + 3*c^2*d^4 + 3*c*d^5 + d^6)
*f*cos(f*x + e)^3 + (3*c^4*d^2 + 10*c^3*d^3 + 12*c^2*d^4 + 6*c*d^5 + d^6)*f
*cos(f*x + e)^2 - (3*c^5*d + 9*c^4*d^2 + 10*c^3*d^3 + 6*c^2*d^4 + 3*c*d^5 +
 d^6)*f*cos(f*x + e) - (c^6 + 6*c^5*d + 15*c^4*d^2 + 20*c^3*d^3 + 15*c^2*d^
4 + 6*c*d^5 + d^6)*f)*sin(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(7/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [B] time = 0.40, size = 793, normalized size = 4.20

$$-\frac{2\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{5}{2}}\sqrt{c+d\sin\left(fx+e\right)}\,\left(480c^{3}\left(\cos^{2}\left(fx+e\right)\right)d^{2}+368\sin\left(fx+e\right)\left(\cos^{2}\left(fx+e\right)\right)d^{5}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(7/2),x)
```

```
[Out] -2/15/f*(a*(1+sin(f*x+e)))^(5/2)*(c+d*sin(f*x+e))^(1/2)*(-355*sin(f*x+e)*co
s(f*x+e)^4*d^5-287*sin(f*x+e)*cos(f*x+e)^4*c*d^4-352*sin(f*x+e)*cos(f*x+e)^
2*c^3*d^2-416*sin(f*x+e)*cos(f*x+e)^2*c^2*d^3+336*sin(f*x+e)*cos(f*x+e)^2*c
*d^4+128*c*d^4-256*c^2*d^3-256*c^3*d^2+128*c^4*d+544*cos(f*x+e)^2*c^2*d^3+3
*cos(f*x+e)^8*c^2*d^3+14*cos(f*x+e)^8*c*d^4-9*cos(f*x+e)^6*c^4*d-27*cos(f*x
+e)^6*c^3*d^2-3*sin(f*x+e)*cos(f*x+e)^4*c^5+115*sin(f*x+e)*cos(f*x+e)^6*d^5
-cos(f*x+e)^6*c^2*d^3-181*cos(f*x+e)^6*c*d^4+7*cos(f*x+e)^4*c^4*d-194*cos(f
*x+e)^4*c^3*d^2+16*sin(f*x+e)*cos(f*x+e)^2*c^5+523*cos(f*x+e)^4*d^5-432*cos
(f*x+e)^2*d^5-262*cos(f*x+e)^6*d^5-128*d^5*sin(f*x+e)+114*sin(f*x+e)*cos(f*
x+e)^4*c^2*d^3+48*sin(f*x+e)*cos(f*x+e)^2*c^4*d+79*sin(f*x+e)*cos(f*x+e)^6*
c*d^4+50*sin(f*x+e)*cos(f*x+e)^4*c^3*d^2+439*c*cos(f*x+e)^4*d^4+480*c^3*cos
(f*x+e)^2*d^2-400*c*cos(f*x+e)^2*d^4+256*c^2*d^3*sin(f*x+e)-128*c*d^4*sin(f
*x+e)-290*cos(f*x+e)^4*c^2*d^3+368*sin(f*x+e)*cos(f*x+e)^2*d^5-112*cos(f*x+
e)^2*c^4*d-128*sin(f*x+e)*c^4*d+256*sin(f*x+e)*c^3*d^2+128*d^5+43*cos(f*x+e
)^8*d^5-5*cos(f*x+e)^4*c^5-80*cos(f*x+e)^2*c^5+sin(f*x+e)*cos(f*x+e)^4*c^4*
d+9*sin(f*x+e)*cos(f*x+e)^6*c^3*d^2+37*sin(f*x+e)*cos(f*x+e)^6*c^2*d^3+128*
c^5-128*c^5*sin(f*x+e))/cos(f*x+e)^5/(cos(f*x+e)^2*d^2+c^2-d^2)^3/(c+d)^3
```

maxima [B] time = 1.14, size = 464, normalized size = 2.46

$$-\frac{2\left(\left(43\,c^3+14\,c^2d+3\,cd^2\right)a^{\frac{5}{2}}-\frac{\left(15\,c^3-256\,c^2d-53\,cd^2-6\,d^3\right)a^{\frac{5}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{\left(113\,c^3-116\,c^2d+493\,cd^2+50\,d^3\right)a^{\frac{5}{2}}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{5\left(17\,c^3\cdots\right.}{\cdots}\right.}{15\left(c^3\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(7/2),x, algorithm="maxim
a")
```

```
[Out] -2/15*((43*c^3 + 14*c^2*d + 3*c*d^2)*a^(5/2) - (15*c^3 - 256*c^2*d - 53*c*d
^2 - 6*d^3)*a^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + (113*c^3 - 116*c^2*d
+ 493*c*d^2 + 50*d^3)*a^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*(17*c
^3 - 82*c^2*d + 65*c*d^2 - 60*d^3)*a^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1
)^3 + 5*(17*c^3 - 82*c^2*d + 65*c*d^2 - 60*d^3)*a^(5/2)*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 - (113*c^3 - 116*c^2*d + 493*c*d^2 + 50*d^3)*a^(5/2)*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 + (15*c^3 - 256*c^2*d - 53*c*d^2 - 6*d^3)*a^
(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - (43*c^3 + 14*c^2*d + 3*c*d^2)*a
^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)*(sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 1)/((c^3 + 3*c^2*d + 3*c*d^2 + d^3 + (c^3 + 3*c^2*d + 3*c*d^2 + d^3
)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(c + 2*d*sin(f*x + e)/(cos(f*x + e)
+ 1) + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)^(7/2)*f)
```

mupad [B] time = 17.46, size = 590, normalized size = 3.12

$$-\frac{\sqrt{c+d\,\sin\left(e+f\,x\right)}\,\left(\frac{8\,a^2\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(15\,c^2-10\,c\,d+7\,d^2\right)}{3\,d^3\,f\,\left(c+d\right)^3}+\frac{4\,a^2\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(5\,c^2+34\,c\,d-3\,d^2\right)}{3\,d^3\,f\,\left(c+d\right)^3}-\frac{8\,a^2}{}\right.}{\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}+\frac{\left(c\,1\mathrm{i}+d\,1\mathrm{i}\right)^3}{\left(c+d\right)^3}-\frac{3\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\left(4\,c^2+2\,c\,d+d^2\right)}{d^2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\left(6\,c+d\right)}{d}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^(7/2),x)
```

```
[Out] -((c + d*sin(e + f*x))^(1/2)*((8*a^2*exp(e*4i + f*x*4i)*(a + a*sin(e + f*x)
)^(1/2)*(15*c^2 - 10*c*d + 7*d^2))/(3*d^3*f*(c + d)^3) + (4*a^2*exp(e*2i +
f*x*2i)*(a + a*sin(e + f*x))^(1/2)*(34*c*d + 5*c^2 - 3*d^2))/(3*d^3*f*(c +
d)^3) - (8*a^2*exp(e*3i + f*x*3i)*(a + a*sin(e + f*x))^(1/2)*(c^2*15i - c*d
*10i + d^2*7i))/(3*d^3*f*(c + d)^3) - (4*a^2*exp(e*5i + f*x*5i)*(a + a*sin(
e + f*x))^(1/2)*(c*d*34i + c^2*5i - d^2*3i))/(3*d^3*f*(c + d)^3) - (4*a^2*e
xp(e*6i + f*x*6i)*(a + a*sin(e + f*x))^(1/2)*(14*c*d + 3*c^2 + 43*d^2))/(15
*d^3*f*(c + d)^3) + (4*a^2*exp(e*1i + f*x*1i)*(a + a*sin(e + f*x))^(1/2)*(c
*d*14i + c^2*3i + d^2*43i))/(15*d^3*f*(c + d)^3)))/(exp(e*7i + f*x*7i) + (c
*1i + d*1i)^3/(c + d)^3 - (3*exp(e*5i + f*x*5i)*(2*c*d + 4*c^2 + d^2))/d^2
- (exp(e*1i + f*x*1i)*(6*c + d))/d + (exp(e*3i + f*x*3i)*(12*c*d^2 + 12*c^2
*d + 8*c^3 + 3*d^3))/d^3 + (exp(e*6i + f*x*6i)*(c*6i + d*1i))/d - (3*exp(e*
2i + f*x*2i)*(c*1i + d*1i)^3*(2*c*d + 4*c^2 + d^2))/(d^2*(c + d)^3) + (exp(
e*4i + f*x*4i)*(c*1i + d*1i)^3*(12*c*d^2 + 12*c^2*d + 8*c^3 + 3*d^3))/(d^3*
(c + d)^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

3.586 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=254

$$-\frac{4a^3\left(3c^2+22cd+115d^2\right)\cos(e+fx)}{105d^2f(c+d)^4\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}-\frac{2a^3\left(3c^2+22cd+115d^2\right)\cos(e+fx)}{105d^2f(c+d)^3\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{3/2}}$$

```
[Out] 6/35*a^3*(c-d)*(c+5*d)*cos(f*x+e)/d^2/(c+d)^2/f/(c+d*sin(f*x+e))^(5/2)/(a+a
*sin(f*x+e))^(1/2)-2/105*a^3*(3*c^2+22*c*d+115*d^2)*cos(f*x+e)/d^2/(c+d)^3/
f/(c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+2/7*a^2*(c-d)*cos(f*x+e)*(a
+a*sin(f*x+e))^(1/2)/d/(c+d)/f/(c+d*sin(f*x+e))^(7/2)-4/105*a^3*(3*c^2+22*c
*d+115*d^2)*cos(f*x+e)/d^2/(c+d)^4/f/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)
)^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2762, 2980, 2772, 2771}

$$-\frac{4a^3\left(3c^2+22cd+115d^2\right)\cos(e+fx)}{105d^2f(c+d)^4\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}-\frac{2a^3\left(3c^2+22cd+115d^2\right)\cos(e+fx)}{105d^2f(c+d)^3\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(9/2),x]
```

```
[Out] (2*a^2*(c - d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(7*d*(c + d)*f*(c + d
*Sin[e + f*x])^(7/2)) + (6*a^3*(c - d)*(c + 5*d)*Cos[e + f*x])/(35*d^2*(c +
 d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2)) - (2*a^3*(3*c^
2 + 22*c*d + 115*d^2)*Cos[e + f*x])/(105*d^2*(c + d)^3*f*Sqrt[a + a*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(3/2)) - (4*a^3*(3*c^2 + 22*c*d + 115*d^2)*Cos[
e + f*x])/(105*d^2*(c + d)^4*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e +
f*x]])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
```

```
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{9/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} - \frac{(2a)\int \frac{\sqrt{a+a\sin(e+fx)}\left(\frac{1}{2}a(c-15d)-\frac{1}{2}a(3c+11d)\cdots\right.}{(c+d\sin(e+fx))^{7/2}}}{7d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} + \frac{6a^3(c-d)(c+5d)\cos(e+\cdots}{35d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}\,(c+\cdots}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} + \frac{6a^3(c-d)(c+5d)\cos(e+\cdots}{35d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}\,(c+\cdots}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{7d(c+d)f(c+d\sin(e+fx))^{7/2}} + \frac{6a^3(c-d)(c+5d)\cos(e+\cdots}{35d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}\,(c+\cdots}$$

Mathematica [A] time = 4.31, size = 216, normalized size = 0.85

$$\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-623c^3+3c^2d\sin(3(e+fx))-495c^2d-\left(196c^3+18\cdots\right.\right.}{105f(c+d)^4\left(\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(9/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(-623
*c^3 - 495*c^2*d - 977*c*d^2 - 145*d^3 + (21*c^3 + 157*c^2*d + 827*c*d^2 +
115*d^3)*Cos[2*(e + f*x)] - (196*c^3 + 1865*c^2*d + 694*c*d^2 + 465*d^3)*Si
n[e + f*x] + 3*c^2*d*Sin[3*(e + f*x)] + 22*c*d^2*Sin[3*(e + f*x)] + 115*d^3
*Sin[3*(e + f*x)]))/(105*(c + d)^4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*
(c + d*Sin[e + f*x])^(7/2))
```

fricas [B] time = 0.59, size = 1033, normalized size = 4.07

$$-\frac{\cdots}{105\left(\left(c^4d^4+4\,c^3d^5+6\,c^2d^6+4\,cd^7+d^8\right)f\cos\left(fx+e\right)^5+\left(4\,c^5d^3+17\,c^4d^4+28\,c^3d^5+22\,c^2d^6+8\,cd^7+d^8\right)f\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(9/2),x, algorithm="frica
s")
```

```
[Out] -2/105*(224*a^2*c^3 - 608*a^2*c^2*d + 544*a^2*c*d^2 - 160*a^2*d^3 - 2*(3*a^
2*c^2*d + 22*a^2*c*d^2 + 115*a^2*d^3)*cos(f*x + e)^4 - (21*a^2*c^3 + 157*a^
2*c^2*d + 827*a^2*c*d^2 + 115*a^2*d^3)*cos(f*x + e)^3 + (77*a^2*c^3 + 783*a
^2*c^2*d - 425*a^2*c*d^2 + 405*a^2*d^3)*cos(f*x + e)^2 + 2*(161*a^2*c^3 + 1
63*a^2*c^2*d + 451*a^2*c*d^2 + 65*a^2*d^3)*cos(f*x + e) - (224*a^2*c^3 - 60
8*a^2*c^2*d + 544*a^2*c*d^2 - 160*a^2*d^3 + 2*(3*a^2*c^2*d + 22*a^2*c*d^2 +
 115*a^2*d^3)*cos(f*x + e)^3 - (21*a^2*c^3 + 151*a^2*c^2*d + 783*a^2*c*d^2
- 115*a^2*d^3)*cos(f*x + e)^2 - 2*(49*a^2*c^3 + 467*a^2*c^2*d + 179*a^2*c*d
^2 + 145*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt
(d*sin(f*x + e) + c)/((c^4*d^4 + 4*c^3*d^5 + 6*c^2*d^6 + 4*c*d^7 + d^8)*f*c
os(f*x + e)^5 + (4*c^5*d^3 + 17*c^4*d^4 + 28*c^3*d^5 + 22*c^2*d^6 + 8*c*d^7
 + d^8)*f*cos(f*x + e)^4 - 2*(3*c^6*d^2 + 12*c^5*d^3 + 19*c^4*d^4 + 16*c^3*
d^5 + 9*c^2*d^6 + 4*c*d^7 + d^8)*f*cos(f*x + e)^3 - 2*(2*c^7*d + 11*c^6*d^2
 + 28*c^5*d^3 + 43*c^4*d^4 + 42*c^3*d^5 + 25*c^2*d^6 + 8*c*d^7 + d^8)*f*cos
(f*x + e)^2 + (c^8 + 4*c^7*d + 12*c^6*d^2 + 28*c^5*d^3 + 38*c^4*d^4 + 28*c^
3*d^5 + 12*c^2*d^6 + 4*c*d^7 + d^8)*f*cos(f*x + e) + (c^8 + 8*c^7*d + 28*c^
6*d^2 + 56*c^5*d^3 + 70*c^4*d^4 + 56*c^3*d^5 + 28*c^2*d^6 + 8*c*d^7 + d^8)*
f + ((c^4*d^4 + 4*c^3*d^5 + 6*c^2*d^6 + 4*c*d^7 + d^8)*f*cos(f*x + e)^4 - 4
*(c^5*d^3 + 4*c^4*d^4 + 6*c^3*d^5 + 4*c^2*d^6 + c*d^7)*f*cos(f*x + e)^3 - 2
*(3*c^6*d^2 + 14*c^5*d^3 + 27*c^4*d^4 + 28*c^3*d^5 + 17*c^2*d^6 + 6*c*d^7 +
 d^8)*f*cos(f*x + e)^2 + 4*(c^7*d + 4*c^6*d^2 + 7*c^5*d^3 + 8*c^4*d^4 + 7*c
^3*d^5 + 4*c^2*d^6 + c*d^7)*f*cos(f*x + e) + (c^8 + 8*c^7*d + 28*c^6*d^2 +
56*c^5*d^3 + 70*c^4*d^4 + 56*c^3*d^5 + 28*c^2*d^6 + 8*c*d^7 + d^8)*f)*sin(f
*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(9/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [B] time = 0.51, size = 1223, normalized size = 4.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(9/2),x)
```

```
[Out] -2/105/f*(a*(1+sin(f*x+e)))^(5/2)*(c+d*sin(f*x+e))^(1/2)*(2944*c^4*d^3*sin(
f*x+e)-1664*c^3*d^4*sin(f*x+e)-2432*c^2*d^5*sin(f*x+e)+384*c*d^6*sin(f*x+e)
```

+225*sin(f*x+e)*cos(f*x+e)^8*c*d^6+1408*sin(f*x+e)*cos(f*x+e)^6*c^3*d^4+239
2*sin(f*x+e)*cos(f*x+e)^6*c^2*d^5-950*sin(f*x+e)*cos(f*x+e)^6*c*d^6+479*sin
(f*x+e)*cos(f*x+e)^4*c^5*d^2+1261*sin(f*x+e)*cos(f*x+e)^4*c^4*d^3-4367*sin(
f*x+e)*cos(f*x+e)^4*c^3*d^4-7117*sin(f*x+e)*cos(f*x+e)^4*c^2*d^5+1669*sin(f
*x+e)*cos(f*x+e)^4*c*d^6+368*sin(f*x+e)*cos(f*x+e)^2*c^6*d-3344*sin(f*x+e)*
cos(f*x+e)^2*c^5*d^2-5008*sin(f*x+e)*cos(f*x+e)^2*c^4*d^3+4560*sin(f*x+e)*c
os(f*x+e)^2*c^3*d^4+7120*sin(f*x+e)*cos(f*x+e)^2*c^2*d^5-1328*sin(f*x+e)*co
s(f*x+e)^2*c*d^6-896*sin(f*x+e)*c^7-1555*cos(f*x+e)^8*d^7+3940*cos(f*x+e)^6
*d^7-4775*cos(f*x+e)^4*d^7-560*cos(f*x+e)^2*c^7+2800*cos(f*x+e)^2*d^7-2176*
c^5*d^2+1664*c^3*d^4+896*c^7-2944*c^4*d^3-384*c*d^6+2432*c^2*d^5+1152*c^6*d
-640*d^7+640*d^7*sin(f*x+e)-944*cos(f*x+e)^2*c^6*d+6480*cos(f*x+e)^2*c^4*d^
3-5392*cos(f*x+e)^2*c^3*d^4-8336*cos(f*x+e)^2*c^2*d^5+1520*cos(f*x+e)^2*c*d
^6-1152*sin(f*x+e)*c^6*d+2176*sin(f*x+e)*c^5*d^2+575*sin(f*x+e)*cos(f*x+e)^
8*d^7+895*cos(f*x+e)^8*c^2*d^5-485*cos(f*x+e)^8*c*d^6-2350*sin(f*x+e)*cos(f
*x+e)^6*d^7-172*cos(f*x+e)^6*c^4*d^3-2968*cos(f*x+e)^6*c^3*d^4-5370*cos(f*x
+e)^6*c^2*d^5+1590*cos(f*x+e)^6*c*d^6+3615*sin(f*x+e)*cos(f*x+e)^4*d^7+39*c
os(f*x+e)^4*c^6*d-1879*cos(f*x+e)^4*c^5*d^2-3397*cos(f*x+e)^4*c^4*d^3+6439*
cos(f*x+e)^4*c^3*d^4+10373*cos(f*x+e)^4*c^2*d^5-2285*cos(f*x+e)^4*c*d^6+112
*sin(f*x+e)*cos(f*x+e)^2*c^7-2480*sin(f*x+e)*cos(f*x+e)^2*d^7+4432*cos(f*x+
e)^2*c^5*d^2+3*sin(f*x+e)*cos(f*x+e)^8*c^3*d^4+37*sin(f*x+e)*cos(f*x+e)^8*c
^2*d^5+102*sin(f*x+e)*cos(f*x+e)^6*c^5*d^2+518*sin(f*x+e)*cos(f*x+e)^6*c^4*
d^3+sin(f*x+e)*cos(f*x+e)^4*c^6*d+6*cos(f*x+e)^10*c^2*d^5+44*cos(f*x+e)^10*
c*d^6+48*cos(f*x+e)^8*c^4*d^3+257*cos(f*x+e)^8*c^3*d^4-78*cos(f*x+e)^6*c^6*
d-302*cos(f*x+e)^6*c^5*d^2-21*sin(f*x+e)*cos(f*x+e)^4*c^7-35*cos(f*x+e)^4*c
^7+230*cos(f*x+e)^10*d^7)/cos(f*x+e)^5/(cos(f*x+e)^2*d^2+c^2-d^2)^4/(c+d)^4

maxima [B] time = 1.21, size = 703, normalized size = 2.77

$$-\frac{2\left(\left(301\,c^{4}+169\,c^{3}d+75\,c^{2}d^{2}+15\,cd^{3}\right)a^{\frac{5}{2}}-\frac{3\left(35\,c^{4}-763\,c^{3}d-297\,c^{2}d^{2}-85\,cd^{3}-10\,d^{4}\right)a^{\frac{5}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{6\left(182\,c^{4}-127\,c^{3}d+1059\,c^{2}d^{2}\cdots\right.}{\left(\cos\left(f\cdots\right.\right.}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(9/2),x, algorithm="maxim
a")

[Out] -2/105*((301*c^4 + 169*c^3*d + 75*c^2*d^2 + 15*c*d^3)*a^(5/2) - 3*(35*c^4 -
763*c^3*d - 297*c^2*d^2 - 85*c*d^3 - 10*d^4)*a^(5/2)*sin(f*x + e)/(cos(f*x
+ e) + 1) + 6*(182*c^4 - 127*c^3*d + 1059*c^2*d^2 + 251*c*d^3 + 35*d^4)*a^
(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 14*(50*c^4 - 421*c^3*d + 201*c^
2*d^2 - 535*c*d^3 - 55*d^4)*a^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1
26*(11*c^4 - 36*c^3*d + 80*c^2*d^2 - 40*c*d^3 + 25*d^4)*a^(5/2)*sin(f*x + e

```
)^4/(cos(f*x + e) + 1)^4 - 126*(11*c^4 - 36*c^3*d + 80*c^2*d^2 - 40*c*d^3 +
 25*d^4)*a^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 14*(50*c^4 - 421*c^3
*d + 201*c^2*d^2 - 535*c*d^3 - 55*d^4)*a^(5/2)*sin(f*x + e)^6/(cos(f*x + e)
 + 1)^6 - 6*(182*c^4 - 127*c^3*d + 1059*c^2*d^2 + 251*c*d^3 + 35*d^4)*a^(5/
2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 3*(35*c^4 - 763*c^3*d - 297*c^2*d^
2 - 85*c*d^3 - 10*d^4)*a^(5/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - (301*c
^4 + 169*c^3*d + 75*c^2*d^2 + 15*c*d^3)*a^(5/2)*sin(f*x + e)^9/(cos(f*x + e
) + 1)^9)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^2/((c^4 + 4*c^3*d + 6*c
^2*d^2 + 4*c*d^3 + d^4 + 2*(c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4)*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + (c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^
4)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*(c + 2*d*sin(f*x + e)/(cos(f*x + e)
 + 1) + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)^(9/2)*f)
```

mupad [B] time = 20.61, size = 862, normalized size = 3.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^(9/2),x)
```

```
[Out] ((c + d*sin(e + f*x))^(1/2)*((8*a^2*exp(e*8i + f*x*8i)*(a + a*sin(e + f*x))
^(1/2)*(22*c*d + 3*c^2 + 115*d^2))/(105*d^3*f*(c + d)^4) + (8*a^2*exp(e*1i
+ f*x*1i)*(a + a*sin(e + f*x))^(1/2)*(c*d*22i + c^2*3i + d^2*115i))/(105*d^
3*f*(c + d)^4) - (8*a^2*exp(e*4i + f*x*4i)*(a + a*sin(e + f*x))^(1/2)*(36*c
*d^2 - 25*c^2*d + 30*c^3 - 5*d^3))/(3*d^4*f*(c + d)^4) - (8*a^2*exp(e*5i +
f*x*5i)*(a + a*sin(e + f*x))^(1/2)*(c*d^2*36i - c^2*d*25i + c^3*30i - d^3*5
i))/(3*d^4*f*(c + d)^4) - (8*a^2*exp(e*6i + f*x*6i)*(a + a*sin(e + f*x))^(1
/2)*(244*c^2*d - 19*c*d^2 + 25*c^3 + 50*d^3))/(15*d^4*f*(c + d)^4) - (8*a^2
*exp(e*3i + f*x*3i)*(a + a*sin(e + f*x))^(1/2)*(c^2*d*244i - c*d^2*19i + c^
3*25i + d^3*50i))/(15*d^4*f*(c + d)^4) + (8*a^2*c*exp(e*2i + f*x*2i)*(a + a
*sin(e + f*x))^(1/2)*(22*c*d + 3*c^2 + 115*d^2))/(15*d^4*f*(c + d)^4) + (8*
a^2*c*exp(e*7i + f*x*7i)*(a + a*sin(e + f*x))^(1/2)*(c*d*22i + c^2*3i + d^2
*115i))/(15*d^4*f*(c + d)^4)))/(exp(e*9i + f*x*9i) + ((c*1i + d*1i)^4*1i)/(
c + d)^4 - (4*exp(e*3i + f*x*3i)*(6*c*d^2 + 6*c^2*d + 8*c^3 + d^3))/d^3 - (
4*exp(e*7i + f*x*7i)*(2*c*d + 6*c^2 + d^2))/d^2 + (exp(e*1i + f*x*1i)*(8*c
+ d))/d + (2*exp(e*5i + f*x*5i)*(12*c*d^3 + 16*c^3*d + 8*c^4 + 3*d^4 + 24*c
^2*d^2))/d^4 - (exp(e*6i + f*x*6i)*(c*1i + d*1i)^4*(6*c*d^2 + 6*c^2*d + 8*c
^3 + d^3)*4i)/(d^3*(c + d)^4) - (exp(e*2i + f*x*2i)*(c*1i + d*1i)^4*(2*c*d
+ 6*c^2 + d^2)*4i)/(d^2*(c + d)^4) + (exp(e*8i + f*x*8i)*(8*c + d)*(c*1i +
d*1i)^4*1i)/(d*(c + d)^4) + (exp(e*4i + f*x*4i)*(c*1i + d*1i)^4*(12*c*d^3 +
 16*c^3*d + 8*c^4 + 3*d^4 + 24*c^2*d^2)*2i)/(d^4*(c + d)^4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

3.587 $\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=317

$$-\frac{16a^3\left(c^2+10cd+73d^2\right)\cos(e+fx)}{315d^2f(c+d)^5\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}-\frac{8a^3\left(c^2+10cd+73d^2\right)\cos(e+fx)}{315d^2f(c+d)^4\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{3/2}}$$

```
[Out] 2/63*a^3*(c-d)*(3*c+19*d)*cos(f*x+e)/d^2/(c+d)^2/f/(c+d*sin(f*x+e))^(7/2)/(
a+a*sin(f*x+e))^(1/2)-2/105*a^3*(c^2+10*c*d+73*d^2)*cos(f*x+e)/d^2/(c+d)^3/
f/(c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)-8/315*a^3*(c^2+10*c*d+73*d^
2)*cos(f*x+e)/d^2/(c+d)^4/f/(c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+2
/9*a^2*(c-d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d/(c+d)/f/(c+d*sin(f*x+e))^(
9/2)-16/315*a^3*(c^2+10*c*d+73*d^2)*cos(f*x+e)/d^2/(c+d)^5/f/(a+a*sin(f*x+e
))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.78, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2762, 2980, 2772, 2771}

$$-\frac{16a^3\left(c^2+10cd+73d^2\right)\cos(e+fx)}{315d^2f(c+d)^5\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}-\frac{8a^3\left(c^2+10cd+73d^2\right)\cos(e+fx)}{315d^2f(c+d)^4\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(11/2),x]
```

```
[Out] (2*a^2*(c - d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(9*d*(c + d)*f*(c + d
*Sin[e + f*x])^(9/2)) + (2*a^3*(c - d)*(3*c + 19*d)*Cos[e + f*x])/(63*d^2*(
c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(7/2)) - (2*a^3*(c
^2 + 10*c*d + 73*d^2)*Cos[e + f*x])/(105*d^2*(c + d)^3*f*Sqrt[a + a*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(5/2)) - (8*a^3*(c^2 + 10*c*d + 73*d^2)*Cos[e +
 f*x])/(315*d^2*(c + d)^4*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(
3/2)) - (16*a^3*(c^2 + 10*c*d + 73*d^2)*Cos[e + f*x])/(315*d^2*(c + d)^5*f*
Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

```
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{11/2}}\,dx = \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{9d(c+d)f(c+d\sin(e+fx))^{9/2}} - \frac{(2a)\int \frac{\sqrt{a+a\sin(e+fx)}\left(\frac{1}{2}a(c-19d)-\frac{3}{2}a(c+5d\cdots}{(c+d\sin(e+fx))^{9/2}}}{9d(c+d)}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{9d(c+d)f(c+d\sin(e+fx))^{9/2}} + \frac{2a^3(c-d)(3c+19d)\cos\cdots}{63d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}\,(c\cdots}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{9d(c+d)f(c+d\sin(e+fx))^{9/2}} + \frac{2a^3(c-d)(3c+19d)\cos\cdots}{63d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}\,(c\cdots}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{9d(c+d)f(c+d\sin(e+fx))^{9/2}} + \frac{2a^3(c-d)(3c+19d)\cos\cdots}{63d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}\,(c\cdots}$$

$$= \frac{2a^2(c-d)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{9d(c+d)f(c+d\sin(e+fx))^{9/2}} + \frac{2a^3(c-d)(3c+19d)\cos\cdots}{63d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}\,(c\cdots}$$

Mathematica [A] time = 6.55, size = 616, normalized size = 1.94

$$\frac{(a(\sin(e+fx)+1))^{5/2}\sqrt{c+d\sin(e+fx)}\left(-\frac{16\left(-c^2\sin\left(\frac{1}{2}(e+fx)\right)+c^2\cos\left(\frac{1}{2}(e+fx)\right)-10cd\sin\left(\frac{1}{2}(e+fx)\right)+10cd\cos\left(\frac{1}{2}(e+fx)\right)-73d^2\sin\cdots}{315d^2(c+d)^5(c+d\sin(e+fx))}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(11/2),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c + d*Sin[e + f*x]]*((-2*(c^2*Cos[(e + f
*x)/2] - 2*c*d*Cos[(e + f*x)/2] + d^2*Cos[(e + f*x)/2] - c^2*Sin[(e + f*x)/
2] + 2*c*d*Sin[(e + f*x)/2] - d^2*Sin[(e + f*x)/2]))/(9*d^2*(c + d)*(c + d*
Sin[e + f*x])^5) - (4*(-5*c^2*Cos[(e + f*x)/2] - 8*c*d*Cos[(e + f*x)/2] + 1
3*d^2*Cos[(e + f*x)/2] + 5*c^2*Sin[(e + f*x)/2] + 8*c*d*Sin[(e + f*x)/2] -
13*d^2*Sin[(e + f*x)/2]))/(63*d^2*(c + d)^2*(c + d*Sin[e + f*x])^4) - (2*(c
^2*Cos[(e + f*x)/2] + 10*c*d*Cos[(e + f*x)/2] + 73*d^2*Cos[(e + f*x)/2] - c
^2*Sin[(e + f*x)/2] - 10*c*d*Sin[(e + f*x)/2] - 73*d^2*Sin[(e + f*x)/2]))/(
105*d^2*(c + d)^3*(c + d*Sin[e + f*x])^3) - (8*(c^2*Cos[(e + f*x)/2] + 10*c
*d*Cos[(e + f*x)/2] + 73*d^2*Cos[(e + f*x)/2] - c^2*Sin[(e + f*x)/2] - 10*c
*d*Sin[(e + f*x)/2] - 73*d^2*Sin[(e + f*x)/2]))/(315*d^2*(c + d)^4*(c + d*S
in[e + f*x])^2) - (16*(c^2*Cos[(e + f*x)/2] + 10*c*d*Cos[(e + f*x)/2] + 73*
d^2*Cos[(e + f*x)/2] - c^2*Sin[(e + f*x)/2] - 10*c*d*Sin[(e + f*x)/2] - 73*
```

```
d^2*Sin[(e + f*x)/2]))/(315*d^2*(c + d)^5*(c + d*Sin[e + f*x]))))/(f*(Cos[(
e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

fricas [B] time = 0.66, size = 1492, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(11/2),x, algorithm="fric
as")
```

```
[Out] 2/315*(672*a^2*c^4 - 2304*a^2*c^3*d + 3008*a^2*c^2*d^2 - 1792*a^2*c*d^3 + 4
16*a^2*d^4 + 8*(a^2*c^2*d^2 + 10*a^2*c*d^3 + 73*a^2*d^4)*cos(f*x + e)^5 - 4
*(9*a^2*c^3*d + 89*a^2*c^2*d^2 + 647*a^2*c*d^3 - 73*a^2*d^4)*cos(f*x + e)^4
 - (63*a^2*c^4 + 648*a^2*c^3*d + 4798*a^2*c^2*d^2 + 1504*a^2*c*d^3 + 1387*a
^2*d^4)*cos(f*x + e)^3 + (231*a^2*c^4 + 3060*a^2*c^3*d - 2158*a^2*c^2*d^2 +
 4580*a^2*c*d^3 - 673*a^2*d^4)*cos(f*x + e)^2 + 2*(483*a^2*c^4 + 684*a^2*c^
3*d + 2642*a^2*c^2*d^2 + 812*a^2*c*d^3 + 419*a^2*d^4)*cos(f*x + e) - (672*a
^2*c^4 - 2304*a^2*c^3*d + 3008*a^2*c^2*d^2 - 1792*a^2*c*d^3 + 416*a^2*d^4 +
 8*(a^2*c^2*d^2 + 10*a^2*c*d^3 + 73*a^2*d^4)*cos(f*x + e)^4 + 4*(9*a^2*c^3*
d + 91*a^2*c^2*d^2 + 667*a^2*c*d^3 + 73*a^2*d^4)*cos(f*x + e)^3 - 3*(21*a^2
*c^4 + 204*a^2*c^3*d + 1478*a^2*c^2*d^2 - 388*a^2*c*d^3 + 365*a^2*d^4)*cos(
f*x + e)^2 - 2*(147*a^2*c^4 + 1836*a^2*c^3*d + 1138*a^2*c^2*d^2 + 1708*a^2*
c*d^3 + 211*a^2*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*s
qrt(d*sin(f*x + e) + c)/((c^5*d^5 + 5*c^4*d^6 + 10*c^3*d^7 + 10*c^2*d^8 + 5
*c*d^9 + d^10)*f*cos(f*x + e)^6 - 5*(c^6*d^4 + 5*c^5*d^5 + 10*c^4*d^6 + 10*
c^3*d^7 + 5*c^2*d^8 + c*d^9)*f*cos(f*x + e)^5 - (10*c^7*d^3 + 55*c^6*d^4 +
128*c^5*d^5 + 165*c^4*d^6 + 130*c^3*d^7 + 65*c^2*d^8 + 20*c*d^9 + 3*d^10)*f
*cos(f*x + e)^4 + 10*(c^8*d^2 + 5*c^7*d^3 + 11*c^6*d^4 + 15*c^5*d^5 + 15*c^
4*d^6 + 11*c^3*d^7 + 5*c^2*d^8 + c*d^9)*f*cos(f*x + e)^3 + (5*c^9*d + 35*c^
8*d^2 + 120*c^7*d^3 + 260*c^6*d^4 + 378*c^5*d^5 + 370*c^4*d^6 + 240*c^3*d^7
 + 100*c^2*d^8 + 25*c*d^9 + 3*d^10)*f*cos(f*x + e)^2 - (c^10 + 5*c^9*d + 20
*c^8*d^2 + 60*c^7*d^3 + 110*c^6*d^4 + 126*c^5*d^5 + 100*c^4*d^6 + 60*c^3*d^
7 + 25*c^2*d^8 + 5*c*d^9)*f*cos(f*x + e) - (c^10 + 10*c^9*d + 45*c^8*d^2 +
120*c^7*d^3 + 210*c^6*d^4 + 252*c^5*d^5 + 210*c^4*d^6 + 120*c^3*d^7 + 45*c^
2*d^8 + 10*c*d^9 + d^10)*f - ((c^5*d^5 + 5*c^4*d^6 + 10*c^3*d^7 + 10*c^2*d^
8 + 5*c*d^9 + d^10)*f*cos(f*x + e)^5 + (5*c^6*d^4 + 26*c^5*d^5 + 55*c^4*d^6
 + 60*c^3*d^7 + 35*c^2*d^8 + 10*c*d^9 + d^10)*f*cos(f*x + e)^4 - 2*(5*c^7*d
^3 + 25*c^6*d^4 + 51*c^5*d^5 + 55*c^4*d^6 + 35*c^3*d^7 + 15*c^2*d^8 + 5*c*d
^9 + d^10)*f*cos(f*x + e)^3 - 2*(5*c^8*d^2 + 30*c^7*d^3 + 80*c^6*d^4 + 126*
c^5*d^5 + 130*c^4*d^6 + 90*c^3*d^7 + 40*c^2*d^8 + 10*c*d^9 + d^10)*f*cos(f*
x + e)^2 + (5*c^9*d + 25*c^8*d^2 + 60*c^7*d^3 + 100*c^6*d^4 + 126*c^5*d^5 +
 110*c^4*d^6 + 60*c^3*d^7 + 20*c^2*d^8 + 5*c*d^9 + d^10)*f*cos(f*x + e) + (
c^10 + 10*c^9*d + 45*c^8*d^2 + 120*c^7*d^3 + 210*c^6*d^4 + 252*c^5*d^5 + 21
0*c^4*d^6 + 120*c^3*d^7 + 45*c^2*d^8 + 10*c*d^9 + d^10)*f)*sin(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(11/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

maple [B] time = 0.56, size = 1730, normalized size = 5.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(11/2),x)
```

```
[Out] -2/315/f*(a*(1+sin(f*x+e)))^(5/2)*(c+d*sin(f*x+e))^(1/2)*(8*cos(f*x+e)^12*c
^2*d^7+13568*c^4*d^5+4224*c^8*d-7168*c^7*d^2+7424*c^5*d^4+15474*sin(f*x+e)*
cos(f*x+e)^6*d^9-12288*c^6*d^3-279*cos(f*x+e)^6*c^8*d-1310*cos(f*x+e)^6*c^7
*d^2-1482*cos(f*x+e)^6*c^6*d^3-17010*cos(f*x+e)^6*c^5*d^4-35980*cos(f*x+e)^
6*c^4*d^5+11406*cos(f*x+e)^6*c^3*d^6+41570*cos(f*x+e)^6*c^2*d^7-11902*cos(f
*x+e)^6*c*d^8-63*sin(f*x+e)*cos(f*x+e)^4*c^9-15847*sin(f*x+e)*cos(f*x+e)^4*
d^9+87*cos(f*x+e)^4*c^8*d-7220*cos(f*x+e)^4*c^7*d^2-15204*cos(f*x+e)^4*c^6*
d^3+31650*cos(f*x+e)^4*c^5*d^4+63090*cos(f*x+e)^4*c^4*d^5-19908*cos(f*x+e)^
4*c^3*d^6-51540*cos(f*x+e)^4*c^2*d^7+11711*cos(f*x+e)^4*c*d^8+336*sin(f*x+e
)*cos(f*x+e)^2*c^9+8112*sin(f*x+e)*cos(f*x+e)^2*d^9+80*cos(f*x+e)^12*c*d^8+
1460*sin(f*x+e)*cos(f*x+e)^10*d^9+37*cos(f*x+e)^10*c^4*d^5+360*cos(f*x+e)^1
0*c^3*d^6+2538*cos(f*x+e)^10*c^2*d^7-1360*cos(f*x+e)^10*c*d^8-7535*sin(f*x+
e)*cos(f*x+e)^8*d^9-3312*cos(f*x+e)^2*c^8*d+16192*cos(f*x+e)^2*c^7*d^2+2886
4*cos(f*x+e)^2*c^6*d^3-23904*cos(f*x+e)^2*c^5*d^4-47520*cos(f*x+e)^2*c^4*d^
5+15168*cos(f*x+e)^2*c^3*d^6+30912*cos(f*x+e)^2*c^2*d^7-5776*cos(f*x+e)^2*c
*d^8-4224*sin(f*x+e)*c^8*d+7168*sin(f*x+e)*c^7*d^2+12288*sin(f*x+e)*c^6*d^3
-7424*sin(f*x+e)*c^5*d^4-13568*sin(f*x+e)*c^4*d^5+4096*sin(f*x+e)*c^3*d^6+7
168*sin(f*x+e)*c^2*d^7-1152*sin(f*x+e)*c*d^8+310*cos(f*x+e)^8*c^6*d^3+1875*
cos(f*x+e)^8*c^5*d^4+6805*cos(f*x+e)^8*c^4*d^5+4*sin(f*x+e)*cos(f*x+e)^10*c
^3*d^6+60*sin(f*x+e)*cos(f*x+e)^10*c^2*d^7+492*sin(f*x+e)*cos(f*x+e)^10*c*d
^8-35*sin(f*x+e)*cos(f*x+e)^8*c^5*d^4+5*sin(f*x+e)*cos(f*x+e)^8*c^4*d^5+165
0*sin(f*x+e)*cos(f*x+e)^8*c^3*d^6+1664*d^9-4096*d^6*c^3-7168*d^7*c^2+1152*d
^8*c+5850*sin(f*x+e)*cos(f*x+e)^8*c^2*d^7-3295*sin(f*x+e)*cos(f*x+e)^8*c*d^
8+1200*sin(f*x+e)*cos(f*x+e)^2*c^8*d-12608*sin(f*x+e)*cos(f*x+e)^2*c^7*d^2-
22720*sin(f*x+e)*cos(f*x+e)^2*c^6*d^3+20192*sin(f*x+e)*cos(f*x+e)^2*c^5*d^4
+40736*sin(f*x+e)*cos(f*x+e)^2*c^4*d^5-13120*sin(f*x+e)*cos(f*x+e)^2*c^3*d^
6-27328*sin(f*x+e)*cos(f*x+e)^2*c^2*d^7+5200*sin(f*x+e)*cos(f*x+e)^2*c*d^8+
```

```
458*sin(f*x+e)*cos(f*x+e)^6*c^7*d^2+2730*sin(f*x+e)*cos(f*x+e)^6*c^6*d^3+87
74*sin(f*x+e)*cos(f*x+e)^6*c^5*d^4+17070*sin(f*x+e)*cos(f*x+e)^6*c^4*d^5-64
90*sin(f*x+e)*cos(f*x+e)^6*c^3*d^6-24522*sin(f*x+e)*cos(f*x+e)^6*c^2*d^7+80
10*sin(f*x+e)*cos(f*x+e)^6*c*d^8-15*sin(f*x+e)*cos(f*x+e)^4*c^8*d+1812*sin(
f*x+e)*cos(f*x+e)^4*c^7*d^2+5380*sin(f*x+e)*cos(f*x+e)^4*c^6*d^3-22482*sin(
f*x+e)*cos(f*x+e)^4*c^5*d^4-44418*sin(f*x+e)*cos(f*x+e)^4*c^4*d^5+13860*sin
(f*x+e)*cos(f*x+e)^4*c^3*d^6+38772*sin(f*x+e)*cos(f*x+e)^4*c^2*d^7-9255*sin
(f*x+e)*cos(f*x+e)^4*c*d^8+2688*c^9-2930*cos(f*x+e)^8*c^3*d^6-16320*cos(f*x
+e)^8*c^2*d^7+6095*cos(f*x+e)^8*c*d^8+584*cos(f*x+e)^12*d^9-4599*cos(f*x+e)
^10*d^9+14245*cos(f*x+e)^8*d^9-22645*cos(f*x+e)^6*d^9-105*cos(f*x+e)^4*c^9+
19695*cos(f*x+e)^4*d^9-1680*cos(f*x+e)^2*c^9-8944*cos(f*x+e)^2*d^9-2688*sin
(f*x+e)*c^9-1664*sin(f*x+e)*d^9)/cos(f*x+e)^5/(cos(f*x+e)^2*d^2+c^2-d^2)^5/
(c+d)^5
```

maxima [B] time = 1.95, size = 984, normalized size = 3.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(11/2),x, algorithm="maxi
ma")
```

```
[Out] -2/315*((903*c^5 + 720*c^4*d + 494*c^3*d^2 + 200*c^2*d^3 + 35*c*d^4)*a^(5/2
) - (315*c^5 - 8358*c^4*d - 4770*c^3*d^2 - 2284*c^2*d^3 - 625*c*d^4 - 70*d^
5)*a^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + (4179*c^5 - 1710*c^4*d + 30878
*c^3*d^2 + 11540*c^2*d^3 + 3383*c*d^4 + 450*d^5)*a^(5/2)*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 - 3*(805*c^5 - 9912*c^4*d + 2330*c^3*d^2 - 18504*c^2*d^3
- 3895*c*d^4 - 504*d^5)*a^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 6*(12
39*c^5 - 3100*c^4*d + 12918*c^3*d^2 - 3560*c^2*d^3 + 8043*c*d^4 + 700*d^5)*
a^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 42*(149*c^5 - 894*c^4*d + 140
2*c^3*d^2 - 2052*c^2*d^3 + 745*c*d^4 - 390*d^5)*a^(5/2)*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 + 42*(149*c^5 - 894*c^4*d + 1402*c^3*d^2 - 2052*c^2*d^3 +
745*c*d^4 - 390*d^5)*a^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 6*(1239*
c^5 - 3100*c^4*d + 12918*c^3*d^2 - 3560*c^2*d^3 + 8043*c*d^4 + 700*d^5)*a^(
5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 3*(805*c^5 - 9912*c^4*d + 2330*c
^3*d^2 - 18504*c^2*d^3 - 3895*c*d^4 - 504*d^5)*a^(5/2)*sin(f*x + e)^8/(cos(
f*x + e) + 1)^8 - (4179*c^5 - 1710*c^4*d + 30878*c^3*d^2 + 11540*c^2*d^3 +
3383*c*d^4 + 450*d^5)*a^(5/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + (315*c^
5 - 8358*c^4*d - 4770*c^3*d^2 - 2284*c^2*d^3 - 625*c*d^4 - 70*d^5)*a^(5/2)*
sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - (903*c^5 + 720*c^4*d + 494*c^3*d^2
+ 200*c^2*d^3 + 35*c*d^4)*a^(5/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11)*(s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^3/((c^5 + 5*c^4*d + 10*c^3*d^2 + 10
*c^2*d^3 + 5*c*d^4 + d^5 + 3*(c^5 + 5*c^4*d + 10*c^3*d^2 + 10*c^2*d^3 + 5*c
*d^4 + d^5)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*(c^5 + 5*c^4*d + 10*c^3
*d^2 + 10*c^2*d^3 + 5*c*d^4 + d^5)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + (c
```

```
^5 + 5*c^4*d + 10*c^3*d^2 + 10*c^2*d^3 + 5*c*d^4 + d^5)*sin(f*x + e)^6/(cos
(f*x + e) + 1)^6)*(c + 2*d*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2)^(11/2)*f)
```

mupad [B] time = 26.14, size = 1155, normalized size = 3.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^(11/2),x)
```

```
[Out] -((c + d*sin(e + f*x))^(1/2)*((32*a^2*exp(e*1i + f*x*1i)*(a + a*sin(e + f*x
))^(1/2)*(c*d*10i + c^2*1i + d^2*73i))/(315*d^3*f*(c + d)^5) - (32*a^2*exp(
e*10i + f*x*10i)*(a + a*sin(e + f*x))^(1/2)*(10*c*d + c^2 + 73*d^2))/(315*d
^3*f*(c + d)^5) - (32*a^2*exp(e*6i + f*x*6i)*(a + a*sin(e + f*x))^(1/2)*(25
*c^4 - 25*c^3*d - 15*c*d^3 + 6*d^4 + 57*c^2*d^2))/(5*d^5*f*(c + d)^5) + (32
*a^2*exp(e*5i + f*x*5i)*(a + a*sin(e + f*x))^(1/2)*(c^4*25i - c^3*d*25i - c
*d^3*15i + d^4*6i + c^2*d^2*57i))/(5*d^5*f*(c + d)^5) - (16*a^2*exp(e*4i +
f*x*4i)*(a + a*sin(e + f*x))^(1/2)*(194*c*d^3 + 318*c^3*d + 25*c^4 - 5*d^4
- 20*c^2*d^2))/(15*d^5*f*(c + d)^5) + (16*a^2*exp(e*7i + f*x*7i)*(a + a*sin
(e + f*x))^(1/2)*(c*d^3*194i + c^3*d*318i + c^4*25i - d^4*5i - c^2*d^2*20i)
)/(15*d^5*f*(c + d)^5) + (16*a^2*exp(e*8i + f*x*8i)*(a + a*sin(e + f*x))^(1
/2)*(10*c*d^3 + 70*c^3*d + 7*c^4 + 73*d^4 + 512*c^2*d^2))/(35*d^5*f*(c + d)
^5) - (16*a^2*exp(e*3i + f*x*3i)*(a + a*sin(e + f*x))^(1/2)*(c*d^3*10i + c^
3*d*70i + c^4*7i + d^4*73i + c^2*d^2*512i))/(35*d^5*f*(c + d)^5) + (32*a^2*
c*exp(e*2i + f*x*2i)*(a + a*sin(e + f*x))^(1/2)*(10*c*d + c^2 + 73*d^2))/(3
5*d^4*f*(c + d)^5) - (32*a^2*c*exp(e*9i + f*x*9i)*(a + a*sin(e + f*x))^(1/2
)*(c*d*10i + c^2*1i + d^2*73i))/(35*d^4*f*(c + d)^5)))/(exp(e*11i + f*x*11i
) - (c*1i + d*1i)^5/(c + d)^5 + (10*exp(e*7i + f*x*7i)*(4*c*d^3 + 8*c^3*d +
 8*c^4 + d^4 + 12*c^2*d^2))/d^4 + (5*exp(e*3i + f*x*3i)*(8*c*d^2 + 8*c^2*d
+ 16*c^3 + d^3))/d^3 - (5*exp(e*9i + f*x*9i)*(2*c*d + 8*c^2 + d^2))/d^2 - (
2*exp(e*5i + f*x*5i)*(30*c*d^4 + 40*c^4*d + 16*c^5 + 5*d^5 + 60*c^2*d^3 + 8
0*c^3*d^2))/d^5 - (exp(e*1i + f*x*1i)*(10*c + d))/d - (5*exp(e*8i + f*x*8i)
*(c*1i + d*1i)^5*(8*c*d^2 + 8*c^2*d + 16*c^3 + d^3))/(d^3*(c + d)^5) + (5*e
xp(e*2i + f*x*2i)*(c*1i + d*1i)^5*(2*c*d + 8*c^2 + d^2))/(d^2*(c + d)^5) +
(2*exp(e*6i + f*x*6i)*(c*1i + d*1i)^5*(30*c*d^4 + 40*c^4*d + 16*c^5 + 5*d^5
 + 60*c^2*d^3 + 80*c^3*d^2))/(d^5*(c + d)^5) + (exp(e*10i + f*x*10i)*(10*c
+ d)*(c*1i + d*1i)^5)/(d*(c + d)^5) - (10*exp(e*4i + f*x*4i)*(c*1i + d*1i)^
5*(4*c*d^3 + 8*c^3*d + 8*c^4 + d^4 + 12*c^2*d^2))/(d^4*(c + d)^5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

3.588 $\int \frac{(c+d\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=249

$$-\frac{\sqrt{d}\left(15c^2-10cd+7d^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{4\sqrt{a}f}-\frac{d\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}-\frac{d(7c-d)\cos(}{4f\sqrt{}}$$

```
[Out] -(c-d)^(5/2)*arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1/2)/(a+a*sin(f*
x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))*2^(1/2)/f/a^(1/2)-1/4*(15*c^2-10*c*d+7*
d^2)*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+
e))^(1/2))*d^(1/2)/f/a^(1/2)-1/2*d*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/f/(a+a
*sin(f*x+e))^(1/2)-1/4*(7*c-d)*d*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+a*s
in(f*x+e))^(1/2)
```

Rubi [A] time = 0.93, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2778, 2983, 2982, 2782, 208, 2775, 205}

$$-\frac{\sqrt{d}\left(15c^2-10cd+7d^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{4\sqrt{a}f}-\frac{d\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}-\frac{d(7c-d)\cos(}{4f\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -(Sqrt[d]*(15*c^2 - 10*c*d + 7*d^2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(
Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(4*Sqrt[a]*f) - (Sqrt[
2]*(c - d)^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a
 + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[a]*f) - ((7*c - d)*d*C
os[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*f*Sqrt[a + a*Sin[e + f*x]]) - (d*C
os[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(2*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{d\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} - \frac{\int \frac{\sqrt{c+d\sin(e+fx)}\left(-a\left(4c^2-cd+3d^2\right)-a(7c-d)d\sin(e+f}{\sqrt{a+a\sin(e+fx)}}}{4a}$$

$$= -\frac{(7c-d)d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f\sqrt{a+a\sin(e+fx)}} - \frac{d\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} -$$

$$= -\frac{(7c-d)d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f\sqrt{a+a\sin(e+fx)}} - \frac{d\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} +$$

$$= -\frac{(7c-d)d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f\sqrt{a+a\sin(e+fx)}} - \frac{d\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} -$$

$$= -\frac{\sqrt{d}\left(15c^2-10cd+7d^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{4\sqrt{a}f} - \frac{\sqrt{2}(c-d)^{5/2}\tanh}{}$$

Mathematica [C] time = 17.61, size = 1893, normalized size = 7.60

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]*((d*(-9*c +
 2*d)*Cos[(e + f*x)/2])/4 - (d^2*Cos[(3*(e + f*x))/2])/4 - (d*(-9*c + 2*d)*
Sin[(e + f*x)/2])/4 - (d^2*Sin[(3*(e + f*x))/2])/4))/(f*Sqrt[a*(1 + Sin[e +
 f*x])]) + ((Sqrt[2]*(c - d)^(5/2)*Log[1 + Tan[(e + f*x)/2]] - Sqrt[2]*(c -
 d)^(5/2)*Log[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c +
d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]] - (I/8)*Sqrt[d]*(15*c^2 - 10*c
*d + 7*d^2)*Log[(2*(c - I*(d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*
x])^(-1)]*Sqrt[c + d*Sin[e + f*x]]) + ((-I)*c + d)*Tan[(e + f*x)/2]))/(d^(3
/2)*(15*c^2 - 10*c*d + 7*d^2)*(I + Tan[(e + f*x)/2]))] + (I/8)*Sqrt[d]*(15*
c^2 - 10*c*d + 7*d^2)*Log[(2*(c + I*d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + C
os[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (I*c + d)*Tan[(e + f*x)/2]))/
(d^(3/2)*(15*c^2 - 10*c*d + 7*d^2)*(-I + Tan[(e + f*x)/2]))])*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])*(c^3/((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c
```

```
 + d*Sin[e + f*x]]) - (9*c^2*d)/(8*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sq
rt[c + d*Sin[e + f*x]]) + (7*c*d^2)/(4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*Sqrt[c + d*Sin[e + f*x]]) - d^3/(8*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*
Sqrt[c + d*Sin[e + f*x]]) + (15*c^2*d*Sin[e + f*x])/(8*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) - (5*c*d^2*Sin[e + f*x])/(4*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) + (7*d^3*Sin[e
 + f*x])/(8*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]])
))/(f*Sqrt[a*(1 + Sin[e + f*x])]*(((c - d)^(5/2)*Sec[(e + f*x)/2]^2)/(Sqrt[
2]*(1 + Tan[(e + f*x)/2])) - (Sqrt[2]*(c - d)^(5/2)*(((-c + d)*Sec[(e + f*x
)/2]^2)/2 + (Sqrt[c - d]*d*Cos[e + f*x]*Sqrt[(1 + Cos[e + f*x])^(-1)])/Sqrt
[c + d*Sin[e + f*x]] + Sqrt[c - d]*((1 + Cos[e + f*x])^(-1))^(3/2)*Sin[e +
f*x]*Sqrt[c + d*Sin[e + f*x]]))/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*
x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]) - ((I/16)*d
^2*(15*c^2 - 10*c*d + 7*d^2)^2*(I + Tan[(e + f*x)/2])*((2*((((-I)*c + d)*Se
c[(e + f*x)/2]^2)/2 - I*(((1 + I)*d^(3/2)*Cos[e + f*x]*Sqrt[(1 + Cos[e + f*
x])^(-1)])/(Sqrt[2]*Sqrt[c + d*Sin[e + f*x]]) + ((1 + I)*Sqrt[d]*((1 + Cos[
e + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[2])))/(d^
(3/2)*(15*c^2 - 10*c*d + 7*d^2)*(I + Tan[(e + f*x)/2])) - (Sec[(e + f*x)/2]
^2*(c - I*(d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c
 + d*Sin[e + f*x]]) + ((-I)*c + d)*Tan[(e + f*x)/2]))/(d^(3/2)*(15*c^2 - 10
*c*d + 7*d^2)*(I + Tan[(e + f*x)/2])^2)))/(c - I*(d + (1 + I)*Sqrt[2]*Sqrt[
d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]]) + ((-I)*c + d)*T
an[(e + f*x)/2]) + ((I/16)*d^2*(15*c^2 - 10*c*d + 7*d^2)^2*(-I + Tan[(e + f
*x)/2])*((2*(((I*c + d)*Sec[(e + f*x)/2]^2)/2 + ((1 + I)*d^(3/2)*Cos[e + f*
x]*Sqrt[(1 + Cos[e + f*x])^(-1)])/(Sqrt[2]*Sqrt[c + d*Sin[e + f*x]]) + ((1
+ I)*Sqrt[d]*((1 + Cos[e + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e
+ f*x]])/Sqrt[2]))/(d^(3/2)*(15*c^2 - 10*c*d + 7*d^2)*(-I + Tan[(e + f*x)/2
])) - (Sec[(e + f*x)/2]^2*(c + I*d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[
e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (I*c + d)*Tan[(e + f*x)/2]))/(d^
(3/2)*(15*c^2 - 10*c*d + 7*d^2)*(-I + Tan[(e + f*x)/2])^2)))/(c + I*d + (1
+ I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]]
 + (I*c + d)*Tan[(e + f*x)/2])))
```

fricas [B] time = 1.23, size = 2925, normalized size = 11.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/32*(16*sqrt(2)*(a*c^2 - 2*a*c*d + a*d^2 + (a*c^2 - 2*a*c*d + a*d^2)*cos(
f*x + e) + (a*c^2 - 2*a*c*d + a*d^2)*sin(f*x + e))*sqrt((c - d)/a)*log((2*s
qrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt((c - d)/a)*(c
os(f*x + e) - sin(f*x + e) + 1) - (c - 3*d)*cos(f*x + e)^2 - (3*c - d)*cos(
```

```
f*x + e) + ((c - 3*d)*cos(f*x + e) - 2*c - 2*d)*sin(f*x + e) - 2*c - 2*d)/(
cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + (15
*a*c^2 - 10*a*c*d + 7*a*d^2 + (15*a*c^2 - 10*a*c*d + 7*a*d^2)*cos(f*x + e)
+ (15*a*c^2 - 10*a*c*d + 7*a*d^2)*sin(f*x + e))*sqrt(-d/a)*log((128*d^4*cos
(f*x + e)^5 + 128*(2*c*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^
2 + 4*c*d^3 + d^4 - 32*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3 - 32*
(c^3*d - 2*c^2*d^2 + 9*c*d^3 - 4*d^4)*cos(f*x + e)^2 - 8*(16*d^3*cos(f*x +
e)^4 + 24*(c*d^2 - d^3)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3
 - 2*(5*c^2*d - 26*c*d^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d
^2 - 25*d^3)*cos(f*x + e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*
d^2 - 51*d^3 - 8*(3*c*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 +
 13*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*
x + e) + c)*sqrt(-d/a) + (c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 + 289*d^
4)*cos(f*x + e) + (128*d^4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c
*d^3 + d^4 - 256*(c*d^3 - d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d^3 + 5
*d^4)*cos(f*x + e)^2 + 32*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(f*x +
e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)) - 8*(2*d^2*cos(f*x + e
)^2 + 9*c*d - 3*d^2 + (9*c*d - d^2)*cos(f*x + e) + (2*d^2*cos(f*x + e) - 9*
c*d + 3*d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c
))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f), 1/16*(8*sqrt(2)*(a*c^2 - 2*
a*c*d + a*d^2 + (a*c^2 - 2*a*c*d + a*d^2)*cos(f*x + e) + (a*c^2 - 2*a*c*d +
 a*d^2)*sin(f*x + e))*sqrt((c - d)/a)*log((2*sqrt(2)*sqrt(a*sin(f*x + e) +
a)*sqrt(d*sin(f*x + e) + c)*sqrt((c - d)/a)*(cos(f*x + e) - sin(f*x + e) +
1) - (c - 3*d)*cos(f*x + e)^2 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x
 + e) - 2*c - 2*d)*sin(f*x + e) - 2*c - 2*d)/(cos(f*x + e)^2 - (cos(f*x + e
) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + (15*a*c^2 - 10*a*c*d + 7*a*d^2 +
 (15*a*c^2 - 10*a*c*d + 7*a*d^2)*cos(f*x + e) + (15*a*c^2 - 10*a*c*d + 7*a*
d^2)*sin(f*x + e))*sqrt(d/a)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d
 - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(
f*x + e) + c)*sqrt(d/a)/(2*d^3*cos(f*x + e)^3 - (3*c*d^2 - d^3)*cos(f*x + e
)*sin(f*x + e) - (c^2*d - c*d^2 + 2*d^3)*cos(f*x + e))) - 4*(2*d^2*cos(f*x
+ e)^2 + 9*c*d - 3*d^2 + (9*c*d - d^2)*cos(f*x + e) + (2*d^2*cos(f*x + e) -
 9*c*d + 3*d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f), -1/32*(32*sqrt(2)*(a*c^2
 - 2*a*c*d + a*d^2 + (a*c^2 - 2*a*c*d + a*d^2)*cos(f*x + e) + (a*c^2 - 2*a*
c*d + a*d^2)*sin(f*x + e))*sqrt(-(c - d)/a)*arctan(-sqrt(2)*sqrt(a*sin(f*x
+ e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-(c - d)/a)/((c - d)*cos(f*x + e)))
 - (15*a*c^2 - 10*a*c*d + 7*a*d^2 + (15*a*c^2 - 10*a*c*d + 7*a*d^2)*cos(f*x
 + e) + (15*a*c^2 - 10*a*c*d + 7*a*d^2)*sin(f*x + e))*sqrt(-d/a)*log((128*d
^4*cos(f*x + e)^5 + 128*(2*c*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*
c^2*d^2 + 4*c*d^3 + d^4 - 32*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3
 - 32*(c^3*d - 2*c^2*d^2 + 9*c*d^3 - 4*d^4)*cos(f*x + e)^2 - 8*(16*d^3*cos(
f*x + e)^4 + 24*(c*d^2 - d^3)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 +
51*d^3 - 2*(5*c^2*d - 26*c*d^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d +
31*c*d^2 - 25*d^3)*cos(f*x + e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d +
```

```
 59*c*d^2 - 51*d^3 - 8*(3*c*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c
*d^2 + 13*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*
sin(f*x + e) + c)*sqrt(-d/a) + (c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 +
289*d^4)*cos(f*x + e) + (128*d^4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2
 + 4*c*d^3 + d^4 - 256*(c*d^3 - d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d
^3 + 5*d^4)*cos(f*x + e)^2 + 32*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(
f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)) + 8*(2*d^2*cos(f
*x + e)^2 + 9*c*d - 3*d^2 + (9*c*d - d^2)*cos(f*x + e) + (2*d^2*cos(f*x + e
) - 9*c*d + 3*d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x +
e) + c))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f), -1/16*(16*sqrt(2)*(a*
c^2 - 2*a*c*d + a*d^2 + (a*c^2 - 2*a*c*d + a*d^2)*cos(f*x + e) + (a*c^2 - 2
*a*c*d + a*d^2)*sin(f*x + e))*sqrt(-(c - d)/a)*arctan(-sqrt(2)*sqrt(a*sin(f
*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-(c - d)/a)/((c - d)*cos(f*x + e
))) - (15*a*c^2 - 10*a*c*d + 7*a*d^2 + (15*a*c^2 - 10*a*c*d + 7*a*d^2)*cos(
f*x + e) + (15*a*c^2 - 10*a*c*d + 7*a*d^2)*sin(f*x + e))*sqrt(d/a)*arctan(1
/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e)
)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(d/a)/(2*d^3*cos(f*
x + e)^3 - (3*c*d^2 - d^3)*cos(f*x + e)*sin(f*x + e) - (c^2*d - c*d^2 + 2*d
^3)*cos(f*x + e))) + 4*(2*d^2*cos(f*x + e)^2 + 9*c*d - 3*d^2 + (9*c*d - d^2
)*cos(f*x + e) + (2*d^2*cos(f*x + e) - 9*c*d + 3*d^2)*sin(f*x + e))*sqrt(a*
sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(a*f*cos(f*x + e) + a*f*sin(f*x
 + e) + a*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\sqrt{a \sin\left(fx+e\right)+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/sqrt(a*sin(f*x + e) + a), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+d \sin\left(fx+e\right)\right)^{\frac{5}{2}}}{\sqrt{a+a \sin\left(fx+e\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] int((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/sqrt(a*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.589 $\int \frac{(c+d\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=188

$$-\frac{d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}-\frac{\sqrt{d}\,(3c-d)\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\,f}-\frac{\sqrt{2}\,(c-d)^{3/2}\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{2}\,\sqrt{a\sin}}\right.}{\sqrt{a}\,f}$$

```
[Out] -(c-d)^(3/2)*arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1/2)/(a+a*sin(f*
x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))*2^(1/2)/f/a^(1/2)-(3*c-d)*arctan(cos(f*
x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))*d^(1/2)
/f/a^(1/2)-d*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.60, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2778, 2982, 2782, 208, 2775, 205}

$$-\frac{d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}-\frac{\sqrt{d}\,(3c-d)\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\,f}-\frac{\sqrt{2}\,(c-d)^{3/2}\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{2}\,\sqrt{a\sin}}\right.}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -(((3*c - d)*Sqrt[d]*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[a]*f)) - (Sqrt[2]*(c - d)^(3/2)
*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[a]*f) - (d*Cos[e + f*x]*Sqrt[c + d*Sin
[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
```

```
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} - \frac{\int \frac{-a(2c^2-cd+d^2)-a(3c-d)d\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{2a}$$

$$= -\frac{d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + (c-d)^2\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+$$

$$= -\frac{d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} - \frac{\left(2a(c-d)^2\right)\text{Subst}\left(\int \frac{1}{2a^2-(ac-ad)x^2}\,dx, x, \frac{}{\sqrt{a+a\,s}}\right.}{f}$$

$$= -\frac{(3c-d)\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\,f} - \frac{\sqrt{2}(c-d)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{}}{\sqrt{2}\sqrt{a+a\sin(}}\right.}{\sqrt{a}\,f}$$

Mathematica [C] time = 17.21, size = 1639, normalized size = 8.72

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-(d*Cos[(e + f*x)/2]) + d*Sin[(e +
f*x)/2])*Sqrt[c + d*Sin[e + f*x]])/(f*Sqrt[a*(1 + Sin[e + f*x])]) + ((Sqrt[
2]*(c - d)^(3/2)*Log[1 + Tan[(e + f*x)/2]] - Sqrt[2]*(c - d)^(3/2)*Log[c -
d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] +
(-c + d)*Tan[(e + f*x)/2]] + (I/2)*Sqrt[d]*(-3*c + d)*(Log[((2*I)*(I*c + d
+ (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e +
f*x]] + (c + I*d)*Tan[(e + f*x)/2]))/(d^(3/2)*(-3*c + d)*(I + Tan[(e + f*x)
/2]))] - Log[(-2*(c + I*d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])
^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (I*c + d)*Tan[(e + f*x)/2]))/(d^(3/2)*(-3
*c + d)*(-I + Tan[(e + f*x)/2]))]))*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(
c^2/((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) - (c*d
)/(2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) + d^2/
(2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) + (3*c*d
*Sin[e + f*x])/(2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e +
f*x]]) - (d^2*Sin[e + f*x])/(2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c
 + d*Sin[e + f*x]])))/(f*Sqrt[a*(1 + Sin[e + f*x])]*(((c - d)^(3/2)*Sec[(e
+ f*x)/2]^2)/(Sqrt[2]*(1 + Tan[(e + f*x)/2])) - (Sqrt[2]*(c - d)^(3/2)*(((-
c + d)*Sec[(e + f*x)/2]^2)/2 + (Sqrt[c - d]*d*Cos[e + f*x]*Sqrt[(1 + Cos[e
+ f*x])^(-1)])/Sqrt[c + d*Sin[e + f*x]] + Sqrt[c - d]*((1 + Cos[e + f*x])^(
```

```
-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]]))/(c - d + 2*Sqrt[c - d]*S
qrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f
*x)/2]) + (I/2)*Sqrt[d]*(-3*c + d)*(((-1/2*I)*d^(3/2)*(-3*c + d)*(I + Tan[(
e + f*x)/2])*(((2*I)*(((c + I*d)*Sec[(e + f*x)/2]^2)/2 + ((1 + I)*d^(3/2)*C
os[e + f*x]*Sqrt[(1 + Cos[e + f*x])^(-1)])/(Sqrt[2]*Sqrt[c + d*Sin[e + f*x]
]) + ((1 + I)*Sqrt[d]*((1 + Cos[e + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c +
 d*Sin[e + f*x]])/Sqrt[2]))/(d^(3/2)*(-3*c + d)*(I + Tan[(e + f*x)/2])) - (
I*Sec[(e + f*x)/2]^2*(I*c + d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f
*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (c + I*d)*Tan[(e + f*x)/2]))/(d^(3/2)
*(-3*c + d)*(I + Tan[(e + f*x)/2])^2)))/(I*c + d + (1 + I)*Sqrt[2]*Sqrt[d]*
Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (c + I*d)*Tan[(e +
 f*x)/2]) + (d^(3/2)*(-3*c + d)*(-I + Tan[(e + f*x)/2])*((-2*(((I*c + d)*Se
c[(e + f*x)/2]^2)/2 + ((1 + I)*d^(3/2)*Cos[e + f*x]*Sqrt[(1 + Cos[e + f*x])
^(-1)])/(Sqrt[2]*Sqrt[c + d*Sin[e + f*x]]) + ((1 + I)*Sqrt[d]*((1 + Cos[e +
 f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[2]))/(d^(3/2
)*(-3*c + d)*(-I + Tan[(e + f*x)/2])) + (Sec[(e + f*x)/2]^2*(c + I*d + (1 +
 I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]]
+ (I*c + d)*Tan[(e + f*x)/2]))/(d^(3/2)*(-3*c + d)*(-I + Tan[(e + f*x)/2])^
2)))/(2*(c + I*d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sq
rt[c + d*Sin[e + f*x]] + (I*c + d)*Tan[(e + f*x)/2]))))
```

fricas [B] time = 1.07, size = 2525, normalized size = 13.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [-1/8*(4*sqrt(2)*(a*c - a*d + (a*c - a*d)*cos(f*x + e) + (a*c - a*d)*sin(f*
x + e))*sqrt((c - d)/a)*log(-(2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin
(f*x + e) + c)*sqrt((c - d)/a)*(cos(f*x + e) - sin(f*x + e) + 1) + (c - 3*d
)*cos(f*x + e)^2 + (3*c - d)*cos(f*x + e) - ((c - 3*d)*cos(f*x + e) - 2*c -
 2*d)*sin(f*x + e) + 2*c + 2*d)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*
x + e) - cos(f*x + e) - 2)) + (3*a*c - a*d + (3*a*c - a*d)*cos(f*x + e) + (
3*a*c - a*d)*sin(f*x + e))*sqrt(-d/a)*log((128*d^4*cos(f*x + e)^5 + 128*(2*
c*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 3
2*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3 - 32*(c^3*d - 2*c^2*d^2 +
9*c*d^3 - 4*d^4)*cos(f*x + e)^2 + 8*(16*d^3*cos(f*x + e)^4 + 24*(c*d^2 - d^
3)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5*c^2*d - 26*c*
d^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 25*d^3)*cos(f*x
+ e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 - 51*d^3 - 8*(3*c
*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3)*cos(f*x + e)
)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-d/a
) + (c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 + 289*d^4)*cos(f*x + e) + (12
```

8*d^4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 256*(c*d^3 - d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d^3 + 5*d^4)*cos(f*x + e)^2 + 32*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)) + 8*(d*cos(f*x + e) - d*sin(f*x + e) + d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f), -1/4*(2*sqrt(2)*(a*c - a*d + (a*c - a*d)*cos(f*x + e) + (a*c - a*d)*sin(f*x + e))*sqrt((c - d)/a)*log(-(2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt((c - d)/a)*(cos(f*x + e) - sin(f*x + e) + 1) + (c - 3*d)*cos(f*x + e)^2 + (3*c - d)*cos(f*x + e) - ((c - 3*d)*cos(f*x + e) - 2*c - 2*d)*sin(f*x + e) + 2*c + 2*d)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - (3*a*c - a*d + (3*a*c - a*d)*cos(f*x + e) + (3*a*c - a*d)*sin(f*x + e))*sqrt(d/a)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(d/a)/(2*d^3*cos(f*x + e)^3 - (3*c*d^2 - d^3)*cos(f*x + e)*sin(f*x + e) - (c^2*d - c*d^2 + 2*d^3)*cos(f*x + e))) + 4*(d*cos(f*x + e) - d*sin(f*x + e) + d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f), -1/8*(8*sqrt(2)*(a*c - a*d + (a*c - a*d)*cos(f*x + e) + (a*c - a*d)*sin(f*x + e))*sqrt(-(c - d)/a)*arctan(-sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-(c - d)/a)/((c - d)*cos(f*x + e))) + (3*a*c - a*d + (3*a*c - a*d)*cos(f*x + e) + (3*a*c - a*d)*sin(f*x + e))*sqrt(-d/a)*log((128*d^4*cos(f*x + e)^5 + 128*(2*c*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 32*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3 - 32*(c^3*d - 2*c^2*d^2 + 9*c*d^3 - 4*d^4)*cos(f*x + e)^2 + 8*(16*d^3*cos(f*x + e)^4 + 24*(c*d^2 - d^3)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5*c^2*d - 26*c*d^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 25*d^3)*cos(f*x + e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 - 51*d^3 - 8*(3*c*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-d/a) + (c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 + 289*d^4)*cos(f*x + e) + (128*d^4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 256*(c*d^3 - d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d^3 + 5*d^4)*cos(f*x + e)^2 + 32*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)) + 8*(d*cos(f*x + e) - d*sin(f*x + e) + d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f), -1/4*(4*sqrt(2)*(a*c - a*d + (a*c - a*d)*cos(f*x + e) + (a*c - a*d)*sin(f*x + e))*sqrt(-(c - d)/a)*arctan(-sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-(c - d)/a)/((c - d)*cos(f*x + e))) - (3*a*c - a*d + (3*a*c - a*d)*cos(f*x + e) + (3*a*c - a*d)*sin(f*x + e))*sqrt(d/a)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(d/a)/(2*d^3*cos(f*x + e)^3 - (3*c*d^2 - d^3)*cos(f*x + e)*sin(f*x + e) - (c^2*d - c*d^2 + 2*d^3)*cos(f*x + e))) + 4*(d*cos(f*x + e) - d*sin(f*x + e) + d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\sqrt{a \sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/sqrt(a*sin(f*x + e) + a), x)
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{\left(c+d \sin\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{a+a \sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] int((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\sqrt{a \sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/sqrt(a*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + d \sin\left(e + f x\right)\right)^{\frac{3}{2}}}{\sqrt{a \left(\sin\left(e + f x\right) + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((c + d*sin(e + f*x))**(3/2)/sqrt(a*(sin(e + f*x) + 1)), x)
```

3.590 $\int \frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=141

$$-\frac{2\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\,f}-\frac{\sqrt{2}\,\sqrt{c-d}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{c-d}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\,f}$$

```
[Out] -arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2)/
(c+d*sin(f*x+e))^(1/2))*2^(1/2)*(c-d)^(1/2)/f/a^(1/2)-2*arctan(cos(f*x+e)*a
^(1/2)*d^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))*d^(1/2)/f/a^(
1/2)
```

Rubi [A] time = 0.30, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2777, 2775, 205, 2782, 208}

$$-\frac{2\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\,f}-\frac{\sqrt{2}\,\sqrt{c-d}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{c-d}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (-2*Sqrt[d]*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[a]*f) - (Sqrt[2]*Sqrt[c - d]*ArcTanh[(Sq
rt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c +
d*Sin[e + f*x]])])/(Sqrt[a]*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
```

```
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2777

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(a_) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c
 + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b, Int[1/(Sqrt[a + b*Sin[e +
f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = (c-d)\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx + \frac{d\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx}{a}$$

$$= -\frac{(2a(c-d))\,\mathrm{Subst}\left(\int \frac{1}{2a^2-(ac-ad)x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{f} - \frac{(2d)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{2\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\,f} - \frac{\sqrt{2}\sqrt{c-d}\,\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+f}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\,s}}\right.}{\sqrt{a}\,f}$$

Mathematica [C] time = 15.15, size = 1251, normalized size = 8.87

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] ((Sqrt[2]*Sqrt[c - d]*Log[1 + Tan[(e + f*x)/2]] - Sqrt[2]*Sqrt[c - d]*Log[c
 - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]]
```

```
 + (-c + d)*Tan[(e + f*x)/2]] - I*Sqrt[d]*(Log[(2*(c - I*d + (1 - I)*Sqrt[2
]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + ((-I)*c
+ d)*Tan[(e + f*x)/2]))/(d^(3/2)*(I + Tan[(e + f*x)/2]))] - Log[(2*(c + I*d
 + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e +
 f*x]] + (I*c + d)*Tan[(e + f*x)/2]))/(d^(3/2)*(-I + Tan[(e + f*x)/2]))]))*
Sqrt[c + d*Sin[e + f*x]])/(f*Sqrt[a*(1 + Sin[e + f*x])]*((Sqrt[c - d]*Sec[(
e + f*x)/2]^2)/(Sqrt[2]*(1 + Tan[(e + f*x)/2])) - (Sqrt[2]*Sqrt[c - d]*(((-
c + d)*Sec[(e + f*x)/2]^2)/2 + (Sqrt[c - d]*d*Cos[e + f*x]*Sqrt[(1 + Cos[e
+ f*x])^(-1)])/Sqrt[c + d*Sin[e + f*x]] + Sqrt[c - d]*((1 + Cos[e + f*x])^(
-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]]))/(c - d + 2*Sqrt[c - d]*S
qrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f
*x)/2]) - I*Sqrt[d]*((d^(3/2)*(I + Tan[(e + f*x)/2])*((2*((((-I)*c + d)*Sec
[(e + f*x)/2]^2)/2 + ((1 - I)*d^(3/2)*Cos[e + f*x]*Sqrt[(1 + Cos[e + f*x])^
(-1)])/(Sqrt[2]*Sqrt[c + d*Sin[e + f*x]]) + ((1 - I)*Sqrt[d]*((1 + Cos[e +
f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[2]))/(d^(3/2)
*(I + Tan[(e + f*x)/2])) - (Sec[(e + f*x)/2]^2*(c - I*d + (1 - I)*Sqrt[2]*S
qrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + ((-I)*c + d
)*Tan[(e + f*x)/2]))/(d^(3/2)*(I + Tan[(e + f*x)/2])^2)))/(2*(c - I*d + (1
- I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]]
 + ((-I)*c + d)*Tan[(e + f*x)/2])) - (d^(3/2)*(-I + Tan[(e + f*x)/2])*((2*(
((I*c + d)*Sec[(e + f*x)/2]^2)/2 + ((1 + I)*d^(3/2)*Cos[e + f*x]*Sqrt[(1 +
Cos[e + f*x])^(-1)])/(Sqrt[2]*Sqrt[c + d*Sin[e + f*x]]) + ((1 + I)*Sqrt[d]*
((1 + Cos[e + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/Sqrt
[2]))/(d^(3/2)*(-I + Tan[(e + f*x)/2])) - (Sec[(e + f*x)/2]^2*(c + I*d + (1
 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]
] + (I*c + d)*Tan[(e + f*x)/2]))/(d^(3/2)*(-I + Tan[(e + f*x)/2])^2)))/(2*(
c + I*d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*
Sin[e + f*x]] + (I*c + d)*Tan[(e + f*x)/2])))))
```

fricas [B] time = 0.91, size = 1944, normalized size = 13.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/4*(2*sqrt(2)*sqrt((c - d)/a)*log((2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqr
t(d*sin(f*x + e) + c)*sqrt((c - d)/a)*(cos(f*x + e) - sin(f*x + e) + 1) - (
c - 3*d)*cos(f*x + e)^2 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e)
- 2*c - 2*d)*sin(f*x + e) - 2*c - 2*d)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)
*sin(f*x + e) - cos(f*x + e) - 2)) + sqrt(-d/a)*log((128*d^4*cos(f*x + e)^5
 + 128*(2*c*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3
 + d^4 - 32*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3 - 32*(c^3*d - 2*
c^2*d^2 + 9*c*d^3 - 4*d^4)*cos(f*x + e)^2 - 8*(16*d^3*cos(f*x + e)^4 + 24*(
```

```
c*d^2 - d^3)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5*c^2
*d - 26*c*d^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 25*d^3
)*cos(f*x + e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 - 51*d^
3 - 8*(3*c*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3)*co
s(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)
*sqrt(-d/a) + (c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 + 289*d^4)*cos(f*x
+ e) + (128*d^4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4
- 256*(c*d^3 - d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d^3 + 5*d^4)*cos(f
*x + e)^2 + 32*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(f*x + e))*sin(f*x
 + e))/(cos(f*x + e) + sin(f*x + e) + 1)))/f, 1/2*(sqrt(2)*sqrt((c - d)/a)*
log((2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt((c -
d)/a)*(cos(f*x + e) - sin(f*x + e) + 1) - (c - 3*d)*cos(f*x + e)^2 - (3*c -
 d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) - 2*c - 2*d)*sin(f*x + e) - 2*c
- 2*d)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2
)) + sqrt(d/a)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(
c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*
sqrt(d/a)/(2*d^3*cos(f*x + e)^3 - (3*c*d^2 - d^3)*cos(f*x + e)*sin(f*x + e)
 - (c^2*d - c*d^2 + 2*d^3)*cos(f*x + e))))/f, -1/4*(4*sqrt(2)*sqrt(-(c - d)
/a)*arctan(-sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(
-(c - d)/a)/((c - d)*cos(f*x + e))) - sqrt(-d/a)*log((128*d^4*cos(f*x + e)^
5 + 128*(2*c*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^
3 + d^4 - 32*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3 - 32*(c^3*d - 2
*c^2*d^2 + 9*c*d^3 - 4*d^4)*cos(f*x + e)^2 - 8*(16*d^3*cos(f*x + e)^4 + 24*
(c*d^2 - d^3)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5*c^
2*d - 26*c*d^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 25*d^
3)*cos(f*x + e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 - 51*d
^3 - 8*(3*c*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3)*c
os(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c
)*sqrt(-d/a) + (c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 + 289*d^4)*cos(f*x
 + e) + (128*d^4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4
 - 256*(c*d^3 - d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d^3 + 5*d^4)*cos(
f*x + e)^2 + 32*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(f*x + e))*sin(f*
x + e))/(cos(f*x + e) + sin(f*x + e) + 1)))/f, -1/2*(2*sqrt(2)*sqrt(-(c - d
)/a)*arctan(-sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt
(-(c - d)/a)/((c - d)*cos(f*x + e))) - sqrt(d/a)*arctan(1/4*(8*d^2*cos(f*x
+ e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x +
 e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(d/a)/(2*d^3*cos(f*x + e)^3 - (3*c*d^
2 - d^3)*cos(f*x + e)*sin(f*x + e) - (c^2*d - c*d^2 + 2*d^3)*cos(f*x + e)))
)/f]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d \sin\left(fx+e\right)+c}}{\sqrt{a \sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/sqrt(a*sin(f*x + e) + a), x)
```

maple [B] time = 0.44, size = 3359, normalized size = 23.82

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*(-2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*(-(d^2/c^2)^(1/2)*c
)^(1/2)*c^2*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))
/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*
cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d*sin(f*x+e)+2*2^(1/2)*((c+d*sin
(f*x+e))/(cos(f*x+e)+1))^(1/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)*c*(2*c-2*d)^(1/2)
*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(
f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+
e)+sin(f*x+e)))*d^2*sin(f*x+e)-2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1
/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2
)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x
+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d^3*sin(f*x+e
)-cos(f*x+e)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*(-(d^2/c^2)^(1
/2)*c)^(1/2)*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e)
)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d
*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2*d+2*cos(f*x+e)*2^(1/2)*((c+
d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)*(2*c-2*d)^(1
/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*s
in(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f
*x+e)+sin(f*x+e)))*c*d^2-cos(f*x+e)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1
))^(1/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2
^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*si
n(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d^3-2*(d
^2/c^2)^(1/2)*c^3*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/
2)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^
2/c^2)^(1/2)*c)^(1/2))*d*cos(f*x+e)+4*(d^2/c^2)^(1/2)*c^2*((c+d*sin(f*x+e))
/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*arctan(((c+d*sin(f*x+e))/((d^2/c
^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*d^2*cos(f*x+
e)-2*(d^2/c^2)^(1/2)*c*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d
)^(1/2)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/
(-(d^2/c^2)^(1/2)*c)^(1/2))*d^3*cos(f*x+e)-2^(1/2)*((c+d*sin(f*x+e))/(cos(f
*x+e)+1))^(1/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^
```

```
(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+
e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c
^2*d+2*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*(-(d^2/c^2)^(1/2)*c)
^(1/2)*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos
(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f
*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d^2-2^(1/2)*((c+d*sin(f*x+e))/(cos(
f*x+e)+1))^(1/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)
^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x
+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*
d^3+2*(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4
*c^2*d^2-4*d^4)*c)^(1/2)*(d^2/c^2)^(1/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)*c*((c+d
*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*arctan(((d^2/c^2)^(1
/2)*c*sin(f*x+e)+d*cos(f*x+e)-d)/((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f
*x+e)+d)*d)^(1/2)/((d^2/c^2)^(1/2)*c*sin(f*x+e)-d*cos(f*x+e)+d)*((d^2/c^2)^
(1/2)*c^2-d^2)*c*((d^2/c^2)^(1/2)-1)/(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2
)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2))*sin(f*x+e)+2*c^2*(
(c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*arctan(((c+d*sin
(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/
2))*d^2*cos(f*x+e)-4*c*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d
)^(1/2)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/
(-(d^2/c^2)^(1/2)*c)^(1/2))*d^3*cos(f*x+e)+2*((c+d*sin(f*x+e))/((d^2/c^2)^(
1/2)*c*sin(f*x+e)+d)*d)^(1/2)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*s
in(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*d^4*cos(f*x+e)+2*(((d^2/c
^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4
)*c)^(1/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*
sin(f*x+e)+d)*d)^(1/2)*arctan(((d^2/c^2)^(1/2)*c*sin(f*x+e)+d*cos(f*x+e)-d)
/((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/((d^2/c^2)^(1/
2)*c*sin(f*x+e)-d*cos(f*x+e)+d)*((d^2/c^2)^(1/2)*c^2-d^2)*c*((d^2/c^2)^(1/2
)-1)/(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*
c^2*d^2-4*d^4)*c)^(1/2))*d*sin(f*x+e)+2*(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(
1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2)*(d^2/c^2)^(1/2)*
(-(d^2/c^2)^(1/2)*c)^(1/2)*c*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e
)+d)*d)^(1/2)*arctan(((d^2/c^2)^(1/2)*c*sin(f*x+e)+d*cos(f*x+e)-d)/((c+d*si
n(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/((d^2/c^2)^(1/2)*c*sin(
f*x+e)-d*cos(f*x+e)+d)*((d^2/c^2)^(1/2)*c^2-d^2)*c*((d^2/c^2)^(1/2)-1)/(((d
^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4
*d^4)*c)^(1/2))+2*(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/
c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)*((c+d*sin(f
*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*arctan(((d^2/c^2)^(1/2)*c*
sin(f*x+e)+d*cos(f*x+e)-d)/((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+
d)*d)^(1/2)/((d^2/c^2)^(1/2)*c*sin(f*x+e)-d*cos(f*x+e)+d)*((d^2/c^2)^(1/2)*
c^2-d^2)*c*((d^2/c^2)^(1/2)-1)/(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*
c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2))*d)/(a*(1+sin(f*x+e)))^(1
/2)/(c+d*sin(f*x+e))^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2)/d/(c^2-2*c*d+d^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\sin\left(fx+e\right)+c}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/sqrt(a*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c+d\,\sin\left(e+f\,x\right)}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+d\sin\left(e+fx\right)}}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*sin(e + f*x))/sqrt(a*(sin(e + f*x) + 1)), x)
```

3.591 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=79

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}f\sqrt{c-d}}$$

```
[Out] -arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2)/
(c+d*sin(f*x+e))^(1/2))*2^(1/2)/f/a^(1/2)/(c-d)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2782, 208}

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}f\sqrt{c-d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*S
in[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])]])/(Sqrt[a]*Sqrt[c - d]*f))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{2a^2-(ac-ad)x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{f}$$
$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\sqrt{c-d}\,f}$$

Mathematica [B] time = 4.10, size = 283, normalized size = 3.58

$$\frac{\log\left(\tan\left(\frac{1}{2}(e+fx)\right)+1\right)-\log\left((d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c-d\right)}{f\sqrt{a(\sin(e+fx)+1)}\sqrt{c+d\sin(e+fx)}\left(\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2\tan\left(\frac{1}{2}(e+fx)\right)+2}-\frac{\frac{\sqrt{c-d}\left(\frac{1}{\cos(e+fx)+1}\right)^{3/2}(c\sin(e+fx)+d\cos(e+fx)+d)}{\sqrt{c+d\sin(e+fx)}}-\frac{1}{2}(c-d)\sec^2\left(\frac{1}{2}(e+fx)\right)}{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c-d}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (Log[1 + Tan[(e + f*x)/2]] - Log[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*
x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]])/(f*Sqrt[a*
(1 + Sin[e + f*x])]*Sqrt[c + d*Sin[e + f*x]]*(Sec[(e + f*x)/2]^2/(2 + 2*Tan
[(e + f*x)/2]) - (-1/2*((c - d)*Sec[(e + f*x)/2]^2) + (Sqrt[c - d]*((1 + Co
s[e + f*x])^(-1))^(3/2)*(d + d*Cos[e + f*x] + c*Sin[e + f*x]))/Sqrt[c + d*S
in[e + f*x]])/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c +
 d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2])))
```

fricas [A] time = 0.68, size = 475, normalized size = 6.01

$$\left[\frac{\sqrt{2}\log\left(\frac{(c^2-14cd+17d^2)\cos(fx+e)^3-(13c^2-22cd-3d^2)\cos(fx+e)^2-\frac{4\sqrt{2}\left((c^2-4cd+3d^2)\cos(fx+e)^2-4c^2+8cd-4d^2-(3c^2-4cd+d^2)\cos(fx+e)+(4c^2-\cdots\right.}{\sqrt{ac-\cdots}}}{\cos(f\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] [1/4*sqrt(2)*log(((c^2 - 14*c*d + 17*d^2)*cos(f*x + e)^3 - (13*c^2 - 22*c*d
 - 3*d^2)*cos(f*x + e)^2 - 4*sqrt(2)*((c^2 - 4*c*d + 3*d^2)*cos(f*x + e)^2
- 4*c^2 + 8*c*d - 4*d^2 - (3*c^2 - 4*c*d + d^2)*cos(f*x + e) + (4*c^2 - 8*c
*d + 4*d^2 + (c^2 - 4*c*d + 3*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f
*x + e) + a)*sqrt(d*sin(f*x + e) + c)/sqrt(a*c - a*d) - 4*c^2 - 8*c*d - 4*d
^2 - 2*(9*c^2 - 14*c*d + 9*d^2)*cos(f*x + e) + ((c^2 - 14*c*d + 17*d^2)*cos
(f*x + e)^2 - 4*c^2 - 8*c*d - 4*d^2 + 2*(7*c^2 - 18*c*d + 7*d^2)*cos(f*x +
e))*sin(f*x + e))/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*
cos(f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x + e) - 4))/(sqrt(a*c - a*d)*f),
1/2*sqrt(2)*sqrt(-1/(a*c - a*d))*arctan(-1/4*sqrt(2)*sqrt(a*sin(f*x + e) +
a)*((c - 3*d)*sin(f*x + e) - 3*c + d)*sqrt(d*sin(f*x + e) + c)*sqrt(-1/(a*c
 - a*d))/(d*cos(f*x + e)*sin(f*x + e) + c*cos(f*x + e)))/f]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)), x)
```

maple [B] time = 0.23, size = 191, normalized size = 2.42

$$-\frac{\left(1-\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)\sqrt{c+d\sin\left(fx+e\right)}\,\ln\left(\frac{2\sqrt{2c-2d}\,\sqrt{2}\,\sqrt{\frac{c+d\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\sin\left(fx+e\right)+2c\sin\left(fx+e\right)-2d\sin\left(fx+e\right)+}{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}\right.}{f\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,\sin\left(fx+e\right)\sqrt{\frac{c+d\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\sqrt{2c-2d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(1-cos(f*x+e)+sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)*ln(2*((2*c-2*d)^(1/2)
*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*
sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))/(a*(1+
sin(f*x+e)))^(1/2)/sin(f*x+e)*2^(1/2)/((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/
2)/(2*c-2*d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(e + f*x) + 1))*sqrt(c + d*sin(e + f*x))), x)
```

3.592 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=131

$$\frac{2d\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{c-d}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\,f(c-d)^{3/2}}$$

```
[Out] -arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2)/
(c+d*sin(f*x+e))^(1/2))*2^(1/2)/(c-d)^(3/2)/f/a^(1/2)+2*d*cos(f*x+e)/(c^2-d
^2)/f/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2779, 12, 2782, 208}

$$\frac{2d\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{c-d}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\,f(c-d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*S
in[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(Sqrt[a]*(c - d)^(3/2)*f)) + (2*d*
Cos[e + f*x])/((c^2 - d^2)*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
```

```
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2d\cos(e+fx)}{\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}} + \frac{\int \frac{}{\sqrt{a+a\sin}}}{}$$

$$= \frac{2d\cos(e+fx)}{\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}} + \frac{\int \frac{}{\sqrt{a+a\sin}}}{}$$

$$= \frac{2d\cos(e+fx)}{\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}} - \frac{(2a)\,\mathrm{Subs}}{}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{c-d}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\,(c-d)^{3/2}f} + \frac{}{\left(c^2-d^2\right)f\sqrt{a}}$$

Mathematica [B] time = 6.48, size = 306, normalized size = 2.34

$$\frac{\frac{2d\cos(e+fx)}{c+d} + \frac{\log\left(\tan\left(\frac{1}{2}(e+fx)\right)+1\right) - \log\left((d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c-d\right)}{\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2\tan\left(\frac{1}{2}(e+fx)\right)+2} - \frac{\frac{\sqrt{c-d}\left(\frac{1}{\cos(e+fx)+1}\right)^{3/2}(c\sin(e+fx)+d\cos(e+fx)+d)}{\sqrt{c+d\sin(e+fx)}} - \frac{1}{2}(c-d)\sec^2\left(\frac{1}{2}(e+fx)\right)}{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c-d}}}{f(c-d)\sqrt{a(\sin(e+fx)+1)}\,\sqrt{c+d\sin(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] ((2*d*Cos[e + f*x])/(c + d) + (Log[1 + Tan[(e + f*x)/2]] - Log[c - d + 2*Sq
rt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)
*Tan[(e + f*x)/2]])/(Sec[(e + f*x)/2]^2/(2 + 2*Tan[(e + f*x)/2]) - (-1/2*((
c - d)*Sec[(e + f*x)/2]^2) + (Sqrt[c - d]*((1 + Cos[e + f*x])^(-1))^(3/2)*(
d + d*Cos[e + f*x] + c*Sin[e + f*x]))/Sqrt[c + d*Sin[e + f*x]])/(c - d + 2*
Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c +
d)*Tan[(e + f*x)/2])))/((c - d)*f*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c + d*Sin
[e + f*x]])
```

fricas [B] time = 0.73, size = 1016, normalized size = 7.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] [-1/4*(8*(d*cos(f*x + e) - d*sin(f*x + e) + d)*sqrt(a*sin(f*x + e) + a)*sqr
t(d*sin(f*x + e) + c) - sqrt(2)*(a*c^2 + 2*a*c*d + a*d^2 - (a*c*d + a*d^2)*
cos(f*x + e)^2 + (a*c^2 + a*c*d)*cos(f*x + e) + (a*c^2 + 2*a*c*d + a*d^2 +
(a*c*d + a*d^2)*cos(f*x + e))*sin(f*x + e))*log(((c^2 - 14*c*d + 17*d^2)*co
s(f*x + e)^3 - (13*c^2 - 22*c*d - 3*d^2)*cos(f*x + e)^2 + 4*sqrt(2)*((c^2 -
 4*c*d + 3*d^2)*cos(f*x + e)^2 - 4*c^2 + 8*c*d - 4*d^2 - (3*c^2 - 4*c*d + d
^2)*cos(f*x + e) + (4*c^2 - 8*c*d + 4*d^2 + (c^2 - 4*c*d + 3*d^2)*cos(f*x +
 e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/sqrt(a
*c - a*d) - 4*c^2 - 8*c*d - 4*d^2 - 2*(9*c^2 - 14*c*d + 9*d^2)*cos(f*x + e)
 + ((c^2 - 14*c*d + 17*d^2)*cos(f*x + e)^2 - 4*c^2 - 8*c*d - 4*d^2 + 2*(7*c
^2 - 18*c*d + 7*d^2)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^3 + 3*cos(f*
x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x +
 e) - 4))/sqrt(a*c - a*d))/((a*c^2*d - a*d^3)*f*cos(f*x + e)^2 - (a*c^3 - a
*c*d^2)*f*cos(f*x + e) - (a*c^3 + a*c^2*d - a*c*d^2 - a*d^3)*f - ((a*c^2*d
- a*d^3)*f*cos(f*x + e) + (a*c^3 + a*c^2*d - a*c*d^2 - a*d^3)*f)*sin(f*x +
e)), -1/2*(sqrt(2)*(a*c^2 + 2*a*c*d + a*d^2 - (a*c*d + a*d^2)*cos(f*x + e)^
2 + (a*c^2 + a*c*d)*cos(f*x + e) + (a*c^2 + 2*a*c*d + a*d^2 + (a*c*d + a*d^
2)*cos(f*x + e))*sin(f*x + e))*sqrt(-1/(a*c - a*d))*arctan(-1/4*sqrt(2)*sqr
t(a*sin(f*x + e) + a)*((c - 3*d)*sin(f*x + e) - 3*c + d)*sqrt(d*sin(f*x + e
) + c)*sqrt(-1/(a*c - a*d))/(d*cos(f*x + e)*sin(f*x + e) + c*cos(f*x + e)))
 + 4*(d*cos(f*x + e) - d*sin(f*x + e) + d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*
sin(f*x + e) + c))/((a*c^2*d - a*d^3)*f*cos(f*x + e)^2 - (a*c^3 - a*c*d^2)*
f*cos(f*x + e) - (a*c^3 + a*c^2*d - a*c*d^2 - a*d^3)*f - ((a*c^2*d - a*d^3)
*f*cos(f*x + e) + (a*c^3 + a*c^2*d - a*c*d^2 - a*d^3)*f)*sin(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2)), x)
```

maple [B] time = 0.34, size = 874, normalized size = 6.67

$$-\frac{\ln\left(\frac{2\sqrt{2c-2d}\,\sqrt{2}\,\sqrt{\frac{c+d\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\sin\left(fx+e\right)+2c\sin\left(fx+e\right)-2d\sin\left(fx+e\right)+2c\cos\left(fx+e\right)-2d\cos\left(fx+e\right)-2c+2d}{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}\right)\sqrt{2}\,\sqrt{\frac{c+d\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,c\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/f*(ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)
*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos
(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c*sin(
f*x+e)+ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2
)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-co
s(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*d*sin
(f*x+e)+cos(f*x+e)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x
+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e
)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))
^(1/2)*c+cos(f*x+e)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*
x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+
e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1)
)^(1/2)*d+ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(
1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1
-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c+
ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f
*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e
)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*d-2*d*(2*c-2
*d)^(1/2)*cos(f*x+e))/(a*(1+sin(f*x+e)))^(1/2)/(c+d*sin(f*x+e))^(1/2)/(c+d)
/(2*c-2*d)^(1/2)/(c-d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(e + f*x) + 1))*(c + d*sin(e + f*x))**(3/2)), x)
```

3.593 $\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=191

$$\frac{2d(5c+d)\cos(e+fx)}{3f\left(c^2-d^2\right)^2\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}+\frac{2d\cos(e+fx)}{3f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{3/2}}-\frac{\sqrt{2}\ldots}{}$$

```
[Out] -arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2)/
(c+d*sin(f*x+e))^(1/2))*2^(1/2)/(c-d)^(5/2)/f/a^(1/2)+2/3*d*cos(f*x+e)/(c^2
-d^2)/f/(c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+2/3*d*(5*c+d)*cos(f*x
+e)/(c^2-d^2)^2/f/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2779, 2984, 12, 2782, 208}

$$\frac{2d(5c+d)\cos(e+fx)}{3f\left(c^2-d^2\right)^2\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}+\frac{2d\cos(e+fx)}{3f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^{3/2}}-\frac{\sqrt{2}\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*S
in[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])]])/(Sqrt[a]*(c - d)^(5/2)*f)) + (2*d*
Cos[e + f*x])/(3*(c^2 - d^2)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]
)^(3/2)) + (2*d*(5*c + d)*Cos[e + f*x])/(3*(c^2 - d^2)^2*f*Sqrt[a + a*Sin[e
 + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2d\cos(e+fx)}{3\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}} + \frac{\int \frac{}{\sqrt{a+}}}{}$$

$$= \frac{2d\cos(e+fx)}{3\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}} + \frac{}{3\left(c^2-\right.}$$

$$= \frac{2d\cos(e+fx)}{3\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}} + \frac{}{3\left(c^2-\right.}$$

$$= \frac{2d\cos(e+fx)}{3\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}} + \frac{}{3\left(c^2-\right.}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{\sqrt{a}\,(c-d)^{5/2}f} + \frac{}{3\left(c^2-d^2\right)f\sqrt{a}}$$

Mathematica [B] time = 6.45, size = 387, normalized size = 2.03

$$\frac{\frac{2d\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(6c^2+d(5c+d)\sin(e+fx)+cd-d^2\right)}{(c+d)^2(c+d\sin(e+fx))} + \frac{3\left(\log\left(\tan\left(\frac{1}{2}(e+fx)\right)+1\right)-\log\left((d-c)\tan\left(\frac{1}{2}(e+\right.\right.\right.}{\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2\tan\left(\frac{1}{2}(e+fx)\right)+2}-\frac{\frac{\sqrt{c-d}\left(\frac{1}{\cos(e+fx)+1}\right)^{3/2}}{\sqrt{c+d}}}{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)}}}{3f(c-d)^2\sqrt{a(\sin(e+fx)+1)}\sqrt{c+d\sin(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] ((2*d*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])*(6*c^2 + c*d - d^2 + d*(5*c + d)*Sin[e + f*x]))/((c + d)^2*(c + d*Si
n[e + f*x])) + (3*(Log[1 + Tan[(e + f*x)/2]] - Log[c - d + 2*Sqrt[c - d]*Sq
rt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*
x)/2]]))/(Sec[(e + f*x)/2]^2/(2 + 2*Tan[(e + f*x)/2]) - (-1/2*((c - d)*Sec[
(e + f*x)/2]^2) + (Sqrt[c - d]*((1 + Cos[e + f*x])^(-1))^(3/2)*(d + d*Cos[e
 + f*x] + c*Sin[e + f*x]))/Sqrt[c + d*Sin[e + f*x]])/(c - d + 2*Sqrt[c - d]
*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e +
 f*x)/2])))/(3*(c - d)^2*f*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c + d*Sin[e + f*
x]])
```

fricas [B] time = 0.92, size = 1855, normalized size = 9.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] [-1/12*(8*(6*c^2*d - 4*c*d^2 - 2*d^3 + (5*c*d^2 + d^3)*cos(f*x + e)^2 + (6*
c^2*d + c*d^2 - d^3)*cos(f*x + e) - (6*c^2*d - 4*c*d^2 - 2*d^3 - (5*c*d^2 +
 d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x +
 e) + c) + 3*sqrt(2)*(a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 -
 (a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e)^3 - (2*a*c^3*d + 5*a*c^2*d^2
+ 4*a*c*d^3 + a*d^4)*cos(f*x + e)^2 + (a*c^4 + 2*a*c^3*d + 2*a*c^2*d^2 + 2*
a*c*d^3 + a*d^4)*cos(f*x + e) + (a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^
3 + a*d^4 - (a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e)^2 + 2*(a*c^3*d + 2
*a*c^2*d^2 + a*c*d^3)*cos(f*x + e))*sin(f*x + e))*log(((c^2 - 14*c*d + 17*d
^2)*cos(f*x + e)^3 - (13*c^2 - 22*c*d - 3*d^2)*cos(f*x + e)^2 - 4*sqrt(2)*(
(c^2 - 4*c*d + 3*d^2)*cos(f*x + e)^2 - 4*c^2 + 8*c*d - 4*d^2 - (3*c^2 - 4*c
*d + d^2)*cos(f*x + e) + (4*c^2 - 8*c*d + 4*d^2 + (c^2 - 4*c*d + 3*d^2)*cos
(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/
sqrt(a*c - a*d) - 4*c^2 - 8*c*d - 4*d^2 - 2*(9*c^2 - 14*c*d + 9*d^2)*cos(f*
x + e) + ((c^2 - 14*c*d + 17*d^2)*cos(f*x + e)^2 - 4*c^2 - 8*c*d - 4*d^2 +
2*(7*c^2 - 18*c*d + 7*d^2)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^3 + 3*
cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x + e) - 2*cos
(f*x + e) - 4))/sqrt(a*c - a*d))/((a*c^4*d^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f
*x + e)^3 + (2*a*c^5*d + a*c^4*d^2 - 4*a*c^3*d^3 - 2*a*c^2*d^4 + 2*a*c*d^5
+ a*d^6)*f*cos(f*x + e)^2 - (a*c^6 - a*c^4*d^2 - a*c^2*d^4 + a*d^6)*f*cos(f
*x + e) - (a*c^6 + 2*a*c^5*d - a*c^4*d^2 - 4*a*c^3*d^3 - a*c^2*d^4 + 2*a*c*
d^5 + a*d^6)*f + ((a*c^4*d^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f*x + e)^2 - 2*(a
*c^5*d - 2*a*c^3*d^3 + a*c*d^5)*f*cos(f*x + e) - (a*c^6 + 2*a*c^5*d - a*c^4
*d^2 - 4*a*c^3*d^3 - a*c^2*d^4 + 2*a*c*d^5 + a*d^6)*f)*sin(f*x + e)), -1/6*
(3*sqrt(2)*(a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - (a*c^2*d^
2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e)^3 - (2*a*c^3*d + 5*a*c^2*d^2 + 4*a*c*d^
3 + a*d^4)*cos(f*x + e)^2 + (a*c^4 + 2*a*c^3*d + 2*a*c^2*d^2 + 2*a*c*d^3 +
a*d^4)*cos(f*x + e) + (a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4
- (a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e)^2 + 2*(a*c^3*d + 2*a*c^2*d^2
 + a*c*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-1/(a*c - a*d))*arctan(-1/4*sq
rt(2)*sqrt(a*sin(f*x + e) + a)*((c - 3*d)*sin(f*x + e) - 3*c + d)*sqrt(d*si
n(f*x + e) + c)*sqrt(-1/(a*c - a*d))/(d*cos(f*x + e)*sin(f*x + e) + c*cos(f
*x + e))) + 4*(6*c^2*d - 4*c*d^2 - 2*d^3 + (5*c*d^2 + d^3)*cos(f*x + e)^2 +
 (6*c^2*d + c*d^2 - d^3)*cos(f*x + e) - (6*c^2*d - 4*c*d^2 - 2*d^3 - (5*c*d
^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f
*x + e) + c))/((a*c^4*d^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f*x + e)^3 + (2*a*c^
```

```
5*d + a*c^4*d^2 - 4*a*c^3*d^3 - 2*a*c^2*d^4 + 2*a*c*d^5 + a*d^6)*f*cos(f*x
+ e)^2 - (a*c^6 - a*c^4*d^2 - a*c^2*d^4 + a*d^6)*f*cos(f*x + e) - (a*c^6 +
2*a*c^5*d - a*c^4*d^2 - 4*a*c^3*d^3 - a*c^2*d^4 + 2*a*c*d^5 + a*d^6)*f + ((
a*c^4*d^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f*x + e)^2 - 2*(a*c^5*d - 2*a*c^3*d^
3 + a*c*d^5)*f*cos(f*x + e) - (a*c^6 + 2*a*c^5*d - a*c^4*d^2 - 4*a*c^3*d^3
- a*c^2*d^4 + 2*a*c*d^5 + a*d^6)*f)*sin(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2)), x)
```

maple [B] time = 0.37, size = 2572, normalized size = 13.47

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/3/f*(-3*sin(f*x+e)*cos(f*x+e)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x
+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e
)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(c
os(f*x+e)+1))^(1/2)*c^2*d-6*sin(f*x+e)*cos(f*x+e)*ln(2*((2*c-2*d)^(1/2)*2^(
1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(
f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((
c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c*d^2-3*sin(f*x+e)*cos(f*x+e)*ln(2*((
2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c
*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f
*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*d^3+3*cos(f*x+e)^2*
ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f
*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e
)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c^2*d+6*cos(
f*x+e)^2*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1
/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-
cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c*d
^2+3*cos(f*x+e)^2*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+
e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)
```

```
-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^
(1/2)*d^3-3*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))
^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/
(1-cos(f*x+e)+sin(f*x+e)))*c^3*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1
/2)*sin(f*x+e)-9*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e
)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-
c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2*d*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)
+1))^(1/2)*sin(f*x+e)-9*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(co
s(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(
f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d^2*2^(1/2)*((c+d*sin(f*x+e))/(cos
(f*x+e)+1))^(1/2)*sin(f*x+e)-3*sin(f*x+e)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c
+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c
*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(
f*x+e))/(cos(f*x+e)+1))^(1/2)*d^3-3*cos(f*x+e)*ln(2*((2*c-2*d)^(1/2)*2^(1/2
)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x
+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d
*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c^3-6*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+
d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*
cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2*d*2^(1/2)*((c+d
*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-3*ln(2*((2*c-2*d)^(1/2)*2^(1/
2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*
x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d^2*2^(1/2
)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)+10*sin(f*x+e)*cos(f*x+
e)*(2*c-2*d)^(1/2)*c*d^2+2*sin(f*x+e)*cos(f*x+e)*(2*c-2*d)^(1/2)*d^3-3*ln(2
*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e
)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+si
n(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c^3-9*ln(2*((2*c
-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*si
n(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+
e)))*c^2*d*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)-9*ln(2*((2*c-2*d
)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*
x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))
*c*d^2*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)-3*ln(2*((2*c-2*d)^(1
/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)
-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d^3
*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)+12*cos(f*x+e)*(2*c-2*d)^(1
/2)*c^2*d+2*cos(f*x+e)*(2*c-2*d)^(1/2)*c*d^2-2*cos(f*x+e)*(2*c-2*d)^(1/2)*d
^3)*(c+d*sin(f*x+e))^(1/2)/(-cos(f*x+e)^2*d^2+2*c*d*sin(f*x+e)+c^2+d^2)/(a*
(1+sin(f*x+e)))^(1/2)/(c+d)^2/(2*c-2*d)^(1/2)/(c-d)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a \left(\sin{\left(e + f x \right)} + 1\right)} \left(c + d \sin{\left(e + f x \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(e + f*x) + 1))*(c + d*sin(e + f*x))**(5/2)), x)
```

3.594 $\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=251

$$-\frac{d^{3/2}(5c-3d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}f}-\frac{(c+9d)(c-d)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f}-\frac{(c-}{}$$

```
[Out] -(5*c-3*d)*d^(3/2)*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*x+e))^(1/2)
/(c+d*sin(f*x+e))^(1/2))/a^(3/2)/f-1/2*(c-d)*cos(f*x+e)*(c+d*sin(f*x+e))^(3
/2)/f/(a+a*sin(f*x+e))^(3/2)-1/4*(c-d)^(3/2)*(c+9*d)*arctanh(1/2*cos(f*x+e)
*a^(1/2)*(c-d)^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))
/a^(3/2)/f*2^(1/2)+1/2*(c-3*d)*d*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/a/f/(a+a
*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.90, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2765, 2983, 2982, 2782, 208, 2775, 205}

$$-\frac{d^{3/2}(5c-3d)\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}f}-\frac{(c+9d)(c-d)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f}-\frac{(c-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(((5*c - 3*d)*d^(3/2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Si
n[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(a^(3/2)*f)) - ((c - d)^(3/2)*(c +
9*d)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e +
 f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*f) + ((c - 3*d)*d*Cos
[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(2*a*f*Sqrt[a + a*Sin[e + f*x]]) - ((c
- d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(2*f*(a + a*Sin[e + f*x])^(3/
2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
```

```
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{\sqrt{c+d\sin(e+fx)}\left(-\frac{1}{2}a\left(c^2+6cd-3d^2\right)+a(c-3d)d\sin\ldots\right.}{\sqrt{a+a\sin(e+fx)}}}{2a^2}$$

$$= \frac{(c-3d)d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{3/2}}$$

$$= \frac{(c-3d)d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{3/2}}$$

$$= \frac{(c-3d)d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{3/2}}$$

$$= -\frac{(5c-3d)d^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}f} - \frac{(c-d)^{3/2}(c+9d)\tanh^{-1}\left(\frac{\ldots}{\sqrt{2}\sqrt{\ldots}}\right.}{2\sqrt{2}\,a^{3/2}\ldots}$$

Mathematica [C] time = 17.02, size = 1844, normalized size = 7.35

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^(3/2),x]

[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(-(d^2*Cos[(e + f*x)/2]) + d^2*Sin
[(e + f*x)/2] - (c - d)^2/(2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])) + (c^2*
Sin[(e + f*x)/2] - 2*c*d*Sin[(e + f*x)/2] + d^2*Sin[(e + f*x)/2])/(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^2)*Sqrt[c + d*Sin[e + f*x]])/(f*(a*(1 + Sin[e
+ f*x]))^(3/2)) + ((((c - d)^(3/2)*(c + 9*d)*Log[1 + Tan[(e + f*x)/2]])/Sqr
t[2] + I*(5*c - 3*d)*d^(3/2)*Log[((-I)*((-I)*c + d + (1 - I)*Sqrt[2]*Sqrt[d
]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (c - I*d)*Tan[(e
 + f*x)/2]))/(d^(5/2)*(-5*c + 3*d)*(-I + Tan[(e + f*x)/2]))] + I*d^(3/2)*(-
5*c + 3*d)*Log[(I*(I*c + d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x]
)^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (c + I*d)*Tan[(e + f*x)/2]))/(d^(5/2)*(-
5*c + 3*d)*(I + Tan[(e + f*x)/2]))] - ((c - d)^(3/2)*(c + 9*d)*Log[c - d +
```

```
2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c
+ d)*Tan[(e + f*x)/2]])/Sqrt[2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c
^3/(4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) + (7*
c^2*d)/(4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) -
 (7*c*d^2)/(4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]
]) + (3*d^3)/(4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*
x]]) + (5*c*d^2*Sin[e + f*x])/(2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt
[c + d*Sin[e + f*x]]) - (3*d^3*Sin[e + f*x])/(2*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])*Sqrt[c + d*Sin[e + f*x]])))/(f*(a*(1 + Sin[e + f*x]))^(3/2)*(((c
 - d)^(3/2)*(c + 9*d)*Sec[(e + f*x)/2]^2)/(2*Sqrt[2]*(1 + Tan[(e + f*x)/2])
) - ((c - d)^(3/2)*(c + 9*d)*(((-c + d)*Sec[(e + f*x)/2]^2)/2 + (Sqrt[c - d
]*d*Cos[e + f*x]*Sqrt[(1 + Cos[e + f*x])^(-1)])/Sqrt[c + d*Sin[e + f*x]] +
Sqrt[c - d]*((1 + Cos[e + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e +
 f*x]]))/(Sqrt[2]*(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt
[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2])) - ((5*c - 3*d)*d^4*(-5*c
 + 3*d)*(-I + Tan[(e + f*x)/2])*(((-I)*(((c - I*d)*Sec[(e + f*x)/2]^2)/2 +
((1 - I)*d^(3/2)*Cos[e + f*x]*Sqrt[(1 + Cos[e + f*x])^(-1)])/(Sqrt[2]*Sqrt[
c + d*Sin[e + f*x]]) + ((1 - I)*Sqrt[d]*((1 + Cos[e + f*x])^(-1))^(3/2)*Sin
[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[2]))/(d^(5/2)*(-5*c + 3*d)*(-I + T
an[(e + f*x)/2])) + ((I/2)*Sec[(e + f*x)/2]^2*((-I)*c + d + (1 - I)*Sqrt[2]
*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (c - I*d)
*Tan[(e + f*x)/2]))/(d^(5/2)*(-5*c + 3*d)*(-I + Tan[(e + f*x)/2])^2)))/((-I
)*c + d + (1 - I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*
Sin[e + f*x]] + (c - I*d)*Tan[(e + f*x)/2]) + (d^4*(-5*c + 3*d)^2*(I + Tan[
(e + f*x)/2])*((I*(((c + I*d)*Sec[(e + f*x)/2]^2)/2 + ((1 + I)*d^(3/2)*Cos[
e + f*x]*Sqrt[(1 + Cos[e + f*x])^(-1)])/(Sqrt[2]*Sqrt[c + d*Sin[e + f*x]])
+ ((1 + I)*Sqrt[d]*((1 + Cos[e + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*
Sin[e + f*x]])/Sqrt[2]))/(d^(5/2)*(-5*c + 3*d)*(I + Tan[(e + f*x)/2])) - ((
I/2)*Sec[(e + f*x)/2]^2*(I*c + d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e
+ f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (c + I*d)*Tan[(e + f*x)/2]))/(d^(5
/2)*(-5*c + 3*d)*(I + Tan[(e + f*x)/2])^2)))/(I*c + d + (1 + I)*Sqrt[2]*Sqr
t[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (c + I*d)*Tan
[(e + f*x)/2])))
```

fricas [B] time = 1.22, size = 3420, normalized size = 13.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/8*(2*sqrt(1/2)*(2*a*c^2 + 16*a*c*d - 18*a*d^2 - (a*c^2 + 8*a*c*d - 9*a*d
^2)*cos(f*x + e)^2 + (a*c^2 + 8*a*c*d - 9*a*d^2)*cos(f*x + e) + (2*a*c^2 +
16*a*c*d - 18*a*d^2 + (a*c^2 + 8*a*c*d - 9*a*d^2)*cos(f*x + e))*sin(f*x + e
```

))*sqrt((c - d)/a)*log(-(4*sqrt(1/2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt((c - d)/a)*(cos(f*x + e) - sin(f*x + e) + 1) + (c - 3*d)*cos(f*x + e)^2 + (3*c - d)*cos(f*x + e) - ((c - 3*d)*cos(f*x + e) - 2*c - 2*d)*sin(f*x + e) + 2*c + 2*d)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + (10*a*c*d - 6*a*d^2 - (5*a*c*d - 3*a*d^2)*cos(f*x + e)^2 + (5*a*c*d - 3*a*d^2)*cos(f*x + e) + (10*a*c*d - 6*a*d^2 + (5*a*c*d - 3*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(-d/a)*log((128*d^4*cos(f*x + e)^5 + 128*(2*c*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 32*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3 - 32*(c^3*d - 2*c^2*d^2 + 9*c*d^3 - 4*d^4)*cos(f*x + e)^2 + 8*(16*d^3*cos(f*x + e)^4 + 24*(c*d^2 - d^3)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5*c^2*d - 26*c*d^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 25*d^3)*cos(f*x + e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 - 51*d^3 - 8*(3*c*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-d/a) + (c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 + 289*d^4)*cos(f*x + e) + (128*d^4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 256*(c*d^3 - d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d^3 + 5*d^4)*cos(f*x + e)^2 + 32*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)) + 4*(2*d^2*cos(f*x + e)^2 + c^2 - 2*c*d + d^2 + (c^2 - 2*c*d + 3*d^2)*cos(f*x + e) + (2*d^2*cos(f*x + e) - c^2 + 2*c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e)), 1/4*(sqrt(1/2)*(2*a*c^2 + 16*a*c*d - 18*a*d^2 - (a*c^2 + 8*a*c*d - 9*a*d^2)*cos(f*x + e)^2 + (a*c^2 + 8*a*c*d - 9*a*d^2)*cos(f*x + e) + (2*a*c^2 + 16*a*c*d - 18*a*d^2 + (a*c^2 + 8*a*c*d - 9*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt((c - d)/a)*log(-(4*sqrt(1/2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt((c - d)/a)*(cos(f*x + e) - sin(f*x + e) + 1) + (c - 3*d)*cos(f*x + e)^2 + (3*c - d)*cos(f*x + e) - ((c - 3*d)*cos(f*x + e) - 2*c - 2*d)*sin(f*x + e) + 2*c + 2*d)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - (10*a*c*d - 6*a*d^2 - (5*a*c*d - 3*a*d^2)*cos(f*x + e)^2 + (5*a*c*d - 3*a*d^2)*cos(f*x + e) + (10*a*c*d - 6*a*d^2 + (5*a*c*d - 3*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(d/a)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(d/a)/(2*d^3*cos(f*x + e)^3 - (3*c*d^2 - d^3)*cos(f*x + e)*sin(f*x + e) - (c^2*d - c*d^2 + 2*d^3)*cos(f*x + e))) + 2*(2*d^2*cos(f*x + e)^2 + c^2 - 2*c*d + d^2 + (c^2 - 2*c*d + 3*d^2)*cos(f*x + e) + (2*d^2*cos(f*x + e) - c^2 + 2*c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e)), 1/8*(4*sqrt(1/2)*(2*a*c^2 + 16*a*c*d - 18*a*d^2 - (a*c^2 + 8*a*c*d - 9*a*d^2)*cos(f*x + e)^2 + (a*c^2 + 8*a*c*d - 9*a*d^2)*cos(f*x + e) + (2*a*c^2 + 16*a*c*d - 18*a*d^2 + (a*c^2 + 8*a*c*d - 9*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(-(c - d)/a)*arctan(-2*sqrt(1/2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-(c - d)/a)/((c - d)*cos(f

```
*x + e))) + (10*a*c*d - 6*a*d^2 - (5*a*c*d - 3*a*d^2)*cos(f*x + e)^2 + (5*a
*c*d - 3*a*d^2)*cos(f*x + e) + (10*a*c*d - 6*a*d^2 + (5*a*c*d - 3*a*d^2)*co
s(f*x + e))*sin(f*x + e))*sqrt(-d/a)*log((128*d^4*cos(f*x + e)^5 + 128*(2*c
*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 32
*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3 - 32*(c^3*d - 2*c^2*d^2 + 9
*c*d^3 - 4*d^4)*cos(f*x + e)^2 + 8*(16*d^3*cos(f*x + e)^4 + 24*(c*d^2 - d^3
)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5*c^2*d - 26*c*d
^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 25*d^3)*cos(f*x +
 e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 - 51*d^3 - 8*(3*c*
d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3)*cos(f*x + e))
*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-d/a)
 + (c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 + 289*d^4)*cos(f*x + e) + (128
*d^4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 256*(c*d^
3 - d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d^3 + 5*d^4)*cos(f*x + e)^2 +
 32*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(f*x + e))*sin(f*x + e))/(cos
(f*x + e) + sin(f*x + e) + 1)) + 4*(2*d^2*cos(f*x + e)^2 + c^2 - 2*c*d + d^
2 + (c^2 - 2*c*d + 3*d^2)*cos(f*x + e) + (2*d^2*cos(f*x + e) - c^2 + 2*c*d
- d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(a^
2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2
*a^2*f)*sin(f*x + e)), 1/4*(2*sqrt(1/2)*(2*a*c^2 + 16*a*c*d - 18*a*d^2 - (a
*c^2 + 8*a*c*d - 9*a*d^2)*cos(f*x + e)^2 + (a*c^2 + 8*a*c*d - 9*a*d^2)*cos(
f*x + e) + (2*a*c^2 + 16*a*c*d - 18*a*d^2 + (a*c^2 + 8*a*c*d - 9*a*d^2)*cos
(f*x + e))*sin(f*x + e))*sqrt(-(c - d)/a)*arctan(-2*sqrt(1/2)*sqrt(a*sin(f*
x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-(c - d)/a)/((c - d)*cos(f*x + e)
)) - (10*a*c*d - 6*a*d^2 - (5*a*c*d - 3*a*d^2)*cos(f*x + e)^2 + (5*a*c*d -
3*a*d^2)*cos(f*x + e) + (10*a*c*d - 6*a*d^2 + (5*a*c*d - 3*a*d^2)*cos(f*x +
 e))*sin(f*x + e))*sqrt(d/a)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d
 - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(
f*x + e) + c)*sqrt(d/a)/(2*d^3*cos(f*x + e)^3 - (3*c*d^2 - d^3)*cos(f*x + e
)*sin(f*x + e) - (c^2*d - c*d^2 + 2*d^3)*cos(f*x + e))) + 2*(2*d^2*cos(f*x
+ e)^2 + c^2 - 2*c*d + d^2 + (c^2 - 2*c*d + 3*d^2)*cos(f*x + e) + (2*d^2*co
s(f*x + e) - c^2 + 2*c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt
(d*sin(f*x + e) + c))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f
- (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(a \sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a)^(3/2), x)
```

maple [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{\left(c+d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.595 $\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=194

$$-\frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}f}-\frac{\sqrt{c-d}\,(c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f}-\frac{(c-d)\cos(e}{2f(a\,s}$$

```
[Out] -2*d^(3/2)*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*si
n(f*x+e))^(1/2))/a^(3/2)/f-1/4*(c+5*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)
^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))*(c-d)^(1/2)/a
^(3/2)/f*2^(1/2)-1/2*(c-d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x
+e))^(3/2)
```

Rubi [A] time = 0.56, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2765, 2982, 2782, 208, 2775, 205}

$$-\frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}f}-\frac{\sqrt{c-d}\,(c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f}-\frac{(c-d)\cos(e}{2f(a\,s}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*d^(3/2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c + d*Sin[e + f*x]])])/(a^(3/2)*f) - (Sqrt[c - d]*(c + 5*d)*ArcTanh[(
Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
+ d*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*f) - ((c - d)*Cos[e + f*x]*Sqrt[c +
 d*Sin[e + f*x]])/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}a\left(c^2+4cd-d^2\right)-2ad^2\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{2a^2}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(a+a\sin(e+fx))^{3/2}} + \frac{d^2\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx}{a^2} + \frac{((c-d)(c+5d}{\cdots}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{\left(2d^2\right)\text{Subst}\left(\int \frac{1}{a+dx^2}\,dx, x, \frac{a\,\text{co}}{\sqrt{a+a\sin(e+f}}\right.}{af}$$

$$= -\frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}f} - \frac{\sqrt{c-d}\,(c+5d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-}}{\sqrt{2}\sqrt{a+a\sin(e+}}\right.}{2\sqrt{2}\,a^{3/2}f}$$

Mathematica [C] time = 17.12, size = 1625, normalized size = 8.38

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*((-c + d)/(2*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])) + (c*Sin[(e + f*x)/2] - d*Sin[(e + f*x)/2])/(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^2)*Sqrt[c + d*Sin[e + f*x]])/(f*(a*(1 + Sin[e + f*
x]))^(3/2)) + ((Sqrt[2]*(c^2 + 4*c*d - 5*d^2)*Log[1 + Tan[(e + f*x)/2]] - S
qrt[2]*(c^2 + 4*c*d - 5*d^2)*Log[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*
x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]] - (4*I)*Sqr
t[c - d]*d^(3/2)*(Log[(c - I*(d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e +
 f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]]) + ((-I)*c + d)*Tan[(e + f*x)/2])/(2*
d^(5/2)*(I + Tan[(e + f*x)/2]))] - Log[(c + I*d + (1 + I)*Sqrt[2]*Sqrt[d]*S
qrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (I*c + d)*Tan[(e +
f*x)/2])/(2*d^(5/2)*(-I + Tan[(e + f*x)/2]))]))*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^3*(c^2/(4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e
 + f*x]]) + (c*d)/((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e +
 f*x]]) - d^2/(4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f
*x]]) + (d^2*Sin[e + f*x])/((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c +
d*Sin[e + f*x]])))/(f*(a*(1 + Sin[e + f*x]))^(3/2)*(((c^2 + 4*c*d - 5*d^2)*
Sec[(e + f*x)/2]^2)/(Sqrt[2]*(1 + Tan[(e + f*x)/2])) - (Sqrt[2]*(c^2 + 4*c*
d - 5*d^2)*(((-c + d)*Sec[(e + f*x)/2]^2)/2 + (Sqrt[c - d]*d*Cos[e + f*x]*S
qrt[(1 + Cos[e + f*x])^(-1)])/Sqrt[c + d*Sin[e + f*x]] + Sqrt[c - d]*((1 +
Cos[e + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]]))/(c - d +
```

```
2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c
+ d)*Tan[(e + f*x)/2]) - (4*I)*Sqrt[c - d]*d^(3/2)*((2*d^(5/2)*(I + Tan[(e
+ f*x)/2])*(((((-I)*c + d)*Sec[(e + f*x)/2]^2)/2 - I*(((1 + I)*d^(3/2)*Cos[
e + f*x]*Sqrt[(1 + Cos[e + f*x])^(-1)])/(Sqrt[2]*Sqrt[c + d*Sin[e + f*x]])
+ ((1 + I)*Sqrt[d]*((1 + Cos[e + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*
Sin[e + f*x]])/Sqrt[2]))/(2*d^(5/2)*(I + Tan[(e + f*x)/2])) - (Sec[(e + f*x
)/2]^2*(c - I*(d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sq
rt[c + d*Sin[e + f*x]]) + ((-I)*c + d)*Tan[(e + f*x)/2]))/(4*d^(5/2)*(I + T
an[(e + f*x)/2])^2)))/(c - I*(d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e +
 f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]]) + ((-I)*c + d)*Tan[(e + f*x)/2]) - (
2*d^(5/2)*(-I + Tan[(e + f*x)/2])*((((I*c + d)*Sec[(e + f*x)/2]^2)/2 + ((1
+ I)*d^(3/2)*Cos[e + f*x]*Sqrt[(1 + Cos[e + f*x])^(-1)])/(Sqrt[2]*Sqrt[c +
d*Sin[e + f*x]]) + ((1 + I)*Sqrt[d]*((1 + Cos[e + f*x])^(-1))^(3/2)*Sin[e +
 f*x]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[2])/(2*d^(5/2)*(-I + Tan[(e + f*x)/2])
) - (Sec[(e + f*x)/2]^2*(c + I*d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e
+ f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (I*c + d)*Tan[(e + f*x)/2]))/(4*d^
(5/2)*(-I + Tan[(e + f*x)/2])^2)))/(c + I*d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[
(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (I*c + d)*Tan[(e + f*x)
/2]))))
```

fricas [B] time = 1.08, size = 2883, normalized size = 14.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/4*(sqrt(1/2)*((a*c + 5*a*d)*cos(f*x + e)^2 - 2*a*c - 10*a*d - (a*c + 5*a
*d)*cos(f*x + e) - (2*a*c + 10*a*d + (a*c + 5*a*d)*cos(f*x + e))*sin(f*x +
e))*sqrt((c - d)/a)*log((4*sqrt(1/2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*
x + e) + c)*sqrt((c - d)/a)*(cos(f*x + e) - sin(f*x + e) + 1) - (c - 3*d)*c
os(f*x + e)^2 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) - 2*c - 2*
d)*sin(f*x + e) - 2*c - 2*d)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x +
 e) - cos(f*x + e) - 2)) + (a*d*cos(f*x + e)^2 - a*d*cos(f*x + e) - 2*a*d -
 (a*d*cos(f*x + e) + 2*a*d)*sin(f*x + e))*sqrt(-d/a)*log((128*d^4*cos(f*x +
 e)^5 + 128*(2*c*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*
c*d^3 + d^4 - 32*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3 - 32*(c^3*d
 - 2*c^2*d^2 + 9*c*d^3 - 4*d^4)*cos(f*x + e)^2 - 8*(16*d^3*cos(f*x + e)^4 +
 24*(c*d^2 - d^3)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(
5*c^2*d - 26*c*d^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 2
5*d^3)*cos(f*x + e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 -
51*d^3 - 8*(3*c*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^
3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
 + c)*sqrt(-d/a) + (c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 + 289*d^4)*cos
```

(f*x + e) + (128*d^4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 +
d^4 - 256*(c*d^3 - d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d^3 + 5*d^4)*
cos(f*x + e)^2 + 32*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(f*x + e))*si
n(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)) + 2*((c - d)*cos(f*x + e) -
(c - d)*sin(f*x + e) + c - d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x
+ e) + 2*a^2*f)*sin(f*x + e)), 1/4*(sqrt(1/2)*((a*c + 5*a*d)*cos(f*x + e)^
2 - 2*a*c - 10*a*d - (a*c + 5*a*d)*cos(f*x + e) - (2*a*c + 10*a*d + (a*c +
5*a*d)*cos(f*x + e))*sin(f*x + e))*sqrt((c - d)/a)*log((4*sqrt(1/2)*sqrt(a*
sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt((c - d)/a)*(cos(f*x + e) -
sin(f*x + e) + 1) - (c - 3*d)*cos(f*x + e)^2 - (3*c - d)*cos(f*x + e) + ((c
- 3*d)*cos(f*x + e) - 2*c - 2*d)*sin(f*x + e) - 2*c - 2*d)/(cos(f*x + e)^2
- (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 2*(a*d*cos(f*x +
e)^2 - a*d*cos(f*x + e) - 2*a*d - (a*d*cos(f*x + e) + 2*a*d)*sin(f*x + e))*
sqrt(d/a)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d -
d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(
d/a)/(2*d^3*cos(f*x + e)^3 - (3*c*d^2 - d^3)*cos(f*x + e)*sin(f*x + e) - (c
^2*d - c*d^2 + 2*d^3)*cos(f*x + e))) + 2*((c - d)*cos(f*x + e) - (c - d)*si
n(f*x + e) + c - d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(a^2
*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*
a^2*f)*sin(f*x + e)), -1/4*(2*sqrt(1/2)*((a*c + 5*a*d)*cos(f*x + e)^2 - 2*a
*c - 10*a*d - (a*c + 5*a*d)*cos(f*x + e) - (2*a*c + 10*a*d + (a*c + 5*a*d)*
cos(f*x + e))*sin(f*x + e))*sqrt(-(c - d)/a)*arctan(-2*sqrt(1/2)*sqrt(a*sin
(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-(c - d)/a)/((c - d)*cos(f*x +
e))) - (a*d*cos(f*x + e)^2 - a*d*cos(f*x + e) - 2*a*d - (a*d*cos(f*x + e)
+ 2*a*d)*sin(f*x + e))*sqrt(-d/a)*log((128*d^4*cos(f*x + e)^5 + 128*(2*c*d^
3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 32*(5
*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3 - 32*(c^3*d - 2*c^2*d^2 + 9*c*
d^3 - 4*d^4)*cos(f*x + e)^2 - 8*(16*d^3*cos(f*x + e)^4 + 24*(c*d^2 - d^3)*c
os(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5*c^2*d - 26*c*d^2
+ 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 25*d^3)*cos(f*x + e)
+ (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 - 51*d^3 - 8*(3*c*d^2
- 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3)*cos(f*x + e))*si
n(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-d/a) +
(c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 + 289*d^4)*cos(f*x + e) + (128*d^
4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 256*(c*d^3 -
d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d^3 + 5*d^4)*cos(f*x + e)^2 + 32
*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(f*x + e))*sin(f*x + e))/(cos(f*
x + e) + sin(f*x + e) + 1)) - 2*((c - d)*cos(f*x + e) - (c - d)*sin(f*x + e
) + c - d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(a^2*f*cos(f*
x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*si
n(f*x + e)), -1/2*(sqrt(1/2)*((a*c + 5*a*d)*cos(f*x + e)^2 - 2*a*c - 10*a*d
- (a*c + 5*a*d)*cos(f*x + e) - (2*a*c + 10*a*d + (a*c + 5*a*d)*cos(f*x + e
))*sin(f*x + e))*sqrt(-(c - d)/a)*arctan(-2*sqrt(1/2)*sqrt(a*sin(f*x + e) +
a)*sqrt(d*sin(f*x + e) + c)*sqrt(-(c - d)/a)/((c - d)*cos(f*x + e))) - (a*

```
d*cos(f*x + e)^2 - a*d*cos(f*x + e) - 2*a*d - (a*d*cos(f*x + e) + 2*a*d)*si
n(f*x + e))*sqrt(d/a)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^
2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e
) + c)*sqrt(d/a)/(2*d^3*cos(f*x + e)^3 - (3*c*d^2 - d^3)*cos(f*x + e)*sin(f
*x + e) - (c^2*d - c*d^2 + 2*d^3)*cos(f*x + e))) - ((c - d)*cos(f*x + e) -
(c - d)*sin(f*x + e) + c - d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x
 + e) + 2*a^2*f)*sin(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(a \sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a)^(3/2), x)
```

maple [B] time = 0.47, size = 6681, normalized size = 34.44

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(a \sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c+d\,\sin\left(e+f\,x\right)\right)^{3/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^(3/2),x)
[Out] int((c + d*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(3/2),x)
[Out] Integral((c + d*sin(e + f*x))**(3/2)/(a*(sin(e + f*x) + 1))**(3/2), x)
```

3.596 $\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=126

$$-\frac{(c+d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f\sqrt{c-d}}-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(a\sin(e+fx)+a)^{3/2}}$$

```
[Out] -1/4*(c+d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1/2)/(a+a*sin(f*x+
e))^(1/2)/(c+d*sin(f*x+e))^(1/2))/a^(3/2)/f*2^(1/2)/(c-d)^(1/2)-1/2*cos(f*x
+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(3/2)
```

Rubi [A] time = 0.22, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2764, 12, 2782, 208}

$$-\frac{(c+d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f\sqrt{c-d}}-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((c + d)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Si
n[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*Sqrt[c - d]*f) -
 (Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2764

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m
*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*
```

```
(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && L
tQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c,
 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{a(c+d)}{2\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{2a^2}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(a+a\sin(e+fx))^{3/2}} + \frac{(c+d)\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{4a}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{(c+d)\,\mathrm{Subst}\left(\int \frac{1}{2a^2-(ac-ad)x^2}\,dx, x, \frac{a\,\mathrm{co}}{\sqrt{a+a\sin(e+f}}\right.}{2f}$$

$$= -\frac{(c+d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}\sqrt{c-d}\,f} - \frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(a+a\sin(e+fx))^{3/2}}$$

Mathematica [B] time = 5.37, size = 372, normalized size = 2.95

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\frac{(c+d)\left(\log\left(\tan\left(\frac{1}{2}(e+fx)\right)+1\right)-\log\left((d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c\right.\right.}{\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2\tan\left(\frac{1}{2}(e+fx)\right)+2}-\frac{\frac{\sqrt{c-d}\left(\frac{1}{\cos(e+fx)+1}\right)^{3/2}(c\sin(e+fx)+d\cos(e+fx)+d)}{\sqrt{c+d\sin(e+fx)}}-\frac{1}{2}(c-d)\sec^2\left(\frac{1}{2}(e+fx)\right)}{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c-d}}\right.}{4f(a(\sin(e+fx)+1))^{3/2}\sqrt{c+d\sin(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*((-2*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(c + d*Sin[e + f*x]))/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + ((c
+ d)*(Log[1 + Tan[(e + f*x)/2]] - Log[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e
 + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]]))/(Sec
[(e + f*x)/2]^2/(2 + 2*Tan[(e + f*x)/2]) - (-1/2*((c - d)*Sec[(e + f*x)/2]^
2) + (Sqrt[c - d]*((1 + Cos[e + f*x])^(-1))^(3/2)*(d + d*Cos[e + f*x] + c*S
in[e + f*x]))/Sqrt[c + d*Sin[e + f*x]])/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Co
s[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]))))/
(4*f*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c + d*Sin[e + f*x]])
```

fricas [B] time = 0.73, size = 896, normalized size = 7.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/16*(((c + d)*cos(f*x + e)^2 - (c + d)*cos(f*x + e) - ((c + d)*cos(f*x +
e) + 2*c + 2*d)*sin(f*x + e) - 2*c - 2*d)*sqrt(2*a*c - 2*a*d)*log(((a*c^2 -
 14*a*c*d + 17*a*d^2)*cos(f*x + e)^3 - 4*a*c^2 - 8*a*c*d - 4*a*d^2 - (13*a*
c^2 - 22*a*c*d - 3*a*d^2)*cos(f*x + e)^2 - 4*((c - 3*d)*cos(f*x + e)^2 - (3
*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) + 4*c - 4*d)*sin(f*x + e) -
4*c + 4*d)*sqrt(2*a*c - 2*a*d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
 + c) - 2*(9*a*c^2 - 14*a*c*d + 9*a*d^2)*cos(f*x + e) - (4*a*c^2 + 8*a*c*d
+ 4*a*d^2 - (a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^2 - 2*(7*a*c^2 - 18*
a*c*d + 7*a*d^2)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^3 + 3*cos(f*x +
e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x + e)
- 4)) + 8*((c - d)*cos(f*x + e) - (c - d)*sin(f*x + e) + c - d)*sqrt(a*sin(
f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((a^2*c - a^2*d)*f*cos(f*x + e)^2 -
 (a^2*c - a^2*d)*f*cos(f*x + e) - 2*(a^2*c - a^2*d)*f - ((a^2*c - a^2*d)*f*
cos(f*x + e) + 2*(a^2*c - a^2*d)*f)*sin(f*x + e)), -1/8*(((c + d)*cos(f*x +
 e)^2 - (c + d)*cos(f*x + e) - ((c + d)*cos(f*x + e) + 2*c + 2*d)*sin(f*x +
 e) - 2*c - 2*d)*sqrt(-2*a*c + 2*a*d)*arctan(1/4*sqrt(-2*a*c + 2*a*d)*sqrt(
a*sin(f*x + e) + a)*((c - 3*d)*sin(f*x + e) - 3*c + d)*sqrt(d*sin(f*x + e)
+ c)/((a*c*d - a*d^2)*cos(f*x + e)*sin(f*x + e) + (a*c^2 - a*c*d)*cos(f*x +
 e))) - 4*((c - d)*cos(f*x + e) - (c - d)*sin(f*x + e) + c - d)*sqrt(a*sin(
f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((a^2*c - a^2*d)*f*cos(f*x + e)^2 -
 (a^2*c - a^2*d)*f*cos(f*x + e) - 2*(a^2*c - a^2*d)*f - ((a^2*c - a^2*d)*f*
cos(f*x + e) + 2*(a^2*c - a^2*d)*f)*sin(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\sin\left(fx+e\right)+c}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(3/2), x)
```

maple [B] time = 0.33, size = 1373, normalized size = 10.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/8/f*(ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2
)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-co
s(f*x+e)+sin(f*x+e)))*2^(1/2)*sin(f*x+e)*cos(f*x+e)*(2*c-2*d)^(1/2)*c+ln(2*
((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)
+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin
(f*x+e)))*2^(1/2)*sin(f*x+e)*cos(f*x+e)*(2*c-2*d)^(1/2)*d+ln(2*((2*c-2*d)^(
1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e
)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^
(1/2)*cos(f*x+e)^2*(2*c-2*d)^(1/2)*c+ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*si
n(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(
f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*cos(f*x+e)^2*(2
*c-2*d)^(1/2)*d-2*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+
e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)
-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*sin(f*x+e)*(2*c-2*d)^(1/2)*c-2*ln(
2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+
e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+s
in(f*x+e)))*2^(1/2)*sin(f*x+e)*(2*c-2*d)^(1/2)*d+ln(2*((2*c-2*d)^(1/2)*2^(1
/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f
*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*cos
(f*x+e)*(2*c-2*d)^(1/2)*c+ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(
cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*co
s(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*cos(f*x+e)*(2*c-2*d)^(1/2)
*d-2*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*
sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(
```

```
f*x+e)+sin(f*x+e)))*2^(1/2)*(2*c-2*d)^(1/2)*c-2*ln(2*((2*c-2*d)^(1/2)*2^(1/
2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*
x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*(2*c
-2*d)^(1/2)*d-4*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+
e)*c+4*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)*d)*(c+
d*sin(f*x+e))^(1/2)/((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)/sin(f*x+e)/(a*(
1+sin(f*x+e)))^(3/2)/(c-d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d \sin\left(fx+e\right)+c}}{\left(a \sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c+d\,\sin\left(e+f\,x\right)}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+d\sin\left(e+fx\right)}}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(c + d*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(3/2), x)
```

3.597 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=135

$$-\frac{(c-3d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^{3/2}}-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}$$

```
[Out] -1/4*(c-3*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1/2)/(a+a*sin(f*
x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))/a^(3/2)/(c-d)^(3/2)/f*2^(1/2)-1/2*cos(f
*x+e)*(c+d*sin(f*x+e))^(1/2)/(c-d)/f/(a+a*sin(f*x+e))^(3/2)
```

Rubi [A] time = 0.24, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2766, 12, 2782, 208}

$$-\frac{(c-3d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^{3/2}}-\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] -((c - 3*d)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*
Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*(c - d)^(3/2)*
f) - (Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(2*(c - d)*f*(a + a*Sin[e + f*
x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
```

```
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2(c-d)f(a+a\sin(e+fx))^{3/2}} - \frac{\int -\frac{a(c-3d)}{2\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+f}}}{2a^2(c-d)}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2(c-d)f(a+a\sin(e+fx))^{3/2}} + \frac{(c-3d)\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d}}}{4a(c-d)}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2(c-d)f(a+a\sin(e+fx))^{3/2}} - \frac{(c-3d)\,\mathrm{Subst}\left(\int \frac{1}{2a^2-(ac-ad)}\right.}{2}$$

$$= -\frac{(c-3d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}(c-d)^{3/2}f} - \frac{\cos(e+fx)}{2(c-d)f(a}$$

Mathematica [B] time = 5.67, size = 381, normalized size = 2.82

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\frac{(c-3d)\left(\log\left(\tan\left(\frac{1}{2}(e+fx)\right)+1\right)-\log\left((d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c-\right.\right.}{\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2\tan\left(\frac{1}{2}(e+fx)\right)+2}-\frac{\frac{\sqrt{c-d}\left(\frac{1}{\cos(e+fx)+1}\right)^{3/2}(c\sin(e+fx)+d\cos(e+fx)+d)}{\sqrt{c+d\sin(e+fx)}}-\frac{1}{2}(c-d)\sec^2\left(\frac{1}{2}(e+fx)\right)}{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c-d}}\right.}{4f(c-d)(a(\sin(e+fx)+1))^{3/2}\sqrt{c+d\sin(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*((-2*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(c + d*Sin[e + f*x]))/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + ((c
- 3*d)*(Log[1 + Tan[(e + f*x)/2]] - Log[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos
[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]]))/(S
ec[(e + f*x)/2]^2/(2 + 2*Tan[(e + f*x)/2]) - (-1/2*((c - d)*Sec[(e + f*x)/2
]^2) + (Sqrt[c - d]*((1 + Cos[e + f*x])^(-1))^(3/2)*(d + d*Cos[e + f*x] + c
*Sin[e + f*x]))/Sqrt[c + d*Sin[e + f*x]])/(c - d + 2*Sqrt[c - d]*Sqrt[(1 +
Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2])))
)/(4*(c - d)*f*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c + d*Sin[e + f*x]])
```

fricas [B] time = 0.83, size = 1008, normalized size = 7.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] [1/16*(((c - 3*d)*cos(f*x + e)^2 - (c - 3*d)*cos(f*x + e) - ((c - 3*d)*cos(
f*x + e) + 2*c - 6*d)*sin(f*x + e) - 2*c + 6*d)*sqrt(2*a*c - 2*a*d)*log(((a
*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^3 - 4*a*c^2 - 8*a*c*d - 4*a*d^2 -
(13*a*c^2 - 22*a*c*d - 3*a*d^2)*cos(f*x + e)^2 - 4*((c - 3*d)*cos(f*x + e)^
2 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) + 4*c - 4*d)*sin(f*x +
 e) - 4*c + 4*d)*sqrt(2*a*c - 2*a*d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*
x + e) + c) - 2*(9*a*c^2 - 14*a*c*d + 9*a*d^2)*cos(f*x + e) - (4*a*c^2 + 8*
a*c*d + 4*a*d^2 - (a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^2 - 2*(7*a*c^2
 - 18*a*c*d + 7*a*d^2)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^3 + 3*cos(
f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x
 + e) - 4)) + 8*((c - d)*cos(f*x + e) - (c - d)*sin(f*x + e) + c - d)*sqrt(
a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((a^2*c^2 - 2*a^2*c*d + a^2*d
^2)*f*cos(f*x + e)^2 - (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e) - 2*(
a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f - ((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f
*x + e) + 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f)*sin(f*x + e)), -1/8*(((c - 3
*d)*cos(f*x + e)^2 - (c - 3*d)*cos(f*x + e) - ((c - 3*d)*cos(f*x + e) + 2*c
 - 6*d)*sin(f*x + e) - 2*c + 6*d)*sqrt(-2*a*c + 2*a*d)*arctan(1/4*sqrt(-2*a
*c + 2*a*d)*sqrt(a*sin(f*x + e) + a)*((c - 3*d)*sin(f*x + e) - 3*c + d)*sqr
t(d*sin(f*x + e) + c)/((a*c*d - a*d^2)*cos(f*x + e)*sin(f*x + e) + (a*c^2 -
 a*c*d)*cos(f*x + e))) - 4*((c - d)*cos(f*x + e) - (c - d)*sin(f*x + e) + c
 - d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((a^2*c^2 - 2*a^2*
c*d + a^2*d^2)*f*cos(f*x + e)^2 - (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x
 + e) - 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f - ((a^2*c^2 - 2*a^2*c*d + a^2*d
^2)*f*cos(f*x + e) + 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f)*sin(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c)), x)
```

maple [B] time = 0.33, size = 1268, normalized size = 9.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/4/f*(-sin(f*x+e)*cos(f*x+e)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*
sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*co
s(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c+3*sin(f*x+e)*cos(f*
x+e)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1)
)^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)
/(1-cos(f*x+e)+sin(f*x+e)))*d-cos(f*x+e)^2*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^
(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin
(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c+3*cos(f
*x+e)^2*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)
+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c
+d)/(1-cos(f*x+e)+sin(f*x+e)))*d+2*sin(f*x+e)*cos(f*x+e)*(2*c-2*d)^(1/2)*((
c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)+2*sin(f*x+e)*2^(1/2)*ln(2*((2*c-2*d)^
(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+
e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c
-6*sin(f*x+e)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(
f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*
x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d-cos(f*x+e)*2^(1/2)*ln(2*((2*c-2*d)^(
1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e
)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c+
3*cos(f*x+e)*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f
*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x
+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d+2*2^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/
2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*
x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c-6*2^(1/2)*
ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f
```

```
*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e
)+sin(f*x+e)))*d)*(c+d*sin(f*x+e))^(1/2)/sin(f*x+e)/(a*(1+sin(f*x+e)))^(3/2
)/((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)/(2*c-2*d)^(1/2)/(c-d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a \sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{d \sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/((a*(sin(e + f*x) + 1))**(3/2)*sqrt(c + d*sin(e + f*x))), x)
```

3.598 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=197

$$-\frac{(c-7d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f(c-d)^{5/2}}-\frac{d(c+5d)\cos(e+fx)}{2af(c-d)^2(c+d)\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{}{2f(c}$$

```
[Out] -1/4*(c-7*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1/2)/(a+a*sin(f*
x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))/a^(3/2)/(c-d)^(5/2)/f*2^(1/2)-1/2*cos(f
*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2)-1/2*d*(c+5*d)*c
os(f*x+e)/a/(c-d)^2/(c+d)/f/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2766, 2984, 12, 2782, 208}

$$-\frac{(c-7d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f(c-d)^{5/2}}-\frac{d(c+5d)\cos(e+fx)}{2af(c-d)^2(c+d)\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\frac{}{2f(c}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -((c - 7*d)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*
Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*(c - d)^(5/2)*
f) - Cos[e + f*x]/(2*(c - d)*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]) - (d*(c + 5*d)*Cos[e + f*x])/(2*a*(c - d)^2*(c + d)*f*Sqrt[a + a*S
in[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
```

```
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}} - \frac{\int \frac{-}{\sqrt{a+a\,s}}}{}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}} - \frac{}{2a(c-}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}} - \frac{}{2a(c-}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}} - \frac{}{2a(c-}$$

$$= -\frac{(c-7d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}(c-d)^{5/2}f} - \frac{}{2(c-d)f(}$$

Mathematica [B] time = 6.24, size = 401, normalized size = 2.04

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\frac{(c-7d)\left(\log\left(\tan\left(\frac{1}{2}(e+fx)\right)+1\right)-\log\left((d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c-\right.\right.}{\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2\tan\left(\frac{1}{2}(e+fx)\right)+2}-\frac{\frac{\sqrt{c-d}\left(\frac{1}{\cos(e+fx)+1}\right)^{3/2}(c\sin(e+fx)+d\cos(e+fx)+d)}{\sqrt{c+d\sin(e+fx)}}-\frac{1}{2}(c-d)\sec^2\left(\frac{1}{2}(e+fx)\right)}{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c-d}}\right.}{4f(c-d)^2(a(\sin(e+fx)+1))^{3/2}\sqrt{c+d\sin(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*((-2*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(c^2 + c*d + 4*d^2 + d*(c + 5*d)*Sin[e + f*x]))/((c + d)*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])) + ((c - 7*d)*(Log[1 + Tan[(e + f*x)/2]] - Log
[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x
]] + (-c + d)*Tan[(e + f*x)/2]]))/(Sec[(e + f*x)/2]^2/(2 + 2*Tan[(e + f*x)/
2]) - (-1/2*((c - d)*Sec[(e + f*x)/2]^2) + (Sqrt[c - d]*((1 + Cos[e + f*x])
^(-1))^(3/2)*(d + d*Cos[e + f*x] + c*Sin[e + f*x]))/Sqrt[c + d*Sin[e + f*x]
])/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e +
```

f*x]] + (-c + d)*Tan[(e + f*x)/2]]))))/(4*(c - d)^2*f*(a*(1 + Sin[e + f*x]))
^(3/2)*Sqrt[c + d*Sin[e + f*x]])

fricas [B] time = 1.07, size = 1954, normalized size = 9.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] [-1/16*(((c^2*d - 6*c*d^2 - 7*d^3)*cos(f*x + e)^3 - 2*c^3 + 10*c^2*d + 26*c
*d^2 + 14*d^3 + (c^3 - 4*c^2*d - 19*c*d^2 - 14*d^3)*cos(f*x + e)^2 - (c^3 -
 5*c^2*d - 13*c*d^2 - 7*d^3)*cos(f*x + e) - (2*c^3 - 10*c^2*d - 26*c*d^2 -
14*d^3 - (c^2*d - 6*c*d^2 - 7*d^3)*cos(f*x + e)^2 + (c^3 - 5*c^2*d - 13*c*d
^2 - 7*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(2*a*c - 2*a*d)*log(((a*c^2 - 1
4*a*c*d + 17*a*d^2)*cos(f*x + e)^3 - 4*a*c^2 - 8*a*c*d - 4*a*d^2 - (13*a*c^
2 - 22*a*c*d - 3*a*d^2)*cos(f*x + e)^2 + 4*((c - 3*d)*cos(f*x + e)^2 - (3*c
 - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) + 4*c - 4*d)*sin(f*x + e) - 4*
c + 4*d)*sqrt(2*a*c - 2*a*d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) +
 c) - 2*(9*a*c^2 - 14*a*c*d + 9*a*d^2)*cos(f*x + e) - (4*a*c^2 + 8*a*c*d +
4*a*d^2 - (a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^2 - 2*(7*a*c^2 - 18*a*
c*d + 7*a*d^2)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^3 + 3*cos(f*x + e)
^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x + e) -
4)) - 8*(c^3 - c^2*d - c*d^2 + d^3 + (c^2*d + 4*c*d^2 - 5*d^3)*cos(f*x + e)
^2 + (c^3 + 3*c*d^2 - 4*d^3)*cos(f*x + e) - (c^3 - c^2*d - c*d^2 + d^3 - (c
^2*d + 4*c*d^2 - 5*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a
)*sqrt(d*sin(f*x + e) + c))/((a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c*d^4 - a^2
*d^5)*f*cos(f*x + e)^3 + (a^2*c^5 - 4*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + 3*a^2*c
*d^4 - 2*a^2*d^5)*f*cos(f*x + e)^2 - (a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 +
 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e) - 2*(a^2*c^5 - a^2*c^4
*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f + ((a^2*c^4*d -
 2*a^2*c^3*d^2 + 2*a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e)^2 - (a^2*c^5 - a^2*c
^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e)
- 2*(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*
d^5)*f)*sin(f*x + e)), -1/8*(((c^2*d - 6*c*d^2 - 7*d^3)*cos(f*x + e)^3 - 2*
c^3 + 10*c^2*d + 26*c*d^2 + 14*d^3 + (c^3 - 4*c^2*d - 19*c*d^2 - 14*d^3)*co
s(f*x + e)^2 - (c^3 - 5*c^2*d - 13*c*d^2 - 7*d^3)*cos(f*x + e) - (2*c^3 - 1
0*c^2*d - 26*c*d^2 - 14*d^3 - (c^2*d - 6*c*d^2 - 7*d^3)*cos(f*x + e)^2 + (c
^3 - 5*c^2*d - 13*c*d^2 - 7*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-2*a*c +
2*a*d)*arctan(1/4*sqrt(-2*a*c + 2*a*d)*sqrt(a*sin(f*x + e) + a)*((c - 3*d)*
sin(f*x + e) - 3*c + d)*sqrt(d*sin(f*x + e) + c)/((a*c*d - a*d^2)*cos(f*x +
 e)*sin(f*x + e) + (a*c^2 - a*c*d)*cos(f*x + e))) - 4*(c^3 - c^2*d - c*d^2
+ d^3 + (c^2*d + 4*c*d^2 - 5*d^3)*cos(f*x + e)^2 + (c^3 + 3*c*d^2 - 4*d^3)*
cos(f*x + e) - (c^3 - c^2*d - c*d^2 + d^3 - (c^2*d + 4*c*d^2 - 5*d^3)*cos(f
```

```
*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(
(a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e)^3 + (a^2
*c^5 - 4*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + 3*a^2*c*d^4 - 2*a^2*d^5)*f*cos(f*x +
 e)^2 - (a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 -
a^2*d^5)*f*cos(f*x + e) - 2*(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^
2*d^3 + a^2*c*d^4 - a^2*d^5)*f + ((a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c*d^4
- a^2*d^5)*f*cos(f*x + e)^2 - (a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*
c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e) - 2*(a^2*c^5 - a^2*c^4*d - 2*
a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f)*sin(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(3/2)), x)
```

maple [B] time = 0.37, size = 2246, normalized size = 11.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/4/f*(sin(f*x+e)*cos(f*x+e)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e
))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-
d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos
(f*x+e)+1))^(1/2)*c^2-6*sin(f*x+e)*cos(f*x+e)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)
*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+
e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*
sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c*d-7*sin(f*x+e)*cos(f*x+e)*ln(2*((2*c-2*
d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f
*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e))
)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*d^2-cos(f*x+e)^2*ln(2*((2
*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*
sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*
x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c^2+6*cos(f*x+e)^2*l
n(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*
x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)
+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c*d+7*cos(f*x
```

```
+e)^2*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)
*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos
(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*d^2+2*
ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f
*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e
)+sin(f*x+e)))*c^2*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+
e)-12*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)
*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos
(f*x+e)+sin(f*x+e)))*c*d*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*si
n(f*x+e)-14*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))
^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/
(1-cos(f*x+e)+sin(f*x+e)))*d^2*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1
/2)*sin(f*x+e)+ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+
1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+
d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))
^(1/2)*cos(f*x+e)-6*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*
x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+
e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e
)+1))^(1/2)*cos(f*x+e)-7*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(c
os(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos
(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d^2*2^(1/2)*((c+d*sin(f*x+e))/(cos(
f*x+e)+1))^(1/2)*cos(f*x+e)+2*sin(f*x+e)*cos(f*x+e)*(2*c-2*d)^(1/2)*c*d+10*
sin(f*x+e)*cos(f*x+e)*(2*c-2*d)^(1/2)*d^2+2*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*(
(c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)
+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*si
n(f*x+e))/(cos(f*x+e)+1))^(1/2)*c^2-12*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*
sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*co
s(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d*2^(1/2)*((c+d*sin
(f*x+e))/(cos(f*x+e)+1))^(1/2)-14*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f
*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x
+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/
(cos(f*x+e)+1))^(1/2)*d^2+2*cos(f*x+e)*(2*c-2*d)^(1/2)*c^2+2*cos(f*x+e)*(2*
c-2*d)^(1/2)*c*d+8*cos(f*x+e)*(2*c-2*d)^(1/2)*d^2)/(a*(1+sin(f*x+e)))^(3/2)
/(c+d*sin(f*x+e))^(1/2)/(c+d)/(2*c-2*d)^(1/2)/(c-d)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+a\sin\left(e+fx\right)\right)^{3/2}\left(c+d\sin\left(e+fx\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(3/2)),x)
[Out] int(1/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(3/2),x)
[Out] Integral(1/((a*(sin(e + f*x) + 1))**(3/2)*(c + d*sin(e + f*x))**(3/2)), x)
```

3.599 $\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=271

$$-\frac{(c-11d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^{7/2}}-\frac{d\left(3c^2+38cd+19d^2\right)\cos(e+fx)}{6af(c-d)^3(c+d)^2\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\ldots$$

```
[Out] -1/2*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2)-1/4*(
c-11*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1/2)/(a+a*sin(f*x+e))
^(1/2)/(c+d*sin(f*x+e))^(1/2))/a^(3/2)/(c-d)^(7/2)/f*2^(1/2)-1/6*d*(3*c+7*d
)*cos(f*x+e)/a/(c-d)^2/(c+d)/f/(c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2
)-1/6*d*(3*c^2+38*c*d+19*d^2)*cos(f*x+e)/a/(c-d)^3/(c+d)^2/f/(a+a*sin(f*x+e
))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.85, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2766, 2984, 12, 2782, 208}

$$-\frac{(c-11d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^{7/2}}-\frac{d\left(3c^2+38cd+19d^2\right)\cos(e+fx)}{6af(c-d)^3(c+d)^2\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] -((c - 11*d)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a
*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*(c - d)^(7/2)
*f) - Cos[e + f*x]/(2*(c - d)*f*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f
*x])^(3/2)) - (d*(3*c + 7*d)*Cos[e + f*x])/(6*a*(c - d)^2*(c + d)*f*Sqrt[a
+ a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2)) - (d*(3*c^2 + 38*c*d + 19*d^2
)*Cos[e + f*x])/(6*a*(c - d)^3*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
+ d*Sin[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}} - \frac{\int \frac{}{\sqrt{}}}{}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}} - \frac{}{6a(c}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}} - \frac{}{6a(c}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}} - \frac{}{6a(c}$$

$$= -\frac{\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}} - \frac{}{6a(c}$$

$$= -\frac{(c-11d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}(c-d)^{7/2}f} - \frac{}{2(c-d)}$$

Mathematica [A] time = 9.45, size = 478, normalized size = 1.76

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\frac{3(c-11d)\left(\log\left(\tan\left(\frac{1}{2}(e+fx)\right)+1\right)-\log\left((d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}\right.\right.}{\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2\tan\left(\frac{1}{2}(e+fx)\right)+2}-\frac{\frac{\sqrt{c-d}\left(\frac{1}{\cos(e+fx)+1}\right)^{3/2}(c\sin(e+fx)+d\cos(e+fx)+d)}{\sqrt{c+d\sin(e+fx)}}-\frac{1}{2}(c-d)\sec^2\left(\frac{1}{2}(e+fx)\right)}{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c-d}}\right.}{12f(c-d)^3(a(\sin(e+fx}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*(-(((Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])*(6*c^4 + 12*c^3*d + 81*c^2*d^2 + 70*c*d^3 + 11*d^4 - d^2*(3*c^2 + 3
8*c*d + 19*d^2)*Cos[2*(e + f*x)] + 12*d*(c^3 + 8*c^2*d + 9*c*d^2 + 2*d^3)*S
in[e + f*x]))/((c + d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c + d*Sin[e
```

```
 + f*x]))) + (3*(c - 11*d)*(Log[1 + Tan[(e + f*x)/2]] - Log[c - d + 2*Sqrt[
c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Ta
n[(e + f*x)/2]]))/(Sec[(e + f*x)/2]^2/(2 + 2*Tan[(e + f*x)/2]) - (-1/2*((c
- d)*Sec[(e + f*x)/2]^2) + (Sqrt[c - d]*((1 + Cos[e + f*x])^(-1))^(3/2)*(d
+ d*Cos[e + f*x] + c*Sin[e + f*x]))/Sqrt[c + d*Sin[e + f*x]])/(c - d + 2*Sq
rt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)
*Tan[(e + f*x)/2]]))))/(12*(c - d)^3*f*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c +
 d*Sin[e + f*x]])
```

fricas [B] time = 1.38, size = 3182, normalized size = 11.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] [1/48*(3*(2*c^5 - 14*c^4*d - 76*c^3*d^2 - 124*c^2*d^3 - 86*c*d^4 - 22*d^5 +
 (c^3*d^2 - 9*c^2*d^3 - 21*c*d^4 - 11*d^5)*cos(f*x + e)^4 - (2*c^4*d - 17*c
^3*d^2 - 51*c^2*d^3 - 43*c*d^4 - 11*d^5)*cos(f*x + e)^3 - (c^5 - 5*c^4*d -
54*c^3*d^2 - 122*c^2*d^3 - 107*c*d^4 - 33*d^5)*cos(f*x + e)^2 + (c^5 - 7*c^
4*d - 38*c^3*d^2 - 62*c^2*d^3 - 43*c*d^4 - 11*d^5)*cos(f*x + e) + (2*c^5 -
14*c^4*d - 76*c^3*d^2 - 124*c^2*d^3 - 86*c*d^4 - 22*d^5 - (c^3*d^2 - 9*c^2*
d^3 - 21*c*d^4 - 11*d^5)*cos(f*x + e)^3 - 2*(c^4*d - 8*c^3*d^2 - 30*c^2*d^3
 - 32*c*d^4 - 11*d^5)*cos(f*x + e)^2 + (c^5 - 7*c^4*d - 38*c^3*d^2 - 62*c^2
*d^3 - 43*c*d^4 - 11*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(2*a*c - 2*a*d)*l
og(((a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^3 - 4*a*c^2 - 8*a*c*d - 4*a*
d^2 - (13*a*c^2 - 22*a*c*d - 3*a*d^2)*cos(f*x + e)^2 - 4*((c - 3*d)*cos(f*x
 + e)^2 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) + 4*c - 4*d)*sin
(f*x + e) - 4*c + 4*d)*sqrt(2*a*c - 2*a*d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*
sin(f*x + e) + c) - 2*(9*a*c^2 - 14*a*c*d + 9*a*d^2)*cos(f*x + e) - (4*a*c^
2 + 8*a*c*d + 4*a*d^2 - (a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^2 - 2*(7
*a*c^2 - 18*a*c*d + 7*a*d^2)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^3 +
3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x + e) - 2*c
os(f*x + e) - 4)) - 8*(3*c^5 - 3*c^4*d - 6*c^3*d^2 + 6*c^2*d^3 + 3*c*d^4 -
3*d^5 - (3*c^3*d^2 + 35*c^2*d^3 - 19*c*d^4 - 19*d^5)*cos(f*x + e)^3 + (6*c^
4*d + 39*c^3*d^2 - 29*c^2*d^3 - 23*c*d^4 + 7*d^5)*cos(f*x + e)^2 + 3*(c^5 +
 c^4*d + 12*c^3*d^2 + 4*c^2*d^3 - 13*c*d^4 - 5*d^5)*cos(f*x + e) - (3*c^5 -
 3*c^4*d - 6*c^3*d^2 + 6*c^2*d^3 + 3*c*d^4 - 3*d^5 - (3*c^3*d^2 + 35*c^2*d^
3 - 19*c*d^4 - 19*d^5)*cos(f*x + e)^2 - 6*(c^4*d + 7*c^3*d^2 + c^2*d^3 - 7*
c*d^4 - 2*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*
sin(f*x + e) + c))/((a^2*c^6*d^2 - 2*a^2*c^5*d^3 - a^2*c^4*d^4 + 4*a^2*c^3*
d^5 - a^2*c^2*d^6 - 2*a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^4 - (2*a^2*c^7*d
- 3*a^2*c^6*d^2 - 4*a^2*c^5*d^3 + 7*a^2*c^4*d^4 + 2*a^2*c^3*d^5 - 5*a^2*c^2
*d^6 + a^2*d^8)*f*cos(f*x + e)^3 - (a^2*c^8 + 2*a^2*c^7*d - 6*a^2*c^6*d^2 -
```

6*a^2*c^5*d^3 + 12*a^2*c^4*d^4 + 6*a^2*c^3*d^5 - 10*a^2*c^2*d^6 - 2*a^2*c*d^7 + 3*a^2*d^8)*f*cos(f*x + e)^2 + (a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f*cos(f*x + e) + 2*(a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f - ((a^2*c^6*d^2 - 2*a^2*c^5*d^3 - a^2*c^4*d^4 + 4*a^2*c^3*d^5 - a^2*c^2*d^6 - 2*a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^3 + 2*(a^2*c^7*d - a^2*c^6*d^2 - 3*a^2*c^5*d^3 + 3*a^2*c^4*d^4 + 3*a^2*c^3*d^5 - 3*a^2*c^2*d^6 - a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^2 - (a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f*cos(f*x + e) - 2*(a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f)*sin(f*x + e)), -1/24*(3*(2*c^5 - 14*c^4*d - 76*c^3*d^2 - 124*c^2*d^3 - 86*c*d^4 - 22*d^5 + (c^3*d^2 - 9*c^2*d^3 - 21*c*d^4 - 11*d^5)*cos(f*x + e)^4 - (2*c^4*d - 17*c^3*d^2 - 51*c^2*d^3 - 43*c*d^4 - 11*d^5)*cos(f*x + e)^3 - (c^5 - 5*c^4*d - 54*c^3*d^2 - 122*c^2*d^3 - 107*c*d^4 - 33*d^5)*cos(f*x + e)^2 + (c^5 - 7*c^4*d - 38*c^3*d^2 - 62*c^2*d^3 - 43*c*d^4 - 11*d^5)*cos(f*x + e) + (2*c^5 - 14*c^4*d - 76*c^3*d^2 - 124*c^2*d^3 - 86*c*d^4 - 22*d^5 - (c^3*d^2 - 9*c^2*d^3 - 21*c*d^4 - 11*d^5)*cos(f*x + e)^3 - 2*(c^4*d - 8*c^3*d^2 - 30*c^2*d^3 - 32*c*d^4 - 11*d^5)*cos(f*x + e)^2 + (c^5 - 7*c^4*d - 38*c^3*d^2 - 62*c^2*d^3 - 43*c*d^4 - 11*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(-2*a*c + 2*a*d)*arctan(1/4*sqrt(-2*a*c + 2*a*d)*sqrt(a*sin(f*x + e) + a)*((c - 3*d)*sin(f*x + e) - 3*c + d)*sqrt(d*sin(f*x + e) + c)/((a*c*d - a*d^2)*cos(f*x + e)*sin(f*x + e) + (a*c^2 - a*c*d)*cos(f*x + e))) + 4*(3*c^5 - 3*c^4*d - 6*c^3*d^2 + 6*c^2*d^3 + 3*c*d^4 - 3*d^5 - (3*c^3*d^2 + 35*c^2*d^3 - 19*c*d^4 - 19*d^5)*cos(f*x + e)^3 + (6*c^4*d + 39*c^3*d^2 - 29*c^2*d^3 - 23*c*d^4 + 7*d^5)*cos(f*x + e)^2 + 3*(c^5 + c^4*d + 12*c^3*d^2 + 4*c^2*d^3 - 13*c*d^4 - 5*d^5)*cos(f*x + e) - (3*c^5 - 3*c^4*d - 6*c^3*d^2 + 6*c^2*d^3 + 3*c*d^4 - 3*d^5 - (3*c^3*d^2 + 35*c^2*d^3 - 19*c*d^4 - 19*d^5)*cos(f*x + e)^2 - 6*(c^4*d + 7*c^3*d^2 + c^2*d^3 - 7*c*d^4 - 2*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((a^2*c^6*d^2 - 2*a^2*c^5*d^3 - a^2*c^4*d^4 + 4*a^2*c^3*d^5 - a^2*c^2*d^6 - 2*a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^4 - (2*a^2*c^7*d - 3*a^2*c^6*d^2 - 4*a^2*c^5*d^3 + 7*a^2*c^4*d^4 + 2*a^2*c^3*d^5 - 5*a^2*c^2*d^6 + a^2*d^8)*f*cos(f*x + e)^3 - (a^2*c^8 + 2*a^2*c^7*d - 6*a^2*c^6*d^2 - 6*a^2*c^5*d^3 + 12*a^2*c^4*d^4 + 6*a^2*c^3*d^5 - 10*a^2*c^2*d^6 - 2*a^2*c*d^7 + 3*a^2*d^8)*f*cos(f*x + e)^2 + (a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f*cos(f*x + e) + 2*(a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f - ((a^2*c^6*d^2 - 2*a^2*c^5*d^3 - a^2*c^4*d^4 + 4*a^2*c^3*d^5 - a^2*c^2*d^6 - 2*a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^3 + 2*(a^2*c^7*d - a^2*c^6*d^2 - 3*a^2*c^5*d^3 + 3*a^2*c^4*d^4 + 3*a^2*c^3*d^5 - 3*a^2*c^2*d^6 - a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^2 - (a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f*cos(f*x + e) - 2*(a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f)*sin(f*x + e))]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(5/2)), x)
```

maple [B] time = 0.41, size = 5040, normalized size = 18.60

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(5/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(5/2),x)
[Out] Timed out
```

3.600 $\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=260

$$-\frac{\sqrt{c-d}\left(3c^2+14cd+43d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f}-\frac{2d^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{a^{5/2}f}-(c$$

```
[Out] -2*d^(5/2)*arctan(cos(f*x+e)*a^(1/2)*d^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*si
n(f*x+e))^(1/2))/a^(5/2)/f-1/4*(c-d)*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/f/(a
+a*sin(f*x+e))^(5/2)-1/32*(3*c^2+14*c*d+43*d^2)*arctanh(1/2*cos(f*x+e)*a^(1
/2)*(c-d)^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))*(c-d
)^(1/2)/a^(5/2)/f*2^(1/2)-1/16*(c-d)*(3*c+11*d)*cos(f*x+e)*(c+d*sin(f*x+e))
^(1/2)/a/f/(a+a*sin(f*x+e))^(3/2)
```

Rubi [A] time = 0.86, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2765, 2977, 2982, 2782, 208, 2775, 205}

$$-\frac{\sqrt{c-d}\left(3c^2+14cd+43d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f}-\frac{2d^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{a^{5/2}f}-(c$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*d^(5/2)*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c + d*Sin[e + f*x]])])/(a^(5/2)*f) - (Sqrt[c - d]*(3*c^2 + 14*c*d + 4
3*d^2)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e
 + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*f) - ((c - d)*(3*c
 + 11*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(16*a*f*(a + a*Sin[e + f*x]
)^(3/2)) - ((c - d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(4*f*(a + a*Si
n[e + f*x])^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
```

```
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(c-d)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\int \frac{\sqrt{c+d\sin(e+fx)}\left(-\frac{1}{2}a(3c-d)(c+3d)-4ad^2\sin(e+\right.}{(a+a\sin(e+fx))^{3/2}}}{4a^2}$$

$$= -\frac{(c-d)(3c+11d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(c-d)\cos(e+fx)(c+d\sin(e}{4f(a+a\sin(e+fx))^5}$$

$$= -\frac{(c-d)(3c+11d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(c-d)\cos(e+fx)(c+d\sin(e}{4f(a+a\sin(e+fx))^5}$$

$$= -\frac{(c-d)(3c+11d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(c-d)\cos(e+fx)(c+d\sin(e}{4f(a+a\sin(e+fx))^5}$$

$$= -\frac{2d^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{a^{5/2}f} - \frac{\sqrt{c-d}\left(3c^2+14cd+43d^2\right)\tanh^{-1}\left(\frac{}{\sqrt{}}\right.}{16\sqrt{2}\,a^{5/2}f}$$

Mathematica [C] time = 17.44, size = 1845, normalized size = 7.10

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(-1/4*(c - d)^2/(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^3 - (3*(c - d)*(c + 5*d))/(16*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])) + (3*(c^2*Sin[(e + f*x)/2] + 4*c*d*Sin[(e + f*x)/2] - 5*d^2*S
in[(e + f*x)/2]))/(8*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2) + (c^2*Sin[(e
 + f*x)/2] - 2*c*d*Sin[(e + f*x)/2] + d^2*Sin[(e + f*x)/2])/(2*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])^4))*Sqrt[c + d*Sin[e + f*x]])/(f*(a*(1 + Sin[e +
f*x]))^(5/2)) + ((Sqrt[2]*(3*c^3 + 11*c^2*d + 29*c*d^2 - 43*d^3)*Log[1 + Ta
n[(e + f*x)/2]] - Sqrt[2]*(3*c^3 + 11*c^2*d + 29*c*d^2 - 43*d^3)*Log[c - d
+ 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-
c + d)*Tan[(e + f*x)/2]] - (32*I)*Sqrt[c - d]*d^(5/2)*(Log[(c - I*(d + (1 +
 I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]])
```

```
 + ((-I)*c + d)*Tan[(e + f*x)/2])/(16*d^(7/2)*(I + Tan[(e + f*x)/2]))] - Lo
g[(c + I*d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c +
 d*Sin[e + f*x]] + (I*c + d)*Tan[(e + f*x)/2])/(16*d^(7/2)*(-I + Tan[(e + f
*x)/2]))])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*((3*c^3)/(32*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) + (11*c^2*d)/(32*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) + (29*c*d^2)/(3
2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) - (11*d^3
)/(32*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f*x]]) + (d^
3*Sin[e + f*x])/((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c + d*Sin[e + f
*x]])))/(f*(a*(1 + Sin[e + f*x]))^(5/2)*(((3*c^3 + 11*c^2*d + 29*c*d^2 - 43
*d^3)*Sec[(e + f*x)/2]^2)/(Sqrt[2]*(1 + Tan[(e + f*x)/2])) - (Sqrt[2]*(3*c^
3 + 11*c^2*d + 29*c*d^2 - 43*d^3)*(((-c + d)*Sec[(e + f*x)/2]^2)/2 + (Sqrt[
c - d]*d*Cos[e + f*x]*Sqrt[(1 + Cos[e + f*x])^(-1)])/Sqrt[c + d*Sin[e + f*x
]] + Sqrt[c - d]*((1 + Cos[e + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Si
n[e + f*x]]))/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c +
 d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]) - (32*I)*Sqrt[c - d]*d^(5/2)*
((16*d^(7/2)*(I + Tan[(e + f*x)/2])*(((((-I)*c + d)*Sec[(e + f*x)/2]^2)/2 -
 I*(((1 + I)*d^(3/2)*Cos[e + f*x]*Sqrt[(1 + Cos[e + f*x])^(-1)])/(Sqrt[2]*S
qrt[c + d*Sin[e + f*x]]) + ((1 + I)*Sqrt[d]*((1 + Cos[e + f*x])^(-1))^(3/2)
*Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[2]))/(16*d^(7/2)*(I + Tan[(e +
 f*x)/2])) - (Sec[(e + f*x)/2]^2*(c - I*(d + (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(
1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]]) + ((-I)*c + d)*Tan[(e + f
*x)/2]))/(32*d^(7/2)*(I + Tan[(e + f*x)/2])^2)))/(c - I*(d + (1 + I)*Sqrt[2
]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]]) + ((-I)*c
 + d)*Tan[(e + f*x)/2]) - (16*d^(7/2)*(-I + Tan[(e + f*x)/2])*((((I*c + d)*
Sec[(e + f*x)/2]^2)/2 + ((1 + I)*d^(3/2)*Cos[e + f*x]*Sqrt[(1 + Cos[e + f*x
])^(-1)])/(Sqrt[2]*Sqrt[c + d*Sin[e + f*x]]) + ((1 + I)*Sqrt[d]*((1 + Cos[e
 + f*x])^(-1))^(3/2)*Sin[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[2])/(16*d^
(7/2)*(-I + Tan[(e + f*x)/2])) - (Sec[(e + f*x)/2]^2*(c + I*d + (1 + I)*Sqr
t[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (I*c
+ d)*Tan[(e + f*x)/2]))/(32*d^(7/2)*(-I + Tan[(e + f*x)/2])^2)))/(c + I*d +
 (1 + I)*Sqrt[2]*Sqrt[d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f
*x]] + (I*c + d)*Tan[(e + f*x)/2]))))
```

fricas [B] time = 1.29, size = 3855, normalized size = 14.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] [1/32*(sqrt(1/2)*((3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e)^3 - 12*a*c^2
 - 56*a*c*d - 172*a*d^2 + 3*(3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e)^2
- 2*(3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e) - (12*a*c^2 + 56*a*c*d + 1
```

72*a*d^2 - (3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e)^2 + 2*(3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt((c - d)/a)*log((4*sqrt(1/2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt((c - d)/a)*(cos(f*x + e) - sin(f*x + e) + 1) - (c - 3*d)*cos(f*x + e)^2 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) - 2*c - 2*d)*sin(f*x + e) - 2*c - 2*d)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 8*(a*d^2*cos(f*x + e)^3 + 3*a*d^2*cos(f*x + e)^2 - 2*a*d^2*cos(f*x + e) - 4*a*d^2 + (a*d^2*cos(f*x + e)^2 - 2*a*d^2*cos(f*x + e) - 4*a*d^2)*sin(f*x + e))*sqrt(-d/a)*log((128*d^4*cos(f*x + e)^5 + 128*(2*c*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 32*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3 - 32*(c^3*d - 2*c^2*d^2 + 9*c*d^3 - 4*d^4)*cos(f*x + e)^2 - 8*(16*d^3*cos(f*x + e)^4 + 24*(c*d^2 - d^3)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5*c^2*d - 26*c*d^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 25*d^3)*cos(f*x + e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 - 51*d^3 - 8*(3*c*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-d/a) + (c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 + 289*d^4)*cos(f*x + e) + (128*d^4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 256*(c*d^3 - d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d^3 + 5*d^4)*cos(f*x + e)^2 + 32*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)) + 2*(3*(c^2 + 4*c*d - 5*d^2)*cos(f*x + e)^2 + 4*c^2 - 8*c*d + 4*d^2 + (7*c^2 + 4*c*d - 11*d^2)*cos(f*x + e) - (4*c^2 - 8*c*d + 4*d^2 - 3*(c^2 + 4*c*d - 5*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e)), 1/32*(sqrt(1/2)*((3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e)^3 - 12*a*c^2 - 56*a*c*d - 172*a*d^2 + 3*(3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e)^2 - 2*(3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e) - (12*a*c^2 + 56*a*c*d + 172*a*d^2 - (3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e)^2 + 2*(3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt((c - d)/a)*log((4*sqrt(1/2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt((c - d)/a)*(cos(f*x + e) - sin(f*x + e) + 1) - (c - 3*d)*cos(f*x + e)^2 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) - 2*c - 2*d)*sin(f*x + e) - 2*c - 2*d)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 16*(a*d^2*cos(f*x + e)^3 + 3*a*d^2*cos(f*x + e)^2 - 2*a*d^2*cos(f*x + e) - 4*a*d^2 + (a*d^2*cos(f*x + e)^2 - 2*a*d^2*cos(f*x + e) - 4*a*d^2)*sin(f*x + e))*sqrt(d/a)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(d/a)/(2*d^3*cos(f*x + e)^3 - (3*c*d^2 - d^3)*cos(f*x + e)*sin(f*x + e) - (c^2*d - c*d^2 + 2*d^3)*cos(f*x + e))) + 2*(3*(c^2 + 4*c*d - 5*d^2)*cos(f*x + e)^2 + 4*c^2 - 8*c*d + 4*d^2 + (7*c^2 + 4*c*d - 11*d^2)*cos(f*x + e) - (4*c^2 - 8*c*d + 4*d^2 - 3*(c^2 + 4*c*d - 5*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f +

(a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e)), -1/
16*(sqrt(1/2)*((3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e)^3 - 12*a*c^2 -
56*a*c*d - 172*a*d^2 + 3*(3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e)^2 - 2
*(3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e) - (12*a*c^2 + 56*a*c*d + 172*
a*d^2 - (3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e)^2 + 2*(3*a*c^2 + 14*a*
c*d + 43*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(-(c - d)/a)*arctan(-2*sqrt
(1/2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-(c - d)/a)/((
c - d)*cos(f*x + e))) - 4*(a*d^2*cos(f*x + e)^3 + 3*a*d^2*cos(f*x + e)^2 -
2*a*d^2*cos(f*x + e) - 4*a*d^2 + (a*d^2*cos(f*x + e)^2 - 2*a*d^2*cos(f*x +
e) - 4*a*d^2)*sin(f*x + e))*sqrt(-d/a)*log((128*d^4*cos(f*x + e)^5 + 128*(2
*c*d^3 - d^4)*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 -
32*(5*c^2*d^2 - 14*c*d^3 + 13*d^4)*cos(f*x + e)^3 - 32*(c^3*d - 2*c^2*d^2 +
9*c*d^3 - 4*d^4)*cos(f*x + e)^2 - 8*(16*d^3*cos(f*x + e)^4 + 24*(c*d^2 - d
^3)*cos(f*x + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5*c^2*d - 26*c
*d^2 + 33*d^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 25*d^3)*cos(f*x
+ e) + (16*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 - 51*d^3 - 8*(3*
c*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3)*cos(f*x + e
))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sqrt(-d/
a) + (c^4 - 28*c^3*d + 230*c^2*d^2 - 476*c*d^3 + 289*d^4)*cos(f*x + e) + (1
28*d^4*cos(f*x + e)^4 + c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4 - 256*(c*
d^3 - d^4)*cos(f*x + e)^3 - 32*(5*c^2*d^2 - 6*c*d^3 + 5*d^4)*cos(f*x + e)^2
+ 32*(c^3*d - 7*c^2*d^2 + 15*c*d^3 - 9*d^4)*cos(f*x + e))*sin(f*x + e))/(c
os(f*x + e) + sin(f*x + e) + 1)) - (3*(c^2 + 4*c*d - 5*d^2)*cos(f*x + e)^2
+ 4*c^2 - 8*c*d + 4*d^2 + (7*c^2 + 4*c*d - 11*d^2)*cos(f*x + e) - (4*c^2 -
8*c*d + 4*d^2 - 3*(c^2 + 4*c*d - 5*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*
sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f
*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 -
2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e)), -1/16*(sqrt(1/2)*((3*a*c^2 +
14*a*c*d + 43*a*d^2)*cos(f*x + e)^3 - 12*a*c^2 - 56*a*c*d - 172*a*d^2 + 3*
(3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x + e)^2 - 2*(3*a*c^2 + 14*a*c*d + 43
*a*d^2)*cos(f*x + e) - (12*a*c^2 + 56*a*c*d + 172*a*d^2 - (3*a*c^2 + 14*a*c
*d + 43*a*d^2)*cos(f*x + e)^2 + 2*(3*a*c^2 + 14*a*c*d + 43*a*d^2)*cos(f*x +
e))*sin(f*x + e))*sqrt(-(c - d)/a)*arctan(-2*sqrt(1/2)*sqrt(a*sin(f*x + e)
+ a)*sqrt(d*sin(f*x + e) + c)*sqrt(-(c - d)/a)/((c - d)*cos(f*x + e))) - 8
*(a*d^2*cos(f*x + e)^3 + 3*a*d^2*cos(f*x + e)^2 - 2*a*d^2*cos(f*x + e) - 4*
a*d^2 + (a*d^2*cos(f*x + e)^2 - 2*a*d^2*cos(f*x + e) - 4*a*d^2)*sin(f*x + e
))*sqrt(d/a)*arctan(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*
d - d^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)*sq
rt(d/a)/(2*d^3*cos(f*x + e)^3 - (3*c*d^2 - d^3)*cos(f*x + e)*sin(f*x + e) -
(c^2*d - c*d^2 + 2*d^3)*cos(f*x + e))) - (3*(c^2 + 4*c*d - 5*d^2)*cos(f*x
+ e)^2 + 4*c^2 - 8*c*d + 4*d^2 + (7*c^2 + 4*c*d - 11*d^2)*cos(f*x + e) - (4
*c^2 - 8*c*d + 4*d^2 - 3*(c^2 + 4*c*d - 5*d^2)*cos(f*x + e))*sin(f*x + e))*
sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/(a^3*f*cos(f*x + e)^3 +
3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x +
e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a)^(5/2), x)
```

maple [B] time = 0.62, size = 10738, normalized size = 41.30

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^(5/2)/(a + a*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.601 $\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=184

$$-\frac{3(c+d)^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f\sqrt{c-d}}-\frac{(3c+7d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16af(a\sin(e+fx)+a)^{3/2}}-\frac{(c-d)\cos(e+fx)\sqrt{}}{4f(a\sin(e+}$$

```
[Out] -3/32*(c+d)^2*arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1/2)/(a+a*sin(f
*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))/a^(5/2)/f*2^(1/2)/(c-d)^(1/2)-1/4*(c-d
)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(5/2)-1/16*(3*c+7*d)
*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+e))^(3/2)
```

Rubi [A] time = 0.54, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2765, 2978, 12, 2782, 208}

$$-\frac{3(c+d)^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f\sqrt{c-d}}-\frac{(3c+7d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16af(a\sin(e+fx)+a)^{3/2}}-\frac{(c-d)\cos(e+fx)\sqrt{}}{4f(a\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-3*(c + d)^2*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a +
a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*Sqrt[c - d]
*f) - ((c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*f*(a + a*Sin[e + f
*x])^(5/2)) - ((3*c + 7*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(16*a*f*(
a + a*Sin[e + f*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
```

```
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\int \frac{-\frac{1}{2}a\left(3c^2+6cd-d^2\right)-ad(c+3d)\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx}{4a^2}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3c+7d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16af(a+a\sin(e+fx))^{3/2}}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3c+7d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16af(a+a\sin(e+fx))^{3/2}}$$

$$= -\frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3c+7d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16af(a+a\sin(e+fx))^{3/2}}$$

$$= -\frac{3(c+d)^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}\sqrt{c-d}f} - \frac{(c-d)\cos(e+fx)\sqrt{c+d\sin(}{4f(a+a\sin(e+fx))^{5/2}}$$

Mathematica [B] time = 7.22, size = 396, normalized size = 2.15

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4\left(\frac{3(c+d)^2\left(\log\left(\tan\left(\frac{1}{2}(e+fx)\right)+1\right)-\log\left((d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c\right.\right.}{\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2\tan\left(\frac{1}{2}(e+fx)\right)+2}-\frac{\frac{\sqrt{c-d}\left(\frac{1}{\cos(e+fx)+1}\right)^{3/2}(c\sin(e+fx)+d\cos(e+fx)+d)}{\sqrt{c+d\sin(e+fx)}}-\frac{1}{2}(c-d)\sec^2\left(\frac{1}{2}(e+fx)\right)}{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c-d}}\right.}{32f(a(\sin(e+fx)+1))^{5/2}\sqrt{c+d\sin(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*((-2*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(c + d*Sin[e + f*x])*(7*c + 3*d + (3*c + 7*d)*Sin[e + f*x]))/(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])^3 + (3*(c + d)^2*(Log[1 + Tan[(e + f*x)/2]
] - Log[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[
e + f*x]] + (-c + d)*Tan[(e + f*x)/2]]))/(Sec[(e + f*x)/2]^2/(2 + 2*Tan[(e
+ f*x)/2]) - (-1/2*((c - d)*Sec[(e + f*x)/2]^2) + (Sqrt[c - d]*((1 + Cos[e
+ f*x])^(-1))^(3/2)*(d + d*Cos[e + f*x] + c*Sin[e + f*x]))/Sqrt[c + d*Sin[e
 + f*x]])/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*S
```

```
in[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]]))))/(32*f*(a*(1 + Sin[e + f*x]))^(
5/2)*Sqrt[c + d*Sin[e + f*x]])
```

fricas [B] time = 0.83, size = 1304, normalized size = 7.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="frica
s")

[Out] [1/128*(3*((c^2 + 2*c*d + d^2)*cos(f*x + e)^3 + 3*(c^2 + 2*c*d + d^2)*cos(f
*x + e)^2 - 4*c^2 - 8*c*d - 4*d^2 - 2*(c^2 + 2*c*d + d^2)*cos(f*x + e) + ((
c^2 + 2*c*d + d^2)*cos(f*x + e)^2 - 4*c^2 - 8*c*d - 4*d^2 - 2*(c^2 + 2*c*d
+ d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(2*a*c - 2*a*d)*log(((a*c^2 - 14*a*c
*d + 17*a*d^2)*cos(f*x + e)^3 - 4*a*c^2 - 8*a*c*d - 4*a*d^2 - (13*a*c^2 - 2
2*a*c*d - 3*a*d^2)*cos(f*x + e)^2 - 4*((c - 3*d)*cos(f*x + e)^2 - (3*c - d)
*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) + 4*c - 4*d)*sin(f*x + e) - 4*c + 4
*d)*sqrt(2*a*c - 2*a*d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c) -
 2*(9*a*c^2 - 14*a*c*d + 9*a*d^2)*cos(f*x + e) - (4*a*c^2 + 8*a*c*d + 4*a*d
^2 - (a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^2 - 2*(7*a*c^2 - 18*a*c*d +
 7*a*d^2)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 +
(cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x + e) - 2*cos(f*x + e) - 4)) +
 8*((3*c^2 + 4*c*d - 7*d^2)*cos(f*x + e)^2 + 4*c^2 - 8*c*d + 4*d^2 + (7*c^2
 - 4*c*d - 3*d^2)*cos(f*x + e) - (4*c^2 - 8*c*d + 4*d^2 - (3*c^2 + 4*c*d -
7*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x
+ e) + c))/((a^3*c - a^3*d)*f*cos(f*x + e)^3 + 3*(a^3*c - a^3*d)*f*cos(f*x
+ e)^2 - 2*(a^3*c - a^3*d)*f*cos(f*x + e) - 4*(a^3*c - a^3*d)*f + ((a^3*c -
 a^3*d)*f*cos(f*x + e)^2 - 2*(a^3*c - a^3*d)*f*cos(f*x + e) - 4*(a^3*c - a^
3*d)*f)*sin(f*x + e)), -1/64*(3*((c^2 + 2*c*d + d^2)*cos(f*x + e)^3 + 3*(c^
2 + 2*c*d + d^2)*cos(f*x + e)^2 - 4*c^2 - 8*c*d - 4*d^2 - 2*(c^2 + 2*c*d +
d^2)*cos(f*x + e) + ((c^2 + 2*c*d + d^2)*cos(f*x + e)^2 - 4*c^2 - 8*c*d - 4
*d^2 - 2*(c^2 + 2*c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(-2*a*c + 2*a*
d)*arctan(1/4*sqrt(-2*a*c + 2*a*d)*sqrt(a*sin(f*x + e) + a)*((c - 3*d)*sin(
f*x + e) - 3*c + d)*sqrt(d*sin(f*x + e) + c)/((a*c*d - a*d^2)*cos(f*x + e)*
sin(f*x + e) + (a*c^2 - a*c*d)*cos(f*x + e))) - 4*((3*c^2 + 4*c*d - 7*d^2)*
cos(f*x + e)^2 + 4*c^2 - 8*c*d + 4*d^2 + (7*c^2 - 4*c*d - 3*d^2)*cos(f*x +
e) - (4*c^2 - 8*c*d + 4*d^2 - (3*c^2 + 4*c*d - 7*d^2)*cos(f*x + e))*sin(f*x
 + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((a^3*c - a^3*d)*
f*cos(f*x + e)^3 + 3*(a^3*c - a^3*d)*f*cos(f*x + e)^2 - 2*(a^3*c - a^3*d)*f
*cos(f*x + e) - 4*(a^3*c - a^3*d)*f + ((a^3*c - a^3*d)*f*cos(f*x + e)^2 - 2
*(a^3*c - a^3*d)*f*cos(f*x + e) - 4*(a^3*c - a^3*d)*f)*sin(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a)^(5/2), x)
```

maple [B] time = 0.36, size = 3050, normalized size = 16.58

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/64/f*(12*cos(f*x+e)^3*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c^2-12*cos(
f*x+e)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c^2-28*sin(f*x+e)*cos(f*x+e)
*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c^2+12*sin(f*x+e)*cos(f*x+e)*((c+d
*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*d^2-12*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(
1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e
)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^
(1/2)*c^2-12*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e)
)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d
*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*d^2-28*cos(f*x+e)^3*((c
+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*d^2+3*sin(f*x+e)*cos(f*x+e)^2*(2*c-2*d
)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/
2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-c
os(f*x+e)+sin(f*x+e)))*2^(1/2)*d^2+3*sin(f*x+e)*cos(f*x+e)^2*(2*c-2*d)^(1/2
)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin
(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x
+e)+sin(f*x+e)))*2^(1/2)*c^2+28*cos(f*x+e)*((c+d*sin(f*x+e))/(cos(f*x+e)+1)
)^(1/2)*d^2+9*cos(f*x+e)^2*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((
c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+
c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c^2+9*cos
(f*x+e)^2*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(
cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*co
s(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*d^2-12*sin(f*x+e)*(2*c-2*d
)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/
2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-c
```

os(f*x+e)+sin(f*x+e)))*2^(1/2)*c^2-12*sin(f*x+e)*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*d^2+6*cos(f*x+e)*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c^2+6*cos(f*x+e)*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*d^2+16*sin(f*x+e)*cos(f*x+e)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c*d+16*cos(f*x+e)^3*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c*d-16*cos(f*x+e)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c*d-6*cos(f*x+e)^3*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c*d+6*sin(f*x+e)*cos(f*x+e)*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c^2+6*sin(f*x+e)*cos(f*x+e)*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*d^2+18*cos(f*x+e)^2*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c*d-24*sin(f*x+e)*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c*d+12*cos(f*x+e)*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c*d+6*sin(f*x+e)*cos(f*x+e)^2*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c*d+12*sin(f*x+e)*cos(f*x+e)*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c*d-24*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c*d-3*cos(f*x+e)^3*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c^2-3*cos(f*x+e)^3*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*d^2)*(c+d*sin(f*x+e))^(1/2)/sin(f*x+e)/(a*(1+sin(f*x+e)))^(5/2)/((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)/(c-d)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c+d\,\sin\left(e+f\,x\right)\right)^{3/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^(3/2)/(a + a*sin(e + f*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((c + d*sin(e + f*x))**(3/2)/(a*(sin(e + f*x) + 1))**(5/2), x)
```

3.602 $\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=191

$$-\frac{(3c-5d)(c+d)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{c-d}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}f(c-d)^{3/2}}-\frac{(3c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16af(c-d)(a\sin(e+fx)+a)^{3/2}}-\frac{\cos(e+fx)\sqrt{}}{4f(a\sin(}$$

```
[Out] -1/32*(3*c-5*d)*(c+d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1/2)/(a
+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))/a^(5/2)/(c-d)^(3/2)/f*2^(1/2)-
1/4*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(5/2)-1/16*(3*c-d)
*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/a/(c-d)/f/(a+a*sin(f*x+e))^(3/2)
```

Rubi [A] time = 0.49, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2764, 2978, 12, 2782, 208}

$$-\frac{(3c-5d)(c+d)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{c-d}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}f(c-d)^{3/2}}-\frac{(3c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16af(c-d)(a\sin(e+fx)+a)^{3/2}}-\frac{\cos(e+fx)\sqrt{}}{4f(a\sin(}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -((3*c - 5*d)*(c + d)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt[2]*S
qrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*(c
- d)^(3/2)*f) - (Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*f*(a + a*Sin[e +
 f*x])^(5/2)) - ((3*c - d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(16*a*(c
- d)*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2764

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m
```

```
*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*
(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && L
tQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c,
 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(a+a\sin(e+fx))^{5/2}} + \frac{\int \frac{\frac{1}{2}a(3c+d)+ad\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx}{4a^2}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16a(c-d)f(a+a\sin(e+fx))^{3/2}} - \frac{\int -\cdots}{\cdots}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16a(c-d)f(a+a\sin(e+fx))^{3/2}} + \frac{((3c \cdots}{\cdots}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3c-d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16a(c-d)f(a+a\sin(e+fx))^{3/2}} - \frac{((3c \cdots}{\cdots}$$

$$= -\frac{(3c-5d)(c+d)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}(c-d)^{3/2}f} - \frac{\cos(e+fx)\sqrt{c+d\sin \cdots}}{4f(a+a\sin(e+f \cdots}$$

Mathematica [B] time = 7.60, size = 412, normalized size = 2.16

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4\left(\frac{(3c^2-2cd-5d^2)\left(\log\left(\tan\left(\frac{1}{2}(e+fx)\right)+1\right)-\log\left((d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin \cdots}\right.\right.}{\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2\tan\left(\frac{1}{2}(e+fx)\right)+2}-\frac{\frac{\sqrt{c-d}\left(\frac{1}{\cos(e+fx)+1}\right)^{3/2}(c\sin(e+fx)+d\cos(e+fx)+d)}{\sqrt{c+d\sin(e+fx)}}-\frac{1}{2}(c-d)\sec^2\left(\frac{1}{2}(e+fx \cdots\right.}{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c-d}} \cdots\right)}{32f(c-d)(a(\sin(e+fx)+1))^{5/2}\sqrt{c+d\sin(e \cdots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*((-2*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(7*c - 5*d + (3*c - d)*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^3 + ((3*c^2 - 2*c*d - 5*d^2)*(Log[1 + Tan[(e
 + f*x)/2]] - Log[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[
c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]]))/(Sec[(e + f*x)/2]^2/(2 +
 2*Tan[(e + f*x)/2]) - (-1/2*((c - d)*Sec[(e + f*x)/2]^2) + (Sqrt[c - d]*((
1 + Cos[e + f*x])^(-1))^(3/2)*(d + d*Cos[e + f*x] + c*Sin[e + f*x]))/Sqrt[c
 + d*Sin[e + f*x]])/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sq
```

```
rt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]]))))/(32*(c - d)*f*(a*(1
+ Sin[e + f*x]))^(5/2)*Sqrt[c + d*Sin[e + f*x]])
```

fricas [B] time = 0.90, size = 1474, normalized size = 7.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] [1/128*(((3*c^2 - 2*c*d - 5*d^2)*cos(f*x + e)^3 + 3*(3*c^2 - 2*c*d - 5*d^2)
*cos(f*x + e)^2 - 12*c^2 + 8*c*d + 20*d^2 - 2*(3*c^2 - 2*c*d - 5*d^2)*cos(f
*x + e) + ((3*c^2 - 2*c*d - 5*d^2)*cos(f*x + e)^2 - 12*c^2 + 8*c*d + 20*d^2
 - 2*(3*c^2 - 2*c*d - 5*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(2*a*c - 2*a*d
)*log(((a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^3 - 4*a*c^2 - 8*a*c*d - 4
*a*d^2 - (13*a*c^2 - 22*a*c*d - 3*a*d^2)*cos(f*x + e)^2 - 4*((c - 3*d)*cos(
f*x + e)^2 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) + 4*c - 4*d)*
sin(f*x + e) - 4*c + 4*d)*sqrt(2*a*c - 2*a*d)*sqrt(a*sin(f*x + e) + a)*sqrt
(d*sin(f*x + e) + c) - 2*(9*a*c^2 - 14*a*c*d + 9*a*d^2)*cos(f*x + e) - (4*a
*c^2 + 8*a*c*d + 4*a*d^2 - (a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^2 - 2
*(7*a*c^2 - 18*a*c*d + 7*a*d^2)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^3
 + 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x + e) -
2*cos(f*x + e) - 4)) + 8*((3*c^2 - 4*c*d + d^2)*cos(f*x + e)^2 + 4*c^2 - 8*
c*d + 4*d^2 + (7*c^2 - 12*c*d + 5*d^2)*cos(f*x + e) - (4*c^2 - 8*c*d + 4*d^
2 - (3*c^2 - 4*c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) +
 a)*sqrt(d*sin(f*x + e) + c))/((a^3*c^2 - 2*a^3*c*d + a^3*d^2)*f*cos(f*x +
e)^3 + 3*(a^3*c^2 - 2*a^3*c*d + a^3*d^2)*f*cos(f*x + e)^2 - 2*(a^3*c^2 - 2*
a^3*c*d + a^3*d^2)*f*cos(f*x + e) - 4*(a^3*c^2 - 2*a^3*c*d + a^3*d^2)*f + (
(a^3*c^2 - 2*a^3*c*d + a^3*d^2)*f*cos(f*x + e)^2 - 2*(a^3*c^2 - 2*a^3*c*d +
 a^3*d^2)*f*cos(f*x + e) - 4*(a^3*c^2 - 2*a^3*c*d + a^3*d^2)*f)*sin(f*x + e
)), -1/64*(((3*c^2 - 2*c*d - 5*d^2)*cos(f*x + e)^3 + 3*(3*c^2 - 2*c*d - 5*d
^2)*cos(f*x + e)^2 - 12*c^2 + 8*c*d + 20*d^2 - 2*(3*c^2 - 2*c*d - 5*d^2)*co
s(f*x + e) + ((3*c^2 - 2*c*d - 5*d^2)*cos(f*x + e)^2 - 12*c^2 + 8*c*d + 20*
d^2 - 2*(3*c^2 - 2*c*d - 5*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(-2*a*c + 2
*a*d)*arctan(1/4*sqrt(-2*a*c + 2*a*d)*sqrt(a*sin(f*x + e) + a)*((c - 3*d)*s
in(f*x + e) - 3*c + d)*sqrt(d*sin(f*x + e) + c)/((a*c*d - a*d^2)*cos(f*x +
e)*sin(f*x + e) + (a*c^2 - a*c*d)*cos(f*x + e))) - 4*((3*c^2 - 4*c*d + d^2)
*cos(f*x + e)^2 + 4*c^2 - 8*c*d + 4*d^2 + (7*c^2 - 12*c*d + 5*d^2)*cos(f*x
+ e) - (4*c^2 - 8*c*d + 4*d^2 - (3*c^2 - 4*c*d + d^2)*cos(f*x + e))*sin(f*x
 + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((a^3*c^2 - 2*a^3
*c*d + a^3*d^2)*f*cos(f*x + e)^3 + 3*(a^3*c^2 - 2*a^3*c*d + a^3*d^2)*f*cos(
f*x + e)^2 - 2*(a^3*c^2 - 2*a^3*c*d + a^3*d^2)*f*cos(f*x + e) - 4*(a^3*c^2
- 2*a^3*c*d + a^3*d^2)*f + ((a^3*c^2 - 2*a^3*c*d + a^3*d^2)*f*cos(f*x + e)^
```

```
2 - 2*(a^3*c^2 - 2*a^3*c*d + a^3*d^2)*f*cos(f*x + e) - 4*(a^3*c^2 - 2*a^3*c
*d + a^3*d^2)*f)*sin(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\sin\left(fx+e\right)+c}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(5/2), x)
```

maple [B] time = 0.36, size = 3050, normalized size = 15.97

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/64/f*(12*cos(f*x+e)^3*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c^2-12*cos(
f*x+e)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c^2-28*sin(f*x+e)*cos(f*x+e)
*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c^2-20*sin(f*x+e)*cos(f*x+e)*((c+d
*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*d^2-12*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(
1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e
)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^
(1/2)*c^2+20*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e)
)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d
*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*d^2+4*cos(f*x+e)^3*((c+
d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*d^2-5*sin(f*x+e)*cos(f*x+e)^2*(2*c-2*d)
^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2
)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-co
s(f*x+e)+sin(f*x+e)))*2^(1/2)*d^2+3*sin(f*x+e)*cos(f*x+e)^2*(2*c-2*d)^(1/2)
*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(
f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+
e)+sin(f*x+e)))*2^(1/2)*c^2-4*cos(f*x+e)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^
(1/2)*d^2+9*cos(f*x+e)^2*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+
d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*
cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c^2-15*cos(
f*x+e)^2*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(c
os(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos
(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*d^2-12*sin(f*x+e)*(2*c-2*d)
```

^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2
)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-co
s(f*x+e)+sin(f*x+e)))*2^(1/2)*c^2+20*sin(f*x+e)*(2*c-2*d)^(1/2)*ln(2*((2*c-
2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin
(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e
)))*2^(1/2)*d^2+6*cos(f*x+e)*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*
((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e
)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c^2-10*
cos(f*x+e)*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/
(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*c
os(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*d^2+48*sin(f*x+e)*cos(f*x
+e)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c*d-16*cos(f*x+e)^3*((c+d*sin(f
*x+e))/(cos(f*x+e)+1))^(1/2)*c*d+16*cos(f*x+e)*((c+d*sin(f*x+e))/(cos(f*x+e
)+1))^(1/2)*c*d+2*cos(f*x+e)^3*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2
)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x
+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c*d+6
*sin(f*x+e)*cos(f*x+e)*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*
sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*co
s(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c^2-10*sin(f*
x+e)*cos(f*x+e)*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x
+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e
)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*d^2-6*cos(f*x+e)^2*(
2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+
1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+
d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c*d+8*sin(f*x+e)*(2*c-2*d)^(1/2)*ln(2
*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e
)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+si
n(f*x+e)))*2^(1/2)*c*d-4*cos(f*x+e)*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2
^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*si
n(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*
c*d-2*sin(f*x+e)*cos(f*x+e)^2*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)
*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+
e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c*d-4*
sin(f*x+e)*cos(f*x+e)*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*s
in(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos
(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c*d+8*(2*c-2*d
)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/
2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-c
os(f*x+e)+sin(f*x+e)))*2^(1/2)*c*d-3*cos(f*x+e)^3*(2*c-2*d)^(1/2)*ln(2*((2*
c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*s
in(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x
+e)))*2^(1/2)*c^2+5*cos(f*x+e)^3*(2*c-2*d)^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1
/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f
*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*d^2
)*(c+d*sin(f*x+e))^(1/2)/sin(f*x+e)/(a*(1+sin(f*x+e)))^(5/2)/((c+d*sin(f*x+

```
e))/(cos(f*x+e)+1))^(1/2)/(c-d)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d \sin\left(fx+e\right)+c}}{\left(a \sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c+d\,\sin\left(e+f\,x\right)}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^(1/2)/(a + a*sin(e + f*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+d\sin\left(e+fx\right)}}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(c + d*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(5/2), x)
```

3.603 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=201

$$-\frac{\left(3c^2-10cd+19d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f(c-d)^{5/2}}-\frac{3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16af(c-d)^2(a\sin(e+fx)+a)^{3/2}}-\frac{\cos(e+}{4f(c-d}$$

```
[Out] -1/32*(3*c^2-10*c*d+19*d^2)*arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1
/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))/a^(5/2)/(c-d)^(5/2)/f*2^
(1/2)-1/4*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(c-d)/f/(a+a*sin(f*x+e))^(5/2)-
3/16*(c-3*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/a/(c-d)^2/f/(a+a*sin(f*x+e))
^(3/2)
```

Rubi [A] time = 0.49, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2766, 2978, 12, 2782, 208}

$$-\frac{\left(3c^2-10cd+19d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f(c-d)^{5/2}}-\frac{3(c-3d)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{16af(c-d)^2(a\sin(e+fx)+a)^{3/2}}-\frac{\cos(e+}{4f(c-d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] -((3*c^2 - 10*c*d + 19*d^2)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqr
t[2]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/
2)*(c - d)^(5/2)*f) - (Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*(c - d)*f*
(a + a*Sin[e + f*x])^(5/2)) - (3*(c - 3*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e +
f*x]])/(16*a*(c - d)^2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{\int \frac{-\frac{1}{2}a(3c-7d)-ad\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx}}}{4a^2(c-d)}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{3(c-3d)\cos(e+fx)\sqrt{c+d}}{16a(c-d)^2f(a+a\sin(e}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{3(c-3d)\cos(e+fx)\sqrt{c+d}}{16a(c-d)^2f(a+a\sin(e}$$

$$= -\frac{\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{3(c-3d)\cos(e+fx)\sqrt{c+d}}{16a(c-d)^2f(a+a\sin(e}$$

$$= -\frac{(3c^2-10cd+19d^2)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}(c-d)^{5/2}f} -$$

Mathematica [B] time = 7.13, size = 411, normalized size = 2.04

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4\left(\frac{(3c^2-10cd+19d^2)\left(\log\left(\tan\left(\frac{1}{2}(e+fx)\right)+1\right)-\log\left((d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin}\right.\right.}{\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2\tan\left(\frac{1}{2}(e+fx)\right)+2}-\frac{\frac{\sqrt{c-d}\left(\frac{1}{\cos(e+fx)+1}\right)^{3/2}(c\sin(e+fx)+d\cos(e+fx)+d)}{\sqrt{c+d\sin(e+fx)}}-\frac{1}{2}(c-d)\sec^2\left(\frac{1}{2}(e+fx)\right)}{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c-d}}\right.}{32f(c-d)^2(a(\sin(e+fx)+1))^{5/2}\sqrt{c+d\sin(e}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*((-2*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(7*c - 13*d + 3*(c - 3*d)*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + ((3*c^2 - 10*c*d + 19*d^2)*(Log[1 + T
an[(e + f*x)/2]] - Log[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*
Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]]))/(Sec[(e + f*x)/2]^2
/(2 + 2*Tan[(e + f*x)/2]) - (-1/2*((c - d)*Sec[(e + f*x)/2]^2) + (Sqrt[c -
d]*((1 + Cos[e + f*x])^(-1))^(3/2)*(d + d*Cos[e + f*x] + c*Sin[e + f*x]))/S
qrt[c + d*Sin[e + f*x]])/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1
```

```
)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]))))/(32*(c - d)^2*f
*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c + d*Sin[e + f*x]])
```

fricas [B] time = 1.14, size = 1644, normalized size = 8.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] [1/128*(((3*c^2 - 10*c*d + 19*d^2)*cos(f*x + e)^3 + 3*(3*c^2 - 10*c*d + 19*
d^2)*cos(f*x + e)^2 - 12*c^2 + 40*c*d - 76*d^2 - 2*(3*c^2 - 10*c*d + 19*d^2
)*cos(f*x + e) + ((3*c^2 - 10*c*d + 19*d^2)*cos(f*x + e)^2 - 12*c^2 + 40*c*
d - 76*d^2 - 2*(3*c^2 - 10*c*d + 19*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(2
*a*c - 2*a*d)*log(((a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^3 - 4*a*c^2 -
 8*a*c*d - 4*a*d^2 - (13*a*c^2 - 22*a*c*d - 3*a*d^2)*cos(f*x + e)^2 - 4*((c
 - 3*d)*cos(f*x + e)^2 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) +
 4*c - 4*d)*sin(f*x + e) - 4*c + 4*d)*sqrt(2*a*c - 2*a*d)*sqrt(a*sin(f*x +
e) + a)*sqrt(d*sin(f*x + e) + c) - 2*(9*a*c^2 - 14*a*c*d + 9*a*d^2)*cos(f*x
 + e) - (4*a*c^2 + 8*a*c*d + 4*a*d^2 - (a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*
x + e)^2 - 2*(7*a*c^2 - 18*a*c*d + 7*a*d^2)*cos(f*x + e))*sin(f*x + e))/(co
s(f*x + e)^3 + 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin
(f*x + e) - 2*cos(f*x + e) - 4)) + 8*(3*(c^2 - 4*c*d + 3*d^2)*cos(f*x + e)^
2 + 4*c^2 - 8*c*d + 4*d^2 + (7*c^2 - 20*c*d + 13*d^2)*cos(f*x + e) - (4*c^2
 - 8*c*d + 4*d^2 - 3*(c^2 - 4*c*d + 3*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt
(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((a^3*c^3 - 3*a^3*c^2*d + 3*
a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^3 + 3*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*
d^2 - a^3*d^3)*f*cos(f*x + e)^2 - 2*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 -
a^3*d^3)*f*cos(f*x + e) - 4*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)
*f + ((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^2 - 2*
(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e) - 4*(a^3*c^3
 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f)*sin(f*x + e)), -1/64*(((3*c^2 -
10*c*d + 19*d^2)*cos(f*x + e)^3 + 3*(3*c^2 - 10*c*d + 19*d^2)*cos(f*x + e)^
2 - 12*c^2 + 40*c*d - 76*d^2 - 2*(3*c^2 - 10*c*d + 19*d^2)*cos(f*x + e) + (
(3*c^2 - 10*c*d + 19*d^2)*cos(f*x + e)^2 - 12*c^2 + 40*c*d - 76*d^2 - 2*(3*
c^2 - 10*c*d + 19*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(-2*a*c + 2*a*d)*arc
tan(1/4*sqrt(-2*a*c + 2*a*d)*sqrt(a*sin(f*x + e) + a)*((c - 3*d)*sin(f*x +
e) - 3*c + d)*sqrt(d*sin(f*x + e) + c)/((a*c*d - a*d^2)*cos(f*x + e)*sin(f*
x + e) + (a*c^2 - a*c*d)*cos(f*x + e))) - 4*(3*(c^2 - 4*c*d + 3*d^2)*cos(f*
x + e)^2 + 4*c^2 - 8*c*d + 4*d^2 + (7*c^2 - 20*c*d + 13*d^2)*cos(f*x + e) -
 (4*c^2 - 8*c*d + 4*d^2 - 3*(c^2 - 4*c*d + 3*d^2)*cos(f*x + e))*sin(f*x + e
))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((a^3*c^3 - 3*a^3*c^2
*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^3 + 3*(a^3*c^3 - 3*a^3*c^2*d + 3
*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^2 - 2*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c
```

```
*d^2 - a^3*d^3)*f*cos(f*x + e) - 4*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a
^3*d^3)*f + ((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)
^2 - 2*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e) - 4*(
a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f)*sin(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a \sin\left(fx+e\right)+a\right)^{\frac{5}{2}} \sqrt{d \sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*sqrt(d*sin(f*x + e) + c)), x)
```

maple [B] time = 0.36, size = 2805, normalized size = 13.96

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/32/f*(-12*sin(f*x+e)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos
(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f
*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c^2-76*sin(f*x+e)*ln(2*((2*c-
2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin
(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e
)))*2^(1/2)*d^2+6*cos(f*x+e)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e)
)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d
*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c^2+38*cos(f*x+e)*ln(2*
((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)
+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin
(f*x+e)))*2^(1/2)*d^2+40*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(c
os(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos
(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c*d+6*cos(f*x+e)^3*(2*c-2*d
)^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c-18*cos(f*x+e)^3*(2*c-2*d)
^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*d-10*sin(f*x+e)*cos(f*x+e)^2
*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(
f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+
e)+sin(f*x+e)))*2^(1/2)*c*d-20*sin(f*x+e)*cos(f*x+e)*ln(2*((2*c-2*d)^(1/2)*
2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*s
in(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)
```

```
*c*d-6*cos(f*x+e)*(2*c-2*d)^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c
+18*cos(f*x+e)*(2*c-2*d)^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*d-3*
cos(f*x+e)^3*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1)
)^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)
/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c^2-19*cos(f*x+e)^3*ln(2*((2*c-2*d)^(1/
2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-
d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1
/2)*d^2+9*cos(f*x+e)^2*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos
(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f
*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c^2+57*cos(f*x+e)^2*ln(2*((2*
c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*s
in(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x
+e)))*2^(1/2)*d^2-12*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f
*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x
+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c^2-76*ln(2*((2*c-2*d)^(1/2)*2^
(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin
(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*d
^2-20*cos(f*x+e)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e
)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-
c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c*d+3*sin(f*x+e)*cos(f*x+e)^2*ln(2*
((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)
+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin
(f*x+e)))*2^(1/2)*c^2+19*sin(f*x+e)*cos(f*x+e)^2*ln(2*((2*c-2*d)^(1/2)*2^(1
/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f
*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*d^2
+10*cos(f*x+e)^3*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e
)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-
c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c*d+6*sin(f*x+e)*cos(f*x+e)*ln(2*((
2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c
*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f
*x+e)))*2^(1/2)*c^2+38*sin(f*x+e)*cos(f*x+e)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*
((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e
)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*d^2-30*
cos(f*x+e)^2*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1)
)^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)
/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*c*d-14*sin(f*x+e)*cos(f*x+e)*(2*c-2*d)^
(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c+26*sin(f*x+e)*cos(f*x+e)*(2
*c-2*d)^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*d+40*sin(f*x+e)*ln(2*
((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)
+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin
(f*x+e)))*2^(1/2)*c*d)*(c+d*sin(f*x+e))^(1/2)/sin(f*x+e)/(a*(1+sin(f*x+e)))
^(5/2)/((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)/(2*c-2*d)^(1/2)/(c-d)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*sqrt(d*sin(f*x + e) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/((a*(sin(e + f*x) + 1))**(5/2)*sqrt(c + d*sin(e + f*x))), x)
```

3.604 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=270

$$-\frac{3\left(c^2-6cd+25d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f(c-d)^{7/2}}-\frac{d(c-7d)(3c+7d)\cos(e+fx)}{16a^2f(c-d)^3(c+d)\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin}}$$

```
[Out] -3/32*(c^2-6*c*d+25*d^2)*arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1/2)
/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))/a^(5/2)/(c-d)^(7/2)/f*2^(1/
2)-1/4*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2)-1/1
6*(3*c-13*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))
^(1/2)-1/16*(c-7*d)*d*(3*c+7*d)*cos(f*x+e)/a^2/(c-d)^3/(c+d)/f/(a+a*sin(f*x
+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.86, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2766, 2978, 2984, 12, 2782, 208}

$$-\frac{3\left(c^2-6cd+25d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f(c-d)^{7/2}}-\frac{d(c-7d)(3c+7d)\cos(e+fx)}{16a^2f(c-d)^3(c+d)\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (-3*(c^2 - 6*c*d + 25*d^2)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sqrt
[2]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2
)*(c - d)^(7/2)*f) - Cos[e + f*x]/(4*(c - d)*f*(a + a*Sin[e + f*x])^(5/2)*S
qrt[c + d*Sin[e + f*x]]) - ((3*c - 13*d)*Cos[e + f*x])/(16*a*(c - d)^2*f*(a
 + a*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]) - ((c - 7*d)*d*(3*c + 7*
d)*Cos[e + f*x])/(16*a^2*(c - d)^3*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[
c + d*Sin[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}} - \frac{\int \frac{}{(a+a}}{}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}} - \frac{}{16a(c}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}} - \frac{}{16a(c}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}} - \frac{}{16a(c}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}} - \frac{}{16a(c}$$

$$= -\frac{3\left(c^2-6cd+25d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}(c-d)^{7/2}f}$$

Mathematica [A] time = 8.96, size = 462, normalized size = 1.71

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4\left(\frac{3\left(c^2-6cd+25d^2\right)\left(\log\left(\tan\left(\frac{1}{2}(e+fx)\right)+1\right)-\log\left((d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin}\right.\right.}{\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2\tan\left(\frac{1}{2}(e+fx)\right)+2}-\frac{\frac{\sqrt{c-d}\left(\frac{1}{\cos(e+fx)+1}\right)^{3/2}(c\sin(e+fx)+d\cos(e+fx)+d)}{\sqrt{c+d\sin(e+fx)}}-\frac{1}{2}(c-d)\sec^2\left(\frac{1}{2}(e+fx\right.}{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2\sqrt{c-d}\sqrt{\frac{1}{\cos(e+fx)+1}}\sqrt{c+d\sin(e+fx)}+c-d}}\right.}{32f(c-d)^3(a(\sin(e+}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(((Cos[(e + f*x)/2] - Sin[(e + f*x
)/2])*(-14*c^3 + 25*c^2*d + 56*c*d^2 + 113*d^3 + d*(3*c^2 - 14*c*d - 49*d^2
)*Cos[2*(e + f*x)] + (-6*c^3 + 14*c^2*d + 62*c*d^2 + 170*d^3)*Sin[e + f*x])
)/((c + d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3) + (3*(c^2 - 6*c*d + 25*
```

```
d^2)*(Log[1 + Tan[(e + f*x)/2]] - Log[c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e
 + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]]))/(Sec
[(e + f*x)/2]^2/(2 + 2*Tan[(e + f*x)/2]) - (-1/2*((c - d)*Sec[(e + f*x)/2]^
2) + (Sqrt[c - d]*((1 + Cos[e + f*x])^(-1))^(3/2)*(d + d*Cos[e + f*x] + c*S
in[e + f*x]))/Sqrt[c + d*Sin[e + f*x]])/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Co
s[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]))))/
(32*(c - d)^3*f*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c + d*Sin[e + f*x]])
```

fricas [B] time = 1.55, size = 2984, normalized size = 11.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] [-1/128*(3*((c^3*d - 5*c^2*d^2 + 19*c*d^3 + 25*d^4)*cos(f*x + e)^4 + 4*c^4
- 16*c^3*d + 56*c^2*d^2 + 176*c*d^3 + 100*d^4 - (c^4 - 3*c^3*d + 9*c^2*d^2
+ 63*c*d^3 + 50*d^4)*cos(f*x + e)^3 - (3*c^4 - 10*c^3*d + 32*c^2*d^2 + 170*
c*d^3 + 125*d^4)*cos(f*x + e)^2 + 2*(c^4 - 4*c^3*d + 14*c^2*d^2 + 44*c*d^3
+ 25*d^4)*cos(f*x + e) + (4*c^4 - 16*c^3*d + 56*c^2*d^2 + 176*c*d^3 + 100*d
^4 - (c^3*d - 5*c^2*d^2 + 19*c*d^3 + 25*d^4)*cos(f*x + e)^3 - (c^4 - 2*c^3*
d + 4*c^2*d^2 + 82*c*d^3 + 75*d^4)*cos(f*x + e)^2 + 2*(c^4 - 4*c^3*d + 14*c
^2*d^2 + 44*c*d^3 + 25*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(2*a*c - 2*a*d)
*log(((a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^3 - 4*a*c^2 - 8*a*c*d - 4*
a*d^2 - (13*a*c^2 - 22*a*c*d - 3*a*d^2)*cos(f*x + e)^2 + 4*((c - 3*d)*cos(f
*x + e)^2 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) + 4*c - 4*d)*s
in(f*x + e) - 4*c + 4*d)*sqrt(2*a*c - 2*a*d)*sqrt(a*sin(f*x + e) + a)*sqrt(
d*sin(f*x + e) + c) - 2*(9*a*c^2 - 14*a*c*d + 9*a*d^2)*cos(f*x + e) - (4*a*
c^2 + 8*a*c*d + 4*a*d^2 - (a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^2 - 2*
(7*a*c^2 - 18*a*c*d + 7*a*d^2)*cos(f*x + e))*sin(f*x + e))/(cos(f*x + e)^3
+ 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x + e) - 2
*cos(f*x + e) - 4)) + 8*(4*c^4 - 8*c^3*d + 8*c*d^3 - 4*d^4 - (3*c^3*d - 17*
c^2*d^2 - 35*c*d^3 + 49*d^4)*cos(f*x + e)^3 + (3*c^4 - 13*c^3*d - 7*c^2*d^2
 - 19*c*d^3 + 36*d^4)*cos(f*x + e)^2 + (7*c^4 - 18*c^3*d - 24*c^2*d^2 - 46*
c*d^3 + 81*d^4)*cos(f*x + e) - (4*c^4 - 8*c^3*d + 8*c*d^3 - 4*d^4 - (3*c^3*
d - 17*c^2*d^2 - 35*c*d^3 + 49*d^4)*cos(f*x + e)^2 - (3*c^4 - 10*c^3*d - 24
*c^2*d^2 - 54*c*d^3 + 85*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x +
e) + a)*sqrt(d*sin(f*x + e) + c))/((a^3*c^5*d - 3*a^3*c^4*d^2 + 2*a^3*c^3*d
^3 + 2*a^3*c^2*d^4 - 3*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e)^4 - (a^3*c^6 - a
^3*c^5*d - 4*a^3*c^4*d^2 + 6*a^3*c^3*d^3 + a^3*c^2*d^4 - 5*a^3*c*d^5 + 2*a^
3*d^6)*f*cos(f*x + e)^3 - (3*a^3*c^6 - 4*a^3*c^5*d - 9*a^3*c^4*d^2 + 16*a^3
*c^3*d^3 + a^3*c^2*d^4 - 12*a^3*c*d^5 + 5*a^3*d^6)*f*cos(f*x + e)^2 + 2*(a^
3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d
^5 + a^3*d^6)*f*cos(f*x + e) + 4*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a
```

```
^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f - ((a^3*c^5*d - 3*a^3*c
^4*d^2 + 2*a^3*c^3*d^3 + 2*a^3*c^2*d^4 - 3*a^3*c*d^5 + a^3*d^6)*f*cos(f*x +
 e)^3 + (a^3*c^6 - 7*a^3*c^4*d^2 + 8*a^3*c^3*d^3 + 3*a^3*c^2*d^4 - 8*a^3*c*
d^5 + 3*a^3*d^6)*f*cos(f*x + e)^2 - 2*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2
+ 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e) - 4*(
a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c
*d^5 + a^3*d^6)*f)*sin(f*x + e)), -1/64*(3*((c^3*d - 5*c^2*d^2 + 19*c*d^3 +
 25*d^4)*cos(f*x + e)^4 + 4*c^4 - 16*c^3*d + 56*c^2*d^2 + 176*c*d^3 + 100*d
^4 - (c^4 - 3*c^3*d + 9*c^2*d^2 + 63*c*d^3 + 50*d^4)*cos(f*x + e)^3 - (3*c^
4 - 10*c^3*d + 32*c^2*d^2 + 170*c*d^3 + 125*d^4)*cos(f*x + e)^2 + 2*(c^4 -
4*c^3*d + 14*c^2*d^2 + 44*c*d^3 + 25*d^4)*cos(f*x + e) + (4*c^4 - 16*c^3*d
+ 56*c^2*d^2 + 176*c*d^3 + 100*d^4 - (c^3*d - 5*c^2*d^2 + 19*c*d^3 + 25*d^4
)*cos(f*x + e)^3 - (c^4 - 2*c^3*d + 4*c^2*d^2 + 82*c*d^3 + 75*d^4)*cos(f*x
+ e)^2 + 2*(c^4 - 4*c^3*d + 14*c^2*d^2 + 44*c*d^3 + 25*d^4)*cos(f*x + e))*s
in(f*x + e))*sqrt(-2*a*c + 2*a*d)*arctan(1/4*sqrt(-2*a*c + 2*a*d)*sqrt(a*si
n(f*x + e) + a)*((c - 3*d)*sin(f*x + e) - 3*c + d)*sqrt(d*sin(f*x + e) + c)
/((a*c*d - a*d^2)*cos(f*x + e)*sin(f*x + e) + (a*c^2 - a*c*d)*cos(f*x + e))
) + 4*(4*c^4 - 8*c^3*d + 8*c*d^3 - 4*d^4 - (3*c^3*d - 17*c^2*d^2 - 35*c*d^3
 + 49*d^4)*cos(f*x + e)^3 + (3*c^4 - 13*c^3*d - 7*c^2*d^2 - 19*c*d^3 + 36*d
^4)*cos(f*x + e)^2 + (7*c^4 - 18*c^3*d - 24*c^2*d^2 - 46*c*d^3 + 81*d^4)*co
s(f*x + e) - (4*c^4 - 8*c^3*d + 8*c*d^3 - 4*d^4 - (3*c^3*d - 17*c^2*d^2 - 3
5*c*d^3 + 49*d^4)*cos(f*x + e)^2 - (3*c^4 - 10*c^3*d - 24*c^2*d^2 - 54*c*d^
3 + 85*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin
(f*x + e) + c))/((a^3*c^5*d - 3*a^3*c^4*d^2 + 2*a^3*c^3*d^3 + 2*a^3*c^2*d^4
 - 3*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e)^4 - (a^3*c^6 - a^3*c^5*d - 4*a^3*c
^4*d^2 + 6*a^3*c^3*d^3 + a^3*c^2*d^4 - 5*a^3*c*d^5 + 2*a^3*d^6)*f*cos(f*x +
 e)^3 - (3*a^3*c^6 - 4*a^3*c^5*d - 9*a^3*c^4*d^2 + 16*a^3*c^3*d^3 + a^3*c^2
*d^4 - 12*a^3*c*d^5 + 5*a^3*d^6)*f*cos(f*x + e)^2 + 2*(a^3*c^6 - 2*a^3*c^5*
d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f*co
s(f*x + e) + 4*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c
^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f - ((a^3*c^5*d - 3*a^3*c^4*d^2 + 2*a^3*c^3
*d^3 + 2*a^3*c^2*d^4 - 3*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e)^3 + (a^3*c^6 -
 7*a^3*c^4*d^2 + 8*a^3*c^3*d^3 + 3*a^3*c^2*d^4 - 8*a^3*c*d^5 + 3*a^3*d^6)*f
*cos(f*x + e)^2 - 2*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 -
a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e) - 4*(a^3*c^6 - 2*a^3*c^
5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f)
*sin(f*x + e))]
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(3/2)), x)
```

maple [B] time = 0.37, size = 4262, normalized size = 15.79

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/32/f*(-15*cos(f*x+e)^3*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*ln
(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x
+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+
sin(f*x+e)))*c^2*d-98*cos(f*x+e)^3*(2*c-2*d)^(1/2)*d^3+162*cos(f*x+e)*(2*c-
2*d)^(1/2)*d^3+22*cos(f*x+e)*(2*c-2*d)^(1/2)*c^2*d+70*cos(f*x+e)*(2*c-2*d)^
(1/2)*c*d^2-150*sin(f*x+e)*cos(f*x+e)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*s
in(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos
(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+
e))/(cos(f*x+e)+1))^(1/2)*d^3-14*cos(f*x+e)*(2*c-2*d)^(1/2)*c^3-45*cos(f*x+
e)^2*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*
sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(
f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c^2*d+1
71*cos(f*x+e)^2*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)
+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c
+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1
/2)*c*d^2+60*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1)
)^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)
/(1-cos(f*x+e)+sin(f*x+e)))*c^2*d*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))
^(1/2)*sin(f*x+e)-228*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(
f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*
x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d^2*2^(1/2)*((c+d*sin(f*x+e))/(cos(f
*x+e)+1))^(1/2)*sin(f*x+e)+30*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e
))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-
d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2*d*2^(1/2)*((c+d*sin(f*x+e)
)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-114*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*
sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*co
s(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d^2*2^(1/2)*((c+d*s
in(f*x+e))/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)+170*sin(f*x+e)*cos(f*x+e)*(2*c-
2*d)^(1/2)*d^3-12*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+
e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)
-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^
(1/2)*c^3-300*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1
))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d
```

)/(1-cos(f*x+e)+sin(f*x+e)))*d^3*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)+6*cos(f*x+e)^3*(2*c-2*d)^(1/2)*c^2*d-28*cos(f*x+e)^3*(2*c-2*d)^(1/2)*c*d^2-6*sin(f*x+e)*cos(f*x+e)*(2*c-2*d)^(1/2)*c^3+57*cos(f*x+e)^3*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d^2-6*sin(f*x+e)*cos(f*x+e)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^3+3*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^3+75*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d^3+3*cos(f*x+e)^3*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^3+75*cos(f*x+e)^3*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d^3+9*cos(f*x+e)^2*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^3+14*sin(f*x+e)*cos(f*x+e)*(2*c-2*d)^(1/2)*c^2*d-150*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*d^3*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)+225*cos(f*x+e)^2*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*d^3-12*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^3*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-300*sin(f*x+e)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*d^3-6*cos(f*x+e)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*c^3+62*sin(f*x+e)*cos(f*x+e)*(2*c-2*d)^(1/2)*c*d^2+60*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2*d*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)-228*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*

```
sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*co
s(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d^2*2^(1/2)*((c+d*s
in(f*x+e))/(cos(f*x+e)+1))^(1/2)-15*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((c+d*s
in(f*x+e))/(cos(f*x+e)+1))^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*
x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+
e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c^2*d+57*sin(f*x+e)*cos(f*x
+e)^2*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*ln(2*((2*c-2*d)^(1/2)
*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*
sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*c*d^2+
30*sin(f*x+e)*cos(f*x+e)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(c
os(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos
(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*sin(f*x+e))/(cos(f*x+
e)+1))^(1/2)*c^2*d-114*sin(f*x+e)*cos(f*x+e)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*
((c+d*sin(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e
)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*2^(1/2)*((c+d*s
in(f*x+e))/(cos(f*x+e)+1))^(1/2)*c*d^2)/(a*(1+sin(f*x+e)))^(5/2)/(c+d*sin(f
*x+e))^(1/2)/(c+d)/(2*c-2*d)^(1/2)/(c-d)^3
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="max
ima")
[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(3/2)), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(3/2)),x)
[Out] int(1/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(3/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(3/2),x)
[Out] Timed out
```

3.605 $\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=355

$$-\frac{\left(3c^2-26cd+163d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{c-d}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}f(c-d)^{9/2}}-\frac{d\left(9c^2-54cd-95d^2\right)\cos(e+fx)}{48a^2f(c-d)^3(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(}$$

```
[Out] -1/4*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2)-1/16*
(3*c-17*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(
3/2)-1/32*(3*c^2-26*c*d+163*d^2)*arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)
*2^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))/a^(5/2)/(c-d)^(9/2)
/f*2^(1/2)-1/48*d*(9*c^2-54*c*d-95*d^2)*cos(f*x+e)/a^2/(c-d)^3/(c+d)/f/(c+d
*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)-1/48*d*(9*c^3-57*c^2*d-493*c*d^2-
299*d^3)*cos(f*x+e)/a^2/(c-d)^4/(c+d)^2/f/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f
*x+e))^(1/2)
```

Rubi [A] time = 1.26, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2766, 2978, 2984, 12, 2782, 208}

$$-\frac{d\left(-57c^2d+9c^3-493cd^2-299d^3\right)\cos(e+fx)}{48a^2f(c-d)^4(c+d)^2\sqrt{a\sin(e+fx)+a}\,\sqrt{c+d\sin(e+fx)}}-\frac{d\left(9c^2-54cd-95d^2\right)\cos(e+fx)}{48a^2f(c-d)^3(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] -((3*c^2 - 26*c*d + 163*d^2)*ArcTanh[(Sqrt[a]*Sqrt[c - d]*Cos[e + f*x])/(Sq
rt[2]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5
/2)*(c - d)^(9/2)*f) - Cos[e + f*x]/(4*(c - d)*f*(a + a*Sin[e + f*x])^(5/2)
*(c + d*Sin[e + f*x])^(3/2)) - ((3*c - 17*d)*Cos[e + f*x])/(16*a*(c - d)^2*
f*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(3/2)) - (d*(9*c^2 - 54*c
*d - 95*d^2)*Cos[e + f*x])/(48*a^2*(c - d)^3*(c + d)*f*Sqrt[a + a*Sin[e + f
*x]]*(c + d*Sin[e + f*x])^(3/2)) - (d*(9*c^3 - 57*c^2*d - 493*c*d^2 - 299*d
^3)*Cos[e + f*x])/(48*a^2*(c - d)^4*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*Sq
rt[c + d*Sin[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
```

```
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}} - \frac{\int \frac{}{(a+a}}{}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}} - \frac{}{16a(c}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}} - \frac{}{16a(c}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}} - \frac{}{16a(c}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}} - \frac{}{16a(c}$$

$$= -\frac{\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}} - \frac{}{16a(c}$$

$$= -\frac{\left(3c^2-26cd+163d^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}(c-d)^{9/2}f}$$

Mathematica [B] time = 10.43, size = 717, normalized size = 2.02

$$\frac{\left(3c^2-26cd+163d^2\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4\left(\log\left(\tan\left(\frac{1}{2}(e+fx)\right)+1\right)-\log\left((d-c)\tan\left(\frac{1}{2}(e+f\right.\right.\right.}{32f(c-d)^4(a(\sin(e+fx)+1))^{5/2}\sqrt{c+d\sin(e+fx)}\left(\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)}{2\tan\left(\frac{1}{2}(e+fx)\right)+2}-\frac{\frac{\sqrt{c-d}\left(\frac{1}{\cos(e+fx)+1}\right)^{3/2}(c\sin}{\sqrt{c+d\sin(e}}}{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)+2}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sqrt[c + d*Sin[e + f*x]]*(Sin[(e +
 f*x)/2]/(2*(c - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) - 1/(4*(c -
d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3) + (-3*c + 25*d)/(16*(c - d)^4
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])) + (3*c*Sin[(e + f*x)/2] - 25*d*Sin[
(e + f*x)/2])/(8*(c - d)^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2) + (2*(d
^3*Cos[(e + f*x)/2] - d^3*Sin[(e + f*x)/2]))/(3*(c - d)^3*(c + d)*(c + d*Si
n[e + f*x])^2) + (2*(11*c*d^3*Cos[(e + f*x)/2] + 7*d^4*Cos[(e + f*x)/2] - 1
1*c*d^3*Sin[(e + f*x)/2] - 7*d^4*Sin[(e + f*x)/2]))/(3*(c - d)^4*(c + d)^2*
(c + d*Sin[e + f*x]))))/(f*(a*(1 + Sin[e + f*x]))^(5/2)) + ((3*c^2 - 26*c*d
 + 163*d^2)*(Log[1 + Tan[(e + f*x)/2]] - Log[c - d + 2*Sqrt[c - d]*Sqrt[(1
+ Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[e + f*x]] + (-c + d)*Tan[(e + f*x)/2]]
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)/(32*(c - d)^4*f*(a*(1 + Sin[e +
f*x]))^(5/2)*Sqrt[c + d*Sin[e + f*x]]*(Sec[(e + f*x)/2]^2/(2 + 2*Tan[(e + f
*x)/2]) - (-1/2*((c - d)*Sec[(e + f*x)/2]^2) + (Sqrt[c - d]*((1 + Cos[e + f
*x])^(-1))^(3/2)*(d + d*Cos[e + f*x] + c*Sin[e + f*x]))/Sqrt[c + d*Sin[e +
f*x]])/(c - d + 2*Sqrt[c - d]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[c + d*Sin[
e + f*x]] + (-c + d)*Tan[(e + f*x)/2])))
```

fricas [B] time = 3.17, size = 4858, normalized size = 13.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] [1/384*(3*(12*c^6 - 56*c^5*d + 308*c^4*d^2 + 2032*c^3*d^3 + 3508*c^2*d^4 +
2504*c*d^5 + 652*d^6 + (3*c^4*d^2 - 20*c^3*d^3 + 114*c^2*d^4 + 300*c*d^5 +
163*d^6)*cos(f*x + e)^5 + (6*c^5*d - 31*c^4*d^2 + 168*c^3*d^3 + 942*c^2*d^4
 + 1226*c*d^5 + 489*d^6)*cos(f*x + e)^4 - (3*c^6 - 8*c^5*d + 43*c^4*d^2 + 6
96*c^3*d^3 + 1705*c^2*d^4 + 1552*c*d^5 + 489*d^6)*cos(f*x + e)^3 - (9*c^6 -
 30*c^5*d + 163*c^4*d^2 + 1900*c^3*d^3 + 4287*c^2*d^4 + 3730*c*d^5 + 1141*d
^6)*cos(f*x + e)^2 + 2*(3*c^6 - 14*c^5*d + 77*c^4*d^2 + 508*c^3*d^3 + 877*c
^2*d^4 + 626*c*d^5 + 163*d^6)*cos(f*x + e) + (12*c^6 - 56*c^5*d + 308*c^4*d
^2 + 2032*c^3*d^3 + 3508*c^2*d^4 + 2504*c*d^5 + 652*d^6 + (3*c^4*d^2 - 20*c
^3*d^3 + 114*c^2*d^4 + 300*c*d^5 + 163*d^6)*cos(f*x + e)^4 - 2*(3*c^5*d - 1
7*c^4*d^2 + 94*c^3*d^3 + 414*c^2*d^4 + 463*c*d^5 + 163*d^6)*cos(f*x + e)^3
- (3*c^6 - 2*c^5*d + 9*c^4*d^2 + 884*c^3*d^3 + 2533*c^2*d^4 + 2478*c*d^5 +
815*d^6)*cos(f*x + e)^2 + 2*(3*c^6 - 14*c^5*d + 77*c^4*d^2 + 508*c^3*d^3 +
877*c^2*d^4 + 626*c*d^5 + 163*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(2*a*c -
 2*a*d)*log(((a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)^3 - 4*a*c^2 - 8*a*c
*d - 4*a*d^2 - (13*a*c^2 - 22*a*c*d - 3*a*d^2)*cos(f*x + e)^2 - 4*((c - 3*d
)*cos(f*x + e)^2 - (3*c - d)*cos(f*x + e) + ((c - 3*d)*cos(f*x + e) + 4*c -
```

4*d)*sin(f*x + e) - 4*c + 4*d)*sqrt(2*a*c - 2*a*d)*sqrt(a*sin(f*x + e) + a
)*sqrt(d*sin(f*x + e) + c) - 2*(9*a*c^2 - 14*a*c*d + 9*a*d^2)*cos(f*x + e)
- (4*a*c^2 + 8*a*c*d + 4*a*d^2 - (a*c^2 - 14*a*c*d + 17*a*d^2)*cos(f*x + e)
^2 - 2*(7*a*c^2 - 18*a*c*d + 7*a*d^2)*cos(f*x + e))*sin(f*x + e))/(cos(f*x
+ e)^3 + 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 4)*sin(f*x +
e) - 2*cos(f*x + e) - 4)) - 8*(12*c^6 - 24*c^5*d - 12*c^4*d^2 + 48*c^3*d^3
- 12*c^2*d^4 - 24*c*d^5 + 12*d^6 - (9*c^4*d^2 - 66*c^3*d^3 - 436*c^2*d^4 +
194*c*d^5 + 299*d^6)*cos(f*x + e)^4 - (18*c^5*d - 111*c^4*d^2 - 618*c^3*d^
3 - 520*c^2*d^4 + 728*c*d^5 + 503*d^6)*cos(f*x + e)^3 + 3*(3*c^6 - 14*c^5*d
- 29*c^4*d^2 - 144*c^3*d^3 - 59*c^2*d^4 + 158*c*d^5 + 85*d^6)*cos(f*x + e)
^2 + 3*(7*c^6 - 16*c^5*d - 73*c^4*d^2 - 312*c^3*d^3 - 91*c^2*d^4 + 328*c*d^
5 + 157*d^6)*cos(f*x + e) - (12*c^6 - 24*c^5*d - 12*c^4*d^2 + 48*c^3*d^3 -
12*c^2*d^4 - 24*c*d^5 + 12*d^6 + (9*c^4*d^2 - 66*c^3*d^3 - 436*c^2*d^4 + 19
4*c*d^5 + 299*d^6)*cos(f*x + e)^3 - 6*(3*c^5*d - 20*c^4*d^2 - 92*c^3*d^3 -
14*c^2*d^4 + 89*c*d^5 + 34*d^6)*cos(f*x + e)^2 - 3*(3*c^6 - 8*c^5*d - 69*c^
4*d^2 - 328*c^3*d^3 - 87*c^2*d^4 + 336*c*d^5 + 153*d^6)*cos(f*x + e))*sin(f
*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((a^3*c^7*d^2 -
3*a^3*c^6*d^3 + a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 5*a^3*c^3*d^6 - a^3*c^2*d^7
+ 3*a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^5 + (2*a^3*c^8*d - 3*a^3*c^7*d^2 -
7*a^3*c^6*d^3 + 13*a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 17*a^3*c^3*d^6 + 3*a^3*c^2
*d^7 + 7*a^3*c*d^8 - 3*a^3*d^9)*f*cos(f*x + e)^4 - (a^3*c^9 + a^3*c^8*d - 8
*a^3*c^7*d^2 + 18*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 16*a^3*c^3*d^6 + 8*a^3*c^2*
d^7 + 5*a^3*c*d^8 - 3*a^3*d^9)*f*cos(f*x + e)^3 - (3*a^3*c^9 + a^3*c^8*d -
20*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 42*a^3*c^5*d^4 - 18*a^3*c^4*d^5 - 36*a^3*c
^3*d^6 + 20*a^3*c^2*d^7 + 11*a^3*c*d^8 - 7*a^3*d^9)*f*cos(f*x + e)^2 + 2*(a
^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*
c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f*cos(f*x +
e) + 4*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4
- 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f +
((a^3*c^7*d^2 - 3*a^3*c^6*d^3 + a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 5*a^3*c^3*d^
6 - a^3*c^2*d^7 + 3*a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^4 - 2*(a^3*c^8*d -
2*a^3*c^7*d^2 - 2*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^3*d^6 + 2*a^3*c^2*d
^7 + 2*a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^3 - (a^3*c^9 + 3*a^3*c^8*d - 12*
a^3*c^7*d^2 - 4*a^3*c^6*d^3 + 30*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 28*a^3*c^3*d
^6 + 12*a^3*c^2*d^7 + 9*a^3*c*d^8 - 5*a^3*d^9)*f*cos(f*x + e)^2 + 2*(a^3*c^
9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d
^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e) +
4*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*
a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f)*sin(f
x + e)), -1/192(3*(12*c^6 - 56*c^5*d + 308*c^4*d^2 + 2032*c^3*d^3 + 3508*
c^2*d^4 + 2504*c*d^5 + 652*d^6 + (3*c^4*d^2 - 20*c^3*d^3 + 114*c^2*d^4 + 30
0*c*d^5 + 163*d^6)*cos(f*x + e)^5 + (6*c^5*d - 31*c^4*d^2 + 168*c^3*d^3 + 9
42*c^2*d^4 + 1226*c*d^5 + 489*d^6)*cos(f*x + e)^4 - (3*c^6 - 8*c^5*d + 43*c
^4*d^2 + 696*c^3*d^3 + 1705*c^2*d^4 + 1552*c*d^5 + 489*d^6)*cos(f*x + e)^3
- (9*c^6 - 30*c^5*d + 163*c^4*d^2 + 1900*c^3*d^3 + 4287*c^2*d^4 + 3730*c*d^

5 + 1141*d^6)*cos(f*x + e)^2 + 2*(3*c^6 - 14*c^5*d + 77*c^4*d^2 + 508*c^3*d^3 + 877*c^2*d^4 + 626*c*d^5 + 163*d^6)*cos(f*x + e) + (12*c^6 - 56*c^5*d + 308*c^4*d^2 + 2032*c^3*d^3 + 3508*c^2*d^4 + 2504*c*d^5 + 652*d^6 + (3*c^4*d^2 - 20*c^3*d^3 + 114*c^2*d^4 + 300*c*d^5 + 163*d^6)*cos(f*x + e)^4 - 2*(3*c^5*d - 17*c^4*d^2 + 94*c^3*d^3 + 414*c^2*d^4 + 463*c*d^5 + 163*d^6)*cos(f*x + e)^3 - (3*c^6 - 2*c^5*d + 9*c^4*d^2 + 884*c^3*d^3 + 2533*c^2*d^4 + 2478*c*d^5 + 815*d^6)*cos(f*x + e)^2 + 2*(3*c^6 - 14*c^5*d + 77*c^4*d^2 + 508*c^3*d^3 + 877*c^2*d^4 + 626*c*d^5 + 163*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(-2*a*c + 2*a*d)*arctan(1/4*sqrt(-2*a*c + 2*a*d)*sqrt(a*sin(f*x + e) + a)*((c - 3*d)*sin(f*x + e) - 3*c + d)*sqrt(d*sin(f*x + e) + c)/((a*c*d - a*d^2)*cos(f*x + e)*sin(f*x + e) + (a*c^2 - a*c*d)*cos(f*x + e))) + 4*(12*c^6 - 24*c^5*d - 12*c^4*d^2 + 48*c^3*d^3 - 12*c^2*d^4 - 24*c*d^5 + 12*d^6 - (9*c^4*d^2 - 66*c^3*d^3 - 436*c^2*d^4 + 194*c*d^5 + 299*d^6)*cos(f*x + e)^4 - (18*c^5*d - 111*c^4*d^2 - 618*c^3*d^3 - 520*c^2*d^4 + 728*c*d^5 + 503*d^6)*cos(f*x + e)^3 + 3*(3*c^6 - 14*c^5*d - 29*c^4*d^2 - 144*c^3*d^3 - 59*c^2*d^4 + 158*c*d^5 + 85*d^6)*cos(f*x + e)^2 + 3*(7*c^6 - 16*c^5*d - 73*c^4*d^2 - 312*c^3*d^3 - 91*c^2*d^4 + 328*c*d^5 + 157*d^6)*cos(f*x + e) - (12*c^6 - 24*c^5*d - 12*c^4*d^2 + 48*c^3*d^3 - 12*c^2*d^4 - 24*c*d^5 + 12*d^6 + (9*c^4*d^2 - 66*c^3*d^3 - 436*c^2*d^4 + 194*c*d^5 + 299*d^6)*cos(f*x + e)^3 - 6*(3*c^5*d - 20*c^4*d^2 - 92*c^3*d^3 - 14*c^2*d^4 + 89*c*d^5 + 34*d^6)*cos(f*x + e)^2 - 3*(3*c^6 - 8*c^5*d - 69*c^4*d^2 - 328*c^3*d^3 - 87*c^2*d^4 + 336*c*d^5 + 153*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c))/((a^3*c^7*d^2 - 3*a^3*c^6*d^3 + a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 5*a^3*c^3*d^6 - a^3*c^2*d^7 + 3*a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^5 + (2*a^3*c^8*d - 3*a^3*c^7*d^2 - 7*a^3*c^6*d^3 + 13*a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 17*a^3*c^3*d^6 + 3*a^3*c^2*d^7 + 7*a^3*c*d^8 - 3*a^3*d^9)*f*cos(f*x + e)^4 - (a^3*c^9 + a^3*c^8*d - 8*a^3*c^7*d^2 + 18*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 16*a^3*c^3*d^6 + 8*a^3*c^2*d^7 + 5*a^3*c*d^8 - 3*a^3*d^9)*f*cos(f*x + e)^3 - (3*a^3*c^9 + a^3*c^8*d - 20*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 42*a^3*c^5*d^4 - 18*a^3*c^4*d^5 - 36*a^3*c^3*d^6 + 20*a^3*c^2*d^7 + 11*a^3*c*d^8 - 7*a^3*d^9)*f*cos(f*x + e)^2 + 2*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e) + 4*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f + ((a^3*c^7*d^2 - 3*a^3*c^6*d^3 + a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 5*a^3*c^3*d^6 - a^3*c^2*d^7 + 3*a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^4 - 2*(a^3*c^8*d - 2*a^3*c^7*d^2 - 2*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^3*d^6 + 2*a^3*c^2*d^7 + 2*a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^3 - (a^3*c^9 + 3*a^3*c^8*d - 12*a^3*c^7*d^2 - 4*a^3*c^6*d^3 + 30*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 28*a^3*c^3*d^6 + 12*a^3*c^2*d^7 + 9*a^3*c*d^8 - 5*a^3*d^9)*f*cos(f*x + e)^2 + 2*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e) + 4*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f)*sin(f*x + e))]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(5/2)), x)
```

maple [B] time = 0.40, size = 8035, normalized size = 22.63

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(5/2),x)
[Out] Timed out
```

3.606 $\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=129

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(m+\frac{1}{2};\frac{1}{2},-n;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1\right.}{f(2m+1)\sqrt{1-\sin(e+fx)}}$$

```
[Out] AppellF1(1/2+m,-n,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos
(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n*2^(1/2)/f/(1+2*m)/(((c+d*sin(
f*x+e))/(c-d))^n)/(1-sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2788, 140, 139, 138}

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(m+\frac{1}{2};\frac{1}{2},-n;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1\right.}{f(2m+1)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, -n, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1
+ Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e
 + f*x])^n)/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c -
d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx = \frac{\left(a^2\cos(e+fx)\right)\operatorname{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}(c+dx)^n}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\operatorname{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}(c+dx)^n}{\sqrt{\frac{1}{2}-\frac{x}{2}}}\,dx,x,s\right.}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\,(c+d\sin(e+fx))^n\left(\frac{a(c+d\sin(e+fx))}{ac-ad}\right.\right.}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{}}$$

$$= \frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;\frac{1}{2},-n;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)}{f(1+2m)\sqrt{}}$$

Mathematica [B] time = 1.41, size = 373, normalized size = 2.89

$$\frac{6(c+d)\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi\right)}{f\left(\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\left(4dnF_1\left(\frac{3}{2};\frac{1}{2}-m,1-n;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+2fx+\pi)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)}{c+d}\right)+(2m-1)(c+\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (6*(c + d)*AppellF1[1/2, 1/2 - m, -n, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*
d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2
 + m)*Cot[(2*e + Pi + 2*f*x)/4]*(a*(1 + Sin[e + f*x]))^m*(c + d*Sin[e + f*x
])^n*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(f*(-3*(c + d)*AppellF1[1/2,
1/2 - m, -n, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/
4]^2)/(c + d)] + (4*d*n*AppellF1[3/2, 1/2 - m, 1 - n, 5/2, Cos[(2*e + Pi +
2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (c + d)*(-1 + 2*m
)*AppellF1[3/2, 3/2 - m, -n, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*
e - Pi + 2*f*x)/4]^2)/(c + d)])*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m\left(d\sin\left(fx+e\right)+c\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^m\left(d\sin\left(fx+e\right)+c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n, x)
```

maple [F] time = 2.02, size = 0, normalized size = 0.00

$$\int\left(a+a\sin\left(fx+e\right)\right)^m\left(c+d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
[Out] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n,x)
[Out] int((a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\sin{\left(e + f x \right)} + 1\right)\right)^{m} \left(c + d \sin{\left(e + f x \right)}\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**n,x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*(c + d*sin(e + f*x))**n, x)
```

3.607 $\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=320

$$-\frac{d\left(2c^2\left(m^2+6m+8\right)-cd\left(-2m^2-3m+5\right)+d^2(m+4)\right)\cos(e+fx)(a\sin(e+fx)+a)^m}{f(m+1)(m+2)(m+3)}-\frac{2^{m+\frac{1}{2}}\left(c^3\left(m^3+6m\right.\right.}{}$$

```
[Out] -d*(d^2*(4+m)-c*d*(-2*m^2-3*m+5)+2*c^2*(m^2+6*m+8))*cos(f*x+e)*(a+a*sin(f*x
+e))^m/f/(3+m)/(m^2+3*m+2)-2^(1/2+m)*(d^3*m*(m^2+3*m+5)+3*c^2*d*m*(m^2+5*m+
6)+3*c*d^2*(m^3+4*m^2+4*m+3)+c^3*(m^3+6*m^2+11*m+6))*cos(f*x+e)*hypergeom([
1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+
e))^m/f/(3+m)/(m^2+3*m+2)-d^2*(d*m+c*(5+m))*cos(f*x+e)*(a+a*sin(f*x+e))^(1+
m)/a/f/(2+m)/(3+m)-d*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2/f/(3+
m)
```

Rubi [A] time = 0.66, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2783, 2968, 3023, 2751, 2652, 2651}

$$-\frac{2^{m+\frac{1}{2}}\left(3c^2dm\left(m^2+5m+6\right)+c^3\left(m^3+6m^2+11m+6\right)+3cd^2\left(m^3+4m^2+4m+3\right)+d^3m\left(m^2+3m+5\right)\right)\mathrm{c}}{f(m+1)(m+2)(m+3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] -((d*(d^2*(4 + m) - c*d*(5 - 3*m - 2*m^2) + 2*c^2*(8 + 6*m + m^2))*Cos[e +
f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m)*(2 + m)*(3 + m))) - (2^(1/2 + m)*(d
^3*m*(5 + 3*m + m^2) + 3*c^2*d*m*(6 + 5*m + m^2) + 3*c*d^2*(3 + 4*m + 4*m^2
 + m^3) + c^3*(6 + 11*m + 6*m^2 + m^3))*Cos[e + f*x]*Hypergeometric2F1[1/2,
 1/2 - m, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*S
in[e + f*x])^m)/(f*(1 + m)*(2 + m)*(3 + m)) - (d^2*(d*m + c*(5 + m))*Cos[e
+ f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(2 + m)*(3 + m)) - (d*Cos[e + f*x
]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2)/(f*(3 + m))
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]
```

Rule 2783

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + n)), x] + Dist[1/(b*(m + n)), Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n - 2)*Simp[d*(a*c*m + b*d*(n -
 1)) + b*c^2*(m + n) + d*(a*d*m + b*c*(m + 2*n - 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^3\,dx = -\frac{d\cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^2}{f(3+m)} + \frac{\int (a+a\ldots}{\ldots}$$

$$= -\frac{d\cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^2}{f(3+m)} + \frac{\int (a+a\ldots}{\ldots}$$

$$= -\frac{d^2(dm+c(5+m))\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(2+m)(3+m)} - \frac{d\cos(e\ldots}{\ldots}$$

$$= -\frac{d\left(d^2(4+m)-cd\left(5-3m-2m^2\right)+2c^2\left(8+6m+m^2\right)\right)\cos(e\ldots}{f(1+m)(2+m)(3+m)}$$

$$= -\frac{d\left(d^2(4+m)-cd\left(5-3m-2m^2\right)+2c^2\left(8+6m+m^2\right)\right)\cos(e\ldots}{f(1+m)(2+m)(3+m)}$$

$$= -\frac{d\left(d^2(4+m)-cd\left(5-3m-2m^2\right)+2c^2\left(8+6m+m^2\right)\right)\cos(e\ldots}{f(1+m)(2+m)(3+m)}$$

Mathematica [B] time = 57.57, size = 3599, normalized size = 11.25

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (-22*(Cos[(-e + Pi/2 - f*x)/2]^2)^(1/2 - m)*(1 - Sin[(-e + Pi/2 - f*x)/2]^2
)^(-1/2 + m)*(c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2)^3*(945*(c + d)^3*Gamm
a[1/2 - m]*Hypergeometric2F1[1/2, 1/2 - m, 9/2, Sin[(-e + Pi/2 - f*x)/2]^2]
 - 1890*d*(c + d)^2*Gamma[1/2 - m]*Hypergeometric2F1[1/2, 1/2 - m, 9/2, Sin
[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^2 + 142*(c + d)^3*Gamma[3
/2 - m]*Hypergeometric2F1[3/2, 3/2 - m, 11/2, Sin[(-e + Pi/2 - f*x)/2]^2]*S
in[(-e + Pi/2 - f*x)/2]^2 + 60*(c + d)^3*Gamma[3/2 - m]*HypergeometricPFQ[{
3/2, 2, 3/2 - m}, {1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f
*x)/2]^2 + 8*(c + d)^3*Gamma[3/2 - m]*HypergeometricPFQ[{3/2, 2, 2, 3/2 - m
}, {1, 1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^2 + 2
268*d^2*(c + d)*Gamma[1/2 - m]*Hypergeometric2F1[1/2, 1/2 - m, 9/2, Sin[(-e
 + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^4 - 564*d*(c + d)^2*Gamma[3/2
 - m]*Hypergeometric2F1[3/2, 3/2 - m, 11/2, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin
[(-e + Pi/2 - f*x)/2]^4 - 312*d*(c + d)^2*Gamma[3/2 - m]*HypergeometricPFQ[
{3/2, 2, 3/2 - m}, {1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 -
f*x)/2]^4 - 48*d*(c + d)^2*Gamma[3/2 - m]*HypergeometricPFQ[{3/2, 2, 2, 3/2
 - m}, {1, 1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^4
```

```
- 1080*d^3*Gamma[1/2 - m]*Hypergeometric2F1[1/2, 1/2 - m, 9/2, Sin[(-e + P
i/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^6 + 744*d^2*(c + d)*Gamma[3/2 - m
]*Hypergeometric2F1[3/2, 3/2 - m, 11/2, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e
 + Pi/2 - f*x)/2]^6 + 528*d^2*(c + d)*Gamma[3/2 - m]*HypergeometricPFQ[{3/2
, 2, 3/2 - m}, {1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)
/2]^6 + 96*d^2*(c + d)*Gamma[3/2 - m]*HypergeometricPFQ[{3/2, 2, 2, 3/2 - m
}, {1, 1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^6 - 3
68*d^3*Gamma[3/2 - m]*Hypergeometric2F1[3/2, 3/2 - m, 11/2, Sin[(-e + Pi/2
- f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^8 - 288*d^3*Gamma[3/2 - m]*Hypergeome
tricPFQ[{3/2, 2, 3/2 - m}, {1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e +
 Pi/2 - f*x)/2]^8 - 64*d^3*Gamma[3/2 - m]*HypergeometricPFQ[{3/2, 2, 2, 3/2
 - m}, {1, 1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^8
)*(a + a*Sin[e + f*x])^m*Tan[(-e + Pi/2 - f*x)/2])/(3*f*(3465*(c + d)^3*Gam
ma[1/2 - m]*Hypergeometric2F1[1/2, 1/2 - m, 9/2, Sin[(-e + Pi/2 - f*x)/2]^2
] - 20790*d*(c + d)^2*Gamma[1/2 - m]*Hypergeometric2F1[1/2, 1/2 - m, 9/2, S
in[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^2 - 385*(c + d)^3*(-1 +
 2*m)*Gamma[1/2 - m]*Hypergeometric2F1[3/2, 3/2 - m, 11/2, Sin[(-e + Pi/2 -
 f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^2 + 1562*(c + d)^3*Gamma[3/2 - m]*Hype
rgeometric2F1[3/2, 3/2 - m, 11/2, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/
2 - f*x)/2]^2 + 660*(c + d)^3*Gamma[3/2 - m]*HypergeometricPFQ[{3/2, 2, 3/2
 - m}, {1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^2 +
88*(c + d)^3*Gamma[3/2 - m]*HypergeometricPFQ[{3/2, 2, 2, 3/2 - m}, {1, 1,
11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^2 + 41580*d^2*(
c + d)*Gamma[1/2 - m]*Hypergeometric2F1[1/2, 1/2 - m, 9/2, Sin[(-e + Pi/2 -
 f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^4 + 770*d*(c + d)^2*(-1 + 2*m)*Gamma[1
/2 - m]*Hypergeometric2F1[3/2, 3/2 - m, 11/2, Sin[(-e + Pi/2 - f*x)/2]^2]*S
in[(-e + Pi/2 - f*x)/2]^4 - 10340*d*(c + d)^2*Gamma[3/2 - m]*Hypergeometric
2F1[3/2, 3/2 - m, 11/2, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2
]^4 - 142*(c + d)^3*(-3 + 2*m)*Gamma[3/2 - m]*Hypergeometric2F1[5/2, 5/2 -
m, 13/2, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^4 - 5720*d*(c
 + d)^2*Gamma[3/2 - m]*HypergeometricPFQ[{3/2, 2, 3/2 - m}, {1, 11/2}, Sin[
(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^4 - 120*(c + d)^3*(-3 + 2*
m)*Gamma[3/2 - m]*HypergeometricPFQ[{5/2, 3, 5/2 - m}, {2, 13/2}, Sin[(-e +
 Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^4 - 880*d*(c + d)^2*Gamma[3/2 -
 m]*HypergeometricPFQ[{3/2, 2, 2, 3/2 - m}, {1, 1, 11/2}, Sin[(-e + Pi/2 -
f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^4 - 32*(c + d)^3*(-3 + 2*m)*Gamma[3/2 -
 m]*HypergeometricPFQ[{5/2, 3, 3, 5/2 - m}, {2, 2, 13/2}, Sin[(-e + Pi/2 -
f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^4 - 27720*d^3*Gamma[1/2 - m]*Hypergeome
tric2F1[1/2, 1/2 - m, 9/2, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x
)/2]^6 - 924*d^2*(c + d)*(-1 + 2*m)*Gamma[1/2 - m]*Hypergeometric2F1[3/2, 3
/2 - m, 11/2, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^6 + 1909
6*d^2*(c + d)*Gamma[3/2 - m]*Hypergeometric2F1[3/2, 3/2 - m, 11/2, Sin[(-e
+ Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^6 + 564*d*(c + d)^2*(-3 + 2*m)
*Gamma[3/2 - m]*Hypergeometric2F1[5/2, 5/2 - m, 13/2, Sin[(-e + Pi/2 - f*x)
/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^6 + 13552*d^2*(c + d)*Gamma[3/2 - m]*Hyperg
```

```
eometricPFQ[{3/2, 2, 3/2 - m}, {1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(
-e + Pi/2 - f*x)/2]^6 + 624*d*(c + d)^2*(-3 + 2*m)*Gamma[3/2 - m]*Hypergeom
etricPFQ[{5/2, 3, 5/2 - m}, {2, 13/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e
+ Pi/2 - f*x)/2]^6 + 2464*d^2*(c + d)*Gamma[3/2 - m]*HypergeometricPFQ[{3/2
, 2, 2, 3/2 - m}, {1, 1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2
- f*x)/2]^6 + 192*d*(c + d)^2*(-3 + 2*m)*Gamma[3/2 - m]*HypergeometricPFQ[{
5/2, 3, 3, 5/2 - m}, {2, 2, 13/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi
/2 - f*x)/2]^6 + 440*d^3*(-1 + 2*m)*Gamma[1/2 - m]*Hypergeometric2F1[3/2, 3
/2 - m, 11/2, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^8 - 1214
4*d^3*Gamma[3/2 - m]*Hypergeometric2F1[3/2, 3/2 - m, 11/2, Sin[(-e + Pi/2 -
 f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^8 - 744*d^2*(c + d)*(-3 + 2*m)*Gamma[3
/2 - m]*Hypergeometric2F1[5/2, 5/2 - m, 13/2, Sin[(-e + Pi/2 - f*x)/2]^2]*S
in[(-e + Pi/2 - f*x)/2]^8 - 9504*d^3*Gamma[3/2 - m]*HypergeometricPFQ[{3/2,
 2, 3/2 - m}, {1, 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/
2]^8 - 1056*d^2*(c + d)*(-3 + 2*m)*Gamma[3/2 - m]*HypergeometricPFQ[{5/2, 3
, 5/2 - m}, {2, 13/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]
^8 - 2112*d^3*Gamma[3/2 - m]*HypergeometricPFQ[{3/2, 2, 2, 3/2 - m}, {1, 1,
 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^8 - 384*d^2*(c
 + d)*(-3 + 2*m)*Gamma[3/2 - m]*HypergeometricPFQ[{5/2, 3, 3, 5/2 - m}, {2,
 2, 13/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^8 + 368*d^3
*(-3 + 2*m)*Gamma[3/2 - m]*Hypergeometric2F1[5/2, 5/2 - m, 13/2, Sin[(-e +
Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^10 + 576*d^3*(-3 + 2*m)*Gamma[3/
2 - m]*HypergeometricPFQ[{5/2, 3, 5/2 - m}, {2, 13/2}, Sin[(-e + Pi/2 - f*x
)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^10 + 256*d^3*(-3 + 2*m)*Gamma[3/2 - m]*Hyp
ergeometricPFQ[{5/2, 3, 3, 5/2 - m}, {2, 2, 13/2}, Sin[(-e + Pi/2 - f*x)/2]
^2]*Sin[(-e + Pi/2 - f*x)/2]^10))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(3\,cd^2\cos\left(fx+e\right)^2 - c^3 - 3\,cd^2 + \left(d^3\cos\left(fx+e\right)^2 - 3\,c^2d - d^3\right)\sin\left(fx+e\right)\right)\left(a\sin\left(fx+e\right)+a\right)^m,$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*c*d^2*cos(f*x + e)^2 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3
*c^2*d - d^3)*sin(f*x + e))*(a*sin(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)+c\right)^3\left(a\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^3*(a*sin(f*x + e) + a)^m, x)
```

maple [F] time = 6.75, size = 0, normalized size = 0.00

$$\int \left(a + a\sin\left(fx+e\right)\right)^{m} \left(c + d\sin\left(fx+e\right)\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^3,x)
[Out] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)+c\right)^{3}\left(a\sin\left(fx+e\right)+a\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^3,x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^3*(a*sin(f*x + e) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^3,x)
[Out] int((a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\sin{\left(e + f x \right)} + 1\right)\right)^{m} \left(c + d \sin{\left(e + f x \right)}\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**3,x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*(c + d*sin(e + f*x))**3, x)
```

3.608 $\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=193

$$-\frac{2^{m+\frac{1}{2}}\left(c^2\left(m^2+3m+2\right)+2cdm(m+2)+d^2\left(m^2+m+1\right)\right)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a}{f(m+1)(m+2)}$$

[Out] d*(d-2*c*(2+m))*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+m)/(2+m)-2^(1/2+m)*(2*c*d*m*(2+m)+d^2*(m^2+m+1)+c^2*(m^2+3*m+2))*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/f/(m^2+3*m+2)-d^2*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(2+m)

Rubi [A] time = 0.27, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2761, 2751, 2652, 2651}

$$-\frac{2^{m+\frac{1}{2}}\left(c^2\left(m^2+3m+2\right)+2cdm(m+2)+d^2\left(m^2+m+1\right)\right)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a}{f(m+1)(m+2)}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2,x]

[Out] (d*(d - 2*c*(2 + m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m)*(2 + m)) - (2^(1/2 + m)*(2*c*d*m*(2 + m) + d^2*(1 + m + m^2) + c^2*(2 + 3*m + m^2))*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m)*(2 + m)) - (d^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(2 + m))

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^2\,dx = -\frac{d^2\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(2+m)}+\frac{\int (a+a\sin(e+fx))^m}{}$$

$$=\frac{d(d-2c(2+m))\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)(2+m)}-\frac{d^2\cos(e+}{}$$

$$=\frac{d(d-2c(2+m))\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)(2+m)}-\frac{d^2\cos(e+}{}$$

$$=\frac{d(d-2c(2+m))\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)(2+m)}-\frac{2^{\frac{1}{2}+m}\left(2cdn\right.}{}$$

Mathematica [B] time = 65.65, size = 1774, normalized size = 9.19

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (-2*(Cos[(-e + Pi/2 - f*x)/2]^2)^(1/2 - m)*(1 - Sin[(-e + Pi/2 - f*x)/2]^2)
^(-1/2 + m)*(c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2)^2*(4*Gamma[3/2 - m]*Hy
pergeometricPFQ[{3/2, 2, 3/2 - m}, {1, 9/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Si
n[(-e + Pi/2 - f*x)/2]^2*(c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2)^2 + 16*Ga
mma[3/2 - m]*Hypergeometric2F1[3/2, 3/2 - m, 9/2, Sin[(-e + Pi/2 - f*x)/2]^
```

```
2]*Sin[(-e + Pi/2 - f*x)/2]^2*(c^2 + c*d*(2 - 3*Sin[(-e + Pi/2 - f*x)/2]^2)
 + d^2*(1 - 3*Sin[(-e + Pi/2 - f*x)/2]^2 + 2*Sin[(-e + Pi/2 - f*x)/2]^4)) +
 7*Gamma[1/2 - m]*Hypergeometric2F1[1/2, 1/2 - m, 7/2, Sin[(-e + Pi/2 - f*x
)/2]^2]*(15*c^2 + 10*c*d*(3 - 2*Sin[(-e + Pi/2 - f*x)/2]^2) + d^2*(15 - 20*
Sin[(-e + Pi/2 - f*x)/2]^2 + 12*Sin[(-e + Pi/2 - f*x)/2]^4)))*(a + a*Sin[e
+ f*x])^m*Tan[(-e + Pi/2 - f*x)/2])/(f*(4*Gamma[3/2 - m]*HypergeometricPFQ[
{3/2, 2, 3/2 - m}, {1, 9/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f
*x)/2]^2*(c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2)^2 + 16*Gamma[3/2 - m]*Hyp
ergeometric2F1[3/2, 3/2 - m, 9/2, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/
2 - f*x)/2]^2*(c^2 + c*d*(2 - 3*Sin[(-e + Pi/2 - f*x)/2]^2) + d^2*(1 - 3*Si
n[(-e + Pi/2 - f*x)/2]^2 + 2*Sin[(-e + Pi/2 - f*x)/2]^4)) + 7*Gamma[1/2 - m
]*Hypergeometric2F1[1/2, 1/2 - m, 7/2, Sin[(-e + Pi/2 - f*x)/2]^2]*(15*c^2
+ 10*c*d*(3 - 2*Sin[(-e + Pi/2 - f*x)/2]^2) + d^2*(15 - 20*Sin[(-e + Pi/2 -
 f*x)/2]^2 + 12*Sin[(-e + Pi/2 - f*x)/2]^4)) + (2*Sin[(-e + Pi/2 - f*x)/2]^
2*(-48*d*Gamma[3/2 - m]*HypergeometricPFQ[{3/2, 2, 3/2 - m}, {1, 9/2}, Sin[
(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^2*(c + d - 2*d*Sin[(-e + P
i/2 - f*x)/2]^2) + 12*Gamma[3/2 - m]*HypergeometricPFQ[{3/2, 2, 3/2 - m}, {
1, 9/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*(c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^
2)^2 - 4*(-3 + 2*m)*Gamma[3/2 - m]*HypergeometricPFQ[{5/2, 3, 5/2 - m}, {2,
 11/2}, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]^2*(c + d - 2*d
*Sin[(-e + Pi/2 - f*x)/2]^2)^2 + 48*d*Gamma[3/2 - m]*Hypergeometric2F1[3/2,
 3/2 - m, 9/2, Sin[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]*(-3*c*S
in[(-e + Pi/2 - f*x)/2] + d*Sin[(-e + Pi/2 - f*x)/2]*(-3 + 4*Sin[(-e + Pi/2
 - f*x)/2]^2)) + 84*d*Gamma[1/2 - m]*Hypergeometric2F1[1/2, 1/2 - m, 7/2, S
in[(-e + Pi/2 - f*x)/2]^2]*(-5*c + d*(-5 + 6*Sin[(-e + Pi/2 - f*x)/2]^2)) +
 48*Gamma[3/2 - m]*Hypergeometric2F1[3/2, 3/2 - m, 9/2, Sin[(-e + Pi/2 - f*
x)/2]^2]*(c^2 + c*d*(2 - 3*Sin[(-e + Pi/2 - f*x)/2]^2) + d^2*(1 - 3*Sin[(-e
 + Pi/2 - f*x)/2]^2 + 2*Sin[(-e + Pi/2 - f*x)/2]^4)) - 8*(-3 + 2*m)*Gamma[3
/2 - m]*Hypergeometric2F1[5/2, 5/2 - m, 11/2, Sin[(-e + Pi/2 - f*x)/2]^2]*S
in[(-e + Pi/2 - f*x)/2]^2*(c^2 + c*d*(2 - 3*Sin[(-e + Pi/2 - f*x)/2]^2) + d
^2*(1 - 3*Sin[(-e + Pi/2 - f*x)/2]^2 + 2*Sin[(-e + Pi/2 - f*x)/2]^4)) + 3*(
1/2 - m)*Gamma[1/2 - m]*Hypergeometric2F1[3/2, 3/2 - m, 9/2, Sin[(-e + Pi/2
 - f*x)/2]^2]*(15*c^2 + 10*c*d*(3 - 2*Sin[(-e + Pi/2 - f*x)/2]^2) + d^2*(15
 - 20*Sin[(-e + Pi/2 - f*x)/2]^2 + 12*Sin[(-e + Pi/2 - f*x)/2]^4))))/3))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2\right)\left(a\sin\left(fx+e\right)+a\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)*(a*sin(f*x
+ e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)+c\right)^{2}\left(a\sin\left(fx+e\right)+a\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*sin(f*x + e) + c)^2*(a*sin(f*x + e) + a)^m, x)
```

maple [F] time = 6.61, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x)
[Out] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)+c\right)^{2}\left(a\sin\left(fx+e\right)+a\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^2*(a*sin(f*x + e) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^2,x)
[Out] int((a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(c+d\sin\left(e+fx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**2,x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*(c + d*sin(e + f*x))**2, x)
```

3.609 $\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=117

$$-\frac{2^{m+\frac{1}{2}}(cm+c+dm)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f(m+1)}-$$

```
[Out] -d*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+m)-2^(1/2+m)*(c*m+d*m+c)*cos(f*x+e)*h
ypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+
a*sin(f*x+e))^m/f/(1+m)
```

Rubi [A] time = 0.08, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2751, 2652, 2651}

$$-\frac{2^{m+\frac{1}{2}}(cm+c+dm)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f(m+1)}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x]),x]
```

```
[Out] -((d*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m))) - (2^(1/2 + m)*(c +
c*m + d*m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f
*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m))
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a + a\sin(e+fx))^m(c + d\sin(e+fx))\,dx = -\frac{d\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)} + \frac{(c+cm+dm)\int(a+a\sin}{1+m}$$

$$= -\frac{d\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)} + \frac{\big((c+cm+dm)(1+\sin(e}{}$$

$$= -\frac{d\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)} - \frac{2^{\frac{1}{2}+m}(c+cm+dm)\cos(e}{}$$

Mathematica [C] time = 1.86, size = 275, normalized size = 2.35

$$-\frac{\sin^{-2m}\left(\frac{1}{4}(2e+2fx+\pi)\right)(a(\sin(e+fx)+1))^m\left(\frac{2\sqrt{2}c\sin\left(\frac{1}{4}(2e+2fx-\pi)\right)\cos^{2m+1}\left(\frac{1}{4}(2e+2fx-\pi)\right){}_2F_1\left(\frac{1}{2},m+\frac{1}{2};m+\frac{3}{2};\sin^2\left(\frac{1}{4}(2e+\right.\right.}{(2m+1)\sqrt{1-\sin(e+fx)}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x]),x]
```

```
[Out] -(((a*(1 + Sin[e + f*x]))^m*(((-1)^(1/4)*2^(-1 - 2*m)*d*(-(((-1)^(3/4)*(I +
 E^(I*(e + f*x))))/E^((I/2)*(e + f*x))))^(1 + 2*m)*(E^((2*I)*(e + f*x))*(-1
 + m)*Hypergeometric2F1[1, m, -m, (-I)/E^(I*(e + f*x))] - (1 + m)*Hypergeom
etric2F1[1, 2 + m, 2 - m, (-I)/E^(I*(e + f*x))]))/(E^(((3*I)/2)*(e + f*x))*
(-1 + m^2)) + (2*Sqrt[2]*c*Cos[(2*e - Pi + 2*f*x)/4]^(1 + 2*m)*Hypergeometr
ic2F1[1/2, 1/2 + m, 3/2 + m, Sin[(2*e + Pi + 2*f*x)/4]^2]*Sin[(2*e - Pi + 2
*f*x)/4])/((1 + 2*m)*Sqrt[1 - Sin[e + f*x]])))/(f*Sin[(2*e + Pi + 2*f*x)/4]
^(2*m)))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d\sin\left(fx+e\right)+c\right)\left(a\sin\left(fx+e\right)+a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e)),x, algorithm="fricas")
[Out] integral((d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \sin \left(f x + e\right) + c\right) \left(a \sin \left(f x + e\right) + a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e)),x, algorithm="giac")
[Out] integrate((d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

maple [F] time = 2.12, size = 0, normalized size = 0.00

$$\int \left(a + a \sin \left(f x + e\right)\right)^{m} \left(c + d \sin \left(f x + e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e)),x)
[Out] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \sin \left(f x + e\right) + c\right) \left(a \sin \left(f x + e\right) + a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a + a\,\sin\left(e + f\,x\right)\right)}^{m}\,\left(c + d\,\sin\left(e + f\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c + d*sin(e + f*x)),x)
[Out] int((a + a*sin(e + f*x))^m*(c + d*sin(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\sin \left(e + f x\right) + 1\right)\right)^{m} \left(c + d \sin \left(e + f x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c+d*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(c + d*sin(e + f*x)), x)
```

3.610 $\int (a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=74

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f}$$

```
[Out] -2^(1/2+m)*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+s
in(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/f
```

Rubi [A] time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2652, 2651}

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 + m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e
 + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/f)
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m\,dx = \left((1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\right)\int (1+\sin(e+fx))^m\,dx$$
$$= -\frac{2^{\frac{1}{2}+m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{2}-m}(a+a$$
$$f$$

Mathematica [A] time = 0.14, size = 90, normalized size = 1.22

$$\frac{\sqrt{2}\cos(e+fx)(a(\sin(e+fx)+1))^m\,{}_2F_1\left(\frac{1}{2},m+\frac{1}{2};m+\frac{3}{2};\frac{1}{4}\cos^2(e+fx)\csc^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)}{(2fm+f)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Sqrt[2]*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 + m, 3/2 + m, (Cos[e + f*x
]^2*Csc[(2*e - Pi + 2*f*x)/4]^2)/4]*(a*(1 + Sin[e + f*x]))^m)/((f + 2*f*m)*
Sqrt[1 - Sin[e + f*x]])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m, x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m,x)
```

```
[Out] int((a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin \left(fx+e\right)+a\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+a\,\sin\left(e+f\,x\right)\right)^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m,x)
```

```
[Out] int((a + a*sin(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin \left(e+fx\right)+a\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*sin(e + f*x) + a)**m, x)
```

3.611 $\int \frac{(a+a\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=100

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},1;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)(c-d)\sqrt{1-\sin(e+fx)}}$$

```
[Out] AppellF1(1/2+m,1,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(
f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)/(c-d)/f/(1+2*m)/(1-sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2788, 137, 136}

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},1;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)(c-d)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c + d*Sin[e + f*x]),x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, 1, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 +
 Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/((c - d)*f*(
1 + 2*m)*Sqrt[1 - Sin[e + f*x]])
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx = \frac{\left(a^2\cos(e+fx)\right)\mathrm{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}\,(c+dx)}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\mathrm{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}\,(c+dx)}\,dx, x, \sin(e+fx)\right)}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;\frac{1}{2},1;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(}{(c-d)f(1+2m)\sqrt{1-\sin(e+fx)}}$$

Mathematica [B] time = 1.13, size = 363, normalized size = 3.63

$$-\frac{6(c+d)\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2}{f(c+d\sin(e+fx))\left(\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\left(4dF_1\left(\frac{3}{2};\frac{1}{2}-m,2;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+2fx+\pi)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)}{c+d}\right)\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c + d*Sin[e + f*x]),x]
```

```
[Out] (-6*(c + d)*AppellF1[1/2, 1/2 - m, 1, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*
d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2
 + m)*Cot[(2*e + Pi + 2*f*x)/4]*(a*(1 + Sin[e + f*x]))^m*(Sin[(2*e + Pi + 2
*f*x)/4]^2)^(1/2 - m))/(f*(c + d*Sin[e + f*x])*(3*(c + d)*AppellF1[1/2, 1/2
 - m, 1, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2
)/(c + d)] + (4*d*AppellF1[3/2, 1/2 - m, 2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^
2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m)*AppellF1
```

```
[3/2, 3/2 - m, 1, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*
f*x)/4]^2)/(c + d)])*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{d\sin\left(fx+e\right)+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e)),x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e)),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c), x)
```

maple [F] time = 1.41, size = 0, normalized size = 0.00

$$\int\frac{\left(a+a\sin\left(fx+e\right)\right)^{m}}{c+d\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e)),x)
[Out] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{c+d\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c + d*sin(e + f*x)),x)
[Out] int((a + a*sin(e + f*x))^m/(c + d*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c+d*sin(f*x+e)),x)
[Out] Timed out
```

3.612 $\int \frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=100

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},2;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)(c-d)^2\sqrt{1-\sin(e+fx)}}$$

```
[Out] AppellF1(1/2+m,2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(
f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)/(c-d)^2/f/(1+2*m)/(1-sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2788, 137, 136}

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},2;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)(c-d)^2\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, 2, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 +
 Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/((c - d)^2*f
*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]])
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^2}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}\,(c+dx)^2}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}\,(c+dx)^2}\,dx, x, \sin(e+fx)\right)}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;\frac{1}{2},2;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(}{(c-d)^2f(1+2m)\sqrt{1-\sin(e+fx)}}$$

Mathematica [B] time = 1.27, size = 363, normalized size = 3.63

$$-\frac{6(c+d)\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2}{f(c+d\sin(e+fx))^2\left(\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)\left(8dF_1\left(\frac{3}{2};\frac{1}{2}-m,3;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+2fx+\pi)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)}{c+d}\right)\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (-6*(c + d)*AppellF1[1/2, 1/2 - m, 2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*
d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2
 + m)*Cot[(2*e + Pi + 2*f*x)/4]*(a*(1 + Sin[e + f*x]))^m*(Sin[(2*e + Pi + 2
*f*x)/4]^2)^(1/2 - m))/(f*(c + d*Sin[e + f*x])^2*(3*(c + d)*AppellF1[1/2, 1
/2 - m, 2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]
^2)/(c + d)] + (8*d*AppellF1[3/2, 1/2 - m, 3, 5/2, Cos[(2*e + Pi + 2*f*x)/4
]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m)*Appell
```

```
F1[3/2, 3/2 - m, 2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi +
2*f*x)/4]^2)/(c + d)])*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{d^{2}\cos\left(fx+e\right)^{2}-2\,cd\sin\left(fx+e\right)-c^{2}-d^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
[Out] integral(-(a*sin(f*x + e) + a)^m/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) -
 c^2 - d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(d\sin\left(fx+e\right)+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^2, x)
```

maple [F] time = 1.84, size = 0, normalized size = 0.00

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^{m}}{\left(c+d\sin\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x)
[Out] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(d\sin\left(fx+e\right)+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(c+d\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c + d*sin(e + f*x))^2,x)
[Out] int((a + a*sin(e + f*x))^m/(c + d*sin(e + f*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c+d*sin(f*x+e))**2,x)
[Out] Timed out
```

3.613 $\int \frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=100

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},3;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)(c-d)^3\sqrt{1-\sin(e+fx)}}$$

```
[Out] AppellF1(1/2+m,3,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(
f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)/(c-d)^3/f/(1+2*m)/(1-sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2788, 137, 136}

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},3;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)(c-d)^3\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, 3, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 +
 Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/((c - d)^3*f
*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]])
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^3}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}\,(c+dx)^3}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}\,(c+dx)^3}\,dx,x,\sin(e+fx)\right)}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;\frac{1}{2},3;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(}{(c-d)^3f(1+2m)\sqrt{1-\sin(e+fx)}}$$

Mathematica [B] time = 1.42, size = 363, normalized size = 3.63

$$-\frac{6(c+d)\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^{}}{f(c+d\sin(e+fx))^3\left(\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\left(12dF_1\left(\frac{3}{2};\frac{1}{2}-m,4;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+2fx+\pi)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)}{c+d}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (-6*(c + d)*AppellF1[1/2, 1/2 - m, 3, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*
d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2
 + m)*Cot[(2*e + Pi + 2*f*x)/4]*(a*(1 + Sin[e + f*x]))^m*(Sin[(2*e + Pi + 2
*f*x)/4]^2)^(1/2 - m))/(f*(c + d*Sin[e + f*x])^3*(3*(c + d)*AppellF1[1/2, 1
/2 - m, 3, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]
^2)/(c + d)] + (12*d*AppellF1[3/2, 1/2 - m, 4, 5/2, Cos[(2*e + Pi + 2*f*x)/
4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m)*Appel
```

```
1F1[3/2, 3/2 - m, 3, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi +
 2*f*x)/4]^2)/(c + d)])*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a\sin\left(fx+e\right)+a\right)^m}{3\,cd^2\cos\left(fx+e\right)^2-c^3-3\,cd^2+\left(d^3\cos\left(fx+e\right)^2-3\,c^2d-d^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^m/(3*c*d^2*cos(f*x + e)^2 - c^3 - 3*c*d^2 +
(d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\sin\left(fx+e\right)+a\right)^m}{\left(d\sin\left(fx+e\right)+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^3, x)
```

maple [F] time = 2.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+a\sin\left(fx+e\right)\right)^m}{\left(c+d\sin\left(fx+e\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x)
```

```
[Out] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\sin\left(fx+e\right)+a\right)^m}{\left(d\sin\left(fx+e\right)+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(c+d\,\sin\left(e+f\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c + d*sin(e + f*x))^3,x)
[Out] int((a + a*sin(e + f*x))^m/(c + d*sin(e + f*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c+d*sin(f*x+e))**3,x)
[Out] Timed out
```

3.614 $\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=138

$$\frac{\sqrt{2}\,(c-d)^2\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}\,F_1\left(m+\frac{1}{2};\frac{1}{2},-\frac{5}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin}{}\right.}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

```
[Out] (c-d)^2*AppellF1(1/2+m,-5/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f
*x+e))*cos(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*(c+d*sin(f*x+e))^(1/2)/f/(1+2*
m)/(1-sin(f*x+e))^(1/2)/((c+d*sin(f*x+e))/(c-d))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2788, 140, 139, 138}

$$\frac{\sqrt{2}\,(c-d)^2\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}\,F_1\left(m+\frac{1}{2};\frac{1}{2},-\frac{5}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin}{}\right.}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[2]*(c - d)^2*AppellF1[1/2 + m, 1/2, -5/2, 3/2 + m, (1 + Sin[e + f*x])
/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*
Sqrt[c + d*Sin[e + f*x]])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[(c + d*S
in[e + f*x])/(c - d)])
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
```

```
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m (c+d\sin(e+fx))^{5/2}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}(c+dx)^{5/2}}{\sqrt{a-ax}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}(c+dx)^{5/2}}{\sqrt{\frac{1}{2}-\frac{x}{2}}}\,dx, x,\right.}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left((ac-ad)^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\,\sqrt{c+d\sin(e+fx)}\right)\text{Subst}}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{a+a\sin(e+}}$$

$$= \frac{\sqrt{2}\,(c-d)^2 F_1\left(\frac{1}{2}+m;\frac{1}{2},-\frac{5}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(}{c-}\right.}{f(1+2m)\sqrt{1-\sin(e+}}$$

Mathematica [B] time = 1.90, size = 365, normalized size = 2.64

$$-\frac{3\sqrt{2}(c+d)\sqrt{\sin(e+fx)+1}\tan\left(\frac{1}{4}(2e+2fx-\pi\right.}{f\sqrt{\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)}\left(\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\left(10dF_1\left(\frac{3}{2};\frac{1}{2}-m,-\frac{3}{2};\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+2fx+\pi)\right),\frac{2d\sin^2\left(\frac{1}{4}\right.}{\ldots}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-3*Sqrt[2]*(c + d)*AppellF1[1/2, 1/2 - m, -5/2, 3/2, Cos[(2*e + Pi + 2*f*x
)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*Sqrt[1 + Sin[e + f*x]]*(
a*(1 + Sin[e + f*x]))^m*(c + d*Sin[e + f*x])^(5/2)*Tan[(2*e - Pi + 2*f*x)/4
])/(f*Sqrt[Cos[(2*e - Pi + 2*f*x)/4]^2]*(-3*(c + d)*AppellF1[1/2, 1/2 - m,
-5/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(
c + d)] + (10*d*AppellF1[3/2, 1/2 - m, -3/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]
^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (c + d)*(-1 + 2*m)*AppellF
1[3/2, 3/2 - m, -5/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi
+ 2*f*x)/4]^2)/(c + d)])*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2\right)\sqrt{d\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)*sqrt(d*sin(
f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\left(a\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)*(a*sin(f*x + e) + a)^m, x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int\left(a+a\sin\left(fx+e\right)\right)^m\left(c+d\sin\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \sin\left(fx+e\right)+c\right)^{\frac{5}{2}} \left(a \sin\left(fx+e\right)+a\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)*(a*sin(f*x + e) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.615 $\int (a + a\sin(e + fx))^m (c + d\sin(e + fx))^{3/2}\, dx$

Optimal. Leaf size=136

$$\frac{\sqrt{2}\,(c-d)\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}\,F_1\left(m+\frac{1}{2};\frac{1}{2},-\frac{3}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin}{}\right.}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

```
[Out] (c-d)*AppellF1(1/2+m,-3/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x
+e))*cos(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*(c+d*sin(f*x+e))^(1/2)/f/(1+2*m)
/(1-sin(f*x+e))^(1/2)/((c+d*sin(f*x+e))/(c-d))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2788, 140, 139, 138}

$$\frac{\sqrt{2}\,(c-d)\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}\,F_1\left(m+\frac{1}{2};\frac{1}{2},-\frac{3}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin}{}\right.}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sqrt[2]*(c - d)*AppellF1[1/2 + m, 1/2, -3/2, 3/2 + m, (1 + Sin[e + f*x])/2
, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sq
rt[c + d*Sin[e + f*x]])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[(c + d*Sin
[e + f*x])/(c - d)])
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
```

```
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{3/2}\,dx = \frac{\left(a^2\cos(e+fx)\right)\operatorname{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}(c+dx)^{3/2}}{\sqrt{a-ax}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\operatorname{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}(c+dx)^{3/2}}{\sqrt{\frac{1}{2}-\frac{x}{2}}}\,dx, x,\right.}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a(ac-ad)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\,\sqrt{c+d\sin(e+fx)}\right)\operatorname{Subst}}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{a+a\sin(e+}}$$

$$= \frac{\sqrt{2}\,(c-d)F_1\left(\frac{1}{2}+m;\frac{1}{2},-\frac{3}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e}{c-d}\right.}{f(1+2m)\sqrt{1-\sin(e+}}$$

Mathematica [B] time = 1.29, size = 365, normalized size = 2.68

$$-\frac{3\sqrt{2}(c+d)\sqrt{\sin(e+fx)+1}\tan\left(\frac{1}{4}(2e+2fx-\pi\right.}{f\sqrt{\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)}\left(\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\left(6dF_1\left(\frac{3}{2};\frac{1}{2}-m,-\frac{1}{2};\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+2fx+\pi)\right),\frac{2d\sin^2\left(\frac{1}{4}(2\right.}{c+}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-3*Sqrt[2]*(c + d)*AppellF1[1/2, 1/2 - m, -3/2, 3/2, Cos[(2*e + Pi + 2*f*x
)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*Sqrt[1 + Sin[e + f*x]]*(
a*(1 + Sin[e + f*x]))^m*(c + d*Sin[e + f*x])^(3/2)*Tan[(2*e - Pi + 2*f*x)/4
])/(f*Sqrt[Cos[(2*e - Pi + 2*f*x)/4]^2]*(-3*(c + d)*AppellF1[1/2, 1/2 - m,
-3/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(
c + d)] + (6*d*AppellF1[3/2, 1/2 - m, -1/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^
2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (c + d)*(-1 + 2*m)*AppellF1
[3/2, 3/2 - m, -3/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi +
 2*f*x)/4]^2)/(c + d)])*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e) + a)^m, x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \sin\left(fx+e\right)+c\right)^{\frac{3}{2}} \left(a \sin\left(fx+e\right)+a\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{3/2} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.616 $\int (a+a\sin(e+fx))^m\sqrt{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=131

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}\,F_1\left(m+\frac{1}{2};\frac{1}{2},-\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

```
[Out] AppellF1(1/2+m,-1/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*c
os(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*(c+d*sin(f*x+e))^(1/2)/f/(1+2*m)/(1-si
n(f*x+e))^(1/2)/((c+d*sin(f*x+e))/(c-d))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2788, 140, 139, 138}

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}\,F_1\left(m+\frac{1}{2};\frac{1}{2},-\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, -1/2, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(
1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[c + d
*Sin[e + f*x]])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[(c + d*Sin[e + f*x
])/(c - d)])
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
```

```
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m\sqrt{c+d\sin(e+fx)}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}\sqrt{c+dx}}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}\sqrt{c+dx}}{\sqrt{\frac{1}{2}-\frac{x}{2}}}\,dx,x,\sin\right.}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\sqrt{c+d\sin(e+fx)}\right)\text{Subst}\left(\int \frac{(a+ax)}{}\right.}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\sqrt{}}$$

$$= \frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;\frac{1}{2},-\frac{1}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\text{co}}{f(1+2m)\sqrt{1-\sin(e+fx)}\sqrt{}}$$

Mathematica [B] time = 1.10, size = 365, normalized size = 2.79

$$-\frac{3\sqrt{2}\,(c+d)\sqrt{\sin(e+fx)+1}\,\tan\left(\frac{1}{4}(2e+2fx-\pi\right.}{f\sqrt{\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)}\left(\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\left(2dF_1\left(\frac{3}{2};\frac{1}{2}-m,\frac{1}{2};\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+2fx+\pi)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e\right.}{c+d}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-3*Sqrt[2]*(c + d)*AppellF1[1/2, 1/2 - m, -1/2, 3/2, Cos[(2*e + Pi + 2*f*x
)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*Sqrt[1 + Sin[e + f*x]]*(
a*(1 + Sin[e + f*x]))^m*Sqrt[c + d*Sin[e + f*x]]*Tan[(2*e - Pi + 2*f*x)/4])
/(f*Sqrt[Cos[(2*e - Pi + 2*f*x)/4]^2]*(-3*(c + d)*AppellF1[1/2, 1/2 - m, -1
/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c
+ d)] + (2*d*AppellF1[3/2, 1/2 - m, 1/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2,
(2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (c + d)*(-1 + 2*m)*AppellF1[3/
2, 3/2 - m, -1/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*
f*x)/4]^2)/(c + d)])*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{d\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{d\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int\left(a+a\sin\left(fx+e\right)\right)^{m}\sqrt{c+d\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d \sin\left(fx+e\right)+c}\left(a \sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\sqrt{c+d\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\sin\left(e+fx\right)+1\right)\right)^{m}\sqrt{c+d\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*sqrt(c + d*sin(e + f*x)), x)
```

3.617 $\int \frac{(a+a\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=131

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\,F_1\left(m+\frac{1}{2};\frac{1}{2},\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

```
[Out] AppellF1(1/2+m,1/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*co
s(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*((c+d*sin(f*x+e))/(c-d))^(1/2)/f/(1+2*m
)/(1-sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2788, 140, 139, 138}

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\,F_1\left(m+\frac{1}{2};\frac{1}{2},\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, 1/2, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1
 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[(c + d
*Sin[e + f*x])/(c - d)])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[c + d*Sin
[e + f*x]])
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
```

```
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}\,\sqrt{c+dx}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}\,\sqrt{c+dx}}\,dx,x,\sin(e+fx)\right)}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\,\sqrt{\frac{a(c+d\sin(e+fx))}{ac-ad}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}\,\sqrt{\frac{ac}{ac-ad}+\frac{adx}{ac-ad}}}\,dx,x,\sin\right.}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;\frac{1}{2},\frac{1}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(}{f(1+2m)\sqrt{1-\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}$$

Mathematica [B] time = 1.17, size = 373, normalized size = 2.85

$$-\frac{6(c+d)\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos}{f\sqrt{c+d\sin(e+fx)}\left(\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\left(2dF_1\left(\frac{3}{2};\frac{1}{2}-m,\frac{3}{2};\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+2fx+\pi)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)}{c+d}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-6*(c + d)*AppellF1[1/2, 1/2 - m, 1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (
2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1
/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*(a*(1 + Sin[e + f*x]))^m*(Sin[(2*e + Pi +
 2*f*x)/4]^2)^(1/2 - m))/(f*Sqrt[c + d*Sin[e + f*x]]*(3*(c + d)*AppellF1[1/
2, 1/2 - m, 1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f
*x)/4]^2)/(c + d)] + (2*d*AppellF1[3/2, 1/2 - m, 3/2, 5/2, Cos[(2*e + Pi +
2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m
)*AppellF1[3/2, 3/2 - m, 1/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2
*e - Pi + 2*f*x)/4]^2)/(c + d)])*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a\sin\left(fx+e\right)+a\right)^m}{\sqrt{d\sin\left(fx+e\right)+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\sin\left(fx+e\right)+a\right)^m}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) + c), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^{m}}{\sqrt{c+d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x)
[Out] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c + d*sin(e + f*x))^(1/2),x)
[Out] int((a + a*sin(e + f*x))^m/(c + d*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c+d*sin(f*x+e))**(1/2),x)
[Out] Integral((a*(sin(e + f*x) + 1))**m/sqrt(c + d*sin(e + f*x)), x)
```

3.618 $\int \frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=138

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\,F_1\left(m+\frac{1}{2};\frac{1}{2},\frac{3}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)(c-d)\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

```
[Out] AppellF1(1/2+m,3/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*co
s(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*((c+d*sin(f*x+e))/(c-d))^(1/2)/(c-d)/f/
(1+2*m)/(1-sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2788, 140, 139, 138}

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\,F_1\left(m+\frac{1}{2};\frac{1}{2},\frac{3}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)(c-d)\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, 3/2, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1
 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[(c + d
*Sin[e + f*x])/(c - d)])/((c - d)*f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[c
 + d*Sin[e + f*x]])
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
```

```
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}\,(c+dx)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}\,(c+dx)^{3/2}}\,dx, x, \sin(e+fx)\right)}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^3\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\,\sqrt{\frac{a(c+d\sin(e+fx))}{ac-ad}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}\left(\frac{ac}{ac-ad}+\frac{adx}{ac-ad}\right)^{3/2}}\,dx, x, s\right.}{\sqrt{2}\,(ac-ad)f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;\frac{1}{2},\frac{3}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin}{(c-d)f(1+2m)\sqrt{1-\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}$$

Mathematica [B] time = 1.31, size = 373, normalized size = 2.70

$$-\frac{6(c+d)\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\text{co}}{f(c+d\sin(e+fx))^{3/2}\left(\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\left(6dF_1\left(\frac{3}{2};\frac{1}{2}-m,\frac{5}{2};\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+2fx+\pi)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e+2fx-\pi\right.}{c+d}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-6*(c + d)*AppellF1[1/2, 1/2 - m, 3/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (
2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1
/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*(a*(1 + Sin[e + f*x]))^m*(Sin[(2*e + Pi +
 2*f*x)/4]^2)^(1/2 - m))/(f*(c + d*Sin[e + f*x])^(3/2)*(3*(c + d)*AppellF1[
1/2, 1/2 - m, 3/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2
*f*x)/4]^2)/(c + d)] + (6*d*AppellF1[3/2, 1/2 - m, 5/2, 5/2, Cos[(2*e + Pi
+ 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2
*m)*AppellF1[3/2, 3/2 - m, 3/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[
(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{d\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^m}{d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(d^2*cos(f*x + e)
^2 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\sin\left(fx+e\right)+a\right)^m}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^(3/2), x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^{m}}{\left(c+d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x)
[Out] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c + d*sin(e + f*x))^(3/2),x)
[Out] int((a + a*sin(e + f*x))^m/(c + d*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}}{\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m/(c+d*sin(f*x+e))**(3/2),x)
[Out] Integral((a*(sin(e + f*x) + 1))**m/(c + d*sin(e + f*x))**(3/2), x)
```

3.619 $\int \frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=138

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\,F_1\left(m+\frac{1}{2};\frac{1}{2},\frac{5}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)(c-d)^2\sqrt{1-\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}$$

```
[Out] AppellF1(1/2+m,5/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*co
s(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*((c+d*sin(f*x+e))/(c-d))^(1/2)/(c-d)^2/
f/(1+2*m)/(1-sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2788, 140, 139, 138}

$$\frac{\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\,F_1\left(m+\frac{1}{2};\frac{1}{2},\frac{5}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)(c-d)^2\sqrt{1-\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, 5/2, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1
 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[(c + d
*Sin[e + f*x])/(c - d)])/((c - d)^2*f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt
[c + d*Sin[e + f*x]])
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
```

```
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^{5/2}}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}\,(c+dx)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}\,(c+dx)^{5/2}}\,dx, x, \sin(e+fx)\right)}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^4\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\,\sqrt{\frac{a(c+d\sin(e+fx))}{ac-ad}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}\left(\frac{ac}{ac-ad}+\frac{adx}{ac-ad}\right)^{5/2}}\,dx, x, s\right.}{\sqrt{2}\,(ac-ad)^2 f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;\frac{1}{2},\frac{5}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin}{(c-d)^2 f(1+2m)\sqrt{1-\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}$$

Mathematica [B] time = 1.53, size = 373, normalized size = 2.70

$$-\frac{6(c+d)\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\operatorname{co}\cdots}{f(c+d\sin(e+fx))^{5/2}\left(\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\left(10dF_1\left(\frac{3}{2};\frac{1}{2}-m,\frac{7}{2};\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+2fx+\pi)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e+2fx-\cdots\right.}{c+d}\cdots\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-6*(c + d)*AppellF1[1/2, 1/2 - m, 5/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (
2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1
/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*(a*(1 + Sin[e + f*x]))^m*(Sin[(2*e + Pi +
 2*f*x)/4]^2)^(1/2 - m))/(f*(c + d*Sin[e + f*x])^(5/2)*(3*(c + d)*AppellF1[
1/2, 1/2 - m, 5/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2
*f*x)/4]^2)/(c + d)] + (10*d*AppellF1[3/2, 1/2 - m, 7/2, 5/2, Cos[(2*e + Pi
 + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 +
2*m)*AppellF1[3/2, 3/2 - m, 5/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin
[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Sin[(2*e - Pi + 2*f*x)/4]^2))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{d\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^m}{3\,cd^2\cos\left(fx+e\right)^2-c^3-3\,cd^2+\left(d^3\cos\left(fx+e\right)^2-3\,c^2d-d^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(3*c*d^2*cos(f*x
+ e)^2 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e))
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\sin\left(fx+e\right)+a\right)^m}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

[Out] integrate((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^(5/2), x)

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^{m}}{\left(c+d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x)

[Out] int((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*sin(e + f*x))^m/(c + d*sin(e + f*x))^(5/2),x)

[Out] int((a + a*sin(e + f*x))^m/(c + d*sin(e + f*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**m/(c+d*sin(f*x+e))**(5/2),x)

[Out] Timed out

3.620 $\int (1+\sin(e+fx))^m(3+5\sin(e+fx))^{-1-m}\,dx$

Optimal. Leaf size=62

$$-\frac{4^{-m-1}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};\frac{1-\sin(e+fx)}{4(\sin(e+fx)+1)}\right)}{f(\sin(e+fx)+1)}$$

```
[Out] -4^(-1-m)*cos(f*x+e)*hypergeom([1/2, 1+m],[3/2],1/4*(1-sin(f*x+e))/(1+sin(f
*x+e)))/f/(1+sin(f*x+e))
```

Rubi [A] time = 0.11, antiderivative size = 111, normalized size of antiderivative = 1.79, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2788, 132}

$$-\frac{2^{-2m-1}\cos(e+fx)(\sin(e+fx)+1)^{m-1}\left(\frac{\sin(e+fx)+1}{5\sin(e+fx)+3}\right)^{\frac{1}{2}-m}(5\sin(e+fx)+3)^{-m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};-\frac{1-\sin(e+fx)}{5\sin(e+fx)+3}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[e + f*x])^m*(3 + 5*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] -((2^(-1 - 2*m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, -((1 - Si
n[e + f*x])/(3 + 5*Sin[e + f*x]))]*(1 + Sin[e + f*x])^(-1 + m)*((1 + Sin[e
+ f*x])/(3 + 5*Sin[e + f*x]))^(1/2 - m))/(f*(3 + 5*Sin[e + f*x])^m))
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (1+\sin(e+fx))^m(3+5\sin(e+fx))^{-1-m}\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(1+x)^{-\frac{1}{2}+m}(3+5x)^{-1-m}}{\sqrt{1-x}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{2^{-1-2m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};-\frac{1-\sin(e+fx)}{3+5\sin(e+fx)}\right)(1+\sin(e}{f}$$

Mathematica [C] time = 1.40, size = 238, normalized size = 3.84

$$\frac{4^m(\cosh(m\log(4))-\sinh(m\log(4)))(\sin(e+fx)+1)^m(5\sin(e+fx)+3)^{-m}(\sin(e+fx)+i\cos(e+fx)+1)\Big(-}{f(2m+1)((2+i)\sin(e+fx)+(-1+2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sin[e + f*x])^m*(3 + 5*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] (4^m*Hypergeometric2F1[1 + m, 1 + 2*m, 2*(1 + m), (4*Cos[(2*e - Pi + 2*f*x)
/4])/(2*Cos[(2*e - Pi + 2*f*x)/4] + Sin[(2*e - Pi + 2*f*x)/4])]*(1 + Sin[e
+ f*x])^m*(1 + I*Cos[e + f*x] + Sin[e + f*x])*(-((2*Cos[(2*e - Pi + 2*f*x)/
4] + Cos[(2*e + Pi + 2*f*x)/4])/(2*Cos[(2*e - Pi + 2*f*x)/4] + Sin[(2*e - P
i + 2*f*x)/4])))^m*(Cosh[m*Log[4]] - Sinh[m*Log[4]]))/(f*(1 + 2*m)*((2 - I)
 - (1 - 2*I)*Cos[e + f*x] + (2 + I)*Sin[e + f*x])*(3 + 5*Sin[e + f*x])^m)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(5\,\sin\left(fx+e\right)+3\right)^{-m-1}\left(\sin\left(fx+e\right)+1\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+5*sin(f*x+e))^(-1-m),x, algorithm="fricas")
```

```
[Out] integral((5*sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(5\,\sin\left(fx+e\right)+3\right)^{-m-1}\left(\sin\left(fx+e\right)+1\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+5*sin(f*x+e))^(-1-m),x, algorithm="giac")
[Out] integrate((5*sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \left(1+\sin \left(f x+e\right)\right)^{m} \left(3+5 \sin \left(f x+e\right)\right)^{-1-m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sin(f*x+e))^m*(3+5*sin(f*x+e))^(-1-m),x)
[Out] int((1+sin(f*x+e))^m*(3+5*sin(f*x+e))^(-1-m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int {\left(5 \, \sin \left(f x + e\right) + 3\right)}^{-m - 1} {\left(\sin \left(f x + e\right) + 1\right)}^{m}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+5*sin(f*x+e))^(-1-m),x, algorithm="maxima")
[Out] integrate((5*sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(\sin\left(e+f\,x\right)+1\right)}^m}{{\left(5\,\sin\left(e+f\,x\right)+3\right)}^{m+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x) + 1)^m/(5*sin(e + f*x) + 3)^(m + 1),x)
[Out] int((sin(e + f*x) + 1)^m/(5*sin(e + f*x) + 3)^(m + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))**m*(3+5*sin(f*x+e))**(-1-m),x)
[Out] Timed out
```

3.621 $\int (1+\sin(e+fx))^m (3+4\sin(e+fx))^{-1-m}\,dx$

Optimal. Leaf size=64

$$-\frac{\left(\frac{7}{2}\right)^{-m-1}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};\frac{1-\sin(e+fx)}{7(\sin(e+fx)+1)}\right)}{f(\sin(e+fx)+1)}$$

```
[Out] -(7/2)^(-1-m)*cos(f*x+e)*hypergeom([1/2, 1+m],[3/2],1/7*(1-sin(f*x+e))/(1+s
in(f*x+e)))/f/(1+sin(f*x+e))
```

Rubi [A] time = 0.12, antiderivative size = 122, normalized size of antiderivative = 1.91, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2788, 132}

$$-\frac{2^{m+\frac{1}{2}}7^{-m-\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{m-1}\left(\frac{\sin(e+fx)+1}{4\sin(e+fx)+3}\right)^{\frac{1}{2}-m}(4\sin(e+fx)+3)^{-m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};-\frac{1-\sin(e+}{2(4\sin(e+f}\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[e + f*x])^m*(3 + 4*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] -((2^(1/2 + m)*7^(-1/2 - m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/
2, -(1 - Sin[e + f*x])/(2*(3 + 4*Sin[e + f*x]))]*(1 + Sin[e + f*x])^(-1 + m
)*((1 + Sin[e + f*x])/(3 + 4*Sin[e + f*x]))^(1/2 - m))/(f*(3 + 4*Sin[e + f*
x])^m))
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (1+\sin(e+fx))^m(3+4\sin(e+fx))^{-1-m}\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(1+x)^{-\frac{1}{2}+m}(3+4x)^{-1-m}}{\sqrt{1-x}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{2^{\frac{1}{2}+m}7^{-\frac{1}{2}-m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};-\frac{1-\sin(e+fx)}{2(3+4\sin(e+fx))}\right)(1}{f}$$

Mathematica [A] time = 0.48, size = 88, normalized size = 1.38

$$-\frac{2\,7^{-m-1}\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)(\sin(e+fx)+1)^m\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{-m}\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};\frac{1}{7}\tan^2\left(\frac{1}{4}(2e+2\right.\right.}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sin[e + f*x])^m*(3 + 4*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] (-2*7^(-1 - m)*Cot[(2*e + Pi + 2*f*x)/4]*Hypergeometric2F1[1/2, 1 + m, 3/2,
 Tan[(2*e - Pi + 2*f*x)/4]^2/7]*(1 + Sin[e + f*x])^m)/(f*(Sin[(2*e + Pi + 2
*f*x)/4]^2)^m)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(4\sin\left(fx+e\right)+3\right)^{-m-1}\left(\sin\left(fx+e\right)+1\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+4*sin(f*x+e))^(-1-m),x, algorithm="fricas")
```

```
[Out] integral((4*sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(4\sin\left(fx+e\right)+3\right)^{-m-1}\left(\sin\left(fx+e\right)+1\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+4*sin(f*x+e))^(-1-m),x, algorithm="giac")
```

```
[Out] integrate((4*sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

maple [F] time = 0.67, size = 0, normalized size = 0.00

$$\int \left(1+\sin \left(f x +e \right)\right)^{m} \left(3+4 \sin \left(f x +e \right)\right)^{-1-m}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sin(f*x+e))^m*(3+4*sin(f*x+e))^(-1-m),x)
```

```
[Out] int((1+sin(f*x+e))^m*(3+4*sin(f*x+e))^(-1-m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (4 \sin \left(fx + e\right) + 3)^{-m-1} (\sin \left(fx + e\right) + 1)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+4*sin(f*x+e))^(-1-m),x, algorithm="maxima")
```

```
[Out] integrate((4*sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(\sin\left(e+f\,x\right)+1\right)}^m}{{\left(4\,\sin\left(e+f\,x\right)+3\right)}^{m+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x) + 1)^m/(4*sin(e + f*x) + 3)^(m + 1),x)
```

```
[Out] int((sin(e + f*x) + 1)^m/(4*sin(e + f*x) + 3)^(m + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))**m*(3+4*sin(f*x+e))**(-1-m),x)
```

```
[Out] Timed out
```

3.622 $\int (1+\sin(e+fx))^m(3+3\sin(e+fx))^{-1-m}\,dx$

Optimal. Leaf size=28

$$-\frac{3^{-m-1}\cos(e+fx)}{f(\sin(e+fx)+1)}$$

```
[Out] -3^(-1-m)*cos(f*x+e)/f/(1+sin(f*x+e))
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {22, 2648}

$$-\frac{3^{-m-1}\cos(e+fx)}{f(\sin(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[e + f*x])^m*(3 + 3*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] -((3^(-1 - m)*Cos[e + f*x])/(f*(1 + Sin[e + f*x])))
```

Rule 22

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x
] && EqQ[b*c - a*d, 0] && GtQ[b/d, 0] &&  !(IntegerQ[m] || IntegerQ[n])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int (1+\sin(e+fx))^m(3+3\sin(e+fx))^{-1-m}\,dx &= 3^{-m}\int \frac{1}{3+3\sin(e+fx)}\,dx\\ &= -\frac{3^{-1-m}\cos(e+fx)}{f(1+\sin(e+fx))}\end{aligned}$$

Mathematica [A] time = 0.04, size = 45, normalized size = 1.61

$$\frac{2\,3^{-m-1}\sin\left(\frac{1}{2}(e+fx)\right)}{f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[e + f*x])^m*(3 + 3*Sin[e + f*x])^(-1 - m),x]
[Out] (2*3^(-1 - m)*Sin[(e + f*x)/2])/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

fricas [A] time = 0.43, size = 54, normalized size = 1.93

$$-\frac{3^{-m-1}\left(\cos\left(fx+e\right)+1\right)-3^{-m-1}\sin\left(fx+e\right)}{f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+3*sin(f*x+e))^(-1-m),x, algorithm="fricas")
[Out] -(3^(-m - 1)*(cos(f*x + e) + 1) - 3^(-m - 1)*sin(f*x + e))/(f*cos(f*x + e)
+ f*sin(f*x + e) + f)
```

giac [A] time = 0.61, size = 45, normalized size = 1.61

$$\frac{3^{-m-1}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-3^{-m-1}}{f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+3*sin(f*x+e))^(-1-m),x, algorithm="giac")
[Out] (3^(-m - 1)*tan(1/2*f*x + 1/2*e) - 3^(-m - 1))/(f*tan(1/2*f*x + 1/2*e) + f)
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int\left(1+\sin\left(fx+e\right)\right)^{m}\left(3+3\sin\left(fx+e\right)\right)^{-1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sin(f*x+e))^m*(3+3*sin(f*x+e))^(-1-m),x)
[Out] int((1+sin(f*x+e))^m*(3+3*sin(f*x+e))^(-1-m),x)
```

maxima [A] time = 0.84, size = 35, normalized size = 1.25

$$-\frac{2}{\left(3^{m+1}+\frac{3^{m+1}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+3*sin(f*x+e))^(-1-m),x, algorithm="maxima")
[Out] -2/((3^(m + 1) + 3^(m + 1)*sin(f*x + e)/(cos(f*x + e) + 1))*f)
```

mupad [B] time = 0.43, size = 41, normalized size = 1.46

$$\frac{\frac{1}{3^{m+1}}\left(-\cos\left(e+f\,x\right)+\sin\left(e+f\,x\right)\,1\mathrm{i}+1\mathrm{i}\right)}{f\,\left(\sin\left(e+f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x) + 1)^m/(3*sin(e + f*x) + 3)^(m + 1),x)
[Out] (1/3^(m + 1)*(sin(e + f*x)*1i - cos(e + f*x) + 1i))/(f*(sin(e + f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))**m*(3+3*sin(f*x+e))**(-1-m),x)
[Out] Timed out
```

3.623 $\int (1+\sin(e+fx))^m(3+2\sin(e+fx))^{-1-m}\,dx$

Optimal. Leaf size=122

$$-\frac{2^{m+\frac{1}{2}}5^{-m-\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{m-1}\left(\frac{\sin(e+fx)+1}{2\sin(e+fx)+3}\right)^{\frac{1}{2}-m}(2\sin(e+fx)+3)^{-m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1-\sin(e+fx}{2(2\sin(e+fx)}\right.}{f}$$

```
[Out] -2^(1/2+m)*5^(-1/2-m)*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2*(1-sin(f*
x+e))/(3+2*sin(f*x+e)))*(1+sin(f*x+e))^(-1+m)*((1+sin(f*x+e))/(3+2*sin(f*x+
e)))^(1/2-m)/f/((3+2*sin(f*x+e))^m)
```

Rubi [A] time = 0.11, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2788, 132}

$$-\frac{2^{m+\frac{1}{2}}5^{-m-\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{m-1}\left(\frac{\sin(e+fx)+1}{2\sin(e+fx)+3}\right)^{\frac{1}{2}-m}(2\sin(e+fx)+3)^{-m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1-\sin(e+fx}{2(2\sin(e+fx)}\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[e + f*x])^m*(3 + 2*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] -((2^(1/2 + m)*5^(-1/2 - m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/
2, (1 - Sin[e + f*x])/(2*(3 + 2*Sin[e + f*x]))]*(1 + Sin[e + f*x])^(-1 + m)
*((1 + Sin[e + f*x])/(3 + 2*Sin[e + f*x]))^(1/2 - m))/(f*(3 + 2*Sin[e + f*x
])^m))
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

!IntegerQ[m]

Rubi steps

$$\int (1+\sin(e+fx))^m(3+2\sin(e+fx))^{-1-m}\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(1+x)^{-\frac{1}{2}+m}(3+2x)^{-1-m}}{\sqrt{1-x}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{2^{\frac{1}{2}+m}5^{-\frac{1}{2}-m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1-\sin(e+fx)}{2(3+2\sin(e+fx))}\right)(1+}{f}$$

Mathematica [A] time = 0.54, size = 131, normalized size = 1.07

$$\frac{2\,5^{-m-1}\tan\left(\frac{1}{4}(2e+2fx-\pi)\right)(\sin(e+fx)+1)^m(2\sin(e+fx)+3)^{-m}\left((2\sin(e+fx)+3)\sec^2\left(\frac{1}{4}(2e+2fx-\right.\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[e + f*x])^m*(3 + 2*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] (2*5^(-1 - m)*Hypergeometric2F1[1/2, 1 + m, 3/2, -1/5*(Cos[(2*e + Pi + 2*f*
x)/4]^2*Sec[(2*e - Pi + 2*f*x)/4]^2)]*(1 + Sin[e + f*x])^m*(Sec[(2*e - Pi +
 2*f*x)/4]^2*(3 + 2*Sin[e + f*x]))^m*Tan[(2*e - Pi + 2*f*x)/4])/(f*(3 + 2*S
in[e + f*x])^m)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(2\sin\left(fx+e\right)+3\right)^{-m-1}\left(\sin\left(fx+e\right)+1\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+2*sin(f*x+e))^(-1-m),x, algorithm="fricas")
```

```
[Out] integral((2*sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2\sin\left(fx+e\right)+3\right)^{-m-1}\left(\sin\left(fx+e\right)+1\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+2*sin(f*x+e))^(-1-m),x, algorithm="giac")
[Out] integrate((2*sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

maple [F] time = 0.68, size = 0, normalized size = 0.00

$$\int \left(1+\sin \left(f x +e \right)\right)^{m} \left(3+2 \sin \left(f x +e \right)\right)^{-1-m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sin(f*x+e))^m*(3+2*sin(f*x+e))^(-1-m),x)
[Out] int((1+sin(f*x+e))^m*(3+2*sin(f*x+e))^(-1-m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int {\left(2 \, \sin \left(f x + e\right) + 3\right)}^{-m - 1} {\left(\sin \left(f x + e\right) + 1\right)}^{m}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+2*sin(f*x+e))^(-1-m),x, algorithm="maxima")
[Out] integrate((2*sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\sin\left(e+f\,x\right)+1\right)}^m}{{\left(2\,\sin\left(e+f\,x\right)+3\right)}^{m+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x) + 1)^m/(2*sin(e + f*x) + 3)^(m + 1),x)
[Out] int((sin(e + f*x) + 1)^m/(2*sin(e + f*x) + 3)^(m + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))**m*(3+2*sin(f*x+e))**(-1-m),x)
[Out] Timed out
```

3.624 $\int (1+\sin(e+fx))^m (3+\sin(e+fx))^{-1-m}\,dx$

Optimal. Leaf size=106

$$-\frac{2^{-m-\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{m-1}\left(\frac{\sin(e+fx)+1}{\sin(e+fx)+3}\right)^{\frac{1}{2}-m}(\sin(e+fx)+3)^{-m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1-\sin(e+fx)}{\sin(e+fx)+3}\right)}{f}$$

```
[Out] -2^(-1/2-m)*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],(1-sin(f*x+e))/(3+sin(f
*x+e)))*(1+sin(f*x+e))^(-1+m)*((1+sin(f*x+e))/(3+sin(f*x+e)))^(1/2-m)/f/((3
+sin(f*x+e))^m)
```

Rubi [A] time = 0.10, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2788, 132}

$$-\frac{2^{-m-\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{m-1}\left(\frac{\sin(e+fx)+1}{\sin(e+fx)+3}\right)^{\frac{1}{2}-m}(\sin(e+fx)+3)^{-m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1-\sin(e+fx)}{\sin(e+fx)+3}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[e + f*x])^m*(3 + Sin[e + f*x])^(-1 - m),x]
```

```
[Out] -((2^(-1/2 - m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[
e + f*x])/(3 + Sin[e + f*x])]*(1 + Sin[e + f*x])^(-1 + m)*((1 + Sin[e + f*x
])/(3 + Sin[e + f*x]))^(1/2 - m))/(f*(3 + Sin[e + f*x])^m))
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (1+\sin(e+fx))^m(3+\sin(e+fx))^{-1-m}\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(1+x)^{-\frac{1}{2}+m}(3+x)^{-1-m}}{\sqrt{1-x}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{2^{-\frac{1}{2}-m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1-\sin(e+fx)}{3+\sin(e+fx)}\right)(1+\sin(e+fx)}{f}$$

Mathematica [A] time = 0.57, size = 167, normalized size = 1.58

$$\frac{2^{-2m-1}\tan\left(\frac{1}{4}(2e+2fx-\pi)\right)(\sin(e+fx)+1)^m\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{-m}\left(\frac{\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)}{\sin(e+fx)+3}\right)^m\left((\sin(e+fx)+3\right.}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sin[e + f*x])^m*(3 + Sin[e + f*x])^(-1 - m),x]
```

```
[Out] (2^(-1 - 2*m)*Hypergeometric2F1[1/2, 1 + m, 3/2, -1/2*(Cos[(2*e + Pi + 2*f*
x)/4]^2*Sec[(2*e - Pi + 2*f*x)/4]^2)]*(1 + Sin[e + f*x])^m*(Cos[(2*e - Pi +
 2*f*x)/4]^2/(3 + Sin[e + f*x]))^m*(Sec[(2*e - Pi + 2*f*x)/4]^2*(3 + Sin[e
+ f*x]))^m*Tan[(2*e - Pi + 2*f*x)/4])/(f*(Cos[(2*e - Pi + 2*f*x)/4]^2)^m)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\sin\left(fx+e\right)+3\right)^{-m-1}\left(\sin\left(fx+e\right)+1\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+sin(f*x+e))^(-1-m),x, algorithm="fricas")
```

```
[Out] integral((sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sin\left(fx+e\right)+3\right)^{-m-1}\left(\sin\left(fx+e\right)+1\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(f*x+e))^m*(3+sin(f*x+e))^(-1-m),x, algorithm="giac")
```

[Out] integrate((sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \left(1+\sin \left(fx+e\right)\right)^{m} \left(3+\sin \left(fx+e\right)\right)^{-1-m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+sin(f*x+e))^m*(3+sin(f*x+e))^(-1-m),x)

[Out] int((1+sin(f*x+e))^m*(3+sin(f*x+e))^(-1-m),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sin \left(fx+e\right)+3\right)^{-m-1} \left(\sin \left(fx+e\right)+1\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+sin(f*x+e))^m*(3+sin(f*x+e))^(-1-m),x, algorithm="maxima")

[Out] integrate((sin(f*x + e) + 3)^(-m - 1)*(sin(f*x + e) + 1)^m, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\sin \left(e+f\,x\right)+1\right)^{m}}{\left(\sin \left(e+f\,x\right)+3\right)^{m+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sin(e + f*x) + 1)^m/(sin(e + f*x) + 3)^(m + 1),x)

[Out] int((sin(e + f*x) + 1)^m/(sin(e + f*x) + 3)^(m + 1), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+sin(f*x+e))**m*(3+sin(f*x+e))**(-1-m),x)

[Out] Timed out

3.625 $\int 3^{-1-m}(1+\sin(e+fx))^m\,dx$

Optimal. Leaf size=65

$$-\frac{2^{m+\frac{1}{2}}3^{-m-1}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -2^(1/2+m)*3^(-1-m)*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x
+e))/f/(1+sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {12, 2651}

$$-\frac{2^{m+\frac{1}{2}}3^{-m-1}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[3^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 + m)*3^(-1 - m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2,
 (1 - Sin[e + f*x])/2])/(f*Sqrt[1 + Sin[e + f*x]]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int 3^{-1-m}(1+\sin(e+fx))^m\,dx = 3^{-1-m}\int (1+\sin(e+fx))^m\,dx$$
$$= -\frac{2^{\frac{1}{2}+m}3^{-1-m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{1+\sin(e+fx)}}$$

Mathematica [A] time = 0.13, size = 95, normalized size = 1.46

$$\frac{\sqrt{2}\,3^{-m-1}\cos(e+fx)(\sin(e+fx)+1)^m\,{}_2F_1\left(\frac{1}{2},m+\frac{1}{2};m+\frac{3}{2};\frac{1}{4}\cos^2(e+fx)\csc^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)}{(2fm+f)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[3^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] (Sqrt[2]*3^(-1 - m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 + m, 3/2 + m, (
Cos[e + f*x]^2*Csc[(2*e - Pi + 2*f*x)/4]^2)/4]*(1 + Sin[e + f*x])^m)/((f +
2*f*m)*Sqrt[1 - Sin[e + f*x]])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(3^{-m-1}\left(\sin\left(fx+e\right)+1\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral(3^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int 3^{-m-1}\left(\sin\left(fx+e\right)+1\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate(3^(-m - 1)*(sin(f*x + e) + 1)^m, x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int 3^{-1-m}\left(1+\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3^(-1-m)*(1+sin(f*x+e))^m,x)
```

```
[Out] int(3^(-1-m)*(1+sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$3^{-m-1}\int\left(\sin\left(fx+e\right)+1\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] 3^(-m - 1)*integrate((sin(f*x + e) + 1)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{3^{m+1}}\left(\sin\left(e+f\,x\right)+1\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/3^(m + 1)*(sin(e + f*x) + 1)^m,x)
```

```
[Out] int(1/3^(m + 1)*(sin(e + f*x) + 1)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$3^{-m-1}\int\left(\sin\left(e+fx\right)+1\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3**(-1-m)*(1+sin(f*x+e))**m,x)
```

```
[Out] 3**(-m - 1)*Integral((sin(e + f*x) + 1)**m, x)
```

3.626 $\int (3-\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx$

Optimal. Leaf size=94

$$-\frac{\cos(e+fx)(3-\sin(e+fx))^{-m-1}\left(\frac{3-\sin(e+fx)}{\sin(e+fx)+1}\right)^{m+1}(\sin(e+fx)+1)^m\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};-\frac{2(1-\sin(e+fx))}{\sin(e+fx)+1}\right)}{f}$$

```
[Out] -cos(f*x+e)*hypergeom([1/2, 1+m],[3/2],-2*(1-sin(f*x+e))/(1+sin(f*x+e)))*(3
-sin(f*x+e))^(-1-m)*((3-sin(f*x+e))/(1+sin(f*x+e)))^(1+m)*(1+sin(f*x+e))^m/
f
```

Rubi [A] time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2788, 132}

$$-\frac{\cos(e+fx)(3-\sin(e+fx))^{-m-1}\left(\frac{3-\sin(e+fx)}{\sin(e+fx)+1}\right)^{m+1}(\sin(e+fx)+1)^m\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};-\frac{2(1-\sin(e+fx))}{\sin(e+fx)+1}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[1/2, 1 + m, 3/2, (-2*(1 - Sin[e + f*x]))/
(1 + Sin[e + f*x])]*(3 - Sin[e + f*x])^(-1 - m)*((3 - Sin[e + f*x])/(1 + Si
n[e + f*x]))^(1 + m)*(1 + Sin[e + f*x])^m)/f)
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (3-\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(3-x)^{-1-m}(1+x)^{-\frac{1}{2}+m}}{\sqrt{1-x}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},1+m;\frac{3}{2};-\frac{2(1-\sin(e+fx))}{1+\sin(e+fx)}\right)(3-\sin(e+fx))^{-1}}{f}$$

Mathematica [A] time = 1.04, size = 182, normalized size = 1.94

$$-\frac{2^{\frac{1}{2}-m}\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)(3-\sin(e+fx))^{-m}(\sin(e+fx)+1)^m\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 - m)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f
*x)/4]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (-4*Sin[(2*e - Pi + 2*f*x)/4]^2
)/(-3 + Sin[e + f*x])]*(-(Cos[(2*e - Pi + 2*f*x)/4]^2/(-3 + Sin[e + f*x])))
^(1/2 - m)*(1 + Sin[e + f*x])^m*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(f
*(3 - Sin[e + f*x])^m))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\sin\left(fx+e\right)+1\right)^{m}\left(-\sin\left(fx+e\right)+3\right)^{-m-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((sin(f*x + e) + 1)^m*(-sin(f*x + e) + 3)^(-m - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sin\left(fx+e\right)+1\right)^{m}\left(-\sin\left(fx+e\right)+3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((sin(f*x + e) + 1)^m*(-sin(f*x + e) + 3)^(-m - 1), x)
```

maple [F] time = 0.87, size = 0, normalized size = 0.00

$$\int \left(3-\sin \left(fx+e\right)\right)^{-1-m} \left(1+\sin \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x)
[Out] int((3-sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sin \left(fx+e\right)+1\right)^{m}\left(-\sin \left(fx+e\right)+3\right)^{-m-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((sin(f*x + e) + 1)^m*(-sin(f*x + e) + 3)^(-m - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\sin \left(e+f\,x\right)+1\right)^{m}}{\left(3-\sin \left(e+f\,x\right)\right)^{m+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x) + 1)^m/(3 - sin(e + f*x))^(m + 1),x)
[Out] int((sin(e + f*x) + 1)^m/(3 - sin(e + f*x))^(m + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-sin(f*x+e))**(-1-m)*(1+sin(f*x+e))**m,x)
[Out] Timed out
```

3.627 $\int (3-2\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx$

Optimal. Leaf size=114

$$\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(3-2\sin(e+fx))^{-m}(\sin(e+fx)+1)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3-2\sin(e+fx))}{\sin(e+fx)+1}\right)}{\sqrt{5}fm(1-\sin(e+fx))}$$

```
[Out] 1/5*cos(f*x+e)*hypergeom([1/2, -m],[1-m],2*(3-2*sin(f*x+e))/(1+sin(f*x+e)))
*(1+sin(f*x+e))^m*((-1+sin(f*x+e))/(1+sin(f*x+e)))^(1/2)/f/m/((3-2*sin(f*x+
e))^m)/(1-sin(f*x+e))*5^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2788, 132}

$$\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(3-2\sin(e+fx))^{-m}(\sin(e+fx)+1)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3-2\sin(e+fx))}{\sin(e+fx)+1}\right)}{\sqrt{5}fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 2*Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (2*(3 - 2*Sin[e + f*x]))/(1
 + Sin[e + f*x])]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(1 + Sin[e
 + f*x])^m)/(Sqrt[5]*f*m*(3 - 2*Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

!IntegerQ[m]

Rubi steps

$$\int (3-2\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(3-2x)^{-1-m}(1+x)^{-\frac{1}{2}+m}}{\sqrt{1-x}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3-2\sin(e+fx))}{1+\sin(e+fx)}\right)(3-2\sin(e+f}{\sqrt{5}\,fm(1-\sin(e+fx))}$$

Mathematica [A] time = 0.87, size = 177, normalized size = 1.55

$$-\frac{2\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)(3-2\sin(e+fx))^{-m}(\sin(e+fx)+1)^m\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx\right.}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - 2*Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] (-2*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*Hype
rgeometric2F1[1/2, 1/2 - m, 3/2, (5*Sin[(2*e - Pi + 2*f*x)/4]^2)/(3 - 2*Sin
[e + f*x])]*(1 + Sin[e + f*x])^m*(-(Cos[(2*e - Pi + 2*f*x)/4]^2/(-3 + 2*Sin
[e + f*x])))^(1/2 - m)*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(f*(3 - 2*S
in[e + f*x])^m)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\sin\left(fx+e\right)+1\right)^m\left(-2\,\sin\left(fx+e\right)+3\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((sin(f*x + e) + 1)^m*(-2*sin(f*x + e) + 3)^(-m - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sin\left(fx+e\right)+1\right)^m\left(-2\,\sin\left(fx+e\right)+3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((sin(f*x + e) + 1)^m*(-2*sin(f*x + e) + 3)^(-m - 1), x)
```

maple [F] time = 0.74, size = 0, normalized size = 0.00

$$\int \left(3-2\sin \left(f x +e \right)\right)^{-1-m} \left(1+\sin \left(f x +e \right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-2*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x)
[Out] int((3-2*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int {\left(\sin \left(f x + e\right) + 1\right)}^{m} {\left(-2 \, \sin \left(f x + e\right) + 3\right)}^{-m - 1}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((sin(f*x + e) + 1)^m*(-2*sin(f*x + e) + 3)^(-m - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\sin \left(e+f\,x\right)+1\right)}^m}{{\left(3-2\,\sin \left(e+f\,x\right)\right)}^{m+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x) + 1)^m/(3 - 2*sin(e + f*x))^(m + 1),x)
[Out] int((sin(e + f*x) + 1)^m/(3 - 2*sin(e + f*x))^(m + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*sin(f*x+e))**(-1-m)*(1+sin(f*x+e))**m,x)
[Out] Timed out
```

3.628 $\int (3-3\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx$

Optimal. Leaf size=43

$$\frac{\cos(e+fx)(3-3\sin(e+fx))^{-m-1}(\sin(e+fx)+1)^m}{f(2m+1)}$$

```
[Out] cos(f*x+e)*(3-3*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m/f/(1+2*m)
```

Rubi [A] time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2742}

$$\frac{\cos(e+fx)(3-3\sin(e+fx))^{-m-1}(\sin(e+fx)+1)^m}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 3*Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*(3 - 3*Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m)/(f*(1 + 2
*m))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int (3-3\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx=\frac{\cos(e+fx)(3-3\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m}{f(1+2m)}$$

Mathematica [B] time = 0.52, size = 97, normalized size = 2.26

$$\frac{\sin\left(\frac{1}{4}(2e+2fx+\pi)\right)(6-6\sin(e+fx))^{-m}(\sin(e+fx)+1)^m\cos^{-2m-1}\left(\frac{1}{4}(2e+2fx+\pi)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\mathrm{s}\right.}{3(2fm+f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 3*Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] (Cos[(2*e + Pi + 2*f*x)/4]^(-1 - 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^(2*m)*(1 + Sin[e + f*x])^m*Sin[(2*e + Pi + 2*f*x)/4])/(3*(f + 2*f*m)*(6 -
6*Sin[e + f*x])^m)
```

fricas [A] time = 0.45, size = 41, normalized size = 0.95

$$\frac{\left(\sin\left(fx+e\right)+1\right)^{m}\left(-3\,\sin\left(fx+e\right)+3\right)^{-m-1}\cos\left(fx+e\right)}{2\,fm+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] (sin(f*x + e) + 1)^m*(-3*sin(f*x + e) + 3)^(-m - 1)*cos(f*x + e)/(2*f*m + f
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\sin\left(fx+e\right)+1\right)^{m}\left(-3\,\sin\left(fx+e\right)+3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((sin(f*x + e) + 1)^m*(-3*sin(f*x + e) + 3)^(-m - 1), x)
```

maple [F] time = 0.78, size = 0, normalized size = 0.00

$$\int\left(3-3\sin\left(fx+e\right)\right)^{-1-m}\left(1+\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-3*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x)
```

```
[Out] int((3-3*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\sin\left(fx+e\right)+1\right)^{m}\left(-3\,\sin\left(fx+e\right)+3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((sin(f*x + e) + 1)^m*(-3*sin(f*x + e) + 3)^(-m - 1), x)
```

mupad [B] time = 7.85, size = 43, normalized size = 1.00

$$\frac{\cos\left(e+f\,x\right)\,{\left(\sin\left(e+f\,x\right)+1\right)}^m}{f\,\left(2\,m+1\right)\,{\left(3-3\,\sin\left(e+f\,x\right)\right)}^{m+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x) + 1)^m/(3 - 3*sin(e + f*x))^(m + 1),x)
```

```
[Out] (cos(e + f*x)*(sin(e + f*x) + 1)^m)/(f*(2*m + 1)*(3 - 3*sin(e + f*x))^(m +
1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sin(f*x+e))**(-1-m)*(1+sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

3.629 $\int (3-4\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx$

Optimal. Leaf size=83

$$\frac{2^{m+1}\cos(e+fx)(3-4\sin(e+fx))^{-m}(4\sin(e+fx)-3)^m\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};\frac{7(1-\sin(e+fx))}{\sin(e+fx)+1}\right)}{f(\sin(e+fx)+1)}$$

```
[Out] 2^(1+m)*cos(f*x+e)*hypergeom([1/2, 1+m],[3/2],7*(1-sin(f*x+e))/(1+sin(f*x+e
)))*(-3+4*sin(f*x+e))^m/f/((3-4*sin(f*x+e))^m)/(1+sin(f*x+e))
```

Rubi [A] time = 0.10, antiderivative size = 113, normalized size of antiderivative = 1.36, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2788, 132}

$$\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(3-4\sin(e+fx))^{-m}(\sin(e+fx)+1)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{2(3-4\sin(e+fx))}{\sin(e+fx)+1}\right)}{\sqrt{7}fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 4*Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (-2*(3 - 4*Sin[e + f*x]))/(
1 + Sin[e + f*x])]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(1 + Sin[e +
 f*x])^m)/(Sqrt[7]*f*m*(3 - 4*Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (3-4\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx = \frac{\cos(e+fx)\,\mathrm{Subst}\left(\int \frac{(3-4x)^{-1-m}(1+x)^{-\frac{1}{2}+m}}{\sqrt{1-x}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}, -m; 1-m; -\frac{2(3-4\sin(e+fx))}{1+\sin(e+fx)}\right)(3-4\sin(e+}{\sqrt{7}\,fm(1-\sin(e+fx))}$$

Mathematica [B] time = 0.87, size = 176, normalized size = 2.12

$$\frac{2\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)(3-4\sin(e+fx))^{-m}(\sin(e+fx)+1)^m\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\right.}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - 4*Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] (2*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*Hyper
geometric2F1[1/2, 1/2 - m, 3/2, (7*Sin[(2*e - Pi + 2*f*x)/4]^2)/(3 - 4*Sin[
e + f*x])]*(1 + Sin[e + f*x])^m*(Cos[(2*e - Pi + 2*f*x)/4]^2/(-3 + 4*Sin[e
+ f*x]))^(1/2 - m)*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(f*(3 - 4*Sin[e
 + f*x])^m)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(\sin\left(fx+e\right)+1\right)^m\left(-4\,\sin\left(fx+e\right)+3\right)^{-m-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((sin(f*x + e) + 1)^m*(-4*sin(f*x + e) + 3)^(-m - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sin\left(fx+e\right)+1\right)^m\left(-4\,\sin\left(fx+e\right)+3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((sin(f*x + e) + 1)^m*(-4*sin(f*x + e) + 3)^(-m - 1), x)
```

maple [F] time = 0.78, size = 0, normalized size = 0.00

$$\int \left(3-4\sin\left(fx+e\right)\right)^{-1-m} \left(1+\sin\left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-4*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x)
[Out] int((3-4*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sin\left(fx+e\right)+1\right)^{m}\left(-4\,\sin\left(fx+e\right)+3\right)^{-m-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((sin(f*x + e) + 1)^m*(-4*sin(f*x + e) + 3)^(-m - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\sin\left(e+f\,x\right)+1\right)^{m}}{\left(3-4\,\sin\left(e+f\,x\right)\right)^{m+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x) + 1)^m/(3 - 4*sin(e + f*x))^(m + 1),x)
[Out] int((sin(e + f*x) + 1)^m/(3 - 4*sin(e + f*x))^(m + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*sin(f*x+e))**(-1-m)*(1+sin(f*x+e))**m,x)
[Out] Timed out
```

3.630 $\int (3-5\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx$

Optimal. Leaf size=78

$$\frac{\cos(e+fx)(3-5\sin(e+fx))^{-m}(5\sin(e+fx)-3)^m\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};\frac{4(1-\sin(e+fx))}{\sin(e+fx)+1}\right)}{f(\sin(e+fx)+1)}$$

```
[Out] cos(f*x+e)*hypergeom([1/2, 1+m],[3/2],4*(1-sin(f*x+e))/(1+sin(f*x+e)))*(-3+
5*sin(f*x+e))^m/f/((3-5*sin(f*x+e))^m)/(1+sin(f*x+e))
```

Rubi [A] time = 0.09, antiderivative size = 111, normalized size of antiderivative = 1.42, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2788, 132}

$$\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(3-5\sin(e+fx))^{-m}(\sin(e+fx)+1)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{3-5\sin(e+fx)}{\sin(e+fx)+1}\right)}{4fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 5*Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, -((3 - 5*Sin[e + f*x])/(1 +
 Sin[e + f*x]))]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(1 + Sin[e + f
*x])^m)/(4*f*m*(3 - 5*Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (3-5\sin(e+fx))^{-1-m}(1+\sin(e+fx))^m\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(3-5x)^{-1-m}(1+x)^{-\frac{1}{2}+m}}{\sqrt{1-x}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{3-5\sin(e+fx)}{1+\sin(e+fx)}\right)(3-5\sin(e+fx)}{4fm(1-\sin(e+fx))}$$

Mathematica [C] time = 1.76, size = 246, normalized size = 3.15

$$-\frac{2^{2m-1}(\cosh(m\log(4))-\sinh(m\log(4)))(3-5\sin(e+fx))^{-m}(\sin(e+fx)+1)^m(\sin(e+fx)+i\cos(e+fx)+1}{f(2m+1)((1+2i)\sin(e+fx)+(-2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - 5*Sin[e + f*x])^(-1 - m)*(1 + Sin[e + f*x])^m,x]
```

```
[Out] -((2^(-1 + 2*m)*Hypergeometric2F1[1 + m, 1 + 2*m, 2*(1 + m), (2*Cos[(2*e -
Pi + 2*f*x)/4])/(Cos[(2*e - Pi + 2*f*x)/4] + 2*Sin[(2*e - Pi + 2*f*x)/4])]*
(1 + Sin[e + f*x])^m*(1 + I*Cos[e + f*x] + Sin[e + f*x])*((-Cos[(2*e - Pi +
 2*f*x)/4] + 2*Sin[(2*e - Pi + 2*f*x)/4])/(Cos[(2*e - Pi + 2*f*x)/4] + 2*Si
n[(2*e - Pi + 2*f*x)/4]))^m*(Cosh[m*Log[4]] - Sinh[m*Log[4]]))/(f*(1 + 2*m)
*(3 - 5*Sin[e + f*x])^m*((1 - 2*I) - (2 - I)*Cos[e + f*x] + (1 + 2*I)*Sin[e
 + f*x])))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\sin\left(fx+e\right)+1\right)^m\left(-5\,\sin\left(fx+e\right)+3\right)^{-m-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-5*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((sin(f*x + e) + 1)^m*(-5*sin(f*x + e) + 3)^(-m - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sin\left(fx+e\right)+1\right)^m\left(-5\,\sin\left(fx+e\right)+3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-5*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((sin(f*x + e) + 1)^m*(-5*sin(f*x + e) + 3)^(-m - 1), x)
```

maple [F] time = 0.68, size = 0, normalized size = 0.00

$$\int \left(3-5 \sin \left(f x +e \right)\right)^{-1-m} \left(1+\sin \left(f x +e \right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-5*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x)
[Out] int((3-5*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int {\left(\sin \left(f x + e\right) + 1\right)}^{m} {\left(-5 \, \sin \left(f x + e\right) + 3\right)}^{-m - 1}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-5*sin(f*x+e))^(-1-m)*(1+sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((sin(f*x + e) + 1)^m*(-5*sin(f*x + e) + 3)^(-m - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\sin\left(e+f\,x\right)+1\right)}^m}{{\left(3-5\,\sin\left(e+f\,x\right)\right)}^{m+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x) + 1)^m/(3 - 5*sin(e + f*x))^(m + 1),x)
[Out] int((sin(e + f*x) + 1)^m/(3 - 5*sin(e + f*x))^(m + 1), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-5*sin(f*x+e))**(-1-m)*(1+sin(f*x+e))**m,x)
[Out] Exception raised: HeuristicGCDFailed
```

3.631 $\int (3+5\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=81

$$-\frac{4^{-m-1}\cos(e+fx)(\sin(e+fx)+1)^{-m-1}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};\frac{a-a\sin(e+fx)}{4(\sin(e+fx)a+a)}\right)}{f}$$

```
[Out] -4^(-1-m)*cos(f*x+e)*hypergeom([1/2, 1+m],[3/2],1/4*(a-a*sin(f*x+e))/(a+a*s
in(f*x+e)))*(1+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m/f
```

Rubi [A] time = 0.11, antiderivative size = 115, normalized size of antiderivative = 1.42, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2788, 132}

$$-\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(5\sin(e+fx)+3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{5\sin(e+fx)+3}{4(\sin(e+fx)+1)}\right)}{4fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 5*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -(Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (3 + 5*Sin[e + f*x])/(4*(1
 + Sin[e + f*x]))]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(a + a*Sin[e
 + f*x])^m)/(4*f*m*(1 - Sin[e + f*x])*(3 + 5*Sin[e + f*x])^m)
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (3+5\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(3+5x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx, x, \sin(e+\right.}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{3+5\sin(e+fx)}{4(1+\sin(e+fx))}\right)\sqrt{\frac{1-\sin(e+fx)}{1+\sin(e+fx)}}\,(}{4fm(1-\sin(e+fx}$$

Mathematica [C] time = 0.55, size = 240, normalized size = 2.96

$$\frac{4^m(\cosh(m\log(4))-\sinh(m\log(4)))(5\sin(e+fx)+3)^{-m}(\sin(e+fx)+i\cos(e+fx)+1)(a(\sin(e+fx)+1))}{f(2m+1)((2+i)\sin(e+fx)+(-1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 + 5*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (4^m*Hypergeometric2F1[1 + m, 1 + 2*m, 2*(1 + m), (4*Cos[(2*e - Pi + 2*f*x)
/4])/(2*Cos[(2*e - Pi + 2*f*x)/4] + Sin[(2*e - Pi + 2*f*x)/4])]*(a*(1 + Sin
[e + f*x]))^m*(1 + I*Cos[e + f*x] + Sin[e + f*x])*(-((2*Cos[(2*e - Pi + 2*f
*x)/4] + Cos[(2*e + Pi + 2*f*x)/4])/(2*Cos[(2*e - Pi + 2*f*x)/4] + Sin[(2*e
 - Pi + 2*f*x)/4])))^m*(Cosh[m*Log[4]] - Sinh[m*Log[4]]))/(f*(1 + 2*m)*((2
- I) - (1 - 2*I)*Cos[e + f*x] + (2 + I)*Sin[e + f*x])*(3 + 5*Sin[e + f*x])^
m)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m\left(5\sin\left(fx+e\right)+3\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(5*sin(f*x + e) + 3)^(-m - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^m\left(5\sin\left(fx+e\right)+3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(5*sin(f*x + e) + 3)^(-m - 1), x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \left(3+5\sin\left(fx+e\right)\right)^{-1-m} \left(a+a\sin\left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
[Out] int((3+5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m} \left(5\,\sin\left(fx+e\right)+3\right)^{-m-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*(5*sin(f*x + e) + 3)^(-m - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^{m}}{\left(5\,\sin\left(e+f\,x\right)+3\right)^{m+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(5*sin(e + f*x) + 3)^(m + 1),x)
[Out] int((a + a*sin(e + f*x))^m/(5*sin(e + f*x) + 3)^(m + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+5*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
[Out] Timed out
```

3.632 $\int (3+4\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=83

$$-\frac{\left(\frac{7}{2}\right)^{-m-1}\cos(e+fx)(\sin(e+fx)+1)^{-m-1}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};\frac{a-a\sin(e+fx)}{7(\sin(e+fx)a+a)}\right)}{f}$$

```
[Out] -(7/2)^(-1-m)*cos(f*x+e)*hypergeom([1/2, 1+m],[3/2],1/7*(a-a*sin(f*x+e))/(a
+a*sin(f*x+e)))*(1+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m/f
```

Rubi [A] time = 0.11, antiderivative size = 118, normalized size of antiderivative = 1.42, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2788, 132}

$$-\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(4\sin(e+fx)+3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(4\sin(e+fx)+3)}{7(\sin(e+fx)+1)}\right)}{\sqrt{7}fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (2*(3 + 4*Sin[e + f*x]))/
(7*(1 + Sin[e + f*x]))]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(a + a*
Sin[e + f*x])^m)/(Sqrt[7]*f*m*(1 - Sin[e + f*x])*(3 + 4*Sin[e + f*x])^m))
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (3+4\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(3+4x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx, x, \sin(e+f\right.}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3+4\sin(e+fx))}{7(1+\sin(e+fx))}\right)\sqrt{\frac{1-\sin(e+fx)}{1+\sin(e+fx)}}\,(}{\sqrt{7}\,fm(1-\sin(e+f}$$

Mathematica [A] time = 0.47, size = 90, normalized size = 1.08

$$-\frac{2\,7^{-m-1}\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{-m}(a(\sin(e+fx)+1))^m\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};\frac{1}{7}\tan^2\left(\frac{1}{4}(2e+2\right.\right.}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 + 4*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (-2*7^(-1 - m)*Cot[(2*e + Pi + 2*f*x)/4]*Hypergeometric2F1[1/2, 1 + m, 3/2,
 Tan[(2*e - Pi + 2*f*x)/4]^2/7]*(a*(1 + Sin[e + f*x]))^m)/(f*(Sin[(2*e + Pi
 + 2*f*x)/4]^2)^m)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m\left(4\,\sin\left(fx+e\right)+3\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(4*sin(f*x + e) + 3)^(-m - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^m\left(4\,\sin\left(fx+e\right)+3\right)^{-m-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(4*sin(f*x + e) + 3)^(-m - 1), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \left(3+4\sin\left(fx+e\right)\right)^{-1-m} \left(a+a\sin\left(fx+e\right)\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((3+4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(4\sin\left(fx+e\right)+3\right)^{-m-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(4*sin(f*x + e) + 3)^(-m - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\sin\left(e+fx\right)\right)^{m}}{\left(4\sin\left(e+fx\right)+3\right)^{m+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(4*sin(e + f*x) + 3)^(m + 1),x)
```

```
[Out] int((a + a*sin(e + f*x))^m/(4*sin(e + f*x) + 3)^(m + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

3.633 $\int (3+3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=39

$$-\frac{\cos(e+fx)(3\sin(e+fx)+3)^{-m-1}(a\sin(e+fx)+a)^m}{f}$$

[Out] -cos(f*x+e)*(3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m/f

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {23, 2648}

$$-\frac{\cos(e+fx)(3\sin(e+fx)+3)^{-m-1}(a\sin(e+fx)+a)^m}{f}$$

Antiderivative was successfully verified.

[In] Int[(3 + 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]

[Out] -((Cos[e + f*x]*(3 + 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m)/f)

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int (3+3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx &= \left((3+3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^{1+m}\right)\int \frac{1}{a+a\sin(\\ &= -\frac{\cos(e+fx)(3+3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m}{f}\end{aligned}$$

Mathematica [B] time = 5.24, size = 104, normalized size = 2.67

$$-\frac{2^{-m}3^{-m-1}\cos\left(\frac{1}{4}(2e+2fx+\pi)\right)(\sin(e+fx)+1)^{-m-1}\sin^{-2m-1}\left(\frac{1}{4}(2e+2fx+\pi)\right)(a(\sin(e+fx)+1))^m\left(\sin\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((3^(-1 - m)*Cos[(2*e + Pi + 2*f*x)/4]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^(2*(1 + m))*(1 + Sin[e + f*x])^(-1 - m)*(a*(1 + Sin[e + f*x]))^m*Sin[(2*
e + Pi + 2*f*x)/4]^(-1 - 2*m))/(2^m*f))
```

fricas [A] time = 0.43, size = 43, normalized size = 1.10

$$-\frac{\left(\frac{1}{3}\,a\right)^m\left(\cos\left(fx+e\right)-\sin\left(fx+e\right)+1\right)}{3\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] -1/3*(1/3*a)^m*(cos(f*x + e) - sin(f*x + e) + 1)/(f*cos(f*x + e) + f*sin(f*
x + e) + f)
```

giac [B] time = 2.58, size = 827, normalized size = 21.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] (e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(-3/4*pi - pi*m*floor(-1/4*sgn(a
) + 1/2) - 1/4*pi*m*sgn(a) - 3/4*pi*m + 1/2*f*x + 1/2*e)^2*tan(1/2*f*x + 1/
2*e)^3 + 3*e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(-3/4*pi - pi*m*floor(
-1/4*sgn(a) + 1/2) - 1/4*pi*m*sgn(a) - 3/4*pi*m + 1/2*f*x + 1/2*e)^2*tan(1/
2*f*x + 1/2*e)^2 - 2*e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(-3/4*pi - p
i*m*floor(-1/4*sgn(a) + 1/2) - 1/4*pi*m*sgn(a) - 3/4*pi*m + 1/2*f*x + 1/2*e
)*tan(1/2*f*x + 1/2*e)^3 - 3*e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(-3/
4*pi - pi*m*floor(-1/4*sgn(a) + 1/2) - 1/4*pi*m*sgn(a) - 3/4*pi*m + 1/2*f*x
 + 1/2*e)^2*tan(1/2*f*x + 1/2*e) + 6*e^(-m*log(3) + m*log(abs(a)) - log(3))
*tan(-3/4*pi - pi*m*floor(-1/4*sgn(a) + 1/2) - 1/4*pi*m*sgn(a) - 3/4*pi*m +
 1/2*f*x + 1/2*e)*tan(1/2*f*x + 1/2*e)^2 - e^(-m*log(3) + m*log(abs(a)) - l
og(3))*tan(1/2*f*x + 1/2*e)^3 - e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(
```

```
-3/4*pi - pi*m*floor(-1/4*sgn(a) + 1/2) - 1/4*pi*m*sgn(a) - 3/4*pi*m + 1/2*
f*x + 1/2*e)^2 + 6*e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(-3/4*pi - pi*
m*floor(-1/4*sgn(a) + 1/2) - 1/4*pi*m*sgn(a) - 3/4*pi*m + 1/2*f*x + 1/2*e)*
tan(1/2*f*x + 1/2*e) - 3*e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(1/2*f*x
 + 1/2*e)^2 - 2*e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(-3/4*pi - pi*m*f
loor(-1/4*sgn(a) + 1/2) - 1/4*pi*m*sgn(a) - 3/4*pi*m + 1/2*f*x + 1/2*e) + 3
*e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(1/2*f*x + 1/2*e) + e^(-m*log(3)
 + m*log(abs(a)) - log(3)))/(f*tan(-3/4*pi - pi*m*floor(-1/4*sgn(a) + 1/2)
- 1/4*pi*m*sgn(a) - 3/4*pi*m + 1/2*f*x + 1/2*e)^2*tan(1/2*f*x + 1/2*e)^3 +
f*tan(-3/4*pi - pi*m*floor(-1/4*sgn(a) + 1/2) - 1/4*pi*m*sgn(a) - 3/4*pi*m
+ 1/2*f*x + 1/2*e)^2*tan(1/2*f*x + 1/2*e)^2 + f*tan(-3/4*pi - pi*m*floor(-1
/4*sgn(a) + 1/2) - 1/4*pi*m*sgn(a) - 3/4*pi*m + 1/2*f*x + 1/2*e)^2*tan(1/2*
f*x + 1/2*e) + f*tan(1/2*f*x + 1/2*e)^3 + f*tan(-3/4*pi - pi*m*floor(-1/4*s
gn(a) + 1/2) - 1/4*pi*m*sgn(a) - 3/4*pi*m + 1/2*f*x + 1/2*e)^2 + f*tan(1/2*
f*x + 1/2*e)^2 + f*tan(1/2*f*x + 1/2*e) + f)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \left(3+3\sin\left(fx+e\right)\right)^{-1-m} \left(a+a\sin\left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
[Out] int((3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

maxima [A] time = 0.88, size = 38, normalized size = 0.97

$$-\frac{2\,a^{m}}{\left(3^{m+1}+\frac{3^{m+1}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] -2*a^m/((3^(m + 1) + 3^(m + 1)*sin(f*x + e)/(cos(f*x + e) + 1))*f)
```

mupad [B] time = 0.43, size = 52, normalized size = 1.33

$$\frac{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)^{m}\,\left(-\cos\left(e+f\,x\right)+\sin\left(e+f\,x\right)\,1{}\mathrm{i}+1{}\mathrm{i}\right)}{f\,\left(3\,\sin\left(e+f\,x\right)+3\right)^{m+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(3*sin(e + f*x) + 3)^(m + 1),x)
```

```
[Out] ((a*(sin(e + f*x) + 1))^m*(sin(e + f*x)*1i - cos(e + f*x) + 1i))/(f*(3*sin(
e + f*x) + 3)^(m + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

3.634 $\int (3+2\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=83

$$-\frac{\left(\frac{5}{2}\right)^{-m-1}\cos(e+fx)(\sin(e+fx)+1)^{-m-1}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};-\frac{a-a\sin(e+fx)}{5(\sin(e+fx)a+a)}\right)}{f}$$

```
[Out] -(5/2)^(-1-m)*cos(f*x+e)*hypergeom([1/2, 1+m],[3/2],1/5*(-a+a*sin(f*x+e))/(
a+a*sin(f*x+e)))*(1+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m/f
```

Rubi [A] time = 0.11, antiderivative size = 118, normalized size of antiderivative = 1.42, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2788, 132}

$$\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(2\sin(e+fx)+3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(2\sin(e+fx)+3)}{5(\sin(e+fx)+1)}\right)}{\sqrt{5}fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (2*(3 + 2*Sin[e + f*x]))/(5
*(1 + Sin[e + f*x]))]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(a + a
*Sin[e + f*x])^m)/(Sqrt[5]*f*m*(1 - Sin[e + f*x])*(3 + 2*Sin[e + f*x])^m)
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (3+2\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(3+2x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx, x, \sin(e+\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}, -m; 1-m; \frac{2(3+2\sin(e+fx))}{5(1+\sin(e+fx))}\right)\sqrt{-\frac{1-\sin(e+fx)}{1+\sin(e+fx)}}}{\sqrt{5}\,fm(1-\sin(e+f}$$

Mathematica [B] time = 0.62, size = 179, normalized size = 2.16

$$-\frac{2\,5^{-m-1}\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)(2\sin(e+fx)+3)^{-m}\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (-2*5^(-1 - m)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f
*x)/4]*Hypergeometric2F1[1/2, 1 + m, 3/2, -1/5*(Cos[(2*e + Pi + 2*f*x)/4]^2
*Sec[(2*e - Pi + 2*f*x)/4]^2)]*(a*(1 + Sin[e + f*x]))^m*(Sec[(2*e - Pi + 2*
f*x)/4]^2*(3 + 2*Sin[e + f*x]))^m*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/
(f*(3 + 2*Sin[e + f*x])^m)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m\left(2\,\sin\left(fx+e\right)+3\right)^{-m-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(2*sin(f*x + e) + 3)^(-m - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^m\left(2\,\sin\left(fx+e\right)+3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(2*sin(f*x + e) + 3)^(-m - 1), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \left(3+2\sin\left(fx+e\right)\right)^{-1-m} \left(a+a\sin\left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
[Out] int((3+2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m} \left(2\,\sin\left(fx+e\right)+3\right)^{-m-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*(2*sin(f*x + e) + 3)^(-m - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(2\,\sin\left(e+f\,x\right)+3\right)^{m+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(2*sin(e + f*x) + 3)^(m + 1),x)
[Out] int((a + a*sin(e + f*x))^m/(2*sin(e + f*x) + 3)^(m + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
[Out] Timed out
```

3.635 $\int (3+\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=81

$$-\frac{2^{-m-1}\cos(e+fx)(\sin(e+fx)+1)^{-m-1}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};-\frac{a-a\sin(e+fx)}{2(\sin(e+fx)a+a)}\right)}{f}$$

```
[Out] -2^(-1-m)*cos(f*x+e)*hypergeom([1/2, 1+m],[3/2],1/2*(-a+a*sin(f*x+e))/(a+a*
sin(f*x+e)))*(1+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m/f
```

Rubi [A] time = 0.11, antiderivative size = 117, normalized size of antiderivative = 1.44, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2788, 132}

$$\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(\sin(e+fx)+3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{\sin(e+fx)+3}{2(\sin(e+fx)+1)}\right)}{2\sqrt{2}fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (3 + Sin[e + f*x])/(2*(1 +
Sin[e + f*x]))]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(a + a*Sin[e
 + f*x])^m)/(2*Sqrt[2]*f*m*(1 - Sin[e + f*x])*(3 + Sin[e + f*x])^m)
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (3+\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(3+x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx, x, \sin(e+fx)\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{3+\sin(e+fx)}{2(1+\sin(e+fx))}\right)\sqrt{-\frac{1-\sin(e+fx)}{1+\sin(e+fx)}}\,(3+}{2\sqrt{2}\,fm(1-\sin(e+fx))}$$

Mathematica [B] time = 0.68, size = 166, normalized size = 2.05

$$-\frac{2^{-2m-1}\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{-m}(a(\sin(e+fx)+1))^m\left(\frac{\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)}{\sin(e+fx)+3}\right)^m\left((\sin(e+fx)\right.}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 + Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((2^(-1 - 2*m)*Cot[(2*e + Pi + 2*f*x)/4]*Hypergeometric2F1[1/2, 1 + m, 3/2
, -1/2*(Cos[(2*e + Pi + 2*f*x)/4]^2*Sec[(2*e - Pi + 2*f*x)/4]^2)]*(a*(1 + S
in[e + f*x]))^m*(Cos[(2*e - Pi + 2*f*x)/4]^2/(3 + Sin[e + f*x]))^m*(Sec[(2*
e - Pi + 2*f*x)/4]^2*(3 + Sin[e + f*x]))^m)/(f*(Sin[(2*e + Pi + 2*f*x)/4]^2
)^m))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m\left(\sin\left(fx+e\right)+3\right)^{-m-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(sin(f*x + e) + 3)^(-m - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^m\left(\sin\left(fx+e\right)+3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(sin(f*x + e) + 3)^(-m - 1), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \left(3+\sin \left(fx+e\right)\right)^{-1-m} \left(a+a\sin \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
[Out] int((3+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin \left(fx+e\right)+a\right)^{m} \left(\sin \left(fx+e\right)+3\right)^{-m-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*(sin(f*x + e) + 3)^(-m - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin \left(e+f\,x\right)\right)^m}{\left(\sin \left(e+f\,x\right)+3\right)^{m+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(sin(e + f*x) + 3)^(m + 1),x)
[Out] int((a + a*sin(e + f*x))^m/(sin(e + f*x) + 3)^(m + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
[Out] Timed out
```

3.636 $\int 3^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=81

$$-\frac{2^{m+\frac{1}{2}}3^{-m-1}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f}$$

```
[Out] -2^(1/2+m)*3^(-1-m)*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x
+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/f
```

Rubi [A] time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {12, 2652, 2651}

$$-\frac{2^{m+\frac{1}{2}}3^{-m-1}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[3^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 + m)*3^(-1 - m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2,
 (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m
)/f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rubi steps

$$\int 3^{-1-m}(a+a\sin(e+fx))^m\,dx = 3^{-1-m}\int (a+a\sin(e+fx))^m\,dx$$
$$= \left(3^{-1-m}(1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\right)\int (1+\sin(e+fx))^m\,dx$$
$$= -\frac{2^{\frac{1}{2}+m}3^{-1-m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx}{f}$$

Mathematica [A] time = 0.16, size = 97, normalized size = 1.20

$$\frac{\sqrt{2}\,3^{-m-1}\cos(e+fx)(a(\sin(e+fx)+1))^m\,{}_2F_1\left(\frac{1}{2},m+\frac{1}{2};m+\frac{3}{2};\frac{1}{4}\cos^2(e+fx)\csc^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)}{(2fm+f)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[3^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Sqrt[2]*3^(-1 - m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 + m, 3/2 + m, (
Cos[e + f*x]^2*Csc[(2*e - Pi + 2*f*x)/4]^2)/4]*(a*(1 + Sin[e + f*x]))^m)/((
f + 2*f*m)*Sqrt[1 - Sin[e + f*x]])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(3^{-m-1}\left(a\sin\left(fx+e\right)+a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral(3^(-m - 1)*(a*sin(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int 3^{-m-1}\left(a\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate(3^(-m - 1)*(a*sin(f*x + e) + a)^m, x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int 3^{-1-m} \left(a + a \sin\left(fx + e\right)\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3^(-1-m)*(a+a*sin(f*x+e))^m,x)
[Out] int(3^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$3^{-m-1} \int \left(a \sin\left(fx + e\right) + a\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] 3^(-m - 1)*integrate((a*sin(f*x + e) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{3^{m+1}} \left(a + a\ \sin\left(e + f\,x\right)\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/3^(m + 1)*(a + a*sin(e + f*x))^m,x)
[Out] int(1/3^(m + 1)*(a + a*sin(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$3^{-m-1} \int \left(a \sin\left(e + fx\right) + a\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3**(-1-m)*(a+a*sin(f*x+e))**m,x)
[Out] 3**(-m - 1)*Integral((a*sin(e + f*x) + a)**m, x)
```

3.637 $\int (3-\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=72

$$-\frac{\cos(e+fx)(\sin(e+fx)+1)^{-m-1}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};-\frac{2(a-a\sin(e+fx))}{\sin(e+fx)a+a}\right)}{f}$$

```
[Out] -cos(f*x+e)*hypergeom([1/2, 1+m],[3/2],-2*(a-a*sin(f*x+e))/(a+a*sin(f*x+e))
)*(1+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m/f
```

Rubi [A] time = 0.11, antiderivative size = 118, normalized size of antiderivative = 1.64, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2788, 132}

$$\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(3-\sin(e+fx))^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{3-\sin(e+fx)}{\sin(e+fx)+1}\right)}{2\sqrt{2}fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (3 - Sin[e + f*x])/(1 + Sin
[e + f*x])]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(a + a*Sin[e + f
*x])^m)/(2*Sqrt[2]*f*m*(1 - Sin[e + f*x])*(3 - Sin[e + f*x])^m)
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (3-\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(3-x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{3-\sin(e+fx)}{1+\sin(e+fx)}\right)(3-\sin(e+fx))^{-m}}{2\sqrt{2}\,fm(1-\sin(e+fx))}$$

Mathematica [B] time = 0.81, size = 184, normalized size = 2.56

$$-\frac{2^{\frac{1}{2}-m}\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)(3-\sin(e+fx))^{-m}\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a(\sin(e}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 - m)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f
*x)/4]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (-4*Sin[(2*e - Pi + 2*f*x)/4]^2
)/(-3 + Sin[e + f*x])]*(-(Cos[(2*e - Pi + 2*f*x)/4]^2/(-3 + Sin[e + f*x])))
^(1/2 - m)*(a*(1 + Sin[e + f*x]))^m*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m)
)/(f*(3 - Sin[e + f*x])^m))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m\left(-\sin\left(fx+e\right)+3\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-sin(f*x + e) + 3)^(-m - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^m\left(-\sin\left(fx+e\right)+3\right)^{-m-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(-sin(f*x + e) + 3)^(-m - 1), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \left(3-\sin \left(f x +e \right)\right)^{-1-m} \left(a +a \sin \left(f x +e \right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
[Out] int((3-sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int {\left(a \sin \left(f x + e\right) + a\right)}^{m} {\left(-\sin \left(f x + e\right) + 3\right)}^{-m - 1}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*(-sin(f*x + e) + 3)^(-m - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{{\left(3-\sin\left(e+f\,x\right)\right)}^{m+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(3 - sin(e + f*x))^(m + 1),x)
[Out] int((a + a*sin(e + f*x))^m/(3 - sin(e + f*x))^(m + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
[Out] Timed out
```

3.638 $\int (3-2\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=77

$$-\frac{2^{m+1}\cos(e+fx)(\sin(e+fx)+1)^{-m-1}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};-\frac{5(a-a\sin(e+fx))}{\sin(e+fx)a+a}\right)}{f}$$

```
[Out] -2^(1+m)*cos(f*x+e)*hypergeom([1/2, 1+m],[3/2],-5*(a-a*sin(f*x+e))/(a+a*sin
(f*x+e)))*(1+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m/f
```

Rubi [A] time = 0.10, antiderivative size = 116, normalized size of antiderivative = 1.51, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2788, 132}

$$\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(3-2\sin(e+fx))^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3-2\sin(e+fx))}{\sin(e+fx)+1}\right)}{\sqrt{5}fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 2*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (2*(3 - 2*Sin[e + f*x]))/(1
 + Sin[e + f*x])]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(a + a*Sin
[e + f*x])^m)/(Sqrt[5]*f*m*(3 - 2*Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (3-2\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(3-2x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx, x, \sin(e+\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3-2\sin(e+fx))}{1+\sin(e+fx)}\right)(3-2\sin(e+}{\sqrt{5}\,fm(1-\sin(e+f}$$

Mathematica [B] time = 0.63, size = 179, normalized size = 2.32

$$-\frac{2\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)(3-2\sin(e+fx))^{-m}\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a(\sin(e}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - 2*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (-2*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*Hype
rgeometric2F1[1/2, 1/2 - m, 3/2, (5*Sin[(2*e - Pi + 2*f*x)/4]^2)/(3 - 2*Sin
[e + f*x])]*(a*(1 + Sin[e + f*x]))^m*(-(Cos[(2*e - Pi + 2*f*x)/4]^2/(-3 + 2
*Sin[e + f*x])))^(1/2 - m)*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(f*(3 -
 2*Sin[e + f*x])^m)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m\left(-2\,\sin\left(fx+e\right)+3\right)^{-m-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-2*sin(f*x + e) + 3)^(-m - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^m\left(-2\,\sin\left(fx+e\right)+3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(-2*sin(f*x + e) + 3)^(-m - 1), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \left(3-2\sin\left(fx+e\right)\right)^{-1-m}\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
[Out] int((3-2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-2\,\sin\left(fx+e\right)+3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*(-2*sin(f*x + e) + 3)^(-m - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^{m}}{\left(3-2\,\sin\left(e+f\,x\right)\right)^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(3 - 2*sin(e + f*x))^(m + 1),x)
[Out] int((a + a*sin(e + f*x))^m/(3 - 2*sin(e + f*x))^(m + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
[Out] Timed out
```

3.639 $\int (3-3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=45

$$\frac{\cos(e+fx)(3-3\sin(e+fx))^{-m-1}(a\sin(e+fx)+a)^m}{f(2m+1)}$$

```
[Out] cos(f*x+e)*(3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m/f/(1+2*m)
```

Rubi [A] time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2742}

$$\frac{\cos(e+fx)(3-3\sin(e+fx))^{-m-1}(a\sin(e+fx)+a)^m}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*(3 - 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 +
 2*m))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int (3-3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx=\frac{\cos(e+fx)(3-3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m}{f(1+2m)}$$

Mathematica [B] time = 0.60, size = 99, normalized size = 2.20

$$\frac{\sin\left(\frac{1}{4}(2e+2fx+\pi)\right)(6-6\sin(e+fx))^{-m}\cos^{-2m-1}\left(\frac{1}{4}(2e+2fx+\pi)\right)(a(\sin(e+fx)+1))^m\left(\cos\left(\frac{1}{2}(e+fx)\right)\right.}{3(2fm+f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[(2*e + Pi + 2*f*x)/4]^(-1 - 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^(2*m)*(a*(1 + Sin[e + f*x]))^m*Sin[(2*e + Pi + 2*f*x)/4])/(3*(f + 2*f*m)*(
6 - 6*Sin[e + f*x])^m)
```

fricas [A] time = 0.47, size = 43, normalized size = 0.96

$$\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-3\,\sin\left(fx+e\right)+3\right)^{-m-1}\cos\left(fx+e\right)}{2\,fm+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] (a*sin(f*x + e) + a)^m*(-3*sin(f*x + e) + 3)^(-m - 1)*cos(f*x + e)/(2*f*m +
 f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-3\,\sin\left(fx+e\right)+3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-3*sin(f*x + e) + 3)^(-m - 1), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int\left(3-3\sin\left(fx+e\right)\right)^{-1-m}\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-3\,\sin\left(fx+e\right)+3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-3*sin(f*x + e) + 3)^(-m - 1), x)
```

mupad [B] time = 0.37, size = 45, normalized size = 1.00

$$\frac{\cos\left(e+f\,x\right)\,{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)}^m}{f\,\left(2\,m+1\right)\,{\left(3-3\,\sin\left(e+f\,x\right)\right)}^{m+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(3 - 3*sin(e + f*x))^(m + 1),x)
[Out] (cos(e + f*x)*(a*(sin(e + f*x) + 1))^m)/(f*(2*m + 1)*(3 - 3*sin(e + f*x))^(
m + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
[Out] Timed out
```

3.640 $\int (3-4\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=115

$$\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(3-4\sin(e+fx))^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{2(3-4\sin(e+fx))}{\sin(e+fx)+1}\right)}{\sqrt{7}fm(1-\sin(e+fx))}$$

```
[Out] 1/7*cos(f*x+e)*hypergeom([1/2, -m],[1-m],-2*(3-4*sin(f*x+e))/(1+sin(f*x+e))
)*(a+a*sin(f*x+e))^m*((1-sin(f*x+e))/(1+sin(f*x+e)))^(1/2)/f/m/((3-4*sin(f*
x+e))^m)/(1-sin(f*x+e))*7^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2788, 132}

$$\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(3-4\sin(e+fx))^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{2(3-4\sin(e+fx))}{\sin(e+fx)+1}\right)}{\sqrt{7}fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 4*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (-2*(3 - 4*Sin[e + f*x]))/(
1 + Sin[e + f*x])]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(a + a*Sin[e
 + f*x])^m)/(Sqrt[7]*f*m*(3 - 4*Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

```
!IntegerQ[m]
```

Rubi steps

$$\int (3-4\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(3-4x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx, x, \sin(e+\right.}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{2(3-4\sin(e+fx))}{1+\sin(e+fx)}\right)(3-4\sin(e}{\sqrt{7}\,fm(1-\sin(e+f}$$

Mathematica [A] time = 0.65, size = 178, normalized size = 1.55

$$\frac{2\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)(3-4\sin(e+fx))^{-m}\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a(\sin(e+}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - 4*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (2*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*Hyper
geometric2F1[1/2, 1/2 - m, 3/2, (7*Sin[(2*e - Pi + 2*f*x)/4]^2)/(3 - 4*Sin[
e + f*x])]*(a*(1 + Sin[e + f*x]))^m*(Cos[(2*e - Pi + 2*f*x)/4]^2/(-3 + 4*Si
n[e + f*x]))^(1/2 - m)*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(f*(3 - 4*S
in[e + f*x])^m)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m\left(-4\,\sin\left(fx+e\right)+3\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-4*sin(f*x + e) + 3)^(-m - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^m\left(-4\,\sin\left(fx+e\right)+3\right)^{-m-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(-4*sin(f*x + e) + 3)^(-m - 1), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \left(3-4\sin\left(fx+e\right)\right)^{-1-m} \left(a+a\sin\left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
[Out] int((3-4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-4\,\sin\left(fx+e\right)+3\right)^{-m-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*(-4*sin(f*x + e) + 3)^(-m - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^{m}}{\left(3-4\,\sin\left(e+f\,x\right)\right)^{m+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(3 - 4*sin(e + f*x))^(m + 1),x)
[Out] int((a + a*sin(e + f*x))^m/(3 - 4*sin(e + f*x))^(m + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
[Out] Timed out
```

3.641 $\int (3-5\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=113

$$\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(3-5\sin(e+fx))^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{3-5\sin(e+fx)}{\sin(e+fx)+1}\right)}{4fm(1-\sin(e+fx))}$$

```
[Out] 1/4*cos(f*x+e)*hypergeom([1/2, -m],[1-m],(-3+5*sin(f*x+e))/(1+sin(f*x+e)))*
(a+a*sin(f*x+e))^m*((1-sin(f*x+e))/(1+sin(f*x+e)))^(1/2)/f/m/((3-5*sin(f*x+
e))^m)/(1-sin(f*x+e))
```

Rubi [A] time = 0.10, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2788, 132}

$$\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(3-5\sin(e+fx))^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{3-5\sin(e+fx)}{\sin(e+fx)+1}\right)}{4fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 5*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, -((3 - 5*Sin[e + f*x])/(1 +
 Sin[e + f*x]))]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(a + a*Sin[e +
 f*x])^m)/(4*f*m*(3 - 5*Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (3-5\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(3-5x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx, x, \sin(e+f$$

$$\overline{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{3-5\sin(e+fx)}{1+\sin(e+fx)}\right)(3-5\sin(e+f}{4fm(1-\sin(e+fx))}$$

Mathematica [C] time = 0.57, size = 248, normalized size = 2.19

$$-\frac{2^{2m-1}(\cosh(m\log(4))-\sinh(m\log(4)))(3-5\sin(e+fx))^{-m}(\sin(e+fx)+i\cos(e+fx)+1)(a(\sin(e+fx)+}{f(2m+1)((1+2i)\sin(e+fx)+(-2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - 5*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((2^(-1 + 2*m)*Hypergeometric2F1[1 + m, 1 + 2*m, 2*(1 + m), (2*Cos[(2*e -
Pi + 2*f*x)/4])/(Cos[(2*e - Pi + 2*f*x)/4] + 2*Sin[(2*e - Pi + 2*f*x)/4])]*
(a*(1 + Sin[e + f*x]))^m*(1 + I*Cos[e + f*x] + Sin[e + f*x])*((-Cos[(2*e -
Pi + 2*f*x)/4] + 2*Sin[(2*e - Pi + 2*f*x)/4])/(Cos[(2*e - Pi + 2*f*x)/4] +
2*Sin[(2*e - Pi + 2*f*x)/4]))^m*(Cosh[m*Log[4]] - Sinh[m*Log[4]]))/(f*(1 +
2*m)*(3 - 5*Sin[e + f*x])^m*((1 - 2*I) - (2 - I)*Cos[e + f*x] + (1 + 2*I)*S
in[e + f*x])))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-5\,\sin\left(fx+e\right)+3\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-5*sin(f*x + e) + 3)^(-m - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(-5\,\sin\left(fx+e\right)+3\right)^{-m-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(-5*sin(f*x + e) + 3)^(-m - 1), x)
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \left(3-5\sin \left(fx+e\right)\right)^{-1-m} \left(a+a\sin \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
[Out] int((3-5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin \left(fx+e\right)+a\right)^{m}\left(-5\,\sin \left(fx+e\right)+3\right)^{-m-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*(-5*sin(f*x + e) + 3)^(-m - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin \left(e+f\,x\right)\right)^m}{\left(3-5\,\sin \left(e+f\,x\right)\right)^{m+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(3 - 5*sin(e + f*x))^(m + 1),x)
[Out] int((a + a*sin(e + f*x))^m/(3 - 5*sin(e + f*x))^(m + 1), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-5*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
[Out] Exception raised: HeuristicGCDFailed
```

3.642 $\int (-3+5\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=72

$$-\frac{\cos(e+fx)(\sin(e+fx)+1)^{-m-1}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};\frac{4(a-a\sin(e+fx))}{\sin(e+fx)a+a}\right)}{f}$$

```
[Out] -cos(f*x+e)*hypergeom([1/2, 1+m],[3/2],4*(a-a*sin(f*x+e))/(a+a*sin(f*x+e)))
*(1+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m/f
```

Rubi [A] time = 0.10, antiderivative size = 113, normalized size of antiderivative = 1.57, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2788, 132}

$$-\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(5\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{3-5\sin(e+fx)}{\sin(e+fx)+1}\right)}{4fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 5*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -(Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, -((3 - 5*Sin[e + f*x])/(1
+ Sin[e + f*x]))]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(a + a*Sin[e
+ f*x])^m)/(4*f*m*(1 - Sin[e + f*x])*(-3 + 5*Sin[e + f*x])^m)
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int(-3+5\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int\frac{(-3+5x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx,x,\sin(e\right.}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$=-\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{3-5\sin(e+fx)}{1+\sin(e+fx)}\right)\sqrt{\frac{1-\sin(e+fx}{1+\sin(e+fx}}}{4fm(1-\sin(e+}$$

Mathematica [C] time = 1.53, size = 247, normalized size = 3.43

$$\frac{2^{2m-1}(\cosh(m\log(4))-\sinh(m\log(4)))(5\sin(e+fx)-3)^{-m}(\sin(e+fx)+i\cos(e+fx)+1)(a(\sin(e+fx)+}{f(2m+1)((1+2i)\sin(e+fx)+(-}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-3 + 5*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (2^(-1 + 2*m)*Hypergeometric2F1[1 + m, 1 + 2*m, 2*(1 + m), (2*Cos[(2*e - Pi
 + 2*f*x)/4])/(Cos[(2*e - Pi + 2*f*x)/4] + 2*Sin[(2*e - Pi + 2*f*x)/4])]*(a
*(1 + Sin[e + f*x]))^m*(1 + I*Cos[e + f*x] + Sin[e + f*x])*((-Cos[(2*e - Pi
 + 2*f*x)/4] + 2*Sin[(2*e - Pi + 2*f*x)/4])/(Cos[(2*e - Pi + 2*f*x)/4] + 2*
Sin[(2*e - Pi + 2*f*x)/4]))^m*(Cosh[m*Log[4]] - Sinh[m*Log[4]]))/(f*(1 + 2*
m)*((1 - 2*I) - (2 - I)*Cos[e + f*x] + (1 + 2*I)*Sin[e + f*x])*(-3 + 5*Sin[
e + f*x])^m)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\left(5\,\sin\left(fx+e\right)-3\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas"
)
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(5*sin(f*x + e) - 3)^(-m - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\left(5\,\sin\left(fx+e\right)-3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(5*sin(f*x + e) - 3)^(-m - 1), x)
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \left(-3+5\sin \left(fx+e\right)\right)^{-1-m} \left(a+a\sin \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((-3+5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int {\left(a \sin \left(f x + e\right) + a\right)}^{m} {\left(5 \, \sin \left(f x + e\right) - 3\right)}^{-m - 1}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(5*sin(f*x + e) - 3)^(-m - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{{\left(5\,\sin\left(e+f\,x\right)-3\right)}^{m+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(5*sin(e + f*x) - 3)^(m + 1),x)
```

```
[Out] int((a + a*sin(e + f*x))^m/(5*sin(e + f*x) - 3)^(m + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+5*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

3.643 $\int (-3+4\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=77

$$-\frac{2^{m+1}\cos(e+fx)(\sin(e+fx)+1)^{-m-1}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},m+1;\frac{3}{2};\frac{7(a-a\sin(e+fx))}{\sin(e+fx)a+a}\right)}{f}$$

```
[Out] -2^(1+m)*cos(f*x+e)*hypergeom([1/2, 1+m],[3/2],7*(a-a*sin(f*x+e))/(a+a*sin(
f*x+e)))*(1+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m/f
```

Rubi [A] time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.51, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2788, 132}

$$-\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(4\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{2(3-4\sin(e+fx))}{\sin(e+fx)+1}\right)}{\sqrt{7}fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 4*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (-2*(3 - 4*Sin[e + f*x]))
/(1 + Sin[e + f*x])]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(a + a*Sin
[e + f*x])^m)/(Sqrt[7]*f*m*(1 - Sin[e + f*x])*(-3 + 4*Sin[e + f*x])^m))
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int(-3+4\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx=\frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int\frac{(-3+4x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx,x,\sin(e+\right.}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$=-\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{2(3-4\sin(e+fx))}{1+\sin(e+fx)}\right)\sqrt{\frac{1-\sin(e+f}{1+\sin(e+f}}}{\sqrt{7}\,fm(1-\sin(e+}$$

Mathematica [A] time = 0.62, size = 154, normalized size = 2.00

$$-\frac{2\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)(4\sin(e+fx)-3)^{-m}\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a(\sin(e+}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-3 + 4*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (-2*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*Hype
rgeometric2F1[1/2, 1 + m, 3/2, 7*Tan[(2*e - Pi + 2*f*x)/4]^2]*(a*(1 + Sin[e
 + f*x]))^m*(Sec[(2*e - Pi + 2*f*x)/4]^2*(-3 + 4*Sin[e + f*x]))^m*(Sin[(2*e
 + Pi + 2*f*x)/4]^2)^(1/2 - m))/(f*(-3 + 4*Sin[e + f*x])^m)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\left(4\,\sin\left(fx+e\right)-3\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas"
)
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(4*sin(f*x + e) - 3)^(-m - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\left(4\,\sin\left(fx+e\right)-3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(4*sin(f*x + e) - 3)^(-m - 1), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \left(-3+4\sin\left(fx+e\right)\right)^{-1-m} \left(a+a\sin\left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((-3+4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(4\,\sin\left(fx+e\right)-3\right)^{-m-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(4*sin(f*x + e) - 3)^(-m - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(4\,\sin\left(e+f\,x\right)-3\right)^{m+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(4*sin(e + f*x) - 3)^(m + 1),x)
```

```
[Out] int((a + a*sin(e + f*x))^m/(4*sin(e + f*x) - 3)^(m + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+4*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

3.644 $\int (-3+3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=45

$$\frac{\cos(e+fx)(3\sin(e+fx)-3)^{-m-1}(a\sin(e+fx)+a)^m}{f(2m+1)}$$

```
[Out] cos(f*x+e)*(-3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m/f/(1+2*m)
```

Rubi [A] time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2742}

$$\frac{\cos(e+fx)(3\sin(e+fx)-3)^{-m-1}(a\sin(e+fx)+a)^m}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*(-3 + 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m)/(f*(1
+ 2*m))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int (-3+3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{\cos(e+fx)(-3+3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m}{f(1+2m)}$$

Mathematica [B] time = 0.66, size = 110, normalized size = 2.44

$$\frac{2^{-m}3^{-m-1}\sin\left(\frac{1}{4}(2e+2fx+\pi)\right)(\sin(e+fx)-1)^{-m-1}\cos^{-2m-1}\left(\frac{1}{4}(2e+2fx+\pi)\right)(a(\sin(e+fx)+1))^m\left(\cos\left(\frac{1}{2}\right.\right.}{2fm+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (3^(-1 - m)*Cos[(2*e + Pi + 2*f*x)/4]^(-1 - 2*m)*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])^(2*(1 + m))*(-1 + Sin[e + f*x])^(-1 - m)*(a*(1 + Sin[e + f*x]))
^m*Sin[(2*e + Pi + 2*f*x)/4])/(2^m*(f + 2*f*m))
```

fricas [A] time = 0.46, size = 43, normalized size = 0.96

$$\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\left(3\,\sin\left(fx+e\right)-3\right)^{-m-1}\cos\left(fx+e\right)}{2\,fm+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas"
)
```

```
[Out] (a*sin(f*x + e) + a)^m*(3*sin(f*x + e) - 3)^(-m - 1)*cos(f*x + e)/(2*f*m +
f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\left(3\,\sin\left(fx+e\right)-3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(3*sin(f*x + e) - 3)^(-m - 1), x)
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int\left(-3+3\sin\left(fx+e\right)\right)^{-1-m}\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((-3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\left(3\,\sin\left(fx+e\right)-3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(3*sin(f*x + e) - 3)^(-m - 1), x)
```

mupad [B] time = 7.83, size = 45, normalized size = 1.00

$$\frac{\cos\left(e+f\,x\right)\,{\left(\frac{a\,\left(\sin\left(e+f\,x\right)+1\right)}{3}\right)}^m}{3\,f\,\left(2\,m+1\right)\,{\left(\sin\left(e+f\,x\right)-1\right)}^{m+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(3*sin(e + f*x) - 3)^(m + 1),x)
```

```
[Out] (cos(e + f*x)*((a*(sin(e + f*x) + 1))/3)^m)/(3*f*(2*m + 1)*(sin(e + f*x) -
1)^(m + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+3*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

3.645 $\int (-3+2\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=117

$$-\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(2\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3-2\sin(e+fx))}{\sin(e+fx)+1}\right)}{\sqrt{5}\,fm(1-\sin(e+fx))}$$

```
[Out] -1/5*cos(f*x+e)*hypergeom([1/2, -m],[1-m],2*(3-2*sin(f*x+e))/(1+sin(f*x+e))
)*(a+a*sin(f*x+e))^m*((-1+sin(f*x+e))/(1+sin(f*x+e)))^(1/2)/f/m/(1-sin(f*x+
e))/((-3+2*sin(f*x+e))^m)*5^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2788, 132}

$$-\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(2\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3-2\sin(e+fx))}{\sin(e+fx)+1}\right)}{\sqrt{5}\,fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 2*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (2*(3 - 2*Sin[e + f*x]))/
(1 + Sin[e + f*x])]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(a + a*S
in[e + f*x])^m)/(Sqrt[5]*f*m*(1 - Sin[e + f*x])*(-3 + 2*Sin[e + f*x])^m))
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

```
!IntegerQ[m]
```

Rubi steps

$$\int (-3+2\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(-3+2x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx,x,\sin(e+\right.}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3-2\sin(e+fx))}{1+\sin(e+fx)}\right)\sqrt{-\frac{1-\sin(e+f}{1+\sin(e+f}}}{\sqrt{5}\,fm(1-\sin(e+}$$

Mathematica [A] time = 0.63, size = 155, normalized size = 1.32

$$\frac{2\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)(2\sin(e+fx)-3)^{-m}\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a(\sin(e+f}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-3 + 2*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (2*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*Hyper
geometric2F1[1/2, 1 + m, 3/2, -5*Tan[(2*e - Pi + 2*f*x)/4]^2]*(a*(1 + Sin[e
 + f*x]))^m*(-(Sec[(2*e - Pi + 2*f*x)/4]^2*(-3 + 2*Sin[e + f*x])))^m*(Sin[(
2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(f*(-3 + 2*Sin[e + f*x])^m)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m\left(2\,\sin\left(fx+e\right)-3\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas"
)
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(2*sin(f*x + e) - 3)^(-m - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^m\left(2\,\sin\left(fx+e\right)-3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(2*sin(f*x + e) - 3)^(-m - 1), x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \left(-3+2\sin\left(fx+e\right)\right)^{-1-m}\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((-3+2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(2\,\sin\left(fx+e\right)-3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(2*sin(f*x + e) - 3)^(-m - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(2\,\sin\left(e+f\,x\right)-3\right)^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(2*sin(e + f*x) - 3)^(m + 1),x)
```

```
[Out] int((a + a*sin(e + f*x))^m/(2*sin(e + f*x) - 3)^(m + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

3.646 $\int (-3+\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=116

$$-\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{3-\sin(e+fx)}{\sin(e+fx)+1}\right)}{2\sqrt{2}fm(1-\sin(e+fx))}$$

```
[Out] -1/4*cos(f*x+e)*hypergeom([1/2, -m],[1-m],(3-sin(f*x+e))/(1+sin(f*x+e)))*(a
+a*sin(f*x+e))^m*((-1+sin(f*x+e))/(1+sin(f*x+e)))^(1/2)/f/m/(1-sin(f*x+e))/
((-3+sin(f*x+e))^m)*2^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2788, 132}

$$-\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{3-\sin(e+fx)}{\sin(e+fx)+1}\right)}{2\sqrt{2}fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -(Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (3 - Sin[e + f*x])/(1 + Si
n[e + f*x])]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(a + a*Sin[e +
f*x])^m)/(2*Sqrt[2]*f*m*(1 - Sin[e + f*x])*(-3 + Sin[e + f*x])^m)
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

```
!IntegerQ[m]
```

Rubi steps

$$\int (-3+\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(-3+x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx, x, \sin(e+\right.}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{3-\sin(e+fx)}{1+\sin(e+fx)}\right)(-3+\sin(e+f}{2\sqrt{2}\,fm(1-\sin(e+f}$$

Mathematica [A] time = 0.71, size = 155, normalized size = 1.34

$$\frac{2^{-m}\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)(\sin(e+fx)-3)^{-m}\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a(\sin(e}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-3 + Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] ((Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*Hyperge
ometric2F1[1/2, 1 + m, 3/2, -2*Tan[(2*e - Pi + 2*f*x)/4]^2]*(-(Sec[(2*e - P
i + 2*f*x)/4]^2*(-3 + Sin[e + f*x])))^m*(a*(1 + Sin[e + f*x]))^m*(Sin[(2*e
+ Pi + 2*f*x)/4]^2)^(1/2 - m))/(2^m*f*(-3 + Sin[e + f*x])^m)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m\left(\sin\left(fx+e\right)-3\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(sin(f*x + e) - 3)^(-m - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^m\left(\sin\left(fx+e\right)-3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(sin(f*x + e) - 3)^(-m - 1), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \left(-3+\sin \left(f x +e \right)\right)^{-1-m} \left(a +a \sin \left(f x +e \right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
[Out] int((-3+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin \left(f x+e\right)+a\right)^{m}\left(\sin \left(f x+e\right)-3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*(sin(f*x + e) - 3)^(-m - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{{\left(\sin\left(e+f\,x\right)-3\right)}^{m+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(sin(e + f*x) - 3)^(m + 1),x)
[Out] int((a + a*sin(e + f*x))^m/(sin(e + f*x) - 3)^(m + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
[Out] Timed out
```

3.647 $\int (-3)^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=81

$$-\frac{(-3)^{-m-1}2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f}$$

```
[Out] -(-3)^(-1-m)*2^(1/2+m)*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(
f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/f
```

Rubi [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {12, 2652, 2651}

$$-\frac{(-3)^{-m-1}2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(-3)^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -(((-3)^(-1 - m)*2^(1/2 + m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3
/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x]
)^m)/f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int (-3)^{-1-m}(a+a\sin(e+fx))^m\,dx &= (-3)^{-1-m}\int (a+a\sin(e+fx))^m\,dx\\ &= \left((-3)^{-1-m}(1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\right)\int (1+\sin(e+fx))^m\,dx\\ &= -\frac{(-3)^{-1-m}2^{\frac{1}{2}+m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e}{f}\end{aligned}$$

Mathematica [A] time = 0.17, size = 97, normalized size = 1.20

$$\frac{\sqrt{2}(-3)^{-m-1}\cos(e+fx)(a(\sin(e+fx)+1))^m\,{}_2F_1\left(\frac{1}{2},m+\frac{1}{2};m+\frac{3}{2};\frac{1}{4}\cos^2(e+fx)\csc^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)}{(2fm+f)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3)^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] ((-3)^(-1 - m)*Sqrt[2]*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 + m, 3/2 + m
, (Cos[e + f*x]^2*Csc[(2*e - Pi + 2*f*x)/4]^2)/4]*(a*(1 + Sin[e + f*x]))^m)
/((f + 2*f*m)*Sqrt[1 - Sin[e + f*x]])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left((-3)^{-m-1}\left(a\sin\left(fx+e\right)+a\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3)^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((-3)^(-m - 1)*(a*sin(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-3)^{-m-1}\left(a\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3)^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((-3)^(-m - 1)*(a*sin(f*x + e) + a)^m, x)
```

maple [F] time = 0.67, size = 0, normalized size = 0.00

$$\int (-3)^{-1-m} \left(a + a\sin\left(fx + e\right)\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3)^(-1-m)*(a+a*sin(f*x+e))^m,x)
[Out] int((-3)^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$(-3)^{-m-1} \int \left(a\sin\left(fx + e\right) + a\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3)^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] (-3)^(-m - 1)*integrate((a*sin(f*x + e) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(-3)^{m+1}} \left(a + a\ \sin\left(e + f\,x\right)\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3)^(m + 1)*(a + a*sin(e + f*x))^m,x)
[Out] int(1/(-3)^(m + 1)*(a + a*sin(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$(-3)^{-m-1} \int \left(a\sin\left(e + fx\right) + a\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3)**(-1-m)*(a+a*sin(f*x+e))**m,x)
[Out] (-3)**(-m - 1)*Integral((a*sin(e + f*x) + a)**m, x)
```

3.648 $\int(-3-\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=119

$$-\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(-\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{\sin(e+fx)+3}{2(\sin(e+fx)+1)}\right)}{2\sqrt{2}fm(1-\sin(e+fx))}$$

```
[Out] -1/4*cos(f*x+e)*hypergeom([1/2, -m],[1-m],1/2*(3+sin(f*x+e))/(1+sin(f*x+e))
)*(a+a*sin(f*x+e))^m*((-1+sin(f*x+e))/(1+sin(f*x+e)))^(1/2)/f/m/((-3-sin(f*
x+e))^m)/(1-sin(f*x+e))*2^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2788, 132}

$$-\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(-\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{\sin(e+fx)+3}{2(\sin(e+fx)+1)}\right)}{2\sqrt{2}fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 - Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -(Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (3 + Sin[e + f*x])/(2*(1 +
 Sin[e + f*x]))]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(a + a*Sin[
e + f*x])^m)/(2*Sqrt[2]*f*m*(-3 - Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

```
!IntegerQ[m]
```

Rubi steps

$$\int (-3-\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(-3-x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx, x, \sin(e+\ldots\right.}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{3+\sin(e+fx)}{2(1+\sin(e+fx))}\right)(-3-\sin(e+\ldots}{2\sqrt{2}\,fm(1-\sin(e+\ldots}$$

Mathematica [A] time = 0.91, size = 131, normalized size = 1.10

$$\frac{4^{-m}\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)(-\sin(e+fx)-3)^{-m}(\sin(e+fx)+3)^{m-\frac{1}{2}}\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}(a(\sin(e+fx)\ldots}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-3 - Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cot[(2*e + Pi + 2*f*x)/4]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (2*Sin[(2*e
 - Pi + 2*f*x)/4]^2)/(3 + Sin[e + f*x])]*(a*(1 + Sin[e + f*x]))^m*(3 + Sin[
e + f*x])^(-1/2 + m)*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(4^m*f*(-3 -
Sin[e + f*x])^m)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m\left(-\sin\left(fx+e\right)-3\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-sin(f*x + e) - 3)^(-m - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^m\left(-\sin\left(fx+e\right)-3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(-sin(f*x + e) - 3)^(-m - 1), x)
```

maple [F] time = 0.73, size = 0, normalized size = 0.00

$$\int \left(-3-\sin \left(fx+e\right)\right)^{-1-m} \left(a+a\sin \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3-sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
[Out] int((-3-sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin \left(fx+e\right)+a\right)^{m}\left(-\sin \left(fx+e\right)-3\right)^{-m-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*(-sin(f*x + e) - 3)^(-m - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{{\left(-\sin\left(e+f\,x\right)-3\right)}^{m+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(- sin(e + f*x) - 3)^(m + 1),x)
[Out] int((a + a*sin(e + f*x))^m/(- sin(e + f*x) - 3)^(m + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
[Out] Timed out
```

3.649 $\int (-3-2\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=119

$$-\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(-2\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(2\sin(e+fx)+3)}{5(\sin(e+fx)+1)}\right)}{\sqrt{5}\,fm(1-\sin(e+fx))}$$

```
[Out] -1/5*cos(f*x+e)*hypergeom([1/2, -m],[1-m],2/5*(3+2*sin(f*x+e))/(1+sin(f*x+e
)))*(a+a*sin(f*x+e))^m*((-1+sin(f*x+e))/(1+sin(f*x+e)))^(1/2)/f/m/((-3-2*si
n(f*x+e))^m)/(1-sin(f*x+e))*5^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2788, 132}

$$-\frac{\sqrt{-\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(-2\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(2\sin(e+fx)+3)}{5(\sin(e+fx)+1)}\right)}{\sqrt{5}\,fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 - 2*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (2*(3 + 2*Sin[e + f*x]))/
(5*(1 + Sin[e + f*x]))]*Sqrt[-((1 - Sin[e + f*x])/(1 + Sin[e + f*x]))]*(a +
 a*Sin[e + f*x])^m)/(Sqrt[5]*f*m*(-3 - 2*Sin[e + f*x])^m*(1 - Sin[e + f*x])
))
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

```
!IntegerQ[m]
```

Rubi steps

$$\int (-3-2\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(-3-2x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx, x, \sin(e+\right.}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(3+2\sin(e+fx))}{5(1+\sin(e+fx))}\right)(-3-2\sin(e}{\sqrt{5}\,fm(1-\sin(e+}$$

Mathematica [A] time = 1.15, size = 186, normalized size = 1.56

$$\frac{2\,5^{-m-\frac{1}{2}}\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)(-2\sin(e+fx)-3)^{-m}\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a(s}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 - 2*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (2*5^(-1/2 - m)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*
f*x)/4]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, Sin[(2*e - Pi + 2*f*x)/4]^2/(3
 + 2*Sin[e + f*x])]*(a*(1 + Sin[e + f*x]))^m*(Cos[(2*e - Pi + 2*f*x)/4]^2/(
3 + 2*Sin[e + f*x]))^(1/2 - m)*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(f*
(-3 - 2*Sin[e + f*x])^m)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m\left(-2\,\sin\left(fx+e\right)-3\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas"
)
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-2*sin(f*x + e) - 3)^(-m - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^m\left(-2\,\sin\left(fx+e\right)-3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-2*sin(f*x + e) - 3)^(-m - 1), x)
```

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \left(-3-2\sin \left(f x +e \right)\right)^{-1-m} \left(a +a \sin \left(f x +e \right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3-2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((-3-2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int {\left(a \sin \left(f x + e\right) + a\right)}^{m} {\left(-2 \, \sin \left(f x + e\right) - 3\right)}^{-m - 1}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-2*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-2*sin(f*x + e) - 3)^(-m - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{{\left(-2\,\sin\left(e+f\,x\right)-3\right)}^{m+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(- 2*sin(e + f*x) - 3)^(m + 1),x)
```

```
[Out] int((a + a*sin(e + f*x))^m/(- 2*sin(e + f*x) - 3)^(m + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-2*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

3.650 $\int (-3-3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=39

$$-\frac{\cos(e+fx)(-3\sin(e+fx)-3)^{-m-1}(a\sin(e+fx)+a)^m}{f}$$

```
[Out] -cos(f*x+e)*(-3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m/f
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {23, 2648}

$$-\frac{\cos(e+fx)(-3\sin(e+fx)-3)^{-m-1}(a\sin(e+fx)+a)^m}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 - 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((Cos[e + f*x]*(-3 - 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m)/f)
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int (-3-3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \left((-3-3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^{1+m}\right)\int \frac{}{a+a\,\text{si}}$$

$$= -\frac{\cos(e+fx)(-3-3\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m}{f}$$

Mathematica [B] time = 0.52, size = 106, normalized size = 2.72

$$-\frac{2^{-m}3^{-m-1}\cos\left(\frac{1}{4}(2e+2fx+\pi)\right)(-\sin(e+fx)-1)^{-m-1}\sin^{-2m-1}\left(\frac{1}{4}(2e+2fx+\pi)\right)(a(\sin(e+fx)+1))^{m}\left(\mathrm{si}\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 - 3*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((3^(-1 - m)*Cos[(2*e + Pi + 2*f*x)/4]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^(2*(1 + m))*(-1 - Sin[e + f*x])^(-1 - m)*(a*(1 + Sin[e + f*x]))^m*Sin[(2
*e + Pi + 2*f*x)/4]^(-1 - 2*m))/(2^m*f))
```

fricas [A] time = 0.48, size = 43, normalized size = 1.10

$$\frac{\left(-\frac{1}{3}\,a\right)^{m}\left(\cos\left(fx+e\right)-\sin\left(fx+e\right)+1\right)}{3\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas"
)
```

```
[Out] 1/3*(-1/3*a)^m*(cos(f*x + e) - sin(f*x + e) + 1)/(f*cos(f*x + e) + f*sin(f*
x + e) + f)
```

giac [B] time = 1.26, size = 827, normalized size = 21.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] (e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(-5/4*pi - pi*m*floor(-1/4*sgn(a
) + 1/2) - 1/4*pi*m*sgn(a) - 5/4*pi*m + 1/2*f*x + 1/2*e)^2*tan(1/2*f*x + 1/
2*e)^3 + 3*e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(-5/4*pi - pi*m*floor(
-1/4*sgn(a) + 1/2) - 1/4*pi*m*sgn(a) - 5/4*pi*m + 1/2*f*x + 1/2*e)^2*tan(1/
2*f*x + 1/2*e)^2 - 2*e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(-5/4*pi - p
i*m*floor(-1/4*sgn(a) + 1/2) - 1/4*pi*m*sgn(a) - 5/4*pi*m + 1/2*f*x + 1/2*e
)*tan(1/2*f*x + 1/2*e)^3 - 3*e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(-5/
4*pi - pi*m*floor(-1/4*sgn(a) + 1/2) - 1/4*pi*m*sgn(a) - 5/4*pi*m + 1/2*f*x
 + 1/2*e)^2*tan(1/2*f*x + 1/2*e) + 6*e^(-m*log(3) + m*log(abs(a)) - log(3))
*tan(-5/4*pi - pi*m*floor(-1/4*sgn(a) + 1/2) - 1/4*pi*m*sgn(a) - 5/4*pi*m +
 1/2*f*x + 1/2*e)*tan(1/2*f*x + 1/2*e)^2 - e^(-m*log(3) + m*log(abs(a)) - l
```

```
og(3))*tan(1/2*f*x + 1/2*e)^3 - e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(
-5/4*pi - pi*m*floor(-1/4*sgn(a) + 1/2) - 1/4*pi*m*sgn(a) - 5/4*pi*m + 1/2*
f*x + 1/2*e)^2 + 6*e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(-5/4*pi - pi*
m*floor(-1/4*sgn(a) + 1/2) - 1/4*pi*m*sgn(a) - 5/4*pi*m + 1/2*f*x + 1/2*e)*
tan(1/2*f*x + 1/2*e) - 3*e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(1/2*f*x
 + 1/2*e)^2 - 2*e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(-5/4*pi - pi*m*f
loor(-1/4*sgn(a) + 1/2) - 1/4*pi*m*sgn(a) - 5/4*pi*m + 1/2*f*x + 1/2*e) + 3
*e^(-m*log(3) + m*log(abs(a)) - log(3))*tan(1/2*f*x + 1/2*e) + e^(-m*log(3)
 + m*log(abs(a)) - log(3)))/(f*tan(-5/4*pi - pi*m*floor(-1/4*sgn(a) + 1/2)
- 1/4*pi*m*sgn(a) - 5/4*pi*m + 1/2*f*x + 1/2*e)^2*tan(1/2*f*x + 1/2*e)^3 +
f*tan(-5/4*pi - pi*m*floor(-1/4*sgn(a) + 1/2) - 1/4*pi*m*sgn(a) - 5/4*pi*m
+ 1/2*f*x + 1/2*e)^2*tan(1/2*f*x + 1/2*e)^2 + f*tan(-5/4*pi - pi*m*floor(-1
/4*sgn(a) + 1/2) - 1/4*pi*m*sgn(a) - 5/4*pi*m + 1/2*f*x + 1/2*e)^2*tan(1/2*
f*x + 1/2*e) + f*tan(1/2*f*x + 1/2*e)^3 + f*tan(-5/4*pi - pi*m*floor(-1/4*s
gn(a) + 1/2) - 1/4*pi*m*sgn(a) - 5/4*pi*m + 1/2*f*x + 1/2*e)^2 + f*tan(1/2*
f*x + 1/2*e)^2 + f*tan(1/2*f*x + 1/2*e) + f)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \left(-3-3\sin\left(fx+e\right)\right)^{-1-m} \left(a+a\sin\left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((-3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

maxima [A] time = 1.23, size = 45, normalized size = 1.15

$$\frac{2\, a^{m}}{\left(3^{m+1} \left(-1\right)^{m} + \frac{3^{m+1} \left(-1\right)^{m} \sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right) f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-3*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima"
)
```

```
[Out] 2*a^m/((3^(m + 1)*(-1)^m + 3^(m + 1)*(-1)^m*sin(f*x + e)/(cos(f*x + e) + 1)
)*f)
```

mupad [B] time = 0.39, size = 52, normalized size = 1.33

$$\frac{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)^{m}\,\left(-\cos\left(e+f\,x\right)+\sin\left(e+f\,x\right)\,1{}\mathrm{i}+1{}\mathrm{i}\right)}{f\,\left(-3\,\sin\left(e+f\,x\right)-3\right)^{m+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(- 3*sin(e + f*x) - 3)^(m + 1),x)
```

```
[Out] ((a*(sin(e + f*x) + 1))^m*(sin(e + f*x)*1i - cos(e + f*x) + 1i))/(f*(- 3*si
n(e + f*x) - 3)^(m + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-3*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

3.651 $\int (-3-4\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=117

$$\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(-4\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(4\sin(e+fx)+3)}{7(\sin(e+fx)+1)}\right)}{\sqrt{7}fm(1-\sin(e+fx))}$$

```
[Out] 1/7*cos(f*x+e)*hypergeom([1/2, -m],[1-m],2/7*(3+4*sin(f*x+e))/(1+sin(f*x+e)
))*(a+a*sin(f*x+e))^m*((1-sin(f*x+e))/(1+sin(f*x+e)))^(1/2)/f/m/((-3-4*sin(
f*x+e))^m)/(1-sin(f*x+e))*7^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2788, 132}

$$\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(-4\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{2(4\sin(e+fx)+3)}{7(\sin(e+fx)+1)}\right)}{\sqrt{7}fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 - 4*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (2*(3 + 4*Sin[e + f*x]))/(7
*(1 + Sin[e + f*x]))]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(a + a*Si
n[e + f*x])^m)/(Sqrt[7]*f*m*(-3 - 4*Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

```
!IntegerQ[m]
```

Rubi steps

$$\int (-3-4\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(-3-4x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx, x, \sin(e}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}, -m; 1-m; \frac{2(3+4\sin(e+fx))}{7(1+\sin(e+fx))}\right)(-3-4\sin(e}{\sqrt{7}\,fm(1-\sin(e+}$$

Mathematica [A] time = 1.16, size = 187, normalized size = 1.60

$$\frac{2\,7^{-m-\frac{1}{2}}\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)(-4\sin(e+fx)-3)^{-m}\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 - 4*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (2*7^(-1/2 - m)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*
f*x)/4]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, -(Sin[(2*e - Pi + 2*f*x)/4]^2/
(3 + 4*Sin[e + f*x]))]*(a*(1 + Sin[e + f*x]))^m*(Cos[(2*e - Pi + 2*f*x)/4]^
2/(3 + 4*Sin[e + f*x]))^(1/2 - m)*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/
(f*(-3 - 4*Sin[e + f*x])^m)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m\left(-4\,\sin\left(fx+e\right)-3\right)^{-m-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas"
)
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-4*sin(f*x + e) - 3)^(-m - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^m\left(-4\,\sin\left(fx+e\right)-3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-4*sin(f*x + e) - 3)^(-m - 1), x)
```

maple [F] time = 0.74, size = 0, normalized size = 0.00

$$\int \left(-3-4 \sin \left(f x +e \right)\right)^{-1-m} \left(a +a \sin \left(f x +e \right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3-4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((-3-4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int {\left(a \sin \left(f x + e\right) + a\right)}^{m} {\left(-4 \, \sin \left(f x + e\right) - 3\right)}^{-m - 1}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-4*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-4*sin(f*x + e) - 3)^(-m - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{{\left(-4\,\sin\left(e+f\,x\right)-3\right)}^{m+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(- 4*sin(e + f*x) - 3)^(m + 1),x)
```

```
[Out] int((a + a*sin(e + f*x))^m/(- 4*sin(e + f*x) - 3)^(m + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-4*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

3.652 $\int(-3-5\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=115

$$\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(-5\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{5\sin(e+fx)+3}{4(\sin(e+fx)+1)}\right)}{4fm(1-\sin(e+fx))}$$

```
[Out] 1/4*cos(f*x+e)*hypergeom([1/2, -m],[1-m],1/4*(3+5*sin(f*x+e))/(1+sin(f*x+e)
))*(a+a*sin(f*x+e))^m*((1-sin(f*x+e))/(1+sin(f*x+e)))^(1/2)/f/m/((-3-5*sin(
f*x+e))^m)/(1-sin(f*x+e))
```

Rubi [A] time = 0.10, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2788, 132}

$$\frac{\sqrt{\frac{1-\sin(e+fx)}{\sin(e+fx)+1}}\cos(e+fx)(-5\sin(e+fx)-3)^{-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{5\sin(e+fx)+3}{4(\sin(e+fx)+1)}\right)}{4fm(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 - 5*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, -m, 1 - m, (3 + 5*Sin[e + f*x])/(4*(1
+ Sin[e + f*x]))]*Sqrt[(1 - Sin[e + f*x])/(1 + Sin[e + f*x])]*(a + a*Sin[e
+ f*x])^m)/(4*f*m*(-3 - 5*Sin[e + f*x])^m*(1 - Sin[e + f*x]))
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int (-3-5\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(-3-5x)^{-1-m}(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}}\,dx,x,\sin(e+\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-m;1-m;\frac{3+5\sin(e+fx)}{4(1+\sin(e+fx))}\right)(-3-5\sin(e+}{4fm(1-\sin(e+fx)}$$

Mathematica [C] time = 1.56, size = 241, normalized size = 2.10

$$-\frac{4^m(\cosh(m\log(4))-\sinh(m\log(4)))(-5\sin(e+fx)-3)^{-m}(\sin(e+fx)+i\cos(e+fx)+1)(a(\sin(e+fx)+1)}{f(2m+1)((2+i)\sin(e+fx)+(-1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-3 - 5*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((4^m*Hypergeometric2F1[1 + m, 1 + 2*m, 2*(1 + m), (4*Cos[(2*e - Pi + 2*f*
x)/4])/(2*Cos[(2*e - Pi + 2*f*x)/4] + Sin[(2*e - Pi + 2*f*x)/4])]*(a*(1 + S
in[e + f*x]))^m*(1 + I*Cos[e + f*x] + Sin[e + f*x])*(-((2*Cos[(2*e - Pi + 2
*f*x)/4] + Cos[(2*e + Pi + 2*f*x)/4])/(2*Cos[(2*e - Pi + 2*f*x)/4] + Sin[(2
*e - Pi + 2*f*x)/4])))^m*(Cosh[m*Log[4]] - Sinh[m*Log[4]]))/(f*(1 + 2*m)*(-
3 - 5*Sin[e + f*x])^m*((2 - I) - (1 - 2*I)*Cos[e + f*x] + (2 + I)*Sin[e + f
*x])))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m\left(-5\,\sin\left(fx+e\right)-3\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas"
)
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-5*sin(f*x + e) - 3)^(-m - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^m\left(-5\,\sin\left(fx+e\right)-3\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-5*sin(f*x + e) - 3)^(-m - 1), x)
```

maple [F] time = 0.68, size = 0, normalized size = 0.00

$$\int \left(-3-5\sin\left(fx+e\right)\right)^{-1-m} \left(a+a\sin\left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3-5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((-3-5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m} \left(-5\,\sin\left(fx+e\right)-3\right)^{-m-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-5*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima"
)
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-5*sin(f*x + e) - 3)^(-m - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(-5\,\sin\left(e+f\,x\right)-3\right)^{m+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(- 5*sin(e + f*x) - 3)^(m + 1),x)
```

```
[Out] int((a + a*sin(e + f*x))^m/(- 5*sin(e + f*x) - 3)^(m + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-5*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

3.653 $\int (d\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=116

$$-\frac{\cos(e+fx)\left(\frac{\sin(e+fx)+1}{1-\sin(e+fx)}\right)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m(d\sin(e+fx))^{-m}\,{}_2F_1\left(\frac{1}{2}-m,-m;1-m;-\frac{2\sin(e+fx)}{1-\sin(e+fx)}\right)}{dfm(\sin(e+fx)+1)}$$

```
[Out] -cos(f*x+e)*hypergeom([-m, 1/2-m],[1-m],-2*sin(f*x+e)/(1-sin(f*x+e)))*((1+s
in(f*x+e))/(1-sin(f*x+e)))^(1/2-m)*(a+a*sin(f*x+e))^m/d/f/m/((d*sin(f*x+e))
^m)/(1+sin(f*x+e))
```

Rubi [A] time = 0.19, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2787, 2786, 2785, 132}

$$-\frac{\cos(e+fx)\left(\frac{\sin(e+fx)+1}{1-\sin(e+fx)}\right)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m(d\sin(e+fx))^{-m}\,{}_2F_1\left(\frac{1}{2}-m,-m;1-m;-\frac{2\sin(e+fx)}{1-\sin(e+fx)}\right)}{dfm(\sin(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[1/2 - m, -m, 1 - m, (-2*Sin[e + f*x])/(1
- Sin[e + f*x])]*((1 + Sin[e + f*x])/(1 - Sin[e + f*x]))^(1/2 - m)*(a + a*S
in[e + f*x])^m)/(d*f*m*(d*Sin[e + f*x])^m*(1 + Sin[e + f*x])))
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2786

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[((d/b)^IntPart[n]*(d*Sin[e + f*x])^FracPart[n
])/(b*Sin[e + f*x])^FracPart[n], Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (d\sin(e+fx))^{-1-m}(a+a\sin(e+fx))^m\,dx = \left((1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\right)\int (d\sin(e+fx))^{-1-m}$$

$$= \frac{\left(\sin^m(e+fx)(d\sin(e+fx))^{-m}(1+\sin(e+fx))^{-m}(a+a\sin(e\right.}{d}$$

$$= -\frac{\left(\cos(e+fx)\sin^m(e+fx)(d\sin(e+fx))^{-m}(1+\sin(e+fx))^{-\frac{1}{2}}\right.}{df\sqrt{}}$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}-m,-m;1-m;-\frac{2\sin(e+fx)}{1-\sin(e+fx)}\right)(d\sin(e+fx)}{dfm(1+\sin(e+fx))}$$

Mathematica [C] time = 1.49, size = 194, normalized size = 1.67

$$\frac{(1-i)2^m(\cosh(m\log(2))-\sinh(m\log(2)))(\cos(e+fx)-i(\sin(e+fx)+1))(a(\sin(e+fx)+1))^m(d\sin(e+fx}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^(-1 - m)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] ((1 - I)*2^m*Hypergeometric2F1[1 + m, 1 + 2*m, 2*(1 + m), Sqrt[2]*Cos[(2*e
- Pi + 2*f*x)/4]*Csc[(e + f*x)/2]]*((1 - I)*(1 + Cos[e + f*x] - I*Sin[e + f
```

```
*x]))^m*(a*(1 + Sin[e + f*x]))^m*(Cos[e + f*x] - I*(1 + Sin[e + f*x]))*(Cos
h[m*Log[2]] - Sinh[m*Log[2]]))/(d*f*(1 + 2*m)*(-1 + Cos[e + f*x] - I*Sin[e
+ f*x])*((1 + I)*(1 - Cos[e + f*x] + I*Sin[e + f*x]))^m*(d*Sin[e + f*x])^m)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^(-m - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^(-m - 1), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int\left(d\sin\left(fx+e\right)\right)^{-1-m}\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
[Out] int((d*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(-1-m)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^(-m - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(d\,\sin\left(e+f\,x\right)\right)^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(d*sin(e + f*x))^(m + 1),x)
[Out] int((a + a*sin(e + f*x))^m/(d*sin(e + f*x))^(m + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**(-1-m)*(a+a*sin(f*x+e))**m,x)
[Out] Timed out
```

3.654 $\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx$

Optimal. Leaf size=129

$$-\frac{a2^{m+\frac{1}{2}}\cos(e+fx)(a\sin(e+fx)+a)^{m-1}\left(\frac{(c+d)(\sin(e+fx)+1)}{c+d\sin(e+fx)}\right)^{\frac{1}{2}-m}(c+d\sin(e+fx))^{-m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{(c-d)(1-\sin(e+fx))}{2(c+d\sin(e+fx))}\right)}{f(c+d)}$$

```
[Out] -2^(1/2+m)*a*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2*(c-d)*(1-sin(f*x+e
))/(c+d*sin(f*x+e)))*(a+a*sin(f*x+e))^(-1+m)*((c+d)*(1+sin(f*x+e))/(c+d*sin
(f*x+e)))^(1/2-m)/(c+d)/f/((c+d*sin(f*x+e))^m)
```

Rubi [A] time = 0.15, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2788, 132}

$$-\frac{a2^{m+\frac{1}{2}}\cos(e+fx)(a\sin(e+fx)+a)^{m-1}\left(\frac{(c+d)(\sin(e+fx)+1)}{c+d\sin(e+fx)}\right)^{\frac{1}{2}-m}(c+d\sin(e+fx))^{-m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{(c-d)(1-\sin(e+fx))}{2(c+d\sin(e+fx))}\right)}{f(c+d)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] -((2^(1/2 + m)*a*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, ((c - d)
*(1 - Sin[e + f*x]))/(2*(c + d*Sin[e + f*x]))]*(a + a*Sin[e + f*x])^(-1 + m
)*(((c + d)*(1 + Sin[e + f*x]))/(c + d*Sin[e + f*x]))^(1/2 - m))/((c + d)*f
*(c + d*Sin[e + f*x])^m))
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

```
!IntegerQ[m]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}(c+dx)^{-1-m}}{\sqrt{a-ax}}\,dx,x,\sin(e+\right.}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2^{\frac{1}{2}+m}a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{(c-d)(1-\sin(e+fx))}{2(c+d\sin(e+fx))}\right)(a+a}{(c}$$

Mathematica [A] time = 1.40, size = 187, normalized size = 1.45

$$-\frac{2\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m+\frac{1}{2}}(a(\sin(e+fx)+1))^m(c+d\sin}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] (-2*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*Hyper
geometric2F1[1/2, 1/2 - m, 3/2, ((c - d)*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c +
d*Sin[e + f*x])]*(a*(1 + Sin[e + f*x]))^m*(((c + d)*Cos[(2*e - Pi + 2*f*x)/
4]^2)/(c + d*Sin[e + f*x]))^(-1/2 - m)*(c + d*Sin[e + f*x])^(-1 - m)*(Sin[(
2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/f
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m\left(d\sin\left(fx+e\right)+c\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^(-m - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^m\left(d\sin\left(fx+e\right)+c\right)^{-m-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^(-m - 1), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \left(a+a\sin \left(fx+e\right)\right)^{m} \left(c+d\sin \left(fx+e\right)\right)^{-1-m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x)
[Out] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin \left(fx+e\right)+a\right)^{m}\left(d\sin \left(fx+e\right)+c\right)^{-m-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^(-m - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin \left(e+f\,x\right)\right)^m}{\left(c+d\,\sin \left(e+f\,x\right)\right)^{m+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m/(c + d*sin(e + f*x))^(m + 1),x)
[Out] int((a + a*sin(e + f*x))^m/(c + d*sin(e + f*x))^(m + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**(-1-m),x)
[Out] Timed out
```

3.655 $\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=107

$$-\frac{8\sqrt{2}\,a^3\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-\frac{5}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -8*a^3*AppellF1(1/2,-n,-5/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*
cos(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*sin(f*x+e))/(c+d))^n)/(1+sin
(f*x+e))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2784, 139, 138}

$$-\frac{8\sqrt{2}\,a^3\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-\frac{5}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-8*Sqrt[2]*a^3*AppellF1[1/2, -5/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 -
Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Si
n[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2784

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^m*Cos[e + f*x])/(f*Sqrt[1 + Sin[e
+ f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[((1 + (b*x)/a)^(m - 1/2)*(c + d*
x)^n)/Sqrt[1 - (b*x)/a], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^n\,dx = \frac{\left(a^3\cos(e+fx)\right)\text{Subst}\left(\int\frac{(1+x)^{5/2}(c+dx)^n}{\sqrt{1-x}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(a^3\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int\frac{(1}{}\right.}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+}}$$

$$= -\frac{8\sqrt{2}\,a^3F_1\left(\frac{1}{2};-\frac{5}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+}{f\sqrt{1+\sin(e+fx)}}$$

Mathematica [F] time = 36.06, size = 0, normalized size = 0.00

$$\int (a+a\sin(e+fx))^3(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^n,x]
[Out] Integrate[(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^n, x]
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)\right)\left(d\sin\left(fx+e\right)+c\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(
f*x + e))*(d*sin(f*x + e) + c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^3 \left(d\sin\left(fx+e\right)+c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^n, x)
```

maple [F] time = 1.29, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^3 \left(c+d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x)
[Out] int((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^3 \left(d\sin\left(fx+e\right)+c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^3\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^n,x)
[Out] int((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

3.656 $\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=107

$$-\frac{4\sqrt{2}\,a^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -4*a^2*AppellF1(1/2,-n,-3/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*
cos(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*sin(f*x+e))/(c+d))^n)/(1+sin
(f*x+e))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2784, 139, 138}

$$-\frac{4\sqrt{2}\,a^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-4*Sqrt[2]*a^2*AppellF1[1/2, -3/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 -
Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Si
n[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2784

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^m*Cos[e + f*x])/(f*Sqrt[1 + Sin[e
+ f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[((1 + (b*x)/a)^(m - 1/2)*(c + d*
x)^n)/Sqrt[1 - (b*x)/a], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(1+x)^{3/2}(c+dx)^n}{\sqrt{1-x}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{(1}{}\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+}}$$

$$= -\frac{4\sqrt{2}\,a^2F_1\left(\frac{1}{2};-\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+}{f\sqrt{1+\sin(e+fx)}}$$

Mathematica [F] time = 16.63, size = 0, normalized size = 0.00

$$\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] Integrate[(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n, x]
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2\right)\left(d\sin\left(fx+e\right)+c\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*(d*sin(f*x + e)
 + c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{2}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^n, x)
```

maple [F] time = 1.38, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{2}\left(c+d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{2}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

3.657 $\int (a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=105

$$-\frac{2\sqrt{2}\,a\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -2*a*AppellF1(1/2,-n,-1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*co
s(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*sin(f*x+e))/(c+d))^n)/(1+sin(f
*x+e))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2755, 139, 138}

$$-\frac{2\sqrt{2}\,a\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-2*Sqrt[2]*a*AppellF1[1/2, -1/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Si
n[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Sin[
e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2755

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(c*Cos[e + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*S
qrt[1 - Sin[e + f*x]]), Subst[Int[((a + b*x)^m*Sqrt[1 + (d*x)/c])/Sqrt[1 -
(d*x)/c], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m] && EqQ[c^2 - d^2, 0
]
```

Rubi steps

$$\int (a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \frac{(a\cos(e+fx))\,\text{Subst}\left(\int \frac{\sqrt{1+x}\,(c+dx)^n}{\sqrt{1-x}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(a\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{\sqrt{1+\cdots}}{\cdots}\right.}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx\cdots}}$$

$$= -\frac{2\sqrt{2}\,aF_1\left(\frac{1}{2};-\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+f\cdots}{f\sqrt{1+\sin(e+fx)}}$$

Mathematica [F] time = 5.66, size = 0, normalized size = 0.00

$$\int (a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
[Out] Integrate[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n, x]
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n, x)
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
[Out] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+a\,\sin\left(e+f\,x\right)\right)\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^n,x)
[Out] int((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

3.658 $\int (c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=104

$$-\frac{\sqrt{2}\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -AppellF1(1/2,-n,1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos(f*x
+e)*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*sin(f*x+e))/(c+d))^n)/(1+sin(f*x+e)
)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2665, 139, 138}

$$-\frac{\sqrt{2}\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^n,x]
```

```
[Out] -((Sqrt[2]*AppellF1[1/2, 1/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e
+ f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Sin[e +
f*x]]*((c + d*Sin[e + f*x])/(c + d))^n))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (c+d\sin(e+fx))^n\,dx = \frac{\cos(e+fx)\,\mathrm{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{1-x}\,\sqrt{1+x}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\mathrm{Subst}\left(\int \frac{\left(-\frac{c}{-c-d}-\frac{dx}{-c-d}\right)^n}{\sqrt{1-x}\,\sqrt{1+x}}\,dx, x, \sin(e\right.}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{\sqrt{2}\,F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n}{f\sqrt{1+\sin(e+fx)}}$$

Mathematica [A] time = 0.28, size = 120, normalized size = 1.15

$$\frac{\sec(e+fx)\sqrt{-\frac{d(\sin(e+fx)-1)}{c+d}}\,\sqrt{\frac{d(\sin(e+fx)+1)}{d-c}}\,(c+d\sin(e+fx))^{n+1}F_1\left(n+1;\frac{1}{2},\frac{1}{2};n+2;\frac{c+d\sin(e+fx)}{c-d},\frac{c+d\sin(e+fx)}{c+d}\right)}{df(n+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (AppellF1[1 + n, 1/2, 1/2, 2 + n, (c + d*Sin[e + f*x])/(c - d), (c + d*Sin[
e + f*x])/(c + d)]*Sec[e + f*x]*Sqrt[-((d*(-1 + Sin[e + f*x]))/(c + d))]*Sq
rt[(d*(1 + Sin[e + f*x]))/(-c + d)]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(1 +
 n))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(d\sin\left(fx+e\right)+c\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e) + c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n, x)
```

maple [F] time = 0.73, size = 0, normalized size = 0.00

$$\int \left(c+d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^n,x)
```

```
[Out] int((c+d*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c+d\,\sin\left(e+f\,x\right)\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^n,x)
```

```
[Out] int((c + d*sin(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c+d\sin\left(e+fx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**n,x)
```

```
[Out] Integral((c + d*sin(e + f*x))**n, x)
```

3.659 $\int \frac{(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=107

$$-\frac{\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{\sqrt{2}\,af\sqrt{\sin(e+fx)+1}}$$

```
[Out] -1/2*AppellF1(1/2,-n,3/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos
(f*x+e)*(c+d*sin(f*x+e))^n/a/f/(((c+d*sin(f*x+e))/(c+d))^n)*2^(1/2)/(1+sin(
f*x+e))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2784, 139, 138}

$$-\frac{\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{\sqrt{2}\,af\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x]),x]
```

```
[Out] -((AppellF1[1/2, 3/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))
/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(Sqrt[2]*a*f*Sqrt[1 + Sin[e
+ f*x]]*((c + d*Sin[e + f*x])/(c + d))^n))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2784

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^m*Cos[e + f*x])/(f*Sqrt[1 + Sin[e
+ f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[((1 + (b*x)/a)^(m - 1/2)*(c + d*
x)^n)/Sqrt[1 - (b*x)/a], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{1-x}\,(1+x)^{3/2}}\,dx, x, \sin(e+fx)\right)}{af\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{\left(-\frac{c}{-c-d}-\frac{dx}{-c-d}\right)^n}{\sqrt{1-x}\,(1+x)^{3/2}}\,dx, x, \sin(\right.}{af\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{F_1\left(\frac{1}{2};\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+}{}\right.}{\sqrt{2}\,af\sqrt{1+\sin(e+fx)}}$$

Mathematica [F] time = 2.70, size = 0, normalized size = 0.00

$$\int \frac{(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x]),x]
[Out] Integrate[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x]), x]
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d\sin\left(fx+e\right)+c\right)^n}{a\sin\left(fx+e\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin\left(fx+e\right)+c\right)^{n}}{a \sin\left(fx+e\right)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a), x)
```

maple [F] time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{\left(c+d \sin\left(fx+e\right)\right)^{n}}{a+a \sin\left(fx+e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

```
[Out] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin\left(fx+e\right)+c\right)^{n}}{a \sin\left(fx+e\right)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c+d\,\sin\left(e+f\,x\right)\right)^{n}}{a+a\,\sin\left(e+f\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^n/(a + a*sin(e + f*x)),x)
```

```
[Out] int((c + d*sin(e + f*x))^n/(a + a*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**n/(a+a*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.660 $\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=109

$$-\frac{\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{5}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{2\sqrt{2}\,a^2f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -1/4*AppellF1(1/2,-n,5/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos
(f*x+e)*(c+d*sin(f*x+e))^n/a^2/f/(((c+d*sin(f*x+e))/(c+d))^n)*2^(1/2)/(1+si
n(f*x+e))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2784, 139, 138}

$$-\frac{\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{5}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{2\sqrt{2}\,a^2f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -(AppellF1[1/2, 5/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/
(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(2*Sqrt[2]*a^2*f*Sqrt[1 + Sin
[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2784

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^m*Cos[e + f*x])/(f*Sqrt[1 + Sin[e
+ f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[((1 + (b*x)/a)^(m - 1/2)*(c + d*
x)^n)/Sqrt[1 - (b*x)/a], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{1-x}\,(1+x)^{5/2}}\,dx, x, \sin(e+fx)\right)}{a^2 f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{\left(-\frac{c}{-c-d}-\frac{dx}{-c-d}\right)^n}{\sqrt{1-x}\,(1+x)^{5/2}}\,dx, x, \sin(\right.}{a^2 f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{F_1\left(\frac{1}{2};\frac{5}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+}{-}\right.}{2\sqrt{2}\,a^2 f\sqrt{1+\sin(e+fx)}}$$

Mathematica [F] time = 7.01, size = 0, normalized size = 0.00

$$\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^2,x]
[Out] Integrate[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^2, x]
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(d\sin\left(fx+e\right)+c\right)^n}{a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(d*sin(f*x + e) + c)^n/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) -
 2*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin\left(fx+e\right)+c\right)^{n}}{\left(a \sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^2, x)
```

maple [F] time = 1.47, size = 0, normalized size = 0.00

$$\int \frac{\left(c+d \sin\left(fx+e\right)\right)^{n}}{\left(a+a \sin\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

```
[Out] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin\left(fx+e\right)+c\right)^{n}}{\left(a \sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^n/(a + a*sin(e + f*x))^2,x)
```

```
[Out] int((c + d*sin(e + f*x))^n/(a + a*sin(e + f*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.661 $\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=109

$$-\frac{\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{7}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{4\sqrt{2}\,a^3f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -1/8*AppellF1(1/2,-n,7/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos
(f*x+e)*(c+d*sin(f*x+e))^n/a^3/f/(((c+d*sin(f*x+e))/(c+d))^n)*2^(1/2)/(1+si
n(f*x+e))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2784, 139, 138}

$$-\frac{\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{7}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{4\sqrt{2}\,a^3f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -(AppellF1[1/2, 7/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/
(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(4*Sqrt[2]*a^3*f*Sqrt[1 + Sin
[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2784

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^m*Cos[e + f*x])/(f*Sqrt[1 + Sin[e
+ f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[((1 + (b*x)/a)^(m - 1/2)*(c + d*
x)^n)/Sqrt[1 - (b*x)/a], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{1-x}\,(1+x)^{7/2}}\,dx, x, \sin(e+fx)\right)}{a^3 f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{\left(-\frac{c}{-c-d}-\frac{dx}{-c-d}\right)^n}{\sqrt{1-x}\,(1+x)^{7/2}}\,dx, x, \sin(\right.}{a^3 f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{F_1\left(\frac{1}{2};\frac{7}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+}{-}\right.}{4\sqrt{2}\,a^3 f\sqrt{1+\sin(e+fx)}}$$

Mathematica [F] time = 13.13, size = 0, normalized size = 0.00

$$\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^3,x]
[Out] Integrate[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^3, x]
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(d\sin\left(fx+e\right)+c\right)^n}{3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(d*sin(f*x + e) + c)^n/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f
*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin\left(fx+e\right)+c\right)^{n}}{\left(a \sin\left(fx+e\right)+a\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^3, x)
```

maple [F] time = 1.47, size = 0, normalized size = 0.00

$$\int \frac{\left(c+d \sin\left(fx+e\right)\right)^{n}}{\left(a+a \sin\left(fx+e\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x)
```

```
[Out] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin\left(fx+e\right)+c\right)^{n}}{\left(a \sin\left(fx+e\right)+a\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^n/(a + a*sin(e + f*x))^3,x)
```

```
[Out] int((c + d*sin(e + f*x))^n/(a + a*sin(e + f*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.662 $\int (a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=257

$$-\frac{2a^3\left(3c^2-2cd(4n+7)+d^2\left(16n^2+56n+43\right)\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\right.}{d^2f(2n+3)(2n+5)\sqrt{a\sin(e+fx)+a}}$$

```
[Out] 2*a^3*(3*c-d*(11+4*n))*cos(f*x+e)*(c+d*sin(f*x+e))^(1+n)/d^2/f/(3+2*n)/(5+2
*n)/(a+a*sin(f*x+e))^(1/2)-2*a^3*(3*c^2-2*c*d*(7+4*n)+d^2*(16*n^2+56*n+43))
*cos(f*x+e)*hypergeom([1/2, -n],[3/2],d*(1-sin(f*x+e))/(c+d))*(c+d*sin(f*x+
e))^n/d^2/f/(3+2*n)/(5+2*n)/(((c+d*sin(f*x+e))/(c+d))^n)/(a+a*sin(f*x+e))^(
1/2)-2*a^2*cos(f*x+e)*(c+d*sin(f*x+e))^(1+n)*(a+a*sin(f*x+e))^(1/2)/d/f/(5+
2*n)
```

Rubi [A] time = 0.48, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2763, 2981, 2776, 70, 69}

$$-\frac{2a^3\left(3c^2-2cd(4n+7)+d^2\left(16n^2+56n+43\right)\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\right.}{d^2f(2n+3)(2n+5)\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (2*a^3*(3*c - d*(11 + 4*n))*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d^2
*f*(3 + 2*n)*(5 + 2*n)*Sqrt[a + a*Sin[e + f*x]]) - (2*a^2*Cos[e + f*x]*Sqrt
[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(5 + 2*n)) - (2*a^3
*(3*c^2 - 2*c*d*(7 + 4*n) + d^2*(43 + 56*n + 16*n^2))*Cos[e + f*x]*Hypergeo
metric2F1[1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*Sin[e + f*x]
)^n)/(d^2*f*(3 + 2*n)*(5 + 2*n)*Sqrt[a + a*Sin[e + f*x]]*((c + d*Sin[e + f*
x])/(c + d))^n)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a + a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^n\,dx = -\frac{2a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^{1+n}}{df(5+2n)} + -$$

$$= \frac{2a^3(3c-d(11+4n))\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{d^2f(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}} - \frac{2a^2 c}{}$$

$$= \frac{2a^3(3c-d(11+4n))\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{d^2f(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}} - \frac{2a^2 c}{}$$

$$= \frac{2a^3(3c-d(11+4n))\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{d^2f(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}} - \frac{2a^2 c}{}$$

$$= \frac{2a^3(3c-d(11+4n))\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{d^2f(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}} - \frac{2a^2 c}{}$$

Mathematica [A] time = 32.78, size = 190, normalized size = 0.74

$$\frac{a^2(\sin(e+fx)-1)\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\,(c+d\sin(e+fx))^n\left(\left(3c^2-2cd(4n+7)+d^2\left(16n^2+56n+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (a^2*Sec[e + f*x]*(-1 + Sin[e + f*x])*Sqrt[a*(1 + Sin[e + f*x])]*(c + d*Sin
[e + f*x])^n*(-((3*c - d*(11 + 4*n))*(c + d*Sin[e + f*x])) + d*(3 + 2*n)*(1
 + Sin[e + f*x])*(c + d*Sin[e + f*x]) + ((3*c^2 - 2*c*d*(7 + 4*n) + d^2*(43
 + 56*n + 16*n^2))*Hypergeometric2F1[1/2, -n, 3/2, -((d*(-1 + Sin[e + f*x])
)/(c + d))])/((c + d*Sin[e + f*x])/(c + d))^n))/(d^2*f*(5/2 + n)*(3 + 2*n))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2\cos\left(fx+e\right)^2 - 2\,a^2\sin\left(fx+e\right) - 2\,a^2\right)\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*sqrt(a*sin(f*x
+ e) + a)*(d*sin(f*x + e) + c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^n,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^n, x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{\frac{5}{2}}\left(c+d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^n,x)
[Out] int((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^n,x)
[Out] int((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

3.663 $\int (a + a\sin(e + fx))^{3/2}(c + d\sin(e + fx))^n \, dx$

Optimal. Leaf size=160

$$\frac{2a^2(c - d(4n + 5))\cos(e + fx)(c + d\sin(e + fx))^n \left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n} {}_2F_1\left(\frac{1}{2}, -n; \frac{3}{2}; \frac{d(1-\sin(e+fx))}{c+d}\right)}{df(2n + 3)\sqrt{a\sin(e + fx) + a}} - \frac{2a^2\cos(e + fx)(c}{df(2n + 3)\sqrt{a}}$$

```
[Out] -2*a^2*cos(f*x+e)*(c+d*sin(f*x+e))^(1+n)/d/f/(3+2*n)/(a+a*sin(f*x+e))^(1/2)
+2*a^2*(c-d*(5+4*n))*cos(f*x+e)*hypergeom([1/2, -n],[3/2],d*(1-sin(f*x+e))/
(c+d))*(c+d*sin(f*x+e))^n/d/f/(3+2*n)/(((c+d*sin(f*x+e))/(c+d))^n)/(a+a*sin
(f*x+e))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2763, 21, 2776, 70, 69}

$$\frac{2a^2(c - d(4n + 5))\cos(e + fx)(c + d\sin(e + fx))^n \left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n} {}_2F_1\left(\frac{1}{2}, -n; \frac{3}{2}; \frac{d(1-\sin(e+fx))}{c+d}\right)}{df(2n + 3)\sqrt{a\sin(e + fx) + a}} - \frac{2a^2\cos(e + fx)(c}{df(2n + 3)\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-2*a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*Sqrt[a +
a*Sin[e + f*x]]) + (2*a^2*(c - d*(5 + 4*n))*Cos[e + f*x]*Hypergeometric2F1[
1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*Sin[e + f*x])^n)/(d*f*
(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^n\,dx = -\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{(c+d\sin(e+fx))^n\left(-\frac{1}{2}a^2 \right.}{}}{}$$

$$= -\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} - \frac{(a(c-d(5+4n)))\int \sqrt{}}{}$$

$$= -\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} - \frac{\left(a^3(c-d(5+4n))\cos\right.}{df(3+2n)\sqrt{a}}$$

$$= -\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} - \frac{\left(a^3(c-d(5+4n))\cos\right.}{}$$

$$= -\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{2a^2(c-d(5+4n))\cos}{}$$

Mathematica [A] time = 7.64, size = 133, normalized size = 0.83

$$-\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\left((d(4n+5)-c)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};-\frac{d(\sin(e+fx)-1)}{c+d}\right)+(c+d)\left(\frac{c+d\,\mathrm{s}}{}\right.\right.}{df(2n+3)\sqrt{a(\sin(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-2*a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^n*((-c + d*(5 + 4*n))*Hypergeomet
ric2F1[1/2, -n, 3/2, -((d*(-1 + Sin[e + f*x]))/(c + d))] + (c + d)*((c + d*
Sin[e + f*x])/(c + d))^(1 + n)))/(d*f*(3 + 2*n)*Sqrt[a*(1 + Sin[e + f*x])]*
((c + d*Sin[e + f*x])/(c + d))^n)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^n,x, algorithm="fricas")
```

[Out] integral((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^n,x, algorithm="giac")

[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^n, x)

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\left(c+d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^n,x)

[Out] int((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^n,x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^n,x)

[Out] int((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^n, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

3.664 $\int \sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=85

$$-\frac{2a\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -2*a*cos(f*x+e)*hypergeom([1/2, -n],[3/2],d*(1-sin(f*x+e))/(c+d))*(c+d*sin(
f*x+e))^n/f/(((c+d*sin(f*x+e))/(c+d))^n)/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2776, 70, 69}

$$-\frac{2a\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-2*a*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(
c + d)]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[a + a*Sin[e + f*x]]*((c + d*Sin[e +
 f*x])/(c + d))^n)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^n\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{a-ax}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{a(c+d\sin(e+fx))}{-ac-ad}\right)^{-n}\right)\text{Subst}\left(\int\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+f}$$

$$= -\frac{2a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d}\right)(c+d\sin(e+fx))^n\left(\right.}{f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [F] time = 4.63, size = 0, normalized size = 0.00

$$\int \sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n,x]
[Out] Integrate[Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x]
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^n,x, algorithm="fricas")
[Out] integral(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^n,x, algorithm="giac")
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n, x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \sqrt{a+a\sin\left(fx+e\right)}\,\left(c+d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^n,x)
[Out] int((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^n,x, algorithm="maxima")
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^n,x)
[Out] int((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\left(c+d\sin\left(e+fx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e))**n,x)
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(c + d*sin(e + f*x))**n, x)
```

3.665 $\int \frac{(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=99

$$-\frac{\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-n,1;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d},\frac{1}{2}(1-\sin(e+fx))\right)}{f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -AppellF1(1/2,-n,1,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos(f*x+e
)*(c+d*sin(f*x+e))^n/f/(((c+d*sin(f*x+e))/(c+d))^n)/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 129, normalized size of antiderivative = 1.30, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2788, 137, 136}

$$-\frac{\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+d\sin(e+fx))^{n+1}F_1\left(n+1;\frac{1}{2},1;n+2;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)}{f(n+1)(c-d)(1-\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(c + d*Sin[e + f*x])^n/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((AppellF1[1 + n, 1/2, 1, 2 + n, (c + d*Sin[e + f*x])/(c + d), (c + d*Sin[
e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1 - Sin[e + f*x]))/(c + d)]*(c + d
*Sin[e + f*x])^(1 + n))/((c - d)*f*(1 + n)*(1 - Sin[e + f*x])*Sqrt[a + a*Si
n[e + f*x]]))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
```

```
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{a-ax}\,(a+ax)}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{d(a-a\sin(e+fx))}{ac+ad}}\right)\text{Subst}\left(\int \frac{(c+dx)^n}{(a+ax)\sqrt{\frac{ad}{ac+ad}-\frac{adx}{ac+ad}}}\,dx, x, \sin(e+fx)\right)}{f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{F_1\left(1+n;\frac{1}{2},1;2+n;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}\,(c+d\sin}{(c-d)f(1+n)(1-\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

Mathematica [B] time = 2.82, size = 236, normalized size = 2.38

$$\frac{\cos(e+fx)\sqrt{a(\sin(e+fx)+1)}\,(c+d\sin(e+fx))^n\left(\frac{4\sqrt{\frac{\sin(e+fx)-1}{\sin(e+fx)+1}}\left(\frac{c-d}{d\sin(e+fx)+d}+1\right)^{-n}F_1\left(-n-\frac{1}{2};-\frac{1}{2},-n;\frac{1}{2}-n;\frac{2}{\sin(e+fx)+1},\frac{d-c}{\sin(e+fx}\right.}{2n+1}\right.}{4af(\sin(e+fx)-1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^n/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(c + d*Sin[e + f*x])^n*(-((AppellF
1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, (d*(1 + Sin[e + f*x]))/(-c + d)]*Sqr
t[2 - 2*Sin[e + f*x]])/((c + d*Sin[e + f*x])/(c - d))^n) + (4*AppellF1[-1/2
 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d + d*Sin[e + f*x]
)]*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])])/((1 + 2*n)*(1 + (c - d)/(d
 + d*Sin[e + f*x]))^n)))/(4*a*f*(-1 + Sin[e + f*x]))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d\sin\left(fx+e\right)+c\right)^{n}}{\sqrt{a\sin\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral((d*sin(f*x + e) + c)^n/sqrt(a*sin(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(d\sin\left(fx+e\right)+c\right)^{n}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((d*sin(f*x + e) + c)^n/sqrt(a*sin(f*x + e) + a), x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int\frac{\left(c+d\sin\left(fx+e\right)\right)^{n}}{\sqrt{a+a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x)
[Out] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(d\sin\left(fx+e\right)+c\right)^{n}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^n/sqrt(a*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c+d\,\sin\left(e+f\,x\right)\right)^n}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^n/(a + a*sin(e + f*x))^(1/2),x)
[Out] int((c + d*sin(e + f*x))^n/(a + a*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+d\sin\left(e+fx\right)\right)^n}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**(1/2),x)
[Out] Integral((c + d*sin(e + f*x))**n/sqrt(a*(sin(e + f*x) + 1)), x)
```

3.666 $\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-n,2;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d},\frac{1}{2}(1-\sin(e+fx))\right)}{2af\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -1/2*AppellF1(1/2,-n,2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos(f
*x+e)*(c+d*sin(f*x+e))^n/a/f/(((c+d*sin(f*x+e))/(c+d))^n)/(a+a*sin(f*x+e))^
(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 130, normalized size of antiderivative = 1.25, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2788, 137, 136}

$$\frac{d\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+d\sin(e+fx))^{n+1}F_1\left(n+1;\frac{1}{2},2;n+2;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)}{f(n+1)(c-d)^2(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (d*AppellF1[1 + n, 1/2, 2, 2 + n, (c + d*Sin[e + f*x])/(c + d), (c + d*Sin[
e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1 - Sin[e + f*x]))/(c + d)]*(c + d
*Sin[e + f*x])^(1 + n))/((c - d)^2*f*(1 + n)*(a - a*Sin[e + f*x])*Sqrt[a +
a*Sin[e + f*x]])
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
```

```
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\left(a^2\cos(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{a-ax}\,(a+ax)^2}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{d(a-a\sin(e+fx))}{ac+ad}}\right)\text{Subst}\left(\int \frac{(c+dx)^n}{(a+ax)^2\sqrt{\frac{ad}{ac+ad}-\frac{adx}{ac+ad}}}\,dx, x, \sin(e+fx)\right)}{f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{dF_1\left(1+n;\frac{1}{2},2;2+n;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}\,(c+d\,\text{s}}{(c-d)^2 f(1+n)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

Mathematica [B] time = 4.81, size = 319, normalized size = 3.07

$$\frac{\sec(e+fx)(c+d\sin(e+fx))^n\left(a^2\sqrt{2-2\sin(e+fx)}\,(\sin(e+fx)+1)^2\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(1;\frac{1}{2},-n;2;\frac{1}{2}(\sin(e\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]*(c + d*Sin[e + f*x])^n*((a^2*AppellF1[1, 1/2, -n, 2, (1 + Sin
[e + f*x])/2, (d*(1 + Sin[e + f*x]))/(-c + d)]*Sqrt[2 - 2*Sin[e + f*x]]*(1
+ Sin[e + f*x])^2)/((c + d*Sin[e + f*x])/(c - d))^n - (4*a*(1 + Sin[e + f*x
])*Sqrt[1 - 2/(1 + Sin[e + f*x])]*(2*a*(1 + 2*n)*AppellF1[1/2 - n, -1/2, -n
, 3/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d + d*Sin[e + f*x])] + a*(-1 + 2
*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d
```

```
 + d*Sin[e + f*x])]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)*(1 + (c - d)/(d + d*
Sin[e + f*x]))^n)))/(8*a^3*f*Sqrt[a*(1 + Sin[e + f*x])])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{n}}{a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n/(a^2*cos(f*x + e)
^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(d\sin\left(fx+e\right)+c\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^(3/2), x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int\frac{\left(c+d\sin\left(fx+e\right)\right)^{n}}{\left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(d\sin\left(fx+e\right)+c\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c+d\,\sin\left(e+f\,x\right)\right)^n}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^n/(a + a*sin(e + f*x))^(3/2),x)
[Out] int((c + d*sin(e + f*x))^n/(a + a*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+d\sin\left(e+fx\right)\right)^n}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**(3/2),x)
[Out] Integral((c + d*sin(e + f*x))**n/(a*(sin(e + f*x) + 1))**(3/2), x)
```

3.667 $\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-n,3;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d},\frac{1}{2}(1-\sin(e+fx))\right)}{4a^2f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -1/4*AppellF1(1/2,-n,3,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos(f
*x+e)*(c+d*sin(f*x+e))^n/a^2/f/(((c+d*sin(f*x+e))/(c+d))^n)/(a+a*sin(f*x+e)
)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 137, normalized size of antiderivative = 1.32, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2788, 137, 136}

$$-\frac{d^2\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+d\sin(e+fx))^{n+1}F_1\left(n+1;\frac{1}{2},3;n+2;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)}{f(n+1)(c-d)^3\sqrt{a\sin(e+fx)+a}\left(a^2-a^2\sin(e+fx)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -((d^2*AppellF1[1 + n, 1/2, 3, 2 + n, (c + d*Sin[e + f*x])/(c + d), (c + d*
Sin[e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1 - Sin[e + f*x]))/(c + d)]*(c
 + d*Sin[e + f*x])^(1 + n))/((c - d)^3*f*(1 + n)*Sqrt[a + a*Sin[e + f*x]]*(
a^2 - a^2*Sin[e + f*x])))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
```

```
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\left(a^2\cos(e+fx)\right)\operatorname{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{a-ax}\,(a+ax)^3}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)\sqrt{\frac{d(a-a\sin(e+fx))}{ac+ad}}\right)\operatorname{Subst}\left(\int \frac{(c+dx)^n}{(a+ax)^3\sqrt{\frac{ad}{ac+ad}-\frac{adx}{ac+ad}}}\,dx, x, \sin(e+fx)\right)}{f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{d^2F_1\left(1+n;\frac{1}{2},3;2+n;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}\,(c+}{(c-d)^3f(1+n)\sqrt{a+a\sin(e+fx)}\,\left(a^2-a^2\sin(e+fx)\right)}$$

Mathematica [B] time = 9.54, size = 414, normalized size = 3.98

$$\frac{\sec(e+fx)(c+d\sin(e+fx))^n\left(a^3\sqrt{2-2\sin(e+fx)}\,(\sin(e+fx)+1)^3\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(1;\frac{1}{2},-n;2;\frac{1}{2}(\sin(e\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^n/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sec[e + f*x]*(c + d*Sin[e + f*x])^n*((a^3*AppellF1[1, 1/2, -n, 2, (1 + Sin
[e + f*x])/2, (d*(1 + Sin[e + f*x]))/(-c + d)]*Sqrt[2 - 2*Sin[e + f*x]]*(1
+ Sin[e + f*x])^3)/((c + d*Sin[e + f*x])/(c - d))^n - (4*a^2*(1 + Sin[e + f
*x])*Sqrt[1 - 2/(1 + Sin[e + f*x])]*(a*(3 - 8*n + 4*n^2)*AppellF1[-1/2 - n,
 -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d + d*Sin[e + f*x])]*(1
 + Sin[e + f*x])^2 + 2*(1 + 2*n)*(2*a*(-1 + 2*n)*AppellF1[3/2 - n, -1/2, -n
```

```
, 5/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d + d*Sin[e + f*x])] + a*(-3 + 2
*n)*AppellF1[1/2 - n, -1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d
+ d*Sin[e + f*x])]*(1 + Sin[e + f*x]))))/((-3 + 2*n)*(-1 + 2*n)*(1 + 2*n)*(
1 + (c - d)/(d + d*Sin[e + f*x]))^n)))/(16*a^4*f*(a*(1 + Sin[e + f*x]))^(3/
2))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{n}}{3\,a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
[Out] integral(-sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n/(3*a^3*cos(f*x +
e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate((d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^(5/2), x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\left(c+d\sin\left(fx+e\right)\right)^{n}}{\left(a+a\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(5/2),x)
[Out] int((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c+d\,\sin\left(e+f\,x\right)\right)^n}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^n/(a + a*sin(e + f*x))^(5/2),x)
[Out] int((c + d*sin(e + f*x))^n/(a + a*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**(5/2),x)
[Out] Timed out
```

3.668 $\int (a + a\sin(e + fx))\sqrt[3]{c + d\sin(e + fx)}\,dx$

Optimal. Leaf size=107

$$-\frac{2\sqrt{2}\,a\cos(e+fx)\sqrt[3]{c+d\sin(e+fx)}\,F_1\left(\frac{1}{2};-\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] -2*a*AppellF1(1/2,-1/3,-1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*
cos(f*x+e)*(c+d*sin(f*x+e))^(1/3)*2^(1/2)/f/((c+d*sin(f*x+e))/(c+d))^(1/3)/
(1+sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2755, 139, 138}

$$-\frac{2\sqrt{2}\,a\cos(e+fx)\sqrt[3]{c+d\sin(e+fx)}\,F_1\left(\frac{1}{2};-\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^(1/3),x]
```

```
[Out] (-2*Sqrt[2]*a*AppellF1[1/2, -1/2, -1/3, 3/2, (1 - Sin[e + f*x])/2, (d*(1 -
Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1/3))/(f*Sqrt[1
+ Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^(1/3))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2755

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(c*Cos[e + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*S
qrt[1 - Sin[e + f*x]]), Subst[Int[((a + b*x)^m*Sqrt[1 + (d*x)/c])/Sqrt[1 -
(d*x)/c], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m] && EqQ[c^2 - d^2, 0
]
```

Rubi steps

$$\int (a+a\sin(e+fx))\sqrt[3]{c+d\sin(e+fx)}\,dx = \frac{(a\cos(e+fx))\,\mathrm{Subst}\left(\int\frac{\sqrt{1+x}\,\sqrt[3]{c+dx}}{\sqrt{1-x}}\,dx,x,\sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(a\cos(e+fx)\sqrt[3]{c+d\sin(e+fx)}\right)\mathrm{Subst}\left(\int\frac{\sqrt{1+x}\,\sqrt[3]{-\frac{c}{-c-d}-\frac{dx}{-c-d}}}{\sqrt{1-x}}\,dx,\right.}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}\,\sqrt[3]{-\frac{c+d\sin(e+fx)}{-c-d}}}$$

$$= -\frac{2\sqrt{2}\,aF_1\left(\frac{1}{2};-\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+}{f\sqrt{1+\sin(e+fx)}\,\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}}$$

Mathematica [B] time = 6.43, size = 1736, normalized size = 16.22

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^(1/3),x]
```

```
[Out] a*((c*Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-2/3, -1/2, -1/2, 1/3, -((Csc[
e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 +
 Cot[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[
f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1
 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/
(Sqrt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]
]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + C
```

```
ot[e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[
e]^2] + c*Csc[e])]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[
e])^(2/3))) - ((3*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]
^2]*Sin[e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e
]]])/Sqrt[1 + Cot[e]^2])/(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2
]*Sin[e])^(2/3)))/(4*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (d*Se
c[e]*(1 + Sin[e + f*x])*(-((AppellF1[-2/3, -1/2, -1/2, 1/3, -((Csc[e]*(c +
d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^
2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x - Ar
cTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*Cs
c[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[1
+ Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1
 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^2]
 - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] +
c*Csc[e])]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e])^(2/3
))) - ((3*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[
e]))/(d^2*Cos[e]^2 + d^2*Sin[e]^2) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqr
t[1 + Cot[e]^2])/(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]
)^(2/3)))/(f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + ((1 + Sin[e + f
*x])*(c + d*Sin[e + f*x])^(1/3)*((-3*Cos[e]*Cos[f*x])/(4*f) + (3*Sin[e]*Sin
[f*x])/(4*f) + (3*(c + 4*d)*Tan[e])/(4*d*f)))/(Cos[e/2 + (f*x)/2] + Sin[e/2
 + (f*x)/2])^2 + (3*AppellF1[1/3, 1/2, 1/2, 4/3, -((Sec[e]*(c + d*Cos[e]*Si
n[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 - (c*
Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan
[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d*Sq
rt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]]]*(1 + Sin[e + f*x])*S
qrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])
/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f*x
+ ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2])]
*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])^(1/3))/(4*f*(C
os[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2*Sqrt[1 + Tan[e]^2]) + (3*c*Appell
F1[1/3, 1/2, 1/2, 4/3, -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sq
rt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]
^2])))), -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^
2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[
e]*Sec[f*x + ArcTan[Tan[e]]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2]
- d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Ta
n[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 +
 Tan[e]^2])/(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2])]*(c + d*Cos[e]*Sin[f*x + A
rcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])^(1/3))/(d*f*(Cos[e/2 + (f*x)/2] + Sin[e/
2 + (f*x)/2])^2*Sqrt[1 + Tan[e]^2]))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin\left(fx + e\right) + a\right)\left(d \sin\left(fx + e\right) + c\right)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(1/3), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \left(a + a\sin\left(fx + e\right)\right)\left(c + d\sin\left(fx + e\right)\right)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(1/3),x)
[Out] int((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin\left(fx + e\right) + a\right)\left(d \sin\left(fx + e\right) + c\right)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + a\,\sin\left(e + f\,x\right)\right)\,\left(c + d\,\sin\left(e + f\,x\right)\right)^{1/3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^(1/3),x)
[Out] int((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sqrt[3]{c+d\sin\left(e+fx\right)}\,\sin\left(e+fx\right)dx+\int \sqrt[3]{c+d\sin\left(e+fx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(c+d*sin(f*x+e))**(1/3),x)
```

```
[Out] a*(Integral((c + d*sin(e + f*x))**(1/3)*sin(e + f*x), x) + Integral((c + d*
sin(e + f*x))**(1/3), x))
```

3.669 $\int \frac{a+a\sin(e+fx)}{\sqrt[3]{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=107

$$-\frac{2\sqrt{2}\,a\cos(e+fx)\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}\,F_1\left(\frac{1}{2};-\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}\,\sqrt[3]{c+d\sin(e+fx)}}$$

```
[Out] -2*a*AppellF1(1/2,1/3,-1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*c
os(f*x+e)*((c+d*sin(f*x+e))/(c+d))^(1/3)*2^(1/2)/f/(c+d*sin(f*x+e))^(1/3)/(
1+sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2755, 139, 138}

$$-\frac{2\sqrt{2}\,a\cos(e+fx)\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}\,F_1\left(\frac{1}{2};-\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}\,\sqrt[3]{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^(1/3),x]
```

```
[Out] (-2*Sqrt[2]*a*AppellF1[1/2, -1/2, 1/3, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - S
in[e + f*x]))/(c + d)]*Cos[e + f*x]*((c + d*Sin[e + f*x])/(c + d))^(1/3))/(
f*Sqrt[1 + Sin[e + f*x]]*(c + d*Sin[e + f*x])^(1/3))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2755

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(c*Cos[e + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*S
qrt[1 - Sin[e + f*x]]), Subst[Int[((a + b*x)^m*Sqrt[1 + (d*x)/c])/Sqrt[1 -
(d*x)/c], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m] && EqQ[c^2 - d^2, 0
]
```

Rubi steps

$$\int \frac{a+a\sin(e+fx)}{\sqrt[3]{c+d\sin(e+fx)}}\,dx = \frac{(a\cos(e+fx))\,\text{Subst}\left(\int \frac{\sqrt{1+x}}{\sqrt{1-x}\,\sqrt[3]{c+dx}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(a\cos(e+fx)\sqrt[3]{-\frac{c+d\sin(e+fx)}{-c-d}}\right)\text{Subst}\left(\int \frac{\sqrt{1+x}}{\sqrt{1-x}\,\sqrt[3]{-\frac{c}{-c-d}-\frac{dx}{-c-d}}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}\,\sqrt[3]{c+d\sin(e+fx)}}$$

$$= -\frac{2\sqrt{2}\,aF_1\left(\frac{1}{2}; -\frac{1}{2}, \frac{1}{3}; \frac{3}{2}; \frac{1}{2}(1-\sin(e+fx)), \frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}}{f\sqrt{1+\sin(e+fx)}\,\sqrt[3]{c+d\sin(e+fx)}}$$

Mathematica [B] time = 6.27, size = 886, normalized size = 8.28

$$a\left(\frac{\sec(e)\left(-\frac{F_1\left(-\frac{1}{3};-\frac{1}{2},-\frac{1}{2};\frac{2}{3};-\frac{\csc(e)\left(c+d\cos\left(fx-\tan^{-1}(\cot(e))\right)\sqrt{\cot^2(e)+1}\sin(e)\right)}{d\sqrt{\cot^2(e)+1}\left(1-\frac{c\csc(e)}{d\sqrt{\cot^2(e)+1}}\right)},-\frac{\csc(e)\left(c+d\cos\left(fx-\tan^{-1}(\cot(e))\right)\sqrt{\cot^2(e)+1}\sin(e)\right)}{d\sqrt{\cot^2(e)+1}\left(-\frac{c\csc(e)}{d\sqrt{\cot^2(e)+1}}-1\right)}\right)\cot(e)\sin(fx\cdots}{\sqrt{\cot^2(e)+1}\sqrt{\frac{\cos\left(fx-\tan^{-1}(\cot(e))\right)\sqrt{\cot^2(e)+1}\,d+\sqrt{\cot^2(e)+1}\,d}{d\sqrt{\cot^2(e)+1}-c\csc(e)}}\sqrt{\frac{d\sqrt{\cot^2(e)+1}-d\cos\left(fx-\tan^{-1}(\cot(e))\right)\sqrt{\cot^2(e)+1}}{\sqrt{\cot^2(e)+1}\,d+c\csc(e)}}\sqrt[3]{c+d\cos\left(fx-\tan^{-1}(\cot(\cdots\right.}}\right.}{f\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+\sin\left(\frac{e}{2}\cdots\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^(1/3),x]
```

```
[Out] a*((Sec[e]*(1 + Sin[e + f*x])*(-((AppellF1[-1/3, -1/2, -1/2, 2/3, -((Csc[e]
*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + C
ot[e]^2]*(1 - (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*
x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 -
 (c*Csc[e])/(d*Sqrt[1 + Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(S
qrt[1 + Cot[e]^2]*Sqrt[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*
Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot
[e]^2] - d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]
^2] + c*Csc[e])]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]
)^(1/3))) - ((3*d*Sin[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2
]*Sin[e]))/(2*(d^2*Cos[e]^2 + d^2*Sin[e]^2)) - (Cot[e]*Sin[f*x - ArcTan[Cot
[e]]])/Sqrt[1 + Cot[e]^2])/(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]
^2]*Sin[e])^(1/3)))/(f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (3*(1
 + Sin[e + f*x])*(c + d*Sin[e + f*x])^(2/3)*Tan[e])/(2*d*f*(Cos[e/2 + (f*x)
/2] + Sin[e/2 + (f*x)/2])^2) + (3*AppellF1[2/3, 1/2, 1/2, 5/3, -((Sec[e]*(c
 + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[
e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(c + d*Cos[e]*Si
n[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(-1 - (c
*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x + ArcTan[Tan[e]]]*(1 + S
in[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[
1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*Sqrt[(d*Sqrt[1 + Tan[e]^2
```

```
] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(c*Sec[e]) + d*Sqrt[1
 + Tan[e]^2])]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])^
(2/3))/(2*d*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2*Sqrt[1 + Tan[e]^2
]))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a\sin\left(fx+e\right)+a}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a\sin\left(fx+e\right)+a}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(1/3), x)
```

maple [F] time = 0.81, size = 0, normalized size = 0.00

$$\int\frac{a+a\sin\left(fx+e\right)}{\left(c+d\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/3),x)
[Out] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a\sin\left(fx+e\right)+a}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + a\sin\left(e + f\,x\right)}{\left(c + d\sin\left(e + f\,x\right)\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c + d*sin(e + f*x))^(1/3),x)
```

```
[Out] int((a + a*sin(e + f*x))/(c + d*sin(e + f*x))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{\sin\left(e + fx\right)}{\sqrt[3]{c + d\sin\left(e + fx\right)}}\,dx + \int \frac{1}{\sqrt[3]{c + d\sin\left(e + fx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))**(1/3),x)
```

```
[Out] a*(Integral(sin(e + f*x)/(c + d*sin(e + f*x))**(1/3), x) + Integral((c + d*
sin(e + f*x))**(-1/3), x))
```

3.670 $\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^{4/3}}\,dx$

Optimal. Leaf size=112

$$-\frac{2\sqrt{2}\,a\cos(e+fx)\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}\,F_1\left(\frac{1}{2};-\frac{1}{2},\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f(c+d)\sqrt{\sin(e+fx)+1}\,\sqrt[3]{c+d\sin(e+fx)}}$$

```
[Out] -2*a*AppellF1(1/2,4/3,-1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*c
os(f*x+e)*((c+d*sin(f*x+e))/(c+d))^(1/3)*2^(1/2)/(c+d)/f/(c+d*sin(f*x+e))^(
1/3)/(1+sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2755, 139, 138}

$$-\frac{2\sqrt{2}\,a\cos(e+fx)\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}\,F_1\left(\frac{1}{2};-\frac{1}{2},\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f(c+d)\sqrt{\sin(e+fx)+1}\,\sqrt[3]{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^(4/3),x]
```

```
[Out] (-2*Sqrt[2]*a*AppellF1[1/2, -1/2, 4/3, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - S
in[e + f*x]))/(c + d)]*Cos[e + f*x]*((c + d*Sin[e + f*x])/(c + d))^(1/3))/(
(c + d)*f*Sqrt[1 + Sin[e + f*x]]*(c + d*Sin[e + f*x])^(1/3))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2755

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(c*Cos[e + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*S
qrt[1 - Sin[e + f*x]]), Subst[Int[((a + b*x)^m*Sqrt[1 + (d*x)/c])/Sqrt[1 -
(d*x)/c], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m] && EqQ[c^2 - d^2, 0
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+a\sin(e+fx)}{(c+d\sin(e+fx))^{4/3}}\,dx &= \frac{(a\cos(e+fx))\,\text{Subst}\left(\int \frac{\sqrt{1+x}}{\sqrt{1-x}\,(c+dx)^{4/3}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}\\
&= \frac{\left(a\cos(e+fx)\sqrt[3]{-\frac{c+d\sin(e+fx)}{-c-d}}\right)\text{Subst}\left(\int \frac{\sqrt{1+x}}{\sqrt{1-x}\left(-\frac{c}{-c-d}-\frac{dx}{-c-d}\right)^{4/3}}\,dx, x, \sin(e+fx)\right)}{(c+d)f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}\,\sqrt[3]{c+d\sin(e+fx)}}\\
&= -\frac{2\sqrt{2}\,aF_1\left(\frac{1}{2};-\frac{1}{2},\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}}{(c+d)f\sqrt{1+\sin(e+fx)}\,\sqrt[3]{c+d\sin(e+fx)}}
\end{aligned}
$$

Mathematica [B] time = 6.42, size = 942, normalized size = 8.41

$$a\left(\frac{(c+d\sin(e+fx))^{2/3}\left(\frac{3\csc(e)(c\cos(e)+d\sin(fx))}{d(c+d)f(c+d\sin(e+fx))}-\frac{3\csc(e)\sec(e)}{d(c+d)f}\right)(\sin(e+fx)+1)}{\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)^2}-\frac{2\sec(e)\left(-\frac{F_1\left(-\frac{1}{3};-\frac{1}{2},-\frac{1}{2};\frac{2}{3};-\frac{\csc \cdots}{\cdots}\right)\cdots}{\sqrt{\cot^2(e)+1}\sqrt{\frac{\cos\left(fx-\tan^{-1}(\cdots\right)\cdots}{a\cdots}}\cdots}\cdots\right)\cdots}{\cdots}\cdots\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])/(c + d*Sin[e + f*x])^(4/3),x]
```

```
[Out] a*(((1 + Sin[e + f*x])*(c + d*Sin[e + f*x])^(2/3)*((-3*Csc[e]*Sec[e])/(d*(c
 + d)*f) + (3*Csc[e]*(c*Cos[e] + d*Sin[f*x]))/(d*(c + d)*f*(c + d*Sin[e + f
*x]))))/(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2 - (2*Sec[e]*(1 + Sin[e
+ f*x])*(-((AppellF1[-1/3, -1/2, -1/2, 2/3, -((Csc[e]*(c + d*Cos[f*x - ArcT
an[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(1 - (c*Csc[e
])/(d*Sqrt[1 + Cot[e]^2])))), -((Csc[e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sq
rt[1 + Cot[e]^2]*Sin[e]))/(d*Sqrt[1 + Cot[e]^2]*(-1 - (c*Csc[e])/(d*Sqrt[1
+ Cot[e]^2]))))]*Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/(Sqrt[1 + Cot[e]^2]*Sqrt
[(d*Sqrt[1 + Cot[e]^2] + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d
*Sqrt[1 + Cot[e]^2] - c*Csc[e])]*Sqrt[(d*Sqrt[1 + Cot[e]^2] - d*Cos[f*x - A
rcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2])/(d*Sqrt[1 + Cot[e]^2] + c*Csc[e])]*(c +
d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e])^(1/3))) - ((3*d*Sin[
e]*(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e]))/(2*(d^2*Cos
[e]^2 + d^2*Sin[e]^2)) - (Cot[e]*Sin[f*x - ArcTan[Cot[e]]])/Sqrt[1 + Cot[e]
^2])/(c + d*Cos[f*x - ArcTan[Cot[e]]]*Sqrt[1 + Cot[e]^2]*Sin[e])^(1/3)))/((
c + d)*f*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (3*AppellF1[2/3, 1/
2, 1/2, 5/3, -((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan
[e]^2]))/(d*Sqrt[1 + Tan[e]^2]*(1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2])))), -
((Sec[e]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(d*Sq
rt[1 + Tan[e]^2]*(-1 - (c*Sec[e])/(d*Sqrt[1 + Tan[e]^2]))))]*Sec[e]*Sec[f*x
 + ArcTan[Tan[e]]]*(1 + Sin[e + f*x])*Sqrt[(d*Sqrt[1 + Tan[e]^2] - d*Sin[f*
```

```
x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(c*Sec[e] + d*Sqrt[1 + Tan[e]^2])]*
Sqrt[(d*Sqrt[1 + Tan[e]^2] + d*Sin[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]
)/(-(c*Sec[e]) + d*Sqrt[1 + Tan[e]^2])]*(c + d*Cos[e]*Sin[f*x + ArcTan[Tan[
e]]]*Sqrt[1 + Tan[e]^2])^(2/3))/(2*d*(c + d)*f*(Cos[e/2 + (f*x)/2] + Sin[e/
2 + (f*x)/2])^2*Sqrt[1 + Tan[e]^2]))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{2}{3}}}{d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(2/3)/(d^2*cos(f*x + e)
^2 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a\sin\left(fx+e\right)+a}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(4/3), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int\frac{a+a\sin\left(fx+e\right)}{\left(c+d\sin\left(fx+e\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(4/3),x)
```

```
[Out] int((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a\sin\left(fx+e\right)+a}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))^(4/3),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+a\,\sin\left(e+f\,x\right)}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))/(c + d*sin(e + f*x))^(4/3),x)
[Out] int((a + a*sin(e + f*x))/(c + d*sin(e + f*x))^(4/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))/(c+d*sin(f*x+e))**(4/3),x)
[Out] Timed out
```

3.671 $\int (a+b\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=171

$$-\frac{d\left(20acd+6bc^2+9bd^2\right)\sin(e+fx)\cos(e+fx)}{24f}-\frac{\left(4ad\left(4c^2+d^2\right)+3b\left(c^3+4cd^2\right)\right)\cos(e+fx)}{6f}+\frac{1}{8}x\left(8ac^3+1\right.$$

```
[Out] 1/8*(8*a*c^3+12*a*c*d^2+12*b*c^2*d+3*b*d^3)*x-1/6*(4*a*d*(4*c^2+d^2)+3*b*(c
^3+4*c*d^2))*cos(f*x+e)/f-1/24*d*(20*a*c*d+6*b*c^2+9*b*d^2)*cos(f*x+e)*sin(
f*x+e)/f-1/12*(4*a*d+3*b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^2/f-1/4*b*cos(f*x+e
)*(c+d*sin(f*x+e))^3/f
```

Rubi [A] time = 0.21, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2753, 2734}

$$-\frac{\left(4ad\left(4c^2+d^2\right)+3b\left(c^3+4cd^2\right)\right)\cos(e+fx)}{6f}-\frac{d\left(20acd+6bc^2+9bd^2\right)\sin(e+fx)\cos(e+fx)}{24f}+\frac{1}{8}x\left(8ac^3+1\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*b*d^3)*x)/8 - ((4*a*d*(4*c^2 + d^2)
 + 3*b*(c^3 + 4*c*d^2))*Cos[e + f*x])/(6*f) - (d*(6*b*c^2 + 20*a*c*d + 9*b*
d^2)*Cos[e + f*x]*Sin[e + f*x])/(24*f) - ((3*b*c + 4*a*d)*Cos[e + f*x]*(c +
 d*Sin[e + f*x])^2)/(12*f) - (b*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(4*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\sin(e+fx))(c+d\sin(e+fx))^3\,dx = -\frac{b\cos(e+fx)(c+d\sin(e+fx))^3}{4f} + \frac{1}{4}\int (c+d\sin(e+fx))^2(4ac$$

$$= -\frac{(3bc+4ad)\cos(e+fx)(c+d\sin(e+fx))^2}{12f} - \frac{b\cos(e+fx)(c+d}{4f}$$

$$= \frac{1}{8}\left(8ac^3+12bc^2d+12acd^2+3bd^3\right)x - \frac{\left(4ad\left(4c^2+d^2\right)+3b\left(c^3+\right.\right.}{6f}$$

Mathematica [A] time = 0.68, size = 143, normalized size = 0.84

$$\frac{-24\left(3ad\left(4c^2+d^2\right)+b\left(4c^3+9cd^2\right)\right)\cos(e+fx)+3\left(-8d\left(3acd+b\left(3c^2+d^2\right)\right)\sin(2(e+fx))+4(e+fx)\left(8ac\right.\right.}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (-24*(3*a*d*(4*c^2 + d^2) + b*(4*c^3 + 9*c*d^2))*Cos[e + f*x] + 8*d^2*(3*b*
c + a*d)*Cos[3*(e + f*x)] + 3*(4*(8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*b*d
^3)*(e + f*x) - 8*d*(3*a*c*d + b*(3*c^2 + d^2))*Sin[2*(e + f*x)] + b*d^3*Si
n[4*(e + f*x)]))/(96*f)
```

fricas [A] time = 0.48, size = 145, normalized size = 0.85

$$\frac{8\left(3\,bcd^2+ad^3\right)\cos\left(fx+e\right)^3+3\left(8\,ac^3+12\,bc^2d+12\,acd^2+3\,bd^3\right)fx-24\left(bc^3+3\,ac^2d+3\,bcd^2+ad^3\right)\cos\left(f\right.}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/24*(8*(3*b*c*d^2 + a*d^3)*cos(f*x + e)^3 + 3*(8*a*c^3 + 12*b*c^2*d + 12*a
*c*d^2 + 3*b*d^3)*f*x - 24*(b*c^3 + 3*a*c^2*d + 3*b*c*d^2 + a*d^3)*cos(f*x
+ e) + 3*(2*b*d^3*cos(f*x + e)^3 - (12*b*c^2*d + 12*a*c*d^2 + 5*b*d^3)*cos(
f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.19, size = 152, normalized size = 0.89

$$\frac{bd^3\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{1}{8}\left(8\,ac^3+12\,bc^2d+12\,acd^2+3\,bd^3\right)x+\frac{\left(3\,bcd^2+ad^3\right)\cos\left(3\,fx+3\,e\right)}{12\,f}-\frac{\left(4\,bc^3+12\,ac^2d+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/32*b*d^3*sin(4*f*x + 4*e)/f + 1/8*(8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*
b*d^3)*x + 1/12*(3*b*c*d^2 + a*d^3)*cos(3*f*x + 3*e)/f - 1/4*(4*b*c^3 + 12*
a*c^2*d + 9*b*c*d^2 + 3*a*d^3)*cos(f*x + e)/f - 1/4*(3*b*c^2*d + 3*a*c*d^2
+ b*d^3)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.26, size = 182, normalized size = 1.06

$$a c^3 \left(fx+e\right) - 3a c^2 d \cos\left(fx+e\right) + 3ac d^2 \left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2} + \frac{fx}{2} + \frac{e}{2}\right) - \frac{a d^3 \left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3} - b c^3 \cos\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(a*c^3*(f*x+e)-3*a*c^2*d*cos(f*x+e)+3*a*c*d^2*(-1/2*sin(f*x+e)*cos(f*x+
e)+1/2*f*x+1/2*e)-1/3*a*d^3*(2+sin(f*x+e)^2)*cos(f*x+e)-b*c^3*cos(f*x+e)+3*
b*c^2*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-c*d^2*b*(2+sin(f*x+e)^2)
*cos(f*x+e)+b*d^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/
8*e))
```

maxima [A] time = 1.17, size = 175, normalized size = 1.02

$$96\left(fx+e\right)ac^3 + 72\left(2fx+2e-\sin\left(2fx+2e\right)\right)bc^2d + 72\left(2fx+2e-\sin\left(2fx+2e\right)\right)acd^2 + 96\left(\cos\left(fx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/96*(96*(f*x + e)*a*c^3 + 72*(2*f*x + 2*e - sin(2*f*x + 2*e))*b*c^2*d + 72
*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*c*d^2 + 96*(cos(f*x + e)^3 - 3*cos(f*x
+ e))*b*c*d^2 + 32*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*d^3 + 3*(12*f*x + 12
*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*b*d^3 - 96*b*c^3*cos(f*x + e) -
 288*a*c^2*d*cos(f*x + e))/f
```

mupad [B] time = 8.08, size = 183, normalized size = 1.07

$$2\,a\,d^3\,\cos\left(3\,e+3\,f\,x\right) - 6\,b\,d^3\,\sin\left(2\,e+2\,f\,x\right) + \frac{3\,b\,d^3\,\sin\left(4\,e+4\,f\,x\right)}{4} - 18\,a\,d^3\,\cos\left(e+f\,x\right) - 24\,b\,c^3\,\cos\left(e+f\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^3,x)
```

```
[Out] (2*a*d^3*cos(3*e + 3*f*x) - 6*b*d^3*sin(2*e + 2*f*x) + (3*b*d^3*sin(4*e + 4
*f*x))/4 - 18*a*d^3*cos(e + f*x) - 24*b*c^3*cos(e + f*x) - 72*a*c^2*d*cos(e
 + f*x) - 54*b*c*d^2*cos(e + f*x) + 24*a*c^3*f*x + 9*b*d^3*f*x + 6*b*c*d^2*
cos(3*e + 3*f*x) - 18*a*c*d^2*sin(2*e + 2*f*x) - 18*b*c^2*d*sin(2*e + 2*f*x
) + 36*a*c*d^2*f*x + 36*b*c^2*d*f*x)/(24*f)
```

sympy [A] time = 1.86, size = 386, normalized size = 2.26

$$\begin{cases} ac^3x - \frac{3ac^2d\cos(e+fx)}{f} + \frac{3acd^2x\sin^2(e+fx)}{2} + \frac{3acd^2x\cos^2(e+fx)}{2} - \frac{3acd^2\sin(e+fx)\cos(e+fx)}{2f} - \frac{ad^3\sin^2(e+fx)\cos(e+fx)}{f} - \frac{2ad^3\mathrm{co}}{} \\ x(a+b\sin(e))(c+d\sin(e))^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((a*c**3*x - 3*a*c**2*d*cos(e + f*x)/f + 3*a*c*d**2*x*sin(e + f*x)
**2/2 + 3*a*c*d**2*x*cos(e + f*x)**2/2 - 3*a*c*d**2*sin(e + f*x)*cos(e + f*
x)/(2*f) - a*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 2*a*d**3*cos(e + f*x)**3
/(3*f) - b*c**3*cos(e + f*x)/f + 3*b*c**2*d*x*sin(e + f*x)**2/2 + 3*b*c**2*
d*x*cos(e + f*x)**2/2 - 3*b*c**2*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 3*b*c*
d**2*sin(e + f*x)**2*cos(e + f*x)/f - 2*b*c*d**2*cos(e + f*x)**3/f + 3*b*d*
*3*x*sin(e + f*x)**4/8 + 3*b*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*b
*d**3*x*cos(e + f*x)**4/8 - 5*b*d**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3
*b*d**3*sin(e + f*x)*cos(e + f*x)**3/(8*f), Ne(f, 0)), (x*(a + b*sin(e))*(c
 + d*sin(e))**3, True))
```

3.672 $\int (a+b\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=106

$$-\frac{2\left(3acd+b\left(c^2+d^2\right)\right)\cos(e+fx)}{3f}+\frac{1}{2}x\left(a\left(2c^2+d^2\right)+2bcd\right)-\frac{d(3ad+2bc)\sin(e+fx)\cos(e+fx)}{6f}-\frac{b\cos(e+$$

```
[Out] 1/2*(2*b*c*d+a*(2*c^2+d^2))*x-2/3*(3*a*c*d+b*(c^2+d^2))*cos(f*x+e)/f-1/6*d*
(3*a*d+2*b*c)*cos(f*x+e)*sin(f*x+e)/f-1/3*b*cos(f*x+e)*(c+d*sin(f*x+e))^2/f
```

Rubi [A] time = 0.10, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2753, 2734}

$$-\frac{2\left(3acd+b\left(c^2+d^2\right)\right)\cos(e+fx)}{3f}+\frac{1}{2}x\left(a\left(2c^2+d^2\right)+2bcd\right)-\frac{d(3ad+2bc)\sin(e+fx)\cos(e+fx)}{6f}-\frac{b\cos(e+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] ((2*b*c*d + a*(2*c^2 + d^2))*x)/2 - (2*(3*a*c*d + b*(c^2 + d^2))*Cos[e + f*
x])/(3*f) - (d*(2*b*c + 3*a*d)*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (b*Cos[e
+ f*x]*(c + d*Sin[e + f*x])^2)/(3*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\sin(e+fx))(c+d\sin(e+fx))^2\,dx = -\frac{b\cos(e+fx)(c+d\sin(e+fx))^2}{3f} + \frac{1}{3}\int (c+d\sin(e+fx))(3ac+$$

$$= \frac{1}{2}\left(2bcd+a\left(2c^2+d^2\right)\right)x - \frac{2\left(3acd+b\left(c^2+d^2\right)\right)\cos(e+fx)}{3f} - \frac{d}{}$$

Mathematica [A] time = 0.31, size = 90, normalized size = 0.85

$$\frac{6(e+fx)\left(a\left(2c^2+d^2\right)+2bcd\right)-3\left(8acd+4bc^2+3bd^2\right)\cos(e+fx)-3d(ad+2bc)\sin(2(e+fx))+bd^2\cos(3(e}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (6*(2*b*c*d + a*(2*c^2 + d^2))*(e + f*x) - 3*(4*b*c^2 + 8*a*c*d + 3*b*d^2)*
Cos[e + f*x] + b*d^2*Cos[3*(e + f*x)] - 3*d*(2*b*c + a*d)*Sin[2*(e + f*x)])
/(12*f)
```

fricas [A] time = 0.44, size = 90, normalized size = 0.85

$$\frac{2\,bd^2\cos\left(fx+e\right)^3+3\left(2\,ac^2+2\,bcd+ad^2\right)fx-3\left(2\,bcd+ad^2\right)\cos\left(fx+e\right)\sin\left(fx+e\right)-6\left(bc^2+2\,acd+bd^2\right.}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*d^2*cos(f*x + e)^3 + 3*(2*a*c^2 + 2*b*c*d + a*d^2)*f*x - 3*(2*b*c*
d + a*d^2)*cos(f*x + e)*sin(f*x + e) - 6*(b*c^2 + 2*a*c*d + b*d^2)*cos(f*x
+ e))/f
```

giac [A] time = 0.19, size = 96, normalized size = 0.91

$$\frac{bd^2\cos\left(3\,fx+3\,e\right)}{12\,f}+\frac{1}{2}\left(2\,ac^2+2\,bcd+ad^2\right)x-\frac{\left(4\,bc^2+8\,acd+3\,bd^2\right)\cos\left(fx+e\right)}{4\,f}-\frac{\left(2\,bcd+ad^2\right)\sin\left(2\,fx+2\,e\right.}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/12*b*d^2*cos(3*f*x + 3*e)/f + 1/2*(2*a*c^2 + 2*b*c*d + a*d^2)*x - 1/4*(4*
b*c^2 + 8*a*c*d + 3*b*d^2)*cos(f*x + e)/f - 1/4*(2*b*c*d + a*d^2)*sin(2*f*x
 + 2*e)/f
```

maple [A] time = 0.19, size = 115, normalized size = 1.08

$$\frac{c^2a\left(fx+e\right)-2acd\cos\left(fx+e\right)+a\,d^2\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-b\,c^2\cos\left(fx+e\right)+2bcd\left(-\frac{\sin\left(fx+e\right)\cos\left(f\right.}{2}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(c^2*a*(f*x+e)-2*a*c*d*cos(f*x+e)+a*d^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2
*f*x+1/2*e)-b*c^2*cos(f*x+e)+2*b*c*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/
2*e)-1/3*d^2*b*(2+sin(f*x+e)^2)*cos(f*x+e))
```

maxima [A] time = 0.40, size = 112, normalized size = 1.06

$$\frac{12\left(fx+e\right)ac^2+6\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)bcd+3\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)ad^2+4\left(\cos\left(fx+e\right)^3-\right.}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(12*(f*x + e)*a*c^2 + 6*(2*f*x + 2*e - sin(2*f*x + 2*e))*b*c*d + 3*(2*
f*x + 2*e - sin(2*f*x + 2*e))*a*d^2 + 4*(cos(f*x + e)^3 - 3*cos(f*x + e))*b
*d^2 - 12*b*c^2*cos(f*x + e) - 24*a*c*d*cos(f*x + e))/f
```

mupad [B] time = 7.89, size = 108, normalized size = 1.02

$$-\frac{\frac{3\,a\,d^2\,\sin\left(2\,e+2\,f\,x\right)}{2}-\frac{b\,d^2\,\cos\left(3\,e+3\,f\,x\right)}{2}+6\,b\,c^2\,\cos\left(e+f\,x\right)+\frac{9\,b\,d^2\,\cos\left(e+f\,x\right)}{2}+3\,b\,c\,d\,\sin\left(2\,e+2\,f\,x\right)-6\,a\,c^2\,f\,x-}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^2,x)
```

```
[Out] -((3*a*d^2*sin(2*e + 2*f*x))/2 - (b*d^2*cos(3*e + 3*f*x))/2 + 6*b*c^2*cos(e
 + f*x) + (9*b*d^2*cos(e + f*x))/2 + 3*b*c*d*sin(2*e + 2*f*x) - 6*a*c^2*f*x
 - 3*a*d^2*f*x + 12*a*c*d*cos(e + f*x) - 6*b*c*d*f*x)/(6*f)
```

sympy [A] time = 0.83, size = 199, normalized size = 1.88

$$\begin{cases} ac^2x-\frac{2acd\cos\left(e+fx\right)}{f}+\frac{ad^2x\sin^2\left(e+fx\right)}{2}+\frac{ad^2x\cos^2\left(e+fx\right)}{2}-\frac{ad^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{bc^2\cos\left(e+fx\right)}{f}+bcdx\sin^2\left(e+fx\right) \\ x\left(a+b\sin\left(e\right)\right)\left(c+d\sin\left(e\right)\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a*c**2*x - 2*a*c*d*cos(e + f*x)/f + a*d**2*x*sin(e + f*x)**2/2 +
 a*d**2*x*cos(e + f*x)**2/2 - a*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - b*c*
*2*cos(e + f*x)/f + b*c*d*x*sin(e + f*x)**2 + b*c*d*x*cos(e + f*x)**2 - b*c
*d*sin(e + f*x)*cos(e + f*x)/f - b*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 2*
b*d**2*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(a + b*sin(e))*(c + d*sin(e))**
2, True))
```

3.673 $\int (a+b\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=53

$$-\frac{(ad+bc)\cos(e+fx)}{f}+\frac{1}{2}x(2ac+bd)-\frac{bd\sin(e+fx)\cos(e+fx)}{2f}$$

```
[Out] 1/2*(2*a*c+b*d)*x-(a*d+b*c)*cos(f*x+e)/f-1/2*b*d*cos(f*x+e)*sin(f*x+e)/f
```

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2734}

$$-\frac{(ad+bc)\cos(e+fx)}{f}+\frac{1}{2}x(2ac+bd)-\frac{bd\sin(e+fx)\cos(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] ((2*a*c + b*d)*x)/2 - ((b*c + a*d)*Cos[e + f*x])/f - (b*d*Cos[e + f*x]*Sin[
e + f*x])/(2*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))(c+d\sin(e+fx))\,dx=\frac{1}{2}(2ac+bd)x-\frac{(bc+ad)\cos(e+fx)}{f}-\frac{bd\cos(e+fx)\sin(e+fx)}{2f}$$

Mathematica [A] time = 0.09, size = 52, normalized size = 0.98

$$\frac{-4(ad+bc)\cos(e+fx)+4acfx-bd\sin(2(e+fx))+2bde+2bdfx}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (2*b*d*e + 4*a*c*f*x + 2*b*d*f*x - 4*(b*c + a*d)*Cos[e + f*x] - b*d*Sin[2*(
e + f*x)])/(4*f)
```

fricas [A] time = 0.44, size = 48, normalized size = 0.91

$$-\frac{bd\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(2\,ac+bd\right)fx+2\,\left(bc+ad\right)\cos\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/2*(b*d*cos(f*x + e)*sin(f*x + e) - (2*a*c + b*d)*f*x + 2*(b*c + a*d)*cos
(f*x + e))/f
```

giac [A] time = 0.82, size = 48, normalized size = 0.91

$$\frac{1}{2}\,\left(2\,ac+bd\right)x-\frac{bd\sin\left(2\,fx+2\,e\right)}{4\,f}-\frac{\left(bc+ad\right)\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*c + b*d)*x - 1/4*b*d*sin(2*f*x + 2*e)/f - (b*c + a*d)*cos(f*x + e)
/f
```

maple [A] time = 0.09, size = 59, normalized size = 1.11

$$\frac{bd\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-da\cos\left(fx+e\right)-cb\cos\left(fx+e\right)+ac\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] 1/f*(b*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-d*a*cos(f*x+e)-c*b*cos(
f*x+e)+a*c*(f*x+e))
```

maxima [A] time = 0.67, size = 57, normalized size = 1.08

$$\frac{4\left(fx+e\right)ac+\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)bd-4\,bc\cos\left(fx+e\right)-4\,ad\cos\left(fx+e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(f*x + e)*a*c + (2*f*x + 2*e - sin(2*f*x + 2*e))*b*d - 4*b*c*cos(f*x
 + e) - 4*a*d*cos(f*x + e))/f
```

mupad [B] time = 7.72, size = 52, normalized size = 0.98

$$a\,c\,x+\frac{b\,d\,x}{2}-\frac{a\,d\,\cos\left(e+f\,x\right)}{f}-\frac{b\,c\,\cos\left(e+f\,x\right)}{f}-\frac{b\,d\,\sin\left(2\,e+2\,f\,x\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))*(c + d*sin(e + f*x)),x)
```

```
[Out] a*c*x + (b*d*x)/2 - (a*d*cos(e + f*x))/f - (b*c*cos(e + f*x))/f - (b*d*sin(
2*e + 2*f*x))/(4*f)
```

sympy [A] time = 0.32, size = 94, normalized size = 1.77

$$\begin{cases} acx - \frac{ad\cos\left(e+fx\right)}{f} - \frac{bc\cos\left(e+fx\right)}{f} + \frac{bdx\sin^{2}\left(e+fx\right)}{2} + \frac{bdx\cos^{2}\left(e+fx\right)}{2} - \frac{bd\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f} & \text{for}\: f \neq 0 \\ x\left(a+b\sin\left(e\right)\right)\left(c+d\sin\left(e\right)\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((a*c*x - a*d*cos(e + f*x)/f - b*c*cos(e + f*x)/f + b*d*x*sin(e +
f*x)**2/2 + b*d*x*cos(e + f*x)**2/2 - b*d*sin(e + f*x)*cos(e + f*x)/(2*f),
Ne(f, 0)), (x*(a + b*sin(e))*(c + d*sin(e)), True))
```

3.674 $\int (a+b\sin(e+fx))\,dx$

Optimal. Leaf size=16

$$ax-\frac{b\cos(e+fx)}{f}$$

```
[Out] a*x-b*cos(f*x+e)/f
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2638}

$$ax-\frac{b\cos(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sin[e + f*x],x]
```

```
[Out] a*x - (b*Cos[e + f*x])/f
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\sin(e+fx))\,dx &= ax+b\int \sin(e+fx)\,dx\\ &= ax-\frac{b\cos(e+fx)}{f}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.69

$$ax+\frac{b\sin(e)\sin(fx)}{f}-\frac{b\cos(e)\cos(fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sin[e + f*x],x]
```

```
[Out] a*x - (b*Cos[e]*Cos[f*x])/f + (b*Sin[e]*Sin[f*x])/f
```

fricas [A] time = 0.44, size = 18, normalized size = 1.12

$$\frac{afx - b\cos\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e),x, algorithm="fricas")
```

```
[Out] (a*f*x - b*cos(f*x + e))/f
```

giac [A] time = 0.24, size = 17, normalized size = 1.06

$$ax - \frac{b\cos\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e),x, algorithm="giac")
```

```
[Out] a*x - b*cos(f*x + e)/f
```

maple [A] time = 0.01, size = 17, normalized size = 1.06

$$ax - \frac{b\cos\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sin(f*x+e),x)
```

```
[Out] a*x-b*cos(f*x+e)/f
```

maxima [A] time = 0.63, size = 16, normalized size = 1.00

$$ax - \frac{b\cos\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e),x, algorithm="maxima")
```

```
[Out] a*x - b*cos(f*x + e)/f
```

mupad [B] time = 7.64, size = 25, normalized size = 1.56

$$a\,x - \frac{2\,b}{f\,\left({\tan\left(\frac{e}{2} + \frac{f\,x}{2}\right)}^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*sin(e + f*x),x)
```

```
[Out] a*x - (2*b)/(f*(tan(e/2 + (f*x)/2)^2 + 1))
```

sympy [A] time = 0.14, size = 19, normalized size = 1.19

$$ax + b\left(\begin{cases} -\frac{\cos\left(e+fx\right)}{f} & \text{for } f \neq 0 \\ x\sin\left(e\right) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e),x)
```

```
[Out] a*x + b*Piecewise((-cos(e + f*x)/f, Ne(f, 0)), (x*sin(e), True))
```

3.675 $\int \frac{a+b\sin(e+fx)}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=65

$$\frac{bx}{d}-\frac{2(bc-ad)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{df\sqrt{c^2-d^2}}$$

```
[Out] b*x/d-2*(-a*d+b*c)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d/f/(c^
2-d^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2735, 2660, 618, 204}

$$\frac{bx}{d}-\frac{2(bc-ad)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{df\sqrt{c^2-d^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x]),x]
```

```
[Out] (b*x)/d - (2*(b*c - a*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/
(d*Sqrt[c^2 - d^2]*f)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\sin(e+fx)}{c+d\sin(e+fx)}\,dx &= \frac{bx}{d}-\frac{(bc-ad)\int \frac{1}{c+d\sin(e+fx)}\,dx}{d}\\
&= \frac{bx}{d}-\frac{(2(bc-ad))\,\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx,x,\tan\left(\frac{1}{2}(e+fx)\right)\right)}{df}\\
&= \frac{bx}{d}+\frac{(4(bc-ad))\,\text{Subst}\left(\int \frac{1}{-4\left(c^2-d^2\right)-x^2}\,dx,x,2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{df}\\
&= \frac{bx}{d}-\frac{2(bc-ad)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d\sqrt{c^2-d^2}\,f}
\end{aligned}
$$

Mathematica [A] time = 0.13, size = 67, normalized size = 1.03

$$
\frac{\frac{(2ad-2bc)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}+b(e+fx)}{df}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x]),x]
```

```
[Out] (b*(e + f*x) + ((-2*b*c + 2*a*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 -
 d^2]])/Sqrt[c^2 - d^2])/(d*f)
```

fricas [A] time = 0.47, size = 255, normalized size = 3.92

$$\left[\frac{2\left(bc^2-bd^2\right)fx+(bc-ad)\sqrt{-c^2+d^2}\,\log\left(\frac{\left(2c^2-d^2\right)\cos\left(fx+e\right)^2-2cd\sin\left(fx+e\right)-c^2-d^2+2\left(c\cos\left(fx+e\right)\sin\left(fx+e\right)+d\cos\left(fx+e\right)\right)\sqrt{\cdot}}{d^2\cos\left(fx+e\right)^2-2cd\sin\left(fx+e\right)-c^2-d^2}\right)}{2\left(c^2d-d^3\right)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b*c^2 - b*d^2)*f*x + (b*c - a*d)*sqrt(-c^2 + d^2)*log(((2*c^2 - d^
2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(
f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*si
n(f*x + e) - c^2 - d^2)))/((c^2*d - d^3)*f), ((b*c^2 - b*d^2)*f*x + (b*c -
a*d)*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x
+ e))))/((c^2*d - d^3)*f)]
```

giac [A] time = 0.18, size = 86, normalized size = 1.32

$$\frac{\frac{(fx+e)b}{d}-\frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)(bc-ad)}{\sqrt{c^2-d^2}\,d}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] ((f*x + e)*b/d - 2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan
(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))*(b*c - a*d)/(sqrt(c^2 - d^2)*d))/f
```

maple [A] time = 0.13, size = 119, normalized size = 1.83

$$\frac{2\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)a}{f\sqrt{c^2-d^2}}-\frac{2\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)cb}{fd\sqrt{c^2-d^2}}+\frac{2b\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] 2/f/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)
)*a-2/f/d/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)
^(1/2))*c*b+2/f*b/d*arctan(tan(1/2*f*x+1/2*e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 9.68, size = 342, normalized size = 5.26

$$\frac{2\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{d\,f}+\frac{c\,\left(b\,\ln\left(\frac{d\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{d^2-c^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\,\sqrt{-\left(c+d\right)\,\left(c-d\right)}-b\,\ln\left(\frac{d\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+c\,\mathrm{si}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))/(c + d*sin(e + f*x)),x)
```

```
[Out] (2*b*atan(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(d*f) + (c*(b*log((d*cos(
e/2 + (f*x)/2) + c*sin(e/2 + (f*x)/2) - cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2
))/cos(e/2 + (f*x)/2))*(-(c + d)*(c - d))^(1/2) - b*log((d*cos(e/2 + (f*x)/
2) + c*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2))/cos(e/2 +
 (f*x)/2))*(d^2 - c^2)^(1/2)) - a*d*log((d*cos(e/2 + (f*x)/2) + c*sin(e/2 +
 (f*x)/2) - cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2))/cos(e/2 + (f*x)/2))*(-(c
+ d)*(c - d))^(1/2) + a*d*log((d*cos(e/2 + (f*x)/2) + c*sin(e/2 + (f*x)/2)
+ cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2))/cos(e/2 + (f*x)/2))*(d^2 - c^2)^(1/
2))/(d*f*(c^2 - d^2))
```

sympy [A] time = 85.45, size = 537, normalized size = 8.26

$$\begin{cases} \frac{\tilde{\infty}x(a+b\sin(e))}{\sin(e)} & \\ \frac{2ad\sqrt{d^2}}{d^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-f(d^2)^{\frac{3}{2}}}+\frac{bd^2fx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{d^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-f(d^2)^{\frac{3}{2}}}+\frac{2bd^2}{d^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-f(d^2)^{\frac{3}{2}}}-\frac{bdfx\sqrt{d^2}}{d^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-f(d^2)^{\frac{3}{2}}} & \\ -\frac{2ad\sqrt{d^2}}{d^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+f(d^2)^{\frac{3}{2}}}+\frac{bd^2fx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{d^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+f(d^2)^{\frac{3}{2}}}+\frac{2bd^2}{d^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+f(d^2)^{\frac{3}{2}}}+\frac{bdfx\sqrt{d^2}}{d^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+f(d^2)^{\frac{3}{2}}} & \\ \frac{\frac{a\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}+bx}{d} & \\ \frac{ax-\frac{b\cos(e+fx)}{f}}{c} & \\ \frac{x(a+b\sin(e))}{c+d\sin(e)} & \\ \frac{a\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{d}{c}-\frac{\sqrt{-c^2+d^2}}{c}\right)}{f\sqrt{-c^2+d^2}}-\frac{a\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{d}{c}+\frac{\sqrt{-c^2+d^2}}{c}\right)}{f\sqrt{-c^2+d^2}}-\frac{bc\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{d}{c}-\frac{\sqrt{-c^2+d^2}}{c}\right)}{df\sqrt{-c^2+d^2}}+\frac{bc\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{d}{c}+\frac{\sqrt{-c^2+d}}{c}\right)}{df\sqrt{-c^2+d^2}} & \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((zoo*x*(a + b*sin(e))/sin(e), Eq(c, 0) & Eq(d, 0) & Eq(f, 0)), (2
*a*d*sqrt(d**2)/(d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)) + b*d**2*f*x*ta
n(e/2 + f*x/2)/(d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)) + 2*b*d**2/(d**3
*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)) - b*d*f*x*sqrt(d**2)/(d**3*f*tan(e/2
 + f*x/2) - f*(d**2)**(3/2)), Eq(c, -sqrt(d**2))), (-2*a*d*sqrt(d**2)/(d**3
*f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)) + b*d**2*f*x*tan(e/2 + f*x/2)/(d**3*
f*tan(e/2 + f*x/2) + f*(d**2)**(3/2)) + 2*b*d**2/(d**3*f*tan(e/2 + f*x/2) +
 f*(d**2)**(3/2)) + b*d*f*x*sqrt(d**2)/(d**3*f*tan(e/2 + f*x/2) + f*(d**2)*
*(3/2)), Eq(c, sqrt(d**2))), ((a*log(tan(e/2 + f*x/2))/f + b*x)/d, Eq(c, 0)
), ((a*x - b*cos(e + f*x)/f)/c, Eq(d, 0)), (x*(a + b*sin(e))/(c + d*sin(e))
, Eq(f, 0)), (a*log(tan(e/2 + f*x/2) + d/c - sqrt(-c**2 + d**2)/c)/(f*sqrt(
-c**2 + d**2)) - a*log(tan(e/2 + f*x/2) + d/c + sqrt(-c**2 + d**2)/c)/(f*sq
rt(-c**2 + d**2)) - b*c*log(tan(e/2 + f*x/2) + d/c - sqrt(-c**2 + d**2)/c)/
(d*f*sqrt(-c**2 + d**2)) + b*c*log(tan(e/2 + f*x/2) + d/c + sqrt(-c**2 + d*
*2)/c)/(d*f*sqrt(-c**2 + d**2)) + b*x/d, True))
```

3.676 $\int \frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=98

$$\frac{2(ac-bd)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{3/2}}-\frac{(bc-ad)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

```
[Out] 2*(a*c-b*d)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(c^2-d^2)^(3/2
)/f-(-a*d+b*c)*cos(f*x+e)/(c^2-d^2)/f/(c+d*sin(f*x+e))
```

Rubi [A] time = 0.10, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2754, 12, 2660, 618, 204}

$$\frac{2(ac-bd)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{3/2}}-\frac{(bc-ad)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (2*(a*c - b*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c^2 - d^
2)^(3/2)*f) - ((b*c - a*d)*Cos[e + f*x])/((c^2 - d^2)*f*(c + d*Sin[e + f*x]
))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^2}\,dx = -\frac{(bc-ad)\cos(e+fx)}{\left(c^2-d^2\right)f(c+d\sin(e+fx))}+\frac{\int \frac{-ac+bd}{c+d\sin(e+fx)}\,dx}{-c^2+d^2}$$

$$= -\frac{(bc-ad)\cos(e+fx)}{\left(c^2-d^2\right)f(c+d\sin(e+fx))}+\frac{(ac-bd)\int \frac{1}{c+d\sin(e+fx)}\,dx}{c^2-d^2}$$

$$= -\frac{(bc-ad)\cos(e+fx)}{\left(c^2-d^2\right)f(c+d\sin(e+fx))}+\frac{(2(ac-bd))\,\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx,x,\tan\left(\frac{1}{2}(e+ \right.\right.}{\left(c^2-d^2\right)f}$$

$$= -\frac{(bc-ad)\cos(e+fx)}{\left(c^2-d^2\right)f(c+d\sin(e+fx))}-\frac{(4(ac-bd))\,\text{Subst}\left(\int \frac{1}{-4\left(c^2-d^2\right)-x^2}\,dx,x,2d+2c\,\text{t}\right.}{\left(c^2-d^2\right)f}$$

$$= \frac{2(ac-bd)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{3/2}f}-\frac{(bc-ad)\cos(e+fx)}{\left(c^2-d^2\right)f(c+d\sin(e+fx))}$$

Mathematica [A] time = 0.30, size = 96, normalized size = 0.98

$$\frac{\frac{2(ac-bd)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{3/2}}+\frac{(ad-bc)\cos(e+fx)}{(c-d)(c+d)(c+d\sin(e+fx))}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] ((2*(a*c - b*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^
2)^(3/2) + ((-(b*c) + a*d)*Cos[e + f*x])/((c - d)*(c + d)*(c + d*Sin[e + f*
x])))/f
```

fricas [A] time = 0.47, size = 394, normalized size = 4.02

$$\left[-\frac{\left(ac^2-bcd+\left(acd-bd^2\right)\sin\left(fx+e\right)\right)\sqrt{-c^2+d^2}\log\left(\frac{\left(2c^2-d^2\right)\cos\left(fx+e\right)^2-2cd\sin\left(fx+e\right)-c^2-d^2+2\left(c\cos\left(fx+e\right)\sin\left(fx+e\right)+d\right.}{d^2\cos\left(fx+e\right)^2-2cd\sin\left(fx+e\right)-c^2-d^2}\right.}{2\left(\left(c^4d-2c^2d^3+d^5\right)f\sin\left(fx+e\right)+\left(c^5-2c^3d^2+cd\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((a*c^2 - b*c*d + (a*c*d - b*d^2)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(
((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f
*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^
2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(b*c^3 - a*c^2*d - b*c*d^2 + a*d^3
)*cos(f*x + e))/((c^4*d - 2*c^2*d^3 + d^5)*f*sin(f*x + e) + (c^5 - 2*c^3*d^
2 + c*d^4)*f), -((a*c^2 - b*c*d + (a*c*d - b*d^2)*sin(f*x + e))*sqrt(c^2 -
d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (b*c^3
- a*c^2*d - b*c*d^2 + a*d^3)*cos(f*x + e))/((c^4*d - 2*c^2*d^3 + d^5)*f*sin
(f*x + e) + (c^5 - 2*c^3*d^2 + c*d^4)*f)]
```

giac [A] time = 0.48, size = 158, normalized size = 1.61

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)(ac-bd)}{\left(c^2-d^2\right)^{\frac{3}{2}}}-\frac{bcd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-ad^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+bc^2-acd}{\left(c^3-cd^2\right)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+c\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e
) + d)/sqrt(c^2 - d^2)))*(a*c - b*d)/(c^2 - d^2)^(3/2) - (b*c*d*tan(1/2*f*x
 + 1/2*e) - a*d^2*tan(1/2*f*x + 1/2*e) + b*c^2 - a*c*d)/((c^3 - c*d^2)*(c*t
an(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)))/f
```

maple [B] time = 0.23, size = 309, normalized size = 3.15

$$\frac{2d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)a}{f\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d+c\right)\left(c^2-d^2\right)c}-\frac{2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)b}{f\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d+c\right)\left(c^2-d^2\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)*d^2/(c^2-d^2)/c*tan(1
/2*f*x+1/2*e)*a-2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)*d/(c^
2-d^2)*tan(1/2*f*x+1/2*e)*b+2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e
)*d+c)/(c^2-d^2)*d*a-2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/
(c^2-d^2)*c*b+2/f/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(
c^2-d^2)^(1/2))*c*a-2/f/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+
2*d)/(c^2-d^2)^(1/2))*b*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 7.92, size = 214, normalized size = 2.18

$$\frac{\frac{2\,(a\,d-b\,c)}{c^2-d^2}+\frac{2\,d\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(a\,d-b\,c)}{c\,\left(c^2-d^2\right)}}{f\,\left(c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+2\,d\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+c\right)}+\frac{2\,\mathrm{atan}\left(\frac{\left(\frac{2\,\left(c^2\,d-d^3\right)(a\,c-b\,d)}{(c+d)^{3/2}\,\left(c^2-d^2\right)(c-d)^{3/2}}+\frac{2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(a\,c-b\,d)}{(c+d)^{3/2}\,(c-d)^{3/2}}\right)\left(c^2-d^2\right)}{2\,(a\,c-b\,d)}\right)(a\,c-b\,d)}{f\,(c+d)^{3/2}\,(c-d)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))/(c + d*sin(e + f*x))^2,x)
```

```
[Out] ((2*(a*d - b*c))/(c^2 - d^2) + (2*d*tan(e/2 + (f*x)/2)*(a*d - b*c))/(c*(c^2
 - d^2)))/(f*(c + 2*d*tan(e/2 + (f*x)/2) + c*tan(e/2 + (f*x)/2)^2)) + (2*at
an((((2*(c^2*d - d^3)*(a*c - b*d))/((c + d)^(3/2)*(c^2 - d^2)*(c - d)^(3/2)
) + (2*c*tan(e/2 + (f*x)/2)*(a*c - b*d))/((c + d)^(3/2)*(c - d)^(3/2)))*(c^
2 - d^2))/(2*(a*c - b*d)))*(a*c - b*d))/(f*(c + d)^(3/2)*(c - d)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.677 $\int \frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=164

$$-\frac{\left(3bcd-a\left(2c^2+d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{5/2}}+\frac{\left(3acd-b\left(c^2+2d^2\right)\right)\cos(e+fx)}{2f\left(c^2-d^2\right)^2(c+d\sin(e+fx))}-\frac{(bc-ad)\cos(e+fx)}{2f\left(c^2-d^2\right)(c+d\sin(e+f}$$

```
[Out] -(3*b*c*d-a*(2*c^2+d^2))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(
c^2-d^2)^(5/2)/f-1/2*(-a*d+b*c)*cos(f*x+e)/(c^2-d^2)/f/(c+d*sin(f*x+e))^2+1
/2*(3*a*c*d-b*(c^2+2*d^2))*cos(f*x+e)/(c^2-d^2)^2/f/(c+d*sin(f*x+e))
```

Rubi [A] time = 0.20, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2754, 12, 2660, 618, 204}

$$-\frac{\left(3bcd-a\left(2c^2+d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{5/2}}+\frac{\left(3acd-b\left(c^2+2d^2\right)\right)\cos(e+fx)}{2f\left(c^2-d^2\right)^2(c+d\sin(e+fx))}-\frac{(bc-ad)\cos(e+fx)}{2f\left(c^2-d^2\right)(c+d\sin(e+f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] -(((3*b*c*d - a*(2*c^2 + d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d
^2]])/((c^2 - d^2)^(5/2)*f)) - ((b*c - a*d)*Cos[e + f*x])/(2*(c^2 - d^2)*f*
(c + d*Sin[e + f*x])^2) + ((3*a*c*d - b*(c^2 + 2*d^2))*Cos[e + f*x])/(2*(c^
2 - d^2)^2*f*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^3}\,dx = -\frac{(bc-ad)\cos(e+fx)}{2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} - \frac{\int \frac{-2(ac-bd)-(bc-ad)\sin(e+fx)}{(c+d\sin(e+fx))^2}\,dx}{2\left(c^2-d^2\right)}$$

$$= -\frac{(bc-ad)\cos(e+fx)}{2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{\left(3acd-b\left(c^2+2d^2\right)\right)\cos(e+fx)}{2\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))} + \frac{\int \frac{-3bcd+a}{c+d\sin}}{2\left(c^2\right.}$$

$$= -\frac{(bc-ad)\cos(e+fx)}{2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{\left(3acd-b\left(c^2+2d^2\right)\right)\cos(e+fx)}{2\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))} - \frac{\left(3bcd-a\right.}{}$$

$$= -\frac{(bc-ad)\cos(e+fx)}{2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{\left(3acd-b\left(c^2+2d^2\right)\right)\cos(e+fx)}{2\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))} - \frac{\left(3bcd-a\right.}{}$$

$$= -\frac{(bc-ad)\cos(e+fx)}{2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{\left(3acd-b\left(c^2+2d^2\right)\right)\cos(e+fx)}{2\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))} + \frac{\left(2\left(3bcd\right.\right.}{}$$

$$= -\frac{\left(3bcd-a\left(2c^2+d^2\right)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{5/2}f} - \frac{(bc-ad)\cos(e+fx)}{2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} +$$

Mathematica [A] time = 0.62, size = 157, normalized size = 0.96

$$\frac{\frac{2\left(a\left(2c^2+d^2\right)-3bcd\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{5/2}} - \frac{\left(b\left(c^2+2d^2\right)-3acd\right)\cos(e+fx)}{(c-d)^2(c+d)^2(c+d\sin(e+fx))} + \frac{(ad-bc)\cos(e+fx)}{(c-d)(c+d)(c+d\sin(e+fx))^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((2*(-3*b*c*d + a*(2*c^2 + d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 -
 d^2]])/(c^2 - d^2)^(5/2) + ((-(b*c) + a*d)*Cos[e + f*x])/((c - d)*(c + d)*
(c + d*Sin[e + f*x])^2) - ((-3*a*c*d + b*(c^2 + 2*d^2))*Cos[e + f*x])/((c -
 d)^2*(c + d)^2*(c + d*Sin[e + f*x])))/(2*f)
```

fricas [B] time = 0.50, size = 793, normalized size = 4.84

$$\left[\frac{2\left(bc^4d-3\,ac^3d^2+bc^2d^3+3\,acd^4-2\,bd^5\right)\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(2\,ac^4-3\,bc^3d+3\,ac^2d^2-3\,bcd^3+ad^4-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b*c^4*d - 3*a*c^3*d^2 + b*c^2*d^3 + 3*a*c*d^4 - 2*b*d^5)*cos(f*x +
 e)*sin(f*x + e) + (2*a*c^4 - 3*b*c^3*d + 3*a*c^2*d^2 - 3*b*c*d^3 + a*d^4 -
 (2*a*c^2*d^2 - 3*b*c*d^3 + a*d^4)*cos(f*x + e)^2 + 2*(2*a*c^3*d - 3*b*c^2*
d^2 + a*c*d^3)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x +
e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*
cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) -
c^2 - d^2)) + 2*(2*b*c^5 - 4*a*c^4*d - b*c^3*d^2 + 5*a*c^2*d^3 - b*c*d^4 -
a*d^5)*cos(f*x + e))/((c^6*d^2 - 3*c^4*d^4 + 3*c^2*d^6 - d^8)*f*cos(f*x + e
)^2 - 2*(c^7*d - 3*c^5*d^3 + 3*c^3*d^5 - c*d^7)*f*sin(f*x + e) - (c^8 - 2*c
^6*d^2 + 2*c^2*d^6 - d^8)*f), 1/2*((b*c^4*d - 3*a*c^3*d^2 + b*c^2*d^3 + 3*a
*c*d^4 - 2*b*d^5)*cos(f*x + e)*sin(f*x + e) + (2*a*c^4 - 3*b*c^3*d + 3*a*c^
2*d^2 - 3*b*c*d^3 + a*d^4 - (2*a*c^2*d^2 - 3*b*c*d^3 + a*d^4)*cos(f*x + e)^
2 + 2*(2*a*c^3*d - 3*b*c^2*d^2 + a*c*d^3)*sin(f*x + e))*sqrt(c^2 - d^2)*arc
tan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (2*b*c^5 - 4*a*
c^4*d - b*c^3*d^2 + 5*a*c^2*d^3 - b*c*d^4 - a*d^5)*cos(f*x + e))/((c^6*d^2
- 3*c^4*d^4 + 3*c^2*d^6 - d^8)*f*cos(f*x + e)^2 - 2*(c^7*d - 3*c^5*d^3 + 3*
c^3*d^5 - c*d^7)*f*sin(f*x + e) - (c^8 - 2*c^6*d^2 + 2*c^2*d^6 - d^8)*f)]
```

giac [B] time = 0.26, size = 428, normalized size = 2.61

$$\frac{\frac{\left(2\,ac^2-3\,bcd+ad^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(c^4-2\,c^2d^2+d^4\right)\sqrt{c^2-d^2}}-\frac{3\,bc^4d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-5\,ac^3d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+2\,acd^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+2\,bc^5\,\mathrm{ta}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((2*a*c^2 - 3*b*c*d + a*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arc
tan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^4 - 2*c^2*d^2 + d^4)
*sqrt(c^2 - d^2)) - (3*b*c^4*d*tan(1/2*f*x + 1/2*e)^3 - 5*a*c^3*d^2*tan(1/2
*f*x + 1/2*e)^3 + 2*a*c*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*b*c^5*tan(1/2*f*x +
```

```
1/2*e)^2 - 4*a*c^4*d*tan(1/2*f*x + 1/2*e)^2 + 5*b*c^3*d^2*tan(1/2*f*x + 1/2
*e)^2 - 7*a*c^2*d^3*tan(1/2*f*x + 1/2*e)^2 + 2*b*c*d^4*tan(1/2*f*x + 1/2*e)
^2 + 2*a*d^5*tan(1/2*f*x + 1/2*e)^2 + 5*b*c^4*d*tan(1/2*f*x + 1/2*e) - 11*a
*c^3*d^2*tan(1/2*f*x + 1/2*e) + 4*b*c^2*d^3*tan(1/2*f*x + 1/2*e) + 2*a*c*d^
4*tan(1/2*f*x + 1/2*e) + 2*b*c^5 - 4*a*c^4*d + b*c^3*d^2 + a*c^2*d^3)/((c^6
 - 2*c^4*d^2 + c^2*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e
) + c)^2))/f
```

maple [B] time = 0.28, size = 1291, normalized size = 7.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

```
[Out] 5/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^2*c/(c^4-2*c^2*d^
2+d^4)*tan(1/2*f*x+1/2*e)^3*a-2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2
*e)*d+c)^2*d^4/c/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)^3*a-3/f/(tan(1/2*f*
x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*d*c^2/(c^4-2*c^2*d^2+d^4)*tan(1/2*
f*x+1/2*e)^3*b+4/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4
-2*c^2*d^2+d^4)*c^2*tan(1/2*f*x+1/2*e)^2*a*d+7/f/(tan(1/2*f*x+1/2*e)^2*c+2*
tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)^2*a*d^3-2/
f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)/c
^2*tan(1/2*f*x+1/2*e)^2*a*d^5-2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2
*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^3*tan(1/2*f*x+1/2*e)^2*b-5/f/(tan(1/2*f*x+
1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c*tan(1/2*f*x+1/
2*e)^2*b*d^2-2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2
*c^2*d^2+d^4)/c*tan(1/2*f*x+1/2*e)^2*b*d^4+11/f/(tan(1/2*f*x+1/2*e)^2*c+2*t
an(1/2*f*x+1/2*e)*d+c)^2*d^2*c/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*a-2/f
/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^4/c/(c^4-2*c^2*d^2+d
^4)*tan(1/2*f*x+1/2*e)*a-5/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d
+c)^2*d*c^2/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*b-4/f/(tan(1/2*f*x+1/2*e
)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^3/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e
)*b+4/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+
d^4)*c^2*d*a-1/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2
*c^2*d^2+d^4)*d^3*a-2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2
/(c^4-2*c^2*d^2+d^4)*b*c^3-1/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)
*d+c)^2/(c^4-2*c^2*d^2+d^4)*c*d^2*b+2/f/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)
*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c^2*a+1/f/(c^4-2*
c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d
^2)^(1/2))*a*d^2-3/f/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*ta
n(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b*c*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 9.61, size = 477, normalized size = 2.91

$$\frac{\operatorname{atan}\left(\frac{\left(\frac{\left(2c^4d-4c^2d^3+2d^5\right)\left(2ac^2-3bcd+ad^2\right)}{2(c+d)^{5/2}(c-d)^{5/2}\left(c^4-2c^2d^2+d^4\right)}+\frac{c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(2ac^2-3bcd+ad^2\right)}{(c+d)^{5/2}(c-d)^{5/2}}\right)\left(c^4-2c^2d^2+d^4\right)}{2ac^2-3bcd+ad^2}\right)\left(2ac^2-3bcd+ad^2\right)}{f(c+d)^{5/2}(c-d)^{5/2}}-\frac{\frac{2bc^3-4ac^2d+bcd^2}{c^4-2c^2d^2+d^4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))/(c + d*sin(e + f*x))^3,x)
```

```
[Out] (atan(((((2*c^4*d + 2*d^5 - 4*c^2*d^3)*(2*a*c^2 + a*d^2 - 3*b*c*d))/(2*(c +
 d)^(5/2)*(c - d)^(5/2)*(c^4 + d^4 - 2*c^2*d^2)) + (c*tan(e/2 + (f*x)/2)*(2
*a*c^2 + a*d^2 - 3*b*c*d))/((c + d)^(5/2)*(c - d)^(5/2)))*(c^4 + d^4 - 2*c^
2*d^2))/(2*a*c^2 + a*d^2 - 3*b*c*d))*(2*a*c^2 + a*d^2 - 3*b*c*d))/(f*(c + d
)^(5/2)*(c - d)^(5/2)) - ((a*d^3 + 2*b*c^3 - 4*a*c^2*d + b*c*d^2)/(c^4 + d^
4 - 2*c^2*d^2) + (d*tan(e/2 + (f*x)/2)^3*(2*a*d^3 + 3*b*c^3 - 5*a*c^2*d))/(
c*(c^4 + d^4 - 2*c^2*d^2)) + (d*tan(e/2 + (f*x)/2)*(2*a*d^3 + 5*b*c^3 - 11*
a*c^2*d + 4*b*c*d^2))/(c*(c^4 + d^4 - 2*c^2*d^2)) + (tan(e/2 + (f*x)/2)^2*(
c^2 + 2*d^2)*(a*d^3 + 2*b*c^3 - 4*a*c^2*d + b*c*d^2))/(c^2*(c^4 + d^4 - 2*c
^2*d^2)))/(f*(tan(e/2 + (f*x)/2)^2*(2*c^2 + 4*d^2) + c^2*tan(e/2 + (f*x)/2)
^4 + c^2 + 4*c*d*tan(e/2 + (f*x)/2)^3 + 4*c*d*tan(e/2 + (f*x)/2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.678 $\int (a+b\sin(e+fx))^2(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=314

$$-\frac{\left(20a^2d^2\left(4c^2+d^2\right)+30abcd\left(c^2+4d^2\right)-\left(b^2\left(3c^4-52c^2d^2-16d^4\right)\right)\right)\cos(e+fx)}{30df}-\frac{\left(100a^2cd^2+30abd\left(2c^2+\right.\right.}{}$$

```
[Out] 1/8*(6*a*b*d*(4*c^2+d^2)+b^2*c*(4*c^2+9*d^2)+4*a^2*(2*c^3+3*c*d^2))*x-1/30*
(20*a^2*d^2*(4*c^2+d^2)+30*a*b*c*d*(c^2+4*d^2)-b^2*(3*c^4-52*c^2*d^2-16*d^4
))*cos(f*x+e)/d/f-1/120*(100*a^2*c*d^2+30*a*b*d*(2*c^2+3*d^2)-b^2*(6*c^3-71
*c*d^2))*cos(f*x+e)*sin(f*x+e)/f-1/60*(4*(5*a^2+4*b^2)*d^2-3*b*c*(-10*a*d+b
*c))*cos(f*x+e)*(c+d*sin(f*x+e))^2/d/f+1/20*b*(-10*a*d+b*c)*cos(f*x+e)*(c+d
*sin(f*x+e))^3/d/f-1/5*b^2*cos(f*x+e)*(c+d*sin(f*x+e))^4/d/f
```

Rubi [A] time = 0.55, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2791, 2753, 2734}

$$-\frac{\left(20a^2d^2\left(4c^2+d^2\right)+30abcd\left(c^2+4d^2\right)+b^2\left(-\left(-52c^2d^2+3c^4-16d^4\right)\right)\right)\cos(e+fx)}{30df}-\frac{\left(100a^2cd^2+30abd\left(2c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((6*a*b*d*(4*c^2 + d^2) + b^2*c*(4*c^2 + 9*d^2) + 4*a^2*(2*c^3 + 3*c*d^2))*
x)/8 - ((20*a^2*d^2*(4*c^2 + d^2) + 30*a*b*c*d*(c^2 + 4*d^2) - b^2*(3*c^4 -
 52*c^2*d^2 - 16*d^4))*Cos[e + f*x])/(30*d*f) - ((100*a^2*c*d^2 + 30*a*b*d*
(2*c^2 + 3*d^2) - b^2*(6*c^3 - 71*c*d^2))*Cos[e + f*x]*Sin[e + f*x])/(120*f
) - ((4*(5*a^2 + 4*b^2)*d^2 - 3*b*c*(b*c - 10*a*d))*Cos[e + f*x]*(c + d*Sin
[e + f*x])^2)/(60*d*f) + (b*(b*c - 10*a*d)*Cos[e + f*x]*(c + d*Sin[e + f*x]
)^3)/(20*d*f) - (b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(5*d*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
```

```
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+b\sin(e+fx))^2(c+d\sin(e+fx))^3\,dx = -\frac{b^2\cos(e+fx)(c+d\sin(e+fx))^4}{5df}+\frac{\int (c+d\sin(e+fx))^3\left(\left(5a^2\right.\right.}{}$$

$$=\frac{b(bc-10ad)\cos(e+fx)(c+d\sin(e+fx))^3}{20df}-\frac{b^2\cos(e+fx)(c+}{5df}$$

$$=-\frac{\left(4\left(5a^2+4b^2\right)d^2-3bc(bc-10ad)\right)\cos(e+fx)(c+d\sin(e+fx}{60df}$$

$$=\frac{1}{8}\left(6abd\left(4c^2+d^2\right)+b^2c\left(4c^2+9d^2\right)+4a^2\left(2c^3+3cd^2\right)\right)x-\frac{(20a}{}$$

Mathematica [A] time = 1.35, size = 249, normalized size = 0.79

$$\frac{10d\left(4a^2d^2+24abcd+b^2\left(12c^2+5d^2\right)\right)\cos(3(e+fx))-60\left(6a^2\left(4c^2d+d^3\right)+4abc\left(4c^2+9d^2\right)+b^2d\left(18c^2+5d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (-60*(b^2*d*(18*c^2 + 5*d^2) + 4*a*b*c*(4*c^2 + 9*d^2) + 6*a^2*(4*c^2*d + d
^3))*Cos[e + f*x] + 10*d*(24*a*b*c*d + 4*a^2*d^2 + b^2*(12*c^2 + 5*d^2))*Co
s[3*(e + f*x)] - 6*b^2*d^3*Cos[5*(e + f*x)] + 15*(4*(6*a*b*d*(4*c^2 + d^2)
+ b^2*c*(4*c^2 + 9*d^2) + 4*a^2*(2*c^3 + 3*c*d^2))*(e + f*x) - 8*(3*a^2*c*d
^2 + 2*a*b*d*(3*c^2 + d^2) + b^2*(c^3 + 3*c*d^2))*Sin[2*(e + f*x)] + b*d^2*
(3*b*c + 2*a*d)*Sin[4*(e + f*x)]))/(480*f)
```

fricas [A] time = 0.49, size = 247, normalized size = 0.79

$$-\frac{24\,b^2d^3\cos\left(fx+e\right)^5-40\left(3\,b^2c^2d+6\,abcd^2+\left(a^2+2\,b^2\right)d^3\right)\cos\left(fx+e\right)^3-15\left(24\,abc^2d+6\,abd^3+4\left(2\,a^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/120*(24*b^2*d^3*cos(f*x + e)^5 - 40*(3*b^2*c^2*d + 6*a*b*c*d^2 + (a^2 +
2*b^2)*d^3)*cos(f*x + e)^3 - 15*(24*a*b*c^2*d + 6*a*b*d^3 + 4*(2*a^2 + b^2)
*c^3 + 3*(4*a^2 + 3*b^2)*c*d^2)*f*x + 120*(2*a*b*c^3 + 6*a*b*c*d^2 + 3*(a^2
 + b^2)*c^2*d + (a^2 + b^2)*d^3)*cos(f*x + e) - 15*(2*(3*b^2*c*d^2 + 2*a*b*
d^3)*cos(f*x + e)^3 - (4*b^2*c^3 + 24*a*b*c^2*d + 10*a*b*d^3 + 3*(4*a^2 + 5
*b^2)*c*d^2)*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.25, size = 274, normalized size = 0.87

$$-\frac{b^2d^3\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{1}{8}\left(8\,a^2c^3+4\,b^2c^3+24\,abc^2d+12\,a^2cd^2+9\,b^2cd^2+6\,abd^3\right)x+\frac{\left(12\,b^2c^2d+24\,abcd^2+4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/80*b^2*d^3*cos(5*f*x + 5*e)/f + 1/8*(8*a^2*c^3 + 4*b^2*c^3 + 24*a*b*c^2*
d + 12*a^2*c*d^2 + 9*b^2*c*d^2 + 6*a*b*d^3)*x + 1/48*(12*b^2*c^2*d + 24*a*b
*c*d^2 + 4*a^2*d^3 + 5*b^2*d^3)*cos(3*f*x + 3*e)/f - 1/8*(16*a*b*c^3 + 24*a
^2*c^2*d + 18*b^2*c^2*d + 36*a*b*c*d^2 + 6*a^2*d^3 + 5*b^2*d^3)*cos(f*x + e
)/f + 1/32*(3*b^2*c*d^2 + 2*a*b*d^3)*sin(4*f*x + 4*e)/f - 1/4*(b^2*c^3 + 6*
a*b*c^2*d + 3*a^2*c*d^2 + 3*b^2*c*d^2 + 2*a*b*d^3)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.31, size = 325, normalized size = 1.04

$$\frac{a^2c^3\left(fx+e\right)-3a^2c^2d\cos\left(fx+e\right)+3a^2c\,d^2\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{a^2d^3\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}-2ab\,c^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(a^2*c^3*(f*x+e)-3*a^2*c^2*d*cos(f*x+e)+3*a^2*c*d^2*(-1/2*sin(f*x+e)*co
s(f*x+e)+1/2*f*x+1/2*e)-1/3*a^2*d^3*(2+sin(f*x+e)^2)*cos(f*x+e)-2*a*b*c^3*c
```

```
os(f*x+e)+6*a*b*c^2*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2*a*b*c*d^
2*(2+sin(f*x+e)^2)*cos(f*x+e)+2*a*b*d^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))
*cos(f*x+e)+3/8*f*x+3/8*e)+b^2*c^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*
e)-b^2*c^2*d*(2+sin(f*x+e)^2)*cos(f*x+e)+3*b^2*c*d^2*(-1/4*(sin(f*x+e)^3+3/
2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/5*b^2*d^3*(8/3+sin(f*x+e)^4+4/3*s
in(f*x+e)^2)*cos(f*x+e))
```

maxima [A] time = 0.85, size = 314, normalized size = 1.00

$$\frac{480\left(fx+e\right)a^2c^3+120\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)b^2c^3+720\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)abc^2d+480\left(\cos\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/480*(480*(f*x + e)*a^2*c^3 + 120*(2*f*x + 2*e - sin(2*f*x + 2*e))*b^2*c^3
 + 720*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*b*c^2*d + 480*(cos(f*x + e)^3 - 3
*cos(f*x + e))*b^2*c^2*d + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*c*d^2 +
 960*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*b*c*d^2 + 45*(12*f*x + 12*e + sin(
4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*b^2*c*d^2 + 160*(cos(f*x + e)^3 - 3*cos(
f*x + e))*a^2*d^3 + 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*
e))*a*b*d^3 - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*b
^2*d^3 - 960*a*b*c^3*cos(f*x + e) - 1440*a^2*c^2*d*cos(f*x + e))/f
```

mupad [B] time = 8.44, size = 358, normalized size = 1.14

$$-\frac{90\,a^2\,d^3\,\cos\left(e+f\,x\right)+75\,b^2\,d^3\,\cos\left(e+f\,x\right)-10\,a^2\,d^3\,\cos\left(3\,e+3\,f\,x\right)-\frac{25\,b^2\,d^3\,\cos\left(3\,e+3\,f\,x\right)}{2}+\frac{3\,b^2\,d^3\,\cos\left(5\,e+5\,f\,x\right.}{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^3,x)
```

```
[Out] -(90*a^2*d^3*cos(e + f*x) + 75*b^2*d^3*cos(e + f*x) - 10*a^2*d^3*cos(3*e +
3*f*x) - (25*b^2*d^3*cos(3*e + 3*f*x))/2 + (3*b^2*d^3*cos(5*e + 5*f*x))/2 +
 30*b^2*c^3*sin(2*e + 2*f*x) - 30*b^2*c^2*d*cos(3*e + 3*f*x) + 90*a^2*c*d^2
*sin(2*e + 2*f*x) + 90*b^2*c*d^2*sin(2*e + 2*f*x) - (45*b^2*c*d^2*sin(4*e +
 4*f*x))/4 + 240*a*b*c^3*cos(e + f*x) + 360*a^2*c^2*d*cos(e + f*x) + 270*b^
2*c^2*d*cos(e + f*x) + 60*a*b*d^3*sin(2*e + 2*f*x) - (15*a*b*d^3*sin(4*e +
4*f*x))/2 - 120*a^2*c^3*f*x - 60*b^2*c^3*f*x - 60*a*b*c*d^2*cos(3*e + 3*f*x
) + 180*a*b*c^2*d*sin(2*e + 2*f*x) - 180*a^2*c*d^2*f*x - 135*b^2*c*d^2*f*x
+ 540*a*b*c*d^2*cos(e + f*x) - 90*a*b*d^3*f*x - 360*a*b*c^2*d*f*x)/(120*f)
```

sympy [A] time = 4.33, size = 729, normalized size = 2.32

$$\begin{cases} a^2c^3x - \frac{3a^2c^2d\cos(e+fx)}{f} + \frac{3a^2cd^2x\sin^2(e+fx)}{2} + \frac{3a^2cd^2x\cos^2(e+fx)}{2} - \frac{3a^2cd^2\sin(e+fx)\cos(e+fx)}{2f} - \frac{a^2d^3\sin^2(e+fx)\cos(e+fx)}{f} - \\ x(a+b\sin(e))^2(c+d\sin(e))^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((a**2*c**3*x - 3*a**2*c**2*d*cos(e + f*x)/f + 3*a**2*c*d**2*x*sin
(e + f*x)**2/2 + 3*a**2*c*d**2*x*cos(e + f*x)**2/2 - 3*a**2*c*d**2*sin(e +
f*x)*cos(e + f*x)/(2*f) - a**2*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 2*a**2
*d**3*cos(e + f*x)**3/(3*f) - 2*a*b*c**3*cos(e + f*x)/f + 3*a*b*c**2*d*x*si
n(e + f*x)**2 + 3*a*b*c**2*d*x*cos(e + f*x)**2 - 3*a*b*c**2*d*sin(e + f*x)*
cos(e + f*x)/f - 6*a*b*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 4*a*b*c*d**2
*cos(e + f*x)**3/f + 3*a*b*d**3*x*sin(e + f*x)**4/4 + 3*a*b*d**3*x*sin(e +
f*x)**2*cos(e + f*x)**2/2 + 3*a*b*d**3*x*cos(e + f*x)**4/4 - 5*a*b*d**3*sin
(e + f*x)**3*cos(e + f*x)/(4*f) - 3*a*b*d**3*sin(e + f*x)*cos(e + f*x)**3/(
4*f) + b**2*c**3*x*sin(e + f*x)**2/2 + b**2*c**3*x*cos(e + f*x)**2/2 - b**2
*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 3*b**2*c**2*d*sin(e + f*x)**2*cos(e
 + f*x)/f - 2*b**2*c**2*d*cos(e + f*x)**3/f + 9*b**2*c*d**2*x*sin(e + f*x)*
*4/8 + 9*b**2*c*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 9*b**2*c*d**2*x*
cos(e + f*x)**4/8 - 15*b**2*c*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 9*b
**2*c*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) - b**2*d**3*sin(e + f*x)**4*c
os(e + f*x)/f - 4*b**2*d**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 8*b**2*
d**3*cos(e + f*x)**5/(15*f), Ne(f, 0)), (x*(a + b*sin(e))**2*(c + d*sin(e))
**3, True))
```

3.679 $\int (a+b\sin(e+fx))^2(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=217

$$\frac{1}{8}x\left(4a^2\left(2c^2+d^2\right)+16abcd+b^2\left(4c^2+3d^2\right)\right)-\frac{\left(a^3\left(-d^2\right)+8a^2bcd+4ab^2\left(3c^2+2d^2\right)+8b^3cd\right)\cos(e+fx)}{6bf}-\frac{(2a$$

```
[Out] 1/8*(16*a*b*c*d+4*a^2*(2*c^2+d^2)+b^2*(4*c^2+3*d^2))*x-1/6*(8*a^2*b*c*d+8*b
^3*c*d-a^3*d^2+4*a*b^2*(3*c^2+2*d^2))*cos(f*x+e)/b/f-1/24*(2*a*d*(-a*d+8*b*
c)+3*b^2*(4*c^2+3*d^2))*cos(f*x+e)*sin(f*x+e)/f-1/12*d*(-a*d+8*b*c)*cos(f*x
+e)*(a+b*sin(f*x+e))^2/b/f-1/4*d^2*cos(f*x+e)*(a+b*sin(f*x+e))^3/b/f
```

Rubi [A] time = 0.28, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2791, 2753, 2734}

$$-\frac{\left(8a^2bcd+a^3\left(-d^2\right)+4ab^2\left(3c^2+2d^2\right)+8b^3cd\right)\cos(e+fx)}{6bf}+\frac{1}{8}x\left(4a^2\left(2c^2+d^2\right)+16abcd+b^2\left(4c^2+3d^2\right)\right)-\frac{(2$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] ((16*a*b*c*d + 4*a^2*(2*c^2 + d^2) + b^2*(4*c^2 + 3*d^2))*x)/8 - ((8*a^2*b*
c*d + 8*b^3*c*d - a^3*d^2 + 4*a*b^2*(3*c^2 + 2*d^2))*Cos[e + f*x])/(6*b*f)
- ((2*a*d*(8*b*c - a*d) + 3*b^2*(4*c^2 + 3*d^2))*Cos[e + f*x]*Sin[e + f*x])
/(24*f) - (d*(8*b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^2)/(12*b*f) -
(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^3)/(4*b*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+b\sin(e+fx))^2(c+d\sin(e+fx))^2\,dx = -\frac{d^2\cos(e+fx)(a+b\sin(e+fx))^3}{4bf} + \frac{\int (a+b\sin(e+fx))^2\left(b\left(\right.\right.}{}$$

$$= -\frac{d(8bc-ad)\cos(e+fx)(a+b\sin(e+fx))^2}{12bf} - \frac{d^2\cos(e+fx)(a}{4}$$

$$= \frac{1}{8}\left(16abcd+4a^2\left(2c^2+d^2\right)+b^2\left(4c^2+3d^2\right)\right)x - \frac{\left(8a^2bcd+8b^3\right.}{}$$

Mathematica [A] time = 0.78, size = 160, normalized size = 0.74

$$\frac{3\left(4(e+fx)\left(4a^2\left(2c^2+d^2\right)+16abcd+b^2\left(4c^2+3d^2\right)\right)-8\left(a^2d^2+4abcd+b^2\left(c^2+d^2\right)\right)\sin(2(e+fx))+b^2d^2\right.}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (-48*(4*a^2*c*d + 3*b^2*c*d + a*b*(4*c^2 + 3*d^2))*Cos[e + f*x] + 16*b*d*(b
*c + a*d)*Cos[3*(e + f*x)] + 3*(4*(16*a*b*c*d + 4*a^2*(2*c^2 + d^2) + b^2*(
4*c^2 + 3*d^2))*(e + f*x) - 8*(4*a*b*c*d + a^2*d^2 + b^2*(c^2 + d^2))*Sin[2
*(e + f*x)] + b^2*d^2*Sin[4*(e + f*x)]))/(96*f)
```

fricas [A] time = 0.44, size = 163, normalized size = 0.75

$$\frac{16\left(b^2cd+abd^2\right)\cos\left(fx+e\right)^3+3\left(16\,abcd+4\left(2\,a^2+b^2\right)c^2+\left(4\,a^2+3\,b^2\right)d^2\right)fx-48\left(abc^2+abd^2+\left(a^2+b^2\right.\right.}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(16*(b^2*c*d + a*b*d^2)*cos(f*x + e)^3 + 3*(16*a*b*c*d + 4*(2*a^2 + b^
2)*c^2 + (4*a^2 + 3*b^2)*d^2)*f*x - 48*(a*b*c^2 + a*b*d^2 + (a^2 + b^2)*c*d
)*cos(f*x + e) + 3*(2*b^2*d^2*cos(f*x + e)^3 - (4*b^2*c^2 + 16*a*b*c*d + (4
*a^2 + 5*b^2)*d^2)*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.21, size = 176, normalized size = 0.81

$$\frac{b^2d^2\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{1}{8}\left(8\,a^2c^2+4\,b^2c^2+16\,abcd+4\,a^2d^2+3\,b^2d^2\right)x+\frac{\left(b^2cd+abd^2\right)\cos\left(3\,fx+3\,e\right)}{6\,f}-\frac{\left(4\,abc^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/32*b^2*d^2*sin(4*f*x + 4*e)/f + 1/8*(8*a^2*c^2 + 4*b^2*c^2 + 16*a*b*c*d +
 4*a^2*d^2 + 3*b^2*d^2)*x + 1/6*(b^2*c*d + a*b*d^2)*cos(3*f*x + 3*e)/f - 1/
2*(4*a*b*c^2 + 4*a^2*c*d + 3*b^2*c*d + 3*a*b*d^2)*cos(f*x + e)/f - 1/4*(b^2
*c^2 + 4*a*b*c*d + a^2*d^2 + b^2*d^2)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.26, size = 216, normalized size = 1.00

$$\frac{a^2c^2\left(fx+e\right)-2a^2cd\cos\left(fx+e\right)+a^2d^2\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-2ab\,c^2\cos\left(fx+e\right)+4abcd\left(-\frac{\sin\left(fx+e\right)}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(a^2*c^2*(f*x+e)-2*a^2*c*d*cos(f*x+e)+a^2*d^2*(-1/2*sin(f*x+e)*cos(f*x+
e)+1/2*f*x+1/2*e)-2*a*b*c^2*cos(f*x+e)+4*a*b*c*d*(-1/2*sin(f*x+e)*cos(f*x+e
)+1/2*f*x+1/2*e)-2/3*a*b*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+b^2*c^2*(-1/2*sin(
f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2/3*b^2*c*d*(2+sin(f*x+e)^2)*cos(f*x+e)+b^
2*d^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e))
```

maxima [A] time = 0.75, size = 208, normalized size = 0.96

$$\frac{96\left(fx+e\right)a^2c^2+24\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)b^2c^2+96\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)abcd+64\left(\cos\left(fx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/96*(96*(f*x + e)*a^2*c^2 + 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*b^2*c^2 +
96*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*b*c*d + 64*(cos(f*x + e)^3 - 3*cos(f*
x + e))*b^2*c*d + 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^2*d^2 + 64*(cos(f*x
 + e)^3 - 3*cos(f*x + e))*a*b*d^2 + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8
*sin(2*f*x + 2*e))*b^2*d^2 - 192*a*b*c^2*cos(f*x + e) - 192*a^2*c*d*cos(f*x
 + e))/f
```

mupad [B] time = 8.14, size = 221, normalized size = 1.02

$$-\frac{6\,a^2\,d^2\,\sin\left(2\,e+2\,f\,x\right)+6\,b^2\,c^2\,\sin\left(2\,e+2\,f\,x\right)+6\,b^2\,d^2\,\sin\left(2\,e+2\,f\,x\right)-\frac{3\,b^2\,d^2\,\sin\left(4\,e+4\,f\,x\right)}{4}+48\,a\,b\,c^2\,\cos\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^2,x)
```

```
[Out] -(6*a^2*d^2*sin(2*e + 2*f*x) + 6*b^2*c^2*sin(2*e + 2*f*x) + 6*b^2*d^2*sin(2
*e + 2*f*x) - (3*b^2*d^2*sin(4*e + 4*f*x))/4 + 48*a*b*c^2*cos(e + f*x) + 36
*a*b*d^2*cos(e + f*x) + 48*a^2*c*d*cos(e + f*x) + 36*b^2*c*d*cos(e + f*x) -
 4*a*b*d^2*cos(3*e + 3*f*x) - 4*b^2*c*d*cos(3*e + 3*f*x) - 24*a^2*c^2*f*x -
 12*a^2*d^2*f*x - 12*b^2*c^2*f*x - 9*b^2*d^2*f*x + 24*a*b*c*d*sin(2*e + 2*f
*x) - 48*a*b*c*d*f*x)/(24*f)
```

sympy [A] time = 1.94, size = 459, normalized size = 2.12

$$\begin{cases} a^2c^2x - \frac{2a^2cd\cos\left(e+fx\right)}{f} + \frac{a^2d^2x\sin^2\left(e+fx\right)}{2} + \frac{a^2d^2x\cos^2\left(e+fx\right)}{2} - \frac{a^2d^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f} - \frac{2abc^2\cos\left(e+fx\right)}{f} + 2abcdx\sin^2\left(\right. \\ x\left(a+b\sin\left(e\right)\right)^2\left(c+d\sin\left(e\right)\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c**2*x - 2*a**2*c*d*cos(e + f*x)/f + a**2*d**2*x*sin(e + f*
x)**2/2 + a**2*d**2*x*cos(e + f*x)**2/2 - a**2*d**2*sin(e + f*x)*cos(e + f*
x)/(2*f) - 2*a*b*c**2*cos(e + f*x)/f + 2*a*b*c*d*x*sin(e + f*x)**2 + 2*a*b*
c*d*x*cos(e + f*x)**2 - 2*a*b*c*d*sin(e + f*x)*cos(e + f*x)/f - 2*a*b*d**2*
sin(e + f*x)**2*cos(e + f*x)/f - 4*a*b*d**2*cos(e + f*x)**3/(3*f) + b**2*c*
*2*x*sin(e + f*x)**2/2 + b**2*c**2*x*cos(e + f*x)**2/2 - b**2*c**2*sin(e +
f*x)*cos(e + f*x)/(2*f) - 2*b**2*c*d*sin(e + f*x)**2*cos(e + f*x)/f - 4*b**
2*c*d*cos(e + f*x)**3/(3*f) + 3*b**2*d**2*x*sin(e + f*x)**4/8 + 3*b**2*d**2
*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*b**2*d**2*x*cos(e + f*x)**4/8 - 5*
b**2*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*b**2*d**2*sin(e + f*x)*cos
(e + f*x)**3/(8*f), Ne(f, 0)), (x*(a + b*sin(e))**2*(c + d*sin(e))**2, True
))
```

3.680 $\int (a+b\sin(e+fx))^2(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=107

$$-\frac{2\left(a^2d+3abc+b^2d\right)\cos(e+fx)}{3f}+\frac{1}{2}x\left(2a^2c+2abd+b^2c\right)-\frac{b(2ad+3bc)\sin(e+fx)\cos(e+fx)}{6f}-\frac{d\cos(e+fx)}{}$$

```
[Out] 1/2*(2*a^2*c+2*a*b*d+b^2*c)*x-2/3*(a^2*d+3*a*b*c+b^2*d)*cos(f*x+e)/f-1/6*b*
(2*a*d+3*b*c)*cos(f*x+e)*sin(f*x+e)/f-1/3*d*cos(f*x+e)*(a+b*sin(f*x+e))^2/f
```

Rubi [A] time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2753, 2734}

$$-\frac{2\left(a^2d+3abc+b^2d\right)\cos(e+fx)}{3f}+\frac{1}{2}x\left(2a^2c+2abd+b^2c\right)-\frac{b(2ad+3bc)\sin(e+fx)\cos(e+fx)}{6f}-\frac{d\cos(e+fx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x]),x]
```

```
[Out] ((2*a^2*c + b^2*c + 2*a*b*d)*x)/2 - (2*(3*a*b*c + a^2*d + b^2*d)*Cos[e + f*
x])/(3*f) - (b*(3*b*c + 2*a*d)*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (d*Cos[e
+ f*x]*(a + b*Sin[e + f*x])^2)/(3*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\sin(e+fx))^2(c+d\sin(e+fx))\,dx = -\frac{d\cos(e+fx)(a+b\sin(e+fx))^2}{3f} + \frac{1}{3}\int (a+b\sin(e+fx))(3ac$$

$$= \frac{1}{2}\left(2a^2c+b^2c+2abd\right)x - \frac{2\left(3abc+a^2d+b^2d\right)\cos(e+fx)}{3f} - \frac{b(3}{}$$

Mathematica [A] time = 0.30, size = 90, normalized size = 0.84

$$\frac{6(e+fx)\left(2a^2c+2abd+b^2c\right)-3\left(4a^2d+8abc+3b^2d\right)\cos(e+fx)-3b(2ad+bc)\sin(2(e+fx))+b^2d\cos(3(e}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x]),x]
```

```
[Out] (6*(2*a^2*c + b^2*c + 2*a*b*d)*(e + f*x) - 3*(8*a*b*c + 4*a^2*d + 3*b^2*d)*
Cos[e + f*x] + b^2*d*Cos[3*(e + f*x)] - 3*b*(b*c + 2*a*d)*Sin[2*(e + f*x)])
/(12*f)
```

fricas [A] time = 0.45, size = 89, normalized size = 0.83

$$\frac{2\,b^2d\cos\left(fx+e\right)^3+3\left(2\,abd+\left(2\,a^2+b^2\right)c\right)fx-3\left(b^2c+2\,abd\right)\cos\left(fx+e\right)\sin\left(fx+e\right)-6\left(2\,abc+\left(a^2+b^2\right.\right.}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^2*d*cos(f*x + e)^3 + 3*(2*a*b*d + (2*a^2 + b^2)*c)*f*x - 3*(b^2*c
+ 2*a*b*d)*cos(f*x + e)*sin(f*x + e) - 6*(2*a*b*c + (a^2 + b^2)*d)*cos(f*x
+ e))/f
```

giac [A] time = 1.37, size = 96, normalized size = 0.90

$$\frac{b^2d\cos\left(3\,fx+3\,e\right)}{12\,f}+\frac{1}{2}\left(2\,a^2c+b^2c+2\,abd\right)x-\frac{\left(8\,abc+4\,a^2d+3\,b^2d\right)\cos\left(fx+e\right)}{4\,f}-\frac{\left(b^2c+2\,abd\right)\sin\left(2\,fx+2\right.}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/12*b^2*d*cos(3*f*x + 3*e)/f + 1/2*(2*a^2*c + b^2*c + 2*a*b*d)*x - 1/4*(8*
a*b*c + 4*a^2*d + 3*b^2*d)*cos(f*x + e)/f - 1/4*(b^2*c + 2*a*b*d)*sin(2*f*x
 + 2*e)/f
```

maple [A] time = 0.20, size = 115, normalized size = 1.07

$$\frac{a^2c\left(fx+e\right)-a^2d\cos\left(fx+e\right)-2abc\cos\left(fx+e\right)+2abd\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b^2c\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e)),x)
```

```
[Out] 1/f*(a^2*c*(f*x+e)-a^2*d*cos(f*x+e)-2*a*b*c*cos(f*x+e)+2*a*b*d*(-1/2*sin(f*
x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+b^2*c*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/
2*e)-1/3*b^2*d*(2+sin(f*x+e)^2)*cos(f*x+e))
```

maxima [A] time = 0.44, size = 112, normalized size = 1.05

$$\frac{12\left(fx+e\right)a^2c+3\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)b^2c+6\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)abd+4\left(\cos\left(fx+e\right)^3-3\right.}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/12*(12*(f*x + e)*a^2*c + 3*(2*f*x + 2*e - sin(2*f*x + 2*e))*b^2*c + 6*(2*
f*x + 2*e - sin(2*f*x + 2*e))*a*b*d + 4*(cos(f*x + e)^3 - 3*cos(f*x + e))*b
^2*d - 24*a*b*c*cos(f*x + e) - 12*a^2*d*cos(f*x + e))/f
```

mupad [B] time = 7.77, size = 108, normalized size = 1.01

$$-\frac{\frac{3\,b^2\,c\,\sin\left(2\,e+2\,f\,x\right)}{2}-\frac{b^2\,d\,\cos\left(3\,e+3\,f\,x\right)}{2}+6\,a^2\,d\,\cos\left(e+f\,x\right)+\frac{9\,b^2\,d\,\cos\left(e+f\,x\right)}{2}+3\,a\,b\,d\,\sin\left(2\,e+2\,f\,x\right)-6\,a^2\,c\,f\,x-3}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x)),x)
```

```
[Out] -((3*b^2*c*sin(2*e + 2*f*x))/2 - (b^2*d*cos(3*e + 3*f*x))/2 + 6*a^2*d*cos(e
 + f*x) + (9*b^2*d*cos(e + f*x))/2 + 3*a*b*d*sin(2*e + 2*f*x) - 6*a^2*c*f*x
 - 3*b^2*c*f*x + 12*a*b*c*cos(e + f*x) - 6*a*b*d*f*x)/(6*f)
```

sympy [A] time = 0.83, size = 199, normalized size = 1.86

$$\begin{cases} a^2cx-\frac{a^2d\cos\left(e+fx\right)}{f}-\frac{2abc\cos\left(e+fx\right)}{f}+abdx\sin^2\left(e+fx\right)+abdx\cos^2\left(e+fx\right)-\frac{abd\sin\left(e+fx\right)\cos\left(e+fx\right)}{f}+\frac{b^2cx\sin^2\left(e+\right.}{2} \\ x\left(a+b\sin\left(e\right)\right)^2\left(c+d\sin\left(e\right)\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((a**2*c*x - a**2*d*cos(e + f*x)/f - 2*a*b*c*cos(e + f*x)/f + a*b*
d*x*sin(e + f*x)**2 + a*b*d*x*cos(e + f*x)**2 - a*b*d*sin(e + f*x)*cos(e +
f*x)/f + b**2*c*x*sin(e + f*x)**2/2 + b**2*c*x*cos(e + f*x)**2/2 - b**2*c*s
in(e + f*x)*cos(e + f*x)/(2*f) - b**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 2*
b**2*d*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(a + b*sin(e))**2*(c + d*sin(e)
), True))
```

3.681 $\int (a+b\sin(e+fx))^2\,dx$

Optimal. Leaf size=50

$$\frac{1}{2}x\left(2a^2+b^2\right)-\frac{2ab\cos(e+fx)}{f}-\frac{b^2\sin(e+fx)\cos(e+fx)}{2f}$$

```
[Out] 1/2*(2*a^2+b^2)*x-2*a*b*cos(f*x+e)/f-1/2*b^2*cos(f*x+e)*sin(f*x+e)/f
```

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2644}

$$\frac{1}{2}x\left(2a^2+b^2\right)-\frac{2ab\cos(e+fx)}{f}-\frac{b^2\sin(e+fx)\cos(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x)/2 - (2*a*b*Cos[e + f*x])/f - (b^2*Cos[e + f*x]*Sin[e + f*
x])/(2*f)
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a+b\sin(e+fx))^2\,dx=\frac{1}{2}\left(2a^2+b^2\right)x-\frac{2ab\cos(e+fx)}{f}-\frac{b^2\cos(e+fx)\sin(e+fx)}{2f}$$

Mathematica [A] time = 0.10, size = 46, normalized size = 0.92

$$-\frac{-2\left(2a^2+b^2\right)(e+fx)+8ab\cos(e+fx)+b^2\sin(2(e+fx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -1/4*(-2*(2*a^2 + b^2)*(e + f*x) + 8*a*b*Cos[e + f*x] + b^2*Sin[2*(e + f*x)
])/f
```

fricas [A] time = 0.46, size = 45, normalized size = 0.90

$$-\frac{b^2 \cos\left(fx+e\right)\sin\left(fx+e\right)-\left(2\,a^2+b^2\right)fx+4\,ab\cos\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*cos(f*x + e)*sin(f*x + e) - (2*a^2 + b^2)*f*x + 4*a*b*cos(f*x + e
))/f
```

giac [A] time = 0.16, size = 45, normalized size = 0.90

$$\frac{1}{2}\left(2\,a^2+b^2\right)x-\frac{2\,ab\cos\left(fx+e\right)}{f}-\frac{b^2\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2 + b^2)*x - 2*a*b*cos(f*x + e)/f - 1/4*b^2*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.10, size = 51, normalized size = 1.02

$$\frac{b^2\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-2ab\cos\left(fx+e\right)+a^2\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(b^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2*a*b*cos(f*x+e)+a^2*(f
*x+e))
```

maxima [A] time = 0.63, size = 46, normalized size = 0.92

$$a^2x+\frac{\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)b^2}{4\,f}-\frac{2\,ab\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/4*(2*f*x + 2*e - sin(2*f*x + 2*e))*b^2/f - 2*a*b*cos(f*x + e)/f
```

mupad [B] time = 7.61, size = 44, normalized size = 0.88

$$-\frac{\frac{b^2\,\sin\left(2\,e+2\,f\,x\right)}{2}+4\,a\,b\,\cos\left(e+f\,x\right)-2\,a^2\,f\,x-b^2\,f\,x}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2,x)
```

```
[Out] -((b^2*sin(2*e + 2*f*x))/2 + 4*a*b*cos(e + f*x) - 2*a^2*f*x - b^2*f*x)/(2*f
)
```

sympy [A] time = 0.31, size = 78, normalized size = 1.56

$$\begin{cases} a^2x - \frac{2ab\cos\left(e+fx\right)}{f} + \frac{b^2x\sin^2\left(e+fx\right)}{2} + \frac{b^2x\cos^2\left(e+fx\right)}{2} - \frac{b^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f} & \text{for}\ f \neq 0 \\ x\left(a+b\sin\left(e\right)\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*x - 2*a*b*cos(e + f*x)/f + b**2*x*sin(e + f*x)**2/2 + b**2*
x*cos(e + f*x)**2/2 - b**2*sin(e + f*x)*cos(e + f*x)/(2*f), Ne(f, 0)), (x*(
a + b*sin(e))**2, True))
```

3.682 $\int \frac{(a+b\sin(e+fx))^2}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=93

$$\frac{2(bc-ad)^2\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f\sqrt{c^2-d^2}}-\frac{bx(bc-2ad)}{d^2}-\frac{b^2\cos(e+fx)}{df}$$

```
[Out] -b*(-2*a*d+b*c)*x/d^2-b^2*cos(f*x+e)/d/f+2*(-a*d+b*c)^2*arctan((d+c*tan(1/2
*f*x+1/2*e))/(c^2-d^2)^(1/2))/d^2/f/(c^2-d^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2746, 2735, 2660, 618, 204}

$$\frac{2(bc-ad)^2\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f\sqrt{c^2-d^2}}-\frac{bx(bc-2ad)}{d^2}-\frac{b^2\cos(e+fx)}{df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x]),x]
```

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (2*(b*c - a*d)^2*ArcTan[(d + c*Tan[(e + f*x)/2
])/Sqrt[c^2 - d^2]])/(d^2*Sqrt[c^2 - d^2]*f) - (b^2*Cos[e + f*x])/(d*f)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b\sin(e+fx))^2}{c+d\sin(e+fx)}\,dx &= -\frac{b^2\cos(e+fx)}{df} + \frac{\int \frac{a^2d-b(bc-2ad)\sin(e+fx)}{c+d\sin(e+fx)}\,dx}{d} \\
&= -\frac{b(bc-2ad)x}{d^2} - \frac{b^2\cos(e+fx)}{df} + \frac{(bc-ad)^2\int \frac{1}{c+d\sin(e+fx)}\,dx}{d^2} \\
&= -\frac{b(bc-2ad)x}{d^2} - \frac{b^2\cos(e+fx)}{df} + \frac{\left(2(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx, x, \tan\left(\frac{1}{2}(e+\right.\right.}{d^2f} \\
&= -\frac{b(bc-2ad)x}{d^2} - \frac{b^2\cos(e+fx)}{df} - \frac{\left(4(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{-4\left(c^2-d^2\right)-x^2}\,dx, x, 2d+2c\,\text{t}\right.}{d^2f} \\
&= -\frac{b(bc-2ad)x}{d^2} + \frac{2(bc-ad)^2\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^2\sqrt{c^2-d^2}\,f} - \frac{b^2\cos(e+fx)}{df}
\end{aligned}$$

Mathematica [A] time = 0.21, size = 89, normalized size = 0.96

$$-\frac{-\frac{2(bc-ad)^2\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + b(e+fx)(bc-2ad) + b^2d\cos(e+fx)}{d^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x]),x]
```

```
[Out] -((b*(b*c - 2*a*d)*(e + f*x) - (2*(b*c - a*d)^2*ArcTan[(d + c*Tan[(e + f*x)
/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2] + b^2*d*Cos[e + f*x])/(d^2*f))
```

fricas [A] time = 0.49, size = 375, normalized size = 4.03

$$\left[-\frac{2\left(b^2c^3-2\,abc^2d-b^2cd^2+2\,abd^3\right)fx+\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-c^2+d^2}\,\log\left(\frac{\left(2\,c^2-d^2\right)\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-\cdots}{d^2\cos\left(fx\cdots\right.}\right.}{2\left(c^2d^2-d^4\right)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(b^2*c^3 - 2*a*b*c^2*d - b^2*c*d^2 + 2*a*b*d^3)*f*x + (b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*
c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x +
 e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2
)) + 2*(b^2*c^2*d - b^2*d^3)*cos(f*x + e))/((c^2*d^2 - d^4)*f), -((b^2*c^3
- 2*a*b*c^2*d - b^2*c*d^2 + 2*a*b*d^3)*f*x + (b^2*c^2 - 2*a*b*c*d + a^2*d^2
)*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e
))) + (b^2*c^2*d - b^2*d^3)*cos(f*x + e))/((c^2*d^2 - d^4)*f)]
```

giac [A] time = 0.34, size = 134, normalized size = 1.44

$$-\frac{\frac{\left(b^2c-2\,abd\right)\left(fx+e\right)}{d^2}+\frac{2\,b^2}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)d}-\frac{2\left(b^2c^2-2\,abcd+a^2d^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(c\right)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\sqrt{c^2-d^2}\,d^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -((b^2*c - 2*a*b*d)*(f*x + e)/d^2 + 2*b^2/((tan(1/2*f*x + 1/2*e)^2 + 1)*d)
- 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c
) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*
d^2))/f
```

maple [B] time = 0.20, size = 226, normalized size = 2.43

$$\frac{2a^2\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{f\sqrt{c^2-d^2}}-\frac{4\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)abc}{fd\sqrt{c^2-d^2}}+\frac{2\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)b^2c^2}{f\,d^2\sqrt{c^2-d^2}}-\frac{2b^2}{fd\left(1+\tan^2\left(\frac{fx}{2}+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e)),x)
```

```
[Out] 2/f*a^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(
1/2))-4/f/d/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^
2)^(1/2))*a*b*c+2/f/d^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+
2*d)/(c^2-d^2)^(1/2))*b^2*c^2-2/f*b^2/d/(1+tan(1/2*f*x+1/2*e)^2)+4/f*b/d*ar
ctan(tan(1/2*f*x+1/2*e))*a-2/f*b^2/d^2*arctan(tan(1/2*f*x+1/2*e))*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 12.43, size = 2629, normalized size = 28.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/(c + d*sin(e + f*x)),x)
```

```
[Out] (2*b*atan((64*b^6*c^4*tan(e/2 + (f*x)/2))/(64*b^6*c^4 + 128*a^2*b^4*c^4 - 5
12*a^3*b^3*c*d^3 - 512*a^3*b^3*c^3*d + 768*a^2*b^4*c^2*d^2 + 576*a^4*b^2*c^
2*d^2 - 384*a*b^5*c^3*d - 128*a^5*b*c*d^3) + (384*a*b^5*c^3*tan(e/2 + (f*x)
/2))/(384*a*b^5*c^3 + 512*a^3*b^3*c^3 - (64*b^6*c^4)/d - 768*a^2*b^4*c^2*d
+ 512*a^3*b^3*c*d^2 - 576*a^4*b^2*c^2*d - (128*a^2*b^4*c^4)/d + 128*a^5*b*c
*d^2) + (768*a^2*b^4*c^2*tan(e/2 + (f*x)/2))/(768*a^2*b^4*c^2 + 576*a^4*b^2
*c^2 + (64*b^6*c^4)/d^2 - (384*a*b^5*c^3)/d - 128*a^5*b*c*d - (512*a^3*b^3*
c^3)/d + (128*a^2*b^4*c^4)/d^2 - 512*a^3*b^3*c*d) + (576*a^4*b^2*c^2*tan(e/
2 + (f*x)/2))/(768*a^2*b^4*c^2 + 576*a^4*b^2*c^2 + (64*b^6*c^4)/d^2 - (384*
a*b^5*c^3)/d - 128*a^5*b*c*d - (512*a^3*b^3*c^3)/d + (128*a^2*b^4*c^4)/d^2
- 512*a^3*b^3*c*d) + (512*a^3*b^3*c^3*tan(e/2 + (f*x)/2))/(384*a*b^5*c^3 +
512*a^3*b^3*c^3 - (64*b^6*c^4)/d - 768*a^2*b^4*c^2*d + 512*a^3*b^3*c*d^2 -
576*a^4*b^2*c^2*d - (128*a^2*b^4*c^4)/d + 128*a^5*b*c*d^2) + (128*a^2*b^4*c
^4*tan(e/2 + (f*x)/2))/(64*b^6*c^4 + 128*a^2*b^4*c^4 - 512*a^3*b^3*c*d^3 -
512*a^3*b^3*c^3*d + 768*a^2*b^4*c^2*d^2 + 576*a^4*b^2*c^2*d^2 - 384*a*b^5*c
^3*d - 128*a^5*b*c*d^3) - (128*a^5*b*c*d*tan(e/2 + (f*x)/2))/(768*a^2*b^4*c
```

```
^2 + 576*a^4*b^2*c^2 + (64*b^6*c^4)/d^2 - (384*a*b^5*c^3)/d - 128*a^5*b*c*d
 - (512*a^3*b^3*c^3)/d + (128*a^2*b^4*c^4)/d^2 - 512*a^3*b^3*c*d) - (512*a^
3*b^3*c*d*tan(e/2 + (f*x)/2))/(768*a^2*b^4*c^2 + 576*a^4*b^2*c^2 + (64*b^6*
c^4)/d^2 - (384*a*b^5*c^3)/d - 128*a^5*b*c*d - (512*a^3*b^3*c^3)/d + (128*a
^2*b^4*c^4)/d^2 - 512*a^3*b^3*c*d))*(2*a*d - b*c))/(d^2*f) - (2*b^2)/(d*f*(
tan(e/2 + (f*x)/2)^2 + 1)) - (atan((((-(c + d)*(c - d))^(1/2)*(a*d - b*c)^2
*((32*(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 4*a^2*b^2*c^2*d^3))/d^2 - (32*tan(e/2
+ (f*x)/2)*(a^4*c*d^5 + 2*b^4*c^5*d - 2*b^4*c^3*d^3 + 8*a*b^3*c^2*d^4 - 8*a
*b^3*c^4*d^2 - 8*a^2*b^2*c*d^5 - 4*a^3*b*c^2*d^4 + 10*a^2*b^2*c^3*d^3))/d^3
 + ((-(c + d)*(c - d))^(1/2)*(a*d - b*c)^2*((32*(a^2*c^2*d^4 + b^2*c^2*d^4
- 2*a*b*c*d^5))/d^2 + (32*tan(e/2 + (f*x)/2)*(2*a^2*c*d^6 + 2*b^2*c^3*d^4 -
 4*a*b*c^2*d^5))/d^3 + ((32*c^2*d^3 + (32*tan(e/2 + (f*x)/2)*(3*c*d^7 - 2*c
^3*d^5))/d^3)*(-(c + d)*(c - d))^(1/2)*(a*d - b*c)^2)/(d^4 - c^2*d^2)))/(d^
4 - c^2*d^2))*1i)/(d^4 - c^2*d^2) - ((-(c + d)*(c - d))^(1/2)*(a*d - b*c)^2
*((32*tan(e/2 + (f*x)/2)*(a^4*c*d^5 + 2*b^4*c^5*d - 2*b^4*c^3*d^3 + 8*a*b^3
*c^2*d^4 - 8*a*b^3*c^4*d^2 - 8*a^2*b^2*c*d^5 - 4*a^3*b*c^2*d^4 + 10*a^2*b^2
*c^3*d^3))/d^3 - (32*(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 4*a^2*b^2*c^2*d^3))/d^2
 + ((-(c + d)*(c - d))^(1/2)*(a*d - b*c)^2*((32*(a^2*c^2*d^4 + b^2*c^2*d^4
- 2*a*b*c*d^5))/d^2 + (32*tan(e/2 + (f*x)/2)*(2*a^2*c*d^6 + 2*b^2*c^3*d^4 -
 4*a*b*c^2*d^5))/d^3 - ((32*c^2*d^3 + (32*tan(e/2 + (f*x)/2)*(3*c*d^7 - 2*c
^3*d^5))/d^3)*(-(c + d)*(c - d))^(1/2)*(a*d - b*c)^2)/(d^4 - c^2*d^2)))/(d^
4 - c^2*d^2))*1i)/(d^4 - c^2*d^2))/((64*tan(e/2 + (f*x)/2)*(2*b^6*c^5 + 8*a
^4*b^2*c*d^4 + 26*a^2*b^4*c^3*d^2 - 24*a^3*b^3*c^2*d^3 - 12*a*b^5*c^4*d))/d
^3 - (64*(a^2*b^4*c^4 - 4*a^3*b^3*c^3*d + 5*a^4*b^2*c^2*d^2 - 2*a^5*b*c*d^3
))/d^2 + ((-(c + d)*(c - d))^(1/2)*(a*d - b*c)^2*((32*(b^4*c^4*d - 4*a*b^3*
c^3*d^2 + 4*a^2*b^2*c^2*d^3))/d^2 - (32*tan(e/2 + (f*x)/2)*(a^4*c*d^5 + 2*b
^4*c^5*d - 2*b^4*c^3*d^3 + 8*a*b^3*c^2*d^4 - 8*a*b^3*c^4*d^2 - 8*a^2*b^2*c*
d^5 - 4*a^3*b*c^2*d^4 + 10*a^2*b^2*c^3*d^3))/d^3 + ((-(c + d)*(c - d))^(1/2
)*(a*d - b*c)^2*((32*(a^2*c^2*d^4 + b^2*c^2*d^4 - 2*a*b*c*d^5))/d^2 + (32*t
an(e/2 + (f*x)/2)*(2*a^2*c*d^6 + 2*b^2*c^3*d^4 - 4*a*b*c^2*d^5))/d^3 + ((32
*c^2*d^3 + (32*tan(e/2 + (f*x)/2)*(3*c*d^7 - 2*c^3*d^5))/d^3)*(-(c + d)*(c
- d))^(1/2)*(a*d - b*c)^2)/(d^4 - c^2*d^2)))/(d^4 - c^2*d^2)))/(d^4 - c^2*d
^2) + ((-(c + d)*(c - d))^(1/2)*(a*d - b*c)^2*((32*tan(e/2 + (f*x)/2)*(a^4*
c*d^5 + 2*b^4*c^5*d - 2*b^4*c^3*d^3 + 8*a*b^3*c^2*d^4 - 8*a*b^3*c^4*d^2 - 8
*a^2*b^2*c*d^5 - 4*a^3*b*c^2*d^4 + 10*a^2*b^2*c^3*d^3))/d^3 - (32*(b^4*c^4*
d - 4*a*b^3*c^3*d^2 + 4*a^2*b^2*c^2*d^3))/d^2 + ((-(c + d)*(c - d))^(1/2)*(
a*d - b*c)^2*((32*(a^2*c^2*d^4 + b^2*c^2*d^4 - 2*a*b*c*d^5))/d^2 + (32*tan(
e/2 + (f*x)/2)*(2*a^2*c*d^6 + 2*b^2*c^3*d^4 - 4*a*b*c^2*d^5))/d^3 - ((32*c^
2*d^3 + (32*tan(e/2 + (f*x)/2)*(3*c*d^7 - 2*c^3*d^5))/d^3)*(-(c + d)*(c - d
))^(1/2)*(a*d - b*c)^2)/(d^4 - c^2*d^2)))/(d^4 - c^2*d^2)))/(d^4 - c^2*d^2)
))*(-(c + d)*(c - d))^(1/2)*(a*d - b*c)^2*2i)/(f*(d^4 - c^2*d^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.683 $\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=129

$$-\frac{2(bc-ad)\left(acd+b\left(c^2-2d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f\left(c^2-d^2\right)^{3/2}}+\frac{(bc-ad)^2\cos(e+fx)}{df\left(c^2-d^2\right)(c+d\sin(e+fx))}+\frac{b^2x}{d^2}$$

```
[Out] b^2*x/d^2-2*(-a*d+b*c)*(a*c*d+b*(c^2-2*d^2))*arctan((d+c*tan(1/2*f*x+1/2*e)
)/(c^2-d^2)^(1/2))/d^2/(c^2-d^2)^(3/2)/f+(-a*d+b*c)^2*cos(f*x+e)/d/(c^2-d^2
)/f/(c+d*sin(f*x+e))
```

Rubi [A] time = 0.23, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2790, 2735, 2660, 618, 204}

$$-\frac{2(bc-ad)\left(acd+b\left(c^2-2d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f\left(c^2-d^2\right)^{3/2}}+\frac{(bc-ad)^2\cos(e+fx)}{df\left(c^2-d^2\right)(c+d\sin(e+fx))}+\frac{b^2x}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (b^2*x)/d^2 - (2*(b*c - a*d)*(a*c*d + b*(c^2 - 2*d^2))*ArcTan[(d + c*Tan[(e
 + f*x)/2])/Sqrt[c^2 - d^2]])/(d^2*(c^2 - d^2)^(3/2)*f) + ((b*c - a*d)^2*Co
s[e + f*x])/(d*(c^2 - d^2)*f*(c + d*Sin[e + f*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^2}\,dx = \frac{(bc-ad)^2\cos(e+fx)}{d\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \frac{\int \frac{d\left(\left(a^2+b^2\right)c-2abd\right)+b^2\left(c^2-d^2\right)\sin(e+fx)}{c+d\sin(e+fx)}\,dx}{d\left(c^2-d^2\right)}$$

$$= \frac{b^2x}{d^2} + \frac{(bc-ad)^2\cos(e+fx)}{d\left(c^2-d^2\right)f(c+d\sin(e+fx))} - \frac{\left(-d^2\left(\left(a^2+b^2\right)c-2abd\right)+b^2c\left(c^2-d^2\right)\right)\int}{d^2\left(c^2-d^2\right)}$$

$$= \frac{b^2x}{d^2} + \frac{(bc-ad)^2\cos(e+fx)}{d\left(c^2-d^2\right)f(c+d\sin(e+fx))} - \frac{\left(2\left(-d^2\left(\left(a^2+b^2\right)c-2abd\right)+b^2c\left(c^2-d^2\right)\right.\right.}{d^2\left(c^2\right.}$$

$$= \frac{b^2x}{d^2} + \frac{(bc-ad)^2\cos(e+fx)}{d\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \frac{\left(4\left(-d^2\left(\left(a^2+b^2\right)c-2abd\right)+b^2c\left(c^2-d^2\right)\right.\right.}{d}$$

$$= \frac{b^2x}{d^2} - \frac{2(bc-ad)\left(bc^2+acd-2bd^2\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^2\left(c^2-d^2\right)^{3/2}f} + \frac{(bc-ad)^2\cos(e+}{d\left(c^2-d^2\right)f(c+d\sin}$$

Mathematica [A] time = 0.54, size = 134, normalized size = 1.04

$$\frac{-\frac{2\left(-a^2cd^2+2abd^3+b^2\left(c^3-2cd^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{3/2}}+\frac{d(bc-ad)^2\cos(e+fx)}{(c-d)(c+d)(c+d\sin(e+fx))}+b^2(e+fx)}{d^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (b^2*(e + f*x) - (2*(-(a^2*c*d^2) + 2*a*b*d^3 + b^2*(c^3 - 2*c*d^2))*ArcTan
[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^2)^(3/2) + (d*(b*c - a
*d)^2*Cos[e + f*x])/((c - d)*(c + d)*(c + d*Sin[e + f*x])))/(d^2*f)
```

fricas [B] time = 0.50, size = 677, normalized size = 5.25

$$\left[\frac{2\left(b^2c^4d-2b^2c^2d^3+b^2d^5\right)fx\sin\left(fx+e\right)+2\left(b^2c^5-2b^2c^3d^2+b^2cd^4\right)fx-\left(b^2c^4+2abcd^3-\left(a^2+2b^2\right)c^2d^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b^2*c^4*d - 2*b^2*c^2*d^3 + b^2*d^5)*f*x*sin(f*x + e) + 2*(b^2*c^5
 - 2*b^2*c^3*d^2 + b^2*c*d^4)*f*x - (b^2*c^4 + 2*a*b*c*d^3 - (a^2 + 2*b^2)*
c^2*d^2 + (b^2*c^3*d + 2*a*b*d^4 - (a^2 + 2*b^2)*c*d^3)*sin(f*x + e))*sqrt(
-c^2 + d^2)*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 -
 d^2 - 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(
d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(b^2*c^4*d - 2*a*
b*c^3*d^2 + 2*a*b*c*d^4 - a^2*d^5 + (a^2 - b^2)*c^2*d^3)*cos(f*x + e))/((c^
4*d^3 - 2*c^2*d^5 + d^7)*f*sin(f*x + e) + (c^5*d^2 - 2*c^3*d^4 + c*d^6)*f),
 ((b^2*c^4*d - 2*b^2*c^2*d^3 + b^2*d^5)*f*x*sin(f*x + e) + (b^2*c^5 - 2*b^2
*c^3*d^2 + b^2*c*d^4)*f*x + (b^2*c^4 + 2*a*b*c*d^3 - (a^2 + 2*b^2)*c^2*d^2
+ (b^2*c^3*d + 2*a*b*d^4 - (a^2 + 2*b^2)*c*d^3)*sin(f*x + e))*sqrt(c^2 - d^
2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (b^2*c^4*
d - 2*a*b*c^3*d^2 + 2*a*b*c*d^4 - a^2*d^5 + (a^2 - b^2)*c^2*d^3)*cos(f*x +
e))/((c^4*d^3 - 2*c^2*d^5 + d^7)*f*sin(f*x + e) + (c^5*d^2 - 2*c^3*d^4 + c*
d^6)*f)]
```

giac [B] time = 0.22, size = 249, normalized size = 1.93

$$\frac{\frac{(fx+e)b^2}{d^2} - \frac{2\left(b^2c^3-a^2cd^2-2\,b^2cd^2+2\,abd^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(c^2d^2-d^4\right)\sqrt{c^2-d^2}} + \frac{2\left(b^2c^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,abcd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+a^2d^3}{\left(c^3d-cd^3\right)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,d\right.}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] ((f*x + e)*b^2/d^2 - 2*(b^2*c^3 - a^2*c*d^2 - 2*b^2*c*d^2 + 2*a*b*d^3)*(pi*
floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/
sqrt(c^2 - d^2)))/((c^2*d^2 - d^4)*sqrt(c^2 - d^2)) + 2*(b^2*c^2*d*tan(1/2*
f*x + 1/2*e) - 2*a*b*c*d^2*tan(1/2*f*x + 1/2*e) + a^2*d^3*tan(1/2*f*x + 1/2
*e) + b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)/((c^3*d - c*d^3)*(c*tan(1/2*f*x +
1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)))/f
```

maple [B] time = 0.26, size = 556, normalized size = 4.31

$$\frac{2d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)a^2}{f\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d+c\right)\left(c^2-d^2\right)c} - \frac{4d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)ab}{f\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d+c\right)\left(c^2-d^2\right)} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2/f*d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)/c*tan(1
/2*f*x+1/2*e)*a^2-4/f*d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(
c^2-d^2)*tan(1/2*f*x+1/2*e)*a*b+2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1
/2*e)*d+c)/(c^2-d^2)*c*tan(1/2*f*x+1/2*e)*b^2+2/f*d/(tan(1/2*f*x+1/2*e)^2*c
+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*a^2-4/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(
1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*a*b*c+2/f/d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2
*f*x+1/2*e)*d+c)/(c^2-d^2)*b^2*c^2+2/f/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(
1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a^2*c-4/f*d/(c^2-d^2)^(3/2)*arctan(1/2
*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a*b-2/f/d^2/(c^2-d^2)^(3/2)*
arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b^2*c^3+4/f/(c^2-d
^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b^2*c+2/
f*b^2/d^2*arctan(tan(1/2*f*x+1/2*e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 15.47, size = 5776, normalized size = 44.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/(c + d*sin(e + f*x))^2,x)
```

```
[Out] ((2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(d*(c^2 - d^2)) + (2*tan(e/2 + (f*x)/2
)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(c*(c^2 - d^2)))/(f*(c + 2*d*tan(e/2 + (
f*x)/2) + c*tan(e/2 + (f*x)/2)^2)) - (2*b^2*atan(((b^2*((b^2*((32*(b^2*c*d^
8 + a^2*c^3*d^6 - a^2*c^5*d^4 - b^2*c^3*d^6 - 2*a*b*c^2*d^7 + 2*a*b*c^4*d^5
))/(d^6 - 2*c^2*d^4 + c^4*d^2) + (32*tan(e/2 + (f*x)/2)*(2*a^2*c^2*d^8 - 2*
a^2*c^4*d^6 + 4*b^2*c^2*d^8 - 6*b^2*c^4*d^6 + 2*b^2*c^6*d^4 - 4*a*b*c*d^9 +
 4*a*b*c^3*d^7))/(d^7 - 2*c^2*d^5 + c^4*d^3) - (b^2*((32*(c^2*d^9 - 2*c^4*d
^7 + c^6*d^5))/(d^6 - 2*c^2*d^4 + c^4*d^2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^
11 - 8*c^3*d^9 + 7*c^5*d^7 - 2*c^7*d^5))/(d^7 - 2*c^2*d^5 + c^4*d^3))*1i)/d
^2)*1i)/d^2 - (32*(b^4*c^6*d + b^4*c^2*d^5 - 2*b^4*c^4*d^3))/(d^6 - 2*c^2*d
^4 + c^4*d^2) + (32*tan(e/2 + (f*x)/2)*(2*b^4*c^7*d - 2*b^4*c*d^7 + a^4*c^3
*d^5 + 9*b^4*c^3*d^5 - 8*b^4*c^5*d^3 - 8*a*b^3*c^2*d^6 + 4*a*b^3*c^4*d^4 +
4*a^2*b^2*c*d^7 - 4*a^3*b*c^2*d^6 + 4*a^2*b^2*c^3*d^5 - 2*a^2*b^2*c^5*d^3))
/(d^7 - 2*c^2*d^5 + c^4*d^3)))/d^2 - (b^2*((32*(b^4*c^6*d + b^4*c^2*d^5 - 2
*b^4*c^4*d^3))/(d^6 - 2*c^2*d^4 + c^4*d^2) + (b^2*((32*(b^2*c*d^8 + a^2*c^3
*d^6 - a^2*c^5*d^4 - b^2*c^3*d^6 - 2*a*b*c^2*d^7 + 2*a*b*c^4*d^5))/(d^6 - 2
*c^2*d^4 + c^4*d^2) + (32*tan(e/2 + (f*x)/2)*(2*a^2*c^2*d^8 - 2*a^2*c^4*d^6
 + 4*b^2*c^2*d^8 - 6*b^2*c^4*d^6 + 2*b^2*c^6*d^4 - 4*a*b*c*d^9 + 4*a*b*c^3*
d^7))/(d^7 - 2*c^2*d^5 + c^4*d^3) + (b^2*((32*(c^2*d^9 - 2*c^4*d^7 + c^6*d^
5))/(d^6 - 2*c^2*d^4 + c^4*d^2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^11 - 8*c^3*
d^9 + 7*c^5*d^7 - 2*c^7*d^5))/(d^7 - 2*c^2*d^5 + c^4*d^3))*1i)/d^2)*1i)/d^2
 - (32*tan(e/2 + (f*x)/2)*(2*b^4*c^7*d - 2*b^4*c*d^7 + a^4*c^3*d^5 + 9*b^4*
c^3*d^5 - 8*b^4*c^5*d^3 - 8*a*b^3*c^2*d^6 + 4*a*b^3*c^4*d^4 + 4*a^2*b^2*c*d
^7 - 4*a^3*b*c^2*d^6 + 4*a^2*b^2*c^3*d^5 - 2*a^2*b^2*c^5*d^3))/(d^7 - 2*c^2
*d^5 + c^4*d^3)))/d^2)/((b^2*((b^2*((32*(b^2*c*d^8 + a^2*c^3*d^6 - a^2*c^5*
d^4 - b^2*c^3*d^6 - 2*a*b*c^2*d^7 + 2*a*b*c^4*d^5))/(d^6 - 2*c^2*d^4 + c^4*
d^2) + (32*tan(e/2 + (f*x)/2)*(2*a^2*c^2*d^8 - 2*a^2*c^4*d^6 + 4*b^2*c^2*d^
8 - 6*b^2*c^4*d^6 + 2*b^2*c^6*d^4 - 4*a*b*c*d^9 + 4*a*b*c^3*d^7))/(d^7 - 2*
c^2*d^5 + c^4*d^3) - (b^2*((32*(c^2*d^9 - 2*c^4*d^7 + c^6*d^5))/(d^6 - 2*c^
```

2*d^4 + c^4*d^2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^11 - 8*c^3*d^9 + 7*c^5*d^7 - 2*c^7*d^5))/(d^7 - 2*c^2*d^5 + c^4*d^3))*1i)/d^2)*1i)/d^2 - (32*(b^4*c^6 *d + b^4*c^2*d^5 - 2*b^4*c^4*d^3))/(d^6 - 2*c^2*d^4 + c^4*d^2) + (32*tan(e/ 2 + (f*x)/2)*(2*b^4*c^7*d - 2*b^4*c*d^7 + a^4*c^3*d^5 + 9*b^4*c^3*d^5 - 8*b ^4*c^5*d^3 - 8*a*b^3*c^2*d^6 + 4*a*b^3*c^4*d^4 + 4*a^2*b^2*c*d^7 - 4*a^3*b* c^2*d^6 + 4*a^2*b^2*c^3*d^5 - 2*a^2*b^2*c^5*d^3))/(d^7 - 2*c^2*d^5 + c^4*d^ 3))*1i)/d^2 - (64*(2*b^6*c^3*d^2 - a^2*b^4*c^5 - b^6*c^5 - 6*a*b^5*c^2*d^3 + 4*a^2*b^4*c*d^4 + 3*a^2*b^4*c^3*d^2 - 4*a^3*b^3*c^2*d^3 + a^4*b^2*c^3*d^2 + 2*a*b^5*c^4*d))/(d^6 - 2*c^2*d^4 + c^4*d^2) + (b^2*((32*(b^4*c^6*d + b^4 *c^2*d^5 - 2*b^4*c^4*d^3))/(d^6 - 2*c^2*d^4 + c^4*d^2) + (b^2*((32*(b^2*c*d ^8 + a^2*c^3*d^6 - a^2*c^5*d^4 - b^2*c^3*d^6 - 2*a*b*c^2*d^7 + 2*a*b*c^4*d^ 5))/(d^6 - 2*c^2*d^4 + c^4*d^2) + (32*tan(e/2 + (f*x)/2)*(2*a^2*c^2*d^8 - 2 *a^2*c^4*d^6 + 4*b^2*c^2*d^8 - 6*b^2*c^4*d^6 + 2*b^2*c^6*d^4 - 4*a*b*c*d^9 + 4*a*b*c^3*d^7))/(d^7 - 2*c^2*d^5 + c^4*d^3) + (b^2*((32*(c^2*d^9 - 2*c^4* d^7 + c^6*d^5))/(d^6 - 2*c^2*d^4 + c^4*d^2) + (32*tan(e/2 + (f*x)/2)*(3*c*d ^11 - 8*c^3*d^9 + 7*c^5*d^7 - 2*c^7*d^5))/(d^7 - 2*c^2*d^5 + c^4*d^3))*1i)/ d^2)*1i)/d^2 - (32*tan(e/2 + (f*x)/2)*(2*b^4*c^7*d - 2*b^4*c*d^7 + a^4*c^3* d^5 + 9*b^4*c^3*d^5 - 8*b^4*c^5*d^3 - 8*a*b^3*c^2*d^6 + 4*a*b^3*c^4*d^4 + 4 *a^2*b^2*c*d^7 - 4*a^3*b*c^2*d^6 + 4*a^2*b^2*c^3*d^5 - 2*a^2*b^2*c^5*d^3))/ (d^7 - 2*c^2*d^5 + c^4*d^3))*1i)/d^2 + (64*tan(e/2 + (f*x)/2)*(2*b^6*c^6 + 4*b^6*c^2*d^4 - 6*b^6*c^4*d^2 + 4*a*b^5*c^3*d^3 + 2*a^2*b^4*c^2*d^4 - 2*a^2 *b^4*c^4*d^2 - 4*a*b^5*c*d^5))/(d^7 - 2*c^2*d^5 + c^4*d^3))))/(d^2*f) + (at an((((a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(b^4*c^6*d + b^4*c^2*d^5 - 2*b^4*c^4*d^3))/(d^6 - 2*c^2*d^4 + c^4*d^2) - (32*tan(e/2 + (f*x)/2)*(2* b^4*c^7*d - 2*b^4*c*d^7 + a^4*c^3*d^5 + 9*b^4*c^3*d^5 - 8*b^4*c^5*d^3 - 8*a *b^3*c^2*d^6 + 4*a*b^3*c^4*d^4 + 4*a^2*b^2*c*d^7 - 4*a^3*b*c^2*d^6 + 4*a^2* b^2*c^3*d^5 - 2*a^2*b^2*c^5*d^3))/(d^7 - 2*c^2*d^5 + c^4*d^3) + ((a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(b^2*c*d^8 + a^2*c^3*d^6 - a^2*c^5*d^4 - b^2*c^3*d^6 - 2*a*b*c^2*d^7 + 2*a*b*c^4*d^5))/(d^6 - 2*c^2*d^4 + c^4*d^2) + (32*tan(e/2 + (f*x)/2)*(2*a^2*c^2*d^8 - 2*a^2*c^4*d^6 + 4*b^2*c^2*d^8 - 6*b^2*c^4*d^6 + 2*b^2*c^6*d^4 - 4*a*b*c*d^9 + 4*a*b*c^3*d^7))/(d^7 - 2*c^2* d^5 + c^4*d^3) + (((32*(c^2*d^9 - 2*c^4*d^7 + c^6*d^5))/(d^6 - 2*c^2*d^4 + c^4*d^2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^11 - 8*c^3*d^9 + 7*c^5*d^7 - 2*c^7 *d^5))/(d^7 - 2*c^2*d^5 + c^4*d^3))*(a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)*(b*c^2 - 2*b*d^2 + a*c*d))/(d^8 - 3*c^2*d^6 + 3*c^4*d^4 - c^6*d^2))*(b*c^ 2 - 2*b*d^2 + a*c*d))/(d^8 - 3*c^2*d^6 + 3*c^4*d^4 - c^6*d^2))*(b*c^2 - 2*b *d^2 + a*c*d)*1i)/(d^8 - 3*c^2*d^6 + 3*c^4*d^4 - c^6*d^2) - ((a*d - b*c)*(- (c + d)^3*(c - d)^3)^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*b^4*c^7*d - 2*b^4*c*d ^7 + a^4*c^3*d^5 + 9*b^4*c^3*d^5 - 8*b^4*c^5*d^3 - 8*a*b^3*c^2*d^6 + 4*a*b^ 3*c^4*d^4 + 4*a^2*b^2*c*d^7 - 4*a^3*b*c^2*d^6 + 4*a^2*b^2*c^3*d^5 - 2*a^2*b ^2*c^5*d^3))/(d^7 - 2*c^2*d^5 + c^4*d^3) - (32*(b^4*c^6*d + b^4*c^2*d^5 - 2 *b^4*c^4*d^3))/(d^6 - 2*c^2*d^4 + c^4*d^2) + ((a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(b^2*c*d^8 + a^2*c^3*d^6 - a^2*c^5*d^4 - b^2*c^3*d^6 - 2*a *b*c^2*d^7 + 2*a*b*c^4*d^5))/(d^6 - 2*c^2*d^4 + c^4*d^2) + (32*tan(e/2 + (f *x)/2)*(2*a^2*c^2*d^8 - 2*a^2*c^4*d^6 + 4*b^2*c^2*d^8 - 6*b^2*c^4*d^6 + 2*b

^2*c^6*d^4 - 4*a*b*c*d^9 + 4*a*b*c^3*d^7))/(d^7 - 2*c^2*d^5 + c^4*d^3) - ((
(32*(c^2*d^9 - 2*c^4*d^7 + c^6*d^5))/(d^6 - 2*c^2*d^4 + c^4*d^2) + (32*tan(
e/2 + (f*x)/2)*(3*c*d^11 - 8*c^3*d^9 + 7*c^5*d^7 - 2*c^7*d^5))/(d^7 - 2*c^2
*d^5 + c^4*d^3))*(a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)*(b*c^2 - 2*b*d^2
+ a*c*d))/(d^8 - 3*c^2*d^6 + 3*c^4*d^4 - c^6*d^2))*(b*c^2 - 2*b*d^2 + a*c*d
))/(d^8 - 3*c^2*d^6 + 3*c^4*d^4 - c^6*d^2))*(b*c^2 - 2*b*d^2 + a*c*d)*1i)/(
d^8 - 3*c^2*d^6 + 3*c^4*d^4 - c^6*d^2))/((64*tan(e/2 + (f*x)/2)*(2*b^6*c^6
+ 4*b^6*c^2*d^4 - 6*b^6*c^4*d^2 + 4*a*b^5*c^3*d^3 + 2*a^2*b^4*c^2*d^4 - 2*a
^2*b^4*c^4*d^2 - 4*a*b^5*c*d^5))/(d^7 - 2*c^2*d^5 + c^4*d^3) - (64*(2*b^6*c
^3*d^2 - a^2*b^4*c^5 - b^6*c^5 - 6*a*b^5*c^2*d^3 + 4*a^2*b^4*c*d^4 + 3*a^2*
b^4*c^3*d^2 - 4*a^3*b^3*c^2*d^3 + a^4*b^2*c^3*d^2 + 2*a*b^5*c^4*d))/(d^6 -
2*c^2*d^4 + c^4*d^2) + ((a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(b^4*
c^6*d + b^4*c^2*d^5 - 2*b^4*c^4*d^3))/(d^6 - 2*c^2*d^4 + c^4*d^2) - (32*tan
(e/2 + (f*x)/2)*(2*b^4*c^7*d - 2*b^4*c*d^7 + a^4*c^3*d^5 + 9*b^4*c^3*d^5 -
8*b^4*c^5*d^3 - 8*a*b^3*c^2*d^6 + 4*a*b^3*c^4*d^4 + 4*a^2*b^2*c*d^7 - 4*a^3
*b*c^2*d^6 + 4*a^2*b^2*c^3*d^5 - 2*a^2*b^2*c^5*d^3))/(d^7 - 2*c^2*d^5 + c^4
*d^3) + ((a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(b^2*c*d^8 + a^2*c^3
*d^6 - a^2*c^5*d^4 - b^2*c^3*d^6 - 2*a*b*c^2*d^7 + 2*a*b*c^4*d^5))/(d^6 - 2
*c^2*d^4 + c^4*d^2) + (32*tan(e/2 + (f*x)/2)*(2*a^2*c^2*d^8 - 2*a^2*c^4*d^6
+ 4*b^2*c^2*d^8 - 6*b^2*c^4*d^6 + 2*b^2*c^6*d^4 - 4*a*b*c*d^9 + 4*a*b*c^3*
d^7))/(d^7 - 2*c^2*d^5 + c^4*d^3) + (((32*(c^2*d^9 - 2*c^4*d^7 + c^6*d^5))/
(d^6 - 2*c^2*d^4 + c^4*d^2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^11 - 8*c^3*d^9
+ 7*c^5*d^7 - 2*c^7*d^5))/(d^7 - 2*c^2*d^5 + c^4*d^3))*(a*d - b*c)*(-(c + d
)^3*(c - d)^3)^(1/2)*(b*c^2 - 2*b*d^2 + a*c*d))/(d^8 - 3*c^2*d^6 + 3*c^4*d^
4 - c^6*d^2))*(b*c^2 - 2*b*d^2 + a*c*d))/(d^8 - 3*c^2*d^6 + 3*c^4*d^4 - c^6
d^2))(b*c^2 - 2*b*d^2 + a*c*d))/(d^8 - 3*c^2*d^6 + 3*c^4*d^4 - c^6*d^2) +
((a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*b^4*c
^7*d - 2*b^4*c*d^7 + a^4*c^3*d^5 + 9*b^4*c^3*d^5 - 8*b^4*c^5*d^3 - 8*a*b^3*
c^2*d^6 + 4*a*b^3*c^4*d^4 + 4*a^2*b^2*c*d^7 - 4*a^3*b*c^2*d^6 + 4*a^2*b^2*c
^3*d^5 - 2*a^2*b^2*c^5*d^3))/(d^7 - 2*c^2*d^5 + c^4*d^3) - (32*(b^4*c^6*d +
b^4*c^2*d^5 - 2*b^4*c^4*d^3))/(d^6 - 2*c^2*d^4 + c^4*d^2) + ((a*d - b*c)*(
-(c + d)^3*(c - d)^3)^(1/2)*((32*(b^2*c*d^8 + a^2*c^3*d^6 - a^2*c^5*d^4 - b
^2*c^3*d^6 - 2*a*b*c^2*d^7 + 2*a*b*c^4*d^5))/(d^6 - 2*c^2*d^4 + c^4*d^2) +
(32*tan(e/2 + (f*x)/2)*(2*a^2*c^2*d^8 - 2*a^2*c^4*d^6 + 4*b^2*c^2*d^8 - 6*b
^2*c^4*d^6 + 2*b^2*c^6*d^4 - 4*a*b*c*d^9 + 4*a*b*c^3*d^7))/(d^7 - 2*c^2*d^5
+ c^4*d^3) - (((32*(c^2*d^9 - 2*c^4*d^7 + c^6*d^5))/(d^6 - 2*c^2*d^4 + c^4
*d^2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^11 - 8*c^3*d^9 + 7*c^5*d^7 - 2*c^7*d^
5))/(d^7 - 2*c^2*d^5 + c^4*d^3))*(a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)*(
b*c^2 - 2*b*d^2 + a*c*d))/(d^8 - 3*c^2*d^6 + 3*c^4*d^4 - c^6*d^2))*(b*c^2 -
2*b*d^2 + a*c*d))/(d^8 - 3*c^2*d^6 + 3*c^4*d^4 - c^6*d^2))*(b*c^2 - 2*b*d^
2 + a*c*d))/(d^8 - 3*c^2*d^6 + 3*c^4*d^4 - c^6*d^2)))*(a*d - b*c)*(-(c + d)
^3*(c - d)^3)^(1/2)*(b*c^2 - 2*b*d^2 + a*c*d)*2i)/(f*(d^8 - 3*c^2*d^6 + 3*c
^4*d^4 - c^6*d^2))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.684 $\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=196

$$-\frac{\left(-\left(a^2\left(2c^2+d^2\right)\right)+6abcd-b^2\left(c^2+2d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{5/2}}+\frac{(bc-ad)^2\cos(e+fx)}{2df\left(c^2-d^2\right)(c+d\sin(e+fx))^2}-\frac{(3acd\ \cdots}{2d\ \cdots}$$

```
[Out] -(6*a*b*c*d-a^2*(2*c^2+d^2)-b^2*(c^2+2*d^2))*arctan((d+c*tan(1/2*f*x+1/2*e)
)/(c^2-d^2)^(1/2))/(c^2-d^2)^(5/2)/f+1/2*(-a*d+b*c)^2*cos(f*x+e)/d/(c^2-d^2
)/f/(c+d*sin(f*x+e))^2-1/2*(-a*d+b*c)*(3*a*c*d+b*(c^2-4*d^2))*cos(f*x+e)/d/
(c^2-d^2)^2/f/(c+d*sin(f*x+e))
```

Rubi [A] time = 0.28, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2790, 2754, 12, 2660, 618, 204}

$$-\frac{\left(a^2\left(-\left(2c^2+d^2\right)\right)+6abcd-b^2\left(c^2+2d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{5/2}}+\frac{(bc-ad)^2\cos(e+fx)}{2df\left(c^2-d^2\right)(c+d\sin(e+fx))^2}-\frac{(3acd\ \cdots}{2d\ \cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] -(((6*a*b*c*d - a^2*(2*c^2 + d^2) - b^2*(c^2 + 2*d^2))*ArcTan[(d + c*Tan[(e
 + f*x)/2])/Sqrt[c^2 - d^2]])/((c^2 - d^2)^(5/2)*f)) + ((b*c - a*d)^2*Cos[e
 + f*x])/(2*d*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^2) - ((b*c - a*d)*(3*a*c*d
 + b*(c^2 - 4*d^2))*Cos[e + f*x])/(2*d*(c^2 - d^2)^2*f*(c + d*Sin[e + f*x])
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^3}\,dx = \frac{(bc-ad)^2\cos(e+fx)}{2d\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{\int \frac{2d\left(\left(a^2+b^2\right)c-2abd\right)+\left(b^2c^2+2abcd-\left(a^2+2b^2\right)d^2\right)\sin(e+fx)}{(c+d\sin(e+fx))^2}}{2d\left(c^2-d^2\right)}$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{2d\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} - \frac{(bc-ad)\left(3acd+b\left(c^2-4d^2\right)\right)\cos(e+fx)}{2d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))} -$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{2d\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} - \frac{(bc-ad)\left(3acd+b\left(c^2-4d^2\right)\right)\cos(e+fx)}{2d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))} -$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{2d\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} - \frac{(bc-ad)\left(3acd+b\left(c^2-4d^2\right)\right)\cos(e+fx)}{2d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))} -$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{2d\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} - \frac{(bc-ad)\left(3acd+b\left(c^2-4d^2\right)\right)\cos(e+fx)}{2d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))} +$$

$$= -\frac{\left(6abcd-a^2\left(2c^2+d^2\right)-b^2\left(c^2+2d^2\right)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{5/2} f} + \frac{(bc-ad)^2}{2d\left(c^2-d^2\right)f(c}$$

Mathematica [**A**] time = 0.94, size = 202, normalized size = 1.03

$$\frac{\frac{2\left(a^2\left(2c^2+d^2\right)-6abcd+b^2\left(c^2+2d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{5/2}} - \frac{\left(-3a^2cd^2+2abd\left(c^2+2d^2\right)+b^2\left(c^3-4cd^2\right)\right)\cos(e+fx)}{d(c-d)^2(c+d)^2(c+d\sin(e+fx))} + \frac{(bc-ad)^2\cos(e+fx)}{d(c-d)(c+d)(c+d\sin(e+fx))^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((2*(-6*a*b*c*d + a^2*(2*c^2 + d^2) + b^2*(c^2 + 2*d^2))*ArcTan[(d + c*Tan[
(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^2)^(5/2) + ((b*c - a*d)^2*Cos[e +
f*x])/((c - d)*d*(c + d)*(c + d*Sin[e + f*x])^2) - ((-3*a^2*c*d^2 + 2*a*b*d
*(c^2 + 2*d^2) + b^2*(c^3 - 4*c*d^2))*Cos[e + f*x])/((c - d)^2*d*(c + d)^2*
(c + d*Sin[e + f*x])))/(2*f)
```

fricas [**B**] time = 0.53, size = 1027, normalized size = 5.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^2*c^5 + 2*a*b*c^4*d + 2*a*b*c^2*d^3 - 4*a*b*d^5 - (3*a^2 + 5*b^2
)*c^3*d^2 + (3*a^2 + 4*b^2)*c*d^4)*cos(f*x + e)*sin(f*x + e) - (6*a*b*c^3*d
 + 6*a*b*c*d^3 - (2*a^2 + b^2)*c^4 - 3*(a^2 + b^2)*c^2*d^2 - (a^2 + 2*b^2)*
d^4 - (6*a*b*c*d^3 - (2*a^2 + b^2)*c^2*d^2 - (a^2 + 2*b^2)*d^4)*cos(f*x + e
)^2 + 2*(6*a*b*c^2*d^2 - (2*a^2 + b^2)*c^3*d - (a^2 + 2*b^2)*c*d^3)*sin(f*x
 + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x +
 e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^
2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(4*a*b
*c^5 - 2*a*b*c^3*d^2 - 2*a*b*c*d^4 - a^2*d^5 - (4*a^2 + 3*b^2)*c^4*d + (5*a
^2 + 3*b^2)*c^2*d^3)*cos(f*x + e))/((c^6*d^2 - 3*c^4*d^4 + 3*c^2*d^6 - d^8)
*f*cos(f*x + e)^2 - 2*(c^7*d - 3*c^5*d^3 + 3*c^3*d^5 - c*d^7)*f*sin(f*x + e
) - (c^8 - 2*c^6*d^2 + 2*c^2*d^6 - d^8)*f), 1/2*((b^2*c^5 + 2*a*b*c^4*d + 2
*a*b*c^2*d^3 - 4*a*b*d^5 - (3*a^2 + 5*b^2)*c^3*d^2 + (3*a^2 + 4*b^2)*c*d^4)
*cos(f*x + e)*sin(f*x + e) - (6*a*b*c^3*d + 6*a*b*c*d^3 - (2*a^2 + b^2)*c^4
 - 3*(a^2 + b^2)*c^2*d^2 - (a^2 + 2*b^2)*d^4 - (6*a*b*c*d^3 - (2*a^2 + b^2)
*c^2*d^2 - (a^2 + 2*b^2)*d^4)*cos(f*x + e)^2 + 2*(6*a*b*c^2*d^2 - (2*a^2 +
b^2)*c^3*d - (a^2 + 2*b^2)*c*d^3)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*
sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (4*a*b*c^5 - 2*a*b*c^3*
d^2 - 2*a*b*c*d^4 - a^2*d^5 - (4*a^2 + 3*b^2)*c^4*d + (5*a^2 + 3*b^2)*c^2*d
^3)*cos(f*x + e))/((c^6*d^2 - 3*c^4*d^4 + 3*c^2*d^6 - d^8)*f*cos(f*x + e)^2
 - 2*(c^7*d - 3*c^5*d^3 + 3*c^3*d^5 - c*d^7)*f*sin(f*x + e) - (c^8 - 2*c^6*
d^2 + 2*c^2*d^6 - d^8)*f)]
```

giac [B] time = 0.29, size = 609, normalized size = 3.11

$$\frac{\left(2\,a^2c^2+b^2c^2-6\,abcd+a^2d^2+2\,b^2d^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(c^4-2\,c^2d^2+d^4\right)\sqrt{c^2-d^2}}+\frac{b^2c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-6\,abc^4d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+5\,a^2c^3d^2\tan\left(\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((2*a^2*c^2 + b^2*c^2 - 6*a*b*c*d + a^2*d^2 + 2*b^2*d^2)*(pi*floor(1/2*(f*x
 + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2
)))/((c^4 - 2*c^2*d^2 + d^4)*sqrt(c^2 - d^2)) + (b^2*c^5*tan(1/2*f*x + 1/2*
e)^3 - 6*a*b*c^4*d*tan(1/2*f*x + 1/2*e)^3 + 5*a^2*c^3*d^2*tan(1/2*f*x + 1/2
*e)^3 + 2*b^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 2*a^2*c*d^4*tan(1/2*f*x + 1/
2*e)^3 - 4*a*b*c^5*tan(1/2*f*x + 1/2*e)^2 + 4*a^2*c^4*d*tan(1/2*f*x + 1/2*e
)^2 + 3*b^2*c^4*d*tan(1/2*f*x + 1/2*e)^2 - 10*a*b*c^3*d^2*tan(1/2*f*x + 1/2
```

```
*e)^2 + 7*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^2 + 6*b^2*c^2*d^3*tan(1/2*f*x +
1/2*e)^2 - 4*a*b*c*d^4*tan(1/2*f*x + 1/2*e)^2 - 2*a^2*d^5*tan(1/2*f*x + 1/2
*e)^2 - b^2*c^5*tan(1/2*f*x + 1/2*e) - 10*a*b*c^4*d*tan(1/2*f*x + 1/2*e) +
11*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e) + 10*b^2*c^3*d^2*tan(1/2*f*x + 1/2*e) -
 8*a*b*c^2*d^3*tan(1/2*f*x + 1/2*e) - 2*a^2*c*d^4*tan(1/2*f*x + 1/2*e) - 4*
a*b*c^5 + 4*a^2*c^4*d + 3*b^2*c^4*d - 2*a*b*c^3*d^2 - a^2*c^2*d^3)/((c^6 -
2*c^4*d^2 + c^2*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) +
 c)^2))/f
```

maple [B] time = 0.28, size = 1923, normalized size = 9.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x)
```

```
[Out] -8/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4
)*tan(1/2*f*x+1/2*e)*a*b*d^3+10/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2
*e)*d+c)^2*c/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*b^2*d^2-2/f/(tan(1/2*f*
x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*a*b*c*d^2-6/f/
(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^2
*tan(1/2*f*x+1/2*e)^3*a*b*d-10/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*
e)*d+c)^2*c^2/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*a*b*d-4/f/(tan(1/2*f*x
+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)/c*tan(1/2*f*x+1
/2*e)^2*a*b*d^4-6/f/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan
(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a*b*c*d-10/f/(tan(1/2*f*x+1/2*e)^2*c+
2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c*tan(1/2*f*x+1/2*e)^2*a*b*
d^2-1/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+
d^4)*a^2*d^3-1/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c^3/(c
^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*b^2+4/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(
1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*a^2*c^2*d-4/f/(tan(1/2*f*x+1/2*e)
^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*a*b*c^3+3/f/(tan(1/2*f
*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*b^2*c^2*d+2/f
/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d
)/(c^2-d^2)^(1/2))*a^2*c^2+1/f/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1
/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a^2*d^2+1/f/(c^4-2*c^2*d^2
+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/
2))*b^2*c^2+2/f/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2
*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b^2*d^2+1/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan
(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^3*tan(1/2*f*x+1/2*e)^3*b^2+7/f
/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*ta
n(1/2*f*x+1/2*e)^2*a^2*d^3+6/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)
*d+c)^2/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)^2*b^2*d^3+5/f/(tan(1/2*f*x+1
/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c*tan(1/2*f*x+1/2
*e)^3*a^2*d^2-2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-
```

```
2*c^2*d^2+d^4)/c*tan(1/2*f*x+1/2*e)^3*a^2*d^4+2/f/(tan(1/2*f*x+1/2*e)^2*c+2
*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c*tan(1/2*f*x+1/2*e)^3*b^2*d
^2+4/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d
^4)*c^2*tan(1/2*f*x+1/2*e)^2*a^2*d-2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*
x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)/c^2*tan(1/2*f*x+1/2*e)^2*a^2*d^5-4/f/(t
an(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^3*t
an(1/2*f*x+1/2*e)^2*a*b+3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+
c)^2/(c^4-2*c^2*d^2+d^4)*c^2*tan(1/2*f*x+1/2*e)^2*b^2*d+11/f/(tan(1/2*f*x+1
/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2
*e)*a^2*d^2-2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^4-
2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*a^2*d^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 10.26, size = 641, normalized size = 3.27

$$\frac{\mathrm{atan}\left(\frac{\left(\frac{\left(2c^4d-4c^2d^3+2d^5\right)\left(2a^2c^2+a^2d^2-6abcd+b^2c^2+2b^2d^2\right)}{2(c+d)^{5/2}(c-d)^{5/2}\left(c^4-2c^2d^2+d^4\right)}+\frac{c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(2a^2c^2+a^2d^2-6abcd+b^2c^2+2b^2d^2\right)}{(c+d)^{5/2}(c-d)^{5/2}}\right)\left(c^4-2c^2d^2+d^4\right)}{2a^2c^2+a^2d^2-6abcd+b^2c^2+2b^2d^2}\right)\left(2a^2c^2+a^2d^2-\right.}{f(c+d)^{5/2}(c-d)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/(c + d*sin(e + f*x))^3,x)
```

```
[Out] (atan(((((2*c^4*d + 2*d^5 - 4*c^2*d^3)*(2*a^2*c^2 + a^2*d^2 + b^2*c^2 + 2*b
^2*d^2 - 6*a*b*c*d))/(2*(c + d)^(5/2)*(c - d)^(5/2)*(c^4 + d^4 - 2*c^2*d^2)
) + (c*tan(e/2 + (f*x)/2)*(2*a^2*c^2 + a^2*d^2 + b^2*c^2 + 2*b^2*d^2 - 6*a*
b*c*d))/((c + d)^(5/2)*(c - d)^(5/2)))*(c^4 + d^4 - 2*c^2*d^2))/(2*a^2*c^2
+ a^2*d^2 + b^2*c^2 + 2*b^2*d^2 - 6*a*b*c*d))*(2*a^2*c^2 + a^2*d^2 + b^2*c^
2 + 2*b^2*d^2 - 6*a*b*c*d))/(f*(c + d)^(5/2)*(c - d)^(5/2)) - ((a^2*d^3 - 4
*a^2*c^2*d - 3*b^2*c^2*d + 4*a*b*c^3 + 2*a*b*c*d^2)/(c^4 + d^4 - 2*c^2*d^2)
```

```
 + (tan(e/2 + (f*x)/2)*(2*a^2*d^4 + b^2*c^4 - 11*a^2*c^2*d^2 - 10*b^2*c^2*d
^2 + 8*a*b*c*d^3 + 10*a*b*c^3*d))/(c*(c^4 + d^4 - 2*c^2*d^2)) - (tan(e/2 +
(f*x)/2)^3*(b^2*c^4 - 2*a^2*d^4 + 5*a^2*c^2*d^2 + 2*b^2*c^2*d^2 - 6*a*b*c^3
*d))/(c*(c^4 + d^4 - 2*c^2*d^2)) + (tan(e/2 + (f*x)/2)^2*(c^2 + 2*d^2)*(a^2
*d^3 - 4*a^2*c^2*d - 3*b^2*c^2*d + 4*a*b*c^3 + 2*a*b*c*d^2))/(c^2*(c^4 + d^
4 - 2*c^2*d^2)))/(f*(tan(e/2 + (f*x)/2)^2*(2*c^2 + 4*d^2) + c^2*tan(e/2 + (
f*x)/2)^4 + c^2 + 4*c*d*tan(e/2 + (f*x)/2)^3 + 4*c*d*tan(e/2 + (f*x)/2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.685 $\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^4}\,dx$

Optimal. Leaf size=305

$$-\frac{\left(-\left(a^2\left(2c^3+3cd^2\right)\right)+2abd\left(4c^2+d^2\right)-b^2c\left(c^2+4d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{7/2}}+\frac{\left(a^2d^2\left(11c^2+4d^2\right)-ab\left(4c^3d\right.\right.}{6df\left(c^2\right.}$$

```
[Out] -(2*a*b*d*(4*c^2+d^2)-b^2*c*(c^2+4*d^2)-a^2*(2*c^3+3*c*d^2))*arctan((d+c*ta
n(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(c^2-d^2)^(7/2)/f+1/3*(-a*d+b*c)^2*cos(f
*x+e)/d/(c^2-d^2)/f/(c+d*sin(f*x+e))^3-1/6*(-a*d+b*c)*(5*a*c*d+b*(c^2-6*d^2
))*cos(f*x+e)/d/(c^2-d^2)^2/f/(c+d*sin(f*x+e))^2+1/6*(a^2*d^2*(11*c^2+4*d^2
)-a*b*(4*c^3*d+26*c*d^3)-b^2*(c^4-10*c^2*d^2-6*d^4))*cos(f*x+e)/d/(c^2-d^2)
^3/f/(c+d*sin(f*x+e))
```

Rubi [A] time = 0.56, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2790, 2754, 12, 2660, 618, 204}

$$-\frac{\left(a^2\left(-\left(2c^3+3cd^2\right)\right)+2abd\left(4c^2+d^2\right)-b^2c\left(c^2+4d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{7/2}}+\frac{\left(a^2d^2\left(11c^2+4d^2\right)-ab\left(4c^3d\right.\right.}{6df\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^4,x]
```

```
[Out] -(((2*a*b*d*(4*c^2 + d^2) - b^2*c*(c^2 + 4*d^2) - a^2*(2*c^3 + 3*c*d^2))*Ar
cTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c^2 - d^2)^(7/2)*f)) + ((
b*c - a*d)^2*Cos[e + f*x])/(3*d*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^3) - ((b
*c - a*d)*(5*a*c*d + b*(c^2 - 6*d^2))*Cos[e + f*x])/(6*d*(c^2 - d^2)^2*f*(c
 + d*Sin[e + f*x])^2) + ((a^2*d^2*(11*c^2 + 4*d^2) - a*b*(4*c^3*d + 26*c*d^
3) - b^2*(c^4 - 10*c^2*d^2 - 6*d^4))*Cos[e + f*x])/(6*d*(c^2 - d^2)^3*f*(c
+ d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^4}\,dx = \frac{(bc-ad)^2\cos(e+fx)}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^3} + \frac{\int \frac{3d\left(\left(a^2+b^2\right)c-2abd\right)+\left(4abcd-2a^2d^2+b^2\left(c^2-3d^2\right)\right)\sin(e+fx)}{(c+d\sin(e+fx))^3}}{3d\left(c^2-d^2\right)}$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^3} - \frac{(bc-ad)\left(5acd+b\left(c^2-6d^2\right)\right)\cos(e+fx)}{6d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^2} - \int$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^3} - \frac{(bc-ad)\left(5acd+b\left(c^2-6d^2\right)\right)\cos(e+fx)}{6d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^2} + (a$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^3} - \frac{(bc-ad)\left(5acd+b\left(c^2-6d^2\right)\right)\cos(e+fx)}{6d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^2} + (a$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^3} - \frac{(bc-ad)\left(5acd+b\left(c^2-6d^2\right)\right)\cos(e+fx)}{6d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^2} + (a$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^3} - \frac{(bc-ad)\left(5acd+b\left(c^2-6d^2\right)\right)\cos(e+fx)}{6d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^2} + (a$$

$$= -\frac{\left(2abd\left(4c^2+d^2\right)-b^2c\left(c^2+4d^2\right)-a^2\left(2c^3+3cd^2\right)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{7/2} f} + \frac{}{3d\,(}$$

Mathematica [A] time = 1.44, size = 346, normalized size = 1.13

$$\frac{12\left(a^2\left(2c^3+3cd^2\right)-2abd\left(4c^2+d^2\right)+b^2c\left(c^2+4d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{7/2}} + \frac{\cos(e+fx)\left(d\left(-a^2d^2\left(11c^2+4d^2\right)+ab\left(4c^3d+26cd^3\right)+b^2\left(c^4-10c^2d^2-6d^4\right)\right)\cos\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^4,x]
```

```
[Out] ((12*(-2*a*b*d*(4*c^2 + d^2) + b^2*c*(c^2 + 4*d^2) + a^2*(2*c^3 + 3*c*d^2))
*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^2)^(7/2) + (Cos
[e + f*x]*(-24*a*b*c^5 + 36*a^2*c^4*d + 25*b^2*c^4*d - 44*a*b*c^3*d^2 + a^2
```

```
*c^2*d^3 + 14*b^2*c^2*d^3 - 22*a*b*c*d^4 + 8*a^2*d^5 + 6*b^2*d^5 + d*(-(a^2
*d^2*(11*c^2 + 4*d^2)) + a*b*(4*c^3*d + 26*c*d^3) + b^2*(c^4 - 10*c^2*d^2 -
 6*d^4))*Cos[2*(e + f*x)] - 6*(-(a^2*c*d^2*(9*c^2 + d^2)) - 2*a*b*d*(-2*c^4
 - 9*c^2*d^2 + d^4) + b^2*(c^5 - 9*c^3*d^2 - 2*c*d^4))*Sin[e + f*x]))/((c^2
 - d^2)^3*(c + d*Sin[e + f*x])^3))/(12*f)
```

fricas [B] time = 0.59, size = 1724, normalized size = 5.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(2*(b^2*c^6*d + 4*a*b*c^5*d^2 + 22*a*b*c^3*d^4 - 26*a*b*c*d^6 - 11*(
a^2 + b^2)*c^4*d^3 + (7*a^2 + 4*b^2)*c^2*d^5 + 2*(2*a^2 + 3*b^2)*d^7)*cos(f
*x + e)^3 - 6*(b^2*c^7 + 4*a*b*c^6*d + 14*a*b*c^4*d^3 - 20*a*b*c^2*d^5 + 2*
a*b*d^7 - (9*a^2 + 10*b^2)*c^5*d^2 + (8*a^2 + 7*b^2)*c^3*d^4 + (a^2 + 2*b^2
)*c*d^6)*cos(f*x + e)*sin(f*x + e) + 3*(8*a*b*c^5*d + 26*a*b*c^3*d^3 + 6*a*
b*c*d^5 - (2*a^2 + b^2)*c^6 - (9*a^2 + 7*b^2)*c^4*d^2 - 3*(3*a^2 + 4*b^2)*c
^2*d^4 - 3*(8*a*b*c^3*d^3 + 2*a*b*c*d^5 - (2*a^2 + b^2)*c^4*d^2 - (3*a^2 +
4*b^2)*c^2*d^4)*cos(f*x + e)^2 + (24*a*b*c^4*d^2 + 14*a*b*c^2*d^4 + 2*a*b*d
^6 - 3*(2*a^2 + b^2)*c^5*d - (11*a^2 + 13*b^2)*c^3*d^3 - (3*a^2 + 4*b^2)*c*
d^5 - (8*a*b*c^2*d^4 + 2*a*b*d^6 - (2*a^2 + b^2)*c^3*d^3 - (3*a^2 + 4*b^2)*
c*d^5)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*co
s(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x +
 e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x
 + e) - c^2 - d^2)) - 12*(2*a*b*c^7 + 2*a*b*c^5*d^2 + 2*a^2*c^4*d^3 + b^2*c
^2*d^5 - 4*a*b*c*d^6 - (3*a^2 + 2*b^2)*c^6*d + (a^2 + b^2)*d^7)*cos(f*x + e
))/(3*(c^9*d^2 - 4*c^7*d^4 + 6*c^5*d^6 - 4*c^3*d^8 + c*d^10)*f*cos(f*x + e)
^2 - (c^11 - c^9*d^2 - 6*c^7*d^4 + 14*c^5*d^6 - 11*c^3*d^8 + 3*c*d^10)*f +
((c^8*d^3 - 4*c^6*d^5 + 6*c^4*d^7 - 4*c^2*d^9 + d^11)*f*cos(f*x + e)^2 - (3
*c^10*d - 11*c^8*d^3 + 14*c^6*d^5 - 6*c^4*d^7 - c^2*d^9 + d^11)*f)*sin(f*x
+ e)), -1/6*((b^2*c^6*d + 4*a*b*c^5*d^2 + 22*a*b*c^3*d^4 - 26*a*b*c*d^6 - 1
1*(a^2 + b^2)*c^4*d^3 + (7*a^2 + 4*b^2)*c^2*d^5 + 2*(2*a^2 + 3*b^2)*d^7)*co
s(f*x + e)^3 - 3*(b^2*c^7 + 4*a*b*c^6*d + 14*a*b*c^4*d^3 - 20*a*b*c^2*d^5 +
 2*a*b*d^7 - (9*a^2 + 10*b^2)*c^5*d^2 + (8*a^2 + 7*b^2)*c^3*d^4 + (a^2 + 2*
b^2)*c*d^6)*cos(f*x + e)*sin(f*x + e) + 3*(8*a*b*c^5*d + 26*a*b*c^3*d^3 + 6
*a*b*c*d^5 - (2*a^2 + b^2)*c^6 - (9*a^2 + 7*b^2)*c^4*d^2 - 3*(3*a^2 + 4*b^2
)*c^2*d^4 - 3*(8*a*b*c^3*d^3 + 2*a*b*c*d^5 - (2*a^2 + b^2)*c^4*d^2 - (3*a^2
 + 4*b^2)*c^2*d^4)*cos(f*x + e)^2 + (24*a*b*c^4*d^2 + 14*a*b*c^2*d^4 + 2*a*
b*d^6 - 3*(2*a^2 + b^2)*c^5*d - (11*a^2 + 13*b^2)*c^3*d^3 - (3*a^2 + 4*b^2)
*c*d^5 - (8*a*b*c^2*d^4 + 2*a*b*d^6 - (2*a^2 + b^2)*c^3*d^3 - (3*a^2 + 4*b^
2)*c*d^5)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x
+ e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) - 6*(2*a*b*c^7 + 2*a*b*c^5*d^2 +
2*a^2*c^4*d^3 + b^2*c^2*d^5 - 4*a*b*c*d^6 - (3*a^2 + 2*b^2)*c^6*d + (a^2 +
```

```
b^2)*d^7)*cos(f*x + e))/(3*(c^9*d^2 - 4*c^7*d^4 + 6*c^5*d^6 - 4*c^3*d^8 + c
*d^10)*f*cos(f*x + e)^2 - (c^11 - c^9*d^2 - 6*c^7*d^4 + 14*c^5*d^6 - 11*c^3
*d^8 + 3*c*d^10)*f + ((c^8*d^3 - 4*c^6*d^5 + 6*c^4*d^7 - 4*c^2*d^9 + d^11)*
f*cos(f*x + e)^2 - (3*c^10*d - 11*c^8*d^3 + 14*c^6*d^5 - 6*c^4*d^7 - c^2*d^
9 + d^11)*f)*sin(f*x + e))]
```

giac [B] time = 3.15, size = 1316, normalized size = 4.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^4,x, algorithm="giac")

[Out] 1/3*(3*(2*a^2*c^3 + b^2*c^3 - 8*a*b*c^2*d + 3*a^2*c*d^2 + 4*b^2*c*d^2 - 2*a
*b*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x +
1/2*e) + d)/sqrt(c^2 - d^2)))/((c^6 - 3*c^4*d^2 + 3*c^2*d^4 - d^6)*sqrt(c^2
 - d^2)) + (3*b^2*c^8*tan(1/2*f*x + 1/2*e)^5 - 24*a*b*c^7*d*tan(1/2*f*x + 1
/2*e)^5 + 27*a^2*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 + 12*b^2*c^6*d^2*tan(1/2*f*
x + 1/2*e)^5 - 6*a*b*c^5*d^3*tan(1/2*f*x + 1/2*e)^5 - 18*a^2*c^4*d^4*tan(1/
2*f*x + 1/2*e)^5 + 6*a^2*c^2*d^6*tan(1/2*f*x + 1/2*e)^5 - 12*a*b*c^8*tan(1/
2*f*x + 1/2*e)^4 + 18*a^2*c^7*d*tan(1/2*f*x + 1/2*e)^4 + 15*b^2*c^7*d*tan(1
/2*f*x + 1/2*e)^4 - 84*a*b*c^6*d^2*tan(1/2*f*x + 1/2*e)^4 + 81*a^2*c^5*d^3*
tan(1/2*f*x + 1/2*e)^4 + 60*b^2*c^5*d^3*tan(1/2*f*x + 1/2*e)^4 - 66*a*b*c^4
*d^4*tan(1/2*f*x + 1/2*e)^4 - 36*a^2*c^3*d^5*tan(1/2*f*x + 1/2*e)^4 + 12*a*
b*c^2*d^6*tan(1/2*f*x + 1/2*e)^4 + 12*a^2*c*d^7*tan(1/2*f*x + 1/2*e)^4 - 72
*a*b*c^7*d*tan(1/2*f*x + 1/2*e)^3 + 108*a^2*c^6*d^2*tan(1/2*f*x + 1/2*e)^3
+ 78*b^2*c^6*d^2*tan(1/2*f*x + 1/2*e)^3 - 168*a*b*c^5*d^3*tan(1/2*f*x + 1/2
*e)^3 + 42*a^2*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 + 64*b^2*c^4*d^4*tan(1/2*f*x
+ 1/2*e)^3 - 68*a*b*c^3*d^5*tan(1/2*f*x + 1/2*e)^3 - 8*a^2*c^2*d^6*tan(1/2*
f*x + 1/2*e)^3 + 8*b^2*c^2*d^6*tan(1/2*f*x + 1/2*e)^3 + 8*a*b*c*d^7*tan(1/2
*f*x + 1/2*e)^3 + 8*a^2*d^8*tan(1/2*f*x + 1/2*e)^3 - 24*a*b*c^8*tan(1/2*f*x
 + 1/2*e)^2 + 36*a^2*c^7*d*tan(1/2*f*x + 1/2*e)^2 + 24*b^2*c^7*d*tan(1/2*f*
x + 1/2*e)^2 - 120*a*b*c^6*d^2*tan(1/2*f*x + 1/2*e)^2 + 120*a^2*c^5*d^3*tan
(1/2*f*x + 1/2*e)^2 + 102*b^2*c^5*d^3*tan(1/2*f*x + 1/2*e)^2 - 168*a*b*c^4*
d^4*tan(1/2*f*x + 1/2*e)^2 - 18*a^2*c^3*d^5*tan(1/2*f*x + 1/2*e)^2 + 24*b^2
*c^3*d^5*tan(1/2*f*x + 1/2*e)^2 + 12*a*b*c^2*d^6*tan(1/2*f*x + 1/2*e)^2 + 1
2*a^2*c*d^7*tan(1/2*f*x + 1/2*e)^2 - 3*b^2*c^8*tan(1/2*f*x + 1/2*e) - 48*a*
b*c^7*d*tan(1/2*f*x + 1/2*e) + 81*a^2*c^6*d^2*tan(1/2*f*x + 1/2*e) + 66*b^2
*c^6*d^2*tan(1/2*f*x + 1/2*e) - 114*a*b*c^5*d^3*tan(1/2*f*x + 1/2*e) - 12*a
^2*c^4*d^4*tan(1/2*f*x + 1/2*e) + 12*b^2*c^4*d^4*tan(1/2*f*x + 1/2*e) + 12*
a*b*c^3*d^5*tan(1/2*f*x + 1/2*e) + 6*a^2*c^2*d^6*tan(1/2*f*x + 1/2*e) - 12*
a*b*c^8 + 18*a^2*c^7*d + 13*b^2*c^7*d - 20*a*b*c^6*d^2 - 5*a^2*c^5*d^3 + 2*
b^2*c^5*d^3 + 2*a*b*c^4*d^4 + 2*a^2*c^3*d^5)/((c^9 - 3*c^7*d^2 + 3*c^5*d^4
- c^3*d^6)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^3))/f
```

maple [B] time = 0.33, size = 4818, normalized size = 15.80

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^4,x)
```

```
[Out] 2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/c/(c^6-3*c^4*d^2+3*
c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)*a^2*d^6+4/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1
/2*f*x+1/2*e)*d+c)^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/c^2*tan(1/2*f*x+1/2*e)^4
*a^2*d^7-4/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^6-3*c^4
*d^2+3*c^2*d^4-d^6)*c^5*tan(1/2*f*x+1/2*e)^4*a*b+5/f/(tan(1/2*f*x+1/2*e)^2*
c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*c^4*tan(1/2*f*x
+1/2*e)^4*b^2*d+20/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c
^6-3*c^4*d^2+3*c^2*d^4-d^6)*c^2*tan(1/2*f*x+1/2*e)^4*b^2*d^3-6/f/(tan(1/2*f
*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*t
an(1/2*f*x+1/2*e)^5*a^2*d^4+2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e
)*d+c)^3/c/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^5*a^2*d^6-16/f/
(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^4/(c^6-3*c^4*d^2+3*c^
2*d^4-d^6)*tan(1/2*f*x+1/2*e)*a*b*d-38/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*
f*x+1/2*e)*d+c)^3*c^2/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)*a*b*
d^3-2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^2/(c^6-3*c^4*
d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^5*a*b*d^3-28/f/(tan(1/2*f*x+1/2*e)^2*
c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*c^3*tan(1/2*f*x
+1/2*e)^4*a*b*d^2-22/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/
(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*c*tan(1/2*f*x+1/2*e)^4*a*b*d^4+4/f/(tan(1/2*f
*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/c*t
an(1/2*f*x+1/2*e)^4*a*b*d^6-24/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*
e)*d+c)^3*c^4*d/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^3*a*b-40/f
/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^3/(c^6-3*c^4*d^2+3*c
^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^2*a*b*d^2-8/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(
1/2*f*x+1/2*e)*d+c)^3*c^4/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^
5*a*b*d+8/3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/c^2*d^7/(
c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^3*a*b-56/f/(tan(1/2*f*x+1/2
*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^2*d^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*t
an(1/2*f*x+1/2*e)^3*a*b-8/f/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/(c^2-d^2)^(1/2)*a
rctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a*b*c^2*d-56/f/(tan
(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c/(c^6-3*c^4*d^2+3*c^2*d^4-
d^6)*tan(1/2*f*x+1/2*e)^2*a*b*d^4+4/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x
+1/2*e)*d+c)^3/c/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^2*a*b*d^6
+2/3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^6-3*c^4*d^2+3
*c^2*d^4-d^6)*b^2*c^2*d^3-12/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)
*d+c)^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^4*a^2*d^5+2/f/(c^6
-3*c^4*d^2+3*c^2*d^4-d^6)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e
```

)+2*d)/(c^2-d^2)^(1/2))*a^2*c^3+1/f/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/(c^2-d^2)
^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b^2*c^3-6/f
/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^6-3*c^4*d^2+3*c^2*d
^4-d^6)*tan(1/2*f*x+1/2*e)^2*a^2*d^5+8/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*
f*x+1/2*e)*d+c)^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^2*b^2*d^
5+1/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^5/(c^6-3*c^4*d^
2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^5*b^2-1/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan
(1/2*f*x+1/2*e)*d+c)^3*c^5/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)
*b^2+6/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^6-3*c^4*d^2
+3*c^2*d^4-d^6)*a^2*c^4*d-5/3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e
)*d+c)^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*a^2*c^2*d^3+4/f/(tan(1/2*f*x+1/2*e)^
2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f
*x+1/2*e)^5*b^2*d^2+6/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3
/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*c^4*tan(1/2*f*x+1/2*e)^4*a^2*d+27/f/(tan(1/2
*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*c
^2*tan(1/2*f*x+1/2*e)^4*a^2*d^3-8/3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x
+1/2*e)*d+c)^3/c*d^6/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^3*a^2
+8/3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/c^3*d^8/(c^6-3*c
^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^3*a^2+26/f/(tan(1/2*f*x+1/2*e)^2*c
+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^3*d^2/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*
f*x+1/2*e)^3*b^2+36/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c
^3*d^2/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^3*a^2+14/f/(tan(1/2
*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c*d^4/(c^6-3*c^4*d^2+3*c^2*d^4-
d^6)*tan(1/2*f*x+1/2*e)^3*a^2+4/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2
*e)*d+c)^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)*a*b*d^5-68/3/f/
(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*d^5/(c^6-3*c^4*d^2+3*c^
2*d^4-d^6)*tan(1/2*f*x+1/2*e)^3*a*b-20/3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/
2*f*x+1/2*e)*d+c)^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*a*b*c^3*d^2+2/3/f/(tan(1/
2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*
a*b*c*d^4+22/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^3/(c^6
-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)*b^2*d^2+4/f/(tan(1/2*f*x+1/2*e
)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f
*x+1/2*e)*b^2*d^4+3/f/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/(c^2-d^2)^(1/2)*arctan(
1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a^2*c*d^2-2/f/(c^6-3*c^4*
d^2+3*c^2*d^4-d^6)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/
(c^2-d^2)^(1/2))*a*b*d^3+64/3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e
)*d+c)^3*c*d^4/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^3*b^2+4/f/(
c^6-3*c^4*d^2+3*c^2*d^4-d^6)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/
2*e)+2*d)/(c^2-d^2)^(1/2))*b^2*c*d^2+9/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*
f*x+1/2*e)*d+c)^3*c^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^5*a^
2*d^2+8/3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/c*d^6/(c^6-
3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^3*b^2+12/f/(tan(1/2*f*x+1/2*e)^
2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^4/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f
*x+1/2*e)^2*a^2*d+40/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*
c^2/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^2*a^2*d^3+4/f/(tan(1/2

```
*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/c^2/(c^6-3*c^4*d^2+3*c^2*d^4-d^
6)*tan(1/2*f*x+1/2*e)^2*a^2*d^7-8/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1
/2*e)*d+c)^3*c^5/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^2*a*b+8/f
/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^4/(c^6-3*c^4*d^2+3*c
^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^2*b^2*d+34/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1
/2*f*x+1/2*e)*d+c)^3*c^2/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)^2
*b^2*d^3+27/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c^3/(c^6-
3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*x+1/2*e)*a^2*d^2-4/f/(tan(1/2*f*x+1/2*e)
^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3*c/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*tan(1/2*f*
x+1/2*e)*a^2*d^4+2/3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/
(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*a^2*d^5-4/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2
*f*x+1/2*e)*d+c)^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*a*b*c^5+13/3/f/(tan(1/2*f*
x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)*b^2*
c^4*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 11.16, size = 1220, normalized size = 4.00

$$\frac{\frac{18\,a^2\,c^4\,d-5\,a^2\,c^2\,d^3+2\,a^2\,d^5-12\,a\,b\,c^5-20\,a\,b\,c^3\,d^2+2\,a\,b\,c\,d^4+13\,b^2\,c^4\,d+2\,b^2\,c^2\,d^3}{3\,\left(c^6-3\,c^4\,d^2+3\,c^2\,d^4-d^6\right)}+\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(6\,a^2\,c^6\,d+27\,a^2\,c^4\,d^3-12\,a^2\,c^2\,d^5+4\,a^2\,d^7-4\,a\,b\,c^7}{c^2\,\left(c^6-3\,c^4\,d^2+3\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/(c + d*sin(e + f*x))^4,x)
```

```
[Out] ((2*a^2*d^5 + 18*a^2*c^4*d + 13*b^2*c^4*d - 5*a^2*c^2*d^3 + 2*b^2*c^2*d^3 -
 12*a*b*c^5 + 2*a*b*c*d^4 - 20*a*b*c^3*d^2)/(3*(c^6 - d^6 + 3*c^2*d^4 - 3*c
^4*d^2)) + (tan(e/2 + (f*x)/2)^4*(4*a^2*d^7 + 6*a^2*c^6*d + 5*b^2*c^6*d - 1
2*a^2*c^2*d^5 + 27*a^2*c^4*d^3 + 20*b^2*c^4*d^3 - 4*a*b*c^7 + 4*a*b*c*d^6 -
```

```
22*a*b*c^3*d^4 - 28*a*b*c^5*d^2))/(c^2*(c^6 - d^6 + 3*c^2*d^4 - 3*c^4*d^2)
) + (tan(e/2 + (f*x)/2)*(2*a^2*d^6 - b^2*c^6 - 4*a^2*c^2*d^4 + 27*a^2*c^4*d
^2 + 4*b^2*c^2*d^4 + 22*b^2*c^4*d^2 + 4*a*b*c*d^5 - 16*a*b*c^5*d - 38*a*b*c
^3*d^3))/(c*(c^6 - d^6 + 3*c^2*d^4 - 3*c^4*d^2)) + (2*tan(e/2 + (f*x)/2)^2*
(2*a^2*d^7 + 6*a^2*c^6*d + 4*b^2*c^6*d - 3*a^2*c^2*d^5 + 20*a^2*c^4*d^3 + 4
*b^2*c^2*d^5 + 17*b^2*c^4*d^3 - 4*a*b*c^7 + 2*a*b*c*d^6 - 28*a*b*c^3*d^4 -
20*a*b*c^5*d^2))/(c^2*(c^6 - d^6 + 3*c^2*d^4 - 3*c^4*d^2)) + (tan(e/2 + (f*
x)/2)^5*(2*a^2*d^6 + b^2*c^6 - 6*a^2*c^2*d^4 + 9*a^2*c^4*d^2 + 4*b^2*c^4*d^
2 - 8*a*b*c^5*d - 2*a*b*c^3*d^3))/(c*(c^6 - d^6 + 3*c^2*d^4 - 3*c^4*d^2)) +
 (2*d*tan(e/2 + (f*x)/2)^3*(3*c^2 + 2*d^2)*(2*a^2*d^5 + 18*a^2*c^4*d + 13*b
^2*c^4*d - 5*a^2*c^2*d^3 + 2*b^2*c^2*d^3 - 12*a*b*c^5 + 2*a*b*c*d^4 - 20*a*
b*c^3*d^2))/(3*c^3*(c^6 - d^6 + 3*c^2*d^4 - 3*c^4*d^2)))/(f*(c^3*tan(e/2 +
(f*x)/2)^6 + tan(e/2 + (f*x)/2)^2*(12*c*d^2 + 3*c^3) + tan(e/2 + (f*x)/2)^4
*(12*c*d^2 + 3*c^3) + tan(e/2 + (f*x)/2)^3*(12*c^2*d + 8*d^3) + c^3 + 6*c^2
*d*tan(e/2 + (f*x)/2) + 6*c^2*d*tan(e/2 + (f*x)/2)^5)) + (atan((((c*tan(e/2
 + (f*x)/2)*(2*a^2*c^3 + b^2*c^3 + 3*a^2*c*d^2 + 4*b^2*c*d^2 - 2*a*b*d^3 -
8*a*b*c^2*d))/((c + d)^(7/2)*(c - d)^(7/2)) + ((2*c^6*d - 2*d^7 + 6*c^2*d^5
 - 6*c^4*d^3)*(2*a^2*c^3 + b^2*c^3 + 3*a^2*c*d^2 + 4*b^2*c*d^2 - 2*a*b*d^3
- 8*a*b*c^2*d))/(2*(c + d)^(7/2)*(c - d)^(7/2)*(c^6 - d^6 + 3*c^2*d^4 - 3*c
^4*d^2)))*(c^6 - d^6 + 3*c^2*d^4 - 3*c^4*d^2))/(2*a^2*c^3 + b^2*c^3 + 3*a^2
*c*d^2 + 4*b^2*c*d^2 - 2*a*b*d^3 - 8*a*b*c^2*d))*(2*a^2*c^3 + b^2*c^3 + 3*a
^2*c*d^2 + 4*b^2*c*d^2 - 2*a*b*d^3 - 8*a*b*c^2*d))/(f*(c + d)^(7/2)*(c - d)
^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**4,x)
```

```
[Out] Timed out
```

3.686 $\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=400

$$\frac{(ad+bc)\left(a^2d^2+8abcd+b^2\left(c^2+6d^2\right)\right)\cos^3(e+fx)}{3f}-\frac{3bd\left(a^2d^2+3abcd+b^2c^2\right)\sin^3(e+fx)\cos(e+fx)}{4f}-\frac{(24$$

```
[Out] 1/16*(18*a^2*b*d*(4*c^2+d^2)+b^3*d*(18*c^2+5*d^2)+6*a*b^2*c*(4*c^2+9*d^2)+8
*a^3*(2*c^3+3*c*d^2))*x-(3*a*b^2*d*(3*c^2+d^2)+3*a^2*b*c*(c^2+3*d^2)+b^3*c*
(c^2+3*d^2)+a^3*(3*c^2*d+d^3))*cos(f*x+e)/f+1/3*(a*d+b*c)*(8*a*b*c*d+a^2*d^
2+b^2*(c^2+6*d^2))*cos(f*x+e)^3/f-3/5*b^2*d^2*(a*d+b*c)*cos(f*x+e)^5/f-1/16
*(24*a^3*c*d^2+18*a^2*b*d*(4*c^2+d^2)+b^3*d*(18*c^2+5*d^2)+6*a*b^2*c*(4*c^2
+9*d^2))*cos(f*x+e)*sin(f*x+e)/f-5/24*b^3*d^3*cos(f*x+e)*sin(f*x+e)^3/f-3/4
*b*d*(a^2*d^2+3*a*b*c*d+b^2*c^2)*cos(f*x+e)*sin(f*x+e)^3/f-1/6*b^3*d^3*cos(
f*x+e)*sin(f*x+e)^5/f
```

Rubi [A] time = 0.95, antiderivative size = 493, normalized size of antiderivative = 1.23, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2793, 3023, 2753, 2734}

$$-\frac{\left(90a^2bcd^2\left(c^2+4d^2\right)+40a^3d^3\left(4c^2+d^2\right)-6ab^2d\left(-52c^2d^2+3c^4-16d^4\right)+b^3\left(17c^3d^2+2c^5+96cd^4\right)\right)\cos(e}{60d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((18*a^2*b*d*(4*c^2 + d^2) + b^3*d*(18*c^2 + 5*d^2) + 6*a*b^2*c*(4*c^2 + 9*
d^2) + 8*a^3*(2*c^3 + 3*c*d^2))*x)/16 - ((40*a^3*d^3*(4*c^2 + d^2) + 90*a^2
*b*c*d^2*(c^2 + 4*d^2) - 6*a*b^2*d*(3*c^4 - 52*c^2*d^2 - 16*d^4) + b^3*(2*c
^5 + 17*c^3*d^2 + 96*c*d^4))*Cos[e + f*x])/(60*d^2*f) - ((200*a^3*c*d^3 + 9
0*a^2*b*d^2*(2*c^2 + 3*d^2) - 6*a*b^2*d*(6*c^3 - 71*c*d^2) + b^3*(4*c^4 + 3
6*c^2*d^2 + 75*d^4))*Cos[e + f*x]*Sin[e + f*x])/(240*d*f) - ((90*a^2*b*c*d^
2 + 40*a^3*d^3 + b^3*(2*c^3 + 21*c*d^2) - a*b^2*(18*c^2*d - 96*d^3))*Cos[e
+ f*x]*(c + d*Sin[e + f*x])^2)/(120*d^2*f) + (b*(18*a*b*c*d - 90*a^2*d^2 -
b^2*(2*c^2 + 25*d^2))*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(120*d^2*f) + (b
^2*(2*b*c - 13*a*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(30*d^2*f) - (b^2*
Cos[e + f*x]*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^4)/(6*d*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))^3\,dx = -\frac{b^2\cos(e+fx)(a+b\sin(e+fx))(c+d\sin(e+fx))^4}{6df} + \frac{\int(c+\ldots}{}$$

$$= \frac{b^2(2bc-13ad)\cos(e+fx)(c+d\sin(e+fx))^4}{30d^2f} - \frac{b^2\cos(e+fx)\ldots}{}$$

$$= \frac{b\left(18abcd-90a^2d^2-b^2\left(2c^2+25d^2\right)\right)\cos(e+fx)(c+d\sin(e+\ldots}{120d^2f}$$

$$= -\frac{\left(90a^2bcd^2+40a^3d^3+b^3\left(2c^3+21cd^2\right)-ab^2\left(18c^2d-96d^3\right)\right)\ldots}{120d^2f}$$

$$= \frac{1}{16}\left(18a^2bd\left(4c^2+d^2\right)+b^3d\left(18c^2+5d^2\right)+6ab^2c\left(4c^2+9d^2\right)+\ldots\right.$$

Mathematica [A] time = 1.18, size = 552, normalized size = 1.38

$$\frac{960a^3c^3e+960a^3c^3fx-720a^3cd^2\sin(2(e+fx))+1440a^3cd^2e+1440a^3cd^2fx-2160a^2bc^2d\sin(2(e+fx))+4\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (960*a^3*c^3*e + 1440*a*b^2*c^3*e + 4320*a^2*b*c^2*d*e + 1080*b^3*c^2*d*e +
 1440*a^3*c*d^2*e + 3240*a*b^2*c*d^2*e + 1080*a^2*b*d^3*e + 300*b^3*d^3*e +
 960*a^3*c^3*f*x + 1440*a*b^2*c^3*f*x + 4320*a^2*b*c^2*d*f*x + 1080*b^3*c^2
*d*f*x + 1440*a^3*c*d^2*f*x + 3240*a*b^2*c*d^2*f*x + 1080*a^2*b*d^3*f*x + 3
00*b^3*d^3*f*x - 360*(b^3*c*(2*c^2 + 5*d^2) + a*b^2*d*(18*c^2 + 5*d^2) + 2*
a^2*b*c*(4*c^2 + 9*d^2) + 2*a^3*(4*c^2*d + d^3))*Cos[e + f*x] + 20*(36*a^2*
b*c*d^2 + 4*a^3*d^3 + 3*a*b^2*d*(12*c^2 + 5*d^2) + b^3*(4*c^3 + 15*c*d^2))*
Cos[3*(e + f*x)] - 36*b^3*c*d^2*Cos[5*(e + f*x)] - 36*a*b^2*d^3*Cos[5*(e +
f*x)] - 720*a*b^2*c^3*Sin[2*(e + f*x)] - 2160*a^2*b*c^2*d*Sin[2*(e + f*x)]
- 720*b^3*c^2*d*Sin[2*(e + f*x)] - 720*a^3*c*d^2*Sin[2*(e + f*x)] - 2160*a*
b^2*c*d^2*Sin[2*(e + f*x)] - 720*a^2*b*d^3*Sin[2*(e + f*x)] - 225*b^3*d^3*S
in[2*(e + f*x)] + 90*b^3*c^2*d*Sin[4*(e + f*x)] + 270*a*b^2*c*d^2*Sin[4*(e
+ f*x)] + 90*a^2*b*d^3*Sin[4*(e + f*x)] + 45*b^3*d^3*Sin[4*(e + f*x)] - 5*b
^3*d^3*Sin[6*(e + f*x)])/(960*f)
```

fricas [A] time = 0.48, size = 375, normalized size = 0.94

$$-\frac{144\left(b^3cd^2+ab^2d^3\right)\cos\left(fx+e\right)^5-80\left(b^3c^3+9\,ab^2c^2d+3\left(3\,a^2b+2\,b^3\right)cd^2+\left(a^3+6\,ab^2\right)d^3\right)\cos\left(fx+e\right)^3\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/240*(144*(b^3*c*d^2 + a*b^2*d^3)*cos(f*x + e)^5 - 80*(b^3*c^3 + 9*a*b^2*
c^2*d + 3*(3*a^2*b + 2*b^3)*c*d^2 + (a^3 + 6*a*b^2)*d^3)*cos(f*x + e)^3 - 1
5*(8*(2*a^3 + 3*a*b^2)*c^3 + 18*(4*a^2*b + b^3)*c^2*d + 6*(4*a^3 + 9*a*b^2)
*c*d^2 + (18*a^2*b + 5*b^3)*d^3)*f*x + 240*((3*a^2*b + b^3)*c^3 + 3*(a^3 +
3*a*b^2)*c^2*d + 3*(3*a^2*b + b^3)*c*d^2 + (a^3 + 3*a*b^2)*d^3)*cos(f*x + e
) + 5*(8*b^3*d^3*cos(f*x + e)^5 - 2*(18*b^3*c^2*d + 54*a*b^2*c*d^2 + (18*a^
2*b + 13*b^3)*d^3)*cos(f*x + e)^3 + 3*(24*a*b^2*c^3 + 6*(12*a^2*b + 5*b^3)*
c^2*d + 6*(4*a^3 + 15*a*b^2)*c*d^2 + (30*a^2*b + 11*b^3)*d^3)*cos(f*x + e))
*sin(f*x + e))/f
```

giac [A] time = 0.22, size = 416, normalized size = 1.04

$$-\frac{b^3d^3\sin\left(6\,fx+6\,e\right)}{192\,f}+\frac{1}{16}\left(16\,a^3c^3+24\,ab^2c^3+72\,a^2bc^2d+18\,b^3c^2d+24\,a^3cd^2+54\,ab^2cd^2+18\,a^2bd^3+5\,b^3d^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/192*b^3*d^3*sin(6*f*x + 6*e)/f + 1/16*(16*a^3*c^3 + 24*a*b^2*c^3 + 72*a^
2*b*c^2*d + 18*b^3*c^2*d + 24*a^3*c*d^2 + 54*a*b^2*c*d^2 + 18*a^2*b*d^3 + 5
*b^3*d^3)*x - 3/80*(b^3*c*d^2 + a*b^2*d^3)*cos(5*f*x + 5*e)/f + 1/48*(4*b^3
*c^3 + 36*a*b^2*c^2*d + 36*a^2*b*c*d^2 + 15*b^3*c*d^2 + 4*a^3*d^3 + 15*a*b^
2*d^3)*cos(3*f*x + 3*e)/f - 3/8*(8*a^2*b*c^3 + 2*b^3*c^3 + 8*a^3*c^2*d + 18
*a*b^2*c^2*d + 18*a^2*b*c*d^2 + 5*b^3*c*d^2 + 2*a^3*d^3 + 5*a*b^2*d^3)*cos(
f*x + e)/f + 3/64*(2*b^3*c^2*d + 6*a*b^2*c*d^2 + 2*a^2*b*d^3 + b^3*d^3)*sin
(4*f*x + 4*e)/f - 3/64*(16*a*b^2*c^3 + 48*a^2*b*c^2*d + 16*b^3*c^2*d + 16*a
^3*c*d^2 + 48*a*b^2*c*d^2 + 16*a^2*b*d^3 + 5*b^3*d^3)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.33, size = 489, normalized size = 1.22

$$c^3a^3\left(fx+e\right)-3a^3c^2d\cos\left(fx+e\right)+3a^3c\,d^2\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{a^3d^3\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}-3a^2b\,c^3\,\mathrm{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(c^3*a^3*(f*x+e)-3*a^3*c^2*d*cos(f*x+e)+3*a^3*c*d^2*(-1/2*sin(f*x+e)*co
s(f*x+e)+1/2*f*x+1/2*e)-1/3*a^3*d^3*(2+sin(f*x+e)^2)*cos(f*x+e)-3*a^2*b*c^3
```

```
*cos(f*x+e)+9*a^2*b*c^2*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-3*a^2*
b*c*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a^2*b*d^3*(-1/4*(sin(f*x+e)^3+3/2*sin
(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+3*a*b^2*c^3*(-1/2*sin(f*x+e)*cos(f*x+e)+
1/2*f*x+1/2*e)-3*a*b^2*c^2*d*(2+sin(f*x+e)^2)*cos(f*x+e)+9*a*b^2*c*d^2*(-1/
4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3/5*a*b^2*d^3*(8/
3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-1/3*b^3*c^3*(2+sin(f*x+e)^2)*co
s(f*x+e)+3*b^3*c^2*d*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x
+3/8*e)-3/5*b^3*c*d^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+b^3*d^
3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x
+5/16*e))
```

maxima [A] time = 0.73, size = 477, normalized size = 1.19

$$\frac{960\left(fx+e\right)a^3c^3+720\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)ab^2c^3+320\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)b^3c^3+2160\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/960*(960*(f*x + e)*a^3*c^3 + 720*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*b^2*c
^3 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*b^3*c^3 + 2160*(2*f*x + 2*e - si
n(2*f*x + 2*e))*a^2*b*c^2*d + 2880*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*b^2*
c^2*d + 90*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*b^3*c^2*
d + 720*(2*f*x + 2*e - sin(2*f*x + 2*e))*a^3*c*d^2 + 2880*(cos(f*x + e)^3 -
 3*cos(f*x + e))*a^2*b*c*d^2 + 270*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*si
n(2*f*x + 2*e))*a*b^2*c*d^2 - 192*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 1
5*cos(f*x + e))*b^3*c*d^2 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*a^3*d^3 +
 90*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*a^2*b*d^3 - 192
*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*a*b^2*d^3 + 5*(4*
sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*
e))*b^3*d^3 - 2880*a^2*b*c^3*cos(f*x + e) - 2880*a^3*c^2*d*cos(f*x + e))/f
```

mupad [B] time = 8.92, size = 574, normalized size = 1.44

$$-\frac{180\,a^3\,d^3\,\cos\left(e+f\,x\right)+180\,b^3\,c^3\,\cos\left(e+f\,x\right)-20\,a^3\,d^3\,\cos\left(3\,e+3\,f\,x\right)-20\,b^3\,c^3\,\cos\left(3\,e+3\,f\,x\right)+\frac{225\,b^3}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x))^3,x)
```

```
[Out] -(180*a^3*d^3*cos(e + f*x) + 180*b^3*c^3*cos(e + f*x) - 20*a^3*d^3*cos(3*e
+ 3*f*x) - 20*b^3*c^3*cos(3*e + 3*f*x) + (225*b^3*d^3*sin(2*e + 2*f*x))/4 -
 (45*b^3*d^3*sin(4*e + 4*f*x))/4 + (5*b^3*d^3*sin(6*e + 6*f*x))/4 - 75*a*b^
```

```
2*d^3*cos(3*e + 3*f*x) + 9*a*b^2*d^3*cos(5*e + 5*f*x) - 75*b^3*c*d^2*cos(3*
e + 3*f*x) + 9*b^3*c*d^2*cos(5*e + 5*f*x) + 180*a*b^2*c^3*sin(2*e + 2*f*x)
+ 180*a^2*b*d^3*sin(2*e + 2*f*x) - (45*a^2*b*d^3*sin(4*e + 4*f*x))/2 + 180*
a^3*c*d^2*sin(2*e + 2*f*x) + 180*b^3*c^2*d*sin(2*e + 2*f*x) - (45*b^3*c^2*d
*sin(4*e + 4*f*x))/2 + 720*a^2*b*c^3*cos(e + f*x) + 450*a*b^2*d^3*cos(e + f
*x) + 720*a^3*c^2*d*cos(e + f*x) + 450*b^3*c*d^2*cos(e + f*x) - 240*a^3*c^3
*f*x - 75*b^3*d^3*f*x + 1620*a*b^2*c^2*d*cos(e + f*x) + 1620*a^2*b*c*d^2*co
s(e + f*x) - 360*a*b^2*c^3*f*x - 270*a^2*b*d^3*f*x - 360*a^3*c*d^2*f*x - 27
0*b^3*c^2*d*f*x - 180*a*b^2*c^2*d*cos(3*e + 3*f*x) - 180*a^2*b*c*d^2*cos(3*
e + 3*f*x) + 540*a*b^2*c*d^2*sin(2*e + 2*f*x) + 540*a^2*b*c^2*d*sin(2*e + 2
*f*x) - (135*a*b^2*c*d^2*sin(4*e + 4*f*x))/2 - 810*a*b^2*c*d^2*f*x - 1080*a
^2*b*c^2*d*f*x)/(240*f)
```

sympy [A] time = 8.03, size = 1217, normalized size = 3.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((a**3*c**3*x - 3*a**3*c**2*d*cos(e + f*x)/f + 3*a**3*c*d**2*x*sin
(e + f*x)**2/2 + 3*a**3*c*d**2*x*cos(e + f*x)**2/2 - 3*a**3*c*d**2*sin(e +
f*x)*cos(e + f*x)/(2*f) - a**3*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 2*a**3
*d**3*cos(e + f*x)**3/(3*f) - 3*a**2*b*c**3*cos(e + f*x)/f + 9*a**2*b*c**2*
d*x*sin(e + f*x)**2/2 + 9*a**2*b*c**2*d*x*cos(e + f*x)**2/2 - 9*a**2*b*c**2
*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 9*a**2*b*c*d**2*sin(e + f*x)**2*cos(e
+ f*x)/f - 6*a**2*b*c*d**2*cos(e + f*x)**3/f + 9*a**2*b*d**3*x*sin(e + f*x)
**4/8 + 9*a**2*b*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 9*a**2*b*d**3*x
*cos(e + f*x)**4/8 - 15*a**2*b*d**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 9*
a**2*b*d**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) + 3*a*b**2*c**3*x*sin(e + f*
x)**2/2 + 3*a*b**2*c**3*x*cos(e + f*x)**2/2 - 3*a*b**2*c**3*sin(e + f*x)*co
s(e + f*x)/(2*f) - 9*a*b**2*c**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 6*a*b**
2*c**2*d*cos(e + f*x)**3/f + 27*a*b**2*c*d**2*x*sin(e + f*x)**4/8 + 27*a*b*
*2*c*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 27*a*b**2*c*d**2*x*cos(e +
f*x)**4/8 - 45*a*b**2*c*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 27*a*b**2
*c*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 3*a*b**2*d**3*sin(e + f*x)**4*
cos(e + f*x)/f - 4*a*b**2*d**3*sin(e + f*x)**2*cos(e + f*x)**3/f - 8*a*b**2
*d**3*cos(e + f*x)**5/(5*f) - b**3*c**3*sin(e + f*x)**2*cos(e + f*x)/f - 2*
b**3*c**3*cos(e + f*x)**3/(3*f) + 9*b**3*c**2*d*x*sin(e + f*x)**4/8 + 9*b**
3*c**2*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 9*b**3*c**2*d*x*cos(e + f*x)
**4/8 - 15*b**3*c**2*d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 9*b**3*c**2*d*s
in(e + f*x)*cos(e + f*x)**3/(8*f) - 3*b**3*c*d**2*sin(e + f*x)**4*cos(e + f
*x)/f - 4*b**3*c*d**2*sin(e + f*x)**2*cos(e + f*x)**3/f - 8*b**3*c*d**2*cos
(e + f*x)**5/(5*f) + 5*b**3*d**3*x*sin(e + f*x)**6/16 + 15*b**3*d**3*x*sin(
e + f*x)**4*cos(e + f*x)**2/16 + 15*b**3*d**3*x*sin(e + f*x)**2*cos(e + f*x
```

```
)**4/16 + 5*b**3*d**3*x*cos(e + f*x)**6/16 - 11*b**3*d**3*sin(e + f*x)**5*c
os(e + f*x)/(16*f) - 5*b**3*d**3*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 5*
b**3*d**3*sin(e + f*x)*cos(e + f*x)**5/(16*f), Ne(f, 0)), (x*(a + b*sin(e))
**3*(c + d*sin(e))**3, True))
```

3.687 $\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=315

$$-\frac{\left(-6a^3d^2+60a^2bcd+ab^2\left(100c^2+71d^2\right)+90b^3cd\right)\sin(e+fx)\cos(e+fx)}{120f}+\frac{1}{8}x\left(4a^3\left(2c^2+d^2\right)+24a^2bcd+3a\right.$$

```
[Out] 1/8*(24*a^2*b*c*d+6*b^3*c*d+4*a^3*(2*c^2+d^2)+3*a*b^2*(4*c^2+3*d^2))*x-1/30
*(30*a^3*b*c*d+120*a*b^3*c*d-3*a^4*d^2+4*b^4*(5*c^2+4*d^2)+4*a^2*b^2*(20*c^
2+13*d^2))*cos(f*x+e)/b/f-1/120*(60*a^2*b*c*d+90*b^3*c*d-6*a^3*d^2+a*b^2*(1
00*c^2+71*d^2))*cos(f*x+e)*sin(f*x+e)/f-1/60*(3*a*d*(-a*d+10*b*c)+4*b^2*(5*
c^2+4*d^2))*cos(f*x+e)*(a+b*sin(f*x+e))^2/b/f-1/20*d*(-a*d+10*b*c)*cos(f*x+
e)*(a+b*sin(f*x+e))^3/b/f-1/5*d^2*cos(f*x+e)*(a+b*sin(f*x+e))^4/b/f
```

Rubi [A] time = 0.46, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2791, 2753, 2734}

$$-\frac{\left(4a^2b^2\left(20c^2+13d^2\right)+30a^3bcd-3a^4d^2+120ab^3cd+4b^4\left(5c^2+4d^2\right)\right)\cos(e+fx)}{30bf}-\frac{\left(60a^2bcd-6a^3d^2+ab^2\left(1\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] ((24*a^2*b*c*d + 6*b^3*c*d + 4*a^3*(2*c^2 + d^2) + 3*a*b^2*(4*c^2 + 3*d^2))
*x)/8 - ((30*a^3*b*c*d + 120*a*b^3*c*d - 3*a^4*d^2 + 4*b^4*(5*c^2 + 4*d^2)
+ 4*a^2*b^2*(20*c^2 + 13*d^2))*Cos[e + f*x])/(30*b*f) - ((60*a^2*b*c*d + 90
*b^3*c*d - 6*a^3*d^2 + a*b^2*(100*c^2 + 71*d^2))*Cos[e + f*x]*Sin[e + f*x])
/(120*f) - ((3*a*d*(10*b*c - a*d) + 4*b^2*(5*c^2 + 4*d^2))*Cos[e + f*x]*(a
+ b*Sin[e + f*x])^2)/(60*b*f) - (d*(10*b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^3)/(20*b*f) - (d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^4)/(5*b*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
```

```
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))^2\,dx = -\frac{d^2\cos(e+fx)(a+b\sin(e+fx))^4}{5bf}+\frac{\int (a+b\sin(e+fx))^3\left(b\left(\right.\right.}{}$$

$$= -\frac{d(10bc-ad)\cos(e+fx)(a+b\sin(e+fx))^3}{20bf}-\frac{d^2\cos(e+fx)(}{}$$

$$= -\frac{\left(3ad(10bc-ad)+4b^2\left(5c^2+4d^2\right)\right)\cos(e+fx)(a+b\sin(e+f}{60bf}$$

$$= \frac{1}{8}\left(24a^2bcd+6b^3cd+4a^3\left(2c^2+d^2\right)+3ab^2\left(4c^2+3d^2\right)\right)x-\frac{(3}{}$$

Mathematica [A] time = 1.59, size = 246, normalized size = 0.78

$$\frac{10b\left(12a^2d^2+24abcd+b^2\left(4c^2+5d^2\right)\right)\cos(3(e+fx))+15\left(4(e+fx)\left(4a^3\left(2c^2+d^2\right)+24a^2bcd+3ab^2\left(4c^2+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (-60*(16*a^3*c*d + 36*a*b^2*c*d + 6*a^2*b*(4*c^2 + 3*d^2) + b^3*(6*c^2 + 5*
d^2))*Cos[e + f*x] + 10*b*(24*a*b*c*d + 12*a^2*d^2 + b^2*(4*c^2 + 5*d^2))*C
os[3*(e + f*x)] - 6*b^3*d^2*Cos[5*(e + f*x)] + 15*(4*(24*a^2*b*c*d + 6*b^3*
c*d + 4*a^3*(2*c^2 + d^2) + 3*a*b^2*(4*c^2 + 3*d^2))*(e + f*x) - 8*(6*a^2*b
*c*d + 2*b^3*c*d + a^3*d^2 + 3*a*b^2*(c^2 + d^2))*Sin[2*(e + f*x)] + b^2*d*
(2*b*c + 3*a*d)*Sin[4*(e + f*x)]))/(480*f)
```

fricas [A] time = 0.46, size = 253, normalized size = 0.80

$$-\frac{24\,b^3d^2\cos\left(fx+e\right)^5-40\left(b^3c^2+6\,ab^2cd+\left(3\,a^2b+2\,b^3\right)d^2\right)\cos\left(fx+e\right)^3-15\left(4\left(2\,a^3+3\,ab^2\right)c^2+6\left(4\,a^2b+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/120*(24*b^3*d^2*cos(f*x + e)^5 - 40*(b^3*c^2 + 6*a*b^2*c*d + (3*a^2*b +
2*b^3)*d^2)*cos(f*x + e)^3 - 15*(4*(2*a^3 + 3*a*b^2)*c^2 + 6*(4*a^2*b + b^3
)*c*d + (4*a^3 + 9*a*b^2)*d^2)*f*x + 120*((3*a^2*b + b^3)*c^2 + 2*(a^3 + 3*
a*b^2)*c*d + (3*a^2*b + b^3)*d^2)*cos(f*x + e) - 15*(2*(2*b^3*c*d + 3*a*b^2
*d^2)*cos(f*x + e)^3 - (12*a*b^2*c^2 + 2*(12*a^2*b + 5*b^3)*c*d + (4*a^3 +
15*a*b^2)*d^2)*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 1.14, size = 274, normalized size = 0.87

$$-\frac{b^3d^2\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{1}{8}\left(8\,a^3c^2+12\,ab^2c^2+24\,a^2bcd+6\,b^3cd+4\,a^3d^2+9\,ab^2d^2\right)x+\frac{\left(4\,b^3c^2+24\,ab^2cd+12\,a^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/80*b^3*d^2*cos(5*f*x + 5*e)/f + 1/8*(8*a^3*c^2 + 12*a*b^2*c^2 + 24*a^2*b
*c*d + 6*b^3*c*d + 4*a^3*d^2 + 9*a*b^2*d^2)*x + 1/48*(4*b^3*c^2 + 24*a*b^2*
c*d + 12*a^2*b*d^2 + 5*b^3*d^2)*cos(3*f*x + 3*e)/f - 1/8*(24*a^2*b*c^2 + 6*
b^3*c^2 + 16*a^3*c*d + 36*a*b^2*c*d + 18*a^2*b*d^2 + 5*b^3*d^2)*cos(f*x + e
)/f + 1/32*(2*b^3*c*d + 3*a*b^2*d^2)*sin(4*f*x + 4*e)/f - 1/4*(3*a*b^2*c^2
+ 6*a^2*b*c*d + 2*b^3*c*d + a^3*d^2 + 3*a*b^2*d^2)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.32, size = 325, normalized size = 1.03

$$\frac{a^3c^2\left(fx+e\right)-2a^3cd\cos\left(fx+e\right)+a^3d^2\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-3a^2b\,c^2\cos\left(fx+e\right)+6a^2bcd\left(-\frac{\sin\left(fx+\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(a^3*c^2*(f*x+e)-2*a^3*c*d*cos(f*x+e)+a^3*d^2*(-1/2*sin(f*x+e)*cos(f*x+
e)+1/2*f*x+1/2*e)-3*a^2*b*c^2*cos(f*x+e)+6*a^2*b*c*d*(-1/2*sin(f*x+e)*cos(f
```

```
*x+e)+1/2*f*x+1/2*e)-a^2*b*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a*b^2*c^2*(-1/
2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2*a*b^2*c*d*(2+sin(f*x+e)^2)*cos(f*x
+e)+3*a*b^2*d^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*
e)-1/3*b^3*c^2*(2+sin(f*x+e)^2)*cos(f*x+e)+2*b^3*c*d*(-1/4*(sin(f*x+e)^3+3/
2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/5*b^3*d^2*(8/3+sin(f*x+e)^4+4/3*s
in(f*x+e)^2)*cos(f*x+e))
```

maxima [A] time = 0.52, size = 314, normalized size = 1.00

$$\frac{480\left(fx+e\right)a^3c^2+360\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)ab^2c^2+160\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)b^3c^2+720\left(2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/480*(480*(f*x + e)*a^3*c^2 + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*b^2*c
^2 + 160*(cos(f*x + e)^3 - 3*cos(f*x + e))*b^3*c^2 + 720*(2*f*x + 2*e - sin
(2*f*x + 2*e))*a^2*b*c*d + 960*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*b^2*c*d
+ 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*b^3*c*d + 120*
(2*f*x + 2*e - sin(2*f*x + 2*e))*a^3*d^2 + 480*(cos(f*x + e)^3 - 3*cos(f*x
+ e))*a^2*b*d^2 + 45*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e)
)*a*b^2*d^2 - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*b
^3*d^2 - 1440*a^2*b*c^2*cos(f*x + e) - 960*a^3*c*d*cos(f*x + e))/f
```

mupad [B] time = 8.52, size = 358, normalized size = 1.14

$$-\frac{90\,b^3\,c^2\,\cos\left(e+f\,x\right)+75\,b^3\,d^2\,\cos\left(e+f\,x\right)-10\,b^3\,c^2\,\cos\left(3\,e+3\,f\,x\right)-\frac{25\,b^3\,d^2\,\cos\left(3\,e+3\,f\,x\right)}{2}+\frac{3\,b^3\,d^2\,\cos\left(5\,e+5\,f\right.}{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x))^2,x)
```

```
[Out] -(90*b^3*c^2*cos(e + f*x) + 75*b^3*d^2*cos(e + f*x) - 10*b^3*c^2*cos(3*e +
3*f*x) - (25*b^3*d^2*cos(3*e + 3*f*x))/2 + (3*b^3*d^2*cos(5*e + 5*f*x))/2 +
 30*a^3*d^2*sin(2*e + 2*f*x) - 30*a^2*b*d^2*cos(3*e + 3*f*x) + 90*a*b^2*c^2
*sin(2*e + 2*f*x) + 90*a*b^2*d^2*sin(2*e + 2*f*x) - (45*a*b^2*d^2*sin(4*e +
 4*f*x))/4 + 240*a^3*c*d*cos(e + f*x) + 360*a^2*b*c^2*cos(e + f*x) + 270*a^
2*b*d^2*cos(e + f*x) + 60*b^3*c*d*sin(2*e + 2*f*x) - (15*b^3*c*d*sin(4*e +
4*f*x))/2 - 120*a^3*c^2*f*x - 60*a^3*d^2*f*x - 60*a*b^2*c*d*cos(3*e + 3*f*x
) + 180*a^2*b*c*d*sin(2*e + 2*f*x) - 180*a*b^2*c^2*f*x - 135*a*b^2*d^2*f*x
+ 540*a*b^2*c*d*cos(e + f*x) - 90*b^3*c*d*f*x - 360*a^2*b*c*d*f*x)/(120*f)
```

sympy [A] time = 4.29, size = 729, normalized size = 2.31

$$\begin{cases} a^3c^2x - \frac{2a^3cd\cos(e+fx)}{f} + \frac{a^3d^2x\sin^2(e+fx)}{2} + \frac{a^3d^2x\cos^2(e+fx)}{2} - \frac{a^3d^2\sin(e+fx)\cos(e+fx)}{2f} - \frac{3a^2bc^2\cos(e+fx)}{f} + 3a^2bcdx\sin^2(\\ x(a+b\sin(e))^3(c+d\sin(e))^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((a**3*c**2*x - 2*a**3*c*d*cos(e + f*x)/f + a**3*d**2*x*sin(e + f*
x)**2/2 + a**3*d**2*x*cos(e + f*x)**2/2 - a**3*d**2*sin(e + f*x)*cos(e + f*
x)/(2*f) - 3*a**2*b*c**2*cos(e + f*x)/f + 3*a**2*b*c*d*x*sin(e + f*x)**2 +
3*a**2*b*c*d*x*cos(e + f*x)**2 - 3*a**2*b*c*d*sin(e + f*x)*cos(e + f*x)/f -
 3*a**2*b*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 2*a**2*b*d**2*cos(e + f*x)*
*3/f + 3*a*b**2*c**2*x*sin(e + f*x)**2/2 + 3*a*b**2*c**2*x*cos(e + f*x)**2/
2 - 3*a*b**2*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 6*a*b**2*c*d*sin(e + f*
x)**2*cos(e + f*x)/f - 4*a*b**2*c*d*cos(e + f*x)**3/f + 9*a*b**2*d**2*x*sin
(e + f*x)**4/8 + 9*a*b**2*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 9*a*b*
*2*d**2*x*cos(e + f*x)**4/8 - 15*a*b**2*d**2*sin(e + f*x)**3*cos(e + f*x)/(
8*f) - 9*a*b**2*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) - b**3*c**2*sin(e +
 f*x)**2*cos(e + f*x)/f - 2*b**3*c**2*cos(e + f*x)**3/(3*f) + 3*b**3*c*d*x*
sin(e + f*x)**4/4 + 3*b**3*c*d*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + 3*b**3
*c*d*x*cos(e + f*x)**4/4 - 5*b**3*c*d*sin(e + f*x)**3*cos(e + f*x)/(4*f) -
3*b**3*c*d*sin(e + f*x)*cos(e + f*x)**3/(4*f) - b**3*d**2*sin(e + f*x)**4*c
os(e + f*x)/f - 4*b**3*d**2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 8*b**3*
d**2*cos(e + f*x)**5/(15*f), Ne(f, 0)), (x*(a + b*sin(e))**3*(c + d*sin(e))
**2, True))
```

3.688 $\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=171

$$-\frac{b\left(6a^2d+20abc+9b^2d\right)\sin(e+fx)\cos(e+fx)}{24f}-\frac{\left(3a^3d+16a^2bc+12ab^2d+4b^3c\right)\cos(e+fx)}{6f}+\frac{1}{8}x\left(8a^3c+1\right.$$

```
[Out] 1/8*(8*a^3*c+12*a^2*b*d+12*a*b^2*c+3*b^3*d)*x-1/6*(3*a^3*d+16*a^2*b*c+12*a*
b^2*d+4*b^3*c)*cos(f*x+e)/f-1/24*b*(6*a^2*d+20*a*b*c+9*b^2*d)*cos(f*x+e)*si
n(f*x+e)/f-1/12*(3*a*d+4*b*c)*cos(f*x+e)*(a+b*sin(f*x+e))^2/f-1/4*d*cos(f*x
+e)*(a+b*sin(f*x+e))^3/f
```

Rubi [A] time = 0.20, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2753, 2734}

$$-\frac{\left(16a^2bc+3a^3d+12ab^2d+4b^3c\right)\cos(e+fx)}{6f}-\frac{b\left(6a^2d+20abc+9b^2d\right)\sin(e+fx)\cos(e+fx)}{24f}+\frac{1}{8}x\left(12a^2bd\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x]),x]
```

```
[Out] ((8*a^3*c + 12*a*b^2*c + 12*a^2*b*d + 3*b^3*d)*x)/8 - ((16*a^2*b*c + 4*b^3*
c + 3*a^3*d + 12*a*b^2*d)*Cos[e + f*x])/(6*f) - (b*(20*a*b*c + 6*a^2*d + 9*
b^2*d)*Cos[e + f*x]*Sin[e + f*x])/(24*f) - ((4*b*c + 3*a*d)*Cos[e + f*x]*(a
 + b*Sin[e + f*x])^2)/(12*f) - (d*Cos[e + f*x]*(a + b*Sin[e + f*x])^3)/(4*f
)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))\,dx = -\frac{d\cos(e+fx)(a+b\sin(e+fx))^3}{4f}+\frac{1}{4}\int (a+b\sin(e+fx))^2(4ac$$

$$=-\frac{(4bc+3ad)\cos(e+fx)(a+b\sin(e+fx))^2}{12f}-\frac{d\cos(e+fx)(a+b}{4f}$$

$$=\frac{1}{8}\left(8a^3c+12ab^2c+12a^2bd+3b^3d\right)x-\frac{\left(16a^2bc+4b^3c+3a^3d+12\right.}{6f}$$

Mathematica [A] time = 0.66, size = 142, normalized size = 0.83

$$\frac{3\left(-8b\left(3a^2d+3abc+b^2d\right)\sin(2(e+fx))+4(e+fx)\left(8a^3c+12a^2bd+12ab^2c+3b^3d\right)+b^3d\sin(4(e+fx))\right)-2}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x]),x]
```

```
[Out] (-24*(12*a^2*b*c + 3*b^3*c + 4*a^3*d + 9*a*b^2*d)*Cos[e + f*x] + 8*b^2*(b*c
 + 3*a*d)*Cos[3*(e + f*x)] + 3*(4*(8*a^3*c + 12*a*b^2*c + 12*a^2*b*d + 3*b^
3*d)*(e + f*x) - 8*b*(3*a*b*c + 3*a^2*d + b^2*d)*Sin[2*(e + f*x)] + b^3*d*S
in[4*(e + f*x)]))/(96*f)
```

fricas [A] time = 0.45, size = 148, normalized size = 0.87

$$\frac{8\left(b^3c+3\,ab^2d\right)\cos\left(fx+e\right)^3+3\left(4\left(2\,a^3+3\,ab^2\right)c+3\left(4\,a^2b+b^3\right)d\right)fx-24\left(\left(3\,a^2b+b^3\right)c+\left(a^3+3\,ab^2\right)d\right)\cos}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/24*(8*(b^3*c + 3*a*b^2*d)*cos(f*x + e)^3 + 3*(4*(2*a^3 + 3*a*b^2)*c + 3*(
4*a^2*b + b^3)*d)*f*x - 24*((3*a^2*b + b^3)*c + (a^3 + 3*a*b^2)*d)*cos(f*x
+ e) + 3*(2*b^3*d*cos(f*x + e)^3 - (12*a*b^2*c + (12*a^2*b + 5*b^3)*d)*cos(
f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.19, size = 152, normalized size = 0.89

$$\frac{b^3d\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{1}{8}\left(8\,a^3c+12\,ab^2c+12\,a^2bd+3\,b^3d\right)x+\frac{\left(b^3c+3\,ab^2d\right)\cos\left(3\,fx+3\,e\right)}{12\,f}-\frac{\left(12\,a^2bc+3\,b^3c+4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/32*b^3*d*sin(4*f*x + 4*e)/f + 1/8*(8*a^3*c + 12*a*b^2*c + 12*a^2*b*d + 3*
b^3*d)*x + 1/12*(b^3*c + 3*a*b^2*d)*cos(3*f*x + 3*e)/f - 1/4*(12*a^2*b*c +
3*b^3*c + 4*a^3*d + 9*a*b^2*d)*cos(f*x + e)/f - 1/4*(3*a*b^2*c + 3*a^2*b*d
+ b^3*d)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.27, size = 182, normalized size = 1.06

$$a^3c\left(fx+e\right)-a^3d\cos\left(fx+e\right)-3a^2bc\cos\left(fx+e\right)+3a^2bd\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+3a\,b^2c\left(-\frac{\sin\left(fx+e\right)\,c}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e)),x)
```

```
[Out] 1/f*(a^3*c*(f*x+e)-a^3*d*cos(f*x+e)-3*a^2*b*c*cos(f*x+e)+3*a^2*b*d*(-1/2*si
n(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+3*a*b^2*c*(-1/2*sin(f*x+e)*cos(f*x+e)+1/
2*f*x+1/2*e)-a*b^2*d*(2+sin(f*x+e)^2)*cos(f*x+e)-1/3*b^3*c*(2+sin(f*x+e)^2)
*cos(f*x+e)+b^3*d*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/
8*e))
```

maxima [A] time = 0.97, size = 175, normalized size = 1.02

$$96\left(fx+e\right)a^3c+72\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)ab^2c+32\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)b^3c+72\left(2\,fx+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/96*(96*(f*x + e)*a^3*c + 72*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*b^2*c + 32
*(cos(f*x + e)^3 - 3*cos(f*x + e))*b^3*c + 72*(2*f*x + 2*e - sin(2*f*x + 2*
e))*a^2*b*d + 96*(cos(f*x + e)^3 - 3*cos(f*x + e))*a*b^2*d + 3*(12*f*x + 12
*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*b^3*d - 288*a^2*b*c*cos(f*x + e
) - 96*a^3*d*cos(f*x + e))/f
```

mupad [B] time = 7.95, size = 183, normalized size = 1.07

$$2\,b^3\,c\,\cos\left(3\,e+3\,f\,x\right)-6\,b^3\,d\,\sin\left(2\,e+2\,f\,x\right)+\frac{3\,b^3\,d\,\sin\left(4\,e+4\,f\,x\right)}{4}-24\,a^3\,d\,\cos\left(e+f\,x\right)-18\,b^3\,c\,\cos\left(e+f\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x)),x)
```

```
[Out] (2*b^3*c*cos(3*e + 3*f*x) - 6*b^3*d*sin(2*e + 2*f*x) + (3*b^3*d*sin(4*e + 4
*f*x))/4 - 24*a^3*d*cos(e + f*x) - 18*b^3*c*cos(e + f*x) - 72*a^2*b*c*cos(e
 + f*x) - 54*a*b^2*d*cos(e + f*x) + 24*a^3*c*f*x + 9*b^3*d*f*x + 6*a*b^2*d*
cos(3*e + 3*f*x) - 18*a*b^2*c*sin(2*e + 2*f*x) - 18*a^2*b*d*sin(2*e + 2*f*x
) + 36*a*b^2*c*f*x + 36*a^2*b*d*f*x)/(24*f)
```

sympy [A] time = 1.87, size = 386, normalized size = 2.26

$$\begin{cases} a^3cx - \frac{a^3d\cos(e+fx)}{f} - \frac{3a^2bc\cos(e+fx)}{f} + \frac{3a^2bdx\sin^2(e+fx)}{2} + \frac{3a^2bdx\cos^2(e+fx)}{2} - \frac{3a^2bd\sin(e+fx)\cos(e+fx)}{2f} + \frac{3ab^2cx\sin^2(e+fx)}{2} \\ x(a+b\sin(e))^3(c+d\sin(e)) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((a**3*c*x - a**3*d*cos(e + f*x)/f - 3*a**2*b*c*cos(e + f*x)/f + 3
*a**2*b*d*x*sin(e + f*x)**2/2 + 3*a**2*b*d*x*cos(e + f*x)**2/2 - 3*a**2*b*d
*sin(e + f*x)*cos(e + f*x)/(2*f) + 3*a*b**2*c*x*sin(e + f*x)**2/2 + 3*a*b**
2*c*x*cos(e + f*x)**2/2 - 3*a*b**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) - 3*a*
b**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 2*a*b**2*d*cos(e + f*x)**3/f - b**3
*c*sin(e + f*x)**2*cos(e + f*x)/f - 2*b**3*c*cos(e + f*x)**3/(3*f) + 3*b**3
*d*x*sin(e + f*x)**4/8 + 3*b**3*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*b
**3*d*x*cos(e + f*x)**4/8 - 5*b**3*d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3
*b**3*d*sin(e + f*x)*cos(e + f*x)**3/(8*f), Ne(f, 0)), (x*(a + b*sin(e))**3
*(c + d*sin(e)), True))
```

3.689 $\int (a+b\sin(e+fx))^3\,dx$

Optimal. Leaf size=90

$$-\frac{2b\left(4a^2+b^2\right)\cos(e+fx)}{3f}+\frac{1}{2}ax\left(2a^2+3b^2\right)-\frac{5ab^2\sin(e+fx)\cos(e+fx)}{6f}-\frac{b\cos(e+fx)(a+b\sin(e+fx))^2}{3f}$$

```
[Out] 1/2*a*(2*a^2+3*b^2)*x-2/3*b*(4*a^2+b^2)*cos(f*x+e)/f-5/6*a*b^2*cos(f*x+e)*s
in(f*x+e)/f-1/3*b*cos(f*x+e)*(a+b*sin(f*x+e))^2/f
```

Rubi [A] time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2656, 2734}

$$-\frac{2b\left(4a^2+b^2\right)\cos(e+fx)}{3f}+\frac{1}{2}ax\left(2a^2+3b^2\right)-\frac{5ab^2\sin(e+fx)\cos(e+fx)}{6f}-\frac{b\cos(e+fx)(a+b\sin(e+fx))^2}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*x)/2 - (2*b*(4*a^2 + b^2)*Cos[e + f*x])/(3*f) - (5*a*b^2
*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (b*Cos[e + f*x]*(a + b*Sin[e + f*x])^2)
/(3*f)
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))^3\,dx = -\frac{b\cos(e+fx)(a+b\sin(e+fx))^2}{3f} + \frac{1}{3}\int (a+b\sin(e+fx))\left(3a^2+2b^2+5ab\sin(e-\right.$$
$$= \frac{1}{2}a\left(2a^2+3b^2\right)x - \frac{2b\left(4a^2+b^2\right)\cos(e+fx)}{3f} - \frac{5ab^2\cos(e+fx)\sin(e+fx)}{6f} - \frac{b\cos(}{}$$

Mathematica [A] time = 0.17, size = 71, normalized size = 0.79

$$\frac{6a\left(2a^2+3b^2\right)(e+fx) - 9b\left(4a^2+b^2\right)\cos(e+fx) - 9ab^2\sin(2(e+fx)) + b^3\cos(3(e+fx))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (6*a*(2*a^2 + 3*b^2)*(e + f*x) - 9*b*(4*a^2 + b^2)*Cos[e + f*x] + b^3*Cos[3
*(e + f*x)] - 9*a*b^2*Sin[2*(e + f*x)])/(12*f)
```

fricas [A] time = 0.43, size = 71, normalized size = 0.79

$$\frac{2\,b^3\cos\left(fx+e\right)^3 - 9\,ab^2\cos\left(fx+e\right)\sin\left(fx+e\right) + 3\left(2\,a^3+3\,ab^2\right)fx - 6\left(3\,a^2b+b^3\right)\cos\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*cos(f*x + e)^3 - 9*a*b^2*cos(f*x + e)*sin(f*x + e) + 3*(2*a^3 +
3*a*b^2)*f*x - 6*(3*a^2*b + b^3)*cos(f*x + e))/f
```

giac [A] time = 0.15, size = 75, normalized size = 0.83

$$\frac{b^3\cos\left(3\,fx+3\,e\right)}{12\,f} - \frac{3\,ab^2\sin\left(2\,fx+2\,e\right)}{4\,f} + \frac{1}{2}\left(2\,a^3+3\,ab^2\right)x - \frac{3\left(4\,a^2b+b^3\right)\cos\left(fx+e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/12*b^3*cos(3*f*x + 3*e)/f - 3/4*a*b^2*sin(2*f*x + 2*e)/f + 1/2*(2*a^3 + 3
*a*b^2)*x - 3/4*(4*a^2*b + b^3)*cos(f*x + e)/f
```

maple [A] time = 0.18, size = 76, normalized size = 0.84

$$\frac{-\frac{b^3\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}+3ab^2\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-3a^2b\cos\left(fx+e\right)+\left(fx+e\right)a^3}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(-1/3*b^3*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a*b^2*(-1/2*sin(f*x+e)*cos(f*x+
e)+1/2*f*x+1/2*e)-3*a^2*b*cos(f*x+e)+(f*x+e)*a^3)
```

maxima [A] time = 0.67, size = 74, normalized size = 0.82

$$a^3x+\frac{3\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)ab^2}{4\,f}+\frac{\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)b^3}{3\,f}-\frac{3\,a^2b\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 3/4*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*b^2/f + 1/3*(cos(f*x + e)^3
- 3*cos(f*x + e))*b^3/f - 3*a^2*b*cos(f*x + e)/f
```

mupad [B] time = 7.69, size = 127, normalized size = 1.41

$$a^3\,x-\frac{4\,b^3\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{f}+\frac{8\,b^3\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6}{3\,f}+\frac{3\,a\,b^2\,x}{2}-\frac{6\,a^2\,b\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{f}-\frac{6\,a\,b^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3,x)
```

```
[Out] a^3*x - (4*b^3*cos(e/2 + (f*x)/2)^4)/f + (8*b^3*cos(e/2 + (f*x)/2)^6)/(3*f)
 + (3*a*b^2*x)/2 - (6*a^2*b*cos(e/2 + (f*x)/2)^2)/f - (6*a*b^2*cos(e/2 + (f
*x)/2)^3*sin(e/2 + (f*x)/2))/f + (3*a*b^2*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x
)/2))/f
```

sympy [A] time = 0.67, size = 128, normalized size = 1.42

$$\begin{cases} a^3x-\frac{3a^2b\cos\left(e+fx\right)}{f}+\frac{3ab^2x\sin^2\left(e+fx\right)}{2}+\frac{3ab^2x\cos^2\left(e+fx\right)}{2}-\frac{3ab^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{b^3\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{2b^3\cos^3\left(e+\right.}{3f} \\ x\left(a+b\sin\left(e\right)\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((a**3*x - 3*a**2*b*cos(e + f*x)/f + 3*a*b**2*x*sin(e + f*x)**2/2
+ 3*a*b**2*x*cos(e + f*x)**2/2 - 3*a*b**2*sin(e + f*x)*cos(e + f*x)/(2*f) -
 b**3*sin(e + f*x)**2*cos(e + f*x)/f - 2*b**3*cos(e + f*x)**3/(3*f), Ne(f,
0)), (x*(a + b*sin(e))**3, True))
```

3.690 $\int \frac{(a+b\sin(e+fx))^3}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=156

$$-\frac{bx\left(-6a^2d^2+6abcd-\left(b^2\left(2c^2+d^2\right)\right)\right)}{2d^3}+\frac{b^2(2bc-5ad)\cos(e+fx)}{2d^2f}-\frac{b^2\cos(e+fx)(a+b\sin(e+fx))}{2df}-\frac{2(bc-}{}$$

```
[Out] -1/2*b*(6*a*b*c*d-6*a^2*d^2-b^2*(2*c^2+d^2))*x/d^3+1/2*b^2*(-5*a*d+2*b*c)*c
os(f*x+e)/d^2/f-1/2*b^2*cos(f*x+e)*(a+b*sin(f*x+e))/d/f-2*(-a*d+b*c)^3*arct
an((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d^3/f/(c^2-d^2)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2793, 3023, 2735, 2660, 618, 204}

$$-\frac{bx\left(-6a^2d^2+6abcd+b^2\left(-\left(2c^2+d^2\right)\right)\right)}{2d^3}+\frac{b^2(2bc-5ad)\cos(e+fx)}{2d^2f}-\frac{b^2\cos(e+fx)(a+b\sin(e+fx))}{2df}-\frac{2(bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x]),x]
```

```
[Out] -(b*(6*a*b*c*d - 6*a^2*d^2 - b^2*(2*c^2 + d^2))*x)/(2*d^3) - (2*(b*c - a*d)
^3*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^3*Sqrt[c^2 - d^2]*f
) + (b^2*(2*b*c - 5*a*d)*Cos[e + f*x])/(2*d^2*f) - (b^2*Cos[e + f*x]*(a + b
*Sin[e + f*x]))/(2*d*f)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^3}{c+d\sin(e+fx)}\,dx = -\frac{b^2\cos(e+fx)(a+b\sin(e+fx))}{2df} + \frac{\int \frac{b^3c+2a^3d-b\left(abc-6a^2d-b^2d\right)\sin(e+fx)-b^2(2bc-5ad)\sin^2\cdots}{c+d\sin(e+fx)}}{2d}$$

$$= \frac{b^2(2bc-5ad)\cos(e+fx)}{2d^2f} - \frac{b^2\cos(e+fx)(a+b\sin(e+fx))}{2df} + \frac{\int \frac{d\left(b^3c+2a^3d\right)-b\left(6abc\cdots\right.}{c\cdots}}{\cdots}$$

$$= -\frac{b\left(6abcd-6a^2d^2-b^2\left(2c^2+d^2\right)\right)x}{2d^3} + \frac{b^2(2bc-5ad)\cos(e+fx)}{2d^2f} - \frac{b^2\cos(e+fx)\cdots}{\cdots}$$

$$= -\frac{b\left(6abcd-6a^2d^2-b^2\left(2c^2+d^2\right)\right)x}{2d^3} + \frac{b^2(2bc-5ad)\cos(e+fx)}{2d^2f} - \frac{b^2\cos(e+fx)\cdots}{\cdots}$$

$$= -\frac{b\left(6abcd-6a^2d^2-b^2\left(2c^2+d^2\right)\right)x}{2d^3} + \frac{b^2(2bc-5ad)\cos(e+fx)}{2d^2f} - \frac{b^2\cos(e+fx)\cdots}{\cdots}$$

$$= -\frac{b\left(6abcd-6a^2d^2-b^2\left(2c^2+d^2\right)\right)x}{2d^3} - \frac{2(bc-ad)^3\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3\sqrt{c^2-d^2}\,f} + \frac{b^2(2b\cdots}{\cdots}$$

Mathematica [A] time = 0.37, size = 137, normalized size = 0.88

$$\frac{2b(e+fx)\left(6a^2d^2-6abcd+b^2\left(2c^2+d^2\right)\right)+4b^2d(bc-3ad)\cos(e+fx)-\frac{8(bc-ad)^3\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}+b^3\left(-d^2\cdots\right.}{4d^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x]),x]
```

```
[Out] (2*b*(-6*a*b*c*d + 6*a^2*d^2 + b^2*(2*c^2 + d^2))*(e + f*x) - (8*(b*c - a*d
)^3*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2] + 4*b
^2*d*(b*c - 3*a*d)*Cos[e + f*x] - b^3*d^2*Sin[2*(e + f*x)])/(4*d^3*f)
```

fricas [A] time = 0.51, size = 578, normalized size = 3.71

$$\left[\frac{\left(2b^3c^4-6ab^2c^3d+6ab^2cd^3+\left(6a^2b-b^3\right)c^2d^2-\left(6a^2b+b^3\right)d^4\right)fx-\left(b^3c^2d^2-b^3d^4\right)\cos\left(fx+e\right)\sin\left(fx+e\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*((2*b^3*c^4 - 6*a*b^2*c^3*d + 6*a*b^2*c*d^3 + (6*a^2*b - b^3)*c^2*d^2
- (6*a^2*b + b^3)*d^4)*f*x - (b^3*c^2*d^2 - b^3*d^4)*cos(f*x + e)*sin(f*x +
 e) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-c^2 + d^2)*
log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*c
os(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x +
 e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 -
 b^3*c*d^3 + 3*a*b^2*d^4)*cos(f*x + e))/((c^2*d^3 - d^5)*f), 1/2*((2*b^3*c^
4 - 6*a*b^2*c^3*d + 6*a*b^2*c*d^3 + (6*a^2*b - b^3)*c^2*d^2 - (6*a^2*b + b^
3)*d^4)*f*x - (b^3*c^2*d^2 - b^3*d^4)*cos(f*x + e)*sin(f*x + e) + 2*(b^3*c^
3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(c^2 - d^2)*arctan(-(c*sin
(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + 2*(b^3*c^3*d - 3*a*b^2*c^2
*d^2 - b^3*c*d^3 + 3*a*b^2*d^4)*cos(f*x + e))/((c^2*d^3 - d^5)*f)]
```

giac [A] time = 0.60, size = 252, normalized size = 1.62

$$\frac{\frac{\left(2\,b^{3}c^{2}-6\,ab^{2}cd+6\,a^{2}bd^{2}+b^{3}d^{2}\right)\left(fx+e\right)}{d^{3}}-\frac{4\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^{2}-d^{2}}}\right)\right)}{\sqrt{c^{2}-d^{2}}\,d^{3}}+\frac{2\left(b^{3}d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}\cdots\right.}{\cdots}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*((2*b^3*c^2 - 6*a*b^2*c*d + 6*a^2*b*d^2 + b^3*d^2)*(f*x + e)/d^3 - 4*(b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(pi*floor(1/2*(f*x + e)/p
i + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/(sq
rt(c^2 - d^2)*d^3) + 2*(b^3*d*tan(1/2*f*x + 1/2*e)^3 + 2*b^3*c*tan(1/2*f*x
+ 1/2*e)^2 - 6*a*b^2*d*tan(1/2*f*x + 1/2*e)^2 - b^3*d*tan(1/2*f*x + 1/2*e)
+ 2*b^3*c - 6*a*b^2*d)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*d^2))/f
```

maple [B] time = 0.24, size = 506, normalized size = 3.24

$$\frac{2\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^{2}-d^{2}}}\right)a^{3}}{f\sqrt{c^{2}-d^{2}}}-\frac{6\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^{2}-d^{2}}}\right)a^{2}bc}{fd\sqrt{c^{2}-d^{2}}}+\frac{6\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^{2}-d^{2}}}\right)a\,b^{2}c^{2}}{f\,d^{2}\sqrt{c^{2}-d^{2}}}-\frac{2\arctan\left(\frac{2c\tan\left(\frac{fx}{2}\cdots\right.}{2\sqrt{c^{2}\cdots}}\right.}{f\,d^{3}\sqrt{c^{2}\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e)),x)
```

```
[Out] 2/f/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)
)*a^3-6/f/d/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^
2)^(1/2))*a^2*b*c+6/f/d^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e
)+2*d)/(c^2-d^2)^(1/2))*a*b^2*c^2-2/f/d^3/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*t
an(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b^3*c^3+1/f/d*b^3/(1+tan(1/2*f*x+1/
2*e)^2)^2*tan(1/2*f*x+1/2*e)^3-6/f/d*b^2/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2
*f*x+1/2*e)^2*a+2/f/d^2*b^3/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2
*c-1/f/d*b^3/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)-6/f/d*b^2/(1+tan
(1/2*f*x+1/2*e)^2)^2*a+2/f/d^2*b^3/(1+tan(1/2*f*x+1/2*e)^2)^2*c+6/f/d*b*arc
tan(tan(1/2*f*x+1/2*e))*a^2-6/f/d^2*b^2*arctan(tan(1/2*f*x+1/2*e))*a*c+2/f/
d^3*b^3*arctan(tan(1/2*f*x+1/2*e))*c^2+1/f/d*b^3*arctan(tan(1/2*f*x+1/2*e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 14.56, size = 5902, normalized size = 37.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3/(c + d*sin(e + f*x)),x)
```

```
[Out] ((2*(b^3*c - 3*a*b^2*d))/d^2 + (b^3*tan(e/2 + (f*x)/2)^3)/d + (2*tan(e/2 +
(f*x)/2)^2*(b^3*c - 3*a*b^2*d))/d^2 - (b^3*tan(e/2 + (f*x)/2))/d)/(f*(2*tan
(e/2 + (f*x)/2)^2 + tan(e/2 + (f*x)/2)^4 + 1)) + (atan((((b^3*c^2*1i + (b*d
^2*(6*a^2 + b^2)*1i)/2 - a*b^2*c*d*3i)*((8*(b^6*c^2*d^6 + 4*b^6*c^4*d^4 + 4
*b^6*c^6*d^2 - 12*a*b^5*c^3*d^5 - 24*a*b^5*c^5*d^3 + 12*a^2*b^4*c^2*d^6 + 6
0*a^2*b^4*c^4*d^4 - 72*a^3*b^3*c^3*d^5 + 36*a^4*b^2*c^2*d^6))/d^5 + (8*tan(
e/2 + (f*x)/2)*(2*b^6*c*d^8 - 4*a^6*c*d^8 + 7*b^6*c^3*d^6 + 4*b^6*c^5*d^4 -
 8*b^6*c^7*d^2 - 24*a*b^5*c^2*d^7 - 36*a*b^5*c^4*d^5 + 48*a*b^5*c^6*d^3 + 2
4*a^2*b^4*c*d^8 + 72*a^4*b^2*c*d^8 + 24*a^5*b*c^2*d^7 + 108*a^2*b^4*c^3*d^6
 - 120*a^2*b^4*c^5*d^4 - 144*a^3*b^3*c^2*d^7 + 152*a^3*b^3*c^4*d^5 - 96*a^4
*b^2*c^3*d^6))/d^6 + ((b^3*c^2*1i + (b*d^2*(6*a^2 + b^2)*1i)/2 - a*b^2*c*d*
3i)*((8*tan(e/2 + (f*x)/2)*(8*a^3*c*d^9 - 8*b^3*c^4*d^6 + 24*a*b^2*c^3*d^7
- 24*a^2*b*c^2*d^8))/d^6 - (8*(2*b^3*c*d^8 - 4*a^3*c^2*d^7 + 2*b^3*c^3*d^6
- 12*a*b^2*c^2*d^7 + 12*a^2*b*c*d^8))/d^5 + ((32*c^2*d^3 + (8*tan(e/2 + (f*
```

x)/2)*(12*c*d^10 - 8*c^3*d^8))/d^6)*(b^3*c^2*1i + (b*d^2*(6*a^2 + b^2)*1i)/ 2 - a*b^2*c*d*3i))/d^3))/d^3)*1i)/d^3 + ((b^3*c^2*1i + (b*d^2*(6*a^2 + b^2) *1i)/2 - a*b^2*c*d*3i)*((8*(b^6*c^2*d^6 + 4*b^6*c^4*d^4 + 4*b^6*c^6*d^2 - 1 2*a*b^5*c^3*d^5 - 24*a*b^5*c^5*d^3 + 12*a^2*b^4*c^2*d^6 + 60*a^2*b^4*c^4*d^ 4 - 72*a^3*b^3*c^3*d^5 + 36*a^4*b^2*c^2*d^6))/d^5 + (8*tan(e/2 + (f*x)/2)*(2*b^6*c*d^8 - 4*a^6*c*d^8 + 7*b^6*c^3*d^6 + 4*b^6*c^5*d^4 - 8*b^6*c^7*d^2 - 24*a*b^5*c^2*d^7 - 36*a*b^5*c^4*d^5 + 48*a*b^5*c^6*d^3 + 24*a^2*b^4*c*d^8 + 72*a^4*b^2*c*d^8 + 24*a^5*b*c^2*d^7 + 108*a^2*b^4*c^3*d^6 - 120*a^2*b^4*c ^5*d^4 - 144*a^3*b^3*c^2*d^7 + 152*a^3*b^3*c^4*d^5 - 96*a^4*b^2*c^3*d^6))/d ^6 + ((b^3*c^2*1i + (b*d^2*(6*a^2 + b^2)*1i)/2 - a*b^2*c*d*3i)*((8*(2*b^3*c *d^8 - 4*a^3*c^2*d^7 + 2*b^3*c^3*d^6 - 12*a*b^2*c^2*d^7 + 12*a^2*b*c*d^8))/ d^5 - (8*tan(e/2 + (f*x)/2)*(8*a^3*c*d^9 - 8*b^3*c^4*d^6 + 24*a*b^2*c^3*d^7 - 24*a^2*b*c^2*d^8))/d^6 + ((32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 - 8*c^3*d^8))/d^6)*(b^3*c^2*1i + (b*d^2*(6*a^2 + b^2)*1i)/2 - a*b^2*c*d*3i))/d^3))/d^3)*1i)/d^3)/((16*(2*b^9*c^7 + b^9*c^5*d^2 - 3*a*b^8*c^4*d^3 + 4* a^3*b^6*c^6*d - 2*a^6*b^3*c*d^6 + 3*a^2*b^7*c^3*d^4 + 30*a^2*b^7*c^5*d^2 - a^3*b^6*c^2*d^5 - 36*a^3*b^6*c^4*d^3 + 18*a^4*b^5*c^3*d^4 - 24*a^4*b^5*c^5* d^2 + 60*a^5*b^4*c^4*d^3 - 76*a^6*b^3*c^3*d^4 + 48*a^7*b^2*c^2*d^5 - 12*a*b ^8*c^6*d - 12*a^8*b*c*d^6))/d^5 - ((b^3*c^2*1i + (b*d^2*(6*a^2 + b^2)*1i)/2 - a*b^2*c*d*3i)*((8*(b^6*c^2*d^6 + 4*b^6*c^4*d^4 + 4*b^6*c^6*d^2 - 12*a*b^ 5*c^3*d^5 - 24*a*b^5*c^5*d^3 + 12*a^2*b^4*c^2*d^6 + 60*a^2*b^4*c^4*d^4 - 72 *a^3*b^3*c^3*d^5 + 36*a^4*b^2*c^2*d^6))/d^5 + (8*tan(e/2 + (f*x)/2)*(2*b^6* c*d^8 - 4*a^6*c*d^8 + 7*b^6*c^3*d^6 + 4*b^6*c^5*d^4 - 8*b^6*c^7*d^2 - 24*a* b^5*c^2*d^7 - 36*a*b^5*c^4*d^5 + 48*a*b^5*c^6*d^3 + 24*a^2*b^4*c*d^8 + 72*a ^4*b^2*c*d^8 + 24*a^5*b*c^2*d^7 + 108*a^2*b^4*c^3*d^6 - 120*a^2*b^4*c^5*d^4 - 144*a^3*b^3*c^2*d^7 + 152*a^3*b^3*c^4*d^5 - 96*a^4*b^2*c^3*d^6))/d^6 + ((b^3*c^2*1i + (b*d^2*(6*a^2 + b^2)*1i)/2 - a*b^2*c*d*3i)*((8*tan(e/2 + (f*x)/2)*(8*a^3*c*d^9 - 8*b^3*c^4*d^6 + 24*a*b^2*c^3*d^7 - 24*a^2*b*c^2*d^8))/d ^6 - (8*(2*b^3*c*d^8 - 4*a^3*c^2*d^7 + 2*b^3*c^3*d^6 - 12*a*b^2*c^2*d^7 + 1 2*a^2*b*c*d^8))/d^5 + ((32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 - 8*c ^3*d^8))/d^6)*(b^3*c^2*1i + (b*d^2*(6*a^2 + b^2)*1i)/2 - a*b^2*c*d*3i))/d^3))/d^3))/d^3 + ((b^3*c^2*1i + (b*d^2*(6*a^2 + b^2)*1i)/2 - a*b^2*c*d*3i)*((8*(b^6*c^2*d^6 + 4*b^6*c^4*d^4 + 4*b^6*c^6*d^2 - 12*a*b^5*c^3*d^5 - 24*a*b^ 5*c^5*d^3 + 12*a^2*b^4*c^2*d^6 + 60*a^2*b^4*c^4*d^4 - 72*a^3*b^3*c^3*d^5 + 36*a^4*b^2*c^2*d^6))/d^5 + (8*tan(e/2 + (f*x)/2)*(2*b^6*c*d^8 - 4*a^6*c*d^8 + 7*b^6*c^3*d^6 + 4*b^6*c^5*d^4 - 8*b^6*c^7*d^2 - 24*a*b^5*c^2*d^7 - 36*a* b^5*c^4*d^5 + 48*a*b^5*c^6*d^3 + 24*a^2*b^4*c*d^8 + 72*a^4*b^2*c*d^8 + 24*a ^5*b*c^2*d^7 + 108*a^2*b^4*c^3*d^6 - 120*a^2*b^4*c^5*d^4 - 144*a^3*b^3*c^2* d^7 + 152*a^3*b^3*c^4*d^5 - 96*a^4*b^2*c^3*d^6))/d^6 + ((b^3*c^2*1i + (b*d^ 2*(6*a^2 + b^2)*1i)/2 - a*b^2*c*d*3i)*((8*(2*b^3*c*d^8 - 4*a^3*c^2*d^7 + 2* b^3*c^3*d^6 - 12*a*b^2*c^2*d^7 + 12*a^2*b*c*d^8))/d^5 - (8*tan(e/2 + (f*x)/ 2)*(8*a^3*c*d^9 - 8*b^3*c^4*d^6 + 24*a*b^2*c^3*d^7 - 24*a^2*b*c^2*d^8))/d^6 + ((32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 - 8*c^3*d^8))/d^6)*(b^3* c^2*1i + (b*d^2*(6*a^2 + b^2)*1i)/2 - a*b^2*c*d*3i))/d^3))/d^3))/d^3 + (16* tan(e/2 + (f*x)/2)*(8*b^9*c^8 + 2*b^9*c^4*d^4 + 8*b^9*c^6*d^2 - 6*a*b^8*c^3

*d^5 - 48*a*b^8*c^5*d^3 - 2*a^3*b^6*c*d^7 - 24*a^5*b^4*c*d^7 - 72*a^7*b^2*c
*d^7 + 6*a^2*b^7*c^2*d^6 + 120*a^2*b^7*c^4*d^4 + 288*a^2*b^7*c^6*d^2 - 152*
a^3*b^6*c^3*d^5 - 656*a^3*b^6*c^5*d^3 + 96*a^4*b^5*c^2*d^6 + 912*a^4*b^5*c^
4*d^4 - 768*a^5*b^4*c^3*d^5 + 360*a^6*b^3*c^2*d^6 - 72*a*b^8*c^7*d))/d^6))*
(b^3*c^2*1i + (b*d^2*(6*a^2 + b^2)*1i)/2 - a*b^2*c*d*3i)*2i)/(d^3*f) + (ata
n((((-(c + d)*(c - d))^(1/2)*(a*d - b*c)^3*((8*(b^6*c^2*d^6 + 4*b^6*c^4*d^4
 + 4*b^6*c^6*d^2 - 12*a*b^5*c^3*d^5 - 24*a*b^5*c^5*d^3 + 12*a^2*b^4*c^2*d^6
 + 60*a^2*b^4*c^4*d^4 - 72*a^3*b^3*c^3*d^5 + 36*a^4*b^2*c^2*d^6))/d^5 + (8*
tan(e/2 + (f*x)/2)*(2*b^6*c*d^8 - 4*a^6*c*d^8 + 7*b^6*c^3*d^6 + 4*b^6*c^5*d
^4 - 8*b^6*c^7*d^2 - 24*a*b^5*c^2*d^7 - 36*a*b^5*c^4*d^5 + 48*a*b^5*c^6*d^3
 + 24*a^2*b^4*c*d^8 + 72*a^4*b^2*c*d^8 + 24*a^5*b*c^2*d^7 + 108*a^2*b^4*c^3
*d^6 - 120*a^2*b^4*c^5*d^4 - 144*a^3*b^3*c^2*d^7 + 152*a^3*b^3*c^4*d^5 - 96
*a^4*b^2*c^3*d^6))/d^6 + ((-(c + d)*(c - d))^(1/2)*(a*d - b*c)^3*((8*tan(e/
2 + (f*x)/2)*(8*a^3*c*d^9 - 8*b^3*c^4*d^6 + 24*a*b^2*c^3*d^7 - 24*a^2*b*c^2
d^8))/d^6 - (8(2*b^3*c*d^8 - 4*a^3*c^2*d^7 + 2*b^3*c^3*d^6 - 12*a*b^2*c^2
*d^7 + 12*a^2*b*c*d^8))/d^5 + ((32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^
10 - 8*c^3*d^8))/d^6)*(-(c + d)*(c - d))^(1/2)*(a*d - b*c)^3)/(d^5 - c^2*d^
3)))/(d^5 - c^2*d^3))*1i)/(d^5 - c^2*d^3) + ((-(c + d)*(c - d))^(1/2)*(a*d
 - b*c)^3*((8*(b^6*c^2*d^6 + 4*b^6*c^4*d^4 + 4*b^6*c^6*d^2 - 12*a*b^5*c^3*d^
5 - 24*a*b^5*c^5*d^3 + 12*a^2*b^4*c^2*d^6 + 60*a^2*b^4*c^4*d^4 - 72*a^3*b^3
*c^3*d^5 + 36*a^4*b^2*c^2*d^6))/d^5 + (8*tan(e/2 + (f*x)/2)*(2*b^6*c*d^8 -
4*a^6*c*d^8 + 7*b^6*c^3*d^6 + 4*b^6*c^5*d^4 - 8*b^6*c^7*d^2 - 24*a*b^5*c^2*
d^7 - 36*a*b^5*c^4*d^5 + 48*a*b^5*c^6*d^3 + 24*a^2*b^4*c*d^8 + 72*a^4*b^2*c
*d^8 + 24*a^5*b*c^2*d^7 + 108*a^2*b^4*c^3*d^6 - 120*a^2*b^4*c^5*d^4 - 144*a
^3*b^3*c^2*d^7 + 152*a^3*b^3*c^4*d^5 - 96*a^4*b^2*c^3*d^6))/d^6 + ((-(c + d
)*(c - d))^(1/2)*(a*d - b*c)^3*((8*(2*b^3*c*d^8 - 4*a^3*c^2*d^7 + 2*b^3*c^3
*d^6 - 12*a*b^2*c^2*d^7 + 12*a^2*b*c*d^8))/d^5 - (8*tan(e/2 + (f*x)/2)*(8*a
^3*c*d^9 - 8*b^3*c^4*d^6 + 24*a*b^2*c^3*d^7 - 24*a^2*b*c^2*d^8))/d^6 + ((32
*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 - 8*c^3*d^8))/d^6)*(-(c + d)*(c
 - d))^(1/2)*(a*d - b*c)^3)/(d^5 - c^2*d^3)))/(d^5 - c^2*d^3))*1i)/(d^5 - c
^2*d^3))/((16*(2*b^9*c^7 + b^9*c^5*d^2 - 3*a*b^8*c^4*d^3 + 4*a^3*b^6*c^6*d
 - 2*a^6*b^3*c*d^6 + 3*a^2*b^7*c^3*d^4 + 30*a^2*b^7*c^5*d^2 - a^3*b^6*c^2*d^
5 - 36*a^3*b^6*c^4*d^3 + 18*a^4*b^5*c^3*d^4 - 24*a^4*b^5*c^5*d^2 + 60*a^5*b
^4*c^4*d^3 - 76*a^6*b^3*c^3*d^4 + 48*a^7*b^2*c^2*d^5 - 12*a*b^8*c^6*d - 12*
a^8*b*c*d^6))/d^5 + (16*tan(e/2 + (f*x)/2)*(8*b^9*c^8 + 2*b^9*c^4*d^4 + 8*b
^9*c^6*d^2 - 6*a*b^8*c^3*d^5 - 48*a*b^8*c^5*d^3 - 2*a^3*b^6*c*d^7 - 24*a^5*
b^4*c*d^7 - 72*a^7*b^2*c*d^7 + 6*a^2*b^7*c^2*d^6 + 120*a^2*b^7*c^4*d^4 + 28
8*a^2*b^7*c^6*d^2 - 152*a^3*b^6*c^3*d^5 - 656*a^3*b^6*c^5*d^3 + 96*a^4*b^5*
c^2*d^6 + 912*a^4*b^5*c^4*d^4 - 768*a^5*b^4*c^3*d^5 + 360*a^6*b^3*c^2*d^6 -
 72*a*b^8*c^7*d))/d^6 - ((-(c + d)*(c - d))^(1/2)*(a*d - b*c)^3*((8*(b^6*c^
2*d^6 + 4*b^6*c^4*d^4 + 4*b^6*c^6*d^2 - 12*a*b^5*c^3*d^5 - 24*a*b^5*c^5*d^3
 + 12*a^2*b^4*c^2*d^6 + 60*a^2*b^4*c^4*d^4 - 72*a^3*b^3*c^3*d^5 + 36*a^4*b^
2*c^2*d^6))/d^5 + (8*tan(e/2 + (f*x)/2)*(2*b^6*c*d^8 - 4*a^6*c*d^8 + 7*b^6*
c^3*d^6 + 4*b^6*c^5*d^4 - 8*b^6*c^7*d^2 - 24*a*b^5*c^2*d^7 - 36*a*b^5*c^4*d
^5 + 48*a*b^5*c^6*d^3 + 24*a^2*b^4*c*d^8 + 72*a^4*b^2*c*d^8 + 24*a^5*b*c^2*

```
d^7 + 108*a^2*b^4*c^3*d^6 - 120*a^2*b^4*c^5*d^4 - 144*a^3*b^3*c^2*d^7 + 152
*a^3*b^3*c^4*d^5 - 96*a^4*b^2*c^3*d^6))/d^6 + ((-(c + d)*(c - d))^(1/2)*(a*
d - b*c)^3*((8*tan(e/2 + (f*x)/2)*(8*a^3*c*d^9 - 8*b^3*c^4*d^6 + 24*a*b^2*c
^3*d^7 - 24*a^2*b*c^2*d^8))/d^6 - (8*(2*b^3*c*d^8 - 4*a^3*c^2*d^7 + 2*b^3*c
^3*d^6 - 12*a*b^2*c^2*d^7 + 12*a^2*b*c*d^8))/d^5 + ((32*c^2*d^3 + (8*tan(e/
2 + (f*x)/2)*(12*c*d^10 - 8*c^3*d^8))/d^6)*(-(c + d)*(c - d))^(1/2)*(a*d -
b*c)^3)/(d^5 - c^2*d^3)))/(d^5 - c^2*d^3)))/(d^5 - c^2*d^3) + ((-(c + d)*(c
 - d))^(1/2)*(a*d - b*c)^3*((8*(b^6*c^2*d^6 + 4*b^6*c^4*d^4 + 4*b^6*c^6*d^2
 - 12*a*b^5*c^3*d^5 - 24*a*b^5*c^5*d^3 + 12*a^2*b^4*c^2*d^6 + 60*a^2*b^4*c^
4*d^4 - 72*a^3*b^3*c^3*d^5 + 36*a^4*b^2*c^2*d^6))/d^5 + (8*tan(e/2 + (f*x)/
2)*(2*b^6*c*d^8 - 4*a^6*c*d^8 + 7*b^6*c^3*d^6 + 4*b^6*c^5*d^4 - 8*b^6*c^7*d
^2 - 24*a*b^5*c^2*d^7 - 36*a*b^5*c^4*d^5 + 48*a*b^5*c^6*d^3 + 24*a^2*b^4*c*
d^8 + 72*a^4*b^2*c*d^8 + 24*a^5*b*c^2*d^7 + 108*a^2*b^4*c^3*d^6 - 120*a^2*b
^4*c^5*d^4 - 144*a^3*b^3*c^2*d^7 + 152*a^3*b^3*c^4*d^5 - 96*a^4*b^2*c^3*d^6
))/d^6 + ((-(c + d)*(c - d))^(1/2)*(a*d - b*c)^3*((8*(2*b^3*c*d^8 - 4*a^3*c
^2*d^7 + 2*b^3*c^3*d^6 - 12*a*b^2*c^2*d^7 + 12*a^2*b*c*d^8))/d^5 - (8*tan(e
/2 + (f*x)/2)*(8*a^3*c*d^9 - 8*b^3*c^4*d^6 + 24*a*b^2*c^3*d^7 - 24*a^2*b*c^
2*d^8))/d^6 + ((32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 - 8*c^3*d^8))
/d^6)*(-(c + d)*(c - d))^(1/2)*(a*d - b*c)^3)/(d^5 - c^2*d^3)))/(d^5 - c^2*
d^3)))/(d^5 - c^2*d^3)))*(-(c + d)*(c - d))^(1/2)*(a*d - b*c)^3*2i)/(f*(d^5
 - c^2*d^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.691 $\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=208

$$\frac{b\left(-a^2d^2+2abcd-\left(b^2\left(2c^2-d^2\right)\right)\right)\cos(e+fx)}{d^2f\left(c^2-d^2\right)}-\frac{b^2x(2bc-3ad)}{d^3}+\frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{df\left(c^2-d^2\right)(c+d\sin(e+fx))}+\frac{2(}{}$$

```
[Out] -b^2*(-3*a*d+2*b*c)*x/d^3+2*(-a*d+b*c)^2*(a*c*d+2*b*c^2-3*b*d^2)*arctan((d+
c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d^3/(c^2-d^2)^(3/2)/f+b*(2*a*b*c*d-a
^2*d^2-b^2*(2*c^2-d^2))*cos(f*x+e)/d^2/(c^2-d^2)/f+(-a*d+b*c)^2*cos(f*x+e)*
(a+b*sin(f*x+e))/d/(c^2-d^2)/f/(c+d*sin(f*x+e))
```

Rubi [A] time = 0.50, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2792, 3023, 2735, 2660, 618, 204}

$$\frac{b\left(-a^2d^2+2abcd+b^2\left(-\left(2c^2-d^2\right)\right)\right)\cos(e+fx)}{d^2f\left(c^2-d^2\right)}-\frac{b^2x(2bc-3ad)}{d^3}+\frac{2(bc-ad)^2\left(acd+2bc^2-3bd^2\right)\tan^{-1}\left(\frac{c\tan}{}\right.}{d^3f\left(c^2-d^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -((b^2*(2*b*c - 3*a*d)*x)/d^3) + (2*(b*c - a*d)^2*(2*b*c^2 + a*c*d - 3*b*d^
2)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^3*(c^2 - d^2)^(3/2)
*f) + (b*(2*a*b*c*d - a^2*d^2 - b^2*(2*c^2 - d^2))*Cos[e + f*x])/(d^2*(c^2
- d^2)*f) + ((b*c - a*d)^2*Cos[e + f*x]*(a + b*Sin[e + f*x]))/(d*(c^2 - d^2
)*f*(c + d*Sin[e + f*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^2}\,dx = \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{d\left(c^2-d^2\right)f(c+d\sin(e+fx))} - \frac{\int \frac{b^3c^2-a^3cd-3ab^2cd+3a^2bd^2-b\left(abc^2+\left(a^2+b^2\right)cd-\cdots\right.}{c+\cdots}}{d\cdots}$$

$$= \frac{b\left(2abcd-a^2d^2-b^2\left(2c^2-d^2\right)\right)\cos(e+fx)}{d^2\left(c^2-d^2\right)f} + \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+\cdots}{d\left(c^2-d^2\right)f(c+d\sin(e+fx)\cdots}$$

$$= -\frac{b^2(2bc-3ad)x}{d^3} + \frac{b\left(2abcd-a^2d^2-b^2\left(2c^2-d^2\right)\right)\cos(e+fx)}{d^2\left(c^2-d^2\right)f} + \frac{(bc-ad)^2\cos(e\cdots}{d\left(c^2-d^2\right)\cdots}$$

$$= -\frac{b^2(2bc-3ad)x}{d^3} + \frac{b\left(2abcd-a^2d^2-b^2\left(2c^2-d^2\right)\right)\cos(e+fx)}{d^2\left(c^2-d^2\right)f} + \frac{(bc-ad)^2\cos(e\cdots}{d\left(c^2-d^2\right)\cdots}$$

$$= -\frac{b^2(2bc-3ad)x}{d^3} + \frac{b\left(2abcd-a^2d^2-b^2\left(2c^2-d^2\right)\right)\cos(e+fx)}{d^2\left(c^2-d^2\right)f} + \frac{(bc-ad)^2\cos(e\cdots}{d\left(c^2-d^2\right)\cdots}$$

$$= -\frac{b^2(2bc-3ad)x}{d^3} + \frac{2(bc-ad)^2\left(2bc^2+acd-3bd^2\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3\left(c^2-d^2\right)^{3/2}f} + \frac{b\left(2a\cdots\right.}{\cdots}$$

Mathematica [A] time = 1.10, size = 152, normalized size = 0.73

$$\frac{-b^2(e+fx)(2bc-3ad) + \frac{2(bc-ad)^2\left(acd+2bc^2-3bd^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{3/2}} + \frac{d(ad-bc)^3\cos(e+fx)}{(c-d)(c+d)(c+d\sin(e+fx))} + b^3(-d)\cos(e+fx)}{d^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (-(b^2*(2*b*c - 3*a*d)*(e + f*x)) + (2*(b*c - a*d)^2*(2*b*c^2 + a*c*d - 3*b
*d^2)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^2)^(3/2) -
 b^3*d*Cos[e + f*x] + (d*(-(b*c) + a*d)^3*Cos[e + f*x])/((c - d)*(c + d)*(c
 + d*Sin[e + f*x])))/(d^3*f)
```

fricas [B] time = 0.55, size = 1062, normalized size = 5.11

$$\left[-\frac{2\left(2\,b^3c^6-3\,ab^2c^5d-4\,b^3c^4d^2+6\,ab^2c^3d^3+2\,b^3c^2d^4-3\,ab^2cd^5\right)fx+\left(2\,b^3c^5-3\,ab^2c^4d-3\,b^3c^3d^2-3\,a^2bcd^4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(2*b^3*c^6 - 3*a*b^2*c^5*d - 4*b^3*c^4*d^2 + 6*a*b^2*c^3*d^3 + 2*b
^3*c^2*d^4 - 3*a*b^2*c*d^5)*f*x + (2*b^3*c^5 - 3*a*b^2*c^4*d - 3*b^3*c^3*d^
2 - 3*a^2*b*c*d^4 + (a^3 + 6*a*b^2)*c^2*d^3 + (2*b^3*c^4*d - 3*a*b^2*c^3*d^
2 - 3*b^3*c^2*d^3 - 3*a^2*b*d^5 + (a^3 + 6*a*b^2)*c*d^4)*sin(f*x + e))*sqrt
(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 -
 d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(
d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(2*b^3*c^5*d - 3*
a*b^2*c^4*d^2 + a^3*d^6 + 3*(a^2*b - b^3)*c^3*d^3 - (a^3 - 3*a*b^2)*c^2*d^4
 - (3*a^2*b - b^3)*c*d^5)*cos(f*x + e) + 2*((2*b^3*c^5*d - 3*a*b^2*c^4*d^2
- 4*b^3*c^3*d^3 + 6*a*b^2*c^2*d^4 + 2*b^3*c*d^5 - 3*a*b^2*d^6)*f*x + (b^3*c
^4*d^2 - 2*b^3*c^2*d^4 + b^3*d^6)*cos(f*x + e))*sin(f*x + e))/((c^4*d^4 - 2
*c^2*d^6 + d^8)*f*sin(f*x + e) + (c^5*d^3 - 2*c^3*d^5 + c*d^7)*f), -((2*b^3
*c^6 - 3*a*b^2*c^5*d - 4*b^3*c^4*d^2 + 6*a*b^2*c^3*d^3 + 2*b^3*c^2*d^4 - 3*
a*b^2*c*d^5)*f*x + (2*b^3*c^5 - 3*a*b^2*c^4*d - 3*b^3*c^3*d^2 - 3*a^2*b*c*d
^4 + (a^3 + 6*a*b^2)*c^2*d^3 + (2*b^3*c^4*d - 3*a*b^2*c^3*d^2 - 3*b^3*c^2*d
^3 - 3*a^2*b*d^5 + (a^3 + 6*a*b^2)*c*d^4)*sin(f*x + e))*sqrt(c^2 - d^2)*arc
tan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (2*b^3*c^5*d -
3*a*b^2*c^4*d^2 + a^3*d^6 + 3*(a^2*b - b^3)*c^3*d^3 - (a^3 - 3*a*b^2)*c^2*d
^4 - (3*a^2*b - b^3)*c*d^5)*cos(f*x + e) + ((2*b^3*c^5*d - 3*a*b^2*c^4*d^2
- 4*b^3*c^3*d^3 + 6*a*b^2*c^2*d^4 + 2*b^3*c*d^5 - 3*a*b^2*d^6)*f*x + (b^3*c
^4*d^2 - 2*b^3*c^2*d^4 + b^3*d^6)*cos(f*x + e))*sin(f*x + e))/((c^4*d^4 - 2
*c^2*d^6 + d^8)*f*sin(f*x + e) + (c^5*d^3 - 2*c^3*d^5 + c*d^7)*f)]
```

giac [B] time = 0.21, size = 585, normalized size = 2.81

$$\frac{\frac{2\left(2\,b^3c^4-3\,ab^2c^3d-3\,b^3c^2d^2+a^3cd^3+6\,ab^2cd^3-3\,a^2bd^4\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(c\right)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(c^2d^3-d^5\right)\sqrt{c^2-d^2}}-\frac{2\left(b^3c^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3\,ab^2c^2d^2\tan\left(\frac{1}{2}\,f\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] (2*(2*b^3*c^4 - 3*a*b^2*c^3*d - 3*b^3*c^2*d^2 + a^3*c*d^3 + 6*a*b^2*c*d^3 -
 3*a^2*b*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*
f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^2*d^3 - d^5)*sqrt(c^2 - d^2)) - 2*(
b^3*c^3*d*tan(1/2*f*x + 1/2*e)^3 - 3*a*b^2*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 +
 3*a^2*b*c*d^3*tan(1/2*f*x + 1/2*e)^3 - a^3*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*
b^3*c^4*tan(1/2*f*x + 1/2*e)^2 - 3*a*b^2*c^3*d*tan(1/2*f*x + 1/2*e)^2 + 3*a
^2*b*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 - b^3*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 -
a^3*c*d^3*tan(1/2*f*x + 1/2*e)^2 + 3*b^3*c^3*d*tan(1/2*f*x + 1/2*e) - 3*a*b
^2*c^2*d^2*tan(1/2*f*x + 1/2*e) + 3*a^2*b*c*d^3*tan(1/2*f*x + 1/2*e) - 2*b^
3*c*d^3*tan(1/2*f*x + 1/2*e) - a^3*d^4*tan(1/2*f*x + 1/2*e) + 2*b^3*c^4 - 3
*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - b^3*c^2*d^2 - a^3*c*d^3)/((c^3*d^2 - c*d^4
)*(c*tan(1/2*f*x + 1/2*e)^4 + 2*d*tan(1/2*f*x + 1/2*e)^3 + 2*c*tan(1/2*f*x
+ 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)) - (2*b^3*c - 3*a*b^2*d)*(f*x +
e)/d^3)/f
```

maple [B] time = 0.29, size = 842, normalized size = 4.05

$$\frac{2d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)a^3}{f\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d+c\right)\left(c^2-d^2\right)c}-\frac{6d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)a^2b}{f\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d+c\right)\left(c^2-d^2\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2/f*d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)/c*tan(1
/2*f*x+1/2*e)*a^3-6/f*d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(
c^2-d^2)*tan(1/2*f*x+1/2*e)*a^2*b+6/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x
+1/2*e)*d+c)/(c^2-d^2)*c*tan(1/2*f*x+1/2*e)*a*b^2-2/f/d/(tan(1/2*f*x+1/2*e)
^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*c^2*tan(1/2*f*x+1/2*e)*b^3+2/f*d/(
tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*a^3-6/f/(tan(1/2
*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*a^2*b*c+6/f/d/(tan(1/2*
f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*a*b^2*c^2-2/f/d^2/(tan(1
/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*c^3*b^3+2/f/(c^2-d^2)
^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a^3*c-6/f*d
/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a
^2*b-6/f/d^2/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d
^2)^(1/2))*a*b^2*c^3+12/f/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e
)+2*d)/(c^2-d^2)^(1/2))*a*b^2*c+4/f/d^3/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan
(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b^3*c^4-6/f/d/(c^2-d^2)^(3/2)*arctan(
1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b^3*c^2-2/f*b^3/d^2/(1+ta
n(1/2*f*x+1/2*e)^2)+6/f*b^2/d^2*arctan(tan(1/2*f*x+1/2*e))*a-4/f*b^3/d^3*ar
ctan(tan(1/2*f*x+1/2*e))*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 17.64, size = 8953, normalized size = 43.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3/(c + d*sin(e + f*x))^2,x)
```

```
[Out] ((2*(a^3*d^3 - 2*b^3*c^3 + b^3*c*d^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(d^2
*(c^2 - d^2)) + (2*tan(e/2 + (f*x)/2)^2*(a^3*d^3 - 2*b^3*c^3 + b^3*c*d^2 +
3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(d^2*(c^2 - d^2)) + (2*tan(e/2 + (f*x)/2)*(
a^3*d^3 - 3*b^3*c^3 + 2*b^3*c*d^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(c*d*(c
^2 - d^2)) + (2*tan(e/2 + (f*x)/2)^3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3
*a^2*b*c*d^2))/(c*d*(c^2 - d^2)))/(f*(c + 2*d*tan(e/2 + (f*x)/2) + 2*c*tan(
e/2 + (f*x)/2)^2 + c*tan(e/2 + (f*x)/2)^4 + 2*d*tan(e/2 + (f*x)/2)^3)) + (2
*b^2*atan(((b^2*(3*a*d - 2*b*c)*((32*(4*b^6*c^4*d^6 - 8*b^6*c^6*d^4 + 4*b^6
*c^8*d^2 - 12*a*b^5*c^3*d^7 + 24*a*b^5*c^5*d^5 - 12*a*b^5*c^7*d^3 + 9*a^2*b
^4*c^2*d^8 - 18*a^2*b^4*c^4*d^6 + 9*a^2*b^4*c^6*d^4))/(d^9 - 2*c^2*d^7 + c^
4*d^5) - (32*tan(e/2 + (f*x)/2)*(a^6*c^3*d^8 - 8*b^6*c^3*d^8 + 29*b^6*c^5*d
^6 - 28*b^6*c^7*d^4 + 8*b^6*c^9*d^2 + 24*a*b^5*c^2*d^9 - 96*a*b^5*c^4*d^7 +
 90*a*b^5*c^6*d^5 - 24*a*b^5*c^8*d^3 - 18*a^2*b^4*c*d^10 + 9*a^4*b^2*c*d^10
 - 6*a^5*b*c^2*d^9 + 99*a^2*b^4*c^3*d^8 - 84*a^2*b^4*c^5*d^6 + 18*a^2*b^4*c
^7*d^4 - 36*a^3*b^3*c^2*d^9 + 12*a^3*b^3*c^4*d^7 + 4*a^3*b^3*c^6*d^5 + 12*a
^4*b^2*c^3*d^8 - 6*a^4*b^2*c^5*d^6))/(d^10 - 2*c^2*d^8 + c^4*d^6) + (b^2*(3
*a*d - 2*b*c)*((32*tan(e/2 + (f*x)/2)*(2*a^3*c^2*d^11 - 2*a^3*c^4*d^9 - 6*b
^3*c^3*d^10 + 10*b^3*c^5*d^8 - 4*b^3*c^7*d^6 + 12*a*b^2*c^2*d^11 - 18*a*b^2
*c^4*d^9 + 6*a*b^2*c^6*d^7 + 6*a^2*b*c^3*d^10 - 6*a^2*b*c*d^12))/(d^10 - 2*
c^2*d^8 + c^4*d^6) - (32*(a^3*c^5*d^7 - a^3*c^3*d^9 + 2*b^3*c^2*d^10 - 3*b^
3*c^4*d^8 + b^3*c^6*d^6 + 3*a*b^2*c^3*d^9 + 3*a^2*b*c^2*d^10 - 3*a^2*b*c^4*
d^8 - 3*a*b^2*c*d^11))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (b^2*((32*(c^2*d^12 -
2*c^4*d^10 + c^6*d^8))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (32*tan(e/2 + (f*x)/2)
*(3*c*d^14 - 8*c^3*d^12 + 7*c^5*d^10 - 2*c^7*d^8))/(d^10 - 2*c^2*d^8 + c^4*
d^6))*(3*a*d - 2*b*c)*1i)/d^3)*1i)/d^3))/d^3 + (b^2*(3*a*d - 2*b*c)*((32*(4
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*b^6*c^4*d^6 - 8*b^6*c^6*d^4 + 4*b^6*c^8*d^2 - 12*a*b^5*c^3*d^7 + 24*a*b^5*
c^5*d^5 - 12*a*b^5*c^7*d^3 + 9*a^2*b^4*c^2*d^8 - 18*a^2*b^4*c^4*d^6 + 9*a^2
*b^4*c^6*d^4))/(d^9 - 2*c^2*d^7 + c^4*d^5) - (32*tan(e/2 + (f*x)/2)*(a^6*c^
3*d^8 - 8*b^6*c^3*d^8 + 29*b^6*c^5*d^6 - 28*b^6*c^7*d^4 + 8*b^6*c^9*d^2 + 2
4*a*b^5*c^2*d^9 - 96*a*b^5*c^4*d^7 + 90*a*b^5*c^6*d^5 - 24*a*b^5*c^8*d^3 -
18*a^2*b^4*c*d^10 + 9*a^4*b^2*c*d^10 - 6*a^5*b*c^2*d^9 + 99*a^2*b^4*c^3*d^8
 - 84*a^2*b^4*c^5*d^6 + 18*a^2*b^4*c^7*d^4 - 36*a^3*b^3*c^2*d^9 + 12*a^3*b^
3*c^4*d^7 + 4*a^3*b^3*c^6*d^5 + 12*a^4*b^2*c^3*d^8 - 6*a^4*b^2*c^5*d^6))/(d
^10 - 2*c^2*d^8 + c^4*d^6) + (b^2*(3*a*d - 2*b*c)*((32*(a^3*c^5*d^7 - a^3*c
^3*d^9 + 2*b^3*c^2*d^10 - 3*b^3*c^4*d^8 + b^3*c^6*d^6 + 3*a*b^2*c^3*d^9 + 3
*a^2*b*c^2*d^10 - 3*a^2*b*c^4*d^8 - 3*a*b^2*c*d^11))/(d^9 - 2*c^2*d^7 + c^4
*d^5) - (32*tan(e/2 + (f*x)/2)*(2*a^3*c^2*d^11 - 2*a^3*c^4*d^9 - 6*b^3*c^3*
d^10 + 10*b^3*c^5*d^8 - 4*b^3*c^7*d^6 + 12*a*b^2*c^2*d^11 - 18*a*b^2*c^4*d^
9 + 6*a*b^2*c^6*d^7 + 6*a^2*b*c^3*d^10 - 6*a^2*b*c*d^12))/(d^10 - 2*c^2*d^8
 + c^4*d^6) + (b^2*((32*(c^2*d^12 - 2*c^4*d^10 + c^6*d^8))/(d^9 - 2*c^2*d^7
 + c^4*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^14 - 8*c^3*d^12 + 7*c^5*d^10 -
2*c^7*d^8))/(d^10 - 2*c^2*d^8 + c^4*d^6))*(3*a*d - 2*b*c)*1i)/d^3)*1i)/d^3)
)/d^3)/((64*(6*b^9*c^6*d^2 - 4*b^9*c^8 - 39*a*b^8*c^5*d^3 + 4*a^3*b^6*c^7*d
 - 27*a^5*b^4*c*d^7 + 105*a^2*b^7*c^4*d^4 - 57*a^2*b^7*c^6*d^2 - 144*a^3*b^
6*c^3*d^5 + 55*a^3*b^6*c^5*d^3 + 99*a^4*b^5*c^2*d^6 + 3*a^4*b^5*c^4*d^4 - 1
2*a^4*b^5*c^6*d^2 - 39*a^5*b^4*c^3*d^5 + 9*a^5*b^4*c^5*d^3 + 18*a^6*b^3*c^2
*d^6 + 2*a^6*b^3*c^4*d^4 - 3*a^7*b^2*c^3*d^5 + 24*a*b^8*c^7*d))/(d^9 - 2*c^
2*d^7 + c^4*d^5) + (64*tan(e/2 + (f*x)/2)*(40*b^9*c^7*d^2 - 24*b^9*c^5*d^4
- 16*b^9*c^9 + 120*a*b^8*c^4*d^5 - 192*a*b^8*c^6*d^3 - 54*a^4*b^5*c*d^8 - 2
22*a^2*b^7*c^3*d^6 + 330*a^2*b^7*c^5*d^4 - 108*a^2*b^7*c^7*d^2 + 180*a^3*b^
6*c^2*d^7 - 226*a^3*b^6*c^4*d^5 + 46*a^3*b^6*c^6*d^3 + 30*a^4*b^5*c^3*d^6 +
 24*a^4*b^5*c^5*d^4 + 18*a^5*b^4*c^2*d^7 - 18*a^5*b^4*c^4*d^5 + 72*a*b^8*c^
8*d))/(d^10 - 2*c^2*d^8 + c^4*d^6) + (b^2*(3*a*d - 2*b*c)*((32*(4*b^6*c^4*d
^6 - 8*b^6*c^6*d^4 + 4*b^6*c^8*d^2 - 12*a*b^5*c^3*d^7 + 24*a*b^5*c^5*d^5 -
12*a*b^5*c^7*d^3 + 9*a^2*b^4*c^2*d^8 - 18*a^2*b^4*c^4*d^6 + 9*a^2*b^4*c^6*d
^4))/(d^9 - 2*c^2*d^7 + c^4*d^5) - (32*tan(e/2 + (f*x)/2)*(a^6*c^3*d^8 - 8*
b^6*c^3*d^8 + 29*b^6*c^5*d^6 - 28*b^6*c^7*d^4 + 8*b^6*c^9*d^2 + 24*a*b^5*c^
2*d^9 - 96*a*b^5*c^4*d^7 + 90*a*b^5*c^6*d^5 - 24*a*b^5*c^8*d^3 - 18*a^2*b^4
*c*d^10 + 9*a^4*b^2*c*d^10 - 6*a^5*b*c^2*d^9 + 99*a^2*b^4*c^3*d^8 - 84*a^2*
b^4*c^5*d^6 + 18*a^2*b^4*c^7*d^4 - 36*a^3*b^3*c^2*d^9 + 12*a^3*b^3*c^4*d^7
+ 4*a^3*b^3*c^6*d^5 + 12*a^4*b^2*c^3*d^8 - 6*a^4*b^2*c^5*d^6))/(d^10 - 2*c^
2*d^8 + c^4*d^6) + (b^2*(3*a*d - 2*b*c)*((32*tan(e/2 + (f*x)/2)*(2*a^3*c^2*
d^11 - 2*a^3*c^4*d^9 - 6*b^3*c^3*d^10 + 10*b^3*c^5*d^8 - 4*b^3*c^7*d^6 + 12
*a*b^2*c^2*d^11 - 18*a*b^2*c^4*d^9 + 6*a*b^2*c^6*d^7 + 6*a^2*b*c^3*d^10 - 6
*a^2*b*c*d^12))/(d^10 - 2*c^2*d^8 + c^4*d^6) - (32*(a^3*c^5*d^7 - a^3*c^3*d
^9 + 2*b^3*c^2*d^10 - 3*b^3*c^4*d^8 + b^3*c^6*d^6 + 3*a*b^2*c^3*d^9 + 3*a^2
*b*c^2*d^10 - 3*a^2*b*c^4*d^8 - 3*a*b^2*c*d^11))/(d^9 - 2*c^2*d^7 + c^4*d^5
) + (b^2*((32*(c^2*d^12 - 2*c^4*d^10 + c^6*d^8))/(d^9 - 2*c^2*d^7 + c^4*d^5
) + (32*tan(e/2 + (f*x)/2)*(3*c*d^14 - 8*c^3*d^12 + 7*c^5*d^10 - 2*c^7*d^8)
)/(d^10 - 2*c^2*d^8 + c^4*d^6))*(3*a*d - 2*b*c)*1i)/d^3)*1i)/d^3)*1i)/d^3 -

(b^2*(3*a*d - 2*b*c)*((32*(4*b^6*c^4*d^6 - 8*b^6*c^6*d^4 + 4*b^6*c^8*d^2 - 12*a*b^5*c^3*d^7 + 24*a*b^5*c^5*d^5 - 12*a*b^5*c^7*d^3 + 9*a^2*b^4*c^2*d^8 - 18*a^2*b^4*c^4*d^6 + 9*a^2*b^4*c^6*d^4))/(d^9 - 2*c^2*d^7 + c^4*d^5) - (32*tan(e/2 + (f*x)/2)*(a^6*c^3*d^8 - 8*b^6*c^3*d^8 + 29*b^6*c^5*d^6 - 28*b^6*c^7*d^4 + 8*b^6*c^9*d^2 + 24*a*b^5*c^2*d^9 - 96*a*b^5*c^4*d^7 + 90*a*b^5*c^6*d^5 - 24*a*b^5*c^8*d^3 - 18*a^2*b^4*c*d^10 + 9*a^4*b^2*c*d^10 - 6*a^5*b*c^2*d^9 + 99*a^2*b^4*c^3*d^8 - 84*a^2*b^4*c^5*d^6 + 18*a^2*b^4*c^7*d^4 - 36*a^3*b^3*c^2*d^9 + 12*a^3*b^3*c^4*d^7 + 4*a^3*b^3*c^6*d^5 + 12*a^4*b^2*c^3*d^8 - 6*a^4*b^2*c^5*d^6))/(d^10 - 2*c^2*d^8 + c^4*d^6) + (b^2*(3*a*d - 2*b*c)*((32*(a^3*c^5*d^7 - a^3*c^3*d^9 + 2*b^3*c^2*d^10 - 3*b^3*c^4*d^8 + b^3*c^6*d^6 + 3*a*b^2*c^3*d^9 + 3*a^2*b*c^2*d^10 - 3*a^2*b*c^4*d^8 - 3*a*b^2*c*d^11))/(d^9 - 2*c^2*d^7 + c^4*d^5) - (32*tan(e/2 + (f*x)/2)*(2*a^3*c^2*d^11 - 2*a^3*c^4*d^9 - 6*b^3*c^3*d^10 + 10*b^3*c^5*d^8 - 4*b^3*c^7*d^6 + 12*a*b^2*c^2*d^11 - 18*a*b^2*c^4*d^9 + 6*a*b^2*c^6*d^7 + 6*a^2*b*c^3*d^10 - 6*a^2*b*c*d^12))/(d^10 - 2*c^2*d^8 + c^4*d^6) + (b^2*((32*(c^2*d^12 - 2*c^4*d^10 + c^6*d^8))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^14 - 8*c^3*d^12 + 7*c^5*d^10 - 2*c^7*d^8))/(d^10 - 2*c^2*d^8 + c^4*d^6))*(3*a*d - 2*b*c)*1i)/d^3)*1i)/d^3)*1i)/d^3))*(3*a*d - 2*b*c))/(d^3*f) + (atan((((a*d - b*c)^2*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(4*b^6*c^4*d^6 - 8*b^6*c^6*d^4 + 4*b^6*c^8*d^2 - 12*a*b^5*c^3*d^7 + 24*a*b^5*c^5*d^5 - 12*a*b^5*c^7*d^3 + 9*a^2*b^4*c^2*d^8 - 18*a^2*b^4*c^4*d^6 + 9*a^2*b^4*c^6*d^4))/(d^9 - 2*c^2*d^7 + c^4*d^5) - (32*tan(e/2 + (f*x)/2)*(a^6*c^3*d^8 - 8*b^6*c^3*d^8 + 29*b^6*c^5*d^6 - 28*b^6*c^7*d^4 + 8*b^6*c^9*d^2 + 24*a*b^5*c^2*d^9 - 96*a*b^5*c^4*d^7 + 90*a*b^5*c^6*d^5 - 24*a*b^5*c^8*d^3 - 18*a^2*b^4*c*d^10 + 9*a^4*b^2*c*d^10 - 6*a^5*b*c^2*d^9 + 99*a^2*b^4*c^3*d^8 - 84*a^2*b^4*c^5*d^6 + 18*a^2*b^4*c^7*d^4 - 36*a^3*b^3*c^2*d^9 + 12*a^3*b^3*c^4*d^7 + 4*a^3*b^3*c^6*d^5 + 12*a^4*b^2*c^3*d^8 - 6*a^4*b^2*c^5*d^6))/(d^10 - 2*c^2*d^8 + c^4*d^6) + ((a*d - b*c)^2*(-(c + d)^3*(c - d)^3)^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*a^3*c^2*d^11 - 2*a^3*c^4*d^9 - 6*b^3*c^3*d^10 + 10*b^3*c^5*d^8 - 4*b^3*c^7*d^6 + 12*a*b^2*c^2*d^11 - 18*a*b^2*c^4*d^9 + 6*a*b^2*c^6*d^7 + 6*a^2*b*c^3*d^10 - 6*a^2*b*c*d^12))/(d^10 - 2*c^2*d^8 + c^4*d^6) - (32*(a^3*c^5*d^7 - a^3*c^3*d^9 + 2*b^3*c^2*d^10 - 3*b^3*c^4*d^8 + b^3*c^6*d^6 + 3*a*b^2*c^3*d^9 + 3*a^2*b*c^2*d^10 - 3*a^2*b*c^4*d^8 - 3*a*b^2*c*d^11))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (((32*(c^2*d^12 - 2*c^4*d^10 + c^6*d^8))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^14 - 8*c^3*d^12 + 7*c^5*d^10 - 2*c^7*d^8))/(d^10 - 2*c^2*d^8 + c^4*d^6))*(a*d - b*c)^2*(-(c + d)^3*(c - d)^3)^(1/2)*(2*b*c^2 - 3*b*d^2 + a*c*d))/(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3))*(2*b*c^2 - 3*b*d^2 + a*c*d))/(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3))*(2*b*c^2 - 3*b*d^2 + a*c*d)*1i)/(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3) + ((a*d - b*c)^2*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(4*b^6*c^4*d^6 - 8*b^6*c^6*d^4 + 4*b^6*c^8*d^2 - 12*a*b^5*c^3*d^7 + 24*a*b^5*c^5*d^5 - 12*a*b^5*c^7*d^3 + 9*a^2*b^4*c^2*d^8 - 18*a^2*b^4*c^4*d^6 + 9*a^2*b^4*c^6*d^4))/(d^9 - 2*c^2*d^7 + c^4*d^5) - (32*tan(e/2 + (f*x)/2)*(a^6*c^3*d^8 - 8*b^6*c^3*d^8 + 29*b^6*c^5*d^6 - 28*b^6*c^7*d^4 + 8*b^6*c^9*d^2 + 24*a*b^5*c^2*d^9 - 96*a*b^5*c^4*d^7 + 90*a*b^5*c^6*d^5 - 24*a*b^5*c^8*d^3 - 18*a^2*b^4*c*d^10 + 9*a^4*b^2*c

*d^10 - 6*a^5*b*c^2*d^9 + 99*a^2*b^4*c^3*d^8 - 84*a^2*b^4*c^5*d^6 + 18*a^2*
b^4*c^7*d^4 - 36*a^3*b^3*c^2*d^9 + 12*a^3*b^3*c^4*d^7 + 4*a^3*b^3*c^6*d^5 +
12*a^4*b^2*c^3*d^8 - 6*a^4*b^2*c^5*d^6))/(d^10 - 2*c^2*d^8 + c^4*d^6) + ((
a*d - b*c)^2*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(a^3*c^5*d^7 - a^3*c^3*d^9 +
2*b^3*c^2*d^10 - 3*b^3*c^4*d^8 + b^3*c^6*d^6 + 3*a*b^2*c^3*d^9 + 3*a^2*b*c
^2*d^10 - 3*a^2*b*c^4*d^8 - 3*a*b^2*c*d^11))/(d^9 - 2*c^2*d^7 + c^4*d^5) -
(32*tan(e/2 + (f*x)/2)*(2*a^3*c^2*d^11 - 2*a^3*c^4*d^9 - 6*b^3*c^3*d^10 + 1
0*b^3*c^5*d^8 - 4*b^3*c^7*d^6 + 12*a*b^2*c^2*d^11 - 18*a*b^2*c^4*d^9 + 6*a*
b^2*c^6*d^7 + 6*a^2*b*c^3*d^10 - 6*a^2*b*c*d^12))/(d^10 - 2*c^2*d^8 + c^4*d
^6) + (((32*(c^2*d^12 - 2*c^4*d^10 + c^6*d^8))/(d^9 - 2*c^2*d^7 + c^4*d^5)
+ (32*tan(e/2 + (f*x)/2)*(3*c*d^14 - 8*c^3*d^12 + 7*c^5*d^10 - 2*c^7*d^8))/
(d^10 - 2*c^2*d^8 + c^4*d^6))*(a*d - b*c)^2*(-(c + d)^3*(c - d)^3)^(1/2)*(2
*b*c^2 - 3*b*d^2 + a*c*d))/(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3))*(2*b*c^
2 - 3*b*d^2 + a*c*d))/(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3))*(2*b*c^2 - 3
*b*d^2 + a*c*d)*1i)/(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3))/((64*(6*b^9*c^
6*d^2 - 4*b^9*c^8 - 39*a*b^8*c^5*d^3 + 4*a^3*b^6*c^7*d - 27*a^5*b^4*c*d^7 +
105*a^2*b^7*c^4*d^4 - 57*a^2*b^7*c^6*d^2 - 144*a^3*b^6*c^3*d^5 + 55*a^3*b^
6*c^5*d^3 + 99*a^4*b^5*c^2*d^6 + 3*a^4*b^5*c^4*d^4 - 12*a^4*b^5*c^6*d^2 - 3
9*a^5*b^4*c^3*d^5 + 9*a^5*b^4*c^5*d^3 + 18*a^6*b^3*c^2*d^6 + 2*a^6*b^3*c^4*
d^4 - 3*a^7*b^2*c^3*d^5 + 24*a*b^8*c^7*d))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (6
4*tan(e/2 + (f*x)/2)*(40*b^9*c^7*d^2 - 24*b^9*c^5*d^4 - 16*b^9*c^9 + 120*a*
b^8*c^4*d^5 - 192*a*b^8*c^6*d^3 - 54*a^4*b^5*c*d^8 - 222*a^2*b^7*c^3*d^6 +
330*a^2*b^7*c^5*d^4 - 108*a^2*b^7*c^7*d^2 + 180*a^3*b^6*c^2*d^7 - 226*a^3*b
^6*c^4*d^5 + 46*a^3*b^6*c^6*d^3 + 30*a^4*b^5*c^3*d^6 + 24*a^4*b^5*c^5*d^4 +
18*a^5*b^4*c^2*d^7 - 18*a^5*b^4*c^4*d^5 + 72*a*b^8*c^8*d))/(d^10 - 2*c^2*d
^8 + c^4*d^6) + ((a*d - b*c)^2*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(4*b^6*c^4
*d^6 - 8*b^6*c^6*d^4 + 4*b^6*c^8*d^2 - 12*a*b^5*c^3*d^7 + 24*a*b^5*c^5*d^5
- 12*a*b^5*c^7*d^3 + 9*a^2*b^4*c^2*d^8 - 18*a^2*b^4*c^4*d^6 + 9*a^2*b^4*c^6
*d^4))/(d^9 - 2*c^2*d^7 + c^4*d^5) - (32*tan(e/2 + (f*x)/2)*(a^6*c^3*d^8 -
8*b^6*c^3*d^8 + 29*b^6*c^5*d^6 - 28*b^6*c^7*d^4 + 8*b^6*c^9*d^2 + 24*a*b^5*
c^2*d^9 - 96*a*b^5*c^4*d^7 + 90*a*b^5*c^6*d^5 - 24*a*b^5*c^8*d^3 - 18*a^2*b
^4*c*d^10 + 9*a^4*b^2*c*d^10 - 6*a^5*b*c^2*d^9 + 99*a^2*b^4*c^3*d^8 - 84*a^
2*b^4*c^5*d^6 + 18*a^2*b^4*c^7*d^4 - 36*a^3*b^3*c^2*d^9 + 12*a^3*b^3*c^4*d^
7 + 4*a^3*b^3*c^6*d^5 + 12*a^4*b^2*c^3*d^8 - 6*a^4*b^2*c^5*d^6))/(d^10 - 2*
c^2*d^8 + c^4*d^6) + ((a*d - b*c)^2*(-(c + d)^3*(c - d)^3)^(1/2)*((32*tan(e
/2 + (f*x)/2)*(2*a^3*c^2*d^11 - 2*a^3*c^4*d^9 - 6*b^3*c^3*d^10 + 10*b^3*c^5
*d^8 - 4*b^3*c^7*d^6 + 12*a*b^2*c^2*d^11 - 18*a*b^2*c^4*d^9 + 6*a*b^2*c^6*d
^7 + 6*a^2*b*c^3*d^10 - 6*a^2*b*c*d^12))/(d^10 - 2*c^2*d^8 + c^4*d^6) - (32
*(a^3*c^5*d^7 - a^3*c^3*d^9 + 2*b^3*c^2*d^10 - 3*b^3*c^4*d^8 + b^3*c^6*d^6
+ 3*a*b^2*c^3*d^9 + 3*a^2*b*c^2*d^10 - 3*a^2*b*c^4*d^8 - 3*a*b^2*c*d^11))/(
d^9 - 2*c^2*d^7 + c^4*d^5) + (((32*(c^2*d^12 - 2*c^4*d^10 + c^6*d^8))/(d^9
- 2*c^2*d^7 + c^4*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^14 - 8*c^3*d^12 + 7*
c^5*d^10 - 2*c^7*d^8))/(d^10 - 2*c^2*d^8 + c^4*d^6))*(a*d - b*c)^2*(-(c + d
)^3*(c - d)^3)^(1/2)*(2*b*c^2 - 3*b*d^2 + a*c*d))/(d^9 - 3*c^2*d^7 + 3*c^4*
d^5 - c^6*d^3))*(2*b*c^2 - 3*b*d^2 + a*c*d))/(d^9 - 3*c^2*d^7 + 3*c^4*d^5 -

```
c^6*d^3))*(2*b*c^2 - 3*b*d^2 + a*c*d))/(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*
d^3) - ((a*d - b*c)^2*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(4*b^6*c^4*d^6 - 8*
b^6*c^6*d^4 + 4*b^6*c^8*d^2 - 12*a*b^5*c^3*d^7 + 24*a*b^5*c^5*d^5 - 12*a*b^
5*c^7*d^3 + 9*a^2*b^4*c^2*d^8 - 18*a^2*b^4*c^4*d^6 + 9*a^2*b^4*c^6*d^4))/(d
^9 - 2*c^2*d^7 + c^4*d^5) - (32*tan(e/2 + (f*x)/2)*(a^6*c^3*d^8 - 8*b^6*c^3
*d^8 + 29*b^6*c^5*d^6 - 28*b^6*c^7*d^4 + 8*b^6*c^9*d^2 + 24*a*b^5*c^2*d^9 -
 96*a*b^5*c^4*d^7 + 90*a*b^5*c^6*d^5 - 24*a*b^5*c^8*d^3 - 18*a^2*b^4*c*d^10
 + 9*a^4*b^2*c*d^10 - 6*a^5*b*c^2*d^9 + 99*a^2*b^4*c^3*d^8 - 84*a^2*b^4*c^5
*d^6 + 18*a^2*b^4*c^7*d^4 - 36*a^3*b^3*c^2*d^9 + 12*a^3*b^3*c^4*d^7 + 4*a^3
*b^3*c^6*d^5 + 12*a^4*b^2*c^3*d^8 - 6*a^4*b^2*c^5*d^6))/(d^10 - 2*c^2*d^8 +
 c^4*d^6) + ((a*d - b*c)^2*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(a^3*c^5*d^7 -
 a^3*c^3*d^9 + 2*b^3*c^2*d^10 - 3*b^3*c^4*d^8 + b^3*c^6*d^6 + 3*a*b^2*c^3*d
^9 + 3*a^2*b*c^2*d^10 - 3*a^2*b*c^4*d^8 - 3*a*b^2*c*d^11))/(d^9 - 2*c^2*d^7
 + c^4*d^5) - (32*tan(e/2 + (f*x)/2)*(2*a^3*c^2*d^11 - 2*a^3*c^4*d^9 - 6*b^
3*c^3*d^10 + 10*b^3*c^5*d^8 - 4*b^3*c^7*d^6 + 12*a*b^2*c^2*d^11 - 18*a*b^2*
c^4*d^9 + 6*a*b^2*c^6*d^7 + 6*a^2*b*c^3*d^10 - 6*a^2*b*c*d^12))/(d^10 - 2*c
^2*d^8 + c^4*d^6) + (((32*(c^2*d^12 - 2*c^4*d^10 + c^6*d^8))/(d^9 - 2*c^2*d
^7 + c^4*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^14 - 8*c^3*d^12 + 7*c^5*d^10
- 2*c^7*d^8))/(d^10 - 2*c^2*d^8 + c^4*d^6))*(a*d - b*c)^2*(-(c + d)^3*(c -
d)^3)^(1/2)*(2*b*c^2 - 3*b*d^2 + a*c*d))/(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6
*d^3))*(2*b*c^2 - 3*b*d^2 + a*c*d))/(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3)
)*(2*b*c^2 - 3*b*d^2 + a*c*d))/(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3)))*(a
*d - b*c)^2*(-(c + d)^3*(c - d)^3)^(1/2)*(2*b*c^2 - 3*b*d^2 + a*c*d)*2i)/(f
*(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.692 $\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=255

$$-\frac{\left(-a^3d^3\left(2c^2+d^2\right)+9a^2bcd^4-3ab^2d^3\left(c^2+2d^2\right)+b^3\left(2c^5-5c^3d^2+6cd^4\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f\left(c^2-d^2\right)^{5/2}}+\frac{(bc-ad)^{2}}{2d^2f}$$

```
[Out] b^3*x/d^3-(9*a^2*b*c*d^4-a^3*d^3*(2*c^2+d^2)-3*a*b^2*d^3*(c^2+2*d^2)+b^3*(2
*c^5-5*c^3*d^2+6*c*d^4))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d
^3/(c^2-d^2)^(5/2)/f+1/2*(-a*d+b*c)^2*cos(f*x+e)*(a+b*sin(f*x+e))/d/(c^2-d^
2)/f/(c+d*sin(f*x+e))^2+1/2*(-a*d+b*c)^2*(3*a*c*d+2*b*c^2-5*b*d^2)*cos(f*x+
e)/d^2/(c^2-d^2)^2/f/(c+d*sin(f*x+e))
```

Rubi [A] time = 0.63, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2792, 3021, 2735, 2660, 618, 204}

$$-\frac{\left(9a^2bcd^4-a^3d^3\left(2c^2+d^2\right)-3ab^2d^3\left(c^2+2d^2\right)+b^3\left(-5c^3d^2+2c^5+6cd^4\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f\left(c^2-d^2\right)^{5/2}}+\frac{(bc-ad)^{2}}{2d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (b^3*x)/d^3 - ((9*a^2*b*c*d^4 - a^3*d^3*(2*c^2 + d^2) - 3*a*b^2*d^3*(c^2 +
2*d^2) + b^3*(2*c^5 - 5*c^3*d^2 + 6*c*d^4))*ArcTan[(d + c*Tan[(e + f*x)/2])
/Sqrt[c^2 - d^2]])/(d^3*(c^2 - d^2)^(5/2)*f) + ((b*c - a*d)^2*Cos[e + f*x]*
(a + b*Sin[e + f*x]))/(2*d*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^2) + ((b*c -
a*d)^2*(2*b*c^2 + 3*a*c*d - 5*b*d^2)*Cos[e + f*x])/(2*d^2*(c^2 - d^2)^2*f*(
c + d*Sin[e + f*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^3}\,dx = \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{2d\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} - \frac{\int \frac{b^3c^2-2a^3cd-4ab^2cd+5a^2bd^2-\left(4a^2bcd+2b^3cd-a^3\right.}{(c+d\sin}}{2d\left(\right.}$$

$$= \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{2d\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{(bc-ad)^2\left(2bc^2+3acd-5bd^2\right)\cos(e}{2d^2\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))}$$

$$= \frac{b^3x}{d^3} + \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{2d\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{(bc-ad)^2\left(2bc^2+3acd-5bd^2\right)}{2d^2\left(c^2-d^2\right)^2 f(c+d\sin(e}$$

$$= \frac{b^3x}{d^3} + \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{2d\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{(bc-ad)^2\left(2bc^2+3acd-5bd^2\right)}{2d^2\left(c^2-d^2\right)^2 f(c+d\sin(e}$$

$$= \frac{b^3x}{d^3} + \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{2d\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{(bc-ad)^2\left(2bc^2+3acd-5bd^2\right)}{2d^2\left(c^2-d^2\right)^2 f(c+d\sin(e}$$

$$= \frac{b^3x}{d^3} - \frac{\left(9a^2bcd^4-a^3d^3\left(2c^2+d^2\right)-3ab^2d^3\left(c^2+2d^2\right)+b^3\left(2c^5-5c^3d^2+6cd^4\right)\right)\text{ta}}{d^3\left(c^2-d^2\right)^{5/2}f}$$

Mathematica [B] time = 2.35, size = 521, normalized size = 2.04

$$\frac{3a^3cd^5\sin(2(e+fx))-3a^2bc^2d^4\sin(2(e+fx))-6a^2bd^6\sin(2(e+fx))-3ab^2c^3d^3\sin(2(e+fx))+12ab^2cd^5\sin(2(e+fx))-2d(bc-ad)^2\left(-4ac^2d+ad^3-2bc^3+5b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((-4*(9*a^2*b*c*d^4 - a^3*d^3*(2*c^2 + d^2) - 3*a*b^2*d^3*(c^2 + 2*d^2) + b
^3*(2*c^5 - 5*c^3*d^2 + 6*c*d^4))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2
- d^2]])/(c^2 - d^2)^(5/2) + (4*b^3*c^6*e - 6*b^3*c^4*d^2*e + 2*b^3*d^6*e +
 4*b^3*c^6*f*x - 6*b^3*c^4*d^2*f*x + 2*b^3*d^6*f*x - 2*d*(b*c - a*d)^2*(-2*
b*c^3 - 4*a*c^2*d + 5*b*c*d^2 + a*d^3)*Cos[e + f*x] - 2*b^3*(-(c^2*d) + d^3
)^2*(e + f*x)*Cos[2*(e + f*x)] + 8*b^3*c^5*d*e*Sin[e + f*x] - 16*b^3*c^3*d^
3*e*Sin[e + f*x] + 8*b^3*c*d^5*e*Sin[e + f*x] + 8*b^3*c^5*d*f*x*Sin[e + f*x
```

```
] - 16*b^3*c^3*d^3*f*x*Sin[e + f*x] + 8*b^3*c*d^5*f*x*Sin[e + f*x] + 3*b^3*
c^4*d^2*Sin[2*(e + f*x)] - 3*a*b^2*c^3*d^3*Sin[2*(e + f*x)] - 3*a^2*b*c^2*d
^4*Sin[2*(e + f*x)] - 6*b^3*c^2*d^4*Sin[2*(e + f*x)] + 3*a^3*c*d^5*Sin[2*(e
 + f*x)] + 12*a*b^2*c*d^5*Sin[2*(e + f*x)] - 6*a^2*b*d^6*Sin[2*(e + f*x)])/
((c^2 - d^2)^2*(c + d*Sin[e + f*x])^2))/(4*d^3*f)
```

fricas [B] time = 0.60, size = 1707, normalized size = 6.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(b^3*c^6*d^2 - 3*b^3*c^4*d^4 + 3*b^3*c^2*d^6 - b^3*d^8)*f*x*cos(f*x
 + e)^2 - 4*(b^3*c^8 - 2*b^3*c^6*d^2 + 2*b^3*c^2*d^6 - b^3*d^8)*f*x - (2*b^
3*c^7 - 3*b^3*c^5*d^2 - (2*a^3 + 3*a*b^2)*c^4*d^3 + (9*a^2*b + b^3)*c^3*d^4
 - 3*(a^3 + 3*a*b^2)*c^2*d^5 + 3*(3*a^2*b + 2*b^3)*c*d^6 - (a^3 + 6*a*b^2)*
d^7 - (2*b^3*c^5*d^2 - 5*b^3*c^3*d^4 - (2*a^3 + 3*a*b^2)*c^2*d^5 + 3*(3*a^2
*b + 2*b^3)*c*d^6 - (a^3 + 6*a*b^2)*d^7)*cos(f*x + e)^2 + 2*(2*b^3*c^6*d -
5*b^3*c^4*d^3 - (2*a^3 + 3*a*b^2)*c^3*d^4 + 3*(3*a^2*b + 2*b^3)*c^2*d^5 - (
a^3 + 6*a*b^2)*c*d^6)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos
(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x +
e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x
+ e) - c^2 - d^2)) - 2*(2*b^3*c^7*d + 3*a^2*b*c*d^7 + a^3*d^8 - (6*a^2*b +
7*b^3)*c^5*d^3 + (4*a^3 + 9*a*b^2)*c^4*d^4 + (3*a^2*b + 5*b^3)*c^3*d^5 - (5
*a^3 + 9*a*b^2)*c^2*d^6)*cos(f*x + e) - 2*(4*(b^3*c^7*d - 3*b^3*c^5*d^3 + 3
*b^3*c^3*d^5 - b^3*c*d^7)*f*x + 3*(b^3*c^6*d^2 - a*b^2*c^5*d^3 + 2*a^2*b*d^
8 - (a^2*b + 3*b^3)*c^4*d^4 + (a^3 + 5*a*b^2)*c^3*d^5 - (a^2*b - 2*b^3)*c^2
*d^6 - (a^3 + 4*a*b^2)*c*d^7)*cos(f*x + e))*sin(f*x + e))/((c^6*d^5 - 3*c^4
*d^7 + 3*c^2*d^9 - d^11)*f*cos(f*x + e)^2 - 2*(c^7*d^4 - 3*c^5*d^6 + 3*c^3*
d^8 - c*d^10)*f*sin(f*x + e) - (c^8*d^3 - 2*c^6*d^5 + 2*c^2*d^9 - d^11)*f),
 1/2*(2*(b^3*c^6*d^2 - 3*b^3*c^4*d^4 + 3*b^3*c^2*d^6 - b^3*d^8)*f*x*cos(f*x
 + e)^2 - 2*(b^3*c^8 - 2*b^3*c^6*d^2 + 2*b^3*c^2*d^6 - b^3*d^8)*f*x - (2*b^
3*c^7 - 3*b^3*c^5*d^2 - (2*a^3 + 3*a*b^2)*c^4*d^3 + (9*a^2*b + b^3)*c^3*d^4
 - 3*(a^3 + 3*a*b^2)*c^2*d^5 + 3*(3*a^2*b + 2*b^3)*c*d^6 - (a^3 + 6*a*b^2)*
d^7 - (2*b^3*c^5*d^2 - 5*b^3*c^3*d^4 - (2*a^3 + 3*a*b^2)*c^2*d^5 + 3*(3*a^2
*b + 2*b^3)*c*d^6 - (a^3 + 6*a*b^2)*d^7)*cos(f*x + e)^2 + 2*(2*b^3*c^6*d -
5*b^3*c^4*d^3 - (2*a^3 + 3*a*b^2)*c^3*d^4 + 3*(3*a^2*b + 2*b^3)*c^2*d^5 - (
a^3 + 6*a*b^2)*c*d^6)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e)
 + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) - (2*b^3*c^7*d + 3*a^2*b*c*d^7 + a^3*
d^8 - (6*a^2*b + 7*b^3)*c^5*d^3 + (4*a^3 + 9*a*b^2)*c^4*d^4 + (3*a^2*b + 5*
b^3)*c^3*d^5 - (5*a^3 + 9*a*b^2)*c^2*d^6)*cos(f*x + e) - (4*(b^3*c^7*d - 3*
b^3*c^5*d^3 + 3*b^3*c^3*d^5 - b^3*c*d^7)*f*x + 3*(b^3*c^6*d^2 - a*b^2*c^5*d
^3 + 2*a^2*b*d^8 - (a^2*b + 3*b^3)*c^4*d^4 + (a^3 + 5*a*b^2)*c^3*d^5 - (a^2
*b - 2*b^3)*c^2*d^6 - (a^3 + 4*a*b^2)*c*d^7)*cos(f*x + e))*sin(f*x + e))/((
```

```
c^6*d^5 - 3*c^4*d^7 + 3*c^2*d^9 - d^11)*f*cos(f*x + e)^2 - 2*(c^7*d^4 - 3*c
^5*d^6 + 3*c^3*d^8 - c*d^10)*f*sin(f*x + e) - (c^8*d^3 - 2*c^6*d^5 + 2*c^2*
d^9 - d^11)*f)]
```

giac [B] time = 0.29, size = 889, normalized size = 3.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((f*x + e)*b^3/d^3 - (2*b^3*c^5 - 5*b^3*c^3*d^2 - 2*a^3*c^2*d^3 - 3*a*b^2*c
^2*d^3 + 9*a^2*b*c*d^4 + 6*b^3*c*d^4 - a^3*d^5 - 6*a*b^2*d^5)*(pi*floor(1/2
*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2
- d^2)))/((c^4*d^3 - 2*c^2*d^5 + d^7)*sqrt(c^2 - d^2)) + (b^3*c^6*d*tan(1/2
*f*x + 1/2*e)^3 + 3*a*b^2*c^5*d^2*tan(1/2*f*x + 1/2*e)^3 - 9*a^2*b*c^4*d^3*
tan(1/2*f*x + 1/2*e)^3 - 4*b^3*c^4*d^3*tan(1/2*f*x + 1/2*e)^3 + 5*a^3*c^3*d
^4*tan(1/2*f*x + 1/2*e)^3 + 6*a*b^2*c^3*d^4*tan(1/2*f*x + 1/2*e)^3 - 2*a^3*
c*d^6*tan(1/2*f*x + 1/2*e)^3 + 2*b^3*c^7*tan(1/2*f*x + 1/2*e)^2 - 6*a^2*b*c
^5*d^2*tan(1/2*f*x + 1/2*e)^2 - b^3*c^5*d^2*tan(1/2*f*x + 1/2*e)^2 + 4*a^3*
c^4*d^3*tan(1/2*f*x + 1/2*e)^2 + 9*a*b^2*c^4*d^3*tan(1/2*f*x + 1/2*e)^2 - 1
5*a^2*b*c^3*d^4*tan(1/2*f*x + 1/2*e)^2 - 10*b^3*c^3*d^4*tan(1/2*f*x + 1/2*e
)^2 + 7*a^3*c^2*d^5*tan(1/2*f*x + 1/2*e)^2 + 18*a*b^2*c^2*d^5*tan(1/2*f*x +
 1/2*e)^2 - 6*a^2*b*c*d^6*tan(1/2*f*x + 1/2*e)^2 - 2*a^3*d^7*tan(1/2*f*x +
1/2*e)^2 + 7*b^3*c^6*d*tan(1/2*f*x + 1/2*e) - 3*a*b^2*c^5*d^2*tan(1/2*f*x +
 1/2*e) - 15*a^2*b*c^4*d^3*tan(1/2*f*x + 1/2*e) - 16*b^3*c^4*d^3*tan(1/2*f*
x + 1/2*e) + 11*a^3*c^3*d^4*tan(1/2*f*x + 1/2*e) + 30*a*b^2*c^3*d^4*tan(1/2
*f*x + 1/2*e) - 12*a^2*b*c^2*d^5*tan(1/2*f*x + 1/2*e) - 2*a^3*c*d^6*tan(1/2
*f*x + 1/2*e) + 2*b^3*c^7 - 6*a^2*b*c^5*d^2 - 5*b^3*c^5*d^2 + 4*a^3*c^4*d^3
 + 9*a*b^2*c^4*d^3 - 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)/((c^6*d^2 - 2*c^4*d^4 +
 c^2*d^6)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2))/f
```

maple [B] time = 0.30, size = 2785, normalized size = 10.92

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x)
```

```
[Out] 2/f*b^3/d^3*arctan(tan(1/2*f*x+1/2*e))+9/f*d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(
1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^2*tan(1/2*f*x+1/2*e)^2*a*b^2-15
/f*d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c^2/(c^4-2*c^2*d^2
+d^4)*tan(1/2*f*x+1/2*e)*a^2*b+30/f*d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f
*x+1/2*e)*d+c)^2*c/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*a*b^2-9/f*d/(tan(
```

1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^2*tan(
1/2*f*x+1/2*e)^3*a^2*b-9/f*d/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2
*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a^2*b*c+6/f*d^2/(tan(1/2*f*x
+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c*tan(1/2*f*x+1
/2*e)^3*a*b^2-6/f*d^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(
c^4-2*c^2*d^2+d^4)/c*tan(1/2*f*x+1/2*e)^2*a^2*b-15/f*d^2/(tan(1/2*f*x+1/2*e
)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c*tan(1/2*f*x+1/2*e)^
2*a^2*b+7/f/d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c^4/(c^4-
2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*b^3-16/f*d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(
1/2*f*x+1/2*e)*d+c)^2*c^2/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*b^3+18/f*d
^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*
tan(1/2*f*x+1/2*e)^2*a*b^2+6/f*d^2/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arct
an(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a*b^2+1/f/d/(tan(1/2*f
*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^4*tan(1/2*f
*x+1/2*e)^3*b^3-3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c^3
/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*a*b^2+3/f/(c^4-2*c^2*d^2+d^4)/(c^2-
d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a*b^2*c
^2+3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d
^4)*c^3*tan(1/2*f*x+1/2*e)^3*a*b^2-6/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*
x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^3*tan(1/2*f*x+1/2*e)^2*a^2*b+5/f/d/(c
^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(
c^2-d^2)^(1/2))*b^3*c^3-6/f*d/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/
2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b^3*c+4/f*d/(tan(1/2*f*x+1/
2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^2*tan(1/2*f*x+1/
2*e)^2*a^3-2/f/d^3/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(
1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b^3*c^5-4/f*d/(tan(1/2*f*x+1/2*e)^2*c+
2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^2*tan(1/2*f*x+1/2*e)^3*b^
3+9/f*d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+
d^4)*a*b^2*c^2-12/f*d^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2
/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*a^2*b+5/f*d^2/(tan(1/2*f*x+1/2*e)^2
*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c*tan(1/2*f*x+1/2*e)^3*a
^3-1/f*d^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d
^2+d^4)*a^3-5/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*
c^2*d^2+d^4)*b^3*c^3-3/f*d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d
+c)^2/(c^4-2*c^2*d^2+d^4)*a^2*b*c-2/f*d^5/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2
*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)/c^2*tan(1/2*f*x+1/2*e)^2*a^3+2/f/d^2
/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^
5*tan(1/2*f*x+1/2*e)^2*b^3-10/f*d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1
/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c*tan(1/2*f*x+1/2*e)^2*b^3+11/f*d^2/(tan(1
/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^4-2*c^2*d^2+d^4)*tan(1/2
*f*x+1/2*e)*a^3-2/f*d^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2
/(c^4-2*c^2*d^2+d^4)/c*tan(1/2*f*x+1/2*e)^3*a^3-2/f*d^4/(tan(1/2*f*x+1/2*e)
^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*a
^3+2/f/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*
e)+2*d)/(c^2-d^2)^(1/2))*a^3*c^2+7/f*d^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*

```
f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)^2*a^3+4/f*d/(tan(1
/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*a^3*c^2+2
/f/d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d
^4)*b^3*c^5+1/f*d^2/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan
(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a^3-6/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan
(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*a^2*b*c^3-1/f/(tan(1/2*f*x+1/2*e
)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^3*tan(1/2*f*x+1/2*e
)^2*b^3
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [**B**] time = 20.46, size = 11848, normalized size = 46.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3/(c + d*sin(e + f*x))^3,x)
```

```
[Out] - ((a^3*d^5 - 2*b^3*c^5 - 4*a^3*c^2*d^3 + 5*b^3*c^3*d^2 - 9*a*b^2*c^2*d^3 +
 6*a^2*b*c^3*d^2 + 3*a^2*b*c*d^4)/(d^2*(c^4 + d^4 - 2*c^2*d^2)) - (tan(e/2
+ (f*x)/2)^3*(b^3*c^5 - 2*a^3*d^5 + 5*a^3*c^2*d^3 - 4*b^3*c^3*d^2 + 6*a*b^2
*c^2*d^3 - 9*a^2*b*c^3*d^2 + 3*a*b^2*c^4*d))/(c*d*(c^4 + d^4 - 2*c^2*d^2))
+ (tan(e/2 + (f*x)/2)*(2*a^3*d^5 - 7*b^3*c^5 - 11*a^3*c^2*d^3 + 16*b^3*c^3*
d^2 - 30*a*b^2*c^2*d^3 + 15*a^2*b*c^3*d^2 + 3*a*b^2*c^4*d + 12*a^2*b*c*d^4)
)/(c*d*(c^4 + d^4 - 2*c^2*d^2)) + (tan(e/2 + (f*x)/2)^2*(c^2 + 2*d^2)*(a^3*
d^5 - 2*b^3*c^5 - 4*a^3*c^2*d^3 + 5*b^3*c^3*d^2 - 9*a*b^2*c^2*d^3 + 6*a^2*b
*c^3*d^2 + 3*a^2*b*c*d^4))/(c^2*d^2*(c^4 + d^4 - 2*c^2*d^2)))/(f*(tan(e/2 +
 (f*x)/2)^2*(2*c^2 + 4*d^2) + c^2*tan(e/2 + (f*x)/2)^4 + c^2 + 4*c*d*tan(e/
2 + (f*x)/2)^3 + 4*c*d*tan(e/2 + (f*x)/2))) - (2*b^3*atan(((b^3*((8*(4*b^6*
c^2*d^10 - 16*b^6*c^4*d^8 + 24*b^6*c^6*d^6 - 16*b^6*c^8*d^4 + 4*b^6*c^10*d^
2))/(d^13 - 4*c^2*d^11 + 6*c^4*d^9 - 4*c^6*d^7 + c^8*d^5) - (8*tan(e/2 + (f
*x)/2)*(a^6*c*d^12 - 8*b^6*c*d^12 + 4*a^6*c^3*d^10 + 4*a^6*c^5*d^8 + 72*b^6
*c^3*d^10 - 124*b^6*c^5*d^8 + 105*b^6*c^7*d^6 - 44*b^6*c^9*d^4 + 8*b^6*c^11
*d^2 - 72*a*b^5*c^2*d^11 + 24*a*b^5*c^4*d^9 + 6*a*b^5*c^6*d^7 - 12*a*b^5*c^
8*d^5 + 36*a^2*b^4*c*d^12 + 12*a^4*b^2*c*d^12 - 18*a^5*b*c^2*d^11 - 36*a^5*
```

b*c^4*d^9 + 144*a^2*b^4*c^3*d^10 - 81*a^2*b^4*c^5*d^8 + 36*a^2*b^4*c^7*d^6 - 120*a^3*b^3*c^2*d^11 - 68*a^3*b^3*c^4*d^9 + 16*a^3*b^3*c^6*d^7 - 8*a^3*b^3*c^8*d^5 + 111*a^4*b^2*c^3*d^10 + 12*a^4*b^2*c^5*d^8))/(d^14 - 4*c^2*d^12 + 6*c^4*d^10 - 4*c^6*d^8 + c^8*d^6) + (b^3*((b^3*((8*(4*c^2*d^16 - 16*c^4*d^14 + 24*c^6*d^12 - 16*c^8*d^10 + 4*c^10*d^8))/(d^13 - 4*c^2*d^11 + 6*c^4*d^9 - 4*c^6*d^7 + c^8*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^18 - 56*c^3*d^16 + 104*c^5*d^14 - 96*c^7*d^12 + 44*c^9*d^10 - 8*c^11*d^8))/(d^14 - 4*c^2*d^12 + 6*c^4*d^10 - 4*c^6*d^8 + c^8*d^6))*1i)/d^3 - (8*(4*b^3*c*d^14 - 2*a^3*c^2*d^13 + 6*a^3*c^6*d^9 - 4*a^3*c^8*d^7 - 8*b^3*c^3*d^12 + 6*b^3*c^5*d^10 - 4*b^3*c^7*d^8 + 2*b^3*c^9*d^6 - 12*a*b^2*c^2*d^13 + 18*a*b^2*c^4*d^11 - 6*a*b^2*c^8*d^7 + 18*a^2*b*c^3*d^12 - 36*a^2*b*c^5*d^10 + 18*a^2*b*c^7*d^8))/(d^13 - 4*c^2*d^11 + 6*c^4*d^9 - 4*c^6*d^7 + c^8*d^5) + (8*tan(e/2 + (f*x)/2)*(4*a^3*c*d^15 - 12*a^3*c^5*d^11 + 8*a^3*c^7*d^9 - 24*b^3*c^2*d^14 + 68*b^3*c^4*d^12 - 72*b^3*c^6*d^10 + 36*b^3*c^8*d^8 - 8*b^3*c^10*d^6 - 36*a*b^2*c^3*d^13 + 12*a*b^2*c^7*d^9 - 36*a^2*b*c^2*d^14 + 72*a^2*b*c^4*d^12 - 36*a^2*b*c^6*d^10 + 24*a*b^2*c*d^15))/(d^14 - 4*c^2*d^12 + 6*c^4*d^10 - 4*c^6*d^8 + c^8*d^6))*1i)/d^3))/d^3 + (b^3*((8*(4*b^6*c^2*d^10 - 16*b^6*c^4*d^8 + 24*b^6*c^6*d^6 - 16*b^6*c^8*d^4 + 4*b^6*c^10*d^2))/(d^13 - 4*c^2*d^11 + 6*c^4*d^9 - 4*c^6*d^7 + c^8*d^5) - (8*tan(e/2 + (f*x)/2)*(a^6*c*d^12 - 8*b^6*c*d^12 + 4*a^6*c^3*d^10 + 4*a^6*c^5*d^8 + 72*b^6*c^3*d^10 - 124*b^6*c^5*d^8 + 105*b^6*c^7*d^6 - 44*b^6*c^9*d^4 + 8*b^6*c^11*d^2 - 72*a*b^5*c^2*d^11 + 24*a*b^5*c^4*d^9 + 6*a*b^5*c^6*d^7 - 12*a*b^5*c^8*d^5 + 36*a^2*b^4*c*d^12 + 12*a^4*b^2*c*d^12 - 18*a^5*b*c^2*d^11 - 36*a^5*b*c^4*d^9 + 144*a^2*b^4*c^3*d^10 - 81*a^2*b^4*c^5*d^8 + 36*a^2*b^4*c^7*d^6 - 120*a^3*b^3*c^2*d^11 - 68*a^3*b^3*c^4*d^9 + 16*a^3*b^3*c^6*d^7 - 8*a^3*b^3*c^8*d^5 + 111*a^4*b^2*c^3*d^10 + 12*a^4*b^2*c^5*d^8))/(d^14 - 4*c^2*d^12 + 6*c^4*d^10 - 4*c^6*d^8 + c^8*d^6) + (b^3*((8*(4*b^3*c*d^14 - 2*a^3*c^2*d^13 + 6*a^3*c^6*d^9 - 4*a^3*c^8*d^7 - 8*b^3*c^3*d^12 + 6*b^3*c^5*d^10 - 4*b^3*c^7*d^8 + 2*b^3*c^9*d^6 - 12*a*b^2*c^2*d^13 + 18*a*b^2*c^4*d^11 - 6*a*b^2*c^8*d^7 + 18*a^2*b*c^3*d^12 - 36*a^2*b*c^5*d^10 + 18*a^2*b*c^7*d^8))/(d^13 - 4*c^2*d^11 + 6*c^4*d^9 - 4*c^6*d^7 + c^8*d^5) + (b^3*((8*(4*c^2*d^16 - 16*c^4*d^14 + 24*c^6*d^12 - 16*c^8*d^10 + 4*c^10*d^8))/(d^13 - 4*c^2*d^11 + 6*c^4*d^9 - 4*c^6*d^7 + c^8*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^18 - 56*c^3*d^16 + 104*c^5*d^14 - 96*c^7*d^12 + 44*c^9*d^10 - 8*c^11*d^8))/(d^14 - 4*c^2*d^12 + 6*c^4*d^10 - 4*c^6*d^8 + c^8*d^6))*1i)/d^3 - (8*tan(e/2 + (f*x)/2)*(4*a^3*c*d^15 - 12*a^3*c^5*d^11 + 8*a^3*c^7*d^9 - 24*b^3*c^2*d^14 + 68*b^3*c^4*d^12 - 72*b^3*c^6*d^10 + 36*b^3*c^8*d^8 - 8*b^3*c^10*d^6 - 36*a*b^2*c^3*d^13 + 12*a*b^2*c^7*d^9 - 36*a^2*b*c^2*d^14 + 72*a^2*b*c^4*d^12 - 36*a^2*b*c^6*d^10 + 24*a*b^2*c*d^15))/(d^14 - 4*c^2*d^12 + 6*c^4*d^10 - 4*c^6*d^8 + c^8*d^6))*1i)/d^3))/d^3)/((16*(24*b^9*c^3*d^6 - 2*b^9*c^9 - 26*b^9*c^5*d^4 + 13*b^9*c^7*d^2 - 60*a*b^8*c^2*d^7 + 6*a*b^8*c^4*d^5 + 6*a*b^8*c^6*d^3 + 36*a^2*b^7*c*d^8 - 4*a^3*b^6*c^8*d + 12*a^4*b^5*c*d^8 + a^6*b^3*c*d^8 + 126*a^2*b^7*c^3*d^6 - 45*a^2*b^7*c^5*d^4 + 18*a^2*b^7*c^7*d^2 - 118*a^3*b^6*c^2*d^7 - 68*a^3*b^6*c^4*d^5 + 10*a^3*b^6*c^6*d^3 + 111*a^4*b^5*c^3*d^6 + 12*a^4*b^5*c^5*d^4 - 18*a^5*b^4*c^2*d^7 - 36*a^5*b^4*c^4*d^5 + 4*a^6*b^3*c^3*d^6 + 4*a^6*b^3*c^5*d^4 - 6

*a*b^8*c^8*d))/(d^13 - 4*c^2*d^11 + 6*c^4*d^9 - 4*c^6*d^7 + c^8*d^5) + (b^3
((8(4*b^6*c^2*d^10 - 16*b^6*c^4*d^8 + 24*b^6*c^6*d^6 - 16*b^6*c^8*d^4 + 4
*b^6*c^10*d^2))/(d^13 - 4*c^2*d^11 + 6*c^4*d^9 - 4*c^6*d^7 + c^8*d^5) - (8*
tan(e/2 + (f*x)/2)*(a^6*c*d^12 - 8*b^6*c*d^12 + 4*a^6*c^3*d^10 + 4*a^6*c^5*
d^8 + 72*b^6*c^3*d^10 - 124*b^6*c^5*d^8 + 105*b^6*c^7*d^6 - 44*b^6*c^9*d^4
+ 8*b^6*c^11*d^2 - 72*a*b^5*c^2*d^11 + 24*a*b^5*c^4*d^9 + 6*a*b^5*c^6*d^7 -
 12*a*b^5*c^8*d^5 + 36*a^2*b^4*c*d^12 + 12*a^4*b^2*c*d^12 - 18*a^5*b*c^2*d^
11 - 36*a^5*b*c^4*d^9 + 144*a^2*b^4*c^3*d^10 - 81*a^2*b^4*c^5*d^8 + 36*a^2*
b^4*c^7*d^6 - 120*a^3*b^3*c^2*d^11 - 68*a^3*b^3*c^4*d^9 + 16*a^3*b^3*c^6*d^
7 - 8*a^3*b^3*c^8*d^5 + 111*a^4*b^2*c^3*d^10 + 12*a^4*b^2*c^5*d^8))/(d^14 -
 4*c^2*d^12 + 6*c^4*d^10 - 4*c^6*d^8 + c^8*d^6) + (b^3*((b^3*((8*(4*c^2*d^1
6 - 16*c^4*d^14 + 24*c^6*d^12 - 16*c^8*d^10 + 4*c^10*d^8))/(d^13 - 4*c^2*d^
11 + 6*c^4*d^9 - 4*c^6*d^7 + c^8*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^18 -
56*c^3*d^16 + 104*c^5*d^14 - 96*c^7*d^12 + 44*c^9*d^10 - 8*c^11*d^8))/(d^14
 - 4*c^2*d^12 + 6*c^4*d^10 - 4*c^6*d^8 + c^8*d^6))*1i)/d^3 - (8*(4*b^3*c*d^
14 - 2*a^3*c^2*d^13 + 6*a^3*c^6*d^9 - 4*a^3*c^8*d^7 - 8*b^3*c^3*d^12 + 6*b^
3*c^5*d^10 - 4*b^3*c^7*d^8 + 2*b^3*c^9*d^6 - 12*a*b^2*c^2*d^13 + 18*a*b^2*c
^4*d^11 - 6*a*b^2*c^8*d^7 + 18*a^2*b*c^3*d^12 - 36*a^2*b*c^5*d^10 + 18*a^2*
b*c^7*d^8))/(d^13 - 4*c^2*d^11 + 6*c^4*d^9 - 4*c^6*d^7 + c^8*d^5) + (8*tan(
e/2 + (f*x)/2)*(4*a^3*c*d^15 - 12*a^3*c^5*d^11 + 8*a^3*c^7*d^9 - 24*b^3*c^2
*d^14 + 68*b^3*c^4*d^12 - 72*b^3*c^6*d^10 + 36*b^3*c^8*d^8 - 8*b^3*c^10*d^6
 - 36*a*b^2*c^3*d^13 + 12*a*b^2*c^7*d^9 - 36*a^2*b*c^2*d^14 + 72*a^2*b*c^4*
d^12 - 36*a^2*b*c^6*d^10 + 24*a*b^2*c*d^15))/(d^14 - 4*c^2*d^12 + 6*c^4*d^1
0 - 4*c^6*d^8 + c^8*d^6))*1i)/d^3)*1i)/d^3 - (b^3*((8*(4*b^6*c^2*d^10 - 16*
b^6*c^4*d^8 + 24*b^6*c^6*d^6 - 16*b^6*c^8*d^4 + 4*b^6*c^10*d^2))/(d^13 - 4*
c^2*d^11 + 6*c^4*d^9 - 4*c^6*d^7 + c^8*d^5) - (8*tan(e/2 + (f*x)/2)*(a^6*c*
d^12 - 8*b^6*c*d^12 + 4*a^6*c^3*d^10 + 4*a^6*c^5*d^8 + 72*b^6*c^3*d^10 - 12
4*b^6*c^5*d^8 + 105*b^6*c^7*d^6 - 44*b^6*c^9*d^4 + 8*b^6*c^11*d^2 - 72*a*b^
5*c^2*d^11 + 24*a*b^5*c^4*d^9 + 6*a*b^5*c^6*d^7 - 12*a*b^5*c^8*d^5 + 36*a^2
*b^4*c*d^12 + 12*a^4*b^2*c*d^12 - 18*a^5*b*c^2*d^11 - 36*a^5*b*c^4*d^9 + 14
4*a^2*b^4*c^3*d^10 - 81*a^2*b^4*c^5*d^8 + 36*a^2*b^4*c^7*d^6 - 120*a^3*b^3*
c^2*d^11 - 68*a^3*b^3*c^4*d^9 + 16*a^3*b^3*c^6*d^7 - 8*a^3*b^3*c^8*d^5 + 11
1*a^4*b^2*c^3*d^10 + 12*a^4*b^2*c^5*d^8))/(d^14 - 4*c^2*d^12 + 6*c^4*d^10 -
 4*c^6*d^8 + c^8*d^6) + (b^3*((8*(4*b^3*c*d^14 - 2*a^3*c^2*d^13 + 6*a^3*c^6
*d^9 - 4*a^3*c^8*d^7 - 8*b^3*c^3*d^12 + 6*b^3*c^5*d^10 - 4*b^3*c^7*d^8 + 2*
b^3*c^9*d^6 - 12*a*b^2*c^2*d^13 + 18*a*b^2*c^4*d^11 - 6*a*b^2*c^8*d^7 + 18*
a^2*b*c^3*d^12 - 36*a^2*b*c^5*d^10 + 18*a^2*b*c^7*d^8))/(d^13 - 4*c^2*d^11
+ 6*c^4*d^9 - 4*c^6*d^7 + c^8*d^5) + (b^3*((8*(4*c^2*d^16 - 16*c^4*d^14 + 2
4*c^6*d^12 - 16*c^8*d^10 + 4*c^10*d^8))/(d^13 - 4*c^2*d^11 + 6*c^4*d^9 - 4*
c^6*d^7 + c^8*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^18 - 56*c^3*d^16 + 104*c
^5*d^14 - 96*c^7*d^12 + 44*c^9*d^10 - 8*c^11*d^8))/(d^14 - 4*c^2*d^12 + 6*c
^4*d^10 - 4*c^6*d^8 + c^8*d^6))*1i)/d^3 - (8*tan(e/2 + (f*x)/2)*(4*a^3*c*d^
15 - 12*a^3*c^5*d^11 + 8*a^3*c^7*d^9 - 24*b^3*c^2*d^14 + 68*b^3*c^4*d^12 -
72*b^3*c^6*d^10 + 36*b^3*c^8*d^8 - 8*b^3*c^10*d^6 - 36*a*b^2*c^3*d^13 + 12*
a*b^2*c^7*d^9 - 36*a^2*b*c^2*d^14 + 72*a^2*b*c^4*d^12 - 36*a^2*b*c^6*d^10 +

24*a*b^2*c*d^15))/(d^14 - 4*c^2*d^12 + 6*c^4*d^10 - 4*c^6*d^8 + c^8*d^6))*
1i)/d^3)*1i)/d^3 - (16*tan(e/2 + (f*x)/2)*(8*b^9*c^10 + 24*b^9*c^2*d^8 - 68
*b^9*c^4*d^6 + 72*b^9*c^6*d^4 - 36*b^9*c^8*d^2 + 36*a*b^8*c^3*d^7 - 12*a*b^
8*c^7*d^3 - 4*a^3*b^6*c*d^9 + 36*a^2*b^7*c^2*d^8 - 72*a^2*b^7*c^4*d^6 + 36*
a^2*b^7*c^6*d^4 + 12*a^3*b^6*c^5*d^5 - 8*a^3*b^6*c^7*d^3 - 24*a*b^8*c*d^9))
/(d^14 - 4*c^2*d^12 + 6*c^4*d^10 - 4*c^6*d^8 + c^8*d^6))))/(d^3*f) - (atan(
(((a*d - b*c)*(-(c + d)^5*(c - d)^5)^(1/2)*((8*(4*b^6*c^2*d^10 - 16*b^6*c^4
*d^8 + 24*b^6*c^6*d^6 - 16*b^6*c^8*d^4 + 4*b^6*c^10*d^2))/(d^13 - 4*c^2*d^1
1 + 6*c^4*d^9 - 4*c^6*d^7 + c^8*d^5) - (8*tan(e/2 + (f*x)/2)*(a^6*c*d^12 -
8*b^6*c*d^12 + 4*a^6*c^3*d^10 + 4*a^6*c^5*d^8 + 72*b^6*c^3*d^10 - 124*b^6*c
^5*d^8 + 105*b^6*c^7*d^6 - 44*b^6*c^9*d^4 + 8*b^6*c^11*d^2 - 72*a*b^5*c^2*d
^11 + 24*a*b^5*c^4*d^9 + 6*a*b^5*c^6*d^7 - 12*a*b^5*c^8*d^5 + 36*a^2*b^4*c*
d^12 + 12*a^4*b^2*c*d^12 - 18*a^5*b*c^2*d^11 - 36*a^5*b*c^4*d^9 + 144*a^2*b
^4*c^3*d^10 - 81*a^2*b^4*c^5*d^8 + 36*a^2*b^4*c^7*d^6 - 120*a^3*b^3*c^2*d^1
1 - 68*a^3*b^3*c^4*d^9 + 16*a^3*b^3*c^6*d^7 - 8*a^3*b^3*c^8*d^5 + 111*a^4*b
^2*c^3*d^10 + 12*a^4*b^2*c^5*d^8))/(d^14 - 4*c^2*d^12 + 6*c^4*d^10 - 4*c^6*
d^8 + c^8*d^6) + ((a*d - b*c)*(-(c + d)^5*(c - d)^5)^(1/2)*((8*tan(e/2 + (f
x)/2)(4*a^3*c*d^15 - 12*a^3*c^5*d^11 + 8*a^3*c^7*d^9 - 24*b^3*c^2*d^14 +
68*b^3*c^4*d^12 - 72*b^3*c^6*d^10 + 36*b^3*c^8*d^8 - 8*b^3*c^10*d^6 - 36*a*
b^2*c^3*d^13 + 12*a*b^2*c^7*d^9 - 36*a^2*b*c^2*d^14 + 72*a^2*b*c^4*d^12 - 3
6*a^2*b*c^6*d^10 + 24*a*b^2*c*d^15))/(d^14 - 4*c^2*d^12 + 6*c^4*d^10 - 4*c^
6*d^8 + c^8*d^6) - (8*(4*b^3*c*d^14 - 2*a^3*c^2*d^13 + 6*a^3*c^6*d^9 - 4*a^
3*c^8*d^7 - 8*b^3*c^3*d^12 + 6*b^3*c^5*d^10 - 4*b^3*c^7*d^8 + 2*b^3*c^9*d^6
- 12*a*b^2*c^2*d^13 + 18*a*b^2*c^4*d^11 - 6*a*b^2*c^8*d^7 + 18*a^2*b*c^3*d
^12 - 36*a^2*b*c^5*d^10 + 18*a^2*b*c^7*d^8))/(d^13 - 4*c^2*d^11 + 6*c^4*d^9
- 4*c^6*d^7 + c^8*d^5) + (((8*(4*c^2*d^16 - 16*c^4*d^14 + 24*c^6*d^12 - 16
*c^8*d^10 + 4*c^10*d^8))/(d^13 - 4*c^2*d^11 + 6*c^4*d^9 - 4*c^6*d^7 + c^8*d
^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^18 - 56*c^3*d^16 + 104*c^5*d^14 - 96*c^
7*d^12 + 44*c^9*d^10 - 8*c^11*d^8))/(d^14 - 4*c^2*d^12 + 6*c^4*d^10 - 4*c^6
*d^8 + c^8*d^6))*(a*d - b*c)*(-(c + d)^5*(c - d)^5)^(1/2)*(a^2*d^4 + 2*b^2*
c^4 + 6*b^2*d^4 + 2*a^2*c^2*d^2 - 5*b^2*c^2*d^2 - 8*a*b*c*d^3 + 2*a*b*c^3*d
))/(2*(d^13 - 5*c^2*d^11 + 10*c^4*d^9 - 10*c^6*d^7 + 5*c^8*d^5 - c^10*d^3))
)*(a^2*d^4 + 2*b^2*c^4 + 6*b^2*d^4 + 2*a^2*c^2*d^2 - 5*b^2*c^2*d^2 - 8*a*b*
c*d^3 + 2*a*b*c^3*d))/(2*(d^13 - 5*c^2*d^11 + 10*c^4*d^9 - 10*c^6*d^7 + 5*c
^8*d^5 - c^10*d^3)))*(a^2*d^4 + 2*b^2*c^4 + 6*b^2*d^4 + 2*a^2*c^2*d^2 - 5*b
^2*c^2*d^2 - 8*a*b*c*d^3 + 2*a*b*c^3*d)*1i)/(2*(d^13 - 5*c^2*d^11 + 10*c^4*
d^9 - 10*c^6*d^7 + 5*c^8*d^5 - c^10*d^3)) + ((a*d - b*c)*(-(c + d)^5*(c - d
)^5)^(1/2)*((8*(4*b^6*c^2*d^10 - 16*b^6*c^4*d^8 + 24*b^6*c^6*d^6 - 16*b^6*c
^8*d^4 + 4*b^6*c^10*d^2))/(d^13 - 4*c^2*d^11 + 6*c^4*d^9 - 4*c^6*d^7 + c^8*
d^5) - (8*tan(e/2 + (f*x)/2)*(a^6*c*d^12 - 8*b^6*c*d^12 + 4*a^6*c^3*d^10 +
4*a^6*c^5*d^8 + 72*b^6*c^3*d^10 - 124*b^6*c^5*d^8 + 105*b^6*c^7*d^6 - 44*b^
6*c^9*d^4 + 8*b^6*c^11*d^2 - 72*a*b^5*c^2*d^11 + 24*a*b^5*c^4*d^9 + 6*a*b^5
*c^6*d^7 - 12*a*b^5*c^8*d^5 + 36*a^2*b^4*c*d^12 + 12*a^4*b^2*c*d^12 - 18*a^
5*b*c^2*d^11 - 36*a^5*b*c^4*d^9 + 144*a^2*b^4*c^3*d^10 - 81*a^2*b^4*c^5*d^8
+ 36*a^2*b^4*c^7*d^6 - 120*a^3*b^3*c^2*d^11 - 68*a^3*b^3*c^4*d^9 + 16*a^3*

b^3*c^6*d^7 - 8*a^3*b^3*c^8*d^5 + 111*a^4*b^2*c^3*d^10 + 12*a^4*b^2*c^5*d^8
))/(d^14 - 4*c^2*d^12 + 6*c^4*d^10 - 4*c^6*d^8 + c^8*d^6) + ((a*d - b*c)*(-
(c + d)^5*(c - d)^5)^(1/2)*((8*(4*b^3*c*d^14 - 2*a^3*c^2*d^13 + 6*a^3*c^6*d
^9 - 4*a^3*c^8*d^7 - 8*b^3*c^3*d^12 + 6*b^3*c^5*d^10 - 4*b^3*c^7*d^8 + 2*b^
3*c^9*d^6 - 12*a*b^2*c^2*d^13 + 18*a*b^2*c^4*d^11 - 6*a*b^2*c^8*d^7 + 18*a^
2*b*c^3*d^12 - 36*a^2*b*c^5*d^10 + 18*a^2*b*c^7*d^8))/(d^13 - 4*c^2*d^11 +
6*c^4*d^9 - 4*c^6*d^7 + c^8*d^5) - (8*tan(e/2 + (f*x)/2)*(4*a^3*c*d^15 - 12
*a^3*c^5*d^11 + 8*a^3*c^7*d^9 - 24*b^3*c^2*d^14 + 68*b^3*c^4*d^12 - 72*b^3*
c^6*d^10 + 36*b^3*c^8*d^8 - 8*b^3*c^10*d^6 - 36*a*b^2*c^3*d^13 + 12*a*b^2*c
^7*d^9 - 36*a^2*b*c^2*d^14 + 72*a^2*b*c^4*d^12 - 36*a^2*b*c^6*d^10 + 24*a*b
^2*c*d^15))/(d^14 - 4*c^2*d^12 + 6*c^4*d^10 - 4*c^6*d^8 + c^8*d^6) + (((8*(
4*c^2*d^16 - 16*c^4*d^14 + 24*c^6*d^12 - 16*c^8*d^10 + 4*c^10*d^8))/(d^13 -
 4*c^2*d^11 + 6*c^4*d^9 - 4*c^6*d^7 + c^8*d^5) + (8*tan(e/2 + (f*x)/2)*(12*
c*d^18 - 56*c^3*d^16 + 104*c^5*d^14 - 96*c^7*d^12 + 44*c^9*d^10 - 8*c^11*d^
8))/(d^14 - 4*c^2*d^12 + 6*c^4*d^10 - 4*c^6*d^8 + c^8*d^6))*(a*d - b*c)*(-(
c + d)^5*(c - d)^5)^(1/2)*(a^2*d^4 + 2*b^2*c^4 + 6*b^2*d^4 + 2*a^2*c^2*d^2
- 5*b^2*c^2*d^2 - 8*a*b*c*d^3 + 2*a*b*c^3*d))/(2*(d^13 - 5*c^2*d^11 + 10*c^
4*d^9 - 10*c^6*d^7 + 5*c^8*d^5 - c^10*d^3)))*(a^2*d^4 + 2*b^2*c^4 + 6*b^2*d
^4 + 2*a^2*c^2*d^2 - 5*b^2*c^2*d^2 - 8*a*b*c*d^3 + 2*a*b*c^3*d))/(2*(d^13 -
 5*c^2*d^11 + 10*c^4*d^9 - 10*c^6*d^7 + 5*c^8*d^5 - c^10*d^3)))*(a^2*d^4 +
2*b^2*c^4 + 6*b^2*d^4 + 2*a^2*c^2*d^2 - 5*b^2*c^2*d^2 - 8*a*b*c*d^3 + 2*a*b
*c^3*d)*1i)/(2*(d^13 - 5*c^2*d^11 + 10*c^4*d^9 - 10*c^6*d^7 + 5*c^8*d^5 - c
^10*d^3)))/((16*(24*b^9*c^3*d^6 - 2*b^9*c^9 - 26*b^9*c^5*d^4 + 13*b^9*c^7*d
^2 - 60*a*b^8*c^2*d^7 + 6*a*b^8*c^4*d^5 + 6*a*b^8*c^6*d^3 + 36*a^2*b^7*c*d^
8 - 4*a^3*b^6*c^8*d + 12*a^4*b^5*c*d^8 + a^6*b^3*c*d^8 + 126*a^2*b^7*c^3*d^
6 - 45*a^2*b^7*c^5*d^4 + 18*a^2*b^7*c^7*d^2 - 118*a^3*b^6*c^2*d^7 - 68*a^3*
b^6*c^4*d^5 + 10*a^3*b^6*c^6*d^3 + 111*a^4*b^5*c^3*d^6 + 12*a^4*b^5*c^5*d^4
 - 18*a^5*b^4*c^2*d^7 - 36*a^5*b^4*c^4*d^5 + 4*a^6*b^3*c^3*d^6 + 4*a^6*b^3*
c^5*d^4 - 6*a*b^8*c^8*d))/(d^13 - 4*c^2*d^11 + 6*c^4*d^9 - 4*c^6*d^7 + c^8*
d^5) - (16*tan(e/2 + (f*x)/2)*(8*b^9*c^10 + 24*b^9*c^2*d^8 - 68*b^9*c^4*d^6
 + 72*b^9*c^6*d^4 - 36*b^9*c^8*d^2 + 36*a*b^8*c^3*d^7 - 12*a*b^8*c^7*d^3 -
4*a^3*b^6*c*d^9 + 36*a^2*b^7*c^2*d^8 - 72*a^2*b^7*c^4*d^6 + 36*a^2*b^7*c^6*
d^4 + 12*a^3*b^6*c^5*d^5 - 8*a^3*b^6*c^7*d^3 - 24*a*b^8*c*d^9))/(d^14 - 4*c
^2*d^12 + 6*c^4*d^10 - 4*c^6*d^8 + c^8*d^6) + ((a*d - b*c)*(-(c + d)^5*(c -
 d)^5)^(1/2)*((8*(4*b^6*c^2*d^10 - 16*b^6*c^4*d^8 + 24*b^6*c^6*d^6 - 16*b^6
*c^8*d^4 + 4*b^6*c^10*d^2))/(d^13 - 4*c^2*d^11 + 6*c^4*d^9 - 4*c^6*d^7 + c^
8*d^5) - (8*tan(e/2 + (f*x)/2)*(a^6*c*d^12 - 8*b^6*c*d^12 + 4*a^6*c^3*d^10
+ 4*a^6*c^5*d^8 + 72*b^6*c^3*d^10 - 124*b^6*c^5*d^8 + 105*b^6*c^7*d^6 - 44*
b^6*c^9*d^4 + 8*b^6*c^11*d^2 - 72*a*b^5*c^2*d^11 + 24*a*b^5*c^4*d^9 + 6*a*b
^5*c^6*d^7 - 12*a*b^5*c^8*d^5 + 36*a^2*b^4*c*d^12 + 12*a^4*b^2*c*d^12 - 18*
a^5*b*c^2*d^11 - 36*a^5*b*c^4*d^9 + 144*a^2*b^4*c^3*d^10 - 81*a^2*b^4*c^5*d
^8 + 36*a^2*b^4*c^7*d^6 - 120*a^3*b^3*c^2*d^11 - 68*a^3*b^3*c^4*d^9 + 16*a^
3*b^3*c^6*d^7 - 8*a^3*b^3*c^8*d^5 + 111*a^4*b^2*c^3*d^10 + 12*a^4*b^2*c^5*d
^8))/(d^14 - 4*c^2*d^12 + 6*c^4*d^10 - 4*c^6*d^8 + c^8*d^6) + ((a*d - b*c)*
(-(c + d)^5*(c - d)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(4*a^3*c*d^15 - 12*a^3*

c^5*d^11 + 8*a^3*c^7*d^9 - 24*b^3*c^2*d^14 + 68*b^3*c^4*d^12 - 72*b^3*c^6*d
^10 + 36*b^3*c^8*d^8 - 8*b^3*c^10*d^6 - 36*a*b^2*c^3*d^13 + 12*a*b^2*c^7*d^
9 - 36*a^2*b*c^2*d^14 + 72*a^2*b*c^4*d^12 - 36*a^2*b*c^6*d^10 + 24*a*b^2*c*
d^15))/(d^14 - 4*c^2*d^12 + 6*c^4*d^10 - 4*c^6*d^8 + c^8*d^6) - (8*(4*b^3*c
*d^14 - 2*a^3*c^2*d^13 + 6*a^3*c^6*d^9 - 4*a^3*c^8*d^7 - 8*b^3*c^3*d^12 + 6
*b^3*c^5*d^10 - 4*b^3*c^7*d^8 + 2*b^3*c^9*d^6 - 12*a*b^2*c^2*d^13 + 18*a*b^
2*c^4*d^11 - 6*a*b^2*c^8*d^7 + 18*a^2*b*c^3*d^12 - 36*a^2*b*c^5*d^10 + 18*a
^2*b*c^7*d^8))/(d^13 - 4*c^2*d^11 + 6*c^4*d^9 - 4*c^6*d^7 + c^8*d^5) + (((8
*(4*c^2*d^16 - 16*c^4*d^14 + 24*c^6*d^12 - 16*c^8*d^10 + 4*c^10*d^8))/(d^13
 - 4*c^2*d^11 + 6*c^4*d^9 - 4*c^6*d^7 + c^8*d^5) + (8*tan(e/2 + (f*x)/2)*(1
2*c*d^18 - 56*c^3*d^16 + 104*c^5*d^14 - 96*c^7*d^12 + 44*c^9*d^10 - 8*c^11*
d^8))/(d^14 - 4*c^2*d^12 + 6*c^4*d^10 - 4*c^6*d^8 + c^8*d^6))*(a*d - b*c)*(
-(c + d)^5*(c - d)^5)^(1/2)*(a^2*d^4 + 2*b^2*c^4 + 6*b^2*d^4 + 2*a^2*c^2*d^
2 - 5*b^2*c^2*d^2 - 8*a*b*c*d^3 + 2*a*b*c^3*d))/(2*(d^13 - 5*c^2*d^11 + 10*
c^4*d^9 - 10*c^6*d^7 + 5*c^8*d^5 - c^10*d^3)))*(a^2*d^4 + 2*b^2*c^4 + 6*b^2
*d^4 + 2*a^2*c^2*d^2 - 5*b^2*c^2*d^2 - 8*a*b*c*d^3 + 2*a*b*c^3*d))/(2*(d^13
 - 5*c^2*d^11 + 10*c^4*d^9 - 10*c^6*d^7 + 5*c^8*d^5 - c^10*d^3)))*(a^2*d^4
+ 2*b^2*c^4 + 6*b^2*d^4 + 2*a^2*c^2*d^2 - 5*b^2*c^2*d^2 - 8*a*b*c*d^3 + 2*a
*b*c^3*d))/(2*(d^13 - 5*c^2*d^11 + 10*c^4*d^9 - 10*c^6*d^7 + 5*c^8*d^5 - c^
10*d^3)) - ((a*d - b*c)*(-(c + d)^5*(c - d)^5)^(1/2)*((8*(4*b^6*c^2*d^10 -
16*b^6*c^4*d^8 + 24*b^6*c^6*d^6 - 16*b^6*c^8*d^4 + 4*b^6*c^10*d^2))/(d^13 -
 4*c^2*d^11 + 6*c^4*d^9 - 4*c^6*d^7 + c^8*d^5) - (8*tan(e/2 + (f*x)/2)*(a^6
*c*d^12 - 8*b^6*c*d^12 + 4*a^6*c^3*d^10 + 4*a^6*c^5*d^8 + 72*b^6*c^3*d^10 -
 124*b^6*c^5*d^8 + 105*b^6*c^7*d^6 - 44*b^6*c^9*d^4 + 8*b^6*c^11*d^2 - 72*a
*b^5*c^2*d^11 + 24*a*b^5*c^4*d^9 + 6*a*b^5*c^6*d^7 - 12*a*b^5*c^8*d^5 + 36*
a^2*b^4*c*d^12 + 12*a^4*b^2*c*d^12 - 18*a^5*b*c^2*d^11 - 36*a^5*b*c^4*d^9 +
 144*a^2*b^4*c^3*d^10 - 81*a^2*b^4*c^5*d^8 + 36*a^2*b^4*c^7*d^6 - 120*a^3*b
^3*c^2*d^11 - 68*a^3*b^3*c^4*d^9 + 16*a^3*b^3*c^6*d^7 - 8*a^3*b^3*c^8*d^5 +
 111*a^4*b^2*c^3*d^10 + 12*a^4*b^2*c^5*d^8))/(d^14 - 4*c^2*d^12 + 6*c^4*d^1
0 - 4*c^6*d^8 + c^8*d^6) + ((a*d - b*c)*(-(c + d)^5*(c - d)^5)^(1/2)*((8*(4
*b^3*c*d^14 - 2*a^3*c^2*d^13 + 6*a^3*c^6*d^9 - 4*a^3*c^8*d^7 - 8*b^3*c^3*d^
12 + 6*b^3*c^5*d^10 - 4*b^3*c^7*d^8 + 2*b^3*c^9*d^6 - 12*a*b^2*c^2*d^13 + 1
8*a*b^2*c^4*d^11 - 6*a*b^2*c^8*d^7 + 18*a^2*b*c^3*d^12 - 36*a^2*b*c^5*d^10
+ 18*a^2*b*c^7*d^8))/(d^13 - 4*c^2*d^11 + 6*c^4*d^9 - 4*c^6*d^7 + c^8*d^5)
- (8*tan(e/2 + (f*x)/2)*(4*a^3*c*d^15 - 12*a^3*c^5*d^11 + 8*a^3*c^7*d^9 - 2
4*b^3*c^2*d^14 + 68*b^3*c^4*d^12 - 72*b^3*c^6*d^10 + 36*b^3*c^8*d^8 - 8*b^3
*c^10*d^6 - 36*a*b^2*c^3*d^13 + 12*a*b^2*c^7*d^9 - 36*a^2*b*c^2*d^14 + 72*a
^2*b*c^4*d^12 - 36*a^2*b*c^6*d^10 + 24*a*b^2*c*d^15))/(d^14 - 4*c^2*d^12 +
6*c^4*d^10 - 4*c^6*d^8 + c^8*d^6) + (((8*(4*c^2*d^16 - 16*c^4*d^14 + 24*c^6
*d^12 - 16*c^8*d^10 + 4*c^10*d^8))/(d^13 - 4*c^2*d^11 + 6*c^4*d^9 - 4*c^6*d
^7 + c^8*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^18 - 56*c^3*d^16 + 104*c^5*d^
14 - 96*c^7*d^12 + 44*c^9*d^10 - 8*c^11*d^8))/(d^14 - 4*c^2*d^12 + 6*c^4*d^
10 - 4*c^6*d^8 + c^8*d^6))*(a*d - b*c)*(-(c + d)^5*(c - d)^5)^(1/2)*(a^2*d^
4 + 2*b^2*c^4 + 6*b^2*d^4 + 2*a^2*c^2*d^2 - 5*b^2*c^2*d^2 - 8*a*b*c*d^3 + 2
*a*b*c^3*d))/(2*(d^13 - 5*c^2*d^11 + 10*c^4*d^9 - 10*c^6*d^7 + 5*c^8*d^5 -

```
c^10*d^3)))*(a^2*d^4 + 2*b^2*c^4 + 6*b^2*d^4 + 2*a^2*c^2*d^2 - 5*b^2*c^2*d^
2 - 8*a*b*c*d^3 + 2*a*b*c^3*d))/(2*(d^13 - 5*c^2*d^11 + 10*c^4*d^9 - 10*c^6
*d^7 + 5*c^8*d^5 - c^10*d^3)))*(a^2*d^4 + 2*b^2*c^4 + 6*b^2*d^4 + 2*a^2*c^2
*d^2 - 5*b^2*c^2*d^2 - 8*a*b*c*d^3 + 2*a*b*c^3*d))/(2*(d^13 - 5*c^2*d^11 +
10*c^4*d^9 - 10*c^6*d^7 + 5*c^8*d^5 - c^10*d^3))))*(a*d - b*c)*(-(c + d)^5*
(c - d)^5)^(1/2)*(a^2*d^4 + 2*b^2*c^4 + 6*b^2*d^4 + 2*a^2*c^2*d^2 - 5*b^2*c
^2*d^2 - 8*a*b*c*d^3 + 2*a*b*c^3*d)*1i)/(f*(d^13 - 5*c^2*d^11 + 10*c^4*d^9
- 10*c^6*d^7 + 5*c^8*d^5 - c^10*d^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.693 $\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^4}\,dx$

Optimal. Leaf size=325

$$-\frac{(ac-bd)\left(-\left(a^2\left(2c^2+3d^2\right)\right)+10abcd-b^2\left(3c^2+2d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{7/2}}-\frac{\left(a^2d^2\left(11c^2+4d^2\right)+5abcd\left(c^2\right.\right.}{6d^2f}$$

```
[Out] -(a*c-b*d)*(10*a*b*c*d-b^2*(3*c^2+2*d^2)-a^2*(2*c^2+3*d^2))*arctan((d+c*tan
(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(c^2-d^2)^(7/2)/f+1/3*(-a*d+b*c)^2*cos(f*
x+e)*(a+b*sin(f*x+e))/d/(c^2-d^2)/f/(c+d*sin(f*x+e))^3+1/6*(-a*d+b*c)^2*(5*
a*c*d+2*b*c^2-7*b*d^2)*cos(f*x+e)/d^2/(c^2-d^2)^2/f/(c+d*sin(f*x+e))^2-1/6*
(-a*d+b*c)*(5*a*b*c*d*(c^2-7*d^2)+a^2*d^2*(11*c^2+4*d^2)+b^2*(2*c^4-5*c^2*d
^2+18*d^4))*cos(f*x+e)/d^2/(c^2-d^2)^3/f/(c+d*sin(f*x+e))
```

Rubi [A] time = 0.72, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2792, 3021, 2754, 12, 2660, 618, 204}

$$-\frac{(ac-bd)\left(a^2\left(-\left(2c^2+3d^2\right)\right)+10abcd-b^2\left(3c^2+2d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{7/2}}-\frac{\left(a^2d^2\left(11c^2+4d^2\right)+5abcd\left(c^2\right.\right.}{6d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^4,x]
```

```
[Out] -(((a*c - b*d)*(10*a*b*c*d - b^2*(3*c^2 + 2*d^2) - a^2*(2*c^2 + 3*d^2))*Arc
Tan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c^2 - d^2)^(7/2)*f)) + ((b
*c - a*d)^2*Cos[e + f*x]*(a + b*Sin[e + f*x]))/(3*d*(c^2 - d^2)*f*(c + d*Si
n[e + f*x])^3) + ((b*c - a*d)^2*(2*b*c^2 + 5*a*c*d - 7*b*d^2)*Cos[e + f*x])
/(6*d^2*(c^2 - d^2)^2*f*(c + d*Sin[e + f*x])^2) - ((b*c - a*d)*(5*a*b*c*d*(
c^2 - 7*d^2) + a^2*d^2*(11*c^2 + 4*d^2) + b^2*(2*c^4 - 5*c^2*d^2 + 18*d^4))
*Cos[e + f*x])/(6*d^2*(c^2 - d^2)^3*f*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
```

```
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^4}\,dx = \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^3} - \frac{\int \frac{b^3c^2-3a^3cd-5ab^2cd+7a^2bd^2-\left(7a^2bcd+3b^3cd-2a^3 \cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^3} + \frac{(bc-ad)^2\left(2bc^2+5acd-7bd^2\right)\cos(e+\cdots}{6d^2\left(c^2-d^2\right)^2f(c+d\sin(e+fx))^2}\cdots$$

$$= \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^3} + \frac{(bc-ad)^2\left(2bc^2+5acd-7bd^2\right)\cos(e+\cdots}{6d^2\left(c^2-d^2\right)^2f(c+d\sin(e+fx))^2}\cdots$$

$$= \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^3} + \frac{(bc-ad)^2\left(2bc^2+5acd-7bd^2\right)\cos(e+\cdots}{6d^2\left(c^2-d^2\right)^2f(c+d\sin(e+fx))^2}\cdots$$

$$= \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^3} + \frac{(bc-ad)^2\left(2bc^2+5acd-7bd^2\right)\cos(e+\cdots}{6d^2\left(c^2-d^2\right)^2f(c+d\sin(e+fx))^2}\cdots$$

$$= \frac{(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^3} + \frac{(bc-ad)^2\left(2bc^2+5acd-7bd^2\right)\cos(e+\cdots}{6d^2\left(c^2-d^2\right)^2f(c+d\sin(e+fx))^2}\cdots$$

$$= -\frac{(ac-bd)\left(10abcd-b^2\left(3c^2+2d^2\right)-a^2\left(2c^2+3d^2\right)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{7/2}f} + \frac{(bc-\cdots}{3\cdots}$$

Mathematica [A] time = 5.27, size = 345, normalized size = 1.06

$$\frac{\frac{6\left(a^3\left(2c^3+3cd^2\right)-3a^2bd\left(4c^2+d^2\right)+3ab^2c\left(c^2+4d^2\right)-b^3d\left(3c^2+2d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{7/2}} + \frac{\left(-a^3d^3\left(11c^2+4d^2\right)+3a^2bcd^2\left(2c^2+13d^2\right)+3ab^2d\left(c^4-10c^2\cdots\right.\right.}{d^2\left(d^2-c^2\right)^3(c+d\sin(\cdots}}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^4,x]
```

```
[Out] ((6*(-3*a^2*b*d*(4*c^2 + d^2) - b^3*d*(3*c^2 + 2*d^2) + 3*a*b^2*c*(c^2 + 4*
d^2) + a^3*(2*c^3 + 3*c*d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^
2]])/(c^2 - d^2)^(7/2) + (2*(b*c - a*d)^3*Cos[e + f*x])/(d^2*(-c^2 + d^2)*(
c + d*Sin[e + f*x])^3) + ((b*c - a*d)^2*(4*b*c^2 + 5*a*c*d - 9*b*d^2)*Cos[e
 + f*x])/(d^2*(c^2 - d^2)^2*(c + d*Sin[e + f*x])^2) + ((-(a^3*d^3*(11*c^2 +
 4*d^2)) + 3*a^2*b*c*d^2*(2*c^2 + 13*d^2) + 3*a*b^2*d*(c^4 - 10*c^2*d^2 - 6
*d^4) + b^3*(2*c^5 - 5*c^3*d^2 + 18*c*d^4))*Cos[e + f*x])/(d^2*(-c^2 + d^2)
^3*(c + d*Sin[e + f*x])))/(6*f)
```

fricas [B] time = 0.67, size = 2136, normalized size = 6.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(2*(2*b^3*c^7 + 3*a*b^2*c^6*d + (6*a^2*b - 7*b^3)*c^5*d^2 - 11*(a^3
+ 3*a*b^2)*c^4*d^3 + (33*a^2*b + 23*b^3)*c^3*d^4 + (7*a^3 + 12*a*b^2)*c^2*d
^5 - 3*(13*a^2*b + 6*b^3)*c*d^6 + 2*(2*a^3 + 9*a*b^2)*d^7)*cos(f*x + e)^3 -
 6*(3*a*b^2*c^7 + 3*a^2*b*d^7 + (6*a^2*b + b^3)*c^6*d - 3*(3*a^3 + 10*a*b^2
)*c^5*d^2 + (21*a^2*b + 8*b^3)*c^4*d^3 + (8*a^3 + 21*a*b^2)*c^3*d^4 - 3*(10
*a^2*b + 3*b^3)*c^2*d^5 + (a^3 + 6*a*b^2)*c*d^6)*cos(f*x + e)*sin(f*x + e)
- 3*((2*a^3 + 3*a*b^2)*c^6 - 3*(4*a^2*b + b^3)*c^5*d + 3*(3*a^3 + 7*a*b^2)*
c^4*d^2 - (39*a^2*b + 11*b^3)*c^3*d^3 + 9*(a^3 + 4*a*b^2)*c^2*d^4 - 3*(3*a^
2*b + 2*b^3)*c*d^5 - 3*((2*a^3 + 3*a*b^2)*c^4*d^2 - 3*(4*a^2*b + b^3)*c^3*d
^3 + 3*(a^3 + 4*a*b^2)*c^2*d^4 - (3*a^2*b + 2*b^3)*c*d^5)*cos(f*x + e)^2 +
(3*(2*a^3 + 3*a*b^2)*c^5*d - 9*(4*a^2*b + b^3)*c^4*d^2 + (11*a^3 + 39*a*b^2
)*c^3*d^3 - 3*(7*a^2*b + 3*b^3)*c^2*d^4 + 3*(a^3 + 4*a*b^2)*c*d^5 - (3*a^2*
b + 2*b^3)*d^6 - ((2*a^3 + 3*a*b^2)*c^3*d^3 - 3*(4*a^2*b + b^3)*c^2*d^4 + 3
*(a^3 + 4*a*b^2)*c*d^5 - (3*a^2*b + 2*b^3)*d^6)*cos(f*x + e)^2)*sin(f*x + e
))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e)
- c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 +
d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 12*(3*a^2*b*
c^5*d^2 + 2*a^3*c^4*d^3 + 2*b^3*c^3*d^4 + 3*a*b^2*c^2*d^5 + (3*a^2*b + b^3)
*c^7 - 3*(a^3 + 2*a*b^2)*c^6*d - 3*(2*a^2*b + b^3)*c*d^6 + (a^3 + 3*a*b^2)*
d^7)*cos(f*x + e))/(3*(c^9*d^2 - 4*c^7*d^4 + 6*c^5*d^6 - 4*c^3*d^8 + c*d^10
)*f*cos(f*x + e)^2 - (c^11 - c^9*d^2 - 6*c^7*d^4 + 14*c^5*d^6 - 11*c^3*d^8
+ 3*c*d^10)*f + ((c^8*d^3 - 4*c^6*d^5 + 6*c^4*d^7 - 4*c^2*d^9 + d^11)*f*cos
(f*x + e)^2 - (3*c^10*d - 11*c^8*d^3 + 14*c^6*d^5 - 6*c^4*d^7 - c^2*d^9 + d
^11)*f)*sin(f*x + e)), -1/6*((2*b^3*c^7 + 3*a*b^2*c^6*d + (6*a^2*b - 7*b^3)
*c^5*d^2 - 11*(a^3 + 3*a*b^2)*c^4*d^3 + (33*a^2*b + 23*b^3)*c^3*d^4 + (7*a^
```

```
3 + 12*a*b^2)*c^2*d^5 - 3*(13*a^2*b + 6*b^3)*c*d^6 + 2*(2*a^3 + 9*a*b^2)*d^
7)*cos(f*x + e)^3 - 3*(3*a*b^2*c^7 + 3*a^2*b*d^7 + (6*a^2*b + b^3)*c^6*d -
3*(3*a^3 + 10*a*b^2)*c^5*d^2 + (21*a^2*b + 8*b^3)*c^4*d^3 + (8*a^3 + 21*a*b
^2)*c^3*d^4 - 3*(10*a^2*b + 3*b^3)*c^2*d^5 + (a^3 + 6*a*b^2)*c*d^6)*cos(f*x
 + e)*sin(f*x + e) - 3*((2*a^3 + 3*a*b^2)*c^6 - 3*(4*a^2*b + b^3)*c^5*d + 3
*(3*a^3 + 7*a*b^2)*c^4*d^2 - (39*a^2*b + 11*b^3)*c^3*d^3 + 9*(a^3 + 4*a*b^2
)*c^2*d^4 - 3*(3*a^2*b + 2*b^3)*c*d^5 - 3*((2*a^3 + 3*a*b^2)*c^4*d^2 - 3*(4
*a^2*b + b^3)*c^3*d^3 + 3*(a^3 + 4*a*b^2)*c^2*d^4 - (3*a^2*b + 2*b^3)*c*d^5
)*cos(f*x + e)^2 + (3*(2*a^3 + 3*a*b^2)*c^5*d - 9*(4*a^2*b + b^3)*c^4*d^2 +
 (11*a^3 + 39*a*b^2)*c^3*d^3 - 3*(7*a^2*b + 3*b^3)*c^2*d^4 + 3*(a^3 + 4*a*b
^2)*c*d^5 - (3*a^2*b + 2*b^3)*d^6 - ((2*a^3 + 3*a*b^2)*c^3*d^3 - 3*(4*a^2*b
 + b^3)*c^2*d^4 + 3*(a^3 + 4*a*b^2)*c*d^5 - (3*a^2*b + 2*b^3)*d^6)*cos(f*x
+ e)^2)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^
2 - d^2)*cos(f*x + e))) - 6*(3*a^2*b*c^5*d^2 + 2*a^3*c^4*d^3 + 2*b^3*c^3*d^
4 + 3*a*b^2*c^2*d^5 + (3*a^2*b + b^3)*c^7 - 3*(a^3 + 2*a*b^2)*c^6*d - 3*(2*
a^2*b + b^3)*c*d^6 + (a^3 + 3*a*b^2)*d^7)*cos(f*x + e))/(3*(c^9*d^2 - 4*c^7
*d^4 + 6*c^5*d^6 - 4*c^3*d^8 + c*d^10)*f*cos(f*x + e)^2 - (c^11 - c^9*d^2 -
 6*c^7*d^4 + 14*c^5*d^6 - 11*c^3*d^8 + 3*c*d^10)*f + ((c^8*d^3 - 4*c^6*d^5
+ 6*c^4*d^7 - 4*c^2*d^9 + d^11)*f*cos(f*x + e)^2 - (3*c^10*d - 11*c^8*d^3 +
 14*c^6*d^5 - 6*c^4*d^7 - c^2*d^9 + d^11)*f)*sin(f*x + e))]
```

giac [B] time = 0.38, size = 1677, normalized size = 5.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(2*a^3*c^3 + 3*a*b^2*c^3 - 12*a^2*b*c^2*d - 3*b^3*c^2*d + 3*a^3*c*d^
2 + 12*a*b^2*c*d^2 - 3*a^2*b*d^3 - 2*b^3*d^3)*(pi*floor(1/2*(f*x + e)/pi +
1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^6 -
 3*c^4*d^2 + 3*c^2*d^4 - d^6)*sqrt(c^2 - d^2)) + (9*a*b^2*c^8*tan(1/2*f*x +
 1/2*e)^5 - 36*a^2*b*c^7*d*tan(1/2*f*x + 1/2*e)^5 - 9*b^3*c^7*d*tan(1/2*f*x
 + 1/2*e)^5 + 27*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 + 36*a*b^2*c^6*d^2*tan(
1/2*f*x + 1/2*e)^5 - 9*a^2*b*c^5*d^3*tan(1/2*f*x + 1/2*e)^5 - 6*b^3*c^5*d^3
*tan(1/2*f*x + 1/2*e)^5 - 18*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 + 6*a^3*c^2
*d^6*tan(1/2*f*x + 1/2*e)^5 - 18*a^2*b*c^8*tan(1/2*f*x + 1/2*e)^4 + 18*a^3*
c^7*d*tan(1/2*f*x + 1/2*e)^4 + 45*a*b^2*c^7*d*tan(1/2*f*x + 1/2*e)^4 - 126*
a^2*b*c^6*d^2*tan(1/2*f*x + 1/2*e)^4 - 45*b^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^
4 + 81*a^3*c^5*d^3*tan(1/2*f*x + 1/2*e)^4 + 180*a*b^2*c^5*d^3*tan(1/2*f*x +
 1/2*e)^4 - 99*a^2*b*c^4*d^4*tan(1/2*f*x + 1/2*e)^4 - 30*b^3*c^4*d^4*tan(1/
2*f*x + 1/2*e)^4 - 36*a^3*c^3*d^5*tan(1/2*f*x + 1/2*e)^4 + 18*a^2*b*c^2*d^6
*tan(1/2*f*x + 1/2*e)^4 + 12*a^3*c*d^7*tan(1/2*f*x + 1/2*e)^4 - 108*a^2*b*c
^7*d*tan(1/2*f*x + 1/2*e)^3 - 24*b^3*c^7*d*tan(1/2*f*x + 1/2*e)^3 + 108*a^3
*c^6*d^2*tan(1/2*f*x + 1/2*e)^3 + 234*a*b^2*c^6*d^2*tan(1/2*f*x + 1/2*e)^3
```

```
- 252*a^2*b*c^5*d^3*tan(1/2*f*x + 1/2*e)^3 - 82*b^3*c^5*d^3*tan(1/2*f*x + 1
/2*e)^3 + 42*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 + 192*a*b^2*c^4*d^4*tan(1/2
*f*x + 1/2*e)^3 - 102*a^2*b*c^3*d^5*tan(1/2*f*x + 1/2*e)^3 - 44*b^3*c^3*d^5
*tan(1/2*f*x + 1/2*e)^3 - 8*a^3*c^2*d^6*tan(1/2*f*x + 1/2*e)^3 + 24*a*b^2*c
^2*d^6*tan(1/2*f*x + 1/2*e)^3 + 12*a^2*b*c*d^7*tan(1/2*f*x + 1/2*e)^3 + 8*a
^3*d^8*tan(1/2*f*x + 1/2*e)^3 - 36*a^2*b*c^8*tan(1/2*f*x + 1/2*e)^2 - 12*b^
3*c^8*tan(1/2*f*x + 1/2*e)^2 + 36*a^3*c^7*d*tan(1/2*f*x + 1/2*e)^2 + 72*a*b
^2*c^7*d*tan(1/2*f*x + 1/2*e)^2 - 180*a^2*b*c^6*d^2*tan(1/2*f*x + 1/2*e)^2
- 36*b^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^2 + 120*a^3*c^5*d^3*tan(1/2*f*x + 1/2
*e)^2 + 306*a*b^2*c^5*d^3*tan(1/2*f*x + 1/2*e)^2 - 252*a^2*b*c^4*d^4*tan(1/
2*f*x + 1/2*e)^2 - 102*b^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^2 - 18*a^3*c^3*d^5*
tan(1/2*f*x + 1/2*e)^2 + 72*a*b^2*c^3*d^5*tan(1/2*f*x + 1/2*e)^2 + 18*a^2*b
*c^2*d^6*tan(1/2*f*x + 1/2*e)^2 + 12*a^3*c*d^7*tan(1/2*f*x + 1/2*e)^2 - 9*a
*b^2*c^8*tan(1/2*f*x + 1/2*e) - 72*a^2*b*c^7*d*tan(1/2*f*x + 1/2*e) - 15*b^
3*c^7*d*tan(1/2*f*x + 1/2*e) + 81*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e) + 198*a*
b^2*c^6*d^2*tan(1/2*f*x + 1/2*e) - 171*a^2*b*c^5*d^3*tan(1/2*f*x + 1/2*e) -
 60*b^3*c^5*d^3*tan(1/2*f*x + 1/2*e) - 12*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e)
+ 36*a*b^2*c^4*d^4*tan(1/2*f*x + 1/2*e) + 18*a^2*b*c^3*d^5*tan(1/2*f*x + 1/
2*e) + 6*a^3*c^2*d^6*tan(1/2*f*x + 1/2*e) - 18*a^2*b*c^8 - 4*b^3*c^8 + 18*a
^3*c^7*d + 39*a*b^2*c^7*d - 30*a^2*b*c^6*d^2 - 11*b^3*c^6*d^2 - 5*a^3*c^5*d
^3 + 6*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 + 2*a^3*c^3*d^5)/((c^9 - 3*c^7*d^2 +
 3*c^5*d^4 - c^3*d^6)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e)
+ c)^3))/f
```

maple [**B**] time = 0.33, size = 6128, normalized size = 18.86

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^4,x)
```

```
[Out] result too large to display
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 11.80, size = 1423, normalized size = 4.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3/(c + d*sin(e + f*x))^4,x)
```

```
[Out] ((2*a^3*d^5 - 4*b^3*c^5 - 18*a^2*b*c^5 + 18*a^3*c^4*d - 5*a^3*c^2*d^3 - 11*
b^3*c^3*d^2 + 6*a*b^2*c^2*d^3 - 30*a^2*b*c^3*d^2 + 39*a*b^2*c^4*d + 3*a^2*b
*c*d^4)/(3*(c^6 - d^6 + 3*c^2*d^4 - 3*c^4*d^2)) - (tan(e/2 + (f*x)/2)^5*(3*
b^3*c^5*d - 3*a*b^2*c^6 - 2*a^3*d^6 + 6*a^3*c^2*d^4 - 9*a^3*c^4*d^2 + 2*b^3
*c^3*d^3 - 12*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 + 12*a^2*b*c^5*d))/(c*(c^6 -
d^6 + 3*c^2*d^4 - 3*c^4*d^2)) + (2*tan(e/2 + (f*x)/2)^2*(2*a^3*d^7 - 2*b^3*
c^7 - 6*a^2*b*c^7 + 6*a^3*c^6*d - 3*a^3*c^2*d^5 + 20*a^3*c^4*d^3 - 17*b^3*c
^3*d^4 - 6*b^3*c^5*d^2 + 12*a*b^2*c^2*d^5 + 51*a*b^2*c^4*d^3 - 42*a^2*b*c^3
*d^4 - 30*a^2*b*c^5*d^2 + 12*a*b^2*c^6*d + 3*a^2*b*c*d^6))/(c^2*(c^6 - d^6
+ 3*c^2*d^4 - 3*c^4*d^2)) - (tan(e/2 + (f*x)/2)*(3*a*b^2*c^6 - 2*a^3*d^6 +
5*b^3*c^5*d + 4*a^3*c^2*d^4 - 27*a^3*c^4*d^2 + 20*b^3*c^3*d^3 - 12*a*b^2*c^
2*d^4 - 66*a*b^2*c^4*d^2 + 57*a^2*b*c^3*d^3 - 6*a^2*b*c*d^5 + 24*a^2*b*c^5*
d))/(c*(c^6 - d^6 + 3*c^2*d^4 - 3*c^4*d^2)) + (tan(e/2 + (f*x)/2)^4*(4*a^3*
d^7 - 6*a^2*b*c^7 + 6*a^3*c^6*d - 12*a^3*c^2*d^5 + 27*a^3*c^4*d^3 - 10*b^3*
c^3*d^4 - 15*b^3*c^5*d^2 + 60*a*b^2*c^4*d^3 - 33*a^2*b*c^3*d^4 - 42*a^2*b*c
^5*d^2 + 15*a*b^2*c^6*d + 6*a^2*b*c*d^6))/(c^2*(c^6 - d^6 + 3*c^2*d^4 - 3*c
^4*d^2)) + (2*d*tan(e/2 + (f*x)/2)^3*(3*c^2 + 2*d^2)*(2*a^3*d^5 - 4*b^3*c^5
 - 18*a^2*b*c^5 + 18*a^3*c^4*d - 5*a^3*c^2*d^3 - 11*b^3*c^3*d^2 + 6*a*b^2*c
^2*d^3 - 30*a^2*b*c^3*d^2 + 39*a*b^2*c^4*d + 3*a^2*b*c*d^4))/(3*c^3*(c^6 -
d^6 + 3*c^2*d^4 - 3*c^4*d^2)))/(f*(c^3*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*
x)/2)^2*(12*c*d^2 + 3*c^3) + tan(e/2 + (f*x)/2)^4*(12*c*d^2 + 3*c^3) + tan(
e/2 + (f*x)/2)^3*(12*c^2*d + 8*d^3) + c^3 + 6*c^2*d*tan(e/2 + (f*x)/2) + 6*
c^2*d*tan(e/2 + (f*x)/2)^5)) + (atan((((c*tan(e/2 + (f*x)/2)*(a*c - b*d)*(2
*a^2*c^2 + 3*a^2*d^2 + 3*b^2*c^2 + 2*b^2*d^2 - 10*a*b*c*d))/((c + d)^(7/2)*
(c - d)^(7/2)) + ((a*c - b*d)*(2*c^6*d - 2*d^7 + 6*c^2*d^5 - 6*c^4*d^3)*(2*
a^2*c^2 + 3*a^2*d^2 + 3*b^2*c^2 + 2*b^2*d^2 - 10*a*b*c*d))/(2*(c + d)^(7/2)
*(c - d)^(7/2)*(c^6 - d^6 + 3*c^2*d^4 - 3*c^4*d^2)))*(c^6 - d^6 + 3*c^2*d^4
 - 3*c^4*d^2))/(2*a^3*c^3 - 2*b^3*d^3 + 3*a*b^2*c^3 - 3*a^2*b*d^3 + 3*a^3*c
*d^2 - 3*b^3*c^2*d + 12*a*b^2*c*d^2 - 12*a^2*b*c^2*d))*(a*c - b*d)*(2*a^2*c
^2 + 3*a^2*d^2 + 3*b^2*c^2 + 2*b^2*d^2 - 10*a*b*c*d))/(f*(c + d)^(7/2)*(c -
 d)^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**4,x)
```

```
[Out] Timed out
```

3.694 $\int \frac{\frac{bB}{a}+B\sin(x)}{a+b\sin(x)}\,dx$

Optimal. Leaf size=54

$$\frac{Bx}{b}-\frac{2B\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{ab}$$

```
[Out] B*x/b-2*B*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a/b
```

Rubi [A] time = 0.08, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2735, 2660, 618, 204}

$$\frac{Bx}{b}-\frac{2B\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Int[((b*B)/a + B*Sin[x])/(a + b*Sin[x]),x]
```

```
[Out] (B*x)/b - (2*Sqrt[a^2 - b^2]*B*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(a
*b)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\frac{bB}{a}+B\sin(x)}{a+b\sin(x)}\,dx &= \frac{Bx}{b}-\frac{\left(aB-\frac{b^2B}{a}\right)\int \frac{1}{a+b\sin(x)}\,dx}{b}\\ &= \frac{Bx}{b}-\frac{\left(2\left(aB-\frac{b^2B}{a}\right)\right)\operatorname{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{b}\\ &= \frac{Bx}{b}+\frac{\left(4\left(aB-\frac{b^2B}{a}\right)\right)\operatorname{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan\left(\frac{x}{2}\right)\right)}{b}\\ &= \frac{Bx}{b}-\frac{2\sqrt{a^2-b^2}\,B\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{ab}\end{aligned}$$

Mathematica [A] time = 0.07, size = 52, normalized size = 0.96

$$\frac{B\left(ax-2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*B)/a + B*Sin[x])/(a + b*Sin[x]),x]
[Out] (B*(a*x - 2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]]))/(a*b
)
```

fricas [A] time = 0.46, size = 163, normalized size = 3.02

$$\left[\frac{2\,Bax+\sqrt{-a^2+b^2}\,B\log\left(\frac{\left(2\,a^2-b^2\right)\cos(x)^2-2\,ab\sin(x)-a^2-b^2+2\,(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2\,ab\sin(x)-a^2-b^2}\right)}{2\,ab},\ \frac{Bax+\sqrt{a^2-b^2}\,B\,\mathrm{arctan}}{ab}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*sin(x))/(a+b*sin(x)),x, algorithm="fricas")
```

[Out] [1/2*(2*B*a*x + sqrt(-a^2 + b^2)*B*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(
x) - a^2 - b^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(
x)^2 - 2*a*b*sin(x) - a^2 - b^2)))/(a*b), (B*a*x + sqrt(a^2 - b^2)*B*arctan
(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))))/(a*b)]

giac [**A**] time = 0.18, size = 73, normalized size = 1.35

$$\frac{Bx}{b} - \frac{2\left(Ba^2 - Bb^2\right)\left(\pi \left\lfloor \frac{x}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(a) + \arctan\left(\frac{a\tan\left(\frac{1}{2}x\right) + b}{\sqrt{a^2 - b^2}}\right)\right)}{\sqrt{a^2 - b^2}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*B/a+B*sin(x))/(a+b*sin(x)),x, algorithm="giac")

[Out] B*x/b - 2*(B*a^2 - B*b^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(
1/2*x) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a*b)

maple [**B**] time = 0.08, size = 99, normalized size = 1.83

$$\frac{2B\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{b} - \frac{2Ba\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right) + 2b}{2\sqrt{a^2 - b^2}}\right)}{b\sqrt{a^2 - b^2}} + \frac{2Bb\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right) + 2b}{2\sqrt{a^2 - b^2}}\right)}{a\sqrt{a^2 - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*B/a+B*sin(x))/(a+b*sin(x)),x)

[Out] 2*B/b*arctan(tan(1/2*x))-2*B*a/b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)
+2*b)/(a^2-b^2)^(1/2))+2*B/a*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2
*b)/(a^2-b^2)^(1/2))

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*B/a+B*sin(x))/(a+b*sin(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?

mupad [B] time = 8.12, size = 94, normalized size = 1.74

$$\frac{2\,B\,\mathrm{atan}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{b}+\frac{2\,B\,\mathrm{atanh}\left(\frac{-\sin\left(\frac{x}{2}\right)\,a^2+\cos\left(\frac{x}{2}\right)\,a\,b+2\,\sin\left(\frac{x}{2}\right)\,b^2}{\sqrt{b^2-a^2}\,\left(2\,b\,\sin\left(\frac{x}{2}\right)+a\,\cos\left(\frac{x}{2}\right)\right)}\right)\,\sqrt{b^2-a^2}}{a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*sin(x) + (B*b)/a)/(a + b*sin(x)),x)
```

```
[Out] (2*B*atan(sin(x/2)/cos(x/2)))/b + (2*B*atanh((2*b^2*sin(x/2) - a^2*sin(x/2)
 + a*b*cos(x/2))/((b^2 - a^2)^(1/2)*(2*b*sin(x/2) + a*cos(x/2)))))*(b^2 - a^
2)^(1/2))/(a*b)
```

sympy [A] time = 50.12, size = 87, normalized size = 1.61

$$\begin{cases} \frac{Bx}{b}+\frac{B\sqrt{-a^2+b^2}\,\log\left(\tan\left(\frac{x}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{ab}-\frac{B\sqrt{-a^2+b^2}\,\log\left(\tan\left(\frac{x}{2}\right)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{ab} & \text{for } b\neq 0 \\ -\frac{B\cos\left(x\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*sin(x))/(a+b*sin(x)),x)
```

```
[Out] Piecewise((B*x/b + B*sqrt(-a**2 + b**2)*log(tan(x/2) + b/a - sqrt(-a**2 + b
**2)/a)/(a*b) - B*sqrt(-a**2 + b**2)*log(tan(x/2) + b/a + sqrt(-a**2 + b**2
)/a)/(a*b), Ne(b, 0)), (-B*cos(x)/a, True))
```

3.695 $\int \frac{\frac{aB}{b}+B\sin(x)}{a+b\sin(x)}\,dx$

Optimal. Leaf size=6

$$\frac{Bx}{b}$$

```
[Out] B*x/b
```

Rubi [A] time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 8}

$$\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B)/b + B*Sin[x])/(a + b*Sin[x]),x]
```

```
[Out] (B*x)/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned}\int \frac{\frac{aB}{b}+B\sin(x)}{a+b\sin(x)}\,dx &= \frac{B\int 1\,dx}{b}\\ &= \frac{Bx}{b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 6, normalized size = 1.00

$$\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B)/b + B*Sin[x])/(a + b*Sin[x]),x]
```

```
[Out] (B*x)/b
```

fricas [A] time = 0.41, size = 6, normalized size = 1.00

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sin(x))/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] B*x/b
```

giac [A] time = 0.23, size = 6, normalized size = 1.00

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sin(x))/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] B*x/b
```

maple [A] time = 0.01, size = 7, normalized size = 1.17

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B/b+B*sin(x))/(a+b*sin(x)),x)
```

```
[Out] B*x/b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sin(x))/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.70, size = 6, normalized size = 1.00

$$\frac{B\,x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*sin(x) + (B*a)/b)/(a + b*sin(x)),x)
```

```
[Out] (B*x)/b
```

sympy [A] time = 0.33, size = 3, normalized size = 0.50

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sin(x))/(a+b*sin(x)),x)
```

```
[Out] B*x/b
```

3.696 $\int \frac{a+b\sin(x)}{(b+a\sin(x))^2}\,dx$

Optimal. Leaf size=12

$$-\frac{\cos(x)}{a\sin(x)+b}$$

```
[Out] -cos(x)/(b+a*sin(x))
```

Rubi [A] time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2754, 8}

$$-\frac{\cos(x)}{a\sin(x)+b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x])/(b + a*Sin[x])^2,x]
```

```
[Out] -(Cos[x]/(b + a*Sin[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sin(x)}{(b+a\sin(x))^2}\,dx &= -\frac{\cos(x)}{b+a\sin(x)}+\frac{\int 0\,dx}{a^2-b^2}\\ &= -\frac{\cos(x)}{b+a\sin(x)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 12, normalized size = 1.00

$$-\frac{\cos(x)}{a\sin(x)+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x])/(b + a*Sin[x])^2,x]
```

```
[Out] -(Cos[x]/(b + a*Sin[x]))
```

fricas [A] time = 0.42, size = 12, normalized size = 1.00

$$-\frac{\cos(x)}{a\sin(x)+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x))/(b+a*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -cos(x)/(a*sin(x) + b)
```

giac [B] time = 0.29, size = 32, normalized size = 2.67

$$-\frac{2\left(a\tan\left(\frac{1}{2}x\right)+b\right)}{\left(b\tan\left(\frac{1}{2}x\right)^2+2a\tan\left(\frac{1}{2}x\right)+b\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x))/(b+a*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*(a*tan(1/2*x) + b)/((b*tan(1/2*x)^2 + 2*a*tan(1/2*x) + b)*b)
```

maple [B] time = 0.11, size = 34, normalized size = 2.83

$$\frac{-\frac{2a\tan\left(\frac{x}{2}\right)}{b}-2}{\left(\tan^2\left(\frac{x}{2}\right)\right)b+2a\tan\left(\frac{x}{2}\right)+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(x))/(b+a*sin(x))^2,x)
```

```
[Out] 2*(-a/b*tan(1/2*x)-1)/(tan(1/2*x)^2*b+2*a*tan(1/2*x)+b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x))/(b+a*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 7.87, size = 24, normalized size = 2.00

$$-\frac{a\sin(x)+b(\cos(x)+1)}{b(b+a\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(x))/(b + a*sin(x))^2,x)
```

```
[Out] -(a*sin(x) + b*(cos(x) + 1))/(b*(b + a*sin(x)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x))/(b+a*sin(x))**2,x)
```

```
[Out] Timed out
```

3.697 $\int \frac{2-\sin(x)}{2+\sin(x)}\,dx$

Optimal. Leaf size=34

$$\frac{4x}{\sqrt{3}}-x+\frac{8\tan^{-1}\left(\frac{\cos(x)}{\sin(x)+\sqrt{3}+2}\right)}{\sqrt{3}}$$

```
[Out] -x+4/3*x*3^(1/2)+8/3*arctan(cos(x)/(2+sin(x)+3^(1/2)))*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2735, 2657}

$$\frac{4x}{\sqrt{3}}-x+\frac{8\tan^{-1}\left(\frac{\cos(x)}{\sin(x)+\sqrt{3}+2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - Sin[x])/(2 + Sin[x]),x]
```

```
[Out] -x + (4*x)/Sqrt[3] + (8*ArcTan[Cos[x]/(2 + Sqrt[3] + Sin[x])])/Sqrt[3]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2-\sin(x)}{2+\sin(x)}\,dx &= -x+4\int \frac{1}{2+\sin(x)}\,dx\\ &= -x+\frac{4x}{\sqrt{3}}+\frac{8\tan^{-1}\left(\frac{\cos(x)}{2+\sqrt{3}+\sin(x)}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 28, normalized size = 0.82

$$\frac{8\tan^{-1}\left(\frac{2\tan\left(\frac{x}{2}\right)+1}{\sqrt{3}}\right)}{\sqrt{3}}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - Sin[x])/(2 + Sin[x]),x]
[Out] -x + (8*ArcTan[(1 + 2*Tan[x/2])/Sqrt[3]])/Sqrt[3]
```

fricas [A] time = 0.44, size = 27, normalized size = 0.79

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{2\sqrt{3}\sin(x)+\sqrt{3}}{3\cos(x)}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-sin(x))/(2+sin(x)),x, algorithm="fricas")
[Out] 4/3*sqrt(3)*arctan(1/3*(2*sqrt(3)*sin(x) + sqrt(3))/cos(x)) - x
```

giac [A] time = 0.36, size = 51, normalized size = 1.50

$$\frac{4}{3}\sqrt{3}\left(x+2\arctan\left(-\frac{\sqrt{3}\sin(x)-\cos(x)-2\sin(x)-1}{\sqrt{3}\cos(x)+\sqrt{3}-2\cos(x)+\sin(x)+2}\right)\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-sin(x))/(2+sin(x)),x, algorithm="giac")
[Out] 4/3*sqrt(3)*(x + 2*arctan(-(sqrt(3)*sin(x) - cos(x) - 2*sin(x) - 1)/(sqrt(3
)*cos(x) + sqrt(3) - 2*cos(x) + sin(x) + 2))) - x
```

maple [A] time = 0.10, size = 24, normalized size = 0.71

$$\frac{8\sqrt{3}\arctan\left(\frac{\left(1+2\tan\left(\frac{x}{2}\right)\right)\sqrt{3}}{3}\right)}{3}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-sin(x))/(2+sin(x)),x)
[Out] 8/3*3^(1/2)*arctan(1/3*(1+2*tan(1/2*x))*3^(1/2))-x
```

maxima [A] time = 1.22, size = 36, normalized size = 1.06

$$\frac{8}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\sin(x)}{\cos(x)+1}+1\right)\right)-2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-sin(x))/(2+sin(x)),x, algorithm="maxima")
```

```
[Out] 8/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sin(x)/(cos(x) + 1) + 1)) - 2*arctan(sin(
x)/(cos(x) + 1))
```

mupad [B] time = 7.83, size = 36, normalized size = 1.06

$$-x-\frac{8\sqrt{3}\,\mathrm{atan}\left(-\frac{\sqrt{3}\tan\left(\frac{x}{2}\right)-\sqrt{3}}{3\tan\left(\frac{x}{2}\right)+3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(sin(x) - 2)/(sin(x) + 2),x)
```

```
[Out] - x - (8*3^(1/2)*atan(-(3^(1/2)*tan(x/2) - 3^(1/2))/(3*tan(x/2) + 3)))/3
```

sympy [A] time = 0.88, size = 42, normalized size = 1.24

$$-x+\frac{8\sqrt{3}\left(\mathrm{atan}\left(\frac{2\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}+\frac{\sqrt{3}}{3}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-sin(x))/(2+sin(x)),x)
```

```
[Out] -x + 8*sqrt(3)*(atan(2*sqrt(3)*tan(x/2)/3 + sqrt(3)/3) + pi*floor((x/2 - pi
/2)/pi))/3
```

3.698 $\int \frac{(c+d\sin(e+fx))^4}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=235

$$\frac{2(bc-ad)^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4f\sqrt{a^2-b^2}}+\frac{d^2\left(-3a^2d^2+12abcd-\left(b^2\left(17c^2+2d^2\right)\right)\right)\cos(e+fx)}{3b^3f}+\frac{dx\left(-2a^3d^3+8a^2bc\right.}{}$$

```
[Out] 1/2*d*(8*a^2*b*c*d^2-2*a^3*d^3+4*b^3*c*(2*c^2+d^2)-a*b^2*d*(12*c^2+d^2))*x/
b^4+1/3*d^2*(12*a*b*c*d-3*a^2*d^2-b^2*(17*c^2+2*d^2))*cos(f*x+e)/b^3/f-1/6*
d^3*(-3*a*d+8*b*c)*cos(f*x+e)*sin(f*x+e)/b^2/f-1/3*d^2*cos(f*x+e)*(c+d*sin(
f*x+e))^2/b/f+2*(-a*d+b*c)^4*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2
))/b^4/f/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.65, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2793, 3033, 3023, 2735, 2660, 618, 204}

$$\frac{d^2\left(-3a^2d^2+12abcd+b^2\left(-\left(17c^2+2d^2\right)\right)\right)\cos(e+fx)}{3b^3f}+\frac{dx\left(8a^2bcd^2-2a^3d^3-ab^2d\left(12c^2+d^2\right)+4b^3c\left(2c^2+\right.\right.}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^4/(a + b*Sin[e + f*x]),x]
```

```
[Out] (d*(8*a^2*b*c*d^2 - 2*a^3*d^3 + 4*b^3*c*(2*c^2 + d^2) - a*b^2*d*(12*c^2 + d
^2))*x)/(2*b^4) + (2*(b*c - a*d)^4*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2
 - b^2]])/(b^4*Sqrt[a^2 - b^2]*f) + (d^2*(12*a*b*c*d - 3*a^2*d^2 - b^2*(17*
c^2 + 2*d^2))*Cos[e + f*x])/(3*b^3*f) - (d^3*(8*b*c - 3*a*d)*Cos[e + f*x]*S
in[e + f*x])/(6*b^2*f) - (d^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(3*b*f)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^4}{a+b\sin(e+fx)}\,dx = -\frac{d^2\cos(e+fx)(c+d\sin(e+fx))^2}{3bf} + \frac{\int \frac{(c+d\sin(e+fx))\left(3bc^3+2ad^3+d\left(9bc^2-acd+2bd^2\right)\sin(e+\ldots}{a+b\sin(e+fx)}}{3b}$$

$$= -\frac{d^3(8bc-3ad)\cos(e+fx)\sin(e+fx)}{6b^2f} - \frac{d^2\cos(e+fx)(c+d\sin(e+fx))^2}{3bf} + \ldots$$

$$= \frac{d^2\left(12abcd-3a^2d^2-b^2\left(17c^2+2d^2\right)\right)\cos(e+fx)}{3b^3f} - \frac{d^3(8bc-3ad)\cos(e+fx)\sin\ldots}{6b^2f}$$

$$= \frac{d\left(8a^2bcd^2-2a^3d^3+4b^3c\left(2c^2+d^2\right)-ab^2d\left(12c^2+d^2\right)\right)x}{2b^4} + \frac{d^2\left(12abcd-3a^2d^2-\ldots\right.}{}$$

$$= \frac{d\left(8a^2bcd^2-2a^3d^3+4b^3c\left(2c^2+d^2\right)-ab^2d\left(12c^2+d^2\right)\right)x}{2b^4} + \frac{d^2\left(12abcd-3a^2d^2-\ldots\right.}{}$$

$$= \frac{d\left(8a^2bcd^2-2a^3d^3+4b^3c\left(2c^2+d^2\right)-ab^2d\left(12c^2+d^2\right)\right)x}{2b^4} + \frac{d^2\left(12abcd-3a^2d^2-\ldots\right.}{}$$

$$= \frac{d\left(8a^2bcd^2-2a^3d^3+4b^3c\left(2c^2+d^2\right)-ab^2d\left(12c^2+d^2\right)\right)x}{2b^4} + \frac{2(bc-ad)^4\tan^{-1}\left(\frac{b+\ldots}{\ldots}\right)}{b^4\sqrt{a^2-b\ldots}}$$

Mathematica [A] time = 0.58, size = 203, normalized size = 0.86

$$\frac{-3bd^2\left(4a^2d^2-16abcd+3b^2\left(8c^2+d^2\right)\right)\cos(e+fx)+\frac{24(bc-ad)^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-6d(e+fx)\left(2a^3d^3-8a^2bc\ldots\right.}{12b^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^4/(a + b*Sin[e + f*x]),x]
```

```
[Out] (-6*d*(-8*a^2*b*c*d^2 + 2*a^3*d^3 - 4*b^3*c*(2*c^2 + d^2) + a*b^2*d*(12*c^2
 + d^2))*(e + f*x) + (24*(b*c - a*d)^4*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt
[a^2 - b^2]])/Sqrt[a^2 - b^2] - 3*b*d^2*(-16*a*b*c*d + 4*a^2*d^2 + 3*b^2*(8
```

```
*c^2 + d^2))*Cos[e + f*x] + b^3*d^4*Cos[3*(e + f*x)] - 3*b^2*d^3*(4*b*c - a
*d)*Sin[2*(e + f*x)])/(12*b^4*f)
```

fricas [A] time = 0.51, size = 795, normalized size = 3.38

$$\left[\frac{2\left(a^{2}b^{3}-b^{5}\right)d^{4}\cos\left(fx+e\right)^{3}+3\left(8\left(a^{2}b^{3}-b^{5}\right)c^{3}d-12\left(a^{3}b^{2}-ab^{4}\right)c^{2}d^{2}+4\left(2\,a^{4}b-a^{2}b^{3}-b^{5}\right)cd^{3}-\left(2\,a^{5}-a^{3}b\cdots\right.\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(a^2*b^3 - b^5)*d^4*cos(f*x + e)^3 + 3*(8*(a^2*b^3 - b^5)*c^3*d - 1
2*(a^3*b^2 - a*b^4)*c^2*d^2 + 4*(2*a^4*b - a^2*b^3 - b^5)*c*d^3 - (2*a^5 -
a^3*b^2 - a*b^4)*d^4)*f*x - 3*(4*(a^2*b^3 - b^5)*c*d^3 - (a^3*b^2 - a*b^4)*
d^4)*cos(f*x + e)*sin(f*x + e) - 3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2
*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(f*x
 + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 + 2*(a*cos(f*x + e)*sin(f*x + e) +
 b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e)
 - a^2 - b^2)) - 6*(6*(a^2*b^3 - b^5)*c^2*d^2 - 4*(a^3*b^2 - a*b^4)*c*d^3 +
 (a^4*b - b^5)*d^4)*cos(f*x + e))/((a^2*b^4 - b^6)*f), 1/6*(2*(a^2*b^3 - b^
5)*d^4*cos(f*x + e)^3 + 3*(8*(a^2*b^3 - b^5)*c^3*d - 12*(a^3*b^2 - a*b^4)*c
^2*d^2 + 4*(2*a^4*b - a^2*b^3 - b^5)*c*d^3 - (2*a^5 - a^3*b^2 - a*b^4)*d^4)
*f*x - 3*(4*(a^2*b^3 - b^5)*c*d^3 - (a^3*b^2 - a*b^4)*d^4)*cos(f*x + e)*sin
(f*x + e) - 6*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
+ a^4*d^4)*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*co
s(f*x + e))) - 6*(6*(a^2*b^3 - b^5)*c^2*d^2 - 4*(a^3*b^2 - a*b^4)*c*d^3 + (
a^4*b - b^5)*d^4)*cos(f*x + e))/((a^2*b^4 - b^6)*f)]
```

giac [B] time = 0.23, size = 465, normalized size = 1.98

$$\frac{\frac{3\left(8\,b^{3}c^{3}d-12\,ab^{2}c^{2}d^{2}+8\,a^{2}bcd^{3}+4\,b^{3}cd^{3}-2\,a^{3}d^{4}-ab^{2}d^{4}\right)\left(fx+e\right)}{b^{4}}+\frac{12\left(b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\cdots\right.}{\sqrt{\cdots}}\right.\right.}{\sqrt{a^{2}-b^{2}}\,b^{4}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/6*(3*(8*b^3*c^3*d - 12*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 + 4*b^3*c*d^3 - 2*a^
3*d^4 - a*b^2*d^4)*(f*x + e)/b^4 + 12*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*
```

```
c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a)
 + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b
^4) + 2*(12*b^2*c*d^3*tan(1/2*f*x + 1/2*e)^5 - 3*a*b*d^4*tan(1/2*f*x + 1/2*
e)^5 - 36*b^2*c^2*d^2*tan(1/2*f*x + 1/2*e)^4 + 24*a*b*c*d^3*tan(1/2*f*x + 1
/2*e)^4 - 6*a^2*d^4*tan(1/2*f*x + 1/2*e)^4 - 72*b^2*c^2*d^2*tan(1/2*f*x + 1
/2*e)^2 + 48*a*b*c*d^3*tan(1/2*f*x + 1/2*e)^2 - 12*a^2*d^4*tan(1/2*f*x + 1/
2*e)^2 - 12*b^2*d^4*tan(1/2*f*x + 1/2*e)^2 - 12*b^2*c*d^3*tan(1/2*f*x + 1/2
*e) + 3*a*b*d^4*tan(1/2*f*x + 1/2*e) - 36*b^2*c^2*d^2 + 24*a*b*c*d^3 - 6*a^
2*d^4 - 4*b^2*d^4)/((tan(1/2*f*x + 1/2*e)^2 + 1)^3*b^3))/f
```

maple [B] time = 0.24, size = 948, normalized size = 4.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e)),x)
```

```
[Out] 8/f*d^3/b^3*arctan(tan(1/2*f*x+1/2*e))*a^2*c-12/f*d^2/b^2*arctan(tan(1/2*f*
x+1/2*e))*a*c^2+2/f/b^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+
2*b)/(a^2-b^2)^(1/2))*a^4*d^4-1/f*d^4/b^2/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/
2*f*x+1/2*e)^5*a-4/3/f*d^4/b/(1+tan(1/2*f*x+1/2*e)^2)^3+2/f/(a^2-b^2)^(1/2)
*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*c^4-1/f*d^4/b^2*a
rctan(tan(1/2*f*x+1/2*e))*a+4/f*d^3/b*arctan(tan(1/2*f*x+1/2*e))*c-4/f*d^4/
b/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2-2/f*d^4/b^3/(1+tan(1/2*f*
x+1/2*e)^2)^3*a^2-12/f*d^2/b/(1+tan(1/2*f*x+1/2*e)^2)^3*c^2-2/f*d^4/b^4*arc
tan(tan(1/2*f*x+1/2*e))*a^3+8/f*d/b*arctan(tan(1/2*f*x+1/2*e))*c^3+8/f*d^3/
b^2/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^4*a*c-8/f/b/(a^2-b^2)^(1/
2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a*c^3*d+16/f*d^
3/b^2/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2*a*c-8/f/b^3/(a^2-b^2)
^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^3*c*d^3+1
2/f/b^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(
1/2))*a^2*c^2*d^2+4/f*d^3/b/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5
*c-4/f*d^3/b/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)*c+8/f*d^3/b^2/(1
+tan(1/2*f*x+1/2*e)^2)^3*a*c-2/f*d^4/b^3/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2
*f*x+1/2*e)^4*a^2-12/f*d^2/b/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^
4*c^2-4/f*d^4/b^3/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2*a^2-24/f*
d^2/b/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2*c^2+1/f*d^4/b^2/(1+ta
n(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)*a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 16.49, size = 8720, normalized size = 37.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^4/(a + b*sin(e + f*x)),x)
```

```
[Out] - ((2*(3*a^2*d^4 + 2*b^2*d^4 + 18*b^2*c^2*d^2 - 12*a*b*c*d^3))/(3*b^3) + (t
an(e/2 + (f*x)/2)^5*(a*d^4 - 4*b*c*d^3))/b^2 + (4*tan(e/2 + (f*x)/2)^2*(a^2
*d^4 + b^2*d^4 + 6*b^2*c^2*d^2 - 4*a*b*c*d^3))/b^3 + (2*tan(e/2 + (f*x)/2)^
4*(a^2*d^4 + 6*b^2*c^2*d^2 - 4*a*b*c*d^3))/b^3 - (tan(e/2 + (f*x)/2)*(a*d^4
 - 4*b*c*d^3))/b^2)/(f*(3*tan(e/2 + (f*x)/2)^2 + 3*tan(e/2 + (f*x)/2)^4 + t
an(e/2 + (f*x)/2)^6 + 1)) - (atan(((((8*(a^4*b^7*d^8 + 4*a^6*b^5*d^8 + 4*a^
8*b^3*d^8 - 8*a^3*b^8*c*d^7 - 32*a^5*b^6*c*d^7 - 32*a^7*b^4*c*d^7 + 16*a^2*
b^9*c^2*d^6 + 64*a^2*b^9*c^4*d^4 + 64*a^2*b^9*c^6*d^2 - 112*a^3*b^8*c^3*d^5
 - 192*a^3*b^8*c^5*d^3 + 88*a^4*b^7*c^2*d^6 + 272*a^4*b^7*c^4*d^4 - 224*a^5
*b^6*c^3*d^5 + 112*a^6*b^5*c^2*d^6))/b^8 + (((8*(4*a^2*b^10*c^4 + 2*a^2*b^1
0*d^4 + 2*a^4*b^8*d^4 - 8*a^3*b^9*c*d^3 + 24*a^2*b^10*c^2*d^2 - 8*a*b^11*c*
d^3 - 16*a*b^11*c^3*d))/b^8 + (8*tan(e/2 + (f*x)/2)*(8*a*b^12*c^4 + 8*a^5*b
^8*d^4 - 32*a^2*b^11*c^3*d - 32*a^4*b^9*c*d^3 + 48*a^3*b^10*c^2*d^2))/b^9 +
 ((32*a^2*b^3 + (8*tan(e/2 + (f*x)/2)*(12*a*b^13 - 8*a^3*b^11))/b^9)*(a^3*d
^4*1i + (b^2*d*(a*d^3 + 12*a*c^2*d)*1i)/2 - (b^3*d*(4*c*d^2 + 8*c^3)*1i)/2
- a^2*b*c*d^3*4i))/b^4)*(a^3*d^4*1i + (b^2*d*(a*d^3 + 12*a*c^2*d)*1i)/2 - (
b^3*d*(4*c*d^2 + 8*c^3)*1i)/2 - a^2*b*c*d^3*4i))/b^4 + (8*tan(e/2 + (f*x)/2
)*(2*a^3*b^9*d^8 - 4*a*b^11*c^8 + 7*a^5*b^7*d^8 + 4*a^7*b^5*d^8 - 8*a^9*b^3
*d^8 + 32*a*b^11*c^2*d^6 + 128*a*b^11*c^4*d^4 + 128*a*b^11*c^6*d^2 - 16*a^2
*b^10*c*d^7 + 32*a^2*b^10*c^7*d - 56*a^4*b^8*c*d^7 - 32*a^6*b^6*c*d^7 + 64*
a^8*b^4*c*d^7 - 224*a^2*b^10*c^3*d^5 - 384*a^2*b^10*c^5*d^3 + 160*a^3*b^9*c
^2*d^6 + 480*a^3*b^9*c^4*d^4 - 176*a^3*b^9*c^6*d^2 - 336*a^4*b^8*c^3*d^5 +
416*a^4*b^8*c^5*d^3 + 136*a^5*b^7*c^2*d^6 - 552*a^5*b^7*c^4*d^4 + 448*a^6*b
^6*c^3*d^5 - 224*a^7*b^5*c^2*d^6))/b^9)*(a^3*d^4*1i + (b^2*d*(a*d^3 + 12*a*
c^2*d)*1i)/2 - (b^3*d*(4*c*d^2 + 8*c^3)*1i)/2 - a^2*b*c*d^3*4i)*1i)/b^4 + (
((8*(a^4*b^7*d^8 + 4*a^6*b^5*d^8 + 4*a^8*b^3*d^8 - 8*a^3*b^8*c*d^7 - 32*a^5
*b^6*c*d^7 - 32*a^7*b^4*c*d^7 + 16*a^2*b^9*c^2*d^6 + 64*a^2*b^9*c^4*d^4 + 6
4*a^2*b^9*c^6*d^2 - 112*a^3*b^8*c^3*d^5 - 192*a^3*b^8*c^5*d^3 + 88*a^4*b^7*
c^2*d^6 + 272*a^4*b^7*c^4*d^4 - 224*a^5*b^6*c^3*d^5 + 112*a^6*b^5*c^2*d^6))
/b^8 - (((8*(4*a^2*b^10*c^4 + 2*a^2*b^10*d^4 + 2*a^4*b^8*d^4 - 8*a^3*b^9*c*
d^3 + 24*a^2*b^10*c^2*d^2 - 8*a*b^11*c*d^3 - 16*a*b^11*c^3*d))/b^8 + (8*tan
(e/2 + (f*x)/2)*(8*a*b^12*c^4 + 8*a^5*b^8*d^4 - 32*a^2*b^11*c^3*d - 32*a^4*
b^9*c*d^3 + 48*a^3*b^10*c^2*d^2))/b^9 - ((32*a^2*b^3 + (8*tan(e/2 + (f*x)/2
```

)*(12*a*b^13 - 8*a^3*b^11))/b^9)*(a^3*d^4*1i + (b^2*d*(a*d^3 + 12*a*c^2*d)*
1i)/2 - (b^3*d*(4*c*d^2 + 8*c^3)*1i)/2 - a^2*b*c*d^3*4i))/b^4)*(a^3*d^4*1i
+ (b^2*d*(a*d^3 + 12*a*c^2*d)*1i)/2 - (b^3*d*(4*c*d^2 + 8*c^3)*1i)/2 - a^2*
b*c*d^3*4i))/b^4 + (8*tan(e/2 + (f*x)/2)*(2*a^3*b^9*d^8 - 4*a*b^11*c^8 + 7*
a^5*b^7*d^8 + 4*a^7*b^5*d^8 - 8*a^9*b^3*d^8 + 32*a*b^11*c^2*d^6 + 128*a*b^1
1*c^4*d^4 + 128*a*b^11*c^6*d^2 - 16*a^2*b^10*c*d^7 + 32*a^2*b^10*c^7*d - 56
*a^4*b^8*c*d^7 - 32*a^6*b^6*c*d^7 + 64*a^8*b^4*c*d^7 - 224*a^2*b^10*c^3*d^5
 - 384*a^2*b^10*c^5*d^3 + 160*a^3*b^9*c^2*d^6 + 480*a^3*b^9*c^4*d^4 - 176*a
^3*b^9*c^6*d^2 - 336*a^4*b^8*c^3*d^5 + 416*a^4*b^8*c^5*d^3 + 136*a^5*b^7*c^
2*d^6 - 552*a^5*b^7*c^4*d^4 + 448*a^6*b^6*c^3*d^5 - 224*a^7*b^5*c^2*d^6))/b
^9)*(a^3*d^4*1i + (b^2*d*(a*d^3 + 12*a*c^2*d)*1i)/2 - (b^3*d*(4*c*d^2 + 8*c
^3)*1i)/2 - a^2*b*c*d^3*4i)*1i)/b^4)/((16*(2*a^10*d^12 + a^8*b^2*d^12 + 8*a
*b^9*c^9*d^3 - 12*a^7*b^3*c*d^11 + 16*a^2*b^8*c^6*d^6 - 2*a^2*b^8*c^8*d^4 -
 88*a^2*b^8*c^10*d^2 - 72*a^3*b^7*c^5*d^7 - 128*a^3*b^7*c^7*d^5 + 208*a^3*b
^7*c^9*d^3 + 129*a^4*b^6*c^4*d^8 + 416*a^4*b^6*c^6*d^6 - 276*a^4*b^6*c^8*d^
4 - 116*a^5*b^5*c^3*d^9 - 640*a^5*b^5*c^5*d^7 + 224*a^5*b^5*c^7*d^5 + 54*a^
6*b^4*c^2*d^10 + 584*a^6*b^4*c^4*d^8 - 112*a^6*b^4*c^6*d^6 - 336*a^7*b^3*c^
3*d^9 + 32*a^7*b^3*c^5*d^7 + 120*a^8*b^2*c^2*d^10 - 4*a^8*b^2*c^4*d^8 + 16*
a*b^9*c^11*d - 24*a^9*b*c*d^11))/b^8 + (((8*(a^4*b^7*d^8 + 4*a^6*b^5*d^8 +
4*a^8*b^3*d^8 - 8*a^3*b^8*c*d^7 - 32*a^5*b^6*c*d^7 - 32*a^7*b^4*c*d^7 + 16*
a^2*b^9*c^2*d^6 + 64*a^2*b^9*c^4*d^4 + 64*a^2*b^9*c^6*d^2 - 112*a^3*b^8*c^3
*d^5 - 192*a^3*b^8*c^5*d^3 + 88*a^4*b^7*c^2*d^6 + 272*a^4*b^7*c^4*d^4 - 224
*a^5*b^6*c^3*d^5 + 112*a^6*b^5*c^2*d^6))/b^8 + (((8*(4*a^2*b^10*c^4 + 2*a^2
*b^10*d^4 + 2*a^4*b^8*d^4 - 8*a^3*b^9*c*d^3 + 24*a^2*b^10*c^2*d^2 - 8*a*b^1
1*c*d^3 - 16*a*b^11*c^3*d))/b^8 + (8*tan(e/2 + (f*x)/2)*(8*a*b^12*c^4 + 8*a
^5*b^8*d^4 - 32*a^2*b^11*c^3*d - 32*a^4*b^9*c*d^3 + 48*a^3*b^10*c^2*d^2))/b
^9 + ((32*a^2*b^3 + (8*tan(e/2 + (f*x)/2)*(12*a*b^13 - 8*a^3*b^11))/b^9)*(a
^3*d^4*1i + (b^2*d*(a*d^3 + 12*a*c^2*d)*1i)/2 - (b^3*d*(4*c*d^2 + 8*c^3)*1i
)/2 - a^2*b*c*d^3*4i))/b^4)*(a^3*d^4*1i + (b^2*d*(a*d^3 + 12*a*c^2*d)*1i)/2
 - (b^3*d*(4*c*d^2 + 8*c^3)*1i)/2 - a^2*b*c*d^3*4i))/b^4 + (8*tan(e/2 + (f*
x)/2)*(2*a^3*b^9*d^8 - 4*a*b^11*c^8 + 7*a^5*b^7*d^8 + 4*a^7*b^5*d^8 - 8*a^9
*b^3*d^8 + 32*a*b^11*c^2*d^6 + 128*a*b^11*c^4*d^4 + 128*a*b^11*c^6*d^2 - 16
*a^2*b^10*c*d^7 + 32*a^2*b^10*c^7*d - 56*a^4*b^8*c*d^7 - 32*a^6*b^6*c*d^7 +
 64*a^8*b^4*c*d^7 - 224*a^2*b^10*c^3*d^5 - 384*a^2*b^10*c^5*d^3 + 160*a^3*b
^9*c^2*d^6 + 480*a^3*b^9*c^4*d^4 - 176*a^3*b^9*c^6*d^2 - 336*a^4*b^8*c^3*d^
5 + 416*a^4*b^8*c^5*d^3 + 136*a^5*b^7*c^2*d^6 - 552*a^5*b^7*c^4*d^4 + 448*a
^6*b^6*c^3*d^5 - 224*a^7*b^5*c^2*d^6))/b^9)*(a^3*d^4*1i + (b^2*d*(a*d^3 + 1
2*a*c^2*d)*1i)/2 - (b^3*d*(4*c*d^2 + 8*c^3)*1i)/2 - a^2*b*c*d^3*4i))/b^4 -
(((8*(a^4*b^7*d^8 + 4*a^6*b^5*d^8 + 4*a^8*b^3*d^8 - 8*a^3*b^8*c*d^7 - 32*a^
5*b^6*c*d^7 - 32*a^7*b^4*c*d^7 + 16*a^2*b^9*c^2*d^6 + 64*a^2*b^9*c^4*d^4 +
64*a^2*b^9*c^6*d^2 - 112*a^3*b^8*c^3*d^5 - 192*a^3*b^8*c^5*d^3 + 88*a^4*b^7
*c^2*d^6 + 272*a^4*b^7*c^4*d^4 - 224*a^5*b^6*c^3*d^5 + 112*a^6*b^5*c^2*d^6)
)/b^8 - (((8*(4*a^2*b^10*c^4 + 2*a^2*b^10*d^4 + 2*a^4*b^8*d^4 - 8*a^3*b^9*c
*d^3 + 24*a^2*b^10*c^2*d^2 - 8*a*b^11*c*d^3 - 16*a*b^11*c^3*d))/b^8 + (8*ta
n(e/2 + (f*x)/2)*(8*a*b^12*c^4 + 8*a^5*b^8*d^4 - 32*a^2*b^11*c^3*d - 32*a^4

*b^9*c*d^3 + 48*a^3*b^10*c^2*d^2))/b^9 - ((32*a^2*b^3 + (8*tan(e/2 + (f*x)/2)*(12*a*b^13 - 8*a^3*b^11))/b^9)*(a^3*d^4*1i + (b^2*d*(a*d^3 + 12*a*c^2*d)*1i)/2 - (b^3*d*(4*c*d^2 + 8*c^3)*1i)/2 - a^2*b*c*d^3*4i))/b^4)*(a^3*d^4*1i + (b^2*d*(a*d^3 + 12*a*c^2*d)*1i)/2 - (b^3*d*(4*c*d^2 + 8*c^3)*1i)/2 - a^2*b*c*d^3*4i))/b^4 + (8*tan(e/2 + (f*x)/2)*(2*a^3*b^9*d^8 - 4*a*b^11*c^8 + 7*a^5*b^7*d^8 + 4*a^7*b^5*d^8 - 8*a^9*b^3*d^8 + 32*a*b^11*c^2*d^6 + 128*a*b^11*c^4*d^4 + 128*a*b^11*c^6*d^2 - 16*a^2*b^10*c*d^7 + 32*a^2*b^10*c^7*d - 56*a^4*b^8*c*d^7 - 32*a^6*b^6*c*d^7 + 64*a^8*b^4*c*d^7 - 224*a^2*b^10*c^3*d^5 - 384*a^2*b^10*c^5*d^3 + 160*a^3*b^9*c^2*d^6 + 480*a^3*b^9*c^4*d^4 - 176*a^3*b^9*c^6*d^2 - 336*a^4*b^8*c^3*d^5 + 416*a^4*b^8*c^5*d^3 + 136*a^5*b^7*c^2*d^6 - 552*a^5*b^7*c^4*d^4 + 448*a^6*b^6*c^3*d^5 - 224*a^7*b^5*c^2*d^6))/b^9)*(a^3*d^4*1i + (b^2*d*(a*d^3 + 12*a*c^2*d)*1i)/2 - (b^3*d*(4*c*d^2 + 8*c^3)*1i)/2 - a^2*b*c*d^3*4i))/b^4 + (16*tan(e/2 + (f*x)/2)*(8*a^11*d^12 + 2*a^7*b^4*d^12 + 8*a^9*b^2*d^12 + 32*a*b^10*c^6*d^6 + 128*a*b^10*c^8*d^4 + 128*a*b^10*c^10*d^2 - 24*a^6*b^5*c*d^11 - 96*a^8*b^3*c*d^11 - 144*a^2*b^9*c^5*d^7 - 736*a^2*b^9*c^7*d^5 - 896*a^2*b^9*c^9*d^3 + 258*a^3*b^8*c^4*d^8 + 1840*a^3*b^8*c^6*d^6 + 2848*a^3*b^8*c^8*d^4 - 232*a^4*b^7*c^3*d^9 - 2624*a^4*b^7*c^5*d^7 - 5440*a^4*b^7*c^7*d^5 + 108*a^5*b^6*c^2*d^10 + 2344*a^5*b^6*c^4*d^8 + 6944*a^5*b^6*c^6*d^6 - 1344*a^6*b^5*c^3*d^9 - 6208*a^6*b^5*c^5*d^7 + 480*a^7*b^4*c^2*d^10 + 3944*a^7*b^4*c^4*d^8 - 1760*a^8*b^3*c^3*d^9 + 528*a^9*b^2*c^2*d^10 - 96*a^10*b*c*d^11))/b^9))*(a^3*d^4*1i + (b^2*d*(a*d^3 + 12*a*c^2*d)*1i)/2 - (b^3*d*(4*c*d^2 + 8*c^3)*1i)/2 - a^2*b*c*d^3*4i)*2i)/(b^4*f) - (atan((((-(a + b)*(a - b))^(1/2)*(a*d - b*c)^4*((8*(a^4*b^7*d^8 + 4*a^6*b^5*d^8 + 4*a^8*b^3*d^8 - 8*a^3*b^8*c*d^7 - 32*a^5*b^6*c*d^7 - 32*a^7*b^4*c*d^7 + 16*a^2*b^9*c^2*d^6 + 64*a^2*b^9*c^4*d^4 + 64*a^2*b^9*c^6*d^2 - 112*a^3*b^8*c^3*d^5 - 192*a^3*b^8*c^5*d^3 + 88*a^4*b^7*c^2*d^6 + 272*a^4*b^7*c^4*d^4 - 224*a^5*b^6*c^3*d^5 + 112*a^6*b^5*c^2*d^6))/b^8 + (8*tan(e/2 + (f*x)/2)*(2*a^3*b^9*d^8 - 4*a*b^11*c^8 + 7*a^5*b^7*d^8 + 4*a^7*b^5*d^8 - 8*a^9*b^3*d^8 + 32*a*b^11*c^2*d^6 + 128*a*b^11*c^4*d^4 + 128*a*b^11*c^6*d^2 - 16*a^2*b^10*c*d^7 + 32*a^2*b^10*c^7*d - 56*a^4*b^8*c*d^7 - 32*a^6*b^6*c*d^7 + 64*a^8*b^4*c*d^7 - 224*a^2*b^10*c^3*d^5 - 384*a^2*b^10*c^5*d^3 + 160*a^3*b^9*c^2*d^6 + 480*a^3*b^9*c^4*d^4 - 176*a^3*b^9*c^6*d^2 - 336*a^4*b^8*c^3*d^5 + 416*a^4*b^8*c^5*d^3 + 136*a^5*b^7*c^2*d^6 - 552*a^5*b^7*c^4*d^4 + 448*a^6*b^6*c^3*d^5 - 224*a^7*b^5*c^2*d^6))/b^9 + ((-(a + b)*(a - b))^(1/2)*(a*d - b*c)^4*((8*(4*a^2*b^10*c^4 + 2*a^2*b^10*d^4 + 2*a^4*b^8*d^4 - 8*a^3*b^9*c*d^3 + 24*a^2*b^10*c^2*d^2 - 8*a*b^11*c*d^3 - 16*a*b^11*c^3*d))/b^8 + (8*tan(e/2 + (f*x)/2)*(8*a*b^12*c^4 + 8*a^5*b^8*d^4 - 32*a^2*b^11*c^3*d - 32*a^4*b^9*c*d^3 + 48*a^3*b^10*c^2*d^2))/b^9 + ((-(a + b)*(a - b))^(1/2)*(a*d - b*c)^4*(32*a^2*b^3 + (8*tan(e/2 + (f*x)/2)*(12*a*b^13 - 8*a^3*b^11))/b^9))/(b^6 - a^2*b^4)))/(b^6 - a^2*b^4))*1i)/(b^6 - a^2*b^4) + ((-(a + b)*(a - b))^(1/2)*(a*d - b*c)^4*((8*(a^4*b^7*d^8 + 4*a^6*b^5*d^8 + 4*a^8*b^3*d^8 - 8*a^3*b^8*c*d^7 - 32*a^5*b^6*c*d^7 - 32*a^7*b^4*c*d^7 + 16*a^2*b^9*c^2*d^6 + 64*a^2*b^9*c^4*d^4 + 64*a^2*b^9*c^6*d^2 - 112*a^3*b^8*c^3*d^5 - 192*a^3*b^8*c^5*d^3 + 88*a^4*b^7*c^2*d^6 + 272*a^4*b^7*c^4*d^4 - 224*a^5*b^6*c^3*d^5 + 112*a^6*b^5*c^2*d^6))/b^8 + (8*tan(e/2 + (f*x)/2)*(2*a^3*b^9*d^8 - 4*a* b^11*c^8 + 7*a^5*b^7*d^8 + 4*a^7*b^5*d^8 - 8*a^9*b^3*d^8 + 32*a*b^11*c^2*d^6 + 128*a*b^11*c^4*d^4 + 128*a*b^11*c^6*d^2 - 16*a^2*b^10*c*d^7 + 32*a^2*b^10*c^7*d - 56*a^4*b^8*c*d^7 - 32*a^6*b^6*c*d^7 + 64*a^8*b^4*c*d^7 - 224*a^2*b^10*c^3*d^5 - 384*a^2*b^10*c^5*d^3 + 160*a^3*b^9*c^2*d^6 + 480*a^3*b^9*c^4*d^4 - 176*a^3*b^9*c^6*d^2 - 336*a^4*b^8*c^3*d^5 + 416*a^4*b^8*c^5*d^3 + 136*a^5*b^7*c^2*d^6 - 552*a^5*b^7*c^4*d^4 + 448*a^6*b^6*c^3*d^5 - 224*a^7*b^5*c^2*d^6))/b^9 - ((-(a + b)*(a - b))^(1/2)*(a*d - b*c)^4*((8*(4*a^2*b^10*c^4 + 2*a^2*b^10*d^4 + 2*a^4*b^8*d^4 - 8*a^3*b^9*c*d^3 + 24*a^2*b^10*c^2*d^2 - 8*a*b^11*c*d^3 - 16*a*b^11*c^3*d))/b^8 + (8*tan(e/2 + (f*x)/2)*(8*a*b^12*c^4 + 8*a^5*b^8*d^4 - 32*a^2*b^11*c^3*d - 32*a^4*b^9*c*d^3 + 48*a^3*b^10*c^2*d^2))/b^9 - ((-(a + b)*(a - b))^(1/2)*(a*d - b*c)^4*(32*a^2*b^3 + (8*tan(e/2 + (f*x)/2)*(12*a*b^13 - 8*a^3*b^11))/b^9))/(b^6 - a^2*b^4)))/(b^6 - a^2*b^4))*1i)/(b^6 - a^2*b^4))/((16*(2*a^10*d^12 + a^8*b^2*d^12 + 8*a*b^9*c^9*d^3 - 12*a^7*b^3*c*d^11 + 16*a^2*b^8*c^6*d^6 - 2*a^2*b^8*c^8*d^4 - 88*a^2*b^8*c^10*d^2 - 72*a^3*b^7*c^5*d^7 - 128*a^3*b^7*c^7*d^5 + 208*a^3*b^7*c^9*d^3 + 129*a^4*b^6*c^4*d^8 + 416*a^4*b^6*c^6*d^6 - 276*a^4*b^6*c^8*d^4 - 116*a^5*b^5*c^3*d^9 - 640*a^5*b^5*c^5*d^7 + 224*a^5*b^5*c^7*d^5 + 54*a^6*b^4*c^2*d^10 + 584*a^6*b^4*c^4*d^8 - 112*a^6*b^4*c^6*d^6 - 336*a^7*b^3*c^3*d^9 + 32*a^7*b^3*c^5*d^7 + 120*a^8*b^2*c^2*d^10 - 4*a^8*b^2*c^4*d^8 + 16*a*b^9*c^11*d - 24*a^9*b*c*d^11))/b^8 + (16*tan(e/2 + (f*x)/2)*(8*a^11*d^12 + 2*a^7*b^4*d^12 + 8*a^9*b^2*d^12 + 32*a*b^10*c^6*d^6 + 128*a*b^10*c^8*d^4 + 128*a*b^10*c^10*d^2 - 24*a^6*b^5*c*d^11 - 96*a^8*b^3*c*d^11 - 144*a^2*b^9*c^5*d^7 - 736*a^2*b^9*c^7*d^5 - 896*a^2*b^9*c^9*d^3 + 258*a^3*b^8*c^4*d^8 + 1840*a^3*b^8*c^6*d^6 + 2848*a^3*b^8*c^8*d^4 - 232*a^4*b^7*c^3*d^9 - 2624*a^4*b^7*c^5*d^7 - 5440*a^4*b^7*c^7*d^5 + 108*a^5*b^6*c^2*d^10 + 2344*a^5*b^6*c^4*d^8 + 6944*a^5*b^6*c^6*d^6 - 1344*a^6*b^5*c^3*d^9 - 6208*a^6*b^5*c^5*d^7 + 480*a^7*b^4*c^2*d^10 + 3944*a^7*b^4*c^4*d^8 - 1760*a^8*b^3*c^3*d^9 + 528*a^9*b^2*c^2*d^10 - 96*a^10*b*c*d^11))/b^9 + ((-(a + b)*(a - b))^(1/2)*(a*d - b*c)^4*((8*(a^4*b^7*d^8 + 4*a^6*b^5*d^8 + 4*a^8*b^3*d^8 - 8*a^3*b^8*c*d^7 - 32*a^5*b^6*c*d^7 - 32*a^7*b^4*c*d^7 + 16*a^2*b^9*c^2*d^6 + 64*a^2*b^9*c^4*d^4 + 64*a^2*b^9*c^6*d^2 - 112*a^3*b^8*c^3*d^5 - 192*a^3*b^8*c^5*d^3 + 88*a^4*b^7*c^2*d^6 + 272*a^4*b^7*c^4*d^4 - 224*a^5*b^6*c^3*d^5 + 112*a^6*b^5*c^2*d^6))/b^8 + (8*tan(e/2 + (f*x)/2)*(2*a^3*b^9*d^8 - 4*a*b^11*c^8 + 7*a^5*b^7*d^8 + 4*a^7*b^5*d^8 - 8*a^9*b^3*d^8 + 32*a*b^11*c^2*d^6 + 128*a*b^11*c^4*d^4 + 128*a*b^11*c^6*d^2 - 16*a^2*b^10*c*d^7 + 32*a^2*b^10*c^7*d - 56*a^4*b^8*c*d^7 - 32*a^6*b^6*c*d^7 + 64*a^8*b^4*c*d^7 - 224*a^2*b^10*c^3*d^5 - 384*a^2*b^10*c^5*d^3 + 160*a^3*b^9*c^2*d^6 + 480*a^3*b^9*c^4*d^4 - 176*a^3*b^9*c^6*d^2 - 336*a^4*b^8*c^3*d^5 + 416*a^4*b^8*c^5*d^3 + 136*a^5*b^7*c^2*d^6 - 552*a^5*b^7*c^4*d^4 + 448*a^6*b^6*c^3*d^5 - 224*a^7*b^5*c^2*d^6))/b^9 + ((-(a + b)*(a - b))^(1/2)*(a*d - b*c)^4*((8*(4*a^2*b^10*c^4 + 2*a^2*b^10*d^4 + 2*a^4*b^8*d^4 - 8*a^3*b^9*c*d^3 + 24*a^2*b^10*c^2*d^2 - 8*a*b^11*c*d^3 - 16*a*b^11*c^3*d))/b^8 + (8*tan(e/2 + (f*x)/2)*(8*a*b^12*c^4 + 8*a^5*b^8*d^4 - 32*a^2*b^11*c^3*d - 32*a^4*b^9*c*d^3 + 48*a^3*b^10*c^2*d^2))/b^9 + ((-(a + b)*(a - b))^(1/2)*(a*d - b*c)^4*(32*a^2*b^3 + (8*tan(e/2 + (f*x)/2)*(12*a*b^13 - 8*a^3*b^11))/b^9))/(b^6 - a^2*b^4)))/(b^6 - a^2*b^4)))/(b^6 - a^2*

```
b^4) - ((-(a + b)*(a - b))^(1/2)*(a*d - b*c)^4*((8*(a^4*b^7*d^8 + 4*a^6*b^5
*d^8 + 4*a^8*b^3*d^8 - 8*a^3*b^8*c*d^7 - 32*a^5*b^6*c*d^7 - 32*a^7*b^4*c*d^
7 + 16*a^2*b^9*c^2*d^6 + 64*a^2*b^9*c^4*d^4 + 64*a^2*b^9*c^6*d^2 - 112*a^3*
b^8*c^3*d^5 - 192*a^3*b^8*c^5*d^3 + 88*a^4*b^7*c^2*d^6 + 272*a^4*b^7*c^4*d^
4 - 224*a^5*b^6*c^3*d^5 + 112*a^6*b^5*c^2*d^6))/b^8 + (8*tan(e/2 + (f*x)/2)
*(2*a^3*b^9*d^8 - 4*a*b^11*c^8 + 7*a^5*b^7*d^8 + 4*a^7*b^5*d^8 - 8*a^9*b^3*
d^8 + 32*a*b^11*c^2*d^6 + 128*a*b^11*c^4*d^4 + 128*a*b^11*c^6*d^2 - 16*a^2*
b^10*c*d^7 + 32*a^2*b^10*c^7*d - 56*a^4*b^8*c*d^7 - 32*a^6*b^6*c*d^7 + 64*a
^8*b^4*c*d^7 - 224*a^2*b^10*c^3*d^5 - 384*a^2*b^10*c^5*d^3 + 160*a^3*b^9*c^
2*d^6 + 480*a^3*b^9*c^4*d^4 - 176*a^3*b^9*c^6*d^2 - 336*a^4*b^8*c^3*d^5 + 4
16*a^4*b^8*c^5*d^3 + 136*a^5*b^7*c^2*d^6 - 552*a^5*b^7*c^4*d^4 + 448*a^6*b^
6*c^3*d^5 - 224*a^7*b^5*c^2*d^6))/b^9 - ((-(a + b)*(a - b))^(1/2)*(a*d - b*
c)^4*((8*(4*a^2*b^10*c^4 + 2*a^2*b^10*d^4 + 2*a^4*b^8*d^4 - 8*a^3*b^9*c*d^3
 + 24*a^2*b^10*c^2*d^2 - 8*a*b^11*c*d^3 - 16*a*b^11*c^3*d))/b^8 + (8*tan(e/
2 + (f*x)/2)*(8*a*b^12*c^4 + 8*a^5*b^8*d^4 - 32*a^2*b^11*c^3*d - 32*a^4*b^9
*c*d^3 + 48*a^3*b^10*c^2*d^2))/b^9 - ((-(a + b)*(a - b))^(1/2)*(a*d - b*c)^
4*(32*a^2*b^3 + (8*tan(e/2 + (f*x)/2)*(12*a*b^13 - 8*a^3*b^11))/b^9))/(b^6
- a^2*b^4)))/(b^6 - a^2*b^4)))/(b^6 - a^2*b^4)))*(-(a + b)*(a - b))^(1/2)*(
a*d - b*c)^4*2i)/(f*(b^6 - a^2*b^4))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**4/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.699 $\int \frac{(c+d\sin(e+fx))^3}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=156

$$-\frac{dx\left(-2a^2d^2+6abcd-\left(b^2\left(6c^2+d^2\right)\right)\right)}{2b^3}+\frac{2(bc-ad)^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3f\sqrt{a^2-b^2}}-\frac{d^2(5bc-2ad)\cos(e+fx)}{2b^2f}-\frac{d^2\cos}{}$$

```
[Out] -1/2*d*(6*a*b*c*d-2*a^2*d^2-b^2*(6*c^2+d^2))*x/b^3-1/2*d^2*(-2*a*d+5*b*c)*c
os(f*x+e)/b^2/f-1/2*d^2*cos(f*x+e)*(c+d*sin(f*x+e))/b/f+2*(-a*d+b*c)^3*arct
an((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/b^3/f/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2793, 3023, 2735, 2660, 618, 204}

$$-\frac{dx\left(-2a^2d^2+6abcd+b^2\left(-\left(6c^2+d^2\right)\right)\right)}{2b^3}+\frac{2(bc-ad)^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3f\sqrt{a^2-b^2}}-\frac{d^2(5bc-2ad)\cos(e+fx)}{2b^2f}-\frac{d^2c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^3/(a + b*Sin[e + f*x]),x]
```

```
[Out] -(d*(6*a*b*c*d - 2*a^2*d^2 - b^2*(6*c^2 + d^2))*x)/(2*b^3) + (2*(b*c - a*d)
^3*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(b^3*Sqrt[a^2 - b^2]*f
) - (d^2*(5*b*c - 2*a*d)*Cos[e + f*x])/(2*b^2*f) - (d^2*Cos[e + f*x]*(c + d
*Sin[e + f*x]))/(2*b*f)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^3}{a+b\sin(e+fx)}\,dx = -\frac{d^2\cos(e+fx)(c+d\sin(e+fx))}{2bf} + \frac{\int \frac{2bc^3+ad^3-d\left(acd-b\left(6c^2+d^2\right)\right)\sin(e+fx)+d^2(5bc-2ad)\sin}{a+b\sin(e+fx)}}{2b}$$

$$= -\frac{d^2(5bc-2ad)\cos(e+fx)}{2b^2f} - \frac{d^2\cos(e+fx)(c+d\sin(e+fx))}{2bf} + \int \frac{b\left(2bc^3+ad^3\right)-d\left(6a\right.}{}$$

$$= -\frac{d\left(6abcd-2a^2d^2-b^2\left(6c^2+d^2\right)\right)x}{2b^3} - \frac{d^2(5bc-2ad)\cos(e+fx)}{2b^2f} - \frac{d^2\cos(e+fx)}{}$$

$$= -\frac{d\left(6abcd-2a^2d^2-b^2\left(6c^2+d^2\right)\right)x}{2b^3} - \frac{d^2(5bc-2ad)\cos(e+fx)}{2b^2f} - \frac{d^2\cos(e+fx)}{}$$

$$= -\frac{d\left(6abcd-2a^2d^2-b^2\left(6c^2+d^2\right)\right)x}{2b^3} - \frac{d^2(5bc-2ad)\cos(e+fx)}{2b^2f} - \frac{d^2\cos(e+fx)}{}$$

$$= -\frac{d\left(6abcd-2a^2d^2-b^2\left(6c^2+d^2\right)\right)x}{2b^3} + \frac{2(bc-ad)^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,f} - \frac{d^2(5b}{}$$

Mathematica [A] time = 0.35, size = 138, normalized size = 0.88

$$\frac{2d(e+fx)\left(2a^2d^2-6abcd+b^2\left(6c^2+d^2\right)\right)+\frac{8(bc-ad)^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-4bd^2(3bc-ad)\cos(e+fx)-b^2d^3\sin}{4b^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^3/(a + b*Sin[e + f*x]),x]
```

```
[Out] (2*d*(-6*a*b*c*d + 2*a^2*d^2 + b^2*(6*c^2 + d^2))*(e + f*x) + (8*(b*c - a*d
)^3*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - 4*b
*d^2*(3*b*c - a*d)*Cos[e + f*x] - b^2*d^3*Sin[2*(e + f*x)])/(4*b^3*f)
```

fricas [A] time = 0.53, size = 566, normalized size = 3.63

$$\left[-\frac{\left(a^2b^2-b^4\right)d^3\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(6\left(a^2b^2-b^4\right)c^2d-6\left(a^3b-ab^3\right)cd^2+\left(2\,a^4-a^2b^2-b^4\right)d^3\right)fx-\left(b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/2*((a^2*b^2 - b^4)*d^3*cos(f*x + e)*sin(f*x + e) - (6*(a^2*b^2 - b^4)*c
^2*d - 6*(a^3*b - a*b^3)*c*d^2 + (2*a^4 - a^2*b^2 - b^4)*d^3)*f*x - (b^3*c^
3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-a^2 + b^2)*log(-((2*a^2
- b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 - 2*(a*cos(f*x + e)*
sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*
b*sin(f*x + e) - a^2 - b^2)) + 2*(3*(a^2*b^2 - b^4)*c*d^2 - (a^3*b - a*b^3)
*d^3)*cos(f*x + e))/((a^2*b^3 - b^5)*f), -1/2*((a^2*b^2 - b^4)*d^3*cos(f*x
+ e)*sin(f*x + e) - (6*(a^2*b^2 - b^4)*c^2*d - 6*(a^3*b - a*b^3)*c*d^2 + (2
*a^4 - a^2*b^2 - b^4)*d^3)*f*x + 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
 - a^3*d^3)*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*c
os(f*x + e))) + 2*(3*(a^2*b^2 - b^4)*c*d^2 - (a^3*b - a*b^3)*d^3)*cos(f*x +
 e))/((a^2*b^3 - b^5)*f)]
```

giac [A] time = 0.20, size = 252, normalized size = 1.62

$$\frac{\frac{\left(6\,b^2c^2d-6\,abcd^2+2\,a^2d^3+b^2d^3\right)\left(fx+e\right)}{b^3}+\frac{4\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^3}+\frac{2\left(bd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right.}{}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*((6*b^2*c^2*d - 6*a*b*c*d^2 + 2*a^2*d^3 + b^2*d^3)*(f*x + e)/b^3 + 4*(b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(pi*floor(1/2*(f*x + e)/p
i + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/(sq
rt(a^2 - b^2)*b^3) + 2*(b*d^3*tan(1/2*f*x + 1/2*e)^3 - 6*b*c*d^2*tan(1/2*f*
x + 1/2*e)^2 + 2*a*d^3*tan(1/2*f*x + 1/2*e)^2 - b*d^3*tan(1/2*f*x + 1/2*e)
- 6*b*c*d^2 + 2*a*d^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*b^2))/f
```

maple [B] time = 0.22, size = 506, normalized size = 3.24

$$\frac{d^3\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fb\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{2d^3\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a}{f\,b^2\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{6d^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c}{fb\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{d^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{fb\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{}{f\,b^2\left(1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e)),x)
```

```
[Out] 1/f*d^3/b/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^3+2/f*d^3/b^2/(1+ta
n(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*a-6/f*d^2/b/(1+tan(1/2*f*x+1/2*e
)^2)^2*tan(1/2*f*x+1/2*e)^2*c-1/f*d^3/b/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*
f*x+1/2*e)+2/f*d^3/b^2/(1+tan(1/2*f*x+1/2*e)^2)^2*a-6/f*d^2/b/(1+tan(1/2*f*
x+1/2*e)^2)^2*c+2/f*d^3/b^3*arctan(tan(1/2*f*x+1/2*e))*a^2-6/f*d^2/b^2*arct
an(tan(1/2*f*x+1/2*e))*a*c+6/f*d/b*arctan(tan(1/2*f*x+1/2*e))*c^2+1/f*d^3/b
*arctan(tan(1/2*f*x+1/2*e))-2/f/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2
*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^3*d^3+6/f/b^2/(a^2-b^2)^(1/2)*arctan(1/
2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*c*d^2-6/f/b/(a^2-b^2)^(
1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a*c^2*d+2/f/(
a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*c^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 14.77, size = 5902, normalized size = 37.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^3/(a + b*sin(e + f*x)),x)
```

```
[Out] ((2*(a*d^3 - 3*b*c*d^2))/b^2 + (d^3*tan(e/2 + (f*x)/2)^3)/b + (2*tan(e/2 +
(f*x)/2)^2*(a*d^3 - 3*b*c*d^2))/b^2 - (d^3*tan(e/2 + (f*x)/2))/b)/(f*(2*tan
(e/2 + (f*x)/2)^2 + tan(e/2 + (f*x)/2)^4 + 1)) + (atan((((a^2*d^3*1i + (b^2
*d*(6*c^2 + d^2)*1i)/2 - a*b*c*d^2*3i)*((8*(a^2*b^6*d^6 + 4*a^4*b^4*d^6 + 4
*a^6*b^2*d^6 - 12*a^3*b^5*c*d^5 - 24*a^5*b^3*c*d^5 + 12*a^2*b^6*c^2*d^4 + 3
6*a^2*b^6*c^4*d^2 - 72*a^3*b^5*c^3*d^3 + 60*a^4*b^4*c^2*d^4))/b^5 + (8*tan(
e/2 + (f*x)/2)*(2*a*b^8*d^6 - 4*a*b^8*c^6 + 7*a^3*b^6*d^6 + 4*a^5*b^4*d^6 -
 8*a^7*b^2*d^6 + 24*a*b^8*c^2*d^4 + 72*a*b^8*c^4*d^2 - 24*a^2*b^7*c*d^5 + 2
4*a^2*b^7*c^5*d - 36*a^4*b^5*c*d^5 + 48*a^6*b^3*c*d^5 - 144*a^2*b^7*c^3*d^3
 + 108*a^3*b^6*c^2*d^4 - 96*a^3*b^6*c^4*d^2 + 152*a^4*b^5*c^3*d^3 - 120*a^5
*b^4*c^2*d^4))/b^6 + ((a^2*d^3*1i + (b^2*d*(6*c^2 + d^2)*1i)/2 - a*b*c*d^2*
3i)*((8*tan(e/2 + (f*x)/2)*(8*a*b^9*c^3 - 8*a^4*b^6*d^3 - 24*a^2*b^8*c^2*d
+ 24*a^3*b^7*c*d^2))/b^6 - (8*(2*a*b^8*d^3 - 4*a^2*b^7*c^3 + 2*a^3*b^6*d^3
- 12*a^2*b^7*c*d^2 + 12*a*b^8*c^2*d))/b^5 + ((32*a^2*b^3 + (8*tan(e/2 + (f*
```

x)/2)*(12*a*b^10 - 8*a^3*b^8))/b^6)*(a^2*d^3*1i + (b^2*d*(6*c^2 + d^2)*1i)/
2 - a*b*c*d^2*3i))/b^3))/b^3)*1i)/b^3 + ((a^2*d^3*1i + (b^2*d*(6*c^2 + d^2)
*1i)/2 - a*b*c*d^2*3i)*((8*(a^2*b^6*d^6 + 4*a^4*b^4*d^6 + 4*a^6*b^2*d^6 - 1
2*a^3*b^5*c*d^5 - 24*a^5*b^3*c*d^5 + 12*a^2*b^6*c^2*d^4 + 36*a^2*b^6*c^4*d^
2 - 72*a^3*b^5*c^3*d^3 + 60*a^4*b^4*c^2*d^4))/b^5 + (8*tan(e/2 + (f*x)/2)*(
2*a*b^8*d^6 - 4*a*b^8*c^6 + 7*a^3*b^6*d^6 + 4*a^5*b^4*d^6 - 8*a^7*b^2*d^6 +
 24*a*b^8*c^2*d^4 + 72*a*b^8*c^4*d^2 - 24*a^2*b^7*c*d^5 + 24*a^2*b^7*c^5*d
 - 36*a^4*b^5*c*d^5 + 48*a^6*b^3*c*d^5 - 144*a^2*b^7*c^3*d^3 + 108*a^3*b^6*c
^2*d^4 - 96*a^3*b^6*c^4*d^2 + 152*a^4*b^5*c^3*d^3 - 120*a^5*b^4*c^2*d^4))/b
^6 + ((a^2*d^3*1i + (b^2*d*(6*c^2 + d^2)*1i)/2 - a*b*c*d^2*3i)*((8*(2*a*b^8
*d^3 - 4*a^2*b^7*c^3 + 2*a^3*b^6*d^3 - 12*a^2*b^7*c*d^2 + 12*a*b^8*c^2*d))/
b^5 - (8*tan(e/2 + (f*x)/2)*(8*a*b^9*c^3 - 8*a^4*b^6*d^3 - 24*a^2*b^8*c^2*d
 + 24*a^3*b^7*c*d^2))/b^6 + ((32*a^2*b^3 + (8*tan(e/2 + (f*x)/2)*(12*a*b^10
 - 8*a^3*b^8))/b^6)*(a^2*d^3*1i + (b^2*d*(6*c^2 + d^2)*1i)/2 - a*b*c*d^2*3i
))/b^3))/b^3)*1i)/b^3)/((16*(2*a^7*d^9 + a^5*b^2*d^9 - 2*a*b^6*c^6*d^3 - 3*
a^4*b^3*c*d^8 + 4*a^6*b*c^3*d^6 - a^2*b^5*c^3*d^6 + 48*a^2*b^5*c^7*d^2 + 3*
a^3*b^4*c^2*d^7 + 18*a^3*b^4*c^4*d^5 - 76*a^3*b^4*c^6*d^3 - 36*a^4*b^3*c^3*
d^6 + 60*a^4*b^3*c^5*d^4 + 30*a^5*b^2*c^2*d^7 - 24*a^5*b^2*c^4*d^5 - 12*a*b
^6*c^8*d - 12*a^6*b*c*d^8))/b^5 - ((a^2*d^3*1i + (b^2*d*(6*c^2 + d^2)*1i)/2
 - a*b*c*d^2*3i)*((8*(a^2*b^6*d^6 + 4*a^4*b^4*d^6 + 4*a^6*b^2*d^6 - 12*a^3*
b^5*c*d^5 - 24*a^5*b^3*c*d^5 + 12*a^2*b^6*c^2*d^4 + 36*a^2*b^6*c^4*d^2 - 72
*a^3*b^5*c^3*d^3 + 60*a^4*b^4*c^2*d^4))/b^5 + (8*tan(e/2 + (f*x)/2)*(2*a*b^
8*d^6 - 4*a*b^8*c^6 + 7*a^3*b^6*d^6 + 4*a^5*b^4*d^6 - 8*a^7*b^2*d^6 + 24*a*
b^8*c^2*d^4 + 72*a*b^8*c^4*d^2 - 24*a^2*b^7*c*d^5 + 24*a^2*b^7*c^5*d - 36*a
^4*b^5*c*d^5 + 48*a^6*b^3*c*d^5 - 144*a^2*b^7*c^3*d^3 + 108*a^3*b^6*c^2*d^4
 - 96*a^3*b^6*c^4*d^2 + 152*a^4*b^5*c^3*d^3 - 120*a^5*b^4*c^2*d^4))/b^6 + (
(a^2*d^3*1i + (b^2*d*(6*c^2 + d^2)*1i)/2 - a*b*c*d^2*3i)*((8*tan(e/2 + (f*x
)/2)*(8*a*b^9*c^3 - 8*a^4*b^6*d^3 - 24*a^2*b^8*c^2*d + 24*a^3*b^7*c*d^2))/b
^6 - (8*(2*a*b^8*d^3 - 4*a^2*b^7*c^3 + 2*a^3*b^6*d^3 - 12*a^2*b^7*c*d^2 + 1
2*a*b^8*c^2*d))/b^5 + ((32*a^2*b^3 + (8*tan(e/2 + (f*x)/2)*(12*a*b^10 - 8*a
^3*b^8))/b^6)*(a^2*d^3*1i + (b^2*d*(6*c^2 + d^2)*1i)/2 - a*b*c*d^2*3i))/b^3
))/b^3))/b^3 + ((a^2*d^3*1i + (b^2*d*(6*c^2 + d^2)*1i)/2 - a*b*c*d^2*3i)*((
8*(a^2*b^6*d^6 + 4*a^4*b^4*d^6 + 4*a^6*b^2*d^6 - 12*a^3*b^5*c*d^5 - 24*a^5*
b^3*c*d^5 + 12*a^2*b^6*c^2*d^4 + 36*a^2*b^6*c^4*d^2 - 72*a^3*b^5*c^3*d^3 +
60*a^4*b^4*c^2*d^4))/b^5 + (8*tan(e/2 + (f*x)/2)*(2*a*b^8*d^6 - 4*a*b^8*c^6
 + 7*a^3*b^6*d^6 + 4*a^5*b^4*d^6 - 8*a^7*b^2*d^6 + 24*a*b^8*c^2*d^4 + 72*a*
b^8*c^4*d^2 - 24*a^2*b^7*c*d^5 + 24*a^2*b^7*c^5*d - 36*a^4*b^5*c*d^5 + 48*a
^6*b^3*c*d^5 - 144*a^2*b^7*c^3*d^3 + 108*a^3*b^6*c^2*d^4 - 96*a^3*b^6*c^4*d
^2 + 152*a^4*b^5*c^3*d^3 - 120*a^5*b^4*c^2*d^4))/b^6 + ((a^2*d^3*1i + (b^2*
d*(6*c^2 + d^2)*1i)/2 - a*b*c*d^2*3i)*((8*(2*a*b^8*d^3 - 4*a^2*b^7*c^3 + 2*
a^3*b^6*d^3 - 12*a^2*b^7*c*d^2 + 12*a*b^8*c^2*d))/b^5 - (8*tan(e/2 + (f*x)/
2)*(8*a*b^9*c^3 - 8*a^4*b^6*d^3 - 24*a^2*b^8*c^2*d + 24*a^3*b^7*c*d^2))/b^6
 + ((32*a^2*b^3 + (8*tan(e/2 + (f*x)/2)*(12*a*b^10 - 8*a^3*b^8))/b^6)*(a^2*
d^3*1i + (b^2*d*(6*c^2 + d^2)*1i)/2 - a*b*c*d^2*3i))/b^3))/b^3))/b^3 + (16*
tan(e/2 + (f*x)/2)*(8*a^8*d^9 + 2*a^4*b^4*d^9 + 8*a^6*b^2*d^9 - 2*a*b^7*c^3

*d^6 - 24*a*b^7*c^5*d^4 - 72*a*b^7*c^7*d^2 - 6*a^3*b^5*c*d^8 - 48*a^5*b^3*c
*d^8 + 6*a^2*b^6*c^2*d^7 + 96*a^2*b^6*c^4*d^5 + 360*a^2*b^6*c^6*d^3 - 152*a
^3*b^5*c^3*d^6 - 768*a^3*b^5*c^5*d^4 + 120*a^4*b^4*c^2*d^7 + 912*a^4*b^4*c^
4*d^5 - 656*a^5*b^3*c^3*d^6 + 288*a^6*b^2*c^2*d^7 - 72*a^7*b*c*d^8))/b^6))*
(a^2*d^3*1i + (b^2*d*(6*c^2 + d^2)*1i)/2 - a*b*c*d^2*3i)*2i)/(b^3*f) + (ata
n((((-(a + b)*(a - b))^(1/2)*(a*d - b*c)^3*((8*(a^2*b^6*d^6 + 4*a^4*b^4*d^6
 + 4*a^6*b^2*d^6 - 12*a^3*b^5*c*d^5 - 24*a^5*b^3*c*d^5 + 12*a^2*b^6*c^2*d^4
 + 36*a^2*b^6*c^4*d^2 - 72*a^3*b^5*c^3*d^3 + 60*a^4*b^4*c^2*d^4))/b^5 + (8*
tan(e/2 + (f*x)/2)*(2*a*b^8*d^6 - 4*a*b^8*c^6 + 7*a^3*b^6*d^6 + 4*a^5*b^4*d
^6 - 8*a^7*b^2*d^6 + 24*a*b^8*c^2*d^4 + 72*a*b^8*c^4*d^2 - 24*a^2*b^7*c*d^5
 + 24*a^2*b^7*c^5*d - 36*a^4*b^5*c*d^5 + 48*a^6*b^3*c*d^5 - 144*a^2*b^7*c^3
*d^3 + 108*a^3*b^6*c^2*d^4 - 96*a^3*b^6*c^4*d^2 + 152*a^4*b^5*c^3*d^3 - 120
*a^5*b^4*c^2*d^4))/b^6 + ((-(a + b)*(a - b))^(1/2)*(a*d - b*c)^3*((8*tan(e/
2 + (f*x)/2)*(8*a*b^9*c^3 - 8*a^4*b^6*d^3 - 24*a^2*b^8*c^2*d + 24*a^3*b^7*c
d^2))/b^6 - (8(2*a*b^8*d^3 - 4*a^2*b^7*c^3 + 2*a^3*b^6*d^3 - 12*a^2*b^7*c
*d^2 + 12*a*b^8*c^2*d))/b^5 + ((-(a + b)*(a - b))^(1/2)*(a*d - b*c)^3*(32*a
^2*b^3 + (8*tan(e/2 + (f*x)/2)*(12*a*b^10 - 8*a^3*b^8))/b^6))/(b^5 - a^2*b^
3)))/(b^5 - a^2*b^3))*1i)/(b^5 - a^2*b^3) + ((-(a + b)*(a - b))^(1/2)*(a*d
 - b*c)^3*((8*(a^2*b^6*d^6 + 4*a^4*b^4*d^6 + 4*a^6*b^2*d^6 - 12*a^3*b^5*c*d^
5 - 24*a^5*b^3*c*d^5 + 12*a^2*b^6*c^2*d^4 + 36*a^2*b^6*c^4*d^2 - 72*a^3*b^5
*c^3*d^3 + 60*a^4*b^4*c^2*d^4))/b^5 + (8*tan(e/2 + (f*x)/2)*(2*a*b^8*d^6 -
4*a*b^8*c^6 + 7*a^3*b^6*d^6 + 4*a^5*b^4*d^6 - 8*a^7*b^2*d^6 + 24*a*b^8*c^2*
d^4 + 72*a*b^8*c^4*d^2 - 24*a^2*b^7*c*d^5 + 24*a^2*b^7*c^5*d - 36*a^4*b^5*c
*d^5 + 48*a^6*b^3*c*d^5 - 144*a^2*b^7*c^3*d^3 + 108*a^3*b^6*c^2*d^4 - 96*a^
3*b^6*c^4*d^2 + 152*a^4*b^5*c^3*d^3 - 120*a^5*b^4*c^2*d^4))/b^6 + ((-(a + b
)*(a - b))^(1/2)*(a*d - b*c)^3*((8*(2*a*b^8*d^3 - 4*a^2*b^7*c^3 + 2*a^3*b^6
*d^3 - 12*a^2*b^7*c*d^2 + 12*a*b^8*c^2*d))/b^5 - (8*tan(e/2 + (f*x)/2)*(8*a
*b^9*c^3 - 8*a^4*b^6*d^3 - 24*a^2*b^8*c^2*d + 24*a^3*b^7*c*d^2))/b^6 + ((-(
a + b)*(a - b))^(1/2)*(a*d - b*c)^3*(32*a^2*b^3 + (8*tan(e/2 + (f*x)/2)*(12
*a*b^10 - 8*a^3*b^8))/b^6))/(b^5 - a^2*b^3)))/(b^5 - a^2*b^3))*1i)/(b^5 - a
^2*b^3))/((16*(2*a^7*d^9 + a^5*b^2*d^9 - 2*a*b^6*c^6*d^3 - 3*a^4*b^3*c*d^8
+ 4*a^6*b*c^3*d^6 - a^2*b^5*c^3*d^6 + 48*a^2*b^5*c^7*d^2 + 3*a^3*b^4*c^2*d^
7 + 18*a^3*b^4*c^4*d^5 - 76*a^3*b^4*c^6*d^3 - 36*a^4*b^3*c^3*d^6 + 60*a^4*b
^3*c^5*d^4 + 30*a^5*b^2*c^2*d^7 - 24*a^5*b^2*c^4*d^5 - 12*a*b^6*c^8*d - 12*
a^6*b*c*d^8))/b^5 + (16*tan(e/2 + (f*x)/2)*(8*a^8*d^9 + 2*a^4*b^4*d^9 + 8*a
^6*b^2*d^9 - 2*a*b^7*c^3*d^6 - 24*a*b^7*c^5*d^4 - 72*a*b^7*c^7*d^2 - 6*a^3*
b^5*c*d^8 - 48*a^5*b^3*c*d^8 + 6*a^2*b^6*c^2*d^7 + 96*a^2*b^6*c^4*d^5 + 360
*a^2*b^6*c^6*d^3 - 152*a^3*b^5*c^3*d^6 - 768*a^3*b^5*c^5*d^4 + 120*a^4*b^4*
c^2*d^7 + 912*a^4*b^4*c^4*d^5 - 656*a^5*b^3*c^3*d^6 + 288*a^6*b^2*c^2*d^7 -
 72*a^7*b*c*d^8))/b^6 - ((-(a + b)*(a - b))^(1/2)*(a*d - b*c)^3*((8*(a^2*b^
6*d^6 + 4*a^4*b^4*d^6 + 4*a^6*b^2*d^6 - 12*a^3*b^5*c*d^5 - 24*a^5*b^3*c*d^5
 + 12*a^2*b^6*c^2*d^4 + 36*a^2*b^6*c^4*d^2 - 72*a^3*b^5*c^3*d^3 + 60*a^4*b^
4*c^2*d^4))/b^5 + (8*tan(e/2 + (f*x)/2)*(2*a*b^8*d^6 - 4*a*b^8*c^6 + 7*a^3*
b^6*d^6 + 4*a^5*b^4*d^6 - 8*a^7*b^2*d^6 + 24*a*b^8*c^2*d^4 + 72*a*b^8*c^4*d
^2 - 24*a^2*b^7*c*d^5 + 24*a^2*b^7*c^5*d - 36*a^4*b^5*c*d^5 + 48*a^6*b^3*c*

```
d^5 - 144*a^2*b^7*c^3*d^3 + 108*a^3*b^6*c^2*d^4 - 96*a^3*b^6*c^4*d^2 + 152*
a^4*b^5*c^3*d^3 - 120*a^5*b^4*c^2*d^4))/b^6 + ((-(a + b)*(a - b))^(1/2)*(a*
d - b*c)^3*((8*tan(e/2 + (f*x)/2)*(8*a*b^9*c^3 - 8*a^4*b^6*d^3 - 24*a^2*b^8
*c^2*d + 24*a^3*b^7*c*d^2))/b^6 - (8*(2*a*b^8*d^3 - 4*a^2*b^7*c^3 + 2*a^3*b
^6*d^3 - 12*a^2*b^7*c*d^2 + 12*a*b^8*c^2*d))/b^5 + ((-(a + b)*(a - b))^(1/2
)*(a*d - b*c)^3*(32*a^2*b^3 + (8*tan(e/2 + (f*x)/2)*(12*a*b^10 - 8*a^3*b^8)
)/b^6))/(b^5 - a^2*b^3)))/(b^5 - a^2*b^3)))/(b^5 - a^2*b^3) + ((-(a + b)*(a
 - b))^(1/2)*(a*d - b*c)^3*((8*(a^2*b^6*d^6 + 4*a^4*b^4*d^6 + 4*a^6*b^2*d^6
 - 12*a^3*b^5*c*d^5 - 24*a^5*b^3*c*d^5 + 12*a^2*b^6*c^2*d^4 + 36*a^2*b^6*c^
4*d^2 - 72*a^3*b^5*c^3*d^3 + 60*a^4*b^4*c^2*d^4))/b^5 + (8*tan(e/2 + (f*x)/
2)*(2*a*b^8*d^6 - 4*a*b^8*c^6 + 7*a^3*b^6*d^6 + 4*a^5*b^4*d^6 - 8*a^7*b^2*d
^6 + 24*a*b^8*c^2*d^4 + 72*a*b^8*c^4*d^2 - 24*a^2*b^7*c*d^5 + 24*a^2*b^7*c^
5*d - 36*a^4*b^5*c*d^5 + 48*a^6*b^3*c*d^5 - 144*a^2*b^7*c^3*d^3 + 108*a^3*b
^6*c^2*d^4 - 96*a^3*b^6*c^4*d^2 + 152*a^4*b^5*c^3*d^3 - 120*a^5*b^4*c^2*d^4
))/b^6 + ((-(a + b)*(a - b))^(1/2)*(a*d - b*c)^3*((8*(2*a*b^8*d^3 - 4*a^2*b
^7*c^3 + 2*a^3*b^6*d^3 - 12*a^2*b^7*c*d^2 + 12*a*b^8*c^2*d))/b^5 - (8*tan(e
/2 + (f*x)/2)*(8*a*b^9*c^3 - 8*a^4*b^6*d^3 - 24*a^2*b^8*c^2*d + 24*a^3*b^7*
c*d^2))/b^6 + ((-(a + b)*(a - b))^(1/2)*(a*d - b*c)^3*(32*a^2*b^3 + (8*tan(
e/2 + (f*x)/2)*(12*a*b^10 - 8*a^3*b^8))/b^6))/(b^5 - a^2*b^3)))/(b^5 - a^2*
b^3)))/(b^5 - a^2*b^3)))*(-(a + b)*(a - b))^(1/2)*(a*d - b*c)^3*2i)/(f*(b^5
 - a^2*b^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**3/(a+b*sin(f*x+e)),x)
[Out] Timed out
```

3.700 $\int \frac{(c+d\sin(e+fx))^2}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=93

$$\frac{2(bc-ad)^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2f\sqrt{a^2-b^2}}+\frac{dx(2bc-ad)}{b^2}-\frac{d^2\cos(e+fx)}{bf}$$

```
[Out] d*(-a*d+2*b*c)*x/b^2-d^2*cos(f*x+e)/b/f+2*(-a*d+b*c)^2*arctan((b+a*tan(1/2*
f*x+1/2*e))/(a^2-b^2)^(1/2))/b^2/f/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2746, 2735, 2660, 618, 204}

$$\frac{2(bc-ad)^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2f\sqrt{a^2-b^2}}+\frac{dx(2bc-ad)}{b^2}-\frac{d^2\cos(e+fx)}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^2/(a + b*Sin[e + f*x]),x]
```

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (2*(b*c - a*d)^2*ArcTan[(b + a*Tan[(e + f*x)/2])/
Sqrt[a^2 - b^2]])/(b^2*Sqrt[a^2 - b^2]*f) - (d^2*Cos[e + f*x])/(b*f)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^2}{a+b\sin(e+fx)}\,dx = -\frac{d^2\cos(e+fx)}{bf}+\frac{\int \frac{bc^2+d(2bc-ad)\sin(e+fx)}{a+b\sin(e+fx)}\,dx}{b}$$

$$= \frac{d(2bc-ad)x}{b^2}-\frac{d^2\cos(e+fx)}{bf}+\frac{(bc-ad)^2\int \frac{1}{a+b\sin(e+fx)}\,dx}{b^2}$$

$$= \frac{d(2bc-ad)x}{b^2}-\frac{d^2\cos(e+fx)}{bf}+\frac{\left(2(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(e+f$$

$$= \frac{d(2bc-ad)x}{b^2}-\frac{d^2\cos(e+fx)}{bf}-\frac{\left(4(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\,\text{ta}$$

$$= \frac{d(2bc-ad)x}{b^2}+\frac{2(bc-ad)^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}\,f}-\frac{d^2\cos(e+fx)}{bf}$$

Mathematica [A] time = 0.16, size = 90, normalized size = 0.97

$$\frac{\frac{2(bc-ad)^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+d(e+fx)(2bc-ad)-bd^2\cos(e+fx)}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^2/(a + b*Sin[e + f*x]),x]
```

```
[Out] (d*(2*b*c - a*d)*(e + f*x) + (2*(b*c - a*d)^2*ArcTan[(b + a*Tan[(e + f*x)/2
])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - b*d^2*Cos[e + f*x])/(b^2*f)
```

fricas [A] time = 0.48, size = 368, normalized size = 3.96

$$\left[-\frac{2\left(a^2b-b^3\right)d^2\cos\left(fx+e\right)-2\left(2\left(a^2b-b^3\right)cd-\left(a^3-ab^2\right)d^2\right)fx+\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-a^2+b^2}\,\log\left(\left((2\cdots\right.\right.}{2\left(a^2b^2-b^4\right)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^2*b - b^3)*d^2*cos(f*x + e) - 2*(2*(a^2*b - b^3)*c*d - (a^3 - a
*b^2)*d^2)*f*x + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a^2 + b^2)*log(((2*a
^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 + 2*(a*cos(f*x +
e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2
*a*b*sin(f*x + e) - a^2 - b^2)))/((a^2*b^2 - b^4)*f), -((a^2*b - b^3)*d^2*c
os(f*x + e) - (2*(a^2*b - b^3)*c*d - (a^3 - a*b^2)*d^2)*f*x + (b^2*c^2 - 2*
a*b*c*d + a^2*d^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 -
 b^2)*cos(f*x + e))))/((a^2*b^2 - b^4)*f)]
```

giac [A] time = 0.30, size = 134, normalized size = 1.44

$$\frac{\frac{\left(2\,bcd-ad^2\right)\left(fx+e\right)}{b^2}-\frac{2\,d^2}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)b}+\frac{2\left(b^2c^2-2\,abcd+a^2d^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] ((2*b*c*d - a*d^2)*(f*x + e)/b^2 - 2*d^2/((tan(1/2*f*x + 1/2*e)^2 + 1)*b) +
 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a)
 + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b
^2))/f
```

maple [B] time = 0.19, size = 226, normalized size = 2.43

$$-\frac{2d^2}{fb\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}-\frac{2d^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a}{f\,b^2}+\frac{4d\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c}{fb}+\frac{2\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)a\cdots}{f\,b^2\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e)),x)
```

```
[Out] -2/f*d^2/b/(1+tan(1/2*f*x+1/2*e)^2)-2/f*d^2/b^2*arctan(tan(1/2*f*x+1/2*e))*
a+4/f*d/b*arctan(tan(1/2*f*x+1/2*e))*c+2/f/b^2/(a^2-b^2)^(1/2)*arctan(1/2*(
2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*d^2-4/f/b/(a^2-b^2)^(1/2)*
arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a*c*d+2/f/(a^2-b^2
)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*c^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.62, size = 2628, normalized size = 28.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^2/(a + b*sin(e + f*x)),x)
```

```
[Out] - (2*d^2)/(b*f*(tan(e/2 + (f*x)/2)^2 + 1)) - (atan((((-(a + b)*(a - b))^(1/
2)*(a*d - b*c)^2*((32*(a^4*b*d^4 - 4*a^3*b^2*c*d^3 + 4*a^2*b^3*c^2*d^2))/b^
2 - (32*tan(e/2 + (f*x)/2)*(a*b^5*c^4 + 2*a^5*b*d^4 - 2*a^3*b^3*d^4 - 8*a*b
^5*c^2*d^2 + 8*a^2*b^4*c*d^3 - 4*a^2*b^4*c^3*d - 8*a^4*b^2*c*d^3 + 10*a^3*b
^3*c^2*d^2))/b^3 + ((-(a + b)*(a - b))^(1/2)*(a*d - b*c)^2*((32*(a^2*b^4*c^
2 + a^2*b^4*d^2 - 2*a*b^5*c*d))/b^2 + (32*tan(e/2 + (f*x)/2)*(2*a*b^6*c^2 +
 2*a^3*b^4*d^2 - 4*a^2*b^5*c*d))/b^3 + ((-(a + b)*(a - b))^(1/2)*(a*d - b*c
)^2*(32*a^2*b^3 + (32*tan(e/2 + (f*x)/2)*(3*a*b^7 - 2*a^3*b^5))/b^3))/(b^4
- a^2*b^2)))/(b^4 - a^2*b^2))*1i)/(b^4 - a^2*b^2) - ((-(a + b)*(a - b))^(1/
2)*(a*d - b*c)^2*((32*tan(e/2 + (f*x)/2)*(a*b^5*c^4 + 2*a^5*b*d^4 - 2*a^3*b
^3*d^4 - 8*a*b^5*c^2*d^2 + 8*a^2*b^4*c*d^3 - 4*a^2*b^4*c^3*d - 8*a^4*b^2*c*
d^3 + 10*a^3*b^3*c^2*d^2))/b^3 - (32*(a^4*b*d^4 - 4*a^3*b^2*c*d^3 + 4*a^2*b
^3*c^2*d^2))/b^2 + ((-(a + b)*(a - b))^(1/2)*(a*d - b*c)^2*((32*(a^2*b^4*c^
2 + a^2*b^4*d^2 - 2*a*b^5*c*d))/b^2 + (32*tan(e/2 + (f*x)/2)*(2*a*b^6*c^2 +
 2*a^3*b^4*d^2 - 4*a^2*b^5*c*d))/b^3 - ((-(a + b)*(a - b))^(1/2)*(a*d - b*c
)^2*(32*a^2*b^3 + (32*tan(e/2 + (f*x)/2)*(3*a*b^7 - 2*a^3*b^5))/b^3))/(b^4
```

```
- a^2*b^2)))/(b^4 - a^2*b^2))*1i)/(b^4 - a^2*b^2))/((64*tan(e/2 + (f*x)/2)*
(2*a^5*d^6 + 8*a*b^4*c^4*d^2 - 24*a^2*b^3*c^3*d^3 + 26*a^3*b^2*c^2*d^4 - 12
*a^4*b*c*d^5))/b^3 - (64*(a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 5*a^2*b^2*c^4*d^2
 - 2*a*b^3*c^5*d))/b^2 + ((-(a + b)*(a - b))^(1/2)*(a*d - b*c)^2*((32*(a^4*
b*d^4 - 4*a^3*b^2*c*d^3 + 4*a^2*b^3*c^2*d^2))/b^2 - (32*tan(e/2 + (f*x)/2)*
(a*b^5*c^4 + 2*a^5*b*d^4 - 2*a^3*b^3*d^4 - 8*a*b^5*c^2*d^2 + 8*a^2*b^4*c*d^
3 - 4*a^2*b^4*c^3*d - 8*a^4*b^2*c*d^3 + 10*a^3*b^3*c^2*d^2))/b^3 + ((-(a +
b)*(a - b))^(1/2)*(a*d - b*c)^2*((32*(a^2*b^4*c^2 + a^2*b^4*d^2 - 2*a*b^5*c
*d))/b^2 + (32*tan(e/2 + (f*x)/2)*(2*a*b^6*c^2 + 2*a^3*b^4*d^2 - 4*a^2*b^5*
c*d))/b^3 + ((-(a + b)*(a - b))^(1/2)*(a*d - b*c)^2*(32*a^2*b^3 + (32*tan(e
/2 + (f*x)/2)*(3*a*b^7 - 2*a^3*b^5))/b^3))/(b^4 - a^2*b^2)))/(b^4 - a^2*b^2
)))/(b^4 - a^2*b^2) + ((-(a + b)*(a - b))^(1/2)*(a*d - b*c)^2*((32*tan(e/2
+ (f*x)/2)*(a*b^5*c^4 + 2*a^5*b*d^4 - 2*a^3*b^3*d^4 - 8*a*b^5*c^2*d^2 + 8*a
^2*b^4*c*d^3 - 4*a^2*b^4*c^3*d - 8*a^4*b^2*c*d^3 + 10*a^3*b^3*c^2*d^2))/b^3
 - (32*(a^4*b*d^4 - 4*a^3*b^2*c*d^3 + 4*a^2*b^3*c^2*d^2))/b^2 + ((-(a + b)*
(a - b))^(1/2)*(a*d - b*c)^2*((32*(a^2*b^4*c^2 + a^2*b^4*d^2 - 2*a*b^5*c*d)
)/b^2 + (32*tan(e/2 + (f*x)/2)*(2*a*b^6*c^2 + 2*a^3*b^4*d^2 - 4*a^2*b^5*c*d
))/b^3 - ((-(a + b)*(a - b))^(1/2)*(a*d - b*c)^2*(32*a^2*b^3 + (32*tan(e/2
+ (f*x)/2)*(3*a*b^7 - 2*a^3*b^5))/b^3))/(b^4 - a^2*b^2)))/(b^4 - a^2*b^2)))
/(b^4 - a^2*b^2)))*(-(a + b)*(a - b))^(1/2)*(a*d - b*c)^2*2i)/(f*(b^4 - a^2
*b^2)) - (2*d*atan((64*a^4*d^6*tan(e/2 + (f*x)/2))/(64*a^4*d^6 + 128*a^4*c^
2*d^4 - 512*a*b^3*c^3*d^3 - 512*a^3*b*c^3*d^3 + 768*a^2*b^2*c^2*d^4 + 576*a
^2*b^2*c^4*d^2 - 128*a*b^3*c^5*d - 384*a^3*b*c*d^5) + (384*a^3*c*d^5*tan(e/
2 + (f*x)/2))/(384*a^3*c*d^5 - (64*a^4*d^6)/b + 512*a^3*c^3*d^3 + 512*a*b^2
*c^3*d^3 - 768*a^2*b*c^2*d^4 - 576*a^2*b*c^4*d^2 - (128*a^4*c^2*d^4)/b + 12
8*a*b^2*c^5*d) + (768*a^2*c^2*d^4*tan(e/2 + (f*x)/2))/((64*a^4*d^6)/b^2 + 7
68*a^2*c^2*d^4 + 576*a^2*c^4*d^2 - (384*a^3*c*d^5)/b - 128*a*b*c^5*d - (512
*a^3*c^3*d^3)/b + (128*a^4*c^2*d^4)/b^2 - 512*a*b*c^3*d^3) + (576*a^2*c^4*d
^2*tan(e/2 + (f*x)/2))/((64*a^4*d^6)/b^2 + 768*a^2*c^2*d^4 + 576*a^2*c^4*d^
2 - (384*a^3*c*d^5)/b - 128*a*b*c^5*d - (512*a^3*c^3*d^3)/b + (128*a^4*c^2*
d^4)/b^2 - 512*a*b*c^3*d^3) + (512*a^3*c^3*d^3*tan(e/2 + (f*x)/2))/(384*a^3
*c*d^5 - (64*a^4*d^6)/b + 512*a^3*c^3*d^3 + 512*a*b^2*c^3*d^3 - 768*a^2*b*c
^2*d^4 - 576*a^2*b*c^4*d^2 - (128*a^4*c^2*d^4)/b + 128*a*b^2*c^5*d) + (128*
a^4*c^2*d^4*tan(e/2 + (f*x)/2))/(64*a^4*d^6 + 128*a^4*c^2*d^4 - 512*a*b^3*c
^3*d^3 - 512*a^3*b*c^3*d^3 + 768*a^2*b^2*c^2*d^4 + 576*a^2*b^2*c^4*d^2 - 12
8*a*b^3*c^5*d - 384*a^3*b*c*d^5) - (128*a*b*c^5*d*tan(e/2 + (f*x)/2))/((64*
a^4*d^6)/b^2 + 768*a^2*c^2*d^4 + 576*a^2*c^4*d^2 - (384*a^3*c*d^5)/b - 128*
a*b*c^5*d - (512*a^3*c^3*d^3)/b + (128*a^4*c^2*d^4)/b^2 - 512*a*b*c^3*d^3)
- (512*a*b*c^3*d^3*tan(e/2 + (f*x)/2))/((64*a^4*d^6)/b^2 + 768*a^2*c^2*d^4
+ 576*a^2*c^4*d^2 - (384*a^3*c*d^5)/b - 128*a*b*c^5*d - (512*a^3*c^3*d^3)/b
 + (128*a^4*c^2*d^4)/b^2 - 512*a*b*c^3*d^3))*(a*d - 2*b*c))/(b^2*f)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**2/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.701 $\int \frac{c+d\sin(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=65

$$\frac{2(bc-ad)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bf\sqrt{a^2-b^2}}+\frac{dx}{b}$$

```
[Out] d*x/b+2*(-a*d+b*c)*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/b/f/(a^
2-b^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2735, 2660, 618, 204}

$$\frac{2(bc-ad)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bf\sqrt{a^2-b^2}}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] (d*x)/b + (2*(b*c - a*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/
(b*Sqrt[a^2 - b^2]*f)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{c+d\sin(e+fx)}{a+b\sin(e+fx)}\,dx = \frac{dx}{b} - \frac{(-bc+ad)\int \frac{1}{a+b\sin(e+fx)}\,dx}{b}$$

$$= \frac{dx}{b} + \frac{(2(bc-ad))\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{bf}$$

$$= \frac{dx}{b} - \frac{(4(bc-ad))\,\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}(e+fx)\right)\right)}{bf}$$

$$= \frac{dx}{b} + \frac{2(bc-ad)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}\,f}$$

Mathematica [A] time = 0.10, size = 67, normalized size = 1.03

$$\frac{\frac{2(bc-ad)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + d(e+fx)}{bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] (d*(e + f*x) + (2*(b*c - a*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^
2]])/Sqrt[a^2 - b^2])/(b*f)
```

fricas [A] time = 0.49, size = 253, normalized size = 3.89

$$\left[\frac{2\left(a^2-b^2\right)dfx+\sqrt{-a^2+b^2}\,(bc-ad)\log\left(-\frac{\left(2a^2-b^2\right)\cos\left(fx+e\right)^2-2ab\sin\left(fx+e\right)-a^2-b^2-2\left(a\cos\left(fx+e\right)\sin\left(fx+e\right)+b\cos\left(fx+e\right)\right)}{b^2\cos\left(fx+e\right)^2-2ab\sin\left(fx+e\right)-a^2-b^2}\right.}{2\left(a^2b-b^3\right)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^2 - b^2)*d*f*x + sqrt(-a^2 + b^2)*(b*c - a*d)*log(-((2*a^2 - b^2
)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 - 2*(a*cos(f*x + e)*sin(f
*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin
(f*x + e) - a^2 - b^2)))/((a^2*b - b^3)*f), ((a^2 - b^2)*d*f*x - sqrt(a^2 -
 b^2)*(b*c - a*d)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e
))))/((a^2*b - b^3)*f)]
```

giac [A] time = 0.32, size = 86, normalized size = 1.32

$$\frac{\frac{(fx+e)d}{b}+\frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)(bc-ad)}{\sqrt{a^2-b^2}\,b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] ((f*x + e)*d/b + 2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan
(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))*(b*c - a*d)/(sqrt(a^2 - b^2)*b))/f
```

maple [A] time = 0.12, size = 119, normalized size = 1.83

$$\frac{2d\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fb}-\frac{2\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)da}{fb\sqrt{a^2-b^2}}+\frac{2\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)c}{f\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))/(a+b*sin(f*x+e)),x)
```

```
[Out] 2/f*d/b*arctan(tan(1/2*f*x+1/2*e))-2/f/b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*ta
n(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*d*a+2/f/(a^2-b^2)^(1/2)*arctan(1/2*(
2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 10.01, size = 343, normalized size = 5.28

$$\frac{2\,d\,\mathrm{atan}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{b\,f}-\frac{a\,\left(d\,\ln\left(\frac{b\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{b^2-a^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\,\sqrt{-\left(a+b\right)\,\left(a-b\right)}-d\,\ln\left(\frac{b\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\,\mathrm{s}}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))/(a + b*sin(e + f*x)),x)
```

```
[Out] (2*d*atan(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(b*f) - (a*(d*log((b*cos(
e/2 + (f*x)/2) + a*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2
))/cos(e/2 + (f*x)/2))*(-(a + b)*(a - b))^(1/2) - d*log((b*cos(e/2 + (f*x)/
2) + a*sin(e/2 + (f*x)/2) - cos(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2))/cos(e/2 +
 (f*x)/2))*(b^2 - a^2)^(1/2)) - b*c*log((b*cos(e/2 + (f*x)/2) + a*sin(e/2 +
 (f*x)/2) + cos(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2))/cos(e/2 + (f*x)/2))*(-(a
+ b)*(a - b))^(1/2) + b*c*log((b*cos(e/2 + (f*x)/2) + a*sin(e/2 + (f*x)/2)
- cos(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2))/cos(e/2 + (f*x)/2))*(b^2 - a^2)^(1/
2))/(b*f*(a^2 - b^2))
```

sympy [A] time = 84.85, size = 537, normalized size = 8.26

$$\begin{cases} \frac{\tilde{\infty}x(c+d\sin(e))}{\sin(e)} \\ \frac{cx-\frac{d\cos(e+fx)}{f}}{a} \\ \frac{b^2dfx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{b^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-f(b^2)^{\frac{3}{2}}}+\frac{2b^2d}{b^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-f(b^2)^{\frac{3}{2}}}+\frac{2bc\sqrt{b^2}}{b^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-f(b^2)^{\frac{3}{2}}}-\frac{bdfx\sqrt{b^2}}{b^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-f(b^2)^{\frac{3}{2}}} \\ \frac{b^2dfx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{b^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+f(b^2)^{\frac{3}{2}}}+\frac{2b^2d}{b^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+f(b^2)^{\frac{3}{2}}}-\frac{2bc\sqrt{b^2}}{b^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+f(b^2)^{\frac{3}{2}}}+\frac{bdfx\sqrt{b^2}}{b^3f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+f(b^2)^{\frac{3}{2}}} \\ \frac{\frac{c\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}+dx}{b} \\ \frac{x(c+d\sin(e))}{a+b\sin(e)} \\ -\frac{ad\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{bf\sqrt{-a^2+b^2}}+\frac{ad\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{bf\sqrt{-a^2+b^2}}+\frac{c\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{f\sqrt{-a^2+b^2}}-\frac{c\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{b}{a}+\frac{\sqrt{-a^2}}{a}\right)}{f\sqrt{-a^2+b^2}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e)),x)
```

```
[Out] Piecewise((zoo*x*(c + d*sin(e))/sin(e), Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((
c*x - d*cos(e + f*x)/f)/a, Eq(b, 0)), (b**2*d*f*x*tan(e/2 + f*x/2)/(b**3*f*
tan(e/2 + f*x/2) - f*(b**2)**(3/2)) + 2*b**2*d/(b**3*f*tan(e/2 + f*x/2) - f
*(b**2)**(3/2)) + 2*b*c*sqrt(b**2)/(b**3*f*tan(e/2 + f*x/2) - f*(b**2)**(3/
2)) - b*d*f*x*sqrt(b**2)/(b**3*f*tan(e/2 + f*x/2) - f*(b**2)**(3/2)), Eq(a,
 -sqrt(b**2))), (b**2*d*f*x*tan(e/2 + f*x/2)/(b**3*f*tan(e/2 + f*x/2) + f*(
b**2)**(3/2)) + 2*b**2*d/(b**3*f*tan(e/2 + f*x/2) + f*(b**2)**(3/2)) - 2*b*
c*sqrt(b**2)/(b**3*f*tan(e/2 + f*x/2) + f*(b**2)**(3/2)) + b*d*f*x*sqrt(b**
2)/(b**3*f*tan(e/2 + f*x/2) + f*(b**2)**(3/2)), Eq(a, sqrt(b**2))), ((c*log
(tan(e/2 + f*x/2))/f + d*x)/b, Eq(a, 0)), (x*(c + d*sin(e))/(a + b*sin(e)),
 Eq(f, 0)), (-a*d*log(tan(e/2 + f*x/2) + b/a - sqrt(-a**2 + b**2)/a)/(b*f*s
qrt(-a**2 + b**2)) + a*d*log(tan(e/2 + f*x/2) + b/a + sqrt(-a**2 + b**2)/a)
/(b*f*sqrt(-a**2 + b**2)) + c*log(tan(e/2 + f*x/2) + b/a - sqrt(-a**2 + b**
2)/a)/(f*sqrt(-a**2 + b**2)) - c*log(tan(e/2 + f*x/2) + b/a + sqrt(-a**2 +
b**2)/a)/(f*sqrt(-a**2 + b**2)) + d*x/b, True))
```

3.702 $\int \frac{1}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=47

$$\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}}$$

```
[Out] 2*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/f/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2660, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(-1),x]
```

```
[Out] (2*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*f)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{a+b\sin(e+fx)}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{f}$$
$$= -\frac{4\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}(e+fx)\right)\right)}{f}$$
$$= \frac{2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f}$$

Mathematica [A] time = 0.03, size = 47, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^(-1),x]
```

```
[Out] (2*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*f)
```

fricas [A] time = 0.45, size = 190, normalized size = 4.04

$$\left[-\frac{\sqrt{-a^2+b^2}\,\log\left(\frac{\left(2\,a^2-b^2\right)\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2+2\,\left(a\cos\left(fx+e\right)\sin\left(fx+e\right)+b\cos\left(fx+e\right)\right)\sqrt{-a^2+b^2}}{b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2}\right)}{2\left(a^2-b^2\right)f}, -\frac{\arctan\left(-\frac{a\sin}{\sqrt{a^2-b}}\right.}{\sqrt{a^2-b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x +
e) - a^2 - b^2 + 2*(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2
 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2))/((a^2 - b^2
)*f), -arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e)))/(sqrt(a
^2 - b^2)*f)]
```

giac [A] time = 0.42, size = 62, normalized size = 1.32

$$\frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e)
 + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*f)
```

maple [A] time = 0.10, size = 47, normalized size = 1.00

$$\frac{2\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e)),x)
```

```
[Out] 2/f/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2)
)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.79, size = 42, normalized size = 0.89

$$\frac{2\,\mathrm{atan}\left(\frac{b+a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{f\,\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(e + f*x)),x)
```

```
[Out] (2*atan((b + a*tan(e/2 + (f*x)/2))/(a^2 - b^2)^(1/2)))/(f*(a^2 - b^2)^(1/2)
)
```

sympy [A] time = 10.63, size = 177, normalized size = 3.77

$$\begin{cases} \frac{2\sqrt{b^2}}{b^2 f \tan\left(\frac{e}{2}+\frac{fx}{2}\right)-bf\sqrt{b^2}} & \text{for } a = -\sqrt{b^2} \\ -\frac{2\sqrt{b^2}}{b^2 f \tan\left(\frac{e}{2}+\frac{fx}{2}\right)+bf\sqrt{b^2}} & \text{for } a = \sqrt{b^2} \\ \frac{\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{bf} & \text{for } a = 0 \\ \frac{x}{a+b\sin(e)} & \text{for } f = 0 \\ \frac{\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{f\sqrt{-a^2+b^2}} - \frac{\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{f\sqrt{-a^2+b^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)),x)
```

```
[Out] Piecewise((2*sqrt(b**2)/(b**2*f*tan(e/2 + f*x/2) - b*f*sqrt(b**2)), Eq(a, -
sqrt(b**2))), (-2*sqrt(b**2)/(b**2*f*tan(e/2 + f*x/2) + b*f*sqrt(b**2)), Eq
(a, sqrt(b**2))), (log(tan(e/2 + f*x/2))/(b*f), Eq(a, 0)), (x/(a + b*sin(e)
), Eq(f, 0)), (log(tan(e/2 + f*x/2) + b/a - sqrt(-a**2 + b**2)/a)/(f*sqrt(-
a**2 + b**2)) - log(tan(e/2 + f*x/2) + b/a + sqrt(-a**2 + b**2)/a)/(f*sqrt(
-a**2 + b**2)), True))
```

3.703 $\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=117

$$\frac{2b\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}\,(bc-ad)}-\frac{2d\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\sqrt{c^2-d^2}\,(bc-ad)}$$

```
[Out] 2*b*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/(-a*d+b*c)/f/(a^2-b^2)
^(1/2)-2*d*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(-a*d+b*c)/f/(c
^2-d^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2747, 2660, 618, 204}

$$\frac{2b\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}\,(bc-ad)}-\frac{2d\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\sqrt{c^2-d^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])),x]
```

```
[Out] (2*b*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*(b*
c - a*d)*f) - (2*d*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((b*c
- a*d)*Sqrt[c^2 - d^2]*f)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))}\,dx &= \frac{b\int \frac{1}{a+b\sin(e+fx)}\,dx}{bc-ad} - \frac{d\int \frac{1}{c+d\sin(e+fx)}\,dx}{bc-ad} \\
&= \frac{(2b)\operatorname{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{(bc-ad)f} - \frac{(2d)\operatorname{Subst}\left(\int \frac{}{c\cdots}\right.}{} \\
&= -\frac{(4b)\operatorname{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}(e+fx)\right)\right)}{(bc-ad)f} + \frac{(4d)\cdots}{} \\
&= \frac{2b\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(bc-ad)f} - \frac{2d\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(bc-ad)\sqrt{c^2-d^2}\,f}
\end{aligned}
$$

Mathematica [A] time = 0.18, size = 104, normalized size = 0.89

$$
\frac{\frac{2b\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{2d\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}}{bcf-adf}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])),x]
```

```
[Out] ((2*b*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (
2*d*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2])/(b*c
*f - a*d*f)
```

fricas [A] time = 1.85, size = 1057, normalized size = 9.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2 - b^2)*sqrt(-c^2 + d^2)*d*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*
c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x +
 e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2
)) + (b*c^2 - b*d^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(f*x + e)^2 -
2*a*b*sin(f*x + e) - a^2 - b^2 - 2*(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*x
 + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b
^2)))/(((a^2*b - b^3)*c^3 - (a^3 - a*b^2)*c^2*d - (a^2*b - b^3)*c*d^2 + (a^
3 - a*b^2)*d^3)*f), 1/2*(2*(a^2 - b^2)*sqrt(c^2 - d^2)*d*arctan(-(c*sin(f*x
 + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (b*c^2 - b*d^2)*sqrt(-a^2 + b^
2)*log(-((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 - 2*
(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f
*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)))/(((a^2*b - b^3)*c^3 - (a^3 -
a*b^2)*c^2*d - (a^2*b - b^3)*c*d^2 + (a^3 - a*b^2)*d^3)*f), 1/2*((a^2 - b^2
)*sqrt(-c^2 + d^2)*d*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e)
 - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 +
 d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 2*(b*c^2 -
b*d^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*
x + e))))/(((a^2*b - b^3)*c^3 - (a^3 - a*b^2)*c^2*d - (a^2*b - b^3)*c*d^2 +
 (a^3 - a*b^2)*d^3)*f), ((a^2 - b^2)*sqrt(c^2 - d^2)*d*arctan(-(c*sin(f*x +
 e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) - (b*c^2 - b*d^2)*sqrt(a^2 - b^2)*
arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))))/(((a^2*b - b^
3)*c^3 - (a^3 - a*b^2)*c^2*d - (a^2*b - b^3)*c*d^2 + (a^3 - a*b^2)*d^3)*f)]
```

giac [A] time = 0.22, size = 145, normalized size = 1.24

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)b}{\sqrt{a^2-b^2}\,(bc-ad)}-\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)d}{(bc-ad)\sqrt{c^2-d^2}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e
) + b)/sqrt(a^2 - b^2)))*b/(sqrt(a^2 - b^2)*(b*c - a*d)) - (pi*floor(1/2*(f
```

```
*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d
^2)))*d/((b*c - a*d)*sqrt(c^2 - d^2)))/f
```

maple [A] time = 0.30, size = 116, normalized size = 0.99

$$\frac{2d\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{f\left(da-cb\right)\sqrt{c^2-d^2}}-\frac{2b\arctan\left(\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{f\left(da-cb\right)\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] 2/f/(a*d-b*c)*d/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^
2-d^2)^(1/2))-2/f*b/(a*d-b*c)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1
/2*e)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 9.99, size = 3281, normalized size = 28.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))),x)
```

```
[Out] (b*d^2*atan((b^4*c^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(3/2)*3i - a^6*d^2*tan(
e/2 + (f*x)/2)*(b^2 - a^2)^(1/2)*1i - a^3*b^3*d^2*(b^2 - a^2)^(1/2)*2i - b^
6*c^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2)*1i - b^4*d^2*tan(e/2 + (f*x)/2)*
(b^2 - a^2)^(3/2)*4i + b^6*d^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2)*4i - b^
4*c*d*(b^2 - a^2)^(3/2)*1i + b^6*c*d*(b^2 - a^2)^(1/2)*1i + a*b^3*c^2*(b^2
- a^2)^(3/2)*1i - a*b^3*d^2*(b^2 - a^2)^(3/2)*1i + a*b^5*d^2*(b^2 - a^2)^(1
/2)*1i + a^5*b*d^2*(b^2 - a^2)^(1/2)*1i - a^2*b^2*c^2*tan(e/2 + (f*x)/2)*(b
^2 - a^2)^(3/2)*2i + a^2*b^4*c^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2)*2i -
```

a^4*b^2*c^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2)*1i + a^2*b^2*d^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(3/2)*3i - a^2*b^4*d^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2)*9i + a^4*b^2*d^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2)*6i + a^2*b^2*c*d*(b^2 - a^2)^(3/2)*1i - a^2*b^4*c*d*(b^2 - a^2)^(1/2)*2i + a^4*b^2*c*d*(b^2 - a^2)^(1/2)*1i)/(a^7*d^2 - a*b^6*c^2 + 2*a^3*b^4*c^2 - a^5*b^2*c^2 + a^3*b^4*d^2 - 2*a^5*b^2*d^2 - 2*b^7*c^2*tan(e/2 + (f*x)/2) + 2*a^6*b*d^2*tan(e/2 + (f*x)/2) + 4*a^2*b^5*c^2*tan(e/2 + (f*x)/2) - 2*a^4*b^3*c^2*tan(e/2 + (f*x)/2) + 2*a^2*b^5*d^2*tan(e/2 + (f*x)/2) - 4*a^4*b^3*d^2*tan(e/2 + (f*x)/2)))*(b^2 - a^2)^(1/2)*2i)/(f*(a^3*d^3 - b^3*c^3 + a^2*b*c^3 - a*b^2*d^3 - a^3*c^2*d + b^3*c*d^2 + a*b^2*c^2*d - a^2*b*c*d^2)) - (b*c^2*atan((b^4*c^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(3/2)*3i - a^6*d^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2)*1i - a^3*b^3*d^2*(b^2 - a^2)^(1/2)*2i - b^6*c^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2)*1i - b^4*d^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(3/2)*4i + b^6*d^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2)*4i - b^4*c*d*(b^2 - a^2)^(3/2)*1i + b^6*c*d*(b^2 - a^2)^(1/2)*1i + a*b^3*c^2*(b^2 - a^2)^(3/2)*1i - a*b^3*d^2*(b^2 - a^2)^(3/2)*1i + a*b^5*d^2*(b^2 - a^2)^(1/2)*1i + a^5*b*d^2*(b^2 - a^2)^(1/2)*1i - a^2*b^2*c^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(3/2)*2i + a^2*b^4*c^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2)*2i - a^4*b^2*c^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2)*1i + a^2*b^2*d^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(3/2)*3i - a^2*b^4*d^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2)*9i + a^4*b^2*d^2*tan(e/2 + (f*x)/2)*(b^2 - a^2)^(1/2)*6i + a^2*b^2*c*d*(b^2 - a^2)^(3/2)*1i - a^2*b^4*c*d*(b^2 - a^2)^(1/2)*2i + a^4*b^2*c*d*(b^2 - a^2)^(1/2)*1i)/(a^7*d^2 - a*b^6*c^2 + 2*a^3*b^4*c^2 - a^5*b^2*c^2 + a^3*b^4*d^2 - 2*a^5*b^2*d^2 - 2*b^7*c^2*tan(e/2 + (f*x)/2) + 2*a^6*b*d^2*tan(e/2 + (f*x)/2) + 4*a^2*b^5*c^2*tan(e/2 + (f*x)/2) - 2*a^4*b^3*c^2*tan(e/2 + (f*x)/2) + 2*a^2*b^5*d^2*tan(e/2 + (f*x)/2) - 4*a^4*b^3*d^2*tan(e/2 + (f*x)/2)))*(b^2 - a^2)^(1/2)*2i)/(f*(a^3*d^3 - b^3*c^3 + a^2*b*c^3 - a*b^2*d^3 - a^3*c^2*d + b^3*c*d^2 + a*b^2*c^2*d - a^2*b*c*d^2)) + (a^2*d*atan((a^2*d^4*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*3i - b^2*c^3*d^3*(d^2 - c^2)^(1/2)*2i - a^2*d^6*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i - b^2*c^6*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i - b^2*d^4*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*4i + b^2*d^6*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*4i - a*b*d^4*(d^2 - c^2)^(3/2)*1i + a*b*d^6*(d^2 - c^2)^(1/2)*1i + a^2*c*d^3*(d^2 - c^2)^(3/2)*1i - b^2*c*d^3*(d^2 - c^2)^(3/2)*1i + b^2*c*d^5*(d^2 - c^2)^(1/2)*1i + b^2*c^5*d*(d^2 - c^2)^(1/2)*1i - a^2*c^2*d^2*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*2i + a^2*c^2*d^4*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - a^2*c^4*d^2*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i + b^2*c^2*d^2*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*3i - b^2*c^2*d^4*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*9i + b^2*c^4*d^2*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i + a*b*c^2*d^2*(d^2 - c^2)^(3/2)*1i - a*b*c^2*d^4*(d^2 - c^2)^(1/2)*2i + a*b*c^4*d^2*(d^2 - c^2)^(1/2)*1i)/(b^2*c^7 - a^2*c*d^6 + 2*a^2*c^3*d^4 - a^2*c^5*d^2 + b^2*c^3*d^4 - 2*b^2*c^5*d^2 - 2*a^2*d^7*tan(e/2 + (f*x)/2) + 2*b^2*c^6*d*tan(e/2 + (f*x)/2) + 4*a^2*c^2*d^5*tan(e/2 + (f*x)/2) - 2*a^2*c^4*d^3*tan(e/2 + (f*x)/2) + 2*b^2*c^2*d^5*tan(e/2 + (f*x)/2) - 4*b^2*c^4*d^3*tan(e/2 + (f*x)/2)))*(d^2 - c^2)^(1/2)*2i)/(f*(a^3*d^3 - b^3*c^3 + a^2*b*c^3 - a*b^2*d^3 - a^3*c^2*d + b^3*c*d^2 + a*b^2*c^2*d

```
- a^2*b*c*d^2)) - (b^2*d*atan((a^2*d^4*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)
*3i - b^2*c^3*d^3*(d^2 - c^2)^(1/2)*2i - a^2*d^6*tan(e/2 + (f*x)/2)*(d^2 -
c^2)^(1/2)*1i - b^2*c^6*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i - b^2*d^4*t
an(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*4i + b^2*d^6*tan(e/2 + (f*x)/2)*(d^2 -
c^2)^(1/2)*4i - a*b*d^4*(d^2 - c^2)^(3/2)*1i + a*b*d^6*(d^2 - c^2)^(1/2)*1i
 + a^2*c*d^3*(d^2 - c^2)^(3/2)*1i - b^2*c*d^3*(d^2 - c^2)^(3/2)*1i + b^2*c*
d^5*(d^2 - c^2)^(1/2)*1i + b^2*c^5*d*(d^2 - c^2)^(1/2)*1i - a^2*c^2*d^2*tan
(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*2i + a^2*c^2*d^4*tan(e/2 + (f*x)/2)*(d^2
- c^2)^(1/2)*2i - a^2*c^4*d^2*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i + b^2
*c^2*d^2*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*3i - b^2*c^2*d^4*tan(e/2 + (f
*x)/2)*(d^2 - c^2)^(1/2)*9i + b^2*c^4*d^2*tan(e/2 + (f*x)/2)*(d^2 - c^2)^(1
/2)*6i + a*b*c^2*d^2*(d^2 - c^2)^(3/2)*1i - a*b*c^2*d^4*(d^2 - c^2)^(1/2)*2
i + a*b*c^4*d^2*(d^2 - c^2)^(1/2)*1i)/(b^2*c^7 - a^2*c*d^6 + 2*a^2*c^3*d^4
- a^2*c^5*d^2 + b^2*c^3*d^4 - 2*b^2*c^5*d^2 - 2*a^2*d^7*tan(e/2 + (f*x)/2)
+ 2*b^2*c^6*d*tan(e/2 + (f*x)/2) + 4*a^2*c^2*d^5*tan(e/2 + (f*x)/2) - 2*a^2
*c^4*d^3*tan(e/2 + (f*x)/2) + 2*b^2*c^2*d^5*tan(e/2 + (f*x)/2) - 4*b^2*c^4*
d^3*tan(e/2 + (f*x)/2)))*(d^2 - c^2)^(1/2)*2i)/(f*(a^3*d^3 - b^3*c^3 + a^2*
b*c^3 - a*b^2*d^3 - a^3*c^2*d + b^3*c*d^2 + a*b^2*c^2*d - a^2*b*c*d^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.704 $\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=185

$$\frac{2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}\,(bc-ad)^2}+\frac{2d\left(acd-b\left(2c^2-d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{3/2}(bc-ad)^2}-\frac{d^2\cos(e+fx)}{f\left(c^2-d^2\right)(bc-ad)(c+d\sin(e+fx))}$$

```
[Out] 2*d*(a*c*d-b*(2*c^2-d^2))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/
(-a*d+b*c)^2/(c^2-d^2)^(3/2)/f-d^2*cos(f*x+e)/(-a*d+b*c)/(c^2-d^2)/f/(c+d*s
in(f*x+e))+2*b^2*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/(-a*d+b*c
)^2/f/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2802, 3001, 2660, 618, 204}

$$\frac{2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}\,(bc-ad)^2}+\frac{2d\left(acd-b\left(2c^2-d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{3/2}(bc-ad)^2}-\frac{d^2\cos(e+fx)}{f\left(c^2-d^2\right)(bc-ad)(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] (2*b^2*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*(
b*c - a*d)^2*f) + (2*d*(a*c*d - b*(2*c^2 - d^2))*ArcTan[(d + c*Tan[(e + f*x
)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^2*(c^2 - d^2)^(3/2)*f) - (d^2*Cos[e +
f*x])/((b*c - a*d)*(c^2 - d^2)*f*(c + d*Sin[e + f*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))^2}\,dx = -\frac{d^2\cos(e+fx)}{(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \frac{\int \frac{-acd+b\left(c^2-d^2\right)-bcd\sin(e}{(a+b\sin(e+fx))(c+d\sin(e}}{(bc-ad)\left(c^2-d^2\right.}$$

$$= -\frac{d^2\cos(e+fx)}{(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \frac{b^2\int \frac{1}{a+b\sin(e+fx)}\,dx}{(bc-ad)^2} +$$

$$= -\frac{d^2\cos(e+fx)}{(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{a+2bx+}\right.}{(bc}$$

$$= -\frac{d^2\cos(e+fx)}{(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))} - \frac{\left(4b^2\right)\text{Subst}\left(\int \frac{1}{-4\left(a^2-b\right.}\right.}{}$$

$$= \frac{2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(bc-ad)^2 f} + \frac{2d\left(acd-b\left(2c^2-d^2\right)\right)\tan^{-1}\left(\frac{d+c\tan}{\sqrt{}}\right.}{(bc-ad)^2\left(c^2-d^2\right)^{3/2} f}$$

Mathematica [A] time = 0.93, size = 165, normalized size = 0.89

$$\frac{\frac{2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{2d\left(acd+b\left(d^2-2c^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{3/2}} + \frac{d^2(ad-bc)\cos(e+fx)}{(c-d)(c+d)(c+d\sin(e+fx))}}{f(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] ((2*b^2*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] +
 (2*d*(a*c*d + b*(-2*c^2 + d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 -
 d^2]])/(c^2 - d^2)^(3/2) + (d^2*(-(b*c) + a*d)*Cos[e + f*x])/((c - d)*(c +
 d)*(c + d*Sin[e + f*x])))/((b*c - a*d)^2*f)
```

fricas [B] time = 174.52, size = 2882, normalized size = 15.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

[Out] [-1/2*((b^2*c^5 - 2*b^2*c^3*d^2 + b^2*c*d^4 + (b^2*c^4*d - 2*b^2*c^2*d^3 + b^2*d^5)*sin(f*x + e))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 + 2*(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)) + (2*(a^2*b - b^3)*c^3*d - (a^3 - a*b^2)*c^2*d^2 - (a^2*b - b^3)*c*d^3 + (2*(a^2*b - b^3)*c^2*d^2 - (a^3 - a*b^2)*c*d^3 - (a^2*b - b^3)*d^4)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 - 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*((a^2*b - b^3)*c^3*d^2 - (a^3 - a*b^2)*c^2*d^3 - (a^2*b - b^3)*c*d^4 + (a^3 - a*b^2)*d^5)*cos(f*x + e))/(((a^2*b^2 - b^4)*c^6*d - 2*(a^3*b - a*b^3)*c^5*d^2 + (a^4 - 3*a^2*b^2 + 2*b^4)*c^4*d^3 + 4*(a^3*b - a*b^3)*c^3*d^4 - (2*a^4 - 3*a^2*b^2 + b^4)*c^2*d^5 - 2*(a^3*b - a*b^3)*c*d^6 + (a^4 - a^2*b^2)*d^7)*f*sin(f*x + e) + ((a^2*b^2 - b^4)*c^7 - 2*(a^3*b - a*b^3)*c^6*d + (a^4 - 3*a^2*b^2 + 2*b^4)*c^5*d^2 + 4*(a^3*b - a*b^3)*c^4*d^3 - (2*a^4 - 3*a^2*b^2 + b^4)*c^3*d^4 - 2*(a^3*b - a*b^3)*c^2*d^5 + (a^4 - a^2*b^2)*c*d^6)*f), 1/2*(2*(2*(a^2*b - b^3)*c^3*d - (a^3 - a*b^2)*c^2*d^2 - (a^2*b - b^3)*c*d^3 + (2*(a^2*b - b^3)*c^2*d^2 - (a^3 - a*b^2)*c*d^3 - (a^2*b - b^3)*d^4)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) - (b^2*c^5 - 2*b^2*c^3*d^2 + b^2*c*d^4 + (b^2*c^4*d - 2*b^2*c^2*d^3 + b^2*d^5)*sin(f*x + e))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 + 2*(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)) - 2*((a^2*b - b^3)*c^3*d^2 - (a^3 - a*b^2)*c^2*d^3 - (a^2*b - b^3)*c*d^4 + (a^3 - a*b^2)*d^5)*cos(f*x + e))/(((a^2*b^2 - b^4)*c^6*d - 2*(a^3*b - a*b^3)*c^5*d^2 + (a^4 - 3*a^2*b^2 + 2*b^4)*c^4*d^3 + 4*(a^3*b - a*b^3)*c^3*d^4 - (2*a^4 - 3*a^2*b^2 + b^4)*c^2*d^5 - 2*(a^3*b - a*b^3)*c*d^6 + (a^4 - a^2*b^2)*d^7)*f*sin(f*x + e) + ((a^2*b^2 - b^4)*c^7 - 2*(a^3*b - a*b^3)*c^6*d + (a^4 - 3*a^2*b^2 + 2*b^4)*c^5*d^2 + 4*(a^3*b - a*b^3)*c^4*d^3 - (2*a^4 - 3*a^2*b^2 + b^4)*c^3*d^4 - 2*(a^3*b - a*b^3)*c^2*d^5 + (a^4 - a^2*b^2)*c*d^6)*f), -1/2*(2*(b^2*c^5 - 2*b^2*c^3*d^2 + b^2*c*d^4 + (b^2*c^4*d - 2*b^2*c^2*d^3 + b^2*d^5)*sin(f*x + e))*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))) + (2*(a^2*b - b^3)*c^3*d - (a^3 - a*b^2)*c^2*d^2 - (a^2*b - b^3)*c*d^3 + (2*(a^2*b - b^3)*c^2*d^2 - (a^3 - a*b^2)*c*d^3 - (a^2*b - b^3)*d^4)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 - 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*((a^2*b - b^3)*c^3*d^2 - (a^3 - a*b^2)*c^2*d^3 - (a^2*b - b^3)*c*d^4 + (a^3 - a*b^2)*d^5)*cos(f*x + e))/(((a^2*b^2 - b^4)*c^6*d - 2*(a^3*b - a*b^3)*c^5*d^2 + (a^4 - 3*a^2*b^2 + 2*b^4)*c^4*d^3 + 4*(a^3*b - a*b^3)*c^3*d^4 - (2*a^4 - 3*a^2*b^2 + b^4)*c^2*d^5 - 2*(a^3*b - a*b^3)*c*d^6 + (a^4 - a^2*b^2)*d^7)*f*sin(f*x + e) + ((a^2*b^2 - b^4)*c^7 - 2*(a^3*b - a*b^3)*c^6*d + (a^4 - 3*a^2*b^2 + 2*b^4)*c^5*d^2 + 4*(a^3*b - a*b^3)*c^4*d^3 - (2*a^4 - 3*a^2*b^2 + b^4)*c^3*d^4 - 2*(a^3*b - a*b^3)*c^2*d^5 + (a^4 - a^2*b^2)*c*d^6)*f), -((b^2*c^5 - 2*b^2*c^3*d^2 + b^2*c*d^

```
4 + (b^2*c^4*d - 2*b^2*c^2*d^3 + b^2*d^5)*sin(f*x + e))*sqrt(a^2 - b^2)*arc
tan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))) - (2*(a^2*b - b^3
)*c^3*d - (a^3 - a*b^2)*c^2*d^2 - (a^2*b - b^3)*c*d^3 + (2*(a^2*b - b^3)*c^
2*d^2 - (a^3 - a*b^2)*c*d^3 - (a^2*b - b^3)*d^4)*sin(f*x + e))*sqrt(c^2 - d
^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + ((a^2*b
- b^3)*c^3*d^2 - (a^3 - a*b^2)*c^2*d^3 - (a^2*b - b^3)*c*d^4 + (a^3 - a*b^2
)*d^5)*cos(f*x + e))/(((a^2*b^2 - b^4)*c^6*d - 2*(a^3*b - a*b^3)*c^5*d^2 +
(a^4 - 3*a^2*b^2 + 2*b^4)*c^4*d^3 + 4*(a^3*b - a*b^3)*c^3*d^4 - (2*a^4 - 3*
a^2*b^2 + b^4)*c^2*d^5 - 2*(a^3*b - a*b^3)*c*d^6 + (a^4 - a^2*b^2)*d^7)*f*s
in(f*x + e) + ((a^2*b^2 - b^4)*c^7 - 2*(a^3*b - a*b^3)*c^6*d + (a^4 - 3*a^2
*b^2 + 2*b^4)*c^5*d^2 + 4*(a^3*b - a*b^3)*c^4*d^3 - (2*a^4 - 3*a^2*b^2 + b^
4)*c^3*d^4 - 2*(a^3*b - a*b^3)*c^2*d^5 + (a^4 - a^2*b^2)*c*d^6)*f)]
```

giac [A] time = 0.22, size = 308, normalized size = 1.66

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^2}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{a^2-b^2}}-\frac{\left(2\,bc^2d-acd^2-bd^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(b^2c^4-2\,abc^3d+a^2c^2d^2-b^2c^2d^2+2\,abcd^3-a^2d^4\right)\sqrt{c^2-d^2}}-\frac{}{\left(bc^4-ac^3d-bc^2d^2+acd\right.}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e
) + b)/sqrt(a^2 - b^2)))*b^2/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a^2 - b^
2)) - (2*b*c^2*d - a*c*d^2 - b*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c
) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((b^2*c^4 - 2*a*b
*c^3*d + a^2*c^2*d^2 - b^2*c^2*d^2 + 2*a*b*c*d^3 - a^2*d^4)*sqrt(c^2 - d^2)
) - (d^3*tan(1/2*f*x + 1/2*e) + c*d^2)/((b*c^4 - a*c^3*d - b*c^2*d^2 + a*c*
d^3)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)))/f
```

maple [B] time = 0.34, size = 514, normalized size = 2.78

$$\frac{2d^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)a}{f\left(da-cb\right)^2\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d+c\right)c\left(c^2-d^2\right)}-\frac{2d^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)b}{f\left(da-cb\right)^2\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2/f*d^4/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/c/(c^
2-d^2)*tan(1/2*f*x+1/2*e)*a-2/f*d^3/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*c+2*t
```

```
an(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*tan(1/2*f*x+1/2*e)*b+2/f*d^3/(a*d-b*c)^2/(
tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*a-2/f*d^2/(a*d-b
*c)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*c*b+2/f*d
^2/(a*d-b*c)^2/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2
-d^2)^(1/2))*a*c-4/f*d/(a*d-b*c)^2/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*
f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b*c^2+2/f*d^3/(a*d-b*c)^2/(c^2-d^2)^(3/2)*
arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b+2/f*b^2/(a^2*d^2
-2*a*b*c*d+b^2*c^2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)
/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 22.74, size = 24122, normalized size = 130.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^2),x)
```

```
[Out] ((2*d^2)/((c^2 - d^2)*(a*d - b*c)) + (2*d^3*tan(e/2 + (f*x)/2))/(c*(c^2 - d
^2)*(a*d - b*c)))/(f*(c + 2*d*tan(e/2 + (f*x)/2) + c*tan(e/2 + (f*x)/2)^2))
 + (b^2*atan(((b^2*(b^2 - a^2)^(1/2)*((32*tan(e/2 + (f*x)/2)*(a^6*c^3*d^5 -
 a*b^5*c^8 + 4*a*b^5*c^2*d^6 - 13*a*b^5*c^4*d^4 + 12*a*b^5*c^6*d^2 - 4*a^2*
b^4*c*d^7 + a^2*b^4*c^7*d + a^4*b^2*c*d^7 + 2*a^5*b*c^2*d^6 - 5*a^5*b*c^4*d
^4 + 17*a^2*b^4*c^3*d^5 - 20*a^2*b^4*c^5*d^3 - 5*a^3*b^3*c^2*d^6 + 14*a^3*b
^3*c^4*d^4 - 4*a^3*b^3*c^6*d^2 - 8*a^4*b^2*c^3*d^5 + 8*a^4*b^2*c^5*d^3))/(a
^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^
2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 +
 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) - (32*(2*a*b^5*c^5*d^3 - a*b^5*c^3*d^5 + a^
3*b^3*c*d^7 + a^5*b*c^3*d^5 + 2*a^2*b^4*c^4*d^4 - 3*a^2*b^4*c^6*d^2 - 6*a^3
*b^3*c^3*d^5 + 8*a^3*b^3*c^5*d^3 + 2*a^4*b^2*c^2*d^6 - 5*a^4*b^2*c^4*d^4 -
a*b^5*c^7*d))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^
4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3
*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) + (b^2*(b^2 - a^2)^(1/2)*((
32*(a^2*b^5*c^10 + a^7*c^3*d^7 - a^7*c^5*d^5 + a*b^6*c^5*d^5 - 3*a*b^6*c^7*
```

d^3 - 5*a^3*b^4*c^9*d + a^5*b^2*c*d^9 + a^6*b*c^2*d^8 - 6*a^6*b*c^4*d^6 + 5*a^6*b*c^6*d^4 - 4*a^2*b^5*c^4*d^6 + 13*a^2*b^5*c^6*d^4 - 10*a^2*b^5*c^8*d^2 + 6*a^3*b^4*c^3*d^7 - 22*a^3*b^4*c^5*d^5 + 21*a^3*b^4*c^7*d^3 - 4*a^4*b^3*c^2*d^8 + 18*a^4*b^3*c^4*d^6 - 24*a^4*b^3*c^6*d^4 + 10*a^4*b^3*c^8*d^2 - 7*a^5*b^2*c^3*d^7 + 16*a^5*b^2*c^5*d^5 - 10*a^5*b^2*c^7*d^3 + 2*a*b^6*c^9*d))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) + (32*tan(e/2 + (f*x)/2)*(2*a*b^6*c^10 + 2*a^7*c^2*d^8 - 2*a^7*c^4*d^6 - 2*a*b^6*c^8*d^2 - 6*a^2*b^5*c^9*d - 12*a^6*b*c^3*d^7 + 10*a^6*b*c^5*d^5 + 2*a^2*b^5*c^5*d^5 + 4*a^2*b^5*c^7*d^3 - 8*a^3*b^4*c^4*d^6 + 6*a^3*b^4*c^6*d^4 + 2*a^3*b^4*c^8*d^2 + 12*a^4*b^3*c^3*d^7 - 24*a^4*b^3*c^5*d^5 + 12*a^4*b^3*c^7*d^3 - 8*a^5*b^2*c^2*d^8 + 26*a^5*b^2*c^4*d^6 - 18*a^5*b^2*c^6*d^4 + 2*a^6*b*c*d^9))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) + (b^2*(b^2 - a^2)^(1/2)*((32*(a^2*b^6*c^12 - a^8*c^2*d^10 + 2*a^8*c^4*d^8 - a^8*c^6*d^6 - a*b^7*c^7*d^5 + 2*a*b^7*c^9*d^3 - 4*a^3*b^5*c^11*d + 2*a^7*b*c^3*d^9 - 7*a^7*b*c^5*d^7 + 4*a^7*b*c^7*d^5 + 4*a^2*b^6*c^6*d^6 - 7*a^2*b^6*c^8*d^4 + 2*a^2*b^6*c^10*d^2 - 5*a^3*b^5*c^5*d^7 + 6*a^3*b^5*c^7*d^5 + 3*a^3*b^5*c^9*d^3 + 5*a^4*b^4*c^6*d^6 - 10*a^4*b^4*c^8*d^4 + 5*a^4*b^4*c^10*d^2 + 5*a^5*b^3*c^3*d^9 - 10*a^5*b^3*c^5*d^7 + 5*a^5*b^3*c^7*d^5 - 4*a^6*b^2*c^2*d^10 + 3*a^6*b^2*c^4*d^8 + 6*a^6*b^2*c^6*d^6 - 5*a^6*b^2*c^8*d^4 - a*b^7*c^11*d + a^7*b*c*d^11))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) + (32*tan(e/2 + (f*x)/2)*(3*a*b^7*c^12 - 3*a^8*c*d^11 - 2*a^3*b^5*c^12 + 8*a^8*c^3*d^9 - 7*a^8*c^5*d^7 + 2*a^8*c^7*d^5 - 4*a*b^7*c^6*d^6 + 11*a*b^7*c^8*d^4 - 10*a*b^7*c^10*d^2 - 15*a^2*b^6*c^11*d + 10*a^4*b^4*c^11*d + 4*a^6*b^2*c*d^11 + 15*a^7*b*c^2*d^10 - 40*a^7*b*c^4*d^8 + 35*a^7*b*c^6*d^6 - 10*a^7*b*c^8*d^4 + 20*a^2*b^6*c^5*d^7 - 55*a^2*b^6*c^7*d^5 + 50*a^2*b^6*c^9*d^3 - 40*a^3*b^5*c^4*d^8 + 113*a^3*b^5*c^6*d^6 - 108*a^3*b^5*c^8*d^4 + 37*a^3*b^5*c^10*d^2 + 40*a^4*b^4*c^3*d^9 - 125*a^4*b^4*c^5*d^7 + 140*a^4*b^4*c^7*d^5 - 65*a^4*b^4*c^9*d^3 - 20*a^5*b^3*c^2*d^10 + 85*a^5*b^3*c^4*d^8 - 130*a^5*b^3*c^6*d^6 + 85*a^5*b^3*c^8*d^4 - 20*a^5*b^3*c^10*d^2 - 41*a^6*b^2*c^3*d^9 + 90*a^6*b^2*c^5*d^7 - 73*a^6*b^2*c^7*d^5 + 20*a^6*b^2*c^9*d^3))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6)))/(a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d)))/(a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d))*1i)/(a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d) - (b^2*(b^2 - a^2)^(1/2)*((32*(2*a*b^5*c^5*d^3 - a*b^5*c^3*d^5 + a^3*b^3*c*d^7 + a^5*b*c^3*d^5 + 2*a^2*b^4*c^4*d^4 - 3*a^2*b^4*c^6*d^2 - 6*a^3*b^3*c^3*d^5 + 8*a^3*b^3*c^5*d^3 + 2*a^4*b^2*c^2*d^6 - 5*a^4*b^2*c^4*d^4 - a*b^5*c^7*d))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4* d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) - (32*tan(e/2 + (f*x)/2)*(a^6*c^3*d^5 - a*b^5*c^8 + 4*a*b^5*c^2*d^6 - 13*a*b^5*c^4*d^4 + 12*a*b^5*c^6*d^2 - 4*a^2*b^4*c*d^7 + a^2*b^4*c^7*d + a^4*b^2*c*d^7 + 2*a^5*b*c^2*d^6 - 5*a^5*b*c^4*d^4 + 17*a^2*b^4*c^3*d^5 - 20*a^2*b^4*c^5*d^3 - 5*a^3*b^3*c^2*d^6 + 14*a^3*b^3*c^4*d^4 - 4*a^3*b^3*c^6*d^2 - 8*a^4*b^2*c^3*d^5 + 8*a^4*b^2*c^5*d^3))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) + (b^2*(b^2 - a^2)^(1/2)*((32*(a^2*b^5*c^10 + a^7*c^3*d^7 - a^7*c^5*d^5 + a*b^6*c^5*d^5 - 3*a*b^6*c^7*d^3 - 5*a^3*b^4*c^9*d + a^5*b^2*c*d^9 + a^6*b*c^2*d^8 - 6*a^6*b*c^4*d^6 + 5*a^6*b*c^6*d^4 - 4*a^2*b^5*c^4*d^6 + 13*a^2*b^5*c^6*d^4 - 10*a^2*b^5*c^8*d^2 + 6*a^3*b^4*c^3*d^7 - 22*a^3*b^4*c^5*d^5 + 21*a^3*b^4*c^7*d^3 - 4*a^4*b^3*c^2*d^8 + 18*a^4*b^3*c^4*d^6 - 24*a^4*b^3*c^6*d^4 + 10*a^4*b^3*c^8*d^2 - 7*a^5*b^2*c^3*d^7 + 16*a^5*b^2*c^5*d^5 - 10*a^5*b^2*c^7*d^3 + 2*a*b^6*c^9*d))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) + (32*tan(e/2 + (f*x)/2)*(2*a*b^6*c^10 + 2*a^7*c^2*d^8 - 2*a^7*c^4*d^6 - 2*a*b^6*c^8*d^2 - 6*a^2*b^5*c^9*d - 12*a^6*b*c^3*d^7 + 10*a^6*b*c^5*d^5 + 2*a^2*b^5*c^5*d^5 + 4*a^2*b^5*c^7*d^3 - 8*a^3*b^4*c^4*d^6 + 6*a^3*b^4*c^6*d^4 + 2*a^3*b^4*c^8*d^2 + 12*a^4*b^3*c^3*d^7 - 24*a^4*b^3*c^5*d^5 + 12*a^4*b^3*c^7*d^3 - 8*a^5*b^2*c^2*d^8 + 26*a^5*b^2*c^4*d^6 - 18*a^5*b^2*c^6*d^4 + 2*a^6*b*c*d^9))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) - (b^2*(b^2 - a^2)^(1/2)*((32*(a^2*b^6*c^12 - a^8*c^2*d^10 + 2*a^8*c^4*d^8 - a^8*c^6*d^6 - a*b^7*c^7*d^5 + 2*a*b^7*c^9*d^3 - 4*a^3*b^5*c^11*d + 2*a^7*b*c^3*d^9 - 7*a^7*b*c^5*d^7 + 4*a^7*b*c^7*d^5 + 4*a^2*b^6*c^6*d^6 - 7*a^2*b^6*c^8*d^4 + 2*a^2*b^6*c^10*d^2 - 5*a^3*b^5*c^5*d^7 + 6*a^3*b^5*c^7*d^5 + 3*a^3*b^5*c^9*d^3 + 5*a^4*b^4*c^6*d^6 - 10*a^4*b^4*c^8*d^4 + 5*a^4*b^4*c^10*d^2 + 5*a^5*b^3*c^3*d^9 - 10*a^5*b^3*c^5*d^7 + 5*a^5*b^3*c^7*d^5 - 4*a^6*b^2*c^2*d^10 + 3*a^6*b^2*c^4*d^8 + 6*a^6*b^2*c^6*d^6 - 5*a^6*b^2*c^8*d^4 - a*b^7*c^11*d + a^7*b*c*d^11))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) + (32*tan(e/2 + (f*x)/2)*(3*a*b^7*c^12 - 3*a^8*c*d^11 - 2*a^3*b^5*c^12 + 8*a^8*c^3*d^9 - 7*a^8*c^5*d^7 + 2*a^8*c^7*d^5 - 4*a*b^7*c^6*d^6 + 11*a*b^7*c^8*d^4 - 10*a*b^7*c^10*d^2 - 15*a^2*b^6*c^11*d + 10*a^4*b^4*c^11*d + 4*a^6*b^2*c*d^11 + 15*a^7*b*c^2*d^10 - 40*a^7*b*c^4*d^8 + 35*a^7*b*c^6*d^6 - 10*a^7*b*c^8*d^4 + 20*a^2*b^6*c^5*d^7 - 55*a^2*b^6*c^7*d^5 + 50*a^2*b^6*c^9*d^3 - 40*a^3*b^5*c^4*d^8 + 113*a^3*b^5*c^6*d^6 - 108*a^3*b^5*c^8*d^4 + 37*a^3*b^5*c^10*d^2 + 40*a^4*b^4*c^3*d^9 - 125*a^4*b^4*c^5*d^7 + 140*a^4*b^4*c^7*d^5 - 65*a^4*b^4*c^9*d^3 - 20*a^5*b^3*c^2*d^10 + 85*a^5*b^3*c^4*d^8 - 130*a^5*b^3*c^6*d^6 + 85*a^5*b^3*c^8*d^4 - 20*a^5*b^3*c^10*d^2 - 41*a^6*b^2*c^3*d^9 + 90*a^6*b^2*c^5*d^7 - 73*a^6*b^2*c^7*d^5 + 20*a^6*b^2*c^9*d^3))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*

b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b
*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6)))/(a^4*d^2 - b^4*c^2 + a^2*b^2*c^
2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d)))/(a^4*d^2 - b^4*c^2 + a^2*b^2
*c^2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d))*1i)/(a^4*d^2 - b^4*c^2 + a
^2*b^2*c^2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d))/((64*(2*a^2*b^3*c^2*
d^4 - 3*a*b^4*c^3*d^3 - 3*a^2*b^3*c^4*d^2 + a^3*b^2*c^3*d^3 + a*b^4*c*d^5 +
2*a*b^4*c^5*d))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3
*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4
- 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) + (64*tan(e/2 + (f*x)/2)
*(2*a*b^4*c^2*d^4 - 4*a*b^4*c^4*d^2 + 2*a^2*b^3*c^3*d^3))/(a^3*d^7 - b^3*c^
7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2
*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d
- 3*a^2*b*c*d^6) - (b^2*(b^2 - a^2)^(1/2)*((32*tan(e/2 + (f*x)/2)*(a^6*c^3*
d^5 - a*b^5*c^8 + 4*a*b^5*c^2*d^6 - 13*a*b^5*c^4*d^4 + 12*a*b^5*c^6*d^2 - 4
*a^2*b^4*c*d^7 + a^2*b^4*c^7*d + a^4*b^2*c*d^7 + 2*a^5*b*c^2*d^6 - 5*a^5*b*
c^4*d^4 + 17*a^2*b^4*c^3*d^5 - 20*a^2*b^4*c^5*d^3 - 5*a^3*b^3*c^2*d^6 + 14*
a^3*b^3*c^4*d^4 - 4*a^3*b^3*c^6*d^2 - 8*a^4*b^2*c^3*d^5 + 8*a^4*b^2*c^5*d^3
))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c
^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*
d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) - (32*(2*a*b^5*c^5*d^3 - a*b^5*c^3*d^5
+ a^3*b^3*c*d^7 + a^5*b*c^3*d^5 + 2*a^2*b^4*c^4*d^4 - 3*a^2*b^4*c^6*d^2 -
6*a^3*b^3*c^3*d^5 + 8*a^3*b^3*c^5*d^3 + 2*a^4*b^2*c^2*d^6 - 5*a^4*b^2*c^4*d
^4 - a*b^5*c^7*d))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c
^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^
4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) + (b^2*(b^2 - a^2)^(1/
2)*((32*(a^2*b^5*c^10 + a^7*c^3*d^7 - a^7*c^5*d^5 + a*b^6*c^5*d^5 - 3*a*b^6
*c^7*d^3 - 5*a^3*b^4*c^9*d + a^5*b^2*c*d^9 + a^6*b*c^2*d^8 - 6*a^6*b*c^4*d^
6 + 5*a^6*b*c^6*d^4 - 4*a^2*b^5*c^4*d^6 + 13*a^2*b^5*c^6*d^4 - 10*a^2*b^5*c
^8*d^2 + 6*a^3*b^4*c^3*d^7 - 22*a^3*b^4*c^5*d^5 + 21*a^3*b^4*c^7*d^3 - 4*a^
4*b^3*c^2*d^8 + 18*a^4*b^3*c^4*d^6 - 24*a^4*b^3*c^6*d^4 + 10*a^4*b^3*c^8*d^
2 - 7*a^5*b^2*c^3*d^7 + 16*a^5*b^2*c^5*d^5 - 10*a^5*b^2*c^7*d^3 + 2*a*b^6*c
^9*d))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b
^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*
c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) + (32*tan(e/2 + (f*x)/2)*(2*a*b^6*
c^10 + 2*a^7*c^2*d^8 - 2*a^7*c^4*d^6 - 2*a*b^6*c^8*d^2 - 6*a^2*b^5*c^9*d -
12*a^6*b*c^3*d^7 + 10*a^6*b*c^5*d^5 + 2*a^2*b^5*c^5*d^5 + 4*a^2*b^5*c^7*d^3
- 8*a^3*b^4*c^4*d^6 + 6*a^3*b^4*c^6*d^4 + 2*a^3*b^4*c^8*d^2 + 12*a^4*b^3*c
^3*d^7 - 24*a^4*b^3*c^5*d^5 + 12*a^4*b^3*c^7*d^3 - 8*a^5*b^2*c^2*d^8 + 26*a
^5*b^2*c^4*d^6 - 18*a^5*b^2*c^6*d^4 + 2*a^6*b*c*d^9))/(a^3*d^7 - b^3*c^7 -
2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5
- 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*
a^2*b*c*d^6) + (b^2*(b^2 - a^2)^(1/2)*((32*(a^2*b^6*c^12 - a^8*c^2*d^10 + 2
*a^8*c^4*d^8 - a^8*c^6*d^6 - a*b^7*c^7*d^5 + 2*a*b^7*c^9*d^3 - 4*a^3*b^5*c^
11*d + 2*a^7*b*c^3*d^9 - 7*a^7*b*c^5*d^7 + 4*a^7*b*c^7*d^5 + 4*a^2*b^6*c^6*
d^6 - 7*a^2*b^6*c^8*d^4 + 2*a^2*b^6*c^10*d^2 - 5*a^3*b^5*c^5*d^7 + 6*a^3*b^

5*c^7*d^5 + 3*a^3*b^5*c^9*d^3 + 5*a^4*b^4*c^6*d^6 - 10*a^4*b^4*c^8*d^4 + 5*a^4*b^4*c^10*d^2 + 5*a^5*b^3*c^3*d^9 - 10*a^5*b^3*c^5*d^7 + 5*a^5*b^3*c^7*d^5 - 4*a^6*b^2*c^2*d^10 + 3*a^6*b^2*c^4*d^8 + 6*a^6*b^2*c^6*d^6 - 5*a^6*b^2*c^8*d^4 - a*b^7*c^11*d + a^7*b*c*d^11))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) + (32*tan(e/2 + (f*x)/2)*(3*a*b^7*c^12 - 3*a^8*c*d^11 - 2*a^3*b^5*c^12 + 8*a^8*c^3*d^9 - 7*a^8*c^5*d^7 + 2*a^8*c^7*d^5 - 4*a*b^7*c^6*d^6 + 11*a*b^7*c^8*d^4 - 10*a*b^7*c^10*d^2 - 15*a^2*b^6*c^11*d + 10*a^4*b^4*c^11*d + 4*a^6*b^2*c*d^11 + 15*a^7*b*c^2*d^10 - 40*a^7*b*c^4*d^8 + 35*a^7*b*c^6*d^6 - 10*a^7*b*c^8*d^4 + 20*a^2*b^6*c^5*d^7 - 55*a^2*b^6*c^7*d^5 + 50*a^2*b^6*c^9*d^3 - 40*a^3*b^5*c^4*d^8 + 113*a^3*b^5*c^6*d^6 - 108*a^3*b^5*c^8*d^4 + 37*a^3*b^5*c^10*d^2 + 40*a^4*b^4*c^3*d^9 - 125*a^4*b^4*c^5*d^7 + 140*a^4*b^4*c^7*d^5 - 65*a^4*b^4*c^9*d^3 - 20*a^5*b^3*c^2*d^10 + 85*a^5*b^3*c^4*d^8 - 130*a^5*b^3*c^6*d^6 + 85*a^5*b^3*c^8*d^4 - 20*a^5*b^3*c^10*d^2 - 41*a^6*b^2*c^3*d^9 + 90*a^6*b^2*c^5*d^7 - 73*a^6*b^2*c^7*d^5 + 20*a^6*b^2*c^9*d^3))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6)))/(a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d)))/(a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d)))/(a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d) - (b^2*(b^2 - a^2)^(1/2)*((32*(2*a*b^5*c^5*d^3 - a*b^5*c^3*d^5 + a^3*b^3*c*d^7 + a^5*b*c^3*d^5 + 2*a^2*b^4*c^4*d^4 - 3*a^2*b^4*c^6*d^2 - 6*a^3*b^3*c^3*d^5 + 8*a^3*b^3*c^5*d^3 + 2*a^4*b^2*c^2*d^6 - 5*a^4*b^2*c^4*d^4 - a*b^5*c^7*d))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) - (32*tan(e/2 + (f*x)/2)*(a^6*c^3*d^5 - a*b^5*c^8 + 4*a*b^5*c^2*d^6 - 13*a*b^5*c^4*d^4 + 12*a*b^5*c^6*d^2 - 4*a^2*b^4*c*d^7 + a^2*b^4*c^7*d + a^4*b^2*c*d^7 + 2*a^5*b*c^2*d^6 - 5*a^5*b*c^4*d^4 + 17*a^2*b^4*c^3*d^5 - 20*a^2*b^4*c^5*d^3 - 5*a^3*b^3*c^2*d^6 + 14*a^3*b^3*c^4*d^4 - 4*a^3*b^3*c^6*d^2 - 8*a^4*b^2*c^3*d^5 + 8*a^4*b^2*c^5*d^3))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) + (b^2*(b^2 - a^2)^(1/2)*((32*(a^2*b^5*c^10 + a^7*c^3*d^7 - a^7*c^5*d^5 + a*b^6*c^5*d^5 - 3*a*b^6*c^7*d^3 - 5*a^3*b^4*c^9*d + a^5*b^2*c*d^9 + a^6*b*c^2*d^8 - 6*a^6*b*c^4*d^6 + 5*a^6*b*c^6*d^4 - 4*a^2*b^5*c^4*d^6 + 13*a^2*b^5*c^6*d^4 - 10*a^2*b^5*c^8*d^2 + 6*a^3*b^4*c^3*d^7 - 22*a^3*b^4*c^5*d^5 + 21*a^3*b^4*c^7*d^3 - 4*a^4*b^3*c^2*d^8 + 18*a^4*b^3*c^4*d^6 - 24*a^4*b^3*c^6*d^4 + 10*a^4*b^3*c^8*d^2 - 7*a^5*b^2*c^3*d^7 + 16*a^5*b^2*c^5*d^5 - 10*a^5*b^2*c^7*d^3 + 2*a*b^6*c^9*d))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) + (32*tan(e/2 + (f*x)/2)*(2*a*b^6*c^10 + 2*a^7*c^2*d^8 - 2*a^7*c^4*d^6 - 2*a*b^6*c^8*d^2 - 6*a^2*b^5*c^9*d - 12*a^6*b*c^3*d^7 + 10*a^6*b*c^5*d^5 + 2*a^2*b^5*c^5*d^5 +

4*a^2*b^5*c^7*d^3 - 8*a^3*b^4*c^4*d^6 + 6*a^3*b^4*c^6*d^4 + 2*a^3*b^4*c^8*d^2 + 12*a^4*b^3*c^3*d^7 - 24*a^4*b^3*c^5*d^5 + 12*a^4*b^3*c^7*d^3 - 8*a^5*b^2*c^2*d^8 + 26*a^5*b^2*c^4*d^6 - 18*a^5*b^2*c^6*d^4 + 2*a^6*b*c*d^9))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) - (b^2*(b^2 - a^2)^(1/2)*((32*(a^2*b^6*c^12 - a^8*c^2*d^10 + 2*a^8*c^4*d^8 - a^8*c^6*d^6 - a*b^7*c^7*d^5 + 2*a*b^7*c^9*d^3 - 4*a^3*b^5*c^11*d + 2*a^7*b*c^3*d^9 - 7*a^7*b*c^5*d^7 + 4*a^7*b*c^7*d^5 + 4*a^2*b^6*c^6*d^6 - 7*a^2*b^6*c^8*d^4 + 2*a^2*b^6*c^10*d^2 - 5*a^3*b^5*c^5*d^7 + 6*a^3*b^5*c^7*d^5 + 3*a^3*b^5*c^9*d^3 + 5*a^4*b^4*c^6*d^6 - 10*a^4*b^4*c^8*d^4 + 5*a^4*b^4*c^10*d^2 + 5*a^5*b^3*c^3*d^9 - 10*a^5*b^3*c^5*d^7 + 5*a^5*b^3*c^7*d^5 - 4*a^6*b^2*c^2*d^10 + 3*a^6*b^2*c^4*d^8 + 6*a^6*b^2*c^6*d^6 - 5*a^6*b^2*c^8*d^4 - a*b^7*c^11*d + a^7*b*c*d^11))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) + (32*tan(e/2 + (f*x)/2)*(3*a*b^7*c^12 - 3*a^8*c*d^11 - 2*a^3*b^5*c^12 + 8*a^8*c^3*d^9 - 7*a^8*c^5*d^7 + 2*a^8*c^7*d^5 - 4*a*b^7*c^6*d^6 + 11*a*b^7*c^8*d^4 - 10*a*b^7*c^10*d^2 - 15*a^2*b^6*c^11*d + 10*a^4*b^4*c^11*d + 4*a^6*b^2*c*d^11 + 15*a^7*b*c^2*d^10 - 40*a^7*b*c^4*d^8 + 35*a^7*b*c^6*d^6 - 10*a^7*b*c^8*d^4 + 20*a^2*b^6*c^5*d^7 - 55*a^2*b^6*c^7*d^5 + 50*a^2*b^6*c^9*d^3 - 40*a^3*b^5*c^4*d^8 + 113*a^3*b^5*c^6*d^6 - 108*a^3*b^5*c^8*d^4 + 37*a^3*b^5*c^10*d^2 + 40*a^4*b^4*c^3*d^9 - 125*a^4*b^4*c^5*d^7 + 140*a^4*b^4*c^7*d^5 - 65*a^4*b^4*c^9*d^3 - 20*a^5*b^3*c^2*d^10 + 85*a^5*b^3*c^4*d^8 - 130*a^5*b^3*c^6*d^6 + 85*a^5*b^3*c^8*d^4 - 20*a^5*b^3*c^10*d^2 - 41*a^6*b^2*c^3*d^9 + 90*a^6*b^2*c^5*d^7 - 73*a^6*b^2*c^7*d^5 + 20*a^6*b^2*c^9*d^3))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6)))/(a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d)))/(a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d)))/(a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d)))*(b^2 - a^2)^(1/2)*2i)/(f*(a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d)) + (d*atan(((d*(-(c + d)^3*(c - d)^3)^(1/2)*((32*tan(e/2 + (f*x)/2)*(a^6*c^3*d^5 - a*b^5*c^8 + 4*a*b^5*c^2*d^6 - 13*a*b^5*c^4*d^4 + 12*a*b^5*c^6*d^2 - 4*a^2*b^4*c*d^7 + a^2*b^4*c^7*d + a^4*b^2*c*d^7 + 2*a^5*b*c^2*d^6 - 5*a^5*b*c^4*d^4 + 17*a^2*b^4*c^3*d^5 - 20*a^2*b^4*c^5*d^3 - 5*a^3*b^3*c^2*d^6 + 14*a^3*b^3*c^4*d^4 - 4*a^3*b^3*c^6*d^2 - 8*a^4*b^2*c^3*d^5 + 8*a^4*b^2*c^5*d^3))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) - (32*(2*a*b^5*c^5*d^3 - a*b^5*c^3*d^5 + a^3*b^3*c*d^7 + a^5*b*c^3*d^5 + 2*a^2*b^4*c^4*d^4 - 3*a^2*b^4*c^6*d^2 - 6*a^3*b^3*c^3*d^5 + 8*a^3*b^3*c^5*d^3 + 2*a^4*b^2*c^2*d^6 - 5*a^4*b^2*c^4*d^4 - a*b^5*c^7*d))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) + (d*(-(c + d)

^3*(c - d)^3)^(1/2)*((32*(a^2*b^5*c^10 + a^7*c^3*d^7 - a^7*c^5*d^5 + a*b^6*c^5*d^5 - 3*a*b^6*c^7*d^3 - 5*a^3*b^4*c^9*d + a^5*b^2*c*d^9 + a^6*b*c^2*d^8 - 6*a^6*b*c^4*d^6 + 5*a^6*b*c^6*d^4 - 4*a^2*b^5*c^4*d^6 + 13*a^2*b^5*c^6*d^4 - 10*a^2*b^5*c^8*d^2 + 6*a^3*b^4*c^3*d^7 - 22*a^3*b^4*c^5*d^5 + 21*a^3*b^4*c^7*d^3 - 4*a^4*b^3*c^2*d^8 + 18*a^4*b^3*c^4*d^6 - 24*a^4*b^3*c^6*d^4 + 10*a^4*b^3*c^8*d^2 - 7*a^5*b^2*c^3*d^7 + 16*a^5*b^2*c^5*d^5 - 10*a^5*b^2*c^7*d^3 + 2*a*b^6*c^9*d))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) + (32*tan(e/2 + (f*x)/2)*(2*a*b^6*c^10 + 2*a^7*c^2*d^8 - 2*a^7*c^4*d^6 - 2*a*b^6*c^8*d^2 - 6*a^2*b^5*c^9*d - 12*a^6*b*c^3*d^7 + 10*a^6*b*c^5*d^5 + 2*a^2*b^5*c^5*d^5 + 4*a^2*b^5*c^7*d^3 - 8*a^3*b^4*c^4*d^6 + 6*a^3*b^4*c^6*d^4 + 2*a^3*b^4*c^8*d^2 + 12*a^4*b^3*c^3*d^7 - 24*a^4*b^3*c^5*d^5 + 12*a^4*b^3*c^7*d^3 - 8*a^5*b^2*c^2*d^8 + 26*a^5*b^2*c^4*d^6 - 18*a^5*b^2*c^6*d^4 + 2*a^6*b*c*d^9))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) + (d*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(a^2*b^6*c^12 - a^8*c^2*d^10 + 2*a^8*c^4*d^8 - a^8*c^6*d^6 - a*b^7*c^7*d^5 + 2*a*b^7*c^9*d^3 - 4*a^3*b^5*c^11*d + 2*a^7*b*c^3*d^9 - 7*a^7*b*c^5*d^7 + 4*a^7*b*c^7*d^5 + 4*a^2*b^6*c^6*d^6 - 7*a^2*b^6*c^8*d^4 + 2*a^2*b^6*c^10*d^2 - 5*a^3*b^5*c^5*d^7 + 6*a^3*b^5*c^7*d^5 + 3*a^3*b^5*c^9*d^3 + 5*a^4*b^4*c^6*d^6 - 10*a^4*b^4*c^8*d^4 + 5*a^4*b^4*c^10*d^2 + 5*a^5*b^3*c^3*d^9 - 10*a^5*b^3*c^5*d^7 + 5*a^5*b^3*c^7*d^5 - 4*a^6*b^2*c^2*d^10 + 3*a^6*b^2*c^4*d^8 + 6*a^6*b^2*c^6*d^6 - 5*a^6*b^2*c^8*d^4 - a*b^7*c^11*d + a^7*b*c*d^11))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) + (32*tan(e/2 + (f*x)/2)*(3*a*b^7*c^12 - 3*a^8*c*d^11 - 2*a^3*b^5*c^12 + 8*a^8*c^3*d^9 - 7*a^8*c^5*d^7 + 2*a^8*c^7*d^5 - 4*a*b^7*c^6*d^6 + 11*a*b^7*c^8*d^4 - 10*a*b^7*c^10*d^2 - 15*a^2*b^6*c^11*d + 10*a^4*b^4*c^11*d + 4*a^6*b^2*c*d^11 + 15*a^7*b*c^2*d^10 - 40*a^7*b*c^4*d^8 + 35*a^7*b*c^6*d^6 - 10*a^7*b*c^8*d^4 + 20*a^2*b^6*c^5*d^7 - 55*a^2*b^6*c^7*d^5 + 50*a^2*b^6*c^9*d^3 - 40*a^3*b^5*c^4*d^8 + 113*a^3*b^5*c^6*d^6 - 108*a^3*b^5*c^8*d^4 + 37*a^3*b^5*c^10*d^2 + 40*a^4*b^4*c^3*d^9 - 125*a^4*b^4*c^5*d^7 + 140*a^4*b^4*c^7*d^5 - 65*a^4*b^4*c^9*d^3 - 20*a^5*b^3*c^2*d^10 + 85*a^5*b^3*c^4*d^8 - 130*a^5*b^3*c^6*d^6 + 85*a^5*b^3*c^8*d^4 - 20*a^5*b^3*c^10*d^2 - 41*a^6*b^2*c^3*d^9 + 90*a^6*b^2*c^5*d^7 - 73*a^6*b^2*c^7*d^5 + 20*a^6*b^2*c^9*d^3))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6))*(b*d^2 - 2*b*c^2 + a*c*d))/(a^2*d^8 - b^2*c^8 - 3*a^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3*b^2*c^4*d^4 + 3*b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 - 6*a*b*c^5*d^3))*(b*d^2 - 2*b*c^2 + a*c*d))/(a^2*d^8 - b^2*c^8 - 3*a^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3*b^2*c^4*d^4 + 3*b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 - 6*a*b*c^5*d^3))*(b*d^2 - 2*b*c^2 + a*c*d)*1i)/(a^2*d^8 - b^2*c^8 - 3*a^2*c^2*d^6 + 3*a^2

*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3*b^2*c^4*d^4 + 3*b^2*c^6*d^2 - 2*a*
b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 - 6*a*b*c^5*d^3) - (d*(-(c + d)^3*(c
- d)^3)^(1/2)*((32*(2*a*b^5*c^5*d^3 - a*b^5*c^3*d^5 + a^3*b^3*c*d^7 + a^5*b
*c^3*d^5 + 2*a^2*b^4*c^4*d^4 - 3*a^2*b^4*c^6*d^2 - 6*a^3*b^3*c^3*d^5 + 8*a^
3*b^3*c^5*d^3 + 2*a^4*b^2*c^2*d^6 - 5*a^4*b^2*c^4*d^4 - a*b^5*c^7*d))/(a^3*
d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 +
3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*
a*b^2*c^6*d - 3*a^2*b*c*d^6) - (32*tan(e/2 + (f*x)/2)*(a^6*c^3*d^5 - a*b^5*
c^8 + 4*a*b^5*c^2*d^6 - 13*a*b^5*c^4*d^4 + 12*a*b^5*c^6*d^2 - 4*a^2*b^4*c*d
^7 + a^2*b^4*c^7*d + a^4*b^2*c*d^7 + 2*a^5*b*c^2*d^6 - 5*a^5*b*c^4*d^4 + 17
*a^2*b^4*c^3*d^5 - 20*a^2*b^4*c^5*d^3 - 5*a^3*b^3*c^2*d^6 + 14*a^3*b^3*c^4*
d^4 - 4*a^3*b^3*c^6*d^2 - 8*a^4*b^2*c^3*d^5 + 8*a^4*b^2*c^5*d^3))/(a^3*d^7
- b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a
*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^
2*c^6*d - 3*a^2*b*c*d^6) + (d*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(a^2*b^5*c^
10 + a^7*c^3*d^7 - a^7*c^5*d^5 + a*b^6*c^5*d^5 - 3*a*b^6*c^7*d^3 - 5*a^3*b^
4*c^9*d + a^5*b^2*c*d^9 + a^6*b*c^2*d^8 - 6*a^6*b*c^4*d^6 + 5*a^6*b*c^6*d^4
- 4*a^2*b^5*c^4*d^6 + 13*a^2*b^5*c^6*d^4 - 10*a^2*b^5*c^8*d^2 + 6*a^3*b^4*
c^3*d^7 - 22*a^3*b^4*c^5*d^5 + 21*a^3*b^4*c^7*d^3 - 4*a^4*b^3*c^2*d^8 + 18*
a^4*b^3*c^4*d^6 - 24*a^4*b^3*c^6*d^4 + 10*a^4*b^3*c^8*d^2 - 7*a^5*b^2*c^3*d
^7 + 16*a^5*b^2*c^5*d^5 - 10*a^5*b^2*c^7*d^3 + 2*a*b^6*c^9*d))/(a^3*d^7 - b
^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^
2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c
^6*d - 3*a^2*b*c*d^6) + (32*tan(e/2 + (f*x)/2)*(2*a*b^6*c^10 + 2*a^7*c^2*d^
8 - 2*a^7*c^4*d^6 - 2*a*b^6*c^8*d^2 - 6*a^2*b^5*c^9*d - 12*a^6*b*c^3*d^7 +
10*a^6*b*c^5*d^5 + 2*a^2*b^5*c^5*d^5 + 4*a^2*b^5*c^7*d^3 - 8*a^3*b^4*c^4*d^
6 + 6*a^3*b^4*c^6*d^4 + 2*a^3*b^4*c^8*d^2 + 12*a^4*b^3*c^3*d^7 - 24*a^4*b^3
*c^5*d^5 + 12*a^4*b^3*c^7*d^3 - 8*a^5*b^2*c^2*d^8 + 26*a^5*b^2*c^4*d^6 - 18
*a^5*b^2*c^6*d^4 + 2*a^6*b*c*d^9))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3
*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3
+ 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) - (d*(
-(c + d)^3*(c - d)^3)^(1/2)*((32*(a^2*b^6*c^12 - a^8*c^2*d^10 + 2*a^8*c^4*d
^8 - a^8*c^6*d^6 - a*b^7*c^7*d^5 + 2*a*b^7*c^9*d^3 - 4*a^3*b^5*c^11*d + 2*a
^7*b*c^3*d^9 - 7*a^7*b*c^5*d^7 + 4*a^7*b*c^7*d^5 + 4*a^2*b^6*c^6*d^6 - 7*a^
2*b^6*c^8*d^4 + 2*a^2*b^6*c^10*d^2 - 5*a^3*b^5*c^5*d^7 + 6*a^3*b^5*c^7*d^5
+ 3*a^3*b^5*c^9*d^3 + 5*a^4*b^4*c^6*d^6 - 10*a^4*b^4*c^8*d^4 + 5*a^4*b^4*c^
10*d^2 + 5*a^5*b^3*c^3*d^9 - 10*a^5*b^3*c^5*d^7 + 5*a^5*b^3*c^7*d^5 - 4*a^6
*b^2*c^2*d^10 + 3*a^6*b^2*c^4*d^8 + 6*a^6*b^2*c^6*d^6 - 5*a^6*b^2*c^8*d^4 -
a*b^7*c^11*d + a^7*b*c*d^11))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4
*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*
a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) + (32*tan(
e/2 + (f*x)/2)*(3*a*b^7*c^12 - 3*a^8*c*d^11 - 2*a^3*b^5*c^12 + 8*a^8*c^3*d^
9 - 7*a^8*c^5*d^7 + 2*a^8*c^7*d^5 - 4*a*b^7*c^6*d^6 + 11*a*b^7*c^8*d^4 - 10
*a*b^7*c^10*d^2 - 15*a^2*b^6*c^11*d + 10*a^4*b^4*c^11*d + 4*a^6*b^2*c*d^11
+ 15*a^7*b*c^2*d^10 - 40*a^7*b*c^4*d^8 + 35*a^7*b*c^6*d^6 - 10*a^7*b*c^8*d^

4 + 20*a^2*b^6*c^5*d^7 - 55*a^2*b^6*c^7*d^5 + 50*a^2*b^6*c^9*d^3 - 40*a^3*b
^5*c^4*d^8 + 113*a^3*b^5*c^6*d^6 - 108*a^3*b^5*c^8*d^4 + 37*a^3*b^5*c^10*d^
2 + 40*a^4*b^4*c^3*d^9 - 125*a^4*b^4*c^5*d^7 + 140*a^4*b^4*c^7*d^5 - 65*a^4
*b^4*c^9*d^3 - 20*a^5*b^3*c^2*d^10 + 85*a^5*b^3*c^4*d^8 - 130*a^5*b^3*c^6*d
^6 + 85*a^5*b^3*c^8*d^4 - 20*a^5*b^3*c^10*d^2 - 41*a^6*b^2*c^3*d^9 + 90*a^6
*b^2*c^5*d^7 - 73*a^6*b^2*c^7*d^5 + 20*a^6*b^2*c^9*d^3))/(a^3*d^7 - b^3*c^7
 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*
d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d -
 3*a^2*b*c*d^6))*(b*d^2 - 2*b*c^2 + a*c*d))/(a^2*d^8 - b^2*c^8 - 3*a^2*c^2*
d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3*b^2*c^4*d^4 + 3*b^2*c^6
*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 - 6*a*b*c^5*d^3))*(b*d^2 -
 2*b*c^2 + a*c*d))/(a^2*d^8 - b^2*c^8 - 3*a^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2
*c^6*d^2 + b^2*c^2*d^6 - 3*b^2*c^4*d^4 + 3*b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*
b*c^7*d + 6*a*b*c^3*d^5 - 6*a*b*c^5*d^3))*(b*d^2 - 2*b*c^2 + a*c*d)*1i)/(a^
2*d^8 - b^2*c^8 - 3*a^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6
 - 3*b^2*c^4*d^4 + 3*b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^
5 - 6*a*b*c^5*d^3))/((64*(2*a^2*b^3*c^2*d^4 - 3*a*b^4*c^3*d^3 - 3*a^2*b^3*c
^4*d^2 + a^3*b^2*c^3*d^3 + a*b^4*c*d^5 + 2*a*b^4*c^5*d))/(a^3*d^7 - b^3*c^7
 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*
d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d -
 3*a^2*b*c*d^6) + (64*tan(e/2 + (f*x)/2)*(2*a*b^4*c^2*d^4 - 4*a*b^4*c^4*d^2
 + 2*a^2*b^3*c^3*d^3))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b
^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^
3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) - (d*(-(c + d)^3*(
c - d)^3)^(1/2)*((32*tan(e/2 + (f*x)/2)*(a^6*c^3*d^5 - a*b^5*c^8 + 4*a*b^5*
c^2*d^6 - 13*a*b^5*c^4*d^4 + 12*a*b^5*c^6*d^2 - 4*a^2*b^4*c*d^7 + a^2*b^4*c
^7*d + a^4*b^2*c*d^7 + 2*a^5*b*c^2*d^6 - 5*a^5*b*c^4*d^4 + 17*a^2*b^4*c^3*d
^5 - 20*a^2*b^4*c^5*d^3 - 5*a^3*b^3*c^2*d^6 + 14*a^3*b^3*c^4*d^4 - 4*a^3*b^
3*c^6*d^2 - 8*a^4*b^2*c^3*d^5 + 8*a^4*b^2*c^5*d^3))/(a^3*d^7 - b^3*c^7 - 2*
a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 -
 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^
2*b*c*d^6) - (32*(2*a*b^5*c^5*d^3 - a*b^5*c^3*d^5 + a^3*b^3*c*d^7 + a^5*b*c
^3*d^5 + 2*a^2*b^4*c^4*d^4 - 3*a^2*b^4*c^6*d^2 - 6*a^3*b^3*c^3*d^5 + 8*a^3*
b^3*c^5*d^3 + 2*a^4*b^2*c^2*d^6 - 5*a^4*b^2*c^4*d^4 - a*b^5*c^7*d))/(a^3*d^
7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3
*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*
b^2*c^6*d - 3*a^2*b*c*d^6) + (d*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(a^2*b^5*
c^10 + a^7*c^3*d^7 - a^7*c^5*d^5 + a*b^6*c^5*d^5 - 3*a*b^6*c^7*d^3 - 5*a^3*
b^4*c^9*d + a^5*b^2*c*d^9 + a^6*b*c^2*d^8 - 6*a^6*b*c^4*d^6 + 5*a^6*b*c^6*d
^4 - 4*a^2*b^5*c^4*d^6 + 13*a^2*b^5*c^6*d^4 - 10*a^2*b^5*c^8*d^2 + 6*a^3*b^
4*c^3*d^7 - 22*a^3*b^4*c^5*d^5 + 21*a^3*b^4*c^7*d^3 - 4*a^4*b^3*c^2*d^8 + 1
8*a^4*b^3*c^4*d^6 - 24*a^4*b^3*c^6*d^4 + 10*a^4*b^3*c^8*d^2 - 7*a^5*b^2*c^3
*d^7 + 16*a^5*b^2*c^5*d^5 - 10*a^5*b^2*c^7*d^3 + 2*a*b^6*c^9*d))/(a^3*d^7 -
 b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*
b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2

*c^6*d - 3*a^2*b*c*d^6) + (32*tan(e/2 + (f*x)/2)*(2*a*b^6*c^10 + 2*a^7*c^2*d^8 - 2*a^7*c^4*d^6 - 2*a*b^6*c^8*d^2 - 6*a^2*b^5*c^9*d - 12*a^6*b*c^3*d^7 + 10*a^6*b*c^5*d^5 + 2*a^2*b^5*c^5*d^5 + 4*a^2*b^5*c^7*d^3 - 8*a^3*b^4*c^4*d^6 + 6*a^3*b^4*c^6*d^4 + 2*a^3*b^4*c^8*d^2 + 12*a^4*b^3*c^3*d^7 - 24*a^4*b^3*c^5*d^5 + 12*a^4*b^3*c^7*d^3 - 8*a^5*b^2*c^2*d^8 + 26*a^5*b^2*c^4*d^6 - 18*a^5*b^2*c^6*d^4 + 2*a^6*b*c*d^9))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) + (d*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(a^2*b^6*c^12 - a^8*c^2*d^10 + 2*a^8*c^4*d^8 - a^8*c^6*d^6 - a*b^7*c^7*d^5 + 2*a*b^7*c^9*d^3 - 4*a^3*b^5*c^11*d + 2*a^7*b*c^3*d^9 - 7*a^7*b*c^5*d^7 + 4*a^7*b*c^7*d^5 + 4*a^2*b^6*c^6*d^6 - 7*a^2*b^6*c^8*d^4 + 2*a^2*b^6*c^10*d^2 - 5*a^3*b^5*c^5*d^7 + 6*a^3*b^5*c^7*d^5 + 3*a^3*b^5*c^9*d^3 + 5*a^4*b^4*c^6*d^6 - 10*a^4*b^4*c^8*d^4 + 5*a^4*b^4*c^10*d^2 + 5*a^5*b^3*c^3*d^9 - 10*a^5*b^3*c^5*d^7 + 5*a^5*b^3*c^7*d^5 - 4*a^6*b^2*c^2*d^10 + 3*a^6*b^2*c^4*d^8 + 6*a^6*b^2*c^6*d^6 - 5*a^6*b^2*c^8*d^4 - a*b^7*c^11*d + a^7*b*c*d^11))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) + (32*tan(e/2 + (f*x)/2)*(3*a*b^7*c^12 - 3*a^8*c*d^11 - 2*a^3*b^5*c^12 + 8*a^8*c^3*d^9 - 7*a^8*c^5*d^7 + 2*a^8*c^7*d^5 - 4*a*b^7*c^6*d^6 + 11*a*b^7*c^8*d^4 - 10*a*b^7*c^10*d^2 - 15*a^2*b^6*c^11*d + 10*a^4*b^4*c^11*d + 4*a^6*b^2*c*d^11 + 15*a^7*b*c^2*d^10 - 40*a^7*b*c^4*d^8 + 35*a^7*b*c^6*d^6 - 10*a^7*b*c^8*d^4 + 20*a^2*b^6*c^5*d^7 - 55*a^2*b^6*c^7*d^5 + 50*a^2*b^6*c^9*d^3 - 40*a^3*b^5*c^4*d^8 + 113*a^3*b^5*c^6*d^6 - 108*a^3*b^5*c^8*d^4 + 37*a^3*b^5*c^10*d^2 + 40*a^4*b^4*c^3*d^9 - 125*a^4*b^4*c^5*d^7 + 140*a^4*b^4*c^7*d^5 - 65*a^4*b^4*c^9*d^3 - 20*a^5*b^3*c^2*d^10 + 85*a^5*b^3*c^4*d^8 - 130*a^5*b^3*c^6*d^6 + 85*a^5*b^3*c^8*d^4 - 20*a^5*b^3*c^10*d^2 - 41*a^6*b^2*c^3*d^9 + 90*a^6*b^2*c^5*d^7 - 73*a^6*b^2*c^7*d^5 + 20*a^6*b^2*c^9*d^3))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6))*(b*d^2 - 2*b*c^2 + a*c*d))/(a^2*d^8 - b^2*c^8 - 3*a^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3*b^2*c^4*d^4 + 3*b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 - 6*a*b*c^5*d^3))*(b*d^2 - 2*b*c^2 + a*c*d))/(a^2*d^8 - b^2*c^8 - 3*a^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3*b^2*c^4*d^4 + 3*b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 - 6*a*b*c^5*d^3))*(b*d^2 - 2*b*c^2 + a*c*d))/(a^2*d^8 - b^2*c^8 - 3*a^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3*b^2*c^4*d^4 + 3*b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 - 6*a*b*c^5*d^3) - (d*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(2*a*b^5*c^5*d^3 - a*b^5*c^3*d^5 + a^3*b^3*c*d^7 + a^5*b*c^3*d^5 + 2*a^2*b^4*c^4*d^4 - 3*a^2*b^4*c^6*d^2 - 6*a^3*b^3*c^3*d^5 + 8*a^3*b^3*c^5*d^3 + 2*a^4*b^2*c^2*d^6 - 5*a^4*b^2*c^4*d^4 - a*b^5*c^7*d))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) - (32*tan(e/2 + (f*x)/2)*(a^6*c^3*d^5 - a*b^5*c^8 + 4*a*b^5*c^2*d^6 - 13*a*b^5*c^4* d^4 + 12*a*b^5*c^6*d^2 - 4*a^2*b^4*c*d^7 + a^2*b^4*c^7*d + a^4*b^2*c*d^7 + 2*a^5*b*c^2*d^6 - 5*a^5*b*c^4*d^4 + 17*a^2*b^4*c^3*d^5 - 20*a^2*b^4*c^5*d^3 - 5*a^3*b^3*c^2*d^6 + 14*a^3*b^3*c^4*d^4 - 4*a^3*b^3*c^6*d^2 - 8*a^4*b^2*c^3*d^5 + 8*a^4*b^2*c^5*d^3))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) + (d*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(a^2*b^5*c^10 + a^7*c^3*d^7 - a^7*c^5*d^5 + a*b^6*c^5*d^5 - 3*a*b^6*c^7*d^3 - 5*a^3*b^4*c^9*d + a^5*b^2*c*d^9 + a^6*b*c^2*d^8 - 6*a^6*b*c^4*d^6 + 5*a^6*b*c^6*d^4 - 4*a^2*b^5*c^4*d^6 + 13*a^2*b^5*c^6*d^4 - 10*a^2*b^5*c^8*d^2 + 6*a^3*b^4*c^3*d^7 - 22*a^3*b^4*c^5*d^5 + 21*a^3*b^4*c^7*d^3 - 4*a^4*b^3*c^2*d^8 + 18*a^4*b^3*c^4*d^6 - 24*a^4*b^3*c^6*d^4 + 10*a^4*b^3*c^8*d^2 - 7*a^5*b^2*c^3*d^7 + 16*a^5*b^2*c^5*d^5 - 10*a^5*b^2*c^7*d^3 + 2*a*b^6*c^9*d))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) + (32*tan(e/2 + (f*x)/2)*(2*a*b^6*c^10 + 2*a^7*c^2*d^8 - 2*a^7*c^4*d^6 - 2*a*b^6*c^8*d^2 - 6*a^2*b^5*c^9*d - 12*a^6*b*c^3*d^7 + 10*a^6*b*c^5*d^5 + 2*a^2*b^5*c^5*d^5 + 4*a^2*b^5*c^7*d^3 - 8*a^3*b^4*c^4*d^6 + 6*a^3*b^4*c^6*d^4 + 2*a^3*b^4*c^8*d^2 + 12*a^4*b^3*c^3*d^7 - 24*a^4*b^3*c^5*d^5 + 12*a^4*b^3*c^7*d^3 - 8*a^5*b^2*c^2*d^8 + 26*a^5*b^2*c^4*d^6 - 18*a^5*b^2*c^6*d^4 + 2*a^6*b*c*d^9))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) - (d*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(a^2*b^6*c^12 - a^8*c^2*d^10 + 2*a^8*c^4*d^8 - a^8*c^6*d^6 - a*b^7*c^7*d^5 + 2*a*b^7*c^9*d^3 - 4*a^3*b^5*c^11*d + 2*a^7*b*c^3*d^9 - 7*a^7*b*c^5*d^7 + 4*a^7*b*c^7*d^5 + 4*a^2*b^6*c^6*d^6 - 7*a^2*b^6*c^8*d^4 + 2*a^2*b^6*c^10*d^2 - 5*a^3*b^5*c^5*d^7 + 6*a^3*b^5*c^7*d^5 + 3*a^3*b^5*c^9*d^3 + 5*a^4*b^4*c^6*d^6 - 10*a^4*b^4*c^8*d^4 + 5*a^4*b^4*c^10*d^2 + 5*a^5*b^3*c^3*d^9 - 10*a^5*b^3*c^5*d^7 + 5*a^5*b^3*c^7*d^5 - 4*a^6*b^2*c^2*d^10 + 3*a^6*b^2*c^4*d^8 + 6*a^6*b^2*c^6*d^6 - 5*a^6*b^2*c^8*d^4 - a*b^7*c^11*d + a^7*b*c*d^11))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6) + (32*tan(e/2 + (f*x)/2)*(3*a*b^7*c^12 - 3*a^8*c*d^11 - 2*a^3*b^5*c^12 + 8*a^8*c^3*d^9 - 7*a^8*c^5*d^7 + 2*a^8*c^7*d^5 - 4*a*b^7*c^6*d^6 + 11*a*b^7*c^8*d^4 - 10*a*b^7*c^10*d^2 - 15*a^2*b^6*c^11*d + 10*a^4*b^4*c^11*d + 4*a^6*b^2*c*d^11 + 15*a^7*b*c^2*d^10 - 40*a^7*b*c^4*d^8 + 35*a^7*b*c^6*d^6 - 10*a^7*b*c^8*d^4 + 20*a^2*b^6*c^5*d^7 - 55*a^2*b^6*c^7*d^5 + 50*a^2*b^6*c^9*d^3 - 40*a^3*b^5*c^4*d^8 + 113*a^3*b^5*c^6*d^6 - 108*a^3*b^5*c^8*d^4 + 37*a^3*b^5*c^10*d^2 + 40*a^4*b^4*c^3*d^9 - 125*a^4*b^4*c^5*d^7 + 140*a^4*b^4*c^7*d^5 - 65*a^4*b^4*c^9*d^3 - 20*a^5*b^3*c^2*d^10 + 85*a^5*b^3*c^4*d^8 - 130*a^5*b^3*c^6*d^6 + 85*a^5*b^3*c^8*d^4 - 20*a^5*b^3*c^10*d^2 - 41*a^6*b^2*c^3*d^9 + 90*a^6*b^2*c^5*d^7 - 73*a^6*b^2*c^7*d^5 + 20*a^6*b^2*c^9*d^3))/(a^3*d^7 - b^3*c^7 - 2*a^3*c^2*d^5 + a^3*c^4*d^3 - b^3*c^3*d^4 + 2*b^3*c^5*d^2 + 3*a*b^2*c^2*d^5 - 6*a*b^2*c^4*d^3 + 6*a^2*b*c^3*d^4 - 3*a^2*b*c^5*d^2 + 3*a*b^2*c^6*d - 3*a^2*b*c*d^6))*(b*d^2 - 2*b*c^2 + a

```
*c*d))/(a^2*d^8 - b^2*c^8 - 3*a^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b
^2*c^2*d^6 - 3*b^2*c^4*d^4 + 3*b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*
a*b*c^3*d^5 - 6*a*b*c^5*d^3))*(b*d^2 - 2*b*c^2 + a*c*d))/(a^2*d^8 - b^2*c^8
 - 3*a^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3*b^2*c^4*d^
4 + 3*b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 - 6*a*b*c^5*d
^3))*(b*d^2 - 2*b*c^2 + a*c*d))/(a^2*d^8 - b^2*c^8 - 3*a^2*c^2*d^6 + 3*a^2*
c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3*b^2*c^4*d^4 + 3*b^2*c^6*d^2 - 2*a*b
*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 - 6*a*b*c^5*d^3)))*(-(c + d)^3*(c - d)
^3)^(1/2)*(b*d^2 - 2*b*c^2 + a*c*d)*2i)/(f*(a^2*d^8 - b^2*c^8 - 3*a^2*c^2*d
^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3*b^2*c^4*d^4 + 3*b^2*c^6*
d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 - 6*a*b*c^5*d^3))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.705 $\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=284

$$\frac{d\left(-a^2d^2\left(2c^2+d^2\right)+6abc^3d-b^2\left(6c^4-5c^2d^2+2d^4\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{5/2}(bc-ad)^3}+\frac{2b^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}\,(bc-ad)^3}-\frac{d}{2f}$$

```
[Out] d*(6*a*b*c^3*d-a^2*d^2*(2*c^2+d^2)-b^2*(6*c^4-5*c^2*d^2+2*d^4))*arctan((d+c
*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(-a*d+b*c)^3/(c^2-d^2)^(5/2)/f-1/2*d^
2*cos(f*x+e)/(-a*d+b*c)/(c^2-d^2)/f/(c+d*sin(f*x+e))^2-1/2*d^2*(-3*a*c*d+5*
b*c^2-2*b*d^2)*cos(f*x+e)/(-a*d+b*c)^2/(c^2-d^2)^2/f/(c+d*sin(f*x+e))+2*b^3
*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/(-a*d+b*c)^3/f/(a^2-b^2)^
(1/2)
```

Rubi [A] time = 1.08, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2802, 3055, 3001, 2660, 618, 204}

$$\frac{d\left(-a^2d^2\left(2c^2+d^2\right)+6abc^3d-b^2\left(-5c^2d^2+6c^4+2d^4\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{5/2}(bc-ad)^3}+\frac{2b^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}\,(bc-ad)^3}-\frac{}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] (2*b^3*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*(
b*c - a*d)^3*f) + (d*(6*a*b*c^3*d - a^2*d^2*(2*c^2 + d^2) - b^2*(6*c^4 - 5*
c^2*d^2 + 2*d^4))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((b*c -
 a*d)^3*(c^2 - d^2)^(5/2)*f) - (d^2*Cos[e + f*x])/(2*(b*c - a*d)*(c^2 - d^2
)*f*(c + d*Sin[e + f*x])^2) - (d^2*(5*b*c^2 - 3*a*c*d - 2*b*d^2)*Cos[e + f*
x])/(2*(b*c - a*d)^2*(c^2 - d^2)^2*f*(c + d*Sin[e + f*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
```

```
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))^3}\,dx = -\frac{d^2\cos(e+fx)}{2(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{\int \frac{-2\left(acd-b\left(c^2-d^2\right)\right)-d}{(a+b\sin(}}{2(bc}$$

$$= -\frac{d^2\cos(e+fx)}{2(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} - \frac{d^2\left(5bc^2-3acd\right.}{2(bc-ad)^2\left(c^2-d\right.}$$

$$= -\frac{d^2\cos(e+fx)}{2(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} - \frac{d^2\left(5bc^2-3acd\right.}{2(bc-ad)^2\left(c^2-d\right.}$$

$$= -\frac{d^2\cos(e+fx)}{2(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} - \frac{d^2\left(5bc^2-3acd\right.}{2(bc-ad)^2\left(c^2-d\right.}$$

$$= -\frac{d^2\cos(e+fx)}{2(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} - \frac{d^2\left(5bc^2-3acd\right.}{2(bc-ad)^2\left(c^2-d\right.}$$

$$= \frac{2b^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(bc-ad)^3 f} + \frac{d\left(6abc^3d-a^2d^2\left(2c^2+d^2\right)-b^2\left(6\right.\right.}{(bc-ad}$$

Mathematica [A] time = 2.22, size = 263, normalized size = 0.93

$$\frac{-\frac{2d\left(a^2d^2\left(2c^2+d^2\right)-6abc^3d+b^2\left(6c^4-5c^2d^2+2d^4\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{5/2}} + \frac{4b^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{d^2(bc-ad)\left(3acd-5bc^2+2bd^2\right)\cos(e+f}{(c-d)^2(c+d)^2(c+d\sin(e+fx))}}{2f(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] ((4*b^3*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] -
 (2*d*(-6*a*b*c^3*d + a^2*d^2*(2*c^2 + d^2) + b^2*(6*c^4 - 5*c^2*d^2 + 2*d^
```

```
4))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^2)^(5/2) - (
d^2*(b*c - a*d)^2*Cos[e + f*x])/((c - d)*(c + d)*(c + d*Sin[e + f*x])^2) +
(d^2*(b*c - a*d)*(-5*b*c^2 + 3*a*c*d + 2*b*d^2)*Cos[e + f*x])/((c - d)^2*(c
 + d)^2*(c + d*Sin[e + f*x])))/(2*(b*c - a*d)^3*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 1.42, size = 786, normalized size = 2.77

$$\frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^3}{\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\sqrt{a^2-b^2}}-\frac{\left(6b^2c^4d-6abc^3d^2+2a^2c^2d^3-5b^2c^2d^3+a^2d^5+2b^2d^5\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan}{}\right.\right.}{\left(b^3c^7-3ab^2c^6d+3a^2bc^5d^2-2b^3c^5d^2-a^3c^4d^3+6ab^2c^4d^3-6a^2bc^3d^4+b^3c^3d^4+2a^3c^2d^5-3ab^2c^2d^5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] (2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e
) + b)/sqrt(a^2 - b^2)))*b^3/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*sqrt(a^2 - b^2)) - (6*b^2*c^4*d - 6*a*b*c^3*d^2 + 2*a^2*c^2*d^3 - 5*
b^2*c^2*d^3 + a^2*d^5 + 2*b^2*d^5)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c)
 + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((b^3*c^7 - 3*a*b^
2*c^6*d + 3*a^2*b*c^5*d^2 - 2*b^3*c^5*d^2 - a^3*c^4*d^3 + 6*a*b^2*c^4*d^3 -
 6*a^2*b*c^3*d^4 + b^3*c^3*d^4 + 2*a^3*c^2*d^5 - 3*a*b^2*c^2*d^5 + 3*a^2*b*
c*d^6 - a^3*d^7)*sqrt(c^2 - d^2)) - (7*b*c^4*d^3*tan(1/2*f*x + 1/2*e)^3 - 5
*a*c^3*d^4*tan(1/2*f*x + 1/2*e)^3 - 4*b*c^2*d^5*tan(1/2*f*x + 1/2*e)^3 + 2*
a*c*d^6*tan(1/2*f*x + 1/2*e)^3 + 6*b*c^5*d^2*tan(1/2*f*x + 1/2*e)^2 - 4*a*c
^4*d^3*tan(1/2*f*x + 1/2*e)^2 + 9*b*c^3*d^4*tan(1/2*f*x + 1/2*e)^2 - 7*a*c^
2*d^5*tan(1/2*f*x + 1/2*e)^2 - 6*b*c*d^6*tan(1/2*f*x + 1/2*e)^2 + 2*a*d^7*t
an(1/2*f*x + 1/2*e)^2 + 17*b*c^4*d^3*tan(1/2*f*x + 1/2*e) - 11*a*c^3*d^4*ta
n(1/2*f*x + 1/2*e) - 8*b*c^2*d^5*tan(1/2*f*x + 1/2*e) + 2*a*c*d^6*tan(1/2*f
*x + 1/2*e) + 6*b*c^5*d^2 - 4*a*c^4*d^3 - 3*b*c^3*d^4 + a*c^2*d^5)/((b^2*c^
8 - 2*a*b*c^7*d + a^2*c^6*d^2 - 2*b^2*c^6*d^2 + 4*a*b*c^5*d^3 - 2*a^2*c^4*d
^4 + b^2*c^4*d^4 - 2*a*b*c^3*d^5 + a^2*c^2*d^6)*(c*tan(1/2*f*x + 1/2*e)^2 +
 2*d*tan(1/2*f*x + 1/2*e) + c)^2))/f
```

maple [B] time = 0.38, size = 2644, normalized size = 9.31

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

```
[Out] -2/f*b^3/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(a^2-b^2)^(1/2)*arct
an(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))+6/f*d^6/(a*d-b*c)^3/(t
an(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*tan(1
/2*f*x+1/2*e)^3*a*b+17/f*d^3/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*
f*x+1/2*e)*d+c)^2*c^3/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*b^2-8/f*d^5/(a
*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^4-2*c^2*
d^2+d^4)*tan(1/2*f*x+1/2*e)*b^2-10/f*d^3/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*
c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*a*b*c^3+4/f*d^5/(a*d-b*c)
^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*
a*b*c+5/f*d^5/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)
^2/(c^4-2*c^2*d^2+d^4)*c*tan(1/2*f*x+1/2*e)^3*a^2-2/f*d^7/(a*d-b*c)^3/(tan(
1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)/c*tan(1/
2*f*x+1/2*e)^3*a^2+7/f*d^3/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*
x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^3*tan(1/2*f*x+1/2*e)^3*b^2-4/f*d^5/(a
*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^
2+d^4)*c*tan(1/2*f*x+1/2*e)^3*b^2+10/f*d^6/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^
2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*a*b+
6/f*d/(a*d-b*c)^3/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1
/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b^2*c^4-5/f*d^3/(a*d-b*c)^3/(c^4-2*c^2*
d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^
(1/2))*b^2*c^2+2/f*d^3/(a*d-b*c)^3/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arct
an(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a^2*c^2+4/f*d^4/(a*d-b
*c)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^
4)*c^2*tan(1/2*f*x+1/2*e)^2*a^2-2/f*d^8/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*c
+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)/c^2*tan(1/2*f*x+1/2*e)^2*a
^2+6/f*d^2/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/
(c^4-2*c^2*d^2+d^4)*c^4*tan(1/2*f*x+1/2*e)^2*b^2+9/f*d^4/(a*d-b*c)^3/(tan(1
/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^2*tan(1
/2*f*x+1/2*e)^2*b^2+11/f*d^5/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*
f*x+1/2*e)*d+c)^2*c/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*a^2-2/f*d^7/(a*d
-b*c)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^4-2*c^2*d^
2+d^4)*tan(1/2*f*x+1/2*e)*a^2-6/f*d^6/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*c+2
*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)^2*b^2+4/f
*d^4/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2
*c^2*d^2+d^4)*a^2*c^2+2/f*d^5/(a*d-b*c)^3/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/
2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b^2+1/f*d^5/(a*
d-b*c)^3/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/
```

```
2*e)+2*d)/(c^2-d^2)^(1/2))*a^2+6/f*d^2/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*c+
2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*b^2*c^4-3/f*d^4/(a*d-b*c)^3
/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*b^
2*c^2+7/f*d^6/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)
^2/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)^2*a^2-6/f*d^2/(a*d-b*c)^3/(c^4-2*
c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d
^2)^(1/2))*a*b*c^3-12/f*d^4/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f
*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^2*tan(1/2*f*x+1/2*e)^3*a*b-10/f*d^3/
(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*
d^2+d^4)*c^3*tan(1/2*f*x+1/2*e)^2*a*b-16/f*d^5/(a*d-b*c)^3/(tan(1/2*f*x+1/2
*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c*tan(1/2*f*x+1/2*e
)^2*a*b+8/f*d^7/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+
c)^2/(c^4-2*c^2*d^2+d^4)/c*tan(1/2*f*x+1/2*e)^2*a*b-28/f*d^4/(a*d-b*c)^3/(t
an(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c^2/(c^4-2*c^2*d^2+d^4)*t
an(1/2*f*x+1/2*e)*a*b-1/f*d^6/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2
*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*a^2
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [**B**] time = 30.30, size = 62873, normalized size = 221.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^3),x)
```

```
[Out] (b^3*atan(((b^3*(b^2 - a^2)^(1/2)*((8*(4*a*b^8*c^4*d^9 - 16*a*b^8*c^6*d^7 +
 24*a*b^8*c^8*d^5 - 16*a*b^8*c^10*d^3 + 4*a^4*b^5*c*d^12 + 4*a^6*b^3*c*d^12
 + 4*a^8*b*c^3*d^10 + 4*a^8*b*c^5*d^8 - 4*a^2*b^7*c^3*d^10 + 12*a^2*b^7*c^5
*d^8 + a^2*b^7*c^7*d^6 - 28*a^2*b^7*c^9*d^4 + 28*a^2*b^7*c^11*d^2 - 4*a^3*b
^6*c^2*d^11 + 24*a^3*b^6*c^4*d^9 - 98*a^3*b^6*c^6*d^7 + 164*a^3*b^6*c^8*d^5
 - 140*a^3*b^6*c^10*d^3 - 16*a^4*b^5*c^3*d^10 + 95*a^4*b^5*c^5*d^8 - 188*a^
4*b^5*c^7*d^6 + 240*a^4*b^5*c^9*d^4 - 8*a^5*b^4*c^2*d^11 - 20*a^5*b^4*c^4*d
^9 + 64*a^5*b^4*c^6*d^7 - 216*a^5*b^4*c^8*d^5 - a^6*b^3*c^3*d^10 + 20*a^6*b
^3*c^5*d^8 + 112*a^6*b^3*c^7*d^6 - 2*a^7*b^2*c^2*d^11 - 20*a^7*b^2*c^4*d^9
```

- 32*a^7*b^2*c^6*d^7 + 4*a*b^8*c^12*d + a^8*b*c*d^12))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) - (8*tan(e/2 + (f*x)/2)*(4*a*b^8*c^13 + a^9*c*d^12 + 4*a^9*c^3*d^10 + 4*a^9*c^5*d^8 - 16*a*b^8*c^3*d^10 + 76*a*b^8*c^5*d^8 - 162*a*b^8*c^7*d^6 + 176*a*b^8*c^9*d^4 - 96*a*b^8*c^11*d^2 - 8*a^2*b^7*c^12*d - 16*a^3*b^6*c*d^12 - 4*a^5*b^4*c*d^12 + 2*a^7*b^2*c*d^12 - 2*a^8*b*c^2*d^11 - 20*a^8*b*c^4*d^9 - 32*a^8*b*c^6*d^7 + 32*a^2*b^7*c^2*d^11 - 152*a^2*b^7*c^4*d^9 + 372*a^2*b^7*c^6*d^7 - 472*a^2*b^7*c^8*d^5 + 336*a^2*b^7*c^10*d^3 + 72*a^3*b^6*c^3*d^10 - 274*a^3*b^6*c^5*d^8 + 481*a^3*b^6*c^7*d^6 - 564*a^3*b^6*c^9*d^4 + 40*a^3*b^6*c^11*d^2 + 8*a^4*b^5*c^2*d^11 + 80*a^4*b^5*c^4*d^9 - 250*a^4*b^5*c^6*d^7 + 612*a^4*b^5*c^8*d^5 - 144*a^4*b^5*c^10*d^3 - 14*a^5*b^4*c^3*d^10 + 55*a^5*b^4*c^5*d^8 - 412*a^5*b^4*c^7*d^6 + 240*a^5*b^4*c^9*d^4 - 4*a^6*b^3*c^2*d^11 + 20*a^6*b^3*c^4*d^9 + 128*a^6*b^3*c^6*d^7 - 216*a^6*b^3*c^8*d^5 - 9*a^7*b^2*c^3*d^10 + 12*a^7*b^2*c^5*d^8 + 112*a^7*b^2*c^7*d^6))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) + (b^3*(b^2 - a^2)^(1/2)*((8*(4*a^2*b^8*c^16 + 2*a^10*c^2*d^14 - 6*a^10*c^6*d^10 + 4*a^10*c^8*d^8 + 4*a*b^9*c^7*d^9 - 18*a*b^9*c^9*d^7 + 36*a*b^9*c^11*d^5 - 34*a*b^9*c^13*d^3 - 32*a^3*b^7*c^15*d + 4*a^7*b^3*c*d^15 - 10*a^9*b*c^3*d^13 - 12*a^9*b*c^5*d^11 + 54*a^9*b*c^7*d^9 - 32*a^9*b*c^9*d^7 - 24*a^2*b^8*c^6*d^10 + 110*a^2*b^8*c^8*d^8 - 232*a^2*b^8*c^10*d^6 + 234*a^2*b^8*c^12*d^4 - 92*a^2*b^8*c^14*d^2 + 60*a^3*b^7*c^5*d^11 - 282*a^3*b^7*c^7*d^9 + 638*a^3*b^7*c^9*d^7 - 702*a^3*b^7*c^11*d^5 + 318*a^3*b^7*c^13*d^3 - 80*a^4*b^6*c^4*d^12 + 390*a^4*b^6*c^6*d^10 - 970*a^4*b^6*c^8*d^8 + 1202*a^4*b^6*c^10*d^6 - 654*a^4*b^6*c^12*d^4 + 112*a^4*b^6*c^14*d^2 + 60*a^5*b^5*c^3*d^13 - 310*a^5*b^5*c^5*d^11 + 878*a^5*b^5*c^7*d^9 - 1290*a^5*b^5*c^9*d^7 + 886*a^5*b^5*c^11*d^5 - 224*a^5*b^5*c^13*d^3 - 24*a^6*b^4*c^2*d^14 + 138*a^6*b^4*c^4*d^12 - 466*a^6*b^4*c^6*d^10 + 894*a^6*b^4*c^8*d^8 - 822*a^6*b^4*c^10*d^6 + 280*a^6*b^4*c^12*d^4 - 30*a^7*b^3*c^3*d^13 + 122*a^7*b^3*c^5*d^11 - 394*a^7*b^3*c^7*d^9 + 522*a^7*b^3*c^9*d^7 - 224*a^7*b^3*c^11*d^5 + 2*a^8*b^2*c^2*d^14 + 2*a^8*b^2*c^4*d^12 + 102*a^8*b^2*c^6*d^10 - 218*a^8*b^

2*c^8*d^8 + 112*a^8*b^2*c^10*d^6 + 12*a*b^9*c^15*d))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) + (8*tan(e/2 + (f*x)/2)*(8*a*b^9*c^16 + 4*a^10*c*d^15 - 12*a^10*c^5*d^11 + 8*a^10*c^7*d^9 + 4*a*b^9*c^8*d^8 - 8*a*b^9*c^10*d^6 + 12*a*b^9*c^12*d^4 - 16*a*b^9*c^14*d^2 - 40*a^2*b^8*c^15*d + 4*a^8*b^2*c*d^15 - 20*a^9*b*c^2*d^14 - 24*a^9*b*c^4*d^12 + 108*a^9*b*c^6*d^10 - 64*a^9*b*c^8*d^8 - 20*a^2*b^8*c^7*d^9 + 16*a^2*b^8*c^9*d^7 - 12*a^2*b^8*c^11*d^5 + 56*a^2*b^8*c^13*d^3 + 36*a^3*b^7*c^6*d^10 + 76*a^3*b^7*c^8*d^8 - 204*a^3*b^7*c^10*d^6 + 36*a^3*b^7*c^12*d^4 + 56*a^3*b^7*c^14*d^2 - 20*a^4*b^6*c^5*d^11 - 340*a^4*b^6*c^7*d^9 + 804*a^4*b^6*c^9*d^7 - 508*a^4*b^6*c^11*d^5 + 64*a^4*b^6*c^13*d^3 - 20*a^5*b^5*c^4*d^12 + 556*a^5*b^5*c^6*d^10 - 1380*a^5*b^5*c^8*d^8 + 1172*a^5*b^5*c^10*d^6 - 328*a^5*b^5*c^12*d^4 + 36*a^6*b^4*c^3*d^13 - 452*a^6*b^4*c^5*d^11 + 1308*a^6*b^4*c^7*d^9 - 1404*a^6*b^4*c^9*d^7 + 512*a^6*b^4*c^11*d^5 - 20*a^7*b^3*c^2*d^14 + 164*a^7*b^3*c^4*d^12 - 708*a^7*b^3*c^6*d^10 + 1004*a^7*b^3*c^8*d^8 - 440*a^7*b^3*c^10*d^6 + 4*a^8*b^2*c^3*d^13 + 204*a^8*b^2*c^5*d^11 - 436*a^8*b^2*c^7*d^9 + 224*a^8*b^2*c^9*d^7))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) + (b^3*(b^2 - a^2)^(1/2)*((8*(4*a^2*b^9*c^19 + 4*a^11*c^2*d^17 - 16*a^11*c^4*d^15 + 24*a^11*c^6*d^13 - 16*a^11*c^8*d^11 + 4*a^11*c^10*d^9 - 4*a*b^10*c^10*d^9 + 16*a*b^10*c^12*d^7 - 24*a*b^10*c^14*d^5 + 16*a*b^10*c^16*d^3 - 28*a^3*b^8*c^18*d - 12*a^10*b*c^3*d^16 + 88*a^10*b*c^5*d^14 - 152*a^10*b*c^7*d^12 + 108*a^10*b*c^9*d^10 - 28*a^10*b*c^11*d^8 + 28*a^2*b^9*c^9*d^10 - 108*a^2*b^9*c^11*d^8 + 152*a^2*b^9*c^13*d^6 - 88*a^2*b^9*c^15*d^4 + 12*a^2*b^9*c^17*d^2 - 80*a^3*b^8*c^8*d^11 + 292*a^3*b^8*c^10*d^9 - 368*a^3*b^8*c^12*d^7 + 152*a^3*b^8*c^14*d^5 + 32*a^3*b^8*c^16*d^3 + 112*a^4*b^7*c^7*d^12 - 368*a^4*b^7*c^9*d^10 + 352*a^4*b^7*c^11*d^8 + 32*a^4*b^7*c^13*d^6 - 208*a^4*b^7*c^15*d^4 + 80*a^4*b^7*c^17*d^2 - 56*a^5*b^6*c^6*d^13 + 112*a^5*b^6*c^8*d^11 + 112*a^5*b^6*c^10*d^9 - 448*a^5*b^6*c^12*d^7 + 392*a^5*b^6*c^14*d^5 - 112*a^5*b^6*c^16*d^3 - 56*a^6*b^5*c^5*d^14 + 280*a^6*b^5*c^7*d^12 - 560*a^6*b^5*c^9*d^10 + 560*a^6*b^5*c^11*d^8 - 280*a^6*b^5*c^13*d^6 + 56*a^6*b^5*c^1

5*d^4 + 112*a^7*b^4*c^4*d^15 - 392*a^7*b^4*c^6*d^13 + 448*a^7*b^4*c^8*d^11 - 112*a^7*b^4*c^10*d^9 - 112*a^7*b^4*c^12*d^7 + 56*a^7*b^4*c^14*d^5 - 80*a^8*b^3*c^3*d^16 + 208*a^8*b^3*c^5*d^14 - 32*a^8*b^3*c^7*d^12 - 352*a^8*b^3*c^9*d^10 + 368*a^8*b^3*c^11*d^8 - 112*a^8*b^3*c^13*d^6 + 28*a^9*b^2*c^2*d^17 - 32*a^9*b^2*c^4*d^15 - 152*a^9*b^2*c^6*d^13 + 368*a^9*b^2*c^8*d^11 - 292*a^9*b^2*c^10*d^9 + 80*a^9*b^2*c^12*d^7 - 4*a*b^10*c^18*d - 4*a^10*b*c*d^18))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) - (8*tan(e/2 + (f*x)/2)*(8*a^3*b^8*c^19 - 12*a^11*c*d^18 - 12*a*b^10*c^19 + 56*a^11*c^3*d^16 - 104*a^11*c^5*d^14 + 96*a^11*c^7*d^12 - 44*a^11*c^9*d^10 + 8*a^11*c^11*d^8 + 16*a*b^10*c^9*d^10 - 76*a*b^10*c^11*d^8 + 144*a*b^10*c^13*d^6 - 136*a*b^10*c^15*d^4 + 64*a*b^10*c^17*d^2 + 96*a^2*b^9*c^18*d - 64*a^4*b^7*c^18*d + 16*a^9*b^2*c*d^18 + 96*a^10*b*c^2*d^17 - 448*a^10*b*c^4*d^15 + 832*a^10*b*c^6*d^13 - 768*a^10*b*c^8*d^11 + 352*a^10*b*c^10*d^9 - 64*a^10*b*c^12*d^7 - 128*a^2*b^9*c^8*d^11 + 608*a^2*b^9*c^10*d^9 - 1152*a^2*b^9*c^12*d^7 + 1088*a^2*b^9*c^14*d^5 - 512*a^2*b^9*c^16*d^3 + 448*a^3*b^8*c^7*d^12 - 2140*a^3*b^8*c^9*d^10 + 4088*a^3*b^8*c^11*d^8 - 3912*a^3*b^8*c^13*d^6 + 1888*a^3*b^8*c^15*d^4 - 380*a^3*b^8*c^17*d^2 - 896*a^4*b^7*c^6*d^13 + 4352*a^4*b^7*c^8*d^11 - 8512*a^4*b^7*c^10*d^9 + 8448*a^4*b^7*c^12*d^7 - 4352*a^4*b^7*c^14*d^5 + 1024*a^4*b^7*c^16*d^3 + 1120*a^5*b^6*c^5*d^14 - 5656*a^5*b^6*c^7*d^12 + 11648*a^5*b^6*c^9*d^10 - 12432*a^5*b^6*c^11*d^8 + 7168*a^5*b^6*c^13*d^6 - 2072*a^5*b^6*c^15*d^4 + 224*a^5*b^6*c^17*d^2 - 896*a^6*b^5*c^4*d^15 + 4928*a^6*b^5*c^6*d^13 - 11200*a^6*b^5*c^8*d^11 + 13440*a^6*b^5*c^10*d^9 - 8960*a^6*b^5*c^12*d^7 + 3136*a^6*b^5*c^14*d^5 - 448*a^6*b^5*c^16*d^3 + 448*a^7*b^4*c^3*d^16 - 2968*a^7*b^4*c^5*d^14 + 7952*a^7*b^4*c^7*d^12 - 11088*a^7*b^4*c^9*d^10 + 8512*a^7*b^4*c^11*d^8 - 3416*a^7*b^4*c^13*d^6 + 560*a^7*b^4*c^15*d^4 - 128*a^8*b^3*c^2*d^17 + 1280*a^8*b^3*c^4*d^15 - 4288*a^8*b^3*c^6*d^13 + 6912*a^8*b^3*c^8*d^11 - 5888*a^8*b^3*c^10*d^9 + 2560*a^8*b^3*c^12*d^7 - 448*a^8*b^3*c^14*d^5 - 412*a^9*b^2*c^3*d^16 + 1712*a^9*b^2*c^5*d^14 - 3048*a^9*b^2*c^7*d^12 + 2752*a^9*b^2*c^9*d^10 - 1244*a^9*b^2*c^11*d^8 + 224*a^9*b^2*c^13*d^6))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8

- 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13)))/(a^5*d^3 + b^5*c^3 - a^2*b^3*c^3 - a^3*b^2*d^3 + 3*a^2*b^3*c*d^2 + 3*a^3*b^2*c^2*d - 3*a*b^4*c^2*d - 3*a^4*b*c*d^2)))/(a^5*d^3 + b^5*c^3 - a^2*b^3*c^3 - a^3*b^2*d^3 + 3*a^2*b^3*c*d^2 + 3*a^3*b^2*c^2*d - 3*a*b^4*c^2*d - 3*a^4*b*c*d^2))*1i)/(a^5*d^3 + b^5*c^3 - a^2*b^3*c^3 - a^3*b^2*d^3 + 3*a^2*b^3*c*d^2 + 3*a^3*b^2*c^2*d - 3*a*b^4*c^2*d - 3*a^4*b*c*d^2) - (b^3*(b^2 - a^2)^(1/2)*((8*tan(e/2 + (f*x)/2)*(4*a*b^8*c^13 + a^9*c*d^12 + 4*a^9*c^3*d^10 + 4*a^9*c^5*d^8 - 16*a*b^8*c^3*d^10 + 76*a*b^8*c^5*d^8 - 162*a*b^8*c^7*d^6 + 176*a*b^8*c^9*d^4 - 96*a*b^8*c^11*d^2 - 8*a^2*b^7*c^12*d - 16*a^3*b^6*c*d^12 - 4*a^5*b^4*c*d^12 + 2*a^7*b^2*c*d^12 - 2*a^8*b*c^2*d^11 - 20*a^8*b*c^4*d^9 - 32*a^8*b*c^6*d^7 + 32*a^2*b^7*c^2*d^11 - 152*a^2*b^7*c^4*d^9 + 372*a^2*b^7*c^6*d^7 - 472*a^2*b^7*c^8*d^5 + 336*a^2*b^7*c^10*d^3 + 72*a^3*b^6*c^3*d^10 - 274*a^3*b^6*c^5*d^8 + 481*a^3*b^6*c^7*d^6 - 564*a^3*b^6*c^9*d^4 + 40*a^3*b^6*c^11*d^2 + 8*a^4*b^5*c^2*d^11 + 80*a^4*b^5*c^4*d^9 - 250*a^4*b^5*c^6*d^7 + 612*a^4*b^5*c^8*d^5 - 144*a^4*b^5*c^10*d^3 - 14*a^5*b^4*c^3*d^10 + 55*a^5*b^4*c^5*d^8 - 412*a^5*b^4*c^7*d^6 + 240*a^5*b^4*c^9*d^4 - 4*a^6*b^3*c^2*d^11 + 20*a^6*b^3*c^4*d^9 + 128*a^6*b^3*c^6*d^7 - 216*a^6*b^3*c^8*d^5 - 9*a^7*b^2*c^3*d^10 + 12*a^7*b^2*c^5*d^8 + 112*a^7*b^2*c^7*d^6))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) - (8*(4*a*b^8*c^4*d^9 - 16*a*b^8*c^6*d^7 + 24*a*b^8*c^8*d^5 - 16*a*b^8*c^10*d^3 + 4*a^4*b^5*c*d^12 + 4*a^6*b^3*c*d^12 + 4*a^8*b*c^3*d^10 + 4*a^8*b*c^5*d^8 - 4*a^2*b^7*c^3*d^10 + 12*a^2*b^7*c^5*d^8 + a^2*b^7*c^7*d^6 - 28*a^2*b^7*c^9*d^4 + 28*a^2*b^7*c^11*d^2 - 4*a^3*b^6*c^2*d^11 + 24*a^3*b^6*c^4*d^9 - 98*a^3*b^6*c^6*d^7 + 164*a^3*b^6*c^8*d^5 - 140*a^3*b^6*c^10*d^3 - 16*a^4*b^5*c^3*d^10 + 95*a^4*b^5*c^5*d^8 - 188*a^4*b^5*c^7*d^6 + 240*a^4*b^5*c^9*d^4 - 8*a^5*b^4*c^2*d^11 - 20*a^5*b^4*c^4*d^9 + 64*a^5*b^4*c^6*d^7 - 216*a^5*b^4*c^8*d^5 - a^6*b^3*c^3*d^10 + 20*a^6*b^3*c^5*d^8 + 112*a^6*b^3*c^7*d^6 - 2*a^7*b^2*c^2*d^11 - 20*a^7*b^2*c^4*d^9 - 32*a^7*b^2*c^6*d^7 + 4*a*b^8*c^12*d + a^8*b*c*d^12))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) + (b^3*(b^2

- a^2)^(1/2)*((8*(4*a^2*b^8*c^16 + 2*a^10*c^2*d^14 - 6*a^10*c^6*d^10 + 4*a^10*c^8*d^8 + 4*a*b^9*c^7*d^9 - 18*a*b^9*c^9*d^7 + 36*a*b^9*c^11*d^5 - 34*a*b^9*c^13*d^3 - 32*a^3*b^7*c^15*d + 4*a^7*b^3*c*d^15 - 10*a^9*b*c^3*d^13 - 12*a^9*b*c^5*d^11 + 54*a^9*b*c^7*d^9 - 32*a^9*b*c^9*d^7 - 24*a^2*b^8*c^6*d^10 + 110*a^2*b^8*c^8*d^8 - 232*a^2*b^8*c^10*d^6 + 234*a^2*b^8*c^12*d^4 - 92*a^2*b^8*c^14*d^2 + 60*a^3*b^7*c^5*d^11 - 282*a^3*b^7*c^7*d^9 + 638*a^3*b^7*c^9*d^7 - 702*a^3*b^7*c^11*d^5 + 318*a^3*b^7*c^13*d^3 - 80*a^4*b^6*c^4*d^12 + 390*a^4*b^6*c^6*d^10 - 970*a^4*b^6*c^8*d^8 + 1202*a^4*b^6*c^10*d^6 - 654*a^4*b^6*c^12*d^4 + 112*a^4*b^6*c^14*d^2 + 60*a^5*b^5*c^3*d^13 - 310*a^5*b^5*c^5*d^11 + 878*a^5*b^5*c^7*d^9 - 1290*a^5*b^5*c^9*d^7 + 886*a^5*b^5*c^11*d^5 - 224*a^5*b^5*c^13*d^3 - 24*a^6*b^4*c^2*d^14 + 138*a^6*b^4*c^4*d^12 - 466*a^6*b^4*c^6*d^10 + 894*a^6*b^4*c^8*d^8 - 822*a^6*b^4*c^10*d^6 + 280*a^6*b^4*c^12*d^4 - 30*a^7*b^3*c^3*d^13 + 122*a^7*b^3*c^5*d^11 - 394*a^7*b^3*c^7*d^9 + 522*a^7*b^3*c^9*d^7 - 224*a^7*b^3*c^11*d^5 + 2*a^8*b^2*c^2*d^14 + 2*a^8*b^2*c^4*d^12 + 102*a^8*b^2*c^6*d^10 - 218*a^8*b^2*c^8*d^8 + 112*a^8*b^2*c^10*d^6 + 12*a*b^9*c^15*d))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) + (8*tan(e/2 + (f*x)/2)*(8*a*b^9*c^16 + 4*a^10*c*d^15 - 12*a^10*c^5*d^11 + 8*a^10*c^7*d^9 + 4*a*b^9*c^8*d^8 - 8*a*b^9*c^10*d^6 + 12*a*b^9*c^12*d^4 - 16*a*b^9*c^14*d^2 - 40*a^2*b^8*c^15*d + 4*a^8*b^2*c*d^15 - 20*a^9*b*c^2*d^14 - 24*a^9*b*c^4*d^12 + 108*a^9*b*c^6*d^10 - 64*a^9*b*c^8*d^8 - 20*a^2*b^8*c^7*d^9 + 16*a^2*b^8*c^9*d^7 - 12*a^2*b^8*c^11*d^5 + 56*a^2*b^8*c^13*d^3 + 36*a^3*b^7*c^6*d^10 + 76*a^3*b^7*c^8*d^8 - 204*a^3*b^7*c^10*d^6 + 36*a^3*b^7*c^12*d^4 + 56*a^3*b^7*c^14*d^2 - 20*a^4*b^6*c^5*d^11 - 340*a^4*b^6*c^7*d^9 + 804*a^4*b^6*c^9*d^7 - 508*a^4*b^6*c^11*d^5 + 64*a^4*b^6*c^13*d^3 - 20*a^5*b^5*c^4*d^12 + 556*a^5*b^5*c^6*d^10 - 1380*a^5*b^5*c^8*d^8 + 1172*a^5*b^5*c^10*d^6 - 328*a^5*b^5*c^12*d^4 + 36*a^6*b^4*c^3*d^13 - 452*a^6*b^4*c^5*d^11 + 1308*a^6*b^4*c^7*d^9 - 1404*a^6*b^4*c^9*d^7 + 512*a^6*b^4*c^11*d^5 - 20*a^7*b^3*c^2*d^14 + 164*a^7*b^3*c^4*d^12 - 708*a^7*b^3*c^6*d^10 + 1004*a^7*b^3*c^8*d^8 - 440*a^7*b^3*c^10*d^6 + 4*a^8*b^2*c^3*d^13 + 204*a^8*b^2*c^5*d^11 - 436*a^8*b^2*c^7*d^9 + 224*a^8*b^2*c^9*d^7))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2* d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) - (b^3*(b^2 - a^2)^(1/2)*((8*(4*a^2*b^9*c^19 + 4*a^11*c^2*d^17 - 16*a^11*c^4*d^15 + 24*a^11*c^6*d^13 - 16*a^11*c^8*d^11 + 4*a^11*c^10*d^9 - 4*a*b^10*c^10*d^9 + 16*a*b^10*c^12*d^7 - 24*a*b^10*c^14*d^5 + 16*a*b^10*c^16*d^3 - 28*a^3*b^8*c^18*d - 12*a^10*b*c^3*d^16 + 88*a^10*b*c^5*d^14 - 152*a^10*b*c^7*d^12 + 108*a^10*b*c^9*d^10 - 28*a^10*b*c^11*d^8 + 28*a^2*b^9*c^9*d^10 - 108*a^2*b^9*c^11*d^8 + 152*a^2*b^9*c^13*d^6 - 88*a^2*b^9*c^15*d^4 + 12*a^2*b^9*c^17*d^2 - 80*a^3*b^8*c^8*d^11 + 292*a^3*b^8*c^10*d^9 - 368*a^3*b^8*c^12*d^7 + 152*a^3*b^8*c^14*d^5 + 32*a^3*b^8*c^16*d^3 + 112*a^4*b^7*c^7*d^12 - 368*a^4*b^7*c^9*d^10 + 352*a^4*b^7*c^11*d^8 + 32*a^4*b^7*c^13*d^6 - 208*a^4*b^7*c^15*d^4 + 80*a^4*b^7*c^17*d^2 - 56*a^5*b^6*c^6*d^13 + 112*a^5*b^6*c^8*d^11 + 112*a^5*b^6*c^10*d^9 - 448*a^5*b^6*c^12*d^7 + 392*a^5*b^6*c^14*d^5 - 112*a^5*b^6*c^16*d^3 - 56*a^6*b^5*c^5*d^14 + 280*a^6*b^5*c^7*d^12 - 560*a^6*b^5*c^9*d^10 + 560*a^6*b^5*c^11*d^8 - 280*a^6*b^5*c^13*d^6 + 56*a^6*b^5*c^15*d^4 + 112*a^7*b^4*c^4*d^15 - 392*a^7*b^4*c^6*d^13 + 448*a^7*b^4*c^8*d^11 - 112*a^7*b^4*c^10*d^9 - 112*a^7*b^4*c^12*d^7 + 56*a^7*b^4*c^14*d^5 - 80*a^8*b^3*c^3*d^16 + 208*a^8*b^3*c^5*d^14 - 32*a^8*b^3*c^7*d^12 - 352*a^8*b^3*c^9*d^10 + 368*a^8*b^3*c^11*d^8 - 112*a^8*b^3*c^13*d^6 + 28*a^9*b^2*c^2*d^17 - 32*a^9*b^2*c^4*d^15 - 152*a^9*b^2*c^6*d^13 + 368*a^9*b^2*c^8*d^11 - 292*a^9*b^2*c^10*d^9 + 80*a^9*b^2*c^12*d^7 - 4*a*b^10*c^18*d - 4*a^10*b*c*d^18))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) - (8*tan(e/2 + (f*x)/2)*(8*a^3*b^8*c^19 - 12*a^11*c*d^18 - 12*a*b^10*c^19 + 56*a^11*c^3*d^16 - 104*a^11*c^5*d^14 + 96*a^11*c^7*d^12 - 44*a^11*c^9*d^10 + 8*a^11*c^11*d^8 + 16*a*b^10*c^9*d^10 - 76*a*b^10*c^11*d^8 + 144*a*b^10*c^13*d^6 - 136*a*b^10*c^15*d^4 + 64*a*b^10*c^17*d^2 + 96*a^2*b^9*c^18*d - 64*a^4*b^7*c^18*d + 16*a^9*b^2*c*d^18 + 96*a^10*b*c^2*d^17 - 448*a^10*b*c^4*d^15 + 832*a^10*b*c^6*d^13 - 768*a^10*b*c^8*d^11 + 352*a^10*b*c^10*d^9 - 64*a^10*b*c^12*d^7 - 128*a^2*b^9*c^8*d^11 + 608*a^2*b^9*c^10*d^9 - 1152*a^2*b^9*c^12*d^7 + 1088*a^2*b^9*c^14*d^5 - 512*a^2*b^9*c^16*d^3 + 448*a^3*b^8*c^7*d^12 - 2140*a^3*b^8*c^9*d^10 + 4088*a^3*b^8*c^11*d^8 - 3912*a^3*b^8*c^13*d^6 + 1888*a^3*b^8*c^15*d^4 - 380*a^3*b^8*c^17*d^2 - 896*a^4*b^7*c^6*d^13 + 4352*a^4*b^7*c^8*d^11 - 8512*a^4*b^7*c^10*d^9 + 8448*a^4*b^7*c^12*d^7 - 4352*a^4*b^7*c^14*d^5 + 1024*a^4*b^7*c^16*d^3 + 1120*a^5*b^6*c^5*d^14 - 5656*a^5*b^6*c^7*d^12 + 11648*a^5*b^6*c^9*d^10 - 12432*a^5*b^6*c^11*d^8 + 7168*a^5*b^6*c^13*d^6 - 2072*a^5*b^6*c^15*d^4 + 224*a^5*b^6*c^17*d^2 - 896*a^6*b^5*c^4*d^15 + 4928*a^6*b^5*c^6*d^13 - 11200*a^6*b^5*c^8*d^11 + 13440*a^6*b^5*c^10*d^9 - 8960*a^6*b^5*c^12*d^7 + 3

136*a^6*b^5*c^14*d^5 - 448*a^6*b^5*c^16*d^3 + 448*a^7*b^4*c^3*d^16 - 2968*a^7*b^4*c^5*d^14 + 7952*a^7*b^4*c^7*d^12 - 11088*a^7*b^4*c^9*d^10 + 8512*a^7*b^4*c^11*d^8 - 3416*a^7*b^4*c^13*d^6 + 560*a^7*b^4*c^15*d^4 - 128*a^8*b^3*c^2*d^17 + 1280*a^8*b^3*c^4*d^15 - 4288*a^8*b^3*c^6*d^13 + 6912*a^8*b^3*c^8*d^11 - 5888*a^8*b^3*c^10*d^9 + 2560*a^8*b^3*c^12*d^7 - 448*a^8*b^3*c^14*d^5 - 412*a^9*b^2*c^3*d^16 + 1712*a^9*b^2*c^5*d^14 - 3048*a^9*b^2*c^7*d^12 + 2752*a^9*b^2*c^9*d^10 - 1244*a^9*b^2*c^11*d^8 + 224*a^9*b^2*c^13*d^6))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13)))/(a^5*d^3 + b^5*c^3 - a^2*b^3*c^3 - a^3*b^2*d^3 + 3*a^2*b^3*c*d^2 + 3*a^3*b^2*c^2*d - 3*a*b^4*c^2*d - 3*a^4*b*c*d^2)))/(a^5*d^3 + b^5*c^3 - a^2*b^3*c^3 - a^3*b^2*d^3 + 3*a^2*b^3*c*d^2 + 3*a^3*b^2*c^2*d - 3*a*b^4*c^2*d - 3*a^4*b*c*d^2))*1i)/(a^5*d^3 + b^5*c^3 - a^2*b^3*c^3 - a^3*b^2*d^3 + 3*a^2*b^3*c*d^2 + 3*a^3*b^2*c^2*d - 3*a*b^4*c^2*d - 3*a^4*b*c*d^2))/((16*(36*a*b^7*c^5*d^5 - 18*a*b^7*c^3*d^7 - 34*a*b^7*c^7*d^3 + 4*a^3*b^5*c*d^9 + a^5*b^3*c*d^9 + 2*a^2*b^6*c^2*d^8 - 25*a^2*b^6*c^4*d^6 + 50*a^2*b^6*c^6*d^4 - 36*a^2*b^6*c^8*d^2 - a^3*b^5*c^3*d^7 - 16*a^3*b^5*c^5*d^5 + 40*a^3*b^5*c^7*d^3 + a^4*b^4*c^2*d^8 - 8*a^4*b^4*c^4*d^6 - 20*a^4*b^4*c^6*d^4 + 4*a^5*b^3*c^3*d^7 + 4*a^5*b^3*c^5*d^5 + 4*a*b^7*c*d^9 + 12*a*b^7*c^9*d))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) + (16*tan(e/2 + (f*x)/2)*(4*a*b^7*c^2*d^8 - 26*a*b^7*c^4*d^6 + 52*a*b^7*c^6*d^4 - 48*a*b^7*c^8*d^2 + 4*a^2*b^6*c*d^9 + 2*a^4*b^4*c*d^9 - 2*a^2*b^6*c^3*d^7 - 20*a^2*b^6*c^5*d^5 + 72*a^2*b^6*c^7*d^3 + 2*a^3*b^5*c^2*d^8 - 16*a^3*b^5*c^4*d^6 - 40*a^3*b^5*c^6*d^4 + 8*a^4*b^4*c^3*d^7 + 8*a^4*b^4*c^5*d^5))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3

*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*
d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c
d^13) - (b^3(b^2 - a^2)^(1/2)*((8*(4*a*b^8*c^4*d^9 - 16*a*b^8*c^6*d^7 + 2
4*a*b^8*c^8*d^5 - 16*a*b^8*c^10*d^3 + 4*a^4*b^5*c*d^12 + 4*a^6*b^3*c*d^12 +
4*a^8*b*c^3*d^10 + 4*a^8*b*c^5*d^8 - 4*a^2*b^7*c^3*d^10 + 12*a^2*b^7*c^5*d
^8 + a^2*b^7*c^7*d^6 - 28*a^2*b^7*c^9*d^4 + 28*a^2*b^7*c^11*d^2 - 4*a^3*b^6
*c^2*d^11 + 24*a^3*b^6*c^4*d^9 - 98*a^3*b^6*c^6*d^7 + 164*a^3*b^6*c^8*d^5 -
140*a^3*b^6*c^10*d^3 - 16*a^4*b^5*c^3*d^10 + 95*a^4*b^5*c^5*d^8 - 188*a^4*
b^5*c^7*d^6 + 240*a^4*b^5*c^9*d^4 - 8*a^5*b^4*c^2*d^11 - 20*a^5*b^4*c^4*d^9
+ 64*a^5*b^4*c^6*d^7 - 216*a^5*b^4*c^8*d^5 - a^6*b^3*c^3*d^10 + 20*a^6*b^3
*c^5*d^8 + 112*a^6*b^3*c^7*d^6 - 2*a^7*b^2*c^2*d^11 - 20*a^7*b^2*c^4*d^9 -
32*a^7*b^2*c^6*d^7 + 4*a*b^8*c^12*d + a^8*b*c*d^12))/(a^6*d^14 + b^6*c^14 -
4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^
8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*
a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 -
36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10
- 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b
^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^
7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4
*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d
^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) - (8*tan(e/2 + (f*x)/2)*(4*a*b^8*c^13
+ a^9*c*d^12 + 4*a^9*c^3*d^10 + 4*a^9*c^5*d^8 - 16*a*b^8*c^3*d^10 + 76*a*b
^8*c^5*d^8 - 162*a*b^8*c^7*d^6 + 176*a*b^8*c^9*d^4 - 96*a*b^8*c^11*d^2 - 8*
a^2*b^7*c^12*d - 16*a^3*b^6*c*d^12 - 4*a^5*b^4*c*d^12 + 2*a^7*b^2*c*d^12 -
2*a^8*b*c^2*d^11 - 20*a^8*b*c^4*d^9 - 32*a^8*b*c^6*d^7 + 32*a^2*b^7*c^2*d^1
1 - 152*a^2*b^7*c^4*d^9 + 372*a^2*b^7*c^6*d^7 - 472*a^2*b^7*c^8*d^5 + 336*a
^2*b^7*c^10*d^3 + 72*a^3*b^6*c^3*d^10 - 274*a^3*b^6*c^5*d^8 + 481*a^3*b^6*c
^7*d^6 - 564*a^3*b^6*c^9*d^4 + 40*a^3*b^6*c^11*d^2 + 8*a^4*b^5*c^2*d^11 + 8
0*a^4*b^5*c^4*d^9 - 250*a^4*b^5*c^6*d^7 + 612*a^4*b^5*c^8*d^5 - 144*a^4*b^5
*c^10*d^3 - 14*a^5*b^4*c^3*d^10 + 55*a^5*b^4*c^5*d^8 - 412*a^5*b^4*c^7*d^6
+ 240*a^5*b^4*c^9*d^4 - 4*a^6*b^3*c^2*d^11 + 20*a^6*b^3*c^4*d^9 + 128*a^6*b
^3*c^6*d^7 - 216*a^6*b^3*c^8*d^5 - 9*a^7*b^2*c^3*d^10 + 12*a^7*b^2*c^5*d^8
+ 112*a^7*b^2*c^7*d^6))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d
^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^1
0*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*
d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c
^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^
2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*
d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a
^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^
6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b
*c*d^13) + (b^3*(b^2 - a^2)^(1/2)*((8*(4*a^2*b^8*c^16 + 2*a^10*c^2*d^14 - 6
*a^10*c^6*d^10 + 4*a^10*c^8*d^8 + 4*a*b^9*c^7*d^9 - 18*a*b^9*c^9*d^7 + 36*a
*b^9*c^11*d^5 - 34*a*b^9*c^13*d^3 - 32*a^3*b^7*c^15*d + 4*a^7*b^3*c*d^15 -
10*a^9*b*c^3*d^13 - 12*a^9*b*c^5*d^11 + 54*a^9*b*c^7*d^9 - 32*a^9*b*c^9*d^7

- 24*a^2*b^8*c^6*d^10 + 110*a^2*b^8*c^8*d^8 - 232*a^2*b^8*c^10*d^6 + 234*a^2*b^8*c^12*d^4 - 92*a^2*b^8*c^14*d^2 + 60*a^3*b^7*c^5*d^11 - 282*a^3*b^7*c^7*d^9 + 638*a^3*b^7*c^9*d^7 - 702*a^3*b^7*c^11*d^5 + 318*a^3*b^7*c^13*d^3 - 80*a^4*b^6*c^4*d^12 + 390*a^4*b^6*c^6*d^10 - 970*a^4*b^6*c^8*d^8 + 1202*a^4*b^6*c^10*d^6 - 654*a^4*b^6*c^12*d^4 + 112*a^4*b^6*c^14*d^2 + 60*a^5*b^5*c^3*d^13 - 310*a^5*b^5*c^5*d^11 + 878*a^5*b^5*c^7*d^9 - 1290*a^5*b^5*c^9*d^7 + 886*a^5*b^5*c^11*d^5 - 224*a^5*b^5*c^13*d^3 - 24*a^6*b^4*c^2*d^14 + 138*a^6*b^4*c^4*d^12 - 466*a^6*b^4*c^6*d^10 + 894*a^6*b^4*c^8*d^8 - 822*a^6*b^4*c^10*d^6 + 280*a^6*b^4*c^12*d^4 - 30*a^7*b^3*c^3*d^13 + 122*a^7*b^3*c^5*d^11 - 394*a^7*b^3*c^7*d^9 + 522*a^7*b^3*c^9*d^7 - 224*a^7*b^3*c^11*d^5 + 2*a^8*b^2*c^2*d^14 + 2*a^8*b^2*c^4*d^12 + 102*a^8*b^2*c^6*d^10 - 218*a^8*b^2*c^8*d^8 + 112*a^8*b^2*c^10*d^6 + 12*a*b^9*c^15*d))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) + (8*tan(e/2 + (f*x)/2)*(8*a*b^9*c^16 + 4*a^10*c*d^15 - 12*a^10*c^5*d^11 + 8*a^10*c^7*d^9 + 4*a*b^9*c^8*d^8 - 8*a*b^9*c^10*d^6 + 12*a*b^9*c^12*d^4 - 16*a*b^9*c^14*d^2 - 40*a^2*b^8*c^15*d + 4*a^8*b^2*c*d^15 - 20*a^9*b*c^2*d^14 - 24*a^9*b*c^4*d^12 + 108*a^9*b*c^6*d^10 - 64*a^9*b*c^8*d^8 - 20*a^2*b^8*c^7*d^9 + 16*a^2*b^8*c^9*d^7 - 12*a^2*b^8*c^11*d^5 + 56*a^2*b^8*c^13*d^3 + 36*a^3*b^7*c^6*d^10 + 76*a^3*b^7*c^8*d^8 - 204*a^3*b^7*c^10*d^6 + 36*a^3*b^7*c^12*d^4 + 56*a^3*b^7*c^14*d^2 - 20*a^4*b^6*c^5*d^11 - 340*a^4*b^6*c^7*d^9 + 804*a^4*b^6*c^9*d^7 - 508*a^4*b^6*c^11*d^5 + 64*a^4*b^6*c^13*d^3 - 20*a^5*b^5*c^4*d^12 + 556*a^5*b^5*c^6*d^10 - 1380*a^5*b^5*c^8*d^8 + 1172*a^5*b^5*c^10*d^6 - 328*a^5*b^5*c^12*d^4 + 36*a^6*b^4*c^3*d^13 - 452*a^6*b^4*c^5*d^11 + 1308*a^6*b^4*c^7*d^9 - 1404*a^6*b^4*c^9*d^7 + 512*a^6*b^4*c^11*d^5 - 20*a^7*b^3*c^2*d^14 + 164*a^7*b^3*c^4*d^12 - 708*a^7*b^3*c^6*d^10 + 1004*a^7*b^3*c^8*d^8 - 440*a^7*b^3*c^10*d^6 + 4*a^8*b^2*c^3*d^13 + 204*a^8*b^2*c^5*d^11 - 436*a^8*b^2*c^7*d^9 + 224*a^8*b^2*c^9*d^7))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) + (b^3*(b^2 - a^2)^(1/2)*((8*(4*a^2*b^9*c^19 + 4*a^11*c^2*d^17 - 16*a^11*c^4*d^15 + 24*a^11*c^6*d^13 - 16*a^11*c^8*d^11 + 4*a^11*c^10*d^9 - 4*a*b^10*c^10 *d^9 + 16*a*b^10*c^12*d^7 - 24*a*b^10*c^14*d^5 + 16*a*b^10*c^16*d^3 - 28*a^3*b^8*c^18*d - 12*a^10*b*c^3*d^16 + 88*a^10*b*c^5*d^14 - 152*a^10*b*c^7*d^12 + 108*a^10*b*c^9*d^10 - 28*a^10*b*c^11*d^8 + 28*a^2*b^9*c^9*d^10 - 108*a^2*b^9*c^11*d^8 + 152*a^2*b^9*c^13*d^6 - 88*a^2*b^9*c^15*d^4 + 12*a^2*b^9*c^17*d^2 - 80*a^3*b^8*c^8*d^11 + 292*a^3*b^8*c^10*d^9 - 368*a^3*b^8*c^12*d^7 + 152*a^3*b^8*c^14*d^5 + 32*a^3*b^8*c^16*d^3 + 112*a^4*b^7*c^7*d^12 - 368*a^4*b^7*c^9*d^10 + 352*a^4*b^7*c^11*d^8 + 32*a^4*b^7*c^13*d^6 - 208*a^4*b^7*c^15*d^4 + 80*a^4*b^7*c^17*d^2 - 56*a^5*b^6*c^6*d^13 + 112*a^5*b^6*c^8*d^11 + 112*a^5*b^6*c^10*d^9 - 448*a^5*b^6*c^12*d^7 + 392*a^5*b^6*c^14*d^5 - 112*a^5*b^6*c^16*d^3 - 56*a^6*b^5*c^5*d^14 + 280*a^6*b^5*c^7*d^12 - 560*a^6*b^5*c^9*d^10 + 560*a^6*b^5*c^11*d^8 - 280*a^6*b^5*c^13*d^6 + 56*a^6*b^5*c^15*d^4 + 112*a^7*b^4*c^4*d^15 - 392*a^7*b^4*c^6*d^13 + 448*a^7*b^4*c^8*d^11 - 112*a^7*b^4*c^10*d^9 - 112*a^7*b^4*c^12*d^7 + 56*a^7*b^4*c^14*d^5 - 80*a^8*b^3*c^3*d^16 + 208*a^8*b^3*c^5*d^14 - 32*a^8*b^3*c^7*d^12 - 352*a^8*b^3*c^9*d^10 + 368*a^8*b^3*c^11*d^8 - 112*a^8*b^3*c^13*d^6 + 28*a^9*b^2*c^2*d^17 - 32*a^9*b^2*c^4*d^15 - 152*a^9*b^2*c^6*d^13 + 368*a^9*b^2*c^8*d^11 - 292*a^9*b^2*c^10*d^9 + 80*a^9*b^2*c^12*d^7 - 4*a*b^10*c^18*d - 4*a^10*b*c*d^18))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) - (8*tan(e/2 + (f*x)/2)*(8*a^3*b^8*c^19 - 12*a^11*c*d^18 - 12*a*b^10*c^19 + 56*a^11*c^3*d^16 - 104*a^11*c^5*d^14 + 96*a^11*c^7*d^12 - 44*a^11*c^9*d^10 + 8*a^11*c^11*d^8 + 16*a*b^10*c^9*d^10 - 76*a*b^10*c^11*d^8 + 144*a*b^10*c^13*d^6 - 136*a*b^10*c^15*d^4 + 64*a*b^10*c^17*d^2 + 96*a^2*b^9*c^18*d - 64*a^4*b^7*c^18*d + 16*a^9*b^2*c*d^18 + 96*a^10*b*c^2*d^17 - 448*a^10*b*c^4*d^15 + 832*a^10*b*c^6*d^13 - 768*a^10*b*c^8*d^11 + 352*a^10*b*c^10*d^9 - 64*a^10*b*c^12*d^7 - 128*a^2*b^9*c^8*d^11 + 608*a^2*b^9*c^10*d^9 - 1152*a^2*b^9*c^12*d^7 + 1088*a^2*b^9*c^14*d^5 - 512*a^2*b^9*c^16*d^3 + 448*a^3*b^8*c^7*d^12 - 2140*a^3*b^8*c^9*d^10 + 4088*a^3*b^8*c^11*d^8 - 3912*a^3*b^8*c^13*d^6 + 1888*a^3*b^8*c^15*d^4 - 380*a^3*b^8*c^17*d^2 - 896*a^4*b^7*c^6*d^13 + 4352*a^4*b^7*c^8*d^11 - 8512*a^4*b^7*c^10*d^9 + 8448*a^4*b^7*c^12*d^7 - 4352*a^4*b^7*c^14*d^5 + 1024*a^4*b^7*c^16*d^3 + 1120*a^5*b^6*c^5*d^14 - 5656*a^5*b^6*c^7*d^12 + 11648*a^5*b^6*c^9*d^10 - 12432*a^5*b^6*c^11*d^8 + 7168*a^5*b^6*c^13*d^6 - 2072*a^5*b^6*c^15*d^4 + 224*a^5*b^6*c^17*d^2 - 896*a^6*b^5*c^4*d^15 + 4928*a^6*b^5*c^6*d^13 - 11200*a^6*b^5*c^8*d^11 + 13440*a^6*b^5*c^10*d^9 - 8960*a^6*b^5*c^12*d^7 + 3136*a^6*b^5*c^14*d^5 - 448*a^6*b^5*c^16*d^3 + 448*a^7*b^4*c^3*d^16 - 2968*a^7*b^4*c^5*d^14 + 7952*a^7*b^4*c^7*d^12 - 11088*a^7*b^4*c^9*d^10 + 8512*a^7*b^4*c^11*d^8 - 3416*a^7*b^4*c^13*d^6 + 560*a^7*b^4*c^15*d^4 - 128*a^8*b^3*c^2*d^17 + 1280*a^8*b^3*c^4*d^15 - 4288*a^8*b^3*c^6*d^

13 + 6912*a^8*b^3*c^8*d^11 - 5888*a^8*b^3*c^10*d^9 + 2560*a^8*b^3*c^12*d^7
- 448*a^8*b^3*c^14*d^5 - 412*a^9*b^2*c^3*d^16 + 1712*a^9*b^2*c^5*d^14 - 304
8*a^9*b^2*c^7*d^12 + 2752*a^9*b^2*c^9*d^10 - 1244*a^9*b^2*c^11*d^8 + 224*a^
9*b^2*c^13*d^6))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4
*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 -
4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 2
4*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7
- 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c
^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 +
80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*
c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 -
60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13
)))/(a^5*d^3 + b^5*c^3 - a^2*b^3*c^3 - a^3*b^2*d^3 + 3*a^2*b^3*c*d^2 + 3*a^
3*b^2*c^2*d - 3*a*b^4*c^2*d - 3*a^4*b*c*d^2)))/(a^5*d^3 + b^5*c^3 - a^2*b^3
*c^3 - a^3*b^2*d^3 + 3*a^2*b^3*c*d^2 + 3*a^3*b^2*c^2*d - 3*a*b^4*c^2*d - 3*
a^4*b*c*d^2)))/(a^5*d^3 + b^5*c^3 - a^2*b^3*c^3 - a^3*b^2*d^3 + 3*a^2*b^3*c
*d^2 + 3*a^3*b^2*c^2*d - 3*a*b^4*c^2*d - 3*a^4*b*c*d^2) - (b^3*(b^2 - a^2)^
(1/2)*((8*tan(e/2 + (f*x)/2)*(4*a*b^8*c^13 + a^9*c*d^12 + 4*a^9*c^3*d^10 +
4*a^9*c^5*d^8 - 16*a*b^8*c^3*d^10 + 76*a*b^8*c^5*d^8 - 162*a*b^8*c^7*d^6 +
176*a*b^8*c^9*d^4 - 96*a*b^8*c^11*d^2 - 8*a^2*b^7*c^12*d - 16*a^3*b^6*c*d^1
2 - 4*a^5*b^4*c*d^12 + 2*a^7*b^2*c*d^12 - 2*a^8*b*c^2*d^11 - 20*a^8*b*c^4*d
^9 - 32*a^8*b*c^6*d^7 + 32*a^2*b^7*c^2*d^11 - 152*a^2*b^7*c^4*d^9 + 372*a^2
*b^7*c^6*d^7 - 472*a^2*b^7*c^8*d^5 + 336*a^2*b^7*c^10*d^3 + 72*a^3*b^6*c^3*
d^10 - 274*a^3*b^6*c^5*d^8 + 481*a^3*b^6*c^7*d^6 - 564*a^3*b^6*c^9*d^4 + 40
*a^3*b^6*c^11*d^2 + 8*a^4*b^5*c^2*d^11 + 80*a^4*b^5*c^4*d^9 - 250*a^4*b^5*c
^6*d^7 + 612*a^4*b^5*c^8*d^5 - 144*a^4*b^5*c^10*d^3 - 14*a^5*b^4*c^3*d^10 +
55*a^5*b^4*c^5*d^8 - 412*a^5*b^4*c^7*d^6 + 240*a^5*b^4*c^9*d^4 - 4*a^6*b^3
*c^2*d^11 + 20*a^6*b^3*c^4*d^9 + 128*a^6*b^3*c^6*d^7 - 216*a^6*b^3*c^8*d^5
- 9*a^7*b^2*c^3*d^10 + 12*a^7*b^2*c^5*d^8 + 112*a^7*b^2*c^7*d^6))/(a^6*d^14
+ b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6
+ b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*
c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*
b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2
*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d
^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a
^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*
d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a
^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) - (8*(4*a*b^8*c^4*d^9 -
16*a*b^8*c^6*d^7 + 24*a*b^8*c^8*d^5 - 16*a*b^8*c^10*d^3 + 4*a^4*b^5*c*d^12
+ 4*a^6*b^3*c*d^12 + 4*a^8*b*c^3*d^10 + 4*a^8*b*c^5*d^8 - 4*a^2*b^7*c^3*d^1
0 + 12*a^2*b^7*c^5*d^8 + a^2*b^7*c^7*d^6 - 28*a^2*b^7*c^9*d^4 + 28*a^2*b^7*
c^11*d^2 - 4*a^3*b^6*c^2*d^11 + 24*a^3*b^6*c^4*d^9 - 98*a^3*b^6*c^6*d^7 + 1
64*a^3*b^6*c^8*d^5 - 140*a^3*b^6*c^10*d^3 - 16*a^4*b^5*c^3*d^10 + 95*a^4*b^
5*c^5*d^8 - 188*a^4*b^5*c^7*d^6 + 240*a^4*b^5*c^9*d^4 - 8*a^5*b^4*c^2*d^11
- 20*a^5*b^4*c^4*d^9 + 64*a^5*b^4*c^6*d^7 - 216*a^5*b^4*c^8*d^5 - a^6*b^3*c

^3*d^10 + 20*a^6*b^3*c^5*d^8 + 112*a^6*b^3*c^7*d^6 - 2*a^7*b^2*c^2*d^11 - 2
0*a^7*b^2*c^4*d^9 - 32*a^7*b^2*c^6*d^7 + 4*a*b^8*c^12*d + a^8*b*c*d^12))/(a
^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*
c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6
*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 +
24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 +
 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4
*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9
- 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b
^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6
 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) + (b^3*(b^2 - a^2
)^(1/2)*((8*(4*a^2*b^8*c^16 + 2*a^10*c^2*d^14 - 6*a^10*c^6*d^10 + 4*a^10*c^
8*d^8 + 4*a*b^9*c^7*d^9 - 18*a*b^9*c^9*d^7 + 36*a*b^9*c^11*d^5 - 34*a*b^9*c
^13*d^3 - 32*a^3*b^7*c^15*d + 4*a^7*b^3*c*d^15 - 10*a^9*b*c^3*d^13 - 12*a^9
*b*c^5*d^11 + 54*a^9*b*c^7*d^9 - 32*a^9*b*c^9*d^7 - 24*a^2*b^8*c^6*d^10 + 1
10*a^2*b^8*c^8*d^8 - 232*a^2*b^8*c^10*d^6 + 234*a^2*b^8*c^12*d^4 - 92*a^2*b
^8*c^14*d^2 + 60*a^3*b^7*c^5*d^11 - 282*a^3*b^7*c^7*d^9 + 638*a^3*b^7*c^9*d
^7 - 702*a^3*b^7*c^11*d^5 + 318*a^3*b^7*c^13*d^3 - 80*a^4*b^6*c^4*d^12 + 39
0*a^4*b^6*c^6*d^10 - 970*a^4*b^6*c^8*d^8 + 1202*a^4*b^6*c^10*d^6 - 654*a^4*
b^6*c^12*d^4 + 112*a^4*b^6*c^14*d^2 + 60*a^5*b^5*c^3*d^13 - 310*a^5*b^5*c^5
*d^11 + 878*a^5*b^5*c^7*d^9 - 1290*a^5*b^5*c^9*d^7 + 886*a^5*b^5*c^11*d^5 -
 224*a^5*b^5*c^13*d^3 - 24*a^6*b^4*c^2*d^14 + 138*a^6*b^4*c^4*d^12 - 466*a^
6*b^4*c^6*d^10 + 894*a^6*b^4*c^8*d^8 - 822*a^6*b^4*c^10*d^6 + 280*a^6*b^4*c
^12*d^4 - 30*a^7*b^3*c^3*d^13 + 122*a^7*b^3*c^5*d^11 - 394*a^7*b^3*c^7*d^9
+ 522*a^7*b^3*c^9*d^7 - 224*a^7*b^3*c^11*d^5 + 2*a^8*b^2*c^2*d^14 + 2*a^8*b
^2*c^4*d^12 + 102*a^8*b^2*c^6*d^10 - 218*a^8*b^2*c^8*d^8 + 112*a^8*b^2*c^10
*d^6 + 12*a*b^9*c^15*d))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*
d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^
10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9
*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*
c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a
^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3
*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*
a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c
^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*
b*c*d^13) + (8*tan(e/2 + (f*x)/2)*(8*a*b^9*c^16 + 4*a^10*c*d^15 - 12*a^10*c
^5*d^11 + 8*a^10*c^7*d^9 + 4*a*b^9*c^8*d^8 - 8*a*b^9*c^10*d^6 + 12*a*b^9*c^
12*d^4 - 16*a*b^9*c^14*d^2 - 40*a^2*b^8*c^15*d + 4*a^8*b^2*c*d^15 - 20*a^9*
b*c^2*d^14 - 24*a^9*b*c^4*d^12 + 108*a^9*b*c^6*d^10 - 64*a^9*b*c^8*d^8 - 20
*a^2*b^8*c^7*d^9 + 16*a^2*b^8*c^9*d^7 - 12*a^2*b^8*c^11*d^5 + 56*a^2*b^8*c^
13*d^3 + 36*a^3*b^7*c^6*d^10 + 76*a^3*b^7*c^8*d^8 - 204*a^3*b^7*c^10*d^6 +
36*a^3*b^7*c^12*d^4 + 56*a^3*b^7*c^14*d^2 - 20*a^4*b^6*c^5*d^11 - 340*a^4*b
^6*c^7*d^9 + 804*a^4*b^6*c^9*d^7 - 508*a^4*b^6*c^11*d^5 + 64*a^4*b^6*c^13*d
^3 - 20*a^5*b^5*c^4*d^12 + 556*a^5*b^5*c^6*d^10 - 1380*a^5*b^5*c^8*d^8 + 11
72*a^5*b^5*c^10*d^6 - 328*a^5*b^5*c^12*d^4 + 36*a^6*b^4*c^3*d^13 - 452*a^6*

b^4*c^5*d^11 + 1308*a^6*b^4*c^7*d^9 - 1404*a^6*b^4*c^9*d^7 + 512*a^6*b^4*c^11*d^5 - 20*a^7*b^3*c^2*d^14 + 164*a^7*b^3*c^4*d^12 - 708*a^7*b^3*c^6*d^10 + 1004*a^7*b^3*c^8*d^8 - 440*a^7*b^3*c^10*d^6 + 4*a^8*b^2*c^3*d^13 + 204*a^8*b^2*c^5*d^11 - 436*a^8*b^2*c^7*d^9 + 224*a^8*b^2*c^9*d^7))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) - (b^3*(b^2 - a^2)^(1/2)*((8*(4*a^2*b^9*c^19 + 4*a^11*c^2*d^17 - 16*a^11*c^4*d^15 + 24*a^11*c^6*d^13 - 16*a^11*c^8*d^11 + 4*a^11*c^10*d^9 - 4*a*b^10*c^10*d^9 + 16*a*b^10*c^12*d^7 - 24*a*b^10*c^14*d^5 + 16*a*b^10*c^16*d^3 - 28*a^3*b^8*c^18*d - 12*a^10*b*c^3*d^16 + 88*a^10*b*c^5*d^14 - 152*a^10*b*c^7*d^12 + 108*a^10*b*c^9*d^10 - 28*a^10*b*c^11*d^8 + 28*a^2*b^9*c^9*d^10 - 108*a^2*b^9*c^11*d^8 + 152*a^2*b^9*c^13*d^6 - 88*a^2*b^9*c^15*d^4 + 12*a^2*b^9*c^17*d^2 - 80*a^3*b^8*c^8*d^11 + 292*a^3*b^8*c^10*d^9 - 368*a^3*b^8*c^12*d^7 + 152*a^3*b^8*c^14*d^5 + 32*a^3*b^8*c^16*d^3 + 112*a^4*b^7*c^7*d^12 - 368*a^4*b^7*c^9*d^10 + 352*a^4*b^7*c^11*d^8 + 32*a^4*b^7*c^13*d^6 - 208*a^4*b^7*c^15*d^4 + 80*a^4*b^7*c^17*d^2 - 56*a^5*b^6*c^6*d^13 + 112*a^5*b^6*c^8*d^11 + 112*a^5*b^6*c^10*d^9 - 448*a^5*b^6*c^12*d^7 + 392*a^5*b^6*c^14*d^5 - 112*a^5*b^6*c^16*d^3 - 56*a^6*b^5*c^5*d^14 + 280*a^6*b^5*c^7*d^12 - 560*a^6*b^5*c^9*d^10 + 560*a^6*b^5*c^11*d^8 - 280*a^6*b^5*c^13*d^6 + 56*a^6*b^5*c^15*d^4 + 112*a^7*b^4*c^4*d^15 - 392*a^7*b^4*c^6*d^13 + 448*a^7*b^4*c^8*d^11 - 112*a^7*b^4*c^10*d^9 - 112*a^7*b^4*c^12*d^7 + 56*a^7*b^4*c^14*d^5 - 80*a^8*b^3*c^3*d^16 + 208*a^8*b^3*c^5*d^14 - 32*a^8*b^3*c^7*d^12 - 352*a^8*b^3*c^9*d^10 + 368*a^8*b^3*c^11*d^8 - 112*a^8*b^3*c^13*d^6 + 28*a^9*b^2*c^2*d^17 - 32*a^9*b^2*c^4*d^15 - 152*a^9*b^2*c^6*d^13 + 368*a^9*b^2*c^8*d^11 - 292*a^9*b^2*c^10*d^9 + 80*a^9*b^2*c^12*d^7 - 4*a*b^10*c^18*d - 4*a^10*b*c*d^18))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) - (8*tan(e/2 + (f*x)/2)*(8*a^3*b^8*c^19 - 12*a^11*c*d^18 - 12*a*b^10*c^19 + 56*a^11*c^3*d^16 - 104*a^11*c^5*d^14 + 96*a^11*c^7*d^12 - 44*a^11*c^9*d^10 + 8*a^11*c^11*d^8 + 16*a*b^10*c^9*d^10 - 76*a*b^10*c^11*d^8 + 144*a*b^10*c^13*d^6 - 136*a*b^10*c^15*d^4 + 64*a*b^10*c^17*d^2 + 96*a^2*b^9*c^18*d - 64*a^4*b^7*c^18*d + 16*a^9*b^2*c*d^18 + 96*

a^10*b*c^2*d^17 - 448*a^10*b*c^4*d^15 + 832*a^10*b*c^6*d^13 - 768*a^10*b*c^
8*d^11 + 352*a^10*b*c^10*d^9 - 64*a^10*b*c^12*d^7 - 128*a^2*b^9*c^8*d^11 +
608*a^2*b^9*c^10*d^9 - 1152*a^2*b^9*c^12*d^7 + 1088*a^2*b^9*c^14*d^5 - 512*
a^2*b^9*c^16*d^3 + 448*a^3*b^8*c^7*d^12 - 2140*a^3*b^8*c^9*d^10 + 4088*a^3*
b^8*c^11*d^8 - 3912*a^3*b^8*c^13*d^6 + 1888*a^3*b^8*c^15*d^4 - 380*a^3*b^8*
c^17*d^2 - 896*a^4*b^7*c^6*d^13 + 4352*a^4*b^7*c^8*d^11 - 8512*a^4*b^7*c^10
*d^9 + 8448*a^4*b^7*c^12*d^7 - 4352*a^4*b^7*c^14*d^5 + 1024*a^4*b^7*c^16*d^
3 + 1120*a^5*b^6*c^5*d^14 - 5656*a^5*b^6*c^7*d^12 + 11648*a^5*b^6*c^9*d^10
- 12432*a^5*b^6*c^11*d^8 + 7168*a^5*b^6*c^13*d^6 - 2072*a^5*b^6*c^15*d^4 +
224*a^5*b^6*c^17*d^2 - 896*a^6*b^5*c^4*d^15 + 4928*a^6*b^5*c^6*d^13 - 11200
*a^6*b^5*c^8*d^11 + 13440*a^6*b^5*c^10*d^9 - 8960*a^6*b^5*c^12*d^7 + 3136*a
^6*b^5*c^14*d^5 - 448*a^6*b^5*c^16*d^3 + 448*a^7*b^4*c^3*d^16 - 2968*a^7*b^
4*c^5*d^14 + 7952*a^7*b^4*c^7*d^12 - 11088*a^7*b^4*c^9*d^10 + 8512*a^7*b^4*
c^11*d^8 - 3416*a^7*b^4*c^13*d^6 + 560*a^7*b^4*c^15*d^4 - 128*a^8*b^3*c^2*d
^17 + 1280*a^8*b^3*c^4*d^15 - 4288*a^8*b^3*c^6*d^13 + 6912*a^8*b^3*c^8*d^11
 - 5888*a^8*b^3*c^10*d^9 + 2560*a^8*b^3*c^12*d^7 - 448*a^8*b^3*c^14*d^5 - 4
12*a^9*b^2*c^3*d^16 + 1712*a^9*b^2*c^5*d^14 - 3048*a^9*b^2*c^7*d^12 + 2752*
a^9*b^2*c^9*d^10 - 1244*a^9*b^2*c^11*d^8 + 224*a^9*b^2*c^13*d^6))/(a^6*d^14
 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6
 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*
c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*
b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2
*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d
^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a
^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*
d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a
^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13)))/(a^5*d^3 + b^5*c^3 - a
^2*b^3*c^3 - a^3*b^2*d^3 + 3*a^2*b^3*c*d^2 + 3*a^3*b^2*c^2*d - 3*a*b^4*c^2*
d - 3*a^4*b*c*d^2)))/(a^5*d^3 + b^5*c^3 - a^2*b^3*c^3 - a^3*b^2*d^3 + 3*a^2
*b^3*c*d^2 + 3*a^3*b^2*c^2*d - 3*a*b^4*c^2*d - 3*a^4*b*c*d^2)))/(a^5*d^3 +
b^5*c^3 - a^2*b^3*c^3 - a^3*b^2*d^3 + 3*a^2*b^3*c*d^2 + 3*a^3*b^2*c^2*d - 3
*a*b^4*c^2*d - 3*a^4*b*c*d^2)))*(b^2 - a^2)^(1/2)*2i)/(f*(a^5*d^3 + b^5*c^3
 - a^2*b^3*c^3 - a^3*b^2*d^3 + 3*a^2*b^3*c*d^2 + 3*a^3*b^2*c^2*d - 3*a*b^4*
c^2*d - 3*a^4*b*c*d^2)) - ((a*d^5 - 4*a*c^2*d^3 + 6*b*c^3*d^2 - 3*b*c*d^4)/
((a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(c^4 + d^4 - 2*c^2*d^2)) + (d*tan(e/2 + (f
x)/2)(2*a*d^5 - 11*a*c^2*d^3 + 17*b*c^3*d^2 - 8*b*c*d^4))/(c*(a^2*d^2 + b
^2*c^2 - 2*a*b*c*d)*(c^4 + d^4 - 2*c^2*d^2)) + (tan(e/2 + (f*x)/2)^2*(c^2 +
 2*d^2)*(a*d^5 - 4*a*c^2*d^3 + 6*b*c^3*d^2 - 3*b*c*d^4))/(c^2*(a^2*d^2 + b^
2*c^2 - 2*a*b*c*d)*(c^4 + d^4 - 2*c^2*d^2)) + (d*tan(e/2 + (f*x)/2)^3*(2*a*
d^5 - 5*a*c^2*d^3 + 7*b*c^3*d^2 - 4*b*c*d^4))/(c*(a^2*d^2 + b^2*c^2 - 2*a*b
*c*d)*(c^4 + d^4 - 2*c^2*d^2)))/(f*(tan(e/2 + (f*x)/2)^2*(2*c^2 + 4*d^2) +
c^2*tan(e/2 + (f*x)/2)^4 + c^2 + 4*c*d*tan(e/2 + (f*x)/2)^3 + 4*c*d*tan(e/2
 + (f*x)/2))) - (d*atan(((d*(-(c + d)^5*(c - d)^5)^(1/2)*((8*tan(e/2 + (f*x
)/2)*(4*a*b^8*c^13 + a^9*c*d^12 + 4*a^9*c^3*d^10 + 4*a^9*c^5*d^8 - 16*a*b^8
*c^3*d^10 + 76*a*b^8*c^5*d^8 - 162*a*b^8*c^7*d^6 + 176*a*b^8*c^9*d^4 - 96*a

*b^8*c^11*d^2 - 8*a^2*b^7*c^12*d - 16*a^3*b^6*c*d^12 - 4*a^5*b^4*c*d^12 + 2
*a^7*b^2*c*d^12 - 2*a^8*b*c^2*d^11 - 20*a^8*b*c^4*d^9 - 32*a^8*b*c^6*d^7 +
32*a^2*b^7*c^2*d^11 - 152*a^2*b^7*c^4*d^9 + 372*a^2*b^7*c^6*d^7 - 472*a^2*b
^7*c^8*d^5 + 336*a^2*b^7*c^10*d^3 + 72*a^3*b^6*c^3*d^10 - 274*a^3*b^6*c^5*d
^8 + 481*a^3*b^6*c^7*d^6 - 564*a^3*b^6*c^9*d^4 + 40*a^3*b^6*c^11*d^2 + 8*a^
4*b^5*c^2*d^11 + 80*a^4*b^5*c^4*d^9 - 250*a^4*b^5*c^6*d^7 + 612*a^4*b^5*c^8
*d^5 - 144*a^4*b^5*c^10*d^3 - 14*a^5*b^4*c^3*d^10 + 55*a^5*b^4*c^5*d^8 - 41
2*a^5*b^4*c^7*d^6 + 240*a^5*b^4*c^9*d^4 - 4*a^6*b^3*c^2*d^11 + 20*a^6*b^3*c
^4*d^9 + 128*a^6*b^3*c^6*d^7 - 216*a^6*b^3*c^8*d^5 - 9*a^7*b^2*c^3*d^10 + 1
2*a^7*b^2*c^5*d^8 + 112*a^7*b^2*c^7*d^6))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*
d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c
^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d
^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^
5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b
^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2
 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*
b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^
10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^
5*c^13*d - 6*a^5*b*c*d^13) - (8*(4*a*b^8*c^4*d^9 - 16*a*b^8*c^6*d^7 + 24*a*
b^8*c^8*d^5 - 16*a*b^8*c^10*d^3 + 4*a^4*b^5*c*d^12 + 4*a^6*b^3*c*d^12 + 4*a
^8*b*c^3*d^10 + 4*a^8*b*c^5*d^8 - 4*a^2*b^7*c^3*d^10 + 12*a^2*b^7*c^5*d^8 +
 a^2*b^7*c^7*d^6 - 28*a^2*b^7*c^9*d^4 + 28*a^2*b^7*c^11*d^2 - 4*a^3*b^6*c^2
*d^11 + 24*a^3*b^6*c^4*d^9 - 98*a^3*b^6*c^6*d^7 + 164*a^3*b^6*c^8*d^5 - 140
*a^3*b^6*c^10*d^3 - 16*a^4*b^5*c^3*d^10 + 95*a^4*b^5*c^5*d^8 - 188*a^4*b^5*
c^7*d^6 + 240*a^4*b^5*c^9*d^4 - 8*a^5*b^4*c^2*d^11 - 20*a^5*b^4*c^4*d^9 + 6
4*a^5*b^4*c^6*d^7 - 216*a^5*b^4*c^8*d^5 - a^6*b^3*c^3*d^10 + 20*a^6*b^3*c^5
*d^8 + 112*a^6*b^3*c^7*d^6 - 2*a^7*b^2*c^2*d^11 - 20*a^7*b^2*c^4*d^9 - 32*a
^7*b^2*c^6*d^7 + 4*a*b^8*c^12*d + a^8*b*c*d^12))/(a^6*d^14 + b^6*c^14 - 4*a
^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 -
4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^
5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a
^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 6
0*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c
^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 +
80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2
*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 -
 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) + (d*(-(c + d)^5*(c - d)^5)^(1/2)*((8*(4*
a^2*b^8*c^16 + 2*a^10*c^2*d^14 - 6*a^10*c^6*d^10 + 4*a^10*c^8*d^8 + 4*a*b^9
*c^7*d^9 - 18*a*b^9*c^9*d^7 + 36*a*b^9*c^11*d^5 - 34*a*b^9*c^13*d^3 - 32*a^
3*b^7*c^15*d + 4*a^7*b^3*c*d^15 - 10*a^9*b*c^3*d^13 - 12*a^9*b*c^5*d^11 + 5
4*a^9*b*c^7*d^9 - 32*a^9*b*c^9*d^7 - 24*a^2*b^8*c^6*d^10 + 110*a^2*b^8*c^8*
d^8 - 232*a^2*b^8*c^10*d^6 + 234*a^2*b^8*c^12*d^4 - 92*a^2*b^8*c^14*d^2 + 6
0*a^3*b^7*c^5*d^11 - 282*a^3*b^7*c^7*d^9 + 638*a^3*b^7*c^9*d^7 - 702*a^3*b^
7*c^11*d^5 + 318*a^3*b^7*c^13*d^3 - 80*a^4*b^6*c^4*d^12 + 390*a^4*b^6*c^6*d
^10 - 970*a^4*b^6*c^8*d^8 + 1202*a^4*b^6*c^10*d^6 - 654*a^4*b^6*c^12*d^4 +

112*a^4*b^6*c^14*d^2 + 60*a^5*b^5*c^3*d^13 - 310*a^5*b^5*c^5*d^11 + 878*a^5*b^5*c^7*d^9 - 1290*a^5*b^5*c^9*d^7 + 886*a^5*b^5*c^11*d^5 - 224*a^5*b^5*c^13*d^3 - 24*a^6*b^4*c^2*d^14 + 138*a^6*b^4*c^4*d^12 - 466*a^6*b^4*c^6*d^10 + 894*a^6*b^4*c^8*d^8 - 822*a^6*b^4*c^10*d^6 + 280*a^6*b^4*c^12*d^4 - 30*a^7*b^3*c^3*d^13 + 122*a^7*b^3*c^5*d^11 - 394*a^7*b^3*c^7*d^9 + 522*a^7*b^3*c^9*d^7 - 224*a^7*b^3*c^11*d^5 + 2*a^8*b^2*c^2*d^14 + 2*a^8*b^2*c^4*d^12 + 102*a^8*b^2*c^6*d^10 - 218*a^8*b^2*c^8*d^8 + 112*a^8*b^2*c^10*d^6 + 12*a*b^9*c^15*d))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) + (8*tan(e/2 + (f*x)/2)*(8*a*b^9*c^16 + 4*a^10*c*d^15 - 12*a^10*c^5*d^11 + 8*a^10*c^7*d^9 + 4*a*b^9*c^8*d^8 - 8*a*b^9*c^10*d^6 + 12*a*b^9*c^12*d^4 - 16*a*b^9*c^14*d^2 - 40*a^2*b^8*c^15*d + 4*a^8*b^2*c*d^15 - 20*a^9*b*c^2*d^14 - 24*a^9*b*c^4*d^12 + 108*a^9*b*c^6*d^10 - 64*a^9*b*c^8*d^8 - 20*a^2*b^8*c^7*d^9 + 16*a^2*b^8*c^9*d^7 - 12*a^2*b^8*c^11*d^5 + 56*a^2*b^8*c^13*d^3 + 36*a^3*b^7*c^6*d^10 + 76*a^3*b^7*c^8*d^8 - 204*a^3*b^7*c^10*d^6 + 36*a^3*b^7*c^12*d^4 + 56*a^3*b^7*c^14*d^2 - 20*a^4*b^6*c^5*d^11 - 340*a^4*b^6*c^7*d^9 + 804*a^4*b^6*c^9*d^7 - 508*a^4*b^6*c^11*d^5 + 64*a^4*b^6*c^13*d^3 - 20*a^5*b^5*c^4*d^12 + 556*a^5*b^5*c^6*d^10 - 1380*a^5*b^5*c^8*d^8 + 1172*a^5*b^5*c^10*d^6 - 328*a^5*b^5*c^12*d^4 + 36*a^6*b^4*c^3*d^13 - 452*a^6*b^4*c^5*d^11 + 1308*a^6*b^4*c^7*d^9 - 1404*a^6*b^4*c^9*d^7 + 512*a^6*b^4*c^11*d^5 - 20*a^7*b^3*c^2*d^14 + 164*a^7*b^3*c^4*d^12 - 708*a^7*b^3*c^6*d^10 + 1004*a^7*b^3*c^8*d^8 - 440*a^7*b^3*c^10*d^6 + 4*a^8*b^2*c^3*d^13 + 204*a^8*b^2*c^5*d^11 - 436*a^8*b^2*c^7*d^9 + 224*a^8*b^2*c^9*d^7))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) - (d*((8*(4*a^2*b^9*c^19 + 4*a^11*c^2*d^17 - 16*a^11*c^4*d^15 + 24*a^11*c^6*d^13 - 16*a^11*c^8*d^11 + 4*a^11*c^10*d^9 - 4*a*b^10*c^10*d^9 + 16*a*b^10*c^12*d^7 - 24*a*b^10*c^14*d^5 + 16*a*b^10*c^16*d^3 - 28*a^3*b^8*c^18*d - 12*a^10*b*c^3*d^16 + 88*a^10*b*c^5*d^14 - 152*a^10*b*c^7*d^12 + 108*a^10*b*c^9*d^10 - 28*a^10*b*c^11*d^8 + 28*a^2*b^9*c^9*d^10 - 108*a^2*b^9*c^11*d^8 + 152*a^2*b^9*c^13*d^6 - 88*a^2*b^9*c^15*d^4 + 12*a^2*b^9*c^17*d^2 - 80*a^3*b^8*c^8*d^11 + 292*a^3*b^8*c^10*d^9 - 368*a^3 *b^8*c^12*d^7 + 152*a^3*b^8*c^14*d^5 + 32*a^3*b^8*c^16*d^3 + 112*a^4*b^7*c^7*d^12 - 368*a^4*b^7*c^9*d^10 + 352*a^4*b^7*c^11*d^8 + 32*a^4*b^7*c^13*d^6 - 208*a^4*b^7*c^15*d^4 + 80*a^4*b^7*c^17*d^2 - 56*a^5*b^6*c^6*d^13 + 112*a^5*b^6*c^8*d^11 + 112*a^5*b^6*c^10*d^9 - 448*a^5*b^6*c^12*d^7 + 392*a^5*b^6*c^14*d^5 - 112*a^5*b^6*c^16*d^3 - 56*a^6*b^5*c^5*d^14 + 280*a^6*b^5*c^7*d^12 - 560*a^6*b^5*c^9*d^10 + 560*a^6*b^5*c^11*d^8 - 280*a^6*b^5*c^13*d^6 + 56*a^6*b^5*c^15*d^4 + 112*a^7*b^4*c^4*d^15 - 392*a^7*b^4*c^6*d^13 + 448*a^7*b^4*c^8*d^11 - 112*a^7*b^4*c^10*d^9 - 112*a^7*b^4*c^12*d^7 + 56*a^7*b^4*c^14*d^5 - 80*a^8*b^3*c^3*d^16 + 208*a^8*b^3*c^5*d^14 - 32*a^8*b^3*c^7*d^12 - 352*a^8*b^3*c^9*d^10 + 368*a^8*b^3*c^11*d^8 - 112*a^8*b^3*c^13*d^6 + 28*a^9*b^2*c^2*d^17 - 32*a^9*b^2*c^4*d^15 - 152*a^9*b^2*c^6*d^13 + 368*a^9*b^2*c^8*d^11 - 292*a^9*b^2*c^10*d^9 + 80*a^9*b^2*c^12*d^7 - 4*a*b^10*c^18*d - 4*a^10*b*c*d^18))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) - (8*tan(e/2 + (f*x)/2)*(8*a^3*b^8*c^19 - 12*a^11*c*d^18 - 12*a*b^10*c^19 + 56*a^11*c^3*d^16 - 104*a^11*c^5*d^14 + 96*a^11*c^7*d^12 - 44*a^11*c^9*d^10 + 8*a^11*c^11*d^8 + 16*a*b^10*c^9*d^10 - 76*a*b^10*c^11*d^8 + 144*a*b^10*c^13*d^6 - 136*a*b^10*c^15*d^4 + 64*a*b^10*c^17*d^2 + 96*a^2*b^9*c^18*d - 64*a^4*b^7*c^18*d + 16*a^9*b^2*c*d^18 + 96*a^10*b*c^2*d^17 - 448*a^10*b*c^4*d^15 + 832*a^10*b*c^6*d^13 - 768*a^10*b*c^8*d^11 + 352*a^10*b*c^10*d^9 - 64*a^10*b*c^12*d^7 - 128*a^2*b^9*c^8*d^11 + 608*a^2*b^9*c^10*d^9 - 1152*a^2*b^9*c^12*d^7 + 1088*a^2*b^9*c^14*d^5 - 512*a^2*b^9*c^16*d^3 + 448*a^3*b^8*c^7*d^12 - 2140*a^3*b^8*c^9*d^10 + 4088*a^3*b^8*c^11*d^8 - 3912*a^3*b^8*c^13*d^6 + 1888*a^3*b^8*c^15*d^4 - 380*a^3*b^8*c^17*d^2 - 896*a^4*b^7*c^6*d^13 + 4352*a^4*b^7*c^8*d^11 - 8512*a^4*b^7*c^10*d^9 + 8448*a^4*b^7*c^12*d^7 - 4352*a^4*b^7*c^14*d^5 + 1024*a^4*b^7*c^16*d^3 + 1120*a^5*b^6*c^5*d^14 - 5656*a^5*b^6*c^7*d^12 + 11648*a^5*b^6*c^9*d^10 - 12432*a^5*b^6*c^11*d^8 + 7168*a^5*b^6*c^13*d^6 - 2072*a^5*b^6*c^15*d^4 + 224*a^5*b^6*c^17*d^2 - 896*a^6*b^5*c^4*d^15 + 4928*a^6*b^5*c^6*d^13 - 11200*a^6*b^5*c^8*d^11 + 13440*a^6*b^5*c^10*d^9 - 8960*a^6*b^5*c^12*d^7 + 3136*a^6*b^5*c^14*d^5 - 448*a^6*b^5*c^16*d^3 + 448*a^7*b^4*c^3*d^16 - 2968*a^7*b^4*c^5*d^14 + 7952*a^7*b^4*c^7*d^12 - 11088*a^7*b^4*c^9*d^10 + 8512*a^7*b^4*c^11*d^8 - 3416*a^7*b^4*c^13*d^6 + 560*a^7*b^4*c^15*d^4 - 128*a^8*b^3*c^2*d^17 + 1280*a^8*b^3*c^4*d^15 - 4288*a^8*b^3*c^6*d^13 + 6912*a^8*b^3*c^8*d^11 - 5888*a^8*b^3*c^10*d^9 + 2560*a^8*b^3*c^12*d^7 - 448*a^8*b^3*c^14*d^5 - 412*a^9*b^2*c^3*d^16 + 1712*a^9*b^2*c^5*d^14 - 3048*a^9*b^2*c^7*d^12 + 2752*a^9*b^2*c^9*d^10 - 1244*a^9*b^2*c^11*d^8 + 224*a^9*b^2*c^13*d^6))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6* b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13))*(-(c + d)^5*(c - d)^5)^(1/2)*(a^2*d^4 + 6*b^2*c^4 + 2*b^2*d^4 + 2*a^2*c^2*d^2 - 5*b^2*c^2*d^2 - 6*a*b*c^3*d))/(2*(a^3*d^13 + b^3*c^13 - 5*a^3*c^2*d^11 + 10*a^3*c^4*d^9 - 10*a^3*c^6*d^7 + 5*a^3*c^8*d^5 - a^3*c^10*d^3 - b^3*c^3*d^10 + 5*b^3*c^5*d^8 - 10*b^3*c^7*d^6 + 10*b^3*c^9*d^4 - 5*b^3*c^11*d^2 + 3*a*b^2*c^2*d^11 - 15*a*b^2*c^4*d^9 + 30*a*b^2*c^6*d^7 - 30*a*b^2*c^8*d^5 + 15*a*b^2*c^10*d^3 + 15*a^2*b*c^3*d^10 - 30*a^2*b*c^5*d^8 + 30*a^2*b*c^7*d^6 - 15*a^2*b*c^9*d^4 + 3*a^2*b*c^11*d^2 - 3*a*b^2*c^12*d - 3*a^2*b*c*d^12)))*(a^2*d^4 + 6*b^2*c^4 + 2*b^2*d^4 + 2*a^2*c^2*d^2 - 5*b^2*c^2*d^2 - 6*a*b*c^3*d))/(2*(a^3*d^13 + b^3*c^13 - 5*a^3*c^2*d^11 + 10*a^3*c^4*d^9 - 10*a^3*c^6*d^7 + 5*a^3*c^8*d^5 - a^3*c^10*d^3 - b^3*c^3*d^10 + 5*b^3*c^5*d^8 - 10*b^3*c^7*d^6 + 10*b^3*c^9*d^4 - 5*b^3*c^11*d^2 + 3*a*b^2*c^2*d^11 - 15*a*b^2*c^4*d^9 + 30*a*b^2*c^6*d^7 - 30*a*b^2*c^8*d^5 + 15*a*b^2*c^10*d^3 + 15*a^2*b*c^3*d^10 - 30*a^2*b*c^5*d^8 + 30*a^2*b*c^7*d^6 - 15*a^2*b*c^9*d^4 + 3*a^2*b*c^11*d^2 - 3*a*b^2*c^12*d - 3*a^2*b*c*d^12)))*(a^2*d^4 + 6*b^2*c^4 + 2*b^2*d^4 + 2*a^2*c^2*d^2 - 5*b^2*c^2*d^2 - 6*a*b*c^3*d)*1i)/(2*(a^3*d^13 + b^3*c^13 - 5*a^3*c^2*d^11 + 10*a^3*c^4*d^9 - 10*a^3*c^6*d^7 + 5*a^3*c^8*d^5 - a^3*c^10*d^3 - b^3*c^3*d^10 + 5*b^3*c^5*d^8 - 10*b^3*c^7*d^6 + 10*b^3*c^9*d^4 - 5*b^3*c^11*d^2 + 3*a*b^2*c^2*d^11 - 15*a*b^2*c^4*d^9 + 30*a*b^2*c^6*d^7 - 30*a*b^2*c^8*d^5 + 15*a*b^2*c^10*d^3 + 15*a^2*b*c^3*d^10 - 30*a^2*b*c^5*d^8 + 30*a^2*b*c^7*d^6 - 15*a^2*b*c^9*d^4 + 3*a^2*b*c^11*d^2 - 3*a*b^2*c^12*d - 3*a^2*b*c*d^12)) - (d*(-(c + d)^5*(c - d)^5)^(1/2)*((8*(4*a*b^8*c^4*d^9 - 16*a*b^8*c^6*d^7 + 24*a*b^8*c^8*d^5 - 16*a*b^8*c^10*d^3 + 4*a^4*b^5*c*d^12 + 4*a^6*b^3*c*d^12 + 4*a^8*b*c^3*d^10 + 4*a^8*b*c^5*d^8 - 4*a^2*b^7*c^3*d^10 + 12*a^2*b^7*c^5*d^8 + a^2*b^7*c^7*d^6 - 28*a^2*b^7*c^9*d^4 + 28*a^2*b^7*c^11*d^2 - 4*a^3*b^6*c^2*d^11 + 24*a^3*b^6*c^4*d^9 - 98*a^3*b^6*c^6*d^7 + 164*a^3*b^6*c^8*d^5 - 140*a^3*b^6*c^10*d^3 - 16*a^4*b^5*c^3*d^10 + 95*a^4*b^5*c^5*d^8 - 188*a^4*b^5*c^7*d^6 + 240*a^4*b^5*c^9*d^4 - 8*a^5*b^4*c^2*d^11 - 20*a^5*b^4*c^4*d^9 + 64*a^5*b^4*c^6*d^7 - 216*a^5*b^4*c^8*d^5 - a^6*b^3*c^3*d^10 + 20*a^6*b^3*c^5*d^8 + 112*a^6*b^3*c^7*d^6 - 2*a^7*b^2*c^2*d^11 - 20*a^7*b^2*c^4*d^9 - 32*a^7*b^2*c^6*d^7 + 4*a*b^8*c^12*d + a^8*b*c*d^12))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*

d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c
*d^13) - (8*tan(e/2 + (f*x)/2)*(4*a*b^8*c^13 + a^9*c*d^12 + 4*a^9*c^3*d^10
+ 4*a^9*c^5*d^8 - 16*a*b^8*c^3*d^10 + 76*a*b^8*c^5*d^8 - 162*a*b^8*c^7*d^6
+ 176*a*b^8*c^9*d^4 - 96*a*b^8*c^11*d^2 - 8*a^2*b^7*c^12*d - 16*a^3*b^6*c*d
^12 - 4*a^5*b^4*c*d^12 + 2*a^7*b^2*c*d^12 - 2*a^8*b*c^2*d^11 - 20*a^8*b*c^4
*d^9 - 32*a^8*b*c^6*d^7 + 32*a^2*b^7*c^2*d^11 - 152*a^2*b^7*c^4*d^9 + 372*a
^2*b^7*c^6*d^7 - 472*a^2*b^7*c^8*d^5 + 336*a^2*b^7*c^10*d^3 + 72*a^3*b^6*c^
3*d^10 - 274*a^3*b^6*c^5*d^8 + 481*a^3*b^6*c^7*d^6 - 564*a^3*b^6*c^9*d^4 +
40*a^3*b^6*c^11*d^2 + 8*a^4*b^5*c^2*d^11 + 80*a^4*b^5*c^4*d^9 - 250*a^4*b^5
*c^6*d^7 + 612*a^4*b^5*c^8*d^5 - 144*a^4*b^5*c^10*d^3 - 14*a^5*b^4*c^3*d^10
 + 55*a^5*b^4*c^5*d^8 - 412*a^5*b^4*c^7*d^6 + 240*a^5*b^4*c^9*d^4 - 4*a^6*b
^3*c^2*d^11 + 20*a^6*b^3*c^4*d^9 + 128*a^6*b^3*c^6*d^7 - 216*a^6*b^3*c^8*d^
5 - 9*a^7*b^2*c^3*d^10 + 12*a^7*b^2*c^5*d^8 + 112*a^7*b^2*c^7*d^6))/(a^6*d^
14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d
^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^
5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^
5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a
^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10
*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120
*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^
2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15
*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) + (d*(-(c + d)^5*(c -
d)^5)^(1/2)*((8*(4*a^2*b^8*c^16 + 2*a^10*c^2*d^14 - 6*a^10*c^6*d^10 + 4*a^1
0*c^8*d^8 + 4*a*b^9*c^7*d^9 - 18*a*b^9*c^9*d^7 + 36*a*b^9*c^11*d^5 - 34*a*b
^9*c^13*d^3 - 32*a^3*b^7*c^15*d + 4*a^7*b^3*c*d^15 - 10*a^9*b*c^3*d^13 - 12
*a^9*b*c^5*d^11 + 54*a^9*b*c^7*d^9 - 32*a^9*b*c^9*d^7 - 24*a^2*b^8*c^6*d^10
 + 110*a^2*b^8*c^8*d^8 - 232*a^2*b^8*c^10*d^6 + 234*a^2*b^8*c^12*d^4 - 92*a
^2*b^8*c^14*d^2 + 60*a^3*b^7*c^5*d^11 - 282*a^3*b^7*c^7*d^9 + 638*a^3*b^7*c
^9*d^7 - 702*a^3*b^7*c^11*d^5 + 318*a^3*b^7*c^13*d^3 - 80*a^4*b^6*c^4*d^12
+ 390*a^4*b^6*c^6*d^10 - 970*a^4*b^6*c^8*d^8 + 1202*a^4*b^6*c^10*d^6 - 654*
a^4*b^6*c^12*d^4 + 112*a^4*b^6*c^14*d^2 + 60*a^5*b^5*c^3*d^13 - 310*a^5*b^5
*c^5*d^11 + 878*a^5*b^5*c^7*d^9 - 1290*a^5*b^5*c^9*d^7 + 886*a^5*b^5*c^11*d
^5 - 224*a^5*b^5*c^13*d^3 - 24*a^6*b^4*c^2*d^14 + 138*a^6*b^4*c^4*d^12 - 46
6*a^6*b^4*c^6*d^10 + 894*a^6*b^4*c^8*d^8 - 822*a^6*b^4*c^10*d^6 + 280*a^6*b
^4*c^12*d^4 - 30*a^7*b^3*c^3*d^13 + 122*a^7*b^3*c^5*d^11 - 394*a^7*b^3*c^7*
d^9 + 522*a^7*b^3*c^9*d^7 - 224*a^7*b^3*c^11*d^5 + 2*a^8*b^2*c^2*d^14 + 2*a
^8*b^2*c^4*d^12 + 102*a^8*b^2*c^6*d^10 - 218*a^8*b^2*c^8*d^8 + 112*a^8*b^2*
c^10*d^6 + 12*a*b^9*c^15*d))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*
c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^
6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5
*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^
5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 +
90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3
*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 -
 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b

^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*
a^5*b*c*d^13) + (8*tan(e/2 + (f*x)/2)*(8*a*b^9*c^16 + 4*a^10*c*d^15 - 12*a^
10*c^5*d^11 + 8*a^10*c^7*d^9 + 4*a*b^9*c^8*d^8 - 8*a*b^9*c^10*d^6 + 12*a*b^
9*c^12*d^4 - 16*a*b^9*c^14*d^2 - 40*a^2*b^8*c^15*d + 4*a^8*b^2*c*d^15 - 20*
a^9*b*c^2*d^14 - 24*a^9*b*c^4*d^12 + 108*a^9*b*c^6*d^10 - 64*a^9*b*c^8*d^8
- 20*a^2*b^8*c^7*d^9 + 16*a^2*b^8*c^9*d^7 - 12*a^2*b^8*c^11*d^5 + 56*a^2*b^
8*c^13*d^3 + 36*a^3*b^7*c^6*d^10 + 76*a^3*b^7*c^8*d^8 - 204*a^3*b^7*c^10*d^
6 + 36*a^3*b^7*c^12*d^4 + 56*a^3*b^7*c^14*d^2 - 20*a^4*b^6*c^5*d^11 - 340*a
^4*b^6*c^7*d^9 + 804*a^4*b^6*c^9*d^7 - 508*a^4*b^6*c^11*d^5 + 64*a^4*b^6*c^
13*d^3 - 20*a^5*b^5*c^4*d^12 + 556*a^5*b^5*c^6*d^10 - 1380*a^5*b^5*c^8*d^8
+ 1172*a^5*b^5*c^10*d^6 - 328*a^5*b^5*c^12*d^4 + 36*a^6*b^4*c^3*d^13 - 452*
a^6*b^4*c^5*d^11 + 1308*a^6*b^4*c^7*d^9 - 1404*a^6*b^4*c^9*d^7 + 512*a^6*b^
4*c^11*d^5 - 20*a^7*b^3*c^2*d^14 + 164*a^7*b^3*c^4*d^12 - 708*a^7*b^3*c^6*d
^10 + 1004*a^7*b^3*c^8*d^8 - 440*a^7*b^3*c^10*d^6 + 4*a^8*b^2*c^3*d^13 + 20
4*a^8*b^2*c^5*d^11 - 436*a^8*b^2*c^7*d^9 + 224*a^8*b^2*c^9*d^7))/(a^6*d^14
+ b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6
+ b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c
^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b
*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*
b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^
4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^
3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d
^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^
4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) + (d*((8*(4*a^2*b^9*c^19
+ 4*a^11*c^2*d^17 - 16*a^11*c^4*d^15 + 24*a^11*c^6*d^13 - 16*a^11*c^8*d^11
+ 4*a^11*c^10*d^9 - 4*a*b^10*c^10*d^9 + 16*a*b^10*c^12*d^7 - 24*a*b^10*c^14
*d^5 + 16*a*b^10*c^16*d^3 - 28*a^3*b^8*c^18*d - 12*a^10*b*c^3*d^16 + 88*a^1
0*b*c^5*d^14 - 152*a^10*b*c^7*d^12 + 108*a^10*b*c^9*d^10 - 28*a^10*b*c^11*d
^8 + 28*a^2*b^9*c^9*d^10 - 108*a^2*b^9*c^11*d^8 + 152*a^2*b^9*c^13*d^6 - 88
*a^2*b^9*c^15*d^4 + 12*a^2*b^9*c^17*d^2 - 80*a^3*b^8*c^8*d^11 + 292*a^3*b^8
*c^10*d^9 - 368*a^3*b^8*c^12*d^7 + 152*a^3*b^8*c^14*d^5 + 32*a^3*b^8*c^16*d
^3 + 112*a^4*b^7*c^7*d^12 - 368*a^4*b^7*c^9*d^10 + 352*a^4*b^7*c^11*d^8 + 3
2*a^4*b^7*c^13*d^6 - 208*a^4*b^7*c^15*d^4 + 80*a^4*b^7*c^17*d^2 - 56*a^5*b^
6*c^6*d^13 + 112*a^5*b^6*c^8*d^11 + 112*a^5*b^6*c^10*d^9 - 448*a^5*b^6*c^12
*d^7 + 392*a^5*b^6*c^14*d^5 - 112*a^5*b^6*c^16*d^3 - 56*a^6*b^5*c^5*d^14 +
280*a^6*b^5*c^7*d^12 - 560*a^6*b^5*c^9*d^10 + 560*a^6*b^5*c^11*d^8 - 280*a^
6*b^5*c^13*d^6 + 56*a^6*b^5*c^15*d^4 + 112*a^7*b^4*c^4*d^15 - 392*a^7*b^4*c
^6*d^13 + 448*a^7*b^4*c^8*d^11 - 112*a^7*b^4*c^10*d^9 - 112*a^7*b^4*c^12*d^
7 + 56*a^7*b^4*c^14*d^5 - 80*a^8*b^3*c^3*d^16 + 208*a^8*b^3*c^5*d^14 - 32*a
^8*b^3*c^7*d^12 - 352*a^8*b^3*c^9*d^10 + 368*a^8*b^3*c^11*d^8 - 112*a^8*b^3
*c^13*d^6 + 28*a^9*b^2*c^2*d^17 - 32*a^9*b^2*c^4*d^15 - 152*a^9*b^2*c^6*d^1
3 + 368*a^9*b^2*c^8*d^11 - 292*a^9*b^2*c^10*d^9 + 80*a^9*b^2*c^12*d^7 - 4*a
*b^10*c^18*d - 4*a^10*b*c*d^18))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*
a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 +
6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a

*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) - (8*tan(e/2 + (f*x)/2)*(8*a^3*b^8*c^19 - 12*a^11*c*d^18 - 12*a*b^10*c^19 + 56*a^11*c^3*d^16 - 104*a^11*c^5*d^14 + 96*a^11*c^7*d^12 - 44*a^11*c^9*d^10 + 8*a^11*c^11*d^8 + 16*a*b^10*c^9*d^10 - 76*a*b^10*c^11*d^8 + 144*a*b^10*c^13*d^6 - 136*a*b^10*c^15*d^4 + 64*a*b^10*c^17*d^2 + 96*a^2*b^9*c^18*d - 64*a^4*b^7*c^18*d + 16*a^9*b^2*c*d^18 + 96*a^10*b*c^2*d^17 - 448*a^10*b*c^4*d^15 + 832*a^10*b*c^6*d^13 - 768*a^10*b*c^8*d^11 + 352*a^10*b*c^10*d^9 - 64*a^10*b*c^12*d^7 - 128*a^2*b^9*c^8*d^11 + 608*a^2*b^9*c^10*d^9 - 1152*a^2*b^9*c^12*d^7 + 1088*a^2*b^9*c^14*d^5 - 512*a^2*b^9*c^16*d^3 + 448*a^3*b^8*c^7*d^12 - 2140*a^3*b^8*c^9*d^10 + 4088*a^3*b^8*c^11*d^8 - 3912*a^3*b^8*c^13*d^6 + 1888*a^3*b^8*c^15*d^4 - 380*a^3*b^8*c^17*d^2 - 896*a^4*b^7*c^6*d^13 + 4352*a^4*b^7*c^8*d^11 - 8512*a^4*b^7*c^10*d^9 + 8448*a^4*b^7*c^12*d^7 - 4352*a^4*b^7*c^14*d^5 + 1024*a^4*b^7*c^16*d^3 + 1120*a^5*b^6*c^5*d^14 - 5656*a^5*b^6*c^7*d^12 + 11648*a^5*b^6*c^9*d^10 - 12432*a^5*b^6*c^11*d^8 + 7168*a^5*b^6*c^13*d^6 - 2072*a^5*b^6*c^15*d^4 + 224*a^5*b^6*c^17*d^2 - 896*a^6*b^5*c^4*d^15 + 4928*a^6*b^5*c^6*d^13 - 11200*a^6*b^5*c^8*d^11 + 13440*a^6*b^5*c^10*d^9 - 8960*a^6*b^5*c^12*d^7 + 3136*a^6*b^5*c^14*d^5 - 448*a^6*b^5*c^16*d^3 + 448*a^7*b^4*c^3*d^16 - 2968*a^7*b^4*c^5*d^14 + 7952*a^7*b^4*c^7*d^12 - 11088*a^7*b^4*c^9*d^10 + 8512*a^7*b^4*c^11*d^8 - 3416*a^7*b^4*c^13*d^6 + 560*a^7*b^4*c^15*d^4 - 128*a^8*b^3*c^2*d^17 + 1280*a^8*b^3*c^4*d^15 - 4288*a^8*b^3*c^6*d^13 + 6912*a^8*b^3*c^8*d^11 - 5888*a^8*b^3*c^10*d^9 + 2560*a^8*b^3*c^12*d^7 - 448*a^8*b^3*c^14*d^5 - 412*a^9*b^2*c^3*d^16 + 1712*a^9*b^2*c^5*d^14 - 3048*a^9*b^2*c^7*d^12 + 2752*a^9*b^2*c^9*d^10 - 1244*a^9*b^2*c^11*d^8 + 224*a^9*b^2*c^13*d^6))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13))*(-(c + d)^5*(c - d)^5)^(1/2)*(a^2*d^4 + 6*b^2*c^4 + 2*b^2*d^4 + 2*a^2*c^2*d^2 - 5*b^2*c^2*d^2 - 6*a*b*c^3*d))/(2*(a^3*d^13 + b^3*c^13 - 5*a^3*c^2*d^11 + 10*a^3*c^4*d^9 - 10*a^3*c^6*d^7 + 5*a^3*c^8*d^5 - a^3*c^10*d^3 - b^3*c^3*d^10 + 5*b^3*c^5*d^8 - 10*b^3*c^7*d^6 + 10*b^3*c^9*d^4 - 5*b^3*c^11*d^2 + 3*a*b^2*c^2*d^11 - 15*a*b^2*c^4*d^9 + 30*a*b^2*c^6*d^7 - 30*a*b^2*c^8*d^5 + 15*a*b^2*c^10*d^3 + 15*a^2*b*c^3*d^10 - 30*a^2*b*c^5*d^8 + 30*a^2*b*c^7*d^6 - 15*a^2*b*c^9*d^4 + 3*a^2*b*c^11*d^2 - 3*a*b^2*c^12*d - 3*a^2*b*c*d^12)))*(a^2*d^4 + 6*b^2*c^4 + 2*b^2*d^4 + 2* a^2*c^2*d^2 - 5*b^2*c^2*d^2 - 6*a*b*c^3*d))/(2*(a^3*d^13 + b^3*c^13 - 5*a^3*c^2*d^11 + 10*a^3*c^4*d^9 - 10*a^3*c^6*d^7 + 5*a^3*c^8*d^5 - a^3*c^10*d^3 - b^3*c^3*d^10 + 5*b^3*c^5*d^8 - 10*b^3*c^7*d^6 + 10*b^3*c^9*d^4 - 5*b^3*c^11*d^2 + 3*a*b^2*c^2*d^11 - 15*a*b^2*c^4*d^9 + 30*a*b^2*c^6*d^7 - 30*a*b^2*c^8*d^5 + 15*a*b^2*c^10*d^3 + 15*a^2*b*c^3*d^10 - 30*a^2*b*c^5*d^8 + 30*a^2*b*c^7*d^6 - 15*a^2*b*c^9*d^4 + 3*a^2*b*c^11*d^2 - 3*a*b^2*c^12*d - 3*a^2*b*c*d^12)))*(a^2*d^4 + 6*b^2*c^4 + 2*b^2*d^4 + 2*a^2*c^2*d^2 - 5*b^2*c^2*d^2 - 6*a*b*c^3*d)*1i)/(2*(a^3*d^13 + b^3*c^13 - 5*a^3*c^2*d^11 + 10*a^3*c^4*d^9 - 10*a^3*c^6*d^7 + 5*a^3*c^8*d^5 - a^3*c^10*d^3 - b^3*c^3*d^10 + 5*b^3*c^5*d^8 - 10*b^3*c^7*d^6 + 10*b^3*c^9*d^4 - 5*b^3*c^11*d^2 + 3*a*b^2*c^2*d^11 - 15*a*b^2*c^4*d^9 + 30*a*b^2*c^6*d^7 - 30*a*b^2*c^8*d^5 + 15*a*b^2*c^10*d^3 + 15*a^2*b*c^3*d^10 - 30*a^2*b*c^5*d^8 + 30*a^2*b*c^7*d^6 - 15*a^2*b*c^9*d^4 + 3*a^2*b*c^11*d^2 - 3*a*b^2*c^12*d - 3*a^2*b*c*d^12)))/((16*(36*a*b^7*c^5*d^5 - 18*a*b^7*c^3*d^7 - 34*a*b^7*c^7*d^3 + 4*a^3*b^5*c*d^9 + a^5*b^3*c*d^9 + 2*a^2*b^6*c^2*d^8 - 25*a^2*b^6*c^4*d^6 + 50*a^2*b^6*c^6*d^4 - 36*a^2*b^6*c^8*d^2 - a^3*b^5*c^3*d^7 - 16*a^3*b^5*c^5*d^5 + 40*a^3*b^5*c^7*d^3 + a^4*b^4*c^2*d^8 - 8*a^4*b^4*c^4*d^6 - 20*a^4*b^4*c^6*d^4 + 4*a^5*b^3*c^3*d^7 + 4*a^5*b^3*c^5*d^5 + 4*a*b^7*c*d^9 + 12*a*b^7*c^9*d))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) + (16*tan(e/2 + (f*x)/2)*(4*a*b^7*c^2*d^8 - 26*a*b^7*c^4*d^6 + 52*a*b^7*c^6*d^4 - 48*a*b^7*c^8*d^2 + 4*a^2*b^6*c*d^9 + 2*a^4*b^4*c*d^9 - 2*a^2*b^6*c^3*d^7 - 20*a^2*b^6*c^5*d^5 + 72*a^2*b^6*c^7*d^3 + 2*a^3*b^5*c^2*d^8 - 16*a^3*b^5*c^4*d^6 - 40*a^3*b^5*c^6*d^4 + 8*a^4*b^4*c^3*d^7 + 8*a^4*b^4*c^5*d^5))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) - (d*(-(c + d)^5*(c - d)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(4*a*b^8*c^13 + a^9*c*d^12 + 4*a^9*c^3*d^10 + 4*a^9*c^5*d^8 - 16*a*b^8*c^3*d^10 + 76*a*b^8*c^5*d^8 - 162*a*b^8*c^7*d^6 + 176*a*b^8*c^9*d^4 - 96*a*b^8*c^11*d^2 - 8*a^2*b^7*c^12*d - 16*a^3*b^6*c*d^12 - 4*a^5*b^4*c*d^12 + 2*a^7*b^2*c*d^12 - 2*a^8*b*c^2*d^11 - 20*a^8*b*c^4*d^9 - 32*a^8*b*c^6*d^7 + 32*a^2*b^7*c^2*d^11 - 152*a^2*b^7*c^4*d^9 + 372*a^2*b^7*c^6*d^7 - 47

2*a^2*b^7*c^8*d^5 + 336*a^2*b^7*c^10*d^3 + 72*a^3*b^6*c^3*d^10 - 274*a^3*b^6*c^5*d^8 + 481*a^3*b^6*c^7*d^6 - 564*a^3*b^6*c^9*d^4 + 40*a^3*b^6*c^11*d^2 + 8*a^4*b^5*c^2*d^11 + 80*a^4*b^5*c^4*d^9 - 250*a^4*b^5*c^6*d^7 + 612*a^4*b^5*c^8*d^5 - 144*a^4*b^5*c^10*d^3 - 14*a^5*b^4*c^3*d^10 + 55*a^5*b^4*c^5*d^8 - 412*a^5*b^4*c^7*d^6 + 240*a^5*b^4*c^9*d^4 - 4*a^6*b^3*c^2*d^11 + 20*a^6*b^3*c^4*d^9 + 128*a^6*b^3*c^6*d^7 - 216*a^6*b^3*c^8*d^5 - 9*a^7*b^2*c^3*d^10 + 12*a^7*b^2*c^5*d^8 + 112*a^7*b^2*c^7*d^6))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) - (8*(4*a*b^8*c^4*d^9 - 16*a*b^8*c^6*d^7 + 24*a*b^8*c^8*d^5 - 16*a*b^8*c^10*d^3 + 4*a^4*b^5*c*d^12 + 4*a^6*b^3*c*d^12 + 4*a^8*b*c^3*d^10 + 4*a^8*b*c^5*d^8 - 4*a^2*b^7*c^3*d^10 + 12*a^2*b^7*c^5*d^8 + a^2*b^7*c^7*d^6 - 28*a^2*b^7*c^9*d^4 + 28*a^2*b^7*c^11*d^2 - 4*a^3*b^6*c^2*d^11 + 24*a^3*b^6*c^4*d^9 - 98*a^3*b^6*c^6*d^7 + 164*a^3*b^6*c^8*d^5 - 140*a^3*b^6*c^10*d^3 - 16*a^4*b^5*c^3*d^10 + 95*a^4*b^5*c^5*d^8 - 188*a^4*b^5*c^7*d^6 + 240*a^4*b^5*c^9*d^4 - 8*a^5*b^4*c^2*d^11 - 20*a^5*b^4*c^4*d^9 + 64*a^5*b^4*c^6*d^7 - 216*a^5*b^4*c^8*d^5 - a^6*b^3*c^3*d^10 + 20*a^6*b^3*c^5*d^8 + 112*a^6*b^3*c^7*d^6 - 2*a^7*b^2*c^2*d^11 - 20*a^7*b^2*c^4*d^9 - 32*a^7*b^2*c^6*d^7 + 4*a*b^8*c^12*d + a^8*b*c*d^12))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) + (d*(-(c + d)^5*(c - d)^5)^(1/2)*((8*(4*a^2*b^8*c^16 + 2*a^10*c^2*d^14 - 6*a^10*c^6*d^10 + 4*a^10*c^8*d^8 + 4*a*b^9*c^7*d^9 - 18*a*b^9*c^9*d^7 + 36*a*b^9*c^11*d^5 - 34*a*b^9*c^13*d^3 - 32*a^3*b^7*c^15*d + 4*a^7*b^3*c*d^15 - 10*a^9*b*c^3*d^13 - 12*a^9*b*c^5*d^11 + 54*a^9*b*c^7*d^9 - 32*a^9*b*c^9*d^7 - 24*a^2*b^8*c^6*d^10 + 110*a^2*b^8*c^8*d^8 - 232*a^2*b^8*c^10*d^6 + 234*a^2*b^8*c^12*d^4 - 92*a^2*b^8*c^14*d^2 + 60*a^3*b^7*c^5*d^11 - 282*a^3*b^7*c^7*d^9 + 638*a^3*b^7*c^9*d^7 - 702*a^3*b^7*c^11*d^5 + 318*a^3*b^7*c^13*d^3 - 80*a^4*b^6*c^4*d^12 + 390*a^4*b^6*c^6*d^10 - 970*a^4*b^6*c^8*d^8 + 1202*a^4*b^6*c^10*d^6 - 654*a^4*b^6*c^12*d^4 + 112*a^4*b^6*c^14*d^2 + 60*a^5*b^5*c^3*d^13 - 310*a^5*b^5*c^5*d^11 + 878*a^5*b^5*c^7*d^9 - 1290*a^5*b^5*c^9*d^7 + 886*a^5*b^5*c^11*d^5 - 224*a^5*b^5*c^13*d^3 - 24*a^6*b^4*c^2*d^14 + 138*a^6*b^4*c^4*d^12 - 466*a^6*b^4*c^

6*d^10 + 894*a^6*b^4*c^8*d^8 - 822*a^6*b^4*c^10*d^6 + 280*a^6*b^4*c^12*d^4 - 30*a^7*b^3*c^3*d^13 + 122*a^7*b^3*c^5*d^11 - 394*a^7*b^3*c^7*d^9 + 522*a^7*b^3*c^9*d^7 - 224*a^7*b^3*c^11*d^5 + 2*a^8*b^2*c^2*d^14 + 2*a^8*b^2*c^4*d^12 + 102*a^8*b^2*c^6*d^10 - 218*a^8*b^2*c^8*d^8 + 112*a^8*b^2*c^10*d^6 + 12*a*b^9*c^15*d))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) + (8*tan(e/2 + (f*x)/2)*(8*a*b^9*c^16 + 4*a^10*c*d^15 - 12*a^10*c^5*d^11 + 8*a^10*c^7*d^9 + 4*a*b^9*c^8*d^8 - 8*a*b^9*c^10*d^6 + 12*a*b^9*c^12*d^4 - 16*a*b^9*c^14*d^2 - 40*a^2*b^8*c^15*d + 4*a^8*b^2*c*d^15 - 20*a^9*b*c^2*d^14 - 24*a^9*b*c^4*d^12 + 108*a^9*b*c^6*d^10 - 64*a^9*b*c^8*d^8 - 20*a^2*b^8*c^7*d^9 + 16*a^2*b^8*c^9*d^7 - 12*a^2*b^8*c^11*d^5 + 56*a^2*b^8*c^13*d^3 + 36*a^3*b^7*c^6*d^10 + 76*a^3*b^7*c^8*d^8 - 204*a^3*b^7*c^10*d^6 + 36*a^3*b^7*c^12*d^4 + 56*a^3*b^7*c^14*d^2 - 20*a^4*b^6*c^5*d^11 - 340*a^4*b^6*c^7*d^9 + 804*a^4*b^6*c^9*d^7 - 508*a^4*b^6*c^11*d^5 + 64*a^4*b^6*c^13*d^3 - 20*a^5*b^5*c^4*d^12 + 556*a^5*b^5*c^6*d^10 - 1380*a^5*b^5*c^8*d^8 + 1172*a^5*b^5*c^10*d^6 - 328*a^5*b^5*c^12*d^4 + 36*a^6*b^4*c^3*d^13 - 452*a^6*b^4*c^5*d^11 + 1308*a^6*b^4*c^7*d^9 - 1404*a^6*b^4*c^9*d^7 + 512*a^6*b^4*c^11*d^5 - 20*a^7*b^3*c^2*d^14 + 164*a^7*b^3*c^4*d^12 - 708*a^7*b^3*c^6*d^10 + 1004*a^7*b^3*c^8*d^8 - 440*a^7*b^3*c^10*d^6 + 4*a^8*b^2*c^3*d^13 + 204*a^8*b^2*c^5*d^11 - 436*a^8*b^2*c^7*d^9 + 224*a^8*b^2*c^9*d^7))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) - (d*((8*(4*a^2*b^9*c^19 + 4*a^11*c^2*d^17 - 16*a^11*c^4*d^15 + 24*a^11*c^6*d^13 - 16*a^11*c^8*d^11 + 4*a^11*c^10*d^9 - 4*a*b^10*c^10*d^9 + 16*a*b^10*c^12*d^7 - 24*a*b^10*c^14*d^5 + 16*a*b^10*c^16*d^3 - 28*a^3*b^8*c^18*d - 12*a^10*b*c^3*d^16 + 88*a^10*b*c^5*d^14 - 152*a^10*b*c^7*d^12 + 108*a^10*b*c^9*d^10 - 28*a^10*b*c^11*d^8 + 28*a^2*b^9*c^9*d^10 - 108*a^2*b^9*c^11*d^8 + 152*a^2*b^9*c^13*d^6 - 88*a^2*b^9*c^15*d^4 + 12*a^2*b^9*c^17*d^2 - 80*a^3*b^8*c^8*d^11 + 292*a^3*b^8*c^10*d^9 - 368*a^3*b^8*c^12*d^7 + 152*a^3*b^8*c^14*d^5 + 32*a^3*b^8*c^16*d^3 + 112*a^4*b^7*c^7*d^12 - 368*a^4*b^7*c^9*d^10 + 352*a^4*b^7*c^11*d^8 + 32*a^4*b^7*c^13*d^6 - 208*a^4*b^7*c^15*d^4 + 80*a^4*b^7*c^17*d^2 - 56*a^5*b^6*c^6*d^13 +

112*a^5*b^6*c^8*d^11 + 112*a^5*b^6*c^10*d^9 - 448*a^5*b^6*c^12*d^7 + 392*a
^5*b^6*c^14*d^5 - 112*a^5*b^6*c^16*d^3 - 56*a^6*b^5*c^5*d^14 + 280*a^6*b^5*
c^7*d^12 - 560*a^6*b^5*c^9*d^10 + 560*a^6*b^5*c^11*d^8 - 280*a^6*b^5*c^13*d
^6 + 56*a^6*b^5*c^15*d^4 + 112*a^7*b^4*c^4*d^15 - 392*a^7*b^4*c^6*d^13 + 44
8*a^7*b^4*c^8*d^11 - 112*a^7*b^4*c^10*d^9 - 112*a^7*b^4*c^12*d^7 + 56*a^7*b
^4*c^14*d^5 - 80*a^8*b^3*c^3*d^16 + 208*a^8*b^3*c^5*d^14 - 32*a^8*b^3*c^7*d
^12 - 352*a^8*b^3*c^9*d^10 + 368*a^8*b^3*c^11*d^8 - 112*a^8*b^3*c^13*d^6 +
28*a^9*b^2*c^2*d^17 - 32*a^9*b^2*c^4*d^15 - 152*a^9*b^2*c^6*d^13 + 368*a^9*
b^2*c^8*d^11 - 292*a^9*b^2*c^10*d^9 + 80*a^9*b^2*c^12*d^7 - 4*a*b^10*c^18*d
 - 4*a^10*b*c*d^18))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10
 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d
^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5
 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*
d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b
^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^1
1 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*
b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d
^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*
d^13) - (8*tan(e/2 + (f*x)/2)*(8*a^3*b^8*c^19 - 12*a^11*c*d^18 - 12*a*b^10*
c^19 + 56*a^11*c^3*d^16 - 104*a^11*c^5*d^14 + 96*a^11*c^7*d^12 - 44*a^11*c^
9*d^10 + 8*a^11*c^11*d^8 + 16*a*b^10*c^9*d^10 - 76*a*b^10*c^11*d^8 + 144*a*
b^10*c^13*d^6 - 136*a*b^10*c^15*d^4 + 64*a*b^10*c^17*d^2 + 96*a^2*b^9*c^18*
d - 64*a^4*b^7*c^18*d + 16*a^9*b^2*c*d^18 + 96*a^10*b*c^2*d^17 - 448*a^10*b
*c^4*d^15 + 832*a^10*b*c^6*d^13 - 768*a^10*b*c^8*d^11 + 352*a^10*b*c^10*d^9
 - 64*a^10*b*c^12*d^7 - 128*a^2*b^9*c^8*d^11 + 608*a^2*b^9*c^10*d^9 - 1152*
a^2*b^9*c^12*d^7 + 1088*a^2*b^9*c^14*d^5 - 512*a^2*b^9*c^16*d^3 + 448*a^3*b
^8*c^7*d^12 - 2140*a^3*b^8*c^9*d^10 + 4088*a^3*b^8*c^11*d^8 - 3912*a^3*b^8*
c^13*d^6 + 1888*a^3*b^8*c^15*d^4 - 380*a^3*b^8*c^17*d^2 - 896*a^4*b^7*c^6*d
^13 + 4352*a^4*b^7*c^8*d^11 - 8512*a^4*b^7*c^10*d^9 + 8448*a^4*b^7*c^12*d^7
 - 4352*a^4*b^7*c^14*d^5 + 1024*a^4*b^7*c^16*d^3 + 1120*a^5*b^6*c^5*d^14 -
5656*a^5*b^6*c^7*d^12 + 11648*a^5*b^6*c^9*d^10 - 12432*a^5*b^6*c^11*d^8 + 7
168*a^5*b^6*c^13*d^6 - 2072*a^5*b^6*c^15*d^4 + 224*a^5*b^6*c^17*d^2 - 896*a
^6*b^5*c^4*d^15 + 4928*a^6*b^5*c^6*d^13 - 11200*a^6*b^5*c^8*d^11 + 13440*a^
6*b^5*c^10*d^9 - 8960*a^6*b^5*c^12*d^7 + 3136*a^6*b^5*c^14*d^5 - 448*a^6*b^
5*c^16*d^3 + 448*a^7*b^4*c^3*d^16 - 2968*a^7*b^4*c^5*d^14 + 7952*a^7*b^4*c^
7*d^12 - 11088*a^7*b^4*c^9*d^10 + 8512*a^7*b^4*c^11*d^8 - 3416*a^7*b^4*c^13
*d^6 + 560*a^7*b^4*c^15*d^4 - 128*a^8*b^3*c^2*d^17 + 1280*a^8*b^3*c^4*d^15
 - 4288*a^8*b^3*c^6*d^13 + 6912*a^8*b^3*c^8*d^11 - 5888*a^8*b^3*c^10*d^9 + 2
560*a^8*b^3*c^12*d^7 - 448*a^8*b^3*c^14*d^5 - 412*a^9*b^2*c^3*d^16 + 1712*a
^9*b^2*c^5*d^14 - 3048*a^9*b^2*c^7*d^12 + 2752*a^9*b^2*c^9*d^10 - 1244*a^9*
b^2*c^11*d^8 + 224*a^9*b^2*c^13*d^6))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12
 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d
^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 -
 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^
9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c

^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 2
0*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*
c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 +
90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^
13*d - 6*a^5*b*c*d^13))*(-(c + d)^5*(c - d)^5)^(1/2)*(a^2*d^4 + 6*b^2*c^4 +
2*b^2*d^4 + 2*a^2*c^2*d^2 - 5*b^2*c^2*d^2 - 6*a*b*c^3*d))/(2*(a^3*d^13 + b
^3*c^13 - 5*a^3*c^2*d^11 + 10*a^3*c^4*d^9 - 10*a^3*c^6*d^7 + 5*a^3*c^8*d^5
- a^3*c^10*d^3 - b^3*c^3*d^10 + 5*b^3*c^5*d^8 - 10*b^3*c^7*d^6 + 10*b^3*c^9
*d^4 - 5*b^3*c^11*d^2 + 3*a*b^2*c^2*d^11 - 15*a*b^2*c^4*d^9 + 30*a*b^2*c^6*
d^7 - 30*a*b^2*c^8*d^5 + 15*a*b^2*c^10*d^3 + 15*a^2*b*c^3*d^10 - 30*a^2*b*c
^5*d^8 + 30*a^2*b*c^7*d^6 - 15*a^2*b*c^9*d^4 + 3*a^2*b*c^11*d^2 - 3*a*b^2*c
^12*d - 3*a^2*b*c*d^12)))*(a^2*d^4 + 6*b^2*c^4 + 2*b^2*d^4 + 2*a^2*c^2*d^2
- 5*b^2*c^2*d^2 - 6*a*b*c^3*d))/(2*(a^3*d^13 + b^3*c^13 - 5*a^3*c^2*d^11 +
10*a^3*c^4*d^9 - 10*a^3*c^6*d^7 + 5*a^3*c^8*d^5 - a^3*c^10*d^3 - b^3*c^3*d^
10 + 5*b^3*c^5*d^8 - 10*b^3*c^7*d^6 + 10*b^3*c^9*d^4 - 5*b^3*c^11*d^2 + 3*a
*b^2*c^2*d^11 - 15*a*b^2*c^4*d^9 + 30*a*b^2*c^6*d^7 - 30*a*b^2*c^8*d^5 + 15
*a*b^2*c^10*d^3 + 15*a^2*b*c^3*d^10 - 30*a^2*b*c^5*d^8 + 30*a^2*b*c^7*d^6 -
15*a^2*b*c^9*d^4 + 3*a^2*b*c^11*d^2 - 3*a*b^2*c^12*d - 3*a^2*b*c*d^12)))*(
a^2*d^4 + 6*b^2*c^4 + 2*b^2*d^4 + 2*a^2*c^2*d^2 - 5*b^2*c^2*d^2 - 6*a*b*c^3
d))/(2(a^3*d^13 + b^3*c^13 - 5*a^3*c^2*d^11 + 10*a^3*c^4*d^9 - 10*a^3*c^6
*d^7 + 5*a^3*c^8*d^5 - a^3*c^10*d^3 - b^3*c^3*d^10 + 5*b^3*c^5*d^8 - 10*b^3
*c^7*d^6 + 10*b^3*c^9*d^4 - 5*b^3*c^11*d^2 + 3*a*b^2*c^2*d^11 - 15*a*b^2*c^
4*d^9 + 30*a*b^2*c^6*d^7 - 30*a*b^2*c^8*d^5 + 15*a*b^2*c^10*d^3 + 15*a^2*b*
c^3*d^10 - 30*a^2*b*c^5*d^8 + 30*a^2*b*c^7*d^6 - 15*a^2*b*c^9*d^4 + 3*a^2*b
*c^11*d^2 - 3*a*b^2*c^12*d - 3*a^2*b*c*d^12)) - (d*(-(c + d)^5*(c - d)^5)^(
1/2)*((8*(4*a*b^8*c^4*d^9 - 16*a*b^8*c^6*d^7 + 24*a*b^8*c^8*d^5 - 16*a*b^8*
c^10*d^3 + 4*a^4*b^5*c*d^12 + 4*a^6*b^3*c*d^12 + 4*a^8*b*c^3*d^10 + 4*a^8*b
*c^5*d^8 - 4*a^2*b^7*c^3*d^10 + 12*a^2*b^7*c^5*d^8 + a^2*b^7*c^7*d^6 - 28*a
^2*b^7*c^9*d^4 + 28*a^2*b^7*c^11*d^2 - 4*a^3*b^6*c^2*d^11 + 24*a^3*b^6*c^4*
d^9 - 98*a^3*b^6*c^6*d^7 + 164*a^3*b^6*c^8*d^5 - 140*a^3*b^6*c^10*d^3 - 16*
a^4*b^5*c^3*d^10 + 95*a^4*b^5*c^5*d^8 - 188*a^4*b^5*c^7*d^6 + 240*a^4*b^5*c
^9*d^4 - 8*a^5*b^4*c^2*d^11 - 20*a^5*b^4*c^4*d^9 + 64*a^5*b^4*c^6*d^7 - 216
*a^5*b^4*c^8*d^5 - a^6*b^3*c^3*d^10 + 20*a^6*b^3*c^5*d^8 + 112*a^6*b^3*c^7*
d^6 - 2*a^7*b^2*c^2*d^11 - 20*a^7*b^2*c^4*d^9 - 32*a^7*b^2*c^6*d^7 + 4*a*b^
8*c^12*d + a^8*b*c*d^12))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4
*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c
^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^
9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b
*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*
a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^
3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20
*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*
c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5
*b*c*d^13) - (8*tan(e/2 + (f*x)/2)*(4*a*b^8*c^13 + a^9*c*d^12 + 4*a^9*c^3*d
^10 + 4*a^9*c^5*d^8 - 16*a*b^8*c^3*d^10 + 76*a*b^8*c^5*d^8 - 162*a*b^8*c^7*

d^6 + 176*a*b^8*c^9*d^4 - 96*a*b^8*c^11*d^2 - 8*a^2*b^7*c^12*d - 16*a^3*b^6 *c*d^12 - 4*a^5*b^4*c*d^12 + 2*a^7*b^2*c*d^12 - 2*a^8*b*c^2*d^11 - 20*a^8*b *c^4*d^9 - 32*a^8*b*c^6*d^7 + 32*a^2*b^7*c^2*d^11 - 152*a^2*b^7*c^4*d^9 + 3 72*a^2*b^7*c^6*d^7 - 472*a^2*b^7*c^8*d^5 + 336*a^2*b^7*c^10*d^3 + 72*a^3*b^ 6*c^3*d^10 - 274*a^3*b^6*c^5*d^8 + 481*a^3*b^6*c^7*d^6 - 564*a^3*b^6*c^9*d^ 4 + 40*a^3*b^6*c^11*d^2 + 8*a^4*b^5*c^2*d^11 + 80*a^4*b^5*c^4*d^9 - 250*a^4 *b^5*c^6*d^7 + 612*a^4*b^5*c^8*d^5 - 144*a^4*b^5*c^10*d^3 - 14*a^5*b^4*c^3* d^10 + 55*a^5*b^4*c^5*d^8 - 412*a^5*b^4*c^7*d^6 + 240*a^5*b^4*c^9*d^4 - 4*a ^6*b^3*c^2*d^11 + 20*a^6*b^3*c^4*d^9 + 128*a^6*b^3*c^6*d^7 - 216*a^6*b^3*c^ 8*d^5 - 9*a^7*b^2*c^3*d^10 + 12*a^7*b^2*c^5*d^8 + 112*a^7*b^2*c^7*d^6))/(a^ 6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c ^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6* a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 2 4*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4* c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^ 2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) + (d*(-(c + d)^5*(c - d)^5)^(1/2)*((8*(4*a^2*b^8*c^16 + 2*a^10*c^2*d^14 - 6*a^10*c^6*d^10 + 4 *a^10*c^8*d^8 + 4*a*b^9*c^7*d^9 - 18*a*b^9*c^9*d^7 + 36*a*b^9*c^11*d^5 - 34 *a*b^9*c^13*d^3 - 32*a^3*b^7*c^15*d + 4*a^7*b^3*c*d^15 - 10*a^9*b*c^3*d^13 - 12*a^9*b*c^5*d^11 + 54*a^9*b*c^7*d^9 - 32*a^9*b*c^9*d^7 - 24*a^2*b^8*c^6* d^10 + 110*a^2*b^8*c^8*d^8 - 232*a^2*b^8*c^10*d^6 + 234*a^2*b^8*c^12*d^4 - 92*a^2*b^8*c^14*d^2 + 60*a^3*b^7*c^5*d^11 - 282*a^3*b^7*c^7*d^9 + 638*a^3*b ^7*c^9*d^7 - 702*a^3*b^7*c^11*d^5 + 318*a^3*b^7*c^13*d^3 - 80*a^4*b^6*c^4*d ^12 + 390*a^4*b^6*c^6*d^10 - 970*a^4*b^6*c^8*d^8 + 1202*a^4*b^6*c^10*d^6 - 654*a^4*b^6*c^12*d^4 + 112*a^4*b^6*c^14*d^2 + 60*a^5*b^5*c^3*d^13 - 310*a^5 *b^5*c^5*d^11 + 878*a^5*b^5*c^7*d^9 - 1290*a^5*b^5*c^9*d^7 + 886*a^5*b^5*c^ 11*d^5 - 224*a^5*b^5*c^13*d^3 - 24*a^6*b^4*c^2*d^14 + 138*a^6*b^4*c^4*d^12 - 466*a^6*b^4*c^6*d^10 + 894*a^6*b^4*c^8*d^8 - 822*a^6*b^4*c^10*d^6 + 280*a ^6*b^4*c^12*d^4 - 30*a^7*b^3*c^3*d^13 + 122*a^7*b^3*c^5*d^11 - 394*a^7*b^3* c^7*d^9 + 522*a^7*b^3*c^9*d^7 - 224*a^7*b^3*c^11*d^5 + 2*a^8*b^2*c^2*d^14 + 2*a^8*b^2*c^4*d^12 + 102*a^8*b^2*c^6*d^10 - 218*a^8*b^2*c^8*d^8 + 112*a^8* b^2*c^10*d^6 + 12*a*b^9*c^15*d))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6* a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a *b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 2 4*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^ 8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3 *b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d ^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a ^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) + (8*tan(e/2 + (f*x)/2)*(8*a*b^9*c^16 + 4*a^10*c*d^15 - 1 2*a^10*c^5*d^11 + 8*a^10*c^7*d^9 + 4*a*b^9*c^8*d^8 - 8*a*b^9*c^10*d^6 + 12* a*b^9*c^12*d^4 - 16*a*b^9*c^14*d^2 - 40*a^2*b^8*c^15*d + 4*a^8*b^2*c*d^15 - 20*a^9*b*c^2*d^14 - 24*a^9*b*c^4*d^12 + 108*a^9*b*c^6*d^10 - 64*a^9*b*c^8*d^8 - 20*a^2*b^8*c^7*d^9 + 16*a^2*b^8*c^9*d^7 - 12*a^2*b^8*c^11*d^5 + 56*a^2*b^8*c^13*d^3 + 36*a^3*b^7*c^6*d^10 + 76*a^3*b^7*c^8*d^8 - 204*a^3*b^7*c^10*d^6 + 36*a^3*b^7*c^12*d^4 + 56*a^3*b^7*c^14*d^2 - 20*a^4*b^6*c^5*d^11 - 340*a^4*b^6*c^7*d^9 + 804*a^4*b^6*c^9*d^7 - 508*a^4*b^6*c^11*d^5 + 64*a^4*b^6*c^13*d^3 - 20*a^5*b^5*c^4*d^12 + 556*a^5*b^5*c^6*d^10 - 1380*a^5*b^5*c^8*d^8 + 1172*a^5*b^5*c^10*d^6 - 328*a^5*b^5*c^12*d^4 + 36*a^6*b^4*c^3*d^13 - 452*a^6*b^4*c^5*d^11 + 1308*a^6*b^4*c^7*d^9 - 1404*a^6*b^4*c^9*d^7 + 512*a^6*b^4*c^11*d^5 - 20*a^7*b^3*c^2*d^14 + 164*a^7*b^3*c^4*d^12 - 708*a^7*b^3*c^6*d^10 + 1004*a^7*b^3*c^8*d^8 - 440*a^7*b^3*c^10*d^6 + 4*a^8*b^2*c^3*d^13 + 204*a^8*b^2*c^5*d^11 - 436*a^8*b^2*c^7*d^9 + 224*a^8*b^2*c^9*d^7))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) + (d*((8*(4*a^2*b^9*c^19 + 4*a^11*c^2*d^17 - 16*a^11*c^4*d^15 + 24*a^11*c^6*d^13 - 16*a^11*c^8*d^11 + 4*a^11*c^10*d^9 - 4*a*b^10*c^10*d^9 + 16*a*b^10*c^12*d^7 - 24*a*b^10*c^14*d^5 + 16*a*b^10*c^16*d^3 - 28*a^3*b^8*c^18*d - 12*a^10*b*c^3*d^16 + 88*a^10*b*c^5*d^14 - 152*a^10*b*c^7*d^12 + 108*a^10*b*c^9*d^10 - 28*a^10*b*c^11*d^8 + 28*a^2*b^9*c^9*d^10 - 108*a^2*b^9*c^11*d^8 + 152*a^2*b^9*c^13*d^6 - 88*a^2*b^9*c^15*d^4 + 12*a^2*b^9*c^17*d^2 - 80*a^3*b^8*c^8*d^11 + 292*a^3*b^8*c^10*d^9 - 368*a^3*b^8*c^12*d^7 + 152*a^3*b^8*c^14*d^5 + 32*a^3*b^8*c^16*d^3 + 112*a^4*b^7*c^7*d^12 - 368*a^4*b^7*c^9*d^10 + 352*a^4*b^7*c^11*d^8 + 32*a^4*b^7*c^13*d^6 - 208*a^4*b^7*c^15*d^4 + 80*a^4*b^7*c^17*d^2 - 56*a^5*b^6*c^6*d^13 + 112*a^5*b^6*c^8*d^11 + 112*a^5*b^6*c^10*d^9 - 448*a^5*b^6*c^12*d^7 + 392*a^5*b^6*c^14*d^5 - 112*a^5*b^6*c^16*d^3 - 56*a^6*b^5*c^5*d^14 + 280*a^6*b^5*c^7*d^12 - 560*a^6*b^5*c^9*d^10 + 560*a^6*b^5*c^11*d^8 - 280*a^6*b^5*c^13*d^6 + 56*a^6*b^5*c^15*d^4 + 112*a^7*b^4*c^4*d^15 - 392*a^7*b^4*c^6*d^13 + 448*a^7*b^4*c^8*d^11 - 112*a^7*b^4*c^10*d^9 - 112*a^7*b^4*c^12*d^7 + 56*a^7*b^4*c^14*d^5 - 80*a^8*b^3*c^3*d^16 + 208*a^8*b^3*c^5*d^14 - 32*a^8*b^3*c^7*d^12 - 352*a^8*b^3*c^9*d^10 + 368*a^8*b^3*c^11*d^8 - 112*a^8*b^3*c^13*d^6 + 28*a^9*b^2*c^2*d^17 - 32*a^9*b^2*c^4*d^15 - 152*a^9*b^2*c^6*d^13 + 368*a^9*b^2*c^8*d^11 - 292*a^9*b^2*c^10*d^9 + 80*a^9*b^2*c^12*d^7 - 4*a*b^10*c^18*d - 4*a^10*b*c*d^18))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20 *a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13) - (8*tan(e/2 + (f*x)/2)*(8*a^3*b^8*c^19 - 12*a^11*c*d^18 - 12*a*b^10*c^19 + 56*a^11*c^3*d^16 - 104*a^11*c^5*d^14 + 96*a^11*c^7*d^12 - 44*a^11*c^9*d^10 + 8*a^11*c^11*d^8 + 16*a*b^10*c^9*d^10 - 76*a*b^10*c^11*d^8 + 144*a*b^10*c^13*d^6 - 136*a*b^10*c^15*d^4 + 64*a*b^10*c^17*d^2 + 96*a^2*b^9*c^18*d - 64*a^4*b^7*c^18*d + 16*a^9*b^2*c*d^18 + 96*a^10*b*c^2*d^17 - 448*a^10*b*c^4*d^15 + 832*a^10*b*c^6*d^13 - 768*a^10*b*c^8*d^11 + 352*a^10*b*c^10*d^9 - 64*a^10*b*c^12*d^7 - 128*a^2*b^9*c^8*d^11 + 608*a^2*b^9*c^10*d^9 - 1152*a^2*b^9*c^12*d^7 + 1088*a^2*b^9*c^14*d^5 - 512*a^2*b^9*c^16*d^3 + 448*a^3*b^8*c^7*d^12 - 2140*a^3*b^8*c^9*d^10 + 4088*a^3*b^8*c^11*d^8 - 3912*a^3*b^8*c^13*d^6 + 1888*a^3*b^8*c^15*d^4 - 380*a^3*b^8*c^17*d^2 - 896*a^4*b^7*c^6*d^13 + 4352*a^4*b^7*c^8*d^11 - 8512*a^4*b^7*c^10*d^9 + 8448*a^4*b^7*c^12*d^7 - 4352*a^4*b^7*c^14*d^5 + 1024*a^4*b^7*c^16*d^3 + 1120*a^5*b^6*c^5*d^14 - 5656*a^5*b^6*c^7*d^12 + 11648*a^5*b^6*c^9*d^10 - 12432*a^5*b^6*c^11*d^8 + 7168*a^5*b^6*c^13*d^6 - 2072*a^5*b^6*c^15*d^4 + 224*a^5*b^6*c^17*d^2 - 896*a^6*b^5*c^4*d^15 + 4928*a^6*b^5*c^6*d^13 - 11200*a^6*b^5*c^8*d^11 + 13440*a^6*b^5*c^10*d^9 - 8960*a^6*b^5*c^12*d^7 + 3136*a^6*b^5*c^14*d^5 - 448*a^6*b^5*c^16*d^3 + 448*a^7*b^4*c^3*d^16 - 2968*a^7*b^4*c^5*d^14 + 7952*a^7*b^4*c^7*d^12 - 11088*a^7*b^4*c^9*d^10 + 8512*a^7*b^4*c^11*d^8 - 3416*a^7*b^4*c^13*d^6 + 560*a^7*b^4*c^15*d^4 - 128*a^8*b^3*c^2*d^17 + 1280*a^8*b^3*c^4*d^15 - 4288*a^8*b^3*c^6*d^13 + 6912*a^8*b^3*c^8*d^11 - 5888*a^8*b^3*c^10*d^9 + 2560*a^8*b^3*c^12*d^7 - 448*a^8*b^3*c^14*d^5 - 412*a^9*b^2*c^3*d^16 + 1712*a^9*b^2*c^5*d^14 - 3048*a^9*b^2*c^7*d^12 + 2752*a^9*b^2*c^9*d^10 - 1244*a^9*b^2*c^11*d^8 + 224*a^9*b^2*c^13*d^6))/(a^6*d^14 + b^6*c^14 - 4*a^6*c^2*d^12 + 6*a^6*c^4*d^10 - 4*a^6*c^6*d^8 + a^6*c^8*d^6 + b^6*c^6*d^8 - 4*b^6*c^8*d^6 + 6*b^6*c^10*d^4 - 4*b^6*c^12*d^2 - 6*a*b^5*c^5*d^9 + 24*a*b^5*c^7*d^7 - 36*a*b^5*c^9*d^5 + 24*a*b^5*c^11*d^3 + 24*a^5*b*c^3*d^11 - 36*a^5*b*c^5*d^9 + 24*a^5*b*c^7*d^7 - 6*a^5*b*c^9*d^5 + 15*a^2*b^4*c^4*d^10 - 60*a^2*b^4*c^6*d^8 + 90*a^2*b^4*c^8*d^6 - 60*a^2*b^4*c^10*d^4 + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^3*d^11 + 80*a^3*b^3*c^5*d^9 - 120*a^3*b^3*c^7*d^7 + 80*a^3*b^3*c^9*d^5 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^2*d^12 - 60*a^4*b^2*c^4*d^10 + 90*a^4*b^2*c^6*d^8 - 60*a^4*b^2*c^8*d^6 + 15*a^4*b^2*c^10*d^4 - 6*a*b^5*c^13*d - 6*a^5*b*c*d^13))*(-(c + d)^5*(c - d)^5)^(1/2)*(a^2*d^4 + 6*b^2*c^4 + 2*b^2*d^4 + 2*a^2*c^2*d^2 - 5*b^2*c^2*d^2 - 6*a*b*c^3*d))/(2*(a^3*d^13 + b^3*c^13 - 5*a^3*c^2*d^11 + 10*a^3*c^4*d^9 - 10*a^3*c^6*d^7 + 5*a^3*c^8*d^5 - a^3*c^10*d^3 - b^3*c^3*d^10 + 5*b^3*c^5*d^8 - 10*b^3*c^7*d^6 + 10*b^3*c^9*d^4 - 5*b^3*c^11*d^2 + 3*a*b^2*c^2*d^11 - 15*a*b^2*c^4*d^9 + 30*a*b^2*c^6*d^7 - 30*a*b^2*c^8*d^5 + 15*a*b^2*c^10*d^3 + 15*a^2*b*c^3*d^10 - 30*a^2*b*c^5*d^8 + 30*a^2*b*c^7*d^6 - 15*a^2*b*c^9*d^4 + 3*a^2*b*c^11*d^2 - 3*a*b^2*c^12*d - 3*a^2*b*c*d^12)))*(a^2*d^4 + 6*b^2*c^4 + 2*b^2*d^4 + 2*a^2*c^2*d^2 - 5*b^2*c^2*d^2 - 6*a*b*c^3*d))/(2*(a^3*d^13 + b^3*c^13 - 5*a^3*c^2*d^11 + 10*a^3*c^4*d^9 - 10*a^3*c^6*d^7 + 5*a^3*c^8*d^5 - a^3*c^10*d^3 - b^3*c^3*d^10 + 5*b^3*c^5*d^8 - 10*b^3*c^7*d^6 + 10*b^3*c^9*d^4 - 5*b^

```
3*c^11*d^2 + 3*a*b^2*c^2*d^11 - 15*a*b^2*c^4*d^9 + 30*a*b^2*c^6*d^7 - 30*a*
b^2*c^8*d^5 + 15*a*b^2*c^10*d^3 + 15*a^2*b*c^3*d^10 - 30*a^2*b*c^5*d^8 + 30
*a^2*b*c^7*d^6 - 15*a^2*b*c^9*d^4 + 3*a^2*b*c^11*d^2 - 3*a*b^2*c^12*d - 3*a
^2*b*c*d^12)))*(a^2*d^4 + 6*b^2*c^4 + 2*b^2*d^4 + 2*a^2*c^2*d^2 - 5*b^2*c^2
*d^2 - 6*a*b*c^3*d))/(2*(a^3*d^13 + b^3*c^13 - 5*a^3*c^2*d^11 + 10*a^3*c^4*
d^9 - 10*a^3*c^6*d^7 + 5*a^3*c^8*d^5 - a^3*c^10*d^3 - b^3*c^3*d^10 + 5*b^3*
c^5*d^8 - 10*b^3*c^7*d^6 + 10*b^3*c^9*d^4 - 5*b^3*c^11*d^2 + 3*a*b^2*c^2*d^
11 - 15*a*b^2*c^4*d^9 + 30*a*b^2*c^6*d^7 - 30*a*b^2*c^8*d^5 + 15*a*b^2*c^10
*d^3 + 15*a^2*b*c^3*d^10 - 30*a^2*b*c^5*d^8 + 30*a^2*b*c^7*d^6 - 15*a^2*b*c
^9*d^4 + 3*a^2*b*c^11*d^2 - 3*a*b^2*c^12*d - 3*a^2*b*c*d^12))))*(-(c + d)^5
*(c - d)^5)^(1/2)*(a^2*d^4 + 6*b^2*c^4 + 2*b^2*d^4 + 2*a^2*c^2*d^2 - 5*b^2*
c^2*d^2 - 6*a*b*c^3*d)*1i)/(f*(a^3*d^13 + b^3*c^13 - 5*a^3*c^2*d^11 + 10*a^
3*c^4*d^9 - 10*a^3*c^6*d^7 + 5*a^3*c^8*d^5 - a^3*c^10*d^3 - b^3*c^3*d^10 +
5*b^3*c^5*d^8 - 10*b^3*c^7*d^6 + 10*b^3*c^9*d^4 - 5*b^3*c^11*d^2 + 3*a*b^2*
c^2*d^11 - 15*a*b^2*c^4*d^9 + 30*a*b^2*c^6*d^7 - 30*a*b^2*c^8*d^5 + 15*a*b^
2*c^10*d^3 + 15*a^2*b*c^3*d^10 - 30*a^2*b*c^5*d^8 + 30*a^2*b*c^7*d^6 - 15*a
^2*b*c^9*d^4 + 3*a^2*b*c^11*d^2 - 3*a*b^2*c^12*d - 3*a^2*b*c*d^12))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
[Out] Timed out
```

3.706 $\int \frac{(c+d\sin(e+fx))^4}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=306

$$\frac{d^2\left(-3a^2d^2+4abcd-\left(b^2\left(2c^2-d^2\right)\right)\right)\sin(e+fx)\cos(e+fx)}{2b^2f\left(a^2-b^2\right)}+\frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))^2}{bf\left(a^2-b^2\right)(a+b\sin(e+fx))}-\frac{d^2x}{}$$

```
[Out] -1/2*d^2*(16*a*b*c*d-6*a^2*d^2-b^2*(12*c^2+d^2))*x/b^4+2*(-a*d+b*c)^3*(3*a^
2*d+a*b*c-4*b^2*d)*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/b^4/(a^
2-b^2)^(3/2)/f+d*(-a*d+2*b*c)*(2*a*b*c*d-3*a^2*d^2-b^2*(c^2-2*d^2))*cos(f*x
+e)/b^3/(a^2-b^2)/f+1/2*d^2*(4*a*b*c*d-3*a^2*d^2-b^2*(2*c^2-d^2))*cos(f*x+e
)*sin(f*x+e)/b^2/(a^2-b^2)/f+(-a*d+b*c)^2*cos(f*x+e)*(c+d*sin(f*x+e))^2/b/(
a^2-b^2)/f/(a+b*sin(f*x+e))
```

Rubi [A] time = 0.94, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2792, 3033, 3023, 2735, 2660, 618, 204}

$$\frac{d(2bc-ad)\left(-3a^2d^2+2abcd+b^2\left(-\left(c^2-2d^2\right)\right)\right)\cos(e+fx)}{b^3f\left(a^2-b^2\right)}+\frac{d^2\left(-3a^2d^2+4abcd+b^2\left(-\left(2c^2-d^2\right)\right)\right)\sin(e+}{2b^2f\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^4/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -(d^2*(16*a*b*c*d - 6*a^2*d^2 - b^2*(12*c^2 + d^2))*x)/(2*b^4) + (2*(b*c -
a*d)^3*(a*b*c + 3*a^2*d - 4*b^2*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2
 - b^2]])/(b^4*(a^2 - b^2)^(3/2)*f) + (d*(2*b*c - a*d)*(2*a*b*c*d - 3*a^2*d
^2 - b^2*(c^2 - 2*d^2))*Cos[e + f*x])/(b^3*(a^2 - b^2)*f) + (d^2*(4*a*b*c*d
 - 3*a^2*d^2 - b^2*(2*c^2 - d^2))*Cos[e + f*x]*Sin[e + f*x])/(2*b^2*(a^2 -
b^2)*f) + ((b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(b*(a^2 - b^2
)*f*(a + b*Sin[e + f*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
```

```
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] && !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^4}{(a+b\sin(e+fx))^2}\,dx = \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))^2}{b\left(a^2-b^2\right)f(a+b\sin(e+fx))} - \frac{\int \frac{(c+d\sin(e+fx))\left(4b^2c^2d+2a^2d^3-abc\left(c^2+5d^2\right)-a \cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{d^2\left(4abcd-3a^2d^2-b^2\left(2c^2-d^2\right)\right)\cos(e+fx)\sin(e+fx)}{2b^2\left(a^2-b^2\right)f} + \frac{(bc-ad)^2\cos(e+fx \cdots}{b\left(a^2-b^2\right)f(a \cdots}$$

$$= \frac{d(2bc-ad)\left(2abcd-3a^2d^2-b^2\left(c^2-2d^2\right)\right)\cos(e+fx)}{b^3\left(a^2-b^2\right)f} + \frac{d^2\left(4abcd-3a^2d^2-b^2 \cdots\right.}{2b \cdots}$$

$$= -\frac{d^2\left(16abcd-6a^2d^2-b^2\left(12c^2+d^2\right)\right)x}{2b^4} + \frac{d(2bc-ad)\left(2abcd-3a^2d^2-b^2\left(c^2-2 \cdots\right.\right.}{b^3\left(a^2-b^2\right)f}$$

$$= -\frac{d^2\left(16abcd-6a^2d^2-b^2\left(12c^2+d^2\right)\right)x}{2b^4} + \frac{d(2bc-ad)\left(2abcd-3a^2d^2-b^2\left(c^2-2 \cdots\right.\right.}{b^3\left(a^2-b^2\right)f}$$

$$= -\frac{d^2\left(16abcd-6a^2d^2-b^2\left(12c^2+d^2\right)\right)x}{2b^4} + \frac{d(2bc-ad)\left(2abcd-3a^2d^2-b^2\left(c^2-2 \cdots\right.\right.}{b^3\left(a^2-b^2\right)f}$$

$$= -\frac{d^2\left(16abcd-6a^2d^2-b^2\left(12c^2+d^2\right)\right)x}{2b^4} + \frac{2(bc-ad)^3\left(abc+3a^2d-4b^2d\right)\tan^{-1}\left(\cdots\right.}{b^4\left(a^2-b^2\right)^{3/2}f}$$

Mathematica [A] time = 2.02, size = 199, normalized size = 0.65

$$-\frac{-2d^2(e+fx)\left(6a^2d^2-16abcd+b^2\left(12c^2+d^2\right)\right)+\frac{8(ad-bc)^3\left(3a^2d+abc-4b^2d\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+8bd^3(2bc-ad)\,\mathrm{co}\cdots}{4b^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^4/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -1/4*(-2*d^2*(-16*a*b*c*d + 6*a^2*d^2 + b^2*(12*c^2 + d^2))*(e + f*x) + (8*
(-(b*c) + a*d)^3*(a*b*c + 3*a^2*d - 4*b^2*d)*ArcTan[(b + a*Tan[(e + f*x)/2]
)/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + 8*b*d^3*(2*b*c - a*d)*Cos[e + f*x]
- (4*b*(b*c - a*d)^4*Cos[e + f*x])/((a - b)*(a + b)*(a + b*Sin[e + f*x])) +
 b^2*d^4*Sin[2*(e + f*x)])/(b^4*f)
```

fricas [**B**] time = 0.83, size = 1451, normalized size = 4.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a^4*b^3 - 2*a^2*b^5 + b^7)*d^4*cos(f*x + e)^3 + (12*(a^5*b^2 - 2*a^3
*b^4 + a*b^6)*c^2*d^2 - 16*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c*d^3 + (6*a^7 - 1
1*a^5*b^2 + 4*a^3*b^4 + a*b^6)*d^4)*f*x + (a^2*b^4*c^4 - 4*a*b^5*c^3*d - 6*
(a^4*b^2 - 2*a^2*b^4)*c^2*d^2 + 4*(2*a^5*b - 3*a^3*b^3)*c*d^3 - (3*a^6 - 4*
a^4*b^2)*d^4 + (a*b^5*c^4 - 4*b^6*c^3*d - 6*(a^3*b^3 - 2*a*b^5)*c^2*d^2 + 4
*(2*a^4*b^2 - 3*a^2*b^4)*c*d^3 - (3*a^5*b - 4*a^3*b^3)*d^4)*sin(f*x + e))*s
qrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a
^2 - b^2 - 2*(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2
))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)) + (2*(a^2*b^5 - b
^7)*c^4 - 8*(a^3*b^4 - a*b^6)*c^3*d + 12*(a^4*b^3 - a^2*b^5)*c^2*d^2 - 8*(2
*a^5*b^2 - 3*a^3*b^4 + a*b^6)*c*d^3 + (6*a^6*b - 11*a^4*b^3 + 6*a^2*b^5 - b
^7)*d^4)*cos(f*x + e) + ((12*(a^4*b^3 - 2*a^2*b^5 + b^7)*c^2*d^2 - 16*(a^5*
b^2 - 2*a^3*b^4 + a*b^6)*c*d^3 + (6*a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + b^7)*d
^4)*f*x - (8*(a^4*b^3 - 2*a^2*b^5 + b^7)*c*d^3 - 3*(a^5*b^2 - 2*a^3*b^4 + a
*b^6)*d^4)*cos(f*x + e))*sin(f*x + e))/((a^4*b^5 - 2*a^2*b^7 + b^9)*f*sin(f
*x + e) + (a^5*b^4 - 2*a^3*b^6 + a*b^8)*f), 1/2*((a^4*b^3 - 2*a^2*b^5 + b^7
)*d^4*cos(f*x + e)^3 + (12*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^2*d^2 - 16*(a^6*
b - 2*a^4*b^3 + a^2*b^5)*c*d^3 + (6*a^7 - 11*a^5*b^2 + 4*a^3*b^4 + a*b^6)*d
^4)*f*x - 2*(a^2*b^4*c^4 - 4*a*b^5*c^3*d - 6*(a^4*b^2 - 2*a^2*b^4)*c^2*d^2
+ 4*(2*a^5*b - 3*a^3*b^3)*c*d^3 - (3*a^6 - 4*a^4*b^2)*d^4 + (a*b^5*c^4 - 4*
b^6*c^3*d - 6*(a^3*b^3 - 2*a*b^5)*c^2*d^2 + 4*(2*a^4*b^2 - 3*a^2*b^4)*c*d^3
 - (3*a^5*b - 4*a^3*b^3)*d^4)*sin(f*x + e))*sqrt(a^2 - b^2)*arctan(-(a*sin(
f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))) + (2*(a^2*b^5 - b^7)*c^4 - 8*
(a^3*b^4 - a*b^6)*c^3*d + 12*(a^4*b^3 - a^2*b^5)*c^2*d^2 - 8*(2*a^5*b^2 - 3
*a^3*b^4 + a*b^6)*c*d^3 + (6*a^6*b - 11*a^4*b^3 + 6*a^2*b^5 - b^7)*d^4)*cos
(f*x + e) + ((12*(a^4*b^3 - 2*a^2*b^5 + b^7)*c^2*d^2 - 16*(a^5*b^2 - 2*a^3*
b^4 + a*b^6)*c*d^3 + (6*a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + b^7)*d^4)*f*x - (8
*(a^4*b^3 - 2*a^2*b^5 + b^7)*c*d^3 - 3*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*d^4)*c
os(f*x + e))*sin(f*x + e))/((a^4*b^5 - 2*a^2*b^7 + b^9)*f*sin(f*x + e) + (a
^5*b^4 - 2*a^3*b^6 + a*b^8)*f)]
```

giac [A] time = 0.41, size = 517, normalized size = 1.69

$$\frac{\frac{4\left(ab^4c^4-4\,b^5c^3d-6\,a^3b^2c^2d^2+12\,ab^4c^2d^2+8\,a^4bcd^3-12\,a^2b^3cd^3-3\,a^5d^4+4\,a^3b^2d^4\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^4-b^6\right)\sqrt{a^2-b^2}}+\frac{4\left(b^5c^4\tan\left(\frac{1}{2}\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*(a*b^4*c^4 - 4*b^5*c^3*d - 6*a^3*b^2*c^2*d^2 + 12*a*b^4*c^2*d^2 + 8*
a^4*b*c*d^3 - 12*a^2*b^3*c*d^3 - 3*a^5*d^4 + 4*a^3*b^2*d^4)*(pi*floor(1/2*(
f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 -
b^2)))/((a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + 4*(b^5*c^4*tan(1/2*f*x + 1/2*e)
- 4*a*b^4*c^3*d*tan(1/2*f*x + 1/2*e) + 6*a^2*b^3*c^2*d^2*tan(1/2*f*x + 1/2*
e) - 4*a^3*b^2*c*d^3*tan(1/2*f*x + 1/2*e) + a^4*b*d^4*tan(1/2*f*x + 1/2*e)
+ a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4
)/((a^3*b^3 - a*b^5)*(a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) +
 a)) + (12*b^2*c^2*d^2 - 16*a*b*c*d^3 + 6*a^2*d^4 + b^2*d^4)*(f*x + e)/b^4
+ 2*(b*d^4*tan(1/2*f*x + 1/2*e)^3 - 8*b*c*d^3*tan(1/2*f*x + 1/2*e)^2 + 4*a*
d^4*tan(1/2*f*x + 1/2*e)^2 - b*d^4*tan(1/2*f*x + 1/2*e) - 8*b*c*d^3 + 4*a*d
^4)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*b^3))/f
```

maple [B] time = 0.32, size = 1303, normalized size = 4.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e))^2,x)
```

```
[Out] 1/f*d^4/b^2*arctan(tan(1/2*f*x+1/2*e))+2/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(
1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*c^4+2/f/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1
/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a*c^4+1/f*d^4/b^2/(1+tan(1/2*f*x+1/2*e)
^2)^2*tan(1/2*f*x+1/2*e)^3-1/f*d^4/b^2/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f
*x+1/2*e)+4/f*d^4/b^3/(1+tan(1/2*f*x+1/2*e)^2)^2*a-8/f*d^3/b^2/(1+tan(1/2*f
*x+1/2*e)^2)^2*c+6/f*d^4/b^4*arctan(tan(1/2*f*x+1/2*e))*a^2-8/f/b/(tan(1/2*
f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*a^2*tan(1/2*f*x+1/2*e)*c
*d^3+12/f*d^2/b^2*arctan(tan(1/2*f*x+1/2*e))*c^2+12/f/(tan(1/2*f*x+1/2*e)^2
*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*a*tan(1/2*f*x+1/2*e)*c^2*d^2+12/f/b/
(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*a^2*c^2*d^2+16/
f/b^3/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/
2))*a^4*c*d^3-12/f/b^2/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2
*b)/(a^2-b^2)^(1/2))*a^3*c^2*d^2-24/f/b/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan
```

```
(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*c*d^3+2/f/b^2/(tan(1/2*f*x+1/2*e)
^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*a^3*tan(1/2*f*x+1/2*e)*d^4-8/f*b/(
tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*tan(1/2*f*x+1/2*
e)*c^3*d+2/f*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2
)/a*tan(1/2*f*x+1/2*e)*c^4-8/f/b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/
2*e)*b+a)/(a^2-b^2)*a^3*c*d^3-16/f*d^3/b^3*arctan(tan(1/2*f*x+1/2*e))*a*c-8
/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*a*c^3*d+24/f
/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a
*c^2*d^2+2/f/b^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2
)*a^4*d^4+4/f*d^4/b^3/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*a-6/f
/b^4/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2
))*a^5*d^4+8/f/b^2/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/
(a^2-b^2)^(1/2))*a^3*d^4-8/f*b/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+
1/2*e)+2*b)/(a^2-b^2)^(1/2))*c^3*d-8/f*d^3/b^2/(1+tan(1/2*f*x+1/2*e)^2)^2*t
an(1/2*f*x+1/2*e)^2*c
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [**B**] time = 20.45, size = 13700, normalized size = 44.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^4/(a + b*sin(e + f*x))^2,x)
```

```
[Out] ((2*(3*a^4*d^4 + b^4*c^4 - 2*a^2*b^2*d^4 + 6*a^2*b^2*c^2*d^2 + 4*a*b^3*c*d^
3 - 4*a*b^3*c^3*d - 8*a^3*b*c*d^3))/(b^3*(a^2 - b^2)) + (2*tan(e/2 + (f*x)/
2)^4*(3*a^4*d^4 + b^4*c^4 - b^4*d^4 - a^2*b^2*d^4 + 6*a^2*b^2*c^2*d^2 + 4*a
*b^3*c*d^3 - 4*a*b^3*c^3*d - 8*a^3*b*c*d^3))/(b^3*(a^2 - b^2)) + (2*tan(e/2
 + (f*x)/2)^2*(6*a^4*d^4 + 2*b^4*c^4 + b^4*d^4 - 5*a^2*b^2*d^4 + 12*a^2*b^2
*c^2*d^2 + 8*a*b^3*c*d^3 - 8*a*b^3*c^3*d - 16*a^3*b*c*d^3))/(b^3*(a^2 - b^2
)) + (tan(e/2 + (f*x)/2)^5*(3*a^4*d^4 + 2*b^4*c^4 - a^2*b^2*d^4 + 12*a^2*b^
2*c^2*d^2 - 8*a*b^3*c^3*d - 8*a^3*b*c*d^3))/(a*b^2*(a^2 - b^2)) + (tan(e/2
+ (f*x)/2)*(9*a^4*d^4 + 2*b^4*c^4 - 7*a^2*b^2*d^4 + 12*a^2*b^2*c^2*d^2 + 16
*a*b^3*c*d^3 - 8*a*b^3*c^3*d - 24*a^3*b*c*d^3))/(a*b^2*(a^2 - b^2)) + (4*ta
```

n(e/2 + (f*x)/2)^3*(3*a^4*d^4 + b^4*c^4 - 2*a^2*b^2*d^4 + 6*a^2*b^2*c^2*d^2 + 4*a*b^3*c*d^3 - 4*a*b^3*c^3*d - 8*a^3*b*c*d^3))/(a*b^2*(a^2 - b^2)))/(f*(a + 2*b*tan(e/2 + (f*x)/2) + 3*a*tan(e/2 + (f*x)/2)^2 + 3*a*tan(e/2 + (f*x)/2)^4 + a*tan(e/2 + (f*x)/2)^6 + 4*b*tan(e/2 + (f*x)/2)^3 + 2*b*tan(e/2 + (f*x)/2)^5)) + (atan(((((8*(a^2*b^11*d^8 + 10*a^4*b^9*d^8 + 13*a^6*b^7*d^8 - 60*a^8*b^5*d^8 + 36*a^10*b^3*d^8 - 32*a^3*b^10*c*d^7 - 128*a^5*b^8*c*d^7 + 352*a^7*b^6*c*d^7 - 192*a^9*b^4*c*d^7 + 24*a^2*b^11*c^2*d^6 + 144*a^2*b^11*c^4*d^4 - 384*a^3*b^10*c^3*d^5 + 352*a^4*b^9*c^2*d^6 - 288*a^4*b^9*c^4*d^4 + 768*a^5*b^8*c^3*d^5 - 776*a^6*b^7*c^2*d^6 + 144*a^6*b^7*c^4*d^4 - 384*a^7*b^6*c^3*d^5 + 400*a^8*b^5*c^2*d^6))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (((8*(2*a*b^14*d^4 + 4*a^3*b^12*c^4 - 4*a^5*b^10*c^4 + 6*a^3*b^12*d^4 - 14*a^5*b^10*d^4 + 6*a^7*b^8*d^4 + 24*a*b^14*c^2*d^2 - 32*a^2*b^13*c*d^3 - 16*a^2*b^13*c^3*d + 48*a^4*b^11*c*d^3 + 16*a^4*b^11*c^3*d - 16*a^6*b^9*c*d^3 - 24*a^3*b^12*c^2*d^2))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(e/2 + (f*x)/2)*(8*a^2*b^14*c^4 - 8*a^4*b^12*c^4 + 32*a^4*b^12*d^4 - 56*a^6*b^10*d^4 + 24*a^8*b^8*d^4 - 96*a^3*b^13*c*d^3 + 32*a^3*b^13*c^3*d + 160*a^5*b^11*c*d^3 - 64*a^7*b^9*c*d^3 + 96*a^2*b^14*c^2*d^2 - 144*a^4*b^12*c^2*d^2 + 48*a^6*b^10*c^2*d^2 - 32*a*b^15*c^3*d))/(b^13 - 2*a^2*b^11 + a^4*b^9) + (((8*(4*a^2*b^15 - 8*a^4*b^13 + 4*a^6*b^11))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(e/2 + (f*x)/2)*(12*a*b^17 - 32*a^3*b^15 + 28*a^5*b^13 - 8*a^7*b^11))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(a^2*d^4*3i + (b^2*d^2*(12*c^2 + d^2)*1i)/2 - a*b*c*d^3*8i))/b^4)*(a^2*d^4*3i + (b^2*d^2*(12*c^2 + d^2)*1i)/2 - a*b*c*d^3*8i))/b^4 + (8*tan(e/2 + (f*x)/2)*(2*a*b^13*d^8 - 4*a^3*b^11*c^8 + 19*a^3*b^11*d^8 + 16*a^5*b^9*d^8 - 197*a^7*b^7*d^8 + 228*a^9*b^5*d^8 - 72*a^11*b^3*d^8 + 48*a*b^13*c^2*d^6 + 288*a*b^13*c^4*d^4 - 64*a*b^13*c^6*d^2 - 64*a^2*b^12*c*d^7 + 32*a^2*b^12*c^7*d - 224*a^4*b^10*c*d^7 + 1216*a^6*b^8*c*d^7 - 1280*a^8*b^6*c*d^7 + 384*a^10*b^4*c*d^7 - 768*a^2*b^12*c^3*d^5 + 384*a^2*b^12*c^5*d^3 + 680*a^3*b^11*c^2*d^6 - 1680*a^3*b^11*c^4*d^4 - 96*a^3*b^11*c^6*d^2 + 3200*a^4*b^10*c^3*d^5 - 96*a^4*b^10*c^5*d^3 - 2864*a^5*b^9*c^2*d^6 + 1376*a^5*b^9*c^4*d^4 + 48*a^5*b^9*c^6*d^2 - 2976*a^6*b^8*c^3*d^5 - 64*a^6*b^8*c^5*d^3 + 2824*a^7*b^7*c^2*d^6 - 264*a^7*b^7*c^4*d^4 + 768*a^8*b^6*c^3*d^5 - 800*a^9*b^5*c^2*d^6))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(a^2*d^4*3i + (b^2*d^2*(12*c^2 + d^2)*1i)/2 - a*b*c*d^3*8i)*1i)/b^4 + (((8*(a^2*b^11*d^8 + 10*a^4*b^9*d^8 + 13*a^6*b^7*d^8 - 60*a^8*b^5*d^8 + 36*a^10*b^3*d^8 - 32*a^3*b^10*c*d^7 - 128*a^5*b^8*c*d^7 + 352*a^7*b^6*c*d^7 - 192*a^9*b^4*c*d^7 + 24*a^2*b^11*c^2*d^6 + 144*a^2*b^11*c^4*d^4 - 384*a^3*b^10*c^3*d^5 + 352*a^4*b^9*c^2*d^6 - 288*a^4*b^9*c^4*d^4 + 768*a^5*b^8*c^3*d^5 - 776*a^6*b^7*c^2*d^6 + 144*a^6*b^7*c^4*d^4 - 384*a^7*b^6*c^3*d^5 + 400*a^8*b^5*c^2*d^6))/(b^12 - 2*a^2*b^10 + a^4*b^8) - (((8*(2*a*b^14*d^4 + 4*a^3*b^12*c^4 - 4*a^5*b^10*c^4 + 6*a^3*b^12*d^4 - 14*a^5*b^10*d^4 + 6*a^7*b^8*d^4 + 24*a*b^14*c^2*d^2 - 32*a^2*b^13*c*d^3 - 16*a^2*b^13*c^3*d + 48*a^4*b^11*c*d^3 + 16*a^4*b^11*c^3*d - 16*a^6*b^9*c*d^3 - 24*a^3*b^12*c^2*d^2))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(e/2 + (f*x)/2)*(8*a^2*b^14*c^4 - 8*a^4*b^12*c^4 + 32*a^4*b^12*d^4 - 56*a^6*b^10*d^4 + 24*a^8*b^8*d^4 - 96*a^3*b^13*c*d^3 + 32*a^3*b^13*c^3*d + 160*a^5*b^11*c*d^3 - 64*a^7*b^9*c*d^3 + 96*a^2*b^14*c^2*d^2 - 144*a^4*b^12*c^2*d^2

+ 48*a^6*b^10*c^2*d^2 - 32*a*b^15*c^3*d))/(b^13 - 2*a^2*b^11 + a^4*b^9) -
(((8*(4*a^2*b^15 - 8*a^4*b^13 + 4*a^6*b^11))/(b^12 - 2*a^2*b^10 + a^4*b^8)
+ (8*tan(e/2 + (f*x)/2)*(12*a*b^17 - 32*a^3*b^15 + 28*a^5*b^13 - 8*a^7*b^11
))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(a^2*d^4*3i + (b^2*d^2*(12*c^2 + d^2)*1i)
/2 - a*b*c*d^3*8i))/b^4)*(a^2*d^4*3i + (b^2*d^2*(12*c^2 + d^2)*1i)/2 - a*b*
c*d^3*8i))/b^4 + (8*tan(e/2 + (f*x)/2)*(2*a*b^13*d^8 - 4*a^3*b^11*c^8 + 19*
a^3*b^11*d^8 + 16*a^5*b^9*d^8 - 197*a^7*b^7*d^8 + 228*a^9*b^5*d^8 - 72*a^11
*b^3*d^8 + 48*a*b^13*c^2*d^6 + 288*a*b^13*c^4*d^4 - 64*a*b^13*c^6*d^2 - 64*
a^2*b^12*c*d^7 + 32*a^2*b^12*c^7*d - 224*a^4*b^10*c*d^7 + 1216*a^6*b^8*c*d^
7 - 1280*a^8*b^6*c*d^7 + 384*a^10*b^4*c*d^7 - 768*a^2*b^12*c^3*d^5 + 384*a^
2*b^12*c^5*d^3 + 680*a^3*b^11*c^2*d^6 - 1680*a^3*b^11*c^4*d^4 - 96*a^3*b^11
*c^6*d^2 + 3200*a^4*b^10*c^3*d^5 - 96*a^4*b^10*c^5*d^3 - 2864*a^5*b^9*c^2*d
^6 + 1376*a^5*b^9*c^4*d^4 + 48*a^5*b^9*c^6*d^2 - 2976*a^6*b^8*c^3*d^5 - 64*
a^6*b^8*c^5*d^3 + 2824*a^7*b^7*c^2*d^6 - 264*a^7*b^7*c^4*d^4 + 768*a^8*b^6*
c^3*d^5 - 800*a^9*b^5*c^2*d^6))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(a^2*d^4*3i
+ (b^2*d^2*(12*c^2 + d^2)*1i)/2 - a*b*c*d^3*8i)*1i)/b^4)/((16*(54*a^11*d^12
+ 4*a^5*b^6*d^12 + 9*a^7*b^4*d^12 - 81*a^9*b^2*d^12 - 32*a*b^10*c^6*d^6 -
384*a*b^10*c^8*d^4 - 12*a^4*b^7*c*d^11 - 60*a^6*b^5*c*d^11 + 648*a^8*b^3*c*
d^11 - 4*a^2*b^9*c^3*d^9 + 96*a^2*b^9*c^5*d^7 + 2256*a^2*b^9*c^7*d^5 + 192*
a^2*b^9*c^9*d^3 + 12*a^3*b^8*c^2*d^10 - 63*a^3*b^8*c^4*d^8 - 5784*a^3*b^8*c
^6*d^6 - 690*a^3*b^8*c^8*d^4 - 24*a^3*b^8*c^10*d^2 - 76*a^4*b^7*c^3*d^9 + 8
592*a^4*b^7*c^5*d^7 + 480*a^4*b^7*c^7*d^5 + 32*a^4*b^7*c^9*d^3 + 126*a^5*b^
6*c^2*d^10 - 8277*a^5*b^6*c^4*d^8 + 1552*a^5*b^6*c^6*d^6 + 132*a^5*b^6*c^8*
d^4 + 5424*a^6*b^5*c^3*d^9 - 4128*a^6*b^5*c^5*d^7 - 384*a^6*b^5*c^7*d^5 - 2
394*a^7*b^4*c^2*d^10 + 4860*a^7*b^4*c^4*d^8 + 400*a^7*b^4*c^6*d^6 - 3472*a^
8*b^3*c^3*d^9 - 192*a^8*b^3*c^5*d^7 + 1584*a^9*b^2*c^2*d^10 + 36*a^9*b^2*c^
4*d^8 - 432*a^10*b*c*d^11))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (((8*(a^2*b^11*
d^8 + 10*a^4*b^9*d^8 + 13*a^6*b^7*d^8 - 60*a^8*b^5*d^8 + 36*a^10*b^3*d^8 -
32*a^3*b^10*c*d^7 - 128*a^5*b^8*c*d^7 + 352*a^7*b^6*c*d^7 - 192*a^9*b^4*c*d
^7 + 24*a^2*b^11*c^2*d^6 + 144*a^2*b^11*c^4*d^4 - 384*a^3*b^10*c^3*d^5 + 35
2*a^4*b^9*c^2*d^6 - 288*a^4*b^9*c^4*d^4 + 768*a^5*b^8*c^3*d^5 - 776*a^6*b^7
*c^2*d^6 + 144*a^6*b^7*c^4*d^4 - 384*a^7*b^6*c^3*d^5 + 400*a^8*b^5*c^2*d^6)
)/(b^12 - 2*a^2*b^10 + a^4*b^8) + (((8*(2*a*b^14*d^4 + 4*a^3*b^12*c^4 - 4*a
^5*b^10*c^4 + 6*a^3*b^12*d^4 - 14*a^5*b^10*d^4 + 6*a^7*b^8*d^4 + 24*a*b^14*
c^2*d^2 - 32*a^2*b^13*c*d^3 - 16*a^2*b^13*c^3*d + 48*a^4*b^11*c*d^3 + 16*a^
4*b^11*c^3*d - 16*a^6*b^9*c*d^3 - 24*a^3*b^12*c^2*d^2))/(b^12 - 2*a^2*b^10
+ a^4*b^8) + (8*tan(e/2 + (f*x)/2)*(8*a^2*b^14*c^4 - 8*a^4*b^12*c^4 + 32*a^
4*b^12*d^4 - 56*a^6*b^10*d^4 + 24*a^8*b^8*d^4 - 96*a^3*b^13*c*d^3 + 32*a^3*
b^13*c^3*d + 160*a^5*b^11*c*d^3 - 64*a^7*b^9*c*d^3 + 96*a^2*b^14*c^2*d^2 -
144*a^4*b^12*c^2*d^2 + 48*a^6*b^10*c^2*d^2 - 32*a*b^15*c^3*d))/(b^13 - 2*a^
2*b^11 + a^4*b^9) + (((8*(4*a^2*b^15 - 8*a^4*b^13 + 4*a^6*b^11))/(b^12 - 2*
a^2*b^10 + a^4*b^8) + (8*tan(e/2 + (f*x)/2)*(12*a*b^17 - 32*a^3*b^15 + 28*a
^5*b^13 - 8*a^7*b^11))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(a^2*d^4*3i + (b^2*d^
2*(12*c^2 + d^2)*1i)/2 - a*b*c*d^3*8i))/b^4)*(a^2*d^4*3i + (b^2*d^2*(12*c^2
+ d^2)*1i)/2 - a*b*c*d^3*8i))/b^4 + (8*tan(e/2 + (f*x)/2)*(2*a*b^13*d^8 -

4*a^3*b^11*c^8 + 19*a^3*b^11*d^8 + 16*a^5*b^9*d^8 - 197*a^7*b^7*d^8 + 228*a^9*b^5*d^8 - 72*a^11*b^3*d^8 + 48*a*b^13*c^2*d^6 + 288*a*b^13*c^4*d^4 - 64*a*b^13*c^6*d^2 - 64*a^2*b^12*c*d^7 + 32*a^2*b^12*c^7*d - 224*a^4*b^10*c*d^7 + 1216*a^6*b^8*c*d^7 - 1280*a^8*b^6*c*d^7 + 384*a^10*b^4*c*d^7 - 768*a^2*b^12*c^3*d^5 + 384*a^2*b^12*c^5*d^3 + 680*a^3*b^11*c^2*d^6 - 1680*a^3*b^11*c^4*d^4 - 96*a^3*b^11*c^6*d^2 + 3200*a^4*b^10*c^3*d^5 - 96*a^4*b^10*c^5*d^3 - 2864*a^5*b^9*c^2*d^6 + 1376*a^5*b^9*c^4*d^4 + 48*a^5*b^9*c^6*d^2 - 2976*a^6*b^8*c^3*d^5 - 64*a^6*b^8*c^5*d^3 + 2824*a^7*b^7*c^2*d^6 - 264*a^7*b^7*c^4*d^4 + 768*a^8*b^6*c^3*d^5 - 800*a^9*b^5*c^2*d^6))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(a^2*d^4*3i + (b^2*d^2*(12*c^2 + d^2)*1i)/2 - a*b*c*d^3*8i))/b^4 - (((8*(a^2*b^11*d^8 + 10*a^4*b^9*d^8 + 13*a^6*b^7*d^8 - 60*a^8*b^5*d^8 + 36*a^10*b^3*d^8 - 32*a^3*b^10*c*d^7 - 128*a^5*b^8*c*d^7 + 352*a^7*b^6*c*d^7 - 192*a^9*b^4*c*d^7 + 24*a^2*b^11*c^2*d^6 + 144*a^2*b^11*c^4*d^4 - 384*a^3*b^10*c^3*d^5 + 352*a^4*b^9*c^2*d^6 - 288*a^4*b^9*c^4*d^4 + 768*a^5*b^8*c^3*d^5 - 776*a^6*b^7*c^2*d^6 + 144*a^6*b^7*c^4*d^4 - 384*a^7*b^6*c^3*d^5 + 400*a^8*b^5*c^2*d^6))/(b^12 - 2*a^2*b^10 + a^4*b^8) - (((8*(2*a*b^14*d^4 + 4*a^3*b^12*c^4 - 4*a^5*b^10*c^4 + 6*a^3*b^12*d^4 - 14*a^5*b^10*d^4 + 6*a^7*b^8*d^4 + 24*a*b^14*c^2*d^2 - 32*a^2*b^13*c*d^3 - 16*a^2*b^13*c^3*d + 48*a^4*b^11*c*d^3 + 16*a^4*b^11*c^3*d - 16*a^6*b^9*c*d^3 - 24*a^3*b^12*c^2*d^2))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(e/2 + (f*x)/2)*(8*a^2*b^14*c^4 - 8*a^4*b^12*c^4 + 32*a^4*b^12*d^4 - 56*a^6*b^10*d^4 + 24*a^8*b^8*d^4 - 96*a^3*b^13*c*d^3 + 32*a^3*b^13*c^3*d + 160*a^5*b^11*c*d^3 - 64*a^7*b^9*c*d^3 + 96*a^2*b^14*c^2*d^2 - 144*a^4*b^12*c^2*d^2 + 48*a^6*b^10*c^2*d^2 - 32*a*b^15*c^3*d))/(b^13 - 2*a^2*b^11 + a^4*b^9) - (((8*(4*a^2*b^15 - 8*a^4*b^13 + 4*a^6*b^11))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(e/2 + (f*x)/2)*(12*a*b^17 - 32*a^3*b^15 + 28*a^5*b^13 - 8*a^7*b^11))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(a^2*d^4*3i + (b^2*d^2*(12*c^2 + d^2)*1i)/2 - a*b*c*d^3*8i))/b^4)*(a^2*d^4*3i + (b^2*d^2*(12*c^2 + d^2)*1i)/2 - a*b*c*d^3*8i))/b^4 + (8*tan(e/2 + (f*x)/2)*(2*a*b^13*d^8 - 4*a^3*b^11*c^8 + 19*a^3*b^11*d^8 + 16*a^5*b^9*d^8 - 197*a^7*b^7*d^8 + 228*a^9*b^5*d^8 - 72*a^11*b^3*d^8 + 48*a*b^13*c^2*d^6 + 288*a*b^13*c^4*d^4 - 64*a*b^13*c^6*d^2 - 64*a^2*b^12*c*d^7 + 32*a^2*b^12*c^7*d - 224*a^4*b^10*c*d^7 + 1216*a^6*b^8*c*d^7 - 1280*a^8*b^6*c*d^7 + 384*a^10*b^4*c*d^7 - 768*a^2*b^12*c^3*d^5 + 384*a^2*b^12*c^5*d^3 + 680*a^3*b^11*c^2*d^6 - 1680*a^3*b^11*c^4*d^4 - 96*a^3*b^11*c^6*d^2 + 3200*a^4*b^10*c^3*d^5 - 96*a^4*b^10*c^5*d^3 - 2864*a^5*b^9*c^2*d^6 + 1376*a^5*b^9*c^4*d^4 + 48*a^5*b^9*c^6*d^2 - 2976*a^6*b^8*c^3*d^5 - 64*a^6*b^8*c^5*d^3 + 2824*a^7*b^7*c^2*d^6 - 264*a^7*b^7*c^4*d^4 + 768*a^8*b^6*c^3*d^5 - 800*a^9*b^5*c^2*d^6))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(a^2*d^4*3i + (b^2*d^2*(12*c^2 + d^2)*1i)/2 - a*b*c*d^3*8i))/b^4 + (16*tan(e/2 + (f*x)/2)*(216*a^12*d^12 + 8*a^4*b^8*d^12 + 82*a^6*b^6*d^12 + 126*a^8*b^4*d^12 - 432*a^10*b^2*d^12 - 8*a*b^11*c^3*d^9 - 192*a*b^11*c^5*d^7 - 1152*a*b^11*c^7*d^5 - 24*a^3*b^9*c*d^11 - 504*a^5*b^7*c*d^11 - 1488*a^7*b^5*c*d^11 + 3744*a^9*b^3*c*d^11 + 24*a^2*b^10*c^2*d^10 + 834*a^2*b^10*c^4*d^8 + 6576*a^2*b^10*c^6*d^6 + 288*a^2*b^10*c^8*d^4 - 1432*a^3*b^9*c^3*d^9 - 15744*a^3*b^9*c^5*d^7 + 384*a^3*b^9*c^7*d^5 + 1212*a^4*b^8*c^2*d^10 + 20406*a^4*b^8*c^4*d^8 - 7504*a^4*b^8*c^6*d^6 - 288*a^4*b^8*c^8* d^4 - 15360*a^5*b^7*c^3*d^9 + 22464*a^5*b^7*c^5*d^7 + 768*a^5*b^7*c^7*d^5 + 6636*a^6*b^6*c^2*d^10 - 32976*a^6*b^6*c^4*d^8 + 928*a^6*b^6*c^6*d^6 + 27808*a^7*b^5*c^3*d^9 - 6528*a^7*b^5*c^5*d^7 - 13776*a^8*b^4*c^2*d^10 + 11736*a^8*b^4*c^4*d^8 - 11008*a^9*b^3*c^3*d^9 + 5904*a^10*b^2*c^2*d^10 - 1728*a^11*b*c*d^11))/(b^13 - 2*a^2*b^11 + a^4*b^9)))*(a^2*d^4*3i + (b^2*d^2*(12*c^2 + d^2)*1i)/2 - a*b*c*d^3*8i)*2i)/(b^4*f) + (atan((((a*d - b*c)^3*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(a^2*b^11*d^8 + 10*a^4*b^9*d^8 + 13*a^6*b^7*d^8 - 60*a^8*b^5*d^8 + 36*a^10*b^3*d^8 - 32*a^3*b^10*c*d^7 - 128*a^5*b^8*c*d^7 + 352*a^7*b^6*c*d^7 - 192*a^9*b^4*c*d^7 + 24*a^2*b^11*c^2*d^6 + 144*a^2*b^11*c^4*d^4 - 384*a^3*b^10*c^3*d^5 + 352*a^4*b^9*c^2*d^6 - 288*a^4*b^9*c^4*d^4 + 768*a^5*b^8*c^3*d^5 - 776*a^6*b^7*c^2*d^6 + 144*a^6*b^7*c^4*d^4 - 384*a^7*b^6*c^3*d^5 + 400*a^8*b^5*c^2*d^6))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(e/2 + (f*x)/2)*(2*a*b^13*d^8 - 4*a^3*b^11*c^8 + 19*a^3*b^11*d^8 + 16*a^5*b^9*d^8 - 197*a^7*b^7*d^8 + 228*a^9*b^5*d^8 - 72*a^11*b^3*d^8 + 48*a*b^13*c^2*d^6 + 288*a*b^13*c^4*d^4 - 64*a*b^13*c^6*d^2 - 64*a^2*b^12*c*d^7 + 32*a^2*b^12*c^7*d - 224*a^4*b^10*c*d^7 + 1216*a^6*b^8*c*d^7 - 1280*a^8*b^6*c*d^7 + 384*a^10*b^4*c*d^7 - 768*a^2*b^12*c^3*d^5 + 384*a^2*b^12*c^5*d^3 + 680*a^3*b^11*c^2*d^6 - 1680*a^3*b^11*c^4*d^4 - 96*a^3*b^11*c^6*d^2 + 3200*a^4*b^10*c^3*d^5 - 96*a^4*b^10*c^5*d^3 - 2864*a^5*b^9*c^2*d^6 + 1376*a^5*b^9*c^4*d^4 + 48*a^5*b^9*c^6*d^2 - 2976*a^6*b^8*c^3*d^5 - 64*a^6*b^8*c^5*d^3 + 2824*a^7*b^7*c^2*d^6 - 264*a^7*b^7*c^4*d^4 + 768*a^8*b^6*c^3*d^5 - 800*a^9*b^5*c^2*d^6))/(b^13 - 2*a^2*b^11 + a^4*b^9) + ((a*d - b*c)^3*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(2*a*b^14*d^4 + 4*a^3*b^12*c^4 - 4*a^5*b^10*c^4 + 6*a^3*b^12*d^4 - 14*a^5*b^10*d^4 + 6*a^7*b^8*d^4 + 24*a*b^14*c^2*d^2 - 32*a^2*b^13*c*d^3 - 16*a^2*b^13*c^3*d + 48*a^4*b^11*c*d^3 + 16*a^4*b^11*c^3*d - 16*a^6*b^9*c*d^3 - 24*a^3*b^12*c^2*d^2))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(e/2 + (f*x)/2)*(8*a^2*b^14*c^4 - 8*a^4*b^12*c^4 + 32*a^4*b^12*d^4 - 56*a^6*b^10*d^4 + 24*a^8*b^8*d^4 - 96*a^3*b^13*c*d^3 + 32*a^3*b^13*c^3*d + 160*a^5*b^11*c*d^3 - 64*a^7*b^9*c*d^3 + 96*a^2*b^14*c^2*d^2 - 144*a^4*b^12*c^2*d^2 + 48*a^6*b^10*c^2*d^2 - 32*a*b^15*c^3*d))/(b^13 - 2*a^2*b^11 + a^4*b^9) + (((8*(4*a^2*b^15 - 8*a^4*b^13 + 4*a^6*b^11))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(e/2 + (f*x)/2)*(12*a*b^17 - 32*a^3*b^15 + 28*a^5*b^13 - 8*a^7*b^11))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(a*d - b*c)^3*(-(a + b)^3*(a - b)^3)^(1/2)*(3*a^2*d - 4*b^2*d + a*b*c))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))*(3*a^2*d - 4*b^2*d + a*b*c))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))*(3*a^2*d - 4*b^2*d + a*b*c)*1i)/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4) + ((a*d - b*c)^3*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(a^2*b^11*d^8 + 10*a^4*b^9*d^8 + 13*a^6*b^7*d^8 - 60*a^8*b^5*d^8 + 36*a^10*b^3*d^8 - 32*a^3*b^10*c*d^7 - 128*a^5*b^8*c*d^7 + 352*a^7*b^6*c*d^7 - 192*a^9*b^4*c*d^7 + 24*a^2*b^11*c^2*d^6 + 144*a^2*b^11*c^4*d^4 - 384*a^3*b^10*c^3*d^5 + 352*a^4*b^9*c^2*d^6 - 288*a^4*b^9*c^4*d^4 + 768*a^5*b^8*c^3*d^5 - 776*a^6*b^7*c^2*d^6 + 144*a^6*b^7*c^4*d^4 - 384*a^7*b^6*c^3*d^5 + 400*a^8*b^5*c^2*d^6))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(e/2 + (f*x)/2)*(2*a*b^13*d^8 - 4*a^3*b^11*c^8 + 19*a^3*b^11*d^8 + 16*a^5*b^9*d^8 - 197*a^7*b^7*d^8 + 228*a^9*b^5*d^8 - 72*a^11*b^3*d^8 + 48*a*b^13*c^2*d^6 + 288*a*b^13*c^4*d^4 - 64*a*b^13*c^6*d^2 - 64*a^2*b^12*c*d^

7 + 32*a^2*b^12*c^7*d - 224*a^4*b^10*c*d^7 + 1216*a^6*b^8*c*d^7 - 1280*a^8*
b^6*c*d^7 + 384*a^10*b^4*c*d^7 - 768*a^2*b^12*c^3*d^5 + 384*a^2*b^12*c^5*d^
3 + 680*a^3*b^11*c^2*d^6 - 1680*a^3*b^11*c^4*d^4 - 96*a^3*b^11*c^6*d^2 + 32
00*a^4*b^10*c^3*d^5 - 96*a^4*b^10*c^5*d^3 - 2864*a^5*b^9*c^2*d^6 + 1376*a^5
*b^9*c^4*d^4 + 48*a^5*b^9*c^6*d^2 - 2976*a^6*b^8*c^3*d^5 - 64*a^6*b^8*c^5*d
^3 + 2824*a^7*b^7*c^2*d^6 - 264*a^7*b^7*c^4*d^4 + 768*a^8*b^6*c^3*d^5 - 800
*a^9*b^5*c^2*d^6))/(b^13 - 2*a^2*b^11 + a^4*b^9) - ((a*d - b*c)^3*(-(a + b)
^3*(a - b)^3)^(1/2)*((8*(2*a*b^14*d^4 + 4*a^3*b^12*c^4 - 4*a^5*b^10*c^4 + 6
*a^3*b^12*d^4 - 14*a^5*b^10*d^4 + 6*a^7*b^8*d^4 + 24*a*b^14*c^2*d^2 - 32*a^
2*b^13*c*d^3 - 16*a^2*b^13*c^3*d + 48*a^4*b^11*c*d^3 + 16*a^4*b^11*c^3*d -
16*a^6*b^9*c*d^3 - 24*a^3*b^12*c^2*d^2))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8
*tan(e/2 + (f*x)/2)*(8*a^2*b^14*c^4 - 8*a^4*b^12*c^4 + 32*a^4*b^12*d^4 - 56
*a^6*b^10*d^4 + 24*a^8*b^8*d^4 - 96*a^3*b^13*c*d^3 + 32*a^3*b^13*c^3*d + 16
0*a^5*b^11*c*d^3 - 64*a^7*b^9*c*d^3 + 96*a^2*b^14*c^2*d^2 - 144*a^4*b^12*c^
2*d^2 + 48*a^6*b^10*c^2*d^2 - 32*a*b^15*c^3*d))/(b^13 - 2*a^2*b^11 + a^4*b^
9) - (((8*(4*a^2*b^15 - 8*a^4*b^13 + 4*a^6*b^11))/(b^12 - 2*a^2*b^10 + a^4*
b^8) + (8*tan(e/2 + (f*x)/2)*(12*a*b^17 - 32*a^3*b^15 + 28*a^5*b^13 - 8*a^7
*b^11))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(a*d - b*c)^3*(-(a + b)^3*(a - b)^3)
^(1/2)*(3*a^2*d - 4*b^2*d + a*b*c))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4
))*(3*a^2*d - 4*b^2*d + a*b*c))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))*(
3*a^2*d - 4*b^2*d + a*b*c)*1i)/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))/((
16*(54*a^11*d^12 + 4*a^5*b^6*d^12 + 9*a^7*b^4*d^12 - 81*a^9*b^2*d^12 - 32*a
*b^10*c^6*d^6 - 384*a*b^10*c^8*d^4 - 12*a^4*b^7*c*d^11 - 60*a^6*b^5*c*d^11
+ 648*a^8*b^3*c*d^11 - 4*a^2*b^9*c^3*d^9 + 96*a^2*b^9*c^5*d^7 + 2256*a^2*b^
9*c^7*d^5 + 192*a^2*b^9*c^9*d^3 + 12*a^3*b^8*c^2*d^10 - 63*a^3*b^8*c^4*d^8
- 5784*a^3*b^8*c^6*d^6 - 690*a^3*b^8*c^8*d^4 - 24*a^3*b^8*c^10*d^2 - 76*a^4
*b^7*c^3*d^9 + 8592*a^4*b^7*c^5*d^7 + 480*a^4*b^7*c^7*d^5 + 32*a^4*b^7*c^9*
d^3 + 126*a^5*b^6*c^2*d^10 - 8277*a^5*b^6*c^4*d^8 + 1552*a^5*b^6*c^6*d^6 +
132*a^5*b^6*c^8*d^4 + 5424*a^6*b^5*c^3*d^9 - 4128*a^6*b^5*c^5*d^7 - 384*a^6
*b^5*c^7*d^5 - 2394*a^7*b^4*c^2*d^10 + 4860*a^7*b^4*c^4*d^8 + 400*a^7*b^4*c
^6*d^6 - 3472*a^8*b^3*c^3*d^9 - 192*a^8*b^3*c^5*d^7 + 1584*a^9*b^2*c^2*d^10
+ 36*a^9*b^2*c^4*d^8 - 432*a^10*b*c*d^11))/(b^12 - 2*a^2*b^10 + a^4*b^8) +
(16*tan(e/2 + (f*x)/2)*(216*a^12*d^12 + 8*a^4*b^8*d^12 + 82*a^6*b^6*d^12 +
126*a^8*b^4*d^12 - 432*a^10*b^2*d^12 - 8*a*b^11*c^3*d^9 - 192*a*b^11*c^5*d
^7 - 1152*a*b^11*c^7*d^5 - 24*a^3*b^9*c*d^11 - 504*a^5*b^7*c*d^11 - 1488*a^
7*b^5*c*d^11 + 3744*a^9*b^3*c*d^11 + 24*a^2*b^10*c^2*d^10 + 834*a^2*b^10*c^
4*d^8 + 6576*a^2*b^10*c^6*d^6 + 288*a^2*b^10*c^8*d^4 - 1432*a^3*b^9*c^3*d^9
- 15744*a^3*b^9*c^5*d^7 + 384*a^3*b^9*c^7*d^5 + 1212*a^4*b^8*c^2*d^10 + 20
406*a^4*b^8*c^4*d^8 - 7504*a^4*b^8*c^6*d^6 - 288*a^4*b^8*c^8*d^4 - 15360*a^
5*b^7*c^3*d^9 + 22464*a^5*b^7*c^5*d^7 + 768*a^5*b^7*c^7*d^5 + 6636*a^6*b^6*
c^2*d^10 - 32976*a^6*b^6*c^4*d^8 + 928*a^6*b^6*c^6*d^6 + 27808*a^7*b^5*c^3*
d^9 - 6528*a^7*b^5*c^5*d^7 - 13776*a^8*b^4*c^2*d^10 + 11736*a^8*b^4*c^4*d^8
- 11008*a^9*b^3*c^3*d^9 + 5904*a^10*b^2*c^2*d^10 - 1728*a^11*b*c*d^11))/(b
^13 - 2*a^2*b^11 + a^4*b^9) + ((a*d - b*c)^3*(-(a + b)^3*(a - b)^3)^(1/2)*(
(8*(a^2*b^11*d^8 + 10*a^4*b^9*d^8 + 13*a^6*b^7*d^8 - 60*a^8*b^5*d^8 + 36*a^

10*b^3*d^8 - 32*a^3*b^10*c*d^7 - 128*a^5*b^8*c*d^7 + 352*a^7*b^6*c*d^7 - 19
2*a^9*b^4*c*d^7 + 24*a^2*b^11*c^2*d^6 + 144*a^2*b^11*c^4*d^4 - 384*a^3*b^10
*c^3*d^5 + 352*a^4*b^9*c^2*d^6 - 288*a^4*b^9*c^4*d^4 + 768*a^5*b^8*c^3*d^5
- 776*a^6*b^7*c^2*d^6 + 144*a^6*b^7*c^4*d^4 - 384*a^7*b^6*c^3*d^5 + 400*a^8
*b^5*c^2*d^6))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(e/2 + (f*x)/2)*(2*a*b
^13*d^8 - 4*a^3*b^11*c^8 + 19*a^3*b^11*d^8 + 16*a^5*b^9*d^8 - 197*a^7*b^7*d
^8 + 228*a^9*b^5*d^8 - 72*a^11*b^3*d^8 + 48*a*b^13*c^2*d^6 + 288*a*b^13*c^4
*d^4 - 64*a*b^13*c^6*d^2 - 64*a^2*b^12*c*d^7 + 32*a^2*b^12*c^7*d - 224*a^4*
b^10*c*d^7 + 1216*a^6*b^8*c*d^7 - 1280*a^8*b^6*c*d^7 + 384*a^10*b^4*c*d^7 -
 768*a^2*b^12*c^3*d^5 + 384*a^2*b^12*c^5*d^3 + 680*a^3*b^11*c^2*d^6 - 1680*
a^3*b^11*c^4*d^4 - 96*a^3*b^11*c^6*d^2 + 3200*a^4*b^10*c^3*d^5 - 96*a^4*b^1
0*c^5*d^3 - 2864*a^5*b^9*c^2*d^6 + 1376*a^5*b^9*c^4*d^4 + 48*a^5*b^9*c^6*d^
2 - 2976*a^6*b^8*c^3*d^5 - 64*a^6*b^8*c^5*d^3 + 2824*a^7*b^7*c^2*d^6 - 264*
a^7*b^7*c^4*d^4 + 768*a^8*b^6*c^3*d^5 - 800*a^9*b^5*c^2*d^6))/(b^13 - 2*a^2
*b^11 + a^4*b^9) + ((a*d - b*c)^3*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(2*a*b^1
4*d^4 + 4*a^3*b^12*c^4 - 4*a^5*b^10*c^4 + 6*a^3*b^12*d^4 - 14*a^5*b^10*d^4
+ 6*a^7*b^8*d^4 + 24*a*b^14*c^2*d^2 - 32*a^2*b^13*c*d^3 - 16*a^2*b^13*c^3*d
 + 48*a^4*b^11*c*d^3 + 16*a^4*b^11*c^3*d - 16*a^6*b^9*c*d^3 - 24*a^3*b^12*c
^2*d^2))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(e/2 + (f*x)/2)*(8*a^2*b^14*
c^4 - 8*a^4*b^12*c^4 + 32*a^4*b^12*d^4 - 56*a^6*b^10*d^4 + 24*a^8*b^8*d^4 -
 96*a^3*b^13*c*d^3 + 32*a^3*b^13*c^3*d + 160*a^5*b^11*c*d^3 - 64*a^7*b^9*c*
d^3 + 96*a^2*b^14*c^2*d^2 - 144*a^4*b^12*c^2*d^2 + 48*a^6*b^10*c^2*d^2 - 32
*a*b^15*c^3*d))/(b^13 - 2*a^2*b^11 + a^4*b^9) + (((8*(4*a^2*b^15 - 8*a^4*b^
13 + 4*a^6*b^11))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(e/2 + (f*x)/2)*(12
*a*b^17 - 32*a^3*b^15 + 28*a^5*b^13 - 8*a^7*b^11))/(b^13 - 2*a^2*b^11 + a^4
b^9))(a*d - b*c)^3*(-(a + b)^3*(a - b)^3)^(1/2)*(3*a^2*d - 4*b^2*d + a*b*
c))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))*(3*a^2*d - 4*b^2*d + a*b*c))/
(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))*(3*a^2*d - 4*b^2*d + a*b*c))/(b^1
0 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4) - ((a*d - b*c)^3*(-(a + b)^3*(a - b)^3
)^(1/2)*((8*(a^2*b^11*d^8 + 10*a^4*b^9*d^8 + 13*a^6*b^7*d^8 - 60*a^8*b^5*d^
8 + 36*a^10*b^3*d^8 - 32*a^3*b^10*c*d^7 - 128*a^5*b^8*c*d^7 + 352*a^7*b^6*c
*d^7 - 192*a^9*b^4*c*d^7 + 24*a^2*b^11*c^2*d^6 + 144*a^2*b^11*c^4*d^4 - 384
*a^3*b^10*c^3*d^5 + 352*a^4*b^9*c^2*d^6 - 288*a^4*b^9*c^4*d^4 + 768*a^5*b^8
*c^3*d^5 - 776*a^6*b^7*c^2*d^6 + 144*a^6*b^7*c^4*d^4 - 384*a^7*b^6*c^3*d^5
+ 400*a^8*b^5*c^2*d^6))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(e/2 + (f*x)/
2)*(2*a*b^13*d^8 - 4*a^3*b^11*c^8 + 19*a^3*b^11*d^8 + 16*a^5*b^9*d^8 - 197*
a^7*b^7*d^8 + 228*a^9*b^5*d^8 - 72*a^11*b^3*d^8 + 48*a*b^13*c^2*d^6 + 288*a
*b^13*c^4*d^4 - 64*a*b^13*c^6*d^2 - 64*a^2*b^12*c*d^7 + 32*a^2*b^12*c^7*d -
 224*a^4*b^10*c*d^7 + 1216*a^6*b^8*c*d^7 - 1280*a^8*b^6*c*d^7 + 384*a^10*b^
4*c*d^7 - 768*a^2*b^12*c^3*d^5 + 384*a^2*b^12*c^5*d^3 + 680*a^3*b^11*c^2*d^
6 - 1680*a^3*b^11*c^4*d^4 - 96*a^3*b^11*c^6*d^2 + 3200*a^4*b^10*c^3*d^5 - 9
6*a^4*b^10*c^5*d^3 - 2864*a^5*b^9*c^2*d^6 + 1376*a^5*b^9*c^4*d^4 + 48*a^5*b
^9*c^6*d^2 - 2976*a^6*b^8*c^3*d^5 - 64*a^6*b^8*c^5*d^3 + 2824*a^7*b^7*c^2*d
^6 - 264*a^7*b^7*c^4*d^4 + 768*a^8*b^6*c^3*d^5 - 800*a^9*b^5*c^2*d^6))/(b^1
3 - 2*a^2*b^11 + a^4*b^9) - ((a*d - b*c)^3*(-(a + b)^3*(a - b)^3)^(1/2)*((8

```
*(2*a*b^14*d^4 + 4*a^3*b^12*c^4 - 4*a^5*b^10*c^4 + 6*a^3*b^12*d^4 - 14*a^5*
b^10*d^4 + 6*a^7*b^8*d^4 + 24*a*b^14*c^2*d^2 - 32*a^2*b^13*c*d^3 - 16*a^2*b
^13*c^3*d + 48*a^4*b^11*c*d^3 + 16*a^4*b^11*c^3*d - 16*a^6*b^9*c*d^3 - 24*a
^3*b^12*c^2*d^2))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(e/2 + (f*x)/2)*(8*
a^2*b^14*c^4 - 8*a^4*b^12*c^4 + 32*a^4*b^12*d^4 - 56*a^6*b^10*d^4 + 24*a^8*
b^8*d^4 - 96*a^3*b^13*c*d^3 + 32*a^3*b^13*c^3*d + 160*a^5*b^11*c*d^3 - 64*a
^7*b^9*c*d^3 + 96*a^2*b^14*c^2*d^2 - 144*a^4*b^12*c^2*d^2 + 48*a^6*b^10*c^2
*d^2 - 32*a*b^15*c^3*d))/(b^13 - 2*a^2*b^11 + a^4*b^9) - (((8*(4*a^2*b^15 -
 8*a^4*b^13 + 4*a^6*b^11))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(e/2 + (f*
x)/2)*(12*a*b^17 - 32*a^3*b^15 + 28*a^5*b^13 - 8*a^7*b^11))/(b^13 - 2*a^2*b
^11 + a^4*b^9))*(a*d - b*c)^3*(-(a + b)^3*(a - b)^3)^(1/2)*(3*a^2*d - 4*b^2
*d + a*b*c))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))*(3*a^2*d - 4*b^2*d +
 a*b*c))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))*(3*a^2*d - 4*b^2*d + a*b
*c))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))*(a*d - b*c)^3*(-(a + b)^3*(
a - b)^3)^(1/2)*(3*a^2*d - 4*b^2*d + a*b*c)*2i)/(f*(b^10 - 3*a^2*b^8 + 3*a^
4*b^6 - a^6*b^4))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**4/(a+b*sin(f*x+e))**2,x)
[Out] Timed out
```

3.707 $\int \frac{(c+d\sin(e+fx))^3}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=205

$$\frac{d\left(-2a^2d^2+2abcd-\left(b^2\left(c^2-d^2\right)\right)\right)\cos(e+fx)}{b^2f\left(a^2-b^2\right)}+\frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))}{bf\left(a^2-b^2\right)(a+b\sin(e+fx))}+\frac{2(bc-ad)^2\left(2a^2d+\right.}{b}$$

```
[Out] d^2*(-2*a*d+3*b*c)*x/b^3+2*(-a*d+b*c)^2*(2*a^2*d+a*b*c-3*b^2*d)*arctan((b+a
*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/b^3/(a^2-b^2)^(3/2)/f+d*(2*a*b*c*d-2*
a^2*d^2-b^2*(c^2-d^2))*cos(f*x+e)/b^2/(a^2-b^2)/f+(-a*d+b*c)^2*cos(f*x+e)*(
c+d*sin(f*x+e))/b/(a^2-b^2)/f/(a+b*sin(f*x+e))
```

Rubi [A] time = 0.46, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2792, 3023, 2735, 2660, 618, 204}

$$\frac{d\left(-2a^2d^2+2abcd+b^2\left(-\left(c^2-d^2\right)\right)\right)\cos(e+fx)}{b^2f\left(a^2-b^2\right)}+\frac{2(bc-ad)^2\left(2a^2d+abc-3b^2d\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3f\left(a^2-b^2\right)^{3/2}}+\frac{(bc-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^3/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (2*(b*c - a*d)^2*(a*b*c + 2*a^2*d - 3*b^2*d)*
ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(b^3*(a^2 - b^2)^(3/2)*f)
 + (d*(2*a*b*c*d - 2*a^2*d^2 - b^2*(c^2 - d^2))*Cos[e + f*x])/(b^2*(a^2 - b
^2)*f) + ((b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x]))/(b*(a^2 - b^2)*f
*(a + b*Sin[e + f*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^3}{(a+b\sin(e+fx))^2}\,dx = \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))}{b\left(a^2-b^2\right)f(a+b\sin(e+fx))} - \frac{\int \frac{3b^2c^2d+a^2d^3-abc\left(c^2+3d^2\right)-d\left(a^2cd-3b^2cd+ab\left(c^2+\right.\right.}{a+b}}{b\left(\right.}$$

$$= \frac{d\left(2abcd-2a^2d^2-b^2\left(c^2-d^2\right)\right)\cos(e+fx)}{b^2\left(a^2-b^2\right)f} + \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+}{b\left(a^2-b^2\right)f(a+b\sin(e+fx))}$$

$$= \frac{d^2(3bc-2ad)x}{b^3} + \frac{d\left(2abcd-2a^2d^2-b^2\left(c^2-d^2\right)\right)\cos(e+fx)}{b^2\left(a^2-b^2\right)f} + \frac{(bc-ad)^2\cos(e+}{b\left(a^2-b^2\right)f(}$$

$$= \frac{d^2(3bc-2ad)x}{b^3} + \frac{d\left(2abcd-2a^2d^2-b^2\left(c^2-d^2\right)\right)\cos(e+fx)}{b^2\left(a^2-b^2\right)f} + \frac{(bc-ad)^2\cos(e+}{b\left(a^2-b^2\right)f(}$$

$$= \frac{d^2(3bc-2ad)x}{b^3} + \frac{d\left(2abcd-2a^2d^2-b^2\left(c^2-d^2\right)\right)\cos(e+fx)}{b^2\left(a^2-b^2\right)f} + \frac{(bc-ad)^2\cos(e+}{b\left(a^2-b^2\right)f(}$$

$$= \frac{d^2(3bc-2ad)x}{b^3} + \frac{2(bc-ad)^2\left(abc+2a^2d-3b^2d\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3\left(a^2-b^2\right)^{3/2}f} + \frac{d\left(2abc\right.}{}$$

Mathematica [A] time = 1.10, size = 151, normalized size = 0.74

$$\frac{\frac{2(bc-ad)^2\left(2a^2d+abc-3b^2d\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + d^2(e+fx)(3bc-2ad) + \frac{b(bc-ad)^3\cos(e+fx)}{(a-b)(a+b)(a+b\sin(e+fx))} - bd^3\cos(e+fx)}{b^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^3/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*(e + f*x) + (2*(b*c - a*d)^2*(a*b*c + 2*a^2*d - 3*b^2*
d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - b*
d^3*Cos[e + f*x] + (b*(b*c - a*d)^3*Cos[e + f*x])/((a - b)*(a + b)*(a + b*S
in[e + f*x])))/(b^3*f)
```

fricas [B] time = 0.68, size = 1006, normalized size = 4.91

$$\left[\frac{2\left(3\left(a^5b-2a^3b^3+ab^5\right)cd^2-2\left(a^6-2a^4b^2+a^2b^4\right)d^3\right)fx-\left(a^2b^3c^3-3ab^4c^2d-3\left(a^4b-2a^2b^3\right)cd^2+\left(2a^5-\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(3*(a^5*b - 2*a^3*b^3 + a*b^5)*c*d^2 - 2*(a^6 - 2*a^4*b^2 + a^2*b^4
)*d^3)*f*x - (a^2*b^3*c^3 - 3*a*b^4*c^2*d - 3*(a^4*b - 2*a^2*b^3)*c*d^2 + (
2*a^5 - 3*a^3*b^2)*d^3 + (a*b^4*c^3 - 3*b^5*c^2*d - 3*(a^3*b^2 - 2*a*b^4)*c
*d^2 + (2*a^4*b - 3*a^2*b^3)*d^3)*sin(f*x + e))*sqrt(-a^2 + b^2)*log(((2*a^
2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 + 2*(a*cos(f*x + e
)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*
a*b*sin(f*x + e) - a^2 - b^2)) + 2*((a^2*b^4 - b^6)*c^3 - 3*(a^3*b^3 - a*b^
5)*c^2*d + 3*(a^4*b^2 - a^2*b^4)*c*d^2 - (2*a^5*b - 3*a^3*b^3 + a*b^5)*d^3)
*cos(f*x + e) - 2*((a^4*b^2 - 2*a^2*b^4 + b^6)*d^3*cos(f*x + e) - (3*(a^4*b
^2 - 2*a^2*b^4 + b^6)*c*d^2 - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d^3)*f*x)*sin(f
*x + e))/((a^4*b^4 - 2*a^2*b^6 + b^8)*f*sin(f*x + e) + (a^5*b^3 - 2*a^3*b^5
 + a*b^7)*f), ((3*(a^5*b - 2*a^3*b^3 + a*b^5)*c*d^2 - 2*(a^6 - 2*a^4*b^2 +
a^2*b^4)*d^3)*f*x - (a^2*b^3*c^3 - 3*a*b^4*c^2*d - 3*(a^4*b - 2*a^2*b^3)*c*
d^2 + (2*a^5 - 3*a^3*b^2)*d^3 + (a*b^4*c^3 - 3*b^5*c^2*d - 3*(a^3*b^2 - 2*a
*b^4)*c*d^2 + (2*a^4*b - 3*a^2*b^3)*d^3)*sin(f*x + e))*sqrt(a^2 - b^2)*arct
an(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))) + ((a^2*b^4 - b^6)
*c^3 - 3*(a^3*b^3 - a*b^5)*c^2*d + 3*(a^4*b^2 - a^2*b^4)*c*d^2 - (2*a^5*b -
 3*a^3*b^3 + a*b^5)*d^3)*cos(f*x + e) - ((a^4*b^2 - 2*a^2*b^4 + b^6)*d^3*co
s(f*x + e) - (3*(a^4*b^2 - 2*a^2*b^4 + b^6)*c*d^2 - 2*(a^5*b - 2*a^3*b^3 +
a*b^5)*d^3)*f*x)*sin(f*x + e))/((a^4*b^4 - 2*a^2*b^6 + b^8)*f*sin(f*x + e)
+ (a^5*b^3 - 2*a^3*b^5 + a*b^7)*f)]
```

giac [B] time = 0.25, size = 579, normalized size = 2.82

$$\frac{\frac{2\left(ab^3c^3-3b^4c^2d-3a^3bcd^2+6ab^3cd^2+2a^4d^3-3a^2b^2d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^3-b^5\right)\sqrt{a^2-b^2}}+\frac{2\left(b^4c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-3ab^3c^2d\tan\left(\frac{1}{2}f\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] (2*(a*b^3*c^3 - 3*b^4*c^2*d - 3*a^3*b*c*d^2 + 6*a*b^3*c*d^2 + 2*a^4*d^3 - 3
*a^2*b^2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*
f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^2*b^3 - b^5)*sqrt(a^2 - b^2)) + 2*(
b^4*c^3*tan(1/2*f*x + 1/2*e)^3 - 3*a*b^3*c^2*d*tan(1/2*f*x + 1/2*e)^3 + 3*a
^2*b^2*c*d^2*tan(1/2*f*x + 1/2*e)^3 - a^3*b*d^3*tan(1/2*f*x + 1/2*e)^3 + a*
b^3*c^3*tan(1/2*f*x + 1/2*e)^2 - 3*a^2*b^2*c^2*d*tan(1/2*f*x + 1/2*e)^2 + 3
*a^3*b*c*d^2*tan(1/2*f*x + 1/2*e)^2 - 2*a^4*d^3*tan(1/2*f*x + 1/2*e)^2 + a^
2*b^2*d^3*tan(1/2*f*x + 1/2*e)^2 + b^4*c^3*tan(1/2*f*x + 1/2*e) - 3*a*b^3*c
^2*d*tan(1/2*f*x + 1/2*e) + 3*a^2*b^2*c*d^2*tan(1/2*f*x + 1/2*e) - 3*a^3*b*
d^3*tan(1/2*f*x + 1/2*e) + 2*a*b^3*d^3*tan(1/2*f*x + 1/2*e) + a*b^3*c^3 - 3
*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - 2*a^4*d^3 + a^2*b^2*d^3)/((a^3*b^2 - a*b^4
)*(a*tan(1/2*f*x + 1/2*e)^4 + 2*b*tan(1/2*f*x + 1/2*e)^3 + 2*a*tan(1/2*f*x
+ 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)) + (3*b*c*d^2 - 2*a*d^3)*(f*x +
e)/b^3)/f
```

maple [B] time = 0.29, size = 842, normalized size = 4.11

$$-\frac{2d^3}{f\,b^2\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}-\frac{4d^3\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a}{f\,b^3}+\frac{6d^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c}{f\,b^2}-\frac{2a^2\tan}{fb\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e))^2,x)
```

```
[Out] -2/f*d^3/b^2/(1+tan(1/2*f*x+1/2*e)^2)-4/f*d^3/b^3*arctan(tan(1/2*f*x+1/2*e)
)*a+6/f*d^2/b^2*arctan(tan(1/2*f*x+1/2*e))*c-2/f/b/(tan(1/2*f*x+1/2*e)^2*a+
2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*a^2*tan(1/2*f*x+1/2*e)*d^3+6/f/(tan(1/2
*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*a*tan(1/2*f*x+1/2*e)*c*
d^2-6/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*tan(1
/2*f*x+1/2*e)*c^2*d+2/f*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+
a)/(a^2-b^2)/a*tan(1/2*f*x+1/2*e)*c^3-2/f/b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan
(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*a^3*d^3+6/f/b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(
1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*a^2*c*d^2-6/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1
/2*f*x+1/2*e)*b+a)/(a^2-b^2)*a*c^2*d+2/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/
2*f*x+1/2*e)*b+a)/(a^2-b^2)*c^3+4/f/b^3/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan
(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^4*d^3-6/f/b^2/(a^2-b^2)^(3/2)*arcta
n(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^3*c*d^2-6/f/b/(a^2-b^
2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*d^3+2
/f/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))
*a*c^3+12/f/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^
2)^(1/2))*a*c*d^2-6/f*b/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+
2*b)/(a^2-b^2)^(1/2))*c^2*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 17.83, size = 8953, normalized size = 43.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^3/(a + b*sin(e + f*x))^2,x)
```

```
[Out] ((2*(b^3*c^3 - 2*a^3*d^3 + a*b^2*d^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2))/(b^2
*(a^2 - b^2)) + (2*tan(e/2 + (f*x)/2)^2*(b^3*c^3 - 2*a^3*d^3 + a*b^2*d^3 -
3*a*b^2*c^2*d + 3*a^2*b*c*d^2))/(b^2*(a^2 - b^2)) + (2*tan(e/2 + (f*x)/2)*(
b^3*c^3 - 3*a^3*d^3 + 2*a*b^2*d^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2))/(a*b*(a
^2 - b^2)) - (2*tan(e/2 + (f*x)/2)^3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3
*a^2*b*c*d^2))/(a*b*(a^2 - b^2)))/(f*(a + 2*b*tan(e/2 + (f*x)/2) + 2*a*tan(
e/2 + (f*x)/2)^2 + a*tan(e/2 + (f*x)/2)^4 + 2*b*tan(e/2 + (f*x)/2)^3)) + (2
*d^2*atan(((d^2*(2*a*d - 3*b*c)*((32*(4*a^4*b^6*d^6 - 8*a^6*b^4*d^6 + 4*a^8
*b^2*d^6 - 12*a^3*b^7*c*d^5 + 24*a^5*b^5*c*d^5 - 12*a^7*b^3*c*d^5 + 9*a^2*b
^8*c^2*d^4 - 18*a^4*b^6*c^2*d^4 + 9*a^6*b^4*c^2*d^4))/(b^9 - 2*a^2*b^7 + a^
4*b^5) - (32*tan(e/2 + (f*x)/2)*(a^3*b^8*c^6 - 8*a^3*b^8*d^6 + 29*a^5*b^6*d
^6 - 28*a^7*b^4*d^6 + 8*a^9*b^2*d^6 - 18*a*b^10*c^2*d^4 + 9*a*b^10*c^4*d^2
+ 24*a^2*b^9*c*d^5 - 6*a^2*b^9*c^5*d - 96*a^4*b^7*c*d^5 + 90*a^6*b^5*c*d^5
- 24*a^8*b^3*c*d^5 - 36*a^2*b^9*c^3*d^3 + 99*a^3*b^8*c^2*d^4 + 12*a^3*b^8*c
^4*d^2 + 12*a^4*b^7*c^3*d^3 - 84*a^5*b^6*c^2*d^4 - 6*a^5*b^6*c^4*d^2 + 4*a^
6*b^5*c^3*d^3 + 18*a^7*b^4*c^2*d^4))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (d^2*(2
*a*d - 3*b*c)*((32*tan(e/2 + (f*x)/2)*(2*a^2*b^11*c^3 - 2*a^4*b^9*c^3 - 6*a
^3*b^10*d^3 + 10*a^5*b^8*d^3 - 4*a^7*b^6*d^3 + 12*a^2*b^11*c*d^2 + 6*a^3*b^
10*c^2*d - 18*a^4*b^9*c*d^2 + 6*a^6*b^7*c*d^2 - 6*a*b^12*c^2*d))/(b^10 - 2*
a^2*b^8 + a^4*b^6) - (32*(a^5*b^7*c^3 - a^3*b^9*c^3 + 2*a^2*b^10*d^3 - 3*a^
4*b^8*d^3 + a^6*b^6*d^3 + 3*a^2*b^10*c^2*d + 3*a^3*b^9*c*d^2 - 3*a^4*b^8*c^
2*d - 3*a*b^11*c*d^2))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (d^2*((32*(a^2*b^12 -
2*a^4*b^10 + a^6*b^8))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(e/2 + (f*x)/2)
*(3*a*b^14 - 8*a^3*b^12 + 7*a^5*b^10 - 2*a^7*b^8))/(b^10 - 2*a^2*b^8 + a^4*
b^6))*(2*a*d - 3*b*c)*1i)/b^3)*1i)/b^3))/b^3 + (d^2*(2*a*d - 3*b*c)*((32*(4
```

*a^4*b^6*d^6 - 8*a^6*b^4*d^6 + 4*a^8*b^2*d^6 - 12*a^3*b^7*c*d^5 + 24*a^5*b^
5*c*d^5 - 12*a^7*b^3*c*d^5 + 9*a^2*b^8*c^2*d^4 - 18*a^4*b^6*c^2*d^4 + 9*a^6
*b^4*c^2*d^4))/(b^9 - 2*a^2*b^7 + a^4*b^5) - (32*tan(e/2 + (f*x)/2)*(a^3*b^
8*c^6 - 8*a^3*b^8*d^6 + 29*a^5*b^6*d^6 - 28*a^7*b^4*d^6 + 8*a^9*b^2*d^6 - 1
8*a*b^10*c^2*d^4 + 9*a*b^10*c^4*d^2 + 24*a^2*b^9*c*d^5 - 6*a^2*b^9*c^5*d -
96*a^4*b^7*c*d^5 + 90*a^6*b^5*c*d^5 - 24*a^8*b^3*c*d^5 - 36*a^2*b^9*c^3*d^3
 + 99*a^3*b^8*c^2*d^4 + 12*a^3*b^8*c^4*d^2 + 12*a^4*b^7*c^3*d^3 - 84*a^5*b^
6*c^2*d^4 - 6*a^5*b^6*c^4*d^2 + 4*a^6*b^5*c^3*d^3 + 18*a^7*b^4*c^2*d^4))/(b
^10 - 2*a^2*b^8 + a^4*b^6) + (d^2*(2*a*d - 3*b*c)*((32*(a^5*b^7*c^3 - a^3*b
^9*c^3 + 2*a^2*b^10*d^3 - 3*a^4*b^8*d^3 + a^6*b^6*d^3 + 3*a^2*b^10*c^2*d +
3*a^3*b^9*c*d^2 - 3*a^4*b^8*c^2*d - 3*a*b^11*c*d^2))/(b^9 - 2*a^2*b^7 + a^4
*b^5) - (32*tan(e/2 + (f*x)/2)*(2*a^2*b^11*c^3 - 2*a^4*b^9*c^3 - 6*a^3*b^10
*d^3 + 10*a^5*b^8*d^3 - 4*a^7*b^6*d^3 + 12*a^2*b^11*c*d^2 + 6*a^3*b^10*c^2*
d - 18*a^4*b^9*c*d^2 + 6*a^6*b^7*c*d^2 - 6*a*b^12*c^2*d))/(b^10 - 2*a^2*b^8
 + a^4*b^6) + (d^2*((32*(a^2*b^12 - 2*a^4*b^10 + a^6*b^8))/(b^9 - 2*a^2*b^7
 + a^4*b^5) + (32*tan(e/2 + (f*x)/2)*(3*a*b^14 - 8*a^3*b^12 + 7*a^5*b^10 -
2*a^7*b^8))/(b^10 - 2*a^2*b^8 + a^4*b^6))*(2*a*d - 3*b*c)*1i)/b^3)*1i)/b^3)
)/b^3)/((64*(6*a^6*b^2*d^9 - 4*a^8*d^9 - 27*a*b^7*c^5*d^4 - 39*a^5*b^3*c*d^
8 + 4*a^7*b*c^3*d^6 + 99*a^2*b^6*c^4*d^5 + 18*a^2*b^6*c^6*d^3 - 144*a^3*b^5
*c^3*d^6 - 39*a^3*b^5*c^5*d^4 - 3*a^3*b^5*c^7*d^2 + 105*a^4*b^4*c^2*d^7 + 3
*a^4*b^4*c^4*d^5 + 2*a^4*b^4*c^6*d^3 + 55*a^5*b^3*c^3*d^6 + 9*a^5*b^3*c^5*d
^4 - 57*a^6*b^2*c^2*d^7 - 12*a^6*b^2*c^4*d^5 + 24*a^7*b*c*d^8))/(b^9 - 2*a^
2*b^7 + a^4*b^5) + (64*tan(e/2 + (f*x)/2)*(40*a^7*b^2*d^9 - 24*a^5*b^4*d^9
- 16*a^9*d^9 - 54*a*b^8*c^4*d^5 + 120*a^4*b^5*c*d^8 - 192*a^6*b^3*c*d^8 + 1
80*a^2*b^7*c^3*d^6 + 18*a^2*b^7*c^5*d^4 - 222*a^3*b^6*c^2*d^7 + 30*a^3*b^6*
c^4*d^5 - 226*a^4*b^5*c^3*d^6 - 18*a^4*b^5*c^5*d^4 + 330*a^5*b^4*c^2*d^7 +
24*a^5*b^4*c^4*d^5 + 46*a^6*b^3*c^3*d^6 - 108*a^7*b^2*c^2*d^7 + 72*a^8*b*c*
d^8))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (d^2*(2*a*d - 3*b*c)*((32*(4*a^4*b^6*d
^6 - 8*a^6*b^4*d^6 + 4*a^8*b^2*d^6 - 12*a^3*b^7*c*d^5 + 24*a^5*b^5*c*d^5 -
12*a^7*b^3*c*d^5 + 9*a^2*b^8*c^2*d^4 - 18*a^4*b^6*c^2*d^4 + 9*a^6*b^4*c^2*d
^4))/(b^9 - 2*a^2*b^7 + a^4*b^5) - (32*tan(e/2 + (f*x)/2)*(a^3*b^8*c^6 - 8*
a^3*b^8*d^6 + 29*a^5*b^6*d^6 - 28*a^7*b^4*d^6 + 8*a^9*b^2*d^6 - 18*a*b^10*c
^2*d^4 + 9*a*b^10*c^4*d^2 + 24*a^2*b^9*c*d^5 - 6*a^2*b^9*c^5*d - 96*a^4*b^7
*c*d^5 + 90*a^6*b^5*c*d^5 - 24*a^8*b^3*c*d^5 - 36*a^2*b^9*c^3*d^3 + 99*a^3*
b^8*c^2*d^4 + 12*a^3*b^8*c^4*d^2 + 12*a^4*b^7*c^3*d^3 - 84*a^5*b^6*c^2*d^4
- 6*a^5*b^6*c^4*d^2 + 4*a^6*b^5*c^3*d^3 + 18*a^7*b^4*c^2*d^4))/(b^10 - 2*a^
2*b^8 + a^4*b^6) + (d^2*(2*a*d - 3*b*c)*((32*tan(e/2 + (f*x)/2)*(2*a^2*b^11
*c^3 - 2*a^4*b^9*c^3 - 6*a^3*b^10*d^3 + 10*a^5*b^8*d^3 - 4*a^7*b^6*d^3 + 12
*a^2*b^11*c*d^2 + 6*a^3*b^10*c^2*d - 18*a^4*b^9*c*d^2 + 6*a^6*b^7*c*d^2 - 6
*a*b^12*c^2*d))/(b^10 - 2*a^2*b^8 + a^4*b^6) - (32*(a^5*b^7*c^3 - a^3*b^9*c
^3 + 2*a^2*b^10*d^3 - 3*a^4*b^8*d^3 + a^6*b^6*d^3 + 3*a^2*b^10*c^2*d + 3*a^
3*b^9*c*d^2 - 3*a^4*b^8*c^2*d - 3*a*b^11*c*d^2))/(b^9 - 2*a^2*b^7 + a^4*b^5
) + (d^2*((32*(a^2*b^12 - 2*a^4*b^10 + a^6*b^8))/(b^9 - 2*a^2*b^7 + a^4*b^5
) + (32*tan(e/2 + (f*x)/2)*(3*a*b^14 - 8*a^3*b^12 + 7*a^5*b^10 - 2*a^7*b^8)
)/(b^10 - 2*a^2*b^8 + a^4*b^6))*(2*a*d - 3*b*c)*1i)/b^3)*1i)/b^3)*1i)/b^3 -

(d^2*(2*a*d - 3*b*c)*((32*(4*a^4*b^6*d^6 - 8*a^6*b^4*d^6 + 4*a^8*b^2*d^6 - 12*a^3*b^7*c*d^5 + 24*a^5*b^5*c*d^5 - 12*a^7*b^3*c*d^5 + 9*a^2*b^8*c^2*d^4 - 18*a^4*b^6*c^2*d^4 + 9*a^6*b^4*c^2*d^4))/(b^9 - 2*a^2*b^7 + a^4*b^5) - (32*tan(e/2 + (f*x)/2)*(a^3*b^8*c^6 - 8*a^3*b^8*d^6 + 29*a^5*b^6*d^6 - 28*a^7*b^4*d^6 + 8*a^9*b^2*d^6 - 18*a*b^10*c^2*d^4 + 9*a*b^10*c^4*d^2 + 24*a^2*b^9*c*d^5 - 6*a^2*b^9*c^5*d - 96*a^4*b^7*c*d^5 + 90*a^6*b^5*c*d^5 - 24*a^8*b^3*c*d^5 - 36*a^2*b^9*c^3*d^3 + 99*a^3*b^8*c^2*d^4 + 12*a^3*b^8*c^4*d^2 + 12*a^4*b^7*c^3*d^3 - 84*a^5*b^6*c^2*d^4 - 6*a^5*b^6*c^4*d^2 + 4*a^6*b^5*c^3*d^3 + 18*a^7*b^4*c^2*d^4))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (d^2*(2*a*d - 3*b*c)*((32*(a^5*b^7*c^3 - a^3*b^9*c^3 + 2*a^2*b^10*d^3 - 3*a^4*b^8*d^3 + a^6*b^6*d^3 + 3*a^2*b^10*c^2*d + 3*a^3*b^9*c*d^2 - 3*a^4*b^8*c^2*d - 3*a*b^11*c*d^2))/(b^9 - 2*a^2*b^7 + a^4*b^5) - (32*tan(e/2 + (f*x)/2)*(2*a^2*b^11*c^3 - 2*a^4*b^9*c^3 - 6*a^3*b^10*d^3 + 10*a^5*b^8*d^3 - 4*a^7*b^6*d^3 + 12*a^2*b^11*c*d^2 + 6*a^3*b^10*c^2*d - 18*a^4*b^9*c*d^2 + 6*a^6*b^7*c*d^2 - 6*a*b^12*c^2*d))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (d^2*((32*(a^2*b^12 - 2*a^4*b^10 + a^6*b^8))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(e/2 + (f*x)/2)*(3*a*b^14 - 8*a^3*b^12 + 7*a^5*b^10 - 2*a^7*b^8))/(b^10 - 2*a^2*b^8 + a^4*b^6))*(2*a*d - 3*b*c)*1i)/b^3)*1i)/b^3)*1i)/b^3))*(2*a*d - 3*b*c))/(b^3*f) + (atan((((a*d - b*c)^2*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(4*a^4*b^6*d^6 - 8*a^6*b^4*d^6 + 4*a^8*b^2*d^6 - 12*a^3*b^7*c*d^5 + 24*a^5*b^5*c*d^5 - 12*a^7*b^3*c*d^5 + 9*a^2*b^8*c^2*d^4 - 18*a^4*b^6*c^2*d^4 + 9*a^6*b^4*c^2*d^4))/(b^9 - 2*a^2*b^7 + a^4*b^5) - (32*tan(e/2 + (f*x)/2)*(a^3*b^8*c^6 - 8*a^3*b^8*d^6 + 29*a^5*b^6*d^6 - 28*a^7*b^4*d^6 + 8*a^9*b^2*d^6 - 18*a*b^10*c^2*d^4 + 9*a*b^10*c^4*d^2 + 24*a^2*b^9*c*d^5 - 6*a^2*b^9*c^5*d - 96*a^4*b^7*c*d^5 + 90*a^6*b^5*c*d^5 - 24*a^8*b^3*c*d^5 - 36*a^2*b^9*c^3*d^3 + 99*a^3*b^8*c^2*d^4 + 12*a^3*b^8*c^4*d^2 + 12*a^4*b^7*c^3*d^3 - 84*a^5*b^6*c^2*d^4 - 6*a^5*b^6*c^4*d^2 + 4*a^6*b^5*c^3*d^3 + 18*a^7*b^4*c^2*d^4))/(b^10 - 2*a^2*b^8 + a^4*b^6) + ((a*d - b*c)^2*(-(a + b)^3*(a - b)^3)^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*a^2*b^11*c^3 - 2*a^4*b^9*c^3 - 6*a^3*b^10*d^3 + 10*a^5*b^8*d^3 - 4*a^7*b^6*d^3 + 12*a^2*b^11*c*d^2 + 6*a^3*b^10*c^2*d - 18*a^4*b^9*c*d^2 + 6*a^6*b^7*c*d^2 - 6*a*b^12*c^2*d))/(b^10 - 2*a^2*b^8 + a^4*b^6) - (32*(a^5*b^7*c^3 - a^3*b^9*c^3 + 2*a^2*b^10*d^3 - 3*a^4*b^8*d^3 + a^6*b^6*d^3 + 3*a^2*b^10*c^2*d + 3*a^3*b^9*c*d^2 - 3*a^4*b^8*c^2*d - 3*a*b^11*c*d^2))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (((32*(a^2*b^12 - 2*a^4*b^10 + a^6*b^8))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(e/2 + (f*x)/2)*(3*a*b^14 - 8*a^3*b^12 + 7*a^5*b^10 - 2*a^7*b^8))/(b^10 - 2*a^2*b^8 + a^4*b^6))*(a*d - b*c)^2*(-(a + b)^3*(a - b)^3)^(1/2)*(2*a^2*d - 3*b^2*d + a*b*c))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(2*a^2*d - 3*b^2*d + a*b*c))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(2*a^2*d - 3*b^2*d + a*b*c)*1i)/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3) + ((a*d - b*c)^2*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(4*a^4*b^6*d^6 - 8*a^6*b^4*d^6 + 4*a^8*b^2*d^6 - 12*a^3*b^7*c*d^5 + 24*a^5*b^5*c*d^5 - 12*a^7*b^3*c*d^5 + 9*a^2*b^8*c^2*d^4 - 18*a^4*b^6*c^2*d^4 + 9*a^6*b^4*c^2*d^4))/(b^9 - 2*a^2*b^7 + a^4*b^5) - (32*tan(e/2 + (f*x)/2)*(a^3*b^8*c^6 - 8*a^3*b^8*d^6 + 29*a^5*b^6*d^6 - 28*a^7*b^4*d^6 + 8*a^9*b^2*d^6 - 18*a*b^10*c^2*d^4 + 9*a*b^10*c^4*d^2 + 24*a^2*b^9*c*d^5 - 6*a^2*b^9*c^5*d - 96*a^4*b^7*c*d^5 + 90*a^6*b^5*c *d^5 - 24*a^8*b^3*c*d^5 - 36*a^2*b^9*c^3*d^3 + 99*a^3*b^8*c^2*d^4 + 12*a^3*b^8*c^4*d^2 + 12*a^4*b^7*c^3*d^3 - 84*a^5*b^6*c^2*d^4 - 6*a^5*b^6*c^4*d^2 + 4*a^6*b^5*c^3*d^3 + 18*a^7*b^4*c^2*d^4))/(b^10 - 2*a^2*b^8 + a^4*b^6) + ((a*d - b*c)^2*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(a^5*b^7*c^3 - a^3*b^9*c^3 + 2*a^2*b^10*d^3 - 3*a^4*b^8*d^3 + a^6*b^6*d^3 + 3*a^2*b^10*c^2*d + 3*a^3*b^9*c*d^2 - 3*a^4*b^8*c^2*d - 3*a*b^11*c*d^2))/(b^9 - 2*a^2*b^7 + a^4*b^5) - (32*tan(e/2 + (f*x)/2)*(2*a^2*b^11*c^3 - 2*a^4*b^9*c^3 - 6*a^3*b^10*d^3 + 10*a^5*b^8*d^3 - 4*a^7*b^6*d^3 + 12*a^2*b^11*c*d^2 + 6*a^3*b^10*c^2*d - 18*a^4*b^9*c*d^2 + 6*a^6*b^7*c*d^2 - 6*a*b^12*c^2*d))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (((32*(a^2*b^12 - 2*a^4*b^10 + a^6*b^8))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(e/2 + (f*x)/2)*(3*a*b^14 - 8*a^3*b^12 + 7*a^5*b^10 - 2*a^7*b^8))/(b^10 - 2*a^2*b^8 + a^4*b^6))*(a*d - b*c)^2*(-(a + b)^3*(a - b)^3)^(1/2)*(2*a^2*d - 3*b^2*d + a*b*c))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(2*a^2*d - 3*b^2*d + a*b*c))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(2*a^2*d - 3*b^2*d + a*b*c)*1i)/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))/((64*(6*a^6*b^2*d^9 - 4*a^8*d^9 - 27*a*b^7*c^5*d^4 - 39*a^5*b^3*c*d^8 + 4*a^7*b*c^3*d^6 + 99*a^2*b^6*c^4*d^5 + 18*a^2*b^6*c^6*d^3 - 144*a^3*b^5*c^3*d^6 - 39*a^3*b^5*c^5*d^4 - 3*a^3*b^5*c^7*d^2 + 105*a^4*b^4*c^2*d^7 + 3*a^4*b^4*c^4*d^5 + 2*a^4*b^4*c^6*d^3 + 55*a^5*b^3*c^3*d^6 + 9*a^5*b^3*c^5*d^4 - 57*a^6*b^2*c^2*d^7 - 12*a^6*b^2*c^4*d^5 + 24*a^7*b*c*d^8))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (64*tan(e/2 + (f*x)/2)*(40*a^7*b^2*d^9 - 24*a^5*b^4*d^9 - 16*a^9*d^9 - 54*a*b^8*c^4*d^5 + 120*a^4*b^5*c*d^8 - 192*a^6*b^3*c*d^8 + 180*a^2*b^7*c^3*d^6 + 18*a^2*b^7*c^5*d^4 - 222*a^3*b^6*c^2*d^7 + 30*a^3*b^6*c^4*d^5 - 226*a^4*b^5*c^3*d^6 - 18*a^4*b^5*c^5*d^4 + 330*a^5*b^4*c^2*d^7 + 24*a^5*b^4*c^4*d^5 + 46*a^6*b^3*c^3*d^6 - 108*a^7*b^2*c^2*d^7 + 72*a^8*b*c*d^8))/(b^10 - 2*a^2*b^8 + a^4*b^6) + ((a*d - b*c)^2*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(4*a^4*b^6*d^6 - 8*a^6*b^4*d^6 + 4*a^8*b^2*d^6 - 12*a^3*b^7*c*d^5 + 24*a^5*b^5*c*d^5 - 12*a^7*b^3*c*d^5 + 9*a^2*b^8*c^2*d^4 - 18*a^4*b^6*c^2*d^4 + 9*a^6*b^4*c^2*d^4))/(b^9 - 2*a^2*b^7 + a^4*b^5) - (32*tan(e/2 + (f*x)/2)*(a^3*b^8*c^6 - 8*a^3*b^8*d^6 + 29*a^5*b^6*d^6 - 28*a^7*b^4*d^6 + 8*a^9*b^2*d^6 - 18*a*b^10*c^2*d^4 + 9*a*b^10*c^4*d^2 + 24*a^2*b^9*c*d^5 - 6*a^2*b^9*c^5*d - 96*a^4*b^7*c*d^5 + 90*a^6*b^5*c*d^5 - 24*a^8*b^3*c*d^5 - 36*a^2*b^9*c^3*d^3 + 99*a^3*b^8*c^2*d^4 + 12*a^3*b^8*c^4*d^2 + 12*a^4*b^7*c^3*d^3 - 84*a^5*b^6*c^2*d^4 - 6*a^5*b^6*c^4*d^2 + 4*a^6*b^5*c^3*d^3 + 18*a^7*b^4*c^2*d^4))/(b^10 - 2*a^2*b^8 + a^4*b^6) + ((a*d - b*c)^2*(-(a + b)^3*(a - b)^3)^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*a^2*b^11*c^3 - 2*a^4*b^9*c^3 - 6*a^3*b^10*d^3 + 10*a^5*b^8*d^3 - 4*a^7*b^6*d^3 + 12*a^2*b^11*c*d^2 + 6*a^3*b^10*c^2*d - 18*a^4*b^9*c*d^2 + 6*a^6*b^7*c*d^2 - 6*a*b^12*c^2*d))/(b^10 - 2*a^2*b^8 + a^4*b^6) - (32*(a^5*b^7*c^3 - a^3*b^9*c^3 + 2*a^2*b^10*d^3 - 3*a^4*b^8*d^3 + a^6*b^6*d^3 + 3*a^2*b^10*c^2*d + 3*a^3*b^9*c*d^2 - 3*a^4*b^8*c^2*d - 3*a*b^11*c*d^2))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (((32*(a^2*b^12 - 2*a^4*b^10 + a^6*b^8))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(e/2 + (f*x)/2)*(3*a*b^14 - 8*a^3*b^12 + 7*a^5*b^10 - 2*a^7*b^8))/(b^10 - 2*a^2*b^8 + a^4*b^6))*(a*d - b*c)^2*(-(a + b)^3*(a - b)^3)^(1/2)*(2*a^2*d - 3*b^2*d + a*b*c))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(2*a^2*d - 3*b^2*d + a*b*c))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 -

```
a^6*b^3))*(2*a^2*d - 3*b^2*d + a*b*c))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*
b^3) - ((a*d - b*c)^2*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(4*a^4*b^6*d^6 - 8*
a^6*b^4*d^6 + 4*a^8*b^2*d^6 - 12*a^3*b^7*c*d^5 + 24*a^5*b^5*c*d^5 - 12*a^7*
b^3*c*d^5 + 9*a^2*b^8*c^2*d^4 - 18*a^4*b^6*c^2*d^4 + 9*a^6*b^4*c^2*d^4))/(b
^9 - 2*a^2*b^7 + a^4*b^5) - (32*tan(e/2 + (f*x)/2)*(a^3*b^8*c^6 - 8*a^3*b^8
*d^6 + 29*a^5*b^6*d^6 - 28*a^7*b^4*d^6 + 8*a^9*b^2*d^6 - 18*a*b^10*c^2*d^4
+ 9*a*b^10*c^4*d^2 + 24*a^2*b^9*c*d^5 - 6*a^2*b^9*c^5*d - 96*a^4*b^7*c*d^5
+ 90*a^6*b^5*c*d^5 - 24*a^8*b^3*c*d^5 - 36*a^2*b^9*c^3*d^3 + 99*a^3*b^8*c^2
*d^4 + 12*a^3*b^8*c^4*d^2 + 12*a^4*b^7*c^3*d^3 - 84*a^5*b^6*c^2*d^4 - 6*a^5
*b^6*c^4*d^2 + 4*a^6*b^5*c^3*d^3 + 18*a^7*b^4*c^2*d^4))/(b^10 - 2*a^2*b^8 +
a^4*b^6) + ((a*d - b*c)^2*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(a^5*b^7*c^3 -
a^3*b^9*c^3 + 2*a^2*b^10*d^3 - 3*a^4*b^8*d^3 + a^6*b^6*d^3 + 3*a^2*b^10*c^
2*d + 3*a^3*b^9*c*d^2 - 3*a^4*b^8*c^2*d - 3*a*b^11*c*d^2))/(b^9 - 2*a^2*b^7
+ a^4*b^5) - (32*tan(e/2 + (f*x)/2)*(2*a^2*b^11*c^3 - 2*a^4*b^9*c^3 - 6*a^
3*b^10*d^3 + 10*a^5*b^8*d^3 - 4*a^7*b^6*d^3 + 12*a^2*b^11*c*d^2 + 6*a^3*b^1
0*c^2*d - 18*a^4*b^9*c*d^2 + 6*a^6*b^7*c*d^2 - 6*a*b^12*c^2*d))/(b^10 - 2*a
^2*b^8 + a^4*b^6) + (((32*(a^2*b^12 - 2*a^4*b^10 + a^6*b^8))/(b^9 - 2*a^2*b
^7 + a^4*b^5) + (32*tan(e/2 + (f*x)/2)*(3*a*b^14 - 8*a^3*b^12 + 7*a^5*b^10
- 2*a^7*b^8))/(b^10 - 2*a^2*b^8 + a^4*b^6))*(a*d - b*c)^2*(-(a + b)^3*(a -
b)^3)^(1/2)*(2*a^2*d - 3*b^2*d + a*b*c))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6
*b^3))*(2*a^2*d - 3*b^2*d + a*b*c))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)
)*(2*a^2*d - 3*b^2*d + a*b*c))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))*(a
*d - b*c)^2*(-(a + b)^3*(a - b)^3)^(1/2)*(2*a^2*d - 3*b^2*d + a*b*c)*2i)/(f
*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**3/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.708 $\int \frac{(c+d\sin(e+fx))^2}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=129

$$\frac{2(bc-ad)\left(a^2d+abc-2b^2d\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2f\left(a^2-b^2\right)^{3/2}}+\frac{(bc-ad)^2\cos(e+fx)}{bf\left(a^2-b^2\right)(a+b\sin(e+fx))}+\frac{d^2x}{b^2}$$

```
[Out] d^2*x/b^2+2*(-a*d+b*c)*(a^2*d+a*b*c-2*b^2*d)*arctan((b+a*tan(1/2*f*x+1/2*e)
)/(a^2-b^2)^(1/2))/b^2/(a^2-b^2)^(3/2)/f+(-a*d+b*c)^2*cos(f*x+e)/b/(a^2-b^2
)/f/(a+b*sin(f*x+e))
```

Rubi [A] time = 0.22, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2790, 2735, 2660, 618, 204}

$$\frac{2(bc-ad)\left(a^2d+abc-2b^2d\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2f\left(a^2-b^2\right)^{3/2}}+\frac{(bc-ad)^2\cos(e+fx)}{bf\left(a^2-b^2\right)(a+b\sin(e+fx))}+\frac{d^2x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^2/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (d^2*x)/b^2 + (2*(b*c - a*d)*(a*b*c + a^2*d - 2*b^2*d)*ArcTan[(b + a*Tan[(e
 + f*x)/2])/Sqrt[a^2 - b^2]])/(b^2*(a^2 - b^2)^(3/2)*f) + ((b*c - a*d)^2*Co
s[e + f*x])/(b*(a^2 - b^2)*f*(a + b*Sin[e + f*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^2}{(a+b\sin(e+fx))^2}\,dx = \frac{(bc-ad)^2\cos(e+fx)}{b\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{\int \frac{-b\left(2bcd-a\left(c^2+d^2\right)\right)+\left(a^2-b^2\right)d^2\sin(e+fx)}{a+b\sin(e+fx)}\,dx}{b\left(a^2-b^2\right)}$$

$$= \frac{d^2x}{b^2} + \frac{(bc-ad)^2\cos(e+fx)}{b\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{\left((bc-ad)\left(abc+a^2d-2b^2d\right)\right)\int \frac{1}{a+b\sin(e+}$$

$$= \frac{d^2x}{b^2} + \frac{(bc-ad)^2\cos(e+fx)}{b\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{\left(2(bc-ad)\left(abc+a^2d-2b^2d\right)\right)\text{Subst}\left(\int\right.}{b^2\left(a^2-b^2\right)}$$

$$= \frac{d^2x}{b^2} + \frac{(bc-ad)^2\cos(e+fx)}{b\left(a^2-b^2\right)f(a+b\sin(e+fx))} - \frac{\left(4(bc-ad)\left(abc+a^2d-2b^2d\right)\right)\text{Subst}\left(\int\right.}{b^2\left(a^2\right.}$$

$$= \frac{d^2x}{b^2} + \frac{2(bc-ad)\left(abc+a^2d-2b^2d\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{3/2}f} + \frac{(bc-ad)^2\cos(e}{b\left(a^2-b^2\right)f(a+b\,\text{s}}$$

Mathematica [A] time = 0.56, size = 133, normalized size = 1.03

$$\frac{-\frac{2\left(a^3d^2-ab^2\left(c^2+2d^2\right)+2b^3cd\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+\frac{b(bc-ad)^2\cos(e+fx)}{(a-b)(a+b)(a+b\sin(e+fx))}+d^2(e+fx)}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^2/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (d^2*(e + f*x) - (2*(2*b^3*c*d + a^3*d^2 - a*b^2*(c^2 + 2*d^2))*ArcTan[(b +
 a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + (b*(b*c - a*d)^2
*Cos[e + f*x])/((a - b)*(a + b)*(a + b*Sin[e + f*x])))/(b^2*f)
```

fricas [B] time = 0.81, size = 665, normalized size = 5.16

$$\left[\frac{2\left(a^4b-2a^2b^3+b^5\right)d^2fx\sin\left(fx+e\right)+2\left(a^5-2a^3b^2+ab^4\right)d^2fx+\left(a^2b^2c^2-2ab^3cd-\left(a^4-2a^2b^2\right)d^2+\left(ab^3\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^4*b - 2*a^2*b^3 + b^5)*d^2*f*x*sin(f*x + e) + 2*(a^5 - 2*a^3*b^2
 + a*b^4)*d^2*f*x + (a^2*b^2*c^2 - 2*a*b^3*c*d - (a^4 - 2*a^2*b^2)*d^2 + (a
*b^3*c^2 - 2*b^4*c*d - (a^3*b - 2*a*b^3)*d^2)*sin(f*x + e))*sqrt(-a^2 + b^2
)*log(-((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 - 2*(
a*cos(f*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*
x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)) + 2*((a^2*b^3 - b^5)*c^2 - 2*(a
^3*b^2 - a*b^4)*c*d + (a^4*b - a^2*b^3)*d^2)*cos(f*x + e))/((a^4*b^3 - 2*a^
2*b^5 + b^7)*f*sin(f*x + e) + (a^5*b^2 - 2*a^3*b^4 + a*b^6)*f), ((a^4*b - 2
*a^2*b^3 + b^5)*d^2*f*x*sin(f*x + e) + (a^5 - 2*a^3*b^2 + a*b^4)*d^2*f*x -
(a^2*b^2*c^2 - 2*a*b^3*c*d - (a^4 - 2*a^2*b^2)*d^2 + (a*b^3*c^2 - 2*b^4*c*d
 - (a^3*b - 2*a*b^3)*d^2)*sin(f*x + e))*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x
+ e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))) + ((a^2*b^3 - b^5)*c^2 - 2*(a^3*b
^2 - a*b^4)*c*d + (a^4*b - a^2*b^3)*d^2)*cos(f*x + e))/((a^4*b^3 - 2*a^2*b^
5 + b^7)*f*sin(f*x + e) + (a^5*b^2 - 2*a^3*b^4 + a*b^6)*f)]
```

giac [B] time = 0.26, size = 249, normalized size = 1.93

$$\frac{\frac{(fx+e)d^2}{b^2} + \frac{2\left(ab^2c^2-2\,b^3cd-a^3d^2+2\,ab^2d^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^2-b^4\right)\sqrt{a^2-b^2}} + \frac{2\left(b^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,ab^2cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+a^2b\right.}{\left(a^3b-ab^3\right)\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\right.}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] ((f*x + e)*d^2/b^2 + 2*(a*b^2*c^2 - 2*b^3*c*d - a^3*d^2 + 2*a*b^2*d^2)*(pi*
floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/
sqrt(a^2 - b^2)))/((a^2*b^2 - b^4)*sqrt(a^2 - b^2)) + 2*(b^3*c^2*tan(1/2*f*
x + 1/2*e) - 2*a*b^2*c*d*tan(1/2*f*x + 1/2*e) + a^2*b*d^2*tan(1/2*f*x + 1/2
*e) + a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)/((a^3*b - a*b^3)*(a*tan(1/2*f*x +
1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)))/f
```

maple [B] time = 0.32, size = 556, normalized size = 4.31

$$\frac{2d^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f\,b^2}+\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d^2}{f\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)b+a\right)\left(a^2-b^2\right)}-\frac{4b\tan\left(\frac{fx}{2}\right.}{f\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a+2\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e))^2,x)
```

```
[Out] 2/f*d^2/b^2*arctan(tan(1/2*f*x+1/2*e))+2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/
2*f*x+1/2*e)*b+a)/(a^2-b^2)*a*tan(1/2*f*x+1/2*e)*d^2-4/f*b/(tan(1/2*f*x+1/2
*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*tan(1/2*f*x+1/2*e)*c*d+2/f*b^2/
(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)/a*tan(1/2*f*x+1
/2*e)*c^2+2/f/b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)
*a^2*d^2-4/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*a*
c*d+2/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*c^2-2
/f/b^2/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1
/2))*a^3*d^2+2/f/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a
^2-b^2)^(1/2))*a*c^2+4/f/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)
+2*b)/(a^2-b^2)^(1/2))*a*d^2-4/f*b/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*
f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*c*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 15.47, size = 5776, normalized size = 44.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^2/(a + b*sin(e + f*x))^2,x)
```

```
[Out] ((2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(b*(a^2 - b^2)) + (2*tan(e/2 + (f*x)/2
)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(a*(a^2 - b^2)))/(f*(a + 2*b*tan(e/2 + (
f*x)/2) + a*tan(e/2 + (f*x)/2)^2)) - (2*d^2*atan(((d^2*((32*tan(e/2 + (f*x)
/2)*(2*a^7*b*d^4 - 2*a*b^7*d^4 + a^3*b^5*c^4 + 9*a^3*b^5*d^4 - 8*a^5*b^3*d^
4 + 4*a*b^7*c^2*d^2 - 8*a^2*b^6*c*d^3 - 4*a^2*b^6*c^3*d + 4*a^4*b^4*c*d^3 +
 4*a^3*b^5*c^2*d^2 - 2*a^5*b^3*c^2*d^2))/(b^7 - 2*a^2*b^5 + a^4*b^3) - (32*
(a^6*b*d^4 + a^2*b^5*d^4 - 2*a^4*b^3*d^4))/(b^6 - 2*a^2*b^4 + a^4*b^2) + (d
^2*((32*(a*b^8*d^2 + a^3*b^6*c^2 - a^5*b^4*c^2 - a^3*b^6*d^2 - 2*a^2*b^7*c*
d + 2*a^4*b^5*c*d))/(b^6 - 2*a^2*b^4 + a^4*b^2) - (d^2*((32*(a^2*b^9 - 2*a^
4*b^7 + a^6*b^5))/(b^6 - 2*a^2*b^4 + a^4*b^2) + (32*tan(e/2 + (f*x)/2)*(3*a
*b^11 - 8*a^3*b^9 + 7*a^5*b^7 - 2*a^7*b^5))/(b^7 - 2*a^2*b^5 + a^4*b^3))*1i
)/b^2 + (32*tan(e/2 + (f*x)/2)*(2*a^2*b^8*c^2 - 2*a^4*b^6*c^2 + 4*a^2*b^8*d
^2 - 6*a^4*b^6*d^2 + 2*a^6*b^4*d^2 - 4*a*b^9*c*d + 4*a^3*b^7*c*d))/(b^7 - 2
*a^2*b^5 + a^4*b^3))*1i)/b^2))/b^2 - (d^2*((32*(a^6*b*d^4 + a^2*b^5*d^4 - 2
*a^4*b^3*d^4))/(b^6 - 2*a^2*b^4 + a^4*b^2) - (32*tan(e/2 + (f*x)/2)*(2*a^7*
b*d^4 - 2*a*b^7*d^4 + a^3*b^5*c^4 + 9*a^3*b^5*d^4 - 8*a^5*b^3*d^4 + 4*a*b^7
*c^2*d^2 - 8*a^2*b^6*c*d^3 - 4*a^2*b^6*c^3*d + 4*a^4*b^4*c*d^3 + 4*a^3*b^5*
c^2*d^2 - 2*a^5*b^3*c^2*d^2))/(b^7 - 2*a^2*b^5 + a^4*b^3) + (d^2*((32*(a*b^
8*d^2 + a^3*b^6*c^2 - a^5*b^4*c^2 - a^3*b^6*d^2 - 2*a^2*b^7*c*d + 2*a^4*b^5
*c*d))/(b^6 - 2*a^2*b^4 + a^4*b^2) + (d^2*((32*(a^2*b^9 - 2*a^4*b^7 + a^6*b
^5))/(b^6 - 2*a^2*b^4 + a^4*b^2) + (32*tan(e/2 + (f*x)/2)*(3*a*b^11 - 8*a^3
*b^9 + 7*a^5*b^7 - 2*a^7*b^5))/(b^7 - 2*a^2*b^5 + a^4*b^3))*1i)/b^2 + (32*t
an(e/2 + (f*x)/2)*(2*a^2*b^8*c^2 - 2*a^4*b^6*c^2 + 4*a^2*b^8*d^2 - 6*a^4*b^
6*d^2 + 2*a^6*b^4*d^2 - 4*a*b^9*c*d + 4*a^3*b^7*c*d))/(b^7 - 2*a^2*b^5 + a^
4*b^3))*1i)/b^2))/b^2)/((64*tan(e/2 + (f*x)/2)*(2*a^6*d^6 + 4*a^2*b^4*d^6 -
 6*a^4*b^2*d^6 + 4*a^3*b^3*c*d^5 + 2*a^2*b^4*c^2*d^4 - 2*a^4*b^2*c^2*d^4 -
4*a*b^5*c*d^5))/(b^7 - 2*a^2*b^5 + a^4*b^3) - (64*(2*a^3*b^2*d^6 - a^5*d^6
- a^5*c^2*d^4 + 4*a*b^4*c^2*d^4 - 6*a^2*b^3*c*d^5 - 4*a^2*b^3*c^3*d^3 + 3*a
^3*b^2*c^2*d^4 + a^3*b^2*c^4*d^2 + 2*a^4*b*c*d^5))/(b^6 - 2*a^2*b^4 + a^4*b
```

```
^2) + (d^2*((32*tan(e/2 + (f*x)/2)*(2*a^7*b*d^4 - 2*a*b^7*d^4 + a^3*b^5*c^4
 + 9*a^3*b^5*d^4 - 8*a^5*b^3*d^4 + 4*a*b^7*c^2*d^2 - 8*a^2*b^6*c*d^3 - 4*a^
2*b^6*c^3*d + 4*a^4*b^4*c*d^3 + 4*a^3*b^5*c^2*d^2 - 2*a^5*b^3*c^2*d^2))/(b^
7 - 2*a^2*b^5 + a^4*b^3) - (32*(a^6*b*d^4 + a^2*b^5*d^4 - 2*a^4*b^3*d^4))/(
b^6 - 2*a^2*b^4 + a^4*b^2) + (d^2*((32*(a*b^8*d^2 + a^3*b^6*c^2 - a^5*b^4*c
^2 - a^3*b^6*d^2 - 2*a^2*b^7*c*d + 2*a^4*b^5*c*d))/(b^6 - 2*a^2*b^4 + a^4*b
^2) - (d^2*((32*(a^2*b^9 - 2*a^4*b^7 + a^6*b^5))/(b^6 - 2*a^2*b^4 + a^4*b^2
) + (32*tan(e/2 + (f*x)/2)*(3*a*b^11 - 8*a^3*b^9 + 7*a^5*b^7 - 2*a^7*b^5))/
(b^7 - 2*a^2*b^5 + a^4*b^3))*1i)/b^2 + (32*tan(e/2 + (f*x)/2)*(2*a^2*b^8*c^
2 - 2*a^4*b^6*c^2 + 4*a^2*b^8*d^2 - 6*a^4*b^6*d^2 + 2*a^6*b^4*d^2 - 4*a*b^9
*c*d + 4*a^3*b^7*c*d))/(b^7 - 2*a^2*b^5 + a^4*b^3))*1i)/b^2)*1i)/b^2 + (d^2
*((32*(a^6*b*d^4 + a^2*b^5*d^4 - 2*a^4*b^3*d^4))/(b^6 - 2*a^2*b^4 + a^4*b^2
) - (32*tan(e/2 + (f*x)/2)*(2*a^7*b*d^4 - 2*a*b^7*d^4 + a^3*b^5*c^4 + 9*a^3
*b^5*d^4 - 8*a^5*b^3*d^4 + 4*a*b^7*c^2*d^2 - 8*a^2*b^6*c*d^3 - 4*a^2*b^6*c^
3*d + 4*a^4*b^4*c*d^3 + 4*a^3*b^5*c^2*d^2 - 2*a^5*b^3*c^2*d^2))/(b^7 - 2*a^
2*b^5 + a^4*b^3) + (d^2*((32*(a*b^8*d^2 + a^3*b^6*c^2 - a^5*b^4*c^2 - a^3*b
^6*d^2 - 2*a^2*b^7*c*d + 2*a^4*b^5*c*d))/(b^6 - 2*a^2*b^4 + a^4*b^2) + (d^2
*((32*(a^2*b^9 - 2*a^4*b^7 + a^6*b^5))/(b^6 - 2*a^2*b^4 + a^4*b^2) + (32*ta
n(e/2 + (f*x)/2)*(3*a*b^11 - 8*a^3*b^9 + 7*a^5*b^7 - 2*a^7*b^5))/(b^7 - 2*a
^2*b^5 + a^4*b^3))*1i)/b^2 + (32*tan(e/2 + (f*x)/2)*(2*a^2*b^8*c^2 - 2*a^4*
b^6*c^2 + 4*a^2*b^8*d^2 - 6*a^4*b^6*d^2 + 2*a^6*b^4*d^2 - 4*a*b^9*c*d + 4*a
^3*b^7*c*d))/(b^7 - 2*a^2*b^5 + a^4*b^3))*1i)/b^2)*1i)/b^2)))/(b^2*f) + (at
an((((a*d - b*c)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(a^6*b*d^4 + a^2*b^5*d^4
 - 2*a^4*b^3*d^4))/(b^6 - 2*a^2*b^4 + a^4*b^2) - (32*tan(e/2 + (f*x)/2)*(2*
a^7*b*d^4 - 2*a*b^7*d^4 + a^3*b^5*c^4 + 9*a^3*b^5*d^4 - 8*a^5*b^3*d^4 + 4*a
*b^7*c^2*d^2 - 8*a^2*b^6*c*d^3 - 4*a^2*b^6*c^3*d + 4*a^4*b^4*c*d^3 + 4*a^3*
b^5*c^2*d^2 - 2*a^5*b^3*c^2*d^2))/(b^7 - 2*a^2*b^5 + a^4*b^3) + ((a*d - b*c
)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(a*b^8*d^2 + a^3*b^6*c^2 - a^5*b^4*c^2
 - a^3*b^6*d^2 - 2*a^2*b^7*c*d + 2*a^4*b^5*c*d))/(b^6 - 2*a^2*b^4 + a^4*b^2)
 + (32*tan(e/2 + (f*x)/2)*(2*a^2*b^8*c^2 - 2*a^4*b^6*c^2 + 4*a^2*b^8*d^2 -
6*a^4*b^6*d^2 + 2*a^6*b^4*d^2 - 4*a*b^9*c*d + 4*a^3*b^7*c*d))/(b^7 - 2*a^2*
b^5 + a^4*b^3) + (((32*(a^2*b^9 - 2*a^4*b^7 + a^6*b^5))/(b^6 - 2*a^2*b^4 +
a^4*b^2) + (32*tan(e/2 + (f*x)/2)*(3*a*b^11 - 8*a^3*b^9 + 7*a^5*b^7 - 2*a^7
*b^5))/(b^7 - 2*a^2*b^5 + a^4*b^3))*(a*d - b*c)*(-(a + b)^3*(a - b)^3)^(1/2
)*(a^2*d - 2*b^2*d + a*b*c))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*(a^2*
d - 2*b^2*d + a*b*c))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*(a^2*d - 2*b
^2*d + a*b*c)*1i)/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2) - ((a*d - b*c)*(-
(a + b)^3*(a - b)^3)^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*a^7*b*d^4 - 2*a*b^7*d
^4 + a^3*b^5*c^4 + 9*a^3*b^5*d^4 - 8*a^5*b^3*d^4 + 4*a*b^7*c^2*d^2 - 8*a^2*
b^6*c*d^3 - 4*a^2*b^6*c^3*d + 4*a^4*b^4*c*d^3 + 4*a^3*b^5*c^2*d^2 - 2*a^5*b
^3*c^2*d^2))/(b^7 - 2*a^2*b^5 + a^4*b^3) - (32*(a^6*b*d^4 + a^2*b^5*d^4 - 2
*a^4*b^3*d^4))/(b^6 - 2*a^2*b^4 + a^4*b^2) + ((a*d - b*c)*(-(a + b)^3*(a -
b)^3)^(1/2)*((32*(a*b^8*d^2 + a^3*b^6*c^2 - a^5*b^4*c^2 - a^3*b^6*d^2 - 2*a
^2*b^7*c*d + 2*a^4*b^5*c*d))/(b^6 - 2*a^2*b^4 + a^4*b^2) + (32*tan(e/2 + (f
*x)/2)*(2*a^2*b^8*c^2 - 2*a^4*b^6*c^2 + 4*a^2*b^8*d^2 - 6*a^4*b^6*d^2 + 2*a
```

```
^6*b^4*d^2 - 4*a*b^9*c*d + 4*a^3*b^7*c*d))/(b^7 - 2*a^2*b^5 + a^4*b^3) - ((
(32*(a^2*b^9 - 2*a^4*b^7 + a^6*b^5))/(b^6 - 2*a^2*b^4 + a^4*b^2) + (32*tan(
e/2 + (f*x)/2)*(3*a*b^11 - 8*a^3*b^9 + 7*a^5*b^7 - 2*a^7*b^5))/(b^7 - 2*a^2
*b^5 + a^4*b^3))*(a*d - b*c)*(-(a + b)^3*(a - b)^3)^(1/2)*(a^2*d - 2*b^2*d
+ a*b*c))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*(a^2*d - 2*b^2*d + a*b*c
))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*(a^2*d - 2*b^2*d + a*b*c)*1i)/(
b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))/((64*tan(e/2 + (f*x)/2)*(2*a^6*d^6
+ 4*a^2*b^4*d^6 - 6*a^4*b^2*d^6 + 4*a^3*b^3*c*d^5 + 2*a^2*b^4*c^2*d^4 - 2*a
^4*b^2*c^2*d^4 - 4*a*b^5*c*d^5))/(b^7 - 2*a^2*b^5 + a^4*b^3) - (64*(2*a^3*b
^2*d^6 - a^5*d^6 - a^5*c^2*d^4 + 4*a*b^4*c^2*d^4 - 6*a^2*b^3*c*d^5 - 4*a^2*
b^3*c^3*d^3 + 3*a^3*b^2*c^2*d^4 + a^3*b^2*c^4*d^2 + 2*a^4*b*c*d^5))/(b^6 -
2*a^2*b^4 + a^4*b^2) + ((a*d - b*c)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(a^6*
b*d^4 + a^2*b^5*d^4 - 2*a^4*b^3*d^4))/(b^6 - 2*a^2*b^4 + a^4*b^2) - (32*tan
(e/2 + (f*x)/2)*(2*a^7*b*d^4 - 2*a*b^7*d^4 + a^3*b^5*c^4 + 9*a^3*b^5*d^4 -
8*a^5*b^3*d^4 + 4*a*b^7*c^2*d^2 - 8*a^2*b^6*c*d^3 - 4*a^2*b^6*c^3*d + 4*a^4
*b^4*c*d^3 + 4*a^3*b^5*c^2*d^2 - 2*a^5*b^3*c^2*d^2))/(b^7 - 2*a^2*b^5 + a^4
*b^3) + ((a*d - b*c)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(a*b^8*d^2 + a^3*b^6
*c^2 - a^5*b^4*c^2 - a^3*b^6*d^2 - 2*a^2*b^7*c*d + 2*a^4*b^5*c*d))/(b^6 - 2
*a^2*b^4 + a^4*b^2) + (32*tan(e/2 + (f*x)/2)*(2*a^2*b^8*c^2 - 2*a^4*b^6*c^2
 + 4*a^2*b^8*d^2 - 6*a^4*b^6*d^2 + 2*a^6*b^4*d^2 - 4*a*b^9*c*d + 4*a^3*b^7*
c*d))/(b^7 - 2*a^2*b^5 + a^4*b^3) + (((32*(a^2*b^9 - 2*a^4*b^7 + a^6*b^5))/
(b^6 - 2*a^2*b^4 + a^4*b^2) + (32*tan(e/2 + (f*x)/2)*(3*a*b^11 - 8*a^3*b^9
+ 7*a^5*b^7 - 2*a^7*b^5))/(b^7 - 2*a^2*b^5 + a^4*b^3))*(a*d - b*c)*(-(a + b
)^3*(a - b)^3)^(1/2)*(a^2*d - 2*b^2*d + a*b*c))/(b^8 - 3*a^2*b^6 + 3*a^4*b^
4 - a^6*b^2))*(a^2*d - 2*b^2*d + a*b*c))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6
*b^2))*(a^2*d - 2*b^2*d + a*b*c))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2) +
 ((a*d - b*c)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*a^7*b
*d^4 - 2*a*b^7*d^4 + a^3*b^5*c^4 + 9*a^3*b^5*d^4 - 8*a^5*b^3*d^4 + 4*a*b^7*
c^2*d^2 - 8*a^2*b^6*c*d^3 - 4*a^2*b^6*c^3*d + 4*a^4*b^4*c*d^3 + 4*a^3*b^5*c
^2*d^2 - 2*a^5*b^3*c^2*d^2))/(b^7 - 2*a^2*b^5 + a^4*b^3) - (32*(a^6*b*d^4 +
 a^2*b^5*d^4 - 2*a^4*b^3*d^4))/(b^6 - 2*a^2*b^4 + a^4*b^2) + ((a*d - b*c)*(
-(a + b)^3*(a - b)^3)^(1/2)*((32*(a*b^8*d^2 + a^3*b^6*c^2 - a^5*b^4*c^2 - a
^3*b^6*d^2 - 2*a^2*b^7*c*d + 2*a^4*b^5*c*d))/(b^6 - 2*a^2*b^4 + a^4*b^2) +
(32*tan(e/2 + (f*x)/2)*(2*a^2*b^8*c^2 - 2*a^4*b^6*c^2 + 4*a^2*b^8*d^2 - 6*a
^4*b^6*d^2 + 2*a^6*b^4*d^2 - 4*a*b^9*c*d + 4*a^3*b^7*c*d))/(b^7 - 2*a^2*b^5
 + a^4*b^3) - (((32*(a^2*b^9 - 2*a^4*b^7 + a^6*b^5))/(b^6 - 2*a^2*b^4 + a^4
*b^2) + (32*tan(e/2 + (f*x)/2)*(3*a*b^11 - 8*a^3*b^9 + 7*a^5*b^7 - 2*a^7*b^
5))/(b^7 - 2*a^2*b^5 + a^4*b^3))*(a*d - b*c)*(-(a + b)^3*(a - b)^3)^(1/2)*(
a^2*d - 2*b^2*d + a*b*c))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*(a^2*d -
 2*b^2*d + a*b*c))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*(a^2*d - 2*b^2*
d + a*b*c))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))*(a*d - b*c)*(-(a + b)
^3*(a - b)^3)^(1/2)*(a^2*d - 2*b^2*d + a*b*c)*2i)/(f*(b^8 - 3*a^2*b^6 + 3*a
^4*b^4 - a^6*b^2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**2/(a+b*sin(f*x+e))**2,x)
[Out] Timed out
```

3.709 $\int \frac{c+d\sin(e+fx)}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=97

$$\frac{2(ac-bd)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{3/2}}+\frac{(bc-ad)\cos(e+fx)}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

```
[Out] 2*(a*c-b*d)*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2
)/f+(-a*d+b*c)*cos(f*x+e)/(a^2-b^2)/f/(a+b*sin(f*x+e))
```

Rubi [A] time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2754, 12, 2660, 618, 204}

$$\frac{2(ac-bd)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{3/2}}+\frac{(bc-ad)\cos(e+fx)}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (2*(a*c - b*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^
2)^(3/2)*f) + ((b*c - a*d)*Cos[e + f*x])/((a^2 - b^2)*f*(a + b*Sin[e + f*x]
))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{c+d\sin(e+fx)}{(a+b\sin(e+fx))^2}\,dx = \frac{(bc-ad)\cos(e+fx)}{\left(a^2-b^2\right)f(a+b\sin(e+fx))}+\frac{\int \frac{-ac+bd}{a+b\sin(e+fx)}\,dx}{-a^2+b^2}$$

$$= \frac{(bc-ad)\cos(e+fx)}{\left(a^2-b^2\right)f(a+b\sin(e+fx))}+\frac{(ac-bd)\int \frac{1}{a+b\sin(e+fx)}\,dx}{a^2-b^2}$$

$$= \frac{(bc-ad)\cos(e+fx)}{\left(a^2-b^2\right)f(a+b\sin(e+fx))}+\frac{(2(ac-bd))\,\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(e+f$$

$$= \frac{(bc-ad)\cos(e+fx)}{\left(a^2-b^2\right)f(a+b\sin(e+fx))}-\frac{(4(ac-bd))\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\,\mathrm{ta}$$

$$= \frac{2(ac-bd)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f}+\frac{(bc-ad)\cos(e+fx)}{\left(a^2-b^2\right)f(a+b\sin(e+fx))}$$

Mathematica [A] time = 0.30, size = 96, normalized size = 0.99

$$\frac{\frac{2(ac-bd)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+\frac{(bc-ad)\cos(e+fx)}{(a-b)(a+b)(a+b\sin(e+fx))}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((2*(a*c - b*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^
2)^(3/2) + ((b*c - a*d)*Cos[e + f*x])/((a - b)*(a + b)*(a + b*Sin[e + f*x])
))/f
```

fricas [A] time = 0.49, size = 399, normalized size = 4.11

$$\left[-\frac{\left(a^2c-abd+\left(abc-b^2d\right)\sin\left(fx+e\right)\right)\sqrt{-a^2+b^2}\log\left(\frac{\left(2a^2-b^2\right)\cos\left(fx+e\right)^2-2ab\sin\left(fx+e\right)-a^2-b^2+2\left(a\cos\left(fx+e\right)\sin\left(fx+e\right)+b\right.}{b^2\cos\left(fx+e\right)^2-2ab\sin\left(fx+e\right)-a^2-b^2}\right.}{2\left(\left(a^4b-2a^2b^3+b^5\right)f\sin\left(fx+e\right)+\left(a^5-2a^3b^2+a\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((a^2*c - a*b*d + (a*b*c - b^2*d)*sin(f*x + e))*sqrt(-a^2 + b^2)*log(
((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 + 2*(a*cos(f
*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^
2 - 2*a*b*sin(f*x + e) - a^2 - b^2)) - 2*((a^2*b - b^3)*c - (a^3 - a*b^2)*d
)*cos(f*x + e))/((a^4*b - 2*a^2*b^3 + b^5)*f*sin(f*x + e) + (a^5 - 2*a^3*b^
2 + a*b^4)*f), -((a^2*c - a*b*d + (a*b*c - b^2*d)*sin(f*x + e))*sqrt(a^2 -
b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))) - ((a^2*b
 - b^3)*c - (a^3 - a*b^2)*d)*cos(f*x + e))/((a^4*b - 2*a^2*b^3 + b^5)*f*sin
(f*x + e) + (a^5 - 2*a^3*b^2 + a*b^4)*f)]
```

giac [A] time = 0.22, size = 157, normalized size = 1.62

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)(ac-bd)}{\left(a^2-b^2\right)^{\frac{3}{2}}}+\frac{b^2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-abd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+abc-a^2d}{\left(a^3-ab^2\right)\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+a\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e
) + b)/sqrt(a^2 - b^2)))*(a*c - b*d)/(a^2 - b^2)^(3/2) + (b^2*c*tan(1/2*f*x
 + 1/2*e) - a*b*d*tan(1/2*f*x + 1/2*e) + a*b*c - a^2*d)/((a^3 - a*b^2)*(a*t
an(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)))/f
```

maple [B] time = 0.24, size = 309, normalized size = 3.19

$$-\frac{2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d}{f\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)b+a\right)\left(a^2-b^2\right)}+\frac{2b^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c}{f\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)b+a\right)\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))/(a+b*sin(f*x+e))^2,x)
```

```
[Out] -2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)*b/(a^2-b^2)*tan(1/2*
f*x+1/2*e)*d+2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)*b^2/(a^2
-b^2)/a*tan(1/2*f*x+1/2*e)*c-2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*
e)*b+a)/(a^2-b^2)*d*a+2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)
/(a^2-b^2)*c*b+2/f/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/
(a^2-b^2)^(1/2))*c*a-2/f/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)
+2*b)/(a^2-b^2)^(1/2))*b*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 8.02, size = 215, normalized size = 2.22

$$\frac{2\,\mathrm{atan}\left(\frac{\left(\frac{2\left(a^2\,b-b^3\right)(a\,c-b\,d)}{(a+b)^{3/2}\left(a^2-b^2\right)(a-b)^{3/2}}+\frac{2\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(a\,c-b\,d)}{(a+b)^{3/2}\,(a-b)^{3/2}}\right)\left(a^2-b^2\right)}{2\,(a\,c-b\,d)}\right)(a\,c-b\,d)}{f\,(a+b)^{3/2}\,(a-b)^{3/2}}-\frac{\frac{2\,(a\,d-b\,c)}{a^2-b^2}+\frac{2\,b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(a\,d-b\,c)}{a\left(a^2-b^2\right)}}{f\left(a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+2\,b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))/(a + b*sin(e + f*x))^2,x)
```

```
[Out] (2*atan((((2*(a^2*b - b^3)*(a*c - b*d))/((a + b)^(3/2)*(a^2 - b^2)*(a - b)^
(3/2)) + (2*a*tan(e/2 + (f*x)/2)*(a*c - b*d))/((a + b)^(3/2)*(a - b)^(3/2))
)*(a^2 - b^2))/(2*(a*c - b*d)))*(a*c - b*d))/(f*(a + b)^(3/2)*(a - b)^(3/2)
) - ((2*(a*d - b*c))/(a^2 - b^2) + (2*b*tan(e/2 + (f*x)/2)*(a*d - b*c))/(a*
(a^2 - b^2)))/(f*(a + 2*b*tan(e/2 + (f*x)/2) + a*tan(e/2 + (f*x)/2)^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.710 $\int \frac{1}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=83

$$\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{3/2}}+\frac{b\cos(e+fx)}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

```
[Out] 2*a*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)/f+b*co
s(f*x+e)/(a^2-b^2)/f/(a+b*sin(f*x+e))
```

Rubi [A] time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2664, 12, 2660, 618, 204}

$$\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{3/2}}+\frac{b\cos(e+fx)}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(-2),x]
```

```
[Out] (2*a*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(3/2)*f
) + (b*Cos[e + f*x])/((a^2 - b^2)*f*(a + b*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+b\sin(e+fx))^2}\,dx &= \frac{b\cos(e+fx)}{\left(a^2-b^2\right)f(a+b\sin(e+fx))}-\frac{\int \frac{a}{a+b\sin(e+fx)}\,dx}{-a^2+b^2}\\
&= \frac{b\cos(e+fx)}{\left(a^2-b^2\right)f(a+b\sin(e+fx))}+\frac{a\int \frac{1}{a+b\sin(e+fx)}\,dx}{a^2-b^2}\\
&= \frac{b\cos(e+fx)}{\left(a^2-b^2\right)f(a+b\sin(e+fx))}+\frac{(2a)\operatorname{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(e+fx)\right)\right)}{\left(a^2-b^2\right)f}\\
&= \frac{b\cos(e+fx)}{\left(a^2-b^2\right)f(a+b\sin(e+fx))}-\frac{(4a)\operatorname{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan\left(\frac{1}{2}(e+fx)\right)\right)}{\left(a^2-b^2\right)f}\\
&= \frac{2a\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}f}+\frac{b\cos(e+fx)}{\left(a^2-b^2\right)f(a+b\sin(e+fx))}
\end{aligned}
$$

Mathematica [A] time = 0.18, size = 82, normalized size = 0.99

$$
\frac{\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+\frac{b\cos(e+fx)}{(a-b)(a+b)(a+b\sin(e+fx))}}{f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^(-2),x]
```

```
[Out] ((2*a*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) +
 (b*Cos[e + f*x])/((a - b)*(a + b)*(a + b*Sin[e + f*x])))/f
```

fricas [A] time = 0.49, size = 336, normalized size = 4.05

$$\left[\frac{\left(ab\sin\left(fx+e\right)+a^{2}\right)\sqrt{-a^{2}+b^{2}}\,\log\left(-\frac{\left(2\,a^{2}-b^{2}\right)\cos\left(fx+e\right)^{2}-2\,ab\sin\left(fx+e\right)-a^{2}-b^{2}-2\left(a\cos\left(fx+e\right)\sin\left(fx+e\right)+b\cos\left(fx+e\right)\right)\sqrt{-a^{2}+\cdots}}{b^{2}\cos\left(fx+e\right)^{2}-2\,ab\sin\left(fx+e\right)-a^{2}-b^{2}}\cdots}{2\left(\left(a^{4}b-2\,a^{2}b^{3}+b^{5}\right)f\sin\left(fx+e\right)+\left(a^{5}-2\,a^{3}b^{2}+ab^{4}\right)f\right)}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a*b*sin(f*x + e) + a^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(f*x
 + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 - 2*(a*cos(f*x + e)*sin(f*x + e) +
 b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e)
 - a^2 - b^2)) + 2*(a^2*b - b^3)*cos(f*x + e))/((a^4*b - 2*a^2*b^3 + b^5)*f
*sin(f*x + e) + (a^5 - 2*a^3*b^2 + a*b^4)*f), -((a*b*sin(f*x + e) + a^2)*sq
rt(a^2 - b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e)))
- (a^2*b - b^3)*cos(f*x + e))/((a^4*b - 2*a^2*b^3 + b^5)*f*sin(f*x + e) + (
a^5 - 2*a^3*b^2 + a*b^4)*f)]
```

giac [A] time = 0.32, size = 127, normalized size = 1.53

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)a}{\left(a^{2}-b^{2}\right)^{\frac{3}{2}}}+\frac{b^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+ab}{\left(a^{3}-ab^{2}\right)\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+2\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+a\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e
) + b)/sqrt(a^2 - b^2)))*a/(a^2 - b^2)^(3/2) + (b^2*tan(1/2*f*x + 1/2*e) +
a*b)/((a^3 - a*b^2)*(a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) +
a)))/f
```

maple [A] time = 0.20, size = 155, normalized size = 1.87

$$\frac{2b^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)b+a\right)a\left(a^2-b^2\right)}+\frac{2b}{f\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)b+a\right)\left(a^2-b^2\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^2,x)
```

```
[Out] 2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)*b^2/a/(a^2-b^2)*tan(1
/2*f*x+1/2*e)+2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)*b/(a^2-
b^2)+2/f*a/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2
)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 8.24, size = 174, normalized size = 2.10

$$\frac{\frac{2\,b}{a^2-b^2}+\frac{2\,b^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a\,\left(a^2-b^2\right)}}{f\,\left(a\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+2\,b\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\right)}+\frac{2\,a\,\mathrm{atan}\left(\frac{\left(a^2-b^2\right)\,\left(\frac{2\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}+\frac{2\,a\,\left(a^2\,b-b^3\right)}{{\left(a+b\right)}^{3/2}\,\left(a^2-b^2\right)\,{\left(a-b\right)}^{3/2}}\right)}{2\,a}\right)}{f\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(e + f*x))^2,x)
```

```
[Out] ((2*b)/(a^2 - b^2) + (2*b^2*tan(e/2 + (f*x)/2))/(a*(a^2 - b^2)))/(f*(a + 2*
b*tan(e/2 + (f*x)/2) + a*tan(e/2 + (f*x)/2)^2)) + (2*a*atan(((a^2 - b^2)*((
```

```
2*a^2*tan(e/2 + (f*x)/2))/((a + b)^(3/2)*(a - b)^(3/2)) + (2*a*(a^2*b - b^3
))/((a + b)^(3/2)*(a^2 - b^2)*(a - b)^(3/2))))/(2*a)))/(f*(a + b)^(3/2)*(a
- b)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sin\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Integral((a + b*sin(e + f*x))**(-2), x)
```

3.711 $\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=181

$$\frac{2b\left(-2a^2d+abc+b^2d\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{3/2}(bc-ad)^2}+\frac{b^2\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)(a+b\sin(e+fx))}+\frac{2d^2\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+}{\sqrt{c^2-d^2}}\right.}{f\sqrt{c^2-d^2}\,(bc-ad)^2}$$

```
[Out] 2*b*(-2*a^2*d+a*b*c+b^2*d)*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))
/(a^2-b^2)^(3/2)/(-a*d+b*c)^2/f+b^2*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(a+b*
sin(f*x+e))+2*d^2*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(-a*d+b*
c)^2/f/(c^2-d^2)^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2802, 3001, 2660, 618, 204}

$$\frac{2b\left(-2a^2d+abc+b^2d\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{3/2}(bc-ad)^2}+\frac{b^2\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)(a+b\sin(e+fx))}+\frac{2d^2\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+}{\sqrt{c^2-d^2}}\right.}{f\sqrt{c^2-d^2}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])),x]
```

```
[Out] (2*b*(a*b*c - 2*a^2*d + b^2*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b
^2]])/((a^2 - b^2)^(3/2)*(b*c - a*d)^2*f) + (2*d^2*ArcTan[(d + c*Tan[(e + f
*x)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^2*Sqrt[c^2 - d^2]*f) + (b^2*Cos[e +
f*x])/((a^2 - b^2)*(b*c - a*d)*f*(a + b*Sin[e + f*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))}\,dx = \frac{b^2\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))} - \frac{\int \frac{-abc+a^2d-b^2d-abd\sin(e+f}{(a+b\sin(e+fx))(c+d\sin(e+f}}{\left(a^2-b^2\right)(bc-ad)}$$

$$= \frac{b^2\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))} + \frac{d^2\int \frac{1}{c+d\sin(e+fx)}\,dx}{(bc-ad)^2} + \frac{(b}{}$$

$$= \frac{b^2\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))} + \frac{\left(2d^2\right)\operatorname{Subst}\left(\int \frac{1}{c+2dx+cx^2}\right.}{(bc-}$$

$$= \frac{b^2\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))} - \frac{\left(4d^2\right)\operatorname{Subst}\left(\int \frac{1}{-4\left(c^2-d^2\right)}\right.}{(}$$

$$= \frac{2b\left(abc-2a^2d+b^2d\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}(bc-ad)^2f} + \frac{2d^2\tan^{-1}\left(\frac{d+c\tan(}{\sqrt{c^2}}\right.}{(bc-ad)^2\sqrt{c^2}}$$

Mathematica [A] time = 0.87, size = 178, normalized size = 0.98

$$\frac{\frac{2b\left(-2a^2d+abc+b^2d\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}(bc-ad)^2} - \frac{b^2\cos(e+fx)}{(a-b)(a+b)(ad-bc)(a+b\sin(e+fx))} + \frac{2d^2\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}\,(bc-ad)^2}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])),x]
```

```
[Out] ((2*b*(a*b*c - 2*a^2*d + b^2*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 -
b^2]])/((a^2 - b^2)^(3/2)*(b*c - a*d)^2) + (2*d^2*ArcTan[(d + c*Tan[(e + f*
x)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^2*Sqrt[c^2 - d^2]) - (b^2*Cos[e + f*x
])/((a - b)*(a + b)*(-(b*c) + a*d)*(a + b*Sin[e + f*x])))/f
```

fricas [B] time = 140.16, size = 2871, normalized size = 15.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

[Out] [1/2*((a^2*b^2*c^3 - a^2*b^2*c*d^2 - (2*a^3*b - a*b^3)*c^2*d + (2*a^3*b - a*b^3)*d^3 + (a*b^3*c^3 - a*b^3*c*d^2 - (2*a^2*b^2 - b^4)*c^2*d + (2*a^2*b^2 - b^4)*d^3)*sin(f*x + e))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 - 2*(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)) - ((a^4*b - 2*a^2*b^3 + b^5)*d^2*sin(f*x + e) + (a^5 - 2*a^3*b^2 + a*b^4)*d^2)*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*((a^2*b^3 - b^5)*c^3 - (a^3*b^2 - a*b^4)*c^2*d - (a^2*b^3 - b^5)*c*d^2 + (a^3*b^2 - a*b^4)*d^3)*cos(f*x + e))/(((a^4*b^3 - 2*a^2*b^5 + b^7)*c^4 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^3*d + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*c^2*d^2 + 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c*d^3 - (a^6*b - 2*a^4*b^3 + a^2*b^5)*d^4)*f*sin(f*x + e) + ((a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^4 - 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^3*d + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c^2*d^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c*d^3 - (a^7 - 2*a^5*b^2 + a^3*b^4)*d^4)*f), -1/2*(2*(a^2*b^2*c^3 - a^2*b^2*c*d^2 - (2*a^3*b - a*b^3)*c^2*d + (2*a^3*b - a*b^3)*d^3 + (a*b^3*c^3 - a*b^3*c*d^2 - (2*a^2*b^2 - b^4)*c^2*d + (2*a^2*b^2 - b^4)*d^3)*sin(f*x + e))*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))) + ((a^4*b - 2*a^2*b^3 + b^5)*d^2*sin(f*x + e) + (a^5 - 2*a^3*b^2 + a*b^4)*d^2)*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 2*((a^2*b^3 - b^5)*c^3 - (a^3*b^2 - a*b^4)*c^2*d - (a^2*b^3 - b^5)*c*d^2 + (a^3*b^2 - a*b^4)*d^3)*cos(f*x + e))/(((a^4*b^3 - 2*a^2*b^5 + b^7)*c^4 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^3*d + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*c^2*d^2 + 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c*d^3 - (a^6*b - 2*a^4*b^3 + a^2*b^5)*d^4)*f*sin(f*x + e) + ((a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^4 - 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^3*d + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c^2*d^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c*d^3 - (a^7 - 2*a^5*b^2 + a^3*b^4)*d^4)*f), -1/2*(2*((a^4*b - 2*a^2*b^3 + b^5)*d^2*sin(f*x + e) + (a^5 - 2*a^3*b^2 + a*b^4)*d^2)*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) - (a^2*b^2*c^3 - a^2*b^2*c*d^2 - (2*a^3*b - a*b^3)*c^2*d + (2*a^3*b - a*b^3)*d^3 + (a*b^3*c^3 - a*b^3*c*d^2 - (2*a^2*b^2 - b^4)*c^2*d + (2*a^2*b^2 - b^4)*d^3)*sin(f*x + e))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 - 2*(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)) - 2*((a^2*b^3 - b^5)*c^3 - (a^3*b^2 - a*b^4)*c^2*d - (a^2*b^3 - b^5)*c*d^2 + (a^3*b^2 - a*b^4)*d^3)*cos(f*x + e))/(((a^4*b^3 - 2*a^2*b^5 + b^7)*c^4 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^3*d + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*c^2*d^2 + 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c*d^3 - (a^6*b - 2*a^4*b^3 + a^2*b^5)*d^4)*f*sin(f*x + e) + ((a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^4 - 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^3*d + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c^2*d^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c*d^3 - (a^7 - 2*a^5*b^2 + a^3*b^4)*d^4)*f), -((a^2*b^2*c^3 - a^

```
2*b^2*c*d^2 - (2*a^3*b - a*b^3)*c^2*d + (2*a^3*b - a*b^3)*d^3 + (a*b^3*c^3
- a*b^3*c*d^2 - (2*a^2*b^2 - b^4)*c^2*d + (2*a^2*b^2 - b^4)*d^3)*sin(f*x +
e))*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x +
 e))) + ((a^4*b - 2*a^2*b^3 + b^5)*d^2*sin(f*x + e) + (a^5 - 2*a^3*b^2 + a*
b^4)*d^2)*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos
(f*x + e))) - ((a^2*b^3 - b^5)*c^3 - (a^3*b^2 - a*b^4)*c^2*d - (a^2*b^3 - b
^5)*c*d^2 + (a^3*b^2 - a*b^4)*d^3)*cos(f*x + e))/(((a^4*b^3 - 2*a^2*b^5 + b
^7)*c^4 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^3*d + (a^6*b - 3*a^4*b^3 + 3*a^
2*b^5 - b^7)*c^2*d^2 + 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c*d^3 - (a^6*b - 2*a
^4*b^3 + a^2*b^5)*d^4)*f*sin(f*x + e) + ((a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^4
- 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^3*d + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*
b^6)*c^2*d^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c*d^3 - (a^7 - 2*a^5*b^2 + a
^3*b^4)*d^4)*f)]
```

giac [A] time = 0.24, size = 304, normalized size = 1.68

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)d^2}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{c^2-d^2}}+\frac{\left(ab^2c-2\,a^2bd+b^3d\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^2c^2-b^4c^2-2\,a^3bcd+2\,ab^3cd+a^4d^2-a^2b^2d^2\right)\sqrt{a^2-b^2}}+\frac{\cdots}{\left(a^3bc-ab^3c-a^4d+a^2b\cdots\right.}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e
) + d)/sqrt(c^2 - d^2)))*d^2/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c^2 - d^
2)) + (a*b^2*c - 2*a^2*b*d + b^3*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a
) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^2*b^2*c^2 - b
^4*c^2 - 2*a^3*b*c*d + 2*a*b^3*c*d + a^4*d^2 - a^2*b^2*d^2)*sqrt(a^2 - b^2)
) + (b^3*tan(1/2*f*x + 1/2*e) + a*b^2)/((a^3*b*c - a*b^3*c - a^4*d + a^2*b^
2*d)*(a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)))/f
```

maple [B] time = 0.38, size = 514, normalized size = 2.84

$$\frac{2d^2\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{f\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{c^2-d^2}}-\frac{2b^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d}{f\,(da-cb)^2\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)b+a\right)\left(a^2-b^2\right)}+\frac{\cdots}{f\,(da-cb)^2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e)),x)
```

```
[Out] 2/f*d^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2
*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))-2/f*b^3/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2
*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*tan(1/2*f*x+1/2*e)*d+2/f*b^4/(a*d-b*
c)^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/a/(a^2-b^2)*tan(1/2*
f*x+1/2*e)*c-2/f*b^2/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*
e)*b+a)/(a^2-b^2)*d*a+2/f*b^3/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2
*f*x+1/2*e)*b+a)/(a^2-b^2)*c-4/f*b/(a*d-b*c)^2/(a^2-b^2)^(3/2)*arctan(1/2*(
2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*d+2/f*b^2/(a*d-b*c)^2/(a^2
-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a*c+2/
f*b^3/(a*d-b*c)^2/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(
a^2-b^2)^(1/2))*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 22.80, size = 24123, normalized size = 133.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))),x)
```

```
[Out] (d^2*atan(((d^2*(d^2 - c^2))^(1/2)*((32*tan(e/2 + (f*x)/2)*(a^3*b^5*c^6 - a^
8*c*d^5 - 4*a*b^7*c^2*d^4 + a*b^7*c^4*d^2 + 4*a^2*b^6*c*d^5 + 2*a^2*b^6*c^5
*d - 13*a^4*b^4*c*d^5 - 5*a^4*b^4*c^5*d + 12*a^6*b^2*c*d^5 + a^7*b*c^2*d^4
- 5*a^2*b^6*c^3*d^3 + 17*a^3*b^5*c^2*d^4 - 8*a^3*b^5*c^4*d^2 + 14*a^4*b^4*c
^3*d^3 - 20*a^5*b^3*c^2*d^4 + 8*a^5*b^3*c^4*d^2 - 4*a^6*b^2*c^3*d^3))/(a^7*
d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 -
 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*
a*b^6*c^2*d - 3*a^6*b*c*d^2) - (32*(a*b^7*c^3*d^3 - a^3*b^5*c*d^5 + a^3*b^5
*c^5*d + 2*a^5*b^3*c*d^5 + 2*a^2*b^6*c^4*d^2 - 6*a^3*b^5*c^3*d^3 + 2*a^4*b^
4*c^2*d^4 - 5*a^4*b^4*c^4*d^2 + 8*a^5*b^3*c^3*d^3 - 3*a^6*b^2*c^2*d^4 - a^7
*b*c*d^5))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 -
 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^
5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (d^2*(d^2 - c^2))^(1/2)*((32*
(a^3*b^7*c^7 - a^5*b^5*c^7 + a^10*c^2*d^5 + a*b^9*c^5*d^2 + a^2*b^8*c^6*d -
```

6*a^4*b^6*c^6*d + a^5*b^5*c*d^6 + 5*a^6*b^4*c^6*d - 3*a^7*b^3*c*d^6 - 5*a^
9*b*c^3*d^4 - 4*a^2*b^8*c^4*d^3 + 6*a^3*b^7*c^3*d^4 - 7*a^3*b^7*c^5*d^2 - 4
*a^4*b^6*c^2*d^5 + 18*a^4*b^6*c^4*d^3 - 22*a^5*b^5*c^3*d^4 + 16*a^5*b^5*c^5
*d^2 + 13*a^6*b^4*c^2*d^5 - 24*a^6*b^4*c^4*d^3 + 21*a^7*b^3*c^3*d^4 - 10*a^
7*b^3*c^5*d^2 - 10*a^8*b^2*c^2*d^5 + 10*a^8*b^2*c^4*d^3 + 2*a^9*b*c*d^6))/(
a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d
^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d
+ 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (32*tan(e/2 + (f*x)/2)*(2*a^10*c*d^6 + 2
*a^2*b^8*c^7 - 2*a^4*b^6*c^7 - 12*a^3*b^7*c^6*d + 10*a^5*b^5*c^6*d - 2*a^8*
b^2*c*d^6 - 6*a^9*b*c^2*d^5 - 8*a^2*b^8*c^5*d^2 + 12*a^3*b^7*c^4*d^3 - 8*a^
4*b^6*c^3*d^4 + 26*a^4*b^6*c^5*d^2 + 2*a^5*b^5*c^2*d^5 - 24*a^5*b^5*c^4*d^3
 + 6*a^6*b^4*c^3*d^4 - 18*a^6*b^4*c^5*d^2 + 4*a^7*b^3*c^2*d^5 + 12*a^7*b^3*
c^4*d^3 + 2*a^8*b^2*c^3*d^4 + 2*a*b^9*c^6*d))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^
5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3
*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*
d^2) + (d^2*(d^2 - c^2)^(1/2)*((32*(2*a^4*b^8*c^8 - a^2*b^10*c^8 - a^6*b^6*
c^8 + a^12*c^2*d^6 + 2*a^3*b^9*c^7*d - 7*a^5*b^7*c^7*d - a^7*b^5*c*d^7 + 4*
a^7*b^5*c^7*d + 2*a^9*b^3*c*d^7 - 4*a^11*b*c^3*d^5 - 4*a^2*b^10*c^6*d^2 + 5
*a^3*b^9*c^5*d^3 + 3*a^4*b^8*c^6*d^2 - 5*a^5*b^7*c^3*d^5 - 10*a^5*b^7*c^5*d
^3 + 4*a^6*b^6*c^2*d^6 + 5*a^6*b^6*c^4*d^4 + 6*a^6*b^6*c^6*d^2 + 6*a^7*b^5*
c^3*d^5 + 5*a^7*b^5*c^5*d^3 - 7*a^8*b^4*c^2*d^6 - 10*a^8*b^4*c^4*d^4 - 5*a^
8*b^4*c^6*d^2 + 3*a^9*b^3*c^3*d^5 + 2*a^10*b^2*c^2*d^6 + 5*a^10*b^2*c^4*d^4
 + a*b^11*c^7*d - a^11*b*c*d^7))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b
^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d +
6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (32*ta
n(e/2 + (f*x)/2)*(3*a*b^11*c^8 - 3*a^12*c*d^7 - 8*a^3*b^9*c^8 + 7*a^5*b^7*c
^8 - 2*a^7*b^5*c^8 + 2*a^12*c^3*d^5 - 4*a*b^11*c^6*d^2 - 15*a^2*b^10*c^7*d
+ 40*a^4*b^8*c^7*d + 4*a^6*b^6*c*d^7 - 35*a^6*b^6*c^7*d - 11*a^8*b^4*c*d^7
+ 10*a^8*b^4*c^7*d + 10*a^10*b^2*c*d^7 + 15*a^11*b*c^2*d^6 - 10*a^11*b*c^4*
d^4 + 20*a^2*b^10*c^5*d^3 - 40*a^3*b^9*c^4*d^4 + 41*a^3*b^9*c^6*d^2 + 40*a^
4*b^8*c^3*d^5 - 85*a^4*b^8*c^5*d^3 - 20*a^5*b^7*c^2*d^6 + 125*a^5*b^7*c^4*d
^4 - 90*a^5*b^7*c^6*d^2 - 113*a^6*b^6*c^3*d^5 + 130*a^6*b^6*c^5*d^3 + 55*a^
7*b^5*c^2*d^6 - 140*a^7*b^5*c^4*d^4 + 73*a^7*b^5*c^6*d^2 + 108*a^8*b^4*c^3*
d^5 - 85*a^8*b^4*c^5*d^3 - 50*a^9*b^3*c^2*d^6 + 65*a^9*b^3*c^4*d^4 - 20*a^9
*b^3*c^6*d^2 - 37*a^10*b^2*c^3*d^5 + 20*a^10*b^2*c^5*d^3))/(a^7*d^3 - b^7*c
^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*
c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d
 - 3*a^6*b*c*d^2)))/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*
c*d^3 + 2*a*b*c^3*d)))/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a
*b*c*d^3 + 2*a*b*c^3*d))*1i)/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2
 - 2*a*b*c*d^3 + 2*a*b*c^3*d) - (d^2*(d^2 - c^2)^(1/2)*((32*(a*b^7*c^3*d^3
- a^3*b^5*c*d^5 + a^3*b^5*c^5*d + 2*a^5*b^3*c*d^5 + 2*a^2*b^6*c^4*d^2 - 6*a
^3*b^5*c^3*d^3 + 2*a^4*b^4*c^2*d^4 - 5*a^4*b^4*c^4*d^2 + 8*a^5*b^3*c^3*d^3
- 3*a^6*b^2*c^2*d^4 - a^7*b*c*d^5))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^
4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d

+ 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (32
*tan(e/2 + (f*x)/2)*(a^3*b^5*c^6 - a^8*c*d^5 - 4*a*b^7*c^2*d^4 + a*b^7*c^4*
d^2 + 4*a^2*b^6*c*d^5 + 2*a^2*b^6*c^5*d - 13*a^4*b^4*c*d^5 - 5*a^4*b^4*c^5*
d + 12*a^6*b^2*c*d^5 + a^7*b*c^2*d^4 - 5*a^2*b^6*c^3*d^3 + 17*a^3*b^5*c^2*d
^4 - 8*a^3*b^5*c^4*d^2 + 14*a^4*b^4*c^3*d^3 - 20*a^5*b^3*c^2*d^4 + 8*a^5*b^
3*c^4*d^2 - 4*a^6*b^2*c^3*d^3))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^
3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6
*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (d^2*(d
^2 - c^2)^(1/2)*((32*(a^3*b^7*c^7 - a^5*b^5*c^7 + a^10*c^2*d^5 + a*b^9*c^5*
d^2 + a^2*b^8*c^6*d - 6*a^4*b^6*c^6*d + a^5*b^5*c*d^6 + 5*a^6*b^4*c^6*d - 3
*a^7*b^3*c*d^6 - 5*a^9*b*c^3*d^4 - 4*a^2*b^8*c^4*d^3 + 6*a^3*b^7*c^3*d^4 -
7*a^3*b^7*c^5*d^2 - 4*a^4*b^6*c^2*d^5 + 18*a^4*b^6*c^4*d^3 - 22*a^5*b^5*c^3
*d^4 + 16*a^5*b^5*c^5*d^2 + 13*a^6*b^4*c^2*d^5 - 24*a^6*b^4*c^4*d^3 + 21*a^
7*b^3*c^3*d^4 - 10*a^7*b^3*c^5*d^2 - 10*a^8*b^2*c^2*d^5 + 10*a^8*b^2*c^4*d^
3 + 2*a^9*b*c*d^6))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*
b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d
^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (32*tan(e/2 + (f*x)
/2)*(2*a^10*c*d^6 + 2*a^2*b^8*c^7 - 2*a^4*b^6*c^7 - 12*a^3*b^7*c^6*d + 10*a
^5*b^5*c^6*d - 2*a^8*b^2*c*d^6 - 6*a^9*b*c^2*d^5 - 8*a^2*b^8*c^5*d^2 + 12*a
^3*b^7*c^4*d^3 - 8*a^4*b^6*c^3*d^4 + 26*a^4*b^6*c^5*d^2 + 2*a^5*b^5*c^2*d^5
 - 24*a^5*b^5*c^4*d^3 + 6*a^6*b^4*c^3*d^4 - 18*a^6*b^4*c^5*d^2 + 4*a^7*b^3*
c^2*d^5 + 12*a^7*b^3*c^4*d^3 + 2*a^8*b^2*c^3*d^4 + 2*a*b^9*c^6*d))/(a^7*d^3
 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*
a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b
^6*c^2*d - 3*a^6*b*c*d^2) - (d^2*(d^2 - c^2)^(1/2)*((32*(2*a^4*b^8*c^8 - a^
2*b^10*c^8 - a^6*b^6*c^8 + a^12*c^2*d^6 + 2*a^3*b^9*c^7*d - 7*a^5*b^7*c^7*d
 - a^7*b^5*c*d^7 + 4*a^7*b^5*c^7*d + 2*a^9*b^3*c*d^7 - 4*a^11*b*c^3*d^5 - 4
*a^2*b^10*c^6*d^2 + 5*a^3*b^9*c^5*d^3 + 3*a^4*b^8*c^6*d^2 - 5*a^5*b^7*c^3*d
^5 - 10*a^5*b^7*c^5*d^3 + 4*a^6*b^6*c^2*d^6 + 5*a^6*b^6*c^4*d^4 + 6*a^6*b^6
*c^6*d^2 + 6*a^7*b^5*c^3*d^5 + 5*a^7*b^5*c^5*d^3 - 7*a^8*b^4*c^2*d^6 - 10*a
^8*b^4*c^4*d^4 - 5*a^8*b^4*c^6*d^2 + 3*a^9*b^3*c^3*d^5 + 2*a^10*b^2*c^2*d^6
 + 5*a^10*b^2*c^4*d^4 + a*b^11*c^7*d - a^11*b*c*d^7))/(a^7*d^3 - b^7*c^3 +
2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2
 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*
a^6*b*c*d^2) - (32*tan(e/2 + (f*x)/2)*(3*a*b^11*c^8 - 3*a^12*c*d^7 - 8*a^3*
b^9*c^8 + 7*a^5*b^7*c^8 - 2*a^7*b^5*c^8 + 2*a^12*c^3*d^5 - 4*a*b^11*c^6*d^2
 - 15*a^2*b^10*c^7*d + 40*a^4*b^8*c^7*d + 4*a^6*b^6*c*d^7 - 35*a^6*b^6*c^7*
d - 11*a^8*b^4*c*d^7 + 10*a^8*b^4*c^7*d + 10*a^10*b^2*c*d^7 + 15*a^11*b*c^2
*d^6 - 10*a^11*b*c^4*d^4 + 20*a^2*b^10*c^5*d^3 - 40*a^3*b^9*c^4*d^4 + 41*a^
3*b^9*c^6*d^2 + 40*a^4*b^8*c^3*d^5 - 85*a^4*b^8*c^5*d^3 - 20*a^5*b^7*c^2*d^
6 + 125*a^5*b^7*c^4*d^4 - 90*a^5*b^7*c^6*d^2 - 113*a^6*b^6*c^3*d^5 + 130*a^
6*b^6*c^5*d^3 + 55*a^7*b^5*c^2*d^6 - 140*a^7*b^5*c^4*d^4 + 73*a^7*b^5*c^6*d
^2 + 108*a^8*b^4*c^3*d^5 - 85*a^8*b^4*c^5*d^3 - 50*a^9*b^3*c^2*d^6 + 65*a^9
*b^3*c^4*d^4 - 20*a^9*b^3*c^6*d^2 - 37*a^10*b^2*c^3*d^5 + 20*a^10*b^2*c^5*d
^3))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5

*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2)))/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d)))/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d))*1i)/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d))/((64*(2*a^2*b^4*c^2*d^3 - 3*a^3*b^3*c*d^4 + a^3*b^3*c^3*d^2 - 3*a^4*b^2*c^2*d^3 + a*b^5*c*d^4 + 2*a^5*b*c*d^4))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (64*tan(e/2 + (f*x)/2)*(2*a^2*b^4*c*d^4 - 4*a^4*b^2*c*d^4 + 2*a^3*b^3*c^2*d^3))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (d^2*(d^2 - c^2)^(1/2)*((32*tan(e/2 + (f*x)/2)*(a^3*b^5*c^6 - a^8*c*d^5 - 4*a*b^7*c^2*d^4 + a*b^7*c^4*d^2 + 4*a^2*b^6*c*d^5 + 2*a^2*b^6*c^5*d - 13*a^4*b^4*c*d^5 - 5*a^4*b^4*c^5*d + 12*a^6*b^2*c*d^5 + a^7*b*c^2*d^4 - 5*a^2*b^6*c^3*d^3 + 17*a^3*b^5*c^2*d^4 - 8*a^3*b^5*c^4*d^2 + 14*a^4*b^4*c^3*d^3 - 20*a^5*b^3*c^2*d^4 + 8*a^5*b^3*c^4*d^2 - 4*a^6*b^2*c^3*d^3))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (32*(a*b^7*c^3*d^3 - a^3*b^5*c*d^5 + a^3*b^5*c^5*d + 2*a^5*b^3*c*d^5 + 2*a^2*b^6*c^4*d^2 - 6*a^3*b^5*c^3*d^3 + 2*a^4*b^4*c^2*d^4 - 5*a^4*b^4*c^4*d^2 + 8*a^5*b^3*c^3*d^3 - 3*a^6*b^2*c^2*d^4 - a^7*b*c*d^5))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (d^2*(d^2 - c^2)^(1/2)*((32*(a^3*b^7*c^7 - a^5*b^5*c^7 + a^10*c^2*d^5 + a*b^9*c^5*d^2 + a^2*b^8*c^6*d - 6*a^4*b^6*c^6*d + a^5*b^5*c*d^6 + 5*a^6*b^4*c^6*d - 3*a^7*b^3*c*d^6 - 5*a^9*b*c^3*d^4 - 4*a^2*b^8*c^4*d^3 + 6*a^3*b^7*c^3*d^4 - 7*a^3*b^7*c^5*d^2 - 4*a^4*b^6*c^2*d^5 + 18*a^4*b^6*c^4*d^3 - 22*a^5*b^5*c^3*d^4 + 16*a^5*b^5*c^5*d^2 + 13*a^6*b^4*c^2*d^5 - 24*a^6*b^4*c^4*d^3 + 21*a^7*b^3*c^3*d^4 - 10*a^7*b^3*c^5*d^2 - 10*a^8*b^2*c^2*d^5 + 10*a^8*b^2*c^4*d^3 + 2*a^9*b*c*d^6))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (32*tan(e/2 + (f*x)/2)*(2*a^10*c*d^6 + 2*a^2*b^8*c^7 - 2*a^4*b^6*c^7 - 12*a^3*b^7*c^6*d + 10*a^5*b^5*c^6*d - 2*a^8*b^2*c*d^6 - 6*a^9*b*c^2*d^5 - 8*a^2*b^8*c^5*d^2 + 12*a^3*b^7*c^4*d^3 - 8*a^4*b^6*c^3*d^4 + 26*a^4*b^6*c^5*d^2 + 2*a^5*b^5*c^2*d^5 - 24*a^5*b^5*c^4*d^3 + 6*a^6*b^4*c^3*d^4 - 18*a^6*b^4*c^5*d^2 + 4*a^7*b^3*c^2*d^5 + 12*a^7*b^3*c^4*d^3 + 2*a^8*b^2*c^3*d^4 + 2*a*b^9*c^6*d))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (d^2*(d^2 - c^2)^(1/2)*((32*(2*a^4*b^8*c^8 - a^2*b^10*c^8 - a^6*b^6*c^8 + a^12*c^2*d^6 + 2*a^3*b^9*c^7*d - 7*a^5*b^7*c^7*d - a^7*b^5*c*d^7 + 4*a^7*b^5*c^7*d + 2*a^9*b^3*c*d^7 - 4*a^11*b*c^3*d^5 - 4*a^2*b^10*c^6*d^2 + 5*a^3*b^9*c^5*d^3 + 3*a^4*b^8*c^6*d^2 - 5*a^5*b^7*c^3*d^5 - 10*a^5*b^7* c^5*d^3 + 4*a^6*b^6*c^2*d^6 + 5*a^6*b^6*c^4*d^4 + 6*a^6*b^6*c^6*d^2 + 6*a^7*b^5*c^3*d^5 + 5*a^7*b^5*c^5*d^3 - 7*a^8*b^4*c^2*d^6 - 10*a^8*b^4*c^4*d^4 - 5*a^8*b^4*c^6*d^2 + 3*a^9*b^3*c^3*d^5 + 2*a^10*b^2*c^2*d^6 + 5*a^10*b^2*c^4*d^4 + a*b^11*c^7*d - a^11*b*c*d^7))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (32*tan(e/2 + (f*x)/2)*(3*a*b^11*c^8 - 3*a^12*c*d^7 - 8*a^3*b^9*c^8 + 7*a^5*b^7*c^8 - 2*a^7*b^5*c^8 + 2*a^12*c^3*d^5 - 4*a*b^11*c^6*d^2 - 15*a^2*b^10*c^7*d + 40*a^4*b^8*c^7*d + 4*a^6*b^6*c*d^7 - 35*a^6*b^6*c^7*d - 11*a^8*b^4*c*d^7 + 10*a^8*b^4*c^7*d + 10*a^10*b^2*c*d^7 + 15*a^11*b*c^2*d^6 - 10*a^11*b*c^4*d^4 + 20*a^2*b^10*c^5*d^3 - 40*a^3*b^9*c^4*d^4 + 41*a^3*b^9*c^6*d^2 + 40*a^4*b^8*c^3*d^5 - 85*a^4*b^8*c^5*d^3 - 20*a^5*b^7*c^2*d^6 + 125*a^5*b^7*c^4*d^4 - 90*a^5*b^7*c^6*d^2 - 113*a^6*b^6*c^3*d^5 + 130*a^6*b^6*c^5*d^3 + 55*a^7*b^5*c^2*d^6 - 140*a^7*b^5*c^4*d^4 + 73*a^7*b^5*c^6*d^2 + 108*a^8*b^4*c^3*d^5 - 85*a^8*b^4*c^5*d^3 - 50*a^9*b^3*c^2*d^6 + 65*a^9*b^3*c^4*d^4 - 20*a^9*b^3*c^6*d^2 - 37*a^10*b^2*c^3*d^5 + 20*a^10*b^2*c^5*d^3))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2)))/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d)))/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d)))/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d) - (d^2*(d^2 - c^2)^(1/2)*((32*(a*b^7*c^3*d^3 - a^3*b^5*c*d^5 + a^3*b^5*c^5*d + 2*a^5*b^3*c*d^5 + 2*a^2*b^6*c^4*d^2 - 6*a^3*b^5*c^3*d^3 + 2*a^4*b^4*c^2*d^4 - 5*a^4*b^4*c^4*d^2 + 8*a^5*b^3*c^3*d^3 - 3*a^6*b^2*c^2*d^4 - a^7*b*c*d^5))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (32*tan(e/2 + (f*x)/2)*(a^3*b^5*c^6 - a^8*c*d^5 - 4*a*b^7*c^2*d^4 + a*b^7*c^4*d^2 + 4*a^2*b^6*c*d^5 + 2*a^2*b^6*c^5*d - 13*a^4*b^4*c*d^5 - 5*a^4*b^4*c^5*d + 12*a^6*b^2*c*d^5 + a^7*b*c^2*d^4 - 5*a^2*b^6*c^3*d^3 + 17*a^3*b^5*c^2*d^4 - 8*a^3*b^5*c^4*d^2 + 14*a^4*b^4*c^3*d^3 - 20*a^5*b^3*c^2*d^4 + 8*a^5*b^3*c^4*d^2 - 4*a^6*b^2*c^3*d^3))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (d^2*(d^2 - c^2)^(1/2)*((32*(a^3*b^7*c^7 - a^5*b^5*c^7 + a^10*c^2*d^5 + a*b^9*c^5*d^2 + a^2*b^8*c^6*d - 6*a^4*b^6*c^6*d + a^5*b^5*c*d^6 + 5*a^6*b^4*c^6*d - 3*a^7*b^3*c*d^6 - 5*a^9*b*c^3*d^4 - 4*a^2*b^8*c^4*d^3 + 6*a^3*b^7*c^3*d^4 - 7*a^3*b^7*c^5*d^2 - 4*a^4*b^6*c^2*d^5 + 18*a^4*b^6*c^4*d^3 - 22*a^5*b^5*c^3*d^4 + 16*a^5*b^5*c^5*d^2 + 13*a^6*b^4*c^2*d^5 - 24*a^6*b^4*c^4*d^3 + 21*a^7*b^3*c^3*d^4 - 10*a^7*b^3*c^5*d^2 - 10*a^8*b^2*c^2*d^5 + 10*a^8*b^2*c^4*d^3 + 2*a^9*b*c*d^6))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (32*tan(e/2 + (f*x)/2)*(2*a^10*c*d^6 + 2*a^2*b^8*c^7 - 2*a^4*b^6*c^7 - 12*a^3*b^7*c^6*d + 10*a^5*b^5*c^6*d - 2*a^8*b^2*c*d^6 - 6*a^9*b*c^2*d^5 - 8*a^2*b^8*c^5*d^2 + 12 *a^3*b^7*c^4*d^3 - 8*a^4*b^6*c^3*d^4 + 26*a^4*b^6*c^5*d^2 + 2*a^5*b^5*c^2*d ^5 - 24*a^5*b^5*c^4*d^3 + 6*a^6*b^4*c^3*d^4 - 18*a^6*b^4*c^5*d^2 + 4*a^7*b^ 3*c^2*d^5 + 12*a^7*b^3*c^4*d^3 + 2*a^8*b^2*c^3*d^4 + 2*a*b^9*c^6*d))/(a^7*d ^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a *b^6*c^2*d - 3*a^6*b*c*d^2) - (d^2*(d^2 - c^2)^(1/2)*((32*(2*a^4*b^8*c^8 - a^2*b^10*c^8 - a^6*b^6*c^8 + a^12*c^2*d^6 + 2*a^3*b^9*c^7*d - 7*a^5*b^7*c^7 *d - a^7*b^5*c*d^7 + 4*a^7*b^5*c^7*d + 2*a^9*b^3*c*d^7 - 4*a^11*b*c^3*d^5 - 4*a^2*b^10*c^6*d^2 + 5*a^3*b^9*c^5*d^3 + 3*a^4*b^8*c^6*d^2 - 5*a^5*b^7*c^3 *d^5 - 10*a^5*b^7*c^5*d^3 + 4*a^6*b^6*c^2*d^6 + 5*a^6*b^6*c^4*d^4 + 6*a^6*b ^6*c^6*d^2 + 6*a^7*b^5*c^3*d^5 + 5*a^7*b^5*c^5*d^3 - 7*a^8*b^4*c^2*d^6 - 10 *a^8*b^4*c^4*d^4 - 5*a^8*b^4*c^6*d^2 + 3*a^9*b^3*c^3*d^5 + 2*a^10*b^2*c^2*d ^6 + 5*a^10*b^2*c^4*d^4 + a*b^11*c^7*d - a^11*b*c*d^7))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d ^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (32*tan(e/2 + (f*x)/2)*(3*a*b^11*c^8 - 3*a^12*c*d^7 - 8*a^ 3*b^9*c^8 + 7*a^5*b^7*c^8 - 2*a^7*b^5*c^8 + 2*a^12*c^3*d^5 - 4*a*b^11*c^6*d ^2 - 15*a^2*b^10*c^7*d + 40*a^4*b^8*c^7*d + 4*a^6*b^6*c*d^7 - 35*a^6*b^6*c^ 7*d - 11*a^8*b^4*c*d^7 + 10*a^8*b^4*c^7*d + 10*a^10*b^2*c*d^7 + 15*a^11*b*c ^2*d^6 - 10*a^11*b*c^4*d^4 + 20*a^2*b^10*c^5*d^3 - 40*a^3*b^9*c^4*d^4 + 41* a^3*b^9*c^6*d^2 + 40*a^4*b^8*c^3*d^5 - 85*a^4*b^8*c^5*d^3 - 20*a^5*b^7*c^2* d^6 + 125*a^5*b^7*c^4*d^4 - 90*a^5*b^7*c^6*d^2 - 113*a^6*b^6*c^3*d^5 + 130* a^6*b^6*c^5*d^3 + 55*a^7*b^5*c^2*d^6 - 140*a^7*b^5*c^4*d^4 + 73*a^7*b^5*c^6 *d^2 + 108*a^8*b^4*c^3*d^5 - 85*a^8*b^4*c^5*d^3 - 50*a^9*b^3*c^2*d^6 + 65*a ^9*b^3*c^4*d^4 - 20*a^9*b^3*c^6*d^2 - 37*a^10*b^2*c^3*d^5 + 20*a^10*b^2*c^5 *d^3))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a ^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^ 2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2)))/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d)))/(a^2*d^4 - b^2*c^4 - a^2*c^2* d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d)))/(a^2*d^4 - b^2*c^4 - a^2*c ^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d)))*(d^2 - c^2)^(1/2)*2i)/(f*(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3* d)) - ((2*b^2)/((a^2 - b^2)*(a*d - b*c)) + (2*b^3*tan(e/2 + (f*x)/2))/(a*(a ^2 - b^2)*(a*d - b*c)))/(f*(a + 2*b*tan(e/2 + (f*x)/2) + a*tan(e/2 + (f*x)/ 2)^2)) + (b*atan(((b*(-(a + b)^3*(a - b)^3)^(1/2)*((32*tan(e/2 + (f*x)/2)*(a^3*b^5*c^6 - a^8*c*d^5 - 4*a*b^7*c^2*d^4 + a*b^7*c^4*d^2 + 4*a^2*b^6*c*d^5 + 2*a^2*b^6*c^5*d - 13*a^4*b^4*c*d^5 - 5*a^4*b^4*c^5*d + 12*a^6*b^2*c*d^5 + a^7*b*c^2*d^4 - 5*a^2*b^6*c^3*d^3 + 17*a^3*b^5*c^2*d^4 - 8*a^3*b^5*c^4*d^ 2 + 14*a^4*b^4*c^3*d^3 - 20*a^5*b^3*c^2*d^4 + 8*a^5*b^3*c^4*d^2 - 4*a^6*b^2 *c^3*d^3))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^ 5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (32*(a*b^7*c^3*d^3 - a^3*b^5 *c*d^5 + a^3*b^5*c^5*d + 2*a^5*b^3*c*d^5 + 2*a^2*b^6*c^4*d^2 - 6*a^3*b^5*c^ 3*d^3 + 2*a^4*b^4*c^2*d^4 - 5*a^4*b^4*c^4*d^2 + 8*a^5*b^3*c^3*d^3 - 3*a^6*b ^2*c^2*d^4 - a^7*b*c*d^5))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3

+ a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*
b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (b*(-(a + b)
^3*(a - b)^3)^(1/2)*((32*(a^3*b^7*c^7 - a^5*b^5*c^7 + a^10*c^2*d^5 + a*b^9*
c^5*d^2 + a^2*b^8*c^6*d - 6*a^4*b^6*c^6*d + a^5*b^5*c*d^6 + 5*a^6*b^4*c^6*d
- 3*a^7*b^3*c*d^6 - 5*a^9*b*c^3*d^4 - 4*a^2*b^8*c^4*d^3 + 6*a^3*b^7*c^3*d^
4 - 7*a^3*b^7*c^5*d^2 - 4*a^4*b^6*c^2*d^5 + 18*a^4*b^6*c^4*d^3 - 22*a^5*b^5
*c^3*d^4 + 16*a^5*b^5*c^5*d^2 + 13*a^6*b^4*c^2*d^5 - 24*a^6*b^4*c^4*d^3 + 2
1*a^7*b^3*c^3*d^4 - 10*a^7*b^3*c^5*d^2 - 10*a^8*b^2*c^2*d^5 + 10*a^8*b^2*c^
4*d^3 + 2*a^9*b*c*d^6))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 +
a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3
*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (32*tan(e/2 + (
f*x)/2)*(2*a^10*c*d^6 + 2*a^2*b^8*c^7 - 2*a^4*b^6*c^7 - 12*a^3*b^7*c^6*d +
10*a^5*b^5*c^6*d - 2*a^8*b^2*c*d^6 - 6*a^9*b*c^2*d^5 - 8*a^2*b^8*c^5*d^2 +
12*a^3*b^7*c^4*d^3 - 8*a^4*b^6*c^3*d^4 + 26*a^4*b^6*c^5*d^2 + 2*a^5*b^5*c^2
*d^5 - 24*a^5*b^5*c^4*d^3 + 6*a^6*b^4*c^3*d^4 - 18*a^6*b^4*c^5*d^2 + 4*a^7*
b^3*c^2*d^5 + 12*a^7*b^3*c^4*d^3 + 2*a^8*b^2*c^3*d^4 + 2*a*b^9*c^6*d))/(a^7
*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3
- 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3
*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (b*((32*(2*a^4*b^8*c^8 - a^2*b^10*c^8 - a^6
*b^6*c^8 + a^12*c^2*d^6 + 2*a^3*b^9*c^7*d - 7*a^5*b^7*c^7*d - a^7*b^5*c*d^7
+ 4*a^7*b^5*c^7*d + 2*a^9*b^3*c*d^7 - 4*a^11*b*c^3*d^5 - 4*a^2*b^10*c^6*d^
2 + 5*a^3*b^9*c^5*d^3 + 3*a^4*b^8*c^6*d^2 - 5*a^5*b^7*c^3*d^5 - 10*a^5*b^7*
c^5*d^3 + 4*a^6*b^6*c^2*d^6 + 5*a^6*b^6*c^4*d^4 + 6*a^6*b^6*c^6*d^2 + 6*a^7
*b^5*c^3*d^5 + 5*a^7*b^5*c^5*d^3 - 7*a^8*b^4*c^2*d^6 - 10*a^8*b^4*c^4*d^4 -
5*a^8*b^4*c^6*d^2 + 3*a^9*b^3*c^3*d^5 + 2*a^10*b^2*c^2*d^6 + 5*a^10*b^2*c^
4*d^4 + a*b^11*c^7*d - a^11*b*c*d^7))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 -
a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2
*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (
32*tan(e/2 + (f*x)/2)*(3*a*b^11*c^8 - 3*a^12*c*d^7 - 8*a^3*b^9*c^8 + 7*a^5*
b^7*c^8 - 2*a^7*b^5*c^8 + 2*a^12*c^3*d^5 - 4*a*b^11*c^6*d^2 - 15*a^2*b^10*c
^7*d + 40*a^4*b^8*c^7*d + 4*a^6*b^6*c*d^7 - 35*a^6*b^6*c^7*d - 11*a^8*b^4*c
*d^7 + 10*a^8*b^4*c^7*d + 10*a^10*b^2*c*d^7 + 15*a^11*b*c^2*d^6 - 10*a^11*b
*c^4*d^4 + 20*a^2*b^10*c^5*d^3 - 40*a^3*b^9*c^4*d^4 + 41*a^3*b^9*c^6*d^2 +
40*a^4*b^8*c^3*d^5 - 85*a^4*b^8*c^5*d^3 - 20*a^5*b^7*c^2*d^6 + 125*a^5*b^7*
c^4*d^4 - 90*a^5*b^7*c^6*d^2 - 113*a^6*b^6*c^3*d^5 + 130*a^6*b^6*c^5*d^3 +
55*a^7*b^5*c^2*d^6 - 140*a^7*b^5*c^4*d^4 + 73*a^7*b^5*c^6*d^2 + 108*a^8*b^4
*c^3*d^5 - 85*a^8*b^4*c^5*d^3 - 50*a^9*b^3*c^2*d^6 + 65*a^9*b^3*c^4*d^4 - 2
0*a^9*b^3*c^6*d^2 - 37*a^10*b^2*c^3*d^5 + 20*a^10*b^2*c^5*d^3))/(a^7*d^3 -
b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2
*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*
c^2*d - 3*a^6*b*c*d^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(b^2*d - 2*a^2*d + a*b
*c))/(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2
*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^
3*b^5*c*d + 6*a^5*b^3*c*d))*(b^2*d - 2*a^2*d + a*b*c))/(a^8*d^2 - b^8*c^2 +
3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2

- 3*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d
))*(b^2*d - 2*a^2*d + a*b*c)*1i)/(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4
*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*
b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d) - (b*(-(a + b)^3*(a
- b)^3)^(1/2)*((32*(a*b^7*c^3*d^3 - a^3*b^5*c*d^5 + a^3*b^5*c^5*d + 2*a^5*b
^3*c*d^5 + 2*a^2*b^6*c^4*d^2 - 6*a^3*b^5*c^3*d^3 + 2*a^4*b^4*c^2*d^4 - 5*a^
4*b^4*c^4*d^2 + 8*a^5*b^3*c^3*d^3 - 3*a^6*b^2*c^2*d^4 - a^7*b*c*d^5))/(a^7*
d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 -
3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*
a*b^6*c^2*d - 3*a^6*b*c*d^2) - (32*tan(e/2 + (f*x)/2)*(a^3*b^5*c^6 - a^8*c*
d^5 - 4*a*b^7*c^2*d^4 + a*b^7*c^4*d^2 + 4*a^2*b^6*c*d^5 + 2*a^2*b^6*c^5*d -
13*a^4*b^4*c*d^5 - 5*a^4*b^4*c^5*d + 12*a^6*b^2*c*d^5 + a^7*b*c^2*d^4 - 5*
a^2*b^6*c^3*d^3 + 17*a^3*b^5*c^2*d^4 - 8*a^3*b^5*c^4*d^2 + 14*a^4*b^4*c^3*d
^3 - 20*a^5*b^3*c^2*d^4 + 8*a^5*b^3*c^4*d^2 - 4*a^6*b^2*c^3*d^3))/(a^7*d^3
- b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a
^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^
6*c^2*d - 3*a^6*b*c*d^2) + (b*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(a^3*b^7*c^
7 - a^5*b^5*c^7 + a^10*c^2*d^5 + a*b^9*c^5*d^2 + a^2*b^8*c^6*d - 6*a^4*b^6*
c^6*d + a^5*b^5*c*d^6 + 5*a^6*b^4*c^6*d - 3*a^7*b^3*c*d^6 - 5*a^9*b*c^3*d^4
- 4*a^2*b^8*c^4*d^3 + 6*a^3*b^7*c^3*d^4 - 7*a^3*b^7*c^5*d^2 - 4*a^4*b^6*c^
2*d^5 + 18*a^4*b^6*c^4*d^3 - 22*a^5*b^5*c^3*d^4 + 16*a^5*b^5*c^5*d^2 + 13*a
^6*b^4*c^2*d^5 - 24*a^6*b^4*c^4*d^3 + 21*a^7*b^3*c^3*d^4 - 10*a^7*b^3*c^5*d
^2 - 10*a^8*b^2*c^2*d^5 + 10*a^8*b^2*c^4*d^3 + 2*a^9*b*c*d^6))/(a^7*d^3 - b
^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*
b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c
^2*d - 3*a^6*b*c*d^2) + (32*tan(e/2 + (f*x)/2)*(2*a^10*c*d^6 + 2*a^2*b^8*c^
7 - 2*a^4*b^6*c^7 - 12*a^3*b^7*c^6*d + 10*a^5*b^5*c^6*d - 2*a^8*b^2*c*d^6 -
6*a^9*b*c^2*d^5 - 8*a^2*b^8*c^5*d^2 + 12*a^3*b^7*c^4*d^3 - 8*a^4*b^6*c^3*d
^4 + 26*a^4*b^6*c^5*d^2 + 2*a^5*b^5*c^2*d^5 - 24*a^5*b^5*c^4*d^3 + 6*a^6*b^
4*c^3*d^4 - 18*a^6*b^4*c^5*d^2 + 4*a^7*b^3*c^2*d^5 + 12*a^7*b^3*c^4*d^3 + 2
*a^8*b^2*c^3*d^4 + 2*a*b^9*c^6*d))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4
*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d
+ 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (b*(
(32*(2*a^4*b^8*c^8 - a^2*b^10*c^8 - a^6*b^6*c^8 + a^12*c^2*d^6 + 2*a^3*b^9*
c^7*d - 7*a^5*b^7*c^7*d - a^7*b^5*c*d^7 + 4*a^7*b^5*c^7*d + 2*a^9*b^3*c*d^7
- 4*a^11*b*c^3*d^5 - 4*a^2*b^10*c^6*d^2 + 5*a^3*b^9*c^5*d^3 + 3*a^4*b^8*c^
6*d^2 - 5*a^5*b^7*c^3*d^5 - 10*a^5*b^7*c^5*d^3 + 4*a^6*b^6*c^2*d^6 + 5*a^6*
b^6*c^4*d^4 + 6*a^6*b^6*c^6*d^2 + 6*a^7*b^5*c^3*d^5 + 5*a^7*b^5*c^5*d^3 - 7
*a^8*b^4*c^2*d^6 - 10*a^8*b^4*c^4*d^4 - 5*a^8*b^4*c^6*d^2 + 3*a^9*b^3*c^3*d
^5 + 2*a^10*b^2*c^2*d^6 + 5*a^10*b^2*c^4*d^4 + a*b^11*c^7*d - a^11*b*c*d^7)
)/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^
2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2
*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (32*tan(e/2 + (f*x)/2)*(3*a*b^11*c^8
- 3*a^12*c*d^7 - 8*a^3*b^9*c^8 + 7*a^5*b^7*c^8 - 2*a^7*b^5*c^8 + 2*a^12*c^3
*d^5 - 4*a*b^11*c^6*d^2 - 15*a^2*b^10*c^7*d + 40*a^4*b^8*c^7*d + 4*a^6*b^6*

c*d^7 - 35*a^6*b^6*c^7*d - 11*a^8*b^4*c*d^7 + 10*a^8*b^4*c^7*d + 10*a^10*b^2*c*d^7 + 15*a^11*b*c^2*d^6 - 10*a^11*b*c^4*d^4 + 20*a^2*b^10*c^5*d^3 - 40*a^3*b^9*c^4*d^4 + 41*a^3*b^9*c^6*d^2 + 40*a^4*b^8*c^3*d^5 - 85*a^4*b^8*c^5*d^3 - 20*a^5*b^7*c^2*d^6 + 125*a^5*b^7*c^4*d^4 - 90*a^5*b^7*c^6*d^2 - 113*a^6*b^6*c^3*d^5 + 130*a^6*b^6*c^5*d^3 + 55*a^7*b^5*c^2*d^6 - 140*a^7*b^5*c^4*d^4 + 73*a^7*b^5*c^6*d^2 + 108*a^8*b^4*c^3*d^5 - 85*a^8*b^4*c^5*d^3 - 50*a^9*b^3*c^2*d^6 + 65*a^9*b^3*c^4*d^4 - 20*a^9*b^3*c^6*d^2 - 37*a^10*b^2*c^3*d^5 + 20*a^10*b^2*c^5*d^3))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(b^2*d - 2*a^2*d + a*b*c))/(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d))*(b^2*d - 2*a^2*d + a*b*c))/(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d))*(b^2*d - 2*a^2*d + a*b*c)*1i)/(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d))/((64*(2*a^2*b^4*c^2*d^3 - 3*a^3*b^3*c*d^4 + a^3*b^3*c^3*d^2 - 3*a^4*b^2*c^2*d^3 + a*b^5*c*d^4 + 2*a^5*b*c*d^4))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (64*tan(e/2 + (f*x)/2)*(2*a^2*b^4*c*d^4 - 4*a^4*b^2*c*d^4 + 2*a^3*b^3*c^2*d^3))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (b*(-(a + b)^3*(a - b)^3)^(1/2)*((32*tan(e/2 + (f*x)/2)*(a^3*b^5*c^6 - a^8*c*d^5 - 4*a*b^7*c^2*d^4 + a*b^7*c^4*d^2 + 4*a^2*b^6*c*d^5 + 2*a^2*b^6*c^5*d - 13*a^4*b^4*c*d^5 - 5*a^4*b^4*c^5*d + 12*a^6*b^2*c*d^5 + a^7*b*c^2*d^4 - 5*a^2*b^6*c^3*d^3 + 17*a^3*b^5*c^2*d^4 - 8*a^3*b^5*c^4*d^2 + 14*a^4*b^4*c^3*d^3 - 20*a^5*b^3*c^2*d^4 + 8*a^5*b^3*c^4*d^2 - 4*a^6*b^2*c^3*d^3))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (32*(a*b^7*c^3*d^3 - a^3*b^5*c*d^5 + a^3*b^5*c^5*d + 2*a^5*b^3*c*d^5 + 2*a^2*b^6*c^4*d^2 - 6*a^3*b^5*c^3*d^3 + 2*a^4*b^4*c^2*d^4 - 5*a^4*b^4*c^4*d^2 + 8*a^5*b^3*c^3*d^3 - 3*a^6*b^2*c^2*d^4 - a^7*b*c*d^5))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (b*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(a^3*b^7*c^7 - a^5*b^5*c^7 + a^10*c^2*d^5 + a*b^9*c^5*d^2 + a^2*b^8*c^6*d - 6*a^4*b^6*c^6*d + a^5*b^5*c*d^6 + 5*a^6*b^4*c^6*d - 3*a^7*b^3*c*d^6 - 5*a^9*b*c^3*d^4 - 4*a^2*b^8*c^4*d^3 + 6*a^3*b^7*c^3*d^4 - 7*a^3*b^7*c^5*d^2 - 4*a^4*b^6*c^2*d^5 + 18*a^4*b^6*c^4*d^3 - 22*a^5*b^5*c^3*d^4 + 16*a^5*b^5*c^5*d^2 + 13*a^6*b^4*c^2*d^5 - 24*a^6*b^4*c^4*d^3 + 21*a^7*b^3*c^3*d^4 - 10*a^7*b^3*c^5*d^2 - 10*a^8*b^2*c^2*d^5 + 10*a^8*b^2*c^4*d^3 + 2*a^9*b*c*d^6))/(a^7*d^3 -

b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (32*tan(e/2 + (f*x)/2)*(2*a^10*c*d^6 + 2*a^2*b^8*c^7 - 2*a^4*b^6*c^7 - 12*a^3*b^7*c^6*d + 10*a^5*b^5*c^6*d - 2*a^8*b^2*c*d^6 - 6*a^9*b*c^2*d^5 - 8*a^2*b^8*c^5*d^2 + 12*a^3*b^7*c^4*d^3 - 8*a^4*b^6*c^3*d^4 + 26*a^4*b^6*c^5*d^2 + 2*a^5*b^5*c^2*d^5 - 24*a^5*b^5*c^4*d^3 + 6*a^6*b^4*c^3*d^4 - 18*a^6*b^4*c^5*d^2 + 4*a^7*b^3*c^2*d^5 + 12*a^7*b^3*c^4*d^3 + 2*a^8*b^2*c^3*d^4 + 2*a*b^9*c^6*d))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (b*((32*(2*a^4*b^8*c^8 - a^2*b^10*c^8 - a^6*b^6*c^8 + a^12*c^2*d^6 + 2*a^3*b^9*c^7*d - 7*a^5*b^7*c^7*d - a^7*b^5*c*d^7 + 4*a^7*b^5*c^7*d + 2*a^9*b^3*c*d^7 - 4*a^11*b*c^3*d^5 - 4*a^2*b^10*c^6*d^2 + 5*a^3*b^9*c^5*d^3 + 3*a^4*b^8*c^6*d^2 - 5*a^5*b^7*c^3*d^5 - 10*a^5*b^7*c^5*d^3 + 4*a^6*b^6*c^2*d^6 + 5*a^6*b^6*c^4*d^4 + 6*a^6*b^6*c^6*d^2 + 6*a^7*b^5*c^3*d^5 + 5*a^7*b^5*c^5*d^3 - 7*a^8*b^4*c^2*d^6 - 10*a^8*b^4*c^4*d^4 - 5*a^8*b^4*c^6*d^2 + 3*a^9*b^3*c^3*d^5 + 2*a^10*b^2*c^2*d^6 + 5*a^10*b^2*c^4*d^4 + a*b^11*c^7*d - a^11*b*c*d^7))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (32*tan(e/2 + (f*x)/2)*(3*a*b^11*c^8 - 3*a^12*c*d^7 - 8*a^3*b^9*c^8 + 7*a^5*b^7*c^8 - 2*a^7*b^5*c^8 + 2*a^12*c^3*d^5 - 4*a*b^11*c^6*d^2 - 15*a^2*b^10*c^7*d + 40*a^4*b^8*c^7*d + 4*a^6*b^6*c*d^7 - 35*a^6*b^6*c^7*d - 11*a^8*b^4*c*d^7 + 10*a^8*b^4*c^7*d + 10*a^10*b^2*c*d^7 + 15*a^11*b*c^2*d^6 - 10*a^11*b*c^4*d^4 + 20*a^2*b^10*c^5*d^3 - 40*a^3*b^9*c^4*d^4 + 41*a^3*b^9*c^6*d^2 + 40*a^4*b^8*c^3*d^5 - 85*a^4*b^8*c^5*d^3 - 20*a^5*b^7*c^2*d^6 + 125*a^5*b^7*c^4*d^4 - 90*a^5*b^7*c^6*d^2 - 113*a^6*b^6*c^3*d^5 + 130*a^6*b^6*c^5*d^3 + 55*a^7*b^5*c^2*d^6 - 140*a^7*b^5*c^4*d^4 + 73*a^7*b^5*c^6*d^2 + 108*a^8*b^4*c^3*d^5 - 85*a^8*b^4*c^5*d^3 - 50*a^9*b^3*c^2*d^6 + 65*a^9*b^3*c^4*d^4 - 20*a^9*b^3*c^6*d^2 - 37*a^10*b^2*c^3*d^5 + 20*a^10*b^2*c^5*d^3))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(b^2*d - 2*a^2*d + a*b*c))/(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d))*(b^2*d - 2*a^2*d + a*b*c))/(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d))*(b^2*d - 2*a^2*d + a*b*c))/(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d) - (b*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(a*b^7*c^3*d^3 - a^3*b^5*c*d^5 + a^3*b^5*c^5*d + 2*a^5*b^3*c*d^5 + 2*a^2*b^6*c^4*d^2 - 6*a^3*b^5*c^3*d^3 + 2*a^4*b^4*c^2*d^4 - 5*a^4*b^4*c^4*d^2 + 8*a^5*b^3*c^3*d^3 - 3*a^6*b^2*c^2*d^4 - a^7*b*c*d^5))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d +

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6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (32*ta
n(e/2 + (f*x)/2)*(a^3*b^5*c^6 - a^8*c*d^5 - 4*a*b^7*c^2*d^4 + a*b^7*c^4*d^2
 + 4*a^2*b^6*c*d^5 + 2*a^2*b^6*c^5*d - 13*a^4*b^4*c*d^5 - 5*a^4*b^4*c^5*d +
 12*a^6*b^2*c*d^5 + a^7*b*c^2*d^4 - 5*a^2*b^6*c^3*d^3 + 17*a^3*b^5*c^2*d^4
- 8*a^3*b^5*c^4*d^2 + 14*a^4*b^4*c^3*d^3 - 20*a^5*b^3*c^2*d^4 + 8*a^5*b^3*c
^4*d^2 - 4*a^6*b^2*c^3*d^3))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c
^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^
4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (b*(-(a +
b)^3*(a - b)^3)^(1/2)*((32*(a^3*b^7*c^7 - a^5*b^5*c^7 + a^10*c^2*d^5 + a*b^
9*c^5*d^2 + a^2*b^8*c^6*d - 6*a^4*b^6*c^6*d + a^5*b^5*c*d^6 + 5*a^6*b^4*c^6
*d - 3*a^7*b^3*c*d^6 - 5*a^9*b*c^3*d^4 - 4*a^2*b^8*c^4*d^3 + 6*a^3*b^7*c^3*
d^4 - 7*a^3*b^7*c^5*d^2 - 4*a^4*b^6*c^2*d^5 + 18*a^4*b^6*c^4*d^3 - 22*a^5*b
^5*c^3*d^4 + 16*a^5*b^5*c^5*d^2 + 13*a^6*b^4*c^2*d^5 - 24*a^6*b^4*c^4*d^3 +
 21*a^7*b^3*c^3*d^4 - 10*a^7*b^3*c^5*d^2 - 10*a^8*b^2*c^2*d^5 + 10*a^8*b^2*
c^4*d^3 + 2*a^9*b*c*d^6))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3
+ a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b
^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) + (32*tan(e/2 +
 (f*x)/2)*(2*a^10*c*d^6 + 2*a^2*b^8*c^7 - 2*a^4*b^6*c^7 - 12*a^3*b^7*c^6*d
+ 10*a^5*b^5*c^6*d - 2*a^8*b^2*c*d^6 - 6*a^9*b*c^2*d^5 - 8*a^2*b^8*c^5*d^2
+ 12*a^3*b^7*c^4*d^3 - 8*a^4*b^6*c^3*d^4 + 26*a^4*b^6*c^5*d^2 + 2*a^5*b^5*c
^2*d^5 - 24*a^5*b^5*c^4*d^3 + 6*a^6*b^4*c^3*d^4 - 18*a^6*b^4*c^5*d^2 + 4*a^
7*b^3*c^2*d^5 + 12*a^7*b^3*c^4*d^3 + 2*a^8*b^2*c^3*d^4 + 2*a*b^9*c^6*d))/(a
^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^
3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d +
 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) - (b*((32*(2*a^4*b^8*c^8 - a^2*b^10*c^8 - a
^6*b^6*c^8 + a^12*c^2*d^6 + 2*a^3*b^9*c^7*d - 7*a^5*b^7*c^7*d - a^7*b^5*c*d
^7 + 4*a^7*b^5*c^7*d + 2*a^9*b^3*c*d^7 - 4*a^11*b*c^3*d^5 - 4*a^2*b^10*c^6*
d^2 + 5*a^3*b^9*c^5*d^3 + 3*a^4*b^8*c^6*d^2 - 5*a^5*b^7*c^3*d^5 - 10*a^5*b^
7*c^5*d^3 + 4*a^6*b^6*c^2*d^6 + 5*a^6*b^6*c^4*d^4 + 6*a^6*b^6*c^6*d^2 + 6*a
^7*b^5*c^3*d^5 + 5*a^7*b^5*c^5*d^3 - 7*a^8*b^4*c^2*d^6 - 10*a^8*b^4*c^4*d^4
 - 5*a^8*b^4*c^6*d^2 + 3*a^9*b^3*c^3*d^5 + 2*a^10*b^2*c^2*d^6 + 5*a^10*b^2*
c^4*d^4 + a*b^11*c^7*d - a^11*b*c*d^7))/(a^7*d^3 - b^7*c^3 + 2*a^2*b^5*c^3
- a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a^2*b^5*c*d^2 - 6*a^3*b^4*c
^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^6*c^2*d - 3*a^6*b*c*d^2) -
 (32*tan(e/2 + (f*x)/2)*(3*a*b^11*c^8 - 3*a^12*c*d^7 - 8*a^3*b^9*c^8 + 7*a^
5*b^7*c^8 - 2*a^7*b^5*c^8 + 2*a^12*c^3*d^5 - 4*a*b^11*c^6*d^2 - 15*a^2*b^10
*c^7*d + 40*a^4*b^8*c^7*d + 4*a^6*b^6*c*d^7 - 35*a^6*b^6*c^7*d - 11*a^8*b^4
*c*d^7 + 10*a^8*b^4*c^7*d + 10*a^10*b^2*c*d^7 + 15*a^11*b*c^2*d^6 - 10*a^11
*b*c^4*d^4 + 20*a^2*b^10*c^5*d^3 - 40*a^3*b^9*c^4*d^4 + 41*a^3*b^9*c^6*d^2
+ 40*a^4*b^8*c^3*d^5 - 85*a^4*b^8*c^5*d^3 - 20*a^5*b^7*c^2*d^6 + 125*a^5*b^
7*c^4*d^4 - 90*a^5*b^7*c^6*d^2 - 113*a^6*b^6*c^3*d^5 + 130*a^6*b^6*c^5*d^3
+ 55*a^7*b^5*c^2*d^6 - 140*a^7*b^5*c^4*d^4 + 73*a^7*b^5*c^6*d^2 + 108*a^8*b
^4*c^3*d^5 - 85*a^8*b^4*c^5*d^3 - 50*a^9*b^3*c^2*d^6 + 65*a^9*b^3*c^4*d^4 -
 20*a^9*b^3*c^6*d^2 - 37*a^10*b^2*c^3*d^5 + 20*a^10*b^2*c^5*d^3))/(a^7*d^3
- b^7*c^3 + 2*a^2*b^5*c^3 - a^4*b^3*c^3 + a^3*b^4*d^3 - 2*a^5*b^2*d^3 - 3*a
```

```
^2*b^5*c*d^2 - 6*a^3*b^4*c^2*d + 6*a^4*b^3*c*d^2 + 3*a^5*b^2*c^2*d + 3*a*b^
6*c^2*d - 3*a^6*b*c*d^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(b^2*d - 2*a^2*d + a
*b*c))/(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a
^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*
a^3*b^5*c*d + 6*a^5*b^3*c*d))*(b^2*d - 2*a^2*d + a*b*c))/(a^8*d^2 - b^8*c^2
 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^
2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c
*d))*(b^2*d - 2*a^2*d + a*b*c))/(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*
b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b
^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d)))*(-(a + b)^3*(a - b)
^3)^(1/2)*(b^2*d - 2*a^2*d + a*b*c)*2i)/(f*(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c
^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*
d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**2/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.712 $\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=290

$$\frac{d\left(a^2d^2+b^2\left(c^2-2d^2\right)\right)\cos(e+fx)}{f\left(a^2-b^2\right)\left(c^2-d^2\right)(bc-ad)^2(c+d\sin(e+fx))}+\frac{2b^2\left(-3a^2d+abc+2b^2d\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{3/2}(bc-ad)^3}+\frac{\dots}{f\left(a^2-b^2\dots\right.}$$

```
[Out] 2*b^2*(-3*a^2*d+a*b*c+2*b^2*d)*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1
/2))/(a^2-b^2)^(3/2)/(-a*d+b*c)^3/f+2*d^2*(-a*c*d+3*b*c^2-2*b*d^2)*arctan((
d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(-a*d+b*c)^3/(c^2-d^2)^(3/2)/f+d*(
a^2*d^2+b^2*(c^2-2*d^2))*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)^2/(c^2-d^2)/f/(c+d
*sin(f*x+e))+b^2*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(a+b*sin(f*x+e))/(c+d*si
n(f*x+e))
```

Rubi [A] time = 1.20, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2802, 3055, 3001, 2660, 618, 204}

$$\frac{d\left(a^2d^2+b^2\left(c^2-2d^2\right)\right)\cos(e+fx)}{f\left(a^2-b^2\right)\left(c^2-d^2\right)(bc-ad)^2(c+d\sin(e+fx))}+\frac{2b^2\left(-3a^2d+abc+2b^2d\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{3/2}(bc-ad)^3}+\frac{\dots}{f\left(a^2-b^2\dots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] (2*b^2*(a*b*c - 3*a^2*d + 2*b^2*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2
 - b^2]])/((a^2 - b^2)^(3/2)*(b*c - a*d)^3*f) + (2*d^2*(3*b*c^2 - a*c*d - 2
*b*d^2)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^3*(c
^2 - d^2)^(3/2)*f) + (d*(a^2*d^2 + b^2*(c^2 - 2*d^2))*Cos[e + f*x])/((a^2 -
 b^2)*(b*c - a*d)^2*(c^2 - d^2)*f*(c + d*Sin[e + f*x])) + (b^2*Cos[e + f*x]
)/((a^2 - b^2)*(b*c - a*d)*f*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
```

```
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx = \frac{b^2\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))(c+d\sin(e+fx))} - \frac{\int}{}$$

$$= \frac{d\left(a^2d^2+b^2\left(c^2-2d^2\right)\right)\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \frac{}{\left(a^2-b^2\right)(}$$

$$= \frac{d\left(a^2d^2+b^2\left(c^2-2d^2\right)\right)\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \frac{}{\left(a^2-b^2\right)(}$$

$$= \frac{d\left(a^2d^2+b^2\left(c^2-2d^2\right)\right)\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \frac{}{\left(a^2-b^2\right)(}$$

$$= \frac{d\left(a^2d^2+b^2\left(c^2-2d^2\right)\right)\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \frac{}{\left(a^2-b^2\right)(}$$

$$= \frac{2b^2\left(abc-3a^2d+2b^2d\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}(bc-ad)^3f} + \frac{2d^2\left(3bc^2-\right.}{(}$$

Mathematica [A] time = 2.91, size = 227, normalized size = 0.78

$$\frac{\frac{2b^2\left(-3a^2d+abc+2b^2d\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{b^3(bc-ad)\cos(e+fx)}{(a-b)(a+b)(a+b\sin(e+fx))} - \frac{2d^2\left(acd-3bc^2+2bd^2\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{3/2}} + \frac{d^3(bc-ad)\cos}{(c-d)(c+d)(c+d}}{f(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] ((2*b^2*(a*b*c - 3*a^2*d + 2*b^2*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^
2 - b^2]])/(a^2 - b^2)^(3/2) - (2*d^2*(-3*b*c^2 + a*c*d + 2*b*d^2)*ArcTan[(
d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^2)^(3/2) + (b^3*(b*c - a
```

```
*d)*Cos[e + f*x])/((a - b)*(a + b)*(a + b*Sin[e + f*x])) + (d^3*(b*c - a*d)
*Cos[e + f*x])/((c - d)*(c + d)*(c + d*Sin[e + f*x]))))/((b*c - a*d)^3*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 4.78, size = 1005, normalized size = 3.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 2*((a*b^3*c - 3*a^2*b^2*d + 2*b^4*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(
a) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^2*b^3*c^3 -
b^5*c^3 - 3*a^3*b^2*c^2*d + 3*a*b^4*c^2*d + 3*a^4*b*c*d^2 - 3*a^2*b^3*c*d^2
 - a^5*d^3 + a^3*b^2*d^3)*sqrt(a^2 - b^2)) + (3*b*c^2*d^2 - a*c*d^3 - 2*b*d
^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*
e) + d)/sqrt(c^2 - d^2)))/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - b^3
*c^3*d^2 - a^3*c^2*d^3 + 3*a*b^2*c^2*d^3 - 3*a^2*b*c*d^4 + a^3*d^5)*sqrt(c^
2 - d^2)) + (b^4*c^4*tan(1/2*f*x + 1/2*e)^3 - b^4*c^2*d^2*tan(1/2*f*x + 1/2
*e)^3 + a^4*d^4*tan(1/2*f*x + 1/2*e)^3 - a^2*b^2*d^4*tan(1/2*f*x + 1/2*e)^3
 + a*b^3*c^4*tan(1/2*f*x + 1/2*e)^2 + 2*b^4*c^3*d*tan(1/2*f*x + 1/2*e)^2 -
a*b^3*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 + a^4*c*d^3*tan(1/2*f*x + 1/2*e)^2 - a
^2*b^2*c*d^3*tan(1/2*f*x + 1/2*e)^2 - 2*b^4*c*d^3*tan(1/2*f*x + 1/2*e)^2 +
2*a^3*b*d^4*tan(1/2*f*x + 1/2*e)^2 - 2*a*b^3*d^4*tan(1/2*f*x + 1/2*e)^2 + b
^4*c^4*tan(1/2*f*x + 1/2*e) + 2*a*b^3*c^3*d*tan(1/2*f*x + 1/2*e) - b^4*c^2*
d^2*tan(1/2*f*x + 1/2*e) + 2*a^3*b*c*d^3*tan(1/2*f*x + 1/2*e) - 4*a*b^3*c*d
^3*tan(1/2*f*x + 1/2*e) + a^4*d^4*tan(1/2*f*x + 1/2*e) - a^2*b^2*d^4*tan(1/
2*f*x + 1/2*e) + a*b^3*c^4 - a*b^3*c^2*d^2 + a^4*c*d^3 - a^2*b^2*c*d^3)/((a
^3*b^2*c^5 - a*b^4*c^5 - 2*a^4*b*c^4*d + 2*a^2*b^3*c^4*d + a^5*c^3*d^2 - 2*
a^3*b^2*c^3*d^2 + a*b^4*c^3*d^2 + 2*a^4*b*c^2*d^3 - 2*a^2*b^3*c^2*d^3 - a^5
*c*d^4 + a^3*b^2*c*d^4)*(a*c*tan(1/2*f*x + 1/2*e)^4 + 2*b*c*tan(1/2*f*x + 1
/2*e)^3 + 2*a*d*tan(1/2*f*x + 1/2*e)^3 + 2*a*c*tan(1/2*f*x + 1/2*e)^2 + 4*b
*d*tan(1/2*f*x + 1/2*e)^2 + 2*b*c*tan(1/2*f*x + 1/2*e) + 2*a*d*tan(1/2*f*x
+ 1/2*e) + a*c)))/f
```

maple [B] time = 0.38, size = 886, normalized size = 3.06

$$\frac{2d^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)a}{f\left(da-cb\right)^3\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d+c\right)c\left(c^2-d^2\right)}-\frac{2d^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)b}{f\left(da-cb\right)^3\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2\tan\left(\frac{fx}{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2/f*d^5/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/c/(c^
2-d^2)*tan(1/2*f*x+1/2*e)*a-2/f*d^4/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*c+2*t
an(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*tan(1/2*f*x+1/2*e)*b+2/f*d^4/(a*d-b*c)^3/(
tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*a-2/f*d^3/(a*d-b
*c)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*c*b+2/f*d
^3/(a*d-b*c)^3/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2
-d^2)^(1/2))*a*c-6/f*d^2/(a*d-b*c)^3/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/
2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b*c^2+4/f*d^4/(a*d-b*c)^3/(c^2-d^2)^(3/2
)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b+2/f*b^4/(a*d-b
*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*tan(1/2*f
*x+1/2*e)*d-2/f*b^5/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e
)*b+a)/a/(a^2-b^2)*tan(1/2*f*x+1/2*e)*c+2/f*b^3/(a*d-b*c)^3/(tan(1/2*f*x+1/
2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*d*a-2/f*b^4/(a*d-b*c)^3/(tan(1
/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*c+6/f*b^2/(a*d-b*c)^3
/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a
^2*d-2/f*b^3/(a*d-b*c)^3/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)
+2*b)/(a^2-b^2)^(1/2))*a*c-4/f*b^4/(a*d-b*c)^3/(a^2-b^2)^(3/2)*arctan(1/2*(
2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 30.89, size = 71320, normalized size = 245.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^2),x)
```

```
[Out] ((2*(a^3*d^3 + b^3*c^3 - a*b^2*d^3 - b^3*c*d^2))/((a^2*d^2 + b^2*c^2 - 2*a*
b*c*d)*(a^2*c^2 - a^2*d^2 - b^2*c^2 + b^2*d^2)) + (2*tan(e/2 + (f*x)/2)^3*(
a^4*d^4 + b^4*c^4 - a^2*b^2*d^4 - b^4*c^2*d^2))/(a*c*(a^2*d^2 + b^2*c^2 - 2
*a*b*c*d)*(a^2*c^2 - a^2*d^2 - b^2*c^2 + b^2*d^2)) + (2*tan(e/2 + (f*x)/2)*
(a^4*d^4 + b^4*c^4 - a^2*b^2*d^4 - b^4*c^2*d^2 - 4*a*b^3*c*d^3 + 2*a*b^3*c^
3*d + 2*a^3*b*c*d^3))/(a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^2*c^2 - a^2*d
^2 - b^2*c^2 + b^2*d^2)) + (2*tan(e/2 + (f*x)/2)^2*(a*c + 2*b*d)*(a^3*d^3 +
 b^3*c^3 - a*b^2*d^3 - b^3*c*d^2))/(a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^
2*c^2 - a^2*d^2 - b^2*c^2 + b^2*d^2)))/(f*(a*c + tan(e/2 + (f*x)/2)^3*(2*a*
d + 2*b*c) + tan(e/2 + (f*x)/2)^2*(2*a*c + 4*b*d) + tan(e/2 + (f*x)/2)*(2*a
*d + 2*b*c) + a*c*tan(e/2 + (f*x)/2)^4)) - (b^2*atan(((b^2*(-(a + b)^3*(a -
 b)^3)^(1/2)*((32*(4*a*b^10*c^4*d^7 - 8*a*b^10*c^6*d^5 + 4*a*b^10*c^8*d^3 +
 a^3*b^8*c^10*d + 4*a^4*b^7*c*d^10 - 8*a^6*b^5*c*d^10 + 4*a^8*b^3*c*d^10 +
a^10*b*c^3*d^8 - 4*a^2*b^9*c^3*d^8 + 8*a^2*b^9*c^5*d^6 - 7*a^2*b^9*c^7*d^4
+ 4*a^2*b^9*c^9*d^2 - 4*a^3*b^8*c^2*d^9 + 21*a^3*b^8*c^6*d^5 - 22*a^3*b^8*c
^8*d^3 - 18*a^4*b^7*c^5*d^6 + 26*a^4*b^7*c^7*d^4 - 8*a^4*b^7*c^9*d^2 + 8*a^
5*b^6*c^2*d^9 - 18*a^5*b^6*c^4*d^7 - 8*a^5*b^6*c^6*d^5 + 22*a^5*b^6*c^8*d^3
 + 21*a^6*b^5*c^3*d^8 - 8*a^6*b^5*c^5*d^6 - 15*a^6*b^5*c^7*d^4 - 7*a^7*b^4*
c^2*d^9 + 26*a^7*b^4*c^4*d^7 - 15*a^7*b^4*c^6*d^5 - 22*a^8*b^3*c^3*d^8 + 22
*a^8*b^3*c^5*d^6 + 4*a^9*b^2*c^2*d^9 - 8*a^9*b^2*c^4*d^7))/(a^10*d^10 + b^1
0*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*
a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d
^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*
d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*
d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3
*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6
 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^
5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 -
60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c
^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*
a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) - (32*tan(e/2 + (f*x)/2)*(
a^3*b^8*c^11 + a^11*c^3*d^8 - 16*a*b^10*c^3*d^8 + 44*a*b^10*c^5*d^6 - 34*a*
b^10*c^7*d^4 + 4*a*b^10*c^9*d^2 + 4*a^2*b^9*c^10*d - 16*a^3*b^8*c*d^10 - 8*
a^4*b^7*c^10*d + 44*a^5*b^6*c*d^10 - 34*a^7*b^4*c*d^10 + 4*a^9*b^2*c*d^10 +
 4*a^10*b*c^2*d^9 - 8*a^10*b*c^4*d^7 + 32*a^2*b^9*c^2*d^9 - 104*a^2*b^9*c^4
*d^7 + 100*a^2*b^9*c^6*d^5 - 24*a^2*b^9*c^8*d^3 + 120*a^3*b^8*c^3*d^8 - 222
*a^3*b^8*c^5*d^6 + 134*a^3*b^8*c^7*d^4 - 24*a^3*b^8*c^9*d^2 - 104*a^4*b^7*c
^2*d^9 + 312*a^4*b^7*c^4*d^7 - 272*a^4*b^7*c^6*d^5 + 60*a^4*b^7*c^8*d^3 - 2
22*a^5*b^6*c^3*d^8 + 316*a^5*b^6*c^5*d^6 - 136*a^5*b^6*c^7*d^4 + 22*a^5*b^6
*c^9*d^2 + 100*a^6*b^5*c^2*d^9 - 272*a^6*b^5*c^4*d^7 + 192*a^6*b^5*c^6*d^5
- 24*a^6*b^5*c^8*d^3 + 134*a^7*b^4*c^3*d^8 - 136*a^7*b^4*c^5*d^6 + 18*a^7*b
```

^4*c^7*d^4 - 24*a^8*b^3*c^2*d^9 + 60*a^8*b^3*c^4*d^7 - 24*a^8*b^3*c^6*d^5 - 24*a^9*b^2*c^3*d^8 + 22*a^9*b^2*c^5*d^6))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) + (b^2*(-(a + b)^3*(a - b)^3)^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*a^4*b^9*c^13 - 2*a^2*b^11*c^13 - 2*a^13*c^2*d^11 + 2*a^13*c^4*d^9 - 2*a*b^12*c^8*d^5 + 6*a*b^12*c^10*d^3 + 20*a^3*b^10*c^12*d - 16*a^5*b^8*c^12*d - 2*a^8*b^5*c*d^12 + 6*a^10*b^3*c*d^12 + 20*a^12*b*c^3*d^10 - 16*a^12*b*c^5*d^8 + 10*a^2*b^11*c^7*d^6 - 34*a^2*b^11*c^9*d^4 + 26*a^2*b^11*c^11*d^2 - 18*a^3*b^10*c^6*d^7 + 80*a^3*b^10*c^8*d^5 - 82*a^3*b^10*c^10*d^3 + 10*a^4*b^9*c^5*d^8 - 96*a^4*b^9*c^7*d^6 + 160*a^4*b^9*c^9*d^4 - 76*a^4*b^9*c^11*d^2 + 10*a^5*b^8*c^4*d^9 + 44*a^5*b^8*c^6*d^7 - 188*a^5*b^8*c^8*d^5 + 150*a^5*b^8*c^10*d^3 - 18*a^6*b^7*c^3*d^10 + 44*a^6*b^7*c^5*d^8 + 88*a^6*b^7*c^7*d^6 - 164*a^6*b^7*c^9*d^4 + 50*a^6*b^7*c^11*d^2 + 10*a^7*b^6*c^2*d^11 - 96*a^7*b^6*c^4*d^9 + 88*a^7*b^6*c^6*d^7 + 72*a^7*b^6*c^8*d^5 - 74*a^7*b^6*c^10*d^3 + 80*a^8*b^5*c^3*d^10 - 188*a^8*b^5*c^5*d^8 + 72*a^8*b^5*c^7*d^6 + 38*a^8*b^5*c^9*d^4 - 34*a^9*b^4*c^2*d^11 + 160*a^9*b^4*c^4*d^9 - 164*a^9*b^4*c^6*d^7 + 38*a^9*b^4*c^8*d^5 - 82*a^10*b^3*c^3*d^10 + 150*a^10*b^3*c^5*d^8 - 74*a^10*b^3*c^7*d^6 + 26*a^11*b^2*c^2*d^11 - 76*a^11*b^2*c^4*d^9 + 50*a^11*b^2*c^6*d^7 - 4*a*b^12*c^12*d - 4*a^12*b*c*d^12))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) - (32*(a^3*b^10*c^13 - a^5*b^8*c^13 + a^13*c^3*d^10 - a^13*c^5*d^8 + 2*a*b^12*c^7*d^6 - 5*a*b^12*c^9*d^4 + 3*a*b^12*c^11*d^2 + 2*a^2*b^11*c^12*d - 10*a^4*b^9*c^12*d + 8*a^6*b^7*c^12*d + 2*a^7*b^6*c*d^12 - 5*a^9*b^4*c*d^12 + 3*a^11*b^2*c*d^12 + 2*a^12*b*c^2*d^11 - 10*a^12*b*c^4*d^9 + 8*a^12*b*c^6*d^7 - 12*a^2*b^11*c^6*d^7 + 32*a^2*b^11*c^8*d^5 - 22*a^2*b^11*c^10*d^3 + 30*a^3*b^10*c^5*d^8 - 92*a^3*b^10*c^7*d^6 + 83*a^3*b^10*c^9*d^4 - 22*a^3*b^10*c^11*d^2 - 40*a^4*b^9* c^4*d^9 + 160*a^4*b^9*c^6*d^7 - 208*a^4*b^9*c^8*d^5 + 98*a^4*b^9*c^10*d^3 + 30*a^5*b^8*c^3*d^10 - 190*a^5*b^8*c^5*d^8 + 362*a^5*b^8*c^7*d^6 - 248*a^5*b^8*c^9*d^4 + 47*a^5*b^8*c^11*d^2 - 12*a^6*b^7*c^2*d^11 + 160*a^6*b^7*c^4*d^9 - 436*a^6*b^7*c^6*d^7 + 412*a^6*b^7*c^8*d^5 - 132*a^6*b^7*c^10*d^3 - 92*a^7*b^6*c^3*d^10 + 362*a^7*b^6*c^5*d^8 - 484*a^7*b^6*c^7*d^6 + 240*a^7*b^6*c^9*d^4 - 28*a^7*b^6*c^11*d^2 + 32*a^8*b^5*c^2*d^11 - 208*a^8*b^5*c^4*d^9 + 412*a^8*b^5*c^6*d^7 - 292*a^8*b^5*c^8*d^5 + 56*a^8*b^5*c^10*d^3 + 83*a^9*b^4*c^3*d^10 - 248*a^9*b^4*c^5*d^8 + 240*a^9*b^4*c^7*d^6 - 70*a^9*b^4*c^9*d^4 - 22*a^10*b^3*c^2*d^11 + 98*a^10*b^3*c^4*d^9 - 132*a^10*b^3*c^6*d^7 + 56*a^10*b^3*c^8*d^5 - 22*a^11*b^2*c^3*d^10 + 47*a^11*b^2*c^5*d^8 - 28*a^11*b^2*c^7*d^6))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) + (b^2*((32*(a^2*b^13*c^15 - 2*a^4*b^11*c^15 + a^6*b^9*c^15 + a^15*c^2*d^13 - 2*a^15*c^4*d^11 + a^15*c^6*d^9 - a*b^14*c^10*d^5 + 2*a*b^14*c^12*d^3 - 5*a^3*b^12*c^14*d + 13*a^5*b^10*c^14*d - 7*a^7*b^8*c^14*d - a^10*b^5*c*d^14 + 2*a^12*b^3*c*d^14 - 5*a^14*b*c^3*d^12 + 13*a^14*b*c^5*d^10 - 7*a^14*b*c^7*d^8 + 7*a^2*b^13*c^9*d^6 - 13*a^2*b^13*c^11*d^4 + 5*a^2*b^13*c^13*d^2 - 20*a^3*b^12*c^8*d^7 + 35*a^3*b^12*c^10*d^5 - 10*a^3*b^12*c^12*d^3 + 28*a^4*b^11*c^7*d^8 - 50*a^4*b^11*c^9*d^6 + 14*a^4*b^11*c^11*d^4 + 10*a^4*b^11*c^13*d^2 - 14*a^5*b^10*c^6*d^9 + 40*a^5*b^10*c^8*d^7 - 25*a^5*b^10*c^10*d^5 - 14*a^5*b^10*c^12*d^3 - 14*a^6*b^9*c^5*d^10 - 14*a^6*b^9*c^7*d^8 + 37*a^6*b^9*c^9*d^6 + 25*a^6*b^9*c^11*d^4 - 35*a^6*b^9*c^13*d^2 + 28*a^7*b^8*c^4*d^11 - 14*a^7*b^8*c^6*d^9 - 20*a^7*b^8*c^8*d^7 - 37*a^7*b^8*c^10*d^5 + 50*a^7*b^8*c^12*d^3 - 20*a^8*b^7*c^3*d^12 + 40*a^8*b^7*c^5*d^10 - 20*a^8*b^7*c^7*d^8 + 20*a^8*b^7*c^9*d^6 - 40*a^8*b^7*c^11*d^4 + 20*a^8*b^7*c^13*d^2 + 7*a^9*b^6*c^2*d^13 - 50*a^9*b^6*c^4*d^11 + 37*a^9*b^6*c^6*d^9 + 20*a^9*b^6*c^8*d^7 + 14*a^9*b^6*c^10*d^5 - 28*a^9*b^6*c^12*d^3 + 35*a^10*b^5*c^3*d^12 - 25*a^10*b^5*c^5*d^10 - 37*a^10*b^5*c^7*d^8 + 14*a^10*b^5*c^9*d^6 + 14*a^10*b^5*c^11*d^4 - 13*a^11*b^4*c^2*d^13 + 14*a^11*b^4*c^4*d^11 + 25*a^11*b^4*c^6*d^9 - 40*a^11*b^4*c^8*d^7 + 14*a^11*b^4*c^10*d^5 - 10*a^12*b^3*c^3*d^12 - 14*a^12*b^3*c^5*d^10 + 50*a^12*b^3*c^7*d^8 - 28*a^12*b^3*c^9*d^6 + 5*a^13*b^2*c^2*d^13 + 10*a^13*b^2*c^4*d^11 - 35*a^13*b^2*c^6*d^9 + 20*a^13*b^2*c^8*d^7 - a*b^14*c^14*d - a^14*b*c*d^14))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12 *a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) - (32*tan(e/2 + (f*x)/2)*(8*a^3*b^12*c^15 - 3*a^15*c*d^14 - 3*a*b^14*c^15 - 7*a^5*b^10*c^15 + 2*a^7*b^8*c^15 + 8*a^15*c^3*d^12 - 7*a^15*c^5*d^10 + 2*a^15*c^7*d^8 + 4*a*b^14*c^9*d^6 - 11*a*b^14*c^11*d^4 + 10*a*b^14*c^13*d^2 + 24*a^2*b^13*c^14*d - 64*a^4*b^11*c^14*d + 56*a^6*b^9*c^14*d - 16*a^8*b^7*c^14*d + 4*a^9*b^6*c*d^14 - 11*a^11*b^4*c*d^14 + 10*a^13*b^2*c*d^14 + 24*a^14*b*c^2*d^13 - 64*a^14*b*c^4*d^11 + 56*a^14*b*c^6*d^9 - 16*a^14*b*c^8*d^7 - 32*a^2*b^13*c^8*d^7 + 88*a^2*b^13*c^10*d^5 - 80*a^2*b^13*c^12*d^3 + 112*a^3*b^12*c^7*d^8 - 319*a^3*b^12*c^9*d^6 + 310*a^3*b^12*c^11*d^4 - 111*a^3*b^12*c^13*d^2 - 224*a^4*b^11*c^6*d^9 + 704*a^4*b^11*c^8*d^7 - 800*a^4*b^11*c^10*d^5 + 384*a^4*b^11*c^12*d^3 + 280*a^5*b^10*c^5*d^10 - 1078*a^5*b^10*c^7*d^8 + 1550*a^5*b^10*c^9*d^6 - 993*a^5*b^10*c^11*d^4 + 248*a^5*b^10*c^13*d^2 - 224*a^6*b^9*c^4*d^11 + 1232*a^6*b^9*c^6*d^9 - 2320*a^6*b^9*c^8*d^7 + 1896*a^6*b^9*c^10*d^5 - 640*a^6*b^9*c^12*d^3 + 112*a^7*b^8*c^3*d^12 - 1078*a^7*b^8*c^5*d^10 + 2660*a^7*b^8*c^7*d^8 - 2733*a^7*b^8*c^9*d^6 + 1240*a^7*b^8*c^11*d^4 - 203*a^7*b^8*c^13*d^2 - 32*a^8*b^7*c^2*d^13 + 704*a^8*b^7*c^4*d^11 - 2320*a^8*b^7*c^6*d^9 + 3072*a^8*b^7*c^8*d^7 - 1856*a^8*b^7*c^10*d^5 + 448*a^8*b^7*c^12*d^3 - 319*a^9*b^6*c^3*d^12 + 1550*a^9*b^6*c^5*d^10 - 2733*a^9*b^6*c^7*d^8 + 2128*a^9*b^6*c^9*d^6 - 686*a^9*b^6*c^11*d^4 + 56*a^9*b^6*c^13*d^2 + 88*a^10*b^5*c^2*d^13 - 800*a^10*b^5*c^4*d^11 + 1896*a^10*b^5*c^6*d^9 - 1856*a^10*b^5*c^8*d^7 + 784*a^10*b^5*c^10*d^5 - 112*a^10*b^5*c^12*d^3 + 310*a^11*b^4*c^3*d^12 - 993*a^11*b^4*c^5*d^10 + 1240*a^11*b^4*c^7*d^8 - 686*a^11*b^4*c^9*d^6 + 140*a^11*b^4*c^11*d^4 - 80*a^12*b^3*c^2*d^13 + 384*a^12*b^3*c^4*d^11 - 640*a^12*b^3*c^6*d^9 + 448*a^12*b^3*c^8*d^7 - 112*a^12*b^3*c^10*d^5 - 111*a^13*b^2*c^3*d^12 + 248*a^13*b^2*c^5*d^10 - 203*a^13*b^2*c^7*d^8 + 56*a^13*b^2*c^9*d^6))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9))*(-(a + b)^3*(a - b)^3)^(1/2)*(2*b^2*d - 3*a^2*d + a*b*c))/(a^9*d^3 + b^9*c^3 - 3*a^2*b^7*c^3 + 3*a^4*b^5*c^3 - a^6*b^3*c^3 - a^3*b^6*d^3 + 3*a^5*b^4*d^3 - 3*a^7*b^2*d^3 + 3*a^2*b^7 *c*d^2 + 9*a^3*b^6*c^2*d - 9*a^4*b^5*c*d^2 - 9*a^5*b^4*c^2*d + 9*a^6*b^3*c*d^2 + 3*a^7*b^2*c^2*d - 3*a*b^8*c^2*d - 3*a^8*b*c*d^2))*(2*b^2*d - 3*a^2*d + a*b*c))/(a^9*d^3 + b^9*c^3 - 3*a^2*b^7*c^3 + 3*a^4*b^5*c^3 - a^6*b^3*c^3 - a^3*b^6*d^3 + 3*a^5*b^4*d^3 - 3*a^7*b^2*d^3 + 3*a^2*b^7*c*d^2 + 9*a^3*b^6*c^2*d - 9*a^4*b^5*c*d^2 - 9*a^5*b^4*c^2*d + 9*a^6*b^3*c*d^2 + 3*a^7*b^2*c^2*d - 3*a*b^8*c^2*d - 3*a^8*b*c*d^2))*(2*b^2*d - 3*a^2*d + a*b*c)*1i)/(a^9*d^3 + b^9*c^3 - 3*a^2*b^7*c^3 + 3*a^4*b^5*c^3 - a^6*b^3*c^3 - a^3*b^6*d^3 + 3*a^5*b^4*d^3 - 3*a^7*b^2*d^3 + 3*a^2*b^7*c*d^2 + 9*a^3*b^6*c^2*d - 9*a^4*b^5*c*d^2 - 9*a^5*b^4*c^2*d + 9*a^6*b^3*c*d^2 + 3*a^7*b^2*c^2*d - 3*a*b^8*c^2*d - 3*a^8*b*c*d^2) + (b^2*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(4*a*b^10*c^4*d^7 - 8*a*b^10*c^6*d^5 + 4*a*b^10*c^8*d^3 + a^3*b^8*c^10*d + 4*a^4*b^7*c*d^10 - 8*a^6*b^5*c*d^10 + 4*a^8*b^3*c*d^10 + a^10*b*c^3*d^8 - 4*a^2*b^9*c^3*d^8 + 8*a^2*b^9*c^5*d^6 - 7*a^2*b^9*c^7*d^4 + 4*a^2*b^9*c^9*d^2 - 4*a^3*b^8*c^2*d^9 + 21*a^3*b^8*c^6*d^5 - 22*a^3*b^8*c^8*d^3 - 18*a^4*b^7*c^5*d^6 + 26*a^4*b^7*c^7*d^4 - 8*a^4*b^7*c^9*d^2 + 8*a^5*b^6*c^2*d^9 - 18*a^5*b^6*c^4*d^7 - 8*a^5*b^6*c^6*d^5 + 22*a^5*b^6*c^8*d^3 + 21*a^6*b^5*c^3*d^8 - 8*a^6*b^5*c^5*d^6 - 15*a^6*b^5*c^7*d^4 - 7*a^7*b^4*c^2*d^9 + 26*a^7*b^4*c^4*d^7 - 15*a^7*b^4*c^6*d^5 - 22*a^8*b^3*c^3*d^8 + 22*a^8*b^3*c^5*d^6 + 4*a^9*b^2*c^2*d^9 - 8*a^9*b^2*c^4*d^7))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) - (32*tan(e/2 + (f*x)/2)*(a^3*b^8*c^11 + a^11*c^3*d^8 - 16*a*b^10*c^3*d^8 + 44*a*b^10*c^5*d^6 - 34*a*b^10*c^7*d^4 + 4*a*b^10*c^9*d^2 + 4*a^2*b^9*c^10*d - 16*a^3*b^8*c*d^10 - 8*a^4*b^7*c^10*d + 44*a^5*b^6*c*d^10 - 34*a^7*b^4*c*d^10 + 4*a^9*b^2*c*d^10 + 4*a^10*b*c^2*d^9 - 8*a^10*b*c^4*d^7 + 32*a^2*b^9*c^2*d^9 - 104*a^2*b^9*c^4*d^7 + 100*a^2*b^9*c^6*d^5 - 24*a^2*b^9*c^8*d^3 + 120*a^3*b^8*c^3*d^8 - 222*a^3*b^8*c^5*d^6 + 134*a^3*b^8*c^7*d^4 - 24*a^3*b^8*c^9*d^2 - 104*a^4*b^7*c^2*d^9 + 312*a^4*b^7*c^4*d^7 - 272*a^4*b^7*c^6*d^5 + 60*a^4*b^7*c^8*d^3 - 222*a^5*b^6*c^3*d^8 + 316*a^5*b^6*c^5*d^6 - 136*a^5*b^6*c^7*d^4 + 22*a^5*b^6*c^9*d^2 + 100*a^6*b^5*c^2*d^9 - 272*a^6*b^5*c^4*d^7 + 192*a^6*b^5*c^6*d^5 - 24*a^6*b^5*c^8*d^3 + 134*a^7*b^4*c^3*d^8 - 136*a^7*b^4*c^5*d^6 + 18*a^7*b^4*c^7*d^4 - 24*a^8*b^3*c^2*d^9 + 60*a^8*b^3*c^4*d^7 - 24*a^8*b^3*c^6*d^5 - 24*a^9*b^2*c^3*d^8 + 22*a^9*b^2*c^5*d^6))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^

9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2
- 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6
*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 5
2*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^
2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a
^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d
^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^
9) + (b^2*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(a^3*b^10*c^13 - a^5*b^8*c^13 +
 a^13*c^3*d^10 - a^13*c^5*d^8 + 2*a*b^12*c^7*d^6 - 5*a*b^12*c^9*d^4 + 3*a*b
^12*c^11*d^2 + 2*a^2*b^11*c^12*d - 10*a^4*b^9*c^12*d + 8*a^6*b^7*c^12*d + 2
*a^7*b^6*c*d^12 - 5*a^9*b^4*c*d^12 + 3*a^11*b^2*c*d^12 + 2*a^12*b*c^2*d^11
- 10*a^12*b*c^4*d^9 + 8*a^12*b*c^6*d^7 - 12*a^2*b^11*c^6*d^7 + 32*a^2*b^11*
c^8*d^5 - 22*a^2*b^11*c^10*d^3 + 30*a^3*b^10*c^5*d^8 - 92*a^3*b^10*c^7*d^6
+ 83*a^3*b^10*c^9*d^4 - 22*a^3*b^10*c^11*d^2 - 40*a^4*b^9*c^4*d^9 + 160*a^4
*b^9*c^6*d^7 - 208*a^4*b^9*c^8*d^5 + 98*a^4*b^9*c^10*d^3 + 30*a^5*b^8*c^3*d
^10 - 190*a^5*b^8*c^5*d^8 + 362*a^5*b^8*c^7*d^6 - 248*a^5*b^8*c^9*d^4 + 47*
a^5*b^8*c^11*d^2 - 12*a^6*b^7*c^2*d^11 + 160*a^6*b^7*c^4*d^9 - 436*a^6*b^7*
c^6*d^7 + 412*a^6*b^7*c^8*d^5 - 132*a^6*b^7*c^10*d^3 - 92*a^7*b^6*c^3*d^10
+ 362*a^7*b^6*c^5*d^8 - 484*a^7*b^6*c^7*d^6 + 240*a^7*b^6*c^9*d^4 - 28*a^7*
b^6*c^11*d^2 + 32*a^8*b^5*c^2*d^11 - 208*a^8*b^5*c^4*d^9 + 412*a^8*b^5*c^6*
d^7 - 292*a^8*b^5*c^8*d^5 + 56*a^8*b^5*c^10*d^3 + 83*a^9*b^4*c^3*d^10 - 248
*a^9*b^4*c^5*d^8 + 240*a^9*b^4*c^7*d^6 - 70*a^9*b^4*c^9*d^4 - 22*a^10*b^3*c
^2*d^11 + 98*a^10*b^3*c^4*d^9 - 132*a^10*b^3*c^6*d^7 + 56*a^10*b^3*c^8*d^5
- 22*a^11*b^2*c^3*d^10 + 47*a^11*b^2*c^5*d^8 - 28*a^11*b^2*c^7*d^6))/(a^10*
d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2
*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a
*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^
5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^
2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^
7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b
^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 -
 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*
c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52
*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4
*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) - (32*tan(e/2 +
(f*x)/2)*(2*a^4*b^9*c^13 - 2*a^2*b^11*c^13 - 2*a^13*c^2*d^11 + 2*a^13*c^4*d
^9 - 2*a*b^12*c^8*d^5 + 6*a*b^12*c^10*d^3 + 20*a^3*b^10*c^12*d - 16*a^5*b^8
*c^12*d - 2*a^8*b^5*c*d^12 + 6*a^10*b^3*c*d^12 + 20*a^12*b*c^3*d^10 - 16*a^
12*b*c^5*d^8 + 10*a^2*b^11*c^7*d^6 - 34*a^2*b^11*c^9*d^4 + 26*a^2*b^11*c^11
*d^2 - 18*a^3*b^10*c^6*d^7 + 80*a^3*b^10*c^8*d^5 - 82*a^3*b^10*c^10*d^3 + 1
0*a^4*b^9*c^5*d^8 - 96*a^4*b^9*c^7*d^6 + 160*a^4*b^9*c^9*d^4 - 76*a^4*b^9*c
^11*d^2 + 10*a^5*b^8*c^4*d^9 + 44*a^5*b^8*c^6*d^7 - 188*a^5*b^8*c^8*d^5 + 1
50*a^5*b^8*c^10*d^3 - 18*a^6*b^7*c^3*d^10 + 44*a^6*b^7*c^5*d^8 + 88*a^6*b^7
*c^7*d^6 - 164*a^6*b^7*c^9*d^4 + 50*a^6*b^7*c^11*d^2 + 10*a^7*b^6*c^2*d^11
- 96*a^7*b^6*c^4*d^9 + 88*a^7*b^6*c^6*d^7 + 72*a^7*b^6*c^8*d^5 - 74*a^7*b^6

*c^10*d^3 + 80*a^8*b^5*c^3*d^10 - 188*a^8*b^5*c^5*d^8 + 72*a^8*b^5*c^7*d^6 + 38*a^8*b^5*c^9*d^4 - 34*a^9*b^4*c^2*d^11 + 160*a^9*b^4*c^4*d^9 - 164*a^9*b^4*c^6*d^7 + 38*a^9*b^4*c^8*d^5 - 82*a^10*b^3*c^3*d^10 + 150*a^10*b^3*c^5*d^8 - 74*a^10*b^3*c^7*d^6 + 26*a^11*b^2*c^2*d^11 - 76*a^11*b^2*c^4*d^9 + 50*a^11*b^2*c^6*d^7 - 4*a*b^12*c^12*d - 4*a^12*b*c*d^12))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) + (b^2*((32*(a^2*b^13*c^15 - 2*a^4*b^11*c^15 + a^6*b^9*c^15 + a^15*c^2*d^13 - 2*a^15*c^4*d^11 + a^15*c^6*d^9 - a*b^14*c^10*d^5 + 2*a*b^14*c^12*d^3 - 5*a^3*b^12*c^14*d + 13*a^5*b^10*c^14*d - 7*a^7*b^8*c^14*d - a^10*b^5*c*d^14 + 2*a^12*b^3*c*d^14 - 5*a^14*b*c^3*d^12 + 13*a^14*b*c^5*d^10 - 7*a^14*b*c^7*d^8 + 7*a^2*b^13*c^9*d^6 - 13*a^2*b^13*c^11*d^4 + 5*a^2*b^13*c^13*d^2 - 20*a^3*b^12*c^8*d^7 + 35*a^3*b^12*c^10*d^5 - 10*a^3*b^12*c^12*d^3 + 28*a^4*b^11*c^7*d^8 - 50*a^4*b^11*c^9*d^6 + 14*a^4*b^11*c^11*d^4 + 10*a^4*b^11*c^13*d^2 - 14*a^5*b^10*c^6*d^9 + 40*a^5*b^10*c^8*d^7 - 25*a^5*b^10*c^10*d^5 - 14*a^5*b^10*c^12*d^3 - 14*a^6*b^9*c^5*d^10 - 14*a^6*b^9*c^7*d^8 + 37*a^6*b^9*c^9*d^6 + 25*a^6*b^9*c^11*d^4 - 35*a^6*b^9*c^13*d^2 + 28*a^7*b^8*c^4*d^11 - 14*a^7*b^8*c^6*d^9 - 20*a^7*b^8*c^8*d^7 - 37*a^7*b^8*c^10*d^5 + 50*a^7*b^8*c^12*d^3 - 20*a^8*b^7*c^3*d^12 + 40*a^8*b^7*c^5*d^10 - 20*a^8*b^7*c^7*d^8 + 20*a^8*b^7*c^9*d^6 - 40*a^8*b^7*c^11*d^4 + 20*a^8*b^7*c^13*d^2 + 7*a^9*b^6*c^2*d^13 - 50*a^9*b^6*c^4*d^11 + 37*a^9*b^6*c^6*d^9 + 20*a^9*b^6*c^8*d^7 + 14*a^9*b^6*c^10*d^5 - 28*a^9*b^6*c^12*d^3 + 35*a^10*b^5*c^3*d^12 - 25*a^10*b^5*c^5*d^10 - 37*a^10*b^5*c^7*d^8 + 14*a^10*b^5*c^9*d^6 + 14*a^10*b^5*c^11*d^4 - 13*a^11*b^4*c^2*d^13 + 14*a^11*b^4*c^4*d^11 + 25*a^11*b^4*c^6*d^9 - 40*a^11*b^4*c^8*d^7 + 14*a^11*b^4*c^10*d^5 - 10*a^12*b^3*c^3*d^12 - 14*a^12*b^3*c^5*d^10 + 50*a^12*b^3*c^7*d^8 - 28*a^12*b^3*c^9*d^6 + 5*a^13*b^2*c^2*d^13 + 10*a^13*b^2*c^4*d^11 - 35*a^13*b^2*c^6*d^9 + 20*a^13*b^2*c^8*d^7 - a*b^14*c^14*d - a^14*b*c*d^14))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^

3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a
^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) - (32*
tan(e/2 + (f*x)/2)*(8*a^3*b^12*c^15 - 3*a^15*c*d^14 - 3*a*b^14*c^15 - 7*a^5
*b^10*c^15 + 2*a^7*b^8*c^15 + 8*a^15*c^3*d^12 - 7*a^15*c^5*d^10 + 2*a^15*c^
7*d^8 + 4*a*b^14*c^9*d^6 - 11*a*b^14*c^11*d^4 + 10*a*b^14*c^13*d^2 + 24*a^2
*b^13*c^14*d - 64*a^4*b^11*c^14*d + 56*a^6*b^9*c^14*d - 16*a^8*b^7*c^14*d +
4*a^9*b^6*c*d^14 - 11*a^11*b^4*c*d^14 + 10*a^13*b^2*c*d^14 + 24*a^14*b*c^2
*d^13 - 64*a^14*b*c^4*d^11 + 56*a^14*b*c^6*d^9 - 16*a^14*b*c^8*d^7 - 32*a^2
*b^13*c^8*d^7 + 88*a^2*b^13*c^10*d^5 - 80*a^2*b^13*c^12*d^3 + 112*a^3*b^12*
c^7*d^8 - 319*a^3*b^12*c^9*d^6 + 310*a^3*b^12*c^11*d^4 - 111*a^3*b^12*c^13*
d^2 - 224*a^4*b^11*c^6*d^9 + 704*a^4*b^11*c^8*d^7 - 800*a^4*b^11*c^10*d^5 +
384*a^4*b^11*c^12*d^3 + 280*a^5*b^10*c^5*d^10 - 1078*a^5*b^10*c^7*d^8 + 15
50*a^5*b^10*c^9*d^6 - 993*a^5*b^10*c^11*d^4 + 248*a^5*b^10*c^13*d^2 - 224*a
^6*b^9*c^4*d^11 + 1232*a^6*b^9*c^6*d^9 - 2320*a^6*b^9*c^8*d^7 + 1896*a^6*b^
9*c^10*d^5 - 640*a^6*b^9*c^12*d^3 + 112*a^7*b^8*c^3*d^12 - 1078*a^7*b^8*c^5
*d^10 + 2660*a^7*b^8*c^7*d^8 - 2733*a^7*b^8*c^9*d^6 + 1240*a^7*b^8*c^11*d^4
- 203*a^7*b^8*c^13*d^2 - 32*a^8*b^7*c^2*d^13 + 704*a^8*b^7*c^4*d^11 - 2320
*a^8*b^7*c^6*d^9 + 3072*a^8*b^7*c^8*d^7 - 1856*a^8*b^7*c^10*d^5 + 448*a^8*b
^7*c^12*d^3 - 319*a^9*b^6*c^3*d^12 + 1550*a^9*b^6*c^5*d^10 - 2733*a^9*b^6*c
^7*d^8 + 2128*a^9*b^6*c^9*d^6 - 686*a^9*b^6*c^11*d^4 + 56*a^9*b^6*c^13*d^2
+ 88*a^10*b^5*c^2*d^13 - 800*a^10*b^5*c^4*d^11 + 1896*a^10*b^5*c^6*d^9 - 18
56*a^10*b^5*c^8*d^7 + 784*a^10*b^5*c^10*d^5 - 112*a^10*b^5*c^12*d^3 + 310*a
^11*b^4*c^3*d^12 - 993*a^11*b^4*c^5*d^10 + 1240*a^11*b^4*c^7*d^8 - 686*a^11
*b^4*c^9*d^6 + 140*a^11*b^4*c^11*d^4 - 80*a^12*b^3*c^2*d^13 + 384*a^12*b^3*
c^4*d^11 - 640*a^12*b^3*c^6*d^9 + 448*a^12*b^3*c^8*d^7 - 112*a^12*b^3*c^10*
d^5 - 111*a^13*b^2*c^3*d^12 + 248*a^13*b^2*c^5*d^10 - 203*a^13*b^2*c^7*d^8
+ 56*a^13*b^2*c^9*d^6))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c
^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*
c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*
c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3
*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b
^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 +
15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*
c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32
*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8
*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^
8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6
*a^9*b*c*d^9))*(-(a + b)^3*(a - b)^3)^(1/2)*(2*b^2*d - 3*a^2*d + a*b*c))/(a
^9*d^3 + b^9*c^3 - 3*a^2*b^7*c^3 + 3*a^4*b^5*c^3 - a^6*b^3*c^3 - a^3*b^6*d^
3 + 3*a^5*b^4*d^3 - 3*a^7*b^2*d^3 + 3*a^2*b^7*c*d^2 + 9*a^3*b^6*c^2*d - 9*a
^4*b^5*c*d^2 - 9*a^5*b^4*c^2*d + 9*a^6*b^3*c*d^2 + 3*a^7*b^2*c^2*d - 3*a*b^
8*c^2*d - 3*a^8*b*c*d^2))*(2*b^2*d - 3*a^2*d + a*b*c))/(a^9*d^3 + b^9*c^3 -
3*a^2*b^7*c^3 + 3*a^4*b^5*c^3 - a^6*b^3*c^3 - a^3*b^6*d^3 + 3*a^5*b^4*d^3
- 3*a^7*b^2*d^3 + 3*a^2*b^7*c*d^2 + 9*a^3*b^6*c^2*d - 9*a^4*b^5*c*d^2 - 9*a
^5*b^4*c^2*d + 9*a^6*b^3*c*d^2 + 3*a^7*b^2*c^2*d - 3*a*b^8*c^2*d - 3*a^8*b*

c*d^2))*(2*b^2*d - 3*a^2*d + a*b*c)*1i)/(a^9*d^3 + b^9*c^3 - 3*a^2*b^7*c^3 + 3*a^4*b^5*c^3 - a^6*b^3*c^3 - a^3*b^6*d^3 + 3*a^5*b^4*d^3 - 3*a^7*b^2*d^3 + 3*a^2*b^7*c*d^2 + 9*a^3*b^6*c^2*d - 9*a^4*b^5*c*d^2 - 9*a^5*b^4*c^2*d + 9*a^6*b^3*c*d^2 + 3*a^7*b^2*c^2*d - 3*a*b^8*c^2*d - 3*a^8*b*c*d^2))/((64*(12*a*b^8*c^5*d^4 - 20*a*b^8*c^3*d^6 - 20*a^3*b^6*c*d^8 + 12*a^5*b^4*c*d^8 + 16*a^2*b^7*c^2*d^7 - 30*a^2*b^7*c^4*d^5 + 12*a^2*b^7*c^6*d^3 + 60*a^3*b^6*c^3*d^6 - 42*a^3*b^6*c^5*d^4 + 3*a^3*b^6*c^7*d^2 - 30*a^4*b^5*c^2*d^7 + 52*a^4*b^5*c^4*d^5 - 16*a^4*b^5*c^6*d^3 - 42*a^5*b^4*c^3*d^6 + 26*a^5*b^4*c^5*d^4 + 12*a^6*b^3*c^2*d^7 - 16*a^6*b^3*c^4*d^5 + 3*a^7*b^2*c^3*d^6 + 8*a*b^8*c*d^8))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) + (64*tan(e/2 + (f*x)/2)*(8*a*b^8*c^2*d^7 - 12*a*b^8*c^4*d^5 + 8*a^2*b^7*c*d^8 - 12*a^4*b^5*c*d^8 - 12*a^2*b^7*c^3*d^6 + 6*a^2*b^7*c^5*d^4 - 12*a^3*b^6*c^2*d^7 + 18*a^3*b^6*c^4*d^5 + 6*a^3*b^6*c^6*d^3 + 18*a^4*b^5*c^3*d^6 - 14*a^4*b^5*c^5*d^4 + 6*a^5*b^4*c^2*d^7 - 14*a^5*b^4*c^4*d^5 + 6*a^6*b^3*c^3*d^6))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) + (b^2*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(4*a*b^10*c^4*d^7 - 8*a*b^10*c^6*d^5 + 4*a*b^10*c^8*d^3 + a^3*b^8*c^10*d + 4*a^4*b^7*c*d^10 - 8*a^6*b^5*c*d^10 + 4*a^8*b^3*c*d^10 + a^10*b*c^3*d^8 - 4*a^2*b^9*c^3*d^8 + 8*a^2*b^9*c^5*d^6 - 7*a^2*b^9*c^7*d^4 + 4*a^2*b^9*c^9*d^2 - 4*a^3*b^8*c^2*d^9 + 21*a^3*b^8*c^6*d^5 - 22*a^3*b^8*c^8*d^3 - 18*a^4*b^7*c^5*d^6 + 26*a^4*b^7*c^7*d^4 - 8*a^4*b^7*c^9*d^2 + 8*a^5*b^6*c^2*d^9 - 18*a^5*b^6*c^4*d^7 - 8*a^5*b^6*c^6*d^5 + 22*a^5*b^6*c^8*d^3 + 21*a^6*b^5*c^3*d^8 - 8*a^6*b^5*c^5*d^6 - 15*a^6*b^5*c^7*d^4 - 7*a^7*b^4*c^2*d^9 + 26*a^7*b^4*c^4*d^7 - 15*a^7*b^4*c^6*d^5 - 22*a^8*b^3*c^3*d^8 + 22*a^8*b^3*c^5*d^6 + 4*a^9*b^2*c^2*d^9 - 8*a^9*b^2*c^4*d^7))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2* a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) - (32*tan(e/2 + (f*x)/2)*(a^3*b^8*c^11 + a^11*c^3*d^8 - 16*a*b^10*c^3*d^8 + 44*a*b^10*c^5*d^6 - 34*a*b^10*c^7*d^4 + 4*a*b^10*c^9*d^2 + 4*a^2*b^9*c^10*d - 16*a^3*b^8*c*d^10 - 8*a^4*b^7*c^10*d + 44*a^5*b^6*c*d^10 - 34*a^7*b^4*c*d^10 + 4*a^9*b^2*c*d^10 + 4*a^10*b*c^2*d^9 - 8*a^10*b*c^4*d^7 + 32*a^2*b^9*c^2*d^9 - 104*a^2*b^9*c^4*d^7 + 100*a^2*b^9*c^6*d^5 - 24*a^2*b^9*c^8*d^3 + 120*a^3*b^8*c^3*d^8 - 222*a^3*b^8*c^5*d^6 + 134*a^3*b^8*c^7*d^4 - 24*a^3*b^8*c^9*d^2 - 104*a^4*b^7*c^2*d^9 + 312*a^4*b^7*c^4*d^7 - 272*a^4*b^7*c^6*d^5 + 60*a^4*b^7*c^8*d^3 - 222*a^5*b^6*c^3*d^8 + 316*a^5*b^6*c^5*d^6 - 136*a^5*b^6*c^7*d^4 + 22*a^5*b^6*c^9*d^2 + 100*a^6*b^5*c^2*d^9 - 272*a^6*b^5*c^4*d^7 + 192*a^6*b^5*c^6*d^5 - 24*a^6*b^5*c^8*d^3 + 134*a^7*b^4*c^3*d^8 - 136*a^7*b^4*c^5*d^6 + 18*a^7*b^4*c^7*d^4 - 24*a^8*b^3*c^2*d^9 + 60*a^8*b^3*c^4*d^7 - 24*a^8*b^3*c^6*d^5 - 24*a^9*b^2*c^3*d^8 + 22*a^9*b^2*c^5*d^6))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) + (b^2*(-(a + b)^3*(a - b)^3)^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*a^4*b^9*c^13 - 2*a^2*b^11*c^13 - 2*a^13*c^2*d^11 + 2*a^13*c^4*d^9 - 2*a*b^12*c^8*d^5 + 6*a*b^12*c^10*d^3 + 20*a^3*b^10*c^12*d - 16*a^5*b^8*c^12*d - 2*a^8*b^5*c*d^12 + 6*a^10*b^3*c*d^12 + 20*a^12*b*c^3*d^10 - 16*a^12*b*c^5*d^8 + 10*a^2*b^11*c^7*d^6 - 34*a^2*b^11*c^9*d^4 + 26*a^2*b^11*c^11*d^2 - 18*a^3*b^10*c^6*d^7 + 80*a^3*b^10*c^8*d^5 - 82*a^3*b^10*c^10*d^3 + 10*a^4*b^9*c^5*d^8 - 96*a^4*b^9*c^7*d^6 + 160*a^4*b^9*c^9*d^4 - 76*a^4*b^9*c^11*d^2 + 10*a^5*b^8*c^4*d^9 + 44*a^5*b^8*c^6*d^7 - 188*a^5*b^8*c^8*d^5 + 150*a^5*b^8*c^10*d^3 - 18*a^6*b^7*c^3*d^10 + 44*a^6*b^7*c^5*d^8 + 88*a^6*b^7*c^7*d^6 - 164*a^6*b^7*c^9*d^4 + 50*a^6*b^7*c^11*d^2 + 10*a^7*b^6*c^2*d^11 - 96*a^7*b^6*c^4*d^9 + 88*a^7*b^6*c^6*d^7 + 72*a^7*b^6*c^8*d^5 - 74*a^7*b^6*c^10*d^3 + 80*a^8*b^5*c^3*d^10 - 188*a^8*b^5*c^5*d^8 + 72*a^8*b^5*c^7*d^6 + 38*a^8*b^5*c^9*d^4 - 34*a^9*b^4*c^2*d^11 + 160*

a^9*b^4*c^4*d^9 - 164*a^9*b^4*c^6*d^7 + 38*a^9*b^4*c^8*d^5 - 82*a^10*b^3*c^
3*d^10 + 150*a^10*b^3*c^5*d^8 - 74*a^10*b^3*c^7*d^6 + 26*a^11*b^2*c^2*d^11
- 76*a^11*b^2*c^4*d^9 + 50*a^11*b^2*c^6*d^7 - 4*a*b^12*c^12*d - 4*a^12*b*c*
d^12))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^1
0 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*
c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5
*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^
5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a
^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d
^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*
b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8
+ 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3
*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 3
2*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) - (
32*(a^3*b^10*c^13 - a^5*b^8*c^13 + a^13*c^3*d^10 - a^13*c^5*d^8 + 2*a*b^12*
c^7*d^6 - 5*a*b^12*c^9*d^4 + 3*a*b^12*c^11*d^2 + 2*a^2*b^11*c^12*d - 10*a^4
*b^9*c^12*d + 8*a^6*b^7*c^12*d + 2*a^7*b^6*c*d^12 - 5*a^9*b^4*c*d^12 + 3*a^
11*b^2*c*d^12 + 2*a^12*b*c^2*d^11 - 10*a^12*b*c^4*d^9 + 8*a^12*b*c^6*d^7 -
12*a^2*b^11*c^6*d^7 + 32*a^2*b^11*c^8*d^5 - 22*a^2*b^11*c^10*d^3 + 30*a^3*b
^10*c^5*d^8 - 92*a^3*b^10*c^7*d^6 + 83*a^3*b^10*c^9*d^4 - 22*a^3*b^10*c^11*
d^2 - 40*a^4*b^9*c^4*d^9 + 160*a^4*b^9*c^6*d^7 - 208*a^4*b^9*c^8*d^5 + 98*a
^4*b^9*c^10*d^3 + 30*a^5*b^8*c^3*d^10 - 190*a^5*b^8*c^5*d^8 + 362*a^5*b^8*c
^7*d^6 - 248*a^5*b^8*c^9*d^4 + 47*a^5*b^8*c^11*d^2 - 12*a^6*b^7*c^2*d^11 +
160*a^6*b^7*c^4*d^9 - 436*a^6*b^7*c^6*d^7 + 412*a^6*b^7*c^8*d^5 - 132*a^6*b
^7*c^10*d^3 - 92*a^7*b^6*c^3*d^10 + 362*a^7*b^6*c^5*d^8 - 484*a^7*b^6*c^7*d
^6 + 240*a^7*b^6*c^9*d^4 - 28*a^7*b^6*c^11*d^2 + 32*a^8*b^5*c^2*d^11 - 208*
a^8*b^5*c^4*d^9 + 412*a^8*b^5*c^6*d^7 - 292*a^8*b^5*c^8*d^5 + 56*a^8*b^5*c^
10*d^3 + 83*a^9*b^4*c^3*d^10 - 248*a^9*b^4*c^5*d^8 + 240*a^9*b^4*c^7*d^6 -
70*a^9*b^4*c^9*d^4 - 22*a^10*b^3*c^2*d^11 + 98*a^10*b^3*c^4*d^9 - 132*a^10*
b^3*c^6*d^7 + 56*a^10*b^3*c^8*d^5 - 22*a^11*b^2*c^3*d^10 + 47*a^11*b^2*c^5*
d^8 - 28*a^11*b^2*c^7*d^6))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b
^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b
^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*
b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b
*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a
^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d
^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*
b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3
- 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4
*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 1
9*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d
- 6*a^9*b*c*d^9) + (b^2*((32*(a^2*b^13*c^15 - 2*a^4*b^11*c^15 + a^6*b^9*c^
15 + a^15*c^2*d^13 - 2*a^15*c^4*d^11 + a^15*c^6*d^9 - a*b^14*c^10*d^5 + 2*a
*b^14*c^12*d^3 - 5*a^3*b^12*c^14*d + 13*a^5*b^10*c^14*d - 7*a^7*b^8*c^14*d
- a^10*b^5*c*d^14 + 2*a^12*b^3*c*d^14 - 5*a^14*b*c^3*d^12 + 13*a^14*b*c^5*d

^10 - 7*a^14*b*c^7*d^8 + 7*a^2*b^13*c^9*d^6 - 13*a^2*b^13*c^11*d^4 + 5*a^2*b^13*c^13*d^2 - 20*a^3*b^12*c^8*d^7 + 35*a^3*b^12*c^10*d^5 - 10*a^3*b^12*c^12*d^3 + 28*a^4*b^11*c^7*d^8 - 50*a^4*b^11*c^9*d^6 + 14*a^4*b^11*c^11*d^4 + 10*a^4*b^11*c^13*d^2 - 14*a^5*b^10*c^6*d^9 + 40*a^5*b^10*c^8*d^7 - 25*a^5*b^10*c^10*d^5 - 14*a^5*b^10*c^12*d^3 - 14*a^6*b^9*c^5*d^10 - 14*a^6*b^9*c^7*d^8 + 37*a^6*b^9*c^9*d^6 + 25*a^6*b^9*c^11*d^4 - 35*a^6*b^9*c^13*d^2 + 28*a^7*b^8*c^4*d^11 - 14*a^7*b^8*c^6*d^9 - 20*a^7*b^8*c^8*d^7 - 37*a^7*b^8*c^10*d^5 + 50*a^7*b^8*c^12*d^3 - 20*a^8*b^7*c^3*d^12 + 40*a^8*b^7*c^5*d^10 - 20*a^8*b^7*c^7*d^8 + 20*a^8*b^7*c^9*d^6 - 40*a^8*b^7*c^11*d^4 + 20*a^8*b^7*c^13*d^2 + 7*a^9*b^6*c^2*d^13 - 50*a^9*b^6*c^4*d^11 + 37*a^9*b^6*c^6*d^9 + 20*a^9*b^6*c^8*d^7 + 14*a^9*b^6*c^10*d^5 - 28*a^9*b^6*c^12*d^3 + 35*a^10*b^5*c^3*d^12 - 25*a^10*b^5*c^5*d^10 - 37*a^10*b^5*c^7*d^8 + 14*a^10*b^5*c^9*d^6 + 14*a^10*b^5*c^11*d^4 - 13*a^11*b^4*c^2*d^13 + 14*a^11*b^4*c^4*d^11 + 25*a^11*b^4*c^6*d^9 - 40*a^11*b^4*c^8*d^7 + 14*a^11*b^4*c^10*d^5 - 10*a^12*b^3*c^3*d^12 - 14*a^12*b^3*c^5*d^10 + 50*a^12*b^3*c^7*d^8 - 28*a^12*b^3*c^9*d^6 + 5*a^13*b^2*c^2*d^13 + 10*a^13*b^2*c^4*d^11 - 35*a^13*b^2*c^6*d^9 + 20*a^13*b^2*c^8*d^7 - a*b^14*c^14*d - a^14*b*c*d^14))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) - (32*tan(e/2 + (f*x)/2)*(8*a^3*b^12*c^15 - 3*a^15*c*d^14 - 3*a*b^14*c^15 - 7*a^5*b^10*c^15 + 2*a^7*b^8*c^15 + 8*a^15*c^3*d^12 - 7*a^15*c^5*d^10 + 2*a^15*c^7*d^8 + 4*a*b^14*c^9*d^6 - 11*a*b^14*c^11*d^4 + 10*a*b^14*c^13*d^2 + 24*a^2*b^13*c^14*d - 64*a^4*b^11*c^14*d + 56*a^6*b^9*c^14*d - 16*a^8*b^7*c^14*d + 4*a^9*b^6*c*d^14 - 11*a^11*b^4*c*d^14 + 10*a^13*b^2*c*d^14 + 24*a^14*b*c^2*d^13 - 64*a^14*b*c^4*d^11 + 56*a^14*b*c^6*d^9 - 16*a^14*b*c^8*d^7 - 32*a^2*b^13*c^8*d^7 + 88*a^2*b^13*c^10*d^5 - 80*a^2*b^13*c^12*d^3 + 112*a^3*b^12*c^7*d^8 - 319*a^3*b^12*c^9*d^6 + 310*a^3*b^12*c^11*d^4 - 111*a^3*b^12*c^13*d^2 - 224*a^4*b^11*c^6*d^9 + 704*a^4*b^11*c^8*d^7 - 800*a^4*b^11*c^10*d^5 + 384*a^4*b^11*c^12*d^3 + 280*a^5*b^10*c^5*d^10 - 1078*a^5*b^10*c^7*d^8 + 1550*a^5*b^10*c^9*d^6 - 993*a^5*b^10*c^11*d^4 + 248*a^5*b^10*c^13*d^2 - 224*a^6*b^9*c^4*d^11 + 1232*a^6*b^9*c^6*d^9 - 2320*a^6*b^9*c^8*d^7 + 1896*a^6*b^9*c^10*d^5 - 640*a^6*b^9*c^12*d^3 + 112*a^7*b^8*c^3*d^12 - 1078*a^7*b^8*c^5*d^10 + 2660*a^7*b^8*c^7*d^8 - 2733*a^7*b^8*c^9*d^6 + 1240*a^7*b^8*c^11*d^4 - 203*a^7*b^8*c^13*d^2 - 32*a^8*b^7*c^2*d^13 + 704*a^8*b^7*c^4*d^11 - 2320*a^8*b^7*c^6*d^9 + 3072*a^8*b^7*c^8*d^7 - 1856*a^8*b^7*c^10*d^5 + 448*a^8*b^7*c^12*d^3 - 319*a^9*b^6*c^3*d^12 + 1550*a^9*b^6*c^5*d^10 - 2733*a^9*b^6*c^7*d^8 + 2128*a^9*b^6*c^9*d^

6 - 686*a^9*b^6*c^11*d^4 + 56*a^9*b^6*c^13*d^2 + 88*a^10*b^5*c^2*d^13 - 800*a^10*b^5*c^4*d^11 + 1896*a^10*b^5*c^6*d^9 - 1856*a^10*b^5*c^8*d^7 + 784*a^10*b^5*c^10*d^5 - 112*a^10*b^5*c^12*d^3 + 310*a^11*b^4*c^3*d^12 - 993*a^11*b^4*c^5*d^10 + 1240*a^11*b^4*c^7*d^8 - 686*a^11*b^4*c^9*d^6 + 140*a^11*b^4*c^11*d^4 - 80*a^12*b^3*c^2*d^13 + 384*a^12*b^3*c^4*d^11 - 640*a^12*b^3*c^6*d^9 + 448*a^12*b^3*c^8*d^7 - 112*a^12*b^3*c^10*d^5 - 111*a^13*b^2*c^3*d^12 + 248*a^13*b^2*c^5*d^10 - 203*a^13*b^2*c^7*d^8 + 56*a^13*b^2*c^9*d^6))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9))*(-(a + b)^3*(a - b)^3)^(1/2)*(2*b^2*d - 3*a^2*d + a*b*c))/(a^9*d^3 + b^9*c^3 - 3*a^2*b^7*c^3 + 3*a^4*b^5*c^3 - a^6*b^3*c^3 - a^3*b^6*d^3 + 3*a^5*b^4*d^3 - 3*a^7*b^2*d^3 + 3*a^2*b^7*c*d^2 + 9*a^3*b^6*c^2*d - 9*a^4*b^5*c*d^2 - 9*a^5*b^4*c^2*d + 9*a^6*b^3*c*d^2 + 3*a^7*b^2*c^2*d - 3*a*b^8*c^2*d - 3*a^8*b*c*d^2))*(2*b^2*d - 3*a^2*d + a*b*c))/(a^9*d^3 + b^9*c^3 - 3*a^2*b^7*c^3 + 3*a^4*b^5*c^3 - a^6*b^3*c^3 - a^3*b^6*d^3 + 3*a^5*b^4*d^3 - 3*a^7*b^2*d^3 + 3*a^2*b^7*c*d^2 + 9*a^3*b^6*c^2*d - 9*a^4*b^5*c*d^2 - 9*a^5*b^4*c^2*d + 9*a^6*b^3*c*d^2 + 3*a^7*b^2*c^2*d - 3*a*b^8*c^2*d - 3*a^8*b*c*d^2))*(2*b^2*d - 3*a^2*d + a*b*c))/(a^9*d^3 + b^9*c^3 - 3*a^2*b^7*c^3 + 3*a^4*b^5*c^3 - a^6*b^3*c^3 - a^3*b^6*d^3 + 3*a^5*b^4*d^3 - 3*a^7*b^2*d^3 + 3*a^2*b^7*c*d^2 + 9*a^3*b^6*c^2*d - 9*a^4*b^5*c*d^2 - 9*a^5*b^4*c^2*d + 9*a^6*b^3*c*d^2 + 3*a^7*b^2*c^2*d - 3*a*b^8*c^2*d - 3*a^8*b*c*d^2) - (b^2*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(4*a*b^10*c^4*d^7 - 8*a*b^10*c^6*d^5 + 4*a*b^10*c^8*d^3 + a^3*b^8*c^10*d + 4*a^4*b^7*c*d^10 - 8*a^6*b^5*c*d^10 + 4*a^8*b^3*c*d^10 + a^10*b*c^3*d^8 - 4*a^2*b^9*c^3*d^8 + 8*a^2*b^9*c^5*d^6 - 7*a^2*b^9*c^7*d^4 + 4*a^2*b^9*c^9*d^2 - 4*a^3*b^8*c^2*d^9 + 21*a^3*b^8*c^6*d^5 - 22*a^3*b^8*c^8*d^3 - 18*a^4*b^7*c^5*d^6 + 26*a^4*b^7*c^7*d^4 - 8*a^4*b^7*c^9*d^2 + 8*a^5*b^6*c^2*d^9 - 18*a^5*b^6*c^4*d^7 - 8*a^5*b^6*c^6*d^5 + 22*a^5*b^6*c^8*d^3 + 21*a^6*b^5*c^3*d^8 - 8*a^6*b^5*c^5*d^6 - 15*a^6*b^5*c^7*d^4 - 7*a^7*b^4*c^2*d^9 + 26*a^7*b^4*c^4*d^7 - 15*a^7*b^4*c^6*d^5 - 22*a^8*b^3*c^3*d^8 + 22*a^8*b^3*c^5*d^6 + 4*a^9*b^2*c^2*d^9 - 8*a^9*b^2*c^4*d^7))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^

6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) - (32*tan(e/2 + (f*x)/2)*(a^3*b^8*c^11 + a^11*c^3*d^8 - 16*a*b^10*c^3*d^8 + 44*a*b^10*c^5*d^6 - 34*a*b^10*c^7*d^4 + 4*a*b^10*c^9*d^2 + 4*a^2*b^9*c^10*d - 16*a^3*b^8*c*d^10 - 8*a^4*b^7*c^10*d + 44*a^5*b^6*c*d^10 - 34*a^7*b^4*c*d^10 + 4*a^9*b^2*c*d^10 + 4*a^10*b*c^2*d^9 - 8*a^10*b*c^4*d^7 + 32*a^2*b^9*c^2*d^9 - 104*a^2*b^9*c^4*d^7 + 100*a^2*b^9*c^6*d^5 - 24*a^2*b^9*c^8*d^3 + 120*a^3*b^8*c^3*d^8 - 222*a^3*b^8*c^5*d^6 + 134*a^3*b^8*c^7*d^4 - 24*a^3*b^8*c^9*d^2 - 104*a^4*b^7*c^2*d^9 + 312*a^4*b^7*c^4*d^7 - 272*a^4*b^7*c^6*d^5 + 60*a^4*b^7*c^8*d^3 - 222*a^5*b^6*c^3*d^8 + 316*a^5*b^6*c^5*d^6 - 136*a^5*b^6*c^7*d^4 + 22*a^5*b^6*c^9*d^2 + 100*a^6*b^5*c^2*d^9 - 272*a^6*b^5*c^4*d^7 + 192*a^6*b^5*c^6*d^5 - 24*a^6*b^5*c^8*d^3 + 134*a^7*b^4*c^3*d^8 - 136*a^7*b^4*c^5*d^6 + 18*a^7*b^4*c^7*d^4 - 24*a^8*b^3*c^2*d^9 + 60*a^8*b^3*c^4*d^7 - 24*a^8*b^3*c^6*d^5 - 24*a^9*b^2*c^3*d^8 + 22*a^9*b^2*c^5*d^6))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) + (b^2*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(a^3*b^10*c^13 - a^5*b^8*c^13 + a^13*c^3*d^10 - a^13*c^5*d^8 + 2*a*b^12*c^7*d^6 - 5*a*b^12*c^9*d^4 + 3*a*b^12*c^11*d^2 + 2*a^2*b^11*c^12*d - 10*a^4*b^9*c^12*d + 8*a^6*b^7*c^12*d + 2*a^7*b^6*c*d^12 - 5*a^9*b^4*c*d^12 + 3*a^11*b^2*c*d^12 + 2*a^12*b*c^2*d^11 - 10*a^12*b*c^4*d^9 + 8*a^12*b*c^6*d^7 - 12*a^2*b^11*c^6*d^7 + 32*a^2*b^11*c^8*d^5 - 22*a^2*b^11*c^10*d^3 + 30*a^3*b^10*c^5*d^8 - 92*a^3*b^10*c^7*d^6 + 83*a^3*b^10*c^9*d^4 - 22*a^3*b^10*c^11*d^2 - 40*a^4*b^9*c^4*d^9 + 160*a^4*b^9*c^6*d^7 - 208*a^4*b^9*c^8*d^5 + 98*a^4*b^9*c^10*d^3 + 30*a^5*b^8*c^3*d^10 - 190*a^5*b^8*c^5*d^8 + 362*a^5*b^8*c^7*d^6 - 248*a^5*b^8*c^9*d^4 + 47*a^5*b^8*c^11*d^2 - 12*a^6*b^7*c^2*d^11 + 160*a^6*b^7*c^4*d^9 - 436*a^6*b^7*c^6*d^7 + 412*a^6*b^7*c^8*d^5 - 132*a^6*b^7*c^10*d^3 - 92*a^7*b^6*c^3*d^10 + 362*a^7*b^6*c^5*d^8 - 484*a^7*b^6*c^7*d^6 + 240*a^7*b^6*c^9*d^4 - 28*a^7*b^6*c^11*d^2 + 32*a^8*b^5*c^2*d^11 - 208*a^8*b^5*c^4*d^9 + 412*a^8*b^5*c^6*d^7 - 292*a^8*b^5*c^8*d^5 + 56*a^8*b^5*c^10*d^3 + 83*a^9*b^4*c^3*d^10 - 248*a^9*b^4*c^5*d^8 + 240*a^9*b^4*c^7*d^6 - 70*a^9*b^4*c^9*d^4 - 22*a^10*b^3*c^2*d^11 + 98*a^10*b^3*c^4*d^9 - 132*a^10*b^3*c^6*d^7 + 56*a^10*b^3*c^8*d^5 - 22*a^11*b^2*c^3*d^10 + 47*a^11*b^2*c^5*d^8 - 28*a^11*b^2*c^7*d^6))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^

10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10
*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^
5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c
^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*
a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*
d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5
*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8
 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^
3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 -
32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) -
(32*tan(e/2 + (f*x)/2)*(2*a^4*b^9*c^13 - 2*a^2*b^11*c^13 - 2*a^13*c^2*d^11
+ 2*a^13*c^4*d^9 - 2*a*b^12*c^8*d^5 + 6*a*b^12*c^10*d^3 + 20*a^3*b^10*c^12*
d - 16*a^5*b^8*c^12*d - 2*a^8*b^5*c*d^12 + 6*a^10*b^3*c*d^12 + 20*a^12*b*c^
3*d^10 - 16*a^12*b*c^5*d^8 + 10*a^2*b^11*c^7*d^6 - 34*a^2*b^11*c^9*d^4 + 26
*a^2*b^11*c^11*d^2 - 18*a^3*b^10*c^6*d^7 + 80*a^3*b^10*c^8*d^5 - 82*a^3*b^1
0*c^10*d^3 + 10*a^4*b^9*c^5*d^8 - 96*a^4*b^9*c^7*d^6 + 160*a^4*b^9*c^9*d^4
- 76*a^4*b^9*c^11*d^2 + 10*a^5*b^8*c^4*d^9 + 44*a^5*b^8*c^6*d^7 - 188*a^5*b
^8*c^8*d^5 + 150*a^5*b^8*c^10*d^3 - 18*a^6*b^7*c^3*d^10 + 44*a^6*b^7*c^5*d^
8 + 88*a^6*b^7*c^7*d^6 - 164*a^6*b^7*c^9*d^4 + 50*a^6*b^7*c^11*d^2 + 10*a^7
*b^6*c^2*d^11 - 96*a^7*b^6*c^4*d^9 + 88*a^7*b^6*c^6*d^7 + 72*a^7*b^6*c^8*d^
5 - 74*a^7*b^6*c^10*d^3 + 80*a^8*b^5*c^3*d^10 - 188*a^8*b^5*c^5*d^8 + 72*a^
8*b^5*c^7*d^6 + 38*a^8*b^5*c^9*d^4 - 34*a^9*b^4*c^2*d^11 + 160*a^9*b^4*c^4*
d^9 - 164*a^9*b^4*c^6*d^7 + 38*a^9*b^4*c^8*d^5 - 82*a^10*b^3*c^3*d^10 + 150
*a^10*b^3*c^5*d^8 - 74*a^10*b^3*c^7*d^6 + 26*a^11*b^2*c^2*d^11 - 76*a^11*b^
2*c^4*d^9 + 50*a^11*b^2*c^6*d^7 - 4*a*b^12*c^12*d - 4*a^12*b*c*d^12))/(a^10
*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^
2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*
a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a
^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a
^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d
^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*
b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7
- 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4
*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 5
2*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^
4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) + (b^2*((32*(a^
2*b^13*c^15 - 2*a^4*b^11*c^15 + a^6*b^9*c^15 + a^15*c^2*d^13 - 2*a^15*c^4*d
^11 + a^15*c^6*d^9 - a*b^14*c^10*d^5 + 2*a*b^14*c^12*d^3 - 5*a^3*b^12*c^14*
d + 13*a^5*b^10*c^14*d - 7*a^7*b^8*c^14*d - a^10*b^5*c*d^14 + 2*a^12*b^3*c*
d^14 - 5*a^14*b*c^3*d^12 + 13*a^14*b*c^5*d^10 - 7*a^14*b*c^7*d^8 + 7*a^2*b^
13*c^9*d^6 - 13*a^2*b^13*c^11*d^4 + 5*a^2*b^13*c^13*d^2 - 20*a^3*b^12*c^8*d
^7 + 35*a^3*b^12*c^10*d^5 - 10*a^3*b^12*c^12*d^3 + 28*a^4*b^11*c^7*d^8 - 50
*a^4*b^11*c^9*d^6 + 14*a^4*b^11*c^11*d^4 + 10*a^4*b^11*c^13*d^2 - 14*a^5*b^
10*c^6*d^9 + 40*a^5*b^10*c^8*d^7 - 25*a^5*b^10*c^10*d^5 - 14*a^5*b^10*c^12*
d^3 - 14*a^6*b^9*c^5*d^10 - 14*a^6*b^9*c^7*d^8 + 37*a^6*b^9*c^9*d^6 + 25*a^

6*b^9*c^11*d^4 - 35*a^6*b^9*c^13*d^2 + 28*a^7*b^8*c^4*d^11 - 14*a^7*b^8*c^6
*d^9 - 20*a^7*b^8*c^8*d^7 - 37*a^7*b^8*c^10*d^5 + 50*a^7*b^8*c^12*d^3 - 20*
a^8*b^7*c^3*d^12 + 40*a^8*b^7*c^5*d^10 - 20*a^8*b^7*c^7*d^8 + 20*a^8*b^7*c^
9*d^6 - 40*a^8*b^7*c^11*d^4 + 20*a^8*b^7*c^13*d^2 + 7*a^9*b^6*c^2*d^13 - 50
*a^9*b^6*c^4*d^11 + 37*a^9*b^6*c^6*d^9 + 20*a^9*b^6*c^8*d^7 + 14*a^9*b^6*c^
10*d^5 - 28*a^9*b^6*c^12*d^3 + 35*a^10*b^5*c^3*d^12 - 25*a^10*b^5*c^5*d^10
- 37*a^10*b^5*c^7*d^8 + 14*a^10*b^5*c^9*d^6 + 14*a^10*b^5*c^11*d^4 - 13*a^1
1*b^4*c^2*d^13 + 14*a^11*b^4*c^4*d^11 + 25*a^11*b^4*c^6*d^9 - 40*a^11*b^4*c
^8*d^7 + 14*a^11*b^4*c^10*d^5 - 10*a^12*b^3*c^3*d^12 - 14*a^12*b^3*c^5*d^10
 + 50*a^12*b^3*c^7*d^8 - 28*a^12*b^3*c^9*d^6 + 5*a^13*b^2*c^2*d^13 + 10*a^1
3*b^2*c^4*d^11 - 35*a^13*b^2*c^6*d^9 + 20*a^13*b^2*c^8*d^7 - a*b^14*c^14*d
- a^14*b*c*d^14))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 +
a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^
4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d
- 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 -
 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8
*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^
4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^
2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b
^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 -
 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*
c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b
*c*d^9) - (32*tan(e/2 + (f*x)/2)*(8*a^3*b^12*c^15 - 3*a^15*c*d^14 - 3*a*b^1
4*c^15 - 7*a^5*b^10*c^15 + 2*a^7*b^8*c^15 + 8*a^15*c^3*d^12 - 7*a^15*c^5*d^
10 + 2*a^15*c^7*d^8 + 4*a*b^14*c^9*d^6 - 11*a*b^14*c^11*d^4 + 10*a*b^14*c^1
3*d^2 + 24*a^2*b^13*c^14*d - 64*a^4*b^11*c^14*d + 56*a^6*b^9*c^14*d - 16*a^
8*b^7*c^14*d + 4*a^9*b^6*c*d^14 - 11*a^11*b^4*c*d^14 + 10*a^13*b^2*c*d^14 +
 24*a^14*b*c^2*d^13 - 64*a^14*b*c^4*d^11 + 56*a^14*b*c^6*d^9 - 16*a^14*b*c^
8*d^7 - 32*a^2*b^13*c^8*d^7 + 88*a^2*b^13*c^10*d^5 - 80*a^2*b^13*c^12*d^3 +
 112*a^3*b^12*c^7*d^8 - 319*a^3*b^12*c^9*d^6 + 310*a^3*b^12*c^11*d^4 - 111*
a^3*b^12*c^13*d^2 - 224*a^4*b^11*c^6*d^9 + 704*a^4*b^11*c^8*d^7 - 800*a^4*b
^11*c^10*d^5 + 384*a^4*b^11*c^12*d^3 + 280*a^5*b^10*c^5*d^10 - 1078*a^5*b^1
0*c^7*d^8 + 1550*a^5*b^10*c^9*d^6 - 993*a^5*b^10*c^11*d^4 + 248*a^5*b^10*c^
13*d^2 - 224*a^6*b^9*c^4*d^11 + 1232*a^6*b^9*c^6*d^9 - 2320*a^6*b^9*c^8*d^7
 + 1896*a^6*b^9*c^10*d^5 - 640*a^6*b^9*c^12*d^3 + 112*a^7*b^8*c^3*d^12 - 10
78*a^7*b^8*c^5*d^10 + 2660*a^7*b^8*c^7*d^8 - 2733*a^7*b^8*c^9*d^6 + 1240*a^
7*b^8*c^11*d^4 - 203*a^7*b^8*c^13*d^2 - 32*a^8*b^7*c^2*d^13 + 704*a^8*b^7*c
^4*d^11 - 2320*a^8*b^7*c^6*d^9 + 3072*a^8*b^7*c^8*d^7 - 1856*a^8*b^7*c^10*d
^5 + 448*a^8*b^7*c^12*d^3 - 319*a^9*b^6*c^3*d^12 + 1550*a^9*b^6*c^5*d^10 -
2733*a^9*b^6*c^7*d^8 + 2128*a^9*b^6*c^9*d^6 - 686*a^9*b^6*c^11*d^4 + 56*a^9
*b^6*c^13*d^2 + 88*a^10*b^5*c^2*d^13 - 800*a^10*b^5*c^4*d^11 + 1896*a^10*b^
5*c^6*d^9 - 1856*a^10*b^5*c^8*d^7 + 784*a^10*b^5*c^10*d^5 - 112*a^10*b^5*c^
12*d^3 + 310*a^11*b^4*c^3*d^12 - 993*a^11*b^4*c^5*d^10 + 1240*a^11*b^4*c^7*
d^8 - 686*a^11*b^4*c^9*d^6 + 140*a^11*b^4*c^11*d^4 - 80*a^12*b^3*c^2*d^13 +
 384*a^12*b^3*c^4*d^11 - 640*a^12*b^3*c^6*d^9 + 448*a^12*b^3*c^8*d^7 - 112*

a^12*b^3*c^10*d^5 - 111*a^13*b^2*c^3*d^12 + 248*a^13*b^2*c^5*d^10 - 203*a^1
3*b^2*c^7*d^8 + 56*a^13*b^2*c^9*d^6))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^
10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c
^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3
 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9
+ 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*
d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3
*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4
 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^
5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 +
15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c
^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a
*b^9*c^9*d - 6*a^9*b*c*d^9))*(-(a + b)^3*(a - b)^3)^(1/2)*(2*b^2*d - 3*a^2*
d + a*b*c))/(a^9*d^3 + b^9*c^3 - 3*a^2*b^7*c^3 + 3*a^4*b^5*c^3 - a^6*b^3*c^
3 - a^3*b^6*d^3 + 3*a^5*b^4*d^3 - 3*a^7*b^2*d^3 + 3*a^2*b^7*c*d^2 + 9*a^3*b
^6*c^2*d - 9*a^4*b^5*c*d^2 - 9*a^5*b^4*c^2*d + 9*a^6*b^3*c*d^2 + 3*a^7*b^2*
c^2*d - 3*a*b^8*c^2*d - 3*a^8*b*c*d^2))*(2*b^2*d - 3*a^2*d + a*b*c))/(a^9*d
^3 + b^9*c^3 - 3*a^2*b^7*c^3 + 3*a^4*b^5*c^3 - a^6*b^3*c^3 - a^3*b^6*d^3 +
3*a^5*b^4*d^3 - 3*a^7*b^2*d^3 + 3*a^2*b^7*c*d^2 + 9*a^3*b^6*c^2*d - 9*a^4*b
^5*c*d^2 - 9*a^5*b^4*c^2*d + 9*a^6*b^3*c*d^2 + 3*a^7*b^2*c^2*d - 3*a*b^8*c^
2*d - 3*a^8*b*c*d^2))*(2*b^2*d - 3*a^2*d + a*b*c))/(a^9*d^3 + b^9*c^3 - 3*a
^2*b^7*c^3 + 3*a^4*b^5*c^3 - a^6*b^3*c^3 - a^3*b^6*d^3 + 3*a^5*b^4*d^3 - 3*
a^7*b^2*d^3 + 3*a^2*b^7*c*d^2 + 9*a^3*b^6*c^2*d - 9*a^4*b^5*c*d^2 - 9*a^5*b
^4*c^2*d + 9*a^6*b^3*c*d^2 + 3*a^7*b^2*c^2*d - 3*a*b^8*c^2*d - 3*a^8*b*c*d^
2)))*(-(a + b)^3*(a - b)^3)^(1/2)*(2*b^2*d - 3*a^2*d + a*b*c)*2i)/(f*(a^9*d
^3 + b^9*c^3 - 3*a^2*b^7*c^3 + 3*a^4*b^5*c^3 - a^6*b^3*c^3 - a^3*b^6*d^3 +
3*a^5*b^4*d^3 - 3*a^7*b^2*d^3 + 3*a^2*b^7*c*d^2 + 9*a^3*b^6*c^2*d - 9*a^4*b
^5*c*d^2 - 9*a^5*b^4*c^2*d + 9*a^6*b^3*c*d^2 + 3*a^7*b^2*c^2*d - 3*a*b^8*c^
2*d - 3*a^8*b*c*d^2)) - (d^2*atan(((d^2*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(
4*a*b^10*c^4*d^7 - 8*a*b^10*c^6*d^5 + 4*a*b^10*c^8*d^3 + a^3*b^8*c^10*d + 4
*a^4*b^7*c*d^10 - 8*a^6*b^5*c*d^10 + 4*a^8*b^3*c*d^10 + a^10*b*c^3*d^8 - 4*
a^2*b^9*c^3*d^8 + 8*a^2*b^9*c^5*d^6 - 7*a^2*b^9*c^7*d^4 + 4*a^2*b^9*c^9*d^2
 - 4*a^3*b^8*c^2*d^9 + 21*a^3*b^8*c^6*d^5 - 22*a^3*b^8*c^8*d^3 - 18*a^4*b^7
*c^5*d^6 + 26*a^4*b^7*c^7*d^4 - 8*a^4*b^7*c^9*d^2 + 8*a^5*b^6*c^2*d^9 - 18*
a^5*b^6*c^4*d^7 - 8*a^5*b^6*c^6*d^5 + 22*a^5*b^6*c^8*d^3 + 21*a^6*b^5*c^3*d
^8 - 8*a^6*b^5*c^5*d^6 - 15*a^6*b^5*c^7*d^4 - 7*a^7*b^4*c^2*d^9 + 26*a^7*b^
4*c^4*d^7 - 15*a^7*b^4*c^6*d^5 - 22*a^8*b^3*c^3*d^8 + 22*a^8*b^3*c^5*d^6 +
4*a^9*b^2*c^2*d^9 - 8*a^9*b^2*c^4*d^7))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*
c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10
*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d
^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^
9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^
6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a
^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d
^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*

b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3 *c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6 *a*b^9*c^9*d - 6*a^9*b*c*d^9) - (32*tan(e/2 + (f*x)/2)*(a^3*b^8*c^11 + a^11 *c^3*d^8 - 16*a*b^10*c^3*d^8 + 44*a*b^10*c^5*d^6 - 34*a*b^10*c^7*d^4 + 4*a* b^10*c^9*d^2 + 4*a^2*b^9*c^10*d - 16*a^3*b^8*c*d^10 - 8*a^4*b^7*c^10*d + 44 *a^5*b^6*c*d^10 - 34*a^7*b^4*c*d^10 + 4*a^9*b^2*c*d^10 + 4*a^10*b*c^2*d^9 - 8*a^10*b*c^4*d^7 + 32*a^2*b^9*c^2*d^9 - 104*a^2*b^9*c^4*d^7 + 100*a^2*b^9* c^6*d^5 - 24*a^2*b^9*c^8*d^3 + 120*a^3*b^8*c^3*d^8 - 222*a^3*b^8*c^5*d^6 + 134*a^3*b^8*c^7*d^4 - 24*a^3*b^8*c^9*d^2 - 104*a^4*b^7*c^2*d^9 + 312*a^4*b^ 7*c^4*d^7 - 272*a^4*b^7*c^6*d^5 + 60*a^4*b^7*c^8*d^3 - 222*a^5*b^6*c^3*d^8 + 316*a^5*b^6*c^5*d^6 - 136*a^5*b^6*c^7*d^4 + 22*a^5*b^6*c^9*d^2 + 100*a^6* b^5*c^2*d^9 - 272*a^6*b^5*c^4*d^7 + 192*a^6*b^5*c^6*d^5 - 24*a^6*b^5*c^8*d^ 3 + 134*a^7*b^4*c^3*d^8 - 136*a^7*b^4*c^5*d^6 + 18*a^7*b^4*c^7*d^4 - 24*a^8 *b^3*c^2*d^9 + 60*a^8*b^3*c^4*d^7 - 24*a^8*b^3*c^6*d^5 - 24*a^9*b^2*c^3*d^8 + 22*a^9*b^2*c^5*d^6))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c ^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10* c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7* c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3 *d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b ^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6* c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32 *a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8 *d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^ 8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6 *a^9*b*c*d^9) + (d^2*(-(c + d)^3*(c - d)^3)^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*a^4*b^9*c^13 - 2*a^2*b^11*c^13 - 2*a^13*c^2*d^11 + 2*a^13*c^4*d^9 - 2*a*b ^12*c^8*d^5 + 6*a*b^12*c^10*d^3 + 20*a^3*b^10*c^12*d - 16*a^5*b^8*c^12*d - 2*a^8*b^5*c*d^12 + 6*a^10*b^3*c*d^12 + 20*a^12*b*c^3*d^10 - 16*a^12*b*c^5*d ^8 + 10*a^2*b^11*c^7*d^6 - 34*a^2*b^11*c^9*d^4 + 26*a^2*b^11*c^11*d^2 - 18* a^3*b^10*c^6*d^7 + 80*a^3*b^10*c^8*d^5 - 82*a^3*b^10*c^10*d^3 + 10*a^4*b^9* c^5*d^8 - 96*a^4*b^9*c^7*d^6 + 160*a^4*b^9*c^9*d^4 - 76*a^4*b^9*c^11*d^2 + 10*a^5*b^8*c^4*d^9 + 44*a^5*b^8*c^6*d^7 - 188*a^5*b^8*c^8*d^5 + 150*a^5*b^8 *c^10*d^3 - 18*a^6*b^7*c^3*d^10 + 44*a^6*b^7*c^5*d^8 + 88*a^6*b^7*c^7*d^6 - 164*a^6*b^7*c^9*d^4 + 50*a^6*b^7*c^11*d^2 + 10*a^7*b^6*c^2*d^11 - 96*a^7*b ^6*c^4*d^9 + 88*a^7*b^6*c^6*d^7 + 72*a^7*b^6*c^8*d^5 - 74*a^7*b^6*c^10*d^3 + 80*a^8*b^5*c^3*d^10 - 188*a^8*b^5*c^5*d^8 + 72*a^8*b^5*c^7*d^6 + 38*a^8*b ^5*c^9*d^4 - 34*a^9*b^4*c^2*d^11 + 160*a^9*b^4*c^4*d^9 - 164*a^9*b^4*c^6*d^ 7 + 38*a^9*b^4*c^8*d^5 - 82*a^10*b^3*c^3*d^10 + 150*a^10*b^3*c^5*d^8 - 74*a ^10*b^3*c^7*d^6 + 26*a^11*b^2*c^2*d^11 - 76*a^11*b^2*c^4*d^9 + 50*a^11*b^2* c^6*d^7 - 4*a*b^12*c^12*d - 4*a^12*b*c*d^12))/(a^10*d^10 + b^10*c^10 - 2*a^ 2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9 *c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^

3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) - (32*(a^3*b^10*c^13 - a^5*b^8*c^13 + a^13*c^3*d^10 - a^13*c^5*d^8 + 2*a*b^12*c^7*d^6 - 5*a*b^12*c^9*d^4 + 3*a*b^12*c^11*d^2 + 2*a^2*b^11*c^12*d - 10*a^4*b^9*c^12*d + 8*a^6*b^7*c^12*d + 2*a^7*b^6*c*d^12 - 5*a^9*b^4*c*d^12 + 3*a^11*b^2*c*d^12 + 2*a^12*b*c^2*d^11 - 10*a^12*b*c^4*d^9 + 8*a^12*b*c^6*d^7 - 12*a^2*b^11*c^6*d^7 + 32*a^2*b^11*c^8*d^5 - 22*a^2*b^11*c^10*d^3 + 30*a^3*b^10*c^5*d^8 - 92*a^3*b^10*c^7*d^6 + 83*a^3*b^10*c^9*d^4 - 22*a^3*b^10*c^11*d^2 - 40*a^4*b^9*c^4*d^9 + 160*a^4*b^9*c^6*d^7 - 208*a^4*b^9*c^8*d^5 + 98*a^4*b^9*c^10*d^3 + 30*a^5*b^8*c^3*d^10 - 190*a^5*b^8*c^5*d^8 + 362*a^5*b^8*c^7*d^6 - 248*a^5*b^8*c^9*d^4 + 47*a^5*b^8*c^11*d^2 - 12*a^6*b^7*c^2*d^11 + 160*a^6*b^7*c^4*d^9 - 436*a^6*b^7*c^6*d^7 + 412*a^6*b^7*c^8*d^5 - 132*a^6*b^7*c^10*d^3 - 92*a^7*b^6*c^3*d^10 + 362*a^7*b^6*c^5*d^8 - 484*a^7*b^6*c^7*d^6 + 240*a^7*b^6*c^9*d^4 - 28*a^7*b^6*c^11*d^2 + 32*a^8*b^5*c^2*d^11 - 208*a^8*b^5*c^4*d^9 + 412*a^8*b^5*c^6*d^7 - 292*a^8*b^5*c^8*d^5 + 56*a^8*b^5*c^10*d^3 + 83*a^9*b^4*c^3*d^10 - 248*a^9*b^4*c^5*d^8 + 240*a^9*b^4*c^7*d^6 - 70*a^9*b^4*c^9*d^4 - 22*a^10*b^3*c^2*d^11 + 98*a^10*b^3*c^4*d^9 - 132*a^10*b^3*c^6*d^7 + 56*a^10*b^3*c^8*d^5 - 22*a^11*b^2*c^3*d^10 + 47*a^11*b^2*c^5*d^8 - 28*a^11*b^2*c^7*d^6))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) + (d^2*((32*(a^2*b^13*c^15 - 2*a^4*b^11*c^15 + a^6*b^9*c^15 + a^15*c^2*d^13 - 2*a^15*c^4*d^11 + a^15*c^6*d^9 - a*b^14*c^10*d^5 + 2*a*b^14*c^12*d^3 - 5*a^3*b^12*c^14*d + 13*a^5*b^10*c^14*d - 7*a^7*b^8*c^14*d - a^10*b^5*c*d^14 + 2*a^12*b^3*c*d^14 - 5*a^14*b*c^3*d^12 + 13*a^14*b*c^5*d^10 - 7*a^14*b*c^7*d^8 + 7*a^2*b^13*c^9*d^6 - 13*a^2*b^13*c^11*d^4 + 5*a^2*b^13*c^13*d^2 - 20*a^3*b^12*c^8*d^7 + 35*a^3*b^12*c^10*d^5 - 10*a^3*b^12*c^12*d^3 + 28*a^4*b^11*c^7*d^8 - 50*a^4*b^11*c^9*d^6 + 14*a^4*b^11*c^11*d^4 + 10*a^4*b^11*c^13*d^2 - 14*a^5*b^10*c^6*d^9 + 40*a^5*b^10*c^8*d^7 - 25*a^5*b^10*c^10*d^5 - 14*a^5*b^10*c^12*d^3 - 14*a^6*b^9*c^5*d^10 - 14*a^6*b^9*c^7*d^8 + 37*a^6*b^9*c^9*d^6 + 25*a^6*b^9*c^11*d^4 - 35*a^6*b^9*c^13*d^2 + 28*a^7*b^8*c^4*d^11 - 14*a^7*b^8*c^6*d^

9 - 20*a^7*b^8*c^8*d^7 - 37*a^7*b^8*c^10*d^5 + 50*a^7*b^8*c^12*d^3 - 20*a^8 *b^7*c^3*d^12 + 40*a^8*b^7*c^5*d^10 - 20*a^8*b^7*c^7*d^8 + 20*a^8*b^7*c^9*d ^6 - 40*a^8*b^7*c^11*d^4 + 20*a^8*b^7*c^13*d^2 + 7*a^9*b^6*c^2*d^13 - 50*a^ 9*b^6*c^4*d^11 + 37*a^9*b^6*c^6*d^9 + 20*a^9*b^6*c^8*d^7 + 14*a^9*b^6*c^10* d^5 - 28*a^9*b^6*c^12*d^3 + 35*a^10*b^5*c^3*d^12 - 25*a^10*b^5*c^5*d^10 - 3 7*a^10*b^5*c^7*d^8 + 14*a^10*b^5*c^9*d^6 + 14*a^10*b^5*c^11*d^4 - 13*a^11*b ^4*c^2*d^13 + 14*a^11*b^4*c^4*d^11 + 25*a^11*b^4*c^6*d^9 - 40*a^11*b^4*c^8* d^7 + 14*a^11*b^4*c^10*d^5 - 10*a^12*b^3*c^3*d^12 - 14*a^12*b^3*c^5*d^10 + 50*a^12*b^3*c^7*d^8 - 28*a^12*b^3*c^9*d^6 + 5*a^13*b^2*c^2*d^13 + 10*a^13*b ^2*c^4*d^11 - 35*a^13*b^2*c^6*d^9 + 20*a^13*b^2*c^8*d^7 - a*b^14*c^14*d - a ^14*b*c*d^14))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6 *b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6 *a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6* a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^ 2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b ^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4* c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44 *a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2 *d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c* d^9) - (32*tan(e/2 + (f*x)/2)*(8*a^3*b^12*c^15 - 3*a^15*c*d^14 - 3*a*b^14*c ^15 - 7*a^5*b^10*c^15 + 2*a^7*b^8*c^15 + 8*a^15*c^3*d^12 - 7*a^15*c^5*d^10 + 2*a^15*c^7*d^8 + 4*a*b^14*c^9*d^6 - 11*a*b^14*c^11*d^4 + 10*a*b^14*c^13*d ^2 + 24*a^2*b^13*c^14*d - 64*a^4*b^11*c^14*d + 56*a^6*b^9*c^14*d - 16*a^8*b ^7*c^14*d + 4*a^9*b^6*c*d^14 - 11*a^11*b^4*c*d^14 + 10*a^13*b^2*c*d^14 + 24 *a^14*b*c^2*d^13 - 64*a^14*b*c^4*d^11 + 56*a^14*b*c^6*d^9 - 16*a^14*b*c^8*d ^7 - 32*a^2*b^13*c^8*d^7 + 88*a^2*b^13*c^10*d^5 - 80*a^2*b^13*c^12*d^3 + 11 2*a^3*b^12*c^7*d^8 - 319*a^3*b^12*c^9*d^6 + 310*a^3*b^12*c^11*d^4 - 111*a^3 *b^12*c^13*d^2 - 224*a^4*b^11*c^6*d^9 + 704*a^4*b^11*c^8*d^7 - 800*a^4*b^11 *c^10*d^5 + 384*a^4*b^11*c^12*d^3 + 280*a^5*b^10*c^5*d^10 - 1078*a^5*b^10*c ^7*d^8 + 1550*a^5*b^10*c^9*d^6 - 993*a^5*b^10*c^11*d^4 + 248*a^5*b^10*c^13* d^2 - 224*a^6*b^9*c^4*d^11 + 1232*a^6*b^9*c^6*d^9 - 2320*a^6*b^9*c^8*d^7 + 1896*a^6*b^9*c^10*d^5 - 640*a^6*b^9*c^12*d^3 + 112*a^7*b^8*c^3*d^12 - 1078* a^7*b^8*c^5*d^10 + 2660*a^7*b^8*c^7*d^8 - 2733*a^7*b^8*c^9*d^6 + 1240*a^7*b ^8*c^11*d^4 - 203*a^7*b^8*c^13*d^2 - 32*a^8*b^7*c^2*d^13 + 704*a^8*b^7*c^4* d^11 - 2320*a^8*b^7*c^6*d^9 + 3072*a^8*b^7*c^8*d^7 - 1856*a^8*b^7*c^10*d^5 + 448*a^8*b^7*c^12*d^3 - 319*a^9*b^6*c^3*d^12 + 1550*a^9*b^6*c^5*d^10 - 273 3*a^9*b^6*c^7*d^8 + 2128*a^9*b^6*c^9*d^6 - 686*a^9*b^6*c^11*d^4 + 56*a^9*b^ 6*c^13*d^2 + 88*a^10*b^5*c^2*d^13 - 800*a^10*b^5*c^4*d^11 + 1896*a^10*b^5*c ^6*d^9 - 1856*a^10*b^5*c^8*d^7 + 784*a^10*b^5*c^10*d^5 - 112*a^10*b^5*c^12* d^3 + 310*a^11*b^4*c^3*d^12 - 993*a^11*b^4*c^5*d^10 + 1240*a^11*b^4*c^7*d^8 - 686*a^11*b^4*c^9*d^6 + 140*a^11*b^4*c^11*d^4 - 80*a^12*b^3*c^2*d^13 + 38 4*a^12*b^3*c^4*d^11 - 640*a^12*b^3*c^6*d^9 + 448*a^12*b^3*c^8*d^7 - 112*a^1 2*b^3*c^10*d^5 - 111*a^13*b^2*c^3*d^12 + 248*a^13*b^2*c^5*d^10 - 203*a^13*b

^2*c^7*d^8 + 56*a^13*b^2*c^9*d^6))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9))*(-(c + d)^3*(c - d)^3)^(1/2)*(2*b*d^2 - 3*b*c^2 + a*c*d))/(a^3*d^9 + b^3*c^9 - 3*a^3*c^2*d^7 + 3*a^3*c^4*d^5 - a^3*c^6*d^3 - b^3*c^3*d^6 + 3*b^3*c^5*d^4 - 3*b^3*c^7*d^2 + 3*a*b^2*c^2*d^7 - 9*a*b^2*c^4*d^5 + 9*a*b^2*c^6*d^3 + 9*a^2*b*c^3*d^6 - 9*a^2*b*c^5*d^4 + 3*a^2*b*c^7*d^2 - 3*a*b^2*c^8*d - 3*a^2*b*c*d^8))*(2*b*d^2 - 3*b*c^2 + a*c*d))/(a^3*d^9 + b^3*c^9 - 3*a^3*c^2*d^7 + 3*a^3*c^4*d^5 - a^3*c^6*d^3 - b^3*c^3*d^6 + 3*b^3*c^5*d^4 - 3*b^3*c^7*d^2 + 3*a*b^2*c^2*d^7 - 9*a*b^2*c^4*d^5 + 9*a*b^2*c^6*d^3 + 9*a^2*b*c^3*d^6 - 9*a^2*b*c^5*d^4 + 3*a^2*b*c^7*d^2 - 3*a*b^2*c^8*d - 3*a^2*b*c*d^8))*(2*b*d^2 - 3*b*c^2 + a*c*d)*1i)/(a^3*d^9 + b^3*c^9 - 3*a^3*c^2*d^7 + 3*a^3*c^4*d^5 - a^3*c^6*d^3 - b^3*c^3*d^6 + 3*b^3*c^5*d^4 - 3*b^3*c^7*d^2 + 3*a*b^2*c^2*d^7 - 9*a*b^2*c^4*d^5 + 9*a*b^2*c^6*d^3 + 9*a^2*b*c^3*d^6 - 9*a^2*b*c^5*d^4 + 3*a^2*b*c^7*d^2 - 3*a*b^2*c^8*d - 3*a^2*b*c*d^8) + (d^2*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(4*a*b^10*c^4*d^7 - 8*a*b^10*c^6*d^5 + 4*a*b^10*c^8*d^3 + a^3*b^8*c^10*d + 4*a^4*b^7*c*d^10 - 8*a^6*b^5*c*d^10 + 4*a^8*b^3*c*d^10 + a^10*b*c^3*d^8 - 4*a^2*b^9*c^3*d^8 + 8*a^2*b^9*c^5*d^6 - 7*a^2*b^9*c^7*d^4 + 4*a^2*b^9*c^9*d^2 - 4*a^3*b^8*c^2*d^9 + 21*a^3*b^8*c^6*d^5 - 22*a^3*b^8*c^8*d^3 - 18*a^4*b^7*c^5*d^6 + 26*a^4*b^7*c^7*d^4 - 8*a^4*b^7*c^9*d^2 + 8*a^5*b^6*c^2*d^9 - 18*a^5*b^6*c^4*d^7 - 8*a^5*b^6*c^6*d^5 + 22*a^5*b^6*c^8*d^3 + 21*a^6*b^5*c^3*d^8 - 8*a^6*b^5*c^5*d^6 - 15*a^6*b^5*c^7*d^4 - 7*a^7*b^4*c^2*d^9 + 26*a^7*b^4*c^4*d^7 - 15*a^7*b^4*c^6*d^5 - 22*a^8*b^3*c^3*d^8 + 22*a^8*b^3*c^5*d^6 + 4*a^9*b^2*c^2*d^9 - 8*a^9*b^2*c^4*d^7))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) - (32*tan(e/2 + (f*x)/2)*(a^3*b^8*c^11 + a^11*c^3*d^8 - 16*a*b^10*c^3*d^8 + 44*a*b^10*c^5*d^6 - 34*a*b^10*c^7*d^4 + 4*a*b^10*c^9*d^2 + 4*a^2*b^9*c^10*

d - 16*a^3*b^8*c*d^10 - 8*a^4*b^7*c^10*d + 44*a^5*b^6*c*d^10 - 34*a^7*b^4*c
*d^10 + 4*a^9*b^2*c*d^10 + 4*a^10*b*c^2*d^9 - 8*a^10*b*c^4*d^7 + 32*a^2*b^9
*c^2*d^9 - 104*a^2*b^9*c^4*d^7 + 100*a^2*b^9*c^6*d^5 - 24*a^2*b^9*c^8*d^3 +
120*a^3*b^8*c^3*d^8 - 222*a^3*b^8*c^5*d^6 + 134*a^3*b^8*c^7*d^4 - 24*a^3*b
^8*c^9*d^2 - 104*a^4*b^7*c^2*d^9 + 312*a^4*b^7*c^4*d^7 - 272*a^4*b^7*c^6*d^
5 + 60*a^4*b^7*c^8*d^3 - 222*a^5*b^6*c^3*d^8 + 316*a^5*b^6*c^5*d^6 - 136*a^
5*b^6*c^7*d^4 + 22*a^5*b^6*c^9*d^2 + 100*a^6*b^5*c^2*d^9 - 272*a^6*b^5*c^4*
d^7 + 192*a^6*b^5*c^6*d^5 - 24*a^6*b^5*c^8*d^3 + 134*a^7*b^4*c^3*d^8 - 136*
a^7*b^4*c^5*d^6 + 18*a^7*b^4*c^7*d^4 - 24*a^8*b^3*c^2*d^9 + 60*a^8*b^3*c^4*
d^7 - 24*a^8*b^3*c^6*d^5 - 24*a^9*b^2*c^3*d^8 + 22*a^9*b^2*c^5*d^6))/(a^10*
d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2
*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a
*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^
5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^
2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^
7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b
^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 -
92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*
c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52
*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4
*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) + (d^2*(-(c + d)
^3*(c - d)^3)^(1/2)*((32*(a^3*b^10*c^13 - a^5*b^8*c^13 + a^13*c^3*d^10 - a^
13*c^5*d^8 + 2*a*b^12*c^7*d^6 - 5*a*b^12*c^9*d^4 + 3*a*b^12*c^11*d^2 + 2*a^
2*b^11*c^12*d - 10*a^4*b^9*c^12*d + 8*a^6*b^7*c^12*d + 2*a^7*b^6*c*d^12 - 5
*a^9*b^4*c*d^12 + 3*a^11*b^2*c*d^12 + 2*a^12*b*c^2*d^11 - 10*a^12*b*c^4*d^9
+ 8*a^12*b*c^6*d^7 - 12*a^2*b^11*c^6*d^7 + 32*a^2*b^11*c^8*d^5 - 22*a^2*b^
11*c^10*d^3 + 30*a^3*b^10*c^5*d^8 - 92*a^3*b^10*c^7*d^6 + 83*a^3*b^10*c^9*d
^4 - 22*a^3*b^10*c^11*d^2 - 40*a^4*b^9*c^4*d^9 + 160*a^4*b^9*c^6*d^7 - 208*
a^4*b^9*c^8*d^5 + 98*a^4*b^9*c^10*d^3 + 30*a^5*b^8*c^3*d^10 - 190*a^5*b^8*c
^5*d^8 + 362*a^5*b^8*c^7*d^6 - 248*a^5*b^8*c^9*d^4 + 47*a^5*b^8*c^11*d^2 -
12*a^6*b^7*c^2*d^11 + 160*a^6*b^7*c^4*d^9 - 436*a^6*b^7*c^6*d^7 + 412*a^6*b
^7*c^8*d^5 - 132*a^6*b^7*c^10*d^3 - 92*a^7*b^6*c^3*d^10 + 362*a^7*b^6*c^5*d
^8 - 484*a^7*b^6*c^7*d^6 + 240*a^7*b^6*c^9*d^4 - 28*a^7*b^6*c^11*d^2 + 32*a
^8*b^5*c^2*d^11 - 208*a^8*b^5*c^4*d^9 + 412*a^8*b^5*c^6*d^7 - 292*a^8*b^5*c
^8*d^5 + 56*a^8*b^5*c^10*d^3 + 83*a^9*b^4*c^3*d^10 - 248*a^9*b^4*c^5*d^8 +
240*a^9*b^4*c^7*d^6 - 70*a^9*b^4*c^9*d^4 - 22*a^10*b^3*c^2*d^11 + 98*a^10*b
^3*c^4*d^9 - 132*a^10*b^3*c^6*d^7 + 56*a^10*b^3*c^8*d^5 - 22*a^11*b^2*c^3*d
^10 + 47*a^11*b^2*c^5*d^8 - 28*a^11*b^2*c^7*d^6))/(a^10*d^10 + b^10*c^10 -
2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*
d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a
*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^
7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*
a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*
d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4
*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5

+ 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) - (32*tan(e/2 + (f*x)/2)*(2*a^4*b^9*c^13 - 2*a^2*b^11*c^13 - 2*a^13*c^2*d^11 + 2*a^13*c^4*d^9 - 2*a*b^12*c^8*d^5 + 6*a*b^12*c^10*d^3 + 20*a^3*b^10*c^12*d - 16*a^5*b^8*c^12*d - 2*a^8*b^5*c*d^12 + 6*a^10*b^3*c*d^12 + 20*a^12*b*c^3*d^10 - 16*a^12*b*c^5*d^8 + 10*a^2*b^11*c^7*d^6 - 34*a^2*b^11*c^9*d^4 + 26*a^2*b^11*c^11*d^2 - 18*a^3*b^10*c^6*d^7 + 80*a^3*b^10*c^8*d^5 - 82*a^3*b^10*c^10*d^3 + 10*a^4*b^9*c^5*d^8 - 96*a^4*b^9*c^7*d^6 + 160*a^4*b^9*c^9*d^4 - 76*a^4*b^9*c^11*d^2 + 10*a^5*b^8*c^4*d^9 + 44*a^5*b^8*c^6*d^7 - 188*a^5*b^8*c^8*d^5 + 150*a^5*b^8*c^10*d^3 - 18*a^6*b^7*c^3*d^10 + 44*a^6*b^7*c^5*d^8 + 88*a^6*b^7*c^7*d^6 - 164*a^6*b^7*c^9*d^4 + 50*a^6*b^7*c^11*d^2 + 10*a^7*b^6*c^2*d^11 - 96*a^7*b^6*c^4*d^9 + 88*a^7*b^6*c^6*d^7 + 72*a^7*b^6*c^8*d^5 - 74*a^7*b^6*c^10*d^3 + 80*a^8*b^5*c^3*d^10 - 188*a^8*b^5*c^5*d^8 + 72*a^8*b^5*c^7*d^6 + 38*a^8*b^5*c^9*d^4 - 34*a^9*b^4*c^2*d^11 + 160*a^9*b^4*c^4*d^9 - 164*a^9*b^4*c^6*d^7 + 38*a^9*b^4*c^8*d^5 - 82*a^10*b^3*c^3*d^10 + 150*a^10*b^3*c^5*d^8 - 74*a^10*b^3*c^7*d^6 + 26*a^11*b^2*c^2*d^11 - 76*a^11*b^2*c^4*d^9 + 50*a^11*b^2*c^6*d^7 - 4*a*b^12*c^12*d - 4*a^12*b*c*d^12))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) + (d^2*((32*(a^2*b^13*c^15 - 2*a^4*b^11*c^15 + a^6*b^9*c^15 + a^15*c^2*d^13 - 2*a^15*c^4*d^11 + a^15*c^6*d^9 - a*b^14*c^10*d^5 + 2*a*b^14*c^12*d^3 - 5*a^3*b^12*c^14*d + 13*a^5*b^10*c^14*d - 7*a^7*b^8*c^14*d - a^10*b^5*c*d^14 + 2*a^12*b^3*c*d^14 - 5*a^14*b*c^3*d^12 + 13*a^14*b*c^5*d^10 - 7*a^14*b*c^7*d^8 + 7*a^2*b^13*c^9*d^6 - 13*a^2*b^13*c^11*d^4 + 5*a^2*b^13*c^13*d^2 - 20*a^3*b^12*c^8*d^7 + 35*a^3*b^12*c^10*d^5 - 10*a^3*b^12*c^12*d^3 + 28*a^4*b^11*c^7*d^8 - 50*a^4*b^11*c^9*d^6 + 14*a^4*b^11*c^11*d^4 + 10*a^4*b^11*c^13*d^2 - 14*a^5*b^10*c^6*d^9 + 40*a^5*b^10*c^8*d^7 - 25*a^5*b^10*c^10*d^5 - 14*a^5*b^10*c^12*d^3 - 14*a^6*b^9*c^5*d^10 - 14*a^6*b^9*c^7*d^8 + 37*a^6*b^9*c^9*d^6 + 25*a^6*b^9*c^11*d^4 - 35*a^6*b^9*c^13*d^2 + 28*a^7*b^8*c^4*d^11 - 14*a^7*b^8*c^6*d^9 - 20*a^7*b^8*c^8*d^7 - 37*a^7*b^8*c^10*d^5 + 50*a^7*b^8*c^12*d^3 - 20*a^8*b^7*c^3*d^12 + 40*a^8*b^7*c^5*d^10 - 20*a^8*b^7*c^7*d^8 + 20*a^8*b^7*c^9*d^6 - 40*a^8*b^7*c^11*d^4 + 20*a^8*b^7*c^13*d^2 + 7*a^9*b^6*c^2*d^13 - 50*a^9*b^6*c^4*d^11 + 37*a^9*b^6*c^6*d^9 + 20*a^9*b^6*c^8*d^7 + 14*a^9*b^6*c^10*d^5 - 28*a^9*b^6*c^12*d^3 + 35*a^10*b^5*c^3*d^12 - 25*a^10*b^5*c^5*d^10 - 37*a^10*b^5*c^7*d^8 + 14*a^10*b

^5*c^9*d^6 + 14*a^10*b^5*c^11*d^4 - 13*a^11*b^4*c^2*d^13 + 14*a^11*b^4*c^4*d^11 + 25*a^11*b^4*c^6*d^9 - 40*a^11*b^4*c^8*d^7 + 14*a^11*b^4*c^10*d^5 - 10*a^12*b^3*c^3*d^12 - 14*a^12*b^3*c^5*d^10 + 50*a^12*b^3*c^7*d^8 - 28*a^12*b^3*c^9*d^6 + 5*a^13*b^2*c^2*d^13 + 10*a^13*b^2*c^4*d^11 - 35*a^13*b^2*c^6*d^9 + 20*a^13*b^2*c^8*d^7 - a*b^14*c^14*d - a^14*b*c*d^14))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) - (32*tan(e/2 + (f*x)/2)*(8*a^3*b^12*c^15 - 3*a^15*c*d^14 - 3*a*b^14*c^15 - 7*a^5*b^10*c^15 + 2*a^7*b^8*c^15 + 8*a^15*c^3*d^12 - 7*a^15*c^5*d^10 + 2*a^15*c^7*d^8 + 4*a*b^14*c^9*d^6 - 11*a*b^14*c^11*d^4 + 10*a*b^14*c^13*d^2 + 24*a^2*b^13*c^14*d - 64*a^4*b^11*c^14*d + 56*a^6*b^9*c^14*d - 16*a^8*b^7*c^14*d + 4*a^9*b^6*c*d^14 - 11*a^11*b^4*c*d^14 + 10*a^13*b^2*c*d^14 + 24*a^14*b*c^2*d^13 - 64*a^14*b*c^4*d^11 + 56*a^14*b*c^6*d^9 - 16*a^14*b*c^8*d^7 - 32*a^2*b^13*c^8*d^7 + 88*a^2*b^13*c^10*d^5 - 80*a^2*b^13*c^12*d^3 + 112*a^3*b^12*c^7*d^8 - 319*a^3*b^12*c^9*d^6 + 310*a^3*b^12*c^11*d^4 - 111*a^3*b^12*c^13*d^2 - 224*a^4*b^11*c^6*d^9 + 704*a^4*b^11*c^8*d^7 - 800*a^4*b^11*c^10*d^5 + 384*a^4*b^11*c^12*d^3 + 280*a^5*b^10*c^5*d^10 - 1078*a^5*b^10*c^7*d^8 + 1550*a^5*b^10*c^9*d^6 - 993*a^5*b^10*c^11*d^4 + 248*a^5*b^10*c^13*d^2 - 224*a^6*b^9*c^4*d^11 + 1232*a^6*b^9*c^6*d^9 - 2320*a^6*b^9*c^8*d^7 + 1896*a^6*b^9*c^10*d^5 - 640*a^6*b^9*c^12*d^3 + 112*a^7*b^8*c^3*d^12 - 1078*a^7*b^8*c^5*d^10 + 2660*a^7*b^8*c^7*d^8 - 2733*a^7*b^8*c^9*d^6 + 1240*a^7*b^8*c^11*d^4 - 203*a^7*b^8*c^13*d^2 - 32*a^8*b^7*c^2*d^13 + 704*a^8*b^7*c^4*d^11 - 2320*a^8*b^7*c^6*d^9 + 3072*a^8*b^7*c^8*d^7 - 1856*a^8*b^7*c^10*d^5 + 448*a^8*b^7*c^12*d^3 - 319*a^9*b^6*c^3*d^12 + 1550*a^9*b^6*c^5*d^10 - 2733*a^9*b^6*c^7*d^8 + 2128*a^9*b^6*c^9*d^6 - 686*a^9*b^6*c^11*d^4 + 56*a^9*b^6*c^13*d^2 + 88*a^10*b^5*c^2*d^13 - 800*a^10*b^5*c^4*d^11 + 1896*a^10*b^5*c^6*d^9 - 1856*a^10*b^5*c^8*d^7 + 784*a^10*b^5*c^10*d^5 - 112*a^10*b^5*c^12*d^3 + 310*a^11*b^4*c^3*d^12 - 993*a^11*b^4*c^5*d^10 + 1240*a^11*b^4*c^7*d^8 - 686*a^11*b^4*c^9*d^6 + 140*a^11*b^4*c^11*d^4 - 80*a^12*b^3*c^2*d^13 + 384*a^12*b^3*c^4*d^11 - 640*a^12*b^3*c^6*d^9 + 448*a^12*b^3*c^8*d^7 - 112*a^12*b^3*c^10*d^5 - 111*a^13*b^2*c^3*d^12 + 248*a^13*b^2*c^5*d^10 - 203*a^13*b^2*c^7*d^8 + 56*a^13*b^2*c^9*d^6))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3

*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8
- 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^
5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 +
76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c
^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*
a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9))*(-(c
+ d)^3*(c - d)^3)^(1/2)*(2*b*d^2 - 3*b*c^2 + a*c*d))/(a^3*d^9 + b^3*c^9 -
3*a^3*c^2*d^7 + 3*a^3*c^4*d^5 - a^3*c^6*d^3 - b^3*c^3*d^6 + 3*b^3*c^5*d^4 -
3*b^3*c^7*d^2 + 3*a*b^2*c^2*d^7 - 9*a*b^2*c^4*d^5 + 9*a*b^2*c^6*d^3 + 9*a^
2*b*c^3*d^6 - 9*a^2*b*c^5*d^4 + 3*a^2*b*c^7*d^2 - 3*a*b^2*c^8*d - 3*a^2*b*c
d^8))(2*b*d^2 - 3*b*c^2 + a*c*d))/(a^3*d^9 + b^3*c^9 - 3*a^3*c^2*d^7 + 3*
a^3*c^4*d^5 - a^3*c^6*d^3 - b^3*c^3*d^6 + 3*b^3*c^5*d^4 - 3*b^3*c^7*d^2 + 3
*a*b^2*c^2*d^7 - 9*a*b^2*c^4*d^5 + 9*a*b^2*c^6*d^3 + 9*a^2*b*c^3*d^6 - 9*a^
2*b*c^5*d^4 + 3*a^2*b*c^7*d^2 - 3*a*b^2*c^8*d - 3*a^2*b*c*d^8))*(2*b*d^2 -
3*b*c^2 + a*c*d)*1i)/(a^3*d^9 + b^3*c^9 - 3*a^3*c^2*d^7 + 3*a^3*c^4*d^5 - a
^3*c^6*d^3 - b^3*c^3*d^6 + 3*b^3*c^5*d^4 - 3*b^3*c^7*d^2 + 3*a*b^2*c^2*d^7
- 9*a*b^2*c^4*d^5 + 9*a*b^2*c^6*d^3 + 9*a^2*b*c^3*d^6 - 9*a^2*b*c^5*d^4 + 3
*a^2*b*c^7*d^2 - 3*a*b^2*c^8*d - 3*a^2*b*c*d^8))/((64*(12*a*b^8*c^5*d^4 - 2
0*a*b^8*c^3*d^6 - 20*a^3*b^6*c*d^8 + 12*a^5*b^4*c*d^8 + 16*a^2*b^7*c^2*d^7
- 30*a^2*b^7*c^4*d^5 + 12*a^2*b^7*c^6*d^3 + 60*a^3*b^6*c^3*d^6 - 42*a^3*b^6
*c^5*d^4 + 3*a^3*b^6*c^7*d^2 - 30*a^4*b^5*c^2*d^7 + 52*a^4*b^5*c^4*d^5 - 16
*a^4*b^5*c^6*d^3 - 42*a^5*b^4*c^3*d^6 + 26*a^5*b^4*c^5*d^4 + 12*a^6*b^3*c^2
*d^7 - 16*a^6*b^3*c^4*d^5 + 3*a^7*b^2*c^3*d^6 + 8*a*b^8*c*d^8))/(a^10*d^10
+ b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10
- 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*
c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5
*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8
*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 5
2*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^
4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a
^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d
^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*
b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6
+ 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) + (64*tan(e/2 + (f*x)
/2)*(8*a*b^8*c^2*d^7 - 12*a*b^8*c^4*d^5 + 8*a^2*b^7*c*d^8 - 12*a^4*b^5*c*d^
8 - 12*a^2*b^7*c^3*d^6 + 6*a^2*b^7*c^5*d^4 - 12*a^3*b^6*c^2*d^7 + 18*a^3*b^
6*c^4*d^5 + 6*a^3*b^6*c^6*d^3 + 18*a^4*b^5*c^3*d^6 - 14*a^4*b^5*c^5*d^4 + 6
*a^5*b^4*c^2*d^7 - 14*a^5*b^4*c^4*d^5 + 6*a^6*b^3*c^3*d^6))/(a^10*d^10 + b^
10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2
*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*
d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9
*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4
*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^
3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^
6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b

^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) + (d^2*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(4*a*b^10*c^4*d^7 - 8*a*b^10*c^6*d^5 + 4*a*b^10*c^8*d^3 + a^3*b^8*c^10*d + 4*a^4*b^7*c*d^10 - 8*a^6*b^5*c*d^10 + 4*a^8*b^3*c*d^10 + a^10*b*c^3*d^8 - 4*a^2*b^9*c^3*d^8 + 8*a^2*b^9*c^5*d^6 - 7*a^2*b^9*c^7*d^4 + 4*a^2*b^9*c^9*d^2 - 4*a^3*b^8*c^2*d^9 + 21*a^3*b^8*c^6*d^5 - 22*a^3*b^8*c^8*d^3 - 18*a^4*b^7*c^5*d^6 + 26*a^4*b^7*c^7*d^4 - 8*a^4*b^7*c^9*d^2 + 8*a^5*b^6*c^2*d^9 - 18*a^5*b^6*c^4*d^7 - 8*a^5*b^6*c^6*d^5 + 22*a^5*b^6*c^8*d^3 + 21*a^6*b^5*c^3*d^8 - 8*a^6*b^5*c^5*d^6 - 15*a^6*b^5*c^7*d^4 - 7*a^7*b^4*c^2*d^9 + 26*a^7*b^4*c^4*d^7 - 15*a^7*b^4*c^6*d^5 - 22*a^8*b^3*c^3*d^8 + 22*a^8*b^3*c^5*d^6 + 4*a^9*b^2*c^2*d^9 - 8*a^9*b^2*c^4*d^7))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) - (32*tan(e/2 + (f*x)/2)*(a^3*b^8*c^11 + a^11*c^3*d^8 - 16*a*b^10*c^3*d^8 + 44*a*b^10*c^5*d^6 - 34*a*b^10*c^7*d^4 + 4*a*b^10*c^9*d^2 + 4*a^2*b^9*c^10*d - 16*a^3*b^8*c*d^10 - 8*a^4*b^7*c^10*d + 44*a^5*b^6*c*d^10 - 34*a^7*b^4*c*d^10 + 4*a^9*b^2*c*d^10 + 4*a^10*b*c^2*d^9 - 8*a^10*b*c^4*d^7 + 32*a^2*b^9*c^2*d^9 - 104*a^2*b^9*c^4*d^7 + 100*a^2*b^9*c^6*d^5 - 24*a^2*b^9*c^8*d^3 + 120*a^3*b^8*c^3*d^8 - 222*a^3*b^8*c^5*d^6 + 134*a^3*b^8*c^7*d^4 - 24*a^3*b^8*c^9*d^2 - 104*a^4*b^7*c^2*d^9 + 312*a^4*b^7*c^4*d^7 - 272*a^4*b^7*c^6*d^5 + 60*a^4*b^7*c^8*d^3 - 222*a^5*b^6*c^3*d^8 + 316*a^5*b^6*c^5*d^6 - 136*a^5*b^6*c^7*d^4 + 22*a^5*b^6*c^9*d^2 + 100*a^6*b^5*c^2*d^9 - 272*a^6*b^5*c^4*d^7 + 192*a^6*b^5*c^6*d^5 - 24*a^6*b^5*c^8*d^3 + 134*a^7*b^4*c^3*d^8 - 136*a^7*b^4*c^5*d^6 + 18*a^7*b^4*c^7*d^4 - 24*a^8*b^3*c^2*d^9 + 60*a^8*b^3*c^4*d^7 - 24*a^8*b^3*c^6*d^5 - 24*a^9*b^2*c^3*d^8 + 22*a^9*b^2*c^5*d^6))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^

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3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 -
6*a*b^9*c^9*d - 6*a^9*b*c*d^9) + (d^2*(-(c + d)^3*(c - d)^3)^(1/2)*((32*tan
(e/2 + (f*x)/2)*(2*a^4*b^9*c^13 - 2*a^2*b^11*c^13 - 2*a^13*c^2*d^11 + 2*a^1
3*c^4*d^9 - 2*a*b^12*c^8*d^5 + 6*a*b^12*c^10*d^3 + 20*a^3*b^10*c^12*d - 16*
a^5*b^8*c^12*d - 2*a^8*b^5*c*d^12 + 6*a^10*b^3*c*d^12 + 20*a^12*b*c^3*d^10
- 16*a^12*b*c^5*d^8 + 10*a^2*b^11*c^7*d^6 - 34*a^2*b^11*c^9*d^4 + 26*a^2*b^
11*c^11*d^2 - 18*a^3*b^10*c^6*d^7 + 80*a^3*b^10*c^8*d^5 - 82*a^3*b^10*c^10*
d^3 + 10*a^4*b^9*c^5*d^8 - 96*a^4*b^9*c^7*d^6 + 160*a^4*b^9*c^9*d^4 - 76*a^
4*b^9*c^11*d^2 + 10*a^5*b^8*c^4*d^9 + 44*a^5*b^8*c^6*d^7 - 188*a^5*b^8*c^8*
d^5 + 150*a^5*b^8*c^10*d^3 - 18*a^6*b^7*c^3*d^10 + 44*a^6*b^7*c^5*d^8 + 88*
a^6*b^7*c^7*d^6 - 164*a^6*b^7*c^9*d^4 + 50*a^6*b^7*c^11*d^2 + 10*a^7*b^6*c^
2*d^11 - 96*a^7*b^6*c^4*d^9 + 88*a^7*b^6*c^6*d^7 + 72*a^7*b^6*c^8*d^5 - 74*
a^7*b^6*c^10*d^3 + 80*a^8*b^5*c^3*d^10 - 188*a^8*b^5*c^5*d^8 + 72*a^8*b^5*c
^7*d^6 + 38*a^8*b^5*c^9*d^4 - 34*a^9*b^4*c^2*d^11 + 160*a^9*b^4*c^4*d^9 - 1
64*a^9*b^4*c^6*d^7 + 38*a^9*b^4*c^8*d^5 - 82*a^10*b^3*c^3*d^10 + 150*a^10*b
^3*c^5*d^8 - 74*a^10*b^3*c^7*d^6 + 26*a^11*b^2*c^2*d^11 - 76*a^11*b^2*c^4*d
^9 + 50*a^11*b^2*c^6*d^7 - 4*a*b^12*c^12*d - 4*a^12*b*c*d^12))/(a^10*d^10 +
 b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10
- 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c
^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*
c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*
c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52
*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4
*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^
5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^
6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b
^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 +
 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) - (32*(a^3*b^10*c^13 -
 a^5*b^8*c^13 + a^13*c^3*d^10 - a^13*c^5*d^8 + 2*a*b^12*c^7*d^6 - 5*a*b^12*
c^9*d^4 + 3*a*b^12*c^11*d^2 + 2*a^2*b^11*c^12*d - 10*a^4*b^9*c^12*d + 8*a^6
*b^7*c^12*d + 2*a^7*b^6*c*d^12 - 5*a^9*b^4*c*d^12 + 3*a^11*b^2*c*d^12 + 2*a
^12*b*c^2*d^11 - 10*a^12*b*c^4*d^9 + 8*a^12*b*c^6*d^7 - 12*a^2*b^11*c^6*d^7
 + 32*a^2*b^11*c^8*d^5 - 22*a^2*b^11*c^10*d^3 + 30*a^3*b^10*c^5*d^8 - 92*a^
3*b^10*c^7*d^6 + 83*a^3*b^10*c^9*d^4 - 22*a^3*b^10*c^11*d^2 - 40*a^4*b^9*c^
4*d^9 + 160*a^4*b^9*c^6*d^7 - 208*a^4*b^9*c^8*d^5 + 98*a^4*b^9*c^10*d^3 + 3
0*a^5*b^8*c^3*d^10 - 190*a^5*b^8*c^5*d^8 + 362*a^5*b^8*c^7*d^6 - 248*a^5*b^
8*c^9*d^4 + 47*a^5*b^8*c^11*d^2 - 12*a^6*b^7*c^2*d^11 + 160*a^6*b^7*c^4*d^9
 - 436*a^6*b^7*c^6*d^7 + 412*a^6*b^7*c^8*d^5 - 132*a^6*b^7*c^10*d^3 - 92*a^
7*b^6*c^3*d^10 + 362*a^7*b^6*c^5*d^8 - 484*a^7*b^6*c^7*d^6 + 240*a^7*b^6*c^
9*d^4 - 28*a^7*b^6*c^11*d^2 + 32*a^8*b^5*c^2*d^11 - 208*a^8*b^5*c^4*d^9 + 4
12*a^8*b^5*c^6*d^7 - 292*a^8*b^5*c^8*d^5 + 56*a^8*b^5*c^10*d^3 + 83*a^9*b^4
*c^3*d^10 - 248*a^9*b^4*c^5*d^8 + 240*a^9*b^4*c^7*d^6 - 70*a^9*b^4*c^9*d^4
- 22*a^10*b^3*c^2*d^11 + 98*a^10*b^3*c^4*d^9 - 132*a^10*b^3*c^6*d^7 + 56*a^
10*b^3*c^8*d^5 - 22*a^11*b^2*c^3*d^10 + 47*a^11*b^2*c^5*d^8 - 28*a^11*b^2*c
^7*d^6))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d
```

^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^1
0*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b
^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*
c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20
*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2
*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^
5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^
8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b
^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 -
32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) +
(d^2*((32*(a^2*b^13*c^15 - 2*a^4*b^11*c^15 + a^6*b^9*c^15 + a^15*c^2*d^13
- 2*a^15*c^4*d^11 + a^15*c^6*d^9 - a*b^14*c^10*d^5 + 2*a*b^14*c^12*d^3 - 5*
a^3*b^12*c^14*d + 13*a^5*b^10*c^14*d - 7*a^7*b^8*c^14*d - a^10*b^5*c*d^14 +
2*a^12*b^3*c*d^14 - 5*a^14*b*c^3*d^12 + 13*a^14*b*c^5*d^10 - 7*a^14*b*c^7*
d^8 + 7*a^2*b^13*c^9*d^6 - 13*a^2*b^13*c^11*d^4 + 5*a^2*b^13*c^13*d^2 - 20*
a^3*b^12*c^8*d^7 + 35*a^3*b^12*c^10*d^5 - 10*a^3*b^12*c^12*d^3 + 28*a^4*b^1
1*c^7*d^8 - 50*a^4*b^11*c^9*d^6 + 14*a^4*b^11*c^11*d^4 + 10*a^4*b^11*c^13*d
^2 - 14*a^5*b^10*c^6*d^9 + 40*a^5*b^10*c^8*d^7 - 25*a^5*b^10*c^10*d^5 - 14*
a^5*b^10*c^12*d^3 - 14*a^6*b^9*c^5*d^10 - 14*a^6*b^9*c^7*d^8 + 37*a^6*b^9*c
^9*d^6 + 25*a^6*b^9*c^11*d^4 - 35*a^6*b^9*c^13*d^2 + 28*a^7*b^8*c^4*d^11 -
14*a^7*b^8*c^6*d^9 - 20*a^7*b^8*c^8*d^7 - 37*a^7*b^8*c^10*d^5 + 50*a^7*b^8*
c^12*d^3 - 20*a^8*b^7*c^3*d^12 + 40*a^8*b^7*c^5*d^10 - 20*a^8*b^7*c^7*d^8 +
20*a^8*b^7*c^9*d^6 - 40*a^8*b^7*c^11*d^4 + 20*a^8*b^7*c^13*d^2 + 7*a^9*b^6
*c^2*d^13 - 50*a^9*b^6*c^4*d^11 + 37*a^9*b^6*c^6*d^9 + 20*a^9*b^6*c^8*d^7 +
14*a^9*b^6*c^10*d^5 - 28*a^9*b^6*c^12*d^3 + 35*a^10*b^5*c^3*d^12 - 25*a^10
*b^5*c^5*d^10 - 37*a^10*b^5*c^7*d^8 + 14*a^10*b^5*c^9*d^6 + 14*a^10*b^5*c^1
1*d^4 - 13*a^11*b^4*c^2*d^13 + 14*a^11*b^4*c^4*d^11 + 25*a^11*b^4*c^6*d^9 -
40*a^11*b^4*c^8*d^7 + 14*a^11*b^4*c^10*d^5 - 10*a^12*b^3*c^3*d^12 - 14*a^1
2*b^3*c^5*d^10 + 50*a^12*b^3*c^7*d^8 - 28*a^12*b^3*c^9*d^6 + 5*a^13*b^2*c^2
*d^13 + 10*a^13*b^2*c^4*d^11 - 35*a^13*b^2*c^6*d^9 + 20*a^13*b^2*c^8*d^7 -
a*b^14*c^14*d - a^14*b*c*d^14))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a
^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6
+ b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*
a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a
^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 +
19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c
^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*
a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*
d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6
*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3
+ 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c
^9*d - 6*a^9*b*c*d^9) - (32*tan(e/2 + (f*x)/2)*(8*a^3*b^12*c^15 - 3*a^15*c*
d^14 - 3*a*b^14*c^15 - 7*a^5*b^10*c^15 + 2*a^7*b^8*c^15 + 8*a^15*c^3*d^12 -
7*a^15*c^5*d^10 + 2*a^15*c^7*d^8 + 4*a*b^14*c^9*d^6 - 11*a*b^14*c^11*d^4 +
10*a*b^14*c^13*d^2 + 24*a^2*b^13*c^14*d - 64*a^4*b^11*c^14*d + 56*a^6*b^9*

c^14*d - 16*a^8*b^7*c^14*d + 4*a^9*b^6*c*d^14 - 11*a^11*b^4*c*d^14 + 10*a^1 3*b^2*c*d^14 + 24*a^14*b*c^2*d^13 - 64*a^14*b*c^4*d^11 + 56*a^14*b*c^6*d^9 - 16*a^14*b*c^8*d^7 - 32*a^2*b^13*c^8*d^7 + 88*a^2*b^13*c^10*d^5 - 80*a^2*b ^13*c^12*d^3 + 112*a^3*b^12*c^7*d^8 - 319*a^3*b^12*c^9*d^6 + 310*a^3*b^12*c ^11*d^4 - 111*a^3*b^12*c^13*d^2 - 224*a^4*b^11*c^6*d^9 + 704*a^4*b^11*c^8*d ^7 - 800*a^4*b^11*c^10*d^5 + 384*a^4*b^11*c^12*d^3 + 280*a^5*b^10*c^5*d^10 - 1078*a^5*b^10*c^7*d^8 + 1550*a^5*b^10*c^9*d^6 - 993*a^5*b^10*c^11*d^4 + 2 48*a^5*b^10*c^13*d^2 - 224*a^6*b^9*c^4*d^11 + 1232*a^6*b^9*c^6*d^9 - 2320*a ^6*b^9*c^8*d^7 + 1896*a^6*b^9*c^10*d^5 - 640*a^6*b^9*c^12*d^3 + 112*a^7*b^8 *c^3*d^12 - 1078*a^7*b^8*c^5*d^10 + 2660*a^7*b^8*c^7*d^8 - 2733*a^7*b^8*c^9 *d^6 + 1240*a^7*b^8*c^11*d^4 - 203*a^7*b^8*c^13*d^2 - 32*a^8*b^7*c^2*d^13 + 704*a^8*b^7*c^4*d^11 - 2320*a^8*b^7*c^6*d^9 + 3072*a^8*b^7*c^8*d^7 - 1856* a^8*b^7*c^10*d^5 + 448*a^8*b^7*c^12*d^3 - 319*a^9*b^6*c^3*d^12 + 1550*a^9*b ^6*c^5*d^10 - 2733*a^9*b^6*c^7*d^8 + 2128*a^9*b^6*c^9*d^6 - 686*a^9*b^6*c^1 1*d^4 + 56*a^9*b^6*c^13*d^2 + 88*a^10*b^5*c^2*d^13 - 800*a^10*b^5*c^4*d^11 + 1896*a^10*b^5*c^6*d^9 - 1856*a^10*b^5*c^8*d^7 + 784*a^10*b^5*c^10*d^5 - 1 12*a^10*b^5*c^12*d^3 + 310*a^11*b^4*c^3*d^12 - 993*a^11*b^4*c^5*d^10 + 1240 *a^11*b^4*c^7*d^8 - 686*a^11*b^4*c^9*d^6 + 140*a^11*b^4*c^11*d^4 - 80*a^12* b^3*c^2*d^13 + 384*a^12*b^3*c^4*d^11 - 640*a^12*b^3*c^6*d^9 + 448*a^12*b^3* c^8*d^7 - 112*a^12*b^3*c^10*d^5 - 111*a^13*b^2*c^3*d^12 + 248*a^13*b^2*c^5* d^10 - 203*a^13*b^2*c^7*d^8 + 56*a^13*b^2*c^9*d^6))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^ 2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12 *a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12* a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 3 2*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^ 5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a ^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d ^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6* b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2 *c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9))*(-(c + d)^3*(c - d)^3)^(1/2)*(2* b*d^2 - 3*b*c^2 + a*c*d))/(a^3*d^9 + b^3*c^9 - 3*a^3*c^2*d^7 + 3*a^3*c^4*d^ 5 - a^3*c^6*d^3 - b^3*c^3*d^6 + 3*b^3*c^5*d^4 - 3*b^3*c^7*d^2 + 3*a*b^2*c^2 *d^7 - 9*a*b^2*c^4*d^5 + 9*a*b^2*c^6*d^3 + 9*a^2*b*c^3*d^6 - 9*a^2*b*c^5*d^ 4 + 3*a^2*b*c^7*d^2 - 3*a*b^2*c^8*d - 3*a^2*b*c*d^8))*(2*b*d^2 - 3*b*c^2 + a*c*d))/(a^3*d^9 + b^3*c^9 - 3*a^3*c^2*d^7 + 3*a^3*c^4*d^5 - a^3*c^6*d^3 - b^3*c^3*d^6 + 3*b^3*c^5*d^4 - 3*b^3*c^7*d^2 + 3*a*b^2*c^2*d^7 - 9*a*b^2*c^4 *d^5 + 9*a*b^2*c^6*d^3 + 9*a^2*b*c^3*d^6 - 9*a^2*b*c^5*d^4 + 3*a^2*b*c^7*d^ 2 - 3*a*b^2*c^8*d - 3*a^2*b*c*d^8))*(2*b*d^2 - 3*b*c^2 + a*c*d))/(a^3*d^9 + b^3*c^9 - 3*a^3*c^2*d^7 + 3*a^3*c^4*d^5 - a^3*c^6*d^3 - b^3*c^3*d^6 + 3*b^ 3*c^5*d^4 - 3*b^3*c^7*d^2 + 3*a*b^2*c^2*d^7 - 9*a*b^2*c^4*d^5 + 9*a*b^2*c^6 *d^3 + 9*a^2*b*c^3*d^6 - 9*a^2*b*c^5*d^4 + 3*a^2*b*c^7*d^2 - 3*a*b^2*c^8*d - 3*a^2*b*c*d^8) - (d^2*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(4*a*b^10*c^4*d^7 - 8*a*b^10*c^6*d^5 + 4*a*b^10*c^8*d^3 + a^3*b^8*c^10*d + 4*a^4*b^7*c*d^10

- 8*a^6*b^5*c*d^10 + 4*a^8*b^3*c*d^10 + a^10*b*c^3*d^8 - 4*a^2*b^9*c^3*d^8 + 8*a^2*b^9*c^5*d^6 - 7*a^2*b^9*c^7*d^4 + 4*a^2*b^9*c^9*d^2 - 4*a^3*b^8*c^2*d^9 + 21*a^3*b^8*c^6*d^5 - 22*a^3*b^8*c^8*d^3 - 18*a^4*b^7*c^5*d^6 + 26*a^4*b^7*c^7*d^4 - 8*a^4*b^7*c^9*d^2 + 8*a^5*b^6*c^2*d^9 - 18*a^5*b^6*c^4*d^7 - 8*a^5*b^6*c^6*d^5 + 22*a^5*b^6*c^8*d^3 + 21*a^6*b^5*c^3*d^8 - 8*a^6*b^5*c^5*d^6 - 15*a^6*b^5*c^7*d^4 - 7*a^7*b^4*c^2*d^9 + 26*a^7*b^4*c^4*d^7 - 15*a^7*b^4*c^6*d^5 - 22*a^8*b^3*c^3*d^8 + 22*a^8*b^3*c^5*d^6 + 4*a^9*b^2*c^2*d^9 - 8*a^9*b^2*c^4*d^7))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) - (32*tan(e/2 + (f*x)/2)*(a^3*b^8*c^11 + a^11*c^3*d^8 - 16*a*b^10*c^3*d^8 + 44*a*b^10*c^5*d^6 - 34*a*b^10*c^7*d^4 + 4*a*b^10*c^9*d^2 + 4*a^2*b^9*c^10*d - 16*a^3*b^8*c*d^10 - 8*a^4*b^7*c^10*d + 44*a^5*b^6*c*d^10 - 34*a^7*b^4*c*d^10 + 4*a^9*b^2*c*d^10 + 4*a^10*b*c^2*d^9 - 8*a^10*b*c^4*d^7 + 32*a^2*b^9*c^2*d^9 - 104*a^2*b^9*c^4*d^7 + 100*a^2*b^9*c^6*d^5 - 24*a^2*b^9*c^8*d^3 + 120*a^3*b^8*c^3*d^8 - 222*a^3*b^8*c^5*d^6 + 134*a^3*b^8*c^7*d^4 - 24*a^3*b^8*c^9*d^2 - 104*a^4*b^7*c^2*d^9 + 312*a^4*b^7*c^4*d^7 - 272*a^4*b^7*c^6*d^5 + 60*a^4*b^7*c^8*d^3 - 222*a^5*b^6*c^3*d^8 + 316*a^5*b^6*c^5*d^6 - 136*a^5*b^6*c^7*d^4 + 22*a^5*b^6*c^9*d^2 + 100*a^6*b^5*c^2*d^9 - 272*a^6*b^5*c^4*d^7 + 192*a^6*b^5*c^6*d^5 - 24*a^6*b^5*c^8*d^3 + 134*a^7*b^4*c^3*d^8 - 136*a^7*b^4*c^5*d^6 + 18*a^7*b^4*c^7*d^4 - 24*a^8*b^3*c^2*d^9 + 60*a^8*b^3*c^4*d^7 - 24*a^8*b^3*c^6*d^5 - 24*a^9*b^2*c^3*d^8 + 22*a^9*b^2*c^5*d^6))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) + (d^2*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(a^3*b^10*c^13 - a^5*b^8*c^13 + a^13*c^3*d^10 - a^13*c^5*d^8 + 2*a*b^12*c^7*d^6 - 5*a*b^12*c^9*d^4 + 3*a*b^12*c^11*d^2 + 2*a^2*b^11*c^12*d - 10*a^4*b^9*c^12*d + 8*a^6*b^7*c^12*d + 2*a^7*b^6*c*d^12 - 5*a^9*b^4*c*d^12 + 3*a^11*b^2*c*d^12 + 2*a^12*b*c^2*d^11 - 10* a^12*b*c^4*d^9 + 8*a^12*b*c^6*d^7 - 12*a^2*b^11*c^6*d^7 + 32*a^2*b^11*c^8*d^5 - 22*a^2*b^11*c^10*d^3 + 30*a^3*b^10*c^5*d^8 - 92*a^3*b^10*c^7*d^6 + 83*a^3*b^10*c^9*d^4 - 22*a^3*b^10*c^11*d^2 - 40*a^4*b^9*c^4*d^9 + 160*a^4*b^9*c^6*d^7 - 208*a^4*b^9*c^8*d^5 + 98*a^4*b^9*c^10*d^3 + 30*a^5*b^8*c^3*d^10 - 190*a^5*b^8*c^5*d^8 + 362*a^5*b^8*c^7*d^6 - 248*a^5*b^8*c^9*d^4 + 47*a^5*b^8*c^11*d^2 - 12*a^6*b^7*c^2*d^11 + 160*a^6*b^7*c^4*d^9 - 436*a^6*b^7*c^6*d^7 + 412*a^6*b^7*c^8*d^5 - 132*a^6*b^7*c^10*d^3 - 92*a^7*b^6*c^3*d^10 + 362*a^7*b^6*c^5*d^8 - 484*a^7*b^6*c^7*d^6 + 240*a^7*b^6*c^9*d^4 - 28*a^7*b^6*c^11*d^2 + 32*a^8*b^5*c^2*d^11 - 208*a^8*b^5*c^4*d^9 + 412*a^8*b^5*c^6*d^7 - 292*a^8*b^5*c^8*d^5 + 56*a^8*b^5*c^10*d^3 + 83*a^9*b^4*c^3*d^10 - 248*a^9*b^4*c^5*d^8 + 240*a^9*b^4*c^7*d^6 - 70*a^9*b^4*c^9*d^4 - 22*a^10*b^3*c^2*d^11 + 98*a^10*b^3*c^4*d^9 - 132*a^10*b^3*c^6*d^7 + 56*a^10*b^3*c^8*d^5 - 22*a^11*b^2*c^3*d^10 + 47*a^11*b^2*c^5*d^8 - 28*a^11*b^2*c^7*d^6))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) - (32*tan(e/2 + (f*x)/2)*(2*a^4*b^9*c^13 - 2*a^2*b^11*c^13 - 2*a^13*c^2*d^11 + 2*a^13*c^4*d^9 - 2*a*b^12*c^8*d^5 + 6*a*b^12*c^10*d^3 + 20*a^3*b^10*c^12*d - 16*a^5*b^8*c^12*d - 2*a^8*b^5*c*d^12 + 6*a^10*b^3*c*d^12 + 20*a^12*b*c^3*d^10 - 16*a^12*b*c^5*d^8 + 10*a^2*b^11*c^7*d^6 - 34*a^2*b^11*c^9*d^4 + 26*a^2*b^11*c^11*d^2 - 18*a^3*b^10*c^6*d^7 + 80*a^3*b^10*c^8*d^5 - 82*a^3*b^10*c^10*d^3 + 10*a^4*b^9*c^5*d^8 - 96*a^4*b^9*c^7*d^6 + 160*a^4*b^9*c^9*d^4 - 76*a^4*b^9*c^11*d^2 + 10*a^5*b^8*c^4*d^9 + 44*a^5*b^8*c^6*d^7 - 188*a^5*b^8*c^8*d^5 + 150*a^5*b^8*c^10*d^3 - 18*a^6*b^7*c^3*d^10 + 44*a^6*b^7*c^5*d^8 + 88*a^6*b^7*c^7*d^6 - 164*a^6*b^7*c^9*d^4 + 50*a^6*b^7*c^11*d^2 + 10*a^7*b^6*c^2*d^11 - 96*a^7*b^6*c^4*d^9 + 88*a^7*b^6*c^6*d^7 + 72*a^7*b^6*c^8*d^5 - 74*a^7*b^6*c^10*d^3 + 80*a^8*b^5*c^3*d^10 - 188*a^8*b^5*c^5*d^8 + 72*a^8*b^5*c^7*d^6 + 38*a^8*b^5*c^9*d^4 - 34*a^9*b^4*c^2*d^11 + 160*a^9*b^4*c^4*d^9 - 164*a^9*b^4*c^6*d^7 + 38*a^9*b^4*c^8*d^5 - 82*a^10*b^3*c^3*d^10 + 150*a^10*b^3*c^5*d^8 - 74*a^10*b^3*c^7*d^6 + 26*a^11*b^2*c^2*d^11 - 76*a^11*b^2*c^4*d^9 + 50*a^11*b^2*c^6*d^7 - 4*a*b^12*c^12*d - 4*a^12*b*c*d^12))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^

4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) + (d^2*((32*(a^2*b^13*c^15 - 2*a^4*b^11*c^15 + a^6*b^9*c^15 + a^15*c^2*d^13 - 2*a^15*c^4*d^11 + a^15*c^6*d^9 - a*b^14*c^10*d^5 + 2*a*b^14*c^12*d^3 - 5*a^3*b^12*c^14*d + 13*a^5*b^10*c^14*d - 7*a^7*b^8*c^14*d - a^10*b^5*c*d^14 + 2*a^12*b^3*c*d^14 - 5*a^14*b*c^3*d^12 + 13*a^14*b*c^5*d^10 - 7*a^14*b*c^7*d^8 + 7*a^2*b^13*c^9*d^6 - 13*a^2*b^13*c^11*d^4 + 5*a^2*b^13*c^13*d^2 - 20*a^3*b^12*c^8*d^7 + 35*a^3*b^12*c^10*d^5 - 10*a^3*b^12*c^12*d^3 + 28*a^4*b^11*c^7*d^8 - 50*a^4*b^11*c^9*d^6 + 14*a^4*b^11*c^11*d^4 + 10*a^4*b^11*c^13*d^2 - 14*a^5*b^10*c^6*d^9 + 40*a^5*b^10*c^8*d^7 - 25*a^5*b^10*c^10*d^5 - 14*a^5*b^10*c^12*d^3 - 14*a^6*b^9*c^5*d^10 - 14*a^6*b^9*c^7*d^8 + 37*a^6*b^9*c^9*d^6 + 25*a^6*b^9*c^11*d^4 - 35*a^6*b^9*c^13*d^2 + 28*a^7*b^8*c^4*d^11 - 14*a^7*b^8*c^6*d^9 - 20*a^7*b^8*c^8*d^7 - 37*a^7*b^8*c^10*d^5 + 50*a^7*b^8*c^12*d^3 - 20*a^8*b^7*c^3*d^12 + 40*a^8*b^7*c^5*d^10 - 20*a^8*b^7*c^7*d^8 + 20*a^8*b^7*c^9*d^6 - 40*a^8*b^7*c^11*d^4 + 20*a^8*b^7*c^13*d^2 + 7*a^9*b^6*c^2*d^13 - 50*a^9*b^6*c^4*d^11 + 37*a^9*b^6*c^6*d^9 + 20*a^9*b^6*c^8*d^7 + 14*a^9*b^6*c^10*d^5 - 28*a^9*b^6*c^12*d^3 + 35*a^10*b^5*c^3*d^12 - 25*a^10*b^5*c^5*d^10 - 37*a^10*b^5*c^7*d^8 + 14*a^10*b^5*c^9*d^6 + 14*a^10*b^5*c^11*d^4 - 13*a^11*b^4*c^2*d^13 + 14*a^11*b^4*c^4*d^11 + 25*a^11*b^4*c^6*d^9 - 40*a^11*b^4*c^8*d^7 + 14*a^11*b^4*c^10*d^5 - 10*a^12*b^3*c^3*d^12 - 14*a^12*b^3*c^5*d^10 + 50*a^12*b^3*c^7*d^8 - 28*a^12*b^3*c^9*d^6 + 5*a^13*b^2*c^2*d^13 + 10*a^13*b^2*c^4*d^11 - 35*a^13*b^2*c^6*d^9 + 20*a^13*b^2*c^8*d^7 - a*b^14*c^14*d - a^14*b*c*d^14))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9) - (32*tan(e/2 + (f*x)/2)*(8*a^3*b^12*c^15 - 3*a^15*c*d^14 - 3*a*b^14*c^15 - 7*a^5*b^10*c^15 + 2*a^7*b^8*c^15 + 8*a^15*c^3*d^12 - 7*a^15*c^5*d^10 + 2*a^15*c^7*d^8 + 4*a*b^14*c^9*d^6 - 11*a*b^14*c^11*d^4 + 10*a*b^14*c^13*d^2 + 24*a^2*b^13*c^14*d - 64*a^4*b^11*c^14*d + 56*a^6*b^9*c^14*d - 16*a^8*b^7*c^14*d + 4*a^9*b^6*c*d^14 - 11*a^11*b^4*c*d^14 + 10*a^13*b^2*c*d^14 + 24*a^14*b*c^2*d^13 - 64*a^14*b*c^4*d^11 + 56*a^14*b*c^6*d^9 - 16*a^14*b*c^8*d^7 - 32*a^2*b^13*c^8*d^7 + 88*a^2*b^13*c^10*d^5 - 80*a^2*b^13*c^12*d^3 + 112*a^3*b^12*c^7*d^8 - 319*a^3*b^12*c^9*d^6 + 310*a^3*b^12*c^11*d^4 - 111*a^3*b^12*c^13*d^2 - 224*a^4*b^11*c^6*d^9 + 704*a^4*b^11*c^8*d^7 - 800*a^4*b^11*c^10*d^5 + 384* a^4*b^11*c^12*d^3 + 280*a^5*b^10*c^5*d^10 - 1078*a^5*b^10*c^7*d^8 + 1550*a^5*b^10*c^9*d^6 - 993*a^5*b^10*c^11*d^4 + 248*a^5*b^10*c^13*d^2 - 224*a^6*b^9*c^4*d^11 + 1232*a^6*b^9*c^6*d^9 - 2320*a^6*b^9*c^8*d^7 + 1896*a^6*b^9*c^10*d^5 - 640*a^6*b^9*c^12*d^3 + 112*a^7*b^8*c^3*d^12 - 1078*a^7*b^8*c^5*d^10 + 2660*a^7*b^8*c^7*d^8 - 2733*a^7*b^8*c^9*d^6 + 1240*a^7*b^8*c^11*d^4 - 203*a^7*b^8*c^13*d^2 - 32*a^8*b^7*c^2*d^13 + 704*a^8*b^7*c^4*d^11 - 2320*a^8*b^7*c^6*d^9 + 3072*a^8*b^7*c^8*d^7 - 1856*a^8*b^7*c^10*d^5 + 448*a^8*b^7*c^12*d^3 - 319*a^9*b^6*c^3*d^12 + 1550*a^9*b^6*c^5*d^10 - 2733*a^9*b^6*c^7*d^8 + 2128*a^9*b^6*c^9*d^6 - 686*a^9*b^6*c^11*d^4 + 56*a^9*b^6*c^13*d^2 + 88*a^10*b^5*c^2*d^13 - 800*a^10*b^5*c^4*d^11 + 1896*a^10*b^5*c^6*d^9 - 1856*a^10*b^5*c^8*d^7 + 784*a^10*b^5*c^10*d^5 - 112*a^10*b^5*c^12*d^3 + 310*a^11*b^4*c^3*d^12 - 993*a^11*b^4*c^5*d^10 + 1240*a^11*b^4*c^7*d^8 - 686*a^11*b^4*c^9*d^6 + 140*a^11*b^4*c^11*d^4 - 80*a^12*b^3*c^2*d^13 + 384*a^12*b^3*c^4*d^11 - 640*a^12*b^3*c^6*d^9 + 448*a^12*b^3*c^8*d^7 - 112*a^12*b^3*c^10*d^5 - 111*a^13*b^2*c^3*d^12 + 248*a^13*b^2*c^5*d^10 - 203*a^13*b^2*c^7*d^8 + 56*a^13*b^2*c^9*d^6))/(a^10*d^10 + b^10*c^10 - 2*a^2*b^8*c^10 + a^4*b^6*c^10 + a^6*b^4*d^10 - 2*a^8*b^2*d^10 - 2*a^10*c^2*d^8 + a^10*c^4*d^6 + b^10*c^6*d^4 - 2*b^10*c^8*d^2 - 6*a*b^9*c^5*d^5 + 12*a*b^9*c^7*d^3 + 12*a^3*b^7*c^9*d - 6*a^5*b^5*c*d^9 - 6*a^5*b^5*c^9*d + 12*a^7*b^3*c*d^9 + 12*a^9*b*c^3*d^7 - 6*a^9*b*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 32*a^2*b^8*c^6*d^4 + 19*a^2*b^8*c^8*d^2 - 20*a^3*b^7*c^3*d^7 + 52*a^3*b^7*c^5*d^5 - 44*a^3*b^7*c^7*d^3 + 15*a^4*b^6*c^2*d^8 - 60*a^4*b^6*c^4*d^6 + 76*a^4*b^6*c^6*d^4 - 32*a^4*b^6*c^8*d^2 + 52*a^5*b^5*c^3*d^7 - 92*a^5*b^5*c^5*d^5 + 52*a^5*b^5*c^7*d^3 - 32*a^6*b^4*c^2*d^8 + 76*a^6*b^4*c^4*d^6 - 60*a^6*b^4*c^6*d^4 + 15*a^6*b^4*c^8*d^2 - 44*a^7*b^3*c^3*d^7 + 52*a^7*b^3*c^5*d^5 - 20*a^7*b^3*c^7*d^3 + 19*a^8*b^2*c^2*d^8 - 32*a^8*b^2*c^4*d^6 + 15*a^8*b^2*c^6*d^4 - 6*a*b^9*c^9*d - 6*a^9*b*c*d^9))*(-(c + d)^3*(c - d)^3)^(1/2)*(2*b*d^2 - 3*b*c^2 + a*c*d))/(a^3*d^9 + b^3*c^9 - 3*a^3*c^2*d^7 + 3*a^3*c^4*d^5 - a^3*c^6*d^3 - b^3*c^3*d^6 + 3*b^3*c^5*d^4 - 3*b^3*c^7*d^2 + 3*a*b^2*c^2*d^7 - 9*a*b^2*c^4*d^5 + 9*a*b^2*c^6*d^3 + 9*a^2*b*c^3*d^6 - 9*a^2*b*c^5*d^4 + 3*a^2*b*c^7*d^2 - 3*a*b^2*c^8*d - 3*a^2*b*c*d^8))*(2*b*d^2 - 3*b*c^2 + a*c*d))/(a^3*d^9 + b^3*c^9 - 3*a^3*c^2*d^7 + 3*a^3*c^4*d^5 - a^3*c^6*d^3 - b^3*c^3*d^6 + 3*b^3*c^5*d^4 - 3*b^3*c^7*d^2 + 3*a*b^2*c^2*d^7 - 9*a*b^2*c^4*d^5 + 9*a*b^2*c^6*d^3 + 9*a^2*b*c^3*d^6 - 9*a^2*b*c^5*d^4 + 3*a^2*b*c^7*d^2 - 3*a*b^2*c^8*d - 3*a^2*b*c*d^8))*(2*b*d^2 - 3*b*c^2 + a*c*d))/(a^3*d^9 + b^3*c^9 - 3*a^3*c^2*d^7 + 3*a^3*c^4*d^5 - a^3*c^6*d^3 - b^3*c^3*d^6 + 3*b^3*c^5*d^4 - 3*b^3*c^7*d^2 + 3*a*b^2*c^2*d^7 - 9*a*b^2*c^4*d^5 + 9*a*b^2*c^6*d^3 + 9*a^2*b*c^3*d^6 - 9*a^2*b*c^5*d^4 + 3*a^2*b*c^7*d^2 - 3*a*b^2*c^8*d - 3*a^2*b*c*d^8)))*(-(c + d)^3*(c - d)^3)^(1/2)*(2*b*d^2 - 3*b*c^2 + a*c*d)*2i)/(f*(a^3*d^9 + b^3*c^9 - 3*a^3*c^2*d^7 + 3*a^3*c^4*d^5 - a^3*c^6*d^3 - b^3*c^3*d^6 + 3*b^3*c^5*d^4 - 3*b^3*c^7*d^2 + 3*a*b^2*c^2*d^7 - 9*a*b^2*c^4*d^5 + 9*a*b^2*c^6*d^3 + 9*a^2*b*c^3*d^6 - 9*a^2*b*c^5*d^4 + 3*a^2*b*c^7*d^2 - 3*a*b^2*c^8*d - 3*a^2*b*c*d^8))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.713 $\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=458

$$\frac{d\left(a^2d^2+b^2\left(2c^2-3d^2\right)\right)\cos(e+fx)}{2f\left(a^2-b^2\right)\left(c^2-d^2\right)(bc-ad)^2(c+d\sin(e+fx))^2}-\frac{d^2\left(-a^2d^2\left(2c^2+d^2\right)+2abcd\left(4c^2-d^2\right)-3b^2\left(4c^4-5c^2d^2\right.\right.}{f\left(c^2-d^2\right)^{5/2}(bc-ad)^4}$$

```
[Out] 2*b^3*(-4*a^2*d+a*b*c+3*b^2*d)*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1
/2))/(a^2-b^2)^(3/2)/(-a*d+b*c)^4/f-d^2*(2*a*b*c*d*(4*c^2-d^2)-a^2*d^2*(2*c
^2+d^2)-3*b^2*(4*c^4-5*c^2*d^2+2*d^4))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2
-d^2)^(1/2))/(-a*d+b*c)^4/(c^2-d^2)^(5/2)/f+1/2*d*(a^2*d^2+b^2*(2*c^2-3*d^2
))*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)^2/(c^2-d^2)/f/(c+d*sin(f*x+e))^2+b^2*cos
(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2-1/2*(3*a
^3*c*d^4-3*a*b^2*c*d^4-a^2*b*d^3*(7*c^2-4*d^2)-b^3*(2*c^4*d-11*c^2*d^3+6*d^
5))*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)^3/(c^2-d^2)^2/f/(c+d*sin(f*x+e))
```

Rubi [A] time = 2.44, antiderivative size = 458, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2802, 3055, 3001, 2660, 618, 204}

$$-\frac{d^2\left(-a^2d^2\left(2c^2+d^2\right)+2abcd\left(4c^2-d^2\right)-3b^2\left(-5c^2d^2+4c^4+2d^4\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{5/2}(bc-ad)^4}-\frac{\left(-a^2bd^3\left(7c^2-4d\right.\right.}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] (2*b^3*(a*b*c - 4*a^2*d + 3*b^2*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2
 - b^2]])/((a^2 - b^2)^(3/2)*(b*c - a*d)^4*f) - (d^2*(2*a*b*c*d*(4*c^2 - d^
2) - a^2*d^2*(2*c^2 + d^2) - 3*b^2*(4*c^4 - 5*c^2*d^2 + 2*d^4))*ArcTan[(d +
 c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^4*(c^2 - d^2)^(5/2)*f)
+ (d*(a^2*d^2 + b^2*(2*c^2 - 3*d^2))*Cos[e + f*x])/(2*(a^2 - b^2)*(b*c - a*
d)^2*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^2) + (b^2*Cos[e + f*x])/((a^2 - b^2
)*(b*c - a*d)*f*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^2) - ((3*a^3*c*d^
4 - 3*a*b^2*c*d^4 - a^2*b*d^3*(7*c^2 - 4*d^2) - b^3*(2*c^4*d - 11*c^2*d^3 +
 6*d^5))*Cos[e + f*x])/(2*(a^2 - b^2)*(b*c - a*d)^3*(c^2 - d^2)^2*f*(c + d*
Sin[e + f*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
```

```
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx = \frac{b^2\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))(c+d\sin(e+fx))^2} - \frac{\int \cdots}{\cdots}$$

$$= \frac{d\left(a^2d^2+b^2\left(2c^2-3d^2\right)\right)\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{\cdots}{\left(a^2-b^2\right)\cdots}$$

$$= \frac{d\left(a^2d^2+b^2\left(2c^2-3d^2\right)\right)\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{\cdots}{\left(a^2-b^2\right)\cdots}$$

$$= \frac{d\left(a^2d^2+b^2\left(2c^2-3d^2\right)\right)\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{\cdots}{\left(a^2-b^2\right)\cdots}$$

$$= \frac{d\left(a^2d^2+b^2\left(2c^2-3d^2\right)\right)\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{\cdots}{\left(a^2-b^2\right)\cdots}$$

$$= \frac{d\left(a^2d^2+b^2\left(2c^2-3d^2\right)\right)\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{\cdots}{\left(a^2-b^2\right)\cdots}$$

$$= \frac{2b^3\left(abc-4a^2d+3b^2d\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}(bc-ad)^4f} - \frac{d^2\left(2abcd\left(4c^2\cdots\right.\right.}{\cdots}$$

Mathematica [A] time = 6.59, size = 346, normalized size = 0.76

$$\frac{\frac{2d^2\left(a^2d^2\left(2c^2+d^2\right)+2abcd\left(d^2-4c^2\right)+3b^2\left(4c^4-5c^2d^2+2d^4\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{5/2}(bc-ad)^4}+\frac{4b^3\left(-4a^2d+abc+3b^2d\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}(bc-ad)^4}-\frac{\cdots}{(a-b)(a+b\cdots}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] ((4*b^3*(a*b*c - 4*a^2*d + 3*b^2*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^
2 - b^2]])/((a^2 - b^2)^(3/2)*(b*c - a*d)^4) + (2*d^2*(2*a*b*c*d*(-4*c^2 +
d^2) + a^2*d^2*(2*c^2 + d^2) + 3*b^2*(4*c^4 - 5*c^2*d^2 + 2*d^4))*ArcTan[(d
 + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^4*(c^2 - d^2)^(5/2))
- (2*b^4*Cos[e + f*x])/((a - b)*(a + b)*(-(b*c) + a*d)^3*(a + b*Sin[e + f*x
])) + (d^3*Cos[e + f*x])/((c - d)*(c + d)*(b*c - a*d)^2*(c + d*Sin[e + f*x]
)^2) + (d^3*(7*b*c^2 - 3*a*c*d - 4*b*d^2)*Cos[e + f*x])/((c - d)^2*(c + d)^
2*(b*c - a*d)^3*(c + d*Sin[e + f*x])))/(2*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.42, size = 1109, normalized size = 2.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] (2*(a*b^4*c - 4*a^2*b^3*d + 3*b^5*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(
a) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^2*b^4*c^4 -
b^6*c^4 - 4*a^3*b^3*c^3*d + 4*a*b^5*c^3*d + 6*a^4*b^2*c^2*d^2 - 6*a^2*b^4*c
^2*d^2 - 4*a^5*b*c*d^3 + 4*a^3*b^3*c*d^3 + a^6*d^4 - a^4*b^2*d^4)*sqrt(a^2
- b^2)) + (12*b^2*c^4*d^2 - 8*a*b*c^3*d^3 + 2*a^2*c^2*d^4 - 15*b^2*c^2*d^4
+ 2*a*b*c*d^5 + a^2*d^6 + 6*b^2*d^6)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(
c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((b^4*c^8 - 4*a*
b^3*c^7*d + 6*a^2*b^2*c^6*d^2 - 2*b^4*c^6*d^2 - 4*a^3*b*c^5*d^3 + 8*a*b^3*c
```

```
^5*d^3 + a^4*c^4*d^4 - 12*a^2*b^2*c^4*d^4 + b^4*c^4*d^4 + 8*a^3*b*c^3*d^5 -
 4*a*b^3*c^3*d^5 - 2*a^4*c^2*d^6 + 6*a^2*b^2*c^2*d^6 - 4*a^3*b*c*d^7 + a^4*
d^8)*sqrt(c^2 - d^2)) + 2*(b^5*tan(1/2*f*x + 1/2*e) + a*b^4)/((a^3*b^3*c^3
- a*b^5*c^3 - 3*a^4*b^2*c^2*d + 3*a^2*b^4*c^2*d + 3*a^5*b*c*d^2 - 3*a^3*b^3
*c*d^2 - a^6*d^3 + a^4*b^2*d^3)*(a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x
 + 1/2*e) + a)) + (9*b*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 - 5*a*c^3*d^5*tan(1/2
*f*x + 1/2*e)^3 - 6*b*c^2*d^6*tan(1/2*f*x + 1/2*e)^3 + 2*a*c*d^7*tan(1/2*f*
x + 1/2*e)^3 + 8*b*c^5*d^3*tan(1/2*f*x + 1/2*e)^2 - 4*a*c^4*d^4*tan(1/2*f*x
 + 1/2*e)^2 + 11*b*c^3*d^5*tan(1/2*f*x + 1/2*e)^2 - 7*a*c^2*d^6*tan(1/2*f*x
 + 1/2*e)^2 - 10*b*c*d^7*tan(1/2*f*x + 1/2*e)^2 + 2*a*d^8*tan(1/2*f*x + 1/2
*e)^2 + 23*b*c^4*d^4*tan(1/2*f*x + 1/2*e) - 11*a*c^3*d^5*tan(1/2*f*x + 1/2*
e) - 14*b*c^2*d^6*tan(1/2*f*x + 1/2*e) + 2*a*c*d^7*tan(1/2*f*x + 1/2*e) + 8
*b*c^5*d^3 - 4*a*c^4*d^4 - 5*b*c^3*d^5 + a*c^2*d^6)/((b^3*c^9 - 3*a*b^2*c^8
*d + 3*a^2*b*c^7*d^2 - 2*b^3*c^7*d^2 - a^3*c^6*d^3 + 6*a*b^2*c^6*d^3 - 6*a^
2*b*c^5*d^4 + b^3*c^5*d^4 + 2*a^3*c^4*d^5 - 3*a*b^2*c^4*d^5 + 3*a^2*b*c^3*d
^6 - a^3*c^2*d^7)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)
^2))/f
```

maple [B] time = 0.44, size = 4023, normalized size = 8.78

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x)
```

```
[Out] -2/f*d^9/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*c+2*
tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)/c^2*tan(1/2*f*x+1/2*e)^2*a^2+
8/f*d^3/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*c+2*t
an(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^4*tan(1/2*f*x+1/2*e)^2*b^2-1
5/f*d^4/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c^4-2*c^2*d^2+d^4)/(c^2-d^
2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b^2*c^2+1
2/f*d^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c^4-2*c^2*d^2+d^4)/(c^2-d^
2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b^2*c^4+1
1/f*d^5/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*c+2*t
an(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^2*tan(1/2*f*x+1/2*e)^2*b^2+1
1/f*d^6/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*c+2*t
an(1/2*f*x+1/2*e)*d+c)^2*c/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*a^2+2/f*d
^4/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1
/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a^2*c^2+8/f*d^
7/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2
*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)^3*a*b+16/f*d^7/(a
^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x
+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*a*b+5/f*d^6/(a^2*d^2-
2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)
*d+c)^2/(c^4-2*c^2*d^2+d^4)*c*tan(1/2*f*x+1/2*e)^3*a^2+2/f*b^6/(a*d-b*c)^2/
```

(a^2*d^2-2*a*b*c*d+b^2*c^2)/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+
a)/a/(a^2-b^2)*tan(1/2*f*x+1/2*e)*c+2/f*b^4/(a*d-b*c)^2/(a^2*d^2-2*a*b*c*d+
b^2*c^2)/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^
(1/2))*a*c-5/f*d^5/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(tan(1/2*f*x+1/2
*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*b^2*c^2+7/f*d^7/(a^
2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+
1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)^2*a^2-10/f*d^7/(a^2*d^
2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*
e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)^2*b^2+1/f*d^6/(a^2*d^2-2*a
*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*
(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a^2+6/f*d^6/(a^2*d^2-2*a*b*c*
d+b^2*c^2)/(a*d-b*c)^2/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*
tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b^2+4/f*d^5/(a^2*d^2-2*a*b*c*d+b^2
*c^2)/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-
2*c^2*d^2+d^4)*a^2*c^2+8/f*d^3/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(tan
(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*b^2*c^4
-2/f*b^5/(a*d-b*c)^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(tan(1/2*f*x+1/2*e)^2*a+2*
tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*tan(1/2*f*x+1/2*e)*d-2/f*b^4/(a*d-b*c)^2/
(a^2*d^2-2*a*b*c*d+b^2*c^2)/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+
a)/(a^2-b^2)*d*a-8/f*b^3/(a*d-b*c)^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a^2-b^2)^
(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*d-18/f*d
^6/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/
2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c*tan(1/2*f*x+1/2*e)^2*a*b+12/f*d^8
/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*
f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)/c*tan(1/2*f*x+1/2*e)^2*a*b-34/f*d^5/(
a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*
x+1/2*e)*d+c)^2*c^2/(c^4-2*c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*a*b-8/f*d^3/(a^2
*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arc
tan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a*b*c^3+2/f*d^5/(a^2*
d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c^4-2*c^2*d^2+d^4)/(c^2-d^2)^(1/2)*arct
an(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a*b*c-14/f*d^5/(a^2*d^
2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*
e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^2*tan(1/2*f*x+1/2*e)^3*a*b-12/f*d^4/(a^2*d^
2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*
e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*c^3*tan(1/2*f*x+1/2*e)^2*a*b-1/f*d^7/(a^2*d^2
-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e
)*d+c)^2/(c^4-2*c^2*d^2+d^4)*a^2+2/f*b^5/(a*d-b*c)^2/(a^2*d^2-2*a*b*c*d+b^2
*c^2)/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*c+6/f*b^5
/(a*d-b*c)^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*ta
n(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*d-2/f*d^8/(a^2*d^2-2*a*b*c*d+b^2*c^2
)/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^4-2*
c^2*d^2+d^4)*tan(1/2*f*x+1/2*e)*a^2+23/f*d^4/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a
*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c^3/(c^4-2*c^
2*d^2+d^4)*tan(1/2*f*x+1/2*e)*b^2-14/f*d^6/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d
-b*c)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^4-2*c^2*d^

```
2+d^4)*tan(1/2*f*x+1/2*e)*b^2-12/f*d^4/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c
)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)
*a*b*c^3+6/f*d^6/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e
)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^4-2*c^2*d^2+d^4)*a*b*c-2/f*d^8/(a^2*d^
2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*
e)*d+c)^2/(c^4-2*c^2*d^2+d^4)/c*tan(1/2*f*x+1/2*e)^3*a^2+9/f*d^4/(a^2*d^2-2
*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*
d+c)^2/(c^4-2*c^2*d^2+d^4)*c^3*tan(1/2*f*x+1/2*e)^3*b^2-6/f*d^6/(a^2*d^2-2*
a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d
+c)^2/(c^4-2*c^2*d^2+d^4)*c*tan(1/2*f*x+1/2*e)^3*b^2+4/f*d^5/(a^2*d^2-2*a*b
*c*d+b^2*c^2)/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)
^2/(c^4-2*c^2*d^2+d^4)*c^2*tan(1/2*f*x+1/2*e)^2*a^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 45.34, size = 137274, normalized size = 299.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^3),x)
```

```
[Out] (d^2*atan(((d^2*(-(c + d)^5*(c - d)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(4*a^3*
b^11*c^16 - a^14*c*d^15 - 4*a^14*c^3*d^13 - 4*a^14*c^5*d^11 - 144*a*b^13*c^
4*d^12 + 684*a*b^13*c^6*d^10 - 1314*a*b^13*c^8*d^8 + 1224*a*b^13*c^10*d^6 -
 504*a*b^13*c^12*d^4 + 36*a*b^13*c^14*d^2 + 24*a^2*b^12*c^15*d + 144*a^4*b^
10*c*d^15 - 44*a^4*b^10*c^15*d - 348*a^6*b^8*c*d^15 + 214*a^8*b^6*c*d^15 +
7*a^10*b^4*c*d^15 - 8*a^12*b^2*c*d^15 - a^13*b*c^2*d^14 + 20*a^13*b*c^4*d^1
2 + 44*a^13*b*c^6*d^10 + 432*a^2*b^12*c^3*d^13 - 2148*a^2*b^12*c^5*d^11 + 4
470*a^2*b^12*c^7*d^9 - 4632*a^2*b^12*c^9*d^7 + 2232*a^2*b^12*c^11*d^5 - 252
*a^2*b^12*c^13*d^3 - 432*a^3*b^11*c^2*d^14 + 2688*a^3*b^11*c^4*d^12 - 7294*
a^3*b^11*c^6*d^10 + 10105*a^3*b^11*c^8*d^8 - 7104*a^3*b^11*c^10*d^6 + 1892*
a^3*b^11*c^12*d^4 - 192*a^3*b^11*c^14*d^2 - 2016*a^4*b^10*c^3*d^13 + 8378*a
^4*b^10*c^5*d^11 - 15815*a^4*b^10*c^7*d^9 + 14976*a^4*b^10*c^9*d^7 - 5932*a
^4*b^10*c^11*d^5 + 624*a^4*b^10*c^13*d^3 + 1140*a^5*b^9*c^2*d^14 - 6574*a^5
```

*b^9*c^4*d^12 + 16053*a^5*b^9*c^6*d^10 - 19912*a^5*b^9*c^8*d^8 + 11320*a^5*
b^9*c^10*d^6 - 1920*a^5*b^9*c^12*d^4 + 172*a^5*b^9*c^14*d^2 + 2938*a^6*b^8*
c^3*d^13 - 10619*a^6*b^8*c^5*d^11 + 18608*a^6*b^8*c^7*d^9 - 15576*a^6*b^8*c
^9*d^7 + 4344*a^6*b^8*c^11*d^5 - 292*a^6*b^8*c^13*d^3 - 818*a^7*b^7*c^2*d^1
4 + 5107*a^7*b^7*c^4*d^12 - 12464*a^7*b^7*c^6*d^10 + 14693*a^7*b^7*c^8*d^8
- 6184*a^7*b^7*c^10*d^6 + 368*a^7*b^7*c^12*d^4 - 1485*a^8*b^6*c^3*d^13 + 50
64*a^8*b^6*c^5*d^11 - 8939*a^8*b^6*c^7*d^9 + 6104*a^8*b^6*c^9*d^7 - 688*a^8
*b^6*c^11*d^5 + 55*a^9*b^5*c^2*d^14 - 1056*a^9*b^5*c^4*d^12 + 3649*a^9*b^5*
c^6*d^10 - 4524*a^9*b^5*c^8*d^8 + 1120*a^9*b^5*c^10*d^6 + 152*a^10*b^4*c^3*
d^13 - 975*a^10*b^4*c^5*d^11 + 2300*a^10*b^4*c^7*d^9 - 1088*a^10*b^4*c^9*d^
7 + 16*a^11*b^3*c^2*d^14 + 59*a^11*b^3*c^4*d^12 - 640*a^11*b^3*c^6*d^10 + 6
28*a^11*b^3*c^8*d^8 + 27*a^12*b^2*c^3*d^13 + 48*a^12*b^2*c^5*d^11 - 220*a^1
2*b^2*c^7*d^9))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a
^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*
c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4
+ 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*
d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^
5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 3
6*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*
c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*
d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 -
444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576
*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b
^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*
d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5
- 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*
a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b
^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d
^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7
- 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a
^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^
5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d
^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 +
126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*
a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b
^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*
c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) - (8*(3
6*a*b^13*c^5*d^11 - 144*a*b^13*c^7*d^9 + 216*a*b^13*c^9*d^7 - 144*a*b^13*c^
11*d^5 + 36*a*b^13*c^13*d^3 + 4*a^3*b^11*c^15*d - 36*a^5*b^9*c*d^15 + 60*a^
7*b^7*c*d^15 - 13*a^9*b^5*c*d^15 - 10*a^11*b^3*c*d^15 - 4*a^13*b*c^3*d^13 -
4*a^13*b*c^5*d^11 - 72*a^2*b^12*c^4*d^12 + 276*a^2*b^12*c^6*d^10 - 375*a^2
*b^12*c^8*d^8 + 216*a^2*b^12*c^10*d^6 - 60*a^2*b^12*c^12*d^4 + 24*a^2*b^12*
c^14*d^2 - 36*a^3*b^11*c^5*d^11 + 61*a^3*b^11*c^7*d^9 - 88*a^3*b^11*c^9*d^7
+ 180*a^3*b^11*c^11*d^5 - 184*a^3*b^11*c^13*d^3 + 72*a^4*b^10*c^2*d^14 - 1
68*a^4*b^10*c^4*d^12 + 233*a^4*b^10*c^6*d^10 - 270*a^4*b^10*c^8*d^8 + 100*a

^4*b^10*c^10*d^6 + 248*a^4*b^10*c^12*d^4 - 44*a^4*b^10*c^14*d^2 + 120*a^5*b^9*c^3*d^13 - 535*a^5*b^9*c^5*d^11 + 1386*a^5*b^9*c^7*d^9 - 1544*a^5*b^9*c^9*d^7 + 248*a^5*b^9*c^11*d^5 + 172*a^5*b^9*c^13*d^3 - 108*a^6*b^8*c^2*d^14 + 699*a^6*b^8*c^4*d^12 - 2046*a^6*b^8*c^6*d^10 + 2885*a^6*b^8*c^8*d^8 - 1336*a^6*b^8*c^10*d^6 - 148*a^6*b^8*c^12*d^4 - 305*a^7*b^7*c^3*d^13 + 1354*a^7*b^7*c^5*d^11 - 2979*a^7*b^7*c^7*d^9 + 2648*a^7*b^7*c^9*d^7 - 400*a^7*b^7*c^11*d^5 + 19*a^8*b^6*c^2*d^14 - 602*a^8*b^6*c^4*d^12 + 2161*a^8*b^6*c^6*d^10 - 3012*a^8*b^6*c^8*d^8 + 1056*a^8*b^6*c^10*d^6 + 190*a^9*b^5*c^3*d^13 - 895*a^9*b^5*c^5*d^11 + 1860*a^9*b^5*c^7*d^9 - 1088*a^9*b^5*c^9*d^7 + 14*a^10*b^4*c^2*d^14 + 99*a^10*b^4*c^4*d^12 - 552*a^10*b^4*c^6*d^10 + 628*a^10*b^4*c^8*d^8 + 19*a^11*b^3*c^3*d^13 + 40*a^11*b^3*c^5*d^11 - 220*a^11*b^3*c^7*d^9 - a^12*b^2*c^2*d^14 + 20*a^12*b^2*c^4*d^12 + 44*a^12*b^2*c^6*d^10 - a^13*b*c*d^15))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) + (d^2*(-(c + d)^5*(c - d)^5)^(1/2)*((8*(4*a^3*b^13*c^19 - 4*a^5*b^11*c^19 + 2*a^16*c^2*d^17 - 6*a^16*c^6*d^13 + 4*a^16*c^8*d^11 + 12*a*b^15*c^9*d^10 - 54*a*b^15*c^11*d^8 + 96*a*b^15*c^13*d^6 - 78*a*b^15*c^15*d^4 + 24*a*b^15*c^17*d^2 + 12*a^2*b^14*c^18*d - 56*a^4*b^12*c^18*d + 44*a^6*b^10*c^18*d + 12*a^9*b^7*c*d^18 - 28*a^11*b^5*c*d^18 + 16*a^13*b^3*c*d^18 - 10*a^15*b*c^3*d^16 - 24*a^15*b*c^5*d^14 + 78*a^15*b*c^7*d^12 - 44*a^15*b*c^9*d^10 - 96*a^2*b^14*c^8*d^11 + 442*a^2*b^14*c^10*d^9 - 816*a^2*b^14*c^12*d^7 + 702*a^2*b^14*c^14*d^5 - 244*a^2*b^14*c^16*d^3 + 336*a^3*b^13*c^7*d^12 - 1620*a^3*b^13*c^9*d^10

+ 3206*a^3*b^13*c^11*d^8 - 3064*a^3*b^13*c^13*d^6 + 1314*a^3*b^13*c^15*d^4
- 176*a^3*b^13*c^17*d^2 - 672*a^4*b^12*c^6*d^13 + 3528*a^4*b^12*c^8*d^11 -
7810*a^4*b^12*c^10*d^9 + 8696*a^4*b^12*c^12*d^7 - 4770*a^4*b^12*c^14*d^5 +
1084*a^4*b^12*c^16*d^3 + 840*a^5*b^11*c^5*d^14 - 5124*a^5*b^11*c^7*d^12 + 1
3320*a^5*b^11*c^9*d^10 - 17850*a^5*b^11*c^11*d^8 + 12400*a^5*b^11*c^13*d^6
- 3954*a^5*b^11*c^15*d^4 + 372*a^5*b^11*c^17*d^2 - 672*a^6*b^10*c^4*d^15 +
5292*a^6*b^10*c^6*d^13 - 16872*a^6*b^10*c^8*d^11 + 27546*a^6*b^10*c^10*d^9
- 23696*a^6*b^10*c^12*d^7 + 9858*a^6*b^10*c^14*d^5 - 1500*a^6*b^10*c^16*d^3
 + 336*a^7*b^9*c^3*d^16 - 4032*a^7*b^9*c^5*d^14 + 16212*a^7*b^9*c^7*d^12 -
32304*a^7*b^9*c^9*d^10 + 34018*a^7*b^9*c^11*d^8 - 18048*a^7*b^9*c^13*d^6 +
4038*a^7*b^9*c^15*d^4 - 220*a^7*b^9*c^17*d^2 - 96*a^8*b^8*c^2*d^17 + 2280*a
^8*b^8*c^4*d^15 - 11772*a^8*b^8*c^6*d^13 + 28848*a^8*b^8*c^8*d^11 - 37338*a
^8*b^8*c^10*d^9 + 25056*a^8*b^8*c^12*d^7 - 7638*a^8*b^8*c^14*d^5 + 660*a^8*
b^8*c^16*d^3 - 918*a^9*b^7*c^3*d^16 + 6360*a^9*b^7*c^5*d^14 - 19602*a^9*b^7
*c^7*d^12 + 31560*a^9*b^7*c^9*d^10 - 26556*a^9*b^7*c^11*d^8 + 10464*a^9*b^7
*c^13*d^6 - 1320*a^9*b^7*c^15*d^4 + 234*a^10*b^6*c^2*d^17 - 2520*a^10*b^6*c
^4*d^15 + 10050*a^10*b^6*c^6*d^13 - 20340*a^10*b^6*c^8*d^11 + 21288*a^10*b^
6*c^10*d^9 - 10560*a^10*b^6*c^12*d^7 + 1848*a^10*b^6*c^14*d^5 + 726*a^11*b^
5*c^3*d^16 - 3768*a^11*b^5*c^5*d^14 + 9670*a^11*b^5*c^7*d^12 - 12648*a^11*b
^5*c^9*d^10 + 7896*a^11*b^5*c^11*d^8 - 1848*a^11*b^5*c^13*d^6 - 146*a^12*b^
4*c^2*d^17 + 952*a^12*b^4*c^4*d^15 - 3174*a^12*b^4*c^6*d^13 + 5396*a^12*b^4
*c^8*d^11 - 4348*a^12*b^4*c^10*d^9 + 1320*a^12*b^4*c^12*d^7 - 134*a^13*b^3*
c^3*d^16 + 624*a^13*b^3*c^5*d^14 - 1570*a^13*b^3*c^7*d^12 + 1724*a^13*b^3*c
^9*d^10 - 660*a^13*b^3*c^11*d^8 + 6*a^14*b^2*c^2*d^17 - 40*a^14*b^2*c^4*d^1
5 + 282*a^14*b^2*c^6*d^13 - 468*a^14*b^2*c^8*d^11 + 220*a^14*b^2*c^10*d^9))
/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2
*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13
*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d
^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12
*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a
^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^
10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a
^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^
10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c
^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^
10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 7
6*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5
*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c
^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^1
0 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 3
6*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*
b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c
^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12
 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 12
6*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b

^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^1
2*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^
10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 1
56*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^
11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) + (8*tan(e/2 + (f*x)/2
)*(4*a^16*c*d^18 + 8*a^2*b^14*c^19 - 8*a^4*b^12*c^19 - 12*a^16*c^5*d^14 + 8
*a^16*c^7*d^12 + 12*a*b^15*c^10*d^9 - 48*a*b^15*c^12*d^7 + 84*a*b^15*c^14*d
^5 - 72*a*b^15*c^16*d^3 - 112*a^3*b^13*c^18*d + 88*a^5*b^11*c^18*d + 12*a^1
0*b^6*c*d^18 - 28*a^12*b^4*c*d^18 + 12*a^14*b^2*c*d^18 - 20*a^15*b*c^2*d^17
 - 48*a^15*b*c^4*d^15 + 156*a^15*b*c^6*d^13 - 88*a^15*b*c^8*d^11 - 84*a^2*b
^14*c^9*d^10 + 328*a^2*b^14*c^11*d^8 - 596*a^2*b^14*c^13*d^6 + 552*a^2*b^14
*c^15*d^4 - 208*a^2*b^14*c^17*d^2 + 240*a^3*b^13*c^8*d^11 - 908*a^3*b^13*c^
10*d^9 + 1792*a^3*b^13*c^12*d^7 - 1932*a^3*b^13*c^14*d^5 + 920*a^3*b^13*c^1
6*d^3 - 336*a^4*b^12*c^7*d^12 + 1188*a^4*b^12*c^9*d^10 - 2808*a^4*b^12*c^11
*d^8 + 3980*a^4*b^12*c^13*d^6 - 2616*a^4*b^12*c^15*d^4 + 600*a^4*b^12*c^17*
d^2 + 168*a^5*b^11*c^6*d^13 - 336*a^5*b^11*c^8*d^11 + 1740*a^5*b^11*c^10*d^
9 - 4720*a^5*b^11*c^12*d^7 + 4812*a^5*b^11*c^14*d^5 - 1752*a^5*b^11*c^16*d^
3 + 168*a^6*b^10*c^5*d^14 - 1344*a^6*b^10*c^7*d^12 + 2292*a^6*b^10*c^9*d^10
 + 1088*a^6*b^10*c^11*d^8 - 4908*a^6*b^10*c^13*d^6 + 3096*a^6*b^10*c^15*d^4
 - 392*a^6*b^10*c^17*d^2 - 336*a^7*b^9*c^4*d^15 + 2520*a^7*b^9*c^6*d^13 - 7
488*a^7*b^9*c^8*d^11 + 7556*a^7*b^9*c^10*d^9 - 144*a^7*b^9*c^12*d^7 - 3012*
a^7*b^9*c^14*d^5 + 904*a^7*b^9*c^16*d^3 + 240*a^8*b^8*c^3*d^16 - 2472*a^8*b
^8*c^5*d^14 + 10416*a^8*b^8*c^7*d^12 - 16596*a^8*b^8*c^9*d^10 + 9600*a^8*b^
8*c^11*d^8 - 156*a^8*b^8*c^13*d^6 - 1032*a^8*b^8*c^15*d^4 - 84*a^9*b^7*c^2*
d^17 + 1632*a^9*b^7*c^4*d^15 - 9204*a^9*b^7*c^6*d^13 + 19800*a^9*b^7*c^8*d^
11 - 18048*a^9*b^7*c^10*d^9 + 5856*a^9*b^7*c^12*d^7 + 48*a^9*b^7*c^14*d^5 -
 744*a^10*b^6*c^3*d^16 + 5460*a^10*b^6*c^5*d^14 - 15960*a^10*b^6*c^7*d^12 +
 20136*a^10*b^6*c^9*d^10 - 10584*a^10*b^6*c^11*d^8 + 1680*a^10*b^6*c^13*d^6
 + 212*a^11*b^5*c^2*d^17 - 2176*a^11*b^5*c^4*d^15 + 9180*a^11*b^5*c^6*d^13
- 15416*a^11*b^5*c^8*d^11 + 10936*a^11*b^5*c^10*d^9 - 2736*a^11*b^5*c^12*d^
7 + 584*a^12*b^4*c^3*d^16 - 3708*a^12*b^4*c^5*d^14 + 8152*a^12*b^4*c^7*d^12
 - 7376*a^12*b^4*c^9*d^10 + 2376*a^12*b^4*c^11*d^8 - 108*a^13*b^3*c^2*d^17
+ 928*a^13*b^3*c^4*d^15 - 2820*a^13*b^3*c^6*d^13 + 3288*a^13*b^3*c^8*d^11 -
 1288*a^13*b^3*c^10*d^9 - 80*a^14*b^2*c^3*d^16 + 564*a^14*b^2*c^5*d^14 - 93
6*a^14*b^2*c^7*d^12 + 440*a^14*b^2*c^9*d^10 + 24*a*b^15*c^18*d))/(a^13*d^17
 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d
^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 -
b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^
12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 -
 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d
^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12
*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^1
1*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11
 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 1
56*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a

^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c
^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^
9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 8
4*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^
6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c
^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^1
1 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 8
4*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8
*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c
^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11
 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156
*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^
10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2
*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10
*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) - (d^2*((8*(4*a^2*b^16*c^22 - 8*a
^4*b^14*c^22 + 4*a^6*b^12*c^22 - 4*a^18*c^2*d^20 + 16*a^18*c^4*d^18 - 24*a^
18*c^6*d^16 + 16*a^18*c^8*d^14 - 4*a^18*c^10*d^12 - 4*a*b^17*c^13*d^9 + 16*
a*b^17*c^15*d^7 - 24*a*b^17*c^17*d^5 + 16*a*b^17*c^19*d^3 - 32*a^3*b^15*c^2
1*d + 76*a^5*b^13*c^21*d - 40*a^7*b^11*c^21*d + 4*a^13*b^5*c*d^21 - 8*a^15*
b^3*c*d^21 + 24*a^17*b*c^3*d^19 - 136*a^17*b*c^5*d^17 + 224*a^17*b*c^7*d^15
 - 156*a^17*b*c^9*d^13 + 40*a^17*b*c^11*d^11 + 40*a^2*b^16*c^12*d^10 - 156*
a^2*b^16*c^14*d^8 + 224*a^2*b^16*c^16*d^6 - 136*a^2*b^16*c^18*d^4 + 24*a^2*
b^16*c^20*d^2 - 176*a^3*b^15*c^11*d^11 + 672*a^3*b^15*c^13*d^9 - 928*a^3*b^
15*c^15*d^7 + 512*a^3*b^15*c^17*d^5 - 48*a^3*b^15*c^19*d^3 + 440*a^4*b^14*c
^10*d^12 - 1664*a^4*b^14*c^12*d^10 + 2248*a^4*b^14*c^14*d^8 - 1152*a^4*b^14
*c^16*d^6 + 8*a^4*b^14*c^18*d^4 + 128*a^4*b^14*c^20*d^2 - 660*a^5*b^13*c^9*
d^13 + 2552*a^5*b^13*c^11*d^11 - 3532*a^5*b^13*c^13*d^9 + 1808*a^5*b^13*c^1
5*d^7 + 148*a^5*b^13*c^17*d^5 - 392*a^5*b^13*c^19*d^3 + 528*a^6*b^12*c^8*d^
14 - 2332*a^6*b^12*c^10*d^12 + 3736*a^6*b^12*c^12*d^10 - 2180*a^6*b^12*c^14
*d^8 - 480*a^6*b^12*c^16*d^6 + 1052*a^6*b^12*c^18*d^4 - 328*a^6*b^12*c^20*d
^2 + 792*a^7*b^11*c^9*d^13 - 2464*a^7*b^11*c^11*d^11 + 1896*a^7*b^11*c^13*d
^9 + 1216*a^7*b^11*c^15*d^7 - 2264*a^7*b^11*c^17*d^5 + 864*a^7*b^11*c^19*d^
3 - 528*a^8*b^10*c^6*d^16 + 1056*a^8*b^10*c^8*d^14 + 176*a^8*b^10*c^10*d^12
 - 528*a^8*b^10*c^12*d^10 - 2288*a^8*b^10*c^14*d^8 + 3520*a^8*b^10*c^16*d^6
 - 1584*a^8*b^10*c^18*d^4 + 176*a^8*b^10*c^20*d^2 + 660*a^9*b^9*c^5*d^17 -
2112*a^9*b^9*c^7*d^15 + 2244*a^9*b^9*c^9*d^13 - 1496*a^9*b^9*c^11*d^11 + 26
84*a^9*b^9*c^13*d^9 - 3696*a^9*b^9*c^15*d^7 + 2156*a^9*b^9*c^17*d^5 - 440*a
^9*b^9*c^19*d^3 - 440*a^10*b^8*c^4*d^18 + 2156*a^10*b^8*c^6*d^16 - 3696*a^1
0*b^8*c^8*d^14 + 2684*a^10*b^8*c^10*d^12 - 1496*a^10*b^8*c^12*d^10 + 2244*a
^10*b^8*c^14*d^8 - 2112*a^10*b^8*c^16*d^6 + 660*a^10*b^8*c^18*d^4 + 176*a^1
1*b^7*c^3*d^19 - 1584*a^11*b^7*c^5*d^17 + 3520*a^11*b^7*c^7*d^15 - 2288*a^1
1*b^7*c^9*d^13 - 528*a^11*b^7*c^11*d^11 + 176*a^11*b^7*c^13*d^9 + 1056*a^11
*b^7*c^15*d^7 - 528*a^11*b^7*c^17*d^5 - 40*a^12*b^6*c^2*d^20 + 864*a^12*b^6
*c^4*d^18 - 2264*a^12*b^6*c^6*d^16 + 1216*a^12*b^6*c^8*d^14 + 1896*a^12*b^6
*c^10*d^12 - 2464*a^12*b^6*c^12*d^10 + 792*a^12*b^6*c^14*d^8 - 328*a^13*b^5

*c^3*d^19 + 1052*a^13*b^5*c^5*d^17 - 480*a^13*b^5*c^7*d^15 - 2180*a^13*b^5*c^9*d^13 + 3736*a^13*b^5*c^11*d^11 - 2332*a^13*b^5*c^13*d^9 + 528*a^13*b^5*c^15*d^7 + 76*a^14*b^4*c^2*d^20 - 392*a^14*b^4*c^4*d^18 + 148*a^14*b^4*c^6*d^16 + 1808*a^14*b^4*c^8*d^14 - 3532*a^14*b^4*c^10*d^12 + 2552*a^14*b^4*c^12*d^10 - 660*a^14*b^4*c^14*d^8 + 128*a^15*b^3*c^3*d^19 + 8*a^15*b^3*c^5*d^17 - 1152*a^15*b^3*c^7*d^15 + 2248*a^15*b^3*c^9*d^13 - 1664*a^15*b^3*c^11*d^11 + 440*a^15*b^3*c^13*d^9 - 32*a^16*b^2*c^2*d^20 - 48*a^16*b^2*c^4*d^18 + 512*a^16*b^2*c^6*d^16 - 928*a^16*b^2*c^8*d^14 + 672*a^16*b^2*c^10*d^12 - 176*a^16*b^2*c^12*d^10 - 4*a*b^17*c^21*d + 4*a^17*b*c*d^21))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) + (8*tan(e/2 + (f*x)/2)*(12*a*b^17*c^22 - 12*a^18*c*d^21 - 32*a^3*b^15*c^22 + 28*a^5*b^13*c^22 - 8*a^7*b^11*c^22 + 56*a^18*c^3*d^19 - 104*a^18*c^5*d^17 + 96*a^18*c^7*d^15 - 44*a^18*c^9*d^13 + 8*a^18*c^11*d^11 - 16*a*b^17*c^12*d^10 + 76*a*b^17*c^14*d^8 - 144*a*b^17*c^16*d^6 + 136*a*b^17*c^18*d^4 - 64*a*b^17*c^20*d^2 - 132*a^2*b^16*c^21*d + 352*a^4*b^14*c^21*d - 308*a^6*b^12*c^21*d + 88*a^8*b^10*c^21*d + 16*a^12*b^6*c*d^21 - 44*a^14*b^4*c*d^21 + 40*a^16*b^2*c*d^21 + 132*a^17*b*c^2*d^20 - 616*a^17*b*c^4*d^18 + 1144*a^17*b*c^6*d^16 - 1056*a^17*b*c^8*d^14 + 484*a^17*b*c^10*d^12 - 88*a^17*b*c^12*d^10 + 176*a^2*b^16*c^11*d^11 - 836*a^2*b^16*c^13*d^9 + 1584*a^2*b^16*c^15*d^7 - 1496*a^2*b^16*c^17*d^5 + 704*a^2*b^16*c^19*d^3 - 880*a^3*b^15*c^10*d^12 + 4224*a^3*b^15*c^12*d^10 - 8128*a^3*b^15*c^14*d^8 + 7872*a^3*b^15*c^16*d^6 - 3888*a^3*b^15*c^18*d^4 + 832*a^3*b^1

5*c^20*d^2 + 2640*a^4*b^14*c^9*d^13 - 13024*a^4*b^14*c^11*d^11 + 26048*a^4*
b^14*c^13*d^9 - 26752*a^4*b^14*c^15*d^7 + 14608*a^4*b^14*c^17*d^5 - 3872*a^
4*b^14*c^19*d^3 - 5280*a^5*b^13*c^8*d^14 + 27500*a^5*b^13*c^10*d^12 - 59000
*a^5*b^13*c^12*d^10 + 66628*a^5*b^13*c^14*d^8 - 41712*a^5*b^13*c^16*d^6 + 1
3748*a^5*b^13*c^18*d^4 - 1912*a^5*b^13*c^20*d^2 + 7392*a^6*b^12*c^7*d^15 -
42372*a^6*b^12*c^9*d^13 + 101288*a^6*b^12*c^11*d^11 - 129580*a^6*b^12*c^13*
d^9 + 94160*a^6*b^12*c^15*d^7 - 37532*a^6*b^12*c^17*d^5 + 6952*a^6*b^12*c^1
9*d^3 - 7392*a^7*b^11*c^6*d^16 + 49632*a^7*b^11*c^8*d^14 - 137368*a^7*b^11*
c^10*d^12 + 202544*a^7*b^11*c^12*d^10 - 170424*a^7*b^11*c^14*d^8 + 80448*a^
7*b^11*c^16*d^6 - 19016*a^7*b^11*c^18*d^4 + 1584*a^7*b^11*c^20*d^2 + 5280*a
^8*b^10*c^5*d^17 - 45408*a^8*b^10*c^7*d^15 + 150216*a^8*b^10*c^9*d^13 - 257
136*a^8*b^10*c^11*d^11 + 249832*a^8*b^10*c^13*d^9 - 138688*a^8*b^10*c^15*d^
7 + 40920*a^8*b^10*c^17*d^5 - 5104*a^8*b^10*c^19*d^3 - 2640*a^9*b^9*c^4*d^1
8 + 32868*a^9*b^9*c^6*d^16 - 133056*a^9*b^9*c^8*d^14 + 266244*a^9*b^9*c^10*
d^12 - 299816*a^9*b^9*c^12*d^10 + 195404*a^9*b^9*c^14*d^8 - 70224*a^9*b^9*c
^16*d^6 + 11660*a^9*b^9*c^18*d^4 - 440*a^9*b^9*c^20*d^2 + 880*a^10*b^8*c^3*
d^19 - 18700*a^10*b^8*c^5*d^17 + 95040*a^10*b^8*c^7*d^15 - 225676*a^10*b^8*
c^9*d^13 + 296824*a^10*b^8*c^11*d^11 - 226116*a^10*b^8*c^13*d^9 + 96624*a^1
0*b^8*c^15*d^7 - 20196*a^10*b^8*c^17*d^5 + 1320*a^10*b^8*c^19*d^3 - 176*a^1
1*b^7*c^2*d^20 + 8096*a^11*b^7*c^4*d^18 - 54384*a^11*b^7*c^6*d^16 + 156992*
a^11*b^7*c^8*d^14 - 242528*a^11*b^7*c^10*d^12 + 214368*a^11*b^7*c^12*d^10 -
 107184*a^11*b^7*c^14*d^8 + 27456*a^11*b^7*c^16*d^6 - 2640*a^11*b^7*c^18*d^
4 - 2496*a^12*b^6*c^3*d^19 + 24784*a^12*b^6*c^5*d^17 - 89280*a^12*b^6*c^7*d
^15 + 162336*a^12*b^6*c^9*d^13 - 165760*a^12*b^6*c^11*d^11 + 96272*a^12*b^6
*c^13*d^9 - 29568*a^12*b^6*c^15*d^7 + 3696*a^12*b^6*c^17*d^5 + 484*a^13*b^5
*c^2*d^20 - 8888*a^13*b^5*c^4*d^18 + 40876*a^13*b^5*c^6*d^16 - 88000*a^13*b
^5*c^8*d^14 + 104060*a^13*b^5*c^10*d^12 - 69784*a^13*b^5*c^12*d^10 + 24948*
a^13*b^5*c^14*d^8 - 3696*a^13*b^5*c^16*d^6 + 2408*a^14*b^4*c^3*d^19 - 14692
*a^14*b^4*c^5*d^17 + 38208*a^14*b^4*c^7*d^15 - 52532*a^14*b^4*c^9*d^13 + 40
072*a^14*b^4*c^11*d^11 - 16060*a^14*b^4*c^13*d^9 + 2640*a^14*b^4*c^15*d^7 -
 440*a^15*b^3*c^2*d^20 + 4048*a^15*b^3*c^4*d^18 - 13112*a^15*b^3*c^6*d^16 +
 20768*a^15*b^3*c^8*d^14 - 17512*a^15*b^3*c^10*d^12 + 7568*a^15*b^3*c^12*d^
10 - 1320*a^15*b^3*c^14*d^8 - 848*a^16*b^2*c^3*d^19 + 3432*a^16*b^2*c^5*d^1
7 - 6048*a^16*b^2*c^7*d^15 + 5432*a^16*b^2*c^9*d^13 - 2448*a^16*b^2*c^11*d^
11 + 440*a^16*b^2*c^13*d^9))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4
*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d
^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b
^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54
*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16
*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b
*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 +
146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a
^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^
10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c
^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^

6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672
*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*
b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5
*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^
6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*
a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*
b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3
*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8
 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*
a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^
4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^
5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^
6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 -
146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*
d^16))*(-(c + d)^5*(c - d)^5)^(1/2)*(a^2*d^4 + 12*b^2*c^4 + 6*b^2*d^4 + 2*a
^2*c^2*d^2 - 15*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*b*c^3*d))/(2*(a^4*d^14 - b^
4*c^14 - 5*a^4*c^2*d^12 + 10*a^4*c^4*d^10 - 10*a^4*c^6*d^8 + 5*a^4*c^8*d^6
- a^4*c^10*d^4 + b^4*c^4*d^10 - 5*b^4*c^6*d^8 + 10*b^4*c^8*d^6 - 10*b^4*c^1
0*d^4 + 5*b^4*c^12*d^2 - 4*a*b^3*c^3*d^11 + 20*a*b^3*c^5*d^9 - 40*a*b^3*c^7
*d^7 + 40*a*b^3*c^9*d^5 - 20*a*b^3*c^11*d^3 + 20*a^3*b*c^3*d^11 - 40*a^3*b*
c^5*d^9 + 40*a^3*b*c^7*d^7 - 20*a^3*b*c^9*d^5 + 4*a^3*b*c^11*d^3 + 6*a^2*b^
2*c^2*d^12 - 30*a^2*b^2*c^4*d^10 + 60*a^2*b^2*c^6*d^8 - 60*a^2*b^2*c^8*d^6
+ 30*a^2*b^2*c^10*d^4 - 6*a^2*b^2*c^12*d^2 + 4*a*b^3*c^13*d - 4*a^3*b*c*d^1
3)))*(a^2*d^4 + 12*b^2*c^4 + 6*b^2*d^4 + 2*a^2*c^2*d^2 - 15*b^2*c^2*d^2 + 2
*a*b*c*d^3 - 8*a*b*c^3*d))/(2*(a^4*d^14 - b^4*c^14 - 5*a^4*c^2*d^12 + 10*a^
4*c^4*d^10 - 10*a^4*c^6*d^8 + 5*a^4*c^8*d^6 - a^4*c^10*d^4 + b^4*c^4*d^10 -
 5*b^4*c^6*d^8 + 10*b^4*c^8*d^6 - 10*b^4*c^10*d^4 + 5*b^4*c^12*d^2 - 4*a*b^
3*c^3*d^11 + 20*a*b^3*c^5*d^9 - 40*a*b^3*c^7*d^7 + 40*a*b^3*c^9*d^5 - 20*a*
b^3*c^11*d^3 + 20*a^3*b*c^3*d^11 - 40*a^3*b*c^5*d^9 + 40*a^3*b*c^7*d^7 - 20
*a^3*b*c^9*d^5 + 4*a^3*b*c^11*d^3 + 6*a^2*b^2*c^2*d^12 - 30*a^2*b^2*c^4*d^1
0 + 60*a^2*b^2*c^6*d^8 - 60*a^2*b^2*c^8*d^6 + 30*a^2*b^2*c^10*d^4 - 6*a^2*b
^2*c^12*d^2 + 4*a*b^3*c^13*d - 4*a^3*b*c*d^13)))*(a^2*d^4 + 12*b^2*c^4 + 6*
b^2*d^4 + 2*a^2*c^2*d^2 - 15*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*b*c^3*d)*1i)/(
2*(a^4*d^14 - b^4*c^14 - 5*a^4*c^2*d^12 + 10*a^4*c^4*d^10 - 10*a^4*c^6*d^8
+ 5*a^4*c^8*d^6 - a^4*c^10*d^4 + b^4*c^4*d^10 - 5*b^4*c^6*d^8 + 10*b^4*c^8*
d^6 - 10*b^4*c^10*d^4 + 5*b^4*c^12*d^2 - 4*a*b^3*c^3*d^11 + 20*a*b^3*c^5*d^
9 - 40*a*b^3*c^7*d^7 + 40*a*b^3*c^9*d^5 - 20*a*b^3*c^11*d^3 + 20*a^3*b*c^3*
d^11 - 40*a^3*b*c^5*d^9 + 40*a^3*b*c^7*d^7 - 20*a^3*b*c^9*d^5 + 4*a^3*b*c^1
1*d^3 + 6*a^2*b^2*c^2*d^12 - 30*a^2*b^2*c^4*d^10 + 60*a^2*b^2*c^6*d^8 - 60*
a^2*b^2*c^8*d^6 + 30*a^2*b^2*c^10*d^4 - 6*a^2*b^2*c^12*d^2 + 4*a*b^3*c^13*d
 - 4*a^3*b*c*d^13)) - (d^2*(-(c + d)^5*(c - d)^5)^(1/2)*((8*(36*a*b^13*c^5*
d^11 - 144*a*b^13*c^7*d^9 + 216*a*b^13*c^9*d^7 - 144*a*b^13*c^11*d^5 + 36*a
*b^13*c^13*d^3 + 4*a^3*b^11*c^15*d - 36*a^5*b^9*c*d^15 + 60*a^7*b^7*c*d^15
- 13*a^9*b^5*c*d^15 - 10*a^11*b^3*c*d^15 - 4*a^13*b*c^3*d^13 - 4*a^13*b*c^5
*d^11 - 72*a^2*b^12*c^4*d^12 + 276*a^2*b^12*c^6*d^10 - 375*a^2*b^12*c^8*d^8

+ 216*a^2*b^12*c^10*d^6 - 60*a^2*b^12*c^12*d^4 + 24*a^2*b^12*c^14*d^2 - 36 *a^3*b^11*c^5*d^11 + 61*a^3*b^11*c^7*d^9 - 88*a^3*b^11*c^9*d^7 + 180*a^3*b^ 11*c^11*d^5 - 184*a^3*b^11*c^13*d^3 + 72*a^4*b^10*c^2*d^14 - 168*a^4*b^10*c ^4*d^12 + 233*a^4*b^10*c^6*d^10 - 270*a^4*b^10*c^8*d^8 + 100*a^4*b^10*c^10* d^6 + 248*a^4*b^10*c^12*d^4 - 44*a^4*b^10*c^14*d^2 + 120*a^5*b^9*c^3*d^13 - 535*a^5*b^9*c^5*d^11 + 1386*a^5*b^9*c^7*d^9 - 1544*a^5*b^9*c^9*d^7 + 248*a ^5*b^9*c^11*d^5 + 172*a^5*b^9*c^13*d^3 - 108*a^6*b^8*c^2*d^14 + 699*a^6*b^8 *c^4*d^12 - 2046*a^6*b^8*c^6*d^10 + 2885*a^6*b^8*c^8*d^8 - 1336*a^6*b^8*c^1 0*d^6 - 148*a^6*b^8*c^12*d^4 - 305*a^7*b^7*c^3*d^13 + 1354*a^7*b^7*c^5*d^11 - 2979*a^7*b^7*c^7*d^9 + 2648*a^7*b^7*c^9*d^7 - 400*a^7*b^7*c^11*d^5 + 19* a^8*b^6*c^2*d^14 - 602*a^8*b^6*c^4*d^12 + 2161*a^8*b^6*c^6*d^10 - 3012*a^8* b^6*c^8*d^8 + 1056*a^8*b^6*c^10*d^6 + 190*a^9*b^5*c^3*d^13 - 895*a^9*b^5*c^ 5*d^11 + 1860*a^9*b^5*c^7*d^9 - 1088*a^9*b^5*c^9*d^7 + 14*a^10*b^4*c^2*d^14 + 99*a^10*b^4*c^4*d^12 - 552*a^10*b^4*c^6*d^10 + 628*a^10*b^4*c^8*d^8 + 19 *a^11*b^3*c^3*d^13 + 40*a^11*b^3*c^5*d^11 - 220*a^11*b^3*c^7*d^9 - a^12*b^2 *c^2*d^14 + 20*a^12*b^2*c^4*d^12 + 44*a^12*b^2*c^6*d^10 - a^13*b*c*d^15))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a ^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c ^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c ^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^1 0*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2 *b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10 *c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^1 2*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76* a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b ^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^1 4*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36* a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^ 6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^1 2*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126* a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4 *c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12* d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156 *a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11 *b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) - (8*tan(e/2 + (f*x)/2)* (4*a^3*b^11*c^16 - a^14*c*d^15 - 4*a^14*c^3*d^13 - 4*a^14*c^5*d^11 - 144*a* b^13*c^4*d^12 + 684*a*b^13*c^6*d^10 - 1314*a*b^13*c^8*d^8 + 1224*a*b^13*c^1 0*d^6 - 504*a*b^13*c^12*d^4 + 36*a*b^13*c^14*d^2 + 24*a^2*b^12*c^15*d + 144 *a^4*b^10*c*d^15 - 44*a^4*b^10*c^15*d - 348*a^6*b^8*c*d^15 + 214*a^8*b^6*c*

d^15 + 7*a^10*b^4*c*d^15 - 8*a^12*b^2*c*d^15 - a^13*b*c^2*d^14 + 20*a^13*b*
c^4*d^12 + 44*a^13*b*c^6*d^10 + 432*a^2*b^12*c^3*d^13 - 2148*a^2*b^12*c^5*d
^11 + 4470*a^2*b^12*c^7*d^9 - 4632*a^2*b^12*c^9*d^7 + 2232*a^2*b^12*c^11*d^
5 - 252*a^2*b^12*c^13*d^3 - 432*a^3*b^11*c^2*d^14 + 2688*a^3*b^11*c^4*d^12
- 7294*a^3*b^11*c^6*d^10 + 10105*a^3*b^11*c^8*d^8 - 7104*a^3*b^11*c^10*d^6
+ 1892*a^3*b^11*c^12*d^4 - 192*a^3*b^11*c^14*d^2 - 2016*a^4*b^10*c^3*d^13 +
8378*a^4*b^10*c^5*d^11 - 15815*a^4*b^10*c^7*d^9 + 14976*a^4*b^10*c^9*d^7 -
5932*a^4*b^10*c^11*d^5 + 624*a^4*b^10*c^13*d^3 + 1140*a^5*b^9*c^2*d^14 - 6
574*a^5*b^9*c^4*d^12 + 16053*a^5*b^9*c^6*d^10 - 19912*a^5*b^9*c^8*d^8 + 113
20*a^5*b^9*c^10*d^6 - 1920*a^5*b^9*c^12*d^4 + 172*a^5*b^9*c^14*d^2 + 2938*a
^6*b^8*c^3*d^13 - 10619*a^6*b^8*c^5*d^11 + 18608*a^6*b^8*c^7*d^9 - 15576*a^
6*b^8*c^9*d^7 + 4344*a^6*b^8*c^11*d^5 - 292*a^6*b^8*c^13*d^3 - 818*a^7*b^7*
c^2*d^14 + 5107*a^7*b^7*c^4*d^12 - 12464*a^7*b^7*c^6*d^10 + 14693*a^7*b^7*c
^8*d^8 - 6184*a^7*b^7*c^10*d^6 + 368*a^7*b^7*c^12*d^4 - 1485*a^8*b^6*c^3*d^
13 + 5064*a^8*b^6*c^5*d^11 - 8939*a^8*b^6*c^7*d^9 + 6104*a^8*b^6*c^9*d^7 -
688*a^8*b^6*c^11*d^5 + 55*a^9*b^5*c^2*d^14 - 1056*a^9*b^5*c^4*d^12 + 3649*a
^9*b^5*c^6*d^10 - 4524*a^9*b^5*c^8*d^8 + 1120*a^9*b^5*c^10*d^6 + 152*a^10*b
^4*c^3*d^13 - 975*a^10*b^4*c^5*d^11 + 2300*a^10*b^4*c^7*d^9 - 1088*a^10*b^4
*c^9*d^7 + 16*a^11*b^3*c^2*d^14 + 59*a^11*b^3*c^4*d^12 - 640*a^11*b^3*c^6*d
^10 + 628*a^11*b^3*c^8*d^8 + 27*a^12*b^2*c^3*d^13 + 48*a^12*b^2*c^5*d^11 -
220*a^12*b^2*c^7*d^9))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c
^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 -
4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^
13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^1
2*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9
*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d
^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^
2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^1
1*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^1
0*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^1
2 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 42
0*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b
^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^
12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12
- 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 39
6*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^
6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^
10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14
- 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672
*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^
4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10
*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12
- 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44
*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^
11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16)

+ (d^2*(-(c + d)^5*(c - d)^5)^(1/2)*((8*(4*a^3*b^13*c^19 - 4*a^5*b^11*c^19
+ 2*a^16*c^2*d^17 - 6*a^16*c^6*d^13 + 4*a^16*c^8*d^11 + 12*a*b^15*c^9*d^10
- 54*a*b^15*c^11*d^8 + 96*a*b^15*c^13*d^6 - 78*a*b^15*c^15*d^4 + 24*a*b^15*
c^17*d^2 + 12*a^2*b^14*c^18*d - 56*a^4*b^12*c^18*d + 44*a^6*b^10*c^18*d + 1
2*a^9*b^7*c*d^18 - 28*a^11*b^5*c*d^18 + 16*a^13*b^3*c*d^18 - 10*a^15*b*c^3*
d^16 - 24*a^15*b*c^5*d^14 + 78*a^15*b*c^7*d^12 - 44*a^15*b*c^9*d^10 - 96*a^
2*b^14*c^8*d^11 + 442*a^2*b^14*c^10*d^9 - 816*a^2*b^14*c^12*d^7 + 702*a^2*b
^14*c^14*d^5 - 244*a^2*b^14*c^16*d^3 + 336*a^3*b^13*c^7*d^12 - 1620*a^3*b^1
3*c^9*d^10 + 3206*a^3*b^13*c^11*d^8 - 3064*a^3*b^13*c^13*d^6 + 1314*a^3*b^1
3*c^15*d^4 - 176*a^3*b^13*c^17*d^2 - 672*a^4*b^12*c^6*d^13 + 3528*a^4*b^12*
c^8*d^11 - 7810*a^4*b^12*c^10*d^9 + 8696*a^4*b^12*c^12*d^7 - 4770*a^4*b^12*
c^14*d^5 + 1084*a^4*b^12*c^16*d^3 + 840*a^5*b^11*c^5*d^14 - 5124*a^5*b^11*c
^7*d^12 + 13320*a^5*b^11*c^9*d^10 - 17850*a^5*b^11*c^11*d^8 + 12400*a^5*b^1
1*c^13*d^6 - 3954*a^5*b^11*c^15*d^4 + 372*a^5*b^11*c^17*d^2 - 672*a^6*b^10*
c^4*d^15 + 5292*a^6*b^10*c^6*d^13 - 16872*a^6*b^10*c^8*d^11 + 27546*a^6*b^1
0*c^10*d^9 - 23696*a^6*b^10*c^12*d^7 + 9858*a^6*b^10*c^14*d^5 - 1500*a^6*b^
10*c^16*d^3 + 336*a^7*b^9*c^3*d^16 - 4032*a^7*b^9*c^5*d^14 + 16212*a^7*b^9*
c^7*d^12 - 32304*a^7*b^9*c^9*d^10 + 34018*a^7*b^9*c^11*d^8 - 18048*a^7*b^9*
c^13*d^6 + 4038*a^7*b^9*c^15*d^4 - 220*a^7*b^9*c^17*d^2 - 96*a^8*b^8*c^2*d^
17 + 2280*a^8*b^8*c^4*d^15 - 11772*a^8*b^8*c^6*d^13 + 28848*a^8*b^8*c^8*d^1
1 - 37338*a^8*b^8*c^10*d^9 + 25056*a^8*b^8*c^12*d^7 - 7638*a^8*b^8*c^14*d^5
 + 660*a^8*b^8*c^16*d^3 - 918*a^9*b^7*c^3*d^16 + 6360*a^9*b^7*c^5*d^14 - 19
602*a^9*b^7*c^7*d^12 + 31560*a^9*b^7*c^9*d^10 - 26556*a^9*b^7*c^11*d^8 + 10
464*a^9*b^7*c^13*d^6 - 1320*a^9*b^7*c^15*d^4 + 234*a^10*b^6*c^2*d^17 - 2520
*a^10*b^6*c^4*d^15 + 10050*a^10*b^6*c^6*d^13 - 20340*a^10*b^6*c^8*d^11 + 21
288*a^10*b^6*c^10*d^9 - 10560*a^10*b^6*c^12*d^7 + 1848*a^10*b^6*c^14*d^5 +
726*a^11*b^5*c^3*d^16 - 3768*a^11*b^5*c^5*d^14 + 9670*a^11*b^5*c^7*d^12 - 1
2648*a^11*b^5*c^9*d^10 + 7896*a^11*b^5*c^11*d^8 - 1848*a^11*b^5*c^13*d^6 -
146*a^12*b^4*c^2*d^17 + 952*a^12*b^4*c^4*d^15 - 3174*a^12*b^4*c^6*d^13 + 53
96*a^12*b^4*c^8*d^11 - 4348*a^12*b^4*c^10*d^9 + 1320*a^12*b^4*c^12*d^7 - 13
4*a^13*b^3*c^3*d^16 + 624*a^13*b^3*c^5*d^14 - 1570*a^13*b^3*c^7*d^12 + 1724
*a^13*b^3*c^9*d^10 - 660*a^13*b^3*c^11*d^8 + 6*a^14*b^2*c^2*d^17 - 40*a^14*
b^2*c^4*d^15 + 282*a^14*b^2*c^6*d^13 - 468*a^14*b^2*c^8*d^11 + 220*a^14*b^2
*c^10*d^9))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b
^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*
d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*
b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5
- 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*
d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^
12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*
d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2
+ 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444
*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4
*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c
^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11

+ 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 20
4*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*
b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c
^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13
+ 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 58
8*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b
^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^
11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13
- 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*
a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10
*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c
^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*
d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) + (8*tan(e/
2 + (f*x)/2)*(4*a^16*c*d^18 + 8*a^2*b^14*c^19 - 8*a^4*b^12*c^19 - 12*a^16*c
^5*d^14 + 8*a^16*c^7*d^12 + 12*a*b^15*c^10*d^9 - 48*a*b^15*c^12*d^7 + 84*a*
b^15*c^14*d^5 - 72*a*b^15*c^16*d^3 - 112*a^3*b^13*c^18*d + 88*a^5*b^11*c^18
*d + 12*a^10*b^6*c*d^18 - 28*a^12*b^4*c*d^18 + 12*a^14*b^2*c*d^18 - 20*a^15
*b*c^2*d^17 - 48*a^15*b*c^4*d^15 + 156*a^15*b*c^6*d^13 - 88*a^15*b*c^8*d^11
- 84*a^2*b^14*c^9*d^10 + 328*a^2*b^14*c^11*d^8 - 596*a^2*b^14*c^13*d^6 + 5
52*a^2*b^14*c^15*d^4 - 208*a^2*b^14*c^17*d^2 + 240*a^3*b^13*c^8*d^11 - 908*
a^3*b^13*c^10*d^9 + 1792*a^3*b^13*c^12*d^7 - 1932*a^3*b^13*c^14*d^5 + 920*a
^3*b^13*c^16*d^3 - 336*a^4*b^12*c^7*d^12 + 1188*a^4*b^12*c^9*d^10 - 2808*a^
4*b^12*c^11*d^8 + 3980*a^4*b^12*c^13*d^6 - 2616*a^4*b^12*c^15*d^4 + 600*a^4
*b^12*c^17*d^2 + 168*a^5*b^11*c^6*d^13 - 336*a^5*b^11*c^8*d^11 + 1740*a^5*b
^11*c^10*d^9 - 4720*a^5*b^11*c^12*d^7 + 4812*a^5*b^11*c^14*d^5 - 1752*a^5*b
^11*c^16*d^3 + 168*a^6*b^10*c^5*d^14 - 1344*a^6*b^10*c^7*d^12 + 2292*a^6*b^
10*c^9*d^10 + 1088*a^6*b^10*c^11*d^8 - 4908*a^6*b^10*c^13*d^6 + 3096*a^6*b^
10*c^15*d^4 - 392*a^6*b^10*c^17*d^2 - 336*a^7*b^9*c^4*d^15 + 2520*a^7*b^9*c
^6*d^13 - 7488*a^7*b^9*c^8*d^11 + 7556*a^7*b^9*c^10*d^9 - 144*a^7*b^9*c^12*
d^7 - 3012*a^7*b^9*c^14*d^5 + 904*a^7*b^9*c^16*d^3 + 240*a^8*b^8*c^3*d^16 -
2472*a^8*b^8*c^5*d^14 + 10416*a^8*b^8*c^7*d^12 - 16596*a^8*b^8*c^9*d^10 +
9600*a^8*b^8*c^11*d^8 - 156*a^8*b^8*c^13*d^6 - 1032*a^8*b^8*c^15*d^4 - 84*a
^9*b^7*c^2*d^17 + 1632*a^9*b^7*c^4*d^15 - 9204*a^9*b^7*c^6*d^13 + 19800*a^9
*b^7*c^8*d^11 - 18048*a^9*b^7*c^10*d^9 + 5856*a^9*b^7*c^12*d^7 + 48*a^9*b^7
*c^14*d^5 - 744*a^10*b^6*c^3*d^16 + 5460*a^10*b^6*c^5*d^14 - 15960*a^10*b^6
*c^7*d^12 + 20136*a^10*b^6*c^9*d^10 - 10584*a^10*b^6*c^11*d^8 + 1680*a^10*b
^6*c^13*d^6 + 212*a^11*b^5*c^2*d^17 - 2176*a^11*b^5*c^4*d^15 + 9180*a^11*b^
5*c^6*d^13 - 15416*a^11*b^5*c^8*d^11 + 10936*a^11*b^5*c^10*d^9 - 2736*a^11*
b^5*c^12*d^7 + 584*a^12*b^4*c^3*d^16 - 3708*a^12*b^4*c^5*d^14 + 8152*a^12*b
^4*c^7*d^12 - 7376*a^12*b^4*c^9*d^10 + 2376*a^12*b^4*c^11*d^8 - 108*a^13*b^
3*c^2*d^17 + 928*a^13*b^3*c^4*d^15 - 2820*a^13*b^3*c^6*d^13 + 3288*a^13*b^3
*c^8*d^11 - 1288*a^13*b^3*c^10*d^9 - 80*a^14*b^2*c^3*d^16 + 564*a^14*b^2*c^
5*d^14 - 936*a^14*b^2*c^7*d^12 + 440*a^14*b^2*c^9*d^10 + 24*a*b^15*c^18*d))
/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2
*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13

*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) + (d^2*((8*(4*a^2*b^16*c^22 - 8*a^4*b^14*c^22 + 4*a^6*b^12*c^22 - 4*a^18*c^2*d^20 + 16*a^18*c^4*d^18 - 24*a^18*c^6*d^16 + 16*a^18*c^8*d^14 - 4*a^18*c^10*d^12 - 4*a*b^17*c^13*d^9 + 16*a*b^17*c^15*d^7 - 24*a*b^17*c^17*d^5 + 16*a*b^17*c^19*d^3 - 32*a^3*b^15*c^21*d + 76*a^5*b^13*c^21*d - 40*a^7*b^11*c^21*d + 4*a^13*b^5*c*d^21 - 8*a^15*b^3*c*d^21 + 24*a^17*b*c^3*d^19 - 136*a^17*b*c^5*d^17 + 224*a^17*b*c^7*d^15 - 156*a^17*b*c^9*d^13 + 40*a^17*b*c^11*d^11 + 40*a^2*b^16*c^12*d^10 - 156*a^2*b^16*c^14*d^8 + 224*a^2*b^16*c^16*d^6 - 136*a^2*b^16*c^18*d^4 + 24*a^2*b^16*c^20*d^2 - 176*a^3*b^15*c^11*d^11 + 672*a^3*b^15*c^13*d^9 - 928*a^3*b^15*c^15*d^7 + 512*a^3*b^15*c^17*d^5 - 48*a^3*b^15*c^19*d^3 + 440*a^4*b^14*c^10*d^12 - 1664*a^4*b^14*c^12*d^10 + 2248*a^4*b^14*c^14*d^8 - 1152*a^4*b^14*c^16*d^6 + 8*a^4*b^14*c^18*d^4 + 128*a^4*b^14*c^20*d^2 - 660*a^5*b^13*c^9*d^13 + 2552*a^5*b^13*c^11*d^11 - 3532*a^5*b^13*c^13*d^9 + 1808*a^5*b^13*c^15*d^7 + 148*a^5*b^13*c^17*d^5 - 392*a^5*b^13*c^19*d^3 + 528*a^6*b^12*c^8*d^14 - 2332*a^6*b^12*c^10*d^12 + 3736*a^6*b^12*c^12*d^10 - 2180*a^6*b^12*c^14*d^8 - 480*a^6*b^12*c^16*d^6 + 1052*a^6*b^12*c^18*d^4 - 328*a^6*b^12*c^20*d^2 + 792*a^7*b^11*c^9*d^13 - 2464*a^7*b^11*c^11*d^11 + 1896*a^7*b^11*c^13*d^9 + 1216*a^7*b^11*c^15*d^7 - 2264*a^7*b^11*c^17*d^5 + 864*a^7*b^11*c^19*d^3 - 528*a^8*b^10*c^6*d^16 + 1056*a^8*b^10*c^8*d^14 + 176*a^8*b^10*c^10*d^12 - 528*a^8*b^10*c^12*d^10 - 2288*a^8*b^10*c^14*d^8 + 3520*a^8*b^10*c^16*d^6 - 1584*a^8*b^10*c^18*d^4 + 176*a^8*b^10*c^20*d^2 + 660*a^9*b^9*c^5*d^17 - 2112*a^9*b^9*c^7*d^15 + 2244*a^9*b^9*c^9*d^13 - 1496*a^9*b^9*c^11*d^11 + 2684*a^9*b^9*c^13*d^9 - 3696*a^9*b^9*c^15*d^7 + 2156*a^9*b^9*c^17* d^5 - 440*a^9*b^9*c^19*d^3 - 440*a^10*b^8*c^4*d^18 + 2156*a^10*b^8*c^6*d^16 - 3696*a^10*b^8*c^8*d^14 + 2684*a^10*b^8*c^10*d^12 - 1496*a^10*b^8*c^12*d^10 + 2244*a^10*b^8*c^14*d^8 - 2112*a^10*b^8*c^16*d^6 + 660*a^10*b^8*c^18*d^4 + 176*a^11*b^7*c^3*d^19 - 1584*a^11*b^7*c^5*d^17 + 3520*a^11*b^7*c^7*d^15 - 2288*a^11*b^7*c^9*d^13 - 528*a^11*b^7*c^11*d^11 + 176*a^11*b^7*c^13*d^9 + 1056*a^11*b^7*c^15*d^7 - 528*a^11*b^7*c^17*d^5 - 40*a^12*b^6*c^2*d^20 + 864*a^12*b^6*c^4*d^18 - 2264*a^12*b^6*c^6*d^16 + 1216*a^12*b^6*c^8*d^14 + 1896*a^12*b^6*c^10*d^12 - 2464*a^12*b^6*c^12*d^10 + 792*a^12*b^6*c^14*d^8 - 328*a^13*b^5*c^3*d^19 + 1052*a^13*b^5*c^5*d^17 - 480*a^13*b^5*c^7*d^15 - 2180*a^13*b^5*c^9*d^13 + 3736*a^13*b^5*c^11*d^11 - 2332*a^13*b^5*c^13*d^9 + 528*a^13*b^5*c^15*d^7 + 76*a^14*b^4*c^2*d^20 - 392*a^14*b^4*c^4*d^18 + 148*a^14*b^4*c^6*d^16 + 1808*a^14*b^4*c^8*d^14 - 3532*a^14*b^4*c^10*d^12 + 2552*a^14*b^4*c^12*d^10 - 660*a^14*b^4*c^14*d^8 + 128*a^15*b^3*c^3*d^19 + 8*a^15*b^3*c^5*d^17 - 1152*a^15*b^3*c^7*d^15 + 2248*a^15*b^3*c^9*d^13 - 1664*a^15*b^3*c^11*d^11 + 440*a^15*b^3*c^13*d^9 - 32*a^16*b^2*c^2*d^20 - 48*a^16*b^2*c^4*d^18 + 512*a^16*b^2*c^6*d^16 - 928*a^16*b^2*c^8*d^14 + 672*a^16*b^2*c^10*d^12 - 176*a^16*b^2*c^12*d^10 - 4*a*b^17*c^21*d + 4*a^17*b*c*d^21))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) + (8*tan(e/2 + (f*x)/2)*(12*a*b^17*c^22 - 12*a^18*c*d^21 - 32*a^3*b^15*c^22 + 28*a^5*b^13*c^22 - 8*a^7*b^11*c^22 + 56*a^18*c^3*d^19 - 104*a^18*c^5*d^17 + 96*a^18*c^7*d^15 - 44*a^18*c^9*d^13 + 8*a^18*c^11*d^11 - 16*a*b^17*c^12*d^10 + 76*a*b^17*c^14*d^8 -

144*a*b^17*c^16*d^6 + 136*a*b^17*c^18*d^4 - 64*a*b^17*c^20*d^2 - 132*a^2*b
^16*c^21*d + 352*a^4*b^14*c^21*d - 308*a^6*b^12*c^21*d + 88*a^8*b^10*c^21*d
 + 16*a^12*b^6*c*d^21 - 44*a^14*b^4*c*d^21 + 40*a^16*b^2*c*d^21 + 132*a^17*
b*c^2*d^20 - 616*a^17*b*c^4*d^18 + 1144*a^17*b*c^6*d^16 - 1056*a^17*b*c^8*d
^14 + 484*a^17*b*c^10*d^12 - 88*a^17*b*c^12*d^10 + 176*a^2*b^16*c^11*d^11 -
 836*a^2*b^16*c^13*d^9 + 1584*a^2*b^16*c^15*d^7 - 1496*a^2*b^16*c^17*d^5 +
704*a^2*b^16*c^19*d^3 - 880*a^3*b^15*c^10*d^12 + 4224*a^3*b^15*c^12*d^10 -
8128*a^3*b^15*c^14*d^8 + 7872*a^3*b^15*c^16*d^6 - 3888*a^3*b^15*c^18*d^4 +
832*a^3*b^15*c^20*d^2 + 2640*a^4*b^14*c^9*d^13 - 13024*a^4*b^14*c^11*d^11 +
 26048*a^4*b^14*c^13*d^9 - 26752*a^4*b^14*c^15*d^7 + 14608*a^4*b^14*c^17*d^
5 - 3872*a^4*b^14*c^19*d^3 - 5280*a^5*b^13*c^8*d^14 + 27500*a^5*b^13*c^10*d
^12 - 59000*a^5*b^13*c^12*d^10 + 66628*a^5*b^13*c^14*d^8 - 41712*a^5*b^13*c
^16*d^6 + 13748*a^5*b^13*c^18*d^4 - 1912*a^5*b^13*c^20*d^2 + 7392*a^6*b^12*
c^7*d^15 - 42372*a^6*b^12*c^9*d^13 + 101288*a^6*b^12*c^11*d^11 - 129580*a^6
*b^12*c^13*d^9 + 94160*a^6*b^12*c^15*d^7 - 37532*a^6*b^12*c^17*d^5 + 6952*a
^6*b^12*c^19*d^3 - 7392*a^7*b^11*c^6*d^16 + 49632*a^7*b^11*c^8*d^14 - 13736
8*a^7*b^11*c^10*d^12 + 202544*a^7*b^11*c^12*d^10 - 170424*a^7*b^11*c^14*d^8
 + 80448*a^7*b^11*c^16*d^6 - 19016*a^7*b^11*c^18*d^4 + 1584*a^7*b^11*c^20*d
^2 + 5280*a^8*b^10*c^5*d^17 - 45408*a^8*b^10*c^7*d^15 + 150216*a^8*b^10*c^9
*d^13 - 257136*a^8*b^10*c^11*d^11 + 249832*a^8*b^10*c^13*d^9 - 138688*a^8*b
^10*c^15*d^7 + 40920*a^8*b^10*c^17*d^5 - 5104*a^8*b^10*c^19*d^3 - 2640*a^9*
b^9*c^4*d^18 + 32868*a^9*b^9*c^6*d^16 - 133056*a^9*b^9*c^8*d^14 + 266244*a^
9*b^9*c^10*d^12 - 299816*a^9*b^9*c^12*d^10 + 195404*a^9*b^9*c^14*d^8 - 7022
4*a^9*b^9*c^16*d^6 + 11660*a^9*b^9*c^18*d^4 - 440*a^9*b^9*c^20*d^2 + 880*a^
10*b^8*c^3*d^19 - 18700*a^10*b^8*c^5*d^17 + 95040*a^10*b^8*c^7*d^15 - 22567
6*a^10*b^8*c^9*d^13 + 296824*a^10*b^8*c^11*d^11 - 226116*a^10*b^8*c^13*d^9
+ 96624*a^10*b^8*c^15*d^7 - 20196*a^10*b^8*c^17*d^5 + 1320*a^10*b^8*c^19*d^
3 - 176*a^11*b^7*c^2*d^20 + 8096*a^11*b^7*c^4*d^18 - 54384*a^11*b^7*c^6*d^1
6 + 156992*a^11*b^7*c^8*d^14 - 242528*a^11*b^7*c^10*d^12 + 214368*a^11*b^7*
c^12*d^10 - 107184*a^11*b^7*c^14*d^8 + 27456*a^11*b^7*c^16*d^6 - 2640*a^11*
b^7*c^18*d^4 - 2496*a^12*b^6*c^3*d^19 + 24784*a^12*b^6*c^5*d^17 - 89280*a^1
2*b^6*c^7*d^15 + 162336*a^12*b^6*c^9*d^13 - 165760*a^12*b^6*c^11*d^11 + 962
72*a^12*b^6*c^13*d^9 - 29568*a^12*b^6*c^15*d^7 + 3696*a^12*b^6*c^17*d^5 + 4
84*a^13*b^5*c^2*d^20 - 8888*a^13*b^5*c^4*d^18 + 40876*a^13*b^5*c^6*d^16 - 8
8000*a^13*b^5*c^8*d^14 + 104060*a^13*b^5*c^10*d^12 - 69784*a^13*b^5*c^12*d^
10 + 24948*a^13*b^5*c^14*d^8 - 3696*a^13*b^5*c^16*d^6 + 2408*a^14*b^4*c^3*d
^19 - 14692*a^14*b^4*c^5*d^17 + 38208*a^14*b^4*c^7*d^15 - 52532*a^14*b^4*c^
9*d^13 + 40072*a^14*b^4*c^11*d^11 - 16060*a^14*b^4*c^13*d^9 + 2640*a^14*b^4
*c^15*d^7 - 440*a^15*b^3*c^2*d^20 + 4048*a^15*b^3*c^4*d^18 - 13112*a^15*b^3
*c^6*d^16 + 20768*a^15*b^3*c^8*d^14 - 17512*a^15*b^3*c^10*d^12 + 7568*a^15*
b^3*c^12*d^10 - 1320*a^15*b^3*c^14*d^8 - 848*a^16*b^2*c^3*d^19 + 3432*a^16*
b^2*c^5*d^17 - 6048*a^16*b^2*c^7*d^15 + 5432*a^16*b^2*c^9*d^13 - 2448*a^16*
b^2*c^11*d^11 + 440*a^16*b^2*c^13*d^9))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11
*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6
*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^1

1*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16))*(-(c + d)^5*(c - d)^5)^(1/2)*(a^2*d^4 + 12*b^2*c^4 + 6*b^2*d^4 + 2*a^2*c^2*d^2 - 15*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*b*c^3*d))/(2*(a^4*d^14 - b^4*c^14 - 5*a^4*c^2*d^12 + 10*a^4*c^4*d^10 - 10*a^4*c^6*d^8 + 5*a^4*c^8*d^6 - a^4*c^10*d^4 + b^4*c^4*d^10 - 5*b^4*c^6*d^8 + 10*b^4*c^8*d^6 - 10*b^4*c^10*d^4 + 5*b^4*c^12*d^2 - 4*a*b^3*c^3*d^11 + 20*a*b^3*c^5*d^9 - 40*a*b^3*c^7*d^7 + 40*a*b^3*c^9*d^5 - 20*a*b^3*c^11*d^3 + 20*a^3*b*c^3*d^11 - 40*a^3*b*c^5*d^9 + 40*a^3*b*c^7*d^7 - 20*a^3*b*c^9*d^5 + 4*a^3*b*c^11*d^3 + 6*a^2*b^2*c^2*d^12 - 30*a^2*b^2*c^4*d^10 + 60*a^2*b^2*c^6*d^8 - 60*a^2*b^2*c^8*d^6 + 30*a^2*b^2*c^10*d^4 - 6*a^2*b^2*c^12*d^2 + 4*a*b^3*c^13*d - 4*a^3*b*c*d^13)))*(a^2*d^4 + 12*b^2*c^4 + 6*b^2*d^4 + 2*a^2*c^2*d^2 - 15*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*b*c^3*d))/(2*(a^4*d^14 - b^4*c^14 - 5*a^4*c^2*d^12 + 10*a^4*c^4*d^10 - 10*a^4*c^6*d^8 + 5*a^4*c^8*d^6 - a^4*c^10*d^4 + b^4*c^4*d^10 - 5*b^4*c^6*d^8 + 10*b^4*c^8*d^6 - 10*b^4*c^10*d^4 + 5*b^4*c^12*d^2 - 4*a*b^3*c^3*d^11 + 20*a*b^3*c^5*d^9 - 40*a*b^3*c^7*d^7 + 40*a*b^3*c^9*d^5 - 20*a*b^3*c^11*d^3 + 20*a^3*b*c^3*d^11 - 40*a^3*b*c^5*d^9 + 40*a^3*b*c^7*d^7 - 20*a^3*b*c^9*d^5 + 4*a^3*b*c^11*d^3 + 6*a^2*b^2*c^2*d^12 - 30*a^2*b^2*c^4*d^10 + 60*a^2*b^2*c^6*d^8 - 60*a^2*b^2*c^8*d^6 + 30*a^2*b^2*c^10*d^4 - 6*a^2*b^2*c^12*d^2 + 4*a*b^3*c^13*d - 4*a^3*b*c*d^13)))*(a^2*d^4 + 12*b^2*c^4 + 6*b^2*d^4 + 2*a^2*c^2*d^2 - 15*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*b*c^3*d)*1i)/(2*(a^4*d^14 - b^4*c^14 - 5*a^4*c^2*d^12 + 10*a^4*c^4*d^10 - 10*a^4*c^6*d^8 + 5*a^4*c^8*d^6 - a^4*c^10*d^4 + b^4*c^4*d^10 - 5*b^4*c^6*d^8 + 10*b^4*c^8*d^6 - 10*b^4*c^10*d^4 + 5*b^4*c^12*d^2 - 4*a*b^3*c^3*d^11 + 20*a*b^3*c^5*d^9 - 40*a*b^3*c^7*d^7 + 40*a*b^3*c^9*d^5 - 20*a*b^3*c^11*d^3 + 20 *a^3*b*c^3*d^11 - 40*a^3*b*c^5*d^9 + 40*a^3*b*c^7*d^7 - 20*a^3*b*c^9*d^5 + 4*a^3*b*c^11*d^3 + 6*a^2*b^2*c^2*d^12 - 30*a^2*b^2*c^4*d^10 + 60*a^2*b^2*c^6*d^8 - 60*a^2*b^2*c^8*d^6 + 30*a^2*b^2*c^10*d^4 - 6*a^2*b^2*c^12*d^2 + 4*a*b^3*c^13*d - 4*a^3*b*c*d^13)))/((16*(864*a*b^11*c^5*d^8 - 486*a*b^11*c^3*d^10 - 702*a*b^11*c^7*d^6 + 216*a*b^11*c^9*d^4 - 216*a^3*b^9*c*d^12 + 63*a^5*b^7*c*d^12 + 41*a^7*b^5*c*d^12 + 4*a^9*b^3*c*d^12 + 162*a^2*b^10*c^2*d^11 - 783*a^2*b^10*c^4*d^9 + 1278*a^2*b^10*c^6*d^7 - 828*a^2*b^10*c^8*d^5 + 144*a^2*b^10*c^10*d^3 + 1197*a^3*b^9*c^3*d^10 - 2511*a^3*b^9*c^5*d^8 + 2328*a^3*b^9*c^7*d^6 - 750*a^3*b^9*c^9*d^4 + 24*a^3*b^9*c^11*d^2 - 261*a^4*b^8*c^2*d^11 + 1444*a^4*b^8*c^4*d^9 - 2508*a^4*b^8*c^6*d^7 + 1518*a^4*b^8*c^8*d^5 - 184*a^4*b^8*c^10*d^3 - 696*a^5*b^7*c^3*d^10 + 1913*a^5*b^7*c^5*d^8 - 1936*a^5*b^7*c^7*d^6 + 476*a^5*b^7*c^9*d^4 + 66*a^6*b^6*c^2*d^11 - 583*a^6*b^6*c^4*d^9 + 1232*a^6*b^6*c^6*d^7 - 580*a^6*b^6*c^8*d^5 - 21*a^7*b^5*c^3*d^10 - 312*a^7*b^5*c^5*d^8 + 364*a^7*b^5*c^7*d^6 + 19*a^8*b^4*c^2*d^11 - 20*a^8*b^4*c^4*d^9 - 116*a^8*b^4*c^6*d^7 + 16*a^9*b^3*c^3*d^10 + 16*a^9*b^3*c^5*d^8 + 108*a*b^11*c*d^12))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) + (16*tan(e/2 + (f*x)/2)*(108*a*b^11*c^2*d^11 - 486*a*b^11*c^4*d^9 + 756*a*b^11*c^6*d^7 - 432*a*b^11*c^8*d^5 + 108*a^2*b^10*c*d^12 - 162*a^4*b^8*c*d^12 + 18*a^6*b^6*c*d^12 + 8*a^8*b^4*c*d^12 - 270*a^2*b^10*c^3*d^10 + 90*a^2*b^10*c^5*d^8 + 216*a^2*b^10*c^7*d^6 - 162*a^3*b^9*c^2*d^11 + 864*a^3*b^9*c^4*d^9 - 1632*a^3*b^9*c^6*d^7 + 900*a^3*b^9*c^8*d^5 + 48*a^3*b^9*c^10*d^3 +

396*a^4*b^8*c^3*d^10 + 82*a^4*b^8*c^5*d^8 - 596*a^4*b^8*c^7*d^6 - 80*a^4*b^8*c^9*d^4 + 36*a^5*b^7*c^2*d^11 - 398*a^5*b^7*c^4*d^9 + 1216*a^5*b^7*c^6*d^7 - 584*a^5*b^7*c^8*d^5 - 42*a^6*b^6*c^3*d^10 - 432*a^6*b^6*c^5*d^8 + 600*a^6*b^6*c^7*d^6 + 38*a^7*b^5*c^2*d^11 - 40*a^7*b^5*c^4*d^9 - 232*a^7*b^5*c^6*d^7 + 32*a^8*b^4*c^3*d^10 + 32*a^8*b^4*c^5*d^8))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) - (d^2*(-(c + d)^5*(c - d)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(4*a^3*b^11*c^16 - a^14*c*d^15 - 4*a^14*c^3*d^13 - 4*a^14*c^5*d^11 - 144*a*b^13*c^4*d^12 + 684*a*b^13*c^6*d^10 - 1314*a*b^13*c^8*d^8 + 1224*a*b^13*c^10*d^6 - 504*a*b^13*c^12*d^4 + 36*a*b^13*c^14*d^2 + 24*a^2*b^12*c^15*d + 144*a^4*b^10*c*d^15 - 44*a^4*b^10*c^15*d - 348*a^6*b^8*c*d^15 + 214*a^8*b^6*c*d^15 + 7*a^10*b^4*c*d^15 - 8*a^12*b^2*c*d^15 - a^13*b*c^2*d^14 + 20*a^13*b*c^4*d^12 + 44*a^13*b*c^6*d^10 + 432*a^2*b^12*c^3*d^13 - 2148*a^2*b^12*c^5*d^11 + 4470*a^2*b^12*c^7*d^9 - 4632*a^2*b^12*c^9*d^7 + 2232*a^2*b^12*c^11*d^5 - 252*a^2*b^12*c^13*d^3 - 432*a^3*b^11*c^2*d^14 + 2688*a^3*b^11*c^4*d^12 - 7294*a^3*b^11*c^6*d^10 + 10105*a^3*b^11*c^8*d^8 - 7104*a^3*b^11*c^10*d^6 + 1892*a^3*b^11*c^12*d^4 - 192*a^3*b^11*c^14*d^2 - 2016*a^4*b^10*c^3*d^13 + 8378*a^4*b^10*c^5*d^11 - 15815*a^4*b^10*c^7*d^9 + 14976*a^4*b^10*c^9*d^7 - 5932*a^4*b^10*c^11*d^5 + 624*a^4*b^10*c^13*d^3 + 1140*a^5*b^9*c^2*d^14 - 6574*a^5*b^9*c^4*d^12 + 16053*a^5*b^9*c^6*d^10 - 19912*a^5*b^9*c^8*d^8 + 11320*a^5*b^9*c^10*d^6 - 1920*a^5*b^9*c^12*d^4 + 172*a^5*b^9*c^14*d^2 + 2938*a^6*b^8*c^3*d^13 - 10619*a^6*b^8*c^5*d^11 + 18608*a^6*b^8*c^7*d^9

- 15576*a^6*b^8*c^9*d^7 + 4344*a^6*b^8*c^11*d^5 - 292*a^6*b^8*c^13*d^3 - 8
18*a^7*b^7*c^2*d^14 + 5107*a^7*b^7*c^4*d^12 - 12464*a^7*b^7*c^6*d^10 + 1469
3*a^7*b^7*c^8*d^8 - 6184*a^7*b^7*c^10*d^6 + 368*a^7*b^7*c^12*d^4 - 1485*a^8
*b^6*c^3*d^13 + 5064*a^8*b^6*c^5*d^11 - 8939*a^8*b^6*c^7*d^9 + 6104*a^8*b^6
*c^9*d^7 - 688*a^8*b^6*c^11*d^5 + 55*a^9*b^5*c^2*d^14 - 1056*a^9*b^5*c^4*d^
12 + 3649*a^9*b^5*c^6*d^10 - 4524*a^9*b^5*c^8*d^8 + 1120*a^9*b^5*c^10*d^6 +
152*a^10*b^4*c^3*d^13 - 975*a^10*b^4*c^5*d^11 + 2300*a^10*b^4*c^7*d^9 - 10
88*a^10*b^4*c^9*d^7 + 16*a^11*b^3*c^2*d^14 + 59*a^11*b^3*c^4*d^12 - 640*a^1
1*b^3*c^6*d^10 + 628*a^11*b^3*c^8*d^8 + 27*a^12*b^2*c^3*d^13 + 48*a^12*b^2*
c^5*d^11 - 220*a^12*b^2*c^7*d^9))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17
- a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*
c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6
- 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7
+ 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8
*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a
^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^
10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 -
44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a
^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*
b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^
11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13
- 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852
*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^
7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^
11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 -
396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548
*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^
5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^
9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 +
420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a
^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b
^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^
11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^
11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12
*b*c*d^16) - (8*(36*a*b^13*c^5*d^11 - 144*a*b^13*c^7*d^9 + 216*a*b^13*c^9*d
^7 - 144*a*b^13*c^11*d^5 + 36*a*b^13*c^13*d^3 + 4*a^3*b^11*c^15*d - 36*a^5*
b^9*c*d^15 + 60*a^7*b^7*c*d^15 - 13*a^9*b^5*c*d^15 - 10*a^11*b^3*c*d^15 - 4
*a^13*b*c^3*d^13 - 4*a^13*b*c^5*d^11 - 72*a^2*b^12*c^4*d^12 + 276*a^2*b^12*
c^6*d^10 - 375*a^2*b^12*c^8*d^8 + 216*a^2*b^12*c^10*d^6 - 60*a^2*b^12*c^12*
d^4 + 24*a^2*b^12*c^14*d^2 - 36*a^3*b^11*c^5*d^11 + 61*a^3*b^11*c^7*d^9 - 8
8*a^3*b^11*c^9*d^7 + 180*a^3*b^11*c^11*d^5 - 184*a^3*b^11*c^13*d^3 + 72*a^4
*b^10*c^2*d^14 - 168*a^4*b^10*c^4*d^12 + 233*a^4*b^10*c^6*d^10 - 270*a^4*b^
10*c^8*d^8 + 100*a^4*b^10*c^10*d^6 + 248*a^4*b^10*c^12*d^4 - 44*a^4*b^10*c^
14*d^2 + 120*a^5*b^9*c^3*d^13 - 535*a^5*b^9*c^5*d^11 + 1386*a^5*b^9*c^7*d^9
- 1544*a^5*b^9*c^9*d^7 + 248*a^5*b^9*c^11*d^5 + 172*a^5*b^9*c^13*d^3 - 108

*a^6*b^8*c^2*d^14 + 699*a^6*b^8*c^4*d^12 - 2046*a^6*b^8*c^6*d^10 + 2885*a^6*b^8*c^8*d^8 - 1336*a^6*b^8*c^10*d^6 - 148*a^6*b^8*c^12*d^4 - 305*a^7*b^7*c^3*d^13 + 1354*a^7*b^7*c^5*d^11 - 2979*a^7*b^7*c^7*d^9 + 2648*a^7*b^7*c^9*d^7 - 400*a^7*b^7*c^11*d^5 + 19*a^8*b^6*c^2*d^14 - 602*a^8*b^6*c^4*d^12 + 2161*a^8*b^6*c^6*d^10 - 3012*a^8*b^6*c^8*d^8 + 1056*a^8*b^6*c^10*d^6 + 190*a^9*b^5*c^3*d^13 - 895*a^9*b^5*c^5*d^11 + 1860*a^9*b^5*c^7*d^9 - 1088*a^9*b^5*c^9*d^7 + 14*a^10*b^4*c^2*d^14 + 99*a^10*b^4*c^4*d^12 - 552*a^10*b^4*c^6*d^10 + 628*a^10*b^4*c^8*d^8 + 19*a^11*b^3*c^3*d^13 + 40*a^11*b^3*c^5*d^11 - 220*a^11*b^3*c^7*d^9 - a^12*b^2*c^2*d^14 + 20*a^12*b^2*c^4*d^12 + 44*a^12*b^2*c^6*d^10 - a^13*b*c*d^15))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) + (d^2*(-(c + d)^5*(c - d)^5)^(1/2)*((8*(4*a^3*b^13*c^19 - 4*a^5*b^11*c^19 + 2*a^16*c^2*d^17 - 6*a^16*c^6*d^13 + 4*a^16*c^8*d^11 + 12*a*b^15*c^9*d^10 - 54*a*b^15*c^11*d^8 + 96*a*b^15*c^13*d^6 - 78*a*b^15*c^15*d^4 + 24*a*b^15*c^17*d^2 + 12*a^2*b^14*c^18*d - 56*a^4*b^12*c^18*d + 44*a^6*b^10*c^18*d + 12*a^9*b^7*c*d^18 - 28*a^11*b^5*c*d^18 + 16*a^13*b^3*c*d^18 - 10*a^15*b*c^3*d^16 - 24*a^15*b*c^5*d^14 + 78*a^15*b*c^7*d^12 - 44*a^15*b*c^9*d^10 - 96*a^2*b^14*c^8*d^11 + 442*a^2*b^14*c^10*d^9 - 816*a^2*b^14*c^12*d^7 + 702*a^2*b^14*c^14*d^5 - 244*a^2*b^14*c^16*d^3 + 336*a^3*b^13*c^7*d^12 - 1620*a^3*b^13*c^9*d^10 + 3206*a^3*b^13*c^11*d^8 - 3064*a^3*b^13*c^13*d^6 + 1314*a^3*b^13*c^15*d^4 - 176*a^3*b^13*c^17*d^2 - 672*a^4*b^12*c^6*d^13 + 3528*a^4*b^12*c^8*d^11 - 7810*a^4*b^12*c^10*d^9 + 8696*a^4*b^12*c^12*d^7 - 4770*a^

4*b^12*c^14*d^5 + 1084*a^4*b^12*c^16*d^3 + 840*a^5*b^11*c^5*d^14 - 5124*a^5
*b^11*c^7*d^12 + 13320*a^5*b^11*c^9*d^10 - 17850*a^5*b^11*c^11*d^8 + 12400*
a^5*b^11*c^13*d^6 - 3954*a^5*b^11*c^15*d^4 + 372*a^5*b^11*c^17*d^2 - 672*a^
6*b^10*c^4*d^15 + 5292*a^6*b^10*c^6*d^13 - 16872*a^6*b^10*c^8*d^11 + 27546*
a^6*b^10*c^10*d^9 - 23696*a^6*b^10*c^12*d^7 + 9858*a^6*b^10*c^14*d^5 - 1500
*a^6*b^10*c^16*d^3 + 336*a^7*b^9*c^3*d^16 - 4032*a^7*b^9*c^5*d^14 + 16212*a
^7*b^9*c^7*d^12 - 32304*a^7*b^9*c^9*d^10 + 34018*a^7*b^9*c^11*d^8 - 18048*a
^7*b^9*c^13*d^6 + 4038*a^7*b^9*c^15*d^4 - 220*a^7*b^9*c^17*d^2 - 96*a^8*b^8
*c^2*d^17 + 2280*a^8*b^8*c^4*d^15 - 11772*a^8*b^8*c^6*d^13 + 28848*a^8*b^8*
c^8*d^11 - 37338*a^8*b^8*c^10*d^9 + 25056*a^8*b^8*c^12*d^7 - 7638*a^8*b^8*c
^14*d^5 + 660*a^8*b^8*c^16*d^3 - 918*a^9*b^7*c^3*d^16 + 6360*a^9*b^7*c^5*d^
14 - 19602*a^9*b^7*c^7*d^12 + 31560*a^9*b^7*c^9*d^10 - 26556*a^9*b^7*c^11*d
^8 + 10464*a^9*b^7*c^13*d^6 - 1320*a^9*b^7*c^15*d^4 + 234*a^10*b^6*c^2*d^17
 - 2520*a^10*b^6*c^4*d^15 + 10050*a^10*b^6*c^6*d^13 - 20340*a^10*b^6*c^8*d^
11 + 21288*a^10*b^6*c^10*d^9 - 10560*a^10*b^6*c^12*d^7 + 1848*a^10*b^6*c^14
*d^5 + 726*a^11*b^5*c^3*d^16 - 3768*a^11*b^5*c^5*d^14 + 9670*a^11*b^5*c^7*d
^12 - 12648*a^11*b^5*c^9*d^10 + 7896*a^11*b^5*c^11*d^8 - 1848*a^11*b^5*c^13
*d^6 - 146*a^12*b^4*c^2*d^17 + 952*a^12*b^4*c^4*d^15 - 3174*a^12*b^4*c^6*d^
13 + 5396*a^12*b^4*c^8*d^11 - 4348*a^12*b^4*c^10*d^9 + 1320*a^12*b^4*c^12*d
^7 - 134*a^13*b^3*c^3*d^16 + 624*a^13*b^3*c^5*d^14 - 1570*a^13*b^3*c^7*d^12
 + 1724*a^13*b^3*c^9*d^10 - 660*a^13*b^3*c^11*d^8 + 6*a^14*b^2*c^2*d^17 - 4
0*a^14*b^2*c^4*d^15 + 282*a^14*b^2*c^6*d^13 - 468*a^14*b^2*c^8*d^11 + 220*a
^14*b^2*c^10*d^9))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17
 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^
13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d
^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^
12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8
*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12
 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^
11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^
15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^
7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 +
576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^
4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c
^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d
^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 15
48*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^
6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^
4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d
^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 85
2*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8
*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^
4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7
 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 5
76*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^1

1*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) + (8*tan(e/2 + (f*x)/2)*(4*a^16*c*d^18 + 8*a^2*b^14*c^19 - 8*a^4*b^12*c^19 - 12*a^16*c^5*d^14 + 8*a^16*c^7*d^12 + 12*a*b^15*c^10*d^9 - 48*a*b^15*c^12*d^7 + 84*a*b^15*c^14*d^5 - 72*a*b^15*c^16*d^3 - 112*a^3*b^13*c^18*d + 88*a^5*b^11*c^18*d + 12*a^10*b^6*c*d^18 - 28*a^12*b^4*c*d^18 + 12*a^14*b^2*c*d^18 - 20*a^15*b*c^2*d^17 - 48*a^15*b*c^4*d^15 + 156*a^15*b*c^6*d^13 - 88*a^15*b*c^8*d^11 - 84*a^2*b^14*c^9*d^10 + 328*a^2*b^14*c^11*d^8 - 596*a^2*b^14*c^13*d^6 + 552*a^2*b^14*c^15*d^4 - 208*a^2*b^14*c^17*d^2 + 240*a^3*b^13*c^8*d^11 - 908*a^3*b^13*c^10*d^9 + 1792*a^3*b^13*c^12*d^7 - 1932*a^3*b^13*c^14*d^5 + 920*a^3*b^13*c^16*d^3 - 336*a^4*b^12*c^7*d^12 + 1188*a^4*b^12*c^9*d^10 - 2808*a^4*b^12*c^11*d^8 + 3980*a^4*b^12*c^13*d^6 - 2616*a^4*b^12*c^15*d^4 + 600*a^4*b^12*c^17*d^2 + 168*a^5*b^11*c^6*d^13 - 336*a^5*b^11*c^8*d^11 + 1740*a^5*b^11*c^10*d^9 - 4720*a^5*b^11*c^12*d^7 + 4812*a^5*b^11*c^14*d^5 - 1752*a^5*b^11*c^16*d^3 + 168*a^6*b^10*c^5*d^14 - 1344*a^6*b^10*c^7*d^12 + 2292*a^6*b^10*c^9*d^10 + 1088*a^6*b^10*c^11*d^8 - 4908*a^6*b^10*c^13*d^6 + 3096*a^6*b^10*c^15*d^4 - 392*a^6*b^10*c^17*d^2 - 336*a^7*b^9*c^4*d^15 + 2520*a^7*b^9*c^6*d^13 - 7488*a^7*b^9*c^8*d^11 + 7556*a^7*b^9*c^10*d^9 - 144*a^7*b^9*c^12*d^7 - 3012*a^7*b^9*c^14*d^5 + 904*a^7*b^9*c^16*d^3 + 240*a^8*b^8*c^3*d^16 - 2472*a^8*b^8*c^5*d^14 + 10416*a^8*b^8*c^7*d^12 - 16596*a^8*b^8*c^9*d^10 + 9600*a^8*b^8*c^11*d^8 - 156*a^8*b^8*c^13*d^6 - 1032*a^8*b^8*c^15*d^4 - 84*a^9*b^7*c^2*d^17 + 1632*a^9*b^7*c^4*d^15 - 9204*a^9*b^7*c^6*d^13 + 19800*a^9*b^7*c^8*d^11 - 18048*a^9*b^7*c^10*d^9 + 5856*a^9*b^7*c^12*d^7 + 48*a^9*b^7*c^14*d^5 - 744*a^10*b^6*c^3*d^16 + 5460*a^10*b^6*c^5*d^14 - 15960*a^10*b^6*c^7*d^12 + 20136*a^10*b^6*c^9*d^10 - 10584*a^10*b^6*c^11*d^8 + 1680*a^10*b^6*c^13*d^6 + 212*a^11*b^5*c^2*d^17 - 2176*a^11*b^5*c^4*d^15 + 9180*a^11*b^5*c^6*d^13 - 15416*a^11*b^5*c^8*d^11 + 10936*a^11*b^5*c^10*d^9 - 2736*a^11*b^5*c^12*d^7 + 584*a^12*b^4*c^3*d^16 - 3708*a^12*b^4*c^5*d^14 + 8152*a^12*b^4*c^7*d^12 - 7376*a^12*b^4*c^9*d^10 + 2376*a^12*b^4*c^11*d^8 - 108*a^13*b^3*c^2*d^17 + 928*a^13*b^3*c^4*d^15 - 2820*a^13*b^3*c^6*d^13 + 3288*a^13*b^3*c^8*d^11 - 1288*a^13*b^3*c^10*d^9 - 80*a^14*b^2*c^3*d^16 + 564*a^14*b^2*c^5*d^14 - 936*a^14*b^2*c^7*d^12 + 440*a^14*b^2*c^9*d^10 + 24*a*b^15*c^18*d))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^

5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) - (d^2*((8*(4*a^2*b^16*c^22 - 8*a^4*b^14*c^22 + 4*a^6*b^12*c^22 - 4*a^18*c^2*d^20 + 16*a^18*c^4*d^18 - 24*a^18*c^6*d^16 + 16*a^18*c^8*d^14 - 4*a^18*c^10*d^12 - 4*a*b^17*c^13*d^9 + 16*a*b^17*c^15*d^7 - 24*a*b^17*c^17*d^5 + 16*a*b^17*c^19*d^3 - 32*a^3*b^15*c^21*d + 76*a^5*b^13*c^21*d - 40*a^7*b^11*c^21*d + 4*a^13*b^5*c*d^21 - 8*a^15*b^3*c*d^21 + 24*a^17*b*c^3*d^19 - 136*a^17*b*c^5*d^17 + 224*a^17*b*c^7*d^15 - 156*a^17*b*c^9*d^13 + 40*a^17*b*c^11*d^11 + 40*a^2*b^16*c^12*d^10 - 156*a^2*b^16*c^14*d^8 + 224*a^2*b^16*c^16*d^6 - 136*a^2*b^16*c^18*d^4 + 24*a^2*b^16*c^20*d^2 - 176*a^3*b^15*c^11*d^11 + 672*a^3*b^15*c^13*d^9 - 928*a^3*b^15*c^15*d^7 + 512*a^3*b^15*c^17*d^5 - 48*a^3*b^15*c^19*d^3 + 440*a^4*b^14*c^10*d^12 - 1664*a^4*b^14*c^12*d^10 + 2248*a^4*b^14*c^14*d^8 - 1152*a^4*b^14*c^16*d^6 + 8*a^4*b^14*c^18*d^4 + 128*a^4*b^14*c^20*d^2 - 660*a^5*b^13*c^9*d^13 + 2552*a^5*b^13*c^11*d^11 - 3532*a^5*b^13*c^13*d^9 + 1808*a^5*b^13*c^15*d^7 + 148*a^5*b^13*c^17*d^5 - 392*a^5*b^13*c^19*d^3 + 528*a^6*b^12*c^8*d^14 - 2332*a^6*b^12*c^10*d^12 + 3736*a^6*b^12*c^12*d^10 - 2180*a^6*b^12*c^14*d^8 - 480*a^6*b^12*c^16*d^6 + 1052*a^6*b^12*c^18*d^4 - 328*a^6*b^12*c^20*d^2 + 792*a^7*b^11*c^9*d^13 - 2464*a^7*b^11*c^11*d^11 + 1896*a^7*b^11*c^13*d^9 + 1216*a^7*b^11*c^15*d^7 - 2264*a^7*b^11*c^17*d^5 + 864*a^7*b^11*c^19*d^3 - 528*a^8*b^10*c^6*d^16 + 1056*a^8*b^10*c^8*d^14 + 176*a^8*b^10*c^10*d^12 - 528*a^8*b^10*c^12*d^10 - 2288*a^8*b^10*c^14*d^8 + 3520*a^8*b^10*c^16*d^6 - 1584*a^8*b^10*c^18*d^4 + 176*a^8*b^10*c^20*d^2 + 660*a^9*b^9*c^5*d^17 - 2112*a^9*b^9*c^7*d^15 + 2244*a^9*b^9*c^9*d^13 - 1496*a^9*b^9*c^11*d^11 + 2684*a^9*b^9*c^13*d^9 - 3696*a^9*b^9*c^15*d^7 + 2156*a^9*b^9*c^17*d^5 - 440*a^9*b^9*c^19*d^3 - 440*a^10*b^8*c^4*d^18 + 2156*a^10*b^8*c^6*d^16 - 3696*a^10*b^8*c^8*d^14 + 2684*a^10*b^8*c^10*d^12 - 1496*a^10*b^8*c^12*d^10 + 2244*a^10*b^8*c^14*d^8 - 2112*a^10*b^8*c^16*d^6 + 660*a^10*b^8*c^18*d^4 + 176*a^11*b^7*c^3*d^19 - 1584*a^11*b^7*c^5*d^17 + 3520*a^11*b^7*c^7*d^15 - 2288*a^11*b^7*c^9*d^13 - 528*a^11*b^7*c^11*d^11 + 176*a^11*b^7*c^13*d^9 + 1056*a^11*b^7*c^15*d^7 - 528*a^11*b^7*c^17*d^5 - 40*a^12*b^6*c^2*d^20 + 864*a^12*b^6*c^4*d^18 - 2264*a^12*b^6*c^6*d^16 + 1216*a^12*b^6*c^8*d^14 + 1896*a^12*b^6*c^10*d^12 - 2464*a^12*b^6*c^12*d^10 + 792*a^12*b^6*c^14*d^8 - 328*a^13*b^5*c^3*d^19 + 1052*a^13*b^5*c^5*d^17 - 480*a^13*b^5*c^7*d^15 - 2180*a^13*b^5*c^9*d^13 + 3736*a^13*b^5*c^11*d^11 - 2332*a^13*b^5*c^13*d^9 + 528*a^13*b^5*c^15*d^7 + 76*a^14*b^4*c^2*d^20 - 392*a^14*b^4*c^4*d^18 +

148*a^14*b^4*c^6*d^16 + 1808*a^14*b^4*c^8*d^14 - 3532*a^14*b^4*c^10*d^12 + 2552*a^14*b^4*c^12*d^10 - 660*a^14*b^4*c^14*d^8 + 128*a^15*b^3*c^3*d^19 + 8*a^15*b^3*c^5*d^17 - 1152*a^15*b^3*c^7*d^15 + 2248*a^15*b^3*c^9*d^13 - 1664*a^15*b^3*c^11*d^11 + 440*a^15*b^3*c^13*d^9 - 32*a^16*b^2*c^2*d^20 - 48*a^16*b^2*c^4*d^18 + 512*a^16*b^2*c^6*d^16 - 928*a^16*b^2*c^8*d^14 + 672*a^16*b^2*c^10*d^12 - 176*a^16*b^2*c^12*d^10 - 4*a*b^17*c^21*d + 4*a^17*b*c*d^21))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) + (8*tan(e/2 + (f*x)/2)*(12*a*b^17*c^22 - 12*a^18*c*d^21 - 32*a^3*b^15*c^22 + 28*a^5*b^13*c^22 - 8*a^7*b^11*c^22 + 56*a^18*c^3*d^19 - 104*a^18*c^5*d^17 + 96*a^18*c^7*d^15 - 44*a^18*c^9*d^13 + 8*a^18*c^11*d^11 - 16*a*b^17*c^12*d^10 + 76*a*b^17*c^14*d^8 - 144*a*b^17*c^16*d^6 + 136*a*b^17*c^18*d^4 - 64*a*b^17*c^20*d^2 - 132*a^2*b^16*c^21*d + 352*a^4*b^14*c^21*d - 308*a^6*b^12*c^21*d + 88*a^8*b^10*c^21*d + 16*a^12*b^6*c*d^21 - 44*a^14*b^4*c*d^21 + 40*a^16*b^2*c*d^21 + 132*a^17*b*c^2*d^20 - 616*a^17*b*c^4*d^18 + 1144*a^17*b*c^6*d^16 - 1056*a^17*b*c^8*d^14 + 484*a^17*b*c^10*d^12 - 88*a^17*b*c^12*d^10 + 176*a^2*b^16*c^11*d^11 - 836*a^2*b^16*c^13*d^9 + 1584*a^2*b^16*c^15*d^7 - 1496*a^2*b^16*c^17*d^5 + 704*a^2*b^16*c^19*d^3 - 880*a^3*b^15*c^10*d^12 + 4224*a^3*b^15*c^12*d^10 - 8128*a^3*b^15*c^14*d^8 + 7872*a^3*b^15*c^16*d^6 - 3888*a^3*b^15*c^18*d^4 + 832*a^3*b^15*c^20*d^2 + 2640*a^4*b^14*c^9*d^13 - 13024*a^4*b^14*c^11*d^11 + 26048*a^4*b^14*c^13*d^9 - 26752*a^4*b^14*c^15*d^7 + 14608*a^4*b^14*c^17*d^5 - 3872*a^4*b^14*c^19*d^3 - 5280*a^5*b^13*c^8*d^14 + 27500*a^5*b^13

*c^10*d^12 - 59000*a^5*b^13*c^12*d^10 + 66628*a^5*b^13*c^14*d^8 - 41712*a^5
*b^13*c^16*d^6 + 13748*a^5*b^13*c^18*d^4 - 1912*a^5*b^13*c^20*d^2 + 7392*a^
6*b^12*c^7*d^15 - 42372*a^6*b^12*c^9*d^13 + 101288*a^6*b^12*c^11*d^11 - 129
580*a^6*b^12*c^13*d^9 + 94160*a^6*b^12*c^15*d^7 - 37532*a^6*b^12*c^17*d^5 +
6952*a^6*b^12*c^19*d^3 - 7392*a^7*b^11*c^6*d^16 + 49632*a^7*b^11*c^8*d^14
- 137368*a^7*b^11*c^10*d^12 + 202544*a^7*b^11*c^12*d^10 - 170424*a^7*b^11*c
^14*d^8 + 80448*a^7*b^11*c^16*d^6 - 19016*a^7*b^11*c^18*d^4 + 1584*a^7*b^11
*c^20*d^2 + 5280*a^8*b^10*c^5*d^17 - 45408*a^8*b^10*c^7*d^15 + 150216*a^8*b
^10*c^9*d^13 - 257136*a^8*b^10*c^11*d^11 + 249832*a^8*b^10*c^13*d^9 - 13868
8*a^8*b^10*c^15*d^7 + 40920*a^8*b^10*c^17*d^5 - 5104*a^8*b^10*c^19*d^3 - 26
40*a^9*b^9*c^4*d^18 + 32868*a^9*b^9*c^6*d^16 - 133056*a^9*b^9*c^8*d^14 + 26
6244*a^9*b^9*c^10*d^12 - 299816*a^9*b^9*c^12*d^10 + 195404*a^9*b^9*c^14*d^8
- 70224*a^9*b^9*c^16*d^6 + 11660*a^9*b^9*c^18*d^4 - 440*a^9*b^9*c^20*d^2 +
880*a^10*b^8*c^3*d^19 - 18700*a^10*b^8*c^5*d^17 + 95040*a^10*b^8*c^7*d^15
- 225676*a^10*b^8*c^9*d^13 + 296824*a^10*b^8*c^11*d^11 - 226116*a^10*b^8*c^
13*d^9 + 96624*a^10*b^8*c^15*d^7 - 20196*a^10*b^8*c^17*d^5 + 1320*a^10*b^8*
c^19*d^3 - 176*a^11*b^7*c^2*d^20 + 8096*a^11*b^7*c^4*d^18 - 54384*a^11*b^7*
c^6*d^16 + 156992*a^11*b^7*c^8*d^14 - 242528*a^11*b^7*c^10*d^12 + 214368*a^
11*b^7*c^12*d^10 - 107184*a^11*b^7*c^14*d^8 + 27456*a^11*b^7*c^16*d^6 - 264
0*a^11*b^7*c^18*d^4 - 2496*a^12*b^6*c^3*d^19 + 24784*a^12*b^6*c^5*d^17 - 89
280*a^12*b^6*c^7*d^15 + 162336*a^12*b^6*c^9*d^13 - 165760*a^12*b^6*c^11*d^1
1 + 96272*a^12*b^6*c^13*d^9 - 29568*a^12*b^6*c^15*d^7 + 3696*a^12*b^6*c^17*
d^5 + 484*a^13*b^5*c^2*d^20 - 8888*a^13*b^5*c^4*d^18 + 40876*a^13*b^5*c^6*d
^16 - 88000*a^13*b^5*c^8*d^14 + 104060*a^13*b^5*c^10*d^12 - 69784*a^13*b^5*
c^12*d^10 + 24948*a^13*b^5*c^14*d^8 - 3696*a^13*b^5*c^16*d^6 + 2408*a^14*b^
4*c^3*d^19 - 14692*a^14*b^4*c^5*d^17 + 38208*a^14*b^4*c^7*d^15 - 52532*a^14
*b^4*c^9*d^13 + 40072*a^14*b^4*c^11*d^11 - 16060*a^14*b^4*c^13*d^9 + 2640*a
^14*b^4*c^15*d^7 - 440*a^15*b^3*c^2*d^20 + 4048*a^15*b^3*c^4*d^18 - 13112*a
^15*b^3*c^6*d^16 + 20768*a^15*b^3*c^8*d^14 - 17512*a^15*b^3*c^10*d^12 + 756
8*a^15*b^3*c^12*d^10 - 1320*a^15*b^3*c^14*d^8 - 848*a^16*b^2*c^3*d^19 + 343
2*a^16*b^2*c^5*d^17 - 6048*a^16*b^2*c^7*d^15 + 5432*a^16*b^2*c^9*d^13 - 244
8*a^16*b^2*c^11*d^11 + 440*a^16*b^2*c^13*d^9))/(a^13*d^17 - b^13*c^17 + 2*a
^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d
^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b
^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*
b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*
d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^
3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^
2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^
11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^
8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d
^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 9
40*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*
b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c
^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14

+ 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 13
08*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b
^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c
^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3
+ 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 14
37*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b
^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^
8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14
+ 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 8
4*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^
11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^
16*d - 9*a^12*b*c*d^16))*(-(c + d)^5*(c - d)^5)^(1/2)*(a^2*d^4 + 12*b^2*c^4
+ 6*b^2*d^4 + 2*a^2*c^2*d^2 - 15*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*b*c^3*d))
/(2*(a^4*d^14 - b^4*c^14 - 5*a^4*c^2*d^12 + 10*a^4*c^4*d^10 - 10*a^4*c^6*d^
8 + 5*a^4*c^8*d^6 - a^4*c^10*d^4 + b^4*c^4*d^10 - 5*b^4*c^6*d^8 + 10*b^4*c^
8*d^6 - 10*b^4*c^10*d^4 + 5*b^4*c^12*d^2 - 4*a*b^3*c^3*d^11 + 20*a*b^3*c^5*
d^9 - 40*a*b^3*c^7*d^7 + 40*a*b^3*c^9*d^5 - 20*a*b^3*c^11*d^3 + 20*a^3*b*c^
3*d^11 - 40*a^3*b*c^5*d^9 + 40*a^3*b*c^7*d^7 - 20*a^3*b*c^9*d^5 + 4*a^3*b*c
^11*d^3 + 6*a^2*b^2*c^2*d^12 - 30*a^2*b^2*c^4*d^10 + 60*a^2*b^2*c^6*d^8 - 6
0*a^2*b^2*c^8*d^6 + 30*a^2*b^2*c^10*d^4 - 6*a^2*b^2*c^12*d^2 + 4*a*b^3*c^13
*d - 4*a^3*b*c*d^13)))*(a^2*d^4 + 12*b^2*c^4 + 6*b^2*d^4 + 2*a^2*c^2*d^2 -
15*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*b*c^3*d))/(2*(a^4*d^14 - b^4*c^14 - 5*a^
4*c^2*d^12 + 10*a^4*c^4*d^10 - 10*a^4*c^6*d^8 + 5*a^4*c^8*d^6 - a^4*c^10*d^
4 + b^4*c^4*d^10 - 5*b^4*c^6*d^8 + 10*b^4*c^8*d^6 - 10*b^4*c^10*d^4 + 5*b^4
*c^12*d^2 - 4*a*b^3*c^3*d^11 + 20*a*b^3*c^5*d^9 - 40*a*b^3*c^7*d^7 + 40*a*b
^3*c^9*d^5 - 20*a*b^3*c^11*d^3 + 20*a^3*b*c^3*d^11 - 40*a^3*b*c^5*d^9 + 40*
a^3*b*c^7*d^7 - 20*a^3*b*c^9*d^5 + 4*a^3*b*c^11*d^3 + 6*a^2*b^2*c^2*d^12 -
30*a^2*b^2*c^4*d^10 + 60*a^2*b^2*c^6*d^8 - 60*a^2*b^2*c^8*d^6 + 30*a^2*b^2*
c^10*d^4 - 6*a^2*b^2*c^12*d^2 + 4*a*b^3*c^13*d - 4*a^3*b*c*d^13)))*(a^2*d^4
+ 12*b^2*c^4 + 6*b^2*d^4 + 2*a^2*c^2*d^2 - 15*b^2*c^2*d^2 + 2*a*b*c*d^3 -
8*a*b*c^3*d))/(2*(a^4*d^14 - b^4*c^14 - 5*a^4*c^2*d^12 + 10*a^4*c^4*d^10 -
10*a^4*c^6*d^8 + 5*a^4*c^8*d^6 - a^4*c^10*d^4 + b^4*c^4*d^10 - 5*b^4*c^6*d^
8 + 10*b^4*c^8*d^6 - 10*b^4*c^10*d^4 + 5*b^4*c^12*d^2 - 4*a*b^3*c^3*d^11 +
20*a*b^3*c^5*d^9 - 40*a*b^3*c^7*d^7 + 40*a*b^3*c^9*d^5 - 20*a*b^3*c^11*d^3
+ 20*a^3*b*c^3*d^11 - 40*a^3*b*c^5*d^9 + 40*a^3*b*c^7*d^7 - 20*a^3*b*c^9*d^
5 + 4*a^3*b*c^11*d^3 + 6*a^2*b^2*c^2*d^12 - 30*a^2*b^2*c^4*d^10 + 60*a^2*b^
2*c^6*d^8 - 60*a^2*b^2*c^8*d^6 + 30*a^2*b^2*c^10*d^4 - 6*a^2*b^2*c^12*d^2 +
4*a*b^3*c^13*d - 4*a^3*b*c*d^13)) - (d^2*(-(c + d)^5*(c - d)^5)^(1/2)*((8*
(36*a*b^13*c^5*d^11 - 144*a*b^13*c^7*d^9 + 216*a*b^13*c^9*d^7 - 144*a*b^13*
c^11*d^5 + 36*a*b^13*c^13*d^3 + 4*a^3*b^11*c^15*d - 36*a^5*b^9*c*d^15 + 60*
a^7*b^7*c*d^15 - 13*a^9*b^5*c*d^15 - 10*a^11*b^3*c*d^15 - 4*a^13*b*c^3*d^13
- 4*a^13*b*c^5*d^11 - 72*a^2*b^12*c^4*d^12 + 276*a^2*b^12*c^6*d^10 - 375*a
^2*b^12*c^8*d^8 + 216*a^2*b^12*c^10*d^6 - 60*a^2*b^12*c^12*d^4 + 24*a^2*b^1
2*c^14*d^2 - 36*a^3*b^11*c^5*d^11 + 61*a^3*b^11*c^7*d^9 - 88*a^3*b^11*c^9*d
^7 + 180*a^3*b^11*c^11*d^5 - 184*a^3*b^11*c^13*d^3 + 72*a^4*b^10*c^2*d^14 -

168*a^4*b^10*c^4*d^12 + 233*a^4*b^10*c^6*d^10 - 270*a^4*b^10*c^8*d^8 + 100*a^4*b^10*c^10*d^6 + 248*a^4*b^10*c^12*d^4 - 44*a^4*b^10*c^14*d^2 + 120*a^5*b^9*c^3*d^13 - 535*a^5*b^9*c^5*d^11 + 1386*a^5*b^9*c^7*d^9 - 1544*a^5*b^9*c^9*d^7 + 248*a^5*b^9*c^11*d^5 + 172*a^5*b^9*c^13*d^3 - 108*a^6*b^8*c^2*d^14 + 699*a^6*b^8*c^4*d^12 - 2046*a^6*b^8*c^6*d^10 + 2885*a^6*b^8*c^8*d^8 - 1336*a^6*b^8*c^10*d^6 - 148*a^6*b^8*c^12*d^4 - 305*a^7*b^7*c^3*d^13 + 1354*a^7*b^7*c^5*d^11 - 2979*a^7*b^7*c^7*d^9 + 2648*a^7*b^7*c^9*d^7 - 400*a^7*b^7*c^11*d^5 + 19*a^8*b^6*c^2*d^14 - 602*a^8*b^6*c^4*d^12 + 2161*a^8*b^6*c^6*d^10 - 3012*a^8*b^6*c^8*d^8 + 1056*a^8*b^6*c^10*d^6 + 190*a^9*b^5*c^3*d^13 - 895*a^9*b^5*c^5*d^11 + 1860*a^9*b^5*c^7*d^9 - 1088*a^9*b^5*c^9*d^7 + 14*a^10*b^4*c^2*d^14 + 99*a^10*b^4*c^4*d^12 - 552*a^10*b^4*c^6*d^10 + 628*a^10*b^4*c^8*d^8 + 19*a^11*b^3*c^3*d^13 + 40*a^11*b^3*c^5*d^11 - 220*a^11*b^3*c^7*d^9 - a^12*b^2*c^2*d^14 + 20*a^12*b^2*c^4*d^12 + 44*a^12*b^2*c^6*d^10 - a^13*b*c*d^15))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) - (8*tan(e/2 + (f*x)/2)*(4*a^3*b^11*c^16 - a^14*c*d^15 - 4*a^14*c^3*d^13 - 4*a^14*c^5*d^11 - 144*a*b^13*c^4*d^12 + 684*a*b^13*c^6*d^10 - 1314*a*b^13*c^8*d^8 + 1224*a*b^13*c^10*d^6 - 504*a*b^13*c^12*d^4 + 36*a*b^13*c^14*d^2 + 24*a^2*b^12*c^15*d + 144*a^4*b^10*c*d^15 - 44*a^4*b^10*c^15*d - 348*a^6*b^8*c*d^15 + 214*a^8*b^6*c*d^15 + 7*a^10*b^4*c*d^15 - 8*a^12*b^2*c*d^15 - a^13*b*c^2*d^14 + 20*a^13*b*c^4*d^12 + 44*a^13*b*c^6*d^10 + 432*a^2*b^12*c^3*d^13 - 2148*a^2*b^12*c^5*d^11 + 4470*a^2*b^12*c^7*d^9 - 4632*a^2*b^12*c^9*d^7 + 2232*a

^2*b^12*c^11*d^5 - 252*a^2*b^12*c^13*d^3 - 432*a^3*b^11*c^2*d^14 + 2688*a^3 *b^11*c^4*d^12 - 7294*a^3*b^11*c^6*d^10 + 10105*a^3*b^11*c^8*d^8 - 7104*a^3 *b^11*c^10*d^6 + 1892*a^3*b^11*c^12*d^4 - 192*a^3*b^11*c^14*d^2 - 2016*a^4* b^10*c^3*d^13 + 8378*a^4*b^10*c^5*d^11 - 15815*a^4*b^10*c^7*d^9 + 14976*a^4 *b^10*c^9*d^7 - 5932*a^4*b^10*c^11*d^5 + 624*a^4*b^10*c^13*d^3 + 1140*a^5*b ^9*c^2*d^14 - 6574*a^5*b^9*c^4*d^12 + 16053*a^5*b^9*c^6*d^10 - 19912*a^5*b^ 9*c^8*d^8 + 11320*a^5*b^9*c^10*d^6 - 1920*a^5*b^9*c^12*d^4 + 172*a^5*b^9*c^ 14*d^2 + 2938*a^6*b^8*c^3*d^13 - 10619*a^6*b^8*c^5*d^11 + 18608*a^6*b^8*c^7 *d^9 - 15576*a^6*b^8*c^9*d^7 + 4344*a^6*b^8*c^11*d^5 - 292*a^6*b^8*c^13*d^3 - 818*a^7*b^7*c^2*d^14 + 5107*a^7*b^7*c^4*d^12 - 12464*a^7*b^7*c^6*d^10 + 14693*a^7*b^7*c^8*d^8 - 6184*a^7*b^7*c^10*d^6 + 368*a^7*b^7*c^12*d^4 - 1485 *a^8*b^6*c^3*d^13 + 5064*a^8*b^6*c^5*d^11 - 8939*a^8*b^6*c^7*d^9 + 6104*a^8 *b^6*c^9*d^7 - 688*a^8*b^6*c^11*d^5 + 55*a^9*b^5*c^2*d^14 - 1056*a^9*b^5*c^ 4*d^12 + 3649*a^9*b^5*c^6*d^10 - 4524*a^9*b^5*c^8*d^8 + 1120*a^9*b^5*c^10*d ^6 + 152*a^10*b^4*c^3*d^13 - 975*a^10*b^4*c^5*d^11 + 2300*a^10*b^4*c^7*d^9 - 1088*a^10*b^4*c^9*d^7 + 16*a^11*b^3*c^2*d^14 + 59*a^11*b^3*c^4*d^12 - 640 *a^11*b^3*c^6*d^10 + 628*a^11*b^3*c^8*d^8 + 27*a^12*b^2*c^3*d^13 + 48*a^12* b^2*c^5*d^11 - 220*a^12*b^2*c^7*d^9))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c ^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a ^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11* d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10 *d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5 *b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^ 7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d ^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 5 76*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126* a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^ 9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d ^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^ 6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^ 7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^ 15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^ 8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^ 5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^ 15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 5 76*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^ 10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^ 3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^ 6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9* a^12*b*c*d^16) + (d^2*(-(c + d)^5*(c - d)^5)^(1/2)*((8*(4*a^3*b^13*c^19 - 4 *a^5*b^11*c^19 + 2*a^16*c^2*d^17 - 6*a^16*c^6*d^13 + 4*a^16*c^8*d^11 + 12*a *b^15*c^9*d^10 - 54*a*b^15*c^11*d^8 + 96*a*b^15*c^13*d^6 - 78*a*b^15*c^15*d

^4 + 24*a*b^15*c^17*d^2 + 12*a^2*b^14*c^18*d - 56*a^4*b^12*c^18*d + 44*a^6*b^10*c^18*d + 12*a^9*b^7*c*d^18 - 28*a^11*b^5*c*d^18 + 16*a^13*b^3*c*d^18 - 10*a^15*b*c^3*d^16 - 24*a^15*b*c^5*d^14 + 78*a^15*b*c^7*d^12 - 44*a^15*b*c^9*d^10 - 96*a^2*b^14*c^8*d^11 + 442*a^2*b^14*c^10*d^9 - 816*a^2*b^14*c^12*d^7 + 702*a^2*b^14*c^14*d^5 - 244*a^2*b^14*c^16*d^3 + 336*a^3*b^13*c^7*d^12 - 1620*a^3*b^13*c^9*d^10 + 3206*a^3*b^13*c^11*d^8 - 3064*a^3*b^13*c^13*d^6 + 1314*a^3*b^13*c^15*d^4 - 176*a^3*b^13*c^17*d^2 - 672*a^4*b^12*c^6*d^13 + 3528*a^4*b^12*c^8*d^11 - 7810*a^4*b^12*c^10*d^9 + 8696*a^4*b^12*c^12*d^7 - 4770*a^4*b^12*c^14*d^5 + 1084*a^4*b^12*c^16*d^3 + 840*a^5*b^11*c^5*d^14 - 5124*a^5*b^11*c^7*d^12 + 13320*a^5*b^11*c^9*d^10 - 17850*a^5*b^11*c^11*d^8 + 12400*a^5*b^11*c^13*d^6 - 3954*a^5*b^11*c^15*d^4 + 372*a^5*b^11*c^17*d^2 - 672*a^6*b^10*c^4*d^15 + 5292*a^6*b^10*c^6*d^13 - 16872*a^6*b^10*c^8*d^11 + 27546*a^6*b^10*c^10*d^9 - 23696*a^6*b^10*c^12*d^7 + 9858*a^6*b^10*c^14*d^5 - 1500*a^6*b^10*c^16*d^3 + 336*a^7*b^9*c^3*d^16 - 4032*a^7*b^9*c^5*d^14 + 16212*a^7*b^9*c^7*d^12 - 32304*a^7*b^9*c^9*d^10 + 34018*a^7*b^9*c^11*d^8 - 18048*a^7*b^9*c^13*d^6 + 4038*a^7*b^9*c^15*d^4 - 220*a^7*b^9*c^17*d^2 - 96*a^8*b^8*c^2*d^17 + 2280*a^8*b^8*c^4*d^15 - 11772*a^8*b^8*c^6*d^13 + 28848*a^8*b^8*c^8*d^11 - 37338*a^8*b^8*c^10*d^9 + 25056*a^8*b^8*c^12*d^7 - 7638*a^8*b^8*c^14*d^5 + 660*a^8*b^8*c^16*d^3 - 918*a^9*b^7*c^3*d^16 + 6360*a^9*b^7*c^5*d^14 - 19602*a^9*b^7*c^7*d^12 + 31560*a^9*b^7*c^9*d^10 - 26556*a^9*b^7*c^11*d^8 + 10464*a^9*b^7*c^13*d^6 - 1320*a^9*b^7*c^15*d^4 + 234*a^10*b^6*c^2*d^17 - 2520*a^10*b^6*c^4*d^15 + 10050*a^10*b^6*c^6*d^13 - 20340*a^10*b^6*c^8*d^11 + 21288*a^10*b^6*c^10*d^9 - 10560*a^10*b^6*c^12*d^7 + 1848*a^10*b^6*c^14*d^5 + 726*a^11*b^5*c^3*d^16 - 3768*a^11*b^5*c^5*d^14 + 9670*a^11*b^5*c^7*d^12 - 12648*a^11*b^5*c^9*d^10 + 7896*a^11*b^5*c^11*d^8 - 1848*a^11*b^5*c^13*d^6 - 146*a^12*b^4*c^2*d^17 + 952*a^12*b^4*c^4*d^15 - 3174*a^12*b^4*c^6*d^13 + 5396*a^12*b^4*c^8*d^11 - 4348*a^12*b^4*c^10*d^9 + 1320*a^12*b^4*c^12*d^7 - 134*a^13*b^3*c^3*d^16 + 624*a^13*b^3*c^5*d^14 - 1570*a^13*b^3*c^7*d^12 + 1724*a^13*b^3*c^9*d^10 - 660*a^13*b^3*c^11*d^8 + 6*a^14*b^2*c^2*d^17 - 40*a^14*b^2*c^4*d^15 + 282*a^14*b^2*c^6*d^13 - 468*a^14*b^2*c^8*d^11 + 220*a^14*b^2*c^10*d^9))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6

+ 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) + (8*tan(e/2 + (f*x)/2)*(4*a^16*c*d^18 + 8*a^2*b^14*c^19 - 8*a^4*b^12*c^19 - 12*a^16*c^5*d^14 + 8*a^16*c^7*d^12 + 12*a*b^15*c^10*d^9 - 48*a*b^15*c^12*d^7 + 84*a*b^15*c^14*d^5 - 72*a*b^15*c^16*d^3 - 112*a^3*b^13*c^18*d + 88*a^5*b^11*c^18*d + 12*a^10*b^6*c*d^18 - 28*a^12*b^4*c*d^18 + 12*a^14*b^2*c*d^18 - 20*a^15*b*c^2*d^17 - 48*a^15*b*c^4*d^15 + 156*a^15*b*c^6*d^13 - 88*a^15*b*c^8*d^11 - 84*a^2*b^14*c^9*d^10 + 328*a^2*b^14*c^11*d^8 - 596*a^2*b^14*c^13*d^6 + 552*a^2*b^14*c^15*d^4 - 208*a^2*b^14*c^17*d^2 + 240*a^3*b^13*c^8*d^11 - 908*a^3*b^13*c^10*d^9 + 1792*a^3*b^13*c^12*d^7 - 1932*a^3*b^13*c^14*d^5 + 920*a^3*b^13*c^16*d^3 - 336*a^4*b^12*c^7*d^12 + 1188*a^4*b^12*c^9*d^10 - 2808*a^4*b^12*c^11*d^8 + 3980*a^4*b^12*c^13*d^6 - 2616*a^4*b^12*c^15*d^4 + 600*a^4*b^12*c^17*d^2 + 168*a^5*b^11*c^6*d^13 - 336*a^5*b^11*c^8*d^11 + 1740*a^5*b^11*c^10*d^9 - 4720*a^5*b^11*c^12*d^7 + 4812*a^5*b^11*c^14*d^5 - 1752*a^5*b^11*c^16*d^3 + 168*a^6*b^10*c^5*d^14 - 1344*a^6*b^10*c^7*d^12 + 2292*a^6*b^10*c^9*d^10 + 1088*a^6*b^10*c^11*d^8 - 4908*a^6*b^10*c^13*d^6 + 3096*a^6*b^10*c^15*d^4 - 392*a^6*b^10*c^17*d^2 - 336*a^7*b^9*c^4*d^15 + 2520*a^7*b^9*c^6*d^13 - 7488*a^7*b^9*c^8*d^11 + 7556*a^7*b^9*c^10*d^9 - 144*a^7*b^9*c^12*d^7 - 3012*a^7*b^9*c^14*d^5 + 904*a^7*b^9*c^16*d^3 + 240*a^8*b^8*c^3*d^16 - 2472*a^8*b^8*c^5*d^14 + 10416*a^8*b^8*c^7*d^12 - 16596*a^8*b^8*c^9*d^10 + 9600*a^8*b^8*c^11*d^8 - 156*a^8*b^8*c^13*d^6 - 1032*a^8*b^8*c^15*d^4 - 84*a^9*b^7*c^2*d^17 + 1632*a^9*b^7*c^4*d^15 - 9204*a^9*b^7*c^6*d^13 + 19800*a^9*b^7*c^8*d^11 - 18048*a^9*b^7*c^10*d^9 + 5856*a^9*b^7*c^12*d^7 + 48*a^9*b^7*c^14*d^5 - 744*a^10*b^6*c^3*d^16 + 5460*a^10*b^6*c^5*d^14 - 15960*a^10*b^6*c^7*d^12 + 20136*a^10*b^6*c^9*d^10 - 10584*a^10*b^6*c^11*d^8 + 1680*a^10*b^6*c^13*d^6 + 212*a^11*b^5*c^2*d^17 - 2176*a^11*b^5*c^4*d^15 + 9180*a^11*b^5*c^6*d^13 - 15416*a^11*b^5*c^8*d^11 + 10936*a^11*b^5*c^10*d^9 - 2736*a^11*b^5*c^12*d^7 + 584*a^12*b^4*c^3*d^16 - 3708*a^12*b^4*c^5*d^14 + 8152*a^12*b^4*c^7*d^12 - 7376*a^12*b^4*c^9*d^10 + 2376*a^12*b^4*c^11*d^8 - 108*a^13*b^3*c^2*d^17 + 928*a^13*b^3*c^4*d^15 - 2820*a^13*b^3*c^6*d^13 + 3288*a^13*b^3*c^8*d^11 - 1288*a^13*b^3*c^10*d^9 - 80*a^14*b^2*c^3*d^16 + 564*a^14*b^2*c^5*d^14 - 936*a^14*b^2*c^7*d^12 + 440*a^14*b^2*c^9*d^10 + 24*a*b^15*c^18*d))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^

5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) + (d^2*((8*(4*a^2*b^16*c^22 - 8*a^4*b^14*c^22 + 4*a^6*b^12*c^22 - 4*a^18*c^2*d^20 + 16*a^18*c^4*d^18 - 24*a^18*c^6*d^16 + 16*a^18*c^8*d^14 - 4*a^18*c^10*d^12 - 4*a*b^17*c^13*d^9 + 16*a*b^17*c^15*d^7 - 24*a*b^17*c^17*d^5 + 16*a*b^17*c^19*d^3 - 32*a^3*b^15*c^21*d + 76*a^5*b^13*c^21*d - 40*a^7*b^11*c^21*d + 4*a^13*b^5*c*d^21 - 8*a^15*b^3*c*d^21 + 24*a^17*b*c^3*d^19 - 136*a^17*b*c^5*d^17 + 224*a^17*b*c^7*d^15 - 156*a^17*b*c^9*d^13 + 40*a^17*b*c^11*d^11 + 40*a^2*b^16*c^12*d^10 - 156*a^2*b^16*c^14*d^8 + 224*a^2*b^16*c^16*d^6 - 136*a^2*b^16*c^18*d^4 + 24*a^2*b^16*c^20*d^2 - 176*a^3*b^15*c^11*d^11 + 672*a^3*b^15*c^13*d^9 - 928*a^3*b^15*c^15*d^7 + 512*a^3*b^15*c^17*d^5 - 48*a^3*b^15*c^19*d^3 + 440*a^4*b^14*c^10*d^12 - 1664*a^4*b^14*c^12*d^10 + 2248*a^4*b^14*c^14*d^8 - 1152*a^4*b^14*c^16*d^6 + 8*a^4*b^14*c^18*d^4 + 128*a^4*b^14*c^20*d^2 - 660*a^5*b^13*c^9*d^13 + 2552*a^5*b^13*c^11*d^11 - 3532*a^5*b^13*c^13*d^9 + 1808*a^5*b^13*c^15*d^7 + 148*a^5*b^13*c^17*d^5 - 392*a^5*b^13*c^19*d^3 + 528*a^6*b^12*c^8*d^14 - 2332*a^6*b^12*c^10*d^12 + 3736*a^6*b^12*c^12*d^10 - 2180*a^6*b^12*c^14*d^8 - 480*a^6*b^12*c^16*d^6 + 1052*a^6*b^12*c^18*d^4 - 328*a^6*b^12*c^20*d^2 + 792*a^7*b^11*c^9*d^13 - 2464*a^7*b^11*c^11*d^11 + 1896*a^7*b^11*c^13*d^9 + 1216*a^7*b^11*c^15*d^7 - 2264*a^7*b^11*c^17*d^5 + 864*a^7*b^11*c^19*d^3 - 528*a^8*b^10*c^6*d^16 + 1056*a^8*b^10*c^8*d^14 + 176*a^8*b^10*c^10*d^12 - 528*a^8*b^10*c^12*d^10 - 2288*a^8*b^10*c^14*d^8 + 3520*a^8*b^10*c^16*d^6 - 1584*a^8*b^10*c^18*d^4 + 176*a^8*b^10*c^20*d^2 + 660*a^9*b^9*c^5*d^17 - 2112*a^9*b^9*c^7*d^15 + 2244*a^9*b^9*c^9*d^13 - 1496*a^9*b^9*c^11*d^11 + 2684*a^9*b^9*c^13*d^9 - 3696*a^9*b^9*c^15*d^7 + 2156*a^9*b^9*c^17*d^5 - 440*a^9*b^9*c^19*d^3 - 440*a^10*b^8*c^4*d^18 + 2156*a^10*b^8*c^6*d^16 - 3696*a^10*b^8*c^8*d^14 + 2684*a^10*b^8*c^10*d^12 - 1496*a^10*b^8*c^12*d^10 + 2244*a^10*b^8*c^14*d^8 - 2112*a^10*b^8*c^16*d^6 + 660*a

^10*b^8*c^18*d^4 + 176*a^11*b^7*c^3*d^19 - 1584*a^11*b^7*c^5*d^17 + 3520*a^
11*b^7*c^7*d^15 - 2288*a^11*b^7*c^9*d^13 - 528*a^11*b^7*c^11*d^11 + 176*a^1
1*b^7*c^13*d^9 + 1056*a^11*b^7*c^15*d^7 - 528*a^11*b^7*c^17*d^5 - 40*a^12*b
^6*c^2*d^20 + 864*a^12*b^6*c^4*d^18 - 2264*a^12*b^6*c^6*d^16 + 1216*a^12*b^
6*c^8*d^14 + 1896*a^12*b^6*c^10*d^12 - 2464*a^12*b^6*c^12*d^10 + 792*a^12*b
^6*c^14*d^8 - 328*a^13*b^5*c^3*d^19 + 1052*a^13*b^5*c^5*d^17 - 480*a^13*b^5
*c^7*d^15 - 2180*a^13*b^5*c^9*d^13 + 3736*a^13*b^5*c^11*d^11 - 2332*a^13*b^
5*c^13*d^9 + 528*a^13*b^5*c^15*d^7 + 76*a^14*b^4*c^2*d^20 - 392*a^14*b^4*c^
4*d^18 + 148*a^14*b^4*c^6*d^16 + 1808*a^14*b^4*c^8*d^14 - 3532*a^14*b^4*c^1
0*d^12 + 2552*a^14*b^4*c^12*d^10 - 660*a^14*b^4*c^14*d^8 + 128*a^15*b^3*c^3
*d^19 + 8*a^15*b^3*c^5*d^17 - 1152*a^15*b^3*c^7*d^15 + 2248*a^15*b^3*c^9*d^
13 - 1664*a^15*b^3*c^11*d^11 + 440*a^15*b^3*c^13*d^9 - 32*a^16*b^2*c^2*d^20
 - 48*a^16*b^2*c^4*d^18 + 512*a^16*b^2*c^6*d^16 - 928*a^16*b^2*c^8*d^14 + 6
72*a^16*b^2*c^10*d^12 - 176*a^16*b^2*c^12*d^10 - 4*a*b^17*c^21*d + 4*a^17*b
*c*d^21))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4
*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^
11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^
13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 -
36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^
16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12
*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^
8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 +
84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a
^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b
^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^1
3*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 +
 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*
a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^
7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^1
3*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 +
1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*
a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5
*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11
*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 -
940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^
9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b
^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2
*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^
9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) + (8*tan(e/2
+ (f*x)/2)*(12*a*b^17*c^22 - 12*a^18*c*d^21 - 32*a^3*b^15*c^22 + 28*a^5*b^1
3*c^22 - 8*a^7*b^11*c^22 + 56*a^18*c^3*d^19 - 104*a^18*c^5*d^17 + 96*a^18*c
^7*d^15 - 44*a^18*c^9*d^13 + 8*a^18*c^11*d^11 - 16*a*b^17*c^12*d^10 + 76*a*
b^17*c^14*d^8 - 144*a*b^17*c^16*d^6 + 136*a*b^17*c^18*d^4 - 64*a*b^17*c^20*
d^2 - 132*a^2*b^16*c^21*d + 352*a^4*b^14*c^21*d - 308*a^6*b^12*c^21*d + 88*
a^8*b^10*c^21*d + 16*a^12*b^6*c*d^21 - 44*a^14*b^4*c*d^21 + 40*a^16*b^2*c*d

^21 + 132*a^17*b*c^2*d^20 - 616*a^17*b*c^4*d^18 + 1144*a^17*b*c^6*d^16 - 10
56*a^17*b*c^8*d^14 + 484*a^17*b*c^10*d^12 - 88*a^17*b*c^12*d^10 + 176*a^2*b
^16*c^11*d^11 - 836*a^2*b^16*c^13*d^9 + 1584*a^2*b^16*c^15*d^7 - 1496*a^2*b
^16*c^17*d^5 + 704*a^2*b^16*c^19*d^3 - 880*a^3*b^15*c^10*d^12 + 4224*a^3*b^
15*c^12*d^10 - 8128*a^3*b^15*c^14*d^8 + 7872*a^3*b^15*c^16*d^6 - 3888*a^3*b
^15*c^18*d^4 + 832*a^3*b^15*c^20*d^2 + 2640*a^4*b^14*c^9*d^13 - 13024*a^4*b
^14*c^11*d^11 + 26048*a^4*b^14*c^13*d^9 - 26752*a^4*b^14*c^15*d^7 + 14608*a
^4*b^14*c^17*d^5 - 3872*a^4*b^14*c^19*d^3 - 5280*a^5*b^13*c^8*d^14 + 27500*
a^5*b^13*c^10*d^12 - 59000*a^5*b^13*c^12*d^10 + 66628*a^5*b^13*c^14*d^8 - 4
1712*a^5*b^13*c^16*d^6 + 13748*a^5*b^13*c^18*d^4 - 1912*a^5*b^13*c^20*d^2 +
 7392*a^6*b^12*c^7*d^15 - 42372*a^6*b^12*c^9*d^13 + 101288*a^6*b^12*c^11*d^
11 - 129580*a^6*b^12*c^13*d^9 + 94160*a^6*b^12*c^15*d^7 - 37532*a^6*b^12*c^
17*d^5 + 6952*a^6*b^12*c^19*d^3 - 7392*a^7*b^11*c^6*d^16 + 49632*a^7*b^11*c
^8*d^14 - 137368*a^7*b^11*c^10*d^12 + 202544*a^7*b^11*c^12*d^10 - 170424*a^
7*b^11*c^14*d^8 + 80448*a^7*b^11*c^16*d^6 - 19016*a^7*b^11*c^18*d^4 + 1584*
a^7*b^11*c^20*d^2 + 5280*a^8*b^10*c^5*d^17 - 45408*a^8*b^10*c^7*d^15 + 1502
16*a^8*b^10*c^9*d^13 - 257136*a^8*b^10*c^11*d^11 + 249832*a^8*b^10*c^13*d^9
 - 138688*a^8*b^10*c^15*d^7 + 40920*a^8*b^10*c^17*d^5 - 5104*a^8*b^10*c^19*
d^3 - 2640*a^9*b^9*c^4*d^18 + 32868*a^9*b^9*c^6*d^16 - 133056*a^9*b^9*c^8*d
^14 + 266244*a^9*b^9*c^10*d^12 - 299816*a^9*b^9*c^12*d^10 + 195404*a^9*b^9*
c^14*d^8 - 70224*a^9*b^9*c^16*d^6 + 11660*a^9*b^9*c^18*d^4 - 440*a^9*b^9*c^
20*d^2 + 880*a^10*b^8*c^3*d^19 - 18700*a^10*b^8*c^5*d^17 + 95040*a^10*b^8*c
^7*d^15 - 225676*a^10*b^8*c^9*d^13 + 296824*a^10*b^8*c^11*d^11 - 226116*a^1
0*b^8*c^13*d^9 + 96624*a^10*b^8*c^15*d^7 - 20196*a^10*b^8*c^17*d^5 + 1320*a
^10*b^8*c^19*d^3 - 176*a^11*b^7*c^2*d^20 + 8096*a^11*b^7*c^4*d^18 - 54384*a
^11*b^7*c^6*d^16 + 156992*a^11*b^7*c^8*d^14 - 242528*a^11*b^7*c^10*d^12 + 2
14368*a^11*b^7*c^12*d^10 - 107184*a^11*b^7*c^14*d^8 + 27456*a^11*b^7*c^16*d
^6 - 2640*a^11*b^7*c^18*d^4 - 2496*a^12*b^6*c^3*d^19 + 24784*a^12*b^6*c^5*d
^17 - 89280*a^12*b^6*c^7*d^15 + 162336*a^12*b^6*c^9*d^13 - 165760*a^12*b^6*
c^11*d^11 + 96272*a^12*b^6*c^13*d^9 - 29568*a^12*b^6*c^15*d^7 + 3696*a^12*b
^6*c^17*d^5 + 484*a^13*b^5*c^2*d^20 - 8888*a^13*b^5*c^4*d^18 + 40876*a^13*b
^5*c^6*d^16 - 88000*a^13*b^5*c^8*d^14 + 104060*a^13*b^5*c^10*d^12 - 69784*a
^13*b^5*c^12*d^10 + 24948*a^13*b^5*c^14*d^8 - 3696*a^13*b^5*c^16*d^6 + 2408
*a^14*b^4*c^3*d^19 - 14692*a^14*b^4*c^5*d^17 + 38208*a^14*b^4*c^7*d^15 - 52
532*a^14*b^4*c^9*d^13 + 40072*a^14*b^4*c^11*d^11 - 16060*a^14*b^4*c^13*d^9
 + 2640*a^14*b^4*c^15*d^7 - 440*a^15*b^3*c^2*d^20 + 4048*a^15*b^3*c^4*d^18 -
 13112*a^15*b^3*c^6*d^16 + 20768*a^15*b^3*c^8*d^14 - 17512*a^15*b^3*c^10*d^
12 + 7568*a^15*b^3*c^12*d^10 - 1320*a^15*b^3*c^14*d^8 - 848*a^16*b^2*c^3*d^
19 + 3432*a^16*b^2*c^5*d^17 - 6048*a^16*b^2*c^7*d^15 + 5432*a^16*b^2*c^9*d^
13 - 2448*a^16*b^2*c^11*d^11 + 440*a^16*b^2*c^13*d^9))/(a^13*d^17 - b^13*c^
17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^
13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d
^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9
 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^
10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a

^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8
 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 15
6*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3
*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^1
0*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9
*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 +
126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a
^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*
c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*
d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 +
36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a
^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*
c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d
^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 -
76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^
9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*
c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9
*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13
+ 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a
*b^12*c^16*d - 9*a^12*b*c*d^16))*(-(c + d)^5*(c - d)^5)^(1/2)*(a^2*d^4 + 12
*b^2*c^4 + 6*b^2*d^4 + 2*a^2*c^2*d^2 - 15*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*b
*c^3*d))/(2*(a^4*d^14 - b^4*c^14 - 5*a^4*c^2*d^12 + 10*a^4*c^4*d^10 - 10*a^
4*c^6*d^8 + 5*a^4*c^8*d^6 - a^4*c^10*d^4 + b^4*c^4*d^10 - 5*b^4*c^6*d^8 + 1
0*b^4*c^8*d^6 - 10*b^4*c^10*d^4 + 5*b^4*c^12*d^2 - 4*a*b^3*c^3*d^11 + 20*a*
b^3*c^5*d^9 - 40*a*b^3*c^7*d^7 + 40*a*b^3*c^9*d^5 - 20*a*b^3*c^11*d^3 + 20*
a^3*b*c^3*d^11 - 40*a^3*b*c^5*d^9 + 40*a^3*b*c^7*d^7 - 20*a^3*b*c^9*d^5 + 4
*a^3*b*c^11*d^3 + 6*a^2*b^2*c^2*d^12 - 30*a^2*b^2*c^4*d^10 + 60*a^2*b^2*c^6
*d^8 - 60*a^2*b^2*c^8*d^6 + 30*a^2*b^2*c^10*d^4 - 6*a^2*b^2*c^12*d^2 + 4*a*
b^3*c^13*d - 4*a^3*b*c*d^13)))*(a^2*d^4 + 12*b^2*c^4 + 6*b^2*d^4 + 2*a^2*c^
2*d^2 - 15*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*b*c^3*d))/(2*(a^4*d^14 - b^4*c^1
4 - 5*a^4*c^2*d^12 + 10*a^4*c^4*d^10 - 10*a^4*c^6*d^8 + 5*a^4*c^8*d^6 - a^4
*c^10*d^4 + b^4*c^4*d^10 - 5*b^4*c^6*d^8 + 10*b^4*c^8*d^6 - 10*b^4*c^10*d^4
 + 5*b^4*c^12*d^2 - 4*a*b^3*c^3*d^11 + 20*a*b^3*c^5*d^9 - 40*a*b^3*c^7*d^7
+ 40*a*b^3*c^9*d^5 - 20*a*b^3*c^11*d^3 + 20*a^3*b*c^3*d^11 - 40*a^3*b*c^5*d
^9 + 40*a^3*b*c^7*d^7 - 20*a^3*b*c^9*d^5 + 4*a^3*b*c^11*d^3 + 6*a^2*b^2*c^2
*d^12 - 30*a^2*b^2*c^4*d^10 + 60*a^2*b^2*c^6*d^8 - 60*a^2*b^2*c^8*d^6 + 30*
a^2*b^2*c^10*d^4 - 6*a^2*b^2*c^12*d^2 + 4*a*b^3*c^13*d - 4*a^3*b*c*d^13)))*
(a^2*d^4 + 12*b^2*c^4 + 6*b^2*d^4 + 2*a^2*c^2*d^2 - 15*b^2*c^2*d^2 + 2*a*b*
c*d^3 - 8*a*b*c^3*d))/(2*(a^4*d^14 - b^4*c^14 - 5*a^4*c^2*d^12 + 10*a^4*c^4
*d^10 - 10*a^4*c^6*d^8 + 5*a^4*c^8*d^6 - a^4*c^10*d^4 + b^4*c^4*d^10 - 5*b^
4*c^6*d^8 + 10*b^4*c^8*d^6 - 10*b^4*c^10*d^4 + 5*b^4*c^12*d^2 - 4*a*b^3*c^3
*d^11 + 20*a*b^3*c^5*d^9 - 40*a*b^3*c^7*d^7 + 40*a*b^3*c^9*d^5 - 20*a*b^3*c
^11*d^3 + 20*a^3*b*c^3*d^11 - 40*a^3*b*c^5*d^9 + 40*a^3*b*c^7*d^7 - 20*a^3*
b*c^9*d^5 + 4*a^3*b*c^11*d^3 + 6*a^2*b^2*c^2*d^12 - 30*a^2*b^2*c^4*d^10 + 6
0*a^2*b^2*c^6*d^8 - 60*a^2*b^2*c^8*d^6 + 30*a^2*b^2*c^10*d^4 - 6*a^2*b^2*c^

12*d^2 + 4*a*b^3*c^13*d - 4*a^3*b*c*d^13))))*(-(c + d)^5*(c - d)^5)^(1/2)*(
a^2*d^4 + 12*b^2*c^4 + 6*b^2*d^4 + 2*a^2*c^2*d^2 - 15*b^2*c^2*d^2 + 2*a*b*c
*d^3 - 8*a*b*c^3*d)*1i)/(f*(a^4*d^14 - b^4*c^14 - 5*a^4*c^2*d^12 + 10*a^4*c
^4*d^10 - 10*a^4*c^6*d^8 + 5*a^4*c^8*d^6 - a^4*c^10*d^4 + b^4*c^4*d^10 - 5*
b^4*c^6*d^8 + 10*b^4*c^8*d^6 - 10*b^4*c^10*d^4 + 5*b^4*c^12*d^2 - 4*a*b^3*c
^3*d^11 + 20*a*b^3*c^5*d^9 - 40*a*b^3*c^7*d^7 + 40*a*b^3*c^9*d^5 - 20*a*b^3
*c^11*d^3 + 20*a^3*b*c^3*d^11 - 40*a^3*b*c^5*d^9 + 40*a^3*b*c^7*d^7 - 20*a^
3*b*c^9*d^5 + 4*a^3*b*c^11*d^3 + 6*a^2*b^2*c^2*d^12 - 30*a^2*b^2*c^4*d^10 +
60*a^2*b^2*c^6*d^8 - 60*a^2*b^2*c^8*d^6 + 30*a^2*b^2*c^10*d^4 - 6*a^2*b^2*
c^12*d^2 + 4*a*b^3*c^13*d - 4*a^3*b*c*d^13)) - ((a^4*d^6 + 2*b^4*c^6 - a^2*
b^2*d^6 - 4*a^4*c^2*d^4 + 2*b^4*c^2*d^4 - 4*b^4*c^4*d^2 - 8*a*b^3*c^3*d^3 +
8*a^3*b*c^3*d^3 + 4*a^2*b^2*c^2*d^4 + 5*a*b^3*c*d^5 - 5*a^3*b*c*d^5)/((a^3
*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(a^2*c^4 + a^2*d^4 - b^2*c^
4 - b^2*d^4 - 2*a^2*c^2*d^2 + 2*b^2*c^2*d^2)) - (tan(e/2 + (f*x)/2)^4*(2*a^
3*b^2*d^8 - 2*a*b^4*c^8 - 8*b^5*c^7*d - 2*a^5*d^8 + 7*a^5*c^2*d^6 + 4*a^5*c
^4*d^4 - 8*b^5*c^3*d^5 + 16*b^5*c^5*d^3 - 12*a*b^4*c^2*d^6 + 16*a*b^4*c^4*d
^4 + 4*a*b^4*c^6*d^2 - 6*a^2*b^3*c*d^7 - a^4*b*c^3*d^5 - 8*a^4*b*c^5*d^3 +
a^2*b^3*c^3*d^5 + 8*a^2*b^3*c^5*d^3 + 5*a^3*b^2*c^2*d^6 - 22*a^3*b^2*c^4*d^
4 + 6*a^4*b*c*d^7))/(a*c^2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d
^2)*(a^2*c^4 + a^2*d^4 - b^2*c^4 - b^2*d^4 - 2*a^2*c^2*d^2 + 2*b^2*c^2*d^2)
) + (tan(e/2 + (f*x)/2)*(2*a^5*d^7 + 2*b^5*c^7 - 2*a^3*b^2*d^7 - 11*a^5*c^2
*d^5 + 2*b^5*c^3*d^4 - 4*b^5*c^5*d^2 + 18*a*b^4*c^2*d^5 - 32*a*b^4*c^4*d^3
+ 12*a^2*b^3*c*d^6 + 15*a^4*b*c^3*d^4 - 15*a^2*b^3*c^3*d^4 + a^3*b^2*c^2*d^
5 + 16*a^3*b^2*c^4*d^3 + 8*a*b^4*c^6*d - 12*a^4*b*c*d^6))/(a*c*(a^3*d^3 - b
^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(a^2*c^4 + a^2*d^4 - b^2*c^4 - b^2*
d^4 - 2*a^2*c^2*d^2 + 2*b^2*c^2*d^2)) + (tan(e/2 + (f*x)/2)^5*(2*a^5*d^7 +
2*b^5*c^7 - 2*a^3*b^2*d^7 - 5*a^5*c^2*d^5 + 2*b^5*c^3*d^4 - 4*b^5*c^5*d^2 +
6*a^2*b^3*c*d^6 + 9*a^4*b*c^3*d^4 - 9*a^2*b^3*c^3*d^4 + 5*a^3*b^2*c^2*d^5
- 6*a^4*b*c*d^6))/(a*c*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*
(a^2*c^4 + a^2*d^4 - b^2*c^4 - b^2*d^4 - 2*a^2*c^2*d^2 + 2*b^2*c^2*d^2)) +
(2*tan(e/2 + (f*x)/2)^2*(a^5*d^8 + 2*a*b^4*c^8 + 4*b^5*c^7*d - a^3*b^2*d^8
- 3*a^5*c^2*d^6 - 4*a^5*c^4*d^4 + 4*b^5*c^3*d^5 - 8*b^5*c^5*d^3 + 18*a*b^4*
c^2*d^6 - 29*a*b^4*c^4*d^4 + 3*a^2*b^3*c*d^7 - 8*a^4*b*c^3*d^5 + 8*a^4*b*c^
5*d^3 + 8*a^2*b^3*c^3*d^5 - 8*a^2*b^3*c^5*d^3 - 11*a^3*b^2*c^2*d^6 + 27*a^3
*b^2*c^4*d^4 - 3*a^4*b*c*d^7))/(a*c^2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d -
3*a^2*b*c*d^2)*(a^2*c^4 + a^2*d^4 - b^2*c^4 - b^2*d^4 - 2*a^2*c^2*d^2 + 2*b
^2*c^2*d^2)) + (2*tan(e/2 + (f*x)/2)^3*(b*c^2 + 2*b*d^2 + 2*a*c*d)*(a^4*d^6
+ 2*b^4*c^6 - a^2*b^2*d^6 - 4*a^4*c^2*d^4 + 2*b^4*c^2*d^4 - 4*b^4*c^4*d^2
- 8*a*b^3*c^3*d^3 + 8*a^3*b*c^3*d^3 + 4*a^2*b^2*c^2*d^4 + 5*a*b^3*c*d^5 - 5
*a^3*b*c*d^5))/(a*c^2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(
a^2*c^4 + a^2*d^4 - b^2*c^4 - b^2*d^4 - 2*a^2*c^2*d^2 + 2*b^2*c^2*d^2)))/(f
*(a*c^2 + tan(e/2 + (f*x)/2)^2*(3*a*c^2 + 4*a*d^2 + 8*b*c*d) + tan(e/2 + (f
x)/2)^4(3*a*c^2 + 4*a*d^2 + 8*b*c*d) + tan(e/2 + (f*x)/2)^3*(4*b*c^2 + 8*
b*d^2 + 8*a*c*d) + tan(e/2 + (f*x)/2)*(2*b*c^2 + 4*a*c*d) + tan(e/2 + (f*x)
/2)^5*(2*b*c^2 + 4*a*c*d) + a*c^2*tan(e/2 + (f*x)/2)^6)) - (b^3*atan(((b^3*

(-(a + b)^3*(a - b)^3)^(1/2)*((8*(36*a*b^13*c^5*d^11 - 144*a*b^13*c^7*d^9 + 216*a*b^13*c^9*d^7 - 144*a*b^13*c^11*d^5 + 36*a*b^13*c^13*d^3 + 4*a^3*b^11*c^15*d - 36*a^5*b^9*c*d^15 + 60*a^7*b^7*c*d^15 - 13*a^9*b^5*c*d^15 - 10*a^11*b^3*c*d^15 - 4*a^13*b*c^3*d^13 - 4*a^13*b*c^5*d^11 - 72*a^2*b^12*c^4*d^12 + 276*a^2*b^12*c^6*d^10 - 375*a^2*b^12*c^8*d^8 + 216*a^2*b^12*c^10*d^6 - 60*a^2*b^12*c^12*d^4 + 24*a^2*b^12*c^14*d^2 - 36*a^3*b^11*c^5*d^11 + 61*a^3*b^11*c^7*d^9 - 88*a^3*b^11*c^9*d^7 + 180*a^3*b^11*c^11*d^5 - 184*a^3*b^11*c^13*d^3 + 72*a^4*b^10*c^2*d^14 - 168*a^4*b^10*c^4*d^12 + 233*a^4*b^10*c^6*d^10 - 270*a^4*b^10*c^8*d^8 + 100*a^4*b^10*c^10*d^6 + 248*a^4*b^10*c^12*d^4 - 44*a^4*b^10*c^14*d^2 + 120*a^5*b^9*c^3*d^13 - 535*a^5*b^9*c^5*d^11 + 1386*a^5*b^9*c^7*d^9 - 1544*a^5*b^9*c^9*d^7 + 248*a^5*b^9*c^11*d^5 + 172*a^5*b^9*c^13*d^3 - 108*a^6*b^8*c^2*d^14 + 699*a^6*b^8*c^4*d^12 - 2046*a^6*b^8*c^6*d^10 + 2885*a^6*b^8*c^8*d^8 - 1336*a^6*b^8*c^10*d^6 - 148*a^6*b^8*c^12*d^4 - 305*a^7*b^7*c^3*d^13 + 1354*a^7*b^7*c^5*d^11 - 2979*a^7*b^7*c^7*d^9 + 2648*a^7*b^7*c^9*d^7 - 400*a^7*b^7*c^11*d^5 + 19*a^8*b^6*c^2*d^14 - 602*a^8*b^6*c^4*d^12 + 2161*a^8*b^6*c^6*d^10 - 3012*a^8*b^6*c^8*d^8 + 1056*a^8*b^6*c^10*d^6 + 190*a^9*b^5*c^3*d^13 - 895*a^9*b^5*c^5*d^11 + 1860*a^9*b^5*c^7*d^9 - 1088*a^9*b^5*c^9*d^7 + 14*a^10*b^4*c^2*d^14 + 99*a^10*b^4*c^4*d^12 - 552*a^10*b^4*c^6*d^10 + 628*a^10*b^4*c^8*d^8 + 19*a^11*b^3*c^3*d^13 + 40*a^11*b^3*c^5*d^11 - 220*a^11*b^3*c^7*d^9 - a^12*b^2*c^2*d^14 + 20*a^12*b^2*c^4*d^12 + 44*a^12*b^2*c^6*d^10 - a^13*b*c*d^15))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^

16*d - 9*a^12*b*c*d^16) - (8*tan(e/2 + (f*x)/2)*(4*a^3*b^11*c^16 - a^14*c*d^15 - 4*a^14*c^3*d^13 - 4*a^14*c^5*d^11 - 144*a*b^13*c^4*d^12 + 684*a*b^13*c^6*d^10 - 1314*a*b^13*c^8*d^8 + 1224*a*b^13*c^10*d^6 - 504*a*b^13*c^12*d^4 + 36*a*b^13*c^14*d^2 + 24*a^2*b^12*c^15*d + 144*a^4*b^10*c*d^15 - 44*a^4*b^10*c^15*d - 348*a^6*b^8*c*d^15 + 214*a^8*b^6*c*d^15 + 7*a^10*b^4*c*d^15 - 8*a^12*b^2*c*d^15 - a^13*b*c^2*d^14 + 20*a^13*b*c^4*d^12 + 44*a^13*b*c^6*d^10 + 432*a^2*b^12*c^3*d^13 - 2148*a^2*b^12*c^5*d^11 + 4470*a^2*b^12*c^7*d^9 - 4632*a^2*b^12*c^9*d^7 + 2232*a^2*b^12*c^11*d^5 - 252*a^2*b^12*c^13*d^3 - 432*a^3*b^11*c^2*d^14 + 2688*a^3*b^11*c^4*d^12 - 7294*a^3*b^11*c^6*d^10 + 10105*a^3*b^11*c^8*d^8 - 7104*a^3*b^11*c^10*d^6 + 1892*a^3*b^11*c^12*d^4 - 192*a^3*b^11*c^14*d^2 - 2016*a^4*b^10*c^3*d^13 + 8378*a^4*b^10*c^5*d^11 - 15815*a^4*b^10*c^7*d^9 + 14976*a^4*b^10*c^9*d^7 - 5932*a^4*b^10*c^11*d^5 + 624*a^4*b^10*c^13*d^3 + 1140*a^5*b^9*c^2*d^14 - 6574*a^5*b^9*c^4*d^12 + 16053*a^5*b^9*c^6*d^10 - 19912*a^5*b^9*c^8*d^8 + 11320*a^5*b^9*c^10*d^6 - 1920*a^5*b^9*c^12*d^4 + 172*a^5*b^9*c^14*d^2 + 2938*a^6*b^8*c^3*d^13 - 10619*a^6*b^8*c^5*d^11 + 18608*a^6*b^8*c^7*d^9 - 15576*a^6*b^8*c^9*d^7 + 4344*a^6*b^8*c^11*d^5 - 292*a^6*b^8*c^13*d^3 - 818*a^7*b^7*c^2*d^14 + 5107*a^7*b^7*c^4*d^12 - 12464*a^7*b^7*c^6*d^10 + 14693*a^7*b^7*c^8*d^8 - 6184*a^7*b^7*c^10*d^6 + 368*a^7*b^7*c^12*d^4 - 1485*a^8*b^6*c^3*d^13 + 5064*a^8*b^6*c^5*d^11 - 8939*a^8*b^6*c^7*d^9 + 6104*a^8*b^6*c^9*d^7 - 688*a^8*b^6*c^11*d^5 + 55*a^9*b^5*c^2*d^14 - 1056*a^9*b^5*c^4*d^12 + 3649*a^9*b^5*c^6*d^10 - 4524*a^9*b^5*c^8*d^8 + 1120*a^9*b^5*c^10*d^6 + 152*a^10*b^4*c^3*d^13 - 975*a^10*b^4*c^5*d^11 + 2300*a^10*b^4*c^7*d^9 - 1088*a^10*b^4*c^9*d^7 + 16*a^11*b^3*c^2*d^14 + 59*a^11*b^3*c^4*d^12 - 640*a^11*b^3*c^6*d^10 + 628*a^11*b^3*c^8*d^8 + 27*a^12*b^2*c^3*d^13 + 48*a^12*b^2*c^5*d^11 - 220*a^12*b^2*c^7*d^9))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^

6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) + (b^3*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a^3*b^13*c^19 - 4*a^5*b^11*c^19 + 2*a^16*c^2*d^17 - 6*a^16*c^6*d^13 + 4*a^16*c^8*d^11 + 12*a*b^15*c^9*d^10 - 54*a*b^15*c^11*d^8 + 96*a*b^15*c^13*d^6 - 78*a*b^15*c^15*d^4 + 24*a*b^15*c^17*d^2 + 12*a^2*b^14*c^18*d - 56*a^4*b^12*c^18*d + 44*a^6*b^10*c^18*d + 12*a^9*b^7*c*d^18 - 28*a^11*b^5*c*d^18 + 16*a^13*b^3*c*d^18 - 10*a^15*b*c^3*d^16 - 24*a^15*b*c^5*d^14 + 78*a^15*b*c^7*d^12 - 44*a^15*b*c^9*d^10 - 96*a^2*b^14*c^8*d^11 + 442*a^2*b^14*c^10*d^9 - 816*a^2*b^14*c^12*d^7 + 702*a^2*b^14*c^14*d^5 - 244*a^2*b^14*c^16*d^3 + 336*a^3*b^13*c^7*d^12 - 1620*a^3*b^13*c^9*d^10 + 3206*a^3*b^13*c^11*d^8 - 3064*a^3*b^13*c^13*d^6 + 1314*a^3*b^13*c^15*d^4 - 176*a^3*b^13*c^17*d^2 - 672*a^4*b^12*c^6*d^13 + 3528*a^4*b^12*c^8*d^11 - 7810*a^4*b^12*c^10*d^9 + 8696*a^4*b^12*c^12*d^7 - 4770*a^4*b^12*c^14*d^5 + 1084*a^4*b^12*c^16*d^3 + 840*a^5*b^11*c^5*d^14 - 5124*a^5*b^11*c^7*d^12 + 13320*a^5*b^11*c^9*d^10 - 17850*a^5*b^11*c^11*d^8 + 12400*a^5*b^11*c^13*d^6 - 3954*a^5*b^11*c^15*d^4 + 372*a^5*b^11*c^17*d^2 - 672*a^6*b^10*c^4*d^15 + 5292*a^6*b^10*c^6*d^13 - 16872*a^6*b^10*c^8*d^11 + 27546*a^6*b^10*c^10*d^9 - 23696*a^6*b^10*c^12*d^7 + 9858*a^6*b^10*c^14*d^5 - 1500*a^6*b^10*c^16*d^3 + 336*a^7*b^9*c^3*d^16 - 4032*a^7*b^9*c^5*d^14 + 16212*a^7*b^9*c^7*d^12 - 32304*a^7*b^9*c^9*d^10 + 34018*a^7*b^9*c^11*d^8 - 18048*a^7*b^9*c^13*d^6 + 4038*a^7*b^9*c^15*d^4 - 220*a^7*b^9*c^17*d^2 - 96*a^8*b^8*c^2*d^17 + 2280*a^8*b^8*c^4*d^15 - 11772*a^8*b^8*c^6*d^13 + 28848*a^8*b^8*c^8*d^11 - 37338*a^8*b^8*c^10*d^9 + 25056*a^8*b^8*c^12*d^7 - 7638*a^8*b^8*c^14*d^5 + 660*a^8*b^8*c^16*d^3 - 918*a^9*b^7*c^3*d^16 + 6360*a^9*b^7*c^5*d^14 - 19602*a^9*b^7*c^7*d^12 + 31560*a^9*b^7*c^9*d^10 - 26556*a^9*b^7*c^11*d^8 + 10464*a^9*b^7*c^13*d^6 - 1320*a^9*b^7*c^15*d^4 + 234*a^10*b^6*c^2*d^17 - 2520*a^10*b^6*c^4*d^15 + 10050*a^10*b^6*c^6*d^13 - 20340*a^10*b^6*c^8*d^11 + 21288*a^10*b^6*c^10*d^9 - 10560*a^10*b^6*c^12*d^7 + 1848*a^10*b^6*c^14*d^5 + 726*a^11*b^5*c^3*d^16 - 3768*a^11*b^5*c^5*d^14 + 9670*a^11*b^5*c^7*d^12 - 12648*a^11*b^5*c^9*d^10 + 7896*a^11*b^5*c^11*d^8 - 1848*a^11*b^5*c^13*d^6 - 146*a^12*b^4*c^2*d^17 + 952*a^12*b^4*c^4*d^15 - 3174*a^12*b^4*c^6*d^13 + 5396*a^12*b^4*c^8*d^11 - 4348*a^12*b^4*c^10*d^9 + 1320*a^12*b^4*c^12*d^7 - 134*a^13*b^3*c^3*d^16 + 624*a^13*b^3*c^5*d^14 - 1570*a^13*b^3*c^7*d^12 + 1724*a^13*b^3*c^9*d^10 - 660*a^13*b^3*c^11*d^8 + 6*a^14*b^2*c^2*d^17 - 40*a^14*b^2*c^4*d^15 + 282*a^14*b^2*c^6*d^13 - 468*a^14*b^2*c^8*d^11 + 220*a^14*b^2*c^10*d^9))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6

+ 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 35
4*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^
3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^
9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d
^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1
548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6
*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7
*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d
^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1
992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7
*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*
c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d
^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 10
45*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10
*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^
3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*
d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7
+ 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) + (8*tan(e/2 + (f*x)/2)*(4*a^16*c*d^18
+ 8*a^2*b^14*c^19 - 8*a^4*b^12*c^19 - 12*a^16*c^5*d^14 + 8*a^16*c^7*d^12 +
12*a*b^15*c^10*d^9 - 48*a*b^15*c^12*d^7 + 84*a*b^15*c^14*d^5 - 72*a*b^15*c
^16*d^3 - 112*a^3*b^13*c^18*d + 88*a^5*b^11*c^18*d + 12*a^10*b^6*c*d^18 - 2
8*a^12*b^4*c*d^18 + 12*a^14*b^2*c*d^18 - 20*a^15*b*c^2*d^17 - 48*a^15*b*c^4
*d^15 + 156*a^15*b*c^6*d^13 - 88*a^15*b*c^8*d^11 - 84*a^2*b^14*c^9*d^10 + 3
28*a^2*b^14*c^11*d^8 - 596*a^2*b^14*c^13*d^6 + 552*a^2*b^14*c^15*d^4 - 208*
a^2*b^14*c^17*d^2 + 240*a^3*b^13*c^8*d^11 - 908*a^3*b^13*c^10*d^9 + 1792*a^
3*b^13*c^12*d^7 - 1932*a^3*b^13*c^14*d^5 + 920*a^3*b^13*c^16*d^3 - 336*a^4*
b^12*c^7*d^12 + 1188*a^4*b^12*c^9*d^10 - 2808*a^4*b^12*c^11*d^8 + 3980*a^4*
b^12*c^13*d^6 - 2616*a^4*b^12*c^15*d^4 + 600*a^4*b^12*c^17*d^2 + 168*a^5*b^
11*c^6*d^13 - 336*a^5*b^11*c^8*d^11 + 1740*a^5*b^11*c^10*d^9 - 4720*a^5*b^1
1*c^12*d^7 + 4812*a^5*b^11*c^14*d^5 - 1752*a^5*b^11*c^16*d^3 + 168*a^6*b^10
*c^5*d^14 - 1344*a^6*b^10*c^7*d^12 + 2292*a^6*b^10*c^9*d^10 + 1088*a^6*b^10
*c^11*d^8 - 4908*a^6*b^10*c^13*d^6 + 3096*a^6*b^10*c^15*d^4 - 392*a^6*b^10*
c^17*d^2 - 336*a^7*b^9*c^4*d^15 + 2520*a^7*b^9*c^6*d^13 - 7488*a^7*b^9*c^8*
d^11 + 7556*a^7*b^9*c^10*d^9 - 144*a^7*b^9*c^12*d^7 - 3012*a^7*b^9*c^14*d^5
+ 904*a^7*b^9*c^16*d^3 + 240*a^8*b^8*c^3*d^16 - 2472*a^8*b^8*c^5*d^14 + 10
416*a^8*b^8*c^7*d^12 - 16596*a^8*b^8*c^9*d^10 + 9600*a^8*b^8*c^11*d^8 - 156
*a^8*b^8*c^13*d^6 - 1032*a^8*b^8*c^15*d^4 - 84*a^9*b^7*c^2*d^17 + 1632*a^9*
b^7*c^4*d^15 - 9204*a^9*b^7*c^6*d^13 + 19800*a^9*b^7*c^8*d^11 - 18048*a^9*b
^7*c^10*d^9 + 5856*a^9*b^7*c^12*d^7 + 48*a^9*b^7*c^14*d^5 - 744*a^10*b^6*c^
3*d^16 + 5460*a^10*b^6*c^5*d^14 - 15960*a^10*b^6*c^7*d^12 + 20136*a^10*b^6*
c^9*d^10 - 10584*a^10*b^6*c^11*d^8 + 1680*a^10*b^6*c^13*d^6 + 212*a^11*b^5*
c^2*d^17 - 2176*a^11*b^5*c^4*d^15 + 9180*a^11*b^5*c^6*d^13 - 15416*a^11*b^5
*c^8*d^11 + 10936*a^11*b^5*c^10*d^9 - 2736*a^11*b^5*c^12*d^7 + 584*a^12*b^4
*c^3*d^16 - 3708*a^12*b^4*c^5*d^14 + 8152*a^12*b^4*c^7*d^12 - 7376*a^12*b^4
*c^9*d^10 + 2376*a^12*b^4*c^11*d^8 - 108*a^13*b^3*c^2*d^17 + 928*a^13*b^3*c

^4*d^15 - 2820*a^13*b^3*c^6*d^13 + 3288*a^13*b^3*c^8*d^11 - 1288*a^13*b^3*c^10*d^9 - 80*a^14*b^2*c^3*d^16 + 564*a^14*b^2*c^5*d^14 - 936*a^14*b^2*c^7*d^12 + 440*a^14*b^2*c^9*d^10 + 24*a*b^15*c^18*d))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) + (b^3*((8*(4*a^2*b^16*c^22 - 8*a^4*b^14*c^22 + 4*a^6*b^12*c^22 - 4*a^18*c^2*d^20 + 16*a^18*c^4*d^18 - 24*a^18*c^6*d^16 + 16*a^18*c^8*d^14 - 4*a^18*c^10*d^12 - 4*a*b^17*c^13*d^9 + 16*a*b^17*c^15*d^7 - 24*a*b^17*c^17*d^5 + 16*a*b^17*c^19*d^3 - 32*a^3*b^15*c^21*d + 76*a^5*b^13*c^21*d - 40*a^7*b^11*c^21*d + 4*a^13*b^5*c*d^21 - 8*a^15*b^3*c*d^21 + 24*a^17*b*c^3*d^19 - 136*a^17*b*c^5*d^17 + 224*a^17*b*c^7*d^15 - 156*a^17*b*c^9*d^13 + 40*a^17*b*c^11*d^11 + 40*a^2*b^16*c^12*d^10 - 156*a^2*b^16*c^14*d^8 + 224*a^2*b^16*c^16*d^6 - 136*a^2*b^16*c^18*d^4 + 24*a^2*b^16*c^20*d^2 - 176*a^3*b^15*c^11*d^11 + 672*a^3*b^15*c^13*d^9 - 928*a^3*b^15*c^15*d^7 + 512*a^3*b^15*c^17*d^5 - 48*a^3*b^15*c^19*d^3 + 440*a^4*b^14*c^10*d^12 - 1664*a^4*b^14*c^12*d^10 + 2248*a^4*b^14*c^14*d^8 - 1152*a^4*b^14*c^16*d^6 + 8*a^4*b^14*c^18*d^4 + 128*a^4*b^14*c^20*d^2 - 660*a^5*b^13*c^9*d^13 + 2552*a^5*b^13*c^11*d^11 - 3532*a^5*b^13*c^13*d^9 + 1808*a^5*b^13*c^15*d^7 + 148*a^5*b^13*c^17*d^5 - 392*a^5*b^13*c^19*d^3 + 528*a^6*b^12*c^8*d^14 - 2332*a^6*b^12*c^10*d^12 + 3736*a^6*b^12*c^12*d^10 - 2180*a^6*b^12*c^14*d^8 - 480*a^6*b^12*c^16*d^6 + 1052*a^6*b^12*c^18*d^4 - 328*a^6*b^12*c^20*d^2 + 792*a^7*b^11*c^9*d^13 - 2464*a^7*b^11*c^11*d^11 + 1896*a^7*b^11*c^13*d^9 + 1216*a^7*b^11*c^15*d^7 - 2264*a^7*b^11*c^17*d^5 + 864*a^7*b^11*c^19*d^3 - 528*a^8*b^10

*c^6*d^16 + 1056*a^8*b^10*c^8*d^14 + 176*a^8*b^10*c^10*d^12 - 528*a^8*b^10*
c^12*d^10 - 2288*a^8*b^10*c^14*d^8 + 3520*a^8*b^10*c^16*d^6 - 1584*a^8*b^10
*c^18*d^4 + 176*a^8*b^10*c^20*d^2 + 660*a^9*b^9*c^5*d^17 - 2112*a^9*b^9*c^7
*d^15 + 2244*a^9*b^9*c^9*d^13 - 1496*a^9*b^9*c^11*d^11 + 2684*a^9*b^9*c^13*
d^9 - 3696*a^9*b^9*c^15*d^7 + 2156*a^9*b^9*c^17*d^5 - 440*a^9*b^9*c^19*d^3
- 440*a^10*b^8*c^4*d^18 + 2156*a^10*b^8*c^6*d^16 - 3696*a^10*b^8*c^8*d^14 +
2684*a^10*b^8*c^10*d^12 - 1496*a^10*b^8*c^12*d^10 + 2244*a^10*b^8*c^14*d^8
- 2112*a^10*b^8*c^16*d^6 + 660*a^10*b^8*c^18*d^4 + 176*a^11*b^7*c^3*d^19 -
1584*a^11*b^7*c^5*d^17 + 3520*a^11*b^7*c^7*d^15 - 2288*a^11*b^7*c^9*d^13 -
528*a^11*b^7*c^11*d^11 + 176*a^11*b^7*c^13*d^9 + 1056*a^11*b^7*c^15*d^7 -
528*a^11*b^7*c^17*d^5 - 40*a^12*b^6*c^2*d^20 + 864*a^12*b^6*c^4*d^18 - 2264
*a^12*b^6*c^6*d^16 + 1216*a^12*b^6*c^8*d^14 + 1896*a^12*b^6*c^10*d^12 - 246
4*a^12*b^6*c^12*d^10 + 792*a^12*b^6*c^14*d^8 - 328*a^13*b^5*c^3*d^19 + 1052
*a^13*b^5*c^5*d^17 - 480*a^13*b^5*c^7*d^15 - 2180*a^13*b^5*c^9*d^13 + 3736*
a^13*b^5*c^11*d^11 - 2332*a^13*b^5*c^13*d^9 + 528*a^13*b^5*c^15*d^7 + 76*a^
14*b^4*c^2*d^20 - 392*a^14*b^4*c^4*d^18 + 148*a^14*b^4*c^6*d^16 + 1808*a^14
*b^4*c^8*d^14 - 3532*a^14*b^4*c^10*d^12 + 2552*a^14*b^4*c^12*d^10 - 660*a^1
4*b^4*c^14*d^8 + 128*a^15*b^3*c^3*d^19 + 8*a^15*b^3*c^5*d^17 - 1152*a^15*b^
3*c^7*d^15 + 2248*a^15*b^3*c^9*d^13 - 1664*a^15*b^3*c^11*d^11 + 440*a^15*b^
3*c^13*d^9 - 32*a^16*b^2*c^2*d^20 - 48*a^16*b^2*c^4*d^18 + 512*a^16*b^2*c^6
*d^16 - 928*a^16*b^2*c^8*d^14 + 672*a^16*b^2*c^10*d^12 - 176*a^16*b^2*c^12*
d^10 - 4*a*b^17*c^21*d + 4*a^17*b*c*d^21))/(a^13*d^17 - b^13*c^17 + 2*a^2*b
^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15
+ 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*
c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12
*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d +
9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^
14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^
11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c
^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^
9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 -
126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a
^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*
c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*
d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 5
88*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a
^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c
^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d
^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 2
04*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a
^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c
^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^
9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 4
44*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^
10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b

^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d
 - 9*a^12*b*c*d^16) + (8*tan(e/2 + (f*x)/2)*(12*a*b^17*c^22 - 12*a^18*c*d^2
1 - 32*a^3*b^15*c^22 + 28*a^5*b^13*c^22 - 8*a^7*b^11*c^22 + 56*a^18*c^3*d^1
9 - 104*a^18*c^5*d^17 + 96*a^18*c^7*d^15 - 44*a^18*c^9*d^13 + 8*a^18*c^11*d
^11 - 16*a*b^17*c^12*d^10 + 76*a*b^17*c^14*d^8 - 144*a*b^17*c^16*d^6 + 136*
a*b^17*c^18*d^4 - 64*a*b^17*c^20*d^2 - 132*a^2*b^16*c^21*d + 352*a^4*b^14*c
^21*d - 308*a^6*b^12*c^21*d + 88*a^8*b^10*c^21*d + 16*a^12*b^6*c*d^21 - 44*
a^14*b^4*c*d^21 + 40*a^16*b^2*c*d^21 + 132*a^17*b*c^2*d^20 - 616*a^17*b*c^4
*d^18 + 1144*a^17*b*c^6*d^16 - 1056*a^17*b*c^8*d^14 + 484*a^17*b*c^10*d^12
 - 88*a^17*b*c^12*d^10 + 176*a^2*b^16*c^11*d^11 - 836*a^2*b^16*c^13*d^9 + 15
84*a^2*b^16*c^15*d^7 - 1496*a^2*b^16*c^17*d^5 + 704*a^2*b^16*c^19*d^3 - 880
*a^3*b^15*c^10*d^12 + 4224*a^3*b^15*c^12*d^10 - 8128*a^3*b^15*c^14*d^8 + 78
72*a^3*b^15*c^16*d^6 - 3888*a^3*b^15*c^18*d^4 + 832*a^3*b^15*c^20*d^2 + 264
0*a^4*b^14*c^9*d^13 - 13024*a^4*b^14*c^11*d^11 + 26048*a^4*b^14*c^13*d^9 -
26752*a^4*b^14*c^15*d^7 + 14608*a^4*b^14*c^17*d^5 - 3872*a^4*b^14*c^19*d^3
 - 5280*a^5*b^13*c^8*d^14 + 27500*a^5*b^13*c^10*d^12 - 59000*a^5*b^13*c^12*d
^10 + 66628*a^5*b^13*c^14*d^8 - 41712*a^5*b^13*c^16*d^6 + 13748*a^5*b^13*c^
18*d^4 - 1912*a^5*b^13*c^20*d^2 + 7392*a^6*b^12*c^7*d^15 - 42372*a^6*b^12*c
^9*d^13 + 101288*a^6*b^12*c^11*d^11 - 129580*a^6*b^12*c^13*d^9 + 94160*a^6*
b^12*c^15*d^7 - 37532*a^6*b^12*c^17*d^5 + 6952*a^6*b^12*c^19*d^3 - 7392*a^7
*b^11*c^6*d^16 + 49632*a^7*b^11*c^8*d^14 - 137368*a^7*b^11*c^10*d^12 + 2025
44*a^7*b^11*c^12*d^10 - 170424*a^7*b^11*c^14*d^8 + 80448*a^7*b^11*c^16*d^6
 - 19016*a^7*b^11*c^18*d^4 + 1584*a^7*b^11*c^20*d^2 + 5280*a^8*b^10*c^5*d^17
 - 45408*a^8*b^10*c^7*d^15 + 150216*a^8*b^10*c^9*d^13 - 257136*a^8*b^10*c^1
1*d^11 + 249832*a^8*b^10*c^13*d^9 - 138688*a^8*b^10*c^15*d^7 + 40920*a^8*b^
10*c^17*d^5 - 5104*a^8*b^10*c^19*d^3 - 2640*a^9*b^9*c^4*d^18 + 32868*a^9*b^
9*c^6*d^16 - 133056*a^9*b^9*c^8*d^14 + 266244*a^9*b^9*c^10*d^12 - 299816*a^
9*b^9*c^12*d^10 + 195404*a^9*b^9*c^14*d^8 - 70224*a^9*b^9*c^16*d^6 + 11660*
a^9*b^9*c^18*d^4 - 440*a^9*b^9*c^20*d^2 + 880*a^10*b^8*c^3*d^19 - 18700*a^1
0*b^8*c^5*d^17 + 95040*a^10*b^8*c^7*d^15 - 225676*a^10*b^8*c^9*d^13 + 29682
4*a^10*b^8*c^11*d^11 - 226116*a^10*b^8*c^13*d^9 + 96624*a^10*b^8*c^15*d^7 -
 20196*a^10*b^8*c^17*d^5 + 1320*a^10*b^8*c^19*d^3 - 176*a^11*b^7*c^2*d^20 +
 8096*a^11*b^7*c^4*d^18 - 54384*a^11*b^7*c^6*d^16 + 156992*a^11*b^7*c^8*d^1
4 - 242528*a^11*b^7*c^10*d^12 + 214368*a^11*b^7*c^12*d^10 - 107184*a^11*b^7
*c^14*d^8 + 27456*a^11*b^7*c^16*d^6 - 2640*a^11*b^7*c^18*d^4 - 2496*a^12*b^
6*c^3*d^19 + 24784*a^12*b^6*c^5*d^17 - 89280*a^12*b^6*c^7*d^15 + 162336*a^1
2*b^6*c^9*d^13 - 165760*a^12*b^6*c^11*d^11 + 96272*a^12*b^6*c^13*d^9 - 2956
8*a^12*b^6*c^15*d^7 + 3696*a^12*b^6*c^17*d^5 + 484*a^13*b^5*c^2*d^20 - 8888
*a^13*b^5*c^4*d^18 + 40876*a^13*b^5*c^6*d^16 - 88000*a^13*b^5*c^8*d^14 + 10
4060*a^13*b^5*c^10*d^12 - 69784*a^13*b^5*c^12*d^10 + 24948*a^13*b^5*c^14*d^
8 - 3696*a^13*b^5*c^16*d^6 + 2408*a^14*b^4*c^3*d^19 - 14692*a^14*b^4*c^5*d^
17 + 38208*a^14*b^4*c^7*d^15 - 52532*a^14*b^4*c^9*d^13 + 40072*a^14*b^4*c^1
1*d^11 - 16060*a^14*b^4*c^13*d^9 + 2640*a^14*b^4*c^15*d^7 - 440*a^15*b^3*c^
2*d^20 + 4048*a^15*b^3*c^4*d^18 - 13112*a^15*b^3*c^6*d^16 + 20768*a^15*b^3*
c^8*d^14 - 17512*a^15*b^3*c^10*d^12 + 7568*a^15*b^3*c^12*d^10 - 1320*a^15*b

^3*c^14*d^8 - 848*a^16*b^2*c^3*d^19 + 3432*a^16*b^2*c^5*d^17 - 6048*a^16*b^2*c^7*d^15 + 5432*a^16*b^2*c^9*d^13 - 2448*a^16*b^2*c^11*d^11 + 440*a^16*b^2*c^13*d^9))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16))*(-(a + b)^3*(a - b)^3)^(1/2)*(3*b^2*d - 4*a^2*d + a*b*c))/(a^10*d^4 - b^10*c^4 + 3*a^2*b^8*c^4 - 3*a^4*b^6*c^4 + a^6*b^4*c^4 - a^4*b^6*d^4 + 3*a^6*b^4*d^4 - 3*a^8*b^2*d^4 + 4*a^3*b^7*c*d^3 - 12*a^3*b^7*c^3*d - 12*a^5*b^5*c*d^3 + 12*a^5*b^5*c^3*d + 12*a^7*b^3*c*d^3 - 4*a^7*b^3*c^3*d - 6*a^2*b^8*c^2*d^2 + 18*a^4*b^6*c^2*d^2 - 18*a^6*b^4*c^2*d^2 + 6*a^8*b^2*c^2*d^2 + 4*a*b^9*c^3*d - 4*a^9*b*c*d^3))*(3*b^2*d - 4*a^2*d + a*b*c))/(a^10*d^4 - b^10*c^4 + 3*a^2*b^8*c^4 - 3*a^4*b^6*c^4 + a^6*b^4*c^4 - a^4*b^6*d^4 + 3*a^6*b^4*d^4 - 3*a^8*b^2*d^4 + 4*a^3*b^7*c*d^3 - 12*a^3*b^7*c^3*d - 12*a^5*b^5*c*d^3 + 12*a^5*b^5*c^3*d + 12*a^7*b^3*c*d^3 - 4*a^7*b^3*c^3*d - 6*a^2*b^8*c^2*d^2 + 18*a^4*b^6*c^2*d^2 - 18*a^6*b^4*c^2*d^2 + 6*a^8*b^2*c^2*d^2 + 4*a*b^9*c^3*d - 4*a^9*b*c*d^3))*(3*b^2*d - 4*a^2*d + a*b*c)*1i)/(a^10*d^4 - b^10*c^4 + 3*a^2*b^8*c^4 - 3*a^4*b^6*c^4 + a^6*b^4*c^4 - a^4*b^6*d^4 + 3*a^6*b^4*d^4 - 3*a^8*b^2*d^4 + 4*a^3*b^7*c*d^3 - 12*a^3*b^7*c^3*d - 12*a^5*b^5*c*d^3 + 12*a^5*b^5*c^3*d + 12*a^7*b^3*c*d^3 - 4*a^7*b^3*c^3*d - 6*a^2*b^8*c^2*d^2 + 18*a^4*b^6*c^2*d^2 - 18*a^6*b^4*c^2*d^2 + 6*a^8*b^2*c^2*d^2 + 4*a*b^9*c^3*d - 4*a^9*b*c*d^3) - (b^3*(-(a + b)^3*(a - b)^3)^(1/2)*((8*tan(e/2 + (f*x)/2)*(4*a^3*b^11*c^16 - a^14*c*d^15 - 4*a^14*c^3*d^13 - 4*a^14*c^5*d^11 - 144*a*b^13*c^4*d^12 + 684*a*b^13*c^6*d^10 - 1314*a*b^13*c^8*d^8 + 1224*a*b^13*c^10*d^6 - 5

04*a*b^13*c^12*d^4 + 36*a*b^13*c^14*d^2 + 24*a^2*b^12*c^15*d + 144*a^4*b^10*c*d^15 - 44*a^4*b^10*c^15*d - 348*a^6*b^8*c*d^15 + 214*a^8*b^6*c*d^15 + 7*a^10*b^4*c*d^15 - 8*a^12*b^2*c*d^15 - a^13*b*c^2*d^14 + 20*a^13*b*c^4*d^12 + 44*a^13*b*c^6*d^10 + 432*a^2*b^12*c^3*d^13 - 2148*a^2*b^12*c^5*d^11 + 4470*a^2*b^12*c^7*d^9 - 4632*a^2*b^12*c^9*d^7 + 2232*a^2*b^12*c^11*d^5 - 252*a^2*b^12*c^13*d^3 - 432*a^3*b^11*c^2*d^14 + 2688*a^3*b^11*c^4*d^12 - 7294*a^3*b^11*c^6*d^10 + 10105*a^3*b^11*c^8*d^8 - 7104*a^3*b^11*c^10*d^6 + 1892*a^3*b^11*c^12*d^4 - 192*a^3*b^11*c^14*d^2 - 2016*a^4*b^10*c^3*d^13 + 8378*a^4*b^10*c^5*d^11 - 15815*a^4*b^10*c^7*d^9 + 14976*a^4*b^10*c^9*d^7 - 5932*a^4*b^10*c^11*d^5 + 624*a^4*b^10*c^13*d^3 + 1140*a^5*b^9*c^2*d^14 - 6574*a^5*b^9*c^4*d^12 + 16053*a^5*b^9*c^6*d^10 - 19912*a^5*b^9*c^8*d^8 + 11320*a^5*b^9*c^10*d^6 - 1920*a^5*b^9*c^12*d^4 + 172*a^5*b^9*c^14*d^2 + 2938*a^6*b^8*c^3*d^13 - 10619*a^6*b^8*c^5*d^11 + 18608*a^6*b^8*c^7*d^9 - 15576*a^6*b^8*c^9*d^7 + 4344*a^6*b^8*c^11*d^5 - 292*a^6*b^8*c^13*d^3 - 818*a^7*b^7*c^2*d^14 + 5107*a^7*b^7*c^4*d^12 - 12464*a^7*b^7*c^6*d^10 + 14693*a^7*b^7*c^8*d^8 - 6184*a^7*b^7*c^10*d^6 + 368*a^7*b^7*c^12*d^4 - 1485*a^8*b^6*c^3*d^13 + 5064*a^8*b^6*c^5*d^11 - 8939*a^8*b^6*c^7*d^9 + 6104*a^8*b^6*c^9*d^7 - 688*a^8*b^6*c^11*d^5 + 55*a^9*b^5*c^2*d^14 - 1056*a^9*b^5*c^4*d^12 + 3649*a^9*b^5*c^6*d^10 - 4524*a^9*b^5*c^8*d^8 + 1120*a^9*b^5*c^10*d^6 + 152*a^10*b^4*c^3*d^13 - 975*a^10*b^4*c^5*d^11 + 2300*a^10*b^4*c^7*d^9 - 1088*a^10*b^4*c^9*d^7 + 16*a^11*b^3*c^2*d^14 + 59*a^11*b^3*c^4*d^12 - 640*a^11*b^3*c^6*d^10 + 628*a^11*b^3*c^8*d^8 + 27*a^12*b^2*c^3*d^13 + 48*a^12*b^2*c^5*d^11 - 220*a^12*b^2*c^7*d^9))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2 *c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) - (8*(36*a*b^13*c^5*d^11 - 144*a*b^13*c^7*d^9 + 216*a*b^13*c^9*d^7 - 144*a*b^13*c^11*d^5 + 36*a*b^13*c^13*d^3 + 4*a^3*b^11*c^15*d - 36*a^5*b^9*c*d^15 + 60*a^7*b^7*c*d^15 - 13*a^9*b^5*c*d^15 - 10*a^11*b^3*c*d^15 - 4*a^13*b*c^3*d^13 - 4*a^13*b*c^5*d^11 - 72*a^2*b^12*c^4*d^12 + 276*a^2*b^12*c^6*d^10 - 375*a^2*b^12*c^8*d^8 + 216*a^2*b^12*c^10*d^6 - 60*a^2*b^12*c^12*d^4 + 24*a^2*b^12*c^14*d^2 - 36*a^3*b^11*c^5*d^11 + 61*a^3*b^11*c^7*d^9 - 88*a^3*b^11*c^9*d^7 + 180*a^3*b^11*c^11*d^5 - 184*a^3*b^11*c^13*d^3 + 72*a^4*b^10*c^2*d^14 - 168*a^4*b^10*c^4*d^12 + 233*a^4*b^10*c^6*d^10 - 270*a^4*b^10*c^8*d^8 + 100*a^4*b^10*c^10*d^6 + 248*a^4*b^10*c^12*d^4 - 44*a^4*b^10*c^14*d^2 + 120*a^5*b^9*c^3*d^13 - 535*a^5*b^9*c^5*d^11 + 1386*a^5*b^9*c^7*d^9 - 1544*a^5*b^9*c^9*d^7 + 248*a^5*b^9*c^11*d^5 + 172*a^5*b^9*c^13*d^3 - 108*a^6*b^8*c^2*d^14 + 699*a^6*b^8*c^4*d^12 - 2046*a^6*b^8*c^6*d^10 + 2885*a^6*b^8*c^8*d^8 - 1336*a^6*b^8*c^10*d^6 - 148*a^6*b^8*c^12*d^4 - 305*a^7*b^7*c^3*d^13 + 1354*a^7*b^7*c^5*d^11 - 2979*a^7*b^7*c^7*d^9 + 2648*a^7*b^7*c^9*d^7 - 400*a^7*b^7*c^11*d^5 + 19*a^8*b^6*c^2*d^14 - 602*a^8*b^6*c^4*d^12 + 2161*a^8*b^6*c^6*d^10 - 3012*a^8*b^6*c^8*d^8 + 1056*a^8*b^6*c^10*d^6 + 190*a^9*b^5*c^3*d^13 - 895*a^9*b^5*c^5*d^11 + 1860*a^9*b^5*c^7*d^9 - 1088*a^9*b^5*c^9*d^7 + 14*a^10*b^4*c^2*d^14 + 99*a^10*b^4*c^4*d^12 - 552*a^10*b^4*c^6*d^10 + 628*a^10*b^4*c^8*d^8 + 19*a^11*b^3*c^3*d^13 + 40*a^11*b^3*c^5*d^11 - 220*a^11*b^3*c^7*d^9 - a^12*b^2*c^2*d^14 + 20*a^12*b^2*c^4*d^12 + 44*a^12*b^2*c^6*d^10 - a^13*b*c*d^15))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2 *d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) + (b^3*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a^3*b^13*c^19 - 4*a^5*b^11*c^19 + 2*a^16*c^2*d^17 - 6*a^16*c^6*d^13 + 4*a^16*c^8*d^11 + 12*a*b^15*c^9*d^10 - 54*a*b^15*c^11*d^8 + 96*a*b^15*c^13*d^6 - 78*a*b^15*c^15*d^4 + 24*a*b^15*c^17*d^2 + 12*a^2*b^14*c^18*d - 56*a^4*b^12*c^18*d + 44*a^6*b^10*c^18*d + 12*a^9*b^7*c*d^18 - 28*a^11*b^5*c*d^18 + 16*a^13*b^3*c*d^18 - 10*a^15*b*c^3*d^16 - 24*a^15*b*c^5*d^14 + 78*a^15*b*c^7*d^12 - 44*a^15*b*c^9*d^10 - 96*a^2*b^14*c^8*d^11 + 442*a^2*b^14*c^10*d^9 - 816*a^2*b^14*c^12*d^7 + 702*a^2*b^14*c^14*d^5 - 244*a^2*b^14*c^16*d^3 + 336*a^3*b^13*c^7*d^12 - 1620*a^3*b^13*c^9*d^10 + 3206*a^3*b^13*c^11*d^8 - 3064*a^3*b^13*c^13*d^6 + 1314*a^3*b^13*c^15*d^4 - 176*a^3*b^13*c^17*d^2 - 672*a^4*b^12*c^6*d^13 + 3528*a^4*b^12*c^8*d^11 - 7810*a^4*b^12*c^10*d^9 + 8696*a^4*b^12*c^12*d^7 - 4770*a^4*b^12*c^14*d^5 + 1084*a^4*b^12*c^16*d^3 + 840*a^5*b^11*c^5*d^14 - 5124*a^5*b^11*c^7*d^12 + 13320*a^5*b^11*c^9*d^10 - 17850*a^5*b^11*c^11*d^8 + 12400*a^5*b^11*c^13*d^6 - 3954*a^5*b^11*c^15*d^4 + 372*a^5*b^11*c^17*d^2 - 672*a^6*b^10*c^4*d^15 + 5292*a^6*b^10*c^6*d^13 - 16872*a^6*b^10*c^8*d^11 + 27546*a^6*b^10*c^10*d^9 - 23696*a^6*b^10*c^12*d^7 + 9858*a^6*b^10*c^14*d^5 - 1500*a^6*b^10*c^16*d^3 + 336*a^7*b^9*c^3*d^16 - 4032*a^7*b^9*c^5*d^14 + 16212*a^7*b^9*c^7*d^12 - 32304*a^7*b^9*c^9*d^10 + 34018*a^7*b^9*c^11*d^8 - 18048*a^7*b^9*c^13*d^6 + 4038*a^7*b^9*c^15*d^4 - 220*a^7*b^9*c^17*d^2 - 96*a^8*b^8*c^2*d^17 + 2280*a^8*b^8*c^4*d^15 - 11772*a^8*b^8*c^6*d^13 + 28848*a^8*b^8*c^8*d^11 - 37338*a^8*b^8*c^10*d^9 + 25056*a^8*b^8*c^12*d^7 - 7638*a^8*b^8*c^14*d^5 + 660*a^8*b^8*c^16*d^3 - 918*a^9*b^7*c^3*d^16 + 6360*a^9*b^7*c^5*d^14 - 19602*a^9*b^7*c^7*d^12 + 31560*a^9*b^7*c^9*d^10 - 26556*a^9*b^7*c^11*d^8 + 10464*a^9*b^7*c^13*d^6 - 1320*a^9*b^7*c^15*d^4 + 234*a^10*b^6*c^2*d^17 - 2520*a^10*b^6*c^4*d^15 + 10050*a^10*b^6*c^6*d^13 - 20340*a^10*b^6*c^8*d^11 + 21288*a^10*b^6*c^10*d^9 - 10560*a^10*b^6*c^12*d^7 + 1848*a^10*b^6*c^14*d^5 + 726*a^11*b^5*c^3*d^16 - 3768*a^11*b^5*c^5*d^14 + 9670*a^11*b^5*c^7*d^12 - 12648*a^11*b^5*c^9*d^10 + 7896*a^11*b^5*c^11*d^8 - 1848*a^11*b^5*c^13*d^6 - 146*a^12*b^4*c^2*d^17 + 952*a^12*b^4*c^4*d^15 - 3174*a^12*b^4*c^6*d^13 + 5396*a^12*b^4*c^8*d^11 - 4348*a^12*b^4*c^10*d^9 + 1320*a^12*b^4*c^12*d^7 - 134*a^13*b^3*c^3*d^16 + 624*a^13*b^3*c^5*d^14 - 1570*a^13*b^3*c^7*d^12 + 1724*a^13*b^3*c^9*d^10 - 660*a^13*b^3*c^11*d^8 + 6*a^14*b^2*c^2*d^17 - 40*a^14*b^2*c^4*d^15 + 282*a^14*b^2*c^6*d^13 - 468*a^14*b^2*c^8*d^11 + 220*a^14*b^2*c^10*d^9))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10

- 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) + (8*tan(e/2 + (f*x)/2)*(4*a^16*c*d^18 + 8*a^2*b^14*c^19 - 8*a^4*b^12*c^19 - 12*a^16*c^5*d^14 + 8*a^16*c^7*d^12 + 12*a*b^15*c^10*d^9 - 48*a*b^15*c^12*d^7 + 84*a*b^15*c^14*d^5 - 72*a*b^15*c^16*d^3 - 112*a^3*b^13*c^18*d + 88*a^5*b^11*c^18*d + 12*a^10*b^6*c*d^18 - 28*a^12*b^4*c*d^18 + 12*a^14*b^2*c*d^18 - 20*a^15*b*c^2*d^17 - 48*a^15*b*c^4*d^15 + 156*a^15*b*c^6*d^13 - 88*a^15*b*c^8*d^11 - 84*a^2*b^14*c^9*d^10 + 328*a^2*b^14*c^11*d^8 - 596*a^2*b^14*c^13*d^6 + 552*a^2*b^14*c^15*d^4 - 208*a^2*b^14*c^17*d^2 + 240*a^3*b^13*c^8*d^11 - 908*a^3*b^13*c^10*d^9 + 1792*a^3*b^13*c^12*d^7 - 1932*a^3*b^13*c^14*d^5 + 920*a^3*b^13*c^16*d^3 - 336*a^4*b^12*c^7*d^12 + 1188*a^4*b^12*c^9*d^10 - 2808*a^4*b^12*c^11*d^8 + 3980*a^4*b^12*c^13*d^6 - 2616*a^4*b^12*c^15*d^4 + 600*a^4*b^12*c^17*d^2 + 168*a^5*b^11*c^6*d^13 - 336*a^5*b^11*c^8*d^11 + 1740*a^5*b^11*c^10*d^9 - 4720*a^5*b^11*c^12*d^7 + 4812*a^5*b^11*c^14*d^5 - 1752*a^5*b^11*c^16*d^3 + 168*a^6*b^10*c^5*d^14 - 1344*a^6*b^10*c^7*d^12 + 2292*a^6*b^10*c^9*d^10 + 1088*a^6*b^10*c^11*d^8 - 4908*a^6*b^10*c^13*d^6 + 3096*a^6*b^10*c^15*d^4 - 392*a^6*b^10*c^17*d^2 - 336*a^7*b^9*c^4*d^15 + 2520*a^7*b^9*c^6*d^13 - 7488*a^7*b^9*c^8*d^11 + 7556*a^7*b^9*c^10*d^9 - 144*a^7*b^9*c^12*d^7 - 3012*a^7*b^9*c^14*d^5 + 904*a^7*b^9*c^16*d^3 + 240*a^8*b^8*c^3*d^16 - 2472*a^8*b^8*c^5*d^14 + 10416*a^8*b^8*c^7*d^12 - 16596*a^8*b^8*c^9*d^10 + 9600*a^8*b^8*c^11*d^8 - 156*a^8*b^8*c^13*d^6 - 1032*a^8*b^8*c^15*d^4 - 84*a^9*b^7*c^2*d^17 + 1632*a^9*b^7*c^4*d^15 - 9204*a^9*b^7*c^6*d^13 + 19800*a^9*b^7*c^8*d^11 - 18048*a^9*b^7*c^10*d^9 + 5856*a^9*b^7*c^12*d^7 + 48*a^9*b^7*c^14*d^5 - 744*a^10*b^6*c^3*d^16 + 5460*a^10*b^6*c^5*d^14 - 15960*a^10*b^6*c^7*d^12 + 20136*a^10*b^6*c^9*d^10 - 10584*a^10*b^6*c^11*d^8 + 1680*a^10*b^6*c^13*d^6 + 212*a^11*b^5*c^2*d^17 - 2176*a^11*b^5*c^4*d^15 + 9180*a^11*b^5*c^6*d^13 - 15416*a^11*b^5*c^8*d^11 + 10936*a^11*b^5*c^10*d^9 - 2736*a^11*b^5*c^12*d^7 + 584*a^12*b^4*c^3*d^16 - 3708*a^12*b^4*c^5*d^14 + 8152*a^12*b^4*c^7*d^12 - 7376*a^12*b^4*c^9*d^10 + 2376*a^12*b^4*c^11*d^8 - 108*a^13*b^3*c^2*d^17 + 928*a^13*b^3*c^4*d^15 - 2820*a^13*b^3*c^6*d^13 + 3288*a^13*b^3*c^8*d^11 - 1288*a^13*b^3*c^10*d^9 - 80*a^14*b^2*c^3*d^16 + 564*a^14*b^2*c^5*d^14 - 936*a^14*b^2*c^7*d^12 + 440*a^14*b^2*c^9*d^10 + 24*a*b^15*c^18*d))/(a^13*d^17 -

b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^1
7 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^
13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12
*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 1
8*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^1
6 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b
*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*
d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 -
354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156
*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4
*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^1
5*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9
- 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*
a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*
b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^1
5*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11
- 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*
a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b
^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^1
3*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 +
1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a
^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10
*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c
^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d
^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) - (b^3*((8*(4*a^2*b^16*c^22 - 8*a^4
*b^14*c^22 + 4*a^6*b^12*c^22 - 4*a^18*c^2*d^20 + 16*a^18*c^4*d^18 - 24*a^18
*c^6*d^16 + 16*a^18*c^8*d^14 - 4*a^18*c^10*d^12 - 4*a*b^17*c^13*d^9 + 16*a*
b^17*c^15*d^7 - 24*a*b^17*c^17*d^5 + 16*a*b^17*c^19*d^3 - 32*a^3*b^15*c^21*
d + 76*a^5*b^13*c^21*d - 40*a^7*b^11*c^21*d + 4*a^13*b^5*c*d^21 - 8*a^15*b^
3*c*d^21 + 24*a^17*b*c^3*d^19 - 136*a^17*b*c^5*d^17 + 224*a^17*b*c^7*d^15 -
156*a^17*b*c^9*d^13 + 40*a^17*b*c^11*d^11 + 40*a^2*b^16*c^12*d^10 - 156*a^
2*b^16*c^14*d^8 + 224*a^2*b^16*c^16*d^6 - 136*a^2*b^16*c^18*d^4 + 24*a^2*b^
16*c^20*d^2 - 176*a^3*b^15*c^11*d^11 + 672*a^3*b^15*c^13*d^9 - 928*a^3*b^15
*c^15*d^7 + 512*a^3*b^15*c^17*d^5 - 48*a^3*b^15*c^19*d^3 + 440*a^4*b^14*c^1
0*d^12 - 1664*a^4*b^14*c^12*d^10 + 2248*a^4*b^14*c^14*d^8 - 1152*a^4*b^14*c
^16*d^6 + 8*a^4*b^14*c^18*d^4 + 128*a^4*b^14*c^20*d^2 - 660*a^5*b^13*c^9*d^
13 + 2552*a^5*b^13*c^11*d^11 - 3532*a^5*b^13*c^13*d^9 + 1808*a^5*b^13*c^15*
d^7 + 148*a^5*b^13*c^17*d^5 - 392*a^5*b^13*c^19*d^3 + 528*a^6*b^12*c^8*d^14
- 2332*a^6*b^12*c^10*d^12 + 3736*a^6*b^12*c^12*d^10 - 2180*a^6*b^12*c^14*d
^8 - 480*a^6*b^12*c^16*d^6 + 1052*a^6*b^12*c^18*d^4 - 328*a^6*b^12*c^20*d^2
+ 792*a^7*b^11*c^9*d^13 - 2464*a^7*b^11*c^11*d^11 + 1896*a^7*b^11*c^13*d^9
+ 1216*a^7*b^11*c^15*d^7 - 2264*a^7*b^11*c^17*d^5 + 864*a^7*b^11*c^19*d^3
- 528*a^8*b^10*c^6*d^16 + 1056*a^8*b^10*c^8*d^14 + 176*a^8*b^10*c^10*d^12 -
528*a^8*b^10*c^12*d^10 - 2288*a^8*b^10*c^14*d^8 + 3520*a^8*b^10*c^16*d^6 -
1584*a^8*b^10*c^18*d^4 + 176*a^8*b^10*c^20*d^2 + 660*a^9*b^9*c^5*d^17 - 21

12*a^9*b^9*c^7*d^15 + 2244*a^9*b^9*c^9*d^13 - 1496*a^9*b^9*c^11*d^11 + 2684
*a^9*b^9*c^13*d^9 - 3696*a^9*b^9*c^15*d^7 + 2156*a^9*b^9*c^17*d^5 - 440*a^9
*b^9*c^19*d^3 - 440*a^10*b^8*c^4*d^18 + 2156*a^10*b^8*c^6*d^16 - 3696*a^10*
b^8*c^8*d^14 + 2684*a^10*b^8*c^10*d^12 - 1496*a^10*b^8*c^12*d^10 + 2244*a^1
0*b^8*c^14*d^8 - 2112*a^10*b^8*c^16*d^6 + 660*a^10*b^8*c^18*d^4 + 176*a^11*
b^7*c^3*d^19 - 1584*a^11*b^7*c^5*d^17 + 3520*a^11*b^7*c^7*d^15 - 2288*a^11*
b^7*c^9*d^13 - 528*a^11*b^7*c^11*d^11 + 176*a^11*b^7*c^13*d^9 + 1056*a^11*b
^7*c^15*d^7 - 528*a^11*b^7*c^17*d^5 - 40*a^12*b^6*c^2*d^20 + 864*a^12*b^6*c
^4*d^18 - 2264*a^12*b^6*c^6*d^16 + 1216*a^12*b^6*c^8*d^14 + 1896*a^12*b^6*c
^10*d^12 - 2464*a^12*b^6*c^12*d^10 + 792*a^12*b^6*c^14*d^8 - 328*a^13*b^5*c
^3*d^19 + 1052*a^13*b^5*c^5*d^17 - 480*a^13*b^5*c^7*d^15 - 2180*a^13*b^5*c^
9*d^13 + 3736*a^13*b^5*c^11*d^11 - 2332*a^13*b^5*c^13*d^9 + 528*a^13*b^5*c^
15*d^7 + 76*a^14*b^4*c^2*d^20 - 392*a^14*b^4*c^4*d^18 + 148*a^14*b^4*c^6*d^
16 + 1808*a^14*b^4*c^8*d^14 - 3532*a^14*b^4*c^10*d^12 + 2552*a^14*b^4*c^12*
d^10 - 660*a^14*b^4*c^14*d^8 + 128*a^15*b^3*c^3*d^19 + 8*a^15*b^3*c^5*d^17
- 1152*a^15*b^3*c^7*d^15 + 2248*a^15*b^3*c^9*d^13 - 1664*a^15*b^3*c^11*d^11
+ 440*a^15*b^3*c^13*d^9 - 32*a^16*b^2*c^2*d^20 - 48*a^16*b^2*c^4*d^18 + 51
2*a^16*b^2*c^6*d^16 - 928*a^16*b^2*c^8*d^14 + 672*a^16*b^2*c^10*d^12 - 176*
a^16*b^2*c^12*d^10 - 4*a*b^17*c^21*d + 4*a^17*b*c*d^21))/(a^13*d^17 - b^13*
c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*
a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9
*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d
^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*
b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36
*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d
^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 +
156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a
^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b
^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c
^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2
+ 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548
*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^
7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^
9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2
+ 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992
*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^
6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7
*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4
- 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*
a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^
3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c
^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^1
3 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9
*a*b^12*c^16*d - 9*a^12*b*c*d^16) + (8*tan(e/2 + (f*x)/2)*(12*a*b^17*c^22 -
12*a^18*c*d^21 - 32*a^3*b^15*c^22 + 28*a^5*b^13*c^22 - 8*a^7*b^11*c^22 + 5

6*a^18*c^3*d^19 - 104*a^18*c^5*d^17 + 96*a^18*c^7*d^15 - 44*a^18*c^9*d^13 + 8*a^18*c^11*d^11 - 16*a*b^17*c^12*d^10 + 76*a*b^17*c^14*d^8 - 144*a*b^17*c^16*d^6 + 136*a*b^17*c^18*d^4 - 64*a*b^17*c^20*d^2 - 132*a^2*b^16*c^21*d + 352*a^4*b^14*c^21*d - 308*a^6*b^12*c^21*d + 88*a^8*b^10*c^21*d + 16*a^12*b^6*c*d^21 - 44*a^14*b^4*c*d^21 + 40*a^16*b^2*c*d^21 + 132*a^17*b*c^2*d^20 - 616*a^17*b*c^4*d^18 + 1144*a^17*b*c^6*d^16 - 1056*a^17*b*c^8*d^14 + 484*a^17*b*c^10*d^12 - 88*a^17*b*c^12*d^10 + 176*a^2*b^16*c^11*d^11 - 836*a^2*b^16*c^13*d^9 + 1584*a^2*b^16*c^15*d^7 - 1496*a^2*b^16*c^17*d^5 + 704*a^2*b^16*c^19*d^3 - 880*a^3*b^15*c^10*d^12 + 4224*a^3*b^15*c^12*d^10 - 8128*a^3*b^15*c^14*d^8 + 7872*a^3*b^15*c^16*d^6 - 3888*a^3*b^15*c^18*d^4 + 832*a^3*b^15*c^20*d^2 + 2640*a^4*b^14*c^9*d^13 - 13024*a^4*b^14*c^11*d^11 + 26048*a^4*b^14*c^13*d^9 - 26752*a^4*b^14*c^15*d^7 + 14608*a^4*b^14*c^17*d^5 - 3872*a^4*b^14*c^19*d^3 - 5280*a^5*b^13*c^8*d^14 + 27500*a^5*b^13*c^10*d^12 - 59000*a^5*b^13*c^12*d^10 + 66628*a^5*b^13*c^14*d^8 - 41712*a^5*b^13*c^16*d^6 + 13748*a^5*b^13*c^18*d^4 - 1912*a^5*b^13*c^20*d^2 + 7392*a^6*b^12*c^7*d^15 - 42372*a^6*b^12*c^9*d^13 + 101288*a^6*b^12*c^11*d^11 - 129580*a^6*b^12*c^13*d^9 + 94160*a^6*b^12*c^15*d^7 - 37532*a^6*b^12*c^17*d^5 + 6952*a^6*b^12*c^19*d^3 - 7392*a^7*b^11*c^6*d^16 + 49632*a^7*b^11*c^8*d^14 - 137368*a^7*b^11*c^10*d^12 + 202544*a^7*b^11*c^12*d^10 - 170424*a^7*b^11*c^14*d^8 + 80448*a^7*b^11*c^16*d^6 - 19016*a^7*b^11*c^18*d^4 + 1584*a^7*b^11*c^20*d^2 + 5280*a^8*b^10*c^5*d^17 - 45408*a^8*b^10*c^7*d^15 + 150216*a^8*b^10*c^9*d^13 - 257136*a^8*b^10*c^11*d^11 + 249832*a^8*b^10*c^13*d^9 - 138688*a^8*b^10*c^15*d^7 + 40920*a^8*b^10*c^17*d^5 - 5104*a^8*b^10*c^19*d^3 - 2640*a^9*b^9*c^4*d^18 + 32868*a^9*b^9*c^6*d^16 - 133056*a^9*b^9*c^8*d^14 + 266244*a^9*b^9*c^10*d^12 - 299816*a^9*b^9*c^12*d^10 + 195404*a^9*b^9*c^14*d^8 - 70224*a^9*b^9*c^16*d^6 + 11660*a^9*b^9*c^18*d^4 - 440*a^9*b^9*c^20*d^2 + 880*a^10*b^8*c^3*d^19 - 18700*a^10*b^8*c^5*d^17 + 95040*a^10*b^8*c^7*d^15 - 225676*a^10*b^8*c^9*d^13 + 296824*a^10*b^8*c^11*d^11 - 226116*a^10*b^8*c^13*d^9 + 96624*a^10*b^8*c^15*d^7 - 20196*a^10*b^8*c^17*d^5 + 1320*a^10*b^8*c^19*d^3 - 176*a^11*b^7*c^2*d^20 + 8096*a^11*b^7*c^4*d^18 - 54384*a^11*b^7*c^6*d^16 + 156992*a^11*b^7*c^8*d^14 - 242528*a^11*b^7*c^10*d^12 + 214368*a^11*b^7*c^12*d^10 - 107184*a^11*b^7*c^14*d^8 + 27456*a^11*b^7*c^16*d^6 - 2640*a^11*b^7*c^18*d^4 - 2496*a^12*b^6*c^3*d^19 + 24784*a^12*b^6*c^5*d^17 - 89280*a^12*b^6*c^7*d^15 + 162336*a^12*b^6*c^9*d^13 - 165760*a^12*b^6*c^11*d^11 + 96272*a^12*b^6*c^13*d^9 - 29568*a^12*b^6*c^15*d^7 + 3696*a^12*b^6*c^17*d^5 + 484*a^13*b^5*c^2*d^20 - 8888*a^13*b^5*c^4*d^18 + 40876*a^13*b^5*c^6*d^16 - 88000*a^13*b^5*c^8*d^14 + 104060*a^13*b^5*c^10*d^12 - 69784*a^13*b^5*c^12*d^10 + 24948*a^13*b^5*c^14*d^8 - 3696*a^13*b^5*c^16*d^6 + 2408*a^14*b^4*c^3*d^19 - 14692*a^14*b^4*c^5*d^17 + 38208*a^14*b^4*c^7*d^15 - 52532*a^14*b^4*c^9*d^13 + 40072*a^14*b^4*c^11*d^11 - 16060*a^14*b^4*c^13*d^9 + 2640*a^14*b^4*c^15*d^7 - 440*a^15*b^3*c^2*d^20 + 4048*a^15*b^3*c^4*d^18 - 13112*a^15*b^3*c^6*d^16 + 20768*a^15*b^3*c^8*d^14 - 17512*a^15*b^3*c^10*d^12 + 7568*a^15*b^3*c^12*d^10 - 1320*a^15*b^3*c^14*d^8 - 848*a^16*b^2*c^3*d^19 + 3432*a^16*b^2*c^5*d^17 - 6048*a^16*b^2*c^7*d^15 + 5432*a^16*b^2*c^9*d^13 - 2448*a^16*b^2*c^11*d^11 + 440*a^16*b^2*c^13*d^9))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b

^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^1
3 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^1
3*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a
*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d
 - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c
^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 14
6*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2
*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10
*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5
*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6
 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a
^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^
8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d
^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6
 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^
7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^
6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d
^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 +
 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^
9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*
c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*
d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6
 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 14
6*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^
16))*(-(a + b)^3*(a - b)^3)^(1/2)*(3*b^2*d - 4*a^2*d + a*b*c))/(a^10*d^4 -
b^10*c^4 + 3*a^2*b^8*c^4 - 3*a^4*b^6*c^4 + a^6*b^4*c^4 - a^4*b^6*d^4 + 3*a^
6*b^4*d^4 - 3*a^8*b^2*d^4 + 4*a^3*b^7*c*d^3 - 12*a^3*b^7*c^3*d - 12*a^5*b^5
*c*d^3 + 12*a^5*b^5*c^3*d + 12*a^7*b^3*c*d^3 - 4*a^7*b^3*c^3*d - 6*a^2*b^8*
c^2*d^2 + 18*a^4*b^6*c^2*d^2 - 18*a^6*b^4*c^2*d^2 + 6*a^8*b^2*c^2*d^2 + 4*a
*b^9*c^3*d - 4*a^9*b*c*d^3))*(3*b^2*d - 4*a^2*d + a*b*c))/(a^10*d^4 - b^10*
c^4 + 3*a^2*b^8*c^4 - 3*a^4*b^6*c^4 + a^6*b^4*c^4 - a^4*b^6*d^4 + 3*a^6*b^4
*d^4 - 3*a^8*b^2*d^4 + 4*a^3*b^7*c*d^3 - 12*a^3*b^7*c^3*d - 12*a^5*b^5*c*d^
3 + 12*a^5*b^5*c^3*d + 12*a^7*b^3*c*d^3 - 4*a^7*b^3*c^3*d - 6*a^2*b^8*c^2*d
^2 + 18*a^4*b^6*c^2*d^2 - 18*a^6*b^4*c^2*d^2 + 6*a^8*b^2*c^2*d^2 + 4*a*b^9*
c^3*d - 4*a^9*b*c*d^3))*(3*b^2*d - 4*a^2*d + a*b*c)*1i)/(a^10*d^4 - b^10*c^
4 + 3*a^2*b^8*c^4 - 3*a^4*b^6*c^4 + a^6*b^4*c^4 - a^4*b^6*d^4 + 3*a^6*b^4*d
^4 - 3*a^8*b^2*d^4 + 4*a^3*b^7*c*d^3 - 12*a^3*b^7*c^3*d - 12*a^5*b^5*c*d^3
 + 12*a^5*b^5*c^3*d + 12*a^7*b^3*c*d^3 - 4*a^7*b^3*c^3*d - 6*a^2*b^8*c^2*d^2
 + 18*a^4*b^6*c^2*d^2 - 18*a^6*b^4*c^2*d^2 + 6*a^8*b^2*c^2*d^2 + 4*a*b^9*c^
3*d - 4*a^9*b*c*d^3))/((16*(864*a*b^11*c^5*d^8 - 486*a*b^11*c^3*d^10 - 702*
a*b^11*c^7*d^6 + 216*a*b^11*c^9*d^4 - 216*a^3*b^9*c*d^12 + 63*a^5*b^7*c*d^1
2 + 41*a^7*b^5*c*d^12 + 4*a^9*b^3*c*d^12 + 162*a^2*b^10*c^2*d^11 - 783*a^2*
b^10*c^4*d^9 + 1278*a^2*b^10*c^6*d^7 - 828*a^2*b^10*c^8*d^5 + 144*a^2*b^10*
c^10*d^3 + 1197*a^3*b^9*c^3*d^10 - 2511*a^3*b^9*c^5*d^8 + 2328*a^3*b^9*c^7*
d^6 - 750*a^3*b^9*c^9*d^4 + 24*a^3*b^9*c^11*d^2 - 261*a^4*b^8*c^2*d^11 + 14

44*a^4*b^8*c^4*d^9 - 2508*a^4*b^8*c^6*d^7 + 1518*a^4*b^8*c^8*d^5 - 184*a^4*
b^8*c^10*d^3 - 696*a^5*b^7*c^3*d^10 + 1913*a^5*b^7*c^5*d^8 - 1936*a^5*b^7*c
^7*d^6 + 476*a^5*b^7*c^9*d^4 + 66*a^6*b^6*c^2*d^11 - 583*a^6*b^6*c^4*d^9 +
1232*a^6*b^6*c^6*d^7 - 580*a^6*b^6*c^8*d^5 - 21*a^7*b^5*c^3*d^10 - 312*a^7*
b^5*c^5*d^8 + 364*a^7*b^5*c^7*d^6 + 19*a^8*b^4*c^2*d^11 - 20*a^8*b^4*c^4*d^
9 - 116*a^8*b^4*c^6*d^7 + 16*a^9*b^3*c^3*d^10 + 16*a^9*b^3*c^5*d^8 + 108*a*
b^11*c*d^12))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9
*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^
6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 +
4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^
5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*
c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*
a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^
9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^
2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 4
44*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a
^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9
*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^
11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 -
204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^
6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7
*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^1
3 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 -
588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8
*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*
c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^1
3 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 12
6*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^
10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2
*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^
8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) + (16*tan
(e/2 + (f*x)/2)*(108*a*b^11*c^2*d^11 - 486*a*b^11*c^4*d^9 + 756*a*b^11*c^6*
d^7 - 432*a*b^11*c^8*d^5 + 108*a^2*b^10*c*d^12 - 162*a^4*b^8*c*d^12 + 18*a^
6*b^6*c*d^12 + 8*a^8*b^4*c*d^12 - 270*a^2*b^10*c^3*d^10 + 90*a^2*b^10*c^5*d
^8 + 216*a^2*b^10*c^7*d^6 - 162*a^3*b^9*c^2*d^11 + 864*a^3*b^9*c^4*d^9 - 16
32*a^3*b^9*c^6*d^7 + 900*a^3*b^9*c^8*d^5 + 48*a^3*b^9*c^10*d^3 + 396*a^4*b^
8*c^3*d^10 + 82*a^4*b^8*c^5*d^8 - 596*a^4*b^8*c^7*d^6 - 80*a^4*b^8*c^9*d^4
+ 36*a^5*b^7*c^2*d^11 - 398*a^5*b^7*c^4*d^9 + 1216*a^5*b^7*c^6*d^7 - 584*a^
5*b^7*c^8*d^5 - 42*a^6*b^6*c^3*d^10 - 432*a^6*b^6*c^5*d^8 + 600*a^6*b^6*c^7
*d^6 + 38*a^7*b^5*c^2*d^11 - 40*a^7*b^5*c^4*d^9 - 232*a^7*b^5*c^6*d^7 + 32*
a^8*b^4*c^3*d^10 + 32*a^8*b^4*c^5*d^8))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11
*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6
*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^1
1*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^
10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a

^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) - (b^3*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(36*a*b^13*c^5*d^11 - 144*a*b^13*c^7*d^9 + 216*a*b^13*c^9*d^7 - 144*a*b^13*c^11*d^5 + 36*a*b^13*c^13*d^3 + 4*a^3*b^11*c^15*d - 36*a^5*b^9*c*d^15 + 60*a^7*b^7*c*d^15 - 13*a^9*b^5*c*d^15 - 10*a^11*b^3*c*d^15 - 4*a^13*b*c^3*d^13 - 4*a^13*b*c^5*d^11 - 72*a^2*b^12*c^4*d^12 + 276*a^2*b^12*c^6*d^10 - 375*a^2*b^12*c^8*d^8 + 216*a^2*b^12*c^10*d^6 - 60*a^2*b^12*c^12*d^4 + 24*a^2*b^12*c^14*d^2 - 36*a^3*b^11*c^5*d^11 + 61*a^3*b^11*c^7*d^9 - 88*a^3*b^11*c^9*d^7 + 180*a^3*b^11*c^11*d^5 - 184*a^3*b^11*c^13*d^3 + 72*a^4*b^10*c^2*d^14 - 168*a^4*b^10*c^4*d^12 + 233*a^4*b^10*c^6*d^10 - 270*a^4*b^10*c^8*d^8 + 100*a^4*b^10*c^10*d^6 + 248*a^4*b^10*c^12*d^4 - 44*a^4*b^10*c^14*d^2 + 120*a^5*b^9*c^3*d^13 - 535*a^5*b^9*c^5*d^11 + 1386*a^5*b^9*c^7*d^9 - 1544*a^5*b^9*c^9*d^7 + 248*a^5*b^9*c^11*d^5 + 172*a^5*b^9*c^13*d^3 - 108*a^6*b^8*c^2*d^14 + 699*a^6*b^8*c^4*d^12 - 2046*a^6*b^8*c^6*d^10 + 2885*a^6*b^8*c^8*d^8 - 1336*a^6*b^8*c^10*d^6 - 148*a^6*b^8*c^12*d^4 - 305*a^7*b^7*c^3*d^13 + 1354*a^7*b^7*c^5*d^11 - 2979*a^7*b^7*c^7*d^9 + 2648*a^7*b^7*c^9*d^7 - 400*a^7*b^7*c^11*d^5 + 19*a^8*b^6*c^2*d^14 - 602*a^8*b^6*c^4*d^12 + 2161*a^8*b^6*c^6*d^10 - 3012*a^8*b^6*c^8*d^8 + 1056*a^8*b^6*c^10*d^6 + 190*a^9*b^5*c^3*d^13 - 895*a^9*b^5*c^5*d^11 + 1860*a^9*b^5*c^7*d^9 - 1088*a^9*b^5*c^9*d^7 + 14*a^10*b^4*c^2*d^14 + 99*a^10*b^4*c^4*d^12 - 552*a^10*b^4*c^6*d^10 + 628*a^10*b^4*c^8*d^8 + 19*a^11*b^3*c^3*d^13 + 40*a^11*b^3*c^5*d^11 - 220*a^11*b^3*c^7*d^9 - a^12*b^2*c^2*d^14 + 20*a^12*b^2*c^4*d^12 + 44*a^12*b^2*c^6*d^10 - a^13*b*c*d^15))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14 *d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) - (8*tan(e/2 + (f*x)/2)*(4*a^3*b^11*c^16 - a^14*c*d^15 - 4*a^14*c^3*d^13 - 4*a^14*c^5*d^11 - 144*a*b^13*c^4*d^12 + 684*a*b^13*c^6*d^10 - 1314*a*b^13*c^8*d^8 + 1224*a*b^13*c^10*d^6 - 504*a*b^13*c^12*d^4 + 36*a*b^13*c^14*d^2 + 24*a^2*b^12*c^15*d + 144*a^4*b^10*c*d^15 - 44*a^4*b^10*c^15*d - 348*a^6*b^8*c*d^15 + 214*a^8*b^6*c*d^15 + 7*a^10*b^4*c*d^15 - 8*a^12*b^2*c*d^15 - a^13*b*c^2*d^14 + 20*a^13*b*c^4*d^12 + 44*a^13*b*c^6*d^10 + 432*a^2*b^12*c^3*d^13 - 2148*a^2*b^12*c^5*d^11 + 4470*a^2*b^12*c^7*d^9 - 4632*a^2*b^12*c^9*d^7 + 2232*a^2*b^12*c^11*d^5 - 252*a^2*b^12*c^13*d^3 - 432*a^3*b^11*c^2*d^14 + 2688*a^3*b^11*c^4*d^12 - 7294*a^3*b^11*c^6*d^10 + 10105*a^3*b^11*c^8*d^8 - 7104*a^3*b^11*c^10*d^6 + 1892*a^3*b^11*c^12*d^4 - 192*a^3*b^11*c^14*d^2 - 2016*a^4*b^10*c^3*d^13 + 8378*a^4*b^10*c^5*d^11 - 15815*a^4*b^10*c^7*d^9 + 14976*a^4*b^10*c^9*d^7 - 5932*a^4*b^10*c^11*d^5 + 624*a^4*b^10*c^13*d^3 + 1140*a^5*b^9*c^2*d^14 - 6574*a^5*b^9*c^4*d^12 + 16053*a^5*b^9*c^6*d^10 - 19912*a^5*b^9*c^8*d^8 + 11320*a^5*b^9*c^10*d^6 - 1920*a^5*b^9*c^12*d^4 + 172*a^5*b^9*c^14*d^2 + 2938*a^6*b^8*c^3*d^13 - 10619*a^6*b^8*c^5*d^11 + 18608*a^6*b^8*c^7*d^9 - 15576*a^6*b^8*c^9*d^7 + 4344*a^6*b^8*c^11*d^5 - 292*a^6*b^8*c^13*d^3 - 818*a^7*b^7*c^2*d^14 + 5107*a^7*b^7*c^4*d^12 - 12464*a^7*b^7*c^6*d^10 + 14693*a^7*b^7*c^8*d^8 - 6184*a^7*b^7*c^10*d^6 + 368*a^7*b^7*c^12*d^4 - 1485*a^8*b^6*c^3*d^13 + 5064*a^8*b^6*c^5*d^11 - 8939*a^8*b^6*c^7*d^9 + 6104*a^8*b^6*c^9*d^7 - 688*a^8*b^6*c^11*d^5 + 55*a^9*b^5*c^2*d^14 - 1056*a^9*b^5*c^4*d^12 + 3649*a^9*b^5*c^6*d^10 - 4524*a^9*b^5*c^8*d^8 + 1120*a^9*b^5*c^10*d^6 + 152*a^10*b^4*c^3*d^13 - 975*a^10*b^4*c^5*d^11 + 2300*a^10*b^4*c^7*d^9 - 1088*a^10*b^4*c^9*d^7 + 16*a^11*b^3*c^2*d^14 + 59*a^11*b^3*c^4*d^12 - 640*a^11*b^3*c^6*d^10 + 628*a^11*b^3*c^8*d^8 + 27*a^12*b^2*c^3*d^13 + 48*a^12*b^2*c^5*d^11 - 220 *a^12*b^2*c^7*d^9))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) + (b^3*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a^3*b^13*c^19 - 4*a^5*b^11*c^19 + 2*a^16*c^2*d^17 - 6*a^16*c^6*d^13 + 4*a^16*c^8*d^11 + 12*a*b^15*c^9*d^10 - 54*a*b^15*c^11*d^8 + 96*a*b^15*c^13*d^6 - 78*a*b^15*c^15*d^4 + 24*a*b^15*c^17*d^2 + 12*a^2*b^14*c^18*d - 56*a^4*b^12*c^18*d + 44*a^6*b^10*c^18*d + 12*a^9*b^7*c*d^18 - 28*a^11*b^5*c*d^18 + 16*a^13*b^3*c*d^18 - 10*a^15*b*c^3*d^16 - 24*a^15*b*c^5*d^14 + 78*a^15*b*c^7*d^12 - 44*a^15*b*c^9*d^10 - 96*a^2*b^14*c^8*d^11 + 442*a^2*b^14*c^10*d^9 - 816*a^2*b^14*c^12*d^7 + 702*a^2*b^14*c^14*d^5 - 244*a^2*b^14*c^16*d^3 + 336*a^3*b^13*c^7*d^12 - 1620*a^3*b^13*c^9*d^10 + 3206*a^3*b^13*c^11*d^8 - 3064*a^3*b^13*c^13*d^6 + 1314*a^3*b^13*c^15*d^4 - 176*a^3*b^13*c^17*d^2 - 672*a^4*b^12*c^6*d^13 + 3528*a^4*b^12*c^8*d^11 - 7810*a^4*b^12*c^10*d^9 + 8696*a^4*b^12*c^12*d^7 - 4770*a^4*b^12*c^14*d^5 + 1084*a^4*b^12*c^16*d^3 + 840*a^5*b^11*c^5*d^14 - 5124*a^5*b^11*c^7*d^12 + 13320*a^5*b^11*c^9*d^10 - 17850*a^5*b^11*c^11*d^8 + 12400*a^5*b^11*c^13*d^6 - 3954*a^5*b^11*c^15*d^4 + 372*a^5*b^11*c^17*d^2 - 672*a^6*b^10*c^4*d^15 + 5292*a^6*b^10*c^6*d^13 - 16872*a^6*b^10*c^8*d^11 + 27546*a^6*b^10*c^10*d^9 - 23696*a^6*b^10*c^12*d^7 + 9858*a^6*b^10*c^14*d^5 - 1500*a^6*b^10*c^16*d^3 + 336*a^7*b^9*c^3*d^16 - 4032*a^7*b^9*c^5*d^14 + 16212*a^7*b^9*c^7*d^12 - 32304*a^7*b^9*c^9*d^10 + 34018*a^7*b^9*c^11*d^8 - 18048*a^7*b^9*c^13*d^6 + 4038*a^7*b^9*c^15*d^4 - 220*a^7*b^9*c^17*d^2 - 96*a^8*b^8*c^2*d^17 + 2280*a^8*b^8*c^4*d^15 - 11772*a^8*b^8*c^6*d^13 + 28848*a^8*b^8*c^8*d^11 -

37338*a^8*b^8*c^10*d^9 + 25056*a^8*b^8*c^12*d^7 - 7638*a^8*b^8*c^14*d^5 + 660*a^8*b^8*c^16*d^3 - 918*a^9*b^7*c^3*d^16 + 6360*a^9*b^7*c^5*d^14 - 19602*a^9*b^7*c^7*d^12 + 31560*a^9*b^7*c^9*d^10 - 26556*a^9*b^7*c^11*d^8 + 10464*a^9*b^7*c^13*d^6 - 1320*a^9*b^7*c^15*d^4 + 234*a^10*b^6*c^2*d^17 - 2520*a^10*b^6*c^4*d^15 + 10050*a^10*b^6*c^6*d^13 - 20340*a^10*b^6*c^8*d^11 + 21288*a^10*b^6*c^10*d^9 - 10560*a^10*b^6*c^12*d^7 + 1848*a^10*b^6*c^14*d^5 + 726*a^11*b^5*c^3*d^16 - 3768*a^11*b^5*c^5*d^14 + 9670*a^11*b^5*c^7*d^12 - 12648*a^11*b^5*c^9*d^10 + 7896*a^11*b^5*c^11*d^8 - 1848*a^11*b^5*c^13*d^6 - 146*a^12*b^4*c^2*d^17 + 952*a^12*b^4*c^4*d^15 - 3174*a^12*b^4*c^6*d^13 + 5396*a^12*b^4*c^8*d^11 - 4348*a^12*b^4*c^10*d^9 + 1320*a^12*b^4*c^12*d^7 - 134*a^13*b^3*c^3*d^16 + 624*a^13*b^3*c^5*d^14 - 1570*a^13*b^3*c^7*d^12 + 1724*a^13*b^3*c^9*d^10 - 660*a^13*b^3*c^11*d^8 + 6*a^14*b^2*c^2*d^17 - 40*a^14*b^2*c^4*d^15 + 282*a^14*b^2*c^6*d^13 - 468*a^14*b^2*c^8*d^11 + 220*a^14*b^2*c^10*d^9))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) + (8*tan(e/2 + (f*x)/2)*(4*a^16*c*d^18 + 8*a^2*b^14*c^19 - 8*a^4*b^12*c^19 - 12*a^16*c^5*d^14 + 8*a^16*c^7*d^12 + 12*a*b^15*c^10*d^9 - 48*a*b^15*c^12*d^7 + 84*a*b^15*c^14*d^5 - 72*a*b^15*c^16*d^3 - 112*a^3*b^13*c^18*d + 88*a^5*b^11*c^18*d + 12*a^10*b^6*c*d^18 - 28*a^12*b^4*c*d^18 + 12*a^14*b^2*c*d^18 - 20*a^15*b*c^2*d^17 - 48*a^15*b*c^4*d^15 + 156*a^15*b*c^6*d^13 - 88*a^15*b*c^8*d^11 - 84*a^2*b^14*c^9*d^10 + 328*a^2*b^14*c^11*d^8 - 596*a^2*b^14*c^13*d^6 + 552*a^2*b^14*c^15*d^4 - 208*a^2*b^14*c^17*d^2 + 240*a^3*b^13*c^8*d^11 - 908*a^3 *b^13*c^10*d^9 + 1792*a^3*b^13*c^12*d^7 - 1932*a^3*b^13*c^14*d^5 + 920*a^3*b^13*c^16*d^3 - 336*a^4*b^12*c^7*d^12 + 1188*a^4*b^12*c^9*d^10 - 2808*a^4*b^12*c^11*d^8 + 3980*a^4*b^12*c^13*d^6 - 2616*a^4*b^12*c^15*d^4 + 600*a^4*b^12*c^17*d^2 + 168*a^5*b^11*c^6*d^13 - 336*a^5*b^11*c^8*d^11 + 1740*a^5*b^11*c^10*d^9 - 4720*a^5*b^11*c^12*d^7 + 4812*a^5*b^11*c^14*d^5 - 1752*a^5*b^11*c^16*d^3 + 168*a^6*b^10*c^5*d^14 - 1344*a^6*b^10*c^7*d^12 + 2292*a^6*b^10*c^9*d^10 + 1088*a^6*b^10*c^11*d^8 - 4908*a^6*b^10*c^13*d^6 + 3096*a^6*b^10*c^15*d^4 - 392*a^6*b^10*c^17*d^2 - 336*a^7*b^9*c^4*d^15 + 2520*a^7*b^9*c^6*d^13 - 7488*a^7*b^9*c^8*d^11 + 7556*a^7*b^9*c^10*d^9 - 144*a^7*b^9*c^12*d^7 - 3012*a^7*b^9*c^14*d^5 + 904*a^7*b^9*c^16*d^3 + 240*a^8*b^8*c^3*d^16 - 2472*a^8*b^8*c^5*d^14 + 10416*a^8*b^8*c^7*d^12 - 16596*a^8*b^8*c^9*d^10 + 9600*a^8*b^8*c^11*d^8 - 156*a^8*b^8*c^13*d^6 - 1032*a^8*b^8*c^15*d^4 - 84*a^9*b^7*c^2*d^17 + 1632*a^9*b^7*c^4*d^15 - 9204*a^9*b^7*c^6*d^13 + 19800*a^9*b^7*c^8*d^11 - 18048*a^9*b^7*c^10*d^9 + 5856*a^9*b^7*c^12*d^7 + 48*a^9*b^7*c^14*d^5 - 744*a^10*b^6*c^3*d^16 + 5460*a^10*b^6*c^5*d^14 - 15960*a^10*b^6*c^7*d^12 + 20136*a^10*b^6*c^9*d^10 - 10584*a^10*b^6*c^11*d^8 + 1680*a^10*b^6*c^13*d^6 + 212*a^11*b^5*c^2*d^17 - 2176*a^11*b^5*c^4*d^15 + 9180*a^11*b^5*c^6*d^13 - 15416*a^11*b^5*c^8*d^11 + 10936*a^11*b^5*c^10*d^9 - 2736*a^11*b^5*c^12*d^7 + 584*a^12*b^4*c^3*d^16 - 3708*a^12*b^4*c^5*d^14 + 8152*a^12*b^4*c^7*d^12 - 7376*a^12*b^4*c^9*d^10 + 2376*a^12*b^4*c^11*d^8 - 108*a^13*b^3*c^2*d^17 + 928*a^13*b^3*c^4*d^15 - 2820*a^13*b^3*c^6*d^13 + 3288*a^13*b^3*c^8*d^11 - 1288*a^13*b^3*c^10*d^9 - 80*a^14*b^2*c^3*d^16 + 564*a^14*b^2*c^5*d^14 - 936*a^14*b^2*c^7*d^12 + 440*a^14*b^2*c^9*d^10 + 24*a*b^15*c^18*d))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10

+ 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) + (b^3*((8*(4*a^2*b^16*c^22 - 8*a^4*b^14*c^22 + 4*a^6*b^12*c^22 - 4*a^18*c^2*d^20 + 16*a^18*c^4*d^18 - 24*a^18*c^6*d^16 + 16*a^18*c^8*d^14 - 4*a^18*c^10*d^12 - 4*a*b^17*c^13*d^9 + 16*a*b^17*c^15*d^7 - 24*a*b^17*c^17*d^5 + 16*a*b^17*c^19*d^3 - 32*a^3*b^15*c^21*d + 76*a^5*b^13*c^21*d - 40*a^7*b^11*c^21*d + 4*a^13*b^5*c*d^21 - 8*a^15*b^3*c*d^21 + 24*a^17*b*c^3*d^19 - 136*a^17*b*c^5*d^17 + 224*a^17*b*c^7*d^15 - 156*a^17*b*c^9*d^13 + 40*a^17*b*c^11*d^11 + 40*a^2*b^16*c^12*d^10 - 156*a^2*b^16*c^14*d^8 + 224*a^2*b^16*c^16*d^6 - 136*a^2*b^16*c^18*d^4 + 24*a^2*b^16*c^20*d^2 - 176*a^3*b^15*c^11*d^11 + 672*a^3*b^15*c^13*d^9 - 928*a^3*b^15*c^15*d^7 + 512*a^3*b^15*c^17*d^5 - 48*a^3*b^15*c^19*d^3 + 440*a^4*b^14*c^10*d^12 - 1664*a^4*b^14*c^12*d^10 + 2248*a^4*b^14*c^14*d^8 - 1152*a^4*b^14*c^16*d^6 + 8*a^4*b^14*c^18*d^4 + 128*a^4*b^14*c^20*d^2 - 660*a^5*b^13*c^9*d^13 + 2552*a^5*b^13*c^11*d^11 - 3532*a^5*b^13*c^13*d^9 + 1808*a^5*b^13*c^15*d^7 + 148*a^5*b^13*c^17*d^5 - 392*a^5*b^13*c^19*d^3 + 528*a^6*b^12*c^8*d^14 - 2332*a^6*b^12*c^10*d^12 + 3736*a^6*b^12*c^12*d^10 - 2180*a^6*b^12*c^14*d^8 - 480*a^6*b^12*c^16*d^6 + 1052*a^6*b^12*c^18*d^4 - 328*a^6*b^12*c^20*d^2 + 792*a^7*b^11*c^9*d^13 - 2464*a^7*b^11*c^11*d^11 + 1896*a^7*b^11*c^13*d^9 + 1216*a^7*b^11*c^15*d^7 - 2264*a^7*b^11*c^17*d^5 + 864*a^7*b^11*c^19*d^3 - 528*a^8*b^10*c^6*d^16 + 1056*a^8*b^10*c^8*d^14 + 176*a^8*b^10*c^10*d^12 - 528*a^8*b^10*c^12*d^10 - 2288*a^8*b^10*c^14*d^8 + 3520*a^8*b^10*c^16*d^6 - 1584*a^8*b^10*c^18*d^4 + 176*a^8*b^10*c^20*d^2 + 660*a^9*b^9*c^5*d^17 - 2112*a^9*b^9*c^7*d^15 + 2244*a^9*b^9*c^9*d^13 - 1496*a^9*b^9*c^11*d^11 + 2684*a^9*b^9*c^13*d^9 - 3696*a^9*b^9*c^15*d^7 + 2156*a^9*b^9*c^17*d^5 - 440*a^9*b^9*c^19*d^3 - 440*a^10*b^8*c^4*d^18 + 2156*a^10*b^8*c^6*d^16 - 3696*a^10*b^8*c^8*d^14 + 2684*a^10*b^8*c^10*d^12 - 1496*a^10*b^8*c^12*d^10 + 2244*a^10*b^8*c^14*d^8 - 2112*a^10*b^8*c^16*d^6 + 660*a^10*b^8*c^18*d^4 + 176*a^11*b^7*c^3*d^19 - 1584*a^11*b^7*c^5*d^17 + 3520*a^11*b^7*c^7*d^15 - 2288*a^11*b^7*c^9*d^13 - 528*a^11*b^7*c^11*d^11 + 176*a^11*b^7*c^13*d^9 + 1056*a^11*b^7*c^15*d^7 - 528*a^11*b^7*c^17*d^5 - 40*a^12*b^6*c^2*d^20 + 864*a^12*b^6*c^4*d^18 - 2264*a^12*b^6*c^6*d^16 + 1216*a^12*b^6*c^8*d^14 + 1896*a^12*b^6*c^10*d^12 - 2464*a^12*b^6*c^12*d^10 + 792*a^12*b^6*c^14*d^8 - 328*a^13*b^5*c^3*d^19 + 1052*a^13*b^5*c^5*d^17 - 480*a^13*b^5*c^7*d^15 - 2180*a^13*b^5*c^9*d^13 + 3736*a^13*b^5*c^11*d^11 - 2332*a^13*b^5*c^13*d^9 + 528*a^13*b^5*c^15*d^7 + 76*a^14*b^4*c^2*d^20 - 392*a^14*b^4*c^4*d^18 + 148*a^14*b^4*c^6*d^16 + 1808*a^14*b^4*c^8*d^14 - 3532*a^14*b^4*c^10*d^12 + 2552*a^14*b^4*c^12*d^10 - 660*a^14*b^4*c^14*d^8 + 128*a^15*b^3*c^3*d^19 + 8*a^15*b^3*c^5*d^17 - 1152*a^15*b^3*c^7*d^15 + 2248*a^15*b^3*c^9*d^13 - 1664*a^15*b^3*c^11*d^11 + 440*a^15*b^3*c^13*d^9 - 32*a^16*b^2*c^2*d^20 - 48*a^16*b^2*c^4*d^18 + 512*a^16*b^2*c^6*d^16 - 928*a^16*b^2*c^8*d^14 + 672*a^16*b^2*c^10*d^12 - 176*a^16*b^2*c^12*d^10 - 4*a*b^17*c^21*d + 4*a^17*b*c*d^21))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a* b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) + (8*tan(e/2 + (f*x)/2)*(12*a*b^17*c^22 - 12*a^18*c*d^21 - 32*a^3*b^15*c^22 + 28*a^5*b^13*c^22 - 8*a^7*b^11*c^22 + 56*a^18*c^3*d^19 - 104*a^18*c^5*d^17 + 96*a^18*c^7*d^15 - 44*a^18*c^9*d^13 + 8*a^18*c^11*d^11 - 16*a*b^17*c^12*d^10 + 76*a*b^17*c^14*d^8 - 144*a*b^17*c^16*d^6 + 136*a*b^17*c^18*d^4 - 64*a*b^17*c^20*d^2 - 132*a^2*b^16*c^21*d + 352*a^4*b^14*c^21*d - 308*a^6*b^12*c^21*d + 88*a^8*b^10*c^21*d + 16*a^12*b^6*c*d^21 - 44*a^14*b^4*c*d^21 + 40*a^16*b^2*c*d^21 + 132*a^17*b*c^2*d^20 - 616*a^17*b*c^4*d^18 + 1144*a^17*b*c^6*d^16 - 1056*a^17*b*c^8*d^14 + 484*a^17*b*c^10*d^12 - 88*a^17*b*c^12*d^10 + 176*a^2*b^16*c^11*d^11 - 836*a^2*b^16*c^13*d^9 + 1584*a^2*b^16*c^15*d^7 - 1496*a^2*b^16*c^17*d^5 + 704*a^2*b^16*c^19*d^3 - 880*a^3*b^15*c^10*d^12 + 4224*a^3*b^15*c^12*d^10 - 8128*a^3*b^15*c^14*d^8 + 7872*a^3*b^15*c^16*d^6 - 3888*a^3*b^15*c^18*d^4 + 832*a^3*b^15*c^20*d^2 + 2640*a^4*b^14*c^9*d^13 - 13024*a^4*b^14*c^11*d^11 + 26048*a^4*b^14*c^13*d^9 - 26752*a^4*b^14*c^15*d^7 + 14608*a^4*b^14*c^17*d^5 - 3872*a^4*b^14*c^19*d^3 - 5280*a^5*b^13*c^8*d^14 + 27500*a^5*b^13*c^10*d^12 - 59000*a^5*b^13*c^12*d^10 + 66628*a^5*b^13*c^14*d^8 - 41712*a^5*b^13*c^16*d^6 + 13748*a^5*b^13*c^18*d^4 - 1912*a^5*b^13*c^20*d^2 + 7392*a^6*b^12*c^7*d^15 - 42372*a^6*b^12*c^9*d^13 + 101288*a^6*b^12*c^11*d^11 - 129580*a^6*b^12*c^13*d^9 + 94160*a^6*b^12*c^15*d^7 - 37532*a^6*b^12*c^17*d^5 + 6952*a^6*b^12*c^19*d^3 - 7392*a^7*b^11*c^6*d^16 + 49632*a^7*b^11*c^8*d^14 - 137368*a^7*b^11*c^10*d^12 + 202544*a^7*b^11*c^12*d^10 - 170424*a^7*b^11*c^14*d^8 + 80448*a^7*b^11*c^16*d^6 - 19016*a^7*b^11*c^18*d^4 + 1584*a^7*b^11*c^20*d^2 + 5280*a^8*b^10*c^5*d^17 - 45408*a^8*b^10*c^7*d^15 + 150216*a^8*b^10*c^9*d^13 - 257136*a^8*b^10*c^11*d^11 + 249832*a^8*b^10*c^13*d^9 - 138688*a^8*b^10

*c^15*d^7 + 40920*a^8*b^10*c^17*d^5 - 5104*a^8*b^10*c^19*d^3 - 2640*a^9*b^9
*c^4*d^18 + 32868*a^9*b^9*c^6*d^16 - 133056*a^9*b^9*c^8*d^14 + 266244*a^9*b
^9*c^10*d^12 - 299816*a^9*b^9*c^12*d^10 + 195404*a^9*b^9*c^14*d^8 - 70224*a
^9*b^9*c^16*d^6 + 11660*a^9*b^9*c^18*d^4 - 440*a^9*b^9*c^20*d^2 + 880*a^10*
b^8*c^3*d^19 - 18700*a^10*b^8*c^5*d^17 + 95040*a^10*b^8*c^7*d^15 - 225676*a
^10*b^8*c^9*d^13 + 296824*a^10*b^8*c^11*d^11 - 226116*a^10*b^8*c^13*d^9 + 9
6624*a^10*b^8*c^15*d^7 - 20196*a^10*b^8*c^17*d^5 + 1320*a^10*b^8*c^19*d^3 -
 176*a^11*b^7*c^2*d^20 + 8096*a^11*b^7*c^4*d^18 - 54384*a^11*b^7*c^6*d^16 +
 156992*a^11*b^7*c^8*d^14 - 242528*a^11*b^7*c^10*d^12 + 214368*a^11*b^7*c^1
2*d^10 - 107184*a^11*b^7*c^14*d^8 + 27456*a^11*b^7*c^16*d^6 - 2640*a^11*b^7
*c^18*d^4 - 2496*a^12*b^6*c^3*d^19 + 24784*a^12*b^6*c^5*d^17 - 89280*a^12*b
^6*c^7*d^15 + 162336*a^12*b^6*c^9*d^13 - 165760*a^12*b^6*c^11*d^11 + 96272*
a^12*b^6*c^13*d^9 - 29568*a^12*b^6*c^15*d^7 + 3696*a^12*b^6*c^17*d^5 + 484*
a^13*b^5*c^2*d^20 - 8888*a^13*b^5*c^4*d^18 + 40876*a^13*b^5*c^6*d^16 - 8800
0*a^13*b^5*c^8*d^14 + 104060*a^13*b^5*c^10*d^12 - 69784*a^13*b^5*c^12*d^10
+ 24948*a^13*b^5*c^14*d^8 - 3696*a^13*b^5*c^16*d^6 + 2408*a^14*b^4*c^3*d^19
 - 14692*a^14*b^4*c^5*d^17 + 38208*a^14*b^4*c^7*d^15 - 52532*a^14*b^4*c^9*d
^13 + 40072*a^14*b^4*c^11*d^11 - 16060*a^14*b^4*c^13*d^9 + 2640*a^14*b^4*c^
15*d^7 - 440*a^15*b^3*c^2*d^20 + 4048*a^15*b^3*c^4*d^18 - 13112*a^15*b^3*c^
6*d^16 + 20768*a^15*b^3*c^8*d^14 - 17512*a^15*b^3*c^10*d^12 + 7568*a^15*b^3
*c^12*d^10 - 1320*a^15*b^3*c^14*d^8 - 848*a^16*b^2*c^3*d^19 + 3432*a^16*b^2
*c^5*d^17 - 6048*a^16*b^2*c^7*d^15 + 5432*a^16*b^2*c^9*d^13 - 2448*a^16*b^2
*c^11*d^11 + 440*a^16*b^2*c^13*d^9))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^
17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^
13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d
^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*
d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*
b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 5
4*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7
*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^
4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 57
6*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a
^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9
*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^
13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 +
852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6
*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7
*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^1
5 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1
548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8
*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5
*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^1
5 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 57
6*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^1
0*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3

*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6
*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a
^12*b*c*d^16))*(-(a + b)^3*(a - b)^3)^(1/2)*(3*b^2*d - 4*a^2*d + a*b*c))/(a
^10*d^4 - b^10*c^4 + 3*a^2*b^8*c^4 - 3*a^4*b^6*c^4 + a^6*b^4*c^4 - a^4*b^6*
d^4 + 3*a^6*b^4*d^4 - 3*a^8*b^2*d^4 + 4*a^3*b^7*c*d^3 - 12*a^3*b^7*c^3*d -
12*a^5*b^5*c*d^3 + 12*a^5*b^5*c^3*d + 12*a^7*b^3*c*d^3 - 4*a^7*b^3*c^3*d -
6*a^2*b^8*c^2*d^2 + 18*a^4*b^6*c^2*d^2 - 18*a^6*b^4*c^2*d^2 + 6*a^8*b^2*c^2
*d^2 + 4*a*b^9*c^3*d - 4*a^9*b*c*d^3))*(3*b^2*d - 4*a^2*d + a*b*c))/(a^10*d
^4 - b^10*c^4 + 3*a^2*b^8*c^4 - 3*a^4*b^6*c^4 + a^6*b^4*c^4 - a^4*b^6*d^4 +
3*a^6*b^4*d^4 - 3*a^8*b^2*d^4 + 4*a^3*b^7*c*d^3 - 12*a^3*b^7*c^3*d - 12*a^
5*b^5*c*d^3 + 12*a^5*b^5*c^3*d + 12*a^7*b^3*c*d^3 - 4*a^7*b^3*c^3*d - 6*a^2
*b^8*c^2*d^2 + 18*a^4*b^6*c^2*d^2 - 18*a^6*b^4*c^2*d^2 + 6*a^8*b^2*c^2*d^2
+ 4*a*b^9*c^3*d - 4*a^9*b*c*d^3))*(3*b^2*d - 4*a^2*d + a*b*c))/(a^10*d^4 -
b^10*c^4 + 3*a^2*b^8*c^4 - 3*a^4*b^6*c^4 + a^6*b^4*c^4 - a^4*b^6*d^4 + 3*a^
6*b^4*d^4 - 3*a^8*b^2*d^4 + 4*a^3*b^7*c*d^3 - 12*a^3*b^7*c^3*d - 12*a^5*b^5
*c*d^3 + 12*a^5*b^5*c^3*d + 12*a^7*b^3*c*d^3 - 4*a^7*b^3*c^3*d - 6*a^2*b^8*
c^2*d^2 + 18*a^4*b^6*c^2*d^2 - 18*a^6*b^4*c^2*d^2 + 6*a^8*b^2*c^2*d^2 + 4*a
*b^9*c^3*d - 4*a^9*b*c*d^3) - (b^3*(-(a + b)^3*(a - b)^3)^(1/2)*((8*tan(e/2
+ (f*x)/2)*(4*a^3*b^11*c^16 - a^14*c*d^15 - 4*a^14*c^3*d^13 - 4*a^14*c^5*d
^11 - 144*a*b^13*c^4*d^12 + 684*a*b^13*c^6*d^10 - 1314*a*b^13*c^8*d^8 + 122
4*a*b^13*c^10*d^6 - 504*a*b^13*c^12*d^4 + 36*a*b^13*c^14*d^2 + 24*a^2*b^12*
c^15*d + 144*a^4*b^10*c*d^15 - 44*a^4*b^10*c^15*d - 348*a^6*b^8*c*d^15 + 21
4*a^8*b^6*c*d^15 + 7*a^10*b^4*c*d^15 - 8*a^12*b^2*c*d^15 - a^13*b*c^2*d^14
+ 20*a^13*b*c^4*d^12 + 44*a^13*b*c^6*d^10 + 432*a^2*b^12*c^3*d^13 - 2148*a^
2*b^12*c^5*d^11 + 4470*a^2*b^12*c^7*d^9 - 4632*a^2*b^12*c^9*d^7 + 2232*a^2*
b^12*c^11*d^5 - 252*a^2*b^12*c^13*d^3 - 432*a^3*b^11*c^2*d^14 + 2688*a^3*b^
11*c^4*d^12 - 7294*a^3*b^11*c^6*d^10 + 10105*a^3*b^11*c^8*d^8 - 7104*a^3*b^
11*c^10*d^6 + 1892*a^3*b^11*c^12*d^4 - 192*a^3*b^11*c^14*d^2 - 2016*a^4*b^1
0*c^3*d^13 + 8378*a^4*b^10*c^5*d^11 - 15815*a^4*b^10*c^7*d^9 + 14976*a^4*b^
10*c^9*d^7 - 5932*a^4*b^10*c^11*d^5 + 624*a^4*b^10*c^13*d^3 + 1140*a^5*b^9*
c^2*d^14 - 6574*a^5*b^9*c^4*d^12 + 16053*a^5*b^9*c^6*d^10 - 19912*a^5*b^9*c
^8*d^8 + 11320*a^5*b^9*c^10*d^6 - 1920*a^5*b^9*c^12*d^4 + 172*a^5*b^9*c^14*
d^2 + 2938*a^6*b^8*c^3*d^13 - 10619*a^6*b^8*c^5*d^11 + 18608*a^6*b^8*c^7*d^
9 - 15576*a^6*b^8*c^9*d^7 + 4344*a^6*b^8*c^11*d^5 - 292*a^6*b^8*c^13*d^3 -
818*a^7*b^7*c^2*d^14 + 5107*a^7*b^7*c^4*d^12 - 12464*a^7*b^7*c^6*d^10 + 146
93*a^7*b^7*c^8*d^8 - 6184*a^7*b^7*c^10*d^6 + 368*a^7*b^7*c^12*d^4 - 1485*a^
8*b^6*c^3*d^13 + 5064*a^8*b^6*c^5*d^11 - 8939*a^8*b^6*c^7*d^9 + 6104*a^8*b^
6*c^9*d^7 - 688*a^8*b^6*c^11*d^5 + 55*a^9*b^5*c^2*d^14 - 1056*a^9*b^5*c^4*d
^12 + 3649*a^9*b^5*c^6*d^10 - 4524*a^9*b^5*c^8*d^8 + 1120*a^9*b^5*c^10*d^6
+ 152*a^10*b^4*c^3*d^13 - 975*a^10*b^4*c^5*d^11 + 2300*a^10*b^4*c^7*d^9 - 1
088*a^10*b^4*c^9*d^7 + 16*a^11*b^3*c^2*d^14 + 59*a^11*b^3*c^4*d^12 - 640*a^
11*b^3*c^6*d^10 + 628*a^11*b^3*c^8*d^8 + 27*a^12*b^2*c^3*d^13 + 48*a^12*b^2
*c^5*d^11 - 220*a^12*b^2*c^7*d^9))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17
- a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13
*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6

- 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^
7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^
8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*
a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d
^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4
- 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*
a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4
*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c
^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13
- 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 85
2*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b
^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c
^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15
- 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 154
8*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b
^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c
^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15
+ 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*
a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*
b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c
^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d
^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^1
2*b*c*d^16) - (8*(36*a*b^13*c^5*d^11 - 144*a*b^13*c^7*d^9 + 216*a*b^13*c^9*
d^7 - 144*a*b^13*c^11*d^5 + 36*a*b^13*c^13*d^3 + 4*a^3*b^11*c^15*d - 36*a^5
*b^9*c*d^15 + 60*a^7*b^7*c*d^15 - 13*a^9*b^5*c*d^15 - 10*a^11*b^3*c*d^15 -
4*a^13*b*c^3*d^13 - 4*a^13*b*c^5*d^11 - 72*a^2*b^12*c^4*d^12 + 276*a^2*b^12
*c^6*d^10 - 375*a^2*b^12*c^8*d^8 + 216*a^2*b^12*c^10*d^6 - 60*a^2*b^12*c^12
*d^4 + 24*a^2*b^12*c^14*d^2 - 36*a^3*b^11*c^5*d^11 + 61*a^3*b^11*c^7*d^9 -
88*a^3*b^11*c^9*d^7 + 180*a^3*b^11*c^11*d^5 - 184*a^3*b^11*c^13*d^3 + 72*a^
4*b^10*c^2*d^14 - 168*a^4*b^10*c^4*d^12 + 233*a^4*b^10*c^6*d^10 - 270*a^4*b
^10*c^8*d^8 + 100*a^4*b^10*c^10*d^6 + 248*a^4*b^10*c^12*d^4 - 44*a^4*b^10*c
^14*d^2 + 120*a^5*b^9*c^3*d^13 - 535*a^5*b^9*c^5*d^11 + 1386*a^5*b^9*c^7*d^
9 - 1544*a^5*b^9*c^9*d^7 + 248*a^5*b^9*c^11*d^5 + 172*a^5*b^9*c^13*d^3 - 10
8*a^6*b^8*c^2*d^14 + 699*a^6*b^8*c^4*d^12 - 2046*a^6*b^8*c^6*d^10 + 2885*a^
6*b^8*c^8*d^8 - 1336*a^6*b^8*c^10*d^6 - 148*a^6*b^8*c^12*d^4 - 305*a^7*b^7*
c^3*d^13 + 1354*a^7*b^7*c^5*d^11 - 2979*a^7*b^7*c^7*d^9 + 2648*a^7*b^7*c^9*
d^7 - 400*a^7*b^7*c^11*d^5 + 19*a^8*b^6*c^2*d^14 - 602*a^8*b^6*c^4*d^12 + 2
161*a^8*b^6*c^6*d^10 - 3012*a^8*b^6*c^8*d^8 + 1056*a^8*b^6*c^10*d^6 + 190*a
^9*b^5*c^3*d^13 - 895*a^9*b^5*c^5*d^11 + 1860*a^9*b^5*c^7*d^9 - 1088*a^9*b^
5*c^9*d^7 + 14*a^10*b^4*c^2*d^14 + 99*a^10*b^4*c^4*d^12 - 552*a^10*b^4*c^6*
d^10 + 628*a^10*b^4*c^8*d^8 + 19*a^11*b^3*c^3*d^13 + 40*a^11*b^3*c^5*d^11 -
220*a^11*b^3*c^7*d^9 - a^12*b^2*c^2*d^14 + 20*a^12*b^2*c^4*d^12 + 44*a^12*
b^2*c^6*d^10 - a^13*b*c*d^15))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a
^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4
*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6

*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) + (b^3*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a^3*b^13*c^19 - 4*a^5*b^11*c^19 + 2*a^16*c^2*d^17 - 6*a^16*c^6*d^13 + 4*a^16*c^8*d^11 + 12*a*b^15*c^9*d^10 - 54*a*b^15*c^11*d^8 + 96*a*b^15*c^13*d^6 - 78*a*b^15*c^15*d^4 + 24*a*b^15*c^17*d^2 + 12*a^2*b^14*c^18*d - 56*a^4*b^12*c^18*d + 44*a^6*b^10*c^18*d + 12*a^9*b^7*c*d^18 - 28*a^11*b^5*c*d^18 + 16*a^13*b^3*c*d^18 - 10*a^15*b*c^3*d^16 - 24*a^15*b*c^5*d^14 + 78*a^15*b*c^7*d^12 - 44*a^15*b*c^9*d^10 - 96*a^2*b^14*c^8*d^11 + 442*a^2*b^14*c^10*d^9 - 816*a^2*b^14*c^12*d^7 + 702*a^2*b^14*c^14*d^5 - 244*a^2*b^14*c^16*d^3 + 336*a^3*b^13*c^7*d^12 - 1620*a^3*b^13*c^9*d^10 + 3206*a^3*b^13*c^11*d^8 - 3064*a^3*b^13*c^13*d^6 + 1314*a^3*b^13*c^15*d^4 - 176*a^3*b^13*c^17*d^2 - 672*a^4*b^12*c^6*d^13 + 3528*a^4*b^12*c^8*d^11 - 7810*a^4*b^12*c^10*d^9 + 8696*a^4*b^12*c^12*d^7 - 4770*a^4*b^12*c^14*d^5 + 1084*a^4*b^12*c^16*d^3 + 840*a^5*b^11*c^5*d^14 - 5124*a^5*b^11*c^7*d^12 + 13320*a^5*b^11*c^9*d^10 - 17850*a^5*b^11*c^11*d^8 + 12400*a^5*b^11*c^13*d^6 - 3954*a^5*b^11*c^15*d^4 + 372*a^5*b^11*c^17*d^2 - 672*a^6*b^10*c^4*d^15 + 5292*a^6*b^10*c^6*d^13 - 16872*a^6*b^10*c^8*d^11 + 27546*a^6*b^10*c^10*d^9 - 23696*a^6*b^10*c^12*d^7 + 9858*a^6*b^10*c^14*d^5 - 1500*a^6*b^10*c^16*d^3 + 336*a^7*b^9*c^3*d^16 - 4032*a^7*b^9*c^5*d^14 + 16212*a^7*b^9*c^7*d^12 - 32304*a^7*b^9*c^9*d^10 + 34018*a^7*b^9*c^11*d^8 - 18048*a^7*b^9*c^13*d^6 + 4038*a^7*b^9*c^15*d^4 - 220*a^7*b^9*c^17*d^2 - 96*a^8*b^8*c^2*d^17 + 2280*a^8*b^8*c^4*d^15 - 11772*a^8*b^8*c^6*d^13 + 28848*a^8*b^8*c^8*d^11 - 37338*a^8*b^8*c^10*d^9 + 25056*a^8*b^8*c^12*d^7 - 7638*a^8*b^8*c^14*d^5 + 660*a^8*b^8*c^16*d^3 - 918*a^9*b^7*c^3*d^16 + 6360*a^9*b^7*c^5*d^14 - 19602*a^9*b^7*c^7*d^12 + 31560*a^9*b^7*c^9*d^10 - 26556*a^9*b^7*c^11*

d^8 + 10464*a^9*b^7*c^13*d^6 - 1320*a^9*b^7*c^15*d^4 + 234*a^10*b^6*c^2*d^1
7 - 2520*a^10*b^6*c^4*d^15 + 10050*a^10*b^6*c^6*d^13 - 20340*a^10*b^6*c^8*d
^11 + 21288*a^10*b^6*c^10*d^9 - 10560*a^10*b^6*c^12*d^7 + 1848*a^10*b^6*c^1
4*d^5 + 726*a^11*b^5*c^3*d^16 - 3768*a^11*b^5*c^5*d^14 + 9670*a^11*b^5*c^7*
d^12 - 12648*a^11*b^5*c^9*d^10 + 7896*a^11*b^5*c^11*d^8 - 1848*a^11*b^5*c^1
3*d^6 - 146*a^12*b^4*c^2*d^17 + 952*a^12*b^4*c^4*d^15 - 3174*a^12*b^4*c^6*d
^13 + 5396*a^12*b^4*c^8*d^11 - 4348*a^12*b^4*c^10*d^9 + 1320*a^12*b^4*c^12*
d^7 - 134*a^13*b^3*c^3*d^16 + 624*a^13*b^3*c^5*d^14 - 1570*a^13*b^3*c^7*d^1
2 + 1724*a^13*b^3*c^9*d^10 - 660*a^13*b^3*c^11*d^8 + 6*a^14*b^2*c^2*d^17 -
40*a^14*b^2*c^4*d^15 + 282*a^14*b^2*c^6*d^13 - 468*a^14*b^2*c^8*d^11 + 220*
a^14*b^2*c^10*d^9))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17
 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a
^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*
d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c
^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^
8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12
 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b
^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c
^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d
^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 +
 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a
^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*
c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*
d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1
548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a
^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c
^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*
d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 8
52*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^
8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c
^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^
7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 -
576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^
11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*
b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) + (
8*tan(e/2 + (f*x)/2)*(4*a^16*c*d^18 + 8*a^2*b^14*c^19 - 8*a^4*b^12*c^19 - 1
2*a^16*c^5*d^14 + 8*a^16*c^7*d^12 + 12*a*b^15*c^10*d^9 - 48*a*b^15*c^12*d^7
 + 84*a*b^15*c^14*d^5 - 72*a*b^15*c^16*d^3 - 112*a^3*b^13*c^18*d + 88*a^5*b
^11*c^18*d + 12*a^10*b^6*c*d^18 - 28*a^12*b^4*c*d^18 + 12*a^14*b^2*c*d^18 -
 20*a^15*b*c^2*d^17 - 48*a^15*b*c^4*d^15 + 156*a^15*b*c^6*d^13 - 88*a^15*b*
c^8*d^11 - 84*a^2*b^14*c^9*d^10 + 328*a^2*b^14*c^11*d^8 - 596*a^2*b^14*c^13
*d^6 + 552*a^2*b^14*c^15*d^4 - 208*a^2*b^14*c^17*d^2 + 240*a^3*b^13*c^8*d^1
1 - 908*a^3*b^13*c^10*d^9 + 1792*a^3*b^13*c^12*d^7 - 1932*a^3*b^13*c^14*d^5
 + 920*a^3*b^13*c^16*d^3 - 336*a^4*b^12*c^7*d^12 + 1188*a^4*b^12*c^9*d^10 -
 2808*a^4*b^12*c^11*d^8 + 3980*a^4*b^12*c^13*d^6 - 2616*a^4*b^12*c^15*d^4 +

600*a^4*b^12*c^17*d^2 + 168*a^5*b^11*c^6*d^13 - 336*a^5*b^11*c^8*d^11 + 17
40*a^5*b^11*c^10*d^9 - 4720*a^5*b^11*c^12*d^7 + 4812*a^5*b^11*c^14*d^5 - 17
52*a^5*b^11*c^16*d^3 + 168*a^6*b^10*c^5*d^14 - 1344*a^6*b^10*c^7*d^12 + 229
2*a^6*b^10*c^9*d^10 + 1088*a^6*b^10*c^11*d^8 - 4908*a^6*b^10*c^13*d^6 + 309
6*a^6*b^10*c^15*d^4 - 392*a^6*b^10*c^17*d^2 - 336*a^7*b^9*c^4*d^15 + 2520*a
^7*b^9*c^6*d^13 - 7488*a^7*b^9*c^8*d^11 + 7556*a^7*b^9*c^10*d^9 - 144*a^7*b
^9*c^12*d^7 - 3012*a^7*b^9*c^14*d^5 + 904*a^7*b^9*c^16*d^3 + 240*a^8*b^8*c^
3*d^16 - 2472*a^8*b^8*c^5*d^14 + 10416*a^8*b^8*c^7*d^12 - 16596*a^8*b^8*c^9
*d^10 + 9600*a^8*b^8*c^11*d^8 - 156*a^8*b^8*c^13*d^6 - 1032*a^8*b^8*c^15*d^
4 - 84*a^9*b^7*c^2*d^17 + 1632*a^9*b^7*c^4*d^15 - 9204*a^9*b^7*c^6*d^13 + 1
9800*a^9*b^7*c^8*d^11 - 18048*a^9*b^7*c^10*d^9 + 5856*a^9*b^7*c^12*d^7 + 48
*a^9*b^7*c^14*d^5 - 744*a^10*b^6*c^3*d^16 + 5460*a^10*b^6*c^5*d^14 - 15960*
a^10*b^6*c^7*d^12 + 20136*a^10*b^6*c^9*d^10 - 10584*a^10*b^6*c^11*d^8 + 168
0*a^10*b^6*c^13*d^6 + 212*a^11*b^5*c^2*d^17 - 2176*a^11*b^5*c^4*d^15 + 9180
*a^11*b^5*c^6*d^13 - 15416*a^11*b^5*c^8*d^11 + 10936*a^11*b^5*c^10*d^9 - 27
36*a^11*b^5*c^12*d^7 + 584*a^12*b^4*c^3*d^16 - 3708*a^12*b^4*c^5*d^14 + 815
2*a^12*b^4*c^7*d^12 - 7376*a^12*b^4*c^9*d^10 + 2376*a^12*b^4*c^11*d^8 - 108
*a^13*b^3*c^2*d^17 + 928*a^13*b^3*c^4*d^15 - 2820*a^13*b^3*c^6*d^13 + 3288*
a^13*b^3*c^8*d^11 - 1288*a^13*b^3*c^10*d^9 - 80*a^14*b^2*c^3*d^16 + 564*a^1
4*b^2*c^5*d^14 - 936*a^14*b^2*c^7*d^12 + 440*a^14*b^2*c^9*d^10 + 24*a*b^15*
c^18*d))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*
d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^1
1 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^1
3*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 3
6*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^1
6 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*
b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8
 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 8
4*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^
3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^
9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13
*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 +
1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a
^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7
*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13
*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1
308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a
^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*
c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*
d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 9
40*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9
*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^
3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*
d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9
 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) - (b^3*((8*(4*

a^2*b^16*c^22 - 8*a^4*b^14*c^22 + 4*a^6*b^12*c^22 - 4*a^18*c^2*d^20 + 16*a^18*c^4*d^18 - 24*a^18*c^6*d^16 + 16*a^18*c^8*d^14 - 4*a^18*c^10*d^12 - 4*a*b^17*c^13*d^9 + 16*a*b^17*c^15*d^7 - 24*a*b^17*c^17*d^5 + 16*a*b^17*c^19*d^3 - 32*a^3*b^15*c^21*d + 76*a^5*b^13*c^21*d - 40*a^7*b^11*c^21*d + 4*a^13*b^5*c*d^21 - 8*a^15*b^3*c*d^21 + 24*a^17*b*c^3*d^19 - 136*a^17*b*c^5*d^17 + 224*a^17*b*c^7*d^15 - 156*a^17*b*c^9*d^13 + 40*a^17*b*c^11*d^11 + 40*a^2*b^16*c^12*d^10 - 156*a^2*b^16*c^14*d^8 + 224*a^2*b^16*c^16*d^6 - 136*a^2*b^16*c^18*d^4 + 24*a^2*b^16*c^20*d^2 - 176*a^3*b^15*c^11*d^11 + 672*a^3*b^15*c^13*d^9 - 928*a^3*b^15*c^15*d^7 + 512*a^3*b^15*c^17*d^5 - 48*a^3*b^15*c^19*d^3 + 440*a^4*b^14*c^10*d^12 - 1664*a^4*b^14*c^12*d^10 + 2248*a^4*b^14*c^14*d^8 - 1152*a^4*b^14*c^16*d^6 + 8*a^4*b^14*c^18*d^4 + 128*a^4*b^14*c^20*d^2 - 660*a^5*b^13*c^9*d^13 + 2552*a^5*b^13*c^11*d^11 - 3532*a^5*b^13*c^13*d^9 + 1808*a^5*b^13*c^15*d^7 + 148*a^5*b^13*c^17*d^5 - 392*a^5*b^13*c^19*d^3 + 528*a^6*b^12*c^8*d^14 - 2332*a^6*b^12*c^10*d^12 + 3736*a^6*b^12*c^12*d^10 - 2180*a^6*b^12*c^14*d^8 - 480*a^6*b^12*c^16*d^6 + 1052*a^6*b^12*c^18*d^4 - 328*a^6*b^12*c^20*d^2 + 792*a^7*b^11*c^9*d^13 - 2464*a^7*b^11*c^11*d^11 + 1896*a^7*b^11*c^13*d^9 + 1216*a^7*b^11*c^15*d^7 - 2264*a^7*b^11*c^17*d^5 + 864*a^7*b^11*c^19*d^3 - 528*a^8*b^10*c^6*d^16 + 1056*a^8*b^10*c^8*d^14 + 176*a^8*b^10*c^10*d^12 - 528*a^8*b^10*c^12*d^10 - 2288*a^8*b^10*c^14*d^8 + 3520*a^8*b^10*c^16*d^6 - 1584*a^8*b^10*c^18*d^4 + 176*a^8*b^10*c^20*d^2 + 660*a^9*b^9*c^5*d^17 - 2112*a^9*b^9*c^7*d^15 + 2244*a^9*b^9*c^9*d^13 - 1496*a^9*b^9*c^11*d^11 + 2684*a^9*b^9*c^13*d^9 - 3696*a^9*b^9*c^15*d^7 + 2156*a^9*b^9*c^17*d^5 - 440*a^9*b^9*c^19*d^3 - 440*a^10*b^8*c^4*d^18 + 2156*a^10*b^8*c^6*d^16 - 3696*a^10*b^8*c^8*d^14 + 2684*a^10*b^8*c^10*d^12 - 1496*a^10*b^8*c^12*d^10 + 2244*a^10*b^8*c^14*d^8 - 2112*a^10*b^8*c^16*d^6 + 660*a^10*b^8*c^18*d^4 + 176*a^11*b^7*c^3*d^19 - 1584*a^11*b^7*c^5*d^17 + 3520*a^11*b^7*c^7*d^15 - 2288*a^11*b^7*c^9*d^13 - 528*a^11*b^7*c^11*d^11 + 176*a^11*b^7*c^13*d^9 + 1056*a^11*b^7*c^15*d^7 - 528*a^11*b^7*c^17*d^5 - 40*a^12*b^6*c^2*d^20 + 864*a^12*b^6*c^4*d^18 - 2264*a^12*b^6*c^6*d^16 + 1216*a^12*b^6*c^8*d^14 + 1896*a^12*b^6*c^10*d^12 - 2464*a^12*b^6*c^12*d^10 + 792*a^12*b^6*c^14*d^8 - 328*a^13*b^5*c^3*d^19 + 1052*a^13*b^5*c^5*d^17 - 480*a^13*b^5*c^7*d^15 - 2180*a^13*b^5*c^9*d^13 + 3736*a^13*b^5*c^11*d^11 - 2332*a^13*b^5*c^13*d^9 + 528*a^13*b^5*c^15*d^7 + 76*a^14*b^4*c^2*d^20 - 392*a^14*b^4*c^4*d^18 + 148*a^14*b^4*c^6*d^16 + 1808*a^14*b^4*c^8*d^14 - 3532*a^14*b^4*c^10*d^12 + 2552*a^14*b^4*c^12*d^10 - 660*a^14*b^4*c^14*d^8 + 128*a^15*b^3*c^3*d^19 + 8*a^15*b^3*c^5*d^17 - 1152*a^15*b^3*c^7*d^15 + 2248*a^15*b^3*c^9*d^13 - 1664*a^15*b^3*c^11*d^11 + 440*a^15*b^3*c^13*d^9 - 32*a^16*b^2*c^2*d^20 - 48*a^16*b^2*c^4*d^18 + 512*a^16*b^2*c^6*d^16 - 928*a^16*b^2*c^8*d^14 + 672*a^16*b^2*c^10*d^12 - 176*a^16*b^2*c^12*d^10 - 4*a*b^17*c^21*d + 4*a^17*b*c*d^21))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*

d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224
*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*
b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10
*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*
d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 +
76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a
^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8
*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d
^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 -
36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^
7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6
*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^
12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 -
126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9
*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c
^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*
d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 -
156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*
a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16) + (8*tan(e/2 + (f*x)
/2)*(12*a*b^17*c^22 - 12*a^18*c*d^21 - 32*a^3*b^15*c^22 + 28*a^5*b^13*c^22
- 8*a^7*b^11*c^22 + 56*a^18*c^3*d^19 - 104*a^18*c^5*d^17 + 96*a^18*c^7*d^15
- 44*a^18*c^9*d^13 + 8*a^18*c^11*d^11 - 16*a*b^17*c^12*d^10 + 76*a*b^17*c^
14*d^8 - 144*a*b^17*c^16*d^6 + 136*a*b^17*c^18*d^4 - 64*a*b^17*c^20*d^2 - 1
32*a^2*b^16*c^21*d + 352*a^4*b^14*c^21*d - 308*a^6*b^12*c^21*d + 88*a^8*b^1
0*c^21*d + 16*a^12*b^6*c*d^21 - 44*a^14*b^4*c*d^21 + 40*a^16*b^2*c*d^21 + 1
32*a^17*b*c^2*d^20 - 616*a^17*b*c^4*d^18 + 1144*a^17*b*c^6*d^16 - 1056*a^17
*b*c^8*d^14 + 484*a^17*b*c^10*d^12 - 88*a^17*b*c^12*d^10 + 176*a^2*b^16*c^1
1*d^11 - 836*a^2*b^16*c^13*d^9 + 1584*a^2*b^16*c^15*d^7 - 1496*a^2*b^16*c^1
7*d^5 + 704*a^2*b^16*c^19*d^3 - 880*a^3*b^15*c^10*d^12 + 4224*a^3*b^15*c^12
*d^10 - 8128*a^3*b^15*c^14*d^8 + 7872*a^3*b^15*c^16*d^6 - 3888*a^3*b^15*c^1
8*d^4 + 832*a^3*b^15*c^20*d^2 + 2640*a^4*b^14*c^9*d^13 - 13024*a^4*b^14*c^1
1*d^11 + 26048*a^4*b^14*c^13*d^9 - 26752*a^4*b^14*c^15*d^7 + 14608*a^4*b^14
*c^17*d^5 - 3872*a^4*b^14*c^19*d^3 - 5280*a^5*b^13*c^8*d^14 + 27500*a^5*b^1
3*c^10*d^12 - 59000*a^5*b^13*c^12*d^10 + 66628*a^5*b^13*c^14*d^8 - 41712*a^
5*b^13*c^16*d^6 + 13748*a^5*b^13*c^18*d^4 - 1912*a^5*b^13*c^20*d^2 + 7392*a
^6*b^12*c^7*d^15 - 42372*a^6*b^12*c^9*d^13 + 101288*a^6*b^12*c^11*d^11 - 12
9580*a^6*b^12*c^13*d^9 + 94160*a^6*b^12*c^15*d^7 - 37532*a^6*b^12*c^17*d^5
+ 6952*a^6*b^12*c^19*d^3 - 7392*a^7*b^11*c^6*d^16 + 49632*a^7*b^11*c^8*d^14
- 137368*a^7*b^11*c^10*d^12 + 202544*a^7*b^11*c^12*d^10 - 170424*a^7*b^11*
c^14*d^8 + 80448*a^7*b^11*c^16*d^6 - 19016*a^7*b^11*c^18*d^4 + 1584*a^7*b^1
1*c^20*d^2 + 5280*a^8*b^10*c^5*d^17 - 45408*a^8*b^10*c^7*d^15 + 150216*a^8*
b^10*c^9*d^13 - 257136*a^8*b^10*c^11*d^11 + 249832*a^8*b^10*c^13*d^9 - 1386
88*a^8*b^10*c^15*d^7 + 40920*a^8*b^10*c^17*d^5 - 5104*a^8*b^10*c^19*d^3 - 2
640*a^9*b^9*c^4*d^18 + 32868*a^9*b^9*c^6*d^16 - 133056*a^9*b^9*c^8*d^14 + 2
66244*a^9*b^9*c^10*d^12 - 299816*a^9*b^9*c^12*d^10 + 195404*a^9*b^9*c^14*d^

8 - 70224*a^9*b^9*c^16*d^6 + 11660*a^9*b^9*c^18*d^4 - 440*a^9*b^9*c^20*d^2 + 880*a^10*b^8*c^3*d^19 - 18700*a^10*b^8*c^5*d^17 + 95040*a^10*b^8*c^7*d^15 - 225676*a^10*b^8*c^9*d^13 + 296824*a^10*b^8*c^11*d^11 - 226116*a^10*b^8*c^13*d^9 + 96624*a^10*b^8*c^15*d^7 - 20196*a^10*b^8*c^17*d^5 + 1320*a^10*b^8*c^19*d^3 - 176*a^11*b^7*c^2*d^20 + 8096*a^11*b^7*c^4*d^18 - 54384*a^11*b^7*c^6*d^16 + 156992*a^11*b^7*c^8*d^14 - 242528*a^11*b^7*c^10*d^12 + 214368*a^11*b^7*c^12*d^10 - 107184*a^11*b^7*c^14*d^8 + 27456*a^11*b^7*c^16*d^6 - 2640*a^11*b^7*c^18*d^4 - 2496*a^12*b^6*c^3*d^19 + 24784*a^12*b^6*c^5*d^17 - 89280*a^12*b^6*c^7*d^15 + 162336*a^12*b^6*c^9*d^13 - 165760*a^12*b^6*c^11*d^11 + 96272*a^12*b^6*c^13*d^9 - 29568*a^12*b^6*c^15*d^7 + 3696*a^12*b^6*c^17*d^5 + 484*a^13*b^5*c^2*d^20 - 8888*a^13*b^5*c^4*d^18 + 40876*a^13*b^5*c^6*d^16 - 88000*a^13*b^5*c^8*d^14 + 104060*a^13*b^5*c^10*d^12 - 69784*a^13*b^5*c^12*d^10 + 24948*a^13*b^5*c^14*d^8 - 3696*a^13*b^5*c^16*d^6 + 2408*a^14*b^4*c^3*d^19 - 14692*a^14*b^4*c^5*d^17 + 38208*a^14*b^4*c^7*d^15 - 52532*a^14*b^4*c^9*d^13 + 40072*a^14*b^4*c^11*d^11 - 16060*a^14*b^4*c^13*d^9 + 2640*a^14*b^4*c^15*d^7 - 440*a^15*b^3*c^2*d^20 + 4048*a^15*b^3*c^4*d^18 - 13112*a^15*b^3*c^6*d^16 + 20768*a^15*b^3*c^8*d^14 - 17512*a^15*b^3*c^10*d^12 + 7568*a^15*b^3*c^12*d^10 - 1320*a^15*b^3*c^14*d^8 - 848*a^16*b^2*c^3*d^19 + 3432*a^16*b^2*c^5*d^17 - 6048*a^16*b^2*c^7*d^15 + 5432*a^16*b^2*c^9*d^13 - 2448*a^16*b^2*c^11*d^11 + 440*a^16*b^2*c^13*d^9))/(a^13*d^17 - b^13*c^17 + 2*a^2*b^11*c^17 - a^4*b^9*c^17 + a^9*b^4*d^17 - 2*a^11*b^2*d^17 - 4*a^13*c^2*d^15 + 6*a^13*c^4*d^13 - 4*a^13*c^6*d^11 + a^13*c^8*d^9 - b^13*c^9*d^8 + 4*b^13*c^11*d^6 - 6*b^13*c^13*d^4 + 4*b^13*c^15*d^2 + 9*a*b^12*c^8*d^9 - 36*a*b^12*c^10*d^7 + 54*a*b^12*c^12*d^5 - 36*a*b^12*c^14*d^3 - 18*a^3*b^10*c^16*d + 9*a^5*b^8*c^16*d - 9*a^8*b^5*c*d^16 + 18*a^10*b^3*c*d^16 + 36*a^12*b*c^3*d^14 - 54*a^12*b*c^5*d^12 + 36*a^12*b*c^7*d^10 - 9*a^12*b*c^9*d^8 - 36*a^2*b^11*c^7*d^10 + 146*a^2*b^11*c^9*d^8 - 224*a^2*b^11*c^11*d^6 + 156*a^2*b^11*c^13*d^4 - 44*a^2*b^11*c^15*d^2 + 84*a^3*b^10*c^6*d^11 - 354*a^3*b^10*c^8*d^9 + 576*a^3*b^10*c^10*d^7 - 444*a^3*b^10*c^12*d^5 + 156*a^3*b^10*c^14*d^3 - 126*a^4*b^9*c^5*d^12 + 576*a^4*b^9*c^7*d^10 - 1045*a^4*b^9*c^9*d^8 + 940*a^4*b^9*c^11*d^6 - 420*a^4*b^9*c^13*d^4 + 76*a^4*b^9*c^15*d^2 + 126*a^5*b^8*c^4*d^13 - 672*a^5*b^8*c^6*d^11 + 1437*a^5*b^8*c^8*d^9 - 1548*a^5*b^8*c^10*d^7 + 852*a^5*b^8*c^12*d^5 - 204*a^5*b^8*c^14*d^3 - 84*a^6*b^7*c^3*d^14 + 588*a^6*b^7*c^5*d^12 - 1548*a^6*b^7*c^7*d^10 + 1992*a^6*b^7*c^9*d^8 - 1308*a^6*b^7*c^11*d^6 + 396*a^6*b^7*c^13*d^4 - 36*a^6*b^7*c^15*d^2 + 36*a^7*b^6*c^2*d^15 - 396*a^7*b^6*c^4*d^13 + 1308*a^7*b^6*c^6*d^11 - 1992*a^7*b^6*c^8*d^9 + 1548*a^7*b^6*c^10*d^7 - 588*a^7*b^6*c^12*d^5 + 84*a^7*b^6*c^14*d^3 + 204*a^8*b^5*c^3*d^14 - 852*a^8*b^5*c^5*d^12 + 1548*a^8*b^5*c^7*d^10 - 1437*a^8*b^5*c^9*d^8 + 672*a^8*b^5*c^11*d^6 - 126*a^8*b^5*c^13*d^4 - 76*a^9*b^4*c^2*d^15 + 420*a^9*b^4*c^4*d^13 - 940*a^9*b^4*c^6*d^11 + 1045*a^9*b^4*c^8*d^9 - 576*a^9*b^4*c^10*d^7 + 126*a^9*b^4*c^12*d^5 - 156*a^10*b^3*c^3*d^14 + 444*a^10*b^3*c^5*d^12 - 576*a^10*b^3*c^7*d^10 + 354*a^10*b^3*c^9*d^8 - 84*a^10*b^3*c^11*d^6 + 44*a^11*b^2*c^2*d^15 - 156*a^11*b^2*c^4*d^13 + 224*a^11*b^2*c^6*d^11 - 146*a^11*b^2*c^8*d^9 + 36*a^11*b^2*c^10*d^7 + 9*a*b^12*c^16*d - 9*a^12*b*c*d^16))*(-(a + b)^3*(a - b)^3)^(1/2)*(3*b^2*d - 4*a^2*d +

```
a*b*c))/(a^10*d^4 - b^10*c^4 + 3*a^2*b^8*c^4 - 3*a^4*b^6*c^4 + a^6*b^4*c^4
 - a^4*b^6*d^4 + 3*a^6*b^4*d^4 - 3*a^8*b^2*d^4 + 4*a^3*b^7*c*d^3 - 12*a^3*b
^7*c^3*d - 12*a^5*b^5*c*d^3 + 12*a^5*b^5*c^3*d + 12*a^7*b^3*c*d^3 - 4*a^7*b
^3*c^3*d - 6*a^2*b^8*c^2*d^2 + 18*a^4*b^6*c^2*d^2 - 18*a^6*b^4*c^2*d^2 + 6*
a^8*b^2*c^2*d^2 + 4*a*b^9*c^3*d - 4*a^9*b*c*d^3))*(3*b^2*d - 4*a^2*d + a*b*
c))/(a^10*d^4 - b^10*c^4 + 3*a^2*b^8*c^4 - 3*a^4*b^6*c^4 + a^6*b^4*c^4 - a^
4*b^6*d^4 + 3*a^6*b^4*d^4 - 3*a^8*b^2*d^4 + 4*a^3*b^7*c*d^3 - 12*a^3*b^7*c^
3*d - 12*a^5*b^5*c*d^3 + 12*a^5*b^5*c^3*d + 12*a^7*b^3*c*d^3 - 4*a^7*b^3*c^
3*d - 6*a^2*b^8*c^2*d^2 + 18*a^4*b^6*c^2*d^2 - 18*a^6*b^4*c^2*d^2 + 6*a^8*b
^2*c^2*d^2 + 4*a*b^9*c^3*d - 4*a^9*b*c*d^3))*(3*b^2*d - 4*a^2*d + a*b*c))/(
a^10*d^4 - b^10*c^4 + 3*a^2*b^8*c^4 - 3*a^4*b^6*c^4 + a^6*b^4*c^4 - a^4*b^6
*d^4 + 3*a^6*b^4*d^4 - 3*a^8*b^2*d^4 + 4*a^3*b^7*c*d^3 - 12*a^3*b^7*c^3*d -
 12*a^5*b^5*c*d^3 + 12*a^5*b^5*c^3*d + 12*a^7*b^3*c*d^3 - 4*a^7*b^3*c^3*d -
 6*a^2*b^8*c^2*d^2 + 18*a^4*b^6*c^2*d^2 - 18*a^6*b^4*c^2*d^2 + 6*a^8*b^2*c^
2*d^2 + 4*a*b^9*c^3*d - 4*a^9*b*c*d^3)))*(-(a + b)^3*(a - b)^3)^(1/2)*(3*b^
2*d - 4*a^2*d + a*b*c)*2i)/(f*(a^10*d^4 - b^10*c^4 + 3*a^2*b^8*c^4 - 3*a^4*
b^6*c^4 + a^6*b^4*c^4 - a^4*b^6*d^4 + 3*a^6*b^4*d^4 - 3*a^8*b^2*d^4 + 4*a^3
*b^7*c*d^3 - 12*a^3*b^7*c^3*d - 12*a^5*b^5*c*d^3 + 12*a^5*b^5*c^3*d + 12*a^
7*b^3*c*d^3 - 4*a^7*b^3*c^3*d - 6*a^2*b^8*c^2*d^2 + 18*a^4*b^6*c^2*d^2 - 18
*a^6*b^4*c^2*d^2 + 6*a^8*b^2*c^2*d^2 + 4*a*b^9*c^3*d - 4*a^9*b*c*d^3))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.714 $\int \frac{(c+d\sin(e+fx))^5}{(a+b\sin(e+fx))^3}\,dx$

Optimal. Leaf size=534

$$\frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))^3}{2bf\left(a^2-b^2\right)(a+b\sin(e+fx))^2}+\frac{(bc-ad)^2\left(4a^2d+3abc-7b^2d\right)\cos(e+fx)(c+d\sin(e+fx))^2}{2b^2f\left(a^2-b^2\right)^2(a+b\sin(e+fx))}-\frac{d^3x}{}$$

```
[Out] -1/2*d^3*(30*a*b*c*d-12*a^2*d^2-b^2*(20*c^2+d^2))*x/b^5+(-a*d+b*c)^3*(6*a^3
*b*c*d-12*a*b^3*c*d+12*a^4*d^2+a^2*b^2*(2*c^2-29*d^2)+b^4*(c^2+20*d^2))*arc
tan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/b^5/(a^2-b^2)^(5/2)/f-1/2*d*(
30*a^4*b*c*d^3-12*a^5*d^4-a^3*b^2*d^2*(16*c^2-21*d^2)-b^5*c*d*(17*c^2-10*d^
2)-a^2*b^3*c*d*(4*c^2+55*d^2)+a*b^4*(6*c^4+43*c^2*d^2-6*d^4))*cos(f*x+e)/b^
4/(a^2-b^2)^2/f+1/2*d^2*(7*a^3*b*c*d^2-6*a^4*d^3+b^4*d*(8*c^2-d^2)+a^2*b^2*
d*(c^2+10*d^2)-a*b^3*c*(3*c^2+16*d^2))*cos(f*x+e)*sin(f*x+e)/b^3/(a^2-b^2)^
2/f+1/2*(-a*d+b*c)^2*(4*a^2*d+3*a*b*c-7*b^2*d)*cos(f*x+e)*(c+d*sin(f*x+e))^
2/b^2/(a^2-b^2)^2/f/(a+b*sin(f*x+e))+1/2*(-a*d+b*c)^2*cos(f*x+e)*(c+d*sin(f
*x+e))^3/b/(a^2-b^2)/f/(a+b*sin(f*x+e))^2
```

Rubi [A] time = 2.16, antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2792, 3047, 3033, 3023, 2735, 2660, 618, 204}

$$-\frac{d\left(-a^3b^2d^2\left(16c^2-21d^2\right)-a^2b^3cd\left(4c^2+55d^2\right)+30a^4bcd^3-12a^5d^4+ab^4\left(43c^2d^2+6c^4-6d^4\right)-b^5cd\left(17c^2-\right.\right.}{2b^4f\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^5/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] -(d^3*(30*a*b*c*d - 12*a^2*d^2 - b^2*(20*c^2 + d^2))*x)/(2*b^5) + ((b*c - a
*d)^3*(6*a^3*b*c*d - 12*a*b^3*c*d + 12*a^4*d^2 + a^2*b^2*(2*c^2 - 29*d^2) +
 b^4*(c^2 + 20*d^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(b^5
*(a^2 - b^2)^(5/2)*f) - (d*(30*a^4*b*c*d^3 - 12*a^5*d^4 - a^3*b^2*d^2*(16*c
^2 - 21*d^2) - b^5*c*d*(17*c^2 - 10*d^2) - a^2*b^3*c*d*(4*c^2 + 55*d^2) + a
*b^4*(6*c^4 + 43*c^2*d^2 - 6*d^4))*Cos[e + f*x])/(2*b^4*(a^2 - b^2)^2*f) +
(d^2*(7*a^3*b*c*d^2 - 6*a^4*d^3 + b^4*d*(8*c^2 - d^2) + a^2*b^2*d*(c^2 + 10
*d^2) - a*b^3*c*(3*c^2 + 16*d^2))*Cos[e + f*x]*Sin[e + f*x])/(2*b^3*(a^2 -
b^2)^2*f) + ((b*c - a*d)^2*(3*a*b*c + 4*a^2*d - 7*b^2*d)*Cos[e + f*x]*(c +
d*Sin[e + f*x])^2)/(2*b^2*(a^2 - b^2)^2*f*(a + b*Sin[e + f*x])) + ((b*c - a
```

```
*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(2*b*(a^2 - b^2)*f*(a + b*Sin[e
+ f*x])^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^5}{(a+b\sin(e+fx))^3}\,dx = \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))^3}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} - \frac{\int \frac{(c+d\sin(e+fx))^2\left(7b^2c^2d+3a^2d^3-2abc\left(c^2+4d^2\right)\right.}{}}{}$$

$$= \frac{(bc-ad)^2\left(3abc+4a^2d-7b^2d\right)\cos(e+fx)(c+d\sin(e+fx))^2}{2b^2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{(bc-ad)^2\cos(e}{2b\left(a^2-b^2\right)}$$

$$= \frac{d^2\left(7a^3bcd^2-6a^4d^3+b^4d\left(8c^2-d^2\right)+a^2b^2d\left(c^2+10d^2\right)-ab^3c\left(3c^2+16d^2\right)\right)\cos(}{2b^3\left(a^2-b^2\right)^2 f}$$

$$= -\frac{d\left(30a^4bcd^3-12a^5d^4-a^3b^2d^2\left(16c^2-21d^2\right)-b^5cd\left(17c^2-10d^2\right)-a^2b^3cd\left(4c^2\right.}{2b^4\left(a^2-b^2\right)^2 f}$$

$$= -\frac{d^3\left(30abcd-12a^2d^2-b^2\left(20c^2+d^2\right)\right)x}{2b^5} - \frac{d\left(30a^4bcd^3-12a^5d^4-a^3b^2d^2\left(16c^2\right.}{}$$

$$= -\frac{d^3\left(30abcd-12a^2d^2-b^2\left(20c^2+d^2\right)\right)x}{2b^5} - \frac{d\left(30a^4bcd^3-12a^5d^4-a^3b^2d^2\left(16c^2\right.}{}$$

$$= -\frac{d^3\left(30abcd-12a^2d^2-b^2\left(20c^2+d^2\right)\right)x}{2b^5} - \frac{d\left(30a^4bcd^3-12a^5d^4-a^3b^2d^2\left(16c^2\right.}{}$$

$$= -\frac{d^3\left(30abcd-12a^2d^2-b^2\left(20c^2+d^2\right)\right)x}{2b^5} + \frac{(bc-ad)^3\left(6a^3bcd-12ab^3cd+12a^4d^2\right.}{}$$

Mathematica [C] time = 3.82, size = 341, normalized size = 0.64

$$\frac{2d^3(e+fx)\left(12a^2d^2-30abcd+b^2\left(20c^2+d^2\right)\right)+\frac{2b(bc-ad)^4\left(7a^2d+3abc-10b^2d\right)\cos(e+fx)}{\left(a^2-b^2\right)^2(a+b\sin(e+fx))}-\frac{2b(bc-ad)^5\cos(e+fx)}{\left(b^2-a^2\right)(a+b\sin(e+fx))^2}+\frac{4(bc-}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^5/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (2*d^3*(-30*a*b*c*d + 12*a^2*d^2 + b^2*(20*c^2 + d^2))*(e + f*x) + (4*(b*c
- a*d)^3*(6*a^3*b*c*d - 12*a*b^3*c*d + 12*a^4*d^2 + a^2*b^2*(2*c^2 - 29*d^2
) + b^4*(c^2 + 20*d^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(
a^2 - b^2)^(5/2) + 2*b*d^4*(-5*b*c + 3*a*d)*(Cos[e + f*x] - I*Sin[e + f*x])
 + 2*b*d^4*(-5*b*c + 3*a*d)*(Cos[e + f*x] + I*Sin[e + f*x]) - (2*b*(b*c - a
*d)^5*Cos[e + f*x])/((-a^2 + b^2)*(a + b*Sin[e + f*x])^2) + (2*b*(b*c - a*d
)^4*(3*a*b*c + 7*a^2*d - 10*b^2*d)*Cos[e + f*x])/((a^2 - b^2)^2*(a + b*Sin[
e + f*x])) - b^2*d^5*Sin[2*(e + f*x)])/(4*b^5*f)
```

fricas [B] time = 1.25, size = 3174, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^5/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(20*(a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*c^2*d^3 - 30*(a^7*b^3
- 3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*c*d^4 + (12*a^8*b^2 - 35*a^6*b^4 + 33*a^4*
b^6 - 9*a^2*b^8 - b^10)*d^5)*f*x*cos(f*x + e)^2 - 4*(5*(a^6*b^4 - 3*a^4*b^6
 + 3*a^2*b^8 - b^10)*c*d^4 - 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*d^
5)*cos(f*x + e)^3 - 2*(20*(a^8*b^2 - 2*a^6*b^4 + 2*a^2*b^8 - b^10)*c^2*d^3
- 30*(a^9*b - 2*a^7*b^3 + 2*a^3*b^7 - a*b^9)*c*d^4 + (12*a^10 - 23*a^8*b^2
- 2*a^6*b^4 + 24*a^4*b^6 - 10*a^2*b^8 - b^10)*d^5)*f*x - ((2*a^4*b^5 + 3*a^
2*b^7 + b^9)*c^5 - 15*(a^3*b^6 + a*b^8)*c^4*d + 10*(a^4*b^5 + 3*a^2*b^7 + 2
*b^9)*c^3*d^2 - 10*(2*a^7*b^2 - 3*a^5*b^4 + a^3*b^6 + 6*a*b^8)*c^2*d^3 + 15
*(2*a^8*b - 3*a^6*b^3 - a^4*b^5 + 4*a^2*b^7)*c*d^4 - (12*a^9 - 17*a^7*b^2 -
 9*a^5*b^4 + 20*a^3*b^6)*d^5 + (15*a*b^8*c^4*d - (2*a^2*b^7 + b^9)*c^5 - 10
*(a^2*b^7 + 2*b^9)*c^3*d^2 + 10*(2*a^5*b^4 - 5*a^3*b^6 + 6*a*b^8)*c^2*d^3 -
 15*(2*a^6*b^3 - 5*a^4*b^5 + 4*a^2*b^7)*c*d^4 + (12*a^7*b^2 - 29*a^5*b^4 +
20*a^3*b^6)*d^5)*cos(f*x + e)^2 - 2*(15*a^2*b^7*c^4*d - (2*a^3*b^6 + a*b^8)
*c^5 - 10*(a^3*b^6 + 2*a*b^8)*c^3*d^2 + 10*(2*a^6*b^3 - 5*a^4*b^5 + 6*a^2*b
^7)*c^2*d^3 - 15*(2*a^7*b^2 - 5*a^5*b^4 + 4*a^3*b^6)*c*d^4 + (12*a^8*b - 29
*a^6*b^3 + 20*a^4*b^5)*d^5)*sin(f*x + e))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b
^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 - 2*(a*cos(f*x + e)*sin
(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*s
in(f*x + e) - a^2 - b^2)) - 2*((4*a^4*b^6 - 5*a^2*b^8 + b^10)*c^5 - 5*(2*a^
5*b^5 - a^3*b^7 - a*b^9)*c^4*d + 30*(a^4*b^6 - a^2*b^8)*c^3*d^2 + 10*(2*a^7
*b^3 - 7*a^5*b^5 + 5*a^3*b^7)*c^2*d^3 - 5*(6*a^8*b^2 - 15*a^6*b^4 + 7*a^4*b
^6 + 4*a^2*b^8 - 2*b^10)*c*d^4 + (12*a^9*b - 29*a^7*b^3 + 15*a^5*b^5 + 6*a^
3*b^7 - 4*a*b^9)*d^5)*cos(f*x + e) - 2*((a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 -
b^10)*d^5*cos(f*x + e)^3 + 2*(20*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*
c^2*d^3 - 30*(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8)*c*d^4 + (12*a^9*b
- 35*a^7*b^3 + 33*a^5*b^5 - 9*a^3*b^7 - a*b^9)*d^5)*f*x + (3*(a^3*b^7 - a*b
^9)*c^5 - 5*(a^4*b^6 + a^2*b^8 - 2*b^10)*c^4*d - 10*(a^5*b^5 - 5*a^3*b^7 +
4*a*b^9)*c^3*d^2 + 30*(a^6*b^4 - 3*a^4*b^6 + 2*a^2*b^8)*c^2*d^3 - 5*(9*a^7*
```

```
b^3 - 25*a^5*b^5 + 20*a^3*b^7 - 4*a*b^9)*c*d^4 + (18*a^8*b^2 - 51*a^6*b^4 +
 46*a^4*b^6 - 14*a^2*b^8 + b^10)*d^5)*cos(f*x + e))*sin(f*x + e))/((a^6*b^7
 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*f*cos(f*x + e)^2 - 2*(a^7*b^6 - 3*a^5*b^8
 + 3*a^3*b^10 - a*b^12)*f*sin(f*x + e) - (a^8*b^5 - 2*a^6*b^7 + 2*a^2*b^11
- b^13)*f), 1/2*((20*(a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*c^2*d^3 - 30*
(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*c*d^4 + (12*a^8*b^2 - 35*a^6*b^4
+ 33*a^4*b^6 - 9*a^2*b^8 - b^10)*d^5)*f*x*cos(f*x + e)^2 - 2*(5*(a^6*b^4 -
3*a^4*b^6 + 3*a^2*b^8 - b^10)*c*d^4 - 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 -
a*b^9)*d^5)*cos(f*x + e)^3 - (20*(a^8*b^2 - 2*a^6*b^4 + 2*a^2*b^8 - b^10)*c
^2*d^3 - 30*(a^9*b - 2*a^7*b^3 + 2*a^3*b^7 - a*b^9)*c*d^4 + (12*a^10 - 23*a
^8*b^2 - 2*a^6*b^4 + 24*a^4*b^6 - 10*a^2*b^8 - b^10)*d^5)*f*x + ((2*a^4*b^5
 + 3*a^2*b^7 + b^9)*c^5 - 15*(a^3*b^6 + a*b^8)*c^4*d + 10*(a^4*b^5 + 3*a^2*
b^7 + 2*b^9)*c^3*d^2 - 10*(2*a^7*b^2 - 3*a^5*b^4 + a^3*b^6 + 6*a*b^8)*c^2*d
^3 + 15*(2*a^8*b - 3*a^6*b^3 - a^4*b^5 + 4*a^2*b^7)*c*d^4 - (12*a^9 - 17*a^
7*b^2 - 9*a^5*b^4 + 20*a^3*b^6)*d^5 + (15*a*b^8*c^4*d - (2*a^2*b^7 + b^9)*c
^5 - 10*(a^2*b^7 + 2*b^9)*c^3*d^2 + 10*(2*a^5*b^4 - 5*a^3*b^6 + 6*a*b^8)*c^
2*d^3 - 15*(2*a^6*b^3 - 5*a^4*b^5 + 4*a^2*b^7)*c*d^4 + (12*a^7*b^2 - 29*a^5
*b^4 + 20*a^3*b^6)*d^5)*cos(f*x + e)^2 - 2*(15*a^2*b^7*c^4*d - (2*a^3*b^6 +
 a*b^8)*c^5 - 10*(a^3*b^6 + 2*a*b^8)*c^3*d^2 + 10*(2*a^6*b^3 - 5*a^4*b^5 +
6*a^2*b^7)*c^2*d^3 - 15*(2*a^7*b^2 - 5*a^5*b^4 + 4*a^3*b^6)*c*d^4 + (12*a^8
*b - 29*a^6*b^3 + 20*a^4*b^5)*d^5)*sin(f*x + e))*sqrt(a^2 - b^2)*arctan(-(a
*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))) - ((4*a^4*b^6 - 5*a^2*b^
8 + b^10)*c^5 - 5*(2*a^5*b^5 - a^3*b^7 - a*b^9)*c^4*d + 30*(a^4*b^6 - a^2*b
^8)*c^3*d^2 + 10*(2*a^7*b^3 - 7*a^5*b^5 + 5*a^3*b^7)*c^2*d^3 - 5*(6*a^8*b^2
 - 15*a^6*b^4 + 7*a^4*b^6 + 4*a^2*b^8 - 2*b^10)*c*d^4 + (12*a^9*b - 29*a^7*
b^3 + 15*a^5*b^5 + 6*a^3*b^7 - 4*a*b^9)*d^5)*cos(f*x + e) - ((a^6*b^4 - 3*a
^4*b^6 + 3*a^2*b^8 - b^10)*d^5*cos(f*x + e)^3 + 2*(20*(a^7*b^3 - 3*a^5*b^5
+ 3*a^3*b^7 - a*b^9)*c^2*d^3 - 30*(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^
8)*c*d^4 + (12*a^9*b - 35*a^7*b^3 + 33*a^5*b^5 - 9*a^3*b^7 - a*b^9)*d^5)*f*
x + (3*(a^3*b^7 - a*b^9)*c^5 - 5*(a^4*b^6 + a^2*b^8 - 2*b^10)*c^4*d - 10*(a
^5*b^5 - 5*a^3*b^7 + 4*a*b^9)*c^3*d^2 + 30*(a^6*b^4 - 3*a^4*b^6 + 2*a^2*b^8
)*c^2*d^3 - 5*(9*a^7*b^3 - 25*a^5*b^5 + 20*a^3*b^7 - 4*a*b^9)*c*d^4 + (18*a
^8*b^2 - 51*a^6*b^4 + 46*a^4*b^6 - 14*a^2*b^8 + b^10)*d^5)*cos(f*x + e))*si
n(f*x + e))/((a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*f*cos(f*x + e)^2 - 2
*(a^7*b^6 - 3*a^5*b^8 + 3*a^3*b^10 - a*b^12)*f*sin(f*x + e) - (a^8*b^5 - 2*
a^6*b^7 + 2*a^2*b^11 - b^13)*f)]
```

giac [B] time = 0.38, size = 3162, normalized size = 5.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^5/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(2*a^2*b^5*c^5 + b^7*c^5 - 15*a*b^6*c^4*d + 10*a^2*b^5*c^3*d^2 + 20*
```

b^7*c^3*d^2 - 20*a^5*b^2*c^2*d^3 + 50*a^3*b^4*c^2*d^3 - 60*a*b^6*c^2*d^3 + 30*a^6*b*c*d^4 - 75*a^4*b^3*c*d^4 + 60*a^2*b^5*c*d^4 - 12*a^7*d^5 + 29*a^5*b^2*d^5 - 20*a^3*b^4*d^5)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^4*b^5 - 2*a^2*b^7 + b^9)*sqrt(a^2 - b^2)) + 2*(5*a^3*b^6*c^5*tan(1/2*f*x + 1/2*e)^7 - 2*a*b^8*c^5*tan(1/2*f*x + 1/2*e)^7 - 15*a^4*b^5*c^4*d*tan(1/2*f*x + 1/2*e)^7 + 10*a^5*b^4*c^3*d^2*tan(1/2*f*x + 1/2*e)^7 + 20*a^3*b^6*c^3*d^2*tan(1/2*f*x + 1/2*e)^7 + 10*a^6*b^3*c^2*d^3*tan(1/2*f*x + 1/2*e)^7 - 40*a^4*b^5*c^2*d^3*tan(1/2*f*x + 1/2*e)^7 - 15*a^7*b^2*c*d^4*tan(1/2*f*x + 1/2*e)^7 + 30*a^5*b^4*c*d^4*tan(1/2*f*x + 1/2*e)^7 + 6*a^8*b*d^5*tan(1/2*f*x + 1/2*e)^7 - 10*a^6*b^3*d^5*tan(1/2*f*x + 1/2*e)^7 + a^4*b^5*d^5*tan(1/2*f*x + 1/2*e)^7 + 4*a^4*b^5*c^5*tan(1/2*f*x + 1/2*e)^6 + 7*a^2*b^7*c^5*tan(1/2*f*x + 1/2*e)^6 - 2*b^9*c^5*tan(1/2*f*x + 1/2*e)^6 - 10*a^5*b^4*c^4*d*tan(1/2*f*x + 1/2*e)^6 - 25*a^3*b^6*c^4*d*tan(1/2*f*x + 1/2*e)^6 - 10*a*b^8*c^4*d*tan(1/2*f*x + 1/2*e)^6 + 30*a^4*b^5*c^3*d^2*tan(1/2*f*x + 1/2*e)^6 + 60*a^2*b^7*c^3*d^2*tan(1/2*f*x + 1/2*e)^6 + 20*a^7*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^6 - 10*a^5*b^4*c^2*d^3*tan(1/2*f*x + 1/2*e)^6 - 100*a^3*b^6*c^2*d^3*tan(1/2*f*x + 1/2*e)^6 - 30*a^8*b*c*d^4*tan(1/2*f*x + 1/2*e)^6 + 15*a^6*b^3*c*d^4*tan(1/2*f*x + 1/2*e)^6 + 60*a^4*b^5*c*d^4*tan(1/2*f*x + 1/2*e)^6 + 12*a^9*d^5*tan(1/2*f*x + 1/2*e)^6 - 5*a^7*b^2*d^5*tan(1/2*f*x + 1/2*e)^6 - 20*a^5*b^4*d^5*tan(1/2*f*x + 1/2*e)^6 + 4*a^3*b^6*d^5*tan(1/2*f*x + 1/2*e)^6 + 21*a^3*b^6*c^5*tan(1/2*f*x + 1/2*e)^5 - 6*a*b^8*c^5*tan(1/2*f*x + 1/2*e)^5 - 55*a^4*b^5*c^4*d*tan(1/2*f*x + 1/2*e)^5 - 20*a^2*b^7*c^4*d*tan(1/2*f*x + 1/2*e)^5 + 10*a^5*b^4*c^3*d^2*tan(1/2*f*x + 1/2*e)^5 + 140*a^3*b^6*c^3*d^2*tan(1/2*f*x + 1/2*e)^5 + 90*a^6*b^3*c^2*d^3*tan(1/2*f*x + 1/2*e)^5 - 240*a^4*b^5*c^2*d^3*tan(1/2*f*x + 1/2*e)^5 - 135*a^7*b^2*c*d^4*tan(1/2*f*x + 1/2*e)^5 + 250*a^5*b^4*c*d^4*tan(1/2*f*x + 1/2*e)^5 - 40*a^3*b^6*c*d^4*tan(1/2*f*x + 1/2*e)^5 + 54*a^8*b*d^5*tan(1/2*f*x + 1/2*e)^5 - 90*a^6*b^3*d^5*tan(1/2*f*x + 1/2*e)^5 + 17*a^4*b^5*d^5*tan(1/2*f*x + 1/2*e)^5 + 4*a^2*b^7*d^5*tan(1/2*f*x + 1/2*e)^5 + 12*a^4*b^5*c^5*tan(1/2*f*x + 1/2*e)^4 + 13*a^2*b^7*c^5*tan(1/2*f*x + 1/2*e)^4 - 4*b^9*c^5*tan(1/2*f*x + 1/2*e)^4 - 30*a^5*b^4*c^4*d*tan(1/2*f*x + 1/2*e)^4 - 55*a^3*b^6*c^4*d*tan(1/2*f*x + 1/2*e)^4 - 20*a*b^8*c^4*d*tan(1/2*f*x + 1/2*e)^4 + 90*a^4*b^5*c^3*d^2*tan(1/2*f*x + 1/2*e)^4 + 120*a^2*b^7*c^3*d^2*tan(1/2*f*x + 1/2*e)^4 + 60*a^7*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^4 - 70*a^5*b^4*c^2*d^3*tan(1/2*f*x + 1/2*e)^4 - 200*a^3*b^6*c^2*d^3*tan(1/2*f*x + 1/2*e)^4 - 90*a^8*b*c*d^4*tan(1/2*f*x + 1/2*e)^4 + 45*a^6*b^3*c*d^4*tan(1/2*f*x + 1/2*e)^4 + 190*a^4*b^5*c*d^4*tan(1/2*f*x + 1/2*e)^4 - 40*a^2*b^7*c*d^4*tan(1/2*f*x + 1/2*e)^4 + 36*a^9*d^5*tan(1/2*f*x + 1/2*e)^4 - 15*a^7*b^2*d^5*tan(1/2*f*x + 1/2*e)^4 - 66*a^5*b^4*d^5*tan(1/2*f*x + 1/2*e)^4 + 24*a^3*b^6*d^5*tan(1/2*f*x + 1/2*e)^4 + 27*a^3*b^6*c^5*tan(1/2*f*x + 1/2*e)^3 - 6*a*b^8*c^5*tan(1/2*f*x + 1/2*e)^3 - 65*a^4*b^5*c^4*d*tan(1/2*f*x + 1/2*e)^3 - 40*a^2*b^7*c^4*d*tan(1/2*f*x + 1/2*e)^3 - 10*a^5*b^4*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 + 220*a^3*b^6*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 + 150*a^6*b^3*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 - 360*a^4*b^5*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 - 225*a^7*b^2*c*d^4*tan(1/2*f*x + 1/2*e)^3 + 410*a^5*b^4*c*d^4*tan(1/2*f*x +

```
1/2*e)^3 - 80*a^3*b^6*c*d^4*tan(1/2*f*x + 1/2*e)^3 + 90*a^8*b*d^5*tan(1/2*f
*x + 1/2*e)^3 - 162*a^6*b^3*d^5*tan(1/2*f*x + 1/2*e)^3 + 55*a^4*b^5*d^5*tan
(1/2*f*x + 1/2*e)^3 - 4*a^2*b^7*d^5*tan(1/2*f*x + 1/2*e)^3 + 12*a^4*b^5*c^5
*tan(1/2*f*x + 1/2*e)^2 + 5*a^2*b^7*c^5*tan(1/2*f*x + 1/2*e)^2 - 2*b^9*c^5*
tan(1/2*f*x + 1/2*e)^2 - 30*a^5*b^4*c^4*d*tan(1/2*f*x + 1/2*e)^2 - 35*a^3*b
^6*c^4*d*tan(1/2*f*x + 1/2*e)^2 - 10*a*b^8*c^4*d*tan(1/2*f*x + 1/2*e)^2 + 9
0*a^4*b^5*c^3*d^2*tan(1/2*f*x + 1/2*e)^2 + 60*a^2*b^7*c^3*d^2*tan(1/2*f*x +
 1/2*e)^2 + 60*a^7*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^2 - 110*a^5*b^4*c^2*d^3
*tan(1/2*f*x + 1/2*e)^2 - 100*a^3*b^6*c^2*d^3*tan(1/2*f*x + 1/2*e)^2 - 90*a
^8*b*c*d^4*tan(1/2*f*x + 1/2*e)^2 + 85*a^6*b^3*c*d^4*tan(1/2*f*x + 1/2*e)^2
 + 120*a^4*b^5*c*d^4*tan(1/2*f*x + 1/2*e)^2 - 40*a^2*b^7*c*d^4*tan(1/2*f*x
+ 1/2*e)^2 + 36*a^9*d^5*tan(1/2*f*x + 1/2*e)^2 - 31*a^7*b^2*d^5*tan(1/2*f*x
 + 1/2*e)^2 - 40*a^5*b^4*d^5*tan(1/2*f*x + 1/2*e)^2 + 20*a^3*b^6*d^5*tan(1/
2*f*x + 1/2*e)^2 + 11*a^3*b^6*c^5*tan(1/2*f*x + 1/2*e) - 2*a*b^8*c^5*tan(1/
2*f*x + 1/2*e) - 25*a^4*b^5*c^4*d*tan(1/2*f*x + 1/2*e) - 20*a^2*b^7*c^4*d*t
an(1/2*f*x + 1/2*e) - 10*a^5*b^4*c^3*d^2*tan(1/2*f*x + 1/2*e) + 100*a^3*b^6
*c^3*d^2*tan(1/2*f*x + 1/2*e) + 70*a^6*b^3*c^2*d^3*tan(1/2*f*x + 1/2*e) - 1
60*a^4*b^5*c^2*d^3*tan(1/2*f*x + 1/2*e) - 105*a^7*b^2*c*d^4*tan(1/2*f*x + 1
/2*e) + 190*a^5*b^4*c*d^4*tan(1/2*f*x + 1/2*e) - 40*a^3*b^6*c*d^4*tan(1/2*f
*x + 1/2*e) + 42*a^8*b*d^5*tan(1/2*f*x + 1/2*e) - 74*a^6*b^3*d^5*tan(1/2*f*
x + 1/2*e) + 23*a^4*b^5*d^5*tan(1/2*f*x + 1/2*e) + 4*a^4*b^5*c^5 - a^2*b^7*
c^5 - 10*a^5*b^4*c^4*d - 5*a^3*b^6*c^4*d + 30*a^4*b^5*c^3*d^2 + 20*a^7*b^2*
c^2*d^3 - 50*a^5*b^4*c^2*d^3 - 30*a^8*b*c*d^4 + 55*a^6*b^3*c*d^4 - 10*a^4*b
^5*c*d^4 + 12*a^9*d^5 - 21*a^7*b^2*d^5 + 6*a^5*b^4*d^5)/((a^6*b^4 - 2*a^4*b
^6 + a^2*b^8)*(a*tan(1/2*f*x + 1/2*e)^4 + 2*b*tan(1/2*f*x + 1/2*e)^3 + 2*a*
tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)^2) + (20*b^2*c^2*d^3
 - 30*a*b*c*d^4 + 12*a^2*d^5 + b^2*d^5)*(f*x + e)/b^5)/f
```

maple [B] time = 0.36, size = 4767, normalized size = 8.93

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^5/(a+b*sin(f*x+e))^3,x)
```

```
[Out] 1/f*d^5/b^3*arctan(tan(1/2*f*x+1/2*e))-15/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan
(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+1/2*e)^3*c^4*d-4
0/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^
4)*a^2*tan(1/2*f*x+1/2*e)^3*c^2*d^3+20/f*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(
1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*tan(1/2*f*x+1/2*e)^3*c^3*d^2-20
/f/b^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b
^4)*a^6*tan(1/2*f*x+1/2*e)^2*c*d^4+20/f/b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1
/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^5*tan(1/2*f*x+1/2*e)^2*c^2*d^3-5
/f/b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4
)*a^4*tan(1/2*f*x+1/2*e)^2*c*d^4+30/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f
```

*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+1/2*e)^2*c^3*d^2+100/f
*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a/(a^4-2*a^2*b^2+b
^4)*tan(1/2*f*x+1/2*e)*c^3*d^2-160/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*
x+1/2*e)*b+a)^2*a^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c^2*d^3+10/f/b/(
tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^4*
tan(1/2*f*x+1/2*e)^3*c^2*d^3-15/f/b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x
+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^5*tan(1/2*f*x+1/2*e)^3*c*d^4+70/f/b/(t
an(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a^4/(a^4-2*a^2*b^2+b^4)*t
an(1/2*f*x+1/2*e)*c^2*d^3-25/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*
e)*b+a)^2*a^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c^4*d+30/f/b^4/(a^4-2*
a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b
^2)^(1/2))*a^6*c*d^4-20/f/b^3/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/
2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^5*c^2*d^3-75/f/b^2/(a^4-2
*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-
b^2)^(1/2))*a^4*c*d^4+50/f/b/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2
*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^3*c^2*d^3-15/f*b/(a^4-2*a^
2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2
)^(1/2))*a*c^4*d-60/f*b/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a
*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a*c^2*d^3+70/f*b/(tan(1/2*f*x+1/2
*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+1/2
*e)^2*c*d^4-25/f*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a
^4-2*a^2*b^2+b^4)*a*tan(1/2*f*x+1/2*e)^2*c^4*d-100/f*b^2/(tan(1/2*f*x+1/2*e
)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*tan(1/2*f*x+1/2*e)^
2*c^2*d^3-10/f*b^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4
-2*a^2*b^2+b^4)/a*tan(1/2*f*x+1/2*e)^2*c^4*d-65/f/b^2/(tan(1/2*f*x+1/2*e)^2
*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a^5/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c
*d^4+60/f/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1
/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*c*d^4-2/f*b^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan
(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)/a*tan(1/2*f*x+1/2*e)^3*c^5+6/f/b
^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*
a^7*tan(1/2*f*x+1/2*e)^2*d^5+3/f/b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+
1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^5*tan(1/2*f*x+1/2*e)^2*d^5+10/f/(tan(1/
2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*tan(1/
2*f*x+1/2*e)^3*c^3*d^2+30/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+
a)^2/(a^4-2*a^2*b^2+b^4)*a^3*tan(1/2*f*x+1/2*e)^3*c*d^4-10/f/(tan(1/2*f*x+1
/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*tan(1/2*f*x+1
/2*e)^2*c^4*d-10/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4
-2*a^2*b^2+b^4)*a^3*tan(1/2*f*x+1/2*e)^2*c^2*d^3-10/f/(tan(1/2*f*x+1/2*e)^2
*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a^3/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c
^3*d^2+110/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a^3/(a^4-2
*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c*d^4+4/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(
1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+1/2*e)^2*c^5-2/f*
b^5/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)
/a^2*tan(1/2*f*x+1/2*e)^2*c^5+19/f/b^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*
x+1/2*e)*b+a)^2*a^6/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*d^5-28/f/b/(tan(

```
1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a^4/(a^4-2*a^2*b^2+b^4)*tan(
1/2*f*x+1/2*e)*d^5+11/f*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+
a)^2*a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c^5-2/f*b^4/(tan(1/2*f*x+1/2*
e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)
*c^5-20/f/b^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^
2*b^2+b^4)*a^6*c*d^4+20/f/b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*
b+a)^2/(a^4-2*a^2*b^2+b^4)*a^5*c^2*d^3+35/f/b/(tan(1/2*f*x+1/2*e)^2*a+2*tan
(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^4*c*d^4+60/f*b^3/(tan(1/2*f*x+
1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*
e)^2*c^3*d^2-20/f*b^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(
a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c^4*d+2/f/(a^4-2*a^2*b^2+b^4)/(a^2-b^
2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*c^5-1
0/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)
*a^3*c^4*d-50/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*
a^2*b^2+b^4)*a^3*c^2*d^3+6/f*d^5/b^4/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x
+1/2*e)^2*a-10/f*d^4/b^3/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*c-
30/f*d^4/b^4*arctan(tan(1/2*f*x+1/2*e))*a*c-1/f*b^3/(tan(1/2*f*x+1/2*e)^2*a
+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*c^5+1/f*d^5/b^3/(1+tan(1/2
*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^3-1/f*d^5/b^3/(1+tan(1/2*f*x+1/2*e)^2)^
2*tan(1/2*f*x+1/2*e)+6/f*d^5/b^4/(1+tan(1/2*f*x+1/2*e)^2)^2*a-10/f*d^4/b^3/
(1+tan(1/2*f*x+1/2*e)^2)^2*c+12/f*d^5/b^5*arctan(tan(1/2*f*x+1/2*e))*a^2+20
/f*d^3/b^3*arctan(tan(1/2*f*x+1/2*e))*c^2-18/f/(tan(1/2*f*x+1/2*e)^2*a+2*ta
n(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*tan(1/2*f*x+1/2*e)^2*d^5+6/
f/b^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^
4)*a^7*d^5-9/f/b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4
-2*a^2*b^2+b^4)*a^5*d^5+4/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*
b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*c^5+7/f*b^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/
2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)^2*c^5+1/f*b^2/(a
^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(
a^2-b^2)^(1/2))*c^5+20/f*b^2/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2
*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*c^3*d^2-12/f/b^5/(a^4-2*a^2*
b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^
(1/2))*a^7*d^5+29/f/b^3/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a
*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^5*d^5-20/f/b/(a^4-2*a^2*b^2+b^4
)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*
a^3*d^5+10/f/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*
x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*c^3*d^2+30/f*b/(tan(1/2*f*x+1/2*e)^2*a+2
*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*c^3*d^2-5/f*b^2/(tan(1/2
*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*c^4*d+5/f
/b^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4
)*a^6*tan(1/2*f*x+1/2*e)^3*d^5-8/f/b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+
1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^4*tan(1/2*f*x+1/2*e)^3*d^5+5/f*b^2/(tan
(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*tan(1
/2*f*x+1/2*e)^3*c^5
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^5/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 25.76, size = 23910, normalized size = 44.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^5/(a + b*sin(e + f*x))^3,x)
```

```
[Out] - ((b^7*c^5 - 12*a^7*d^5 - 4*a^2*b^5*c^5 - 6*a^3*b^4*d^5 + 21*a^5*b^2*d^5 +
 10*a^2*b^5*c*d^4 + 10*a^3*b^4*c^4*d - 55*a^4*b^3*c*d^4 - 30*a^2*b^5*c^3*d^
2 + 50*a^3*b^4*c^2*d^3 - 20*a^5*b^2*c^2*d^3 + 5*a*b^6*c^4*d + 30*a^6*b*c*d^
4)/(b^4*(a^4 + b^4 - 2*a^2*b^2)) - (tan(e/2 + (f*x)/2)^6*(12*a^9*d^5 - 2*b^
9*c^5 + 7*a^2*b^7*c^5 + 4*a^4*b^5*c^5 + 4*a^3*b^6*d^5 - 20*a^5*b^4*d^5 - 5*
a^7*b^2*d^5 - 25*a^3*b^6*c^4*d + 60*a^4*b^5*c*d^4 - 10*a^5*b^4*c^4*d + 15*a
^6*b^3*c*d^4 + 60*a^2*b^7*c^3*d^2 - 100*a^3*b^6*c^2*d^3 + 30*a^4*b^5*c^3*d^
2 - 10*a^5*b^4*c^2*d^3 + 20*a^7*b^2*c^2*d^3 - 10*a*b^8*c^4*d - 30*a^8*b*c*d
^4))/(a^2*b^4*(a^4 + b^4 - 2*a^2*b^2)) + (tan(e/2 + (f*x)/2)^2*(2*b^9*c^5 -
 36*a^9*d^5 - 5*a^2*b^7*c^5 - 12*a^4*b^5*c^5 - 20*a^3*b^6*d^5 + 40*a^5*b^4*
d^5 + 31*a^7*b^2*d^5 + 40*a^2*b^7*c*d^4 + 35*a^3*b^6*c^4*d - 120*a^4*b^5*c*
d^4 + 30*a^5*b^4*c^4*d - 85*a^6*b^3*c*d^4 - 60*a^2*b^7*c^3*d^2 + 100*a^3*b^
6*c^2*d^3 - 90*a^4*b^5*c^3*d^2 + 110*a^5*b^4*c^2*d^3 - 60*a^7*b^2*c^2*d^3 +
 10*a*b^8*c^4*d + 90*a^8*b*c*d^4))/(a^2*b^4*(a^4 + b^4 - 2*a^2*b^2)) - (tan
(e/2 + (f*x)/2)^5*(54*a^7*d^5 - 6*b^7*c^5 + 4*a*b^6*d^5 + 21*a^2*b^5*c^5 +
17*a^3*b^4*d^5 - 90*a^5*b^2*d^5 - 40*a^2*b^5*c*d^4 - 55*a^3*b^4*c^4*d + 250
*a^4*b^3*c*d^4 + 140*a^2*b^5*c^3*d^2 - 240*a^3*b^4*c^2*d^3 + 10*a^4*b^3*c^3
*d^2 + 90*a^5*b^2*c^2*d^3 - 20*a*b^6*c^4*d - 135*a^6*b*c*d^4))/(a*b^3*(a^4
+ b^4 - 2*a^2*b^2)) + (tan(e/2 + (f*x)/2)^3*(6*b^7*c^5 - 90*a^7*d^5 + 4*a*b
^6*d^5 - 27*a^2*b^5*c^5 - 55*a^3*b^4*d^5 + 162*a^5*b^2*d^5 + 80*a^2*b^5*c*d
^4 + 65*a^3*b^4*c^4*d - 410*a^4*b^3*c*d^4 - 220*a^2*b^5*c^3*d^2 + 360*a^3*b
^4*c^2*d^3 + 10*a^4*b^3*c^3*d^2 - 150*a^5*b^2*c^2*d^3 + 40*a*b^6*c^4*d + 22
5*a^6*b*c*d^4))/(a*b^3*(a^4 + b^4 - 2*a^2*b^2)) - (tan(e/2 + (f*x)/2)^7*(6*
a^7*d^5 - 2*b^7*c^5 + 5*a^2*b^5*c^5 + a^3*b^4*d^5 - 10*a^5*b^2*d^5 - 15*a^3
*b^4*c^4*d + 30*a^4*b^3*c*d^4 + 20*a^2*b^5*c^3*d^2 - 40*a^3*b^4*c^2*d^3 + 1
```

0*a^4*b^3*c^3*d^2 + 10*a^5*b^2*c^2*d^3 - 15*a^6*b*c*d^4))/(a*b^3*(a^4 + b^4 - 2*a^2*b^2)) + (tan(e/2 + (f*x)/2)*(2*b^7*c^5 - 42*a^7*d^5 - 11*a^2*b^5*c^5 - 23*a^3*b^4*d^5 + 74*a^5*b^2*d^5 + 40*a^2*b^5*c*d^4 + 25*a^3*b^4*c^4*d - 190*a^4*b^3*c*d^4 - 100*a^2*b^5*c^3*d^2 + 160*a^3*b^4*c^2*d^3 + 10*a^4*b^3*c^3*d^2 - 70*a^5*b^2*c^2*d^3 + 20*a*b^6*c^4*d + 105*a^6*b*c*d^4))/(a*b^3*(a^4 + b^4 - 2*a^2*b^2)) + (tan(e/2 + (f*x)/2)^4*(3*a^2 + 4*b^2)*(b^7*c^5 - 12*a^7*d^5 - 4*a^2*b^5*c^5 - 6*a^3*b^4*d^5 + 21*a^5*b^2*d^5 + 10*a^2*b^5*c*d^4 + 10*a^3*b^4*c^4*d - 55*a^4*b^3*c*d^4 - 30*a^2*b^5*c^3*d^2 + 50*a^3*b^4*c^2*d^3 - 20*a^5*b^2*c^2*d^3 + 5*a*b^6*c^4*d + 30*a^6*b*c*d^4))/(a^2*b^4*(a^4 + b^4 - 2*a^2*b^2)))/(f*(tan(e/2 + (f*x)/2)^2*(4*a^2 + 4*b^2) + tan(e/2 + (f*x)/2)^6*(4*a^2 + 4*b^2) + tan(e/2 + (f*x)/2)^4*(6*a^2 + 8*b^2) + a^2*tan(e/2 + (f*x)/2)^8 + a^2 + 12*a*b*tan(e/2 + (f*x)/2)^3 + 12*a*b*tan(e/2 + (f*x)/2)^5 + 4*a*b*tan(e/2 + (f*x)/2)^7 + 4*a*b*tan(e/2 + (f*x)/2))) - (a tan(((((4*(2*a^2*b^16*d^10 + 40*a^4*b^14*d^10 + 108*a^6*b^12*d^10 - 872*a^8*b^10*d^10 + 1538*a^10*b^8*d^10 - 1104*a^12*b^6*d^10 + 288*a^14*b^4*d^10 - 120*a^3*b^15*c*d^9 - 960*a^5*b^13*c*d^9 + 5040*a^7*b^11*c*d^9 - 8160*a^9*b^9*c*d^9 + 5640*a^11*b^7*c*d^9 - 1440*a^13*b^5*c*d^9 + 80*a^2*b^16*c^2*d^8 + 800*a^2*b^16*c^4*d^6 - 2400*a^3*b^15*c^3*d^7 + 2440*a^4*b^14*c^2*d^8 - 3200*a^4*b^14*c^4*d^6 + 9600*a^5*b^13*c^3*d^7 - 10560*a^6*b^12*c^2*d^8 + 4800*a^6*b^12*c^4*d^6 - 14400*a^7*b^11*c^3*d^7 + 16240*a^8*b^10*c^2*d^8 - 3200*a^8*b^10*c^4*d^6 + 9600*a^9*b^9*c^3*d^7 - 10960*a^10*b^8*c^2*d^8 + 800*a^10*b^8*c^4*d^6 - 2400*a^11*b^7*c^3*d^7 + 2760*a^12*b^6*c^2*d^8))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) - (8*tan(e/2 + (f*x)/2)*(a*b^18*c^10 - 2*a*b^18*d^10 + 4*a^3*b^16*c^10 + 4*a^5*b^14*c^10 - 39*a^3*b^16*d^10 - 88*a^5*b^14*d^10 + 1326*a^7*b^12*d^10 - 3134*a^9*b^10*d^10 + 3194*a^11*b^8*d^10 - 1536*a^13*b^6*d^10 + 288*a^15*b^4*d^10 - 80*a*b^18*c^2*d^8 - 800*a*b^18*c^4*d^6 + 400*a*b^18*c^6*d^4 + 40*a*b^18*c^8*d^2 + 120*a^2*b^17*c*d^9 - 30*a^2*b^17*c^9*d + 900*a^4*b^15*c*d^9 - 60*a^4*b^15*c^9*d - 7920*a^6*b^13*c*d^9 + 17160*a^8*b^11*c*d^9 - 16710*a^10*b^9*c*d^9 + 7800*a^12*b^7*c*d^9 - 1440*a^14*b^5*c*d^9 + 2400*a^2*b^17*c^3*d^7 - 2400*a^2*b^17*c^5*d^5 - 720*a^2*b^17*c^7*d^3 - 2400*a^3*b^16*c^2*d^8 + 9600*a^3*b^16*c^4*d^6 + 2320*a^3*b^16*c^6*d^4 + 325*a^3*b^16*c^8*d^2 - 18800*a^4*b^15*c^3*d^7 - 1040*a^4*b^15*c^5*d^5 - 440*a^4*b^15*c^7*d^3 + 17780*a^5*b^14*c^2*d^8 - 13600*a^5*b^14*c^4*d^6 - 1310*a^5*b^14*c^6*d^4 + 40*a^5*b^14*c^8*d^2 + 34960*a^6*b^13*c^3*d^7 + 2428*a^6*b^13*c^5*d^5 + 160*a^6*b^13*c^7*d^3 - 36000*a^7*b^12*c^2*d^8 + 9330*a^7*b^12*c^4*d^6 + 360*a^7*b^12*c^6*d^4 - 30200*a^8*b^11*c^3*d^7 - 1208*a^8*b^11*c^5*d^5 - 80*a^8*b^11*c^7*d^3 + 33445*a^9*b^10*c^2*d^8 - 3440*a^9*b^10*c^4*d^6 + 120*a^9*b^10*c^6*d^4 + 12960*a^10*b^9*c^3*d^7 - 48*a^10*b^9*c^5*d^5 - 15100*a^11*b^8*c^2*d^8 + 800*a^11*b^8*c^4*d^6 - 2400*a^12*b^7*c^3*d^7 + 2760*a^13*b^6*c^2*d^8))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12) + ((a^2*d^5*6i + (b^2*d^3*(20*c^2 + d^2)*1i)/2 - a*b*c*d^4*15i)*((8*tan(e/2 + (f*x)/2)*(4*a*b^21*c^5 - 12*a^5*b^17*c^5 + 8*a^7*b^15*c^5 - 80*a^4*b^18*d^5 + 276*a^6*b^16*d^5 - 360*a^8*b^14*d^5 + 212*a^10*b^12*d^5 - 48*a^12*b^10*d^5 + 80*a*b^21*c^3*d^2 - 60*a^2*b^20*c^4*d + 240*a^3*b^19*c*d^4 + 120*a^4*b^18*c^4*d - 780*a^5*b^17*c*d^4 - 60*a^6*b^16*c^4 *d + 960*a^7*b^15*c*d^4 - 540*a^9*b^13*c*d^4 + 120*a^11*b^11*c*d^4 - 240*a^2*b^20*c^2*d^3 - 120*a^3*b^19*c^3*d^2 + 680*a^4*b^18*c^2*d^3 - 720*a^6*b^16*c^2*d^3 + 40*a^7*b^15*c^3*d^2 + 360*a^8*b^14*c^2*d^3 - 80*a^10*b^12*c^2*d^3))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12) - (4*(4*a*b^20*d^5 - 4*a^2*b^19*c^5 + 12*a^6*b^15*c^5 - 8*a^8*b^13*c^5 + 28*a^3*b^18*d^5 - 120*a^5*b^16*d^5 + 164*a^7*b^14*d^5 - 100*a^9*b^12*d^5 + 24*a^11*b^10*d^5 + 80*a*b^20*c^2*d^3 - 120*a^2*b^19*c*d^4 + 60*a^3*b^18*c^4*d + 360*a^4*b^17*c*d^4 - 120*a^5*b^16*c^4*d - 420*a^6*b^15*c*d^4 + 60*a^7*b^14*c^4*d + 240*a^8*b^13*c*d^4 - 60*a^10*b^11*c*d^4 - 80*a^2*b^19*c^3*d^2 - 160*a^3*b^18*c^2*d^3 + 120*a^4*b^17*c^3*d^2 + 120*a^5*b^16*c^2*d^3 - 80*a^7*b^14*c^2*d^3 - 40*a^8*b^13*c^3*d^2 + 40*a^9*b^12*c^2*d^3))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) + (((4*(8*a^2*b^22 - 32*a^4*b^20 + 48*a^6*b^18 - 32*a^8*b^16 + 8*a^10*b^14))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) + (8*tan(e/2 + (f*x)/2)*(12*a*b^24 - 56*a^3*b^22 + 104*a^5*b^20 - 96*a^7*b^18 + 44*a^9*b^16 - 8*a^11*b^14))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12))*(a^2*d^5*6i + (b^2*d^3*(20*c^2 + d^2)*1i)/2 - a*b*c*d^4*15i))/b^5))/b^5)*(a^2*d^5*6i + (b^2*d^3*(20*c^2 + d^2)*1i)/2 - a*b*c*d^4*15i)*1i)/b^5 + (((4*(2*a^2*b^16*d^10 + 40*a^4*b^14*d^10 + 108*a^6*b^12*d^10 - 872*a^8*b^10*d^10 + 1538*a^10*b^8*d^10 - 1104*a^12*b^6*d^10 + 288*a^14*b^4*d^10 - 120*a^3*b^15*c*d^9 - 960*a^5*b^13*c*d^9 + 5040*a^7*b^11*c*d^9 - 8160*a^9*b^9*c*d^9 + 5640*a^11*b^7*c*d^9 - 1440*a^13*b^5*c*d^9 + 80*a^2*b^16*c^2*d^8 + 800*a^2*b^16*c^4*d^6 - 2400*a^3*b^15*c^3*d^7 + 2440*a^4*b^14*c^2*d^8 - 3200*a^4*b^14*c^4*d^6 + 9600*a^5*b^13*c^3*d^7 - 10560*a^6*b^12*c^2*d^8 + 4800*a^6*b^12*c^4*d^6 - 14400*a^7*b^11*c^3*d^7 + 16240*a^8*b^10*c^2*d^8 - 3200*a^8*b^10*c^4*d^6 + 9600*a^9*b^9*c^3*d^7 - 10960*a^10*b^8*c^2*d^8 + 800*a^10*b^8*c^4*d^6 - 2400*a^11*b^7*c^3*d^7 + 2760*a^12*b^6*c^2*d^8))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) - (8*tan(e/2 + (f*x)/2)*(a*b^18*c^10 - 2*a*b^18*d^10 + 4*a^3*b^16*c^10 + 4*a^5*b^14*c^10 - 39*a^3*b^16*d^10 - 88*a^5*b^14*d^10 + 1326*a^7*b^12*d^10 - 3134*a^9*b^10*d^10 + 3194*a^11*b^8*d^10 - 1536*a^13*b^6*d^10 + 288*a^15*b^4*d^10 - 80*a*b^18*c^2*d^8 - 800*a*b^18*c^4*d^6 + 400*a*b^18*c^6*d^4 + 40*a*b^18*c^8*d^2 + 120*a^2*b^17*c*d^9 - 30*a^2*b^17*c^9*d + 900*a^4*b^15*c*d^9 - 60*a^4*b^15*c^9*d - 7920*a^6*b^13*c*d^9 + 17160*a^8*b^11*c*d^9 - 16710*a^10*b^9*c*d^9 + 7800*a^12*b^7*c*d^9 - 1440*a^14*b^5*c*d^9 + 2400*a^2*b^17*c^3*d^7 - 2400*a^2*b^17*c^5*d^5 - 720*a^2*b^17*c^7*d^3 - 2400*a^3*b^16*c^2*d^8 + 9600*a^3*b^16*c^4*d^6 + 2320*a^3*b^16*c^6*d^4 + 325*a^3*b^16*c^8*d^2 - 18800*a^4*b^15*c^3*d^7 - 1040*a^4*b^15*c^5*d^5 - 440*a^4*b^15*c^7*d^3 + 17780*a^5*b^14*c^2*d^8 - 13600*a^5*b^14*c^4*d^6 - 1310*a^5*b^14*c^6*d^4 + 40*a^5*b^14*c^8*d^2 + 34960*a^6*b^13*c^3*d^7 + 2428*a^6*b^13*c^5*d^5 + 160*a^6*b^13*c^7*d^3 - 36000*a^7*b^12*c^2*d^8 + 9330*a^7*b^12*c^4*d^6 + 360*a^7*b^12*c^6*d^4 - 30200*a^8*b^11*c^3*d^7 - 1208*a^8*b^11*c^5*d^5 - 80*a^8*b^11*c^7*d^3 + 33445*a^9*b^10*c^2*d^8 - 3440*a^9*b^10*c^4*d^6 + 120*a^9*b^10*c^6*d^4 + 12960*a^10*b^9*c^3*d^7 - 48*a^10*b^9*c^5*d^5 - 15100*a^11*b^8*c^2*d^8 + 800*a^11*b^8*c^4*d^6 - 2400*a^12*b^7*c^3*d^7 + 2760*a^13*b^6*c^2*d^8))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12) + ((a^2*d^5*6i + (b^2*d^3*(20*c^2

+ d^2)*1i)/2 - a*b*c*d^4*15i)*((4*(4*a*b^20*d^5 - 4*a^2*b^19*c^5 + 12*a^6*b^15*c^5 - 8*a^8*b^13*c^5 + 28*a^3*b^18*d^5 - 120*a^5*b^16*d^5 + 164*a^7*b^14*d^5 - 100*a^9*b^12*d^5 + 24*a^11*b^10*d^5 + 80*a*b^20*c^2*d^3 - 120*a^2*b^19*c*d^4 + 60*a^3*b^18*c^4*d + 360*a^4*b^17*c*d^4 - 120*a^5*b^16*c^4*d - 420*a^6*b^15*c*d^4 + 60*a^7*b^14*c^4*d + 240*a^8*b^13*c*d^4 - 60*a^10*b^11*c*d^4 - 80*a^2*b^19*c^3*d^2 - 160*a^3*b^18*c^2*d^3 + 120*a^4*b^17*c^3*d^2 + 120*a^5*b^16*c^2*d^3 - 80*a^7*b^14*c^2*d^3 - 40*a^8*b^13*c^3*d^2 + 40*a^9*b^12*c^2*d^3))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) - (8*tan(e/2 + (f*x)/2)*(4*a*b^21*c^5 - 12*a^5*b^17*c^5 + 8*a^7*b^15*c^5 - 80*a^4*b^18*d^5 + 276*a^6*b^16*d^5 - 360*a^8*b^14*d^5 + 212*a^10*b^12*d^5 - 48*a^12*b^10*d^5 + 80*a*b^21*c^3*d^2 - 60*a^2*b^20*c^4*d + 240*a^3*b^19*c*d^4 + 120*a^4*b^18*c^4*d - 780*a^5*b^17*c*d^4 - 60*a^6*b^16*c^4*d + 960*a^7*b^15*c*d^4 - 540*a^9*b^13*c*d^4 + 120*a^11*b^11*c*d^4 - 240*a^2*b^20*c^2*d^3 - 120*a^3*b^19*c^3*d^2 + 680*a^4*b^18*c^2*d^3 - 720*a^6*b^16*c^2*d^3 + 40*a^7*b^15*c^3*d^2 + 360*a^8*b^14*c^2*d^3 - 80*a^10*b^12*c^2*d^3))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12) + (((4*(8*a^2*b^22 - 32*a^4*b^20 + 48*a^6*b^18 - 32*a^8*b^16 + 8*a^10*b^14))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) + (8*tan(e/2 + (f*x)/2)*(12*a*b^24 - 56*a^3*b^22 + 104*a^5*b^20 - 96*a^7*b^18 + 44*a^9*b^16 - 8*a^11*b^14))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12))*(a^2*d^5*6i + (b^2*d^3*(20*c^2 + d^2)*1i)/2 - a*b*c*d^4*15i))/b^5))/b^5)*(a^2*d^5*6i + (b^2*d^3*(20*c^2 + d^2)*1i)/2 - a*b*c*d^4*15i)*1i)/b^5)/((8*(2326*a^9*b^6*d^15 - 20*a^5*b^10*d^15 - 11*a^7*b^8*d^15 - 864*a^15*d^15 - 4770*a^11*b^4*d^15 + 3456*a^13*b^2*d^15 + 400*a*b^14*c^6*d^9 + 8040*a*b^14*c^8*d^7 + 801*a*b^14*c^10*d^5 + 20*a*b^14*c^12*d^3 + 60*a^4*b^11*c*d^14 + 45*a^6*b^9*c*d^14 - 19860*a^8*b^7*c*d^14 + 38835*a^10*b^5*c*d^14 - 27000*a^12*b^3*c*d^14 + 20*a^2*b^13*c^3*d^12 - 1599*a^2*b^13*c^5*d^10 - 52680*a^2*b^13*c^7*d^8 - 15230*a^2*b^13*c^9*d^6 - 630*a^2*b^13*c^11*d^4 - 60*a^3*b^12*c^2*d^13 + 2385*a^3*b^12*c^4*d^11 + 150460*a^3*b^12*c^6*d^9 + 61605*a^3*b^12*c^8*d^7 + 7416*a^3*b^12*c^10*d^5 + 80*a^3*b^12*c^12*d^3 - 1550*a^4*b^11*c^3*d^12 - 246516*a^4*b^11*c^5*d^10 - 92100*a^4*b^11*c^7*d^8 - 18970*a^4*b^11*c^9*d^6 - 1320*a^4*b^11*c^11*d^4 + 330*a^5*b^10*c^2*d^13 + 255870*a^5*b^10*c^4*d^11 - 2490*a^5*b^10*c^6*d^9 + 2940*a^5*b^10*c^8*d^7 + 2652*a^5*b^10*c^10*d^5 + 80*a^5*b^10*c^12*d^3 - 174080*a^6*b^9*c^3*d^12 + 206889*a^6*b^9*c^5*d^10 + 46620*a^6*b^9*c^7*d^8 + 80*a^6*b^9*c^9*d^6 - 120*a^6*b^9*c^11*d^4 + 76440*a^7*b^8*c^2*d^13 - 335925*a^7*b^8*c^4*d^11 - 44620*a^7*b^8*c^6*d^9 + 480*a^7*b^8*c^8*d^7 + 48*a^7*b^8*c^10*d^5 + 281510*a^8*b^7*c^3*d^12 - 60342*a^8*b^7*c^5*d^10 - 15180*a^8*b^7*c^7*d^8 - 800*a^8*b^7*c^9*d^6 - 139125*a^9*b^6*c^2*d^13 + 167580*a^9*b^6*c^4*d^11 + 25220*a^9*b^6*c^6*d^9 + 2400*a^9*b^6*c^8*d^7 - 167550*a^10*b^5*c^3*d^12 - 5928*a^10*b^5*c^5*d^10 - 2760*a^10*b^5*c^7*d^8 + 91080*a^11*b^4*c^2*d^13 - 24840*a^11*b^4*c^4*d^11 + 1440*a^11*b^4*c^6*d^9 + 33660*a^12*b^3*c^3*d^12 - 288*a^12*b^3*c^5*d^10 - 20520*a^13*b^2*c^2*d^13 + 6480*a^14*b*c*d^14))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) + (16*tan(e/2 + (f*x)/2)*(7829*a^10*b^6*d^15 - 20*a^4*b^12*d^15 - 411*a^6*b^10*d^15 - 1314*a^8*b^8*d^15 - 1728*a^16*d^15 - 11700*a^12*b^4*d^15 + 7344*a^14*b^2*d

^15 + 20*a*b^15*c^3*d^12 + 801*a*b^15*c^5*d^10 + 8040*a*b^15*c^7*d^8 + 400*
a*b^15*c^9*d^6 + 60*a^3*b^13*c*d^14 + 2445*a^5*b^11*c*d^14 + 14460*a^7*b^9*
c*d^14 - 66735*a^9*b^7*c*d^14 + 92970*a^11*b^5*c*d^14 - 56160*a^13*b^3*c*d^
14 - 60*a^2*b^14*c^2*d^13 - 3615*a^2*b^14*c^4*d^11 - 48660*a^2*b^14*c^6*d^9
 - 7200*a^2*b^14*c^8*d^7 + 6450*a^3*b^13*c^3*d^12 + 123324*a^3*b^13*c^5*d^1
0 + 7380*a^3*b^13*c^7*d^8 - 5670*a^4*b^12*c^2*d^13 - 168930*a^4*b^12*c^4*d^
11 + 83780*a^4*b^12*c^6*d^9 + 12000*a^4*b^12*c^8*d^7 + 134160*a^5*b^11*c^3*
d^12 - 314259*a^5*b^11*c^5*d^10 - 36120*a^5*b^11*c^7*d^8 - 1200*a^5*b^11*c^
9*d^6 - 61080*a^6*b^10*c^2*d^13 + 509145*a^6*b^10*c^4*d^11 - 31020*a^6*b^10
*c^6*d^9 - 2400*a^6*b^10*c^8*d^7 - 458210*a^7*b^9*c^3*d^12 + 291630*a^7*b^9
*c^5*d^10 + 17940*a^7*b^9*c^7*d^8 + 800*a^7*b^9*c^9*d^6 + 237870*a^8*b^8*c^
2*d^13 - 565440*a^8*b^8*c^4*d^11 + 5340*a^8*b^8*c^6*d^9 - 2400*a^8*b^8*c^8*
d^7 + 558240*a^9*b^7*c^3*d^12 - 137784*a^9*b^7*c^5*d^10 + 2760*a^9*b^7*c^7*
d^8 - 310560*a^10*b^6*c^2*d^13 + 297240*a^10*b^6*c^4*d^11 - 9440*a^10*b^6*c
^6*d^9 - 310860*a^11*b^5*c^3*d^12 + 36288*a^11*b^5*c^5*d^10 + 180540*a^12*b
^4*c^2*d^13 - 68400*a^12*b^4*c^4*d^11 + 70200*a^13*b^3*c^3*d^12 - 41040*a^1
4*b^2*c^2*d^13 + 12960*a^15*b*c*d^14))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*
a^6*b^14 + a^8*b^12) + (((4*(2*a^2*b^16*d^10 + 40*a^4*b^14*d^10 + 108*a^6*b
^12*d^10 - 872*a^8*b^10*d^10 + 1538*a^10*b^8*d^10 - 1104*a^12*b^6*d^10 + 28
8*a^14*b^4*d^10 - 120*a^3*b^15*c*d^9 - 960*a^5*b^13*c*d^9 + 5040*a^7*b^11*c
*d^9 - 8160*a^9*b^9*c*d^9 + 5640*a^11*b^7*c*d^9 - 1440*a^13*b^5*c*d^9 + 80*
a^2*b^16*c^2*d^8 + 800*a^2*b^16*c^4*d^6 - 2400*a^3*b^15*c^3*d^7 + 2440*a^4*
b^14*c^2*d^8 - 3200*a^4*b^14*c^4*d^6 + 9600*a^5*b^13*c^3*d^7 - 10560*a^6*b^
12*c^2*d^8 + 4800*a^6*b^12*c^4*d^6 - 14400*a^7*b^11*c^3*d^7 + 16240*a^8*b^1
0*c^2*d^8 - 3200*a^8*b^10*c^4*d^6 + 9600*a^9*b^9*c^3*d^7 - 10960*a^10*b^8*c
^2*d^8 + 800*a^10*b^8*c^4*d^6 - 2400*a^11*b^7*c^3*d^7 + 2760*a^12*b^6*c^2*d
^8))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) - (8*tan(e/2
 + (f*x)/2)*(a*b^18*c^10 - 2*a*b^18*d^10 + 4*a^3*b^16*c^10 + 4*a^5*b^14*c^10
 - 39*a^3*b^16*d^10 - 88*a^5*b^14*d^10 + 1326*a^7*b^12*d^10 - 3134*a^9*b^10
*d^10 + 3194*a^11*b^8*d^10 - 1536*a^13*b^6*d^10 + 288*a^15*b^4*d^10 - 80*a*
b^18*c^2*d^8 - 800*a*b^18*c^4*d^6 + 400*a*b^18*c^6*d^4 + 40*a*b^18*c^8*d^2
 + 120*a^2*b^17*c*d^9 - 30*a^2*b^17*c^9*d + 900*a^4*b^15*c*d^9 - 60*a^4*b^15
*c^9*d - 7920*a^6*b^13*c*d^9 + 17160*a^8*b^11*c*d^9 - 16710*a^10*b^9*c*d^9
 + 7800*a^12*b^7*c*d^9 - 1440*a^14*b^5*c*d^9 + 2400*a^2*b^17*c^3*d^7 - 2400*
a^2*b^17*c^5*d^5 - 720*a^2*b^17*c^7*d^3 - 2400*a^3*b^16*c^2*d^8 + 9600*a^3*
b^16*c^4*d^6 + 2320*a^3*b^16*c^6*d^4 + 325*a^3*b^16*c^8*d^2 - 18800*a^4*b^1
5*c^3*d^7 - 1040*a^4*b^15*c^5*d^5 - 440*a^4*b^15*c^7*d^3 + 17780*a^5*b^14*c
^2*d^8 - 13600*a^5*b^14*c^4*d^6 - 1310*a^5*b^14*c^6*d^4 + 40*a^5*b^14*c^8*d
^2 + 34960*a^6*b^13*c^3*d^7 + 2428*a^6*b^13*c^5*d^5 + 160*a^6*b^13*c^7*d^3
 - 36000*a^7*b^12*c^2*d^8 + 9330*a^7*b^12*c^4*d^6 + 360*a^7*b^12*c^6*d^4 - 3
0200*a^8*b^11*c^3*d^7 - 1208*a^8*b^11*c^5*d^5 - 80*a^8*b^11*c^7*d^3 + 33445
*a^9*b^10*c^2*d^8 - 3440*a^9*b^10*c^4*d^6 + 120*a^9*b^10*c^6*d^4 + 12960*a^
10*b^9*c^3*d^7 - 48*a^10*b^9*c^5*d^5 - 15100*a^11*b^8*c^2*d^8 + 800*a^11*b^
8*c^4*d^6 - 2400*a^12*b^7*c^3*d^7 + 2760*a^13*b^6*c^2*d^8))/(b^20 - 4*a^2*b
^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12) + ((a^2*d^5*6i + (b^2*d^3*(20*c^2

+ d^2)*1i)/2 - a*b*c*d^4*15i)*((8*tan(e/2 + (f*x)/2)*(4*a*b^21*c^5 - 12*a^5*b^17*c^5 + 8*a^7*b^15*c^5 - 80*a^4*b^18*d^5 + 276*a^6*b^16*d^5 - 360*a^8*b^14*d^5 + 212*a^10*b^12*d^5 - 48*a^12*b^10*d^5 + 80*a*b^21*c^3*d^2 - 60*a^2*b^20*c^4*d + 240*a^3*b^19*c*d^4 + 120*a^4*b^18*c^4*d - 780*a^5*b^17*c*d^4 - 60*a^6*b^16*c^4*d + 960*a^7*b^15*c*d^4 - 540*a^9*b^13*c*d^4 + 120*a^11*b^11*c*d^4 - 240*a^2*b^20*c^2*d^3 - 120*a^3*b^19*c^3*d^2 + 680*a^4*b^18*c^2*d^3 - 720*a^6*b^16*c^2*d^3 + 40*a^7*b^15*c^3*d^2 + 360*a^8*b^14*c^2*d^3 - 80*a^10*b^12*c^2*d^3))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12) - (4*(4*a*b^20*d^5 - 4*a^2*b^19*c^5 + 12*a^6*b^15*c^5 - 8*a^8*b^13*c^5 + 28*a^3*b^18*d^5 - 120*a^5*b^16*d^5 + 164*a^7*b^14*d^5 - 100*a^9*b^12*d^5 + 24*a^11*b^10*d^5 + 80*a*b^20*c^2*d^3 - 120*a^2*b^19*c*d^4 + 60*a^3*b^18*c^4*d + 360*a^4*b^17*c*d^4 - 120*a^5*b^16*c^4*d - 420*a^6*b^15*c*d^4 + 60*a^7*b^14*c^4*d + 240*a^8*b^13*c*d^4 - 60*a^10*b^11*c*d^4 - 80*a^2*b^19*c^3*d^2 - 160*a^3*b^18*c^2*d^3 + 120*a^4*b^17*c^3*d^2 + 120*a^5*b^16*c^2*d^3 - 80*a^7*b^14*c^2*d^3 - 40*a^8*b^13*c^3*d^2 + 40*a^9*b^12*c^2*d^3))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) + (((4*(8*a^2*b^22 - 32*a^4*b^20 + 48*a^6*b^18 - 32*a^8*b^16 + 8*a^10*b^14))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) + (8*tan(e/2 + (f*x)/2)*(12*a*b^24 - 56*a^3*b^22 + 104*a^5*b^20 - 96*a^7*b^18 + 44*a^9*b^16 - 8*a^11*b^14))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12))*(a^2*d^5*6i + (b^2*d^3*(20*c^2 + d^2)*1i)/2 - a*b*c*d^4*15i))/b^5))/b^5)*(a^2*d^5*6i + (b^2*d^3*(20*c^2 + d^2)*1i)/2 - a*b*c*d^4*15i))/b^5 - (((4*(2*a^2*b^16*d^10 + 40*a^4*b^14*d^10 + 108*a^6*b^12*d^10 - 872*a^8*b^10*d^10 + 1538*a^10*b^8*d^10 - 1104*a^12*b^6*d^10 + 288*a^14*b^4*d^10 - 120*a^3*b^15*c*d^9 - 960*a^5*b^13*c*d^9 + 5040*a^7*b^11*c*d^9 - 8160*a^9*b^9*c*d^9 + 5640*a^11*b^7*c*d^9 - 1440*a^13*b^5*c*d^9 + 80*a^2*b^16*c^2*d^8 + 800*a^2*b^16*c^4*d^6 - 2400*a^3*b^15*c^3*d^7 + 2440*a^4*b^14*c^2*d^8 - 3200*a^4*b^14*c^4*d^6 + 9600*a^5*b^13*c^3*d^7 - 10560*a^6*b^12*c^2*d^8 + 4800*a^6*b^12*c^4*d^6 - 14400*a^7*b^11*c^3*d^7 + 16240*a^8*b^10*c^2*d^8 - 3200*a^8*b^10*c^4*d^6 + 9600*a^9*b^9*c^3*d^7 - 10960*a^10*b^8*c^2*d^8 + 800*a^10*b^8*c^4*d^6 - 2400*a^11*b^7*c^3*d^7 + 2760*a^12*b^6*c^2*d^8))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) - (8*tan(e/2 + (f*x)/2)*(a*b^18*c^10 - 2*a*b^18*d^10 + 4*a^3*b^16*c^10 + 4*a^5*b^14*c^10 - 39*a^3*b^16*d^10 - 88*a^5*b^14*d^10 + 1326*a^7*b^12*d^10 - 3134*a^9*b^10*d^10 + 3194*a^11*b^8*d^10 - 1536*a^13*b^6*d^10 + 288*a^15*b^4*d^10 - 80*a*b^18*c^2*d^8 - 800*a*b^18*c^4*d^6 + 400*a*b^18*c^6*d^4 + 40*a*b^18*c^8*d^2 + 120*a^2*b^17*c*d^9 - 30*a^2*b^17*c^9*d + 900*a^4*b^15*c*d^9 - 60*a^4*b^15*c^9*d - 7920*a^6*b^13*c*d^9 + 17160*a^8*b^11*c*d^9 - 16710*a^10*b^9*c*d^9 + 7800*a^12*b^7*c*d^9 - 1440*a^14*b^5*c*d^9 + 2400*a^2*b^17*c^3*d^7 - 2400*a^2*b^17*c^5*d^5 - 720*a^2*b^17*c^7*d^3 - 2400*a^3*b^16*c^2*d^8 + 9600*a^3*b^16*c^4*d^6 + 2320*a^3*b^16*c^6*d^4 + 325*a^3*b^16*c^8*d^2 - 18800*a^4*b^15*c^3*d^7 - 1040*a^4*b^15*c^5*d^5 - 440*a^4*b^15*c^7*d^3 + 17780*a^5*b^14*c^2*d^8 - 13600*a^5*b^14*c^4*d^6 - 1310*a^5*b^14*c^6*d^4 + 40*a^5*b^14*c^8*d^2 + 34960*a^6*b^13*c^3*d^7 + 2428*a^6*b^13*c^5*d^5 + 160*a^6*b^13*c^7*d^3 - 36000*a^7*b^12*c^2*d^8 + 9330*a^7*b^12*c^4*d^6 + 360*a^7*b^12*c^6*d^4 - 30200*a^8*b^11*c^3*d^7 - 1208*a^8*b^11*c^5*d^5 - 80*a^8*b^11*c

^7*d^3 + 33445*a^9*b^10*c^2*d^8 - 3440*a^9*b^10*c^4*d^6 + 120*a^9*b^10*c^6*d^4 + 12960*a^10*b^9*c^3*d^7 - 48*a^10*b^9*c^5*d^5 - 15100*a^11*b^8*c^2*d^8 + 800*a^11*b^8*c^4*d^6 - 2400*a^12*b^7*c^3*d^7 + 2760*a^13*b^6*c^2*d^8))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12) + ((a^2*d^5*6i + (b^2*d^3*(20*c^2 + d^2)*1i)/2 - a*b*c*d^4*15i)*((4*(4*a*b^20*d^5 - 4*a^2*b^19*c^5 + 12*a^6*b^15*c^5 - 8*a^8*b^13*c^5 + 28*a^3*b^18*d^5 - 120*a^5*b^16*d^5 + 164*a^7*b^14*d^5 - 100*a^9*b^12*d^5 + 24*a^11*b^10*d^5 + 80*a*b^20*c^2*d^3 - 120*a^2*b^19*c*d^4 + 60*a^3*b^18*c^4*d + 360*a^4*b^17*c*d^4 - 120*a^5*b^16*c^4*d - 420*a^6*b^15*c*d^4 + 60*a^7*b^14*c^4*d + 240*a^8*b^13*c*d^4 - 60*a^10*b^11*c*d^4 - 80*a^2*b^19*c^3*d^2 - 160*a^3*b^18*c^2*d^3 + 120*a^4*b^17*c^3*d^2 + 120*a^5*b^16*c^2*d^3 - 80*a^7*b^14*c^2*d^3 - 40*a^8*b^13*c^3*d^2 + 40*a^9*b^12*c^2*d^3))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) - (8*tan(e/2 + (f*x)/2)*(4*a*b^21*c^5 - 12*a^5*b^17*c^5 + 8*a^7*b^15*c^5 - 80*a^4*b^18*d^5 + 276*a^6*b^16*d^5 - 360*a^8*b^14*d^5 + 212*a^10*b^12*d^5 - 48*a^12*b^10*d^5 + 80*a*b^21*c^3*d^2 - 60*a^2*b^20*c^4*d + 240*a^3*b^19*c*d^4 + 120*a^4*b^18*c^4*d - 780*a^5*b^17*c*d^4 - 60*a^6*b^16*c^4*d + 960*a^7*b^15*c*d^4 - 540*a^9*b^13*c*d^4 + 120*a^11*b^11*c*d^4 - 240*a^2*b^20*c^2*d^3 - 120*a^3*b^19*c^3*d^2 + 680*a^4*b^18*c^2*d^3 - 720*a^6*b^16*c^2*d^3 + 40*a^7*b^15*c^3*d^2 + 360*a^8*b^14*c^2*d^3 - 80*a^10*b^12*c^2*d^3))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12) + (((4*(8*a^2*b^22 - 32*a^4*b^20 + 48*a^6*b^18 - 32*a^8*b^16 + 8*a^10*b^14))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) + (8*tan(e/2 + (f*x)/2)*(12*a*b^24 - 56*a^3*b^22 + 104*a^5*b^20 - 96*a^7*b^18 + 44*a^9*b^16 - 8*a^11*b^14))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12))*(a^2*d^5*6i + (b^2*d^3*(20*c^2 + d^2)*1i)/2 - a*b*c*d^4*15i))/b^5))/b^5)*(a^2*d^5*6i + (b^2*d^3*(20*c^2 + d^2)*1i)/2 - a*b*c*d^4*15i))/b^5))*(a^2*d^5*6i + (b^2*d^3*(20*c^2 + d^2)*1i)/2 - a*b*c*d^4*15i)*2i)/(b^5*f) - (atan((((a*d - b*c)^3*(-(a + b)^5*(a - b)^5)^(1/2)*((4*(2*a^2*b^16*d^10 + 40*a^4*b^14*d^10 + 108*a^6*b^12*d^10 - 872*a^8*b^10*d^10 + 1538*a^10*b^8*d^10 - 1104*a^12*b^6*d^10 + 288*a^14*b^4*d^10 - 120*a^3*b^15*c*d^9 - 960*a^5*b^13*c*d^9 + 5040*a^7*b^11*c*d^9 - 8160*a^9*b^9*c*d^9 + 5640*a^11*b^7*c*d^9 - 1440*a^13*b^5*c*d^9 + 80*a^2*b^16*c^2*d^8 + 800*a^2*b^16*c^4*d^6 - 2400*a^3*b^15*c^3*d^7 + 2440*a^4*b^14*c^2*d^8 - 3200*a^4*b^14*c^4*d^6 + 9600*a^5*b^13*c^3*d^7 - 10560*a^6*b^12*c^2*d^8 + 4800*a^6*b^12*c^4*d^6 - 14400*a^7*b^11*c^3*d^7 + 16240*a^8*b^10*c^2*d^8 - 3200*a^8*b^10*c^4*d^6 + 9600*a^9*b^9*c^3*d^7 - 10960*a^10*b^8*c^2*d^8 + 800*a^10*b^8*c^4*d^6 - 2400*a^11*b^7*c^3*d^7 + 2760*a^12*b^6*c^2*d^8))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) - (8*tan(e/2 + (f*x)/2)*(a*b^18*c^10 - 2*a*b^18*d^10 + 4*a^3*b^16*c^10 + 4*a^5*b^14*c^10 - 39*a^3*b^16*d^10 - 88*a^5*b^14*d^10 + 1326*a^7*b^12*d^10 - 3134*a^9*b^10*d^10 + 3194*a^11*b^8*d^10 - 1536*a^13*b^6*d^10 + 288*a^15*b^4*d^10 - 80*a*b^18*c^2*d^8 - 800*a*b^18*c^4*d^6 + 400*a*b^18*c^6*d^4 + 40*a*b^18*c^8*d^2 + 120*a^2*b^17*c*d^9 - 30*a^2*b^17*c^9*d + 900*a^4*b^15*c*d^9 - 60*a^4*b^15*c^9*d - 7920*a^6*b^13*c*d^9 + 17160*a^8*b^11*c*d^9 - 16710*a^10*b^9*c*d^9 + 7800*a^12*b^7*c*d^9 - 1440*a^14*b^5*c*d^9 + 2400*a^2*b^17*c^3*d^7 - 2400*a^2*b^17*c^5*d^5 - 720*a^2*b^17*c^7*d^3 - 2400*a^3*b^16*c^2*d^8 + 9600*a

^3*b^16*c^4*d^6 + 2320*a^3*b^16*c^6*d^4 + 325*a^3*b^16*c^8*d^2 - 18800*a^4*
b^15*c^3*d^7 - 1040*a^4*b^15*c^5*d^5 - 440*a^4*b^15*c^7*d^3 + 17780*a^5*b^1
4*c^2*d^8 - 13600*a^5*b^14*c^4*d^6 - 1310*a^5*b^14*c^6*d^4 + 40*a^5*b^14*c^
8*d^2 + 34960*a^6*b^13*c^3*d^7 + 2428*a^6*b^13*c^5*d^5 + 160*a^6*b^13*c^7*d
^3 - 36000*a^7*b^12*c^2*d^8 + 9330*a^7*b^12*c^4*d^6 + 360*a^7*b^12*c^6*d^4
- 30200*a^8*b^11*c^3*d^7 - 1208*a^8*b^11*c^5*d^5 - 80*a^8*b^11*c^7*d^3 + 33
445*a^9*b^10*c^2*d^8 - 3440*a^9*b^10*c^4*d^6 + 120*a^9*b^10*c^6*d^4 + 12960
*a^10*b^9*c^3*d^7 - 48*a^10*b^9*c^5*d^5 - 15100*a^11*b^8*c^2*d^8 + 800*a^11
*b^8*c^4*d^6 - 2400*a^12*b^7*c^3*d^7 + 2760*a^13*b^6*c^2*d^8))/(b^20 - 4*a^
2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12) + ((a*d - b*c)^3*(-(a + b)^5*(
a - b)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(4*a*b^21*c^5 - 12*a^5*b^17*c^5 + 8*
a^7*b^15*c^5 - 80*a^4*b^18*d^5 + 276*a^6*b^16*d^5 - 360*a^8*b^14*d^5 + 212*
a^10*b^12*d^5 - 48*a^12*b^10*d^5 + 80*a*b^21*c^3*d^2 - 60*a^2*b^20*c^4*d +
240*a^3*b^19*c*d^4 + 120*a^4*b^18*c^4*d - 780*a^5*b^17*c*d^4 - 60*a^6*b^16*
c^4*d + 960*a^7*b^15*c*d^4 - 540*a^9*b^13*c*d^4 + 120*a^11*b^11*c*d^4 - 240
*a^2*b^20*c^2*d^3 - 120*a^3*b^19*c^3*d^2 + 680*a^4*b^18*c^2*d^3 - 720*a^6*b
^16*c^2*d^3 + 40*a^7*b^15*c^3*d^2 + 360*a^8*b^14*c^2*d^3 - 80*a^10*b^12*c^2
*d^3))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12) - (4*(4*a*b
^20*d^5 - 4*a^2*b^19*c^5 + 12*a^6*b^15*c^5 - 8*a^8*b^13*c^5 + 28*a^3*b^18*d
^5 - 120*a^5*b^16*d^5 + 164*a^7*b^14*d^5 - 100*a^9*b^12*d^5 + 24*a^11*b^10*
d^5 + 80*a*b^20*c^2*d^3 - 120*a^2*b^19*c*d^4 + 60*a^3*b^18*c^4*d + 360*a^4*
b^17*c*d^4 - 120*a^5*b^16*c^4*d - 420*a^6*b^15*c*d^4 + 60*a^7*b^14*c^4*d +
240*a^8*b^13*c*d^4 - 60*a^10*b^11*c*d^4 - 80*a^2*b^19*c^3*d^2 - 160*a^3*b^1
8*c^2*d^3 + 120*a^4*b^17*c^3*d^2 + 120*a^5*b^16*c^2*d^3 - 80*a^7*b^14*c^2*d
^3 - 40*a^8*b^13*c^3*d^2 + 40*a^9*b^12*c^2*d^3))/(b^19 - 4*a^2*b^17 + 6*a^4
*b^15 - 4*a^6*b^13 + a^8*b^11) + (((4*(8*a^2*b^22 - 32*a^4*b^20 + 48*a^6*b^
18 - 32*a^8*b^16 + 8*a^10*b^14))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^
13 + a^8*b^11) + (8*tan(e/2 + (f*x)/2)*(12*a*b^24 - 56*a^3*b^22 + 104*a^5*b
^20 - 96*a^7*b^18 + 44*a^9*b^16 - 8*a^11*b^14))/(b^20 - 4*a^2*b^18 + 6*a^4*
b^16 - 4*a^6*b^14 + a^8*b^12))*(a*d - b*c)^3*(-(a + b)^5*(a - b)^5)^(1/2)*(
12*a^4*d^2 + b^4*c^2 + 20*b^4*d^2 + 2*a^2*b^2*c^2 - 29*a^2*b^2*d^2 - 12*a*b
^3*c*d + 6*a^3*b*c*d))/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5
*a^8*b^7 - a^10*b^5)))*(12*a^4*d^2 + b^4*c^2 + 20*b^4*d^2 + 2*a^2*b^2*c^2 -
 29*a^2*b^2*d^2 - 12*a*b^3*c*d + 6*a^3*b*c*d))/(2*(b^15 - 5*a^2*b^13 + 10*a
^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)))*(12*a^4*d^2 + b^4*c^2 + 20*b
^4*d^2 + 2*a^2*b^2*c^2 - 29*a^2*b^2*d^2 - 12*a*b^3*c*d + 6*a^3*b*c*d)*1i)/(
2*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)) +
((a*d - b*c)^3*(-(a + b)^5*(a - b)^5)^(1/2)*((4*(2*a^2*b^16*d^10 + 40*a^4*b
^14*d^10 + 108*a^6*b^12*d^10 - 872*a^8*b^10*d^10 + 1538*a^10*b^8*d^10 - 110
4*a^12*b^6*d^10 + 288*a^14*b^4*d^10 - 120*a^3*b^15*c*d^9 - 960*a^5*b^13*c*d
^9 + 5040*a^7*b^11*c*d^9 - 8160*a^9*b^9*c*d^9 + 5640*a^11*b^7*c*d^9 - 1440*
a^13*b^5*c*d^9 + 80*a^2*b^16*c^2*d^8 + 800*a^2*b^16*c^4*d^6 - 2400*a^3*b^15
*c^3*d^7 + 2440*a^4*b^14*c^2*d^8 - 3200*a^4*b^14*c^4*d^6 + 9600*a^5*b^13*c^
3*d^7 - 10560*a^6*b^12*c^2*d^8 + 4800*a^6*b^12*c^4*d^6 - 14400*a^7*b^11*c^3
*d^7 + 16240*a^8*b^10*c^2*d^8 - 3200*a^8*b^10*c^4*d^6 + 9600*a^9*b^9*c^3*d^

7 - 10960*a^10*b^8*c^2*d^8 + 800*a^10*b^8*c^4*d^6 - 2400*a^11*b^7*c^3*d^7 + 2760*a^12*b^6*c^2*d^8))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) - (8*tan(e/2 + (f*x)/2)*(a*b^18*c^10 - 2*a*b^18*d^10 + 4*a^3*b^16*c^10 + 4*a^5*b^14*c^10 - 39*a^3*b^16*d^10 - 88*a^5*b^14*d^10 + 1326*a^7*b^12*d^10 - 3134*a^9*b^10*d^10 + 3194*a^11*b^8*d^10 - 1536*a^13*b^6*d^10 + 288*a^15*b^4*d^10 - 80*a*b^18*c^2*d^8 - 800*a*b^18*c^4*d^6 + 400*a*b^18*c^6*d^4 + 40*a*b^18*c^8*d^2 + 120*a^2*b^17*c*d^9 - 30*a^2*b^17*c^9*d + 900*a^4*b^15*c*d^9 - 60*a^4*b^15*c^9*d - 7920*a^6*b^13*c*d^9 + 17160*a^8*b^11*c*d^9 - 16710*a^10*b^9*c*d^9 + 7800*a^12*b^7*c*d^9 - 1440*a^14*b^5*c*d^9 + 2400*a^2*b^17*c^3*d^7 - 2400*a^2*b^17*c^5*d^5 - 720*a^2*b^17*c^7*d^3 - 2400*a^3*b^16*c^2*d^8 + 9600*a^3*b^16*c^4*d^6 + 2320*a^3*b^16*c^6*d^4 + 325*a^3*b^16*c^8*d^2 - 18800*a^4*b^15*c^3*d^7 - 1040*a^4*b^15*c^5*d^5 - 440*a^4*b^15*c^7*d^3 + 17780*a^5*b^14*c^2*d^8 - 13600*a^5*b^14*c^4*d^6 - 1310*a^5*b^14*c^6*d^4 + 40*a^5*b^14*c^8*d^2 + 34960*a^6*b^13*c^3*d^7 + 2428*a^6*b^13*c^5*d^5 + 160*a^6*b^13*c^7*d^3 - 36000*a^7*b^12*c^2*d^8 + 9330*a^7*b^12*c^4*d^6 + 360*a^7*b^12*c^6*d^4 - 30200*a^8*b^11*c^3*d^7 - 1208*a^8*b^11*c^5*d^5 - 80*a^8*b^11*c^7*d^3 + 33445*a^9*b^10*c^2*d^8 - 3440*a^9*b^10*c^4*d^6 + 120*a^9*b^10*c^6*d^4 + 12960*a^10*b^9*c^3*d^7 - 48*a^10*b^9*c^5*d^5 - 15100*a^11*b^8*c^2*d^8 + 800*a^11*b^8*c^4*d^6 - 2400*a^12*b^7*c^3*d^7 + 2760*a^13*b^6*c^2*d^8))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12) + ((a*d - b*c)^3*(-(a + b)^5*(a - b)^5)^(1/2)*((4*(4*a*b^20*d^5 - 4*a^2*b^19*c^5 + 12*a^6*b^15*c^5 - 8*a^8*b^13*c^5 + 28*a^3*b^18*d^5 - 120*a^5*b^16*d^5 + 164*a^7*b^14*d^5 - 100*a^9*b^12*d^5 + 24*a^11*b^10*d^5 + 80*a*b^20*c^2*d^3 - 120*a^2*b^19*c*d^4 + 60*a^3*b^18*c^4*d + 360*a^4*b^17*c*d^4 - 120*a^5*b^16*c^4*d - 420*a^6*b^15*c*d^4 + 60*a^7*b^14*c^4*d + 240*a^8*b^13*c*d^4 - 60*a^10*b^11*c*d^4 - 80*a^2*b^19*c^3*d^2 - 160*a^3*b^18*c^2*d^3 + 120*a^4*b^17*c^3*d^2 + 120*a^5*b^16*c^2*d^3 - 80*a^7*b^14*c^2*d^3 - 40*a^8*b^13*c^3*d^2 + 40*a^9*b^12*c^2*d^3))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) - (8*tan(e/2 + (f*x)/2)*(4*a*b^21*c^5 - 12*a^5*b^17*c^5 + 8*a^7*b^15*c^5 - 80*a^4*b^18*d^5 + 276*a^6*b^16*d^5 - 360*a^8*b^14*d^5 + 212*a^10*b^12*d^5 - 48*a^12*b^10*d^5 + 80*a*b^21*c^3*d^2 - 60*a^2*b^20*c^4*d + 240*a^3*b^19*c*d^4 + 120*a^4*b^18*c^4*d - 780*a^5*b^17*c*d^4 - 60*a^6*b^16*c^4*d + 960*a^7*b^15*c*d^4 - 540*a^9*b^13*c*d^4 + 120*a^11*b^11*c*d^4 - 240*a^2*b^20*c^2*d^3 - 120*a^3*b^19*c^3*d^2 + 680*a^4*b^18*c^2*d^3 - 720*a^6*b^16*c^2*d^3 + 40*a^7*b^15*c^3*d^2 + 360*a^8*b^14*c^2*d^3 - 80*a^10*b^12*c^2*d^3))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12) + (((4*(8*a^2*b^22 - 32*a^4*b^20 + 48*a^6*b^18 - 32*a^8*b^16 + 8*a^10*b^14))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) + (8*tan(e/2 + (f*x)/2)*(12*a*b^24 - 56*a^3*b^22 + 104*a^5*b^20 - 96*a^7*b^18 + 44*a^9*b^16 - 8*a^11*b^14))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12))*(a*d - b*c)^3*(-(a + b)^5*(a - b)^5)^(1/2)*(12*a^4*d^2 + b^4*c^2 + 20*b^4*d^2 + 2*a^2*b^2*c^2 - 29*a^2*b^2*d^2 - 12*a*b^3*c*d + 6*a^3*b*c*d))/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)))*(12*a^4*d^2 + b^4*c^2 + 20*b^4*d^2 + 2*a^2*b^2*c^2 - 29*a^2*b^2*d^2 - 12*a*b^3*c*d + 6*a^3*b*c*d))/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)))*(12*a^4*d^2

+ b^4*c^2 + 20*b^4*d^2 + 2*a^2*b^2*c^2 - 29*a^2*b^2*d^2 - 12*a*b^3*c*d + 6
*a^3*b*c*d)*1i)/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^
7 - a^10*b^5)))/((8*(2326*a^9*b^6*d^15 - 20*a^5*b^10*d^15 - 11*a^7*b^8*d^15
 - 864*a^15*d^15 - 4770*a^11*b^4*d^15 + 3456*a^13*b^2*d^15 + 400*a*b^14*c^6
*d^9 + 8040*a*b^14*c^8*d^7 + 801*a*b^14*c^10*d^5 + 20*a*b^14*c^12*d^3 + 60*
a^4*b^11*c*d^14 + 45*a^6*b^9*c*d^14 - 19860*a^8*b^7*c*d^14 + 38835*a^10*b^5
*c*d^14 - 27000*a^12*b^3*c*d^14 + 20*a^2*b^13*c^3*d^12 - 1599*a^2*b^13*c^5*
d^10 - 52680*a^2*b^13*c^7*d^8 - 15230*a^2*b^13*c^9*d^6 - 630*a^2*b^13*c^11*
d^4 - 60*a^3*b^12*c^2*d^13 + 2385*a^3*b^12*c^4*d^11 + 150460*a^3*b^12*c^6*d
^9 + 61605*a^3*b^12*c^8*d^7 + 7416*a^3*b^12*c^10*d^5 + 80*a^3*b^12*c^12*d^3
 - 1550*a^4*b^11*c^3*d^12 - 246516*a^4*b^11*c^5*d^10 - 92100*a^4*b^11*c^7*d
^8 - 18970*a^4*b^11*c^9*d^6 - 1320*a^4*b^11*c^11*d^4 + 330*a^5*b^10*c^2*d^1
3 + 255870*a^5*b^10*c^4*d^11 - 2490*a^5*b^10*c^6*d^9 + 2940*a^5*b^10*c^8*d^
7 + 2652*a^5*b^10*c^10*d^5 + 80*a^5*b^10*c^12*d^3 - 174080*a^6*b^9*c^3*d^12
 + 206889*a^6*b^9*c^5*d^10 + 46620*a^6*b^9*c^7*d^8 + 80*a^6*b^9*c^9*d^6 - 1
20*a^6*b^9*c^11*d^4 + 76440*a^7*b^8*c^2*d^13 - 335925*a^7*b^8*c^4*d^11 - 44
620*a^7*b^8*c^6*d^9 + 480*a^7*b^8*c^8*d^7 + 48*a^7*b^8*c^10*d^5 + 281510*a^
8*b^7*c^3*d^12 - 60342*a^8*b^7*c^5*d^10 - 15180*a^8*b^7*c^7*d^8 - 800*a^8*b
^7*c^9*d^6 - 139125*a^9*b^6*c^2*d^13 + 167580*a^9*b^6*c^4*d^11 + 25220*a^9*
b^6*c^6*d^9 + 2400*a^9*b^6*c^8*d^7 - 167550*a^10*b^5*c^3*d^12 - 5928*a^10*b
^5*c^5*d^10 - 2760*a^10*b^5*c^7*d^8 + 91080*a^11*b^4*c^2*d^13 - 24840*a^11*
b^4*c^4*d^11 + 1440*a^11*b^4*c^6*d^9 + 33660*a^12*b^3*c^3*d^12 - 288*a^12*b
^3*c^5*d^10 - 20520*a^13*b^2*c^2*d^13 + 6480*a^14*b*c*d^14))/(b^19 - 4*a^2*
b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) + (16*tan(e/2 + (f*x)/2)*(7829*a
^10*b^6*d^15 - 20*a^4*b^12*d^15 - 411*a^6*b^10*d^15 - 1314*a^8*b^8*d^15 - 1
728*a^16*d^15 - 11700*a^12*b^4*d^15 + 7344*a^14*b^2*d^15 + 20*a*b^15*c^3*d^
12 + 801*a*b^15*c^5*d^10 + 8040*a*b^15*c^7*d^8 + 400*a*b^15*c^9*d^6 + 60*a^
3*b^13*c*d^14 + 2445*a^5*b^11*c*d^14 + 14460*a^7*b^9*c*d^14 - 66735*a^9*b^7
*c*d^14 + 92970*a^11*b^5*c*d^14 - 56160*a^13*b^3*c*d^14 - 60*a^2*b^14*c^2*d
^13 - 3615*a^2*b^14*c^4*d^11 - 48660*a^2*b^14*c^6*d^9 - 7200*a^2*b^14*c^8*d
^7 + 6450*a^3*b^13*c^3*d^12 + 123324*a^3*b^13*c^5*d^10 + 7380*a^3*b^13*c^7*
d^8 - 5670*a^4*b^12*c^2*d^13 - 168930*a^4*b^12*c^4*d^11 + 83780*a^4*b^12*c^
6*d^9 + 12000*a^4*b^12*c^8*d^7 + 134160*a^5*b^11*c^3*d^12 - 314259*a^5*b^11
*c^5*d^10 - 36120*a^5*b^11*c^7*d^8 - 1200*a^5*b^11*c^9*d^6 - 61080*a^6*b^10
*c^2*d^13 + 509145*a^6*b^10*c^4*d^11 - 31020*a^6*b^10*c^6*d^9 - 2400*a^6*b^
10*c^8*d^7 - 458210*a^7*b^9*c^3*d^12 + 291630*a^7*b^9*c^5*d^10 + 17940*a^7*
b^9*c^7*d^8 + 800*a^7*b^9*c^9*d^6 + 237870*a^8*b^8*c^2*d^13 - 565440*a^8*b^
8*c^4*d^11 + 5340*a^8*b^8*c^6*d^9 - 2400*a^8*b^8*c^8*d^7 + 558240*a^9*b^7*c
^3*d^12 - 137784*a^9*b^7*c^5*d^10 + 2760*a^9*b^7*c^7*d^8 - 310560*a^10*b^6*
c^2*d^13 + 297240*a^10*b^6*c^4*d^11 - 9440*a^10*b^6*c^6*d^9 - 310860*a^11*b
^5*c^3*d^12 + 36288*a^11*b^5*c^5*d^10 + 180540*a^12*b^4*c^2*d^13 - 68400*a^
12*b^4*c^4*d^11 + 70200*a^13*b^3*c^3*d^12 - 41040*a^14*b^2*c^2*d^13 + 12960
*a^15*b*c*d^14))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12) +
 ((a*d - b*c)^3*(-(a + b)^5*(a - b)^5)^(1/2)*((4*(2*a^2*b^16*d^10 + 40*a^4*
b^14*d^10 + 108*a^6*b^12*d^10 - 872*a^8*b^10*d^10 + 1538*a^10*b^8*d^10 - 11

04*a^12*b^6*d^10 + 288*a^14*b^4*d^10 - 120*a^3*b^15*c*d^9 - 960*a^5*b^13*c*
d^9 + 5040*a^7*b^11*c*d^9 - 8160*a^9*b^9*c*d^9 + 5640*a^11*b^7*c*d^9 - 1440
*a^13*b^5*c*d^9 + 80*a^2*b^16*c^2*d^8 + 800*a^2*b^16*c^4*d^6 - 2400*a^3*b^1
5*c^3*d^7 + 2440*a^4*b^14*c^2*d^8 - 3200*a^4*b^14*c^4*d^6 + 9600*a^5*b^13*c
^3*d^7 - 10560*a^6*b^12*c^2*d^8 + 4800*a^6*b^12*c^4*d^6 - 14400*a^7*b^11*c^
3*d^7 + 16240*a^8*b^10*c^2*d^8 - 3200*a^8*b^10*c^4*d^6 + 9600*a^9*b^9*c^3*d
^7 - 10960*a^10*b^8*c^2*d^8 + 800*a^10*b^8*c^4*d^6 - 2400*a^11*b^7*c^3*d^7
+ 2760*a^12*b^6*c^2*d^8))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^
8*b^11) - (8*tan(e/2 + (f*x)/2)*(a*b^18*c^10 - 2*a*b^18*d^10 + 4*a^3*b^16*c
^10 + 4*a^5*b^14*c^10 - 39*a^3*b^16*d^10 - 88*a^5*b^14*d^10 + 1326*a^7*b^12
*d^10 - 3134*a^9*b^10*d^10 + 3194*a^11*b^8*d^10 - 1536*a^13*b^6*d^10 + 288*
a^15*b^4*d^10 - 80*a*b^18*c^2*d^8 - 800*a*b^18*c^4*d^6 + 400*a*b^18*c^6*d^4
 + 40*a*b^18*c^8*d^2 + 120*a^2*b^17*c*d^9 - 30*a^2*b^17*c^9*d + 900*a^4*b^1
5*c*d^9 - 60*a^4*b^15*c^9*d - 7920*a^6*b^13*c*d^9 + 17160*a^8*b^11*c*d^9 -
16710*a^10*b^9*c*d^9 + 7800*a^12*b^7*c*d^9 - 1440*a^14*b^5*c*d^9 + 2400*a^2
*b^17*c^3*d^7 - 2400*a^2*b^17*c^5*d^5 - 720*a^2*b^17*c^7*d^3 - 2400*a^3*b^1
6*c^2*d^8 + 9600*a^3*b^16*c^4*d^6 + 2320*a^3*b^16*c^6*d^4 + 325*a^3*b^16*c^
8*d^2 - 18800*a^4*b^15*c^3*d^7 - 1040*a^4*b^15*c^5*d^5 - 440*a^4*b^15*c^7*d
^3 + 17780*a^5*b^14*c^2*d^8 - 13600*a^5*b^14*c^4*d^6 - 1310*a^5*b^14*c^6*d^
4 + 40*a^5*b^14*c^8*d^2 + 34960*a^6*b^13*c^3*d^7 + 2428*a^6*b^13*c^5*d^5 +
160*a^6*b^13*c^7*d^3 - 36000*a^7*b^12*c^2*d^8 + 9330*a^7*b^12*c^4*d^6 + 360
*a^7*b^12*c^6*d^4 - 30200*a^8*b^11*c^3*d^7 - 1208*a^8*b^11*c^5*d^5 - 80*a^8
*b^11*c^7*d^3 + 33445*a^9*b^10*c^2*d^8 - 3440*a^9*b^10*c^4*d^6 + 120*a^9*b^
10*c^6*d^4 + 12960*a^10*b^9*c^3*d^7 - 48*a^10*b^9*c^5*d^5 - 15100*a^11*b^8*
c^2*d^8 + 800*a^11*b^8*c^4*d^6 - 2400*a^12*b^7*c^3*d^7 + 2760*a^13*b^6*c^2*
d^8))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12) + ((a*d - b*
c)^3*(-(a + b)^5*(a - b)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(4*a*b^21*c^5 - 12
*a^5*b^17*c^5 + 8*a^7*b^15*c^5 - 80*a^4*b^18*d^5 + 276*a^6*b^16*d^5 - 360*a
^8*b^14*d^5 + 212*a^10*b^12*d^5 - 48*a^12*b^10*d^5 + 80*a*b^21*c^3*d^2 - 60
*a^2*b^20*c^4*d + 240*a^3*b^19*c*d^4 + 120*a^4*b^18*c^4*d - 780*a^5*b^17*c*
d^4 - 60*a^6*b^16*c^4*d + 960*a^7*b^15*c*d^4 - 540*a^9*b^13*c*d^4 + 120*a^1
1*b^11*c*d^4 - 240*a^2*b^20*c^2*d^3 - 120*a^3*b^19*c^3*d^2 + 680*a^4*b^18*c
^2*d^3 - 720*a^6*b^16*c^2*d^3 + 40*a^7*b^15*c^3*d^2 + 360*a^8*b^14*c^2*d^3
- 80*a^10*b^12*c^2*d^3))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8
b^12) - (4(4*a*b^20*d^5 - 4*a^2*b^19*c^5 + 12*a^6*b^15*c^5 - 8*a^8*b^13*c
^5 + 28*a^3*b^18*d^5 - 120*a^5*b^16*d^5 + 164*a^7*b^14*d^5 - 100*a^9*b^12*d
^5 + 24*a^11*b^10*d^5 + 80*a*b^20*c^2*d^3 - 120*a^2*b^19*c*d^4 + 60*a^3*b^1
8*c^4*d + 360*a^4*b^17*c*d^4 - 120*a^5*b^16*c^4*d - 420*a^6*b^15*c*d^4 + 60
*a^7*b^14*c^4*d + 240*a^8*b^13*c*d^4 - 60*a^10*b^11*c*d^4 - 80*a^2*b^19*c^3
*d^2 - 160*a^3*b^18*c^2*d^3 + 120*a^4*b^17*c^3*d^2 + 120*a^5*b^16*c^2*d^3 -
 80*a^7*b^14*c^2*d^3 - 40*a^8*b^13*c^3*d^2 + 40*a^9*b^12*c^2*d^3))/(b^19 -
4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) + (((4*(8*a^2*b^22 - 32*a^
4*b^20 + 48*a^6*b^18 - 32*a^8*b^16 + 8*a^10*b^14))/(b^19 - 4*a^2*b^17 + 6*a
^4*b^15 - 4*a^6*b^13 + a^8*b^11) + (8*tan(e/2 + (f*x)/2)*(12*a*b^24 - 56*a^
3*b^22 + 104*a^5*b^20 - 96*a^7*b^18 + 44*a^9*b^16 - 8*a^11*b^14))/(b^20 - 4

*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12))*(a*d - b*c)^3*(-(a + b)^5*(a - b)^5)^(1/2)*(12*a^4*d^2 + b^4*c^2 + 20*b^4*d^2 + 2*a^2*b^2*c^2 - 29*a^2*b^2*d^2 - 12*a*b^3*c*d + 6*a^3*b*c*d))/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)))*(12*a^4*d^2 + b^4*c^2 + 20*b^4*d^2 + 2*a^2*b^2*c^2 - 29*a^2*b^2*d^2 - 12*a*b^3*c*d + 6*a^3*b*c*d))/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)))*(12*a^4*d^2 + b^4*c^2 + 20*b^4*d^2 + 2*a^2*b^2*c^2 - 29*a^2*b^2*d^2 - 12*a*b^3*c*d + 6*a^3*b*c*d))/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)) - ((a*d - b*c)^3*(-(a + b)^5*(a - b)^5)^(1/2)*((4*(2*a^2*b^16*d^10 + 40*a^4*b^14*d^10 + 108*a^6*b^12*d^10 - 872*a^8*b^10*d^10 + 1538*a^10*b^8*d^10 - 1104*a^12*b^6*d^10 + 288*a^14*b^4*d^10 - 120*a^3*b^15*c*d^9 - 960*a^5*b^13*c*d^9 + 5040*a^7*b^11*c*d^9 - 8160*a^9*b^9*c*d^9 + 5640*a^11*b^7*c*d^9 - 1440*a^13*b^5*c*d^9 + 80*a^2*b^16*c^2*d^8 + 800*a^2*b^16*c^4*d^6 - 2400*a^3*b^15*c^3*d^7 + 2440*a^4*b^14*c^2*d^8 - 3200*a^4*b^14*c^4*d^6 + 9600*a^5*b^13*c^3*d^7 - 10560*a^6*b^12*c^2*d^8 + 4800*a^6*b^12*c^4*d^6 - 14400*a^7*b^11*c^3*d^7 + 16240*a^8*b^10*c^2*d^8 - 3200*a^8*b^10*c^4*d^6 + 9600*a^9*b^9*c^3*d^7 - 10960*a^10*b^8*c^2*d^8 + 800*a^10*b^8*c^4*d^6 - 2400*a^11*b^7*c^3*d^7 + 2760*a^12*b^6*c^2*d^8))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) - (8*tan(e/2 + (f*x)/2)*(a*b^18*c^10 - 2*a*b^18*d^10 + 4*a^3*b^16*c^10 + 4*a^5*b^14*c^10 - 39*a^3*b^16*d^10 - 88*a^5*b^14*d^10 + 1326*a^7*b^12*d^10 - 3134*a^9*b^10*d^10 + 3194*a^11*b^8*d^10 - 1536*a^13*b^6*d^10 + 288*a^15*b^4*d^10 - 80*a*b^18*c^2*d^8 - 800*a*b^18*c^4*d^6 + 400*a*b^18*c^6*d^4 + 40*a*b^18*c^8*d^2 + 120*a^2*b^17*c*d^9 - 30*a^2*b^17*c^9*d + 900*a^4*b^15*c*d^9 - 60*a^4*b^15*c^9*d - 7920*a^6*b^13*c*d^9 + 17160*a^8*b^11*c*d^9 - 16710*a^10*b^9*c*d^9 + 7800*a^12*b^7*c*d^9 - 1440*a^14*b^5*c*d^9 + 2400*a^2*b^17*c^3*d^7 - 2400*a^2*b^17*c^5*d^5 - 720*a^2*b^17*c^7*d^3 - 2400*a^3*b^16*c^2*d^8 + 9600*a^3*b^16*c^4*d^6 + 2320*a^3*b^16*c^6*d^4 + 325*a^3*b^16*c^8*d^2 - 18800*a^4*b^15*c^3*d^7 - 1040*a^4*b^15*c^5*d^5 - 440*a^4*b^15*c^7*d^3 + 17780*a^5*b^14*c^2*d^8 - 13600*a^5*b^14*c^4*d^6 - 1310*a^5*b^14*c^6*d^4 + 40*a^5*b^14*c^8*d^2 + 34960*a^6*b^13*c^3*d^7 + 2428*a^6*b^13*c^5*d^5 + 160*a^6*b^13*c^7*d^3 - 36000*a^7*b^12*c^2*d^8 + 9330*a^7*b^12*c^4*d^6 + 360*a^7*b^12*c^6*d^4 - 30200*a^8*b^11*c^3*d^7 - 1208*a^8*b^11*c^5*d^5 - 80*a^8*b^11*c^7*d^3 + 33445*a^9*b^10*c^2*d^8 - 3440*a^9*b^10*c^4*d^6 + 120*a^9*b^10*c^6*d^4 + 12960*a^10*b^9*c^3*d^7 - 48*a^10*b^9*c^5*d^5 - 15100*a^11*b^8*c^2*d^8 + 800*a^11*b^8*c^4*d^6 - 2400*a^12*b^7*c^3*d^7 + 2760*a^13*b^6*c^2*d^8))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12) + ((a*d - b*c)^3*(-(a + b)^5*(a - b)^5)^(1/2)*((4*(4*a*b^20*d^5 - 4*a^2*b^19*c^5 + 12*a^6*b^15*c^5 - 8*a^8*b^13*c^5 + 28*a^3*b^18*d^5 - 120*a^5*b^16*d^5 + 164*a^7*b^14*d^5 - 100*a^9*b^12*d^5 + 24*a^11*b^10*d^5 + 80*a*b^20*c^2*d^3 - 120*a^2*b^19*c*d^4 + 60*a^3*b^18*c^4*d + 360*a^4*b^17*c*d^4 - 120*a^5*b^16*c^4*d - 420*a^6*b^15*c*d^4 + 60*a^7*b^14*c^4*d + 240*a^8*b^13*c*d^4 - 60*a^10*b^11*c*d^4 - 80*a^2*b^19*c^3*d^2 - 160*a^3*b^18*c^2*d^3 + 120*a^4*b^17*c^3*d^2 + 120*a^5*b^16*c^2*d^3 - 80*a^7*b^14*c^2*d^3 - 40*a^8*b^13*c^3*d^2 + 40*a^9*b^12*c^2*d^3))/(b^19 - 4*a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) - (8*tan(e/2 + (f*x)/2)*(4*a*b^21*c^5 - 12*a^5*b^17*c^5 + 8*a

```
^7*b^15*c^5 - 80*a^4*b^18*d^5 + 276*a^6*b^16*d^5 - 360*a^8*b^14*d^5 + 212*a
^10*b^12*d^5 - 48*a^12*b^10*d^5 + 80*a*b^21*c^3*d^2 - 60*a^2*b^20*c^4*d + 2
40*a^3*b^19*c*d^4 + 120*a^4*b^18*c^4*d - 780*a^5*b^17*c*d^4 - 60*a^6*b^16*c
^4*d + 960*a^7*b^15*c*d^4 - 540*a^9*b^13*c*d^4 + 120*a^11*b^11*c*d^4 - 240*
a^2*b^20*c^2*d^3 - 120*a^3*b^19*c^3*d^2 + 680*a^4*b^18*c^2*d^3 - 720*a^6*b^
16*c^2*d^3 + 40*a^7*b^15*c^3*d^2 + 360*a^8*b^14*c^2*d^3 - 80*a^10*b^12*c^2*
d^3))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12) + (((4*(8*a^
2*b^22 - 32*a^4*b^20 + 48*a^6*b^18 - 32*a^8*b^16 + 8*a^10*b^14))/(b^19 - 4*
a^2*b^17 + 6*a^4*b^15 - 4*a^6*b^13 + a^8*b^11) + (8*tan(e/2 + (f*x)/2)*(12*
a*b^24 - 56*a^3*b^22 + 104*a^5*b^20 - 96*a^7*b^18 + 44*a^9*b^16 - 8*a^11*b^
14))/(b^20 - 4*a^2*b^18 + 6*a^4*b^16 - 4*a^6*b^14 + a^8*b^12))*(a*d - b*c)^
3*(-(a + b)^5*(a - b)^5)^(1/2)*(12*a^4*d^2 + b^4*c^2 + 20*b^4*d^2 + 2*a^2*b
^2*c^2 - 29*a^2*b^2*d^2 - 12*a*b^3*c*d + 6*a^3*b*c*d))/(2*(b^15 - 5*a^2*b^1
3 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)))*(12*a^4*d^2 + b^4*c^
2 + 20*b^4*d^2 + 2*a^2*b^2*c^2 - 29*a^2*b^2*d^2 - 12*a*b^3*c*d + 6*a^3*b*c*
d))/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5
)))*(12*a^4*d^2 + b^4*c^2 + 20*b^4*d^2 + 2*a^2*b^2*c^2 - 29*a^2*b^2*d^2 - 1
2*a*b^3*c*d + 6*a^3*b*c*d))/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^
9 + 5*a^8*b^7 - a^10*b^5))))*(a*d - b*c)^3*(-(a + b)^5*(a - b)^5)^(1/2)*(12
*a^4*d^2 + b^4*c^2 + 20*b^4*d^2 + 2*a^2*b^2*c^2 - 29*a^2*b^2*d^2 - 12*a*b^3
*c*d + 6*a^3*b*c*d)*1i)/(f*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 +
5*a^8*b^7 - a^10*b^5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**5/(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.715 $\int \frac{(c+d\sin(e+fx))^4}{(a+b\sin(e+fx))^3}\,dx$

Optimal. Leaf size=318

$$\frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))^2}{2bf\left(a^2-b^2\right)(a+b\sin(e+fx))^2}+\frac{d^2\left(-3a^2d^2+2abcd-\left(b^2\left(c^2-2d^2\right)\right)\right)\cos(e+fx)}{2b^3f\left(a^2-b^2\right)}+\frac{3(bc-ad)^3\left(a^2c\right.}{2b^3f\left(a^2-\right.}$$

```
[Out] d^3*(-3*a*d+4*b*c)*x/b^4+(-a*d+b*c)^2*(4*a^3*b*c*d-10*a*b^3*c*d+6*a^4*d^2+a
^2*b^2*(2*c^2-15*d^2)+b^4*(c^2+12*d^2))*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^
2-b^2)^(1/2))/b^4/(a^2-b^2)^(5/2)/f+1/2*d^2*(2*a*b*c*d-3*a^2*d^2-b^2*(c^2-2
*d^2))*cos(f*x+e)/b^3/(a^2-b^2)/f+3/2*(-a*d+b*c)^3*(a^2*d+a*b*c-2*b^2*d)*co
s(f*x+e)/b^3/(a^2-b^2)^2/f/(a+b*sin(f*x+e))+1/2*(-a*d+b*c)^2*cos(f*x+e)*(c+
d*sin(f*x+e))^2/b/(a^2-b^2)/f/(a+b*sin(f*x+e))^2
```

Rubi [A] time = 0.97, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2792, 3031, 3023, 2735, 2660, 618, 204}

$$\frac{d^2\left(-3a^2d^2+2abcd+b^2\left(-\left(c^2-2d^2\right)\right)\right)\cos(e+fx)}{2b^3f\left(a^2-b^2\right)}+\frac{(bc-ad)^2\left(a^2b^2\left(2c^2-15d^2\right)+4a^3bcd+6a^4d^2-10ab^3cd\right.}{b^4f\left(a^2-b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^4/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (d^3*(4*b*c - 3*a*d)*x)/b^4 + ((b*c - a*d)^2*(4*a^3*b*c*d - 10*a*b^3*c*d +
6*a^4*d^2 + a^2*b^2*(2*c^2 - 15*d^2) + b^4*(c^2 + 12*d^2))*ArcTan[(b + a*Ta
n[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(b^4*(a^2 - b^2)^(5/2)*f) + (d^2*(2*a*b*c
*d - 3*a^2*d^2 - b^2*(c^2 - 2*d^2))*Cos[e + f*x])/(2*b^3*(a^2 - b^2)*f) + (
3*(b*c - a*d)^3*(a*b*c + a^2*d - 2*b^2*d)*Cos[e + f*x])/(2*b^3*(a^2 - b^2)^
2*f*(a + b*Sin[e + f*x])) + ((b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x]
)^2)/(2*b*(a^2 - b^2)*f*(a + b*Sin[e + f*x])^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
```

```
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^4}{(a+b\sin(e+fx))^3}\,dx = \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))^2}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} - \frac{\int \frac{(c+d\sin(e+fx))\left(2\left(3b^2c^2d+a^2d^3-abc\left(c^2+3d^2\right)\right)\cdots}{\cdots}}{\cdots}$$

$$= \frac{3(bc-ad)^3\left(abc+a^2d-2b^2d\right)\cos(e+fx)}{2b^3\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+\cdots}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))\cdots}$$

$$= \frac{d^2\left(2abcd-3a^2d^2-b^2\left(c^2-2d^2\right)\right)\cos(e+fx)}{2b^3\left(a^2-b^2\right)f} + \frac{3(bc-ad)^3\left(abc+a^2d-2b^2d\right)\cos\cdots}{2b^3\left(a^2-b^2\right)^2 f(a+b\sin(e+\cdots}$$

$$= \frac{d^3(4bc-3ad)x}{b^4} + \frac{d^2\left(2abcd-3a^2d^2-b^2\left(c^2-2d^2\right)\right)\cos(e+fx)}{2b^3\left(a^2-b^2\right)f} + \frac{3(bc-ad)^3\left(ab\cdots}{2b^3\left(a^2-b\cdots}$$

$$= \frac{d^3(4bc-3ad)x}{b^4} + \frac{d^2\left(2abcd-3a^2d^2-b^2\left(c^2-2d^2\right)\right)\cos(e+fx)}{2b^3\left(a^2-b^2\right)f} + \frac{3(bc-ad)^3\left(ab\cdots}{2b^3\left(a^2-b\cdots}$$

$$= \frac{d^3(4bc-3ad)x}{b^4} + \frac{d^2\left(2abcd-3a^2d^2-b^2\left(c^2-2d^2\right)\right)\cos(e+fx)}{2b^3\left(a^2-b^2\right)f} + \frac{3(bc-ad)^3\left(ab\cdots}{2b^3\left(a^2-b\cdots}$$

$$= \frac{d^3(4bc-3ad)x}{b^4} + \frac{(bc-ad)^2\left(4a^3bcd-10ab^3cd+6a^4d^2+a^2b^2\left(2c^2-15d^2\right)+b^4\left(\cdots\right.\right.}{b^4\left(a^2-b^2\right)^{5/2}f}$$

Mathematica [B] time = 4.16, size = 894, normalized size = 2.81

$$\frac{\frac{4\left(6d^2a^4+4bcda^3+b^2\left(2c^2-15d^2\right)a^2-10b^3cda+b^4\left(c^2+12d^2\right)\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)(bc-ad)^2}{\left(a^2-b^2\right)^{5/2}}+\frac{-12d^4ea^7-12d^4fxa^7+16bcd^3ea^6+16bcd^3fxa^6-24bd^4e}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^4/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((4*(b*c - a*d)^2*(4*a^3*b*c*d - 10*a*b^3*c*d + 6*a^4*d^2 + a^2*b^2*(2*c^2
- 15*d^2) + b^4*(c^2 + 12*d^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 -
b^2]])/(a^2 - b^2)^(5/2) + (16*a^6*b*c*d^3*e - 24*a^4*b^3*c*d^3*e + 8*b^7*c
*d^3*e - 12*a^7*d^4*e + 18*a^5*b^2*d^4*e - 6*a*b^6*d^4*e + 16*a^6*b*c*d^3*f
*x - 24*a^4*b^3*c*d^3*f*x + 8*b^7*c*d^3*f*x - 12*a^7*d^4*f*x + 18*a^5*b^2*d
^4*f*x - 6*a*b^6*d^4*f*x - b*(8*a*b^5*c^3*d - 16*a^5*b*c*d^3 + 12*a^6*d^4 -
 21*a^4*b^2*d^4 + 8*a^3*b^3*c*d*(2*c^2 + 5*d^2) + b^6*(2*c^4 + d^4) + 2*a^2
*b^4*(-4*c^4 - 18*c^2*d^2 + d^4))*Cos[e + f*x] + 2*b^2*(a^2 - b^2)^2*d^3*(-
4*b*c + 3*a*d)*(e + f*x)*Cos[2*(e + f*x)] + a^4*b^3*d^4*Cos[3*(e + f*x)] -
2*a^2*b^5*d^4*Cos[3*(e + f*x)] + b^7*d^4*Cos[3*(e + f*x)] + 32*a^5*b^2*c*d^
3*e*Sin[e + f*x] - 64*a^3*b^4*c*d^3*e*Sin[e + f*x] + 32*a*b^6*c*d^3*e*Sin[e
 + f*x] - 24*a^6*b*d^4*e*Sin[e + f*x] + 48*a^4*b^3*d^4*e*Sin[e + f*x] - 24*
a^2*b^5*d^4*e*Sin[e + f*x] + 32*a^5*b^2*c*d^3*f*x*Sin[e + f*x] - 64*a^3*b^4
*c*d^3*f*x*Sin[e + f*x] + 32*a*b^6*c*d^3*f*x*Sin[e + f*x] - 24*a^6*b*d^4*f*
x*Sin[e + f*x] + 48*a^4*b^3*d^4*f*x*Sin[e + f*x] - 24*a^2*b^5*d^4*f*x*Sin[e
 + f*x] + 3*a*b^6*c^4*Sin[2*(e + f*x)] - 4*a^2*b^5*c^3*d*Sin[2*(e + f*x)] -
 8*b^7*c^3*d*Sin[2*(e + f*x)] - 6*a^3*b^4*c^2*d^2*Sin[2*(e + f*x)] + 24*a*b
^6*c^2*d^2*Sin[2*(e + f*x)] + 12*a^4*b^3*c*d^3*Sin[2*(e + f*x)] - 24*a^2*b^
5*c*d^3*Sin[2*(e + f*x)] - 9*a^5*b^2*d^4*Sin[2*(e + f*x)] + 16*a^3*b^4*d^4*
Sin[2*(e + f*x)] - 4*a*b^6*d^4*Sin[2*(e + f*x)])/((a^2 - b^2)^2*(a + b*Sin[
e + f*x])^2))/(4*b^4*f)
```

fricas [B] time = 1.04, size = 2335, normalized size = 7.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*d^4*cos(f*x + e)^3 - 4*(4*
(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*c*d^3 - 3*(a^7*b^2 - 3*a^5*b^4 + 3*
a^3*b^6 - a*b^8)*d^4)*f*x*cos(f*x + e)^2 + 4*(4*(a^8*b - 2*a^6*b^3 + 2*a^2*
b^7 - b^9)*c*d^3 - 3*(a^9 - 2*a^7*b^2 + 2*a^3*b^6 - a*b^8)*d^4)*f*x - ((2*a
^4*b^4 + 3*a^2*b^6 + b^8)*c^4 - 12*(a^3*b^5 + a*b^7)*c^3*d + 6*(a^4*b^4 + 3
```

```
*a^2*b^6 + 2*b^8)*c^2*d^2 - 4*(2*a^7*b - 3*a^5*b^3 + a^3*b^5 + 6*a*b^7)*c*d
^3 + 3*(2*a^8 - 3*a^6*b^2 - a^4*b^4 + 4*a^2*b^6)*d^4 + (12*a*b^7*c^3*d - (2
*a^2*b^6 + b^8)*c^4 - 6*(a^2*b^6 + 2*b^8)*c^2*d^2 + 4*(2*a^5*b^3 - 5*a^3*b^
5 + 6*a*b^7)*c*d^3 - 3*(2*a^6*b^2 - 5*a^4*b^4 + 4*a^2*b^6)*d^4)*cos(f*x + e
)^2 - 2*(12*a^2*b^6*c^3*d - (2*a^3*b^5 + a*b^7)*c^4 - 6*(a^3*b^5 + 2*a*b^7)
*c^2*d^2 + 4*(2*a^6*b^2 - 5*a^4*b^4 + 6*a^2*b^6)*c*d^3 - 3*(2*a^7*b - 5*a^5
*b^3 + 4*a^3*b^5)*d^4)*sin(f*x + e))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*co
s(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 + 2*(a*cos(f*x + e)*sin(f*x +
 e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x
 + e) - a^2 - b^2)) + 2*((4*a^4*b^5 - 5*a^2*b^7 + b^9)*c^4 - 4*(2*a^5*b^4 -
 a^3*b^6 - a*b^8)*c^3*d + 18*(a^4*b^5 - a^2*b^7)*c^2*d^2 + 4*(2*a^7*b^2 - 7
*a^5*b^4 + 5*a^3*b^6)*c*d^3 - (6*a^8*b - 15*a^6*b^3 + 7*a^4*b^5 + 4*a^2*b^7
 - 2*b^9)*d^4)*cos(f*x + e) + 2*(4*(4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*
b^8)*c*d^3 - 3*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d^4)*f*x + (3*(a^3
*b^6 - a*b^8)*c^4 - 4*(a^4*b^5 + a^2*b^7 - 2*b^9)*c^3*d - 6*(a^5*b^4 - 5*a^
3*b^6 + 4*a*b^8)*c^2*d^2 + 12*(a^6*b^3 - 3*a^4*b^5 + 2*a^2*b^7)*c*d^3 - (9*
a^7*b^2 - 25*a^5*b^4 + 20*a^3*b^6 - 4*a*b^8)*d^4)*cos(f*x + e))*sin(f*x + e
))/((a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*f*cos(f*x + e)^2 - 2*(a^7*b^5
 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*f*sin(f*x + e) - (a^8*b^4 - 2*a^6*b^6 +
2*a^2*b^10 - b^12)*f), -1/2*(2*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*d^4*
cos(f*x + e)^3 - 2*(4*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*c*d^3 - 3*(a^
7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*d^4)*f*x*cos(f*x + e)^2 + 2*(4*(a^8*
b - 2*a^6*b^3 + 2*a^2*b^7 - b^9)*c*d^3 - 3*(a^9 - 2*a^7*b^2 + 2*a^3*b^6 - a
*b^8)*d^4)*f*x - ((2*a^4*b^4 + 3*a^2*b^6 + b^8)*c^4 - 12*(a^3*b^5 + a*b^7)*
c^3*d + 6*(a^4*b^4 + 3*a^2*b^6 + 2*b^8)*c^2*d^2 - 4*(2*a^7*b - 3*a^5*b^3 +
a^3*b^5 + 6*a*b^7)*c*d^3 + 3*(2*a^8 - 3*a^6*b^2 - a^4*b^4 + 4*a^2*b^6)*d^4
+ (12*a*b^7*c^3*d - (2*a^2*b^6 + b^8)*c^4 - 6*(a^2*b^6 + 2*b^8)*c^2*d^2 + 4
*(2*a^5*b^3 - 5*a^3*b^5 + 6*a*b^7)*c*d^3 - 3*(2*a^6*b^2 - 5*a^4*b^4 + 4*a^2
*b^6)*d^4)*cos(f*x + e)^2 - 2*(12*a^2*b^6*c^3*d - (2*a^3*b^5 + a*b^7)*c^4 -
 6*(a^3*b^5 + 2*a*b^7)*c^2*d^2 + 4*(2*a^6*b^2 - 5*a^4*b^4 + 6*a^2*b^6)*c*d^
3 - 3*(2*a^7*b - 5*a^5*b^3 + 4*a^3*b^5)*d^4)*sin(f*x + e))*sqrt(a^2 - b^2)*
arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))) + ((4*a^4*b^5
- 5*a^2*b^7 + b^9)*c^4 - 4*(2*a^5*b^4 - a^3*b^6 - a*b^8)*c^3*d + 18*(a^4*b^
5 - a^2*b^7)*c^2*d^2 + 4*(2*a^7*b^2 - 7*a^5*b^4 + 5*a^3*b^6)*c*d^3 - (6*a^8
*b - 15*a^6*b^3 + 7*a^4*b^5 + 4*a^2*b^7 - 2*b^9)*d^4)*cos(f*x + e) + (4*(4*
(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*c*d^3 - 3*(a^8*b - 3*a^6*b^3 + 3*
a^4*b^5 - a^2*b^7)*d^4)*f*x + (3*(a^3*b^6 - a*b^8)*c^4 - 4*(a^4*b^5 + a^2*b
^7 - 2*b^9)*c^3*d - 6*(a^5*b^4 - 5*a^3*b^6 + 4*a*b^8)*c^2*d^2 + 12*(a^6*b^3
 - 3*a^4*b^5 + 2*a^2*b^7)*c*d^3 - (9*a^7*b^2 - 25*a^5*b^4 + 20*a^3*b^6 - 4*
a*b^8)*d^4)*cos(f*x + e))*sin(f*x + e))/((a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10
- b^12)*f*cos(f*x + e)^2 - 2*(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*f*s
in(f*x + e) - (a^8*b^4 - 2*a^6*b^6 + 2*a^2*b^10 - b^12)*f)]
```

giac [B] time = 2.21, size = 1159, normalized size = 3.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((2*a^2*b^4*c^4 + b^6*c^4 - 12*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 + 12*b^6*c^2
*d^2 - 8*a^5*b*c*d^3 + 20*a^3*b^3*c*d^3 - 24*a*b^5*c*d^3 + 6*a^6*d^4 - 15*a
^4*b^2*d^4 + 12*a^2*b^4*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arc
tan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^4*b^4 - 2*a^2*b^6 +
b^8)*sqrt(a^2 - b^2)) - 2*d^4/((tan(1/2*f*x + 1/2*e)^2 + 1)*b^3) + (5*a^3*b
^5*c^4*tan(1/2*f*x + 1/2*e)^3 - 2*a*b^7*c^4*tan(1/2*f*x + 1/2*e)^3 - 12*a^4
*b^4*c^3*d*tan(1/2*f*x + 1/2*e)^3 + 6*a^5*b^3*c^2*d^2*tan(1/2*f*x + 1/2*e)^
3 + 12*a^3*b^5*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 + 4*a^6*b^2*c*d^3*tan(1/2*f*x
 + 1/2*e)^3 - 16*a^4*b^4*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 3*a^7*b*d^4*tan(1/2
*f*x + 1/2*e)^3 + 6*a^5*b^3*d^4*tan(1/2*f*x + 1/2*e)^3 + 4*a^4*b^4*c^4*tan(
1/2*f*x + 1/2*e)^2 + 7*a^2*b^6*c^4*tan(1/2*f*x + 1/2*e)^2 - 2*b^8*c^4*tan(1
/2*f*x + 1/2*e)^2 - 8*a^5*b^3*c^3*d*tan(1/2*f*x + 1/2*e)^2 - 20*a^3*b^5*c^3
*d*tan(1/2*f*x + 1/2*e)^2 - 8*a*b^7*c^3*d*tan(1/2*f*x + 1/2*e)^2 + 18*a^4*b
^4*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 + 36*a^2*b^6*c^2*d^2*tan(1/2*f*x + 1/2*e)
^2 + 8*a^7*b*c*d^3*tan(1/2*f*x + 1/2*e)^2 - 4*a^5*b^3*c*d^3*tan(1/2*f*x + 1
/2*e)^2 - 40*a^3*b^5*c*d^3*tan(1/2*f*x + 1/2*e)^2 - 4*a^8*d^4*tan(1/2*f*x +
 1/2*e)^2 - a^6*b^2*d^4*tan(1/2*f*x + 1/2*e)^2 + 14*a^4*b^4*d^4*tan(1/2*f*x
 + 1/2*e)^2 + 11*a^3*b^5*c^4*tan(1/2*f*x + 1/2*e) - 2*a*b^7*c^4*tan(1/2*f*x
 + 1/2*e) - 20*a^4*b^4*c^3*d*tan(1/2*f*x + 1/2*e) - 16*a^2*b^6*c^3*d*tan(1/
2*f*x + 1/2*e) - 6*a^5*b^3*c^2*d^2*tan(1/2*f*x + 1/2*e) + 60*a^3*b^5*c^2*d^
2*tan(1/2*f*x + 1/2*e) + 28*a^6*b^2*c*d^3*tan(1/2*f*x + 1/2*e) - 64*a^4*b^4
*c*d^3*tan(1/2*f*x + 1/2*e) - 13*a^7*b*d^4*tan(1/2*f*x + 1/2*e) + 22*a^5*b^
3*d^4*tan(1/2*f*x + 1/2*e) + 4*a^4*b^4*c^4 - a^2*b^6*c^4 - 8*a^5*b^3*c^3*d
- 4*a^3*b^5*c^3*d + 18*a^4*b^4*c^2*d^2 + 8*a^7*b*c*d^3 - 20*a^5*b^3*c*d^3 -
 4*a^8*d^4 + 7*a^6*b^2*d^4)/((a^6*b^3 - 2*a^4*b^5 + a^2*b^7)*(a*tan(1/2*f*x
 + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)^2) + (4*b*c*d^3 - 3*a*d^4)*(f*x
 + e)/b^4)/f
```

maple [B] time = 0.34, size = 3683, normalized size = 11.58

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e))^3,x)
```

```
[Out] -2/f*d^4/b^3/(1+tan(1/2*f*x+1/2*e)^2)-12/f*b/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^
(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a*c^3*d-24/f
*b/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2
*b)/(a^2-b^2)^(1/2))*a*c*d^3-8/f/b^3/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*ar
ctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^5*c*d^3+20/f/b/(a^
4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a
```

^2-b^2)^(1/2))*a^3*c*d^3+8/f/b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^5*tan(1/2*f*x+1/2*e)^2*c*d^3+18/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+1/2*e)^2*c^2*d^2-20/f*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*tan(1/2*f*x+1/2*e)^2*c^3*d-40/f*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*tan(1/2*f*x+1/2*e)^2*c*d^3+28/f/b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a^4/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c*d^3+60/f*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c^2*d^2-12/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+1/2*e)^3*c^3*d-16/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+1/2*e)^3*c*d^3+12/f*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*tan(1/2*f*x+1/2*e)^3*c^2*d^2-20/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c^3*d-64/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c*d^3-8/f*b^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)/a*tan(1/2*f*x+1/2*e)^2*c^3*d+4/f/b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^4*tan(1/2*f*x+1/2*e)^3*c*d^3+6/f/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*c^2*d^2-3/f/b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^5*tan(1/2*f*x+1/2*e)^3*d^4-6/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a^3/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c^2*d^2+5/f*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*tan(1/2*f*x+1/2*e)^3*c^4-1/f*b^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*c^4-6/f*d^4/b^4*arctan(tan(1/2*f*x+1/2*e))*a+8/f*d^3/b^3*arctan(tan(1/2*f*x+1/2*e))*c-2/f*b^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)/a*tan(1/2*f*x+1/2*e)^3*c^4-4/f/b^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^6*tan(1/2*f*x+1/2*e)^2*d^4-1/f/b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^4*tan(1/2*f*x+1/2*e)^2*d^4+6/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*tan(1/2*f*x+1/2*e)^3*c^2*d^2-8/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*tan(1/2*f*x+1/2*e)^2*c^3*d-4/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*tan(1/2*f*x+1/2*e)^2*c*d^3-15/f/b^2/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^4*d^4+12/f*b^2/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*c^2*d^2+6/f/b^4/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^6*d^4+4/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+1/2*e)^2*c^4+36/f*b^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)^2*c^2*d^2-16/f*b^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c^3*d+14/f*b/(tan(1/2*f*x+1/2*e)^

```
2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+1/2*e)^
2*d^4-2/f*b^5/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^
2*b^2+b^4)/a^2*tan(1/2*f*x+1/2*e)^2*c^4-13/f/b^2/(tan(1/2*f*x+1/2*e)^2*a+2*
tan(1/2*f*x+1/2*e)*b+a)^2*a^5/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*d^4+11
/f*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a/(a^4-2*a^2*b^2
+b^4)*tan(1/2*f*x+1/2*e)*c^4-2/f*b^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+
1/2*e)*b+a)^2/a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c^4+8/f/b^2/(tan(1/2
*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^5*c*d^3+1
8/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^
4)*a^2*c^2*d^2-4/f*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/
(a^4-2*a^2*b^2+b^4)*a*c^3*d+2/f/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(
1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*c^4+22/f/(tan(1/2*f*x
+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a^3/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x
+1/2*e)*d^4-8/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*
a^2*b^2+b^4)*a^3*c^3*d-20/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+
a)^2/(a^4-2*a^2*b^2+b^4)*a^3*c*d^3+1/f*b^2/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1
/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*c^4-4/f/b^3/(t
an(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^6*d
^4+7/f/b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2
+b^4)*a^4*d^4+4/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^
4-2*a^2*b^2+b^4)*a^2*c^4+7/f*b^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*
e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)^2*c^4+6/f/(tan(1/2*f*x+1/2
*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*tan(1/2*f*x+1/2
*e)^3*d^4+12/f/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*
f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*d^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^4/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 21.74, size = 16958, normalized size = 53.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^4/(a + b*sin(e + f*x))^3,x)
```

```
[Out] (2*d^3*atan(((d^3*(3*a*d - 4*b*c)*((8*tan(e/2 + (f*x)/2)*(a*b^15*c^8 + 4*a^
3*b^13*c^8 + 4*a^5*b^11*c^8 - 72*a^3*b^13*d^8 + 468*a^5*b^11*d^8 - 936*a^7*
b^9*d^8 + 873*a^9*b^7*d^8 - 396*a^11*b^5*d^8 + 72*a^13*b^3*d^8 - 128*a*b^15
*c^2*d^6 + 144*a*b^15*c^4*d^4 + 24*a*b^15*c^6*d^2 + 192*a^2*b^14*c*d^7 - 24
*a^2*b^14*c^7*d - 1440*a^4*b^12*c*d^7 - 48*a^4*b^12*c^7*d + 2736*a^6*b^10*c
*d^7 - 2424*a^8*b^8*c*d^7 + 1056*a^10*b^6*c*d^7 - 192*a^12*b^4*c*d^7 - 576*
a^2*b^14*c^3*d^5 - 336*a^2*b^14*c^5*d^3 + 1440*a^3*b^13*c^2*d^6 + 744*a^3*b
^13*c^4*d^4 + 204*a^3*b^13*c^6*d^2 - 96*a^4*b^12*c^3*d^5 - 200*a^4*b^12*c^5
*d^3 - 2200*a^5*b^11*c^2*d^6 - 426*a^5*b^11*c^4*d^4 + 24*a^5*b^11*c^6*d^2 +
 408*a^6*b^10*c^3*d^5 + 64*a^6*b^10*c^5*d^3 + 1644*a^7*b^9*c^2*d^6 + 144*a^
7*b^9*c^4*d^4 - 240*a^8*b^8*c^3*d^5 - 32*a^8*b^8*c^5*d^3 - 632*a^9*b^7*c^2*
d^6 + 24*a^9*b^7*c^4*d^4 + 128*a^11*b^5*c^2*d^6))/(b^17 - 4*a^2*b^15 + 6*a^
4*b^13 - 4*a^6*b^11 + a^8*b^9) - (8*(36*a^4*b^11*d^8 - 144*a^6*b^9*d^8 + 21
6*a^8*b^7*d^8 - 144*a^10*b^5*d^8 + 36*a^12*b^3*d^8 - 96*a^3*b^12*c*d^7 + 38
4*a^5*b^10*c*d^7 - 576*a^7*b^8*c*d^7 + 384*a^9*b^6*c*d^7 - 96*a^11*b^4*c*d^
7 + 64*a^2*b^13*c^2*d^6 - 256*a^4*b^11*c^2*d^6 + 384*a^6*b^9*c^2*d^6 - 256*
a^8*b^7*c^2*d^6 + 64*a^10*b^5*c^2*d^6))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4
*a^6*b^10 + a^8*b^8) + (d^3*(3*a*d - 4*b*c)*((8*(2*a^2*b^16*c^4 - 6*a^6*b^1
2*c^4 + 4*a^8*b^10*c^4 + 12*a^2*b^16*d^4 - 36*a^4*b^14*d^4 + 42*a^6*b^12*d^
4 - 24*a^8*b^10*d^4 + 6*a^10*b^8*d^4 + 32*a^3*b^15*c*d^3 - 24*a^3*b^15*c^3*
d - 24*a^5*b^13*c*d^3 + 48*a^5*b^13*c^3*d + 16*a^7*b^11*c*d^3 - 24*a^7*b^11
*c^3*d - 8*a^9*b^9*c*d^3 + 24*a^2*b^16*c^2*d^2 - 36*a^4*b^14*c^2*d^2 + 12*a
^8*b^10*c^2*d^2 - 16*a*b^17*c*d^3))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6
*b^10 + a^8*b^8) + (8*tan(e/2 + (f*x)/2)*(4*a*b^18*c^4 - 12*a^5*b^14*c^4 +
8*a^7*b^12*c^4 + 48*a^3*b^16*d^4 - 156*a^5*b^14*d^4 + 192*a^7*b^12*d^4 - 10
8*a^9*b^10*d^4 + 24*a^11*b^8*d^4 + 48*a*b^18*c^2*d^2 - 96*a^2*b^17*c*d^3 -
48*a^2*b^17*c^3*d + 272*a^4*b^15*c*d^3 + 96*a^4*b^15*c^3*d - 288*a^6*b^13*c
*d^3 - 48*a^6*b^13*c^3*d + 144*a^8*b^11*c*d^3 - 32*a^10*b^9*c*d^3 - 72*a^3*
b^16*c^2*d^2 + 24*a^7*b^12*c^2*d^2))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^
6*b^11 + a^8*b^9) - (d^3*((8*(4*a^2*b^19 - 16*a^4*b^17 + 24*a^6*b^15 - 16*a
^8*b^13 + 4*a^10*b^11))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*
b^8) + (8*tan(e/2 + (f*x)/2)*(12*a*b^21 - 56*a^3*b^19 + 104*a^5*b^17 - 96*a
^7*b^15 + 44*a^9*b^13 - 8*a^11*b^11))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a
^6*b^11 + a^8*b^9))*(3*a*d - 4*b*c)*1i)/b^4)*1i)/b^4))/b^4 - (d^3*(3*a*d -
4*b*c)*((8*(36*a^4*b^11*d^8 - 144*a^6*b^9*d^8 + 216*a^8*b^7*d^8 - 144*a^10*
b^5*d^8 + 36*a^12*b^3*d^8 - 96*a^3*b^12*c*d^7 + 384*a^5*b^10*c*d^7 - 576*a^
7*b^8*c*d^7 + 384*a^9*b^6*c*d^7 - 96*a^11*b^4*c*d^7 + 64*a^2*b^13*c^2*d^6 -
 256*a^4*b^11*c^2*d^6 + 384*a^6*b^9*c^2*d^6 - 256*a^8*b^7*c^2*d^6 + 64*a^10
*b^5*c^2*d^6))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) - (8
*tan(e/2 + (f*x)/2)*(a*b^15*c^8 + 4*a^3*b^13*c^8 + 4*a^5*b^11*c^8 - 72*a^3*
b^13*d^8 + 468*a^5*b^11*d^8 - 936*a^7*b^9*d^8 + 873*a^9*b^7*d^8 - 396*a^11*
b^5*d^8 + 72*a^13*b^3*d^8 - 128*a*b^15*c^2*d^6 + 144*a*b^15*c^4*d^4 + 24*a*
b^15*c^6*d^2 + 192*a^2*b^14*c*d^7 - 24*a^2*b^14*c^7*d - 1440*a^4*b^12*c*d^7
 - 48*a^4*b^12*c^7*d + 2736*a^6*b^10*c*d^7 - 2424*a^8*b^8*c*d^7 + 1056*a^10
*b^6*c*d^7 - 192*a^12*b^4*c*d^7 - 576*a^2*b^14*c^3*d^5 - 336*a^2*b^14*c^5*d
```

^3 + 1440*a^3*b^13*c^2*d^6 + 744*a^3*b^13*c^4*d^4 + 204*a^3*b^13*c^6*d^2 - 96*a^4*b^12*c^3*d^5 - 200*a^4*b^12*c^5*d^3 - 2200*a^5*b^11*c^2*d^6 - 426*a^5*b^11*c^4*d^4 + 24*a^5*b^11*c^6*d^2 + 408*a^6*b^10*c^3*d^5 + 64*a^6*b^10*c^5*d^3 + 1644*a^7*b^9*c^2*d^6 + 144*a^7*b^9*c^4*d^4 - 240*a^8*b^8*c^3*d^5 - 32*a^8*b^8*c^5*d^3 - 632*a^9*b^7*c^2*d^6 + 24*a^9*b^7*c^4*d^4 + 128*a^11*b^5*c^2*d^6))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9) + (d^3*(3*a*d - 4*b*c)*((8*(2*a^2*b^16*c^4 - 6*a^6*b^12*c^4 + 4*a^8*b^10*c^4 + 12*a^2*b^16*d^4 - 36*a^4*b^14*d^4 + 42*a^6*b^12*d^4 - 24*a^8*b^10*d^4 + 6*a^10*b^8*d^4 + 32*a^3*b^15*c*d^3 - 24*a^3*b^15*c^3*d - 24*a^5*b^13*c*d^3 + 48*a^5*b^13*c^3*d + 16*a^7*b^11*c*d^3 - 24*a^7*b^11*c^3*d - 8*a^9*b^9*c*d^3 + 24*a^2*b^16*c^2*d^2 - 36*a^4*b^14*c^2*d^2 + 12*a^8*b^10*c^2*d^2 - 16*a*b^17*c*d^3))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(e/2 + (f*x)/2)*(4*a*b^18*c^4 - 12*a^5*b^14*c^4 + 8*a^7*b^12*c^4 + 48*a^3*b^16*d^4 - 156*a^5*b^14*d^4 + 192*a^7*b^12*d^4 - 108*a^9*b^10*d^4 + 24*a^11*b^8*d^4 + 48*a*b^18*c^2*d^2 - 96*a^2*b^17*c*d^3 - 48*a^2*b^17*c^3*d + 272*a^4*b^15*c*d^3 + 96*a^4*b^15*c^3*d - 288*a^6*b^13*c*d^3 - 48*a^6*b^13*c^3*d + 144*a^8*b^11*c*d^3 - 32*a^10*b^9*c*d^3 - 72*a^3*b^16*c^2*d^2 + 24*a^7*b^12*c^2*d^2))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9) + (d^3*((8*(4*a^2*b^19 - 16*a^4*b^17 + 24*a^6*b^15 - 16*a^8*b^13 + 4*a^10*b^11))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(e/2 + (f*x)/2)*(12*a*b^21 - 56*a^3*b^19 + 104*a^5*b^17 - 96*a^7*b^15 + 44*a^9*b^13 - 8*a^11*b^11))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9))*(3*a*d - 4*b*c)*1i)/b^4)*1i)/b^4))/b^4)/((16*(54*a^12*d^12 - 216*a^6*b^6*d^12 + 378*a^8*b^4*d^12 - 243*a^10*b^2*d^12 + 576*a*b^11*c^5*d^7 + 96*a*b^11*c^7*d^5 + 4*a*b^11*c^9*d^3 + 1296*a^5*b^7*c*d^11 - 1944*a^7*b^5*c*d^11 + 1116*a^9*b^3*c*d^11 - 2352*a^2*b^10*c^4*d^8 - 1384*a^2*b^10*c^6*d^6 - 99*a^2*b^10*c^8*d^4 + 3840*a^3*b^9*c^3*d^9 + 3552*a^3*b^9*c^5*d^7 + 888*a^3*b^9*c^7*d^5 + 16*a^3*b^9*c^9*d^3 - 3144*a^4*b^8*c^2*d^10 - 2598*a^4*b^8*c^4*d^8 - 1412*a^4*b^8*c^6*d^6 - 204*a^4*b^8*c^8*d^4 - 1592*a^5*b^7*c^3*d^9 - 336*a^5*b^7*c^5*d^7 + 240*a^5*b^7*c^7*d^5 + 16*a^5*b^7*c^9*d^3 + 3492*a^6*b^6*c^2*d^10 + 1758*a^6*b^6*c^4*d^8 + 88*a^6*b^6*c^6*d^6 - 12*a^6*b^6*c^8*d^4 - 104*a^7*b^5*c^3*d^9 + 144*a^7*b^5*c^5*d^7 - 1572*a^8*b^4*c^2*d^10 - 678*a^8*b^4*c^4*d^8 - 64*a^8*b^4*c^6*d^6 + 376*a^9*b^3*c^3*d^9 + 96*a^9*b^3*c^5*d^7 + 180*a^10*b^2*c^2*d^10 - 36*a^10*b^2*c^4*d^8 - 216*a^11*b*c*d^11))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (16*tan(e/2 + (f*x)/2)*(216*a^13*d^12 + 432*a^5*b^8*d^12 - 1404*a^7*b^6*d^12 + 1728*a^9*b^4*d^12 - 972*a^11*b^2*d^12 + 768*a*b^12*c^4*d^8 + 64*a*b^12*c^6*d^6 - 2016*a^4*b^9*c*d^11 + 6192*a^6*b^7*c*d^11 - 7200*a^8*b^5*c*d^11 + 3888*a^10*b^3*c*d^11 - 2688*a^2*b^11*c^3*d^9 - 864*a^2*b^11*c^5*d^7 + 3504*a^3*b^10*c^2*d^10 + 36*a^3*b^10*c^4*d^8 + 5648*a^4*b^9*c^3*d^9 + 1536*a^4*b^9*c^5*d^7 - 9672*a^5*b^8*c^2*d^10 - 2304*a^5*b^8*c^4*d^8 - 192*a^5*b^8*c^6*d^6 - 3744*a^6*b^7*c^3*d^9 - 480*a^6*b^7*c^5*d^7 + 9984*a^7*b^6*c^2*d^10 + 1428*a^7*b^6*c^4*d^8 + 128*a^7*b^6*c^6*d^6 + 1296*a^8*b^5*c^3*d^9 - 192*a^8*b^5*c^5*d^7 - 4968*a^9*b^4*c^2*d^10 + 72*a^9*b^4*c^4*d^8 - 512*a^10*b^3*c^3*d^9 + 1152*a^11*b^2*c^2*d^10 - 864*a^12*b*c*d^11))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8* b^9) + (d^3*(3*a*d - 4*b*c)*((8*tan(e/2 + (f*x)/2)*(a*b^15*c^8 + 4*a^3*b^13*c^8 + 4*a^5*b^11*c^8 - 72*a^3*b^13*d^8 + 468*a^5*b^11*d^8 - 936*a^7*b^9*d^8 + 873*a^9*b^7*d^8 - 396*a^11*b^5*d^8 + 72*a^13*b^3*d^8 - 128*a*b^15*c^2*d^6 + 144*a*b^15*c^4*d^4 + 24*a*b^15*c^6*d^2 + 192*a^2*b^14*c*d^7 - 24*a^2*b^14*c^7*d - 1440*a^4*b^12*c*d^7 - 48*a^4*b^12*c^7*d + 2736*a^6*b^10*c*d^7 - 2424*a^8*b^8*c*d^7 + 1056*a^10*b^6*c*d^7 - 192*a^12*b^4*c*d^7 - 576*a^2*b^14*c^3*d^5 - 336*a^2*b^14*c^5*d^3 + 1440*a^3*b^13*c^2*d^6 + 744*a^3*b^13*c^4*d^4 + 204*a^3*b^13*c^6*d^2 - 96*a^4*b^12*c^3*d^5 - 200*a^4*b^12*c^5*d^3 - 2200*a^5*b^11*c^2*d^6 - 426*a^5*b^11*c^4*d^4 + 24*a^5*b^11*c^6*d^2 + 408*a^6*b^10*c^3*d^5 + 64*a^6*b^10*c^5*d^3 + 1644*a^7*b^9*c^2*d^6 + 144*a^7*b^9*c^4*d^4 - 240*a^8*b^8*c^3*d^5 - 32*a^8*b^8*c^5*d^3 - 632*a^9*b^7*c^2*d^6 + 24*a^9*b^7*c^4*d^4 + 128*a^11*b^5*c^2*d^6))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9) - (8*(36*a^4*b^11*d^8 - 144*a^6*b^9*d^8 + 216*a^8*b^7*d^8 - 144*a^10*b^5*d^8 + 36*a^12*b^3*d^8 - 96*a^3*b^12*c*d^7 + 384*a^5*b^10*c*d^7 - 576*a^7*b^8*c*d^7 + 384*a^9*b^6*c*d^7 - 96*a^11*b^4*c*d^7 + 64*a^2*b^13*c^2*d^6 - 256*a^4*b^11*c^2*d^6 + 384*a^6*b^9*c^2*d^6 - 256*a^8*b^7*c^2*d^6 + 64*a^10*b^5*c^2*d^6))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (d^3*(3*a*d - 4*b*c)*((8*(2*a^2*b^16*c^4 - 6*a^6*b^12*c^4 + 4*a^8*b^10*c^4 + 12*a^2*b^16*d^4 - 36*a^4*b^14*d^4 + 42*a^6*b^12*d^4 - 24*a^8*b^10*d^4 + 6*a^10*b^8*d^4 + 32*a^3*b^15*c*d^3 - 24*a^3*b^15*c^3*d - 24*a^5*b^13*c*d^3 + 48*a^5*b^13*c^3*d + 16*a^7*b^11*c*d^3 - 24*a^7*b^11*c^3*d - 8*a^9*b^9*c*d^3 + 24*a^2*b^16*c^2*d^2 - 36*a^4*b^14*c^2*d^2 + 12*a^8*b^10*c^2*d^2 - 16*a*b^17*c*d^3))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(e/2 + (f*x)/2)*(4*a*b^18*c^4 - 12*a^5*b^14*c^4 + 8*a^7*b^12*c^4 + 48*a^3*b^16*d^4 - 156*a^5*b^14*d^4 + 192*a^7*b^12*d^4 - 108*a^9*b^10*d^4 + 24*a^11*b^8*d^4 + 48*a*b^18*c^2*d^2 - 96*a^2*b^17*c*d^3 - 48*a^2*b^17*c^3*d + 272*a^4*b^15*c*d^3 + 96*a^4*b^15*c^3*d - 288*a^6*b^13*c*d^3 - 48*a^6*b^13*c^3*d + 144*a^8*b^11*c*d^3 - 32*a^10*b^9*c*d^3 - 72*a^3*b^16*c^2*d^2 + 24*a^7*b^12*c^2*d^2))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9) - (d^3*((8*(4*a^2*b^19 - 16*a^4*b^17 + 24*a^6*b^15 - 16*a^8*b^13 + 4*a^10*b^11))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(e/2 + (f*x)/2)*(12*a*b^21 - 56*a^3*b^19 + 104*a^5*b^17 - 96*a^7*b^15 + 44*a^9*b^13 - 8*a^11*b^11))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9))*(3*a*d - 4*b*c)*1i)/b^4)*1i)/b^4)*1i)/b^4 + (d^3*(3*a*d - 4*b*c)*((8*(36*a^4*b^11*d^8 - 144*a^6*b^9*d^8 + 216*a^8*b^7*d^8 - 144*a^10*b^5*d^8 + 36*a^12*b^3*d^8 - 96*a^3*b^12*c*d^7 + 384*a^5*b^10*c*d^7 - 576*a^7*b^8*c*d^7 + 384*a^9*b^6*c*d^7 - 96*a^11*b^4*c*d^7 + 64*a^2*b^13*c^2*d^6 - 256*a^4*b^11*c^2*d^6 + 384*a^6*b^9*c^2*d^6 - 256*a^8*b^7*c^2*d^6 + 64*a^10*b^5*c^2*d^6))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) - (8*tan(e/2 + (f*x)/2)*(a*b^15*c^8 + 4*a^3*b^13*c^8 + 4*a^5*b^11*c^8 - 72*a^3*b^13*d^8 + 468*a^5*b^11*d^8 - 936*a^7*b^9*d^8 + 873*a^9*b^7*d^8 - 396*a^11*b^5*d^8 + 72*a^13*b^3*d^8 - 128*a*b^15*c^2*d^6 + 144*a*b^15*c^4*d^4 + 24*a*b^15*c^6*d^2 + 192*a^2*b^14*c*d^7 - 24*a^2*b^14*c^7*d - 1440*a^4*b^12*c*d^7 - 48*a^4*b^12*c^7*d + 2736*a^6*b^10*c*d^7 - 2424*a^8*b^8*c*d^7 + 1056*a^10*b^6*c*d^7 - 192*a^12*b^4*c*d^7 - 576*a^2*b^14*c^3*d^5 - 336*a^2*b^14*c^5*d^3

+ 1440*a^3*b^13*c^2*d^6 + 744*a^3*b^13*c^4*d^4 + 204*a^3*b^13*c^6*d^2 - 96*a^4*b^12*c^3*d^5 - 200*a^4*b^12*c^5*d^3 - 2200*a^5*b^11*c^2*d^6 - 426*a^5*b^11*c^4*d^4 + 24*a^5*b^11*c^6*d^2 + 408*a^6*b^10*c^3*d^5 + 64*a^6*b^10*c^5*d^3 + 1644*a^7*b^9*c^2*d^6 + 144*a^7*b^9*c^4*d^4 - 240*a^8*b^8*c^3*d^5 - 32*a^8*b^8*c^5*d^3 - 632*a^9*b^7*c^2*d^6 + 24*a^9*b^7*c^4*d^4 + 128*a^11*b^5*c^2*d^6))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9) + (d^3*(3*a*d - 4*b*c)*((8*(2*a^2*b^16*c^4 - 6*a^6*b^12*c^4 + 4*a^8*b^10*c^4 + 12*a^2*b^16*d^4 - 36*a^4*b^14*d^4 + 42*a^6*b^12*d^4 - 24*a^8*b^10*d^4 + 6*a^10*b^8*d^4 + 32*a^3*b^15*c*d^3 - 24*a^3*b^15*c^3*d - 24*a^5*b^13*c*d^3 + 48*a^5*b^13*c^3*d + 16*a^7*b^11*c*d^3 - 24*a^7*b^11*c^3*d - 8*a^9*b^9*c*d^3 + 24*a^2*b^16*c^2*d^2 - 36*a^4*b^14*c^2*d^2 + 12*a^8*b^10*c^2*d^2 - 16*a*b^17*c*d^3))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(e/2 + (f*x)/2)*(4*a*b^18*c^4 - 12*a^5*b^14*c^4 + 8*a^7*b^12*c^4 + 48*a^3*b^16*d^4 - 156*a^5*b^14*d^4 + 192*a^7*b^12*d^4 - 108*a^9*b^10*d^4 + 24*a^11*b^8*d^4 + 48*a*b^18*c^2*d^2 - 96*a^2*b^17*c*d^3 - 48*a^2*b^17*c^3*d + 272*a^4*b^15*c*d^3 + 96*a^4*b^15*c^3*d - 288*a^6*b^13*c*d^3 - 48*a^6*b^13*c^3*d + 144*a^8*b^11*c*d^3 - 32*a^10*b^9*c*d^3 - 72*a^3*b^16*c^2*d^2 + 24*a^7*b^12*c^2*d^2))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9) + (d^3*((8*(4*a^2*b^19 - 16*a^4*b^17 + 24*a^6*b^15 - 16*a^8*b^13 + 4*a^10*b^11))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(e/2 + (f*x)/2)*(12*a*b^21 - 56*a^3*b^19 + 104*a^5*b^17 - 96*a^7*b^15 + 44*a^9*b^13 - 8*a^11*b^11))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9))*(3*a*d - 4*b*c)*1i)/b^4)*1i)/b^4)*1i)/b^4))*(3*a*d - 4*b*c))/(b^4*f) - ((6*a^6*d^4 + b^6*c^4 - 4*a^2*b^4*c^4 + 2*a^2*b^4*d^4 - 11*a^4*b^2*d^4 + 20*a^3*b^3*c*d^3 + 8*a^3*b^3*c^3*d - 18*a^2*b^4*c^2*d^2 + 4*a*b^5*c^3*d - 8*a^5*b*c*d^3)/(b^3*(a^2 - b^2)^2) + (2*tan(e/2 + (f*x)/2)^2*(6*a^8*d^4 + b^8*c^4 - 3*a^2*b^6*c^4 - 4*a^4*b^4*c^4 + 4*a^2*b^6*d^4 - 13*a^4*b^4*d^4 - 3*a^6*b^2*d^4 + 20*a^3*b^5*c*d^3 + 12*a^3*b^5*c^3*d + 12*a^5*b^3*c*d^3 + 8*a^5*b^3*c^3*d - 18*a^2*b^6*c^2*d^2 - 18*a^4*b^4*c^2*d^2 + 4*a*b^7*c^3*d - 8*a^7*b*c*d^3))/(a^2*b^3*(a^2 - b^2)^2) + (4*tan(e/2 + (f*x)/2)^3*(6*a^6*d^4 + b^6*c^4 - 4*a^2*b^4*c^4 + 2*a^2*b^4*d^4 - 11*a^4*b^2*d^4 + 20*a^3*b^3*c*d^3 + 8*a^3*b^3*c^3*d - 18*a^2*b^4*c^2*d^2 + 4*a*b^5*c^3*d - 8*a^5*b*c*d^3))/(a*b^2*(a^2 - b^2)^2) + (tan(e/2 + (f*x)/2)*(21*a^6*d^4 + 2*b^6*c^4 - 11*a^2*b^4*c^4 + 8*a^2*b^4*d^4 - 38*a^4*b^2*d^4 + 64*a^3*b^3*c*d^3 + 20*a^3*b^3*c^3*d - 60*a^2*b^4*c^2*d^2 + 6*a^4*b^2*c^2*d^2 + 16*a*b^5*c^3*d - 28*a^5*b*c*d^3))/(a*b^2*(a^2 - b^2)^2) - (tan(e/2 + (f*x)/2)^5*(5*a^2*b^4*c^4 - 2*b^6*c^4 - 3*a^6*d^4 + 6*a^4*b^2*d^4 - 16*a^3*b^3*c*d^3 - 12*a^3*b^3*c^3*d + 12*a^2*b^4*c^2*d^2 + 6*a^4*b^2*c^2*d^2 + 4*a^5*b*c*d^3))/(a*b^2*(a^2 - b^2)^2) + (tan(e/2 + (f*x)/2)^4*(6*a^8*d^4 + 2*b^8*c^4 - 7*a^2*b^6*c^4 - 4*a^4*b^4*c^4 - 12*a^4*b^4*d^4 - 3*a^6*b^2*d^4 + 40*a^3*b^5*c*d^3 + 20*a^3*b^5*c^3*d + 4*a^5*b^3*c*d^3 + 8*a^5*b^3*c^3*d - 36*a^2*b^6*c^2*d^2 - 18*a^4*b^4*c^2*d^2 + 8*a*b^7*c^3*d - 8*a^7*b*c*d^3))/(a^2*b^3*(a^2 - b^2)^2))/(f*(tan(e/2 + (f*x)/2)^2*(3*a^2 + 4*b^2) + tan(e/2 + (f*x)/2)^4*(3*a^2 + 4*b^2) + a^2*tan(e/2 + (f*x)/2)^6 + a^2 + 8*a*b*tan(e/2 + (f*x)/2)^3 + 4*a*b*tan(e/2 + (f*x)/2)^5 + 4*a*b*tan(e/2 + (f*x)/2))) + (atan((((a*d - b*c)^2*(-(a + b)^5*(a - b)^5)^(1/2

)*((8*tan(e/2 + (f*x)/2)*(a*b^15*c^8 + 4*a^3*b^13*c^8 + 4*a^5*b^11*c^8 - 72
*a^3*b^13*d^8 + 468*a^5*b^11*d^8 - 936*a^7*b^9*d^8 + 873*a^9*b^7*d^8 - 396*
a^11*b^5*d^8 + 72*a^13*b^3*d^8 - 128*a*b^15*c^2*d^6 + 144*a*b^15*c^4*d^4 +
24*a*b^15*c^6*d^2 + 192*a^2*b^14*c*d^7 - 24*a^2*b^14*c^7*d - 1440*a^4*b^12*
c*d^7 - 48*a^4*b^12*c^7*d + 2736*a^6*b^10*c*d^7 - 2424*a^8*b^8*c*d^7 + 1056
*a^10*b^6*c*d^7 - 192*a^12*b^4*c*d^7 - 576*a^2*b^14*c^3*d^5 - 336*a^2*b^14*
c^5*d^3 + 1440*a^3*b^13*c^2*d^6 + 744*a^3*b^13*c^4*d^4 + 204*a^3*b^13*c^6*d
^2 - 96*a^4*b^12*c^3*d^5 - 200*a^4*b^12*c^5*d^3 - 2200*a^5*b^11*c^2*d^6 - 4
26*a^5*b^11*c^4*d^4 + 24*a^5*b^11*c^6*d^2 + 408*a^6*b^10*c^3*d^5 + 64*a^6*b
^10*c^5*d^3 + 1644*a^7*b^9*c^2*d^6 + 144*a^7*b^9*c^4*d^4 - 240*a^8*b^8*c^3*
d^5 - 32*a^8*b^8*c^5*d^3 - 632*a^9*b^7*c^2*d^6 + 24*a^9*b^7*c^4*d^4 + 128*a
^11*b^5*c^2*d^6))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9) -
(8*(36*a^4*b^11*d^8 - 144*a^6*b^9*d^8 + 216*a^8*b^7*d^8 - 144*a^10*b^5*d^8
+ 36*a^12*b^3*d^8 - 96*a^3*b^12*c*d^7 + 384*a^5*b^10*c*d^7 - 576*a^7*b^8*c
*d^7 + 384*a^9*b^6*c*d^7 - 96*a^11*b^4*c*d^7 + 64*a^2*b^13*c^2*d^6 - 256*a^
4*b^11*c^2*d^6 + 384*a^6*b^9*c^2*d^6 - 256*a^8*b^7*c^2*d^6 + 64*a^10*b^5*c^
2*d^6))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + ((a*d - b
c)^2(-(a + b)^5*(a - b)^5)^(1/2)*((8*(2*a^2*b^16*c^4 - 6*a^6*b^12*c^4 + 4
*a^8*b^10*c^4 + 12*a^2*b^16*d^4 - 36*a^4*b^14*d^4 + 42*a^6*b^12*d^4 - 24*a^
8*b^10*d^4 + 6*a^10*b^8*d^4 + 32*a^3*b^15*c*d^3 - 24*a^3*b^15*c^3*d - 24*a^
5*b^13*c*d^3 + 48*a^5*b^13*c^3*d + 16*a^7*b^11*c*d^3 - 24*a^7*b^11*c^3*d -
8*a^9*b^9*c*d^3 + 24*a^2*b^16*c^2*d^2 - 36*a^4*b^14*c^2*d^2 + 12*a^8*b^10*c
^2*d^2 - 16*a*b^17*c*d^3))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a
^8*b^8) + (8*tan(e/2 + (f*x)/2)*(4*a*b^18*c^4 - 12*a^5*b^14*c^4 + 8*a^7*b^1
2*c^4 + 48*a^3*b^16*d^4 - 156*a^5*b^14*d^4 + 192*a^7*b^12*d^4 - 108*a^9*b^1
0*d^4 + 24*a^11*b^8*d^4 + 48*a*b^18*c^2*d^2 - 96*a^2*b^17*c*d^3 - 48*a^2*b^
17*c^3*d + 272*a^4*b^15*c*d^3 + 96*a^4*b^15*c^3*d - 288*a^6*b^13*c*d^3 - 48
*a^6*b^13*c^3*d + 144*a^8*b^11*c*d^3 - 32*a^10*b^9*c*d^3 - 72*a^3*b^16*c^2*
d^2 + 24*a^7*b^12*c^2*d^2))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 +
a^8*b^9) - (((8*(4*a^2*b^19 - 16*a^4*b^17 + 24*a^6*b^15 - 16*a^8*b^13 + 4*a
^10*b^11))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan
(e/2 + (f*x)/2)*(12*a*b^21 - 56*a^3*b^19 + 104*a^5*b^17 - 96*a^7*b^15 + 44*
a^9*b^13 - 8*a^11*b^11))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8
b^9))(a*d - b*c)^2*(-(a + b)^5*(a - b)^5)^(1/2)*(6*a^4*d^2 + b^4*c^2 + 12
*b^4*d^2 + 2*a^2*b^2*c^2 - 15*a^2*b^2*d^2 - 10*a*b^3*c*d + 4*a^3*b*c*d))/(2
*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)))*(6
*a^4*d^2 + b^4*c^2 + 12*b^4*d^2 + 2*a^2*b^2*c^2 - 15*a^2*b^2*d^2 - 10*a*b^3
*c*d + 4*a^3*b*c*d))/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a
^8*b^6 - a^10*b^4)))*(6*a^4*d^2 + b^4*c^2 + 12*b^4*d^2 + 2*a^2*b^2*c^2 - 15
*a^2*b^2*d^2 - 10*a*b^3*c*d + 4*a^3*b*c*d)*1i)/(2*(b^14 - 5*a^2*b^12 + 10*a
^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)) - ((a*d - b*c)^2*(-(a + b)^5*
(a - b)^5)^(1/2)*((8*(36*a^4*b^11*d^8 - 144*a^6*b^9*d^8 + 216*a^8*b^7*d^8 -
144*a^10*b^5*d^8 + 36*a^12*b^3*d^8 - 96*a^3*b^12*c*d^7 + 384*a^5*b^10*c*d^
7 - 576*a^7*b^8*c*d^7 + 384*a^9*b^6*c*d^7 - 96*a^11*b^4*c*d^7 + 64*a^2*b^13
*c^2*d^6 - 256*a^4*b^11*c^2*d^6 + 384*a^6*b^9*c^2*d^6 - 256*a^8*b^7*c^2*d^6

+ 64*a^10*b^5*c^2*d^6))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) - (8*tan(e/2 + (f*x)/2)*(a*b^15*c^8 + 4*a^3*b^13*c^8 + 4*a^5*b^11*c^8 - 72*a^3*b^13*d^8 + 468*a^5*b^11*d^8 - 936*a^7*b^9*d^8 + 873*a^9*b^7*d^8 - 396*a^11*b^5*d^8 + 72*a^13*b^3*d^8 - 128*a*b^15*c^2*d^6 + 144*a*b^15*c^4*d^4 + 24*a*b^15*c^6*d^2 + 192*a^2*b^14*c*d^7 - 24*a^2*b^14*c^7*d - 1440*a^4*b^12*c*d^7 - 48*a^4*b^12*c^7*d + 2736*a^6*b^10*c*d^7 - 2424*a^8*b^8*c*d^7 + 1056*a^10*b^6*c*d^7 - 192*a^12*b^4*c*d^7 - 576*a^2*b^14*c^3*d^5 - 336*a^2*b^14*c^5*d^3 + 1440*a^3*b^13*c^2*d^6 + 744*a^3*b^13*c^4*d^4 + 204*a^3*b^13*c^6*d^2 - 96*a^4*b^12*c^3*d^5 - 200*a^4*b^12*c^5*d^3 - 2200*a^5*b^11*c^2*d^6 - 426*a^5*b^11*c^4*d^4 + 24*a^5*b^11*c^6*d^2 + 408*a^6*b^10*c^3*d^5 + 64*a^6*b^10*c^5*d^3 + 1644*a^7*b^9*c^2*d^6 + 144*a^7*b^9*c^4*d^4 - 240*a^8*b^8*c^3*d^5 - 32*a^8*b^8*c^5*d^3 - 632*a^9*b^7*c^2*d^6 + 24*a^9*b^7*c^4*d^4 + 128*a^11*b^5*c^2*d^6))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9) + ((a*d - b*c)^2*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(2*a^2*b^16*c^4 - 6*a^6*b^12*c^4 + 4*a^8*b^10*c^4 + 12*a^2*b^16*d^4 - 36*a^4*b^14*d^4 + 42*a^6*b^12*d^4 - 24*a^8*b^10*d^4 + 6*a^10*b^8*d^4 + 32*a^3*b^15*c*d^3 - 24*a^3*b^15*c^3*d - 24*a^5*b^13*c*d^3 + 48*a^5*b^13*c^3*d + 16*a^7*b^11*c*d^3 - 24*a^7*b^11*c^3*d - 8*a^9*b^9*c*d^3 + 24*a^2*b^16*c^2*d^2 - 36*a^4*b^14*c^2*d^2 + 12*a^8*b^10*c^2*d^2 - 16*a*b^17*c*d^3))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(e/2 + (f*x)/2)*(4*a*b^18*c^4 - 12*a^5*b^14*c^4 + 8*a^7*b^12*c^4 + 48*a^3*b^16*d^4 - 156*a^5*b^14*d^4 + 192*a^7*b^12*d^4 - 108*a^9*b^10*d^4 + 24*a^11*b^8*d^4 + 48*a*b^18*c^2*d^2 - 96*a^2*b^17*c*d^3 - 48*a^2*b^17*c^3*d + 272*a^4*b^15*c*d^3 + 96*a^4*b^15*c^3*d - 288*a^6*b^13*c*d^3 - 48*a^6*b^13*c^3*d + 144*a^8*b^11*c*d^3 - 32*a^10*b^9*c*d^3 - 72*a^3*b^16*c^2*d^2 + 24*a^7*b^12*c^2*d^2))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9) + (((8*(4*a^2*b^19 - 16*a^4*b^17 + 24*a^6*b^15 - 16*a^8*b^13 + 4*a^10*b^11))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(e/2 + (f*x)/2)*(12*a*b^21 - 56*a^3*b^19 + 104*a^5*b^17 - 96*a^7*b^15 + 44*a^9*b^13 - 8*a^11*b^11))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9))*(a*d - b*c)^2*(-(a + b)^5*(a - b)^5)^(1/2)*(6*a^4*d^2 + b^4*c^2 + 12*b^4*d^2 + 2*a^2*b^2*c^2 - 15*a^2*b^2*d^2 - 10*a*b^3*c*d + 4*a^3*b*c*d))/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)))*(6*a^4*d^2 + b^4*c^2 + 12*b^4*d^2 + 2*a^2*b^2*c^2 - 15*a^2*b^2*d^2 - 10*a*b^3*c*d + 4*a^3*b*c*d))/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)))*(6*a^4*d^2 + b^4*c^2 + 12*b^4*d^2 + 2*a^2*b^2*c^2 - 15*a^2*b^2*d^2 - 10*a*b^3*c*d + 4*a^3*b*c*d)*1i)/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)))/((16*(54*a^12*d^12 - 216*a^6*b^6*d^12 + 378*a^8*b^4*d^12 - 243*a^10*b^2*d^12 + 576*a*b^11*c^5*d^7 + 96*a*b^11*c^7*d^5 + 4*a*b^11*c^9*d^3 + 1296*a^5*b^7*c*d^11 - 1944*a^7*b^5*c*d^11 + 1116*a^9*b^3*c*d^11 - 2352*a^2*b^10*c^4*d^8 - 1384*a^2*b^10*c^6*d^6 - 99*a^2*b^10*c^8*d^4 + 3840*a^3*b^9*c^3*d^9 + 3552*a^3*b^9*c^5*d^7 + 888*a^3*b^9*c^7*d^5 + 16*a^3*b^9*c^9*d^3 - 3144*a^4*b^8*c^2*d^10 - 2598*a^4*b^8*c^4*d^8 - 1412*a^4*b^8*c^6*d^6 - 204*a^4*b^8*c^8*d^4 - 1592*a^5*b^7*c^3*d^9 - 336*a^5*b^7*c^5*d^7 + 240*a^5*b^7*c^7*d^5 + 16*a^5*b^7*c^9*d^3 + 3492*a^6*b^6*c^2*d^10 + 1758*a^6*b^6*c^4*d^8 + 88*a^6*b^6*c^6*d^6 - 1

2*a^6*b^6*c^8*d^4 - 104*a^7*b^5*c^3*d^9 + 144*a^7*b^5*c^5*d^7 - 1572*a^8*b^4*c^2*d^10 - 678*a^8*b^4*c^4*d^8 - 64*a^8*b^4*c^6*d^6 + 376*a^9*b^3*c^3*d^9 + 96*a^9*b^3*c^5*d^7 + 180*a^10*b^2*c^2*d^10 - 36*a^10*b^2*c^4*d^8 - 216*a^11*b*c*d^11))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (16*tan(e/2 + (f*x)/2)*(216*a^13*d^12 + 432*a^5*b^8*d^12 - 1404*a^7*b^6*d^12 + 1728*a^9*b^4*d^12 - 972*a^11*b^2*d^12 + 768*a*b^12*c^4*d^8 + 64*a*b^12*c^6*d^6 - 2016*a^4*b^9*c*d^11 + 6192*a^6*b^7*c*d^11 - 7200*a^8*b^5*c*d^11 + 3888*a^10*b^3*c*d^11 - 2688*a^2*b^11*c^3*d^9 - 864*a^2*b^11*c^5*d^7 + 3504*a^3*b^10*c^2*d^10 + 36*a^3*b^10*c^4*d^8 + 5648*a^4*b^9*c^3*d^9 + 1536*a^4*b^9*c^5*d^7 - 9672*a^5*b^8*c^2*d^10 - 2304*a^5*b^8*c^4*d^8 - 192*a^5*b^8*c^6*d^6 - 3744*a^6*b^7*c^3*d^9 - 480*a^6*b^7*c^5*d^7 + 9984*a^7*b^6*c^2*d^10 + 1428*a^7*b^6*c^4*d^8 + 128*a^7*b^6*c^6*d^6 + 1296*a^8*b^5*c^3*d^9 - 192*a^8*b^5*c^5*d^7 - 4968*a^9*b^4*c^2*d^10 + 72*a^9*b^4*c^4*d^8 - 512*a^10*b^3*c^3*d^9 + 1152*a^11*b^2*c^2*d^10 - 864*a^12*b*c*d^11))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9) + ((a*d - b*c)^2*(-(a + b)^5*(a - b)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(a*b^15*c^8 + 4*a^3*b^13*c^8 + 4*a^5*b^11*c^8 - 72*a^3*b^13*d^8 + 468*a^5*b^11*d^8 - 936*a^7*b^9*d^8 + 873*a^9*b^7*d^8 - 396*a^11*b^5*d^8 + 72*a^13*b^3*d^8 - 128*a*b^15*c^2*d^6 + 144*a*b^15*c^4*d^4 + 24*a*b^15*c^6*d^2 + 192*a^2*b^14*c*d^7 - 24*a^2*b^14*c^7*d - 1440*a^4*b^12*c*d^7 - 48*a^4*b^12*c^7*d + 2736*a^6*b^10*c*d^7 - 2424*a^8*b^8*c*d^7 + 1056*a^10*b^6*c*d^7 - 192*a^12*b^4*c*d^7 - 576*a^2*b^14*c^3*d^5 - 336*a^2*b^14*c^5*d^3 + 1440*a^3*b^13*c^2*d^6 + 744*a^3*b^13*c^4*d^4 + 204*a^3*b^13*c^6*d^2 - 96*a^4*b^12*c^3*d^5 - 200*a^4*b^12*c^5*d^3 - 2200*a^5*b^11*c^2*d^6 - 426*a^5*b^11*c^4*d^4 + 24*a^5*b^11*c^6*d^2 + 408*a^6*b^10*c^3*d^5 + 64*a^6*b^10*c^5*d^3 + 1644*a^7*b^9*c^2*d^6 + 144*a^7*b^9*c^4*d^4 - 240*a^8*b^8*c^3*d^5 - 32*a^8*b^8*c^5*d^3 - 632*a^9*b^7*c^2*d^6 + 24*a^9*b^7*c^4*d^4 + 128*a^11*b^5*c^2*d^6))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9) - (8*(36*a^4*b^11*d^8 - 144*a^6*b^9*d^8 + 216*a^8*b^7*d^8 - 144*a^10*b^5*d^8 + 36*a^12*b^3*d^8 - 96*a^3*b^12*c*d^7 + 384*a^5*b^10*c*d^7 - 576*a^7*b^8*c*d^7 + 384*a^9*b^6*c*d^7 - 96*a^11*b^4*c*d^7 + 64*a^2*b^13*c^2*d^6 - 256*a^4*b^11*c^2*d^6 + 384*a^6*b^9*c^2*d^6 - 256*a^8*b^7*c^2*d^6 + 64*a^10*b^5*c^2*d^6))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + ((a*d - b*c)^2*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(2*a^2*b^16*c^4 - 6*a^6*b^12*c^4 + 4*a^8*b^10*c^4 + 12*a^2*b^16*d^4 - 36*a^4*b^14*d^4 + 42*a^6*b^12*d^4 - 24*a^8*b^10*d^4 + 6*a^10*b^8*d^4 + 32*a^3*b^15*c*d^3 - 24*a^3*b^15*c^3*d - 24*a^5*b^13*c*d^3 + 48*a^5*b^13*c^3*d + 16*a^7*b^11*c*d^3 - 24*a^7*b^11*c^3*d - 8*a^9*b^9*c*d^3 + 24*a^2*b^16*c^2*d^2 - 36*a^4*b^14*c^2*d^2 + 12*a^8*b^10*c^2*d^2 - 16*a*b^17*c*d^3))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*tan(e/2 + (f*x)/2)*(4*a*b^18*c^4 - 12*a^5*b^14*c^4 + 8*a^7*b^12*c^4 + 48*a^3*b^16*d^4 - 156*a^5*b^14*d^4 + 192*a^7*b^12*d^4 - 108*a^9*b^10*d^4 + 24*a^11*b^8*d^4 + 48*a*b^18*c^2*d^2 - 96*a^2*b^17*c*d^3 - 48*a^2*b^17*c^3*d + 272*a^4*b^15*c*d^3 + 96*a^4*b^15*c^3*d - 288*a^6*b^13*c*d^3 - 48*a^6*b^13*c^3*d + 144*a^8*b^11*c*d^3 - 32*a^10*b^9*c*d^3 - 72*a^3*b^16*c^2*d^2 + 24*a^7*b^12*c^2*d^2))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b^9) - (((8*(4*a^2*b^19 - 16*a^4*b^17 + 24*a^6*b^15 - 16*a^8*b^13 +

4*a^10*b^11))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8*b^8) + (8*
tan(e/2 + (f*x)/2)*(12*a*b^21 - 56*a^3*b^19 + 104*a^5*b^17 - 96*a^7*b^15 +
44*a^9*b^13 - 8*a^11*b^11))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 +
a^8*b^9))*(a*d - b*c)^2*(-(a + b)^5*(a - b)^5)^(1/2)*(6*a^4*d^2 + b^4*c^2 +
12*b^4*d^2 + 2*a^2*b^2*c^2 - 15*a^2*b^2*d^2 - 10*a*b^3*c*d + 4*a^3*b*c*d))
/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)))
*(6*a^4*d^2 + b^4*c^2 + 12*b^4*d^2 + 2*a^2*b^2*c^2 - 15*a^2*b^2*d^2 - 10*a*
b^3*c*d + 4*a^3*b*c*d))/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 +
5*a^8*b^6 - a^10*b^4)))*(6*a^4*d^2 + b^4*c^2 + 12*b^4*d^2 + 2*a^2*b^2*c^2 -
15*a^2*b^2*d^2 - 10*a*b^3*c*d + 4*a^3*b*c*d))/(2*(b^14 - 5*a^2*b^12 + 10*a
^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)) + ((a*d - b*c)^2*(-(a + b)^5*
(a - b)^5)^(1/2)*((8*(36*a^4*b^11*d^8 - 144*a^6*b^9*d^8 + 216*a^8*b^7*d^8 -
144*a^10*b^5*d^8 + 36*a^12*b^3*d^8 - 96*a^3*b^12*c*d^7 + 384*a^5*b^10*c*d^
7 - 576*a^7*b^8*c*d^7 + 384*a^9*b^6*c*d^7 - 96*a^11*b^4*c*d^7 + 64*a^2*b^13
*c^2*d^6 - 256*a^4*b^11*c^2*d^6 + 384*a^6*b^9*c^2*d^6 - 256*a^8*b^7*c^2*d^6
+ 64*a^10*b^5*c^2*d^6))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^8
*b^8) - (8*tan(e/2 + (f*x)/2)*(a*b^15*c^8 + 4*a^3*b^13*c^8 + 4*a^5*b^11*c^8
- 72*a^3*b^13*d^8 + 468*a^5*b^11*d^8 - 936*a^7*b^9*d^8 + 873*a^9*b^7*d^8 -
396*a^11*b^5*d^8 + 72*a^13*b^3*d^8 - 128*a*b^15*c^2*d^6 + 144*a*b^15*c^4*d
^4 + 24*a*b^15*c^6*d^2 + 192*a^2*b^14*c*d^7 - 24*a^2*b^14*c^7*d - 1440*a^4*
b^12*c*d^7 - 48*a^4*b^12*c^7*d + 2736*a^6*b^10*c*d^7 - 2424*a^8*b^8*c*d^7 +
1056*a^10*b^6*c*d^7 - 192*a^12*b^4*c*d^7 - 576*a^2*b^14*c^3*d^5 - 336*a^2*
b^14*c^5*d^3 + 1440*a^3*b^13*c^2*d^6 + 744*a^3*b^13*c^4*d^4 + 204*a^3*b^13*
c^6*d^2 - 96*a^4*b^12*c^3*d^5 - 200*a^4*b^12*c^5*d^3 - 2200*a^5*b^11*c^2*d^
6 - 426*a^5*b^11*c^4*d^4 + 24*a^5*b^11*c^6*d^2 + 408*a^6*b^10*c^3*d^5 + 64*
a^6*b^10*c^5*d^3 + 1644*a^7*b^9*c^2*d^6 + 144*a^7*b^9*c^4*d^4 - 240*a^8*b^8
*c^3*d^5 - 32*a^8*b^8*c^5*d^3 - 632*a^9*b^7*c^2*d^6 + 24*a^9*b^7*c^4*d^4 +
128*a^11*b^5*c^2*d^6))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4*a^6*b^11 + a^8*b
^9) + ((a*d - b*c)^2*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(2*a^2*b^16*c^4 - 6*a
^6*b^12*c^4 + 4*a^8*b^10*c^4 + 12*a^2*b^16*d^4 - 36*a^4*b^14*d^4 + 42*a^6*b
^12*d^4 - 24*a^8*b^10*d^4 + 6*a^10*b^8*d^4 + 32*a^3*b^15*c*d^3 - 24*a^3*b^1
5*c^3*d - 24*a^5*b^13*c*d^3 + 48*a^5*b^13*c^3*d + 16*a^7*b^11*c*d^3 - 24*a^
7*b^11*c^3*d - 8*a^9*b^9*c*d^3 + 24*a^2*b^16*c^2*d^2 - 36*a^4*b^14*c^2*d^2
+ 12*a^8*b^10*c^2*d^2 - 16*a*b^17*c*d^3))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 -
4*a^6*b^10 + a^8*b^8) + (8*tan(e/2 + (f*x)/2)*(4*a*b^18*c^4 - 12*a^5*b^14*
c^4 + 8*a^7*b^12*c^4 + 48*a^3*b^16*d^4 - 156*a^5*b^14*d^4 + 192*a^7*b^12*d^
4 - 108*a^9*b^10*d^4 + 24*a^11*b^8*d^4 + 48*a*b^18*c^2*d^2 - 96*a^2*b^17*c*
d^3 - 48*a^2*b^17*c^3*d + 272*a^4*b^15*c*d^3 + 96*a^4*b^15*c^3*d - 288*a^6*
b^13*c*d^3 - 48*a^6*b^13*c^3*d + 144*a^8*b^11*c*d^3 - 32*a^10*b^9*c*d^3 - 7
2*a^3*b^16*c^2*d^2 + 24*a^7*b^12*c^2*d^2))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13
- 4*a^6*b^11 + a^8*b^9) + (((8*(4*a^2*b^19 - 16*a^4*b^17 + 24*a^6*b^15 - 16
*a^8*b^13 + 4*a^10*b^11))/(b^16 - 4*a^2*b^14 + 6*a^4*b^12 - 4*a^6*b^10 + a^
8*b^8) + (8*tan(e/2 + (f*x)/2)*(12*a*b^21 - 56*a^3*b^19 + 104*a^5*b^17 - 96
*a^7*b^15 + 44*a^9*b^13 - 8*a^11*b^11))/(b^17 - 4*a^2*b^15 + 6*a^4*b^13 - 4
*a^6*b^11 + a^8*b^9))*(a*d - b*c)^2*(-(a + b)^5*(a - b)^5)^(1/2)*(6*a^4*d^2

```
 + b^4*c^2 + 12*b^4*d^2 + 2*a^2*b^2*c^2 - 15*a^2*b^2*d^2 - 10*a*b^3*c*d + 4
*a^3*b*c*d))/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 -
 a^10*b^4)))*(6*a^4*d^2 + b^4*c^2 + 12*b^4*d^2 + 2*a^2*b^2*c^2 - 15*a^2*b^2
*d^2 - 10*a*b^3*c*d + 4*a^3*b*c*d))/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 1
0*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)))*(6*a^4*d^2 + b^4*c^2 + 12*b^4*d^2 + 2*a
^2*b^2*c^2 - 15*a^2*b^2*d^2 - 10*a*b^3*c*d + 4*a^3*b*c*d))/(2*(b^14 - 5*a^2
*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4))))*(a*d - b*c)^2*(
-(a + b)^5*(a - b)^5)^(1/2)*(6*a^4*d^2 + b^4*c^2 + 12*b^4*d^2 + 2*a^2*b^2*c
^2 - 15*a^2*b^2*d^2 - 10*a*b^3*c*d + 4*a^3*b*c*d)*1i)/(f*(b^14 - 5*a^2*b^12
 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**4/(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.716 $\int \frac{(c+d\sin(e+fx))^3}{(a+b\sin(e+fx))^3}\,dx$

Optimal. Leaf size=248

$$\frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))}{2bf\left(a^2-b^2\right)(a+b\sin(e+fx))^2}+\frac{(bc-ad)^2\left(2a^2d+3abc-5b^2d\right)\cos(e+fx)}{2b^2f\left(a^2-b^2\right)^2(a+b\sin(e+fx))}+\frac{(bc-ad)\left(2a^4d^2+2a^3bc\right.}{}$$

```
[Out] d^3*x/b^3+(-a*d+b*c)*(2*a^3*b*c*d-8*a*b^3*c*d+2*a^4*d^2+a^2*b^2*(2*c^2-5*d^
2)+b^4*(c^2+6*d^2))*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/b^3/(a
^2-b^2)^(5/2)/f+1/2*(-a*d+b*c)^2*(2*a^2*d+3*a*b*c-5*b^2*d)*cos(f*x+e)/b^2/(
a^2-b^2)^2/f/(a+b*sin(f*x+e))+1/2*(-a*d+b*c)^2*cos(f*x+e)*(c+d*sin(f*x+e))/
b/(a^2-b^2)/f/(a+b*sin(f*x+e))^2
```

Rubi [A] time = 0.81, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2792, 3021, 2735, 2660, 618, 204}

$$\frac{(bc-ad)\left(a^2b^2\left(2c^2-5d^2\right)+2a^3bcd+2a^4d^2-8ab^3cd+b^4\left(c^2+6d^2\right)\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3f\left(a^2-b^2\right)^{5/2}}+\frac{(bc-ad)^2\cos(e}{2bf\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^3/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (d^3*x)/b^3 + ((b*c - a*d)*(2*a^3*b*c*d - 8*a*b^3*c*d + 2*a^4*d^2 + a^2*b^2
*(2*c^2 - 5*d^2) + b^4*(c^2 + 6*d^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[
a^2 - b^2]])/(b^3*(a^2 - b^2)^(5/2)*f) + ((b*c - a*d)^2*(3*a*b*c + 2*a^2*d
- 5*b^2*d)*Cos[e + f*x])/(2*b^2*(a^2 - b^2)^2*f*(a + b*Sin[e + f*x])) + ((b
*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x]))/(2*b*(a^2 - b^2)*f*(a + b*Si
n[e + f*x])^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^3}{(a+b\sin(e+fx))^3}\,dx = \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} - \frac{\int \frac{5b^2c^2d+a^2d^3-2abc\left(c^2+2d^2\right)-\left(a^2cd^2+2abd\left(2c^2+d^2\right.\right.}{(a+b\sin}}{2b\left(a^2\right.}$$

$$= \frac{(bc-ad)^2\left(3abc+2a^2d-5b^2d\right)\cos(e+fx)}{2b^2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2}$$

$$= \frac{d^3x}{b^3} + \frac{(bc-ad)^2\left(3abc+2a^2d-5b^2d\right)\cos(e+fx)}{2b^2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{(bc-ad)^2\cos(e+fx)(c+d\sin}{2b\left(a^2-b^2\right)f(a+b\sin(e-}$$

$$= \frac{d^3x}{b^3} + \frac{(bc-ad)^2\left(3abc+2a^2d-5b^2d\right)\cos(e+fx)}{2b^2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{(bc-ad)^2\cos(e+fx)(c+d\sin}{2b\left(a^2-b^2\right)f(a+b\sin(e-}$$

$$= \frac{d^3x}{b^3} + \frac{(bc-ad)^2\left(3abc+2a^2d-5b^2d\right)\cos(e+fx)}{2b^2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{(bc-ad)^2\cos(e+fx)(c+d\sin}{2b\left(a^2-b^2\right)f(a+b\sin(e-}$$

$$= \frac{d^3x}{b^3} + \frac{(bc-ad)\left(2a^2b^2c^2+b^4c^2+2a^3bcd-8ab^3cd+2a^4d^2-5a^2b^2d^2+6b^4d^2\right)\tan^{}}{b^3\left(a^2-b^2\right)^{5/2} f}$$

Mathematica [B] time = 2.36, size = 524, normalized size = 2.11

$$\frac{4a^6d^3e+4a^6d^3fx+8a^5bd^3e\sin(e+fx)+8a^5bd^3fx\sin(e+fx)+3a^4b^2d^3\sin(2(e+fx))-6a^4b^2d^3e-6a^4b^2d^3fx-3a^3b^3cd^2\sin(2(e+fx))-16a^3b^3d^3e\sin(e+fx)-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^3/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((-4*(2*a^5*d^3 - 5*a^3*b^2*d^3 + 3*a*b^4*d*(3*c^2 + 2*d^2) - a^2*b^3*c*(2*
c^2 + 3*d^2) - b^5*c*(c^2 + 6*d^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^
2 - b^2]])/(a^2 - b^2)^(5/2) + (4*a^6*d^3*e - 6*a^4*b^2*d^3*e + 2*b^6*d^3*e
 + 4*a^6*d^3*f*x - 6*a^4*b^2*d^3*f*x + 2*b^6*d^3*f*x - 2*b*(b*c - a*d)^2*(-
4*a^2*b*c + b^3*c - 2*a^3*d + 5*a*b^2*d)*Cos[e + f*x] - 2*(-(a^2*b) + b^3)^
2*d^3*(e + f*x)*Cos[2*(e + f*x)] + 8*a^5*b*d^3*e*Sin[e + f*x] - 16*a^3*b^3*
d^3*e*Sin[e + f*x] + 8*a*b^5*d^3*e*Sin[e + f*x] + 8*a^5*b*d^3*f*x*Sin[e + f
```

```
*x] - 16*a^3*b^3*d^3*f*x*Sin[e + f*x] + 8*a*b^5*d^3*f*x*Sin[e + f*x] + 3*a*
b^5*c^3*Sin[2*(e + f*x)] - 3*a^2*b^4*c^2*d*Sin[2*(e + f*x)] - 6*b^6*c^2*d*S
in[2*(e + f*x)] - 3*a^3*b^3*c*d^2*Sin[2*(e + f*x)] + 12*a*b^5*c*d^2*Sin[2*(
e + f*x)] + 3*a^4*b^2*d^3*Sin[2*(e + f*x)] - 6*a^2*b^4*d^3*Sin[2*(e + f*x)]
)/((a^2 - b^2)^2*(a + b*Sin[e + f*x])^2))/(4*b^3*f)
```

fricas [B] time = 0.78, size = 1631, normalized size = 6.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d^3*f*x*cos(f*x + e)^2 - 4*
(a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8)*d^3*f*x - ((2*a^4*b^3 + 3*a^2*b^5 + b^7
)*c^3 - 9*(a^3*b^4 + a*b^6)*c^2*d + 3*(a^4*b^3 + 3*a^2*b^5 + 2*b^7)*c*d^2 -
 (2*a^7 - 3*a^5*b^2 + a^3*b^4 + 6*a*b^6)*d^3 + (9*a*b^6*c^2*d - (2*a^2*b^5
+ b^7)*c^3 - 3*(a^2*b^5 + 2*b^7)*c*d^2 + (2*a^5*b^2 - 5*a^3*b^4 + 6*a*b^6)*
d^3)*cos(f*x + e)^2 - 2*(9*a^2*b^5*c^2*d - (2*a^3*b^4 + a*b^6)*c^3 - 3*(a^3
*b^4 + 2*a*b^6)*c*d^2 + (2*a^6*b - 5*a^4*b^3 + 6*a^2*b^5)*d^3)*sin(f*x + e)
)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e)
- a^2 - b^2 - 2*(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 +
b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)) - 2*((4*a^4*b^
4 - 5*a^2*b^6 + b^8)*c^3 - 3*(2*a^5*b^3 - a^3*b^5 - a*b^7)*c^2*d + 9*(a^4*b
^4 - a^2*b^6)*c*d^2 + (2*a^7*b - 7*a^5*b^3 + 5*a^3*b^5)*d^3)*cos(f*x + e) -
 2*(4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d^3*f*x + 3*((a^3*b^5 - a*b^7
)*c^3 - (a^4*b^4 + a^2*b^6 - 2*b^8)*c^2*d - (a^5*b^3 - 5*a^3*b^5 + 4*a*b^7)
*c*d^2 + (a^6*b^2 - 3*a^4*b^4 + 2*a^2*b^6)*d^3)*cos(f*x + e))*sin(f*x + e))
/((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*f*cos(f*x + e)^2 - 2*(a^7*b^4 -
3*a^5*b^6 + 3*a^3*b^8 - a*b^10)*f*sin(f*x + e) - (a^8*b^3 - 2*a^6*b^5 + 2*a
^2*b^9 - b^11)*f), 1/2*(2*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d^3*f*x*c
os(f*x + e)^2 - 2*(a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8)*d^3*f*x + ((2*a^4*b^3
 + 3*a^2*b^5 + b^7)*c^3 - 9*(a^3*b^4 + a*b^6)*c^2*d + 3*(a^4*b^3 + 3*a^2*b^
5 + 2*b^7)*c*d^2 - (2*a^7 - 3*a^5*b^2 + a^3*b^4 + 6*a*b^6)*d^3 + (9*a*b^6*c
^2*d - (2*a^2*b^5 + b^7)*c^3 - 3*(a^2*b^5 + 2*b^7)*c*d^2 + (2*a^5*b^2 - 5*a
^3*b^4 + 6*a*b^6)*d^3)*cos(f*x + e)^2 - 2*(9*a^2*b^5*c^2*d - (2*a^3*b^4 + a
*b^6)*c^3 - 3*(a^3*b^4 + 2*a*b^6)*c*d^2 + (2*a^6*b - 5*a^4*b^3 + 6*a^2*b^5)
*d^3)*sin(f*x + e))*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2
- b^2)*cos(f*x + e))) - ((4*a^4*b^4 - 5*a^2*b^6 + b^8)*c^3 - 3*(2*a^5*b^3 -
 a^3*b^5 - a*b^7)*c^2*d + 9*(a^4*b^4 - a^2*b^6)*c*d^2 + (2*a^7*b - 7*a^5*b^
3 + 5*a^3*b^5)*d^3)*cos(f*x + e) - (4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^
7)*d^3*f*x + 3*((a^3*b^5 - a*b^7)*c^3 - (a^4*b^4 + a^2*b^6 - 2*b^8)*c^2*d -
 (a^5*b^3 - 5*a^3*b^5 + 4*a*b^7)*c*d^2 + (a^6*b^2 - 3*a^4*b^4 + 2*a^2*b^6)*
d^3)*cos(f*x + e))*sin(f*x + e))/((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*
```

```
f*cos(f*x + e)^2 - 2*(a^7*b^4 - 3*a^5*b^6 + 3*a^3*b^8 - a*b^10)*f*sin(f*x +
 e) - (a^8*b^3 - 2*a^6*b^5 + 2*a^2*b^9 - b^11)*f)]
```

giac [B] time = 1.19, size = 887, normalized size = 3.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((f*x + e)*d^3/b^3 + (2*a^2*b^3*c^3 + b^5*c^3 - 9*a*b^4*c^2*d + 3*a^2*b^3*c
*d^2 + 6*b^5*c*d^2 - 2*a^5*d^3 + 5*a^3*b^2*d^3 - 6*a*b^4*d^3)*(pi*floor(1/2
*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2
- b^2)))/((a^4*b^3 - 2*a^2*b^5 + b^7)*sqrt(a^2 - b^2)) + (5*a^3*b^4*c^3*tan
(1/2*f*x + 1/2*e)^3 - 2*a*b^6*c^3*tan(1/2*f*x + 1/2*e)^3 - 9*a^4*b^3*c^2*d*
tan(1/2*f*x + 1/2*e)^3 + 3*a^5*b^2*c*d^2*tan(1/2*f*x + 1/2*e)^3 + 6*a^3*b^4
*c*d^2*tan(1/2*f*x + 1/2*e)^3 + a^6*b*d^3*tan(1/2*f*x + 1/2*e)^3 - 4*a^4*b^
3*d^3*tan(1/2*f*x + 1/2*e)^3 + 4*a^4*b^3*c^3*tan(1/2*f*x + 1/2*e)^2 + 7*a^2
*b^5*c^3*tan(1/2*f*x + 1/2*e)^2 - 2*b^7*c^3*tan(1/2*f*x + 1/2*e)^2 - 6*a^5*
b^2*c^2*d*tan(1/2*f*x + 1/2*e)^2 - 15*a^3*b^4*c^2*d*tan(1/2*f*x + 1/2*e)^2
- 6*a*b^6*c^2*d*tan(1/2*f*x + 1/2*e)^2 + 9*a^4*b^3*c*d^2*tan(1/2*f*x + 1/2*
e)^2 + 18*a^2*b^5*c*d^2*tan(1/2*f*x + 1/2*e)^2 + 2*a^7*d^3*tan(1/2*f*x + 1/
2*e)^2 - a^5*b^2*d^3*tan(1/2*f*x + 1/2*e)^2 - 10*a^3*b^4*d^3*tan(1/2*f*x +
1/2*e)^2 + 11*a^3*b^4*c^3*tan(1/2*f*x + 1/2*e) - 2*a*b^6*c^3*tan(1/2*f*x +
1/2*e) - 15*a^4*b^3*c^2*d*tan(1/2*f*x + 1/2*e) - 12*a^2*b^5*c^2*d*tan(1/2*f
*x + 1/2*e) - 3*a^5*b^2*c*d^2*tan(1/2*f*x + 1/2*e) + 30*a^3*b^4*c*d^2*tan(1
/2*f*x + 1/2*e) + 7*a^6*b*d^3*tan(1/2*f*x + 1/2*e) - 16*a^4*b^3*d^3*tan(1/2
*f*x + 1/2*e) + 4*a^4*b^3*c^3 - a^2*b^5*c^3 - 6*a^5*b^2*c^2*d - 3*a^3*b^4*c
^2*d + 9*a^4*b^3*c*d^2 + 2*a^7*d^3 - 5*a^5*b^2*d^3)/((a^6*b^2 - 2*a^4*b^4 +
 a^2*b^6)*(a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)^2))/f
```

maple [B] time = 0.31, size = 2785, normalized size = 11.23

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e))^3,x)
```

```
[Out] -5/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4
)*a^3*d^3+2/f*d^3/b^3*arctan(tan(1/2*f*x+1/2*e))+6/f*b^2/(tan(1/2*f*x+1/2*e
)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*tan(1/2*f*x+1/2*e)^
3*c*d^2-9/f*b/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f
*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a*c^2*d-6/f*b^4/(tan(1/2*f*x+1/2*e)^2*a+2*t
an(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)/a*tan(1/2*f*x+1/2*e)^2*c^2*d-1
```

5/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a^2/(a^4-2*a^2*b^
2+b^4)*tan(1/2*f*x+1/2*e)*c^2*d+30/f*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*
f*x+1/2*e)*b+a)^2*a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c*d^2+9/f*b/(tan
(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan
(1/2*f*x+1/2*e)^2*c*d^2-15/f*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*
e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*tan(1/2*f*x+1/2*e)^2*c^2*d-9/f*b/(tan(1/2*f
*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f
*x+1/2*e)^3*c^2*d+4/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2
/(a^4-2*a^2*b^2+b^4)*a^2*c^3+2/f/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan
(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*c^3-6/f/(tan(1/2*f*x
+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*c^2*d-1/f/(
tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*
tan(1/2*f*x+1/2*e)^2*d^3+7/f*b^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*
e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)^2*c^3+1/f*b^2/(a^4-2*a^2*b
^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(
1/2))*c^3+2/f/b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-
2*a^2*b^2+b^4)*a^5*d^3-1/f*b^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)
*b+a)^2/(a^4-2*a^2*b^2+b^4)*c^3+9/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x
+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*c*d^2-3/f*b^2/(tan(1/2*f*x+1/2*e)^2*
a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*c^2*d+11/f*b^2/(tan(1/2
*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f
*x+1/2*e)*c^3-2/f*b^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/a
/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c^3+2/f/b^2/(tan(1/2*f*x+1/2*e)^2*a
+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^5*tan(1/2*f*x+1/2*e)^2*d
^3+4/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2
+b^4)*a^2*tan(1/2*f*x+1/2*e)^2*c^3-10/f*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1
/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*tan(1/2*f*x+1/2*e)^2*d^3-2/f*b^5
/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)/a^
2*tan(1/2*f*x+1/2*e)^2*c^3+18/f*b^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1
/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)^2*c*d^2-12/f*b^3/(tan(1
/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f
*x+1/2*e)*c^2*d+1/f/b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(
a^4-2*a^2*b^2+b^4)*a^4*tan(1/2*f*x+1/2*e)^3*d^3-3/f/(tan(1/2*f*x+1/2*e)^2*a
+2*tan(1/2*f*x+1/2*e)*b+a)^2*a^3/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c*d
^2+5/f*b^2/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b
^2+b^4)*a*tan(1/2*f*x+1/2*e)^3*c^3-2/f*b^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/
2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)/a*tan(1/2*f*x+1/2*e)^3*c^3+7/f/b/(t
an(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a^4/(a^4-2*a^2*b^2+b^4)*t
an(1/2*f*x+1/2*e)*d^3-16/f*b/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b
+a)^2*a^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*d^3-6/f/(tan(1/2*f*x+1/2*e
)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*tan(1/2*f*x+1/2*e
)^2*c^2*d+3/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^
2*b^2+b^4)*a^3*tan(1/2*f*x+1/2*e)^3*c*d^2+3/f/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)
^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*c*d^2+5
/f/b/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)

```
+2*b)/(a^2-b^2)^(1/2))*a^3*d^3-2/f/b^3/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*
arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^5*d^3-6/f*b/(a^4
-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^
2-b^2)^(1/2))*a*d^3+6/f*b^2/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*
(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*c*d^2-4/f*b/(tan(1/2*f*x+1/2*
e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+1/2*
e)^3*d^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^3/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 20.93, size = 11848, normalized size = 47.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^3/(a + b*sin(e + f*x))^3,x)
```

```
[Out] - ((b^5*c^3 - 2*a^5*d^3 - 4*a^2*b^3*c^3 + 5*a^3*b^2*d^3 - 9*a^2*b^3*c*d^2 +
 6*a^3*b^2*c^2*d + 3*a*b^4*c^2*d)/(b^2*(a^4 + b^4 - 2*a^2*b^2)) - (tan(e/2
+ (f*x)/2)^3*(a^5*d^3 - 2*b^5*c^3 + 5*a^2*b^3*c^3 - 4*a^3*b^2*d^3 + 6*a^2*b
^3*c*d^2 - 9*a^3*b^2*c^2*d + 3*a^4*b*c*d^2))/(a*b*(a^4 + b^4 - 2*a^2*b^2))
+ (tan(e/2 + (f*x)/2)*(2*b^5*c^3 - 7*a^5*d^3 - 11*a^2*b^3*c^3 + 16*a^3*b^2*
d^3 - 30*a^2*b^3*c*d^2 + 15*a^3*b^2*c^2*d + 12*a*b^4*c^2*d + 3*a^4*b*c*d^2)
)/(a*b*(a^4 + b^4 - 2*a^2*b^2)) + (tan(e/2 + (f*x)/2)^2*(a^2 + 2*b^2)*(b^5*
c^3 - 2*a^5*d^3 - 4*a^2*b^3*c^3 + 5*a^3*b^2*d^3 - 9*a^2*b^3*c*d^2 + 6*a^3*b
^2*c^2*d + 3*a*b^4*c^2*d))/(a^2*b^2*(a^4 + b^4 - 2*a^2*b^2)))/(f*(tan(e/2 +
 (f*x)/2)^2*(2*a^2 + 4*b^2) + a^2*tan(e/2 + (f*x)/2)^4 + a^2 + 4*a*b*tan(e/
2 + (f*x)/2)^3 + 4*a*b*tan(e/2 + (f*x)/2))) - (2*d^3*atan(((d^3*((8*(4*a^2*
b^10*d^6 - 16*a^4*b^8*d^6 + 24*a^6*b^6*d^6 - 16*a^8*b^4*d^6 + 4*a^10*b^2*d^
6))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (d^3*((8*tan(e/
2 + (f*x)/2)*(4*a*b^15*c^3 - 12*a^5*b^11*c^3 + 8*a^7*b^9*c^3 - 24*a^2*b^14*
d^3 + 68*a^4*b^12*d^3 - 72*a^6*b^10*d^3 + 36*a^8*b^8*d^3 - 8*a^10*b^6*d^3 -
 36*a^2*b^14*c^2*d - 36*a^3*b^13*c*d^2 + 72*a^4*b^12*c^2*d - 36*a^6*b^10*c^
2*d + 12*a^7*b^9*c*d^2 + 24*a*b^15*c*d^2))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10
- 4*a^6*b^8 + a^8*b^6) - (8*(4*a*b^14*d^3 - 2*a^2*b^13*c^3 + 6*a^6*b^9*c^3
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- 4*a^8*b^7*c^3 - 8*a^3*b^12*d^3 + 6*a^5*b^10*d^3 - 4*a^7*b^8*d^3 + 2*a^9*b
^6*d^3 - 12*a^2*b^13*c*d^2 + 18*a^3*b^12*c^2*d + 18*a^4*b^11*c*d^2 - 36*a^5
*b^10*c^2*d + 18*a^7*b^8*c^2*d - 6*a^8*b^7*c*d^2))/(b^13 - 4*a^2*b^11 + 6*a
^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (d^3*((8*(4*a^2*b^16 - 16*a^4*b^14 + 24*a^6
*b^12 - 16*a^8*b^10 + 4*a^10*b^8))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b
^7 + a^8*b^5) + (8*tan(e/2 + (f*x)/2)*(12*a*b^18 - 56*a^3*b^16 + 104*a^5*b^
14 - 96*a^7*b^12 + 44*a^9*b^10 - 8*a^11*b^8))/(b^14 - 4*a^2*b^12 + 6*a^4*b^
10 - 4*a^6*b^8 + a^8*b^6))*1i)/b^3)*1i)/b^3 - (8*tan(e/2 + (f*x)/2)*(a*b^12
*c^6 - 8*a*b^12*d^6 + 4*a^3*b^10*c^6 + 4*a^5*b^8*c^6 + 72*a^3*b^10*d^6 - 12
4*a^5*b^8*d^6 + 105*a^7*b^6*d^6 - 44*a^9*b^4*d^6 + 8*a^11*b^2*d^6 + 36*a*b^
12*c^2*d^4 + 12*a*b^12*c^4*d^2 - 72*a^2*b^11*c*d^5 - 18*a^2*b^11*c^5*d + 24
*a^4*b^9*c*d^5 - 36*a^4*b^9*c^5*d + 6*a^6*b^7*c*d^5 - 12*a^8*b^5*c*d^5 - 12
0*a^2*b^11*c^3*d^3 + 144*a^3*b^10*c^2*d^4 + 111*a^3*b^10*c^4*d^2 - 68*a^4*b
^9*c^3*d^3 - 81*a^5*b^8*c^2*d^4 + 12*a^5*b^8*c^4*d^2 + 16*a^6*b^7*c^3*d^3 +
 36*a^7*b^6*c^2*d^4 - 8*a^8*b^5*c^3*d^3))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 -
 4*a^6*b^8 + a^8*b^6)))/b^3 + (d^3*((8*(4*a^2*b^10*d^6 - 16*a^4*b^8*d^6 + 2
4*a^6*b^6*d^6 - 16*a^8*b^4*d^6 + 4*a^10*b^2*d^6))/(b^13 - 4*a^2*b^11 + 6*a^
4*b^9 - 4*a^6*b^7 + a^8*b^5) + (d^3*((8*(4*a*b^14*d^3 - 2*a^2*b^13*c^3 + 6*
a^6*b^9*c^3 - 4*a^8*b^7*c^3 - 8*a^3*b^12*d^3 + 6*a^5*b^10*d^3 - 4*a^7*b^8*d
^3 + 2*a^9*b^6*d^3 - 12*a^2*b^13*c*d^2 + 18*a^3*b^12*c^2*d + 18*a^4*b^11*c*
d^2 - 36*a^5*b^10*c^2*d + 18*a^7*b^8*c^2*d - 6*a^8*b^7*c*d^2))/(b^13 - 4*a^
2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) - (8*tan(e/2 + (f*x)/2)*(4*a*b^15
*c^3 - 12*a^5*b^11*c^3 + 8*a^7*b^9*c^3 - 24*a^2*b^14*d^3 + 68*a^4*b^12*d^3
- 72*a^6*b^10*d^3 + 36*a^8*b^8*d^3 - 8*a^10*b^6*d^3 - 36*a^2*b^14*c^2*d - 3
6*a^3*b^13*c*d^2 + 72*a^4*b^12*c^2*d - 36*a^6*b^10*c^2*d + 12*a^7*b^9*c*d^2
 + 24*a*b^15*c*d^2))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6)
 + (d^3*((8*(4*a^2*b^16 - 16*a^4*b^14 + 24*a^6*b^12 - 16*a^8*b^10 + 4*a^10*
b^8))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (8*tan(e/2 +
(f*x)/2)*(12*a*b^18 - 56*a^3*b^16 + 104*a^5*b^14 - 96*a^7*b^12 + 44*a^9*b^1
0 - 8*a^11*b^8))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))*1i
)/b^3)*1i)/b^3 - (8*tan(e/2 + (f*x)/2)*(a*b^12*c^6 - 8*a*b^12*d^6 + 4*a^3*b
^10*c^6 + 4*a^5*b^8*c^6 + 72*a^3*b^10*d^6 - 124*a^5*b^8*d^6 + 105*a^7*b^6*d
^6 - 44*a^9*b^4*d^6 + 8*a^11*b^2*d^6 + 36*a*b^12*c^2*d^4 + 12*a*b^12*c^4*d^
2 - 72*a^2*b^11*c*d^5 - 18*a^2*b^11*c^5*d + 24*a^4*b^9*c*d^5 - 36*a^4*b^9*c
^5*d + 6*a^6*b^7*c*d^5 - 12*a^8*b^5*c*d^5 - 120*a^2*b^11*c^3*d^3 + 144*a^3*
b^10*c^2*d^4 + 111*a^3*b^10*c^4*d^2 - 68*a^4*b^9*c^3*d^3 - 81*a^5*b^8*c^2*d
^4 + 12*a^5*b^8*c^4*d^2 + 16*a^6*b^7*c^3*d^3 + 36*a^7*b^6*c^2*d^4 - 8*a^8*b
^5*c^3*d^3))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6)))/b^3)/
((16*(24*a^3*b^6*d^9 - 2*a^9*d^9 - 26*a^5*b^4*d^9 + 13*a^7*b^2*d^9 + 36*a*b
^8*c^2*d^7 + 12*a*b^8*c^4*d^5 + a*b^8*c^6*d^3 - 60*a^2*b^7*c*d^8 + 6*a^4*b^
5*c*d^8 + 6*a^6*b^3*c*d^8 - 4*a^8*b*c^3*d^6 - 118*a^2*b^7*c^3*d^6 - 18*a^2*
b^7*c^5*d^4 + 126*a^3*b^6*c^2*d^7 + 111*a^3*b^6*c^4*d^5 + 4*a^3*b^6*c^6*d^3
 - 68*a^4*b^5*c^3*d^6 - 36*a^4*b^5*c^5*d^4 - 45*a^5*b^4*c^2*d^7 + 12*a^5*b^
4*c^4*d^5 + 4*a^5*b^4*c^6*d^3 + 10*a^6*b^3*c^3*d^6 + 18*a^7*b^2*c^2*d^7 - 6
*a^8*b*c*d^8))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) - (16*

tan(e/2 + (f*x)/2)*(8*a^10*d^9 + 24*a^2*b^8*d^9 - 68*a^4*b^6*d^9 + 72*a^6*b^4*d^9 - 36*a^8*b^2*d^9 - 4*a*b^9*c^3*d^6 + 36*a^3*b^7*c*d^8 - 12*a^7*b^3*c*d^8 + 36*a^2*b^8*c^2*d^7 - 72*a^4*b^6*c^2*d^7 + 12*a^5*b^5*c^3*d^6 + 36*a^6*b^4*c^2*d^7 - 8*a^7*b^3*c^3*d^6 - 24*a*b^9*c*d^8))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6) + (d^3*((8*(4*a^2*b^10*d^6 - 16*a^4*b^8*d^6 + 24*a^6*b^6*d^6 - 16*a^8*b^4*d^6 + 4*a^10*b^2*d^6))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (d^3*((8*tan(e/2 + (f*x)/2)*(4*a*b^15*c^3 - 12*a^5*b^11*c^3 + 8*a^7*b^9*c^3 - 24*a^2*b^14*d^3 + 68*a^4*b^12*d^3 - 72*a^6*b^10*d^3 + 36*a^8*b^8*d^3 - 8*a^10*b^6*d^3 - 36*a^2*b^14*c^2*d - 36*a^3*b^13*c*d^2 + 72*a^4*b^12*c^2*d - 36*a^6*b^10*c^2*d + 12*a^7*b^9*c*d^2 + 24*a*b^15*c*d^2))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6) - (8*(4*a*b^14*d^3 - 2*a^2*b^13*c^3 + 6*a^6*b^9*c^3 - 4*a^8*b^7*c^3 - 8*a^3*b^12*d^3 + 6*a^5*b^10*d^3 - 4*a^7*b^8*d^3 + 2*a^9*b^6*d^3 - 12*a^2*b^13*c*d^2 + 18*a^3*b^12*c^2*d + 18*a^4*b^11*c*d^2 - 36*a^5*b^10*c^2*d + 18*a^7*b^8*c^2*d - 6*a^8*b^7*c*d^2))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (d^3*((8*(4*a^2*b^16 - 16*a^4*b^14 + 24*a^6*b^12 - 16*a^8*b^10 + 4*a^10*b^8))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (8*tan(e/2 + (f*x)/2)*(12*a*b^18 - 56*a^3*b^16 + 104*a^5*b^14 - 96*a^7*b^12 + 44*a^9*b^10 - 8*a^11*b^8))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))*1i)/b^3)*1i)/b^3 - (8*tan(e/2 + (f*x)/2)*(a*b^12*c^6 - 8*a*b^12*d^6 + 4*a^3*b^10*c^6 + 4*a^5*b^8*c^6 + 72*a^3*b^10*d^6 - 124*a^5*b^8*d^6 + 105*a^7*b^6*d^6 - 44*a^9*b^4*d^6 + 8*a^11*b^2*d^6 + 36*a*b^12*c^2*d^4 + 12*a*b^12*c^4*d^2 - 72*a^2*b^11*c*d^5 - 18*a^2*b^11*c^5*d + 24*a^4*b^9*c*d^5 - 36*a^4*b^9*c^5*d + 6*a^6*b^7*c*d^5 - 12*a^8*b^5*c*d^5 - 120*a^2*b^11*c^3*d^3 + 144*a^3*b^10*c^2*d^4 + 111*a^3*b^10*c^4*d^2 - 68*a^4*b^9*c^3*d^3 - 81*a^5*b^8*c^2*d^4 + 12*a^5*b^8*c^4*d^2 + 16*a^6*b^7*c^3*d^3 + 36*a^7*b^6*c^2*d^4 - 8*a^8*b^5*c^3*d^3))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))*1i)/b^3 - (d^3*((8*(4*a^2*b^10*d^6 - 16*a^4*b^8*d^6 + 24*a^6*b^6*d^6 - 16*a^8*b^4*d^6 + 4*a^10*b^2*d^6))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (d^3*((8*(4*a*b^14*d^3 - 2*a^2*b^13*c^3 + 6*a^6*b^9*c^3 - 4*a^8*b^7*c^3 - 8*a^3*b^12*d^3 + 6*a^5*b^10*d^3 - 4*a^7*b^8*d^3 + 2*a^9*b^6*d^3 - 12*a^2*b^13*c*d^2 + 18*a^3*b^12*c^2*d + 18*a^4*b^11*c*d^2 - 36*a^5*b^10*c^2*d + 18*a^7*b^8*c^2*d - 6*a^8*b^7*c*d^2))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) - (8*tan(e/2 + (f*x)/2)*(4*a*b^15*c^3 - 12*a^5*b^11*c^3 + 8*a^7*b^9*c^3 - 24*a^2*b^14*d^3 + 68*a^4*b^12*d^3 - 72*a^6*b^10*d^3 + 36*a^8*b^8*d^3 - 8*a^10*b^6*d^3 - 36*a^2*b^14*c^2*d - 36*a^3*b^13*c*d^2 + 72*a^4*b^12*c^2*d - 36*a^6*b^10*c^2*d + 12*a^7*b^9*c*d^2 + 24*a*b^15*c*d^2))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6) + (d^3*((8*(4*a^2*b^16 - 16*a^4*b^14 + 24*a^6*b^12 - 16*a^8*b^10 + 4*a^10*b^8))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (8*tan(e/2 + (f*x)/2)*(12*a*b^18 - 56*a^3*b^16 + 104*a^5*b^14 - 96*a^7*b^12 + 44*a^9*b^10 - 8*a^11*b^8))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))*1i)/b^3)*1i)/b^3 - (8*tan(e/2 + (f*x)/2)*(a*b^12*c^6 - 8*a*b^12*d^6 + 4*a^3*b^10*c^6 + 4*a^5*b^8*c^6 + 72*a^3*b^10*d^6 - 124*a^5*b^8*d^6 + 105*a^7*b^6*d^6 - 44*a^9*b^4*d^6 + 8*a^11*b^2*d^6 + 36*a*b^12*c^2*d^4 + 12*a*b^12*c^4*d^2 - 72*a^2*b^11*c*d^5

- 18*a^2*b^11*c^5*d + 24*a^4*b^9*c*d^5 - 36*a^4*b^9*c^5*d + 6*a^6*b^7*c*d^5 - 12*a^8*b^5*c*d^5 - 120*a^2*b^11*c^3*d^3 + 144*a^3*b^10*c^2*d^4 + 111*a^3*b^10*c^4*d^2 - 68*a^4*b^9*c^3*d^3 - 81*a^5*b^8*c^2*d^4 + 12*a^5*b^8*c^4*d^2 + 16*a^6*b^7*c^3*d^3 + 36*a^7*b^6*c^2*d^4 - 8*a^8*b^5*c^3*d^3))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))*1i)/b^3)))/(b^3*f) - (atan((((a*d - b*c)*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(4*a^2*b^10*d^6 - 16*a^4*b^8*d^6 + 24*a^6*b^6*d^6 - 16*a^8*b^4*d^6 + 4*a^10*b^2*d^6))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) - (8*tan(e/2 + (f*x)/2)*(a*b^12*c^6 - 8*a*b^12*d^6 + 4*a^3*b^10*c^6 + 4*a^5*b^8*c^6 + 72*a^3*b^10*d^6 - 124*a^5*b^8*d^6 + 105*a^7*b^6*d^6 - 44*a^9*b^4*d^6 + 8*a^11*b^2*d^6 + 36*a*b^12*c^2*d^4 + 12*a*b^12*c^4*d^2 - 72*a^2*b^11*c*d^5 - 18*a^2*b^11*c^5*d + 24*a^4*b^9*c*d^5 - 36*a^4*b^9*c^5*d + 6*a^6*b^7*c*d^5 - 12*a^8*b^5*c*d^5 - 120*a^2*b^11*c^3*d^3 + 144*a^3*b^10*c^2*d^4 + 111*a^3*b^10*c^4*d^2 - 68*a^4*b^9*c^3*d^3 - 81*a^5*b^8*c^2*d^4 + 12*a^5*b^8*c^4*d^2 + 16*a^6*b^7*c^3*d^3 + 36*a^7*b^6*c^2*d^4 - 8*a^8*b^5*c^3*d^3))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6) + ((a*d - b*c)*(-(a + b)^5*(a - b)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(4*a*b^15*c^3 - 12*a^5*b^11*c^3 + 8*a^7*b^9*c^3 - 24*a^2*b^14*d^3 + 68*a^4*b^12*d^3 - 72*a^6*b^10*d^3 + 36*a^8*b^8*d^3 - 8*a^10*b^6*d^3 - 36*a^2*b^14*c^2*d - 36*a^3*b^13*c*d^2 + 72*a^4*b^12*c^2*d - 36*a^6*b^10*c^2*d + 12*a^7*b^9*c*d^2 + 24*a*b^15*c*d^2))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6) - (8*(4*a*b^14*d^3 - 2*a^2*b^13*c^3 + 6*a^6*b^9*c^3 - 4*a^8*b^7*c^3 - 8*a^3*b^12*d^3 + 6*a^5*b^10*d^3 - 4*a^7*b^8*d^3 + 2*a^9*b^6*d^3 - 12*a^2*b^13*c*d^2 + 18*a^3*b^12*c^2*d + 18*a^4*b^11*c*d^2 - 36*a^5*b^10*c^2*d + 18*a^7*b^8*c^2*d - 6*a^8*b^7*c*d^2))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (((8*(4*a^2*b^16 - 16*a^4*b^14 + 24*a^6*b^12 - 16*a^8*b^10 + 4*a^10*b^8))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (8*tan(e/2 + (f*x)/2)*(12*a*b^18 - 56*a^3*b^16 + 104*a^5*b^14 - 96*a^7*b^12 + 44*a^9*b^10 - 8*a^11*b^8))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))*(a*d - b*c)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4*d^2 + b^4*c^2 + 6*b^4*d^2 + 2*a^2*b^2*c^2 - 5*a^2*b^2*d^2 - 8*a*b^3*c*d + 2*a^3*b*c*d))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)))*(2*a^4*d^2 + b^4*c^2 + 6*b^4*d^2 + 2*a^2*b^2*c^2 - 5*a^2*b^2*d^2 - 8*a*b^3*c*d + 2*a^3*b*c*d))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)))*(2*a^4*d^2 + b^4*c^2 + 6*b^4*d^2 + 2*a^2*b^2*c^2 - 5*a^2*b^2*d^2 - 8*a*b^3*c*d + 2*a^3*b*c*d)*1i)/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)) + ((a*d - b*c)*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(4*a^2*b^10*d^6 - 16*a^4*b^8*d^6 + 24*a^6*b^6*d^6 - 16*a^8*b^4*d^6 + 4*a^10*b^2*d^6))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) - (8*tan(e/2 + (f*x)/2)*(a*b^12*c^6 - 8*a*b^12*d^6 + 4*a^3*b^10*c^6 + 4*a^5*b^8*c^6 + 72*a^3*b^10*d^6 - 124*a^5*b^8*d^6 + 105*a^7*b^6*d^6 - 44*a^9*b^4*d^6 + 8*a^11*b^2*d^6 + 36*a*b^12*c^2*d^4 + 12*a*b^12*c^4*d^2 - 72*a^2*b^11*c*d^5 - 18*a^2*b^11*c^5*d + 24*a^4*b^9*c*d^5 - 36*a^4*b^9*c^5*d + 6*a^6*b^7*c*d^5 - 12*a^8*b^5*c*d^5 - 120*a^2*b^11*c^3*d^3 + 144*a^3*b^10*c^2*d^4 + 111*a^3*b^10*c^4*d^2 - 68*a^4*b^9*c^3*d^3 - 81*a^5*b^8*c^2*d^4 + 12*a^5*b^8*c^4*d^2 + 16*a^6*b^7*c^3*d^3 + 36*a^7*b^6*c^2*d^4 - 8*a^8*b^5*c^3*d^3

))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6) + ((a*d - b*c)*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(4*a*b^14*d^3 - 2*a^2*b^13*c^3 + 6*a^6*b^9*c^3 - 4*a^8*b^7*c^3 - 8*a^3*b^12*d^3 + 6*a^5*b^10*d^3 - 4*a^7*b^8*d^3 + 2*a^9*b^6*d^3 - 12*a^2*b^13*c*d^2 + 18*a^3*b^12*c^2*d + 18*a^4*b^11*c*d^2 - 36*a^5*b^10*c^2*d + 18*a^7*b^8*c^2*d - 6*a^8*b^7*c*d^2))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) - (8*tan(e/2 + (f*x)/2)*(4*a*b^15*c^3 - 12*a^5*b^11*c^3 + 8*a^7*b^9*c^3 - 24*a^2*b^14*d^3 + 68*a^4*b^12*d^3 - 72*a^6*b^10*d^3 + 36*a^8*b^8*d^3 - 8*a^10*b^6*d^3 - 36*a^2*b^14*c^2*d - 36*a^3*b^13*c*d^2 + 72*a^4*b^12*c^2*d - 36*a^6*b^10*c^2*d + 12*a^7*b^9*c*d^2 + 24*a*b^15*c*d^2))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6) + (((8*(4*a^2*b^16 - 16*a^4*b^14 + 24*a^6*b^12 - 16*a^8*b^10 + 4*a^10*b^8))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (8*tan(e/2 + (f*x)/2)*(12*a*b^18 - 56*a^3*b^16 + 104*a^5*b^14 - 96*a^7*b^12 + 44*a^9*b^10 - 8*a^11*b^8))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))*(a*d - b*c)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4*d^2 + b^4*c^2 + 6*b^4*d^2 + 2*a^2*b^2*c^2 - 5*a^2*b^2*d^2 - 8*a*b^3*c*d + 2*a^3*b*c*d))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)))*(2*a^4*d^2 + b^4*c^2 + 6*b^4*d^2 + 2*a^2*b^2*c^2 - 5*a^2*b^2*d^2 - 8*a*b^3*c*d + 2*a^3*b*c*d))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)))*(2*a^4*d^2 + b^4*c^2 + 6*b^4*d^2 + 2*a^2*b^2*c^2 - 5*a^2*b^2*d^2 - 8*a*b^3*c*d + 2*a^3*b*c*d)*1i)/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)))/((16*(24*a^3*b^6*d^9 - 2*a^9*d^9 - 26*a^5*b^4*d^9 + 13*a^7*b^2*d^9 + 36*a*b^8*c^2*d^7 + 12*a*b^8*c^4*d^5 + a*b^8*c^6*d^3 - 60*a^2*b^7*c*d^8 + 6*a^4*b^5*c*d^8 + 6*a^6*b^3*c*d^8 - 4*a^8*b*c^3*d^6 - 118*a^2*b^7*c^3*d^6 - 18*a^2*b^7*c^5*d^4 + 126*a^3*b^6*c^2*d^7 + 111*a^3*b^6*c^4*d^5 + 4*a^3*b^6*c^6*d^3 - 68*a^4*b^5*c^3*d^6 - 36*a^4*b^5*c^5*d^4 - 45*a^5*b^4*c^2*d^7 + 12*a^5*b^4*c^4*d^5 + 4*a^5*b^4*c^6*d^3 + 10*a^6*b^3*c^3*d^6 + 18*a^7*b^2*c^2*d^7 - 6*a^8*b*c*d^8))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) - (16*tan(e/2 + (f*x)/2)*(8*a^10*d^9 + 24*a^2*b^8*d^9 - 68*a^4*b^6*d^9 + 72*a^6*b^4*d^9 - 36*a^8*b^2*d^9 - 4*a*b^9*c^3*d^6 + 36*a^3*b^7*c*d^8 - 12*a^7*b^3*c*d^8 + 36*a^2*b^8*c^2*d^7 - 72*a^4*b^6*c^2*d^7 + 12*a^5*b^5*c^3*d^6 + 36*a^6*b^4*c^2*d^7 - 8*a^7*b^3*c^3*d^6 - 24*a*b^9*c*d^8))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6) + ((a*d - b*c)*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(4*a^2*b^10*d^6 - 16*a^4*b^8*d^6 + 24*a^6*b^6*d^6 - 16*a^8*b^4*d^6 + 4*a^10*b^2*d^6))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) - (8*tan(e/2 + (f*x)/2)*(a*b^12*c^6 - 8*a*b^12*d^6 + 4*a^3*b^10*c^6 + 4*a^5*b^8*c^6 + 72*a^3*b^10*d^6 - 124*a^5*b^8*d^6 + 105*a^7*b^6*d^6 - 44*a^9*b^4*d^6 + 8*a^11*b^2*d^6 + 36*a*b^12*c^2*d^4 + 12*a*b^12*c^4*d^2 - 72*a^2*b^11*c*d^5 - 18*a^2*b^11*c^5*d + 24*a^4*b^9*c*d^5 - 36*a^4*b^9*c^5*d + 6*a^6*b^7*c*d^5 - 12*a^8*b^5*c*d^5 - 120*a^2*b^11*c^3*d^3 + 144*a^3*b^10*c^2*d^4 + 111*a^3*b^10*c^4*d^2 - 68*a^4*b^9*c^3*d^3 - 81*a^5*b^8*c^2*d^4 + 12*a^5*b^8*c^4*d^2 + 16*a^6*b^7*c^3*d^3 + 36*a^7*b^6*c^2*d^4 - 8*a^8*b^5*c^3*d^3))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6) + ((a*d - b*c)*(-(a + b)^5*(a - b)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(4*a*b^15*c^3 - 12*a^5*b^11*c^3 + 8*a^7*b^9*c^3 - 24*a^2*b^14*d^3 + 68*a^4*b^12*d^3 - 72*a^6*b^10*

d^3 + 36*a^8*b^8*d^3 - 8*a^10*b^6*d^3 - 36*a^2*b^14*c^2*d - 36*a^3*b^13*c*d
^2 + 72*a^4*b^12*c^2*d - 36*a^6*b^10*c^2*d + 12*a^7*b^9*c*d^2 + 24*a*b^15*c
*d^2))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6) - (8*(4*a*b^1
4*d^3 - 2*a^2*b^13*c^3 + 6*a^6*b^9*c^3 - 4*a^8*b^7*c^3 - 8*a^3*b^12*d^3 + 6
*a^5*b^10*d^3 - 4*a^7*b^8*d^3 + 2*a^9*b^6*d^3 - 12*a^2*b^13*c*d^2 + 18*a^3*
b^12*c^2*d + 18*a^4*b^11*c*d^2 - 36*a^5*b^10*c^2*d + 18*a^7*b^8*c^2*d - 6*a
^8*b^7*c*d^2))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (((8
*(4*a^2*b^16 - 16*a^4*b^14 + 24*a^6*b^12 - 16*a^8*b^10 + 4*a^10*b^8))/(b^13
 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) + (8*tan(e/2 + (f*x)/2)*(1
2*a*b^18 - 56*a^3*b^16 + 104*a^5*b^14 - 96*a^7*b^12 + 44*a^9*b^10 - 8*a^11*
b^8))/(b^14 - 4*a^2*b^12 + 6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6))*(a*d - b*c)*(
-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4*d^2 + b^4*c^2 + 6*b^4*d^2 + 2*a^2*b^2*c^
2 - 5*a^2*b^2*d^2 - 8*a*b^3*c*d + 2*a^3*b*c*d))/(2*(b^13 - 5*a^2*b^11 + 10*
a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)))*(2*a^4*d^2 + b^4*c^2 + 6*b^4
*d^2 + 2*a^2*b^2*c^2 - 5*a^2*b^2*d^2 - 8*a*b^3*c*d + 2*a^3*b*c*d))/(2*(b^13
 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)))*(2*a^4*d^
2 + b^4*c^2 + 6*b^4*d^2 + 2*a^2*b^2*c^2 - 5*a^2*b^2*d^2 - 8*a*b^3*c*d + 2*a
^3*b*c*d))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^
10*b^3)) - ((a*d - b*c)*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(4*a^2*b^10*d^6 -
16*a^4*b^8*d^6 + 24*a^6*b^6*d^6 - 16*a^8*b^4*d^6 + 4*a^10*b^2*d^6))/(b^13 -
 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5) - (8*tan(e/2 + (f*x)/2)*(a*b
^12*c^6 - 8*a*b^12*d^6 + 4*a^3*b^10*c^6 + 4*a^5*b^8*c^6 + 72*a^3*b^10*d^6 -
 124*a^5*b^8*d^6 + 105*a^7*b^6*d^6 - 44*a^9*b^4*d^6 + 8*a^11*b^2*d^6 + 36*a
*b^12*c^2*d^4 + 12*a*b^12*c^4*d^2 - 72*a^2*b^11*c*d^5 - 18*a^2*b^11*c^5*d +
 24*a^4*b^9*c*d^5 - 36*a^4*b^9*c^5*d + 6*a^6*b^7*c*d^5 - 12*a^8*b^5*c*d^5 -
 120*a^2*b^11*c^3*d^3 + 144*a^3*b^10*c^2*d^4 + 111*a^3*b^10*c^4*d^2 - 68*a^
4*b^9*c^3*d^3 - 81*a^5*b^8*c^2*d^4 + 12*a^5*b^8*c^4*d^2 + 16*a^6*b^7*c^3*d^
3 + 36*a^7*b^6*c^2*d^4 - 8*a^8*b^5*c^3*d^3))/(b^14 - 4*a^2*b^12 + 6*a^4*b^1
0 - 4*a^6*b^8 + a^8*b^6) + ((a*d - b*c)*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(4
*a*b^14*d^3 - 2*a^2*b^13*c^3 + 6*a^6*b^9*c^3 - 4*a^8*b^7*c^3 - 8*a^3*b^12*d
^3 + 6*a^5*b^10*d^3 - 4*a^7*b^8*d^3 + 2*a^9*b^6*d^3 - 12*a^2*b^13*c*d^2 + 1
8*a^3*b^12*c^2*d + 18*a^4*b^11*c*d^2 - 36*a^5*b^10*c^2*d + 18*a^7*b^8*c^2*d
 - 6*a^8*b^7*c*d^2))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b^7 + a^8*b^5)
- (8*tan(e/2 + (f*x)/2)*(4*a*b^15*c^3 - 12*a^5*b^11*c^3 + 8*a^7*b^9*c^3 - 2
4*a^2*b^14*d^3 + 68*a^4*b^12*d^3 - 72*a^6*b^10*d^3 + 36*a^8*b^8*d^3 - 8*a^1
0*b^6*d^3 - 36*a^2*b^14*c^2*d - 36*a^3*b^13*c*d^2 + 72*a^4*b^12*c^2*d - 36*
a^6*b^10*c^2*d + 12*a^7*b^9*c*d^2 + 24*a*b^15*c*d^2))/(b^14 - 4*a^2*b^12 +
6*a^4*b^10 - 4*a^6*b^8 + a^8*b^6) + (((8*(4*a^2*b^16 - 16*a^4*b^14 + 24*a^6
*b^12 - 16*a^8*b^10 + 4*a^10*b^8))/(b^13 - 4*a^2*b^11 + 6*a^4*b^9 - 4*a^6*b
^7 + a^8*b^5) + (8*tan(e/2 + (f*x)/2)*(12*a*b^18 - 56*a^3*b^16 + 104*a^5*b^
14 - 96*a^7*b^12 + 44*a^9*b^10 - 8*a^11*b^8))/(b^14 - 4*a^2*b^12 + 6*a^4*b^
10 - 4*a^6*b^8 + a^8*b^6))*(a*d - b*c)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4*
d^2 + b^4*c^2 + 6*b^4*d^2 + 2*a^2*b^2*c^2 - 5*a^2*b^2*d^2 - 8*a*b^3*c*d + 2
*a^3*b*c*d))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 -
a^10*b^3)))*(2*a^4*d^2 + b^4*c^2 + 6*b^4*d^2 + 2*a^2*b^2*c^2 - 5*a^2*b^2*d^

```
2 - 8*a*b^3*c*d + 2*a^3*b*c*d))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6
*b^7 + 5*a^8*b^5 - a^10*b^3)))*(2*a^4*d^2 + b^4*c^2 + 6*b^4*d^2 + 2*a^2*b^2
*c^2 - 5*a^2*b^2*d^2 - 8*a*b^3*c*d + 2*a^3*b*c*d))/(2*(b^13 - 5*a^2*b^11 +
10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3))))*(a*d - b*c)*(-(a + b)^5*
(a - b)^5)^(1/2)*(2*a^4*d^2 + b^4*c^2 + 6*b^4*d^2 + 2*a^2*b^2*c^2 - 5*a^2*b
^2*d^2 - 8*a*b^3*c*d + 2*a^3*b*c*d)*1i)/(f*(b^13 - 5*a^2*b^11 + 10*a^4*b^9
- 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**3/(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.717 $\int \frac{(c+d\sin(e+fx))^2}{(a+b\sin(e+fx))^3}\,dx$

Optimal. Leaf size=196

$$-\frac{\left(-\left(a^2\left(2c^2+d^2\right)\right)+6abcd-b^2\left(c^2+2d^2\right)\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{5/2}}+\frac{(bc-ad)^2\cos(e+fx)}{2bf\left(a^2-b^2\right)(a+b\sin(e+fx))^2}+\frac{\left(a^2d+\right.}{2b}$$

```
[Out] -(6*a*b*c*d-a^2*(2*c^2+d^2)-b^2*(c^2+2*d^2))*arctan((b+a*tan(1/2*f*x+1/2*e)
)/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/f+1/2*(-a*d+b*c)^2*cos(f*x+e)/b/(a^2-b^2
)/f/(a+b*sin(f*x+e))^2+1/2*(-a*d+b*c)*(a^2*d+3*a*b*c-4*b^2*d)*cos(f*x+e)/b/
(a^2-b^2)^2/f/(a+b*sin(f*x+e))
```

Rubi [A] time = 0.29, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2790, 2754, 12, 2660, 618, 204}

$$-\frac{\left(a^2\left(-\left(2c^2+d^2\right)\right)+6abcd-b^2\left(c^2+2d^2\right)\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{5/2}}+\frac{(bc-ad)^2\cos(e+fx)}{2bf\left(a^2-b^2\right)(a+b\sin(e+fx))^2}+\frac{\left(a^2d+\right.}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^2/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] -(((6*a*b*c*d - a^2*(2*c^2 + d^2) - b^2*(c^2 + 2*d^2))*ArcTan[(b + a*Tan[(e
 + f*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2)*f)) + ((b*c - a*d)^2*Cos[e
 + f*x])/(2*b*(a^2 - b^2)*f*(a + b*Sin[e + f*x])^2) + ((b*c - a*d)*(3*a*b*c
 + a^2*d - 4*b^2*d)*Cos[e + f*x])/(2*b*(a^2 - b^2)^2*f*(a + b*Sin[e + f*x])
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^2}{(a+b\sin(e+fx))^3}\,dx = \frac{(bc-ad)^2\cos(e+fx)}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{\int \frac{-2b\left(2bcd-a\left(c^2+d^2\right)\right)+\left(2abcd+a^2d^2-b^2\left(c^2+2d^2\right)\right)\sin(e+f}{(a+b\sin(e+fx))^2}}{2b\left(a^2-b^2\right)}$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{(bc-ad)\left(3abc+a^2d-4b^2d\right)\cos(e+fx)}{2b\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} - \int$$

$$= \frac{(bc-ad)^2\cos(e+fx)}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{(bc-ad)\left(3abc+a^2d-4b^2d\right)\cos(e+fx)}{2b\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} - ($$

$$= \frac{(bc-ad)^2\cos(e+fx)}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{(bc-ad)\left(3abc+a^2d-4b^2d\right)\cos(e+fx)}{2b\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} - ($$

$$= \frac{(bc-ad)^2\cos(e+fx)}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{(bc-ad)\left(3abc+a^2d-4b^2d\right)\cos(e+fx)}{2b\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + ($$

$$= -\frac{\left(6abcd-a^2\left(2c^2+d^2\right)-b^2\left(c^2+2d^2\right)\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2} f} + \frac{(bc-ad)^2}{2b\left(a^2-b^2\right)f(a}$$

Mathematica [**A**] time = 0.95, size = 204, normalized size = 1.04

$$\frac{\frac{2\left(a^2\left(2c^2+d^2\right)-6abcd+b^2\left(c^2+2d^2\right)\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} - \frac{\left(a^3d^2+2a^2bcd-ab^2\left(3c^2+4d^2\right)+4b^3cd\right)\cos(e+fx)}{b(a-b)^2(a+b)^2(a+b\sin(e+fx))} + \frac{(bc-ad)^2\cos(e+fx)}{b(a-b)(a+b)(a+b\sin(e+fx))^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^2/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((2*(-6*a*b*c*d + a^2*(2*c^2 + d^2) + b^2*(c^2 + 2*d^2))*ArcTan[(b + a*Tan[
(e + f*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + ((b*c - a*d)^2*Cos[e +
f*x])/((a - b)*b*(a + b)*(a + b*Sin[e + f*x])^2) - ((2*a^2*b*c*d + 4*b^3*c*
d + a^3*d^2 - a*b^2*(3*c^2 + 4*d^2))*Cos[e + f*x])/((a - b)^2*b*(a + b)^2*(
a + b*Sin[e + f*x])))/(2*f)
```

fricas [**B**] time = 1.04, size = 1025, normalized size = 5.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(3*(a^3*b^2 - a*b^4)*c^2 - 2*(a^4*b + a^2*b^3 - 2*b^5)*c*d - (a^5
- 5*a^3*b^2 + 4*a*b^4)*d^2)*cos(f*x + e)*sin(f*x + e) - ((2*a^4 + 3*a^2*b^2
 + b^4)*c^2 - 6*(a^3*b + a*b^3)*c*d + (a^4 + 3*a^2*b^2 + 2*b^4)*d^2 + (6*a*
b^3*c*d - (2*a^2*b^2 + b^4)*c^2 - (a^2*b^2 + 2*b^4)*d^2)*cos(f*x + e)^2 - 2
*(6*a^2*b^2*c*d - (2*a^3*b + a*b^3)*c^2 - (a^3*b + 2*a*b^3)*d^2)*sin(f*x +
e))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e)
 - a^2 - b^2 + 2*(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 +
 b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)) + 2*((4*a^4*b
 - 5*a^2*b^3 + b^5)*c^2 - 2*(2*a^5 - a^3*b^2 - a*b^4)*c*d + 3*(a^4*b - a^2*
b^3)*d^2)*cos(f*x + e))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*f*cos(f*x
+ e)^2 - 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*f*sin(f*x + e) - (a^8 -
2*a^6*b^2 + 2*a^2*b^6 - b^8)*f), -1/2*((3*(a^3*b^2 - a*b^4)*c^2 - 2*(a^4*b
+ a^2*b^3 - 2*b^5)*c*d - (a^5 - 5*a^3*b^2 + 4*a*b^4)*d^2)*cos(f*x + e)*sin(
f*x + e) - ((2*a^4 + 3*a^2*b^2 + b^4)*c^2 - 6*(a^3*b + a*b^3)*c*d + (a^4 +
3*a^2*b^2 + 2*b^4)*d^2 + (6*a*b^3*c*d - (2*a^2*b^2 + b^4)*c^2 - (a^2*b^2 +
2*b^4)*d^2)*cos(f*x + e)^2 - 2*(6*a^2*b^2*c*d - (2*a^3*b + a*b^3)*c^2 - (a^
3*b + 2*a*b^3)*d^2)*sin(f*x + e))*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x + e) +
 b)/(sqrt(a^2 - b^2)*cos(f*x + e))) + ((4*a^4*b - 5*a^2*b^3 + b^5)*c^2 - 2*
(2*a^5 - a^3*b^2 - a*b^4)*c*d + 3*(a^4*b - a^2*b^3)*d^2)*cos(f*x + e))/((a^
6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*f*cos(f*x + e)^2 - 2*(a^7*b - 3*a^5*b^
3 + 3*a^3*b^5 - a*b^7)*f*sin(f*x + e) - (a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8)
*f)]
```

giac [B] time = 0.27, size = 609, normalized size = 3.11

$$\frac{\frac{\left(2\,a^2c^2+b^2c^2-6\,abcd+a^2d^2+2\,b^2d^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{5\,a^3b^2c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,ab^4c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-6\,a^4bcd\tan(}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((2*a^2*c^2 + b^2*c^2 - 6*a*b*c*d + a^2*d^2 + 2*b^2*d^2)*(pi*floor(1/2*(f*x
 + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2
)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) + (5*a^3*b^2*c^2*tan(1/2*f*x
+ 1/2*e)^3 - 2*a*b^4*c^2*tan(1/2*f*x + 1/2*e)^3 - 6*a^4*b*c*d*tan(1/2*f*x +
 1/2*e)^3 + a^5*d^2*tan(1/2*f*x + 1/2*e)^3 + 2*a^3*b^2*d^2*tan(1/2*f*x + 1/
2*e)^3 + 4*a^4*b*c^2*tan(1/2*f*x + 1/2*e)^2 + 7*a^2*b^3*c^2*tan(1/2*f*x + 1
/2*e)^2 - 2*b^5*c^2*tan(1/2*f*x + 1/2*e)^2 - 4*a^5*c*d*tan(1/2*f*x + 1/2*e)
```

```
^2 - 10*a^3*b^2*c*d*tan(1/2*f*x + 1/2*e)^2 - 4*a*b^4*c*d*tan(1/2*f*x + 1/2*
e)^2 + 3*a^4*b*d^2*tan(1/2*f*x + 1/2*e)^2 + 6*a^2*b^3*d^2*tan(1/2*f*x + 1/2
*e)^2 + 11*a^3*b^2*c^2*tan(1/2*f*x + 1/2*e) - 2*a*b^4*c^2*tan(1/2*f*x + 1/2
*e) - 10*a^4*b*c*d*tan(1/2*f*x + 1/2*e) - 8*a^2*b^3*c*d*tan(1/2*f*x + 1/2*e
) - a^5*d^2*tan(1/2*f*x + 1/2*e) + 10*a^3*b^2*d^2*tan(1/2*f*x + 1/2*e) + 4*
a^4*b*c^2 - a^2*b^3*c^2 - 4*a^5*c*d - 2*a^3*b^2*c*d + 3*a^4*b*d^2)/((a^6 -
2*a^4*b^2 + a^2*b^4)*(a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) +
 a)^2))/f
```

maple [B] time = 0.27, size = 1923, normalized size = 9.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e))^3,x)
```

```
[Out] -4/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4
)*a^3*tan(1/2*f*x+1/2*e)^2*c*d+4/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/
2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+1/2*e)^2*b*c^2-10/f/(tan(1/
2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*tan(1/2*
f*x+1/2*e)^2*b^2*c*d-10/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)
^2*a^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*b*c*d-4/f/(tan(1/2*f*x+1/2*e)
^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)/a*tan(1/2*f*x+1/2*e)^2
*b^4*c*d-6/f/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*
x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a*b*c*d-6/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/
2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+1/2*e)^3*b*c*d-1/f/
(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*b^3
*c^2+5/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2
+b^4)*a*tan(1/2*f*x+1/2*e)^3*b^2*c^2+2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*
f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*tan(1/2*f*x+1/2*e)^3*b^2*d^2+3/f/(t
an(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*t
an(1/2*f*x+1/2*e)^2*b*d^2-2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*
b+a)^2/(a^4-2*a^2*b^2+b^4)/a^2*tan(1/2*f*x+1/2*e)^2*b^5*c^2+11/f/(tan(1/2*f
*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x
+1/2*e)*b^2*c^2+10/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a/
(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*b^2*d^2-8/f/(tan(1/2*f*x+1/2*e)^2*a+
2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*b^3*c*d-
2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/a/(a^4-2*a^2*b^2+b^
4)*tan(1/2*f*x+1/2*e)*b^4*c^2-2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2
*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*b^2*c*d-2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(
1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)/a*tan(1/2*f*x+1/2*e)^3*b^4*c^2+1/
f/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*
b)/(a^2-b^2)^(1/2))*a^2*d^2+1/f/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(
1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*b^2*c^2+2/f/(a^4-2*a^2*b^
2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1
```

```
/2))*b^2*d^2+1/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2
*a^2*b^2+b^4)*a^3*tan(1/2*f*x+1/2*e)^3*d^2+7/f/(tan(1/2*f*x+1/2*e)^2*a+2*ta
n(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)^2*b^3*c^2+6/
f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*t
an(1/2*f*x+1/2*e)^2*b^3*d^2-1/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e
)*b+a)^2*a^3/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*d^2-4/f/(tan(1/2*f*x+1/
2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*c*d+4/f/(tan(1
/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*b*c^2
+3/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4
)*a^2*b*d^2+2/f/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2
*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*c^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^2/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 10.44, size = 641, normalized size = 3.27

$$\frac{\operatorname{atan}\left(\frac{\left(\frac{\left(2\,a^4\,b-4\,a^2\,b^3+2\,b^5\right)\left(2\,a^2\,c^2+a^2\,d^2-6\,a\,b\,c\,d+b^2\,c^2+2\,b^2\,d^2\right)}{2\,{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{a\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(2\,a^2\,c^2+a^2\,d^2-6\,a\,b\,c\,d+b^2\,c^2+2\,b^2\,d^2\right)}{{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}\right)\left(a^4-2\,a^2\,b^2+b^4\right)}{2\,a^2\,c^2+a^2\,d^2-6\,a\,b\,c\,d+b^2\,c^2+2\,b^2\,d^2}\right)\left(2\,a^2\,c^2+a^2\,d^2 \cdots\right.}{f\,{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^2/(a + b*sin(e + f*x))^3,x)
```

```
[Out] (atan(((((2*a^4*b + 2*b^5 - 4*a^2*b^3)*(2*a^2*c^2 + a^2*d^2 + b^2*c^2 + 2*b
^2*d^2 - 6*a*b*c*d))/(2*(a + b)^(5/2)*(a - b)^(5/2)*(a^4 + b^4 - 2*a^2*b^2)
) + (a*tan(e/2 + (f*x)/2)*(2*a^2*c^2 + a^2*d^2 + b^2*c^2 + 2*b^2*d^2 - 6*a*
b*c*d))/((a + b)^(5/2)*(a - b)^(5/2)))*(a^4 + b^4 - 2*a^2*b^2))/(2*a^2*c^2
+ a^2*d^2 + b^2*c^2 + 2*b^2*d^2 - 6*a*b*c*d))*(2*a^2*c^2 + a^2*d^2 + b^2*c^
2 + 2*b^2*d^2 - 6*a*b*c*d))/(f*(a + b)^(5/2)*(a - b)^(5/2)) - ((b^3*c^2 - 4
*a^2*b*c^2 - 3*a^2*b*d^2 + 4*a^3*c*d + 2*a*b^2*c*d)/(a^4 + b^4 - 2*a^2*b^2)
```

```
 + (tan(e/2 + (f*x)/2)*(a^4*d^2 + 2*b^4*c^2 - 11*a^2*b^2*c^2 - 10*a^2*b^2*d
^2 + 8*a*b^3*c*d + 10*a^3*b*c*d))/(a*(a^4 + b^4 - 2*a^2*b^2)) - (tan(e/2 +
(f*x)/2)^3*(a^4*d^2 - 2*b^4*c^2 + 5*a^2*b^2*c^2 + 2*a^2*b^2*d^2 - 6*a^3*b*c
*d))/(a*(a^4 + b^4 - 2*a^2*b^2)) + (tan(e/2 + (f*x)/2)^2*(a^2 + 2*b^2)*(b^3
*c^2 - 4*a^2*b*c^2 - 3*a^2*b*d^2 + 4*a^3*c*d + 2*a*b^2*c*d))/(a^2*(a^4 + b^
4 - 2*a^2*b^2)))/(f*(tan(e/2 + (f*x)/2)^2*(2*a^2 + 4*b^2) + a^2*tan(e/2 + (
f*x)/2)^4 + a^2 + 4*a*b*tan(e/2 + (f*x)/2)^3 + 4*a*b*tan(e/2 + (f*x)/2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**2/(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.718 $\int \frac{c+d\sin(e+fx)}{(a+b\sin(e+fx))^3}\,dx$

Optimal. Leaf size=162

$$\frac{(2a^2c-3abd+b^2c)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{5/2}}+\frac{\left(a^2(-d)+3abc-2b^2d\right)\cos(e+fx)}{2f\left(a^2-b^2\right)^2(a+b\sin(e+fx))}+\frac{(bc-ad)\cos(e+fx)}{2f\left(a^2-b^2\right)(a+b\sin(e+fx))^2}$$

```
[Out] (2*a^2*c-3*a*b*d+b^2*c)*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/(a
^2-b^2)^(5/2)/f+1/2*(-a*d+b*c)*cos(f*x+e)/(a^2-b^2)/f/(a+b*sin(f*x+e))^2+1/
2*(-a^2*d+3*a*b*c-2*b^2*d)*cos(f*x+e)/(a^2-b^2)^2/f/(a+b*sin(f*x+e))
```

Rubi [A] time = 0.17, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2754, 12, 2660, 618, 204}

$$\frac{(2a^2c-3abd+b^2c)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{5/2}}+\frac{\left(a^2(-d)+3abc-2b^2d\right)\cos(e+fx)}{2f\left(a^2-b^2\right)^2(a+b\sin(e+fx))}+\frac{(bc-ad)\cos(e+fx)}{2f\left(a^2-b^2\right)(a+b\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((2*a^2*c + b^2*c - 3*a*b*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2
]])/((a^2 - b^2)^(5/2)*f) + ((b*c - a*d)*Cos[e + f*x])/(2*(a^2 - b^2)*f*(a
+ b*Sin[e + f*x])^2) + ((3*a*b*c - a^2*d - 2*b^2*d)*Cos[e + f*x])/(2*(a^2 -
 b^2)^2*f*(a + b*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{c+d\sin(e+fx)}{(a+b\sin(e+fx))^3}\,dx = \frac{(bc-ad)\cos(e+fx)}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} - \frac{\int \frac{-2(ac-bd)+(bc-ad)\sin(e+fx)}{(a+b\sin(e+fx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= \frac{(bc-ad)\cos(e+fx)}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{\left(3abc-a^2d-2b^2d\right)\cos(e+fx)}{2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{\int \frac{2a^2c+b^2c-3ab}{a+b\sin(e+fx}}{2\left(a^2-b^2\right)}$$

$$= \frac{(bc-ad)\cos(e+fx)}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{\left(3abc-a^2d-2b^2d\right)\cos(e+fx)}{2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{\left(2a^2c+b^2c-\right.}{2}$$

$$= \frac{(bc-ad)\cos(e+fx)}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{\left(3abc-a^2d-2b^2d\right)\cos(e+fx)}{2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{\left(2a^2c+b^2c-\right.}{}$$

$$= \frac{(bc-ad)\cos(e+fx)}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{\left(3abc-a^2d-2b^2d\right)\cos(e+fx)}{2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} - \frac{\left(2\left(2a^2c+b^2\right.\right.}{}$$

$$= \frac{\left(2a^2c+b^2c-3abd\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2} f} + \frac{(bc-ad)\cos(e+fx)}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{(3a}{2(}$$

Mathematica [A] time = 0.63, size = 157, normalized size = 0.97

$$\frac{\frac{2\left(2a^2c-3abd+b^2c\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} - \frac{\left(a^2d-3abc+2b^2d\right)\cos(e+fx)}{(a-b)^2(a+b)^2(a+b\sin(e+fx))} + \frac{(bc-ad)\cos(e+fx)}{(a-b)(a+b)(a+b\sin(e+fx))^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((2*(2*a^2*c + b^2*c - 3*a*b*d)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 -
b^2]])/(a^2 - b^2)^(5/2) + ((b*c - a*d)*Cos[e + f*x])/((a - b)*(a + b)*(a +
 b*Sin[e + f*x])^2) - ((-3*a*b*c + a^2*d + 2*b^2*d)*Cos[e + f*x])/((a - b)^
2*(a + b)^2*(a + b*Sin[e + f*x])))/(2*f)
```

fricas [B] time = 0.91, size = 799, normalized size = 4.93

$$\left[-\frac{2\left(3\left(a^{3}b^{2}-ab^{4}\right)c-\left(a^{4}b+a^{2}b^{3}-2\,b^{5}\right)d\right)\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(\left(3\,ab^{3}d-\left(2\,a^{2}b^{2}+b^{4}\right)c\right)\cos\left(fx+e\right)^{2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(3*(a^3*b^2 - a*b^4)*c - (a^4*b + a^2*b^3 - 2*b^5)*d)*cos(f*x + e)
*sin(f*x + e) + ((3*a*b^3*d - (2*a^2*b^2 + b^4)*c)*cos(f*x + e)^2 + (2*a^4
+ 3*a^2*b^2 + b^4)*c - 3*(a^3*b + a*b^3)*d - 2*(3*a^2*b^2*d - (2*a^3*b + a*
b^3)*c)*sin(f*x + e))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(f*x + e)^2 -
 2*a*b*sin(f*x + e) - a^2 - b^2 - 2*(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*
x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 -
b^2)) + 2*((4*a^4*b - 5*a^2*b^3 + b^5)*c - (2*a^5 - a^3*b^2 - a*b^4)*d)*cos
(f*x + e))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*f*cos(f*x + e)^2 - 2*(a
^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*f*sin(f*x + e) - (a^8 - 2*a^6*b^2 + 2
*a^2*b^6 - b^8)*f), -1/2*((3*(a^3*b^2 - a*b^4)*c - (a^4*b + a^2*b^3 - 2*b^5
)*d)*cos(f*x + e)*sin(f*x + e) - ((3*a*b^3*d - (2*a^2*b^2 + b^4)*c)*cos(f*x
 + e)^2 + (2*a^4 + 3*a^2*b^2 + b^4)*c - 3*(a^3*b + a*b^3)*d - 2*(3*a^2*b^2*
d - (2*a^3*b + a*b^3)*c)*sin(f*x + e))*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x +
 e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))) + ((4*a^4*b - 5*a^2*b^3 + b^5)*c -
 (2*a^5 - a^3*b^2 - a*b^4)*d)*cos(f*x + e))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b
^6 - b^8)*f*cos(f*x + e)^2 - 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*f*si
n(f*x + e) - (a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8)*f)]
```

giac [B] time = 0.24, size = 429, normalized size = 2.65

$$\frac{\frac{\left(2\,a^{2}c+b^{2}c-3\,abd\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\sqrt{a^{2}-b^{2}}}+\frac{5\,a^{3}b^{2}c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-2\,ab^{4}c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-3\,a^{4}bd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+4\,a^{4}b}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((2*a^2*c + b^2*c - 3*a*b*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arc
tan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^4 - 2*a^2*b^2 + b^4)
*sqrt(a^2 - b^2)) + (5*a^3*b^2*c*tan(1/2*f*x + 1/2*e)^3 - 2*a*b^4*c*tan(1/2
*f*x + 1/2*e)^3 - 3*a^4*b*d*tan(1/2*f*x + 1/2*e)^3 + 4*a^4*b*c*tan(1/2*f*x
```

```
+ 1/2*e)^2 + 7*a^2*b^3*c*tan(1/2*f*x + 1/2*e)^2 - 2*b^5*c*tan(1/2*f*x + 1/2
*e)^2 - 2*a^5*d*tan(1/2*f*x + 1/2*e)^2 - 5*a^3*b^2*d*tan(1/2*f*x + 1/2*e)^2
 - 2*a*b^4*d*tan(1/2*f*x + 1/2*e)^2 + 11*a^3*b^2*c*tan(1/2*f*x + 1/2*e) - 2
*a*b^4*c*tan(1/2*f*x + 1/2*e) - 5*a^4*b*d*tan(1/2*f*x + 1/2*e) - 4*a^2*b^3*
d*tan(1/2*f*x + 1/2*e) + 4*a^4*b*c - a^2*b^3*c - 2*a^5*d - a^3*b^2*d)/((a^6
 - 2*a^4*b^2 + a^2*b^4)*(a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e
) + a)^2))/f
```

maple [B] time = 0.24, size = 1291, normalized size = 7.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))/(a+b*sin(f*x+e))^3,x)
```

```
[Out] -3/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*b*a^2/(a^4-2*a^2*b
^2+b^4)*tan(1/2*f*x+1/2*e)^3*d+5/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/
2*e)*b+a)^2*b^2*a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)^3*c-2/f/(tan(1/2*f
*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*b^4/a/(a^4-2*a^2*b^2+b^4)*tan(1/2
*f*x+1/2*e)^3*c-2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^
4-2*a^2*b^2+b^4)*a^3*tan(1/2*f*x+1/2*e)^2*d+4/f/(tan(1/2*f*x+1/2*e)^2*a+2*t
an(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+1/2*e)^2*b*c-5
/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*
a*tan(1/2*f*x+1/2*e)^2*b^2*d+7/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*
e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)^2*b^3*c-2/f/(tan(1/2*f*x+1
/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)/a*tan(1/2*f*x+1/2
*e)^2*b^4*d-2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*
a^2*b^2+b^4)/a^2*tan(1/2*f*x+1/2*e)^2*b^5*c-5/f/(tan(1/2*f*x+1/2*e)^2*a+2*t
an(1/2*f*x+1/2*e)*b+a)^2*b*a^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*d+11/
f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*b^2*a/(a^4-2*a^2*b^2+
b^4)*tan(1/2*f*x+1/2*e)*c-4/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*
b+a)^2*b^3/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*d-2/f/(tan(1/2*f*x+1/2*e)
^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*b^4/a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*
e)*c-2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2
+b^4)*a^3*d+4/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*
a^2*b^2+b^4)*a^2*b*c-1/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^
2/(a^4-2*a^2*b^2+b^4)*a*b^2*d-1/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2
*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*b^3*c+2/f/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2
)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*c-3/f/(a^4-2
*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-
b^2)^(1/2))*a*b*d+1/f/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*t
an(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*b^2*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 9.79, size = 477, normalized size = 2.94

$$\frac{\operatorname{atan}\left(\frac{\left(\frac{\left(2a^4b-4a^2b^3+2b^5\right)\left(2ca^2-3dab+cb^2\right)}{2(a+b)^{5/2}(a-b)^{5/2}\left(a^4-2a^2b^2+b^4\right)}+\frac{a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(2ca^2-3dab+cb^2\right)}{(a+b)^{5/2}(a-b)^{5/2}}\right)\left(a^4-2a^2b^2+b^4\right)}{2ca^2-3dab+cb^2}\right)\left(2ca^2-3dab+cb^2\right)}{f(a+b)^{5/2}(a-b)^{5/2}}-\frac{2da^3-4ca^2b+da}{a^4-2a^2b^2+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))/(a + b*sin(e + f*x))^3,x)
```

```
[Out] (atan(((((2*a^4*b + 2*b^5 - 4*a^2*b^3)*(2*a^2*c + b^2*c - 3*a*b*d))/(2*(a +
 b)^(5/2)*(a - b)^(5/2)*(a^4 + b^4 - 2*a^2*b^2)) + (a*tan(e/2 + (f*x)/2)*(2
*a^2*c + b^2*c - 3*a*b*d))/((a + b)^(5/2)*(a - b)^(5/2)))*(a^4 + b^4 - 2*a^
2*b^2))/(2*a^2*c + b^2*c - 3*a*b*d))*(2*a^2*c + b^2*c - 3*a*b*d))/(f*(a + b
)^(5/2)*(a - b)^(5/2)) - ((2*a^3*d + b^3*c - 4*a^2*b*c + a*b^2*d)/(a^4 + b^
4 - 2*a^2*b^2) + (b*tan(e/2 + (f*x)/2)^3*(3*a^3*d + 2*b^3*c - 5*a^2*b*c))/(
a*(a^4 + b^4 - 2*a^2*b^2)) + (b*tan(e/2 + (f*x)/2)*(5*a^3*d + 2*b^3*c - 11*
a^2*b*c + 4*a*b^2*d))/(a*(a^4 + b^4 - 2*a^2*b^2)) + (tan(e/2 + (f*x)/2)^2*(
a^2 + 2*b^2)*(2*a^3*d + b^3*c - 4*a^2*b*c + a*b^2*d))/(a^2*(a^4 + b^4 - 2*a
^2*b^2)))/(f*(tan(e/2 + (f*x)/2)^2*(2*a^2 + 4*b^2) + a^2*tan(e/2 + (f*x)/2)
^4 + a^2 + 4*a*b*tan(e/2 + (f*x)/2)^3 + 4*a*b*tan(e/2 + (f*x)/2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))/(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.719 $\int \frac{1}{(a+b\sin(e+fx))^3}\,dx$

Optimal. Leaf size=131

$$\frac{(2a^2+b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f(a^2-b^2)^{5/2}}+\frac{3ab\cos(e+fx)}{2f(a^2-b^2)^2(a+b\sin(e+fx))}+\frac{b\cos(e+fx)}{2f(a^2-b^2)(a+b\sin(e+fx))^2}$$

```
[Out] (2*a^2+b^2)*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2
)/f+1/2*b*cos(f*x+e)/(a^2-b^2)/f/(a+b*sin(f*x+e))^2+3/2*a*b*cos(f*x+e)/(a^2
-b^2)^2/f/(a+b*sin(f*x+e))
```

Rubi [A] time = 0.11, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2664, 2754, 12, 2660, 618, 204}

$$\frac{(2a^2+b^2)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f(a^2-b^2)^{5/2}}+\frac{3ab\cos(e+fx)}{2f(a^2-b^2)^2(a+b\sin(e+fx))}+\frac{b\cos(e+fx)}{2f(a^2-b^2)(a+b\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(-3),x]
```

```
[Out] ((2*a^2 + b^2)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^
2)^(5/2)*f) + (b*Cos[e + f*x])/(2*(a^2 - b^2)*f*(a + b*Sin[e + f*x])^2) + (
3*a*b*Cos[e + f*x])/(2*(a^2 - b^2)^2*f*(a + b*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^3}\,dx = \frac{b\cos(e+fx)}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} - \frac{\int \frac{-2a+b\sin(e+fx)}{(a+b\sin(e+fx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= \frac{b\cos(e+fx)}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{3ab\cos(e+fx)}{2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{\int \frac{2a^2+b^2}{a+b\sin(e+fx)}}{2\left(a^2-b^2\right)}$$

$$= \frac{b\cos(e+fx)}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{3ab\cos(e+fx)}{2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{\left(2a^2+b^2\right)\int}{2\left(a^2\right.}$$

$$= \frac{b\cos(e+fx)}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{3ab\cos(e+fx)}{2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{\left(2a^2+b^2\right)\mathrm{Su}}{}$$

$$= \frac{b\cos(e+fx)}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{3ab\cos(e+fx)}{2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} - \frac{\left(2\left(2a^2+b^2\right)\right.}{}$$

$$= \frac{\left(2a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}f} + \frac{b\cos(e+fx)}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{3ab}{2\left(a^2-b^2\right)^2}$$

Mathematica [A] time = 0.40, size = 114, normalized size = 0.87

$$\frac{\frac{2\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{b\cos(e+fx)\left(4a^2+3ab\sin(e+fx)-b^2\right)}{(a-b)^2(a+b)^2(a+b\sin(e+fx))^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^(-3),x]
```

```
[Out] ((2*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(a^2 -
b^2)^(5/2) + (b*Cos[e + f*x]*(4*a^2 - b^2 + 3*a*b*Sin[e + f*x]))/((a - b)^2
*(a + b)^2*(a + b*Sin[e + f*x])^2))/(2*f)
```

fricas [B] time = 0.71, size = 618, normalized size = 4.72

$$\left[-\frac{6\left(a^3b^2-ab^4\right)\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(2\,a^4+3\,a^2b^2+b^4-\left(2\,a^2b^2+b^4\right)\cos\left(fx+e\right)^2+2\left(2\,a^3b+ab^3\right)\mathrm{s}\right.}{4\left(\left(a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8\right)f\cos\left(fx+e\right)^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*(a^3*b^2 - a*b^4)*cos(f*x + e)*sin(f*x + e) - (2*a^4 + 3*a^2*b^2 +
 b^4 - (2*a^2*b^2 + b^4)*cos(f*x + e)^2 + 2*(2*a^3*b + a*b^3)*sin(f*x + e))
*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) -
a^2 - b^2 + 2*(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^
2))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)) + 2*(4*a^4*b - 5
*a^2*b^3 + b^5)*cos(f*x + e))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*f*co
s(f*x + e)^2 - 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*f*sin(f*x + e) - (
a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8)*f), -1/2*(3*(a^3*b^2 - a*b^4)*cos(f*x +
e)*sin(f*x + e) - (2*a^4 + 3*a^2*b^2 + b^4 - (2*a^2*b^2 + b^4)*cos(f*x + e)
^2 + 2*(2*a^3*b + a*b^3)*sin(f*x + e))*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x +
 e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))) + (4*a^4*b - 5*a^2*b^3 + b^5)*cos(
f*x + e))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*f*cos(f*x + e)^2 - 2*(a^
7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*f*sin(f*x + e) - (a^8 - 2*a^6*b^2 + 2*
a^2*b^6 - b^8)*f)]
```

giac [B] time = 0.18, size = 284, normalized size = 2.17

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+b}{\sqrt{a^2-b^2}}\right)\right)\left(2\,a^2+b^2\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{5\,a^3b^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2\,ab^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+4\,a^4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+7\,a^2b^3\tan\left(\frac{1}{2}fx-\right.}{\left(a^6-2\,a^4b^2+a^2b^4\right)\left(a\tan\left(\frac{1}{2}fx+\right.\right.}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e)
+ b)/sqrt(a^2 - b^2)))*(2*a^2 + b^2)/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^
2)) + (5*a^3*b^2*tan(1/2*f*x + 1/2*e)^3 - 2*a*b^4*tan(1/2*f*x + 1/2*e)^3 +
4*a^4*b*tan(1/2*f*x + 1/2*e)^2 + 7*a^2*b^3*tan(1/2*f*x + 1/2*e)^2 - 2*b^5*t
an(1/2*f*x + 1/2*e)^2 + 11*a^3*b^2*tan(1/2*f*x + 1/2*e) - 2*a*b^4*tan(1/2*f
*x + 1/2*e) + 4*a^4*b - a^2*b^3)/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a*tan(1/2*f*
x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)^2))/f
```

maple [B] time = 0.20, size = 705, normalized size = 5.38

$$\frac{5b^2a\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)b+a\right)^2\left(a^4-2a^2b^2+b^4\right)}-\frac{2b^4\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)b+a\right)^2\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^3,x)
```

```
[Out] 5/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*b^2/(a^4-2*a^2*b^2+
b^4)*a*tan(1/2*f*x+1/2*e)^3-2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e
)*b+a)^2*b^4/(a^4-2*a^2*b^2+b^4)/a*tan(1/2*f*x+1/2*e)^3+4/f/(tan(1/2*f*x+1/
2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*b/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+
1/2*e)^2+7/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*b^3/(a^4-2
*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)^2-2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*
x+1/2*e)*b+a)^2*b^5/(a^4-2*a^2*b^2+b^4)/a^2*tan(1/2*f*x+1/2*e)^2+11/f/(tan(
1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*b^2*a/(a^4-2*a^2*b^2+b^4)*ta
n(1/2*f*x+1/2*e)-2/f/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*b^
4/a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)+4/f/(tan(1/2*f*x+1/2*e)^2*a+2*ta
n(1/2*f*x+1/2*e)*b+a)^2*b/(a^4-2*a^2*b^2+b^4)*a^2-1/f/(tan(1/2*f*x+1/2*e)^2
*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*b^3/(a^4-2*a^2*b^2+b^4)+2/f/(a^4-2*a^2*b^2+b
^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2)
)*a^2+1/f/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1
/2*e)+2*b)/(a^2-b^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 10.15, size = 395, normalized size = 3.02

$$\frac{\frac{4a^2b-b^3}{a^4-2a^2b^2+b^4}+\frac{b\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(11a^2b-2b^3\right)}{a\left(a^4-2a^2b^2+b^4\right)}+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(4a^2b-b^3\right)\left(a^2+2b^2\right)}{a^2\left(a^4-2a^2b^2+b^4\right)}+\frac{b\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(5a^2b-2b^3\right)}{a\left(a^4-2a^2b^2+b^4\right)}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(2a^2+4b^2\right)+a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+a^2+4ab\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3+4ab\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}+\frac{\mathrm{atan}\left(\left(\left(\frac{(2\ldots}{2(a+\ldots}\right.\right.\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(e + f*x))^3,x)
```

```
[Out] ((4*a^2*b - b^3)/(a^4 + b^4 - 2*a^2*b^2) + (b*tan(e/2 + (f*x)/2)*(11*a^2*b
- 2*b^3))/(a*(a^4 + b^4 - 2*a^2*b^2)) + (tan(e/2 + (f*x)/2)^2*(4*a^2*b - b^
3)*(a^2 + 2*b^2))/(a^2*(a^4 + b^4 - 2*a^2*b^2)) + (b*tan(e/2 + (f*x)/2)^3*(
5*a^2*b - 2*b^3))/(a*(a^4 + b^4 - 2*a^2*b^2)))/(f*(tan(e/2 + (f*x)/2)^2*(2*
a^2 + 4*b^2) + a^2*tan(e/2 + (f*x)/2)^4 + a^2 + 4*a*b*tan(e/2 + (f*x)/2)^3
+ 4*a*b*tan(e/2 + (f*x)/2))) + (atan(((((2*a^2 + b^2)*(2*a^4*b + 2*b^5 - 4*
a^2*b^3))/(2*(a + b)^(5/2)*(a - b)^(5/2)*(a^4 + b^4 - 2*a^2*b^2)) + (a*tan(
e/2 + (f*x)/2)*(2*a^2 + b^2))/((a + b)^(5/2)*(a - b)^(5/2)))*(a^4 + b^4 - 2
*a^2*b^2))/(2*a^2 + b^2))*(2*a^2 + b^2))/(f*(a + b)^(5/2)*(a - b)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.720 $\int \frac{1}{(a+b\sin(e+fx))^3(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=285

$$\frac{b^2\left(-5a^2d+3abc+2b^2d\right)\cos(e+fx)}{2f\left(a^2-b^2\right)^2(bc-ad)^2(a+b\sin(e+fx))}+\frac{b^2\cos(e+fx)}{2f\left(a^2-b^2\right)(bc-ad)(a+b\sin(e+fx))^2}-\frac{b\left(-6a^4d^2+6a^3bcd-a^2b^2\right.}{}$$

```
[Out] -b*(6*a^3*b*c*d-6*a^4*d^2-a^2*b^2*(2*c^2-5*d^2)-b^4*(c^2+2*d^2))*arctan((b+
a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/(-a*d+b*c)^3/f+1/2*b
^2*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(a+b*sin(f*x+e))^2+1/2*b^2*(-5*a^2*d+3
*a*b*c+2*b^2*d)*cos(f*x+e)/(a^2-b^2)^2/(-a*d+b*c)^2/f/(a+b*sin(f*x+e))-2*d^
3*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(-a*d+b*c)^3/f/(c^2-d^2)
^(1/2)
```

Rubi [A] time = 1.04, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2802, 3055, 3001, 2660, 618, 204}

$$-\frac{b\left(-a^2b^2\left(2c^2-5d^2\right)+6a^3bcd-6a^4d^2-b^4\left(c^2+2d^2\right)\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{5/2}(bc-ad)^3}+\frac{b^2\left(-5a^2d+3abc+2b^2d\right)\cos}{2f\left(a^2-b^2\right)^2(bc-ad)^2(a+b\sin}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])),x]
```

```
[Out] -((b*(6*a^3*b*c*d - 6*a^4*d^2 - a^2*b^2*(2*c^2 - 5*d^2) - b^4*(c^2 + 2*d^2)
)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2)*(b*c
 - a*d)^3*f)) - (2*d^3*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((
b*c - a*d)^3*Sqrt[c^2 - d^2]*f) + (b^2*Cos[e + f*x])/(2*(a^2 - b^2)*(b*c -
a*d)*f*(a + b*Sin[e + f*x])^2) + (b^2*(3*a*b*c - 5*a^2*d + 2*b^2*d)*Cos[e +
 f*x])/(2*(a^2 - b^2)^2*(b*c - a*d)^2*f*(a + b*Sin[e + f*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
```

```
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^3(c+d\sin(e+fx))}\,dx = \frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2} - \frac{\int \frac{-2\left(abc-a^2d+b^2d\right)+b(bc-}{(a+b\sin(e+f}}{2\left(a^2-\right.}$$

$$= \frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2} + \frac{b^2\left(3abc-5a^2d+\right.}{2\left(a^2-b^2\right)^2(bc-ad)}$$

$$= \frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2} + \frac{b^2\left(3abc-5a^2d+\right.}{2\left(a^2-b^2\right)^2(bc-ad)}$$

$$= \frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2} + \frac{b^2\left(3abc-5a^2d+\right.}{2\left(a^2-b^2\right)^2(bc-ad)}$$

$$= \frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2} + \frac{b^2\left(3abc-5a^2d+\right.}{2\left(a^2-b^2\right)^2(bc-ad)}$$

$$= -\frac{b\left(6a^3bcd-6a^4d^2-a^2b^2\left(2c^2-5d^2\right)-b^4\left(c^2+2d^2\right)\right)\tan^{-1}\left(\frac{b+a}{}\right.}{\left(a^2-b^2\right)^{5/2}(bc-ad)^3f}$$

Mathematica [A] time = 2.35, size = 275, normalized size = 0.96

$$\frac{\frac{b^2\left(-5a^2d+3abc+2b^2d\right)\cos(e+fx)}{(a-b)^2(a+b)^2(bc-ad)^2(a+b\sin(e+fx))} - \frac{2b\left(6a^4d^2-6a^3bcd+a^2b^2\left(2c^2-5d^2\right)+b^4\left(c^2+2d^2\right)\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}(ad-bc)^3} - \frac{b^2\cos(e+fx)}{(a-b)(a+b)(ad-bc)(a+b\sin(e+f}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])),x]
```

```
[Out] ((-2*b*(-6*a^3*b*c*d + 6*a^4*d^2 + a^2*b^2*(2*c^2 - 5*d^2) + b^4*(c^2 + 2*d
^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2)*(
```

```
-(b*c) + a*d)^3) + (4*d^3*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])
/((-(b*c) + a*d)^3*Sqrt[c^2 - d^2]) - (b^2*Cos[e + f*x])/((a - b)*(a + b)*(
-(b*c) + a*d)*(a + b*Sin[e + f*x])^2) + (b^2*(3*a*b*c - 5*a^2*d + 2*b^2*d)*
Cos[e + f*x])/((a - b)^2*(a + b)^2*(b*c - a*d)^2*(a + b*Sin[e + f*x])))/(2*
f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 1.34, size = 788, normalized size = 2.76

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)d^3}{\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\sqrt{c^2-d^2}}-\frac{\left(2a^2b^3c^2+b^5c^2-6a^3b^2cd+6a^4bd^2-5a^2b^3d^2+2b^5d^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\right.}{\left(a^4b^3c^3-2a^2b^5c^3+b^7c^3-3a^5b^2c^2d+6a^3b^4c^2d-3ab^6c^2d+3a^6bcd^2-6a^4b^3cd^2+3a^2b^5cd^2-a^7\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -(2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*
e) + d)/sqrt(c^2 - d^2)))*d^3/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3)*sqrt(c^2 - d^2)) - (2*a^2*b^3*c^2 + b^5*c^2 - 6*a^3*b^2*c*d + 6*a^4
*b*d^2 - 5*a^2*b^3*d^2 + 2*b^5*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a
) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^4*b^3*c^3 - 2
*a^2*b^5*c^3 + b^7*c^3 - 3*a^5*b^2*c^2*d + 6*a^3*b^4*c^2*d - 3*a*b^6*c^2*d
+ 3*a^6*b*c*d^2 - 6*a^4*b^3*c*d^2 + 3*a^2*b^5*c*d^2 - a^7*d^3 + 2*a^5*b^2*d
^3 - a^3*b^4*d^3)*sqrt(a^2 - b^2)) - (5*a^3*b^4*c*tan(1/2*f*x + 1/2*e)^3 -
2*a*b^6*c*tan(1/2*f*x + 1/2*e)^3 - 7*a^4*b^3*d*tan(1/2*f*x + 1/2*e)^3 + 4*a
^2*b^5*d*tan(1/2*f*x + 1/2*e)^3 + 4*a^4*b^3*c*tan(1/2*f*x + 1/2*e)^2 + 7*a^
2*b^5*c*tan(1/2*f*x + 1/2*e)^2 - 2*b^7*c*tan(1/2*f*x + 1/2*e)^2 - 6*a^5*b^2
*d*tan(1/2*f*x + 1/2*e)^2 - 9*a^3*b^4*d*tan(1/2*f*x + 1/2*e)^2 + 6*a*b^6*d*
tan(1/2*f*x + 1/2*e)^2 + 11*a^3*b^4*c*tan(1/2*f*x + 1/2*e) - 2*a*b^6*c*tan(
1/2*f*x + 1/2*e) - 17*a^4*b^3*d*tan(1/2*f*x + 1/2*e) + 8*a^2*b^5*d*tan(1/2*
f*x + 1/2*e) + 4*a^4*b^3*c - a^2*b^5*c - 6*a^5*b^2*d + 3*a^3*b^4*d)/((a^6*b
^2*c^2 - 2*a^4*b^4*c^2 + a^2*b^6*c^2 - 2*a^7*b*c*d + 4*a^5*b^3*c*d - 2*a^3*
b^5*c*d + a^8*d^2 - 2*a^6*b^2*d^2 + a^4*b^4*d^2)*(a*tan(1/2*f*x + 1/2*e)^2
+ 2*b*tan(1/2*f*x + 1/2*e) + a)^2))/f
```

maple [B] time = 0.37, size = 2644, normalized size = 9.28

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e)),x)
```

```
[Out] 16/f*b^5/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a
^4-2*a^2*b^2+b^4)*a*tan(1/2*f*x+1/2*e)^2*c*d-8/f*b^7/(a*d-b*c)^3/(tan(1/2*f
*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)/a*tan(1/2*f*x
+1/2*e)^2*c*d+28/f*b^4/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/
2*e)*b+a)^2*a^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c*d+1/f*b^6/(a*d-b*c
)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)
*c^2-6/f*b^2/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^
2/(a^4-2*a^2*b^2+b^4)*a^4*tan(1/2*f*x+1/2*e)^2*d^2-4/f*b^4/(a*d-b*c)^3/(tan
(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan
(1/2*f*x+1/2*e)^2*c^2-9/f*b^4/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2
*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+1/2*e)^2*d^2+2/f*b^8
/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2
*b^2+b^4)/a^2*tan(1/2*f*x+1/2*e)^2*c^2+4/f*b^5/(a*d-b*c)^3/(tan(1/2*f*x+1/2
*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*tan(1/2*f*x+1/2*e
)^3*d^2-17/f*b^3/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b
+a)^2*a^3/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*d^2-11/f*b^5/(a*d-b*c)^3/(
tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a/(a^4-2*a^2*b^2+b^4)*ta
n(1/2*f*x+1/2*e)*c^2+8/f*b^5/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*
f*x+1/2*e)*b+a)^2*a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*d^2+2/f*b^7/(a*d
-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/a/(a^4-2*a^2*b^
2+b^4)*tan(1/2*f*x+1/2*e)*c^2+10/f*b^3/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+
2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*c*d-4/f*b^5/(a*d-b*c)^3
/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*
c*d-6/f*b^6/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2
/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)^3*c*d-2/f*b^3/(a*d-b*c)^3/(a^4-2*a^
2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2
)^(1/2))*c^2*a^2-10/f*b^6/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x
+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c*d+2/f*b^7/(a*d-b*c)
^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)/
a*tan(1/2*f*x+1/2*e)^3*c^2+5/f*b^3/(a*d-b*c)^3/(a^4-2*a^2*b^2+b^4)/(a^2-b^2
)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^2*d^2-6/
f*b/(a*d-b*c)^3/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2
*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^4*d^2-7/f*b^3/(a*d-b*c)^3/(tan(1/2*f*x+
1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*tan(1/2*f*x+
1/2*e)^3*d^2-5/f*b^5/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*
e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*tan(1/2*f*x+1/2*e)^3*c^2-7/f*b^6/(a*d-b*c)^
3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*t
```

```
an(1/2*f*x+1/2*e)^2*c^2+6/f*b^6/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1
/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)^2*d^2-6/f*b^2/(
a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b
^2+b^4)*a^4*d^2-4/f*b^4/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1
/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*c^2*a^2+3/f*b^4/(a*d-b*c)^3/(tan(1/2*f*x+1
/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*d^2-1/f*b^5/(
a*d-b*c)^3/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+
1/2*e)+2*b)/(a^2-b^2)^(1/2))*c^2-2/f*b^5/(a*d-b*c)^3/(a^4-2*a^2*b^2+b^4)/(a
^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*d^2+
6/f*b^2/(a*d-b*c)^3/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan
(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^3*c*d+12/f*b^4/(a*d-b*c)^3/(tan(1/2
*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2
*f*x+1/2*e)^3*c*d+10/f*b^3/(a*d-b*c)^3/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*
x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*tan(1/2*f*x+1/2*e)^2*c*d+2/f*d^3/(a
^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c
*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [**B**] time = 30.17, size = 62873, normalized size = 220.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x))),x)
```

```
[Out] (d^3*atan(((d^3*(d^2 - c^2)^(1/2)*((8*(4*a*b^12*c^4*d^5 + 4*a*b^12*c^6*d^3
+ 4*a^3*b^10*c^8*d + 4*a^4*b^9*c*d^8 + 4*a^5*b^8*c^8*d - 16*a^6*b^7*c*d^8 +
 24*a^8*b^5*c*d^8 - 16*a^10*b^3*c*d^8 - 4*a^2*b^11*c^3*d^6 - 8*a^2*b^11*c^5
*d^4 - 2*a^2*b^11*c^7*d^2 - 4*a^3*b^10*c^2*d^7 - 16*a^3*b^10*c^4*d^5 - a^3*
b^10*c^6*d^3 + 24*a^4*b^9*c^3*d^6 - 20*a^4*b^9*c^5*d^4 - 20*a^4*b^9*c^7*d^2
 + 12*a^5*b^8*c^2*d^7 + 95*a^5*b^8*c^4*d^5 + 20*a^5*b^8*c^6*d^3 - 98*a^6*b^
7*c^3*d^6 + 64*a^6*b^7*c^5*d^4 - 32*a^6*b^7*c^7*d^2 + a^7*b^6*c^2*d^7 - 188
*a^7*b^6*c^4*d^5 + 112*a^7*b^6*c^6*d^3 + 164*a^8*b^5*c^3*d^6 - 216*a^8*b^5*
c^5*d^4 - 28*a^9*b^4*c^2*d^7 + 240*a^9*b^4*c^4*d^5 - 140*a^10*b^3*c^3*d^6 +
```

28*a^11*b^2*c^2*d^7 + a*b^12*c^8*d + 4*a^12*b*c*d^8))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) - (8*tan(e/2 + (f*x)/2)*(a*b^12*c^9 + 4*a^13*c*d^8 + 4*a^3*b^10*c^9 + 4*a^5*b^8*c^9 - 16*a*b^12*c^3*d^6 - 4*a*b^12*c^5*d^4 + 2*a*b^12*c^7*d^2 - 2*a^2*b^11*c^8*d - 16*a^3*b^10*c*d^8 - 20*a^4*b^9*c^8*d + 76*a^5*b^8*c*d^8 - 32*a^6*b^7*c^8*d - 162*a^7*b^6*c*d^8 + 176*a^9*b^4*c*d^8 - 96*a^11*b^2*c*d^8 - 8*a^12*b*c^2*d^7 + 32*a^2*b^11*c^2*d^7 + 8*a^2*b^11*c^4*d^5 - 4*a^2*b^11*c^6*d^3 + 72*a^3*b^10*c^3*d^6 - 14*a^3*b^10*c^5*d^4 - 9*a^3*b^10*c^7*d^2 - 152*a^4*b^9*c^2*d^7 + 80*a^4*b^9*c^4*d^5 + 20*a^4*b^9*c^6*d^3 - 274*a^5*b^8*c^3*d^6 + 55*a^5*b^8*c^5*d^4 + 12*a^5*b^8*c^7*d^2 + 372*a^6*b^7*c^2*d^7 - 250*a^6*b^7*c^4*d^5 + 128*a^6*b^7*c^6*d^3 + 481*a^7*b^6*c^3*d^6 - 412*a^7*b^6*c^5*d^4 + 112*a^7*b^6*c^7*d^2 - 472*a^8*b^5*c^2*d^7 + 612*a^8*b^5*c^4*d^5 - 216*a^8*b^5*c^6*d^3 - 564*a^9*b^4*c^3*d^6 + 240*a^9*b^4*c^5*d^4 + 336*a^10*b^3*c^2*d^7 - 144*a^10*b^3*c^4*d^5 + 40*a^11*b^2*c^3*d^6))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) + (d^3*(d^2 - c^2)^(1/2)*((8*(2*a^2*b^14*c^10 - 6*a^6*b^10*c^10 + 4*a^8*b^8*c^10 + 4*a^16*c^2*d^8 + 4*a*b^15*c^7*d^3 - 10*a^3*b^13*c^9*d - 12*a^5*b^11*c^9*d + 4*a^7*b^9*c*d^9 + 54*a^7*b^9*c^9*d - 18*a^9*b^7*c*d^9 - 32*a^9*b^7*c^9*d + 36*a^11*b^5*c*d^9 - 34*a^13*b^3*c*d^9 - 32*a^15*b*c^3*d^7 - 24*a^2*b^14*c^6*d^4 + 2*a^2*b^14*c^8*d^2 + 60*a^3*b^13*c^5*d^5 - 30*a^3*b^13*c^7*d^3 - 80*a^4*b^12*c^4*d^6 + 138*a^4*b^12*c^6*d^4 + 2*a^4*b^12*c^8*d^2 + 60*a^5*b^11*c^3*d^7 - 310*a^5*b^11*c^5*d^5 + 122*a^5*b^11*c^7*d^3 - 24*a^6*b^10*c^2*d^8 + 390*a^6*b^10*c^4*d^6 - 466*a^6*b^10*c^6*d^4 + 102*a^6*b^10*c^8*d^2 - 282*a^7*b^9*c^3*d^7 + 878*a^7*b^9*c^5*d^5 - 394*a^7*b^9*c^7*d^3 + 110*a^8*b^8*c^2*d^8 - 970*a^8*b^8*c^4*d^6 + 894*a^8*b^8*c^6*d^4 - 218*a^8*b^8*c^8*d^2 + 638*a^9*b^7*c^3*d^7 - 1290*a^9*b^7*c^5*d^5 + 522*a^9*b^7*c^7*d^3 - 232*a^10*b^6*c^2*d^8 + 1202*a^10*b^6*c^4*d^6 - 822*a^10*b^6*c^6*d^4 + 112*a^10*b^6*c^8*d^2 - 702*a^11*b^5*c^3*d^7 + 886*a^11*b^5*c^5*d^5 - 224*a^11*b^5*c^7*d^3 + 234*a^12*b^4*c^2*d^8 - 654*a^12*b^4*c^4*d^6 + 280*a^12*b^4*c^6*d^4 + 318*a^13*b^3*c^3*d^7 - 224*a^13*b^3*c^5*d^5 - 92*a^14*b^

2*c^2*d^8 + 112*a^14*b^2*c^4*d^6 + 12*a^15*b*c*d^9))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) + (8*tan(e/2 + (f*x)/2)*(4*a*b^15*c^10 + 8*a^16*c*d^9 - 12*a^5*b^11*c^10 + 8*a^7*b^9*c^10 + 4*a*b^15*c^8*d^2 - 20*a^2*b^14*c^9*d - 24*a^4*b^12*c^9*d + 108*a^6*b^10*c^9*d + 4*a^8*b^8*c*d^9 - 64*a^8*b^8*c^9*d - 8*a^10*b^6*c*d^9 + 12*a^12*b^4*c*d^9 - 16*a^14*b^2*c*d^9 - 40*a^15*b*c^2*d^8 - 20*a^2*b^14*c^7*d^3 + 36*a^3*b^13*c^6*d^4 + 4*a^3*b^13*c^8*d^2 - 20*a^4*b^12*c^5*d^5 + 164*a^4*b^12*c^7*d^3 - 20*a^5*b^11*c^4*d^6 - 452*a^5*b^11*c^6*d^4 + 204*a^5*b^11*c^8*d^2 + 36*a^6*b^10*c^3*d^7 + 556*a^6*b^10*c^5*d^5 - 708*a^6*b^10*c^7*d^3 - 20*a^7*b^9*c^2*d^8 - 340*a^7*b^9*c^4*d^6 + 1308*a^7*b^9*c^6*d^4 - 436*a^7*b^9*c^8*d^2 + 76*a^8*b^8*c^3*d^7 - 1380*a^8*b^8*c^5*d^5 + 1004*a^8*b^8*c^7*d^3 + 16*a^9*b^7*c^2*d^8 + 804*a^9*b^7*c^4*d^6 - 1404*a^9*b^7*c^6*d^4 + 224*a^9*b^7*c^8*d^2 - 204*a^10*b^6*c^3*d^7 + 1172*a^10*b^6*c^5*d^5 - 440*a^10*b^6*c^7*d^3 - 12*a^11*b^5*c^2*d^8 - 508*a^11*b^5*c^4*d^6 + 512*a^11*b^5*c^6*d^4 + 36*a^12*b^4*c^3*d^7 - 328*a^12*b^4*c^5*d^5 + 56*a^13*b^3*c^2*d^8 + 64*a^13*b^3*c^4*d^6 + 56*a^14*b^2*c^3*d^7))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) + (d^3*(d^2 - c^2)^(1/2)*((8*(4*a^2*b^17*c^11 - 16*a^4*b^15*c^11 + 24*a^6*b^13*c^11 - 16*a^8*b^11*c^11 + 4*a^10*b^9*c^11 + 4*a^19*c^2*d^9 - 12*a^3*b^16*c^10*d + 88*a^5*b^14*c^10*d - 152*a^7*b^12*c^10*d + 108*a^9*b^10*c^10*d - 4*a^10*b^9*c*d^10 - 28*a^11*b^8*c^10*d + 16*a^12*b^7*c*d^10 - 24*a^14*b^5*c*d^10 + 16*a^16*b^3*c*d^10 - 28*a^18*b*c^3*d^8 + 28*a^2*b^17*c^9*d^2 - 80*a^3*b^16*c^8*d^3 + 112*a^4*b^15*c^7*d^4 - 32*a^4*b^15*c^9*d^2 - 56*a^5*b^14*c^6*d^5 + 208*a^5*b^14*c^8*d^3 - 56*a^6*b^13*c^5*d^6 - 392*a^6*b^13*c^7*d^4 - 152*a^6*b^13*c^9*d^2 + 112*a^7*b^12*c^4*d^7 + 280*a^7*b^12*c^6*d^5 - 32*a^7*b^12*c^8*d^3 - 80*a^8*b^11*c^3*d^8 + 112*a^8*b^11*c^5*d^6 + 448*a^8*b^11*c^7*d^4 + 368*a^8*b^11*c^9*d^2 + 28*a^9*b^10*c^2*d^9 - 368*a^9*b^10*c^4*d^7 - 560*a^9*b^10*c^6*d^5 - 352*a^9*b^10*c^8*d^3 + 292*a^10*b^9*c^3*d^8 + 112*a^10*b^9*c^5*d^6 - 112*a^10*b^9*c^7*d^4 - 292*a^10*b^9*c^9*d^2 - 108*a^11*b^8*c^2*d^9 + 352*a^11*b^8*c^4*d^7 + 560*a^11*b^8*c^6*d^5 + 368*a^11*b^8* c^8*d^3 - 368*a^12*b^7*c^3*d^8 - 448*a^12*b^7*c^5*d^6 - 112*a^12*b^7*c^7*d^4 + 80*a^12*b^7*c^9*d^2 + 152*a^13*b^6*c^2*d^9 + 32*a^13*b^6*c^4*d^7 - 280*a^13*b^6*c^6*d^5 - 112*a^13*b^6*c^8*d^3 + 152*a^14*b^5*c^3*d^8 + 392*a^14*b^5*c^5*d^6 + 56*a^14*b^5*c^7*d^4 - 88*a^15*b^4*c^2*d^9 - 208*a^15*b^4*c^4*d^7 + 56*a^15*b^4*c^6*d^5 + 32*a^16*b^3*c^3*d^8 - 112*a^16*b^3*c^5*d^6 + 12*a^17*b^2*c^2*d^9 + 80*a^17*b^2*c^4*d^7 - 4*a*b^18*c^10*d - 4*a^18*b*c*d^10))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) - (8*tan(e/2 + (f*x)/2)*(56*a^3*b^16*c^11 - 12*a^19*c*d^10 - 12*a*b^18*c^11 - 104*a^5*b^14*c^11 + 96*a^7*b^12*c^11 - 44*a^9*b^10*c^11 + 8*a^11*b^8*c^11 + 8*a^19*c^3*d^8 + 16*a*b^18*c^9*d^2 + 96*a^2*b^17*c^10*d - 448*a^4*b^15*c^10*d + 832*a^6*b^13*c^10*d - 768*a^8*b^11*c^10*d + 16*a^9*b^10*c*d^10 + 352*a^10*b^9*c^10*d - 76*a^11*b^8*c*d^10 - 64*a^12*b^7*c^10*d + 144*a^13*b^6*c*d^10 - 136*a^15*b^4*c*d^10 + 64*a^17*b^2*c*d^10 + 96*a^18*b*c^2*d^9 - 64*a^18*b*c^4*d^7 - 128*a^2*b^17*c^8*d^3 + 448*a^3*b^16*c^7*d^4 - 412*a^3*b^16*c^9*d^2 - 896*a^4*b^15*c^6*d^5 + 1280*a^4*b^15*c^8*d^3 + 1120*a^5*b^14*c^5*d^6 - 2968*a^5*b^14*c^7*d^4 + 1712*a^5*b^14*c^9*d^2 - 896*a^6*b^13*c^4*d^7 + 4928*a^6*b^13*c^6*d^5 - 4288*a^6*b^13*c^8*d^3 + 448*a^7*b^12*c^3*d^8 - 5656*a^7*b^12*c^5*d^6 + 7952*a^7*b^12*c^7*d^4 - 3048*a^7*b^12*c^9*d^2 - 128*a^8*b^11*c^2*d^9 + 4352*a^8*b^11*c^4*d^7 - 11200*a^8*b^11*c^6*d^5 + 6912*a^8*b^11*c^8*d^3 - 2140*a^9*b^10*c^3*d^8 + 11648*a^9*b^10*c^5*d^6 - 11088*a^9*b^10*c^7*d^4 + 2752*a^9*b^10*c^9*d^2 + 608*a^10*b^9*c^2*d^9 - 8512*a^10*b^9*c^4*d^7 + 13440*a^10*b^9*c^6*d^5 - 5888*a^10*b^9*c^8*d^3 + 4088*a^11*b^8*c^3*d^8 - 12432*a^11*b^8*c^5*d^6 + 8512*a^11*b^8*c^7*d^4 - 1244*a^11*b^8*c^9*d^2 - 1152*a^12*b^7*c^2*d^9 + 8448*a^12*b^7*c^4*d^7 - 8960*a^12*b^7*c^6*d^5 + 2560*a^12*b^7*c^8*d^3 - 3912*a^13*b^6*c^3*d^8 + 7168*a^13*b^6*c^5*d^6 - 3416*a^13*b^6*c^7*d^4 + 224*a^13*b^6*c^9*d^2 + 1088*a^14*b^5*c^2*d^9 - 4352*a^14*b^5*c^4*d^7 + 3136*a^14*b^5*c^6*d^5 - 448*a^14*b^5*c^8*d^3 + 1888*a^15*b^4*c^3*d^8 - 2072*a^15*b^4*c^5*d^6 + 560*a^15*b^4*c^7*d^4 - 512*a^16*b^3*c^2*d^9 + 1024*a^16*b^3*c^4*d^7 - 448*a^16*b^3*c^6*d^5 - 380*a^17*b^2*c^3*d^8 + 224*a^17*b^2*c^5*d^6))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2

- 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d
^5)))/(a^3*d^5 + b^3*c^5 - a^3*c^2*d^3 - b^3*c^3*d^2 + 3*a*b^2*c^2*d^3 + 3*
a^2*b*c^3*d^2 - 3*a*b^2*c^4*d - 3*a^2*b*c*d^4)))/(a^3*d^5 + b^3*c^5 - a^3*c
^2*d^3 - b^3*c^3*d^2 + 3*a*b^2*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d -
3*a^2*b*c*d^4))*1i)/(a^3*d^5 + b^3*c^5 - a^3*c^2*d^3 - b^3*c^3*d^2 + 3*a*b^
2*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d - 3*a^2*b*c*d^4) - (d^3*(d^2 -
c^2)^(1/2)*((8*tan(e/2 + (f*x)/2)*(a*b^12*c^9 + 4*a^13*c*d^8 + 4*a^3*b^10*c
^9 + 4*a^5*b^8*c^9 - 16*a*b^12*c^3*d^6 - 4*a*b^12*c^5*d^4 + 2*a*b^12*c^7*d^
2 - 2*a^2*b^11*c^8*d - 16*a^3*b^10*c*d^8 - 20*a^4*b^9*c^8*d + 76*a^5*b^8*c*
d^8 - 32*a^6*b^7*c^8*d - 162*a^7*b^6*c*d^8 + 176*a^9*b^4*c*d^8 - 96*a^11*b^
2*c*d^8 - 8*a^12*b*c^2*d^7 + 32*a^2*b^11*c^2*d^7 + 8*a^2*b^11*c^4*d^5 - 4*a
^2*b^11*c^6*d^3 + 72*a^3*b^10*c^3*d^6 - 14*a^3*b^10*c^5*d^4 - 9*a^3*b^10*c^
7*d^2 - 152*a^4*b^9*c^2*d^7 + 80*a^4*b^9*c^4*d^5 + 20*a^4*b^9*c^6*d^3 - 274
*a^5*b^8*c^3*d^6 + 55*a^5*b^8*c^5*d^4 + 12*a^5*b^8*c^7*d^2 + 372*a^6*b^7*c^
2*d^7 - 250*a^6*b^7*c^4*d^5 + 128*a^6*b^7*c^6*d^3 + 481*a^7*b^6*c^3*d^6 - 4
12*a^7*b^6*c^5*d^4 + 112*a^7*b^6*c^7*d^2 - 472*a^8*b^5*c^2*d^7 + 612*a^8*b^
5*c^4*d^5 - 216*a^8*b^5*c^6*d^3 - 564*a^9*b^4*c^3*d^6 + 240*a^9*b^4*c^5*d^4
 + 336*a^10*b^3*c^2*d^7 - 144*a^10*b^3*c^4*d^5 + 40*a^11*b^2*c^3*d^6))/(a^1
4*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^
6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*
a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24
*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 1
5*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^1
0*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 -
120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*
c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 +
 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) - (8*(4*a*b^12*c^4*
d^5 + 4*a*b^12*c^6*d^3 + 4*a^3*b^10*c^8*d + 4*a^4*b^9*c*d^8 + 4*a^5*b^8*c^8
*d - 16*a^6*b^7*c*d^8 + 24*a^8*b^5*c*d^8 - 16*a^10*b^3*c*d^8 - 4*a^2*b^11*c
^3*d^6 - 8*a^2*b^11*c^5*d^4 - 2*a^2*b^11*c^7*d^2 - 4*a^3*b^10*c^2*d^7 - 16*
a^3*b^10*c^4*d^5 - a^3*b^10*c^6*d^3 + 24*a^4*b^9*c^3*d^6 - 20*a^4*b^9*c^5*d
^4 - 20*a^4*b^9*c^7*d^2 + 12*a^5*b^8*c^2*d^7 + 95*a^5*b^8*c^4*d^5 + 20*a^5*
b^8*c^6*d^3 - 98*a^6*b^7*c^3*d^6 + 64*a^6*b^7*c^5*d^4 - 32*a^6*b^7*c^7*d^2
+ a^7*b^6*c^2*d^7 - 188*a^7*b^6*c^4*d^5 + 112*a^7*b^6*c^6*d^3 + 164*a^8*b^5
*c^3*d^6 - 216*a^8*b^5*c^5*d^4 - 28*a^9*b^4*c^2*d^7 + 240*a^9*b^4*c^4*d^5 -
 140*a^10*b^3*c^3*d^6 + 28*a^11*b^2*c^2*d^7 + a*b^12*c^8*d + 4*a^12*b*c*d^8
))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 +
 a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^
6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d
^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*
d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*
a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4
*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a
^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^
3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) + (d^3*(d^2

- c^2)^(1/2)*((8*(2*a^2*b^14*c^10 - 6*a^6*b^10*c^10 + 4*a^8*b^8*c^10 + 4*a^16*c^2*d^8 + 4*a*b^15*c^7*d^3 - 10*a^3*b^13*c^9*d - 12*a^5*b^11*c^9*d + 4*a^7*b^9*c*d^9 + 54*a^7*b^9*c^9*d - 18*a^9*b^7*c*d^9 - 32*a^9*b^7*c^9*d + 36*a^11*b^5*c*d^9 - 34*a^13*b^3*c*d^9 - 32*a^15*b*c^3*d^7 - 24*a^2*b^14*c^6*d^4 + 2*a^2*b^14*c^8*d^2 + 60*a^3*b^13*c^5*d^5 - 30*a^3*b^13*c^7*d^3 - 80*a^4*b^12*c^4*d^6 + 138*a^4*b^12*c^6*d^4 + 2*a^4*b^12*c^8*d^2 + 60*a^5*b^11*c^3*d^7 - 310*a^5*b^11*c^5*d^5 + 122*a^5*b^11*c^7*d^3 - 24*a^6*b^10*c^2*d^8 + 390*a^6*b^10*c^4*d^6 - 466*a^6*b^10*c^6*d^4 + 102*a^6*b^10*c^8*d^2 - 282*a^7*b^9*c^3*d^7 + 878*a^7*b^9*c^5*d^5 - 394*a^7*b^9*c^7*d^3 + 110*a^8*b^8*c^2*d^8 - 970*a^8*b^8*c^4*d^6 + 894*a^8*b^8*c^6*d^4 - 218*a^8*b^8*c^8*d^2 + 638*a^9*b^7*c^3*d^7 - 1290*a^9*b^7*c^5*d^5 + 522*a^9*b^7*c^7*d^3 - 232*a^10*b^6*c^2*d^8 + 1202*a^10*b^6*c^4*d^6 - 822*a^10*b^6*c^6*d^4 + 112*a^10*b^6*c^8*d^2 - 702*a^11*b^5*c^3*d^7 + 886*a^11*b^5*c^5*d^5 - 224*a^11*b^5*c^7*d^3 + 234*a^12*b^4*c^2*d^8 - 654*a^12*b^4*c^4*d^6 + 280*a^12*b^4*c^6*d^4 + 318*a^13*b^3*c^3*d^7 - 224*a^13*b^3*c^5*d^5 - 92*a^14*b^2*c^2*d^8 + 112*a^14*b^2*c^4*d^6 + 12*a^15*b*c*d^9))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) + (8*tan(e/2 + (f*x)/2)*(4*a*b^15*c^10 + 8*a^16*c*d^9 - 12*a^5*b^11*c^10 + 8*a^7*b^9*c^10 + 4*a*b^15*c^8*d^2 - 20*a^2*b^14*c^9*d - 24*a^4*b^12*c^9*d + 108*a^6*b^10*c^9*d + 4*a^8*b^8*c*d^9 - 64*a^8*b^8*c^9*d - 8*a^10*b^6*c*d^9 + 12*a^12*b^4*c*d^9 - 16*a^14*b^2*c*d^9 - 40*a^15*b*c^2*d^8 - 20*a^2*b^14*c^7*d^3 + 36*a^3*b^13*c^6*d^4 + 4*a^3*b^13*c^8*d^2 - 20*a^4*b^12*c^5*d^5 + 164*a^4*b^12*c^7*d^3 - 20*a^5*b^11*c^4*d^6 - 452*a^5*b^11*c^6*d^4 + 204*a^5*b^11*c^8*d^2 + 36*a^6*b^10*c^3*d^7 + 556*a^6*b^10*c^5*d^5 - 708*a^6*b^10*c^7*d^3 - 20*a^7*b^9*c^2*d^8 - 340*a^7*b^9*c^4*d^6 + 1308*a^7*b^9*c^6*d^4 - 436*a^7*b^9*c^8*d^2 + 76*a^8*b^8*c^3*d^7 - 1380*a^8*b^8*c^5*d^5 + 1004*a^8*b^8*c^7*d^3 + 16*a^9*b^7*c^2*d^8 + 804*a^9*b^7*c^4*d^6 - 1404*a^9*b^7*c^6*d^4 + 224*a^9*b^7*c^8*d^2 - 204*a^10*b^6*c^3*d^7 + 1172*a^10*b^6*c^5*d^5 - 440*a^10*b^6*c^7*d^3 - 12*a^11*b^5*c^2*d^8 - 508*a^11*b^5*c^4*d^6 + 512*a^11*b^5*c^6*d^4 + 36*a^12*b^4*c^3*d^7 - 328*a^12*b^4*c^5*d^5 + 56*a^13*b^3*c^2*d^8 + 64*a^13*b^3*c^4*d^6 + 56*a^14*b^2*c^3*d^7))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d

^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a
^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) - (d^3*(d^2 - c^2)^(1/2)
((8(4*a^2*b^17*c^11 - 16*a^4*b^15*c^11 + 24*a^6*b^13*c^11 - 16*a^8*b^11*c
^11 + 4*a^10*b^9*c^11 + 4*a^19*c^2*d^9 - 12*a^3*b^16*c^10*d + 88*a^5*b^14*c
^10*d - 152*a^7*b^12*c^10*d + 108*a^9*b^10*c^10*d - 4*a^10*b^9*c*d^10 - 28*
a^11*b^8*c^10*d + 16*a^12*b^7*c*d^10 - 24*a^14*b^5*c*d^10 + 16*a^16*b^3*c*d
^10 - 28*a^18*b*c^3*d^8 + 28*a^2*b^17*c^9*d^2 - 80*a^3*b^16*c^8*d^3 + 112*a
^4*b^15*c^7*d^4 - 32*a^4*b^15*c^9*d^2 - 56*a^5*b^14*c^6*d^5 + 208*a^5*b^14*
c^8*d^3 - 56*a^6*b^13*c^5*d^6 - 392*a^6*b^13*c^7*d^4 - 152*a^6*b^13*c^9*d^2
 + 112*a^7*b^12*c^4*d^7 + 280*a^7*b^12*c^6*d^5 - 32*a^7*b^12*c^8*d^3 - 80*a
^8*b^11*c^3*d^8 + 112*a^8*b^11*c^5*d^6 + 448*a^8*b^11*c^7*d^4 + 368*a^8*b^1
1*c^9*d^2 + 28*a^9*b^10*c^2*d^9 - 368*a^9*b^10*c^4*d^7 - 560*a^9*b^10*c^6*d
^5 - 352*a^9*b^10*c^8*d^3 + 292*a^10*b^9*c^3*d^8 + 112*a^10*b^9*c^5*d^6 - 1
12*a^10*b^9*c^7*d^4 - 292*a^10*b^9*c^9*d^2 - 108*a^11*b^8*c^2*d^9 + 352*a^1
1*b^8*c^4*d^7 + 560*a^11*b^8*c^6*d^5 + 368*a^11*b^8*c^8*d^3 - 368*a^12*b^7*
c^3*d^8 - 448*a^12*b^7*c^5*d^6 - 112*a^12*b^7*c^7*d^4 + 80*a^12*b^7*c^9*d^2
 + 152*a^13*b^6*c^2*d^9 + 32*a^13*b^6*c^4*d^7 - 280*a^13*b^6*c^6*d^5 - 112*
a^13*b^6*c^8*d^3 + 152*a^14*b^5*c^3*d^8 + 392*a^14*b^5*c^5*d^6 + 56*a^14*b^
5*c^7*d^4 - 88*a^15*b^4*c^2*d^9 - 208*a^15*b^4*c^4*d^7 + 56*a^15*b^4*c^6*d^
5 + 32*a^16*b^3*c^3*d^8 - 112*a^16*b^3*c^5*d^6 + 12*a^17*b^2*c^2*d^9 + 80*a
^17*b^2*c^4*d^7 - 4*a*b^18*c^10*d - 4*a^18*b*c*d^10))/(a^14*d^6 + b^14*c^6
- 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d
^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d -
6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d -
36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^
2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^
5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d
^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10
*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*
d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) - (8*tan(e/2 + (f*x)/2)*(56*a^3*b^16
*c^11 - 12*a^19*c*d^10 - 12*a*b^18*c^11 - 104*a^5*b^14*c^11 + 96*a^7*b^12*c
^11 - 44*a^9*b^10*c^11 + 8*a^11*b^8*c^11 + 8*a^19*c^3*d^8 + 16*a*b^18*c^9*d
^2 + 96*a^2*b^17*c^10*d - 448*a^4*b^15*c^10*d + 832*a^6*b^13*c^10*d - 768*a
^8*b^11*c^10*d + 16*a^9*b^10*c*d^10 + 352*a^10*b^9*c^10*d - 76*a^11*b^8*c*d
^10 - 64*a^12*b^7*c^10*d + 144*a^13*b^6*c*d^10 - 136*a^15*b^4*c*d^10 + 64*a
^17*b^2*c*d^10 + 96*a^18*b*c^2*d^9 - 64*a^18*b*c^4*d^7 - 128*a^2*b^17*c^8*d
^3 + 448*a^3*b^16*c^7*d^4 - 412*a^3*b^16*c^9*d^2 - 896*a^4*b^15*c^6*d^5 + 1
280*a^4*b^15*c^8*d^3 + 1120*a^5*b^14*c^5*d^6 - 2968*a^5*b^14*c^7*d^4 + 1712
*a^5*b^14*c^9*d^2 - 896*a^6*b^13*c^4*d^7 + 4928*a^6*b^13*c^6*d^5 - 4288*a^6
*b^13*c^8*d^3 + 448*a^7*b^12*c^3*d^8 - 5656*a^7*b^12*c^5*d^6 + 7952*a^7*b^1
2*c^7*d^4 - 3048*a^7*b^12*c^9*d^2 - 128*a^8*b^11*c^2*d^9 + 4352*a^8*b^11*c^
4*d^7 - 11200*a^8*b^11*c^6*d^5 + 6912*a^8*b^11*c^8*d^3 - 2140*a^9*b^10*c^3*
d^8 + 11648*a^9*b^10*c^5*d^6 - 11088*a^9*b^10*c^7*d^4 + 2752*a^9*b^10*c^9*d
^2 + 608*a^10*b^9*c^2*d^9 - 8512*a^10*b^9*c^4*d^7 + 13440*a^10*b^9*c^6*d^5
- 5888*a^10*b^9*c^8*d^3 + 4088*a^11*b^8*c^3*d^8 - 12432*a^11*b^8*c^5*d^6 +

8512*a^11*b^8*c^7*d^4 - 1244*a^11*b^8*c^9*d^2 - 1152*a^12*b^7*c^2*d^9 + 844
8*a^12*b^7*c^4*d^7 - 8960*a^12*b^7*c^6*d^5 + 2560*a^12*b^7*c^8*d^3 - 3912*a
^13*b^6*c^3*d^8 + 7168*a^13*b^6*c^5*d^6 - 3416*a^13*b^6*c^7*d^4 + 224*a^13*
b^6*c^9*d^2 + 1088*a^14*b^5*c^2*d^9 - 4352*a^14*b^5*c^4*d^7 + 3136*a^14*b^5
*c^6*d^5 - 448*a^14*b^5*c^8*d^3 + 1888*a^15*b^4*c^3*d^8 - 2072*a^15*b^4*c^5
*d^6 + 560*a^15*b^4*c^7*d^4 - 512*a^16*b^3*c^2*d^9 + 1024*a^16*b^3*c^4*d^7
- 448*a^16*b^3*c^6*d^5 - 380*a^17*b^2*c^3*d^8 + 224*a^17*b^2*c^5*d^6))/(a^1
4*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^
6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*
a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24
*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 1
5*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^1
0*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 -
120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*
c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 +
15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5)))/(a^3*d^5 + b^3*c^
5 - a^3*c^2*d^3 - b^3*c^3*d^2 + 3*a*b^2*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2
*c^4*d - 3*a^2*b*c*d^4)))/(a^3*d^5 + b^3*c^5 - a^3*c^2*d^3 - b^3*c^3*d^2 +
3*a*b^2*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d - 3*a^2*b*c*d^4))*1i)/(a^
3*d^5 + b^3*c^5 - a^3*c^2*d^3 - b^3*c^3*d^2 + 3*a*b^2*c^2*d^3 + 3*a^2*b*c^3
*d^2 - 3*a*b^2*c^4*d - 3*a^2*b*c*d^4))/((16*(4*a*b^9*c^3*d^5 + a*b^9*c^5*d^
3 - 18*a^3*b^7*c*d^7 + 36*a^5*b^5*c*d^7 - 34*a^7*b^3*c*d^7 + 2*a^2*b^8*c^2*
d^6 + a^2*b^8*c^4*d^4 - a^3*b^7*c^3*d^5 + 4*a^3*b^7*c^5*d^3 - 25*a^4*b^6*c^
2*d^6 - 8*a^4*b^6*c^4*d^4 - 16*a^5*b^5*c^3*d^5 + 4*a^5*b^5*c^5*d^3 + 50*a^6
*b^4*c^2*d^6 - 20*a^6*b^4*c^4*d^4 + 40*a^7*b^3*c^3*d^5 - 36*a^8*b^2*c^2*d^6
+ 4*a*b^9*c*d^7 + 12*a^9*b*c*d^7))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 +
6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6
+ 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 -
36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 -
6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^
3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60
*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^
2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*
a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5
*d - 6*a^13*b*c*d^5) + (16*tan(e/2 + (f*x)/2)*(4*a*b^9*c^2*d^6 + 2*a*b^9*c^
4*d^4 + 4*a^2*b^8*c*d^7 - 26*a^4*b^6*c*d^7 + 52*a^6*b^4*c*d^7 - 48*a^8*b^2*
c*d^7 + 2*a^2*b^8*c^3*d^5 - 2*a^3*b^7*c^2*d^6 + 8*a^3*b^7*c^4*d^4 - 16*a^4*
b^6*c^3*d^5 - 20*a^5*b^5*c^2*d^6 + 8*a^5*b^5*c^4*d^4 - 40*a^6*b^4*c^3*d^5 +
72*a^7*b^3*c^2*d^6))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^
6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*
d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5
*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5
*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4
*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d
^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8

*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d
^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*
c*d^5) - (d^3*(d^2 - c^2)^(1/2)*((8*(4*a*b^12*c^4*d^5 + 4*a*b^12*c^6*d^3 +
4*a^3*b^10*c^8*d + 4*a^4*b^9*c*d^8 + 4*a^5*b^8*c^8*d - 16*a^6*b^7*c*d^8 + 2
4*a^8*b^5*c*d^8 - 16*a^10*b^3*c*d^8 - 4*a^2*b^11*c^3*d^6 - 8*a^2*b^11*c^5*d
^4 - 2*a^2*b^11*c^7*d^2 - 4*a^3*b^10*c^2*d^7 - 16*a^3*b^10*c^4*d^5 - a^3*b^
10*c^6*d^3 + 24*a^4*b^9*c^3*d^6 - 20*a^4*b^9*c^5*d^4 - 20*a^4*b^9*c^7*d^2 +
12*a^5*b^8*c^2*d^7 + 95*a^5*b^8*c^4*d^5 + 20*a^5*b^8*c^6*d^3 - 98*a^6*b^7*
c^3*d^6 + 64*a^6*b^7*c^5*d^4 - 32*a^6*b^7*c^7*d^2 + a^7*b^6*c^2*d^7 - 188*a
^7*b^6*c^4*d^5 + 112*a^7*b^6*c^6*d^3 + 164*a^8*b^5*c^3*d^6 - 216*a^8*b^5*c^
5*d^4 - 28*a^9*b^4*c^2*d^7 + 240*a^9*b^4*c^4*d^5 - 140*a^10*b^3*c^3*d^6 + 2
8*a^11*b^2*c^2*d^7 + a*b^12*c^8*d + 4*a^12*b*c*d^8))/(a^14*d^6 + b^14*c^6 -
4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^
6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6
*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 3
6*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2
- 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5
*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^
3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*
b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d
^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) - (8*tan(e/2 + (f*x)/2)*(a*b^12*c^9 +
4*a^13*c*d^8 + 4*a^3*b^10*c^9 + 4*a^5*b^8*c^9 - 16*a*b^12*c^3*d^6 - 4*a*b^
12*c^5*d^4 + 2*a*b^12*c^7*d^2 - 2*a^2*b^11*c^8*d - 16*a^3*b^10*c*d^8 - 20*a
^4*b^9*c^8*d + 76*a^5*b^8*c*d^8 - 32*a^6*b^7*c^8*d - 162*a^7*b^6*c*d^8 + 17
6*a^9*b^4*c*d^8 - 96*a^11*b^2*c*d^8 - 8*a^12*b*c^2*d^7 + 32*a^2*b^11*c^2*d^
7 + 8*a^2*b^11*c^4*d^5 - 4*a^2*b^11*c^6*d^3 + 72*a^3*b^10*c^3*d^6 - 14*a^3*
b^10*c^5*d^4 - 9*a^3*b^10*c^7*d^2 - 152*a^4*b^9*c^2*d^7 + 80*a^4*b^9*c^4*d^
5 + 20*a^4*b^9*c^6*d^3 - 274*a^5*b^8*c^3*d^6 + 55*a^5*b^8*c^5*d^4 + 12*a^5*
b^8*c^7*d^2 + 372*a^6*b^7*c^2*d^7 - 250*a^6*b^7*c^4*d^5 + 128*a^6*b^7*c^6*d
^3 + 481*a^7*b^6*c^3*d^6 - 412*a^7*b^6*c^5*d^4 + 112*a^7*b^6*c^7*d^2 - 472*
a^8*b^5*c^2*d^7 + 612*a^8*b^5*c^4*d^5 - 216*a^8*b^5*c^6*d^3 - 564*a^9*b^4*c
^3*d^6 + 240*a^9*b^4*c^5*d^4 + 336*a^10*b^3*c^2*d^7 - 144*a^10*b^3*c^4*d^5
+ 40*a^11*b^2*c^3*d^6))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*
c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^
4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c
^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c
^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a
^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2
*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a
^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4
*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*
b*c*d^5) + (d^3*(d^2 - c^2)^(1/2)*((8*(2*a^2*b^14*c^10 - 6*a^6*b^10*c^10 +
4*a^8*b^8*c^10 + 4*a^16*c^2*d^8 + 4*a*b^15*c^7*d^3 - 10*a^3*b^13*c^9*d - 12
*a^5*b^11*c^9*d + 4*a^7*b^9*c*d^9 + 54*a^7*b^9*c^9*d - 18*a^9*b^7*c*d^9 - 3
2*a^9*b^7*c^9*d + 36*a^11*b^5*c*d^9 - 34*a^13*b^3*c*d^9 - 32*a^15*b*c^3*d^7

- 24*a^2*b^14*c^6*d^4 + 2*a^2*b^14*c^8*d^2 + 60*a^3*b^13*c^5*d^5 - 30*a^3*
b^13*c^7*d^3 - 80*a^4*b^12*c^4*d^6 + 138*a^4*b^12*c^6*d^4 + 2*a^4*b^12*c^8*
d^2 + 60*a^5*b^11*c^3*d^7 - 310*a^5*b^11*c^5*d^5 + 122*a^5*b^11*c^7*d^3 - 2
4*a^6*b^10*c^2*d^8 + 390*a^6*b^10*c^4*d^6 - 466*a^6*b^10*c^6*d^4 + 102*a^6*
b^10*c^8*d^2 - 282*a^7*b^9*c^3*d^7 + 878*a^7*b^9*c^5*d^5 - 394*a^7*b^9*c^7*
d^3 + 110*a^8*b^8*c^2*d^8 - 970*a^8*b^8*c^4*d^6 + 894*a^8*b^8*c^6*d^4 - 218
*a^8*b^8*c^8*d^2 + 638*a^9*b^7*c^3*d^7 - 1290*a^9*b^7*c^5*d^5 + 522*a^9*b^7
*c^7*d^3 - 232*a^10*b^6*c^2*d^8 + 1202*a^10*b^6*c^4*d^6 - 822*a^10*b^6*c^6*
d^4 + 112*a^10*b^6*c^8*d^2 - 702*a^11*b^5*c^3*d^7 + 886*a^11*b^5*c^5*d^5 -
224*a^11*b^5*c^7*d^3 + 234*a^12*b^4*c^2*d^8 - 654*a^12*b^4*c^4*d^6 + 280*a^
12*b^4*c^6*d^4 + 318*a^13*b^3*c^3*d^7 - 224*a^13*b^3*c^5*d^5 - 92*a^14*b^2*
c^2*d^8 + 112*a^14*b^2*c^4*d^6 + 12*a^15*b*c*d^9))/(a^14*d^6 + b^14*c^6 - 4
*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6
- 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a
^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*
a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 -
20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b
^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3
+ 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^
4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4
- 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) + (8*tan(e/2 + (f*x)/2)*(4*a*b^15*c^10
+ 8*a^16*c*d^9 - 12*a^5*b^11*c^10 + 8*a^7*b^9*c^10 + 4*a*b^15*c^8*d^2 - 20*
a^2*b^14*c^9*d - 24*a^4*b^12*c^9*d + 108*a^6*b^10*c^9*d + 4*a^8*b^8*c*d^9 -
64*a^8*b^8*c^9*d - 8*a^10*b^6*c*d^9 + 12*a^12*b^4*c*d^9 - 16*a^14*b^2*c*d^
9 - 40*a^15*b*c^2*d^8 - 20*a^2*b^14*c^7*d^3 + 36*a^3*b^13*c^6*d^4 + 4*a^3*b
^13*c^8*d^2 - 20*a^4*b^12*c^5*d^5 + 164*a^4*b^12*c^7*d^3 - 20*a^5*b^11*c^4*
d^6 - 452*a^5*b^11*c^6*d^4 + 204*a^5*b^11*c^8*d^2 + 36*a^6*b^10*c^3*d^7 + 5
56*a^6*b^10*c^5*d^5 - 708*a^6*b^10*c^7*d^3 - 20*a^7*b^9*c^2*d^8 - 340*a^7*b
^9*c^4*d^6 + 1308*a^7*b^9*c^6*d^4 - 436*a^7*b^9*c^8*d^2 + 76*a^8*b^8*c^3*d^
7 - 1380*a^8*b^8*c^5*d^5 + 1004*a^8*b^8*c^7*d^3 + 16*a^9*b^7*c^2*d^8 + 804*
a^9*b^7*c^4*d^6 - 1404*a^9*b^7*c^6*d^4 + 224*a^9*b^7*c^8*d^2 - 204*a^10*b^6
*c^3*d^7 + 1172*a^10*b^6*c^5*d^5 - 440*a^10*b^6*c^7*d^3 - 12*a^11*b^5*c^2*d
^8 - 508*a^11*b^5*c^4*d^6 + 512*a^11*b^5*c^6*d^4 + 36*a^12*b^4*c^3*d^7 - 32
8*a^12*b^4*c^5*d^5 + 56*a^13*b^3*c^2*d^8 + 64*a^13*b^3*c^4*d^6 + 56*a^14*b^
2*c^3*d^7))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*
b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^
12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^
7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^
11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*
d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^
6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d
^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^
11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) + (
d^3*(d^2 - c^2)^(1/2)*((8*(4*a^2*b^17*c^11 - 16*a^4*b^15*c^11 + 24*a^6*b^13
*c^11 - 16*a^8*b^11*c^11 + 4*a^10*b^9*c^11 + 4*a^19*c^2*d^9 - 12*a^3*b^16*c

^10*d + 88*a^5*b^14*c^10*d - 152*a^7*b^12*c^10*d + 108*a^9*b^10*c^10*d - 4*a^10*b^9*c*d^10 - 28*a^11*b^8*c^10*d + 16*a^12*b^7*c*d^10 - 24*a^14*b^5*c*d^10 + 16*a^16*b^3*c*d^10 - 28*a^18*b*c^3*d^8 + 28*a^2*b^17*c^9*d^2 - 80*a^3*b^16*c^8*d^3 + 112*a^4*b^15*c^7*d^4 - 32*a^4*b^15*c^9*d^2 - 56*a^5*b^14*c^6*d^5 + 208*a^5*b^14*c^8*d^3 - 56*a^6*b^13*c^5*d^6 - 392*a^6*b^13*c^7*d^4 - 152*a^6*b^13*c^9*d^2 + 112*a^7*b^12*c^4*d^7 + 280*a^7*b^12*c^6*d^5 - 32*a^7*b^12*c^8*d^3 - 80*a^8*b^11*c^3*d^8 + 112*a^8*b^11*c^5*d^6 + 448*a^8*b^11*c^7*d^4 + 368*a^8*b^11*c^9*d^2 + 28*a^9*b^10*c^2*d^9 - 368*a^9*b^10*c^4*d^7 - 560*a^9*b^10*c^6*d^5 - 352*a^9*b^10*c^8*d^3 + 292*a^10*b^9*c^3*d^8 + 112*a^10*b^9*c^5*d^6 - 112*a^10*b^9*c^7*d^4 - 292*a^10*b^9*c^9*d^2 - 108*a^11*b^8*c^2*d^9 + 352*a^11*b^8*c^4*d^7 + 560*a^11*b^8*c^6*d^5 + 368*a^11*b^8*c^8*d^3 - 368*a^12*b^7*c^3*d^8 - 448*a^12*b^7*c^5*d^6 - 112*a^12*b^7*c^7*d^4 + 80*a^12*b^7*c^9*d^2 + 152*a^13*b^6*c^2*d^9 + 32*a^13*b^6*c^4*d^7 - 280*a^13*b^6*c^6*d^5 - 112*a^13*b^6*c^8*d^3 + 152*a^14*b^5*c^3*d^8 + 392*a^14*b^5*c^5*d^6 + 56*a^14*b^5*c^7*d^4 - 88*a^15*b^4*c^2*d^9 - 208*a^15*b^4*c^4*d^7 + 56*a^15*b^4*c^6*d^5 + 32*a^16*b^3*c^3*d^8 - 112*a^16*b^3*c^5*d^6 + 12*a^17*b^2*c^2*d^9 + 80*a^17*b^2*c^4*d^7 - 4*a*b^18*c^10*d - 4*a^18*b*c*d^10))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) - (8*tan(e/2 + (f*x)/2)*(56*a^3*b^16*c^11 - 12*a^19*c*d^10 - 12*a*b^18*c^11 - 104*a^5*b^14*c^11 + 96*a^7*b^12*c^11 - 44*a^9*b^10*c^11 + 8*a^11*b^8*c^11 + 8*a^19*c^3*d^8 + 16*a*b^18*c^9*d^2 + 96*a^2*b^17*c^10*d - 448*a^4*b^15*c^10*d + 832*a^6*b^13*c^10*d - 768*a^8*b^11*c^10*d + 16*a^9*b^10*c*d^10 + 352*a^10*b^9*c^10*d - 76*a^11*b^8*c*d^10 - 64*a^12*b^7*c^10*d + 144*a^13*b^6*c*d^10 - 136*a^15*b^4*c*d^10 + 64*a^17*b^2*c*d^10 + 96*a^18*b*c^2*d^9 - 64*a^18*b*c^4*d^7 - 128*a^2*b^17*c^8*d^3 + 448*a^3*b^16*c^7*d^4 - 412*a^3*b^16*c^9*d^2 - 896*a^4*b^15*c^6*d^5 + 1280*a^4*b^15*c^8*d^3 + 1120*a^5*b^14*c^5*d^6 - 2968*a^5*b^14*c^7*d^4 + 1712*a^5*b^14*c^9*d^2 - 896*a^6*b^13*c^4*d^7 + 4928*a^6*b^13*c^6*d^5 - 4288*a^6*b^13*c^8*d^3 + 448*a^7*b^12*c^3*d^8 - 5656*a^7*b^12*c^5*d^6 + 7952*a^7*b^12*c^7*d^4 - 3048*a^7*b^12*c^9*d^2 - 128*a^8*b^11*c^2*d^9 + 4352*a^8*b^11*c^4*d^7 - 11200*a^8*b^11*c^6*d^5 + 6912*a^8*b^11*c^8*d^3 - 2140*a^9*b^10*c^3*d^8 + 11648*a^9*b^10*c^5*d^6 - 11088*a^9*b^10*c^7*d^4 + 2752*a^9*b^10*c^9*d^2 + 608*a^10*b^9*c^2*d^9 - 8512*a^10*b^9*c^4*d^7 + 13440*a^10*b^9*c^6*d^5 - 5888*a^10*b^9*c^8*d^3 + 4088*a^11*b^8*c^3*d^8 - 12432*a^11*b^8*c^5*d^6 + 8512*a^11*b^8*c^7*d^4 - 1244*a^11*b^8*c^9*d^2 - 1152*a^12*b^7*c^2*d^9 + 8448*a^12*b^7*c^4*d^7 - 8960*a^12*b^7*c^6*d^5 + 2560*a^12*b^7*c^8*d^3 - 3912*a^13*b^6*c^3*d^8 + 7168*a^13*b^6*c^5*d^6 - 3416*a^13*b^6*c^7*d^4 + 224*a^13*b^6*c^9*d^2 + 1088*a^14*b^5*c^2*d^9 - 4352*a^14*b^5*c^

4*d^7 + 3136*a^14*b^5*c^6*d^5 - 448*a^14*b^5*c^8*d^3 + 1888*a^15*b^4*c^3*d^
8 - 2072*a^15*b^4*c^5*d^6 + 560*a^15*b^4*c^7*d^4 - 512*a^16*b^3*c^2*d^9 + 1
024*a^16*b^3*c^4*d^7 - 448*a^16*b^3*c^6*d^5 - 380*a^17*b^2*c^3*d^8 + 224*a^
17*b^2*c^5*d^6))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4
*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 -
 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d +
24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d +
24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10
*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 +
90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*
c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 -
20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5
)))/(a^3*d^5 + b^3*c^5 - a^3*c^2*d^3 - b^3*c^3*d^2 + 3*a*b^2*c^2*d^3 + 3*a^
2*b*c^3*d^2 - 3*a*b^2*c^4*d - 3*a^2*b*c*d^4)))/(a^3*d^5 + b^3*c^5 - a^3*c^2
*d^3 - b^3*c^3*d^2 + 3*a*b^2*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d - 3*
a^2*b*c*d^4)))/(a^3*d^5 + b^3*c^5 - a^3*c^2*d^3 - b^3*c^3*d^2 + 3*a*b^2*c^2
*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d - 3*a^2*b*c*d^4) - (d^3*(d^2 - c^2)^
(1/2)*((8*tan(e/2 + (f*x)/2)*(a*b^12*c^9 + 4*a^13*c*d^8 + 4*a^3*b^10*c^9 +
4*a^5*b^8*c^9 - 16*a*b^12*c^3*d^6 - 4*a*b^12*c^5*d^4 + 2*a*b^12*c^7*d^2 - 2
*a^2*b^11*c^8*d - 16*a^3*b^10*c*d^8 - 20*a^4*b^9*c^8*d + 76*a^5*b^8*c*d^8 -
 32*a^6*b^7*c^8*d - 162*a^7*b^6*c*d^8 + 176*a^9*b^4*c*d^8 - 96*a^11*b^2*c*d
^8 - 8*a^12*b*c^2*d^7 + 32*a^2*b^11*c^2*d^7 + 8*a^2*b^11*c^4*d^5 - 4*a^2*b^
11*c^6*d^3 + 72*a^3*b^10*c^3*d^6 - 14*a^3*b^10*c^5*d^4 - 9*a^3*b^10*c^7*d^2
 - 152*a^4*b^9*c^2*d^7 + 80*a^4*b^9*c^4*d^5 + 20*a^4*b^9*c^6*d^3 - 274*a^5*
b^8*c^3*d^6 + 55*a^5*b^8*c^5*d^4 + 12*a^5*b^8*c^7*d^2 + 372*a^6*b^7*c^2*d^7
 - 250*a^6*b^7*c^4*d^5 + 128*a^6*b^7*c^6*d^3 + 481*a^7*b^6*c^3*d^6 - 412*a^
7*b^6*c^5*d^4 + 112*a^7*b^6*c^7*d^2 - 472*a^8*b^5*c^2*d^7 + 612*a^8*b^5*c^4
*d^5 - 216*a^8*b^5*c^6*d^3 - 564*a^9*b^4*c^3*d^6 + 240*a^9*b^4*c^5*d^4 + 33
6*a^10*b^3*c^2*d^7 - 144*a^10*b^3*c^4*d^5 + 40*a^11*b^2*c^3*d^6))/(a^14*d^6
 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6
 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b
^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*
b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2
*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4
*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a
^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d
^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a
^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) - (8*(4*a*b^12*c^4*d^5 +
 4*a*b^12*c^6*d^3 + 4*a^3*b^10*c^8*d + 4*a^4*b^9*c*d^8 + 4*a^5*b^8*c^8*d -
16*a^6*b^7*c*d^8 + 24*a^8*b^5*c*d^8 - 16*a^10*b^3*c*d^8 - 4*a^2*b^11*c^3*d^
6 - 8*a^2*b^11*c^5*d^4 - 2*a^2*b^11*c^7*d^2 - 4*a^3*b^10*c^2*d^7 - 16*a^3*b
^10*c^4*d^5 - a^3*b^10*c^6*d^3 + 24*a^4*b^9*c^3*d^6 - 20*a^4*b^9*c^5*d^4 -
20*a^4*b^9*c^7*d^2 + 12*a^5*b^8*c^2*d^7 + 95*a^5*b^8*c^4*d^5 + 20*a^5*b^8*c
^6*d^3 - 98*a^6*b^7*c^3*d^6 + 64*a^6*b^7*c^5*d^4 - 32*a^6*b^7*c^7*d^2 + a^7
*b^6*c^2*d^7 - 188*a^7*b^6*c^4*d^5 + 112*a^7*b^6*c^6*d^3 + 164*a^8*b^5*c^3*

d^6 - 216*a^8*b^5*c^5*d^4 - 28*a^9*b^4*c^2*d^7 + 240*a^9*b^4*c^4*d^5 - 140*
a^10*b^3*c^3*d^6 + 28*a^11*b^2*c^2*d^7 + a*b^12*c^8*d + 4*a^12*b*c*d^8))/(a
^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*
b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 2
4*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 +
24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 +
15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b
^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2
- 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^
5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3
+ 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) + (d^3*(d^2 - c^2
)^(1/2)*((8*(2*a^2*b^14*c^10 - 6*a^6*b^10*c^10 + 4*a^8*b^8*c^10 + 4*a^16*c^
2*d^8 + 4*a*b^15*c^7*d^3 - 10*a^3*b^13*c^9*d - 12*a^5*b^11*c^9*d + 4*a^7*b^
9*c*d^9 + 54*a^7*b^9*c^9*d - 18*a^9*b^7*c*d^9 - 32*a^9*b^7*c^9*d + 36*a^11*
b^5*c*d^9 - 34*a^13*b^3*c*d^9 - 32*a^15*b*c^3*d^7 - 24*a^2*b^14*c^6*d^4 + 2
*a^2*b^14*c^8*d^2 + 60*a^3*b^13*c^5*d^5 - 30*a^3*b^13*c^7*d^3 - 80*a^4*b^12
*c^4*d^6 + 138*a^4*b^12*c^6*d^4 + 2*a^4*b^12*c^8*d^2 + 60*a^5*b^11*c^3*d^7
- 310*a^5*b^11*c^5*d^5 + 122*a^5*b^11*c^7*d^3 - 24*a^6*b^10*c^2*d^8 + 390*a
^6*b^10*c^4*d^6 - 466*a^6*b^10*c^6*d^4 + 102*a^6*b^10*c^8*d^2 - 282*a^7*b^9
*c^3*d^7 + 878*a^7*b^9*c^5*d^5 - 394*a^7*b^9*c^7*d^3 + 110*a^8*b^8*c^2*d^8
- 970*a^8*b^8*c^4*d^6 + 894*a^8*b^8*c^6*d^4 - 218*a^8*b^8*c^8*d^2 + 638*a^9
*b^7*c^3*d^7 - 1290*a^9*b^7*c^5*d^5 + 522*a^9*b^7*c^7*d^3 - 232*a^10*b^6*c^
2*d^8 + 1202*a^10*b^6*c^4*d^6 - 822*a^10*b^6*c^6*d^4 + 112*a^10*b^6*c^8*d^2
- 702*a^11*b^5*c^3*d^7 + 886*a^11*b^5*c^5*d^5 - 224*a^11*b^5*c^7*d^3 + 234
*a^12*b^4*c^2*d^8 - 654*a^12*b^4*c^4*d^6 + 280*a^12*b^4*c^6*d^4 + 318*a^13*
b^3*c^3*d^7 - 224*a^13*b^3*c^5*d^5 - 92*a^14*b^2*c^2*d^8 + 112*a^14*b^2*c^4
*d^6 + 12*a^15*b*c*d^9))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10
*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b
^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*
c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*
c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*
a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^
2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*
a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^
4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13
*b*c*d^5) + (8*tan(e/2 + (f*x)/2)*(4*a*b^15*c^10 + 8*a^16*c*d^9 - 12*a^5*b^
11*c^10 + 8*a^7*b^9*c^10 + 4*a*b^15*c^8*d^2 - 20*a^2*b^14*c^9*d - 24*a^4*b^
12*c^9*d + 108*a^6*b^10*c^9*d + 4*a^8*b^8*c*d^9 - 64*a^8*b^8*c^9*d - 8*a^10
*b^6*c*d^9 + 12*a^12*b^4*c*d^9 - 16*a^14*b^2*c*d^9 - 40*a^15*b*c^2*d^8 - 20
*a^2*b^14*c^7*d^3 + 36*a^3*b^13*c^6*d^4 + 4*a^3*b^13*c^8*d^2 - 20*a^4*b^12*
c^5*d^5 + 164*a^4*b^12*c^7*d^3 - 20*a^5*b^11*c^4*d^6 - 452*a^5*b^11*c^6*d^4
+ 204*a^5*b^11*c^8*d^2 + 36*a^6*b^10*c^3*d^7 + 556*a^6*b^10*c^5*d^5 - 708*
a^6*b^10*c^7*d^3 - 20*a^7*b^9*c^2*d^8 - 340*a^7*b^9*c^4*d^6 + 1308*a^7*b^9*
c^6*d^4 - 436*a^7*b^9*c^8*d^2 + 76*a^8*b^8*c^3*d^7 - 1380*a^8*b^8*c^5*d^5 +
1004*a^8*b^8*c^7*d^3 + 16*a^9*b^7*c^2*d^8 + 804*a^9*b^7*c^4*d^6 - 1404*a^9

*b^7*c^6*d^4 + 224*a^9*b^7*c^8*d^2 - 204*a^10*b^6*c^3*d^7 + 1172*a^10*b^6*c^5*d^5 - 440*a^10*b^6*c^7*d^3 - 12*a^11*b^5*c^2*d^8 - 508*a^11*b^5*c^4*d^6 + 512*a^11*b^5*c^6*d^4 + 36*a^12*b^4*c^3*d^7 - 328*a^12*b^4*c^5*d^5 + 56*a^13*b^3*c^2*d^8 + 64*a^13*b^3*c^4*d^6 + 56*a^14*b^2*c^3*d^7))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) - (d^3*(d^2 - c^2)^(1/2)*((8*(4*a^2*b^17*c^11 - 16*a^4*b^15*c^11 + 24*a^6*b^13*c^11 - 16*a^8*b^11*c^11 + 4*a^10*b^9*c^11 + 4*a^19*c^2*d^9 - 12*a^3*b^16*c^10*d + 88*a^5*b^14*c^10*d - 152*a^7*b^12*c^10*d + 108*a^9*b^10*c^10*d - 4*a^10*b^9*c*d^10 - 28*a^11*b^8*c^10*d + 16*a^12*b^7*c*d^10 - 24*a^14*b^5*c*d^10 + 16*a^16*b^3*c*d^10 - 28*a^18*b*c^3*d^8 + 28*a^2*b^17*c^9*d^2 - 80*a^3*b^16*c^8*d^3 + 112*a^4*b^15*c^7*d^4 - 32*a^4*b^15*c^9*d^2 - 56*a^5*b^14*c^6*d^5 + 208*a^5*b^14*c^8*d^3 - 56*a^6*b^13*c^5*d^6 - 392*a^6*b^13*c^7*d^4 - 152*a^6*b^13*c^9*d^2 + 112*a^7*b^12*c^4*d^7 + 280*a^7*b^12*c^6*d^5 - 32*a^7*b^12*c^8*d^3 - 80*a^8*b^11*c^3*d^8 + 112*a^8*b^11*c^5*d^6 + 448*a^8*b^11*c^7*d^4 + 368*a^8*b^11*c^9*d^2 + 28*a^9*b^10*c^2*d^9 - 368*a^9*b^10*c^4*d^7 - 560*a^9*b^10*c^6*d^5 - 352*a^9*b^10*c^8*d^3 + 292*a^10*b^9*c^3*d^8 + 112*a^10*b^9*c^5*d^6 - 112*a^10*b^9*c^7*d^4 - 292*a^10*b^9*c^9*d^2 - 108*a^11*b^8*c^2*d^9 + 352*a^11*b^8*c^4*d^7 + 560*a^11*b^8*c^6*d^5 + 368*a^11*b^8*c^8*d^3 - 368*a^12*b^7*c^3*d^8 - 448*a^12*b^7*c^5*d^6 - 112*a^12*b^7*c^7*d^4 + 80*a^12*b^7*c^9*d^2 + 152*a^13*b^6*c^2*d^9 + 32*a^13*b^6*c^4*d^7 - 280*a^13*b^6*c^6*d^5 - 112*a^13*b^6*c^8*d^3 + 152*a^14*b^5*c^3*d^8 + 392*a^14*b^5*c^5*d^6 + 56*a^14*b^5*c^7*d^4 - 88*a^15*b^4*c^2*d^9 - 208*a^15*b^4*c^4*d^7 + 56*a^15*b^4*c^6*d^5 + 32*a^16*b^3*c^3*d^8 - 112*a^16*b^3*c^5*d^6 + 12*a^17*b^2*c^2*d^9 + 80*a^17*b^2*c^4*d^7 - 4*a*b^18*c^10*d - 4*a^18*b*c*d^10))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) - (8*tan(e/2 + (f*x)/2)*(56*a^3*b^16*c^11 - 12*a^19*c*d^10 - 12*a*b^18*c^11 - 104*a^5*b^14*c^11 + 96*a^7*b^12*c^11 - 44*a^9*b^10*c^11 + 8*a^11*b^8*c^11 + 8*a^19*c^3*d^8 + 16*a*b^18*c^9*d^2 + 96*a^2*b^17*c^10*d - 448*a^4*b^15*c^10*d + 832*a^6*b^13*c^10*d - 768*a^8*b^11*c^10*d + 16*a^9*b^10*c*d^10 + 352*a^10*b^9*c^10*d - 76*a^11*b^8*c*d^10 -

64*a^12*b^7*c^10*d + 144*a^13*b^6*c*d^10 - 136*a^15*b^4*c*d^10 + 64*a^17*b^2*c*d^10 + 96*a^18*b*c^2*d^9 - 64*a^18*b*c^4*d^7 - 128*a^2*b^17*c^8*d^3 + 448*a^3*b^16*c^7*d^4 - 412*a^3*b^16*c^9*d^2 - 896*a^4*b^15*c^6*d^5 + 1280*a^4*b^15*c^8*d^3 + 1120*a^5*b^14*c^5*d^6 - 2968*a^5*b^14*c^7*d^4 + 1712*a^5*b^14*c^9*d^2 - 896*a^6*b^13*c^4*d^7 + 4928*a^6*b^13*c^6*d^5 - 4288*a^6*b^13*c^8*d^3 + 448*a^7*b^12*c^3*d^8 - 5656*a^7*b^12*c^5*d^6 + 7952*a^7*b^12*c^7*d^4 - 3048*a^7*b^12*c^9*d^2 - 128*a^8*b^11*c^2*d^9 + 4352*a^8*b^11*c^4*d^7 - 11200*a^8*b^11*c^6*d^5 + 6912*a^8*b^11*c^8*d^3 - 2140*a^9*b^10*c^3*d^8 + 11648*a^9*b^10*c^5*d^6 - 11088*a^9*b^10*c^7*d^4 + 2752*a^9*b^10*c^9*d^2 + 608*a^10*b^9*c^2*d^9 - 8512*a^10*b^9*c^4*d^7 + 13440*a^10*b^9*c^6*d^5 - 5888*a^10*b^9*c^8*d^3 + 4088*a^11*b^8*c^3*d^8 - 12432*a^11*b^8*c^5*d^6 + 8512*a^11*b^8*c^7*d^4 - 1244*a^11*b^8*c^9*d^2 - 1152*a^12*b^7*c^2*d^9 + 8448*a^12*b^7*c^4*d^7 - 8960*a^12*b^7*c^6*d^5 + 2560*a^12*b^7*c^8*d^3 - 3912*a^13*b^6*c^3*d^8 + 7168*a^13*b^6*c^5*d^6 - 3416*a^13*b^6*c^7*d^4 + 224*a^13*b^6*c^9*d^2 + 1088*a^14*b^5*c^2*d^9 - 4352*a^14*b^5*c^4*d^7 + 3136*a^14*b^5*c^6*d^5 - 448*a^14*b^5*c^8*d^3 + 1888*a^15*b^4*c^3*d^8 - 2072*a^15*b^4*c^5*d^6 + 560*a^15*b^4*c^7*d^4 - 512*a^16*b^3*c^2*d^9 + 1024*a^16*b^3*c^4*d^7 - 448*a^16*b^3*c^6*d^5 - 380*a^17*b^2*c^3*d^8 + 224*a^17*b^2*c^5*d^6))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5)))/(a^3*d^5 + b^3*c^5 - a^3*c^2*d^3 - b^3*c^3*d^2 + 3*a*b^2*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d - 3*a^2*b*c*d^4)))/(a^3*d^5 + b^3*c^5 - a^3*c^2*d^3 - b^3*c^3*d^2 + 3*a*b^2*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d - 3*a^2*b*c*d^4)))/(a^3*d^5 + b^3*c^5 - a^3*c^2*d^3 - b^3*c^3*d^2 + 3*a*b^2*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d - 3*a^2*b*c*d^4)))*(d^2 - c^2)^(1/2)*2i)/(f*(a^3*d^5 + b^3*c^5 - a^3*c^2*d^3 - b^3*c^3*d^2 + 3*a*b^2*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d - 3*a^2*b*c*d^4)) - ((b^5*c - 4*a^2*b^3*c + 6*a^3*b^2*d - 3*a*b^4*d)/((a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^4 + b^4 - 2*a^2*b^2)) + (b*tan(e/2 + (f*x)/2)*(2*b^5*c - 11*a^2*b^3*c + 17*a^3*b^2*d - 8*a*b^4*d))/(a*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^4 + b^4 - 2*a^2*b^2)) + (tan(e/2 + (f*x)/2)^2*(a^2 + 2*b^2)*(b^5*c - 4*a^2*b^3*c + 6*a^3*b^2*d - 3*a*b^4*d))/(a^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^4 + b^4 - 2*a^2*b^2)) + (b*tan(e/2 + (f*x)/2)^3*(2*b^5*c - 5*a^2*b^3*c + 7*a^3*b^2*d - 4*a*b^4*d))/(a*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^4 + b^4 - 2*a^2*b^2)))/(f*(tan(e/2 + (f*x)/2)^2*(2*a^2 + 4*b^2) + a^2*tan(e/2 + (f*x)/2)^4 + a^2 + 4*a*b*tan(e/2 + (f*x)/2)^3 + 4*a*b*tan(e/2 + (f*x)/2))) - (b*atan(((b*(-(a + b)^5*(a - b)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(a*b^12*c^9 + 4*a^13*c*d^8 + 4*a^3*b^10*c^9 + 4*a^5*b^8*c^9 - 16*a*b^12*c^3*d^6 - 4*a*b^12*c^5*d^4 + 2*a*b^12*c^7*d^2 - 2*a^2*b^11*c^8*d - 16*a^3

*b^10*c*d^8 - 20*a^4*b^9*c^8*d + 76*a^5*b^8*c*d^8 - 32*a^6*b^7*c^8*d - 162*
a^7*b^6*c*d^8 + 176*a^9*b^4*c*d^8 - 96*a^11*b^2*c*d^8 - 8*a^12*b*c^2*d^7 +
32*a^2*b^11*c^2*d^7 + 8*a^2*b^11*c^4*d^5 - 4*a^2*b^11*c^6*d^3 + 72*a^3*b^10
*c^3*d^6 - 14*a^3*b^10*c^5*d^4 - 9*a^3*b^10*c^7*d^2 - 152*a^4*b^9*c^2*d^7 +
80*a^4*b^9*c^4*d^5 + 20*a^4*b^9*c^6*d^3 - 274*a^5*b^8*c^3*d^6 + 55*a^5*b^8
*c^5*d^4 + 12*a^5*b^8*c^7*d^2 + 372*a^6*b^7*c^2*d^7 - 250*a^6*b^7*c^4*d^5 +
128*a^6*b^7*c^6*d^3 + 481*a^7*b^6*c^3*d^6 - 412*a^7*b^6*c^5*d^4 + 112*a^7*
b^6*c^7*d^2 - 472*a^8*b^5*c^2*d^7 + 612*a^8*b^5*c^4*d^5 - 216*a^8*b^5*c^6*d
^3 - 564*a^9*b^4*c^3*d^6 + 240*a^9*b^4*c^5*d^4 + 336*a^10*b^3*c^2*d^7 - 144
*a^10*b^3*c^4*d^5 + 40*a^11*b^2*c^3*d^6))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12
*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b
^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*
d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c
*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b
^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^
3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*
b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4
+ 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^
13*c^5*d - 6*a^13*b*c*d^5) - (8*(4*a*b^12*c^4*d^5 + 4*a*b^12*c^6*d^3 + 4*a^
3*b^10*c^8*d + 4*a^4*b^9*c*d^8 + 4*a^5*b^8*c^8*d - 16*a^6*b^7*c*d^8 + 24*a^
8*b^5*c*d^8 - 16*a^10*b^3*c*d^8 - 4*a^2*b^11*c^3*d^6 - 8*a^2*b^11*c^5*d^4 -
2*a^2*b^11*c^7*d^2 - 4*a^3*b^10*c^2*d^7 - 16*a^3*b^10*c^4*d^5 - a^3*b^10*c
^6*d^3 + 24*a^4*b^9*c^3*d^6 - 20*a^4*b^9*c^5*d^4 - 20*a^4*b^9*c^7*d^2 + 12*
a^5*b^8*c^2*d^7 + 95*a^5*b^8*c^4*d^5 + 20*a^5*b^8*c^6*d^3 - 98*a^6*b^7*c^3*
d^6 + 64*a^6*b^7*c^5*d^4 - 32*a^6*b^7*c^7*d^2 + a^7*b^6*c^2*d^7 - 188*a^7*b
^6*c^4*d^5 + 112*a^7*b^6*c^6*d^3 + 164*a^8*b^5*c^3*d^6 - 216*a^8*b^5*c^5*d^
4 - 28*a^9*b^4*c^2*d^7 + 240*a^9*b^4*c^4*d^5 - 140*a^10*b^3*c^3*d^6 + 28*a^
11*b^2*c^2*d^7 + a*b^12*c^8*d + 4*a^12*b*c*d^8))/(a^14*d^6 + b^14*c^6 - 4*a
^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 -
4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5
*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^
9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 2
0*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9
*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 +
90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*
c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 -
6*a*b^13*c^5*d - 6*a^13*b*c*d^5) + (b*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(2*
a^2*b^14*c^10 - 6*a^6*b^10*c^10 + 4*a^8*b^8*c^10 + 4*a^16*c^2*d^8 + 4*a*b^1
5*c^7*d^3 - 10*a^3*b^13*c^9*d - 12*a^5*b^11*c^9*d + 4*a^7*b^9*c*d^9 + 54*a^
7*b^9*c^9*d - 18*a^9*b^7*c*d^9 - 32*a^9*b^7*c^9*d + 36*a^11*b^5*c*d^9 - 34*
a^13*b^3*c*d^9 - 32*a^15*b*c^3*d^7 - 24*a^2*b^14*c^6*d^4 + 2*a^2*b^14*c^8*d
^2 + 60*a^3*b^13*c^5*d^5 - 30*a^3*b^13*c^7*d^3 - 80*a^4*b^12*c^4*d^6 + 138*
a^4*b^12*c^6*d^4 + 2*a^4*b^12*c^8*d^2 + 60*a^5*b^11*c^3*d^7 - 310*a^5*b^11*
c^5*d^5 + 122*a^5*b^11*c^7*d^3 - 24*a^6*b^10*c^2*d^8 + 390*a^6*b^10*c^4*d^6
- 466*a^6*b^10*c^6*d^4 + 102*a^6*b^10*c^8*d^2 - 282*a^7*b^9*c^3*d^7 + 878*

a^7*b^9*c^5*d^5 - 394*a^7*b^9*c^7*d^3 + 110*a^8*b^8*c^2*d^8 - 970*a^8*b^8*c
^4*d^6 + 894*a^8*b^8*c^6*d^4 - 218*a^8*b^8*c^8*d^2 + 638*a^9*b^7*c^3*d^7 -
1290*a^9*b^7*c^5*d^5 + 522*a^9*b^7*c^7*d^3 - 232*a^10*b^6*c^2*d^8 + 1202*a^
10*b^6*c^4*d^6 - 822*a^10*b^6*c^6*d^4 + 112*a^10*b^6*c^8*d^2 - 702*a^11*b^5
*c^3*d^7 + 886*a^11*b^5*c^5*d^5 - 224*a^11*b^5*c^7*d^3 + 234*a^12*b^4*c^2*d
^8 - 654*a^12*b^4*c^4*d^6 + 280*a^12*b^4*c^6*d^4 + 318*a^13*b^3*c^3*d^7 - 2
24*a^13*b^3*c^5*d^5 - 92*a^14*b^2*c^2*d^8 + 112*a^14*b^2*c^4*d^6 + 12*a^15*
b*c*d^9))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^
8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12
*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*
b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11
*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^
4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*
b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2
 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11
*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) + (8*
tan(e/2 + (f*x)/2)*(4*a*b^15*c^10 + 8*a^16*c*d^9 - 12*a^5*b^11*c^10 + 8*a^7
*b^9*c^10 + 4*a*b^15*c^8*d^2 - 20*a^2*b^14*c^9*d - 24*a^4*b^12*c^9*d + 108*
a^6*b^10*c^9*d + 4*a^8*b^8*c*d^9 - 64*a^8*b^8*c^9*d - 8*a^10*b^6*c*d^9 + 12
*a^12*b^4*c*d^9 - 16*a^14*b^2*c*d^9 - 40*a^15*b*c^2*d^8 - 20*a^2*b^14*c^7*d
^3 + 36*a^3*b^13*c^6*d^4 + 4*a^3*b^13*c^8*d^2 - 20*a^4*b^12*c^5*d^5 + 164*a
^4*b^12*c^7*d^3 - 20*a^5*b^11*c^4*d^6 - 452*a^5*b^11*c^6*d^4 + 204*a^5*b^11
*c^8*d^2 + 36*a^6*b^10*c^3*d^7 + 556*a^6*b^10*c^5*d^5 - 708*a^6*b^10*c^7*d^
3 - 20*a^7*b^9*c^2*d^8 - 340*a^7*b^9*c^4*d^6 + 1308*a^7*b^9*c^6*d^4 - 436*a
^7*b^9*c^8*d^2 + 76*a^8*b^8*c^3*d^7 - 1380*a^8*b^8*c^5*d^5 + 1004*a^8*b^8*c
^7*d^3 + 16*a^9*b^7*c^2*d^8 + 804*a^9*b^7*c^4*d^6 - 1404*a^9*b^7*c^6*d^4 +
224*a^9*b^7*c^8*d^2 - 204*a^10*b^6*c^3*d^7 + 1172*a^10*b^6*c^5*d^5 - 440*a^
10*b^6*c^7*d^3 - 12*a^11*b^5*c^2*d^8 - 508*a^11*b^5*c^4*d^6 + 512*a^11*b^5*
c^6*d^4 + 36*a^12*b^4*c^3*d^7 - 328*a^12*b^4*c^5*d^5 + 56*a^13*b^3*c^2*d^8
 + 64*a^13*b^3*c^4*d^6 + 56*a^14*b^2*c^3*d^7))/(a^14*d^6 + b^14*c^6 - 4*a^2*
b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a
^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^
9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b
^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a
^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^
3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*
a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2
*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*
a*b^13*c^5*d - 6*a^13*b*c*d^5) - (b*((8*(4*a^2*b^17*c^11 - 16*a^4*b^15*c^11
 + 24*a^6*b^13*c^11 - 16*a^8*b^11*c^11 + 4*a^10*b^9*c^11 + 4*a^19*c^2*d^9 -
 12*a^3*b^16*c^10*d + 88*a^5*b^14*c^10*d - 152*a^7*b^12*c^10*d + 108*a^9*b^
10*c^10*d - 4*a^10*b^9*c*d^10 - 28*a^11*b^8*c^10*d + 16*a^12*b^7*c*d^10 - 2
4*a^14*b^5*c*d^10 + 16*a^16*b^3*c*d^10 - 28*a^18*b*c^3*d^8 + 28*a^2*b^17*c^
9*d^2 - 80*a^3*b^16*c^8*d^3 + 112*a^4*b^15*c^7*d^4 - 32*a^4*b^15*c^9*d^2 -
56*a^5*b^14*c^6*d^5 + 208*a^5*b^14*c^8*d^3 - 56*a^6*b^13*c^5*d^6 - 392*a^6*

b^13*c^7*d^4 - 152*a^6*b^13*c^9*d^2 + 112*a^7*b^12*c^4*d^7 + 280*a^7*b^12*c^6*d^5 - 32*a^7*b^12*c^8*d^3 - 80*a^8*b^11*c^3*d^8 + 112*a^8*b^11*c^5*d^6 + 448*a^8*b^11*c^7*d^4 + 368*a^8*b^11*c^9*d^2 + 28*a^9*b^10*c^2*d^9 - 368*a^9*b^10*c^4*d^7 - 560*a^9*b^10*c^6*d^5 - 352*a^9*b^10*c^8*d^3 + 292*a^10*b^9*c^3*d^8 + 112*a^10*b^9*c^5*d^6 - 112*a^10*b^9*c^7*d^4 - 292*a^10*b^9*c^9*d^2 - 108*a^11*b^8*c^2*d^9 + 352*a^11*b^8*c^4*d^7 + 560*a^11*b^8*c^6*d^5 + 368*a^11*b^8*c^8*d^3 - 368*a^12*b^7*c^3*d^8 - 448*a^12*b^7*c^5*d^6 - 112*a^12*b^7*c^7*d^4 + 80*a^12*b^7*c^9*d^2 + 152*a^13*b^6*c^2*d^9 + 32*a^13*b^6*c^4*d^7 - 280*a^13*b^6*c^6*d^5 - 112*a^13*b^6*c^8*d^3 + 152*a^14*b^5*c^3*d^8 + 392*a^14*b^5*c^5*d^6 + 56*a^14*b^5*c^7*d^4 - 88*a^15*b^4*c^2*d^9 - 208*a^15*b^4*c^4*d^7 + 56*a^15*b^4*c^6*d^5 + 32*a^16*b^3*c^3*d^8 - 112*a^16*b^3*c^5*d^6 + 12*a^17*b^2*c^2*d^9 + 80*a^17*b^2*c^4*d^7 - 4*a*b^18*c^10*d - 4*a^18*b*c*d^10))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) - (8*tan(e/2 + (f*x)/2)*(56*a^3*b^16*c^11 - 12*a^19*c*d^10 - 12*a*b^18*c^11 - 104*a^5*b^14*c^11 + 96*a^7*b^12*c^11 - 44*a^9*b^10*c^11 + 8*a^11*b^8*c^11 + 8*a^19*c^3*d^8 + 16*a*b^18*c^9*d^2 + 96*a^2*b^17*c^10*d - 448*a^4*b^15*c^10*d + 832*a^6*b^13*c^10*d - 768*a^8*b^11*c^10*d + 16*a^9*b^10*c*d^10 + 352*a^10*b^9*c^10*d - 76*a^11*b^8*c*d^10 - 64*a^12*b^7*c^10*d + 144*a^13*b^6*c*d^10 - 136*a^15*b^4*c*d^10 + 64*a^17*b^2*c*d^10 + 96*a^18*b*c^2*d^9 - 64*a^18*b*c^4*d^7 - 128*a^2*b^17*c^8*d^3 + 448*a^3*b^16*c^7*d^4 - 412*a^3*b^16*c^9*d^2 - 896*a^4*b^15*c^6*d^5 + 1280*a^4*b^15*c^8*d^3 + 1120*a^5*b^14*c^5*d^6 - 2968*a^5*b^14*c^7*d^4 + 1712*a^5*b^14*c^9*d^2 - 896*a^6*b^13*c^4*d^7 + 4928*a^6*b^13*c^6*d^5 - 4288*a^6*b^13*c^8*d^3 + 448*a^7*b^12*c^3*d^8 - 5656*a^7*b^12*c^5*d^6 + 7952*a^7*b^12*c^7*d^4 - 3048*a^7*b^12*c^9*d^2 - 128*a^8*b^11*c^2*d^9 + 4352*a^8*b^11*c^4*d^7 - 11200*a^8*b^11*c^6*d^5 + 6912*a^8*b^11*c^8*d^3 - 2140*a^9*b^10*c^3*d^8 + 11648*a^9*b^10*c^5*d^6 - 11088*a^9*b^10*c^7*d^4 + 2752*a^9*b^10*c^9*d^2 + 608*a^10*b^9*c^2*d^9 - 8512*a^10*b^9*c^4*d^7 + 13440*a^10*b^9*c^6*d^5 - 5888*a^10*b^9*c^8*d^3 + 4088*a^11*b^8*c^3*d^8 - 12432*a^11*b^8*c^5*d^6 + 8512*a^11*b^8*c^7*d^4 - 1244*a^11*b^8*c^9*d^2 - 1152*a^12*b^7*c^2*d^9 + 8448*a^12*b^7*c^4*d^7 - 8960*a^12*b^7*c^6*d^5 + 2560*a^12*b^7*c^8*d^3 - 3912*a^13*b^6*c^3*d^8 + 7168*a^13*b^6*c^5*d^6 - 3416*a^13*b^6*c^7*d^4 + 224*a^13*b^6*c^9*d^2 + 1088*a^14*b^5*c^2*d^9 - 4352*a^14*b^5*c^4*d^7 + 3136*a^14*b^5*c^6*d^5 - 448*a^14*b^5*c^8*d^3 + 1888*a^15*b^4*c^3*d^8 - 2072*a^15*b^4*c^5*d^6 + 560*a^15*b^4*c^7*d^4 - 512*a^16*b^3*c^2*d^9 + 1024*a^16*b^3*c^4*d^7 - 448*a^16*b^3*c^6*d^5 - 380*a^17*b^2*c^3*d^8 + 224*a^17*b^2*c^5*d^6))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6* a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5))*(-(a + b)^5*(a - b)^5)^(1/2)*(6*a^4*d^2 + b^4*c^2 + 2*b^4*d^2 + 2*a^2*b^2*c^2 - 5*a^2*b^2*d^2 - 6*a^3*b*c*d))/(2*(a^13*d^3 + b^13*c^3 - 5*a^2*b^11*c^3 + 10*a^4*b^9*c^3 - 10*a^6*b^7*c^3 + 5*a^8*b^5*c^3 - a^10*b^3*c^3 - a^3*b^10*d^3 + 5*a^5*b^8*d^3 - 10*a^7*b^6*d^3 + 10*a^9*b^4*d^3 - 5*a^11*b^2*d^3 + 3*a^2*b^11*c*d^2 + 15*a^3*b^10*c^2*d - 15*a^4*b^9*c*d^2 - 30*a^5*b^8*c^2*d + 30*a^6*b^7*c*d^2 + 30*a^7*b^6*c^2*d - 30*a^8*b^5*c*d^2 - 15*a^9*b^4*c^2*d + 15*a^10*b^3*c*d^2 + 3*a^11*b^2*c^2*d - 3*a*b^12*c^2*d - 3*a^12*b*c*d^2)))*(6*a^4*d^2 + b^4*c^2 + 2*b^4*d^2 + 2*a^2*b^2*c^2 - 5*a^2*b^2*d^2 - 6*a^3*b*c*d))/(2*(a^13*d^3 + b^13*c^3 - 5*a^2*b^11*c^3 + 10*a^4*b^9*c^3 - 10*a^6*b^7*c^3 + 5*a^8*b^5*c^3 - a^10*b^3*c^3 - a^3*b^10*d^3 + 5*a^5*b^8*d^3 - 10*a^7*b^6*d^3 + 10*a^9*b^4*d^3 - 5*a^11*b^2*d^3 + 3*a^2*b^11*c*d^2 + 15*a^3*b^10*c^2*d - 15*a^4*b^9*c*d^2 - 30*a^5*b^8*c^2*d + 30*a^6*b^7*c*d^2 + 30*a^7*b^6*c^2*d - 30*a^8*b^5*c*d^2 - 15*a^9*b^4*c^2*d + 15*a^10*b^3*c*d^2 + 3*a^11*b^2*c^2*d - 3*a*b^12*c^2*d - 3*a^12*b*c*d^2)))*(6*a^4*d^2 + b^4*c^2 + 2*b^4*d^2 + 2*a^2*b^2*c^2 - 5*a^2*b^2*d^2 - 6*a^3*b*c*d)*1i)/(2*(a^13*d^3 + b^13*c^3 - 5*a^2*b^11*c^3 + 10*a^4*b^9*c^3 - 10*a^6*b^7*c^3 + 5*a^8*b^5*c^3 - a^10*b^3*c^3 - a^3*b^10*d^3 + 5*a^5*b^8*d^3 - 10*a^7*b^6*d^3 + 10*a^9*b^4*d^3 - 5*a^11*b^2*d^3 + 3*a^2*b^11*c*d^2 + 15*a^3*b^10*c^2*d - 15*a^4*b^9*c*d^2 - 30*a^5*b^8*c^2*d + 30*a^6*b^7*c*d^2 + 30*a^7*b^6*c^2*d - 30*a^8*b^5*c*d^2 - 15*a^9*b^4*c^2*d + 15*a^10*b^3*c*d^2 + 3*a^11*b^2*c^2*d - 3*a*b^12*c^2*d - 3*a^12*b*c*d^2)) - (b*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(4*a*b^12*c^4*d^5 + 4*a*b^12*c^6*d^3 + 4*a^3*b^10*c^8*d + 4*a^4*b^9*c*d^8 + 4*a^5*b^8*c^8*d - 16*a^6*b^7*c*d^8 + 24*a^8*b^5*c*d^8 - 16*a^10*b^3*c*d^8 - 4*a^2*b^11*c^3*d^6 - 8*a^2*b^11*c^5*d^4 - 2*a^2*b^11*c^7*d^2 - 4*a^3*b^10*c^2*d^7 - 16*a^3*b^10*c^4*d^5 - a^3*b^10*c^6*d^3 + 24*a^4*b^9*c^3*d^6 - 20*a^4*b^9*c^5*d^4 - 20*a^4*b^9*c^7*d^2 + 12*a^5*b^8*c^2*d^7 + 95*a^5*b^8*c^4*d^5 + 20*a^5*b^8*c^6*d^3 - 98*a^6*b^7*c^3*d^6 + 64*a^6*b^7*c^5*d^4 - 32*a^6*b^7*c^7*d^2 + a^7*b^6*c^2*d^7 - 188*a^7*b^6*c^4*d^5 + 112*a^7*b^6*c^6*d^3 + 164*a^8*b^5*c^3*d^6 - 216*a^8*b^5*c^5*d^4 - 28*a^9*b^4*c^2*d^7 + 240*a^9*b^4*c^4*d^5 - 140*a^10*b^3*c^3*d^6 + 28*a^11*b^2*c^2*d^7 + a*b^12*c^8*d + 4*a^12*b*c*d^8))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d

^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) - (8*tan(e/2 + (f*x)/2)*(a*b^12*c^9 + 4*a^13*c*d^8 + 4*a^3*b^10*c^9 + 4*a^5*b^8*c^9 - 16*a*b^12*c^3*d^6 - 4*a*b^12*c^5*d^4 + 2*a*b^12*c^7*d^2 - 2*a^2*b^11*c^8*d - 16*a^3*b^10*c*d^8 - 20*a^4*b^9*c^8*d + 76*a^5*b^8*c*d^8 - 32*a^6*b^7*c^8*d - 162*a^7*b^6*c*d^8 + 176*a^9*b^4*c*d^8 - 96*a^11*b^2*c*d^8 - 8*a^12*b*c^2*d^7 + 32*a^2*b^11*c^2*d^7 + 8*a^2*b^11*c^4*d^5 - 4*a^2*b^11*c^6*d^3 + 72*a^3*b^10*c^3*d^6 - 14*a^3*b^10*c^5*d^4 - 9*a^3*b^10*c^7*d^2 - 152*a^4*b^9*c^2*d^7 + 80*a^4*b^9*c^4*d^5 + 20*a^4*b^9*c^6*d^3 - 274*a^5*b^8*c^3*d^6 + 55*a^5*b^8*c^5*d^4 + 12*a^5*b^8*c^7*d^2 + 372*a^6*b^7*c^2*d^7 - 250*a^6*b^7*c^4*d^5 + 128*a^6*b^7*c^6*d^3 + 481*a^7*b^6*c^3*d^6 - 412*a^7*b^6*c^5*d^4 + 112*a^7*b^6*c^7*d^2 - 472*a^8*b^5*c^2*d^7 + 612*a^8*b^5*c^4*d^5 - 216*a^8*b^5*c^6*d^3 - 564*a^9*b^4*c^3*d^6 + 240*a^9*b^4*c^5*d^4 + 336*a^10*b^3*c^2*d^7 - 144*a^10*b^3*c^4*d^5 + 40*a^11*b^2*c^3*d^6))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) + (b*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(2*a^2*b^14*c^10 - 6*a^6*b^10*c^10 + 4*a^8*b^8*c^10 + 4*a^16*c^2*d^8 + 4*a*b^15*c^7*d^3 - 10*a^3*b^13*c^9*d - 12*a^5*b^11*c^9*d + 4*a^7*b^9*c*d^9 + 54*a^7*b^9*c^9*d - 18*a^9*b^7*c*d^9 - 32*a^9*b^7*c^9*d + 36*a^11*b^5*c*d^9 - 34*a^13*b^3*c*d^9 - 32*a^15*b*c^3*d^7 - 24*a^2*b^14*c^6*d^4 + 2*a^2*b^14*c^8*d^2 + 60*a^3*b^13*c^5*d^5 - 30*a^3*b^13*c^7*d^3 - 80*a^4*b^12*c^4*d^6 + 138*a^4*b^12*c^6*d^4 + 2*a^4*b^12*c^8*d^2 + 60*a^5*b^11*c^3*d^7 - 310*a^5*b^11*c^5*d^5 + 122*a^5*b^11*c^7*d^3 - 24*a^6*b^10*c^2*d^8 + 390*a^6*b^10*c^4*d^6 - 466*a^6*b^10*c^6*d^4 + 102*a^6*b^10*c^8*d^2 - 282*a^7*b^9*c^3*d^7 + 878*a^7*b^9*c^5*d^5 - 394*a^7*b^9*c^7*d^3 + 110*a^8*b^8*c^2*d^8 - 970*a^8*b^8*c^4*d^6 + 894*a^8*b^8*c^6*d^4 - 218*a^8*b^8*c^8*d^2 + 638*a^9*b^7*c^3*d^7 - 1290*a^9*b^7*c^5*d^5 + 522*a^9*b^7*c^7*d^3 - 232*a^10*b^6*c^2*d^8 + 1202*a^10*b^6*c^4*d^6 - 822*a^10*b^6*c^6*d^4 + 112*a^10*b^6*c^8*d^2 - 702*a^11*b^5*c^3*d^7 + 886*a^11*b^5*c^5*d^5 - 224*a^11*b^5*c^7*d^3 + 234*a^12*b^4*c^2*d^8 - 654*a^12*b^4*c^4*d^6 + 280*a^12*b^4*c^6*d^4 + 318*a^13*b^3*c^3*d^7 - 224*a^13*b^3*c^5*d^5 - 92*a^14*b^2*c^2*d^8 + 112*a^14*b^2*c^4*d^6 + 12*a^15*b*c*d^9))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^

4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) + (8*tan(e/2 + (f*x)/2)*(4*a*b^15*c^10 + 8*a^16*c*d^9 - 12*a^5*b^11*c^10 + 8*a^7*b^9*c^10 + 4*a*b^15*c^8*d^2 - 20*a^2*b^14*c^9*d - 24*a^4*b^12*c^9*d + 108*a^6*b^10*c^9*d + 4*a^8*b^8*c*d^9 - 64*a^8*b^8*c^9*d - 8*a^10*b^6*c*d^9 + 12*a^12*b^4*c*d^9 - 16*a^14*b^2*c*d^9 - 40*a^15*b*c^2*d^8 - 20*a^2*b^14*c^7*d^3 + 36*a^3*b^13*c^6*d^4 + 4*a^3*b^13*c^8*d^2 - 20*a^4*b^12*c^5*d^5 + 164*a^4*b^12*c^7*d^3 - 20*a^5*b^11*c^4*d^6 - 452*a^5*b^11*c^6*d^4 + 204*a^5*b^11*c^8*d^2 + 36*a^6*b^10*c^3*d^7 + 556*a^6*b^10*c^5*d^5 - 708*a^6*b^10*c^7*d^3 - 20*a^7*b^9*c^2*d^8 - 340*a^7*b^9*c^4*d^6 + 1308*a^7*b^9*c^6*d^4 - 436*a^7*b^9*c^8*d^2 + 76*a^8*b^8*c^3*d^7 - 1380*a^8*b^8*c^5*d^5 + 1004*a^8*b^8*c^7*d^3 + 16*a^9*b^7*c^2*d^8 + 804*a^9*b^7*c^4*d^6 - 1404*a^9*b^7*c^6*d^4 + 224*a^9*b^7*c^8*d^2 - 204*a^10*b^6*c^3*d^7 + 1172*a^10*b^6*c^5*d^5 - 440*a^10*b^6*c^7*d^3 - 12*a^11*b^5*c^2*d^8 - 508*a^11*b^5*c^4*d^6 + 512*a^11*b^5*c^6*d^4 + 36*a^12*b^4*c^3*d^7 - 328*a^12*b^4*c^5*d^5 + 56*a^13*b^3*c^2*d^8 + 64*a^13*b^3*c^4*d^6 + 56*a^14*b^2*c^3*d^7))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) + (b*((8*(4*a^2*b^17*c^11 - 16*a^4*b^15*c^11 + 24*a^6*b^13*c^11 - 16*a^8*b^11*c^11 + 4*a^10*b^9*c^11 + 4*a^19*c^2*d^9 - 12*a^3*b^16*c^10*d + 88*a^5*b^14*c^10*d - 152*a^7*b^12*c^10*d + 108*a^9*b^10*c^10*d - 4*a^10*b^9*c*d^10 - 28*a^11*b^8*c^10*d + 16*a^12*b^7*c*d^10 - 24*a^14*b^5*c*d^10 + 16*a^16*b^3*c*d^10 - 28*a^18*b*c^3*d^8 + 28*a^2*b^17*c^9*d^2 - 80*a^3*b^16*c^8*d^3 + 112*a^4*b^15*c^7*d^4 - 32*a^4*b^15*c^9*d^2 - 56*a^5*b^14*c^6*d^5 + 208*a^5*b^14*c^8*d^3 - 56*a^6*b^13*c^5*d^6 - 392*a^6*b^13*c^7*d^4 - 152*a^6*b^13*c^9*d^2 + 112*a^7*b^12*c^4*d^7 + 280*a^7*b^12*c^6*d^5 - 32*a^7*b^12*c^8*d^3 - 80*a^8*b^11*c^3*d^8 + 112*a^8*b^11*c^5*d^6 + 448*a^8*b^11*c^7*d^4 + 368*a^8*b^11*c^9*d^2 + 28*a^9*b^10*c^2*d^9 - 368*a^9*b^10*c^4*d^7 - 560*a^9*b^10*c^6*d^5 - 352*a^9*b^10*c^8*d^3 + 292*a^10*b^9*c^3*d^8 + 112*a^10*b^9*c^5*d^6 - 112*a^10*b^9*c^7*d^4 - 292*a^10*b^9*c^9*d^2 - 108*a^11*b^8*c^2*d^9 + 352*a^11*b^8*c^4*d^7 + 560*a^11*b^8*c^6*d^5 + 368*a^11*b^8*c^8*d^3 - 368*a^12*b^7*c^3*d^8 - 448*a^12*b^7*c^5*d^6 - 112*a^12*b^7*c^7*d^4 + 80*a^12*b^7*c^9*d^2 + 152*a^13*b^6*c^2*d^9 + 32*a^13*b^6*c^4*d^7 - 280*a^13*b^6*c^6*d^5 - 112*a^13*b^6*c^8*d^3 + 152*a^14*b^5*c^3*d^8 + 392*a^14*b^5*c^5*d^6 + 56*a^14*b^5*c^7*d^4 - 88*a^15*b^4*c^2*d^9 - 208*a^15*b^4*c^4*d^7 + 56*a^15*b^4*c^6*d^5 + 32*a^16*b^3*c^3*d^8 - 112*a^16*b^3*c^5*d^6 + 12*a^17*b^2*c^2*d^9 + 80*a^17*b^2*c^4*d^7 - 4*a*b^18*c^10*d - 4*a^18*b*c*d^10))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*

a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*
a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d
^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^
6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d
^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^1
0*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d
- 6*a^13*b*c*d^5) - (8*tan(e/2 + (f*x)/2)*(56*a^3*b^16*c^11 - 12*a^19*c*d^1
0 - 12*a*b^18*c^11 - 104*a^5*b^14*c^11 + 96*a^7*b^12*c^11 - 44*a^9*b^10*c^1
1 + 8*a^11*b^8*c^11 + 8*a^19*c^3*d^8 + 16*a*b^18*c^9*d^2 + 96*a^2*b^17*c^10
*d - 448*a^4*b^15*c^10*d + 832*a^6*b^13*c^10*d - 768*a^8*b^11*c^10*d + 16*a
^9*b^10*c*d^10 + 352*a^10*b^9*c^10*d - 76*a^11*b^8*c*d^10 - 64*a^12*b^7*c^1
0*d + 144*a^13*b^6*c*d^10 - 136*a^15*b^4*c*d^10 + 64*a^17*b^2*c*d^10 + 96*a
^18*b*c^2*d^9 - 64*a^18*b*c^4*d^7 - 128*a^2*b^17*c^8*d^3 + 448*a^3*b^16*c^7
*d^4 - 412*a^3*b^16*c^9*d^2 - 896*a^4*b^15*c^6*d^5 + 1280*a^4*b^15*c^8*d^3
+ 1120*a^5*b^14*c^5*d^6 - 2968*a^5*b^14*c^7*d^4 + 1712*a^5*b^14*c^9*d^2 - 8
96*a^6*b^13*c^4*d^7 + 4928*a^6*b^13*c^6*d^5 - 4288*a^6*b^13*c^8*d^3 + 448*a
^7*b^12*c^3*d^8 - 5656*a^7*b^12*c^5*d^6 + 7952*a^7*b^12*c^7*d^4 - 3048*a^7*
b^12*c^9*d^2 - 128*a^8*b^11*c^2*d^9 + 4352*a^8*b^11*c^4*d^7 - 11200*a^8*b^1
1*c^6*d^5 + 6912*a^8*b^11*c^8*d^3 - 2140*a^9*b^10*c^3*d^8 + 11648*a^9*b^10*
c^5*d^6 - 11088*a^9*b^10*c^7*d^4 + 2752*a^9*b^10*c^9*d^2 + 608*a^10*b^9*c^2
*d^9 - 8512*a^10*b^9*c^4*d^7 + 13440*a^10*b^9*c^6*d^5 - 5888*a^10*b^9*c^8*d
^3 + 4088*a^11*b^8*c^3*d^8 - 12432*a^11*b^8*c^5*d^6 + 8512*a^11*b^8*c^7*d^4
- 1244*a^11*b^8*c^9*d^2 - 1152*a^12*b^7*c^2*d^9 + 8448*a^12*b^7*c^4*d^7 -
8960*a^12*b^7*c^6*d^5 + 2560*a^12*b^7*c^8*d^3 - 3912*a^13*b^6*c^3*d^8 + 716
8*a^13*b^6*c^5*d^6 - 3416*a^13*b^6*c^7*d^4 + 224*a^13*b^6*c^9*d^2 + 1088*a^
14*b^5*c^2*d^9 - 4352*a^14*b^5*c^4*d^7 + 3136*a^14*b^5*c^6*d^5 - 448*a^14*b
^5*c^8*d^3 + 1888*a^15*b^4*c^3*d^8 - 2072*a^15*b^4*c^5*d^6 + 560*a^15*b^4*c
^7*d^4 - 512*a^16*b^3*c^2*d^9 + 1024*a^16*b^3*c^4*d^7 - 448*a^16*b^3*c^6*d^
5 - 380*a^17*b^2*c^3*d^8 + 224*a^17*b^2*c^5*d^6))/(a^14*d^6 + b^14*c^6 - 4*
a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 -
4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^
5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a
^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 -
20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^
9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 +
90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4
*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4
- 6*a*b^13*c^5*d - 6*a^13*b*c*d^5))*(-(a + b)^5*(a - b)^5)^(1/2)*(6*a^4*d^2
+ b^4*c^2 + 2*b^4*d^2 + 2*a^2*b^2*c^2 - 5*a^2*b^2*d^2 - 6*a^3*b*c*d))/(2*(
a^13*d^3 + b^13*c^3 - 5*a^2*b^11*c^3 + 10*a^4*b^9*c^3 - 10*a^6*b^7*c^3 + 5*
a^8*b^5*c^3 - a^10*b^3*c^3 - a^3*b^10*d^3 + 5*a^5*b^8*d^3 - 10*a^7*b^6*d^3
+ 10*a^9*b^4*d^3 - 5*a^11*b^2*d^3 + 3*a^2*b^11*c*d^2 + 15*a^3*b^10*c^2*d -
15*a^4*b^9*c*d^2 - 30*a^5*b^8*c^2*d + 30*a^6*b^7*c*d^2 + 30*a^7*b^6*c^2*d -
30*a^8*b^5*c*d^2 - 15*a^9*b^4*c^2*d + 15*a^10*b^3*c*d^2 + 3*a^11*b^2*c^2*d
- 3*a*b^12*c^2*d - 3*a^12*b*c*d^2)))*(6*a^4*d^2 + b^4*c^2 + 2*b^4*d^2 + 2*

a^2*b^2*c^2 - 5*a^2*b^2*d^2 - 6*a^3*b*c*d))/(2*(a^13*d^3 + b^13*c^3 - 5*a^2
*b^11*c^3 + 10*a^4*b^9*c^3 - 10*a^6*b^7*c^3 + 5*a^8*b^5*c^3 - a^10*b^3*c^3
- a^3*b^10*d^3 + 5*a^5*b^8*d^3 - 10*a^7*b^6*d^3 + 10*a^9*b^4*d^3 - 5*a^11*b
^2*d^3 + 3*a^2*b^11*c*d^2 + 15*a^3*b^10*c^2*d - 15*a^4*b^9*c*d^2 - 30*a^5*b
^8*c^2*d + 30*a^6*b^7*c*d^2 + 30*a^7*b^6*c^2*d - 30*a^8*b^5*c*d^2 - 15*a^9*
b^4*c^2*d + 15*a^10*b^3*c*d^2 + 3*a^11*b^2*c^2*d - 3*a*b^12*c^2*d - 3*a^12*
b*c*d^2)))*(6*a^4*d^2 + b^4*c^2 + 2*b^4*d^2 + 2*a^2*b^2*c^2 - 5*a^2*b^2*d^2
 - 6*a^3*b*c*d)*1i)/(2*(a^13*d^3 + b^13*c^3 - 5*a^2*b^11*c^3 + 10*a^4*b^9*c
^3 - 10*a^6*b^7*c^3 + 5*a^8*b^5*c^3 - a^10*b^3*c^3 - a^3*b^10*d^3 + 5*a^5*b
^8*d^3 - 10*a^7*b^6*d^3 + 10*a^9*b^4*d^3 - 5*a^11*b^2*d^3 + 3*a^2*b^11*c*d^
2 + 15*a^3*b^10*c^2*d - 15*a^4*b^9*c*d^2 - 30*a^5*b^8*c^2*d + 30*a^6*b^7*c*
d^2 + 30*a^7*b^6*c^2*d - 30*a^8*b^5*c*d^2 - 15*a^9*b^4*c^2*d + 15*a^10*b^3*
c*d^2 + 3*a^11*b^2*c^2*d - 3*a*b^12*c^2*d - 3*a^12*b*c*d^2)))/((16*(4*a*b^9
*c^3*d^5 + a*b^9*c^5*d^3 - 18*a^3*b^7*c*d^7 + 36*a^5*b^5*c*d^7 - 34*a^7*b^3
*c*d^7 + 2*a^2*b^8*c^2*d^6 + a^2*b^8*c^4*d^4 - a^3*b^7*c^3*d^5 + 4*a^3*b^7*
c^5*d^3 - 25*a^4*b^6*c^2*d^6 - 8*a^4*b^6*c^4*d^4 - 16*a^5*b^5*c^3*d^5 + 4*a
^5*b^5*c^5*d^3 + 50*a^6*b^4*c^2*d^6 - 20*a^6*b^4*c^4*d^4 + 40*a^7*b^3*c^3*d
^5 - 36*a^8*b^2*c^2*d^6 + 4*a*b^9*c*d^7 + 12*a^9*b*c*d^7))/(a^14*d^6 + b^14
*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*
b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5
*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5
*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c
^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 +
80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*
c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60
*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2
*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) + (16*tan(e/2 + (f*x)/2)*(4*a*b
^9*c^2*d^6 + 2*a*b^9*c^4*d^4 + 4*a^2*b^8*c*d^7 - 26*a^4*b^6*c*d^7 + 52*a^6*
b^4*c*d^7 - 48*a^8*b^2*c*d^7 + 2*a^2*b^8*c^3*d^5 - 2*a^3*b^7*c^2*d^6 + 8*a^
3*b^7*c^4*d^4 - 16*a^4*b^6*c^3*d^5 - 20*a^5*b^5*c^2*d^6 + 8*a^5*b^5*c^4*d^4
 - 40*a^6*b^4*c^3*d^5 + 72*a^7*b^3*c^2*d^6))/(a^14*d^6 + b^14*c^6 - 4*a^2*b
^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^
8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9
*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^
5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^
3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3
*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a
^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*
d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a
*b^13*c^5*d - 6*a^13*b*c*d^5) - (b*(-(a + b)^5*(a - b)^5)^(1/2)*((8*tan(e/2
 + (f*x)/2)*(a*b^12*c^9 + 4*a^13*c*d^8 + 4*a^3*b^10*c^9 + 4*a^5*b^8*c^9 - 1
6*a*b^12*c^3*d^6 - 4*a*b^12*c^5*d^4 + 2*a*b^12*c^7*d^2 - 2*a^2*b^11*c^8*d -
 16*a^3*b^10*c*d^8 - 20*a^4*b^9*c^8*d + 76*a^5*b^8*c*d^8 - 32*a^6*b^7*c^8*d
 - 162*a^7*b^6*c*d^8 + 176*a^9*b^4*c*d^8 - 96*a^11*b^2*c*d^8 - 8*a^12*b*c^2
*d^7 + 32*a^2*b^11*c^2*d^7 + 8*a^2*b^11*c^4*d^5 - 4*a^2*b^11*c^6*d^3 + 72*a

^3*b^10*c^3*d^6 - 14*a^3*b^10*c^5*d^4 - 9*a^3*b^10*c^7*d^2 - 152*a^4*b^9*c^2*d^7 + 80*a^4*b^9*c^4*d^5 + 20*a^4*b^9*c^6*d^3 - 274*a^5*b^8*c^3*d^6 + 55*a^5*b^8*c^5*d^4 + 12*a^5*b^8*c^7*d^2 + 372*a^6*b^7*c^2*d^7 - 250*a^6*b^7*c^4*d^5 + 128*a^6*b^7*c^6*d^3 + 481*a^7*b^6*c^3*d^6 - 412*a^7*b^6*c^5*d^4 + 112*a^7*b^6*c^7*d^2 - 472*a^8*b^5*c^2*d^7 + 612*a^8*b^5*c^4*d^5 - 216*a^8*b^5*c^6*d^3 - 564*a^9*b^4*c^3*d^6 + 240*a^9*b^4*c^5*d^4 + 336*a^10*b^3*c^2*d^7 - 144*a^10*b^3*c^4*d^5 + 40*a^11*b^2*c^3*d^6))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) - (8*(4*a*b^12*c^4*d^5 + 4*a*b^12*c^6*d^3 + 4*a^3*b^10*c^8*d + 4*a^4*b^9*c*d^8 + 4*a^5*b^8*c^8*d - 16*a^6*b^7*c*d^8 + 24*a^8*b^5*c*d^8 - 16*a^10*b^3*c*d^8 - 4*a^2*b^11*c^3*d^6 - 8*a^2*b^11*c^5*d^4 - 2*a^2*b^11*c^7*d^2 - 4*a^3*b^10*c^2*d^7 - 16*a^3*b^10*c^4*d^5 - a^3*b^10*c^6*d^3 + 24*a^4*b^9*c^3*d^6 - 20*a^4*b^9*c^5*d^4 - 20*a^4*b^9*c^7*d^2 + 12*a^5*b^8*c^2*d^7 + 95*a^5*b^8*c^4*d^5 + 20*a^5*b^8*c^6*d^3 - 98*a^6*b^7*c^3*d^6 + 64*a^6*b^7*c^5*d^4 - 32*a^6*b^7*c^7*d^2 + a^7*b^6*c^2*d^7 - 188*a^7*b^6*c^4*d^5 + 112*a^7*b^6*c^6*d^3 + 164*a^8*b^5*c^3*d^6 - 216*a^8*b^5*c^5*d^4 - 28*a^9*b^4*c^2*d^7 + 240*a^9*b^4*c^4*d^5 - 140*a^10*b^3*c^3*d^6 + 28*a^11*b^2*c^2*d^7 + a*b^12*c^8*d + 4*a^12*b*c*d^8))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) + (b*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(2*a^2*b^14*c^10 - 6*a^6*b^10*c^10 + 4*a^8*b^8*c^10 + 4*a^16*c^2*d^8 + 4*a*b^15*c^7*d^3 - 10*a^3*b^13*c^9*d - 12*a^5*b^11*c^9*d + 4*a^7*b^9*c*d^9 + 54*a^7*b^9*c^9*d - 18*a^9*b^7*c*d^9 - 32*a^9*b^7*c^9*d + 36*a^11*b^5*c*d^9 - 34*a^13*b^3*c*d^9 - 32*a^15*b*c^3*d^7 - 24*a^2*b^14*c^6*d^4 + 2*a^2*b^14*c^8*d^2 + 60*a^3*b^13*c^5*d^5 - 30*a^3*b^13*c^7*d^3 - 80*a^4*b^12*c^4*d^6 + 138*a^4*b^12*c^6*d^4 + 2*a^4*b^12*c^8*d^2 + 60*a^5*b^11*c^3*d^7 - 310*a^5*b^11*c^5*d^5 + 122*a^5*b^11*c^7*d^3 - 24*a^6*b^10*c^2*d^8 + 390*a^6*b^10*c^4*d^6 - 466*a^6*b^10*c^6*d^4 + 102*a^6*b^10*c^8*d^2 - 282*a^7*b^9*c^3*d^7 + 878*a^7*b^9*c^5*d^5 - 394*a^7*b^9*c^7*d^3 + 110*a^8*b^8*c^2*d^8 - 970*a^8*b^8*c^4*d^6 + 894*a^8*b^8*c^6*d^4 - 218*a^8*b^8*c^8*d^2 + 638*a^9*b^7*c^3*d^7 - 1290*a^9*b^7*c^5*d^5 + 522*a^9*b^7*c^7*d^3 - 232*a^10*b^6*c^2*d^8 +

1202*a^10*b^6*c^4*d^6 - 822*a^10*b^6*c^6*d^4 + 112*a^10*b^6*c^8*d^2 - 702*a^11*b^5*c^3*d^7 + 886*a^11*b^5*c^5*d^5 - 224*a^11*b^5*c^7*d^3 + 234*a^12*b^4*c^2*d^8 - 654*a^12*b^4*c^4*d^6 + 280*a^12*b^4*c^6*d^4 + 318*a^13*b^3*c^3*d^7 - 224*a^13*b^3*c^5*d^5 - 92*a^14*b^2*c^2*d^8 + 112*a^14*b^2*c^4*d^6 + 12*a^15*b*c*d^9))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) + (8*tan(e/2 + (f*x)/2)*(4*a*b^15*c^10 + 8*a^16*c*d^9 - 12*a^5*b^11*c^10 + 8*a^7*b^9*c^10 + 4*a*b^15*c^8*d^2 - 20*a^2*b^14*c^9*d - 24*a^4*b^12*c^9*d + 108*a^6*b^10*c^9*d + 4*a^8*b^8*c*d^9 - 64*a^8*b^8*c^9*d - 8*a^10*b^6*c*d^9 + 12*a^12*b^4*c*d^9 - 16*a^14*b^2*c*d^9 - 40*a^15*b*c^2*d^8 - 20*a^2*b^14*c^7*d^3 + 36*a^3*b^13*c^6*d^4 + 4*a^3*b^13*c^8*d^2 - 20*a^4*b^12*c^5*d^5 + 164*a^4*b^12*c^7*d^3 - 20*a^5*b^11*c^4*d^6 - 452*a^5*b^11*c^6*d^4 + 204*a^5*b^11*c^8*d^2 + 36*a^6*b^10*c^3*d^7 + 556*a^6*b^10*c^5*d^5 - 708*a^6*b^10*c^7*d^3 - 20*a^7*b^9*c^2*d^8 - 340*a^7*b^9*c^4*d^6 + 1308*a^7*b^9*c^6*d^4 - 436*a^7*b^9*c^8*d^2 + 76*a^8*b^8*c^3*d^7 - 1380*a^8*b^8*c^5*d^5 + 1004*a^8*b^8*c^7*d^3 + 16*a^9*b^7*c^2*d^8 + 804*a^9*b^7*c^4*d^6 - 1404*a^9*b^7*c^6*d^4 + 224*a^9*b^7*c^8*d^2 - 204*a^10*b^6*c^3*d^7 + 1172*a^10*b^6*c^5*d^5 - 440*a^10*b^6*c^7*d^3 - 12*a^11*b^5*c^2*d^8 - 508*a^11*b^5*c^4*d^6 + 512*a^11*b^5*c^6*d^4 + 36*a^12*b^4*c^3*d^7 - 328*a^12*b^4*c^5*d^5 + 56*a^13*b^3*c^2*d^8 + 64*a^13*b^3*c^4*d^6 + 56*a^14*b^2*c^3*d^7))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) - (b*((8*(4*a^2*b^17*c^11 - 16*a^4*b^15*c^11 + 24*a^6*b^13*c^11 - 16*a^8*b^11*c^11 + 4*a^10*b^9*c^11 + 4*a^19*c^2*d^9 - 12*a^3*b^16*c^10*d + 88*a^5*b^14*c^10*d - 152*a^7*b^12*c^10*d + 108*a^9*b^10*c^10*d - 4*a^10*b^9*c*d^10 - 28*a^11*b^8*c^10*d + 16*a^12*b^7*c*d^10 - 24*a^14*b^5*c*d^10 + 16*a^16*b^3*c*d^10 - 28*a^18*b*c^3*d^8 + 28*a^2*b^17*c^9*d^2 - 80*a^3*b^16*c^8*d^3 + 112*a^4*b^15*c^7*d^4 - 32*a^4*b^15*c^9*d^2 - 56*a^5*b^14*c^6*d^5 + 208*a^5*b^14*c^8*d^3 - 56*a^6*b^13*c^5*d^6 - 392*a^6*b^13*c^7*d^4 - 152*a^6*b^13*c^9*d^2 + 112*a^7*b^12*c^4*d^7 + 280*a^7*b^12*c^6*d^5 - 32*a^7*b^12*c^8*d^3 - 80*a^8*b^11*c^3*d^8 + 112*a^8*b^11*c^5*d^6 + 448*a^8*b^11*c^7*d^4 + 368*a^8*b^11*c^9*d^2 + 28*a^9*b^10*c^2*d^9 -

368*a^9*b^10*c^4*d^7 - 560*a^9*b^10*c^6*d^5 - 352*a^9*b^10*c^8*d^3 + 292*a
^10*b^9*c^3*d^8 + 112*a^10*b^9*c^5*d^6 - 112*a^10*b^9*c^7*d^4 - 292*a^10*b^
9*c^9*d^2 - 108*a^11*b^8*c^2*d^9 + 352*a^11*b^8*c^4*d^7 + 560*a^11*b^8*c^6*
d^5 + 368*a^11*b^8*c^8*d^3 - 368*a^12*b^7*c^3*d^8 - 448*a^12*b^7*c^5*d^6 -
112*a^12*b^7*c^7*d^4 + 80*a^12*b^7*c^9*d^2 + 152*a^13*b^6*c^2*d^9 + 32*a^13
*b^6*c^4*d^7 - 280*a^13*b^6*c^6*d^5 - 112*a^13*b^6*c^8*d^3 + 152*a^14*b^5*c
^3*d^8 + 392*a^14*b^5*c^5*d^6 + 56*a^14*b^5*c^7*d^4 - 88*a^15*b^4*c^2*d^9 -
208*a^15*b^4*c^4*d^7 + 56*a^15*b^4*c^6*d^5 + 32*a^16*b^3*c^3*d^8 - 112*a^1
6*b^3*c^5*d^6 + 12*a^17*b^2*c^2*d^9 + 80*a^17*b^2*c^4*d^7 - 4*a*b^18*c^10*d
- 4*a^18*b*c*d^10))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6
- 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d
^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*
d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*
d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*
b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^
4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*
b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^
2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c
*d^5) - (8*tan(e/2 + (f*x)/2)*(56*a^3*b^16*c^11 - 12*a^19*c*d^10 - 12*a*b^1
8*c^11 - 104*a^5*b^14*c^11 + 96*a^7*b^12*c^11 - 44*a^9*b^10*c^11 + 8*a^11*b
^8*c^11 + 8*a^19*c^3*d^8 + 16*a*b^18*c^9*d^2 + 96*a^2*b^17*c^10*d - 448*a^4
*b^15*c^10*d + 832*a^6*b^13*c^10*d - 768*a^8*b^11*c^10*d + 16*a^9*b^10*c*d^
10 + 352*a^10*b^9*c^10*d - 76*a^11*b^8*c*d^10 - 64*a^12*b^7*c^10*d + 144*a^
13*b^6*c*d^10 - 136*a^15*b^4*c*d^10 + 64*a^17*b^2*c*d^10 + 96*a^18*b*c^2*d^
9 - 64*a^18*b*c^4*d^7 - 128*a^2*b^17*c^8*d^3 + 448*a^3*b^16*c^7*d^4 - 412*a
^3*b^16*c^9*d^2 - 896*a^4*b^15*c^6*d^5 + 1280*a^4*b^15*c^8*d^3 + 1120*a^5*b
^14*c^5*d^6 - 2968*a^5*b^14*c^7*d^4 + 1712*a^5*b^14*c^9*d^2 - 896*a^6*b^13*
c^4*d^7 + 4928*a^6*b^13*c^6*d^5 - 4288*a^6*b^13*c^8*d^3 + 448*a^7*b^12*c^3*
d^8 - 5656*a^7*b^12*c^5*d^6 + 7952*a^7*b^12*c^7*d^4 - 3048*a^7*b^12*c^9*d^2
- 128*a^8*b^11*c^2*d^9 + 4352*a^8*b^11*c^4*d^7 - 11200*a^8*b^11*c^6*d^5 +
6912*a^8*b^11*c^8*d^3 - 2140*a^9*b^10*c^3*d^8 + 11648*a^9*b^10*c^5*d^6 - 11
088*a^9*b^10*c^7*d^4 + 2752*a^9*b^10*c^9*d^2 + 608*a^10*b^9*c^2*d^9 - 8512*
a^10*b^9*c^4*d^7 + 13440*a^10*b^9*c^6*d^5 - 5888*a^10*b^9*c^8*d^3 + 4088*a^
11*b^8*c^3*d^8 - 12432*a^11*b^8*c^5*d^6 + 8512*a^11*b^8*c^7*d^4 - 1244*a^11
*b^8*c^9*d^2 - 1152*a^12*b^7*c^2*d^9 + 8448*a^12*b^7*c^4*d^7 - 8960*a^12*b^
7*c^6*d^5 + 2560*a^12*b^7*c^8*d^3 - 3912*a^13*b^6*c^3*d^8 + 7168*a^13*b^6*c
^5*d^6 - 3416*a^13*b^6*c^7*d^4 + 224*a^13*b^6*c^9*d^2 + 1088*a^14*b^5*c^2*d
^9 - 4352*a^14*b^5*c^4*d^7 + 3136*a^14*b^5*c^6*d^5 - 448*a^14*b^5*c^8*d^3 +
1888*a^15*b^4*c^3*d^8 - 2072*a^15*b^4*c^5*d^6 + 560*a^15*b^4*c^7*d^4 - 512
*a^16*b^3*c^2*d^9 + 1024*a^16*b^3*c^4*d^7 - 448*a^16*b^3*c^6*d^5 - 380*a^17
*b^2*c^3*d^8 + 224*a^17*b^2*c^5*d^6))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6
+ 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d
^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5
- 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5
- 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*

c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5))*(-(a + b)^5*(a - b)^5)^(1/2)*(6*a^4*d^2 + b^4*c^2 + 2*b^4*d^2 + 2*a^2*b^2*c^2 - 5*a^2*b^2*d^2 - 6*a^3*b*c*d))/(2*(a^13*d^3 + b^13*c^3 - 5*a^2*b^11*c^3 + 10*a^4*b^9*c^3 - 10*a^6*b^7*c^3 + 5*a^8*b^5*c^3 - a^10*b^3*c^3 - a^3*b^10*d^3 + 5*a^5*b^8*d^3 - 10*a^7*b^6*d^3 + 10*a^9*b^4*d^3 - 5*a^11*b^2*d^3 + 3*a^2*b^11*c*d^2 + 15*a^3*b^10*c^2*d - 15*a^4*b^9*c*d^2 - 30*a^5*b^8*c^2*d + 30*a^6*b^7*c*d^2 + 30*a^7*b^6*c^2*d - 30*a^8*b^5*c*d^2 - 15*a^9*b^4*c^2*d + 15*a^10*b^3*c*d^2 + 3*a^11*b^2*c^2*d - 3*a*b^12*c^2*d - 3*a^12*b*c*d^2)))*(6*a^4*d^2 + b^4*c^2 + 2*b^4*d^2 + 2*a^2*b^2*c^2 - 5*a^2*b^2*d^2 - 6*a^3*b*c*d))/(2*(a^13*d^3 + b^13*c^3 - 5*a^2*b^11*c^3 + 10*a^4*b^9*c^3 - 10*a^6*b^7*c^3 + 5*a^8*b^5*c^3 - a^10*b^3*c^3 - a^3*b^10*d^3 + 5*a^5*b^8*d^3 - 10*a^7*b^6*d^3 + 10*a^9*b^4*d^3 - 5*a^11*b^2*d^3 + 3*a^2*b^11*c*d^2 + 15*a^3*b^10*c^2*d - 15*a^4*b^9*c*d^2 - 30*a^5*b^8*c^2*d + 30*a^6*b^7*c*d^2 + 30*a^7*b^6*c^2*d - 30*a^8*b^5*c*d^2 - 15*a^9*b^4*c^2*d + 15*a^10*b^3*c*d^2 + 3*a^11*b^2*c^2*d - 3*a*b^12*c^2*d - 3*a^12*b*c*d^2)))*(6*a^4*d^2 + b^4*c^2 + 2*b^4*d^2 + 2*a^2*b^2*c^2 - 5*a^2*b^2*d^2 - 6*a^3*b*c*d))/(2*(a^13*d^3 + b^13*c^3 - 5*a^2*b^11*c^3 + 10*a^4*b^9*c^3 - 10*a^6*b^7*c^3 + 5*a^8*b^5*c^3 - a^10*b^3*c^3 - a^3*b^10*d^3 + 5*a^5*b^8*d^3 - 10*a^7*b^6*d^3 + 10*a^9*b^4*d^3 - 5*a^11*b^2*d^3 + 3*a^2*b^11*c*d^2 + 15*a^3*b^10*c^2*d - 15*a^4*b^9*c*d^2 - 30*a^5*b^8*c^2*d + 30*a^6*b^7*c*d^2 + 30*a^7*b^6*c^2*d - 30*a^8*b^5*c*d^2 - 15*a^9*b^4*c^2*d + 15*a^10*b^3*c*d^2 + 3*a^11*b^2*c^2*d - 3*a*b^12*c^2*d - 3*a^12*b*c*d^2)) - (b*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(4*a*b^12*c^4*d^5 + 4*a*b^12*c^6*d^3 + 4*a^3*b^10*c^8*d + 4*a^4*b^9*c*d^8 + 4*a^5*b^8*c^8*d - 16*a^6*b^7*c*d^8 + 24*a^8*b^5*c*d^8 - 16*a^10*b^3*c*d^8 - 4*a^2*b^11*c^3*d^6 - 8*a^2*b^11*c^5*d^4 - 2*a^2*b^11*c^7*d^2 - 4*a^3*b^10*c^2*d^7 - 16*a^3*b^10*c^4*d^5 - a^3*b^10*c^6*d^3 + 24*a^4*b^9*c^3*d^6 - 20*a^4*b^9*c^5*d^4 - 20*a^4*b^9*c^7*d^2 + 12*a^5*b^8*c^2*d^7 + 95*a^5*b^8*c^4*d^5 + 20*a^5*b^8*c^6*d^3 - 98*a^6*b^7*c^3*d^6 + 64*a^6*b^7*c^5*d^4 - 32*a^6*b^7*c^7*d^2 + a^7*b^6*c^2*d^7 - 188*a^7*b^6*c^4*d^5 + 112*a^7*b^6*c^6*d^3 + 164*a^8*b^5*c^3*d^6 - 216*a^8*b^5*c^5*d^4 - 28*a^9*b^4*c^2*d^7 + 240*a^9*b^4*c^4*d^5 - 140*a^10*b^3*c^3*d^6 + 28*a^11*b^2*c^2*d^7 + a*b^12*c^8*d + 4*a^12*b*c*d^8))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) - (8*tan(e/2 + (f*x)/2)*(a*b^12*c^9 + 4*a^13*c*d^8 + 4*a^3*b^10*c^9 + 4*a^5*b^8*c^9 - 16*a*b^12*c^3*d^6 - 4*a*b^12*c^5*d^4 + 2*a*b^12*c^7*d

^2 - 2*a^2*b^11*c^8*d - 16*a^3*b^10*c*d^8 - 20*a^4*b^9*c^8*d + 76*a^5*b^8*c *d^8 - 32*a^6*b^7*c^8*d - 162*a^7*b^6*c*d^8 + 176*a^9*b^4*c*d^8 - 96*a^11*b ^2*c*d^8 - 8*a^12*b*c^2*d^7 + 32*a^2*b^11*c^2*d^7 + 8*a^2*b^11*c^4*d^5 - 4* a^2*b^11*c^6*d^3 + 72*a^3*b^10*c^3*d^6 - 14*a^3*b^10*c^5*d^4 - 9*a^3*b^10*c ^7*d^2 - 152*a^4*b^9*c^2*d^7 + 80*a^4*b^9*c^4*d^5 + 20*a^4*b^9*c^6*d^3 - 27 4*a^5*b^8*c^3*d^6 + 55*a^5*b^8*c^5*d^4 + 12*a^5*b^8*c^7*d^2 + 372*a^6*b^7*c ^2*d^7 - 250*a^6*b^7*c^4*d^5 + 128*a^6*b^7*c^6*d^3 + 481*a^7*b^6*c^3*d^6 - 412*a^7*b^6*c^5*d^4 + 112*a^7*b^6*c^7*d^2 - 472*a^8*b^5*c^2*d^7 + 612*a^8*b ^5*c^4*d^5 - 216*a^8*b^5*c^6*d^3 - 564*a^9*b^4*c^3*d^6 + 240*a^9*b^4*c^5*d^ 4 + 336*a^10*b^3*c^2*d^7 - 144*a^10*b^3*c^4*d^5 + 40*a^11*b^2*c^3*d^6))/(a^ 14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b ^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24 *a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 2 4*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^ 10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5 *c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) + (b*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(2*a^2*b^14*c^10 - 6*a^6*b^10*c^10 + 4*a^8*b^8*c^10 + 4 *a^16*c^2*d^8 + 4*a*b^15*c^7*d^3 - 10*a^3*b^13*c^9*d - 12*a^5*b^11*c^9*d + 4*a^7*b^9*c*d^9 + 54*a^7*b^9*c^9*d - 18*a^9*b^7*c*d^9 - 32*a^9*b^7*c^9*d + 36*a^11*b^5*c*d^9 - 34*a^13*b^3*c*d^9 - 32*a^15*b*c^3*d^7 - 24*a^2*b^14*c^6 *d^4 + 2*a^2*b^14*c^8*d^2 + 60*a^3*b^13*c^5*d^5 - 30*a^3*b^13*c^7*d^3 - 80* a^4*b^12*c^4*d^6 + 138*a^4*b^12*c^6*d^4 + 2*a^4*b^12*c^8*d^2 + 60*a^5*b^11* c^3*d^7 - 310*a^5*b^11*c^5*d^5 + 122*a^5*b^11*c^7*d^3 - 24*a^6*b^10*c^2*d^8 + 390*a^6*b^10*c^4*d^6 - 466*a^6*b^10*c^6*d^4 + 102*a^6*b^10*c^8*d^2 - 282 *a^7*b^9*c^3*d^7 + 878*a^7*b^9*c^5*d^5 - 394*a^7*b^9*c^7*d^3 + 110*a^8*b^8* c^2*d^8 - 970*a^8*b^8*c^4*d^6 + 894*a^8*b^8*c^6*d^4 - 218*a^8*b^8*c^8*d^2 + 638*a^9*b^7*c^3*d^7 - 1290*a^9*b^7*c^5*d^5 + 522*a^9*b^7*c^7*d^3 - 232*a^1 0*b^6*c^2*d^8 + 1202*a^10*b^6*c^4*d^6 - 822*a^10*b^6*c^6*d^4 + 112*a^10*b^6 *c^8*d^2 - 702*a^11*b^5*c^3*d^7 + 886*a^11*b^5*c^5*d^5 - 224*a^11*b^5*c^7*d ^3 + 234*a^12*b^4*c^2*d^8 - 654*a^12*b^4*c^4*d^6 + 280*a^12*b^4*c^6*d^4 + 3 18*a^13*b^3*c^3*d^7 - 224*a^13*b^3*c^5*d^5 - 92*a^14*b^2*c^2*d^8 + 112*a^14 *b^2*c^4*d^6 + 12*a^15*b*c*d^9))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6* a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36* a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6* a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d ^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^ 6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d ^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^1 0*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) + (8*tan(e/2 + (f*x)/2)*(4*a*b^15*c^10 + 8*a^16*c*d^9 - 1 2*a^5*b^11*c^10 + 8*a^7*b^9*c^10 + 4*a*b^15*c^8*d^2 - 20*a^2*b^14*c^9*d - 2

4*a^4*b^12*c^9*d + 108*a^6*b^10*c^9*d + 4*a^8*b^8*c*d^9 - 64*a^8*b^8*c^9*d
- 8*a^10*b^6*c*d^9 + 12*a^12*b^4*c*d^9 - 16*a^14*b^2*c*d^9 - 40*a^15*b*c^2*
d^8 - 20*a^2*b^14*c^7*d^3 + 36*a^3*b^13*c^6*d^4 + 4*a^3*b^13*c^8*d^2 - 20*a
^4*b^12*c^5*d^5 + 164*a^4*b^12*c^7*d^3 - 20*a^5*b^11*c^4*d^6 - 452*a^5*b^11
*c^6*d^4 + 204*a^5*b^11*c^8*d^2 + 36*a^6*b^10*c^3*d^7 + 556*a^6*b^10*c^5*d^
5 - 708*a^6*b^10*c^7*d^3 - 20*a^7*b^9*c^2*d^8 - 340*a^7*b^9*c^4*d^6 + 1308*
a^7*b^9*c^6*d^4 - 436*a^7*b^9*c^8*d^2 + 76*a^8*b^8*c^3*d^7 - 1380*a^8*b^8*c
^5*d^5 + 1004*a^8*b^8*c^7*d^3 + 16*a^9*b^7*c^2*d^8 + 804*a^9*b^7*c^4*d^6 -
1404*a^9*b^7*c^6*d^4 + 224*a^9*b^7*c^8*d^2 - 204*a^10*b^6*c^3*d^7 + 1172*a^
10*b^6*c^5*d^5 - 440*a^10*b^6*c^7*d^3 - 12*a^11*b^5*c^2*d^8 - 508*a^11*b^5*
c^4*d^6 + 512*a^11*b^5*c^6*d^4 + 36*a^12*b^4*c^3*d^7 - 328*a^12*b^4*c^5*d^5
 + 56*a^13*b^3*c^2*d^8 + 64*a^13*b^3*c^4*d^6 + 56*a^14*b^2*c^3*d^7))/(a^14*
d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*
c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^
3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a
^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*
a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*
c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 12
0*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^
3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 1
5*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) + (b*((8*(4*a^2*b^17*
c^11 - 16*a^4*b^15*c^11 + 24*a^6*b^13*c^11 - 16*a^8*b^11*c^11 + 4*a^10*b^9*
c^11 + 4*a^19*c^2*d^9 - 12*a^3*b^16*c^10*d + 88*a^5*b^14*c^10*d - 152*a^7*b
^12*c^10*d + 108*a^9*b^10*c^10*d - 4*a^10*b^9*c*d^10 - 28*a^11*b^8*c^10*d +
 16*a^12*b^7*c*d^10 - 24*a^14*b^5*c*d^10 + 16*a^16*b^3*c*d^10 - 28*a^18*b*c
^3*d^8 + 28*a^2*b^17*c^9*d^2 - 80*a^3*b^16*c^8*d^3 + 112*a^4*b^15*c^7*d^4 -
 32*a^4*b^15*c^9*d^2 - 56*a^5*b^14*c^6*d^5 + 208*a^5*b^14*c^8*d^3 - 56*a^6*
b^13*c^5*d^6 - 392*a^6*b^13*c^7*d^4 - 152*a^6*b^13*c^9*d^2 + 112*a^7*b^12*c
^4*d^7 + 280*a^7*b^12*c^6*d^5 - 32*a^7*b^12*c^8*d^3 - 80*a^8*b^11*c^3*d^8 +
 112*a^8*b^11*c^5*d^6 + 448*a^8*b^11*c^7*d^4 + 368*a^8*b^11*c^9*d^2 + 28*a^
9*b^10*c^2*d^9 - 368*a^9*b^10*c^4*d^7 - 560*a^9*b^10*c^6*d^5 - 352*a^9*b^10
*c^8*d^3 + 292*a^10*b^9*c^3*d^8 + 112*a^10*b^9*c^5*d^6 - 112*a^10*b^9*c^7*d
^4 - 292*a^10*b^9*c^9*d^2 - 108*a^11*b^8*c^2*d^9 + 352*a^11*b^8*c^4*d^7 + 5
60*a^11*b^8*c^6*d^5 + 368*a^11*b^8*c^8*d^3 - 368*a^12*b^7*c^3*d^8 - 448*a^1
2*b^7*c^5*d^6 - 112*a^12*b^7*c^7*d^4 + 80*a^12*b^7*c^9*d^2 + 152*a^13*b^6*c
^2*d^9 + 32*a^13*b^6*c^4*d^7 - 280*a^13*b^6*c^6*d^5 - 112*a^13*b^6*c^8*d^3
+ 152*a^14*b^5*c^3*d^8 + 392*a^14*b^5*c^5*d^6 + 56*a^14*b^5*c^7*d^4 - 88*a^
15*b^4*c^2*d^9 - 208*a^15*b^4*c^4*d^7 + 56*a^15*b^4*c^6*d^5 + 32*a^16*b^3*c
^3*d^8 - 112*a^16*b^3*c^5*d^6 + 12*a^17*b^2*c^2*d^9 + 80*a^17*b^2*c^4*d^7 -
 4*a*b^18*c^10*d - 4*a^18*b*c*d^10))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6
+ 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^
6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 -
 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5
- 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c
^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 6

0*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5) - (8*tan(e/2 + (f*x)/2)*(56*a^3*b^16*c^11 - 12*a^19*c*d^10 - 12*a*b^18*c^11 - 104*a^5*b^14*c^11 + 96*a^7*b^12*c^11 - 44*a^9*b^10*c^11 + 8*a^11*b^8*c^11 + 8*a^19*c^3*d^8 + 16*a*b^18*c^9*d^2 + 96*a^2*b^17*c^10*d - 448*a^4*b^15*c^10*d + 832*a^6*b^13*c^10*d - 768*a^8*b^11*c^10*d + 16*a^9*b^10*c*d^10 + 352*a^10*b^9*c^10*d - 76*a^11*b^8*c*d^10 - 64*a^12*b^7*c^10*d + 144*a^13*b^6*c*d^10 - 136*a^15*b^4*c*d^10 + 64*a^17*b^2*c*d^10 + 96*a^18*b*c^2*d^9 - 64*a^18*b*c^4*d^7 - 128*a^2*b^17*c^8*d^3 + 448*a^3*b^16*c^7*d^4 - 412*a^3*b^16*c^9*d^2 - 896*a^4*b^15*c^6*d^5 + 1280*a^4*b^15*c^8*d^3 + 1120*a^5*b^14*c^5*d^6 - 2968*a^5*b^14*c^7*d^4 + 1712*a^5*b^14*c^9*d^2 - 896*a^6*b^13*c^4*d^7 + 4928*a^6*b^13*c^6*d^5 - 4288*a^6*b^13*c^8*d^3 + 448*a^7*b^12*c^3*d^8 - 5656*a^7*b^12*c^5*d^6 + 7952*a^7*b^12*c^7*d^4 - 3048*a^7*b^12*c^9*d^2 - 128*a^8*b^11*c^2*d^9 + 4352*a^8*b^11*c^4*d^7 - 11200*a^8*b^11*c^6*d^5 + 6912*a^8*b^11*c^8*d^3 - 2140*a^9*b^10*c^3*d^8 + 11648*a^9*b^10*c^5*d^6 - 11088*a^9*b^10*c^7*d^4 + 2752*a^9*b^10*c^9*d^2 + 608*a^10*b^9*c^2*d^9 - 8512*a^10*b^9*c^4*d^7 + 13440*a^10*b^9*c^6*d^5 - 5888*a^10*b^9*c^8*d^3 + 4088*a^11*b^8*c^3*d^8 - 12432*a^11*b^8*c^5*d^6 + 8512*a^11*b^8*c^7*d^4 - 1244*a^11*b^8*c^9*d^2 - 1152*a^12*b^7*c^2*d^9 + 8448*a^12*b^7*c^4*d^7 - 8960*a^12*b^7*c^6*d^5 + 2560*a^12*b^7*c^8*d^3 - 3912*a^13*b^6*c^3*d^8 + 7168*a^13*b^6*c^5*d^6 - 3416*a^13*b^6*c^7*d^4 + 224*a^13*b^6*c^9*d^2 + 1088*a^14*b^5*c^2*d^9 - 4352*a^14*b^5*c^4*d^7 + 3136*a^14*b^5*c^6*d^5 - 448*a^14*b^5*c^8*d^3 + 1888*a^15*b^4*c^3*d^8 - 2072*a^15*b^4*c^5*d^6 + 560*a^15*b^4*c^7*d^4 - 512*a^16*b^3*c^2*d^9 + 1024*a^16*b^3*c^4*d^7 - 448*a^16*b^3*c^6*d^5 - 380*a^17*b^2*c^3*d^8 + 224*a^17*b^2*c^5*d^6))/(a^14*d^6 + b^14*c^6 - 4*a^2*b^12*c^6 + 6*a^4*b^10*c^6 - 4*a^6*b^8*c^6 + a^8*b^6*c^6 + a^6*b^8*d^6 - 4*a^8*b^6*d^6 + 6*a^10*b^4*d^6 - 4*a^12*b^2*d^6 + 24*a^3*b^11*c^5*d - 6*a^5*b^9*c*d^5 - 36*a^5*b^9*c^5*d + 24*a^7*b^7*c*d^5 + 24*a^7*b^7*c^5*d - 36*a^9*b^5*c*d^5 - 6*a^9*b^5*c^5*d + 24*a^11*b^3*c*d^5 + 15*a^2*b^12*c^4*d^2 - 20*a^3*b^11*c^3*d^3 + 15*a^4*b^10*c^2*d^4 - 60*a^4*b^10*c^4*d^2 + 80*a^5*b^9*c^3*d^3 - 60*a^6*b^8*c^2*d^4 + 90*a^6*b^8*c^4*d^2 - 120*a^7*b^7*c^3*d^3 + 90*a^8*b^6*c^2*d^4 - 60*a^8*b^6*c^4*d^2 + 80*a^9*b^5*c^3*d^3 - 60*a^10*b^4*c^2*d^4 + 15*a^10*b^4*c^4*d^2 - 20*a^11*b^3*c^3*d^3 + 15*a^12*b^2*c^2*d^4 - 6*a*b^13*c^5*d - 6*a^13*b*c*d^5))*(-(a + b)^5*(a - b)^5)^(1/2)*(6*a^4*d^2 + b^4*c^2 + 2*b^4*d^2 + 2*a^2*b^2*c^2 - 5*a^2*b^2*d^2 - 6*a^3*b*c*d))/(2*(a^13*d^3 + b^13*c^3 - 5*a^2*b^11*c^3 + 10*a^4*b^9*c^3 - 10*a^6*b^7*c^3 + 5*a^8*b^5*c^3 - a^10*b^3*c^3 - a^3*b^10*d^3 + 5*a^5*b^8*d^3 - 10*a^7*b^6*d^3 + 10*a^9*b^4*d^3 - 5*a^11*b^2*d^3 + 3*a^2*b^11*c*d^2 + 15*a^3*b^10*c^2*d - 15*a^4*b^9*c*d^2 - 30*a^5*b^8*c^2*d + 30*a^6*b^7*c*d^2 + 30*a^7*b^6*c^2*d - 30*a^8*b^5*c*d^2 - 15*a^9*b^4*c^2*d + 15*a^10*b^3*c*d^2 + 3*a^11*b^2*c^2*d - 3*a*b^12*c^2*d - 3*a^12*b*c*d^2)))*(6*a^4*d^2 + b^4*c^2 + 2*b^4*d^2 + 2*a^2*b^2*c^2 - 5*a^2*b^2*d^2 - 6*a^3*b*c*d))/(2*(a^13*d^3 + b^13*c^3 - 5*a^2*b^11*c^3 + 10*a^4*b^9*c^3 - 10*a^6*b^7*c^3 + 5*a^8*b^5*c^3 - a^10*b^3*c^3 - a^3*b^10*d^3 + 5*a^5*b^8*d^3 - 10*a^7*b^6*d^3 + 10*a^9*b^4*d^3 - 5*a^

```
11*b^2*d^3 + 3*a^2*b^11*c*d^2 + 15*a^3*b^10*c^2*d - 15*a^4*b^9*c*d^2 - 30*a
^5*b^8*c^2*d + 30*a^6*b^7*c*d^2 + 30*a^7*b^6*c^2*d - 30*a^8*b^5*c*d^2 - 15*
a^9*b^4*c^2*d + 15*a^10*b^3*c*d^2 + 3*a^11*b^2*c^2*d - 3*a*b^12*c^2*d - 3*a
^12*b*c*d^2)))*(6*a^4*d^2 + b^4*c^2 + 2*b^4*d^2 + 2*a^2*b^2*c^2 - 5*a^2*b^2
*d^2 - 6*a^3*b*c*d))/(2*(a^13*d^3 + b^13*c^3 - 5*a^2*b^11*c^3 + 10*a^4*b^9*
c^3 - 10*a^6*b^7*c^3 + 5*a^8*b^5*c^3 - a^10*b^3*c^3 - a^3*b^10*d^3 + 5*a^5*
b^8*d^3 - 10*a^7*b^6*d^3 + 10*a^9*b^4*d^3 - 5*a^11*b^2*d^3 + 3*a^2*b^11*c*d
^2 + 15*a^3*b^10*c^2*d - 15*a^4*b^9*c*d^2 - 30*a^5*b^8*c^2*d + 30*a^6*b^7*c
*d^2 + 30*a^7*b^6*c^2*d - 30*a^8*b^5*c*d^2 - 15*a^9*b^4*c^2*d + 15*a^10*b^3
*c*d^2 + 3*a^11*b^2*c^2*d - 3*a*b^12*c^2*d - 3*a^12*b*c*d^2))))*(-(a + b)^5
*(a - b)^5)^(1/2)*(6*a^4*d^2 + b^4*c^2 + 2*b^4*d^2 + 2*a^2*b^2*c^2 - 5*a^2*
b^2*d^2 - 6*a^3*b*c*d)*1i)/(f*(a^13*d^3 + b^13*c^3 - 5*a^2*b^11*c^3 + 10*a^
4*b^9*c^3 - 10*a^6*b^7*c^3 + 5*a^8*b^5*c^3 - a^10*b^3*c^3 - a^3*b^10*d^3 +
5*a^5*b^8*d^3 - 10*a^7*b^6*d^3 + 10*a^9*b^4*d^3 - 5*a^11*b^2*d^3 + 3*a^2*b^
11*c*d^2 + 15*a^3*b^10*c^2*d - 15*a^4*b^9*c*d^2 - 30*a^5*b^8*c^2*d + 30*a^6
*b^7*c*d^2 + 30*a^7*b^6*c^2*d - 30*a^8*b^5*c*d^2 - 15*a^9*b^4*c^2*d + 15*a^
10*b^3*c*d^2 + 3*a^11*b^2*c^2*d - 3*a*b^12*c^2*d - 3*a^12*b*c*d^2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**3/(c+d*sin(f*x+e)),x)
[Out] Timed out
```

3.721 $\int \frac{1}{(a+b\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=454

$$\frac{3b^2\left(-2a^2d+abc+b^2d\right)\cos(e+fx)}{2f\left(a^2-b^2\right)^2(bc-ad)^2(a+b\sin(e+fx))(c+d\sin(e+fx))}+\frac{b^2\cos(e+fx)}{2f\left(a^2-b^2\right)(bc-ad)(a+b\sin(e+fx))^2(c+d\sin \cdots}$$

```
[Out] -b^2*(8*a^3*b*c*d-2*a*b^3*c*d-12*a^4*d^2-a^2*b^2*(2*c^2-15*d^2)-b^4*(c^2+6*
d^2))*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/(-a*
d+b*c)^4/f-2*d^3*(-a*c*d+4*b*c^2-3*b*d^2)*arctan((d+c*tan(1/2*f*x+1/2*e))/(
c^2-d^2)^(1/2))/(-a*d+b*c)^4/(c^2-d^2)^(3/2)/f-1/2*d*(2*a^4*d^3+a^2*b^2*d*(
7*c^2-11*d^2)-2*b^4*d*(2*c^2-3*d^2)-3*a*b^3*c*(c^2-d^2))*cos(f*x+e)/(a^2-b^
2)^2/(-a*d+b*c)^3/(c^2-d^2)/f/(c+d*sin(f*x+e))+1/2*b^2*cos(f*x+e)/(a^2-b^2)
/(-a*d+b*c)/f/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))+3/2*b^2*(-2*a^2*d+a*b*c+b
^2*d)*cos(f*x+e)/(a^2-b^2)^2/(-a*d+b*c)^2/f/(a+b*sin(f*x+e))/(c+d*sin(f*x+e
))
```

Rubi [A] time = 2.44, antiderivative size = 454, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2802, 3055, 3001, 2660, 618, 204}

$$-\frac{b^2\left(-a^2b^2\left(2c^2-15d^2\right)+8a^3bcd-12a^4d^2-2ab^3cd-b^4\left(c^2+6d^2\right)\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{5/2}(bc-ad)^4}-\frac{d\left(a^2b^2d\left(7c^2-11d \cdots\right.\right.}{2f\left(a \cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] -((b^2*(8*a^3*b*c*d - 2*a*b^3*c*d - 12*a^4*d^2 - a^2*b^2*(2*c^2 - 15*d^2) -
 b^4*(c^2 + 6*d^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/((a^2
 - b^2)^(5/2)*(b*c - a*d)^4*f)) - (2*d^3*(4*b*c^2 - a*c*d - 3*b*d^2)*ArcTan
[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^4*(c^2 - d^2)^(3/2
)*f) - (d*(2*a^4*d^3 + a^2*b^2*d*(7*c^2 - 11*d^2) - 2*b^4*d*(2*c^2 - 3*d^2)
 - 3*a*b^3*c*(c^2 - d^2))*Cos[e + f*x])/(2*(a^2 - b^2)^2*(b*c - a*d)^3*(c^2
 - d^2)*f*(c + d*Sin[e + f*x])) + (b^2*Cos[e + f*x])/(2*(a^2 - b^2)*(b*c -
a*d)*f*(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])) + (3*b^2*(a*b*c - 2*a^2
*d + b^2*d)*Cos[e + f*x])/(2*(a^2 - b^2)^2*(b*c - a*d)^2*f*(a + b*Sin[e + f
*x])*(c + d*Sin[e + f*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
```

```
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx = \frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2(c+d\sin(e+fx))} - \frac{\int}{}$$

$$= \frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2(c+d\sin(e+fx))} + \frac{}{2}$$

$$= -\frac{d\left(2a^4d^3+a^2b^2d\left(7c^2-11d^2\right)-2b^4d\left(2c^2-3d^2\right)-3ab^3c\left(c^2-\right.\right.}{2\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f(c+d\sin(e+f}$$

$$= -\frac{d\left(2a^4d^3+a^2b^2d\left(7c^2-11d^2\right)-2b^4d\left(2c^2-3d^2\right)-3ab^3c\left(c^2-\right.\right.}{2\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f(c+d\sin(e+f}$$

$$= -\frac{d\left(2a^4d^3+a^2b^2d\left(7c^2-11d^2\right)-2b^4d\left(2c^2-3d^2\right)-3ab^3c\left(c^2-\right.\right.}{2\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f(c+d\sin(e+f}$$

$$= -\frac{d\left(2a^4d^3+a^2b^2d\left(7c^2-11d^2\right)-2b^4d\left(2c^2-3d^2\right)-3ab^3c\left(c^2-\right.\right.}{2\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f(c+d\sin(e+f}$$

$$= -\frac{b^2\left(8a^3bcd-2ab^3cd-12a^4d^2-a^2b^2\left(2c^2-15d^2\right)-b^4\left(c^2+6d\right.\right.}{\left(a^2-b^2\right)^{5/2}(bc-ad)^4f}$$

Mathematica [A] time = 5.47, size = 346, normalized size = 0.76

$$\frac{\frac{b^3\left(7a^2d-3abc-4b^2d\right)\cos(e+fx)}{(a-b)^2(a+b)^2(ad-bc)^3(a+b\sin(e+fx))}+\frac{2b^2\left(12a^4d^2-8a^3bcd+a^2b^2\left(2c^2-15d^2\right)+2ab^3cd+b^4\left(c^2+6d^2\right)\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}(bc-ad)^4}+\frac{b^3\cos}{(a-b)(a+b)(bc-ad}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] ((2*b^2*(-8*a^3*b*c*d + 2*a*b^3*c*d + 12*a^4*d^2 + a^2*b^2*(2*c^2 - 15*d^2)
 + b^4*(c^2 + 6*d^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/((a
^2 - b^2)^(5/2)*(b*c - a*d)^4) + (4*d^3*(-4*b*c^2 + a*c*d + 3*b*d^2)*ArcTan
[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^4*(c^2 - d^2)^(3/2
)) + (b^3*Cos[e + f*x])/((a - b)*(a + b)*(b*c - a*d)^2*(a + b*Sin[e + f*x])
^2) + (b^3*(-3*a*b*c + 7*a^2*d - 4*b^2*d)*Cos[e + f*x])/((a - b)^2*(a + b)^
2*(-(b*c) + a*d)^3*(a + b*Sin[e + f*x])) - (2*d^4*Cos[e + f*x])/((c - d)*(c
 + d)*(b*c - a*d)^3*(c + d*Sin[e + f*x])))/(2*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 1.38, size = 1111, normalized size = 2.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] ((2*a^2*b^4*c^2 + b^6*c^2 - 8*a^3*b^3*c*d + 2*a*b^5*c*d + 12*a^4*b^2*d^2 -
15*a^2*b^4*d^2 + 6*b^6*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arct
an((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^4*b^4*c^4 - 2*a^2*b^6
*c^4 + b^8*c^4 - 4*a^5*b^3*c^3*d + 8*a^3*b^5*c^3*d - 4*a*b^7*c^3*d + 6*a^6*
b^2*c^2*d^2 - 12*a^4*b^4*c^2*d^2 + 6*a^2*b^6*c^2*d^2 - 4*a^7*b*c*d^3 + 8*a^
5*b^3*c*d^3 - 4*a^3*b^5*c*d^3 + a^8*d^4 - 2*a^6*b^2*d^4 + a^4*b^4*d^4)*sqrt
(a^2 - b^2)) - 2*(4*b*c^2*d^3 - a*c*d^4 - 3*b*d^5)*(pi*floor(1/2*(f*x + e)/
pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((
```

```
b^4*c^6 - 4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - b^4*c^4*d^2 - 4*a^3*b*c^3*d^3
 + 4*a*b^3*c^3*d^3 + a^4*c^2*d^4 - 6*a^2*b^2*c^2*d^4 + 4*a^3*b*c*d^5 - a^4*
d^6)*sqrt(c^2 - d^2)) - 2*(d^5*tan(1/2*f*x + 1/2*e) + c*d^4)/((b^3*c^6 - 3*
a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - b^3*c^4*d^2 - a^3*c^3*d^3 + 3*a*b^2*c^3*d^3
 - 3*a^2*b*c^2*d^4 + a^3*c*d^5)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x
 + 1/2*e) + c)) + (5*a^3*b^5*c*tan(1/2*f*x + 1/2*e)^3 - 2*a*b^7*c*tan(1/2*f
*x + 1/2*e)^3 - 9*a^4*b^4*d*tan(1/2*f*x + 1/2*e)^3 + 6*a^2*b^6*d*tan(1/2*f*
x + 1/2*e)^3 + 4*a^4*b^4*c*tan(1/2*f*x + 1/2*e)^2 + 7*a^2*b^6*c*tan(1/2*f*x
 + 1/2*e)^2 - 2*b^8*c*tan(1/2*f*x + 1/2*e)^2 - 8*a^5*b^3*d*tan(1/2*f*x + 1/
2*e)^2 - 11*a^3*b^5*d*tan(1/2*f*x + 1/2*e)^2 + 10*a*b^7*d*tan(1/2*f*x + 1/2
*e)^2 + 11*a^3*b^5*c*tan(1/2*f*x + 1/2*e) - 2*a*b^7*c*tan(1/2*f*x + 1/2*e)
- 23*a^4*b^4*d*tan(1/2*f*x + 1/2*e) + 14*a^2*b^6*d*tan(1/2*f*x + 1/2*e) + 4
*a^4*b^4*c - a^2*b^6*c - 8*a^5*b^3*d + 5*a^3*b^5*d)/((a^6*b^3*c^3 - 2*a^4*b
^5*c^3 + a^2*b^7*c^3 - 3*a^7*b^2*c^2*d + 6*a^5*b^4*c^2*d - 3*a^3*b^6*c^2*d
+ 3*a^8*b*c*d^2 - 6*a^6*b^3*c*d^2 + 3*a^4*b^5*c*d^2 - a^9*d^3 + 2*a^7*b^2*d
^3 - a^5*b^4*d^3)*(a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)
^2))/f
```

maple [B] time = 0.44, size = 3241, normalized size = 7.14

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x)
```

```
[Out] 8/f*b^7/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^
4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)^3*c*d+16/f*b^7/(a*d-b*c)^4/(tan(1/2*f*x
+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2
*e)*c*d+2/f*b^4/(a*d-b*c)^4/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*
(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*c^2*a^2-15/f*b^4/(a*d-b*c)^4/
(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)
/(a^2-b^2)^(1/2))*a^2*d^2+9/f*b^4/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan
(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*tan(1/2*f*x+1/2*e)^3*d^2+5/f
*b^6/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2
*a^2*b^2+b^4)*a*tan(1/2*f*x+1/2*e)^3*c^2-6/f*b^6/(a*d-b*c)^4/(tan(1/2*f*x+1
/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*tan(1/2*f*x+1/2
*e)^3*d^2-2/f*b^8/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*
b+a)^2/(a^4-2*a^2*b^2+b^4)/a*tan(1/2*f*x+1/2*e)^3*c^2+8/f*b^3/(a*d-b*c)^4/(
tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^4*
tan(1/2*f*x+1/2*e)^2*d^2+4/f*b^5/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(
1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+1/2*e)^2*c^2+11/f
*b^5/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2
*a^2*b^2+b^4)*a^2*tan(1/2*f*x+1/2*e)^2*d^2-2/f*b^9/(a*d-b*c)^4/(tan(1/2*f*x
+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)/a^2*tan(1/2*f*x
+1/2*e)^2*c^2+23/f*b^4/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/
```

2*e)*b+a)^2*a^3/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*d^2+11/f*b^6/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c^2-14/f*b^6/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*d^2-2/f*b^8/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/a/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c^2-12/f*b^4/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*c*d+6/f*b^6/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*c*d+12/f*b^2/(a*d-b*c)^4/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^4*d^2-2/f*d^5/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*tan(1/2*f*x+1/2*e)*b-2/f*d^4/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*c*b+2/f*d^4/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*a*c-8/f*d^3/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b*c^2-1/f*b^7/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*c^2-18/f*b^6/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a*tan(1/2*f*x+1/2*e)^2*c*d+12/f*b^8/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)/a*tan(1/2*f*x+1/2*e)^2*c*d+2/f*d^5/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*a+6/f*d^5/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*b-5/f*b^5/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*d^2+6/f*b^6/(a*d-b*c)^4/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*d^2+7/f*b^7/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)^2*c^2-10/f*b^7/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)^2*d^2+8/f*b^3/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^4*d^2+4/f*b^5/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*c^2*a^2+1/f*b^6/(a*d-b*c)^4/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*c^2-34/f*b^5/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2*a^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*f*x+1/2*e)*c*d-8/f*b^3/(a*d-b*c)^4/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a^3*c*d+2/f*b^5/(a*d-b*c)^4/(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*a*c*d+2/f*d^6/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/c/(c^2-d^2)*tan(1/2*f*x+1/2*e)*a-14/f*b^5/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^2*tan(1/2*f*x+1/2*e)^3*c*d-12/f*b^4/(a*d-b*c)^4/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)^2/(a^4-2*a^2*b^2+b^4)*a^3*tan(1/2*f*x+1/2*e)^2*c*d

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 43.67, size = 137273, normalized size = 302.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x))^2),x)
```

```
[Out] ((2*a^6*d^4 + b^6*c^4 - 4*a^2*b^4*c^4 + 2*a^2*b^4*d^4 - 4*a^4*b^2*d^4 - b^6
*c^2*d^2 - 8*a^3*b^3*c*d^3 + 8*a^3*b^3*c^3*d + 4*a^2*b^4*c^2*d^2 + 5*a*b^5*
c*d^3 - 5*a*b^5*c^3*d)/((a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)
*(a^4*c^2 - a^4*d^2 + b^4*c^2 - b^4*d^2 - 2*a^2*b^2*c^2 + 2*a^2*b^2*d^2)) +
 (tan(e/2 + (f*x)/2)*(2*a^7*d^5 + 2*b^7*c^5 - 11*a^2*b^5*c^5 + 2*a^3*b^4*d^
5 - 4*a^5*b^2*d^5 - 2*b^7*c^3*d^2 + 12*a*b^6*c^2*d^3 + 18*a^2*b^5*c*d^4 + 1
5*a^3*b^4*c^4*d - 32*a^4*b^3*c*d^4 + a^2*b^5*c^3*d^2 - 15*a^3*b^4*c^2*d^3 +
 16*a^4*b^3*c^3*d^2 - 12*a*b^6*c^4*d + 8*a^6*b*c*d^4))/(a*c*(a^3*d^3 - b^3*
c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(a^4*c^2 - a^4*d^2 + b^4*c^2 - b^4*d^2
 - 2*a^2*b^2*c^2 + 2*a^2*b^2*d^2)) + (tan(e/2 + (f*x)/2)^5*(2*a^7*d^5 + 2*b
^7*c^5 - 5*a^2*b^5*c^5 + 2*a^3*b^4*d^5 - 4*a^5*b^2*d^5 - 2*b^7*c^3*d^2 + 6*
a*b^6*c^2*d^3 + 9*a^3*b^4*c^4*d + 5*a^2*b^5*c^3*d^2 - 9*a^3*b^4*c^2*d^3 - 6
*a*b^6*c^4*d))/(a*c*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(a^
4*c^2 - a^4*d^2 + b^4*c^2 - b^4*d^2 - 2*a^2*b^2*c^2 + 2*a^2*b^2*d^2)) + (2*
tan(e/2 + (f*x)/2)^2*(b^8*c^5 + 4*a^7*b*d^5 + 2*a^8*c*d^4 - 3*a^2*b^6*c^5 -
 4*a^4*b^4*c^5 + 4*a^3*b^5*d^5 - 8*a^5*b^3*d^5 - b^8*c^3*d^2 + 3*a*b^7*c^2*
d^3 + 18*a^2*b^6*c*d^4 - 8*a^3*b^5*c^4*d - 29*a^4*b^4*c*d^4 + 8*a^5*b^3*c^4
*d - 11*a^2*b^6*c^3*d^2 + 8*a^3*b^5*c^2*d^3 + 27*a^4*b^4*c^3*d^2 - 8*a^5*b^
3*c^2*d^3 - 3*a*b^7*c^4*d))/(a^2*c*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2)*(a^4*c^2 - a^4*d^2 + b^4*c^2 - b^4*d^2 - 2*a^2*b^2*c^2 + 2*a^2*
b^2*d^2)) - (tan(e/2 + (f*x)/2)^4*(7*a^2*b^6*c^5 - 8*a^7*b*d^5 - 2*a^8*c*d^
4 - 2*b^8*c^5 + 4*a^4*b^4*c^5 - 8*a^3*b^5*d^5 + 16*a^5*b^3*d^5 + 2*b^8*c^3*
d^2 - 6*a*b^7*c^2*d^3 - 12*a^2*b^6*c*d^4 - a^3*b^5*c^4*d + 16*a^4*b^4*c*d^4
 - 8*a^5*b^3*c^4*d + 4*a^6*b^2*c*d^4 + 5*a^2*b^6*c^3*d^2 + a^3*b^5*c^2*d^3
- 22*a^4*b^4*c^3*d^2 + 8*a^5*b^3*c^2*d^3 + 6*a*b^7*c^4*d))/(a^2*c*(a^3*d^3
```

- b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(a^4*c^2 - a^4*d^2 + b^4*c^2 - b^4*d^2 - 2*a^2*b^2*c^2 + 2*a^2*b^2*d^2)) + (2*tan(e/2 + (f*x)/2)^3*(a^2*d + 2*b^2*d + 2*a*b*c)*(2*a^6*d^4 + b^6*c^4 - 4*a^2*b^4*c^4 + 2*a^2*b^4*d^4 - 4*a^4*b^2*d^4 - b^6*c^2*d^2 - 8*a^3*b^3*c*d^3 + 8*a^3*b^3*c^3*d + 4*a^2*b^4*c^2*d^2 + 5*a*b^5*c*d^3 - 5*a*b^5*c^3*d))/(a^2*c*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(a^4*c^2 - a^4*d^2 + b^4*c^2 - b^4*d^2 - 2*a^2*b^2*c^2 + 2*a^2*b^2*d^2)))/(f*(tan(e/2 + (f*x)/2)^2*(3*a^2*c + 4*b^2*c + 8*a*b*d) + tan(e/2 + (f*x)/2)^4*(3*a^2*c + 4*b^2*c + 8*a*b*d) + tan(e/2 + (f*x)/2)^3*(4*a^2*d + 8*b^2*d + 8*a*b*c) + a^2*c + tan(e/2 + (f*x)/2)*(2*a^2*d + 4*a*b*c) + tan(e/2 + (f*x)/2)^5*(2*a^2*d + 4*a*b*c) + a^2*c*tan(e/2 + (f*x)/2)^6)) - (d^3*atan(((d^3*(-(c + d)^3*(c - d)^3)^(1/2)*((8*(60*a*b^15*c^7*d^7 - 36*a*b^15*c^5*d^9 - 13*a*b^15*c^9*d^5 - 10*a*b^15*c^11*d^3 - 4*a^3*b^13*c^13*d + 36*a^5*b^11*c*d^13 - 4*a^5*b^11*c^13*d - 144*a^7*b^9*c*d^13 + 216*a^9*b^7*c*d^13 - 144*a^11*b^5*c*d^13 + 36*a^13*b^3*c*d^13 + 4*a^15*b*c^3*d^11 + 72*a^2*b^14*c^4*d^10 - 108*a^2*b^14*c^6*d^8 + 19*a^2*b^14*c^8*d^6 + 14*a^2*b^14*c^10*d^4 - a^2*b^14*c^12*d^2 + 120*a^3*b^13*c^5*d^9 - 305*a^3*b^13*c^7*d^7 + 190*a^3*b^13*c^9*d^5 + 19*a^3*b^13*c^11*d^3 - 72*a^4*b^12*c^2*d^12 - 168*a^4*b^12*c^4*d^10 + 699*a^4*b^12*c^6*d^8 - 602*a^4*b^12*c^8*d^6 + 99*a^4*b^12*c^10*d^4 + 20*a^4*b^12*c^12*d^2 - 36*a^5*b^11*c^3*d^11 - 535*a^5*b^11*c^5*d^9 + 1354*a^5*b^11*c^7*d^7 - 895*a^5*b^11*c^9*d^5 + 40*a^5*b^11*c^11*d^3 + 276*a^6*b^10*c^2*d^12 + 233*a^6*b^10*c^4*d^10 - 2046*a^6*b^10*c^6*d^8 + 2161*a^6*b^10*c^8*d^6 - 552*a^6*b^10*c^10*d^4 + 44*a^6*b^10*c^12*d^2 + 61*a^7*b^9*c^3*d^11 + 1386*a^7*b^9*c^5*d^9 - 2979*a^7*b^9*c^7*d^7 + 1860*a^7*b^9*c^9*d^5 - 220*a^7*b^9*c^11*d^3 - 375*a^8*b^8*c^2*d^12 - 270*a^8*b^8*c^4*d^10 + 2885*a^8*b^8*c^6*d^8 - 3012*a^8*b^8*c^8*d^6 + 628*a^8*b^8*c^10*d^4 - 88*a^9*b^7*c^3*d^11 - 1544*a^9*b^7*c^5*d^9 + 2648*a^9*b^7*c^7*d^7 - 1088*a^9*b^7*c^9*d^5 + 216*a^10*b^6*c^2*d^12 + 100*a^10*b^6*c^4*d^10 - 1336*a^10*b^6*c^6*d^8 + 1056*a^10*b^6*c^8*d^6 + 180*a^11*b^5*c^3*d^11 + 248*a^11*b^5*c^5*d^9 - 400*a^11*b^5*c^7*d^7 - 60*a^12*b^4*c^2*d^12 + 248*a^12*b^4*c^4*d^10 - 148*a^12*b^4*c^6*d^8 - 184*a^13*b^3*c^3*d^11 + 172*a^13*b^3*c^5*d^9 + 24*a^14*b^2*c^2*d^12 - 44*a^14*b^2*c^4*d^10 - a*b^15*c^13*d))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10

- 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146
*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*
b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^
3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d
^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 +
1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a
^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b
^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^
6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8
- 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^
15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12) - (8*tan(e/2 + (f*x)/2)
*(4*a^16*c^3*d^11 - 4*a^3*b^13*c^14 - 4*a^5*b^11*c^14 - a*b^15*c^14 + 144*a
*b^15*c^4*d^10 - 348*a*b^15*c^6*d^8 + 214*a*b^15*c^8*d^6 + 7*a*b^15*c^10*d^
4 - 8*a*b^15*c^12*d^2 - a^2*b^14*c^13*d - 144*a^4*b^12*c*d^13 + 20*a^4*b^12
*c^13*d + 684*a^6*b^10*c*d^13 + 44*a^6*b^10*c^13*d - 1314*a^8*b^8*c*d^13 +
1224*a^10*b^6*c*d^13 - 504*a^12*b^4*c*d^13 + 36*a^14*b^2*c*d^13 + 24*a^15*b
*c^2*d^12 - 44*a^15*b*c^4*d^10 - 432*a^2*b^14*c^3*d^11 + 1140*a^2*b^14*c^5*
d^9 - 818*a^2*b^14*c^7*d^7 + 55*a^2*b^14*c^9*d^5 + 16*a^2*b^14*c^11*d^3 + 4
32*a^3*b^13*c^2*d^12 - 2016*a^3*b^13*c^4*d^10 + 2938*a^3*b^13*c^6*d^8 - 148
5*a^3*b^13*c^8*d^6 + 152*a^3*b^13*c^10*d^4 + 27*a^3*b^13*c^12*d^2 + 2688*a^
4*b^12*c^3*d^11 - 6574*a^4*b^12*c^5*d^9 + 5107*a^4*b^12*c^7*d^7 - 1056*a^4*
b^12*c^9*d^5 + 59*a^4*b^12*c^11*d^3 - 2148*a^5*b^11*c^2*d^12 + 8378*a^5*b^1
1*c^4*d^10 - 10619*a^5*b^11*c^6*d^8 + 5064*a^5*b^11*c^8*d^6 - 975*a^5*b^11*
c^10*d^4 + 48*a^5*b^11*c^12*d^2 - 7294*a^6*b^10*c^3*d^11 + 16053*a^6*b^10*c
^5*d^9 - 12464*a^6*b^10*c^7*d^7 + 3649*a^6*b^10*c^9*d^5 - 640*a^6*b^10*c^11
*d^3 + 4470*a^7*b^9*c^2*d^12 - 15815*a^7*b^9*c^4*d^10 + 18608*a^7*b^9*c^6*d
^8 - 8939*a^7*b^9*c^8*d^6 + 2300*a^7*b^9*c^10*d^4 - 220*a^7*b^9*c^12*d^2 +
10105*a^8*b^8*c^3*d^11 - 19912*a^8*b^8*c^5*d^9 + 14693*a^8*b^8*c^7*d^7 - 45
24*a^8*b^8*c^9*d^5 + 628*a^8*b^8*c^11*d^3 - 4632*a^9*b^7*c^2*d^12 + 14976*a
^9*b^7*c^4*d^10 - 15576*a^9*b^7*c^6*d^8 + 6104*a^9*b^7*c^8*d^6 - 1088*a^9*b
^7*c^10*d^4 - 7104*a^10*b^6*c^3*d^11 + 11320*a^10*b^6*c^5*d^9 - 6184*a^10*b
^6*c^7*d^7 + 1120*a^10*b^6*c^9*d^5 + 2232*a^11*b^5*c^2*d^12 - 5932*a^11*b^5
*c^4*d^10 + 4344*a^11*b^5*c^6*d^8 - 688*a^11*b^5*c^8*d^6 + 1892*a^12*b^4*c^
3*d^11 - 1920*a^12*b^4*c^5*d^9 + 368*a^12*b^4*c^7*d^7 - 252*a^13*b^3*c^2*d^
12 + 624*a^13*b^3*c^4*d^10 - 292*a^13*b^3*c^6*d^8 - 192*a^14*b^2*c^3*d^11 +
172*a^14*b^2*c^5*d^9))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^
13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 +
6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*
c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*
b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 +
9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*
d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2
*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c
^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^
6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 5

88*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6
*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^1
1*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4
*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^
3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 10
45*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9
*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^
10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*
d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 -
 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a
^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*
b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c
^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^1
0 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*
a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12)
 + (d^3*(-(c + d)^3*(c - d)^3)^(1/2)*((8*(2*a^2*b^17*c^16 - 6*a^6*b^13*c^16
 + 4*a^8*b^11*c^16 + 4*a^19*c^3*d^13 - 4*a^19*c^5*d^11 + 12*a*b^18*c^9*d^7
- 28*a*b^18*c^11*d^5 + 16*a*b^18*c^13*d^3 - 10*a^3*b^16*c^15*d - 24*a^5*b^1
4*c^15*d + 78*a^7*b^12*c^15*d + 12*a^9*b^10*c*d^15 - 44*a^9*b^10*c^15*d - 5
4*a^11*b^8*c*d^15 + 96*a^13*b^6*c*d^15 - 78*a^15*b^4*c*d^15 + 24*a^17*b^2*c
*d^15 + 12*a^18*b*c^2*d^14 - 56*a^18*b*c^4*d^12 + 44*a^18*b*c^6*d^10 - 96*a
^2*b^17*c^8*d^8 + 234*a^2*b^17*c^10*d^6 - 146*a^2*b^17*c^12*d^4 + 6*a^2*b^1
7*c^14*d^2 + 336*a^3*b^16*c^7*d^9 - 918*a^3*b^16*c^9*d^7 + 726*a^3*b^16*c^1
1*d^5 - 134*a^3*b^16*c^13*d^3 - 672*a^4*b^15*c^6*d^10 + 2280*a^4*b^15*c^8*d
^8 - 2520*a^4*b^15*c^10*d^6 + 952*a^4*b^15*c^12*d^4 - 40*a^4*b^15*c^14*d^2
+ 840*a^5*b^14*c^5*d^11 - 4032*a^5*b^14*c^7*d^9 + 6360*a^5*b^14*c^9*d^7 - 3
768*a^5*b^14*c^11*d^5 + 624*a^5*b^14*c^13*d^3 - 672*a^6*b^13*c^4*d^12 + 529
2*a^6*b^13*c^6*d^10 - 11772*a^6*b^13*c^8*d^8 + 10050*a^6*b^13*c^10*d^6 - 31
74*a^6*b^13*c^12*d^4 + 282*a^6*b^13*c^14*d^2 + 336*a^7*b^12*c^3*d^13 - 5124
*a^7*b^12*c^5*d^11 + 16212*a^7*b^12*c^7*d^9 - 19602*a^7*b^12*c^9*d^7 + 9670
*a^7*b^12*c^11*d^5 - 1570*a^7*b^12*c^13*d^3 - 96*a^8*b^11*c^2*d^14 + 3528*a
^8*b^11*c^4*d^12 - 16872*a^8*b^11*c^6*d^10 + 28848*a^8*b^11*c^8*d^8 - 20340
*a^8*b^11*c^10*d^6 + 5396*a^8*b^11*c^12*d^4 - 468*a^8*b^11*c^14*d^2 - 1620*
a^9*b^10*c^3*d^13 + 13320*a^9*b^10*c^5*d^11 - 32304*a^9*b^10*c^7*d^9 + 3156
0*a^9*b^10*c^9*d^7 - 12648*a^9*b^10*c^11*d^5 + 1724*a^9*b^10*c^13*d^3 + 442
*a^10*b^9*c^2*d^14 - 7810*a^10*b^9*c^4*d^12 + 27546*a^10*b^9*c^6*d^10 - 373
38*a^10*b^9*c^8*d^8 + 21288*a^10*b^9*c^10*d^6 - 4348*a^10*b^9*c^12*d^4 + 22
0*a^10*b^9*c^14*d^2 + 3206*a^11*b^8*c^3*d^13 - 17850*a^11*b^8*c^5*d^11 + 34
018*a^11*b^8*c^7*d^9 - 26556*a^11*b^8*c^9*d^7 + 7896*a^11*b^8*c^11*d^5 - 66
0*a^11*b^8*c^13*d^3 - 816*a^12*b^7*c^2*d^14 + 8696*a^12*b^7*c^4*d^12 - 2369
6*a^12*b^7*c^6*d^10 + 25056*a^12*b^7*c^8*d^8 - 10560*a^12*b^7*c^10*d^6 + 13
20*a^12*b^7*c^12*d^4 - 3064*a^13*b^6*c^3*d^13 + 12400*a^13*b^6*c^5*d^11 - 1
8048*a^13*b^6*c^7*d^9 + 10464*a^13*b^6*c^9*d^7 - 1848*a^13*b^6*c^11*d^5 + 7
02*a^14*b^5*c^2*d^14 - 4770*a^14*b^5*c^4*d^12 + 9858*a^14*b^5*c^6*d^10 - 76
38*a^14*b^5*c^8*d^8 + 1848*a^14*b^5*c^10*d^6 + 1314*a^15*b^4*c^3*d^13 - 395

4*a^15*b^4*c^5*d^11 + 4038*a^15*b^4*c^7*d^9 - 1320*a^15*b^4*c^9*d^7 - 244*a
^16*b^3*c^2*d^14 + 1084*a^16*b^3*c^4*d^12 - 1500*a^16*b^3*c^6*d^10 + 660*a^
16*b^3*c^8*d^8 - 176*a^17*b^2*c^3*d^13 + 372*a^17*b^2*c^5*d^11 - 220*a^17*b
^2*c^7*d^9))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4
*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4
*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2
*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d
 + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c
^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a
^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d
^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 15
6*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4
*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12
*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d
^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 -
 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548
*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8
*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*
c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^
9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 57
6*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a
^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b
^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^1
0*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6
 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 39
6*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^1
4*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^
4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12) + (8*tan(e
/2 + (f*x)/2)*(4*a*b^18*c^16 - 12*a^5*b^14*c^16 + 8*a^7*b^12*c^16 + 8*a^19*
c^2*d^14 - 8*a^19*c^4*d^12 + 12*a*b^18*c^10*d^6 - 28*a*b^18*c^12*d^4 + 12*a
*b^18*c^14*d^2 - 20*a^2*b^17*c^15*d - 48*a^4*b^15*c^15*d + 156*a^6*b^13*c^1
5*d - 88*a^8*b^11*c^15*d + 12*a^10*b^9*c*d^15 - 48*a^12*b^7*c*d^15 + 84*a^1
4*b^5*c*d^15 - 72*a^16*b^3*c*d^15 - 112*a^18*b*c^3*d^13 + 88*a^18*b*c^5*d^1
1 - 84*a^2*b^17*c^9*d^7 + 212*a^2*b^17*c^11*d^5 - 108*a^2*b^17*c^13*d^3 + 2
40*a^3*b^16*c^8*d^8 - 744*a^3*b^16*c^10*d^6 + 584*a^3*b^16*c^12*d^4 - 80*a^
3*b^16*c^14*d^2 - 336*a^4*b^15*c^7*d^9 + 1632*a^4*b^15*c^9*d^7 - 2176*a^4*b
^15*c^11*d^5 + 928*a^4*b^15*c^13*d^3 + 168*a^5*b^14*c^6*d^10 - 2472*a^5*b^1
4*c^8*d^8 + 5460*a^5*b^14*c^10*d^6 - 3708*a^5*b^14*c^12*d^4 + 564*a^5*b^14*
c^14*d^2 + 168*a^6*b^13*c^5*d^11 + 2520*a^6*b^13*c^7*d^9 - 9204*a^6*b^13*c^
9*d^7 + 9180*a^6*b^13*c^11*d^5 - 2820*a^6*b^13*c^13*d^3 - 336*a^7*b^12*c^4*
d^12 - 1344*a^7*b^12*c^6*d^10 + 10416*a^7*b^12*c^8*d^8 - 15960*a^7*b^12*c^1
0*d^6 + 8152*a^7*b^12*c^12*d^4 - 936*a^7*b^12*c^14*d^2 + 240*a^8*b^11*c^3*d
^13 - 336*a^8*b^11*c^5*d^11 - 7488*a^8*b^11*c^7*d^9 + 19800*a^8*b^11*c^9*d^
7 - 15416*a^8*b^11*c^11*d^5 + 3288*a^8*b^11*c^13*d^3 - 84*a^9*b^10*c^2*d^14
 + 1188*a^9*b^10*c^4*d^12 + 2292*a^9*b^10*c^6*d^10 - 16596*a^9*b^10*c^8*d^8

+ 20136*a^9*b^10*c^10*d^6 - 7376*a^9*b^10*c^12*d^4 + 440*a^9*b^10*c^14*d^2
- 908*a^10*b^9*c^3*d^13 + 1740*a^10*b^9*c^5*d^11 + 7556*a^10*b^9*c^7*d^9 -
18048*a^10*b^9*c^9*d^7 + 10936*a^10*b^9*c^11*d^5 - 1288*a^10*b^9*c^13*d^3
+ 328*a^11*b^8*c^2*d^14 - 2808*a^11*b^8*c^4*d^12 + 1088*a^11*b^8*c^6*d^10 +
9600*a^11*b^8*c^8*d^8 - 10584*a^11*b^8*c^10*d^6 + 2376*a^11*b^8*c^12*d^4 +
1792*a^12*b^7*c^3*d^13 - 4720*a^12*b^7*c^5*d^11 - 144*a^12*b^7*c^7*d^9 + 5
856*a^12*b^7*c^9*d^7 - 2736*a^12*b^7*c^11*d^5 - 596*a^13*b^6*c^2*d^14 + 398
0*a^13*b^6*c^4*d^12 - 4908*a^13*b^6*c^6*d^10 - 156*a^13*b^6*c^8*d^8 + 1680*
a^13*b^6*c^10*d^6 - 1932*a^14*b^5*c^3*d^13 + 4812*a^14*b^5*c^5*d^11 - 3012*
a^14*b^5*c^7*d^9 + 48*a^14*b^5*c^9*d^7 + 552*a^15*b^4*c^2*d^14 - 2616*a^15*
b^4*c^4*d^12 + 3096*a^15*b^4*c^6*d^10 - 1032*a^15*b^4*c^8*d^8 + 920*a^16*b^
3*c^3*d^13 - 1752*a^16*b^3*c^5*d^11 + 904*a^16*b^3*c^7*d^9 - 208*a^17*b^2*c
^2*d^14 + 600*a^17*b^2*c^4*d^12 - 392*a^17*b^2*c^6*d^10 + 24*a^18*b*c*d^15)
)/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c
^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a
^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*
d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b
^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*
a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d
^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2
*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*
c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d
^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 +
852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a
^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^
11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c
^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^
10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 1
46*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^
9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*
c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9
*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9
+ 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444
*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12
*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*
c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d
^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*
a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12) + (d^3*((8*(16*a^4*b^
18*c^18 - 4*a^2*b^20*c^18 - 24*a^6*b^16*c^18 + 16*a^8*b^14*c^18 - 4*a^10*b^
12*c^18 + 4*a^22*c^2*d^16 - 8*a^22*c^4*d^14 + 4*a^22*c^6*d^12 + 4*a*b^21*c^
13*d^5 - 8*a*b^21*c^15*d^3 + 24*a^3*b^19*c^17*d - 136*a^5*b^17*c^17*d + 224
*a^7*b^15*c^17*d - 156*a^9*b^13*c^17*d + 40*a^11*b^11*c^17*d - 4*a^13*b^9*c
*d^17 + 16*a^15*b^7*c*d^17 - 24*a^17*b^5*c*d^17 + 16*a^19*b^3*c*d^17 - 32*a
^21*b*c^3*d^15 + 76*a^21*b*c^5*d^13 - 40*a^21*b*c^7*d^11 - 40*a^2*b^20*c^12
*d^6 + 76*a^2*b^20*c^14*d^4 - 32*a^2*b^20*c^16*d^2 + 176*a^3*b^19*c^11*d^7

- 328*a^3*b^19*c^13*d^5 + 128*a^3*b^19*c^15*d^3 - 440*a^4*b^18*c^10*d^8 + 8
64*a^4*b^18*c^12*d^6 - 392*a^4*b^18*c^14*d^4 - 48*a^4*b^18*c^16*d^2 + 660*a
^5*b^17*c^9*d^9 - 1584*a^5*b^17*c^11*d^7 + 1052*a^5*b^17*c^13*d^5 + 8*a^5*b
^17*c^15*d^3 - 528*a^6*b^16*c^8*d^10 + 2156*a^6*b^16*c^10*d^8 - 2264*a^6*b^
16*c^12*d^6 + 148*a^6*b^16*c^14*d^4 + 512*a^6*b^16*c^16*d^2 - 2112*a^7*b^15
*c^9*d^9 + 3520*a^7*b^15*c^11*d^7 - 480*a^7*b^15*c^13*d^5 - 1152*a^7*b^15*c
^15*d^3 + 528*a^8*b^14*c^6*d^12 + 1056*a^8*b^14*c^8*d^10 - 3696*a^8*b^14*c^
10*d^8 + 1216*a^8*b^14*c^12*d^6 + 1808*a^8*b^14*c^14*d^4 - 928*a^8*b^14*c^1
6*d^2 - 660*a^9*b^13*c^5*d^13 + 792*a^9*b^13*c^7*d^11 + 2244*a^9*b^13*c^9*d
^9 - 2288*a^9*b^13*c^11*d^7 - 2180*a^9*b^13*c^13*d^5 + 2248*a^9*b^13*c^15*d
^3 + 440*a^10*b^12*c^4*d^14 - 2332*a^10*b^12*c^6*d^12 + 176*a^10*b^12*c^8*d
^10 + 2684*a^10*b^12*c^10*d^8 + 1896*a^10*b^12*c^12*d^6 - 3532*a^10*b^12*c^
14*d^4 + 672*a^10*b^12*c^16*d^2 - 176*a^11*b^11*c^3*d^15 + 2552*a^11*b^11*c
^5*d^13 - 2464*a^11*b^11*c^7*d^11 - 1496*a^11*b^11*c^9*d^9 - 528*a^11*b^11*
c^11*d^7 + 3736*a^11*b^11*c^13*d^5 - 1664*a^11*b^11*c^15*d^3 + 40*a^12*b^10
*c^2*d^16 - 1664*a^12*b^10*c^4*d^14 + 3736*a^12*b^10*c^6*d^12 - 528*a^12*b^
10*c^8*d^10 - 1496*a^12*b^10*c^10*d^8 - 2464*a^12*b^10*c^12*d^6 + 2552*a^12
*b^10*c^14*d^4 - 176*a^12*b^10*c^16*d^2 + 672*a^13*b^9*c^3*d^15 - 3532*a^13
*b^9*c^5*d^13 + 1896*a^13*b^9*c^7*d^11 + 2684*a^13*b^9*c^9*d^9 + 176*a^13*b
^9*c^11*d^7 - 2332*a^13*b^9*c^13*d^5 + 440*a^13*b^9*c^15*d^3 - 156*a^14*b^8
*c^2*d^16 + 2248*a^14*b^8*c^4*d^14 - 2180*a^14*b^8*c^6*d^12 - 2288*a^14*b^8
*c^8*d^10 + 2244*a^14*b^8*c^10*d^8 + 792*a^14*b^8*c^12*d^6 - 660*a^14*b^8*c
^14*d^4 - 928*a^15*b^7*c^3*d^15 + 1808*a^15*b^7*c^5*d^13 + 1216*a^15*b^7*c^
7*d^11 - 3696*a^15*b^7*c^9*d^9 + 1056*a^15*b^7*c^11*d^7 + 528*a^15*b^7*c^13
*d^5 + 224*a^16*b^6*c^2*d^16 - 1152*a^16*b^6*c^4*d^14 - 480*a^16*b^6*c^6*d^
12 + 3520*a^16*b^6*c^8*d^10 - 2112*a^16*b^6*c^10*d^8 + 512*a^17*b^5*c^3*d^1
5 + 148*a^17*b^5*c^5*d^13 - 2264*a^17*b^5*c^7*d^11 + 2156*a^17*b^5*c^9*d^9
- 528*a^17*b^5*c^11*d^7 - 136*a^18*b^4*c^2*d^16 + 8*a^18*b^4*c^4*d^14 + 105
2*a^18*b^4*c^6*d^12 - 1584*a^18*b^4*c^8*d^10 + 660*a^18*b^4*c^10*d^8 - 48*a
^19*b^3*c^3*d^15 - 392*a^19*b^3*c^5*d^13 + 864*a^19*b^3*c^7*d^11 - 440*a^19
*b^3*c^9*d^9 + 24*a^20*b^2*c^2*d^16 + 128*a^20*b^2*c^4*d^14 - 328*a^20*b^2*
c^6*d^12 + 176*a^20*b^2*c^8*d^10 + 4*a*b^21*c^17*d - 4*a^21*b*c*d^17))/(a^1
7*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 -
a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^
2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 +
9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^
12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b
^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 -
9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*
c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d
^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 1
56*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^
5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^1
1*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^1
1*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7

- 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12) - (8*tan(e/2 + (f*x)/2)*(12*a*b^21*c^18 - 12*a^22*c*d^17 - 56*a^3*b^19*c^18 + 104*a^5*b^17*c^18 - 96*a^7*b^15*c^18 + 44*a^9*b^13*c^18 - 8*a^11*b^11*c^18 + 32*a^22*c^3*d^15 - 28*a^22*c^5*d^13 + 8*a^22*c^7*d^11 - 16*a*b^21*c^12*d^6 + 44*a*b^21*c^14*d^4 - 40*a*b^21*c^16*d^2 - 132*a^2*b^20*c^17*d + 616*a^4*b^18*c^17*d - 1144*a^6*b^16*c^17*d + 1056*a^8*b^14*c^17*d - 484*a^10*b^12*c^17*d + 16*a^12*b^10*c*d^17 + 88*a^12*b^10*c^17*d - 76*a^14*b^8*c*d^17 + 144*a^16*b^6*c*d^17 - 136*a^18*b^4*c*d^17 + 64*a^20*b^2*c*d^17 + 132*a^21*b*c^2*d^16 - 352*a^21*b*c^4*d^14 + 308*a^21*b*c^6*d^12 - 88*a^21*b*c^8*d^10 + 176*a^2*b^20*c^11*d^7 - 484*a^2*b^20*c^13*d^5 + 440*a^2*b^20*c^15*d^3 - 880*a^3*b^19*c^10*d^8 + 2496*a^3*b^19*c^12*d^6 - 2408*a^3*b^19*c^14*d^4 + 848*a^3*b^19*c^16*d^2 + 2640*a^4*b^18*c^9*d^9 - 8096*a^4*b^18*c^11*d^7 + 8888*a^4*b^18*c^13*d^5 - 4048*a^4*b^18*c^15*d^3 - 5280*a^5*b^17*c^8*d^10 + 18700*a^5*b^17*c^10*d^8 - 24784*a^5*b^17*c^12*d^6 + 14692*a^5*b^17*c^14*d^4 - 3432*a^5*b^17*c^16*d^2 + 7392*a^6*b^16*c^7*d^11 - 32868*a^6*b^16*c^9*d^9 + 54384*a^6*b^16*c^11*d^7 - 40876*a^6*b^16*c^13*d^5 + 13112*a^6*b^16*c^15*d^3 - 7392*a^7*b^15*c^6*d^12 + 45408*a^7*b^15*c^8*d^10 - 95040*a^7*b^15*c^10*d^8 + 89280*a^7*b^15*c^12*d^6 - 38208*a^7*b^15*c^14*d^4 + 6048*a^7*b^15*c^16*d^2 + 5280*a^8*b^14*c^5*d^13 - 49632*a^8*b^14*c^7*d^11 + 133056*a^8*b^14*c^9*d^9 - 156992*a^8*b^14*c^11*d^7 + 88000*a^8*b^14*c^13*d^5 - 20768*a^8*b^14*c^15*d^3 - 2640*a^9*b^13*c^4*d^14 + 42372*a^9*b^13*c^6*d^12 - 150216*a^9*b^13*c^8*d^10 + 225676*a^9*b^13*c^10*d^8 - 162336*a^9*b^13*c^12*d^6 + 52532*a^9*b^13*c^14*d^4 - 5432*a^9*b^13*c^16*d^2 + 880*a^10*b^12*c^3*d^15 - 27500*a^10*b^12*c^5*d^13 + 137368*a^10*b^12*c^7*d^11 - 266244*a^10*b^12*c^9*d^9 + 242528*a^10*b^12*c^11*d^7 - 104060*a^10*b^12*c^13*d^5 + 17512*a^10*b^12*c^15*d^3 - 176*a^11*b^11*c^2*d^16 + 13024*a^11*b^11*c^4*d^14 - 101288*a^11*b^11*c^6*d^12 + 257136*a^11*b^11*c^8*d^10 - 296824*a^11*b^11*c^10*d^8 + 165760*a^11*b^11*c^12*d^6 - 40072*a^11*b^11*c^14*d^4 + 2448*a^11*b^11*c^16*d^2 - 4224*a^12*b^10*c^3*d^15 + 59000*a^12*b^10*c^5*d^13 - 202544*a^12*b^10*c^7*d^11 + 299816*a^12*b^10*c^9*d^9 - 214368*a^12*b^10*c^11*d^7 + 69784*a^12*b^10*c^13*d^5 - 7568*a^12*b^10*c^15*d^3 + 836*a^13*b^9*c^2*d^16 - 26048*a^13*b^9*c^4*d^14 + 129580*a^13*b^9*c^6*d^12 - 249832*a^13*b^9*c^8*d^10 + 226116*a^13*b^9*c^10*d^8 - 96272*a^13*b^9*c^12*d^6 + 16060*a^13*b^9*c^14*d^4 - 440*a^13*b^9*c^16*d^2 + 8128*a^14*b^8*c^3*d^15 - 66628*a^14*b^8*c^5*d^13 + 170424*a^14*b^8*c^7*d^11 -

195404*a^14*b^8*c^9*d^9 + 107184*a^14*b^8*c^11*d^7 - 24948*a^14*b^8*c^13*d
^5 + 1320*a^14*b^8*c^15*d^3 - 1584*a^15*b^7*c^2*d^16 + 26752*a^15*b^7*c^4*d
^14 - 94160*a^15*b^7*c^6*d^12 + 138688*a^15*b^7*c^8*d^10 - 96624*a^15*b^7*c
^10*d^8 + 29568*a^15*b^7*c^12*d^6 - 2640*a^15*b^7*c^14*d^4 - 7872*a^16*b^6*
c^3*d^15 + 41712*a^16*b^6*c^5*d^13 - 80448*a^16*b^6*c^7*d^11 + 70224*a^16*b
^6*c^9*d^9 - 27456*a^16*b^6*c^11*d^7 + 3696*a^16*b^6*c^13*d^5 + 1496*a^17*b
^5*c^2*d^16 - 14608*a^17*b^5*c^4*d^14 + 37532*a^17*b^5*c^6*d^12 - 40920*a^1
7*b^5*c^8*d^10 + 20196*a^17*b^5*c^10*d^8 - 3696*a^17*b^5*c^12*d^6 + 3888*a^
18*b^4*c^3*d^15 - 13748*a^18*b^4*c^5*d^13 + 19016*a^18*b^4*c^7*d^11 - 11660
*a^18*b^4*c^9*d^9 + 2640*a^18*b^4*c^11*d^7 - 704*a^19*b^3*c^2*d^16 + 3872*a
^19*b^3*c^4*d^14 - 6952*a^19*b^3*c^6*d^12 + 5104*a^19*b^3*c^8*d^10 - 1320*a
^19*b^3*c^10*d^8 - 832*a^20*b^2*c^3*d^15 + 1912*a^20*b^2*c^5*d^13 - 1584*a^
20*b^2*c^7*d^11 + 440*a^20*b^2*c^9*d^9))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^1
5*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 -
4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^1
7*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c
^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*
a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12
 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15
*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7
 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 39
6*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5
*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*
c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*
d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 -
 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576
*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8
*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^
2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5
 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1
548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^
11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b
^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^
5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11
 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156
*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b
^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d -
 9*a^16*b*c*d^12))*(-(c + d)^3*(c - d)^3)^(1/2)*(3*b*d^2 - 4*b*c^2 + a*c*d)
)/(a^4*d^10 - b^4*c^10 - 3*a^4*c^2*d^8 + 3*a^4*c^4*d^6 - a^4*c^6*d^4 + b^4*
c^4*d^6 - 3*b^4*c^6*d^4 + 3*b^4*c^8*d^2 - 4*a*b^3*c^3*d^7 + 12*a*b^3*c^5*d^
5 - 12*a*b^3*c^7*d^3 + 12*a^3*b*c^3*d^7 - 12*a^3*b*c^5*d^5 + 4*a^3*b*c^7*d^
3 + 6*a^2*b^2*c^2*d^8 - 18*a^2*b^2*c^4*d^6 + 18*a^2*b^2*c^6*d^4 - 6*a^2*b^2
*c^8*d^2 + 4*a*b^3*c^9*d - 4*a^3*b*c*d^9))*(3*b*d^2 - 4*b*c^2 + a*c*d))/(a^
4*d^10 - b^4*c^10 - 3*a^4*c^2*d^8 + 3*a^4*c^4*d^6 - a^4*c^6*d^4 + b^4*c^4*d
^6 - 3*b^4*c^6*d^4 + 3*b^4*c^8*d^2 - 4*a*b^3*c^3*d^7 + 12*a*b^3*c^5*d^5 - 1

2*a*b^3*c^7*d^3 + 12*a^3*b*c^3*d^7 - 12*a^3*b*c^5*d^5 + 4*a^3*b*c^7*d^3 + 6*a^2*b^2*c^2*d^8 - 18*a^2*b^2*c^4*d^6 + 18*a^2*b^2*c^6*d^4 - 6*a^2*b^2*c^8*d^2 + 4*a*b^3*c^9*d - 4*a^3*b*c*d^9))*(3*b*d^2 - 4*b*c^2 + a*c*d)*1i)/(a^4*d^10 - b^4*c^10 - 3*a^4*c^2*d^8 + 3*a^4*c^4*d^6 - a^4*c^6*d^4 + b^4*c^4*d^6 - 3*b^4*c^6*d^4 + 3*b^4*c^8*d^2 - 4*a*b^3*c^3*d^7 + 12*a*b^3*c^5*d^5 - 12*a*b^3*c^7*d^3 + 12*a^3*b*c^3*d^7 - 12*a^3*b*c^5*d^5 + 4*a^3*b*c^7*d^3 + 6*a^2*b^2*c^2*d^8 - 18*a^2*b^2*c^4*d^6 + 18*a^2*b^2*c^6*d^4 - 6*a^2*b^2*c^8*d^2 + 4*a*b^3*c^9*d - 4*a^3*b*c*d^9) - (d^3*(-(c + d)^3*(c - d)^3)^(1/2)*((8*tan(e/2 + (f*x)/2)*(4*a^16*c^3*d^11 - 4*a^3*b^13*c^14 - 4*a^5*b^11*c^14 - a*b^15*c^14 + 144*a*b^15*c^4*d^10 - 348*a*b^15*c^6*d^8 + 214*a*b^15*c^8*d^6 + 7*a*b^15*c^10*d^4 - 8*a*b^15*c^12*d^2 - a^2*b^14*c^13*d - 144*a^4*b^12*c*d^13 + 20*a^4*b^12*c^13*d + 684*a^6*b^10*c*d^13 + 44*a^6*b^10*c^13*d - 1314*a^8*b^8*c*d^13 + 1224*a^10*b^6*c*d^13 - 504*a^12*b^4*c*d^13 + 36*a^14*b^2*c*d^13 + 24*a^15*b*c^2*d^12 - 44*a^15*b*c^4*d^10 - 432*a^2*b^14*c^3*d^11 + 1140*a^2*b^14*c^5*d^9 - 818*a^2*b^14*c^7*d^7 + 55*a^2*b^14*c^9*d^5 + 16*a^2*b^14*c^11*d^3 + 432*a^3*b^13*c^2*d^12 - 2016*a^3*b^13*c^4*d^10 + 2938*a^3*b^13*c^6*d^8 - 1485*a^3*b^13*c^8*d^6 + 152*a^3*b^13*c^10*d^4 + 27*a^3*b^13*c^12*d^2 + 2688*a^4*b^12*c^3*d^11 - 6574*a^4*b^12*c^5*d^9 + 5107*a^4*b^12*c^7*d^7 - 1056*a^4*b^12*c^9*d^5 + 59*a^4*b^12*c^11*d^3 - 2148*a^5*b^11*c^2*d^12 + 8378*a^5*b^11*c^4*d^10 - 10619*a^5*b^11*c^6*d^8 + 5064*a^5*b^11*c^8*d^6 - 975*a^5*b^11*c^10*d^4 + 48*a^5*b^11*c^12*d^2 - 7294*a^6*b^10*c^3*d^11 + 16053*a^6*b^10*c^5*d^9 - 12464*a^6*b^10*c^7*d^7 + 3649*a^6*b^10*c^9*d^5 - 640*a^6*b^10*c^11*d^3 + 4470*a^7*b^9*c^2*d^12 - 15815*a^7*b^9*c^4*d^10 + 18608*a^7*b^9*c^6*d^8 - 8939*a^7*b^9*c^8*d^6 + 2300*a^7*b^9*c^10*d^4 - 220*a^7*b^9*c^12*d^2 + 10105*a^8*b^8*c^3*d^11 - 19912*a^8*b^8*c^5*d^9 + 14693*a^8*b^8*c^7*d^7 - 4524*a^8*b^8*c^9*d^5 + 628*a^8*b^8*c^11*d^3 - 4632*a^9*b^7*c^2*d^12 + 14976*a^9*b^7*c^4*d^10 - 15576*a^9*b^7*c^6*d^8 + 6104*a^9*b^7*c^8*d^6 - 1088*a^9*b^7*c^10*d^4 - 7104*a^10*b^6*c^3*d^11 + 11320*a^10*b^6*c^5*d^9 - 6184*a^10*b^6*c^7*d^7 + 1120*a^10*b^6*c^9*d^5 + 2232*a^11*b^5*c^2*d^12 - 5932*a^11*b^5*c^4*d^10 + 4344*a^11*b^5*c^6*d^8 - 688*a^11*b^5*c^8*d^6 + 1892*a^12*b^4*c^3*d^11 - 1920*a^12*b^4*c^5*d^9 + 368*a^12*b^4*c^7*d^7 - 252*a^13*b^3*c^2*d^12 + 624*a^13*b^3*c^4*d^10 - 292*a^13*b^3*c^6*d^8 - 192*a^14*b^2*c^3*d^11 + 172*a^14*b^2*c^5*d^9))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11

- 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12) - (8*(60*a*b^15*c^7*d^7 - 36*a*b^15*c^5*d^9 - 13*a*b^15*c^9*d^5 - 10*a*b^15*c^11*d^3 - 4*a^3*b^13*c^13*d + 36*a^5*b^11*c*d^13 - 4*a^5*b^11*c^13*d - 144*a^7*b^9*c*d^13 + 216*a^9*b^7*c*d^13 - 144*a^11*b^5*c*d^13 + 36*a^13*b^3*c*d^13 + 4*a^15*b*c^3*d^11 + 72*a^2*b^14*c^4*d^10 - 108*a^2*b^14*c^6*d^8 + 19*a^2*b^14*c^8*d^6 + 14*a^2*b^14*c^10*d^4 - a^2*b^14*c^12*d^2 + 120*a^3*b^13*c^5*d^9 - 305*a^3*b^13*c^7*d^7 + 190*a^3*b^13*c^9*d^5 + 19*a^3*b^13*c^11*d^3 - 72*a^4*b^12*c^2*d^12 - 168*a^4*b^12*c^4*d^10 + 699*a^4*b^12*c^6*d^8 - 602*a^4*b^12*c^8*d^6 + 99*a^4*b^12*c^10*d^4 + 20*a^4*b^12*c^12*d^2 - 36*a^5*b^11*c^3*d^11 - 535*a^5*b^11*c^5*d^9 + 1354*a^5*b^11*c^7*d^7 - 895*a^5*b^11*c^9*d^5 + 40*a^5*b^11*c^11*d^3 + 276*a^6*b^10*c^2*d^12 + 233*a^6*b^10*c^4*d^10 - 2046*a^6*b^10*c^6*d^8 + 2161*a^6*b^10*c^8*d^6 - 552*a^6*b^10*c^10*d^4 + 44*a^6*b^10*c^12*d^2 + 61*a^7*b^9*c^3*d^11 + 1386*a^7*b^9*c^5*d^9 - 2979*a^7*b^9*c^7*d^7 + 1860*a^7*b^9*c^9*d^5 - 220*a^7*b^9*c^11*d^3 - 375*a^8*b^8*c^2*d^12 - 270*a^8*b^8*c^4*d^10 + 2885*a^8*b^8*c^6*d^8 - 3012*a^8*b^8*c^8*d^6 + 628*a^8*b^8*c^10*d^4 - 88*a^9*b^7*c^3*d^11 - 1544*a^9*b^7*c^5*d^9 + 2648*a^9*b^7*c^7*d^7 - 1088*a^9*b^7*c^9*d^5 + 216*a^10*b^6*c^2*d^12 + 100*a^10*b^6*c^4*d^10 - 1336*a^10*b^6*c^6*d^8 + 1056*a^10*b^6*c^8*d^6 + 180*a^11*b^5*c^3*d^11 + 248*a^11*b^5*c^5*d^9 - 400*a^11*b^5*c^7*d^7 - 60*a^12*b^4*c^2*d^12 + 248*a^12*b^4*c^4*d^10 - 148*a^12*b^4*c^6*d^8 - 184*a^13*b^3*c^3*d^11 + 172*a^13*b^3*c^5*d^9 + 24*a^14*b^2*c^2*d^12 - 44*a^14*b^2*c^4*d^10 - a*b^15*c^13*d))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 57

6*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12) + (d^3*(-(c + d)^3*(c - d)^3)^(1/2)*((8*(2*a^2*b^17*c^16 - 6*a^6*b^13*c^16 + 4*a^8*b^11*c^16 + 4*a^19*c^3*d^13 - 4*a^19*c^5*d^11 + 12*a*b^18*c^9*d^7 - 28*a*b^18*c^11*d^5 + 16*a*b^18*c^13*d^3 - 10*a^3*b^16*c^15*d - 24*a^5*b^14*c^15*d + 78*a^7*b^12*c^15*d + 12*a^9*b^10*c*d^15 - 44*a^9*b^10*c^15*d - 54*a^11*b^8*c*d^15 + 96*a^13*b^6*c*d^15 - 78*a^15*b^4*c*d^15 + 24*a^17*b^2*c*d^15 + 12*a^18*b*c^2*d^14 - 56*a^18*b*c^4*d^12 + 44*a^18*b*c^6*d^10 - 96*a^2*b^17*c^8*d^8 + 234*a^2*b^17*c^10*d^6 - 146*a^2*b^17*c^12*d^4 + 6*a^2*b^17*c^14*d^2 + 336*a^3*b^16*c^7*d^9 - 918*a^3*b^16*c^9*d^7 + 726*a^3*b^16*c^11*d^5 - 134*a^3*b^16*c^13*d^3 - 672*a^4*b^15*c^6*d^10 + 2280*a^4*b^15*c^8*d^8 - 2520*a^4*b^15*c^10*d^6 + 952*a^4*b^15*c^12*d^4 - 40*a^4*b^15*c^14*d^2 + 840*a^5*b^14*c^5*d^11 - 4032*a^5*b^14*c^7*d^9 + 6360*a^5*b^14*c^9*d^7 - 3768*a^5*b^14*c^11*d^5 + 624*a^5*b^14*c^13*d^3 - 672*a^6*b^13*c^4*d^12 + 5292*a^6*b^13*c^6*d^10 - 11772*a^6*b^13*c^8*d^8 + 10050*a^6*b^13*c^10*d^6 - 3174*a^6*b^13*c^12*d^4 + 282*a^6*b^13*c^14*d^2 + 336*a^7*b^12*c^3*d^13 - 5124*a^7*b^12*c^5*d^11 + 16212*a^7*b^12*c^7*d^9 - 19602*a^7*b^12*c^9*d^7 + 9670*a^7*b^12*c^11*d^5 - 1570*a^7*b^12*c^13*d^3 - 96*a^8*b^11*c^2*d^14 + 3528*a^8*b^11*c^4*d^12 - 16872*a^8*b^11*c^6*d^10 + 28848*a^8*b^11*c^8*d^8 - 20340*a^8*b^11*c^10*d^6 + 5396*a^8*b^11*c^12*d^4 - 468*a^8*b^11*c^14*d^2 - 1620*a^9*b^10*c^3*d^13 + 13320*a^9*b^10*c^5*d^11 - 32304*a^9*b^10*c^7*d^9 + 31560*a^9*b^10*c^9*d^7 - 12648*a^9*b^10*c^11*d^5 + 1724*a^9*b^10*c^13*d^3 + 442*a^10*b^9*c^2*d^14 - 7810*a^10*b^9*c^4*d^12 + 27546*a^10*b^9*c^6*d^10 - 37338*a^10*b^9*c^8*d^8 + 21288*a^10*b^9*c^10*d^6 - 4348*a^10*b^9*c^12*d^4 + 220*a^10*b^9*c^14*d^2 + 3206*a^11*b^8*c^3*d^13 - 17850*a^11*b^8*c^5*d^11 + 34018*a^11*b^8*c^7*d^9 - 26556*a^11*b^8*c^9*d^7 + 7896*a^11*b^8*c^11*d^5 - 660*a^11*b^8*c^13*d^3 - 816*a^12*b^7*c^2*d^14 + 8696*a^12*b^7*c^4*d^12 - 23696*a^12*b^7*c^6*d^10 + 25056*a^12*b^7*c^8*d^8 - 10560*a^12*b^7*c^10*d^6 + 1320*a^12*b^7*c^12*d^4 - 3064*a^13*b^6*c^3*d^13 + 12400*a^13*b^6*c^5*d^11 - 18048*a^13*b^6*c^7*d^9 + 10464*a^13*b^6*c^9*d^7 - 1848*a^13*b^6*c^11*d^5 + 702*a^14*b^5*c^2*d^14 - 4770*a^14*b^5*c^4*d^12 + 9858*a^14*b^5*c^6*d^10 - 7638*a^14*b^5*c^8*d^8 + 1848*a^14*b^5*c^10*d^6 + 1314*a^15*b^4*c^3*d^13 - 3954*a^15*b^4*c^5*d^11 + 4038*a^15*b^4*c^7*d^9 - 1320*a^15*b^4*c^9*d^7 - 244*a^16*b^3*c^2*d^14 + 1084*a^16*b^3*c^4*d^12 - 1500*a^16*b^3*c^6*d^10 + 660*a^16*b^3*c^8*d^8 - 176*a^17*b^2*c^3*d^13 + 372*a^17*b^2*c^5*d^1

1 - 220*a^17*b^2*c^7*d^9))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4
*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^1
3 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^
17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a
^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12
 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3
*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*
a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^1
4*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7
*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9
- 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*
a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*
b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*
c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10
*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 -
 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*
a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8
*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c
^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^1
1 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 8
4*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^
12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^
4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*
d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 -
76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^
12) + (8*tan(e/2 + (f*x)/2)*(4*a*b^18*c^16 - 12*a^5*b^14*c^16 + 8*a^7*b^12*
c^16 + 8*a^19*c^2*d^14 - 8*a^19*c^4*d^12 + 12*a*b^18*c^10*d^6 - 28*a*b^18*c
^12*d^4 + 12*a*b^18*c^14*d^2 - 20*a^2*b^17*c^15*d - 48*a^4*b^15*c^15*d + 15
6*a^6*b^13*c^15*d - 88*a^8*b^11*c^15*d + 12*a^10*b^9*c*d^15 - 48*a^12*b^7*c
*d^15 + 84*a^14*b^5*c*d^15 - 72*a^16*b^3*c*d^15 - 112*a^18*b*c^3*d^13 + 88*
a^18*b*c^5*d^11 - 84*a^2*b^17*c^9*d^7 + 212*a^2*b^17*c^11*d^5 - 108*a^2*b^1
7*c^13*d^3 + 240*a^3*b^16*c^8*d^8 - 744*a^3*b^16*c^10*d^6 + 584*a^3*b^16*c^
12*d^4 - 80*a^3*b^16*c^14*d^2 - 336*a^4*b^15*c^7*d^9 + 1632*a^4*b^15*c^9*d^
7 - 2176*a^4*b^15*c^11*d^5 + 928*a^4*b^15*c^13*d^3 + 168*a^5*b^14*c^6*d^10
- 2472*a^5*b^14*c^8*d^8 + 5460*a^5*b^14*c^10*d^6 - 3708*a^5*b^14*c^12*d^4 +
 564*a^5*b^14*c^14*d^2 + 168*a^6*b^13*c^5*d^11 + 2520*a^6*b^13*c^7*d^9 - 92
04*a^6*b^13*c^9*d^7 + 9180*a^6*b^13*c^11*d^5 - 2820*a^6*b^13*c^13*d^3 - 336
*a^7*b^12*c^4*d^12 - 1344*a^7*b^12*c^6*d^10 + 10416*a^7*b^12*c^8*d^8 - 1596
0*a^7*b^12*c^10*d^6 + 8152*a^7*b^12*c^12*d^4 - 936*a^7*b^12*c^14*d^2 + 240*
a^8*b^11*c^3*d^13 - 336*a^8*b^11*c^5*d^11 - 7488*a^8*b^11*c^7*d^9 + 19800*a
^8*b^11*c^9*d^7 - 15416*a^8*b^11*c^11*d^5 + 3288*a^8*b^11*c^13*d^3 - 84*a^9
*b^10*c^2*d^14 + 1188*a^9*b^10*c^4*d^12 + 2292*a^9*b^10*c^6*d^10 - 16596*a^
9*b^10*c^8*d^8 + 20136*a^9*b^10*c^10*d^6 - 7376*a^9*b^10*c^12*d^4 + 440*a^9
*b^10*c^14*d^2 - 908*a^10*b^9*c^3*d^13 + 1740*a^10*b^9*c^5*d^11 + 7556*a^10
*b^9*c^7*d^9 - 18048*a^10*b^9*c^9*d^7 + 10936*a^10*b^9*c^11*d^5 - 1288*a^10

*b^9*c^13*d^3 + 328*a^11*b^8*c^2*d^14 - 2808*a^11*b^8*c^4*d^12 + 1088*a^11*b^8*c^6*d^10 + 9600*a^11*b^8*c^8*d^8 - 10584*a^11*b^8*c^10*d^6 + 2376*a^11*b^8*c^12*d^4 + 1792*a^12*b^7*c^3*d^13 - 4720*a^12*b^7*c^5*d^11 - 144*a^12*b^7*c^7*d^9 + 5856*a^12*b^7*c^9*d^7 - 2736*a^12*b^7*c^11*d^5 - 596*a^13*b^6*c^2*d^14 + 3980*a^13*b^6*c^4*d^12 - 4908*a^13*b^6*c^6*d^10 - 156*a^13*b^6*c^8*d^8 + 1680*a^13*b^6*c^10*d^6 - 1932*a^14*b^5*c^3*d^13 + 4812*a^14*b^5*c^5*d^11 - 3012*a^14*b^5*c^7*d^9 + 48*a^14*b^5*c^9*d^7 + 552*a^15*b^4*c^2*d^14 - 2616*a^15*b^4*c^4*d^12 + 3096*a^15*b^4*c^6*d^10 - 1032*a^15*b^4*c^8*d^8 + 920*a^16*b^3*c^3*d^13 - 1752*a^16*b^3*c^5*d^11 + 904*a^16*b^3*c^7*d^9 - 208*a^17*b^2*c^2*d^14 + 600*a^17*b^2*c^4*d^12 - 392*a^17*b^2*c^6*d^10 + 24*a^18*b*c*d^15))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12) - (d^3*((8*(16*a^4*b^18*c^18 - 4*a^2*b^20*c^18 - 24*a^6*b^16*c^18 + 16*a^8*b^14*c^18 - 4*a^10*b^12*c^18 + 4*a^22*c^2*d^16 - 8*a^22*c^4*d^14 + 4*a^22*c^6*d^12 + 4*a*b^21*c^13*d^5 - 8*a*b^21*c^15*d^3 + 24*a^3*b^19*c^17*d - 136*a^5*b^17*c^17*d + 224*a^7*b^15*c^17*d - 156*a^9*b^13*c^17*d + 40*a^11*b^11*c^17*d - 4*a^13*b^9*c*d^17 + 16*a^15*b^7*c*d^17 - 24*a^17*b^5*c*d^17 + 16*a^19*b^3*c*d^17 - 32*a^21*b*c^3*d^15 + 76*a^21*b*c^5*d^13 - 40*a^21*b*c^7*d^11 - 40*a^2*b^20*c^12*d^6 + 76*a^2*b^20*c^14*d^4 - 32*a^2*b^20*c^16*d^2 + 176*a^3*b^19*c^11*d^7 - 328*a^3*b^19*c^13*d^5 + 128*a^3*b^19*c^15*d^3 - 440*a^4*b^18*c^10*d^8 + 864*a^4*b^18*c^12*d^6 - 392*a^4*b^18*c^14*d^4 - 48*a^4*b^18*c^16*d^2 + 660*a^5*b^17*c^9*d^9 - 1584*a^5*b^17*c^11*d^7 + 1052*a^5*b^17*c^13 *d^5 + 8*a^5*b^17*c^15*d^3 - 528*a^6*b^16*c^8*d^10 + 2156*a^6*b^16*c^10*d^8 - 2264*a^6*b^16*c^12*d^6 + 148*a^6*b^16*c^14*d^4 + 512*a^6*b^16*c^16*d^2 - 2112*a^7*b^15*c^9*d^9 + 3520*a^7*b^15*c^11*d^7 - 480*a^7*b^15*c^13*d^5 - 1152*a^7*b^15*c^15*d^3 + 528*a^8*b^14*c^6*d^12 + 1056*a^8*b^14*c^8*d^10 - 3696*a^8*b^14*c^10*d^8 + 1216*a^8*b^14*c^12*d^6 + 1808*a^8*b^14*c^14*d^4 - 928*a^8*b^14*c^16*d^2 - 660*a^9*b^13*c^5*d^13 + 792*a^9*b^13*c^7*d^11 + 2244*a^9*b^13*c^9*d^9 - 2288*a^9*b^13*c^11*d^7 - 2180*a^9*b^13*c^13*d^5 + 2248*a^9*b^13*c^15*d^3 + 440*a^10*b^12*c^4*d^14 - 2332*a^10*b^12*c^6*d^12 + 176*a^10*b^12*c^8*d^10 + 2684*a^10*b^12*c^10*d^8 + 1896*a^10*b^12*c^12*d^6 - 3532*a^10*b^12*c^14*d^4 + 672*a^10*b^12*c^16*d^2 - 176*a^11*b^11*c^3*d^15 + 2552*a^11*b^11*c^5*d^13 - 2464*a^11*b^11*c^7*d^11 - 1496*a^11*b^11*c^9*d^9 - 528*a^11*b^11*c^11*d^7 + 3736*a^11*b^11*c^13*d^5 - 1664*a^11*b^11*c^15*d^3 + 40*a^12*b^10*c^2*d^16 - 1664*a^12*b^10*c^4*d^14 + 3736*a^12*b^10*c^6*d^12 - 528*a^12*b^10*c^8*d^10 - 1496*a^12*b^10*c^10*d^8 - 2464*a^12*b^10*c^12*d^6 + 2552*a^12*b^10*c^14*d^4 - 176*a^12*b^10*c^16*d^2 + 672*a^13*b^9*c^3*d^15 - 3532*a^13*b^9*c^5*d^13 + 1896*a^13*b^9*c^7*d^11 + 2684*a^13*b^9*c^9*d^9 + 176*a^13*b^9*c^11*d^7 - 2332*a^13*b^9*c^13*d^5 + 440*a^13*b^9*c^15*d^3 - 156*a^14*b^8*c^2*d^16 + 2248*a^14*b^8*c^4*d^14 - 2180*a^14*b^8*c^6*d^12 - 2288*a^14*b^8*c^8*d^10 + 2244*a^14*b^8*c^10*d^8 + 792*a^14*b^8*c^12*d^6 - 660*a^14*b^8*c^14*d^4 - 928*a^15*b^7*c^3*d^15 + 1808*a^15*b^7*c^5*d^13 + 1216*a^15*b^7*c^7*d^11 - 3696*a^15*b^7*c^9*d^9 + 1056*a^15*b^7*c^11*d^7 + 528*a^15*b^7*c^13*d^5 + 224*a^16*b^6*c^2*d^16 - 1152*a^16*b^6*c^4*d^14 - 480*a^16*b^6*c^6*d^12 + 3520*a^16*b^6*c^8*d^10 - 2112*a^16*b^6*c^10*d^8 + 512*a^17*b^5*c^3*d^15 + 148*a^17*b^5*c^5*d^13 - 2264*a^17*b^5*c^7*d^11 + 2156*a^17*b^5*c^9*d^9 - 528*a^17*b^5*c^11*d^7 - 136*a^18*b^4*c^2*d^16 + 8*a^18*b^4*c^4*d^14 + 1052*a^18*b^4*c^6*d^12 - 1584*a^18*b^4*c^8*d^10 + 660*a^18*b^4*c^10*d^8 - 48*a^19*b^3*c^3*d^15 - 392*a^19*b^3*c^5*d^13 + 864*a^19*b^3*c^7*d^11 - 440*a^19*b^3*c^9*d^9 + 24*a^20*b^2*c^2*d^16 + 128*a^20*b^2*c^4*d^14 - 328*a^20*b^2*c^6*d^12 + 176*a^20*b^2*c^8*d^10 + 4*a*b^21*c^17*d - 4*a^21*b*c*d^17))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 -

1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12) - (8*tan(e/2 + (f*x)/2)*(12*a*b^21*c^18 - 12*a^22*c*d^17 - 56*a^3*b^19*c^18 + 104*a^5*b^17*c^18 - 96*a^7*b^15*c^18 + 44*a^9*b^13*c^18 - 8*a^11*b^11*c^18 + 32*a^22*c^3*d^15 - 28*a^22*c^5*d^13 + 8*a^22*c^7*d^11 - 16*a*b^21*c^12*d^6 + 44*a*b^21*c^14*d^4 - 40*a*b^21*c^16*d^2 - 132*a^2*b^20*c^17*d + 616*a^4*b^18*c^17*d - 1144*a^6*b^16*c^17*d + 1056*a^8*b^14*c^17*d - 484*a^10*b^12*c^17*d + 16*a^12*b^10*c*d^17 + 88*a^12*b^10*c^17*d - 76*a^14*b^8*c*d^17 + 144*a^16*b^6*c*d^17 - 136*a^18*b^4*c*d^17 + 64*a^20*b^2*c*d^17 + 132*a^21*b*c^2*d^16 - 352*a^21*b*c^4*d^14 + 308*a^21*b*c^6*d^12 - 88*a^21*b*c^8*d^10 + 176*a^2*b^20*c^11*d^7 - 484*a^2*b^20*c^13*d^5 + 440*a^2*b^20*c^15*d^3 - 880*a^3*b^19*c^10*d^8 + 2496*a^3*b^19*c^12*d^6 - 2408*a^3*b^19*c^14*d^4 + 848*a^3*b^19*c^16*d^2 + 2640*a^4*b^18*c^9*d^9 - 8096*a^4*b^18*c^11*d^7 + 8888*a^4*b^18*c^13*d^5 - 4048*a^4*b^18*c^15*d^3 - 5280*a^5*b^17*c^8*d^10 + 18700*a^5*b^17*c^10*d^8 - 24784*a^5*b^17*c^12*d^6 + 14692*a^5*b^17*c^14*d^4 - 3432*a^5*b^17*c^16*d^2 + 7392*a^6*b^16*c^7*d^11 - 32868*a^6*b^16*c^9*d^9 + 54384*a^6*b^16*c^11*d^7 - 40876*a^6*b^16*c^13*d^5 + 13112*a^6*b^16*c^15*d^3 - 7392*a^7*b^15*c^6*d^12 + 45408*a^7*b^15*c^8*d^10 - 95040*a^7*b^15*c^10*d^8 + 89280*a^7*b^15*c^12*d^6 - 38208*a^7*b^15*c^14*d^4 + 6048*a^7*b^15*c^16*d^2 + 5280*a^8*b^14*c^5*d^13 - 49632*a^8*b^14*c^7*d^11 + 133056*a^8*b^14*c^9*d^9 - 156992*a^8*b^14*c^11*d^7 + 88000*a^8*b^14*c^13*d^5 - 20768*a^8*b^14*c^15*d^3 - 2640*a^9*b^13*c^4*d^14 + 42372*a^9*b^13*c^6*d^12 - 150216*a^9*b^13*c^8*d^10 + 225676*a^9*b^13*c^10*d^8 - 162336*a^9*b^13*c^12*d^6 + 52532*a^9*b^13*c^14*d^4 - 5432*a^9*b^13*c^16*d^2 + 880*a^10*b^12*c^3*d^15 - 27500*a^10*b^12*c^5*d^13 + 137368*a^10*b^12*c^7*d^11 - 266244*a^10*b^12*c^9*d^9 + 242528*a^10*b^12*c^11*d^7 - 104060*a^10*b^12*c^13*d^5 + 17512*a^10*b^12*c^15*d^3 - 176*a^11*b^11*c^2*d^16 + 13024*a^11*b^11*c^4*d^14 - 101288*a^11*b^11*c^6*d^12 + 257136*a^11*b^11*c^8*d^10 - 296824*a^11*b^11*c^10*d^8 + 165760*a^11*b^11*c^12*d^6 - 40072*a^11*b^11*c^14*d^4 + 2448*a^11*b^11*c^16*d^2 - 4224*a^12*b^10*c^3*d^15 + 59000*a^12*b^10*c^5*d^13 - 202544*a^12*b^10*c^7*d^11 + 299816*a^12*b^10*c^9*d^9 - 214368*a^12*b^10*c^11*d^7 + 69784*a^12*b^10*c^13*d^5 - 7568*a^12*b^10*c^15*d^3 + 836*a^13*b^9*c^2*d^16 - 26048*a^13*b^9*c^4*d^14 + 129580*a^13*b^9*c^6*d^12 - 249832*a^13*b^9*c^8*d^10 + 226116*a^13*b^9*c^10*d^8 - 96272*a^13*b^9*c^12*d^6 + 16060*a^13*b^9*c^14*d^4 - 440*a^13*b^9*c^16*d^2 + 8128*a^14*b^8*c^3*d^15 - 66628*a^14*b^8*c^5*d^13 + 170424*a^14*b^8*c^7*d^11 - 195404*a^14*b^8*c^9*d^9 + 107184*a^14*b^8*c^11*d^7 - 24948*a^14*b^8*c^13*d^5 + 1320*a^14*b^8*c^15*d^3 - 1584*a^15*b^7*c^2*d^16 + 26752*a^15*b^7*c^4*d^14 - 94160*a^15*b^7*c^6*d^12 + 138688*a^15*b^7*c^8*d^10 - 96

624*a^15*b^7*c^10*d^8 + 29568*a^15*b^7*c^12*d^6 - 2640*a^15*b^7*c^14*d^4 -
7872*a^16*b^6*c^3*d^15 + 41712*a^16*b^6*c^5*d^13 - 80448*a^16*b^6*c^7*d^11
+ 70224*a^16*b^6*c^9*d^9 - 27456*a^16*b^6*c^11*d^7 + 3696*a^16*b^6*c^13*d^5
+ 1496*a^17*b^5*c^2*d^16 - 14608*a^17*b^5*c^4*d^14 + 37532*a^17*b^5*c^6*d^
12 - 40920*a^17*b^5*c^8*d^10 + 20196*a^17*b^5*c^10*d^8 - 3696*a^17*b^5*c^12
*d^6 + 3888*a^18*b^4*c^3*d^15 - 13748*a^18*b^4*c^5*d^13 + 19016*a^18*b^4*c^
7*d^11 - 11660*a^18*b^4*c^9*d^9 + 2640*a^18*b^4*c^11*d^7 - 704*a^19*b^3*c^2
*d^16 + 3872*a^19*b^3*c^4*d^14 - 6952*a^19*b^3*c^6*d^12 + 5104*a^19*b^3*c^8
*d^10 - 1320*a^19*b^3*c^10*d^8 - 832*a^20*b^2*c^3*d^15 + 1912*a^20*b^2*c^5*
d^13 - 1584*a^20*b^2*c^7*d^11 + 440*a^20*b^2*c^9*d^9))/(a^17*d^13 - b^17*c^
13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a
^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*
c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5
- 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^
10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a
^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8
- 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^
3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^1
3*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11
*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 -
444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308
*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b
^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10
*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*
d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 -
146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a
^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b
^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^1
1*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^
7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 8
52*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^1
3*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4
*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d
^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a
*b^16*c^12*d - 9*a^16*b*c*d^12))*(-(c + d)^3*(c - d)^3)^(1/2)*(3*b*d^2 - 4*
b*c^2 + a*c*d))/(a^4*d^10 - b^4*c^10 - 3*a^4*c^2*d^8 + 3*a^4*c^4*d^6 - a^4*
c^6*d^4 + b^4*c^4*d^6 - 3*b^4*c^6*d^4 + 3*b^4*c^8*d^2 - 4*a*b^3*c^3*d^7 + 1
2*a*b^3*c^5*d^5 - 12*a*b^3*c^7*d^3 + 12*a^3*b*c^3*d^7 - 12*a^3*b*c^5*d^5 +
4*a^3*b*c^7*d^3 + 6*a^2*b^2*c^2*d^8 - 18*a^2*b^2*c^4*d^6 + 18*a^2*b^2*c^6*d
^4 - 6*a^2*b^2*c^8*d^2 + 4*a*b^3*c^9*d - 4*a^3*b*c*d^9))*(3*b*d^2 - 4*b*c^2
+ a*c*d))/(a^4*d^10 - b^4*c^10 - 3*a^4*c^2*d^8 + 3*a^4*c^4*d^6 - a^4*c^6*d
^4 + b^4*c^4*d^6 - 3*b^4*c^6*d^4 + 3*b^4*c^8*d^2 - 4*a*b^3*c^3*d^7 + 12*a*b
^3*c^5*d^5 - 12*a*b^3*c^7*d^3 + 12*a^3*b*c^3*d^7 - 12*a^3*b*c^5*d^5 + 4*a^3
*b*c^7*d^3 + 6*a^2*b^2*c^2*d^8 - 18*a^2*b^2*c^4*d^6 + 18*a^2*b^2*c^6*d^4 -
6*a^2*b^2*c^8*d^2 + 4*a*b^3*c^9*d - 4*a^3*b*c*d^9))*(3*b*d^2 - 4*b*c^2 + a*

c*d)*1i)/(a^4*d^10 - b^4*c^10 - 3*a^4*c^2*d^8 + 3*a^4*c^4*d^6 - a^4*c^6*d^4 + b^4*c^4*d^6 - 3*b^4*c^6*d^4 + 3*b^4*c^8*d^2 - 4*a*b^3*c^3*d^7 + 12*a*b^3*c^5*d^5 - 12*a*b^3*c^7*d^3 + 12*a^3*b*c^3*d^7 - 12*a^3*b*c^5*d^5 + 4*a^3*b*c^7*d^3 + 6*a^2*b^2*c^2*d^8 - 18*a^2*b^2*c^4*d^6 + 18*a^2*b^2*c^6*d^4 - 6*a^2*b^2*c^8*d^2 + 4*a*b^3*c^9*d - 4*a^3*b*c*d^9))/((16*(63*a*b^12*c^5*d^7 - 216*a*b^12*c^3*d^9 + 41*a*b^12*c^7*d^5 + 4*a*b^12*c^9*d^3 - 486*a^3*b^10*c*d^11 + 864*a^5*b^8*c*d^11 - 702*a^7*b^6*c*d^11 + 216*a^9*b^4*c*d^11 + 162*a^2*b^11*c^2*d^10 - 261*a^2*b^11*c^4*d^8 + 66*a^2*b^11*c^6*d^6 + 19*a^2*b^11*c^8*d^4 + 1197*a^3*b^10*c^3*d^9 - 696*a^3*b^10*c^5*d^7 - 21*a^3*b^10*c^7*d^5 + 16*a^3*b^10*c^9*d^3 - 783*a^4*b^9*c^2*d^10 + 1444*a^4*b^9*c^4*d^8 - 583*a^4*b^9*c^6*d^6 - 20*a^4*b^9*c^8*d^4 - 2511*a^5*b^8*c^3*d^9 + 1913*a^5*b^8*c^5*d^7 - 312*a^5*b^8*c^7*d^5 + 16*a^5*b^8*c^9*d^3 + 1278*a^6*b^7*c^2*d^10 - 2508*a^6*b^7*c^4*d^8 + 1232*a^6*b^7*c^6*d^6 - 116*a^6*b^7*c^8*d^4 + 2328*a^7*b^6*c^3*d^9 - 1936*a^7*b^6*c^5*d^7 + 364*a^7*b^6*c^7*d^5 - 828*a^8*b^5*c^2*d^10 + 1518*a^8*b^5*c^4*d^8 - 580*a^8*b^5*c^6*d^6 - 750*a^9*b^4*c^3*d^9 + 476*a^9*b^4*c^5*d^7 + 144*a^10*b^3*c^2*d^10 - 184*a^10*b^3*c^4*d^8 + 24*a^11*b^2*c^3*d^9 + 108*a*b^12*c*d^11))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12) + (16*tan(e/2 + (f*x)/2)*(108*a*b^12*c^2*d^10 - 162*a*b^12*c^4*d^8 + 18*a*b^12*c^6*d^6 + 8*a*b^12*c^8*d^4 + 108*a^2*b^11*c*d^11 - 486*a^4*b^9*c*d^11 + 756*a^6*b^7*c*d^11 - 432*a^8*b^5*c*d^11 - 162*a^2*b^11*c^3*d^9 + 36*a^2*b^11*c^5*d^7 + 38*a^2*b^11*c^7*d^5 - 270*a^3*b^10*c^2*d^10

+ 396*a^3*b^10*c^4*d^8 - 42*a^3*b^10*c^6*d^6 + 32*a^3*b^10*c^8*d^4 + 864*a
^4*b^9*c^3*d^9 - 398*a^4*b^9*c^5*d^7 - 40*a^4*b^9*c^7*d^5 + 90*a^5*b^8*c^2*
d^10 + 82*a^5*b^8*c^4*d^8 - 432*a^5*b^8*c^6*d^6 + 32*a^5*b^8*c^8*d^4 - 1632
*a^6*b^7*c^3*d^9 + 1216*a^6*b^7*c^5*d^7 - 232*a^6*b^7*c^7*d^5 + 216*a^7*b^6
*c^2*d^10 - 596*a^7*b^6*c^4*d^8 + 600*a^7*b^6*c^6*d^6 + 900*a^8*b^5*c^3*d^9
- 584*a^8*b^5*c^5*d^7 - 80*a^9*b^4*c^4*d^8 + 48*a^10*b^3*c^3*d^9))/(a^17*d
^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8
*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d
^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a
*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*
d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*
c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a
^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^1
1*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3
- 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*
a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b
^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c
^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d
^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 -
1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437
*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^
9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6
*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10
+ 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36
*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^
11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5
*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*
d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 +
126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a
^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*
c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12) - (d^3*(-(c + d)^3*(c - d)^3)^
(1/2)*((8*(60*a*b^15*c^7*d^7 - 36*a*b^15*c^5*d^9 - 13*a*b^15*c^9*d^5 - 10*a
*b^15*c^11*d^3 - 4*a^3*b^13*c^13*d + 36*a^5*b^11*c*d^13 - 4*a^5*b^11*c^13*d
- 144*a^7*b^9*c*d^13 + 216*a^9*b^7*c*d^13 - 144*a^11*b^5*c*d^13 + 36*a^13*
b^3*c*d^13 + 4*a^15*b*c^3*d^11 + 72*a^2*b^14*c^4*d^10 - 108*a^2*b^14*c^6*d^
8 + 19*a^2*b^14*c^8*d^6 + 14*a^2*b^14*c^10*d^4 - a^2*b^14*c^12*d^2 + 120*a^
3*b^13*c^5*d^9 - 305*a^3*b^13*c^7*d^7 + 190*a^3*b^13*c^9*d^5 + 19*a^3*b^13*
c^11*d^3 - 72*a^4*b^12*c^2*d^12 - 168*a^4*b^12*c^4*d^10 + 699*a^4*b^12*c^6*
d^8 - 602*a^4*b^12*c^8*d^6 + 99*a^4*b^12*c^10*d^4 + 20*a^4*b^12*c^12*d^2 -
36*a^5*b^11*c^3*d^11 - 535*a^5*b^11*c^5*d^9 + 1354*a^5*b^11*c^7*d^7 - 895*a
^5*b^11*c^9*d^5 + 40*a^5*b^11*c^11*d^3 + 276*a^6*b^10*c^2*d^12 + 233*a^6*b^
10*c^4*d^10 - 2046*a^6*b^10*c^6*d^8 + 2161*a^6*b^10*c^8*d^6 - 552*a^6*b^10*
c^10*d^4 + 44*a^6*b^10*c^12*d^2 + 61*a^7*b^9*c^3*d^11 + 1386*a^7*b^9*c^5*d^
9 - 2979*a^7*b^9*c^7*d^7 + 1860*a^7*b^9*c^9*d^5 - 220*a^7*b^9*c^11*d^3 - 37
5*a^8*b^8*c^2*d^12 - 270*a^8*b^8*c^4*d^10 + 2885*a^8*b^8*c^6*d^8 - 3012*a^8

*b^8*c^8*d^6 + 628*a^8*b^8*c^10*d^4 - 88*a^9*b^7*c^3*d^11 - 1544*a^9*b^7*c^5*d^9 + 2648*a^9*b^7*c^7*d^7 - 1088*a^9*b^7*c^9*d^5 + 216*a^10*b^6*c^2*d^12 + 100*a^10*b^6*c^4*d^10 - 1336*a^10*b^6*c^6*d^8 + 1056*a^10*b^6*c^8*d^6 + 180*a^11*b^5*c^3*d^11 + 248*a^11*b^5*c^5*d^9 - 400*a^11*b^5*c^7*d^7 - 60*a^12*b^4*c^2*d^12 + 248*a^12*b^4*c^4*d^10 - 148*a^12*b^4*c^6*d^8 - 184*a^13*b^3*c^3*d^11 + 172*a^13*b^3*c^5*d^9 + 24*a^14*b^2*c^2*d^12 - 44*a^14*b^2*c^4*d^10 - a*b^15*c^13*d))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12) - (8*tan(e/2 + (f*x)/2)*(4*a^16*c^3*d^11 - 4*a^3*b^13*c^14 - 4*a^5*b^11*c^14 - a*b^15*c^14 + 144*a*b^15*c^4*d^10 - 348*a*b^15*c^6*d^8 + 214*a*b^15*c^8*d^6 + 7*a*b^15*c^10*d^4 - 8*a*b^15*c^12*d^2 - a^2*b^14*c^13*d - 144*a^4*b^12*c*d^13 + 20*a^4*b^12*c^13*d + 684*a^6*b^10*c*d^13 + 44*a^6*b^10*c^13*d - 1314*a^8*b^8*c*d^13 + 1224*a^10*b^6*c*d^13 - 504*a^12*b^4*c*d^13 + 36*a^14*b^2*c*d^13 + 24*a^15*b*c^2*d^12 - 44*a^15*b*c^4*d^10 - 432*a^2*b^14*c^3*d^11 + 1140*a^2*b^14*c^5*d^9 - 818*a^2*b^14*c^7*d^7 + 55*a^2*b^14*c^9*d^5 + 16*a^2*b^14*c^11*d^3 + 432*a^3*b^13*c^2*d^12 - 2016*a^3*b^13*c^4*d^10 + 2938*a^3*b^13*c^6*d^8 - 1485*a^3*b^13*c^8*d^6 + 152*a^3*b^13*c^10*d^4 + 27*a^3*b^13*c^12*d^2 + 2688*a^4*b^12*c^3*d^11 - 6574*a^4*b^12*c^5*d^9 + 5107*a^4*b^12*c^7*d^7 - 1056*a^4*b^12*c^9*d^5 + 59*a^4*b^12*c^11*d^3 - 2148*a^5*b^11*c^2*d^12 + 8378*a^5*b^11*c^4*d^10 - 10619*a^5*b^11*c^6*d^8 + 5064*a^5*b^11*c^8*d^6 - 975*a^5*b^11*c^10*d^4 + 48*a^5*b^11*c^12*d^2 - 7294*a^6*b^10*c^3*d^11 + 16053*a^6*b^10*c^5*d^9 - 12464*a^6*b^10*c^7*d^7 + 3649*a^6*b^10*c^9

*d^5 - 640*a^6*b^10*c^11*d^3 + 4470*a^7*b^9*c^2*d^12 - 15815*a^7*b^9*c^4*d^
10 + 18608*a^7*b^9*c^6*d^8 - 8939*a^7*b^9*c^8*d^6 + 2300*a^7*b^9*c^10*d^4 -
 220*a^7*b^9*c^12*d^2 + 10105*a^8*b^8*c^3*d^11 - 19912*a^8*b^8*c^5*d^9 + 14
693*a^8*b^8*c^7*d^7 - 4524*a^8*b^8*c^9*d^5 + 628*a^8*b^8*c^11*d^3 - 4632*a^
9*b^7*c^2*d^12 + 14976*a^9*b^7*c^4*d^10 - 15576*a^9*b^7*c^6*d^8 + 6104*a^9*
b^7*c^8*d^6 - 1088*a^9*b^7*c^10*d^4 - 7104*a^10*b^6*c^3*d^11 + 11320*a^10*b
^6*c^5*d^9 - 6184*a^10*b^6*c^7*d^7 + 1120*a^10*b^6*c^9*d^5 + 2232*a^11*b^5*
c^2*d^12 - 5932*a^11*b^5*c^4*d^10 + 4344*a^11*b^5*c^6*d^8 - 688*a^11*b^5*c^
8*d^6 + 1892*a^12*b^4*c^3*d^11 - 1920*a^12*b^4*c^5*d^9 + 368*a^12*b^4*c^7*d
^7 - 252*a^13*b^3*c^2*d^12 + 624*a^13*b^3*c^4*d^10 - 292*a^13*b^3*c^6*d^8 -
 192*a^14*b^2*c^3*d^11 + 172*a^14*b^2*c^5*d^9))/(a^17*d^13 - b^17*c^13 + 4*
a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*
d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^1
1 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a
*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12
*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5
*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a
^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*
c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d
^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 +
126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^
5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^
11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2
*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^
5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1
992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9
*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*
c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*
d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 +
 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672
*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12
*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c
^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^
5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44
*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c
^12*d - 9*a^16*b*c*d^12) + (d^3*(-(c + d)^3*(c - d)^3)^(1/2)*((8*(2*a^2*b^1
7*c^16 - 6*a^6*b^13*c^16 + 4*a^8*b^11*c^16 + 4*a^19*c^3*d^13 - 4*a^19*c^5*d
^11 + 12*a*b^18*c^9*d^7 - 28*a*b^18*c^11*d^5 + 16*a*b^18*c^13*d^3 - 10*a^3*
b^16*c^15*d - 24*a^5*b^14*c^15*d + 78*a^7*b^12*c^15*d + 12*a^9*b^10*c*d^15
 - 44*a^9*b^10*c^15*d - 54*a^11*b^8*c*d^15 + 96*a^13*b^6*c*d^15 - 78*a^15*b^
4*c*d^15 + 24*a^17*b^2*c*d^15 + 12*a^18*b*c^2*d^14 - 56*a^18*b*c^4*d^12 + 4
4*a^18*b*c^6*d^10 - 96*a^2*b^17*c^8*d^8 + 234*a^2*b^17*c^10*d^6 - 146*a^2*b
^17*c^12*d^4 + 6*a^2*b^17*c^14*d^2 + 336*a^3*b^16*c^7*d^9 - 918*a^3*b^16*c^
9*d^7 + 726*a^3*b^16*c^11*d^5 - 134*a^3*b^16*c^13*d^3 - 672*a^4*b^15*c^6*d^
10 + 2280*a^4*b^15*c^8*d^8 - 2520*a^4*b^15*c^10*d^6 + 952*a^4*b^15*c^12*d^4

- 40*a^4*b^15*c^14*d^2 + 840*a^5*b^14*c^5*d^11 - 4032*a^5*b^14*c^7*d^9 + 6360*a^5*b^14*c^9*d^7 - 3768*a^5*b^14*c^11*d^5 + 624*a^5*b^14*c^13*d^3 - 672*a^6*b^13*c^4*d^12 + 5292*a^6*b^13*c^6*d^10 - 11772*a^6*b^13*c^8*d^8 + 10050*a^6*b^13*c^10*d^6 - 3174*a^6*b^13*c^12*d^4 + 282*a^6*b^13*c^14*d^2 + 336*a^7*b^12*c^3*d^13 - 5124*a^7*b^12*c^5*d^11 + 16212*a^7*b^12*c^7*d^9 - 19602*a^7*b^12*c^9*d^7 + 9670*a^7*b^12*c^11*d^5 - 1570*a^7*b^12*c^13*d^3 - 96*a^8*b^11*c^2*d^14 + 3528*a^8*b^11*c^4*d^12 - 16872*a^8*b^11*c^6*d^10 + 28848*a^8*b^11*c^8*d^8 - 20340*a^8*b^11*c^10*d^6 + 5396*a^8*b^11*c^12*d^4 - 468*a^8*b^11*c^14*d^2 - 1620*a^9*b^10*c^3*d^13 + 13320*a^9*b^10*c^5*d^11 - 32304*a^9*b^10*c^7*d^9 + 31560*a^9*b^10*c^9*d^7 - 12648*a^9*b^10*c^11*d^5 + 1724*a^9*b^10*c^13*d^3 + 442*a^10*b^9*c^2*d^14 - 7810*a^10*b^9*c^4*d^12 + 27546*a^10*b^9*c^6*d^10 - 37338*a^10*b^9*c^8*d^8 + 21288*a^10*b^9*c^10*d^6 - 4348*a^10*b^9*c^12*d^4 + 220*a^10*b^9*c^14*d^2 + 3206*a^11*b^8*c^3*d^13 - 17850*a^11*b^8*c^5*d^11 + 34018*a^11*b^8*c^7*d^9 - 26556*a^11*b^8*c^9*d^7 + 7896*a^11*b^8*c^11*d^5 - 660*a^11*b^8*c^13*d^3 - 816*a^12*b^7*c^2*d^14 + 8696*a^12*b^7*c^4*d^12 - 23696*a^12*b^7*c^6*d^10 + 25056*a^12*b^7*c^8*d^8 - 10560*a^12*b^7*c^10*d^6 + 1320*a^12*b^7*c^12*d^4 - 3064*a^13*b^6*c^3*d^13 + 12400*a^13*b^6*c^5*d^11 - 18048*a^13*b^6*c^7*d^9 + 10464*a^13*b^6*c^9*d^7 - 1848*a^13*b^6*c^11*d^5 + 702*a^14*b^5*c^2*d^14 - 4770*a^14*b^5*c^4*d^12 + 9858*a^14*b^5*c^6*d^10 - 7638*a^14*b^5*c^8*d^8 + 1848*a^14*b^5*c^10*d^6 + 1314*a^15*b^4*c^3*d^13 - 3954*a^15*b^4*c^5*d^11 + 4038*a^15*b^4*c^7*d^9 - 1320*a^15*b^4*c^9*d^7 - 244*a^16*b^3*c^2*d^14 + 1084*a^16*b^3*c^4*d^12 - 1500*a^16*b^3*c^6*d^10 + 660*a^16*b^3*c^8*d^8 - 176*a^17*b^2*c^3*d^13 + 372*a^17*b^2*c^5*d^11 - 220*a^17*b^2*c^7*d^9))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 4

20*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^1
4*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c
^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a
^16*b*c*d^12) + (8*tan(e/2 + (f*x)/2)*(4*a*b^18*c^16 - 12*a^5*b^14*c^16 + 8
*a^7*b^12*c^16 + 8*a^19*c^2*d^14 - 8*a^19*c^4*d^12 + 12*a*b^18*c^10*d^6 - 2
8*a*b^18*c^12*d^4 + 12*a*b^18*c^14*d^2 - 20*a^2*b^17*c^15*d - 48*a^4*b^15*c
^15*d + 156*a^6*b^13*c^15*d - 88*a^8*b^11*c^15*d + 12*a^10*b^9*c*d^15 - 48*
a^12*b^7*c*d^15 + 84*a^14*b^5*c*d^15 - 72*a^16*b^3*c*d^15 - 112*a^18*b*c^3*
d^13 + 88*a^18*b*c^5*d^11 - 84*a^2*b^17*c^9*d^7 + 212*a^2*b^17*c^11*d^5 - 1
08*a^2*b^17*c^13*d^3 + 240*a^3*b^16*c^8*d^8 - 744*a^3*b^16*c^10*d^6 + 584*a
^3*b^16*c^12*d^4 - 80*a^3*b^16*c^14*d^2 - 336*a^4*b^15*c^7*d^9 + 1632*a^4*b
^15*c^9*d^7 - 2176*a^4*b^15*c^11*d^5 + 928*a^4*b^15*c^13*d^3 + 168*a^5*b^14
*c^6*d^10 - 2472*a^5*b^14*c^8*d^8 + 5460*a^5*b^14*c^10*d^6 - 3708*a^5*b^14*
c^12*d^4 + 564*a^5*b^14*c^14*d^2 + 168*a^6*b^13*c^5*d^11 + 2520*a^6*b^13*c^
7*d^9 - 9204*a^6*b^13*c^9*d^7 + 9180*a^6*b^13*c^11*d^5 - 2820*a^6*b^13*c^13
*d^3 - 336*a^7*b^12*c^4*d^12 - 1344*a^7*b^12*c^6*d^10 + 10416*a^7*b^12*c^8*
d^8 - 15960*a^7*b^12*c^10*d^6 + 8152*a^7*b^12*c^12*d^4 - 936*a^7*b^12*c^14*
d^2 + 240*a^8*b^11*c^3*d^13 - 336*a^8*b^11*c^5*d^11 - 7488*a^8*b^11*c^7*d^9
 + 19800*a^8*b^11*c^9*d^7 - 15416*a^8*b^11*c^11*d^5 + 3288*a^8*b^11*c^13*d^
3 - 84*a^9*b^10*c^2*d^14 + 1188*a^9*b^10*c^4*d^12 + 2292*a^9*b^10*c^6*d^10
- 16596*a^9*b^10*c^8*d^8 + 20136*a^9*b^10*c^10*d^6 - 7376*a^9*b^10*c^12*d^4
 + 440*a^9*b^10*c^14*d^2 - 908*a^10*b^9*c^3*d^13 + 1740*a^10*b^9*c^5*d^11 +
 7556*a^10*b^9*c^7*d^9 - 18048*a^10*b^9*c^9*d^7 + 10936*a^10*b^9*c^11*d^5 -
 1288*a^10*b^9*c^13*d^3 + 328*a^11*b^8*c^2*d^14 - 2808*a^11*b^8*c^4*d^12 +
1088*a^11*b^8*c^6*d^10 + 9600*a^11*b^8*c^8*d^8 - 10584*a^11*b^8*c^10*d^6 +
2376*a^11*b^8*c^12*d^4 + 1792*a^12*b^7*c^3*d^13 - 4720*a^12*b^7*c^5*d^11 -
144*a^12*b^7*c^7*d^9 + 5856*a^12*b^7*c^9*d^7 - 2736*a^12*b^7*c^11*d^5 - 596
*a^13*b^6*c^2*d^14 + 3980*a^13*b^6*c^4*d^12 - 4908*a^13*b^6*c^6*d^10 - 156*
a^13*b^6*c^8*d^8 + 1680*a^13*b^6*c^10*d^6 - 1932*a^14*b^5*c^3*d^13 + 4812*a
^14*b^5*c^5*d^11 - 3012*a^14*b^5*c^7*d^9 + 48*a^14*b^5*c^9*d^7 + 552*a^15*b
^4*c^2*d^14 - 2616*a^15*b^4*c^4*d^12 + 3096*a^15*b^4*c^6*d^10 - 1032*a^15*b
^4*c^8*d^8 + 920*a^16*b^3*c^3*d^13 - 1752*a^16*b^3*c^5*d^11 + 904*a^16*b^3*
c^7*d^9 - 208*a^17*b^2*c^2*d^14 + 600*a^17*b^2*c^4*d^12 - 392*a^17*b^2*c^6*
d^10 + 24*a^18*b*c*d^15))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*
b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13
 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^1
7*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^
3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12
+ 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*
c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a
^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14
*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*
d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 -
 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a
^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b

^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12) + (d^3*((8*(16*a^4*b^18*c^18 - 4*a^2*b^20*c^18 - 24*a^6*b^16*c^18 + 16*a^8*b^14*c^18 - 4*a^10*b^12*c^18 + 4*a^22*c^2*d^16 - 8*a^22*c^4*d^14 + 4*a^22*c^6*d^12 + 4*a*b^21*c^13*d^5 - 8*a*b^21*c^15*d^3 + 24*a^3*b^19*c^17*d - 136*a^5*b^17*c^17*d + 224*a^7*b^15*c^17*d - 156*a^9*b^13*c^17*d + 40*a^11*b^11*c^17*d - 4*a^13*b^9*c*d^17 + 16*a^15*b^7*c*d^17 - 24*a^17*b^5*c*d^17 + 16*a^19*b^3*c*d^17 - 32*a^21*b*c^3*d^15 + 76*a^21*b*c^5*d^13 - 40*a^21*b*c^7*d^11 - 40*a^2*b^20*c^12*d^6 + 76*a^2*b^20*c^14*d^4 - 32*a^2*b^20*c^16*d^2 + 176*a^3*b^19*c^11*d^7 - 328*a^3*b^19*c^13*d^5 + 128*a^3*b^19*c^15*d^3 - 440*a^4*b^18*c^10*d^8 + 864*a^4*b^18*c^12*d^6 - 392*a^4*b^18*c^14*d^4 - 48*a^4*b^18*c^16*d^2 + 660*a^5*b^17*c^9*d^9 - 1584*a^5*b^17*c^11*d^7 + 1052*a^5*b^17*c^13*d^5 + 8*a^5*b^17*c^15*d^3 - 528*a^6*b^16*c^8*d^10 + 2156*a^6*b^16*c^10*d^8 - 2264*a^6*b^16*c^12*d^6 + 148*a^6*b^16*c^14*d^4 + 512*a^6*b^16*c^16*d^2 - 2112*a^7*b^15*c^9*d^9 + 3520*a^7*b^15*c^11*d^7 - 480*a^7*b^15*c^13*d^5 - 1152*a^7*b^15*c^15*d^3 + 528*a^8*b^14*c^6*d^12 + 1056*a^8*b^14*c^8*d^10 - 3696*a^8*b^14*c^10*d^8 + 1216*a^8*b^14*c^12*d^6 + 1808*a^8*b^14*c^14*d^4 - 928*a^8*b^14*c^16*d^2 - 660*a^9*b^13*c^5*d^13 + 792*a^9*b^13*c^7*d^11 + 2244*a^9*b^13*c^9*d^9 - 2288*a^9*b^13*c^11*d^7 - 2180*a^9*b^13*c^13*d^5 + 2248*a^9*b^13*c^15*d^3 + 440*a^10*b^12*c^4*d^14 - 2332*a^10*b^12*c^6*d^12 + 176*a^10*b^12*c^8*d^10 + 2684*a^10*b^12*c^10*d^8 + 1896*a^10*b^12*c^12*d^6 - 3532*a^10*b^12*c^14*d^4 + 672*a^10*b^12*c^16*d^2 - 176*a^11*b^11*c^3*d^15 + 2552*a^11*b^11*c^5*d^13 - 2464*a^11*b^11*c^7*d^11 - 1496*a^11*b^11*c^9*d^9 - 528*a^11*b^11*c^11*d^7 + 3736*a^11*b^11*c^13*d^5 - 1664*a^11*b^11*c^15*d^3 + 40*a^12*b^10*c^2*d^16 - 1664*a^12*b^10*c^4*d^14 + 3736*a^12*b^10*c^6*d^12 - 528*a^12*b^10*c^8*d^10 - 1496*a^12*b^10*c^10*d^8 - 2464*a^12*b^10*c^12*d^6 + 2552*a^12*b^10*c^14*d^4 - 176*a^12*b^10*c^16*d^2 + 672*a^13*b^9*c^3*d^15 - 3532*a^13*b^9*c^5*d^13 + 1896*a^13*b^9*c^7*d^11 + 2684*a^13*b^9*c^9*d^9 + 176*a^13*b^9*c^11*d^7 - 2332*a^13*b^9*c^13*d^5 + 440*a^13*b^9*c^15*d^3 - 156*a^14*b^8*c^2*d^16 + 2248*a^14*b^8*c^4*d^14 - 2180*a^14*b^8*c^6*d^12 - 2288*a^14*b^8*c^8*d^10 + 2244*a^14*b^8*c^10*d^8 + 792*a^14*b^8*c^12*d^6 - 660*a^14*b^8*c^14*d^4 - 928*a^15*b^7*c^3*d^15 + 1808*a^15*b^7*c^5*d^13 + 1216*a^15*b^7*c^7*d^11 - 3696*a^15*b^7*c^9*d^9 + 1056*a^15*b^7*c^11*d^7 + 528*a^15*b^7*c^13*d^5 + 224*a^16*b^6*c^2*d^16 - 1152*a^16*b^6*c^4*d^

14 - 480*a^16*b^6*c^6*d^12 + 3520*a^16*b^6*c^8*d^10 - 2112*a^16*b^6*c^10*d^
8 + 512*a^17*b^5*c^3*d^15 + 148*a^17*b^5*c^5*d^13 - 2264*a^17*b^5*c^7*d^11
+ 2156*a^17*b^5*c^9*d^9 - 528*a^17*b^5*c^11*d^7 - 136*a^18*b^4*c^2*d^16 + 8
*a^18*b^4*c^4*d^14 + 1052*a^18*b^4*c^6*d^12 - 1584*a^18*b^4*c^8*d^10 + 660*
a^18*b^4*c^10*d^8 - 48*a^19*b^3*c^3*d^15 - 392*a^19*b^3*c^5*d^13 + 864*a^19
*b^3*c^7*d^11 - 440*a^19*b^3*c^9*d^9 + 24*a^20*b^2*c^2*d^16 + 128*a^20*b^2*
c^4*d^14 - 328*a^20*b^2*c^6*d^12 + 176*a^20*b^2*c^8*d^10 + 4*a*b^21*c^17*d
- 4*a^21*b*c*d^17))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c
^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a
^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*
d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14
*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^
9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12
+ 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^1
5*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d
^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 -
420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a
^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^1
1*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^
9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9
+ 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 +
354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a
^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8
*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d
^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6
+ 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940
*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*
b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*
c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d
^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 +
204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15
*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12) - (
8*tan(e/2 + (f*x)/2)*(12*a*b^21*c^18 - 12*a^22*c*d^17 - 56*a^3*b^19*c^18 +
104*a^5*b^17*c^18 - 96*a^7*b^15*c^18 + 44*a^9*b^13*c^18 - 8*a^11*b^11*c^18
+ 32*a^22*c^3*d^15 - 28*a^22*c^5*d^13 + 8*a^22*c^7*d^11 - 16*a*b^21*c^12*d^
6 + 44*a*b^21*c^14*d^4 - 40*a*b^21*c^16*d^2 - 132*a^2*b^20*c^17*d + 616*a^4
*b^18*c^17*d - 1144*a^6*b^16*c^17*d + 1056*a^8*b^14*c^17*d - 484*a^10*b^12*
c^17*d + 16*a^12*b^10*c*d^17 + 88*a^12*b^10*c^17*d - 76*a^14*b^8*c*d^17 + 1
44*a^16*b^6*c*d^17 - 136*a^18*b^4*c*d^17 + 64*a^20*b^2*c*d^17 + 132*a^21*b*
c^2*d^16 - 352*a^21*b*c^4*d^14 + 308*a^21*b*c^6*d^12 - 88*a^21*b*c^8*d^10 +
176*a^2*b^20*c^11*d^7 - 484*a^2*b^20*c^13*d^5 + 440*a^2*b^20*c^15*d^3 - 88
0*a^3*b^19*c^10*d^8 + 2496*a^3*b^19*c^12*d^6 - 2408*a^3*b^19*c^14*d^4 + 848
*a^3*b^19*c^16*d^2 + 2640*a^4*b^18*c^9*d^9 - 8096*a^4*b^18*c^11*d^7 + 8888*
a^4*b^18*c^13*d^5 - 4048*a^4*b^18*c^15*d^3 - 5280*a^5*b^17*c^8*d^10 + 18700
*a^5*b^17*c^10*d^8 - 24784*a^5*b^17*c^12*d^6 + 14692*a^5*b^17*c^14*d^4 - 34

32*a^5*b^17*c^16*d^2 + 7392*a^6*b^16*c^7*d^11 - 32868*a^6*b^16*c^9*d^9 + 54
384*a^6*b^16*c^11*d^7 - 40876*a^6*b^16*c^13*d^5 + 13112*a^6*b^16*c^15*d^3 -
 7392*a^7*b^15*c^6*d^12 + 45408*a^7*b^15*c^8*d^10 - 95040*a^7*b^15*c^10*d^8
 + 89280*a^7*b^15*c^12*d^6 - 38208*a^7*b^15*c^14*d^4 + 6048*a^7*b^15*c^16*d
^2 + 5280*a^8*b^14*c^5*d^13 - 49632*a^8*b^14*c^7*d^11 + 133056*a^8*b^14*c^9
*d^9 - 156992*a^8*b^14*c^11*d^7 + 88000*a^8*b^14*c^13*d^5 - 20768*a^8*b^14*
c^15*d^3 - 2640*a^9*b^13*c^4*d^14 + 42372*a^9*b^13*c^6*d^12 - 150216*a^9*b^
13*c^8*d^10 + 225676*a^9*b^13*c^10*d^8 - 162336*a^9*b^13*c^12*d^6 + 52532*a
^9*b^13*c^14*d^4 - 5432*a^9*b^13*c^16*d^2 + 880*a^10*b^12*c^3*d^15 - 27500*
a^10*b^12*c^5*d^13 + 137368*a^10*b^12*c^7*d^11 - 266244*a^10*b^12*c^9*d^9 +
 242528*a^10*b^12*c^11*d^7 - 104060*a^10*b^12*c^13*d^5 + 17512*a^10*b^12*c^
15*d^3 - 176*a^11*b^11*c^2*d^16 + 13024*a^11*b^11*c^4*d^14 - 101288*a^11*b^
11*c^6*d^12 + 257136*a^11*b^11*c^8*d^10 - 296824*a^11*b^11*c^10*d^8 + 16576
0*a^11*b^11*c^12*d^6 - 40072*a^11*b^11*c^14*d^4 + 2448*a^11*b^11*c^16*d^2 -
 4224*a^12*b^10*c^3*d^15 + 59000*a^12*b^10*c^5*d^13 - 202544*a^12*b^10*c^7*
d^11 + 299816*a^12*b^10*c^9*d^9 - 214368*a^12*b^10*c^11*d^7 + 69784*a^12*b^
10*c^13*d^5 - 7568*a^12*b^10*c^15*d^3 + 836*a^13*b^9*c^2*d^16 - 26048*a^13*
b^9*c^4*d^14 + 129580*a^13*b^9*c^6*d^12 - 249832*a^13*b^9*c^8*d^10 + 226116
*a^13*b^9*c^10*d^8 - 96272*a^13*b^9*c^12*d^6 + 16060*a^13*b^9*c^14*d^4 - 44
0*a^13*b^9*c^16*d^2 + 8128*a^14*b^8*c^3*d^15 - 66628*a^14*b^8*c^5*d^13 + 17
0424*a^14*b^8*c^7*d^11 - 195404*a^14*b^8*c^9*d^9 + 107184*a^14*b^8*c^11*d^7
 - 24948*a^14*b^8*c^13*d^5 + 1320*a^14*b^8*c^15*d^3 - 1584*a^15*b^7*c^2*d^1
6 + 26752*a^15*b^7*c^4*d^14 - 94160*a^15*b^7*c^6*d^12 + 138688*a^15*b^7*c^8
*d^10 - 96624*a^15*b^7*c^10*d^8 + 29568*a^15*b^7*c^12*d^6 - 2640*a^15*b^7*c
^14*d^4 - 7872*a^16*b^6*c^3*d^15 + 41712*a^16*b^6*c^5*d^13 - 80448*a^16*b^6
*c^7*d^11 + 70224*a^16*b^6*c^9*d^9 - 27456*a^16*b^6*c^11*d^7 + 3696*a^16*b^
6*c^13*d^5 + 1496*a^17*b^5*c^2*d^16 - 14608*a^17*b^5*c^4*d^14 + 37532*a^17*
b^5*c^6*d^12 - 40920*a^17*b^5*c^8*d^10 + 20196*a^17*b^5*c^10*d^8 - 3696*a^1
7*b^5*c^12*d^6 + 3888*a^18*b^4*c^3*d^15 - 13748*a^18*b^4*c^5*d^13 + 19016*a
^18*b^4*c^7*d^11 - 11660*a^18*b^4*c^9*d^9 + 2640*a^18*b^4*c^11*d^7 - 704*a^
19*b^3*c^2*d^16 + 3872*a^19*b^3*c^4*d^14 - 6952*a^19*b^3*c^6*d^12 + 5104*a^
19*b^3*c^8*d^10 - 1320*a^19*b^3*c^10*d^8 - 832*a^20*b^2*c^3*d^15 + 1912*a^2
0*b^2*c^5*d^13 - 1584*a^20*b^2*c^7*d^11 + 440*a^20*b^2*c^9*d^9))/(a^17*d^13
 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^
9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13
 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^
16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d -
 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d
^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16
*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d
^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 1
26*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4
*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12
*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*
d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2

+ 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 154
8*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^
8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c
^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^
7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1
548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^
10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*
b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^
3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4
 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 12
6*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14
*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6
*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12))*(-(c + d)^3*(c - d)^3)^(1/2)*(3*
b*d^2 - 4*b*c^2 + a*c*d))/(a^4*d^10 - b^4*c^10 - 3*a^4*c^2*d^8 + 3*a^4*c^4*
d^6 - a^4*c^6*d^4 + b^4*c^4*d^6 - 3*b^4*c^6*d^4 + 3*b^4*c^8*d^2 - 4*a*b^3*c
^3*d^7 + 12*a*b^3*c^5*d^5 - 12*a*b^3*c^7*d^3 + 12*a^3*b*c^3*d^7 - 12*a^3*b*
c^5*d^5 + 4*a^3*b*c^7*d^3 + 6*a^2*b^2*c^2*d^8 - 18*a^2*b^2*c^4*d^6 + 18*a^2
*b^2*c^6*d^4 - 6*a^2*b^2*c^8*d^2 + 4*a*b^3*c^9*d - 4*a^3*b*c*d^9))*(3*b*d^2
 - 4*b*c^2 + a*c*d))/(a^4*d^10 - b^4*c^10 - 3*a^4*c^2*d^8 + 3*a^4*c^4*d^6 -
 a^4*c^6*d^4 + b^4*c^4*d^6 - 3*b^4*c^6*d^4 + 3*b^4*c^8*d^2 - 4*a*b^3*c^3*d^
7 + 12*a*b^3*c^5*d^5 - 12*a*b^3*c^7*d^3 + 12*a^3*b*c^3*d^7 - 12*a^3*b*c^5*d
^5 + 4*a^3*b*c^7*d^3 + 6*a^2*b^2*c^2*d^8 - 18*a^2*b^2*c^4*d^6 + 18*a^2*b^2*
c^6*d^4 - 6*a^2*b^2*c^8*d^2 + 4*a*b^3*c^9*d - 4*a^3*b*c*d^9))*(3*b*d^2 - 4*
b*c^2 + a*c*d))/(a^4*d^10 - b^4*c^10 - 3*a^4*c^2*d^8 + 3*a^4*c^4*d^6 - a^4*
c^6*d^4 + b^4*c^4*d^6 - 3*b^4*c^6*d^4 + 3*b^4*c^8*d^2 - 4*a*b^3*c^3*d^7 + 1
2*a*b^3*c^5*d^5 - 12*a*b^3*c^7*d^3 + 12*a^3*b*c^3*d^7 - 12*a^3*b*c^5*d^5 +
4*a^3*b*c^7*d^3 + 6*a^2*b^2*c^2*d^8 - 18*a^2*b^2*c^4*d^6 + 18*a^2*b^2*c^6*d
^4 - 6*a^2*b^2*c^8*d^2 + 4*a*b^3*c^9*d - 4*a^3*b*c*d^9) - (d^3*(-(c + d)^3*
(c - d)^3)^(1/2)*((8*tan(e/2 + (f*x)/2)*(4*a^16*c^3*d^11 - 4*a^3*b^13*c^14
- 4*a^5*b^11*c^14 - a*b^15*c^14 + 144*a*b^15*c^4*d^10 - 348*a*b^15*c^6*d^8
+ 214*a*b^15*c^8*d^6 + 7*a*b^15*c^10*d^4 - 8*a*b^15*c^12*d^2 - a^2*b^14*c^1
3*d - 144*a^4*b^12*c*d^13 + 20*a^4*b^12*c^13*d + 684*a^6*b^10*c*d^13 + 44*a
^6*b^10*c^13*d - 1314*a^8*b^8*c*d^13 + 1224*a^10*b^6*c*d^13 - 504*a^12*b^4*
c*d^13 + 36*a^14*b^2*c*d^13 + 24*a^15*b*c^2*d^12 - 44*a^15*b*c^4*d^10 - 432
*a^2*b^14*c^3*d^11 + 1140*a^2*b^14*c^5*d^9 - 818*a^2*b^14*c^7*d^7 + 55*a^2*
b^14*c^9*d^5 + 16*a^2*b^14*c^11*d^3 + 432*a^3*b^13*c^2*d^12 - 2016*a^3*b^13
*c^4*d^10 + 2938*a^3*b^13*c^6*d^8 - 1485*a^3*b^13*c^8*d^6 + 152*a^3*b^13*c^
10*d^4 + 27*a^3*b^13*c^12*d^2 + 2688*a^4*b^12*c^3*d^11 - 6574*a^4*b^12*c^5*
d^9 + 5107*a^4*b^12*c^7*d^7 - 1056*a^4*b^12*c^9*d^5 + 59*a^4*b^12*c^11*d^3
- 2148*a^5*b^11*c^2*d^12 + 8378*a^5*b^11*c^4*d^10 - 10619*a^5*b^11*c^6*d^8
+ 5064*a^5*b^11*c^8*d^6 - 975*a^5*b^11*c^10*d^4 + 48*a^5*b^11*c^12*d^2 - 72
94*a^6*b^10*c^3*d^11 + 16053*a^6*b^10*c^5*d^9 - 12464*a^6*b^10*c^7*d^7 + 36
49*a^6*b^10*c^9*d^5 - 640*a^6*b^10*c^11*d^3 + 4470*a^7*b^9*c^2*d^12 - 15815
*a^7*b^9*c^4*d^10 + 18608*a^7*b^9*c^6*d^8 - 8939*a^7*b^9*c^8*d^6 + 2300*a^7
*b^9*c^10*d^4 - 220*a^7*b^9*c^12*d^2 + 10105*a^8*b^8*c^3*d^11 - 19912*a^8*b

^8*c^5*d^9 + 14693*a^8*b^8*c^7*d^7 - 4524*a^8*b^8*c^9*d^5 + 628*a^8*b^8*c^1
1*d^3 - 4632*a^9*b^7*c^2*d^12 + 14976*a^9*b^7*c^4*d^10 - 15576*a^9*b^7*c^6*
d^8 + 6104*a^9*b^7*c^8*d^6 - 1088*a^9*b^7*c^10*d^4 - 7104*a^10*b^6*c^3*d^11
 + 11320*a^10*b^6*c^5*d^9 - 6184*a^10*b^6*c^7*d^7 + 1120*a^10*b^6*c^9*d^5 +
 2232*a^11*b^5*c^2*d^12 - 5932*a^11*b^5*c^4*d^10 + 4344*a^11*b^5*c^6*d^8 -
688*a^11*b^5*c^8*d^6 + 1892*a^12*b^4*c^3*d^11 - 1920*a^12*b^4*c^5*d^9 + 368
*a^12*b^4*c^7*d^7 - 252*a^13*b^3*c^2*d^12 + 624*a^13*b^3*c^4*d^10 - 292*a^1
3*b^3*c^6*d^8 - 192*a^14*b^2*c^3*d^11 + 172*a^14*b^2*c^5*d^9))/(a^17*d^13 -
 b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*
c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 -
 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16
*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 3
6*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^1
2 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b
*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2
 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126
*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b
^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c
^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^
8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 +
36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*
a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*
b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^1
1*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7
+ 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 154
8*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10
*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^
6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*
d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 -
 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*
a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b
^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d
^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12) - (8*(60*a*b^15*c^7*d^7 - 36*a*b^15
*c^5*d^9 - 13*a*b^15*c^9*d^5 - 10*a*b^15*c^11*d^3 - 4*a^3*b^13*c^13*d + 36*
a^5*b^11*c*d^13 - 4*a^5*b^11*c^13*d - 144*a^7*b^9*c*d^13 + 216*a^9*b^7*c*d^
13 - 144*a^11*b^5*c*d^13 + 36*a^13*b^3*c*d^13 + 4*a^15*b*c^3*d^11 + 72*a^2*
b^14*c^4*d^10 - 108*a^2*b^14*c^6*d^8 + 19*a^2*b^14*c^8*d^6 + 14*a^2*b^14*c^
10*d^4 - a^2*b^14*c^12*d^2 + 120*a^3*b^13*c^5*d^9 - 305*a^3*b^13*c^7*d^7 +
190*a^3*b^13*c^9*d^5 + 19*a^3*b^13*c^11*d^3 - 72*a^4*b^12*c^2*d^12 - 168*a^
4*b^12*c^4*d^10 + 699*a^4*b^12*c^6*d^8 - 602*a^4*b^12*c^8*d^6 + 99*a^4*b^12
*c^10*d^4 + 20*a^4*b^12*c^12*d^2 - 36*a^5*b^11*c^3*d^11 - 535*a^5*b^11*c^5*
d^9 + 1354*a^5*b^11*c^7*d^7 - 895*a^5*b^11*c^9*d^5 + 40*a^5*b^11*c^11*d^3 +
 276*a^6*b^10*c^2*d^12 + 233*a^6*b^10*c^4*d^10 - 2046*a^6*b^10*c^6*d^8 + 21
61*a^6*b^10*c^8*d^6 - 552*a^6*b^10*c^10*d^4 + 44*a^6*b^10*c^12*d^2 + 61*a^7
*b^9*c^3*d^11 + 1386*a^7*b^9*c^5*d^9 - 2979*a^7*b^9*c^7*d^7 + 1860*a^7*b^9*

c^9*d^5 - 220*a^7*b^9*c^11*d^3 - 375*a^8*b^8*c^2*d^12 - 270*a^8*b^8*c^4*d^1
0 + 2885*a^8*b^8*c^6*d^8 - 3012*a^8*b^8*c^8*d^6 + 628*a^8*b^8*c^10*d^4 - 88
*a^9*b^7*c^3*d^11 - 1544*a^9*b^7*c^5*d^9 + 2648*a^9*b^7*c^7*d^7 - 1088*a^9*
b^7*c^9*d^5 + 216*a^10*b^6*c^2*d^12 + 100*a^10*b^6*c^4*d^10 - 1336*a^10*b^6
*c^6*d^8 + 1056*a^10*b^6*c^8*d^6 + 180*a^11*b^5*c^3*d^11 + 248*a^11*b^5*c^5
*d^9 - 400*a^11*b^5*c^7*d^7 - 60*a^12*b^4*c^2*d^12 + 248*a^12*b^4*c^4*d^10
- 148*a^12*b^4*c^6*d^8 - 184*a^13*b^3*c^3*d^11 + 172*a^13*b^3*c^5*d^9 + 24*
a^14*b^2*c^2*d^12 - 44*a^14*b^2*c^4*d^10 - a*b^15*c^13*d))/(a^17*d^13 - b^1
7*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13
 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a
^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8
*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^
7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 -
54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5
*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 8
4*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4
*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*
c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d
^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 -
1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a
^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*
b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*
c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^
2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 14
37*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^
10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7
*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^
6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10
 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156
*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13
*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c
^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 +
 9*a*b^16*c^12*d - 9*a^16*b*c*d^12) + (d^3*(-(c + d)^3*(c - d)^3)^(1/2)*((8
*(2*a^2*b^17*c^16 - 6*a^6*b^13*c^16 + 4*a^8*b^11*c^16 + 4*a^19*c^3*d^13 - 4
*a^19*c^5*d^11 + 12*a*b^18*c^9*d^7 - 28*a*b^18*c^11*d^5 + 16*a*b^18*c^13*d^
3 - 10*a^3*b^16*c^15*d - 24*a^5*b^14*c^15*d + 78*a^7*b^12*c^15*d + 12*a^9*b
^10*c*d^15 - 44*a^9*b^10*c^15*d - 54*a^11*b^8*c*d^15 + 96*a^13*b^6*c*d^15 -
 78*a^15*b^4*c*d^15 + 24*a^17*b^2*c*d^15 + 12*a^18*b*c^2*d^14 - 56*a^18*b*c
^4*d^12 + 44*a^18*b*c^6*d^10 - 96*a^2*b^17*c^8*d^8 + 234*a^2*b^17*c^10*d^6
- 146*a^2*b^17*c^12*d^4 + 6*a^2*b^17*c^14*d^2 + 336*a^3*b^16*c^7*d^9 - 918*
a^3*b^16*c^9*d^7 + 726*a^3*b^16*c^11*d^5 - 134*a^3*b^16*c^13*d^3 - 672*a^4*
b^15*c^6*d^10 + 2280*a^4*b^15*c^8*d^8 - 2520*a^4*b^15*c^10*d^6 + 952*a^4*b^
15*c^12*d^4 - 40*a^4*b^15*c^14*d^2 + 840*a^5*b^14*c^5*d^11 - 4032*a^5*b^14*
c^7*d^9 + 6360*a^5*b^14*c^9*d^7 - 3768*a^5*b^14*c^11*d^5 + 624*a^5*b^14*c^1
3*d^3 - 672*a^6*b^13*c^4*d^12 + 5292*a^6*b^13*c^6*d^10 - 11772*a^6*b^13*c^8

*d^8 + 10050*a^6*b^13*c^10*d^6 - 3174*a^6*b^13*c^12*d^4 + 282*a^6*b^13*c^14
*d^2 + 336*a^7*b^12*c^3*d^13 - 5124*a^7*b^12*c^5*d^11 + 16212*a^7*b^12*c^7*
d^9 - 19602*a^7*b^12*c^9*d^7 + 9670*a^7*b^12*c^11*d^5 - 1570*a^7*b^12*c^13*
d^3 - 96*a^8*b^11*c^2*d^14 + 3528*a^8*b^11*c^4*d^12 - 16872*a^8*b^11*c^6*d^
10 + 28848*a^8*b^11*c^8*d^8 - 20340*a^8*b^11*c^10*d^6 + 5396*a^8*b^11*c^12*
d^4 - 468*a^8*b^11*c^14*d^2 - 1620*a^9*b^10*c^3*d^13 + 13320*a^9*b^10*c^5*d
^11 - 32304*a^9*b^10*c^7*d^9 + 31560*a^9*b^10*c^9*d^7 - 12648*a^9*b^10*c^11
*d^5 + 1724*a^9*b^10*c^13*d^3 + 442*a^10*b^9*c^2*d^14 - 7810*a^10*b^9*c^4*d
^12 + 27546*a^10*b^9*c^6*d^10 - 37338*a^10*b^9*c^8*d^8 + 21288*a^10*b^9*c^1
0*d^6 - 4348*a^10*b^9*c^12*d^4 + 220*a^10*b^9*c^14*d^2 + 3206*a^11*b^8*c^3*
d^13 - 17850*a^11*b^8*c^5*d^11 + 34018*a^11*b^8*c^7*d^9 - 26556*a^11*b^8*c^
9*d^7 + 7896*a^11*b^8*c^11*d^5 - 660*a^11*b^8*c^13*d^3 - 816*a^12*b^7*c^2*d
^14 + 8696*a^12*b^7*c^4*d^12 - 23696*a^12*b^7*c^6*d^10 + 25056*a^12*b^7*c^8
*d^8 - 10560*a^12*b^7*c^10*d^6 + 1320*a^12*b^7*c^12*d^4 - 3064*a^13*b^6*c^3
*d^13 + 12400*a^13*b^6*c^5*d^11 - 18048*a^13*b^6*c^7*d^9 + 10464*a^13*b^6*c
^9*d^7 - 1848*a^13*b^6*c^11*d^5 + 702*a^14*b^5*c^2*d^14 - 4770*a^14*b^5*c^4
*d^12 + 9858*a^14*b^5*c^6*d^10 - 7638*a^14*b^5*c^8*d^8 + 1848*a^14*b^5*c^10
*d^6 + 1314*a^15*b^4*c^3*d^13 - 3954*a^15*b^4*c^5*d^11 + 4038*a^15*b^4*c^7*
d^9 - 1320*a^15*b^4*c^9*d^7 - 244*a^16*b^3*c^2*d^14 + 1084*a^16*b^3*c^4*d^1
2 - 1500*a^16*b^3*c^6*d^10 + 660*a^16*b^3*c^8*d^8 - 176*a^17*b^2*c^3*d^13 +
372*a^17*b^2*c^5*d^11 - 220*a^17*b^2*c^7*d^9))/(a^17*d^13 - b^17*c^13 + 4*
a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*
d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^1
1 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a
*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12
*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5
*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a
^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*
c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d
^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 +
126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^
5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^
11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2
*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^
5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1
992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9
*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*
c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*
d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 +
224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672
*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12
*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c
^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^
5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44
*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c

^12*d - 9*a^16*b*c*d^12) + (8*tan(e/2 + (f*x)/2)*(4*a*b^18*c^16 - 12*a^5*b^14*c^16 + 8*a^7*b^12*c^16 + 8*a^19*c^2*d^14 - 8*a^19*c^4*d^12 + 12*a*b^18*c^10*d^6 - 28*a*b^18*c^12*d^4 + 12*a*b^18*c^14*d^2 - 20*a^2*b^17*c^15*d - 48*a^4*b^15*c^15*d + 156*a^6*b^13*c^15*d - 88*a^8*b^11*c^15*d + 12*a^10*b^9*c*d^15 - 48*a^12*b^7*c*d^15 + 84*a^14*b^5*c*d^15 - 72*a^16*b^3*c*d^15 - 112*a^18*b*c^3*d^13 + 88*a^18*b*c^5*d^11 - 84*a^2*b^17*c^9*d^7 + 212*a^2*b^17*c^11*d^5 - 108*a^2*b^17*c^13*d^3 + 240*a^3*b^16*c^8*d^8 - 744*a^3*b^16*c^10*d^6 + 584*a^3*b^16*c^12*d^4 - 80*a^3*b^16*c^14*d^2 - 336*a^4*b^15*c^7*d^9 + 1632*a^4*b^15*c^9*d^7 - 2176*a^4*b^15*c^11*d^5 + 928*a^4*b^15*c^13*d^3 + 168*a^5*b^14*c^6*d^10 - 2472*a^5*b^14*c^8*d^8 + 5460*a^5*b^14*c^10*d^6 - 3708*a^5*b^14*c^12*d^4 + 564*a^5*b^14*c^14*d^2 + 168*a^6*b^13*c^5*d^11 + 2520*a^6*b^13*c^7*d^9 - 9204*a^6*b^13*c^9*d^7 + 9180*a^6*b^13*c^11*d^5 - 2820*a^6*b^13*c^13*d^3 - 336*a^7*b^12*c^4*d^12 - 1344*a^7*b^12*c^6*d^10 + 10416*a^7*b^12*c^8*d^8 - 15960*a^7*b^12*c^10*d^6 + 8152*a^7*b^12*c^12*d^4 - 936*a^7*b^12*c^14*d^2 + 240*a^8*b^11*c^3*d^13 - 336*a^8*b^11*c^5*d^11 - 7488*a^8*b^11*c^7*d^9 + 19800*a^8*b^11*c^9*d^7 - 15416*a^8*b^11*c^11*d^5 + 3288*a^8*b^11*c^13*d^3 - 84*a^9*b^10*c^2*d^14 + 1188*a^9*b^10*c^4*d^12 + 2292*a^9*b^10*c^6*d^10 - 16596*a^9*b^10*c^8*d^8 + 20136*a^9*b^10*c^10*d^6 - 7376*a^9*b^10*c^12*d^4 + 440*a^9*b^10*c^14*d^2 - 908*a^10*b^9*c^3*d^13 + 1740*a^10*b^9*c^5*d^11 + 7556*a^10*b^9*c^7*d^9 - 18048*a^10*b^9*c^9*d^7 + 10936*a^10*b^9*c^11*d^5 - 1288*a^10*b^9*c^13*d^3 + 328*a^11*b^8*c^2*d^14 - 2808*a^11*b^8*c^4*d^12 + 1088*a^11*b^8*c^6*d^10 + 9600*a^11*b^8*c^8*d^8 - 10584*a^11*b^8*c^10*d^6 + 2376*a^11*b^8*c^12*d^4 + 1792*a^12*b^7*c^3*d^13 - 4720*a^12*b^7*c^5*d^11 - 144*a^12*b^7*c^7*d^9 + 5856*a^12*b^7*c^9*d^7 - 2736*a^12*b^7*c^11*d^5 - 596*a^13*b^6*c^2*d^14 + 3980*a^13*b^6*c^4*d^12 - 4908*a^13*b^6*c^6*d^10 - 156*a^13*b^6*c^8*d^8 + 1680*a^13*b^6*c^10*d^6 - 1932*a^14*b^5*c^3*d^13 + 4812*a^14*b^5*c^5*d^11 - 3012*a^14*b^5*c^7*d^9 + 48*a^14*b^5*c^9*d^7 + 552*a^15*b^4*c^2*d^14 - 2616*a^15*b^4*c^4*d^12 + 3096*a^15*b^4*c^6*d^10 - 1032*a^15*b^4*c^8*d^8 + 920*a^16*b^3*c^3*d^13 - 1752*a^16*b^3*c^5*d^11 + 904*a^16*b^3*c^7*d^9 - 208*a^17*b^2*c^2*d^14 + 600*a^17*b^2*c^4*d^12 - 392*a^17*b^2*c^6*d^10 + 24*a^18*b*c*d^15))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9

*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^
11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 3
54*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*
a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b
^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c
^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^
8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 4
20*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^1
4*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c
^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a
^16*b*c*d^12) - (d^3*((8*(16*a^4*b^18*c^18 - 4*a^2*b^20*c^18 - 24*a^6*b^16*
c^18 + 16*a^8*b^14*c^18 - 4*a^10*b^12*c^18 + 4*a^22*c^2*d^16 - 8*a^22*c^4*d
^14 + 4*a^22*c^6*d^12 + 4*a*b^21*c^13*d^5 - 8*a*b^21*c^15*d^3 + 24*a^3*b^19
*c^17*d - 136*a^5*b^17*c^17*d + 224*a^7*b^15*c^17*d - 156*a^9*b^13*c^17*d +
40*a^11*b^11*c^17*d - 4*a^13*b^9*c*d^17 + 16*a^15*b^7*c*d^17 - 24*a^17*b^5
*c*d^17 + 16*a^19*b^3*c*d^17 - 32*a^21*b*c^3*d^15 + 76*a^21*b*c^5*d^13 - 40
*a^21*b*c^7*d^11 - 40*a^2*b^20*c^12*d^6 + 76*a^2*b^20*c^14*d^4 - 32*a^2*b^2
0*c^16*d^2 + 176*a^3*b^19*c^11*d^7 - 328*a^3*b^19*c^13*d^5 + 128*a^3*b^19*c
^15*d^3 - 440*a^4*b^18*c^10*d^8 + 864*a^4*b^18*c^12*d^6 - 392*a^4*b^18*c^14
*d^4 - 48*a^4*b^18*c^16*d^2 + 660*a^5*b^17*c^9*d^9 - 1584*a^5*b^17*c^11*d^7
+ 1052*a^5*b^17*c^13*d^5 + 8*a^5*b^17*c^15*d^3 - 528*a^6*b^16*c^8*d^10 + 2
156*a^6*b^16*c^10*d^8 - 2264*a^6*b^16*c^12*d^6 + 148*a^6*b^16*c^14*d^4 + 51
2*a^6*b^16*c^16*d^2 - 2112*a^7*b^15*c^9*d^9 + 3520*a^7*b^15*c^11*d^7 - 480*
a^7*b^15*c^13*d^5 - 1152*a^7*b^15*c^15*d^3 + 528*a^8*b^14*c^6*d^12 + 1056*a
^8*b^14*c^8*d^10 - 3696*a^8*b^14*c^10*d^8 + 1216*a^8*b^14*c^12*d^6 + 1808*a
^8*b^14*c^14*d^4 - 928*a^8*b^14*c^16*d^2 - 660*a^9*b^13*c^5*d^13 + 792*a^9*
b^13*c^7*d^11 + 2244*a^9*b^13*c^9*d^9 - 2288*a^9*b^13*c^11*d^7 - 2180*a^9*b
^13*c^13*d^5 + 2248*a^9*b^13*c^15*d^3 + 440*a^10*b^12*c^4*d^14 - 2332*a^10*
b^12*c^6*d^12 + 176*a^10*b^12*c^8*d^10 + 2684*a^10*b^12*c^10*d^8 + 1896*a^1
0*b^12*c^12*d^6 - 3532*a^10*b^12*c^14*d^4 + 672*a^10*b^12*c^16*d^2 - 176*a^
11*b^11*c^3*d^15 + 2552*a^11*b^11*c^5*d^13 - 2464*a^11*b^11*c^7*d^11 - 1496
*a^11*b^11*c^9*d^9 - 528*a^11*b^11*c^11*d^7 + 3736*a^11*b^11*c^13*d^5 - 166
4*a^11*b^11*c^15*d^3 + 40*a^12*b^10*c^2*d^16 - 1664*a^12*b^10*c^4*d^14 + 37
36*a^12*b^10*c^6*d^12 - 528*a^12*b^10*c^8*d^10 - 1496*a^12*b^10*c^10*d^8 -
2464*a^12*b^10*c^12*d^6 + 2552*a^12*b^10*c^14*d^4 - 176*a^12*b^10*c^16*d^2
+ 672*a^13*b^9*c^3*d^15 - 3532*a^13*b^9*c^5*d^13 + 1896*a^13*b^9*c^7*d^11 +
2684*a^13*b^9*c^9*d^9 + 176*a^13*b^9*c^11*d^7 - 2332*a^13*b^9*c^13*d^5 + 4
40*a^13*b^9*c^15*d^3 - 156*a^14*b^8*c^2*d^16 + 2248*a^14*b^8*c^4*d^14 - 218
0*a^14*b^8*c^6*d^12 - 2288*a^14*b^8*c^8*d^10 + 2244*a^14*b^8*c^10*d^8 + 792
*a^14*b^8*c^12*d^6 - 660*a^14*b^8*c^14*d^4 - 928*a^15*b^7*c^3*d^15 + 1808*a
^15*b^7*c^5*d^13 + 1216*a^15*b^7*c^7*d^11 - 3696*a^15*b^7*c^9*d^9 + 1056*a^
15*b^7*c^11*d^7 + 528*a^15*b^7*c^13*d^5 + 224*a^16*b^6*c^2*d^16 - 1152*a^16
*b^6*c^4*d^14 - 480*a^16*b^6*c^6*d^12 + 3520*a^16*b^6*c^8*d^10 - 2112*a^16*
b^6*c^10*d^8 + 512*a^17*b^5*c^3*d^15 + 148*a^17*b^5*c^5*d^13 - 2264*a^17*b^
5*c^7*d^11 + 2156*a^17*b^5*c^9*d^9 - 528*a^17*b^5*c^11*d^7 - 136*a^18*b^4*c

^2*d^16 + 8*a^18*b^4*c^4*d^14 + 1052*a^18*b^4*c^6*d^12 - 1584*a^18*b^4*c^8*
d^10 + 660*a^18*b^4*c^10*d^8 - 48*a^19*b^3*c^3*d^15 - 392*a^19*b^3*c^5*d^13
 + 864*a^19*b^3*c^7*d^11 - 440*a^19*b^3*c^9*d^9 + 24*a^20*b^2*c^2*d^16 + 12
8*a^20*b^2*c^4*d^14 - 328*a^20*b^2*c^6*d^12 + 176*a^20*b^2*c^8*d^10 + 4*a*b
^21*c^17*d - 4*a^21*b*c*d^17))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6
*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6
*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9
- b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 -
36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*
d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14
*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 +
 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3
*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13
*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*
d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 -
 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*
a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b
^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*
c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d
^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1
045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9
*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b
^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2
*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5
 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 58
8*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^1
3*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*
c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^1
1 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*
c*d^12) - (8*tan(e/2 + (f*x)/2)*(12*a*b^21*c^18 - 12*a^22*c*d^17 - 56*a^3*b
^19*c^18 + 104*a^5*b^17*c^18 - 96*a^7*b^15*c^18 + 44*a^9*b^13*c^18 - 8*a^11
*b^11*c^18 + 32*a^22*c^3*d^15 - 28*a^22*c^5*d^13 + 8*a^22*c^7*d^11 - 16*a*b
^21*c^12*d^6 + 44*a*b^21*c^14*d^4 - 40*a*b^21*c^16*d^2 - 132*a^2*b^20*c^17*
d + 616*a^4*b^18*c^17*d - 1144*a^6*b^16*c^17*d + 1056*a^8*b^14*c^17*d - 484
*a^10*b^12*c^17*d + 16*a^12*b^10*c*d^17 + 88*a^12*b^10*c^17*d - 76*a^14*b^8
*c*d^17 + 144*a^16*b^6*c*d^17 - 136*a^18*b^4*c*d^17 + 64*a^20*b^2*c*d^17 +
132*a^21*b*c^2*d^16 - 352*a^21*b*c^4*d^14 + 308*a^21*b*c^6*d^12 - 88*a^21*b
*c^8*d^10 + 176*a^2*b^20*c^11*d^7 - 484*a^2*b^20*c^13*d^5 + 440*a^2*b^20*c^
15*d^3 - 880*a^3*b^19*c^10*d^8 + 2496*a^3*b^19*c^12*d^6 - 2408*a^3*b^19*c^1
4*d^4 + 848*a^3*b^19*c^16*d^2 + 2640*a^4*b^18*c^9*d^9 - 8096*a^4*b^18*c^11*
d^7 + 8888*a^4*b^18*c^13*d^5 - 4048*a^4*b^18*c^15*d^3 - 5280*a^5*b^17*c^8*d
^10 + 18700*a^5*b^17*c^10*d^8 - 24784*a^5*b^17*c^12*d^6 + 14692*a^5*b^17*c^
14*d^4 - 3432*a^5*b^17*c^16*d^2 + 7392*a^6*b^16*c^7*d^11 - 32868*a^6*b^16*c
^9*d^9 + 54384*a^6*b^16*c^11*d^7 - 40876*a^6*b^16*c^13*d^5 + 13112*a^6*b^16
*c^15*d^3 - 7392*a^7*b^15*c^6*d^12 + 45408*a^7*b^15*c^8*d^10 - 95040*a^7*b^

15*c^10*d^8 + 89280*a^7*b^15*c^12*d^6 - 38208*a^7*b^15*c^14*d^4 + 6048*a^7*
b^15*c^16*d^2 + 5280*a^8*b^14*c^5*d^13 - 49632*a^8*b^14*c^7*d^11 + 133056*a
^8*b^14*c^9*d^9 - 156992*a^8*b^14*c^11*d^7 + 88000*a^8*b^14*c^13*d^5 - 2076
8*a^8*b^14*c^15*d^3 - 2640*a^9*b^13*c^4*d^14 + 42372*a^9*b^13*c^6*d^12 - 15
0216*a^9*b^13*c^8*d^10 + 225676*a^9*b^13*c^10*d^8 - 162336*a^9*b^13*c^12*d^
6 + 52532*a^9*b^13*c^14*d^4 - 5432*a^9*b^13*c^16*d^2 + 880*a^10*b^12*c^3*d^
15 - 27500*a^10*b^12*c^5*d^13 + 137368*a^10*b^12*c^7*d^11 - 266244*a^10*b^1
2*c^9*d^9 + 242528*a^10*b^12*c^11*d^7 - 104060*a^10*b^12*c^13*d^5 + 17512*a
^10*b^12*c^15*d^3 - 176*a^11*b^11*c^2*d^16 + 13024*a^11*b^11*c^4*d^14 - 101
288*a^11*b^11*c^6*d^12 + 257136*a^11*b^11*c^8*d^10 - 296824*a^11*b^11*c^10*
d^8 + 165760*a^11*b^11*c^12*d^6 - 40072*a^11*b^11*c^14*d^4 + 2448*a^11*b^11
*c^16*d^2 - 4224*a^12*b^10*c^3*d^15 + 59000*a^12*b^10*c^5*d^13 - 202544*a^1
2*b^10*c^7*d^11 + 299816*a^12*b^10*c^9*d^9 - 214368*a^12*b^10*c^11*d^7 + 69
784*a^12*b^10*c^13*d^5 - 7568*a^12*b^10*c^15*d^3 + 836*a^13*b^9*c^2*d^16 -
26048*a^13*b^9*c^4*d^14 + 129580*a^13*b^9*c^6*d^12 - 249832*a^13*b^9*c^8*d^
10 + 226116*a^13*b^9*c^10*d^8 - 96272*a^13*b^9*c^12*d^6 + 16060*a^13*b^9*c^
14*d^4 - 440*a^13*b^9*c^16*d^2 + 8128*a^14*b^8*c^3*d^15 - 66628*a^14*b^8*c^
5*d^13 + 170424*a^14*b^8*c^7*d^11 - 195404*a^14*b^8*c^9*d^9 + 107184*a^14*b
^8*c^11*d^7 - 24948*a^14*b^8*c^13*d^5 + 1320*a^14*b^8*c^15*d^3 - 1584*a^15*
b^7*c^2*d^16 + 26752*a^15*b^7*c^4*d^14 - 94160*a^15*b^7*c^6*d^12 + 138688*a
^15*b^7*c^8*d^10 - 96624*a^15*b^7*c^10*d^8 + 29568*a^15*b^7*c^12*d^6 - 2640
*a^15*b^7*c^14*d^4 - 7872*a^16*b^6*c^3*d^15 + 41712*a^16*b^6*c^5*d^13 - 804
48*a^16*b^6*c^7*d^11 + 70224*a^16*b^6*c^9*d^9 - 27456*a^16*b^6*c^11*d^7 + 3
696*a^16*b^6*c^13*d^5 + 1496*a^17*b^5*c^2*d^16 - 14608*a^17*b^5*c^4*d^14 +
37532*a^17*b^5*c^6*d^12 - 40920*a^17*b^5*c^8*d^10 + 20196*a^17*b^5*c^10*d^8
 - 3696*a^17*b^5*c^12*d^6 + 3888*a^18*b^4*c^3*d^15 - 13748*a^18*b^4*c^5*d^1
3 + 19016*a^18*b^4*c^7*d^11 - 11660*a^18*b^4*c^9*d^9 + 2640*a^18*b^4*c^11*d
^7 - 704*a^19*b^3*c^2*d^16 + 3872*a^19*b^3*c^4*d^14 - 6952*a^19*b^3*c^6*d^1
2 + 5104*a^19*b^3*c^8*d^10 - 1320*a^19*b^3*c^10*d^8 - 832*a^20*b^2*c^3*d^15
 + 1912*a^20*b^2*c^5*d^13 - 1584*a^20*b^2*c^7*d^11 + 440*a^20*b^2*c^9*d^9))
/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^
13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^
15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d
^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^
12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a
^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^
10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*
b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c
^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^
4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 8
52*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^
6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^1
1*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^
6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^1
0 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 14

6*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9
*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c
^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*
d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 +
1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*
a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*
b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c
^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^
8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a
^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12))*(-(c + d)^3*(c - d)^3
)^(1/2)*(3*b*d^2 - 4*b*c^2 + a*c*d))/(a^4*d^10 - b^4*c^10 - 3*a^4*c^2*d^8 +
3*a^4*c^4*d^6 - a^4*c^6*d^4 + b^4*c^4*d^6 - 3*b^4*c^6*d^4 + 3*b^4*c^8*d^2
- 4*a*b^3*c^3*d^7 + 12*a*b^3*c^5*d^5 - 12*a*b^3*c^7*d^3 + 12*a^3*b*c^3*d^7
- 12*a^3*b*c^5*d^5 + 4*a^3*b*c^7*d^3 + 6*a^2*b^2*c^2*d^8 - 18*a^2*b^2*c^4*d
^6 + 18*a^2*b^2*c^6*d^4 - 6*a^2*b^2*c^8*d^2 + 4*a*b^3*c^9*d - 4*a^3*b*c*d^9
))*(3*b*d^2 - 4*b*c^2 + a*c*d))/(a^4*d^10 - b^4*c^10 - 3*a^4*c^2*d^8 + 3*a^
4*c^4*d^6 - a^4*c^6*d^4 + b^4*c^4*d^6 - 3*b^4*c^6*d^4 + 3*b^4*c^8*d^2 - 4*a
*b^3*c^3*d^7 + 12*a*b^3*c^5*d^5 - 12*a*b^3*c^7*d^3 + 12*a^3*b*c^3*d^7 - 12*
a^3*b*c^5*d^5 + 4*a^3*b*c^7*d^3 + 6*a^2*b^2*c^2*d^8 - 18*a^2*b^2*c^4*d^6 +
18*a^2*b^2*c^6*d^4 - 6*a^2*b^2*c^8*d^2 + 4*a*b^3*c^9*d - 4*a^3*b*c*d^9))*(3
*b*d^2 - 4*b*c^2 + a*c*d))/(a^4*d^10 - b^4*c^10 - 3*a^4*c^2*d^8 + 3*a^4*c^4
*d^6 - a^4*c^6*d^4 + b^4*c^4*d^6 - 3*b^4*c^6*d^4 + 3*b^4*c^8*d^2 - 4*a*b^3*
c^3*d^7 + 12*a*b^3*c^5*d^5 - 12*a*b^3*c^7*d^3 + 12*a^3*b*c^3*d^7 - 12*a^3*b
*c^5*d^5 + 4*a^3*b*c^7*d^3 + 6*a^2*b^2*c^2*d^8 - 18*a^2*b^2*c^4*d^6 + 18*a^
2*b^2*c^6*d^4 - 6*a^2*b^2*c^8*d^2 + 4*a*b^3*c^9*d - 4*a^3*b*c*d^9)))*(-(c +
d)^3*(c - d)^3)^(1/2)*(3*b*d^2 - 4*b*c^2 + a*c*d)*2i)/(f*(a^4*d^10 - b^4*c
^10 - 3*a^4*c^2*d^8 + 3*a^4*c^4*d^6 - a^4*c^6*d^4 + b^4*c^4*d^6 - 3*b^4*c^6
*d^4 + 3*b^4*c^8*d^2 - 4*a*b^3*c^3*d^7 + 12*a*b^3*c^5*d^5 - 12*a*b^3*c^7*d^
3 + 12*a^3*b*c^3*d^7 - 12*a^3*b*c^5*d^5 + 4*a^3*b*c^7*d^3 + 6*a^2*b^2*c^2*d
^8 - 18*a^2*b^2*c^4*d^6 + 18*a^2*b^2*c^6*d^4 - 6*a^2*b^2*c^8*d^2 + 4*a*b^3*
c^9*d - 4*a^3*b*c*d^9)) + (b^2*atan(((b^2*(-(a + b)^5*(a - b)^5)^(1/2)*((8*
tan(e/2 + (f*x)/2)*(4*a^16*c^3*d^11 - 4*a^3*b^13*c^14 - 4*a^5*b^11*c^14 - a
*b^15*c^14 + 144*a*b^15*c^4*d^10 - 348*a*b^15*c^6*d^8 + 214*a*b^15*c^8*d^6
+ 7*a*b^15*c^10*d^4 - 8*a*b^15*c^12*d^2 - a^2*b^14*c^13*d - 144*a^4*b^12*c*
d^13 + 20*a^4*b^12*c^13*d + 684*a^6*b^10*c*d^13 + 44*a^6*b^10*c^13*d - 1314
*a^8*b^8*c*d^13 + 1224*a^10*b^6*c*d^13 - 504*a^12*b^4*c*d^13 + 36*a^14*b^2*
c*d^13 + 24*a^15*b*c^2*d^12 - 44*a^15*b*c^4*d^10 - 432*a^2*b^14*c^3*d^11 +
1140*a^2*b^14*c^5*d^9 - 818*a^2*b^14*c^7*d^7 + 55*a^2*b^14*c^9*d^5 + 16*a^2
*b^14*c^11*d^3 + 432*a^3*b^13*c^2*d^12 - 2016*a^3*b^13*c^4*d^10 + 2938*a^3*
b^13*c^6*d^8 - 1485*a^3*b^13*c^8*d^6 + 152*a^3*b^13*c^10*d^4 + 27*a^3*b^13*
c^12*d^2 + 2688*a^4*b^12*c^3*d^11 - 6574*a^4*b^12*c^5*d^9 + 5107*a^4*b^12*c
^7*d^7 - 1056*a^4*b^12*c^9*d^5 + 59*a^4*b^12*c^11*d^3 - 2148*a^5*b^11*c^2*d
^12 + 8378*a^5*b^11*c^4*d^10 - 10619*a^5*b^11*c^6*d^8 + 5064*a^5*b^11*c^8*d
^6 - 975*a^5*b^11*c^10*d^4 + 48*a^5*b^11*c^12*d^2 - 7294*a^6*b^10*c^3*d^11
+ 16053*a^6*b^10*c^5*d^9 - 12464*a^6*b^10*c^7*d^7 + 3649*a^6*b^10*c^9*d^5 -

640*a^6*b^10*c^11*d^3 + 4470*a^7*b^9*c^2*d^12 - 15815*a^7*b^9*c^4*d^10 + 1
8608*a^7*b^9*c^6*d^8 - 8939*a^7*b^9*c^8*d^6 + 2300*a^7*b^9*c^10*d^4 - 220*a
^7*b^9*c^12*d^2 + 10105*a^8*b^8*c^3*d^11 - 19912*a^8*b^8*c^5*d^9 + 14693*a^
8*b^8*c^7*d^7 - 4524*a^8*b^8*c^9*d^5 + 628*a^8*b^8*c^11*d^3 - 4632*a^9*b^7*
c^2*d^12 + 14976*a^9*b^7*c^4*d^10 - 15576*a^9*b^7*c^6*d^8 + 6104*a^9*b^7*c^
8*d^6 - 1088*a^9*b^7*c^10*d^4 - 7104*a^10*b^6*c^3*d^11 + 11320*a^10*b^6*c^5
*d^9 - 6184*a^10*b^6*c^7*d^7 + 1120*a^10*b^6*c^9*d^5 + 2232*a^11*b^5*c^2*d^
12 - 5932*a^11*b^5*c^4*d^10 + 4344*a^11*b^5*c^6*d^8 - 688*a^11*b^5*c^8*d^6
+ 1892*a^12*b^4*c^3*d^11 - 1920*a^12*b^4*c^5*d^9 + 368*a^12*b^4*c^7*d^7 - 2
52*a^13*b^3*c^2*d^12 + 624*a^13*b^3*c^4*d^10 - 292*a^13*b^3*c^6*d^8 - 192*a
^14*b^2*c^3*d^11 + 172*a^14*b^2*c^5*d^9))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^
15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 -
4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^
17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*
c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9
*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^1
2 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^1
5*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^
7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 3
96*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^
5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12
*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7
*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11
- 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 57
6*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^
8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c
^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^
5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 -
1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a
^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*
b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c
^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^1
1 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 15
6*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*
b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d
- 9*a^16*b*c*d^12) - (8*(60*a*b^15*c^7*d^7 - 36*a*b^15*c^5*d^9 - 13*a*b^15*
c^9*d^5 - 10*a*b^15*c^11*d^3 - 4*a^3*b^13*c^13*d + 36*a^5*b^11*c*d^13 - 4*a
^5*b^11*c^13*d - 144*a^7*b^9*c*d^13 + 216*a^9*b^7*c*d^13 - 144*a^11*b^5*c*d
^13 + 36*a^13*b^3*c*d^13 + 4*a^15*b*c^3*d^11 + 72*a^2*b^14*c^4*d^10 - 108*a
^2*b^14*c^6*d^8 + 19*a^2*b^14*c^8*d^6 + 14*a^2*b^14*c^10*d^4 - a^2*b^14*c^1
2*d^2 + 120*a^3*b^13*c^5*d^9 - 305*a^3*b^13*c^7*d^7 + 190*a^3*b^13*c^9*d^5
+ 19*a^3*b^13*c^11*d^3 - 72*a^4*b^12*c^2*d^12 - 168*a^4*b^12*c^4*d^10 + 699
*a^4*b^12*c^6*d^8 - 602*a^4*b^12*c^8*d^6 + 99*a^4*b^12*c^10*d^4 + 20*a^4*b^
12*c^12*d^2 - 36*a^5*b^11*c^3*d^11 - 535*a^5*b^11*c^5*d^9 + 1354*a^5*b^11*c
^7*d^7 - 895*a^5*b^11*c^9*d^5 + 40*a^5*b^11*c^11*d^3 + 276*a^6*b^10*c^2*d^1

2 + 233*a^6*b^10*c^4*d^10 - 2046*a^6*b^10*c^6*d^8 + 2161*a^6*b^10*c^8*d^6 - 552*a^6*b^10*c^10*d^4 + 44*a^6*b^10*c^12*d^2 + 61*a^7*b^9*c^3*d^11 + 1386*a^7*b^9*c^5*d^9 - 2979*a^7*b^9*c^7*d^7 + 1860*a^7*b^9*c^9*d^5 - 220*a^7*b^9*c^11*d^3 - 375*a^8*b^8*c^2*d^12 - 270*a^8*b^8*c^4*d^10 + 2885*a^8*b^8*c^6*d^8 - 3012*a^8*b^8*c^8*d^6 + 628*a^8*b^8*c^10*d^4 - 88*a^9*b^7*c^3*d^11 - 1544*a^9*b^7*c^5*d^9 + 2648*a^9*b^7*c^7*d^7 - 1088*a^9*b^7*c^9*d^5 + 216*a^10*b^6*c^2*d^12 + 100*a^10*b^6*c^4*d^10 - 1336*a^10*b^6*c^6*d^8 + 1056*a^10*b^6*c^8*d^6 + 180*a^11*b^5*c^3*d^11 + 248*a^11*b^5*c^5*d^9 - 400*a^11*b^5*c^7*d^7 - 60*a^12*b^4*c^2*d^12 + 248*a^12*b^4*c^4*d^10 - 148*a^12*b^4*c^6*d^8 - 184*a^13*b^3*c^3*d^11 + 172*a^13*b^3*c^5*d^9 + 24*a^14*b^2*c^2*d^12 - 44*a^14*b^2*c^4*d^10 - a*b^15*c^13*d))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12) + (b^2*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(2*a^2*b^17*c^16 - 6*a^6*b^13*c^16 + 4*a^8*b^11*c^16 + 4*a^19*c^3*d^13 - 4*a^19*c^5*d^11 + 12*a*b^18*c^9*d^7 - 28*a*b^18*c^11*d^5 + 16*a*b^18*c^13*d^3 - 10*a^3*b^16*c^15*d - 24*a^5*b^14*c^15*d + 78*a^7*b^12*c^15*d + 12*a^9*b^10*c*d^15 - 44*a^9*b^10*c^15*d - 54*a^11*b^8*c*d^15 + 96*a^13*b^6*c*d^15 - 78*a^15*b^4*c*d^15 + 24*a^17*b^2*c*d^15 + 12*a^18*b*c^2*d^14 - 56*a^18*b*c^4*d^12 + 44*a^18*b*c^6*d^10 - 96*a^2*b^17*c^8*d^8 + 234*a^2*b^17*c^10*d^6 - 146*a^2*b^17*c^12*d^4 + 6*a^2*b^17*c^14*d^2 + 336*a^3*b^16*c^7*d^9 - 918*a^3*b^16*c^9*d^7 + 726*a^3*b^16*c^11*d^5 - 134*a^3*b^16*c^13*d^3 - 672*a^4*b^15*c^6*d^10 + 2280*a^4*b^15*c^8*d^8 - 2520*a^4*b^15*c^10*d^6 + 952*a^4*b^15*c^12*d^4 - 40*a^4*

b^15*c^14*d^2 + 840*a^5*b^14*c^5*d^11 - 4032*a^5*b^14*c^7*d^9 + 6360*a^5*b^14*c^9*d^7 - 3768*a^5*b^14*c^11*d^5 + 624*a^5*b^14*c^13*d^3 - 672*a^6*b^13*c^4*d^12 + 5292*a^6*b^13*c^6*d^10 - 11772*a^6*b^13*c^8*d^8 + 10050*a^6*b^13*c^10*d^6 - 3174*a^6*b^13*c^12*d^4 + 282*a^6*b^13*c^14*d^2 + 336*a^7*b^12*c^3*d^13 - 5124*a^7*b^12*c^5*d^11 + 16212*a^7*b^12*c^7*d^9 - 19602*a^7*b^12*c^9*d^7 + 9670*a^7*b^12*c^11*d^5 - 1570*a^7*b^12*c^13*d^3 - 96*a^8*b^11*c^2*d^14 + 3528*a^8*b^11*c^4*d^12 - 16872*a^8*b^11*c^6*d^10 + 28848*a^8*b^11*c^8*d^8 - 20340*a^8*b^11*c^10*d^6 + 5396*a^8*b^11*c^12*d^4 - 468*a^8*b^11*c^14*d^2 - 1620*a^9*b^10*c^3*d^13 + 13320*a^9*b^10*c^5*d^11 - 32304*a^9*b^10*c^7*d^9 + 31560*a^9*b^10*c^9*d^7 - 12648*a^9*b^10*c^11*d^5 + 1724*a^9*b^10*c^13*d^3 + 442*a^10*b^9*c^2*d^14 - 7810*a^10*b^9*c^4*d^12 + 27546*a^10*b^9*c^6*d^10 - 37338*a^10*b^9*c^8*d^8 + 21288*a^10*b^9*c^10*d^6 - 4348*a^10*b^9*c^12*d^4 + 220*a^10*b^9*c^14*d^2 + 3206*a^11*b^8*c^3*d^13 - 17850*a^11*b^8*c^5*d^11 + 34018*a^11*b^8*c^7*d^9 - 26556*a^11*b^8*c^9*d^7 + 7896*a^11*b^8*c^11*d^5 - 660*a^11*b^8*c^13*d^3 - 816*a^12*b^7*c^2*d^14 + 8696*a^12*b^7*c^4*d^12 - 23696*a^12*b^7*c^6*d^10 + 25056*a^12*b^7*c^8*d^8 - 10560*a^12*b^7*c^10*d^6 + 1320*a^12*b^7*c^12*d^4 - 3064*a^13*b^6*c^3*d^13 + 12400*a^13*b^6*c^5*d^11 - 18048*a^13*b^6*c^7*d^9 + 10464*a^13*b^6*c^9*d^7 - 1848*a^13*b^6*c^11*d^5 + 702*a^14*b^5*c^2*d^14 - 4770*a^14*b^5*c^4*d^12 + 9858*a^14*b^5*c^6*d^10 - 7638*a^14*b^5*c^8*d^8 + 1848*a^14*b^5*c^10*d^6 + 1314*a^15*b^4*c^3*d^13 - 3954*a^15*b^4*c^5*d^11 + 4038*a^15*b^4*c^7*d^9 - 1320*a^15*b^4*c^9*d^7 - 244*a^16*b^3*c^2*d^14 + 1084*a^16*b^3*c^4*d^12 - 1500*a^16*b^3*c^6*d^10 + 660*a^16*b^3*c^8*d^8 - 176*a^17*b^2*c^3*d^13 + 372*a^17*b^2*c^5*d^11 - 220*a^17*b^2*c^7*d^9))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^

4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12) + (8*tan(e/2 + (f*x)/2)*(4*a*b^18*c^16 - 12*a^5*b^14*c^16 + 8*a^7*b^12*c^16 + 8*a^19*c^2*d^14 - 8*a^19*c^4*d^12 + 12*a*b^18*c^10*d^6 - 28*a*b^18*c^12*d^4 + 12*a*b^18*c^14*d^2 - 20*a^2*b^17*c^15*d - 48*a^4*b^15*c^15*d + 156*a^6*b^13*c^15*d - 88*a^8*b^11*c^15*d + 12*a^10*b^9*c*d^15 - 48*a^12*b^7*c*d^15 + 84*a^14*b^5*c*d^15 - 72*a^16*b^3*c*d^15 - 112*a^18*b*c^3*d^13 + 88*a^18*b*c^5*d^11 - 84*a^2*b^17*c^9*d^7 + 212*a^2*b^17*c^11*d^5 - 108*a^2*b^17*c^13*d^3 + 240*a^3*b^16*c^8*d^8 - 744*a^3*b^16*c^10*d^6 + 584*a^3*b^16*c^12*d^4 - 80*a^3*b^16*c^14*d^2 - 336*a^4*b^15*c^7*d^9 + 1632*a^4*b^15*c^9*d^7 - 2176*a^4*b^15*c^11*d^5 + 928*a^4*b^15*c^13*d^3 + 168*a^5*b^14*c^6*d^10 - 2472*a^5*b^14*c^8*d^8 + 5460*a^5*b^14*c^10*d^6 - 3708*a^5*b^14*c^12*d^4 + 564*a^5*b^14*c^14*d^2 + 168*a^6*b^13*c^5*d^11 + 2520*a^6*b^13*c^7*d^9 - 9204*a^6*b^13*c^9*d^7 + 9180*a^6*b^13*c^11*d^5 - 2820*a^6*b^13*c^13*d^3 - 336*a^7*b^12*c^4*d^12 - 1344*a^7*b^12*c^6*d^10 + 10416*a^7*b^12*c^8*d^8 - 15960*a^7*b^12*c^10*d^6 + 8152*a^7*b^12*c^12*d^4 - 936*a^7*b^12*c^14*d^2 + 240*a^8*b^11*c^3*d^13 - 336*a^8*b^11*c^5*d^11 - 7488*a^8*b^11*c^7*d^9 + 19800*a^8*b^11*c^9*d^7 - 15416*a^8*b^11*c^11*d^5 + 3288*a^8*b^11*c^13*d^3 - 84*a^9*b^10*c^2*d^14 + 1188*a^9*b^10*c^4*d^12 + 2292*a^9*b^10*c^6*d^10 - 16596*a^9*b^10*c^8*d^8 + 20136*a^9*b^10*c^10*d^6 - 7376*a^9*b^10*c^12*d^4 + 440*a^9*b^10*c^14*d^2 - 908*a^10*b^9*c^3*d^13 + 1740*a^10*b^9*c^5*d^11 + 7556*a^10*b^9*c^7*d^9 - 18048*a^10*b^9*c^9*d^7 + 10936*a^10*b^9*c^11*d^5 - 1288*a^10*b^9*c^13*d^3 + 328*a^11*b^8*c^2*d^14 - 2808*a^11*b^8*c^4*d^12 + 1088*a^11*b^8*c^6*d^10 + 9600*a^11*b^8*c^8*d^8 - 10584*a^11*b^8*c^10*d^6 + 2376*a^11*b^8*c^12*d^4 + 1792*a^12*b^7*c^3*d^13 - 4720*a^12*b^7*c^5*d^11 - 144*a^12*b^7*c^7*d^9 + 5856*a^12*b^7*c^9*d^7 - 2736*a^12*b^7*c^11*d^5 - 596*a^13*b^6*c^2*d^14 + 3980*a^13*b^6*c^4*d^12 - 4908*a^13*b^6*c^6*d^10 - 156*a^13*b^6*c^8*d^8 + 1680*a^13*b^6*c^10*d^6 - 1932*a^14*b^5*c^3*d^13 + 4812*a^14*b^5*c^5*d^11 - 3012*a^14*b^5*c^7*d^9 + 48*a^14*b^5*c^9*d^7 + 552*a^15*b^4*c^2*d^14 - 2616*a^15*b^4*c^4*d^12 + 3096*a^15*b^4*c^6*d^10 - 1032*a^15*b^4*c^8*d^8 + 920*a^16*b^3*c^3*d^13 - 1752*a^16*b^3*c^5*d^11 + 904*a^16*b^3*c^7*d^9 - 208*a^17*b^2*c^2*d^14 + 600*a^17*b^2*c^4*d^12 - 392*a^17*b^2*c^6*d^10 + 24*a^18*b*c*d^15))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^

4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12) - (b^2*((8*(16*a^4*b^18*c^18 - 4*a^2*b^20*c^18 - 24*a^6*b^16*c^18 + 16*a^8*b^14*c^18 - 4*a^10*b^12*c^18 + 4*a^22*c^2*d^16 - 8*a^22*c^4*d^14 + 4*a^22*c^6*d^12 + 4*a*b^21*c^13*d^5 - 8*a*b^21*c^15*d^3 + 24*a^3*b^19*c^17*d - 136*a^5*b^17*c^17*d + 224*a^7*b^15*c^17*d - 156*a^9*b^13*c^17*d + 40*a^11*b^11*c^17*d - 4*a^13*b^9*c*d^17 + 16*a^15*b^7*c*d^17 - 24*a^17*b^5*c*d^17 + 16*a^19*b^3*c*d^17 - 32*a^21*b*c^3*d^15 + 76*a^21*b*c^5*d^13 - 40*a^21*b*c^7*d^11 - 40*a^2*b^20*c^12*d^6 + 76*a^2*b^20*c^14*d^4 - 32*a^2*b^20*c^16*d^2 + 176*a^3*b^19*c^11*d^7 - 328*a^3*b^19*c^13*d^5 + 128*a^3*b^19*c^15*d^3 - 440*a^4*b^18*c^10*d^8 + 864*a^4*b^18*c^12*d^6 - 392*a^4*b^18*c^14*d^4 - 48*a^4*b^18*c^16*d^2 + 660*a^5*b^17*c^9*d^9 - 1584*a^5*b^17*c^11*d^7 + 1052*a^5*b^17*c^13*d^5 + 8*a^5*b^17*c^15*d^3 - 528*a^6*b^16*c^8*d^10 + 2156*a^6*b^16*c^10*d^8 - 2264*a^6*b^16*c^12*d^6 + 148*a^6*b^16*c^14*d^4 + 512*a^6*b^16*c^16*d^2 - 2112*a^7*b^15*c^9*d^9 + 3520*a^7*b^15*c^11*d^7 - 480*a^7*b^15*c^13*d^5 - 1152*a^7*b^15*c^15*d^3 + 528*a^8*b^14*c^6*d^12 + 1056*a^8*b^14*c^8*d^10 - 3696*a^8*b^14*c^10*d^8 + 1216*a^8*b^14*c^12*d^6 + 1808*a^8*b^14*c^14*d^4 - 928*a^8*b^14*c^16*d^2 - 660*a^9*b^13*c^5*d^13 + 792*a^9*b^13*c^7*d^11 + 2244*a^9*b^13*c^9*d^9 - 2288*a^9*b^13*c^11*d^7 - 2180*a^9*b^13*c^13*d^5 + 2248*a^9*b^13*c^15*d^3 + 440*a^10*b^12*c^4*d^14 - 2332*a^10*b^12*c^6*d^12 + 176*a^10*b^12*c^8*d^10 + 2684*a^10*b^12*c^10*d^8 + 1896*a^10*b^12*c^12*d^6 - 3532*a^10*b^12*c^14*d^4 + 672*a^10*b^12*c^16*d^2 - 176*a^11*b^11*c^3*d^15 + 2552*a^11*b^11*c^5*d^13 - 2464*a^11*b^11*c^7*d^11 - 1496*a^11*b^11*c^9*d^9 - 528*a^11*b^11*c^11*d^7 + 3736*a^11*b^11*c^13*d^5 - 1664*a^11*b^11*c^15*d^3 + 40*a^12*b^10*c^2*d^16 - 1664*a^12*b^10*c^4*d^14 + 3736*a^12*b^10*c^6*d^12 - 528*a^12*b^10*c^8*d^10 - 1496*a^12*b^10*c^10*d^8 - 2464*a^12*b^10*c^12*d^6 + 2552*a^12*b^10*c^14*d^4 - 176*a^12*b^10*c^16*d^2 + 672*a^13*b^9*c^3*d^15 - 3532*a^13*b^9*c^5*d^13 + 1896*a^13*b^9*c^7*d^11 + 2684*a^13*b^9*c^9*d^9 + 176*a^13*b^9*c^11*d^7 - 2332*a^13*b^9*c^13*d^5 + 440*a^13*b^9*c^15*d^3 - 156*a^14*b^8*c^2*d^16 + 2248*a^14*b^8*c^4*d^14 - 2180*a^14*b^8*c^6*d^12 - 2288*a^14*b^8*c^8*d^10 + 2244*a^14*b^8*c^10*d^8 + 792*a^14*b^8*c^12*d^6 - 660*a^14*b^8*c^14*d^4 - 928*a^15*b^7*c^3*d^15 + 1808*a^15*b^7*c^5*d^13 + 1216*a^15*b^7*c^7*d^11 - 3696*a^15*b^7*c^9*d^9 + 1056*a^15*b^7*c^11*d^7 + 528*a^15*b^7*c^13*d^5 + 224*a^16*b^6*c^2*d^16 - 1152*a^16*b^6*c^4*d^14 - 480*a

^16*b^6*c^6*d^12 + 3520*a^16*b^6*c^8*d^10 - 2112*a^16*b^6*c^10*d^8 + 512*a^17*b^5*c^3*d^15 + 148*a^17*b^5*c^5*d^13 - 2264*a^17*b^5*c^7*d^11 + 2156*a^17*b^5*c^9*d^9 - 528*a^17*b^5*c^11*d^7 - 136*a^18*b^4*c^2*d^16 + 8*a^18*b^4*c^4*d^14 + 1052*a^18*b^4*c^6*d^12 - 1584*a^18*b^4*c^8*d^10 + 660*a^18*b^4*c^10*d^8 - 48*a^19*b^3*c^3*d^15 - 392*a^19*b^3*c^5*d^13 + 864*a^19*b^3*c^7*d^11 - 440*a^19*b^3*c^9*d^9 + 24*a^20*b^2*c^2*d^16 + 128*a^20*b^2*c^4*d^14 - 328*a^20*b^2*c^6*d^12 + 176*a^20*b^2*c^8*d^10 + 4*a*b^21*c^17*d - 4*a^21*b*c*d^17))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12) - (8*tan(e/2 + (f*x)/2)*(12*a*b^21*c^18 - 12*a^22*c*d^17 - 56*a^3*b^19*c^18 + 104*a^5*b^17*c^18 - 96*a^7*b^15*c^18 + 44*a^9*b^13*c^18 - 8*a^11*b^11*c^18 + 32*a^22*c^3*d^15 - 28*a^22*c^5*d^13 + 8*a^22*c^7*d^11 - 16*a*b^21*c^12*d^6 + 44*a*b^21*c^14*d^4 - 40*a*b^21*c^16*d^2 - 132*a^2*b^20*c^17*d + 616*a^4*b^18*c^17*d - 1144*a^6*b^16*c^17*d + 1056*a^8*b^14*c^17*d - 484*a^10*b^12*c^17*d + 16*a^12*b^10*c*d^17 + 88*a^12*b^10*c^17*d - 76*a^14*b^8*c*d^17 + 144*a^16*b^6*c*d^17 - 136*a^18*b^4*c*d^17 + 64*a^20*b^2*c*d^17 + 132*a^21*b*c^2*d^16 - 352*a^21*b*c^4*d^14 + 308*a^21*b*c^6*d^12 - 88*a^21*b*c^8*d^10 + 176*a^2*b^20*c^11*d^7 - 484*a^2*b^20*c^13*d^5 + 440*a^2*b^20*c^15*d^3 - 880*a^3*b^19*c^10*d^8 + 2496*a^3*b^19*c^12*d^6 - 2408*a^3*b^19*c^14*d^4 + 848*a^3*b^19*c^16*d^2 + 2640*a^4*b^18*c^9*d^9 - 8096*a^4*b^18*c^11*d^7 + 8888*a^4*b^18*c^13*d^5 - 4048*a^4*b^18*c^15*d^3 - 5280*a^5*b^17*c^8*d^10 + 18700*a^5*b^17*c^10*d^8 - 24784*a^5*b^17*c^12*d^6 + 14692*a^5*b^17*c^14*d^4 - 3432*a^5*b^1

7*c^16*d^2 + 7392*a^6*b^16*c^7*d^11 - 32868*a^6*b^16*c^9*d^9 + 54384*a^6*b^16*c^11*d^7 - 40876*a^6*b^16*c^13*d^5 + 13112*a^6*b^16*c^15*d^3 - 7392*a^7*b^15*c^6*d^12 + 45408*a^7*b^15*c^8*d^10 - 95040*a^7*b^15*c^10*d^8 + 89280*a^7*b^15*c^12*d^6 - 38208*a^7*b^15*c^14*d^4 + 6048*a^7*b^15*c^16*d^2 + 5280*a^8*b^14*c^5*d^13 - 49632*a^8*b^14*c^7*d^11 + 133056*a^8*b^14*c^9*d^9 - 156992*a^8*b^14*c^11*d^7 + 88000*a^8*b^14*c^13*d^5 - 20768*a^8*b^14*c^15*d^3 - 2640*a^9*b^13*c^4*d^14 + 42372*a^9*b^13*c^6*d^12 - 150216*a^9*b^13*c^8*d^10 + 225676*a^9*b^13*c^10*d^8 - 162336*a^9*b^13*c^12*d^6 + 52532*a^9*b^13*c^14*d^4 - 5432*a^9*b^13*c^16*d^2 + 880*a^10*b^12*c^3*d^15 - 27500*a^10*b^12*c^5*d^13 + 137368*a^10*b^12*c^7*d^11 - 266244*a^10*b^12*c^9*d^9 + 242528*a^10*b^12*c^11*d^7 - 104060*a^10*b^12*c^13*d^5 + 17512*a^10*b^12*c^15*d^3 - 176*a^11*b^11*c^2*d^16 + 13024*a^11*b^11*c^4*d^14 - 101288*a^11*b^11*c^6*d^12 + 257136*a^11*b^11*c^8*d^10 - 296824*a^11*b^11*c^10*d^8 + 165760*a^11*b^11*c^12*d^6 - 40072*a^11*b^11*c^14*d^4 + 2448*a^11*b^11*c^16*d^2 - 4224*a^12*b^10*c^3*d^15 + 59000*a^12*b^10*c^5*d^13 - 202544*a^12*b^10*c^7*d^11 + 299816*a^12*b^10*c^9*d^9 - 214368*a^12*b^10*c^11*d^7 + 69784*a^12*b^10*c^13*d^5 - 7568*a^12*b^10*c^15*d^3 + 836*a^13*b^9*c^2*d^16 - 26048*a^13*b^9*c^4*d^14 + 129580*a^13*b^9*c^6*d^12 - 249832*a^13*b^9*c^8*d^10 + 226116*a^13*b^9*c^10*d^8 - 96272*a^13*b^9*c^12*d^6 + 16060*a^13*b^9*c^14*d^4 - 440*a^13*b^9*c^16*d^2 + 8128*a^14*b^8*c^3*d^15 - 66628*a^14*b^8*c^5*d^13 + 170424*a^14*b^8*c^7*d^11 - 195404*a^14*b^8*c^9*d^9 + 107184*a^14*b^8*c^11*d^7 - 24948*a^14*b^8*c^13*d^5 + 1320*a^14*b^8*c^15*d^3 - 1584*a^15*b^7*c^2*d^16 + 26752*a^15*b^7*c^4*d^14 - 94160*a^15*b^7*c^6*d^12 + 138688*a^15*b^7*c^8*d^10 - 96624*a^15*b^7*c^10*d^8 + 29568*a^15*b^7*c^12*d^6 - 2640*a^15*b^7*c^14*d^4 - 7872*a^16*b^6*c^3*d^15 + 41712*a^16*b^6*c^5*d^13 - 80448*a^16*b^6*c^7*d^11 + 70224*a^16*b^6*c^9*d^9 - 27456*a^16*b^6*c^11*d^7 + 3696*a^16*b^6*c^13*d^5 + 1496*a^17*b^5*c^2*d^16 - 14608*a^17*b^5*c^4*d^14 + 37532*a^17*b^5*c^6*d^12 - 40920*a^17*b^5*c^8*d^10 + 20196*a^17*b^5*c^10*d^8 - 3696*a^17*b^5*c^12*d^6 + 3888*a^18*b^4*c^3*d^15 - 13748*a^18*b^4*c^5*d^13 + 19016*a^18*b^4*c^7*d^11 - 11660*a^18*b^4*c^9*d^9 + 2640*a^18*b^4*c^11*d^7 - 704*a^19*b^3*c^2*d^16 + 3872*a^19*b^3*c^4*d^14 - 6952*a^19*b^3*c^6*d^12 + 5104*a^19*b^3*c^8*d^10 - 1320*a^19*b^3*c^10*d^8 - 832*a^20*b^2*c^3*d^15 + 1912*a^20*b^2*c^5*d^13 - 1584*a^20*b^2*c^7*d^11 + 440*a^20*b^2*c^9*d^9))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b

^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12))*(-(a + b)^5*(a - b)^5)^(1/2)*(12*a^4*d^2 + b^4*c^2 + 6*b^4*d^2 + 2*a^2*b^2*c^2 - 15*a^2*b^2*d^2 + 2*a*b^3*c*d - 8*a^3*b*c*d))/(2*(a^14*d^4 - b^14*c^4 + 5*a^2*b^12*c^4 - 10*a^4*b^10*c^4 + 10*a^6*b^8*c^4 - 5*a^8*b^6*c^4 + a^10*b^4*c^4 - a^4*b^10*d^4 + 5*a^6*b^8*d^4 - 10*a^8*b^6*d^4 + 10*a^10*b^4*d^4 - 5*a^12*b^2*d^4 + 4*a^3*b^11*c*d^3 - 20*a^3*b^11*c^3*d - 20*a^5*b^9*c*d^3 + 40*a^5*b^9*c^3*d + 40*a^7*b^7*c*d^3 - 40*a^7*b^7*c^3*d - 40*a^9*b^5*c*d^3 + 20*a^9*b^5*c^3*d + 20*a^11*b^3*c*d^3 - 4*a^11*b^3*c^3*d - 6*a^2*b^12*c^2*d^2 + 30*a^4*b^10*c^2*d^2 - 60*a^6*b^8*c^2*d^2 + 60*a^8*b^6*c^2*d^2 - 30*a^10*b^4*c^2*d^2 + 6*a^12*b^2*c^2*d^2 + 4*a*b^13*c^3*d - 4*a^13*b*c*d^3)))*(12*a^4*d^2 + b^4*c^2 + 6*b^4*d^2 + 2*a^2*b^2*c^2 - 15*a^2*b^2*d^2 + 2*a*b^3*c*d - 8*a^3*b*c*d))/(2*(a^14*d^4 - b^14*c^4 + 5*a^2*b^12*c^4 - 10*a^4*b^10*c^4 + 10*a^6*b^8*c^4 - 5*a^8*b^6*c^4 + a^10*b^4*c^4 - a^4*b^10*d^4 + 5*a^6*b^8*d^4 - 10*a^8*b^6*d^4 + 10*a^10*b^4*d^4 - 5*a^12*b^2*d^4 + 4*a^3*b^11*c*d^3 - 20*a^3*b^11*c^3*d - 20*a^5*b^9*c*d^3 + 40*a^5*b^9*c^3*d + 40*a^7*b^7*c*d^3 - 40*a^7*b^7*c^3*d - 40*a^9*b^5*c*d^3 + 20*a^9*b^5*c^3*d + 20*a^11*b^3*c*d^3 - 4*a^11*b^3*c^3*d - 6*a^2*b^12*c^2*d^2 + 30*a^4*b^10*c^2*d^2 - 60*a^6*b^8*c^2*d^2 + 60*a^8*b^6*c^2*d^2 - 30*a^10*b^4*c^2*d^2 + 6*a^12*b^2*c^2*d^2 + 4*a*b^13*c^3*d - 4*a^13*b*c*d^3)))*(12*a^4*d^2 + b^4*c^2 + 6*b^4*d^2 + 2*a^2*b^2*c^2 - 15*a^2*b^2*d^2 + 2*a*b^3*c*d - 8*a^3*b*c*d)*1i)/(2*(a^14*d^4 - b^14*c^4 + 5*a^2*b^12*c^4 - 10*a^4*b^10*c^4 + 10*a^6*b^8*c^4 - 5*a^8*b^6*c^4 + a^10*b^4*c^4 - a^4*b^10*d^4 + 5*a^6*b^8*d^4 - 10*a^8*b^6*d^4 + 10*a^10*b^4*d^4 - 5*a^12*b^2*d^4 + 4*a^3*b^11*c*d^3 - 20*a^3*b^11*c^3*d - 20*a^5*b^9*c*d^3 + 40*a^5*b^9*c^3*d + 40*a^7*b^7*c*d^3 - 40*a^7*b^7*c^3*d - 40*a^9*b^5*c*d^3 + 20*a^9*b^5*c^3*d + 20*a^11*b^3*c*d^3 - 4*a^11*b^3*c^3*d - 6*a^2*b^12*c^2*d^2 + 30*a^4*b^10*c^2*d^2 - 60*a^6*b^8*c^2*d^2 + 60*a^8*b^6*c^2*d^2 - 30*a^10*b^4*c^2*d^2 + 6*a^12*b^2*c^2*d^2 + 4*a*b^13*c^3*d - 4*a^13*b*c*d^3)) - (b^2*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(60*a*b^15*c^7*d^7 - 36*a*b^15*c^5*d^9 - 13*a*b^15*c^9*d^5 - 10*a*b^15*c^11*d^3 - 4*a^3*b^13*c^13*d + 36*a^5*b^11*c*d^13 - 4*a^5*b^11*c^13*d - 144*a^7*b^9*c*d^13 + 216*a^9*b^7*c*d^13 - 144*a^11*b^5*c*d^13 + 36*a^13*b^3*c*d^13 + 4*a^15*b*c^3*d^11 + 72*a^2*b^14*c^4*d^10 - 108*a^2*b^14*c^6*d^8 + 19*a^2*b^14*c^8*d^6 + 14*a^2*b^14*c^10*d^4 - a^2*b^14*c^12*d^2 + 120*a^3*b^13*c^5*d^9 - 305*a^3*b^13*c^7*d^7 + 190*a^3*b^13*c^9*d^5 + 19*a^3*b^13 *c^11*d^3 - 72*a^4*b^12*c^2*d^12 - 168*a^4*b^12*c^4*d^10 + 699*a^4*b^12*c^6*d^8 - 602*a^4*b^12*c^8*d^6 + 99*a^4*b^12*c^10*d^4 + 20*a^4*b^12*c^12*d^2 - 36*a^5*b^11*c^3*d^11 - 535*a^5*b^11*c^5*d^9 + 1354*a^5*b^11*c^7*d^7 - 895*a^5*b^11*c^9*d^5 + 40*a^5*b^11*c^11*d^3 + 276*a^6*b^10*c^2*d^12 + 233*a^6*b^10*c^4*d^10 - 2046*a^6*b^10*c^6*d^8 + 2161*a^6*b^10*c^8*d^6 - 552*a^6*b^10*c^10*d^4 + 44*a^6*b^10*c^12*d^2 + 61*a^7*b^9*c^3*d^11 + 1386*a^7*b^9*c^5*d^9 - 2979*a^7*b^9*c^7*d^7 + 1860*a^7*b^9*c^9*d^5 - 220*a^7*b^9*c^11*d^3 - 375*a^8*b^8*c^2*d^12 - 270*a^8*b^8*c^4*d^10 + 2885*a^8*b^8*c^6*d^8 - 3012*a^8*b^8*c^8*d^6 + 628*a^8*b^8*c^10*d^4 - 88*a^9*b^7*c^3*d^11 - 1544*a^9*b^7*c^5*d^9 + 2648*a^9*b^7*c^7*d^7 - 1088*a^9*b^7*c^9*d^5 + 216*a^10*b^6*c^2*d^12 + 100*a^10*b^6*c^4*d^10 - 1336*a^10*b^6*c^6*d^8 + 1056*a^10*b^6*c^8*d^6 + 180*a^11*b^5*c^3*d^11 + 248*a^11*b^5*c^5*d^9 - 400*a^11*b^5*c^7*d^7 - 60*a^12*b^4*c^2*d^12 + 248*a^12*b^4*c^4*d^10 - 148*a^12*b^4*c^6*d^8 - 184*a^13*b^3*c^3*d^11 + 172*a^13*b^3*c^5*d^9 + 24*a^14*b^2*c^2*d^12 - 44*a^14*b^2*c^4*d^10 - a*b^15*c^13*d))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12) - (8*tan(e/2 + (f*x)/2)*(4*a^16*c^3*d^11 - 4*a^3*b^13*c^14 - 4*a^5*b^11*c^14 - a*b^15*c^14 + 144*a*b^15*c^4*d^10 - 348*a*b^15*c^6*d^8 + 214*a*b^15*c^8*d^6 + 7*a*b^15*c^10*d^4 - 8*a*b^15*c^12*d^2 - a^2*b^14*c^13*d - 144*a^4*b^12*c*d^13 + 20*a^4*b^12*c^13*d + 684*a^6*b^10*c*d^13 + 44*a^6*b^10*c^13*d - 1314*a^8*b^8*c*d^13 + 1224*a^10*b^6*c*d^13 - 504*a^12*b^4*c*d^13 + 36*a^14*b^2*c*d^13 + 24*a^15*b*c^2*d^12 - 44*a^15*b*c^4*d^10 - 432*a^2*b^14*c^3* d^11 + 1140*a^2*b^14*c^5*d^9 - 818*a^2*b^14*c^7*d^7 + 55*a^2*b^14*c^9*d^5 + 16*a^2*b^14*c^11*d^3 + 432*a^3*b^13*c^2*d^12 - 2016*a^3*b^13*c^4*d^10 + 2938*a^3*b^13*c^6*d^8 - 1485*a^3*b^13*c^8*d^6 + 152*a^3*b^13*c^10*d^4 + 27*a^3*b^13*c^12*d^2 + 2688*a^4*b^12*c^3*d^11 - 6574*a^4*b^12*c^5*d^9 + 5107*a^4*b^12*c^7*d^7 - 1056*a^4*b^12*c^9*d^5 + 59*a^4*b^12*c^11*d^3 - 2148*a^5*b^11*c^2*d^12 + 8378*a^5*b^11*c^4*d^10 - 10619*a^5*b^11*c^6*d^8 + 5064*a^5*b^11*c^8*d^6 - 975*a^5*b^11*c^10*d^4 + 48*a^5*b^11*c^12*d^2 - 7294*a^6*b^10*c^3*d^11 + 16053*a^6*b^10*c^5*d^9 - 12464*a^6*b^10*c^7*d^7 + 3649*a^6*b^10*c^9*d^5 - 640*a^6*b^10*c^11*d^3 + 4470*a^7*b^9*c^2*d^12 - 15815*a^7*b^9*c^4*d^10 + 18608*a^7*b^9*c^6*d^8 - 8939*a^7*b^9*c^8*d^6 + 2300*a^7*b^9*c^10*d^4 - 220*a^7*b^9*c^12*d^2 + 10105*a^8*b^8*c^3*d^11 - 19912*a^8*b^8*c^5*d^9 + 14693*a^8*b^8*c^7*d^7 - 4524*a^8*b^8*c^9*d^5 + 628*a^8*b^8*c^11*d^3 - 4632*a^9*b^7*c^2*d^12 + 14976*a^9*b^7*c^4*d^10 - 15576*a^9*b^7*c^6*d^8 + 6104*a^9*b^7*c^8*d^6 - 1088*a^9*b^7*c^10*d^4 - 7104*a^10*b^6*c^3*d^11 + 11320*a^10*b^6*c^5*d^9 - 6184*a^10*b^6*c^7*d^7 + 1120*a^10*b^6*c^9*d^5 + 2232*a^11*b^5*c^2*d^12 - 5932*a^11*b^5*c^4*d^10 + 4344*a^11*b^5*c^6*d^8 - 688*a^11*b^5*c^8*d^6 + 1892*a^12*b^4*c^3*d^11 - 1920*a^12*b^4*c^5*d^9 + 368*a^12*b^4*c^7*d^7 - 252*a^13*b^3*c^2*d^12 + 624*a^13*b^3*c^4*d^10 - 292*a^13*b^3*c^6*d^8 - 192*a^14*b^2*c^3*d^11 + 172*a^14*b^2*c^5*d^9))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12) + (b^2*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(2*a^2*b^17*c^16 - 6*a^6*b^13*c^16 + 4*a^8*b^11*c^16 + 4*a^19*c^3*d^13 - 4*a^19*c^5*

d^11 + 12*a*b^18*c^9*d^7 - 28*a*b^18*c^11*d^5 + 16*a*b^18*c^13*d^3 - 10*a^3
*b^16*c^15*d - 24*a^5*b^14*c^15*d + 78*a^7*b^12*c^15*d + 12*a^9*b^10*c*d^15
 - 44*a^9*b^10*c^15*d - 54*a^11*b^8*c*d^15 + 96*a^13*b^6*c*d^15 - 78*a^15*b
^4*c*d^15 + 24*a^17*b^2*c*d^15 + 12*a^18*b*c^2*d^14 - 56*a^18*b*c^4*d^12 +
44*a^18*b*c^6*d^10 - 96*a^2*b^17*c^8*d^8 + 234*a^2*b^17*c^10*d^6 - 146*a^2*
b^17*c^12*d^4 + 6*a^2*b^17*c^14*d^2 + 336*a^3*b^16*c^7*d^9 - 918*a^3*b^16*c
^9*d^7 + 726*a^3*b^16*c^11*d^5 - 134*a^3*b^16*c^13*d^3 - 672*a^4*b^15*c^6*d
^10 + 2280*a^4*b^15*c^8*d^8 - 2520*a^4*b^15*c^10*d^6 + 952*a^4*b^15*c^12*d^
4 - 40*a^4*b^15*c^14*d^2 + 840*a^5*b^14*c^5*d^11 - 4032*a^5*b^14*c^7*d^9 +
6360*a^5*b^14*c^9*d^7 - 3768*a^5*b^14*c^11*d^5 + 624*a^5*b^14*c^13*d^3 - 67
2*a^6*b^13*c^4*d^12 + 5292*a^6*b^13*c^6*d^10 - 11772*a^6*b^13*c^8*d^8 + 100
50*a^6*b^13*c^10*d^6 - 3174*a^6*b^13*c^12*d^4 + 282*a^6*b^13*c^14*d^2 + 336
*a^7*b^12*c^3*d^13 - 5124*a^7*b^12*c^5*d^11 + 16212*a^7*b^12*c^7*d^9 - 1960
2*a^7*b^12*c^9*d^7 + 9670*a^7*b^12*c^11*d^5 - 1570*a^7*b^12*c^13*d^3 - 96*a
^8*b^11*c^2*d^14 + 3528*a^8*b^11*c^4*d^12 - 16872*a^8*b^11*c^6*d^10 + 28848
*a^8*b^11*c^8*d^8 - 20340*a^8*b^11*c^10*d^6 + 5396*a^8*b^11*c^12*d^4 - 468*
a^8*b^11*c^14*d^2 - 1620*a^9*b^10*c^3*d^13 + 13320*a^9*b^10*c^5*d^11 - 3230
4*a^9*b^10*c^7*d^9 + 31560*a^9*b^10*c^9*d^7 - 12648*a^9*b^10*c^11*d^5 + 172
4*a^9*b^10*c^13*d^3 + 442*a^10*b^9*c^2*d^14 - 7810*a^10*b^9*c^4*d^12 + 2754
6*a^10*b^9*c^6*d^10 - 37338*a^10*b^9*c^8*d^8 + 21288*a^10*b^9*c^10*d^6 - 43
48*a^10*b^9*c^12*d^4 + 220*a^10*b^9*c^14*d^2 + 3206*a^11*b^8*c^3*d^13 - 178
50*a^11*b^8*c^5*d^11 + 34018*a^11*b^8*c^7*d^9 - 26556*a^11*b^8*c^9*d^7 + 78
96*a^11*b^8*c^11*d^5 - 660*a^11*b^8*c^13*d^3 - 816*a^12*b^7*c^2*d^14 + 8696
*a^12*b^7*c^4*d^12 - 23696*a^12*b^7*c^6*d^10 + 25056*a^12*b^7*c^8*d^8 - 105
60*a^12*b^7*c^10*d^6 + 1320*a^12*b^7*c^12*d^4 - 3064*a^13*b^6*c^3*d^13 + 12
400*a^13*b^6*c^5*d^11 - 18048*a^13*b^6*c^7*d^9 + 10464*a^13*b^6*c^9*d^7 - 1
848*a^13*b^6*c^11*d^5 + 702*a^14*b^5*c^2*d^14 - 4770*a^14*b^5*c^4*d^12 + 98
58*a^14*b^5*c^6*d^10 - 7638*a^14*b^5*c^8*d^8 + 1848*a^14*b^5*c^10*d^6 + 131
4*a^15*b^4*c^3*d^13 - 3954*a^15*b^4*c^5*d^11 + 4038*a^15*b^4*c^7*d^9 - 1320
*a^15*b^4*c^9*d^7 - 244*a^16*b^3*c^2*d^14 + 1084*a^16*b^3*c^4*d^12 - 1500*a
^16*b^3*c^6*d^10 + 660*a^16*b^3*c^8*d^8 - 176*a^17*b^2*c^3*d^13 + 372*a^17*
b^2*c^5*d^11 - 220*a^17*b^2*c^7*d^9))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c
^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a
^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c
^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10
*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8
*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 +
36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^
7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 -
204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a
^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^
12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^1
0*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6
 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 57
6*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^

7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12) + (8*tan(e/2 + (f*x)/2)*(4*a*b^18*c^16 - 12*a^5*b^14*c^16 + 8*a^7*b^12*c^16 + 8*a^19*c^2*d^14 - 8*a^19*c^4*d^12 + 12*a*b^18*c^10*d^6 - 28*a*b^18*c^12*d^4 + 12*a*b^18*c^14*d^2 - 20*a^2*b^17*c^15*d - 48*a^4*b^15*c^15*d + 156*a^6*b^13*c^15*d - 88*a^8*b^11*c^15*d + 12*a^10*b^9*c*d^15 - 48*a^12*b^7*c*d^15 + 84*a^14*b^5*c*d^15 - 72*a^16*b^3*c*d^15 - 112*a^18*b*c^3*d^13 + 88*a^18*b*c^5*d^11 - 84*a^2*b^17*c^9*d^7 + 212*a^2*b^17*c^11*d^5 - 108*a^2*b^17*c^13*d^3 + 240*a^3*b^16*c^8*d^8 - 744*a^3*b^16*c^10*d^6 + 584*a^3*b^16*c^12*d^4 - 80*a^3*b^16*c^14*d^2 - 336*a^4*b^15*c^7*d^9 + 1632*a^4*b^15*c^9*d^7 - 2176*a^4*b^15*c^11*d^5 + 928*a^4*b^15*c^13*d^3 + 168*a^5*b^14*c^6*d^10 - 2472*a^5*b^14*c^8*d^8 + 5460*a^5*b^14*c^10*d^6 - 3708*a^5*b^14*c^12*d^4 + 564*a^5*b^14*c^14*d^2 + 168*a^6*b^13*c^5*d^11 + 2520*a^6*b^13*c^7*d^9 - 9204*a^6*b^13*c^9*d^7 + 9180*a^6*b^13*c^11*d^5 - 2820*a^6*b^13*c^13*d^3 - 336*a^7*b^12*c^4*d^12 - 1344*a^7*b^12*c^6*d^10 + 10416*a^7*b^12*c^8*d^8 - 15960*a^7*b^12*c^10*d^6 + 8152*a^7*b^12*c^12*d^4 - 936*a^7*b^12*c^14*d^2 + 240*a^8*b^11*c^3*d^13 - 336*a^8*b^11*c^5*d^11 - 7488*a^8*b^11*c^7*d^9 + 19800*a^8*b^11*c^9*d^7 - 15416*a^8*b^11*c^11*d^5 + 3288*a^8*b^11*c^13*d^3 - 84*a^9*b^10*c^2*d^14 + 1188*a^9*b^10*c^4*d^12 + 2292*a^9*b^10*c^6*d^10 - 16596*a^9*b^10*c^8*d^8 + 20136*a^9*b^10*c^10*d^6 - 7376*a^9*b^10*c^12*d^4 + 440*a^9*b^10*c^14*d^2 - 908*a^10*b^9*c^3*d^13 + 1740*a^10*b^9*c^5*d^11 + 7556*a^10*b^9*c^7*d^9 - 18048*a^10*b^9*c^9*d^7 + 10936*a^10*b^9*c^11*d^5 - 1288*a^10*b^9*c^13*d^3 + 328*a^11*b^8*c^2*d^14 - 2808*a^11*b^8*c^4*d^12 + 1088*a^11*b^8*c^6*d^10 + 9600*a^11*b^8*c^8*d^8 - 10584*a^11*b^8*c^10*d^6 + 2376*a^11*b^8*c^12*d^4 + 1792*a^12*b^7*c^3*d^13 - 4720*a^12*b^7*c^5*d^11 - 144*a^12*b^7*c^7*d^9 + 5856*a^12*b^7*c^9*d^7 - 2736*a^12*b^7*c^11*d^5 - 596*a^13*b^6*c^2*d^14 + 3980*a^13*b^6*c^4*d^12 - 4908*a^13*b^6*c^6*d^10 - 156*a^13*b^6*c^8*d^8 + 1680*a^13*b^6*c^10*d^6 - 1932*a^14*b^5*c^3*d^13 + 4812*a^14*b^5*c^5*d^11 - 3012*a^14*b^5*c^7*d^9 + 48*a^14*b^5*c^9*d^7 + 552*a^15*b^4*c^2*d^14 - 2616*a^15*b^4*c^4*d^12 + 3096*a^15*b^4*c^6*d^10 - 1032*a^15*b^4*c^8*d^8 + 920*a^16*b^3*c^3*d^13 - 1752*a^16*b^3*c^5*d^11 + 904*a^16*b^3*c^7*d^9 - 208*a^17*b^2*c^2*d^14 + 600*a^17*b^2*c^4*d^12 - 392*a^17*b^2*c^6*d^10 + 24*a^18*b*c*d^15))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a

^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12) + (b^2*((8*(16*a^4*b^18*c^18 - 4*a^2*b^20*c^18 - 24*a^6*b^16*c^18 + 16*a^8*b^14*c^18 - 4*a^10*b^12*c^18 + 4*a^22*c^2*d^16 - 8*a^22*c^4*d^14 + 4*a^22*c^6*d^12 + 4*a*b^21*c^13*d^5 - 8*a*b^21*c^15*d^3 + 24*a^3*b^19*c^17*d - 136*a^5*b^17*c^17*d + 224*a^7*b^15*c^17*d - 156*a^9*b^13*c^17*d + 40*a^11*b^11*c^17*d - 4*a^13*b^9*c*d^17 + 16*a^15*b^7*c*d^17 - 24*a^17*b^5*c*d^17 + 16*a^19*b^3*c*d^17 - 32*a^21*b*c^3*d^15 + 76*a^21*b*c^5*d^13 - 40*a^21*b*c^7*d^11 - 40*a^2*b^20*c^12*d^6 + 76*a^2*b^20*c^14*d^4 - 32*a^2*b^20*c^16*d^2 + 176*a^3*b^19*c^11*d^7 - 328*a^3*b^19*c^13*d^5 + 128*a^3*b^19*c^15*d^3 - 440*a^4*b^18*c^10*d^8 + 864*a^4*b^18*c^12*d^6 - 392*a^4*b^18*c^14*d^4 - 48*a^4*b^18*c^16*d^2 + 660*a^5*b^17*c^9*d^9 - 1584*a^5*b^17*c^11*d^7 + 1052*a^5*b^17*c^13*d^5 + 8*a^5*b^17*c^15*d^3 - 528*a^6*b^16*c^8*d^10 + 2156*a^6*b^16*c^10*d^8 - 2264*a^6*b^16*c^12*d^6 + 148*a^6*b^16*c^14*d^4 + 512*a^6*b^16*c^16*d^2 - 2112*a^7*b^15*c^9*d^9 + 3520*a^7*b^15*c^11*d^7 - 480*a^7*b^15*c^13*d^5 - 1152*a^7*b^15*c^15*d^3 + 528*a^8*b^14*c^6*d^12 + 1056*a^8*b^14*c^8*d^10 - 3696*a^8*b^14*c^10*d^8 + 1216*a^8*b^14*c^12*d^6 + 1808*a^8*b^14*c^14*d^4 - 928*a^8*b^14*c^16*d^2 - 660*a^9*b^13*c^5*d^13 + 792*a^9*b^13*c^7*d^11 + 2244*a^9*b^13*c^9*d^9 - 2288*a^9*b^13*c^11*d^7 - 2180*a^9*b^13*c^13*d^5 + 2248*a^9*b^13*c^15*d^3 + 440*a^10*b^12*c^4*d^14 - 2332*a^10*b^12*c^6*d^12 + 176*a^10*b^12*c^8*d^10 + 2684*a^10*b^12*c^10*d^8 + 1896*a^10*b^12*c^12*d^6 - 3532*a^10*b^12*c^14*d^4 + 672*a^10*b^12*c^16*d^2 - 176*a^11*b^11*c^3*d^15 + 2552*a^11*b^11*c^5*d^13 - 2464*a^11*b^11*c^7*d^11 - 1496*a^11*b^11*c^9*d^9 - 528*a^11*b^11*c^11*d^7 + 3736*a^11*b^11*c^13*d^5 - 1664*a^11*b^11*c^15*d^3 + 40*a^12*b^10*c^2*d^16 - 1664*a^12*b^10*c^4*d^14 + 3736*a^12*b^10*c^6*d^12 - 528*a^12*b^10*c^8*d^10 - 1496*a^12*b^10*c^10*d^8 - 2464*a^12* b^10*c^12*d^6 + 2552*a^12*b^10*c^14*d^4 - 176*a^12*b^10*c^16*d^2 + 672*a^13*b^9*c^3*d^15 - 3532*a^13*b^9*c^5*d^13 + 1896*a^13*b^9*c^7*d^11 + 2684*a^13*b^9*c^9*d^9 + 176*a^13*b^9*c^11*d^7 - 2332*a^13*b^9*c^13*d^5 + 440*a^13*b^9*c^15*d^3 - 156*a^14*b^8*c^2*d^16 + 2248*a^14*b^8*c^4*d^14 - 2180*a^14*b^8*c^6*d^12 - 2288*a^14*b^8*c^8*d^10 + 2244*a^14*b^8*c^10*d^8 + 792*a^14*b^8*c^12*d^6 - 660*a^14*b^8*c^14*d^4 - 928*a^15*b^7*c^3*d^15 + 1808*a^15*b^7*c^5*d^13 + 1216*a^15*b^7*c^7*d^11 - 3696*a^15*b^7*c^9*d^9 + 1056*a^15*b^7*c^11*d^7 + 528*a^15*b^7*c^13*d^5 + 224*a^16*b^6*c^2*d^16 - 1152*a^16*b^6*c^4*d^14 - 480*a^16*b^6*c^6*d^12 + 3520*a^16*b^6*c^8*d^10 - 2112*a^16*b^6*c^10*d^8 + 512*a^17*b^5*c^3*d^15 + 148*a^17*b^5*c^5*d^13 - 2264*a^17*b^5*c^7*d^11 + 2156*a^17*b^5*c^9*d^9 - 528*a^17*b^5*c^11*d^7 - 136*a^18*b^4*c^2*d^16 + 8*a^18*b^4*c^4*d^14 + 1052*a^18*b^4*c^6*d^12 - 1584*a^18*b^4*c^8*d^10 + 660*a^18*b^4*c^10*d^8 - 48*a^19*b^3*c^3*d^15 - 392*a^19*b^3*c^5*d^13 + 864*a^19*b^3*c^7*d^11 - 440*a^19*b^3*c^9*d^9 + 24*a^20*b^2*c^2*d^16 + 128*a^20*b^2*c^4*d^14 - 328*a^20*b^2*c^6*d^12 + 176*a^20*b^2*c^8*d^10 + 4*a*b^21*c^17*d - 4*a^21*b*c*d^17))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12) - (8*tan(e/2 + (f*x)/2)*(12*a*b^21*c^18 - 12*a^22*c*d^17 - 56*a^3*b^19*c^18 + 104*a^5*b^17*c^18 - 96*a^7*b^15*c^18 + 44*a^9*b^13*c^18 - 8*a^11*b^11*c^18 + 32*a^22*c^3*d^15 - 28*a^22*c^5*d^13 + 8*a^22*c^7*d^11 - 16*a*b^21*c^12*d^6 + 44*a*b^21*c^14*d^4 - 40*a*b^21*c^16*d^2 - 132*a^2*b^20*c^17*d + 616*a^4*b^18*c^17*d - 1144*a^6*b^16*c^17*d + 1056*a^8*b^14*c^17*d - 484*a^10*b^12 *c^17*d + 16*a^12*b^10*c*d^17 + 88*a^12*b^10*c^17*d - 76*a^14*b^8*c*d^17 + 144*a^16*b^6*c*d^17 - 136*a^18*b^4*c*d^17 + 64*a^20*b^2*c*d^17 + 132*a^21*b*c^2*d^16 - 352*a^21*b*c^4*d^14 + 308*a^21*b*c^6*d^12 - 88*a^21*b*c^8*d^10 + 176*a^2*b^20*c^11*d^7 - 484*a^2*b^20*c^13*d^5 + 440*a^2*b^20*c^15*d^3 - 880*a^3*b^19*c^10*d^8 + 2496*a^3*b^19*c^12*d^6 - 2408*a^3*b^19*c^14*d^4 + 848*a^3*b^19*c^16*d^2 + 2640*a^4*b^18*c^9*d^9 - 8096*a^4*b^18*c^11*d^7 + 8888*a^4*b^18*c^13*d^5 - 4048*a^4*b^18*c^15*d^3 - 5280*a^5*b^17*c^8*d^10 + 18700*a^5*b^17*c^10*d^8 - 24784*a^5*b^17*c^12*d^6 + 14692*a^5*b^17*c^14*d^4 - 3432*a^5*b^17*c^16*d^2 + 7392*a^6*b^16*c^7*d^11 - 32868*a^6*b^16*c^9*d^9 + 54384*a^6*b^16*c^11*d^7 - 40876*a^6*b^16*c^13*d^5 + 13112*a^6*b^16*c^15*d^3 - 7392*a^7*b^15*c^6*d^12 + 45408*a^7*b^15*c^8*d^10 - 95040*a^7*b^15*c^10*d^8 + 89280*a^7*b^15*c^12*d^6 - 38208*a^7*b^15*c^14*d^4 + 6048*a^7*b^15*c^16*d^2 + 5280*a^8*b^14*c^5*d^13 - 49632*a^8*b^14*c^7*d^11 + 133056*a^8*b^14*c^9*d^9 - 156992*a^8*b^14*c^11*d^7 + 88000*a^8*b^14*c^13*d^5 - 20768*a^8*b^14*c^15*d^3 - 2640*a^9*b^13*c^4*d^14 + 42372*a^9*b^13*c^6*d^12 - 150216*a^9*b^13*c^8*d^10 + 225676*a^9*b^13*c^10*d^8 - 162336*a^9*b^13*c^12*d^6 + 52532*a^9*b^13*c^14*d^4 - 5432*a^9*b^13*c^16*d^2 + 880*a^10*b^12*c^3*d^15 - 27500*a^10*b^12*c^5*d^13 + 137368*a^10*b^12*c^7*d^11 - 266244*a^10*b^12*c^9*d^9 + 242528*a^10*b^12*c^11*d^7 - 104060*a^10*b^12*c^13*d^5 + 17512*a^10*b^12*c^15*d^3 - 176*a^11*b^11*c^2*d^16 + 13024*a^11*b^11*c^4*d^14 - 101288*a^11*b^11*c^6*d^12 + 257136*a^11*b^11*c^8*d^10 - 296824*a^11*b^11*c^10*d^8 + 165760*a^11*b^11*c^12*d^6 - 40072*a^11*b^11*c^14*d^4 + 2448*a^11*b^11*c^16*d^2 - 4224*a^12*b^10*c^3*d^15 + 59000*a^12*b^10*c^5*d^13 - 202544*a^12*b^10*c^7*d^11 + 299816*a^12*b^10*c^9*d^9 - 214368*a^12*b^10*c^11*d^7 + 69784*a^12*b^10*c^13*d^5 - 7568*a^12*b^10*c^15*d^3 + 836*a^13*b^9*c^2*d^16 - 26048*a^13*b^9*c^4*d^14 + 129580*a^13*b^9*c^6*d^12 - 249832*a^13*b^9*c^8*d^10 + 226116*a^13*b^9*c^10*d^8 - 96272*a^13*b^9*c^12*d^6 + 16060*a^13*b^9*c^14*d^4 - 440*a^13*b^9*c^16*d^2 + 8128*a^14*b^8*c^3*d^15 - 66628*a^14*b^8*c^5*d^13 + 170424*a^14*b^8*c^7*d^11 - 195404*a^14*b^8*c^9*d^9 + 107184*a^14*b^8*c^11*d^7 - 24948*a^14*b^8*c^13*d^5 + 1320*a^14*b^8*c^15*d^3 - 1584*a^15*b^7*c^2*d^16 + 26752*a^15*b^7*c^4*d^14 - 94160*a^15*b^7*c^6*d^12 + 138688*a^15*b^7*c^8*d^10 - 96624*a^15*b^7*c^10*d^8 + 29568*a^15*b^7*c^12*d^6 - 2640*a^15*b^7*c^14*d^4 - 7872*a^16*b^6*c^3*d^15 + 41712*a^16*b^6*c^5*d^13 - 80448*a^16*b^6*c^7*d^11 + 70224*a^16*b^6*c^9*d^9 - 27456*a^16*b^6*c^11*d^7 + 3696*a^16*b^6*c^13*d^5 + 1496*a^17*b^5*c^2*d^16 - 14608*a^17*b^5*c^4*d^14 + 37532*a^17*b^5*c^6*d^12 - 40920*a^17*b^5*c^8*d^10 + 20196*a^17*b^5*c^10*d^8 - 3696*a^17*b^5*c^12*d^6 + 3888*a^18*b^4*c^3*d^15 - 13748*a^18*b^4*c^5*d^13 + 19016*a^18*b^4*c^7*d^11 - 11660*a^18*b^4*c^9*d^9 + 2640*a^18*b^4*c^11*d^7 - 704*a^19*b^3*c^2*d^16 + 3872*a^19*b^3*c^4*d^14 - 6952*a^19*b^3*c^6*d^12 + 5104*a^19*b^3*c^8*d^10 - 1320*a^19*b^3*c^10*d^8 - 832*a^20*b^2*c^3*d^15 + 1912*a^20*b^2*c^5*d^13 - 1584*a^20*b^2*c^7*d^11 + 440*a^20*b^2*c^9*d^9))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d

- 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12))*(-(a + b)^5*(a - b)^5)^(1/2)*(12*a^4*d^2 + b^4*c^2 + 6*b^4*d^2 + 2*a^2*b^2*c^2 - 15*a^2*b^2*d^2 + 2*a*b^3*c*d - 8*a^3*b*c*d))/(2*(a^14*d^4 - b^14*c^4 + 5*a^2*b^12*c^4 - 10*a^4*b^10*c^4 + 10*a^6*b^8*c^4 - 5*a^8*b^6*c^4 + a^10*b^4*c^4 - a^4*b^10*d^4 + 5*a^6*b^8*d^4 - 10*a^8*b^6*d^4 + 10*a^10*b^4*d^4 - 5*a^12*b^2*d^4 + 4*a^3*b^11*c*d^3 - 20*a^3*b^11*c^3*d - 20*a^5*b^9*c*d^3 + 40*a^5*b^9*c^3*d + 40*a^7*b^7*c*d^3 - 40*a^7*b^7*c^3*d - 40*a^9*b^5*c*d^3 + 20*a^9*b^5*c^3*d + 20*a^11*b^3*c*d^3 - 4*a^11*b^3*c^3*d - 6*a^2*b^12*c^2*d^2 + 30*a^4*b^10*c^2*d^2 - 60*a^6*b^8*c^2*d^2 + 60*a^8*b^6*c^2*d^2 - 30*a^10*b^4*c^2*d^2 + 6*a^12*b^2*c^2*d^2 + 4*a*b^13*c^3*d - 4*a^13*b*c*d^3)))*(12*a^4*d^2 + b^4*c^2 + 6*b^4*d^2 + 2*a^2*b^2*c^2 - 15*a^2*b^2*d^2 + 2*a*b^3*c*d - 8*a^3*b*c*d))/(2*(a^14*d^4 - b^14*c^4 + 5*a^2*b^12*c^4 - 10*a^4*b^10*c^4 + 10*a^6*b^8*c^4 - 5*a^8*b^6*c^4 + a^10*b^4*c^4 - a^4*b^10*d^4 + 5*a^6*b^8*d^4 - 10*a^8*b^6*d^4 + 10*a^10*b^4*d^4 - 5*a^12*b^2*d^4 + 4*a^3*b^11*c*d^3 - 20*a^3*b^11*c^3*d - 20*a^5*b^9*c*d^3 + 40*a^5*b^9*c^3*d + 40*a^7*b^7*c*d^3 - 40*a^7*b^7*c^3*d - 40*a^9*b^5*c*d^3 + 20*a^9*b^5*c^3*d + 20*a^11*b^3*c*d^3 - 4*a^11*b^3*c^3*d - 6*a^2*b^12*c^2*d^2 + 30*a^4*b^10*c^2*d^2 - 60*a^6*b^8*c^2*d^2 + 60*a^8*b^6*c^2*d^2 - 30*a^10*b^4*c^2*d^2 + 6*a^12*b^2*c^2*d^2 + 4*a*b^13*c^3*d - 4*a^13*b*c*d^3)))*(12*a^4*d^2 + b^4*c^2 + 6*b^4*d^2 + 2*a^2*b^2*c^2 - 15*a^2*b^2*d^2 + 2*a*b^3*c*d - 8*a^3*b*c*d)*1i)/(2*(a^14*d^4 - b^14*c^4 + 5*a^2*b^12*c^4 - 10*a^4*b^10*c^4 + 10*a^6*b^8*c^4 - 5*a^8*b^6*c^4 + a^10*b^4*c^4 - a^4*b^10*d^4 + 5*a^6*b^8*d^4 - 10*a^8*b^6*d^4 + 10*a^10*b^4*d^4 - 5*a^12*b^2*d^4 + 4*a^3*b^11*c*d^3 - 20*a^3*b^11*c^3*d - 20*a^5*b^9*c*d^3 + 40*a^5*b^9*c^3*d + 40*a^7*b^7*c*d^3 - 40*a^7*b^7*c^3*d - 40*a^9*b^5*c*d^3 + 20*a^9*b^5*c^3*d + 20*a^11*b^3*c*d^3 - 4*a^11*b^3*c^3*d - 6*a^2*b^12*c^2*d^2 + 30*a^4*b^

10*c^2*d^2 - 60*a^6*b^8*c^2*d^2 + 60*a^8*b^6*c^2*d^2 - 30*a^10*b^4*c^2*d^2 + 6*a^12*b^2*c^2*d^2 + 4*a*b^13*c^3*d - 4*a^13*b*c*d^3)))/((16*(63*a*b^12*c^5*d^7 - 216*a*b^12*c^3*d^9 + 41*a*b^12*c^7*d^5 + 4*a*b^12*c^9*d^3 - 486*a^3*b^10*c*d^11 + 864*a^5*b^8*c*d^11 - 702*a^7*b^6*c*d^11 + 216*a^9*b^4*c*d^11 + 162*a^2*b^11*c^2*d^10 - 261*a^2*b^11*c^4*d^8 + 66*a^2*b^11*c^6*d^6 + 19*a^2*b^11*c^8*d^4 + 1197*a^3*b^10*c^3*d^9 - 696*a^3*b^10*c^5*d^7 - 21*a^3*b^10*c^7*d^5 + 16*a^3*b^10*c^9*d^3 - 783*a^4*b^9*c^2*d^10 + 1444*a^4*b^9*c^4*d^8 - 583*a^4*b^9*c^6*d^6 - 20*a^4*b^9*c^8*d^4 - 2511*a^5*b^8*c^3*d^9 + 1913*a^5*b^8*c^5*d^7 - 312*a^5*b^8*c^7*d^5 + 16*a^5*b^8*c^9*d^3 + 1278*a^6*b^7*c^2*d^10 - 2508*a^6*b^7*c^4*d^8 + 1232*a^6*b^7*c^6*d^6 - 116*a^6*b^7*c^8*d^4 + 2328*a^7*b^6*c^3*d^9 - 1936*a^7*b^6*c^5*d^7 + 364*a^7*b^6*c^7*d^5 - 828*a^8*b^5*c^2*d^10 + 1518*a^8*b^5*c^4*d^8 - 580*a^8*b^5*c^6*d^6 - 750*a^9*b^4*c^3*d^9 + 476*a^9*b^4*c^5*d^7 + 144*a^10*b^3*c^2*d^10 - 184*a^10*b^3*c^4*d^8 + 24*a^11*b^2*c^3*d^9 + 108*a*b^12*c*d^11))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12) + (16*tan(e/2 + (f*x)/2)*(108*a*b^12*c^2*d^10 - 162*a*b^12*c^4*d^8 + 18*a*b^12*c^6*d^6 + 8*a*b^12*c^8*d^4 + 108*a^2*b^11*c*d^11 - 486*a^4*b^9*c*d^11 + 756*a^6*b^7*c*d^11 - 432*a^8*b^5*c*d^11 - 162*a^2*b^11*c^3*d^9 + 36*a^2*b^11*c^5*d^7 + 38*a^2*b^11*c^7*d^5 - 270*a^3*b^10*c^2*d^10 + 396*a^3*b^10*c^4*d^8 - 42*a^3*b^10*c^6*d^6 + 32*a^3*b^10*c^8*d^4 + 864*a^4*b^9*c^3*d^9 - 398*a^4*b^9*c^5*d^7 - 40*a^4*b^9*c^7*d^5 + 90*a^5*b^8*c^2*d^10 + 82*a^5*b^8*c^4*d^8 - 432*a^5*b^8*c^6*d^6 + 32*a^5*b^8*c^8*d^

4 - 1632*a^6*b^7*c^3*d^9 + 1216*a^6*b^7*c^5*d^7 - 232*a^6*b^7*c^7*d^5 + 216
*a^7*b^6*c^2*d^10 - 596*a^7*b^6*c^4*d^8 + 600*a^7*b^6*c^6*d^6 + 900*a^8*b^5
*c^3*d^9 - 584*a^8*b^5*c^5*d^7 - 80*a^9*b^4*c^4*d^8 + 48*a^10*b^3*c^3*d^9))
/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^
13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^
15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d
^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^
12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a
^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^
10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*
b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c
^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^
4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 8
52*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^
6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^1
1*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^
6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^1
0 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 14
6*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9
*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c
^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*
d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 +
1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*
a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*
b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c
^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^
8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a
^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12) - (b^2*(-(a + b)^5*(a
- b)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(4*a^16*c^3*d^11 - 4*a^3*b^13*c^14 - 4
*a^5*b^11*c^14 - a*b^15*c^14 + 144*a*b^15*c^4*d^10 - 348*a*b^15*c^6*d^8 + 2
14*a*b^15*c^8*d^6 + 7*a*b^15*c^10*d^4 - 8*a*b^15*c^12*d^2 - a^2*b^14*c^13*d
- 144*a^4*b^12*c*d^13 + 20*a^4*b^12*c^13*d + 684*a^6*b^10*c*d^13 + 44*a^6*
b^10*c^13*d - 1314*a^8*b^8*c*d^13 + 1224*a^10*b^6*c*d^13 - 504*a^12*b^4*c*d
^13 + 36*a^14*b^2*c*d^13 + 24*a^15*b*c^2*d^12 - 44*a^15*b*c^4*d^10 - 432*a^
2*b^14*c^3*d^11 + 1140*a^2*b^14*c^5*d^9 - 818*a^2*b^14*c^7*d^7 + 55*a^2*b^1
4*c^9*d^5 + 16*a^2*b^14*c^11*d^3 + 432*a^3*b^13*c^2*d^12 - 2016*a^3*b^13*c^
4*d^10 + 2938*a^3*b^13*c^6*d^8 - 1485*a^3*b^13*c^8*d^6 + 152*a^3*b^13*c^10*
d^4 + 27*a^3*b^13*c^12*d^2 + 2688*a^4*b^12*c^3*d^11 - 6574*a^4*b^12*c^5*d^9
+ 5107*a^4*b^12*c^7*d^7 - 1056*a^4*b^12*c^9*d^5 + 59*a^4*b^12*c^11*d^3 - 2
148*a^5*b^11*c^2*d^12 + 8378*a^5*b^11*c^4*d^10 - 10619*a^5*b^11*c^6*d^8 + 5
064*a^5*b^11*c^8*d^6 - 975*a^5*b^11*c^10*d^4 + 48*a^5*b^11*c^12*d^2 - 7294*
a^6*b^10*c^3*d^11 + 16053*a^6*b^10*c^5*d^9 - 12464*a^6*b^10*c^7*d^7 + 3649*
a^6*b^10*c^9*d^5 - 640*a^6*b^10*c^11*d^3 + 4470*a^7*b^9*c^2*d^12 - 15815*a^
7*b^9*c^4*d^10 + 18608*a^7*b^9*c^6*d^8 - 8939*a^7*b^9*c^8*d^6 + 2300*a^7*b^
9*c^10*d^4 - 220*a^7*b^9*c^12*d^2 + 10105*a^8*b^8*c^3*d^11 - 19912*a^8*b^8*

c^5*d^9 + 14693*a^8*b^8*c^7*d^7 - 4524*a^8*b^8*c^9*d^5 + 628*a^8*b^8*c^11*d
^3 - 4632*a^9*b^7*c^2*d^12 + 14976*a^9*b^7*c^4*d^10 - 15576*a^9*b^7*c^6*d^8
 + 6104*a^9*b^7*c^8*d^6 - 1088*a^9*b^7*c^10*d^4 - 7104*a^10*b^6*c^3*d^11 +
11320*a^10*b^6*c^5*d^9 - 6184*a^10*b^6*c^7*d^7 + 1120*a^10*b^6*c^9*d^5 + 22
32*a^11*b^5*c^2*d^12 - 5932*a^11*b^5*c^4*d^10 + 4344*a^11*b^5*c^6*d^8 - 688
*a^11*b^5*c^8*d^6 + 1892*a^12*b^4*c^3*d^11 - 1920*a^12*b^4*c^5*d^9 + 368*a^
12*b^4*c^7*d^7 - 252*a^13*b^3*c^2*d^12 + 624*a^13*b^3*c^4*d^10 - 292*a^13*b
^3*c^6*d^8 - 192*a^14*b^2*c^3*d^11 + 172*a^14*b^2*c^5*d^9))/(a^17*d^13 - b^
17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^1
3 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*
a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^
8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a
^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 -
 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^
5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 +
84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^
4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13
*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*
d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 -
 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*
a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7
*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9
*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d
^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1
437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a
^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^
7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c
^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^1
0 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 15
6*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^1
3*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*
c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7
 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12) - (8*(60*a*b^15*c^7*d^7 - 36*a*b^15*c^
5*d^9 - 13*a*b^15*c^9*d^5 - 10*a*b^15*c^11*d^3 - 4*a^3*b^13*c^13*d + 36*a^5
*b^11*c*d^13 - 4*a^5*b^11*c^13*d - 144*a^7*b^9*c*d^13 + 216*a^9*b^7*c*d^13
 - 144*a^11*b^5*c*d^13 + 36*a^13*b^3*c*d^13 + 4*a^15*b*c^3*d^11 + 72*a^2*b^1
4*c^4*d^10 - 108*a^2*b^14*c^6*d^8 + 19*a^2*b^14*c^8*d^6 + 14*a^2*b^14*c^10*
d^4 - a^2*b^14*c^12*d^2 + 120*a^3*b^13*c^5*d^9 - 305*a^3*b^13*c^7*d^7 + 190
*a^3*b^13*c^9*d^5 + 19*a^3*b^13*c^11*d^3 - 72*a^4*b^12*c^2*d^12 - 168*a^4*b
^12*c^4*d^10 + 699*a^4*b^12*c^6*d^8 - 602*a^4*b^12*c^8*d^6 + 99*a^4*b^12*c^
10*d^4 + 20*a^4*b^12*c^12*d^2 - 36*a^5*b^11*c^3*d^11 - 535*a^5*b^11*c^5*d^9
 + 1354*a^5*b^11*c^7*d^7 - 895*a^5*b^11*c^9*d^5 + 40*a^5*b^11*c^11*d^3 + 27
6*a^6*b^10*c^2*d^12 + 233*a^6*b^10*c^4*d^10 - 2046*a^6*b^10*c^6*d^8 + 2161*
a^6*b^10*c^8*d^6 - 552*a^6*b^10*c^10*d^4 + 44*a^6*b^10*c^12*d^2 + 61*a^7*b^
9*c^3*d^11 + 1386*a^7*b^9*c^5*d^9 - 2979*a^7*b^9*c^7*d^7 + 1860*a^7*b^9*c^9

*d^5 - 220*a^7*b^9*c^11*d^3 - 375*a^8*b^8*c^2*d^12 - 270*a^8*b^8*c^4*d^10 +
 2885*a^8*b^8*c^6*d^8 - 3012*a^8*b^8*c^8*d^6 + 628*a^8*b^8*c^10*d^4 - 88*a^
9*b^7*c^3*d^11 - 1544*a^9*b^7*c^5*d^9 + 2648*a^9*b^7*c^7*d^7 - 1088*a^9*b^7
*c^9*d^5 + 216*a^10*b^6*c^2*d^12 + 100*a^10*b^6*c^4*d^10 - 1336*a^10*b^6*c^
6*d^8 + 1056*a^10*b^6*c^8*d^6 + 180*a^11*b^5*c^3*d^11 + 248*a^11*b^5*c^5*d^
9 - 400*a^11*b^5*c^7*d^7 - 60*a^12*b^4*c^2*d^12 + 248*a^12*b^4*c^4*d^10 - 1
48*a^12*b^4*c^6*d^8 - 184*a^13*b^3*c^3*d^11 + 172*a^13*b^3*c^5*d^9 + 24*a^1
4*b^2*c^2*d^12 - 44*a^14*b^2*c^4*d^10 - a*b^15*c^13*d))/(a^17*d^13 - b^17*c
^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 +
a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17
*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^
5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b
^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*
a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^
8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a
^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^
13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^1
1*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5
 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 130
8*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*
b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^1
0*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5
*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 -
 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*
a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*
b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^
11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d
^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 -
852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^
13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^
4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*
d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*
a*b^16*c^12*d - 9*a^16*b*c*d^12) + (b^2*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(2
*a^2*b^17*c^16 - 6*a^6*b^13*c^16 + 4*a^8*b^11*c^16 + 4*a^19*c^3*d^13 - 4*a^
19*c^5*d^11 + 12*a*b^18*c^9*d^7 - 28*a*b^18*c^11*d^5 + 16*a*b^18*c^13*d^3 -
 10*a^3*b^16*c^15*d - 24*a^5*b^14*c^15*d + 78*a^7*b^12*c^15*d + 12*a^9*b^10
*c*d^15 - 44*a^9*b^10*c^15*d - 54*a^11*b^8*c*d^15 + 96*a^13*b^6*c*d^15 - 78
*a^15*b^4*c*d^15 + 24*a^17*b^2*c*d^15 + 12*a^18*b*c^2*d^14 - 56*a^18*b*c^4*
d^12 + 44*a^18*b*c^6*d^10 - 96*a^2*b^17*c^8*d^8 + 234*a^2*b^17*c^10*d^6 - 1
46*a^2*b^17*c^12*d^4 + 6*a^2*b^17*c^14*d^2 + 336*a^3*b^16*c^7*d^9 - 918*a^3
*b^16*c^9*d^7 + 726*a^3*b^16*c^11*d^5 - 134*a^3*b^16*c^13*d^3 - 672*a^4*b^1
5*c^6*d^10 + 2280*a^4*b^15*c^8*d^8 - 2520*a^4*b^15*c^10*d^6 + 952*a^4*b^15*
c^12*d^4 - 40*a^4*b^15*c^14*d^2 + 840*a^5*b^14*c^5*d^11 - 4032*a^5*b^14*c^7
*d^9 + 6360*a^5*b^14*c^9*d^7 - 3768*a^5*b^14*c^11*d^5 + 624*a^5*b^14*c^13*d
^3 - 672*a^6*b^13*c^4*d^12 + 5292*a^6*b^13*c^6*d^10 - 11772*a^6*b^13*c^8*d^

8 + 10050*a^6*b^13*c^10*d^6 - 3174*a^6*b^13*c^12*d^4 + 282*a^6*b^13*c^14*d^2 + 336*a^7*b^12*c^3*d^13 - 5124*a^7*b^12*c^5*d^11 + 16212*a^7*b^12*c^7*d^9 - 19602*a^7*b^12*c^9*d^7 + 9670*a^7*b^12*c^11*d^5 - 1570*a^7*b^12*c^13*d^3 - 96*a^8*b^11*c^2*d^14 + 3528*a^8*b^11*c^4*d^12 - 16872*a^8*b^11*c^6*d^10 + 28848*a^8*b^11*c^8*d^8 - 20340*a^8*b^11*c^10*d^6 + 5396*a^8*b^11*c^12*d^4 - 468*a^8*b^11*c^14*d^2 - 1620*a^9*b^10*c^3*d^13 + 13320*a^9*b^10*c^5*d^11 - 32304*a^9*b^10*c^7*d^9 + 31560*a^9*b^10*c^9*d^7 - 12648*a^9*b^10*c^11*d^5 + 1724*a^9*b^10*c^13*d^3 + 442*a^10*b^9*c^2*d^14 - 7810*a^10*b^9*c^4*d^12 + 27546*a^10*b^9*c^6*d^10 - 37338*a^10*b^9*c^8*d^8 + 21288*a^10*b^9*c^10*d^6 - 4348*a^10*b^9*c^12*d^4 + 220*a^10*b^9*c^14*d^2 + 3206*a^11*b^8*c^3*d^13 - 17850*a^11*b^8*c^5*d^11 + 34018*a^11*b^8*c^7*d^9 - 26556*a^11*b^8*c^9*d^7 + 7896*a^11*b^8*c^11*d^5 - 660*a^11*b^8*c^13*d^3 - 816*a^12*b^7*c^2*d^14 + 8696*a^12*b^7*c^4*d^12 - 23696*a^12*b^7*c^6*d^10 + 25056*a^12*b^7*c^8*d^8 - 10560*a^12*b^7*c^10*d^6 + 1320*a^12*b^7*c^12*d^4 - 3064*a^13*b^6*c^3*d^13 + 12400*a^13*b^6*c^5*d^11 - 18048*a^13*b^6*c^7*d^9 + 10464*a^13*b^6*c^9*d^7 - 1848*a^13*b^6*c^11*d^5 + 702*a^14*b^5*c^2*d^14 - 4770*a^14*b^5*c^4*d^12 + 9858*a^14*b^5*c^6*d^10 - 7638*a^14*b^5*c^8*d^8 + 1848*a^14*b^5*c^10*d^6 + 1314*a^15*b^4*c^3*d^13 - 3954*a^15*b^4*c^5*d^11 + 4038*a^15*b^4*c^7*d^9 - 1320*a^15*b^4*c^9*d^7 - 244*a^16*b^3*c^2*d^14 + 1084*a^16*b^3*c^4*d^12 - 1500*a^16*b^3*c^6*d^10 + 660*a^16*b^3*c^8*d^8 - 176*a^17*b^2*c^3*d^13 + 372*a^17*b^2*c^5*d^11 - 220*a^17*b^2*c^7*d^9))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12

*d - 9*a^16*b*c*d^12) + (8*tan(e/2 + (f*x)/2)*(4*a*b^18*c^16 - 12*a^5*b^14*c^16 + 8*a^7*b^12*c^16 + 8*a^19*c^2*d^14 - 8*a^19*c^4*d^12 + 12*a*b^18*c^10*d^6 - 28*a*b^18*c^12*d^4 + 12*a*b^18*c^14*d^2 - 20*a^2*b^17*c^15*d - 48*a^4*b^15*c^15*d + 156*a^6*b^13*c^15*d - 88*a^8*b^11*c^15*d + 12*a^10*b^9*c*d^15 - 48*a^12*b^7*c*d^15 + 84*a^14*b^5*c*d^15 - 72*a^16*b^3*c*d^15 - 112*a^18*b*c^3*d^13 + 88*a^18*b*c^5*d^11 - 84*a^2*b^17*c^9*d^7 + 212*a^2*b^17*c^11*d^5 - 108*a^2*b^17*c^13*d^3 + 240*a^3*b^16*c^8*d^8 - 744*a^3*b^16*c^10*d^6 + 584*a^3*b^16*c^12*d^4 - 80*a^3*b^16*c^14*d^2 - 336*a^4*b^15*c^7*d^9 + 1632*a^4*b^15*c^9*d^7 - 2176*a^4*b^15*c^11*d^5 + 928*a^4*b^15*c^13*d^3 + 168*a^5*b^14*c^6*d^10 - 2472*a^5*b^14*c^8*d^8 + 5460*a^5*b^14*c^10*d^6 - 3708*a^5*b^14*c^12*d^4 + 564*a^5*b^14*c^14*d^2 + 168*a^6*b^13*c^5*d^11 + 2520*a^6*b^13*c^7*d^9 - 9204*a^6*b^13*c^9*d^7 + 9180*a^6*b^13*c^11*d^5 - 2820*a^6*b^13*c^13*d^3 - 336*a^7*b^12*c^4*d^12 - 1344*a^7*b^12*c^6*d^10 + 10416*a^7*b^12*c^8*d^8 - 15960*a^7*b^12*c^10*d^6 + 8152*a^7*b^12*c^12*d^4 - 936*a^7*b^12*c^14*d^2 + 240*a^8*b^11*c^3*d^13 - 336*a^8*b^11*c^5*d^11 - 7488*a^8*b^11*c^7*d^9 + 19800*a^8*b^11*c^9*d^7 - 15416*a^8*b^11*c^11*d^5 + 3288*a^8*b^11*c^13*d^3 - 84*a^9*b^10*c^2*d^14 + 1188*a^9*b^10*c^4*d^12 + 2292*a^9*b^10*c^6*d^10 - 16596*a^9*b^10*c^8*d^8 + 20136*a^9*b^10*c^10*d^6 - 7376*a^9*b^10*c^12*d^4 + 440*a^9*b^10*c^14*d^2 - 908*a^10*b^9*c^3*d^13 + 1740*a^10*b^9*c^5*d^11 + 7556*a^10*b^9*c^7*d^9 - 18048*a^10*b^9*c^9*d^7 + 10936*a^10*b^9*c^11*d^5 - 1288*a^10*b^9*c^13*d^3 + 328*a^11*b^8*c^2*d^14 - 2808*a^11*b^8*c^4*d^12 + 1088*a^11*b^8*c^6*d^10 + 9600*a^11*b^8*c^8*d^8 - 10584*a^11*b^8*c^10*d^6 + 2376*a^11*b^8*c^12*d^4 + 1792*a^12*b^7*c^3*d^13 - 4720*a^12*b^7*c^5*d^11 - 144*a^12*b^7*c^7*d^9 + 5856*a^12*b^7*c^9*d^7 - 2736*a^12*b^7*c^11*d^5 - 596*a^13*b^6*c^2*d^14 + 3980*a^13*b^6*c^4*d^12 - 4908*a^13*b^6*c^6*d^10 - 156*a^13*b^6*c^8*d^8 + 1680*a^13*b^6*c^10*d^6 - 1932*a^14*b^5*c^3*d^13 + 4812*a^14*b^5*c^5*d^11 - 3012*a^14*b^5*c^7*d^9 + 48*a^14*b^5*c^9*d^7 + 552*a^15*b^4*c^2*d^14 - 2616*a^15*b^4*c^4*d^12 + 3096*a^15*b^4*c^6*d^10 - 1032*a^15*b^4*c^8*d^8 + 920*a^16*b^3*c^3*d^13 - 1752*a^16*b^3*c^5*d^11 + 904*a^16*b^3*c^7*d^9 - 208*a^17*b^2*c^2*d^14 + 600*a^17*b^2*c^4*d^12 - 392*a^17*b^2*c^6*d^10 + 24*a^18*b*c*d^15))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^

7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12) - (b^2*((8*(16*a^4*b^18*c^18 - 4*a^2*b^20*c^18 - 24*a^6*b^16*c^18 + 16*a^8*b^14*c^18 - 4*a^10*b^12*c^18 + 4*a^22*c^2*d^16 - 8*a^22*c^4*d^14 + 4*a^22*c^6*d^12 + 4*a*b^21*c^13*d^5 - 8*a*b^21*c^15*d^3 + 24*a^3*b^19*c^17*d - 136*a^5*b^17*c^17*d + 224*a^7*b^15*c^17*d - 156*a^9*b^13*c^17*d + 40*a^11*b^11*c^17*d - 4*a^13*b^9*c*d^17 + 16*a^15*b^7*c*d^17 - 24*a^17*b^5*c*d^17 + 16*a^19*b^3*c*d^17 - 32*a^21*b*c^3*d^15 + 76*a^21*b*c^5*d^13 - 40*a^21*b*c^7*d^11 - 40*a^2*b^20*c^12*d^6 + 76*a^2*b^20*c^14*d^4 - 32*a^2*b^20*c^16*d^2 + 176*a^3*b^19*c^11*d^7 - 328*a^3*b^19*c^13*d^5 + 128*a^3*b^19*c^15*d^3 - 440*a^4*b^18*c^10*d^8 + 864*a^4*b^18*c^12*d^6 - 392*a^4*b^18*c^14*d^4 - 48*a^4*b^18*c^16*d^2 + 660*a^5*b^17*c^9*d^9 - 1584*a^5*b^17*c^11*d^7 + 1052*a^5*b^17*c^13*d^5 + 8*a^5*b^17*c^15*d^3 - 528*a^6*b^16*c^8*d^10 + 2156*a^6*b^16*c^10*d^8 - 2264*a^6*b^16*c^12*d^6 + 148*a^6*b^16*c^14*d^4 + 512*a^6*b^16*c^16*d^2 - 2112*a^7*b^15*c^9*d^9 + 3520*a^7*b^15*c^11*d^7 - 480*a^7*b^15*c^13*d^5 - 1152*a^7*b^15*c^15*d^3 + 528*a^8*b^14*c^6*d^12 + 1056*a^8*b^14*c^8*d^10 - 3696*a^8*b^14*c^10*d^8 + 1216*a^8*b^14*c^12*d^6 + 1808*a^8*b^14*c^14*d^4 - 928*a^8*b^14*c^16*d^2 - 660*a^9*b^13*c^5*d^13 + 792*a^9*b^13*c^7*d^11 + 2244*a^9*b^13*c^9*d^9 - 2288*a^9*b^13*c^11*d^7 - 2180*a^9*b^13*c^13*d^5 + 2248*a^9*b^13*c^15*d^3 + 440*a^10*b^12*c^4*d^14 - 2332*a^10*b^12*c^6*d^12 + 176*a^10*b^12*c^8*d^10 + 2684*a^10*b^12*c^10*d^8 + 1896*a^10*b^12*c^12*d^6 - 3532*a^10*b^12*c^14*d^4 + 672*a^10*b^12*c^16*d^2 - 176*a^11*b^11*c^3*d^15 + 2552*a^11*b^11*c^5*d^13 - 2464*a^11*b^11*c^7*d^11 - 1496*a^11*b^11*c^9*d^9 - 528*a^11*b^11*c^11*d^7 + 3736*a^11*b^11*c^13*d^5 - 1664*a^11*b^11*c^15*d^3 + 40*a^12*b^10*c^2*d^16 - 1664*a^12*b^10*c^4*d^14 + 3736*a^12*b^10*c^6*d^12 - 528*a^12*b^10*c^8*d^10 - 1496*a^12*b^10*c^10*d^8 - 2464*a^12*b^10*c^12*d^6 + 2552*a^12*b^10*c^14*d^4 - 176*a^12*b^10*c^16*d^2 + 672*a^13*b^9*c^3*d^15 - 3532*a^13*b^9*c^5*d^13 + 1896*a^13*b^9*c^7*d^11 + 2684*a^13*b^9*c^9*d^9 + 176*a^13*b^9*c^11*d^7 - 2332*a^13*b^9*c^13*d^5 + 440*a^13*b^9*c^15*d^3 - 156*a^14*b^8*c^2*d^16 + 2248*a^14*b^8*c^4*d^14 - 2180*a^14*b^8*c^6*d^12 - 2288*a^14*b^8*c^8*d^10 + 2244*a^14*b^8*c^10*d^8 + 792*a^14*b^8*c^12*d^6 - 660*a^14*b^8*c^14*d^4 - 928*a^15*b^7*c^3*d^15 + 1808*a^15*b^7*c^5*d^13 + 1216*a^15*b^7*c^7*d^11 - 3696*a^15*b^7*c^9*d^9 + 1056*a^15*b^7*c^11*d^7 + 528*a^15*b^7*c^13*d^5 + 224*a^16*b^6*c^2*d^16 - 1152*a^16*b^6*c^4*d^14 - 480*a^16*b^6*c^6*d^12 + 3520*a^16*b^6*c^8*d^10 - 2112*a^16*b^6*c^10*d^8 + 512*a^17*b^5*c^3*d^15 + 148*a^17*b^5*c^5*d^13 - 2264*a^17*b^5*c^7*d^11 + 2156*a^17*b^5*c^9*d^9 - 528*a^17*b^5*c^11*d^7 - 136*a^18*b^4*c^2*

d^16 + 8*a^18*b^4*c^4*d^14 + 1052*a^18*b^4*c^6*d^12 - 1584*a^18*b^4*c^8*d^1
0 + 660*a^18*b^4*c^10*d^8 - 48*a^19*b^3*c^3*d^15 - 392*a^19*b^3*c^5*d^13 +
864*a^19*b^3*c^7*d^11 - 440*a^19*b^3*c^9*d^9 + 24*a^20*b^2*c^2*d^16 + 128*a
^20*b^2*c^4*d^14 - 328*a^20*b^2*c^6*d^12 + 176*a^20*b^2*c^8*d^10 + 4*a*b^21
*c^17*d - 4*a^21*b*c*d^17))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^
4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^
13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b
^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*
a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^1
2 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^
3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76
*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^
14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^
7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9
 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84
*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6
*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10
*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^1
0*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6
- 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045
*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^
8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*
c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^
11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 +
84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a
^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b
^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3
*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 -
 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d
^12) - (8*tan(e/2 + (f*x)/2)*(12*a*b^21*c^18 - 12*a^22*c*d^17 - 56*a^3*b^19
*c^18 + 104*a^5*b^17*c^18 - 96*a^7*b^15*c^18 + 44*a^9*b^13*c^18 - 8*a^11*b^
11*c^18 + 32*a^22*c^3*d^15 - 28*a^22*c^5*d^13 + 8*a^22*c^7*d^11 - 16*a*b^21
*c^12*d^6 + 44*a*b^21*c^14*d^4 - 40*a*b^21*c^16*d^2 - 132*a^2*b^20*c^17*d +
 616*a^4*b^18*c^17*d - 1144*a^6*b^16*c^17*d + 1056*a^8*b^14*c^17*d - 484*a^
10*b^12*c^17*d + 16*a^12*b^10*c*d^17 + 88*a^12*b^10*c^17*d - 76*a^14*b^8*c*
d^17 + 144*a^16*b^6*c*d^17 - 136*a^18*b^4*c*d^17 + 64*a^20*b^2*c*d^17 + 132
*a^21*b*c^2*d^16 - 352*a^21*b*c^4*d^14 + 308*a^21*b*c^6*d^12 - 88*a^21*b*c^
8*d^10 + 176*a^2*b^20*c^11*d^7 - 484*a^2*b^20*c^13*d^5 + 440*a^2*b^20*c^15*
d^3 - 880*a^3*b^19*c^10*d^8 + 2496*a^3*b^19*c^12*d^6 - 2408*a^3*b^19*c^14*d
^4 + 848*a^3*b^19*c^16*d^2 + 2640*a^4*b^18*c^9*d^9 - 8096*a^4*b^18*c^11*d^7
 + 8888*a^4*b^18*c^13*d^5 - 4048*a^4*b^18*c^15*d^3 - 5280*a^5*b^17*c^8*d^10
 + 18700*a^5*b^17*c^10*d^8 - 24784*a^5*b^17*c^12*d^6 + 14692*a^5*b^17*c^14*
d^4 - 3432*a^5*b^17*c^16*d^2 + 7392*a^6*b^16*c^7*d^11 - 32868*a^6*b^16*c^9*
d^9 + 54384*a^6*b^16*c^11*d^7 - 40876*a^6*b^16*c^13*d^5 + 13112*a^6*b^16*c^
15*d^3 - 7392*a^7*b^15*c^6*d^12 + 45408*a^7*b^15*c^8*d^10 - 95040*a^7*b^15*

c^10*d^8 + 89280*a^7*b^15*c^12*d^6 - 38208*a^7*b^15*c^14*d^4 + 6048*a^7*b^15*c^16*d^2 + 5280*a^8*b^14*c^5*d^13 - 49632*a^8*b^14*c^7*d^11 + 133056*a^8*b^14*c^9*d^9 - 156992*a^8*b^14*c^11*d^7 + 88000*a^8*b^14*c^13*d^5 - 20768*a^8*b^14*c^15*d^3 - 2640*a^9*b^13*c^4*d^14 + 42372*a^9*b^13*c^6*d^12 - 150216*a^9*b^13*c^8*d^10 + 225676*a^9*b^13*c^10*d^8 - 162336*a^9*b^13*c^12*d^6 + 52532*a^9*b^13*c^14*d^4 - 5432*a^9*b^13*c^16*d^2 + 880*a^10*b^12*c^3*d^15 - 27500*a^10*b^12*c^5*d^13 + 137368*a^10*b^12*c^7*d^11 - 266244*a^10*b^12*c^9*d^9 + 242528*a^10*b^12*c^11*d^7 - 104060*a^10*b^12*c^13*d^5 + 17512*a^10*b^12*c^15*d^3 - 176*a^11*b^11*c^2*d^16 + 13024*a^11*b^11*c^4*d^14 - 101288*a^11*b^11*c^6*d^12 + 257136*a^11*b^11*c^8*d^10 - 296824*a^11*b^11*c^10*d^8 + 165760*a^11*b^11*c^12*d^6 - 40072*a^11*b^11*c^14*d^4 + 2448*a^11*b^11*c^16*d^2 - 4224*a^12*b^10*c^3*d^15 + 59000*a^12*b^10*c^5*d^13 - 202544*a^12*b^10*c^7*d^11 + 299816*a^12*b^10*c^9*d^9 - 214368*a^12*b^10*c^11*d^7 + 69784*a^12*b^10*c^13*d^5 - 7568*a^12*b^10*c^15*d^3 + 836*a^13*b^9*c^2*d^16 - 26048*a^13*b^9*c^4*d^14 + 129580*a^13*b^9*c^6*d^12 - 249832*a^13*b^9*c^8*d^10 + 226116*a^13*b^9*c^10*d^8 - 96272*a^13*b^9*c^12*d^6 + 16060*a^13*b^9*c^14*d^4 - 440*a^13*b^9*c^16*d^2 + 8128*a^14*b^8*c^3*d^15 - 66628*a^14*b^8*c^5*d^13 + 170424*a^14*b^8*c^7*d^11 - 195404*a^14*b^8*c^9*d^9 + 107184*a^14*b^8*c^11*d^7 - 24948*a^14*b^8*c^13*d^5 + 1320*a^14*b^8*c^15*d^3 - 1584*a^15*b^7*c^2*d^16 + 26752*a^15*b^7*c^4*d^14 - 94160*a^15*b^7*c^6*d^12 + 138688*a^15*b^7*c^8*d^10 - 96624*a^15*b^7*c^10*d^8 + 29568*a^15*b^7*c^12*d^6 - 2640*a^15*b^7*c^14*d^4 - 7872*a^16*b^6*c^3*d^15 + 41712*a^16*b^6*c^5*d^13 - 80448*a^16*b^6*c^7*d^11 + 70224*a^16*b^6*c^9*d^9 - 27456*a^16*b^6*c^11*d^7 + 3696*a^16*b^6*c^13*d^5 + 1496*a^17*b^5*c^2*d^16 - 14608*a^17*b^5*c^4*d^14 + 37532*a^17*b^5*c^6*d^12 - 40920*a^17*b^5*c^8*d^10 + 20196*a^17*b^5*c^10*d^8 - 3696*a^17*b^5*c^12*d^6 + 3888*a^18*b^4*c^3*d^15 - 13748*a^18*b^4*c^5*d^13 + 19016*a^18*b^4*c^7*d^11 - 11660*a^18*b^4*c^9*d^9 + 2640*a^18*b^4*c^11*d^7 - 704*a^19*b^3*c^2*d^16 + 3872*a^19*b^3*c^4*d^14 - 6952*a^19*b^3*c^6*d^12 + 5104*a^19*b^3*c^8*d^10 - 1320*a^19*b^3*c^10*d^8 - 832*a^20*b^2*c^3*d^15 + 1912*a^20*b^2*c^5*d^13 - 1584*a^20*b^2*c^7*d^11 + 440*a^20*b^2*c^9*d^9))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a

^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12))*(-(a + b)^5*(a - b)^5)^(1/2)*(12*a^4*d^2 + b^4*c^2 + 6*b^4*d^2 + 2*a^2*b^2*c^2 - 15*a^2*b^2*d^2 + 2*a*b^3*c*d - 8*a^3*b*c*d))/(2*(a^14*d^4 - b^14*c^4 + 5*a^2*b^12*c^4 - 10*a^4*b^10*c^4 + 10*a^6*b^8*c^4 - 5*a^8*b^6*c^4 + a^10*b^4*c^4 - a^4*b^10*d^4 + 5*a^6*b^8*d^4 - 10*a^8*b^6*d^4 + 10*a^10*b^4*d^4 - 5*a^12*b^2*d^4 + 4*a^3*b^11*c*d^3 - 20*a^3*b^11*c^3*d - 20*a^5*b^9*c*d^3 + 40*a^5*b^9*c^3*d + 40*a^7*b^7*c*d^3 - 40*a^7*b^7*c^3*d - 40*a^9*b^5*c*d^3 + 20*a^9*b^5*c^3*d + 20*a^11*b^3*c*d^3 - 4*a^11*b^3*c^3*d - 6*a^2*b^12*c^2*d^2 + 30*a^4*b^10*c^2*d^2 - 60*a^6*b^8*c^2*d^2 + 60*a^8*b^6*c^2*d^2 - 30*a^10*b^4*c^2*d^2 + 6*a^12*b^2*c^2*d^2 + 4*a*b^13*c^3*d - 4*a^13*b*c*d^3)))*(12*a^4*d^2 + b^4*c^2 + 6*b^4*d^2 + 2*a^2*b^2*c^2 - 15*a^2*b^2*d^2 + 2*a*b^3*c*d - 8*a^3*b*c*d))/(2*(a^14*d^4 - b^14*c^4 + 5*a^2*b^12*c^4 - 10*a^4*b^10*c^4 + 10*a^6*b^8*c^4 - 5*a^8*b^6*c^4 + a^10*b^4*c^4 - a^4*b^10*d^4 + 5*a^6*b^8*d^4 - 10*a^8*b^6*d^4 + 10*a^10*b^4*d^4 - 5*a^12*b^2*d^4 + 4*a^3*b^11*c*d^3 - 20*a^3*b^11*c^3*d - 20*a^5*b^9*c*d^3 + 40*a^5*b^9*c^3*d + 40*a^7*b^7*c*d^3 - 40*a^7*b^7*c^3*d - 40*a^9*b^5*c*d^3 + 20*a^9*b^5*c^3*d + 20*a^11*b^3*c*d^3 - 4*a^11*b^3*c^3*d - 6*a^2*b^12*c^2*d^2 + 30*a^4*b^10*c^2*d^2 - 60*a^6*b^8*c^2*d^2 + 60*a^8*b^6*c^2*d^2 - 30*a^10*b^4*c^2*d^2 + 6*a^12*b^2*c^2*d^2 + 4*a*b^13*c^3*d - 4*a^13*b*c*d^3)))*(12*a^4*d^2 + b^4*c^2 + 6*b^4*d^2 + 2*a^2*b^2*c^2 - 15*a^2*b^2*d^2 + 2*a*b^3*c*d - 8*a^3*b*c*d))/(2*(a^14*d^4 - b^14*c^4 + 5*a^2*b^12*c^4 - 10*a^4*b^10*c^4 + 10*a^6*b^8*c^4 - 5*a^8*b^6*c^4 + a^10*b^4*c^4 - a^4*b^10*d^4 + 5*a^6*b^8*d^4 - 10*a^8*b^6*d^4 + 10*a^10*b^4*d^4 - 5*a^12*b^2*d^4 + 4*a^3*b^11*c*d^3 - 20*a^3*b^11*c^3*d - 20*a^5*b^9*c*d^3 + 40*a^5*b^9*c^3*d + 40*a^7*b^7*c*d^3 - 40*a^7*b^7*c^3*d - 40*a^9*b^5*c*d^3 + 20*a^9*b^5*c^3*d + 20*a^11*b^3*c*d^3 - 4*a^11*b^3*c^3*d - 6*a^2*b^12*c^2*d^2 + 30*a^4*b^10*c^2*d^2 - 60*a^6*b^8*c^2*d^2 + 60*a^8*b^6*c^2*d^2 - 30*a^10*b^4*c^2*d^2 + 6*a^12*b^2*c^2*d^2 + 4*a*b^13*c^3*d - 4*a^13*b*c*d^3)) - (b^2*(-(a + b)^5*(a - b)^5)^(1/2)*((8*(60*a*b^15*c^7*d^7 - 36*a*b^15*c^5*d^9 - 13*a*b^15*c^9*d^5 - 10*a*b^15*c^11*d^3 - 4*a^3*b^13*c^13*d + 36*a^5*b^11*c*d^13 - 4*a^5*b^11*c^13*d - 144*a^7*b^9*c*d^13 + 216*a^9*b^7*c*d^13 - 144*a^11*b^5*c*d^13 + 36*a^13*b^3*c*d^13 + 4*a^15*b*c^3*d^11 + 72*a^2*b^14*c^4*d^10 - 108*a^2*b^14*c^6*d^8 + 19*a^2*b^14*c^8*d^6 + 14*a^2*b^14*c^10*d^4 - a^2*b^14*c^12*d^2 + 120*a^3*b^13*c^5*d^9 - 305*a^3*b^13*c^7*d^7 + 190*a^3*b^13*c^9*d^5 + 19*a^3*b^13*c^11*d^3 - 72*a^4*b^12*c^2*d^12 - 168*a^4*b^12*c^4*d^10 + 699*a^4*b^12*c^6*d^8 - 602*a^4*b^12*c^8*d^6 + 99*a^4*b^12*c^10*d^4 + 20*a^4*b^12*c^12*d^2 - 36*a^5*b^11*c^3*d^11 - 535*a^5*b^11*c^5*d^9 + 1354*a^5*b^11 *c^7*d^7 - 895*a^5*b^11*c^9*d^5 + 40*a^5*b^11*c^11*d^3 + 276*a^6*b^10*c^2*d^12 + 233*a^6*b^10*c^4*d^10 - 2046*a^6*b^10*c^6*d^8 + 2161*a^6*b^10*c^8*d^6 - 552*a^6*b^10*c^10*d^4 + 44*a^6*b^10*c^12*d^2 + 61*a^7*b^9*c^3*d^11 + 1386*a^7*b^9*c^5*d^9 - 2979*a^7*b^9*c^7*d^7 + 1860*a^7*b^9*c^9*d^5 - 220*a^7*b^9*c^11*d^3 - 375*a^8*b^8*c^2*d^12 - 270*a^8*b^8*c^4*d^10 + 2885*a^8*b^8*c^6*d^8 - 3012*a^8*b^8*c^8*d^6 + 628*a^8*b^8*c^10*d^4 - 88*a^9*b^7*c^3*d^11 - 1544*a^9*b^7*c^5*d^9 + 2648*a^9*b^7*c^7*d^7 - 1088*a^9*b^7*c^9*d^5 + 216*a^10*b^6*c^2*d^12 + 100*a^10*b^6*c^4*d^10 - 1336*a^10*b^6*c^6*d^8 + 1056*a^10*b^6*c^8*d^6 + 180*a^11*b^5*c^3*d^11 + 248*a^11*b^5*c^5*d^9 - 400*a^11*b^5*c^7*d^7 - 60*a^12*b^4*c^2*d^12 + 248*a^12*b^4*c^4*d^10 - 148*a^12*b^4*c^6*d^8 - 184*a^13*b^3*c^3*d^11 + 172*a^13*b^3*c^5*d^9 + 24*a^14*b^2*c^2*d^12 - 44*a^14*b^2*c^4*d^10 - a*b^15*c^13*d))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12) - (8*tan(e/2 + (f*x)/2)*(4*a^16*c^3*d^11 - 4*a^3*b^13*c^14 - 4*a^5*b^11*c^14 - a*b^15*c^14 + 144*a*b^15*c^4*d^10 - 348*a*b^15*c^6*d^8 + 214*a*b^15*c^8*d^6 + 7*a*b^15*c^10*d^4 - 8*a*b^15*c^12*d^2 - a^2*b^14*c^13*d - 144*a^4*b^12*c*d^13 + 20*a^4*b^12*c^13*d + 684*a^6*b^10*c*d^13 + 44*a^6*b^10*c^13*d - 1314*a^8*b^8*c*d^13 + 1224*a^10*b^6*c*d^13 - 504*a^12*b^4*c*d^13 + 36*a^14*b^2*c*d^13 + 24*a^15*b*c^2*d^12 - 44*a^15*b*c^4*d^10 - 432*a^2*b^14*c^3*d^11 + 1140*a^2*b^14*c^5*d^9 - 818*a^2*b^14*c^7*d^7 + 55*a^2*b^14*c^9*d^5 + 16*a^2*b^14*c^11*d^3 + 432*a^3*b^13*c^2*d^12 - 2016*a^3*b^13*c^4*d^10 + 2938*a^3*b^13*c^6*d^8 - 1485*a^3*b^13*c^8*d^6 + 152*a^3*b^13*c

^10*d^4 + 27*a^3*b^13*c^12*d^2 + 2688*a^4*b^12*c^3*d^11 - 6574*a^4*b^12*c^5
*d^9 + 5107*a^4*b^12*c^7*d^7 - 1056*a^4*b^12*c^9*d^5 + 59*a^4*b^12*c^11*d^3
 - 2148*a^5*b^11*c^2*d^12 + 8378*a^5*b^11*c^4*d^10 - 10619*a^5*b^11*c^6*d^8
 + 5064*a^5*b^11*c^8*d^6 - 975*a^5*b^11*c^10*d^4 + 48*a^5*b^11*c^12*d^2 - 7
294*a^6*b^10*c^3*d^11 + 16053*a^6*b^10*c^5*d^9 - 12464*a^6*b^10*c^7*d^7 + 3
649*a^6*b^10*c^9*d^5 - 640*a^6*b^10*c^11*d^3 + 4470*a^7*b^9*c^2*d^12 - 1581
5*a^7*b^9*c^4*d^10 + 18608*a^7*b^9*c^6*d^8 - 8939*a^7*b^9*c^8*d^6 + 2300*a^
7*b^9*c^10*d^4 - 220*a^7*b^9*c^12*d^2 + 10105*a^8*b^8*c^3*d^11 - 19912*a^8*
b^8*c^5*d^9 + 14693*a^8*b^8*c^7*d^7 - 4524*a^8*b^8*c^9*d^5 + 628*a^8*b^8*c^
11*d^3 - 4632*a^9*b^7*c^2*d^12 + 14976*a^9*b^7*c^4*d^10 - 15576*a^9*b^7*c^6
*d^8 + 6104*a^9*b^7*c^8*d^6 - 1088*a^9*b^7*c^10*d^4 - 7104*a^10*b^6*c^3*d^1
1 + 11320*a^10*b^6*c^5*d^9 - 6184*a^10*b^6*c^7*d^7 + 1120*a^10*b^6*c^9*d^5
+ 2232*a^11*b^5*c^2*d^12 - 5932*a^11*b^5*c^4*d^10 + 4344*a^11*b^5*c^6*d^8 -
 688*a^11*b^5*c^8*d^6 + 1892*a^12*b^4*c^3*d^11 - 1920*a^12*b^4*c^5*d^9 + 36
8*a^12*b^4*c^7*d^7 - 252*a^13*b^3*c^2*d^12 + 624*a^13*b^3*c^4*d^10 - 292*a^
13*b^3*c^6*d^8 - 192*a^14*b^2*c^3*d^11 + 172*a^14*b^2*c^5*d^9))/(a^17*d^13
- b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9
*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13
- 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^1
6*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d -
36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^
12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*
b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^
2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 12
6*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*
b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*
c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d
^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 +
 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548
*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8
*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^
11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7
 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 15
48*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^1
0*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b
^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3
*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4
- 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126
*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*
b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*
d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12) + (b^2*(-(a + b)^5*(a - b)^5)^(1/2
)*((8*(2*a^2*b^17*c^16 - 6*a^6*b^13*c^16 + 4*a^8*b^11*c^16 + 4*a^19*c^3*d^1
3 - 4*a^19*c^5*d^11 + 12*a*b^18*c^9*d^7 - 28*a*b^18*c^11*d^5 + 16*a*b^18*c^
13*d^3 - 10*a^3*b^16*c^15*d - 24*a^5*b^14*c^15*d + 78*a^7*b^12*c^15*d + 12*
a^9*b^10*c*d^15 - 44*a^9*b^10*c^15*d - 54*a^11*b^8*c*d^15 + 96*a^13*b^6*c*d

^15 - 78*a^15*b^4*c*d^15 + 24*a^17*b^2*c*d^15 + 12*a^18*b*c^2*d^14 - 56*a^1
8*b*c^4*d^12 + 44*a^18*b*c^6*d^10 - 96*a^2*b^17*c^8*d^8 + 234*a^2*b^17*c^10
*d^6 - 146*a^2*b^17*c^12*d^4 + 6*a^2*b^17*c^14*d^2 + 336*a^3*b^16*c^7*d^9 -
 918*a^3*b^16*c^9*d^7 + 726*a^3*b^16*c^11*d^5 - 134*a^3*b^16*c^13*d^3 - 672
*a^4*b^15*c^6*d^10 + 2280*a^4*b^15*c^8*d^8 - 2520*a^4*b^15*c^10*d^6 + 952*a
^4*b^15*c^12*d^4 - 40*a^4*b^15*c^14*d^2 + 840*a^5*b^14*c^5*d^11 - 4032*a^5*
b^14*c^7*d^9 + 6360*a^5*b^14*c^9*d^7 - 3768*a^5*b^14*c^11*d^5 + 624*a^5*b^1
4*c^13*d^3 - 672*a^6*b^13*c^4*d^12 + 5292*a^6*b^13*c^6*d^10 - 11772*a^6*b^1
3*c^8*d^8 + 10050*a^6*b^13*c^10*d^6 - 3174*a^6*b^13*c^12*d^4 + 282*a^6*b^13
*c^14*d^2 + 336*a^7*b^12*c^3*d^13 - 5124*a^7*b^12*c^5*d^11 + 16212*a^7*b^12
*c^7*d^9 - 19602*a^7*b^12*c^9*d^7 + 9670*a^7*b^12*c^11*d^5 - 1570*a^7*b^12*
c^13*d^3 - 96*a^8*b^11*c^2*d^14 + 3528*a^8*b^11*c^4*d^12 - 16872*a^8*b^11*c
^6*d^10 + 28848*a^8*b^11*c^8*d^8 - 20340*a^8*b^11*c^10*d^6 + 5396*a^8*b^11*
c^12*d^4 - 468*a^8*b^11*c^14*d^2 - 1620*a^9*b^10*c^3*d^13 + 13320*a^9*b^10*
c^5*d^11 - 32304*a^9*b^10*c^7*d^9 + 31560*a^9*b^10*c^9*d^7 - 12648*a^9*b^10
*c^11*d^5 + 1724*a^9*b^10*c^13*d^3 + 442*a^10*b^9*c^2*d^14 - 7810*a^10*b^9*
c^4*d^12 + 27546*a^10*b^9*c^6*d^10 - 37338*a^10*b^9*c^8*d^8 + 21288*a^10*b^
9*c^10*d^6 - 4348*a^10*b^9*c^12*d^4 + 220*a^10*b^9*c^14*d^2 + 3206*a^11*b^8
*c^3*d^13 - 17850*a^11*b^8*c^5*d^11 + 34018*a^11*b^8*c^7*d^9 - 26556*a^11*b
^8*c^9*d^7 + 7896*a^11*b^8*c^11*d^5 - 660*a^11*b^8*c^13*d^3 - 816*a^12*b^7*
c^2*d^14 + 8696*a^12*b^7*c^4*d^12 - 23696*a^12*b^7*c^6*d^10 + 25056*a^12*b^
7*c^8*d^8 - 10560*a^12*b^7*c^10*d^6 + 1320*a^12*b^7*c^12*d^4 - 3064*a^13*b^
6*c^3*d^13 + 12400*a^13*b^6*c^5*d^11 - 18048*a^13*b^6*c^7*d^9 + 10464*a^13*
b^6*c^9*d^7 - 1848*a^13*b^6*c^11*d^5 + 702*a^14*b^5*c^2*d^14 - 4770*a^14*b^
5*c^4*d^12 + 9858*a^14*b^5*c^6*d^10 - 7638*a^14*b^5*c^8*d^8 + 1848*a^14*b^5
*c^10*d^6 + 1314*a^15*b^4*c^3*d^13 - 3954*a^15*b^4*c^5*d^11 + 4038*a^15*b^4
*c^7*d^9 - 1320*a^15*b^4*c^9*d^7 - 244*a^16*b^3*c^2*d^14 + 1084*a^16*b^3*c^
4*d^12 - 1500*a^16*b^3*c^6*d^10 + 660*a^16*b^3*c^8*d^8 - 176*a^17*b^2*c^3*d
^13 + 372*a^17*b^2*c^5*d^11 - 220*a^17*b^2*c^7*d^9))/(a^17*d^13 - b^17*c^13
 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9
*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^
2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 -
 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10
*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^1
2*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 -
 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*
b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*
c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d
^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 4
44*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a
^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^1
0*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c
^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^
8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 14
6*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9

*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7
*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*
d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7
- 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852
*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*
b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c
^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6
 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b
^16*c^12*d - 9*a^16*b*c*d^12) + (8*tan(e/2 + (f*x)/2)*(4*a*b^18*c^16 - 12*a
^5*b^14*c^16 + 8*a^7*b^12*c^16 + 8*a^19*c^2*d^14 - 8*a^19*c^4*d^12 + 12*a*b
^18*c^10*d^6 - 28*a*b^18*c^12*d^4 + 12*a*b^18*c^14*d^2 - 20*a^2*b^17*c^15*d
 - 48*a^4*b^15*c^15*d + 156*a^6*b^13*c^15*d - 88*a^8*b^11*c^15*d + 12*a^10*
b^9*c*d^15 - 48*a^12*b^7*c*d^15 + 84*a^14*b^5*c*d^15 - 72*a^16*b^3*c*d^15 -
 112*a^18*b*c^3*d^13 + 88*a^18*b*c^5*d^11 - 84*a^2*b^17*c^9*d^7 + 212*a^2*b
^17*c^11*d^5 - 108*a^2*b^17*c^13*d^3 + 240*a^3*b^16*c^8*d^8 - 744*a^3*b^16*
c^10*d^6 + 584*a^3*b^16*c^12*d^4 - 80*a^3*b^16*c^14*d^2 - 336*a^4*b^15*c^7*
d^9 + 1632*a^4*b^15*c^9*d^7 - 2176*a^4*b^15*c^11*d^5 + 928*a^4*b^15*c^13*d^
3 + 168*a^5*b^14*c^6*d^10 - 2472*a^5*b^14*c^8*d^8 + 5460*a^5*b^14*c^10*d^6
- 3708*a^5*b^14*c^12*d^4 + 564*a^5*b^14*c^14*d^2 + 168*a^6*b^13*c^5*d^11 +
2520*a^6*b^13*c^7*d^9 - 9204*a^6*b^13*c^9*d^7 + 9180*a^6*b^13*c^11*d^5 - 28
20*a^6*b^13*c^13*d^3 - 336*a^7*b^12*c^4*d^12 - 1344*a^7*b^12*c^6*d^10 + 104
16*a^7*b^12*c^8*d^8 - 15960*a^7*b^12*c^10*d^6 + 8152*a^7*b^12*c^12*d^4 - 93
6*a^7*b^12*c^14*d^2 + 240*a^8*b^11*c^3*d^13 - 336*a^8*b^11*c^5*d^11 - 7488*
a^8*b^11*c^7*d^9 + 19800*a^8*b^11*c^9*d^7 - 15416*a^8*b^11*c^11*d^5 + 3288*
a^8*b^11*c^13*d^3 - 84*a^9*b^10*c^2*d^14 + 1188*a^9*b^10*c^4*d^12 + 2292*a^
9*b^10*c^6*d^10 - 16596*a^9*b^10*c^8*d^8 + 20136*a^9*b^10*c^10*d^6 - 7376*a
^9*b^10*c^12*d^4 + 440*a^9*b^10*c^14*d^2 - 908*a^10*b^9*c^3*d^13 + 1740*a^1
0*b^9*c^5*d^11 + 7556*a^10*b^9*c^7*d^9 - 18048*a^10*b^9*c^9*d^7 + 10936*a^1
0*b^9*c^11*d^5 - 1288*a^10*b^9*c^13*d^3 + 328*a^11*b^8*c^2*d^14 - 2808*a^11
*b^8*c^4*d^12 + 1088*a^11*b^8*c^6*d^10 + 9600*a^11*b^8*c^8*d^8 - 10584*a^11
*b^8*c^10*d^6 + 2376*a^11*b^8*c^12*d^4 + 1792*a^12*b^7*c^3*d^13 - 4720*a^12
*b^7*c^5*d^11 - 144*a^12*b^7*c^7*d^9 + 5856*a^12*b^7*c^9*d^7 - 2736*a^12*b^
7*c^11*d^5 - 596*a^13*b^6*c^2*d^14 + 3980*a^13*b^6*c^4*d^12 - 4908*a^13*b^6
*c^6*d^10 - 156*a^13*b^6*c^8*d^8 + 1680*a^13*b^6*c^10*d^6 - 1932*a^14*b^5*c
^3*d^13 + 4812*a^14*b^5*c^5*d^11 - 3012*a^14*b^5*c^7*d^9 + 48*a^14*b^5*c^9*
d^7 + 552*a^15*b^4*c^2*d^14 - 2616*a^15*b^4*c^4*d^12 + 3096*a^15*b^4*c^6*d^
10 - 1032*a^15*b^4*c^8*d^8 + 920*a^16*b^3*c^3*d^13 - 1752*a^16*b^3*c^5*d^11
 + 904*a^16*b^3*c^7*d^9 - 208*a^17*b^2*c^2*d^14 + 600*a^17*b^2*c^4*d^12 - 3
92*a^17*b^2*c^6*d^10 + 24*a^18*b*c*d^15))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^
15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 -
 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^
17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*
c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9
*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^1
2 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^1

5*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^
7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 3
96*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^
5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12
*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7
*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11
- 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 57
6*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^
8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c
^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^
5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 -
1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a
^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*
b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c
^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^1
1 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 15
6*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*
b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d
- 9*a^16*b*c*d^12) + (b^2*((8*(16*a^4*b^18*c^18 - 4*a^2*b^20*c^18 - 24*a^6*
b^16*c^18 + 16*a^8*b^14*c^18 - 4*a^10*b^12*c^18 + 4*a^22*c^2*d^16 - 8*a^22*
c^4*d^14 + 4*a^22*c^6*d^12 + 4*a*b^21*c^13*d^5 - 8*a*b^21*c^15*d^3 + 24*a^3
*b^19*c^17*d - 136*a^5*b^17*c^17*d + 224*a^7*b^15*c^17*d - 156*a^9*b^13*c^1
7*d + 40*a^11*b^11*c^17*d - 4*a^13*b^9*c*d^17 + 16*a^15*b^7*c*d^17 - 24*a^1
7*b^5*c*d^17 + 16*a^19*b^3*c*d^17 - 32*a^21*b*c^3*d^15 + 76*a^21*b*c^5*d^13
 - 40*a^21*b*c^7*d^11 - 40*a^2*b^20*c^12*d^6 + 76*a^2*b^20*c^14*d^4 - 32*a^
2*b^20*c^16*d^2 + 176*a^3*b^19*c^11*d^7 - 328*a^3*b^19*c^13*d^5 + 128*a^3*b
^19*c^15*d^3 - 440*a^4*b^18*c^10*d^8 + 864*a^4*b^18*c^12*d^6 - 392*a^4*b^18
*c^14*d^4 - 48*a^4*b^18*c^16*d^2 + 660*a^5*b^17*c^9*d^9 - 1584*a^5*b^17*c^1
1*d^7 + 1052*a^5*b^17*c^13*d^5 + 8*a^5*b^17*c^15*d^3 - 528*a^6*b^16*c^8*d^1
0 + 2156*a^6*b^16*c^10*d^8 - 2264*a^6*b^16*c^12*d^6 + 148*a^6*b^16*c^14*d^4
 + 512*a^6*b^16*c^16*d^2 - 2112*a^7*b^15*c^9*d^9 + 3520*a^7*b^15*c^11*d^7 -
 480*a^7*b^15*c^13*d^5 - 1152*a^7*b^15*c^15*d^3 + 528*a^8*b^14*c^6*d^12 + 1
056*a^8*b^14*c^8*d^10 - 3696*a^8*b^14*c^10*d^8 + 1216*a^8*b^14*c^12*d^6 + 1
808*a^8*b^14*c^14*d^4 - 928*a^8*b^14*c^16*d^2 - 660*a^9*b^13*c^5*d^13 + 792
*a^9*b^13*c^7*d^11 + 2244*a^9*b^13*c^9*d^9 - 2288*a^9*b^13*c^11*d^7 - 2180*
a^9*b^13*c^13*d^5 + 2248*a^9*b^13*c^15*d^3 + 440*a^10*b^12*c^4*d^14 - 2332*
a^10*b^12*c^6*d^12 + 176*a^10*b^12*c^8*d^10 + 2684*a^10*b^12*c^10*d^8 + 189
6*a^10*b^12*c^12*d^6 - 3532*a^10*b^12*c^14*d^4 + 672*a^10*b^12*c^16*d^2 - 1
76*a^11*b^11*c^3*d^15 + 2552*a^11*b^11*c^5*d^13 - 2464*a^11*b^11*c^7*d^11 -
 1496*a^11*b^11*c^9*d^9 - 528*a^11*b^11*c^11*d^7 + 3736*a^11*b^11*c^13*d^5
- 1664*a^11*b^11*c^15*d^3 + 40*a^12*b^10*c^2*d^16 - 1664*a^12*b^10*c^4*d^14
 + 3736*a^12*b^10*c^6*d^12 - 528*a^12*b^10*c^8*d^10 - 1496*a^12*b^10*c^10*d
^8 - 2464*a^12*b^10*c^12*d^6 + 2552*a^12*b^10*c^14*d^4 - 176*a^12*b^10*c^16
*d^2 + 672*a^13*b^9*c^3*d^15 - 3532*a^13*b^9*c^5*d^13 + 1896*a^13*b^9*c^7*d
^11 + 2684*a^13*b^9*c^9*d^9 + 176*a^13*b^9*c^11*d^7 - 2332*a^13*b^9*c^13*d^

5 + 440*a^13*b^9*c^15*d^3 - 156*a^14*b^8*c^2*d^16 + 2248*a^14*b^8*c^4*d^14 - 2180*a^14*b^8*c^6*d^12 - 2288*a^14*b^8*c^8*d^10 + 2244*a^14*b^8*c^10*d^8 + 792*a^14*b^8*c^12*d^6 - 660*a^14*b^8*c^14*d^4 - 928*a^15*b^7*c^3*d^15 + 1808*a^15*b^7*c^5*d^13 + 1216*a^15*b^7*c^7*d^11 - 3696*a^15*b^7*c^9*d^9 + 1056*a^15*b^7*c^11*d^7 + 528*a^15*b^7*c^13*d^5 + 224*a^16*b^6*c^2*d^16 - 1152*a^16*b^6*c^4*d^14 - 480*a^16*b^6*c^6*d^12 + 3520*a^16*b^6*c^8*d^10 - 2112*a^16*b^6*c^10*d^8 + 512*a^17*b^5*c^3*d^15 + 148*a^17*b^5*c^5*d^13 - 2264*a^17*b^5*c^7*d^11 + 2156*a^17*b^5*c^9*d^9 - 528*a^17*b^5*c^11*d^7 - 136*a^18*b^4*c^2*d^16 + 8*a^18*b^4*c^4*d^14 + 1052*a^18*b^4*c^6*d^12 - 1584*a^18*b^4*c^8*d^10 + 660*a^18*b^4*c^10*d^8 - 48*a^19*b^3*c^3*d^15 - 392*a^19*b^3*c^5*d^13 + 864*a^19*b^3*c^7*d^11 - 440*a^19*b^3*c^9*d^9 + 24*a^20*b^2*c^2*d^16 + 128*a^20*b^2*c^4*d^14 - 328*a^20*b^2*c^6*d^12 + 176*a^20*b^2*c^8*d^10 + 4*a*b^21*c^17*d - 4*a^21*b*c*d^17))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 672*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 1992*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 + 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 + 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12) - (8*tan(e/2 + (f*x)/2)*(12*a*b^21*c^18 - 12*a^22*c*d^17 - 56*a^3*b^19*c^18 + 104*a^5*b^17*c^18 - 96*a^7*b^15*c^18 + 44*a^9*b^13*c^18 - 8*a^11*b^11*c^18 + 32*a^22*c^3*d^15 - 28*a^22*c^5*d^13 + 8*a^22*c^7*d^11 - 16*a*b^21*c^12*d^6 + 44*a*b^21*c^14*d^4 - 40*a*b^21*c^16*d^2 - 132*a^2*b^20*c^17*d + 616*a^4*b^18*c^17*d - 1144*a^6*b^16*c^17*d + 1056*a^8*b^14*c^17*d - 484*a^10*b^12*c^17*d + 16*a^12*b^10*c*d^17 + 88*a^12*b^10*c^17*d - 76*a^14*b^8*c*d^17 + 144*a^16*b^6*c*d^17 - 136*a^18*b^4*c*d^17 + 64*a^20*b^2*c*d^17 + 132*a^21*b*c^2*d^16 - 352*a^21*b*c^4*d^14 + 308*a^21*b*c^6*d^12 - 88*a

^21*b*c^8*d^10 + 176*a^2*b^20*c^11*d^7 - 484*a^2*b^20*c^13*d^5 + 440*a^2*b^20*c^15*d^3 - 880*a^3*b^19*c^10*d^8 + 2496*a^3*b^19*c^12*d^6 - 2408*a^3*b^19*c^14*d^4 + 848*a^3*b^19*c^16*d^2 + 2640*a^4*b^18*c^9*d^9 - 8096*a^4*b^18*c^11*d^7 + 8888*a^4*b^18*c^13*d^5 - 4048*a^4*b^18*c^15*d^3 - 5280*a^5*b^17*c^8*d^10 + 18700*a^5*b^17*c^10*d^8 - 24784*a^5*b^17*c^12*d^6 + 14692*a^5*b^17*c^14*d^4 - 3432*a^5*b^17*c^16*d^2 + 7392*a^6*b^16*c^7*d^11 - 32868*a^6*b^16*c^9*d^9 + 54384*a^6*b^16*c^11*d^7 - 40876*a^6*b^16*c^13*d^5 + 13112*a^6*b^16*c^15*d^3 - 7392*a^7*b^15*c^6*d^12 + 45408*a^7*b^15*c^8*d^10 - 95040*a^7*b^15*c^10*d^8 + 89280*a^7*b^15*c^12*d^6 - 38208*a^7*b^15*c^14*d^4 + 6048*a^7*b^15*c^16*d^2 + 5280*a^8*b^14*c^5*d^13 - 49632*a^8*b^14*c^7*d^11 + 133056*a^8*b^14*c^9*d^9 - 156992*a^8*b^14*c^11*d^7 + 88000*a^8*b^14*c^13*d^5 - 20768*a^8*b^14*c^15*d^3 - 2640*a^9*b^13*c^4*d^14 + 42372*a^9*b^13*c^6*d^12 - 150216*a^9*b^13*c^8*d^10 + 225676*a^9*b^13*c^10*d^8 - 162336*a^9*b^13*c^12*d^6 + 52532*a^9*b^13*c^14*d^4 - 5432*a^9*b^13*c^16*d^2 + 880*a^10*b^12*c^3*d^15 - 27500*a^10*b^12*c^5*d^13 + 137368*a^10*b^12*c^7*d^11 - 266244*a^10*b^12*c^9*d^9 + 242528*a^10*b^12*c^11*d^7 - 104060*a^10*b^12*c^13*d^5 + 17512*a^10*b^12*c^15*d^3 - 176*a^11*b^11*c^2*d^16 + 13024*a^11*b^11*c^4*d^14 - 101288*a^11*b^11*c^6*d^12 + 257136*a^11*b^11*c^8*d^10 - 296824*a^11*b^11*c^10*d^8 + 165760*a^11*b^11*c^12*d^6 - 40072*a^11*b^11*c^14*d^4 + 2448*a^11*b^11*c^16*d^2 - 4224*a^12*b^10*c^3*d^15 + 59000*a^12*b^10*c^5*d^13 - 202544*a^12*b^10*c^7*d^11 + 299816*a^12*b^10*c^9*d^9 - 214368*a^12*b^10*c^11*d^7 + 69784*a^12*b^10*c^13*d^5 - 7568*a^12*b^10*c^15*d^3 + 836*a^13*b^9*c^2*d^16 - 26048*a^13*b^9*c^4*d^14 + 129580*a^13*b^9*c^6*d^12 - 249832*a^13*b^9*c^8*d^10 + 226116*a^13*b^9*c^10*d^8 - 96272*a^13*b^9*c^12*d^6 + 16060*a^13*b^9*c^14*d^4 - 440*a^13*b^9*c^16*d^2 + 8128*a^14*b^8*c^3*d^15 - 66628*a^14*b^8*c^5*d^13 + 170424*a^14*b^8*c^7*d^11 - 195404*a^14*b^8*c^9*d^9 + 107184*a^14*b^8*c^11*d^7 - 24948*a^14*b^8*c^13*d^5 + 1320*a^14*b^8*c^15*d^3 - 1584*a^15*b^7*c^2*d^16 + 26752*a^15*b^7*c^4*d^14 - 94160*a^15*b^7*c^6*d^12 + 138688*a^15*b^7*c^8*d^10 - 96624*a^15*b^7*c^10*d^8 + 29568*a^15*b^7*c^12*d^6 - 2640*a^15*b^7*c^14*d^4 - 7872*a^16*b^6*c^3*d^15 + 41712*a^16*b^6*c^5*d^13 - 80448*a^16*b^6*c^7*d^11 + 70224*a^16*b^6*c^9*d^9 - 27456*a^16*b^6*c^11*d^7 + 3696*a^16*b^6*c^13*d^5 + 1496*a^17*b^5*c^2*d^16 - 14608*a^17*b^5*c^4*d^14 + 37532*a^17*b^5*c^6*d^12 - 40920*a^17*b^5*c^8*d^10 + 20196*a^17*b^5*c^10*d^8 - 3696*a^17*b^5*c^12*d^6 + 3888*a^18*b^4*c^3*d^15 - 13748*a^18*b^4*c^5*d^13 + 19016*a^18*b^4*c^7*d^11 - 11660*a^18*b^4*c^9*d^9 + 2640*a^18*b^4*c^11*d^7 - 704*a^19*b^3*c^2*d^16 + 3872*a^19*b^3*c^4*d^14 - 6952*a^19*b^3*c^6*d^12 + 5104*a^19*b^3*c^8*d^10 - 1320*a^19*b^3*c^10*d^8 - 832*a^20*b^2*c^3*d^15 + 1912*a^20*b^2*c^5*d^13 - 1584*a^20*b^2*c^7*d^11 + 440*a^20*b^2*c^9*d^9))/(a^17*d^13 - b^17*c^13 + 4*a^2*b^15*c^13 - 6*a^4*b^13*c^13 + 4*a^6*b^11*c^13 - a^8*b^9*c^13 + a^9*b^8*d^13 - 4*a^11*b^6*d^13 + 6*a^13*b^4*d^13 - 4*a^15*b^2*d^13 - 2*a^17*c^2*d^11 + a^17*c^4*d^9 - b^17*c^9*d^4 + 2*b^17*c^11*d^2 + 9*a*b^16*c^8*d^5 - 18*a*b^16*c^10*d^3 - 36*a^3*b^14*c^12*d + 54*a^5*b^12*c^12*d - 36*a^7*b^10*c^12*d - 9*a^8*b^9*c*d^12 + 9*a^9*b^8*c^12*d + 36*a^10*b^7*c*d^12 - 54*a^12*b^5*c*d^12 + 36*a^14*b^3*c*d^12 + 18*a^16*b*c^3*d^10 - 9*a^16*b*c^5*d^8 - 36*a^2*b^15*c^7*d^6 + 76*a^2*b^15*c^9*d^4 - 44

*a^2*b^15*c^11*d^2 + 84*a^3*b^14*c^6*d^7 - 204*a^3*b^14*c^8*d^5 + 156*a^3*b
^14*c^10*d^3 - 126*a^4*b^13*c^5*d^8 + 396*a^4*b^13*c^7*d^6 - 420*a^4*b^13*c
^9*d^4 + 156*a^4*b^13*c^11*d^2 + 126*a^5*b^12*c^4*d^9 - 588*a^5*b^12*c^6*d^
7 + 852*a^5*b^12*c^8*d^5 - 444*a^5*b^12*c^10*d^3 - 84*a^6*b^11*c^3*d^10 + 6
72*a^6*b^11*c^5*d^8 - 1308*a^6*b^11*c^7*d^6 + 940*a^6*b^11*c^9*d^4 - 224*a^
6*b^11*c^11*d^2 + 36*a^7*b^10*c^2*d^11 - 576*a^7*b^10*c^4*d^9 + 1548*a^7*b^
10*c^6*d^7 - 1548*a^7*b^10*c^8*d^5 + 576*a^7*b^10*c^10*d^3 + 354*a^8*b^9*c^
3*d^10 - 1437*a^8*b^9*c^5*d^8 + 1992*a^8*b^9*c^7*d^6 - 1045*a^8*b^9*c^9*d^4
 + 146*a^8*b^9*c^11*d^2 - 146*a^9*b^8*c^2*d^11 + 1045*a^9*b^8*c^4*d^9 - 199
2*a^9*b^8*c^6*d^7 + 1437*a^9*b^8*c^8*d^5 - 354*a^9*b^8*c^10*d^3 - 576*a^10*
b^7*c^3*d^10 + 1548*a^10*b^7*c^5*d^8 - 1548*a^10*b^7*c^7*d^6 + 576*a^10*b^7
*c^9*d^4 - 36*a^10*b^7*c^11*d^2 + 224*a^11*b^6*c^2*d^11 - 940*a^11*b^6*c^4*
d^9 + 1308*a^11*b^6*c^6*d^7 - 672*a^11*b^6*c^8*d^5 + 84*a^11*b^6*c^10*d^3 +
 444*a^12*b^5*c^3*d^10 - 852*a^12*b^5*c^5*d^8 + 588*a^12*b^5*c^7*d^6 - 126*
a^12*b^5*c^9*d^4 - 156*a^13*b^4*c^2*d^11 + 420*a^13*b^4*c^4*d^9 - 396*a^13*
b^4*c^6*d^7 + 126*a^13*b^4*c^8*d^5 - 156*a^14*b^3*c^3*d^10 + 204*a^14*b^3*c
^5*d^8 - 84*a^14*b^3*c^7*d^6 + 44*a^15*b^2*c^2*d^11 - 76*a^15*b^2*c^4*d^9 +
 36*a^15*b^2*c^6*d^7 + 9*a*b^16*c^12*d - 9*a^16*b*c*d^12))*(-(a + b)^5*(a -
 b)^5)^(1/2)*(12*a^4*d^2 + b^4*c^2 + 6*b^4*d^2 + 2*a^2*b^2*c^2 - 15*a^2*b^2
*d^2 + 2*a*b^3*c*d - 8*a^3*b*c*d))/(2*(a^14*d^4 - b^14*c^4 + 5*a^2*b^12*c^4
 - 10*a^4*b^10*c^4 + 10*a^6*b^8*c^4 - 5*a^8*b^6*c^4 + a^10*b^4*c^4 - a^4*b^
10*d^4 + 5*a^6*b^8*d^4 - 10*a^8*b^6*d^4 + 10*a^10*b^4*d^4 - 5*a^12*b^2*d^4
+ 4*a^3*b^11*c*d^3 - 20*a^3*b^11*c^3*d - 20*a^5*b^9*c*d^3 + 40*a^5*b^9*c^3*
d + 40*a^7*b^7*c*d^3 - 40*a^7*b^7*c^3*d - 40*a^9*b^5*c*d^3 + 20*a^9*b^5*c^3
*d + 20*a^11*b^3*c*d^3 - 4*a^11*b^3*c^3*d - 6*a^2*b^12*c^2*d^2 + 30*a^4*b^1
0*c^2*d^2 - 60*a^6*b^8*c^2*d^2 + 60*a^8*b^6*c^2*d^2 - 30*a^10*b^4*c^2*d^2 +
 6*a^12*b^2*c^2*d^2 + 4*a*b^13*c^3*d - 4*a^13*b*c*d^3)))*(12*a^4*d^2 + b^4*
c^2 + 6*b^4*d^2 + 2*a^2*b^2*c^2 - 15*a^2*b^2*d^2 + 2*a*b^3*c*d - 8*a^3*b*c*
d))/(2*(a^14*d^4 - b^14*c^4 + 5*a^2*b^12*c^4 - 10*a^4*b^10*c^4 + 10*a^6*b^8
*c^4 - 5*a^8*b^6*c^4 + a^10*b^4*c^4 - a^4*b^10*d^4 + 5*a^6*b^8*d^4 - 10*a^8
*b^6*d^4 + 10*a^10*b^4*d^4 - 5*a^12*b^2*d^4 + 4*a^3*b^11*c*d^3 - 20*a^3*b^1
1*c^3*d - 20*a^5*b^9*c*d^3 + 40*a^5*b^9*c^3*d + 40*a^7*b^7*c*d^3 - 40*a^7*b
^7*c^3*d - 40*a^9*b^5*c*d^3 + 20*a^9*b^5*c^3*d + 20*a^11*b^3*c*d^3 - 4*a^11
*b^3*c^3*d - 6*a^2*b^12*c^2*d^2 + 30*a^4*b^10*c^2*d^2 - 60*a^6*b^8*c^2*d^2
+ 60*a^8*b^6*c^2*d^2 - 30*a^10*b^4*c^2*d^2 + 6*a^12*b^2*c^2*d^2 + 4*a*b^13*
c^3*d - 4*a^13*b*c*d^3)))*(12*a^4*d^2 + b^4*c^2 + 6*b^4*d^2 + 2*a^2*b^2*c^2
 - 15*a^2*b^2*d^2 + 2*a*b^3*c*d - 8*a^3*b*c*d))/(2*(a^14*d^4 - b^14*c^4 + 5
*a^2*b^12*c^4 - 10*a^4*b^10*c^4 + 10*a^6*b^8*c^4 - 5*a^8*b^6*c^4 + a^10*b^4
*c^4 - a^4*b^10*d^4 + 5*a^6*b^8*d^4 - 10*a^8*b^6*d^4 + 10*a^10*b^4*d^4 - 5*
a^12*b^2*d^4 + 4*a^3*b^11*c*d^3 - 20*a^3*b^11*c^3*d - 20*a^5*b^9*c*d^3 + 40
*a^5*b^9*c^3*d + 40*a^7*b^7*c*d^3 - 40*a^7*b^7*c^3*d - 40*a^9*b^5*c*d^3 + 2
0*a^9*b^5*c^3*d + 20*a^11*b^3*c*d^3 - 4*a^11*b^3*c^3*d - 6*a^2*b^12*c^2*d^2
 + 30*a^4*b^10*c^2*d^2 - 60*a^6*b^8*c^2*d^2 + 60*a^8*b^6*c^2*d^2 - 30*a^10*
b^4*c^2*d^2 + 6*a^12*b^2*c^2*d^2 + 4*a*b^13*c^3*d - 4*a^13*b*c*d^3))))*(-(a
 + b)^5*(a - b)^5)^(1/2)*(12*a^4*d^2 + b^4*c^2 + 6*b^4*d^2 + 2*a^2*b^2*c^2

```
- 15*a^2*b^2*d^2 + 2*a*b^3*c*d - 8*a^3*b*c*d)*1i)/(f*(a^14*d^4 - b^14*c^4 +
 5*a^2*b^12*c^4 - 10*a^4*b^10*c^4 + 10*a^6*b^8*c^4 - 5*a^8*b^6*c^4 + a^10*b
^4*c^4 - a^4*b^10*d^4 + 5*a^6*b^8*d^4 - 10*a^8*b^6*d^4 + 10*a^10*b^4*d^4 -
5*a^12*b^2*d^4 + 4*a^3*b^11*c*d^3 - 20*a^3*b^11*c^3*d - 20*a^5*b^9*c*d^3 +
40*a^5*b^9*c^3*d + 40*a^7*b^7*c*d^3 - 40*a^7*b^7*c^3*d - 40*a^9*b^5*c*d^3 +
 20*a^9*b^5*c^3*d + 20*a^11*b^3*c*d^3 - 4*a^11*b^3*c^3*d - 6*a^2*b^12*c^2*d
^2 + 30*a^4*b^10*c^2*d^2 - 60*a^6*b^8*c^2*d^2 + 60*a^8*b^6*c^2*d^2 - 30*a^1
0*b^4*c^2*d^2 + 6*a^12*b^2*c^2*d^2 + 4*a*b^13*c^3*d - 4*a^13*b*c*d^3))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.722 $\int \frac{1}{(a+b\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=669

$$-\frac{d^3\left(a^2d^2\left(2c^2+d^2\right)-ab\left(10c^3d-4cd^3\right)+b^2\left(20c^4-29c^2d^2+12d^4\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{5/2}(bc-ad)^5}+\frac{b^2\left(-7\right.}{2f\left(a^2-b^2\right)^2(bc-}$$

```
[Out] -b^3*(10*a^3*b*c*d-4*a*b^3*c*d-20*a^4*d^2-a^2*b^2*(2*c^2-29*d^2)-b^4*(c^2+1
2*d^2))*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/(-
a*d+b*c)^5/f-d^3*(a^2*d^2*(2*c^2+d^2)-a*b*(10*c^3*d-4*c*d^3)+b^2*(20*c^4-29
*c^2*d^2+12*d^4))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(-a*d+b*
c)^5/(c^2-d^2)^(5/2)/f-1/2*d*(a^4*d^3-b^4*d*(5*c^2-6*d^2)+2*a^2*b^2*d*(4*c^
2-5*d^2)-3*a*b^3*c*(c^2-d^2))*cos(f*x+e)/(a^2-b^2)^2/(-a*d+b*c)^3/(c^2-d^2)
/f/(c+d*sin(f*x+e))^2+1/2*b^2*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(a+b*sin(f*
x+e))^2/(c+d*sin(f*x+e))^2+1/2*b^2*(-7*a^2*d+3*a*b*c+4*b^2*d)*cos(f*x+e)/(a
^2-b^2)^2/(-a*d+b*c)^2/f/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2+3/2*d*(a^5*c*d
^4-2*a^3*b^2*c*d^4+a*b^4*c*(c^4-2*c^2*d^2+2*d^4)+b^5*d*(2*c^4-7*c^2*d^2+4*d
^4)-a^2*b^3*d*(3*c^4-12*c^2*d^2+7*d^4)-a^4*b*(3*c^2*d^3-2*d^5))*cos(f*x+e)/
(a^2-b^2)^2/(-a*d+b*c)^4/(c^2-d^2)^2/f/(c+d*sin(f*x+e))
```

Rubi [A] time = 3.31, antiderivative size = 669, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2802, 3055, 3001, 2660, 618, 204}

$$-\frac{b^3\left(-a^2b^2\left(2c^2-29d^2\right)+10a^3bcd-20a^4d^2-4ab^3cd-b^4\left(c^2+12d^2\right)\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{f\left(a^2-b^2\right)^{5/2}(bc-ad)^5}-\frac{d^3\left(a^2d^2\left(2c^2+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] -((b^3*(10*a^3*b*c*d - 4*a*b^3*c*d - 20*a^4*d^2 - a^2*b^2*(2*c^2 - 29*d^2)
- b^4*(c^2 + 12*d^2))*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/((a
^2 - b^2)^(5/2)*(b*c - a*d)^5*f)) - (d^3*(a^2*d^2*(2*c^2 + d^2) - a*b*(10*c
^3*d - 4*c*d^3) + b^2*(20*c^4 - 29*c^2*d^2 + 12*d^4))*ArcTan[(d + c*Tan[(e
+ f*x)/2])/Sqrt[c^2 - d^2]])/((b*c - a*d)^5*(c^2 - d^2)^(5/2)*f) - (d*(a^4*
d^3 - b^4*d*(5*c^2 - 6*d^2) + 2*a^2*b^2*d*(4*c^2 - 5*d^2) - 3*a*b^3*c*(c^2
- d^2))*Cos[e + f*x])/(2*(a^2 - b^2)^2*(b*c - a*d)^3*(c^2 - d^2)*f*(c + d*S
in[e + f*x])^2) + (b^2*Cos[e + f*x])/(2*(a^2 - b^2)*(b*c - a*d)*f*(a + b*Si
n[e + f*x])^2*(c + d*Sin[e + f*x])^2) + (b^2*(3*a*b*c - 7*a^2*d + 4*b^2*d)*
```

```
Cos[e + f*x])/(2*(a^2 - b^2)^2*(b*c - a*d)^2*f*(a + b*Sin[e + f*x])*(c + d*
Sin[e + f*x])^2) + (3*d*(a^5*c*d^4 - 2*a^3*b^2*c*d^4 + a*b^4*c*(c^4 - 2*c^2
*d^2 + 2*d^4) + b^5*d*(2*c^4 - 7*c^2*d^2 + 4*d^4) - a^2*b^3*d*(3*c^4 - 12*c
^2*d^2 + 7*d^4) - a^4*b*(3*c^2*d^3 - 2*d^5))*Cos[e + f*x])/(2*(a^2 - b^2)^2
*(b*c - a*d)^4*(c^2 - d^2)^2*f*(c + d*Sin[e + f*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx = \frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2(c+d\sin(e+fx))^2} -$$

$$= \frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2(c+d\sin(e+fx))^2} +$$

$$= -\frac{d\left(a^4d^3-b^4d\left(5c^2-6d^2\right)+2a^2b^2d\left(4c^2-5d^2\right)-3ab^3c\left(c^2-d\right.\right.}{2\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f(c+d\sin(e+f}$$

$$= -\frac{d\left(a^4d^3-b^4d\left(5c^2-6d^2\right)+2a^2b^2d\left(4c^2-5d^2\right)-3ab^3c\left(c^2-d\right.\right.}{2\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f(c+d\sin(e+f}$$

$$= -\frac{d\left(a^4d^3-b^4d\left(5c^2-6d^2\right)+2a^2b^2d\left(4c^2-5d^2\right)-3ab^3c\left(c^2-d\right.\right.}{2\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f(c+d\sin(e+f}$$

$$= -\frac{d\left(a^4d^3-b^4d\left(5c^2-6d^2\right)+2a^2b^2d\left(4c^2-5d^2\right)-3ab^3c\left(c^2-d\right.\right.}{2\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f(c+d\sin(e+f}$$

$$= -\frac{d\left(a^4d^3-b^4d\left(5c^2-6d^2\right)+2a^2b^2d\left(4c^2-5d^2\right)-3ab^3c\left(c^2-d\right.\right.}{2\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f(c+d\sin(e+f}$$

$$= -\frac{b^3\left(10a^3bcd-4ab^3cd-20a^4d^2-a^2b^2\left(2c^2-29d^2\right)-b^4\left(c^2+\right.\right.}{\left(a^2-b^2\right)^{5/2}(bc-ad)^5f}$$

Mathematica [B] time = 8.53, size = 1815, normalized size = 2.71

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] -((b^3*(2*a^2*b^2*c^2 + b^4*c^2 - 10*a^3*b*c*d + 4*a*b^3*c*d + 20*a^4*d^2 -
 29*a^2*b^2*d^2 + 12*b^4*d^2)*ArcTan[(Sec[(e + f*x)/2]*(b*Cos[(e + f*x)/2]
```

```
+ a*Sin[(e + f*x)/2]))/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2)*(-(b*c) + a*d)^
5*f)) - (d^3*(20*b^2*c^4 - 10*a*b*c^3*d + 2*a^2*c^2*d^2 - 29*b^2*c^2*d^2 +
4*a*b*c*d^3 + a^2*d^4 + 12*b^2*d^4)*ArcTan[(Sec[(e + f*x)/2]*(d*Cos[(e + f*
x)/2] + c*Sin[(e + f*x)/2]))/Sqrt[c^2 - d^2]])/((b*c - a*d)^5*(c^2 - d^2)^(
5/2)*f) + (32*a^2*b^5*c^7*Cos[e + f*x] - 8*b^7*c^7*Cos[e + f*x] - 80*a^3*b^
4*c^6*d*Cos[e + f*x] + 68*a*b^6*c^6*d*Cos[e + f*x] - 92*a^2*b^5*c^5*d^2*Cos
[e + f*x] + 38*b^7*c^5*d^2*Cos[e + f*x] + 140*a^3*b^4*c^4*d^3*Cos[e + f*x]
- 122*a*b^6*c^4*d^3*Cos[e + f*x] - 80*a^6*b*c^3*d^4*Cos[e + f*x] + 140*a^4*
b^3*c^3*d^4*Cos[e + f*x] + 48*a^2*b^5*c^3*d^4*Cos[e + f*x] - 72*b^7*c^3*d^4
*Cos[e + f*x] + 32*a^7*c^2*d^5*Cos[e + f*x] - 92*a^5*b^2*c^2*d^5*Cos[e + f*
x] + 48*a^3*b^4*c^2*d^5*Cos[e + f*x] + 12*a*b^6*c^2*d^5*Cos[e + f*x] + 68*a
^6*b*c*d^6*Cos[e + f*x] - 122*a^4*b^3*c*d^6*Cos[e + f*x] + 12*a^2*b^5*c*d^6
*Cos[e + f*x] + 36*b^7*c*d^6*Cos[e + f*x] - 8*a^7*d^7*Cos[e + f*x] + 38*a^5
*b^2*d^7*Cos[e + f*x] - 72*a^3*b^4*d^7*Cos[e + f*x] + 36*a*b^6*d^7*Cos[e +
f*x] - 12*a*b^6*c^6*d*Cos[3*(e + f*x)] + 28*a^2*b^5*c^5*d^2*Cos[3*(e + f*x)
] - 22*b^7*c^5*d^2*Cos[3*(e + f*x)] + 20*a^3*b^4*c^4*d^3*Cos[3*(e + f*x)] +
 10*a*b^6*c^4*d^3*Cos[3*(e + f*x)] + 20*a^4*b^3*c^3*d^4*Cos[3*(e + f*x)] -
96*a^2*b^5*c^3*d^4*Cos[3*(e + f*x)] + 64*b^7*c^3*d^4*Cos[3*(e + f*x)] + 28*
a^5*b^2*c^2*d^5*Cos[3*(e + f*x)] - 96*a^3*b^4*c^2*d^5*Cos[3*(e + f*x)] + 44
*a*b^6*c^2*d^5*Cos[3*(e + f*x)] - 12*a^6*b*c*d^6*Cos[3*(e + f*x)] + 10*a^4*
b^3*c*d^6*Cos[3*(e + f*x)] + 44*a^2*b^5*c*d^6*Cos[3*(e + f*x)] - 36*b^7*c*d
^6*Cos[3*(e + f*x)] - 22*a^5*b^2*d^7*Cos[3*(e + f*x)] + 64*a^3*b^4*d^7*Cos[
3*(e + f*x)] - 36*a*b^6*d^7*Cos[3*(e + f*x)] + 12*a*b^6*c^7*Sin[2*(e + f*x)
] - 4*a^2*b^5*c^6*d*Sin[2*(e + f*x)] + 16*b^7*c^6*d*Sin[2*(e + f*x)] - 80*a
^3*b^4*c^5*d^2*Sin[2*(e + f*x)] + 38*a*b^6*c^5*d^2*Sin[2*(e + f*x)] - 10*a^
2*b^5*c^4*d^3*Sin[2*(e + f*x)] - 20*b^7*c^4*d^3*Sin[2*(e + f*x)] - 80*a^5*b
^2*c^3*d^4*Sin[2*(e + f*x)] + 320*a^3*b^4*c^3*d^4*Sin[2*(e + f*x)] - 192*a*
b^6*c^3*d^4*Sin[2*(e + f*x)] - 4*a^6*b*c^2*d^5*Sin[2*(e + f*x)] - 10*a^4*b^
3*c^2*d^5*Sin[2*(e + f*x)] + 64*a^2*b^5*c^2*d^5*Sin[2*(e + f*x)] - 26*b^7*c
^2*d^5*Sin[2*(e + f*x)] + 12*a^7*c*d^6*Sin[2*(e + f*x)] + 38*a^5*b^2*c*d^6*
Sin[2*(e + f*x)] - 192*a^3*b^4*c*d^6*Sin[2*(e + f*x)] + 124*a*b^6*c*d^6*Sin
[2*(e + f*x)] + 16*a^6*b*d^7*Sin[2*(e + f*x)] - 20*a^4*b^3*d^7*Sin[2*(e + f
*x)] - 26*a^2*b^5*d^7*Sin[2*(e + f*x)] + 24*b^7*d^7*Sin[2*(e + f*x)] - 3*a*
b^6*c^5*d^2*Sin[4*(e + f*x)] + 9*a^2*b^5*c^4*d^3*Sin[4*(e + f*x)] - 6*b^7*c
^4*d^3*Sin[4*(e + f*x)] + 6*a*b^6*c^3*d^4*Sin[4*(e + f*x)] + 9*a^4*b^3*c^2*
d^5*Sin[4*(e + f*x)] - 36*a^2*b^5*c^2*d^5*Sin[4*(e + f*x)] + 21*b^7*c^2*d^5
*Sin[4*(e + f*x)] - 3*a^5*b^2*c*d^6*Sin[4*(e + f*x)] + 6*a^3*b^4*c*d^6*Sin[
4*(e + f*x)] - 6*a*b^6*c*d^6*Sin[4*(e + f*x)] - 6*a^4*b^3*d^7*Sin[4*(e + f*
x)] + 21*a^2*b^5*d^7*Sin[4*(e + f*x)] - 12*b^7*d^7*Sin[4*(e + f*x)])/(16*(a
^2 - b^2)^2*(-(b*c) + a*d)^4*(c^2 - d^2)^2*f*(a + b*Sin[e + f*x])^2*(c + d*
Sin[e + f*x])^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 16.26, size = 7128, normalized size = 10.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] ((2*a^2*b^5*c^2 + b^7*c^2 - 10*a^3*b^4*c*d + 4*a*b^6*c*d + 20*a^4*b^3*d^2 -
 29*a^2*b^5*d^2 + 12*b^7*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + ar
ctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^4*b^5*c^5 - 2*a^2*b
^7*c^5 + b^9*c^5 - 5*a^5*b^4*c^4*d + 10*a^3*b^6*c^4*d - 5*a*b^8*c^4*d + 10*
a^6*b^3*c^3*d^2 - 20*a^4*b^5*c^3*d^2 + 10*a^2*b^7*c^3*d^2 - 10*a^7*b^2*c^2*
d^3 + 20*a^5*b^4*c^2*d^3 - 10*a^3*b^6*c^2*d^3 + 5*a^8*b*c*d^4 - 10*a^6*b^3*
c*d^4 + 5*a^4*b^5*c*d^4 - a^9*d^5 + 2*a^7*b^2*d^5 - a^5*b^4*d^5)*sqrt(a^2 -
 b^2)) - (20*b^2*c^4*d^3 - 10*a*b*c^3*d^4 + 2*a^2*c^2*d^5 - 29*b^2*c^2*d^5
+ 4*a*b*c*d^6 + a^2*d^7 + 12*b^2*d^7)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn
(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((b^5*c^9 - 5*a
*b^4*c^8*d + 10*a^2*b^3*c^7*d^2 - 2*b^5*c^7*d^2 - 10*a^3*b^2*c^6*d^3 + 10*a
*b^4*c^6*d^3 + 5*a^4*b*c^5*d^4 - 20*a^2*b^3*c^5*d^4 + b^5*c^5*d^4 - a^5*c^4
*d^5 + 20*a^3*b^2*c^4*d^5 - 5*a*b^4*c^4*d^5 - 10*a^4*b*c^3*d^6 + 10*a^2*b^3
*c^3*d^6 + 2*a^5*c^2*d^7 - 10*a^3*b^2*c^2*d^7 + 5*a^4*b*c*d^8 - a^5*d^9)*sq
rt(c^2 - d^2)) + (5*a^3*b^6*c^9*tan(1/2*f*x + 1/2*e)^7 - 2*a*b^8*c^9*tan(1/
2*f*x + 1/2*e)^7 - 11*a^4*b^5*c^8*d*tan(1/2*f*x + 1/2*e)^7 + 8*a^2*b^7*c^8*
d*tan(1/2*f*x + 1/2*e)^7 - 10*a^3*b^6*c^7*d^2*tan(1/2*f*x + 1/2*e)^7 + 4*a*
b^8*c^7*d^2*tan(1/2*f*x + 1/2*e)^7 + 22*a^4*b^5*c^6*d^3*tan(1/2*f*x + 1/2*e
)^7 - 16*a^2*b^7*c^6*d^3*tan(1/2*f*x + 1/2*e)^7 + 5*a^3*b^6*c^5*d^4*tan(1/2
*f*x + 1/2*e)^7 - 2*a*b^8*c^5*d^4*tan(1/2*f*x + 1/2*e)^7 - 11*a^8*b*c^4*d^5
*tan(1/2*f*x + 1/2*e)^7 + 22*a^6*b^3*c^4*d^5*tan(1/2*f*x + 1/2*e)^7 - 22*a^
4*b^5*c^4*d^5*tan(1/2*f*x + 1/2*e)^7 + 8*a^2*b^7*c^4*d^5*tan(1/2*f*x + 1/2*
e)^7 + 5*a^9*c^3*d^6*tan(1/2*f*x + 1/2*e)^7 - 10*a^7*b^2*c^3*d^6*tan(1/2*f*
x + 1/2*e)^7 + 5*a^5*b^4*c^3*d^6*tan(1/2*f*x + 1/2*e)^7 + 8*a^8*b*c^2*d^7*t
an(1/2*f*x + 1/2*e)^7 - 16*a^6*b^3*c^2*d^7*tan(1/2*f*x + 1/2*e)^7 + 8*a^4*b
^5*c^2*d^7*tan(1/2*f*x + 1/2*e)^7 - 2*a^9*c*d^8*tan(1/2*f*x + 1/2*e)^7 + 4*
a^7*b^2*c*d^8*tan(1/2*f*x + 1/2*e)^7 - 2*a^5*b^4*c*d^8*tan(1/2*f*x + 1/2*e)
^7 + 4*a^4*b^5*c^9*tan(1/2*f*x + 1/2*e)^6 + 7*a^2*b^7*c^9*tan(1/2*f*x + 1/2
*e)^6 - 2*b^9*c^9*tan(1/2*f*x + 1/2*e)^6 - 10*a^5*b^4*c^8*d*tan(1/2*f*x + 1
/2*e)^6 + 7*a^3*b^6*c^8*d*tan(1/2*f*x + 1/2*e)^6 + 6*a*b^8*c^8*d*tan(1/2*f*
x + 1/2*e)^6 - 52*a^4*b^5*c^7*d^2*tan(1/2*f*x + 1/2*e)^6 + 18*a^2*b^7*c^7*d
^2*tan(1/2*f*x + 1/2*e)^6 + 4*b^9*c^7*d^2*tan(1/2*f*x + 1/2*e)^6 + 20*a^5*b
```

^4*c^6*d^3*tan(1/2*f*x + 1/2*e)^6 - 14*a^3*b^6*c^6*d^3*tan(1/2*f*x + 1/2*e)^6 - 12*a*b^8*c^6*d^3*tan(1/2*f*x + 1/2*e)^6 - 10*a^8*b*c^5*d^4*tan(1/2*f*x + 1/2*e)^6 + 20*a^6*b^3*c^5*d^4*tan(1/2*f*x + 1/2*e)^6 + 82*a^4*b^5*c^5*d^4*tan(1/2*f*x + 1/2*e)^6 - 57*a^2*b^7*c^5*d^4*tan(1/2*f*x + 1/2*e)^6 - 2*b^9*c^5*d^4*tan(1/2*f*x + 1/2*e)^6 + 4*a^9*c^4*d^5*tan(1/2*f*x + 1/2*e)^6 - 52*a^7*b^2*c^4*d^5*tan(1/2*f*x + 1/2*e)^6 + 82*a^5*b^4*c^4*d^5*tan(1/2*f*x + 1/2*e)^6 - 37*a^3*b^6*c^4*d^5*tan(1/2*f*x + 1/2*e)^6 + 6*a*b^8*c^4*d^5*tan(1/2*f*x + 1/2*e)^6 + 7*a^8*b*c^3*d^6*tan(1/2*f*x + 1/2*e)^6 - 14*a^6*b^3*c^3*d^6*tan(1/2*f*x + 1/2*e)^6 - 37*a^4*b^5*c^3*d^6*tan(1/2*f*x + 1/2*e)^6 + 32*a^2*b^7*c^3*d^6*tan(1/2*f*x + 1/2*e)^6 + 7*a^9*c^2*d^7*tan(1/2*f*x + 1/2*e)^6 + 18*a^7*b^2*c^2*d^7*tan(1/2*f*x + 1/2*e)^6 - 57*a^5*b^4*c^2*d^7*tan(1/2*f*x + 1/2*e)^6 + 32*a^3*b^6*c^2*d^7*tan(1/2*f*x + 1/2*e)^6 + 6*a^8*b*c*d^8*tan(1/2*f*x + 1/2*e)^6 - 12*a^6*b^3*c*d^8*tan(1/2*f*x + 1/2*e)^6 + 6*a^4*b^5*c*d^8*tan(1/2*f*x + 1/2*e)^6 - 2*a^9*d^9*tan(1/2*f*x + 1/2*e)^6 + 4*a^7*b^2*d^9*tan(1/2*f*x + 1/2*e)^6 - 2*a^5*b^4*d^9*tan(1/2*f*x + 1/2*e)^6 + 21*a^3*b^6*c^9*tan(1/2*f*x + 1/2*e)^5 - 6*a*b^8*c^9*tan(1/2*f*x + 1/2*e)^5 - 35*a^4*b^5*c^8*d*tan(1/2*f*x + 1/2*e)^5 + 64*a^2*b^7*c^8*d*tan(1/2*f*x + 1/2*e)^5 - 8*b^9*c^8*d*tan(1/2*f*x + 1/2*e)^5 - 40*a^5*b^4*c^7*d^2*tan(1/2*f*x + 1/2*e)^5 - 74*a^3*b^6*c^7*d^2*tan(1/2*f*x + 1/2*e)^5 + 60*a*b^8*c^7*d^2*tan(1/2*f*x + 1/2*e)^5 + 26*a^4*b^5*c^6*d^3*tan(1/2*f*x + 1/2*e)^5 - 96*a^2*b^7*c^6*d^3*tan(1/2*f*x + 1/2*e)^5 + 16*b^9*c^6*d^3*tan(1/2*f*x + 1/2*e)^5 - 40*a^7*b^2*c^5*d^4*tan(1/2*f*x + 1/2*e)^5 + 160*a^5*b^4*c^5*d^4*tan(1/2*f*x + 1/2*e)^5 + 45*a^3*b^6*c^5*d^4*tan(1/2*f*x + 1/2*e)^5 - 102*a*b^8*c^5*d^4*tan(1/2*f*x + 1/2*e)^5 - 35*a^8*b*c^4*d^5*tan(1/2*f*x + 1/2*e)^5 + 26*a^6*b^3*c^4*d^5*tan(1/2*f*x + 1/2*e)^5 + 106*a^4*b^5*c^4*d^5*tan(1/2*f*x + 1/2*e)^5 - 44*a^2*b^7*c^4*d^5*tan(1/2*f*x + 1/2*e)^5 - 8*b^9*c^4*d^5*tan(1/2*f*x + 1/2*e)^5 + 21*a^9*c^3*d^6*tan(1/2*f*x + 1/2*e)^5 - 74*a^7*b^2*c^3*d^6*tan(1/2*f*x + 1/2*e)^5 + 45*a^5*b^4*c^3*d^6*tan(1/2*f*x + 1/2*e)^5 - 64*a^3*b^6*c^3*d^6*tan(1/2*f*x + 1/2*e)^5 + 48*a*b^8*c^3*d^6*tan(1/2*f*x + 1/2*e)^5 + 64*a^8*b*c^2*d^7*tan(1/2*f*x + 1/2*e)^5 - 96*a^6*b^3*c^2*d^7*tan(1/2*f*x + 1/2*e)^5 - 44*a^4*b^5*c^2*d^7*tan(1/2*f*x + 1/2*e)^5 + 64*a^2*b^7*c^2*d^7*tan(1/2*f*x + 1/2*e)^5 - 6*a^9*c*d^8*tan(1/2*f*x + 1/2*e)^5 + 60*a^7*b^2*c*d^8*tan(1/2*f*x + 1/2*e)^5 - 102*a^5*b^4*c*d^8*tan(1/2*f*x + 1/2*e)^5 + 48*a^3*b^6*c*d^8*tan(1/2*f*x + 1/2*e)^5 - 8*a^8*b*d^9*tan(1/2*f*x + 1/2*e)^5 + 16*a^6*b^3*d^9*tan(1/2*f*x + 1/2*e)^5 - 8*a^4*b^5*d^9*tan(1/2*f*x + 1/2*e)^5 + 12*a^4*b^5*c^9*tan(1/2*f*x + 1/2*e)^4 + 13*a^2*b^7*c^9*tan(1/2*f*x + 1/2*e)^4 - 4*b^9*c^9*tan(1/2*f*x + 1/2*e)^4 - 30*a^5*b^4*c^8*d*tan(1/2*f*x + 1/2*e)^4 + 45*a^3*b^6*c^8*d*tan(1/2*f*x + 1/2*e)^4 + 12*a*b^8*c^8*d*tan(1/2*f*x + 1/2*e)^4 - 168*a^4*b^5*c^7*d^2*tan(1/2*f*x + 1/2*e)^4 + 114*a^2*b^7*c^7*d^2*tan(1/2*f*x + 1/2*e)^4 + 20*a^5*b^4*c^6*d^3*tan(1/2*f*x + 1/2*e)^4 - 142*a^3*b^6*c^6*d^3*tan(1/2*f*x + 1/2*e)^4 + 32*a*b^8*c^6*d^3*tan(1/2*f*x + 1/2*e)^4 - 30*a^8*b*c^5*d^4*tan(1/2*f*x + 1/2*e)^4 + 20*a^6*b^3*c^5*d^4*tan(1/2*f*x + 1/2*e)^4 + 350*a^4*b^5*c^5*d^4*tan(1/2*f*x + 1/2*e)^4 - 307*a^2*b^7*c^5*d^4*tan(1/2*f*x + 1/2*e)^4 + 12*b^9*c^5*d^4*tan(1/2*f*x + 1/2*e)^4 + 12*a^9*c^4*d^5*tan(1/2*f*x + 1/2*e)^4 - 168*a^7*b^2*c^4*d^5 *tan(1/2*f*x + 1/2*e)^4 + 350*a^5*b^4*c^4*d^5*tan(1/2*f*x + 1/2*e)^4 + 5*a^3*b^6*c^4*d^5*tan(1/2*f*x + 1/2*e)^4 - 100*a*b^8*c^4*d^5*tan(1/2*f*x + 1/2*e)^4 + 45*a^8*b*c^3*d^6*tan(1/2*f*x + 1/2*e)^4 - 142*a^6*b^3*c^3*d^6*tan(1/2*f*x + 1/2*e)^4 + 5*a^4*b^5*c^3*d^6*tan(1/2*f*x + 1/2*e)^4 + 88*a^2*b^7*c^3*d^6*tan(1/2*f*x + 1/2*e)^4 - 8*b^9*c^3*d^6*tan(1/2*f*x + 1/2*e)^4 + 13*a^9*c^2*d^7*tan(1/2*f*x + 1/2*e)^4 + 114*a^7*b^2*c^2*d^7*tan(1/2*f*x + 1/2*e)^4 - 307*a^5*b^4*c^2*d^7*tan(1/2*f*x + 1/2*e)^4 + 88*a^3*b^6*c^2*d^7*tan(1/2*f*x + 1/2*e)^4 + 56*a*b^8*c^2*d^7*tan(1/2*f*x + 1/2*e)^4 + 12*a^8*b*c*d^8*tan(1/2*f*x + 1/2*e)^4 + 32*a^6*b^3*c*d^8*tan(1/2*f*x + 1/2*e)^4 - 100*a^4*b^5*c*d^8*tan(1/2*f*x + 1/2*e)^4 + 56*a^2*b^7*c*d^8*tan(1/2*f*x + 1/2*e)^4 - 4*a^9*d^9*tan(1/2*f*x + 1/2*e)^4 + 12*a^5*b^4*d^9*tan(1/2*f*x + 1/2*e)^4 - 8*a^3*b^6*d^9*tan(1/2*f*x + 1/2*e)^4 + 27*a^3*b^6*c^9*tan(1/2*f*x + 1/2*e)^3 - 6*a*b^8*c^9*tan(1/2*f*x + 1/2*e)^3 - 37*a^4*b^5*c^8*d*tan(1/2*f*x + 1/2*e)^3 + 72*a^2*b^7*c^8*d*tan(1/2*f*x + 1/2*e)^3 - 8*b^9*c^8*d*tan(1/2*f*x + 1/2*e)^3 - 80*a^5*b^4*c^7*d^2*tan(1/2*f*x + 1/2*e)^3 - 34*a^3*b^6*c^7*d^2*tan(1/2*f*x + 1/2*e)^3 + 60*a*b^8*c^7*d^2*tan(1/2*f*x + 1/2*e)^3 - 42*a^4*b^5*c^6*d^3*tan(1/2*f*x + 1/2*e)^3 - 64*a^2*b^7*c^6*d^3*tan(1/2*f*x + 1/2*e)^3 + 16*b^9*c^6*d^3*tan(1/2*f*x + 1/2*e)^3 - 80*a^7*b^2*c^5*d^4*tan(1/2*f*x + 1/2*e)^3 + 320*a^5*b^4*c^5*d^4*tan(1/2*f*x + 1/2*e)^3 - 93*a^3*b^6*c^5*d^4*tan(1/2*f*x + 1/2*e)^3 - 102*a*b^8*c^5*d^4*tan(1/2*f*x + 1/2*e)^3 - 37*a^8*b*c^4*d^5*tan(1/2*f*x + 1/2*e)^3 - 42*a^6*b^3*c^4*d^5*tan(1/2*f*x + 1/2*e)^3 + 390*a^4*b^5*c^4*d^5*tan(1/2*f*x + 1/2*e)^3 - 204*a^2*b^7*c^4*d^5*tan(1/2*f*x + 1/2*e)^3 - 8*b^9*c^4*d^5*tan(1/2*f*x + 1/2*e)^3 + 27*a^9*c^3*d^6*tan(1/2*f*x + 1/2*e)^3 - 34*a^7*b^2*c^3*d^6*tan(1/2*f*x + 1/2*e)^3 - 93*a^5*b^4*c^3*d^6*tan(1/2*f*x + 1/2*e)^3 + 40*a^3*b^6*c^3*d^6*tan(1/2*f*x + 1/2*e)^3 + 48*a*b^8*c^3*d^6*tan(1/2*f*x + 1/2*e)^3 + 72*a^8*b*c^2*d^7*tan(1/2*f*x + 1/2*e)^3 - 64*a^6*b^3*c^2*d^7*tan(1/2*f*x + 1/2*e)^3 - 204*a^4*b^5*c^2*d^7*tan(1/2*f*x + 1/2*e)^3 + 160*a^2*b^7*c^2*d^7*tan(1/2*f*x + 1/2*e)^3 - 6*a^9*c*d^8*tan(1/2*f*x + 1/2*e)^3 + 60*a^7*b^2*c*d^8*tan(1/2*f*x + 1/2*e)^3 - 102*a^5*b^4*c*d^8*tan(1/2*f*x + 1/2*e)^3 + 48*a^3*b^6*c*d^8*tan(1/2*f*x + 1/2*e)^3 - 8*a^8*b*d^9*tan(1/2*f*x + 1/2*e)^3 + 16*a^6*b^3*d^9*tan(1/2*f*x + 1/2*e)^3 - 8*a^4*b^5*d^9*tan(1/2*f*x + 1/2*e)^3 + 12*a^4*b^5*c^9*tan(1/2*f*x + 1/2*e)^2 + 5*a^2*b^7*c^9*tan(1/2*f*x + 1/2*e)^2 - 2*b^9*c^9*tan(1/2*f*x + 1/2*e)^2 - 30*a^5*b^4*c^8*d*tan(1/2*f*x + 1/2*e)^2 + 45*a^3*b^6*c^8*d*tan(1/2*f*x + 1/2*e)^2 + 6*a*b^8*c^8*d*tan(1/2*f*x + 1/2*e)^2 - 124*a^4*b^5*c^7*d^2*tan(1/2*f*x + 1/2*e)^2 + 66*a^2*b^7*c^7*d^2*tan(1/2*f*x + 1/2*e)^2 + 4*b^9*c^7*d^2*tan(1/2*f*x + 1/2*e)^2 + 20*a^5*b^4*c^6*d^3*tan(1/2*f*x + 1/2*e)^2 - 62*a^3*b^6*c^6*d^3*tan(1/2*f*x + 1/2*e)^2 - 12*a*b^8*c^6*d^3*tan(1/2*f*x + 1/2*e)^2 - 30*a^8*b*c^5*d^4*tan(1/2*f*x + 1/2*e)^2 + 20*a^6*b^3*c^5*d^4*tan(1/2*f*x + 1/2*e)^2 + 262*a^4*b^5*c^5*d^4*tan(1/2*f*x + 1/2*e)^2 - 187*a^2*b^7*c^5*d^4*tan(1/2*f*x + 1/2*e)^2 - 2*b^9*c^5*d^4*tan(1/2*f*x + 1/2*e)^2 + 12*a^9*c^4*d^5*tan(1/2*f*x + 1/2*e)^2 - 124*a^7*b^2*c^4*d^5*tan(1/2*f*x + 1/2*e)^2 + 262*a^5*b^4*c^4*d^5*tan(1/2*f*x + 1/2*e)^2 - 111*a^3*b^6*c^4*d^5*tan(1/2*f*x + 1/2*e)^2 + 6*a*b^8*c^4*d^5*tan(1/2*f*x + 1/2*e)^2 + 45*a^8*b*c^3*d^6*tan(1/2*f*x + 1/2*e)^2 - 62*a^6*b^3*c^3*d^6*tan(

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1/2*f*x + 1/2*e)^2 - 111*a^4*b^5*c^3*d^6*tan(1/2*f*x + 1/2*e)^2 + 104*a^2*b
^7*c^3*d^6*tan(1/2*f*x + 1/2*e)^2 + 5*a^9*c^2*d^7*tan(1/2*f*x + 1/2*e)^2 +
66*a^7*b^2*c^2*d^7*tan(1/2*f*x + 1/2*e)^2 - 187*a^5*b^4*c^2*d^7*tan(1/2*f*x
 + 1/2*e)^2 + 104*a^3*b^6*c^2*d^7*tan(1/2*f*x + 1/2*e)^2 + 6*a^8*b*c*d^8*ta
n(1/2*f*x + 1/2*e)^2 - 12*a^6*b^3*c*d^8*tan(1/2*f*x + 1/2*e)^2 + 6*a^4*b^5*
c*d^8*tan(1/2*f*x + 1/2*e)^2 - 2*a^9*d^9*tan(1/2*f*x + 1/2*e)^2 + 4*a^7*b^2
*d^9*tan(1/2*f*x + 1/2*e)^2 - 2*a^5*b^4*d^9*tan(1/2*f*x + 1/2*e)^2 + 11*a^3
*b^6*c^9*tan(1/2*f*x + 1/2*e) - 2*a*b^8*c^9*tan(1/2*f*x + 1/2*e) - 13*a^4*b
^5*c^8*d*tan(1/2*f*x + 1/2*e) + 16*a^2*b^7*c^8*d*tan(1/2*f*x + 1/2*e) - 40*
a^5*b^4*c^7*d^2*tan(1/2*f*x + 1/2*e) + 6*a^3*b^6*c^7*d^2*tan(1/2*f*x + 1/2*
e) + 4*a*b^8*c^7*d^2*tan(1/2*f*x + 1/2*e) + 26*a^4*b^5*c^6*d^3*tan(1/2*f*x
+ 1/2*e) - 32*a^2*b^7*c^6*d^3*tan(1/2*f*x + 1/2*e) - 40*a^7*b^2*c^5*d^4*tan
(1/2*f*x + 1/2*e) + 160*a^5*b^4*c^5*d^4*tan(1/2*f*x + 1/2*e) - 85*a^3*b^6*c
^5*d^4*tan(1/2*f*x + 1/2*e) - 2*a*b^8*c^5*d^4*tan(1/2*f*x + 1/2*e) - 13*a^8
*b*c^4*d^5*tan(1/2*f*x + 1/2*e) + 26*a^6*b^3*c^4*d^5*tan(1/2*f*x + 1/2*e) -
 26*a^4*b^5*c^4*d^5*tan(1/2*f*x + 1/2*e) + 16*a^2*b^7*c^4*d^5*tan(1/2*f*x +
 1/2*e) + 11*a^9*c^3*d^6*tan(1/2*f*x + 1/2*e) + 6*a^7*b^2*c^3*d^6*tan(1/2*f
*x + 1/2*e) - 85*a^5*b^4*c^3*d^6*tan(1/2*f*x + 1/2*e) + 56*a^3*b^6*c^3*d^6*
tan(1/2*f*x + 1/2*e) + 16*a^8*b*c^2*d^7*tan(1/2*f*x + 1/2*e) - 32*a^6*b^3*c
^2*d^7*tan(1/2*f*x + 1/2*e) + 16*a^4*b^5*c^2*d^7*tan(1/2*f*x + 1/2*e) - 2*a
^9*c*d^8*tan(1/2*f*x + 1/2*e) + 4*a^7*b^2*c*d^8*tan(1/2*f*x + 1/2*e) - 2*a^
5*b^4*c*d^8*tan(1/2*f*x + 1/2*e) + 4*a^4*b^5*c^9 - a^2*b^7*c^9 - 10*a^5*b^4
*c^8*d + 7*a^3*b^6*c^8*d - 8*a^4*b^5*c^7*d^2 + 2*a^2*b^7*c^7*d^2 + 20*a^5*b
^4*c^6*d^3 - 14*a^3*b^6*c^6*d^3 - 10*a^8*b*c^5*d^4 + 20*a^6*b^3*c^5*d^4 - 6
*a^4*b^5*c^5*d^4 - a^2*b^7*c^5*d^4 + 4*a^9*c^4*d^5 - 8*a^7*b^2*c^4*d^5 - 6*
a^5*b^4*c^4*d^5 + 7*a^3*b^6*c^4*d^5 + 7*a^8*b*c^3*d^6 - 14*a^6*b^3*c^3*d^6
+ 7*a^4*b^5*c^3*d^6 - a^9*c^2*d^7 + 2*a^7*b^2*c^2*d^7 - a^5*b^4*c^2*d^7)/((
a^6*b^4*c^10 - 2*a^4*b^6*c^10 + a^2*b^8*c^10 - 4*a^7*b^3*c^9*d + 8*a^5*b^5*
c^9*d - 4*a^3*b^7*c^9*d + 6*a^8*b^2*c^8*d^2 - 14*a^6*b^4*c^8*d^2 + 10*a^4*b
^6*c^8*d^2 - 2*a^2*b^8*c^8*d^2 - 4*a^9*b*c^7*d^3 + 16*a^7*b^3*c^7*d^3 - 20*
a^5*b^5*c^7*d^3 + 8*a^3*b^7*c^7*d^3 + a^10*c^6*d^4 - 14*a^8*b^2*c^6*d^4 + 2
6*a^6*b^4*c^6*d^4 - 14*a^4*b^6*c^6*d^4 + a^2*b^8*c^6*d^4 + 8*a^9*b*c^5*d^5
- 20*a^7*b^3*c^5*d^5 + 16*a^5*b^5*c^5*d^5 - 4*a^3*b^7*c^5*d^5 - 2*a^10*c^4*
d^6 + 10*a^8*b^2*c^4*d^6 - 14*a^6*b^4*c^4*d^6 + 6*a^4*b^6*c^4*d^6 - 4*a^9*b
*c^3*d^7 + 8*a^7*b^3*c^3*d^7 - 4*a^5*b^5*c^3*d^7 + a^10*c^2*d^8 - 2*a^8*b^2
*c^2*d^8 + a^6*b^4*c^2*d^8)*(a*c*tan(1/2*f*x + 1/2*e)^4 + 2*b*c*tan(1/2*f*x
 + 1/2*e)^3 + 2*a*d*tan(1/2*f*x + 1/2*e)^3 + 2*a*c*tan(1/2*f*x + 1/2*e)^2 +
 4*b*d*tan(1/2*f*x + 1/2*e)^2 + 2*b*c*tan(1/2*f*x + 1/2*e) + 2*a*d*tan(1/2*
f*x + 1/2*e) + a*c)^2))/f
```

maple [B] time = 0.52, size = 7348, normalized size = 10.98

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 80.30, size = 571173, normalized size = 853.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x))^3),x)
```

```
[Out] (atan((((((4*a^24*d^24 + 4*b^24*c^24 + 16*a^2*b^22*c^24 + 16*a^4*b^20*c^24
- 1152*a^10*b^14*d^24 + 5568*a^12*b^12*d^24 - 10568*a^14*b^10*d^24 + 9460*a
^16*b^8*d^24 - 3560*a^18*b^6*d^24 + 136*a^20*b^4*d^24 + 76*a^22*b^2*d^24 +
16*a^24*c^2*d^22 + 16*a^24*c^4*d^20 - 1152*b^24*c^10*d^14 + 5568*b^24*c^12*
d^12 - 10568*b^24*c^14*d^10 + 9460*b^24*c^16*d^8 - 3560*b^24*c^18*d^6 + 136
*b^24*c^20*d^4 + 76*b^24*c^22*d^2 + 11520*a*b^23*c^9*d^15 - 56448*a*b^23*c^
11*d^13 + 109456*a*b^23*c^13*d^11 - 101240*a*b^23*c^15*d^9 + 40720*a*b^23*c
^17*d^7 - 2960*a*b^23*c^19*d^5 - 536*a*b^23*c^21*d^3 - 176*a^3*b^21*c^23*d
- 320*a^5*b^19*c^23*d + 11520*a^9*b^15*c*d^23 - 56448*a^11*b^13*c*d^23 + 10
9456*a^13*b^11*c*d^23 - 101240*a^15*b^9*c*d^23 + 40720*a^17*b^7*c*d^23 - 29
60*a^19*b^5*c*d^23 - 536*a^21*b^3*c*d^23 - 176*a^23*b*c^3*d^21 - 320*a^23*b
*c^5*d^19 - 51840*a^2*b^22*c^8*d^16 + 263808*a^2*b^22*c^10*d^14 - 541208*a^
2*b^22*c^12*d^12 + 547088*a^2*b^22*c^14*d^10 - 263320*a^2*b^22*c^16*d^8 + 4
4120*a^2*b^22*c^18*d^6 - 1564*a^2*b^22*c^20*d^4 - 196*a^2*b^22*c^22*d^2 + 1
38240*a^3*b^21*c^7*d^17 - 758400*a^3*b^21*c^9*d^15 + 1720736*a^3*b^21*c^11*
d^13 - 2002728*a^3*b^21*c^13*d^11 + 1210560*a^3*b^21*c^15*d^9 - 335040*a^3*
b^21*c^17*d^7 + 37680*a^3*b^21*c^19*d^5 - 288*a^3*b^21*c^21*d^3 - 241920*a^
4*b^20*c^6*d^18 + 1512000*a^4*b^20*c^8*d^16 - 3975688*a^4*b^20*c^10*d^14 +
5501328*a^4*b^20*c^12*d^12 - 4147952*a^4*b^20*c^14*d^10 + 1586920*a^4*b^20*
c^16*d^8 - 276020*a^4*b^20*c^18*d^6 + 21124*a^4*b^20*c^20*d^4 + 176*a^4*b^2
0*c^22*d^2 + 290304*a^5*b^19*c^5*d^19 - 2232576*a^5*b^19*c^7*d^17 + 7078256
*a^5*b^19*c^9*d^15 - 11781560*a^5*b^19*c^11*d^13 + 10875200*a^5*b^19*c^13*d
```

^11 - 5365072*a^5*b^19*c^15*d^9 + 1310168*a^5*b^19*c^17*d^7 - 170968*a^5*b^19*c^19*d^5 + 8160*a^5*b^19*c^21*d^3 - 241920*a^6*b^18*c^4*d^20 + 2532096*a^6*b^18*c^6*d^18 - 9955992*a^6*b^18*c^8*d^16 + 20019440*a^6*b^18*c^10*d^14 - 22419600*a^6*b^18*c^12*d^12 + 13887520*a^6*b^18*c^14*d^10 - 4506428*a^6*b^18*c^16*d^8 + 793756*a^6*b^18*c^18*d^6 - 72240*a^6*b^18*c^20*d^4 + 3040*a^6*b^18*c^22*d^2 + 138240*a^7*b^17*c^3*d^21 - 2232576*a^7*b^17*c^5*d^19 + 11150016*a^7*b^17*c^7*d^17 - 27336616*a^7*b^17*c^9*d^15 + 37153600*a^7*b^17*c^11*d^13 - 28461040*a^7*b^17*c^13*d^11 + 11779808*a^7*b^17*c^15*d^9 - 2621008*a^7*b^17*c^17*d^7 + 336688*a^7*b^17*c^19*d^5 - 17920*a^7*b^17*c^21*d^3 - 51840*a^8*b^16*c^2*d^22 + 1512000*a^8*b^16*c^4*d^20 - 9955992*a^8*b^16*c^6*d^18 + 30289656*a^8*b^16*c^8*d^16 - 50137600*a^8*b^16*c^10*d^14 + 46972560*a^8*b^16*c^12*d^12 - 24199280*a^8*b^16*c^14*d^10 + 6661036*a^8*b^16*c^16*d^8 - 1058448*a^8*b^16*c^18*d^6 + 72560*a^8*b^16*c^20*d^4 - 758400*a^9*b^15*c^3*d^21 + 7078256*a^9*b^15*c^5*d^19 - 27336616*a^9*b^15*c^7*d^17 + 55383904*a^9*b^15*c^9*d^15 - 63124080*a^9*b^15*c^11*d^13 + 39987520*a^9*b^15*c^13*d^11 - 13462088*a^9*b^15*c^15*d^9 + 2478528*a^9*b^15*c^17*d^7 - 212032*a^9*b^15*c^19*d^5 + 263808*a^10*b^14*c^2*d^22 - 3975688*a^10*b^14*c^4*d^20 + 20019440*a^10*b^14*c^6*d^18 - 50137600*a^10*b^14*c^8*d^16 + 69593872*a^10*b^14*c^10*d^14 - 53854288*a^10*b^14*c^12*d^12 + 21989928*a^10*b^14*c^14*d^10 - 4591360*a^10*b^14*c^16*d^8 + 460480*a^10*b^14*c^18*d^6 + 1720736*a^11*b^13*c^3*d^21 - 11781560*a^11*b^13*c^5*d^19 + 37153600*a^11*b^13*c^7*d^17 - 63124080*a^11*b^13*c^9*d^15 + 59445728*a^11*b^13*c^11*d^13 - 29358696*a^11*b^13*c^13*d^11 + 6995840*a^11*b^13*c^15*d^9 - 762560*a^11*b^13*c^17*d^7 - 541208*a^12*b^12*c^2*d^22 + 5501328*a^12*b^12*c^4*d^20 - 22419600*a^12*b^12*c^6*d^18 + 46972560*a^12*b^12*c^8*d^16 - 53854288*a^12*b^12*c^10*d^14 + 32294808*a^12*b^12*c^12*d^12 - 8958208*a^12*b^12*c^14*d^10 + 999040*a^12*b^12*c^16*d^8 - 2002728*a^13*b^11*c^3*d^21 + 10875200*a^13*b^11*c^5*d^19 - 28461040*a^13*b^11*c^7*d^17 + 39987520*a^13*b^11*c^9*d^15 - 29358696*a^13*b^11*c^11*d^13 + 9722048*a^13*b^11*c^13*d^11 - 1104320*a^13*b^11*c^15*d^9 + 547088*a^14*b^10*c^2*d^22 - 4147952*a^14*b^10*c^4*d^20 + 13887520*a^14*b^10*c^6*d^18 - 24199280*a^14*b^10*c^8*d^16 + 21989928*a^14*b^10*c^10*d^14 - 8958208*a^14*b^10*c^12*d^12 + 1124032*a^14*b^10*c^14*d^10 + 1210560*a^15*b^9*c^3*d^21 - 5365072*a^15*b^9*c^5*d^19 + 11779808*a^15*b^9*c^7*d^17 - 13462088*a^15*b^9*c^9*d^15 + 6995840*a^15*b^9*c^11*d^13 - 1104320*a^15*b^9*c^13*d^11 - 263320*a^16*b^8*c^2*d^22 + 1586920*a^16*b^8*c^4*d^20 - 4506428*a^16*b^8*c^6*d^18 + 6661036*a^16*b^8*c^8*d^16 - 4591360*a^16*b^8*c^10*d^14 + 999040*a^16*b^8*c^12*d^12 - 335040*a^17*b^7*c^3*d^21 + 1310168*a^17*b^7*c^5*d^19 - 2621008*a^17*b^7*c^7*d^17 + 2478528*a^17*b^7*c^9*d^15 - 762560*a^17*b^7*c^11*d^13 + 44120*a^18*b^6*c^2*d^22 - 276020*a^18*b^6*c^4*d^20 + 793756*a^18*b^6*c^6*d^18 - 1058448*a^18*b^6*c^8*d^16 + 460480*a^18*b^6*c^10*d^14 + 37680*a^19*b^5*c^3*d^21 - 170968*a^19*b^5*c^5*d^19 + 336688*a^19*b^5*c^7*d^17 - 212032*a^19*b^5*c^9*d^15 - 1564*a^20*b^4*c^2*d^22 + 21124*a^20*b^4*c^4*d^20 - 72240*a^20*b^4*c^6*d^18 + 72560*a^20*b^4*c^8*d^16 - 288*a^21*b^3*c^3*d^21 + 8160*a^21*b^3*c^5*d^19 - 17920*a^21*b^3*c^7*d^17 - 196*a^22*b^2*c^2*d^22 + 176*a^22*b^2*c^4*d^20 + 3040*a^22*b^2*c^6*d^18 - 8*a*b^23*c^23*d - 8*a^23*b* c*d^23)^2/4 - (20736*b^18*d^18 - 96768*a^2*b^16*d^18 + 173664*a^4*b^14*d^18 - 136032*a^6*b^12*d^18 + 31081*a^8*b^10*d^18 + 8440*a^10*b^8*d^18 + 400*a^12*b^6*d^18 - 96768*b^18*c^2*d^16 + 173664*b^18*c^4*d^14 - 136032*b^18*c^6*d^12 + 31081*b^18*c^8*d^10 + 8440*b^18*c^10*d^8 + 400*b^18*c^12*d^6 - 131328*a*b^17*c^3*d^15 + 216576*a*b^17*c^5*d^13 - 141104*a*b^17*c^7*d^11 + 20260*a*b^17*c^9*d^9 + 2800*a*b^17*c^11*d^7 - 131328*a^3*b^15*c*d^17 + 216576*a^5*b^13*c*d^17 - 141104*a^7*b^11*c*d^17 + 20260*a^9*b^9*c*d^17 + 2800*a^11*b^7*c*d^17 + 495936*a^2*b^16*c^2*d^16 - 989856*a^2*b^16*c^4*d^14 + 901948*a^2*b^16*c^6*d^12 - 308392*a^2*b^16*c^8*d^10 - 5260*a^2*b^16*c^10*d^8 + 1600*a^2*b^16*c^12*d^6 + 657408*a^3*b^15*c^3*d^15 - 1158992*a^3*b^15*c^5*d^13 + 838256*a^3*b^15*c^7*d^11 - 182200*a^3*b^15*c^9*d^9 - 3200*a^3*b^15*c^11*d^7 - 989856*a^4*b^14*c^2*d^16 + 2185654*a^4*b^14*c^4*d^14 - 2218576*a^4*b^14*c^6*d^12 + 900624*a^4*b^14*c^8*d^10 - 64720*a^4*b^14*c^10*d^8 + 1600*a^4*b^14*c^12*d^6 - 1158992*a^5*b^13*c^3*d^15 + 2158808*a^5*b^13*c^5*d^13 - 1641528*a^5*b^13*c^7*d^11 + 406880*a^5*b^13*c^9*d^9 - 17600*a^5*b^13*c^11*d^7 + 901948*a^6*b^12*c^2*d^16 - 2218576*a^6*b^12*c^4*d^14 + 2430936*a^6*b^12*c^6*d^12 - 1026928*a^6*b^12*c^8*d^10 + 88720*a^6*b^12*c^10*d^8 + 838256*a^7*b^11*c^3*d^15 - 1641528*a^7*b^11*c^5*d^13 + 1206848*a^7*b^11*c^7*d^11 - 239360*a^7*b^11*c^9*d^9 - 308392*a^8*b^10*c^2*d^16 + 900624*a^8*b^10*c^4*d^14 - 1026928*a^8*b^10*c^6*d^12 + 354016*a^8*b^10*c^8*d^10 - 182200*a^9*b^9*c^3*d^15 + 406880*a^9*b^9*c^5*d^13 - 239360*a^9*b^9*c^7*d^11 - 5260*a^10*b^8*c^2*d^16 - 64720*a^10*b^8*c^4*d^14 + 88720*a^10*b^8*c^6*d^12 - 3200*a^11*b^7*c^3*d^15 - 17600*a^11*b^7*c^5*d^13 + 1600*a^12*b^6*c^2*d^16 + 1600*a^12*b^6*c^4*d^14 + 27648*a*b^17*c*d^17)*(80*a^2*b^28*c^30 - 16*b^30*c^30 - 16*a^30*d^30 - 160*a^4*b^26*c^30 + 160*a^6*b^24*c^30 - 80*a^8*b^22*c^30 + 16*a^10*b^20*c^30 + 16*a^20*b^10*d^30 - 80*a^22*b^8*d^30 + 160*a^24*b^6*d^30 - 160*a^26*b^4*d^30 + 80*a^28*b^2*d^30 + 80*a^30*c^2*d^28 - 160*a^30*c^4*d^26 + 160*a^30*c^6*d^24 - 80*a^30*c^8*d^22 + 16*a^30*c^10*d^20 + 16*b^30*c^20*d^10 - 80*b^30*c^22*d^8 + 160*b^30*c^24*d^6 - 160*b^30*c^26*d^4 + 80*b^30*c^28*d^2 - 320*a*b^29*c^19*d^11 + 1600*a*b^29*c^21*d^9 - 3200*a*b^29*c^23*d^7 + 3200*a*b^29*c^25*d^5 - 1600*a*b^29*c^27*d^3 - 1600*a^3*b^27*c^29*d + 3200*a^5*b^25*c^29*d - 3200*a^7*b^23*c^29*d + 1600*a^9*b^21*c^29*d - 320*a^11*b^19*c^29*d - 320*a^19*b^11*c*d^29 + 1600*a^21*b^9*c*d^29 - 3200*a^23*b^7*c*d^29 + 3200*a^25*b^5*c*d^29 - 1600*a^27*b^3*c*d^29 - 1600*a^29*b*c^3*d^27 + 3200*a^29*b*c^5*d^25 - 3200*a^29*b*c^7*d^23 + 1600*a^29*b*c^9*d^21 - 320*a^29*b*c^11*d^19 + 3040*a^2*b^28*c^18*d^12 - 15280*a^2*b^28*c^20*d^10 + 30800*a^2*b^28*c^22*d^8 - 31200*a^2*b^28*c^24*d^6 + 16000*a^2*b^28*c^26*d^4 - 3440*a^2*b^28*c^28*d^2 - 18240*a^3*b^27*c^17*d^13 + 92800*a^3*b^27*c^19*d^11 - 190400*a^3*b^27*c^21*d^9 + 198400*a^3*b^27*c^23*d^7 - 107200*a^3*b^27*c^25*d^5 + 26240*a^3*b^27*c^27*d^3 + 77520*a^4*b^26*c^16*d^14 - 402800*a^4*b^26*c^18*d^12 + 851360*a^4*b^26*c^20*d^10 - 928000*a^4*b^26*c^22*d^8 + 541200*a^4*b^26*c^24*d^6 - 155120*a^4*b^26*c^26*d^4 + 16000*a^4*b^26*c^28*d^2 - 248064*a^5*b^25*c^15*d^15 + 1331520*a^5*b^25*c^17*d^13 - 2939840*a^5*b^25*c^19*d^11 + 3408640*a^5*b^25*c^21*d^9 - 2184320*a^5*b^25*c^23*d^7 + 736064*a^5*b^25*c^25*d^5 - 107200*a^5*b^25*c^27*d^3 + 620160*a^6*b^24*c^14*d^16 - 3488

400*a^6*b^24*c^16*d^14 + 8170000*a^6*b^24*c^18*d^12 - 10229760*a^6*b^24*c^2
0*d^10 + 7281600*a^6*b^24*c^22*d^8 - 2863760*a^6*b^24*c^24*d^6 + 541200*a^6
*b^24*c^26*d^4 - 31200*a^6*b^24*c^28*d^2 - 1240320*a^7*b^23*c^13*d^17 + 744
1920*a^7*b^23*c^15*d^15 - 18787200*a^7*b^23*c^17*d^13 + 25721600*a^7*b^23*c
^19*d^11 - 20444800*a^7*b^23*c^21*d^9 + 9297920*a^7*b^23*c^23*d^7 - 2184320
*a^7*b^23*c^25*d^5 + 198400*a^7*b^23*c^27*d^3 + 2015520*a^8*b^22*c^12*d^18
- 13178400*a^8*b^22*c^14*d^16 + 36434400*a^8*b^22*c^16*d^14 - 55069600*a^8*
b^22*c^18*d^12 + 48989680*a^8*b^22*c^20*d^10 - 25575920*a^8*b^22*c^22*d^8 +
7281600*a^8*b^22*c^24*d^6 - 928000*a^8*b^22*c^26*d^4 + 30800*a^8*b^22*c^28
*d^2 - 2687360*a^9*b^21*c^11*d^19 + 19638400*a^9*b^21*c^13*d^17 - 60362240*
a^9*b^21*c^15*d^15 + 101475200*a^9*b^21*c^17*d^13 - 101172800*a^9*b^21*c^19
*d^11 + 60333760*a^9*b^21*c^21*d^9 - 20444800*a^9*b^21*c^23*d^7 + 3408640*a
^9*b^21*c^25*d^5 - 190400*a^9*b^21*c^27*d^3 + 2956096*a^10*b^20*c^10*d^20 -
24858080*a^10*b^20*c^12*d^18 + 86150560*a^10*b^20*c^14*d^16 - 162120160*a^
10*b^20*c^16*d^14 + 181463680*a^10*b^20*c^18*d^12 - 123188112*a^10*b^20*c^2
0*d^10 + 48989680*a^10*b^20*c^22*d^8 - 10229760*a^10*b^20*c^24*d^6 + 851360
*a^10*b^20*c^26*d^4 - 15280*a^10*b^20*c^28*d^2 - 2687360*a^11*b^19*c^9*d^21
+ 26873600*a^11*b^19*c^11*d^19 - 106460800*a^11*b^19*c^13*d^17 + 225738240
*a^11*b^19*c^15*d^15 - 284331200*a^11*b^19*c^17*d^13 + 219166080*a^11*b^19*
c^19*d^11 - 101172800*a^11*b^19*c^21*d^9 + 25721600*a^11*b^19*c^23*d^7 - 29
39840*a^11*b^19*c^25*d^5 + 92800*a^11*b^19*c^27*d^3 + 2015520*a^12*b^18*c^8
*d^22 - 24858080*a^12*b^18*c^10*d^20 + 114212800*a^12*b^18*c^12*d^18 - 2749
37600*a^12*b^18*c^14*d^16 + 390830000*a^12*b^18*c^16*d^14 - 341426960*a^12*
b^18*c^18*d^12 + 181463680*a^12*b^18*c^20*d^10 - 55069600*a^12*b^18*c^22*d^
8 + 8170000*a^12*b^18*c^24*d^6 - 402800*a^12*b^18*c^26*d^4 + 3040*a^12*b^18
*c^28*d^2 - 1240320*a^13*b^17*c^7*d^23 + 19638400*a^13*b^17*c^9*d^21 - 1064
60800*a^13*b^17*c^11*d^19 + 293542400*a^13*b^17*c^13*d^17 - 472561920*a^13*
b^17*c^15*d^15 + 467412160*a^13*b^17*c^17*d^13 - 284331200*a^13*b^17*c^19*d
^11 + 101475200*a^13*b^17*c^21*d^9 - 18787200*a^13*b^17*c^23*d^7 + 1331520*
a^13*b^17*c^25*d^5 - 18240*a^13*b^17*c^27*d^3 + 620160*a^14*b^16*c^6*d^24 -
13178400*a^14*b^16*c^8*d^22 + 86150560*a^14*b^16*c^10*d^20 - 274937600*a^1
4*b^16*c^12*d^18 + 503363200*a^14*b^16*c^14*d^16 - 563751280*a^14*b^16*c^16
*d^14 + 390830000*a^14*b^16*c^18*d^12 - 162120160*a^14*b^16*c^20*d^10 + 364
34400*a^14*b^16*c^22*d^8 - 3488400*a^14*b^16*c^24*d^6 + 77520*a^14*b^16*c^2
6*d^4 - 248064*a^15*b^15*c^5*d^25 + 7441920*a^15*b^15*c^7*d^23 - 60362240*a
^15*b^15*c^9*d^21 + 225738240*a^15*b^15*c^11*d^19 - 472561920*a^15*b^15*c^1
3*d^17 + 599984128*a^15*b^15*c^15*d^15 - 472561920*a^15*b^15*c^17*d^13 + 22
5738240*a^15*b^15*c^19*d^11 - 60362240*a^15*b^15*c^21*d^9 + 7441920*a^15*b^
15*c^23*d^7 - 248064*a^15*b^15*c^25*d^5 + 77520*a^16*b^14*c^4*d^26 - 348840
0*a^16*b^14*c^6*d^24 + 36434400*a^16*b^14*c^8*d^22 - 162120160*a^16*b^14*c^
10*d^20 + 390830000*a^16*b^14*c^12*d^18 - 563751280*a^16*b^14*c^14*d^16 + 5
03363200*a^16*b^14*c^16*d^14 - 274937600*a^16*b^14*c^18*d^12 + 86150560*a^1
6*b^14*c^20*d^10 - 13178400*a^16*b^14*c^22*d^8 + 620160*a^16*b^14*c^24*d^6
- 18240*a^17*b^13*c^3*d^27 + 1331520*a^17*b^13*c^5*d^25 - 18787200*a^17*b^1
3*c^7*d^23 + 101475200*a^17*b^13*c^9*d^21 - 284331200*a^17*b^13*c^11*d^19 +

467412160*a^17*b^13*c^13*d^17 - 472561920*a^17*b^13*c^15*d^15 + 293542400*a^17*b^13*c^17*d^13 - 106460800*a^17*b^13*c^19*d^11 + 19638400*a^17*b^13*c^21*d^9 - 1240320*a^17*b^13*c^23*d^7 + 3040*a^18*b^12*c^2*d^28 - 402800*a^18*b^12*c^4*d^26 + 8170000*a^18*b^12*c^6*d^24 - 55069600*a^18*b^12*c^8*d^22 + 181463680*a^18*b^12*c^10*d^20 - 341426960*a^18*b^12*c^12*d^18 + 390830000*a^18*b^12*c^14*d^16 - 274937600*a^18*b^12*c^16*d^14 + 114212800*a^18*b^12*c^18*d^12 - 24858080*a^18*b^12*c^20*d^10 + 2015520*a^18*b^12*c^22*d^8 + 92800*a^19*b^11*c^3*d^27 - 2939840*a^19*b^11*c^5*d^25 + 25721600*a^19*b^11*c^7*d^23 - 101172800*a^19*b^11*c^9*d^21 + 219166080*a^19*b^11*c^11*d^19 - 284331200*a^19*b^11*c^13*d^17 + 225738240*a^19*b^11*c^15*d^15 - 106460800*a^19*b^11*c^17*d^13 + 26873600*a^19*b^11*c^19*d^11 - 2687360*a^19*b^11*c^21*d^9 - 15280*a^20*b^10*c^2*d^28 + 851360*a^20*b^10*c^4*d^26 - 10229760*a^20*b^10*c^6*d^24 + 48989680*a^20*b^10*c^8*d^22 - 123188112*a^20*b^10*c^10*d^20 + 181463680*a^20*b^10*c^12*d^18 - 162120160*a^20*b^10*c^14*d^16 + 86150560*a^20*b^10*c^16*d^14 - 24858080*a^20*b^10*c^18*d^12 + 2956096*a^20*b^10*c^20*d^10 - 190400*a^21*b^9*c^3*d^27 + 3408640*a^21*b^9*c^5*d^25 - 20444800*a^21*b^9*c^7*d^23 + 60333760*a^21*b^9*c^9*d^21 - 101172800*a^21*b^9*c^11*d^19 + 101475200*a^21*b^9*c^13*d^17 - 60362240*a^21*b^9*c^15*d^15 + 19638400*a^21*b^9*c^17*d^13 - 2687360*a^21*b^9*c^19*d^11 + 30800*a^22*b^8*c^2*d^28 - 928000*a^22*b^8*c^4*d^26 + 7281600*a^22*b^8*c^6*d^24 - 25575920*a^22*b^8*c^8*d^22 + 48989680*a^22*b^8*c^10*d^20 - 55069600*a^22*b^8*c^12*d^18 + 36434400*a^22*b^8*c^14*d^16 - 13178400*a^22*b^8*c^16*d^14 + 2015520*a^22*b^8*c^18*d^12 + 198400*a^23*b^7*c^3*d^27 - 2184320*a^23*b^7*c^5*d^25 + 9297920*a^23*b^7*c^7*d^23 - 20444800*a^23*b^7*c^9*d^21 + 25721600*a^23*b^7*c^11*d^19 - 18787200*a^23*b^7*c^13*d^17 + 7441920*a^23*b^7*c^15*d^15 - 1240320*a^23*b^7*c^17*d^13 - 31200*a^24*b^6*c^2*d^28 + 541200*a^24*b^6*c^4*d^26 - 2863760*a^24*b^6*c^6*d^24 + 7281600*a^24*b^6*c^8*d^22 - 10229760*a^24*b^6*c^10*d^20 + 8170000*a^24*b^6*c^12*d^18 - 3488400*a^24*b^6*c^14*d^16 + 620160*a^24*b^6*c^16*d^14 - 107200*a^25*b^5*c^3*d^27 + 736064*a^25*b^5*c^5*d^25 - 2184320*a^25*b^5*c^7*d^23 + 3408640*a^25*b^5*c^9*d^21 - 2939840*a^25*b^5*c^11*d^19 + 1331520*a^25*b^5*c^13*d^17 - 248064*a^25*b^5*c^15*d^15 + 16000*a^26*b^4*c^2*d^28 - 155120*a^26*b^4*c^4*d^26 + 541200*a^26*b^4*c^6*d^24 - 928000*a^26*b^4*c^8*d^22 + 851360*a^26*b^4*c^10*d^20 - 402800*a^26*b^4*c^12*d^18 + 77520*a^26*b^4*c^14*d^16 + 26240*a^27*b^3*c^3*d^27 - 107200*a^27*b^3*c^5*d^25 + 198400*a^27*b^3*c^7*d^23 - 190400*a^27*b^3*c^9*d^21 + 92800*a^27*b^3*c^11*d^19 - 18240*a^27*b^3*c^13*d^17 - 3440*a^28*b^2*c^2*d^28 + 16000*a^28*b^2*c^4*d^26 - 31200*a^28*b^2*c^6*d^24 + 30800*a^28*b^2*c^8*d^22 - 15280*a^28*b^2*c^10*d^20 + 3040*a^28*b^2*c^12*d^18 + 320*a*b^29*c^29*d + 320*a^29*b*c*d^29))^(1/2) - 2*a^24*d^24 - 2*b^24*c^24 - 8*a^2*b^22*c^24 - 8*a^4*b^20*c^24 + 576*a^10*b^14*d^24 - 2784*a^12*b^12*d^24 + 5284*a^14*b^10*d^24 - 4730*a^16*b^8*d^24 + 1780*a^18*b^6*d^24 - 68*a^20*b^4*d^24 - 38*a^22*b^2*d^24 - 8*a^24*c^2*d^22 - 8*a^24*c^4*d^20 + 576*b^24*c^10*d^14 - 2784*b^24*c^12*d^12 + 5284*b^24*c^14*d^10 - 4730*b^24*c^16*d^8 + 1780*b^24*c^18*d^6 - 68*b^24*c^20*d^4 - 38*b^24*c^22*d^2 - 5760*a*b^23*c^9*d^15 + 28224*a*b^23*c^11*d^13 - 54728*a*b^23*c^13*d^11 + 50620*a*b^23*c^15*d^9 - 20360*a*b^23*c^17*d^7 + 1480*a *b^23*c^19*d^5 + 268*a*b^23*c^21*d^3 + 88*a^3*b^21*c^23*d + 160*a^5*b^19*c^23*d - 5760*a^9*b^15*c*d^23 + 28224*a^11*b^13*c*d^23 - 54728*a^13*b^11*c*d^23 + 50620*a^15*b^9*c*d^23 - 20360*a^17*b^7*c*d^23 + 1480*a^19*b^5*c*d^23 + 268*a^21*b^3*c*d^23 + 88*a^23*b*c^3*d^21 + 160*a^23*b*c^5*d^19 + 25920*a^2*b^22*c^8*d^16 - 131904*a^2*b^22*c^10*d^14 + 270604*a^2*b^22*c^12*d^12 - 273544*a^2*b^22*c^14*d^10 + 131660*a^2*b^22*c^16*d^8 - 22060*a^2*b^22*c^18*d^6 + 782*a^2*b^22*c^20*d^4 + 98*a^2*b^22*c^22*d^2 - 69120*a^3*b^21*c^7*d^17 + 379200*a^3*b^21*c^9*d^15 - 860368*a^3*b^21*c^11*d^13 + 1001364*a^3*b^21*c^13*d^11 - 605280*a^3*b^21*c^15*d^9 + 167520*a^3*b^21*c^17*d^7 - 18840*a^3*b^21*c^19*d^5 + 144*a^3*b^21*c^21*d^3 + 120960*a^4*b^20*c^6*d^18 - 756000*a^4*b^20*c^8*d^16 + 1987844*a^4*b^20*c^10*d^14 - 2750664*a^4*b^20*c^12*d^12 + 2073976*a^4*b^20*c^14*d^10 - 793460*a^4*b^20*c^16*d^8 + 138010*a^4*b^20*c^18*d^6 - 10562*a^4*b^20*c^20*d^4 - 88*a^4*b^20*c^22*d^2 - 145152*a^5*b^19*c^5*d^19 + 1116288*a^5*b^19*c^7*d^17 - 3539128*a^5*b^19*c^9*d^15 + 5890780*a^5*b^19*c^11*d^13 - 5437600*a^5*b^19*c^13*d^11 + 2682536*a^5*b^19*c^15*d^9 - 655084*a^5*b^19*c^17*d^7 + 85484*a^5*b^19*c^19*d^5 - 4080*a^5*b^19*c^21*d^3 + 120960*a^6*b^18*c^4*d^20 - 1266048*a^6*b^18*c^6*d^18 + 4977996*a^6*b^18*c^8*d^16 - 10009720*a^6*b^18*c^10*d^14 + 11209800*a^6*b^18*c^12*d^12 - 6943760*a^6*b^18*c^14*d^10 + 2253214*a^6*b^18*c^16*d^8 - 396878*a^6*b^18*c^18*d^6 + 36120*a^6*b^18*c^20*d^4 - 1520*a^6*b^18*c^22*d^2 - 69120*a^7*b^17*c^3*d^21 + 1116288*a^7*b^17*c^5*d^19 - 5575008*a^7*b^17*c^7*d^17 + 13668308*a^7*b^17*c^9*d^15 - 18576800*a^7*b^17*c^11*d^13 + 14230520*a^7*b^17*c^13*d^11 - 5889904*a^7*b^17*c^15*d^9 + 1310504*a^7*b^17*c^17*d^7 - 168344*a^7*b^17*c^19*d^5 + 8960*a^7*b^17*c^21*d^3 + 25920*a^8*b^16*c^2*d^22 - 756000*a^8*b^16*c^4*d^20 + 4977996*a^8*b^16*c^6*d^18 - 15144828*a^8*b^16*c^8*d^16 + 25068800*a^8*b^16*c^10*d^14 - 23486280*a^8*b^16*c^12*d^12 + 12099640*a^8*b^16*c^14*d^10 - 3330518*a^8*b^16*c^16*d^8 + 529224*a^8*b^16*c^18*d^6 - 36280*a^8*b^16*c^20*d^4 + 379200*a^9*b^15*c^3*d^21 - 3539128*a^9*b^15*c^5*d^19 + 13668308*a^9*b^15*c^7*d^17 - 27691952*a^9*b^15*c^9*d^15 + 31562040*a^9*b^15*c^11*d^13 - 19993760*a^9*b^15*c^13*d^11 + 6731044*a^9*b^15*c^15*d^9 - 1239264*a^9*b^15*c^17*d^7 + 106016*a^9*b^15*c^19*d^5 - 131904*a^10*b^14*c^2*d^22 + 1987844*a^10*b^14*c^4*d^20 - 10009720*a^10*b^14*c^6*d^18 + 25068800*a^10*b^14*c^8*d^16 - 34796936*a^10*b^14*c^10*d^14 + 26927144*a^10*b^14*c^12*d^12 - 10994964*a^10*b^14*c^14*d^10 + 2295680*a^10*b^14*c^16*d^8 - 230240*a^10*b^14*c^18*d^6 - 860368*a^11*b^13*c^3*d^21 + 5890780*a^11*b^13*c^5*d^19 - 18576800*a^11*b^13*c^7*d^17 + 31562040*a^11*b^13*c^9*d^15 - 29722864*a^11*b^13*c^11*d^13 + 14679348*a^11*b^13*c^13*d^11 - 3497920*a^11*b^13*c^15*d^9 + 381280*a^11*b^13*c^17*d^7 + 270604*a^12*b^12*c^2*d^22 - 2750664*a^12*b^12*c^4*d^20 + 11209800*a^12*b^12*c^6*d^18 - 23486280*a^12*b^12*c^8*d^16 + 26927144*a^12*b^12*c^10*d^14 - 16147404*a^12*b^12*c^12*d^12 + 4479104*a^12*b^12*c^14*d^10 - 499520*a^12*b^12*c^16*d^8 + 1001364*a^13*b^11*c^3*d^21 - 5437600*a^13*b^11*c^5*d^19 + 14230520*a^13*b^11*c^7*d^17 - 19993760*a^13*b^11*c^9*d^15 + 14679348*a^13*b^11*c^11*d^13 - 4861024*a^13*b^11*c^13*d^11 + 552160*a^13*b^11*c^15*d^9 - 273544*a^14*b^10*c^2*d^22 + 2073976*a^14*b^10*c^4*d^20 - 6943760*a^14*b^10*c^6*d^18 + 12099640*a^14*b^10*c^8*d^16 - 10994964*a^1

4*b^10*c^10*d^14 + 4479104*a^14*b^10*c^12*d^12 - 562016*a^14*b^10*c^14*d^10 - 605280*a^15*b^9*c^3*d^21 + 2682536*a^15*b^9*c^5*d^19 - 5889904*a^15*b^9*c^7*d^17 + 6731044*a^15*b^9*c^9*d^15 - 3497920*a^15*b^9*c^11*d^13 + 552160*a^15*b^9*c^13*d^11 + 131660*a^16*b^8*c^2*d^22 - 793460*a^16*b^8*c^4*d^20 + 2253214*a^16*b^8*c^6*d^18 - 3330518*a^16*b^8*c^8*d^16 + 2295680*a^16*b^8*c^10*d^14 - 499520*a^16*b^8*c^12*d^12 + 167520*a^17*b^7*c^3*d^21 - 655084*a^17*b^7*c^5*d^19 + 1310504*a^17*b^7*c^7*d^17 - 1239264*a^17*b^7*c^9*d^15 + 381280*a^17*b^7*c^11*d^13 - 22060*a^18*b^6*c^2*d^22 + 138010*a^18*b^6*c^4*d^20 - 396878*a^18*b^6*c^6*d^18 + 529224*a^18*b^6*c^8*d^16 - 230240*a^18*b^6*c^10*d^14 - 18840*a^19*b^5*c^3*d^21 + 85484*a^19*b^5*c^5*d^19 - 168344*a^19*b^5*c^7*d^17 + 106016*a^19*b^5*c^9*d^15 + 782*a^20*b^4*c^2*d^22 - 10562*a^20*b^4*c^4*d^20 + 36120*a^20*b^4*c^6*d^18 - 36280*a^20*b^4*c^8*d^16 + 144*a^21*b^3*c^3*d^21 - 4080*a^21*b^3*c^5*d^19 + 8960*a^21*b^3*c^7*d^17 + 98*a^22*b^2*c^2*d^22 - 88*a^22*b^2*c^4*d^20 - 1520*a^22*b^2*c^6*d^18 + 4*a*b^23*c^23*d + 4*a^23*b*c*d^23)/(16*(5*a^2*b^28*c^30 - b^30*c^30 - a^30*d^30 - 10*a^4*b^26*c^30 + 10*a^6*b^24*c^30 - 5*a^8*b^22*c^30 + a^10*b^20*c^30 + a^20*b^10*d^30 - 5*a^22*b^8*d^30 + 10*a^24*b^6*d^30 - 10*a^26*b^4*d^30 + 5*a^28*b^2*d^30 + 5*a^30*c^2*d^28 - 10*a^30*c^4*d^26 + 10*a^30*c^6*d^24 - 5*a^30*c^8*d^22 + a^30*c^10*d^20 + b^30*c^20*d^10 - 5*b^30*c^22*d^8 + 10*b^30*c^24*d^6 - 10*b^30*c^26*d^4 + 5*b^30*c^28*d^2 - 20*a*b^29*c^19*d^11 + 100*a*b^29*c^21*d^9 - 200*a*b^29*c^23*d^7 + 200*a*b^29*c^25*d^5 - 100*a*b^29*c^27*d^3 - 100*a^3*b^27*c^29*d + 200*a^5*b^25*c^29*d - 200*a^7*b^23*c^29*d + 100*a^9*b^21*c^29*d - 20*a^11*b^19*c^29*d - 20*a^19*b^11*c*d^29 + 100*a^21*b^9*c*d^29 - 200*a^23*b^7*c*d^29 + 200*a^25*b^5*c*d^29 - 100*a^27*b^3*c*d^29 - 100*a^29*b*c^3*d^27 + 200*a^29*b*c^5*d^25 - 200*a^29*b*c^7*d^23 + 100*a^29*b*c^9*d^21 - 20*a^29*b*c^11*d^19 + 190*a^2*b^28*c^18*d^12 - 955*a^2*b^28*c^20*d^10 + 1925*a^2*b^28*c^22*d^8 - 1950*a^2*b^28*c^24*d^6 + 1000*a^2*b^28*c^26*d^4 - 215*a^2*b^28*c^28*d^2 - 1140*a^3*b^27*c^17*d^13 + 5800*a^3*b^27*c^19*d^11 - 11900*a^3*b^27*c^21*d^9 + 12400*a^3*b^27*c^23*d^7 - 6700*a^3*b^27*c^25*d^5 + 1640*a^3*b^27*c^27*d^3 + 4845*a^4*b^26*c^16*d^14 - 25175*a^4*b^26*c^18*d^12 + 53210*a^4*b^26*c^20*d^10 - 58000*a^4*b^26*c^22*d^8 + 33825*a^4*b^26*c^24*d^6 - 9695*a^4*b^26*c^26*d^4 + 1000*a^4*b^26*c^28*d^2 - 15504*a^5*b^25*c^15*d^15 + 83220*a^5*b^25*c^17*d^13 - 183740*a^5*b^25*c^19*d^11 + 213040*a^5*b^25*c^21*d^9 - 136520*a^5*b^25*c^23*d^7 + 46004*a^5*b^25*c^25*d^5 - 6700*a^5*b^25*c^27*d^3 + 38760*a^6*b^24*c^14*d^16 - 218025*a^6*b^24*c^16*d^14 + 510625*a^6*b^24*c^18*d^12 - 639360*a^6*b^24*c^20*d^10 + 455100*a^6*b^24*c^22*d^8 - 178985*a^6*b^24*c^24*d^6 + 33825*a^6*b^24*c^26*d^4 - 1950*a^6*b^24*c^28*d^2 - 77520*a^7*b^23*c^13*d^17 + 465120*a^7*b^23*c^15*d^15 - 1174200*a^7*b^23*c^17*d^13 + 1607600*a^7*b^23*c^19*d^11 - 1277800*a^7*b^23*c^21*d^9 + 581120*a^7*b^23*c^23*d^7 - 136520*a^7*b^23*c^25*d^5 + 12400*a^7*b^23*c^27*d^3 + 125970*a^8*b^22*c^12*d^18 - 823650*a^8*b^22*c^14*d^16 + 2277150*a^8*b^22*c^16*d^14 - 3441850*a^8*b^22*c^18*d^12 + 3061855*a^8*b^22*c^20*d^10 - 1598495*a^8*b^22*c^22*d^8 + 455100*a^8*b^22*c^24*d^6 - 58000*a^8*b^22*c^26*d^4 + 1925*a^8*b^22*c^28*d^2 - 167960*a^9*b^21*c^11*d^19 + 1227400*a^9*b^21*c^13*d^17 - 3772640*a^9*b^21*c^15*d^15 + 6342200*a^9*b^21*c^17*d^

13 - 6323300*a^9*b^21*c^19*d^11 + 3770860*a^9*b^21*c^21*d^9 - 1277800*a^9*b
^21*c^23*d^7 + 213040*a^9*b^21*c^25*d^5 - 11900*a^9*b^21*c^27*d^3 + 184756*
a^10*b^20*c^10*d^20 - 1553630*a^10*b^20*c^12*d^18 + 5384410*a^10*b^20*c^14*
d^16 - 10132510*a^10*b^20*c^16*d^14 + 11341480*a^10*b^20*c^18*d^12 - 769925
7*a^10*b^20*c^20*d^10 + 3061855*a^10*b^20*c^22*d^8 - 639360*a^10*b^20*c^24*
d^6 + 53210*a^10*b^20*c^26*d^4 - 955*a^10*b^20*c^28*d^2 - 167960*a^11*b^19*
c^9*d^21 + 1679600*a^11*b^19*c^11*d^19 - 6653800*a^11*b^19*c^13*d^17 + 1410
8640*a^11*b^19*c^15*d^15 - 17770700*a^11*b^19*c^17*d^13 + 13697880*a^11*b^1
9*c^19*d^11 - 6323300*a^11*b^19*c^21*d^9 + 1607600*a^11*b^19*c^23*d^7 - 183
740*a^11*b^19*c^25*d^5 + 5800*a^11*b^19*c^27*d^3 + 125970*a^12*b^18*c^8*d^2
2 - 1553630*a^12*b^18*c^10*d^20 + 7138300*a^12*b^18*c^12*d^18 - 17183600*a^
12*b^18*c^14*d^16 + 24426875*a^12*b^18*c^16*d^14 - 21339185*a^12*b^18*c^18*
d^12 + 11341480*a^12*b^18*c^20*d^10 - 3441850*a^12*b^18*c^22*d^8 + 510625*a
^12*b^18*c^24*d^6 - 25175*a^12*b^18*c^26*d^4 + 190*a^12*b^18*c^28*d^2 - 775
20*a^13*b^17*c^7*d^23 + 1227400*a^13*b^17*c^9*d^21 - 6653800*a^13*b^17*c^11
*d^19 + 18346400*a^13*b^17*c^13*d^17 - 29535120*a^13*b^17*c^15*d^15 + 29213
260*a^13*b^17*c^17*d^13 - 17770700*a^13*b^17*c^19*d^11 + 6342200*a^13*b^17*
c^21*d^9 - 1174200*a^13*b^17*c^23*d^7 + 83220*a^13*b^17*c^25*d^5 - 1140*a^1
3*b^17*c^27*d^3 + 38760*a^14*b^16*c^6*d^24 - 823650*a^14*b^16*c^8*d^22 + 53
84410*a^14*b^16*c^10*d^20 - 17183600*a^14*b^16*c^12*d^18 + 31460200*a^14*b^
16*c^14*d^16 - 35234455*a^14*b^16*c^16*d^14 + 24426875*a^14*b^16*c^18*d^12
- 10132510*a^14*b^16*c^20*d^10 + 2277150*a^14*b^16*c^22*d^8 - 218025*a^14*b
^16*c^24*d^6 + 4845*a^14*b^16*c^26*d^4 - 15504*a^15*b^15*c^5*d^25 + 465120*
a^15*b^15*c^7*d^23 - 3772640*a^15*b^15*c^9*d^21 + 14108640*a^15*b^15*c^11*d
^19 - 29535120*a^15*b^15*c^13*d^17 + 37499008*a^15*b^15*c^15*d^15 - 2953512
0*a^15*b^15*c^17*d^13 + 14108640*a^15*b^15*c^19*d^11 - 3772640*a^15*b^15*c^
21*d^9 + 465120*a^15*b^15*c^23*d^7 - 15504*a^15*b^15*c^25*d^5 + 4845*a^16*b
^14*c^4*d^26 - 218025*a^16*b^14*c^6*d^24 + 2277150*a^16*b^14*c^8*d^22 - 101
32510*a^16*b^14*c^10*d^20 + 24426875*a^16*b^14*c^12*d^18 - 35234455*a^16*b^
14*c^14*d^16 + 31460200*a^16*b^14*c^16*d^14 - 17183600*a^16*b^14*c^18*d^12
+ 5384410*a^16*b^14*c^20*d^10 - 823650*a^16*b^14*c^22*d^8 + 38760*a^16*b^14
*c^24*d^6 - 1140*a^17*b^13*c^3*d^27 + 83220*a^17*b^13*c^5*d^25 - 1174200*a^
17*b^13*c^7*d^23 + 6342200*a^17*b^13*c^9*d^21 - 17770700*a^17*b^13*c^11*d^1
9 + 29213260*a^17*b^13*c^13*d^17 - 29535120*a^17*b^13*c^15*d^15 + 18346400*
a^17*b^13*c^17*d^13 - 6653800*a^17*b^13*c^19*d^11 + 1227400*a^17*b^13*c^21*
d^9 - 77520*a^17*b^13*c^23*d^7 + 190*a^18*b^12*c^2*d^28 - 25175*a^18*b^12*c
^4*d^26 + 510625*a^18*b^12*c^6*d^24 - 3441850*a^18*b^12*c^8*d^22 + 11341480
*a^18*b^12*c^10*d^20 - 21339185*a^18*b^12*c^12*d^18 + 24426875*a^18*b^12*c^
14*d^16 - 17183600*a^18*b^12*c^16*d^14 + 7138300*a^18*b^12*c^18*d^12 - 1553
630*a^18*b^12*c^20*d^10 + 125970*a^18*b^12*c^22*d^8 + 5800*a^19*b^11*c^3*d^
27 - 183740*a^19*b^11*c^5*d^25 + 1607600*a^19*b^11*c^7*d^23 - 6323300*a^19*
b^11*c^9*d^21 + 13697880*a^19*b^11*c^11*d^19 - 17770700*a^19*b^11*c^13*d^17
 + 14108640*a^19*b^11*c^15*d^15 - 6653800*a^19*b^11*c^17*d^13 + 1679600*a^1
9*b^11*c^19*d^11 - 167960*a^19*b^11*c^21*d^9 - 955*a^20*b^10*c^2*d^28 + 532
10*a^20*b^10*c^4*d^26 - 639360*a^20*b^10*c^6*d^24 + 3061855*a^20*b^10*c^8*d

^22 - 7699257*a^20*b^10*c^10*d^20 + 11341480*a^20*b^10*c^12*d^18 - 10132510
*a^20*b^10*c^14*d^16 + 5384410*a^20*b^10*c^16*d^14 - 1553630*a^20*b^10*c^18
*d^12 + 184756*a^20*b^10*c^20*d^10 - 11900*a^21*b^9*c^3*d^27 + 213040*a^21*
b^9*c^5*d^25 - 1277800*a^21*b^9*c^7*d^23 + 3770860*a^21*b^9*c^9*d^21 - 6323
300*a^21*b^9*c^11*d^19 + 6342200*a^21*b^9*c^13*d^17 - 3772640*a^21*b^9*c^15
*d^15 + 1227400*a^21*b^9*c^17*d^13 - 167960*a^21*b^9*c^19*d^11 + 1925*a^22*
b^8*c^2*d^28 - 58000*a^22*b^8*c^4*d^26 + 455100*a^22*b^8*c^6*d^24 - 1598495
*a^22*b^8*c^8*d^22 + 3061855*a^22*b^8*c^10*d^20 - 3441850*a^22*b^8*c^12*d^1
8 + 2277150*a^22*b^8*c^14*d^16 - 823650*a^22*b^8*c^16*d^14 + 125970*a^22*b^
8*c^18*d^12 + 12400*a^23*b^7*c^3*d^27 - 136520*a^23*b^7*c^5*d^25 + 581120*a
^23*b^7*c^7*d^23 - 1277800*a^23*b^7*c^9*d^21 + 1607600*a^23*b^7*c^11*d^19 -
1174200*a^23*b^7*c^13*d^17 + 465120*a^23*b^7*c^15*d^15 - 77520*a^23*b^7*c^
17*d^13 - 1950*a^24*b^6*c^2*d^28 + 33825*a^24*b^6*c^4*d^26 - 178985*a^24*b^
6*c^6*d^24 + 455100*a^24*b^6*c^8*d^22 - 639360*a^24*b^6*c^10*d^20 + 510625*
a^24*b^6*c^12*d^18 - 218025*a^24*b^6*c^14*d^16 + 38760*a^24*b^6*c^16*d^14 -
6700*a^25*b^5*c^3*d^27 + 46004*a^25*b^5*c^5*d^25 - 136520*a^25*b^5*c^7*d^2
3 + 213040*a^25*b^5*c^9*d^21 - 183740*a^25*b^5*c^11*d^19 + 83220*a^25*b^5*c
^13*d^17 - 15504*a^25*b^5*c^15*d^15 + 1000*a^26*b^4*c^2*d^28 - 9695*a^26*b^
4*c^4*d^26 + 33825*a^26*b^4*c^6*d^24 - 58000*a^26*b^4*c^8*d^22 + 53210*a^26
*b^4*c^10*d^20 - 25175*a^26*b^4*c^12*d^18 + 4845*a^26*b^4*c^14*d^16 + 1640*
a^27*b^3*c^3*d^27 - 6700*a^27*b^3*c^5*d^25 + 12400*a^27*b^3*c^7*d^23 - 1190
0*a^27*b^3*c^9*d^21 + 5800*a^27*b^3*c^11*d^19 - 1140*a^27*b^3*c^13*d^17 - 2
15*a^28*b^2*c^2*d^28 + 1000*a^28*b^2*c^4*d^26 - 1950*a^28*b^2*c^6*d^24 + 19
25*a^28*b^2*c^8*d^22 - 955*a^28*b^2*c^10*d^20 + 190*a^28*b^2*c^12*d^18 + 20
*a*b^29*c^29*d + 20*a^29*b*c*d^29)))^(1/2)*(((((4*a^24*d^24 + 4*b^24*c^24 +
16*a^2*b^22*c^24 + 16*a^4*b^20*c^24 - 1152*a^10*b^14*d^24 + 5568*a^12*b^12
*d^24 - 10568*a^14*b^10*d^24 + 9460*a^16*b^8*d^24 - 3560*a^18*b^6*d^24 + 13
6*a^20*b^4*d^24 + 76*a^22*b^2*d^24 + 16*a^24*c^2*d^22 + 16*a^24*c^4*d^20 -
1152*b^24*c^10*d^14 + 5568*b^24*c^12*d^12 - 10568*b^24*c^14*d^10 + 9460*b^2
4*c^16*d^8 - 3560*b^24*c^18*d^6 + 136*b^24*c^20*d^4 + 76*b^24*c^22*d^2 + 11
520*a*b^23*c^9*d^15 - 56448*a*b^23*c^11*d^13 + 109456*a*b^23*c^13*d^11 - 10
1240*a*b^23*c^15*d^9 + 40720*a*b^23*c^17*d^7 - 2960*a*b^23*c^19*d^5 - 536*a
*b^23*c^21*d^3 - 176*a^3*b^21*c^23*d - 320*a^5*b^19*c^23*d + 11520*a^9*b^15
*c*d^23 - 56448*a^11*b^13*c*d^23 + 109456*a^13*b^11*c*d^23 - 101240*a^15*b^
9*c*d^23 + 40720*a^17*b^7*c*d^23 - 2960*a^19*b^5*c*d^23 - 536*a^21*b^3*c*d^
23 - 176*a^23*b*c^3*d^21 - 320*a^23*b*c^5*d^19 - 51840*a^2*b^22*c^8*d^16 +
263808*a^2*b^22*c^10*d^14 - 541208*a^2*b^22*c^12*d^12 + 547088*a^2*b^22*c^1
4*d^10 - 263320*a^2*b^22*c^16*d^8 + 44120*a^2*b^22*c^18*d^6 - 1564*a^2*b^22
*c^20*d^4 - 196*a^2*b^22*c^22*d^2 + 138240*a^3*b^21*c^7*d^17 - 758400*a^3*b
^21*c^9*d^15 + 1720736*a^3*b^21*c^11*d^13 - 2002728*a^3*b^21*c^13*d^11 + 12
10560*a^3*b^21*c^15*d^9 - 335040*a^3*b^21*c^17*d^7 + 37680*a^3*b^21*c^19*d^
5 - 288*a^3*b^21*c^21*d^3 - 241920*a^4*b^20*c^6*d^18 + 1512000*a^4*b^20*c^8
*d^16 - 3975688*a^4*b^20*c^10*d^14 + 5501328*a^4*b^20*c^12*d^12 - 4147952*a
^4*b^20*c^14*d^10 + 1586920*a^4*b^20*c^16*d^8 - 276020*a^4*b^20*c^18*d^6 +
21124*a^4*b^20*c^20*d^4 + 176*a^4*b^20*c^22*d^2 + 290304*a^5*b^19*c^5*d^19

- 2232576*a^5*b^19*c^7*d^17 + 7078256*a^5*b^19*c^9*d^15 - 11781560*a^5*b^19*c^11*d^13 + 10875200*a^5*b^19*c^13*d^11 - 5365072*a^5*b^19*c^15*d^9 + 1310168*a^5*b^19*c^17*d^7 - 170968*a^5*b^19*c^19*d^5 + 8160*a^5*b^19*c^21*d^3 - 241920*a^6*b^18*c^4*d^20 + 2532096*a^6*b^18*c^6*d^18 - 9955992*a^6*b^18*c^8*d^16 + 20019440*a^6*b^18*c^10*d^14 - 22419600*a^6*b^18*c^12*d^12 + 13887520*a^6*b^18*c^14*d^10 - 4506428*a^6*b^18*c^16*d^8 + 793756*a^6*b^18*c^18*d^6 - 72240*a^6*b^18*c^20*d^4 + 3040*a^6*b^18*c^22*d^2 + 138240*a^7*b^17*c^3*d^21 - 2232576*a^7*b^17*c^5*d^19 + 11150016*a^7*b^17*c^7*d^17 - 27336616*a^7*b^17*c^9*d^15 + 37153600*a^7*b^17*c^11*d^13 - 28461040*a^7*b^17*c^13*d^11 + 11779808*a^7*b^17*c^15*d^9 - 2621008*a^7*b^17*c^17*d^7 + 336688*a^7*b^17*c^19*d^5 - 17920*a^7*b^17*c^21*d^3 - 51840*a^8*b^16*c^2*d^22 + 1512000*a^8*b^16*c^4*d^20 - 9955992*a^8*b^16*c^6*d^18 + 30289656*a^8*b^16*c^8*d^16 - 50137600*a^8*b^16*c^10*d^14 + 46972560*a^8*b^16*c^12*d^12 - 24199280*a^8*b^16*c^14*d^10 + 6661036*a^8*b^16*c^16*d^8 - 1058448*a^8*b^16*c^18*d^6 + 72560*a^8*b^16*c^20*d^4 - 758400*a^9*b^15*c^3*d^21 + 7078256*a^9*b^15*c^5*d^19 - 27336616*a^9*b^15*c^7*d^17 + 55383904*a^9*b^15*c^9*d^15 - 63124080*a^9*b^15*c^11*d^13 + 39987520*a^9*b^15*c^13*d^11 - 13462088*a^9*b^15*c^15*d^9 + 2478528*a^9*b^15*c^17*d^7 - 212032*a^9*b^15*c^19*d^5 + 263808*a^10*b^14*c^2*d^22 - 3975688*a^10*b^14*c^4*d^20 + 20019440*a^10*b^14*c^6*d^18 - 50137600*a^10*b^14*c^8*d^16 + 69593872*a^10*b^14*c^10*d^14 - 53854288*a^10*b^14*c^12*d^12 + 21989928*a^10*b^14*c^14*d^10 - 4591360*a^10*b^14*c^16*d^8 + 460480*a^10*b^14*c^18*d^6 + 1720736*a^11*b^13*c^3*d^21 - 11781560*a^11*b^13*c^5*d^19 + 37153600*a^11*b^13*c^7*d^17 - 63124080*a^11*b^13*c^9*d^15 + 59445728*a^11*b^13*c^11*d^13 - 29358696*a^11*b^13*c^13*d^11 + 6995840*a^11*b^13*c^15*d^9 - 762560*a^11*b^13*c^17*d^7 - 541208*a^12*b^12*c^2*d^22 + 5501328*a^12*b^12*c^4*d^20 - 22419600*a^12*b^12*c^6*d^18 + 46972560*a^12*b^12*c^8*d^16 - 53854288*a^12*b^12*c^10*d^14 + 32294808*a^12*b^12*c^12*d^12 - 8958208*a^12*b^12*c^14*d^10 + 999040*a^12*b^12*c^16*d^8 - 2002728*a^13*b^11*c^3*d^21 + 10875200*a^13*b^11*c^5*d^19 - 28461040*a^13*b^11*c^7*d^17 + 39987520*a^13*b^11*c^9*d^15 - 29358696*a^13*b^11*c^11*d^13 + 9722048*a^13*b^11*c^13*d^11 - 1104320*a^13*b^11*c^15*d^9 + 547088*a^14*b^10*c^2*d^22 - 4147952*a^14*b^10*c^4*d^20 + 13887520*a^14*b^10*c^6*d^18 - 24199280*a^14*b^10*c^8*d^16 + 21989928*a^14*b^10*c^10*d^14 - 8958208*a^14*b^10*c^12*d^12 + 1124032*a^14*b^10*c^14*d^10 + 1210560*a^15*b^9*c^3*d^21 - 5365072*a^15*b^9*c^5*d^19 + 11779808*a^15*b^9*c^7*d^17 - 13462088*a^15*b^9*c^9*d^15 + 6995840*a^15*b^9*c^11*d^13 - 1104320*a^15*b^9*c^13*d^11 - 263320*a^16*b^8*c^2*d^22 + 1586920*a^16*b^8*c^4*d^20 - 4506428*a^16*b^8*c^6*d^18 + 6661036*a^16*b^8*c^8*d^16 - 4591360*a^16*b^8*c^10*d^14 + 999040*a^16*b^8*c^12*d^12 - 335040*a^17*b^7*c^3*d^21 + 1310168*a^17*b^7*c^5*d^19 - 2621008*a^17*b^7*c^7*d^17 + 2478528*a^17*b^7*c^9*d^15 - 762560*a^17*b^7*c^11*d^13 + 44120*a^18*b^6*c^2*d^22 - 276020*a^18*b^6*c^4*d^20 + 793756*a^18*b^6*c^6*d^18 - 1058448*a^18*b^6*c^8*d^16 + 460480*a^18*b^6*c^10*d^14 + 37680*a^19*b^5*c^3*d^21 - 170968*a^19*b^5*c^5*d^19 + 336688*a^19*b^5*c^7*d^17 - 212032*a^19*b^5*c^9*d^15 - 1564*a^20*b^4*c^2*d^22 + 21124*a^20*b^4*c^4*d^20 - 72240*a^20*b^4*c^6*d^18 + 72560*a^20*b^4*c^8*d^16 - 288*a^21*b^3*c^3*d^21 + 8160*a^21*b^3*c^5*d^19 - 17920*a^21*b^3

*c^7*d^17 - 196*a^22*b^2*c^2*d^22 + 176*a^22*b^2*c^4*d^20 + 3040*a^22*b^2*c
^6*d^18 - 8*a*b^23*c^23*d - 8*a^23*b*c*d^23)^2/4 - (20736*b^18*d^18 - 96768
*a^2*b^16*d^18 + 173664*a^4*b^14*d^18 - 136032*a^6*b^12*d^18 + 31081*a^8*b^
10*d^18 + 8440*a^10*b^8*d^18 + 400*a^12*b^6*d^18 - 96768*b^18*c^2*d^16 + 17
3664*b^18*c^4*d^14 - 136032*b^18*c^6*d^12 + 31081*b^18*c^8*d^10 + 8440*b^18
*c^10*d^8 + 400*b^18*c^12*d^6 - 131328*a*b^17*c^3*d^15 + 216576*a*b^17*c^5*
d^13 - 141104*a*b^17*c^7*d^11 + 20260*a*b^17*c^9*d^9 + 2800*a*b^17*c^11*d^7
 - 131328*a^3*b^15*c*d^17 + 216576*a^5*b^13*c*d^17 - 141104*a^7*b^11*c*d^17
 + 20260*a^9*b^9*c*d^17 + 2800*a^11*b^7*c*d^17 + 495936*a^2*b^16*c^2*d^16 -
 989856*a^2*b^16*c^4*d^14 + 901948*a^2*b^16*c^6*d^12 - 308392*a^2*b^16*c^8*
d^10 - 5260*a^2*b^16*c^10*d^8 + 1600*a^2*b^16*c^12*d^6 + 657408*a^3*b^15*c^
3*d^15 - 1158992*a^3*b^15*c^5*d^13 + 838256*a^3*b^15*c^7*d^11 - 182200*a^3*
b^15*c^9*d^9 - 3200*a^3*b^15*c^11*d^7 - 989856*a^4*b^14*c^2*d^16 + 2185654*
a^4*b^14*c^4*d^14 - 2218576*a^4*b^14*c^6*d^12 + 900624*a^4*b^14*c^8*d^10 -
64720*a^4*b^14*c^10*d^8 + 1600*a^4*b^14*c^12*d^6 - 1158992*a^5*b^13*c^3*d^1
5 + 2158808*a^5*b^13*c^5*d^13 - 1641528*a^5*b^13*c^7*d^11 + 406880*a^5*b^13
*c^9*d^9 - 17600*a^5*b^13*c^11*d^7 + 901948*a^6*b^12*c^2*d^16 - 2218576*a^6
*b^12*c^4*d^14 + 2430936*a^6*b^12*c^6*d^12 - 1026928*a^6*b^12*c^8*d^10 + 88
720*a^6*b^12*c^10*d^8 + 838256*a^7*b^11*c^3*d^15 - 1641528*a^7*b^11*c^5*d^1
3 + 1206848*a^7*b^11*c^7*d^11 - 239360*a^7*b^11*c^9*d^9 - 308392*a^8*b^10*c
^2*d^16 + 900624*a^8*b^10*c^4*d^14 - 1026928*a^8*b^10*c^6*d^12 + 354016*a^8
*b^10*c^8*d^10 - 182200*a^9*b^9*c^3*d^15 + 406880*a^9*b^9*c^5*d^13 - 239360
*a^9*b^9*c^7*d^11 - 5260*a^10*b^8*c^2*d^16 - 64720*a^10*b^8*c^4*d^14 + 8872
0*a^10*b^8*c^6*d^12 - 3200*a^11*b^7*c^3*d^15 - 17600*a^11*b^7*c^5*d^13 + 16
00*a^12*b^6*c^2*d^16 + 1600*a^12*b^6*c^4*d^14 + 27648*a*b^17*c*d^17)*(80*a^
2*b^28*c^30 - 16*b^30*c^30 - 16*a^30*d^30 - 160*a^4*b^26*c^30 + 160*a^6*b^2
4*c^30 - 80*a^8*b^22*c^30 + 16*a^10*b^20*c^30 + 16*a^20*b^10*d^30 - 80*a^22
*b^8*d^30 + 160*a^24*b^6*d^30 - 160*a^26*b^4*d^30 + 80*a^28*b^2*d^30 + 80*a
^30*c^2*d^28 - 160*a^30*c^4*d^26 + 160*a^30*c^6*d^24 - 80*a^30*c^8*d^22 + 1
6*a^30*c^10*d^20 + 16*b^30*c^20*d^10 - 80*b^30*c^22*d^8 + 160*b^30*c^24*d^6
 - 160*b^30*c^26*d^4 + 80*b^30*c^28*d^2 - 320*a*b^29*c^19*d^11 + 1600*a*b^2
9*c^21*d^9 - 3200*a*b^29*c^23*d^7 + 3200*a*b^29*c^25*d^5 - 1600*a*b^29*c^27
*d^3 - 1600*a^3*b^27*c^29*d + 3200*a^5*b^25*c^29*d - 3200*a^7*b^23*c^29*d +
 1600*a^9*b^21*c^29*d - 320*a^11*b^19*c^29*d - 320*a^19*b^11*c*d^29 + 1600*
a^21*b^9*c*d^29 - 3200*a^23*b^7*c*d^29 + 3200*a^25*b^5*c*d^29 - 1600*a^27*b
^3*c*d^29 - 1600*a^29*b*c^3*d^27 + 3200*a^29*b*c^5*d^25 - 3200*a^29*b*c^7*d
^23 + 1600*a^29*b*c^9*d^21 - 320*a^29*b*c^11*d^19 + 3040*a^2*b^28*c^18*d^12
 - 15280*a^2*b^28*c^20*d^10 + 30800*a^2*b^28*c^22*d^8 - 31200*a^2*b^28*c^24
*d^6 + 16000*a^2*b^28*c^26*d^4 - 3440*a^2*b^28*c^28*d^2 - 18240*a^3*b^27*c^
17*d^13 + 92800*a^3*b^27*c^19*d^11 - 190400*a^3*b^27*c^21*d^9 + 198400*a^3*
b^27*c^23*d^7 - 107200*a^3*b^27*c^25*d^5 + 26240*a^3*b^27*c^27*d^3 + 77520*
a^4*b^26*c^16*d^14 - 402800*a^4*b^26*c^18*d^12 + 851360*a^4*b^26*c^20*d^10
- 928000*a^4*b^26*c^22*d^8 + 541200*a^4*b^26*c^24*d^6 - 155120*a^4*b^26*c^2
6*d^4 + 16000*a^4*b^26*c^28*d^2 - 248064*a^5*b^25*c^15*d^15 + 1331520*a^5*b
^25*c^17*d^13 - 2939840*a^5*b^25*c^19*d^11 + 3408640*a^5*b^25*c^21*d^9 - 21

84320*a^5*b^25*c^23*d^7 + 736064*a^5*b^25*c^25*d^5 - 107200*a^5*b^25*c^27*d^3 + 620160*a^6*b^24*c^14*d^16 - 3488400*a^6*b^24*c^16*d^14 + 8170000*a^6*b^24*c^18*d^12 - 10229760*a^6*b^24*c^20*d^10 + 7281600*a^6*b^24*c^22*d^8 - 2863760*a^6*b^24*c^24*d^6 + 541200*a^6*b^24*c^26*d^4 - 31200*a^6*b^24*c^28*d^2 - 1240320*a^7*b^23*c^13*d^17 + 7441920*a^7*b^23*c^15*d^15 - 18787200*a^7*b^23*c^17*d^13 + 25721600*a^7*b^23*c^19*d^11 - 20444800*a^7*b^23*c^21*d^9 + 9297920*a^7*b^23*c^23*d^7 - 2184320*a^7*b^23*c^25*d^5 + 198400*a^7*b^23*c^27*d^3 + 2015520*a^8*b^22*c^12*d^18 - 13178400*a^8*b^22*c^14*d^16 + 36434400*a^8*b^22*c^16*d^14 - 55069600*a^8*b^22*c^18*d^12 + 48989680*a^8*b^22*c^20*d^10 - 25575920*a^8*b^22*c^22*d^8 + 7281600*a^8*b^22*c^24*d^6 - 928000*a^8*b^22*c^26*d^4 + 30800*a^8*b^22*c^28*d^2 - 2687360*a^9*b^21*c^11*d^19 + 19638400*a^9*b^21*c^13*d^17 - 60362240*a^9*b^21*c^15*d^15 + 101475200*a^9*b^21*c^17*d^13 - 101172800*a^9*b^21*c^19*d^11 + 60333760*a^9*b^21*c^21*d^9 - 20444800*a^9*b^21*c^23*d^7 + 3408640*a^9*b^21*c^25*d^5 - 190400*a^9*b^21*c^27*d^3 + 2956096*a^10*b^20*c^10*d^20 - 24858080*a^10*b^20*c^12*d^18 + 86150560*a^10*b^20*c^14*d^16 - 162120160*a^10*b^20*c^16*d^14 + 181463680*a^10*b^20*c^18*d^12 - 123188112*a^10*b^20*c^20*d^10 + 48989680*a^10*b^20*c^22*d^8 - 10229760*a^10*b^20*c^24*d^6 + 851360*a^10*b^20*c^26*d^4 - 15280*a^10*b^20*c^28*d^2 - 2687360*a^11*b^19*c^9*d^21 + 26873600*a^11*b^19*c^11*d^19 - 106460800*a^11*b^19*c^13*d^17 + 225738240*a^11*b^19*c^15*d^15 - 284331200*a^11*b^19*c^17*d^13 + 219166080*a^11*b^19*c^19*d^11 - 101172800*a^11*b^19*c^21*d^9 + 25721600*a^11*b^19*c^23*d^7 - 2939840*a^11*b^19*c^25*d^5 + 92800*a^11*b^19*c^27*d^3 + 2015520*a^12*b^18*c^8*d^22 - 24858080*a^12*b^18*c^10*d^20 + 114212800*a^12*b^18*c^12*d^18 - 274937600*a^12*b^18*c^14*d^16 + 390830000*a^12*b^18*c^16*d^14 - 341426960*a^12*b^18*c^18*d^12 + 181463680*a^12*b^18*c^20*d^10 - 55069600*a^12*b^18*c^22*d^8 + 8170000*a^12*b^18*c^24*d^6 - 402800*a^12*b^18*c^26*d^4 + 3040*a^12*b^18*c^28*d^2 - 1240320*a^13*b^17*c^7*d^23 + 19638400*a^13*b^17*c^9*d^21 - 106460800*a^13*b^17*c^11*d^19 + 293542400*a^13*b^17*c^13*d^17 - 472561920*a^13*b^17*c^15*d^15 + 467412160*a^13*b^17*c^17*d^13 - 284331200*a^13*b^17*c^19*d^11 + 101475200*a^13*b^17*c^21*d^9 - 18787200*a^13*b^17*c^23*d^7 + 1331520*a^13*b^17*c^25*d^5 - 18240*a^13*b^17*c^27*d^3 + 620160*a^14*b^16*c^6*d^24 - 13178400*a^14*b^16*c^8*d^22 + 86150560*a^14*b^16*c^10*d^20 - 274937600*a^14*b^16*c^12*d^18 + 503363200*a^14*b^16*c^14*d^16 - 563751280*a^14*b^16*c^16*d^14 + 390830000*a^14*b^16*c^18*d^12 - 162120160*a^14*b^16*c^20*d^10 + 36434400*a^14*b^16*c^22*d^8 - 3488400*a^14*b^16*c^24*d^6 + 77520*a^14*b^16*c^26*d^4 - 248064*a^15*b^15*c^5*d^25 + 7441920*a^15*b^15*c^7*d^23 - 60362240*a^15*b^15*c^9*d^21 + 225738240*a^15*b^15*c^11*d^19 - 472561920*a^15*b^15*c^13*d^17 + 599984128*a^15*b^15*c^15*d^15 - 472561920*a^15*b^15*c^17*d^13 + 225738240*a^15*b^15*c^19*d^11 - 60362240*a^15*b^15*c^21*d^9 + 7441920*a^15*b^15*c^23*d^7 - 248064*a^15*b^15*c^25*d^5 + 77520*a^16*b^14*c^4*d^26 - 3488400*a^16*b^14*c^6*d^24 + 36434400*a^16*b^14*c^8*d^22 - 162120160*a^16*b^14*c^10*d^20 + 390830000*a^16*b^14*c^12*d^18 - 563751280*a^16*b^14*c^14*d^16 + 503363200*a^16*b^14*c^16*d^14 - 274937600*a^16*b^14*c^18*d^12 + 86150560*a^16*b^14*c^20*d^10 - 13178400*a^16*b^14*c^22*d^8 + 620160*a^16*b^14*c^24*d^6 - 18240*a^17*b^13*c^3*d^27 + 1331520*a

^17*b^13*c^5*d^25 - 18787200*a^17*b^13*c^7*d^23 + 101475200*a^17*b^13*c^9*d^21 - 284331200*a^17*b^13*c^11*d^19 + 467412160*a^17*b^13*c^13*d^17 - 472561920*a^17*b^13*c^15*d^15 + 293542400*a^17*b^13*c^17*d^13 - 106460800*a^17*b^13*c^19*d^11 + 19638400*a^17*b^13*c^21*d^9 - 1240320*a^17*b^13*c^23*d^7 + 3040*a^18*b^12*c^2*d^28 - 402800*a^18*b^12*c^4*d^26 + 8170000*a^18*b^12*c^6*d^24 - 55069600*a^18*b^12*c^8*d^22 + 181463680*a^18*b^12*c^10*d^20 - 341426960*a^18*b^12*c^12*d^18 + 390830000*a^18*b^12*c^14*d^16 - 274937600*a^18*b^12*c^16*d^14 + 114212800*a^18*b^12*c^18*d^12 - 24858080*a^18*b^12*c^20*d^10 + 2015520*a^18*b^12*c^22*d^8 + 92800*a^19*b^11*c^3*d^27 - 2939840*a^19*b^11*c^5*d^25 + 25721600*a^19*b^11*c^7*d^23 - 101172800*a^19*b^11*c^9*d^21 + 219166080*a^19*b^11*c^11*d^19 - 284331200*a^19*b^11*c^13*d^17 + 225738240*a^19*b^11*c^15*d^15 - 106460800*a^19*b^11*c^17*d^13 + 26873600*a^19*b^11*c^19*d^11 - 2687360*a^19*b^11*c^21*d^9 - 15280*a^20*b^10*c^2*d^28 + 851360*a^20*b^10*c^4*d^26 - 10229760*a^20*b^10*c^6*d^24 + 48989680*a^20*b^10*c^8*d^22 - 123188112*a^20*b^10*c^10*d^20 + 181463680*a^20*b^10*c^12*d^18 - 162120160*a^20*b^10*c^14*d^16 + 86150560*a^20*b^10*c^16*d^14 - 24858080*a^20*b^10*c^18*d^12 + 2956096*a^20*b^10*c^20*d^10 - 190400*a^21*b^9*c^3*d^27 + 3408640*a^21*b^9*c^5*d^25 - 20444800*a^21*b^9*c^7*d^23 + 60333760*a^21*b^9*c^9*d^21 - 101172800*a^21*b^9*c^11*d^19 + 101475200*a^21*b^9*c^13*d^17 - 60362240*a^21*b^9*c^15*d^15 + 19638400*a^21*b^9*c^17*d^13 - 2687360*a^21*b^9*c^19*d^11 + 30800*a^22*b^8*c^2*d^28 - 928000*a^22*b^8*c^4*d^26 + 7281600*a^22*b^8*c^6*d^24 - 25575920*a^22*b^8*c^8*d^22 + 48989680*a^22*b^8*c^10*d^20 - 55069600*a^22*b^8*c^12*d^18 + 36434400*a^22*b^8*c^14*d^16 - 13178400*a^22*b^8*c^16*d^14 + 2015520*a^22*b^8*c^18*d^12 + 198400*a^23*b^7*c^3*d^27 - 2184320*a^23*b^7*c^5*d^25 + 9297920*a^23*b^7*c^7*d^23 - 20444800*a^23*b^7*c^9*d^21 + 25721600*a^23*b^7*c^11*d^19 - 18787200*a^23*b^7*c^13*d^17 + 7441920*a^23*b^7*c^15*d^15 - 1240320*a^23*b^7*c^17*d^13 - 31200*a^24*b^6*c^2*d^28 + 541200*a^24*b^6*c^4*d^26 - 2863760*a^24*b^6*c^6*d^24 + 7281600*a^24*b^6*c^8*d^22 - 10229760*a^24*b^6*c^10*d^20 + 8170000*a^24*b^6*c^12*d^18 - 3488400*a^24*b^6*c^14*d^16 + 620160*a^24*b^6*c^16*d^14 - 107200*a^25*b^5*c^3*d^27 + 736064*a^25*b^5*c^5*d^25 - 2184320*a^25*b^5*c^7*d^23 + 3408640*a^25*b^5*c^9*d^21 - 2939840*a^25*b^5*c^11*d^19 + 1331520*a^25*b^5*c^13*d^17 - 248064*a^25*b^5*c^15*d^15 + 16000*a^26*b^4*c^2*d^28 - 155120*a^26*b^4*c^4*d^26 + 541200*a^26*b^4*c^6*d^24 - 928000*a^26*b^4*c^8*d^22 + 851360*a^26*b^4*c^10*d^20 - 402800*a^26*b^4*c^12*d^18 + 77520*a^26*b^4*c^14*d^16 + 26240*a^27*b^3*c^3*d^27 - 107200*a^27*b^3*c^5*d^25 + 198400*a^27*b^3*c^7*d^23 - 190400*a^27*b^3*c^9*d^21 + 92800*a^27*b^3*c^11*d^19 - 18240*a^27*b^3*c^13*d^17 - 3440*a^28*b^2*c^2*d^28 + 16000*a^28*b^2*c^4*d^26 - 31200*a^28*b^2*c^6*d^24 + 30800*a^28*b^2*c^8*d^22 - 15280*a^28*b^2*c^10*d^20 + 3040*a^28*b^2*c^12*d^18 + 320*a*b^29*c^29*d + 320*a^29*b*c*d^29))^(1/2) - 2*a^24*d^24 - 2*b^24*c^24 - 8*a^2*b^22*c^24 - 8*a^4*b^20*c^24 + 576*a^10*b^14*d^24 - 2784*a^12*b^12*d^24 + 5284*a^14*b^10*d^24 - 4730*a^16*b^8*d^24 + 1780*a^18*b^6*d^24 - 68*a^20*b^4*d^24 - 38*a^22*b^2*d^24 - 8*a^24*c^2*d^22 - 8*a^24*c^4*d^20 + 576*b^24*c^10*d^14 - 2784*b^24*c^12*d^12 + 5284*b^24*c^14*d^10 - 4730*b^24*c^16*d^8 + 1780*b^24*c^18*d^6 - 68*b^24*c^20*d^4 - 38*b^24*c^22*d^2 - 5760*a*b^23*c^9

*d^15 + 28224*a*b^23*c^11*d^13 - 54728*a*b^23*c^13*d^11 + 50620*a*b^23*c^15
*d^9 - 20360*a*b^23*c^17*d^7 + 1480*a*b^23*c^19*d^5 + 268*a*b^23*c^21*d^3 +
88*a^3*b^21*c^23*d + 160*a^5*b^19*c^23*d - 5760*a^9*b^15*c*d^23 + 28224*a^
11*b^13*c*d^23 - 54728*a^13*b^11*c*d^23 + 50620*a^15*b^9*c*d^23 - 20360*a^1
7*b^7*c*d^23 + 1480*a^19*b^5*c*d^23 + 268*a^21*b^3*c*d^23 + 88*a^23*b*c^3*d
^21 + 160*a^23*b*c^5*d^19 + 25920*a^2*b^22*c^8*d^16 - 131904*a^2*b^22*c^10*
d^14 + 270604*a^2*b^22*c^12*d^12 - 273544*a^2*b^22*c^14*d^10 + 131660*a^2*b
^22*c^16*d^8 - 22060*a^2*b^22*c^18*d^6 + 782*a^2*b^22*c^20*d^4 + 98*a^2*b^2
2*c^22*d^2 - 69120*a^3*b^21*c^7*d^17 + 379200*a^3*b^21*c^9*d^15 - 860368*a^
3*b^21*c^11*d^13 + 1001364*a^3*b^21*c^13*d^11 - 605280*a^3*b^21*c^15*d^9 +
167520*a^3*b^21*c^17*d^7 - 18840*a^3*b^21*c^19*d^5 + 144*a^3*b^21*c^21*d^3
+ 120960*a^4*b^20*c^6*d^18 - 756000*a^4*b^20*c^8*d^16 + 1987844*a^4*b^20*c^
10*d^14 - 2750664*a^4*b^20*c^12*d^12 + 2073976*a^4*b^20*c^14*d^10 - 793460*
a^4*b^20*c^16*d^8 + 138010*a^4*b^20*c^18*d^6 - 10562*a^4*b^20*c^20*d^4 - 88
*a^4*b^20*c^22*d^2 - 145152*a^5*b^19*c^5*d^19 + 1116288*a^5*b^19*c^7*d^17 -
3539128*a^5*b^19*c^9*d^15 + 5890780*a^5*b^19*c^11*d^13 - 5437600*a^5*b^19*
c^13*d^11 + 2682536*a^5*b^19*c^15*d^9 - 655084*a^5*b^19*c^17*d^7 + 85484*a^
5*b^19*c^19*d^5 - 4080*a^5*b^19*c^21*d^3 + 120960*a^6*b^18*c^4*d^20 - 12660
48*a^6*b^18*c^6*d^18 + 4977996*a^6*b^18*c^8*d^16 - 10009720*a^6*b^18*c^10*d
^14 + 11209800*a^6*b^18*c^12*d^12 - 6943760*a^6*b^18*c^14*d^10 + 2253214*a^
6*b^18*c^16*d^8 - 396878*a^6*b^18*c^18*d^6 + 36120*a^6*b^18*c^20*d^4 - 1520
*a^6*b^18*c^22*d^2 - 69120*a^7*b^17*c^3*d^21 + 1116288*a^7*b^17*c^5*d^19 -
5575008*a^7*b^17*c^7*d^17 + 13668308*a^7*b^17*c^9*d^15 - 18576800*a^7*b^17*
c^11*d^13 + 14230520*a^7*b^17*c^13*d^11 - 5889904*a^7*b^17*c^15*d^9 + 13105
04*a^7*b^17*c^17*d^7 - 168344*a^7*b^17*c^19*d^5 + 8960*a^7*b^17*c^21*d^3 +
25920*a^8*b^16*c^2*d^22 - 756000*a^8*b^16*c^4*d^20 + 4977996*a^8*b^16*c^6*d
^18 - 15144828*a^8*b^16*c^8*d^16 + 25068800*a^8*b^16*c^10*d^14 - 23486280*a
^8*b^16*c^12*d^12 + 12099640*a^8*b^16*c^14*d^10 - 3330518*a^8*b^16*c^16*d^8
+ 529224*a^8*b^16*c^18*d^6 - 36280*a^8*b^16*c^20*d^4 + 379200*a^9*b^15*c^3
*d^21 - 3539128*a^9*b^15*c^5*d^19 + 13668308*a^9*b^15*c^7*d^17 - 27691952*a
^9*b^15*c^9*d^15 + 31562040*a^9*b^15*c^11*d^13 - 19993760*a^9*b^15*c^13*d^1
1 + 6731044*a^9*b^15*c^15*d^9 - 1239264*a^9*b^15*c^17*d^7 + 106016*a^9*b^15
*c^19*d^5 - 131904*a^10*b^14*c^2*d^22 + 1987844*a^10*b^14*c^4*d^20 - 100097
20*a^10*b^14*c^6*d^18 + 25068800*a^10*b^14*c^8*d^16 - 34796936*a^10*b^14*c^
10*d^14 + 26927144*a^10*b^14*c^12*d^12 - 10994964*a^10*b^14*c^14*d^10 + 229
5680*a^10*b^14*c^16*d^8 - 230240*a^10*b^14*c^18*d^6 - 860368*a^11*b^13*c^3*
d^21 + 5890780*a^11*b^13*c^5*d^19 - 18576800*a^11*b^13*c^7*d^17 + 31562040*
a^11*b^13*c^9*d^15 - 29722864*a^11*b^13*c^11*d^13 + 14679348*a^11*b^13*c^13
*d^11 - 3497920*a^11*b^13*c^15*d^9 + 381280*a^11*b^13*c^17*d^7 + 270604*a^1
2*b^12*c^2*d^22 - 2750664*a^12*b^12*c^4*d^20 + 11209800*a^12*b^12*c^6*d^18
- 23486280*a^12*b^12*c^8*d^16 + 26927144*a^12*b^12*c^10*d^14 - 16147404*a^1
2*b^12*c^12*d^12 + 4479104*a^12*b^12*c^14*d^10 - 499520*a^12*b^12*c^16*d^8
+ 1001364*a^13*b^11*c^3*d^21 - 5437600*a^13*b^11*c^5*d^19 + 14230520*a^13*b
^11*c^7*d^17 - 19993760*a^13*b^11*c^9*d^15 + 14679348*a^13*b^11*c^11*d^13 -
4861024*a^13*b^11*c^13*d^11 + 552160*a^13*b^11*c^15*d^9 - 273544*a^14*b^10

*c^2*d^22 + 2073976*a^14*b^10*c^4*d^20 - 6943760*a^14*b^10*c^6*d^18 + 12099
640*a^14*b^10*c^8*d^16 - 10994964*a^14*b^10*c^10*d^14 + 4479104*a^14*b^10*c
^12*d^12 - 562016*a^14*b^10*c^14*d^10 - 605280*a^15*b^9*c^3*d^21 + 2682536*
a^15*b^9*c^5*d^19 - 5889904*a^15*b^9*c^7*d^17 + 6731044*a^15*b^9*c^9*d^15 -
3497920*a^15*b^9*c^11*d^13 + 552160*a^15*b^9*c^13*d^11 + 131660*a^16*b^8*c
^2*d^22 - 793460*a^16*b^8*c^4*d^20 + 2253214*a^16*b^8*c^6*d^18 - 3330518*a^
16*b^8*c^8*d^16 + 2295680*a^16*b^8*c^10*d^14 - 499520*a^16*b^8*c^12*d^12 +
167520*a^17*b^7*c^3*d^21 - 655084*a^17*b^7*c^5*d^19 + 1310504*a^17*b^7*c^7*
d^17 - 1239264*a^17*b^7*c^9*d^15 + 381280*a^17*b^7*c^11*d^13 - 22060*a^18*b
^6*c^2*d^22 + 138010*a^18*b^6*c^4*d^20 - 396878*a^18*b^6*c^6*d^18 + 529224*
a^18*b^6*c^8*d^16 - 230240*a^18*b^6*c^10*d^14 - 18840*a^19*b^5*c^3*d^21 + 8
5484*a^19*b^5*c^5*d^19 - 168344*a^19*b^5*c^7*d^17 + 106016*a^19*b^5*c^9*d^1
5 + 782*a^20*b^4*c^2*d^22 - 10562*a^20*b^4*c^4*d^20 + 36120*a^20*b^4*c^6*d^
18 - 36280*a^20*b^4*c^8*d^16 + 144*a^21*b^3*c^3*d^21 - 4080*a^21*b^3*c^5*d^
19 + 8960*a^21*b^3*c^7*d^17 + 98*a^22*b^2*c^2*d^22 - 88*a^22*b^2*c^4*d^20 -
1520*a^22*b^2*c^6*d^18 + 4*a*b^23*c^23*d + 4*a^23*b*c*d^23)/(16*(5*a^2*b^2
8*c^30 - b^30*c^30 - a^30*d^30 - 10*a^4*b^26*c^30 + 10*a^6*b^24*c^30 - 5*a^
8*b^22*c^30 + a^10*b^20*c^30 + a^20*b^10*d^30 - 5*a^22*b^8*d^30 + 10*a^24*b
^6*d^30 - 10*a^26*b^4*d^30 + 5*a^28*b^2*d^30 + 5*a^30*c^2*d^28 - 10*a^30*c^
4*d^26 + 10*a^30*c^6*d^24 - 5*a^30*c^8*d^22 + a^30*c^10*d^20 + b^30*c^20*d^
10 - 5*b^30*c^22*d^8 + 10*b^30*c^24*d^6 - 10*b^30*c^26*d^4 + 5*b^30*c^28*d^
2 - 20*a*b^29*c^19*d^11 + 100*a*b^29*c^21*d^9 - 200*a*b^29*c^23*d^7 + 200*a
*b^29*c^25*d^5 - 100*a*b^29*c^27*d^3 - 100*a^3*b^27*c^29*d + 200*a^5*b^25*c
^29*d - 200*a^7*b^23*c^29*d + 100*a^9*b^21*c^29*d - 20*a^11*b^19*c^29*d - 2
0*a^19*b^11*c*d^29 + 100*a^21*b^9*c*d^29 - 200*a^23*b^7*c*d^29 + 200*a^25*b
^5*c*d^29 - 100*a^27*b^3*c*d^29 - 100*a^29*b*c^3*d^27 + 200*a^29*b*c^5*d^25
- 200*a^29*b*c^7*d^23 + 100*a^29*b*c^9*d^21 - 20*a^29*b*c^11*d^19 + 190*a^
2*b^28*c^18*d^12 - 955*a^2*b^28*c^20*d^10 + 1925*a^2*b^28*c^22*d^8 - 1950*a
^2*b^28*c^24*d^6 + 1000*a^2*b^28*c^26*d^4 - 215*a^2*b^28*c^28*d^2 - 1140*a^
3*b^27*c^17*d^13 + 5800*a^3*b^27*c^19*d^11 - 11900*a^3*b^27*c^21*d^9 + 1240
0*a^3*b^27*c^23*d^7 - 6700*a^3*b^27*c^25*d^5 + 1640*a^3*b^27*c^27*d^3 + 484
5*a^4*b^26*c^16*d^14 - 25175*a^4*b^26*c^18*d^12 + 53210*a^4*b^26*c^20*d^10
- 58000*a^4*b^26*c^22*d^8 + 33825*a^4*b^26*c^24*d^6 - 9695*a^4*b^26*c^26*d^
4 + 1000*a^4*b^26*c^28*d^2 - 15504*a^5*b^25*c^15*d^15 + 83220*a^5*b^25*c^17
*d^13 - 183740*a^5*b^25*c^19*d^11 + 213040*a^5*b^25*c^21*d^9 - 136520*a^5*b
^25*c^23*d^7 + 46004*a^5*b^25*c^25*d^5 - 6700*a^5*b^25*c^27*d^3 + 38760*a^6
*b^24*c^14*d^16 - 218025*a^6*b^24*c^16*d^14 + 510625*a^6*b^24*c^18*d^12 - 6
39360*a^6*b^24*c^20*d^10 + 455100*a^6*b^24*c^22*d^8 - 178985*a^6*b^24*c^24*
d^6 + 33825*a^6*b^24*c^26*d^4 - 1950*a^6*b^24*c^28*d^2 - 77520*a^7*b^23*c^1
3*d^17 + 465120*a^7*b^23*c^15*d^15 - 1174200*a^7*b^23*c^17*d^13 + 1607600*a
^7*b^23*c^19*d^11 - 1277800*a^7*b^23*c^21*d^9 + 581120*a^7*b^23*c^23*d^7 -
136520*a^7*b^23*c^25*d^5 + 12400*a^7*b^23*c^27*d^3 + 125970*a^8*b^22*c^12*d
^18 - 823650*a^8*b^22*c^14*d^16 + 2277150*a^8*b^22*c^16*d^14 - 3441850*a^8*
b^22*c^18*d^12 + 3061855*a^8*b^22*c^20*d^10 - 1598495*a^8*b^22*c^22*d^8 + 4
55100*a^8*b^22*c^24*d^6 - 58000*a^8*b^22*c^26*d^4 + 1925*a^8*b^22*c^28*d^2

- 167960*a^9*b^21*c^11*d^19 + 1227400*a^9*b^21*c^13*d^17 - 3772640*a^9*b^21
*c^15*d^15 + 6342200*a^9*b^21*c^17*d^13 - 6323300*a^9*b^21*c^19*d^11 + 3770
860*a^9*b^21*c^21*d^9 - 1277800*a^9*b^21*c^23*d^7 + 213040*a^9*b^21*c^25*d^
5 - 11900*a^9*b^21*c^27*d^3 + 184756*a^10*b^20*c^10*d^20 - 1553630*a^10*b^2
0*c^12*d^18 + 5384410*a^10*b^20*c^14*d^16 - 10132510*a^10*b^20*c^16*d^14 +
11341480*a^10*b^20*c^18*d^12 - 7699257*a^10*b^20*c^20*d^10 + 3061855*a^10*b
^20*c^22*d^8 - 639360*a^10*b^20*c^24*d^6 + 53210*a^10*b^20*c^26*d^4 - 955*a
^10*b^20*c^28*d^2 - 167960*a^11*b^19*c^9*d^21 + 1679600*a^11*b^19*c^11*d^19
 - 6653800*a^11*b^19*c^13*d^17 + 14108640*a^11*b^19*c^15*d^15 - 17770700*a^
11*b^19*c^17*d^13 + 13697880*a^11*b^19*c^19*d^11 - 6323300*a^11*b^19*c^21*d
^9 + 1607600*a^11*b^19*c^23*d^7 - 183740*a^11*b^19*c^25*d^5 + 5800*a^11*b^1
9*c^27*d^3 + 125970*a^12*b^18*c^8*d^22 - 1553630*a^12*b^18*c^10*d^20 + 7138
300*a^12*b^18*c^12*d^18 - 17183600*a^12*b^18*c^14*d^16 + 24426875*a^12*b^18
*c^16*d^14 - 21339185*a^12*b^18*c^18*d^12 + 11341480*a^12*b^18*c^20*d^10 -
3441850*a^12*b^18*c^22*d^8 + 510625*a^12*b^18*c^24*d^6 - 25175*a^12*b^18*c^
26*d^4 + 190*a^12*b^18*c^28*d^2 - 77520*a^13*b^17*c^7*d^23 + 1227400*a^13*b
^17*c^9*d^21 - 6653800*a^13*b^17*c^11*d^19 + 18346400*a^13*b^17*c^13*d^17 -
 29535120*a^13*b^17*c^15*d^15 + 29213260*a^13*b^17*c^17*d^13 - 17770700*a^1
3*b^17*c^19*d^11 + 6342200*a^13*b^17*c^21*d^9 - 1174200*a^13*b^17*c^23*d^7
+ 83220*a^13*b^17*c^25*d^5 - 1140*a^13*b^17*c^27*d^3 + 38760*a^14*b^16*c^6*
d^24 - 823650*a^14*b^16*c^8*d^22 + 5384410*a^14*b^16*c^10*d^20 - 17183600*a
^14*b^16*c^12*d^18 + 31460200*a^14*b^16*c^14*d^16 - 35234455*a^14*b^16*c^16
*d^14 + 24426875*a^14*b^16*c^18*d^12 - 10132510*a^14*b^16*c^20*d^10 + 22771
50*a^14*b^16*c^22*d^8 - 218025*a^14*b^16*c^24*d^6 + 4845*a^14*b^16*c^26*d^4
 - 15504*a^15*b^15*c^5*d^25 + 465120*a^15*b^15*c^7*d^23 - 3772640*a^15*b^15
*c^9*d^21 + 14108640*a^15*b^15*c^11*d^19 - 29535120*a^15*b^15*c^13*d^17 + 3
7499008*a^15*b^15*c^15*d^15 - 29535120*a^15*b^15*c^17*d^13 + 14108640*a^15*
b^15*c^19*d^11 - 3772640*a^15*b^15*c^21*d^9 + 465120*a^15*b^15*c^23*d^7 - 1
5504*a^15*b^15*c^25*d^5 + 4845*a^16*b^14*c^4*d^26 - 218025*a^16*b^14*c^6*d^
24 + 2277150*a^16*b^14*c^8*d^22 - 10132510*a^16*b^14*c^10*d^20 + 24426875*a
^16*b^14*c^12*d^18 - 35234455*a^16*b^14*c^14*d^16 + 31460200*a^16*b^14*c^16
*d^14 - 17183600*a^16*b^14*c^18*d^12 + 5384410*a^16*b^14*c^20*d^10 - 823650
*a^16*b^14*c^22*d^8 + 38760*a^16*b^14*c^24*d^6 - 1140*a^17*b^13*c^3*d^27 +
83220*a^17*b^13*c^5*d^25 - 1174200*a^17*b^13*c^7*d^23 + 6342200*a^17*b^13*c
^9*d^21 - 17770700*a^17*b^13*c^11*d^19 + 29213260*a^17*b^13*c^13*d^17 - 295
35120*a^17*b^13*c^15*d^15 + 18346400*a^17*b^13*c^17*d^13 - 6653800*a^17*b^1
3*c^19*d^11 + 1227400*a^17*b^13*c^21*d^9 - 77520*a^17*b^13*c^23*d^7 + 190*a
^18*b^12*c^2*d^28 - 25175*a^18*b^12*c^4*d^26 + 510625*a^18*b^12*c^6*d^24 -
3441850*a^18*b^12*c^8*d^22 + 11341480*a^18*b^12*c^10*d^20 - 21339185*a^18*b
^12*c^12*d^18 + 24426875*a^18*b^12*c^14*d^16 - 17183600*a^18*b^12*c^16*d^14
 + 7138300*a^18*b^12*c^18*d^12 - 1553630*a^18*b^12*c^20*d^10 + 125970*a^18*
b^12*c^22*d^8 + 5800*a^19*b^11*c^3*d^27 - 183740*a^19*b^11*c^5*d^25 + 16076
00*a^19*b^11*c^7*d^23 - 6323300*a^19*b^11*c^9*d^21 + 13697880*a^19*b^11*c^1
1*d^19 - 17770700*a^19*b^11*c^13*d^17 + 14108640*a^19*b^11*c^15*d^15 - 6653
800*a^19*b^11*c^17*d^13 + 1679600*a^19*b^11*c^19*d^11 - 167960*a^19*b^11*c^

21*d^9 - 955*a^20*b^10*c^2*d^28 + 53210*a^20*b^10*c^4*d^26 - 639360*a^20*b^10*c^6*d^24 + 3061855*a^20*b^10*c^8*d^22 - 7699257*a^20*b^10*c^10*d^20 + 11341480*a^20*b^10*c^12*d^18 - 10132510*a^20*b^10*c^14*d^16 + 5384410*a^20*b^10*c^16*d^14 - 1553630*a^20*b^10*c^18*d^12 + 184756*a^20*b^10*c^20*d^10 - 11900*a^21*b^9*c^3*d^27 + 213040*a^21*b^9*c^5*d^25 - 1277800*a^21*b^9*c^7*d^23 + 3770860*a^21*b^9*c^9*d^21 - 6323300*a^21*b^9*c^11*d^19 + 6342200*a^21*b^9*c^13*d^17 - 3772640*a^21*b^9*c^15*d^15 + 1227400*a^21*b^9*c^17*d^13 - 167960*a^21*b^9*c^19*d^11 + 1925*a^22*b^8*c^2*d^28 - 58000*a^22*b^8*c^4*d^26 + 455100*a^22*b^8*c^6*d^24 - 1598495*a^22*b^8*c^8*d^22 + 3061855*a^22*b^8*c^10*d^20 - 3441850*a^22*b^8*c^12*d^18 + 2277150*a^22*b^8*c^14*d^16 - 823650*a^22*b^8*c^16*d^14 + 125970*a^22*b^8*c^18*d^12 + 12400*a^23*b^7*c^3*d^27 - 136520*a^23*b^7*c^5*d^25 + 581120*a^23*b^7*c^7*d^23 - 1277800*a^23*b^7*c^9*d^21 + 1607600*a^23*b^7*c^11*d^19 - 1174200*a^23*b^7*c^13*d^17 + 465120*a^23*b^7*c^15*d^15 - 77520*a^23*b^7*c^17*d^13 - 1950*a^24*b^6*c^2*d^28 + 33825*a^24*b^6*c^4*d^26 - 178985*a^24*b^6*c^6*d^24 + 455100*a^24*b^6*c^8*d^22 - 639360*a^24*b^6*c^10*d^20 + 510625*a^24*b^6*c^12*d^18 - 218025*a^24*b^6*c^14*d^16 + 38760*a^24*b^6*c^16*d^14 - 6700*a^25*b^5*c^3*d^27 + 46004*a^25*b^5*c^5*d^25 - 136520*a^25*b^5*c^7*d^23 + 213040*a^25*b^5*c^9*d^21 - 183740*a^25*b^5*c^11*d^19 + 83220*a^25*b^5*c^13*d^17 - 15504*a^25*b^5*c^15*d^15 + 1000*a^26*b^4*c^2*d^28 - 9695*a^26*b^4*c^4*d^26 + 33825*a^26*b^4*c^6*d^24 - 58000*a^26*b^4*c^8*d^22 + 53210*a^26*b^4*c^10*d^20 - 25175*a^26*b^4*c^12*d^18 + 4845*a^26*b^4*c^14*d^16 + 1640*a^27*b^3*c^3*d^27 - 6700*a^27*b^3*c^5*d^25 + 12400*a^27*b^3*c^7*d^23 - 11900*a^27*b^3*c^9*d^21 + 5800*a^27*b^3*c^11*d^19 - 1140*a^27*b^3*c^13*d^17 - 215*a^28*b^2*c^2*d^28 + 1000*a^28*b^2*c^4*d^26 - 1950*a^28*b^2*c^6*d^24 + 1925*a^28*b^2*c^8*d^22 - 955*a^28*b^2*c^10*d^20 + 190*a^28*b^2*c^12*d^18 + 20*a*b^29*c^29*d + 20*a^29*b*c*d^29)))^(1/2)*(((4*(8*a^2*b^23*c^25 - 32*a^4*b^21*c^25 + 48*a^6*b^19*c^25 - 32*a^8*b^17*c^25 + 8*a^10*b^15*c^25 + 8*a^25*c^2*d^23 - 32*a^25*c^4*d^21 + 48*a^25*c^6*d^19 - 32*a^25*c^8*d^17 + 8*a^25*c^10*d^15 - 8*a*b^24*c^16*d^9 + 32*a*b^24*c^18*d^7 - 48*a*b^24*c^20*d^5 + 32*a*b^24*c^22*d^3 - 72*a^3*b^22*c^24*d + 368*a^5*b^20*c^24*d - 592*a^7*b^18*c^24*d + 408*a^9*b^16*c^24*d - 104*a^11*b^14*c^24*d - 8*a^16*b^9*c*d^24 + 32*a^18*b^7*c*d^24 - 48*a^20*b^5*c*d^24 + 32*a^22*b^3*c*d^24 - 72*a^24*b*c^3*d^22 + 368*a^24*b*c^5*d^20 - 592*a^24*b*c^7*d^18 + 408*a^24*b*c^9*d^16 - 104*a^24*b*c^11*d^14 + 104*a^2*b^23*c^15*d^10 - 408*a^2*b^23*c^17*d^8 + 592*a^2*b^23*c^19*d^6 - 368*a^2*b^23*c^21*d^4 + 72*a^2*b^23*c^23*d^2 - 616*a^3*b^22*c^14*d^11 + 2392*a^3*b^22*c^16*d^9 - 3408*a^3*b^22*c^18*d^7 + 2032*a^3*b^22*c^20*d^5 - 328*a^3*b^22*c^22*d^3 + 2184*a^4*b^21*c^13*d^12 - 8536*a^4*b^21*c^15*d^10 + 12272*a^4*b^21*c^17*d^8 - 7408*a^4*b^21*c^19*d^6 + 1192*a^4*b^21*c^21*d^4 + 328*a^4*b^21*c^23*d^2 - 5096*a^5*b^20*c^12*d^13 + 20664*a^5*b^20*c^14*d^11 - 31328*a^5*b^20*c^16*d^9 + 20592*a^5*b^20*c^18*d^7 - 4008*a^5*b^20*c^20*d^5 - 1192*a^5*b^20*c^22*d^3 + 8008*a^6*b^19*c^11*d^14 - 35672*a^6*b^19*c^13*d^12 + 60768*a^6*b^19*c^15*d^10 - 46464*a^6*b^19*c^17*d^8 + 11336*a^6*b^19*c^19*d^6 + 4008*a^6*b^19*c^21*d^4 - 2032*a^6*b^19*c^23*d^2 - 8008*a^7*b^18*c^10*d^15 + 44408*a^7*b^18*c^12*d^13 - 92512*a^7*b^18*c^14*d^11 + 85536*a^7*b^18*c^16*d^9 - 2

4904*a^7*b^18*c^18*d^7 - 11336*a^7*b^18*c^20*d^5 + 7408*a^7*b^18*c^22*d^3 +
 3432*a^8*b^17*c^9*d^16 - 37752*a^8*b^17*c^11*d^14 + 109408*a^8*b^17*c^13*d
^12 - 125472*a^8*b^17*c^15*d^10 + 42696*a^8*b^17*c^17*d^8 + 24904*a^8*b^17*
c^19*d^6 - 20592*a^8*b^17*c^21*d^4 + 3408*a^8*b^17*c^23*d^2 + 3432*a^9*b^16
*c^8*d^17 + 14872*a^9*b^16*c^10*d^15 - 92352*a^9*b^16*c^12*d^13 + 141408*a^
9*b^16*c^14*d^11 - 59264*a^9*b^16*c^16*d^9 - 42696*a^9*b^16*c^18*d^7 + 4646
4*a^9*b^16*c^20*d^5 - 12272*a^9*b^16*c^22*d^3 - 8008*a^10*b^15*c^7*d^18 + 1
4872*a^10*b^15*c^9*d^16 + 36608*a^10*b^15*c^11*d^14 - 113152*a^10*b^15*c^13
*d^12 + 67008*a^10*b^15*c^15*d^10 + 59264*a^10*b^15*c^17*d^8 - 85536*a^10*b
^15*c^19*d^6 + 31328*a^10*b^15*c^21*d^4 - 2392*a^10*b^15*c^23*d^2 + 8008*a^
11*b^14*c^6*d^19 - 37752*a^11*b^14*c^8*d^17 + 36608*a^11*b^14*c^10*d^15 + 4
3264*a^11*b^14*c^12*d^13 - 56256*a^11*b^14*c^14*d^11 - 67008*a^11*b^14*c^16
*d^9 + 125472*a^11*b^14*c^18*d^7 - 60768*a^11*b^14*c^20*d^5 + 8536*a^11*b^1
4*c^22*d^3 - 5096*a^12*b^13*c^5*d^20 + 44408*a^12*b^13*c^7*d^18 - 92352*a^1
2*b^13*c^9*d^16 + 43264*a^12*b^13*c^11*d^14 + 22464*a^12*b^13*c^13*d^12 + 5
6256*a^12*b^13*c^15*d^10 - 141408*a^12*b^13*c^17*d^8 + 92512*a^12*b^13*c^19
*d^6 - 20664*a^12*b^13*c^21*d^4 + 616*a^12*b^13*c^23*d^2 + 2184*a^13*b^12*c
^4*d^21 - 35672*a^13*b^12*c^6*d^19 + 109408*a^13*b^12*c^8*d^17 - 113152*a^1
3*b^12*c^10*d^15 + 22464*a^13*b^12*c^12*d^13 - 22464*a^13*b^12*c^14*d^11 +
113152*a^13*b^12*c^16*d^9 - 109408*a^13*b^12*c^18*d^7 + 35672*a^13*b^12*c^2
0*d^5 - 2184*a^13*b^12*c^22*d^3 - 616*a^14*b^11*c^3*d^22 + 20664*a^14*b^11*
c^5*d^20 - 92512*a^14*b^11*c^7*d^18 + 141408*a^14*b^11*c^9*d^16 - 56256*a^1
4*b^11*c^11*d^14 - 22464*a^14*b^11*c^13*d^12 - 43264*a^14*b^11*c^15*d^10 +
92352*a^14*b^11*c^17*d^8 - 44408*a^14*b^11*c^19*d^6 + 5096*a^14*b^11*c^21*d
^4 + 104*a^15*b^10*c^2*d^23 - 8536*a^15*b^10*c^4*d^21 + 60768*a^15*b^10*c^6
*d^19 - 125472*a^15*b^10*c^8*d^17 + 67008*a^15*b^10*c^10*d^15 + 56256*a^15*
b^10*c^12*d^13 - 43264*a^15*b^10*c^14*d^11 - 36608*a^15*b^10*c^16*d^9 + 377
52*a^15*b^10*c^18*d^7 - 8008*a^15*b^10*c^20*d^5 + 2392*a^16*b^9*c^3*d^22 -
31328*a^16*b^9*c^5*d^20 + 85536*a^16*b^9*c^7*d^18 - 59264*a^16*b^9*c^9*d^16
 - 67008*a^16*b^9*c^11*d^14 + 113152*a^16*b^9*c^13*d^12 - 36608*a^16*b^9*c^
15*d^10 - 14872*a^16*b^9*c^17*d^8 + 8008*a^16*b^9*c^19*d^6 - 408*a^17*b^8*c
^2*d^23 + 12272*a^17*b^8*c^4*d^21 - 46464*a^17*b^8*c^6*d^19 + 42696*a^17*b^
8*c^8*d^17 + 59264*a^17*b^8*c^10*d^15 - 141408*a^17*b^8*c^12*d^13 + 92352*a
^17*b^8*c^14*d^11 - 14872*a^17*b^8*c^16*d^9 - 3432*a^17*b^8*c^18*d^7 - 3408
*a^18*b^7*c^3*d^22 + 20592*a^18*b^7*c^5*d^20 - 24904*a^18*b^7*c^7*d^18 - 42
696*a^18*b^7*c^9*d^16 + 125472*a^18*b^7*c^11*d^14 - 109408*a^18*b^7*c^13*d^
12 + 37752*a^18*b^7*c^15*d^10 - 3432*a^18*b^7*c^17*d^8 + 592*a^19*b^6*c^2*d
^23 - 7408*a^19*b^6*c^4*d^21 + 11336*a^19*b^6*c^6*d^19 + 24904*a^19*b^6*c^8
*d^17 - 85536*a^19*b^6*c^10*d^15 + 92512*a^19*b^6*c^12*d^13 - 44408*a^19*b^
6*c^14*d^11 + 8008*a^19*b^6*c^16*d^9 + 2032*a^20*b^5*c^3*d^22 - 4008*a^20*b
^5*c^5*d^20 - 11336*a^20*b^5*c^7*d^18 + 46464*a^20*b^5*c^9*d^16 - 60768*a^2
0*b^5*c^11*d^14 + 35672*a^20*b^5*c^13*d^12 - 8008*a^20*b^5*c^15*d^10 - 368*
a^21*b^4*c^2*d^23 + 1192*a^21*b^4*c^4*d^21 + 4008*a^21*b^4*c^6*d^19 - 20592
*a^21*b^4*c^8*d^17 + 31328*a^21*b^4*c^10*d^15 - 20664*a^21*b^4*c^12*d^13 +
5096*a^21*b^4*c^14*d^11 - 328*a^22*b^3*c^3*d^22 - 1192*a^22*b^3*c^5*d^20 +

7408*a^22*b^3*c^7*d^18 - 12272*a^22*b^3*c^9*d^16 + 8536*a^22*b^3*c^11*d^14 - 2184*a^22*b^3*c^13*d^12 + 72*a^23*b^2*c^2*d^23 + 328*a^23*b^2*c^4*d^21 - 2032*a^23*b^2*c^6*d^19 + 3408*a^23*b^2*c^8*d^17 - 2392*a^23*b^2*c^10*d^15 + 616*a^23*b^2*c^12*d^13 - 8*a*b^24*c^24*d - 8*a^24*b*c*d^24))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a

^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a
^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^
17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^1
7*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^
2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^
10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19) - (8*tan(e/2 + (f*x)/2)*(56*
a^3*b^22*c^25 - 12*a^25*c*d^24 - 12*a*b^24*c^25 - 104*a^5*b^20*c^25 + 96*a^
7*b^18*c^25 - 44*a^9*b^16*c^25 + 8*a^11*b^14*c^25 + 56*a^25*c^3*d^22 - 104*
a^25*c^5*d^20 + 96*a^25*c^7*d^18 - 44*a^25*c^9*d^16 + 8*a^25*c^11*d^14 + 16
*a*b^24*c^15*d^10 - 76*a*b^24*c^17*d^8 + 144*a*b^24*c^19*d^6 - 136*a*b^24*c
^21*d^4 + 64*a*b^24*c^23*d^2 + 168*a^2*b^23*c^24*d - 784*a^4*b^21*c^24*d +
1456*a^6*b^19*c^24*d - 1344*a^8*b^17*c^24*d + 616*a^10*b^15*c^24*d - 112*a^
12*b^13*c^24*d + 16*a^15*b^10*c*d^24 - 76*a^17*b^8*c*d^24 + 144*a^19*b^6*c*
d^24 - 136*a^21*b^4*c*d^24 + 64*a^23*b^2*c*d^24 + 168*a^24*b*c^2*d^23 - 784
*a^24*b*c^4*d^21 + 1456*a^24*b*c^6*d^19 - 1344*a^24*b*c^8*d^17 + 616*a^24*b
*c^10*d^15 - 112*a^24*b*c^12*d^13 - 224*a^2*b^23*c^14*d^11 + 1064*a^2*b^23*
c^16*d^9 - 2016*a^2*b^23*c^18*d^7 + 1904*a^2*b^23*c^20*d^5 - 896*a^2*b^23*c
^22*d^3 + 1456*a^3*b^22*c^13*d^12 - 6992*a^3*b^22*c^15*d^10 + 13464*a^3*b^2
2*c^17*d^8 - 13056*a^3*b^22*c^19*d^6 + 6464*a^3*b^22*c^21*d^4 - 1392*a^3*b^
22*c^23*d^2 - 5824*a^4*b^21*c^12*d^13 + 28728*a^4*b^21*c^14*d^11 - 57456*a^
4*b^21*c^16*d^9 + 59024*a^4*b^21*c^18*d^7 - 32256*a^4*b^21*c^20*d^5 + 8568*
a^4*b^21*c^22*d^3 + 16016*a^5*b^20*c^11*d^14 - 82992*a^5*b^20*c^13*d^12 + 1
77048*a^5*b^20*c^15*d^10 - 198696*a^5*b^20*c^17*d^8 + 123584*a^5*b^20*c^19*
d^6 - 40512*a^5*b^20*c^21*d^4 + 5656*a^5*b^20*c^23*d^2 - 32032*a^6*b^19*c^1
0*d^15 + 179816*a^6*b^19*c^12*d^13 - 421344*a^6*b^19*c^14*d^11 + 529312*a^6
*b^19*c^16*d^9 - 379008*a^6*b^19*c^18*d^7 + 150024*a^6*b^19*c^20*d^5 - 2822
4*a^6*b^19*c^22*d^3 + 48048*a^7*b^18*c^9*d^16 - 304304*a^7*b^18*c^11*d^14 +
805896*a^7*b^18*c^13*d^12 - 1151104*a^7*b^18*c^15*d^10 + 949952*a^7*b^18*c
^17*d^8 - 446736*a^7*b^18*c^19*d^6 + 108136*a^7*b^18*c^21*d^4 - 9984*a^7*b^
18*c^23*d^2 - 54912*a^8*b^17*c^8*d^17 + 412984*a^8*b^17*c^10*d^15 - 1267344
*a^8*b^17*c^12*d^13 + 2077536*a^8*b^17*c^14*d^11 - 1975808*a^8*b^17*c^16*d^
9 + 1095384*a^8*b^17*c^18*d^7 - 331632*a^8*b^17*c^20*d^5 + 45136*a^8*b^17*c
^22*d^3 + 48048*a^9*b^16*c^7*d^18 - 456456*a^9*b^16*c^9*d^16 + 1657656*a^9*
b^16*c^11*d^14 - 3143504*a^9*b^16*c^13*d^12 + 3453696*a^9*b^16*c^15*d^10 -
2247636*a^9*b^16*c^17*d^8 + 831208*a^9*b^16*c^19*d^6 - 151944*a^9*b^16*c^21
*d^4 + 8976*a^9*b^16*c^23*d^2 - 32032*a^10*b^15*c^6*d^19 + 412984*a^10*b^15
*c^8*d^17 - 1812096*a^10*b^15*c^10*d^15 + 4016896*a^10*b^15*c^12*d^13 - 512
1024*a^10*b^15*c^14*d^11 + 3897024*a^10*b^15*c^16*d^9 - 1728832*a^10*b^15*c
^18*d^7 + 404768*a^10*b^15*c^20*d^5 - 38304*a^10*b^15*c^22*d^3 + 16016*a^11
*b^14*c^5*d^20 - 304304*a^11*b^14*c^7*d^18 + 1657656*a^11*b^14*c^9*d^16 - 4
356352*a^11*b^14*c^11*d^14 + 6476288*a^11*b^14*c^13*d^12 - 5745024*a^11*b^1
4*c^15*d^10 + 3021984*a^11*b^14*c^17*d^8 - 880256*a^11*b^14*c^19*d^6 + 1180
32*a^11*b^14*c^21*d^4 - 4048*a^11*b^14*c^23*d^2 - 5824*a^12*b^13*c^4*d^21 +
179816*a^12*b^13*c^6*d^19 - 1267344*a^12*b^13*c^8*d^17 + 4016896*a^12*b^13
*c^10*d^15 - 7002112*a^12*b^13*c^12*d^13 + 7235136*a^12*b^13*c^14*d^11 - 44

80896*a^12*b^13*c^16*d^9 + 1588704*a^12*b^13*c^18*d^7 - 280896*a^12*b^13*c^20*d^5 + 16632*a^12*b^13*c^22*d^3 + 1456*a^13*b^12*c^3*d^22 - 82992*a^13*b^12*c^5*d^20 + 805896*a^13*b^12*c^7*d^18 - 3143504*a^13*b^12*c^9*d^16 + 6476288*a^13*b^12*c^11*d^14 - 7809984*a^13*b^12*c^13*d^12 + 5666752*a^13*b^12*c^15*d^10 - 2403856*a^13*b^12*c^17*d^8 + 537264*a^13*b^12*c^19*d^6 - 48048*a^13*b^12*c^21*d^4 + 728*a^13*b^12*c^23*d^2 - 224*a^14*b^11*c^2*d^23 + 28728*a^14*b^11*c^4*d^21 - 421344*a^14*b^11*c^6*d^19 + 2077536*a^14*b^11*c^8*d^17 - 5121024*a^14*b^11*c^10*d^15 + 7235136*a^14*b^11*c^12*d^13 - 6126848*a^14*b^11*c^14*d^11 + 3071744*a^14*b^11*c^16*d^9 - 844896*a^14*b^11*c^18*d^7 + 104104*a^14*b^11*c^20*d^5 - 2912*a^14*b^11*c^22*d^3 - 6992*a^15*b^10*c^3*d^22 + 177048*a^15*b^10*c^5*d^20 - 1151104*a^15*b^10*c^7*d^18 + 3453696*a^15*b^10*c^9*d^16 - 5745024*a^15*b^10*c^11*d^14 + 5666752*a^15*b^10*c^13*d^12 - 3331328*a^15*b^10*c^15*d^10 + 1105104*a^15*b^10*c^17*d^8 - 176176*a^15*b^10*c^19*d^6 + 8008*a^15*b^10*c^21*d^4 + 1064*a^16*b^9*c^2*d^23 - 57456*a^16*b^9*c^4*d^21 + 529312*a^16*b^9*c^6*d^19 - 1975808*a^16*b^9*c^8*d^17 + 3897024*a^16*b^9*c^10*d^15 - 4480896*a^16*b^9*c^12*d^13 + 3071744*a^16*b^9*c^14*d^11 - 1208064*a^16*b^9*c^16*d^9 + 239096*a^16*b^9*c^18*d^7 - 16016*a^16*b^9*c^20*d^5 + 13464*a^17*b^8*c^3*d^22 - 198696*a^17*b^8*c^5*d^20 + 949952*a^17*b^8*c^7*d^18 - 2247636*a^17*b^8*c^9*d^16 + 3021984*a^17*b^8*c^11*d^14 - 2403856*a^17*b^8*c^13*d^12 + 1105104*a^17*b^8*c^15*d^10 - 264264*a^17*b^8*c^17*d^8 + 24024*a^17*b^8*c^19*d^6 - 2016*a^18*b^7*c^2*d^23 + 59024*a^18*b^7*c^4*d^21 - 379008*a^18*b^7*c^6*d^19 + 1095384*a^18*b^7*c^8*d^17 - 1728832*a^18*b^7*c^10*d^15 + 1588704*a^18*b^7*c^12*d^13 - 844896*a^18*b^7*c^14*d^11 + 239096*a^18*b^7*c^16*d^9 - 27456*a^18*b^7*c^18*d^7 - 13056*a^19*b^6*c^3*d^22 + 123584*a^19*b^6*c^5*d^20 - 446736*a^19*b^6*c^7*d^18 + 831208*a^19*b^6*c^9*d^16 - 880256*a^19*b^6*c^11*d^14 + 537264*a^19*b^6*c^13*d^12 - 176176*a^19*b^6*c^15*d^10 + 24024*a^19*b^6*c^17*d^8 + 1904*a^20*b^5*c^2*d^23 - 32256*a^20*b^5*c^4*d^21 + 150024*a^20*b^5*c^6*d^19 - 331632*a^20*b^5*c^8*d^17 + 404768*a^20*b^5*c^10*d^15 - 280896*a^20*b^5*c^12*d^13 + 104104*a^20*b^5*c^14*d^11 - 16016*a^20*b^5*c^16*d^9 + 6464*a^21*b^4*c^3*d^22 - 40512*a^21*b^4*c^5*d^20 + 108136*a^21*b^4*c^7*d^18 - 151944*a^21*b^4*c^9*d^16 + 118032*a^21*b^4*c^11*d^14 - 48048*a^21*b^4*c^13*d^12 + 8008*a^21*b^4*c^15*d^10 - 896*a^22*b^3*c^2*d^23 + 8568*a^22*b^3*c^4*d^21 - 28224*a^22*b^3*c^6*d^19 + 45136*a^22*b^3*c^8*d^17 - 38304*a^22*b^3*c^10*d^15 + 16632*a^22*b^3*c^12*d^13 - 2912*a^22*b^3*c^14*d^11 - 1392*a^23*b^2*c^3*d^22 + 5656*a^23*b^2*c^5*d^20 - 9984*a^23*b^2*c^7*d^18 + 8976*a^23*b^2*c^9*d^16 - 4048*a^23*b^2*c^11*d^14 + 728*a^23*b^2*c^13*d^12))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66

*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a
^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b
^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^
17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^
16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^1
6*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15
*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15
*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14
*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b
^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^1
3*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*
b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b
^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8
*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*
a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^
9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936
*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928
*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 1386
0*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10
 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^1
6*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d
^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^
11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^
17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^
6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^
8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7
*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b
^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b
^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b
^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^1
4*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15
*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15
*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b
^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b
^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^
3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*
c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*
d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 1
2*a^19*b*c*d^19))*((((4*a^24*d^24 + 4*b^24*c^24 + 16*a^2*b^22*c^24 + 16*a^4
*b^20*c^24 - 1152*a^10*b^14*d^24 + 5568*a^12*b^12*d^24 - 10568*a^14*b^10*d^
24 + 9460*a^16*b^8*d^24 - 3560*a^18*b^6*d^24 + 136*a^20*b^4*d^24 + 76*a^22*
b^2*d^24 + 16*a^24*c^2*d^22 + 16*a^24*c^4*d^20 - 1152*b^24*c^10*d^14 + 5568
*b^24*c^12*d^12 - 10568*b^24*c^14*d^10 + 9460*b^24*c^16*d^8 - 3560*b^24*c^1
8*d^6 + 136*b^24*c^20*d^4 + 76*b^24*c^22*d^2 + 11520*a*b^23*c^9*d^15 - 5644
8*a*b^23*c^11*d^13 + 109456*a*b^23*c^13*d^11 - 101240*a*b^23*c^15*d^9 + 407

20*a*b^23*c^17*d^7 - 2960*a*b^23*c^19*d^5 - 536*a*b^23*c^21*d^3 - 176*a^3*b
^21*c^23*d - 320*a^5*b^19*c^23*d + 11520*a^9*b^15*c*d^23 - 56448*a^11*b^13*
c*d^23 + 109456*a^13*b^11*c*d^23 - 101240*a^15*b^9*c*d^23 + 40720*a^17*b^7*
c*d^23 - 2960*a^19*b^5*c*d^23 - 536*a^21*b^3*c*d^23 - 176*a^23*b*c^3*d^21 -
 320*a^23*b*c^5*d^19 - 51840*a^2*b^22*c^8*d^16 + 263808*a^2*b^22*c^10*d^14
- 541208*a^2*b^22*c^12*d^12 + 547088*a^2*b^22*c^14*d^10 - 263320*a^2*b^22*c
^16*d^8 + 44120*a^2*b^22*c^18*d^6 - 1564*a^2*b^22*c^20*d^4 - 196*a^2*b^22*c
^22*d^2 + 138240*a^3*b^21*c^7*d^17 - 758400*a^3*b^21*c^9*d^15 + 1720736*a^3
*b^21*c^11*d^13 - 2002728*a^3*b^21*c^13*d^11 + 1210560*a^3*b^21*c^15*d^9 -
335040*a^3*b^21*c^17*d^7 + 37680*a^3*b^21*c^19*d^5 - 288*a^3*b^21*c^21*d^3
- 241920*a^4*b^20*c^6*d^18 + 1512000*a^4*b^20*c^8*d^16 - 3975688*a^4*b^20*c
^10*d^14 + 5501328*a^4*b^20*c^12*d^12 - 4147952*a^4*b^20*c^14*d^10 + 158692
0*a^4*b^20*c^16*d^8 - 276020*a^4*b^20*c^18*d^6 + 21124*a^4*b^20*c^20*d^4 +
176*a^4*b^20*c^22*d^2 + 290304*a^5*b^19*c^5*d^19 - 2232576*a^5*b^19*c^7*d^1
7 + 7078256*a^5*b^19*c^9*d^15 - 11781560*a^5*b^19*c^11*d^13 + 10875200*a^5*
b^19*c^13*d^11 - 5365072*a^5*b^19*c^15*d^9 + 1310168*a^5*b^19*c^17*d^7 - 17
0968*a^5*b^19*c^19*d^5 + 8160*a^5*b^19*c^21*d^3 - 241920*a^6*b^18*c^4*d^20
+ 2532096*a^6*b^18*c^6*d^18 - 9955992*a^6*b^18*c^8*d^16 + 20019440*a^6*b^18
*c^10*d^14 - 22419600*a^6*b^18*c^12*d^12 + 13887520*a^6*b^18*c^14*d^10 - 45
06428*a^6*b^18*c^16*d^8 + 793756*a^6*b^18*c^18*d^6 - 72240*a^6*b^18*c^20*d^
4 + 3040*a^6*b^18*c^22*d^2 + 138240*a^7*b^17*c^3*d^21 - 2232576*a^7*b^17*c^
5*d^19 + 11150016*a^7*b^17*c^7*d^17 - 27336616*a^7*b^17*c^9*d^15 + 37153600
*a^7*b^17*c^11*d^13 - 28461040*a^7*b^17*c^13*d^11 + 11779808*a^7*b^17*c^15*
d^9 - 2621008*a^7*b^17*c^17*d^7 + 336688*a^7*b^17*c^19*d^5 - 17920*a^7*b^17
*c^21*d^3 - 51840*a^8*b^16*c^2*d^22 + 1512000*a^8*b^16*c^4*d^20 - 9955992*a
^8*b^16*c^6*d^18 + 30289656*a^8*b^16*c^8*d^16 - 50137600*a^8*b^16*c^10*d^14
 + 46972560*a^8*b^16*c^12*d^12 - 24199280*a^8*b^16*c^14*d^10 + 6661036*a^8*
b^16*c^16*d^8 - 1058448*a^8*b^16*c^18*d^6 + 72560*a^8*b^16*c^20*d^4 - 75840
0*a^9*b^15*c^3*d^21 + 7078256*a^9*b^15*c^5*d^19 - 27336616*a^9*b^15*c^7*d^1
7 + 55383904*a^9*b^15*c^9*d^15 - 63124080*a^9*b^15*c^11*d^13 + 39987520*a^9
*b^15*c^13*d^11 - 13462088*a^9*b^15*c^15*d^9 + 2478528*a^9*b^15*c^17*d^7 -
212032*a^9*b^15*c^19*d^5 + 263808*a^10*b^14*c^2*d^22 - 3975688*a^10*b^14*c^
4*d^20 + 20019440*a^10*b^14*c^6*d^18 - 50137600*a^10*b^14*c^8*d^16 + 695938
72*a^10*b^14*c^10*d^14 - 53854288*a^10*b^14*c^12*d^12 + 21989928*a^10*b^14*
c^14*d^10 - 4591360*a^10*b^14*c^16*d^8 + 460480*a^10*b^14*c^18*d^6 + 172073
6*a^11*b^13*c^3*d^21 - 11781560*a^11*b^13*c^5*d^19 + 37153600*a^11*b^13*c^7
*d^17 - 63124080*a^11*b^13*c^9*d^15 + 59445728*a^11*b^13*c^11*d^13 - 293586
96*a^11*b^13*c^13*d^11 + 6995840*a^11*b^13*c^15*d^9 - 762560*a^11*b^13*c^17
*d^7 - 541208*a^12*b^12*c^2*d^22 + 5501328*a^12*b^12*c^4*d^20 - 22419600*a^
12*b^12*c^6*d^18 + 46972560*a^12*b^12*c^8*d^16 - 53854288*a^12*b^12*c^10*d^
14 + 32294808*a^12*b^12*c^12*d^12 - 8958208*a^12*b^12*c^14*d^10 + 999040*a^
12*b^12*c^16*d^8 - 2002728*a^13*b^11*c^3*d^21 + 10875200*a^13*b^11*c^5*d^19
 - 28461040*a^13*b^11*c^7*d^17 + 39987520*a^13*b^11*c^9*d^15 - 29358696*a^1
3*b^11*c^11*d^13 + 9722048*a^13*b^11*c^13*d^11 - 1104320*a^13*b^11*c^15*d^9
 + 547088*a^14*b^10*c^2*d^22 - 4147952*a^14*b^10*c^4*d^20 + 13887520*a^14*b

^10*c^6*d^18 - 24199280*a^14*b^10*c^8*d^16 + 21989928*a^14*b^10*c^10*d^14 - 8958208*a^14*b^10*c^12*d^12 + 1124032*a^14*b^10*c^14*d^10 + 1210560*a^15*b^9*c^3*d^21 - 5365072*a^15*b^9*c^5*d^19 + 11779808*a^15*b^9*c^7*d^17 - 13462088*a^15*b^9*c^9*d^15 + 6995840*a^15*b^9*c^11*d^13 - 1104320*a^15*b^9*c^13*d^11 - 263320*a^16*b^8*c^2*d^22 + 1586920*a^16*b^8*c^4*d^20 - 4506428*a^16*b^8*c^6*d^18 + 6661036*a^16*b^8*c^8*d^16 - 4591360*a^16*b^8*c^10*d^14 + 999040*a^16*b^8*c^12*d^12 - 335040*a^17*b^7*c^3*d^21 + 1310168*a^17*b^7*c^5*d^19 - 2621008*a^17*b^7*c^7*d^17 + 2478528*a^17*b^7*c^9*d^15 - 762560*a^17*b^7*c^11*d^13 + 44120*a^18*b^6*c^2*d^22 - 276020*a^18*b^6*c^4*d^20 + 793756*a^18*b^6*c^6*d^18 - 1058448*a^18*b^6*c^8*d^16 + 460480*a^18*b^6*c^10*d^14 + 37680*a^19*b^5*c^3*d^21 - 170968*a^19*b^5*c^5*d^19 + 336688*a^19*b^5*c^7*d^17 - 212032*a^19*b^5*c^9*d^15 - 1564*a^20*b^4*c^2*d^22 + 21124*a^20*b^4*c^4*d^20 - 72240*a^20*b^4*c^6*d^18 + 72560*a^20*b^4*c^8*d^16 - 288*a^21*b^3*c^3*d^21 + 8160*a^21*b^3*c^5*d^19 - 17920*a^21*b^3*c^7*d^17 - 196*a^22*b^2*c^2*d^22 + 176*a^22*b^2*c^4*d^20 + 3040*a^22*b^2*c^6*d^18 - 8*a*b^23*c^23*d - 8*a^23*b*c*d^23)^2/4 - (20736*b^18*d^18 - 96768*a^2*b^16*d^18 + 173664*a^4*b^14*d^18 - 136032*a^6*b^12*d^18 + 31081*a^8*b^10*d^18 + 8440*a^10*b^8*d^18 + 400*a^12*b^6*d^18 - 96768*b^18*c^2*d^16 + 173664*b^18*c^4*d^14 - 136032*b^18*c^6*d^12 + 31081*b^18*c^8*d^10 + 8440*b^18*c^10*d^8 + 400*b^18*c^12*d^6 - 131328*a*b^17*c^3*d^15 + 216576*a*b^17*c^5*d^13 - 141104*a*b^17*c^7*d^11 + 20260*a*b^17*c^9*d^9 + 2800*a*b^17*c^11*d^7 - 131328*a^3*b^15*c*d^17 + 216576*a^5*b^13*c*d^17 - 141104*a^7*b^11*c*d^17 + 20260*a^9*b^9*c*d^17 + 2800*a^11*b^7*c*d^17 + 495936*a^2*b^16*c^2*d^16 - 989856*a^2*b^16*c^4*d^14 + 901948*a^2*b^16*c^6*d^12 - 308392*a^2*b^16*c^8*d^10 - 5260*a^2*b^16*c^10*d^8 + 1600*a^2*b^16*c^12*d^6 + 657408*a^3*b^15*c^3*d^15 - 1158992*a^3*b^15*c^5*d^13 + 838256*a^3*b^15*c^7*d^11 - 182200*a^3*b^15*c^9*d^9 - 3200*a^3*b^15*c^11*d^7 - 989856*a^4*b^14*c^2*d^16 + 2185654*a^4*b^14*c^4*d^14 - 2218576*a^4*b^14*c^6*d^12 + 900624*a^4*b^14*c^8*d^10 - 64720*a^4*b^14*c^10*d^8 + 1600*a^4*b^14*c^12*d^6 - 1158992*a^5*b^13*c^3*d^15 + 2158808*a^5*b^13*c^5*d^13 - 1641528*a^5*b^13*c^7*d^11 + 406880*a^5*b^13*c^9*d^9 - 17600*a^5*b^13*c^11*d^7 + 901948*a^6*b^12*c^2*d^16 - 2218576*a^6*b^12*c^4*d^14 + 2430936*a^6*b^12*c^6*d^12 - 1026928*a^6*b^12*c^8*d^10 + 88720*a^6*b^12*c^10*d^8 + 838256*a^7*b^11*c^3*d^15 - 1641528*a^7*b^11*c^5*d^13 + 1206848*a^7*b^11*c^7*d^11 - 239360*a^7*b^11*c^9*d^9 - 308392*a^8*b^10*c^2*d^16 + 900624*a^8*b^10*c^4*d^14 - 1026928*a^8*b^10*c^6*d^12 + 354016*a^8*b^10*c^8*d^10 - 182200*a^9*b^9*c^3*d^15 + 406880*a^9*b^9*c^5*d^13 - 239360*a^9*b^9*c^7*d^11 - 5260*a^10*b^8*c^2*d^16 - 64720*a^10*b^8*c^4*d^14 + 88720*a^10*b^8*c^6*d^12 - 3200*a^11*b^7*c^3*d^15 - 17600*a^11*b^7*c^5*d^13 + 1600*a^12*b^6*c^2*d^16 + 1600*a^12*b^6*c^4*d^14 + 27648*a*b^17*c*d^17)*(80*a^2*b^28*c^30 - 16*b^30*c^30 - 16*a^30*d^30 - 160*a^4*b^26*c^30 + 160*a^6*b^24*c^30 - 80*a^8*b^22*c^30 + 16*a^10*b^20*c^30 + 16*a^20*b^10*d^30 - 80*a^22*b^8*d^30 + 160*a^24*b^6*d^30 - 160*a^26*b^4*d^30 + 80*a^28*b^2*d^30 + 80*a^30*c^2*d^28 - 160*a^30*c^4*d^26 + 160*a^30*c^6*d^24 - 80*a^30*c^8*d^22 + 16*a^30*c^10*d^20 + 16*b^30*c^20*d^10 - 80*b^30*c^22*d^8 + 160*b^30*c^24*d^6 - 160*b^30*c^26*d^4 + 80*b^30*c^28*d^2 - 320*a*b^29*c^19*d^11 + 1600*a*b^29*c^21*d^9 - 3200*a*b^29*c

^23*d^7 + 3200*a*b^29*c^25*d^5 - 1600*a*b^29*c^27*d^3 - 1600*a^3*b^27*c^29*
d + 3200*a^5*b^25*c^29*d - 3200*a^7*b^23*c^29*d + 1600*a^9*b^21*c^29*d - 32
0*a^11*b^19*c^29*d - 320*a^19*b^11*c*d^29 + 1600*a^21*b^9*c*d^29 - 3200*a^2
3*b^7*c*d^29 + 3200*a^25*b^5*c*d^29 - 1600*a^27*b^3*c*d^29 - 1600*a^29*b*c^
3*d^27 + 3200*a^29*b*c^5*d^25 - 3200*a^29*b*c^7*d^23 + 1600*a^29*b*c^9*d^21
 - 320*a^29*b*c^11*d^19 + 3040*a^2*b^28*c^18*d^12 - 15280*a^2*b^28*c^20*d^1
0 + 30800*a^2*b^28*c^22*d^8 - 31200*a^2*b^28*c^24*d^6 + 16000*a^2*b^28*c^26
*d^4 - 3440*a^2*b^28*c^28*d^2 - 18240*a^3*b^27*c^17*d^13 + 92800*a^3*b^27*c
^19*d^11 - 190400*a^3*b^27*c^21*d^9 + 198400*a^3*b^27*c^23*d^7 - 107200*a^3
*b^27*c^25*d^5 + 26240*a^3*b^27*c^27*d^3 + 77520*a^4*b^26*c^16*d^14 - 40280
0*a^4*b^26*c^18*d^12 + 851360*a^4*b^26*c^20*d^10 - 928000*a^4*b^26*c^22*d^8
 + 541200*a^4*b^26*c^24*d^6 - 155120*a^4*b^26*c^26*d^4 + 16000*a^4*b^26*c^2
8*d^2 - 248064*a^5*b^25*c^15*d^15 + 1331520*a^5*b^25*c^17*d^13 - 2939840*a^
5*b^25*c^19*d^11 + 3408640*a^5*b^25*c^21*d^9 - 2184320*a^5*b^25*c^23*d^7 +
736064*a^5*b^25*c^25*d^5 - 107200*a^5*b^25*c^27*d^3 + 620160*a^6*b^24*c^14*
d^16 - 3488400*a^6*b^24*c^16*d^14 + 8170000*a^6*b^24*c^18*d^12 - 10229760*a
^6*b^24*c^20*d^10 + 7281600*a^6*b^24*c^22*d^8 - 2863760*a^6*b^24*c^24*d^6 +
 541200*a^6*b^24*c^26*d^4 - 31200*a^6*b^24*c^28*d^2 - 1240320*a^7*b^23*c^13
*d^17 + 7441920*a^7*b^23*c^15*d^15 - 18787200*a^7*b^23*c^17*d^13 + 25721600
*a^7*b^23*c^19*d^11 - 20444800*a^7*b^23*c^21*d^9 + 9297920*a^7*b^23*c^23*d^
7 - 2184320*a^7*b^23*c^25*d^5 + 198400*a^7*b^23*c^27*d^3 + 2015520*a^8*b^22
*c^12*d^18 - 13178400*a^8*b^22*c^14*d^16 + 36434400*a^8*b^22*c^16*d^14 - 55
069600*a^8*b^22*c^18*d^12 + 48989680*a^8*b^22*c^20*d^10 - 25575920*a^8*b^22
*c^22*d^8 + 7281600*a^8*b^22*c^24*d^6 - 928000*a^8*b^22*c^26*d^4 + 30800*a^
8*b^22*c^28*d^2 - 2687360*a^9*b^21*c^11*d^19 + 19638400*a^9*b^21*c^13*d^17
- 60362240*a^9*b^21*c^15*d^15 + 101475200*a^9*b^21*c^17*d^13 - 101172800*a^
9*b^21*c^19*d^11 + 60333760*a^9*b^21*c^21*d^9 - 20444800*a^9*b^21*c^23*d^7
+ 3408640*a^9*b^21*c^25*d^5 - 190400*a^9*b^21*c^27*d^3 + 2956096*a^10*b^20*
c^10*d^20 - 24858080*a^10*b^20*c^12*d^18 + 86150560*a^10*b^20*c^14*d^16 - 1
62120160*a^10*b^20*c^16*d^14 + 181463680*a^10*b^20*c^18*d^12 - 123188112*a^
10*b^20*c^20*d^10 + 48989680*a^10*b^20*c^22*d^8 - 10229760*a^10*b^20*c^24*d
^6 + 851360*a^10*b^20*c^26*d^4 - 15280*a^10*b^20*c^28*d^2 - 2687360*a^11*b^
19*c^9*d^21 + 26873600*a^11*b^19*c^11*d^19 - 106460800*a^11*b^19*c^13*d^17
+ 225738240*a^11*b^19*c^15*d^15 - 284331200*a^11*b^19*c^17*d^13 + 219166080
*a^11*b^19*c^19*d^11 - 101172800*a^11*b^19*c^21*d^9 + 25721600*a^11*b^19*c^
23*d^7 - 2939840*a^11*b^19*c^25*d^5 + 92800*a^11*b^19*c^27*d^3 + 2015520*a^
12*b^18*c^8*d^22 - 24858080*a^12*b^18*c^10*d^20 + 114212800*a^12*b^18*c^12*
d^18 - 274937600*a^12*b^18*c^14*d^16 + 390830000*a^12*b^18*c^16*d^14 - 3414
26960*a^12*b^18*c^18*d^12 + 181463680*a^12*b^18*c^20*d^10 - 55069600*a^12*b
^18*c^22*d^8 + 8170000*a^12*b^18*c^24*d^6 - 402800*a^12*b^18*c^26*d^4 + 304
0*a^12*b^18*c^28*d^2 - 1240320*a^13*b^17*c^7*d^23 + 19638400*a^13*b^17*c^9*
d^21 - 106460800*a^13*b^17*c^11*d^19 + 293542400*a^13*b^17*c^13*d^17 - 4725
61920*a^13*b^17*c^15*d^15 + 467412160*a^13*b^17*c^17*d^13 - 284331200*a^13*
b^17*c^19*d^11 + 101475200*a^13*b^17*c^21*d^9 - 18787200*a^13*b^17*c^23*d^7
 + 1331520*a^13*b^17*c^25*d^5 - 18240*a^13*b^17*c^27*d^3 + 620160*a^14*b^16

*c^6*d^24 - 13178400*a^14*b^16*c^8*d^22 + 86150560*a^14*b^16*c^10*d^20 - 27
4937600*a^14*b^16*c^12*d^18 + 503363200*a^14*b^16*c^14*d^16 - 563751280*a^1
4*b^16*c^16*d^14 + 390830000*a^14*b^16*c^18*d^12 - 162120160*a^14*b^16*c^20
*d^10 + 36434400*a^14*b^16*c^22*d^8 - 3488400*a^14*b^16*c^24*d^6 + 77520*a^
14*b^16*c^26*d^4 - 248064*a^15*b^15*c^5*d^25 + 7441920*a^15*b^15*c^7*d^23 -
60362240*a^15*b^15*c^9*d^21 + 225738240*a^15*b^15*c^11*d^19 - 472561920*a^
15*b^15*c^13*d^17 + 599984128*a^15*b^15*c^15*d^15 - 472561920*a^15*b^15*c^1
7*d^13 + 225738240*a^15*b^15*c^19*d^11 - 60362240*a^15*b^15*c^21*d^9 + 7441
920*a^15*b^15*c^23*d^7 - 248064*a^15*b^15*c^25*d^5 + 77520*a^16*b^14*c^4*d^
26 - 3488400*a^16*b^14*c^6*d^24 + 36434400*a^16*b^14*c^8*d^22 - 162120160*a
^16*b^14*c^10*d^20 + 390830000*a^16*b^14*c^12*d^18 - 563751280*a^16*b^14*c^
14*d^16 + 503363200*a^16*b^14*c^16*d^14 - 274937600*a^16*b^14*c^18*d^12 + 8
6150560*a^16*b^14*c^20*d^10 - 13178400*a^16*b^14*c^22*d^8 + 620160*a^16*b^1
4*c^24*d^6 - 18240*a^17*b^13*c^3*d^27 + 1331520*a^17*b^13*c^5*d^25 - 187872
00*a^17*b^13*c^7*d^23 + 101475200*a^17*b^13*c^9*d^21 - 284331200*a^17*b^13*
c^11*d^19 + 467412160*a^17*b^13*c^13*d^17 - 472561920*a^17*b^13*c^15*d^15 +
293542400*a^17*b^13*c^17*d^13 - 106460800*a^17*b^13*c^19*d^11 + 19638400*a
^17*b^13*c^21*d^9 - 1240320*a^17*b^13*c^23*d^7 + 3040*a^18*b^12*c^2*d^28 -
402800*a^18*b^12*c^4*d^26 + 8170000*a^18*b^12*c^6*d^24 - 55069600*a^18*b^12
*c^8*d^22 + 181463680*a^18*b^12*c^10*d^20 - 341426960*a^18*b^12*c^12*d^18 +
390830000*a^18*b^12*c^14*d^16 - 274937600*a^18*b^12*c^16*d^14 + 114212800*
a^18*b^12*c^18*d^12 - 24858080*a^18*b^12*c^20*d^10 + 2015520*a^18*b^12*c^22
*d^8 + 92800*a^19*b^11*c^3*d^27 - 2939840*a^19*b^11*c^5*d^25 + 25721600*a^1
9*b^11*c^7*d^23 - 101172800*a^19*b^11*c^9*d^21 + 219166080*a^19*b^11*c^11*d
^19 - 284331200*a^19*b^11*c^13*d^17 + 225738240*a^19*b^11*c^15*d^15 - 10646
0800*a^19*b^11*c^17*d^13 + 26873600*a^19*b^11*c^19*d^11 - 2687360*a^19*b^11
*c^21*d^9 - 15280*a^20*b^10*c^2*d^28 + 851360*a^20*b^10*c^4*d^26 - 10229760
*a^20*b^10*c^6*d^24 + 48989680*a^20*b^10*c^8*d^22 - 123188112*a^20*b^10*c^1
0*d^20 + 181463680*a^20*b^10*c^12*d^18 - 162120160*a^20*b^10*c^14*d^16 + 86
150560*a^20*b^10*c^16*d^14 - 24858080*a^20*b^10*c^18*d^12 + 2956096*a^20*b^
10*c^20*d^10 - 190400*a^21*b^9*c^3*d^27 + 3408640*a^21*b^9*c^5*d^25 - 20444
800*a^21*b^9*c^7*d^23 + 60333760*a^21*b^9*c^9*d^21 - 101172800*a^21*b^9*c^1
1*d^19 + 101475200*a^21*b^9*c^13*d^17 - 60362240*a^21*b^9*c^15*d^15 + 19638
400*a^21*b^9*c^17*d^13 - 2687360*a^21*b^9*c^19*d^11 + 30800*a^22*b^8*c^2*d^
28 - 928000*a^22*b^8*c^4*d^26 + 7281600*a^22*b^8*c^6*d^24 - 25575920*a^22*b
^8*c^8*d^22 + 48989680*a^22*b^8*c^10*d^20 - 55069600*a^22*b^8*c^12*d^18 + 3
6434400*a^22*b^8*c^14*d^16 - 13178400*a^22*b^8*c^16*d^14 + 2015520*a^22*b^8
*c^18*d^12 + 198400*a^23*b^7*c^3*d^27 - 2184320*a^23*b^7*c^5*d^25 + 9297920
*a^23*b^7*c^7*d^23 - 20444800*a^23*b^7*c^9*d^21 + 25721600*a^23*b^7*c^11*d^
19 - 18787200*a^23*b^7*c^13*d^17 + 7441920*a^23*b^7*c^15*d^15 - 1240320*a^2
3*b^7*c^17*d^13 - 31200*a^24*b^6*c^2*d^28 + 541200*a^24*b^6*c^4*d^26 - 2863
760*a^24*b^6*c^6*d^24 + 7281600*a^24*b^6*c^8*d^22 - 10229760*a^24*b^6*c^10*
d^20 + 8170000*a^24*b^6*c^12*d^18 - 3488400*a^24*b^6*c^14*d^16 + 620160*a^2
4*b^6*c^16*d^14 - 107200*a^25*b^5*c^3*d^27 + 736064*a^25*b^5*c^5*d^25 - 218
4320*a^25*b^5*c^7*d^23 + 3408640*a^25*b^5*c^9*d^21 - 2939840*a^25*b^5*c^11*

d^19 + 1331520*a^25*b^5*c^13*d^17 - 248064*a^25*b^5*c^15*d^15 + 16000*a^26*b^4*c^2*d^28 - 155120*a^26*b^4*c^4*d^26 + 541200*a^26*b^4*c^6*d^24 - 928000*a^26*b^4*c^8*d^22 + 851360*a^26*b^4*c^10*d^20 - 402800*a^26*b^4*c^12*d^18 + 77520*a^26*b^4*c^14*d^16 + 26240*a^27*b^3*c^3*d^27 - 107200*a^27*b^3*c^5*d^25 + 198400*a^27*b^3*c^7*d^23 - 190400*a^27*b^3*c^9*d^21 + 92800*a^27*b^3*c^11*d^19 - 18240*a^27*b^3*c^13*d^17 - 3440*a^28*b^2*c^2*d^28 + 16000*a^28*b^2*c^4*d^26 - 31200*a^28*b^2*c^6*d^24 + 30800*a^28*b^2*c^8*d^22 - 15280*a^28*b^2*c^10*d^20 + 3040*a^28*b^2*c^12*d^18 + 320*a*b^29*c^29*d + 320*a^29*b*c*d^29))^(1/2) - 2*a^24*d^24 - 2*b^24*c^24 - 8*a^2*b^22*c^24 - 8*a^4*b^20*c^24 + 576*a^10*b^14*d^24 - 2784*a^12*b^12*d^24 + 5284*a^14*b^10*d^24 - 4730*a^16*b^8*d^24 + 1780*a^18*b^6*d^24 - 68*a^20*b^4*d^24 - 38*a^22*b^2*d^24 - 8*a^24*c^2*d^22 - 8*a^24*c^4*d^20 + 576*b^24*c^10*d^14 - 2784*b^24*c^12*d^12 + 5284*b^24*c^14*d^10 - 4730*b^24*c^16*d^8 + 1780*b^24*c^18*d^6 - 68*b^24*c^20*d^4 - 38*b^24*c^22*d^2 - 5760*a*b^23*c^9*d^15 + 28224*a*b^23*c^11*d^13 - 54728*a*b^23*c^13*d^11 + 50620*a*b^23*c^15*d^9 - 20360*a*b^23*c^17*d^7 + 1480*a*b^23*c^19*d^5 + 268*a*b^23*c^21*d^3 + 88*a^3*b^21*c^23*d + 160*a^5*b^19*c^23*d - 5760*a^9*b^15*c*d^23 + 28224*a^11*b^13*c*d^23 - 54728*a^13*b^11*c*d^23 + 50620*a^15*b^9*c*d^23 - 20360*a^17*b^7*c*d^23 + 1480*a^19*b^5*c*d^23 + 268*a^21*b^3*c*d^23 + 88*a^23*b*c^3*d^21 + 160*a^23*b*c^5*d^19 + 25920*a^2*b^22*c^8*d^16 - 131904*a^2*b^22*c^10*d^14 + 270604*a^2*b^22*c^12*d^12 - 273544*a^2*b^22*c^14*d^10 + 131660*a^2*b^22*c^16*d^8 - 22060*a^2*b^22*c^18*d^6 + 782*a^2*b^22*c^20*d^4 + 98*a^2*b^22*c^22*d^2 - 69120*a^3*b^21*c^7*d^17 + 379200*a^3*b^21*c^9*d^15 - 860368*a^3*b^21*c^11*d^13 + 1001364*a^3*b^21*c^13*d^11 - 605280*a^3*b^21*c^15*d^9 + 167520*a^3*b^21*c^17*d^7 - 18840*a^3*b^21*c^19*d^5 + 144*a^3*b^21*c^21*d^3 + 120960*a^4*b^20*c^6*d^18 - 756000*a^4*b^20*c^8*d^16 + 1987844*a^4*b^20*c^10*d^14 - 2750664*a^4*b^20*c^12*d^12 + 2073976*a^4*b^20*c^14*d^10 - 793460*a^4*b^20*c^16*d^8 + 138010*a^4*b^20*c^18*d^6 - 10562*a^4*b^20*c^20*d^4 - 88*a^4*b^20*c^22*d^2 - 145152*a^5*b^19*c^5*d^19 + 1116288*a^5*b^19*c^7*d^17 - 3539128*a^5*b^19*c^9*d^15 + 5890780*a^5*b^19*c^11*d^13 - 5437600*a^5*b^19*c^13*d^11 + 2682536*a^5*b^19*c^15*d^9 - 655084*a^5*b^19*c^17*d^7 + 85484*a^5*b^19*c^19*d^5 - 4080*a^5*b^19*c^21*d^3 + 120960*a^6*b^18*c^4*d^20 - 1266048*a^6*b^18*c^6*d^18 + 4977996*a^6*b^18*c^8*d^16 - 10009720*a^6*b^18*c^10*d^14 + 11209800*a^6*b^18*c^12*d^12 - 6943760*a^6*b^18*c^14*d^10 + 2253214*a^6*b^18*c^16*d^8 - 396878*a^6*b^18*c^18*d^6 + 36120*a^6*b^18*c^20*d^4 - 1520*a^6*b^18*c^22*d^2 - 69120*a^7*b^17*c^3*d^21 + 1116288*a^7*b^17*c^5*d^19 - 5575008*a^7*b^17*c^7*d^17 + 13668308*a^7*b^17*c^9*d^15 - 18576800*a^7*b^17*c^11*d^13 + 14230520*a^7*b^17*c^13*d^11 - 5889904*a^7*b^17*c^15*d^9 + 1310504*a^7*b^17*c^17*d^7 - 168344*a^7*b^17*c^19*d^5 + 8960*a^7*b^17*c^21*d^3 + 25920*a^8*b^16*c^2*d^22 - 756000*a^8*b^16*c^4*d^20 + 4977996*a^8*b^16*c^6*d^18 - 15144828*a^8*b^16*c^8*d^16 + 25068800*a^8*b^16*c^10*d^14 - 23486280*a^8*b^16*c^12*d^12 + 12099640*a^8*b^16*c^14*d^10 - 3330518*a^8*b^16*c^16*d^8 + 529224*a^8*b^16*c^18*d^6 - 36280*a^8*b^16*c^20*d^4 + 379200*a^9*b^15*c^3*d^21 - 3539128*a^9*b^15*c^5*d^19 + 13668308*a^9*b^15*c^7*d^17 - 27691952*a^9*b^15*c^9*d^15 + 31562040*a^9*b^15*c^11*d^13 - 19993760*a^9*b^15*c^13*d^11 + 6731044*a^9*b^15*c^15* d^9 - 1239264*a^9*b^15*c^17*d^7 + 106016*a^9*b^15*c^19*d^5 - 131904*a^10*b^14*c^2*d^22 + 1987844*a^10*b^14*c^4*d^20 - 10009720*a^10*b^14*c^6*d^18 + 25068800*a^10*b^14*c^8*d^16 - 34796936*a^10*b^14*c^10*d^14 + 26927144*a^10*b^14*c^12*d^12 - 10994964*a^10*b^14*c^14*d^10 + 2295680*a^10*b^14*c^16*d^8 - 230240*a^10*b^14*c^18*d^6 - 860368*a^11*b^13*c^3*d^21 + 5890780*a^11*b^13*c^5*d^19 - 18576800*a^11*b^13*c^7*d^17 + 31562040*a^11*b^13*c^9*d^15 - 29722864*a^11*b^13*c^11*d^13 + 14679348*a^11*b^13*c^13*d^11 - 3497920*a^11*b^13*c^15*d^9 + 381280*a^11*b^13*c^17*d^7 + 270604*a^12*b^12*c^2*d^22 - 2750664*a^12*b^12*c^4*d^20 + 11209800*a^12*b^12*c^6*d^18 - 23486280*a^12*b^12*c^8*d^16 + 26927144*a^12*b^12*c^10*d^14 - 16147404*a^12*b^12*c^12*d^12 + 4479104*a^12*b^12*c^14*d^10 - 499520*a^12*b^12*c^16*d^8 + 1001364*a^13*b^11*c^3*d^21 - 5437600*a^13*b^11*c^5*d^19 + 14230520*a^13*b^11*c^7*d^17 - 19993760*a^13*b^11*c^9*d^15 + 14679348*a^13*b^11*c^11*d^13 - 4861024*a^13*b^11*c^13*d^11 + 552160*a^13*b^11*c^15*d^9 - 273544*a^14*b^10*c^2*d^22 + 2073976*a^14*b^10*c^4*d^20 - 6943760*a^14*b^10*c^6*d^18 + 12099640*a^14*b^10*c^8*d^16 - 10994964*a^14*b^10*c^10*d^14 + 4479104*a^14*b^10*c^12*d^12 - 562016*a^14*b^10*c^14*d^10 - 605280*a^15*b^9*c^3*d^21 + 2682536*a^15*b^9*c^5*d^19 - 5889904*a^15*b^9*c^7*d^17 + 6731044*a^15*b^9*c^9*d^15 - 3497920*a^15*b^9*c^11*d^13 + 552160*a^15*b^9*c^13*d^11 + 131660*a^16*b^8*c^2*d^22 - 793460*a^16*b^8*c^4*d^20 + 2253214*a^16*b^8*c^6*d^18 - 3330518*a^16*b^8*c^8*d^16 + 2295680*a^16*b^8*c^10*d^14 - 499520*a^16*b^8*c^12*d^12 + 167520*a^17*b^7*c^3*d^21 - 655084*a^17*b^7*c^5*d^19 + 1310504*a^17*b^7*c^7*d^17 - 1239264*a^17*b^7*c^9*d^15 + 381280*a^17*b^7*c^11*d^13 - 22060*a^18*b^6*c^2*d^22 + 138010*a^18*b^6*c^4*d^20 - 396878*a^18*b^6*c^6*d^18 + 529224*a^18*b^6*c^8*d^16 - 230240*a^18*b^6*c^10*d^14 - 18840*a^19*b^5*c^3*d^21 + 85484*a^19*b^5*c^5*d^19 - 168344*a^19*b^5*c^7*d^17 + 106016*a^19*b^5*c^9*d^15 + 782*a^20*b^4*c^2*d^22 - 10562*a^20*b^4*c^4*d^20 + 36120*a^20*b^4*c^6*d^18 - 36280*a^20*b^4*c^8*d^16 + 144*a^21*b^3*c^3*d^21 - 4080*a^21*b^3*c^5*d^19 + 8960*a^21*b^3*c^7*d^17 + 98*a^22*b^2*c^2*d^22 - 88*a^22*b^2*c^4*d^20 - 1520*a^22*b^2*c^6*d^18 + 4*a*b^23*c^23*d + 4*a^23*b*c*d^23)/(16*(5*a^2*b^28*c^30 - b^30*c^30 - a^30*d^30 - 10*a^4*b^26*c^30 + 10*a^6*b^24*c^30 - 5*a^8*b^22*c^30 + a^10*b^20*c^30 + a^20*b^10*d^30 - 5*a^22*b^8*d^30 + 10*a^24*b^6*d^30 - 10*a^26*b^4*d^30 + 5*a^28*b^2*d^30 + 5*a^30*c^2*d^28 - 10*a^30*c^4*d^26 + 10*a^30*c^6*d^24 - 5*a^30*c^8*d^22 + a^30*c^10*d^20 + b^30*c^20*d^10 - 5*b^30*c^22*d^8 + 10*b^30*c^24*d^6 - 10*b^30*c^26*d^4 + 5*b^30*c^28*d^2 - 20*a*b^29*c^19*d^11 + 100*a*b^29*c^21*d^9 - 200*a*b^29*c^23*d^7 + 200*a*b^29*c^25*d^5 - 100*a*b^29*c^27*d^3 - 100*a^3*b^27*c^29*d + 200*a^5*b^25*c^29*d - 200*a^7*b^23*c^29*d + 100*a^9*b^21*c^29*d - 20*a^11*b^19*c^29*d - 20*a^19*b^11*c*d^29 + 100*a^21*b^9*c*d^29 - 200*a^23*b^7*c*d^29 + 200*a^25*b^5*c*d^29 - 100*a^27*b^3*c*d^29 - 100*a^29*b*c^3*d^27 + 200*a^29*b*c^5*d^25 - 200*a^29*b*c^7*d^23 + 100*a^29*b*c^9*d^21 - 20*a^29*b*c^11*d^19 + 190*a^2*b^28*c^18*d^12 - 955*a^2*b^28*c^20*d^10 + 1925*a^2*b^28*c^22*d^8 - 1950*a^2*b^28*c^24*d^6 + 1000*a^2*b^28*c^26*d^4 - 215*a^2*b^28*c^28*d^2 - 1140*a^3*b^27*c^17*d^13 + 5800*a^3*b^27*c^19*d^11 - 11900*a^3*b^27*c^21*d^9 + 12400*a^3*b^27*c^23*d^7 - 6700*a^3*b^27*c^25*d^5 + 1640*a^3*b^27*c^27*d^3 + 4845*a^4*b^26*c^16*d^14 - 251

75*a^4*b^26*c^18*d^12 + 53210*a^4*b^26*c^20*d^10 - 58000*a^4*b^26*c^22*d^8 + 33825*a^4*b^26*c^24*d^6 - 9695*a^4*b^26*c^26*d^4 + 1000*a^4*b^26*c^28*d^2 - 15504*a^5*b^25*c^15*d^15 + 83220*a^5*b^25*c^17*d^13 - 183740*a^5*b^25*c^19*d^11 + 213040*a^5*b^25*c^21*d^9 - 136520*a^5*b^25*c^23*d^7 + 46004*a^5*b^25*c^25*d^5 - 6700*a^5*b^25*c^27*d^3 + 38760*a^6*b^24*c^14*d^16 - 218025*a^6*b^24*c^16*d^14 + 510625*a^6*b^24*c^18*d^12 - 639360*a^6*b^24*c^20*d^10 + 455100*a^6*b^24*c^22*d^8 - 178985*a^6*b^24*c^24*d^6 + 33825*a^6*b^24*c^26*d^4 - 1950*a^6*b^24*c^28*d^2 - 77520*a^7*b^23*c^13*d^17 + 465120*a^7*b^23*c^15*d^15 - 1174200*a^7*b^23*c^17*d^13 + 1607600*a^7*b^23*c^19*d^11 - 1277800*a^7*b^23*c^21*d^9 + 581120*a^7*b^23*c^23*d^7 - 136520*a^7*b^23*c^25*d^5 + 12400*a^7*b^23*c^27*d^3 + 125970*a^8*b^22*c^12*d^18 - 823650*a^8*b^22*c^14*d^16 + 2277150*a^8*b^22*c^16*d^14 - 3441850*a^8*b^22*c^18*d^12 + 3061855*a^8*b^22*c^20*d^10 - 1598495*a^8*b^22*c^22*d^8 + 455100*a^8*b^22*c^24*d^6 - 58000*a^8*b^22*c^26*d^4 + 1925*a^8*b^22*c^28*d^2 - 167960*a^9*b^21*c^11*d^19 + 1227400*a^9*b^21*c^13*d^17 - 3772640*a^9*b^21*c^15*d^15 + 6342200*a^9*b^21*c^17*d^13 - 6323300*a^9*b^21*c^19*d^11 + 3770860*a^9*b^21*c^21*d^9 - 1277800*a^9*b^21*c^23*d^7 + 213040*a^9*b^21*c^25*d^5 - 11900*a^9*b^21*c^27*d^3 + 184756*a^10*b^20*c^10*d^20 - 1553630*a^10*b^20*c^12*d^18 + 5384410*a^10*b^20*c^14*d^16 - 10132510*a^10*b^20*c^16*d^14 + 11341480*a^10*b^20*c^18*d^12 - 7699257*a^10*b^20*c^20*d^10 + 3061855*a^10*b^20*c^22*d^8 - 639360*a^10*b^20*c^24*d^6 + 53210*a^10*b^20*c^26*d^4 - 955*a^10*b^20*c^28*d^2 - 167960*a^11*b^19*c^9*d^21 + 1679600*a^11*b^19*c^11*d^19 - 6653800*a^11*b^19*c^13*d^17 + 14108640*a^11*b^19*c^15*d^15 - 17770700*a^11*b^19*c^17*d^13 + 13697880*a^11*b^19*c^19*d^11 - 6323300*a^11*b^19*c^21*d^9 + 1607600*a^11*b^19*c^23*d^7 - 183740*a^11*b^19*c^25*d^5 + 5800*a^11*b^19*c^27*d^3 + 125970*a^12*b^18*c^8*d^22 - 1553630*a^12*b^18*c^10*d^20 + 7138300*a^12*b^18*c^12*d^18 - 17183600*a^12*b^18*c^14*d^16 + 24426875*a^12*b^18*c^16*d^14 - 21339185*a^12*b^18*c^18*d^12 + 11341480*a^12*b^18*c^20*d^10 - 3441850*a^12*b^18*c^22*d^8 + 510625*a^12*b^18*c^24*d^6 - 25175*a^12*b^18*c^26*d^4 + 190*a^12*b^18*c^28*d^2 - 77520*a^13*b^17*c^7*d^23 + 1227400*a^13*b^17*c^9*d^21 - 6653800*a^13*b^17*c^11*d^19 + 18346400*a^13*b^17*c^13*d^17 - 29535120*a^13*b^17*c^15*d^15 + 29213260*a^13*b^17*c^17*d^13 - 17770700*a^13*b^17*c^19*d^11 + 6342200*a^13*b^17*c^21*d^9 - 1174200*a^13*b^17*c^23*d^7 + 83220*a^13*b^17*c^25*d^5 - 1140*a^13*b^17*c^27*d^3 + 38760*a^14*b^16*c^6*d^24 - 823650*a^14*b^16*c^8*d^22 + 5384410*a^14*b^16*c^10*d^20 - 17183600*a^14*b^16*c^12*d^18 + 31460200*a^14*b^16*c^14*d^16 - 35234455*a^14*b^16*c^16*d^14 + 24426875*a^14*b^16*c^18*d^12 - 10132510*a^14*b^16*c^20*d^10 + 2277150*a^14*b^16*c^22*d^8 - 218025*a^14*b^16*c^24*d^6 + 4845*a^14*b^16*c^26*d^4 - 15504*a^15*b^15*c^5*d^25 + 465120*a^15*b^15*c^7*d^23 - 3772640*a^15*b^15*c^9*d^21 + 14108640*a^15*b^15*c^11*d^19 - 29535120*a^15*b^15*c^13*d^17 + 37499008*a^15*b^15*c^15*d^15 - 29535120*a^15*b^15*c^17*d^13 + 14108640*a^15*b^15*c^19*d^11 - 3772640*a^15*b^15*c^21*d^9 + 465120*a^15*b^15*c^23*d^7 - 15504*a^15*b^15*c^25*d^5 + 4845*a^16*b^14*c^4*d^26 - 218025*a^16*b^14*c^6*d^24 + 2277150*a^16*b^14*c^8*d^22 - 10132510*a^16*b^14*c^10*d^20 + 24426875*a^16*b^14*c^12*d^18 - 35234455*a^16*b^14*c^14*d^16 + 31460200*a^16*b^14*c^16*d^14 - 17183600*a^16*b^14

*c^18*d^12 + 5384410*a^16*b^14*c^20*d^10 - 823650*a^16*b^14*c^22*d^8 + 3876
0*a^16*b^14*c^24*d^6 - 1140*a^17*b^13*c^3*d^27 + 83220*a^17*b^13*c^5*d^25 -
1174200*a^17*b^13*c^7*d^23 + 6342200*a^17*b^13*c^9*d^21 - 17770700*a^17*b^
13*c^11*d^19 + 29213260*a^17*b^13*c^13*d^17 - 29535120*a^17*b^13*c^15*d^15
+ 18346400*a^17*b^13*c^17*d^13 - 6653800*a^17*b^13*c^19*d^11 + 1227400*a^17
*b^13*c^21*d^9 - 77520*a^17*b^13*c^23*d^7 + 190*a^18*b^12*c^2*d^28 - 25175*
a^18*b^12*c^4*d^26 + 510625*a^18*b^12*c^6*d^24 - 3441850*a^18*b^12*c^8*d^22
+ 11341480*a^18*b^12*c^10*d^20 - 21339185*a^18*b^12*c^12*d^18 + 24426875*a
^18*b^12*c^14*d^16 - 17183600*a^18*b^12*c^16*d^14 + 7138300*a^18*b^12*c^18*
d^12 - 1553630*a^18*b^12*c^20*d^10 + 125970*a^18*b^12*c^22*d^8 + 5800*a^19*
b^11*c^3*d^27 - 183740*a^19*b^11*c^5*d^25 + 1607600*a^19*b^11*c^7*d^23 - 63
23300*a^19*b^11*c^9*d^21 + 13697880*a^19*b^11*c^11*d^19 - 17770700*a^19*b^1
1*c^13*d^17 + 14108640*a^19*b^11*c^15*d^15 - 6653800*a^19*b^11*c^17*d^13 +
1679600*a^19*b^11*c^19*d^11 - 167960*a^19*b^11*c^21*d^9 - 955*a^20*b^10*c^2
*d^28 + 53210*a^20*b^10*c^4*d^26 - 639360*a^20*b^10*c^6*d^24 + 3061855*a^20
*b^10*c^8*d^22 - 7699257*a^20*b^10*c^10*d^20 + 11341480*a^20*b^10*c^12*d^18
- 10132510*a^20*b^10*c^14*d^16 + 5384410*a^20*b^10*c^16*d^14 - 1553630*a^2
0*b^10*c^18*d^12 + 184756*a^20*b^10*c^20*d^10 - 11900*a^21*b^9*c^3*d^27 + 2
13040*a^21*b^9*c^5*d^25 - 1277800*a^21*b^9*c^7*d^23 + 3770860*a^21*b^9*c^9*
d^21 - 6323300*a^21*b^9*c^11*d^19 + 6342200*a^21*b^9*c^13*d^17 - 3772640*a^
21*b^9*c^15*d^15 + 1227400*a^21*b^9*c^17*d^13 - 167960*a^21*b^9*c^19*d^11 +
1925*a^22*b^8*c^2*d^28 - 58000*a^22*b^8*c^4*d^26 + 455100*a^22*b^8*c^6*d^2
4 - 1598495*a^22*b^8*c^8*d^22 + 3061855*a^22*b^8*c^10*d^20 - 3441850*a^22*b
^8*c^12*d^18 + 2277150*a^22*b^8*c^14*d^16 - 823650*a^22*b^8*c^16*d^14 + 125
970*a^22*b^8*c^18*d^12 + 12400*a^23*b^7*c^3*d^27 - 136520*a^23*b^7*c^5*d^25
+ 581120*a^23*b^7*c^7*d^23 - 1277800*a^23*b^7*c^9*d^21 + 1607600*a^23*b^7*
c^11*d^19 - 1174200*a^23*b^7*c^13*d^17 + 465120*a^23*b^7*c^15*d^15 - 77520*
a^23*b^7*c^17*d^13 - 1950*a^24*b^6*c^2*d^28 + 33825*a^24*b^6*c^4*d^26 - 178
985*a^24*b^6*c^6*d^24 + 455100*a^24*b^6*c^8*d^22 - 639360*a^24*b^6*c^10*d^2
0 + 510625*a^24*b^6*c^12*d^18 - 218025*a^24*b^6*c^14*d^16 + 38760*a^24*b^6*
c^16*d^14 - 6700*a^25*b^5*c^3*d^27 + 46004*a^25*b^5*c^5*d^25 - 136520*a^25*
b^5*c^7*d^23 + 213040*a^25*b^5*c^9*d^21 - 183740*a^25*b^5*c^11*d^19 + 83220
*a^25*b^5*c^13*d^17 - 15504*a^25*b^5*c^15*d^15 + 1000*a^26*b^4*c^2*d^28 - 9
695*a^26*b^4*c^4*d^26 + 33825*a^26*b^4*c^6*d^24 - 58000*a^26*b^4*c^8*d^22 +
53210*a^26*b^4*c^10*d^20 - 25175*a^26*b^4*c^12*d^18 + 4845*a^26*b^4*c^14*d
^16 + 1640*a^27*b^3*c^3*d^27 - 6700*a^27*b^3*c^5*d^25 + 12400*a^27*b^3*c^7*
d^23 - 11900*a^27*b^3*c^9*d^21 + 5800*a^27*b^3*c^11*d^19 - 1140*a^27*b^3*c^
13*d^17 - 215*a^28*b^2*c^2*d^28 + 1000*a^28*b^2*c^4*d^26 - 1950*a^28*b^2*c^
6*d^24 + 1925*a^28*b^2*c^8*d^22 - 955*a^28*b^2*c^10*d^20 + 190*a^28*b^2*c^1
2*d^18 + 20*a*b^29*c^29*d + 20*a^29*b*c*d^29)))^(1/2) - (4*(4*a^2*b^20*c^22
- 12*a^6*b^16*c^22 + 8*a^8*b^14*c^22 + 4*a^22*c^2*d^20 - 12*a^22*c^6*d^16
+ 8*a^22*c^8*d^14 + 48*a*b^21*c^11*d^11 - 212*a*b^21*c^13*d^9 + 360*a*b^21*
c^15*d^7 - 276*a*b^21*c^17*d^5 + 80*a*b^21*c^19*d^3 - 20*a^3*b^19*c^21*d -
72*a^5*b^17*c^21*d + 204*a^7*b^15*c^21*d - 112*a^9*b^13*c^21*d + 48*a^11*b^
11*c*d^21 - 212*a^13*b^9*c*d^21 + 360*a^15*b^7*c*d^21 - 276*a^17*b^5*c*d^21

+ 80*a^19*b^3*c*d^21 - 20*a^21*b*c^3*d^19 - 72*a^21*b*c^5*d^17 + 204*a^21*
b*c^7*d^15 - 112*a^21*b*c^9*d^13 - 480*a^2*b^20*c^10*d^12 + 2160*a^2*b^20*c
^12*d^10 - 3772*a^2*b^20*c^14*d^8 + 3020*a^2*b^20*c^16*d^6 - 960*a^2*b^20*c
^18*d^4 + 28*a^2*b^20*c^20*d^2 + 2160*a^3*b^19*c^9*d^13 - 10152*a^3*b^19*c^
11*d^11 + 18888*a^3*b^19*c^13*d^9 - 16732*a^3*b^19*c^15*d^7 + 6588*a^3*b^19
*c^17*d^5 - 732*a^3*b^19*c^19*d^3 - 5760*a^4*b^18*c^8*d^14 + 29360*a^4*b^18
*c^10*d^12 - 60792*a^4*b^18*c^12*d^10 + 62708*a^4*b^18*c^14*d^8 - 31892*a^4
*b^18*c^16*d^6 + 6588*a^4*b^18*c^18*d^4 - 212*a^4*b^18*c^20*d^2 + 10080*a^5
*b^17*c^7*d^15 - 58860*a^5*b^17*c^9*d^13 + 141880*a^5*b^17*c^11*d^11 - 1755
92*a^5*b^17*c^13*d^9 + 113748*a^5*b^17*c^15*d^7 - 34492*a^5*b^17*c^17*d^5 +
3308*a^5*b^17*c^19*d^3 - 12096*a^6*b^16*c^6*d^16 + 87264*a^6*b^16*c^8*d^14
- 254340*a^6*b^16*c^10*d^12 + 381532*a^6*b^16*c^12*d^10 - 307752*a^6*b^16*
c^14*d^8 + 125568*a^6*b^16*c^16*d^6 - 21232*a^6*b^16*c^18*d^4 + 1068*a^6*b^
16*c^20*d^2 + 10080*a^7*b^15*c^5*d^17 - 99120*a^7*b^15*c^7*d^15 + 359064*a^
7*b^15*c^9*d^13 - 655076*a^7*b^15*c^11*d^11 + 650108*a^7*b^15*c^13*d^9 - 34
3368*a^7*b^15*c^15*d^7 + 85760*a^7*b^15*c^17*d^5 - 7652*a^7*b^15*c^19*d^3 -
5760*a^8*b^14*c^4*d^18 + 87264*a^8*b^14*c^6*d^16 - 402576*a^8*b^14*c^8*d^1
4 + 900324*a^8*b^14*c^10*d^12 - 1096236*a^8*b^14*c^12*d^10 + 731392*a^8*b^1
4*c^14*d^8 - 247352*a^8*b^14*c^16*d^6 + 34548*a^8*b^14*c^18*d^4 - 1612*a^8*
b^14*c^20*d^2 + 2160*a^9*b^13*c^3*d^19 - 58860*a^9*b^13*c^5*d^17 + 359064*a
^9*b^13*c^7*d^15 - 999816*a^9*b^13*c^9*d^13 + 1494564*a^9*b^13*c^11*d^11 -
1238148*a^9*b^13*c^13*d^9 + 542272*a^9*b^13*c^15*d^7 - 109032*a^9*b^13*c^17
*d^5 + 7908*a^9*b^13*c^19*d^3 - 480*a^10*b^12*c^2*d^20 + 29360*a^10*b^12*c^
4*d^18 - 254340*a^10*b^12*c^6*d^16 + 900324*a^10*b^12*c^8*d^14 - 1656496*a^
10*b^12*c^10*d^12 + 1688232*a^10*b^12*c^12*d^10 - 934868*a^10*b^12*c^14*d^8
+ 254492*a^10*b^12*c^16*d^6 - 26952*a^10*b^12*c^18*d^4 + 728*a^10*b^12*c^2
0*d^2 - 10152*a^11*b^11*c^3*d^19 + 141880*a^11*b^11*c^5*d^17 - 655076*a^11*
b^11*c^7*d^15 + 1494564*a^11*b^11*c^9*d^13 - 1870136*a^11*b^11*c^11*d^11 +
1289704*a^11*b^11*c^13*d^9 - 455388*a^11*b^11*c^15*d^7 + 67468*a^11*b^11*c^
17*d^5 - 2912*a^11*b^11*c^19*d^3 + 2160*a^12*b^10*c^2*d^20 - 60792*a^12*b^1
0*c^4*d^18 + 381532*a^12*b^10*c^6*d^16 - 1096236*a^12*b^10*c^8*d^14 + 16882
32*a^12*b^10*c^10*d^12 - 1434728*a^12*b^10*c^12*d^10 + 639684*a^12*b^10*c^1
4*d^8 - 127860*a^12*b^10*c^16*d^6 + 8008*a^12*b^10*c^18*d^4 + 18888*a^13*b^
9*c^3*d^19 - 175592*a^13*b^9*c^5*d^17 + 650108*a^13*b^9*c^7*d^15 - 1238148*
a^13*b^9*c^9*d^13 + 1289704*a^13*b^9*c^11*d^11 - 715296*a^13*b^9*c^13*d^9 +
186564*a^13*b^9*c^15*d^7 - 16016*a^13*b^9*c^17*d^5 - 3772*a^14*b^8*c^2*d^2
0 + 62708*a^14*b^8*c^4*d^18 - 307752*a^14*b^8*c^6*d^16 + 731392*a^14*b^8*c^
8*d^14 - 934868*a^14*b^8*c^10*d^12 + 639684*a^14*b^8*c^12*d^10 - 211416*a^1
4*b^8*c^14*d^8 + 24024*a^14*b^8*c^16*d^6 - 16732*a^15*b^7*c^3*d^19 + 113748
*a^15*b^7*c^5*d^17 - 343368*a^15*b^7*c^7*d^15 + 542272*a^15*b^7*c^9*d^13 -
455388*a^15*b^7*c^11*d^11 + 186564*a^15*b^7*c^13*d^9 - 27456*a^15*b^7*c^15*
d^7 + 3020*a^16*b^6*c^2*d^20 - 31892*a^16*b^6*c^4*d^18 + 125568*a^16*b^6*c^
6*d^16 - 247352*a^16*b^6*c^8*d^14 + 254492*a^16*b^6*c^10*d^12 - 127860*a^16
*b^6*c^12*d^10 + 24024*a^16*b^6*c^14*d^8 + 6588*a^17*b^5*c^3*d^19 - 34492*a
^17*b^5*c^5*d^17 + 85760*a^17*b^5*c^7*d^15 - 109032*a^17*b^5*c^9*d^13 + 674

68*a^17*b^5*c^11*d^11 - 16016*a^17*b^5*c^13*d^9 - 960*a^18*b^4*c^2*d^20 + 6
588*a^18*b^4*c^4*d^18 - 21232*a^18*b^4*c^6*d^16 + 34548*a^18*b^4*c^8*d^14 -
 26952*a^18*b^4*c^10*d^12 + 8008*a^18*b^4*c^12*d^10 - 732*a^19*b^3*c^3*d^19
 + 3308*a^19*b^3*c^5*d^17 - 7652*a^19*b^3*c^7*d^15 + 7908*a^19*b^3*c^9*d^13
 - 2912*a^19*b^3*c^11*d^11 + 28*a^20*b^2*c^2*d^20 - 212*a^20*b^2*c^4*d^18 +
 1068*a^20*b^2*c^6*d^16 - 1612*a^20*b^2*c^8*d^14 + 728*a^20*b^2*c^10*d^12))
/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^
20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*
a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*
c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*
d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b
^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d
 - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b
^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 +
48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b
^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*
c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^
13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*
d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^1
4*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d
^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*
d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^
14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^1
2*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^1
8*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9
*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c
^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^
6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^1
2*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^1
2*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^1
1*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*
b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^
10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^
10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 1
3860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 +
 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 -
 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^
7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18
+ 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^1
2 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14
*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d
^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^
11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*
d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^
8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c

^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19) + (8*tan(e/2 + (f*x)/2)*(12*a^5*b^17*c^22 - 4*a^22*c*d^21 - 4*a*b^21*c^22 - 8*a^7*b^15*c^22 + 12*a^22*c^5*d^17 - 8*a^22*c^7*d^15 - 24*a*b^21*c^12*d^10 + 100*a*b^21*c^14*d^8 - 164*a*b^21*c^16*d^6 + 120*a*b^21*c^18*d^4 - 28*a*b^21*c^20*d^2 + 20*a^2*b^20*c^21*d + 72*a^4*b^18*c^21*d - 204*a^6*b^16*c^21*d + 112*a^8*b^14*c^21*d - 24*a^12*b^10*c*d^21 + 100*a^14*b^8*c*d^21 - 164*a^16*b^6*c*d^21 + 120*a^18*b^4*c*d^21 - 28*a^20*b^2*c*d^21 + 20*a^21*b*c^2*d^20 + 72*a^21*b*c^4*d^18 - 204*a^21*b*c^6*d^16 + 112*a^21*b*c^8*d^14 + 216*a^2*b^20*c^11*d^11 - 908*a^2*b^20*c^13*d^9 + 1540*a^2*b^20*c^15*d^7 - 1200*a^2*b^20*c^17*d^5 + 332*a^2*b^20*c^19*d^3 - 840*a^3*b^19*c^10*d^12 + 3672*a^3*b^19*c^12*d^10 - 6788*a^3*b^19*c^14*d^8 + 6132*a^3*b^19*c^16*d^6 - 2388*a^3*b^19*c^18*d^4 + 212*a^3*b^19*c^20*d^2 + 1800*a^4*b^18*c^9*d^13 - 8680*a^4*b^18*c^11*d^11 + 18852*a^4*b^18*c^13*d^9 - 21228*a^4*b^18*c^15*d^7 + 11692*a^4*b^18*c^17*d^5 - 2508*a^4*b^18*c^19*d^3 - 2160*a^5*b^17*c^8*d^14 + 13100*a^5*b^17*c^10*d^12 - 36820*a^5*b^17*c^12*d^10 + 53712*a^5*b^17*c^14*d^8 - 39608*a^5*b^17*c^16*d^6 + 12832*a^5*b^17*c^18*d^4 - 1068*a^5*b^17*c^20*d^2 + 1008*a^6*b^16*c^7*d^15 - 12420*a^6*b^16*c^9*d^13 + 51764*a^6*b^16*c^11*d^11 - 100128*a^6*b^16*c^13*d^9 + 96048*a^6*b^16*c^15*d^7 - 42920*a^6*b^16*c^17*d^5 + 6852*a^6*b^16*c^19*d^3 + 1008*a^7*b^15*c^6*d^16 + 5136*a^7*b^15*c^8*d^14 - 48820*a^7*b^15*c^10*d^12 + 134700*a^7*b^15*c^12*d^10 - 171472*a^7*b^15*c^14*d^8 + 103992*a^7*b^15*c^16*d^6 - 26148*a^7*b^15*c^18*d^4 + 1612*a^7*b^15*c^20*d^2 - 2160*a^8*b^14*c^5*d^17 + 5136*a^8*b^14*c^7*d^15 + 20436*a^8*b^14*c^9*d^13 - 121524*a^8*b^14*c^11*d^11 + 224888*a^8*b^14*c^13*d^9 - 186952*a^8*b^14*c^15*d^7 + 67572*a^8*b^14*c^17*d^5 - 7508*a^8*b^14*c^19*d^3 + 1800*a^9*b^13*c^4*d^18 - 12420*a^9*b^13*c^6*d^16 + 20436*a^9*b^13*c^8*d^14 + 49416*a^9*b^13*c^10*d^12 - 201552*a^9*b^13*c^12*d^10 + 245708*a^9*b^13*c^14*d^8 - 125412*a^9*b^13*c^16*d^6 + 22752*a^9*b^13*c^18*d^4 - 728*a^9*b^13*c^20*d^2 - 840*a^10*b^12*c^3*d^19 + 13100*a^10*b^12*c^5*d^17 - 48820*a^10*b^12*c^7*d^15 + 49416*a^10*b^12*c^9*d^13 + 82088*a^10*b^12*c^11*d^11 - 219092*a^10*b^12*c^13*d^9 + 168468*a^10*b^12*c^15*d^7 - 47152*a^10*b^12*c^17*d^5 + 2832*a^10*b^12*c^19*d^3 + 216*a^11*b^11*c^2*d^20 - 8680*a^11*b^11*c^4*d^18 + 51764*a^11*b^11*c^6*d^16 - 121524*a^11*b^11*c^8*d^14 + 82088*a^11*b^11*c^10*d^12 + 88712*a^11*b^11*c^12*d^10 - 153012*a^11*b^11*c^14*d^8 + 67604*a^11*b^11*c^16*d^6 - 7168*a^11*b^11*c^18*d^4 + 3672*a^12*b^10*c^3*d^19 - 36820*a^12*b^10*c^5*d^17 + 134700*a^12*b^10*c^7*d^15 - 201552*a^12*b^10*c^9*d^13 + 88712*a^12*b^10*c^11*d^11 + 62676*a^12*b^10*c^13*d^9 - 63372*a^12*b^10*c^15*d^7 + 12008*a^12*b^10*c^17*d^5 - 908*a^13*b^9*c^2*d^20 + 18852*a^13*b^9*c^4*d^18 - 100128*a^13*b^9*c^6*d^16 + 224888*a^13*b^9*c^8*d^14 - 219092*a^13*b^

9*c^10*d^12 + 62676*a^13*b^9*c^12*d^10 + 26256*a^13*b^9*c^14*d^8 - 12544*a^13*b^9*c^16*d^6 - 6788*a^14*b^8*c^3*d^19 + 53712*a^14*b^8*c^5*d^17 - 171472*a^14*b^8*c^7*d^15 + 245708*a^14*b^8*c^9*d^13 - 153012*a^14*b^8*c^11*d^11 + 26256*a^14*b^8*c^13*d^9 + 5496*a^14*b^8*c^15*d^7 + 1540*a^15*b^7*c^2*d^20 - 21228*a^15*b^7*c^4*d^18 + 96048*a^15*b^7*c^6*d^16 - 186952*a^15*b^7*c^8*d^14 + 168468*a^15*b^7*c^10*d^12 - 63372*a^15*b^7*c^12*d^10 + 5496*a^15*b^7*c^14*d^8 + 6132*a^16*b^6*c^3*d^19 - 39608*a^16*b^6*c^5*d^17 + 103992*a^16*b^6*c^7*d^15 - 125412*a^16*b^6*c^9*d^13 + 67604*a^16*b^6*c^11*d^11 - 12544*a^16*b^6*c^13*d^9 - 1200*a^17*b^5*c^2*d^20 + 11692*a^17*b^5*c^4*d^18 - 42920*a^17*b^5*c^6*d^16 + 67572*a^17*b^5*c^8*d^14 - 47152*a^17*b^5*c^10*d^12 + 12008*a^17*b^5*c^12*d^10 - 2388*a^18*b^4*c^3*d^19 + 12832*a^18*b^4*c^5*d^17 - 26148*a^18*b^4*c^7*d^15 + 22752*a^18*b^4*c^9*d^13 - 7168*a^18*b^4*c^11*d^11 + 332*a^19*b^3*c^2*d^20 - 2508*a^19*b^3*c^4*d^18 + 6852*a^19*b^3*c^6*d^16 - 7508*a^19*b^3*c^8*d^14 + 2832*a^19*b^3*c^10*d^12 + 212*a^20*b^2*c^3*d^19 - 1068*a^20*b^2*c^5*d^17 + 1612*a^20*b^2*c^7*d^15 - 728*a^20*b^2*c^9*d^13))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13

- 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19)) + (4*(288*a*b^18*c^6*d^13 - 1104*a*b^18*c^8*d^11 + 1538*a*b^18*c^10*d^9 - 872*a*b^18*c^12*d^7 + 108*a*b^18*c^14*d^5 + 40*a*b^18*c^16*d^3 + 8*a^3*b^16*c^18*d + 8*a^5*b^14*c^18*d + 288*a^6*b^13*c*d^18 - 1104*a^8*b^11*c*d^18 + 1538*a^10*b^9*c*d^18 - 872*a^12*b^7*c*d^18 + 108*a^14*b^5*c*d^18 + 40*a^16*b^3*c*d^18 + 8*a^18*b*c^3*d^16 + 8*a^18*b*c^5*d^14 - 864*a^2*b^17*c^5*d^14 + 3216*a^2*b^17*c^7*d^12 - 4262*a^2*b^17*c^9*d^10 + 2256*a^2*b^17*c^11*d^8 - 304*a^2*b^17*c^13*d^6 - 32*a^2*b^17*c^15*d^4 + 8*a^2*b^17*c^17*d^2 + 576*a^3*b^16*c^4*d^15 - 3024*a^3*b^16*c^6*d^13 + 6304*a^3*b^16*c^8*d^11 - 7216*a^3*b^16*c^10*d^9 + 4944*a^3*b^16*c^12*d^7 - 1664*a^3*b^16*c^14*d^5 - 72*a^3*b^16*c^16*d^3 + 576*a^4*b^15*c^3*d^16 + 912*a^4*b^15*c^5*d^14 - 8720*a^4*b^15*c^7*d^12 + 16632*a^4*b^15*c^9*d^10 - 14888*a^4*b^15*c^11*d^8 + 6704*a^4*b^15*c^13*d^6 - 744*a^4*b^15*c^15*d^4 - 40*a^4*b^15*c^17*d^2 - 864*a^5*b^14*c^2*d^17 + 912*a^5*b^14*c^4*d^15 + 5140*a^5*b^14*c^6*d^13 - 16080*a^5*b^14*c^8*d^11 + 23520*a^5*b^14*c^10*d^9 - 20208*a^5*b^14*c^12*d^7 + 7404*a^5*b^14*c^14*d^5 - 264*a^5*b^14*c^16*d^3 - 3024*a^6*b^13*c^3*d^16 + 5140*a^6*b^13*c^5*d^14 + 5280*a^6*b^13*c^7*d^12 - 28380*a^6*b^13*c^9*d^10 + 39792*a^6*b^13*c^11*d^8 - 22728*a^6*b^13*c^13*d^6 + 3096*a^6*b^13*c^15*d^4 - 112*a^6*b^13*c^17*d^2 + 3216*a^7*b^12*c^2*d^17 - 8720*a^7*b^12*c^4*d^15 + 5280*a^7*b^12*c^6*d^13 + 15000*a^7*b^12*c^8*d^11 - 40656*a^7*b^12*c^10*d^9 + 40296*a^7*b^12*c^12*d^7 - 12984*a^7*b^12*c^14*d^5 + 728*a^7*b^12*c^16*d^3 + 6304*a^8*b^11*c^3*d^16 - 16080*a^8*b^11*c^5*d^14 + 15000*a^8*b^11*c^7*d^12 + 16024*a^8*b^11*c^9*d^10 - 46184*a^8*b^11*c^11*d^8 + 27208*a^8*b^11*c^13*d^6 - 2752*a^8*b^11*c^15*d^4 - 4262*a^9*b^10*c^2*d^17 + 16632*a^9*b^10*c^4*d^15 - 28380*a^9*b^10*c^6*d^13 + 16024*a^9*b^10*c^8*d^11 + 22018*a^9*b^10*c^10*d^9 - 30104*a^9*b^10*c^12*d^7 + 6488*a^9*b^10*c^14*d^5 - 7216*a^10*b^9*c^3*d^16 + 23520*a^10*b^9*c^5*d^14 - 40656*a^10*b^9*c^7*d^12 + 22018*a^10*b^9*c^9*d^10 + 13080*a^10*b^9*c^11*d^8 - 8720*a^10*b^9*c^13*d^6 + 2256*a^11*b^8*c^2*d^17 - 14888*a^11*b^8*c^4*d^15 + 39792*a^11*b^8*c^6*d^13 - 46184*a^11*b^8*c^8*d^11 +

13080*a^11*b^8*c^10*d^9 + 4360*a^11*b^8*c^12*d^7 + 4944*a^12*b^7*c^3*d^16 - 20208*a^12*b^7*c^5*d^14 + 40296*a^12*b^7*c^7*d^12 - 30104*a^12*b^7*c^9*d^10 + 4360*a^12*b^7*c^11*d^8 - 304*a^13*b^6*c^2*d^17 + 6704*a^13*b^6*c^4*d^15 - 22728*a^13*b^6*c^6*d^13 + 27208*a^13*b^6*c^8*d^11 - 8720*a^13*b^6*c^10*d^9 - 1664*a^14*b^5*c^3*d^16 + 7404*a^14*b^5*c^5*d^14 - 12984*a^14*b^5*c^7*d^12 + 6488*a^14*b^5*c^9*d^10 - 32*a^15*b^4*c^2*d^17 - 744*a^15*b^4*c^4*d^15 + 3096*a^15*b^4*c^6*d^13 - 2752*a^15*b^4*c^8*d^11 - 72*a^16*b^3*c^3*d^16 - 264*a^16*b^3*c^5*d^14 + 728*a^16*b^3*c^7*d^12 + 8*a^17*b^2*c^2*d^17 - 40*a^17*b^2*c^4*d^15 - 112*a^17*b^2*c^6*d^13 + 2*a*b^18*c^18*d + 2*a^18*b*c*d^18))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7

*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c
^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6
*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^
6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^
5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5
*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*
c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*
c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c
^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d
^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12
 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19) - (8*tan(e/
2 + (f*x)/2)*(a*b^18*c^19 + a^19*c*d^18 + 4*a^3*b^16*c^19 + 4*a^5*b^14*c^19
 + 4*a^19*c^3*d^16 + 4*a^19*c^5*d^14 - 576*a*b^18*c^5*d^14 + 2640*a*b^18*c^
7*d^12 - 4732*a*b^18*c^9*d^10 + 3961*a*b^18*c^11*d^8 - 1344*a*b^18*c^13*d^6
 + 14*a*b^18*c^15*d^4 + 18*a*b^18*c^17*d^2 + 4*a^2*b^17*c^18*d - 20*a^4*b^1
5*c^18*d - 576*a^5*b^14*c*d^18 - 56*a^6*b^13*c^18*d + 2640*a^7*b^12*c*d^18
- 4732*a^9*b^10*c*d^18 + 3961*a^11*b^8*c*d^18 - 1344*a^13*b^6*c*d^18 + 14*a
^15*b^4*c*d^18 + 18*a^17*b^2*c*d^18 + 4*a^18*b*c^2*d^17 - 20*a^18*b*c^4*d^1
5 - 56*a^18*b*c^6*d^13 + 2304*a^2*b^17*c^4*d^15 - 10944*a^2*b^17*c^6*d^13 +
 20720*a^2*b^17*c^8*d^11 - 18788*a^2*b^17*c^10*d^9 + 7392*a^2*b^17*c^12*d^7
 - 520*a^2*b^17*c^14*d^5 - 24*a^2*b^17*c^16*d^3 - 3456*a^3*b^16*c^3*d^16 +
20016*a^3*b^16*c^5*d^14 - 48112*a^3*b^16*c^7*d^12 + 58925*a^3*b^16*c^9*d^10
 - 36732*a^3*b^16*c^11*d^8 + 9736*a^3*b^16*c^13*d^6 - 760*a^3*b^16*c^15*d^4
 - 44*a^3*b^16*c^17*d^2 + 2304*a^4*b^15*c^2*d^17 - 23424*a^4*b^15*c^4*d^15
+ 81680*a^4*b^15*c^6*d^13 - 135520*a^4*b^15*c^8*d^11 + 114144*a^4*b^15*c^10
*d^9 - 44168*a^4*b^15*c^12*d^7 + 5696*a^4*b^15*c^14*d^5 - 332*a^4*b^15*c^16
*d^3 + 20016*a^5*b^14*c^3*d^16 - 99112*a^5*b^14*c^5*d^14 + 213338*a^5*b^14*
c^7*d^12 - 235152*a^5*b^14*c^9*d^10 + 130428*a^5*b^14*c^11*d^8 - 31908*a^5*
b^14*c^13*d^6 + 3966*a^5*b^14*c^15*d^4 - 140*a^5*b^14*c^17*d^2 - 10944*a^6*
b^13*c^2*d^17 + 81680*a^6*b^13*c^4*d^15 - 243832*a^6*b^13*c^6*d^13 + 364608
*a^6*b^13*c^8*d^11 - 281736*a^6*b^13*c^10*d^9 + 103104*a^6*b^13*c^12*d^7 -
16860*a^6*b^13*c^14*d^5 + 1660*a^6*b^13*c^16*d^3 - 48112*a^7*b^12*c^3*d^16
+ 213338*a^7*b^12*c^5*d^14 - 425832*a^7*b^12*c^7*d^12 + 434414*a^7*b^12*c^9
*d^10 - 219064*a^7*b^12*c^11*d^8 + 50732*a^7*b^12*c^13*d^6 - 7220*a^7*b^12*
c^15*d^4 + 364*a^7*b^12*c^17*d^2 + 20720*a^8*b^11*c^2*d^17 - 135520*a^8*b^1
1*c^4*d^15 + 364608*a^8*b^11*c^6*d^13 - 496336*a^8*b^11*c^8*d^11 + 343832*a
^8*b^11*c^10*d^9 - 111220*a^8*b^11*c^12*d^7 + 17956*a^8*b^11*c^14*d^5 - 137
6*a^8*b^11*c^16*d^3 + 58925*a^9*b^10*c^3*d^16 - 235152*a^9*b^10*c^5*d^14 +
434414*a^9*b^10*c^7*d^12 - 401788*a^9*b^10*c^9*d^10 + 172673*a^9*b^10*c^11*
d^8 - 31940*a^9*b^10*c^13*d^6 + 3244*a^9*b^10*c^15*d^4 - 18788*a^10*b^9*c^2
*d^17 + 114144*a^10*b^9*c^4*d^15 - 281736*a^10*b^9*c^6*d^13 + 343832*a^10*b
^9*c^8*d^11 - 197840*a^10*b^9*c^10*d^9 + 45940*a^10*b^9*c^12*d^7 - 4760*a^1
0*b^9*c^14*d^5 - 36732*a^11*b^8*c^3*d^16 + 130428*a^11*b^8*c^5*d^14 - 21906
4*a^11*b^8*c^7*d^12 + 172673*a^11*b^8*c^9*d^10 - 52480*a^11*b^8*c^11*d^8 +
4580*a^11*b^8*c^13*d^6 + 7392*a^12*b^7*c^2*d^17 - 44168*a^12*b^7*c^4*d^15 +

103104*a^12*b^7*c^6*d^13 - 111220*a^12*b^7*c^8*d^11 + 45940*a^12*b^7*c^10*d^9 - 4000*a^12*b^7*c^12*d^7 + 9736*a^13*b^6*c^3*d^16 - 31908*a^13*b^6*c^5*d^14 + 50732*a^13*b^6*c^7*d^12 - 31940*a^13*b^6*c^9*d^10 + 4580*a^13*b^6*c^11*d^8 - 520*a^14*b^5*c^2*d^17 + 5696*a^14*b^5*c^4*d^15 - 16860*a^14*b^5*c^6*d^13 + 17956*a^14*b^5*c^8*d^11 - 4760*a^14*b^5*c^10*d^9 - 760*a^15*b^4*c^3*d^16 + 3966*a^15*b^4*c^5*d^14 - 7220*a^15*b^4*c^7*d^12 + 3244*a^15*b^4*c^9*d^10 - 24*a^16*b^3*c^2*d^17 - 332*a^16*b^3*c^4*d^15 + 1660*a^16*b^3*c^6*d^13 - 1376*a^16*b^3*c^8*d^11 - 44*a^17*b^2*c^3*d^16 - 140*a^17*b^2*c^5*d^14 + 364*a^17*b^2*c^7*d^12))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^

15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19))*1i + ((((4*a^24*d^24 + 4*b^24*c^24 + 16*a^2*b^22*c^24 + 16*a^4*b^20*c^24 - 1152*a^10*b^14*d^24 + 5568*a^12*b^12*d^24 - 10568*a^14*b^10*d^24 + 9460*a^16*b^8*d^24 - 3560*a^18*b^6*d^24 + 136*a^20*b^4*d^24 + 76*a^22*b^2*d^24 + 16*a^24*c^2*d^22 + 16*a^24*c^4*d^20 - 1152*b^24*c^10*d^14 + 5568*b^24*c^12*d^12 - 10568*b^24*c^14*d^10 + 9460*b^24*c^16*d^8 - 3560*b^24*c^18*d^6 + 136*b^24*c^20*d^4 + 76*b^24*c^22*d^2 + 11520*a*b^23*c^9*d^15 - 56448*a*b^23*c^11*d^13 + 109456*a*b^23*c^13*d^11 - 101240*a*b^23*c^15*d^9 + 40720*a*b^23*c^17*d^7 - 2960*a*b^23*c^19*d^5 - 536*a*b^23*c^21*d^3 - 176*a^3*b^21*c^23*d - 320*a^5*b^19*c^23*d + 11520*a^9*b^15*c*d^23 - 56448*a^11*b^13*c*d^23 + 109456*a^13*b^11*c*d^23 - 101240*a^15*b^9*c*d^23 + 40720*a^17*b^7*c*d^23 - 2960*a^19*b^5*c*d^23 - 536*a^21*b^3*c*d^23 - 176*a^23*b*c^3*d^21 - 320*a^23*b*c^5*d^19 - 51840*a^2*b^22*c^8*d^16 + 263808*a^2*b^22*c^10*d^14 - 541208*a^2*b^22*c^12*d^12 + 547088*a^2*b^22*c^14*d^10 - 263320*a^2*b^22*c^16*d^8 + 44120*a^2*b^22*c^18*d^6 - 1564*a^2*b^22*c^20*d^4 - 196*a^2*b^22*c^22*d^2 + 138240*a^3*b^21*c^7*d^17 - 758400*a^3*b^21*c^9*d^15 + 1720736*a^3*b^21*c^11*d^13 - 2002728*a^3*b^21*c^13*d^11 + 1210560*a^3*b^21*c^15*d^9 - 335040*a^3*b^21*c^17*d^7 + 37680*a^3*b^21*c^19*d^5 - 288*a^3*b^21*c^21*d^3 - 241920*a^4*b^20*c^6*d^18 + 1512000*a^4*b^20*c^8*d^16 - 3975688*a^4*b^20*c^10*d^14 + 5501328*a^4*b^20*c^12*d^12 - 4147952*a^4*b^20*c^14*d^10 + 1586920*a^4*b^20*c^16*d^8 - 276020*a^4*b^20*c^18*d^6 + 21124*a^4*b^20*c^20*d^4 + 176*a^4*b^20*c^22*d^2 + 290304*a^5*b^19*c^5*d^19 - 2232576*a^5*b^19*c^7*d^17 + 7078256*a^5*b^19*c^9*d^15 - 11781560*a^5*b^19*c^11*d^13 + 10875200*a^5*b^19*c^13*d^11 - 5365072*a^5*b^19*c^15*d^9 + 1310168*a^5*b^19*c^17*d^7 - 170968*a^5*b^19*c^19*d^5 + 8160*a^5*b^19*c^21*d^3 - 241920*a^6*b^18*c^4*d^20 + 2532096*a^6*b^18*c^6*d^18 - 9955992*a^6*b^18*c^8*d^16 + 20019440*a^6*b^18*c^10*d^14 - 22419600*a^6*b^18*c^12*d^12 + 13887520*a^6*b^18*c^14*d^10 - 4506428*a^6*b^18*c^16*d^8 + 793756*a^6*b^18*c^18*d^6 - 72240*a^6*b^18*c^20*d^4 + 3040*a^6*b^18*c^22*d^2 + 138240*a^7*b^17*c^3*d^21 - 2232576*a^7*b^17*c^5*d^19 + 11150016*a^7*b^17*c^7*d^17 - 27336616*a^7*b^17*c^9*d^15 + 37153600*a^7*b^17*c^11*d^13 - 28461040*a^7*b^17*c^13*d^11 + 11779808*a^7*b^17*c^15*d^9 - 2621008*a^7*b^17*c^17*d^7 + 336688*a^7*b^17*c^19*d^5 - 17920*a^7*b^17*c^21*d^3 - 51840*a^8*b^16*c^2*d^22 + 1512000*a^8*b^16*c^4*d^20 - 9955992*a^8*b^16*c^6*d^18 + 30289656*a^8*b^16*c^8*d^16 - 50137600*a^8*b^16*c^10*d^14 + 46972560*a^8*b^16*c^12*d^12 - 24199280*a^8*b^16*c^14*d^10 + 6661036*a^8*b^16*c^16*d^8 - 1058448*a^8*b^16*c^18*d^6 + 72560*a^8*b^16*c^20*d^4 - 75840

0*a^9*b^15*c^3*d^21 + 7078256*a^9*b^15*c^5*d^19 - 27336616*a^9*b^15*c^7*d^1
7 + 55383904*a^9*b^15*c^9*d^15 - 63124080*a^9*b^15*c^11*d^13 + 39987520*a^9
*b^15*c^13*d^11 - 13462088*a^9*b^15*c^15*d^9 + 2478528*a^9*b^15*c^17*d^7 -
212032*a^9*b^15*c^19*d^5 + 263808*a^10*b^14*c^2*d^22 - 3975688*a^10*b^14*c^
4*d^20 + 20019440*a^10*b^14*c^6*d^18 - 50137600*a^10*b^14*c^8*d^16 + 695938
72*a^10*b^14*c^10*d^14 - 53854288*a^10*b^14*c^12*d^12 + 21989928*a^10*b^14*
c^14*d^10 - 4591360*a^10*b^14*c^16*d^8 + 460480*a^10*b^14*c^18*d^6 + 172073
6*a^11*b^13*c^3*d^21 - 11781560*a^11*b^13*c^5*d^19 + 37153600*a^11*b^13*c^7
*d^17 - 63124080*a^11*b^13*c^9*d^15 + 59445728*a^11*b^13*c^11*d^13 - 293586
96*a^11*b^13*c^13*d^11 + 6995840*a^11*b^13*c^15*d^9 - 762560*a^11*b^13*c^17
*d^7 - 541208*a^12*b^12*c^2*d^22 + 5501328*a^12*b^12*c^4*d^20 - 22419600*a^
12*b^12*c^6*d^18 + 46972560*a^12*b^12*c^8*d^16 - 53854288*a^12*b^12*c^10*d^
14 + 32294808*a^12*b^12*c^12*d^12 - 8958208*a^12*b^12*c^14*d^10 + 999040*a^
12*b^12*c^16*d^8 - 2002728*a^13*b^11*c^3*d^21 + 10875200*a^13*b^11*c^5*d^19
 - 28461040*a^13*b^11*c^7*d^17 + 39987520*a^13*b^11*c^9*d^15 - 29358696*a^1
3*b^11*c^11*d^13 + 9722048*a^13*b^11*c^13*d^11 - 1104320*a^13*b^11*c^15*d^9
 + 547088*a^14*b^10*c^2*d^22 - 4147952*a^14*b^10*c^4*d^20 + 13887520*a^14*b
^10*c^6*d^18 - 24199280*a^14*b^10*c^8*d^16 + 21989928*a^14*b^10*c^10*d^14 -
 8958208*a^14*b^10*c^12*d^12 + 1124032*a^14*b^10*c^14*d^10 + 1210560*a^15*b
^9*c^3*d^21 - 5365072*a^15*b^9*c^5*d^19 + 11779808*a^15*b^9*c^7*d^17 - 1346
2088*a^15*b^9*c^9*d^15 + 6995840*a^15*b^9*c^11*d^13 - 1104320*a^15*b^9*c^13
*d^11 - 263320*a^16*b^8*c^2*d^22 + 1586920*a^16*b^8*c^4*d^20 - 4506428*a^16
*b^8*c^6*d^18 + 6661036*a^16*b^8*c^8*d^16 - 4591360*a^16*b^8*c^10*d^14 + 99
9040*a^16*b^8*c^12*d^12 - 335040*a^17*b^7*c^3*d^21 + 1310168*a^17*b^7*c^5*d
^19 - 2621008*a^17*b^7*c^7*d^17 + 2478528*a^17*b^7*c^9*d^15 - 762560*a^17*b
^7*c^11*d^13 + 44120*a^18*b^6*c^2*d^22 - 276020*a^18*b^6*c^4*d^20 + 793756*
a^18*b^6*c^6*d^18 - 1058448*a^18*b^6*c^8*d^16 + 460480*a^18*b^6*c^10*d^14 +
 37680*a^19*b^5*c^3*d^21 - 170968*a^19*b^5*c^5*d^19 + 336688*a^19*b^5*c^7*d
^17 - 212032*a^19*b^5*c^9*d^15 - 1564*a^20*b^4*c^2*d^22 + 21124*a^20*b^4*c^
4*d^20 - 72240*a^20*b^4*c^6*d^18 + 72560*a^20*b^4*c^8*d^16 - 288*a^21*b^3*c
^3*d^21 + 8160*a^21*b^3*c^5*d^19 - 17920*a^21*b^3*c^7*d^17 - 196*a^22*b^2*c
^2*d^22 + 176*a^22*b^2*c^4*d^20 + 3040*a^22*b^2*c^6*d^18 - 8*a*b^23*c^23*d
- 8*a^23*b*c*d^23)^2/4 - (20736*b^18*d^18 - 96768*a^2*b^16*d^18 + 173664*a^
4*b^14*d^18 - 136032*a^6*b^12*d^18 + 31081*a^8*b^10*d^18 + 8440*a^10*b^8*d^
18 + 400*a^12*b^6*d^18 - 96768*b^18*c^2*d^16 + 173664*b^18*c^4*d^14 - 13603
2*b^18*c^6*d^12 + 31081*b^18*c^8*d^10 + 8440*b^18*c^10*d^8 + 400*b^18*c^12*
d^6 - 131328*a*b^17*c^3*d^15 + 216576*a*b^17*c^5*d^13 - 141104*a*b^17*c^7*d
^11 + 20260*a*b^17*c^9*d^9 + 2800*a*b^17*c^11*d^7 - 131328*a^3*b^15*c*d^17
+ 216576*a^5*b^13*c*d^17 - 141104*a^7*b^11*c*d^17 + 20260*a^9*b^9*c*d^17 +
2800*a^11*b^7*c*d^17 + 495936*a^2*b^16*c^2*d^16 - 989856*a^2*b^16*c^4*d^14
+ 901948*a^2*b^16*c^6*d^12 - 308392*a^2*b^16*c^8*d^10 - 5260*a^2*b^16*c^10*
d^8 + 1600*a^2*b^16*c^12*d^6 + 657408*a^3*b^15*c^3*d^15 - 1158992*a^3*b^15*
c^5*d^13 + 838256*a^3*b^15*c^7*d^11 - 182200*a^3*b^15*c^9*d^9 - 3200*a^3*b^
15*c^11*d^7 - 989856*a^4*b^14*c^2*d^16 + 2185654*a^4*b^14*c^4*d^14 - 221857
6*a^4*b^14*c^6*d^12 + 900624*a^4*b^14*c^8*d^10 - 64720*a^4*b^14*c^10*d^8 +

1600*a^4*b^14*c^12*d^6 - 1158992*a^5*b^13*c^3*d^15 + 2158808*a^5*b^13*c^5*d
^13 - 1641528*a^5*b^13*c^7*d^11 + 406880*a^5*b^13*c^9*d^9 - 17600*a^5*b^13*
c^11*d^7 + 901948*a^6*b^12*c^2*d^16 - 2218576*a^6*b^12*c^4*d^14 + 2430936*a
^6*b^12*c^6*d^12 - 1026928*a^6*b^12*c^8*d^10 + 88720*a^6*b^12*c^10*d^8 + 83
8256*a^7*b^11*c^3*d^15 - 1641528*a^7*b^11*c^5*d^13 + 1206848*a^7*b^11*c^7*d
^11 - 239360*a^7*b^11*c^9*d^9 - 308392*a^8*b^10*c^2*d^16 + 900624*a^8*b^10*
c^4*d^14 - 1026928*a^8*b^10*c^6*d^12 + 354016*a^8*b^10*c^8*d^10 - 182200*a^
9*b^9*c^3*d^15 + 406880*a^9*b^9*c^5*d^13 - 239360*a^9*b^9*c^7*d^11 - 5260*a
^10*b^8*c^2*d^16 - 64720*a^10*b^8*c^4*d^14 + 88720*a^10*b^8*c^6*d^12 - 3200
*a^11*b^7*c^3*d^15 - 17600*a^11*b^7*c^5*d^13 + 1600*a^12*b^6*c^2*d^16 + 160
0*a^12*b^6*c^4*d^14 + 27648*a*b^17*c*d^17)*(80*a^2*b^28*c^30 - 16*b^30*c^30
 - 16*a^30*d^30 - 160*a^4*b^26*c^30 + 160*a^6*b^24*c^30 - 80*a^8*b^22*c^30
+ 16*a^10*b^20*c^30 + 16*a^20*b^10*d^30 - 80*a^22*b^8*d^30 + 160*a^24*b^6*d
^30 - 160*a^26*b^4*d^30 + 80*a^28*b^2*d^30 + 80*a^30*c^2*d^28 - 160*a^30*c^
4*d^26 + 160*a^30*c^6*d^24 - 80*a^30*c^8*d^22 + 16*a^30*c^10*d^20 + 16*b^30
*c^20*d^10 - 80*b^30*c^22*d^8 + 160*b^30*c^24*d^6 - 160*b^30*c^26*d^4 + 80*
b^30*c^28*d^2 - 320*a*b^29*c^19*d^11 + 1600*a*b^29*c^21*d^9 - 3200*a*b^29*c
^23*d^7 + 3200*a*b^29*c^25*d^5 - 1600*a*b^29*c^27*d^3 - 1600*a^3*b^27*c^29*
d + 3200*a^5*b^25*c^29*d - 3200*a^7*b^23*c^29*d + 1600*a^9*b^21*c^29*d - 32
0*a^11*b^19*c^29*d - 320*a^19*b^11*c*d^29 + 1600*a^21*b^9*c*d^29 - 3200*a^2
3*b^7*c*d^29 + 3200*a^25*b^5*c*d^29 - 1600*a^27*b^3*c*d^29 - 1600*a^29*b*c^
3*d^27 + 3200*a^29*b*c^5*d^25 - 3200*a^29*b*c^7*d^23 + 1600*a^29*b*c^9*d^21
 - 320*a^29*b*c^11*d^19 + 3040*a^2*b^28*c^18*d^12 - 15280*a^2*b^28*c^20*d^1
0 + 30800*a^2*b^28*c^22*d^8 - 31200*a^2*b^28*c^24*d^6 + 16000*a^2*b^28*c^26
*d^4 - 3440*a^2*b^28*c^28*d^2 - 18240*a^3*b^27*c^17*d^13 + 92800*a^3*b^27*c
^19*d^11 - 190400*a^3*b^27*c^21*d^9 + 198400*a^3*b^27*c^23*d^7 - 107200*a^3
*b^27*c^25*d^5 + 26240*a^3*b^27*c^27*d^3 + 77520*a^4*b^26*c^16*d^14 - 40280
0*a^4*b^26*c^18*d^12 + 851360*a^4*b^26*c^20*d^10 - 928000*a^4*b^26*c^22*d^8
 + 541200*a^4*b^26*c^24*d^6 - 155120*a^4*b^26*c^26*d^4 + 16000*a^4*b^26*c^2
8*d^2 - 248064*a^5*b^25*c^15*d^15 + 1331520*a^5*b^25*c^17*d^13 - 2939840*a^
5*b^25*c^19*d^11 + 3408640*a^5*b^25*c^21*d^9 - 2184320*a^5*b^25*c^23*d^7 +
736064*a^5*b^25*c^25*d^5 - 107200*a^5*b^25*c^27*d^3 + 620160*a^6*b^24*c^14*
d^16 - 3488400*a^6*b^24*c^16*d^14 + 8170000*a^6*b^24*c^18*d^12 - 10229760*a
^6*b^24*c^20*d^10 + 7281600*a^6*b^24*c^22*d^8 - 2863760*a^6*b^24*c^24*d^6 +
 541200*a^6*b^24*c^26*d^4 - 31200*a^6*b^24*c^28*d^2 - 1240320*a^7*b^23*c^13
*d^17 + 7441920*a^7*b^23*c^15*d^15 - 18787200*a^7*b^23*c^17*d^13 + 25721600
*a^7*b^23*c^19*d^11 - 20444800*a^7*b^23*c^21*d^9 + 9297920*a^7*b^23*c^23*d^
7 - 2184320*a^7*b^23*c^25*d^5 + 198400*a^7*b^23*c^27*d^3 + 2015520*a^8*b^22
*c^12*d^18 - 13178400*a^8*b^22*c^14*d^16 + 36434400*a^8*b^22*c^16*d^14 - 55
069600*a^8*b^22*c^18*d^12 + 48989680*a^8*b^22*c^20*d^10 - 25575920*a^8*b^22
*c^22*d^8 + 7281600*a^8*b^22*c^24*d^6 - 928000*a^8*b^22*c^26*d^4 + 30800*a^
8*b^22*c^28*d^2 - 2687360*a^9*b^21*c^11*d^19 + 19638400*a^9*b^21*c^13*d^17
- 60362240*a^9*b^21*c^15*d^15 + 101475200*a^9*b^21*c^17*d^13 - 101172800*a^
9*b^21*c^19*d^11 + 60333760*a^9*b^21*c^21*d^9 - 20444800*a^9*b^21*c^23*d^7
+ 3408640*a^9*b^21*c^25*d^5 - 190400*a^9*b^21*c^27*d^3 + 2956096*a^10*b^20*

c^10*d^20 - 24858080*a^10*b^20*c^12*d^18 + 86150560*a^10*b^20*c^14*d^16 - 1
62120160*a^10*b^20*c^16*d^14 + 181463680*a^10*b^20*c^18*d^12 - 123188112*a^
10*b^20*c^20*d^10 + 48989680*a^10*b^20*c^22*d^8 - 10229760*a^10*b^20*c^24*d
^6 + 851360*a^10*b^20*c^26*d^4 - 15280*a^10*b^20*c^28*d^2 - 2687360*a^11*b^
19*c^9*d^21 + 26873600*a^11*b^19*c^11*d^19 - 106460800*a^11*b^19*c^13*d^17
+ 225738240*a^11*b^19*c^15*d^15 - 284331200*a^11*b^19*c^17*d^13 + 219166080
*a^11*b^19*c^19*d^11 - 101172800*a^11*b^19*c^21*d^9 + 25721600*a^11*b^19*c^
23*d^7 - 2939840*a^11*b^19*c^25*d^5 + 92800*a^11*b^19*c^27*d^3 + 2015520*a^
12*b^18*c^8*d^22 - 24858080*a^12*b^18*c^10*d^20 + 114212800*a^12*b^18*c^12*
d^18 - 274937600*a^12*b^18*c^14*d^16 + 390830000*a^12*b^18*c^16*d^14 - 3414
26960*a^12*b^18*c^18*d^12 + 181463680*a^12*b^18*c^20*d^10 - 55069600*a^12*b
^18*c^22*d^8 + 8170000*a^12*b^18*c^24*d^6 - 402800*a^12*b^18*c^26*d^4 + 304
0*a^12*b^18*c^28*d^2 - 1240320*a^13*b^17*c^7*d^23 + 19638400*a^13*b^17*c^9*
d^21 - 106460800*a^13*b^17*c^11*d^19 + 293542400*a^13*b^17*c^13*d^17 - 4725
61920*a^13*b^17*c^15*d^15 + 467412160*a^13*b^17*c^17*d^13 - 284331200*a^13*
b^17*c^19*d^11 + 101475200*a^13*b^17*c^21*d^9 - 18787200*a^13*b^17*c^23*d^7
 + 1331520*a^13*b^17*c^25*d^5 - 18240*a^13*b^17*c^27*d^3 + 620160*a^14*b^16
*c^6*d^24 - 13178400*a^14*b^16*c^8*d^22 + 86150560*a^14*b^16*c^10*d^20 - 27
4937600*a^14*b^16*c^12*d^18 + 503363200*a^14*b^16*c^14*d^16 - 563751280*a^1
4*b^16*c^16*d^14 + 390830000*a^14*b^16*c^18*d^12 - 162120160*a^14*b^16*c^20
*d^10 + 36434400*a^14*b^16*c^22*d^8 - 3488400*a^14*b^16*c^24*d^6 + 77520*a^
14*b^16*c^26*d^4 - 248064*a^15*b^15*c^5*d^25 + 7441920*a^15*b^15*c^7*d^23 -
 60362240*a^15*b^15*c^9*d^21 + 225738240*a^15*b^15*c^11*d^19 - 472561920*a^
15*b^15*c^13*d^17 + 599984128*a^15*b^15*c^15*d^15 - 472561920*a^15*b^15*c^1
7*d^13 + 225738240*a^15*b^15*c^19*d^11 - 60362240*a^15*b^15*c^21*d^9 + 7441
920*a^15*b^15*c^23*d^7 - 248064*a^15*b^15*c^25*d^5 + 77520*a^16*b^14*c^4*d^
26 - 3488400*a^16*b^14*c^6*d^24 + 36434400*a^16*b^14*c^8*d^22 - 162120160*a
^16*b^14*c^10*d^20 + 390830000*a^16*b^14*c^12*d^18 - 563751280*a^16*b^14*c^
14*d^16 + 503363200*a^16*b^14*c^16*d^14 - 274937600*a^16*b^14*c^18*d^12 + 8
6150560*a^16*b^14*c^20*d^10 - 13178400*a^16*b^14*c^22*d^8 + 620160*a^16*b^1
4*c^24*d^6 - 18240*a^17*b^13*c^3*d^27 + 1331520*a^17*b^13*c^5*d^25 - 187872
00*a^17*b^13*c^7*d^23 + 101475200*a^17*b^13*c^9*d^21 - 284331200*a^17*b^13*
c^11*d^19 + 467412160*a^17*b^13*c^13*d^17 - 472561920*a^17*b^13*c^15*d^15 +
 293542400*a^17*b^13*c^17*d^13 - 106460800*a^17*b^13*c^19*d^11 + 19638400*a
^17*b^13*c^21*d^9 - 1240320*a^17*b^13*c^23*d^7 + 3040*a^18*b^12*c^2*d^28 -
402800*a^18*b^12*c^4*d^26 + 8170000*a^18*b^12*c^6*d^24 - 55069600*a^18*b^12
*c^8*d^22 + 181463680*a^18*b^12*c^10*d^20 - 341426960*a^18*b^12*c^12*d^18 +
 390830000*a^18*b^12*c^14*d^16 - 274937600*a^18*b^12*c^16*d^14 + 114212800*
a^18*b^12*c^18*d^12 - 24858080*a^18*b^12*c^20*d^10 + 2015520*a^18*b^12*c^22
*d^8 + 92800*a^19*b^11*c^3*d^27 - 2939840*a^19*b^11*c^5*d^25 + 25721600*a^1
9*b^11*c^7*d^23 - 101172800*a^19*b^11*c^9*d^21 + 219166080*a^19*b^11*c^11*d
^19 - 284331200*a^19*b^11*c^13*d^17 + 225738240*a^19*b^11*c^15*d^15 - 10646
0800*a^19*b^11*c^17*d^13 + 26873600*a^19*b^11*c^19*d^11 - 2687360*a^19*b^11
*c^21*d^9 - 15280*a^20*b^10*c^2*d^28 + 851360*a^20*b^10*c^4*d^26 - 10229760
*a^20*b^10*c^6*d^24 + 48989680*a^20*b^10*c^8*d^22 - 123188112*a^20*b^10*c^1

0*d^20 + 181463680*a^20*b^10*c^12*d^18 - 162120160*a^20*b^10*c^14*d^16 + 86150560*a^20*b^10*c^16*d^14 - 24858080*a^20*b^10*c^18*d^12 + 2956096*a^20*b^10*c^20*d^10 - 190400*a^21*b^9*c^3*d^27 + 3408640*a^21*b^9*c^5*d^25 - 20444800*a^21*b^9*c^7*d^23 + 60333760*a^21*b^9*c^9*d^21 - 101172800*a^21*b^9*c^11*d^19 + 101475200*a^21*b^9*c^13*d^17 - 60362240*a^21*b^9*c^15*d^15 + 19638400*a^21*b^9*c^17*d^13 - 2687360*a^21*b^9*c^19*d^11 + 30800*a^22*b^8*c^2*d^28 - 928000*a^22*b^8*c^4*d^26 + 7281600*a^22*b^8*c^6*d^24 - 25575920*a^22*b^8*c^8*d^22 + 48989680*a^22*b^8*c^10*d^20 - 55069600*a^22*b^8*c^12*d^18 + 36434400*a^22*b^8*c^14*d^16 - 13178400*a^22*b^8*c^16*d^14 + 2015520*a^22*b^8*c^18*d^12 + 198400*a^23*b^7*c^3*d^27 - 2184320*a^23*b^7*c^5*d^25 + 9297920*a^23*b^7*c^7*d^23 - 20444800*a^23*b^7*c^9*d^21 + 25721600*a^23*b^7*c^11*d^19 - 18787200*a^23*b^7*c^13*d^17 + 7441920*a^23*b^7*c^15*d^15 - 1240320*a^23*b^7*c^17*d^13 - 31200*a^24*b^6*c^2*d^28 + 541200*a^24*b^6*c^4*d^26 - 2863760*a^24*b^6*c^6*d^24 + 7281600*a^24*b^6*c^8*d^22 - 10229760*a^24*b^6*c^10*d^20 + 8170000*a^24*b^6*c^12*d^18 - 3488400*a^24*b^6*c^14*d^16 + 620160*a^24*b^6*c^16*d^14 - 107200*a^25*b^5*c^3*d^27 + 736064*a^25*b^5*c^5*d^25 - 2184320*a^25*b^5*c^7*d^23 + 3408640*a^25*b^5*c^9*d^21 - 2939840*a^25*b^5*c^11*d^19 + 1331520*a^25*b^5*c^13*d^17 - 248064*a^25*b^5*c^15*d^15 + 16000*a^26*b^4*c^2*d^28 - 155120*a^26*b^4*c^4*d^26 + 541200*a^26*b^4*c^6*d^24 - 928000*a^26*b^4*c^8*d^22 + 851360*a^26*b^4*c^10*d^20 - 402800*a^26*b^4*c^12*d^18 + 77520*a^26*b^4*c^14*d^16 + 26240*a^27*b^3*c^3*d^27 - 107200*a^27*b^3*c^5*d^25 + 198400*a^27*b^3*c^7*d^23 - 190400*a^27*b^3*c^9*d^21 + 92800*a^27*b^3*c^11*d^19 - 18240*a^27*b^3*c^13*d^17 - 3440*a^28*b^2*c^2*d^28 + 16000*a^28*b^2*c^4*d^26 - 31200*a^28*b^2*c^6*d^24 + 30800*a^28*b^2*c^8*d^22 - 15280*a^28*b^2*c^10*d^20 + 3040*a^28*b^2*c^12*d^18 + 320*a*b^29*c^29*d + 320*a^29*b*c*d^29))^(1/2) - 2*a^24*d^24 - 2*b^24*c^24 - 8*a^2*b^22*c^24 - 8*a^4*b^20*c^24 + 576*a^10*b^14*d^24 - 2784*a^12*b^12*d^24 + 5284*a^14*b^10*d^24 - 4730*a^16*b^8*d^24 + 1780*a^18*b^6*d^24 - 68*a^20*b^4*d^24 - 38*a^22*b^2*d^24 - 8*a^24*c^2*d^22 - 8*a^24*c^4*d^20 + 576*b^24*c^10*d^14 - 2784*b^24*c^12*d^12 + 5284*b^24*c^14*d^10 - 4730*b^24*c^16*d^8 + 1780*b^24*c^18*d^6 - 68*b^24*c^20*d^4 - 38*b^24*c^22*d^2 - 5760*a*b^23*c^9*d^15 + 28224*a*b^23*c^11*d^13 - 54728*a*b^23*c^13*d^11 + 50620*a*b^23*c^15*d^9 - 20360*a*b^23*c^17*d^7 + 1480*a*b^23*c^19*d^5 + 268*a*b^23*c^21*d^3 + 88*a^3*b^21*c^23*d + 160*a^5*b^19*c^23*d - 5760*a^9*b^15*c*d^23 + 28224*a^11*b^13*c*d^23 - 54728*a^13*b^11*c*d^23 + 50620*a^15*b^9*c*d^23 - 20360*a^17*b^7*c*d^23 + 1480*a^19*b^5*c*d^23 + 268*a^21*b^3*c*d^23 + 88*a^23*b*c^3*d^21 + 160*a^23*b*c^5*d^19 + 25920*a^2*b^22*c^8*d^16 - 131904*a^2*b^22*c^10*d^14 + 270604*a^2*b^22*c^12*d^12 - 273544*a^2*b^22*c^14*d^10 + 131660*a^2*b^22*c^16*d^8 - 22060*a^2*b^22*c^18*d^6 + 782*a^2*b^22*c^20*d^4 + 98*a^2*b^22*c^22*d^2 - 69120*a^3*b^21*c^7*d^17 + 379200*a^3*b^21*c^9*d^15 - 860368*a^3*b^21*c^11*d^13 + 1001364*a^3*b^21*c^13*d^11 - 605280*a^3*b^21*c^15*d^9 + 167520*a^3*b^21*c^17*d^7 - 18840*a^3*b^21*c^19*d^5 + 144*a^3*b^21*c^21*d^3 + 120960*a^4*b^20*c^6*d^18 - 756000*a^4*b^20*c^8*d^16 + 1987844*a^4*b^20*c^10*d^14 - 2750664*a^4*b^20*c^12*d^12 + 2073976*a^4*b^20*c^14*d^10 - 793460*a^4*b^20*c^16*d^8 + 138010*a^4*b^20*c^18*d^6 - 10562*a^4*b^20*c^20*d^4 - 88*a^4*b^20*c^22*d^2 - 14515

2*a^5*b^19*c^5*d^19 + 1116288*a^5*b^19*c^7*d^17 - 3539128*a^5*b^19*c^9*d^15 + 5890780*a^5*b^19*c^11*d^13 - 5437600*a^5*b^19*c^13*d^11 + 2682536*a^5*b^19*c^15*d^9 - 655084*a^5*b^19*c^17*d^7 + 85484*a^5*b^19*c^19*d^5 - 4080*a^5*b^19*c^21*d^3 + 120960*a^6*b^18*c^4*d^20 - 1266048*a^6*b^18*c^6*d^18 + 4977996*a^6*b^18*c^8*d^16 - 10009720*a^6*b^18*c^10*d^14 + 11209800*a^6*b^18*c^12*d^12 - 6943760*a^6*b^18*c^14*d^10 + 2253214*a^6*b^18*c^16*d^8 - 396878*a^6*b^18*c^18*d^6 + 36120*a^6*b^18*c^20*d^4 - 1520*a^6*b^18*c^22*d^2 - 69120*a^7*b^17*c^3*d^21 + 1116288*a^7*b^17*c^5*d^19 - 5575008*a^7*b^17*c^7*d^17 + 13668308*a^7*b^17*c^9*d^15 - 18576800*a^7*b^17*c^11*d^13 + 14230520*a^7*b^17*c^13*d^11 - 5889904*a^7*b^17*c^15*d^9 + 1310504*a^7*b^17*c^17*d^7 - 168344*a^7*b^17*c^19*d^5 + 8960*a^7*b^17*c^21*d^3 + 25920*a^8*b^16*c^2*d^22 - 756000*a^8*b^16*c^4*d^20 + 4977996*a^8*b^16*c^6*d^18 - 15144828*a^8*b^16*c^8*d^16 + 25068800*a^8*b^16*c^10*d^14 - 23486280*a^8*b^16*c^12*d^12 + 12099640*a^8*b^16*c^14*d^10 - 3330518*a^8*b^16*c^16*d^8 + 529224*a^8*b^16*c^18*d^6 - 36280*a^8*b^16*c^20*d^4 + 379200*a^9*b^15*c^3*d^21 - 3539128*a^9*b^15*c^5*d^19 + 13668308*a^9*b^15*c^7*d^17 - 27691952*a^9*b^15*c^9*d^15 + 31562040*a^9*b^15*c^11*d^13 - 19993760*a^9*b^15*c^13*d^11 + 6731044*a^9*b^15*c^15*d^9 - 1239264*a^9*b^15*c^17*d^7 + 106016*a^9*b^15*c^19*d^5 - 131904*a^10*b^14*c^2*d^22 + 1987844*a^10*b^14*c^4*d^20 - 10009720*a^10*b^14*c^6*d^18 + 25068800*a^10*b^14*c^8*d^16 - 34796936*a^10*b^14*c^10*d^14 + 26927144*a^10*b^14*c^12*d^12 - 10994964*a^10*b^14*c^14*d^10 + 2295680*a^10*b^14*c^16*d^8 - 230240*a^10*b^14*c^18*d^6 - 860368*a^11*b^13*c^3*d^21 + 5890780*a^11*b^13*c^5*d^19 - 18576800*a^11*b^13*c^7*d^17 + 31562040*a^11*b^13*c^9*d^15 - 29722864*a^11*b^13*c^11*d^13 + 14679348*a^11*b^13*c^13*d^11 - 3497920*a^11*b^13*c^15*d^9 + 381280*a^11*b^13*c^17*d^7 + 270604*a^12*b^12*c^2*d^22 - 2750664*a^12*b^12*c^4*d^20 + 11209800*a^12*b^12*c^6*d^18 - 23486280*a^12*b^12*c^8*d^16 + 26927144*a^12*b^12*c^10*d^14 - 16147404*a^12*b^12*c^12*d^12 + 4479104*a^12*b^12*c^14*d^10 - 499520*a^12*b^12*c^16*d^8 + 1001364*a^13*b^11*c^3*d^21 - 5437600*a^13*b^11*c^5*d^19 + 14230520*a^13*b^11*c^7*d^17 - 19993760*a^13*b^11*c^9*d^15 + 14679348*a^13*b^11*c^11*d^13 - 4861024*a^13*b^11*c^13*d^11 + 552160*a^13*b^11*c^15*d^9 - 273544*a^14*b^10*c^2*d^22 + 2073976*a^14*b^10*c^4*d^20 - 6943760*a^14*b^10*c^6*d^18 + 12099640*a^14*b^10*c^8*d^16 - 10994964*a^14*b^10*c^10*d^14 + 4479104*a^14*b^10*c^12*d^12 - 562016*a^14*b^10*c^14*d^10 - 605280*a^15*b^9*c^3*d^21 + 2682536*a^15*b^9*c^5*d^19 - 5889904*a^15*b^9*c^7*d^17 + 6731044*a^15*b^9*c^9*d^15 - 3497920*a^15*b^9*c^11*d^13 + 552160*a^15*b^9*c^13*d^11 + 131660*a^16*b^8*c^2*d^22 - 793460*a^16*b^8*c^4*d^20 + 2253214*a^16*b^8*c^6*d^18 - 3330518*a^16*b^8*c^8*d^16 + 2295680*a^16*b^8*c^10*d^14 - 499520*a^16*b^8*c^12*d^12 + 167520*a^17*b^7*c^3*d^21 - 655084*a^17*b^7*c^5*d^19 + 1310504*a^17*b^7*c^7*d^17 - 1239264*a^17*b^7*c^9*d^15 + 381280*a^17*b^7*c^11*d^13 - 22060*a^18*b^6*c^2*d^22 + 138010*a^18*b^6*c^4*d^20 - 396878*a^18*b^6*c^6*d^18 + 529224*a^18*b^6*c^8*d^16 - 230240*a^18*b^6*c^10*d^14 - 18840*a^19*b^5*c^3*d^21 + 85484*a^19*b^5*c^5*d^19 - 168344*a^19*b^5*c^7*d^17 + 106016*a^19*b^5*c^9*d^15 + 782*a^20*b^4*c^2*d^22 - 10562*a^20*b^4*c^4*d^20 + 36120*a^20*b^4*c^6*d^18 - 36280*a^20*b^4*c^8*d^16 + 144*a^21*b^3*c^3*d^21 - 4080*a^21*b^3*c^5*d^19 + 8960*a^21*b^3*c^7*d^1

7 + 98*a^22*b^2*c^2*d^22 - 88*a^22*b^2*c^4*d^20 - 1520*a^22*b^2*c^6*d^18 + 4*a*b^23*c^23*d + 4*a^23*b*c*d^23)/(16*(5*a^2*b^28*c^30 - b^30*c^30 - a^30*d^30 - 10*a^4*b^26*c^30 + 10*a^6*b^24*c^30 - 5*a^8*b^22*c^30 + a^10*b^20*c^30 + a^20*b^10*d^30 - 5*a^22*b^8*d^30 + 10*a^24*b^6*d^30 - 10*a^26*b^4*d^30 + 5*a^28*b^2*d^30 + 5*a^30*c^2*d^28 - 10*a^30*c^4*d^26 + 10*a^30*c^6*d^24 - 5*a^30*c^8*d^22 + a^30*c^10*d^20 + b^30*c^20*d^10 - 5*b^30*c^22*d^8 + 10*b^30*c^24*d^6 - 10*b^30*c^26*d^4 + 5*b^30*c^28*d^2 - 20*a*b^29*c^19*d^11 + 100*a*b^29*c^21*d^9 - 200*a*b^29*c^23*d^7 + 200*a*b^29*c^25*d^5 - 100*a*b^29*c^27*d^3 - 100*a^3*b^27*c^29*d + 200*a^5*b^25*c^29*d - 200*a^7*b^23*c^29*d + 100*a^9*b^21*c^29*d - 20*a^11*b^19*c^29*d - 20*a^19*b^11*c*d^29 + 100*a^21*b^9*c*d^29 - 200*a^23*b^7*c*d^29 + 200*a^25*b^5*c*d^29 - 100*a^27*b^3*c*d^29 - 100*a^29*b*c^3*d^27 + 200*a^29*b*c^5*d^25 - 200*a^29*b*c^7*d^23 + 100*a^29*b*c^9*d^21 - 20*a^29*b*c^11*d^19 + 190*a^2*b^28*c^18*d^12 - 955*a^2*b^28*c^20*d^10 + 1925*a^2*b^28*c^22*d^8 - 1950*a^2*b^28*c^24*d^6 + 1000*a^2*b^28*c^26*d^4 - 215*a^2*b^28*c^28*d^2 - 1140*a^3*b^27*c^17*d^13 + 5800*a^3*b^27*c^19*d^11 - 11900*a^3*b^27*c^21*d^9 + 12400*a^3*b^27*c^23*d^7 - 6700*a^3*b^27*c^25*d^5 + 1640*a^3*b^27*c^27*d^3 + 4845*a^4*b^26*c^16*d^14 - 25175*a^4*b^26*c^18*d^12 + 53210*a^4*b^26*c^20*d^10 - 58000*a^4*b^26*c^22*d^8 + 33825*a^4*b^26*c^24*d^6 - 9695*a^4*b^26*c^26*d^4 + 1000*a^4*b^26*c^28*d^2 - 15504*a^5*b^25*c^15*d^15 + 83220*a^5*b^25*c^17*d^13 - 183740*a^5*b^25*c^19*d^11 + 213040*a^5*b^25*c^21*d^9 - 136520*a^5*b^25*c^23*d^7 + 46004*a^5*b^25*c^25*d^5 - 6700*a^5*b^25*c^27*d^3 + 38760*a^6*b^24*c^14*d^16 - 218025*a^6*b^24*c^16*d^14 + 510625*a^6*b^24*c^18*d^12 - 639360*a^6*b^24*c^20*d^10 + 455100*a^6*b^24*c^22*d^8 - 178985*a^6*b^24*c^24*d^6 + 33825*a^6*b^24*c^26*d^4 - 1950*a^6*b^24*c^28*d^2 - 77520*a^7*b^23*c^13*d^17 + 465120*a^7*b^23*c^15*d^15 - 1174200*a^7*b^23*c^17*d^13 + 1607600*a^7*b^23*c^19*d^11 - 1277800*a^7*b^23*c^21*d^9 + 581120*a^7*b^23*c^23*d^7 - 136520*a^7*b^23*c^25*d^5 + 12400*a^7*b^23*c^27*d^3 + 125970*a^8*b^22*c^12*d^18 - 823650*a^8*b^22*c^14*d^16 + 2277150*a^8*b^22*c^16*d^14 - 3441850*a^8*b^22*c^18*d^12 + 3061855*a^8*b^22*c^20*d^10 - 1598495*a^8*b^22*c^22*d^8 + 455100*a^8*b^22*c^24*d^6 - 58000*a^8*b^22*c^26*d^4 + 1925*a^8*b^22*c^28*d^2 - 167960*a^9*b^21*c^11*d^19 + 1227400*a^9*b^21*c^13*d^17 - 3772640*a^9*b^21*c^15*d^15 + 6342200*a^9*b^21*c^17*d^13 - 6323300*a^9*b^21*c^19*d^11 + 3770860*a^9*b^21*c^21*d^9 - 1277800*a^9*b^21*c^23*d^7 + 213040*a^9*b^21*c^25*d^5 - 11900*a^9*b^21*c^27*d^3 + 184756*a^10*b^20*c^10*d^20 - 1553630*a^10*b^20*c^12*d^18 + 5384410*a^10*b^20*c^14*d^16 - 10132510*a^10*b^20*c^16*d^14 + 11341480*a^10*b^20*c^18*d^12 - 7699257*a^10*b^20*c^20*d^10 + 3061855*a^10*b^20*c^22*d^8 - 639360*a^10*b^20*c^24*d^6 + 53210*a^10*b^20*c^26*d^4 - 955*a^10*b^20*c^28*d^2 - 167960*a^11*b^19*c^9*d^21 + 1679600*a^11*b^19*c^11*d^19 - 6653800*a^11*b^19*c^13*d^17 + 14108640*a^11*b^19*c^15*d^15 - 17770700*a^11*b^19*c^17*d^13 + 13697880*a^11*b^19*c^19*d^11 - 6323300*a^11*b^19*c^21*d^9 + 1607600*a^11*b^19*c^23*d^7 - 183740*a^11*b^19*c^25*d^5 + 5800*a^11*b^19*c^27*d^3 + 125970*a^12*b^18*c^8*d^22 - 1553630*a^12*b^18*c^10*d^20 + 7138300*a^12*b^18*c^12*d^18 - 17183600*a^12*b^18*c^14*d^16 + 24426875*a^12*b^18*c^16*d^14 - 21339185*a^12*b^18*c^18*d^12 + 11341480*a^12*b^18*c^20*d^10 - 3441850*a^12*b^18*c^22*d^8

+ 510625*a^12*b^18*c^24*d^6 - 25175*a^12*b^18*c^26*d^4 + 190*a^12*b^18*c^2
8*d^2 - 77520*a^13*b^17*c^7*d^23 + 1227400*a^13*b^17*c^9*d^21 - 6653800*a^1
3*b^17*c^11*d^19 + 18346400*a^13*b^17*c^13*d^17 - 29535120*a^13*b^17*c^15*d
^15 + 29213260*a^13*b^17*c^17*d^13 - 17770700*a^13*b^17*c^19*d^11 + 6342200
*a^13*b^17*c^21*d^9 - 1174200*a^13*b^17*c^23*d^7 + 83220*a^13*b^17*c^25*d^5
 - 1140*a^13*b^17*c^27*d^3 + 38760*a^14*b^16*c^6*d^24 - 823650*a^14*b^16*c^
8*d^22 + 5384410*a^14*b^16*c^10*d^20 - 17183600*a^14*b^16*c^12*d^18 + 31460
200*a^14*b^16*c^14*d^16 - 35234455*a^14*b^16*c^16*d^14 + 24426875*a^14*b^16
*c^18*d^12 - 10132510*a^14*b^16*c^20*d^10 + 2277150*a^14*b^16*c^22*d^8 - 21
8025*a^14*b^16*c^24*d^6 + 4845*a^14*b^16*c^26*d^4 - 15504*a^15*b^15*c^5*d^2
5 + 465120*a^15*b^15*c^7*d^23 - 3772640*a^15*b^15*c^9*d^21 + 14108640*a^15*
b^15*c^11*d^19 - 29535120*a^15*b^15*c^13*d^17 + 37499008*a^15*b^15*c^15*d^1
5 - 29535120*a^15*b^15*c^17*d^13 + 14108640*a^15*b^15*c^19*d^11 - 3772640*a
^15*b^15*c^21*d^9 + 465120*a^15*b^15*c^23*d^7 - 15504*a^15*b^15*c^25*d^5 +
4845*a^16*b^14*c^4*d^26 - 218025*a^16*b^14*c^6*d^24 + 2277150*a^16*b^14*c^8
*d^22 - 10132510*a^16*b^14*c^10*d^20 + 24426875*a^16*b^14*c^12*d^18 - 35234
455*a^16*b^14*c^14*d^16 + 31460200*a^16*b^14*c^16*d^14 - 17183600*a^16*b^14
*c^18*d^12 + 5384410*a^16*b^14*c^20*d^10 - 823650*a^16*b^14*c^22*d^8 + 3876
0*a^16*b^14*c^24*d^6 - 1140*a^17*b^13*c^3*d^27 + 83220*a^17*b^13*c^5*d^25 -
 1174200*a^17*b^13*c^7*d^23 + 6342200*a^17*b^13*c^9*d^21 - 17770700*a^17*b^
13*c^11*d^19 + 29213260*a^17*b^13*c^13*d^17 - 29535120*a^17*b^13*c^15*d^15
+ 18346400*a^17*b^13*c^17*d^13 - 6653800*a^17*b^13*c^19*d^11 + 1227400*a^17
*b^13*c^21*d^9 - 77520*a^17*b^13*c^23*d^7 + 190*a^18*b^12*c^2*d^28 - 25175*
a^18*b^12*c^4*d^26 + 510625*a^18*b^12*c^6*d^24 - 3441850*a^18*b^12*c^8*d^22
 + 11341480*a^18*b^12*c^10*d^20 - 21339185*a^18*b^12*c^12*d^18 + 24426875*a
^18*b^12*c^14*d^16 - 17183600*a^18*b^12*c^16*d^14 + 7138300*a^18*b^12*c^18*
d^12 - 1553630*a^18*b^12*c^20*d^10 + 125970*a^18*b^12*c^22*d^8 + 5800*a^19*
b^11*c^3*d^27 - 183740*a^19*b^11*c^5*d^25 + 1607600*a^19*b^11*c^7*d^23 - 63
23300*a^19*b^11*c^9*d^21 + 13697880*a^19*b^11*c^11*d^19 - 17770700*a^19*b^1
1*c^13*d^17 + 14108640*a^19*b^11*c^15*d^15 - 6653800*a^19*b^11*c^17*d^13 +
1679600*a^19*b^11*c^19*d^11 - 167960*a^19*b^11*c^21*d^9 - 955*a^20*b^10*c^2
*d^28 + 53210*a^20*b^10*c^4*d^26 - 639360*a^20*b^10*c^6*d^24 + 3061855*a^20
*b^10*c^8*d^22 - 7699257*a^20*b^10*c^10*d^20 + 11341480*a^20*b^10*c^12*d^18
 - 10132510*a^20*b^10*c^14*d^16 + 5384410*a^20*b^10*c^16*d^14 - 1553630*a^2
0*b^10*c^18*d^12 + 184756*a^20*b^10*c^20*d^10 - 11900*a^21*b^9*c^3*d^27 + 2
13040*a^21*b^9*c^5*d^25 - 1277800*a^21*b^9*c^7*d^23 + 3770860*a^21*b^9*c^9*
d^21 - 6323300*a^21*b^9*c^11*d^19 + 6342200*a^21*b^9*c^13*d^17 - 3772640*a^
21*b^9*c^15*d^15 + 1227400*a^21*b^9*c^17*d^13 - 167960*a^21*b^9*c^19*d^11 +
 1925*a^22*b^8*c^2*d^28 - 58000*a^22*b^8*c^4*d^26 + 455100*a^22*b^8*c^6*d^2
4 - 1598495*a^22*b^8*c^8*d^22 + 3061855*a^22*b^8*c^10*d^20 - 3441850*a^22*b
^8*c^12*d^18 + 2277150*a^22*b^8*c^14*d^16 - 823650*a^22*b^8*c^16*d^14 + 125
970*a^22*b^8*c^18*d^12 + 12400*a^23*b^7*c^3*d^27 - 136520*a^23*b^7*c^5*d^25
 + 581120*a^23*b^7*c^7*d^23 - 1277800*a^23*b^7*c^9*d^21 + 1607600*a^23*b^7*
c^11*d^19 - 1174200*a^23*b^7*c^13*d^17 + 465120*a^23*b^7*c^15*d^15 - 77520*
a^23*b^7*c^17*d^13 - 1950*a^24*b^6*c^2*d^28 + 33825*a^24*b^6*c^4*d^26 - 178

985*a^24*b^6*c^6*d^24 + 455100*a^24*b^6*c^8*d^22 - 639360*a^24*b^6*c^10*d^2
0 + 510625*a^24*b^6*c^12*d^18 - 218025*a^24*b^6*c^14*d^16 + 38760*a^24*b^6*
c^16*d^14 - 6700*a^25*b^5*c^3*d^27 + 46004*a^25*b^5*c^5*d^25 - 136520*a^25*
b^5*c^7*d^23 + 213040*a^25*b^5*c^9*d^21 - 183740*a^25*b^5*c^11*d^19 + 83220
*a^25*b^5*c^13*d^17 - 15504*a^25*b^5*c^15*d^15 + 1000*a^26*b^4*c^2*d^28 - 9
695*a^26*b^4*c^4*d^26 + 33825*a^26*b^4*c^6*d^24 - 58000*a^26*b^4*c^8*d^22 +
 53210*a^26*b^4*c^10*d^20 - 25175*a^26*b^4*c^12*d^18 + 4845*a^26*b^4*c^14*d
^16 + 1640*a^27*b^3*c^3*d^27 - 6700*a^27*b^3*c^5*d^25 + 12400*a^27*b^3*c^7*
d^23 - 11900*a^27*b^3*c^9*d^21 + 5800*a^27*b^3*c^11*d^19 - 1140*a^27*b^3*c^
13*d^17 - 215*a^28*b^2*c^2*d^28 + 1000*a^28*b^2*c^4*d^26 - 1950*a^28*b^2*c^
6*d^24 + 1925*a^28*b^2*c^8*d^22 - 955*a^28*b^2*c^10*d^20 + 190*a^28*b^2*c^1
2*d^18 + 20*a*b^29*c^29*d + 20*a^29*b*c*d^29)))^(1/2)*(((((4*a^24*d^24 + 4*
b^24*c^24 + 16*a^2*b^22*c^24 + 16*a^4*b^20*c^24 - 1152*a^10*b^14*d^24 + 556
8*a^12*b^12*d^24 - 10568*a^14*b^10*d^24 + 9460*a^16*b^8*d^24 - 3560*a^18*b^
6*d^24 + 136*a^20*b^4*d^24 + 76*a^22*b^2*d^24 + 16*a^24*c^2*d^22 + 16*a^24*
c^4*d^20 - 1152*b^24*c^10*d^14 + 5568*b^24*c^12*d^12 - 10568*b^24*c^14*d^10
 + 9460*b^24*c^16*d^8 - 3560*b^24*c^18*d^6 + 136*b^24*c^20*d^4 + 76*b^24*c^
22*d^2 + 11520*a*b^23*c^9*d^15 - 56448*a*b^23*c^11*d^13 + 109456*a*b^23*c^1
3*d^11 - 101240*a*b^23*c^15*d^9 + 40720*a*b^23*c^17*d^7 - 2960*a*b^23*c^19*
d^5 - 536*a*b^23*c^21*d^3 - 176*a^3*b^21*c^23*d - 320*a^5*b^19*c^23*d + 115
20*a^9*b^15*c*d^23 - 56448*a^11*b^13*c*d^23 + 109456*a^13*b^11*c*d^23 - 101
240*a^15*b^9*c*d^23 + 40720*a^17*b^7*c*d^23 - 2960*a^19*b^5*c*d^23 - 536*a^
21*b^3*c*d^23 - 176*a^23*b*c^3*d^21 - 320*a^23*b*c^5*d^19 - 51840*a^2*b^22*
c^8*d^16 + 263808*a^2*b^22*c^10*d^14 - 541208*a^2*b^22*c^12*d^12 + 547088*a
^2*b^22*c^14*d^10 - 263320*a^2*b^22*c^16*d^8 + 44120*a^2*b^22*c^18*d^6 - 15
64*a^2*b^22*c^20*d^4 - 196*a^2*b^22*c^22*d^2 + 138240*a^3*b^21*c^7*d^17 - 7
58400*a^3*b^21*c^9*d^15 + 1720736*a^3*b^21*c^11*d^13 - 2002728*a^3*b^21*c^1
3*d^11 + 1210560*a^3*b^21*c^15*d^9 - 335040*a^3*b^21*c^17*d^7 + 37680*a^3*b
^21*c^19*d^5 - 288*a^3*b^21*c^21*d^3 - 241920*a^4*b^20*c^6*d^18 + 1512000*a
^4*b^20*c^8*d^16 - 3975688*a^4*b^20*c^10*d^14 + 5501328*a^4*b^20*c^12*d^12
 - 4147952*a^4*b^20*c^14*d^10 + 1586920*a^4*b^20*c^16*d^8 - 276020*a^4*b^20*
c^18*d^6 + 21124*a^4*b^20*c^20*d^4 + 176*a^4*b^20*c^22*d^2 + 290304*a^5*b^1
9*c^5*d^19 - 2232576*a^5*b^19*c^7*d^17 + 7078256*a^5*b^19*c^9*d^15 - 117815
60*a^5*b^19*c^11*d^13 + 10875200*a^5*b^19*c^13*d^11 - 5365072*a^5*b^19*c^15
*d^9 + 1310168*a^5*b^19*c^17*d^7 - 170968*a^5*b^19*c^19*d^5 + 8160*a^5*b^19
*c^21*d^3 - 241920*a^6*b^18*c^4*d^20 + 2532096*a^6*b^18*c^6*d^18 - 9955992*
a^6*b^18*c^8*d^16 + 20019440*a^6*b^18*c^10*d^14 - 22419600*a^6*b^18*c^12*d^
12 + 13887520*a^6*b^18*c^14*d^10 - 4506428*a^6*b^18*c^16*d^8 + 793756*a^6*b
^18*c^18*d^6 - 72240*a^6*b^18*c^20*d^4 + 3040*a^6*b^18*c^22*d^2 + 138240*a^
7*b^17*c^3*d^21 - 2232576*a^7*b^17*c^5*d^19 + 11150016*a^7*b^17*c^7*d^17 -
27336616*a^7*b^17*c^9*d^15 + 37153600*a^7*b^17*c^11*d^13 - 28461040*a^7*b^1
7*c^13*d^11 + 11779808*a^7*b^17*c^15*d^9 - 2621008*a^7*b^17*c^17*d^7 + 3366
88*a^7*b^17*c^19*d^5 - 17920*a^7*b^17*c^21*d^3 - 51840*a^8*b^16*c^2*d^22 +
1512000*a^8*b^16*c^4*d^20 - 9955992*a^8*b^16*c^6*d^18 + 30289656*a^8*b^16*c
^8*d^16 - 50137600*a^8*b^16*c^10*d^14 + 46972560*a^8*b^16*c^12*d^12 - 24199

280*a^8*b^16*c^14*d^10 + 6661036*a^8*b^16*c^16*d^8 - 1058448*a^8*b^16*c^18*d^6 + 72560*a^8*b^16*c^20*d^4 - 758400*a^9*b^15*c^3*d^21 + 7078256*a^9*b^15*c^5*d^19 - 27336616*a^9*b^15*c^7*d^17 + 55383904*a^9*b^15*c^9*d^15 - 63124080*a^9*b^15*c^11*d^13 + 39987520*a^9*b^15*c^13*d^11 - 13462088*a^9*b^15*c^15*d^9 + 2478528*a^9*b^15*c^17*d^7 - 212032*a^9*b^15*c^19*d^5 + 263808*a^10*b^14*c^2*d^22 - 3975688*a^10*b^14*c^4*d^20 + 20019440*a^10*b^14*c^6*d^18 - 50137600*a^10*b^14*c^8*d^16 + 69593872*a^10*b^14*c^10*d^14 - 53854288*a^10*b^14*c^12*d^12 + 21989928*a^10*b^14*c^14*d^10 - 4591360*a^10*b^14*c^16*d^8 + 460480*a^10*b^14*c^18*d^6 + 1720736*a^11*b^13*c^3*d^21 - 11781560*a^11*b^13*c^5*d^19 + 37153600*a^11*b^13*c^7*d^17 - 63124080*a^11*b^13*c^9*d^15 + 59445728*a^11*b^13*c^11*d^13 - 29358696*a^11*b^13*c^13*d^11 + 6995840*a^11*b^13*c^15*d^9 - 762560*a^11*b^13*c^17*d^7 - 541208*a^12*b^12*c^2*d^22 + 5501328*a^12*b^12*c^4*d^20 - 22419600*a^12*b^12*c^6*d^18 + 46972560*a^12*b^12*c^8*d^16 - 53854288*a^12*b^12*c^10*d^14 + 32294808*a^12*b^12*c^12*d^12 - 8958208*a^12*b^12*c^14*d^10 + 999040*a^12*b^12*c^16*d^8 - 2002728*a^13*b^11*c^3*d^21 + 10875200*a^13*b^11*c^5*d^19 - 28461040*a^13*b^11*c^7*d^17 + 39987520*a^13*b^11*c^9*d^15 - 29358696*a^13*b^11*c^11*d^13 + 9722048*a^13*b^11*c^13*d^11 - 1104320*a^13*b^11*c^15*d^9 + 547088*a^14*b^10*c^2*d^22 - 4147952*a^14*b^10*c^4*d^20 + 13887520*a^14*b^10*c^6*d^18 - 24199280*a^14*b^10*c^8*d^16 + 21989928*a^14*b^10*c^10*d^14 - 8958208*a^14*b^10*c^12*d^12 + 1124032*a^14*b^10*c^14*d^10 + 1210560*a^15*b^9*c^3*d^21 - 5365072*a^15*b^9*c^5*d^19 + 11779808*a^15*b^9*c^7*d^17 - 13462088*a^15*b^9*c^9*d^15 + 6995840*a^15*b^9*c^11*d^13 - 1104320*a^15*b^9*c^13*d^11 - 263320*a^16*b^8*c^2*d^22 + 1586920*a^16*b^8*c^4*d^20 - 4506428*a^16*b^8*c^6*d^18 + 6661036*a^16*b^8*c^8*d^16 - 4591360*a^16*b^8*c^10*d^14 + 999040*a^16*b^8*c^12*d^12 - 335040*a^17*b^7*c^3*d^21 + 1310168*a^17*b^7*c^5*d^19 - 2621008*a^17*b^7*c^7*d^17 + 2478528*a^17*b^7*c^9*d^15 - 762560*a^17*b^7*c^11*d^13 + 44120*a^18*b^6*c^2*d^22 - 276020*a^18*b^6*c^4*d^20 + 793756*a^18*b^6*c^6*d^18 - 1058448*a^18*b^6*c^8*d^16 + 460480*a^18*b^6*c^10*d^14 + 37680*a^19*b^5*c^3*d^21 - 170968*a^19*b^5*c^5*d^19 + 336688*a^19*b^5*c^7*d^17 - 212032*a^19*b^5*c^9*d^15 - 1564*a^20*b^4*c^2*d^22 + 21124*a^20*b^4*c^4*d^20 - 72240*a^20*b^4*c^6*d^18 + 72560*a^20*b^4*c^8*d^16 - 288*a^21*b^3*c^3*d^21 + 8160*a^21*b^3*c^5*d^19 - 17920*a^21*b^3*c^7*d^17 - 196*a^22*b^2*c^2*d^22 + 176*a^22*b^2*c^4*d^20 + 3040*a^22*b^2*c^6*d^18 - 8*a*b^23*c^23*d - 8*a^23*b*c*d^23)^2/4 - (20736*b^18*d^18 - 96768*a^2*b^16*d^18 + 173664*a^4*b^14*d^18 - 136032*a^6*b^12*d^18 + 31081*a^8*b^10*d^18 + 8440*a^10*b^8*d^18 + 400*a^12*b^6*d^18 - 96768*b^18*c^2*d^16 + 173664*b^18*c^4*d^14 - 136032*b^18*c^6*d^12 + 31081*b^18*c^8*d^10 + 8440*b^18*c^10*d^8 + 400*b^18*c^12*d^6 - 131328*a*b^17*c^3*d^15 + 216576*a*b^17*c^5*d^13 - 141104*a*b^17*c^7*d^11 + 20260*a*b^17*c^9*d^9 + 2800*a*b^17*c^11*d^7 - 131328*a^3*b^15*c*d^17 + 216576*a^5*b^13*c*d^17 - 141104*a^7*b^11*c*d^17 + 20260*a^9*b^9*c*d^17 + 2800*a^11*b^7*c*d^17 + 495936*a^2*b^16*c^2*d^16 - 989856*a^2*b^16*c^4*d^14 + 901948*a^2*b^16*c^6*d^12 - 308392*a^2*b^16*c^8*d^10 - 5260*a^2*b^16*c^10*d^8 + 1600*a^2*b^16*c^12*d^6 + 657408*a^3*b^15*c^3*d^15 - 1158992*a^3*b^15*c^5*d^13 + 838256*a^3*b^15*c^7*d^11 - 182200*a^3*b^15*c^9*d^9 - 3200*a^3*b^15*c^11*d^7 - 989856*a^4*b^14*c^2*d^16

+ 2185654*a^4*b^14*c^4*d^14 - 2218576*a^4*b^14*c^6*d^12 + 900624*a^4*b^14*c^8*d^10 - 64720*a^4*b^14*c^10*d^8 + 1600*a^4*b^14*c^12*d^6 - 1158992*a^5*b^13*c^3*d^15 + 2158808*a^5*b^13*c^5*d^13 - 1641528*a^5*b^13*c^7*d^11 + 406880*a^5*b^13*c^9*d^9 - 17600*a^5*b^13*c^11*d^7 + 901948*a^6*b^12*c^2*d^16 - 2218576*a^6*b^12*c^4*d^14 + 2430936*a^6*b^12*c^6*d^12 - 1026928*a^6*b^12*c^8*d^10 + 88720*a^6*b^12*c^10*d^8 + 838256*a^7*b^11*c^3*d^15 - 1641528*a^7*b^11*c^5*d^13 + 1206848*a^7*b^11*c^7*d^11 - 239360*a^7*b^11*c^9*d^9 - 308392*a^8*b^10*c^2*d^16 + 900624*a^8*b^10*c^4*d^14 - 1026928*a^8*b^10*c^6*d^12 + 354016*a^8*b^10*c^8*d^10 - 182200*a^9*b^9*c^3*d^15 + 406880*a^9*b^9*c^5*d^13 - 239360*a^9*b^9*c^7*d^11 - 5260*a^10*b^8*c^2*d^16 - 64720*a^10*b^8*c^4*d^14 + 88720*a^10*b^8*c^6*d^12 - 3200*a^11*b^7*c^3*d^15 - 17600*a^11*b^7*c^5*d^13 + 1600*a^12*b^6*c^2*d^16 + 1600*a^12*b^6*c^4*d^14 + 27648*a*b^17*c*d^17)*(80*a^2*b^28*c^30 - 16*b^30*c^30 - 16*a^30*d^30 - 160*a^4*b^26*c^30 + 160*a^6*b^24*c^30 - 80*a^8*b^22*c^30 + 16*a^10*b^20*c^30 + 16*a^20*b^10*d^30 - 80*a^22*b^8*d^30 + 160*a^24*b^6*d^30 - 160*a^26*b^4*d^30 + 80*a^28*b^2*d^30 + 80*a^30*c^2*d^28 - 160*a^30*c^4*d^26 + 160*a^30*c^6*d^24 - 80*a^30*c^8*d^22 + 16*a^30*c^10*d^20 + 16*b^30*c^20*d^10 - 80*b^30*c^22*d^8 + 160*b^30*c^24*d^6 - 160*b^30*c^26*d^4 + 80*b^30*c^28*d^2 - 320*a*b^29*c^19*d^11 + 1600*a*b^29*c^21*d^9 - 3200*a*b^29*c^23*d^7 + 3200*a*b^29*c^25*d^5 - 1600*a*b^29*c^27*d^3 - 1600*a^3*b^27*c^29*d + 3200*a^5*b^25*c^29*d - 3200*a^7*b^23*c^29*d + 1600*a^9*b^21*c^29*d - 320*a^11*b^19*c^29*d - 320*a^19*b^11*c*d^29 + 1600*a^21*b^9*c*d^29 - 3200*a^23*b^7*c*d^29 + 3200*a^25*b^5*c*d^29 - 1600*a^27*b^3*c*d^29 - 1600*a^29*b*c^3*d^27 + 3200*a^29*b*c^5*d^25 - 3200*a^29*b*c^7*d^23 + 1600*a^29*b*c^9*d^21 - 320*a^29*b*c^11*d^19 + 3040*a^2*b^28*c^18*d^12 - 15280*a^2*b^28*c^20*d^10 + 30800*a^2*b^28*c^22*d^8 - 31200*a^2*b^28*c^24*d^6 + 16000*a^2*b^28*c^26*d^4 - 3440*a^2*b^28*c^28*d^2 - 18240*a^3*b^27*c^17*d^13 + 92800*a^3*b^27*c^19*d^11 - 190400*a^3*b^27*c^21*d^9 + 198400*a^3*b^27*c^23*d^7 - 107200*a^3*b^27*c^25*d^5 + 26240*a^3*b^27*c^27*d^3 + 77520*a^4*b^26*c^16*d^14 - 402800*a^4*b^26*c^18*d^12 + 851360*a^4*b^26*c^20*d^10 - 928000*a^4*b^26*c^22*d^8 + 541200*a^4*b^26*c^24*d^6 - 155120*a^4*b^26*c^26*d^4 + 16000*a^4*b^26*c^28*d^2 - 248064*a^5*b^25*c^15*d^15 + 1331520*a^5*b^25*c^17*d^13 - 2939840*a^5*b^25*c^19*d^11 + 3408640*a^5*b^25*c^21*d^9 - 2184320*a^5*b^25*c^23*d^7 + 736064*a^5*b^25*c^25*d^5 - 107200*a^5*b^25*c^27*d^3 + 620160*a^6*b^24*c^14*d^16 - 3488400*a^6*b^24*c^16*d^14 + 8170000*a^6*b^24*c^18*d^12 - 10229760*a^6*b^24*c^20*d^10 + 7281600*a^6*b^24*c^22*d^8 - 2863760*a^6*b^24*c^24*d^6 + 541200*a^6*b^24*c^26*d^4 - 31200*a^6*b^24*c^28*d^2 - 1240320*a^7*b^23*c^13*d^17 + 7441920*a^7*b^23*c^15*d^15 - 18787200*a^7*b^23*c^17*d^13 + 25721600*a^7*b^23*c^19*d^11 - 20444800*a^7*b^23*c^21*d^9 + 9297920*a^7*b^23*c^23*d^7 - 2184320*a^7*b^23*c^25*d^5 + 198400*a^7*b^23*c^27*d^3 + 2015520*a^8*b^22*c^12*d^18 - 13178400*a^8*b^22*c^14*d^16 + 36434400*a^8*b^22*c^16*d^14 - 55069600*a^8*b^22*c^18*d^12 + 48989680*a^8*b^22*c^20*d^10 - 25575920*a^8*b^22*c^22*d^8 + 7281600*a^8*b^22*c^24*d^6 - 928000*a^8*b^22*c^26*d^4 + 30800*a^8*b^22*c^28*d^2 - 2687360*a^9*b^21*c^11*d^19 + 19638400*a^9*b^21*c^13*d^17 - 60362240*a^9*b^21*c^15*d^15 + 101475200*a^9*b^21*c^17*d^13 - 101172800*a^9*b^21*c^19*d^11 + 60333760*a^9*b^21*c

^21*d^9 - 20444800*a^9*b^21*c^23*d^7 + 3408640*a^9*b^21*c^25*d^5 - 190400*a
^9*b^21*c^27*d^3 + 2956096*a^10*b^20*c^10*d^20 - 24858080*a^10*b^20*c^12*d^
18 + 86150560*a^10*b^20*c^14*d^16 - 162120160*a^10*b^20*c^16*d^14 + 1814636
80*a^10*b^20*c^18*d^12 - 123188112*a^10*b^20*c^20*d^10 + 48989680*a^10*b^20
*c^22*d^8 - 10229760*a^10*b^20*c^24*d^6 + 851360*a^10*b^20*c^26*d^4 - 15280
*a^10*b^20*c^28*d^2 - 2687360*a^11*b^19*c^9*d^21 + 26873600*a^11*b^19*c^11*
d^19 - 106460800*a^11*b^19*c^13*d^17 + 225738240*a^11*b^19*c^15*d^15 - 2843
31200*a^11*b^19*c^17*d^13 + 219166080*a^11*b^19*c^19*d^11 - 101172800*a^11*
b^19*c^21*d^9 + 25721600*a^11*b^19*c^23*d^7 - 2939840*a^11*b^19*c^25*d^5 +
92800*a^11*b^19*c^27*d^3 + 2015520*a^12*b^18*c^8*d^22 - 24858080*a^12*b^18*
c^10*d^20 + 114212800*a^12*b^18*c^12*d^18 - 274937600*a^12*b^18*c^14*d^16 +
 390830000*a^12*b^18*c^16*d^14 - 341426960*a^12*b^18*c^18*d^12 + 181463680*
a^12*b^18*c^20*d^10 - 55069600*a^12*b^18*c^22*d^8 + 8170000*a^12*b^18*c^24*
d^6 - 402800*a^12*b^18*c^26*d^4 + 3040*a^12*b^18*c^28*d^2 - 1240320*a^13*b^
17*c^7*d^23 + 19638400*a^13*b^17*c^9*d^21 - 106460800*a^13*b^17*c^11*d^19 +
 293542400*a^13*b^17*c^13*d^17 - 472561920*a^13*b^17*c^15*d^15 + 467412160*
a^13*b^17*c^17*d^13 - 284331200*a^13*b^17*c^19*d^11 + 101475200*a^13*b^17*c
^21*d^9 - 18787200*a^13*b^17*c^23*d^7 + 1331520*a^13*b^17*c^25*d^5 - 18240*
a^13*b^17*c^27*d^3 + 620160*a^14*b^16*c^6*d^24 - 13178400*a^14*b^16*c^8*d^2
2 + 86150560*a^14*b^16*c^10*d^20 - 274937600*a^14*b^16*c^12*d^18 + 50336320
0*a^14*b^16*c^14*d^16 - 563751280*a^14*b^16*c^16*d^14 + 390830000*a^14*b^16
*c^18*d^12 - 162120160*a^14*b^16*c^20*d^10 + 36434400*a^14*b^16*c^22*d^8 -
3488400*a^14*b^16*c^24*d^6 + 77520*a^14*b^16*c^26*d^4 - 248064*a^15*b^15*c^
5*d^25 + 7441920*a^15*b^15*c^7*d^23 - 60362240*a^15*b^15*c^9*d^21 + 2257382
40*a^15*b^15*c^11*d^19 - 472561920*a^15*b^15*c^13*d^17 + 599984128*a^15*b^1
5*c^15*d^15 - 472561920*a^15*b^15*c^17*d^13 + 225738240*a^15*b^15*c^19*d^11
 - 60362240*a^15*b^15*c^21*d^9 + 7441920*a^15*b^15*c^23*d^7 - 248064*a^15*b
^15*c^25*d^5 + 77520*a^16*b^14*c^4*d^26 - 3488400*a^16*b^14*c^6*d^24 + 3643
4400*a^16*b^14*c^8*d^22 - 162120160*a^16*b^14*c^10*d^20 + 390830000*a^16*b^
14*c^12*d^18 - 563751280*a^16*b^14*c^14*d^16 + 503363200*a^16*b^14*c^16*d^1
4 - 274937600*a^16*b^14*c^18*d^12 + 86150560*a^16*b^14*c^20*d^10 - 13178400
*a^16*b^14*c^22*d^8 + 620160*a^16*b^14*c^24*d^6 - 18240*a^17*b^13*c^3*d^27
+ 1331520*a^17*b^13*c^5*d^25 - 18787200*a^17*b^13*c^7*d^23 + 101475200*a^17
*b^13*c^9*d^21 - 284331200*a^17*b^13*c^11*d^19 + 467412160*a^17*b^13*c^13*d
^17 - 472561920*a^17*b^13*c^15*d^15 + 293542400*a^17*b^13*c^17*d^13 - 10646
0800*a^17*b^13*c^19*d^11 + 19638400*a^17*b^13*c^21*d^9 - 1240320*a^17*b^13*
c^23*d^7 + 3040*a^18*b^12*c^2*d^28 - 402800*a^18*b^12*c^4*d^26 + 8170000*a^
18*b^12*c^6*d^24 - 55069600*a^18*b^12*c^8*d^22 + 181463680*a^18*b^12*c^10*d
^20 - 341426960*a^18*b^12*c^12*d^18 + 390830000*a^18*b^12*c^14*d^16 - 27493
7600*a^18*b^12*c^16*d^14 + 114212800*a^18*b^12*c^18*d^12 - 24858080*a^18*b^
12*c^20*d^10 + 2015520*a^18*b^12*c^22*d^8 + 92800*a^19*b^11*c^3*d^27 - 2939
840*a^19*b^11*c^5*d^25 + 25721600*a^19*b^11*c^7*d^23 - 101172800*a^19*b^11*
c^9*d^21 + 219166080*a^19*b^11*c^11*d^19 - 284331200*a^19*b^11*c^13*d^17 +
225738240*a^19*b^11*c^15*d^15 - 106460800*a^19*b^11*c^17*d^13 + 26873600*a^
19*b^11*c^19*d^11 - 2687360*a^19*b^11*c^21*d^9 - 15280*a^20*b^10*c^2*d^28 +

851360*a^20*b^10*c^4*d^26 - 10229760*a^20*b^10*c^6*d^24 + 48989680*a^20*b^10*c^8*d^22 - 123188112*a^20*b^10*c^10*d^20 + 181463680*a^20*b^10*c^12*d^18 - 162120160*a^20*b^10*c^14*d^16 + 86150560*a^20*b^10*c^16*d^14 - 24858080*a^20*b^10*c^18*d^12 + 2956096*a^20*b^10*c^20*d^10 - 190400*a^21*b^9*c^3*d^27 + 3408640*a^21*b^9*c^5*d^25 - 20444800*a^21*b^9*c^7*d^23 + 60333760*a^21*b^9*c^9*d^21 - 101172800*a^21*b^9*c^11*d^19 + 101475200*a^21*b^9*c^13*d^17 - 60362240*a^21*b^9*c^15*d^15 + 19638400*a^21*b^9*c^17*d^13 - 2687360*a^21*b^9*c^19*d^11 + 30800*a^22*b^8*c^2*d^28 - 928000*a^22*b^8*c^4*d^26 + 7281600*a^22*b^8*c^6*d^24 - 25575920*a^22*b^8*c^8*d^22 + 48989680*a^22*b^8*c^10*d^20 - 55069600*a^22*b^8*c^12*d^18 + 36434400*a^22*b^8*c^14*d^16 - 13178400*a^22*b^8*c^16*d^14 + 2015520*a^22*b^8*c^18*d^12 + 198400*a^23*b^7*c^3*d^27 - 2184320*a^23*b^7*c^5*d^25 + 9297920*a^23*b^7*c^7*d^23 - 20444800*a^23*b^7*c^9*d^21 + 25721600*a^23*b^7*c^11*d^19 - 18787200*a^23*b^7*c^13*d^17 + 7441920*a^23*b^7*c^15*d^15 - 1240320*a^23*b^7*c^17*d^13 - 31200*a^24*b^6*c^2*d^28 + 541200*a^24*b^6*c^4*d^26 - 2863760*a^24*b^6*c^6*d^24 + 7281600*a^24*b^6*c^8*d^22 - 10229760*a^24*b^6*c^10*d^20 + 8170000*a^24*b^6*c^12*d^18 - 3488400*a^24*b^6*c^14*d^16 + 620160*a^24*b^6*c^16*d^14 - 107200*a^25*b^5*c^3*d^27 + 736064*a^25*b^5*c^5*d^25 - 2184320*a^25*b^5*c^7*d^23 + 3408640*a^25*b^5*c^9*d^21 - 2939840*a^25*b^5*c^11*d^19 + 1331520*a^25*b^5*c^13*d^17 - 248064*a^25*b^5*c^15*d^15 + 16000*a^26*b^4*c^2*d^28 - 155120*a^26*b^4*c^4*d^26 + 541200*a^26*b^4*c^6*d^24 - 928000*a^26*b^4*c^8*d^22 + 851360*a^26*b^4*c^10*d^20 - 402800*a^26*b^4*c^12*d^18 + 77520*a^26*b^4*c^14*d^16 + 26240*a^27*b^3*c^3*d^27 - 107200*a^27*b^3*c^5*d^25 + 198400*a^27*b^3*c^7*d^23 - 190400*a^27*b^3*c^9*d^21 + 92800*a^27*b^3*c^11*d^19 - 18240*a^27*b^3*c^13*d^17 - 3440*a^28*b^2*c^2*d^28 + 16000*a^28*b^2*c^4*d^26 - 31200*a^28*b^2*c^6*d^24 + 30800*a^28*b^2*c^8*d^22 - 15280*a^28*b^2*c^10*d^20 + 3040*a^28*b^2*c^12*d^18 + 320*a*b^29*c^29*d + 320*a^29*b*c*d^29))^(1/2) - 2*a^24*d^24 - 2*b^24*c^24 - 8*a^2*b^22*c^24 - 8*a^4*b^20*c^24 + 576*a^10*b^14*d^24 - 2784*a^12*b^12*d^24 + 5284*a^14*b^10*d^24 - 4730*a^16*b^8*d^24 + 1780*a^18*b^6*d^24 - 68*a^20*b^4*d^24 - 38*a^22*b^2*d^24 - 8*a^24*c^2*d^22 - 8*a^24*c^4*d^20 + 576*b^24*c^10*d^14 - 2784*b^24*c^12*d^12 + 5284*b^24*c^14*d^10 - 4730*b^24*c^16*d^8 + 1780*b^24*c^18*d^6 - 68*b^24*c^20*d^4 - 38*b^24*c^22*d^2 - 5760*a*b^23*c^9*d^15 + 28224*a*b^23*c^11*d^13 - 54728*a*b^23*c^13*d^11 + 50620*a*b^23*c^15*d^9 - 20360*a*b^23*c^17*d^7 + 1480*a*b^23*c^19*d^5 + 268*a*b^23*c^21*d^3 + 88*a^3*b^21*c^23*d + 160*a^5*b^19*c^23*d - 5760*a^9*b^15*c*d^23 + 28224*a^11*b^13*c*d^23 - 54728*a^13*b^11*c*d^23 + 50620*a^15*b^9*c*d^23 - 20360*a^17*b^7*c*d^23 + 1480*a^19*b^5*c*d^23 + 268*a^21*b^3*c*d^23 + 88*a^23*b*c^3*d^21 + 160*a^23*b*c^5*d^19 + 25920*a^2*b^22*c^8*d^16 - 131904*a^2*b^22*c^10*d^14 + 270604*a^2*b^22*c^12*d^12 - 273544*a^2*b^22*c^14*d^10 + 131660*a^2*b^22*c^16*d^8 - 22060*a^2*b^22*c^18*d^6 + 782*a^2*b^22*c^20*d^4 + 98*a^2*b^22*c^22*d^2 - 69120*a^3*b^21*c^7*d^17 + 379200*a^3*b^21*c^9*d^15 - 860368*a^3*b^21*c^11*d^13 + 1001364*a^3*b^21*c^13*d^11 - 605280*a^3*b^21*c^15*d^9 + 167520*a^3*b^21*c^17*d^7 - 18840*a^3*b^21*c^19*d^5 + 144*a^3*b^21*c^21*d^3 + 120960*a^4*b^20*c^6*d^18 - 756000*a^4*b^20*c^8*d^16 + 1987844*a^4*b^20*c^10*d^14 - 2750664*a^4*b^20*c^12*d^12 + 2073976*a^4*b^20*c^14*d^1

0 - 793460*a^4*b^20*c^16*d^8 + 138010*a^4*b^20*c^18*d^6 - 10562*a^4*b^20*c^20*d^4 - 88*a^4*b^20*c^22*d^2 - 145152*a^5*b^19*c^5*d^19 + 1116288*a^5*b^19*c^7*d^17 - 3539128*a^5*b^19*c^9*d^15 + 5890780*a^5*b^19*c^11*d^13 - 5437600*a^5*b^19*c^13*d^11 + 2682536*a^5*b^19*c^15*d^9 - 655084*a^5*b^19*c^17*d^7 + 85484*a^5*b^19*c^19*d^5 - 4080*a^5*b^19*c^21*d^3 + 120960*a^6*b^18*c^4*d^20 - 1266048*a^6*b^18*c^6*d^18 + 4977996*a^6*b^18*c^8*d^16 - 10009720*a^6*b^18*c^10*d^14 + 11209800*a^6*b^18*c^12*d^12 - 6943760*a^6*b^18*c^14*d^10 + 2253214*a^6*b^18*c^16*d^8 - 396878*a^6*b^18*c^18*d^6 + 36120*a^6*b^18*c^20*d^4 - 1520*a^6*b^18*c^22*d^2 - 69120*a^7*b^17*c^3*d^21 + 1116288*a^7*b^17*c^5*d^19 - 5575008*a^7*b^17*c^7*d^17 + 13668308*a^7*b^17*c^9*d^15 - 18576800*a^7*b^17*c^11*d^13 + 14230520*a^7*b^17*c^13*d^11 - 5889904*a^7*b^17*c^15*d^9 + 1310504*a^7*b^17*c^17*d^7 - 168344*a^7*b^17*c^19*d^5 + 8960*a^7*b^17*c^21*d^3 + 25920*a^8*b^16*c^2*d^22 - 756000*a^8*b^16*c^4*d^20 + 4977996*a^8*b^16*c^6*d^18 - 15144828*a^8*b^16*c^8*d^16 + 25068800*a^8*b^16*c^10*d^14 - 23486280*a^8*b^16*c^12*d^12 + 12099640*a^8*b^16*c^14*d^10 - 3330518*a^8*b^16*c^16*d^8 + 529224*a^8*b^16*c^18*d^6 - 36280*a^8*b^16*c^20*d^4 + 379200*a^9*b^15*c^3*d^21 - 3539128*a^9*b^15*c^5*d^19 + 13668308*a^9*b^15*c^7*d^17 - 27691952*a^9*b^15*c^9*d^15 + 31562040*a^9*b^15*c^11*d^13 - 19993760*a^9*b^15*c^13*d^11 + 6731044*a^9*b^15*c^15*d^9 - 1239264*a^9*b^15*c^17*d^7 + 106016*a^9*b^15*c^19*d^5 - 131904*a^10*b^14*c^2*d^22 + 1987844*a^10*b^14*c^4*d^20 - 10009720*a^10*b^14*c^6*d^18 + 25068800*a^10*b^14*c^8*d^16 - 34796936*a^10*b^14*c^10*d^14 + 26927144*a^10*b^14*c^12*d^12 - 10994964*a^10*b^14*c^14*d^10 + 2295680*a^10*b^14*c^16*d^8 - 230240*a^10*b^14*c^18*d^6 - 860368*a^11*b^13*c^3*d^21 + 5890780*a^11*b^13*c^5*d^19 - 18576800*a^11*b^13*c^7*d^17 + 31562040*a^11*b^13*c^9*d^15 - 29722864*a^11*b^13*c^11*d^13 + 14679348*a^11*b^13*c^13*d^11 - 3497920*a^11*b^13*c^15*d^9 + 381280*a^11*b^13*c^17*d^7 + 270604*a^12*b^12*c^2*d^22 - 2750664*a^12*b^12*c^4*d^20 + 11209800*a^12*b^12*c^6*d^18 - 23486280*a^12*b^12*c^8*d^16 + 26927144*a^12*b^12*c^10*d^14 - 16147404*a^12*b^12*c^12*d^12 + 4479104*a^12*b^12*c^14*d^10 - 499520*a^12*b^12*c^16*d^8 + 1001364*a^13*b^11*c^3*d^21 - 5437600*a^13*b^11*c^5*d^19 + 14230520*a^13*b^11*c^7*d^17 - 19993760*a^13*b^11*c^9*d^15 + 14679348*a^13*b^11*c^11*d^13 - 4861024*a^13*b^11*c^13*d^11 + 552160*a^13*b^11*c^15*d^9 - 273544*a^14*b^10*c^2*d^22 + 2073976*a^14*b^10*c^4*d^20 - 6943760*a^14*b^10*c^6*d^18 + 12099640*a^14*b^10*c^8*d^16 - 10994964*a^14*b^10*c^10*d^14 + 4479104*a^14*b^10*c^12*d^12 - 562016*a^14*b^10*c^14*d^10 - 605280*a^15*b^9*c^3*d^21 + 2682536*a^15*b^9*c^5*d^19 - 5889904*a^15*b^9*c^7*d^17 + 6731044*a^15*b^9*c^9*d^15 - 3497920*a^15*b^9*c^11*d^13 + 552160*a^15*b^9*c^13*d^11 + 131660*a^16*b^8*c^2*d^22 - 793460*a^16*b^8*c^4*d^20 + 2253214*a^16*b^8*c^6*d^18 - 3330518*a^16*b^8*c^8*d^16 + 2295680*a^16*b^8*c^10*d^14 - 499520*a^16*b^8*c^12*d^12 + 167520*a^17*b^7*c^3*d^21 - 655084*a^17*b^7*c^5*d^19 + 1310504*a^17*b^7*c^7*d^17 - 1239264*a^17*b^7*c^9*d^15 + 381280*a^17*b^7*c^11*d^13 - 22060*a^18*b^6*c^2*d^22 + 138010*a^18*b^6*c^4*d^20 - 396878*a^18*b^6*c^6*d^18 + 529224*a^18*b^6*c^8*d^16 - 230240*a^18*b^6*c^10*d^14 - 18840*a^19*b^5*c^3*d^21 + 85484*a^19*b^5*c^5*d^19 - 168344*a^19*b^5*c^7*d^17 + 106016*a^19*b^5*c^9*d^15 + 782*a^20*b^4*c^2*d^22 - 10562*a^20*b^4*c^4*d^20 + 36120*a^20

*b^4*c^6*d^18 - 36280*a^20*b^4*c^8*d^16 + 144*a^21*b^3*c^3*d^21 - 4080*a^21
*b^3*c^5*d^19 + 8960*a^21*b^3*c^7*d^17 + 98*a^22*b^2*c^2*d^22 - 88*a^22*b^2
*c^4*d^20 - 1520*a^22*b^2*c^6*d^18 + 4*a*b^23*c^23*d + 4*a^23*b*c*d^23)/(16
*(5*a^2*b^28*c^30 - b^30*c^30 - a^30*d^30 - 10*a^4*b^26*c^30 + 10*a^6*b^24*
c^30 - 5*a^8*b^22*c^30 + a^10*b^20*c^30 + a^20*b^10*d^30 - 5*a^22*b^8*d^30
+ 10*a^24*b^6*d^30 - 10*a^26*b^4*d^30 + 5*a^28*b^2*d^30 + 5*a^30*c^2*d^28 -
10*a^30*c^4*d^26 + 10*a^30*c^6*d^24 - 5*a^30*c^8*d^22 + a^30*c^10*d^20 + b
^30*c^20*d^10 - 5*b^30*c^22*d^8 + 10*b^30*c^24*d^6 - 10*b^30*c^26*d^4 + 5*b
^30*c^28*d^2 - 20*a*b^29*c^19*d^11 + 100*a*b^29*c^21*d^9 - 200*a*b^29*c^23*
d^7 + 200*a*b^29*c^25*d^5 - 100*a*b^29*c^27*d^3 - 100*a^3*b^27*c^29*d + 200
*a^5*b^25*c^29*d - 200*a^7*b^23*c^29*d + 100*a^9*b^21*c^29*d - 20*a^11*b^19
*c^29*d - 20*a^19*b^11*c*d^29 + 100*a^21*b^9*c*d^29 - 200*a^23*b^7*c*d^29 +
200*a^25*b^5*c*d^29 - 100*a^27*b^3*c*d^29 - 100*a^29*b*c^3*d^27 + 200*a^29
*b*c^5*d^25 - 200*a^29*b*c^7*d^23 + 100*a^29*b*c^9*d^21 - 20*a^29*b*c^11*d^
19 + 190*a^2*b^28*c^18*d^12 - 955*a^2*b^28*c^20*d^10 + 1925*a^2*b^28*c^22*d
^8 - 1950*a^2*b^28*c^24*d^6 + 1000*a^2*b^28*c^26*d^4 - 215*a^2*b^28*c^28*d^
2 - 1140*a^3*b^27*c^17*d^13 + 5800*a^3*b^27*c^19*d^11 - 11900*a^3*b^27*c^21
*d^9 + 12400*a^3*b^27*c^23*d^7 - 6700*a^3*b^27*c^25*d^5 + 1640*a^3*b^27*c^2
7*d^3 + 4845*a^4*b^26*c^16*d^14 - 25175*a^4*b^26*c^18*d^12 + 53210*a^4*b^26
*c^20*d^10 - 58000*a^4*b^26*c^22*d^8 + 33825*a^4*b^26*c^24*d^6 - 9695*a^4*b
^26*c^26*d^4 + 1000*a^4*b^26*c^28*d^2 - 15504*a^5*b^25*c^15*d^15 + 83220*a^
5*b^25*c^17*d^13 - 183740*a^5*b^25*c^19*d^11 + 213040*a^5*b^25*c^21*d^9 - 1
36520*a^5*b^25*c^23*d^7 + 46004*a^5*b^25*c^25*d^5 - 6700*a^5*b^25*c^27*d^3
+ 38760*a^6*b^24*c^14*d^16 - 218025*a^6*b^24*c^16*d^14 + 510625*a^6*b^24*c^
18*d^12 - 639360*a^6*b^24*c^20*d^10 + 455100*a^6*b^24*c^22*d^8 - 178985*a^6
*b^24*c^24*d^6 + 33825*a^6*b^24*c^26*d^4 - 1950*a^6*b^24*c^28*d^2 - 77520*a
^7*b^23*c^13*d^17 + 465120*a^7*b^23*c^15*d^15 - 1174200*a^7*b^23*c^17*d^13
+ 1607600*a^7*b^23*c^19*d^11 - 1277800*a^7*b^23*c^21*d^9 + 581120*a^7*b^23*
c^23*d^7 - 136520*a^7*b^23*c^25*d^5 + 12400*a^7*b^23*c^27*d^3 + 125970*a^8*
b^22*c^12*d^18 - 823650*a^8*b^22*c^14*d^16 + 2277150*a^8*b^22*c^16*d^14 - 3
441850*a^8*b^22*c^18*d^12 + 3061855*a^8*b^22*c^20*d^10 - 1598495*a^8*b^22*c
^22*d^8 + 455100*a^8*b^22*c^24*d^6 - 58000*a^8*b^22*c^26*d^4 + 1925*a^8*b^2
2*c^28*d^2 - 167960*a^9*b^21*c^11*d^19 + 1227400*a^9*b^21*c^13*d^17 - 37726
40*a^9*b^21*c^15*d^15 + 6342200*a^9*b^21*c^17*d^13 - 6323300*a^9*b^21*c^19*
d^11 + 3770860*a^9*b^21*c^21*d^9 - 1277800*a^9*b^21*c^23*d^7 + 213040*a^9*b
^21*c^25*d^5 - 11900*a^9*b^21*c^27*d^3 + 184756*a^10*b^20*c^10*d^20 - 15536
30*a^10*b^20*c^12*d^18 + 5384410*a^10*b^20*c^14*d^16 - 10132510*a^10*b^20*c
^16*d^14 + 11341480*a^10*b^20*c^18*d^12 - 7699257*a^10*b^20*c^20*d^10 + 306
1855*a^10*b^20*c^22*d^8 - 639360*a^10*b^20*c^24*d^6 + 53210*a^10*b^20*c^26*
d^4 - 955*a^10*b^20*c^28*d^2 - 167960*a^11*b^19*c^9*d^21 + 1679600*a^11*b^1
9*c^11*d^19 - 6653800*a^11*b^19*c^13*d^17 + 14108640*a^11*b^19*c^15*d^15 -
17770700*a^11*b^19*c^17*d^13 + 13697880*a^11*b^19*c^19*d^11 - 6323300*a^11*
b^19*c^21*d^9 + 1607600*a^11*b^19*c^23*d^7 - 183740*a^11*b^19*c^25*d^5 + 58
00*a^11*b^19*c^27*d^3 + 125970*a^12*b^18*c^8*d^22 - 1553630*a^12*b^18*c^10*
d^20 + 7138300*a^12*b^18*c^12*d^18 - 17183600*a^12*b^18*c^14*d^16 + 2442687

5*a^12*b^18*c^16*d^14 - 21339185*a^12*b^18*c^18*d^12 + 11341480*a^12*b^18*c
^20*d^10 - 3441850*a^12*b^18*c^22*d^8 + 510625*a^12*b^18*c^24*d^6 - 25175*a
^12*b^18*c^26*d^4 + 190*a^12*b^18*c^28*d^2 - 77520*a^13*b^17*c^7*d^23 + 122
7400*a^13*b^17*c^9*d^21 - 6653800*a^13*b^17*c^11*d^19 + 18346400*a^13*b^17*
c^13*d^17 - 29535120*a^13*b^17*c^15*d^15 + 29213260*a^13*b^17*c^17*d^13 - 1
7770700*a^13*b^17*c^19*d^11 + 6342200*a^13*b^17*c^21*d^9 - 1174200*a^13*b^1
7*c^23*d^7 + 83220*a^13*b^17*c^25*d^5 - 1140*a^13*b^17*c^27*d^3 + 38760*a^1
4*b^16*c^6*d^24 - 823650*a^14*b^16*c^8*d^22 + 5384410*a^14*b^16*c^10*d^20 -
17183600*a^14*b^16*c^12*d^18 + 31460200*a^14*b^16*c^14*d^16 - 35234455*a^1
4*b^16*c^16*d^14 + 24426875*a^14*b^16*c^18*d^12 - 10132510*a^14*b^16*c^20*d
^10 + 2277150*a^14*b^16*c^22*d^8 - 218025*a^14*b^16*c^24*d^6 + 4845*a^14*b^
16*c^26*d^4 - 15504*a^15*b^15*c^5*d^25 + 465120*a^15*b^15*c^7*d^23 - 377264
0*a^15*b^15*c^9*d^21 + 14108640*a^15*b^15*c^11*d^19 - 29535120*a^15*b^15*c^
13*d^17 + 37499008*a^15*b^15*c^15*d^15 - 29535120*a^15*b^15*c^17*d^13 + 141
08640*a^15*b^15*c^19*d^11 - 3772640*a^15*b^15*c^21*d^9 + 465120*a^15*b^15*c
^23*d^7 - 15504*a^15*b^15*c^25*d^5 + 4845*a^16*b^14*c^4*d^26 - 218025*a^16*
b^14*c^6*d^24 + 2277150*a^16*b^14*c^8*d^22 - 10132510*a^16*b^14*c^10*d^20 +
24426875*a^16*b^14*c^12*d^18 - 35234455*a^16*b^14*c^14*d^16 + 31460200*a^1
6*b^14*c^16*d^14 - 17183600*a^16*b^14*c^18*d^12 + 5384410*a^16*b^14*c^20*d^
10 - 823650*a^16*b^14*c^22*d^8 + 38760*a^16*b^14*c^24*d^6 - 1140*a^17*b^13*
c^3*d^27 + 83220*a^17*b^13*c^5*d^25 - 1174200*a^17*b^13*c^7*d^23 + 6342200*
a^17*b^13*c^9*d^21 - 17770700*a^17*b^13*c^11*d^19 + 29213260*a^17*b^13*c^13
*d^17 - 29535120*a^17*b^13*c^15*d^15 + 18346400*a^17*b^13*c^17*d^13 - 66538
00*a^17*b^13*c^19*d^11 + 1227400*a^17*b^13*c^21*d^9 - 77520*a^17*b^13*c^23*
d^7 + 190*a^18*b^12*c^2*d^28 - 25175*a^18*b^12*c^4*d^26 + 510625*a^18*b^12*
c^6*d^24 - 3441850*a^18*b^12*c^8*d^22 + 11341480*a^18*b^12*c^10*d^20 - 2133
9185*a^18*b^12*c^12*d^18 + 24426875*a^18*b^12*c^14*d^16 - 17183600*a^18*b^1
2*c^16*d^14 + 7138300*a^18*b^12*c^18*d^12 - 1553630*a^18*b^12*c^20*d^10 + 1
25970*a^18*b^12*c^22*d^8 + 5800*a^19*b^11*c^3*d^27 - 183740*a^19*b^11*c^5*d
^25 + 1607600*a^19*b^11*c^7*d^23 - 6323300*a^19*b^11*c^9*d^21 + 13697880*a^
19*b^11*c^11*d^19 - 17770700*a^19*b^11*c^13*d^17 + 14108640*a^19*b^11*c^15*
d^15 - 6653800*a^19*b^11*c^17*d^13 + 1679600*a^19*b^11*c^19*d^11 - 167960*a
^19*b^11*c^21*d^9 - 955*a^20*b^10*c^2*d^28 + 53210*a^20*b^10*c^4*d^26 - 639
360*a^20*b^10*c^6*d^24 + 3061855*a^20*b^10*c^8*d^22 - 7699257*a^20*b^10*c^1
0*d^20 + 11341480*a^20*b^10*c^12*d^18 - 10132510*a^20*b^10*c^14*d^16 + 5384
410*a^20*b^10*c^16*d^14 - 1553630*a^20*b^10*c^18*d^12 + 184756*a^20*b^10*c^
20*d^10 - 11900*a^21*b^9*c^3*d^27 + 213040*a^21*b^9*c^5*d^25 - 1277800*a^21
*b^9*c^7*d^23 + 3770860*a^21*b^9*c^9*d^21 - 6323300*a^21*b^9*c^11*d^19 + 63
42200*a^21*b^9*c^13*d^17 - 3772640*a^21*b^9*c^15*d^15 + 1227400*a^21*b^9*c^
17*d^13 - 167960*a^21*b^9*c^19*d^11 + 1925*a^22*b^8*c^2*d^28 - 58000*a^22*b
^8*c^4*d^26 + 455100*a^22*b^8*c^6*d^24 - 1598495*a^22*b^8*c^8*d^22 + 306185
5*a^22*b^8*c^10*d^20 - 3441850*a^22*b^8*c^12*d^18 + 2277150*a^22*b^8*c^14*d
^16 - 823650*a^22*b^8*c^16*d^14 + 125970*a^22*b^8*c^18*d^12 + 12400*a^23*b^
7*c^3*d^27 - 136520*a^23*b^7*c^5*d^25 + 581120*a^23*b^7*c^7*d^23 - 1277800*
a^23*b^7*c^9*d^21 + 1607600*a^23*b^7*c^11*d^19 - 1174200*a^23*b^7*c^13*d^17

+ 465120*a^23*b^7*c^15*d^15 - 77520*a^23*b^7*c^17*d^13 - 1950*a^24*b^6*c^2*d^28 + 33825*a^24*b^6*c^4*d^26 - 178985*a^24*b^6*c^6*d^24 + 455100*a^24*b^6*c^8*d^22 - 639360*a^24*b^6*c^10*d^20 + 510625*a^24*b^6*c^12*d^18 - 218025*a^24*b^6*c^14*d^16 + 38760*a^24*b^6*c^16*d^14 - 6700*a^25*b^5*c^3*d^27 + 46004*a^25*b^5*c^5*d^25 - 136520*a^25*b^5*c^7*d^23 + 213040*a^25*b^5*c^9*d^21 - 183740*a^25*b^5*c^11*d^19 + 83220*a^25*b^5*c^13*d^17 - 15504*a^25*b^5*c^15*d^15 + 1000*a^26*b^4*c^2*d^28 - 9695*a^26*b^4*c^4*d^26 + 33825*a^26*b^4*c^6*d^24 - 58000*a^26*b^4*c^8*d^22 + 53210*a^26*b^4*c^10*d^20 - 25175*a^26*b^4*c^12*d^18 + 4845*a^26*b^4*c^14*d^16 + 1640*a^27*b^3*c^3*d^27 - 6700*a^27*b^3*c^5*d^25 + 12400*a^27*b^3*c^7*d^23 - 11900*a^27*b^3*c^9*d^21 + 5800*a^27*b^3*c^11*d^19 - 1140*a^27*b^3*c^13*d^17 - 215*a^28*b^2*c^2*d^28 + 1000*a^28*b^2*c^4*d^26 - 1950*a^28*b^2*c^6*d^24 + 1925*a^28*b^2*c^8*d^22 - 955*a^28*b^2*c^10*d^20 + 190*a^28*b^2*c^12*d^18 + 20*a*b^29*c^29*d + 20*a^29*b*c*d^29)))^(1/2)*(((4*(8*a^2*b^23*c^25 - 32*a^4*b^21*c^25 + 48*a^6*b^19*c^25 - 32*a^8*b^17*c^25 + 8*a^10*b^15*c^25 + 8*a^25*c^2*d^23 - 32*a^25*c^4*d^21 + 48*a^25*c^6*d^19 - 32*a^25*c^8*d^17 + 8*a^25*c^10*d^15 - 8*a*b^24*c^16*d^9 + 32*a*b^24*c^18*d^7 - 48*a*b^24*c^20*d^5 + 32*a*b^24*c^22*d^3 - 72*a^3*b^22*c^24*d + 368*a^5*b^20*c^24*d - 592*a^7*b^18*c^24*d + 408*a^9*b^16*c^24*d - 104*a^11*b^14*c^24*d - 8*a^16*b^9*c*d^24 + 32*a^18*b^7*c*d^24 - 48*a^20*b^5*c*d^24 + 32*a^22*b^3*c*d^24 - 72*a^24*b*c^3*d^22 + 368*a^24*b*c^5*d^20 - 592*a^24*b*c^7*d^18 + 408*a^24*b*c^9*d^16 - 104*a^24*b*c^11*d^14 + 104*a^2*b^23*c^15*d^10 - 408*a^2*b^23*c^17*d^8 + 592*a^2*b^23*c^19*d^6 - 368*a^2*b^23*c^21*d^4 + 72*a^2*b^23*c^23*d^2 - 616*a^3*b^22*c^14*d^11 + 2392*a^3*b^22*c^16*d^9 - 3408*a^3*b^22*c^18*d^7 + 2032*a^3*b^22*c^20*d^5 - 328*a^3*b^22*c^22*d^3 + 2184*a^4*b^21*c^13*d^12 - 8536*a^4*b^21*c^15*d^10 + 12272*a^4*b^21*c^17*d^8 - 7408*a^4*b^21*c^19*d^6 + 1192*a^4*b^21*c^21*d^4 + 328*a^4*b^21*c^23*d^2 - 5096*a^5*b^20*c^12*d^13 + 20664*a^5*b^20*c^14*d^11 - 31328*a^5*b^20*c^16*d^9 + 20592*a^5*b^20*c^18*d^7 - 4008*a^5*b^20*c^20*d^5 - 1192*a^5*b^20*c^22*d^3 + 8008*a^6*b^19*c^11*d^14 - 35672*a^6*b^19*c^13*d^12 + 60768*a^6*b^19*c^15*d^10 - 46464*a^6*b^19*c^17*d^8 + 11336*a^6*b^19*c^19*d^6 + 4008*a^6*b^19*c^21*d^4 - 2032*a^6*b^19*c^23*d^2 - 8008*a^7*b^18*c^10*d^15 + 44408*a^7*b^18*c^12*d^13 - 92512*a^7*b^18*c^14*d^11 + 85536*a^7*b^18*c^16*d^9 - 24904*a^7*b^18*c^18*d^7 - 11336*a^7*b^18*c^20*d^5 + 7408*a^7*b^18*c^22*d^3 + 3432*a^8*b^17*c^9*d^16 - 37752*a^8*b^17*c^11*d^14 + 109408*a^8*b^17*c^13*d^12 - 125472*a^8*b^17*c^15*d^10 + 42696*a^8*b^17*c^17*d^8 + 24904*a^8*b^17*c^19*d^6 - 20592*a^8*b^17*c^21*d^4 + 3408*a^8*b^17*c^23*d^2 + 3432*a^9*b^16*c^8*d^17 + 14872*a^9*b^16*c^10*d^15 - 92352*a^9*b^16*c^12*d^13 + 141408*a^9*b^16*c^14*d^11 - 59264*a^9*b^16*c^16*d^9 - 42696*a^9*b^16*c^18*d^7 + 46464*a^9*b^16*c^20*d^5 - 12272*a^9*b^16*c^22*d^3 - 8008*a^10*b^15*c^7*d^18 + 14872*a^10*b^15*c^9*d^16 + 36608*a^10*b^15*c^11*d^14 - 113152*a^10*b^15*c^13*d^12 + 67008*a^10*b^15*c^15*d^10 + 59264*a^10*b^15*c^17*d^8 - 85536*a^10*b^15*c^19*d^6 + 31328*a^10*b^15*c^21*d^4 - 2392*a^10*b^15*c^23*d^2 + 8008*a^11*b^14*c^6*d^19 - 37752*a^11*b^14*c^8*d^17 + 36608*a^11*b^14*c^10*d^15 + 43264*a^11*b^14*c^12*d^13 - 56256*a^11*b^14*c^14*d^11 - 67008*a^11*b^14*c^16*d^9 + 125472*a^11*b^14*c^18*d^7 - 60768*a^11*b^14*c^20*d^5 + 85

36*a^11*b^14*c^22*d^3 - 5096*a^12*b^13*c^5*d^20 + 44408*a^12*b^13*c^7*d^18 - 92352*a^12*b^13*c^9*d^16 + 43264*a^12*b^13*c^11*d^14 + 22464*a^12*b^13*c^13*d^12 + 56256*a^12*b^13*c^15*d^10 - 141408*a^12*b^13*c^17*d^8 + 92512*a^12*b^13*c^19*d^6 - 20664*a^12*b^13*c^21*d^4 + 616*a^12*b^13*c^23*d^2 + 2184*a^13*b^12*c^4*d^21 - 35672*a^13*b^12*c^6*d^19 + 109408*a^13*b^12*c^8*d^17 - 113152*a^13*b^12*c^10*d^15 + 22464*a^13*b^12*c^12*d^13 - 22464*a^13*b^12*c^14*d^11 + 113152*a^13*b^12*c^16*d^9 - 109408*a^13*b^12*c^18*d^7 + 35672*a^13*b^12*c^20*d^5 - 2184*a^13*b^12*c^22*d^3 - 616*a^14*b^11*c^3*d^22 + 20664*a^14*b^11*c^5*d^20 - 92512*a^14*b^11*c^7*d^18 + 141408*a^14*b^11*c^9*d^16 - 56256*a^14*b^11*c^11*d^14 - 22464*a^14*b^11*c^13*d^12 - 43264*a^14*b^11*c^15*d^10 + 92352*a^14*b^11*c^17*d^8 - 44408*a^14*b^11*c^19*d^6 + 5096*a^14*b^11*c^21*d^4 + 104*a^15*b^10*c^2*d^23 - 8536*a^15*b^10*c^4*d^21 + 60768*a^15*b^10*c^6*d^19 - 125472*a^15*b^10*c^8*d^17 + 67008*a^15*b^10*c^10*d^15 + 56256*a^15*b^10*c^12*d^13 - 43264*a^15*b^10*c^14*d^11 - 36608*a^15*b^10*c^16*d^9 + 37752*a^15*b^10*c^18*d^7 - 8008*a^15*b^10*c^20*d^5 + 2392*a^16*b^9*c^3*d^22 - 31328*a^16*b^9*c^5*d^20 + 85536*a^16*b^9*c^7*d^18 - 59264*a^16*b^9*c^9*d^16 - 67008*a^16*b^9*c^11*d^14 + 113152*a^16*b^9*c^13*d^12 - 36608*a^16*b^9*c^15*d^10 - 14872*a^16*b^9*c^17*d^8 + 8008*a^16*b^9*c^19*d^6 - 408*a^17*b^8*c^2*d^23 + 12272*a^17*b^8*c^4*d^21 - 46464*a^17*b^8*c^6*d^19 + 42696*a^17*b^8*c^8*d^17 + 59264*a^17*b^8*c^10*d^15 - 141408*a^17*b^8*c^12*d^13 + 92352*a^17*b^8*c^14*d^11 - 14872*a^17*b^8*c^16*d^9 - 3432*a^17*b^8*c^18*d^7 - 3408*a^18*b^7*c^3*d^22 + 20592*a^18*b^7*c^5*d^20 - 24904*a^18*b^7*c^7*d^18 - 42696*a^18*b^7*c^9*d^16 + 125472*a^18*b^7*c^11*d^14 - 109408*a^18*b^7*c^13*d^12 + 37752*a^18*b^7*c^15*d^10 - 3432*a^18*b^7*c^17*d^8 + 592*a^19*b^6*c^2*d^23 - 7408*a^19*b^6*c^4*d^21 + 11336*a^19*b^6*c^6*d^19 + 24904*a^19*b^6*c^8*d^17 - 85536*a^19*b^6*c^10*d^15 + 92512*a^19*b^6*c^12*d^13 - 44408*a^19*b^6*c^14*d^11 + 8008*a^19*b^6*c^16*d^9 + 2032*a^20*b^5*c^3*d^22 - 4008*a^20*b^5*c^5*d^20 - 11336*a^20*b^5*c^7*d^18 + 46464*a^20*b^5*c^9*d^16 - 60768*a^20*b^5*c^11*d^14 + 35672*a^20*b^5*c^13*d^12 - 8008*a^20*b^5*c^15*d^10 - 368*a^21*b^4*c^2*d^23 + 1192*a^21*b^4*c^4*d^21 + 4008*a^21*b^4*c^6*d^19 - 20592*a^21*b^4*c^8*d^17 + 31328*a^21*b^4*c^10*d^15 - 20664*a^21*b^4*c^12*d^13 + 5096*a^21*b^4*c^14*d^11 - 328*a^22*b^3*c^3*d^22 - 1192*a^22*b^3*c^5*d^20 + 7408*a^22*b^3*c^7*d^18 - 12272*a^22*b^3*c^9*d^16 + 8536*a^22*b^3*c^11*d^14 - 2184*a^22*b^3*c^13*d^12 + 72*a^23*b^2*c^2*d^23 + 328*a^23*b^2*c^4*d^21 - 2032*a^23*b^2*c^6*d^19 + 3408*a^23*b^2*c^8*d^17 - 2392*a^23*b^2*c^10*d^15 + 616*a^23*b^2*c^12*d^13 - 8*a*b^24*c^24*d - 8*a^24*b*c*d^24))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18

*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^1
8*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*
d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^1
2 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d
^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13
 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7
 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14
- 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d
^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d
^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^
11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15
*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d
^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c
^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c
^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c
^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^1
1*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*
c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*
b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 1386
0*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 92
8*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44
936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 +
 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4
032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 -
 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^
6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15
 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*
d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^1
8 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d
^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14
*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*
d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*
d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^
14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d
^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^1
3 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 -
288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*
a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19) - (8*tan(e/2 + (f
x)/2)(56*a^3*b^22*c^25 - 12*a^25*c*d^24 - 12*a*b^24*c^25 - 104*a^5*b^20*c
^25 + 96*a^7*b^18*c^25 - 44*a^9*b^16*c^25 + 8*a^11*b^14*c^25 + 56*a^25*c^3*
d^22 - 104*a^25*c^5*d^20 + 96*a^25*c^7*d^18 - 44*a^25*c^9*d^16 + 8*a^25*c^1
1*d^14 + 16*a*b^24*c^15*d^10 - 76*a*b^24*c^17*d^8 + 144*a*b^24*c^19*d^6 - 1
36*a*b^24*c^21*d^4 + 64*a*b^24*c^23*d^2 + 168*a^2*b^23*c^24*d - 784*a^4*b^2
1*c^24*d + 1456*a^6*b^19*c^24*d - 1344*a^8*b^17*c^24*d + 616*a^10*b^15*c^24
*d - 112*a^12*b^13*c^24*d + 16*a^15*b^10*c*d^24 - 76*a^17*b^8*c*d^24 + 144*

a^19*b^6*c*d^24 - 136*a^21*b^4*c*d^24 + 64*a^23*b^2*c*d^24 + 168*a^24*b*c^2
*d^23 - 784*a^24*b*c^4*d^21 + 1456*a^24*b*c^6*d^19 - 1344*a^24*b*c^8*d^17 +
616*a^24*b*c^10*d^15 - 112*a^24*b*c^12*d^13 - 224*a^2*b^23*c^14*d^11 + 106
4*a^2*b^23*c^16*d^9 - 2016*a^2*b^23*c^18*d^7 + 1904*a^2*b^23*c^20*d^5 - 896
*a^2*b^23*c^22*d^3 + 1456*a^3*b^22*c^13*d^12 - 6992*a^3*b^22*c^15*d^10 + 13
464*a^3*b^22*c^17*d^8 - 13056*a^3*b^22*c^19*d^6 + 6464*a^3*b^22*c^21*d^4 -
1392*a^3*b^22*c^23*d^2 - 5824*a^4*b^21*c^12*d^13 + 28728*a^4*b^21*c^14*d^11
- 57456*a^4*b^21*c^16*d^9 + 59024*a^4*b^21*c^18*d^7 - 32256*a^4*b^21*c^20*
d^5 + 8568*a^4*b^21*c^22*d^3 + 16016*a^5*b^20*c^11*d^14 - 82992*a^5*b^20*c^
13*d^12 + 177048*a^5*b^20*c^15*d^10 - 198696*a^5*b^20*c^17*d^8 + 123584*a^5
*b^20*c^19*d^6 - 40512*a^5*b^20*c^21*d^4 + 5656*a^5*b^20*c^23*d^2 - 32032*a
^6*b^19*c^10*d^15 + 179816*a^6*b^19*c^12*d^13 - 421344*a^6*b^19*c^14*d^11 +
529312*a^6*b^19*c^16*d^9 - 379008*a^6*b^19*c^18*d^7 + 150024*a^6*b^19*c^20
*d^5 - 28224*a^6*b^19*c^22*d^3 + 48048*a^7*b^18*c^9*d^16 - 304304*a^7*b^18*
c^11*d^14 + 805896*a^7*b^18*c^13*d^12 - 1151104*a^7*b^18*c^15*d^10 + 949952
*a^7*b^18*c^17*d^8 - 446736*a^7*b^18*c^19*d^6 + 108136*a^7*b^18*c^21*d^4 -
9984*a^7*b^18*c^23*d^2 - 54912*a^8*b^17*c^8*d^17 + 412984*a^8*b^17*c^10*d^1
5 - 1267344*a^8*b^17*c^12*d^13 + 2077536*a^8*b^17*c^14*d^11 - 1975808*a^8*b
^17*c^16*d^9 + 1095384*a^8*b^17*c^18*d^7 - 331632*a^8*b^17*c^20*d^5 + 45136
*a^8*b^17*c^22*d^3 + 48048*a^9*b^16*c^7*d^18 - 456456*a^9*b^16*c^9*d^16 + 1
657656*a^9*b^16*c^11*d^14 - 3143504*a^9*b^16*c^13*d^12 + 3453696*a^9*b^16*c
^15*d^10 - 2247636*a^9*b^16*c^17*d^8 + 831208*a^9*b^16*c^19*d^6 - 151944*a^
9*b^16*c^21*d^4 + 8976*a^9*b^16*c^23*d^2 - 32032*a^10*b^15*c^6*d^19 + 41298
4*a^10*b^15*c^8*d^17 - 1812096*a^10*b^15*c^10*d^15 + 4016896*a^10*b^15*c^12
*d^13 - 5121024*a^10*b^15*c^14*d^11 + 3897024*a^10*b^15*c^16*d^9 - 1728832*
a^10*b^15*c^18*d^7 + 404768*a^10*b^15*c^20*d^5 - 38304*a^10*b^15*c^22*d^3 +
16016*a^11*b^14*c^5*d^20 - 304304*a^11*b^14*c^7*d^18 + 1657656*a^11*b^14*c
^9*d^16 - 4356352*a^11*b^14*c^11*d^14 + 6476288*a^11*b^14*c^13*d^12 - 57450
24*a^11*b^14*c^15*d^10 + 3021984*a^11*b^14*c^17*d^8 - 880256*a^11*b^14*c^19
*d^6 + 118032*a^11*b^14*c^21*d^4 - 4048*a^11*b^14*c^23*d^2 - 5824*a^12*b^13
*c^4*d^21 + 179816*a^12*b^13*c^6*d^19 - 1267344*a^12*b^13*c^8*d^17 + 401689
6*a^12*b^13*c^10*d^15 - 7002112*a^12*b^13*c^12*d^13 + 7235136*a^12*b^13*c^1
4*d^11 - 4480896*a^12*b^13*c^16*d^9 + 1588704*a^12*b^13*c^18*d^7 - 280896*a
^12*b^13*c^20*d^5 + 16632*a^12*b^13*c^22*d^3 + 1456*a^13*b^12*c^3*d^22 - 82
992*a^13*b^12*c^5*d^20 + 805896*a^13*b^12*c^7*d^18 - 3143504*a^13*b^12*c^9*
d^16 + 6476288*a^13*b^12*c^11*d^14 - 7809984*a^13*b^12*c^13*d^12 + 5666752*
a^13*b^12*c^15*d^10 - 2403856*a^13*b^12*c^17*d^8 + 537264*a^13*b^12*c^19*d^
6 - 48048*a^13*b^12*c^21*d^4 + 728*a^13*b^12*c^23*d^2 - 224*a^14*b^11*c^2*d
^23 + 28728*a^14*b^11*c^4*d^21 - 421344*a^14*b^11*c^6*d^19 + 2077536*a^14*b
^11*c^8*d^17 - 5121024*a^14*b^11*c^10*d^15 + 7235136*a^14*b^11*c^12*d^13 -
6126848*a^14*b^11*c^14*d^11 + 3071744*a^14*b^11*c^16*d^9 - 844896*a^14*b^11
*c^18*d^7 + 104104*a^14*b^11*c^20*d^5 - 2912*a^14*b^11*c^22*d^3 - 6992*a^15
*b^10*c^3*d^22 + 177048*a^15*b^10*c^5*d^20 - 1151104*a^15*b^10*c^7*d^18 + 3
453696*a^15*b^10*c^9*d^16 - 5745024*a^15*b^10*c^11*d^14 + 5666752*a^15*b^10
*c^13*d^12 - 3331328*a^15*b^10*c^15*d^10 + 1105104*a^15*b^10*c^17*d^8 - 176

176*a^15*b^10*c^19*d^6 + 8008*a^15*b^10*c^21*d^4 + 1064*a^16*b^9*c^2*d^23 -
57456*a^16*b^9*c^4*d^21 + 529312*a^16*b^9*c^6*d^19 - 1975808*a^16*b^9*c^8*
d^17 + 3897024*a^16*b^9*c^10*d^15 - 4480896*a^16*b^9*c^12*d^13 + 3071744*a^
16*b^9*c^14*d^11 - 1208064*a^16*b^9*c^16*d^9 + 239096*a^16*b^9*c^18*d^7 - 1
6016*a^16*b^9*c^20*d^5 + 13464*a^17*b^8*c^3*d^22 - 198696*a^17*b^8*c^5*d^20
+ 949952*a^17*b^8*c^7*d^18 - 2247636*a^17*b^8*c^9*d^16 + 3021984*a^17*b^8*
c^11*d^14 - 2403856*a^17*b^8*c^13*d^12 + 1105104*a^17*b^8*c^15*d^10 - 26426
4*a^17*b^8*c^17*d^8 + 24024*a^17*b^8*c^19*d^6 - 2016*a^18*b^7*c^2*d^23 + 59
024*a^18*b^7*c^4*d^21 - 379008*a^18*b^7*c^6*d^19 + 1095384*a^18*b^7*c^8*d^1
7 - 1728832*a^18*b^7*c^10*d^15 + 1588704*a^18*b^7*c^12*d^13 - 844896*a^18*b
^7*c^14*d^11 + 239096*a^18*b^7*c^16*d^9 - 27456*a^18*b^7*c^18*d^7 - 13056*a
^19*b^6*c^3*d^22 + 123584*a^19*b^6*c^5*d^20 - 446736*a^19*b^6*c^7*d^18 + 83
1208*a^19*b^6*c^9*d^16 - 880256*a^19*b^6*c^11*d^14 + 537264*a^19*b^6*c^13*d
^12 - 176176*a^19*b^6*c^15*d^10 + 24024*a^19*b^6*c^17*d^8 + 1904*a^20*b^5*c
^2*d^23 - 32256*a^20*b^5*c^4*d^21 + 150024*a^20*b^5*c^6*d^19 - 331632*a^20*
b^5*c^8*d^17 + 404768*a^20*b^5*c^10*d^15 - 280896*a^20*b^5*c^12*d^13 + 1041
04*a^20*b^5*c^14*d^11 - 16016*a^20*b^5*c^16*d^9 + 6464*a^21*b^4*c^3*d^22 -
40512*a^21*b^4*c^5*d^20 + 108136*a^21*b^4*c^7*d^18 - 151944*a^21*b^4*c^9*d^
16 + 118032*a^21*b^4*c^11*d^14 - 48048*a^21*b^4*c^13*d^12 + 8008*a^21*b^4*c
^15*d^10 - 896*a^22*b^3*c^2*d^23 + 8568*a^22*b^3*c^4*d^21 - 28224*a^22*b^3*
c^6*d^19 + 45136*a^22*b^3*c^8*d^17 - 38304*a^22*b^3*c^10*d^15 + 16632*a^22*
b^3*c^12*d^13 - 2912*a^22*b^3*c^14*d^11 - 1392*a^23*b^2*c^3*d^22 + 5656*a^2
3*b^2*c^5*d^20 - 9984*a^23*b^2*c^7*d^18 + 8976*a^23*b^2*c^9*d^16 - 4048*a^2
3*b^2*c^11*d^14 + 728*a^23*b^2*c^13*d^12))/(a^20*d^20 + b^20*c^20 - 4*a^2*b
^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^2
0 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 +
6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20
*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b
^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d
- 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b
^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 +
48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^
9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*
d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11
+ 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 -
412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 +
4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 -
288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 83
44*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 11
68*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 138
60*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 -
3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6
336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 -
21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3
+ 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12

- 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*
d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^1
7 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d
^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^1
5*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*
d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^1
0*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^
10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^1
1*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*
a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*
a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*
a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20
724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 15
12*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 2
7504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 -
 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 1
1236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10
 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17
- 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11
+ 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 +
1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 -
2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1
168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 22
0*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^
18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19
*c^19*d - 12*a^19*b*c*d^19))*((((4*a^24*d^24 + 4*b^24*c^24 + 16*a^2*b^22*c^
24 + 16*a^4*b^20*c^24 - 1152*a^10*b^14*d^24 + 5568*a^12*b^12*d^24 - 10568*a
^14*b^10*d^24 + 9460*a^16*b^8*d^24 - 3560*a^18*b^6*d^24 + 136*a^20*b^4*d^24
 + 76*a^22*b^2*d^24 + 16*a^24*c^2*d^22 + 16*a^24*c^4*d^20 - 1152*b^24*c^10*
d^14 + 5568*b^24*c^12*d^12 - 10568*b^24*c^14*d^10 + 9460*b^24*c^16*d^8 - 35
60*b^24*c^18*d^6 + 136*b^24*c^20*d^4 + 76*b^24*c^22*d^2 + 11520*a*b^23*c^9*
d^15 - 56448*a*b^23*c^11*d^13 + 109456*a*b^23*c^13*d^11 - 101240*a*b^23*c^1
5*d^9 + 40720*a*b^23*c^17*d^7 - 2960*a*b^23*c^19*d^5 - 536*a*b^23*c^21*d^3
- 176*a^3*b^21*c^23*d - 320*a^5*b^19*c^23*d + 11520*a^9*b^15*c*d^23 - 56448
*a^11*b^13*c*d^23 + 109456*a^13*b^11*c*d^23 - 101240*a^15*b^9*c*d^23 + 4072
0*a^17*b^7*c*d^23 - 2960*a^19*b^5*c*d^23 - 536*a^21*b^3*c*d^23 - 176*a^23*b
*c^3*d^21 - 320*a^23*b*c^5*d^19 - 51840*a^2*b^22*c^8*d^16 + 263808*a^2*b^22
*c^10*d^14 - 541208*a^2*b^22*c^12*d^12 + 547088*a^2*b^22*c^14*d^10 - 263320
*a^2*b^22*c^16*d^8 + 44120*a^2*b^22*c^18*d^6 - 1564*a^2*b^22*c^20*d^4 - 196
*a^2*b^22*c^22*d^2 + 138240*a^3*b^21*c^7*d^17 - 758400*a^3*b^21*c^9*d^15 +
1720736*a^3*b^21*c^11*d^13 - 2002728*a^3*b^21*c^13*d^11 + 1210560*a^3*b^21*
c^15*d^9 - 335040*a^3*b^21*c^17*d^7 + 37680*a^3*b^21*c^19*d^5 - 288*a^3*b^2
1*c^21*d^3 - 241920*a^4*b^20*c^6*d^18 + 1512000*a^4*b^20*c^8*d^16 - 3975688
*a^4*b^20*c^10*d^14 + 5501328*a^4*b^20*c^12*d^12 - 4147952*a^4*b^20*c^14*d^
10 + 1586920*a^4*b^20*c^16*d^8 - 276020*a^4*b^20*c^18*d^6 + 21124*a^4*b^20*

c^20*d^4 + 176*a^4*b^20*c^22*d^2 + 290304*a^5*b^19*c^5*d^19 - 2232576*a^5*b^19*c^7*d^17 + 7078256*a^5*b^19*c^9*d^15 - 11781560*a^5*b^19*c^11*d^13 + 10875200*a^5*b^19*c^13*d^11 - 5365072*a^5*b^19*c^15*d^9 + 1310168*a^5*b^19*c^17*d^7 - 170968*a^5*b^19*c^19*d^5 + 8160*a^5*b^19*c^21*d^3 - 241920*a^6*b^18*c^4*d^20 + 2532096*a^6*b^18*c^6*d^18 - 9955992*a^6*b^18*c^8*d^16 + 20019440*a^6*b^18*c^10*d^14 - 22419600*a^6*b^18*c^12*d^12 + 13887520*a^6*b^18*c^14*d^10 - 4506428*a^6*b^18*c^16*d^8 + 793756*a^6*b^18*c^18*d^6 - 72240*a^6*b^18*c^20*d^4 + 3040*a^6*b^18*c^22*d^2 + 138240*a^7*b^17*c^3*d^21 - 2232576*a^7*b^17*c^5*d^19 + 11150016*a^7*b^17*c^7*d^17 - 27336616*a^7*b^17*c^9*d^15 + 37153600*a^7*b^17*c^11*d^13 - 28461040*a^7*b^17*c^13*d^11 + 11779808*a^7*b^17*c^15*d^9 - 2621008*a^7*b^17*c^17*d^7 + 336688*a^7*b^17*c^19*d^5 - 17920*a^7*b^17*c^21*d^3 - 51840*a^8*b^16*c^2*d^22 + 1512000*a^8*b^16*c^4*d^20 - 9955992*a^8*b^16*c^6*d^18 + 30289656*a^8*b^16*c^8*d^16 - 50137600*a^8*b^16*c^10*d^14 + 46972560*a^8*b^16*c^12*d^12 - 24199280*a^8*b^16*c^14*d^10 + 6661036*a^8*b^16*c^16*d^8 - 1058448*a^8*b^16*c^18*d^6 + 72560*a^8*b^16*c^20*d^4 - 758400*a^9*b^15*c^3*d^21 + 7078256*a^9*b^15*c^5*d^19 - 27336616*a^9*b^15*c^7*d^17 + 55383904*a^9*b^15*c^9*d^15 - 63124080*a^9*b^15*c^11*d^13 + 39987520*a^9*b^15*c^13*d^11 - 13462088*a^9*b^15*c^15*d^9 + 2478528*a^9*b^15*c^17*d^7 - 212032*a^9*b^15*c^19*d^5 + 263808*a^10*b^14*c^2*d^22 - 3975688*a^10*b^14*c^4*d^20 + 20019440*a^10*b^14*c^6*d^18 - 50137600*a^10*b^14*c^8*d^16 + 69593872*a^10*b^14*c^10*d^14 - 53854288*a^10*b^14*c^12*d^12 + 21989928*a^10*b^14*c^14*d^10 - 4591360*a^10*b^14*c^16*d^8 + 460480*a^10*b^14*c^18*d^6 + 1720736*a^11*b^13*c^3*d^21 - 11781560*a^11*b^13*c^5*d^19 + 37153600*a^11*b^13*c^7*d^17 - 63124080*a^11*b^13*c^9*d^15 + 59445728*a^11*b^13*c^11*d^13 - 29358696*a^11*b^13*c^13*d^11 + 6995840*a^11*b^13*c^15*d^9 - 762560*a^11*b^13*c^17*d^7 - 541208*a^12*b^12*c^2*d^22 + 5501328*a^12*b^12*c^4*d^20 - 22419600*a^12*b^12*c^6*d^18 + 46972560*a^12*b^12*c^8*d^16 - 53854288*a^12*b^12*c^10*d^14 + 32294808*a^12*b^12*c^12*d^12 - 8958208*a^12*b^12*c^14*d^10 + 999040*a^12*b^12*c^16*d^8 - 2002728*a^13*b^11*c^3*d^21 + 10875200*a^13*b^11*c^5*d^19 - 28461040*a^13*b^11*c^7*d^17 + 39987520*a^13*b^11*c^9*d^15 - 29358696*a^13*b^11*c^11*d^13 + 9722048*a^13*b^11*c^13*d^11 - 1104320*a^13*b^11*c^15*d^9 + 547088*a^14*b^10*c^2*d^22 - 4147952*a^14*b^10*c^4*d^20 + 13887520*a^14*b^10*c^6*d^18 - 24199280*a^14*b^10*c^8*d^16 + 21989928*a^14*b^10*c^10*d^14 - 8958208*a^14*b^10*c^12*d^12 + 1124032*a^14*b^10*c^14*d^10 + 1210560*a^15*b^9*c^3*d^21 - 5365072*a^15*b^9*c^5*d^19 + 11779808*a^15*b^9*c^7*d^17 - 13462088*a^15*b^9*c^9*d^15 + 6995840*a^15*b^9*c^11*d^13 - 1104320*a^15*b^9*c^13*d^11 - 263320*a^16*b^8*c^2*d^22 + 1586920*a^16*b^8*c^4*d^20 - 4506428*a^16*b^8*c^6*d^18 + 6661036*a^16*b^8*c^8*d^16 - 4591360*a^16*b^8*c^10*d^14 + 999040*a^16*b^8*c^12*d^12 - 335040*a^17*b^7*c^3*d^21 + 1310168*a^17*b^7*c^5*d^19 - 2621008*a^17*b^7*c^7*d^17 + 2478528*a^17*b^7*c^9*d^15 - 762560*a^17*b^7*c^11*d^13 + 44120*a^18*b^6*c^2*d^22 - 276020*a^18*b^6*c^4*d^20 + 793756*a^18*b^6*c^6*d^18 - 1058448*a^18*b^6*c^8*d^16 + 460480*a^18*b^6*c^10*d^14 + 37680*a^19*b^5*c^3*d^21 - 170968*a^19*b^5*c^5*d^19 + 336688*a^19*b^5*c^7*d^17 - 212032*a^19*b^5*c^9*d^15 - 1564*a^20*b^4*c^2*d^22 + 21124*a^20*b^4*c^4*d^20 - 72240*a^20*b^4*c^6*d^18 + 72560*a^20*b^4*c^8*d^16 - 288

*a^21*b^3*c^3*d^21 + 8160*a^21*b^3*c^5*d^19 - 17920*a^21*b^3*c^7*d^17 - 196
*a^22*b^2*c^2*d^22 + 176*a^22*b^2*c^4*d^20 + 3040*a^22*b^2*c^6*d^18 - 8*a*b
^23*c^23*d - 8*a^23*b*c*d^23)^2/4 - (20736*b^18*d^18 - 96768*a^2*b^16*d^18
+ 173664*a^4*b^14*d^18 - 136032*a^6*b^12*d^18 + 31081*a^8*b^10*d^18 + 8440*
a^10*b^8*d^18 + 400*a^12*b^6*d^18 - 96768*b^18*c^2*d^16 + 173664*b^18*c^4*d
^14 - 136032*b^18*c^6*d^12 + 31081*b^18*c^8*d^10 + 8440*b^18*c^10*d^8 + 400
*b^18*c^12*d^6 - 131328*a*b^17*c^3*d^15 + 216576*a*b^17*c^5*d^13 - 141104*a
*b^17*c^7*d^11 + 20260*a*b^17*c^9*d^9 + 2800*a*b^17*c^11*d^7 - 131328*a^3*b
^15*c*d^17 + 216576*a^5*b^13*c*d^17 - 141104*a^7*b^11*c*d^17 + 20260*a^9*b^
9*c*d^17 + 2800*a^11*b^7*c*d^17 + 495936*a^2*b^16*c^2*d^16 - 989856*a^2*b^1
6*c^4*d^14 + 901948*a^2*b^16*c^6*d^12 - 308392*a^2*b^16*c^8*d^10 - 5260*a^2
*b^16*c^10*d^8 + 1600*a^2*b^16*c^12*d^6 + 657408*a^3*b^15*c^3*d^15 - 115899
2*a^3*b^15*c^5*d^13 + 838256*a^3*b^15*c^7*d^11 - 182200*a^3*b^15*c^9*d^9 -
3200*a^3*b^15*c^11*d^7 - 989856*a^4*b^14*c^2*d^16 + 2185654*a^4*b^14*c^4*d^
14 - 2218576*a^4*b^14*c^6*d^12 + 900624*a^4*b^14*c^8*d^10 - 64720*a^4*b^14*
c^10*d^8 + 1600*a^4*b^14*c^12*d^6 - 1158992*a^5*b^13*c^3*d^15 + 2158808*a^5
*b^13*c^5*d^13 - 1641528*a^5*b^13*c^7*d^11 + 406880*a^5*b^13*c^9*d^9 - 1760
0*a^5*b^13*c^11*d^7 + 901948*a^6*b^12*c^2*d^16 - 2218576*a^6*b^12*c^4*d^14
+ 2430936*a^6*b^12*c^6*d^12 - 1026928*a^6*b^12*c^8*d^10 + 88720*a^6*b^12*c^
10*d^8 + 838256*a^7*b^11*c^3*d^15 - 1641528*a^7*b^11*c^5*d^13 + 1206848*a^7
*b^11*c^7*d^11 - 239360*a^7*b^11*c^9*d^9 - 308392*a^8*b^10*c^2*d^16 + 90062
4*a^8*b^10*c^4*d^14 - 1026928*a^8*b^10*c^6*d^12 + 354016*a^8*b^10*c^8*d^10
- 182200*a^9*b^9*c^3*d^15 + 406880*a^9*b^9*c^5*d^13 - 239360*a^9*b^9*c^7*d^
11 - 5260*a^10*b^8*c^2*d^16 - 64720*a^10*b^8*c^4*d^14 + 88720*a^10*b^8*c^6*
d^12 - 3200*a^11*b^7*c^3*d^15 - 17600*a^11*b^7*c^5*d^13 + 1600*a^12*b^6*c^2
*d^16 + 1600*a^12*b^6*c^4*d^14 + 27648*a*b^17*c*d^17)*(80*a^2*b^28*c^30 - 1
6*b^30*c^30 - 16*a^30*d^30 - 160*a^4*b^26*c^30 + 160*a^6*b^24*c^30 - 80*a^8
*b^22*c^30 + 16*a^10*b^20*c^30 + 16*a^20*b^10*d^30 - 80*a^22*b^8*d^30 + 160
*a^24*b^6*d^30 - 160*a^26*b^4*d^30 + 80*a^28*b^2*d^30 + 80*a^30*c^2*d^28 -
160*a^30*c^4*d^26 + 160*a^30*c^6*d^24 - 80*a^30*c^8*d^22 + 16*a^30*c^10*d^2
0 + 16*b^30*c^20*d^10 - 80*b^30*c^22*d^8 + 160*b^30*c^24*d^6 - 160*b^30*c^2
6*d^4 + 80*b^30*c^28*d^2 - 320*a*b^29*c^19*d^11 + 1600*a*b^29*c^21*d^9 - 32
00*a*b^29*c^23*d^7 + 3200*a*b^29*c^25*d^5 - 1600*a*b^29*c^27*d^3 - 1600*a^3
*b^27*c^29*d + 3200*a^5*b^25*c^29*d - 3200*a^7*b^23*c^29*d + 1600*a^9*b^21*
c^29*d - 320*a^11*b^19*c^29*d - 320*a^19*b^11*c*d^29 + 1600*a^21*b^9*c*d^29
 - 3200*a^23*b^7*c*d^29 + 3200*a^25*b^5*c*d^29 - 1600*a^27*b^3*c*d^29 - 160
0*a^29*b*c^3*d^27 + 3200*a^29*b*c^5*d^25 - 3200*a^29*b*c^7*d^23 + 1600*a^29
*b*c^9*d^21 - 320*a^29*b*c^11*d^19 + 3040*a^2*b^28*c^18*d^12 - 15280*a^2*b^
28*c^20*d^10 + 30800*a^2*b^28*c^22*d^8 - 31200*a^2*b^28*c^24*d^6 + 16000*a^
2*b^28*c^26*d^4 - 3440*a^2*b^28*c^28*d^2 - 18240*a^3*b^27*c^17*d^13 + 92800
*a^3*b^27*c^19*d^11 - 190400*a^3*b^27*c^21*d^9 + 198400*a^3*b^27*c^23*d^7 -
 107200*a^3*b^27*c^25*d^5 + 26240*a^3*b^27*c^27*d^3 + 77520*a^4*b^26*c^16*d
^14 - 402800*a^4*b^26*c^18*d^12 + 851360*a^4*b^26*c^20*d^10 - 928000*a^4*b^
26*c^22*d^8 + 541200*a^4*b^26*c^24*d^6 - 155120*a^4*b^26*c^26*d^4 + 16000*a
^4*b^26*c^28*d^2 - 248064*a^5*b^25*c^15*d^15 + 1331520*a^5*b^25*c^17*d^13 -

2939840*a^5*b^25*c^19*d^11 + 3408640*a^5*b^25*c^21*d^9 - 2184320*a^5*b^25*c^23*d^7 + 736064*a^5*b^25*c^25*d^5 - 107200*a^5*b^25*c^27*d^3 + 620160*a^6*b^24*c^14*d^16 - 3488400*a^6*b^24*c^16*d^14 + 8170000*a^6*b^24*c^18*d^12 - 10229760*a^6*b^24*c^20*d^10 + 7281600*a^6*b^24*c^22*d^8 - 2863760*a^6*b^24*c^24*d^6 + 541200*a^6*b^24*c^26*d^4 - 31200*a^6*b^24*c^28*d^2 - 1240320*a^7*b^23*c^13*d^17 + 7441920*a^7*b^23*c^15*d^15 - 18787200*a^7*b^23*c^17*d^13 + 25721600*a^7*b^23*c^19*d^11 - 20444800*a^7*b^23*c^21*d^9 + 9297920*a^7*b^23*c^23*d^7 - 2184320*a^7*b^23*c^25*d^5 + 198400*a^7*b^23*c^27*d^3 + 2015520*a^8*b^22*c^12*d^18 - 13178400*a^8*b^22*c^14*d^16 + 36434400*a^8*b^22*c^16*d^14 - 55069600*a^8*b^22*c^18*d^12 + 48989680*a^8*b^22*c^20*d^10 - 25575920*a^8*b^22*c^22*d^8 + 7281600*a^8*b^22*c^24*d^6 - 928000*a^8*b^22*c^26*d^4 + 30800*a^8*b^22*c^28*d^2 - 2687360*a^9*b^21*c^11*d^19 + 19638400*a^9*b^21*c^13*d^17 - 60362240*a^9*b^21*c^15*d^15 + 101475200*a^9*b^21*c^17*d^13 - 101172800*a^9*b^21*c^19*d^11 + 60333760*a^9*b^21*c^21*d^9 - 20444800*a^9*b^21*c^23*d^7 + 3408640*a^9*b^21*c^25*d^5 - 190400*a^9*b^21*c^27*d^3 + 2956096*a^10*b^20*c^10*d^20 - 24858080*a^10*b^20*c^12*d^18 + 86150560*a^10*b^20*c^14*d^16 - 162120160*a^10*b^20*c^16*d^14 + 181463680*a^10*b^20*c^18*d^12 - 123188112*a^10*b^20*c^20*d^10 + 48989680*a^10*b^20*c^22*d^8 - 10229760*a^10*b^20*c^24*d^6 + 851360*a^10*b^20*c^26*d^4 - 15280*a^10*b^20*c^28*d^2 - 2687360*a^11*b^19*c^9*d^21 + 26873600*a^11*b^19*c^11*d^19 - 106460800*a^11*b^19*c^13*d^17 + 225738240*a^11*b^19*c^15*d^15 - 284331200*a^11*b^19*c^17*d^13 + 219166080*a^11*b^19*c^19*d^11 - 101172800*a^11*b^19*c^21*d^9 + 25721600*a^11*b^19*c^23*d^7 - 2939840*a^11*b^19*c^25*d^5 + 92800*a^11*b^19*c^27*d^3 + 2015520*a^12*b^18*c^8*d^22 - 24858080*a^12*b^18*c^10*d^20 + 114212800*a^12*b^18*c^12*d^18 - 274937600*a^12*b^18*c^14*d^16 + 390830000*a^12*b^18*c^16*d^14 - 341426960*a^12*b^18*c^18*d^12 + 181463680*a^12*b^18*c^20*d^10 - 55069600*a^12*b^18*c^22*d^8 + 8170000*a^12*b^18*c^24*d^6 - 402800*a^12*b^18*c^26*d^4 + 3040*a^12*b^18*c^28*d^2 - 1240320*a^13*b^17*c^7*d^23 + 19638400*a^13*b^17*c^9*d^21 - 106460800*a^13*b^17*c^11*d^19 + 293542400*a^13*b^17*c^13*d^17 - 472561920*a^13*b^17*c^15*d^15 + 467412160*a^13*b^17*c^17*d^13 - 284331200*a^13*b^17*c^19*d^11 + 101475200*a^13*b^17*c^21*d^9 - 18787200*a^13*b^17*c^23*d^7 + 1331520*a^13*b^17*c^25*d^5 - 18240*a^13*b^17*c^27*d^3 + 620160*a^14*b^16*c^6*d^24 - 13178400*a^14*b^16*c^8*d^22 + 86150560*a^14*b^16*c^10*d^20 - 274937600*a^14*b^16*c^12*d^18 + 503363200*a^14*b^16*c^14*d^16 - 563751280*a^14*b^16*c^16*d^14 + 390830000*a^14*b^16*c^18*d^12 - 162120160*a^14*b^16*c^20*d^10 + 36434400*a^14*b^16*c^22*d^8 - 3488400*a^14*b^16*c^24*d^6 + 77520*a^14*b^16*c^26*d^4 - 248064*a^15*b^15*c^5*d^25 + 7441920*a^15*b^15*c^7*d^23 - 60362240*a^15*b^15*c^9*d^21 + 225738240*a^15*b^15*c^11*d^19 - 472561920*a^15*b^15*c^13*d^17 + 599984128*a^15*b^15*c^15*d^15 - 472561920*a^15*b^15*c^17*d^13 + 225738240*a^15*b^15*c^19*d^11 - 60362240*a^15*b^15*c^21*d^9 + 7441920*a^15*b^15*c^23*d^7 - 248064*a^15*b^15*c^25*d^5 + 77520*a^16*b^14*c^4*d^26 - 3488400*a^16*b^14*c^6*d^24 + 36434400*a^16*b^14*c^8*d^22 - 162120160*a^16*b^14*c^10*d^20 + 390830000*a^16*b^14*c^12*d^18 - 563751280*a^16*b^14*c^14*d^16 + 503363200*a^16*b^14*c^16*d^14 - 274937600*a^16*b^14*c^18*d^12 + 86150560*a^16*b^14*c^20*d^10 - 13178400*a^16*b^14*c^22*d^8 + 6201

60*a^16*b^14*c^24*d^6 - 18240*a^17*b^13*c^3*d^27 + 1331520*a^17*b^13*c^5*d^
25 - 18787200*a^17*b^13*c^7*d^23 + 101475200*a^17*b^13*c^9*d^21 - 284331200
*a^17*b^13*c^11*d^19 + 467412160*a^17*b^13*c^13*d^17 - 472561920*a^17*b^13*
c^15*d^15 + 293542400*a^17*b^13*c^17*d^13 - 106460800*a^17*b^13*c^19*d^11 +
19638400*a^17*b^13*c^21*d^9 - 1240320*a^17*b^13*c^23*d^7 + 3040*a^18*b^12*
c^2*d^28 - 402800*a^18*b^12*c^4*d^26 + 8170000*a^18*b^12*c^6*d^24 - 5506960
0*a^18*b^12*c^8*d^22 + 181463680*a^18*b^12*c^10*d^20 - 341426960*a^18*b^12*
c^12*d^18 + 390830000*a^18*b^12*c^14*d^16 - 274937600*a^18*b^12*c^16*d^14 +
114212800*a^18*b^12*c^18*d^12 - 24858080*a^18*b^12*c^20*d^10 + 2015520*a^1
8*b^12*c^22*d^8 + 92800*a^19*b^11*c^3*d^27 - 2939840*a^19*b^11*c^5*d^25 + 2
5721600*a^19*b^11*c^7*d^23 - 101172800*a^19*b^11*c^9*d^21 + 219166080*a^19*
b^11*c^11*d^19 - 284331200*a^19*b^11*c^13*d^17 + 225738240*a^19*b^11*c^15*d
^15 - 106460800*a^19*b^11*c^17*d^13 + 26873600*a^19*b^11*c^19*d^11 - 268736
0*a^19*b^11*c^21*d^9 - 15280*a^20*b^10*c^2*d^28 + 851360*a^20*b^10*c^4*d^26
- 10229760*a^20*b^10*c^6*d^24 + 48989680*a^20*b^10*c^8*d^22 - 123188112*a^
20*b^10*c^10*d^20 + 181463680*a^20*b^10*c^12*d^18 - 162120160*a^20*b^10*c^1
4*d^16 + 86150560*a^20*b^10*c^16*d^14 - 24858080*a^20*b^10*c^18*d^12 + 2956
096*a^20*b^10*c^20*d^10 - 190400*a^21*b^9*c^3*d^27 + 3408640*a^21*b^9*c^5*d
^25 - 20444800*a^21*b^9*c^7*d^23 + 60333760*a^21*b^9*c^9*d^21 - 101172800*a
^21*b^9*c^11*d^19 + 101475200*a^21*b^9*c^13*d^17 - 60362240*a^21*b^9*c^15*d
^15 + 19638400*a^21*b^9*c^17*d^13 - 2687360*a^21*b^9*c^19*d^11 + 30800*a^22
*b^8*c^2*d^28 - 928000*a^22*b^8*c^4*d^26 + 7281600*a^22*b^8*c^6*d^24 - 2557
5920*a^22*b^8*c^8*d^22 + 48989680*a^22*b^8*c^10*d^20 - 55069600*a^22*b^8*c^
12*d^18 + 36434400*a^22*b^8*c^14*d^16 - 13178400*a^22*b^8*c^16*d^14 + 20155
20*a^22*b^8*c^18*d^12 + 198400*a^23*b^7*c^3*d^27 - 2184320*a^23*b^7*c^5*d^2
5 + 9297920*a^23*b^7*c^7*d^23 - 20444800*a^23*b^7*c^9*d^21 + 25721600*a^23*
b^7*c^11*d^19 - 18787200*a^23*b^7*c^13*d^17 + 7441920*a^23*b^7*c^15*d^15 -
1240320*a^23*b^7*c^17*d^13 - 31200*a^24*b^6*c^2*d^28 + 541200*a^24*b^6*c^4*
d^26 - 2863760*a^24*b^6*c^6*d^24 + 7281600*a^24*b^6*c^8*d^22 - 10229760*a^2
4*b^6*c^10*d^20 + 8170000*a^24*b^6*c^12*d^18 - 3488400*a^24*b^6*c^14*d^16 +
620160*a^24*b^6*c^16*d^14 - 107200*a^25*b^5*c^3*d^27 + 736064*a^25*b^5*c^5
*d^25 - 2184320*a^25*b^5*c^7*d^23 + 3408640*a^25*b^5*c^9*d^21 - 2939840*a^2
5*b^5*c^11*d^19 + 1331520*a^25*b^5*c^13*d^17 - 248064*a^25*b^5*c^15*d^15 +
16000*a^26*b^4*c^2*d^28 - 155120*a^26*b^4*c^4*d^26 + 541200*a^26*b^4*c^6*d^
24 - 928000*a^26*b^4*c^8*d^22 + 851360*a^26*b^4*c^10*d^20 - 402800*a^26*b^4
*c^12*d^18 + 77520*a^26*b^4*c^14*d^16 + 26240*a^27*b^3*c^3*d^27 - 107200*a^
27*b^3*c^5*d^25 + 198400*a^27*b^3*c^7*d^23 - 190400*a^27*b^3*c^9*d^21 + 928
00*a^27*b^3*c^11*d^19 - 18240*a^27*b^3*c^13*d^17 - 3440*a^28*b^2*c^2*d^28 +
16000*a^28*b^2*c^4*d^26 - 31200*a^28*b^2*c^6*d^24 + 30800*a^28*b^2*c^8*d^2
2 - 15280*a^28*b^2*c^10*d^20 + 3040*a^28*b^2*c^12*d^18 + 320*a*b^29*c^29*d
+ 320*a^29*b*c*d^29))^(1/2) - 2*a^24*d^24 - 2*b^24*c^24 - 8*a^2*b^22*c^24 -
8*a^4*b^20*c^24 + 576*a^10*b^14*d^24 - 2784*a^12*b^12*d^24 + 5284*a^14*b^1
0*d^24 - 4730*a^16*b^8*d^24 + 1780*a^18*b^6*d^24 - 68*a^20*b^4*d^24 - 38*a^
22*b^2*d^24 - 8*a^24*c^2*d^22 - 8*a^24*c^4*d^20 + 576*b^24*c^10*d^14 - 2784
*b^24*c^12*d^12 + 5284*b^24*c^14*d^10 - 4730*b^24*c^16*d^8 + 1780*b^24*c^18

*d^6 - 68*b^24*c^20*d^4 - 38*b^24*c^22*d^2 - 5760*a*b^23*c^9*d^15 + 28224*a
*b^23*c^11*d^13 - 54728*a*b^23*c^13*d^11 + 50620*a*b^23*c^15*d^9 - 20360*a*
b^23*c^17*d^7 + 1480*a*b^23*c^19*d^5 + 268*a*b^23*c^21*d^3 + 88*a^3*b^21*c^
23*d + 160*a^5*b^19*c^23*d - 5760*a^9*b^15*c*d^23 + 28224*a^11*b^13*c*d^23
- 54728*a^13*b^11*c*d^23 + 50620*a^15*b^9*c*d^23 - 20360*a^17*b^7*c*d^23 +
1480*a^19*b^5*c*d^23 + 268*a^21*b^3*c*d^23 + 88*a^23*b*c^3*d^21 + 160*a^23*
b*c^5*d^19 + 25920*a^2*b^22*c^8*d^16 - 131904*a^2*b^22*c^10*d^14 + 270604*a
^2*b^22*c^12*d^12 - 273544*a^2*b^22*c^14*d^10 + 131660*a^2*b^22*c^16*d^8 -
22060*a^2*b^22*c^18*d^6 + 782*a^2*b^22*c^20*d^4 + 98*a^2*b^22*c^22*d^2 - 69
120*a^3*b^21*c^7*d^17 + 379200*a^3*b^21*c^9*d^15 - 860368*a^3*b^21*c^11*d^1
3 + 1001364*a^3*b^21*c^13*d^11 - 605280*a^3*b^21*c^15*d^9 + 167520*a^3*b^21
*c^17*d^7 - 18840*a^3*b^21*c^19*d^5 + 144*a^3*b^21*c^21*d^3 + 120960*a^4*b^
20*c^6*d^18 - 756000*a^4*b^20*c^8*d^16 + 1987844*a^4*b^20*c^10*d^14 - 27506
64*a^4*b^20*c^12*d^12 + 2073976*a^4*b^20*c^14*d^10 - 793460*a^4*b^20*c^16*d
^8 + 138010*a^4*b^20*c^18*d^6 - 10562*a^4*b^20*c^20*d^4 - 88*a^4*b^20*c^22*
d^2 - 145152*a^5*b^19*c^5*d^19 + 1116288*a^5*b^19*c^7*d^17 - 3539128*a^5*b^
19*c^9*d^15 + 5890780*a^5*b^19*c^11*d^13 - 5437600*a^5*b^19*c^13*d^11 + 268
2536*a^5*b^19*c^15*d^9 - 655084*a^5*b^19*c^17*d^7 + 85484*a^5*b^19*c^19*d^5
 - 4080*a^5*b^19*c^21*d^3 + 120960*a^6*b^18*c^4*d^20 - 1266048*a^6*b^18*c^6
*d^18 + 4977996*a^6*b^18*c^8*d^16 - 10009720*a^6*b^18*c^10*d^14 + 11209800*
a^6*b^18*c^12*d^12 - 6943760*a^6*b^18*c^14*d^10 + 2253214*a^6*b^18*c^16*d^8
 - 396878*a^6*b^18*c^18*d^6 + 36120*a^6*b^18*c^20*d^4 - 1520*a^6*b^18*c^22*
d^2 - 69120*a^7*b^17*c^3*d^21 + 1116288*a^7*b^17*c^5*d^19 - 5575008*a^7*b^1
7*c^7*d^17 + 13668308*a^7*b^17*c^9*d^15 - 18576800*a^7*b^17*c^11*d^13 + 142
30520*a^7*b^17*c^13*d^11 - 5889904*a^7*b^17*c^15*d^9 + 1310504*a^7*b^17*c^1
7*d^7 - 168344*a^7*b^17*c^19*d^5 + 8960*a^7*b^17*c^21*d^3 + 25920*a^8*b^16*
c^2*d^22 - 756000*a^8*b^16*c^4*d^20 + 4977996*a^8*b^16*c^6*d^18 - 15144828*
a^8*b^16*c^8*d^16 + 25068800*a^8*b^16*c^10*d^14 - 23486280*a^8*b^16*c^12*d^
12 + 12099640*a^8*b^16*c^14*d^10 - 3330518*a^8*b^16*c^16*d^8 + 529224*a^8*b
^16*c^18*d^6 - 36280*a^8*b^16*c^20*d^4 + 379200*a^9*b^15*c^3*d^21 - 3539128
*a^9*b^15*c^5*d^19 + 13668308*a^9*b^15*c^7*d^17 - 27691952*a^9*b^15*c^9*d^1
5 + 31562040*a^9*b^15*c^11*d^13 - 19993760*a^9*b^15*c^13*d^11 + 6731044*a^9
*b^15*c^15*d^9 - 1239264*a^9*b^15*c^17*d^7 + 106016*a^9*b^15*c^19*d^5 - 131
904*a^10*b^14*c^2*d^22 + 1987844*a^10*b^14*c^4*d^20 - 10009720*a^10*b^14*c^
6*d^18 + 25068800*a^10*b^14*c^8*d^16 - 34796936*a^10*b^14*c^10*d^14 + 26927
144*a^10*b^14*c^12*d^12 - 10994964*a^10*b^14*c^14*d^10 + 2295680*a^10*b^14*
c^16*d^8 - 230240*a^10*b^14*c^18*d^6 - 860368*a^11*b^13*c^3*d^21 + 5890780*
a^11*b^13*c^5*d^19 - 18576800*a^11*b^13*c^7*d^17 + 31562040*a^11*b^13*c^9*d
^15 - 29722864*a^11*b^13*c^11*d^13 + 14679348*a^11*b^13*c^13*d^11 - 3497920
*a^11*b^13*c^15*d^9 + 381280*a^11*b^13*c^17*d^7 + 270604*a^12*b^12*c^2*d^22
 - 2750664*a^12*b^12*c^4*d^20 + 11209800*a^12*b^12*c^6*d^18 - 23486280*a^12
*b^12*c^8*d^16 + 26927144*a^12*b^12*c^10*d^14 - 16147404*a^12*b^12*c^12*d^1
2 + 4479104*a^12*b^12*c^14*d^10 - 499520*a^12*b^12*c^16*d^8 + 1001364*a^13*
b^11*c^3*d^21 - 5437600*a^13*b^11*c^5*d^19 + 14230520*a^13*b^11*c^7*d^17 -
19993760*a^13*b^11*c^9*d^15 + 14679348*a^13*b^11*c^11*d^13 - 4861024*a^13*b

^11*c^13*d^11 + 552160*a^13*b^11*c^15*d^9 - 273544*a^14*b^10*c^2*d^22 + 207
3976*a^14*b^10*c^4*d^20 - 6943760*a^14*b^10*c^6*d^18 + 12099640*a^14*b^10*c
^8*d^16 - 10994964*a^14*b^10*c^10*d^14 + 4479104*a^14*b^10*c^12*d^12 - 5620
16*a^14*b^10*c^14*d^10 - 605280*a^15*b^9*c^3*d^21 + 2682536*a^15*b^9*c^5*d^
19 - 5889904*a^15*b^9*c^7*d^17 + 6731044*a^15*b^9*c^9*d^15 - 3497920*a^15*b
^9*c^11*d^13 + 552160*a^15*b^9*c^13*d^11 + 131660*a^16*b^8*c^2*d^22 - 79346
0*a^16*b^8*c^4*d^20 + 2253214*a^16*b^8*c^6*d^18 - 3330518*a^16*b^8*c^8*d^16
 + 2295680*a^16*b^8*c^10*d^14 - 499520*a^16*b^8*c^12*d^12 + 167520*a^17*b^7
*c^3*d^21 - 655084*a^17*b^7*c^5*d^19 + 1310504*a^17*b^7*c^7*d^17 - 1239264*
a^17*b^7*c^9*d^15 + 381280*a^17*b^7*c^11*d^13 - 22060*a^18*b^6*c^2*d^22 + 1
38010*a^18*b^6*c^4*d^20 - 396878*a^18*b^6*c^6*d^18 + 529224*a^18*b^6*c^8*d^
16 - 230240*a^18*b^6*c^10*d^14 - 18840*a^19*b^5*c^3*d^21 + 85484*a^19*b^5*c
^5*d^19 - 168344*a^19*b^5*c^7*d^17 + 106016*a^19*b^5*c^9*d^15 + 782*a^20*b^
4*c^2*d^22 - 10562*a^20*b^4*c^4*d^20 + 36120*a^20*b^4*c^6*d^18 - 36280*a^20
*b^4*c^8*d^16 + 144*a^21*b^3*c^3*d^21 - 4080*a^21*b^3*c^5*d^19 + 8960*a^21*
b^3*c^7*d^17 + 98*a^22*b^2*c^2*d^22 - 88*a^22*b^2*c^4*d^20 - 1520*a^22*b^2*
c^6*d^18 + 4*a*b^23*c^23*d + 4*a^23*b*c*d^23)/(16*(5*a^2*b^28*c^30 - b^30*c
^30 - a^30*d^30 - 10*a^4*b^26*c^30 + 10*a^6*b^24*c^30 - 5*a^8*b^22*c^30 + a
^10*b^20*c^30 + a^20*b^10*d^30 - 5*a^22*b^8*d^30 + 10*a^24*b^6*d^30 - 10*a^
26*b^4*d^30 + 5*a^28*b^2*d^30 + 5*a^30*c^2*d^28 - 10*a^30*c^4*d^26 + 10*a^3
0*c^6*d^24 - 5*a^30*c^8*d^22 + a^30*c^10*d^20 + b^30*c^20*d^10 - 5*b^30*c^2
2*d^8 + 10*b^30*c^24*d^6 - 10*b^30*c^26*d^4 + 5*b^30*c^28*d^2 - 20*a*b^29*c
^19*d^11 + 100*a*b^29*c^21*d^9 - 200*a*b^29*c^23*d^7 + 200*a*b^29*c^25*d^5
 - 100*a*b^29*c^27*d^3 - 100*a^3*b^27*c^29*d + 200*a^5*b^25*c^29*d - 200*a^7
*b^23*c^29*d + 100*a^9*b^21*c^29*d - 20*a^11*b^19*c^29*d - 20*a^19*b^11*c*d
^29 + 100*a^21*b^9*c*d^29 - 200*a^23*b^7*c*d^29 + 200*a^25*b^5*c*d^29 - 100
*a^27*b^3*c*d^29 - 100*a^29*b*c^3*d^27 + 200*a^29*b*c^5*d^25 - 200*a^29*b*c
^7*d^23 + 100*a^29*b*c^9*d^21 - 20*a^29*b*c^11*d^19 + 190*a^2*b^28*c^18*d^1
2 - 955*a^2*b^28*c^20*d^10 + 1925*a^2*b^28*c^22*d^8 - 1950*a^2*b^28*c^24*d^
6 + 1000*a^2*b^28*c^26*d^4 - 215*a^2*b^28*c^28*d^2 - 1140*a^3*b^27*c^17*d^1
3 + 5800*a^3*b^27*c^19*d^11 - 11900*a^3*b^27*c^21*d^9 + 12400*a^3*b^27*c^23
*d^7 - 6700*a^3*b^27*c^25*d^5 + 1640*a^3*b^27*c^27*d^3 + 4845*a^4*b^26*c^16
*d^14 - 25175*a^4*b^26*c^18*d^12 + 53210*a^4*b^26*c^20*d^10 - 58000*a^4*b^2
6*c^22*d^8 + 33825*a^4*b^26*c^24*d^6 - 9695*a^4*b^26*c^26*d^4 + 1000*a^4*b^
26*c^28*d^2 - 15504*a^5*b^25*c^15*d^15 + 83220*a^5*b^25*c^17*d^13 - 183740*
a^5*b^25*c^19*d^11 + 213040*a^5*b^25*c^21*d^9 - 136520*a^5*b^25*c^23*d^7 +
46004*a^5*b^25*c^25*d^5 - 6700*a^5*b^25*c^27*d^3 + 38760*a^6*b^24*c^14*d^16
 - 218025*a^6*b^24*c^16*d^14 + 510625*a^6*b^24*c^18*d^12 - 639360*a^6*b^24*
c^20*d^10 + 455100*a^6*b^24*c^22*d^8 - 178985*a^6*b^24*c^24*d^6 + 33825*a^6
*b^24*c^26*d^4 - 1950*a^6*b^24*c^28*d^2 - 77520*a^7*b^23*c^13*d^17 + 465120
*a^7*b^23*c^15*d^15 - 1174200*a^7*b^23*c^17*d^13 + 1607600*a^7*b^23*c^19*d^
11 - 1277800*a^7*b^23*c^21*d^9 + 581120*a^7*b^23*c^23*d^7 - 136520*a^7*b^23
*c^25*d^5 + 12400*a^7*b^23*c^27*d^3 + 125970*a^8*b^22*c^12*d^18 - 823650*a^
8*b^22*c^14*d^16 + 2277150*a^8*b^22*c^16*d^14 - 3441850*a^8*b^22*c^18*d^12
+ 3061855*a^8*b^22*c^20*d^10 - 1598495*a^8*b^22*c^22*d^8 + 455100*a^8*b^22*

c^24*d^6 - 58000*a^8*b^22*c^26*d^4 + 1925*a^8*b^22*c^28*d^2 - 167960*a^9*b^21*c^11*d^19 + 1227400*a^9*b^21*c^13*d^17 - 3772640*a^9*b^21*c^15*d^15 + 6342200*a^9*b^21*c^17*d^13 - 6323300*a^9*b^21*c^19*d^11 + 3770860*a^9*b^21*c^21*d^9 - 1277800*a^9*b^21*c^23*d^7 + 213040*a^9*b^21*c^25*d^5 - 11900*a^9*b^21*c^27*d^3 + 184756*a^10*b^20*c^10*d^20 - 1553630*a^10*b^20*c^12*d^18 + 5384410*a^10*b^20*c^14*d^16 - 10132510*a^10*b^20*c^16*d^14 + 11341480*a^10*b^20*c^18*d^12 - 7699257*a^10*b^20*c^20*d^10 + 3061855*a^10*b^20*c^22*d^8 - 639360*a^10*b^20*c^24*d^6 + 53210*a^10*b^20*c^26*d^4 - 955*a^10*b^20*c^28*d^2 - 167960*a^11*b^19*c^9*d^21 + 1679600*a^11*b^19*c^11*d^19 - 6653800*a^11*b^19*c^13*d^17 + 14108640*a^11*b^19*c^15*d^15 - 17770700*a^11*b^19*c^17*d^13 + 13697880*a^11*b^19*c^19*d^11 - 6323300*a^11*b^19*c^21*d^9 + 1607600*a^11*b^19*c^23*d^7 - 183740*a^11*b^19*c^25*d^5 + 5800*a^11*b^19*c^27*d^3 + 125970*a^12*b^18*c^8*d^22 - 1553630*a^12*b^18*c^10*d^20 + 7138300*a^12*b^18*c^12*d^18 - 17183600*a^12*b^18*c^14*d^16 + 24426875*a^12*b^18*c^16*d^14 - 21339185*a^12*b^18*c^18*d^12 + 11341480*a^12*b^18*c^20*d^10 - 3441850*a^12*b^18*c^22*d^8 + 510625*a^12*b^18*c^24*d^6 - 25175*a^12*b^18*c^26*d^4 + 190*a^12*b^18*c^28*d^2 - 77520*a^13*b^17*c^7*d^23 + 1227400*a^13*b^17*c^9*d^21 - 6653800*a^13*b^17*c^11*d^19 + 18346400*a^13*b^17*c^13*d^17 - 29535120*a^13*b^17*c^15*d^15 + 29213260*a^13*b^17*c^17*d^13 - 17770700*a^13*b^17*c^19*d^11 + 6342200*a^13*b^17*c^21*d^9 - 1174200*a^13*b^17*c^23*d^7 + 83220*a^13*b^17*c^25*d^5 - 1140*a^13*b^17*c^27*d^3 + 38760*a^14*b^16*c^6*d^24 - 823650*a^14*b^16*c^8*d^22 + 5384410*a^14*b^16*c^10*d^20 - 17183600*a^14*b^16*c^12*d^18 + 31460200*a^14*b^16*c^14*d^16 - 35234455*a^14*b^16*c^16*d^14 + 24426875*a^14*b^16*c^18*d^12 - 10132510*a^14*b^16*c^20*d^10 + 2277150*a^14*b^16*c^22*d^8 - 218025*a^14*b^16*c^24*d^6 + 4845*a^14*b^16*c^26*d^4 - 15504*a^15*b^15*c^5*d^25 + 465120*a^15*b^15*c^7*d^23 - 3772640*a^15*b^15*c^9*d^21 + 14108640*a^15*b^15*c^11*d^19 - 29535120*a^15*b^15*c^13*d^17 + 37499008*a^15*b^15*c^15*d^15 - 29535120*a^15*b^15*c^17*d^13 + 14108640*a^15*b^15*c^19*d^11 - 3772640*a^15*b^15*c^21*d^9 + 465120*a^15*b^15*c^23*d^7 - 15504*a^15*b^15*c^25*d^5 + 4845*a^16*b^14*c^4*d^26 - 218025*a^16*b^14*c^6*d^24 + 2277150*a^16*b^14*c^8*d^22 - 10132510*a^16*b^14*c^10*d^20 + 24426875*a^16*b^14*c^12*d^18 - 35234455*a^16*b^14*c^14*d^16 + 31460200*a^16*b^14*c^16*d^14 - 17183600*a^16*b^14*c^18*d^12 + 5384410*a^16*b^14*c^20*d^10 - 823650*a^16*b^14*c^22*d^8 + 38760*a^16*b^14*c^24*d^6 - 1140*a^17*b^13*c^3*d^27 + 83220*a^17*b^13*c^5*d^25 - 1174200*a^17*b^13*c^7*d^23 + 6342200*a^17*b^13*c^9*d^21 - 17770700*a^17*b^13*c^11*d^19 + 29213260*a^17*b^13*c^13*d^17 - 29535120*a^17*b^13*c^15*d^15 + 18346400*a^17*b^13*c^17*d^13 - 6653800*a^17*b^13*c^19*d^11 + 1227400*a^17*b^13*c^21*d^9 - 77520*a^17*b^13*c^23*d^7 + 190*a^18*b^12*c^2*d^28 - 25175*a^18*b^12*c^4*d^26 + 510625*a^18*b^12*c^6*d^24 - 3441850*a^18*b^12*c^8*d^22 + 11341480*a^18*b^12*c^10*d^20 - 21339185*a^18*b^12*c^12*d^18 + 24426875*a^18*b^12*c^14*d^16 - 17183600*a^18*b^12*c^16*d^14 + 7138300*a^18*b^12*c^18*d^12 - 1553630*a^18*b^12*c^20*d^10 + 125970*a^18*b^12*c^22*d^8 + 5800*a^19*b^11*c^3*d^27 - 183740*a^19*b^11*c^5*d^25 + 1607600*a^19*b^11*c^7*d^23 - 6323300*a^19*b^11*c^9*d^21 + 13697880*a^19*b^11*c^11*d^19 - 17770700*a^19*b^11*c^13*d^17 + 14108640*a^19*b^11*c^15*d^15 - 6653800*a^19*b^11*c

^17*d^13 + 1679600*a^19*b^11*c^19*d^11 - 167960*a^19*b^11*c^21*d^9 - 955*a^20*b^10*c^2*d^28 + 53210*a^20*b^10*c^4*d^26 - 639360*a^20*b^10*c^6*d^24 + 3061855*a^20*b^10*c^8*d^22 - 7699257*a^20*b^10*c^10*d^20 + 11341480*a^20*b^10*c^12*d^18 - 10132510*a^20*b^10*c^14*d^16 + 5384410*a^20*b^10*c^16*d^14 - 1553630*a^20*b^10*c^18*d^12 + 184756*a^20*b^10*c^20*d^10 - 11900*a^21*b^9*c^3*d^27 + 213040*a^21*b^9*c^5*d^25 - 1277800*a^21*b^9*c^7*d^23 + 3770860*a^21*b^9*c^9*d^21 - 6323300*a^21*b^9*c^11*d^19 + 6342200*a^21*b^9*c^13*d^17 - 3772640*a^21*b^9*c^15*d^15 + 1227400*a^21*b^9*c^17*d^13 - 167960*a^21*b^9*c^19*d^11 + 1925*a^22*b^8*c^2*d^28 - 58000*a^22*b^8*c^4*d^26 + 455100*a^22*b^8*c^6*d^24 - 1598495*a^22*b^8*c^8*d^22 + 3061855*a^22*b^8*c^10*d^20 - 3441850*a^22*b^8*c^12*d^18 + 2277150*a^22*b^8*c^14*d^16 - 823650*a^22*b^8*c^16*d^14 + 125970*a^22*b^8*c^18*d^12 + 12400*a^23*b^7*c^3*d^27 - 136520*a^23*b^7*c^5*d^25 + 581120*a^23*b^7*c^7*d^23 - 1277800*a^23*b^7*c^9*d^21 + 1607600*a^23*b^7*c^11*d^19 - 1174200*a^23*b^7*c^13*d^17 + 465120*a^23*b^7*c^15*d^15 - 77520*a^23*b^7*c^17*d^13 - 1950*a^24*b^6*c^2*d^28 + 33825*a^24*b^6*c^4*d^26 - 178985*a^24*b^6*c^6*d^24 + 455100*a^24*b^6*c^8*d^22 - 639360*a^24*b^6*c^10*d^20 + 510625*a^24*b^6*c^12*d^18 - 218025*a^24*b^6*c^14*d^16 + 38760*a^24*b^6*c^16*d^14 - 6700*a^25*b^5*c^3*d^27 + 46004*a^25*b^5*c^5*d^25 - 136520*a^25*b^5*c^7*d^23 + 213040*a^25*b^5*c^9*d^21 - 183740*a^25*b^5*c^11*d^19 + 83220*a^25*b^5*c^13*d^17 - 15504*a^25*b^5*c^15*d^15 + 1000*a^26*b^4*c^2*d^28 - 9695*a^26*b^4*c^4*d^26 + 33825*a^26*b^4*c^6*d^24 - 58000*a^26*b^4*c^8*d^22 + 53210*a^26*b^4*c^10*d^20 - 25175*a^26*b^4*c^12*d^18 + 4845*a^26*b^4*c^14*d^16 + 1640*a^27*b^3*c^3*d^27 - 6700*a^27*b^3*c^5*d^25 + 12400*a^27*b^3*c^7*d^23 - 11900*a^27*b^3*c^9*d^21 + 5800*a^27*b^3*c^11*d^19 - 1140*a^27*b^3*c^13*d^17 - 215*a^28*b^2*c^2*d^28 + 1000*a^28*b^2*c^4*d^26 - 1950*a^28*b^2*c^6*d^24 + 1925*a^28*b^2*c^8*d^22 - 955*a^28*b^2*c^10*d^20 + 190*a^28*b^2*c^12*d^18 + 20*a*b^29*c^29*d + 20*a^29*b*c*d^29)))^(1/2) + (4*(4*a^2*b^20*c^22 - 12*a^6*b^16*c^22 + 8*a^8*b^14*c^22 + 4*a^22*c^2*d^20 - 12*a^22*c^6*d^16 + 8*a^22*c^8*d^14 + 48*a*b^21*c^11*d^11 - 212*a*b^21*c^13*d^9 + 360*a*b^21*c^15*d^7 - 276*a*b^21*c^17*d^5 + 80*a*b^21*c^19*d^3 - 20*a^3*b^19*c^21*d - 72*a^5*b^17*c^21*d + 204*a^7*b^15*c^21*d - 112*a^9*b^13*c^21*d + 48*a^11*b^11*c*d^21 - 212*a^13*b^9*c*d^21 + 360*a^15*b^7*c*d^21 - 276*a^17*b^5*c*d^21 + 80*a^19*b^3*c*d^21 - 20*a^21*b*c^3*d^19 - 72*a^21*b*c^5*d^17 + 204*a^21*b*c^7*d^15 - 112*a^21*b*c^9*d^13 - 480*a^2*b^20*c^10*d^12 + 2160*a^2*b^20*c^12*d^10 - 3772*a^2*b^20*c^14*d^8 + 3020*a^2*b^20*c^16*d^6 - 960*a^2*b^20*c^18*d^4 + 28*a^2*b^20*c^20*d^2 + 2160*a^3*b^19*c^9*d^13 - 10152*a^3*b^19*c^11*d^11 + 18888*a^3*b^19*c^13*d^9 - 16732*a^3*b^19*c^15*d^7 + 6588*a^3*b^19*c^17*d^5 - 732*a^3*b^19*c^19*d^3 - 5760*a^4*b^18*c^8*d^14 + 29360*a^4*b^18*c^10*d^12 - 60792*a^4*b^18*c^12*d^10 + 62708*a^4*b^18*c^14*d^8 - 31892*a^4*b^18*c^16*d^6 + 6588*a^4*b^18*c^18*d^4 - 212*a^4*b^18*c^20*d^2 + 10080*a^5*b^17*c^7*d^15 - 58860*a^5*b^17*c^9*d^13 + 141880*a^5*b^17*c^11*d^11 - 175592*a^5*b^17*c^13*d^9 + 113748*a^5*b^17*c^15*d^7 - 34492*a^5*b^17*c^17*d^5 + 3308*a^5*b^17*c^19*d^3 - 12096*a^6*b^16*c^6*d^16 + 87264*a^6*b^16*c^8*d^14 - 254340*a^6*b^16*c^10*d^12 + 381532*a^6*b^16*c^12*d^10 - 307752*a^6*b^16*c^14*d^8 + 125568*a^6*b^16*c^16*d^6 - 21232*a^6*b^16*c^18*d^4 +

1068*a^6*b^16*c^20*d^2 + 10080*a^7*b^15*c^5*d^17 - 99120*a^7*b^15*c^7*d^15 + 359064*a^7*b^15*c^9*d^13 - 655076*a^7*b^15*c^11*d^11 + 650108*a^7*b^15*c^13*d^9 - 343368*a^7*b^15*c^15*d^7 + 85760*a^7*b^15*c^17*d^5 - 7652*a^7*b^15*c^19*d^3 - 5760*a^8*b^14*c^4*d^18 + 87264*a^8*b^14*c^6*d^16 - 402576*a^8*b^14*c^8*d^14 + 900324*a^8*b^14*c^10*d^12 - 1096236*a^8*b^14*c^12*d^10 + 731392*a^8*b^14*c^14*d^8 - 247352*a^8*b^14*c^16*d^6 + 34548*a^8*b^14*c^18*d^4 - 1612*a^8*b^14*c^20*d^2 + 2160*a^9*b^13*c^3*d^19 - 58860*a^9*b^13*c^5*d^17 + 359064*a^9*b^13*c^7*d^15 - 999816*a^9*b^13*c^9*d^13 + 1494564*a^9*b^13*c^11*d^11 - 1238148*a^9*b^13*c^13*d^9 + 542272*a^9*b^13*c^15*d^7 - 109032*a^9*b^13*c^17*d^5 + 7908*a^9*b^13*c^19*d^3 - 480*a^10*b^12*c^2*d^20 + 29360*a^10*b^12*c^4*d^18 - 254340*a^10*b^12*c^6*d^16 + 900324*a^10*b^12*c^8*d^14 - 1656496*a^10*b^12*c^10*d^12 + 1688232*a^10*b^12*c^12*d^10 - 934868*a^10*b^12*c^14*d^8 + 254492*a^10*b^12*c^16*d^6 - 26952*a^10*b^12*c^18*d^4 + 728*a^10*b^12*c^20*d^2 - 10152*a^11*b^11*c^3*d^19 + 141880*a^11*b^11*c^5*d^17 - 655076*a^11*b^11*c^7*d^15 + 1494564*a^11*b^11*c^9*d^13 - 1870136*a^11*b^11*c^11*d^11 + 1289704*a^11*b^11*c^13*d^9 - 455388*a^11*b^11*c^15*d^7 + 67468*a^11*b^11*c^17*d^5 - 2912*a^11*b^11*c^19*d^3 + 2160*a^12*b^10*c^2*d^20 - 60792*a^12*b^10*c^4*d^18 + 381532*a^12*b^10*c^6*d^16 - 1096236*a^12*b^10*c^8*d^14 + 1688232*a^12*b^10*c^10*d^12 - 1434728*a^12*b^10*c^12*d^10 + 639684*a^12*b^10*c^14*d^8 - 127860*a^12*b^10*c^16*d^6 + 8008*a^12*b^10*c^18*d^4 + 18888*a^13*b^9*c^3*d^19 - 175592*a^13*b^9*c^5*d^17 + 650108*a^13*b^9*c^7*d^15 - 1238148*a^13*b^9*c^9*d^13 + 1289704*a^13*b^9*c^11*d^11 - 715296*a^13*b^9*c^13*d^9 + 186564*a^13*b^9*c^15*d^7 - 16016*a^13*b^9*c^17*d^5 - 3772*a^14*b^8*c^2*d^20 + 62708*a^14*b^8*c^4*d^18 - 307752*a^14*b^8*c^6*d^16 + 731392*a^14*b^8*c^8*d^14 - 934868*a^14*b^8*c^10*d^12 + 639684*a^14*b^8*c^12*d^10 - 211416*a^14*b^8*c^14*d^8 + 24024*a^14*b^8*c^16*d^6 - 16732*a^15*b^7*c^3*d^19 + 113748*a^15*b^7*c^5*d^17 - 343368*a^15*b^7*c^7*d^15 + 542272*a^15*b^7*c^9*d^13 - 455388*a^15*b^7*c^11*d^11 + 186564*a^15*b^7*c^13*d^9 - 27456*a^15*b^7*c^15*d^7 + 3020*a^16*b^6*c^2*d^20 - 31892*a^16*b^6*c^4*d^18 + 125568*a^16*b^6*c^6*d^16 - 247352*a^16*b^6*c^8*d^14 + 254492*a^16*b^6*c^10*d^12 - 127860*a^16*b^6*c^12*d^10 + 24024*a^16*b^6*c^14*d^8 + 6588*a^17*b^5*c^3*d^19 - 34492*a^17*b^5*c^5*d^17 + 85760*a^17*b^5*c^7*d^15 - 109032*a^17*b^5*c^9*d^13 + 67468*a^17*b^5*c^11*d^11 - 16016*a^17*b^5*c^13*d^9 - 960*a^18*b^4*c^2*d^20 + 6588*a^18*b^4*c^4*d^18 - 21232*a^18*b^4*c^6*d^16 + 34548*a^18*b^4*c^8*d^14 - 26952*a^18*b^4*c^10*d^12 + 8008*a^18*b^4*c^12*d^10 - 732*a^19*b^3*c^3*d^19 + 3308*a^19*b^3*c^5*d^17 - 7652*a^19*b^3*c^7*d^15 + 7908*a^19*b^3*c^9*d^13 - 2912*a^19*b^3*c^11*d^11 + 28*a^20*b^2*c^2*d^20 - 212*a^20*b^2*c^4*d^18 + 1068*a^20*b^2*c^6*d^16 - 1612*a^20*b^2*c^8*d^14 + 728*a^20*b^2*c^10*d^12))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19

- 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19) - (8*tan(e/2 + (f*x)/2)*(12*a^5*b^17*c^22 - 4*a^22*c*d^21 - 4*a*b^21*c^22 - 8*a^7*b^15*c^22 + 12*a^22*c^5*d^17 - 8*a^22*c^7*d^15 - 24*a*b^21*c^12*d^10 + 100*a*b^21*c^14*d^8 - 164*a*b^21*c^16*d^6 + 120*a*b^21*c^18*d^4 - 28*a*b^21*c^20*d^2 + 20*a^2*b^20*c^21*d + 72*a^4*b^18*c^21*d - 204*a^6*b^16*c^21*d + 112*a^8*b^14*c^21*d - 24*a^12*b^10*c*d^21 + 100*a^14*b^8*c*d^21 - 164*a^16 *b^6*c*d^21 + 120*a^18*b^4*c*d^21 - 28*a^20*b^2*c*d^21 + 20*a^21*b*c^2*d^20 + 72*a^21*b*c^4*d^18 - 204*a^21*b*c^6*d^16 + 112*a^21*b*c^8*d^14 + 216*a^2*b^20*c^11*d^11 - 908*a^2*b^20*c^13*d^9 + 1540*a^2*b^20*c^15*d^7 - 1200*a^2*b^20*c^17*d^5 + 332*a^2*b^20*c^19*d^3 - 840*a^3*b^19*c^10*d^12 + 3672*a^3*b^19*c^12*d^10 - 6788*a^3*b^19*c^14*d^8 + 6132*a^3*b^19*c^16*d^6 - 2388*a^3*b^19*c^18*d^4 + 212*a^3*b^19*c^20*d^2 + 1800*a^4*b^18*c^9*d^13 - 8680*a^4*b^18*c^11*d^11 + 18852*a^4*b^18*c^13*d^9 - 21228*a^4*b^18*c^15*d^7 + 11692*a^4*b^18*c^17*d^5 - 2508*a^4*b^18*c^19*d^3 - 2160*a^5*b^17*c^8*d^14 + 13100*a^5*b^17*c^10*d^12 - 36820*a^5*b^17*c^12*d^10 + 53712*a^5*b^17*c^14*d^8 - 39608*a^5*b^17*c^16*d^6 + 12832*a^5*b^17*c^18*d^4 - 1068*a^5*b^17*c^20*d^2 + 1008*a^6*b^16*c^7*d^15 - 12420*a^6*b^16*c^9*d^13 + 51764*a^6*b^16*c^11*d^11 - 100128*a^6*b^16*c^13*d^9 + 96048*a^6*b^16*c^15*d^7 - 42920*a^6*b^16*c^17*d^5 + 6852*a^6*b^16*c^19*d^3 + 1008*a^7*b^15*c^6*d^16 + 5136*a^7*b^15*c^8*d^14 - 48820*a^7*b^15*c^10*d^12 + 134700*a^7*b^15*c^12*d^10 - 171472*a^7*b^15*c^14*d^8 + 103992*a^7*b^15*c^16*d^6 - 26148*a^7*b^15*c^18*d^4 + 1612*a^7*b^15*c^20*d^2 - 2160*a^8*b^14*c^5*d^17 + 5136*a^8*b^14*c^7*d^15 + 20436*a^8*b^14*c^9*d^13 - 121524*a^8*b^14*c^11*d^11 + 224888*a^8*b^14*c^13*d^9 - 186952*a^8*b^14*c^15*d^7 + 67572*a^8*b^14*c^17*d^5 - 7508*a^8*b^14*c^19*d^3 + 1800*a^9*b^13*c^4*d^18 - 12420*a^9*b^13*c^6*d^16 + 20436*a^9*b^13*c^8*d^14 + 49416*a^9*b^13*c^10*d^12 - 201552*a^9*b^13*c^12*d^10 + 245708*a^9*b^13*c^14*d^8 - 125412*a^9*b^13*c^16*d^6 + 22752*a^9*b^13*c^18*d^4 - 728*a^9*b^13*c^20*d^2 - 840*a^10*b^12*c^3*d^19 + 13100*a^10*b^12*c^5*d^17 - 48820*a^10*b^12*c^7*d^15 + 49416*a^10*b^12*c^9*d^13 + 82088*a^10*b^12*c^11*d^11 - 219092*a^10*b^12*c^13*d^9 + 168468*a^10*b^12*c^15*d^7 - 47152*a^10*b^12*c^17*d^5 + 2832*a^10*b^12*c^19*d^3 + 216*a^11*b^11*c^2*d^20 - 8680*a^11*b^11*c^4*d^18 + 51764*a^11*b^11*c^6*d^16 - 121524*a^11*b^11*c^8*d^14 + 82088*a^11*b^11*c^10*d^12 + 88712*a^11*b^11*c^12*d^10 - 153012*a^11*b^11*c^14*d^8 + 67604*a^11*b^11*c^16*d^6 - 7168*a^11*b^11*c^18*d^4 + 3672*a^12*b^10*c^3*d^19 - 36820*a^12*b^10*c^5*d^17 + 134700*a^12*b^10*c^7*d^15 - 201552*a^12*b^10*c^9*d^13 + 88712*a^12*b^10*c^11*d^11 + 62676*a^12*b^10*c^13*d^9 - 63372*a^12*b^10*c^15*d^7 + 12008*a^12*b^10*c^17*d^5 - 908*a^13*b^9*c^2*d^20 + 18852*a^13*b^9*c^4*d^18 - 100128*a^13*b^9*c^6*d^16 + 224888*a^13*b^9*c^8*d^14 - 219092*a^13*b^9*c^10*d^12 + 62676*a^13*b^9*c^12*d^10 + 26256*a^13*b^9*c^14*d^8 - 12544*a^13*b^9*c^16*d^6 - 6788*a^14*b^8*c^3*d^19 + 53712*a^14*b^8*c^5*d^17 - 171472*a^14*b^8*c^7*d^15 + 245708*a^14*b^8*c^9*d^13 - 153012*a^14*b^8*c^11*d^11 + 26256*a^14*b^8*c^13*d^9 + 5496*a^14*b^8*c^15*d^7 + 1540*a^15*b^7*c^2*d^20 - 21228*a^15*b^7*c^4*d^18 + 96048*a^15*b^7*c^6*d^16 - 186952*a^15*b^7*c^8*d^14 + 168468*a^15*b^7*c^10*d^12 - 63372*a^15*b^7*c^12*d^10 + 5496*a^15*b^7*c^14*d^8 + 6132*a^16*b^6*c^3*d^19 - 39608*a^16*b^6*c^5*d^17 + 103992*a^16*b^6*c^7*d^15 - 125412*a^16*b^6*c^9*d^13 + 67604*a^16*b^6*c^11*d^11 - 12544*a^16*b^6*c^13*d^9 - 1200*a^17*b^5*c^2*d^20 + 11692*a^17*b^5*c^4*d^18 - 42920*a^17*b^5*c^6*d^16 + 67572*a^17*b^5*c^8*d^14 - 47152*a^17*b^5*c^10*d^12 + 12008*a^17*b^5*c^12*d^10 - 2388*a^18*b^4*c^3*d^19 + 12832*a^18*b^4*c^5*d^17 - 26148*a^18*b^4*c^7*d^15 + 22752*a^18*b^4*c^9*d^13 - 7168*a^18*b^4*c^11*d^11 + 332*a^19*b^3*c^2*d^20 - 2508*a^19*b^3*c^4*d^18 + 6852*a^19* b^3*c^6*d^16 - 7508*a^19*b^3*c^8*d^14 + 2832*a^19*b^3*c^10*d^12 + 212*a^20*b^2*c^3*d^19 - 1068*a^20*b^2*c^5*d^17 + 1612*a^20*b^2*c^7*d^15 - 728*a^20*b^2*c^9*d^13))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495

*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*
a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18
*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^
2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19))
+ (4*(288*a*b^18*c^6*d^13 - 1104*a*b^18*c^8*d^11 + 1538*a*b^18*c^10*d^9 - 8
72*a*b^18*c^12*d^7 + 108*a*b^18*c^14*d^5 + 40*a*b^18*c^16*d^3 + 8*a^3*b^16*
c^18*d + 8*a^5*b^14*c^18*d + 288*a^6*b^13*c*d^18 - 1104*a^8*b^11*c*d^18 + 1
538*a^10*b^9*c*d^18 - 872*a^12*b^7*c*d^18 + 108*a^14*b^5*c*d^18 + 40*a^16*b
^3*c*d^18 + 8*a^18*b*c^3*d^16 + 8*a^18*b*c^5*d^14 - 864*a^2*b^17*c^5*d^14 +
 3216*a^2*b^17*c^7*d^12 - 4262*a^2*b^17*c^9*d^10 + 2256*a^2*b^17*c^11*d^8 -
 304*a^2*b^17*c^13*d^6 - 32*a^2*b^17*c^15*d^4 + 8*a^2*b^17*c^17*d^2 + 576*a
^3*b^16*c^4*d^15 - 3024*a^3*b^16*c^6*d^13 + 6304*a^3*b^16*c^8*d^11 - 7216*a
^3*b^16*c^10*d^9 + 4944*a^3*b^16*c^12*d^7 - 1664*a^3*b^16*c^14*d^5 - 72*a^3
*b^16*c^16*d^3 + 576*a^4*b^15*c^3*d^16 + 912*a^4*b^15*c^5*d^14 - 8720*a^4*b
^15*c^7*d^12 + 16632*a^4*b^15*c^9*d^10 - 14888*a^4*b^15*c^11*d^8 + 6704*a^4
*b^15*c^13*d^6 - 744*a^4*b^15*c^15*d^4 - 40*a^4*b^15*c^17*d^2 - 864*a^5*b^1
4*c^2*d^17 + 912*a^5*b^14*c^4*d^15 + 5140*a^5*b^14*c^6*d^13 - 16080*a^5*b^1
4*c^8*d^11 + 23520*a^5*b^14*c^10*d^9 - 20208*a^5*b^14*c^12*d^7 + 7404*a^5*b
^14*c^14*d^5 - 264*a^5*b^14*c^16*d^3 - 3024*a^6*b^13*c^3*d^16 + 5140*a^6*b^
13*c^5*d^14 + 5280*a^6*b^13*c^7*d^12 - 28380*a^6*b^13*c^9*d^10 + 39792*a^6*
b^13*c^11*d^8 - 22728*a^6*b^13*c^13*d^6 + 3096*a^6*b^13*c^15*d^4 - 112*a^6*
b^13*c^17*d^2 + 3216*a^7*b^12*c^2*d^17 - 8720*a^7*b^12*c^4*d^15 + 5280*a^7*
b^12*c^6*d^13 + 15000*a^7*b^12*c^8*d^11 - 40656*a^7*b^12*c^10*d^9 + 40296*a
^7*b^12*c^12*d^7 - 12984*a^7*b^12*c^14*d^5 + 728*a^7*b^12*c^16*d^3 + 6304*a
^8*b^11*c^3*d^16 - 16080*a^8*b^11*c^5*d^14 + 15000*a^8*b^11*c^7*d^12 + 1602
4*a^8*b^11*c^9*d^10 - 46184*a^8*b^11*c^11*d^8 + 27208*a^8*b^11*c^13*d^6 - 2
752*a^8*b^11*c^15*d^4 - 4262*a^9*b^10*c^2*d^17 + 16632*a^9*b^10*c^4*d^15 -
28380*a^9*b^10*c^6*d^13 + 16024*a^9*b^10*c^8*d^11 + 22018*a^9*b^10*c^10*d^9
 - 30104*a^9*b^10*c^12*d^7 + 6488*a^9*b^10*c^14*d^5 - 7216*a^10*b^9*c^3*d^1
6 + 23520*a^10*b^9*c^5*d^14 - 40656*a^10*b^9*c^7*d^12 + 22018*a^10*b^9*c^9*
d^10 + 13080*a^10*b^9*c^11*d^8 - 8720*a^10*b^9*c^13*d^6 + 2256*a^11*b^8*c^2
*d^17 - 14888*a^11*b^8*c^4*d^15 + 39792*a^11*b^8*c^6*d^13 - 46184*a^11*b^8*
c^8*d^11 + 13080*a^11*b^8*c^10*d^9 + 4360*a^11*b^8*c^12*d^7 + 4944*a^12*b^7
*c^3*d^16 - 20208*a^12*b^7*c^5*d^14 + 40296*a^12*b^7*c^7*d^12 - 30104*a^12*
b^7*c^9*d^10 + 4360*a^12*b^7*c^11*d^8 - 304*a^13*b^6*c^2*d^17 + 6704*a^13*b
^6*c^4*d^15 - 22728*a^13*b^6*c^6*d^13 + 27208*a^13*b^6*c^8*d^11 - 8720*a^13
*b^6*c^10*d^9 - 1664*a^14*b^5*c^3*d^16 + 7404*a^14*b^5*c^5*d^14 - 12984*a^1
4*b^5*c^7*d^12 + 6488*a^14*b^5*c^9*d^10 - 32*a^15*b^4*c^2*d^17 - 744*a^15*b
^4*c^4*d^15 + 3096*a^15*b^4*c^6*d^13 - 2752*a^15*b^4*c^8*d^11 - 72*a^16*b^3
*c^3*d^16 - 264*a^16*b^3*c^5*d^14 + 728*a^16*b^3*c^7*d^12 + 8*a^17*b^2*c^2*
d^17 - 40*a^17*b^2*c^4*d^15 - 112*a^17*b^2*c^6*d^13 + 2*a*b^18*c^18*d + 2*a
^18*b*c*d^18))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 -
 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16
*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^
6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4

- 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^1
5*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a
^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^
19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19
*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^
10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4
+ 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 15
12*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495
*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 358
8*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*
a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*
a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a
^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 1716
4*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 41
2*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 1874
4*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8
736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 56
76*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 +
34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4
- 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 -
18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^
9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^
3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d
^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^1
0*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b
^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*
b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^
11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12
*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^
12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676
*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*
a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 187
44*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412
*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 171
64*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 +
924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8
736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 7
92*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 358
8*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 49
5*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512
*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^1
8*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b
^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19)
- (8*tan(e/2 + (f*x)/2)*(a*b^18*c^19 + a^19*c*d^18 + 4*a^3*b^16*c^19 + 4*a^
5*b^14*c^19 + 4*a^19*c^3*d^16 + 4*a^19*c^5*d^14 - 576*a*b^18*c^5*d^14 + 264

0*a*b^18*c^7*d^12 - 4732*a*b^18*c^9*d^10 + 3961*a*b^18*c^11*d^8 - 1344*a*b^18*c^13*d^6 + 14*a*b^18*c^15*d^4 + 18*a*b^18*c^17*d^2 + 4*a^2*b^17*c^18*d - 20*a^4*b^15*c^18*d - 576*a^5*b^14*c*d^18 - 56*a^6*b^13*c^18*d + 2640*a^7*b^12*c*d^18 - 4732*a^9*b^10*c*d^18 + 3961*a^11*b^8*c*d^18 - 1344*a^13*b^6*c*d^18 + 14*a^15*b^4*c*d^18 + 18*a^17*b^2*c*d^18 + 4*a^18*b*c^2*d^17 - 20*a^18*b*c^4*d^15 - 56*a^18*b*c^6*d^13 + 2304*a^2*b^17*c^4*d^15 - 10944*a^2*b^17*c^6*d^13 + 20720*a^2*b^17*c^8*d^11 - 18788*a^2*b^17*c^10*d^9 + 7392*a^2*b^17*c^12*d^7 - 520*a^2*b^17*c^14*d^5 - 24*a^2*b^17*c^16*d^3 - 3456*a^3*b^16*c^3*d^16 + 20016*a^3*b^16*c^5*d^14 - 48112*a^3*b^16*c^7*d^12 + 58925*a^3*b^16*c^9*d^10 - 36732*a^3*b^16*c^11*d^8 + 9736*a^3*b^16*c^13*d^6 - 760*a^3*b^16*c^15*d^4 - 44*a^3*b^16*c^17*d^2 + 2304*a^4*b^15*c^2*d^17 - 23424*a^4*b^15*c^4*d^15 + 81680*a^4*b^15*c^6*d^13 - 135520*a^4*b^15*c^8*d^11 + 114144*a^4*b^15*c^10*d^9 - 44168*a^4*b^15*c^12*d^7 + 5696*a^4*b^15*c^14*d^5 - 332*a^4*b^15*c^16*d^3 + 20016*a^5*b^14*c^3*d^16 - 99112*a^5*b^14*c^5*d^14 + 213338*a^5*b^14*c^7*d^12 - 235152*a^5*b^14*c^9*d^10 + 130428*a^5*b^14*c^11*d^8 - 31908*a^5*b^14*c^13*d^6 + 3966*a^5*b^14*c^15*d^4 - 140*a^5*b^14*c^17*d^2 - 10944*a^6*b^13*c^2*d^17 + 81680*a^6*b^13*c^4*d^15 - 243832*a^6*b^13*c^6*d^13 + 364608*a^6*b^13*c^8*d^11 - 281736*a^6*b^13*c^10*d^9 + 103104*a^6*b^13*c^12*d^7 - 16860*a^6*b^13*c^14*d^5 + 1660*a^6*b^13*c^16*d^3 - 48112*a^7*b^12*c^3*d^16 + 213338*a^7*b^12*c^5*d^14 - 425832*a^7*b^12*c^7*d^12 + 434414*a^7*b^12*c^9*d^10 - 219064*a^7*b^12*c^11*d^8 + 50732*a^7*b^12*c^13*d^6 - 7220*a^7*b^12*c^15*d^4 + 364*a^7*b^12*c^17*d^2 + 20720*a^8*b^11*c^2*d^17 - 135520*a^8*b^11*c^4*d^15 + 364608*a^8*b^11*c^6*d^13 - 496336*a^8*b^11*c^8*d^11 + 343832*a^8*b^11*c^10*d^9 - 111220*a^8*b^11*c^12*d^7 + 17956*a^8*b^11*c^14*d^5 - 1376*a^8*b^11*c^16*d^3 + 58925*a^9*b^10*c^3*d^16 - 235152*a^9*b^10*c^5*d^14 + 434414*a^9*b^10*c^7*d^12 - 401788*a^9*b^10*c^9*d^10 + 172673*a^9*b^10*c^11*d^8 - 31940*a^9*b^10*c^13*d^6 + 3244*a^9*b^10*c^15*d^4 - 18788*a^10*b^9*c^2*d^17 + 114144*a^10*b^9*c^4*d^15 - 281736*a^10*b^9*c^6*d^13 + 343832*a^10*b^9*c^8*d^11 - 197840*a^10*b^9*c^10*d^9 + 45940*a^10*b^9*c^12*d^7 - 4760*a^10*b^9*c^14*d^5 - 36732*a^11*b^8*c^3*d^16 + 130428*a^11*b^8*c^5*d^14 - 219064*a^11*b^8*c^7*d^12 + 172673*a^11*b^8*c^9*d^10 - 52480*a^11*b^8*c^11*d^8 + 4580*a^11*b^8*c^13*d^6 + 7392*a^12*b^7*c^2*d^17 - 44168*a^12*b^7*c^4*d^15 + 103104*a^12*b^7*c^6*d^13 - 111220*a^12*b^7*c^8*d^11 + 45940*a^12*b^7*c^10*d^9 - 4000*a^12*b^7*c^12*d^7 + 9736*a^13*b^6*c^3*d^16 - 31908*a^13*b^6*c^5*d^14 + 50732*a^13*b^6*c^7*d^12 - 31940*a^13*b^6*c^9*d^10 + 4580*a^13*b^6*c^11*d^8 - 520*a^14*b^5*c^2*d^17 + 5696*a^14*b^5*c^4*d^15 - 16860*a^14*b^5*c^6*d^13 + 17956*a^14*b^5*c^8*d^11 - 4760*a^14*b^5*c^10*d^9 - 760*a^15*b^4*c^3*d^16 + 3966*a^15*b^4*c^5*d^14 - 7220*a^15*b^4*c^7*d^12 + 3244*a^15*b^4*c^9*d^10 - 24*a^16*b^3*c^2*d^17 - 332*a^16*b^3*c^4*d^15 + 1660*a^16*b^3*c^6*d^13 - 1376*a^16*b^3*c^8*d^11 - 44*a^17*b^2*c^3*d^16 - 140*a^17*b^2*c^5*d^14 + 364*a^17*b^2*c^7*d^12))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c

^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72 *a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c* d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^ 19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^1 1 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928 *a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412* a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032* a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a ^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^ 5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^ 5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^ 6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588 *a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a ^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 2157 6*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495 *a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 363 00*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4 048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^1 0*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^ 10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9 *c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11* b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11* b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12* b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a ^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^ 13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504* a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792* a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236* a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 56 76*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 474 4*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 404 8*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587* a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244* a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a ^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^1 7*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^ 2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19 *d - 12*a^19*b*c*d^19))*1i)/((8*(11700*a*b^15*c^5*d^11 - 7344*a*b^15*c^3*d^ 13 - 7829*a*b^15*c^7*d^9 + 1314*a*b^15*c^9*d^7 + 411*a*b^15*c^11*d^5 + 20*a *b^15*c^13*d^3 - 7344*a^3*b^13*c*d^15 + 11700*a^5*b^11*c*d^15 - 7829*a^7*b^

9*c*d^15 + 1314*a^9*b^7*c*d^15 + 411*a^11*b^5*c*d^15 + 20*a^13*b^3*c*d^15 + 2592*a^2*b^14*c^2*d^14 - 10944*a^2*b^14*c^4*d^12 + 16014*a^2*b^14*c^6*d^10 - 9112*a^2*b^14*c^8*d^8 + 1163*a^2*b^14*c^10*d^6 + 170*a^2*b^14*c^12*d^4 + 34776*a^3*b^13*c^3*d^13 - 62715*a^3*b^13*c^5*d^11 + 50316*a^3*b^13*c^7*d^9 - 14734*a^3*b^13*c^9*d^7 - 64*a^3*b^13*c^11*d^5 + 80*a^3*b^13*c^13*d^3 - 1
0944*a^4*b^12*c^2*d^14 + 48580*a^4*b^12*c^4*d^12 - 75480*a^4*b^12*c^6*d^10
+ 47294*a^4*b^12*c^8*d^8 - 9176*a^4*b^12*c^10*d^6 - 40*a^4*b^12*c^12*d^4 -
62715*a^5*b^11*c^3*d^13 + 126404*a^5*b^11*c^5*d^11 - 113533*a^5*b^11*c^7*d^
9 + 40192*a^5*b^11*c^9*d^7 - 3388*a^5*b^11*c^11*d^5 + 80*a^5*b^11*c^13*d^3
+ 16014*a^6*b^10*c^2*d^14 - 75480*a^6*b^10*c^4*d^12 + 122510*a^6*b^10*c^6*d
^10 - 79106*a^6*b^10*c^8*d^8 + 17020*a^6*b^10*c^10*d^6 - 760*a^6*b^10*c^12*
d^4 + 50316*a^7*b^9*c^3*d^13 - 113533*a^7*b^9*c^5*d^11 + 108024*a^7*b^9*c^7
*d^9 - 38084*a^7*b^9*c^9*d^7 + 3248*a^7*b^9*c^11*d^5 - 9112*a^8*b^8*c^2*d^1
4 + 47294*a^8*b^8*c^4*d^12 - 79106*a^8*b^8*c^6*d^10 + 47096*a^8*b^8*c^8*d^8
 - 7432*a^8*b^8*c^10*d^6 - 14734*a^9*b^7*c^3*d^13 + 40192*a^9*b^7*c^5*d^11
- 38084*a^9*b^7*c^7*d^9 + 9728*a^9*b^7*c^9*d^7 + 1163*a^10*b^6*c^2*d^14 - 9
176*a^10*b^6*c^4*d^12 + 17020*a^10*b^6*c^6*d^10 - 7432*a^10*b^6*c^8*d^8 - 6
4*a^11*b^5*c^3*d^13 - 3388*a^11*b^5*c^5*d^11 + 3248*a^11*b^5*c^7*d^9 + 170*
a^12*b^4*c^2*d^14 - 40*a^12*b^4*c^4*d^12 - 760*a^12*b^4*c^6*d^10 + 80*a^13*
b^3*c^3*d^13 + 80*a^13*b^3*c^5*d^11 + 1728*a*b^15*c*d^15))/(a^20*d^20 + b^2
0*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^2
0 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4
*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c
^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c
^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48
*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^
19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^
17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^1
3 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 41
2*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^
3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3
*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b
^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*
b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^
15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^
15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^1
4*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6
*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b
^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7
*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^
7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^
8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 1716
4*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*
a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 3977
6*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8

344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 224
4*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12
+ 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^
14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3
*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c
^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9
*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c
^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^
8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b
^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*
b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^1
3*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*
b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^
14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^
15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^
15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16
*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16
*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*
b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b
^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c
^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*
d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19) + ((((4*a^24*d^24 + 4*b^24*c^24
 + 16*a^2*b^22*c^24 + 16*a^4*b^20*c^24 - 1152*a^10*b^14*d^24 + 5568*a^12*b^
12*d^24 - 10568*a^14*b^10*d^24 + 9460*a^16*b^8*d^24 - 3560*a^18*b^6*d^24 +
136*a^20*b^4*d^24 + 76*a^22*b^2*d^24 + 16*a^24*c^2*d^22 + 16*a^24*c^4*d^20
- 1152*b^24*c^10*d^14 + 5568*b^24*c^12*d^12 - 10568*b^24*c^14*d^10 + 9460*b
^24*c^16*d^8 - 3560*b^24*c^18*d^6 + 136*b^24*c^20*d^4 + 76*b^24*c^22*d^2 +
11520*a*b^23*c^9*d^15 - 56448*a*b^23*c^11*d^13 + 109456*a*b^23*c^13*d^11 -
101240*a*b^23*c^15*d^9 + 40720*a*b^23*c^17*d^7 - 2960*a*b^23*c^19*d^5 - 536
*a*b^23*c^21*d^3 - 176*a^3*b^21*c^23*d - 320*a^5*b^19*c^23*d + 11520*a^9*b^
15*c*d^23 - 56448*a^11*b^13*c*d^23 + 109456*a^13*b^11*c*d^23 - 101240*a^15*
b^9*c*d^23 + 40720*a^17*b^7*c*d^23 - 2960*a^19*b^5*c*d^23 - 536*a^21*b^3*c*
d^23 - 176*a^23*b*c^3*d^21 - 320*a^23*b*c^5*d^19 - 51840*a^2*b^22*c^8*d^16
+ 263808*a^2*b^22*c^10*d^14 - 541208*a^2*b^22*c^12*d^12 + 547088*a^2*b^22*c
^14*d^10 - 263320*a^2*b^22*c^16*d^8 + 44120*a^2*b^22*c^18*d^6 - 1564*a^2*b^
22*c^20*d^4 - 196*a^2*b^22*c^22*d^2 + 138240*a^3*b^21*c^7*d^17 - 758400*a^3
*b^21*c^9*d^15 + 1720736*a^3*b^21*c^11*d^13 - 2002728*a^3*b^21*c^13*d^11 +
1210560*a^3*b^21*c^15*d^9 - 335040*a^3*b^21*c^17*d^7 + 37680*a^3*b^21*c^19*
d^5 - 288*a^3*b^21*c^21*d^3 - 241920*a^4*b^20*c^6*d^18 + 1512000*a^4*b^20*c
^8*d^16 - 3975688*a^4*b^20*c^10*d^14 + 5501328*a^4*b^20*c^12*d^12 - 4147952
*a^4*b^20*c^14*d^10 + 1586920*a^4*b^20*c^16*d^8 - 276020*a^4*b^20*c^18*d^6
+ 21124*a^4*b^20*c^20*d^4 + 176*a^4*b^20*c^22*d^2 + 290304*a^5*b^19*c^5*d^1
9 - 2232576*a^5*b^19*c^7*d^17 + 7078256*a^5*b^19*c^9*d^15 - 11781560*a^5*b^
19*c^11*d^13 + 10875200*a^5*b^19*c^13*d^11 - 5365072*a^5*b^19*c^15*d^9 + 13
10168*a^5*b^19*c^17*d^7 - 170968*a^5*b^19*c^19*d^5 + 8160*a^5*b^19*c^21*d^3

- 241920*a^6*b^18*c^4*d^20 + 2532096*a^6*b^18*c^6*d^18 - 9955992*a^6*b^18*c^8*d^16 + 20019440*a^6*b^18*c^10*d^14 - 22419600*a^6*b^18*c^12*d^12 + 13887520*a^6*b^18*c^14*d^10 - 4506428*a^6*b^18*c^16*d^8 + 793756*a^6*b^18*c^18*d^6 - 72240*a^6*b^18*c^20*d^4 + 3040*a^6*b^18*c^22*d^2 + 138240*a^7*b^17*c^3*d^21 - 2232576*a^7*b^17*c^5*d^19 + 11150016*a^7*b^17*c^7*d^17 - 27336616*a^7*b^17*c^9*d^15 + 37153600*a^7*b^17*c^11*d^13 - 28461040*a^7*b^17*c^13*d^11 + 11779808*a^7*b^17*c^15*d^9 - 2621008*a^7*b^17*c^17*d^7 + 336688*a^7*b^17*c^19*d^5 - 17920*a^7*b^17*c^21*d^3 - 51840*a^8*b^16*c^2*d^22 + 1512000*a^8*b^16*c^4*d^20 - 9955992*a^8*b^16*c^6*d^18 + 30289656*a^8*b^16*c^8*d^16 - 50137600*a^8*b^16*c^10*d^14 + 46972560*a^8*b^16*c^12*d^12 - 24199280*a^8*b^16*c^14*d^10 + 6661036*a^8*b^16*c^16*d^8 - 1058448*a^8*b^16*c^18*d^6 + 72560*a^8*b^16*c^20*d^4 - 758400*a^9*b^15*c^3*d^21 + 7078256*a^9*b^15*c^5*d^19 - 27336616*a^9*b^15*c^7*d^17 + 55383904*a^9*b^15*c^9*d^15 - 63124080*a^9*b^15*c^11*d^13 + 39987520*a^9*b^15*c^13*d^11 - 13462088*a^9*b^15*c^15*d^9 + 2478528*a^9*b^15*c^17*d^7 - 212032*a^9*b^15*c^19*d^5 + 263808*a^10*b^14*c^2*d^22 - 3975688*a^10*b^14*c^4*d^20 + 20019440*a^10*b^14*c^6*d^18 - 50137600*a^10*b^14*c^8*d^16 + 69593872*a^10*b^14*c^10*d^14 - 53854288*a^10*b^14*c^12*d^12 + 21989928*a^10*b^14*c^14*d^10 - 4591360*a^10*b^14*c^16*d^8 + 460480*a^10*b^14*c^18*d^6 + 1720736*a^11*b^13*c^3*d^21 - 11781560*a^11*b^13*c^5*d^19 + 37153600*a^11*b^13*c^7*d^17 - 63124080*a^11*b^13*c^9*d^15 + 59445728*a^11*b^13*c^11*d^13 - 29358696*a^11*b^13*c^13*d^11 + 6995840*a^11*b^13*c^15*d^9 - 762560*a^11*b^13*c^17*d^7 - 541208*a^12*b^12*c^2*d^22 + 5501328*a^12*b^12*c^4*d^20 - 22419600*a^12*b^12*c^6*d^18 + 46972560*a^12*b^12*c^8*d^16 - 53854288*a^12*b^12*c^10*d^14 + 32294808*a^12*b^12*c^12*d^12 - 8958208*a^12*b^12*c^14*d^10 + 999040*a^12*b^12*c^16*d^8 - 2002728*a^13*b^11*c^3*d^21 + 10875200*a^13*b^11*c^5*d^19 - 28461040*a^13*b^11*c^7*d^17 + 39987520*a^13*b^11*c^9*d^15 - 29358696*a^13*b^11*c^11*d^13 + 9722048*a^13*b^11*c^13*d^11 - 1104320*a^13*b^11*c^15*d^9 + 547088*a^14*b^10*c^2*d^22 - 4147952*a^14*b^10*c^4*d^20 + 13887520*a^14*b^10*c^6*d^18 - 24199280*a^14*b^10*c^8*d^16 + 21989928*a^14*b^10*c^10*d^14 - 8958208*a^14*b^10*c^12*d^12 + 1124032*a^14*b^10*c^14*d^10 + 1210560*a^15*b^9*c^3*d^21 - 5365072*a^15*b^9*c^5*d^19 + 11779808*a^15*b^9*c^7*d^17 - 13462088*a^15*b^9*c^9*d^15 + 6995840*a^15*b^9*c^11*d^13 - 1104320*a^15*b^9*c^13*d^11 - 263320*a^16*b^8*c^2*d^22 + 1586920*a^16*b^8*c^4*d^20 - 4506428*a^16*b^8*c^6*d^18 + 6661036*a^16*b^8*c^8*d^16 - 4591360*a^16*b^8*c^10*d^14 + 999040*a^16*b^8*c^12*d^12 - 335040*a^17*b^7*c^3*d^21 + 1310168*a^17*b^7*c^5*d^19 - 2621008*a^17*b^7*c^7*d^17 + 2478528*a^17*b^7*c^9*d^15 - 762560*a^17*b^7*c^11*d^13 + 44120*a^18*b^6*c^2*d^22 - 276020*a^18*b^6*c^4*d^20 + 793756*a^18*b^6*c^6*d^18 - 1058448*a^18*b^6*c^8*d^16 + 460480*a^18*b^6*c^10*d^14 + 37680*a^19*b^5*c^3*d^21 - 170968*a^19*b^5*c^5*d^19 + 336688*a^19*b^5*c^7*d^17 - 212032*a^19*b^5*c^9*d^15 - 1564*a^20*b^4*c^2*d^22 + 21124*a^20*b^4*c^4*d^20 - 72240*a^20*b^4*c^6*d^18 + 72560*a^20*b^4*c^8*d^16 - 288*a^21*b^3*c^3*d^21 + 8160*a^21*b^3*c^5*d^19 - 17920*a^21*b^3*c^7*d^17 - 196*a^22*b^2*c^2*d^22 + 176*a^22*b^2*c^4*d^20 + 3040*a^22*b^2*c^6*d^18 - 8*a*b^23*c^23*d - 8*a^23*b*c*d^23)^2/4 - (20736*b^18*d^18 - 96768*a^2*b^16*d^18 + 173664*a^4*b^14*d^18 - 136032*a^6*b^12*d^18 + 31081*a^8* b^10*d^18 + 8440*a^10*b^8*d^18 + 400*a^12*b^6*d^18 - 96768*b^18*c^2*d^16 + 173664*b^18*c^4*d^14 - 136032*b^18*c^6*d^12 + 31081*b^18*c^8*d^10 + 8440*b^18*c^10*d^8 + 400*b^18*c^12*d^6 - 131328*a*b^17*c^3*d^15 + 216576*a*b^17*c^5*d^13 - 141104*a*b^17*c^7*d^11 + 20260*a*b^17*c^9*d^9 + 2800*a*b^17*c^11*d^7 - 131328*a^3*b^15*c*d^17 + 216576*a^5*b^13*c*d^17 - 141104*a^7*b^11*c*d^17 + 20260*a^9*b^9*c*d^17 + 2800*a^11*b^7*c*d^17 + 495936*a^2*b^16*c^2*d^16 - 989856*a^2*b^16*c^4*d^14 + 901948*a^2*b^16*c^6*d^12 - 308392*a^2*b^16*c^8*d^10 - 5260*a^2*b^16*c^10*d^8 + 1600*a^2*b^16*c^12*d^6 + 657408*a^3*b^15*c^3*d^15 - 1158992*a^3*b^15*c^5*d^13 + 838256*a^3*b^15*c^7*d^11 - 182200*a^3*b^15*c^9*d^9 - 3200*a^3*b^15*c^11*d^7 - 989856*a^4*b^14*c^2*d^16 + 2185654*a^4*b^14*c^4*d^14 - 2218576*a^4*b^14*c^6*d^12 + 900624*a^4*b^14*c^8*d^10 - 64720*a^4*b^14*c^10*d^8 + 1600*a^4*b^14*c^12*d^6 - 1158992*a^5*b^13*c^3*d^15 + 2158808*a^5*b^13*c^5*d^13 - 1641528*a^5*b^13*c^7*d^11 + 406880*a^5*b^13*c^9*d^9 - 17600*a^5*b^13*c^11*d^7 + 901948*a^6*b^12*c^2*d^16 - 2218576*a^6*b^12*c^4*d^14 + 2430936*a^6*b^12*c^6*d^12 - 1026928*a^6*b^12*c^8*d^10 + 88720*a^6*b^12*c^10*d^8 + 838256*a^7*b^11*c^3*d^15 - 1641528*a^7*b^11*c^5*d^13 + 1206848*a^7*b^11*c^7*d^11 - 239360*a^7*b^11*c^9*d^9 - 308392*a^8*b^10*c^2*d^16 + 900624*a^8*b^10*c^4*d^14 - 1026928*a^8*b^10*c^6*d^12 + 354016*a^8*b^10*c^8*d^10 - 182200*a^9*b^9*c^3*d^15 + 406880*a^9*b^9*c^5*d^13 - 239360*a^9*b^9*c^7*d^11 - 5260*a^10*b^8*c^2*d^16 - 64720*a^10*b^8*c^4*d^14 + 88720*a^10*b^8*c^6*d^12 - 3200*a^11*b^7*c^3*d^15 - 17600*a^11*b^7*c^5*d^13 + 1600*a^12*b^6*c^2*d^16 + 1600*a^12*b^6*c^4*d^14 + 27648*a*b^17*c*d^17)*(80*a^2*b^28*c^30 - 16*b^30*c^30 - 16*a^30*d^30 - 160*a^4*b^26*c^30 + 160*a^6*b^24*c^30 - 80*a^8*b^22*c^30 + 16*a^10*b^20*c^30 + 16*a^20*b^10*d^30 - 80*a^22*b^8*d^30 + 160*a^24*b^6*d^30 - 160*a^26*b^4*d^30 + 80*a^28*b^2*d^30 + 80*a^30*c^2*d^28 - 160*a^30*c^4*d^26 + 160*a^30*c^6*d^24 - 80*a^30*c^8*d^22 + 16*a^30*c^10*d^20 + 16*b^30*c^20*d^10 - 80*b^30*c^22*d^8 + 160*b^30*c^24*d^6 - 160*b^30*c^26*d^4 + 80*b^30*c^28*d^2 - 320*a*b^29*c^19*d^11 + 1600*a*b^29*c^21*d^9 - 3200*a*b^29*c^23*d^7 + 3200*a*b^29*c^25*d^5 - 1600*a*b^29*c^27*d^3 - 1600*a^3*b^27*c^29*d + 3200*a^5*b^25*c^29*d - 3200*a^7*b^23*c^29*d + 1600*a^9*b^21*c^29*d - 320*a^11*b^19*c^29*d - 320*a^19*b^11*c*d^29 + 1600*a^21*b^9*c*d^29 - 3200*a^23*b^7*c*d^29 + 3200*a^25*b^5*c*d^29 - 1600*a^27*b^3*c*d^29 - 1600*a^29*b*c^3*d^27 + 3200*a^29*b*c^5*d^25 - 3200*a^29*b*c^7*d^23 + 1600*a^29*b*c^9*d^21 - 320*a^29*b*c^11*d^19 + 3040*a^2*b^28*c^18*d^12 - 15280*a^2*b^28*c^20*d^10 + 30800*a^2*b^28*c^22*d^8 - 31200*a^2*b^28*c^24*d^6 + 16000*a^2*b^28*c^26*d^4 - 3440*a^2*b^28*c^28*d^2 - 18240*a^3*b^27*c^17*d^13 + 92800*a^3*b^27*c^19*d^11 - 190400*a^3*b^27*c^21*d^9 + 198400*a^3*b^27*c^23*d^7 - 107200*a^3*b^27*c^25*d^5 + 26240*a^3*b^27*c^27*d^3 + 77520*a^4*b^26*c^16*d^14 - 402800*a^4*b^26*c^18*d^12 + 851360*a^4*b^26*c^20*d^10 - 928000*a^4*b^26*c^22*d^8 + 541200*a^4*b^26*c^24*d^6 - 155120*a^4*b^26*c^26*d^4 + 16000*a^4*b^26*c^28*d^2 - 248064*a^5*b^25*c^15*d^15 + 1331520*a^5*b^25*c^17*d^13 - 2939840*a^5*b^25*c^19*d^11 + 3408640*a^5*b^25*c^21*d^9 - 2184320*a^5*b^25*c^23*d^7 + 736064*a^5*b^25*c^25*d^5 - 107200*a^5*b^25*c^27*d^3 + 620160*a^6*b^24*c^14*d^16 - 3488400*a^6*b^24*c^16*d^14 + 8170000*a^6*b^24*c^18*d^12 - 10229760*a^6*b^24*c^20*d^10 + 7281600*a^6*b^24*c^22*d^8 -

2863760*a^6*b^24*c^24*d^6 + 541200*a^6*b^24*c^26*d^4 - 31200*a^6*b^24*c^28
*d^2 - 1240320*a^7*b^23*c^13*d^17 + 7441920*a^7*b^23*c^15*d^15 - 18787200*a
^7*b^23*c^17*d^13 + 25721600*a^7*b^23*c^19*d^11 - 20444800*a^7*b^23*c^21*d^
9 + 9297920*a^7*b^23*c^23*d^7 - 2184320*a^7*b^23*c^25*d^5 + 198400*a^7*b^23
*c^27*d^3 + 2015520*a^8*b^22*c^12*d^18 - 13178400*a^8*b^22*c^14*d^16 + 3643
4400*a^8*b^22*c^16*d^14 - 55069600*a^8*b^22*c^18*d^12 + 48989680*a^8*b^22*c
^20*d^10 - 25575920*a^8*b^22*c^22*d^8 + 7281600*a^8*b^22*c^24*d^6 - 928000*
a^8*b^22*c^26*d^4 + 30800*a^8*b^22*c^28*d^2 - 2687360*a^9*b^21*c^11*d^19 +
19638400*a^9*b^21*c^13*d^17 - 60362240*a^9*b^21*c^15*d^15 + 101475200*a^9*b
^21*c^17*d^13 - 101172800*a^9*b^21*c^19*d^11 + 60333760*a^9*b^21*c^21*d^9 -
20444800*a^9*b^21*c^23*d^7 + 3408640*a^9*b^21*c^25*d^5 - 190400*a^9*b^21*c
^27*d^3 + 2956096*a^10*b^20*c^10*d^20 - 24858080*a^10*b^20*c^12*d^18 + 8615
0560*a^10*b^20*c^14*d^16 - 162120160*a^10*b^20*c^16*d^14 + 181463680*a^10*b
^20*c^18*d^12 - 123188112*a^10*b^20*c^20*d^10 + 48989680*a^10*b^20*c^22*d^8
- 10229760*a^10*b^20*c^24*d^6 + 851360*a^10*b^20*c^26*d^4 - 15280*a^10*b^2
0*c^28*d^2 - 2687360*a^11*b^19*c^9*d^21 + 26873600*a^11*b^19*c^11*d^19 - 10
6460800*a^11*b^19*c^13*d^17 + 225738240*a^11*b^19*c^15*d^15 - 284331200*a^1
1*b^19*c^17*d^13 + 219166080*a^11*b^19*c^19*d^11 - 101172800*a^11*b^19*c^21
*d^9 + 25721600*a^11*b^19*c^23*d^7 - 2939840*a^11*b^19*c^25*d^5 + 92800*a^1
1*b^19*c^27*d^3 + 2015520*a^12*b^18*c^8*d^22 - 24858080*a^12*b^18*c^10*d^20
+ 114212800*a^12*b^18*c^12*d^18 - 274937600*a^12*b^18*c^14*d^16 + 39083000
0*a^12*b^18*c^16*d^14 - 341426960*a^12*b^18*c^18*d^12 + 181463680*a^12*b^18
*c^20*d^10 - 55069600*a^12*b^18*c^22*d^8 + 8170000*a^12*b^18*c^24*d^6 - 402
800*a^12*b^18*c^26*d^4 + 3040*a^12*b^18*c^28*d^2 - 1240320*a^13*b^17*c^7*d^
23 + 19638400*a^13*b^17*c^9*d^21 - 106460800*a^13*b^17*c^11*d^19 + 29354240
0*a^13*b^17*c^13*d^17 - 472561920*a^13*b^17*c^15*d^15 + 467412160*a^13*b^17
*c^17*d^13 - 284331200*a^13*b^17*c^19*d^11 + 101475200*a^13*b^17*c^21*d^9 -
18787200*a^13*b^17*c^23*d^7 + 1331520*a^13*b^17*c^25*d^5 - 18240*a^13*b^17
*c^27*d^3 + 620160*a^14*b^16*c^6*d^24 - 13178400*a^14*b^16*c^8*d^22 + 86150
560*a^14*b^16*c^10*d^20 - 274937600*a^14*b^16*c^12*d^18 + 503363200*a^14*b^
16*c^14*d^16 - 563751280*a^14*b^16*c^16*d^14 + 390830000*a^14*b^16*c^18*d^1
2 - 162120160*a^14*b^16*c^20*d^10 + 36434400*a^14*b^16*c^22*d^8 - 3488400*a
^14*b^16*c^24*d^6 + 77520*a^14*b^16*c^26*d^4 - 248064*a^15*b^15*c^5*d^25 +
7441920*a^15*b^15*c^7*d^23 - 60362240*a^15*b^15*c^9*d^21 + 225738240*a^15*b
^15*c^11*d^19 - 472561920*a^15*b^15*c^13*d^17 + 599984128*a^15*b^15*c^15*d^
15 - 472561920*a^15*b^15*c^17*d^13 + 225738240*a^15*b^15*c^19*d^11 - 603622
40*a^15*b^15*c^21*d^9 + 7441920*a^15*b^15*c^23*d^7 - 248064*a^15*b^15*c^25*
d^5 + 77520*a^16*b^14*c^4*d^26 - 3488400*a^16*b^14*c^6*d^24 + 36434400*a^16
*b^14*c^8*d^22 - 162120160*a^16*b^14*c^10*d^20 + 390830000*a^16*b^14*c^12*d
^18 - 563751280*a^16*b^14*c^14*d^16 + 503363200*a^16*b^14*c^16*d^14 - 27493
7600*a^16*b^14*c^18*d^12 + 86150560*a^16*b^14*c^20*d^10 - 13178400*a^16*b^1
4*c^22*d^8 + 620160*a^16*b^14*c^24*d^6 - 18240*a^17*b^13*c^3*d^27 + 1331520
*a^17*b^13*c^5*d^25 - 18787200*a^17*b^13*c^7*d^23 + 101475200*a^17*b^13*c^9
*d^21 - 284331200*a^17*b^13*c^11*d^19 + 467412160*a^17*b^13*c^13*d^17 - 472
561920*a^17*b^13*c^15*d^15 + 293542400*a^17*b^13*c^17*d^13 - 106460800*a^17

*b^13*c^19*d^11 + 19638400*a^17*b^13*c^21*d^9 - 1240320*a^17*b^13*c^23*d^7 + 3040*a^18*b^12*c^2*d^28 - 402800*a^18*b^12*c^4*d^26 + 8170000*a^18*b^12*c^6*d^24 - 55069600*a^18*b^12*c^8*d^22 + 181463680*a^18*b^12*c^10*d^20 - 341426960*a^18*b^12*c^12*d^18 + 390830000*a^18*b^12*c^14*d^16 - 274937600*a^18*b^12*c^16*d^14 + 114212800*a^18*b^12*c^18*d^12 - 24858080*a^18*b^12*c^20*d^10 + 2015520*a^18*b^12*c^22*d^8 + 92800*a^19*b^11*c^3*d^27 - 2939840*a^19*b^11*c^5*d^25 + 25721600*a^19*b^11*c^7*d^23 - 101172800*a^19*b^11*c^9*d^21 + 219166080*a^19*b^11*c^11*d^19 - 284331200*a^19*b^11*c^13*d^17 + 225738240*a^19*b^11*c^15*d^15 - 106460800*a^19*b^11*c^17*d^13 + 26873600*a^19*b^11*c^19*d^11 - 2687360*a^19*b^11*c^21*d^9 - 15280*a^20*b^10*c^2*d^28 + 851360*a^20*b^10*c^4*d^26 - 10229760*a^20*b^10*c^6*d^24 + 48989680*a^20*b^10*c^8*d^22 - 123188112*a^20*b^10*c^10*d^20 + 181463680*a^20*b^10*c^12*d^18 - 162120160*a^20*b^10*c^14*d^16 + 86150560*a^20*b^10*c^16*d^14 - 24858080*a^20*b^10*c^18*d^12 + 2956096*a^20*b^10*c^20*d^10 - 190400*a^21*b^9*c^3*d^27 + 3408640*a^21*b^9*c^5*d^25 - 20444800*a^21*b^9*c^7*d^23 + 60333760*a^21*b^9*c^9*d^21 - 101172800*a^21*b^9*c^11*d^19 + 101475200*a^21*b^9*c^13*d^17 - 60362240*a^21*b^9*c^15*d^15 + 19638400*a^21*b^9*c^17*d^13 - 2687360*a^21*b^9*c^19*d^11 + 30800*a^22*b^8*c^2*d^28 - 928000*a^22*b^8*c^4*d^26 + 7281600*a^22*b^8*c^6*d^24 - 25575920*a^22*b^8*c^8*d^22 + 48989680*a^22*b^8*c^10*d^20 - 55069600*a^22*b^8*c^12*d^18 + 36434400*a^22*b^8*c^14*d^16 - 13178400*a^22*b^8*c^16*d^14 + 2015520*a^22*b^8*c^18*d^12 + 198400*a^23*b^7*c^3*d^27 - 2184320*a^23*b^7*c^5*d^25 + 9297920*a^23*b^7*c^7*d^23 - 20444800*a^23*b^7*c^9*d^21 + 25721600*a^23*b^7*c^11*d^19 - 18787200*a^23*b^7*c^13*d^17 + 7441920*a^23*b^7*c^15*d^15 - 1240320*a^23*b^7*c^17*d^13 - 31200*a^24*b^6*c^2*d^28 + 541200*a^24*b^6*c^4*d^26 - 2863760*a^24*b^6*c^6*d^24 + 7281600*a^24*b^6*c^8*d^22 - 10229760*a^24*b^6*c^10*d^20 + 8170000*a^24*b^6*c^12*d^18 - 3488400*a^24*b^6*c^14*d^16 + 620160*a^24*b^6*c^16*d^14 - 107200*a^25*b^5*c^3*d^27 + 736064*a^25*b^5*c^5*d^25 - 2184320*a^25*b^5*c^7*d^23 + 3408640*a^25*b^5*c^9*d^21 - 2939840*a^25*b^5*c^11*d^19 + 1331520*a^25*b^5*c^13*d^17 - 248064*a^25*b^5*c^15*d^15 + 16000*a^26*b^4*c^2*d^28 - 155120*a^26*b^4*c^4*d^26 + 541200*a^26*b^4*c^6*d^24 - 928000*a^26*b^4*c^8*d^22 + 851360*a^26*b^4*c^10*d^20 - 402800*a^26*b^4*c^12*d^18 + 77520*a^26*b^4*c^14*d^16 + 26240*a^27*b^3*c^3*d^27 - 107200*a^27*b^3*c^5*d^25 + 198400*a^27*b^3*c^7*d^23 - 190400*a^27*b^3*c^9*d^21 + 92800*a^27*b^3*c^11*d^19 - 18240*a^27*b^3*c^13*d^17 - 3440*a^28*b^2*c^2*d^28 + 16000*a^28*b^2*c^4*d^26 - 31200*a^28*b^2*c^6*d^24 + 30800*a^28*b^2*c^8*d^22 - 15280*a^28*b^2*c^10*d^20 + 3040*a^28*b^2*c^12*d^18 + 320*a*b^29*c^29*d + 320*a^29*b*c*d^29))^(1/2) - 2*a^24*d^24 - 2*b^24*c^24 - 8*a^2*b^22*c^24 - 8*a^4*b^20*c^24 + 576*a^10*b^14*d^24 - 2784*a^12*b^12*d^24 + 5284*a^14*b^10*d^24 - 4730*a^16*b^8*d^24 + 1780*a^18*b^6*d^24 - 68*a^20*b^4*d^24 - 38*a^22*b^2*d^24 - 8*a^24*c^2*d^22 - 8*a^24*c^4*d^20 + 576*b^24*c^10*d^14 - 2784*b^24*c^12*d^12 + 5284*b^24*c^14*d^10 - 4730*b^24*c^16*d^8 + 1780*b^24*c^18*d^6 - 68*b^24*c^20*d^4 - 38*b^24*c^22*d^2 - 5760*a*b^23*c^9*d^15 + 28224*a*b^23*c^11*d^13 - 54728*a*b^23*c^13*d^11 + 50620*a*b^23*c^15*d^9 - 20360*a*b^23*c^17*d^7 + 1480*a*b^23*c^19*d^5 + 268*a*b^23*c^21*d^3 + 88*a^3*b^21*c^23*d + 160*a^5*b^19*c^23*d - 5760*a^9*b^15*c*d^23 + 28224*

a^11*b^13*c*d^23 - 54728*a^13*b^11*c*d^23 + 50620*a^15*b^9*c*d^23 - 20360*a
^17*b^7*c*d^23 + 1480*a^19*b^5*c*d^23 + 268*a^21*b^3*c*d^23 + 88*a^23*b*c^3
*d^21 + 160*a^23*b*c^5*d^19 + 25920*a^2*b^22*c^8*d^16 - 131904*a^2*b^22*c^1
0*d^14 + 270604*a^2*b^22*c^12*d^12 - 273544*a^2*b^22*c^14*d^10 + 131660*a^2
*b^22*c^16*d^8 - 22060*a^2*b^22*c^18*d^6 + 782*a^2*b^22*c^20*d^4 + 98*a^2*b
^22*c^22*d^2 - 69120*a^3*b^21*c^7*d^17 + 379200*a^3*b^21*c^9*d^15 - 860368*
a^3*b^21*c^11*d^13 + 1001364*a^3*b^21*c^13*d^11 - 605280*a^3*b^21*c^15*d^9
+ 167520*a^3*b^21*c^17*d^7 - 18840*a^3*b^21*c^19*d^5 + 144*a^3*b^21*c^21*d^
3 + 120960*a^4*b^20*c^6*d^18 - 756000*a^4*b^20*c^8*d^16 + 1987844*a^4*b^20*
c^10*d^14 - 2750664*a^4*b^20*c^12*d^12 + 2073976*a^4*b^20*c^14*d^10 - 79346
0*a^4*b^20*c^16*d^8 + 138010*a^4*b^20*c^18*d^6 - 10562*a^4*b^20*c^20*d^4 -
88*a^4*b^20*c^22*d^2 - 145152*a^5*b^19*c^5*d^19 + 1116288*a^5*b^19*c^7*d^17
 - 3539128*a^5*b^19*c^9*d^15 + 5890780*a^5*b^19*c^11*d^13 - 5437600*a^5*b^1
9*c^13*d^11 + 2682536*a^5*b^19*c^15*d^9 - 655084*a^5*b^19*c^17*d^7 + 85484*
a^5*b^19*c^19*d^5 - 4080*a^5*b^19*c^21*d^3 + 120960*a^6*b^18*c^4*d^20 - 126
6048*a^6*b^18*c^6*d^18 + 4977996*a^6*b^18*c^8*d^16 - 10009720*a^6*b^18*c^10
*d^14 + 11209800*a^6*b^18*c^12*d^12 - 6943760*a^6*b^18*c^14*d^10 + 2253214*
a^6*b^18*c^16*d^8 - 396878*a^6*b^18*c^18*d^6 + 36120*a^6*b^18*c^20*d^4 - 15
20*a^6*b^18*c^22*d^2 - 69120*a^7*b^17*c^3*d^21 + 1116288*a^7*b^17*c^5*d^19
- 5575008*a^7*b^17*c^7*d^17 + 13668308*a^7*b^17*c^9*d^15 - 18576800*a^7*b^1
7*c^11*d^13 + 14230520*a^7*b^17*c^13*d^11 - 5889904*a^7*b^17*c^15*d^9 + 131
0504*a^7*b^17*c^17*d^7 - 168344*a^7*b^17*c^19*d^5 + 8960*a^7*b^17*c^21*d^3
+ 25920*a^8*b^16*c^2*d^22 - 756000*a^8*b^16*c^4*d^20 + 4977996*a^8*b^16*c^6
*d^18 - 15144828*a^8*b^16*c^8*d^16 + 25068800*a^8*b^16*c^10*d^14 - 23486280
*a^8*b^16*c^12*d^12 + 12099640*a^8*b^16*c^14*d^10 - 3330518*a^8*b^16*c^16*d
^8 + 529224*a^8*b^16*c^18*d^6 - 36280*a^8*b^16*c^20*d^4 + 379200*a^9*b^15*c
^3*d^21 - 3539128*a^9*b^15*c^5*d^19 + 13668308*a^9*b^15*c^7*d^17 - 27691952
*a^9*b^15*c^9*d^15 + 31562040*a^9*b^15*c^11*d^13 - 19993760*a^9*b^15*c^13*d
^11 + 6731044*a^9*b^15*c^15*d^9 - 1239264*a^9*b^15*c^17*d^7 + 106016*a^9*b^
15*c^19*d^5 - 131904*a^10*b^14*c^2*d^22 + 1987844*a^10*b^14*c^4*d^20 - 1000
9720*a^10*b^14*c^6*d^18 + 25068800*a^10*b^14*c^8*d^16 - 34796936*a^10*b^14*
c^10*d^14 + 26927144*a^10*b^14*c^12*d^12 - 10994964*a^10*b^14*c^14*d^10 + 2
295680*a^10*b^14*c^16*d^8 - 230240*a^10*b^14*c^18*d^6 - 860368*a^11*b^13*c^
3*d^21 + 5890780*a^11*b^13*c^5*d^19 - 18576800*a^11*b^13*c^7*d^17 + 3156204
0*a^11*b^13*c^9*d^15 - 29722864*a^11*b^13*c^11*d^13 + 14679348*a^11*b^13*c^
13*d^11 - 3497920*a^11*b^13*c^15*d^9 + 381280*a^11*b^13*c^17*d^7 + 270604*a
^12*b^12*c^2*d^22 - 2750664*a^12*b^12*c^4*d^20 + 11209800*a^12*b^12*c^6*d^1
8 - 23486280*a^12*b^12*c^8*d^16 + 26927144*a^12*b^12*c^10*d^14 - 16147404*a
^12*b^12*c^12*d^12 + 4479104*a^12*b^12*c^14*d^10 - 499520*a^12*b^12*c^16*d^
8 + 1001364*a^13*b^11*c^3*d^21 - 5437600*a^13*b^11*c^5*d^19 + 14230520*a^13
*b^11*c^7*d^17 - 19993760*a^13*b^11*c^9*d^15 + 14679348*a^13*b^11*c^11*d^13
 - 4861024*a^13*b^11*c^13*d^11 + 552160*a^13*b^11*c^15*d^9 - 273544*a^14*b^
10*c^2*d^22 + 2073976*a^14*b^10*c^4*d^20 - 6943760*a^14*b^10*c^6*d^18 + 120
99640*a^14*b^10*c^8*d^16 - 10994964*a^14*b^10*c^10*d^14 + 4479104*a^14*b^10
*c^12*d^12 - 562016*a^14*b^10*c^14*d^10 - 605280*a^15*b^9*c^3*d^21 + 268253

6*a^15*b^9*c^5*d^19 - 5889904*a^15*b^9*c^7*d^17 + 6731044*a^15*b^9*c^9*d^15 - 3497920*a^15*b^9*c^11*d^13 + 552160*a^15*b^9*c^13*d^11 + 131660*a^16*b^8*c^2*d^22 - 793460*a^16*b^8*c^4*d^20 + 2253214*a^16*b^8*c^6*d^18 - 3330518*a^16*b^8*c^8*d^16 + 2295680*a^16*b^8*c^10*d^14 - 499520*a^16*b^8*c^12*d^12 + 167520*a^17*b^7*c^3*d^21 - 655084*a^17*b^7*c^5*d^19 + 1310504*a^17*b^7*c^7*d^17 - 1239264*a^17*b^7*c^9*d^15 + 381280*a^17*b^7*c^11*d^13 - 22060*a^18*b^6*c^2*d^22 + 138010*a^18*b^6*c^4*d^20 - 396878*a^18*b^6*c^6*d^18 + 529224*a^18*b^6*c^8*d^16 - 230240*a^18*b^6*c^10*d^14 - 18840*a^19*b^5*c^3*d^21 + 85484*a^19*b^5*c^5*d^19 - 168344*a^19*b^5*c^7*d^17 + 106016*a^19*b^5*c^9*d^15 + 782*a^20*b^4*c^2*d^22 - 10562*a^20*b^4*c^4*d^20 + 36120*a^20*b^4*c^6*d^18 - 36280*a^20*b^4*c^8*d^16 + 144*a^21*b^3*c^3*d^21 - 4080*a^21*b^3*c^5*d^19 + 8960*a^21*b^3*c^7*d^17 + 98*a^22*b^2*c^2*d^22 - 88*a^22*b^2*c^4*d^20 - 1520*a^22*b^2*c^6*d^18 + 4*a*b^23*c^23*d + 4*a^23*b*c*d^23)/(16*(5*a^2*b^28*c^30 - b^30*c^30 - a^30*d^30 - 10*a^4*b^26*c^30 + 10*a^6*b^24*c^30 - 5*a^8*b^22*c^30 + a^10*b^20*c^30 + a^20*b^10*d^30 - 5*a^22*b^8*d^30 + 10*a^24*b^6*d^30 - 10*a^26*b^4*d^30 + 5*a^28*b^2*d^30 + 5*a^30*c^2*d^28 - 10*a^30*c^4*d^26 + 10*a^30*c^6*d^24 - 5*a^30*c^8*d^22 + a^30*c^10*d^20 + b^30*c^20*d^10 - 5*b^30*c^22*d^8 + 10*b^30*c^24*d^6 - 10*b^30*c^26*d^4 + 5*b^30*c^28*d^2 - 20*a*b^29*c^19*d^11 + 100*a*b^29*c^21*d^9 - 200*a*b^29*c^23*d^7 + 200*a*b^29*c^25*d^5 - 100*a*b^29*c^27*d^3 - 100*a^3*b^27*c^29*d + 200*a^5*b^25*c^29*d - 200*a^7*b^23*c^29*d + 100*a^9*b^21*c^29*d - 20*a^11*b^19*c^29*d - 20*a^19*b^11*c*d^29 + 100*a^21*b^9*c*d^29 - 200*a^23*b^7*c*d^29 + 200*a^25*b^5*c*d^29 - 100*a^27*b^3*c*d^29 - 100*a^29*b*c^3*d^27 + 200*a^29*b*c^5*d^25 - 200*a^29*b*c^7*d^23 + 100*a^29*b*c^9*d^21 - 20*a^29*b*c^11*d^19 + 190*a^2*b^28*c^18*d^12 - 955*a^2*b^28*c^20*d^10 + 1925*a^2*b^28*c^22*d^8 - 1950*a^2*b^28*c^24*d^6 + 1000*a^2*b^28*c^26*d^4 - 215*a^2*b^28*c^28*d^2 - 1140*a^3*b^27*c^17*d^13 + 5800*a^3*b^27*c^19*d^11 - 11900*a^3*b^27*c^21*d^9 + 12400*a^3*b^27*c^23*d^7 - 6700*a^3*b^27*c^25*d^5 + 1640*a^3*b^27*c^27*d^3 + 4845*a^4*b^26*c^16*d^14 - 25175*a^4*b^26*c^18*d^12 + 53210*a^4*b^26*c^20*d^10 - 58000*a^4*b^26*c^22*d^8 + 33825*a^4*b^26*c^24*d^6 - 9695*a^4*b^26*c^26*d^4 + 1000*a^4*b^26*c^28*d^2 - 15504*a^5*b^25*c^15*d^15 + 83220*a^5*b^25*c^17*d^13 - 183740*a^5*b^25*c^19*d^11 + 213040*a^5*b^25*c^21*d^9 - 136520*a^5*b^25*c^23*d^7 + 46004*a^5*b^25*c^25*d^5 - 6700*a^5*b^25*c^27*d^3 + 38760*a^6*b^24*c^14*d^16 - 218025*a^6*b^24*c^16*d^14 + 510625*a^6*b^24*c^18*d^12 - 639360*a^6*b^24*c^20*d^10 + 455100*a^6*b^24*c^22*d^8 - 178985*a^6*b^24*c^24*d^6 + 33825*a^6*b^24*c^26*d^4 - 1950*a^6*b^24*c^28*d^2 - 77520*a^7*b^23*c^13*d^17 + 465120*a^7*b^23*c^15*d^15 - 1174200*a^7*b^23*c^17*d^13 + 1607600*a^7*b^23*c^19*d^11 - 1277800*a^7*b^23*c^21*d^9 + 581120*a^7*b^23*c^23*d^7 - 136520*a^7*b^23*c^25*d^5 + 12400*a^7*b^23*c^27*d^3 + 125970*a^8*b^22*c^12*d^18 - 823650*a^8*b^22*c^14*d^16 + 2277150*a^8*b^22*c^16*d^14 - 3441850*a^8*b^22*c^18*d^12 + 3061855*a^8*b^22*c^20*d^10 - 1598495*a^8*b^22*c^22*d^8 + 455100*a^8*b^22*c^24*d^6 - 58000*a^8*b^22*c^26*d^4 + 1925*a^8*b^22*c^28*d^2 - 167960*a^9*b^21*c^11*d^19 + 1227400*a^9*b^21*c^13*d^17 - 3772640*a^9*b^21*c^15*d^15 + 6342200*a^9*b^21*c^17*d^13 - 6323300*a^9*b^21*c^19*d^11 + 3770860*a^9*b^21*c^21*d^9 - 1277800*a^9*b^21*c^23*d^7 + 213040*a^9*b^21*c^25* d^5 - 11900*a^9*b^21*c^27*d^3 + 184756*a^10*b^20*c^10*d^20 - 1553630*a^10*b^20*c^12*d^18 + 5384410*a^10*b^20*c^14*d^16 - 10132510*a^10*b^20*c^16*d^14 + 11341480*a^10*b^20*c^18*d^12 - 7699257*a^10*b^20*c^20*d^10 + 3061855*a^10*b^20*c^22*d^8 - 639360*a^10*b^20*c^24*d^6 + 53210*a^10*b^20*c^26*d^4 - 955*a^10*b^20*c^28*d^2 - 167960*a^11*b^19*c^9*d^21 + 1679600*a^11*b^19*c^11*d^19 - 6653800*a^11*b^19*c^13*d^17 + 14108640*a^11*b^19*c^15*d^15 - 17770700*a^11*b^19*c^17*d^13 + 13697880*a^11*b^19*c^19*d^11 - 6323300*a^11*b^19*c^21*d^9 + 1607600*a^11*b^19*c^23*d^7 - 183740*a^11*b^19*c^25*d^5 + 5800*a^11*b^19*c^27*d^3 + 125970*a^12*b^18*c^8*d^22 - 1553630*a^12*b^18*c^10*d^20 + 7138300*a^12*b^18*c^12*d^18 - 17183600*a^12*b^18*c^14*d^16 + 24426875*a^12*b^18*c^16*d^14 - 21339185*a^12*b^18*c^18*d^12 + 11341480*a^12*b^18*c^20*d^10 - 3441850*a^12*b^18*c^22*d^8 + 510625*a^12*b^18*c^24*d^6 - 25175*a^12*b^18*c^26*d^4 + 190*a^12*b^18*c^28*d^2 - 77520*a^13*b^17*c^7*d^23 + 1227400*a^13*b^17*c^9*d^21 - 6653800*a^13*b^17*c^11*d^19 + 18346400*a^13*b^17*c^13*d^17 - 29535120*a^13*b^17*c^15*d^15 + 29213260*a^13*b^17*c^17*d^13 - 17770700*a^13*b^17*c^19*d^11 + 6342200*a^13*b^17*c^21*d^9 - 1174200*a^13*b^17*c^23*d^7 + 83220*a^13*b^17*c^25*d^5 - 1140*a^13*b^17*c^27*d^3 + 38760*a^14*b^16*c^6*d^24 - 823650*a^14*b^16*c^8*d^22 + 5384410*a^14*b^16*c^10*d^20 - 17183600*a^14*b^16*c^12*d^18 + 31460200*a^14*b^16*c^14*d^16 - 35234455*a^14*b^16*c^16*d^14 + 24426875*a^14*b^16*c^18*d^12 - 10132510*a^14*b^16*c^20*d^10 + 2277150*a^14*b^16*c^22*d^8 - 218025*a^14*b^16*c^24*d^6 + 4845*a^14*b^16*c^26*d^4 - 15504*a^15*b^15*c^5*d^25 + 465120*a^15*b^15*c^7*d^23 - 3772640*a^15*b^15*c^9*d^21 + 14108640*a^15*b^15*c^11*d^19 - 29535120*a^15*b^15*c^13*d^17 + 37499008*a^15*b^15*c^15*d^15 - 29535120*a^15*b^15*c^17*d^13 + 14108640*a^15*b^15*c^19*d^11 - 3772640*a^15*b^15*c^21*d^9 + 465120*a^15*b^15*c^23*d^7 - 15504*a^15*b^15*c^25*d^5 + 4845*a^16*b^14*c^4*d^26 - 218025*a^16*b^14*c^6*d^24 + 2277150*a^16*b^14*c^8*d^22 - 10132510*a^16*b^14*c^10*d^20 + 24426875*a^16*b^14*c^12*d^18 - 35234455*a^16*b^14*c^14*d^16 + 31460200*a^16*b^14*c^16*d^14 - 17183600*a^16*b^14*c^18*d^12 + 5384410*a^16*b^14*c^20*d^10 - 823650*a^16*b^14*c^22*d^8 + 38760*a^16*b^14*c^24*d^6 - 1140*a^17*b^13*c^3*d^27 + 83220*a^17*b^13*c^5*d^25 - 1174200*a^17*b^13*c^7*d^23 + 6342200*a^17*b^13*c^9*d^21 - 17770700*a^17*b^13*c^11*d^19 + 29213260*a^17*b^13*c^13*d^17 - 29535120*a^17*b^13*c^15*d^15 + 18346400*a^17*b^13*c^17*d^13 - 6653800*a^17*b^13*c^19*d^11 + 1227400*a^17*b^13*c^21*d^9 - 77520*a^17*b^13*c^23*d^7 + 190*a^18*b^12*c^2*d^28 - 25175*a^18*b^12*c^4*d^26 + 510625*a^18*b^12*c^6*d^24 - 3441850*a^18*b^12*c^8*d^22 + 11341480*a^18*b^12*c^10*d^20 - 21339185*a^18*b^12*c^12*d^18 + 24426875*a^18*b^12*c^14*d^16 - 17183600*a^18*b^12*c^16*d^14 + 7138300*a^18*b^12*c^18*d^12 - 1553630*a^18*b^12*c^20*d^10 + 125970*a^18*b^12*c^22*d^8 + 5800*a^19*b^11*c^3*d^27 - 183740*a^19*b^11*c^5*d^25 + 1607600*a^19*b^11*c^7*d^23 - 6323300*a^19*b^11*c^9*d^21 + 13697880*a^19*b^11*c^11*d^19 - 17770700*a^19*b^11*c^13*d^17 + 14108640*a^19*b^11*c^15*d^15 - 6653800*a^19*b^11*c^17*d^13 + 1679600*a^19*b^11*c^19*d^11 - 167960*a^19*b^11*c^21*d^9 - 955*a^20*b^10*c^2*d^28 + 53210*a^20*b^10*c^4*d^26 - 639360*a^20*b^10*c^6*d^24 + 3061855*a^20*b^10*c^8*d^22 - 7699257*a^20*b^10*c^10*d^20 + 11341480*a^20*b^10*c^12*d^18 - 10132510*a^20*b^10*c^14*d^16 + 5384410*a^20* b^10*c^16*d^14 - 1553630*a^20*b^10*c^18*d^12 + 184756*a^20*b^10*c^20*d^10 - 11900*a^21*b^9*c^3*d^27 + 213040*a^21*b^9*c^5*d^25 - 1277800*a^21*b^9*c^7*d^23 + 3770860*a^21*b^9*c^9*d^21 - 6323300*a^21*b^9*c^11*d^19 + 6342200*a^21*b^9*c^13*d^17 - 3772640*a^21*b^9*c^15*d^15 + 1227400*a^21*b^9*c^17*d^13 - 167960*a^21*b^9*c^19*d^11 + 1925*a^22*b^8*c^2*d^28 - 58000*a^22*b^8*c^4*d^26 + 455100*a^22*b^8*c^6*d^24 - 1598495*a^22*b^8*c^8*d^22 + 3061855*a^22*b^8*c^10*d^20 - 3441850*a^22*b^8*c^12*d^18 + 2277150*a^22*b^8*c^14*d^16 - 823650*a^22*b^8*c^16*d^14 + 125970*a^22*b^8*c^18*d^12 + 12400*a^23*b^7*c^3*d^27 - 136520*a^23*b^7*c^5*d^25 + 581120*a^23*b^7*c^7*d^23 - 1277800*a^23*b^7*c^9*d^21 + 1607600*a^23*b^7*c^11*d^19 - 1174200*a^23*b^7*c^13*d^17 + 465120*a^23*b^7*c^15*d^15 - 77520*a^23*b^7*c^17*d^13 - 1950*a^24*b^6*c^2*d^28 + 33825*a^24*b^6*c^4*d^26 - 178985*a^24*b^6*c^6*d^24 + 455100*a^24*b^6*c^8*d^22 - 639360*a^24*b^6*c^10*d^20 + 510625*a^24*b^6*c^12*d^18 - 218025*a^24*b^6*c^14*d^16 + 38760*a^24*b^6*c^16*d^14 - 6700*a^25*b^5*c^3*d^27 + 46004*a^25*b^5*c^5*d^25 - 136520*a^25*b^5*c^7*d^23 + 213040*a^25*b^5*c^9*d^21 - 183740*a^25*b^5*c^11*d^19 + 83220*a^25*b^5*c^13*d^17 - 15504*a^25*b^5*c^15*d^15 + 1000*a^26*b^4*c^2*d^28 - 9695*a^26*b^4*c^4*d^26 + 33825*a^26*b^4*c^6*d^24 - 58000*a^26*b^4*c^8*d^22 + 53210*a^26*b^4*c^10*d^20 - 25175*a^26*b^4*c^12*d^18 + 4845*a^26*b^4*c^14*d^16 + 1640*a^27*b^3*c^3*d^27 - 6700*a^27*b^3*c^5*d^25 + 12400*a^27*b^3*c^7*d^23 - 11900*a^27*b^3*c^9*d^21 + 5800*a^27*b^3*c^11*d^19 - 1140*a^27*b^3*c^13*d^17 - 215*a^28*b^2*c^2*d^28 + 1000*a^28*b^2*c^4*d^26 - 1950*a^28*b^2*c^6*d^24 + 1925*a^28*b^2*c^8*d^22 - 955*a^28*b^2*c^10*d^20 + 190*a^28*b^2*c^12*d^18 + 20*a*b^29*c^29*d + 20*a^29*b*c*d^29)))^(1/2)*(((((4*a^24*d^24 + 4*b^24*c^24 + 16*a^2*b^22*c^24 + 16*a^4*b^20*c^24 - 1152*a^10*b^14*d^24 + 5568*a^12*b^12*d^24 - 10568*a^14*b^10*d^24 + 9460*a^16*b^8*d^24 - 3560*a^18*b^6*d^24 + 136*a^20*b^4*d^24 + 76*a^22*b^2*d^24 + 16*a^24*c^2*d^22 + 16*a^24*c^4*d^20 - 1152*b^24*c^10*d^14 + 5568*b^24*c^12*d^12 - 10568*b^24*c^14*d^10 + 9460*b^24*c^16*d^8 - 3560*b^24*c^18*d^6 + 136*b^24*c^20*d^4 + 76*b^24*c^22*d^2 + 11520*a*b^23*c^9*d^15 - 56448*a*b^23*c^11*d^13 + 109456*a*b^23*c^13*d^11 - 101240*a*b^23*c^15*d^9 + 40720*a*b^23*c^17*d^7 - 2960*a*b^23*c^19*d^5 - 536*a*b^23*c^21*d^3 - 176*a^3*b^21*c^23*d - 320*a^5*b^19*c^23*d + 11520*a^9*b^15*c*d^23 - 56448*a^11*b^13*c*d^23 + 109456*a^13*b^11*c*d^23 - 101240*a^15*b^9*c*d^23 + 40720*a^17*b^7*c*d^23 - 2960*a^19*b^5*c*d^23 - 536*a^21*b^3*c*d^23 - 176*a^23*b*c^3*d^21 - 320*a^23*b*c^5*d^19 - 51840*a^2*b^22*c^8*d^16 + 263808*a^2*b^22*c^10*d^14 - 541208*a^2*b^22*c^12*d^12 + 547088*a^2*b^22*c^14*d^10 - 263320*a^2*b^22*c^16*d^8 + 44120*a^2*b^22*c^18*d^6 - 1564*a^2*b^22*c^20*d^4 - 196*a^2*b^22*c^22*d^2 + 138240*a^3*b^21*c^7*d^17 - 758400*a^3*b^21*c^9*d^15 + 1720736*a^3*b^21*c^11*d^13 - 2002728*a^3*b^21*c^13*d^11 + 1210560*a^3*b^21*c^15*d^9 - 335040*a^3*b^21*c^17*d^7 + 37680*a^3*b^21*c^19*d^5 - 288*a^3*b^21*c^21*d^3 - 241920*a^4*b^20*c^6*d^18 + 1512000*a^4*b^20*c^8*d^16 - 3975688*a^4*b^20*c^10*d^14 + 5501328*a^4*b^20*c^12*d^12 - 4147952*a^4*b^20*c^14*d^10 + 1586920*a^4*b^20*c^16*d^8 - 276020*a^4*b^20*c^18*d^6 + 21124*a^4*b^20*c^20*d^4 + 176*a^4*b^20*c^22*d^2 + 290304*a^5*b^19*c^5*d^19 - 2232576*a^5*b^19*c^7*d^17 + 7078256*a^5*b^19*c^9*d^15 - 11781560*a^5*b^19*c^11*d^13 + 10875200*a^5*b^19*c^13*

d^11 - 5365072*a^5*b^19*c^15*d^9 + 1310168*a^5*b^19*c^17*d^7 - 170968*a^5*b
^19*c^19*d^5 + 8160*a^5*b^19*c^21*d^3 - 241920*a^6*b^18*c^4*d^20 + 2532096*
a^6*b^18*c^6*d^18 - 9955992*a^6*b^18*c^8*d^16 + 20019440*a^6*b^18*c^10*d^14
 - 22419600*a^6*b^18*c^12*d^12 + 13887520*a^6*b^18*c^14*d^10 - 4506428*a^6*
b^18*c^16*d^8 + 793756*a^6*b^18*c^18*d^6 - 72240*a^6*b^18*c^20*d^4 + 3040*a
^6*b^18*c^22*d^2 + 138240*a^7*b^17*c^3*d^21 - 2232576*a^7*b^17*c^5*d^19 + 1
1150016*a^7*b^17*c^7*d^17 - 27336616*a^7*b^17*c^9*d^15 + 37153600*a^7*b^17*
c^11*d^13 - 28461040*a^7*b^17*c^13*d^11 + 11779808*a^7*b^17*c^15*d^9 - 2621
008*a^7*b^17*c^17*d^7 + 336688*a^7*b^17*c^19*d^5 - 17920*a^7*b^17*c^21*d^3
- 51840*a^8*b^16*c^2*d^22 + 1512000*a^8*b^16*c^4*d^20 - 9955992*a^8*b^16*c^
6*d^18 + 30289656*a^8*b^16*c^8*d^16 - 50137600*a^8*b^16*c^10*d^14 + 4697256
0*a^8*b^16*c^12*d^12 - 24199280*a^8*b^16*c^14*d^10 + 6661036*a^8*b^16*c^16*
d^8 - 1058448*a^8*b^16*c^18*d^6 + 72560*a^8*b^16*c^20*d^4 - 758400*a^9*b^15
*c^3*d^21 + 7078256*a^9*b^15*c^5*d^19 - 27336616*a^9*b^15*c^7*d^17 + 553839
04*a^9*b^15*c^9*d^15 - 63124080*a^9*b^15*c^11*d^13 + 39987520*a^9*b^15*c^13
*d^11 - 13462088*a^9*b^15*c^15*d^9 + 2478528*a^9*b^15*c^17*d^7 - 212032*a^9
*b^15*c^19*d^5 + 263808*a^10*b^14*c^2*d^22 - 3975688*a^10*b^14*c^4*d^20 + 2
0019440*a^10*b^14*c^6*d^18 - 50137600*a^10*b^14*c^8*d^16 + 69593872*a^10*b^
14*c^10*d^14 - 53854288*a^10*b^14*c^12*d^12 + 21989928*a^10*b^14*c^14*d^10
- 4591360*a^10*b^14*c^16*d^8 + 460480*a^10*b^14*c^18*d^6 + 1720736*a^11*b^1
3*c^3*d^21 - 11781560*a^11*b^13*c^5*d^19 + 37153600*a^11*b^13*c^7*d^17 - 63
124080*a^11*b^13*c^9*d^15 + 59445728*a^11*b^13*c^11*d^13 - 29358696*a^11*b^
13*c^13*d^11 + 6995840*a^11*b^13*c^15*d^9 - 762560*a^11*b^13*c^17*d^7 - 541
208*a^12*b^12*c^2*d^22 + 5501328*a^12*b^12*c^4*d^20 - 22419600*a^12*b^12*c^
6*d^18 + 46972560*a^12*b^12*c^8*d^16 - 53854288*a^12*b^12*c^10*d^14 + 32294
808*a^12*b^12*c^12*d^12 - 8958208*a^12*b^12*c^14*d^10 + 999040*a^12*b^12*c^
16*d^8 - 2002728*a^13*b^11*c^3*d^21 + 10875200*a^13*b^11*c^5*d^19 - 2846104
0*a^13*b^11*c^7*d^17 + 39987520*a^13*b^11*c^9*d^15 - 29358696*a^13*b^11*c^1
1*d^13 + 9722048*a^13*b^11*c^13*d^11 - 1104320*a^13*b^11*c^15*d^9 + 547088*
a^14*b^10*c^2*d^22 - 4147952*a^14*b^10*c^4*d^20 + 13887520*a^14*b^10*c^6*d^
18 - 24199280*a^14*b^10*c^8*d^16 + 21989928*a^14*b^10*c^10*d^14 - 8958208*a
^14*b^10*c^12*d^12 + 1124032*a^14*b^10*c^14*d^10 + 1210560*a^15*b^9*c^3*d^2
1 - 5365072*a^15*b^9*c^5*d^19 + 11779808*a^15*b^9*c^7*d^17 - 13462088*a^15*
b^9*c^9*d^15 + 6995840*a^15*b^9*c^11*d^13 - 1104320*a^15*b^9*c^13*d^11 - 26
3320*a^16*b^8*c^2*d^22 + 1586920*a^16*b^8*c^4*d^20 - 4506428*a^16*b^8*c^6*d
^18 + 6661036*a^16*b^8*c^8*d^16 - 4591360*a^16*b^8*c^10*d^14 + 999040*a^16*
b^8*c^12*d^12 - 335040*a^17*b^7*c^3*d^21 + 1310168*a^17*b^7*c^5*d^19 - 2621
008*a^17*b^7*c^7*d^17 + 2478528*a^17*b^7*c^9*d^15 - 762560*a^17*b^7*c^11*d^
13 + 44120*a^18*b^6*c^2*d^22 - 276020*a^18*b^6*c^4*d^20 + 793756*a^18*b^6*c
^6*d^18 - 1058448*a^18*b^6*c^8*d^16 + 460480*a^18*b^6*c^10*d^14 + 37680*a^1
9*b^5*c^3*d^21 - 170968*a^19*b^5*c^5*d^19 + 336688*a^19*b^5*c^7*d^17 - 2120
32*a^19*b^5*c^9*d^15 - 1564*a^20*b^4*c^2*d^22 + 21124*a^20*b^4*c^4*d^20 - 7
2240*a^20*b^4*c^6*d^18 + 72560*a^20*b^4*c^8*d^16 - 288*a^21*b^3*c^3*d^21 +
8160*a^21*b^3*c^5*d^19 - 17920*a^21*b^3*c^7*d^17 - 196*a^22*b^2*c^2*d^22 +
176*a^22*b^2*c^4*d^20 + 3040*a^22*b^2*c^6*d^18 - 8*a*b^23*c^23*d - 8*a^23*b

*c*d^23)^2/4 - (20736*b^18*d^18 - 96768*a^2*b^16*d^18 + 173664*a^4*b^14*d^1
8 - 136032*a^6*b^12*d^18 + 31081*a^8*b^10*d^18 + 8440*a^10*b^8*d^18 + 400*a
^12*b^6*d^18 - 96768*b^18*c^2*d^16 + 173664*b^18*c^4*d^14 - 136032*b^18*c^6
*d^12 + 31081*b^18*c^8*d^10 + 8440*b^18*c^10*d^8 + 400*b^18*c^12*d^6 - 1313
28*a*b^17*c^3*d^15 + 216576*a*b^17*c^5*d^13 - 141104*a*b^17*c^7*d^11 + 2026
0*a*b^17*c^9*d^9 + 2800*a*b^17*c^11*d^7 - 131328*a^3*b^15*c*d^17 + 216576*a
^5*b^13*c*d^17 - 141104*a^7*b^11*c*d^17 + 20260*a^9*b^9*c*d^17 + 2800*a^11*
b^7*c*d^17 + 495936*a^2*b^16*c^2*d^16 - 989856*a^2*b^16*c^4*d^14 + 901948*a
^2*b^16*c^6*d^12 - 308392*a^2*b^16*c^8*d^10 - 5260*a^2*b^16*c^10*d^8 + 1600
*a^2*b^16*c^12*d^6 + 657408*a^3*b^15*c^3*d^15 - 1158992*a^3*b^15*c^5*d^13 +
 838256*a^3*b^15*c^7*d^11 - 182200*a^3*b^15*c^9*d^9 - 3200*a^3*b^15*c^11*d^
7 - 989856*a^4*b^14*c^2*d^16 + 2185654*a^4*b^14*c^4*d^14 - 2218576*a^4*b^14
*c^6*d^12 + 900624*a^4*b^14*c^8*d^10 - 64720*a^4*b^14*c^10*d^8 + 1600*a^4*b
^14*c^12*d^6 - 1158992*a^5*b^13*c^3*d^15 + 2158808*a^5*b^13*c^5*d^13 - 1641
528*a^5*b^13*c^7*d^11 + 406880*a^5*b^13*c^9*d^9 - 17600*a^5*b^13*c^11*d^7 +
 901948*a^6*b^12*c^2*d^16 - 2218576*a^6*b^12*c^4*d^14 + 2430936*a^6*b^12*c^
6*d^12 - 1026928*a^6*b^12*c^8*d^10 + 88720*a^6*b^12*c^10*d^8 + 838256*a^7*b
^11*c^3*d^15 - 1641528*a^7*b^11*c^5*d^13 + 1206848*a^7*b^11*c^7*d^11 - 2393
60*a^7*b^11*c^9*d^9 - 308392*a^8*b^10*c^2*d^16 + 900624*a^8*b^10*c^4*d^14 -
 1026928*a^8*b^10*c^6*d^12 + 354016*a^8*b^10*c^8*d^10 - 182200*a^9*b^9*c^3*
d^15 + 406880*a^9*b^9*c^5*d^13 - 239360*a^9*b^9*c^7*d^11 - 5260*a^10*b^8*c^
2*d^16 - 64720*a^10*b^8*c^4*d^14 + 88720*a^10*b^8*c^6*d^12 - 3200*a^11*b^7*
c^3*d^15 - 17600*a^11*b^7*c^5*d^13 + 1600*a^12*b^6*c^2*d^16 + 1600*a^12*b^6
*c^4*d^14 + 27648*a*b^17*c*d^17)*(80*a^2*b^28*c^30 - 16*b^30*c^30 - 16*a^30
*d^30 - 160*a^4*b^26*c^30 + 160*a^6*b^24*c^30 - 80*a^8*b^22*c^30 + 16*a^10*
b^20*c^30 + 16*a^20*b^10*d^30 - 80*a^22*b^8*d^30 + 160*a^24*b^6*d^30 - 160*
a^26*b^4*d^30 + 80*a^28*b^2*d^30 + 80*a^30*c^2*d^28 - 160*a^30*c^4*d^26 + 1
60*a^30*c^6*d^24 - 80*a^30*c^8*d^22 + 16*a^30*c^10*d^20 + 16*b^30*c^20*d^10
 - 80*b^30*c^22*d^8 + 160*b^30*c^24*d^6 - 160*b^30*c^26*d^4 + 80*b^30*c^28*
d^2 - 320*a*b^29*c^19*d^11 + 1600*a*b^29*c^21*d^9 - 3200*a*b^29*c^23*d^7 +
3200*a*b^29*c^25*d^5 - 1600*a*b^29*c^27*d^3 - 1600*a^3*b^27*c^29*d + 3200*a
^5*b^25*c^29*d - 3200*a^7*b^23*c^29*d + 1600*a^9*b^21*c^29*d - 320*a^11*b^1
9*c^29*d - 320*a^19*b^11*c*d^29 + 1600*a^21*b^9*c*d^29 - 3200*a^23*b^7*c*d^
29 + 3200*a^25*b^5*c*d^29 - 1600*a^27*b^3*c*d^29 - 1600*a^29*b*c^3*d^27 + 3
200*a^29*b*c^5*d^25 - 3200*a^29*b*c^7*d^23 + 1600*a^29*b*c^9*d^21 - 320*a^2
9*b*c^11*d^19 + 3040*a^2*b^28*c^18*d^12 - 15280*a^2*b^28*c^20*d^10 + 30800*
a^2*b^28*c^22*d^8 - 31200*a^2*b^28*c^24*d^6 + 16000*a^2*b^28*c^26*d^4 - 344
0*a^2*b^28*c^28*d^2 - 18240*a^3*b^27*c^17*d^13 + 92800*a^3*b^27*c^19*d^11 -
 190400*a^3*b^27*c^21*d^9 + 198400*a^3*b^27*c^23*d^7 - 107200*a^3*b^27*c^25
*d^5 + 26240*a^3*b^27*c^27*d^3 + 77520*a^4*b^26*c^16*d^14 - 402800*a^4*b^26
*c^18*d^12 + 851360*a^4*b^26*c^20*d^10 - 928000*a^4*b^26*c^22*d^8 + 541200*
a^4*b^26*c^24*d^6 - 155120*a^4*b^26*c^26*d^4 + 16000*a^4*b^26*c^28*d^2 - 24
8064*a^5*b^25*c^15*d^15 + 1331520*a^5*b^25*c^17*d^13 - 2939840*a^5*b^25*c^1
9*d^11 + 3408640*a^5*b^25*c^21*d^9 - 2184320*a^5*b^25*c^23*d^7 + 736064*a^5
*b^25*c^25*d^5 - 107200*a^5*b^25*c^27*d^3 + 620160*a^6*b^24*c^14*d^16 - 348

8400*a^6*b^24*c^16*d^14 + 8170000*a^6*b^24*c^18*d^12 - 10229760*a^6*b^24*c^20*d^10 + 7281600*a^6*b^24*c^22*d^8 - 2863760*a^6*b^24*c^24*d^6 + 541200*a^6*b^24*c^26*d^4 - 31200*a^6*b^24*c^28*d^2 - 1240320*a^7*b^23*c^13*d^17 + 7441920*a^7*b^23*c^15*d^15 - 18787200*a^7*b^23*c^17*d^13 + 25721600*a^7*b^23*c^19*d^11 - 20444800*a^7*b^23*c^21*d^9 + 9297920*a^7*b^23*c^23*d^7 - 2184320*a^7*b^23*c^25*d^5 + 198400*a^7*b^23*c^27*d^3 + 2015520*a^8*b^22*c^12*d^18 - 13178400*a^8*b^22*c^14*d^16 + 36434400*a^8*b^22*c^16*d^14 - 55069600*a^8*b^22*c^18*d^12 + 48989680*a^8*b^22*c^20*d^10 - 25575920*a^8*b^22*c^22*d^8 + 7281600*a^8*b^22*c^24*d^6 - 928000*a^8*b^22*c^26*d^4 + 30800*a^8*b^22*c^28*d^2 - 2687360*a^9*b^21*c^11*d^19 + 19638400*a^9*b^21*c^13*d^17 - 60362240*a^9*b^21*c^15*d^15 + 101475200*a^9*b^21*c^17*d^13 - 101172800*a^9*b^21*c^19*d^11 + 60333760*a^9*b^21*c^21*d^9 - 20444800*a^9*b^21*c^23*d^7 + 3408640*a^9*b^21*c^25*d^5 - 190400*a^9*b^21*c^27*d^3 + 2956096*a^10*b^20*c^10*d^20 - 24858080*a^10*b^20*c^12*d^18 + 86150560*a^10*b^20*c^14*d^16 - 162120160*a^10*b^20*c^16*d^14 + 181463680*a^10*b^20*c^18*d^12 - 123188112*a^10*b^20*c^20*d^10 + 48989680*a^10*b^20*c^22*d^8 - 10229760*a^10*b^20*c^24*d^6 + 851360*a^10*b^20*c^26*d^4 - 15280*a^10*b^20*c^28*d^2 - 2687360*a^11*b^19*c^9*d^21 + 26873600*a^11*b^19*c^11*d^19 - 106460800*a^11*b^19*c^13*d^17 + 225738240*a^11*b^19*c^15*d^15 - 284331200*a^11*b^19*c^17*d^13 + 219166080*a^11*b^19*c^19*d^11 - 101172800*a^11*b^19*c^21*d^9 + 25721600*a^11*b^19*c^23*d^7 - 2939840*a^11*b^19*c^25*d^5 + 92800*a^11*b^19*c^27*d^3 + 2015520*a^12*b^18*c^8*d^22 - 24858080*a^12*b^18*c^10*d^20 + 114212800*a^12*b^18*c^12*d^18 - 274937600*a^12*b^18*c^14*d^16 + 390830000*a^12*b^18*c^16*d^14 - 341426960*a^12*b^18*c^18*d^12 + 181463680*a^12*b^18*c^20*d^10 - 55069600*a^12*b^18*c^22*d^8 + 8170000*a^12*b^18*c^24*d^6 - 402800*a^12*b^18*c^26*d^4 + 3040*a^12*b^18*c^28*d^2 - 1240320*a^13*b^17*c^7*d^23 + 19638400*a^13*b^17*c^9*d^21 - 106460800*a^13*b^17*c^11*d^19 + 293542400*a^13*b^17*c^13*d^17 - 472561920*a^13*b^17*c^15*d^15 + 467412160*a^13*b^17*c^17*d^13 - 284331200*a^13*b^17*c^19*d^11 + 101475200*a^13*b^17*c^21*d^9 - 18787200*a^13*b^17*c^23*d^7 + 1331520*a^13*b^17*c^25*d^5 - 18240*a^13*b^17*c^27*d^3 + 620160*a^14*b^16*c^6*d^24 - 13178400*a^14*b^16*c^8*d^22 + 86150560*a^14*b^16*c^10*d^20 - 274937600*a^14*b^16*c^12*d^18 + 503363200*a^14*b^16*c^14*d^16 - 563751280*a^14*b^16*c^16*d^14 + 390830000*a^14*b^16*c^18*d^12 - 162120160*a^14*b^16*c^20*d^10 + 36434400*a^14*b^16*c^22*d^8 - 3488400*a^14*b^16*c^24*d^6 + 77520*a^14*b^16*c^26*d^4 - 248064*a^15*b^15*c^5*d^25 + 7441920*a^15*b^15*c^7*d^23 - 60362240*a^15*b^15*c^9*d^21 + 225738240*a^15*b^15*c^11*d^19 - 472561920*a^15*b^15*c^13*d^17 + 599984128*a^15*b^15*c^15*d^15 - 472561920*a^15*b^15*c^17*d^13 + 225738240*a^15*b^15*c^19*d^11 - 60362240*a^15*b^15*c^21*d^9 + 7441920*a^15*b^15*c^23*d^7 - 248064*a^15*b^15*c^25*d^5 + 77520*a^16*b^14*c^4*d^26 - 3488400*a^16*b^14*c^6*d^24 + 36434400*a^16*b^14*c^8*d^22 - 162120160*a^16*b^14*c^10*d^20 + 390830000*a^16*b^14*c^12*d^18 - 563751280*a^16*b^14*c^14*d^16 + 503363200*a^16*b^14*c^16*d^14 - 274937600*a^16*b^14*c^18*d^12 + 86150560*a^16*b^14*c^20*d^10 - 13178400*a^16*b^14*c^22*d^8 + 620160*a^16*b^14*c^24*d^6 - 18240*a^17*b^13*c^3*d^27 + 1331520*a^17*b^13*c^5*d^25 - 18787200*a^17*b^13*c^7*d^23 + 101475200*a^17*b^13*c^9*d^21 - 284331200*a^17*b^13*c^11*d^19

+ 467412160*a^17*b^13*c^13*d^17 - 472561920*a^17*b^13*c^15*d^15 + 293542400
*a^17*b^13*c^17*d^13 - 106460800*a^17*b^13*c^19*d^11 + 19638400*a^17*b^13*c
^21*d^9 - 1240320*a^17*b^13*c^23*d^7 + 3040*a^18*b^12*c^2*d^28 - 402800*a^1
8*b^12*c^4*d^26 + 8170000*a^18*b^12*c^6*d^24 - 55069600*a^18*b^12*c^8*d^22
+ 181463680*a^18*b^12*c^10*d^20 - 341426960*a^18*b^12*c^12*d^18 + 390830000
*a^18*b^12*c^14*d^16 - 274937600*a^18*b^12*c^16*d^14 + 114212800*a^18*b^12*
c^18*d^12 - 24858080*a^18*b^12*c^20*d^10 + 2015520*a^18*b^12*c^22*d^8 + 928
00*a^19*b^11*c^3*d^27 - 2939840*a^19*b^11*c^5*d^25 + 25721600*a^19*b^11*c^7
*d^23 - 101172800*a^19*b^11*c^9*d^21 + 219166080*a^19*b^11*c^11*d^19 - 2843
31200*a^19*b^11*c^13*d^17 + 225738240*a^19*b^11*c^15*d^15 - 106460800*a^19*
b^11*c^17*d^13 + 26873600*a^19*b^11*c^19*d^11 - 2687360*a^19*b^11*c^21*d^9
- 15280*a^20*b^10*c^2*d^28 + 851360*a^20*b^10*c^4*d^26 - 10229760*a^20*b^10
*c^6*d^24 + 48989680*a^20*b^10*c^8*d^22 - 123188112*a^20*b^10*c^10*d^20 + 1
81463680*a^20*b^10*c^12*d^18 - 162120160*a^20*b^10*c^14*d^16 + 86150560*a^2
0*b^10*c^16*d^14 - 24858080*a^20*b^10*c^18*d^12 + 2956096*a^20*b^10*c^20*d^
10 - 190400*a^21*b^9*c^3*d^27 + 3408640*a^21*b^9*c^5*d^25 - 20444800*a^21*b
^9*c^7*d^23 + 60333760*a^21*b^9*c^9*d^21 - 101172800*a^21*b^9*c^11*d^19 + 1
01475200*a^21*b^9*c^13*d^17 - 60362240*a^21*b^9*c^15*d^15 + 19638400*a^21*b
^9*c^17*d^13 - 2687360*a^21*b^9*c^19*d^11 + 30800*a^22*b^8*c^2*d^28 - 92800
0*a^22*b^8*c^4*d^26 + 7281600*a^22*b^8*c^6*d^24 - 25575920*a^22*b^8*c^8*d^2
2 + 48989680*a^22*b^8*c^10*d^20 - 55069600*a^22*b^8*c^12*d^18 + 36434400*a^
22*b^8*c^14*d^16 - 13178400*a^22*b^8*c^16*d^14 + 2015520*a^22*b^8*c^18*d^12
 + 198400*a^23*b^7*c^3*d^27 - 2184320*a^23*b^7*c^5*d^25 + 9297920*a^23*b^7*
c^7*d^23 - 20444800*a^23*b^7*c^9*d^21 + 25721600*a^23*b^7*c^11*d^19 - 18787
200*a^23*b^7*c^13*d^17 + 7441920*a^23*b^7*c^15*d^15 - 1240320*a^23*b^7*c^17
*d^13 - 31200*a^24*b^6*c^2*d^28 + 541200*a^24*b^6*c^4*d^26 - 2863760*a^24*b
^6*c^6*d^24 + 7281600*a^24*b^6*c^8*d^22 - 10229760*a^24*b^6*c^10*d^20 + 817
0000*a^24*b^6*c^12*d^18 - 3488400*a^24*b^6*c^14*d^16 + 620160*a^24*b^6*c^16
*d^14 - 107200*a^25*b^5*c^3*d^27 + 736064*a^25*b^5*c^5*d^25 - 2184320*a^25*
b^5*c^7*d^23 + 3408640*a^25*b^5*c^9*d^21 - 2939840*a^25*b^5*c^11*d^19 + 133
1520*a^25*b^5*c^13*d^17 - 248064*a^25*b^5*c^15*d^15 + 16000*a^26*b^4*c^2*d^
28 - 155120*a^26*b^4*c^4*d^26 + 541200*a^26*b^4*c^6*d^24 - 928000*a^26*b^4*
c^8*d^22 + 851360*a^26*b^4*c^10*d^20 - 402800*a^26*b^4*c^12*d^18 + 77520*a^
26*b^4*c^14*d^16 + 26240*a^27*b^3*c^3*d^27 - 107200*a^27*b^3*c^5*d^25 + 198
400*a^27*b^3*c^7*d^23 - 190400*a^27*b^3*c^9*d^21 + 92800*a^27*b^3*c^11*d^19
 - 18240*a^27*b^3*c^13*d^17 - 3440*a^28*b^2*c^2*d^28 + 16000*a^28*b^2*c^4*d
^26 - 31200*a^28*b^2*c^6*d^24 + 30800*a^28*b^2*c^8*d^22 - 15280*a^28*b^2*c^
10*d^20 + 3040*a^28*b^2*c^12*d^18 + 320*a*b^29*c^29*d + 320*a^29*b*c*d^29))
^(1/2) - 2*a^24*d^24 - 2*b^24*c^24 - 8*a^2*b^22*c^24 - 8*a^4*b^20*c^24 + 57
6*a^10*b^14*d^24 - 2784*a^12*b^12*d^24 + 5284*a^14*b^10*d^24 - 4730*a^16*b^
8*d^24 + 1780*a^18*b^6*d^24 - 68*a^20*b^4*d^24 - 38*a^22*b^2*d^24 - 8*a^24*
c^2*d^22 - 8*a^24*c^4*d^20 + 576*b^24*c^10*d^14 - 2784*b^24*c^12*d^12 + 528
4*b^24*c^14*d^10 - 4730*b^24*c^16*d^8 + 1780*b^24*c^18*d^6 - 68*b^24*c^20*d
^4 - 38*b^24*c^22*d^2 - 5760*a*b^23*c^9*d^15 + 28224*a*b^23*c^11*d^13 - 547
28*a*b^23*c^13*d^11 + 50620*a*b^23*c^15*d^9 - 20360*a*b^23*c^17*d^7 + 1480*

a*b^23*c^19*d^5 + 268*a*b^23*c^21*d^3 + 88*a^3*b^21*c^23*d + 160*a^5*b^19*c
^23*d - 5760*a^9*b^15*c*d^23 + 28224*a^11*b^13*c*d^23 - 54728*a^13*b^11*c*d
^23 + 50620*a^15*b^9*c*d^23 - 20360*a^17*b^7*c*d^23 + 1480*a^19*b^5*c*d^23
+ 268*a^21*b^3*c*d^23 + 88*a^23*b*c^3*d^21 + 160*a^23*b*c^5*d^19 + 25920*a^
2*b^22*c^8*d^16 - 131904*a^2*b^22*c^10*d^14 + 270604*a^2*b^22*c^12*d^12 - 2
73544*a^2*b^22*c^14*d^10 + 131660*a^2*b^22*c^16*d^8 - 22060*a^2*b^22*c^18*d
^6 + 782*a^2*b^22*c^20*d^4 + 98*a^2*b^22*c^22*d^2 - 69120*a^3*b^21*c^7*d^17
 + 379200*a^3*b^21*c^9*d^15 - 860368*a^3*b^21*c^11*d^13 + 1001364*a^3*b^21*
c^13*d^11 - 605280*a^3*b^21*c^15*d^9 + 167520*a^3*b^21*c^17*d^7 - 18840*a^3
*b^21*c^19*d^5 + 144*a^3*b^21*c^21*d^3 + 120960*a^4*b^20*c^6*d^18 - 756000*
a^4*b^20*c^8*d^16 + 1987844*a^4*b^20*c^10*d^14 - 2750664*a^4*b^20*c^12*d^12
 + 2073976*a^4*b^20*c^14*d^10 - 793460*a^4*b^20*c^16*d^8 + 138010*a^4*b^20*
c^18*d^6 - 10562*a^4*b^20*c^20*d^4 - 88*a^4*b^20*c^22*d^2 - 145152*a^5*b^19
*c^5*d^19 + 1116288*a^5*b^19*c^7*d^17 - 3539128*a^5*b^19*c^9*d^15 + 5890780
*a^5*b^19*c^11*d^13 - 5437600*a^5*b^19*c^13*d^11 + 2682536*a^5*b^19*c^15*d^
9 - 655084*a^5*b^19*c^17*d^7 + 85484*a^5*b^19*c^19*d^5 - 4080*a^5*b^19*c^21
*d^3 + 120960*a^6*b^18*c^4*d^20 - 1266048*a^6*b^18*c^6*d^18 + 4977996*a^6*b
^18*c^8*d^16 - 10009720*a^6*b^18*c^10*d^14 + 11209800*a^6*b^18*c^12*d^12 -
6943760*a^6*b^18*c^14*d^10 + 2253214*a^6*b^18*c^16*d^8 - 396878*a^6*b^18*c^
18*d^6 + 36120*a^6*b^18*c^20*d^4 - 1520*a^6*b^18*c^22*d^2 - 69120*a^7*b^17*
c^3*d^21 + 1116288*a^7*b^17*c^5*d^19 - 5575008*a^7*b^17*c^7*d^17 + 13668308
*a^7*b^17*c^9*d^15 - 18576800*a^7*b^17*c^11*d^13 + 14230520*a^7*b^17*c^13*d
^11 - 5889904*a^7*b^17*c^15*d^9 + 1310504*a^7*b^17*c^17*d^7 - 168344*a^7*b^
17*c^19*d^5 + 8960*a^7*b^17*c^21*d^3 + 25920*a^8*b^16*c^2*d^22 - 756000*a^8
*b^16*c^4*d^20 + 4977996*a^8*b^16*c^6*d^18 - 15144828*a^8*b^16*c^8*d^16 + 2
5068800*a^8*b^16*c^10*d^14 - 23486280*a^8*b^16*c^12*d^12 + 12099640*a^8*b^1
6*c^14*d^10 - 3330518*a^8*b^16*c^16*d^8 + 529224*a^8*b^16*c^18*d^6 - 36280*
a^8*b^16*c^20*d^4 + 379200*a^9*b^15*c^3*d^21 - 3539128*a^9*b^15*c^5*d^19 +
13668308*a^9*b^15*c^7*d^17 - 27691952*a^9*b^15*c^9*d^15 + 31562040*a^9*b^15
*c^11*d^13 - 19993760*a^9*b^15*c^13*d^11 + 6731044*a^9*b^15*c^15*d^9 - 1239
264*a^9*b^15*c^17*d^7 + 106016*a^9*b^15*c^19*d^5 - 131904*a^10*b^14*c^2*d^2
2 + 1987844*a^10*b^14*c^4*d^20 - 10009720*a^10*b^14*c^6*d^18 + 25068800*a^1
0*b^14*c^8*d^16 - 34796936*a^10*b^14*c^10*d^14 + 26927144*a^10*b^14*c^12*d^
12 - 10994964*a^10*b^14*c^14*d^10 + 2295680*a^10*b^14*c^16*d^8 - 230240*a^1
0*b^14*c^18*d^6 - 860368*a^11*b^13*c^3*d^21 + 5890780*a^11*b^13*c^5*d^19 -
18576800*a^11*b^13*c^7*d^17 + 31562040*a^11*b^13*c^9*d^15 - 29722864*a^11*b
^13*c^11*d^13 + 14679348*a^11*b^13*c^13*d^11 - 3497920*a^11*b^13*c^15*d^9 +
 381280*a^11*b^13*c^17*d^7 + 270604*a^12*b^12*c^2*d^22 - 2750664*a^12*b^12*
c^4*d^20 + 11209800*a^12*b^12*c^6*d^18 - 23486280*a^12*b^12*c^8*d^16 + 2692
7144*a^12*b^12*c^10*d^14 - 16147404*a^12*b^12*c^12*d^12 + 4479104*a^12*b^12
*c^14*d^10 - 499520*a^12*b^12*c^16*d^8 + 1001364*a^13*b^11*c^3*d^21 - 54376
00*a^13*b^11*c^5*d^19 + 14230520*a^13*b^11*c^7*d^17 - 19993760*a^13*b^11*c^
9*d^15 + 14679348*a^13*b^11*c^11*d^13 - 4861024*a^13*b^11*c^13*d^11 + 55216
0*a^13*b^11*c^15*d^9 - 273544*a^14*b^10*c^2*d^22 + 2073976*a^14*b^10*c^4*d^
20 - 6943760*a^14*b^10*c^6*d^18 + 12099640*a^14*b^10*c^8*d^16 - 10994964*a^

14*b^10*c^10*d^14 + 4479104*a^14*b^10*c^12*d^12 - 562016*a^14*b^10*c^14*d^10 - 605280*a^15*b^9*c^3*d^21 + 2682536*a^15*b^9*c^5*d^19 - 5889904*a^15*b^9*c^7*d^17 + 6731044*a^15*b^9*c^9*d^15 - 3497920*a^15*b^9*c^11*d^13 + 552160*a^15*b^9*c^13*d^11 + 131660*a^16*b^8*c^2*d^22 - 793460*a^16*b^8*c^4*d^20 + 2253214*a^16*b^8*c^6*d^18 - 3330518*a^16*b^8*c^8*d^16 + 2295680*a^16*b^8*c^10*d^14 - 499520*a^16*b^8*c^12*d^12 + 167520*a^17*b^7*c^3*d^21 - 655084*a^17*b^7*c^5*d^19 + 1310504*a^17*b^7*c^7*d^17 - 1239264*a^17*b^7*c^9*d^15 + 381280*a^17*b^7*c^11*d^13 - 22060*a^18*b^6*c^2*d^22 + 138010*a^18*b^6*c^4*d^20 - 396878*a^18*b^6*c^6*d^18 + 529224*a^18*b^6*c^8*d^16 - 230240*a^18*b^6*c^10*d^14 - 18840*a^19*b^5*c^3*d^21 + 85484*a^19*b^5*c^5*d^19 - 168344*a^19*b^5*c^7*d^17 + 106016*a^19*b^5*c^9*d^15 + 782*a^20*b^4*c^2*d^22 - 10562*a^20*b^4*c^4*d^20 + 36120*a^20*b^4*c^6*d^18 - 36280*a^20*b^4*c^8*d^16 + 144*a^21*b^3*c^3*d^21 - 4080*a^21*b^3*c^5*d^19 + 8960*a^21*b^3*c^7*d^17 + 98*a^22*b^2*c^2*d^22 - 88*a^22*b^2*c^4*d^20 - 1520*a^22*b^2*c^6*d^18 + 4*a*b^23*c^23*d + 4*a^23*b*c*d^23)/(16*(5*a^2*b^28*c^30 - b^30*c^30 - a^30*d^30 - 10*a^4*b^26*c^30 + 10*a^6*b^24*c^30 - 5*a^8*b^22*c^30 + a^10*b^20*c^30 + a^20*b^10*d^30 - 5*a^22*b^8*d^30 + 10*a^24*b^6*d^30 - 10*a^26*b^4*d^30 + 5*a^28*b^2*d^30 + 5*a^30*c^2*d^28 - 10*a^30*c^4*d^26 + 10*a^30*c^6*d^24 - 5*a^30*c^8*d^22 + a^30*c^10*d^20 + b^30*c^20*d^10 - 5*b^30*c^22*d^8 + 10*b^30*c^24*d^6 - 10*b^30*c^26*d^4 + 5*b^30*c^28*d^2 - 20*a*b^29*c^19*d^11 + 100*a*b^29*c^21*d^9 - 200*a*b^29*c^23*d^7 + 200*a*b^29*c^25*d^5 - 100*a*b^29*c^27*d^3 - 100*a^3*b^27*c^29*d + 200*a^5*b^25*c^29*d - 200*a^7*b^23*c^29*d + 100*a^9*b^21*c^29*d - 20*a^11*b^19*c^29*d - 20*a^19*b^11*c*d^29 + 100*a^21*b^9*c*d^29 - 200*a^23*b^7*c*d^29 + 200*a^25*b^5*c*d^29 - 100*a^27*b^3*c*d^29 - 100*a^29*b*c^3*d^27 + 200*a^29*b*c^5*d^25 - 200*a^29*b*c^7*d^23 + 100*a^29*b*c^9*d^21 - 20*a^29*b*c^11*d^19 + 190*a^2*b^28*c^18*d^12 - 955*a^2*b^28*c^20*d^10 + 1925*a^2*b^28*c^22*d^8 - 1950*a^2*b^28*c^24*d^6 + 1000*a^2*b^28*c^26*d^4 - 215*a^2*b^28*c^28*d^2 - 1140*a^3*b^27*c^17*d^13 + 5800*a^3*b^27*c^19*d^11 - 11900*a^3*b^27*c^21*d^9 + 12400*a^3*b^27*c^23*d^7 - 6700*a^3*b^27*c^25*d^5 + 1640*a^3*b^27*c^27*d^3 + 4845*a^4*b^26*c^16*d^14 - 25175*a^4*b^26*c^18*d^12 + 53210*a^4*b^26*c^20*d^10 - 58000*a^4*b^26*c^22*d^8 + 33825*a^4*b^26*c^24*d^6 - 9695*a^4*b^26*c^26*d^4 + 1000*a^4*b^26*c^28*d^2 - 15504*a^5*b^25*c^15*d^15 + 83220*a^5*b^25*c^17*d^13 - 183740*a^5*b^25*c^19*d^11 + 213040*a^5*b^25*c^21*d^9 - 136520*a^5*b^25*c^23*d^7 + 46004*a^5*b^25*c^25*d^5 - 6700*a^5*b^25*c^27*d^3 + 38760*a^6*b^24*c^14*d^16 - 218025*a^6*b^24*c^16*d^14 + 510625*a^6*b^24*c^18*d^12 - 639360*a^6*b^24*c^20*d^10 + 455100*a^6*b^24*c^22*d^8 - 178985*a^6*b^24*c^24*d^6 + 33825*a^6*b^24*c^26*d^4 - 1950*a^6*b^24*c^28*d^2 - 77520*a^7*b^23*c^13*d^17 + 465120*a^7*b^23*c^15*d^15 - 1174200*a^7*b^23*c^17*d^13 + 1607600*a^7*b^23*c^19*d^11 - 1277800*a^7*b^23*c^21*d^9 + 581120*a^7*b^23*c^23*d^7 - 136520*a^7*b^23*c^25*d^5 + 12400*a^7*b^23*c^27*d^3 + 125970*a^8*b^22*c^12*d^18 - 823650*a^8*b^22*c^14*d^16 + 2277150*a^8*b^22*c^16*d^14 - 3441850*a^8*b^22*c^18*d^12 + 3061855*a^8*b^22*c^20*d^10 - 1598495*a^8*b^22*c^22*d^8 + 455100*a^8*b^22*c^24*d^6 - 58000*a^8*b^22*c^26*d^4 + 1925*a^8*b^22*c^28*d^2 - 167960*a^9*b^21*c^11*d^19 + 1227400*a^9*b^21*c^13*d^17 - 3772640*a^9*b^21*c^15*d^15 + 6342200*a^9*b^21*c^17*d

^13 - 6323300*a^9*b^21*c^19*d^11 + 3770860*a^9*b^21*c^21*d^9 - 1277800*a^9*b^21*c^23*d^7 + 213040*a^9*b^21*c^25*d^5 - 11900*a^9*b^21*c^27*d^3 + 184756*a^10*b^20*c^10*d^20 - 1553630*a^10*b^20*c^12*d^18 + 5384410*a^10*b^20*c^14*d^16 - 10132510*a^10*b^20*c^16*d^14 + 11341480*a^10*b^20*c^18*d^12 - 7699257*a^10*b^20*c^20*d^10 + 3061855*a^10*b^20*c^22*d^8 - 639360*a^10*b^20*c^24*d^6 + 53210*a^10*b^20*c^26*d^4 - 955*a^10*b^20*c^28*d^2 - 167960*a^11*b^19*c^9*d^21 + 1679600*a^11*b^19*c^11*d^19 - 6653800*a^11*b^19*c^13*d^17 + 14108640*a^11*b^19*c^15*d^15 - 17770700*a^11*b^19*c^17*d^13 + 13697880*a^11*b^19*c^19*d^11 - 6323300*a^11*b^19*c^21*d^9 + 1607600*a^11*b^19*c^23*d^7 - 183740*a^11*b^19*c^25*d^5 + 5800*a^11*b^19*c^27*d^3 + 125970*a^12*b^18*c^8*d^22 - 1553630*a^12*b^18*c^10*d^20 + 7138300*a^12*b^18*c^12*d^18 - 17183600*a^12*b^18*c^14*d^16 + 24426875*a^12*b^18*c^16*d^14 - 21339185*a^12*b^18*c^18*d^12 + 11341480*a^12*b^18*c^20*d^10 - 3441850*a^12*b^18*c^22*d^8 + 510625*a^12*b^18*c^24*d^6 - 25175*a^12*b^18*c^26*d^4 + 190*a^12*b^18*c^28*d^2 - 77520*a^13*b^17*c^7*d^23 + 1227400*a^13*b^17*c^9*d^21 - 6653800*a^13*b^17*c^11*d^19 + 18346400*a^13*b^17*c^13*d^17 - 29535120*a^13*b^17*c^15*d^15 + 29213260*a^13*b^17*c^17*d^13 - 17770700*a^13*b^17*c^19*d^11 + 6342200*a^13*b^17*c^21*d^9 - 1174200*a^13*b^17*c^23*d^7 + 83220*a^13*b^17*c^25*d^5 - 1140*a^13*b^17*c^27*d^3 + 38760*a^14*b^16*c^6*d^24 - 823650*a^14*b^16*c^8*d^22 + 5384410*a^14*b^16*c^10*d^20 - 17183600*a^14*b^16*c^12*d^18 + 31460200*a^14*b^16*c^14*d^16 - 35234455*a^14*b^16*c^16*d^14 + 24426875*a^14*b^16*c^18*d^12 - 10132510*a^14*b^16*c^20*d^10 + 2277150*a^14*b^16*c^22*d^8 - 218025*a^14*b^16*c^24*d^6 + 4845*a^14*b^16*c^26*d^4 - 15504*a^15*b^15*c^5*d^25 + 465120*a^15*b^15*c^7*d^23 - 3772640*a^15*b^15*c^9*d^21 + 14108640*a^15*b^15*c^11*d^19 - 29535120*a^15*b^15*c^13*d^17 + 37499008*a^15*b^15*c^15*d^15 - 29535120*a^15*b^15*c^17*d^13 + 14108640*a^15*b^15*c^19*d^11 - 3772640*a^15*b^15*c^21*d^9 + 465120*a^15*b^15*c^23*d^7 - 15504*a^15*b^15*c^25*d^5 + 4845*a^16*b^14*c^4*d^26 - 218025*a^16*b^14*c^6*d^24 + 2277150*a^16*b^14*c^8*d^22 - 10132510*a^16*b^14*c^10*d^20 + 24426875*a^16*b^14*c^12*d^18 - 35234455*a^16*b^14*c^14*d^16 + 31460200*a^16*b^14*c^16*d^14 - 17183600*a^16*b^14*c^18*d^12 + 5384410*a^16*b^14*c^20*d^10 - 823650*a^16*b^14*c^22*d^8 + 38760*a^16*b^14*c^24*d^6 - 1140*a^17*b^13*c^3*d^27 + 83220*a^17*b^13*c^5*d^25 - 1174200*a^17*b^13*c^7*d^23 + 6342200*a^17*b^13*c^9*d^21 - 17770700*a^17*b^13*c^11*d^19 + 29213260*a^17*b^13*c^13*d^17 - 29535120*a^17*b^13*c^15*d^15 + 18346400*a^17*b^13*c^17*d^13 - 6653800*a^17*b^13*c^19*d^11 + 1227400*a^17*b^13*c^21*d^9 - 77520*a^17*b^13*c^23*d^7 + 190*a^18*b^12*c^2*d^28 - 25175*a^18*b^12*c^4*d^26 + 510625*a^18*b^12*c^6*d^24 - 3441850*a^18*b^12*c^8*d^22 + 11341480*a^18*b^12*c^10*d^20 - 21339185*a^18*b^12*c^12*d^18 + 24426875*a^18*b^12*c^14*d^16 - 17183600*a^18*b^12*c^16*d^14 + 7138300*a^18*b^12*c^18*d^12 - 1553630*a^18*b^12*c^20*d^10 + 125970*a^18*b^12*c^22*d^8 + 5800*a^19*b^11*c^3*d^27 - 183740*a^19*b^11*c^5*d^25 + 1607600*a^19*b^11*c^7*d^23 - 6323300*a^19*b^11*c^9*d^21 + 13697880*a^19*b^11*c^11*d^19 - 17770700*a^19*b^11*c^13*d^17 + 14108640*a^19*b^11*c^15*d^15 - 6653800*a^19*b^11*c^17*d^13 + 1679600*a^19*b^11*c^19*d^11 - 167960*a^19*b^11*c^21*d^9 - 955*a^20*b^10*c^2*d^28 + 53210*a^20*b^10*c^4*d^26 - 639360*a^20*b^10*c^6*d^24 + 3061855*a^20*b^10*c^8*

d^22 - 7699257*a^20*b^10*c^10*d^20 + 11341480*a^20*b^10*c^12*d^18 - 1013251
0*a^20*b^10*c^14*d^16 + 5384410*a^20*b^10*c^16*d^14 - 1553630*a^20*b^10*c^1
8*d^12 + 184756*a^20*b^10*c^20*d^10 - 11900*a^21*b^9*c^3*d^27 + 213040*a^21
*b^9*c^5*d^25 - 1277800*a^21*b^9*c^7*d^23 + 3770860*a^21*b^9*c^9*d^21 - 632
3300*a^21*b^9*c^11*d^19 + 6342200*a^21*b^9*c^13*d^17 - 3772640*a^21*b^9*c^1
5*d^15 + 1227400*a^21*b^9*c^17*d^13 - 167960*a^21*b^9*c^19*d^11 + 1925*a^22
*b^8*c^2*d^28 - 58000*a^22*b^8*c^4*d^26 + 455100*a^22*b^8*c^6*d^24 - 159849
5*a^22*b^8*c^8*d^22 + 3061855*a^22*b^8*c^10*d^20 - 3441850*a^22*b^8*c^12*d^
18 + 2277150*a^22*b^8*c^14*d^16 - 823650*a^22*b^8*c^16*d^14 + 125970*a^22*b
^8*c^18*d^12 + 12400*a^23*b^7*c^3*d^27 - 136520*a^23*b^7*c^5*d^25 + 581120*
a^23*b^7*c^7*d^23 - 1277800*a^23*b^7*c^9*d^21 + 1607600*a^23*b^7*c^11*d^19
- 1174200*a^23*b^7*c^13*d^17 + 465120*a^23*b^7*c^15*d^15 - 77520*a^23*b^7*c
^17*d^13 - 1950*a^24*b^6*c^2*d^28 + 33825*a^24*b^6*c^4*d^26 - 178985*a^24*b
^6*c^6*d^24 + 455100*a^24*b^6*c^8*d^22 - 639360*a^24*b^6*c^10*d^20 + 510625
*a^24*b^6*c^12*d^18 - 218025*a^24*b^6*c^14*d^16 + 38760*a^24*b^6*c^16*d^14
- 6700*a^25*b^5*c^3*d^27 + 46004*a^25*b^5*c^5*d^25 - 136520*a^25*b^5*c^7*d^
23 + 213040*a^25*b^5*c^9*d^21 - 183740*a^25*b^5*c^11*d^19 + 83220*a^25*b^5*
c^13*d^17 - 15504*a^25*b^5*c^15*d^15 + 1000*a^26*b^4*c^2*d^28 - 9695*a^26*b
^4*c^4*d^26 + 33825*a^26*b^4*c^6*d^24 - 58000*a^26*b^4*c^8*d^22 + 53210*a^2
6*b^4*c^10*d^20 - 25175*a^26*b^4*c^12*d^18 + 4845*a^26*b^4*c^14*d^16 + 1640
*a^27*b^3*c^3*d^27 - 6700*a^27*b^3*c^5*d^25 + 12400*a^27*b^3*c^7*d^23 - 119
00*a^27*b^3*c^9*d^21 + 5800*a^27*b^3*c^11*d^19 - 1140*a^27*b^3*c^13*d^17 -
215*a^28*b^2*c^2*d^28 + 1000*a^28*b^2*c^4*d^26 - 1950*a^28*b^2*c^6*d^24 + 1
925*a^28*b^2*c^8*d^22 - 955*a^28*b^2*c^10*d^20 + 190*a^28*b^2*c^12*d^18 + 2
0*a*b^29*c^29*d + 20*a^29*b*c*d^29)))^(1/2)*(((4*(8*a^2*b^23*c^25 - 32*a^4*
b^21*c^25 + 48*a^6*b^19*c^25 - 32*a^8*b^17*c^25 + 8*a^10*b^15*c^25 + 8*a^25
*c^2*d^23 - 32*a^25*c^4*d^21 + 48*a^25*c^6*d^19 - 32*a^25*c^8*d^17 + 8*a^25
*c^10*d^15 - 8*a*b^24*c^16*d^9 + 32*a*b^24*c^18*d^7 - 48*a*b^24*c^20*d^5 +
32*a*b^24*c^22*d^3 - 72*a^3*b^22*c^24*d + 368*a^5*b^20*c^24*d - 592*a^7*b^1
8*c^24*d + 408*a^9*b^16*c^24*d - 104*a^11*b^14*c^24*d - 8*a^16*b^9*c*d^24 +
32*a^18*b^7*c*d^24 - 48*a^20*b^5*c*d^24 + 32*a^22*b^3*c*d^24 - 72*a^24*b*c
^3*d^22 + 368*a^24*b*c^5*d^20 - 592*a^24*b*c^7*d^18 + 408*a^24*b*c^9*d^16 -
104*a^24*b*c^11*d^14 + 104*a^2*b^23*c^15*d^10 - 408*a^2*b^23*c^17*d^8 + 59
2*a^2*b^23*c^19*d^6 - 368*a^2*b^23*c^21*d^4 + 72*a^2*b^23*c^23*d^2 - 616*a^
3*b^22*c^14*d^11 + 2392*a^3*b^22*c^16*d^9 - 3408*a^3*b^22*c^18*d^7 + 2032*a
^3*b^22*c^20*d^5 - 328*a^3*b^22*c^22*d^3 + 2184*a^4*b^21*c^13*d^12 - 8536*a
^4*b^21*c^15*d^10 + 12272*a^4*b^21*c^17*d^8 - 7408*a^4*b^21*c^19*d^6 + 1192
*a^4*b^21*c^21*d^4 + 328*a^4*b^21*c^23*d^2 - 5096*a^5*b^20*c^12*d^13 + 2066
4*a^5*b^20*c^14*d^11 - 31328*a^5*b^20*c^16*d^9 + 20592*a^5*b^20*c^18*d^7 -
4008*a^5*b^20*c^20*d^5 - 1192*a^5*b^20*c^22*d^3 + 8008*a^6*b^19*c^11*d^14 -
35672*a^6*b^19*c^13*d^12 + 60768*a^6*b^19*c^15*d^10 - 46464*a^6*b^19*c^17*
d^8 + 11336*a^6*b^19*c^19*d^6 + 4008*a^6*b^19*c^21*d^4 - 2032*a^6*b^19*c^23
*d^2 - 8008*a^7*b^18*c^10*d^15 + 44408*a^7*b^18*c^12*d^13 - 92512*a^7*b^18*
c^14*d^11 + 85536*a^7*b^18*c^16*d^9 - 24904*a^7*b^18*c^18*d^7 - 11336*a^7*b
^18*c^20*d^5 + 7408*a^7*b^18*c^22*d^3 + 3432*a^8*b^17*c^9*d^16 - 37752*a^8*

b^17*c^11*d^14 + 109408*a^8*b^17*c^13*d^12 - 125472*a^8*b^17*c^15*d^10 + 42
696*a^8*b^17*c^17*d^8 + 24904*a^8*b^17*c^19*d^6 - 20592*a^8*b^17*c^21*d^4 +
3408*a^8*b^17*c^23*d^2 + 3432*a^9*b^16*c^8*d^17 + 14872*a^9*b^16*c^10*d^15
- 92352*a^9*b^16*c^12*d^13 + 141408*a^9*b^16*c^14*d^11 - 59264*a^9*b^16*c^
16*d^9 - 42696*a^9*b^16*c^18*d^7 + 46464*a^9*b^16*c^20*d^5 - 12272*a^9*b^16
*c^22*d^3 - 8008*a^10*b^15*c^7*d^18 + 14872*a^10*b^15*c^9*d^16 + 36608*a^10
*b^15*c^11*d^14 - 113152*a^10*b^15*c^13*d^12 + 67008*a^10*b^15*c^15*d^10 +
59264*a^10*b^15*c^17*d^8 - 85536*a^10*b^15*c^19*d^6 + 31328*a^10*b^15*c^21*
d^4 - 2392*a^10*b^15*c^23*d^2 + 8008*a^11*b^14*c^6*d^19 - 37752*a^11*b^14*c
^8*d^17 + 36608*a^11*b^14*c^10*d^15 + 43264*a^11*b^14*c^12*d^13 - 56256*a^1
1*b^14*c^14*d^11 - 67008*a^11*b^14*c^16*d^9 + 125472*a^11*b^14*c^18*d^7 - 6
0768*a^11*b^14*c^20*d^5 + 8536*a^11*b^14*c^22*d^3 - 5096*a^12*b^13*c^5*d^20
+ 44408*a^12*b^13*c^7*d^18 - 92352*a^12*b^13*c^9*d^16 + 43264*a^12*b^13*c^
11*d^14 + 22464*a^12*b^13*c^13*d^12 + 56256*a^12*b^13*c^15*d^10 - 141408*a^
12*b^13*c^17*d^8 + 92512*a^12*b^13*c^19*d^6 - 20664*a^12*b^13*c^21*d^4 + 61
6*a^12*b^13*c^23*d^2 + 2184*a^13*b^12*c^4*d^21 - 35672*a^13*b^12*c^6*d^19 +
109408*a^13*b^12*c^8*d^17 - 113152*a^13*b^12*c^10*d^15 + 22464*a^13*b^12*c
^12*d^13 - 22464*a^13*b^12*c^14*d^11 + 113152*a^13*b^12*c^16*d^9 - 109408*a
^13*b^12*c^18*d^7 + 35672*a^13*b^12*c^20*d^5 - 2184*a^13*b^12*c^22*d^3 - 61
6*a^14*b^11*c^3*d^22 + 20664*a^14*b^11*c^5*d^20 - 92512*a^14*b^11*c^7*d^18
+ 141408*a^14*b^11*c^9*d^16 - 56256*a^14*b^11*c^11*d^14 - 22464*a^14*b^11*c
^13*d^12 - 43264*a^14*b^11*c^15*d^10 + 92352*a^14*b^11*c^17*d^8 - 44408*a^1
4*b^11*c^19*d^6 + 5096*a^14*b^11*c^21*d^4 + 104*a^15*b^10*c^2*d^23 - 8536*a
^15*b^10*c^4*d^21 + 60768*a^15*b^10*c^6*d^19 - 125472*a^15*b^10*c^8*d^17 +
67008*a^15*b^10*c^10*d^15 + 56256*a^15*b^10*c^12*d^13 - 43264*a^15*b^10*c^1
4*d^11 - 36608*a^15*b^10*c^16*d^9 + 37752*a^15*b^10*c^18*d^7 - 8008*a^15*b^
10*c^20*d^5 + 2392*a^16*b^9*c^3*d^22 - 31328*a^16*b^9*c^5*d^20 + 85536*a^16
*b^9*c^7*d^18 - 59264*a^16*b^9*c^9*d^16 - 67008*a^16*b^9*c^11*d^14 + 113152
*a^16*b^9*c^13*d^12 - 36608*a^16*b^9*c^15*d^10 - 14872*a^16*b^9*c^17*d^8 +
8008*a^16*b^9*c^19*d^6 - 408*a^17*b^8*c^2*d^23 + 12272*a^17*b^8*c^4*d^21 -
46464*a^17*b^8*c^6*d^19 + 42696*a^17*b^8*c^8*d^17 + 59264*a^17*b^8*c^10*d^1
5 - 141408*a^17*b^8*c^12*d^13 + 92352*a^17*b^8*c^14*d^11 - 14872*a^17*b^8*c
^16*d^9 - 3432*a^17*b^8*c^18*d^7 - 3408*a^18*b^7*c^3*d^22 + 20592*a^18*b^7*
c^5*d^20 - 24904*a^18*b^7*c^7*d^18 - 42696*a^18*b^7*c^9*d^16 + 125472*a^18*
b^7*c^11*d^14 - 109408*a^18*b^7*c^13*d^12 + 37752*a^18*b^7*c^15*d^10 - 3432
*a^18*b^7*c^17*d^8 + 592*a^19*b^6*c^2*d^23 - 7408*a^19*b^6*c^4*d^21 + 11336
*a^19*b^6*c^6*d^19 + 24904*a^19*b^6*c^8*d^17 - 85536*a^19*b^6*c^10*d^15 + 9
2512*a^19*b^6*c^12*d^13 - 44408*a^19*b^6*c^14*d^11 + 8008*a^19*b^6*c^16*d^9
+ 2032*a^20*b^5*c^3*d^22 - 4008*a^20*b^5*c^5*d^20 - 11336*a^20*b^5*c^7*d^1
8 + 46464*a^20*b^5*c^9*d^16 - 60768*a^20*b^5*c^11*d^14 + 35672*a^20*b^5*c^1
3*d^12 - 8008*a^20*b^5*c^15*d^10 - 368*a^21*b^4*c^2*d^23 + 1192*a^21*b^4*c^
4*d^21 + 4008*a^21*b^4*c^6*d^19 - 20592*a^21*b^4*c^8*d^17 + 31328*a^21*b^4*
c^10*d^15 - 20664*a^21*b^4*c^12*d^13 + 5096*a^21*b^4*c^14*d^11 - 328*a^22*b
^3*c^3*d^22 - 1192*a^22*b^3*c^5*d^20 + 7408*a^22*b^3*c^7*d^18 - 12272*a^22*
b^3*c^9*d^16 + 8536*a^22*b^3*c^11*d^14 - 2184*a^22*b^3*c^13*d^12 + 72*a^23*

b^2*c^2*d^23 + 328*a^23*b^2*c^4*d^21 - 2032*a^23*b^2*c^6*d^19 + 3408*a^23*b
^2*c^8*d^17 - 2392*a^23*b^2*c^10*d^15 + 616*a^23*b^2*c^12*d^13 - 8*a*b^24*c
^24*d - 8*a^24*b*c*d^24))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*
b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^
20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16
- 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^2
0*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72
*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^
19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^
13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^1
7 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b
^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^1
8*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^
11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^1
7*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^1
2*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18
*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*
d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*
d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*
d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^
16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*
d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c
^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^
4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*
c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^
12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11
*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b
^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b
^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10
*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 363
00*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4
+ 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 +
27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9
- 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3
- 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14
+ 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12
*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d
^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9
*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^1
5*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6
*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*
c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c
^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c
^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4
*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^1

0*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19) - (8*tan(e/2 + (f*x)/2)*(56*a^3*b^22*c^25 - 12*a^25*c*d^24 - 12*a*b^24*c^25 - 104*a^5*b^20*c^25 + 96*a^7*b^18*c^25 - 44*a^9*b^16*c^25 + 8*a^11*b^14*c^25 + 56*a^25*c^3*d^22 - 104*a^25*c^5*d^20 + 96*a^25*c^7*d^18 - 44*a^25*c^9*d^16 + 8*a^25*c^11*d^14 + 16*a*b^24*c^15*d^10 - 76*a*b^24*c^17*d^8 + 144*a*b^24*c^19*d^6 - 136*a*b^24*c^21*d^4 + 64*a*b^24*c^23*d^2 + 168*a^2*b^23*c^24*d - 784*a^4*b^21*c^24*d + 1456*a^6*b^19*c^24*d - 1344*a^8*b^17*c^24*d + 616*a^10*b^15*c^24*d - 112*a^12*b^13*c^24*d + 16*a^15*b^10*c*d^24 - 76*a^17*b^8*c*d^24 + 144*a^19*b^6*c*d^24 - 136*a^21*b^4*c*d^24 + 64*a^23*b^2*c*d^24 + 168*a^24*b*c^2*d^23 - 784*a^24*b*c^4*d^21 + 1456*a^24*b*c^6*d^19 - 1344*a^24*b*c^8*d^17 + 616*a^24*b*c^10*d^15 - 112*a^24*b*c^12*d^13 - 224*a^2*b^23*c^14*d^11 + 1064*a^2*b^23*c^16*d^9 - 2016*a^2*b^23*c^18*d^7 + 1904*a^2*b^23*c^20*d^5 - 896*a^2*b^23*c^22*d^3 + 1456*a^3*b^22*c^13*d^12 - 6992*a^3*b^22*c^15*d^10 + 13464*a^3*b^22*c^17*d^8 - 13056*a^3*b^22*c^19*d^6 + 6464*a^3*b^22*c^21*d^4 - 1392*a^3*b^22*c^23*d^2 - 5824*a^4*b^21*c^12*d^13 + 28728*a^4*b^21*c^14*d^11 - 57456*a^4*b^21*c^16*d^9 + 59024*a^4*b^21*c^18*d^7 - 32256*a^4*b^21*c^20*d^5 + 8568*a^4*b^21*c^22*d^3 + 16016*a^5*b^20*c^11*d^14 - 82992*a^5*b^20*c^13*d^12 + 177048*a^5*b^20*c^15*d^10 - 198696*a^5*b^20*c^17*d^8 + 123584*a^5*b^20*c^19*d^6 - 40512*a^5*b^20*c^21*d^4 + 5656*a^5*b^20*c^23*d^2 - 32032*a^6*b^19*c^10*d^15 + 179816*a^6*b^19*c^12*d^13 - 421344*a^6*b^19*c^14*d^11 + 529312*a^6*b^19*c^16*d^9 - 379008*a^6*b^19*c^18*d^7 + 150024*a^6*b^19*c^20*d^5 - 28224*a^6*b^19*c^22*d^3 + 48048*a^7*b^18*c^9*d^16 - 304304*a^7*b^18*c^11*d^14 + 805896*a^7*b^18*c^13*d^12 - 1151104*a^7*b^18*c^15*d^10 + 949952*a^7*b^18*c^17*d^8 - 446736*a^7*b^18*c^19*d^6 + 108136*a^7*b^18*c^21*d^4 - 9984*a^7*b^18*c^23*d^2 - 54912*a^8*b^17*c^8*d^17 + 412984*a^8*b^17*c^10*d^15 - 1267344*a^8*b^17*c^12*d^13 + 2077536*a^8*b^17*c^14*d^11 - 1975808*a^8*b^17*c^16*d^9 + 1095384*a^8*b^17*c^18*d^7 - 331632*a^8*b^17*c^20*d^5 + 45136*a^8*b^17*c^22*d^3 + 48048*a^9*b^16*c^7*d^18 - 456456*a^9*b^16*c^9*d^16 + 1657656*a^9*b^16*c^11*d^14 - 3143504*a^9*b^16*c^13*d^12 + 3453696*a^9*b^16*c^15*d^10 - 2247636*a^9*b^16*c^17*d^8 + 831208*a^9*b^16*c^19*d^6 - 151944*a^9*b^16*c^21*d^4 + 8976*a^9*b^16*c^23*d^2 - 32032*a^10*b^15*c^6*d^19 + 412984*a^10*b^15*c^8*d^17 - 1812096*a^10*b^15*c^10*d^15 + 4016896*a^10*b^15*c^12*d^13 - 5121024*a^10*b^15*c^14*d^11 + 3897024*a^10*b^15*c^16*d^9 - 1728832*a^10*b^15*c^18*d^7 + 404768*a^10*b^15*c^20*d^5 - 38304*a^10*b^15*c^22*d^3 + 16016*a^11*b^14*c^5*d^20 - 304304*a^11*b^14*c^7*d^18 + 1657656*a^11*b^14*c^9*d^16 - 4356352*a^11*b^14*c^11*d^14 + 6476288*a^11*b^14*c^13*d^12 - 5745024*a^11*b^14*c^15*d^10 + 3021984*a^11*b^14*c^17*d^8 - 880256*a^11*b^14*c^19*d^6 + 118032*a^11*b^14*c^21*d^4 - 4048*a^11*b^14*c^23*d^2 - 5824*a^12*b^13*c^4*d^21 + 179816*a^12*b^13*c^6*d^19 - 1267344*a^12*b^13*c^8*d^17 + 4016896*a^12*b^13*c^10*d^15 - 7002112*a^12*b^13*c^12*d^13 + 7235136*a^12*b^13*c^14*d^11 - 4480896*a^12*b^13*c^16*d^9 + 1588704*a^12*b^13*c^18*d^7 - 280896*a^12*b^13*c^20*d^5 + 16632*a^12*b^13*c^22*d^3 +

1456*a^13*b^12*c^3*d^22 - 82992*a^13*b^12*c^5*d^20 + 805896*a^13*b^12*c^7*d^18 - 3143504*a^13*b^12*c^9*d^16 + 6476288*a^13*b^12*c^11*d^14 - 7809984*a^13*b^12*c^13*d^12 + 5666752*a^13*b^12*c^15*d^10 - 2403856*a^13*b^12*c^17*d^8 + 537264*a^13*b^12*c^19*d^6 - 48048*a^13*b^12*c^21*d^4 + 728*a^13*b^12*c^23*d^2 - 224*a^14*b^11*c^2*d^23 + 28728*a^14*b^11*c^4*d^21 - 421344*a^14*b^11*c^6*d^19 + 2077536*a^14*b^11*c^8*d^17 - 5121024*a^14*b^11*c^10*d^15 + 7235136*a^14*b^11*c^12*d^13 - 6126848*a^14*b^11*c^14*d^11 + 3071744*a^14*b^11*c^16*d^9 - 844896*a^14*b^11*c^18*d^7 + 104104*a^14*b^11*c^20*d^5 - 2912*a^14*b^11*c^22*d^3 - 6992*a^15*b^10*c^3*d^22 + 177048*a^15*b^10*c^5*d^20 - 1151104*a^15*b^10*c^7*d^18 + 3453696*a^15*b^10*c^9*d^16 - 5745024*a^15*b^10*c^11*d^14 + 5666752*a^15*b^10*c^13*d^12 - 3331328*a^15*b^10*c^15*d^10 + 1105104*a^15*b^10*c^17*d^8 - 176176*a^15*b^10*c^19*d^6 + 8008*a^15*b^10*c^21*d^4 + 1064*a^16*b^9*c^2*d^23 - 57456*a^16*b^9*c^4*d^21 + 529312*a^16*b^9*c^6*d^19 - 1975808*a^16*b^9*c^8*d^17 + 3897024*a^16*b^9*c^10*d^15 - 4480896*a^16*b^9*c^12*d^13 + 3071744*a^16*b^9*c^14*d^11 - 1208064*a^16*b^9*c^16*d^9 + 239096*a^16*b^9*c^18*d^7 - 16016*a^16*b^9*c^20*d^5 + 13464*a^17*b^8*c^3*d^22 - 198696*a^17*b^8*c^5*d^20 + 949952*a^17*b^8*c^7*d^18 - 2247636*a^17*b^8*c^9*d^16 + 3021984*a^17*b^8*c^11*d^14 - 2403856*a^17*b^8*c^13*d^12 + 1105104*a^17*b^8*c^15*d^10 - 264264*a^17*b^8*c^17*d^8 + 24024*a^17*b^8*c^19*d^6 - 2016*a^18*b^7*c^2*d^23 + 59024*a^18*b^7*c^4*d^21 - 379008*a^18*b^7*c^6*d^19 + 1095384*a^18*b^7*c^8*d^17 - 1728832*a^18*b^7*c^10*d^15 + 1588704*a^18*b^7*c^12*d^13 - 844896*a^18*b^7*c^14*d^11 + 239096*a^18*b^7*c^16*d^9 - 27456*a^18*b^7*c^18*d^7 - 13056*a^19*b^6*c^3*d^22 + 123584*a^19*b^6*c^5*d^20 - 446736*a^19*b^6*c^7*d^18 + 831208*a^19*b^6*c^9*d^16 - 880256*a^19*b^6*c^11*d^14 + 537264*a^19*b^6*c^13*d^12 - 176176*a^19*b^6*c^15*d^10 + 24024*a^19*b^6*c^17*d^8 + 1904*a^20*b^5*c^2*d^23 - 32256*a^20*b^5*c^4*d^21 + 150024*a^20*b^5*c^6*d^19 - 331632*a^20*b^5*c^8*d^17 + 404768*a^20*b^5*c^10*d^15 - 280896*a^20*b^5*c^12*d^13 + 104104*a^20*b^5*c^14*d^11 - 16016*a^20*b^5*c^16*d^9 + 6464*a^21*b^4*c^3*d^22 - 40512*a^21*b^4*c^5*d^20 + 108136*a^21*b^4*c^7*d^18 - 151944*a^21*b^4*c^9*d^16 + 118032*a^21*b^4*c^11*d^14 - 48048*a^21*b^4*c^13*d^12 + 8008*a^21*b^4*c^15*d^10 - 896*a^22*b^3*c^2*d^23 + 8568*a^22*b^3*c^4*d^21 - 28224*a^22*b^3*c^6*d^19 + 45136*a^22*b^3*c^8*d^17 - 38304*a^22*b^3*c^10*d^15 + 16632*a^22*b^3*c^12*d^13 - 2912*a^22*b^3*c^14*d^11 - 1392*a^23*b^2*c^3*d^22 + 5656*a^23*b^2*c^5*d^20 - 9984*a^23*b^2*c^7*d^18 + 8976*a^23*b^2*c^9*d^16 - 4048*a^23*b^2*c^11*d^14 + 728*a^23*b^2*c^13*d^12))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18* d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19))*((((4*a^24*d^24 + 4*b^24*c^24 + 16*a^2*b^22*c^24 + 16*a^4*b^20*c^24 - 1152*a^10*b^14*d^24 + 5568*a^12*b^12*d^24 - 10568*a^14*b^10*d^24 + 9460*a^16*b^8*d^24 - 3560*a^18*b^6*d^24 + 136*a^20*b^4*d^24 + 76*a^22*b^2*d^24 + 16*a^24*c^2*d^22 + 16*a^24*c^4*d^20 - 1152*b^24*c^10*d^14 + 5568*b^24*c^12*d^12 - 10568*b^24*c^14*d^10 + 9460*b^24*c^16*d^8 - 3560*b^24*c^18*d^6 + 136*b^24*c^20*d^4 + 76*b^24*c^22*d^2 + 11520*a*b^23*c^9*d^15 - 56448*a*b^23*c^11*d^13 + 109456*a*b^23*c^13*d^11 - 101240*a*b^23*c^15*d^9 + 40720*a*b^23*c^17*d^7 - 2960*a*b^23*c^19*d^5 - 536*a*b^23*c^21*d^3 - 176*a^3*b^21*c^23*d - 320*a^5*b^19*c^23*d + 1

1520*a^9*b^15*c*d^23 - 56448*a^11*b^13*c*d^23 + 109456*a^13*b^11*c*d^23 - 1
01240*a^15*b^9*c*d^23 + 40720*a^17*b^7*c*d^23 - 2960*a^19*b^5*c*d^23 - 536*
a^21*b^3*c*d^23 - 176*a^23*b*c^3*d^21 - 320*a^23*b*c^5*d^19 - 51840*a^2*b^2
2*c^8*d^16 + 263808*a^2*b^22*c^10*d^14 - 541208*a^2*b^22*c^12*d^12 + 547088
*a^2*b^22*c^14*d^10 - 263320*a^2*b^22*c^16*d^8 + 44120*a^2*b^22*c^18*d^6 -
1564*a^2*b^22*c^20*d^4 - 196*a^2*b^22*c^22*d^2 + 138240*a^3*b^21*c^7*d^17 -
758400*a^3*b^21*c^9*d^15 + 1720736*a^3*b^21*c^11*d^13 - 2002728*a^3*b^21*c
^13*d^11 + 1210560*a^3*b^21*c^15*d^9 - 335040*a^3*b^21*c^17*d^7 + 37680*a^3
*b^21*c^19*d^5 - 288*a^3*b^21*c^21*d^3 - 241920*a^4*b^20*c^6*d^18 + 1512000
*a^4*b^20*c^8*d^16 - 3975688*a^4*b^20*c^10*d^14 + 5501328*a^4*b^20*c^12*d^1
2 - 4147952*a^4*b^20*c^14*d^10 + 1586920*a^4*b^20*c^16*d^8 - 276020*a^4*b^2
0*c^18*d^6 + 21124*a^4*b^20*c^20*d^4 + 176*a^4*b^20*c^22*d^2 + 290304*a^5*b
^19*c^5*d^19 - 2232576*a^5*b^19*c^7*d^17 + 7078256*a^5*b^19*c^9*d^15 - 1178
1560*a^5*b^19*c^11*d^13 + 10875200*a^5*b^19*c^13*d^11 - 5365072*a^5*b^19*c^
15*d^9 + 1310168*a^5*b^19*c^17*d^7 - 170968*a^5*b^19*c^19*d^5 + 8160*a^5*b^
19*c^21*d^3 - 241920*a^6*b^18*c^4*d^20 + 2532096*a^6*b^18*c^6*d^18 - 995599
2*a^6*b^18*c^8*d^16 + 20019440*a^6*b^18*c^10*d^14 - 22419600*a^6*b^18*c^12*
d^12 + 13887520*a^6*b^18*c^14*d^10 - 4506428*a^6*b^18*c^16*d^8 + 793756*a^6
*b^18*c^18*d^6 - 72240*a^6*b^18*c^20*d^4 + 3040*a^6*b^18*c^22*d^2 + 138240*
a^7*b^17*c^3*d^21 - 2232576*a^7*b^17*c^5*d^19 + 11150016*a^7*b^17*c^7*d^17
- 27336616*a^7*b^17*c^9*d^15 + 37153600*a^7*b^17*c^11*d^13 - 28461040*a^7*b
^17*c^13*d^11 + 11779808*a^7*b^17*c^15*d^9 - 2621008*a^7*b^17*c^17*d^7 + 33
6688*a^7*b^17*c^19*d^5 - 17920*a^7*b^17*c^21*d^3 - 51840*a^8*b^16*c^2*d^22
+ 1512000*a^8*b^16*c^4*d^20 - 9955992*a^8*b^16*c^6*d^18 + 30289656*a^8*b^16
*c^8*d^16 - 50137600*a^8*b^16*c^10*d^14 + 46972560*a^8*b^16*c^12*d^12 - 241
99280*a^8*b^16*c^14*d^10 + 6661036*a^8*b^16*c^16*d^8 - 1058448*a^8*b^16*c^1
8*d^6 + 72560*a^8*b^16*c^20*d^4 - 758400*a^9*b^15*c^3*d^21 + 7078256*a^9*b^
15*c^5*d^19 - 27336616*a^9*b^15*c^7*d^17 + 55383904*a^9*b^15*c^9*d^15 - 631
24080*a^9*b^15*c^11*d^13 + 39987520*a^9*b^15*c^13*d^11 - 13462088*a^9*b^15*
c^15*d^9 + 2478528*a^9*b^15*c^17*d^7 - 212032*a^9*b^15*c^19*d^5 + 263808*a^
10*b^14*c^2*d^22 - 3975688*a^10*b^14*c^4*d^20 + 20019440*a^10*b^14*c^6*d^18
- 50137600*a^10*b^14*c^8*d^16 + 69593872*a^10*b^14*c^10*d^14 - 53854288*a^
10*b^14*c^12*d^12 + 21989928*a^10*b^14*c^14*d^10 - 4591360*a^10*b^14*c^16*d
^8 + 460480*a^10*b^14*c^18*d^6 + 1720736*a^11*b^13*c^3*d^21 - 11781560*a^11
*b^13*c^5*d^19 + 37153600*a^11*b^13*c^7*d^17 - 63124080*a^11*b^13*c^9*d^15
+ 59445728*a^11*b^13*c^11*d^13 - 29358696*a^11*b^13*c^13*d^11 + 6995840*a^1
1*b^13*c^15*d^9 - 762560*a^11*b^13*c^17*d^7 - 541208*a^12*b^12*c^2*d^22 + 5
501328*a^12*b^12*c^4*d^20 - 22419600*a^12*b^12*c^6*d^18 + 46972560*a^12*b^1
2*c^8*d^16 - 53854288*a^12*b^12*c^10*d^14 + 32294808*a^12*b^12*c^12*d^12 -
8958208*a^12*b^12*c^14*d^10 + 999040*a^12*b^12*c^16*d^8 - 2002728*a^13*b^11
*c^3*d^21 + 10875200*a^13*b^11*c^5*d^19 - 28461040*a^13*b^11*c^7*d^17 + 399
87520*a^13*b^11*c^9*d^15 - 29358696*a^13*b^11*c^11*d^13 + 9722048*a^13*b^11
*c^13*d^11 - 1104320*a^13*b^11*c^15*d^9 + 547088*a^14*b^10*c^2*d^22 - 41479
52*a^14*b^10*c^4*d^20 + 13887520*a^14*b^10*c^6*d^18 - 24199280*a^14*b^10*c^
8*d^16 + 21989928*a^14*b^10*c^10*d^14 - 8958208*a^14*b^10*c^12*d^12 + 11240

32*a^14*b^10*c^14*d^10 + 1210560*a^15*b^9*c^3*d^21 - 5365072*a^15*b^9*c^5*d^19 + 11779808*a^15*b^9*c^7*d^17 - 13462088*a^15*b^9*c^9*d^15 + 6995840*a^15*b^9*c^11*d^13 - 1104320*a^15*b^9*c^13*d^11 - 263320*a^16*b^8*c^2*d^22 + 1586920*a^16*b^8*c^4*d^20 - 4506428*a^16*b^8*c^6*d^18 + 6661036*a^16*b^8*c^8*d^16 - 4591360*a^16*b^8*c^10*d^14 + 999040*a^16*b^8*c^12*d^12 - 335040*a^17*b^7*c^3*d^21 + 1310168*a^17*b^7*c^5*d^19 - 2621008*a^17*b^7*c^7*d^17 + 2478528*a^17*b^7*c^9*d^15 - 762560*a^17*b^7*c^11*d^13 + 44120*a^18*b^6*c^2*d^22 - 276020*a^18*b^6*c^4*d^20 + 793756*a^18*b^6*c^6*d^18 - 1058448*a^18*b^6*c^8*d^16 + 460480*a^18*b^6*c^10*d^14 + 37680*a^19*b^5*c^3*d^21 - 170968*a^19*b^5*c^5*d^19 + 336688*a^19*b^5*c^7*d^17 - 212032*a^19*b^5*c^9*d^15 - 1564*a^20*b^4*c^2*d^22 + 21124*a^20*b^4*c^4*d^20 - 72240*a^20*b^4*c^6*d^18 + 72560*a^20*b^4*c^8*d^16 - 288*a^21*b^3*c^3*d^21 + 8160*a^21*b^3*c^5*d^19 - 17920*a^21*b^3*c^7*d^17 - 196*a^22*b^2*c^2*d^22 + 176*a^22*b^2*c^4*d^20 + 3040*a^22*b^2*c^6*d^18 - 8*a*b^23*c^23*d - 8*a^23*b*c*d^23)^2/4 - (20736*b^18*d^18 - 96768*a^2*b^16*d^18 + 173664*a^4*b^14*d^18 - 136032*a^6*b^12*d^18 + 31081*a^8*b^10*d^18 + 8440*a^10*b^8*d^18 + 400*a^12*b^6*d^18 - 96768*b^18*c^2*d^16 + 173664*b^18*c^4*d^14 - 136032*b^18*c^6*d^12 + 31081*b^18*c^8*d^10 + 8440*b^18*c^10*d^8 + 400*b^18*c^12*d^6 - 131328*a*b^17*c^3*d^15 + 216576*a*b^17*c^5*d^13 - 141104*a*b^17*c^7*d^11 + 20260*a*b^17*c^9*d^9 + 2800*a*b^17*c^11*d^7 - 131328*a^3*b^15*c*d^17 + 216576*a^5*b^13*c*d^17 - 141104*a^7*b^11*c*d^17 + 20260*a^9*b^9*c*d^17 + 2800*a^11*b^7*c*d^17 + 495936*a^2*b^16*c^2*d^16 - 989856*a^2*b^16*c^4*d^14 + 901948*a^2*b^16*c^6*d^12 - 308392*a^2*b^16*c^8*d^10 - 5260*a^2*b^16*c^10*d^8 + 1600*a^2*b^16*c^12*d^6 + 657408*a^3*b^15*c^3*d^15 - 1158992*a^3*b^15*c^5*d^13 + 838256*a^3*b^15*c^7*d^11 - 182200*a^3*b^15*c^9*d^9 - 3200*a^3*b^15*c^11*d^7 - 989856*a^4*b^14*c^2*d^16 + 2185654*a^4*b^14*c^4*d^14 - 2218576*a^4*b^14*c^6*d^12 + 900624*a^4*b^14*c^8*d^10 - 64720*a^4*b^14*c^10*d^8 + 1600*a^4*b^14*c^12*d^6 - 1158992*a^5*b^13*c^3*d^15 + 2158808*a^5*b^13*c^5*d^13 - 1641528*a^5*b^13*c^7*d^11 + 406880*a^5*b^13*c^9*d^9 - 17600*a^5*b^13*c^11*d^7 + 901948*a^6*b^12*c^2*d^16 - 2218576*a^6*b^12*c^4*d^14 + 2430936*a^6*b^12*c^6*d^12 - 1026928*a^6*b^12*c^8*d^10 + 88720*a^6*b^12*c^10*d^8 + 838256*a^7*b^11*c^3*d^15 - 1641528*a^7*b^11*c^5*d^13 + 1206848*a^7*b^11*c^7*d^11 - 239360*a^7*b^11*c^9*d^9 - 308392*a^8*b^10*c^2*d^16 + 900624*a^8*b^10*c^4*d^14 - 1026928*a^8*b^10*c^6*d^12 + 354016*a^8*b^10*c^8*d^10 - 182200*a^9*b^9*c^3*d^15 + 406880*a^9*b^9*c^5*d^13 - 239360*a^9*b^9*c^7*d^11 - 5260*a^10*b^8*c^2*d^16 - 64720*a^10*b^8*c^4*d^14 + 88720*a^10*b^8*c^6*d^12 - 3200*a^11*b^7*c^3*d^15 - 17600*a^11*b^7*c^5*d^13 + 1600*a^12*b^6*c^2*d^16 + 1600*a^12*b^6*c^4*d^14 + 27648*a*b^17*c*d^17)*(80*a^2*b^28*c^30 - 16*b^30*c^30 - 16*a^30*d^30 - 160*a^4*b^26*c^30 + 160*a^6*b^24*c^30 - 80*a^8*b^22*c^30 + 16*a^10*b^20*c^30 + 16*a^20*b^10*d^30 - 80*a^22*b^8*d^30 + 160*a^24*b^6*d^30 - 160*a^26*b^4*d^30 + 80*a^28*b^2*d^30 + 80*a^30*c^2*d^28 - 160*a^30*c^4*d^26 + 160*a^30*c^6*d^24 - 80*a^30*c^8*d^22 + 16*a^30*c^10*d^20 + 16*b^30*c^20*d^10 - 80*b^30*c^22*d^8 + 160*b^30*c^24*d^6 - 160*b^30*c^26*d^4 + 80*b^30*c^28*d^2 - 320*a*b^29*c^19*d^11 + 1600*a*b^29*c^21*d^9 - 3200*a*b^29*c^23*d^7 + 3200*a*b^29*c^25*d^5 - 1600*a*b^29*c^27*d^3 - 1600*a^3*b^27*c^29*d + 3200*a^5*b^25*c^29*d - 3200*a^7*

b^23*c^29*d + 1600*a^9*b^21*c^29*d - 320*a^11*b^19*c^29*d - 320*a^19*b^11*c
*d^29 + 1600*a^21*b^9*c*d^29 - 3200*a^23*b^7*c*d^29 + 3200*a^25*b^5*c*d^29
- 1600*a^27*b^3*c*d^29 - 1600*a^29*b*c^3*d^27 + 3200*a^29*b*c^5*d^25 - 3200
*a^29*b*c^7*d^23 + 1600*a^29*b*c^9*d^21 - 320*a^29*b*c^11*d^19 + 3040*a^2*b
^28*c^18*d^12 - 15280*a^2*b^28*c^20*d^10 + 30800*a^2*b^28*c^22*d^8 - 31200*
a^2*b^28*c^24*d^6 + 16000*a^2*b^28*c^26*d^4 - 3440*a^2*b^28*c^28*d^2 - 1824
0*a^3*b^27*c^17*d^13 + 92800*a^3*b^27*c^19*d^11 - 190400*a^3*b^27*c^21*d^9
+ 198400*a^3*b^27*c^23*d^7 - 107200*a^3*b^27*c^25*d^5 + 26240*a^3*b^27*c^27
*d^3 + 77520*a^4*b^26*c^16*d^14 - 402800*a^4*b^26*c^18*d^12 + 851360*a^4*b^
26*c^20*d^10 - 928000*a^4*b^26*c^22*d^8 + 541200*a^4*b^26*c^24*d^6 - 155120
*a^4*b^26*c^26*d^4 + 16000*a^4*b^26*c^28*d^2 - 248064*a^5*b^25*c^15*d^15 +
1331520*a^5*b^25*c^17*d^13 - 2939840*a^5*b^25*c^19*d^11 + 3408640*a^5*b^25*
c^21*d^9 - 2184320*a^5*b^25*c^23*d^7 + 736064*a^5*b^25*c^25*d^5 - 107200*a^
5*b^25*c^27*d^3 + 620160*a^6*b^24*c^14*d^16 - 3488400*a^6*b^24*c^16*d^14 +
8170000*a^6*b^24*c^18*d^12 - 10229760*a^6*b^24*c^20*d^10 + 7281600*a^6*b^24
*c^22*d^8 - 2863760*a^6*b^24*c^24*d^6 + 541200*a^6*b^24*c^26*d^4 - 31200*a^
6*b^24*c^28*d^2 - 1240320*a^7*b^23*c^13*d^17 + 7441920*a^7*b^23*c^15*d^15 -
18787200*a^7*b^23*c^17*d^13 + 25721600*a^7*b^23*c^19*d^11 - 20444800*a^7*b
^23*c^21*d^9 + 9297920*a^7*b^23*c^23*d^7 - 2184320*a^7*b^23*c^25*d^5 + 1984
00*a^7*b^23*c^27*d^3 + 2015520*a^8*b^22*c^12*d^18 - 13178400*a^8*b^22*c^14*
d^16 + 36434400*a^8*b^22*c^16*d^14 - 55069600*a^8*b^22*c^18*d^12 + 48989680
*a^8*b^22*c^20*d^10 - 25575920*a^8*b^22*c^22*d^8 + 7281600*a^8*b^22*c^24*d^
6 - 928000*a^8*b^22*c^26*d^4 + 30800*a^8*b^22*c^28*d^2 - 2687360*a^9*b^21*c
^11*d^19 + 19638400*a^9*b^21*c^13*d^17 - 60362240*a^9*b^21*c^15*d^15 + 1014
75200*a^9*b^21*c^17*d^13 - 101172800*a^9*b^21*c^19*d^11 + 60333760*a^9*b^21
*c^21*d^9 - 20444800*a^9*b^21*c^23*d^7 + 3408640*a^9*b^21*c^25*d^5 - 190400
*a^9*b^21*c^27*d^3 + 2956096*a^10*b^20*c^10*d^20 - 24858080*a^10*b^20*c^12*
d^18 + 86150560*a^10*b^20*c^14*d^16 - 162120160*a^10*b^20*c^16*d^14 + 18146
3680*a^10*b^20*c^18*d^12 - 123188112*a^10*b^20*c^20*d^10 + 48989680*a^10*b^
20*c^22*d^8 - 10229760*a^10*b^20*c^24*d^6 + 851360*a^10*b^20*c^26*d^4 - 152
80*a^10*b^20*c^28*d^2 - 2687360*a^11*b^19*c^9*d^21 + 26873600*a^11*b^19*c^1
1*d^19 - 106460800*a^11*b^19*c^13*d^17 + 225738240*a^11*b^19*c^15*d^15 - 28
4331200*a^11*b^19*c^17*d^13 + 219166080*a^11*b^19*c^19*d^11 - 101172800*a^1
1*b^19*c^21*d^9 + 25721600*a^11*b^19*c^23*d^7 - 2939840*a^11*b^19*c^25*d^5
+ 92800*a^11*b^19*c^27*d^3 + 2015520*a^12*b^18*c^8*d^22 - 24858080*a^12*b^1
8*c^10*d^20 + 114212800*a^12*b^18*c^12*d^18 - 274937600*a^12*b^18*c^14*d^16
+ 390830000*a^12*b^18*c^16*d^14 - 341426960*a^12*b^18*c^18*d^12 + 18146368
0*a^12*b^18*c^20*d^10 - 55069600*a^12*b^18*c^22*d^8 + 8170000*a^12*b^18*c^2
4*d^6 - 402800*a^12*b^18*c^26*d^4 + 3040*a^12*b^18*c^28*d^2 - 1240320*a^13*
b^17*c^7*d^23 + 19638400*a^13*b^17*c^9*d^21 - 106460800*a^13*b^17*c^11*d^19
+ 293542400*a^13*b^17*c^13*d^17 - 472561920*a^13*b^17*c^15*d^15 + 46741216
0*a^13*b^17*c^17*d^13 - 284331200*a^13*b^17*c^19*d^11 + 101475200*a^13*b^17
*c^21*d^9 - 18787200*a^13*b^17*c^23*d^7 + 1331520*a^13*b^17*c^25*d^5 - 1824
0*a^13*b^17*c^27*d^3 + 620160*a^14*b^16*c^6*d^24 - 13178400*a^14*b^16*c^8*d
^22 + 86150560*a^14*b^16*c^10*d^20 - 274937600*a^14*b^16*c^12*d^18 + 503363

200*a^14*b^16*c^14*d^16 - 563751280*a^14*b^16*c^16*d^14 + 390830000*a^14*b^16*c^18*d^12 - 162120160*a^14*b^16*c^20*d^10 + 36434400*a^14*b^16*c^22*d^8 - 3488400*a^14*b^16*c^24*d^6 + 77520*a^14*b^16*c^26*d^4 - 248064*a^15*b^15*c^5*d^25 + 7441920*a^15*b^15*c^7*d^23 - 60362240*a^15*b^15*c^9*d^21 + 225738240*a^15*b^15*c^11*d^19 - 472561920*a^15*b^15*c^13*d^17 + 599984128*a^15*b^15*c^15*d^15 - 472561920*a^15*b^15*c^17*d^13 + 225738240*a^15*b^15*c^19*d^11 - 60362240*a^15*b^15*c^21*d^9 + 7441920*a^15*b^15*c^23*d^7 - 248064*a^15*b^15*c^25*d^5 + 77520*a^16*b^14*c^4*d^26 - 3488400*a^16*b^14*c^6*d^24 + 36434400*a^16*b^14*c^8*d^22 - 162120160*a^16*b^14*c^10*d^20 + 390830000*a^16*b^14*c^12*d^18 - 563751280*a^16*b^14*c^14*d^16 + 503363200*a^16*b^14*c^16*d^14 - 274937600*a^16*b^14*c^18*d^12 + 86150560*a^16*b^14*c^20*d^10 - 13178400*a^16*b^14*c^22*d^8 + 620160*a^16*b^14*c^24*d^6 - 18240*a^17*b^13*c^3*d^27 + 1331520*a^17*b^13*c^5*d^25 - 18787200*a^17*b^13*c^7*d^23 + 101475200*a^17*b^13*c^9*d^21 - 284331200*a^17*b^13*c^11*d^19 + 467412160*a^17*b^13*c^13*d^17 - 472561920*a^17*b^13*c^15*d^15 + 293542400*a^17*b^13*c^17*d^13 - 106460800*a^17*b^13*c^19*d^11 + 19638400*a^17*b^13*c^21*d^9 - 1240320*a^17*b^13*c^23*d^7 + 3040*a^18*b^12*c^2*d^28 - 402800*a^18*b^12*c^4*d^26 + 8170000*a^18*b^12*c^6*d^24 - 55069600*a^18*b^12*c^8*d^22 + 181463680*a^18*b^12*c^10*d^20 - 341426960*a^18*b^12*c^12*d^18 + 390830000*a^18*b^12*c^14*d^16 - 274937600*a^18*b^12*c^16*d^14 + 114212800*a^18*b^12*c^18*d^12 - 24858080*a^18*b^12*c^20*d^10 + 2015520*a^18*b^12*c^22*d^8 + 92800*a^19*b^11*c^3*d^27 - 2939840*a^19*b^11*c^5*d^25 + 25721600*a^19*b^11*c^7*d^23 - 101172800*a^19*b^11*c^9*d^21 + 219166080*a^19*b^11*c^11*d^19 - 284331200*a^19*b^11*c^13*d^17 + 225738240*a^19*b^11*c^15*d^15 - 106460800*a^19*b^11*c^17*d^13 + 26873600*a^19*b^11*c^19*d^11 - 2687360*a^19*b^11*c^21*d^9 - 15280*a^20*b^10*c^2*d^28 + 851360*a^20*b^10*c^4*d^26 - 10229760*a^20*b^10*c^6*d^24 + 48989680*a^20*b^10*c^8*d^22 - 123188112*a^20*b^10*c^10*d^20 + 181463680*a^20*b^10*c^12*d^18 - 162120160*a^20*b^10*c^14*d^16 + 86150560*a^20*b^10*c^16*d^14 - 24858080*a^20*b^10*c^18*d^12 + 2956096*a^20*b^10*c^20*d^10 - 190400*a^21*b^9*c^3*d^27 + 3408640*a^21*b^9*c^5*d^25 - 20444800*a^21*b^9*c^7*d^23 + 60333760*a^21*b^9*c^9*d^21 - 101172800*a^21*b^9*c^11*d^19 + 101475200*a^21*b^9*c^13*d^17 - 60362240*a^21*b^9*c^15*d^15 + 19638400*a^21*b^9*c^17*d^13 - 2687360*a^21*b^9*c^19*d^11 + 30800*a^22*b^8*c^2*d^28 - 928000*a^22*b^8*c^4*d^26 + 7281600*a^22*b^8*c^6*d^24 - 25575920*a^22*b^8*c^8*d^22 + 48989680*a^22*b^8*c^10*d^20 - 55069600*a^22*b^8*c^12*d^18 + 36434400*a^22*b^8*c^14*d^16 - 13178400*a^22*b^8*c^16*d^14 + 2015520*a^22*b^8*c^18*d^12 + 198400*a^23*b^7*c^3*d^27 - 2184320*a^23*b^7*c^5*d^25 + 9297920*a^23*b^7*c^7*d^23 - 20444800*a^23*b^7*c^9*d^21 + 25721600*a^23*b^7*c^11*d^19 - 18787200*a^23*b^7*c^13*d^17 + 7441920*a^23*b^7*c^15*d^15 - 1240320*a^23*b^7*c^17*d^13 - 31200*a^24*b^6*c^2*d^28 + 541200*a^24*b^6*c^4*d^26 - 2863760*a^24*b^6*c^6*d^24 + 7281600*a^24*b^6*c^8*d^22 - 10229760*a^24*b^6*c^10*d^20 + 8170000*a^24*b^6*c^12*d^18 - 3488400*a^24*b^6*c^14*d^16 + 620160*a^24*b^6*c^16*d^14 - 107200*a^25*b^5*c^3*d^27 + 736064*a^25*b^5*c^5*d^25 - 2184320*a^25*b^5*c^7*d^23 + 3408640*a^25*b^5*c^9*d^21 - 2939840*a^25*b^5*c^11*d^19 + 1331520*a^25*b^5*c^13*d^17 - 248064*a^25*b^5*c^15*d^15 + 16000*a^26*b^4*c^2*d^28 - 155120*a^26*b^4*c^4*d

^26 + 541200*a^26*b^4*c^6*d^24 - 928000*a^26*b^4*c^8*d^22 + 851360*a^26*b^4
*c^10*d^20 - 402800*a^26*b^4*c^12*d^18 + 77520*a^26*b^4*c^14*d^16 + 26240*a
^27*b^3*c^3*d^27 - 107200*a^27*b^3*c^5*d^25 + 198400*a^27*b^3*c^7*d^23 - 19
0400*a^27*b^3*c^9*d^21 + 92800*a^27*b^3*c^11*d^19 - 18240*a^27*b^3*c^13*d^1
7 - 3440*a^28*b^2*c^2*d^28 + 16000*a^28*b^2*c^4*d^26 - 31200*a^28*b^2*c^6*d
^24 + 30800*a^28*b^2*c^8*d^22 - 15280*a^28*b^2*c^10*d^20 + 3040*a^28*b^2*c^
12*d^18 + 320*a*b^29*c^29*d + 320*a^29*b*c*d^29))^(1/2) - 2*a^24*d^24 - 2*b
^24*c^24 - 8*a^2*b^22*c^24 - 8*a^4*b^20*c^24 + 576*a^10*b^14*d^24 - 2784*a^
12*b^12*d^24 + 5284*a^14*b^10*d^24 - 4730*a^16*b^8*d^24 + 1780*a^18*b^6*d^2
4 - 68*a^20*b^4*d^24 - 38*a^22*b^2*d^24 - 8*a^24*c^2*d^22 - 8*a^24*c^4*d^20
 + 576*b^24*c^10*d^14 - 2784*b^24*c^12*d^12 + 5284*b^24*c^14*d^10 - 4730*b^
24*c^16*d^8 + 1780*b^24*c^18*d^6 - 68*b^24*c^20*d^4 - 38*b^24*c^22*d^2 - 57
60*a*b^23*c^9*d^15 + 28224*a*b^23*c^11*d^13 - 54728*a*b^23*c^13*d^11 + 5062
0*a*b^23*c^15*d^9 - 20360*a*b^23*c^17*d^7 + 1480*a*b^23*c^19*d^5 + 268*a*b^
23*c^21*d^3 + 88*a^3*b^21*c^23*d + 160*a^5*b^19*c^23*d - 5760*a^9*b^15*c*d^
23 + 28224*a^11*b^13*c*d^23 - 54728*a^13*b^11*c*d^23 + 50620*a^15*b^9*c*d^2
3 - 20360*a^17*b^7*c*d^23 + 1480*a^19*b^5*c*d^23 + 268*a^21*b^3*c*d^23 + 88
*a^23*b*c^3*d^21 + 160*a^23*b*c^5*d^19 + 25920*a^2*b^22*c^8*d^16 - 131904*a
^2*b^22*c^10*d^14 + 270604*a^2*b^22*c^12*d^12 - 273544*a^2*b^22*c^14*d^10 +
 131660*a^2*b^22*c^16*d^8 - 22060*a^2*b^22*c^18*d^6 + 782*a^2*b^22*c^20*d^4
 + 98*a^2*b^22*c^22*d^2 - 69120*a^3*b^21*c^7*d^17 + 379200*a^3*b^21*c^9*d^1
5 - 860368*a^3*b^21*c^11*d^13 + 1001364*a^3*b^21*c^13*d^11 - 605280*a^3*b^2
1*c^15*d^9 + 167520*a^3*b^21*c^17*d^7 - 18840*a^3*b^21*c^19*d^5 + 144*a^3*b
^21*c^21*d^3 + 120960*a^4*b^20*c^6*d^18 - 756000*a^4*b^20*c^8*d^16 + 198784
4*a^4*b^20*c^10*d^14 - 2750664*a^4*b^20*c^12*d^12 + 2073976*a^4*b^20*c^14*d
^10 - 793460*a^4*b^20*c^16*d^8 + 138010*a^4*b^20*c^18*d^6 - 10562*a^4*b^20*
c^20*d^4 - 88*a^4*b^20*c^22*d^2 - 145152*a^5*b^19*c^5*d^19 + 1116288*a^5*b^
19*c^7*d^17 - 3539128*a^5*b^19*c^9*d^15 + 5890780*a^5*b^19*c^11*d^13 - 5437
600*a^5*b^19*c^13*d^11 + 2682536*a^5*b^19*c^15*d^9 - 655084*a^5*b^19*c^17*d
^7 + 85484*a^5*b^19*c^19*d^5 - 4080*a^5*b^19*c^21*d^3 + 120960*a^6*b^18*c^4
*d^20 - 1266048*a^6*b^18*c^6*d^18 + 4977996*a^6*b^18*c^8*d^16 - 10009720*a^
6*b^18*c^10*d^14 + 11209800*a^6*b^18*c^12*d^12 - 6943760*a^6*b^18*c^14*d^10
 + 2253214*a^6*b^18*c^16*d^8 - 396878*a^6*b^18*c^18*d^6 + 36120*a^6*b^18*c^
20*d^4 - 1520*a^6*b^18*c^22*d^2 - 69120*a^7*b^17*c^3*d^21 + 1116288*a^7*b^1
7*c^5*d^19 - 5575008*a^7*b^17*c^7*d^17 + 13668308*a^7*b^17*c^9*d^15 - 18576
800*a^7*b^17*c^11*d^13 + 14230520*a^7*b^17*c^13*d^11 - 5889904*a^7*b^17*c^1
5*d^9 + 1310504*a^7*b^17*c^17*d^7 - 168344*a^7*b^17*c^19*d^5 + 8960*a^7*b^1
7*c^21*d^3 + 25920*a^8*b^16*c^2*d^22 - 756000*a^8*b^16*c^4*d^20 + 4977996*a
^8*b^16*c^6*d^18 - 15144828*a^8*b^16*c^8*d^16 + 25068800*a^8*b^16*c^10*d^14
 - 23486280*a^8*b^16*c^12*d^12 + 12099640*a^8*b^16*c^14*d^10 - 3330518*a^8*
b^16*c^16*d^8 + 529224*a^8*b^16*c^18*d^6 - 36280*a^8*b^16*c^20*d^4 + 379200
*a^9*b^15*c^3*d^21 - 3539128*a^9*b^15*c^5*d^19 + 13668308*a^9*b^15*c^7*d^17
 - 27691952*a^9*b^15*c^9*d^15 + 31562040*a^9*b^15*c^11*d^13 - 19993760*a^9*
b^15*c^13*d^11 + 6731044*a^9*b^15*c^15*d^9 - 1239264*a^9*b^15*c^17*d^7 + 10
6016*a^9*b^15*c^19*d^5 - 131904*a^10*b^14*c^2*d^22 + 1987844*a^10*b^14*c^4*

d^20 - 10009720*a^10*b^14*c^6*d^18 + 25068800*a^10*b^14*c^8*d^16 - 34796936
*a^10*b^14*c^10*d^14 + 26927144*a^10*b^14*c^12*d^12 - 10994964*a^10*b^14*c^
14*d^10 + 2295680*a^10*b^14*c^16*d^8 - 230240*a^10*b^14*c^18*d^6 - 860368*a
^11*b^13*c^3*d^21 + 5890780*a^11*b^13*c^5*d^19 - 18576800*a^11*b^13*c^7*d^1
7 + 31562040*a^11*b^13*c^9*d^15 - 29722864*a^11*b^13*c^11*d^13 + 14679348*a
^11*b^13*c^13*d^11 - 3497920*a^11*b^13*c^15*d^9 + 381280*a^11*b^13*c^17*d^7
 + 270604*a^12*b^12*c^2*d^22 - 2750664*a^12*b^12*c^4*d^20 + 11209800*a^12*b
^12*c^6*d^18 - 23486280*a^12*b^12*c^8*d^16 + 26927144*a^12*b^12*c^10*d^14 -
 16147404*a^12*b^12*c^12*d^12 + 4479104*a^12*b^12*c^14*d^10 - 499520*a^12*b
^12*c^16*d^8 + 1001364*a^13*b^11*c^3*d^21 - 5437600*a^13*b^11*c^5*d^19 + 14
230520*a^13*b^11*c^7*d^17 - 19993760*a^13*b^11*c^9*d^15 + 14679348*a^13*b^1
1*c^11*d^13 - 4861024*a^13*b^11*c^13*d^11 + 552160*a^13*b^11*c^15*d^9 - 273
544*a^14*b^10*c^2*d^22 + 2073976*a^14*b^10*c^4*d^20 - 6943760*a^14*b^10*c^6
*d^18 + 12099640*a^14*b^10*c^8*d^16 - 10994964*a^14*b^10*c^10*d^14 + 447910
4*a^14*b^10*c^12*d^12 - 562016*a^14*b^10*c^14*d^10 - 605280*a^15*b^9*c^3*d^
21 + 2682536*a^15*b^9*c^5*d^19 - 5889904*a^15*b^9*c^7*d^17 + 6731044*a^15*b
^9*c^9*d^15 - 3497920*a^15*b^9*c^11*d^13 + 552160*a^15*b^9*c^13*d^11 + 1316
60*a^16*b^8*c^2*d^22 - 793460*a^16*b^8*c^4*d^20 + 2253214*a^16*b^8*c^6*d^18
 - 3330518*a^16*b^8*c^8*d^16 + 2295680*a^16*b^8*c^10*d^14 - 499520*a^16*b^8
*c^12*d^12 + 167520*a^17*b^7*c^3*d^21 - 655084*a^17*b^7*c^5*d^19 + 1310504*
a^17*b^7*c^7*d^17 - 1239264*a^17*b^7*c^9*d^15 + 381280*a^17*b^7*c^11*d^13 -
 22060*a^18*b^6*c^2*d^22 + 138010*a^18*b^6*c^4*d^20 - 396878*a^18*b^6*c^6*d
^18 + 529224*a^18*b^6*c^8*d^16 - 230240*a^18*b^6*c^10*d^14 - 18840*a^19*b^5
*c^3*d^21 + 85484*a^19*b^5*c^5*d^19 - 168344*a^19*b^5*c^7*d^17 + 106016*a^1
9*b^5*c^9*d^15 + 782*a^20*b^4*c^2*d^22 - 10562*a^20*b^4*c^4*d^20 + 36120*a^
20*b^4*c^6*d^18 - 36280*a^20*b^4*c^8*d^16 + 144*a^21*b^3*c^3*d^21 - 4080*a^
21*b^3*c^5*d^19 + 8960*a^21*b^3*c^7*d^17 + 98*a^22*b^2*c^2*d^22 - 88*a^22*b
^2*c^4*d^20 - 1520*a^22*b^2*c^6*d^18 + 4*a*b^23*c^23*d + 4*a^23*b*c*d^23)/(
16*(5*a^2*b^28*c^30 - b^30*c^30 - a^30*d^30 - 10*a^4*b^26*c^30 + 10*a^6*b^2
4*c^30 - 5*a^8*b^22*c^30 + a^10*b^20*c^30 + a^20*b^10*d^30 - 5*a^22*b^8*d^3
0 + 10*a^24*b^6*d^30 - 10*a^26*b^4*d^30 + 5*a^28*b^2*d^30 + 5*a^30*c^2*d^28
 - 10*a^30*c^4*d^26 + 10*a^30*c^6*d^24 - 5*a^30*c^8*d^22 + a^30*c^10*d^20 +
 b^30*c^20*d^10 - 5*b^30*c^22*d^8 + 10*b^30*c^24*d^6 - 10*b^30*c^26*d^4 + 5
*b^30*c^28*d^2 - 20*a*b^29*c^19*d^11 + 100*a*b^29*c^21*d^9 - 200*a*b^29*c^2
3*d^7 + 200*a*b^29*c^25*d^5 - 100*a*b^29*c^27*d^3 - 100*a^3*b^27*c^29*d + 2
00*a^5*b^25*c^29*d - 200*a^7*b^23*c^29*d + 100*a^9*b^21*c^29*d - 20*a^11*b^
19*c^29*d - 20*a^19*b^11*c*d^29 + 100*a^21*b^9*c*d^29 - 200*a^23*b^7*c*d^29
 + 200*a^25*b^5*c*d^29 - 100*a^27*b^3*c*d^29 - 100*a^29*b*c^3*d^27 + 200*a^
29*b*c^5*d^25 - 200*a^29*b*c^7*d^23 + 100*a^29*b*c^9*d^21 - 20*a^29*b*c^11*
d^19 + 190*a^2*b^28*c^18*d^12 - 955*a^2*b^28*c^20*d^10 + 1925*a^2*b^28*c^22
*d^8 - 1950*a^2*b^28*c^24*d^6 + 1000*a^2*b^28*c^26*d^4 - 215*a^2*b^28*c^28*
d^2 - 1140*a^3*b^27*c^17*d^13 + 5800*a^3*b^27*c^19*d^11 - 11900*a^3*b^27*c^
21*d^9 + 12400*a^3*b^27*c^23*d^7 - 6700*a^3*b^27*c^25*d^5 + 1640*a^3*b^27*c
^27*d^3 + 4845*a^4*b^26*c^16*d^14 - 25175*a^4*b^26*c^18*d^12 + 53210*a^4*b^
26*c^20*d^10 - 58000*a^4*b^26*c^22*d^8 + 33825*a^4*b^26*c^24*d^6 - 9695*a^4

*b^26*c^26*d^4 + 1000*a^4*b^26*c^28*d^2 - 15504*a^5*b^25*c^15*d^15 + 83220*a^5*b^25*c^17*d^13 - 183740*a^5*b^25*c^19*d^11 + 213040*a^5*b^25*c^21*d^9 - 136520*a^5*b^25*c^23*d^7 + 46004*a^5*b^25*c^25*d^5 - 6700*a^5*b^25*c^27*d^3 + 38760*a^6*b^24*c^14*d^16 - 218025*a^6*b^24*c^16*d^14 + 510625*a^6*b^24*c^18*d^12 - 639360*a^6*b^24*c^20*d^10 + 455100*a^6*b^24*c^22*d^8 - 178985*a^6*b^24*c^24*d^6 + 33825*a^6*b^24*c^26*d^4 - 1950*a^6*b^24*c^28*d^2 - 77520*a^7*b^23*c^13*d^17 + 465120*a^7*b^23*c^15*d^15 - 1174200*a^7*b^23*c^17*d^13 + 1607600*a^7*b^23*c^19*d^11 - 1277800*a^7*b^23*c^21*d^9 + 581120*a^7*b^23*c^23*d^7 - 136520*a^7*b^23*c^25*d^5 + 12400*a^7*b^23*c^27*d^3 + 125970*a^8*b^22*c^12*d^18 - 823650*a^8*b^22*c^14*d^16 + 2277150*a^8*b^22*c^16*d^14 - 3441850*a^8*b^22*c^18*d^12 + 3061855*a^8*b^22*c^20*d^10 - 1598495*a^8*b^22*c^22*d^8 + 455100*a^8*b^22*c^24*d^6 - 58000*a^8*b^22*c^26*d^4 + 1925*a^8*b^22*c^28*d^2 - 167960*a^9*b^21*c^11*d^19 + 1227400*a^9*b^21*c^13*d^17 - 3772640*a^9*b^21*c^15*d^15 + 6342200*a^9*b^21*c^17*d^13 - 6323300*a^9*b^21*c^19*d^11 + 3770860*a^9*b^21*c^21*d^9 - 1277800*a^9*b^21*c^23*d^7 + 213040*a^9*b^21*c^25*d^5 - 11900*a^9*b^21*c^27*d^3 + 184756*a^10*b^20*c^10*d^20 - 1553630*a^10*b^20*c^12*d^18 + 5384410*a^10*b^20*c^14*d^16 - 10132510*a^10*b^20*c^16*d^14 + 11341480*a^10*b^20*c^18*d^12 - 7699257*a^10*b^20*c^20*d^10 + 3061855*a^10*b^20*c^22*d^8 - 639360*a^10*b^20*c^24*d^6 + 53210*a^10*b^20*c^26*d^4 - 955*a^10*b^20*c^28*d^2 - 167960*a^11*b^19*c^9*d^21 + 1679600*a^11*b^19*c^11*d^19 - 6653800*a^11*b^19*c^13*d^17 + 14108640*a^11*b^19*c^15*d^15 - 17770700*a^11*b^19*c^17*d^13 + 13697880*a^11*b^19*c^19*d^11 - 6323300*a^11*b^19*c^21*d^9 + 1607600*a^11*b^19*c^23*d^7 - 183740*a^11*b^19*c^25*d^5 + 5800*a^11*b^19*c^27*d^3 + 125970*a^12*b^18*c^8*d^22 - 1553630*a^12*b^18*c^10*d^20 + 7138300*a^12*b^18*c^12*d^18 - 17183600*a^12*b^18*c^14*d^16 + 24426875*a^12*b^18*c^16*d^14 - 21339185*a^12*b^18*c^18*d^12 + 11341480*a^12*b^18*c^20*d^10 - 3441850*a^12*b^18*c^22*d^8 + 510625*a^12*b^18*c^24*d^6 - 25175*a^12*b^18*c^26*d^4 + 190*a^12*b^18*c^28*d^2 - 77520*a^13*b^17*c^7*d^23 + 1227400*a^13*b^17*c^9*d^21 - 6653800*a^13*b^17*c^11*d^19 + 18346400*a^13*b^17*c^13*d^17 - 29535120*a^13*b^17*c^15*d^15 + 29213260*a^13*b^17*c^17*d^13 - 17770700*a^13*b^17*c^19*d^11 + 6342200*a^13*b^17*c^21*d^9 - 1174200*a^13*b^17*c^23*d^7 + 83220*a^13*b^17*c^25*d^5 - 1140*a^13*b^17*c^27*d^3 + 38760*a^14*b^16*c^6*d^24 - 823650*a^14*b^16*c^8*d^22 + 5384410*a^14*b^16*c^10*d^20 - 17183600*a^14*b^16*c^12*d^18 + 31460200*a^14*b^16*c^14*d^16 - 35234455*a^14*b^16*c^16*d^14 + 24426875*a^14*b^16*c^18*d^12 - 10132510*a^14*b^16*c^20*d^10 + 2277150*a^14*b^16*c^22*d^8 - 218025*a^14*b^16*c^24*d^6 + 4845*a^14*b^16*c^26*d^4 - 15504*a^15*b^15*c^5*d^25 + 465120*a^15*b^15*c^7*d^23 - 3772640*a^15*b^15*c^9*d^21 + 14108640*a^15*b^15*c^11*d^19 - 29535120*a^15*b^15*c^13*d^17 + 37499008*a^15*b^15*c^15*d^15 - 29535120*a^15*b^15*c^17*d^13 + 14108640*a^15*b^15*c^19*d^11 - 3772640*a^15*b^15*c^21*d^9 + 465120*a^15*b^15*c^23*d^7 - 15504*a^15*b^15*c^25*d^5 + 4845*a^16*b^14*c^4*d^26 - 218025*a^16*b^14*c^6*d^24 + 2277150*a^16*b^14*c^8*d^22 - 10132510*a^16*b^14*c^10*d^20 + 24426875*a^16*b^14*c^12*d^18 - 35234455*a^16*b^14*c^14*d^16 + 31460200*a^16*b^14*c^16*d^14 - 17183600*a^16*b^14*c^18*d^12 + 5384410*a^16*b^14*c^20*d^10 - 823650*a^16*b^14*c^22*d^8 + 38760*a^16*b^14*c^24*d^6 - 1140*a^17*b^1

3*c^3*d^27 + 83220*a^17*b^13*c^5*d^25 - 1174200*a^17*b^13*c^7*d^23 + 634220 0*a^17*b^13*c^9*d^21 - 17770700*a^17*b^13*c^11*d^19 + 29213260*a^17*b^13*c^ 13*d^17 - 29535120*a^17*b^13*c^15*d^15 + 18346400*a^17*b^13*c^17*d^13 - 665 3800*a^17*b^13*c^19*d^11 + 1227400*a^17*b^13*c^21*d^9 - 77520*a^17*b^13*c^2 3*d^7 + 190*a^18*b^12*c^2*d^28 - 25175*a^18*b^12*c^4*d^26 + 510625*a^18*b^1 2*c^6*d^24 - 3441850*a^18*b^12*c^8*d^22 + 11341480*a^18*b^12*c^10*d^20 - 21 339185*a^18*b^12*c^12*d^18 + 24426875*a^18*b^12*c^14*d^16 - 17183600*a^18*b ^12*c^16*d^14 + 7138300*a^18*b^12*c^18*d^12 - 1553630*a^18*b^12*c^20*d^10 + 125970*a^18*b^12*c^22*d^8 + 5800*a^19*b^11*c^3*d^27 - 183740*a^19*b^11*c^5 *d^25 + 1607600*a^19*b^11*c^7*d^23 - 6323300*a^19*b^11*c^9*d^21 + 13697880* a^19*b^11*c^11*d^19 - 17770700*a^19*b^11*c^13*d^17 + 14108640*a^19*b^11*c^1 5*d^15 - 6653800*a^19*b^11*c^17*d^13 + 1679600*a^19*b^11*c^19*d^11 - 167960 *a^19*b^11*c^21*d^9 - 955*a^20*b^10*c^2*d^28 + 53210*a^20*b^10*c^4*d^26 - 6 39360*a^20*b^10*c^6*d^24 + 3061855*a^20*b^10*c^8*d^22 - 7699257*a^20*b^10*c ^10*d^20 + 11341480*a^20*b^10*c^12*d^18 - 10132510*a^20*b^10*c^14*d^16 + 53 84410*a^20*b^10*c^16*d^14 - 1553630*a^20*b^10*c^18*d^12 + 184756*a^20*b^10* c^20*d^10 - 11900*a^21*b^9*c^3*d^27 + 213040*a^21*b^9*c^5*d^25 - 1277800*a^ 21*b^9*c^7*d^23 + 3770860*a^21*b^9*c^9*d^21 - 6323300*a^21*b^9*c^11*d^19 + 6342200*a^21*b^9*c^13*d^17 - 3772640*a^21*b^9*c^15*d^15 + 1227400*a^21*b^9* c^17*d^13 - 167960*a^21*b^9*c^19*d^11 + 1925*a^22*b^8*c^2*d^28 - 58000*a^22 *b^8*c^4*d^26 + 455100*a^22*b^8*c^6*d^24 - 1598495*a^22*b^8*c^8*d^22 + 3061 855*a^22*b^8*c^10*d^20 - 3441850*a^22*b^8*c^12*d^18 + 2277150*a^22*b^8*c^14 *d^16 - 823650*a^22*b^8*c^16*d^14 + 125970*a^22*b^8*c^18*d^12 + 12400*a^23* b^7*c^3*d^27 - 136520*a^23*b^7*c^5*d^25 + 581120*a^23*b^7*c^7*d^23 - 127780 0*a^23*b^7*c^9*d^21 + 1607600*a^23*b^7*c^11*d^19 - 1174200*a^23*b^7*c^13*d^ 17 + 465120*a^23*b^7*c^15*d^15 - 77520*a^23*b^7*c^17*d^13 - 1950*a^24*b^6*c ^2*d^28 + 33825*a^24*b^6*c^4*d^26 - 178985*a^24*b^6*c^6*d^24 + 455100*a^24* b^6*c^8*d^22 - 639360*a^24*b^6*c^10*d^20 + 510625*a^24*b^6*c^12*d^18 - 2180 25*a^24*b^6*c^14*d^16 + 38760*a^24*b^6*c^16*d^14 - 6700*a^25*b^5*c^3*d^27 + 46004*a^25*b^5*c^5*d^25 - 136520*a^25*b^5*c^7*d^23 + 213040*a^25*b^5*c^9*d ^21 - 183740*a^25*b^5*c^11*d^19 + 83220*a^25*b^5*c^13*d^17 - 15504*a^25*b^5 *c^15*d^15 + 1000*a^26*b^4*c^2*d^28 - 9695*a^26*b^4*c^4*d^26 + 33825*a^26*b ^4*c^6*d^24 - 58000*a^26*b^4*c^8*d^22 + 53210*a^26*b^4*c^10*d^20 - 25175*a^ 26*b^4*c^12*d^18 + 4845*a^26*b^4*c^14*d^16 + 1640*a^27*b^3*c^3*d^27 - 6700* a^27*b^3*c^5*d^25 + 12400*a^27*b^3*c^7*d^23 - 11900*a^27*b^3*c^9*d^21 + 580 0*a^27*b^3*c^11*d^19 - 1140*a^27*b^3*c^13*d^17 - 215*a^28*b^2*c^2*d^28 + 10 00*a^28*b^2*c^4*d^26 - 1950*a^28*b^2*c^6*d^24 + 1925*a^28*b^2*c^8*d^22 - 95 5*a^28*b^2*c^10*d^20 + 190*a^28*b^2*c^12*d^18 + 20*a*b^29*c^29*d + 20*a^29* b*c*d^29)))^(1/2) - (4*(4*a^2*b^20*c^22 - 12*a^6*b^16*c^22 + 8*a^8*b^14*c^2 2 + 4*a^22*c^2*d^20 - 12*a^22*c^6*d^16 + 8*a^22*c^8*d^14 + 48*a*b^21*c^11*d ^11 - 212*a*b^21*c^13*d^9 + 360*a*b^21*c^15*d^7 - 276*a*b^21*c^17*d^5 + 80* a*b^21*c^19*d^3 - 20*a^3*b^19*c^21*d - 72*a^5*b^17*c^21*d + 204*a^7*b^15*c^ 21*d - 112*a^9*b^13*c^21*d + 48*a^11*b^11*c*d^21 - 212*a^13*b^9*c*d^21 + 36 0*a^15*b^7*c*d^21 - 276*a^17*b^5*c*d^21 + 80*a^19*b^3*c*d^21 - 20*a^21*b*c^ 3*d^19 - 72*a^21*b*c^5*d^17 + 204*a^21*b*c^7*d^15 - 112*a^21*b*c^9*d^13 - 4

80*a^2*b^20*c^10*d^12 + 2160*a^2*b^20*c^12*d^10 - 3772*a^2*b^20*c^14*d^8 +
3020*a^2*b^20*c^16*d^6 - 960*a^2*b^20*c^18*d^4 + 28*a^2*b^20*c^20*d^2 + 216
0*a^3*b^19*c^9*d^13 - 10152*a^3*b^19*c^11*d^11 + 18888*a^3*b^19*c^13*d^9 -
16732*a^3*b^19*c^15*d^7 + 6588*a^3*b^19*c^17*d^5 - 732*a^3*b^19*c^19*d^3 -
5760*a^4*b^18*c^8*d^14 + 29360*a^4*b^18*c^10*d^12 - 60792*a^4*b^18*c^12*d^1
0 + 62708*a^4*b^18*c^14*d^8 - 31892*a^4*b^18*c^16*d^6 + 6588*a^4*b^18*c^18*
d^4 - 212*a^4*b^18*c^20*d^2 + 10080*a^5*b^17*c^7*d^15 - 58860*a^5*b^17*c^9*
d^13 + 141880*a^5*b^17*c^11*d^11 - 175592*a^5*b^17*c^13*d^9 + 113748*a^5*b^
17*c^15*d^7 - 34492*a^5*b^17*c^17*d^5 + 3308*a^5*b^17*c^19*d^3 - 12096*a^6*
b^16*c^6*d^16 + 87264*a^6*b^16*c^8*d^14 - 254340*a^6*b^16*c^10*d^12 + 38153
2*a^6*b^16*c^12*d^10 - 307752*a^6*b^16*c^14*d^8 + 125568*a^6*b^16*c^16*d^6
- 21232*a^6*b^16*c^18*d^4 + 1068*a^6*b^16*c^20*d^2 + 10080*a^7*b^15*c^5*d^1
7 - 99120*a^7*b^15*c^7*d^15 + 359064*a^7*b^15*c^9*d^13 - 655076*a^7*b^15*c^
11*d^11 + 650108*a^7*b^15*c^13*d^9 - 343368*a^7*b^15*c^15*d^7 + 85760*a^7*b
^15*c^17*d^5 - 7652*a^7*b^15*c^19*d^3 - 5760*a^8*b^14*c^4*d^18 + 87264*a^8*
b^14*c^6*d^16 - 402576*a^8*b^14*c^8*d^14 + 900324*a^8*b^14*c^10*d^12 - 1096
236*a^8*b^14*c^12*d^10 + 731392*a^8*b^14*c^14*d^8 - 247352*a^8*b^14*c^16*d^
6 + 34548*a^8*b^14*c^18*d^4 - 1612*a^8*b^14*c^20*d^2 + 2160*a^9*b^13*c^3*d^
19 - 58860*a^9*b^13*c^5*d^17 + 359064*a^9*b^13*c^7*d^15 - 999816*a^9*b^13*c
^9*d^13 + 1494564*a^9*b^13*c^11*d^11 - 1238148*a^9*b^13*c^13*d^9 + 542272*a
^9*b^13*c^15*d^7 - 109032*a^9*b^13*c^17*d^5 + 7908*a^9*b^13*c^19*d^3 - 480*
a^10*b^12*c^2*d^20 + 29360*a^10*b^12*c^4*d^18 - 254340*a^10*b^12*c^6*d^16 +
900324*a^10*b^12*c^8*d^14 - 1656496*a^10*b^12*c^10*d^12 + 1688232*a^10*b^1
2*c^12*d^10 - 934868*a^10*b^12*c^14*d^8 + 254492*a^10*b^12*c^16*d^6 - 26952
*a^10*b^12*c^18*d^4 + 728*a^10*b^12*c^20*d^2 - 10152*a^11*b^11*c^3*d^19 + 1
41880*a^11*b^11*c^5*d^17 - 655076*a^11*b^11*c^7*d^15 + 1494564*a^11*b^11*c^
9*d^13 - 1870136*a^11*b^11*c^11*d^11 + 1289704*a^11*b^11*c^13*d^9 - 455388*
a^11*b^11*c^15*d^7 + 67468*a^11*b^11*c^17*d^5 - 2912*a^11*b^11*c^19*d^3 + 2
160*a^12*b^10*c^2*d^20 - 60792*a^12*b^10*c^4*d^18 + 381532*a^12*b^10*c^6*d^
16 - 1096236*a^12*b^10*c^8*d^14 + 1688232*a^12*b^10*c^10*d^12 - 1434728*a^1
2*b^10*c^12*d^10 + 639684*a^12*b^10*c^14*d^8 - 127860*a^12*b^10*c^16*d^6 +
8008*a^12*b^10*c^18*d^4 + 18888*a^13*b^9*c^3*d^19 - 175592*a^13*b^9*c^5*d^1
7 + 650108*a^13*b^9*c^7*d^15 - 1238148*a^13*b^9*c^9*d^13 + 1289704*a^13*b^9
*c^11*d^11 - 715296*a^13*b^9*c^13*d^9 + 186564*a^13*b^9*c^15*d^7 - 16016*a^
13*b^9*c^17*d^5 - 3772*a^14*b^8*c^2*d^20 + 62708*a^14*b^8*c^4*d^18 - 307752
*a^14*b^8*c^6*d^16 + 731392*a^14*b^8*c^8*d^14 - 934868*a^14*b^8*c^10*d^12 +
639684*a^14*b^8*c^12*d^10 - 211416*a^14*b^8*c^14*d^8 + 24024*a^14*b^8*c^16
*d^6 - 16732*a^15*b^7*c^3*d^19 + 113748*a^15*b^7*c^5*d^17 - 343368*a^15*b^7
*c^7*d^15 + 542272*a^15*b^7*c^9*d^13 - 455388*a^15*b^7*c^11*d^11 + 186564*a
^15*b^7*c^13*d^9 - 27456*a^15*b^7*c^15*d^7 + 3020*a^16*b^6*c^2*d^20 - 31892
*a^16*b^6*c^4*d^18 + 125568*a^16*b^6*c^6*d^16 - 247352*a^16*b^6*c^8*d^14 +
254492*a^16*b^6*c^10*d^12 - 127860*a^16*b^6*c^12*d^10 + 24024*a^16*b^6*c^14
*d^8 + 6588*a^17*b^5*c^3*d^19 - 34492*a^17*b^5*c^5*d^17 + 85760*a^17*b^5*c^
7*d^15 - 109032*a^17*b^5*c^9*d^13 + 67468*a^17*b^5*c^11*d^11 - 16016*a^17*b
^5*c^13*d^9 - 960*a^18*b^4*c^2*d^20 + 6588*a^18*b^4*c^4*d^18 - 21232*a^18*b

^4*c^6*d^16 + 34548*a^18*b^4*c^8*d^14 - 26952*a^18*b^4*c^10*d^12 + 8008*a^18*b^4*c^12*d^10 - 732*a^19*b^3*c^3*d^19 + 3308*a^19*b^3*c^5*d^17 - 7652*a^19*b^3*c^7*d^15 + 7908*a^19*b^3*c^9*d^13 - 2912*a^19*b^3*c^11*d^11 + 28*a^20*b^2*c^2*d^20 - 212*a^20*b^2*c^4*d^18 + 1068*a^20*b^2*c^6*d^16 - 1612*a^20*b^2*c^8*d^14 + 728*a^20*b^2*c^10*d^12))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4

048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 158
7*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 224
4*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168
*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a
^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*
b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^
19*d - 12*a^19*b*c*d^19) + (8*tan(e/2 + (f*x)/2)*(12*a^5*b^17*c^22 - 4*a^22
*c*d^21 - 4*a*b^21*c^22 - 8*a^7*b^15*c^22 + 12*a^22*c^5*d^17 - 8*a^22*c^7*d
^15 - 24*a*b^21*c^12*d^10 + 100*a*b^21*c^14*d^8 - 164*a*b^21*c^16*d^6 + 120
*a*b^21*c^18*d^4 - 28*a*b^21*c^20*d^2 + 20*a^2*b^20*c^21*d + 72*a^4*b^18*c^
21*d - 204*a^6*b^16*c^21*d + 112*a^8*b^14*c^21*d - 24*a^12*b^10*c*d^21 + 10
0*a^14*b^8*c*d^21 - 164*a^16*b^6*c*d^21 + 120*a^18*b^4*c*d^21 - 28*a^20*b^2
*c*d^21 + 20*a^21*b*c^2*d^20 + 72*a^21*b*c^4*d^18 - 204*a^21*b*c^6*d^16 + 1
12*a^21*b*c^8*d^14 + 216*a^2*b^20*c^11*d^11 - 908*a^2*b^20*c^13*d^9 + 1540*
a^2*b^20*c^15*d^7 - 1200*a^2*b^20*c^17*d^5 + 332*a^2*b^20*c^19*d^3 - 840*a^
3*b^19*c^10*d^12 + 3672*a^3*b^19*c^12*d^10 - 6788*a^3*b^19*c^14*d^8 + 6132*
a^3*b^19*c^16*d^6 - 2388*a^3*b^19*c^18*d^4 + 212*a^3*b^19*c^20*d^2 + 1800*a
^4*b^18*c^9*d^13 - 8680*a^4*b^18*c^11*d^11 + 18852*a^4*b^18*c^13*d^9 - 2122
8*a^4*b^18*c^15*d^7 + 11692*a^4*b^18*c^17*d^5 - 2508*a^4*b^18*c^19*d^3 - 21
60*a^5*b^17*c^8*d^14 + 13100*a^5*b^17*c^10*d^12 - 36820*a^5*b^17*c^12*d^10
+ 53712*a^5*b^17*c^14*d^8 - 39608*a^5*b^17*c^16*d^6 + 12832*a^5*b^17*c^18*d
^4 - 1068*a^5*b^17*c^20*d^2 + 1008*a^6*b^16*c^7*d^15 - 12420*a^6*b^16*c^9*d
^13 + 51764*a^6*b^16*c^11*d^11 - 100128*a^6*b^16*c^13*d^9 + 96048*a^6*b^16*
c^15*d^7 - 42920*a^6*b^16*c^17*d^5 + 6852*a^6*b^16*c^19*d^3 + 1008*a^7*b^15
*c^6*d^16 + 5136*a^7*b^15*c^8*d^14 - 48820*a^7*b^15*c^10*d^12 + 134700*a^7*
b^15*c^12*d^10 - 171472*a^7*b^15*c^14*d^8 + 103992*a^7*b^15*c^16*d^6 - 2614
8*a^7*b^15*c^18*d^4 + 1612*a^7*b^15*c^20*d^2 - 2160*a^8*b^14*c^5*d^17 + 513
6*a^8*b^14*c^7*d^15 + 20436*a^8*b^14*c^9*d^13 - 121524*a^8*b^14*c^11*d^11 +
224888*a^8*b^14*c^13*d^9 - 186952*a^8*b^14*c^15*d^7 + 67572*a^8*b^14*c^17*
d^5 - 7508*a^8*b^14*c^19*d^3 + 1800*a^9*b^13*c^4*d^18 - 12420*a^9*b^13*c^6*
d^16 + 20436*a^9*b^13*c^8*d^14 + 49416*a^9*b^13*c^10*d^12 - 201552*a^9*b^13
*c^12*d^10 + 245708*a^9*b^13*c^14*d^8 - 125412*a^9*b^13*c^16*d^6 + 22752*a^
9*b^13*c^18*d^4 - 728*a^9*b^13*c^20*d^2 - 840*a^10*b^12*c^3*d^19 + 13100*a^
10*b^12*c^5*d^17 - 48820*a^10*b^12*c^7*d^15 + 49416*a^10*b^12*c^9*d^13 + 82
088*a^10*b^12*c^11*d^11 - 219092*a^10*b^12*c^13*d^9 + 168468*a^10*b^12*c^15
*d^7 - 47152*a^10*b^12*c^17*d^5 + 2832*a^10*b^12*c^19*d^3 + 216*a^11*b^11*c
^2*d^20 - 8680*a^11*b^11*c^4*d^18 + 51764*a^11*b^11*c^6*d^16 - 121524*a^11*
b^11*c^8*d^14 + 82088*a^11*b^11*c^10*d^12 + 88712*a^11*b^11*c^12*d^10 - 153
012*a^11*b^11*c^14*d^8 + 67604*a^11*b^11*c^16*d^6 - 7168*a^11*b^11*c^18*d^4
+ 3672*a^12*b^10*c^3*d^19 - 36820*a^12*b^10*c^5*d^17 + 134700*a^12*b^10*c^
7*d^15 - 201552*a^12*b^10*c^9*d^13 + 88712*a^12*b^10*c^11*d^11 + 62676*a^12
*b^10*c^13*d^9 - 63372*a^12*b^10*c^15*d^7 + 12008*a^12*b^10*c^17*d^5 - 908*
a^13*b^9*c^2*d^20 + 18852*a^13*b^9*c^4*d^18 - 100128*a^13*b^9*c^6*d^16 + 22
4888*a^13*b^9*c^8*d^14 - 219092*a^13*b^9*c^10*d^12 + 62676*a^13*b^9*c^12*d^
10 + 26256*a^13*b^9*c^14*d^8 - 12544*a^13*b^9*c^16*d^6 - 6788*a^14*b^8*c^3*

d^19 + 53712*a^14*b^8*c^5*d^17 - 171472*a^14*b^8*c^7*d^15 + 245708*a^14*b^8
*c^9*d^13 - 153012*a^14*b^8*c^11*d^11 + 26256*a^14*b^8*c^13*d^9 + 5496*a^14
*b^8*c^15*d^7 + 1540*a^15*b^7*c^2*d^20 - 21228*a^15*b^7*c^4*d^18 + 96048*a^
15*b^7*c^6*d^16 - 186952*a^15*b^7*c^8*d^14 + 168468*a^15*b^7*c^10*d^12 - 63
372*a^15*b^7*c^12*d^10 + 5496*a^15*b^7*c^14*d^8 + 6132*a^16*b^6*c^3*d^19 -
39608*a^16*b^6*c^5*d^17 + 103992*a^16*b^6*c^7*d^15 - 125412*a^16*b^6*c^9*d^
13 + 67604*a^16*b^6*c^11*d^11 - 12544*a^16*b^6*c^13*d^9 - 1200*a^17*b^5*c^2
*d^20 + 11692*a^17*b^5*c^4*d^18 - 42920*a^17*b^5*c^6*d^16 + 67572*a^17*b^5*
c^8*d^14 - 47152*a^17*b^5*c^10*d^12 + 12008*a^17*b^5*c^12*d^10 - 2388*a^18*
b^4*c^3*d^19 + 12832*a^18*b^4*c^5*d^17 - 26148*a^18*b^4*c^7*d^15 + 22752*a^
18*b^4*c^9*d^13 - 7168*a^18*b^4*c^11*d^11 + 332*a^19*b^3*c^2*d^20 - 2508*a^
19*b^3*c^4*d^18 + 6852*a^19*b^3*c^6*d^16 - 7508*a^19*b^3*c^8*d^14 + 2832*a^
19*b^3*c^10*d^12 + 212*a^20*b^2*c^3*d^19 - 1068*a^20*b^2*c^5*d^17 + 1612*a^
20*b^2*c^7*d^15 - 728*a^20*b^2*c^9*d^13))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^
18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20
 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 +
6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*
c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^
19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d
 - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^
9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 4
8*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9
*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d
^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 +
 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 -
412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4
032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 2
88*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 834
4*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 116
8*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 1386
0*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 -
3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 63
36*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 -
21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 +
 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 -
 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d
^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17
 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^
11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15
*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d
^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10
*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^1
0*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11
*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a
^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a

^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a
^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 207
24*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 151
2*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27
504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 -
792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11
236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10
- 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 -
4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 +
4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1
587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2
244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 11
68*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220
*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^1
8*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*
c^19*d - 12*a^19*b*c*d^19)) + (4*(288*a*b^18*c^6*d^13 - 1104*a*b^18*c^8*d^1
1 + 1538*a*b^18*c^10*d^9 - 872*a*b^18*c^12*d^7 + 108*a*b^18*c^14*d^5 + 40*a
*b^18*c^16*d^3 + 8*a^3*b^16*c^18*d + 8*a^5*b^14*c^18*d + 288*a^6*b^13*c*d^1
8 - 1104*a^8*b^11*c*d^18 + 1538*a^10*b^9*c*d^18 - 872*a^12*b^7*c*d^18 + 108
*a^14*b^5*c*d^18 + 40*a^16*b^3*c*d^18 + 8*a^18*b*c^3*d^16 + 8*a^18*b*c^5*d^
14 - 864*a^2*b^17*c^5*d^14 + 3216*a^2*b^17*c^7*d^12 - 4262*a^2*b^17*c^9*d^1
0 + 2256*a^2*b^17*c^11*d^8 - 304*a^2*b^17*c^13*d^6 - 32*a^2*b^17*c^15*d^4 +
8*a^2*b^17*c^17*d^2 + 576*a^3*b^16*c^4*d^15 - 3024*a^3*b^16*c^6*d^13 + 630
4*a^3*b^16*c^8*d^11 - 7216*a^3*b^16*c^10*d^9 + 4944*a^3*b^16*c^12*d^7 - 166
4*a^3*b^16*c^14*d^5 - 72*a^3*b^16*c^16*d^3 + 576*a^4*b^15*c^3*d^16 + 912*a^
4*b^15*c^5*d^14 - 8720*a^4*b^15*c^7*d^12 + 16632*a^4*b^15*c^9*d^10 - 14888*
a^4*b^15*c^11*d^8 + 6704*a^4*b^15*c^13*d^6 - 744*a^4*b^15*c^15*d^4 - 40*a^4
*b^15*c^17*d^2 - 864*a^5*b^14*c^2*d^17 + 912*a^5*b^14*c^4*d^15 + 5140*a^5*b
^14*c^6*d^13 - 16080*a^5*b^14*c^8*d^11 + 23520*a^5*b^14*c^10*d^9 - 20208*a^
5*b^14*c^12*d^7 + 7404*a^5*b^14*c^14*d^5 - 264*a^5*b^14*c^16*d^3 - 3024*a^6
*b^13*c^3*d^16 + 5140*a^6*b^13*c^5*d^14 + 5280*a^6*b^13*c^7*d^12 - 28380*a^
6*b^13*c^9*d^10 + 39792*a^6*b^13*c^11*d^8 - 22728*a^6*b^13*c^13*d^6 + 3096*
a^6*b^13*c^15*d^4 - 112*a^6*b^13*c^17*d^2 + 3216*a^7*b^12*c^2*d^17 - 8720*a
^7*b^12*c^4*d^15 + 5280*a^7*b^12*c^6*d^13 + 15000*a^7*b^12*c^8*d^11 - 40656
*a^7*b^12*c^10*d^9 + 40296*a^7*b^12*c^12*d^7 - 12984*a^7*b^12*c^14*d^5 + 72
8*a^7*b^12*c^16*d^3 + 6304*a^8*b^11*c^3*d^16 - 16080*a^8*b^11*c^5*d^14 + 15
000*a^8*b^11*c^7*d^12 + 16024*a^8*b^11*c^9*d^10 - 46184*a^8*b^11*c^11*d^8 +
27208*a^8*b^11*c^13*d^6 - 2752*a^8*b^11*c^15*d^4 - 4262*a^9*b^10*c^2*d^17
+ 16632*a^9*b^10*c^4*d^15 - 28380*a^9*b^10*c^6*d^13 + 16024*a^9*b^10*c^8*d^
11 + 22018*a^9*b^10*c^10*d^9 - 30104*a^9*b^10*c^12*d^7 + 6488*a^9*b^10*c^14
*d^5 - 7216*a^10*b^9*c^3*d^16 + 23520*a^10*b^9*c^5*d^14 - 40656*a^10*b^9*c^
7*d^12 + 22018*a^10*b^9*c^9*d^10 + 13080*a^10*b^9*c^11*d^8 - 8720*a^10*b^9*
c^13*d^6 + 2256*a^11*b^8*c^2*d^17 - 14888*a^11*b^8*c^4*d^15 + 39792*a^11*b^
8*c^6*d^13 - 46184*a^11*b^8*c^8*d^11 + 13080*a^11*b^8*c^10*d^9 + 4360*a^11*
b^8*c^12*d^7 + 4944*a^12*b^7*c^3*d^16 - 20208*a^12*b^7*c^5*d^14 + 40296*a^1

2*b^7*c^7*d^12 - 30104*a^12*b^7*c^9*d^10 + 4360*a^12*b^7*c^11*d^8 - 304*a^1
3*b^6*c^2*d^17 + 6704*a^13*b^6*c^4*d^15 - 22728*a^13*b^6*c^6*d^13 + 27208*a
^13*b^6*c^8*d^11 - 8720*a^13*b^6*c^10*d^9 - 1664*a^14*b^5*c^3*d^16 + 7404*a
^14*b^5*c^5*d^14 - 12984*a^14*b^5*c^7*d^12 + 6488*a^14*b^5*c^9*d^10 - 32*a^
15*b^4*c^2*d^17 - 744*a^15*b^4*c^4*d^15 + 3096*a^15*b^4*c^6*d^13 - 2752*a^1
5*b^4*c^8*d^11 - 72*a^16*b^3*c^3*d^16 - 264*a^16*b^3*c^5*d^14 + 728*a^16*b^
3*c^7*d^12 + 8*a^17*b^2*c^2*d^17 - 40*a^17*b^2*c^4*d^15 - 112*a^17*b^2*c^6*
d^13 + 2*a*b^18*c^18*d + 2*a^18*b*c*d^18))/(a^20*d^20 + b^20*c^20 - 4*a^2*b
^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^2
0 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 +
6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20
*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b
^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d
- 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b
^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 +
48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^
9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*
d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11
+ 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 -
412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 +
4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 -
288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 83
44*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 11
68*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 138
60*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 -
3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6
336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 -
21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3
+ 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12
- 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*
d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^1
7 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d
^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^1
5*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*
d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^1
0*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^
10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^1
1*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*
a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*
a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*
a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20
724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 15
12*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 2
7504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 -
792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 1

1236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19) - (8*tan(e/2 + (f*x)/2)*(a*b^18*c^19 + a^19*c*d^18 + 4*a^3*b^16*c^19 + 4*a^5*b^14*c^19 + 4*a^19*c^3*d^16 + 4*a^19*c^5*d^14 - 576*a*b^18*c^5*d^14 + 2640*a*b^18*c^7*d^12 - 4732*a*b^18*c^9*d^10 + 3961*a*b^18*c^11*d^8 - 1344*a*b^18*c^13*d^6 + 14*a*b^18*c^15*d^4 + 18*a*b^18*c^17*d^2 + 4*a^2*b^17*c^18*d - 20*a^4*b^15*c^18*d - 576*a^5*b^14*c*d^18 - 56*a^6*b^13*c^18*d + 2640*a^7*b^12*c*d^18 - 4732*a^9*b^10*c*d^18 + 3961*a^11*b^8*c*d^18 - 1344*a^13*b^6*c*d^18 + 14*a^15*b^4*c*d^18 + 18*a^17*b^2*c*d^18 + 4*a^18*b*c^2*d^17 - 20*a^18*b*c^4*d^15 - 56*a^18*b*c^6*d^13 + 2304*a^2*b^17*c^4*d^15 - 10944*a^2*b^17*c^6*d^13 + 20720*a^2*b^17*c^8*d^11 - 18788*a^2*b^17*c^10*d^9 + 7392*a^2*b^17*c^12*d^7 - 520*a^2*b^17*c^14*d^5 - 24*a^2*b^17*c^16*d^3 - 3456*a^3*b^16*c^3*d^16 + 20016*a^3*b^16*c^5*d^14 - 48112*a^3*b^16*c^7*d^12 + 58925*a^3*b^16*c^9*d^10 - 36732*a^3*b^16*c^11*d^8 + 9736*a^3*b^16*c^13*d^6 - 760*a^3*b^16*c^15*d^4 - 44*a^3*b^16*c^17*d^2 + 2304*a^4*b^15*c^2*d^17 - 23424*a^4*b^15*c^4*d^15 + 81680*a^4*b^15*c^6*d^13 - 135520*a^4*b^15*c^8*d^11 + 114144*a^4*b^15*c^10*d^9 - 44168*a^4*b^15*c^12*d^7 + 5696*a^4*b^15*c^14*d^5 - 332*a^4*b^15*c^16*d^3 + 20016*a^5*b^14*c^3*d^16 - 99112*a^5*b^14*c^5*d^14 + 213338*a^5*b^14*c^7*d^12 - 235152*a^5*b^14*c^9*d^10 + 130428*a^5*b^14*c^11*d^8 - 31908*a^5*b^14*c^13*d^6 + 3966*a^5*b^14*c^15*d^4 - 140*a^5*b^14*c^17*d^2 - 10944*a^6*b^13*c^2*d^17 + 81680*a^6*b^13*c^4*d^15 - 243832*a^6*b^13*c^6*d^13 + 364608*a^6*b^13*c^8*d^11 - 281736*a^6*b^13*c^10*d^9 + 103104*a^6*b^13*c^12*d^7 - 16860*a^6*b^13*c^14*d^5 + 1660*a^6*b^13*c^16*d^3 - 48112*a^7*b^12*c^3*d^16 + 213338*a^7*b^12*c^5*d^14 - 425832*a^7*b^12*c^7*d^12 + 434414*a^7*b^12*c^9*d^10 - 219064*a^7*b^12*c^11*d^8 + 50732*a^7*b^12*c^13*d^6 - 7220*a^7*b^12*c^15*d^4 + 364*a^7*b^12*c^17*d^2 + 20720*a^8*b^11*c^2*d^17 - 135520*a^8*b^11*c^4*d^15 + 364608*a^8*b^11*c^6*d^13 - 496336*a^8*b^11*c^8*d^11 + 343832*a^8*b^11*c^10*d^9 - 111220*a^8*b^11*c^12*d^7 + 17956*a^8*b^11*c^14*d^5 - 1376*a^8*b^11*c^16*d^3 + 58925*a^9*b^10*c^3*d^16 - 235152*a^9*b^10*c^5*d^14 + 434414*a^9*b^10*c^7*d^12 - 401788*a^9*b^10*c^9*d^10 + 172673*a^9*b^10*c^11*d^8 - 31940*a^9*b^10*c^13*d^6 + 3244*a^9*b^10*c^15*d^4 - 18788*a^10*b^9*c^2*d^17 + 114144*a^10*b^9*c^4*d^15 - 281736*a^10*b^9*c^6*d^13 + 343832*a^10*b^9*c^8*d^11 - 197840*a^10*b^9*c^10*d^9 + 45940*a^10*b^9*c^12*d^7 - 4760*a^10*b^9*c^14*d^5 - 36732*a^11*b^8*c^3*d^16 + 130428*a^11*b^8*c^5*d^14 - 219064*a^11*b^8*c^7*d^12 + 172673*a^11*b^8*c^9*d^10 - 52480*a^11*b^8*c^11*d^8 + 4580*a^11*b^8*c^13*d^6 + 7392*a^12*b^7*c^2*d^17 - 44168*a^12*b^7*c^4*d^15 + 103104*a^12*b^7*c^6*d^13 - 111220*a^12*b^7*c^8*d^11 + 45940*a^12*b^7*c^10*d^9 - 4000*a^12*b^7*c^12*d^7 + 9736*

a^13*b^6*c^3*d^16 - 31908*a^13*b^6*c^5*d^14 + 50732*a^13*b^6*c^7*d^12 - 319
40*a^13*b^6*c^9*d^10 + 4580*a^13*b^6*c^11*d^8 - 520*a^14*b^5*c^2*d^17 + 569
6*a^14*b^5*c^4*d^15 - 16860*a^14*b^5*c^6*d^13 + 17956*a^14*b^5*c^8*d^11 - 4
760*a^14*b^5*c^10*d^9 - 760*a^15*b^4*c^3*d^16 + 3966*a^15*b^4*c^5*d^14 - 72
20*a^15*b^4*c^7*d^12 + 3244*a^15*b^4*c^9*d^10 - 24*a^16*b^3*c^2*d^17 - 332*
a^16*b^3*c^4*d^15 + 1660*a^16*b^3*c^6*d^13 - 1376*a^16*b^3*c^8*d^11 - 44*a^
17*b^2*c^3*d^16 - 140*a^17*b^2*c^5*d^14 + 364*a^17*b^2*c^7*d^12))/(a^20*d^2
0 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b
^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d
^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 +
b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a
*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d
^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*
b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19
+ 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*
c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d
^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 -
220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1
168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 224
4*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 15
87*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048
*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744
*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*
a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11
236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 79
2*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27
504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 -
1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 2
0724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8
- 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2
- 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13
+ 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*
d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^1
8 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^
8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*
b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*
b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^1
1*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a
^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^1
2*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*
a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495
*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 2157
6*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6
336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 358
8*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 1

3860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 +
1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 -
8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 2
88*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 40
32*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 41
2*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928
*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^1
8*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^
2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19)) - ((((4*a^24*d^24 + 4*b
^24*c^24 + 16*a^2*b^22*c^24 + 16*a^4*b^20*c^24 - 1152*a^10*b^14*d^24 + 5568
*a^12*b^12*d^24 - 10568*a^14*b^10*d^24 + 9460*a^16*b^8*d^24 - 3560*a^18*b^6
*d^24 + 136*a^20*b^4*d^24 + 76*a^22*b^2*d^24 + 16*a^24*c^2*d^22 + 16*a^24*c
^4*d^20 - 1152*b^24*c^10*d^14 + 5568*b^24*c^12*d^12 - 10568*b^24*c^14*d^10
+ 9460*b^24*c^16*d^8 - 3560*b^24*c^18*d^6 + 136*b^24*c^20*d^4 + 76*b^24*c^2
2*d^2 + 11520*a*b^23*c^9*d^15 - 56448*a*b^23*c^11*d^13 + 109456*a*b^23*c^13
*d^11 - 101240*a*b^23*c^15*d^9 + 40720*a*b^23*c^17*d^7 - 2960*a*b^23*c^19*d
^5 - 536*a*b^23*c^21*d^3 - 176*a^3*b^21*c^23*d - 320*a^5*b^19*c^23*d + 1152
0*a^9*b^15*c*d^23 - 56448*a^11*b^13*c*d^23 + 109456*a^13*b^11*c*d^23 - 1012
40*a^15*b^9*c*d^23 + 40720*a^17*b^7*c*d^23 - 2960*a^19*b^5*c*d^23 - 536*a^2
1*b^3*c*d^23 - 176*a^23*b*c^3*d^21 - 320*a^23*b*c^5*d^19 - 51840*a^2*b^22*c
^8*d^16 + 263808*a^2*b^22*c^10*d^14 - 541208*a^2*b^22*c^12*d^12 + 547088*a^
2*b^22*c^14*d^10 - 263320*a^2*b^22*c^16*d^8 + 44120*a^2*b^22*c^18*d^6 - 156
4*a^2*b^22*c^20*d^4 - 196*a^2*b^22*c^22*d^2 + 138240*a^3*b^21*c^7*d^17 - 75
8400*a^3*b^21*c^9*d^15 + 1720736*a^3*b^21*c^11*d^13 - 2002728*a^3*b^21*c^13
*d^11 + 1210560*a^3*b^21*c^15*d^9 - 335040*a^3*b^21*c^17*d^7 + 37680*a^3*b^
21*c^19*d^5 - 288*a^3*b^21*c^21*d^3 - 241920*a^4*b^20*c^6*d^18 + 1512000*a^
4*b^20*c^8*d^16 - 3975688*a^4*b^20*c^10*d^14 + 5501328*a^4*b^20*c^12*d^12 -
4147952*a^4*b^20*c^14*d^10 + 1586920*a^4*b^20*c^16*d^8 - 276020*a^4*b^20*c
^18*d^6 + 21124*a^4*b^20*c^20*d^4 + 176*a^4*b^20*c^22*d^2 + 290304*a^5*b^19
*c^5*d^19 - 2232576*a^5*b^19*c^7*d^17 + 7078256*a^5*b^19*c^9*d^15 - 1178156
0*a^5*b^19*c^11*d^13 + 10875200*a^5*b^19*c^13*d^11 - 5365072*a^5*b^19*c^15*
d^9 + 1310168*a^5*b^19*c^17*d^7 - 170968*a^5*b^19*c^19*d^5 + 8160*a^5*b^19*
c^21*d^3 - 241920*a^6*b^18*c^4*d^20 + 2532096*a^6*b^18*c^6*d^18 - 9955992*a
^6*b^18*c^8*d^16 + 20019440*a^6*b^18*c^10*d^14 - 22419600*a^6*b^18*c^12*d^1
2 + 13887520*a^6*b^18*c^14*d^10 - 4506428*a^6*b^18*c^16*d^8 + 793756*a^6*b^
18*c^18*d^6 - 72240*a^6*b^18*c^20*d^4 + 3040*a^6*b^18*c^22*d^2 + 138240*a^7
*b^17*c^3*d^21 - 2232576*a^7*b^17*c^5*d^19 + 11150016*a^7*b^17*c^7*d^17 - 2
7336616*a^7*b^17*c^9*d^15 + 37153600*a^7*b^17*c^11*d^13 - 28461040*a^7*b^17
*c^13*d^11 + 11779808*a^7*b^17*c^15*d^9 - 2621008*a^7*b^17*c^17*d^7 + 33668
8*a^7*b^17*c^19*d^5 - 17920*a^7*b^17*c^21*d^3 - 51840*a^8*b^16*c^2*d^22 + 1
512000*a^8*b^16*c^4*d^20 - 9955992*a^8*b^16*c^6*d^18 + 30289656*a^8*b^16*c^
8*d^16 - 50137600*a^8*b^16*c^10*d^14 + 46972560*a^8*b^16*c^12*d^12 - 241992
80*a^8*b^16*c^14*d^10 + 6661036*a^8*b^16*c^16*d^8 - 1058448*a^8*b^16*c^18*d
^6 + 72560*a^8*b^16*c^20*d^4 - 758400*a^9*b^15*c^3*d^21 + 7078256*a^9*b^15*
c^5*d^19 - 27336616*a^9*b^15*c^7*d^17 + 55383904*a^9*b^15*c^9*d^15 - 631240

80*a^9*b^15*c^11*d^13 + 39987520*a^9*b^15*c^13*d^11 - 13462088*a^9*b^15*c^15*d^9 + 2478528*a^9*b^15*c^17*d^7 - 212032*a^9*b^15*c^19*d^5 + 263808*a^10*b^14*c^2*d^22 - 3975688*a^10*b^14*c^4*d^20 + 20019440*a^10*b^14*c^6*d^18 - 50137600*a^10*b^14*c^8*d^16 + 69593872*a^10*b^14*c^10*d^14 - 53854288*a^10*b^14*c^12*d^12 + 21989928*a^10*b^14*c^14*d^10 - 4591360*a^10*b^14*c^16*d^8 + 460480*a^10*b^14*c^18*d^6 + 1720736*a^11*b^13*c^3*d^21 - 11781560*a^11*b^13*c^5*d^19 + 37153600*a^11*b^13*c^7*d^17 - 63124080*a^11*b^13*c^9*d^15 + 59445728*a^11*b^13*c^11*d^13 - 29358696*a^11*b^13*c^13*d^11 + 6995840*a^11*b^13*c^15*d^9 - 762560*a^11*b^13*c^17*d^7 - 541208*a^12*b^12*c^2*d^22 + 5501328*a^12*b^12*c^4*d^20 - 22419600*a^12*b^12*c^6*d^18 + 46972560*a^12*b^12*c^8*d^16 - 53854288*a^12*b^12*c^10*d^14 + 32294808*a^12*b^12*c^12*d^12 - 8958208*a^12*b^12*c^14*d^10 + 999040*a^12*b^12*c^16*d^8 - 2002728*a^13*b^11*c^3*d^21 + 10875200*a^13*b^11*c^5*d^19 - 28461040*a^13*b^11*c^7*d^17 + 39987520*a^13*b^11*c^9*d^15 - 29358696*a^13*b^11*c^11*d^13 + 9722048*a^13*b^11*c^13*d^11 - 1104320*a^13*b^11*c^15*d^9 + 547088*a^14*b^10*c^2*d^22 - 4147952*a^14*b^10*c^4*d^20 + 13887520*a^14*b^10*c^6*d^18 - 24199280*a^14*b^10*c^8*d^16 + 21989928*a^14*b^10*c^10*d^14 - 8958208*a^14*b^10*c^12*d^12 + 1124032*a^14*b^10*c^14*d^10 + 1210560*a^15*b^9*c^3*d^21 - 5365072*a^15*b^9*c^5*d^19 + 11779808*a^15*b^9*c^7*d^17 - 13462088*a^15*b^9*c^9*d^15 + 6995840*a^15*b^9*c^11*d^13 - 1104320*a^15*b^9*c^13*d^11 - 263320*a^16*b^8*c^2*d^22 + 1586920*a^16*b^8*c^4*d^20 - 4506428*a^16*b^8*c^6*d^18 + 6661036*a^16*b^8*c^8*d^16 - 4591360*a^16*b^8*c^10*d^14 + 999040*a^16*b^8*c^12*d^12 - 335040*a^17*b^7*c^3*d^21 + 1310168*a^17*b^7*c^5*d^19 - 2621008*a^17*b^7*c^7*d^17 + 2478528*a^17*b^7*c^9*d^15 - 762560*a^17*b^7*c^11*d^13 + 44120*a^18*b^6*c^2*d^22 - 276020*a^18*b^6*c^4*d^20 + 793756*a^18*b^6*c^6*d^18 - 1058448*a^18*b^6*c^8*d^16 + 460480*a^18*b^6*c^10*d^14 + 37680*a^19*b^5*c^3*d^21 - 170968*a^19*b^5*c^5*d^19 + 336688*a^19*b^5*c^7*d^17 - 212032*a^19*b^5*c^9*d^15 - 1564*a^20*b^4*c^2*d^22 + 21124*a^20*b^4*c^4*d^20 - 72240*a^20*b^4*c^6*d^18 + 72560*a^20*b^4*c^8*d^16 - 288*a^21*b^3*c^3*d^21 + 8160*a^21*b^3*c^5*d^19 - 17920*a^21*b^3*c^7*d^17 - 196*a^22*b^2*c^2*d^22 + 176*a^22*b^2*c^4*d^20 + 3040*a^22*b^2*c^6*d^18 - 8*a*b^23*c^23*d - 8*a^23*b*c*d^23)^2/4 - (20736*b^18*d^18 - 96768*a^2*b^16*d^18 + 173664*a^4*b^14*d^18 - 136032*a^6*b^12*d^18 + 31081*a^8*b^10*d^18 + 8440*a^10*b^8*d^18 + 400*a^12*b^6*d^18 - 96768*b^18*c^2*d^16 + 173664*b^18*c^4*d^14 - 136032*b^18*c^6*d^12 + 31081*b^18*c^8*d^10 + 8440*b^18*c^10*d^8 + 400*b^18*c^12*d^6 - 131328*a*b^17*c^3*d^15 + 216576*a*b^17*c^5*d^13 - 141104*a*b^17*c^7*d^11 + 20260*a*b^17*c^9*d^9 + 2800*a*b^17*c^11*d^7 - 131328*a^3*b^15*c*d^17 + 216576*a^5*b^13*c*d^17 - 141104*a^7*b^11*c*d^17 + 20260*a^9*b^9*c*d^17 + 2800*a^11*b^7*c*d^17 + 495936*a^2*b^16*c^2*d^16 - 989856*a^2*b^16*c^4*d^14 + 901948*a^2*b^16*c^6*d^12 - 308392*a^2*b^16*c^8*d^10 - 5260*a^2*b^16*c^10*d^8 + 1600*a^2*b^16*c^12*d^6 + 657408*a^3*b^15*c^3*d^15 - 1158992*a^3*b^15*c^5*d^13 + 838256*a^3*b^15*c^7*d^11 - 182200*a^3*b^15*c^9*d^9 - 3200*a^3*b^15*c^11*d^7 - 989856*a^4*b^14*c^2*d^16 + 2185654*a^4*b^14*c^4*d^14 - 2218576*a^4*b^14*c^6*d^12 + 900624*a^4*b^14*c^8*d^10 - 64720*a^4*b^14*c^10*d^8 + 1600*a^4*b^14*c^12*d^6 - 1158992*a^5*b^13*c^3*d^15 + 2158808*a^5*b^13*c^5*d^13 - 1641528*a^5*b^13*c^7*d^11 + 40688

0*a^5*b^13*c^9*d^9 - 17600*a^5*b^13*c^11*d^7 + 901948*a^6*b^12*c^2*d^16 - 2
218576*a^6*b^12*c^4*d^14 + 2430936*a^6*b^12*c^6*d^12 - 1026928*a^6*b^12*c^8
*d^10 + 88720*a^6*b^12*c^10*d^8 + 838256*a^7*b^11*c^3*d^15 - 1641528*a^7*b^
11*c^5*d^13 + 1206848*a^7*b^11*c^7*d^11 - 239360*a^7*b^11*c^9*d^9 - 308392*
a^8*b^10*c^2*d^16 + 900624*a^8*b^10*c^4*d^14 - 1026928*a^8*b^10*c^6*d^12 +
354016*a^8*b^10*c^8*d^10 - 182200*a^9*b^9*c^3*d^15 + 406880*a^9*b^9*c^5*d^1
3 - 239360*a^9*b^9*c^7*d^11 - 5260*a^10*b^8*c^2*d^16 - 64720*a^10*b^8*c^4*d
^14 + 88720*a^10*b^8*c^6*d^12 - 3200*a^11*b^7*c^3*d^15 - 17600*a^11*b^7*c^5
*d^13 + 1600*a^12*b^6*c^2*d^16 + 1600*a^12*b^6*c^4*d^14 + 27648*a*b^17*c*d^
17)*(80*a^2*b^28*c^30 - 16*b^30*c^30 - 16*a^30*d^30 - 160*a^4*b^26*c^30 + 1
60*a^6*b^24*c^30 - 80*a^8*b^22*c^30 + 16*a^10*b^20*c^30 + 16*a^20*b^10*d^30
 - 80*a^22*b^8*d^30 + 160*a^24*b^6*d^30 - 160*a^26*b^4*d^30 + 80*a^28*b^2*d
^30 + 80*a^30*c^2*d^28 - 160*a^30*c^4*d^26 + 160*a^30*c^6*d^24 - 80*a^30*c^
8*d^22 + 16*a^30*c^10*d^20 + 16*b^30*c^20*d^10 - 80*b^30*c^22*d^8 + 160*b^3
0*c^24*d^6 - 160*b^30*c^26*d^4 + 80*b^30*c^28*d^2 - 320*a*b^29*c^19*d^11 +
1600*a*b^29*c^21*d^9 - 3200*a*b^29*c^23*d^7 + 3200*a*b^29*c^25*d^5 - 1600*a
*b^29*c^27*d^3 - 1600*a^3*b^27*c^29*d + 3200*a^5*b^25*c^29*d - 3200*a^7*b^2
3*c^29*d + 1600*a^9*b^21*c^29*d - 320*a^11*b^19*c^29*d - 320*a^19*b^11*c*d^
29 + 1600*a^21*b^9*c*d^29 - 3200*a^23*b^7*c*d^29 + 3200*a^25*b^5*c*d^29 - 1
600*a^27*b^3*c*d^29 - 1600*a^29*b*c^3*d^27 + 3200*a^29*b*c^5*d^25 - 3200*a^
29*b*c^7*d^23 + 1600*a^29*b*c^9*d^21 - 320*a^29*b*c^11*d^19 + 3040*a^2*b^28
*c^18*d^12 - 15280*a^2*b^28*c^20*d^10 + 30800*a^2*b^28*c^22*d^8 - 31200*a^2
*b^28*c^24*d^6 + 16000*a^2*b^28*c^26*d^4 - 3440*a^2*b^28*c^28*d^2 - 18240*a
^3*b^27*c^17*d^13 + 92800*a^3*b^27*c^19*d^11 - 190400*a^3*b^27*c^21*d^9 + 1
98400*a^3*b^27*c^23*d^7 - 107200*a^3*b^27*c^25*d^5 + 26240*a^3*b^27*c^27*d^
3 + 77520*a^4*b^26*c^16*d^14 - 402800*a^4*b^26*c^18*d^12 + 851360*a^4*b^26*
c^20*d^10 - 928000*a^4*b^26*c^22*d^8 + 541200*a^4*b^26*c^24*d^6 - 155120*a^
4*b^26*c^26*d^4 + 16000*a^4*b^26*c^28*d^2 - 248064*a^5*b^25*c^15*d^15 + 133
1520*a^5*b^25*c^17*d^13 - 2939840*a^5*b^25*c^19*d^11 + 3408640*a^5*b^25*c^2
1*d^9 - 2184320*a^5*b^25*c^23*d^7 + 736064*a^5*b^25*c^25*d^5 - 107200*a^5*b
^25*c^27*d^3 + 620160*a^6*b^24*c^14*d^16 - 3488400*a^6*b^24*c^16*d^14 + 817
0000*a^6*b^24*c^18*d^12 - 10229760*a^6*b^24*c^20*d^10 + 7281600*a^6*b^24*c^
22*d^8 - 2863760*a^6*b^24*c^24*d^6 + 541200*a^6*b^24*c^26*d^4 - 31200*a^6*b
^24*c^28*d^2 - 1240320*a^7*b^23*c^13*d^17 + 7441920*a^7*b^23*c^15*d^15 - 18
787200*a^7*b^23*c^17*d^13 + 25721600*a^7*b^23*c^19*d^11 - 20444800*a^7*b^23
*c^21*d^9 + 9297920*a^7*b^23*c^23*d^7 - 2184320*a^7*b^23*c^25*d^5 + 198400*
a^7*b^23*c^27*d^3 + 2015520*a^8*b^22*c^12*d^18 - 13178400*a^8*b^22*c^14*d^1
6 + 36434400*a^8*b^22*c^16*d^14 - 55069600*a^8*b^22*c^18*d^12 + 48989680*a^
8*b^22*c^20*d^10 - 25575920*a^8*b^22*c^22*d^8 + 7281600*a^8*b^22*c^24*d^6 -
 928000*a^8*b^22*c^26*d^4 + 30800*a^8*b^22*c^28*d^2 - 2687360*a^9*b^21*c^11
*d^19 + 19638400*a^9*b^21*c^13*d^17 - 60362240*a^9*b^21*c^15*d^15 + 1014752
00*a^9*b^21*c^17*d^13 - 101172800*a^9*b^21*c^19*d^11 + 60333760*a^9*b^21*c^
21*d^9 - 20444800*a^9*b^21*c^23*d^7 + 3408640*a^9*b^21*c^25*d^5 - 190400*a^
9*b^21*c^27*d^3 + 2956096*a^10*b^20*c^10*d^20 - 24858080*a^10*b^20*c^12*d^1
8 + 86150560*a^10*b^20*c^14*d^16 - 162120160*a^10*b^20*c^16*d^14 + 18146368

0*a^10*b^20*c^18*d^12 - 123188112*a^10*b^20*c^20*d^10 + 48989680*a^10*b^20*c^22*d^8 - 10229760*a^10*b^20*c^24*d^6 + 851360*a^10*b^20*c^26*d^4 - 15280*a^10*b^20*c^28*d^2 - 2687360*a^11*b^19*c^9*d^21 + 26873600*a^11*b^19*c^11*d^19 - 106460800*a^11*b^19*c^13*d^17 + 225738240*a^11*b^19*c^15*d^15 - 284331200*a^11*b^19*c^17*d^13 + 219166080*a^11*b^19*c^19*d^11 - 101172800*a^11*b^19*c^21*d^9 + 25721600*a^11*b^19*c^23*d^7 - 2939840*a^11*b^19*c^25*d^5 + 92800*a^11*b^19*c^27*d^3 + 2015520*a^12*b^18*c^8*d^22 - 24858080*a^12*b^18*c^10*d^20 + 114212800*a^12*b^18*c^12*d^18 - 274937600*a^12*b^18*c^14*d^16 + 390830000*a^12*b^18*c^16*d^14 - 341426960*a^12*b^18*c^18*d^12 + 181463680*a^12*b^18*c^20*d^10 - 55069600*a^12*b^18*c^22*d^8 + 8170000*a^12*b^18*c^24*d^6 - 402800*a^12*b^18*c^26*d^4 + 3040*a^12*b^18*c^28*d^2 - 1240320*a^13*b^17*c^7*d^23 + 19638400*a^13*b^17*c^9*d^21 - 106460800*a^13*b^17*c^11*d^19 + 293542400*a^13*b^17*c^13*d^17 - 472561920*a^13*b^17*c^15*d^15 + 467412160*a^13*b^17*c^17*d^13 - 284331200*a^13*b^17*c^19*d^11 + 101475200*a^13*b^17*c^21*d^9 - 18787200*a^13*b^17*c^23*d^7 + 1331520*a^13*b^17*c^25*d^5 - 18240*a^13*b^17*c^27*d^3 + 620160*a^14*b^16*c^6*d^24 - 13178400*a^14*b^16*c^8*d^22 + 86150560*a^14*b^16*c^10*d^20 - 274937600*a^14*b^16*c^12*d^18 + 503363200*a^14*b^16*c^14*d^16 - 563751280*a^14*b^16*c^16*d^14 + 390830000*a^14*b^16*c^18*d^12 - 162120160*a^14*b^16*c^20*d^10 + 36434400*a^14*b^16*c^22*d^8 - 3488400*a^14*b^16*c^24*d^6 + 77520*a^14*b^16*c^26*d^4 - 248064*a^15*b^15*c^5*d^25 + 7441920*a^15*b^15*c^7*d^23 - 60362240*a^15*b^15*c^9*d^21 + 225738240*a^15*b^15*c^11*d^19 - 472561920*a^15*b^15*c^13*d^17 + 599984128*a^15*b^15*c^15*d^15 - 472561920*a^15*b^15*c^17*d^13 + 225738240*a^15*b^15*c^19*d^11 - 60362240*a^15*b^15*c^21*d^9 + 7441920*a^15*b^15*c^23*d^7 - 248064*a^15*b^15*c^25*d^5 + 77520*a^16*b^14*c^4*d^26 - 3488400*a^16*b^14*c^6*d^24 + 36434400*a^16*b^14*c^8*d^22 - 162120160*a^16*b^14*c^10*d^20 + 390830000*a^16*b^14*c^12*d^18 - 563751280*a^16*b^14*c^14*d^16 + 503363200*a^16*b^14*c^16*d^14 - 274937600*a^16*b^14*c^18*d^12 + 86150560*a^16*b^14*c^20*d^10 - 13178400*a^16*b^14*c^22*d^8 + 620160*a^16*b^14*c^24*d^6 - 18240*a^17*b^13*c^3*d^27 + 1331520*a^17*b^13*c^5*d^25 - 18787200*a^17*b^13*c^7*d^23 + 101475200*a^17*b^13*c^9*d^21 - 284331200*a^17*b^13*c^11*d^19 + 467412160*a^17*b^13*c^13*d^17 - 472561920*a^17*b^13*c^15*d^15 + 293542400*a^17*b^13*c^17*d^13 - 106460800*a^17*b^13*c^19*d^11 + 19638400*a^17*b^13*c^21*d^9 - 1240320*a^17*b^13*c^23*d^7 + 3040*a^18*b^12*c^2*d^28 - 402800*a^18*b^12*c^4*d^26 + 8170000*a^18*b^12*c^6*d^24 - 55069600*a^18*b^12*c^8*d^22 + 181463680*a^18*b^12*c^10*d^20 - 341426960*a^18*b^12*c^12*d^18 + 390830000*a^18*b^12*c^14*d^16 - 274937600*a^18*b^12*c^16*d^14 + 114212800*a^18*b^12*c^18*d^12 - 24858080*a^18*b^12*c^20*d^10 + 2015520*a^18*b^12*c^22*d^8 + 92800*a^19*b^11*c^3*d^27 - 2939840*a^19*b^11*c^5*d^25 + 25721600*a^19*b^11*c^7*d^23 - 101172800*a^19*b^11*c^9*d^21 + 219166080*a^19*b^11*c^11*d^19 - 284331200*a^19*b^11*c^13*d^17 + 225738240*a^19*b^11*c^15*d^15 - 106460800*a^19*b^11*c^17*d^13 + 26873600*a^19*b^11*c^19*d^11 - 2687360*a^19*b^11*c^21*d^9 - 15280*a^20*b^10*c^2*d^28 + 851360*a^20*b^10*c^4*d^26 - 10229760*a^20*b^10*c^6*d^24 + 48989680*a^20*b^10*c^8*d^22 - 123188112*a^20*b^10*c^10*d^20 + 181463680*a^20*b^10*c^12*d^18 - 162120160*a^20*b^10*c^14*d^16 + 86150560*a^20*b^10*c^16*d^14 - 24858080*a

^20*b^10*c^18*d^12 + 2956096*a^20*b^10*c^20*d^10 - 190400*a^21*b^9*c^3*d^27 + 3408640*a^21*b^9*c^5*d^25 - 20444800*a^21*b^9*c^7*d^23 + 60333760*a^21*b^9*c^9*d^21 - 101172800*a^21*b^9*c^11*d^19 + 101475200*a^21*b^9*c^13*d^17 - 60362240*a^21*b^9*c^15*d^15 + 19638400*a^21*b^9*c^17*d^13 - 2687360*a^21*b^9*c^19*d^11 + 30800*a^22*b^8*c^2*d^28 - 928000*a^22*b^8*c^4*d^26 + 7281600*a^22*b^8*c^6*d^24 - 25575920*a^22*b^8*c^8*d^22 + 48989680*a^22*b^8*c^10*d^20 - 55069600*a^22*b^8*c^12*d^18 + 36434400*a^22*b^8*c^14*d^16 - 13178400*a^22*b^8*c^16*d^14 + 2015520*a^22*b^8*c^18*d^12 + 198400*a^23*b^7*c^3*d^27 - 2184320*a^23*b^7*c^5*d^25 + 9297920*a^23*b^7*c^7*d^23 - 20444800*a^23*b^7*c^9*d^21 + 25721600*a^23*b^7*c^11*d^19 - 18787200*a^23*b^7*c^13*d^17 + 7441920*a^23*b^7*c^15*d^15 - 1240320*a^23*b^7*c^17*d^13 - 31200*a^24*b^6*c^2*d^28 + 541200*a^24*b^6*c^4*d^26 - 2863760*a^24*b^6*c^6*d^24 + 7281600*a^24*b^6*c^8*d^22 - 10229760*a^24*b^6*c^10*d^20 + 8170000*a^24*b^6*c^12*d^18 - 3488400*a^24*b^6*c^14*d^16 + 620160*a^24*b^6*c^16*d^14 - 107200*a^25*b^5*c^3*d^27 + 736064*a^25*b^5*c^5*d^25 - 2184320*a^25*b^5*c^7*d^23 + 3408640*a^25*b^5*c^9*d^21 - 2939840*a^25*b^5*c^11*d^19 + 1331520*a^25*b^5*c^13*d^17 - 248064*a^25*b^5*c^15*d^15 + 16000*a^26*b^4*c^2*d^28 - 155120*a^26*b^4*c^4*d^26 + 541200*a^26*b^4*c^6*d^24 - 928000*a^26*b^4*c^8*d^22 + 851360*a^26*b^4*c^10*d^20 - 402800*a^26*b^4*c^12*d^18 + 77520*a^26*b^4*c^14*d^16 + 26240*a^27*b^3*c^3*d^27 - 107200*a^27*b^3*c^5*d^25 + 198400*a^27*b^3*c^7*d^23 - 190400*a^27*b^3*c^9*d^21 + 92800*a^27*b^3*c^11*d^19 - 18240*a^27*b^3*c^13*d^17 - 3440*a^28*b^2*c^2*d^28 + 16000*a^28*b^2*c^4*d^26 - 31200*a^28*b^2*c^6*d^24 + 30800*a^28*b^2*c^8*d^22 - 15280*a^28*b^2*c^10*d^20 + 3040*a^28*b^2*c^12*d^18 + 320*a*b^29*c^29*d + 320*a^29*b*c*d^29))^(1/2) - 2*a^24*d^24 - 2*b^24*c^24 - 8*a^2*b^22*c^24 - 8*a^4*b^20*c^24 + 576*a^10*b^14*d^24 - 2784*a^12*b^12*d^24 + 5284*a^14*b^10*d^24 - 4730*a^16*b^8*d^24 + 1780*a^18*b^6*d^24 - 68*a^20*b^4*d^24 - 38*a^22*b^2*d^24 - 8*a^24*c^2*d^22 - 8*a^24*c^4*d^20 + 576*b^24*c^10*d^14 - 2784*b^24*c^12*d^12 + 5284*b^24*c^14*d^10 - 4730*b^24*c^16*d^8 + 1780*b^24*c^18*d^6 - 68*b^24*c^20*d^4 - 38*b^24*c^22*d^2 - 5760*a*b^23*c^9*d^15 + 28224*a*b^23*c^11*d^13 - 54728*a*b^23*c^13*d^11 + 50620*a*b^23*c^15*d^9 - 20360*a*b^23*c^17*d^7 + 1480*a*b^23*c^19*d^5 + 268*a*b^23*c^21*d^3 + 88*a^3*b^21*c^23*d + 160*a^5*b^19*c^23*d - 5760*a^9*b^15*c*d^23 + 28224*a^11*b^13*c*d^23 - 54728*a^13*b^11*c*d^23 + 50620*a^15*b^9*c*d^23 - 20360*a^17*b^7*c*d^23 + 1480*a^19*b^5*c*d^23 + 268*a^21*b^3*c*d^23 + 88*a^23*b*c^3*d^21 + 160*a^23*b*c^5*d^19 + 25920*a^2*b^22*c^8*d^16 - 131904*a^2*b^22*c^10*d^14 + 270604*a^2*b^22*c^12*d^12 - 273544*a^2*b^22*c^14*d^10 + 131660*a^2*b^22*c^16*d^8 - 22060*a^2*b^22*c^18*d^6 + 782*a^2*b^22*c^20*d^4 + 98*a^2*b^22*c^22*d^2 - 69120*a^3*b^21*c^7*d^17 + 379200*a^3*b^21*c^9*d^15 - 860368*a^3*b^21*c^11*d^13 + 1001364*a^3*b^21*c^13*d^11 - 605280*a^3*b^21*c^15*d^9 + 167520*a^3*b^21*c^17*d^7 - 18840*a^3*b^21*c^19*d^5 + 144*a^3*b^21*c^21*d^3 + 120960*a^4*b^20*c^6*d^18 - 756000*a^4*b^20*c^8*d^16 + 1987844*a^4*b^20*c^10*d^14 - 2750664*a^4*b^20*c^12*d^12 + 2073976*a^4*b^20*c^14*d^10 - 793460*a^4*b^20*c^16*d^8 + 138010*a^4*b^20*c^18*d^6 - 10562*a^4*b^20*c^20*d^4 - 88*a^4*b^20*c^22*d^2 - 145152*a^5*b^19*c^5*d^19 + 1116288*a^5*b^19*c^7*d^17 - 3539128*a^5*b^19*c^9*d^15 + 5890780*a^5*b^19*c^11*d^13 - 5437600 *a^5*b^19*c^13*d^11 + 2682536*a^5*b^19*c^15*d^9 - 655084*a^5*b^19*c^17*d^7 + 85484*a^5*b^19*c^19*d^5 - 4080*a^5*b^19*c^21*d^3 + 120960*a^6*b^18*c^4*d^20 - 1266048*a^6*b^18*c^6*d^18 + 4977996*a^6*b^18*c^8*d^16 - 10009720*a^6*b^18*c^10*d^14 + 11209800*a^6*b^18*c^12*d^12 - 6943760*a^6*b^18*c^14*d^10 + 2253214*a^6*b^18*c^16*d^8 - 396878*a^6*b^18*c^18*d^6 + 36120*a^6*b^18*c^20*d^4 - 1520*a^6*b^18*c^22*d^2 - 69120*a^7*b^17*c^3*d^21 + 1116288*a^7*b^17*c^5*d^19 - 5575008*a^7*b^17*c^7*d^17 + 13668308*a^7*b^17*c^9*d^15 - 18576800*a^7*b^17*c^11*d^13 + 14230520*a^7*b^17*c^13*d^11 - 5889904*a^7*b^17*c^15*d^9 + 1310504*a^7*b^17*c^17*d^7 - 168344*a^7*b^17*c^19*d^5 + 8960*a^7*b^17*c^21*d^3 + 25920*a^8*b^16*c^2*d^22 - 756000*a^8*b^16*c^4*d^20 + 4977996*a^8*b^16*c^6*d^18 - 15144828*a^8*b^16*c^8*d^16 + 25068800*a^8*b^16*c^10*d^14 - 23486280*a^8*b^16*c^12*d^12 + 12099640*a^8*b^16*c^14*d^10 - 3330518*a^8*b^16*c^16*d^8 + 529224*a^8*b^16*c^18*d^6 - 36280*a^8*b^16*c^20*d^4 + 379200*a^9*b^15*c^3*d^21 - 3539128*a^9*b^15*c^5*d^19 + 13668308*a^9*b^15*c^7*d^17 - 27691952*a^9*b^15*c^9*d^15 + 31562040*a^9*b^15*c^11*d^13 - 19993760*a^9*b^15*c^13*d^11 + 6731044*a^9*b^15*c^15*d^9 - 1239264*a^9*b^15*c^17*d^7 + 106016*a^9*b^15*c^19*d^5 - 131904*a^10*b^14*c^2*d^22 + 1987844*a^10*b^14*c^4*d^20 - 10009720*a^10*b^14*c^6*d^18 + 25068800*a^10*b^14*c^8*d^16 - 34796936*a^10*b^14*c^10*d^14 + 26927144*a^10*b^14*c^12*d^12 - 10994964*a^10*b^14*c^14*d^10 + 2295680*a^10*b^14*c^16*d^8 - 230240*a^10*b^14*c^18*d^6 - 860368*a^11*b^13*c^3*d^21 + 5890780*a^11*b^13*c^5*d^19 - 18576800*a^11*b^13*c^7*d^17 + 31562040*a^11*b^13*c^9*d^15 - 29722864*a^11*b^13*c^11*d^13 + 14679348*a^11*b^13*c^13*d^11 - 3497920*a^11*b^13*c^15*d^9 + 381280*a^11*b^13*c^17*d^7 + 270604*a^12*b^12*c^2*d^22 - 2750664*a^12*b^12*c^4*d^20 + 11209800*a^12*b^12*c^6*d^18 - 23486280*a^12*b^12*c^8*d^16 + 26927144*a^12*b^12*c^10*d^14 - 16147404*a^12*b^12*c^12*d^12 + 4479104*a^12*b^12*c^14*d^10 - 499520*a^12*b^12*c^16*d^8 + 1001364*a^13*b^11*c^3*d^21 - 5437600*a^13*b^11*c^5*d^19 + 14230520*a^13*b^11*c^7*d^17 - 19993760*a^13*b^11*c^9*d^15 + 14679348*a^13*b^11*c^11*d^13 - 4861024*a^13*b^11*c^13*d^11 + 552160*a^13*b^11*c^15*d^9 - 273544*a^14*b^10*c^2*d^22 + 2073976*a^14*b^10*c^4*d^20 - 6943760*a^14*b^10*c^6*d^18 + 12099640*a^14*b^10*c^8*d^16 - 10994964*a^14*b^10*c^10*d^14 + 4479104*a^14*b^10*c^12*d^12 - 562016*a^14*b^10*c^14*d^10 - 605280*a^15*b^9*c^3*d^21 + 2682536*a^15*b^9*c^5*d^19 - 5889904*a^15*b^9*c^7*d^17 + 6731044*a^15*b^9*c^9*d^15 - 3497920*a^15*b^9*c^11*d^13 + 552160*a^15*b^9*c^13*d^11 + 131660*a^16*b^8*c^2*d^22 - 793460*a^16*b^8*c^4*d^20 + 2253214*a^16*b^8*c^6*d^18 - 3330518*a^16*b^8*c^8*d^16 + 2295680*a^16*b^8*c^10*d^14 - 499520*a^16*b^8*c^12*d^12 + 167520*a^17*b^7*c^3*d^21 - 655084*a^17*b^7*c^5*d^19 + 1310504*a^17*b^7*c^7*d^17 - 1239264*a^17*b^7*c^9*d^15 + 381280*a^17*b^7*c^11*d^13 - 22060*a^18*b^6*c^2*d^22 + 138010*a^18*b^6*c^4*d^20 - 396878*a^18*b^6*c^6*d^18 + 529224*a^18*b^6*c^8*d^16 - 230240*a^18*b^6*c^10*d^14 - 18840*a^19*b^5*c^3*d^21 + 85484*a^19*b^5*c^5*d^19 - 168344*a^19*b^5*c^7*d^17 + 106016*a^19*b^5*c^9*d^15 + 782*a^20*b^4*c^2*d^22 - 10562*a^20*b^4*c^4*d^20 + 36120*a^20*b^4*c^6*d^18 - 36280*a^20*b^4*c^8*d^16 + 144*a^21*b^3*c^3*d^21 - 4080*a^21*b^3*c^5*d^19 + 8960*a^21*b^3*c^7*d^17 + 98*a^22*b^2*c^2*d^22 - 88*a^22*b^2*c^4*d^20 - 1520*a^22*b^2*c^6*d^18 + 4*a*b^23*c^23*d + 4*a^23*b*c*d^23)/(16*

(5*a^2*b^28*c^30 - b^30*c^30 - a^30*d^30 - 10*a^4*b^26*c^30 + 10*a^6*b^24*c
^30 - 5*a^8*b^22*c^30 + a^10*b^20*c^30 + a^20*b^10*d^30 - 5*a^22*b^8*d^30 +
 10*a^24*b^6*d^30 - 10*a^26*b^4*d^30 + 5*a^28*b^2*d^30 + 5*a^30*c^2*d^28 -
10*a^30*c^4*d^26 + 10*a^30*c^6*d^24 - 5*a^30*c^8*d^22 + a^30*c^10*d^20 + b^
30*c^20*d^10 - 5*b^30*c^22*d^8 + 10*b^30*c^24*d^6 - 10*b^30*c^26*d^4 + 5*b^
30*c^28*d^2 - 20*a*b^29*c^19*d^11 + 100*a*b^29*c^21*d^9 - 200*a*b^29*c^23*d
^7 + 200*a*b^29*c^25*d^5 - 100*a*b^29*c^27*d^3 - 100*a^3*b^27*c^29*d + 200*
a^5*b^25*c^29*d - 200*a^7*b^23*c^29*d + 100*a^9*b^21*c^29*d - 20*a^11*b^19*
c^29*d - 20*a^19*b^11*c*d^29 + 100*a^21*b^9*c*d^29 - 200*a^23*b^7*c*d^29 +
200*a^25*b^5*c*d^29 - 100*a^27*b^3*c*d^29 - 100*a^29*b*c^3*d^27 + 200*a^29*
b*c^5*d^25 - 200*a^29*b*c^7*d^23 + 100*a^29*b*c^9*d^21 - 20*a^29*b*c^11*d^1
9 + 190*a^2*b^28*c^18*d^12 - 955*a^2*b^28*c^20*d^10 + 1925*a^2*b^28*c^22*d^
8 - 1950*a^2*b^28*c^24*d^6 + 1000*a^2*b^28*c^26*d^4 - 215*a^2*b^28*c^28*d^2
 - 1140*a^3*b^27*c^17*d^13 + 5800*a^3*b^27*c^19*d^11 - 11900*a^3*b^27*c^21*
d^9 + 12400*a^3*b^27*c^23*d^7 - 6700*a^3*b^27*c^25*d^5 + 1640*a^3*b^27*c^27
*d^3 + 4845*a^4*b^26*c^16*d^14 - 25175*a^4*b^26*c^18*d^12 + 53210*a^4*b^26*
c^20*d^10 - 58000*a^4*b^26*c^22*d^8 + 33825*a^4*b^26*c^24*d^6 - 9695*a^4*b^
26*c^26*d^4 + 1000*a^4*b^26*c^28*d^2 - 15504*a^5*b^25*c^15*d^15 + 83220*a^5
*b^25*c^17*d^13 - 183740*a^5*b^25*c^19*d^11 + 213040*a^5*b^25*c^21*d^9 - 13
6520*a^5*b^25*c^23*d^7 + 46004*a^5*b^25*c^25*d^5 - 6700*a^5*b^25*c^27*d^3 +
 38760*a^6*b^24*c^14*d^16 - 218025*a^6*b^24*c^16*d^14 + 510625*a^6*b^24*c^1
8*d^12 - 639360*a^6*b^24*c^20*d^10 + 455100*a^6*b^24*c^22*d^8 - 178985*a^6*
b^24*c^24*d^6 + 33825*a^6*b^24*c^26*d^4 - 1950*a^6*b^24*c^28*d^2 - 77520*a^
7*b^23*c^13*d^17 + 465120*a^7*b^23*c^15*d^15 - 1174200*a^7*b^23*c^17*d^13 +
 1607600*a^7*b^23*c^19*d^11 - 1277800*a^7*b^23*c^21*d^9 + 581120*a^7*b^23*c
^23*d^7 - 136520*a^7*b^23*c^25*d^5 + 12400*a^7*b^23*c^27*d^3 + 125970*a^8*b
^22*c^12*d^18 - 823650*a^8*b^22*c^14*d^16 + 2277150*a^8*b^22*c^16*d^14 - 34
41850*a^8*b^22*c^18*d^12 + 3061855*a^8*b^22*c^20*d^10 - 1598495*a^8*b^22*c^
22*d^8 + 455100*a^8*b^22*c^24*d^6 - 58000*a^8*b^22*c^26*d^4 + 1925*a^8*b^22
*c^28*d^2 - 167960*a^9*b^21*c^11*d^19 + 1227400*a^9*b^21*c^13*d^17 - 377264
0*a^9*b^21*c^15*d^15 + 6342200*a^9*b^21*c^17*d^13 - 6323300*a^9*b^21*c^19*d
^11 + 3770860*a^9*b^21*c^21*d^9 - 1277800*a^9*b^21*c^23*d^7 + 213040*a^9*b^
21*c^25*d^5 - 11900*a^9*b^21*c^27*d^3 + 184756*a^10*b^20*c^10*d^20 - 155363
0*a^10*b^20*c^12*d^18 + 5384410*a^10*b^20*c^14*d^16 - 10132510*a^10*b^20*c^
16*d^14 + 11341480*a^10*b^20*c^18*d^12 - 7699257*a^10*b^20*c^20*d^10 + 3061
855*a^10*b^20*c^22*d^8 - 639360*a^10*b^20*c^24*d^6 + 53210*a^10*b^20*c^26*d
^4 - 955*a^10*b^20*c^28*d^2 - 167960*a^11*b^19*c^9*d^21 + 1679600*a^11*b^19
*c^11*d^19 - 6653800*a^11*b^19*c^13*d^17 + 14108640*a^11*b^19*c^15*d^15 - 1
7770700*a^11*b^19*c^17*d^13 + 13697880*a^11*b^19*c^19*d^11 - 6323300*a^11*b
^19*c^21*d^9 + 1607600*a^11*b^19*c^23*d^7 - 183740*a^11*b^19*c^25*d^5 + 580
0*a^11*b^19*c^27*d^3 + 125970*a^12*b^18*c^8*d^22 - 1553630*a^12*b^18*c^10*d
^20 + 7138300*a^12*b^18*c^12*d^18 - 17183600*a^12*b^18*c^14*d^16 + 24426875
*a^12*b^18*c^16*d^14 - 21339185*a^12*b^18*c^18*d^12 + 11341480*a^12*b^18*c^
20*d^10 - 3441850*a^12*b^18*c^22*d^8 + 510625*a^12*b^18*c^24*d^6 - 25175*a^
12*b^18*c^26*d^4 + 190*a^12*b^18*c^28*d^2 - 77520*a^13*b^17*c^7*d^23 + 1227

400*a^13*b^17*c^9*d^21 - 6653800*a^13*b^17*c^11*d^19 + 18346400*a^13*b^17*c^13*d^17 - 29535120*a^13*b^17*c^15*d^15 + 29213260*a^13*b^17*c^17*d^13 - 17770700*a^13*b^17*c^19*d^11 + 6342200*a^13*b^17*c^21*d^9 - 1174200*a^13*b^17*c^23*d^7 + 83220*a^13*b^17*c^25*d^5 - 1140*a^13*b^17*c^27*d^3 + 38760*a^14*b^16*c^6*d^24 - 823650*a^14*b^16*c^8*d^22 + 5384410*a^14*b^16*c^10*d^20 - 17183600*a^14*b^16*c^12*d^18 + 31460200*a^14*b^16*c^14*d^16 - 35234455*a^14*b^16*c^16*d^14 + 24426875*a^14*b^16*c^18*d^12 - 10132510*a^14*b^16*c^20*d^10 + 2277150*a^14*b^16*c^22*d^8 - 218025*a^14*b^16*c^24*d^6 + 4845*a^14*b^16*c^26*d^4 - 15504*a^15*b^15*c^5*d^25 + 465120*a^15*b^15*c^7*d^23 - 3772640*a^15*b^15*c^9*d^21 + 14108640*a^15*b^15*c^11*d^19 - 29535120*a^15*b^15*c^13*d^17 + 37499008*a^15*b^15*c^15*d^15 - 29535120*a^15*b^15*c^17*d^13 + 14108640*a^15*b^15*c^19*d^11 - 3772640*a^15*b^15*c^21*d^9 + 465120*a^15*b^15*c^23*d^7 - 15504*a^15*b^15*c^25*d^5 + 4845*a^16*b^14*c^4*d^26 - 218025*a^16*b^14*c^6*d^24 + 2277150*a^16*b^14*c^8*d^22 - 10132510*a^16*b^14*c^10*d^20 + 24426875*a^16*b^14*c^12*d^18 - 35234455*a^16*b^14*c^14*d^16 + 31460200*a^16*b^14*c^16*d^14 - 17183600*a^16*b^14*c^18*d^12 + 5384410*a^16*b^14*c^20*d^10 - 823650*a^16*b^14*c^22*d^8 + 38760*a^16*b^14*c^24*d^6 - 1140*a^17*b^13*c^3*d^27 + 83220*a^17*b^13*c^5*d^25 - 1174200*a^17*b^13*c^7*d^23 + 6342200*a^17*b^13*c^9*d^21 - 17770700*a^17*b^13*c^11*d^19 + 29213260*a^17*b^13*c^13*d^17 - 29535120*a^17*b^13*c^15*d^15 + 18346400*a^17*b^13*c^17*d^13 - 6653800*a^17*b^13*c^19*d^11 + 1227400*a^17*b^13*c^21*d^9 - 77520*a^17*b^13*c^23*d^7 + 190*a^18*b^12*c^2*d^28 - 25175*a^18*b^12*c^4*d^26 + 510625*a^18*b^12*c^6*d^24 - 3441850*a^18*b^12*c^8*d^22 + 11341480*a^18*b^12*c^10*d^20 - 21339185*a^18*b^12*c^12*d^18 + 24426875*a^18*b^12*c^14*d^16 - 17183600*a^18*b^12*c^16*d^14 + 7138300*a^18*b^12*c^18*d^12 - 1553630*a^18*b^12*c^20*d^10 + 125970*a^18*b^12*c^22*d^8 + 5800*a^19*b^11*c^3*d^27 - 183740*a^19*b^11*c^5*d^25 + 1607600*a^19*b^11*c^7*d^23 - 6323300*a^19*b^11*c^9*d^21 + 13697880*a^19*b^11*c^11*d^19 - 17770700*a^19*b^11*c^13*d^17 + 14108640*a^19*b^11*c^15*d^15 - 6653800*a^19*b^11*c^17*d^13 + 1679600*a^19*b^11*c^19*d^11 - 167960*a^19*b^11*c^21*d^9 - 955*a^20*b^10*c^2*d^28 + 53210*a^20*b^10*c^4*d^26 - 639360*a^20*b^10*c^6*d^24 + 3061855*a^20*b^10*c^8*d^22 - 7699257*a^20*b^10*c^10*d^20 + 11341480*a^20*b^10*c^12*d^18 - 10132510*a^20*b^10*c^14*d^16 + 5384410*a^20*b^10*c^16*d^14 - 1553630*a^20*b^10*c^18*d^12 + 184756*a^20*b^10*c^20*d^10 - 11900*a^21*b^9*c^3*d^27 + 213040*a^21*b^9*c^5*d^25 - 1277800*a^21*b^9*c^7*d^23 + 3770860*a^21*b^9*c^9*d^21 - 6323300*a^21*b^9*c^11*d^19 + 6342200*a^21*b^9*c^13*d^17 - 3772640*a^21*b^9*c^15*d^15 + 1227400*a^21*b^9*c^17*d^13 - 167960*a^21*b^9*c^19*d^11 + 1925*a^22*b^8*c^2*d^28 - 58000*a^22*b^8*c^4*d^26 + 455100*a^22*b^8*c^6*d^24 - 1598495*a^22*b^8*c^8*d^22 + 3061855*a^22*b^8*c^10*d^20 - 3441850*a^22*b^8*c^12*d^18 + 2277150*a^22*b^8*c^14*d^16 - 823650*a^22*b^8*c^16*d^14 + 125970*a^22*b^8*c^18*d^12 + 12400*a^23*b^7*c^3*d^27 - 136520*a^23*b^7*c^5*d^25 + 581120*a^23*b^7*c^7*d^23 - 1277800*a^23*b^7*c^9*d^21 + 1607600*a^23*b^7*c^11*d^19 - 1174200*a^23*b^7*c^13*d^17 + 465120*a^23*b^7*c^15*d^15 - 77520*a^23*b^7*c^17*d^13 - 1950*a^24*b^6*c^2*d^28 + 33825*a^24*b^6*c^4*d^26 - 178985*a^24*b^6*c^6*d^24 + 455100*a^24*b^6*c^8*d^22 - 639360*a^24*b^6*c^10*d^20 + 510625*a^24*b^6*c^12*d^18 - 218025* a^24*b^6*c^14*d^16 + 38760*a^24*b^6*c^16*d^14 - 6700*a^25*b^5*c^3*d^27 + 46004*a^25*b^5*c^5*d^25 - 136520*a^25*b^5*c^7*d^23 + 213040*a^25*b^5*c^9*d^21 - 183740*a^25*b^5*c^11*d^19 + 83220*a^25*b^5*c^13*d^17 - 15504*a^25*b^5*c^15*d^15 + 1000*a^26*b^4*c^2*d^28 - 9695*a^26*b^4*c^4*d^26 + 33825*a^26*b^4*c^6*d^24 - 58000*a^26*b^4*c^8*d^22 + 53210*a^26*b^4*c^10*d^20 - 25175*a^26*b^4*c^12*d^18 + 4845*a^26*b^4*c^14*d^16 + 1640*a^27*b^3*c^3*d^27 - 6700*a^27*b^3*c^5*d^25 + 12400*a^27*b^3*c^7*d^23 - 11900*a^27*b^3*c^9*d^21 + 5800*a^27*b^3*c^11*d^19 - 1140*a^27*b^3*c^13*d^17 - 215*a^28*b^2*c^2*d^28 + 1000*a^28*b^2*c^4*d^26 - 1950*a^28*b^2*c^6*d^24 + 1925*a^28*b^2*c^8*d^22 - 955*a^28*b^2*c^10*d^20 + 190*a^28*b^2*c^12*d^18 + 20*a*b^29*c^29*d + 20*a^29*b*c*d^29)))^(1/2)*(((((4*a^24*d^24 + 4*b^24*c^24 + 16*a^2*b^22*c^24 + 16*a^4*b^20*c^24 - 1152*a^10*b^14*d^24 + 5568*a^12*b^12*d^24 - 10568*a^14*b^10*d^24 + 9460*a^16*b^8*d^24 - 3560*a^18*b^6*d^24 + 136*a^20*b^4*d^24 + 76*a^22*b^2*d^24 + 16*a^24*c^2*d^22 + 16*a^24*c^4*d^20 - 1152*b^24*c^10*d^14 + 5568*b^24*c^12*d^12 - 10568*b^24*c^14*d^10 + 9460*b^24*c^16*d^8 - 3560*b^24*c^18*d^6 + 136*b^24*c^20*d^4 + 76*b^24*c^22*d^2 + 11520*a*b^23*c^9*d^15 - 56448*a*b^23*c^11*d^13 + 109456*a*b^23*c^13*d^11 - 101240*a*b^23*c^15*d^9 + 40720*a*b^23*c^17*d^7 - 2960*a*b^23*c^19*d^5 - 536*a*b^23*c^21*d^3 - 176*a^3*b^21*c^23*d - 320*a^5*b^19*c^23*d + 11520*a^9*b^15*c*d^23 - 56448*a^11*b^13*c*d^23 + 109456*a^13*b^11*c*d^23 - 101240*a^15*b^9*c*d^23 + 40720*a^17*b^7*c*d^23 - 2960*a^19*b^5*c*d^23 - 536*a^21*b^3*c*d^23 - 176*a^23*b*c^3*d^21 - 320*a^23*b*c^5*d^19 - 51840*a^2*b^22*c^8*d^16 + 263808*a^2*b^22*c^10*d^14 - 541208*a^2*b^22*c^12*d^12 + 547088*a^2*b^22*c^14*d^10 - 263320*a^2*b^22*c^16*d^8 + 44120*a^2*b^22*c^18*d^6 - 1564*a^2*b^22*c^20*d^4 - 196*a^2*b^22*c^22*d^2 + 138240*a^3*b^21*c^7*d^17 - 758400*a^3*b^21*c^9*d^15 + 1720736*a^3*b^21*c^11*d^13 - 2002728*a^3*b^21*c^13*d^11 + 1210560*a^3*b^21*c^15*d^9 - 335040*a^3*b^21*c^17*d^7 + 37680*a^3*b^21*c^19*d^5 - 288*a^3*b^21*c^21*d^3 - 241920*a^4*b^20*c^6*d^18 + 1512000*a^4*b^20*c^8*d^16 - 3975688*a^4*b^20*c^10*d^14 + 5501328*a^4*b^20*c^12*d^12 - 4147952*a^4*b^20*c^14*d^10 + 1586920*a^4*b^20*c^16*d^8 - 276020*a^4*b^20*c^18*d^6 + 21124*a^4*b^20*c^20*d^4 + 176*a^4*b^20*c^22*d^2 + 290304*a^5*b^19*c^5*d^19 - 2232576*a^5*b^19*c^7*d^17 + 7078256*a^5*b^19*c^9*d^15 - 11781560*a^5*b^19*c^11*d^13 + 10875200*a^5*b^19*c^13*d^11 - 5365072*a^5*b^19*c^15*d^9 + 1310168*a^5*b^19*c^17*d^7 - 170968*a^5*b^19*c^19*d^5 + 8160*a^5*b^19*c^21*d^3 - 241920*a^6*b^18*c^4*d^20 + 2532096*a^6*b^18*c^6*d^18 - 9955992*a^6*b^18*c^8*d^16 + 20019440*a^6*b^18*c^10*d^14 - 22419600*a^6*b^18*c^12*d^12 + 13887520*a^6*b^18*c^14*d^10 - 4506428*a^6*b^18*c^16*d^8 + 793756*a^6*b^18*c^18*d^6 - 72240*a^6*b^18*c^20*d^4 + 3040*a^6*b^18*c^22*d^2 + 138240*a^7*b^17*c^3*d^21 - 2232576*a^7*b^17*c^5*d^19 + 11150016*a^7*b^17*c^7*d^17 - 27336616*a^7*b^17*c^9*d^15 + 37153600*a^7*b^17*c^11*d^13 - 28461040*a^7*b^17*c^13*d^11 + 11779808*a^7*b^17*c^15*d^9 - 2621008*a^7*b^17*c^17*d^7 + 336688*a^7*b^17*c^19*d^5 - 17920*a^7*b^17*c^21*d^3 - 51840*a^8*b^16*c^2*d^22 + 1512000*a^8*b^16*c^4*d^20 - 9955992*a^8*b^16*c^6*d^18 + 30289656*a^8*b^16*c^8*d^16 - 50137600*a^8*b^16*c^10*d^14 + 46972560*a^8*b^16*c^12*d^12 - 24199280*a^8*b^16*c^14*d^10 + 6661036*a^8*b^16*c^16*d^8 - 1058448*a^8*b^16*c^18*d^6 + 72560*a^8*b^16*c^20*d^4 - 758400* a^9*b^15*c^3*d^21 + 7078256*a^9*b^15*c^5*d^19 - 27336616*a^9*b^15*c^7*d^17 + 55383904*a^9*b^15*c^9*d^15 - 63124080*a^9*b^15*c^11*d^13 + 39987520*a^9*b^15*c^13*d^11 - 13462088*a^9*b^15*c^15*d^9 + 2478528*a^9*b^15*c^17*d^7 - 212032*a^9*b^15*c^19*d^5 + 263808*a^10*b^14*c^2*d^22 - 3975688*a^10*b^14*c^4*d^20 + 20019440*a^10*b^14*c^6*d^18 - 50137600*a^10*b^14*c^8*d^16 + 69593872*a^10*b^14*c^10*d^14 - 53854288*a^10*b^14*c^12*d^12 + 21989928*a^10*b^14*c^14*d^10 - 4591360*a^10*b^14*c^16*d^8 + 460480*a^10*b^14*c^18*d^6 + 1720736*a^11*b^13*c^3*d^21 - 11781560*a^11*b^13*c^5*d^19 + 37153600*a^11*b^13*c^7*d^17 - 63124080*a^11*b^13*c^9*d^15 + 59445728*a^11*b^13*c^11*d^13 - 29358696*a^11*b^13*c^13*d^11 + 6995840*a^11*b^13*c^15*d^9 - 762560*a^11*b^13*c^17*d^7 - 541208*a^12*b^12*c^2*d^22 + 5501328*a^12*b^12*c^4*d^20 - 22419600*a^12*b^12*c^6*d^18 + 46972560*a^12*b^12*c^8*d^16 - 53854288*a^12*b^12*c^10*d^14 + 32294808*a^12*b^12*c^12*d^12 - 8958208*a^12*b^12*c^14*d^10 + 999040*a^12*b^12*c^16*d^8 - 2002728*a^13*b^11*c^3*d^21 + 10875200*a^13*b^11*c^5*d^19 - 28461040*a^13*b^11*c^7*d^17 + 39987520*a^13*b^11*c^9*d^15 - 29358696*a^13*b^11*c^11*d^13 + 9722048*a^13*b^11*c^13*d^11 - 1104320*a^13*b^11*c^15*d^9 + 547088*a^14*b^10*c^2*d^22 - 4147952*a^14*b^10*c^4*d^20 + 13887520*a^14*b^10*c^6*d^18 - 24199280*a^14*b^10*c^8*d^16 + 21989928*a^14*b^10*c^10*d^14 - 8958208*a^14*b^10*c^12*d^12 + 1124032*a^14*b^10*c^14*d^10 + 1210560*a^15*b^9*c^3*d^21 - 5365072*a^15*b^9*c^5*d^19 + 11779808*a^15*b^9*c^7*d^17 - 13462088*a^15*b^9*c^9*d^15 + 6995840*a^15*b^9*c^11*d^13 - 1104320*a^15*b^9*c^13*d^11 - 263320*a^16*b^8*c^2*d^22 + 1586920*a^16*b^8*c^4*d^20 - 4506428*a^16*b^8*c^6*d^18 + 6661036*a^16*b^8*c^8*d^16 - 4591360*a^16*b^8*c^10*d^14 + 999040*a^16*b^8*c^12*d^12 - 335040*a^17*b^7*c^3*d^21 + 1310168*a^17*b^7*c^5*d^19 - 2621008*a^17*b^7*c^7*d^17 + 2478528*a^17*b^7*c^9*d^15 - 762560*a^17*b^7*c^11*d^13 + 44120*a^18*b^6*c^2*d^22 - 276020*a^18*b^6*c^4*d^20 + 793756*a^18*b^6*c^6*d^18 - 1058448*a^18*b^6*c^8*d^16 + 460480*a^18*b^6*c^10*d^14 + 37680*a^19*b^5*c^3*d^21 - 170968*a^19*b^5*c^5*d^19 + 336688*a^19*b^5*c^7*d^17 - 212032*a^19*b^5*c^9*d^15 - 1564*a^20*b^4*c^2*d^22 + 21124*a^20*b^4*c^4*d^20 - 72240*a^20*b^4*c^6*d^18 + 72560*a^20*b^4*c^8*d^16 - 288*a^21*b^3*c^3*d^21 + 8160*a^21*b^3*c^5*d^19 - 17920*a^21*b^3*c^7*d^17 - 196*a^22*b^2*c^2*d^22 + 176*a^22*b^2*c^4*d^20 + 3040*a^22*b^2*c^6*d^18 - 8*a*b^23*c^23*d - 8*a^23*b*c*d^23)^2/4 - (20736*b^18*d^18 - 96768*a^2*b^16*d^18 + 173664*a^4*b^14*d^18 - 136032*a^6*b^12*d^18 + 31081*a^8*b^10*d^18 + 8440*a^10*b^8*d^18 + 400*a^12*b^6*d^18 - 96768*b^18*c^2*d^16 + 173664*b^18*c^4*d^14 - 136032*b^18*c^6*d^12 + 31081*b^18*c^8*d^10 + 8440*b^18*c^10*d^8 + 400*b^18*c^12*d^6 - 131328*a*b^17*c^3*d^15 + 216576*a*b^17*c^5*d^13 - 141104*a*b^17*c^7*d^11 + 20260*a*b^17*c^9*d^9 + 2800*a*b^17*c^11*d^7 - 131328*a^3*b^15*c*d^17 + 216576*a^5*b^13*c*d^17 - 141104*a^7*b^11*c*d^17 + 20260*a^9*b^9*c*d^17 + 2800*a^11*b^7*c*d^17 + 495936*a^2*b^16*c^2*d^16 - 989856*a^2*b^16*c^4*d^14 + 901948*a^2*b^16*c^6*d^12 - 308392*a^2*b^16*c^8*d^10 - 5260*a^2*b^16*c^10*d^8 + 1600*a^2*b^16*c^12*d^6 + 657408*a^3*b^15*c^3*d^15 - 1158992*a^3*b^15*c^5*d^13 + 838256*a^3*b^15*c^7*d^11 - 182200*a^3*b^15*c^9*d^9 - 3200*a^3*b^15*c^11*d^7 - 989856*a^4*b^14*c^2*d^16 + 2185654*a^4*b^14*c^4*d^14 - 2218576*a^4*b^14*c^6*d^12 + 900624*a^4*b^14*c^8*d^10 - 64720*a^4*b^14*c^10*d^8 + 16

00*a^4*b^14*c^12*d^6 - 1158992*a^5*b^13*c^3*d^15 + 2158808*a^5*b^13*c^5*d^1
3 - 1641528*a^5*b^13*c^7*d^11 + 406880*a^5*b^13*c^9*d^9 - 17600*a^5*b^13*c^
11*d^7 + 901948*a^6*b^12*c^2*d^16 - 2218576*a^6*b^12*c^4*d^14 + 2430936*a^6
*b^12*c^6*d^12 - 1026928*a^6*b^12*c^8*d^10 + 88720*a^6*b^12*c^10*d^8 + 8382
56*a^7*b^11*c^3*d^15 - 1641528*a^7*b^11*c^5*d^13 + 1206848*a^7*b^11*c^7*d^1
1 - 239360*a^7*b^11*c^9*d^9 - 308392*a^8*b^10*c^2*d^16 + 900624*a^8*b^10*c^
4*d^14 - 1026928*a^8*b^10*c^6*d^12 + 354016*a^8*b^10*c^8*d^10 - 182200*a^9*
b^9*c^3*d^15 + 406880*a^9*b^9*c^5*d^13 - 239360*a^9*b^9*c^7*d^11 - 5260*a^1
0*b^8*c^2*d^16 - 64720*a^10*b^8*c^4*d^14 + 88720*a^10*b^8*c^6*d^12 - 3200*a
^11*b^7*c^3*d^15 - 17600*a^11*b^7*c^5*d^13 + 1600*a^12*b^6*c^2*d^16 + 1600*
a^12*b^6*c^4*d^14 + 27648*a*b^17*c*d^17)*(80*a^2*b^28*c^30 - 16*b^30*c^30 -
 16*a^30*d^30 - 160*a^4*b^26*c^30 + 160*a^6*b^24*c^30 - 80*a^8*b^22*c^30 +
16*a^10*b^20*c^30 + 16*a^20*b^10*d^30 - 80*a^22*b^8*d^30 + 160*a^24*b^6*d^3
0 - 160*a^26*b^4*d^30 + 80*a^28*b^2*d^30 + 80*a^30*c^2*d^28 - 160*a^30*c^4*
d^26 + 160*a^30*c^6*d^24 - 80*a^30*c^8*d^22 + 16*a^30*c^10*d^20 + 16*b^30*c
^20*d^10 - 80*b^30*c^22*d^8 + 160*b^30*c^24*d^6 - 160*b^30*c^26*d^4 + 80*b^
30*c^28*d^2 - 320*a*b^29*c^19*d^11 + 1600*a*b^29*c^21*d^9 - 3200*a*b^29*c^2
3*d^7 + 3200*a*b^29*c^25*d^5 - 1600*a*b^29*c^27*d^3 - 1600*a^3*b^27*c^29*d
+ 3200*a^5*b^25*c^29*d - 3200*a^7*b^23*c^29*d + 1600*a^9*b^21*c^29*d - 320*
a^11*b^19*c^29*d - 320*a^19*b^11*c*d^29 + 1600*a^21*b^9*c*d^29 - 3200*a^23*
b^7*c*d^29 + 3200*a^25*b^5*c*d^29 - 1600*a^27*b^3*c*d^29 - 1600*a^29*b*c^3*
d^27 + 3200*a^29*b*c^5*d^25 - 3200*a^29*b*c^7*d^23 + 1600*a^29*b*c^9*d^21 -
 320*a^29*b*c^11*d^19 + 3040*a^2*b^28*c^18*d^12 - 15280*a^2*b^28*c^20*d^10
+ 30800*a^2*b^28*c^22*d^8 - 31200*a^2*b^28*c^24*d^6 + 16000*a^2*b^28*c^26*d
^4 - 3440*a^2*b^28*c^28*d^2 - 18240*a^3*b^27*c^17*d^13 + 92800*a^3*b^27*c^1
9*d^11 - 190400*a^3*b^27*c^21*d^9 + 198400*a^3*b^27*c^23*d^7 - 107200*a^3*b
^27*c^25*d^5 + 26240*a^3*b^27*c^27*d^3 + 77520*a^4*b^26*c^16*d^14 - 402800*
a^4*b^26*c^18*d^12 + 851360*a^4*b^26*c^20*d^10 - 928000*a^4*b^26*c^22*d^8 +
 541200*a^4*b^26*c^24*d^6 - 155120*a^4*b^26*c^26*d^4 + 16000*a^4*b^26*c^28*
d^2 - 248064*a^5*b^25*c^15*d^15 + 1331520*a^5*b^25*c^17*d^13 - 2939840*a^5*
b^25*c^19*d^11 + 3408640*a^5*b^25*c^21*d^9 - 2184320*a^5*b^25*c^23*d^7 + 73
6064*a^5*b^25*c^25*d^5 - 107200*a^5*b^25*c^27*d^3 + 620160*a^6*b^24*c^14*d^
16 - 3488400*a^6*b^24*c^16*d^14 + 8170000*a^6*b^24*c^18*d^12 - 10229760*a^6
*b^24*c^20*d^10 + 7281600*a^6*b^24*c^22*d^8 - 2863760*a^6*b^24*c^24*d^6 + 5
41200*a^6*b^24*c^26*d^4 - 31200*a^6*b^24*c^28*d^2 - 1240320*a^7*b^23*c^13*d
^17 + 7441920*a^7*b^23*c^15*d^15 - 18787200*a^7*b^23*c^17*d^13 + 25721600*a
^7*b^23*c^19*d^11 - 20444800*a^7*b^23*c^21*d^9 + 9297920*a^7*b^23*c^23*d^7
- 2184320*a^7*b^23*c^25*d^5 + 198400*a^7*b^23*c^27*d^3 + 2015520*a^8*b^22*c
^12*d^18 - 13178400*a^8*b^22*c^14*d^16 + 36434400*a^8*b^22*c^16*d^14 - 5506
9600*a^8*b^22*c^18*d^12 + 48989680*a^8*b^22*c^20*d^10 - 25575920*a^8*b^22*c
^22*d^8 + 7281600*a^8*b^22*c^24*d^6 - 928000*a^8*b^22*c^26*d^4 + 30800*a^8*
b^22*c^28*d^2 - 2687360*a^9*b^21*c^11*d^19 + 19638400*a^9*b^21*c^13*d^17 -
60362240*a^9*b^21*c^15*d^15 + 101475200*a^9*b^21*c^17*d^13 - 101172800*a^9*
b^21*c^19*d^11 + 60333760*a^9*b^21*c^21*d^9 - 20444800*a^9*b^21*c^23*d^7 +
3408640*a^9*b^21*c^25*d^5 - 190400*a^9*b^21*c^27*d^3 + 2956096*a^10*b^20*c^

10*d^20 - 24858080*a^10*b^20*c^12*d^18 + 86150560*a^10*b^20*c^14*d^16 - 162
120160*a^10*b^20*c^16*d^14 + 181463680*a^10*b^20*c^18*d^12 - 123188112*a^10
*b^20*c^20*d^10 + 48989680*a^10*b^20*c^22*d^8 - 10229760*a^10*b^20*c^24*d^6
 + 851360*a^10*b^20*c^26*d^4 - 15280*a^10*b^20*c^28*d^2 - 2687360*a^11*b^19
*c^9*d^21 + 26873600*a^11*b^19*c^11*d^19 - 106460800*a^11*b^19*c^13*d^17 +
225738240*a^11*b^19*c^15*d^15 - 284331200*a^11*b^19*c^17*d^13 + 219166080*a
^11*b^19*c^19*d^11 - 101172800*a^11*b^19*c^21*d^9 + 25721600*a^11*b^19*c^23
*d^7 - 2939840*a^11*b^19*c^25*d^5 + 92800*a^11*b^19*c^27*d^3 + 2015520*a^12
*b^18*c^8*d^22 - 24858080*a^12*b^18*c^10*d^20 + 114212800*a^12*b^18*c^12*d^
18 - 274937600*a^12*b^18*c^14*d^16 + 390830000*a^12*b^18*c^16*d^14 - 341426
960*a^12*b^18*c^18*d^12 + 181463680*a^12*b^18*c^20*d^10 - 55069600*a^12*b^1
8*c^22*d^8 + 8170000*a^12*b^18*c^24*d^6 - 402800*a^12*b^18*c^26*d^4 + 3040*
a^12*b^18*c^28*d^2 - 1240320*a^13*b^17*c^7*d^23 + 19638400*a^13*b^17*c^9*d^
21 - 106460800*a^13*b^17*c^11*d^19 + 293542400*a^13*b^17*c^13*d^17 - 472561
920*a^13*b^17*c^15*d^15 + 467412160*a^13*b^17*c^17*d^13 - 284331200*a^13*b^
17*c^19*d^11 + 101475200*a^13*b^17*c^21*d^9 - 18787200*a^13*b^17*c^23*d^7 +
 1331520*a^13*b^17*c^25*d^5 - 18240*a^13*b^17*c^27*d^3 + 620160*a^14*b^16*c
^6*d^24 - 13178400*a^14*b^16*c^8*d^22 + 86150560*a^14*b^16*c^10*d^20 - 2749
37600*a^14*b^16*c^12*d^18 + 503363200*a^14*b^16*c^14*d^16 - 563751280*a^14*
b^16*c^16*d^14 + 390830000*a^14*b^16*c^18*d^12 - 162120160*a^14*b^16*c^20*d
^10 + 36434400*a^14*b^16*c^22*d^8 - 3488400*a^14*b^16*c^24*d^6 + 77520*a^14
*b^16*c^26*d^4 - 248064*a^15*b^15*c^5*d^25 + 7441920*a^15*b^15*c^7*d^23 - 6
0362240*a^15*b^15*c^9*d^21 + 225738240*a^15*b^15*c^11*d^19 - 472561920*a^15
*b^15*c^13*d^17 + 599984128*a^15*b^15*c^15*d^15 - 472561920*a^15*b^15*c^17*
d^13 + 225738240*a^15*b^15*c^19*d^11 - 60362240*a^15*b^15*c^21*d^9 + 744192
0*a^15*b^15*c^23*d^7 - 248064*a^15*b^15*c^25*d^5 + 77520*a^16*b^14*c^4*d^26
 - 3488400*a^16*b^14*c^6*d^24 + 36434400*a^16*b^14*c^8*d^22 - 162120160*a^1
6*b^14*c^10*d^20 + 390830000*a^16*b^14*c^12*d^18 - 563751280*a^16*b^14*c^14
*d^16 + 503363200*a^16*b^14*c^16*d^14 - 274937600*a^16*b^14*c^18*d^12 + 861
50560*a^16*b^14*c^20*d^10 - 13178400*a^16*b^14*c^22*d^8 + 620160*a^16*b^14*
c^24*d^6 - 18240*a^17*b^13*c^3*d^27 + 1331520*a^17*b^13*c^5*d^25 - 18787200
*a^17*b^13*c^7*d^23 + 101475200*a^17*b^13*c^9*d^21 - 284331200*a^17*b^13*c^
11*d^19 + 467412160*a^17*b^13*c^13*d^17 - 472561920*a^17*b^13*c^15*d^15 + 2
93542400*a^17*b^13*c^17*d^13 - 106460800*a^17*b^13*c^19*d^11 + 19638400*a^1
7*b^13*c^21*d^9 - 1240320*a^17*b^13*c^23*d^7 + 3040*a^18*b^12*c^2*d^28 - 40
2800*a^18*b^12*c^4*d^26 + 8170000*a^18*b^12*c^6*d^24 - 55069600*a^18*b^12*c
^8*d^22 + 181463680*a^18*b^12*c^10*d^20 - 341426960*a^18*b^12*c^12*d^18 + 3
90830000*a^18*b^12*c^14*d^16 - 274937600*a^18*b^12*c^16*d^14 + 114212800*a^
18*b^12*c^18*d^12 - 24858080*a^18*b^12*c^20*d^10 + 2015520*a^18*b^12*c^22*d
^8 + 92800*a^19*b^11*c^3*d^27 - 2939840*a^19*b^11*c^5*d^25 + 25721600*a^19*
b^11*c^7*d^23 - 101172800*a^19*b^11*c^9*d^21 + 219166080*a^19*b^11*c^11*d^1
9 - 284331200*a^19*b^11*c^13*d^17 + 225738240*a^19*b^11*c^15*d^15 - 1064608
00*a^19*b^11*c^17*d^13 + 26873600*a^19*b^11*c^19*d^11 - 2687360*a^19*b^11*c
^21*d^9 - 15280*a^20*b^10*c^2*d^28 + 851360*a^20*b^10*c^4*d^26 - 10229760*a
^20*b^10*c^6*d^24 + 48989680*a^20*b^10*c^8*d^22 - 123188112*a^20*b^10*c^10*

d^20 + 181463680*a^20*b^10*c^12*d^18 - 162120160*a^20*b^10*c^14*d^16 + 8615
0560*a^20*b^10*c^16*d^14 - 24858080*a^20*b^10*c^18*d^12 + 2956096*a^20*b^10
*c^20*d^10 - 190400*a^21*b^9*c^3*d^27 + 3408640*a^21*b^9*c^5*d^25 - 2044480
0*a^21*b^9*c^7*d^23 + 60333760*a^21*b^9*c^9*d^21 - 101172800*a^21*b^9*c^11*
d^19 + 101475200*a^21*b^9*c^13*d^17 - 60362240*a^21*b^9*c^15*d^15 + 1963840
0*a^21*b^9*c^17*d^13 - 2687360*a^21*b^9*c^19*d^11 + 30800*a^22*b^8*c^2*d^28
 - 928000*a^22*b^8*c^4*d^26 + 7281600*a^22*b^8*c^6*d^24 - 25575920*a^22*b^8
*c^8*d^22 + 48989680*a^22*b^8*c^10*d^20 - 55069600*a^22*b^8*c^12*d^18 + 364
34400*a^22*b^8*c^14*d^16 - 13178400*a^22*b^8*c^16*d^14 + 2015520*a^22*b^8*c
^18*d^12 + 198400*a^23*b^7*c^3*d^27 - 2184320*a^23*b^7*c^5*d^25 + 9297920*a
^23*b^7*c^7*d^23 - 20444800*a^23*b^7*c^9*d^21 + 25721600*a^23*b^7*c^11*d^19
 - 18787200*a^23*b^7*c^13*d^17 + 7441920*a^23*b^7*c^15*d^15 - 1240320*a^23*
b^7*c^17*d^13 - 31200*a^24*b^6*c^2*d^28 + 541200*a^24*b^6*c^4*d^26 - 286376
0*a^24*b^6*c^6*d^24 + 7281600*a^24*b^6*c^8*d^22 - 10229760*a^24*b^6*c^10*d^
20 + 8170000*a^24*b^6*c^12*d^18 - 3488400*a^24*b^6*c^14*d^16 + 620160*a^24*
b^6*c^16*d^14 - 107200*a^25*b^5*c^3*d^27 + 736064*a^25*b^5*c^5*d^25 - 21843
20*a^25*b^5*c^7*d^23 + 3408640*a^25*b^5*c^9*d^21 - 2939840*a^25*b^5*c^11*d^
19 + 1331520*a^25*b^5*c^13*d^17 - 248064*a^25*b^5*c^15*d^15 + 16000*a^26*b^
4*c^2*d^28 - 155120*a^26*b^4*c^4*d^26 + 541200*a^26*b^4*c^6*d^24 - 928000*a
^26*b^4*c^8*d^22 + 851360*a^26*b^4*c^10*d^20 - 402800*a^26*b^4*c^12*d^18 +
77520*a^26*b^4*c^14*d^16 + 26240*a^27*b^3*c^3*d^27 - 107200*a^27*b^3*c^5*d^
25 + 198400*a^27*b^3*c^7*d^23 - 190400*a^27*b^3*c^9*d^21 + 92800*a^27*b^3*c
^11*d^19 - 18240*a^27*b^3*c^13*d^17 - 3440*a^28*b^2*c^2*d^28 + 16000*a^28*b
^2*c^4*d^26 - 31200*a^28*b^2*c^6*d^24 + 30800*a^28*b^2*c^8*d^22 - 15280*a^2
8*b^2*c^10*d^20 + 3040*a^28*b^2*c^12*d^18 + 320*a*b^29*c^29*d + 320*a^29*b*
c*d^29))^(1/2) - 2*a^24*d^24 - 2*b^24*c^24 - 8*a^2*b^22*c^24 - 8*a^4*b^20*c
^24 + 576*a^10*b^14*d^24 - 2784*a^12*b^12*d^24 + 5284*a^14*b^10*d^24 - 4730
*a^16*b^8*d^24 + 1780*a^18*b^6*d^24 - 68*a^20*b^4*d^24 - 38*a^22*b^2*d^24 -
 8*a^24*c^2*d^22 - 8*a^24*c^4*d^20 + 576*b^24*c^10*d^14 - 2784*b^24*c^12*d^
12 + 5284*b^24*c^14*d^10 - 4730*b^24*c^16*d^8 + 1780*b^24*c^18*d^6 - 68*b^2
4*c^20*d^4 - 38*b^24*c^22*d^2 - 5760*a*b^23*c^9*d^15 + 28224*a*b^23*c^11*d^
13 - 54728*a*b^23*c^13*d^11 + 50620*a*b^23*c^15*d^9 - 20360*a*b^23*c^17*d^7
 + 1480*a*b^23*c^19*d^5 + 268*a*b^23*c^21*d^3 + 88*a^3*b^21*c^23*d + 160*a^
5*b^19*c^23*d - 5760*a^9*b^15*c*d^23 + 28224*a^11*b^13*c*d^23 - 54728*a^13*
b^11*c*d^23 + 50620*a^15*b^9*c*d^23 - 20360*a^17*b^7*c*d^23 + 1480*a^19*b^5
*c*d^23 + 268*a^21*b^3*c*d^23 + 88*a^23*b*c^3*d^21 + 160*a^23*b*c^5*d^19 +
25920*a^2*b^22*c^8*d^16 - 131904*a^2*b^22*c^10*d^14 + 270604*a^2*b^22*c^12*
d^12 - 273544*a^2*b^22*c^14*d^10 + 131660*a^2*b^22*c^16*d^8 - 22060*a^2*b^2
2*c^18*d^6 + 782*a^2*b^22*c^20*d^4 + 98*a^2*b^22*c^22*d^2 - 69120*a^3*b^21*
c^7*d^17 + 379200*a^3*b^21*c^9*d^15 - 860368*a^3*b^21*c^11*d^13 + 1001364*a
^3*b^21*c^13*d^11 - 605280*a^3*b^21*c^15*d^9 + 167520*a^3*b^21*c^17*d^7 - 1
8840*a^3*b^21*c^19*d^5 + 144*a^3*b^21*c^21*d^3 + 120960*a^4*b^20*c^6*d^18 -
 756000*a^4*b^20*c^8*d^16 + 1987844*a^4*b^20*c^10*d^14 - 2750664*a^4*b^20*c
^12*d^12 + 2073976*a^4*b^20*c^14*d^10 - 793460*a^4*b^20*c^16*d^8 + 138010*a
^4*b^20*c^18*d^6 - 10562*a^4*b^20*c^20*d^4 - 88*a^4*b^20*c^22*d^2 - 145152*

a^5*b^19*c^5*d^19 + 1116288*a^5*b^19*c^7*d^17 - 3539128*a^5*b^19*c^9*d^15 +
5890780*a^5*b^19*c^11*d^13 - 5437600*a^5*b^19*c^13*d^11 + 2682536*a^5*b^19
*c^15*d^9 - 655084*a^5*b^19*c^17*d^7 + 85484*a^5*b^19*c^19*d^5 - 4080*a^5*b
^19*c^21*d^3 + 120960*a^6*b^18*c^4*d^20 - 1266048*a^6*b^18*c^6*d^18 + 49779
96*a^6*b^18*c^8*d^16 - 10009720*a^6*b^18*c^10*d^14 + 11209800*a^6*b^18*c^12
*d^12 - 6943760*a^6*b^18*c^14*d^10 + 2253214*a^6*b^18*c^16*d^8 - 396878*a^6
*b^18*c^18*d^6 + 36120*a^6*b^18*c^20*d^4 - 1520*a^6*b^18*c^22*d^2 - 69120*a
^7*b^17*c^3*d^21 + 1116288*a^7*b^17*c^5*d^19 - 5575008*a^7*b^17*c^7*d^17 +
13668308*a^7*b^17*c^9*d^15 - 18576800*a^7*b^17*c^11*d^13 + 14230520*a^7*b^1
7*c^13*d^11 - 5889904*a^7*b^17*c^15*d^9 + 1310504*a^7*b^17*c^17*d^7 - 16834
4*a^7*b^17*c^19*d^5 + 8960*a^7*b^17*c^21*d^3 + 25920*a^8*b^16*c^2*d^22 - 75
6000*a^8*b^16*c^4*d^20 + 4977996*a^8*b^16*c^6*d^18 - 15144828*a^8*b^16*c^8*
d^16 + 25068800*a^8*b^16*c^10*d^14 - 23486280*a^8*b^16*c^12*d^12 + 12099640
*a^8*b^16*c^14*d^10 - 3330518*a^8*b^16*c^16*d^8 + 529224*a^8*b^16*c^18*d^6
- 36280*a^8*b^16*c^20*d^4 + 379200*a^9*b^15*c^3*d^21 - 3539128*a^9*b^15*c^5
*d^19 + 13668308*a^9*b^15*c^7*d^17 - 27691952*a^9*b^15*c^9*d^15 + 31562040*
a^9*b^15*c^11*d^13 - 19993760*a^9*b^15*c^13*d^11 + 6731044*a^9*b^15*c^15*d^
9 - 1239264*a^9*b^15*c^17*d^7 + 106016*a^9*b^15*c^19*d^5 - 131904*a^10*b^14
*c^2*d^22 + 1987844*a^10*b^14*c^4*d^20 - 10009720*a^10*b^14*c^6*d^18 + 2506
8800*a^10*b^14*c^8*d^16 - 34796936*a^10*b^14*c^10*d^14 + 26927144*a^10*b^14
*c^12*d^12 - 10994964*a^10*b^14*c^14*d^10 + 2295680*a^10*b^14*c^16*d^8 - 23
0240*a^10*b^14*c^18*d^6 - 860368*a^11*b^13*c^3*d^21 + 5890780*a^11*b^13*c^5
*d^19 - 18576800*a^11*b^13*c^7*d^17 + 31562040*a^11*b^13*c^9*d^15 - 2972286
4*a^11*b^13*c^11*d^13 + 14679348*a^11*b^13*c^13*d^11 - 3497920*a^11*b^13*c^
15*d^9 + 381280*a^11*b^13*c^17*d^7 + 270604*a^12*b^12*c^2*d^22 - 2750664*a^
12*b^12*c^4*d^20 + 11209800*a^12*b^12*c^6*d^18 - 23486280*a^12*b^12*c^8*d^1
6 + 26927144*a^12*b^12*c^10*d^14 - 16147404*a^12*b^12*c^12*d^12 + 4479104*a
^12*b^12*c^14*d^10 - 499520*a^12*b^12*c^16*d^8 + 1001364*a^13*b^11*c^3*d^21
- 5437600*a^13*b^11*c^5*d^19 + 14230520*a^13*b^11*c^7*d^17 - 19993760*a^13
*b^11*c^9*d^15 + 14679348*a^13*b^11*c^11*d^13 - 4861024*a^13*b^11*c^13*d^11
+ 552160*a^13*b^11*c^15*d^9 - 273544*a^14*b^10*c^2*d^22 + 2073976*a^14*b^1
0*c^4*d^20 - 6943760*a^14*b^10*c^6*d^18 + 12099640*a^14*b^10*c^8*d^16 - 109
94964*a^14*b^10*c^10*d^14 + 4479104*a^14*b^10*c^12*d^12 - 562016*a^14*b^10*
c^14*d^10 - 605280*a^15*b^9*c^3*d^21 + 2682536*a^15*b^9*c^5*d^19 - 5889904*
a^15*b^9*c^7*d^17 + 6731044*a^15*b^9*c^9*d^15 - 3497920*a^15*b^9*c^11*d^13
+ 552160*a^15*b^9*c^13*d^11 + 131660*a^16*b^8*c^2*d^22 - 793460*a^16*b^8*c^
4*d^20 + 2253214*a^16*b^8*c^6*d^18 - 3330518*a^16*b^8*c^8*d^16 + 2295680*a^
16*b^8*c^10*d^14 - 499520*a^16*b^8*c^12*d^12 + 167520*a^17*b^7*c^3*d^21 - 6
55084*a^17*b^7*c^5*d^19 + 1310504*a^17*b^7*c^7*d^17 - 1239264*a^17*b^7*c^9*
d^15 + 381280*a^17*b^7*c^11*d^13 - 22060*a^18*b^6*c^2*d^22 + 138010*a^18*b^
6*c^4*d^20 - 396878*a^18*b^6*c^6*d^18 + 529224*a^18*b^6*c^8*d^16 - 230240*a
^18*b^6*c^10*d^14 - 18840*a^19*b^5*c^3*d^21 + 85484*a^19*b^5*c^5*d^19 - 168
344*a^19*b^5*c^7*d^17 + 106016*a^19*b^5*c^9*d^15 + 782*a^20*b^4*c^2*d^22 -
10562*a^20*b^4*c^4*d^20 + 36120*a^20*b^4*c^6*d^18 - 36280*a^20*b^4*c^8*d^16
+ 144*a^21*b^3*c^3*d^21 - 4080*a^21*b^3*c^5*d^19 + 8960*a^21*b^3*c^7*d^17

+ 98*a^22*b^2*c^2*d^22 - 88*a^22*b^2*c^4*d^20 - 1520*a^22*b^2*c^6*d^18 + 4*a*b^23*c^23*d + 4*a^23*b*c*d^23)/(16*(5*a^2*b^28*c^30 - b^30*c^30 - a^30*d^30 - 10*a^4*b^26*c^30 + 10*a^6*b^24*c^30 - 5*a^8*b^22*c^30 + a^10*b^20*c^30 + a^20*b^10*d^30 - 5*a^22*b^8*d^30 + 10*a^24*b^6*d^30 - 10*a^26*b^4*d^30 + 5*a^28*b^2*d^30 + 5*a^30*c^2*d^28 - 10*a^30*c^4*d^26 + 10*a^30*c^6*d^24 - 5*a^30*c^8*d^22 + a^30*c^10*d^20 + b^30*c^20*d^10 - 5*b^30*c^22*d^8 + 10*b^30*c^24*d^6 - 10*b^30*c^26*d^4 + 5*b^30*c^28*d^2 - 20*a*b^29*c^19*d^11 + 100*a*b^29*c^21*d^9 - 200*a*b^29*c^23*d^7 + 200*a*b^29*c^25*d^5 - 100*a*b^29*c^27*d^3 - 100*a^3*b^27*c^29*d + 200*a^5*b^25*c^29*d - 200*a^7*b^23*c^29*d + 100*a^9*b^21*c^29*d - 20*a^11*b^19*c^29*d - 20*a^19*b^11*c*d^29 + 100*a^21*b^9*c*d^29 - 200*a^23*b^7*c*d^29 + 200*a^25*b^5*c*d^29 - 100*a^27*b^3*c*d^29 - 100*a^29*b*c^3*d^27 + 200*a^29*b*c^5*d^25 - 200*a^29*b*c^7*d^23 + 100*a^29*b*c^9*d^21 - 20*a^29*b*c^11*d^19 + 190*a^2*b^28*c^18*d^12 - 955*a^2*b^28*c^20*d^10 + 1925*a^2*b^28*c^22*d^8 - 1950*a^2*b^28*c^24*d^6 + 1000*a^2*b^28*c^26*d^4 - 215*a^2*b^28*c^28*d^2 - 1140*a^3*b^27*c^17*d^13 + 5800*a^3*b^27*c^19*d^11 - 11900*a^3*b^27*c^21*d^9 + 12400*a^3*b^27*c^23*d^7 - 6700*a^3*b^27*c^25*d^5 + 1640*a^3*b^27*c^27*d^3 + 4845*a^4*b^26*c^16*d^14 - 25175*a^4*b^26*c^18*d^12 + 53210*a^4*b^26*c^20*d^10 - 58000*a^4*b^26*c^22*d^8 + 33825*a^4*b^26*c^24*d^6 - 9695*a^4*b^26*c^26*d^4 + 1000*a^4*b^26*c^28*d^2 - 15504*a^5*b^25*c^15*d^15 + 83220*a^5*b^25*c^17*d^13 - 183740*a^5*b^25*c^19*d^11 + 213040*a^5*b^25*c^21*d^9 - 136520*a^5*b^25*c^23*d^7 + 46004*a^5*b^25*c^25*d^5 - 6700*a^5*b^25*c^27*d^3 + 38760*a^6*b^24*c^14*d^16 - 218025*a^6*b^24*c^16*d^14 + 510625*a^6*b^24*c^18*d^12 - 639360*a^6*b^24*c^20*d^10 + 455100*a^6*b^24*c^22*d^8 - 178985*a^6*b^24*c^24*d^6 + 33825*a^6*b^24*c^26*d^4 - 1950*a^6*b^24*c^28*d^2 - 77520*a^7*b^23*c^13*d^17 + 465120*a^7*b^23*c^15*d^15 - 1174200*a^7*b^23*c^17*d^13 + 1607600*a^7*b^23*c^19*d^11 - 1277800*a^7*b^23*c^21*d^9 + 581120*a^7*b^23*c^23*d^7 - 136520*a^7*b^23*c^25*d^5 + 12400*a^7*b^23*c^27*d^3 + 125970*a^8*b^22*c^12*d^18 - 823650*a^8*b^22*c^14*d^16 + 2277150*a^8*b^22*c^16*d^14 - 3441850*a^8*b^22*c^18*d^12 + 3061855*a^8*b^22*c^20*d^10 - 1598495*a^8*b^22*c^22*d^8 + 455100*a^8*b^22*c^24*d^6 - 58000*a^8*b^22*c^26*d^4 + 1925*a^8*b^22*c^28*d^2 - 167960*a^9*b^21*c^11*d^19 + 1227400*a^9*b^21*c^13*d^17 - 3772640*a^9*b^21*c^15*d^15 + 6342200*a^9*b^21*c^17*d^13 - 6323300*a^9*b^21*c^19*d^11 + 3770860*a^9*b^21*c^21*d^9 - 1277800*a^9*b^21*c^23*d^7 + 213040*a^9*b^21*c^25*d^5 - 11900*a^9*b^21*c^27*d^3 + 184756*a^10*b^20*c^10*d^20 - 1553630*a^10*b^20*c^12*d^18 + 5384410*a^10*b^20*c^14*d^16 - 10132510*a^10*b^20*c^16*d^14 + 11341480*a^10*b^20*c^18*d^12 - 7699257*a^10*b^20*c^20*d^10 + 3061855*a^10*b^20*c^22*d^8 - 639360*a^10*b^20*c^24*d^6 + 53210*a^10*b^20*c^26*d^4 - 955*a^10*b^20*c^28*d^2 - 167960*a^11*b^19*c^9*d^21 + 1679600*a^11*b^19*c^11*d^19 - 6653800*a^11*b^19*c^13*d^17 + 14108640*a^11*b^19*c^15*d^15 - 17770700*a^11*b^19*c^17*d^13 + 13697880*a^11*b^19*c^19*d^11 - 6323300*a^11*b^19*c^21*d^9 + 1607600*a^11*b^19*c^23*d^7 - 183740*a^11*b^19*c^25*d^5 + 5800*a^11*b^19*c^27*d^3 + 125970*a^12*b^18*c^8*d^22 - 1553630*a^12*b^18*c^10*d^20 + 7138300*a^12*b^18*c^12*d^18 - 17183600*a^12*b^18*c^14*d^16 + 24426875*a^12*b^18*c^16*d^14 - 21339185*a^12*b^18*c^18*d^12 + 11341480*a^12*b^18*c^20*d^10 - 3441850*a^12*b^18*c^22*d^8 +

510625*a^12*b^18*c^24*d^6 - 25175*a^12*b^18*c^26*d^4 + 190*a^12*b^18*c^28*
d^2 - 77520*a^13*b^17*c^7*d^23 + 1227400*a^13*b^17*c^9*d^21 - 6653800*a^13*
b^17*c^11*d^19 + 18346400*a^13*b^17*c^13*d^17 - 29535120*a^13*b^17*c^15*d^1
5 + 29213260*a^13*b^17*c^17*d^13 - 17770700*a^13*b^17*c^19*d^11 + 6342200*a
^13*b^17*c^21*d^9 - 1174200*a^13*b^17*c^23*d^7 + 83220*a^13*b^17*c^25*d^5 -
1140*a^13*b^17*c^27*d^3 + 38760*a^14*b^16*c^6*d^24 - 823650*a^14*b^16*c^8*
d^22 + 5384410*a^14*b^16*c^10*d^20 - 17183600*a^14*b^16*c^12*d^18 + 3146020
0*a^14*b^16*c^14*d^16 - 35234455*a^14*b^16*c^16*d^14 + 24426875*a^14*b^16*c
^18*d^12 - 10132510*a^14*b^16*c^20*d^10 + 2277150*a^14*b^16*c^22*d^8 - 2180
25*a^14*b^16*c^24*d^6 + 4845*a^14*b^16*c^26*d^4 - 15504*a^15*b^15*c^5*d^25
+ 465120*a^15*b^15*c^7*d^23 - 3772640*a^15*b^15*c^9*d^21 + 14108640*a^15*b^
15*c^11*d^19 - 29535120*a^15*b^15*c^13*d^17 + 37499008*a^15*b^15*c^15*d^15
- 29535120*a^15*b^15*c^17*d^13 + 14108640*a^15*b^15*c^19*d^11 - 3772640*a^1
5*b^15*c^21*d^9 + 465120*a^15*b^15*c^23*d^7 - 15504*a^15*b^15*c^25*d^5 + 48
45*a^16*b^14*c^4*d^26 - 218025*a^16*b^14*c^6*d^24 + 2277150*a^16*b^14*c^8*d
^22 - 10132510*a^16*b^14*c^10*d^20 + 24426875*a^16*b^14*c^12*d^18 - 3523445
5*a^16*b^14*c^14*d^16 + 31460200*a^16*b^14*c^16*d^14 - 17183600*a^16*b^14*c
^18*d^12 + 5384410*a^16*b^14*c^20*d^10 - 823650*a^16*b^14*c^22*d^8 + 38760*
a^16*b^14*c^24*d^6 - 1140*a^17*b^13*c^3*d^27 + 83220*a^17*b^13*c^5*d^25 - 1
174200*a^17*b^13*c^7*d^23 + 6342200*a^17*b^13*c^9*d^21 - 17770700*a^17*b^13
*c^11*d^19 + 29213260*a^17*b^13*c^13*d^17 - 29535120*a^17*b^13*c^15*d^15 +
18346400*a^17*b^13*c^17*d^13 - 6653800*a^17*b^13*c^19*d^11 + 1227400*a^17*b
^13*c^21*d^9 - 77520*a^17*b^13*c^23*d^7 + 190*a^18*b^12*c^2*d^28 - 25175*a^
18*b^12*c^4*d^26 + 510625*a^18*b^12*c^6*d^24 - 3441850*a^18*b^12*c^8*d^22 +
11341480*a^18*b^12*c^10*d^20 - 21339185*a^18*b^12*c^12*d^18 + 24426875*a^1
8*b^12*c^14*d^16 - 17183600*a^18*b^12*c^16*d^14 + 7138300*a^18*b^12*c^18*d^
12 - 1553630*a^18*b^12*c^20*d^10 + 125970*a^18*b^12*c^22*d^8 + 5800*a^19*b^
11*c^3*d^27 - 183740*a^19*b^11*c^5*d^25 + 1607600*a^19*b^11*c^7*d^23 - 6323
300*a^19*b^11*c^9*d^21 + 13697880*a^19*b^11*c^11*d^19 - 17770700*a^19*b^11*
c^13*d^17 + 14108640*a^19*b^11*c^15*d^15 - 6653800*a^19*b^11*c^17*d^13 + 16
79600*a^19*b^11*c^19*d^11 - 167960*a^19*b^11*c^21*d^9 - 955*a^20*b^10*c^2*d
^28 + 53210*a^20*b^10*c^4*d^26 - 639360*a^20*b^10*c^6*d^24 + 3061855*a^20*b
^10*c^8*d^22 - 7699257*a^20*b^10*c^10*d^20 + 11341480*a^20*b^10*c^12*d^18 -
10132510*a^20*b^10*c^14*d^16 + 5384410*a^20*b^10*c^16*d^14 - 1553630*a^20*
b^10*c^18*d^12 + 184756*a^20*b^10*c^20*d^10 - 11900*a^21*b^9*c^3*d^27 + 213
040*a^21*b^9*c^5*d^25 - 1277800*a^21*b^9*c^7*d^23 + 3770860*a^21*b^9*c^9*d^
21 - 6323300*a^21*b^9*c^11*d^19 + 6342200*a^21*b^9*c^13*d^17 - 3772640*a^21
*b^9*c^15*d^15 + 1227400*a^21*b^9*c^17*d^13 - 167960*a^21*b^9*c^19*d^11 + 1
925*a^22*b^8*c^2*d^28 - 58000*a^22*b^8*c^4*d^26 + 455100*a^22*b^8*c^6*d^24
- 1598495*a^22*b^8*c^8*d^22 + 3061855*a^22*b^8*c^10*d^20 - 3441850*a^22*b^8
*c^12*d^18 + 2277150*a^22*b^8*c^14*d^16 - 823650*a^22*b^8*c^16*d^14 + 12597
0*a^22*b^8*c^18*d^12 + 12400*a^23*b^7*c^3*d^27 - 136520*a^23*b^7*c^5*d^25 +
581120*a^23*b^7*c^7*d^23 - 1277800*a^23*b^7*c^9*d^21 + 1607600*a^23*b^7*c^
11*d^19 - 1174200*a^23*b^7*c^13*d^17 + 465120*a^23*b^7*c^15*d^15 - 77520*a^
23*b^7*c^17*d^13 - 1950*a^24*b^6*c^2*d^28 + 33825*a^24*b^6*c^4*d^26 - 17898

5*a^24*b^6*c^6*d^24 + 455100*a^24*b^6*c^8*d^22 - 639360*a^24*b^6*c^10*d^20 + 510625*a^24*b^6*c^12*d^18 - 218025*a^24*b^6*c^14*d^16 + 38760*a^24*b^6*c^16*d^14 - 6700*a^25*b^5*c^3*d^27 + 46004*a^25*b^5*c^5*d^25 - 136520*a^25*b^5*c^7*d^23 + 213040*a^25*b^5*c^9*d^21 - 183740*a^25*b^5*c^11*d^19 + 83220*a^25*b^5*c^13*d^17 - 15504*a^25*b^5*c^15*d^15 + 1000*a^26*b^4*c^2*d^28 - 9695*a^26*b^4*c^4*d^26 + 33825*a^26*b^4*c^6*d^24 - 58000*a^26*b^4*c^8*d^22 + 53210*a^26*b^4*c^10*d^20 - 25175*a^26*b^4*c^12*d^18 + 4845*a^26*b^4*c^14*d^16 + 1640*a^27*b^3*c^3*d^27 - 6700*a^27*b^3*c^5*d^25 + 12400*a^27*b^3*c^7*d^23 - 11900*a^27*b^3*c^9*d^21 + 5800*a^27*b^3*c^11*d^19 - 1140*a^27*b^3*c^13*d^17 - 215*a^28*b^2*c^2*d^28 + 1000*a^28*b^2*c^4*d^26 - 1950*a^28*b^2*c^6*d^24 + 1925*a^28*b^2*c^8*d^22 - 955*a^28*b^2*c^10*d^20 + 190*a^28*b^2*c^12*d^18 + 20*a*b^29*c^29*d + 20*a^29*b*c*d^29)))^(1/2)*(((4*(8*a^2*b^23*c^25 - 32*a^4*b^21*c^25 + 48*a^6*b^19*c^25 - 32*a^8*b^17*c^25 + 8*a^10*b^15*c^25 + 8*a^25*c^2*d^23 - 32*a^25*c^4*d^21 + 48*a^25*c^6*d^19 - 32*a^25*c^8*d^17 + 8*a^25*c^10*d^15 - 8*a*b^24*c^16*d^9 + 32*a*b^24*c^18*d^7 - 48*a*b^24*c^20*d^5 + 32*a*b^24*c^22*d^3 - 72*a^3*b^22*c^24*d + 368*a^5*b^20*c^24*d - 592*a^7*b^18*c^24*d + 408*a^9*b^16*c^24*d - 104*a^11*b^14*c^24*d - 8*a^16*b^9*c*d^24 + 32*a^18*b^7*c*d^24 - 48*a^20*b^5*c*d^24 + 32*a^22*b^3*c*d^24 - 72*a^24*b*c^3*d^22 + 368*a^24*b*c^5*d^20 - 592*a^24*b*c^7*d^18 + 408*a^24*b*c^9*d^16 - 104*a^24*b*c^11*d^14 + 104*a^2*b^23*c^15*d^10 - 408*a^2*b^23*c^17*d^8 + 592*a^2*b^23*c^19*d^6 - 368*a^2*b^23*c^21*d^4 + 72*a^2*b^23*c^23*d^2 - 616*a^3*b^22*c^14*d^11 + 2392*a^3*b^22*c^16*d^9 - 3408*a^3*b^22*c^18*d^7 + 2032*a^3*b^22*c^20*d^5 - 328*a^3*b^22*c^22*d^3 + 2184*a^4*b^21*c^13*d^12 - 8536*a^4*b^21*c^15*d^10 + 12272*a^4*b^21*c^17*d^8 - 7408*a^4*b^21*c^19*d^6 + 1192*a^4*b^21*c^21*d^4 + 328*a^4*b^21*c^23*d^2 - 5096*a^5*b^20*c^12*d^13 + 20664*a^5*b^20*c^14*d^11 - 31328*a^5*b^20*c^16*d^9 + 20592*a^5*b^20*c^18*d^7 - 4008*a^5*b^20*c^20*d^5 - 1192*a^5*b^20*c^22*d^3 + 8008*a^6*b^19*c^11*d^14 - 35672*a^6*b^19*c^13*d^12 + 60768*a^6*b^19*c^15*d^10 - 46464*a^6*b^19*c^17*d^8 + 11336*a^6*b^19*c^19*d^6 + 4008*a^6*b^19*c^21*d^4 - 2032*a^6*b^19*c^23*d^2 - 8008*a^7*b^18*c^10*d^15 + 44408*a^7*b^18*c^12*d^13 - 92512*a^7*b^18*c^14*d^11 + 85536*a^7*b^18*c^16*d^9 - 24904*a^7*b^18*c^18*d^7 - 11336*a^7*b^18*c^20*d^5 + 7408*a^7*b^18*c^22*d^3 + 3432*a^8*b^17*c^9*d^16 - 37752*a^8*b^17*c^11*d^14 + 109408*a^8*b^17*c^13*d^12 - 125472*a^8*b^17*c^15*d^10 + 42696*a^8*b^17*c^17*d^8 + 24904*a^8*b^17*c^19*d^6 - 20592*a^8*b^17*c^21*d^4 + 3408*a^8*b^17*c^23*d^2 + 3432*a^9*b^16*c^8*d^17 + 14872*a^9*b^16*c^10*d^15 - 92352*a^9*b^16*c^12*d^13 + 141408*a^9*b^16*c^14*d^11 - 59264*a^9*b^16*c^16*d^9 - 42696*a^9*b^16*c^18*d^7 + 46464*a^9*b^16*c^20*d^5 - 12272*a^9*b^16*c^22*d^3 - 8008*a^10*b^15*c^7*d^18 + 14872*a^10*b^15*c^9*d^16 + 36608*a^10*b^15*c^11*d^14 - 113152*a^10*b^15*c^13*d^12 + 67008*a^10*b^15*c^15*d^10 + 59264*a^10*b^15*c^17*d^8 - 85536*a^10*b^15*c^19*d^6 + 31328*a^10*b^15*c^21*d^4 - 2392*a^10*b^15*c^23*d^2 + 8008*a^11*b^14*c^6*d^19 - 37752*a^11*b^14*c^8*d^17 + 36608*a^11*b^14*c^10*d^15 + 43264*a^11*b^14*c^12*d^13 - 56256*a^11*b^14*c^14*d^11 - 67008*a^11*b^14*c^16*d^9 + 125472*a^11*b^14*c^18*d^7 - 60768*a^11*b^14*c^20*d^5 + 8536*a^11*b^14*c^22*d^3 - 5096*a^12*b^13*c^5*d^20 + 44408*a^12*b^13*c^7*d^18 - 92352*a^12*b^13*c^9*d^16 + 43264*a^12

*b^13*c^11*d^14 + 22464*a^12*b^13*c^13*d^12 + 56256*a^12*b^13*c^15*d^10 - 1
41408*a^12*b^13*c^17*d^8 + 92512*a^12*b^13*c^19*d^6 - 20664*a^12*b^13*c^21*
d^4 + 616*a^12*b^13*c^23*d^2 + 2184*a^13*b^12*c^4*d^21 - 35672*a^13*b^12*c^
6*d^19 + 109408*a^13*b^12*c^8*d^17 - 113152*a^13*b^12*c^10*d^15 + 22464*a^1
3*b^12*c^12*d^13 - 22464*a^13*b^12*c^14*d^11 + 113152*a^13*b^12*c^16*d^9 -
109408*a^13*b^12*c^18*d^7 + 35672*a^13*b^12*c^20*d^5 - 2184*a^13*b^12*c^22*
d^3 - 616*a^14*b^11*c^3*d^22 + 20664*a^14*b^11*c^5*d^20 - 92512*a^14*b^11*c
^7*d^18 + 141408*a^14*b^11*c^9*d^16 - 56256*a^14*b^11*c^11*d^14 - 22464*a^1
4*b^11*c^13*d^12 - 43264*a^14*b^11*c^15*d^10 + 92352*a^14*b^11*c^17*d^8 - 4
4408*a^14*b^11*c^19*d^6 + 5096*a^14*b^11*c^21*d^4 + 104*a^15*b^10*c^2*d^23
- 8536*a^15*b^10*c^4*d^21 + 60768*a^15*b^10*c^6*d^19 - 125472*a^15*b^10*c^8
*d^17 + 67008*a^15*b^10*c^10*d^15 + 56256*a^15*b^10*c^12*d^13 - 43264*a^15*
b^10*c^14*d^11 - 36608*a^15*b^10*c^16*d^9 + 37752*a^15*b^10*c^18*d^7 - 8008
*a^15*b^10*c^20*d^5 + 2392*a^16*b^9*c^3*d^22 - 31328*a^16*b^9*c^5*d^20 + 85
536*a^16*b^9*c^7*d^18 - 59264*a^16*b^9*c^9*d^16 - 67008*a^16*b^9*c^11*d^14
+ 113152*a^16*b^9*c^13*d^12 - 36608*a^16*b^9*c^15*d^10 - 14872*a^16*b^9*c^1
7*d^8 + 8008*a^16*b^9*c^19*d^6 - 408*a^17*b^8*c^2*d^23 + 12272*a^17*b^8*c^4
*d^21 - 46464*a^17*b^8*c^6*d^19 + 42696*a^17*b^8*c^8*d^17 + 59264*a^17*b^8*
c^10*d^15 - 141408*a^17*b^8*c^12*d^13 + 92352*a^17*b^8*c^14*d^11 - 14872*a^
17*b^8*c^16*d^9 - 3432*a^17*b^8*c^18*d^7 - 3408*a^18*b^7*c^3*d^22 + 20592*a
^18*b^7*c^5*d^20 - 24904*a^18*b^7*c^7*d^18 - 42696*a^18*b^7*c^9*d^16 + 1254
72*a^18*b^7*c^11*d^14 - 109408*a^18*b^7*c^13*d^12 + 37752*a^18*b^7*c^15*d^1
0 - 3432*a^18*b^7*c^17*d^8 + 592*a^19*b^6*c^2*d^23 - 7408*a^19*b^6*c^4*d^21
 + 11336*a^19*b^6*c^6*d^19 + 24904*a^19*b^6*c^8*d^17 - 85536*a^19*b^6*c^10*
d^15 + 92512*a^19*b^6*c^12*d^13 - 44408*a^19*b^6*c^14*d^11 + 8008*a^19*b^6*
c^16*d^9 + 2032*a^20*b^5*c^3*d^22 - 4008*a^20*b^5*c^5*d^20 - 11336*a^20*b^5
*c^7*d^18 + 46464*a^20*b^5*c^9*d^16 - 60768*a^20*b^5*c^11*d^14 + 35672*a^20
*b^5*c^13*d^12 - 8008*a^20*b^5*c^15*d^10 - 368*a^21*b^4*c^2*d^23 + 1192*a^2
1*b^4*c^4*d^21 + 4008*a^21*b^4*c^6*d^19 - 20592*a^21*b^4*c^8*d^17 + 31328*a
^21*b^4*c^10*d^15 - 20664*a^21*b^4*c^12*d^13 + 5096*a^21*b^4*c^14*d^11 - 32
8*a^22*b^3*c^3*d^22 - 1192*a^22*b^3*c^5*d^20 + 7408*a^22*b^3*c^7*d^18 - 122
72*a^22*b^3*c^9*d^16 + 8536*a^22*b^3*c^11*d^14 - 2184*a^22*b^3*c^13*d^12 +
72*a^23*b^2*c^2*d^23 + 328*a^23*b^2*c^4*d^21 - 2032*a^23*b^2*c^6*d^19 + 340
8*a^23*b^2*c^8*d^17 - 2392*a^23*b^2*c^10*d^15 + 616*a^23*b^2*c^12*d^13 - 8*
a*b^24*c^24*d - 8*a^24*b*c*d^24))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20
+ 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^1
4*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c
^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6
 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*
d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5
*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19
 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b
*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 +
66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288
*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3

*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19) - (8*tan(e/2 + (f*x)/2)*(56*a^3*b^22*c^25 - 12*a^25*c*d^24 - 12*a*b^24*c^25 - 104*a^5*b^20*c^25 + 96*a^7*b^18*c^25 - 44*a^9*b^16*c^25 + 8*a^11*b^14*c^25 + 56*a^25*c^3*d^22 - 104*a^25*c^5*d^20 + 96*a^25*c^7*d^18 - 44*a^25*c^9*d^16 + 8*a^25*c^11*d^14 + 16*a*b^24*c^15*d^10 - 76*a*b^24*c^17*d^8 + 144*a*b^24*c^19*d^6 - 136*a*b^24*c^21*d^4 + 64*a*b^24*c^23*d^2 + 168*a^2*b^23*c^24*d - 784*a^4*b^21*c^24*d + 1456*a^6*b^19*c^24*d - 1344*a^8*b^17*c^24*d + 616*a^10*b^15*c^24*d - 112*a^12*b^13*c^24*d + 16*a^15*b^10*c*d^24 - 76*a^17*b^8*c*d^24 + 144*a^19*b^6*c*d^24 - 136*a^21*b^4*c*d^24 + 64*a^23*b^2*c*d^24 + 168*a^24*b*c^2*d^23 - 784*a^24*b*c^4*d^21 + 1456*a^24

*b*c^6*d^19 - 1344*a^24*b*c^8*d^17 + 616*a^24*b*c^10*d^15 - 112*a^24*b*c^12
*d^13 - 224*a^2*b^23*c^14*d^11 + 1064*a^2*b^23*c^16*d^9 - 2016*a^2*b^23*c^1
8*d^7 + 1904*a^2*b^23*c^20*d^5 - 896*a^2*b^23*c^22*d^3 + 1456*a^3*b^22*c^13
*d^12 - 6992*a^3*b^22*c^15*d^10 + 13464*a^3*b^22*c^17*d^8 - 13056*a^3*b^22*
c^19*d^6 + 6464*a^3*b^22*c^21*d^4 - 1392*a^3*b^22*c^23*d^2 - 5824*a^4*b^21*
c^12*d^13 + 28728*a^4*b^21*c^14*d^11 - 57456*a^4*b^21*c^16*d^9 + 59024*a^4*
b^21*c^18*d^7 - 32256*a^4*b^21*c^20*d^5 + 8568*a^4*b^21*c^22*d^3 + 16016*a^
5*b^20*c^11*d^14 - 82992*a^5*b^20*c^13*d^12 + 177048*a^5*b^20*c^15*d^10 - 1
98696*a^5*b^20*c^17*d^8 + 123584*a^5*b^20*c^19*d^6 - 40512*a^5*b^20*c^21*d^
4 + 5656*a^5*b^20*c^23*d^2 - 32032*a^6*b^19*c^10*d^15 + 179816*a^6*b^19*c^1
2*d^13 - 421344*a^6*b^19*c^14*d^11 + 529312*a^6*b^19*c^16*d^9 - 379008*a^6*
b^19*c^18*d^7 + 150024*a^6*b^19*c^20*d^5 - 28224*a^6*b^19*c^22*d^3 + 48048*
a^7*b^18*c^9*d^16 - 304304*a^7*b^18*c^11*d^14 + 805896*a^7*b^18*c^13*d^12 -
1151104*a^7*b^18*c^15*d^10 + 949952*a^7*b^18*c^17*d^8 - 446736*a^7*b^18*c^
19*d^6 + 108136*a^7*b^18*c^21*d^4 - 9984*a^7*b^18*c^23*d^2 - 54912*a^8*b^17
*c^8*d^17 + 412984*a^8*b^17*c^10*d^15 - 1267344*a^8*b^17*c^12*d^13 + 207753
6*a^8*b^17*c^14*d^11 - 1975808*a^8*b^17*c^16*d^9 + 1095384*a^8*b^17*c^18*d^
7 - 331632*a^8*b^17*c^20*d^5 + 45136*a^8*b^17*c^22*d^3 + 48048*a^9*b^16*c^7
*d^18 - 456456*a^9*b^16*c^9*d^16 + 1657656*a^9*b^16*c^11*d^14 - 3143504*a^9
*b^16*c^13*d^12 + 3453696*a^9*b^16*c^15*d^10 - 2247636*a^9*b^16*c^17*d^8 +
831208*a^9*b^16*c^19*d^6 - 151944*a^9*b^16*c^21*d^4 + 8976*a^9*b^16*c^23*d^
2 - 32032*a^10*b^15*c^6*d^19 + 412984*a^10*b^15*c^8*d^17 - 1812096*a^10*b^1
5*c^10*d^15 + 4016896*a^10*b^15*c^12*d^13 - 5121024*a^10*b^15*c^14*d^11 + 3
897024*a^10*b^15*c^16*d^9 - 1728832*a^10*b^15*c^18*d^7 + 404768*a^10*b^15*c
^20*d^5 - 38304*a^10*b^15*c^22*d^3 + 16016*a^11*b^14*c^5*d^20 - 304304*a^11
*b^14*c^7*d^18 + 1657656*a^11*b^14*c^9*d^16 - 4356352*a^11*b^14*c^11*d^14 +
6476288*a^11*b^14*c^13*d^12 - 5745024*a^11*b^14*c^15*d^10 + 3021984*a^11*b
^14*c^17*d^8 - 880256*a^11*b^14*c^19*d^6 + 118032*a^11*b^14*c^21*d^4 - 4048
*a^11*b^14*c^23*d^2 - 5824*a^12*b^13*c^4*d^21 + 179816*a^12*b^13*c^6*d^19 -
1267344*a^12*b^13*c^8*d^17 + 4016896*a^12*b^13*c^10*d^15 - 7002112*a^12*b^
13*c^12*d^13 + 7235136*a^12*b^13*c^14*d^11 - 4480896*a^12*b^13*c^16*d^9 + 1
588704*a^12*b^13*c^18*d^7 - 280896*a^12*b^13*c^20*d^5 + 16632*a^12*b^13*c^2
2*d^3 + 1456*a^13*b^12*c^3*d^22 - 82992*a^13*b^12*c^5*d^20 + 805896*a^13*b^
12*c^7*d^18 - 3143504*a^13*b^12*c^9*d^16 + 6476288*a^13*b^12*c^11*d^14 - 78
09984*a^13*b^12*c^13*d^12 + 5666752*a^13*b^12*c^15*d^10 - 2403856*a^13*b^12
*c^17*d^8 + 537264*a^13*b^12*c^19*d^6 - 48048*a^13*b^12*c^21*d^4 + 728*a^13
*b^12*c^23*d^2 - 224*a^14*b^11*c^2*d^23 + 28728*a^14*b^11*c^4*d^21 - 421344
*a^14*b^11*c^6*d^19 + 2077536*a^14*b^11*c^8*d^17 - 5121024*a^14*b^11*c^10*d
^15 + 7235136*a^14*b^11*c^12*d^13 - 6126848*a^14*b^11*c^14*d^11 + 3071744*a
^14*b^11*c^16*d^9 - 844896*a^14*b^11*c^18*d^7 + 104104*a^14*b^11*c^20*d^5 -
2912*a^14*b^11*c^22*d^3 - 6992*a^15*b^10*c^3*d^22 + 177048*a^15*b^10*c^5*d
^20 - 1151104*a^15*b^10*c^7*d^18 + 3453696*a^15*b^10*c^9*d^16 - 5745024*a^1
5*b^10*c^11*d^14 + 5666752*a^15*b^10*c^13*d^12 - 3331328*a^15*b^10*c^15*d^1
0 + 1105104*a^15*b^10*c^17*d^8 - 176176*a^15*b^10*c^19*d^6 + 8008*a^15*b^10
*c^21*d^4 + 1064*a^16*b^9*c^2*d^23 - 57456*a^16*b^9*c^4*d^21 + 529312*a^16*

b^9*c^6*d^19 - 1975808*a^16*b^9*c^8*d^17 + 3897024*a^16*b^9*c^10*d^15 - 448
0896*a^16*b^9*c^12*d^13 + 3071744*a^16*b^9*c^14*d^11 - 1208064*a^16*b^9*c^1
6*d^9 + 239096*a^16*b^9*c^18*d^7 - 16016*a^16*b^9*c^20*d^5 + 13464*a^17*b^8
*c^3*d^22 - 198696*a^17*b^8*c^5*d^20 + 949952*a^17*b^8*c^7*d^18 - 2247636*a
^17*b^8*c^9*d^16 + 3021984*a^17*b^8*c^11*d^14 - 2403856*a^17*b^8*c^13*d^12
+ 1105104*a^17*b^8*c^15*d^10 - 264264*a^17*b^8*c^17*d^8 + 24024*a^17*b^8*c^
19*d^6 - 2016*a^18*b^7*c^2*d^23 + 59024*a^18*b^7*c^4*d^21 - 379008*a^18*b^7
*c^6*d^19 + 1095384*a^18*b^7*c^8*d^17 - 1728832*a^18*b^7*c^10*d^15 + 158870
4*a^18*b^7*c^12*d^13 - 844896*a^18*b^7*c^14*d^11 + 239096*a^18*b^7*c^16*d^9
 - 27456*a^18*b^7*c^18*d^7 - 13056*a^19*b^6*c^3*d^22 + 123584*a^19*b^6*c^5*
d^20 - 446736*a^19*b^6*c^7*d^18 + 831208*a^19*b^6*c^9*d^16 - 880256*a^19*b^
6*c^11*d^14 + 537264*a^19*b^6*c^13*d^12 - 176176*a^19*b^6*c^15*d^10 + 24024
*a^19*b^6*c^17*d^8 + 1904*a^20*b^5*c^2*d^23 - 32256*a^20*b^5*c^4*d^21 + 150
024*a^20*b^5*c^6*d^19 - 331632*a^20*b^5*c^8*d^17 + 404768*a^20*b^5*c^10*d^1
5 - 280896*a^20*b^5*c^12*d^13 + 104104*a^20*b^5*c^14*d^11 - 16016*a^20*b^5*
c^16*d^9 + 6464*a^21*b^4*c^3*d^22 - 40512*a^21*b^4*c^5*d^20 + 108136*a^21*b
^4*c^7*d^18 - 151944*a^21*b^4*c^9*d^16 + 118032*a^21*b^4*c^11*d^14 - 48048*
a^21*b^4*c^13*d^12 + 8008*a^21*b^4*c^15*d^10 - 896*a^22*b^3*c^2*d^23 + 8568
*a^22*b^3*c^4*d^21 - 28224*a^22*b^3*c^6*d^19 + 45136*a^22*b^3*c^8*d^17 - 38
304*a^22*b^3*c^10*d^15 + 16632*a^22*b^3*c^12*d^13 - 2912*a^22*b^3*c^14*d^11
 - 1392*a^23*b^2*c^3*d^22 + 5656*a^23*b^2*c^5*d^20 - 9984*a^23*b^2*c^7*d^18
 + 8976*a^23*b^2*c^9*d^16 - 4048*a^23*b^2*c^11*d^14 + 728*a^23*b^2*c^13*d^1
2))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14
*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 -
 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^
20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^
18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*
a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^1
9*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^1
5*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15
 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^
2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^
18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17
*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c
^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*
c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^
7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^
13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6
*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*
c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*
c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*
c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^1
3*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12
*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*
b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*

b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*
b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a
^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10
*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300
*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8
+ 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^
2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^1
3 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13
*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^
18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*
d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c
^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^
5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7
*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c
^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6
*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^
6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^
5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5
*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*
c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*
c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c
^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d
^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12
+ 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19))*((((4*a^24
*d^24 + 4*b^24*c^24 + 16*a^2*b^22*c^24 + 16*a^4*b^20*c^24 - 1152*a^10*b^14*
d^24 + 5568*a^12*b^12*d^24 - 10568*a^14*b^10*d^24 + 9460*a^16*b^8*d^24 - 35
60*a^18*b^6*d^24 + 136*a^20*b^4*d^24 + 76*a^22*b^2*d^24 + 16*a^24*c^2*d^22
+ 16*a^24*c^4*d^20 - 1152*b^24*c^10*d^14 + 5568*b^24*c^12*d^12 - 10568*b^24
*c^14*d^10 + 9460*b^24*c^16*d^8 - 3560*b^24*c^18*d^6 + 136*b^24*c^20*d^4 +
76*b^24*c^22*d^2 + 11520*a*b^23*c^9*d^15 - 56448*a*b^23*c^11*d^13 + 109456*
a*b^23*c^13*d^11 - 101240*a*b^23*c^15*d^9 + 40720*a*b^23*c^17*d^7 - 2960*a*
b^23*c^19*d^5 - 536*a*b^23*c^21*d^3 - 176*a^3*b^21*c^23*d - 320*a^5*b^19*c^
23*d + 11520*a^9*b^15*c*d^23 - 56448*a^11*b^13*c*d^23 + 109456*a^13*b^11*c*
d^23 - 101240*a^15*b^9*c*d^23 + 40720*a^17*b^7*c*d^23 - 2960*a^19*b^5*c*d^2
3 - 536*a^21*b^3*c*d^23 - 176*a^23*b*c^3*d^21 - 320*a^23*b*c^5*d^19 - 51840
*a^2*b^22*c^8*d^16 + 263808*a^2*b^22*c^10*d^14 - 541208*a^2*b^22*c^12*d^12
+ 547088*a^2*b^22*c^14*d^10 - 263320*a^2*b^22*c^16*d^8 + 44120*a^2*b^22*c^1
8*d^6 - 1564*a^2*b^22*c^20*d^4 - 196*a^2*b^22*c^22*d^2 + 138240*a^3*b^21*c^
7*d^17 - 758400*a^3*b^21*c^9*d^15 + 1720736*a^3*b^21*c^11*d^13 - 2002728*a^
3*b^21*c^13*d^11 + 1210560*a^3*b^21*c^15*d^9 - 335040*a^3*b^21*c^17*d^7 + 3
7680*a^3*b^21*c^19*d^5 - 288*a^3*b^21*c^21*d^3 - 241920*a^4*b^20*c^6*d^18 +
1512000*a^4*b^20*c^8*d^16 - 3975688*a^4*b^20*c^10*d^14 + 5501328*a^4*b^20*
c^12*d^12 - 4147952*a^4*b^20*c^14*d^10 + 1586920*a^4*b^20*c^16*d^8 - 276020
*a^4*b^20*c^18*d^6 + 21124*a^4*b^20*c^20*d^4 + 176*a^4*b^20*c^22*d^2 + 2903
04*a^5*b^19*c^5*d^19 - 2232576*a^5*b^19*c^7*d^17 + 7078256*a^5*b^19*c^9*d^1

5 - 11781560*a^5*b^19*c^11*d^13 + 10875200*a^5*b^19*c^13*d^11 - 5365072*a^5
*b^19*c^15*d^9 + 1310168*a^5*b^19*c^17*d^7 - 170968*a^5*b^19*c^19*d^5 + 816
0*a^5*b^19*c^21*d^3 - 241920*a^6*b^18*c^4*d^20 + 2532096*a^6*b^18*c^6*d^18
- 9955992*a^6*b^18*c^8*d^16 + 20019440*a^6*b^18*c^10*d^14 - 22419600*a^6*b^
18*c^12*d^12 + 13887520*a^6*b^18*c^14*d^10 - 4506428*a^6*b^18*c^16*d^8 + 79
3756*a^6*b^18*c^18*d^6 - 72240*a^6*b^18*c^20*d^4 + 3040*a^6*b^18*c^22*d^2 +
138240*a^7*b^17*c^3*d^21 - 2232576*a^7*b^17*c^5*d^19 + 11150016*a^7*b^17*c
^7*d^17 - 27336616*a^7*b^17*c^9*d^15 + 37153600*a^7*b^17*c^11*d^13 - 284610
40*a^7*b^17*c^13*d^11 + 11779808*a^7*b^17*c^15*d^9 - 2621008*a^7*b^17*c^17*
d^7 + 336688*a^7*b^17*c^19*d^5 - 17920*a^7*b^17*c^21*d^3 - 51840*a^8*b^16*c
^2*d^22 + 1512000*a^8*b^16*c^4*d^20 - 9955992*a^8*b^16*c^6*d^18 + 30289656*
a^8*b^16*c^8*d^16 - 50137600*a^8*b^16*c^10*d^14 + 46972560*a^8*b^16*c^12*d^
12 - 24199280*a^8*b^16*c^14*d^10 + 6661036*a^8*b^16*c^16*d^8 - 1058448*a^8*
b^16*c^18*d^6 + 72560*a^8*b^16*c^20*d^4 - 758400*a^9*b^15*c^3*d^21 + 707825
6*a^9*b^15*c^5*d^19 - 27336616*a^9*b^15*c^7*d^17 + 55383904*a^9*b^15*c^9*d^
15 - 63124080*a^9*b^15*c^11*d^13 + 39987520*a^9*b^15*c^13*d^11 - 13462088*a
^9*b^15*c^15*d^9 + 2478528*a^9*b^15*c^17*d^7 - 212032*a^9*b^15*c^19*d^5 + 2
63808*a^10*b^14*c^2*d^22 - 3975688*a^10*b^14*c^4*d^20 + 20019440*a^10*b^14*
c^6*d^18 - 50137600*a^10*b^14*c^8*d^16 + 69593872*a^10*b^14*c^10*d^14 - 538
54288*a^10*b^14*c^12*d^12 + 21989928*a^10*b^14*c^14*d^10 - 4591360*a^10*b^1
4*c^16*d^8 + 460480*a^10*b^14*c^18*d^6 + 1720736*a^11*b^13*c^3*d^21 - 11781
560*a^11*b^13*c^5*d^19 + 37153600*a^11*b^13*c^7*d^17 - 63124080*a^11*b^13*c
^9*d^15 + 59445728*a^11*b^13*c^11*d^13 - 29358696*a^11*b^13*c^13*d^11 + 699
5840*a^11*b^13*c^15*d^9 - 762560*a^11*b^13*c^17*d^7 - 541208*a^12*b^12*c^2*
d^22 + 5501328*a^12*b^12*c^4*d^20 - 22419600*a^12*b^12*c^6*d^18 + 46972560*
a^12*b^12*c^8*d^16 - 53854288*a^12*b^12*c^10*d^14 + 32294808*a^12*b^12*c^12
*d^12 - 8958208*a^12*b^12*c^14*d^10 + 999040*a^12*b^12*c^16*d^8 - 2002728*a
^13*b^11*c^3*d^21 + 10875200*a^13*b^11*c^5*d^19 - 28461040*a^13*b^11*c^7*d^
17 + 39987520*a^13*b^11*c^9*d^15 - 29358696*a^13*b^11*c^11*d^13 + 9722048*a
^13*b^11*c^13*d^11 - 1104320*a^13*b^11*c^15*d^9 + 547088*a^14*b^10*c^2*d^22
- 4147952*a^14*b^10*c^4*d^20 + 13887520*a^14*b^10*c^6*d^18 - 24199280*a^14
*b^10*c^8*d^16 + 21989928*a^14*b^10*c^10*d^14 - 8958208*a^14*b^10*c^12*d^12
+ 1124032*a^14*b^10*c^14*d^10 + 1210560*a^15*b^9*c^3*d^21 - 5365072*a^15*b
^9*c^5*d^19 + 11779808*a^15*b^9*c^7*d^17 - 13462088*a^15*b^9*c^9*d^15 + 699
5840*a^15*b^9*c^11*d^13 - 1104320*a^15*b^9*c^13*d^11 - 263320*a^16*b^8*c^2*
d^22 + 1586920*a^16*b^8*c^4*d^20 - 4506428*a^16*b^8*c^6*d^18 + 6661036*a^16
*b^8*c^8*d^16 - 4591360*a^16*b^8*c^10*d^14 + 999040*a^16*b^8*c^12*d^12 - 33
5040*a^17*b^7*c^3*d^21 + 1310168*a^17*b^7*c^5*d^19 - 2621008*a^17*b^7*c^7*d
^17 + 2478528*a^17*b^7*c^9*d^15 - 762560*a^17*b^7*c^11*d^13 + 44120*a^18*b^
6*c^2*d^22 - 276020*a^18*b^6*c^4*d^20 + 793756*a^18*b^6*c^6*d^18 - 1058448*
a^18*b^6*c^8*d^16 + 460480*a^18*b^6*c^10*d^14 + 37680*a^19*b^5*c^3*d^21 - 1
70968*a^19*b^5*c^5*d^19 + 336688*a^19*b^5*c^7*d^17 - 212032*a^19*b^5*c^9*d^
15 - 1564*a^20*b^4*c^2*d^22 + 21124*a^20*b^4*c^4*d^20 - 72240*a^20*b^4*c^6*
d^18 + 72560*a^20*b^4*c^8*d^16 - 288*a^21*b^3*c^3*d^21 + 8160*a^21*b^3*c^5*
d^19 - 17920*a^21*b^3*c^7*d^17 - 196*a^22*b^2*c^2*d^22 + 176*a^22*b^2*c^4*d

^20 + 3040*a^22*b^2*c^6*d^18 - 8*a*b^23*c^23*d - 8*a^23*b*c*d^23)^2/4 - (20
736*b^18*d^18 - 96768*a^2*b^16*d^18 + 173664*a^4*b^14*d^18 - 136032*a^6*b^1
2*d^18 + 31081*a^8*b^10*d^18 + 8440*a^10*b^8*d^18 + 400*a^12*b^6*d^18 - 967
68*b^18*c^2*d^16 + 173664*b^18*c^4*d^14 - 136032*b^18*c^6*d^12 + 31081*b^18
*c^8*d^10 + 8440*b^18*c^10*d^8 + 400*b^18*c^12*d^6 - 131328*a*b^17*c^3*d^15
 + 216576*a*b^17*c^5*d^13 - 141104*a*b^17*c^7*d^11 + 20260*a*b^17*c^9*d^9 +
 2800*a*b^17*c^11*d^7 - 131328*a^3*b^15*c*d^17 + 216576*a^5*b^13*c*d^17 - 1
41104*a^7*b^11*c*d^17 + 20260*a^9*b^9*c*d^17 + 2800*a^11*b^7*c*d^17 + 49593
6*a^2*b^16*c^2*d^16 - 989856*a^2*b^16*c^4*d^14 + 901948*a^2*b^16*c^6*d^12 -
 308392*a^2*b^16*c^8*d^10 - 5260*a^2*b^16*c^10*d^8 + 1600*a^2*b^16*c^12*d^6
 + 657408*a^3*b^15*c^3*d^15 - 1158992*a^3*b^15*c^5*d^13 + 838256*a^3*b^15*c
^7*d^11 - 182200*a^3*b^15*c^9*d^9 - 3200*a^3*b^15*c^11*d^7 - 989856*a^4*b^1
4*c^2*d^16 + 2185654*a^4*b^14*c^4*d^14 - 2218576*a^4*b^14*c^6*d^12 + 900624
*a^4*b^14*c^8*d^10 - 64720*a^4*b^14*c^10*d^8 + 1600*a^4*b^14*c^12*d^6 - 115
8992*a^5*b^13*c^3*d^15 + 2158808*a^5*b^13*c^5*d^13 - 1641528*a^5*b^13*c^7*d
^11 + 406880*a^5*b^13*c^9*d^9 - 17600*a^5*b^13*c^11*d^7 + 901948*a^6*b^12*c
^2*d^16 - 2218576*a^6*b^12*c^4*d^14 + 2430936*a^6*b^12*c^6*d^12 - 1026928*a
^6*b^12*c^8*d^10 + 88720*a^6*b^12*c^10*d^8 + 838256*a^7*b^11*c^3*d^15 - 164
1528*a^7*b^11*c^5*d^13 + 1206848*a^7*b^11*c^7*d^11 - 239360*a^7*b^11*c^9*d^
9 - 308392*a^8*b^10*c^2*d^16 + 900624*a^8*b^10*c^4*d^14 - 1026928*a^8*b^10*
c^6*d^12 + 354016*a^8*b^10*c^8*d^10 - 182200*a^9*b^9*c^3*d^15 + 406880*a^9*
b^9*c^5*d^13 - 239360*a^9*b^9*c^7*d^11 - 5260*a^10*b^8*c^2*d^16 - 64720*a^1
0*b^8*c^4*d^14 + 88720*a^10*b^8*c^6*d^12 - 3200*a^11*b^7*c^3*d^15 - 17600*a
^11*b^7*c^5*d^13 + 1600*a^12*b^6*c^2*d^16 + 1600*a^12*b^6*c^4*d^14 + 27648*
a*b^17*c*d^17)*(80*a^2*b^28*c^30 - 16*b^30*c^30 - 16*a^30*d^30 - 160*a^4*b^
26*c^30 + 160*a^6*b^24*c^30 - 80*a^8*b^22*c^30 + 16*a^10*b^20*c^30 + 16*a^2
0*b^10*d^30 - 80*a^22*b^8*d^30 + 160*a^24*b^6*d^30 - 160*a^26*b^4*d^30 + 80
*a^28*b^2*d^30 + 80*a^30*c^2*d^28 - 160*a^30*c^4*d^26 + 160*a^30*c^6*d^24 -
 80*a^30*c^8*d^22 + 16*a^30*c^10*d^20 + 16*b^30*c^20*d^10 - 80*b^30*c^22*d^
8 + 160*b^30*c^24*d^6 - 160*b^30*c^26*d^4 + 80*b^30*c^28*d^2 - 320*a*b^29*c
^19*d^11 + 1600*a*b^29*c^21*d^9 - 3200*a*b^29*c^23*d^7 + 3200*a*b^29*c^25*d
^5 - 1600*a*b^29*c^27*d^3 - 1600*a^3*b^27*c^29*d + 3200*a^5*b^25*c^29*d - 3
200*a^7*b^23*c^29*d + 1600*a^9*b^21*c^29*d - 320*a^11*b^19*c^29*d - 320*a^1
9*b^11*c*d^29 + 1600*a^21*b^9*c*d^29 - 3200*a^23*b^7*c*d^29 + 3200*a^25*b^5
*c*d^29 - 1600*a^27*b^3*c*d^29 - 1600*a^29*b*c^3*d^27 + 3200*a^29*b*c^5*d^2
5 - 3200*a^29*b*c^7*d^23 + 1600*a^29*b*c^9*d^21 - 320*a^29*b*c^11*d^19 + 30
40*a^2*b^28*c^18*d^12 - 15280*a^2*b^28*c^20*d^10 + 30800*a^2*b^28*c^22*d^8
 - 31200*a^2*b^28*c^24*d^6 + 16000*a^2*b^28*c^26*d^4 - 3440*a^2*b^28*c^28*d^
2 - 18240*a^3*b^27*c^17*d^13 + 92800*a^3*b^27*c^19*d^11 - 190400*a^3*b^27*c
^21*d^9 + 198400*a^3*b^27*c^23*d^7 - 107200*a^3*b^27*c^25*d^5 + 26240*a^3*b
^27*c^27*d^3 + 77520*a^4*b^26*c^16*d^14 - 402800*a^4*b^26*c^18*d^12 + 85136
0*a^4*b^26*c^20*d^10 - 928000*a^4*b^26*c^22*d^8 + 541200*a^4*b^26*c^24*d^6
 - 155120*a^4*b^26*c^26*d^4 + 16000*a^4*b^26*c^28*d^2 - 248064*a^5*b^25*c^15
*d^15 + 1331520*a^5*b^25*c^17*d^13 - 2939840*a^5*b^25*c^19*d^11 + 3408640*a
^5*b^25*c^21*d^9 - 2184320*a^5*b^25*c^23*d^7 + 736064*a^5*b^25*c^25*d^5 - 1

07200*a^5*b^25*c^27*d^3 + 620160*a^6*b^24*c^14*d^16 - 3488400*a^6*b^24*c^16
*d^14 + 8170000*a^6*b^24*c^18*d^12 - 10229760*a^6*b^24*c^20*d^10 + 7281600*
a^6*b^24*c^22*d^8 - 2863760*a^6*b^24*c^24*d^6 + 541200*a^6*b^24*c^26*d^4 -
31200*a^6*b^24*c^28*d^2 - 1240320*a^7*b^23*c^13*d^17 + 7441920*a^7*b^23*c^1
5*d^15 - 18787200*a^7*b^23*c^17*d^13 + 25721600*a^7*b^23*c^19*d^11 - 204448
00*a^7*b^23*c^21*d^9 + 9297920*a^7*b^23*c^23*d^7 - 2184320*a^7*b^23*c^25*d^
5 + 198400*a^7*b^23*c^27*d^3 + 2015520*a^8*b^22*c^12*d^18 - 13178400*a^8*b^
22*c^14*d^16 + 36434400*a^8*b^22*c^16*d^14 - 55069600*a^8*b^22*c^18*d^12 +
48989680*a^8*b^22*c^20*d^10 - 25575920*a^8*b^22*c^22*d^8 + 7281600*a^8*b^22
*c^24*d^6 - 928000*a^8*b^22*c^26*d^4 + 30800*a^8*b^22*c^28*d^2 - 2687360*a^
9*b^21*c^11*d^19 + 19638400*a^9*b^21*c^13*d^17 - 60362240*a^9*b^21*c^15*d^1
5 + 101475200*a^9*b^21*c^17*d^13 - 101172800*a^9*b^21*c^19*d^11 + 60333760*
a^9*b^21*c^21*d^9 - 20444800*a^9*b^21*c^23*d^7 + 3408640*a^9*b^21*c^25*d^5
- 190400*a^9*b^21*c^27*d^3 + 2956096*a^10*b^20*c^10*d^20 - 24858080*a^10*b^
20*c^12*d^18 + 86150560*a^10*b^20*c^14*d^16 - 162120160*a^10*b^20*c^16*d^14
 + 181463680*a^10*b^20*c^18*d^12 - 123188112*a^10*b^20*c^20*d^10 + 48989680
*a^10*b^20*c^22*d^8 - 10229760*a^10*b^20*c^24*d^6 + 851360*a^10*b^20*c^26*d
^4 - 15280*a^10*b^20*c^28*d^2 - 2687360*a^11*b^19*c^9*d^21 + 26873600*a^11*
b^19*c^11*d^19 - 106460800*a^11*b^19*c^13*d^17 + 225738240*a^11*b^19*c^15*d
^15 - 284331200*a^11*b^19*c^17*d^13 + 219166080*a^11*b^19*c^19*d^11 - 10117
2800*a^11*b^19*c^21*d^9 + 25721600*a^11*b^19*c^23*d^7 - 2939840*a^11*b^19*c
^25*d^5 + 92800*a^11*b^19*c^27*d^3 + 2015520*a^12*b^18*c^8*d^22 - 24858080*
a^12*b^18*c^10*d^20 + 114212800*a^12*b^18*c^12*d^18 - 274937600*a^12*b^18*c
^14*d^16 + 390830000*a^12*b^18*c^16*d^14 - 341426960*a^12*b^18*c^18*d^12 +
181463680*a^12*b^18*c^20*d^10 - 55069600*a^12*b^18*c^22*d^8 + 8170000*a^12*
b^18*c^24*d^6 - 402800*a^12*b^18*c^26*d^4 + 3040*a^12*b^18*c^28*d^2 - 12403
20*a^13*b^17*c^7*d^23 + 19638400*a^13*b^17*c^9*d^21 - 106460800*a^13*b^17*c
^11*d^19 + 293542400*a^13*b^17*c^13*d^17 - 472561920*a^13*b^17*c^15*d^15 +
467412160*a^13*b^17*c^17*d^13 - 284331200*a^13*b^17*c^19*d^11 + 101475200*a
^13*b^17*c^21*d^9 - 18787200*a^13*b^17*c^23*d^7 + 1331520*a^13*b^17*c^25*d^
5 - 18240*a^13*b^17*c^27*d^3 + 620160*a^14*b^16*c^6*d^24 - 13178400*a^14*b^
16*c^8*d^22 + 86150560*a^14*b^16*c^10*d^20 - 274937600*a^14*b^16*c^12*d^18
+ 503363200*a^14*b^16*c^14*d^16 - 563751280*a^14*b^16*c^16*d^14 + 390830000
*a^14*b^16*c^18*d^12 - 162120160*a^14*b^16*c^20*d^10 + 36434400*a^14*b^16*c
^22*d^8 - 3488400*a^14*b^16*c^24*d^6 + 77520*a^14*b^16*c^26*d^4 - 248064*a^
15*b^15*c^5*d^25 + 7441920*a^15*b^15*c^7*d^23 - 60362240*a^15*b^15*c^9*d^21
 + 225738240*a^15*b^15*c^11*d^19 - 472561920*a^15*b^15*c^13*d^17 + 59998412
8*a^15*b^15*c^15*d^15 - 472561920*a^15*b^15*c^17*d^13 + 225738240*a^15*b^15
*c^19*d^11 - 60362240*a^15*b^15*c^21*d^9 + 7441920*a^15*b^15*c^23*d^7 - 248
064*a^15*b^15*c^25*d^5 + 77520*a^16*b^14*c^4*d^26 - 3488400*a^16*b^14*c^6*d
^24 + 36434400*a^16*b^14*c^8*d^22 - 162120160*a^16*b^14*c^10*d^20 + 3908300
00*a^16*b^14*c^12*d^18 - 563751280*a^16*b^14*c^14*d^16 + 503363200*a^16*b^1
4*c^16*d^14 - 274937600*a^16*b^14*c^18*d^12 + 86150560*a^16*b^14*c^20*d^10
- 13178400*a^16*b^14*c^22*d^8 + 620160*a^16*b^14*c^24*d^6 - 18240*a^17*b^13
*c^3*d^27 + 1331520*a^17*b^13*c^5*d^25 - 18787200*a^17*b^13*c^7*d^23 + 1014

75200*a^17*b^13*c^9*d^21 - 284331200*a^17*b^13*c^11*d^19 + 467412160*a^17*b^13*c^13*d^17 - 472561920*a^17*b^13*c^15*d^15 + 293542400*a^17*b^13*c^17*d^13 - 106460800*a^17*b^13*c^19*d^11 + 19638400*a^17*b^13*c^21*d^9 - 1240320*a^17*b^13*c^23*d^7 + 3040*a^18*b^12*c^2*d^28 - 402800*a^18*b^12*c^4*d^26 + 8170000*a^18*b^12*c^6*d^24 - 55069600*a^18*b^12*c^8*d^22 + 181463680*a^18*b^12*c^10*d^20 - 341426960*a^18*b^12*c^12*d^18 + 390830000*a^18*b^12*c^14*d^16 - 274937600*a^18*b^12*c^16*d^14 + 114212800*a^18*b^12*c^18*d^12 - 24858080*a^18*b^12*c^20*d^10 + 2015520*a^18*b^12*c^22*d^8 + 92800*a^19*b^11*c^3*d^27 - 2939840*a^19*b^11*c^5*d^25 + 25721600*a^19*b^11*c^7*d^23 - 101172800*a^19*b^11*c^9*d^21 + 219166080*a^19*b^11*c^11*d^19 - 284331200*a^19*b^11*c^13*d^17 + 225738240*a^19*b^11*c^15*d^15 - 106460800*a^19*b^11*c^17*d^13 + 26873600*a^19*b^11*c^19*d^11 - 2687360*a^19*b^11*c^21*d^9 - 15280*a^20*b^10*c^2*d^28 + 851360*a^20*b^10*c^4*d^26 - 10229760*a^20*b^10*c^6*d^24 + 48989680*a^20*b^10*c^8*d^22 - 123188112*a^20*b^10*c^10*d^20 + 181463680*a^20*b^10*c^12*d^18 - 162120160*a^20*b^10*c^14*d^16 + 86150560*a^20*b^10*c^16*d^14 - 24858080*a^20*b^10*c^18*d^12 + 2956096*a^20*b^10*c^20*d^10 - 190400*a^21*b^9*c^3*d^27 + 3408640*a^21*b^9*c^5*d^25 - 20444800*a^21*b^9*c^7*d^23 + 60333760*a^21*b^9*c^9*d^21 - 101172800*a^21*b^9*c^11*d^19 + 101475200*a^21*b^9*c^13*d^17 - 60362240*a^21*b^9*c^15*d^15 + 19638400*a^21*b^9*c^17*d^13 - 2687360*a^21*b^9*c^19*d^11 + 30800*a^22*b^8*c^2*d^28 - 928000*a^22*b^8*c^4*d^26 + 7281600*a^22*b^8*c^6*d^24 - 25575920*a^22*b^8*c^8*d^22 + 48989680*a^22*b^8*c^10*d^20 - 55069600*a^22*b^8*c^12*d^18 + 36434400*a^22*b^8*c^14*d^16 - 13178400*a^22*b^8*c^16*d^14 + 2015520*a^22*b^8*c^18*d^12 + 198400*a^23*b^7*c^3*d^27 - 2184320*a^23*b^7*c^5*d^25 + 9297920*a^23*b^7*c^7*d^23 - 20444800*a^23*b^7*c^9*d^21 + 25721600*a^23*b^7*c^11*d^19 - 18787200*a^23*b^7*c^13*d^17 + 7441920*a^23*b^7*c^15*d^15 - 1240320*a^23*b^7*c^17*d^13 - 31200*a^24*b^6*c^2*d^28 + 541200*a^24*b^6*c^4*d^26 - 2863760*a^24*b^6*c^6*d^24 + 7281600*a^24*b^6*c^8*d^22 - 10229760*a^24*b^6*c^10*d^20 + 8170000*a^24*b^6*c^12*d^18 - 3488400*a^24*b^6*c^14*d^16 + 620160*a^24*b^6*c^16*d^14 - 107200*a^25*b^5*c^3*d^27 + 736064*a^25*b^5*c^5*d^25 - 2184320*a^25*b^5*c^7*d^23 + 3408640*a^25*b^5*c^9*d^21 - 2939840*a^25*b^5*c^11*d^19 + 1331520*a^25*b^5*c^13*d^17 - 248064*a^25*b^5*c^15*d^15 + 16000*a^26*b^4*c^2*d^28 - 155120*a^26*b^4*c^4*d^26 + 541200*a^26*b^4*c^6*d^24 - 928000*a^26*b^4*c^8*d^22 + 851360*a^26*b^4*c^10*d^20 - 402800*a^26*b^4*c^12*d^18 + 77520*a^26*b^4*c^14*d^16 + 26240*a^27*b^3*c^3*d^27 - 107200*a^27*b^3*c^5*d^25 + 198400*a^27*b^3*c^7*d^23 - 190400*a^27*b^3*c^9*d^21 + 92800*a^27*b^3*c^11*d^19 - 18240*a^27*b^3*c^13*d^17 - 3440*a^28*b^2*c^2*d^28 + 16000*a^28*b^2*c^4*d^26 - 31200*a^28*b^2*c^6*d^24 + 30800*a^28*b^2*c^8*d^22 - 15280*a^28*b^2*c^10*d^20 + 3040*a^28*b^2*c^12*d^18 + 320*a*b^29*c^29*d + 320*a^29*b*c*d^29))^(1/2) - 2*a^24*d^24 - 2*b^24*c^24 - 8*a^2*b^22*c^24 - 8*a^4*b^20*c^24 + 576*a^10*b^14*d^24 - 2784*a^12*b^12*d^24 + 5284*a^14*b^10*d^24 - 4730*a^16*b^8*d^24 + 1780*a^18*b^6*d^24 - 68*a^20*b^4*d^24 - 38*a^22*b^2*d^24 - 8*a^24*c^2*d^22 - 8*a^24*c^4*d^20 + 576*b^24*c^10*d^14 - 2784*b^24*c^12*d^12 + 5284*b^24*c^14*d^10 - 4730*b^24*c^16*d^8 + 1780*b^24*c^18*d^6 - 68*b^24*c^20*d^4 - 38*b^24*c^22*d^2 - 5760*a*b^23*c^9*d^15 + 28224*a*b^23*c^11*d^13 - 54728*a*b^23*c^13*d^1

1 + 50620*a*b^23*c^15*d^9 - 20360*a*b^23*c^17*d^7 + 1480*a*b^23*c^19*d^5 + 268*a*b^23*c^21*d^3 + 88*a^3*b^21*c^23*d + 160*a^5*b^19*c^23*d - 5760*a^9*b^15*c*d^23 + 28224*a^11*b^13*c*d^23 - 54728*a^13*b^11*c*d^23 + 50620*a^15*b^9*c*d^23 - 20360*a^17*b^7*c*d^23 + 1480*a^19*b^5*c*d^23 + 268*a^21*b^3*c*d^23 + 88*a^23*b*c^3*d^21 + 160*a^23*b*c^5*d^19 + 25920*a^2*b^22*c^8*d^16 - 131904*a^2*b^22*c^10*d^14 + 270604*a^2*b^22*c^12*d^12 - 273544*a^2*b^22*c^14*d^10 + 131660*a^2*b^22*c^16*d^8 - 22060*a^2*b^22*c^18*d^6 + 782*a^2*b^22*c^20*d^4 + 98*a^2*b^22*c^22*d^2 - 69120*a^3*b^21*c^7*d^17 + 379200*a^3*b^21*c^9*d^15 - 860368*a^3*b^21*c^11*d^13 + 1001364*a^3*b^21*c^13*d^11 - 605280*a^3*b^21*c^15*d^9 + 167520*a^3*b^21*c^17*d^7 - 18840*a^3*b^21*c^19*d^5 + 144*a^3*b^21*c^21*d^3 + 120960*a^4*b^20*c^6*d^18 - 756000*a^4*b^20*c^8*d^16 + 1987844*a^4*b^20*c^10*d^14 - 2750664*a^4*b^20*c^12*d^12 + 2073976*a^4*b^20*c^14*d^10 - 793460*a^4*b^20*c^16*d^8 + 138010*a^4*b^20*c^18*d^6 - 10562*a^4*b^20*c^20*d^4 - 88*a^4*b^20*c^22*d^2 - 145152*a^5*b^19*c^5*d^19 + 1116288*a^5*b^19*c^7*d^17 - 3539128*a^5*b^19*c^9*d^15 + 5890780*a^5*b^19*c^11*d^13 - 5437600*a^5*b^19*c^13*d^11 + 2682536*a^5*b^19*c^15*d^9 - 655084*a^5*b^19*c^17*d^7 + 85484*a^5*b^19*c^19*d^5 - 4080*a^5*b^19*c^21*d^3 + 120960*a^6*b^18*c^4*d^20 - 1266048*a^6*b^18*c^6*d^18 + 4977996*a^6*b^18*c^8*d^16 - 10009720*a^6*b^18*c^10*d^14 + 11209800*a^6*b^18*c^12*d^12 - 6943760*a^6*b^18*c^14*d^10 + 2253214*a^6*b^18*c^16*d^8 - 396878*a^6*b^18*c^18*d^6 + 36120*a^6*b^18*c^20*d^4 - 1520*a^6*b^18*c^22*d^2 - 69120*a^7*b^17*c^3*d^21 + 1116288*a^7*b^17*c^5*d^19 - 5575008*a^7*b^17*c^7*d^17 + 13668308*a^7*b^17*c^9*d^15 - 18576800*a^7*b^17*c^11*d^13 + 14230520*a^7*b^17*c^13*d^11 - 5889904*a^7*b^17*c^15*d^9 + 1310504*a^7*b^17*c^17*d^7 - 168344*a^7*b^17*c^19*d^5 + 8960*a^7*b^17*c^21*d^3 + 25920*a^8*b^16*c^2*d^22 - 756000*a^8*b^16*c^4*d^20 + 4977996*a^8*b^16*c^6*d^18 - 15144828*a^8*b^16*c^8*d^16 + 25068800*a^8*b^16*c^10*d^14 - 23486280*a^8*b^16*c^12*d^12 + 12099640*a^8*b^16*c^14*d^10 - 3330518*a^8*b^16*c^16*d^8 + 529224*a^8*b^16*c^18*d^6 - 36280*a^8*b^16*c^20*d^4 + 379200*a^9*b^15*c^3*d^21 - 3539128*a^9*b^15*c^5*d^19 + 13668308*a^9*b^15*c^7*d^17 - 27691952*a^9*b^15*c^9*d^15 + 31562040*a^9*b^15*c^11*d^13 - 19993760*a^9*b^15*c^13*d^11 + 6731044*a^9*b^15*c^15*d^9 - 1239264*a^9*b^15*c^17*d^7 + 106016*a^9*b^15*c^19*d^5 - 131904*a^10*b^14*c^2*d^22 + 1987844*a^10*b^14*c^4*d^20 - 10009720*a^10*b^14*c^6*d^18 + 25068800*a^10*b^14*c^8*d^16 - 34796936*a^10*b^14*c^10*d^14 + 26927144*a^10*b^14*c^12*d^12 - 10994964*a^10*b^14*c^14*d^10 + 2295680*a^10*b^14*c^16*d^8 - 230240*a^10*b^14*c^18*d^6 - 860368*a^11*b^13*c^3*d^21 + 5890780*a^11*b^13*c^5*d^19 - 18576800*a^11*b^13*c^7*d^17 + 31562040*a^11*b^13*c^9*d^15 - 29722864*a^11*b^13*c^11*d^13 + 14679348*a^11*b^13*c^13*d^11 - 3497920*a^11*b^13*c^15*d^9 + 381280*a^11*b^13*c^17*d^7 + 270604*a^12*b^12*c^2*d^22 - 2750664*a^12*b^12*c^4*d^20 + 11209800*a^12*b^12*c^6*d^18 - 23486280*a^12*b^12*c^8*d^16 + 26927144*a^12*b^12*c^10*d^14 - 16147404*a^12*b^12*c^12*d^12 + 4479104*a^12*b^12*c^14*d^10 - 499520*a^12*b^12*c^16*d^8 + 1001364*a^13*b^11*c^3*d^21 - 5437600*a^13*b^11*c^5*d^19 + 14230520*a^13*b^11*c^7*d^17 - 19993760*a^13*b^11*c^9*d^15 + 14679348*a^13*b^11*c^11*d^13 - 4861024*a^13*b^11*c^13*d^11 + 552160*a^13*b^11*c^15*d^9 - 273544*a^14*b^10*c^2*d^22 + 2073976*a^14*b^10*c^4*d^20 - 6943760*a^14*

b^10*c^6*d^18 + 12099640*a^14*b^10*c^8*d^16 - 10994964*a^14*b^10*c^10*d^14 + 4479104*a^14*b^10*c^12*d^12 - 562016*a^14*b^10*c^14*d^10 - 605280*a^15*b^9*c^3*d^21 + 2682536*a^15*b^9*c^5*d^19 - 5889904*a^15*b^9*c^7*d^17 + 6731044*a^15*b^9*c^9*d^15 - 3497920*a^15*b^9*c^11*d^13 + 552160*a^15*b^9*c^13*d^11 + 131660*a^16*b^8*c^2*d^22 - 793460*a^16*b^8*c^4*d^20 + 2253214*a^16*b^8*c^6*d^18 - 3330518*a^16*b^8*c^8*d^16 + 2295680*a^16*b^8*c^10*d^14 - 499520*a^16*b^8*c^12*d^12 + 167520*a^17*b^7*c^3*d^21 - 655084*a^17*b^7*c^5*d^19 + 1310504*a^17*b^7*c^7*d^17 - 1239264*a^17*b^7*c^9*d^15 + 381280*a^17*b^7*c^11*d^13 - 22060*a^18*b^6*c^2*d^22 + 138010*a^18*b^6*c^4*d^20 - 396878*a^18*b^6*c^6*d^18 + 529224*a^18*b^6*c^8*d^16 - 230240*a^18*b^6*c^10*d^14 - 18840*a^19*b^5*c^3*d^21 + 85484*a^19*b^5*c^5*d^19 - 168344*a^19*b^5*c^7*d^17 + 106016*a^19*b^5*c^9*d^15 + 782*a^20*b^4*c^2*d^22 - 10562*a^20*b^4*c^4*d^20 + 36120*a^20*b^4*c^6*d^18 - 36280*a^20*b^4*c^8*d^16 + 144*a^21*b^3*c^3*d^21 - 4080*a^21*b^3*c^5*d^19 + 8960*a^21*b^3*c^7*d^17 + 98*a^22*b^2*c^2*d^22 - 88*a^22*b^2*c^4*d^20 - 1520*a^22*b^2*c^6*d^18 + 4*a*b^23*c^23*d + 4*a^23*b*c*d^23)/(16*(5*a^2*b^28*c^30 - b^30*c^30 - a^30*d^30 - 10*a^4*b^26*c^30 + 10*a^6*b^24*c^30 - 5*a^8*b^22*c^30 + a^10*b^20*c^30 + a^20*b^10*d^30 - 5*a^22*b^8*d^30 + 10*a^24*b^6*d^30 - 10*a^26*b^4*d^30 + 5*a^28*b^2*d^30 + 5*a^30*c^2*d^28 - 10*a^30*c^4*d^26 + 10*a^30*c^6*d^24 - 5*a^30*c^8*d^22 + a^30*c^10*d^20 + b^30*c^20*d^10 - 5*b^30*c^22*d^8 + 10*b^30*c^24*d^6 - 10*b^30*c^26*d^4 + 5*b^30*c^28*d^2 - 20*a*b^29*c^19*d^11 + 100*a*b^29*c^21*d^9 - 200*a*b^29*c^23*d^7 + 200*a*b^29*c^25*d^5 - 100*a*b^29*c^27*d^3 - 100*a^3*b^27*c^29*d + 200*a^5*b^25*c^29*d - 200*a^7*b^23*c^29*d + 100*a^9*b^21*c^29*d - 20*a^11*b^19*c^29*d - 20*a^19*b^11*c*d^29 + 100*a^21*b^9*c*d^29 - 200*a^23*b^7*c*d^29 + 200*a^25*b^5*c*d^29 - 100*a^27*b^3*c*d^29 - 100*a^29*b*c^3*d^27 + 200*a^29*b*c^5*d^25 - 200*a^29*b*c^7*d^23 + 100*a^29*b*c^9*d^21 - 20*a^29*b*c^11*d^19 + 190*a^2*b^28*c^18*d^12 - 955*a^2*b^28*c^20*d^10 + 1925*a^2*b^28*c^22*d^8 - 1950*a^2*b^28*c^24*d^6 + 1000*a^2*b^28*c^26*d^4 - 215*a^2*b^28*c^28*d^2 - 1140*a^3*b^27*c^17*d^13 + 5800*a^3*b^27*c^19*d^11 - 11900*a^3*b^27*c^21*d^9 + 12400*a^3*b^27*c^23*d^7 - 6700*a^3*b^27*c^25*d^5 + 1640*a^3*b^27*c^27*d^3 + 4845*a^4*b^26*c^16*d^14 - 25175*a^4*b^26*c^18*d^12 + 53210*a^4*b^26*c^20*d^10 - 58000*a^4*b^26*c^22*d^8 + 33825*a^4*b^26*c^24*d^6 - 9695*a^4*b^26*c^26*d^4 + 1000*a^4*b^26*c^28*d^2 - 15504*a^5*b^25*c^15*d^15 + 83220*a^5*b^25*c^17*d^13 - 183740*a^5*b^25*c^19*d^11 + 213040*a^5*b^25*c^21*d^9 - 136520*a^5*b^25*c^23*d^7 + 46004*a^5*b^25*c^25*d^5 - 6700*a^5*b^25*c^27*d^3 + 38760*a^6*b^24*c^14*d^16 - 218025*a^6*b^24*c^16*d^14 + 510625*a^6*b^24*c^18*d^12 - 639360*a^6*b^24*c^20*d^10 + 455100*a^6*b^24*c^22*d^8 - 178985*a^6*b^24*c^24*d^6 + 33825*a^6*b^24*c^26*d^4 - 1950*a^6*b^24*c^28*d^2 - 77520*a^7*b^23*c^13*d^17 + 465120*a^7*b^23*c^15*d^15 - 1174200*a^7*b^23*c^17*d^13 + 1607600*a^7*b^23*c^19*d^11 - 1277800*a^7*b^23*c^21*d^9 + 581120*a^7*b^23*c^23*d^7 - 136520*a^7*b^23*c^25*d^5 + 12400*a^7*b^23*c^27*d^3 + 125970*a^8*b^22*c^12*d^18 - 823650*a^8*b^22*c^14*d^16 + 2277150*a^8*b^22*c^16*d^14 - 3441850*a^8*b^22*c^18*d^12 + 3061855*a^8*b^22*c^20*d^10 - 1598495*a^8*b^22*c^22*d^8 + 455100*a^8*b^22*c^24*d^6 - 58000*a^8*b^22*c^26*d^4 + 1925*a^8*b^22*c^28*d^2 - 167960*a^9*b^21*c^11*d^19 + 1227400*a^9*b^21*c^13*d^

17 - 3772640*a^9*b^21*c^15*d^15 + 6342200*a^9*b^21*c^17*d^13 - 6323300*a^9*
b^21*c^19*d^11 + 3770860*a^9*b^21*c^21*d^9 - 1277800*a^9*b^21*c^23*d^7 + 21
3040*a^9*b^21*c^25*d^5 - 11900*a^9*b^21*c^27*d^3 + 184756*a^10*b^20*c^10*d^
20 - 1553630*a^10*b^20*c^12*d^18 + 5384410*a^10*b^20*c^14*d^16 - 10132510*a
^10*b^20*c^16*d^14 + 11341480*a^10*b^20*c^18*d^12 - 7699257*a^10*b^20*c^20*
d^10 + 3061855*a^10*b^20*c^22*d^8 - 639360*a^10*b^20*c^24*d^6 + 53210*a^10*
b^20*c^26*d^4 - 955*a^10*b^20*c^28*d^2 - 167960*a^11*b^19*c^9*d^21 + 167960
0*a^11*b^19*c^11*d^19 - 6653800*a^11*b^19*c^13*d^17 + 14108640*a^11*b^19*c^
15*d^15 - 17770700*a^11*b^19*c^17*d^13 + 13697880*a^11*b^19*c^19*d^11 - 632
3300*a^11*b^19*c^21*d^9 + 1607600*a^11*b^19*c^23*d^7 - 183740*a^11*b^19*c^2
5*d^5 + 5800*a^11*b^19*c^27*d^3 + 125970*a^12*b^18*c^8*d^22 - 1553630*a^12*
b^18*c^10*d^20 + 7138300*a^12*b^18*c^12*d^18 - 17183600*a^12*b^18*c^14*d^16
 + 24426875*a^12*b^18*c^16*d^14 - 21339185*a^12*b^18*c^18*d^12 + 11341480*a
^12*b^18*c^20*d^10 - 3441850*a^12*b^18*c^22*d^8 + 510625*a^12*b^18*c^24*d^6
 - 25175*a^12*b^18*c^26*d^4 + 190*a^12*b^18*c^28*d^2 - 77520*a^13*b^17*c^7*
d^23 + 1227400*a^13*b^17*c^9*d^21 - 6653800*a^13*b^17*c^11*d^19 + 18346400*
a^13*b^17*c^13*d^17 - 29535120*a^13*b^17*c^15*d^15 + 29213260*a^13*b^17*c^1
7*d^13 - 17770700*a^13*b^17*c^19*d^11 + 6342200*a^13*b^17*c^21*d^9 - 117420
0*a^13*b^17*c^23*d^7 + 83220*a^13*b^17*c^25*d^5 - 1140*a^13*b^17*c^27*d^3 +
 38760*a^14*b^16*c^6*d^24 - 823650*a^14*b^16*c^8*d^22 + 5384410*a^14*b^16*c
^10*d^20 - 17183600*a^14*b^16*c^12*d^18 + 31460200*a^14*b^16*c^14*d^16 - 35
234455*a^14*b^16*c^16*d^14 + 24426875*a^14*b^16*c^18*d^12 - 10132510*a^14*b
^16*c^20*d^10 + 2277150*a^14*b^16*c^22*d^8 - 218025*a^14*b^16*c^24*d^6 + 48
45*a^14*b^16*c^26*d^4 - 15504*a^15*b^15*c^5*d^25 + 465120*a^15*b^15*c^7*d^2
3 - 3772640*a^15*b^15*c^9*d^21 + 14108640*a^15*b^15*c^11*d^19 - 29535120*a^
15*b^15*c^13*d^17 + 37499008*a^15*b^15*c^15*d^15 - 29535120*a^15*b^15*c^17*
d^13 + 14108640*a^15*b^15*c^19*d^11 - 3772640*a^15*b^15*c^21*d^9 + 465120*a
^15*b^15*c^23*d^7 - 15504*a^15*b^15*c^25*d^5 + 4845*a^16*b^14*c^4*d^26 - 21
8025*a^16*b^14*c^6*d^24 + 2277150*a^16*b^14*c^8*d^22 - 10132510*a^16*b^14*c
^10*d^20 + 24426875*a^16*b^14*c^12*d^18 - 35234455*a^16*b^14*c^14*d^16 + 31
460200*a^16*b^14*c^16*d^14 - 17183600*a^16*b^14*c^18*d^12 + 5384410*a^16*b^
14*c^20*d^10 - 823650*a^16*b^14*c^22*d^8 + 38760*a^16*b^14*c^24*d^6 - 1140*
a^17*b^13*c^3*d^27 + 83220*a^17*b^13*c^5*d^25 - 1174200*a^17*b^13*c^7*d^23
+ 6342200*a^17*b^13*c^9*d^21 - 17770700*a^17*b^13*c^11*d^19 + 29213260*a^17
*b^13*c^13*d^17 - 29535120*a^17*b^13*c^15*d^15 + 18346400*a^17*b^13*c^17*d^
13 - 6653800*a^17*b^13*c^19*d^11 + 1227400*a^17*b^13*c^21*d^9 - 77520*a^17*
b^13*c^23*d^7 + 190*a^18*b^12*c^2*d^28 - 25175*a^18*b^12*c^4*d^26 + 510625*
a^18*b^12*c^6*d^24 - 3441850*a^18*b^12*c^8*d^22 + 11341480*a^18*b^12*c^10*d
^20 - 21339185*a^18*b^12*c^12*d^18 + 24426875*a^18*b^12*c^14*d^16 - 1718360
0*a^18*b^12*c^16*d^14 + 7138300*a^18*b^12*c^18*d^12 - 1553630*a^18*b^12*c^2
0*d^10 + 125970*a^18*b^12*c^22*d^8 + 5800*a^19*b^11*c^3*d^27 - 183740*a^19*
b^11*c^5*d^25 + 1607600*a^19*b^11*c^7*d^23 - 6323300*a^19*b^11*c^9*d^21 + 1
3697880*a^19*b^11*c^11*d^19 - 17770700*a^19*b^11*c^13*d^17 + 14108640*a^19*
b^11*c^15*d^15 - 6653800*a^19*b^11*c^17*d^13 + 1679600*a^19*b^11*c^19*d^11
- 167960*a^19*b^11*c^21*d^9 - 955*a^20*b^10*c^2*d^28 + 53210*a^20*b^10*c^4*

d^26 - 639360*a^20*b^10*c^6*d^24 + 3061855*a^20*b^10*c^8*d^22 - 7699257*a^2
0*b^10*c^10*d^20 + 11341480*a^20*b^10*c^12*d^18 - 10132510*a^20*b^10*c^14*d
^16 + 5384410*a^20*b^10*c^16*d^14 - 1553630*a^20*b^10*c^18*d^12 + 184756*a^
20*b^10*c^20*d^10 - 11900*a^21*b^9*c^3*d^27 + 213040*a^21*b^9*c^5*d^25 - 12
77800*a^21*b^9*c^7*d^23 + 3770860*a^21*b^9*c^9*d^21 - 6323300*a^21*b^9*c^11
*d^19 + 6342200*a^21*b^9*c^13*d^17 - 3772640*a^21*b^9*c^15*d^15 + 1227400*a
^21*b^9*c^17*d^13 - 167960*a^21*b^9*c^19*d^11 + 1925*a^22*b^8*c^2*d^28 - 58
000*a^22*b^8*c^4*d^26 + 455100*a^22*b^8*c^6*d^24 - 1598495*a^22*b^8*c^8*d^2
2 + 3061855*a^22*b^8*c^10*d^20 - 3441850*a^22*b^8*c^12*d^18 + 2277150*a^22*
b^8*c^14*d^16 - 823650*a^22*b^8*c^16*d^14 + 125970*a^22*b^8*c^18*d^12 + 124
00*a^23*b^7*c^3*d^27 - 136520*a^23*b^7*c^5*d^25 + 581120*a^23*b^7*c^7*d^23
- 1277800*a^23*b^7*c^9*d^21 + 1607600*a^23*b^7*c^11*d^19 - 1174200*a^23*b^7
*c^13*d^17 + 465120*a^23*b^7*c^15*d^15 - 77520*a^23*b^7*c^17*d^13 - 1950*a^
24*b^6*c^2*d^28 + 33825*a^24*b^6*c^4*d^26 - 178985*a^24*b^6*c^6*d^24 + 4551
00*a^24*b^6*c^8*d^22 - 639360*a^24*b^6*c^10*d^20 + 510625*a^24*b^6*c^12*d^1
8 - 218025*a^24*b^6*c^14*d^16 + 38760*a^24*b^6*c^16*d^14 - 6700*a^25*b^5*c^
3*d^27 + 46004*a^25*b^5*c^5*d^25 - 136520*a^25*b^5*c^7*d^23 + 213040*a^25*b
^5*c^9*d^21 - 183740*a^25*b^5*c^11*d^19 + 83220*a^25*b^5*c^13*d^17 - 15504*
a^25*b^5*c^15*d^15 + 1000*a^26*b^4*c^2*d^28 - 9695*a^26*b^4*c^4*d^26 + 3382
5*a^26*b^4*c^6*d^24 - 58000*a^26*b^4*c^8*d^22 + 53210*a^26*b^4*c^10*d^20 -
25175*a^26*b^4*c^12*d^18 + 4845*a^26*b^4*c^14*d^16 + 1640*a^27*b^3*c^3*d^27
- 6700*a^27*b^3*c^5*d^25 + 12400*a^27*b^3*c^7*d^23 - 11900*a^27*b^3*c^9*d^
21 + 5800*a^27*b^3*c^11*d^19 - 1140*a^27*b^3*c^13*d^17 - 215*a^28*b^2*c^2*d
^28 + 1000*a^28*b^2*c^4*d^26 - 1950*a^28*b^2*c^6*d^24 + 1925*a^28*b^2*c^8*d
^22 - 955*a^28*b^2*c^10*d^20 + 190*a^28*b^2*c^12*d^18 + 20*a*b^29*c^29*d +
20*a^29*b*c*d^29)))^(1/2) + (4*(4*a^2*b^20*c^22 - 12*a^6*b^16*c^22 + 8*a^8*
b^14*c^22 + 4*a^22*c^2*d^20 - 12*a^22*c^6*d^16 + 8*a^22*c^8*d^14 + 48*a*b^2
1*c^11*d^11 - 212*a*b^21*c^13*d^9 + 360*a*b^21*c^15*d^7 - 276*a*b^21*c^17*d
^5 + 80*a*b^21*c^19*d^3 - 20*a^3*b^19*c^21*d - 72*a^5*b^17*c^21*d + 204*a^7
*b^15*c^21*d - 112*a^9*b^13*c^21*d + 48*a^11*b^11*c*d^21 - 212*a^13*b^9*c*d
^21 + 360*a^15*b^7*c*d^21 - 276*a^17*b^5*c*d^21 + 80*a^19*b^3*c*d^21 - 20*a
^21*b*c^3*d^19 - 72*a^21*b*c^5*d^17 + 204*a^21*b*c^7*d^15 - 112*a^21*b*c^9*
d^13 - 480*a^2*b^20*c^10*d^12 + 2160*a^2*b^20*c^12*d^10 - 3772*a^2*b^20*c^1
4*d^8 + 3020*a^2*b^20*c^16*d^6 - 960*a^2*b^20*c^18*d^4 + 28*a^2*b^20*c^20*d
^2 + 2160*a^3*b^19*c^9*d^13 - 10152*a^3*b^19*c^11*d^11 + 18888*a^3*b^19*c^1
3*d^9 - 16732*a^3*b^19*c^15*d^7 + 6588*a^3*b^19*c^17*d^5 - 732*a^3*b^19*c^1
9*d^3 - 5760*a^4*b^18*c^8*d^14 + 29360*a^4*b^18*c^10*d^12 - 60792*a^4*b^18*
c^12*d^10 + 62708*a^4*b^18*c^14*d^8 - 31892*a^4*b^18*c^16*d^6 + 6588*a^4*b^
18*c^18*d^4 - 212*a^4*b^18*c^20*d^2 + 10080*a^5*b^17*c^7*d^15 - 58860*a^5*b
^17*c^9*d^13 + 141880*a^5*b^17*c^11*d^11 - 175592*a^5*b^17*c^13*d^9 + 11374
8*a^5*b^17*c^15*d^7 - 34492*a^5*b^17*c^17*d^5 + 3308*a^5*b^17*c^19*d^3 - 12
096*a^6*b^16*c^6*d^16 + 87264*a^6*b^16*c^8*d^14 - 254340*a^6*b^16*c^10*d^12
+ 381532*a^6*b^16*c^12*d^10 - 307752*a^6*b^16*c^14*d^8 + 125568*a^6*b^16*c
^16*d^6 - 21232*a^6*b^16*c^18*d^4 + 1068*a^6*b^16*c^20*d^2 + 10080*a^7*b^15
*c^5*d^17 - 99120*a^7*b^15*c^7*d^15 + 359064*a^7*b^15*c^9*d^13 - 655076*a^7

*b^15*c^11*d^11 + 650108*a^7*b^15*c^13*d^9 - 343368*a^7*b^15*c^15*d^7 + 857
60*a^7*b^15*c^17*d^5 - 7652*a^7*b^15*c^19*d^3 - 5760*a^8*b^14*c^4*d^18 + 87
264*a^8*b^14*c^6*d^16 - 402576*a^8*b^14*c^8*d^14 + 900324*a^8*b^14*c^10*d^1
2 - 1096236*a^8*b^14*c^12*d^10 + 731392*a^8*b^14*c^14*d^8 - 247352*a^8*b^14
*c^16*d^6 + 34548*a^8*b^14*c^18*d^4 - 1612*a^8*b^14*c^20*d^2 + 2160*a^9*b^1
3*c^3*d^19 - 58860*a^9*b^13*c^5*d^17 + 359064*a^9*b^13*c^7*d^15 - 999816*a^
9*b^13*c^9*d^13 + 1494564*a^9*b^13*c^11*d^11 - 1238148*a^9*b^13*c^13*d^9 +
542272*a^9*b^13*c^15*d^7 - 109032*a^9*b^13*c^17*d^5 + 7908*a^9*b^13*c^19*d^
3 - 480*a^10*b^12*c^2*d^20 + 29360*a^10*b^12*c^4*d^18 - 254340*a^10*b^12*c^
6*d^16 + 900324*a^10*b^12*c^8*d^14 - 1656496*a^10*b^12*c^10*d^12 + 1688232*
a^10*b^12*c^12*d^10 - 934868*a^10*b^12*c^14*d^8 + 254492*a^10*b^12*c^16*d^6
 - 26952*a^10*b^12*c^18*d^4 + 728*a^10*b^12*c^20*d^2 - 10152*a^11*b^11*c^3*
d^19 + 141880*a^11*b^11*c^5*d^17 - 655076*a^11*b^11*c^7*d^15 + 1494564*a^11
*b^11*c^9*d^13 - 1870136*a^11*b^11*c^11*d^11 + 1289704*a^11*b^11*c^13*d^9 -
 455388*a^11*b^11*c^15*d^7 + 67468*a^11*b^11*c^17*d^5 - 2912*a^11*b^11*c^19
*d^3 + 2160*a^12*b^10*c^2*d^20 - 60792*a^12*b^10*c^4*d^18 + 381532*a^12*b^1
0*c^6*d^16 - 1096236*a^12*b^10*c^8*d^14 + 1688232*a^12*b^10*c^10*d^12 - 143
4728*a^12*b^10*c^12*d^10 + 639684*a^12*b^10*c^14*d^8 - 127860*a^12*b^10*c^1
6*d^6 + 8008*a^12*b^10*c^18*d^4 + 18888*a^13*b^9*c^3*d^19 - 175592*a^13*b^9
*c^5*d^17 + 650108*a^13*b^9*c^7*d^15 - 1238148*a^13*b^9*c^9*d^13 + 1289704*
a^13*b^9*c^11*d^11 - 715296*a^13*b^9*c^13*d^9 + 186564*a^13*b^9*c^15*d^7 -
16016*a^13*b^9*c^17*d^5 - 3772*a^14*b^8*c^2*d^20 + 62708*a^14*b^8*c^4*d^18
- 307752*a^14*b^8*c^6*d^16 + 731392*a^14*b^8*c^8*d^14 - 934868*a^14*b^8*c^1
0*d^12 + 639684*a^14*b^8*c^12*d^10 - 211416*a^14*b^8*c^14*d^8 + 24024*a^14*
b^8*c^16*d^6 - 16732*a^15*b^7*c^3*d^19 + 113748*a^15*b^7*c^5*d^17 - 343368*
a^15*b^7*c^7*d^15 + 542272*a^15*b^7*c^9*d^13 - 455388*a^15*b^7*c^11*d^11 +
186564*a^15*b^7*c^13*d^9 - 27456*a^15*b^7*c^15*d^7 + 3020*a^16*b^6*c^2*d^20
 - 31892*a^16*b^6*c^4*d^18 + 125568*a^16*b^6*c^6*d^16 - 247352*a^16*b^6*c^8
*d^14 + 254492*a^16*b^6*c^10*d^12 - 127860*a^16*b^6*c^12*d^10 + 24024*a^16*
b^6*c^14*d^8 + 6588*a^17*b^5*c^3*d^19 - 34492*a^17*b^5*c^5*d^17 + 85760*a^1
7*b^5*c^7*d^15 - 109032*a^17*b^5*c^9*d^13 + 67468*a^17*b^5*c^11*d^11 - 1601
6*a^17*b^5*c^13*d^9 - 960*a^18*b^4*c^2*d^20 + 6588*a^18*b^4*c^4*d^18 - 2123
2*a^18*b^4*c^6*d^16 + 34548*a^18*b^4*c^8*d^14 - 26952*a^18*b^4*c^10*d^12 +
8008*a^18*b^4*c^12*d^10 - 732*a^19*b^3*c^3*d^19 + 3308*a^19*b^3*c^5*d^17 -
7652*a^19*b^3*c^7*d^15 + 7908*a^19*b^3*c^9*d^13 - 2912*a^19*b^3*c^11*d^11 +
 28*a^20*b^2*c^2*d^20 - 212*a^20*b^2*c^4*d^18 + 1068*a^20*b^2*c^6*d^16 - 16
12*a^20*b^2*c^8*d^14 + 728*a^20*b^2*c^10*d^12))/(a^20*d^20 + b^20*c^20 - 4*
a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^
8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d
^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4
*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 4
8*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c
^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a
^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^
19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19

*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19) - (8*tan(e/2 + (f*x)/2)*(12*a^5*b^17*c^22 - 4*a^22*c*d^21 - 4*a*b^21*c^22 - 8*a^7*b^15*c^22 + 12*a^22*c^5*d^17 - 8*a^22*c^7*d^15 - 24*a*b^21*c^12*d^10 + 100*a*b^21*c^14*d^8 - 164*a*b^21*c^16*d^6 + 120*a*b^21*c^18*d^4 - 28*a*b^21*c^20*d^2 + 20*a^2*b^20*c^21*d + 72*a^4*b^18*c^21*d - 204*a^6*b^16*c^21*d + 112*a^8*b^14*c^21*d - 24*a^12*b^10*c*d^21 + 100*a^14*b^8*c*d^21 - 164*a^16*b^6*c*d^21 + 120*a^18*b^4*c*d^21 - 28*a^20*b^2*c*d^21 + 20*a^21*b*c^2*d^20 + 72*a^21*b*c^4*d^18 - 204*a^21*b*c^6* d^16 + 112*a^21*b*c^8*d^14 + 216*a^2*b^20*c^11*d^11 - 908*a^2*b^20*c^13*d^9 + 1540*a^2*b^20*c^15*d^7 - 1200*a^2*b^20*c^17*d^5 + 332*a^2*b^20*c^19*d^3 - 840*a^3*b^19*c^10*d^12 + 3672*a^3*b^19*c^12*d^10 - 6788*a^3*b^19*c^14*d^8 + 6132*a^3*b^19*c^16*d^6 - 2388*a^3*b^19*c^18*d^4 + 212*a^3*b^19*c^20*d^2 + 1800*a^4*b^18*c^9*d^13 - 8680*a^4*b^18*c^11*d^11 + 18852*a^4*b^18*c^13*d^9 - 21228*a^4*b^18*c^15*d^7 + 11692*a^4*b^18*c^17*d^5 - 2508*a^4*b^18*c^19*d^3 - 2160*a^5*b^17*c^8*d^14 + 13100*a^5*b^17*c^10*d^12 - 36820*a^5*b^17*c^12*d^10 + 53712*a^5*b^17*c^14*d^8 - 39608*a^5*b^17*c^16*d^6 + 12832*a^5*b^17*c^18*d^4 - 1068*a^5*b^17*c^20*d^2 + 1008*a^6*b^16*c^7*d^15 - 12420*a^6*b^16*c^9*d^13 + 51764*a^6*b^16*c^11*d^11 - 100128*a^6*b^16*c^13*d^9 + 96048*a^6*b^16*c^15*d^7 - 42920*a^6*b^16*c^17*d^5 + 6852*a^6*b^16*c^19*d^3 + 1008*a^7*b^15*c^6*d^16 + 5136*a^7*b^15*c^8*d^14 - 48820*a^7*b^15*c^10*d^12 + 134700*a^7*b^15*c^12*d^10 - 171472*a^7*b^15*c^14*d^8 + 103992*a^7*b^15*c^16*d^6 - 26148*a^7*b^15*c^18*d^4 + 1612*a^7*b^15*c^20*d^2 - 2160*a^8*b^14*c^5*d^17 + 5136*a^8*b^14*c^7*d^15 + 20436*a^8*b^14*c^9*d^13 - 121524*a^8*b^14*c^11*d^11 + 224888*a^8*b^14*c^13*d^9 - 186952*a^8*b^14*c^15*d^7 + 67572*a^8*b^14*c^17*d^5 - 7508*a^8*b^14*c^19*d^3 + 1800*a^9*b^13*c^4*d^18 - 12420*a^9*b^13*c^6*d^16 + 20436*a^9*b^13*c^8*d^14 + 49416*a^9*b^13*c^10*d^12 - 201552*a^9*b^13*c^12*d^10 + 245708*a^9*b^13*c^14*d^8 - 125412*a^9*b^13*c^16*d^6 + 22752*a^9*b^13*c^18*d^4 - 728*a^9*b^13*c^20*d^2 - 840*a^10*b^12*c^3*d^19 + 13100*a^10*b^12*c^5*d^17 - 48820*a^10*b^12*c^7*d^15 + 49416*a^10*b^12*c^9*d^13 + 82088*a^10*b^12*c^11*d^11 - 219092*a^10*b^12*c^13*d^9 + 168468*a^10*b^12*c^15*d^7 - 47152*a^10*b^12*c^17*d^5 + 2832*a^10*b^12*c^19*d^3 + 216*a^11*b^11*c^2*d^20 - 8680*a^11*b^11*c^4*d^18 + 51764*a^11*b^11*c^6*d^16 - 121524*a^11*b^11*c^8*d^14 + 82088*a^11*b^11*c^10*d^12 + 88712*a^11*b^11*c^12*d^10 - 153012*a^11*b^11*c^14*d^8 + 67604*a^11*b^11*c^16*d^6 - 7168*a^11*b^11*c^18*d^4 + 3672*a^12*b^10*c^3*d^19 - 36820*a^12*b^10*c^5*d^17 + 134700*a^12*b^10*c^7*d^15 - 201552*a^12*b^10*c^9*d^13 + 88712*a^12*b^10*c^11*d^11 + 62676*a^12*b^10*c^13*d^9 - 63372*a^12*b^10*c^15*d^7 + 12008*a^12*b^10*c^17*d^5 - 908*a^13*b^9*c^2*d^20 + 18852*a^13*b^9*c^4*d^18 - 100128*a^13*b^9*c^6*d^16 + 224888*a^13*b^9*c^8*d^14 - 219092*a^13*b^9*c^10*d^12 + 62676*a^13*b^9*c^12*d^10 + 26256*a^13*b^9*c^14*d^8 - 12544*a^13*b^9*c^16*d^6 - 6788*a^14*b^8*c^3*d^19 + 53712*a^14*b^8*c^5*d^17 - 171472*a^14*b^8*c^7*d^15 + 245708*a^14*b^8*c^9*d^13 - 153012*a^14*b^8*c^11*d^11 + 26256*a^14*b^8*c^13*d^9 + 5496*a^14*b^8*c^15*d^7 + 1540*a^15*b^7*c^2*d^20 - 21228*a^15*b^7*c^4*d^18 + 96048*a^15*b^7*c^6*d^16 - 186952*a^15*b^7*c^8*d^14 + 168468*a^15*b^7*c^10*d^12 - 63372*a^15*b^7*c^12*d^10 + 5496*a^15*b^7*c^14*d^8 + 6132*a^16*b^6*c^3*d^19 - 39608*a^16*b^6*c^5*d^17 + 103992*a^16*b^6*c^7*d^15 - 125412*a^16*b^6*c^9*d^13 + 67604*a^16*b^6*c^11*d^11 - 12544*a^16*b^6*c^13*d^9 - 1200*a^17*b^5*c^2*d^20 + 11692*a^17*b^5*c^4*d^18 - 42920*a^17*b^5*c^6*d^16 + 67572*a^17*b^5*c^8*d^14 - 47152*a^17*b^5*c^10*d^12 + 12008*a^17*b^5*c^12*d^10 - 2388*a^18*b^4*c^3*d^19 + 12832*a^18*b^4*c^5*d^17 - 26148*a^18*b^4*c^7*d^15 + 22752*a^18*b^4*c^9*d^13 - 7168*a^18*b^4*c^11*d^11 + 332*a^19*b^3*c^2*d^20 - 2508*a^19*b^3*c^4*d^18 + 6852*a^19*b^3*c^6*d^16 - 7508*a^19*b^3*c^8*d^14 + 2832*a^19*b^3*c^10*d^12 + 212*a^20*b^2*c^3*d^19 - 1068*a^20*b^2*c^5*d^17 +

1612*a^20*b^2*c^7*d^15 - 728*a^20*b^2*c^9*d^13))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^

11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12 *a*b^19*c^19*d - 12*a^19*b*c*d^19)) + (4*(288*a*b^18*c^6*d^13 - 1104*a*b^18 *c^8*d^11 + 1538*a*b^18*c^10*d^9 - 872*a*b^18*c^12*d^7 + 108*a*b^18*c^14*d^ 5 + 40*a*b^18*c^16*d^3 + 8*a^3*b^16*c^18*d + 8*a^5*b^14*c^18*d + 288*a^6*b^ 13*c*d^18 - 1104*a^8*b^11*c*d^18 + 1538*a^10*b^9*c*d^18 - 872*a^12*b^7*c*d^ 18 + 108*a^14*b^5*c*d^18 + 40*a^16*b^3*c*d^18 + 8*a^18*b*c^3*d^16 + 8*a^18* b*c^5*d^14 - 864*a^2*b^17*c^5*d^14 + 3216*a^2*b^17*c^7*d^12 - 4262*a^2*b^17 *c^9*d^10 + 2256*a^2*b^17*c^11*d^8 - 304*a^2*b^17*c^13*d^6 - 32*a^2*b^17*c^ 15*d^4 + 8*a^2*b^17*c^17*d^2 + 576*a^3*b^16*c^4*d^15 - 3024*a^3*b^16*c^6*d^ 13 + 6304*a^3*b^16*c^8*d^11 - 7216*a^3*b^16*c^10*d^9 + 4944*a^3*b^16*c^12*d ^7 - 1664*a^3*b^16*c^14*d^5 - 72*a^3*b^16*c^16*d^3 + 576*a^4*b^15*c^3*d^16 + 912*a^4*b^15*c^5*d^14 - 8720*a^4*b^15*c^7*d^12 + 16632*a^4*b^15*c^9*d^10 - 14888*a^4*b^15*c^11*d^8 + 6704*a^4*b^15*c^13*d^6 - 744*a^4*b^15*c^15*d^4 - 40*a^4*b^15*c^17*d^2 - 864*a^5*b^14*c^2*d^17 + 912*a^5*b^14*c^4*d^15 + 51 40*a^5*b^14*c^6*d^13 - 16080*a^5*b^14*c^8*d^11 + 23520*a^5*b^14*c^10*d^9 - 20208*a^5*b^14*c^12*d^7 + 7404*a^5*b^14*c^14*d^5 - 264*a^5*b^14*c^16*d^3 - 3024*a^6*b^13*c^3*d^16 + 5140*a^6*b^13*c^5*d^14 + 5280*a^6*b^13*c^7*d^12 - 28380*a^6*b^13*c^9*d^10 + 39792*a^6*b^13*c^11*d^8 - 22728*a^6*b^13*c^13*d^6 + 3096*a^6*b^13*c^15*d^4 - 112*a^6*b^13*c^17*d^2 + 3216*a^7*b^12*c^2*d^17 - 8720*a^7*b^12*c^4*d^15 + 5280*a^7*b^12*c^6*d^13 + 15000*a^7*b^12*c^8*d^11 - 40656*a^7*b^12*c^10*d^9 + 40296*a^7*b^12*c^12*d^7 - 12984*a^7*b^12*c^14* d^5 + 728*a^7*b^12*c^16*d^3 + 6304*a^8*b^11*c^3*d^16 - 16080*a^8*b^11*c^5*d ^14 + 15000*a^8*b^11*c^7*d^12 + 16024*a^8*b^11*c^9*d^10 - 46184*a^8*b^11*c^ 11*d^8 + 27208*a^8*b^11*c^13*d^6 - 2752*a^8*b^11*c^15*d^4 - 4262*a^9*b^10*c ^2*d^17 + 16632*a^9*b^10*c^4*d^15 - 28380*a^9*b^10*c^6*d^13 + 16024*a^9*b^1 0*c^8*d^11 + 22018*a^9*b^10*c^10*d^9 - 30104*a^9*b^10*c^12*d^7 + 6488*a^9*b ^10*c^14*d^5 - 7216*a^10*b^9*c^3*d^16 + 23520*a^10*b^9*c^5*d^14 - 40656*a^1 0*b^9*c^7*d^12 + 22018*a^10*b^9*c^9*d^10 + 13080*a^10*b^9*c^11*d^8 - 8720*a ^10*b^9*c^13*d^6 + 2256*a^11*b^8*c^2*d^17 - 14888*a^11*b^8*c^4*d^15 + 39792 *a^11*b^8*c^6*d^13 - 46184*a^11*b^8*c^8*d^11 + 13080*a^11*b^8*c^10*d^9 + 43 60*a^11*b^8*c^12*d^7 + 4944*a^12*b^7*c^3*d^16 - 20208*a^12*b^7*c^5*d^14 + 4 0296*a^12*b^7*c^7*d^12 - 30104*a^12*b^7*c^9*d^10 + 4360*a^12*b^7*c^11*d^8 - 304*a^13*b^6*c^2*d^17 + 6704*a^13*b^6*c^4*d^15 - 22728*a^13*b^6*c^6*d^13 + 27208*a^13*b^6*c^8*d^11 - 8720*a^13*b^6*c^10*d^9 - 1664*a^14*b^5*c^3*d^16 + 7404*a^14*b^5*c^5*d^14 - 12984*a^14*b^5*c^7*d^12 + 6488*a^14*b^5*c^9*d^10 - 32*a^15*b^4*c^2*d^17 - 744*a^15*b^4*c^4*d^15 + 3096*a^15*b^4*c^6*d^13 - 2752*a^15*b^4*c^8*d^11 - 72*a^16*b^3*c^3*d^16 - 264*a^16*b^3*c^5*d^14 + 728 *a^16*b^3*c^7*d^12 + 8*a^17*b^2*c^2*d^17 - 40*a^17*b^2*c^4*d^15 - 112*a^17* b^2*c^6*d^13 + 2*a*b^18*c^18*d + 2*a^18*b*c*d^18))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12 *b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^ 2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^1

7*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19) - (8*tan(e/2 + (f*x)/2)*(a*b^18*c^19 + a^19*c*d^18 + 4*a^3*b^16*c^19 + 4*a^5*b^14*c^19 + 4*a^19*c^3*d^16 + 4*a^19*c^5*d^14 - 576*a*b^18*c^5*d^14 + 2640*a*b^18*c^7*d^12 - 4732*a*b^18*c^9*d^10 + 3961*a*b^18*c^11*d^8 - 1344*a*b^18*c^13*d^6 + 14*a*b^18*c^15*d^4 + 18*a

*b^18*c^17*d^2 + 4*a^2*b^17*c^18*d - 20*a^4*b^15*c^18*d - 576*a^5*b^14*c*d^
18 - 56*a^6*b^13*c^18*d + 2640*a^7*b^12*c*d^18 - 4732*a^9*b^10*c*d^18 + 396
1*a^11*b^8*c*d^18 - 1344*a^13*b^6*c*d^18 + 14*a^15*b^4*c*d^18 + 18*a^17*b^2
*c*d^18 + 4*a^18*b*c^2*d^17 - 20*a^18*b*c^4*d^15 - 56*a^18*b*c^6*d^13 + 230
4*a^2*b^17*c^4*d^15 - 10944*a^2*b^17*c^6*d^13 + 20720*a^2*b^17*c^8*d^11 - 1
8788*a^2*b^17*c^10*d^9 + 7392*a^2*b^17*c^12*d^7 - 520*a^2*b^17*c^14*d^5 - 2
4*a^2*b^17*c^16*d^3 - 3456*a^3*b^16*c^3*d^16 + 20016*a^3*b^16*c^5*d^14 - 48
112*a^3*b^16*c^7*d^12 + 58925*a^3*b^16*c^9*d^10 - 36732*a^3*b^16*c^11*d^8 +
 9736*a^3*b^16*c^13*d^6 - 760*a^3*b^16*c^15*d^4 - 44*a^3*b^16*c^17*d^2 + 23
04*a^4*b^15*c^2*d^17 - 23424*a^4*b^15*c^4*d^15 + 81680*a^4*b^15*c^6*d^13 -
135520*a^4*b^15*c^8*d^11 + 114144*a^4*b^15*c^10*d^9 - 44168*a^4*b^15*c^12*d
^7 + 5696*a^4*b^15*c^14*d^5 - 332*a^4*b^15*c^16*d^3 + 20016*a^5*b^14*c^3*d^
16 - 99112*a^5*b^14*c^5*d^14 + 213338*a^5*b^14*c^7*d^12 - 235152*a^5*b^14*c
^9*d^10 + 130428*a^5*b^14*c^11*d^8 - 31908*a^5*b^14*c^13*d^6 + 3966*a^5*b^1
4*c^15*d^4 - 140*a^5*b^14*c^17*d^2 - 10944*a^6*b^13*c^2*d^17 + 81680*a^6*b^
13*c^4*d^15 - 243832*a^6*b^13*c^6*d^13 + 364608*a^6*b^13*c^8*d^11 - 281736*
a^6*b^13*c^10*d^9 + 103104*a^6*b^13*c^12*d^7 - 16860*a^6*b^13*c^14*d^5 + 16
60*a^6*b^13*c^16*d^3 - 48112*a^7*b^12*c^3*d^16 + 213338*a^7*b^12*c^5*d^14 -
 425832*a^7*b^12*c^7*d^12 + 434414*a^7*b^12*c^9*d^10 - 219064*a^7*b^12*c^11
*d^8 + 50732*a^7*b^12*c^13*d^6 - 7220*a^7*b^12*c^15*d^4 + 364*a^7*b^12*c^17
*d^2 + 20720*a^8*b^11*c^2*d^17 - 135520*a^8*b^11*c^4*d^15 + 364608*a^8*b^11
*c^6*d^13 - 496336*a^8*b^11*c^8*d^11 + 343832*a^8*b^11*c^10*d^9 - 111220*a^
8*b^11*c^12*d^7 + 17956*a^8*b^11*c^14*d^5 - 1376*a^8*b^11*c^16*d^3 + 58925*
a^9*b^10*c^3*d^16 - 235152*a^9*b^10*c^5*d^14 + 434414*a^9*b^10*c^7*d^12 - 4
01788*a^9*b^10*c^9*d^10 + 172673*a^9*b^10*c^11*d^8 - 31940*a^9*b^10*c^13*d^
6 + 3244*a^9*b^10*c^15*d^4 - 18788*a^10*b^9*c^2*d^17 + 114144*a^10*b^9*c^4*
d^15 - 281736*a^10*b^9*c^6*d^13 + 343832*a^10*b^9*c^8*d^11 - 197840*a^10*b^
9*c^10*d^9 + 45940*a^10*b^9*c^12*d^7 - 4760*a^10*b^9*c^14*d^5 - 36732*a^11*
b^8*c^3*d^16 + 130428*a^11*b^8*c^5*d^14 - 219064*a^11*b^8*c^7*d^12 + 172673
*a^11*b^8*c^9*d^10 - 52480*a^11*b^8*c^11*d^8 + 4580*a^11*b^8*c^13*d^6 + 739
2*a^12*b^7*c^2*d^17 - 44168*a^12*b^7*c^4*d^15 + 103104*a^12*b^7*c^6*d^13 -
111220*a^12*b^7*c^8*d^11 + 45940*a^12*b^7*c^10*d^9 - 4000*a^12*b^7*c^12*d^7
 + 9736*a^13*b^6*c^3*d^16 - 31908*a^13*b^6*c^5*d^14 + 50732*a^13*b^6*c^7*d^
12 - 31940*a^13*b^6*c^9*d^10 + 4580*a^13*b^6*c^11*d^8 - 520*a^14*b^5*c^2*d^
17 + 5696*a^14*b^5*c^4*d^15 - 16860*a^14*b^5*c^6*d^13 + 17956*a^14*b^5*c^8*
d^11 - 4760*a^14*b^5*c^10*d^9 - 760*a^15*b^4*c^3*d^16 + 3966*a^15*b^4*c^5*d
^14 - 7220*a^15*b^4*c^7*d^12 + 3244*a^15*b^4*c^9*d^10 - 24*a^16*b^3*c^2*d^1
7 - 332*a^16*b^3*c^4*d^15 + 1660*a^16*b^3*c^6*d^13 - 1376*a^16*b^3*c^8*d^11
 - 44*a^17*b^2*c^3*d^16 - 140*a^17*b^2*c^5*d^14 + 364*a^17*b^2*c^7*d^12))/(
a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20
 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^
18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^
8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^
2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^1
9*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d -

12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19)) + (16*tan(e/2 + (f*x)/2)*(864*a*b^15*c^2*d^14 - 3456*a*b^15*c^4*d^12 + 4770*a*b^15*c^6*d^10 - 2326*a*b^15*c^8*d^8 + 11*a*b^15*c^10*d^6 + 20*a*b^15*c^12*d^4 + 864*a^2*b^14*c*d^15 - 3456*a^4*b^12*c*d^15 + 4770*a^6*b^10*c*d^15 - 2326*a^8*b^8*c*d^15 + 11*a^10*b^6*c*d^15 + 20*a^12*b^4*c*d^15 - 2592*a^2*b^14*c^3*d^13 +

2322*a^2*b^14*c^5*d^11 - 412*a^2*b^14*c^7*d^9 - 37*a^2*b^14*c^9*d^7 + 170*a
^2*b^14*c^11*d^5 - 2592*a^3*b^13*c^2*d^14 + 11052*a^3*b^13*c^4*d^12 - 16128
*a^3*b^13*c^6*d^10 + 7886*a^3*b^13*c^8*d^8 - 64*a^3*b^13*c^10*d^6 + 80*a^3*
b^13*c^12*d^4 + 11052*a^4*b^12*c^3*d^13 - 11374*a^4*b^12*c^5*d^11 + 4534*a^
4*b^12*c^7*d^9 - 1976*a^4*b^12*c^9*d^7 - 40*a^4*b^12*c^11*d^5 + 2322*a^5*b^
11*c^2*d^14 - 11374*a^5*b^11*c^4*d^12 + 17846*a^5*b^11*c^6*d^10 - 6488*a^5*
b^11*c^8*d^8 - 2188*a^5*b^11*c^10*d^6 + 80*a^5*b^11*c^12*d^4 - 16128*a^6*b^
10*c^3*d^13 + 17846*a^6*b^10*c^5*d^11 - 9806*a^6*b^10*c^7*d^9 + 6220*a^6*b^
10*c^9*d^7 - 760*a^6*b^10*c^11*d^5 - 412*a^7*b^9*c^2*d^14 + 4534*a^7*b^9*c^
4*d^12 - 9806*a^7*b^9*c^6*d^10 + 1976*a^7*b^9*c^8*d^8 + 2448*a^7*b^9*c^10*d
^6 + 7886*a^8*b^8*c^3*d^13 - 6488*a^8*b^8*c^5*d^11 + 1976*a^8*b^8*c^7*d^9 -
2632*a^8*b^8*c^9*d^7 - 37*a^9*b^7*c^2*d^14 - 1976*a^9*b^7*c^4*d^12 + 6220*
a^9*b^7*c^6*d^10 - 2632*a^9*b^7*c^8*d^8 - 64*a^10*b^6*c^3*d^13 - 2188*a^10*
b^6*c^5*d^11 + 2448*a^10*b^6*c^7*d^9 + 170*a^11*b^5*c^2*d^14 - 40*a^11*b^5*
c^4*d^12 - 760*a^11*b^5*c^6*d^10 + 80*a^12*b^4*c^3*d^13 + 80*a^12*b^4*c^5*d
^11))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^
14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20
- 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 +
a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*
c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 4
8*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c
^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a
^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^
15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*
a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*
b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^
17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16
*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^1
6*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*
c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*
c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c
^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^1
4*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^1
4*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^1
3*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b
^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^
12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^
8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^
8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^
9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504
*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^
10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 363
00*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^
8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*
d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d

^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19)))*((((4*a^24*d^24 + 4*b^24*c^24 + 16*a^2*b^22*c^24 + 16*a^4*b^20*c^24 - 1152*a^10*b^14*d^24 + 5568*a^12*b^12*d^24 - 10568*a^14*b^10*d^24 + 9460*a^16*b^8*d^24 - 3560*a^18*b^6*d^24 + 136*a^20*b^4*d^24 + 76*a^22*b^2*d^24 + 16*a^24*c^2*d^22 + 16*a^24*c^4*d^20 - 1152*b^24*c^10*d^14 + 5568*b^24*c^12*d^12 - 10568*b^24*c^14*d^10 + 9460*b^24*c^16*d^8 - 3560*b^24*c^18*d^6 + 136*b^24*c^20*d^4 + 76*b^24*c^22*d^2 + 11520*a*b^23*c^9*d^15 - 56448*a*b^23*c^11*d^13 + 109456*a*b^23*c^13*d^11 - 101240*a*b^23*c^15*d^9 + 40720*a*b^23*c^17*d^7 - 2960*a*b^23*c^19*d^5 - 536*a*b^23*c^21*d^3 - 176*a^3*b^21*c^23*d - 320*a^5*b^19*c^23*d + 11520*a^9*b^15*c*d^23 - 56448*a^11*b^13*c*d^23 + 109456*a^13*b^11*c*d^23 - 101240*a^15*b^9*c*d^23 + 40720*a^17*b^7*c*d^23 - 2960*a^19*b^5*c*d^23 - 536*a^21*b^3*c*d^23 - 176*a^23*b*c^3*d^21 - 320*a^23*b*c^5*d^19 - 51840*a^2*b^22*c^8*d^16 + 263808*a^2*b^22*c^10*d^14 - 541208*a^2*b^22*c^12*d^12 + 547088*a^2*b^22*c^14*d^10 - 263320*a^2*b^22*c^16*d^8 + 44120*a^2*b^22*c^18*d^6 - 1564*a^2*b^22*c^20*d^4 - 196*a^2*b^22*c^22*d^2 + 138240*a^3*b^21*c^7*d^17 - 758400*a^3*b^21*c^9*d^15 + 1720736*a^3*b^21*c^11*d^13 - 2002728*a^3*b^21*c^13*d^11 + 1210560*a^3*b^21*c^15*d^9 - 335040*a^3*b^21*c^17*d^7 + 37680*a^3*b^21*c^19*d^5 - 288*a^3*b^21*c^21*d^3 - 241920*a^4*b^20*c^6*d^18 + 1512000*a^4*b^20*c^8*d^16 - 3975688*a^4*b^20*c^10*d^14 + 5501328*a^4*b^20*c^12*d^12 - 4147952*a^4*b^20*c^14*d^10 + 1586920*a^4*b^20*c^16*d^8 - 276020*a^4*b^20*c^18*d^6 + 21124*a^4*b^20*c^20*d^4 + 176*a^4*b^20*c^22*d^2 + 290304*a^5*b^19*c^5*d^19 - 2232576*a^5*b^19*c^7*d^17 + 7078256*a^5*b^19*c^9*d^15 - 11781560*a^5*b^19*c^11*d^13 + 10875200*a^5*b^19*c^13*d^11 - 5365072*a^5*b^19*c^15*d^9 + 1310168*a^5*b^19*c^17*d^7 - 170968*a^5*b^19*c^19*d^5 + 8160*a^5*b^19*c^21*d^3 - 241920*a^6*b^18*c^4*d^20 + 2532096*a^6*b^18*c^6*d^18 - 9955992*a^6*b^18*c^8*d^16 + 20019440*a^6*b^18*c^10*d^14 - 22419600*a^6*b^18*c^12*d^12 + 13887520*a^6*b^18*c^14*d^10 - 4506428*a^6*b^18*c^16*d^8 + 793756*a^6*b^18*c^18*d^6 - 72240*a^6*b^18*c^20*d^4 + 3040*a^6*b^18*c^22*d^2 + 138240*a^7*b^17*c^3*d^21 - 2232576*a^7*b^17*c^5*d^19 + 11150016*a^7*b^1

7*c^7*d^17 - 27336616*a^7*b^17*c^9*d^15 + 37153600*a^7*b^17*c^11*d^13 - 284
61040*a^7*b^17*c^13*d^11 + 11779808*a^7*b^17*c^15*d^9 - 2621008*a^7*b^17*c^
17*d^7 + 336688*a^7*b^17*c^19*d^5 - 17920*a^7*b^17*c^21*d^3 - 51840*a^8*b^1
6*c^2*d^22 + 1512000*a^8*b^16*c^4*d^20 - 9955992*a^8*b^16*c^6*d^18 + 302896
56*a^8*b^16*c^8*d^16 - 50137600*a^8*b^16*c^10*d^14 + 46972560*a^8*b^16*c^12
*d^12 - 24199280*a^8*b^16*c^14*d^10 + 6661036*a^8*b^16*c^16*d^8 - 1058448*a
^8*b^16*c^18*d^6 + 72560*a^8*b^16*c^20*d^4 - 758400*a^9*b^15*c^3*d^21 + 707
8256*a^9*b^15*c^5*d^19 - 27336616*a^9*b^15*c^7*d^17 + 55383904*a^9*b^15*c^9
*d^15 - 63124080*a^9*b^15*c^11*d^13 + 39987520*a^9*b^15*c^13*d^11 - 1346208
8*a^9*b^15*c^15*d^9 + 2478528*a^9*b^15*c^17*d^7 - 212032*a^9*b^15*c^19*d^5
+ 263808*a^10*b^14*c^2*d^22 - 3975688*a^10*b^14*c^4*d^20 + 20019440*a^10*b^
14*c^6*d^18 - 50137600*a^10*b^14*c^8*d^16 + 69593872*a^10*b^14*c^10*d^14 -
53854288*a^10*b^14*c^12*d^12 + 21989928*a^10*b^14*c^14*d^10 - 4591360*a^10*
b^14*c^16*d^8 + 460480*a^10*b^14*c^18*d^6 + 1720736*a^11*b^13*c^3*d^21 - 11
781560*a^11*b^13*c^5*d^19 + 37153600*a^11*b^13*c^7*d^17 - 63124080*a^11*b^1
3*c^9*d^15 + 59445728*a^11*b^13*c^11*d^13 - 29358696*a^11*b^13*c^13*d^11 +
6995840*a^11*b^13*c^15*d^9 - 762560*a^11*b^13*c^17*d^7 - 541208*a^12*b^12*c
^2*d^22 + 5501328*a^12*b^12*c^4*d^20 - 22419600*a^12*b^12*c^6*d^18 + 469725
60*a^12*b^12*c^8*d^16 - 53854288*a^12*b^12*c^10*d^14 + 32294808*a^12*b^12*c
^12*d^12 - 8958208*a^12*b^12*c^14*d^10 + 999040*a^12*b^12*c^16*d^8 - 200272
8*a^13*b^11*c^3*d^21 + 10875200*a^13*b^11*c^5*d^19 - 28461040*a^13*b^11*c^7
*d^17 + 39987520*a^13*b^11*c^9*d^15 - 29358696*a^13*b^11*c^11*d^13 + 972204
8*a^13*b^11*c^13*d^11 - 1104320*a^13*b^11*c^15*d^9 + 547088*a^14*b^10*c^2*d
^22 - 4147952*a^14*b^10*c^4*d^20 + 13887520*a^14*b^10*c^6*d^18 - 24199280*a
^14*b^10*c^8*d^16 + 21989928*a^14*b^10*c^10*d^14 - 8958208*a^14*b^10*c^12*d
^12 + 1124032*a^14*b^10*c^14*d^10 + 1210560*a^15*b^9*c^3*d^21 - 5365072*a^1
5*b^9*c^5*d^19 + 11779808*a^15*b^9*c^7*d^17 - 13462088*a^15*b^9*c^9*d^15 +
6995840*a^15*b^9*c^11*d^13 - 1104320*a^15*b^9*c^13*d^11 - 263320*a^16*b^8*c
^2*d^22 + 1586920*a^16*b^8*c^4*d^20 - 4506428*a^16*b^8*c^6*d^18 + 6661036*a
^16*b^8*c^8*d^16 - 4591360*a^16*b^8*c^10*d^14 + 999040*a^16*b^8*c^12*d^12 -
335040*a^17*b^7*c^3*d^21 + 1310168*a^17*b^7*c^5*d^19 - 2621008*a^17*b^7*c^
7*d^17 + 2478528*a^17*b^7*c^9*d^15 - 762560*a^17*b^7*c^11*d^13 + 44120*a^18
*b^6*c^2*d^22 - 276020*a^18*b^6*c^4*d^20 + 793756*a^18*b^6*c^6*d^18 - 10584
48*a^18*b^6*c^8*d^16 + 460480*a^18*b^6*c^10*d^14 + 37680*a^19*b^5*c^3*d^21
- 170968*a^19*b^5*c^5*d^19 + 336688*a^19*b^5*c^7*d^17 - 212032*a^19*b^5*c^9
*d^15 - 1564*a^20*b^4*c^2*d^22 + 21124*a^20*b^4*c^4*d^20 - 72240*a^20*b^4*c
^6*d^18 + 72560*a^20*b^4*c^8*d^16 - 288*a^21*b^3*c^3*d^21 + 8160*a^21*b^3*c
^5*d^19 - 17920*a^21*b^3*c^7*d^17 - 196*a^22*b^2*c^2*d^22 + 176*a^22*b^2*c^
4*d^20 + 3040*a^22*b^2*c^6*d^18 - 8*a*b^23*c^23*d - 8*a^23*b*c*d^23)^2/4 -
(20736*b^18*d^18 - 96768*a^2*b^16*d^18 + 173664*a^4*b^14*d^18 - 136032*a^6*
b^12*d^18 + 31081*a^8*b^10*d^18 + 8440*a^10*b^8*d^18 + 400*a^12*b^6*d^18 -
96768*b^18*c^2*d^16 + 173664*b^18*c^4*d^14 - 136032*b^18*c^6*d^12 + 31081*b
^18*c^8*d^10 + 8440*b^18*c^10*d^8 + 400*b^18*c^12*d^6 - 131328*a*b^17*c^3*d
^15 + 216576*a*b^17*c^5*d^13 - 141104*a*b^17*c^7*d^11 + 20260*a*b^17*c^9*d^
9 + 2800*a*b^17*c^11*d^7 - 131328*a^3*b^15*c*d^17 + 216576*a^5*b^13*c*d^17

- 141104*a^7*b^11*c*d^17 + 20260*a^9*b^9*c*d^17 + 2800*a^11*b^7*c*d^17 + 49
5936*a^2*b^16*c^2*d^16 - 989856*a^2*b^16*c^4*d^14 + 901948*a^2*b^16*c^6*d^1
2 - 308392*a^2*b^16*c^8*d^10 - 5260*a^2*b^16*c^10*d^8 + 1600*a^2*b^16*c^12*
d^6 + 657408*a^3*b^15*c^3*d^15 - 1158992*a^3*b^15*c^5*d^13 + 838256*a^3*b^1
5*c^7*d^11 - 182200*a^3*b^15*c^9*d^9 - 3200*a^3*b^15*c^11*d^7 - 989856*a^4*
b^14*c^2*d^16 + 2185654*a^4*b^14*c^4*d^14 - 2218576*a^4*b^14*c^6*d^12 + 900
624*a^4*b^14*c^8*d^10 - 64720*a^4*b^14*c^10*d^8 + 1600*a^4*b^14*c^12*d^6 -
1158992*a^5*b^13*c^3*d^15 + 2158808*a^5*b^13*c^5*d^13 - 1641528*a^5*b^13*c^
7*d^11 + 406880*a^5*b^13*c^9*d^9 - 17600*a^5*b^13*c^11*d^7 + 901948*a^6*b^1
2*c^2*d^16 - 2218576*a^6*b^12*c^4*d^14 + 2430936*a^6*b^12*c^6*d^12 - 102692
8*a^6*b^12*c^8*d^10 + 88720*a^6*b^12*c^10*d^8 + 838256*a^7*b^11*c^3*d^15 -
1641528*a^7*b^11*c^5*d^13 + 1206848*a^7*b^11*c^7*d^11 - 239360*a^7*b^11*c^9
*d^9 - 308392*a^8*b^10*c^2*d^16 + 900624*a^8*b^10*c^4*d^14 - 1026928*a^8*b^
10*c^6*d^12 + 354016*a^8*b^10*c^8*d^10 - 182200*a^9*b^9*c^3*d^15 + 406880*a
^9*b^9*c^5*d^13 - 239360*a^9*b^9*c^7*d^11 - 5260*a^10*b^8*c^2*d^16 - 64720*
a^10*b^8*c^4*d^14 + 88720*a^10*b^8*c^6*d^12 - 3200*a^11*b^7*c^3*d^15 - 1760
0*a^11*b^7*c^5*d^13 + 1600*a^12*b^6*c^2*d^16 + 1600*a^12*b^6*c^4*d^14 + 276
48*a*b^17*c*d^17)*(80*a^2*b^28*c^30 - 16*b^30*c^30 - 16*a^30*d^30 - 160*a^4
*b^26*c^30 + 160*a^6*b^24*c^30 - 80*a^8*b^22*c^30 + 16*a^10*b^20*c^30 + 16*
a^20*b^10*d^30 - 80*a^22*b^8*d^30 + 160*a^24*b^6*d^30 - 160*a^26*b^4*d^30 +
80*a^28*b^2*d^30 + 80*a^30*c^2*d^28 - 160*a^30*c^4*d^26 + 160*a^30*c^6*d^2
4 - 80*a^30*c^8*d^22 + 16*a^30*c^10*d^20 + 16*b^30*c^20*d^10 - 80*b^30*c^22
*d^8 + 160*b^30*c^24*d^6 - 160*b^30*c^26*d^4 + 80*b^30*c^28*d^2 - 320*a*b^2
9*c^19*d^11 + 1600*a*b^29*c^21*d^9 - 3200*a*b^29*c^23*d^7 + 3200*a*b^29*c^2
5*d^5 - 1600*a*b^29*c^27*d^3 - 1600*a^3*b^27*c^29*d + 3200*a^5*b^25*c^29*d
- 3200*a^7*b^23*c^29*d + 1600*a^9*b^21*c^29*d - 320*a^11*b^19*c^29*d - 320*
a^19*b^11*c*d^29 + 1600*a^21*b^9*c*d^29 - 3200*a^23*b^7*c*d^29 + 3200*a^25*
b^5*c*d^29 - 1600*a^27*b^3*c*d^29 - 1600*a^29*b*c^3*d^27 + 3200*a^29*b*c^5*
d^25 - 3200*a^29*b*c^7*d^23 + 1600*a^29*b*c^9*d^21 - 320*a^29*b*c^11*d^19 +
3040*a^2*b^28*c^18*d^12 - 15280*a^2*b^28*c^20*d^10 + 30800*a^2*b^28*c^22*d
^8 - 31200*a^2*b^28*c^24*d^6 + 16000*a^2*b^28*c^26*d^4 - 3440*a^2*b^28*c^28
*d^2 - 18240*a^3*b^27*c^17*d^13 + 92800*a^3*b^27*c^19*d^11 - 190400*a^3*b^2
7*c^21*d^9 + 198400*a^3*b^27*c^23*d^7 - 107200*a^3*b^27*c^25*d^5 + 26240*a^
3*b^27*c^27*d^3 + 77520*a^4*b^26*c^16*d^14 - 402800*a^4*b^26*c^18*d^12 + 85
1360*a^4*b^26*c^20*d^10 - 928000*a^4*b^26*c^22*d^8 + 541200*a^4*b^26*c^24*d
^6 - 155120*a^4*b^26*c^26*d^4 + 16000*a^4*b^26*c^28*d^2 - 248064*a^5*b^25*c
^15*d^15 + 1331520*a^5*b^25*c^17*d^13 - 2939840*a^5*b^25*c^19*d^11 + 340864
0*a^5*b^25*c^21*d^9 - 2184320*a^5*b^25*c^23*d^7 + 736064*a^5*b^25*c^25*d^5
- 107200*a^5*b^25*c^27*d^3 + 620160*a^6*b^24*c^14*d^16 - 3488400*a^6*b^24*c
^16*d^14 + 8170000*a^6*b^24*c^18*d^12 - 10229760*a^6*b^24*c^20*d^10 + 72816
00*a^6*b^24*c^22*d^8 - 2863760*a^6*b^24*c^24*d^6 + 541200*a^6*b^24*c^26*d^4
- 31200*a^6*b^24*c^28*d^2 - 1240320*a^7*b^23*c^13*d^17 + 7441920*a^7*b^23*
c^15*d^15 - 18787200*a^7*b^23*c^17*d^13 + 25721600*a^7*b^23*c^19*d^11 - 204
44800*a^7*b^23*c^21*d^9 + 9297920*a^7*b^23*c^23*d^7 - 2184320*a^7*b^23*c^25
*d^5 + 198400*a^7*b^23*c^27*d^3 + 2015520*a^8*b^22*c^12*d^18 - 13178400*a^8

*b^22*c^14*d^16 + 36434400*a^8*b^22*c^16*d^14 - 55069600*a^8*b^22*c^18*d^12
 + 48989680*a^8*b^22*c^20*d^10 - 25575920*a^8*b^22*c^22*d^8 + 7281600*a^8*b
^22*c^24*d^6 - 928000*a^8*b^22*c^26*d^4 + 30800*a^8*b^22*c^28*d^2 - 2687360
*a^9*b^21*c^11*d^19 + 19638400*a^9*b^21*c^13*d^17 - 60362240*a^9*b^21*c^15*
d^15 + 101475200*a^9*b^21*c^17*d^13 - 101172800*a^9*b^21*c^19*d^11 + 603337
60*a^9*b^21*c^21*d^9 - 20444800*a^9*b^21*c^23*d^7 + 3408640*a^9*b^21*c^25*d
^5 - 190400*a^9*b^21*c^27*d^3 + 2956096*a^10*b^20*c^10*d^20 - 24858080*a^10
*b^20*c^12*d^18 + 86150560*a^10*b^20*c^14*d^16 - 162120160*a^10*b^20*c^16*d
^14 + 181463680*a^10*b^20*c^18*d^12 - 123188112*a^10*b^20*c^20*d^10 + 48989
680*a^10*b^20*c^22*d^8 - 10229760*a^10*b^20*c^24*d^6 + 851360*a^10*b^20*c^2
6*d^4 - 15280*a^10*b^20*c^28*d^2 - 2687360*a^11*b^19*c^9*d^21 + 26873600*a^
11*b^19*c^11*d^19 - 106460800*a^11*b^19*c^13*d^17 + 225738240*a^11*b^19*c^1
5*d^15 - 284331200*a^11*b^19*c^17*d^13 + 219166080*a^11*b^19*c^19*d^11 - 10
1172800*a^11*b^19*c^21*d^9 + 25721600*a^11*b^19*c^23*d^7 - 2939840*a^11*b^1
9*c^25*d^5 + 92800*a^11*b^19*c^27*d^3 + 2015520*a^12*b^18*c^8*d^22 - 248580
80*a^12*b^18*c^10*d^20 + 114212800*a^12*b^18*c^12*d^18 - 274937600*a^12*b^1
8*c^14*d^16 + 390830000*a^12*b^18*c^16*d^14 - 341426960*a^12*b^18*c^18*d^12
 + 181463680*a^12*b^18*c^20*d^10 - 55069600*a^12*b^18*c^22*d^8 + 8170000*a^
12*b^18*c^24*d^6 - 402800*a^12*b^18*c^26*d^4 + 3040*a^12*b^18*c^28*d^2 - 12
40320*a^13*b^17*c^7*d^23 + 19638400*a^13*b^17*c^9*d^21 - 106460800*a^13*b^1
7*c^11*d^19 + 293542400*a^13*b^17*c^13*d^17 - 472561920*a^13*b^17*c^15*d^15
 + 467412160*a^13*b^17*c^17*d^13 - 284331200*a^13*b^17*c^19*d^11 + 10147520
0*a^13*b^17*c^21*d^9 - 18787200*a^13*b^17*c^23*d^7 + 1331520*a^13*b^17*c^25
*d^5 - 18240*a^13*b^17*c^27*d^3 + 620160*a^14*b^16*c^6*d^24 - 13178400*a^14
*b^16*c^8*d^22 + 86150560*a^14*b^16*c^10*d^20 - 274937600*a^14*b^16*c^12*d^
18 + 503363200*a^14*b^16*c^14*d^16 - 563751280*a^14*b^16*c^16*d^14 + 390830
000*a^14*b^16*c^18*d^12 - 162120160*a^14*b^16*c^20*d^10 + 36434400*a^14*b^1
6*c^22*d^8 - 3488400*a^14*b^16*c^24*d^6 + 77520*a^14*b^16*c^26*d^4 - 248064
*a^15*b^15*c^5*d^25 + 7441920*a^15*b^15*c^7*d^23 - 60362240*a^15*b^15*c^9*d
^21 + 225738240*a^15*b^15*c^11*d^19 - 472561920*a^15*b^15*c^13*d^17 + 59998
4128*a^15*b^15*c^15*d^15 - 472561920*a^15*b^15*c^17*d^13 + 225738240*a^15*b
^15*c^19*d^11 - 60362240*a^15*b^15*c^21*d^9 + 7441920*a^15*b^15*c^23*d^7 -
248064*a^15*b^15*c^25*d^5 + 77520*a^16*b^14*c^4*d^26 - 3488400*a^16*b^14*c^
6*d^24 + 36434400*a^16*b^14*c^8*d^22 - 162120160*a^16*b^14*c^10*d^20 + 3908
30000*a^16*b^14*c^12*d^18 - 563751280*a^16*b^14*c^14*d^16 + 503363200*a^16*
b^14*c^16*d^14 - 274937600*a^16*b^14*c^18*d^12 + 86150560*a^16*b^14*c^20*d^
10 - 13178400*a^16*b^14*c^22*d^8 + 620160*a^16*b^14*c^24*d^6 - 18240*a^17*b
^13*c^3*d^27 + 1331520*a^17*b^13*c^5*d^25 - 18787200*a^17*b^13*c^7*d^23 + 1
01475200*a^17*b^13*c^9*d^21 - 284331200*a^17*b^13*c^11*d^19 + 467412160*a^1
7*b^13*c^13*d^17 - 472561920*a^17*b^13*c^15*d^15 + 293542400*a^17*b^13*c^17
*d^13 - 106460800*a^17*b^13*c^19*d^11 + 19638400*a^17*b^13*c^21*d^9 - 12403
20*a^17*b^13*c^23*d^7 + 3040*a^18*b^12*c^2*d^28 - 402800*a^18*b^12*c^4*d^26
 + 8170000*a^18*b^12*c^6*d^24 - 55069600*a^18*b^12*c^8*d^22 + 181463680*a^1
8*b^12*c^10*d^20 - 341426960*a^18*b^12*c^12*d^18 + 390830000*a^18*b^12*c^14
*d^16 - 274937600*a^18*b^12*c^16*d^14 + 114212800*a^18*b^12*c^18*d^12 - 248

58080*a^18*b^12*c^20*d^10 + 2015520*a^18*b^12*c^22*d^8 + 92800*a^19*b^11*c^3*d^27 - 2939840*a^19*b^11*c^5*d^25 + 25721600*a^19*b^11*c^7*d^23 - 101172800*a^19*b^11*c^9*d^21 + 219166080*a^19*b^11*c^11*d^19 - 284331200*a^19*b^11*c^13*d^17 + 225738240*a^19*b^11*c^15*d^15 - 106460800*a^19*b^11*c^17*d^13 + 26873600*a^19*b^11*c^19*d^11 - 2687360*a^19*b^11*c^21*d^9 - 15280*a^20*b^10*c^2*d^28 + 851360*a^20*b^10*c^4*d^26 - 10229760*a^20*b^10*c^6*d^24 + 48989680*a^20*b^10*c^8*d^22 - 123188112*a^20*b^10*c^10*d^20 + 181463680*a^20*b^10*c^12*d^18 - 162120160*a^20*b^10*c^14*d^16 + 86150560*a^20*b^10*c^16*d^14 - 24858080*a^20*b^10*c^18*d^12 + 2956096*a^20*b^10*c^20*d^10 - 190400*a^21*b^9*c^3*d^27 + 3408640*a^21*b^9*c^5*d^25 - 20444800*a^21*b^9*c^7*d^23 + 60333760*a^21*b^9*c^9*d^21 - 101172800*a^21*b^9*c^11*d^19 + 101475200*a^21*b^9*c^13*d^17 - 60362240*a^21*b^9*c^15*d^15 + 19638400*a^21*b^9*c^17*d^13 - 2687360*a^21*b^9*c^19*d^11 + 30800*a^22*b^8*c^2*d^28 - 928000*a^22*b^8*c^4*d^26 + 7281600*a^22*b^8*c^6*d^24 - 25575920*a^22*b^8*c^8*d^22 + 48989680*a^22*b^8*c^10*d^20 - 55069600*a^22*b^8*c^12*d^18 + 36434400*a^22*b^8*c^14*d^16 - 13178400*a^22*b^8*c^16*d^14 + 2015520*a^22*b^8*c^18*d^12 + 198400*a^23*b^7*c^3*d^27 - 2184320*a^23*b^7*c^5*d^25 + 9297920*a^23*b^7*c^7*d^23 - 20444800*a^23*b^7*c^9*d^21 + 25721600*a^23*b^7*c^11*d^19 - 18787200*a^23*b^7*c^13*d^17 + 7441920*a^23*b^7*c^15*d^15 - 1240320*a^23*b^7*c^17*d^13 - 31200*a^24*b^6*c^2*d^28 + 541200*a^24*b^6*c^4*d^26 - 2863760*a^24*b^6*c^6*d^24 + 7281600*a^24*b^6*c^8*d^22 - 10229760*a^24*b^6*c^10*d^20 + 8170000*a^24*b^6*c^12*d^18 - 3488400*a^24*b^6*c^14*d^16 + 620160*a^24*b^6*c^16*d^14 - 107200*a^25*b^5*c^3*d^27 + 736064*a^25*b^5*c^5*d^25 - 2184320*a^25*b^5*c^7*d^23 + 3408640*a^25*b^5*c^9*d^21 - 2939840*a^25*b^5*c^11*d^19 + 1331520*a^25*b^5*c^13*d^17 - 248064*a^25*b^5*c^15*d^15 + 16000*a^26*b^4*c^2*d^28 - 155120*a^26*b^4*c^4*d^26 + 541200*a^26*b^4*c^6*d^24 - 928000*a^26*b^4*c^8*d^22 + 851360*a^26*b^4*c^10*d^20 - 402800*a^26*b^4*c^12*d^18 + 77520*a^26*b^4*c^14*d^16 + 26240*a^27*b^3*c^3*d^27 - 107200*a^27*b^3*c^5*d^25 + 198400*a^27*b^3*c^7*d^23 - 190400*a^27*b^3*c^9*d^21 + 92800*a^27*b^3*c^11*d^19 - 18240*a^27*b^3*c^13*d^17 - 3440*a^28*b^2*c^2*d^28 + 16000*a^28*b^2*c^4*d^26 - 31200*a^28*b^2*c^6*d^24 + 30800*a^28*b^2*c^8*d^22 - 15280*a^28*b^2*c^10*d^20 + 3040*a^28*b^2*c^12*d^18 + 320*a*b^29*c^29*d + 320*a^29*b*c*d^29))^(1/2) - 2*a^24*d^24 - 2*b^24*c^24 - 8*a^2*b^22*c^24 - 8*a^4*b^20*c^24 + 576*a^10*b^14*d^24 - 2784*a^12*b^12*d^24 + 5284*a^14*b^10*d^24 - 4730*a^16*b^8*d^24 + 1780*a^18*b^6*d^24 - 68*a^20*b^4*d^24 - 38*a^22*b^2*d^24 - 8*a^24*c^2*d^22 - 8*a^24*c^4*d^20 + 576*b^24*c^10*d^14 - 2784*b^24*c^12*d^12 + 5284*b^24*c^14*d^10 - 4730*b^24*c^16*d^8 + 1780*b^24*c^18*d^6 - 68*b^24*c^20*d^4 - 38*b^24*c^22*d^2 - 5760*a*b^23*c^9*d^15 + 28224*a*b^23*c^11*d^13 - 54728*a*b^23*c^13*d^11 + 50620*a*b^23*c^15*d^9 - 20360*a*b^23*c^17*d^7 + 1480*a*b^23*c^19*d^5 + 268*a*b^23*c^21*d^3 + 88*a^3*b^21*c^23*d + 160*a^5*b^19*c^23*d - 5760*a^9*b^15*c*d^23 + 28224*a^11*b^13*c*d^23 - 54728*a^13*b^11*c*d^23 + 50620*a^15*b^9*c*d^23 - 20360*a^17*b^7*c*d^23 + 1480*a^19*b^5*c*d^23 + 268*a^21*b^3*c*d^23 + 88*a^23*b*c^3*d^21 + 160*a^23*b*c^5*d^19 + 25920*a^2*b^22*c^8*d^16 - 131904*a^2*b^22*c^10*d^14 + 270604*a^2*b^22*c^12*d^12 - 273544*a^2*b^22*c^14*d^10 + 131660*a^2*b^22*c^16*d^8 - 22060*a^2*b^22*c^18*d^6 + 782*a^2*b^

22*c^20*d^4 + 98*a^2*b^22*c^22*d^2 - 69120*a^3*b^21*c^7*d^17 + 379200*a^3*b
^21*c^9*d^15 - 860368*a^3*b^21*c^11*d^13 + 1001364*a^3*b^21*c^13*d^11 - 605
280*a^3*b^21*c^15*d^9 + 167520*a^3*b^21*c^17*d^7 - 18840*a^3*b^21*c^19*d^5
+ 144*a^3*b^21*c^21*d^3 + 120960*a^4*b^20*c^6*d^18 - 756000*a^4*b^20*c^8*d^
16 + 1987844*a^4*b^20*c^10*d^14 - 2750664*a^4*b^20*c^12*d^12 + 2073976*a^4*
b^20*c^14*d^10 - 793460*a^4*b^20*c^16*d^8 + 138010*a^4*b^20*c^18*d^6 - 1056
2*a^4*b^20*c^20*d^4 - 88*a^4*b^20*c^22*d^2 - 145152*a^5*b^19*c^5*d^19 + 111
6288*a^5*b^19*c^7*d^17 - 3539128*a^5*b^19*c^9*d^15 + 5890780*a^5*b^19*c^11*
d^13 - 5437600*a^5*b^19*c^13*d^11 + 2682536*a^5*b^19*c^15*d^9 - 655084*a^5*
b^19*c^17*d^7 + 85484*a^5*b^19*c^19*d^5 - 4080*a^5*b^19*c^21*d^3 + 120960*a
^6*b^18*c^4*d^20 - 1266048*a^6*b^18*c^6*d^18 + 4977996*a^6*b^18*c^8*d^16 -
10009720*a^6*b^18*c^10*d^14 + 11209800*a^6*b^18*c^12*d^12 - 6943760*a^6*b^1
8*c^14*d^10 + 2253214*a^6*b^18*c^16*d^8 - 396878*a^6*b^18*c^18*d^6 + 36120*
a^6*b^18*c^20*d^4 - 1520*a^6*b^18*c^22*d^2 - 69120*a^7*b^17*c^3*d^21 + 1116
288*a^7*b^17*c^5*d^19 - 5575008*a^7*b^17*c^7*d^17 + 13668308*a^7*b^17*c^9*d
^15 - 18576800*a^7*b^17*c^11*d^13 + 14230520*a^7*b^17*c^13*d^11 - 5889904*a
^7*b^17*c^15*d^9 + 1310504*a^7*b^17*c^17*d^7 - 168344*a^7*b^17*c^19*d^5 + 8
960*a^7*b^17*c^21*d^3 + 25920*a^8*b^16*c^2*d^22 - 756000*a^8*b^16*c^4*d^20
+ 4977996*a^8*b^16*c^6*d^18 - 15144828*a^8*b^16*c^8*d^16 + 25068800*a^8*b^1
6*c^10*d^14 - 23486280*a^8*b^16*c^12*d^12 + 12099640*a^8*b^16*c^14*d^10 - 3
330518*a^8*b^16*c^16*d^8 + 529224*a^8*b^16*c^18*d^6 - 36280*a^8*b^16*c^20*d
^4 + 379200*a^9*b^15*c^3*d^21 - 3539128*a^9*b^15*c^5*d^19 + 13668308*a^9*b^
15*c^7*d^17 - 27691952*a^9*b^15*c^9*d^15 + 31562040*a^9*b^15*c^11*d^13 - 19
993760*a^9*b^15*c^13*d^11 + 6731044*a^9*b^15*c^15*d^9 - 1239264*a^9*b^15*c^
17*d^7 + 106016*a^9*b^15*c^19*d^5 - 131904*a^10*b^14*c^2*d^22 + 1987844*a^1
0*b^14*c^4*d^20 - 10009720*a^10*b^14*c^6*d^18 + 25068800*a^10*b^14*c^8*d^16
 - 34796936*a^10*b^14*c^10*d^14 + 26927144*a^10*b^14*c^12*d^12 - 10994964*a
^10*b^14*c^14*d^10 + 2295680*a^10*b^14*c^16*d^8 - 230240*a^10*b^14*c^18*d^6
 - 860368*a^11*b^13*c^3*d^21 + 5890780*a^11*b^13*c^5*d^19 - 18576800*a^11*b
^13*c^7*d^17 + 31562040*a^11*b^13*c^9*d^15 - 29722864*a^11*b^13*c^11*d^13 +
 14679348*a^11*b^13*c^13*d^11 - 3497920*a^11*b^13*c^15*d^9 + 381280*a^11*b^
13*c^17*d^7 + 270604*a^12*b^12*c^2*d^22 - 2750664*a^12*b^12*c^4*d^20 + 1120
9800*a^12*b^12*c^6*d^18 - 23486280*a^12*b^12*c^8*d^16 + 26927144*a^12*b^12*
c^10*d^14 - 16147404*a^12*b^12*c^12*d^12 + 4479104*a^12*b^12*c^14*d^10 - 49
9520*a^12*b^12*c^16*d^8 + 1001364*a^13*b^11*c^3*d^21 - 5437600*a^13*b^11*c^
5*d^19 + 14230520*a^13*b^11*c^7*d^17 - 19993760*a^13*b^11*c^9*d^15 + 146793
48*a^13*b^11*c^11*d^13 - 4861024*a^13*b^11*c^13*d^11 + 552160*a^13*b^11*c^1
5*d^9 - 273544*a^14*b^10*c^2*d^22 + 2073976*a^14*b^10*c^4*d^20 - 6943760*a^
14*b^10*c^6*d^18 + 12099640*a^14*b^10*c^8*d^16 - 10994964*a^14*b^10*c^10*d^
14 + 4479104*a^14*b^10*c^12*d^12 - 562016*a^14*b^10*c^14*d^10 - 605280*a^15
*b^9*c^3*d^21 + 2682536*a^15*b^9*c^5*d^19 - 5889904*a^15*b^9*c^7*d^17 + 673
1044*a^15*b^9*c^9*d^15 - 3497920*a^15*b^9*c^11*d^13 + 552160*a^15*b^9*c^13*
d^11 + 131660*a^16*b^8*c^2*d^22 - 793460*a^16*b^8*c^4*d^20 + 2253214*a^16*b
^8*c^6*d^18 - 3330518*a^16*b^8*c^8*d^16 + 2295680*a^16*b^8*c^10*d^14 - 4995
20*a^16*b^8*c^12*d^12 + 167520*a^17*b^7*c^3*d^21 - 655084*a^17*b^7*c^5*d^19

+ 1310504*a^17*b^7*c^7*d^17 - 1239264*a^17*b^7*c^9*d^15 + 381280*a^17*b^7*c^11*d^13 - 22060*a^18*b^6*c^2*d^22 + 138010*a^18*b^6*c^4*d^20 - 396878*a^18*b^6*c^6*d^18 + 529224*a^18*b^6*c^8*d^16 - 230240*a^18*b^6*c^10*d^14 - 18840*a^19*b^5*c^3*d^21 + 85484*a^19*b^5*c^5*d^19 - 168344*a^19*b^5*c^7*d^17 + 106016*a^19*b^5*c^9*d^15 + 782*a^20*b^4*c^2*d^22 - 10562*a^20*b^4*c^4*d^20 + 36120*a^20*b^4*c^6*d^18 - 36280*a^20*b^4*c^8*d^16 + 144*a^21*b^3*c^3*d^21 - 4080*a^21*b^3*c^5*d^19 + 8960*a^21*b^3*c^7*d^17 + 98*a^22*b^2*c^2*d^22 - 88*a^22*b^2*c^4*d^20 - 1520*a^22*b^2*c^6*d^18 + 4*a*b^23*c^23*d + 4*a^23*b*c*d^23)/(16*(5*a^2*b^28*c^30 - b^30*c^30 - a^30*d^30 - 10*a^4*b^26*c^30 + 10*a^6*b^24*c^30 - 5*a^8*b^22*c^30 + a^10*b^20*c^30 + a^20*b^10*d^30 - 5*a^22*b^8*d^30 + 10*a^24*b^6*d^30 - 10*a^26*b^4*d^30 + 5*a^28*b^2*d^30 + 5*a^30*c^2*d^28 - 10*a^30*c^4*d^26 + 10*a^30*c^6*d^24 - 5*a^30*c^8*d^22 + a^30*c^10*d^20 + b^30*c^20*d^10 - 5*b^30*c^22*d^8 + 10*b^30*c^24*d^6 - 10*b^30*c^26*d^4 + 5*b^30*c^28*d^2 - 20*a*b^29*c^19*d^11 + 100*a*b^29*c^21*d^9 - 200*a*b^29*c^23*d^7 + 200*a*b^29*c^25*d^5 - 100*a*b^29*c^27*d^3 - 100*a^3*b^27*c^29*d + 200*a^5*b^25*c^29*d - 200*a^7*b^23*c^29*d + 100*a^9*b^21*c^29*d - 20*a^11*b^19*c^29*d - 20*a^19*b^11*c*d^29 + 100*a^21*b^9*c*d^29 - 200*a^23*b^7*c*d^29 + 200*a^25*b^5*c*d^29 - 100*a^27*b^3*c*d^29 - 100*a^29*b*c^3*d^27 + 200*a^29*b*c^5*d^25 - 200*a^29*b*c^7*d^23 + 100*a^29*b*c^9*d^21 - 20*a^29*b*c^11*d^19 + 190*a^2*b^28*c^18*d^12 - 955*a^2*b^28*c^20*d^10 + 1925*a^2*b^28*c^22*d^8 - 1950*a^2*b^28*c^24*d^6 + 1000*a^2*b^28*c^26*d^4 - 215*a^2*b^28*c^28*d^2 - 1140*a^3*b^27*c^17*d^13 + 5800*a^3*b^27*c^19*d^11 - 11900*a^3*b^27*c^21*d^9 + 12400*a^3*b^27*c^23*d^7 - 6700*a^3*b^27*c^25*d^5 + 1640*a^3*b^27*c^27*d^3 + 4845*a^4*b^26*c^16*d^14 - 25175*a^4*b^26*c^18*d^12 + 53210*a^4*b^26*c^20*d^10 - 58000*a^4*b^26*c^22*d^8 + 33825*a^4*b^26*c^24*d^6 - 9695*a^4*b^26*c^26*d^4 + 1000*a^4*b^26*c^28*d^2 - 15504*a^5*b^25*c^15*d^15 + 83220*a^5*b^25*c^17*d^13 - 183740*a^5*b^25*c^19*d^11 + 213040*a^5*b^25*c^21*d^9 - 136520*a^5*b^25*c^23*d^7 + 46004*a^5*b^25*c^25*d^5 - 6700*a^5*b^25*c^27*d^3 + 38760*a^6*b^24*c^14*d^16 - 218025*a^6*b^24*c^16*d^14 + 510625*a^6*b^24*c^18*d^12 - 639360*a^6*b^24*c^20*d^10 + 455100*a^6*b^24*c^22*d^8 - 178985*a^6*b^24*c^24*d^6 + 33825*a^6*b^24*c^26*d^4 - 1950*a^6*b^24*c^28*d^2 - 77520*a^7*b^23*c^13*d^17 + 465120*a^7*b^23*c^15*d^15 - 1174200*a^7*b^23*c^17*d^13 + 1607600*a^7*b^23*c^19*d^11 - 1277800*a^7*b^23*c^21*d^9 + 581120*a^7*b^23*c^23*d^7 - 136520*a^7*b^23*c^25*d^5 + 12400*a^7*b^23*c^27*d^3 + 125970*a^8*b^22*c^12*d^18 - 823650*a^8*b^22*c^14*d^16 + 2277150*a^8*b^22*c^16*d^14 - 3441850*a^8*b^22*c^18*d^12 + 3061855*a^8*b^22*c^20*d^10 - 1598495*a^8*b^22*c^22*d^8 + 455100*a^8*b^22*c^24*d^6 - 58000*a^8*b^22*c^26*d^4 + 1925*a^8*b^22*c^28*d^2 - 167960*a^9*b^21*c^11*d^19 + 1227400*a^9*b^21*c^13*d^17 - 3772640*a^9*b^21*c^15*d^15 + 6342200*a^9*b^21*c^17*d^13 - 6323300*a^9*b^21*c^19*d^11 + 3770860*a^9*b^21*c^21*d^9 - 1277800*a^9*b^21*c^23*d^7 + 213040*a^9*b^21*c^25*d^5 - 11900*a^9*b^21*c^27*d^3 + 184756*a^10*b^20*c^10*d^20 - 1553630*a^10*b^20*c^12*d^18 + 5384410*a^10*b^20*c^14*d^16 - 10132510*a^10*b^20*c^16*d^14 + 11341480*a^10*b^20*c^18*d^12 - 7699257*a^10*b^20*c^20*d^10 + 3061855*a^10*b^20*c^22*d^8 - 639360*a^10*b^20*c^24*d^6 + 53210*a^10*b^20*c^26*d^4 - 955*a^10*b^20*c^28*d^2 - 167960*a^11*b^19*c^9*d^21 + 167

9600*a^11*b^19*c^11*d^19 - 6653800*a^11*b^19*c^13*d^17 + 14108640*a^11*b^19*c^15*d^15 - 17770700*a^11*b^19*c^17*d^13 + 13697880*a^11*b^19*c^19*d^11 - 6323300*a^11*b^19*c^21*d^9 + 1607600*a^11*b^19*c^23*d^7 - 183740*a^11*b^19*c^25*d^5 + 5800*a^11*b^19*c^27*d^3 + 125970*a^12*b^18*c^8*d^22 - 1553630*a^12*b^18*c^10*d^20 + 7138300*a^12*b^18*c^12*d^18 - 17183600*a^12*b^18*c^14*d^16 + 24426875*a^12*b^18*c^16*d^14 - 21339185*a^12*b^18*c^18*d^12 + 11341480*a^12*b^18*c^20*d^10 - 3441850*a^12*b^18*c^22*d^8 + 510625*a^12*b^18*c^24*d^6 - 25175*a^12*b^18*c^26*d^4 + 190*a^12*b^18*c^28*d^2 - 77520*a^13*b^17*c^7*d^23 + 1227400*a^13*b^17*c^9*d^21 - 6653800*a^13*b^17*c^11*d^19 + 18346400*a^13*b^17*c^13*d^17 - 29535120*a^13*b^17*c^15*d^15 + 29213260*a^13*b^17*c^17*d^13 - 17770700*a^13*b^17*c^19*d^11 + 6342200*a^13*b^17*c^21*d^9 - 1174200*a^13*b^17*c^23*d^7 + 83220*a^13*b^17*c^25*d^5 - 1140*a^13*b^17*c^27*d^3 + 38760*a^14*b^16*c^6*d^24 - 823650*a^14*b^16*c^8*d^22 + 5384410*a^14*b^16*c^10*d^20 - 17183600*a^14*b^16*c^12*d^18 + 31460200*a^14*b^16*c^14*d^16 - 35234455*a^14*b^16*c^16*d^14 + 24426875*a^14*b^16*c^18*d^12 - 10132510*a^14*b^16*c^20*d^10 + 2277150*a^14*b^16*c^22*d^8 - 218025*a^14*b^16*c^24*d^6 + 4845*a^14*b^16*c^26*d^4 - 15504*a^15*b^15*c^5*d^25 + 465120*a^15*b^15*c^7*d^23 - 3772640*a^15*b^15*c^9*d^21 + 14108640*a^15*b^15*c^11*d^19 - 29535120*a^15*b^15*c^13*d^17 + 37499008*a^15*b^15*c^15*d^15 - 29535120*a^15*b^15*c^17*d^13 + 14108640*a^15*b^15*c^19*d^11 - 3772640*a^15*b^15*c^21*d^9 + 465120*a^15*b^15*c^23*d^7 - 15504*a^15*b^15*c^25*d^5 + 4845*a^16*b^14*c^4*d^26 - 218025*a^16*b^14*c^6*d^24 + 2277150*a^16*b^14*c^8*d^22 - 10132510*a^16*b^14*c^10*d^20 + 24426875*a^16*b^14*c^12*d^18 - 35234455*a^16*b^14*c^14*d^16 + 31460200*a^16*b^14*c^16*d^14 - 17183600*a^16*b^14*c^18*d^12 + 5384410*a^16*b^14*c^20*d^10 - 823650*a^16*b^14*c^22*d^8 + 38760*a^16*b^14*c^24*d^6 - 1140*a^17*b^13*c^3*d^27 + 83220*a^17*b^13*c^5*d^25 - 1174200*a^17*b^13*c^7*d^23 + 6342200*a^17*b^13*c^9*d^21 - 17770700*a^17*b^13*c^11*d^19 + 29213260*a^17*b^13*c^13*d^17 - 29535120*a^17*b^13*c^15*d^15 + 18346400*a^17*b^13*c^17*d^13 - 6653800*a^17*b^13*c^19*d^11 + 1227400*a^17*b^13*c^21*d^9 - 77520*a^17*b^13*c^23*d^7 + 190*a^18*b^12*c^2*d^28 - 25175*a^18*b^12*c^4*d^26 + 510625*a^18*b^12*c^6*d^24 - 3441850*a^18*b^12*c^8*d^22 + 11341480*a^18*b^12*c^10*d^20 - 21339185*a^18*b^12*c^12*d^18 + 24426875*a^18*b^12*c^14*d^16 - 17183600*a^18*b^12*c^16*d^14 + 7138300*a^18*b^12*c^18*d^12 - 1553630*a^18*b^12*c^20*d^10 + 125970*a^18*b^12*c^22*d^8 + 5800*a^19*b^11*c^3*d^27 - 183740*a^19*b^11*c^5*d^25 + 1607600*a^19*b^11*c^7*d^23 - 6323300*a^19*b^11*c^9*d^21 + 13697880*a^19*b^11*c^11*d^19 - 17770700*a^19*b^11*c^13*d^17 + 14108640*a^19*b^11*c^15*d^15 - 6653800*a^19*b^11*c^17*d^13 + 1679600*a^19*b^11*c^19*d^11 - 167960*a^19*b^11*c^21*d^9 - 955*a^20*b^10*c^2*d^28 + 53210*a^20*b^10*c^4*d^26 - 639360*a^20*b^10*c^6*d^24 + 3061855*a^20*b^10*c^8*d^22 - 7699257*a^20*b^10*c^10*d^20 + 11341480*a^20*b^10*c^12*d^18 - 10132510*a^20*b^10*c^14*d^16 + 5384410*a^20*b^10*c^16*d^14 - 1553630*a^20*b^10*c^18*d^12 + 184756*a^20*b^10*c^20*d^10 - 11900*a^21*b^9*c^3*d^27 + 213040*a^21*b^9*c^5*d^25 - 1277800*a^21*b^9*c^7*d^23 + 3770860*a^21*b^9*c^9*d^21 - 6323300*a^21*b^9*c^11*d^19 + 6342200*a^21*b^9*c^13*d^17 - 3772640*a^21*b^9*c^15*d^15 + 1227400*a^21*b^9*c^17*d^13 - 167960*a^21*b^9*c^19*d^11 + 1925*a^22*b^8*c^2*d^28 -

58000*a^22*b^8*c^4*d^26 + 455100*a^22*b^8*c^6*d^24 - 1598495*a^22*b^8*c^8*d^22 + 3061855*a^22*b^8*c^10*d^20 - 3441850*a^22*b^8*c^12*d^18 + 2277150*a^22*b^8*c^14*d^16 - 823650*a^22*b^8*c^16*d^14 + 125970*a^22*b^8*c^18*d^12 + 12400*a^23*b^7*c^3*d^27 - 136520*a^23*b^7*c^5*d^25 + 581120*a^23*b^7*c^7*d^23 - 1277800*a^23*b^7*c^9*d^21 + 1607600*a^23*b^7*c^11*d^19 - 1174200*a^23*b^7*c^13*d^17 + 465120*a^23*b^7*c^15*d^15 - 77520*a^23*b^7*c^17*d^13 - 1950*a^24*b^6*c^2*d^28 + 33825*a^24*b^6*c^4*d^26 - 178985*a^24*b^6*c^6*d^24 + 455100*a^24*b^6*c^8*d^22 - 639360*a^24*b^6*c^10*d^20 + 510625*a^24*b^6*c^12*d^18 - 218025*a^24*b^6*c^14*d^16 + 38760*a^24*b^6*c^16*d^14 - 6700*a^25*b^5*c^3*d^27 + 46004*a^25*b^5*c^5*d^25 - 136520*a^25*b^5*c^7*d^23 + 213040*a^25*b^5*c^9*d^21 - 183740*a^25*b^5*c^11*d^19 + 83220*a^25*b^5*c^13*d^17 - 15504*a^25*b^5*c^15*d^15 + 1000*a^26*b^4*c^2*d^28 - 9695*a^26*b^4*c^4*d^26 + 33825*a^26*b^4*c^6*d^24 - 58000*a^26*b^4*c^8*d^22 + 53210*a^26*b^4*c^10*d^20 - 25175*a^26*b^4*c^12*d^18 + 4845*a^26*b^4*c^14*d^16 + 1640*a^27*b^3*c^3*d^27 - 6700*a^27*b^3*c^5*d^25 + 12400*a^27*b^3*c^7*d^23 - 11900*a^27*b^3*c^9*d^21 + 5800*a^27*b^3*c^11*d^19 - 1140*a^27*b^3*c^13*d^17 - 215*a^28*b^2*c^2*d^28 + 1000*a^28*b^2*c^4*d^26 - 1950*a^28*b^2*c^6*d^24 + 1925*a^28*b^2*c^8*d^22 - 955*a^28*b^2*c^10*d^20 + 190*a^28*b^2*c^12*d^18 + 20*a*b^29*c^29*d + 20*a^29*b*c*d^29)))^(1/2)*2i)/f - ((a^7*d^7 + b^7*c^7 - 4*a^2*b^5*c^7 + a^3*b^4*d^7 - 2*a^5*b^2*d^7 - 4*a^7*c^2*d^5 + b^7*c^3*d^4 - 2*b^7*c^5*d^2 - 7*a*b^6*c^2*d^5 + 14*a*b^6*c^4*d^3 - 7*a^2*b^5*c*d^6 + 10*a^3*b^4*c^6*d + 14*a^4*b^3*c*d^6 + 10*a^6*b*c^3*d^4 + 6*a^2*b^5*c^3*d^4 + 8*a^2*b^5*c^5*d^2 + 6*a^3*b^4*c^2*d^5 - 20*a^3*b^4*c^4*d^3 - 20*a^4*b^3*c^3*d^4 + 8*a^5*b^2*c^2*d^5 - 7*a*b^6*c^6*d - 7*a^6*b*c*d^6)/((a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)*(a^4*c^4 + a^4*d^4 + b^4*c^4 + b^4*d^4 - 2*a^2*b^2*c^4 - 2*a^2*b^2*d^4 - 2*a^4*c^2*d^2 - 2*b^4*c^2*d^2 + 4*a^2*b^2*c^2*d^2)) - (tan(e/2 + (f*x)/2)*(11*a^2*b^6*c^8 - 2*b^8*c^8 - 2*a^8*d^8 - 2*a^4*b^4*d^8 + 4*a^6*b^2*d^8 + 11*a^8*c^2*d^6 - 2*b^8*c^4*d^4 + 4*b^8*c^6*d^2 + 16*a*b^7*c^3*d^5 - 32*a*b^7*c^5*d^3 + 16*a^3*b^5*c*d^7 - 13*a^3*b^5*c^7*d - 32*a^5*b^3*c*d^7 - 13*a^7*b*c^3*d^5 + 56*a^2*b^6*c^2*d^6 - 85*a^2*b^6*c^4*d^4 + 6*a^2*b^6*c^6*d^2 - 26*a^3*b^5*c^3*d^5 + 26*a^3*b^5*c^5*d^3 - 85*a^4*b^4*c^2*d^6 + 160*a^4*b^4*c^4*d^4 - 40*a^4*b^4*c^6*d^2 + 26*a^5*b^3*c^3*d^5 + 6*a^6*b^2*c^2*d^6 - 40*a^6*b^2*c^4*d^4 + 16*a*b^7*c^7*d + 16*a^7*b*c*d^7))/(a*c*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)*(a^4*c^4 + a^4*d^4 + b^4*c^4 + b^4*d^4 - 2*a^2*b^2*c^4 - 2*a^2*b^2*d^4 - 2*a^4*c^2*d^2 - 2*b^4*c^2*d^2 + 4*a^2*b^2*c^2*d^2)) + (tan(e/2 + (f*x)/2)^5*(6*a*b^8*c^9 + 8*a^8*b*d^9 + 6*a^9*c*d^8 + 8*b^9*c^8*d - 21*a^3*b^6*c^9 + 8*a^4*b^5*d^9 - 16*a^6*b^3*d^9 - 21*a^9*c^3*d^6 + 8*b^9*c^4*d^5 - 16*b^9*c^6*d^3 - 48*a*b^8*c^3*d^6 + 102*a*b^8*c^5*d^4 - 60*a*b^8*c^7*d^2 - 64*a^2*b^7*c^8*d - 48*a^3*b^6*c*d^8 + 35*a^4*b^5*c^8*d + 102*a^5*b^4*c*d^8 - 60*a^7*b^2*c*d^8 - 64*a^8*b*c^2*d^7 + 35*a^8*b*c^4*d^5 - 64*a^2*b^7*c^2*d^7 + 44*a^2*b^7*c^4*d^5 + 96*a^2*b^7*c^6*d^3 + 64*a^3*b^6*c^3*d^6 - 45*a^3*b^6*c^5*d^4 + 74*a^3*b^6*c^7*d^2 + 44*a^4*b^5*c^2*d^7 - 106*a^4*b^5*c^4*d^5 - 26*a^4*b^5*c^6*d^3 - 45*a^5*b^4*c^3*d^6 - 160*a^5*b^4*c^5*d^4 + 40*a^5*b^4*c^7*d^2 + 96*a^6*b^3*c^2*d^7 - 26*a^6*b^3*c^4*d^5 + 74*a^7*b^2*c^3*d^

6 + 40*a^7*b^2*c^5*d^4))/(a^2*c^2*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)*(a^4*c^4 + a^4*d^4 + b^4*c^4 + b^4*d^4 - 2*a^2*b^2*c^4 - 2*a^2*b^2*d^4 - 2*a^4*c^2*d^2 - 2*b^4*c^2*d^2 + 4*a^2*b^2*c^2*d^2)) + (tan(e/2 + (f*x)/2)^3*(6*a*b^8*c^9 + 8*a^8*b*d^9 + 6*a^9*c*d^8 + 8*b^9*c^8*d - 27*a^3*b^6*c^9 + 8*a^4*b^5*d^9 - 16*a^6*b^3*d^9 - 27*a^9*c^3*d^6 + 8*b^9*c^4*d^5 - 16*b^9*c^6*d^3 - 48*a*b^8*c^3*d^6 + 102*a*b^8*c^5*d^4 - 60*a*b^8*c^7*d^2 - 72*a^2*b^7*c^8*d - 48*a^3*b^6*c*d^8 + 37*a^4*b^5*c^8*d + 102*a^5*b^4*c*d^8 - 60*a^7*b^2*c*d^8 - 72*a^8*b*c^2*d^7 + 37*a^8*b*c^4*d^5 - 160*a^2*b^7*c^2*d^7 + 204*a^2*b^7*c^4*d^5 + 64*a^2*b^7*c^6*d^3 - 40*a^3*b^6*c^3*d^6 + 93*a^3*b^6*c^5*d^4 + 34*a^3*b^6*c^7*d^2 + 204*a^4*b^5*c^2*d^7 - 390*a^4*b^5*c^4*d^5 + 42*a^4*b^5*c^6*d^3 + 93*a^5*b^4*c^3*d^6 - 320*a^5*b^4*c^5*d^4 + 80*a^5*b^4*c^7*d^2 + 64*a^6*b^3*c^2*d^7 + 42*a^6*b^3*c^4*d^5 + 34*a^7*b^2*c^3*d^6 + 80*a^7*b^2*c^5*d^4))/(a^2*c^2*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)*(a^4*c^4 + a^4*d^4 + b^4*c^4 + b^4*d^4 - 2*a^2*b^2*c^4 - 2*a^2*b^2*d^4 - 2*a^4*c^2*d^2 - 2*b^4*c^2*d^2 + 4*a^2*b^2*c^2*d^2)) - (tan(e/2 + (f*x)/2)^6*(7*a^2*b^7*c^9 - 2*b^9*c^9 - 2*a^9*d^9 + 4*a^4*b^5*c^9 - 2*a^5*b^4*d^9 + 4*a^7*b^2*d^9 + 7*a^9*c^2*d^7 + 4*a^9*c^4*d^5 - 2*b^9*c^5*d^4 + 4*b^9*c^7*d^2 + 6*a*b^8*c^4*d^5 - 12*a*b^8*c^6*d^3 + 7*a^3*b^6*c^8*d + 6*a^4*b^5*c*d^8 - 10*a^5*b^4*c^8*d - 12*a^6*b^3*c*d^8 + 7*a^8*b*c^3*d^6 - 10*a^8*b*c^5*d^4 + 32*a^2*b^7*c^3*d^6 - 57*a^2*b^7*c^5*d^4 + 18*a^2*b^7*c^7*d^2 + 32*a^3*b^6*c^2*d^7 - 37*a^3*b^6*c^4*d^5 - 14*a^3*b^6*c^6*d^3 - 37*a^4*b^5*c^3*d^6 + 82*a^4*b^5*c^5*d^4 - 52*a^4*b^5*c^7*d^2 - 57*a^5*b^4*c^2*d^7 + 82*a^5*b^4*c^4*d^5 + 20*a^5*b^4*c^6*d^3 - 14*a^6*b^3*c^3*d^6 + 20*a^6*b^3*c^5*d^4 + 18*a^7*b^2*c^2*d^7 - 52*a^7*b^2*c^4*d^5 + 6*a*b^8*c^8*d + 6*a^8*b*c*d^8))/(a^2*c^2*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)*(a^4*c^4 + a^4*d^4 + b^4*c^4 + b^4*d^4 - 2*a^2*b^2*c^4 - 2*a^2*b^2*d^4 - 2*a^4*c^2*d^2 - 2*b^4*c^2*d^2 + 4*a^2*b^2*c^2*d^2)) - (tan(e/2 + (f*x)/2)^2*(5*a^2*b^7*c^9 - 2*b^9*c^9 - 2*a^9*d^9 + 12*a^4*b^5*c^9 - 2*a^5*b^4*d^9 + 4*a^7*b^2*d^9 + 5*a^9*c^2*d^7 + 12*a^9*c^4*d^5 - 2*b^9*c^5*d^4 + 4*b^9*c^7*d^2 + 6*a*b^8*c^4*d^5 - 12*a*b^8*c^6*d^3 + 45*a^3*b^6*c^8*d + 6*a^4*b^5*c*d^8 - 30*a^5*b^4*c^8*d - 12*a^6*b^3*c*d^8 + 45*a^8*b*c^3*d^6 - 30*a^8*b*c^5*d^4 + 104*a^2*b^7*c^3*d^6 - 187*a^2*b^7*c^5*d^4 + 66*a^2*b^7*c^7*d^2 + 104*a^3*b^6*c^2*d^7 - 111*a^3*b^6*c^4*d^5 - 62*a^3*b^6*c^6*d^3 - 111*a^4*b^5*c^3*d^6 + 262*a^4*b^5*c^5*d^4 - 124*a^4*b^5*c^7*d^2 - 187*a^5*b^4*c^2*d^7 + 262*a^5*b^4*c^4*d^5 + 20*a^5*b^4*c^6*d^3 - 62*a^6*b^3*c^3*d^6 + 20*a^6*b^3*c^5*d^4 + 66*a^7*b^2*c^2*d^7 - 124*a^7*b^2*c^4*d^5 + 6*a*b^8*c^8*d + 6*a^8*b*c*d^8))/(a^2*c^2*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)*(a^4*c^4 + a^4*d^4 + b^4*c^4 + b^4*d^4 - 2*a^2*b^2*c^4 - 2*a^2*b^2*d^4 - 2*a^4*c^2*d^2 - 2*b^4*c^2*d^2 + 4*a^2*b^2*c^2*d^2)) - (tan(e/2 + (f*x)/2)^7*(5*a^2*b^6*c^8 - 2*b^8*c^8 - 2*a^8*d^8 - 2*a^4*b^4*d^8 + 4*a^6*b^2*d^8 + 5*a^8*c^2*d^6 - 2*b^8*c^4*d^4 + 4*b^8*c^6*d^2 + 8*a*b^7*c^3*d^5 - 16*a*b^7*c^5*d^3 + 8*a^3*b^5*c*d^7 - 11*a^3*b^5*c^7*d - 16*a^5*b^3*c*d^7 - 11*a^7*b*c^3*d^5 + 5*a^2*b^6*c^4*d^4 - 10*a^2*b^6*c^6*d^2 - 22*a^3*b^5*c^3*d^5 + 22*a^3*b^5*c^5*d^3 + 5*a^4*b^4*c^2*d^6 + 22*a^5*b^3*c^3*d^5 - 10*a^6*b^2*c^2*d^6 + 8*a*b^7*c^

7*d + 8*a^7*b*c*d^7))/(a*c*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)*(a^4*c^4 + a^4*d^4 + b^4*c^4 + b^4*d^4 - 2*a^2*b^2*c^4 - 2*a^2*b^2*d^4 - 2*a^4*c^2*d^2 - 2*b^4*c^2*d^2 + 4*a^2*b^2*c^2*d^2)) + (tan(e/2 + (f*x)/2)^4*(3*a^2*c^2 + 4*a^2*d^2 + 4*b^2*c^2 + 8*b^2*d^2 + 16*a*b*c*d)*(a^7*d^7 + b^7*c^7 - 4*a^2*b^5*c^7 + a^3*b^4*d^7 - 2*a^5*b^2*d^7 - 4*a^7*c^2*d^5 + b^7*c^3*d^4 - 2*b^7*c^5*d^2 - 7*a*b^6*c^2*d^5 + 14*a*b^6*c^4*d^3 - 7*a^2*b^5*c*d^6 + 10*a^3*b^4*c^6*d + 14*a^4*b^3*c*d^6 + 10*a^6*b*c^3*d^4 + 6*a^2*b^5*c^3*d^4 + 8*a^2*b^5*c^5*d^2 + 6*a^3*b^4*c^2*d^5 - 20*a^3*b^4*c^4*d^3 - 20*a^4*b^3*c^3*d^4 + 8*a^5*b^2*c^2*d^5 - 7*a*b^6*c^6*d - 7*a^6*b*c*d^6))/(a^2*c^2*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)*(a^4*c^4 + a^4*d^4 + b^4*c^4 + b^4*d^4 - 2*a^2*b^2*c^4 - 2*a^2*b^2*d^4 - 2*a^4*c^2*d^2 - 2*b^4*c^2*d^2 + 4*a^2*b^2*c^2*d^2)))/(f*(tan(e/2 + (f*x)/2)*(4*a*b*c^2 + 4*a^2*c*d) + tan(e/2 + (f*x)/2)^4*(6*a^2*c^2 + 8*a^2*d^2 + 8*b^2*c^2 + 16*b^2*d^2 + 32*a*b*c*d) + tan(e/2 + (f*x)/2)^2*(4*a^2*c^2 + 4*a^2*d^2 + 4*b^2*c^2 + 16*a*b*c*d) + tan(e/2 + (f*x)/2)^6*(4*a^2*c^2 + 4*a^2*d^2 + 4*b^2*c^2 + 16*a*b*c*d) + tan(e/2 + (f*x)/2)^3*(12*a*b*c^2 + 16*a*b*d^2 + 12*a^2*c*d + 16*b^2*c*d) + tan(e/2 + (f*x)/2)^5*(12*a*b*c^2 + 16*a*b*d^2 + 12*a^2*c*d + 16*b^2*c*d) + tan(e/2 + (f*x)/2)^7*(4*a*b*c^2 + 4*a^2*c*d) + a^2*c^2 + a^2*c^2*tan(e/2 + (f*x)/2)^8)) + (atan(((-(((4*a^24*d^24 + 4*b^24*c^24 + 16*a^2*b^22*c^24 + 16*a^4*b^20*c^24 - 1152*a^10*b^14*d^24 + 5568*a^12*b^12*d^24 - 10568*a^14*b^10*d^24 + 9460*a^16*b^8*d^24 - 3560*a^18*b^6*d^24 + 136*a^20*b^4*d^24 + 76*a^22*b^2*d^24 + 16*a^24*c^2*d^22 + 16*a^24*c^4*d^20 - 1152*b^24*c^10*d^14 + 5568*b^24*c^12*d^12 - 10568*b^24*c^14*d^10 + 9460*b^24*c^16*d^8 - 3560*b^24*c^18*d^6 + 136*b^24*c^20*d^4 + 76*b^24*c^22*d^2 + 11520*a*b^23*c^9*d^15 - 56448*a*b^23*c^11*d^13 + 109456*a*b^23*c^13*d^11 - 101240*a*b^23*c^15*d^9 + 40720*a*b^23*c^17*d^7 - 2960*a*b^23*c^19*d^5 - 536*a*b^23*c^21*d^3 - 176*a^3*b^21*c^23*d - 320*a^5*b^19*c^23*d + 11520*a^9*b^15*c*d^23 - 56448*a^11*b^13*c*d^23 + 109456*a^13*b^11*c*d^23 - 101240*a^15*b^9*c*d^23 + 40720*a^17*b^7*c*d^23 - 2960*a^19*b^5*c*d^23 - 536*a^21*b^3*c*d^23 - 176*a^23*b*c^3*d^21 - 320*a^23*b*c^5*d^19 - 51840*a^2*b^22*c^8*d^16 + 263808*a^2*b^22*c^10*d^14 - 541208*a^2*b^22*c^12*d^12 + 547088*a^2*b^22*c^14*d^10 - 263320*a^2*b^22*c^16*d^8 + 44120*a^2*b^22*c^18*d^6 - 1564*a^2*b^22*c^20*d^4 - 196*a^2*b^22*c^22*d^2 + 138240*a^3*b^21*c^7*d^17 - 758400*a^3*b^21*c^9*d^15 + 1720736*a^3*b^21*c^11*d^13 - 2002728*a^3*b^21*c^13*d^11 + 1210560*a^3*b^21*c^15*d^9 - 335040*a^3*b^21*c^17*d^7 + 37680*a^3*b^21*c^19*d^5 - 288*a^3*b^21*c^21*d^3 - 241920*a^4*b^20*c^6*d^18 + 1512000*a^4*b^20*c^8*d^16 - 3975688*a^4*b^20*c^10*d^14 + 5501328*a^4*b^20*c^12*d^12 - 4147952*a^4*b^20*c^14*d^10 + 1586920*a^4*b^20*c^16*d^8 - 276020*a^4*b^20*c^18*d^6 + 21124*a^4*b^20*c^20*d^4 + 176*a^4*b^20*c^22*d^2 + 290304*a^5*b^19*c^5*d^19 - 2232576*a^5*b^19*c^7*d^17 + 7078256*a^5*b^19*c^9*d^15 - 11781560*a^5*b^19*c^11*d^13 + 10875200*a^5*b^19*c^13*d^11 - 5365072*a^5*b^19*c^15*d^9 + 1310168*a^5*b^19*c^17*d^7 - 170968*a^5*b^19*c^19*d^5 + 8160*a^5*b^19*c^21*d^3 - 241920*a^6*b^18*c^4*d^20 + 2532096*a^6*b^18*c^6*d^18 - 9955992*a^6*b^18*c^8*d^16 + 20019440*a^6*b^18*c^10*d^14 - 22419600*a^6*b^18*c^12*d^12 + 13887520*a^6*b^18*c^14*d^10 - 4506428*a^6*b^18*c^16*d^

8 + 793756*a^6*b^18*c^18*d^6 - 72240*a^6*b^18*c^20*d^4 + 3040*a^6*b^18*c^22
*d^2 + 138240*a^7*b^17*c^3*d^21 - 2232576*a^7*b^17*c^5*d^19 + 11150016*a^7*
b^17*c^7*d^17 - 27336616*a^7*b^17*c^9*d^15 + 37153600*a^7*b^17*c^11*d^13 -
28461040*a^7*b^17*c^13*d^11 + 11779808*a^7*b^17*c^15*d^9 - 2621008*a^7*b^17
*c^17*d^7 + 336688*a^7*b^17*c^19*d^5 - 17920*a^7*b^17*c^21*d^3 - 51840*a^8*
b^16*c^2*d^22 + 1512000*a^8*b^16*c^4*d^20 - 9955992*a^8*b^16*c^6*d^18 + 302
89656*a^8*b^16*c^8*d^16 - 50137600*a^8*b^16*c^10*d^14 + 46972560*a^8*b^16*c
^12*d^12 - 24199280*a^8*b^16*c^14*d^10 + 6661036*a^8*b^16*c^16*d^8 - 105844
8*a^8*b^16*c^18*d^6 + 72560*a^8*b^16*c^20*d^4 - 758400*a^9*b^15*c^3*d^21 +
7078256*a^9*b^15*c^5*d^19 - 27336616*a^9*b^15*c^7*d^17 + 55383904*a^9*b^15*
c^9*d^15 - 63124080*a^9*b^15*c^11*d^13 + 39987520*a^9*b^15*c^13*d^11 - 1346
2088*a^9*b^15*c^15*d^9 + 2478528*a^9*b^15*c^17*d^7 - 212032*a^9*b^15*c^19*d
^5 + 263808*a^10*b^14*c^2*d^22 - 3975688*a^10*b^14*c^4*d^20 + 20019440*a^10
*b^14*c^6*d^18 - 50137600*a^10*b^14*c^8*d^16 + 69593872*a^10*b^14*c^10*d^14
 - 53854288*a^10*b^14*c^12*d^12 + 21989928*a^10*b^14*c^14*d^10 - 4591360*a^
10*b^14*c^16*d^8 + 460480*a^10*b^14*c^18*d^6 + 1720736*a^11*b^13*c^3*d^21 -
 11781560*a^11*b^13*c^5*d^19 + 37153600*a^11*b^13*c^7*d^17 - 63124080*a^11*
b^13*c^9*d^15 + 59445728*a^11*b^13*c^11*d^13 - 29358696*a^11*b^13*c^13*d^11
 + 6995840*a^11*b^13*c^15*d^9 - 762560*a^11*b^13*c^17*d^7 - 541208*a^12*b^1
2*c^2*d^22 + 5501328*a^12*b^12*c^4*d^20 - 22419600*a^12*b^12*c^6*d^18 + 469
72560*a^12*b^12*c^8*d^16 - 53854288*a^12*b^12*c^10*d^14 + 32294808*a^12*b^1
2*c^12*d^12 - 8958208*a^12*b^12*c^14*d^10 + 999040*a^12*b^12*c^16*d^8 - 200
2728*a^13*b^11*c^3*d^21 + 10875200*a^13*b^11*c^5*d^19 - 28461040*a^13*b^11*
c^7*d^17 + 39987520*a^13*b^11*c^9*d^15 - 29358696*a^13*b^11*c^11*d^13 + 972
2048*a^13*b^11*c^13*d^11 - 1104320*a^13*b^11*c^15*d^9 + 547088*a^14*b^10*c^
2*d^22 - 4147952*a^14*b^10*c^4*d^20 + 13887520*a^14*b^10*c^6*d^18 - 2419928
0*a^14*b^10*c^8*d^16 + 21989928*a^14*b^10*c^10*d^14 - 8958208*a^14*b^10*c^1
2*d^12 + 1124032*a^14*b^10*c^14*d^10 + 1210560*a^15*b^9*c^3*d^21 - 5365072*
a^15*b^9*c^5*d^19 + 11779808*a^15*b^9*c^7*d^17 - 13462088*a^15*b^9*c^9*d^15
 + 6995840*a^15*b^9*c^11*d^13 - 1104320*a^15*b^9*c^13*d^11 - 263320*a^16*b^
8*c^2*d^22 + 1586920*a^16*b^8*c^4*d^20 - 4506428*a^16*b^8*c^6*d^18 + 666103
6*a^16*b^8*c^8*d^16 - 4591360*a^16*b^8*c^10*d^14 + 999040*a^16*b^8*c^12*d^1
2 - 335040*a^17*b^7*c^3*d^21 + 1310168*a^17*b^7*c^5*d^19 - 2621008*a^17*b^7
*c^7*d^17 + 2478528*a^17*b^7*c^9*d^15 - 762560*a^17*b^7*c^11*d^13 + 44120*a
^18*b^6*c^2*d^22 - 276020*a^18*b^6*c^4*d^20 + 793756*a^18*b^6*c^6*d^18 - 10
58448*a^18*b^6*c^8*d^16 + 460480*a^18*b^6*c^10*d^14 + 37680*a^19*b^5*c^3*d^
21 - 170968*a^19*b^5*c^5*d^19 + 336688*a^19*b^5*c^7*d^17 - 212032*a^19*b^5*
c^9*d^15 - 1564*a^20*b^4*c^2*d^22 + 21124*a^20*b^4*c^4*d^20 - 72240*a^20*b^
4*c^6*d^18 + 72560*a^20*b^4*c^8*d^16 - 288*a^21*b^3*c^3*d^21 + 8160*a^21*b^
3*c^5*d^19 - 17920*a^21*b^3*c^7*d^17 - 196*a^22*b^2*c^2*d^22 + 176*a^22*b^2
*c^4*d^20 + 3040*a^22*b^2*c^6*d^18 - 8*a*b^23*c^23*d - 8*a^23*b*c*d^23)^2/4
 - (20736*b^18*d^18 - 96768*a^2*b^16*d^18 + 173664*a^4*b^14*d^18 - 136032*a
^6*b^12*d^18 + 31081*a^8*b^10*d^18 + 8440*a^10*b^8*d^18 + 400*a^12*b^6*d^18
 - 96768*b^18*c^2*d^16 + 173664*b^18*c^4*d^14 - 136032*b^18*c^6*d^12 + 3108
1*b^18*c^8*d^10 + 8440*b^18*c^10*d^8 + 400*b^18*c^12*d^6 - 131328*a*b^17*c^

3*d^15 + 216576*a*b^17*c^5*d^13 - 141104*a*b^17*c^7*d^11 + 20260*a*b^17*c^9
*d^9 + 2800*a*b^17*c^11*d^7 - 131328*a^3*b^15*c*d^17 + 216576*a^5*b^13*c*d^
17 - 141104*a^7*b^11*c*d^17 + 20260*a^9*b^9*c*d^17 + 2800*a^11*b^7*c*d^17 +
495936*a^2*b^16*c^2*d^16 - 989856*a^2*b^16*c^4*d^14 + 901948*a^2*b^16*c^6*
d^12 - 308392*a^2*b^16*c^8*d^10 - 5260*a^2*b^16*c^10*d^8 + 1600*a^2*b^16*c^
12*d^6 + 657408*a^3*b^15*c^3*d^15 - 1158992*a^3*b^15*c^5*d^13 + 838256*a^3*
b^15*c^7*d^11 - 182200*a^3*b^15*c^9*d^9 - 3200*a^3*b^15*c^11*d^7 - 989856*a
^4*b^14*c^2*d^16 + 2185654*a^4*b^14*c^4*d^14 - 2218576*a^4*b^14*c^6*d^12 +
900624*a^4*b^14*c^8*d^10 - 64720*a^4*b^14*c^10*d^8 + 1600*a^4*b^14*c^12*d^6
- 1158992*a^5*b^13*c^3*d^15 + 2158808*a^5*b^13*c^5*d^13 - 1641528*a^5*b^13
*c^7*d^11 + 406880*a^5*b^13*c^9*d^9 - 17600*a^5*b^13*c^11*d^7 + 901948*a^6*
b^12*c^2*d^16 - 2218576*a^6*b^12*c^4*d^14 + 2430936*a^6*b^12*c^6*d^12 - 102
6928*a^6*b^12*c^8*d^10 + 88720*a^6*b^12*c^10*d^8 + 838256*a^7*b^11*c^3*d^15
- 1641528*a^7*b^11*c^5*d^13 + 1206848*a^7*b^11*c^7*d^11 - 239360*a^7*b^11*
c^9*d^9 - 308392*a^8*b^10*c^2*d^16 + 900624*a^8*b^10*c^4*d^14 - 1026928*a^8
*b^10*c^6*d^12 + 354016*a^8*b^10*c^8*d^10 - 182200*a^9*b^9*c^3*d^15 + 40688
0*a^9*b^9*c^5*d^13 - 239360*a^9*b^9*c^7*d^11 - 5260*a^10*b^8*c^2*d^16 - 647
20*a^10*b^8*c^4*d^14 + 88720*a^10*b^8*c^6*d^12 - 3200*a^11*b^7*c^3*d^15 - 1
7600*a^11*b^7*c^5*d^13 + 1600*a^12*b^6*c^2*d^16 + 1600*a^12*b^6*c^4*d^14 +
27648*a*b^17*c*d^17)*(80*a^2*b^28*c^30 - 16*b^30*c^30 - 16*a^30*d^30 - 160*
a^4*b^26*c^30 + 160*a^6*b^24*c^30 - 80*a^8*b^22*c^30 + 16*a^10*b^20*c^30 +
16*a^20*b^10*d^30 - 80*a^22*b^8*d^30 + 160*a^24*b^6*d^30 - 160*a^26*b^4*d^3
0 + 80*a^28*b^2*d^30 + 80*a^30*c^2*d^28 - 160*a^30*c^4*d^26 + 160*a^30*c^6*
d^24 - 80*a^30*c^8*d^22 + 16*a^30*c^10*d^20 + 16*b^30*c^20*d^10 - 80*b^30*c
^22*d^8 + 160*b^30*c^24*d^6 - 160*b^30*c^26*d^4 + 80*b^30*c^28*d^2 - 320*a*
b^29*c^19*d^11 + 1600*a*b^29*c^21*d^9 - 3200*a*b^29*c^23*d^7 + 3200*a*b^29*
c^25*d^5 - 1600*a*b^29*c^27*d^3 - 1600*a^3*b^27*c^29*d + 3200*a^5*b^25*c^29
*d - 3200*a^7*b^23*c^29*d + 1600*a^9*b^21*c^29*d - 320*a^11*b^19*c^29*d - 3
20*a^19*b^11*c*d^29 + 1600*a^21*b^9*c*d^29 - 3200*a^23*b^7*c*d^29 + 3200*a^
25*b^5*c*d^29 - 1600*a^27*b^3*c*d^29 - 1600*a^29*b*c^3*d^27 + 3200*a^29*b*c
^5*d^25 - 3200*a^29*b*c^7*d^23 + 1600*a^29*b*c^9*d^21 - 320*a^29*b*c^11*d^1
9 + 3040*a^2*b^28*c^18*d^12 - 15280*a^2*b^28*c^20*d^10 + 30800*a^2*b^28*c^2
2*d^8 - 31200*a^2*b^28*c^24*d^6 + 16000*a^2*b^28*c^26*d^4 - 3440*a^2*b^28*c
^28*d^2 - 18240*a^3*b^27*c^17*d^13 + 92800*a^3*b^27*c^19*d^11 - 190400*a^3*
b^27*c^21*d^9 + 198400*a^3*b^27*c^23*d^7 - 107200*a^3*b^27*c^25*d^5 + 26240
*a^3*b^27*c^27*d^3 + 77520*a^4*b^26*c^16*d^14 - 402800*a^4*b^26*c^18*d^12 +
851360*a^4*b^26*c^20*d^10 - 928000*a^4*b^26*c^22*d^8 + 541200*a^4*b^26*c^2
4*d^6 - 155120*a^4*b^26*c^26*d^4 + 16000*a^4*b^26*c^28*d^2 - 248064*a^5*b^2
5*c^15*d^15 + 1331520*a^5*b^25*c^17*d^13 - 2939840*a^5*b^25*c^19*d^11 + 340
8640*a^5*b^25*c^21*d^9 - 2184320*a^5*b^25*c^23*d^7 + 736064*a^5*b^25*c^25*d
^5 - 107200*a^5*b^25*c^27*d^3 + 620160*a^6*b^24*c^14*d^16 - 3488400*a^6*b^2
4*c^16*d^14 + 8170000*a^6*b^24*c^18*d^12 - 10229760*a^6*b^24*c^20*d^10 + 72
81600*a^6*b^24*c^22*d^8 - 2863760*a^6*b^24*c^24*d^6 + 541200*a^6*b^24*c^26*
d^4 - 31200*a^6*b^24*c^28*d^2 - 1240320*a^7*b^23*c^13*d^17 + 7441920*a^7*b^
23*c^15*d^15 - 18787200*a^7*b^23*c^17*d^13 + 25721600*a^7*b^23*c^19*d^11 -

20444800*a^7*b^23*c^21*d^9 + 9297920*a^7*b^23*c^23*d^7 - 2184320*a^7*b^23*c
^25*d^5 + 198400*a^7*b^23*c^27*d^3 + 2015520*a^8*b^22*c^12*d^18 - 13178400*
a^8*b^22*c^14*d^16 + 36434400*a^8*b^22*c^16*d^14 - 55069600*a^8*b^22*c^18*d
^12 + 48989680*a^8*b^22*c^20*d^10 - 25575920*a^8*b^22*c^22*d^8 + 7281600*a^
8*b^22*c^24*d^6 - 928000*a^8*b^22*c^26*d^4 + 30800*a^8*b^22*c^28*d^2 - 2687
360*a^9*b^21*c^11*d^19 + 19638400*a^9*b^21*c^13*d^17 - 60362240*a^9*b^21*c^
15*d^15 + 101475200*a^9*b^21*c^17*d^13 - 101172800*a^9*b^21*c^19*d^11 + 603
33760*a^9*b^21*c^21*d^9 - 20444800*a^9*b^21*c^23*d^7 + 3408640*a^9*b^21*c^2
5*d^5 - 190400*a^9*b^21*c^27*d^3 + 2956096*a^10*b^20*c^10*d^20 - 24858080*a
^10*b^20*c^12*d^18 + 86150560*a^10*b^20*c^14*d^16 - 162120160*a^10*b^20*c^1
6*d^14 + 181463680*a^10*b^20*c^18*d^12 - 123188112*a^10*b^20*c^20*d^10 + 48
989680*a^10*b^20*c^22*d^8 - 10229760*a^10*b^20*c^24*d^6 + 851360*a^10*b^20*
c^26*d^4 - 15280*a^10*b^20*c^28*d^2 - 2687360*a^11*b^19*c^9*d^21 + 26873600
*a^11*b^19*c^11*d^19 - 106460800*a^11*b^19*c^13*d^17 + 225738240*a^11*b^19*
c^15*d^15 - 284331200*a^11*b^19*c^17*d^13 + 219166080*a^11*b^19*c^19*d^11 -
101172800*a^11*b^19*c^21*d^9 + 25721600*a^11*b^19*c^23*d^7 - 2939840*a^11*
b^19*c^25*d^5 + 92800*a^11*b^19*c^27*d^3 + 2015520*a^12*b^18*c^8*d^22 - 248
58080*a^12*b^18*c^10*d^20 + 114212800*a^12*b^18*c^12*d^18 - 274937600*a^12*
b^18*c^14*d^16 + 390830000*a^12*b^18*c^16*d^14 - 341426960*a^12*b^18*c^18*d
^12 + 181463680*a^12*b^18*c^20*d^10 - 55069600*a^12*b^18*c^22*d^8 + 8170000
*a^12*b^18*c^24*d^6 - 402800*a^12*b^18*c^26*d^4 + 3040*a^12*b^18*c^28*d^2 -
1240320*a^13*b^17*c^7*d^23 + 19638400*a^13*b^17*c^9*d^21 - 106460800*a^13*
b^17*c^11*d^19 + 293542400*a^13*b^17*c^13*d^17 - 472561920*a^13*b^17*c^15*d
^15 + 467412160*a^13*b^17*c^17*d^13 - 284331200*a^13*b^17*c^19*d^11 + 10147
5200*a^13*b^17*c^21*d^9 - 18787200*a^13*b^17*c^23*d^7 + 1331520*a^13*b^17*c
^25*d^5 - 18240*a^13*b^17*c^27*d^3 + 620160*a^14*b^16*c^6*d^24 - 13178400*a
^14*b^16*c^8*d^22 + 86150560*a^14*b^16*c^10*d^20 - 274937600*a^14*b^16*c^12
*d^18 + 503363200*a^14*b^16*c^14*d^16 - 563751280*a^14*b^16*c^16*d^14 + 390
830000*a^14*b^16*c^18*d^12 - 162120160*a^14*b^16*c^20*d^10 + 36434400*a^14*
b^16*c^22*d^8 - 3488400*a^14*b^16*c^24*d^6 + 77520*a^14*b^16*c^26*d^4 - 248
064*a^15*b^15*c^5*d^25 + 7441920*a^15*b^15*c^7*d^23 - 60362240*a^15*b^15*c^
9*d^21 + 225738240*a^15*b^15*c^11*d^19 - 472561920*a^15*b^15*c^13*d^17 + 59
9984128*a^15*b^15*c^15*d^15 - 472561920*a^15*b^15*c^17*d^13 + 225738240*a^1
5*b^15*c^19*d^11 - 60362240*a^15*b^15*c^21*d^9 + 7441920*a^15*b^15*c^23*d^7
- 248064*a^15*b^15*c^25*d^5 + 77520*a^16*b^14*c^4*d^26 - 3488400*a^16*b^14
*c^6*d^24 + 36434400*a^16*b^14*c^8*d^22 - 162120160*a^16*b^14*c^10*d^20 + 3
90830000*a^16*b^14*c^12*d^18 - 563751280*a^16*b^14*c^14*d^16 + 503363200*a^
16*b^14*c^16*d^14 - 274937600*a^16*b^14*c^18*d^12 + 86150560*a^16*b^14*c^20
*d^10 - 13178400*a^16*b^14*c^22*d^8 + 620160*a^16*b^14*c^24*d^6 - 18240*a^1
7*b^13*c^3*d^27 + 1331520*a^17*b^13*c^5*d^25 - 18787200*a^17*b^13*c^7*d^23
+ 101475200*a^17*b^13*c^9*d^21 - 284331200*a^17*b^13*c^11*d^19 + 467412160*
a^17*b^13*c^13*d^17 - 472561920*a^17*b^13*c^15*d^15 + 293542400*a^17*b^13*c
^17*d^13 - 106460800*a^17*b^13*c^19*d^11 + 19638400*a^17*b^13*c^21*d^9 - 12
40320*a^17*b^13*c^23*d^7 + 3040*a^18*b^12*c^2*d^28 - 402800*a^18*b^12*c^4*d
^26 + 8170000*a^18*b^12*c^6*d^24 - 55069600*a^18*b^12*c^8*d^22 + 181463680*

a^18*b^12*c^10*d^20 - 341426960*a^18*b^12*c^12*d^18 + 390830000*a^18*b^12*c
^14*d^16 - 274937600*a^18*b^12*c^16*d^14 + 114212800*a^18*b^12*c^18*d^12 -
24858080*a^18*b^12*c^20*d^10 + 2015520*a^18*b^12*c^22*d^8 + 92800*a^19*b^11
*c^3*d^27 - 2939840*a^19*b^11*c^5*d^25 + 25721600*a^19*b^11*c^7*d^23 - 1011
72800*a^19*b^11*c^9*d^21 + 219166080*a^19*b^11*c^11*d^19 - 284331200*a^19*b
^11*c^13*d^17 + 225738240*a^19*b^11*c^15*d^15 - 106460800*a^19*b^11*c^17*d^
13 + 26873600*a^19*b^11*c^19*d^11 - 2687360*a^19*b^11*c^21*d^9 - 15280*a^20
*b^10*c^2*d^28 + 851360*a^20*b^10*c^4*d^26 - 10229760*a^20*b^10*c^6*d^24 +
48989680*a^20*b^10*c^8*d^22 - 123188112*a^20*b^10*c^10*d^20 + 181463680*a^2
0*b^10*c^12*d^18 - 162120160*a^20*b^10*c^14*d^16 + 86150560*a^20*b^10*c^16*
d^14 - 24858080*a^20*b^10*c^18*d^12 + 2956096*a^20*b^10*c^20*d^10 - 190400*
a^21*b^9*c^3*d^27 + 3408640*a^21*b^9*c^5*d^25 - 20444800*a^21*b^9*c^7*d^23
+ 60333760*a^21*b^9*c^9*d^21 - 101172800*a^21*b^9*c^11*d^19 + 101475200*a^2
1*b^9*c^13*d^17 - 60362240*a^21*b^9*c^15*d^15 + 19638400*a^21*b^9*c^17*d^13
 - 2687360*a^21*b^9*c^19*d^11 + 30800*a^22*b^8*c^2*d^28 - 928000*a^22*b^8*c
^4*d^26 + 7281600*a^22*b^8*c^6*d^24 - 25575920*a^22*b^8*c^8*d^22 + 48989680
*a^22*b^8*c^10*d^20 - 55069600*a^22*b^8*c^12*d^18 + 36434400*a^22*b^8*c^14*
d^16 - 13178400*a^22*b^8*c^16*d^14 + 2015520*a^22*b^8*c^18*d^12 + 198400*a^
23*b^7*c^3*d^27 - 2184320*a^23*b^7*c^5*d^25 + 9297920*a^23*b^7*c^7*d^23 - 2
0444800*a^23*b^7*c^9*d^21 + 25721600*a^23*b^7*c^11*d^19 - 18787200*a^23*b^7
*c^13*d^17 + 7441920*a^23*b^7*c^15*d^15 - 1240320*a^23*b^7*c^17*d^13 - 3120
0*a^24*b^6*c^2*d^28 + 541200*a^24*b^6*c^4*d^26 - 2863760*a^24*b^6*c^6*d^24
+ 7281600*a^24*b^6*c^8*d^22 - 10229760*a^24*b^6*c^10*d^20 + 8170000*a^24*b^
6*c^12*d^18 - 3488400*a^24*b^6*c^14*d^16 + 620160*a^24*b^6*c^16*d^14 - 1072
00*a^25*b^5*c^3*d^27 + 736064*a^25*b^5*c^5*d^25 - 2184320*a^25*b^5*c^7*d^23
 + 3408640*a^25*b^5*c^9*d^21 - 2939840*a^25*b^5*c^11*d^19 + 1331520*a^25*b^
5*c^13*d^17 - 248064*a^25*b^5*c^15*d^15 + 16000*a^26*b^4*c^2*d^28 - 155120*
a^26*b^4*c^4*d^26 + 541200*a^26*b^4*c^6*d^24 - 928000*a^26*b^4*c^8*d^22 + 8
51360*a^26*b^4*c^10*d^20 - 402800*a^26*b^4*c^12*d^18 + 77520*a^26*b^4*c^14*
d^16 + 26240*a^27*b^3*c^3*d^27 - 107200*a^27*b^3*c^5*d^25 + 198400*a^27*b^3
*c^7*d^23 - 190400*a^27*b^3*c^9*d^21 + 92800*a^27*b^3*c^11*d^19 - 18240*a^2
7*b^3*c^13*d^17 - 3440*a^28*b^2*c^2*d^28 + 16000*a^28*b^2*c^4*d^26 - 31200*
a^28*b^2*c^6*d^24 + 30800*a^28*b^2*c^8*d^22 - 15280*a^28*b^2*c^10*d^20 + 30
40*a^28*b^2*c^12*d^18 + 320*a*b^29*c^29*d + 320*a^29*b*c*d^29))^(1/2) + 2*a
^24*d^24 + 2*b^24*c^24 + 8*a^2*b^22*c^24 + 8*a^4*b^20*c^24 - 576*a^10*b^14*
d^24 + 2784*a^12*b^12*d^24 - 5284*a^14*b^10*d^24 + 4730*a^16*b^8*d^24 - 178
0*a^18*b^6*d^24 + 68*a^20*b^4*d^24 + 38*a^22*b^2*d^24 + 8*a^24*c^2*d^22 + 8
*a^24*c^4*d^20 - 576*b^24*c^10*d^14 + 2784*b^24*c^12*d^12 - 5284*b^24*c^14*
d^10 + 4730*b^24*c^16*d^8 - 1780*b^24*c^18*d^6 + 68*b^24*c^20*d^4 + 38*b^24
*c^22*d^2 + 5760*a*b^23*c^9*d^15 - 28224*a*b^23*c^11*d^13 + 54728*a*b^23*c^
13*d^11 - 50620*a*b^23*c^15*d^9 + 20360*a*b^23*c^17*d^7 - 1480*a*b^23*c^19*
d^5 - 268*a*b^23*c^21*d^3 - 88*a^3*b^21*c^23*d - 160*a^5*b^19*c^23*d + 5760
*a^9*b^15*c*d^23 - 28224*a^11*b^13*c*d^23 + 54728*a^13*b^11*c*d^23 - 50620*
a^15*b^9*c*d^23 + 20360*a^17*b^7*c*d^23 - 1480*a^19*b^5*c*d^23 - 268*a^21*b
^3*c*d^23 - 88*a^23*b*c^3*d^21 - 160*a^23*b*c^5*d^19 - 25920*a^2*b^22*c^8*d

^16 + 131904*a^2*b^22*c^10*d^14 - 270604*a^2*b^22*c^12*d^12 + 273544*a^2*b^22*c^14*d^10 - 131660*a^2*b^22*c^16*d^8 + 22060*a^2*b^22*c^18*d^6 - 782*a^2*b^22*c^20*d^4 - 98*a^2*b^22*c^22*d^2 + 69120*a^3*b^21*c^7*d^17 - 379200*a^3*b^21*c^9*d^15 + 860368*a^3*b^21*c^11*d^13 - 1001364*a^3*b^21*c^13*d^11 + 605280*a^3*b^21*c^15*d^9 - 167520*a^3*b^21*c^17*d^7 + 18840*a^3*b^21*c^19*d^5 - 144*a^3*b^21*c^21*d^3 - 120960*a^4*b^20*c^6*d^18 + 756000*a^4*b^20*c^8*d^16 - 1987844*a^4*b^20*c^10*d^14 + 2750664*a^4*b^20*c^12*d^12 - 2073976*a^4*b^20*c^14*d^10 + 793460*a^4*b^20*c^16*d^8 - 138010*a^4*b^20*c^18*d^6 + 10562*a^4*b^20*c^20*d^4 + 88*a^4*b^20*c^22*d^2 + 145152*a^5*b^19*c^5*d^19 - 1116288*a^5*b^19*c^7*d^17 + 3539128*a^5*b^19*c^9*d^15 - 5890780*a^5*b^19*c^11*d^13 + 5437600*a^5*b^19*c^13*d^11 - 2682536*a^5*b^19*c^15*d^9 + 655084*a^5*b^19*c^17*d^7 - 85484*a^5*b^19*c^19*d^5 + 4080*a^5*b^19*c^21*d^3 - 120960*a^6*b^18*c^4*d^20 + 1266048*a^6*b^18*c^6*d^18 - 4977996*a^6*b^18*c^8*d^16 + 10009720*a^6*b^18*c^10*d^14 - 11209800*a^6*b^18*c^12*d^12 + 6943760*a^6*b^18*c^14*d^10 - 2253214*a^6*b^18*c^16*d^8 + 396878*a^6*b^18*c^18*d^6 - 36120*a^6*b^18*c^20*d^4 + 1520*a^6*b^18*c^22*d^2 + 69120*a^7*b^17*c^3*d^21 - 1116288*a^7*b^17*c^5*d^19 + 5575008*a^7*b^17*c^7*d^17 - 13668308*a^7*b^17*c^9*d^15 + 18576800*a^7*b^17*c^11*d^13 - 14230520*a^7*b^17*c^13*d^11 + 5889904*a^7*b^17*c^15*d^9 - 1310504*a^7*b^17*c^17*d^7 + 168344*a^7*b^17*c^19*d^5 - 8960*a^7*b^17*c^21*d^3 - 25920*a^8*b^16*c^2*d^22 + 756000*a^8*b^16*c^4*d^20 - 4977996*a^8*b^16*c^6*d^18 + 15144828*a^8*b^16*c^8*d^16 - 25068800*a^8*b^16*c^10*d^14 + 23486280*a^8*b^16*c^12*d^12 - 12099640*a^8*b^16*c^14*d^10 + 3330518*a^8*b^16*c^16*d^8 - 529224*a^8*b^16*c^18*d^6 + 36280*a^8*b^16*c^20*d^4 - 379200*a^9*b^15*c^3*d^21 + 3539128*a^9*b^15*c^5*d^19 - 13668308*a^9*b^15*c^7*d^17 + 27691952*a^9*b^15*c^9*d^15 - 31562040*a^9*b^15*c^11*d^13 + 19993760*a^9*b^15*c^13*d^11 - 6731044*a^9*b^15*c^15*d^9 + 1239264*a^9*b^15*c^17*d^7 - 106016*a^9*b^15*c^19*d^5 + 131904*a^10*b^14*c^2*d^22 - 1987844*a^10*b^14*c^4*d^20 + 10009720*a^10*b^14*c^6*d^18 - 25068800*a^10*b^14*c^8*d^16 + 34796936*a^10*b^14*c^10*d^14 - 26927144*a^10*b^14*c^12*d^12 + 10994964*a^10*b^14*c^14*d^10 - 2295680*a^10*b^14*c^16*d^8 + 230240*a^10*b^14*c^18*d^6 + 860368*a^11*b^13*c^3*d^21 - 5890780*a^11*b^13*c^5*d^19 + 18576800*a^11*b^13*c^7*d^17 - 31562040*a^11*b^13*c^9*d^15 + 29722864*a^11*b^13*c^11*d^13 - 14679348*a^11*b^13*c^13*d^11 + 3497920*a^11*b^13*c^15*d^9 - 381280*a^11*b^13*c^17*d^7 - 270604*a^12*b^12*c^2*d^22 + 2750664*a^12*b^12*c^4*d^20 - 11209800*a^12*b^12*c^6*d^18 + 23486280*a^12*b^12*c^8*d^16 - 26927144*a^12*b^12*c^10*d^14 + 16147404*a^12*b^12*c^12*d^12 - 4479104*a^12*b^12*c^14*d^10 + 499520*a^12*b^12*c^16*d^8 - 1001364*a^13*b^11*c^3*d^21 + 5437600*a^13*b^11*c^5*d^19 - 14230520*a^13*b^11*c^7*d^17 + 19993760*a^13*b^11*c^9*d^15 - 14679348*a^13*b^11*c^11*d^13 + 4861024*a^13*b^11*c^13*d^11 - 552160*a^13*b^11*c^15*d^9 + 273544*a^14*b^10*c^2*d^22 - 2073976*a^14*b^10*c^4*d^20 + 6943760*a^14*b^10*c^6*d^18 - 12099640*a^14*b^10*c^8*d^16 + 10994964*a^14*b^10*c^10*d^14 - 4479104*a^14*b^10*c^12*d^12 + 562016*a^14*b^10*c^14*d^10 + 605280*a^15*b^9*c^3*d^21 - 2682536*a^15*b^9*c^5*d^19 + 5889904*a^15*b^9*c^7*d^17 - 6731044*a^15*b^9*c^9*d^15 + 3497920*a^15*b^9*c^11*d^13 - 552160*a^15*b^9*c^13*d^11 - 131660*a^16*b^8*c^2*d^22 + 793460*a^16*b^8*c^4*d^20 - 2253214*a^1

6*b^8*c^6*d^18 + 3330518*a^16*b^8*c^8*d^16 - 2295680*a^16*b^8*c^10*d^14 + 4
99520*a^16*b^8*c^12*d^12 - 167520*a^17*b^7*c^3*d^21 + 655084*a^17*b^7*c^5*d
^19 - 1310504*a^17*b^7*c^7*d^17 + 1239264*a^17*b^7*c^9*d^15 - 381280*a^17*b
^7*c^11*d^13 + 22060*a^18*b^6*c^2*d^22 - 138010*a^18*b^6*c^4*d^20 + 396878*
a^18*b^6*c^6*d^18 - 529224*a^18*b^6*c^8*d^16 + 230240*a^18*b^6*c^10*d^14 +
18840*a^19*b^5*c^3*d^21 - 85484*a^19*b^5*c^5*d^19 + 168344*a^19*b^5*c^7*d^1
7 - 106016*a^19*b^5*c^9*d^15 - 782*a^20*b^4*c^2*d^22 + 10562*a^20*b^4*c^4*d
^20 - 36120*a^20*b^4*c^6*d^18 + 36280*a^20*b^4*c^8*d^16 - 144*a^21*b^3*c^3*
d^21 + 4080*a^21*b^3*c^5*d^19 - 8960*a^21*b^3*c^7*d^17 - 98*a^22*b^2*c^2*d^
22 + 88*a^22*b^2*c^4*d^20 + 1520*a^22*b^2*c^6*d^18 - 4*a*b^23*c^23*d - 4*a^
23*b*c*d^23)/(16*(5*a^2*b^28*c^30 - b^30*c^30 - a^30*d^30 - 10*a^4*b^26*c^3
0 + 10*a^6*b^24*c^30 - 5*a^8*b^22*c^30 + a^10*b^20*c^30 + a^20*b^10*d^30 -
5*a^22*b^8*d^30 + 10*a^24*b^6*d^30 - 10*a^26*b^4*d^30 + 5*a^28*b^2*d^30 + 5
*a^30*c^2*d^28 - 10*a^30*c^4*d^26 + 10*a^30*c^6*d^24 - 5*a^30*c^8*d^22 + a^
30*c^10*d^20 + b^30*c^20*d^10 - 5*b^30*c^22*d^8 + 10*b^30*c^24*d^6 - 10*b^3
0*c^26*d^4 + 5*b^30*c^28*d^2 - 20*a*b^29*c^19*d^11 + 100*a*b^29*c^21*d^9 -
200*a*b^29*c^23*d^7 + 200*a*b^29*c^25*d^5 - 100*a*b^29*c^27*d^3 - 100*a^3*b
^27*c^29*d + 200*a^5*b^25*c^29*d - 200*a^7*b^23*c^29*d + 100*a^9*b^21*c^29*
d - 20*a^11*b^19*c^29*d - 20*a^19*b^11*c*d^29 + 100*a^21*b^9*c*d^29 - 200*a
^23*b^7*c*d^29 + 200*a^25*b^5*c*d^29 - 100*a^27*b^3*c*d^29 - 100*a^29*b*c^3
*d^27 + 200*a^29*b*c^5*d^25 - 200*a^29*b*c^7*d^23 + 100*a^29*b*c^9*d^21 - 2
0*a^29*b*c^11*d^19 + 190*a^2*b^28*c^18*d^12 - 955*a^2*b^28*c^20*d^10 + 1925
*a^2*b^28*c^22*d^8 - 1950*a^2*b^28*c^24*d^6 + 1000*a^2*b^28*c^26*d^4 - 215*
a^2*b^28*c^28*d^2 - 1140*a^3*b^27*c^17*d^13 + 5800*a^3*b^27*c^19*d^11 - 119
00*a^3*b^27*c^21*d^9 + 12400*a^3*b^27*c^23*d^7 - 6700*a^3*b^27*c^25*d^5 + 1
640*a^3*b^27*c^27*d^3 + 4845*a^4*b^26*c^16*d^14 - 25175*a^4*b^26*c^18*d^12
+ 53210*a^4*b^26*c^20*d^10 - 58000*a^4*b^26*c^22*d^8 + 33825*a^4*b^26*c^24*
d^6 - 9695*a^4*b^26*c^26*d^4 + 1000*a^4*b^26*c^28*d^2 - 15504*a^5*b^25*c^15
*d^15 + 83220*a^5*b^25*c^17*d^13 - 183740*a^5*b^25*c^19*d^11 + 213040*a^5*b
^25*c^21*d^9 - 136520*a^5*b^25*c^23*d^7 + 46004*a^5*b^25*c^25*d^5 - 6700*a^
5*b^25*c^27*d^3 + 38760*a^6*b^24*c^14*d^16 - 218025*a^6*b^24*c^16*d^14 + 51
0625*a^6*b^24*c^18*d^12 - 639360*a^6*b^24*c^20*d^10 + 455100*a^6*b^24*c^22*
d^8 - 178985*a^6*b^24*c^24*d^6 + 33825*a^6*b^24*c^26*d^4 - 1950*a^6*b^24*c^
28*d^2 - 77520*a^7*b^23*c^13*d^17 + 465120*a^7*b^23*c^15*d^15 - 1174200*a^7
*b^23*c^17*d^13 + 1607600*a^7*b^23*c^19*d^11 - 1277800*a^7*b^23*c^21*d^9 +
581120*a^7*b^23*c^23*d^7 - 136520*a^7*b^23*c^25*d^5 + 12400*a^7*b^23*c^27*d
^3 + 125970*a^8*b^22*c^12*d^18 - 823650*a^8*b^22*c^14*d^16 + 2277150*a^8*b^
22*c^16*d^14 - 3441850*a^8*b^22*c^18*d^12 + 3061855*a^8*b^22*c^20*d^10 - 15
98495*a^8*b^22*c^22*d^8 + 455100*a^8*b^22*c^24*d^6 - 58000*a^8*b^22*c^26*d^
4 + 1925*a^8*b^22*c^28*d^2 - 167960*a^9*b^21*c^11*d^19 + 1227400*a^9*b^21*c
^13*d^17 - 3772640*a^9*b^21*c^15*d^15 + 6342200*a^9*b^21*c^17*d^13 - 632330
0*a^9*b^21*c^19*d^11 + 3770860*a^9*b^21*c^21*d^9 - 1277800*a^9*b^21*c^23*d^
7 + 213040*a^9*b^21*c^25*d^5 - 11900*a^9*b^21*c^27*d^3 + 184756*a^10*b^20*c
^10*d^20 - 1553630*a^10*b^20*c^12*d^18 + 5384410*a^10*b^20*c^14*d^16 - 1013
2510*a^10*b^20*c^16*d^14 + 11341480*a^10*b^20*c^18*d^12 - 7699257*a^10*b^20

*c^20*d^10 + 3061855*a^10*b^20*c^22*d^8 - 639360*a^10*b^20*c^24*d^6 + 53210*a^10*b^20*c^26*d^4 - 955*a^10*b^20*c^28*d^2 - 167960*a^11*b^19*c^9*d^21 + 1679600*a^11*b^19*c^11*d^19 - 6653800*a^11*b^19*c^13*d^17 + 14108640*a^11*b^19*c^15*d^15 - 17770700*a^11*b^19*c^17*d^13 + 13697880*a^11*b^19*c^19*d^11 - 6323300*a^11*b^19*c^21*d^9 + 1607600*a^11*b^19*c^23*d^7 - 183740*a^11*b^19*c^25*d^5 + 5800*a^11*b^19*c^27*d^3 + 125970*a^12*b^18*c^8*d^22 - 1553630*a^12*b^18*c^10*d^20 + 7138300*a^12*b^18*c^12*d^18 - 17183600*a^12*b^18*c^14*d^16 + 24426875*a^12*b^18*c^16*d^14 - 21339185*a^12*b^18*c^18*d^12 + 11341480*a^12*b^18*c^20*d^10 - 3441850*a^12*b^18*c^22*d^8 + 510625*a^12*b^18*c^24*d^6 - 25175*a^12*b^18*c^26*d^4 + 190*a^12*b^18*c^28*d^2 - 77520*a^13*b^17*c^7*d^23 + 1227400*a^13*b^17*c^9*d^21 - 6653800*a^13*b^17*c^11*d^19 + 18346400*a^13*b^17*c^13*d^17 - 29535120*a^13*b^17*c^15*d^15 + 29213260*a^13*b^17*c^17*d^13 - 17770700*a^13*b^17*c^19*d^11 + 6342200*a^13*b^17*c^21*d^9 - 1174200*a^13*b^17*c^23*d^7 + 83220*a^13*b^17*c^25*d^5 - 1140*a^13*b^17*c^27*d^3 + 38760*a^14*b^16*c^6*d^24 - 823650*a^14*b^16*c^8*d^22 + 5384410*a^14*b^16*c^10*d^20 - 17183600*a^14*b^16*c^12*d^18 + 31460200*a^14*b^16*c^14*d^16 - 35234455*a^14*b^16*c^16*d^14 + 24426875*a^14*b^16*c^18*d^12 - 10132510*a^14*b^16*c^20*d^10 + 2277150*a^14*b^16*c^22*d^8 - 218025*a^14*b^16*c^24*d^6 + 4845*a^14*b^16*c^26*d^4 - 15504*a^15*b^15*c^5*d^25 + 465120*a^15*b^15*c^7*d^23 - 3772640*a^15*b^15*c^9*d^21 + 14108640*a^15*b^15*c^11*d^19 - 29535120*a^15*b^15*c^13*d^17 + 37499008*a^15*b^15*c^15*d^15 - 29535120*a^15*b^15*c^17*d^13 + 14108640*a^15*b^15*c^19*d^11 - 3772640*a^15*b^15*c^21*d^9 + 465120*a^15*b^15*c^23*d^7 - 15504*a^15*b^15*c^25*d^5 + 4845*a^16*b^14*c^4*d^26 - 218025*a^16*b^14*c^6*d^24 + 2277150*a^16*b^14*c^8*d^22 - 10132510*a^16*b^14*c^10*d^20 + 24426875*a^16*b^14*c^12*d^18 - 35234455*a^16*b^14*c^14*d^16 + 31460200*a^16*b^14*c^16*d^14 - 17183600*a^16*b^14*c^18*d^12 + 5384410*a^16*b^14*c^20*d^10 - 823650*a^16*b^14*c^22*d^8 + 38760*a^16*b^14*c^24*d^6 - 1140*a^17*b^13*c^3*d^27 + 83220*a^17*b^13*c^5*d^25 - 1174200*a^17*b^13*c^7*d^23 + 6342200*a^17*b^13*c^9*d^21 - 17770700*a^17*b^13*c^11*d^19 + 29213260*a^17*b^13*c^13*d^17 - 29535120*a^17*b^13*c^15*d^15 + 18346400*a^17*b^13*c^17*d^13 - 6653800*a^17*b^13*c^19*d^11 + 1227400*a^17*b^13*c^21*d^9 - 77520*a^17*b^13*c^23*d^7 + 190*a^18*b^12*c^2*d^28 - 25175*a^18*b^12*c^4*d^26 + 510625*a^18*b^12*c^6*d^24 - 3441850*a^18*b^12*c^8*d^22 + 11341480*a^18*b^12*c^10*d^20 - 21339185*a^18*b^12*c^12*d^18 + 24426875*a^18*b^12*c^14*d^16 - 17183600*a^18*b^12*c^16*d^14 + 7138300*a^18*b^12*c^18*d^12 - 1553630*a^18*b^12*c^20*d^10 + 125970*a^18*b^12*c^22*d^8 + 5800*a^19*b^11*c^3*d^27 - 183740*a^19*b^11*c^5*d^25 + 1607600*a^19*b^11*c^7*d^23 - 6323300*a^19*b^11*c^9*d^21 + 13697880*a^19*b^11*c^11*d^19 - 17770700*a^19*b^11*c^13*d^17 + 14108640*a^19*b^11*c^15*d^15 - 6653800*a^19*b^11*c^17*d^13 + 1679600*a^19*b^11*c^19*d^11 - 167960*a^19*b^11*c^21*d^9 - 955*a^20*b^10*c^2*d^28 + 53210*a^20*b^10*c^4*d^26 - 639360*a^20*b^10*c^6*d^24 + 3061855*a^20*b^10*c^8*d^22 - 7699257*a^20*b^10*c^10*d^20 + 11341480*a^20*b^10*c^12*d^18 - 10132510*a^20*b^10*c^14*d^16 + 5384410*a^20*b^10*c^16*d^14 - 1553630*a^20*b^10*c^18*d^12 + 184756*a^20*b^10*c^20*d^10 - 11900*a^21*b^9*c^3*d^27 + 213040*a^21*b^9*c^5*d^25 - 1277800*a^21*b^9*c^7*d^23 + 3770860*a^21*b^9*c^9*d^21 - 6323300*a^21*b^

9*c^11*d^19 + 6342200*a^21*b^9*c^13*d^17 - 3772640*a^21*b^9*c^15*d^15 + 122
7400*a^21*b^9*c^17*d^13 - 167960*a^21*b^9*c^19*d^11 + 1925*a^22*b^8*c^2*d^2
8 - 58000*a^22*b^8*c^4*d^26 + 455100*a^22*b^8*c^6*d^24 - 1598495*a^22*b^8*c
^8*d^22 + 3061855*a^22*b^8*c^10*d^20 - 3441850*a^22*b^8*c^12*d^18 + 2277150
*a^22*b^8*c^14*d^16 - 823650*a^22*b^8*c^16*d^14 + 125970*a^22*b^8*c^18*d^12
 + 12400*a^23*b^7*c^3*d^27 - 136520*a^23*b^7*c^5*d^25 + 581120*a^23*b^7*c^7
*d^23 - 1277800*a^23*b^7*c^9*d^21 + 1607600*a^23*b^7*c^11*d^19 - 1174200*a^
23*b^7*c^13*d^17 + 465120*a^23*b^7*c^15*d^15 - 77520*a^23*b^7*c^17*d^13 - 1
950*a^24*b^6*c^2*d^28 + 33825*a^24*b^6*c^4*d^26 - 178985*a^24*b^6*c^6*d^24
+ 455100*a^24*b^6*c^8*d^22 - 639360*a^24*b^6*c^10*d^20 + 510625*a^24*b^6*c^
12*d^18 - 218025*a^24*b^6*c^14*d^16 + 38760*a^24*b^6*c^16*d^14 - 6700*a^25*
b^5*c^3*d^27 + 46004*a^25*b^5*c^5*d^25 - 136520*a^25*b^5*c^7*d^23 + 213040*
a^25*b^5*c^9*d^21 - 183740*a^25*b^5*c^11*d^19 + 83220*a^25*b^5*c^13*d^17 -
15504*a^25*b^5*c^15*d^15 + 1000*a^26*b^4*c^2*d^28 - 9695*a^26*b^4*c^4*d^26
+ 33825*a^26*b^4*c^6*d^24 - 58000*a^26*b^4*c^8*d^22 + 53210*a^26*b^4*c^10*d
^20 - 25175*a^26*b^4*c^12*d^18 + 4845*a^26*b^4*c^14*d^16 + 1640*a^27*b^3*c^
3*d^27 - 6700*a^27*b^3*c^5*d^25 + 12400*a^27*b^3*c^7*d^23 - 11900*a^27*b^3*
c^9*d^21 + 5800*a^27*b^3*c^11*d^19 - 1140*a^27*b^3*c^13*d^17 - 215*a^28*b^2
*c^2*d^28 + 1000*a^28*b^2*c^4*d^26 - 1950*a^28*b^2*c^6*d^24 + 1925*a^28*b^2
*c^8*d^22 - 955*a^28*b^2*c^10*d^20 + 190*a^28*b^2*c^12*d^18 + 20*a*b^29*c^2
9*d + 20*a^29*b*c*d^29)))^(1/2)*((-(((4*a^24*d^24 + 4*b^24*c^24 + 16*a^2*b^
22*c^24 + 16*a^4*b^20*c^24 - 1152*a^10*b^14*d^24 + 5568*a^12*b^12*d^24 - 10
568*a^14*b^10*d^24 + 9460*a^16*b^8*d^24 - 3560*a^18*b^6*d^24 + 136*a^20*b^4
*d^24 + 76*a^22*b^2*d^24 + 16*a^24*c^2*d^22 + 16*a^24*c^4*d^20 - 1152*b^24*
c^10*d^14 + 5568*b^24*c^12*d^12 - 10568*b^24*c^14*d^10 + 9460*b^24*c^16*d^8
 - 3560*b^24*c^18*d^6 + 136*b^24*c^20*d^4 + 76*b^24*c^22*d^2 + 11520*a*b^23
*c^9*d^15 - 56448*a*b^23*c^11*d^13 + 109456*a*b^23*c^13*d^11 - 101240*a*b^2
3*c^15*d^9 + 40720*a*b^23*c^17*d^7 - 2960*a*b^23*c^19*d^5 - 536*a*b^23*c^21
*d^3 - 176*a^3*b^21*c^23*d - 320*a^5*b^19*c^23*d + 11520*a^9*b^15*c*d^23 -
56448*a^11*b^13*c*d^23 + 109456*a^13*b^11*c*d^23 - 101240*a^15*b^9*c*d^23 +
 40720*a^17*b^7*c*d^23 - 2960*a^19*b^5*c*d^23 - 536*a^21*b^3*c*d^23 - 176*a
^23*b*c^3*d^21 - 320*a^23*b*c^5*d^19 - 51840*a^2*b^22*c^8*d^16 + 263808*a^2
*b^22*c^10*d^14 - 541208*a^2*b^22*c^12*d^12 + 547088*a^2*b^22*c^14*d^10 - 2
63320*a^2*b^22*c^16*d^8 + 44120*a^2*b^22*c^18*d^6 - 1564*a^2*b^22*c^20*d^4
- 196*a^2*b^22*c^22*d^2 + 138240*a^3*b^21*c^7*d^17 - 758400*a^3*b^21*c^9*d^
15 + 1720736*a^3*b^21*c^11*d^13 - 2002728*a^3*b^21*c^13*d^11 + 1210560*a^3*
b^21*c^15*d^9 - 335040*a^3*b^21*c^17*d^7 + 37680*a^3*b^21*c^19*d^5 - 288*a^
3*b^21*c^21*d^3 - 241920*a^4*b^20*c^6*d^18 + 1512000*a^4*b^20*c^8*d^16 - 39
75688*a^4*b^20*c^10*d^14 + 5501328*a^4*b^20*c^12*d^12 - 4147952*a^4*b^20*c^
14*d^10 + 1586920*a^4*b^20*c^16*d^8 - 276020*a^4*b^20*c^18*d^6 + 21124*a^4*
b^20*c^20*d^4 + 176*a^4*b^20*c^22*d^2 + 290304*a^5*b^19*c^5*d^19 - 2232576*
a^5*b^19*c^7*d^17 + 7078256*a^5*b^19*c^9*d^15 - 11781560*a^5*b^19*c^11*d^13
 + 10875200*a^5*b^19*c^13*d^11 - 5365072*a^5*b^19*c^15*d^9 + 1310168*a^5*b^
19*c^17*d^7 - 170968*a^5*b^19*c^19*d^5 + 8160*a^5*b^19*c^21*d^3 - 241920*a^
6*b^18*c^4*d^20 + 2532096*a^6*b^18*c^6*d^18 - 9955992*a^6*b^18*c^8*d^16 + 2

0019440*a^6*b^18*c^10*d^14 - 22419600*a^6*b^18*c^12*d^12 + 13887520*a^6*b^1
8*c^14*d^10 - 4506428*a^6*b^18*c^16*d^8 + 793756*a^6*b^18*c^18*d^6 - 72240*
a^6*b^18*c^20*d^4 + 3040*a^6*b^18*c^22*d^2 + 138240*a^7*b^17*c^3*d^21 - 223
2576*a^7*b^17*c^5*d^19 + 11150016*a^7*b^17*c^7*d^17 - 27336616*a^7*b^17*c^9
*d^15 + 37153600*a^7*b^17*c^11*d^13 - 28461040*a^7*b^17*c^13*d^11 + 1177980
8*a^7*b^17*c^15*d^9 - 2621008*a^7*b^17*c^17*d^7 + 336688*a^7*b^17*c^19*d^5
- 17920*a^7*b^17*c^21*d^3 - 51840*a^8*b^16*c^2*d^22 + 1512000*a^8*b^16*c^4*
d^20 - 9955992*a^8*b^16*c^6*d^18 + 30289656*a^8*b^16*c^8*d^16 - 50137600*a^
8*b^16*c^10*d^14 + 46972560*a^8*b^16*c^12*d^12 - 24199280*a^8*b^16*c^14*d^1
0 + 6661036*a^8*b^16*c^16*d^8 - 1058448*a^8*b^16*c^18*d^6 + 72560*a^8*b^16*
c^20*d^4 - 758400*a^9*b^15*c^3*d^21 + 7078256*a^9*b^15*c^5*d^19 - 27336616*
a^9*b^15*c^7*d^17 + 55383904*a^9*b^15*c^9*d^15 - 63124080*a^9*b^15*c^11*d^1
3 + 39987520*a^9*b^15*c^13*d^11 - 13462088*a^9*b^15*c^15*d^9 + 2478528*a^9*
b^15*c^17*d^7 - 212032*a^9*b^15*c^19*d^5 + 263808*a^10*b^14*c^2*d^22 - 3975
688*a^10*b^14*c^4*d^20 + 20019440*a^10*b^14*c^6*d^18 - 50137600*a^10*b^14*c
^8*d^16 + 69593872*a^10*b^14*c^10*d^14 - 53854288*a^10*b^14*c^12*d^12 + 219
89928*a^10*b^14*c^14*d^10 - 4591360*a^10*b^14*c^16*d^8 + 460480*a^10*b^14*c
^18*d^6 + 1720736*a^11*b^13*c^3*d^21 - 11781560*a^11*b^13*c^5*d^19 + 371536
00*a^11*b^13*c^7*d^17 - 63124080*a^11*b^13*c^9*d^15 + 59445728*a^11*b^13*c^
11*d^13 - 29358696*a^11*b^13*c^13*d^11 + 6995840*a^11*b^13*c^15*d^9 - 76256
0*a^11*b^13*c^17*d^7 - 541208*a^12*b^12*c^2*d^22 + 5501328*a^12*b^12*c^4*d^
20 - 22419600*a^12*b^12*c^6*d^18 + 46972560*a^12*b^12*c^8*d^16 - 53854288*a
^12*b^12*c^10*d^14 + 32294808*a^12*b^12*c^12*d^12 - 8958208*a^12*b^12*c^14*
d^10 + 999040*a^12*b^12*c^16*d^8 - 2002728*a^13*b^11*c^3*d^21 + 10875200*a^
13*b^11*c^5*d^19 - 28461040*a^13*b^11*c^7*d^17 + 39987520*a^13*b^11*c^9*d^1
5 - 29358696*a^13*b^11*c^11*d^13 + 9722048*a^13*b^11*c^13*d^11 - 1104320*a^
13*b^11*c^15*d^9 + 547088*a^14*b^10*c^2*d^22 - 4147952*a^14*b^10*c^4*d^20 +
13887520*a^14*b^10*c^6*d^18 - 24199280*a^14*b^10*c^8*d^16 + 21989928*a^14*
b^10*c^10*d^14 - 8958208*a^14*b^10*c^12*d^12 + 1124032*a^14*b^10*c^14*d^10
+ 1210560*a^15*b^9*c^3*d^21 - 5365072*a^15*b^9*c^5*d^19 + 11779808*a^15*b^9
*c^7*d^17 - 13462088*a^15*b^9*c^9*d^15 + 6995840*a^15*b^9*c^11*d^13 - 11043
20*a^15*b^9*c^13*d^11 - 263320*a^16*b^8*c^2*d^22 + 1586920*a^16*b^8*c^4*d^2
0 - 4506428*a^16*b^8*c^6*d^18 + 6661036*a^16*b^8*c^8*d^16 - 4591360*a^16*b^
8*c^10*d^14 + 999040*a^16*b^8*c^12*d^12 - 335040*a^17*b^7*c^3*d^21 + 131016
8*a^17*b^7*c^5*d^19 - 2621008*a^17*b^7*c^7*d^17 + 2478528*a^17*b^7*c^9*d^15
 - 762560*a^17*b^7*c^11*d^13 + 44120*a^18*b^6*c^2*d^22 - 276020*a^18*b^6*c^
4*d^20 + 793756*a^18*b^6*c^6*d^18 - 1058448*a^18*b^6*c^8*d^16 + 460480*a^18
*b^6*c^10*d^14 + 37680*a^19*b^5*c^3*d^21 - 170968*a^19*b^5*c^5*d^19 + 33668
8*a^19*b^5*c^7*d^17 - 212032*a^19*b^5*c^9*d^15 - 1564*a^20*b^4*c^2*d^22 + 2
1124*a^20*b^4*c^4*d^20 - 72240*a^20*b^4*c^6*d^18 + 72560*a^20*b^4*c^8*d^16
- 288*a^21*b^3*c^3*d^21 + 8160*a^21*b^3*c^5*d^19 - 17920*a^21*b^3*c^7*d^17
- 196*a^22*b^2*c^2*d^22 + 176*a^22*b^2*c^4*d^20 + 3040*a^22*b^2*c^6*d^18 -
8*a*b^23*c^23*d - 8*a^23*b*c*d^23)^2/4 - (20736*b^18*d^18 - 96768*a^2*b^16*
d^18 + 173664*a^4*b^14*d^18 - 136032*a^6*b^12*d^18 + 31081*a^8*b^10*d^18 +
8440*a^10*b^8*d^18 + 400*a^12*b^6*d^18 - 96768*b^18*c^2*d^16 + 173664*b^18*

c^4*d^14 - 136032*b^18*c^6*d^12 + 31081*b^18*c^8*d^10 + 8440*b^18*c^10*d^8 + 400*b^18*c^12*d^6 - 131328*a*b^17*c^3*d^15 + 216576*a*b^17*c^5*d^13 - 141104*a*b^17*c^7*d^11 + 20260*a*b^17*c^9*d^9 + 2800*a*b^17*c^11*d^7 - 131328*a^3*b^15*c*d^17 + 216576*a^5*b^13*c*d^17 - 141104*a^7*b^11*c*d^17 + 20260*a^9*b^9*c*d^17 + 2800*a^11*b^7*c*d^17 + 495936*a^2*b^16*c^2*d^16 - 989856*a^2*b^16*c^4*d^14 + 901948*a^2*b^16*c^6*d^12 - 308392*a^2*b^16*c^8*d^10 - 5260*a^2*b^16*c^10*d^8 + 1600*a^2*b^16*c^12*d^6 + 657408*a^3*b^15*c^3*d^15 - 1158992*a^3*b^15*c^5*d^13 + 838256*a^3*b^15*c^7*d^11 - 182200*a^3*b^15*c^9*d^9 - 3200*a^3*b^15*c^11*d^7 - 989856*a^4*b^14*c^2*d^16 + 2185654*a^4*b^14*c^4*d^14 - 2218576*a^4*b^14*c^6*d^12 + 900624*a^4*b^14*c^8*d^10 - 64720*a^4*b^14*c^10*d^8 + 1600*a^4*b^14*c^12*d^6 - 1158992*a^5*b^13*c^3*d^15 + 2158808*a^5*b^13*c^5*d^13 - 1641528*a^5*b^13*c^7*d^11 + 406880*a^5*b^13*c^9*d^9 - 17600*a^5*b^13*c^11*d^7 + 901948*a^6*b^12*c^2*d^16 - 2218576*a^6*b^12*c^4*d^14 + 2430936*a^6*b^12*c^6*d^12 - 1026928*a^6*b^12*c^8*d^10 + 88720*a^6*b^12*c^10*d^8 + 838256*a^7*b^11*c^3*d^15 - 1641528*a^7*b^11*c^5*d^13 + 1206848*a^7*b^11*c^7*d^11 - 239360*a^7*b^11*c^9*d^9 - 308392*a^8*b^10*c^2*d^16 + 900624*a^8*b^10*c^4*d^14 - 1026928*a^8*b^10*c^6*d^12 + 354016*a^8*b^10*c^8*d^10 - 182200*a^9*b^9*c^3*d^15 + 406880*a^9*b^9*c^5*d^13 - 239360*a^9*b^9*c^7*d^11 - 5260*a^10*b^8*c^2*d^16 - 64720*a^10*b^8*c^4*d^14 + 88720*a^10*b^8*c^6*d^12 - 3200*a^11*b^7*c^3*d^15 - 17600*a^11*b^7*c^5*d^13 + 1600*a^12*b^6*c^2*d^16 + 1600*a^12*b^6*c^4*d^14 + 27648*a*b^17*c*d^17)*(80*a^2*b^28*c^30 - 16*b^30*c^30 - 16*a^30*d^30 - 160*a^4*b^26*c^30 + 160*a^6*b^24*c^30 - 80*a^8*b^22*c^30 + 16*a^10*b^20*c^30 + 16*a^20*b^10*d^30 - 80*a^22*b^8*d^30 + 160*a^24*b^6*d^30 - 160*a^26*b^4*d^30 + 80*a^28*b^2*d^30 + 80*a^30*c^2*d^28 - 160*a^30*c^4*d^26 + 160*a^30*c^6*d^24 - 80*a^30*c^8*d^22 + 16*a^30*c^10*d^20 + 16*b^30*c^20*d^10 - 80*b^30*c^22*d^8 + 160*b^30*c^24*d^6 - 160*b^30*c^26*d^4 + 80*b^30*c^28*d^2 - 320*a*b^29*c^19*d^11 + 1600*a*b^29*c^21*d^9 - 3200*a*b^29*c^23*d^7 + 3200*a*b^29*c^25*d^5 - 1600*a*b^29*c^27*d^3 - 1600*a^3*b^27*c^29*d + 3200*a^5*b^25*c^29*d - 3200*a^7*b^23*c^29*d + 1600*a^9*b^21*c^29*d - 320*a^11*b^19*c^29*d - 320*a^19*b^11*c*d^29 + 1600*a^21*b^9*c*d^29 - 3200*a^23*b^7*c*d^29 + 3200*a^25*b^5*c*d^29 - 1600*a^27*b^3*c*d^29 - 1600*a^29*b*c^3*d^27 + 3200*a^29*b*c^5*d^25 - 3200*a^29*b*c^7*d^23 + 1600*a^29*b*c^9*d^21 - 320*a^29*b*c^11*d^19 + 3040*a^2*b^28*c^18*d^12 - 15280*a^2*b^28*c^20*d^10 + 30800*a^2*b^28*c^22*d^8 - 31200*a^2*b^28*c^24*d^6 + 16000*a^2*b^28*c^26*d^4 - 3440*a^2*b^28*c^28*d^2 - 18240*a^3*b^27*c^17*d^13 + 92800*a^3*b^27*c^19*d^11 - 190400*a^3*b^27*c^21*d^9 + 198400*a^3*b^27*c^23*d^7 - 107200*a^3*b^27*c^25*d^5 + 26240*a^3*b^27*c^27*d^3 + 77520*a^4*b^26*c^16*d^14 - 402800*a^4*b^26*c^18*d^12 + 851360*a^4*b^26*c^20*d^10 - 928000*a^4*b^26*c^22*d^8 + 541200*a^4*b^26*c^24*d^6 - 155120*a^4*b^26*c^26*d^4 + 16000*a^4*b^26*c^28*d^2 - 248064*a^5*b^25*c^15*d^15 + 1331520*a^5*b^25*c^17*d^13 - 2939840*a^5*b^25*c^19*d^11 + 3408640*a^5*b^25*c^21*d^9 - 2184320*a^5*b^25*c^23*d^7 + 736064*a^5*b^25*c^25*d^5 - 107200*a^5*b^25*c^27*d^3 + 620160*a^6*b^24*c^14*d^16 - 3488400*a^6*b^24*c^16*d^14 + 8170000*a^6*b^24*c^18*d^12 - 10229760*a^6*b^24*c^20*d^10 + 7281600*a^6*b^24*c^22*d^8 - 2863760*a^6*b^24*c^24*d^6 + 541200*a^6*b^24*c^26*d^4 - 31200*a^6*b^24*c^28*d^2 - 12403

20*a^7*b^23*c^13*d^17 + 7441920*a^7*b^23*c^15*d^15 - 18787200*a^7*b^23*c^17
*d^13 + 25721600*a^7*b^23*c^19*d^11 - 20444800*a^7*b^23*c^21*d^9 + 9297920*
a^7*b^23*c^23*d^7 - 2184320*a^7*b^23*c^25*d^5 + 198400*a^7*b^23*c^27*d^3 +
2015520*a^8*b^22*c^12*d^18 - 13178400*a^8*b^22*c^14*d^16 + 36434400*a^8*b^2
2*c^16*d^14 - 55069600*a^8*b^22*c^18*d^12 + 48989680*a^8*b^22*c^20*d^10 - 2
5575920*a^8*b^22*c^22*d^8 + 7281600*a^8*b^22*c^24*d^6 - 928000*a^8*b^22*c^2
6*d^4 + 30800*a^8*b^22*c^28*d^2 - 2687360*a^9*b^21*c^11*d^19 + 19638400*a^9
*b^21*c^13*d^17 - 60362240*a^9*b^21*c^15*d^15 + 101475200*a^9*b^21*c^17*d^1
3 - 101172800*a^9*b^21*c^19*d^11 + 60333760*a^9*b^21*c^21*d^9 - 20444800*a^
9*b^21*c^23*d^7 + 3408640*a^9*b^21*c^25*d^5 - 190400*a^9*b^21*c^27*d^3 + 29
56096*a^10*b^20*c^10*d^20 - 24858080*a^10*b^20*c^12*d^18 + 86150560*a^10*b^
20*c^14*d^16 - 162120160*a^10*b^20*c^16*d^14 + 181463680*a^10*b^20*c^18*d^1
2 - 123188112*a^10*b^20*c^20*d^10 + 48989680*a^10*b^20*c^22*d^8 - 10229760*
a^10*b^20*c^24*d^6 + 851360*a^10*b^20*c^26*d^4 - 15280*a^10*b^20*c^28*d^2 -
2687360*a^11*b^19*c^9*d^21 + 26873600*a^11*b^19*c^11*d^19 - 106460800*a^11
*b^19*c^13*d^17 + 225738240*a^11*b^19*c^15*d^15 - 284331200*a^11*b^19*c^17*
d^13 + 219166080*a^11*b^19*c^19*d^11 - 101172800*a^11*b^19*c^21*d^9 + 25721
600*a^11*b^19*c^23*d^7 - 2939840*a^11*b^19*c^25*d^5 + 92800*a^11*b^19*c^27*
d^3 + 2015520*a^12*b^18*c^8*d^22 - 24858080*a^12*b^18*c^10*d^20 + 114212800
*a^12*b^18*c^12*d^18 - 274937600*a^12*b^18*c^14*d^16 + 390830000*a^12*b^18*
c^16*d^14 - 341426960*a^12*b^18*c^18*d^12 + 181463680*a^12*b^18*c^20*d^10 -
55069600*a^12*b^18*c^22*d^8 + 8170000*a^12*b^18*c^24*d^6 - 402800*a^12*b^1
8*c^26*d^4 + 3040*a^12*b^18*c^28*d^2 - 1240320*a^13*b^17*c^7*d^23 + 1963840
0*a^13*b^17*c^9*d^21 - 106460800*a^13*b^17*c^11*d^19 + 293542400*a^13*b^17*
c^13*d^17 - 472561920*a^13*b^17*c^15*d^15 + 467412160*a^13*b^17*c^17*d^13 -
284331200*a^13*b^17*c^19*d^11 + 101475200*a^13*b^17*c^21*d^9 - 18787200*a^
13*b^17*c^23*d^7 + 1331520*a^13*b^17*c^25*d^5 - 18240*a^13*b^17*c^27*d^3 +
620160*a^14*b^16*c^6*d^24 - 13178400*a^14*b^16*c^8*d^22 + 86150560*a^14*b^1
6*c^10*d^20 - 274937600*a^14*b^16*c^12*d^18 + 503363200*a^14*b^16*c^14*d^16
- 563751280*a^14*b^16*c^16*d^14 + 390830000*a^14*b^16*c^18*d^12 - 16212016
0*a^14*b^16*c^20*d^10 + 36434400*a^14*b^16*c^22*d^8 - 3488400*a^14*b^16*c^2
4*d^6 + 77520*a^14*b^16*c^26*d^4 - 248064*a^15*b^15*c^5*d^25 + 7441920*a^15
*b^15*c^7*d^23 - 60362240*a^15*b^15*c^9*d^21 + 225738240*a^15*b^15*c^11*d^1
9 - 472561920*a^15*b^15*c^13*d^17 + 599984128*a^15*b^15*c^15*d^15 - 4725619
20*a^15*b^15*c^17*d^13 + 225738240*a^15*b^15*c^19*d^11 - 60362240*a^15*b^15
*c^21*d^9 + 7441920*a^15*b^15*c^23*d^7 - 248064*a^15*b^15*c^25*d^5 + 77520*
a^16*b^14*c^4*d^26 - 3488400*a^16*b^14*c^6*d^24 + 36434400*a^16*b^14*c^8*d^
22 - 162120160*a^16*b^14*c^10*d^20 + 390830000*a^16*b^14*c^12*d^18 - 563751
280*a^16*b^14*c^14*d^16 + 503363200*a^16*b^14*c^16*d^14 - 274937600*a^16*b^
14*c^18*d^12 + 86150560*a^16*b^14*c^20*d^10 - 13178400*a^16*b^14*c^22*d^8 +
620160*a^16*b^14*c^24*d^6 - 18240*a^17*b^13*c^3*d^27 + 1331520*a^17*b^13*c
^5*d^25 - 18787200*a^17*b^13*c^7*d^23 + 101475200*a^17*b^13*c^9*d^21 - 2843
31200*a^17*b^13*c^11*d^19 + 467412160*a^17*b^13*c^13*d^17 - 472561920*a^17*
b^13*c^15*d^15 + 293542400*a^17*b^13*c^17*d^13 - 106460800*a^17*b^13*c^19*d
^11 + 19638400*a^17*b^13*c^21*d^9 - 1240320*a^17*b^13*c^23*d^7 + 3040*a^18*

b^12*c^2*d^28 - 402800*a^18*b^12*c^4*d^26 + 8170000*a^18*b^12*c^6*d^24 - 55069600*a^18*b^12*c^8*d^22 + 181463680*a^18*b^12*c^10*d^20 - 341426960*a^18*b^12*c^12*d^18 + 390830000*a^18*b^12*c^14*d^16 - 274937600*a^18*b^12*c^16*d^14 + 114212800*a^18*b^12*c^18*d^12 - 24858080*a^18*b^12*c^20*d^10 + 2015520*a^18*b^12*c^22*d^8 + 92800*a^19*b^11*c^3*d^27 - 2939840*a^19*b^11*c^5*d^25 + 25721600*a^19*b^11*c^7*d^23 - 101172800*a^19*b^11*c^9*d^21 + 219166080*a^19*b^11*c^11*d^19 - 284331200*a^19*b^11*c^13*d^17 + 225738240*a^19*b^11*c^15*d^15 - 106460800*a^19*b^11*c^17*d^13 + 26873600*a^19*b^11*c^19*d^11 - 2687360*a^19*b^11*c^21*d^9 - 15280*a^20*b^10*c^2*d^28 + 851360*a^20*b^10*c^4*d^26 - 10229760*a^20*b^10*c^6*d^24 + 48989680*a^20*b^10*c^8*d^22 - 123188112*a^20*b^10*c^10*d^20 + 181463680*a^20*b^10*c^12*d^18 - 162120160*a^20*b^10*c^14*d^16 + 86150560*a^20*b^10*c^16*d^14 - 24858080*a^20*b^10*c^18*d^12 + 2956096*a^20*b^10*c^20*d^10 - 190400*a^21*b^9*c^3*d^27 + 3408640*a^21*b^9*c^5*d^25 - 20444800*a^21*b^9*c^7*d^23 + 60333760*a^21*b^9*c^9*d^21 - 101172800*a^21*b^9*c^11*d^19 + 101475200*a^21*b^9*c^13*d^17 - 60362240*a^21*b^9*c^15*d^15 + 19638400*a^21*b^9*c^17*d^13 - 2687360*a^21*b^9*c^19*d^11 + 30800*a^22*b^8*c^2*d^28 - 928000*a^22*b^8*c^4*d^26 + 7281600*a^22*b^8*c^6*d^24 - 25575920*a^22*b^8*c^8*d^22 + 48989680*a^22*b^8*c^10*d^20 - 55069600*a^22*b^8*c^12*d^18 + 36434400*a^22*b^8*c^14*d^16 - 13178400*a^22*b^8*c^16*d^14 + 2015520*a^22*b^8*c^18*d^12 + 198400*a^23*b^7*c^3*d^27 - 2184320*a^23*b^7*c^5*d^25 + 9297920*a^23*b^7*c^7*d^23 - 20444800*a^23*b^7*c^9*d^21 + 25721600*a^23*b^7*c^11*d^19 - 18787200*a^23*b^7*c^13*d^17 + 7441920*a^23*b^7*c^15*d^15 - 1240320*a^23*b^7*c^17*d^13 - 31200*a^24*b^6*c^2*d^28 + 541200*a^24*b^6*c^4*d^26 - 2863760*a^24*b^6*c^6*d^24 + 7281600*a^24*b^6*c^8*d^22 - 10229760*a^24*b^6*c^10*d^20 + 8170000*a^24*b^6*c^12*d^18 - 3488400*a^24*b^6*c^14*d^16 + 620160*a^24*b^6*c^16*d^14 - 107200*a^25*b^5*c^3*d^27 + 736064*a^25*b^5*c^5*d^25 - 2184320*a^25*b^5*c^7*d^23 + 3408640*a^25*b^5*c^9*d^21 - 2939840*a^25*b^5*c^11*d^19 + 1331520*a^25*b^5*c^13*d^17 - 248064*a^25*b^5*c^15*d^15 + 16000*a^26*b^4*c^2*d^28 - 155120*a^26*b^4*c^4*d^26 + 541200*a^26*b^4*c^6*d^24 - 928000*a^26*b^4*c^8*d^22 + 851360*a^26*b^4*c^10*d^20 - 402800*a^26*b^4*c^12*d^18 + 77520*a^26*b^4*c^14*d^16 + 26240*a^27*b^3*c^3*d^27 - 107200*a^27*b^3*c^5*d^25 + 198400*a^27*b^3*c^7*d^23 - 190400*a^27*b^3*c^9*d^21 + 92800*a^27*b^3*c^11*d^19 - 18240*a^27*b^3*c^13*d^17 - 3440*a^28*b^2*c^2*d^28 + 16000*a^28*b^2*c^4*d^26 - 31200*a^28*b^2*c^6*d^24 + 30800*a^28*b^2*c^8*d^22 - 15280*a^28*b^2*c^10*d^20 + 3040*a^28*b^2*c^12*d^18 + 320*a*b^29*c^29*d + 320*a^29*b*c*d^29))^(1/2) + 2*a^24*d^24 + 2*b^24*c^24 + 8*a^2*b^22*c^24 + 8*a^4*b^20*c^24 - 576*a^10*b^14*d^24 + 2784*a^12*b^12*d^24 - 5284*a^14*b^10*d^24 + 4730*a^16*b^8*d^24 - 1780*a^18*b^6*d^24 + 68*a^20*b^4*d^24 + 38*a^22*b^2*d^24 + 8*a^24*c^2*d^22 + 8*a^24*c^4*d^20 - 576*b^24*c^10*d^14 + 2784*b^24*c^12*d^12 - 5284*b^24*c^14*d^10 + 4730*b^24*c^16*d^8 - 1780*b^24*c^18*d^6 + 68*b^24*c^20*d^4 + 38*b^24*c^22*d^2 + 5760*a*b^23*c^9*d^15 - 28224*a*b^23*c^11*d^13 + 54728*a*b^23*c^13*d^11 - 50620*a*b^23*c^15*d^9 + 20360*a*b^23*c^17*d^7 - 1480*a*b^23*c^19*d^5 - 268*a*b^23*c^21*d^3 - 88*a^3*b^21*c^23*d - 160*a^5*b^19*c^23*d + 5760*a^9*b^15*c*d^23 - 28224*a^11*b^13*c*d^23 + 54728*a^13*b^11*c*d^23 - 50620*a^15*b^9*c*d^23 + 20360*a^17*b^7*c*d^

23 - 1480*a^19*b^5*c*d^23 - 268*a^21*b^3*c*d^23 - 88*a^23*b*c^3*d^21 - 160*
a^23*b*c^5*d^19 - 25920*a^2*b^22*c^8*d^16 + 131904*a^2*b^22*c^10*d^14 - 270
604*a^2*b^22*c^12*d^12 + 273544*a^2*b^22*c^14*d^10 - 131660*a^2*b^22*c^16*d
^8 + 22060*a^2*b^22*c^18*d^6 - 782*a^2*b^22*c^20*d^4 - 98*a^2*b^22*c^22*d^2
 + 69120*a^3*b^21*c^7*d^17 - 379200*a^3*b^21*c^9*d^15 + 860368*a^3*b^21*c^1
1*d^13 - 1001364*a^3*b^21*c^13*d^11 + 605280*a^3*b^21*c^15*d^9 - 167520*a^3
*b^21*c^17*d^7 + 18840*a^3*b^21*c^19*d^5 - 144*a^3*b^21*c^21*d^3 - 120960*a
^4*b^20*c^6*d^18 + 756000*a^4*b^20*c^8*d^16 - 1987844*a^4*b^20*c^10*d^14 +
2750664*a^4*b^20*c^12*d^12 - 2073976*a^4*b^20*c^14*d^10 + 793460*a^4*b^20*c
^16*d^8 - 138010*a^4*b^20*c^18*d^6 + 10562*a^4*b^20*c^20*d^4 + 88*a^4*b^20*
c^22*d^2 + 145152*a^5*b^19*c^5*d^19 - 1116288*a^5*b^19*c^7*d^17 + 3539128*a
^5*b^19*c^9*d^15 - 5890780*a^5*b^19*c^11*d^13 + 5437600*a^5*b^19*c^13*d^11
- 2682536*a^5*b^19*c^15*d^9 + 655084*a^5*b^19*c^17*d^7 - 85484*a^5*b^19*c^1
9*d^5 + 4080*a^5*b^19*c^21*d^3 - 120960*a^6*b^18*c^4*d^20 + 1266048*a^6*b^1
8*c^6*d^18 - 4977996*a^6*b^18*c^8*d^16 + 10009720*a^6*b^18*c^10*d^14 - 1120
9800*a^6*b^18*c^12*d^12 + 6943760*a^6*b^18*c^14*d^10 - 2253214*a^6*b^18*c^1
6*d^8 + 396878*a^6*b^18*c^18*d^6 - 36120*a^6*b^18*c^20*d^4 + 1520*a^6*b^18*
c^22*d^2 + 69120*a^7*b^17*c^3*d^21 - 1116288*a^7*b^17*c^5*d^19 + 5575008*a^
7*b^17*c^7*d^17 - 13668308*a^7*b^17*c^9*d^15 + 18576800*a^7*b^17*c^11*d^13
- 14230520*a^7*b^17*c^13*d^11 + 5889904*a^7*b^17*c^15*d^9 - 1310504*a^7*b^1
7*c^17*d^7 + 168344*a^7*b^17*c^19*d^5 - 8960*a^7*b^17*c^21*d^3 - 25920*a^8*
b^16*c^2*d^22 + 756000*a^8*b^16*c^4*d^20 - 4977996*a^8*b^16*c^6*d^18 + 1514
4828*a^8*b^16*c^8*d^16 - 25068800*a^8*b^16*c^10*d^14 + 23486280*a^8*b^16*c^
12*d^12 - 12099640*a^8*b^16*c^14*d^10 + 3330518*a^8*b^16*c^16*d^8 - 529224*
a^8*b^16*c^18*d^6 + 36280*a^8*b^16*c^20*d^4 - 379200*a^9*b^15*c^3*d^21 + 35
39128*a^9*b^15*c^5*d^19 - 13668308*a^9*b^15*c^7*d^17 + 27691952*a^9*b^15*c^
9*d^15 - 31562040*a^9*b^15*c^11*d^13 + 19993760*a^9*b^15*c^13*d^11 - 673104
4*a^9*b^15*c^15*d^9 + 1239264*a^9*b^15*c^17*d^7 - 106016*a^9*b^15*c^19*d^5
+ 131904*a^10*b^14*c^2*d^22 - 1987844*a^10*b^14*c^4*d^20 + 10009720*a^10*b^
14*c^6*d^18 - 25068800*a^10*b^14*c^8*d^16 + 34796936*a^10*b^14*c^10*d^14 -
26927144*a^10*b^14*c^12*d^12 + 10994964*a^10*b^14*c^14*d^10 - 2295680*a^10*
b^14*c^16*d^8 + 230240*a^10*b^14*c^18*d^6 + 860368*a^11*b^13*c^3*d^21 - 589
0780*a^11*b^13*c^5*d^19 + 18576800*a^11*b^13*c^7*d^17 - 31562040*a^11*b^13*
c^9*d^15 + 29722864*a^11*b^13*c^11*d^13 - 14679348*a^11*b^13*c^13*d^11 + 34
97920*a^11*b^13*c^15*d^9 - 381280*a^11*b^13*c^17*d^7 - 270604*a^12*b^12*c^2
*d^22 + 2750664*a^12*b^12*c^4*d^20 - 11209800*a^12*b^12*c^6*d^18 + 23486280
*a^12*b^12*c^8*d^16 - 26927144*a^12*b^12*c^10*d^14 + 16147404*a^12*b^12*c^1
2*d^12 - 4479104*a^12*b^12*c^14*d^10 + 499520*a^12*b^12*c^16*d^8 - 1001364*
a^13*b^11*c^3*d^21 + 5437600*a^13*b^11*c^5*d^19 - 14230520*a^13*b^11*c^7*d^
17 + 19993760*a^13*b^11*c^9*d^15 - 14679348*a^13*b^11*c^11*d^13 + 4861024*a
^13*b^11*c^13*d^11 - 552160*a^13*b^11*c^15*d^9 + 273544*a^14*b^10*c^2*d^22
- 2073976*a^14*b^10*c^4*d^20 + 6943760*a^14*b^10*c^6*d^18 - 12099640*a^14*b
^10*c^8*d^16 + 10994964*a^14*b^10*c^10*d^14 - 4479104*a^14*b^10*c^12*d^12 +
 562016*a^14*b^10*c^14*d^10 + 605280*a^15*b^9*c^3*d^21 - 2682536*a^15*b^9*c
^5*d^19 + 5889904*a^15*b^9*c^7*d^17 - 6731044*a^15*b^9*c^9*d^15 + 3497920*a

^15*b^9*c^11*d^13 - 552160*a^15*b^9*c^13*d^11 - 131660*a^16*b^8*c^2*d^22 + 793460*a^16*b^8*c^4*d^20 - 2253214*a^16*b^8*c^6*d^18 + 3330518*a^16*b^8*c^8*d^16 - 2295680*a^16*b^8*c^10*d^14 + 499520*a^16*b^8*c^12*d^12 - 167520*a^17*b^7*c^3*d^21 + 655084*a^17*b^7*c^5*d^19 - 1310504*a^17*b^7*c^7*d^17 + 1239264*a^17*b^7*c^9*d^15 - 381280*a^17*b^7*c^11*d^13 + 22060*a^18*b^6*c^2*d^22 - 138010*a^18*b^6*c^4*d^20 + 396878*a^18*b^6*c^6*d^18 - 529224*a^18*b^6*c^8*d^16 + 230240*a^18*b^6*c^10*d^14 + 18840*a^19*b^5*c^3*d^21 - 85484*a^19*b^5*c^5*d^19 + 168344*a^19*b^5*c^7*d^17 - 106016*a^19*b^5*c^9*d^15 - 782*a^20*b^4*c^2*d^22 + 10562*a^20*b^4*c^4*d^20 - 36120*a^20*b^4*c^6*d^18 + 36280*a^20*b^4*c^8*d^16 - 144*a^21*b^3*c^3*d^21 + 4080*a^21*b^3*c^5*d^19 - 8960*a^21*b^3*c^7*d^17 - 98*a^22*b^2*c^2*d^22 + 88*a^22*b^2*c^4*d^20 + 1520*a^22*b^2*c^6*d^18 - 4*a*b^23*c^23*d - 4*a^23*b*c*d^23)/(16*(5*a^2*b^28*c^30 - b^30*c^30 - a^30*d^30 - 10*a^4*b^26*c^30 + 10*a^6*b^24*c^30 - 5*a^8*b^22*c^30 + a^10*b^20*c^30 + a^20*b^10*d^30 - 5*a^22*b^8*d^30 + 10*a^24*b^6*d^30 - 10*a^26*b^4*d^30 + 5*a^28*b^2*d^30 + 5*a^30*c^2*d^28 - 10*a^30*c^4*d^26 + 10*a^30*c^6*d^24 - 5*a^30*c^8*d^22 + a^30*c^10*d^20 + b^30*c^20*d^10 - 5*b^30*c^22*d^8 + 10*b^30*c^24*d^6 - 10*b^30*c^26*d^4 + 5*b^30*c^28*d^2 - 20*a*b^29*c^19*d^11 + 100*a*b^29*c^21*d^9 - 200*a*b^29*c^23*d^7 + 200*a*b^29*c^25*d^5 - 100*a*b^29*c^27*d^3 - 100*a^3*b^27*c^29*d + 200*a^5*b^25*c^29*d - 200*a^7*b^23*c^29*d + 100*a^9*b^21*c^29*d - 20*a^11*b^19*c^29*d - 20*a^19*b^11*c*d^29 + 100*a^21*b^9*c*d^29 - 200*a^23*b^7*c*d^29 + 200*a^25*b^5*c*d^29 - 100*a^27*b^3*c*d^29 - 100*a^29*b*c^3*d^27 + 200*a^29*b*c^5*d^25 - 200*a^29*b*c^7*d^23 + 100*a^29*b*c^9*d^21 - 20*a^29*b*c^11*d^19 + 190*a^2*b^28*c^18*d^12 - 955*a^2*b^28*c^20*d^10 + 1925*a^2*b^28*c^22*d^8 - 1950*a^2*b^28*c^24*d^6 + 1000*a^2*b^28*c^26*d^4 - 215*a^2*b^28*c^28*d^2 - 1140*a^3*b^27*c^17*d^13 + 5800*a^3*b^27*c^19*d^11 - 11900*a^3*b^27*c^21*d^9 + 12400*a^3*b^27*c^23*d^7 - 6700*a^3*b^27*c^25*d^5 + 1640*a^3*b^27*c^27*d^3 + 4845*a^4*b^26*c^16*d^14 - 25175*a^4*b^26*c^18*d^12 + 53210*a^4*b^26*c^20*d^10 - 58000*a^4*b^26*c^22*d^8 + 33825*a^4*b^26*c^24*d^6 - 9695*a^4*b^26*c^26*d^4 + 1000*a^4*b^26*c^28*d^2 - 15504*a^5*b^25*c^15*d^15 + 83220*a^5*b^25*c^17*d^13 - 183740*a^5*b^25*c^19*d^11 + 213040*a^5*b^25*c^21*d^9 - 136520*a^5*b^25*c^23*d^7 + 46004*a^5*b^25*c^25*d^5 - 6700*a^5*b^25*c^27*d^3 + 38760*a^6*b^24*c^14*d^16 - 218025*a^6*b^24*c^16*d^14 + 510625*a^6*b^24*c^18*d^12 - 639360*a^6*b^24*c^20*d^10 + 455100*a^6*b^24*c^22*d^8 - 178985*a^6*b^24*c^24*d^6 + 33825*a^6*b^24*c^26*d^4 - 1950*a^6*b^24*c^28*d^2 - 77520*a^7*b^23*c^13*d^17 + 465120*a^7*b^23*c^15*d^15 - 1174200*a^7*b^23*c^17*d^13 + 1607600*a^7*b^23*c^19*d^11 - 1277800*a^7*b^23*c^21*d^9 + 581120*a^7*b^23*c^23*d^7 - 136520*a^7*b^23*c^25*d^5 + 12400*a^7*b^23*c^27*d^3 + 125970*a^8*b^22*c^12*d^18 - 823650*a^8*b^22*c^14*d^16 + 2277150*a^8*b^22*c^16*d^14 - 3441850*a^8*b^22*c^18*d^12 + 3061855*a^8*b^22*c^20*d^10 - 1598495*a^8*b^22*c^22*d^8 + 455100*a^8*b^22*c^24*d^6 - 58000*a^8*b^22*c^26*d^4 + 1925*a^8*b^22*c^28*d^2 - 167960*a^9*b^21*c^11*d^19 + 1227400*a^9*b^21*c^13*d^17 - 3772640*a^9*b^21*c^15*d^15 + 6342200*a^9*b^21*c^17*d^13 - 6323300*a^9*b^21*c^19*d^11 + 3770860*a^9*b^21*c^21*d^9 - 1277800*a^9*b^21*c^23*d^7 + 213040*a^9*b^21*c^25*d^5 - 11900*a^9*b^21*c^27*d^3 + 184756*a^10*b^20*c^10*d^20 - 1553630*a^10*b^20*c^12*d^1

8 + 5384410*a^10*b^20*c^14*d^16 - 10132510*a^10*b^20*c^16*d^14 + 11341480*a
^10*b^20*c^18*d^12 - 7699257*a^10*b^20*c^20*d^10 + 3061855*a^10*b^20*c^22*d
^8 - 639360*a^10*b^20*c^24*d^6 + 53210*a^10*b^20*c^26*d^4 - 955*a^10*b^20*c
^28*d^2 - 167960*a^11*b^19*c^9*d^21 + 1679600*a^11*b^19*c^11*d^19 - 6653800
*a^11*b^19*c^13*d^17 + 14108640*a^11*b^19*c^15*d^15 - 17770700*a^11*b^19*c^
17*d^13 + 13697880*a^11*b^19*c^19*d^11 - 6323300*a^11*b^19*c^21*d^9 + 16076
00*a^11*b^19*c^23*d^7 - 183740*a^11*b^19*c^25*d^5 + 5800*a^11*b^19*c^27*d^3
 + 125970*a^12*b^18*c^8*d^22 - 1553630*a^12*b^18*c^10*d^20 + 7138300*a^12*b
^18*c^12*d^18 - 17183600*a^12*b^18*c^14*d^16 + 24426875*a^12*b^18*c^16*d^14
 - 21339185*a^12*b^18*c^18*d^12 + 11341480*a^12*b^18*c^20*d^10 - 3441850*a^
12*b^18*c^22*d^8 + 510625*a^12*b^18*c^24*d^6 - 25175*a^12*b^18*c^26*d^4 + 1
90*a^12*b^18*c^28*d^2 - 77520*a^13*b^17*c^7*d^23 + 1227400*a^13*b^17*c^9*d^
21 - 6653800*a^13*b^17*c^11*d^19 + 18346400*a^13*b^17*c^13*d^17 - 29535120*
a^13*b^17*c^15*d^15 + 29213260*a^13*b^17*c^17*d^13 - 17770700*a^13*b^17*c^1
9*d^11 + 6342200*a^13*b^17*c^21*d^9 - 1174200*a^13*b^17*c^23*d^7 + 83220*a^
13*b^17*c^25*d^5 - 1140*a^13*b^17*c^27*d^3 + 38760*a^14*b^16*c^6*d^24 - 823
650*a^14*b^16*c^8*d^22 + 5384410*a^14*b^16*c^10*d^20 - 17183600*a^14*b^16*c
^12*d^18 + 31460200*a^14*b^16*c^14*d^16 - 35234455*a^14*b^16*c^16*d^14 + 24
426875*a^14*b^16*c^18*d^12 - 10132510*a^14*b^16*c^20*d^10 + 2277150*a^14*b^
16*c^22*d^8 - 218025*a^14*b^16*c^24*d^6 + 4845*a^14*b^16*c^26*d^4 - 15504*a
^15*b^15*c^5*d^25 + 465120*a^15*b^15*c^7*d^23 - 3772640*a^15*b^15*c^9*d^21
+ 14108640*a^15*b^15*c^11*d^19 - 29535120*a^15*b^15*c^13*d^17 + 37499008*a^
15*b^15*c^15*d^15 - 29535120*a^15*b^15*c^17*d^13 + 14108640*a^15*b^15*c^19*
d^11 - 3772640*a^15*b^15*c^21*d^9 + 465120*a^15*b^15*c^23*d^7 - 15504*a^15*
b^15*c^25*d^5 + 4845*a^16*b^14*c^4*d^26 - 218025*a^16*b^14*c^6*d^24 + 22771
50*a^16*b^14*c^8*d^22 - 10132510*a^16*b^14*c^10*d^20 + 24426875*a^16*b^14*c
^12*d^18 - 35234455*a^16*b^14*c^14*d^16 + 31460200*a^16*b^14*c^16*d^14 - 17
183600*a^16*b^14*c^18*d^12 + 5384410*a^16*b^14*c^20*d^10 - 823650*a^16*b^14
*c^22*d^8 + 38760*a^16*b^14*c^24*d^6 - 1140*a^17*b^13*c^3*d^27 + 83220*a^17
*b^13*c^5*d^25 - 1174200*a^17*b^13*c^7*d^23 + 6342200*a^17*b^13*c^9*d^21 -
17770700*a^17*b^13*c^11*d^19 + 29213260*a^17*b^13*c^13*d^17 - 29535120*a^17
*b^13*c^15*d^15 + 18346400*a^17*b^13*c^17*d^13 - 6653800*a^17*b^13*c^19*d^1
1 + 1227400*a^17*b^13*c^21*d^9 - 77520*a^17*b^13*c^23*d^7 + 190*a^18*b^12*c
^2*d^28 - 25175*a^18*b^12*c^4*d^26 + 510625*a^18*b^12*c^6*d^24 - 3441850*a^
18*b^12*c^8*d^22 + 11341480*a^18*b^12*c^10*d^20 - 21339185*a^18*b^12*c^12*d
^18 + 24426875*a^18*b^12*c^14*d^16 - 17183600*a^18*b^12*c^16*d^14 + 7138300
*a^18*b^12*c^18*d^12 - 1553630*a^18*b^12*c^20*d^10 + 125970*a^18*b^12*c^22*
d^8 + 5800*a^19*b^11*c^3*d^27 - 183740*a^19*b^11*c^5*d^25 + 1607600*a^19*b^
11*c^7*d^23 - 6323300*a^19*b^11*c^9*d^21 + 13697880*a^19*b^11*c^11*d^19 - 1
7770700*a^19*b^11*c^13*d^17 + 14108640*a^19*b^11*c^15*d^15 - 6653800*a^19*b
^11*c^17*d^13 + 1679600*a^19*b^11*c^19*d^11 - 167960*a^19*b^11*c^21*d^9 - 9
55*a^20*b^10*c^2*d^28 + 53210*a^20*b^10*c^4*d^26 - 639360*a^20*b^10*c^6*d^2
4 + 3061855*a^20*b^10*c^8*d^22 - 7699257*a^20*b^10*c^10*d^20 + 11341480*a^2
0*b^10*c^12*d^18 - 10132510*a^20*b^10*c^14*d^16 + 5384410*a^20*b^10*c^16*d^
14 - 1553630*a^20*b^10*c^18*d^12 + 184756*a^20*b^10*c^20*d^10 - 11900*a^21*

b^9*c^3*d^27 + 213040*a^21*b^9*c^5*d^25 - 1277800*a^21*b^9*c^7*d^23 + 37708
60*a^21*b^9*c^9*d^21 - 6323300*a^21*b^9*c^11*d^19 + 6342200*a^21*b^9*c^13*d
^17 - 3772640*a^21*b^9*c^15*d^15 + 1227400*a^21*b^9*c^17*d^13 - 167960*a^21
*b^9*c^19*d^11 + 1925*a^22*b^8*c^2*d^28 - 58000*a^22*b^8*c^4*d^26 + 455100*
a^22*b^8*c^6*d^24 - 1598495*a^22*b^8*c^8*d^22 + 3061855*a^22*b^8*c^10*d^20
- 3441850*a^22*b^8*c^12*d^18 + 2277150*a^22*b^8*c^14*d^16 - 823650*a^22*b^8
*c^16*d^14 + 125970*a^22*b^8*c^18*d^12 + 12400*a^23*b^7*c^3*d^27 - 136520*a
^23*b^7*c^5*d^25 + 581120*a^23*b^7*c^7*d^23 - 1277800*a^23*b^7*c^9*d^21 + 1
607600*a^23*b^7*c^11*d^19 - 1174200*a^23*b^7*c^13*d^17 + 465120*a^23*b^7*c^
15*d^15 - 77520*a^23*b^7*c^17*d^13 - 1950*a^24*b^6*c^2*d^28 + 33825*a^24*b^
6*c^4*d^26 - 178985*a^24*b^6*c^6*d^24 + 455100*a^24*b^6*c^8*d^22 - 639360*a
^24*b^6*c^10*d^20 + 510625*a^24*b^6*c^12*d^18 - 218025*a^24*b^6*c^14*d^16 +
38760*a^24*b^6*c^16*d^14 - 6700*a^25*b^5*c^3*d^27 + 46004*a^25*b^5*c^5*d^2
5 - 136520*a^25*b^5*c^7*d^23 + 213040*a^25*b^5*c^9*d^21 - 183740*a^25*b^5*c
^11*d^19 + 83220*a^25*b^5*c^13*d^17 - 15504*a^25*b^5*c^15*d^15 + 1000*a^26*
b^4*c^2*d^28 - 9695*a^26*b^4*c^4*d^26 + 33825*a^26*b^4*c^6*d^24 - 58000*a^2
6*b^4*c^8*d^22 + 53210*a^26*b^4*c^10*d^20 - 25175*a^26*b^4*c^12*d^18 + 4845
*a^26*b^4*c^14*d^16 + 1640*a^27*b^3*c^3*d^27 - 6700*a^27*b^3*c^5*d^25 + 124
00*a^27*b^3*c^7*d^23 - 11900*a^27*b^3*c^9*d^21 + 5800*a^27*b^3*c^11*d^19 -
1140*a^27*b^3*c^13*d^17 - 215*a^28*b^2*c^2*d^28 + 1000*a^28*b^2*c^4*d^26 -
1950*a^28*b^2*c^6*d^24 + 1925*a^28*b^2*c^8*d^22 - 955*a^28*b^2*c^10*d^20 +
190*a^28*b^2*c^12*d^18 + 20*a*b^29*c^29*d + 20*a^29*b*c*d^29)))^(1/2)*(((4*
(8*a^2*b^23*c^25 - 32*a^4*b^21*c^25 + 48*a^6*b^19*c^25 - 32*a^8*b^17*c^25 +
8*a^10*b^15*c^25 + 8*a^25*c^2*d^23 - 32*a^25*c^4*d^21 + 48*a^25*c^6*d^19 -
32*a^25*c^8*d^17 + 8*a^25*c^10*d^15 - 8*a*b^24*c^16*d^9 + 32*a*b^24*c^18*d
^7 - 48*a*b^24*c^20*d^5 + 32*a*b^24*c^22*d^3 - 72*a^3*b^22*c^24*d + 368*a^5
*b^20*c^24*d - 592*a^7*b^18*c^24*d + 408*a^9*b^16*c^24*d - 104*a^11*b^14*c^
24*d - 8*a^16*b^9*c*d^24 + 32*a^18*b^7*c*d^24 - 48*a^20*b^5*c*d^24 + 32*a^2
2*b^3*c*d^24 - 72*a^24*b*c^3*d^22 + 368*a^24*b*c^5*d^20 - 592*a^24*b*c^7*d^
18 + 408*a^24*b*c^9*d^16 - 104*a^24*b*c^11*d^14 + 104*a^2*b^23*c^15*d^10 -
408*a^2*b^23*c^17*d^8 + 592*a^2*b^23*c^19*d^6 - 368*a^2*b^23*c^21*d^4 + 72*
a^2*b^23*c^23*d^2 - 616*a^3*b^22*c^14*d^11 + 2392*a^3*b^22*c^16*d^9 - 3408*
a^3*b^22*c^18*d^7 + 2032*a^3*b^22*c^20*d^5 - 328*a^3*b^22*c^22*d^3 + 2184*a
^4*b^21*c^13*d^12 - 8536*a^4*b^21*c^15*d^10 + 12272*a^4*b^21*c^17*d^8 - 740
8*a^4*b^21*c^19*d^6 + 1192*a^4*b^21*c^21*d^4 + 328*a^4*b^21*c^23*d^2 - 5096
*a^5*b^20*c^12*d^13 + 20664*a^5*b^20*c^14*d^11 - 31328*a^5*b^20*c^16*d^9 +
20592*a^5*b^20*c^18*d^7 - 4008*a^5*b^20*c^20*d^5 - 1192*a^5*b^20*c^22*d^3 +
8008*a^6*b^19*c^11*d^14 - 35672*a^6*b^19*c^13*d^12 + 60768*a^6*b^19*c^15*d
^10 - 46464*a^6*b^19*c^17*d^8 + 11336*a^6*b^19*c^19*d^6 + 4008*a^6*b^19*c^2
1*d^4 - 2032*a^6*b^19*c^23*d^2 - 8008*a^7*b^18*c^10*d^15 + 44408*a^7*b^18*c
^12*d^13 - 92512*a^7*b^18*c^14*d^11 + 85536*a^7*b^18*c^16*d^9 - 24904*a^7*b
^18*c^18*d^7 - 11336*a^7*b^18*c^20*d^5 + 7408*a^7*b^18*c^22*d^3 + 3432*a^8*
b^17*c^9*d^16 - 37752*a^8*b^17*c^11*d^14 + 109408*a^8*b^17*c^13*d^12 - 1254
72*a^8*b^17*c^15*d^10 + 42696*a^8*b^17*c^17*d^8 + 24904*a^8*b^17*c^19*d^6 -
20592*a^8*b^17*c^21*d^4 + 3408*a^8*b^17*c^23*d^2 + 3432*a^9*b^16*c^8*d^17

+ 14872*a^9*b^16*c^10*d^15 - 92352*a^9*b^16*c^12*d^13 + 141408*a^9*b^16*c^14*d^11 - 59264*a^9*b^16*c^16*d^9 - 42696*a^9*b^16*c^18*d^7 + 46464*a^9*b^16*c^20*d^5 - 12272*a^9*b^16*c^22*d^3 - 8008*a^10*b^15*c^7*d^18 + 14872*a^10*b^15*c^9*d^16 + 36608*a^10*b^15*c^11*d^14 - 113152*a^10*b^15*c^13*d^12 + 67008*a^10*b^15*c^15*d^10 + 59264*a^10*b^15*c^17*d^8 - 85536*a^10*b^15*c^19*d^6 + 31328*a^10*b^15*c^21*d^4 - 2392*a^10*b^15*c^23*d^2 + 8008*a^11*b^14*c^6*d^19 - 37752*a^11*b^14*c^8*d^17 + 36608*a^11*b^14*c^10*d^15 + 43264*a^11*b^14*c^12*d^13 - 56256*a^11*b^14*c^14*d^11 - 67008*a^11*b^14*c^16*d^9 + 125472*a^11*b^14*c^18*d^7 - 60768*a^11*b^14*c^20*d^5 + 8536*a^11*b^14*c^22*d^3 - 5096*a^12*b^13*c^5*d^20 + 44408*a^12*b^13*c^7*d^18 - 92352*a^12*b^13*c^9*d^16 + 43264*a^12*b^13*c^11*d^14 + 22464*a^12*b^13*c^13*d^12 + 56256*a^12*b^13*c^15*d^10 - 141408*a^12*b^13*c^17*d^8 + 92512*a^12*b^13*c^19*d^6 - 20664*a^12*b^13*c^21*d^4 + 616*a^12*b^13*c^23*d^2 + 2184*a^13*b^12*c^4*d^21 - 35672*a^13*b^12*c^6*d^19 + 109408*a^13*b^12*c^8*d^17 - 113152*a^13*b^12*c^10*d^15 + 22464*a^13*b^12*c^12*d^13 - 22464*a^13*b^12*c^14*d^11 + 113152*a^13*b^12*c^16*d^9 - 109408*a^13*b^12*c^18*d^7 + 35672*a^13*b^12*c^20*d^5 - 2184*a^13*b^12*c^22*d^3 - 616*a^14*b^11*c^3*d^22 + 20664*a^14*b^11*c^5*d^20 - 92512*a^14*b^11*c^7*d^18 + 141408*a^14*b^11*c^9*d^16 - 56256*a^14*b^11*c^11*d^14 - 22464*a^14*b^11*c^13*d^12 - 43264*a^14*b^11*c^15*d^10 + 92352*a^14*b^11*c^17*d^8 - 44408*a^14*b^11*c^19*d^6 + 5096*a^14*b^11*c^21*d^4 + 104*a^15*b^10*c^2*d^23 - 8536*a^15*b^10*c^4*d^21 + 60768*a^15*b^10*c^6*d^19 - 125472*a^15*b^10*c^8*d^17 + 67008*a^15*b^10*c^10*d^15 + 56256*a^15*b^10*c^12*d^13 - 43264*a^15*b^10*c^14*d^11 - 36608*a^15*b^10*c^16*d^9 + 37752*a^15*b^10*c^18*d^7 - 8008*a^15*b^10*c^20*d^5 + 2392*a^16*b^9*c^3*d^22 - 31328*a^16*b^9*c^5*d^20 + 85536*a^16*b^9*c^7*d^18 - 59264*a^16*b^9*c^9*d^16 - 67008*a^16*b^9*c^11*d^14 + 113152*a^16*b^9*c^13*d^12 - 36608*a^16*b^9*c^15*d^10 - 14872*a^16*b^9*c^17*d^8 + 8008*a^16*b^9*c^19*d^6 - 408*a^17*b^8*c^2*d^23 + 12272*a^17*b^8*c^4*d^21 - 46464*a^17*b^8*c^6*d^19 + 42696*a^17*b^8*c^8*d^17 + 59264*a^17*b^8*c^10*d^15 - 141408*a^17*b^8*c^12*d^13 + 92352*a^17*b^8*c^14*d^11 - 14872*a^17*b^8*c^16*d^9 - 3432*a^17*b^8*c^18*d^7 - 3408*a^18*b^7*c^3*d^22 + 20592*a^18*b^7*c^5*d^20 - 24904*a^18*b^7*c^7*d^18 - 42696*a^18*b^7*c^9*d^16 + 125472*a^18*b^7*c^11*d^14 - 109408*a^18*b^7*c^13*d^12 + 37752*a^18*b^7*c^15*d^10 - 3432*a^18*b^7*c^17*d^8 + 592*a^19*b^6*c^2*d^23 - 7408*a^19*b^6*c^4*d^21 + 11336*a^19*b^6*c^6*d^19 + 24904*a^19*b^6*c^8*d^17 - 85536*a^19*b^6*c^10*d^15 + 92512*a^19*b^6*c^12*d^13 - 44408*a^19*b^6*c^14*d^11 + 8008*a^19*b^6*c^16*d^9 + 2032*a^20*b^5*c^3*d^22 - 4008*a^20*b^5*c^5*d^20 - 11336*a^20*b^5*c^7*d^18 + 46464*a^20*b^5*c^9*d^16 - 60768*a^20*b^5*c^11*d^14 + 35672*a^20*b^5*c^13*d^12 - 8008*a^20*b^5*c^15*d^10 - 368*a^21*b^4*c^2*d^23 + 1192*a^21*b^4*c^4*d^21 + 4008*a^21*b^4*c^6*d^19 - 20592*a^21*b^4*c^8*d^17 + 31328*a^21*b^4*c^10*d^15 - 20664*a^21*b^4*c^12*d^13 + 5096*a^21*b^4*c^14*d^11 - 328*a^22*b^3*c^3*d^22 - 1192*a^22*b^3*c^5*d^20 + 7408*a^22*b^3*c^7*d^18 - 12272*a^22*b^3*c^9*d^16 + 8536*a^22*b^3*c^11*d^14 - 2184*a^22*b^3*c^13*d^12 + 72*a^23*b^2*c^2*d^23 + 328*a^23*b^2*c^4*d^21 - 2032*a^23*b^2*c^6*d^19 + 3408*a^23*b^2*c^8*d^17 - 2392*a^23*b^2*c^10*d^15 + 616*a^23*b^2*c^12*d^13 - 8*a*b^24*c^24*d - 8*a^24*b*c*d^24))/(a^20*d^20 + b^20*c^20

- 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^1
2*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c
^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8
 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9
 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^
17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d -
12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*
c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*
a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b
^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*
c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c
^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^1
0*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^
16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*
d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15
*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d
^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c
^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5
*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c
^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*
c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*
c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b
^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^1
1*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b
^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9
*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*
b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236
*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6
- 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 -
 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11
 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d
^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16
 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*
d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16
*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7
*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c
^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4
*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*
c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*
c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*
c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^
2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^
8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3
*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*
d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16

+ 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19) - (8*tan(e/2 + (f*x)/2)*(56*a^3*b^22*c^25 - 12*a^25*c*d^24 - 12*a*b^24*c^25 - 104*a^5*b^20*c^25 + 96*a^7*b^18*c^25 - 44*a^9*b^16*c^25 + 8*a^11*b^14*c^25 + 56*a^25*c^3*d^22 - 104*a^25*c^5*d^20 + 96*a^25*c^7*d^18 - 44*a^25*c^9*d^16 + 8*a^25*c^11*d^14 + 16*a*b^24*c^15*d^10 - 76*a*b^24*c^17*d^8 + 144*a*b^24*c^19*d^6 - 136*a*b^24*c^21*d^4 + 64*a*b^24*c^23*d^2 + 168*a^2*b^23*c^24*d - 784*a^4*b^21*c^24*d + 1456*a^6*b^19*c^24*d - 1344*a^8*b^17*c^24*d + 616*a^10*b^15*c^24*d - 112*a^12*b^13*c^24*d + 16*a^15*b^10*c*d^24 - 76*a^17*b^8*c*d^24 + 144*a^19*b^6*c*d^24 - 136*a^21*b^4*c*d^24 + 64*a^23*b^2*c*d^24 + 168*a^24*b*c^2*d^23 - 784*a^24*b*c^4*d^21 + 1456*a^24*b*c^6*d^19 - 1344*a^24*b*c^8*d^17 + 616*a^24*b*c^10*d^15 - 112*a^24*b*c^12*d^13 - 224*a^2*b^23*c^14*d^11 + 1064*a^2*b^23*c^16*d^9 - 2016*a^2*b^23*c^18*d^7 + 1904*a^2*b^23*c^20*d^5 - 896*a^2*b^23*c^22*d^3 + 1456*a^3*b^22*c^13*d^12 - 6992*a^3*b^22*c^15*d^10 + 13464*a^3*b^22*c^17*d^8 - 13056*a^3*b^22*c^19*d^6 + 6464*a^3*b^22*c^21*d^4 - 1392*a^3*b^22*c^23*d^2 - 5824*a^4*b^21*c^12*d^13 + 28728*a^4*b^21*c^14*d^11 - 57456*a^4*b^21*c^16*d^9 + 59024*a^4*b^21*c^18*d^7 - 32256*a^4*b^21*c^20*d^5 + 8568*a^4*b^21*c^22*d^3 + 16016*a^5*b^20*c^11*d^14 - 82992*a^5*b^20*c^13*d^12 + 177048*a^5*b^20*c^15*d^10 - 198696*a^5*b^20*c^17*d^8 + 123584*a^5*b^20*c^19*d^6 - 40512*a^5*b^20*c^21*d^4 + 5656*a^5*b^20*c^23*d^2 - 32032*a^6*b^19*c^10*d^15 + 179816*a^6*b^19*c^12*d^13 - 421344*a^6*b^19*c^14*d^11 + 529312*a^6*b^19*c^16*d^9 - 379008*a^6*b^19*c^18*d^7 + 150024*a^6*b^19*c^20*d^5 - 28224*a^6*b^19*c^22*d^3 + 48048*a^7*b^18*c^9*d^16 - 304304*a^7*b^18*c^11*d^14 + 805896*a^7*b^18*c^13*d^12 - 1151104*a^7*b^18*c^15*d^10 + 949952*a^7*b^18*c^17*d^8 - 446736*a^7*b^18*c^19*d^6 + 108136*a^7*b^18*c^21*d^4 - 9984*a^7*b^18*c^23*d^2 - 54912*a^8*b^17*c^8*d^17 + 412984*a^8*b^17*c^10*d^15 - 1267344*a^8*b^17*c^12*d^13 + 2077536*a^8*b^17*c^14*d^11 - 1975808*a^8*b^17*c^16*d^9 + 1095384*a^8*b^17*c^18*d^7 - 331632*a^8*b^17*c^20*d^5 + 45136*a^8*b^17*c^22*d^3 + 48048*a^9*b^16*c^7*d^18 - 456456*a^9*b^16*c^9*d^16 + 1657656*a^9*b^16*c^11*d^14 - 3143504*a^9*b^16*c^13*d^12 + 3453696*a^9*b^16*c^15*d^10 - 2247636*a^9*b^16*c^17*d^8 + 831208*a^9*b^16*c^19*d^6 - 151944*a^9*b^16*c^21*d^4 + 8976*a^9*b^16*c^23*d^2 - 32032*a^10*b^15*c^6*d^19 + 412984*a^10*b^15*c^8*d^17 - 1812096*a^10*b^15*c^10*d^15 + 4016896*a^10*b^15*c^12*d^13 - 5121024*a^10*b^15*c^14*d^11 + 3897024*a^10*b^15*c^16*d^9 - 1728832*a^10*b^15*c^18*d^7 + 404768*a^10*b^15*c^20*d^5 - 38304*a^10*b^15*c^22*d^3 + 16016*a^11*b^14*c^5*d^20 - 304304*a^11*b^14*c^7*d^18 + 1657656*a^11*b^14*c^9*d^16 - 4356352*a^11*b^14*c^11*d^14 + 6476288*a^11*b^14*c^13*d^12 - 5745024*a^11*b^14*c^15*d^10 + 3021984*a^11*b^14*c^17*d^8 - 880256*a^11*b^14*c^19*d^6 + 118032*a^11*b^14*c^21*d^4 - 4048*a^11*b^14*c^23*d^2 - 5824*a^12*b^13*c^4*d^21 + 179816*a^12*b^13*c^6*d^19 - 1267344*a^12*b^13*c^8*d^17 + 4016896*a^12*b^13*c^10*d^15 - 7002112*a^12*b^13*c^12*d^13 + 7235136*a^12*b^13*c^14*d^11 - 4480896*a^12*b^13*c^16*d^9 + 1588704*a^12*b^13*c^18*d^7 - 280896*a^12*b^13*c^20*d^5 + 16632*a^12*b^13*c^22*d^3 + 1456*a^13*b^12*c^3*d^22 - 82992*a^13*b^12*c^5*d^20 + 805896*a^13*b^12*c^7*d^18 - 3143504*a^13*b^12*c^9*d^16 + 6476288*a^13*b^12*c^11*d^14 - 7809984*a^13*b^12*c^13*d^12 + 5666752*a^13*b^12*c^15*d^10 -

2403856*a^13*b^12*c^17*d^8 + 537264*a^13*b^12*c^19*d^6 - 48048*a^13*b^12*c^21*d^4 + 728*a^13*b^12*c^23*d^2 - 224*a^14*b^11*c^2*d^23 + 28728*a^14*b^11*c^4*d^21 - 421344*a^14*b^11*c^6*d^19 + 2077536*a^14*b^11*c^8*d^17 - 5121024*a^14*b^11*c^10*d^15 + 7235136*a^14*b^11*c^12*d^13 - 6126848*a^14*b^11*c^14*d^11 + 3071744*a^14*b^11*c^16*d^9 - 844896*a^14*b^11*c^18*d^7 + 104104*a^14*b^11*c^20*d^5 - 2912*a^14*b^11*c^22*d^3 - 6992*a^15*b^10*c^3*d^22 + 177048*a^15*b^10*c^5*d^20 - 1151104*a^15*b^10*c^7*d^18 + 3453696*a^15*b^10*c^9*d^16 - 5745024*a^15*b^10*c^11*d^14 + 5666752*a^15*b^10*c^13*d^12 - 3331328*a^15*b^10*c^15*d^10 + 1105104*a^15*b^10*c^17*d^8 - 176176*a^15*b^10*c^19*d^6 + 8008*a^15*b^10*c^21*d^4 + 1064*a^16*b^9*c^2*d^23 - 57456*a^16*b^9*c^4*d^21 + 529312*a^16*b^9*c^6*d^19 - 1975808*a^16*b^9*c^8*d^17 + 3897024*a^16*b^9*c^10*d^15 - 4480896*a^16*b^9*c^12*d^13 + 3071744*a^16*b^9*c^14*d^11 - 1208064*a^16*b^9*c^16*d^9 + 239096*a^16*b^9*c^18*d^7 - 16016*a^16*b^9*c^20*d^5 + 13464*a^17*b^8*c^3*d^22 - 198696*a^17*b^8*c^5*d^20 + 949952*a^17*b^8*c^7*d^18 - 2247636*a^17*b^8*c^9*d^16 + 3021984*a^17*b^8*c^11*d^14 - 2403856*a^17*b^8*c^13*d^12 + 1105104*a^17*b^8*c^15*d^10 - 264264*a^17*b^8*c^17*d^8 + 24024*a^17*b^8*c^19*d^6 - 2016*a^18*b^7*c^2*d^23 + 59024*a^18*b^7*c^4*d^21 - 379008*a^18*b^7*c^6*d^19 + 1095384*a^18*b^7*c^8*d^17 - 1728832*a^18*b^7*c^10*d^15 + 1588704*a^18*b^7*c^12*d^13 - 844896*a^18*b^7*c^14*d^11 + 239096*a^18*b^7*c^16*d^9 - 27456*a^18*b^7*c^18*d^7 - 13056*a^19*b^6*c^3*d^22 + 123584*a^19*b^6*c^5*d^20 - 446736*a^19*b^6*c^7*d^18 + 831208*a^19*b^6*c^9*d^16 - 880256*a^19*b^6*c^11*d^14 + 537264*a^19*b^6*c^13*d^12 - 176176*a^19*b^6*c^15*d^10 + 24024*a^19*b^6*c^17*d^8 + 1904*a^20*b^5*c^2*d^23 - 32256*a^20*b^5*c^4*d^21 + 150024*a^20*b^5*c^6*d^19 - 331632*a^20*b^5*c^8*d^17 + 404768*a^20*b^5*c^10*d^15 - 280896*a^20*b^5*c^12*d^13 + 104104*a^20*b^5*c^14*d^11 - 16016*a^20*b^5*c^16*d^9 + 6464*a^21*b^4*c^3*d^22 - 40512*a^21*b^4*c^5*d^20 + 108136*a^21*b^4*c^7*d^18 - 151944*a^21*b^4*c^9*d^16 + 118032*a^21*b^4*c^11*d^14 - 48048*a^21*b^4*c^13*d^12 + 8008*a^21*b^4*c^15*d^10 - 896*a^22*b^3*c^2*d^23 + 8568*a^22*b^3*c^4*d^21 - 28224*a^22*b^3*c^6*d^19 + 45136*a^22*b^3*c^8*d^17 - 38304*a^22*b^3*c^10*d^15 + 16632*a^22*b^3*c^12*d^13 - 2912*a^22*b^3*c^14*d^11 - 1392*a^23*b^2*c^3*d^22 + 5656*a^23*b^2*c^5*d^20 - 9984*a^23*b^2*c^7*d^18 + 8976*a^23*b^2*c^9*d^16 - 4048*a^23*b^2*c^11*d^14 + 728*a^23*b^2*c^13*d^12))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^

8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2
 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9
+ 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3
+ 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10
 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d
^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13
 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*
d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^
16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10
*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c
^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5
*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*
c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*
c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^1
0*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a
^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66
*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 2750
4*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 1
8744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 2
68*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 3
4156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8
 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17
+ 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^1
1 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^
5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^1
4 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12
*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d
^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*
d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^1
6 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^
10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15
 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 +
 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268
*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c
d^19))(-(((4*a^24*d^24 + 4*b^24*c^24 + 16*a^2*b^22*c^24 + 16*a^4*b^20*c^2
4 - 1152*a^10*b^14*d^24 + 5568*a^12*b^12*d^24 - 10568*a^14*b^10*d^24 + 9460
*a^16*b^8*d^24 - 3560*a^18*b^6*d^24 + 136*a^20*b^4*d^24 + 76*a^22*b^2*d^24
+ 16*a^24*c^2*d^22 + 16*a^24*c^4*d^20 - 1152*b^24*c^10*d^14 + 5568*b^24*c^1
2*d^12 - 10568*b^24*c^14*d^10 + 9460*b^24*c^16*d^8 - 3560*b^24*c^18*d^6 + 1
36*b^24*c^20*d^4 + 76*b^24*c^22*d^2 + 11520*a*b^23*c^9*d^15 - 56448*a*b^23*
c^11*d^13 + 109456*a*b^23*c^13*d^11 - 101240*a*b^23*c^15*d^9 + 40720*a*b^23
*c^17*d^7 - 2960*a*b^23*c^19*d^5 - 536*a*b^23*c^21*d^3 - 176*a^3*b^21*c^23*
d - 320*a^5*b^19*c^23*d + 11520*a^9*b^15*c*d^23 - 56448*a^11*b^13*c*d^23 +
109456*a^13*b^11*c*d^23 - 101240*a^15*b^9*c*d^23 + 40720*a^17*b^7*c*d^23 -
2960*a^19*b^5*c*d^23 - 536*a^21*b^3*c*d^23 - 176*a^23*b*c^3*d^21 - 320*a^23

*b*c^5*d^19 - 51840*a^2*b^22*c^8*d^16 + 263808*a^2*b^22*c^10*d^14 - 541208*
a^2*b^22*c^12*d^12 + 547088*a^2*b^22*c^14*d^10 - 263320*a^2*b^22*c^16*d^8 +
44120*a^2*b^22*c^18*d^6 - 1564*a^2*b^22*c^20*d^4 - 196*a^2*b^22*c^22*d^2 +
138240*a^3*b^21*c^7*d^17 - 758400*a^3*b^21*c^9*d^15 + 1720736*a^3*b^21*c^1
1*d^13 - 2002728*a^3*b^21*c^13*d^11 + 1210560*a^3*b^21*c^15*d^9 - 335040*a^
3*b^21*c^17*d^7 + 37680*a^3*b^21*c^19*d^5 - 288*a^3*b^21*c^21*d^3 - 241920*
a^4*b^20*c^6*d^18 + 1512000*a^4*b^20*c^8*d^16 - 3975688*a^4*b^20*c^10*d^14
+ 5501328*a^4*b^20*c^12*d^12 - 4147952*a^4*b^20*c^14*d^10 + 1586920*a^4*b^2
0*c^16*d^8 - 276020*a^4*b^20*c^18*d^6 + 21124*a^4*b^20*c^20*d^4 + 176*a^4*b
^20*c^22*d^2 + 290304*a^5*b^19*c^5*d^19 - 2232576*a^5*b^19*c^7*d^17 + 70782
56*a^5*b^19*c^9*d^15 - 11781560*a^5*b^19*c^11*d^13 + 10875200*a^5*b^19*c^13
*d^11 - 5365072*a^5*b^19*c^15*d^9 + 1310168*a^5*b^19*c^17*d^7 - 170968*a^5*
b^19*c^19*d^5 + 8160*a^5*b^19*c^21*d^3 - 241920*a^6*b^18*c^4*d^20 + 2532096
*a^6*b^18*c^6*d^18 - 9955992*a^6*b^18*c^8*d^16 + 20019440*a^6*b^18*c^10*d^1
4 - 22419600*a^6*b^18*c^12*d^12 + 13887520*a^6*b^18*c^14*d^10 - 4506428*a^6
*b^18*c^16*d^8 + 793756*a^6*b^18*c^18*d^6 - 72240*a^6*b^18*c^20*d^4 + 3040*
a^6*b^18*c^22*d^2 + 138240*a^7*b^17*c^3*d^21 - 2232576*a^7*b^17*c^5*d^19 +
11150016*a^7*b^17*c^7*d^17 - 27336616*a^7*b^17*c^9*d^15 + 37153600*a^7*b^17
*c^11*d^13 - 28461040*a^7*b^17*c^13*d^11 + 11779808*a^7*b^17*c^15*d^9 - 262
1008*a^7*b^17*c^17*d^7 + 336688*a^7*b^17*c^19*d^5 - 17920*a^7*b^17*c^21*d^3
- 51840*a^8*b^16*c^2*d^22 + 1512000*a^8*b^16*c^4*d^20 - 9955992*a^8*b^16*c
^6*d^18 + 30289656*a^8*b^16*c^8*d^16 - 50137600*a^8*b^16*c^10*d^14 + 469725
60*a^8*b^16*c^12*d^12 - 24199280*a^8*b^16*c^14*d^10 + 6661036*a^8*b^16*c^16
*d^8 - 1058448*a^8*b^16*c^18*d^6 + 72560*a^8*b^16*c^20*d^4 - 758400*a^9*b^1
5*c^3*d^21 + 7078256*a^9*b^15*c^5*d^19 - 27336616*a^9*b^15*c^7*d^17 + 55383
904*a^9*b^15*c^9*d^15 - 63124080*a^9*b^15*c^11*d^13 + 39987520*a^9*b^15*c^1
3*d^11 - 13462088*a^9*b^15*c^15*d^9 + 2478528*a^9*b^15*c^17*d^7 - 212032*a^
9*b^15*c^19*d^5 + 263808*a^10*b^14*c^2*d^22 - 3975688*a^10*b^14*c^4*d^20 +
20019440*a^10*b^14*c^6*d^18 - 50137600*a^10*b^14*c^8*d^16 + 69593872*a^10*b
^14*c^10*d^14 - 53854288*a^10*b^14*c^12*d^12 + 21989928*a^10*b^14*c^14*d^10
- 4591360*a^10*b^14*c^16*d^8 + 460480*a^10*b^14*c^18*d^6 + 1720736*a^11*b^
13*c^3*d^21 - 11781560*a^11*b^13*c^5*d^19 + 37153600*a^11*b^13*c^7*d^17 - 6
3124080*a^11*b^13*c^9*d^15 + 59445728*a^11*b^13*c^11*d^13 - 29358696*a^11*b
^13*c^13*d^11 + 6995840*a^11*b^13*c^15*d^9 - 762560*a^11*b^13*c^17*d^7 - 54
1208*a^12*b^12*c^2*d^22 + 5501328*a^12*b^12*c^4*d^20 - 22419600*a^12*b^12*c
^6*d^18 + 46972560*a^12*b^12*c^8*d^16 - 53854288*a^12*b^12*c^10*d^14 + 3229
4808*a^12*b^12*c^12*d^12 - 8958208*a^12*b^12*c^14*d^10 + 999040*a^12*b^12*c
^16*d^8 - 2002728*a^13*b^11*c^3*d^21 + 10875200*a^13*b^11*c^5*d^19 - 284610
40*a^13*b^11*c^7*d^17 + 39987520*a^13*b^11*c^9*d^15 - 29358696*a^13*b^11*c^
11*d^13 + 9722048*a^13*b^11*c^13*d^11 - 1104320*a^13*b^11*c^15*d^9 + 547088
*a^14*b^10*c^2*d^22 - 4147952*a^14*b^10*c^4*d^20 + 13887520*a^14*b^10*c^6*d
^18 - 24199280*a^14*b^10*c^8*d^16 + 21989928*a^14*b^10*c^10*d^14 - 8958208*
a^14*b^10*c^12*d^12 + 1124032*a^14*b^10*c^14*d^10 + 1210560*a^15*b^9*c^3*d^
21 - 5365072*a^15*b^9*c^5*d^19 + 11779808*a^15*b^9*c^7*d^17 - 13462088*a^15
*b^9*c^9*d^15 + 6995840*a^15*b^9*c^11*d^13 - 1104320*a^15*b^9*c^13*d^11 - 2

63320*a^16*b^8*c^2*d^22 + 1586920*a^16*b^8*c^4*d^20 - 4506428*a^16*b^8*c^6*d^18 + 6661036*a^16*b^8*c^8*d^16 - 4591360*a^16*b^8*c^10*d^14 + 999040*a^16*b^8*c^12*d^12 - 335040*a^17*b^7*c^3*d^21 + 1310168*a^17*b^7*c^5*d^19 - 2621008*a^17*b^7*c^7*d^17 + 2478528*a^17*b^7*c^9*d^15 - 762560*a^17*b^7*c^11*d^13 + 44120*a^18*b^6*c^2*d^22 - 276020*a^18*b^6*c^4*d^20 + 793756*a^18*b^6*c^6*d^18 - 1058448*a^18*b^6*c^8*d^16 + 460480*a^18*b^6*c^10*d^14 + 37680*a^19*b^5*c^3*d^21 - 170968*a^19*b^5*c^5*d^19 + 336688*a^19*b^5*c^7*d^17 - 212032*a^19*b^5*c^9*d^15 - 1564*a^20*b^4*c^2*d^22 + 21124*a^20*b^4*c^4*d^20 - 72240*a^20*b^4*c^6*d^18 + 72560*a^20*b^4*c^8*d^16 - 288*a^21*b^3*c^3*d^21 + 8160*a^21*b^3*c^5*d^19 - 17920*a^21*b^3*c^7*d^17 - 196*a^22*b^2*c^2*d^22 + 176*a^22*b^2*c^4*d^20 + 3040*a^22*b^2*c^6*d^18 - 8*a*b^23*c^23*d - 8*a^23*b*c*d^23)^2/4 - (20736*b^18*d^18 - 96768*a^2*b^16*d^18 + 173664*a^4*b^14*d^18 - 136032*a^6*b^12*d^18 + 31081*a^8*b^10*d^18 + 8440*a^10*b^8*d^18 + 400*a^12*b^6*d^18 - 96768*b^18*c^2*d^16 + 173664*b^18*c^4*d^14 - 136032*b^18*c^6*d^12 + 31081*b^18*c^8*d^10 + 8440*b^18*c^10*d^8 + 400*b^18*c^12*d^6 - 131328*a*b^17*c^3*d^15 + 216576*a*b^17*c^5*d^13 - 141104*a*b^17*c^7*d^11 + 20260*a*b^17*c^9*d^9 + 2800*a*b^17*c^11*d^7 - 131328*a^3*b^15*c*d^17 + 216576*a^5*b^13*c*d^17 - 141104*a^7*b^11*c*d^17 + 20260*a^9*b^9*c*d^17 + 2800*a^11*b^7*c*d^17 + 495936*a^2*b^16*c^2*d^16 - 989856*a^2*b^16*c^4*d^14 + 901948*a^2*b^16*c^6*d^12 - 308392*a^2*b^16*c^8*d^10 - 5260*a^2*b^16*c^10*d^8 + 1600*a^2*b^16*c^12*d^6 + 657408*a^3*b^15*c^3*d^15 - 1158992*a^3*b^15*c^5*d^13 + 838256*a^3*b^15*c^7*d^11 - 182200*a^3*b^15*c^9*d^9 - 3200*a^3*b^15*c^11*d^7 - 989856*a^4*b^14*c^2*d^16 + 2185654*a^4*b^14*c^4*d^14 - 2218576*a^4*b^14*c^6*d^12 + 900624*a^4*b^14*c^8*d^10 - 64720*a^4*b^14*c^10*d^8 + 1600*a^4*b^14*c^12*d^6 - 1158992*a^5*b^13*c^3*d^15 + 2158808*a^5*b^13*c^5*d^13 - 1641528*a^5*b^13*c^7*d^11 + 406880*a^5*b^13*c^9*d^9 - 17600*a^5*b^13*c^11*d^7 + 901948*a^6*b^12*c^2*d^16 - 2218576*a^6*b^12*c^4*d^14 + 2430936*a^6*b^12*c^6*d^12 - 1026928*a^6*b^12*c^8*d^10 + 88720*a^6*b^12*c^10*d^8 + 838256*a^7*b^11*c^3*d^15 - 1641528*a^7*b^11*c^5*d^13 + 1206848*a^7*b^11*c^7*d^11 - 239360*a^7*b^11*c^9*d^9 - 308392*a^8*b^10*c^2*d^16 + 900624*a^8*b^10*c^4*d^14 - 1026928*a^8*b^10*c^6*d^12 + 354016*a^8*b^10*c^8*d^10 - 182200*a^9*b^9*c^3*d^15 + 406880*a^9*b^9*c^5*d^13 - 239360*a^9*b^9*c^7*d^11 - 5260*a^10*b^8*c^2*d^16 - 64720*a^10*b^8*c^4*d^14 + 88720*a^10*b^8*c^6*d^12 - 3200*a^11*b^7*c^3*d^15 - 17600*a^11*b^7*c^5*d^13 + 1600*a^12*b^6*c^2*d^16 + 1600*a^12*b^6*c^4*d^14 + 27648*a*b^17*c*d^17)*(80*a^2*b^28*c^30 - 16*b^30*c^30 - 16*a^30*d^30 - 160*a^4*b^26*c^30 + 160*a^6*b^24*c^30 - 80*a^8*b^22*c^30 + 16*a^10*b^20*c^30 + 16*a^20*b^10*d^30 - 80*a^22*b^8*d^30 + 160*a^24*b^6*d^30 - 160*a^26*b^4*d^30 + 80*a^28*b^2*d^30 + 80*a^30*c^2*d^28 - 160*a^30*c^4*d^26 + 160*a^30*c^6*d^24 - 80*a^30*c^8*d^22 + 16*a^30*c^10*d^20 + 16*b^30*c^20*d^10 - 80*b^30*c^22*d^8 + 160*b^30*c^24*d^6 - 160*b^30*c^26*d^4 + 80*b^30*c^28*d^2 - 320*a*b^29*c^19*d^11 + 1600*a*b^29*c^21*d^9 - 3200*a*b^29*c^23*d^7 + 3200*a*b^29*c^25*d^5 - 1600*a*b^29*c^27*d^3 - 1600*a^3*b^27*c^29*d + 3200*a^5*b^25*c^29*d - 3200*a^7*b^23*c^29*d + 1600*a^9*b^21*c^29*d - 320*a^11*b^19*c^29*d - 320*a^19*b^11*c*d^29 + 1600*a^21*b^9*c*d^29 - 3200*a^23*b^7*c*d^29 + 3200*a^25*b^5*c*d^29 - 1600*a^27*b^3*c*d^29 - 1600*a^29*b*c^3*d^27 +

3200*a^29*b*c^5*d^25 - 3200*a^29*b*c^7*d^23 + 1600*a^29*b*c^9*d^21 - 320*a^29*b*c^11*d^19 + 3040*a^2*b^28*c^18*d^12 - 15280*a^2*b^28*c^20*d^10 + 30800*a^2*b^28*c^22*d^8 - 31200*a^2*b^28*c^24*d^6 + 16000*a^2*b^28*c^26*d^4 - 3440*a^2*b^28*c^28*d^2 - 18240*a^3*b^27*c^17*d^13 + 92800*a^3*b^27*c^19*d^11 - 190400*a^3*b^27*c^21*d^9 + 198400*a^3*b^27*c^23*d^7 - 107200*a^3*b^27*c^25*d^5 + 26240*a^3*b^27*c^27*d^3 + 77520*a^4*b^26*c^16*d^14 - 402800*a^4*b^26*c^18*d^12 + 851360*a^4*b^26*c^20*d^10 - 928000*a^4*b^26*c^22*d^8 + 541200*a^4*b^26*c^24*d^6 - 155120*a^4*b^26*c^26*d^4 + 16000*a^4*b^26*c^28*d^2 - 248064*a^5*b^25*c^15*d^15 + 1331520*a^5*b^25*c^17*d^13 - 2939840*a^5*b^25*c^19*d^11 + 3408640*a^5*b^25*c^21*d^9 - 2184320*a^5*b^25*c^23*d^7 + 736064*a^5*b^25*c^25*d^5 - 107200*a^5*b^25*c^27*d^3 + 620160*a^6*b^24*c^14*d^16 - 3488400*a^6*b^24*c^16*d^14 + 8170000*a^6*b^24*c^18*d^12 - 10229760*a^6*b^24*c^20*d^10 + 7281600*a^6*b^24*c^22*d^8 - 2863760*a^6*b^24*c^24*d^6 + 541200*a^6*b^24*c^26*d^4 - 31200*a^6*b^24*c^28*d^2 - 1240320*a^7*b^23*c^13*d^17 + 7441920*a^7*b^23*c^15*d^15 - 18787200*a^7*b^23*c^17*d^13 + 25721600*a^7*b^23*c^19*d^11 - 20444800*a^7*b^23*c^21*d^9 + 9297920*a^7*b^23*c^23*d^7 - 2184320*a^7*b^23*c^25*d^5 + 198400*a^7*b^23*c^27*d^3 + 2015520*a^8*b^22*c^12*d^18 - 13178400*a^8*b^22*c^14*d^16 + 36434400*a^8*b^22*c^16*d^14 - 55069600*a^8*b^22*c^18*d^12 + 48989680*a^8*b^22*c^20*d^10 - 25575920*a^8*b^22*c^22*d^8 + 7281600*a^8*b^22*c^24*d^6 - 928000*a^8*b^22*c^26*d^4 + 30800*a^8*b^22*c^28*d^2 - 2687360*a^9*b^21*c^11*d^19 + 19638400*a^9*b^21*c^13*d^17 - 60362240*a^9*b^21*c^15*d^15 + 101475200*a^9*b^21*c^17*d^13 - 101172800*a^9*b^21*c^19*d^11 + 60333760*a^9*b^21*c^21*d^9 - 20444800*a^9*b^21*c^23*d^7 + 3408640*a^9*b^21*c^25*d^5 - 190400*a^9*b^21*c^27*d^3 + 2956096*a^10*b^20*c^10*d^20 - 24858080*a^10*b^20*c^12*d^18 + 86150560*a^10*b^20*c^14*d^16 - 162120160*a^10*b^20*c^16*d^14 + 181463680*a^10*b^20*c^18*d^12 - 123188112*a^10*b^20*c^20*d^10 + 48989680*a^10*b^20*c^22*d^8 - 10229760*a^10*b^20*c^24*d^6 + 851360*a^10*b^20*c^26*d^4 - 15280*a^10*b^20*c^28*d^2 - 2687360*a^11*b^19*c^9*d^21 + 26873600*a^11*b^19*c^11*d^19 - 106460800*a^11*b^19*c^13*d^17 + 225738240*a^11*b^19*c^15*d^15 - 284331200*a^11*b^19*c^17*d^13 + 219166080*a^11*b^19*c^19*d^11 - 101172800*a^11*b^19*c^21*d^9 + 25721600*a^11*b^19*c^23*d^7 - 2939840*a^11*b^19*c^25*d^5 + 92800*a^11*b^19*c^27*d^3 + 2015520*a^12*b^18*c^8*d^22 - 24858080*a^12*b^18*c^10*d^20 + 114212800*a^12*b^18*c^12*d^18 - 274937600*a^12*b^18*c^14*d^16 + 390830000*a^12*b^18*c^16*d^14 - 341426960*a^12*b^18*c^18*d^12 + 181463680*a^12*b^18*c^20*d^10 - 55069600*a^12*b^18*c^22*d^8 + 8170000*a^12*b^18*c^24*d^6 - 402800*a^12*b^18*c^26*d^4 + 3040*a^12*b^18*c^28*d^2 - 1240320*a^13*b^17*c^7*d^23 + 19638400*a^13*b^17*c^9*d^21 - 106460800*a^13*b^17*c^11*d^19 + 293542400*a^13*b^17*c^13*d^17 - 472561920*a^13*b^17*c^15*d^15 + 467412160*a^13*b^17*c^17*d^13 - 284331200*a^13*b^17*c^19*d^11 + 101475200*a^13*b^17*c^21*d^9 - 18787200*a^13*b^17*c^23*d^7 + 1331520*a^13*b^17*c^25*d^5 - 18240*a^13*b^17*c^27*d^3 + 620160*a^14*b^16*c^6*d^24 - 13178400*a^14*b^16*c^8*d^22 + 86150560*a^14*b^16*c^10*d^20 - 274937600*a^14*b^16*c^12*d^18 + 503363200*a^14*b^16*c^14*d^16 - 563751280*a^14*b^16*c^16*d^14 + 390830000*a^14*b^16*c^18*d^12 - 162120160*a^14*b^16*c^20*d^10 + 36434400*a^14*b^16*c^22*d^8 - 3488400*a^14*b^16*c^24*d^6 + 77520*a^14*b^16*c

^26*d^4 - 248064*a^15*b^15*c^5*d^25 + 7441920*a^15*b^15*c^7*d^23 - 60362240
*a^15*b^15*c^9*d^21 + 225738240*a^15*b^15*c^11*d^19 - 472561920*a^15*b^15*c
^13*d^17 + 599984128*a^15*b^15*c^15*d^15 - 472561920*a^15*b^15*c^17*d^13 +
225738240*a^15*b^15*c^19*d^11 - 60362240*a^15*b^15*c^21*d^9 + 7441920*a^15*
b^15*c^23*d^7 - 248064*a^15*b^15*c^25*d^5 + 77520*a^16*b^14*c^4*d^26 - 3488
400*a^16*b^14*c^6*d^24 + 36434400*a^16*b^14*c^8*d^22 - 162120160*a^16*b^14*
c^10*d^20 + 390830000*a^16*b^14*c^12*d^18 - 563751280*a^16*b^14*c^14*d^16 +
503363200*a^16*b^14*c^16*d^14 - 274937600*a^16*b^14*c^18*d^12 + 86150560*a
^16*b^14*c^20*d^10 - 13178400*a^16*b^14*c^22*d^8 + 620160*a^16*b^14*c^24*d^
6 - 18240*a^17*b^13*c^3*d^27 + 1331520*a^17*b^13*c^5*d^25 - 18787200*a^17*b
^13*c^7*d^23 + 101475200*a^17*b^13*c^9*d^21 - 284331200*a^17*b^13*c^11*d^19
+ 467412160*a^17*b^13*c^13*d^17 - 472561920*a^17*b^13*c^15*d^15 + 29354240
0*a^17*b^13*c^17*d^13 - 106460800*a^17*b^13*c^19*d^11 + 19638400*a^17*b^13*
c^21*d^9 - 1240320*a^17*b^13*c^23*d^7 + 3040*a^18*b^12*c^2*d^28 - 402800*a^
18*b^12*c^4*d^26 + 8170000*a^18*b^12*c^6*d^24 - 55069600*a^18*b^12*c^8*d^22
+ 181463680*a^18*b^12*c^10*d^20 - 341426960*a^18*b^12*c^12*d^18 + 39083000
0*a^18*b^12*c^14*d^16 - 274937600*a^18*b^12*c^16*d^14 + 114212800*a^18*b^12
*c^18*d^12 - 24858080*a^18*b^12*c^20*d^10 + 2015520*a^18*b^12*c^22*d^8 + 92
800*a^19*b^11*c^3*d^27 - 2939840*a^19*b^11*c^5*d^25 + 25721600*a^19*b^11*c^
7*d^23 - 101172800*a^19*b^11*c^9*d^21 + 219166080*a^19*b^11*c^11*d^19 - 284
331200*a^19*b^11*c^13*d^17 + 225738240*a^19*b^11*c^15*d^15 - 106460800*a^19
*b^11*c^17*d^13 + 26873600*a^19*b^11*c^19*d^11 - 2687360*a^19*b^11*c^21*d^9
- 15280*a^20*b^10*c^2*d^28 + 851360*a^20*b^10*c^4*d^26 - 10229760*a^20*b^1
0*c^6*d^24 + 48989680*a^20*b^10*c^8*d^22 - 123188112*a^20*b^10*c^10*d^20 +
181463680*a^20*b^10*c^12*d^18 - 162120160*a^20*b^10*c^14*d^16 + 86150560*a^
20*b^10*c^16*d^14 - 24858080*a^20*b^10*c^18*d^12 + 2956096*a^20*b^10*c^20*d
^10 - 190400*a^21*b^9*c^3*d^27 + 3408640*a^21*b^9*c^5*d^25 - 20444800*a^21*
b^9*c^7*d^23 + 60333760*a^21*b^9*c^9*d^21 - 101172800*a^21*b^9*c^11*d^19 +
101475200*a^21*b^9*c^13*d^17 - 60362240*a^21*b^9*c^15*d^15 + 19638400*a^21*
b^9*c^17*d^13 - 2687360*a^21*b^9*c^19*d^11 + 30800*a^22*b^8*c^2*d^28 - 9280
00*a^22*b^8*c^4*d^26 + 7281600*a^22*b^8*c^6*d^24 - 25575920*a^22*b^8*c^8*d^
22 + 48989680*a^22*b^8*c^10*d^20 - 55069600*a^22*b^8*c^12*d^18 + 36434400*a
^22*b^8*c^14*d^16 - 13178400*a^22*b^8*c^16*d^14 + 2015520*a^22*b^8*c^18*d^1
2 + 198400*a^23*b^7*c^3*d^27 - 2184320*a^23*b^7*c^5*d^25 + 9297920*a^23*b^7
*c^7*d^23 - 20444800*a^23*b^7*c^9*d^21 + 25721600*a^23*b^7*c^11*d^19 - 1878
7200*a^23*b^7*c^13*d^17 + 7441920*a^23*b^7*c^15*d^15 - 1240320*a^23*b^7*c^1
7*d^13 - 31200*a^24*b^6*c^2*d^28 + 541200*a^24*b^6*c^4*d^26 - 2863760*a^24*
b^6*c^6*d^24 + 7281600*a^24*b^6*c^8*d^22 - 10229760*a^24*b^6*c^10*d^20 + 81
70000*a^24*b^6*c^12*d^18 - 3488400*a^24*b^6*c^14*d^16 + 620160*a^24*b^6*c^1
6*d^14 - 107200*a^25*b^5*c^3*d^27 + 736064*a^25*b^5*c^5*d^25 - 2184320*a^25
*b^5*c^7*d^23 + 3408640*a^25*b^5*c^9*d^21 - 2939840*a^25*b^5*c^11*d^19 + 13
31520*a^25*b^5*c^13*d^17 - 248064*a^25*b^5*c^15*d^15 + 16000*a^26*b^4*c^2*d
^28 - 155120*a^26*b^4*c^4*d^26 + 541200*a^26*b^4*c^6*d^24 - 928000*a^26*b^4
*c^8*d^22 + 851360*a^26*b^4*c^10*d^20 - 402800*a^26*b^4*c^12*d^18 + 77520*a
^26*b^4*c^14*d^16 + 26240*a^27*b^3*c^3*d^27 - 107200*a^27*b^3*c^5*d^25 + 19

8400*a^27*b^3*c^7*d^23 - 190400*a^27*b^3*c^9*d^21 + 92800*a^27*b^3*c^11*d^1
9 - 18240*a^27*b^3*c^13*d^17 - 3440*a^28*b^2*c^2*d^28 + 16000*a^28*b^2*c^4*
d^26 - 31200*a^28*b^2*c^6*d^24 + 30800*a^28*b^2*c^8*d^22 - 15280*a^28*b^2*c
^10*d^20 + 3040*a^28*b^2*c^12*d^18 + 320*a*b^29*c^29*d + 320*a^29*b*c*d^29)
)^(1/2) + 2*a^24*d^24 + 2*b^24*c^24 + 8*a^2*b^22*c^24 + 8*a^4*b^20*c^24 - 5
76*a^10*b^14*d^24 + 2784*a^12*b^12*d^24 - 5284*a^14*b^10*d^24 + 4730*a^16*b
^8*d^24 - 1780*a^18*b^6*d^24 + 68*a^20*b^4*d^24 + 38*a^22*b^2*d^24 + 8*a^24
*c^2*d^22 + 8*a^24*c^4*d^20 - 576*b^24*c^10*d^14 + 2784*b^24*c^12*d^12 - 52
84*b^24*c^14*d^10 + 4730*b^24*c^16*d^8 - 1780*b^24*c^18*d^6 + 68*b^24*c^20*
d^4 + 38*b^24*c^22*d^2 + 5760*a*b^23*c^9*d^15 - 28224*a*b^23*c^11*d^13 + 54
728*a*b^23*c^13*d^11 - 50620*a*b^23*c^15*d^9 + 20360*a*b^23*c^17*d^7 - 1480
*a*b^23*c^19*d^5 - 268*a*b^23*c^21*d^3 - 88*a^3*b^21*c^23*d - 160*a^5*b^19*
c^23*d + 5760*a^9*b^15*c*d^23 - 28224*a^11*b^13*c*d^23 + 54728*a^13*b^11*c*
d^23 - 50620*a^15*b^9*c*d^23 + 20360*a^17*b^7*c*d^23 - 1480*a^19*b^5*c*d^23
 - 268*a^21*b^3*c*d^23 - 88*a^23*b*c^3*d^21 - 160*a^23*b*c^5*d^19 - 25920*a
^2*b^22*c^8*d^16 + 131904*a^2*b^22*c^10*d^14 - 270604*a^2*b^22*c^12*d^12 +
273544*a^2*b^22*c^14*d^10 - 131660*a^2*b^22*c^16*d^8 + 22060*a^2*b^22*c^18*
d^6 - 782*a^2*b^22*c^20*d^4 - 98*a^2*b^22*c^22*d^2 + 69120*a^3*b^21*c^7*d^1
7 - 379200*a^3*b^21*c^9*d^15 + 860368*a^3*b^21*c^11*d^13 - 1001364*a^3*b^21
*c^13*d^11 + 605280*a^3*b^21*c^15*d^9 - 167520*a^3*b^21*c^17*d^7 + 18840*a^
3*b^21*c^19*d^5 - 144*a^3*b^21*c^21*d^3 - 120960*a^4*b^20*c^6*d^18 + 756000
*a^4*b^20*c^8*d^16 - 1987844*a^4*b^20*c^10*d^14 + 2750664*a^4*b^20*c^12*d^1
2 - 2073976*a^4*b^20*c^14*d^10 + 793460*a^4*b^20*c^16*d^8 - 138010*a^4*b^20
*c^18*d^6 + 10562*a^4*b^20*c^20*d^4 + 88*a^4*b^20*c^22*d^2 + 145152*a^5*b^1
9*c^5*d^19 - 1116288*a^5*b^19*c^7*d^17 + 3539128*a^5*b^19*c^9*d^15 - 589078
0*a^5*b^19*c^11*d^13 + 5437600*a^5*b^19*c^13*d^11 - 2682536*a^5*b^19*c^15*d
^9 + 655084*a^5*b^19*c^17*d^7 - 85484*a^5*b^19*c^19*d^5 + 4080*a^5*b^19*c^2
1*d^3 - 120960*a^6*b^18*c^4*d^20 + 1266048*a^6*b^18*c^6*d^18 - 4977996*a^6*
b^18*c^8*d^16 + 10009720*a^6*b^18*c^10*d^14 - 11209800*a^6*b^18*c^12*d^12 +
 6943760*a^6*b^18*c^14*d^10 - 2253214*a^6*b^18*c^16*d^8 + 396878*a^6*b^18*c
^18*d^6 - 36120*a^6*b^18*c^20*d^4 + 1520*a^6*b^18*c^22*d^2 + 69120*a^7*b^17
*c^3*d^21 - 1116288*a^7*b^17*c^5*d^19 + 5575008*a^7*b^17*c^7*d^17 - 1366830
8*a^7*b^17*c^9*d^15 + 18576800*a^7*b^17*c^11*d^13 - 14230520*a^7*b^17*c^13*
d^11 + 5889904*a^7*b^17*c^15*d^9 - 1310504*a^7*b^17*c^17*d^7 + 168344*a^7*b
^17*c^19*d^5 - 8960*a^7*b^17*c^21*d^3 - 25920*a^8*b^16*c^2*d^22 + 756000*a^
8*b^16*c^4*d^20 - 4977996*a^8*b^16*c^6*d^18 + 15144828*a^8*b^16*c^8*d^16 -
25068800*a^8*b^16*c^10*d^14 + 23486280*a^8*b^16*c^12*d^12 - 12099640*a^8*b^
16*c^14*d^10 + 3330518*a^8*b^16*c^16*d^8 - 529224*a^8*b^16*c^18*d^6 + 36280
*a^8*b^16*c^20*d^4 - 379200*a^9*b^15*c^3*d^21 + 3539128*a^9*b^15*c^5*d^19 -
 13668308*a^9*b^15*c^7*d^17 + 27691952*a^9*b^15*c^9*d^15 - 31562040*a^9*b^1
5*c^11*d^13 + 19993760*a^9*b^15*c^13*d^11 - 6731044*a^9*b^15*c^15*d^9 + 123
9264*a^9*b^15*c^17*d^7 - 106016*a^9*b^15*c^19*d^5 + 131904*a^10*b^14*c^2*d^
22 - 1987844*a^10*b^14*c^4*d^20 + 10009720*a^10*b^14*c^6*d^18 - 25068800*a^
10*b^14*c^8*d^16 + 34796936*a^10*b^14*c^10*d^14 - 26927144*a^10*b^14*c^12*d
^12 + 10994964*a^10*b^14*c^14*d^10 - 2295680*a^10*b^14*c^16*d^8 + 230240*a^

10*b^14*c^18*d^6 + 860368*a^11*b^13*c^3*d^21 - 5890780*a^11*b^13*c^5*d^19 + 18576800*a^11*b^13*c^7*d^17 - 31562040*a^11*b^13*c^9*d^15 + 29722864*a^11*b^13*c^11*d^13 - 14679348*a^11*b^13*c^13*d^11 + 3497920*a^11*b^13*c^15*d^9 - 381280*a^11*b^13*c^17*d^7 - 270604*a^12*b^12*c^2*d^22 + 2750664*a^12*b^12*c^4*d^20 - 11209800*a^12*b^12*c^6*d^18 + 23486280*a^12*b^12*c^8*d^16 - 26927144*a^12*b^12*c^10*d^14 + 16147404*a^12*b^12*c^12*d^12 - 4479104*a^12*b^12*c^14*d^10 + 499520*a^12*b^12*c^16*d^8 - 1001364*a^13*b^11*c^3*d^21 + 5437600*a^13*b^11*c^5*d^19 - 14230520*a^13*b^11*c^7*d^17 + 19993760*a^13*b^11*c^9*d^15 - 14679348*a^13*b^11*c^11*d^13 + 4861024*a^13*b^11*c^13*d^11 - 552160*a^13*b^11*c^15*d^9 + 273544*a^14*b^10*c^2*d^22 - 2073976*a^14*b^10*c^4*d^20 + 6943760*a^14*b^10*c^6*d^18 - 12099640*a^14*b^10*c^8*d^16 + 10994964*a^14*b^10*c^10*d^14 - 4479104*a^14*b^10*c^12*d^12 + 562016*a^14*b^10*c^14*d^10 + 605280*a^15*b^9*c^3*d^21 - 2682536*a^15*b^9*c^5*d^19 + 5889904*a^15*b^9*c^7*d^17 - 6731044*a^15*b^9*c^9*d^15 + 3497920*a^15*b^9*c^11*d^13 - 552160*a^15*b^9*c^13*d^11 - 131660*a^16*b^8*c^2*d^22 + 793460*a^16*b^8*c^4*d^20 - 2253214*a^16*b^8*c^6*d^18 + 3330518*a^16*b^8*c^8*d^16 - 2295680*a^16*b^8*c^10*d^14 + 499520*a^16*b^8*c^12*d^12 - 167520*a^17*b^7*c^3*d^21 + 655084*a^17*b^7*c^5*d^19 - 1310504*a^17*b^7*c^7*d^17 + 1239264*a^17*b^7*c^9*d^15 - 381280*a^17*b^7*c^11*d^13 + 22060*a^18*b^6*c^2*d^22 - 138010*a^18*b^6*c^4*d^20 + 396878*a^18*b^6*c^6*d^18 - 529224*a^18*b^6*c^8*d^16 + 230240*a^18*b^6*c^10*d^14 + 18840*a^19*b^5*c^3*d^21 - 85484*a^19*b^5*c^5*d^19 + 168344*a^19*b^5*c^7*d^17 - 106016*a^19*b^5*c^9*d^15 - 782*a^20*b^4*c^2*d^22 + 10562*a^20*b^4*c^4*d^20 - 36120*a^20*b^4*c^6*d^18 + 36280*a^20*b^4*c^8*d^16 - 144*a^21*b^3*c^3*d^21 + 4080*a^21*b^3*c^5*d^19 - 8960*a^21*b^3*c^7*d^17 - 98*a^22*b^2*c^2*d^22 + 88*a^22*b^2*c^4*d^20 + 1520*a^22*b^2*c^6*d^18 - 4*a*b^23*c^23*d - 4*a^23*b*c*d^23)/(16*(5*a^2*b^28*c^30 - b^30*c^30 - a^30*d^30 - 10*a^4*b^26*c^30 + 10*a^6*b^24*c^30 - 5*a^8*b^22*c^30 + a^10*b^20*c^30 + a^20*b^10*d^30 - 5*a^22*b^8*d^30 + 10*a^24*b^6*d^30 - 10*a^26*b^4*d^30 + 5*a^28*b^2*d^30 + 5*a^30*c^2*d^28 - 10*a^30*c^4*d^26 + 10*a^30*c^6*d^24 - 5*a^30*c^8*d^22 + a^30*c^10*d^20 + b^30*c^20*d^10 - 5*b^30*c^22*d^8 + 10*b^30*c^24*d^6 - 10*b^30*c^26*d^4 + 5*b^30*c^28*d^2 - 20*a*b^29*c^19*d^11 + 100*a*b^29*c^21*d^9 - 200*a*b^29*c^23*d^7 + 200*a*b^29*c^25*d^5 - 100*a*b^29*c^27*d^3 - 100*a^3*b^27*c^29*d + 200*a^5*b^25*c^29*d - 200*a^7*b^23*c^29*d + 100*a^9*b^21*c^29*d - 20*a^11*b^19*c^29*d - 20*a^19*b^11*c*d^29 + 100*a^21*b^9*c*d^29 - 200*a^23*b^7*c*d^29 + 200*a^25*b^5*c*d^29 - 100*a^27*b^3*c*d^29 - 100*a^29*b*c^3*d^27 + 200*a^29*b*c^5*d^25 - 200*a^29*b*c^7*d^23 + 100*a^29*b*c^9*d^21 - 20*a^29*b*c^11*d^19 + 190*a^2*b^28*c^18*d^12 - 955*a^2*b^28*c^20*d^10 + 1925*a^2*b^28*c^22*d^8 - 1950*a^2*b^28*c^24*d^6 + 1000*a^2*b^28*c^26*d^4 - 215*a^2*b^28*c^28*d^2 - 1140*a^3*b^27*c^17*d^13 + 5800*a^3*b^27*c^19*d^11 - 11900*a^3*b^27*c^21*d^9 + 12400*a^3*b^27*c^23*d^7 - 6700*a^3*b^27*c^25*d^5 + 1640*a^3*b^27*c^27*d^3 + 4845*a^4*b^26*c^16*d^14 - 25175*a^4*b^26*c^18*d^12 + 53210*a^4*b^26*c^20*d^10 - 58000*a^4*b^26*c^22*d^8 + 33825*a^4*b^26*c^24*d^6 - 9695*a^4*b^26*c^26*d^4 + 1000*a^4*b^26*c^28*d^2 - 15504*a^5*b^25*c^15*d^15 + 83220*a^5*b^25*c^17*d^13 - 183740*a^5*b^25*c^19*d^11 + 213040*a^5*b^25*c^21*d^9 - 136520*a^5*b^25*c^23*d^7 + 46004*a^5*b^25*c^25* d^5 - 6700*a^5*b^25*c^27*d^3 + 38760*a^6*b^24*c^14*d^16 - 218025*a^6*b^24*c^16*d^14 + 510625*a^6*b^24*c^18*d^12 - 639360*a^6*b^24*c^20*d^10 + 455100*a^6*b^24*c^22*d^8 - 178985*a^6*b^24*c^24*d^6 + 33825*a^6*b^24*c^26*d^4 - 1950*a^6*b^24*c^28*d^2 - 77520*a^7*b^23*c^13*d^17 + 465120*a^7*b^23*c^15*d^15 - 1174200*a^7*b^23*c^17*d^13 + 1607600*a^7*b^23*c^19*d^11 - 1277800*a^7*b^23*c^21*d^9 + 581120*a^7*b^23*c^23*d^7 - 136520*a^7*b^23*c^25*d^5 + 12400*a^7*b^23*c^27*d^3 + 125970*a^8*b^22*c^12*d^18 - 823650*a^8*b^22*c^14*d^16 + 2277150*a^8*b^22*c^16*d^14 - 3441850*a^8*b^22*c^18*d^12 + 3061855*a^8*b^22*c^20*d^10 - 1598495*a^8*b^22*c^22*d^8 + 455100*a^8*b^22*c^24*d^6 - 58000*a^8*b^22*c^26*d^4 + 1925*a^8*b^22*c^28*d^2 - 167960*a^9*b^21*c^11*d^19 + 1227400*a^9*b^21*c^13*d^17 - 3772640*a^9*b^21*c^15*d^15 + 6342200*a^9*b^21*c^17*d^13 - 6323300*a^9*b^21*c^19*d^11 + 3770860*a^9*b^21*c^21*d^9 - 1277800*a^9*b^21*c^23*d^7 + 213040*a^9*b^21*c^25*d^5 - 11900*a^9*b^21*c^27*d^3 + 184756*a^10*b^20*c^10*d^20 - 1553630*a^10*b^20*c^12*d^18 + 5384410*a^10*b^20*c^14*d^16 - 10132510*a^10*b^20*c^16*d^14 + 11341480*a^10*b^20*c^18*d^12 - 7699257*a^10*b^20*c^20*d^10 + 3061855*a^10*b^20*c^22*d^8 - 639360*a^10*b^20*c^24*d^6 + 53210*a^10*b^20*c^26*d^4 - 955*a^10*b^20*c^28*d^2 - 167960*a^11*b^19*c^9*d^21 + 1679600*a^11*b^19*c^11*d^19 - 6653800*a^11*b^19*c^13*d^17 + 14108640*a^11*b^19*c^15*d^15 - 17770700*a^11*b^19*c^17*d^13 + 13697880*a^11*b^19*c^19*d^11 - 6323300*a^11*b^19*c^21*d^9 + 1607600*a^11*b^19*c^23*d^7 - 183740*a^11*b^19*c^25*d^5 + 5800*a^11*b^19*c^27*d^3 + 125970*a^12*b^18*c^8*d^22 - 1553630*a^12*b^18*c^10*d^20 + 7138300*a^12*b^18*c^12*d^18 - 17183600*a^12*b^18*c^14*d^16 + 24426875*a^12*b^18*c^16*d^14 - 21339185*a^12*b^18*c^18*d^12 + 11341480*a^12*b^18*c^20*d^10 - 3441850*a^12*b^18*c^22*d^8 + 510625*a^12*b^18*c^24*d^6 - 25175*a^12*b^18*c^26*d^4 + 190*a^12*b^18*c^28*d^2 - 77520*a^13*b^17*c^7*d^23 + 1227400*a^13*b^17*c^9*d^21 - 6653800*a^13*b^17*c^11*d^19 + 18346400*a^13*b^17*c^13*d^17 - 29535120*a^13*b^17*c^15*d^15 + 29213260*a^13*b^17*c^17*d^13 - 17770700*a^13*b^17*c^19*d^11 + 6342200*a^13*b^17*c^21*d^9 - 1174200*a^13*b^17*c^23*d^7 + 83220*a^13*b^17*c^25*d^5 - 1140*a^13*b^17*c^27*d^3 + 38760*a^14*b^16*c^6*d^24 - 823650*a^14*b^16*c^8*d^22 + 5384410*a^14*b^16*c^10*d^20 - 17183600*a^14*b^16*c^12*d^18 + 31460200*a^14*b^16*c^14*d^16 - 35234455*a^14*b^16*c^16*d^14 + 24426875*a^14*b^16*c^18*d^12 - 10132510*a^14*b^16*c^20*d^10 + 2277150*a^14*b^16*c^22*d^8 - 218025*a^14*b^16*c^24*d^6 + 4845*a^14*b^16*c^26*d^4 - 15504*a^15*b^15*c^5*d^25 + 465120*a^15*b^15*c^7*d^23 - 3772640*a^15*b^15*c^9*d^21 + 14108640*a^15*b^15*c^11*d^19 - 29535120*a^15*b^15*c^13*d^17 + 37499008*a^15*b^15*c^15*d^15 - 29535120*a^15*b^15*c^17*d^13 + 14108640*a^15*b^15*c^19*d^11 - 3772640*a^15*b^15*c^21*d^9 + 465120*a^15*b^15*c^23*d^7 - 15504*a^15*b^15*c^25*d^5 + 4845*a^16*b^14*c^4*d^26 - 218025*a^16*b^14*c^6*d^24 + 2277150*a^16*b^14*c^8*d^22 - 10132510*a^16*b^14*c^10*d^20 + 24426875*a^16*b^14*c^12*d^18 - 35234455*a^16*b^14*c^14*d^16 + 31460200*a^16*b^14*c^16*d^14 - 17183600*a^16*b^14*c^18*d^12 + 5384410*a^16*b^14*c^20*d^10 - 823650*a^16*b^14*c^22*d^8 + 38760*a^16*b^14*c^24*d^6 - 1140*a^17*b^13*c^3*d^27 + 83220*a^17*b^13*c^5*d^25 - 1174200*a^17*b^13*c^7*d^23 + 6342200*a^17*b^13*c^9*d^21 - 17770700*a^17*b^13*c^11*d^19 + 29213260*a^17*b^13*c^13*d^17 - 29535120*a^17*b^13*c^15*d^15 + 1834640

0*a^17*b^13*c^17*d^13 - 6653800*a^17*b^13*c^19*d^11 + 1227400*a^17*b^13*c^2
1*d^9 - 77520*a^17*b^13*c^23*d^7 + 190*a^18*b^12*c^2*d^28 - 25175*a^18*b^12
*c^4*d^26 + 510625*a^18*b^12*c^6*d^24 - 3441850*a^18*b^12*c^8*d^22 + 113414
80*a^18*b^12*c^10*d^20 - 21339185*a^18*b^12*c^12*d^18 + 24426875*a^18*b^12*
c^14*d^16 - 17183600*a^18*b^12*c^16*d^14 + 7138300*a^18*b^12*c^18*d^12 - 15
53630*a^18*b^12*c^20*d^10 + 125970*a^18*b^12*c^22*d^8 + 5800*a^19*b^11*c^3*
d^27 - 183740*a^19*b^11*c^5*d^25 + 1607600*a^19*b^11*c^7*d^23 - 6323300*a^1
9*b^11*c^9*d^21 + 13697880*a^19*b^11*c^11*d^19 - 17770700*a^19*b^11*c^13*d^
17 + 14108640*a^19*b^11*c^15*d^15 - 6653800*a^19*b^11*c^17*d^13 + 1679600*a
^19*b^11*c^19*d^11 - 167960*a^19*b^11*c^21*d^9 - 955*a^20*b^10*c^2*d^28 + 5
3210*a^20*b^10*c^4*d^26 - 639360*a^20*b^10*c^6*d^24 + 3061855*a^20*b^10*c^8
*d^22 - 7699257*a^20*b^10*c^10*d^20 + 11341480*a^20*b^10*c^12*d^18 - 101325
10*a^20*b^10*c^14*d^16 + 5384410*a^20*b^10*c^16*d^14 - 1553630*a^20*b^10*c^
18*d^12 + 184756*a^20*b^10*c^20*d^10 - 11900*a^21*b^9*c^3*d^27 + 213040*a^2
1*b^9*c^5*d^25 - 1277800*a^21*b^9*c^7*d^23 + 3770860*a^21*b^9*c^9*d^21 - 63
23300*a^21*b^9*c^11*d^19 + 6342200*a^21*b^9*c^13*d^17 - 3772640*a^21*b^9*c^
15*d^15 + 1227400*a^21*b^9*c^17*d^13 - 167960*a^21*b^9*c^19*d^11 + 1925*a^2
2*b^8*c^2*d^28 - 58000*a^22*b^8*c^4*d^26 + 455100*a^22*b^8*c^6*d^24 - 15984
95*a^22*b^8*c^8*d^22 + 3061855*a^22*b^8*c^10*d^20 - 3441850*a^22*b^8*c^12*d
^18 + 2277150*a^22*b^8*c^14*d^16 - 823650*a^22*b^8*c^16*d^14 + 125970*a^22*
b^8*c^18*d^12 + 12400*a^23*b^7*c^3*d^27 - 136520*a^23*b^7*c^5*d^25 + 581120
*a^23*b^7*c^7*d^23 - 1277800*a^23*b^7*c^9*d^21 + 1607600*a^23*b^7*c^11*d^19
 - 1174200*a^23*b^7*c^13*d^17 + 465120*a^23*b^7*c^15*d^15 - 77520*a^23*b^7*
c^17*d^13 - 1950*a^24*b^6*c^2*d^28 + 33825*a^24*b^6*c^4*d^26 - 178985*a^24*
b^6*c^6*d^24 + 455100*a^24*b^6*c^8*d^22 - 639360*a^24*b^6*c^10*d^20 + 51062
5*a^24*b^6*c^12*d^18 - 218025*a^24*b^6*c^14*d^16 + 38760*a^24*b^6*c^16*d^14
 - 6700*a^25*b^5*c^3*d^27 + 46004*a^25*b^5*c^5*d^25 - 136520*a^25*b^5*c^7*d
^23 + 213040*a^25*b^5*c^9*d^21 - 183740*a^25*b^5*c^11*d^19 + 83220*a^25*b^5
*c^13*d^17 - 15504*a^25*b^5*c^15*d^15 + 1000*a^26*b^4*c^2*d^28 - 9695*a^26*
b^4*c^4*d^26 + 33825*a^26*b^4*c^6*d^24 - 58000*a^26*b^4*c^8*d^22 + 53210*a^
26*b^4*c^10*d^20 - 25175*a^26*b^4*c^12*d^18 + 4845*a^26*b^4*c^14*d^16 + 164
0*a^27*b^3*c^3*d^27 - 6700*a^27*b^3*c^5*d^25 + 12400*a^27*b^3*c^7*d^23 - 11
900*a^27*b^3*c^9*d^21 + 5800*a^27*b^3*c^11*d^19 - 1140*a^27*b^3*c^13*d^17 -
 215*a^28*b^2*c^2*d^28 + 1000*a^28*b^2*c^4*d^26 - 1950*a^28*b^2*c^6*d^24 +
1925*a^28*b^2*c^8*d^22 - 955*a^28*b^2*c^10*d^20 + 190*a^28*b^2*c^12*d^18 +
20*a*b^29*c^29*d + 20*a^29*b*c*d^29)))^(1/2) - (4*(4*a^2*b^20*c^22 - 12*a^6
*b^16*c^22 + 8*a^8*b^14*c^22 + 4*a^22*c^2*d^20 - 12*a^22*c^6*d^16 + 8*a^22*
c^8*d^14 + 48*a*b^21*c^11*d^11 - 212*a*b^21*c^13*d^9 + 360*a*b^21*c^15*d^7
- 276*a*b^21*c^17*d^5 + 80*a*b^21*c^19*d^3 - 20*a^3*b^19*c^21*d - 72*a^5*b^
17*c^21*d + 204*a^7*b^15*c^21*d - 112*a^9*b^13*c^21*d + 48*a^11*b^11*c*d^21
 - 212*a^13*b^9*c*d^21 + 360*a^15*b^7*c*d^21 - 276*a^17*b^5*c*d^21 + 80*a^1
9*b^3*c*d^21 - 20*a^21*b*c^3*d^19 - 72*a^21*b*c^5*d^17 + 204*a^21*b*c^7*d^1
5 - 112*a^21*b*c^9*d^13 - 480*a^2*b^20*c^10*d^12 + 2160*a^2*b^20*c^12*d^10
- 3772*a^2*b^20*c^14*d^8 + 3020*a^2*b^20*c^16*d^6 - 960*a^2*b^20*c^18*d^4 +
 28*a^2*b^20*c^20*d^2 + 2160*a^3*b^19*c^9*d^13 - 10152*a^3*b^19*c^11*d^11 +

18888*a^3*b^19*c^13*d^9 - 16732*a^3*b^19*c^15*d^7 + 6588*a^3*b^19*c^17*d^5 - 732*a^3*b^19*c^19*d^3 - 5760*a^4*b^18*c^8*d^14 + 29360*a^4*b^18*c^10*d^12 - 60792*a^4*b^18*c^12*d^10 + 62708*a^4*b^18*c^14*d^8 - 31892*a^4*b^18*c^16*d^6 + 6588*a^4*b^18*c^18*d^4 - 212*a^4*b^18*c^20*d^2 + 10080*a^5*b^17*c^7*d^15 - 58860*a^5*b^17*c^9*d^13 + 141880*a^5*b^17*c^11*d^11 - 175592*a^5*b^17*c^13*d^9 + 113748*a^5*b^17*c^15*d^7 - 34492*a^5*b^17*c^17*d^5 + 3308*a^5*b^17*c^19*d^3 - 12096*a^6*b^16*c^6*d^16 + 87264*a^6*b^16*c^8*d^14 - 254340*a^6*b^16*c^10*d^12 + 381532*a^6*b^16*c^12*d^10 - 307752*a^6*b^16*c^14*d^8 + 125568*a^6*b^16*c^16*d^6 - 21232*a^6*b^16*c^18*d^4 + 1068*a^6*b^16*c^20*d^2 + 10080*a^7*b^15*c^5*d^17 - 99120*a^7*b^15*c^7*d^15 + 359064*a^7*b^15*c^9*d^13 - 655076*a^7*b^15*c^11*d^11 + 650108*a^7*b^15*c^13*d^9 - 343368*a^7*b^15*c^15*d^7 + 85760*a^7*b^15*c^17*d^5 - 7652*a^7*b^15*c^19*d^3 - 5760*a^8*b^14*c^4*d^18 + 87264*a^8*b^14*c^6*d^16 - 402576*a^8*b^14*c^8*d^14 + 900324*a^8*b^14*c^10*d^12 - 1096236*a^8*b^14*c^12*d^10 + 731392*a^8*b^14*c^14*d^8 - 247352*a^8*b^14*c^16*d^6 + 34548*a^8*b^14*c^18*d^4 - 1612*a^8*b^14*c^20*d^2 + 2160*a^9*b^13*c^3*d^19 - 58860*a^9*b^13*c^5*d^17 + 359064*a^9*b^13*c^7*d^15 - 999816*a^9*b^13*c^9*d^13 + 1494564*a^9*b^13*c^11*d^11 - 1238148*a^9*b^13*c^13*d^9 + 542272*a^9*b^13*c^15*d^7 - 109032*a^9*b^13*c^17*d^5 + 7908*a^9*b^13*c^19*d^3 - 480*a^10*b^12*c^2*d^20 + 29360*a^10*b^12*c^4*d^18 - 254340*a^10*b^12*c^6*d^16 + 900324*a^10*b^12*c^8*d^14 - 1656496*a^10*b^12*c^10*d^12 + 1688232*a^10*b^12*c^12*d^10 - 934868*a^10*b^12*c^14*d^8 + 254492*a^10*b^12*c^16*d^6 - 26952*a^10*b^12*c^18*d^4 + 728*a^10*b^12*c^20*d^2 - 10152*a^11*b^11*c^3*d^19 + 141880*a^11*b^11*c^5*d^17 - 655076*a^11*b^11*c^7*d^15 + 1494564*a^11*b^11*c^9*d^13 - 1870136*a^11*b^11*c^11*d^11 + 1289704*a^11*b^11*c^13*d^9 - 455388*a^11*b^11*c^15*d^7 + 67468*a^11*b^11*c^17*d^5 - 2912*a^11*b^11*c^19*d^3 + 2160*a^12*b^10*c^2*d^20 - 60792*a^12*b^10*c^4*d^18 + 381532*a^12*b^10*c^6*d^16 - 1096236*a^12*b^10*c^8*d^14 + 1688232*a^12*b^10*c^10*d^12 - 1434728*a^12*b^10*c^12*d^10 + 639684*a^12*b^10*c^14*d^8 - 127860*a^12*b^10*c^16*d^6 + 8008*a^12*b^10*c^18*d^4 + 18888*a^13*b^9*c^3*d^19 - 175592*a^13*b^9*c^5*d^17 + 650108*a^13*b^9*c^7*d^15 - 1238148*a^13*b^9*c^9*d^13 + 1289704*a^13*b^9*c^11*d^11 - 715296*a^13*b^9*c^13*d^9 + 186564*a^13*b^9*c^15*d^7 - 16016*a^13*b^9*c^17*d^5 - 3772*a^14*b^8*c^2*d^20 + 62708*a^14*b^8*c^4*d^18 - 307752*a^14*b^8*c^6*d^16 + 731392*a^14*b^8*c^8*d^14 - 934868*a^14*b^8*c^10*d^12 + 639684*a^14*b^8*c^12*d^10 - 211416*a^14*b^8*c^14*d^8 + 24024*a^14*b^8*c^16*d^6 - 16732*a^15*b^7*c^3*d^19 + 113748*a^15*b^7*c^5*d^17 - 343368*a^15*b^7*c^7*d^15 + 542272*a^15*b^7*c^9*d^13 - 455388*a^15*b^7*c^11*d^11 + 186564*a^15*b^7*c^13*d^9 - 27456*a^15*b^7*c^15*d^7 + 3020*a^16*b^6*c^2*d^20 - 31892*a^16*b^6*c^4*d^18 + 125568*a^16*b^6*c^6*d^16 - 247352*a^16*b^6*c^8*d^14 + 254492*a^16*b^6*c^10*d^12 - 127860*a^16*b^6*c^12*d^10 + 24024*a^16*b^6*c^14*d^8 + 6588*a^17*b^5*c^3*d^19 - 34492*a^17*b^5*c^5*d^17 + 85760*a^17*b^5*c^7*d^15 - 109032*a^17*b^5*c^9*d^13 + 67468*a^17*b^5*c^11*d^11 - 16016*a^17*b^5*c^13*d^9 - 960*a^18*b^4*c^2*d^20 + 6588*a^18*b^4*c^4*d^18 - 21232*a^18*b^4*c^6*d^16 + 34548*a^18*b^4*c^8*d^14 - 26952*a^18*b^4*c^10*d^12 + 8008*a^18*b^4*c^12*d^10 - 732*a^19*b^3*c^3*d^19 + 3308*a^19*b^3*c^5*d^17 - 7652*a^19*b^3*c^7*d^15 + 7908*a^19*b^3*c^9*d^13 - 2912*a

^19*b^3*c^11*d^11 + 28*a^20*b^2*c^2*d^20 - 212*a^20*b^2*c^4*d^18 + 1068*a^2
0*b^2*c^6*d^16 - 1612*a^20*b^2*c^8*d^14 + 728*a^20*b^2*c^10*d^12))/(a^20*d^
20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*
b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*
d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12
+ b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*
a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*
d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9
*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19
 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b
*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*
d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2
- 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 +
1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 22
44*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1
587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 404
8*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 474
4*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676
*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 1
1236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 7
92*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 2
7504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 -
 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 +
20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^
8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^
2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^1
3 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13
*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^
18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c
^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10
*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11
*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^
11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*
a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^
12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300
*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 49
5*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 215
76*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 +
6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 35
88*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 +
13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 +
 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 -
 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 -
288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4
032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 4

12*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19) + (8*tan(e/2 + (f*x)/2)*(12*a^5*b^17*c^22 - 4*a^22*c*d^21 - 4*a*b^21*c^22 - 8*a^7*b^15*c^22 + 12*a^22*c^5*d^17 - 8*a^22*c^7*d^15 - 24*a*b^21*c^12*d^10 + 100*a*b^21*c^14*d^8 - 164*a*b^21*c^16*d^6 + 120*a*b^21*c^18*d^4 - 28*a*b^21*c^20*d^2 + 20*a^2*b^20*c^21*d + 72*a^4*b^18*c^21*d - 204*a^6*b^16*c^21*d + 112*a^8*b^14*c^21*d - 24*a^12*b^10*c*d^21 + 100*a^14*b^8*c*d^21 - 164*a^16*b^6*c*d^21 + 120*a^18*b^4*c*d^21 - 28*a^20*b^2*c*d^21 + 20*a^21*b*c^2*d^20 + 72*a^21*b*c^4*d^18 - 204*a^21*b*c^6*d^16 + 112*a^21*b*c^8*d^14 + 216*a^2*b^20*c^11*d^11 - 908*a^2*b^20*c^13*d^9 + 1540*a^2*b^20*c^15*d^7 - 1200*a^2*b^20*c^17*d^5 + 332*a^2*b^20*c^19*d^3 - 840*a^3*b^19*c^10*d^12 + 3672*a^3*b^19*c^12*d^10 - 6788*a^3*b^19*c^14*d^8 + 6132*a^3*b^19*c^16*d^6 - 2388*a^3*b^19*c^18*d^4 + 212*a^3*b^19*c^20*d^2 + 1800*a^4*b^18*c^9*d^13 - 8680*a^4*b^18*c^11*d^11 + 18852*a^4*b^18*c^13*d^9 - 21228*a^4*b^18*c^15*d^7 + 11692*a^4*b^18*c^17*d^5 - 2508*a^4*b^18*c^19*d^3 - 2160*a^5*b^17*c^8*d^14 + 13100*a^5*b^17*c^10*d^12 - 36820*a^5*b^17*c^12*d^10 + 53712*a^5*b^17*c^14*d^8 - 39608*a^5*b^17*c^16*d^6 + 12832*a^5*b^17*c^18*d^4 - 1068*a^5*b^17*c^20*d^2 + 1008*a^6*b^16*c^7*d^15 - 12420*a^6*b^16*c^9*d^13 + 51764*a^6*b^16*c^11*d^11 - 100128*a^6*b^16*c^13*d^9 + 96048*a^6*b^16*c^15*d^7 - 42920*a^6*b^16*c^17*d^5 + 6852*a^6*b^16*c^19*d^3 + 1008*a^7*b^15*c^6*d^16 + 5136*a^7*b^15*c^8*d^14 - 48820*a^7*b^15*c^10*d^12 + 134700*a^7*b^15*c^12*d^10 - 171472*a^7*b^15*c^14*d^8 + 103992*a^7*b^15*c^16*d^6 - 26148*a^7*b^15*c^18*d^4 + 1612*a^7*b^15*c^20*d^2 - 2160*a^8*b^14*c^5*d^17 + 5136*a^8*b^14*c^7*d^15 + 20436*a^8*b^14*c^9*d^13 - 121524*a^8*b^14*c^11*d^11 + 224888*a^8*b^14*c^13*d^9 - 186952*a^8*b^14*c^15*d^7 + 67572*a^8*b^14*c^17*d^5 - 7508*a^8*b^14*c^19*d^3 + 1800*a^9*b^13*c^4*d^18 - 12420*a^9*b^13*c^6*d^16 + 20436*a^9*b^13*c^8*d^14 + 49416*a^9*b^13*c^10*d^12 - 201552*a^9*b^13*c^12*d^10 + 245708*a^9*b^13*c^14*d^8 - 125412*a^9*b^13*c^16*d^6 + 22752*a^9*b^13*c^18*d^4 - 728*a^9*b^13*c^20*d^2 - 840*a^10*b^12*c^3*d^19 + 13100*a^10*b^12*c^5*d^17 - 48820*a^10*b^12*c^7*d^15 + 49416*a^10*b^12*c^9*d^13 + 82088*a^10*b^12*c^11*d^11 - 219092*a^10*b^12*c^13*d^9 + 168468*a^10*b^12*c^15*d^7 - 47152*a^10*b^12*c^17*d^5 + 2832*a^10*b^12*c^19*d^3 + 216*a^11*b^11*c^2*d^20 - 8680*a^11*b^11*c^4*d^18 + 51764*a^11*b^11*c^6*d^16 - 121524*a^11*b^11*c^8*d^14 + 82088*a^11*b^11*c^10*d^12 + 88712*a^11*b^11*c^12*d^10 - 153012*a^11*b^11*c^14*d^8 + 67604*a^11*b^11*c^16*d^6 - 7168*a^11*b^11*c^18*d^4 + 3672*a^12*b^10*c^3*d^19 - 36820*a^12*b^10*c^5*d^17 + 134700*a^12*b^10*c^7*d^15 - 201552*a^12*b^10*c^9*d^13 + 88712*a^12*b^10*c^11*d^11 + 62676*a^12*b^10*c^13*d^9 - 63372*a^12*b^10*c^15*d^7 + 12008*a^12*b^10*c^17*d^5 - 908*a^13*b^9*c^2*d^20 + 18852*a^13*b^9*c^4*d^18 - 100128*a^13*b^9*c^6*d^16 + 224888*a^13*b^9*c^8*d^14 - 219092*a^13*b^9*c^10*d^12 + 62676*a^13*b^9*c^12*d^10 + 26256*a^13*b^9*c^14*d^8 - 12544*a^13*b^9*c^16*d^6 - 6788*a^14*b^8*c^3*d^19 + 53712*a^14*b^8*c^5*d^17 - 171472*a^14*b^8*c^7*d^15 + 245708*a^14*b^8*c^9*d^13 - 153012*a^14*b^8*c^11*d^11 + 26256*a^14*b^8*c^13*d^9 + 5496*a^14*b^8*c^15*d^7 + 1540*a^15*b^7*c^2*d^20 - 21228*a

^15*b^7*c^4*d^18 + 96048*a^15*b^7*c^6*d^16 - 186952*a^15*b^7*c^8*d^14 + 168
468*a^15*b^7*c^10*d^12 - 63372*a^15*b^7*c^12*d^10 + 5496*a^15*b^7*c^14*d^8
+ 6132*a^16*b^6*c^3*d^19 - 39608*a^16*b^6*c^5*d^17 + 103992*a^16*b^6*c^7*d^
15 - 125412*a^16*b^6*c^9*d^13 + 67604*a^16*b^6*c^11*d^11 - 12544*a^16*b^6*c
^13*d^9 - 1200*a^17*b^5*c^2*d^20 + 11692*a^17*b^5*c^4*d^18 - 42920*a^17*b^5
*c^6*d^16 + 67572*a^17*b^5*c^8*d^14 - 47152*a^17*b^5*c^10*d^12 + 12008*a^17
*b^5*c^12*d^10 - 2388*a^18*b^4*c^3*d^19 + 12832*a^18*b^4*c^5*d^17 - 26148*a
^18*b^4*c^7*d^15 + 22752*a^18*b^4*c^9*d^13 - 7168*a^18*b^4*c^11*d^11 + 332*
a^19*b^3*c^2*d^20 - 2508*a^19*b^3*c^4*d^18 + 6852*a^19*b^3*c^6*d^16 - 7508*
a^19*b^3*c^8*d^14 + 2832*a^19*b^3*c^10*d^12 + 212*a^20*b^2*c^3*d^19 - 1068*
a^20*b^2*c^5*d^17 + 1612*a^20*b^2*c^7*d^15 - 728*a^20*b^2*c^9*d^13))/(a^20*
d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^
8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^
2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^1
2 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 1
2*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^1
7*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a
^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^
19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19
*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^1
2*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^
2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7
+ 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 -
2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 +
1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4
048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4
744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 56
76*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 +
11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 -
792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 +
27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5
- 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14
+ 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*
d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*
d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d
^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^
13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*
d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10
*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^
10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^
11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*
a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 404
8*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*
a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 363
00*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 +

495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19)) + (4*(288*a*b^18*c^6*d^13 - 1104*a*b^18*c^8*d^11 + 1538*a*b^18*c^10*d^9 - 872*a*b^18*c^12*d^7 + 108*a*b^18*c^14*d^5 + 40*a*b^18*c^16*d^3 + 8*a^3*b^16*c^18*d + 8*a^5*b^14*c^18*d + 288*a^6*b^13*c*d^18 - 1104*a^8*b^11*c*d^18 + 1538*a^10*b^9*c*d^18 - 872*a^12*b^7*c*d^18 + 108*a^14*b^5*c*d^18 + 40*a^16*b^3*c*d^18 + 8*a^18*b*c^3*d^16 + 8*a^18*b*c^5*d^14 - 864*a^2*b^17*c^5*d^14 + 3216*a^2*b^17*c^7*d^12 - 4262*a^2*b^17*c^9*d^10 + 2256*a^2*b^17*c^11*d^8 - 304*a^2*b^17*c^13*d^6 - 32*a^2*b^17*c^15*d^4 + 8*a^2*b^17*c^17*d^2 + 576*a^3*b^16*c^4*d^15 - 3024*a^3*b^16*c^6*d^13 + 6304*a^3*b^16*c^8*d^11 - 7216*a^3*b^16*c^10*d^9 + 4944*a^3*b^16*c^12*d^7 - 1664*a^3*b^16*c^14*d^5 - 72*a^3*b^16*c^16*d^3 + 576*a^4*b^15*c^3*d^16 + 912*a^4*b^15*c^5*d^14 - 8720*a^4*b^15*c^7*d^12 + 16632*a^4*b^15*c^9*d^10 - 14888*a^4*b^15*c^11*d^8 + 6704*a^4*b^15*c^13*d^6 - 744*a^4*b^15*c^15*d^4 - 40*a^4*b^15*c^17*d^2 - 864*a^5*b^14*c^2*d^17 + 912*a^5*b^14*c^4*d^15 + 5140*a^5*b^14*c^6*d^13 - 16080*a^5*b^14*c^8*d^11 + 23520*a^5*b^14*c^10*d^9 - 20208*a^5*b^14*c^12*d^7 + 7404*a^5*b^14*c^14*d^5 - 264*a^5*b^14*c^16*d^3 - 3024*a^6*b^13*c^3*d^16 + 5140*a^6*b^13*c^5*d^14 + 5280*a^6*b^13*c^7*d^12 - 28380*a^6*b^13*c^9*d^10 + 39792*a^6*b^13*c^11*d^8 - 22728*a^6*b^13*c^13*d^6 + 3096*a^6*b^13*c^15*d^4 - 112*a^6*b^13*c^17*d^2 + 3216*a^7*b^12*c^2*d^17 - 8720*a^7*b^12*c^4*d^15 + 5280*a^7*b^12*c^6*d^13 + 15000*a^7*b^12*c^8*d^11 - 40656*a^7*b^12*c^10*d^9 + 40296*a^7*b^12*c^12*d^7 - 12984*a^7*b^12*c^14*d^5 + 728*a^7*b^12*c^16*d^3 + 6304*a^8*b^11*c^3*d^16 - 16080*a^8*b^11*c^5*d^14 + 15000*a^8*b^11*c^7*d^12 + 16024*a^8*b^11*c^9*d^10 - 46184*a^8*b^11*c^11*d^8 + 27208*a^8*b^11*c^13*d^6 - 2752*a^8*b^11*c^15*d^4 - 4262*a^9*b^10*c^2*d^17 + 16632*a^9*b^10*c^4*d^15 - 28380*a^9*b^10*c^6*d^13 + 16024*a^9*b^10*c^8*d^11 + 22018*a^9*b^10*c^10*d^9 - 30104*a^9*b^10*c^12*d^7 + 6488*a^9*b^10*c^14*d^5 - 7216*a^10*b^9*c^3*d^16 + 23520*a^10*b^9*c^5*d^14 - 40656*a^10*b^9*c^7*d^12 + 22018*a^10*b^9*c^9*d^10 + 13080*a^10*b^9*c^11*d^8 - 8720*a^10*b^9*c^13*d^6 + 2256*a^11*b^8*c^2*d^17 - 14888*a^11*b^8*c^4*d^15 + 39792*a^11*b^8*c^6*d^13 - 46184*a^11*b^8*c^8*d^11 + 13080*a^11*b^8*c^10*d^9 + 4360*a^11*b^8*c^12*d^7 + 4944*a^12*b^7*c^3*d^16 - 20208*a^12*b^7*c^5*d^14 + 40296*a^12*b^7*c^7*d^12 - 30104*a^12*b^7*c^9*d^10 + 4360*a^12*b^7*c^11*d^8 - 304*a^13*b^6*c^2*d^17 + 6704*a^13*b^6*c^4*d^15 - 22728*a^13*b^6*c^6*d^13 + 27208*a^13*b^6*c^8*d^11 - 8720*a^13*b^6*c^10*d^9 - 1664

*a^14*b^5*c^3*d^16 + 7404*a^14*b^5*c^5*d^14 - 12984*a^14*b^5*c^7*d^12 + 648
8*a^14*b^5*c^9*d^10 - 32*a^15*b^4*c^2*d^17 - 744*a^15*b^4*c^4*d^15 + 3096*a
^15*b^4*c^6*d^13 - 2752*a^15*b^4*c^8*d^11 - 72*a^16*b^3*c^3*d^16 - 264*a^16
*b^3*c^5*d^14 + 728*a^16*b^3*c^7*d^12 + 8*a^17*b^2*c^2*d^17 - 40*a^17*b^2*c
^4*d^15 - 112*a^17*b^2*c^6*d^13 + 2*a*b^18*c^18*d + 2*a^18*b*c*d^18))/(a^20
*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a
^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b
^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^
12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 -
12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^
17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*
a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d
^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^1
9*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^
12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d
^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7
 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 -
 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6
+ 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 +
4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 -
4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5
676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8
+ 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2
- 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11
+ 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^
5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14
 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12
*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18
*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*
d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c
^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2
*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^1
0*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a
^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a
^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936
*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 40
48*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032
*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36
300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 +
 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 -
21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9
 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 -
 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12
 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^
6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^1

3 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19) - (8*tan(e/2 + (f*x)/2)*(a*b^18*c^19 + a^19*c*d^18 + 4*a^3*b^16*c^19 + 4*a^5*b^14*c^19 + 4*a^19*c^3*d^16 + 4*a^19*c^5*d^14 - 576*a*b^18*c^5*d^14 + 2640*a*b^18*c^7*d^12 - 4732*a*b^18*c^9*d^10 + 3961*a*b^18*c^11*d^8 - 1344*a*b^18*c^13*d^6 + 14*a*b^18*c^15*d^4 + 18*a*b^18*c^17*d^2 + 4*a^2*b^17*c^18*d - 20*a^4*b^15*c^18*d - 576*a^5*b^14*c*d^18 - 56*a^6*b^13*c^18*d + 2640*a^7*b^12*c*d^18 - 4732*a^9*b^10*c*d^18 + 3961*a^11*b^8*c*d^18 - 1344*a^13*b^6*c*d^18 + 14*a^15*b^4*c*d^18 + 18*a^17*b^2*c*d^18 + 4*a^18*b*c^2*d^17 - 20*a^18*b*c^4*d^15 - 56*a^18*b*c^6*d^13 + 2304*a^2*b^17*c^4*d^15 - 10944*a^2*b^17*c^6*d^13 + 20720*a^2*b^17*c^8*d^11 - 18788*a^2*b^17*c^10*d^9 + 7392*a^2*b^17*c^12*d^7 - 520*a^2*b^17*c^14*d^5 - 24*a^2*b^17*c^16*d^3 - 3456*a^3*b^16*c^3*d^16 + 20016*a^3*b^16*c^5*d^14 - 48112*a^3*b^16*c^7*d^12 + 58925*a^3*b^16*c^9*d^10 - 36732*a^3*b^16*c^11*d^8 + 9736*a^3*b^16*c^13*d^6 - 760*a^3*b^16*c^15*d^4 - 44*a^3*b^16*c^17*d^2 + 2304*a^4*b^15*c^2*d^17 - 23424*a^4*b^15*c^4*d^15 + 81680*a^4*b^15*c^6*d^13 - 135520*a^4*b^15*c^8*d^11 + 114144*a^4*b^15*c^10*d^9 - 44168*a^4*b^15*c^12*d^7 + 5696*a^4*b^15*c^14*d^5 - 332*a^4*b^15*c^16*d^3 + 20016*a^5*b^14*c^3*d^16 - 99112*a^5*b^14*c^5*d^14 + 213338*a^5*b^14*c^7*d^12 - 235152*a^5*b^14*c^9*d^10 + 130428*a^5*b^14*c^11*d^8 - 31908*a^5*b^14*c^13*d^6 + 3966*a^5*b^14*c^15*d^4 - 140*a^5*b^14*c^17*d^2 - 10944*a^6*b^13*c^2*d^17 + 81680*a^6*b^13*c^4*d^15 - 243832*a^6*b^13*c^6*d^13 + 364608*a^6*b^13*c^8*d^11 - 281736*a^6*b^13*c^10*d^9 + 103104*a^6*b^13*c^12*d^7 - 16860*a^6*b^13*c^14*d^5 + 1660*a^6*b^13*c^16*d^3 - 48112*a^7*b^12*c^3*d^16 + 213338*a^7*b^12*c^5*d^14 - 425832*a^7*b^12*c^7*d^12 + 434414*a^7*b^12*c^9*d^10 - 219064*a^7*b^12*c^11*d^8 + 50732*a^7*b^12*c^13*d^6 - 7220*a^7*b^12*c^15*d^4 + 364*a^7*b^12*c^17*d^2 + 20720*a^8*b^11*c^2*d^17 - 135520*a^8*b^11*c^4*d^15 + 364608*a^8*b^11*c^6*d^13 - 496336*a^8*b^11*c^8*d^11 + 343832*a^8*b^11*c^10*d^9 - 111220*a^8*b^11*c^12*d^7 + 17956*a^8*b^11*c^14*d^5 - 1376*a^8*b^11*c^16*d^3 + 58925*a^9*b^10*c^3*d^16 - 235152*a^9*b^10*c^5*d^14 + 434414*a^9*b^10*c^7*d^12 - 401788*a^9*b^10*c^9*d^10 + 172673*a^9*b^10*c^11*d^8 - 31940*a^9*b^10*c^13*d^6 + 3244*a^9*b^10*c^15*d^4 - 18788*a^10*b^9*c^2*d^17 + 114144*a^10*b^9*c^4*d^15 - 281736*a^10*b^9*c^6*d^13 + 343832*a^10*b^9*c^8*d^11 - 197840*a^10*b^9*c^10*d^9 + 45940*a^10*b^9*c^12*d^7 - 4760*a^10*b^9*c^14*d^5 - 36732*a^11*b^8*c^3*d^16 + 130428*a^11*b^8*c^5*d^14 - 219064*a^11*b^8*c^7*d^12 + 172673*a^11*b^8*c^9*d^10 - 52480*a^11*b^8*c^11*d^8 + 4580*a^11*b^8*c^13*d^6 + 7392*a^12*b^7*c^2*d^17 - 44168*a^12*b^7*c^4*d^15 + 103104*a^12*b^7*c^6*d^13 - 111220*a^12*b^7*c^8*d^11 + 45940*a^12*b^7*c^10*d^9 - 4000*a^12*b^7*c^12*d^7 + 9736*a^13*b^6*c^3*d^16 - 31908*a^13*b^6*c^5*d^14 + 50732*a^13*b^6*c^7*d^12 - 31940*a^13*b^6*c^9*d^10 + 4580*a^13*b^6*c^11*d^8 - 520*a^14*b^5*c^2*d^17 + 5696*a^14*b^5*c^4*d^15 - 16860*a^14*b^5*c^6*d^13 +

17956*a^14*b^5*c^8*d^11 - 4760*a^14*b^5*c^10*d^9 - 760*a^15*b^4*c^3*d^16 +
3966*a^15*b^4*c^5*d^14 - 7220*a^15*b^4*c^7*d^12 + 3244*a^15*b^4*c^9*d^10 -
24*a^16*b^3*c^2*d^17 - 332*a^16*b^3*c^4*d^15 + 1660*a^16*b^3*c^6*d^13 - 137
6*a^16*b^3*c^8*d^11 - 44*a^17*b^2*c^3*d^16 - 140*a^17*b^2*c^5*d^14 + 364*a^
17*b^2*c^7*d^12))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^2
0 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a
^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20
*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d
^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*
c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 4
8*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c
*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a
^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10
*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d
^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 -
1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 +
495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 -
3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 7
92*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 87
36*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 92
4*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 1
7164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 +
412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 1
8744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7
+ 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 -
5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^1
0 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*
d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^1
5 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11
*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17
*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^
6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*
b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^1
0*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^
11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744
*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a
^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156
*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5
676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 87
36*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 -
18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 +
412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 -
17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8
+ 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15
+ 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9

- 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19))*1i + (-(((4*a^24*d^24 + 4*b^24*c^24 + 16*a^2*b^22*c^24 + 16*a^4*b^20*c^24 - 1152*a^10*b^14*d^24 + 5568*a^12*b^12*d^24 - 10568*a^14*b^10*d^24 + 9460*a^16*b^8*d^24 - 3560*a^18*b^6*d^24 + 136*a^20*b^4*d^24 + 76*a^22*b^2*d^24 + 16*a^24*c^2*d^22 + 16*a^24*c^4*d^20 - 1152*b^24*c^10*d^14 + 5568*b^24*c^12*d^12 - 10568*b^24*c^14*d^10 + 9460*b^24*c^16*d^8 - 3560*b^24*c^18*d^6 + 136*b^24*c^20*d^4 + 76*b^24*c^22*d^2 + 11520*a*b^23*c^9*d^15 - 56448*a*b^23*c^11*d^13 + 109456*a*b^23*c^13*d^11 - 101240*a*b^23*c^15*d^9 + 40720*a*b^23*c^17*d^7 - 2960*a*b^23*c^19*d^5 - 536*a*b^23*c^21*d^3 - 176*a^3*b^21*c^23*d - 320*a^5*b^19*c^23*d + 11520*a^9*b^15*c*d^23 - 56448*a^11*b^13*c*d^23 + 109456*a^13*b^11*c*d^23 - 101240*a^15*b^9*c*d^23 + 40720*a^17*b^7*c*d^23 - 2960*a^19*b^5*c*d^23 - 536*a^21*b^3*c*d^23 - 176*a^23*b*c^3*d^21 - 320*a^23*b*c^5*d^19 - 51840*a^2*b^22*c^8*d^16 + 263808*a^2*b^22*c^10*d^14 - 541208*a^2*b^22*c^12*d^12 + 547088*a^2*b^22*c^14*d^10 - 263320*a^2*b^22*c^16*d^8 + 44120*a^2*b^22*c^18*d^6 - 1564*a^2*b^22*c^20*d^4 - 196*a^2*b^22*c^22*d^2 + 138240*a^3*b^21*c^7*d^17 - 758400*a^3*b^21*c^9*d^15 + 1720736*a^3*b^21*c^11*d^13 - 2002728*a^3*b^21*c^13*d^11 + 1210560*a^3*b^21*c^15*d^9 - 335040*a^3*b^21*c^17*d^7 + 37680*a^3*b^21*c^19*d^5 - 288*a^3*b^21*c^21*d^3 - 241920*a^4*b^20*c^6*d^18 + 1512000*a^4*b^20*c^8*d^16 - 3975688*a^4*b^20*c^10*d^14 + 5501328*a^4*b^20*c^12*d^12 - 4147952*a^4*b^20*c^14*d^10 + 1586920*a^4*b^20*c^16*d^8 - 276020*a^4*b^20*c^18*d^6 + 21124*a^4*b^20*c^20*d^4 + 176*a^4*b^20*c^22*d^2 + 290304*a^5*b^19*c^5*d^19 - 2232576*a^5*b^19*c^7*d^17 + 7078256*a^5*b^19*c^9*d^15 - 11781560*a^5*b^19*c^11*d^13 + 10875200*a^5*b^19*c^13*d^11 - 5365072*a^5*b^19*c^15*d^9 + 1310168*a^5*b^19*c^17*d^7 - 170968*a^5*b^19*c^19*d^5 + 8160*a^5*b^19*c^21*d^3 - 241920*a^6*b^18*c^4*d^20 + 2532096*a^6*b^18*c^6*d^18 - 9955992*a^6*b^18*c^8*d^16 + 20019440*a^6*b^18*c^10*d^14 - 22419600*a^6*b^18*c^12*d^12 + 13887520*a^6*b^18*c^14*d^10 - 4506428*a^6*b^18*c^16*d^8 + 793756*a^6*b^18*c^18*d^6 - 72240*a^6*b^18*c^20*d^4 + 3040*a^6*b^18*c^22*d^2 + 138240*a^7*b^17*c^3*d^21 - 2232576*a^7*b^17*c^5*d^19 + 11150016*a^7*b^17*c^7*d^17 - 27336616*a^7*b^17*c^9*d^15 + 37153600*a^7*b^17*c^11*d^13 - 28461040*a^7*b^17*c^13*d^11 + 11779808*a^7*b^17*c^15*d^9 - 2621008*a^7*b^17*c^17*d^7 + 336688*a^7*b^17*c^19*d^5 - 17920*a^7*b^17*c^21*d^3 - 51840*a^8*b^16*c^2*d^22 + 1512000*a^8*b^16*c^4*d^20 - 9955992*a^8*b^16*c^6*d^18 + 30289656*a^8*b^16*c^8*d^16 - 50137600*a^8*b^16*c^10*d^14 + 46972560*a^8*b^16*c^12*d^12 - 24199280*a^8*b^16*c^14*d^10 + 6661036*a^8*b^16*c^16*d^8 - 1058448*a^8*b^16*c^18*d^6 + 72560*a^8*b^16*c^20*d^4 - 758400*a^9*b^15*c^3*d^21 + 7078256*a^9*b^15*c^5*d^19 - 27336616*a^9*b^15*c^7*d^17 + 55383904*a^9*b^15*c^9*d^15 - 63124080*a^9*b^15*c^11*d^13 + 39987520*a^9*b^15*c^13*d^11 - 13462088*a^9*b^15*c^15*d^9 + 2478528*a^9*b^15*c^17*d^7 - 212032*a^9*b^15*c^19*d^5 + 263808*a^10*b^14*c^2*d^22 - 3975688*a^10*b^14*c^4*d^20 +

20019440*a^10*b^14*c^6*d^18 - 50137600*a^10*b^14*c^8*d^16 + 69593872*a^10*b^14*c^10*d^14 - 53854288*a^10*b^14*c^12*d^12 + 21989928*a^10*b^14*c^14*d^10 - 4591360*a^10*b^14*c^16*d^8 + 460480*a^10*b^14*c^18*d^6 + 1720736*a^11*b^13*c^3*d^21 - 11781560*a^11*b^13*c^5*d^19 + 37153600*a^11*b^13*c^7*d^17 - 63124080*a^11*b^13*c^9*d^15 + 59445728*a^11*b^13*c^11*d^13 - 29358696*a^11*b^13*c^13*d^11 + 6995840*a^11*b^13*c^15*d^9 - 762560*a^11*b^13*c^17*d^7 - 541208*a^12*b^12*c^2*d^22 + 5501328*a^12*b^12*c^4*d^20 - 22419600*a^12*b^12*c^6*d^18 + 46972560*a^12*b^12*c^8*d^16 - 53854288*a^12*b^12*c^10*d^14 + 32294808*a^12*b^12*c^12*d^12 - 8958208*a^12*b^12*c^14*d^10 + 999040*a^12*b^12*c^16*d^8 - 2002728*a^13*b^11*c^3*d^21 + 10875200*a^13*b^11*c^5*d^19 - 28461040*a^13*b^11*c^7*d^17 + 39987520*a^13*b^11*c^9*d^15 - 29358696*a^13*b^11*c^11*d^13 + 9722048*a^13*b^11*c^13*d^11 - 1104320*a^13*b^11*c^15*d^9 + 547088*a^14*b^10*c^2*d^22 - 4147952*a^14*b^10*c^4*d^20 + 13887520*a^14*b^10*c^6*d^18 - 24199280*a^14*b^10*c^8*d^16 + 21989928*a^14*b^10*c^10*d^14 - 8958208*a^14*b^10*c^12*d^12 + 1124032*a^14*b^10*c^14*d^10 + 1210560*a^15*b^9*c^3*d^21 - 5365072*a^15*b^9*c^5*d^19 + 11779808*a^15*b^9*c^7*d^17 - 13462088*a^15*b^9*c^9*d^15 + 6995840*a^15*b^9*c^11*d^13 - 1104320*a^15*b^9*c^13*d^11 - 263320*a^16*b^8*c^2*d^22 + 1586920*a^16*b^8*c^4*d^20 - 4506428*a^16*b^8*c^6*d^18 + 6661036*a^16*b^8*c^8*d^16 - 4591360*a^16*b^8*c^10*d^14 + 999040*a^16*b^8*c^12*d^12 - 335040*a^17*b^7*c^3*d^21 + 1310168*a^17*b^7*c^5*d^19 - 2621008*a^17*b^7*c^7*d^17 + 2478528*a^17*b^7*c^9*d^15 - 762560*a^17*b^7*c^11*d^13 + 44120*a^18*b^6*c^2*d^22 - 276020*a^18*b^6*c^4*d^20 + 793756*a^18*b^6*c^6*d^18 - 1058448*a^18*b^6*c^8*d^16 + 460480*a^18*b^6*c^10*d^14 + 37680*a^19*b^5*c^3*d^21 - 170968*a^19*b^5*c^5*d^19 + 336688*a^19*b^5*c^7*d^17 - 212032*a^19*b^5*c^9*d^15 - 1564*a^20*b^4*c^2*d^22 + 21124*a^20*b^4*c^4*d^20 - 72240*a^20*b^4*c^6*d^18 + 72560*a^20*b^4*c^8*d^16 - 288*a^21*b^3*c^3*d^21 + 8160*a^21*b^3*c^5*d^19 - 17920*a^21*b^3*c^7*d^17 - 196*a^22*b^2*c^2*d^22 + 176*a^22*b^2*c^4*d^20 + 3040*a^22*b^2*c^6*d^18 - 8*a*b^23*c^23*d - 8*a^23*b*c*d^23)^2/4 - (20736*b^18*d^18 - 96768*a^2*b^16*d^18 + 173664*a^4*b^14*d^18 - 136032*a^6*b^12*d^18 + 31081*a^8*b^10*d^18 + 8440*a^10*b^8*d^18 + 400*a^12*b^6*d^18 - 96768*b^18*c^2*d^16 + 173664*b^18*c^4*d^14 - 136032*b^18*c^6*d^12 + 31081*b^18*c^8*d^10 + 8440*b^18*c^10*d^8 + 400*b^18*c^12*d^6 - 131328*a*b^17*c^3*d^15 + 216576*a*b^17*c^5*d^13 - 141104*a*b^17*c^7*d^11 + 20260*a*b^17*c^9*d^9 + 2800*a*b^17*c^11*d^7 - 131328*a^3*b^15*c*d^17 + 216576*a^5*b^13*c*d^17 - 141104*a^7*b^11*c*d^17 + 20260*a^9*b^9*c*d^17 + 2800*a^11*b^7*c*d^17 + 495936*a^2*b^16*c^2*d^16 - 989856*a^2*b^16*c^4*d^14 + 901948*a^2*b^16*c^6*d^12 - 308392*a^2*b^16*c^8*d^10 - 5260*a^2*b^16*c^10*d^8 + 1600*a^2*b^16*c^12*d^6 + 657408*a^3*b^15*c^3*d^15 - 1158992*a^3*b^15*c^5*d^13 + 838256*a^3*b^15*c^7*d^11 - 182200*a^3*b^15*c^9*d^9 - 3200*a^3*b^15*c^11*d^7 - 989856*a^4*b^14*c^2*d^16 + 2185654*a^4*b^14*c^4*d^14 - 2218576*a^4*b^14*c^6*d^12 + 900624*a^4*b^14*c^8*d^10 - 64720*a^4*b^14*c^10*d^8 + 1600*a^4*b^14*c^12*d^6 - 1158992*a^5*b^13*c^3*d^15 + 2158808*a^5*b^13*c^5*d^13 - 1641528*a^5*b^13*c^7*d^11 + 406880*a^5*b^13*c^9*d^9 - 17600*a^5*b^13*c^11*d^7 + 901948*a^6*b^12*c^2*d^16 - 2218576*a^6*b^12*c^4*d^14 + 2430936*a^6*b^12*c^6*d^12 - 1026928*a^6*b^12*c^8*d^10 + 88720*a^6*b^12*c^10*d^8 + 838256*a^7

*b^11*c^3*d^15 - 1641528*a^7*b^11*c^5*d^13 + 1206848*a^7*b^11*c^7*d^11 - 23
9360*a^7*b^11*c^9*d^9 - 308392*a^8*b^10*c^2*d^16 + 900624*a^8*b^10*c^4*d^14
 - 1026928*a^8*b^10*c^6*d^12 + 354016*a^8*b^10*c^8*d^10 - 182200*a^9*b^9*c^
3*d^15 + 406880*a^9*b^9*c^5*d^13 - 239360*a^9*b^9*c^7*d^11 - 5260*a^10*b^8*
c^2*d^16 - 64720*a^10*b^8*c^4*d^14 + 88720*a^10*b^8*c^6*d^12 - 3200*a^11*b^
7*c^3*d^15 - 17600*a^11*b^7*c^5*d^13 + 1600*a^12*b^6*c^2*d^16 + 1600*a^12*b
^6*c^4*d^14 + 27648*a*b^17*c*d^17)*(80*a^2*b^28*c^30 - 16*b^30*c^30 - 16*a^
30*d^30 - 160*a^4*b^26*c^30 + 160*a^6*b^24*c^30 - 80*a^8*b^22*c^30 + 16*a^1
0*b^20*c^30 + 16*a^20*b^10*d^30 - 80*a^22*b^8*d^30 + 160*a^24*b^6*d^30 - 16
0*a^26*b^4*d^30 + 80*a^28*b^2*d^30 + 80*a^30*c^2*d^28 - 160*a^30*c^4*d^26 +
 160*a^30*c^6*d^24 - 80*a^30*c^8*d^22 + 16*a^30*c^10*d^20 + 16*b^30*c^20*d^
10 - 80*b^30*c^22*d^8 + 160*b^30*c^24*d^6 - 160*b^30*c^26*d^4 + 80*b^30*c^2
8*d^2 - 320*a*b^29*c^19*d^11 + 1600*a*b^29*c^21*d^9 - 3200*a*b^29*c^23*d^7
 + 3200*a*b^29*c^25*d^5 - 1600*a*b^29*c^27*d^3 - 1600*a^3*b^27*c^29*d + 3200
*a^5*b^25*c^29*d - 3200*a^7*b^23*c^29*d + 1600*a^9*b^21*c^29*d - 320*a^11*b
^19*c^29*d - 320*a^19*b^11*c*d^29 + 1600*a^21*b^9*c*d^29 - 3200*a^23*b^7*c*
d^29 + 3200*a^25*b^5*c*d^29 - 1600*a^27*b^3*c*d^29 - 1600*a^29*b*c^3*d^27 +
 3200*a^29*b*c^5*d^25 - 3200*a^29*b*c^7*d^23 + 1600*a^29*b*c^9*d^21 - 320*a
^29*b*c^11*d^19 + 3040*a^2*b^28*c^18*d^12 - 15280*a^2*b^28*c^20*d^10 + 3080
0*a^2*b^28*c^22*d^8 - 31200*a^2*b^28*c^24*d^6 + 16000*a^2*b^28*c^26*d^4 - 3
440*a^2*b^28*c^28*d^2 - 18240*a^3*b^27*c^17*d^13 + 92800*a^3*b^27*c^19*d^11
 - 190400*a^3*b^27*c^21*d^9 + 198400*a^3*b^27*c^23*d^7 - 107200*a^3*b^27*c^
25*d^5 + 26240*a^3*b^27*c^27*d^3 + 77520*a^4*b^26*c^16*d^14 - 402800*a^4*b^
26*c^18*d^12 + 851360*a^4*b^26*c^20*d^10 - 928000*a^4*b^26*c^22*d^8 + 54120
0*a^4*b^26*c^24*d^6 - 155120*a^4*b^26*c^26*d^4 + 16000*a^4*b^26*c^28*d^2 -
248064*a^5*b^25*c^15*d^15 + 1331520*a^5*b^25*c^17*d^13 - 2939840*a^5*b^25*c
^19*d^11 + 3408640*a^5*b^25*c^21*d^9 - 2184320*a^5*b^25*c^23*d^7 + 736064*a
^5*b^25*c^25*d^5 - 107200*a^5*b^25*c^27*d^3 + 620160*a^6*b^24*c^14*d^16 - 3
488400*a^6*b^24*c^16*d^14 + 8170000*a^6*b^24*c^18*d^12 - 10229760*a^6*b^24*
c^20*d^10 + 7281600*a^6*b^24*c^22*d^8 - 2863760*a^6*b^24*c^24*d^6 + 541200*
a^6*b^24*c^26*d^4 - 31200*a^6*b^24*c^28*d^2 - 1240320*a^7*b^23*c^13*d^17 +
7441920*a^7*b^23*c^15*d^15 - 18787200*a^7*b^23*c^17*d^13 + 25721600*a^7*b^2
3*c^19*d^11 - 20444800*a^7*b^23*c^21*d^9 + 9297920*a^7*b^23*c^23*d^7 - 2184
320*a^7*b^23*c^25*d^5 + 198400*a^7*b^23*c^27*d^3 + 2015520*a^8*b^22*c^12*d^
18 - 13178400*a^8*b^22*c^14*d^16 + 36434400*a^8*b^22*c^16*d^14 - 55069600*a
^8*b^22*c^18*d^12 + 48989680*a^8*b^22*c^20*d^10 - 25575920*a^8*b^22*c^22*d^
8 + 7281600*a^8*b^22*c^24*d^6 - 928000*a^8*b^22*c^26*d^4 + 30800*a^8*b^22*c
^28*d^2 - 2687360*a^9*b^21*c^11*d^19 + 19638400*a^9*b^21*c^13*d^17 - 603622
40*a^9*b^21*c^15*d^15 + 101475200*a^9*b^21*c^17*d^13 - 101172800*a^9*b^21*c
^19*d^11 + 60333760*a^9*b^21*c^21*d^9 - 20444800*a^9*b^21*c^23*d^7 + 340864
0*a^9*b^21*c^25*d^5 - 190400*a^9*b^21*c^27*d^3 + 2956096*a^10*b^20*c^10*d^2
0 - 24858080*a^10*b^20*c^12*d^18 + 86150560*a^10*b^20*c^14*d^16 - 162120160
*a^10*b^20*c^16*d^14 + 181463680*a^10*b^20*c^18*d^12 - 123188112*a^10*b^20*
c^20*d^10 + 48989680*a^10*b^20*c^22*d^8 - 10229760*a^10*b^20*c^24*d^6 + 851
360*a^10*b^20*c^26*d^4 - 15280*a^10*b^20*c^28*d^2 - 2687360*a^11*b^19*c^9*d

^21 + 26873600*a^11*b^19*c^11*d^19 - 106460800*a^11*b^19*c^13*d^17 + 225738
240*a^11*b^19*c^15*d^15 - 284331200*a^11*b^19*c^17*d^13 + 219166080*a^11*b^
19*c^19*d^11 - 101172800*a^11*b^19*c^21*d^9 + 25721600*a^11*b^19*c^23*d^7 -
 2939840*a^11*b^19*c^25*d^5 + 92800*a^11*b^19*c^27*d^3 + 2015520*a^12*b^18*
c^8*d^22 - 24858080*a^12*b^18*c^10*d^20 + 114212800*a^12*b^18*c^12*d^18 - 2
74937600*a^12*b^18*c^14*d^16 + 390830000*a^12*b^18*c^16*d^14 - 341426960*a^
12*b^18*c^18*d^12 + 181463680*a^12*b^18*c^20*d^10 - 55069600*a^12*b^18*c^22
*d^8 + 8170000*a^12*b^18*c^24*d^6 - 402800*a^12*b^18*c^26*d^4 + 3040*a^12*b
^18*c^28*d^2 - 1240320*a^13*b^17*c^7*d^23 + 19638400*a^13*b^17*c^9*d^21 - 1
06460800*a^13*b^17*c^11*d^19 + 293542400*a^13*b^17*c^13*d^17 - 472561920*a^
13*b^17*c^15*d^15 + 467412160*a^13*b^17*c^17*d^13 - 284331200*a^13*b^17*c^1
9*d^11 + 101475200*a^13*b^17*c^21*d^9 - 18787200*a^13*b^17*c^23*d^7 + 13315
20*a^13*b^17*c^25*d^5 - 18240*a^13*b^17*c^27*d^3 + 620160*a^14*b^16*c^6*d^2
4 - 13178400*a^14*b^16*c^8*d^22 + 86150560*a^14*b^16*c^10*d^20 - 274937600*
a^14*b^16*c^12*d^18 + 503363200*a^14*b^16*c^14*d^16 - 563751280*a^14*b^16*c
^16*d^14 + 390830000*a^14*b^16*c^18*d^12 - 162120160*a^14*b^16*c^20*d^10 +
36434400*a^14*b^16*c^22*d^8 - 3488400*a^14*b^16*c^24*d^6 + 77520*a^14*b^16*
c^26*d^4 - 248064*a^15*b^15*c^5*d^25 + 7441920*a^15*b^15*c^7*d^23 - 6036224
0*a^15*b^15*c^9*d^21 + 225738240*a^15*b^15*c^11*d^19 - 472561920*a^15*b^15*
c^13*d^17 + 599984128*a^15*b^15*c^15*d^15 - 472561920*a^15*b^15*c^17*d^13 +
 225738240*a^15*b^15*c^19*d^11 - 60362240*a^15*b^15*c^21*d^9 + 7441920*a^15
*b^15*c^23*d^7 - 248064*a^15*b^15*c^25*d^5 + 77520*a^16*b^14*c^4*d^26 - 348
8400*a^16*b^14*c^6*d^24 + 36434400*a^16*b^14*c^8*d^22 - 162120160*a^16*b^14
*c^10*d^20 + 390830000*a^16*b^14*c^12*d^18 - 563751280*a^16*b^14*c^14*d^16
+ 503363200*a^16*b^14*c^16*d^14 - 274937600*a^16*b^14*c^18*d^12 + 86150560*
a^16*b^14*c^20*d^10 - 13178400*a^16*b^14*c^22*d^8 + 620160*a^16*b^14*c^24*d
^6 - 18240*a^17*b^13*c^3*d^27 + 1331520*a^17*b^13*c^5*d^25 - 18787200*a^17*
b^13*c^7*d^23 + 101475200*a^17*b^13*c^9*d^21 - 284331200*a^17*b^13*c^11*d^1
9 + 467412160*a^17*b^13*c^13*d^17 - 472561920*a^17*b^13*c^15*d^15 + 2935424
00*a^17*b^13*c^17*d^13 - 106460800*a^17*b^13*c^19*d^11 + 19638400*a^17*b^13
*c^21*d^9 - 1240320*a^17*b^13*c^23*d^7 + 3040*a^18*b^12*c^2*d^28 - 402800*a
^18*b^12*c^4*d^26 + 8170000*a^18*b^12*c^6*d^24 - 55069600*a^18*b^12*c^8*d^2
2 + 181463680*a^18*b^12*c^10*d^20 - 341426960*a^18*b^12*c^12*d^18 + 3908300
00*a^18*b^12*c^14*d^16 - 274937600*a^18*b^12*c^16*d^14 + 114212800*a^18*b^1
2*c^18*d^12 - 24858080*a^18*b^12*c^20*d^10 + 2015520*a^18*b^12*c^22*d^8 + 9
2800*a^19*b^11*c^3*d^27 - 2939840*a^19*b^11*c^5*d^25 + 25721600*a^19*b^11*c
^7*d^23 - 101172800*a^19*b^11*c^9*d^21 + 219166080*a^19*b^11*c^11*d^19 - 28
4331200*a^19*b^11*c^13*d^17 + 225738240*a^19*b^11*c^15*d^15 - 106460800*a^1
9*b^11*c^17*d^13 + 26873600*a^19*b^11*c^19*d^11 - 2687360*a^19*b^11*c^21*d^
9 - 15280*a^20*b^10*c^2*d^28 + 851360*a^20*b^10*c^4*d^26 - 10229760*a^20*b^
10*c^6*d^24 + 48989680*a^20*b^10*c^8*d^22 - 123188112*a^20*b^10*c^10*d^20 +
 181463680*a^20*b^10*c^12*d^18 - 162120160*a^20*b^10*c^14*d^16 + 86150560*a
^20*b^10*c^16*d^14 - 24858080*a^20*b^10*c^18*d^12 + 2956096*a^20*b^10*c^20*
d^10 - 190400*a^21*b^9*c^3*d^27 + 3408640*a^21*b^9*c^5*d^25 - 20444800*a^21
*b^9*c^7*d^23 + 60333760*a^21*b^9*c^9*d^21 - 101172800*a^21*b^9*c^11*d^19 +

101475200*a^21*b^9*c^13*d^17 - 60362240*a^21*b^9*c^15*d^15 + 19638400*a^21*b^9*c^17*d^13 - 2687360*a^21*b^9*c^19*d^11 + 30800*a^22*b^8*c^2*d^28 - 928000*a^22*b^8*c^4*d^26 + 7281600*a^22*b^8*c^6*d^24 - 25575920*a^22*b^8*c^8*d^22 + 48989680*a^22*b^8*c^10*d^20 - 55069600*a^22*b^8*c^12*d^18 + 36434400*a^22*b^8*c^14*d^16 - 13178400*a^22*b^8*c^16*d^14 + 2015520*a^22*b^8*c^18*d^12 + 198400*a^23*b^7*c^3*d^27 - 2184320*a^23*b^7*c^5*d^25 + 9297920*a^23*b^7*c^7*d^23 - 20444800*a^23*b^7*c^9*d^21 + 25721600*a^23*b^7*c^11*d^19 - 18787200*a^23*b^7*c^13*d^17 + 7441920*a^23*b^7*c^15*d^15 - 1240320*a^23*b^7*c^17*d^13 - 31200*a^24*b^6*c^2*d^28 + 541200*a^24*b^6*c^4*d^26 - 2863760*a^24*b^6*c^6*d^24 + 7281600*a^24*b^6*c^8*d^22 - 10229760*a^24*b^6*c^10*d^20 + 8170000*a^24*b^6*c^12*d^18 - 3488400*a^24*b^6*c^14*d^16 + 620160*a^24*b^6*c^16*d^14 - 107200*a^25*b^5*c^3*d^27 + 736064*a^25*b^5*c^5*d^25 - 2184320*a^25*b^5*c^7*d^23 + 3408640*a^25*b^5*c^9*d^21 - 2939840*a^25*b^5*c^11*d^19 + 1331520*a^25*b^5*c^13*d^17 - 248064*a^25*b^5*c^15*d^15 + 16000*a^26*b^4*c^2*d^28 - 155120*a^26*b^4*c^4*d^26 + 541200*a^26*b^4*c^6*d^24 - 928000*a^26*b^4*c^8*d^22 + 851360*a^26*b^4*c^10*d^20 - 402800*a^26*b^4*c^12*d^18 + 77520*a^26*b^4*c^14*d^16 + 26240*a^27*b^3*c^3*d^27 - 107200*a^27*b^3*c^5*d^25 + 198400*a^27*b^3*c^7*d^23 - 190400*a^27*b^3*c^9*d^21 + 92800*a^27*b^3*c^11*d^19 - 18240*a^27*b^3*c^13*d^17 - 3440*a^28*b^2*c^2*d^28 + 16000*a^28*b^2*c^4*d^26 - 31200*a^28*b^2*c^6*d^24 + 30800*a^28*b^2*c^8*d^22 - 15280*a^28*b^2*c^10*d^20 + 3040*a^28*b^2*c^12*d^18 + 320*a*b^29*c^29*d + 320*a^29*b*c*d^29))^(1/2) + 2*a^24*d^24 + 2*b^24*c^24 + 8*a^2*b^22*c^24 + 8*a^4*b^20*c^24 - 576*a^10*b^14*d^24 + 2784*a^12*b^12*d^24 - 5284*a^14*b^10*d^24 + 4730*a^16*b^8*d^24 - 1780*a^18*b^6*d^24 + 68*a^20*b^4*d^24 + 38*a^22*b^2*d^24 + 8*a^24*c^2*d^22 + 8*a^24*c^4*d^20 - 576*b^24*c^10*d^14 + 2784*b^24*c^12*d^12 - 5284*b^24*c^14*d^10 + 4730*b^24*c^16*d^8 - 1780*b^24*c^18*d^6 + 68*b^24*c^20*d^4 + 38*b^24*c^22*d^2 + 5760*a*b^23*c^9*d^15 - 28224*a*b^23*c^11*d^13 + 54728*a*b^23*c^13*d^11 - 50620*a*b^23*c^15*d^9 + 20360*a*b^23*c^17*d^7 - 1480*a*b^23*c^19*d^5 - 268*a*b^23*c^21*d^3 - 88*a^3*b^21*c^23*d - 160*a^5*b^19*c^23*d + 5760*a^9*b^15*c*d^23 - 28224*a^11*b^13*c*d^23 + 54728*a^13*b^11*c*d^23 - 50620*a^15*b^9*c*d^23 + 20360*a^17*b^7*c*d^23 - 1480*a^19*b^5*c*d^23 - 268*a^21*b^3*c*d^23 - 88*a^23*b*c^3*d^21 - 160*a^23*b*c^5*d^19 - 25920*a^2*b^22*c^8*d^16 + 131904*a^2*b^22*c^10*d^14 - 270604*a^2*b^22*c^12*d^12 + 273544*a^2*b^22*c^14*d^10 - 131660*a^2*b^22*c^16*d^8 + 22060*a^2*b^22*c^18*d^6 - 782*a^2*b^22*c^20*d^4 - 98*a^2*b^22*c^22*d^2 + 69120*a^3*b^21*c^7*d^17 - 379200*a^3*b^21*c^9*d^15 + 860368*a^3*b^21*c^11*d^13 - 1001364*a^3*b^21*c^13*d^11 + 605280*a^3*b^21*c^15*d^9 - 167520*a^3*b^21*c^17*d^7 + 18840*a^3*b^21*c^19*d^5 - 144*a^3*b^21*c^21*d^3 - 120960*a^4*b^20*c^6*d^18 + 756000*a^4*b^20*c^8*d^16 - 1987844*a^4*b^20*c^10*d^14 + 2750664*a^4*b^20*c^12*d^12 - 2073976*a^4*b^20*c^14*d^10 + 793460*a^4*b^20*c^16*d^8 - 138010*a^4*b^20*c^18*d^6 + 10562*a^4*b^20*c^20*d^4 + 88*a^4*b^20*c^22*d^2 + 145152*a^5*b^19*c^5*d^19 - 1116288*a^5*b^19*c^7*d^17 + 3539128*a^5*b^19*c^9*d^15 - 5890780*a^5*b^19*c^11*d^13 + 5437600*a^5*b^19*c^13*d^11 - 2682536*a^5*b^19*c^15*d^9 + 655084*a^5*b^19*c^17*d^7 - 85484*a^5*b^19*c^19*d^5 + 4080*a^5*b^19*c^21*d^3 - 120960*a^6*b^18*c^4*d^20 + 1266048*a^6*b^18*c^6*d^18 - 4977996*a^6 *b^18*c^8*d^16 + 10009720*a^6*b^18*c^10*d^14 - 11209800*a^6*b^18*c^12*d^12 + 6943760*a^6*b^18*c^14*d^10 - 2253214*a^6*b^18*c^16*d^8 + 396878*a^6*b^18*c^18*d^6 - 36120*a^6*b^18*c^20*d^4 + 1520*a^6*b^18*c^22*d^2 + 69120*a^7*b^17*c^3*d^21 - 1116288*a^7*b^17*c^5*d^19 + 5575008*a^7*b^17*c^7*d^17 - 13668308*a^7*b^17*c^9*d^15 + 18576800*a^7*b^17*c^11*d^13 - 14230520*a^7*b^17*c^13*d^11 + 5889904*a^7*b^17*c^15*d^9 - 1310504*a^7*b^17*c^17*d^7 + 168344*a^7*b^17*c^19*d^5 - 8960*a^7*b^17*c^21*d^3 - 25920*a^8*b^16*c^2*d^22 + 756000*a^8*b^16*c^4*d^20 - 4977996*a^8*b^16*c^6*d^18 + 15144828*a^8*b^16*c^8*d^16 - 25068800*a^8*b^16*c^10*d^14 + 23486280*a^8*b^16*c^12*d^12 - 12099640*a^8*b^16*c^14*d^10 + 3330518*a^8*b^16*c^16*d^8 - 529224*a^8*b^16*c^18*d^6 + 36280*a^8*b^16*c^20*d^4 - 379200*a^9*b^15*c^3*d^21 + 3539128*a^9*b^15*c^5*d^19 - 13668308*a^9*b^15*c^7*d^17 + 27691952*a^9*b^15*c^9*d^15 - 31562040*a^9*b^15*c^11*d^13 + 19993760*a^9*b^15*c^13*d^11 - 6731044*a^9*b^15*c^15*d^9 + 1239264*a^9*b^15*c^17*d^7 - 106016*a^9*b^15*c^19*d^5 + 131904*a^10*b^14*c^2*d^22 - 1987844*a^10*b^14*c^4*d^20 + 10009720*a^10*b^14*c^6*d^18 - 25068800*a^10*b^14*c^8*d^16 + 34796936*a^10*b^14*c^10*d^14 - 26927144*a^10*b^14*c^12*d^12 + 10994964*a^10*b^14*c^14*d^10 - 2295680*a^10*b^14*c^16*d^8 + 230240*a^10*b^14*c^18*d^6 + 860368*a^11*b^13*c^3*d^21 - 5890780*a^11*b^13*c^5*d^19 + 18576800*a^11*b^13*c^7*d^17 - 31562040*a^11*b^13*c^9*d^15 + 29722864*a^11*b^13*c^11*d^13 - 14679348*a^11*b^13*c^13*d^11 + 3497920*a^11*b^13*c^15*d^9 - 381280*a^11*b^13*c^17*d^7 - 270604*a^12*b^12*c^2*d^22 + 2750664*a^12*b^12*c^4*d^20 - 11209800*a^12*b^12*c^6*d^18 + 23486280*a^12*b^12*c^8*d^16 - 26927144*a^12*b^12*c^10*d^14 + 16147404*a^12*b^12*c^12*d^12 - 4479104*a^12*b^12*c^14*d^10 + 499520*a^12*b^12*c^16*d^8 - 1001364*a^13*b^11*c^3*d^21 + 5437600*a^13*b^11*c^5*d^19 - 14230520*a^13*b^11*c^7*d^17 + 19993760*a^13*b^11*c^9*d^15 - 14679348*a^13*b^11*c^11*d^13 + 4861024*a^13*b^11*c^13*d^11 - 552160*a^13*b^11*c^15*d^9 + 273544*a^14*b^10*c^2*d^22 - 2073976*a^14*b^10*c^4*d^20 + 6943760*a^14*b^10*c^6*d^18 - 12099640*a^14*b^10*c^8*d^16 + 10994964*a^14*b^10*c^10*d^14 - 4479104*a^14*b^10*c^12*d^12 + 562016*a^14*b^10*c^14*d^10 + 605280*a^15*b^9*c^3*d^21 - 2682536*a^15*b^9*c^5*d^19 + 5889904*a^15*b^9*c^7*d^17 - 6731044*a^15*b^9*c^9*d^15 + 3497920*a^15*b^9*c^11*d^13 - 552160*a^15*b^9*c^13*d^11 - 131660*a^16*b^8*c^2*d^22 + 793460*a^16*b^8*c^4*d^20 - 2253214*a^16*b^8*c^6*d^18 + 3330518*a^16*b^8*c^8*d^16 - 2295680*a^16*b^8*c^10*d^14 + 499520*a^16*b^8*c^12*d^12 - 167520*a^17*b^7*c^3*d^21 + 655084*a^17*b^7*c^5*d^19 - 1310504*a^17*b^7*c^7*d^17 + 1239264*a^17*b^7*c^9*d^15 - 381280*a^17*b^7*c^11*d^13 + 22060*a^18*b^6*c^2*d^22 - 138010*a^18*b^6*c^4*d^20 + 396878*a^18*b^6*c^6*d^18 - 529224*a^18*b^6*c^8*d^16 + 230240*a^18*b^6*c^10*d^14 + 18840*a^19*b^5*c^3*d^21 - 85484*a^19*b^5*c^5*d^19 + 168344*a^19*b^5*c^7*d^17 - 106016*a^19*b^5*c^9*d^15 - 782*a^20*b^4*c^2*d^22 + 10562*a^20*b^4*c^4*d^20 - 36120*a^20*b^4*c^6*d^18 + 36280*a^20*b^4*c^8*d^16 - 144*a^21*b^3*c^3*d^21 + 4080*a^21*b^3*c^5*d^19 - 8960*a^21*b^3*c^7*d^17 - 98*a^22*b^2*c^2*d^22 + 88*a^22*b^2*c^4*d^20 + 1520*a^22*b^2*c^6*d^18 - 4*a*b^23*c^23*d - 4*a^23*b*c*d^23)/(16*(5*a^2*b^28*c^30 - b^30*c^30 - a^30*d^30 - 10*a^4*b^26*c^30 + 10*a^6*b^24*c^30 - 5*a^8*b^22*c^30 + a^10*b^20*c^30 + a^20*b^10*d^30 - 5*a^22*b^8*d^30 + 10*a^24*b^6*d^30 - 10*a^26*b^4*d^30 + 5*a^2

8*b^2*d^30 + 5*a^30*c^2*d^28 - 10*a^30*c^4*d^26 + 10*a^30*c^6*d^24 - 5*a^30*c^8*d^22 + a^30*c^10*d^20 + b^30*c^20*d^10 - 5*b^30*c^22*d^8 + 10*b^30*c^24*d^6 - 10*b^30*c^26*d^4 + 5*b^30*c^28*d^2 - 20*a*b^29*c^19*d^11 + 100*a*b^29*c^21*d^9 - 200*a*b^29*c^23*d^7 + 200*a*b^29*c^25*d^5 - 100*a*b^29*c^27*d^3 - 100*a^3*b^27*c^29*d + 200*a^5*b^25*c^29*d - 200*a^7*b^23*c^29*d + 100*a^9*b^21*c^29*d - 20*a^11*b^19*c^29*d - 20*a^19*b^11*c*d^29 + 100*a^21*b^9*c*d^29 - 200*a^23*b^7*c*d^29 + 200*a^25*b^5*c*d^29 - 100*a^27*b^3*c*d^29 - 100*a^29*b*c^3*d^27 + 200*a^29*b*c^5*d^25 - 200*a^29*b*c^7*d^23 + 100*a^29*b*c^9*d^21 - 20*a^29*b*c^11*d^19 + 190*a^2*b^28*c^18*d^12 - 955*a^2*b^28*c^20*d^10 + 1925*a^2*b^28*c^22*d^8 - 1950*a^2*b^28*c^24*d^6 + 1000*a^2*b^28*c^26*d^4 - 215*a^2*b^28*c^28*d^2 - 1140*a^3*b^27*c^17*d^13 + 5800*a^3*b^27*c^19*d^11 - 11900*a^3*b^27*c^21*d^9 + 12400*a^3*b^27*c^23*d^7 - 6700*a^3*b^27*c^25*d^5 + 1640*a^3*b^27*c^27*d^3 + 4845*a^4*b^26*c^16*d^14 - 25175*a^4*b^26*c^18*d^12 + 53210*a^4*b^26*c^20*d^10 - 58000*a^4*b^26*c^22*d^8 + 33825*a^4*b^26*c^24*d^6 - 9695*a^4*b^26*c^26*d^4 + 1000*a^4*b^26*c^28*d^2 - 15504*a^5*b^25*c^15*d^15 + 83220*a^5*b^25*c^17*d^13 - 183740*a^5*b^25*c^19*d^11 + 213040*a^5*b^25*c^21*d^9 - 136520*a^5*b^25*c^23*d^7 + 46004*a^5*b^25*c^25*d^5 - 6700*a^5*b^25*c^27*d^3 + 38760*a^6*b^24*c^14*d^16 - 218025*a^6*b^24*c^16*d^14 + 510625*a^6*b^24*c^18*d^12 - 639360*a^6*b^24*c^20*d^10 + 455100*a^6*b^24*c^22*d^8 - 178985*a^6*b^24*c^24*d^6 + 33825*a^6*b^24*c^26*d^4 - 1950*a^6*b^24*c^28*d^2 - 77520*a^7*b^23*c^13*d^17 + 465120*a^7*b^23*c^15*d^15 - 1174200*a^7*b^23*c^17*d^13 + 1607600*a^7*b^23*c^19*d^11 - 1277800*a^7*b^23*c^21*d^9 + 581120*a^7*b^23*c^23*d^7 - 136520*a^7*b^23*c^25*d^5 + 12400*a^7*b^23*c^27*d^3 + 125970*a^8*b^22*c^12*d^18 - 823650*a^8*b^22*c^14*d^16 + 2277150*a^8*b^22*c^16*d^14 - 3441850*a^8*b^22*c^18*d^12 + 3061855*a^8*b^22*c^20*d^10 - 1598495*a^8*b^22*c^22*d^8 + 455100*a^8*b^22*c^24*d^6 - 58000*a^8*b^22*c^26*d^4 + 1925*a^8*b^22*c^28*d^2 - 167960*a^9*b^21*c^11*d^19 + 1227400*a^9*b^21*c^13*d^17 - 3772640*a^9*b^21*c^15*d^15 + 6342200*a^9*b^21*c^17*d^13 - 6323300*a^9*b^21*c^19*d^11 + 3770860*a^9*b^21*c^21*d^9 - 1277800*a^9*b^21*c^23*d^7 + 213040*a^9*b^21*c^25*d^5 - 11900*a^9*b^21*c^27*d^3 + 184756*a^10*b^20*c^10*d^20 - 1553630*a^10*b^20*c^12*d^18 + 5384410*a^10*b^20*c^14*d^16 - 10132510*a^10*b^20*c^16*d^14 + 11341480*a^10*b^20*c^18*d^12 - 7699257*a^10*b^20*c^20*d^10 + 3061855*a^10*b^20*c^22*d^8 - 639360*a^10*b^20*c^24*d^6 + 53210*a^10*b^20*c^26*d^4 - 955*a^10*b^20*c^28*d^2 - 167960*a^11*b^19*c^9*d^21 + 1679600*a^11*b^19*c^11*d^19 - 6653800*a^11*b^19*c^13*d^17 + 14108640*a^11*b^19*c^15*d^15 - 17770700*a^11*b^19*c^17*d^13 + 13697880*a^11*b^19*c^19*d^11 - 6323300*a^11*b^19*c^21*d^9 + 1607600*a^11*b^19*c^23*d^7 - 183740*a^11*b^19*c^25*d^5 + 5800*a^11*b^19*c^27*d^3 + 125970*a^12*b^18*c^8*d^22 - 1553630*a^12*b^18*c^10*d^20 + 7138300*a^12*b^18*c^12*d^18 - 17183600*a^12*b^18*c^14*d^16 + 24426875*a^12*b^18*c^16*d^14 - 21339185*a^12*b^18*c^18*d^12 + 11341480*a^12*b^18*c^20*d^10 - 3441850*a^12*b^18*c^22*d^8 + 510625*a^12*b^18*c^24*d^6 - 25175*a^12*b^18*c^26*d^4 + 190*a^12*b^18*c^28*d^2 - 77520*a^13*b^17*c^7*d^23 + 1227400*a^13*b^17*c^9*d^21 - 6653800*a^13*b^17*c^11*d^19 + 18346400*a^13*b^17*c^13*d^17 - 29535120*a^13*b^17*c^15*d^15 + 29213260*a^13*b^17*c^17*d^13 - 17770700*a^13*b^17*c^19*d^11 + 6342200*a^13*b^

17*c^21*d^9 - 1174200*a^13*b^17*c^23*d^7 + 83220*a^13*b^17*c^25*d^5 - 1140*a^13*b^17*c^27*d^3 + 38760*a^14*b^16*c^6*d^24 - 823650*a^14*b^16*c^8*d^22 + 5384410*a^14*b^16*c^10*d^20 - 17183600*a^14*b^16*c^12*d^18 + 31460200*a^14*b^16*c^14*d^16 - 35234455*a^14*b^16*c^16*d^14 + 24426875*a^14*b^16*c^18*d^12 - 10132510*a^14*b^16*c^20*d^10 + 2277150*a^14*b^16*c^22*d^8 - 218025*a^14*b^16*c^24*d^6 + 4845*a^14*b^16*c^26*d^4 - 15504*a^15*b^15*c^5*d^25 + 465120*a^15*b^15*c^7*d^23 - 3772640*a^15*b^15*c^9*d^21 + 14108640*a^15*b^15*c^11*d^19 - 29535120*a^15*b^15*c^13*d^17 + 37499008*a^15*b^15*c^15*d^15 - 29535120*a^15*b^15*c^17*d^13 + 14108640*a^15*b^15*c^19*d^11 - 3772640*a^15*b^15*c^21*d^9 + 465120*a^15*b^15*c^23*d^7 - 15504*a^15*b^15*c^25*d^5 + 4845*a^16*b^14*c^4*d^26 - 218025*a^16*b^14*c^6*d^24 + 2277150*a^16*b^14*c^8*d^22 - 10132510*a^16*b^14*c^10*d^20 + 24426875*a^16*b^14*c^12*d^18 - 35234455*a^16*b^14*c^14*d^16 + 31460200*a^16*b^14*c^16*d^14 - 17183600*a^16*b^14*c^18*d^12 + 5384410*a^16*b^14*c^20*d^10 - 823650*a^16*b^14*c^22*d^8 + 38760*a^16*b^14*c^24*d^6 - 1140*a^17*b^13*c^3*d^27 + 83220*a^17*b^13*c^5*d^25 - 1174200*a^17*b^13*c^7*d^23 + 6342200*a^17*b^13*c^9*d^21 - 17770700*a^17*b^13*c^11*d^19 + 29213260*a^17*b^13*c^13*d^17 - 29535120*a^17*b^13*c^15*d^15 + 18346400*a^17*b^13*c^17*d^13 - 6653800*a^17*b^13*c^19*d^11 + 1227400*a^17*b^13*c^21*d^9 - 77520*a^17*b^13*c^23*d^7 + 190*a^18*b^12*c^2*d^28 - 25175*a^18*b^12*c^4*d^26 + 510625*a^18*b^12*c^6*d^24 - 3441850*a^18*b^12*c^8*d^22 + 11341480*a^18*b^12*c^10*d^20 - 21339185*a^18*b^12*c^12*d^18 + 24426875*a^18*b^12*c^14*d^16 - 17183600*a^18*b^12*c^16*d^14 + 7138300*a^18*b^12*c^18*d^12 - 1553630*a^18*b^12*c^20*d^10 + 125970*a^18*b^12*c^22*d^8 + 5800*a^19*b^11*c^3*d^27 - 183740*a^19*b^11*c^5*d^25 + 1607600*a^19*b^11*c^7*d^23 - 6323300*a^19*b^11*c^9*d^21 + 13697880*a^19*b^11*c^11*d^19 - 17770700*a^19*b^11*c^13*d^17 + 14108640*a^19*b^11*c^15*d^15 - 6653800*a^19*b^11*c^17*d^13 + 1679600*a^19*b^11*c^19*d^11 - 167960*a^19*b^11*c^21*d^9 - 955*a^20*b^10*c^2*d^28 + 53210*a^20*b^10*c^4*d^26 - 639360*a^20*b^10*c^6*d^24 + 3061855*a^20*b^10*c^8*d^22 - 7699257*a^20*b^10*c^10*d^20 + 11341480*a^20*b^10*c^12*d^18 - 10132510*a^20*b^10*c^14*d^16 + 5384410*a^20*b^10*c^16*d^14 - 1553630*a^20*b^10*c^18*d^12 + 184756*a^20*b^10*c^20*d^10 - 11900*a^21*b^9*c^3*d^27 + 213040*a^21*b^9*c^5*d^25 - 1277800*a^21*b^9*c^7*d^23 + 3770860*a^21*b^9*c^9*d^21 - 6323300*a^21*b^9*c^11*d^19 + 6342200*a^21*b^9*c^13*d^17 - 3772640*a^21*b^9*c^15*d^15 + 1227400*a^21*b^9*c^17*d^13 - 167960*a^21*b^9*c^19*d^11 + 1925*a^22*b^8*c^2*d^28 - 58000*a^22*b^8*c^4*d^26 + 455100*a^22*b^8*c^6*d^24 - 1598495*a^22*b^8*c^8*d^22 + 3061855*a^22*b^8*c^10*d^20 - 3441850*a^22*b^8*c^12*d^18 + 2277150*a^22*b^8*c^14*d^16 - 823650*a^22*b^8*c^16*d^14 + 125970*a^22*b^8*c^18*d^12 + 12400*a^23*b^7*c^3*d^27 - 136520*a^23*b^7*c^5*d^25 + 581120*a^23*b^7*c^7*d^23 - 1277800*a^23*b^7*c^9*d^21 + 1607600*a^23*b^7*c^11*d^19 - 1174200*a^23*b^7*c^13*d^17 + 465120*a^23*b^7*c^15*d^15 - 77520*a^23*b^7*c^17*d^13 - 1950*a^24*b^6*c^2*d^28 + 33825*a^24*b^6*c^4*d^26 - 178985*a^24*b^6*c^6*d^24 + 455100*a^24*b^6*c^8*d^22 - 639360*a^24*b^6*c^10*d^20 + 510625*a^24*b^6*c^12*d^18 - 218025*a^24*b^6*c^14*d^16 + 38760*a^24*b^6*c^16*d^14 - 6700*a^25*b^5*c^3*d^27 + 46004*a^25*b^5*c^5*d^25 - 136520*a^25*b^5*c^7*d^23 + 213040*a^25*b^5*c^9*d^21 - 183740*a^25*b^5*c^11*d^19 + 83220*a^25*b^

5*c^13*d^17 - 15504*a^25*b^5*c^15*d^15 + 1000*a^26*b^4*c^2*d^28 - 9695*a^26
*b^4*c^4*d^26 + 33825*a^26*b^4*c^6*d^24 - 58000*a^26*b^4*c^8*d^22 + 53210*a
^26*b^4*c^10*d^20 - 25175*a^26*b^4*c^12*d^18 + 4845*a^26*b^4*c^14*d^16 + 16
40*a^27*b^3*c^3*d^27 - 6700*a^27*b^3*c^5*d^25 + 12400*a^27*b^3*c^7*d^23 - 1
1900*a^27*b^3*c^9*d^21 + 5800*a^27*b^3*c^11*d^19 - 1140*a^27*b^3*c^13*d^17
- 215*a^28*b^2*c^2*d^28 + 1000*a^28*b^2*c^4*d^26 - 1950*a^28*b^2*c^6*d^24 +
1925*a^28*b^2*c^8*d^22 - 955*a^28*b^2*c^10*d^20 + 190*a^28*b^2*c^12*d^18 +
20*a*b^29*c^29*d + 20*a^29*b*c*d^29)))^(1/2)*((-(((4*a^24*d^24 + 4*b^24*c^
24 + 16*a^2*b^22*c^24 + 16*a^4*b^20*c^24 - 1152*a^10*b^14*d^24 + 5568*a^12*
b^12*d^24 - 10568*a^14*b^10*d^24 + 9460*a^16*b^8*d^24 - 3560*a^18*b^6*d^24
+ 136*a^20*b^4*d^24 + 76*a^22*b^2*d^24 + 16*a^24*c^2*d^22 + 16*a^24*c^4*d^2
0 - 1152*b^24*c^10*d^14 + 5568*b^24*c^12*d^12 - 10568*b^24*c^14*d^10 + 9460
*b^24*c^16*d^8 - 3560*b^24*c^18*d^6 + 136*b^24*c^20*d^4 + 76*b^24*c^22*d^2
+ 11520*a*b^23*c^9*d^15 - 56448*a*b^23*c^11*d^13 + 109456*a*b^23*c^13*d^11
- 101240*a*b^23*c^15*d^9 + 40720*a*b^23*c^17*d^7 - 2960*a*b^23*c^19*d^5 - 5
36*a*b^23*c^21*d^3 - 176*a^3*b^21*c^23*d - 320*a^5*b^19*c^23*d + 11520*a^9*
b^15*c*d^23 - 56448*a^11*b^13*c*d^23 + 109456*a^13*b^11*c*d^23 - 101240*a^1
5*b^9*c*d^23 + 40720*a^17*b^7*c*d^23 - 2960*a^19*b^5*c*d^23 - 536*a^21*b^3*
c*d^23 - 176*a^23*b*c^3*d^21 - 320*a^23*b*c^5*d^19 - 51840*a^2*b^22*c^8*d^1
6 + 263808*a^2*b^22*c^10*d^14 - 541208*a^2*b^22*c^12*d^12 + 547088*a^2*b^22
*c^14*d^10 - 263320*a^2*b^22*c^16*d^8 + 44120*a^2*b^22*c^18*d^6 - 1564*a^2*
b^22*c^20*d^4 - 196*a^2*b^22*c^22*d^2 + 138240*a^3*b^21*c^7*d^17 - 758400*a
^3*b^21*c^9*d^15 + 1720736*a^3*b^21*c^11*d^13 - 2002728*a^3*b^21*c^13*d^11
+ 1210560*a^3*b^21*c^15*d^9 - 335040*a^3*b^21*c^17*d^7 + 37680*a^3*b^21*c^1
9*d^5 - 288*a^3*b^21*c^21*d^3 - 241920*a^4*b^20*c^6*d^18 + 1512000*a^4*b^20
*c^8*d^16 - 3975688*a^4*b^20*c^10*d^14 + 5501328*a^4*b^20*c^12*d^12 - 41479
52*a^4*b^20*c^14*d^10 + 1586920*a^4*b^20*c^16*d^8 - 276020*a^4*b^20*c^18*d^
6 + 21124*a^4*b^20*c^20*d^4 + 176*a^4*b^20*c^22*d^2 + 290304*a^5*b^19*c^5*d
^19 - 2232576*a^5*b^19*c^7*d^17 + 7078256*a^5*b^19*c^9*d^15 - 11781560*a^5*
b^19*c^11*d^13 + 10875200*a^5*b^19*c^13*d^11 - 5365072*a^5*b^19*c^15*d^9 +
1310168*a^5*b^19*c^17*d^7 - 170968*a^5*b^19*c^19*d^5 + 8160*a^5*b^19*c^21*d
^3 - 241920*a^6*b^18*c^4*d^20 + 2532096*a^6*b^18*c^6*d^18 - 9955992*a^6*b^1
8*c^8*d^16 + 20019440*a^6*b^18*c^10*d^14 - 22419600*a^6*b^18*c^12*d^12 + 13
887520*a^6*b^18*c^14*d^10 - 4506428*a^6*b^18*c^16*d^8 + 793756*a^6*b^18*c^1
8*d^6 - 72240*a^6*b^18*c^20*d^4 + 3040*a^6*b^18*c^22*d^2 + 138240*a^7*b^17*
c^3*d^21 - 2232576*a^7*b^17*c^5*d^19 + 11150016*a^7*b^17*c^7*d^17 - 2733661
6*a^7*b^17*c^9*d^15 + 37153600*a^7*b^17*c^11*d^13 - 28461040*a^7*b^17*c^13*
d^11 + 11779808*a^7*b^17*c^15*d^9 - 2621008*a^7*b^17*c^17*d^7 + 336688*a^7*
b^17*c^19*d^5 - 17920*a^7*b^17*c^21*d^3 - 51840*a^8*b^16*c^2*d^22 + 1512000
*a^8*b^16*c^4*d^20 - 9955992*a^8*b^16*c^6*d^18 + 30289656*a^8*b^16*c^8*d^16
- 50137600*a^8*b^16*c^10*d^14 + 46972560*a^8*b^16*c^12*d^12 - 24199280*a^8
*b^16*c^14*d^10 + 6661036*a^8*b^16*c^16*d^8 - 1058448*a^8*b^16*c^18*d^6 + 7
2560*a^8*b^16*c^20*d^4 - 758400*a^9*b^15*c^3*d^21 + 7078256*a^9*b^15*c^5*d^
19 - 27336616*a^9*b^15*c^7*d^17 + 55383904*a^9*b^15*c^9*d^15 - 63124080*a^9
*b^15*c^11*d^13 + 39987520*a^9*b^15*c^13*d^11 - 13462088*a^9*b^15*c^15*d^9

+ 2478528*a^9*b^15*c^17*d^7 - 212032*a^9*b^15*c^19*d^5 + 263808*a^10*b^14*c
^2*d^22 - 3975688*a^10*b^14*c^4*d^20 + 20019440*a^10*b^14*c^6*d^18 - 501376
00*a^10*b^14*c^8*d^16 + 69593872*a^10*b^14*c^10*d^14 - 53854288*a^10*b^14*c
^12*d^12 + 21989928*a^10*b^14*c^14*d^10 - 4591360*a^10*b^14*c^16*d^8 + 4604
80*a^10*b^14*c^18*d^6 + 1720736*a^11*b^13*c^3*d^21 - 11781560*a^11*b^13*c^5
*d^19 + 37153600*a^11*b^13*c^7*d^17 - 63124080*a^11*b^13*c^9*d^15 + 5944572
8*a^11*b^13*c^11*d^13 - 29358696*a^11*b^13*c^13*d^11 + 6995840*a^11*b^13*c^
15*d^9 - 762560*a^11*b^13*c^17*d^7 - 541208*a^12*b^12*c^2*d^22 + 5501328*a^
12*b^12*c^4*d^20 - 22419600*a^12*b^12*c^6*d^18 + 46972560*a^12*b^12*c^8*d^1
6 - 53854288*a^12*b^12*c^10*d^14 + 32294808*a^12*b^12*c^12*d^12 - 8958208*a
^12*b^12*c^14*d^10 + 999040*a^12*b^12*c^16*d^8 - 2002728*a^13*b^11*c^3*d^21
 + 10875200*a^13*b^11*c^5*d^19 - 28461040*a^13*b^11*c^7*d^17 + 39987520*a^1
3*b^11*c^9*d^15 - 29358696*a^13*b^11*c^11*d^13 + 9722048*a^13*b^11*c^13*d^1
1 - 1104320*a^13*b^11*c^15*d^9 + 547088*a^14*b^10*c^2*d^22 - 4147952*a^14*b
^10*c^4*d^20 + 13887520*a^14*b^10*c^6*d^18 - 24199280*a^14*b^10*c^8*d^16 +
21989928*a^14*b^10*c^10*d^14 - 8958208*a^14*b^10*c^12*d^12 + 1124032*a^14*b
^10*c^14*d^10 + 1210560*a^15*b^9*c^3*d^21 - 5365072*a^15*b^9*c^5*d^19 + 117
79808*a^15*b^9*c^7*d^17 - 13462088*a^15*b^9*c^9*d^15 + 6995840*a^15*b^9*c^1
1*d^13 - 1104320*a^15*b^9*c^13*d^11 - 263320*a^16*b^8*c^2*d^22 + 1586920*a^
16*b^8*c^4*d^20 - 4506428*a^16*b^8*c^6*d^18 + 6661036*a^16*b^8*c^8*d^16 - 4
591360*a^16*b^8*c^10*d^14 + 999040*a^16*b^8*c^12*d^12 - 335040*a^17*b^7*c^3
*d^21 + 1310168*a^17*b^7*c^5*d^19 - 2621008*a^17*b^7*c^7*d^17 + 2478528*a^1
7*b^7*c^9*d^15 - 762560*a^17*b^7*c^11*d^13 + 44120*a^18*b^6*c^2*d^22 - 2760
20*a^18*b^6*c^4*d^20 + 793756*a^18*b^6*c^6*d^18 - 1058448*a^18*b^6*c^8*d^16
 + 460480*a^18*b^6*c^10*d^14 + 37680*a^19*b^5*c^3*d^21 - 170968*a^19*b^5*c^
5*d^19 + 336688*a^19*b^5*c^7*d^17 - 212032*a^19*b^5*c^9*d^15 - 1564*a^20*b^
4*c^2*d^22 + 21124*a^20*b^4*c^4*d^20 - 72240*a^20*b^4*c^6*d^18 + 72560*a^20
*b^4*c^8*d^16 - 288*a^21*b^3*c^3*d^21 + 8160*a^21*b^3*c^5*d^19 - 17920*a^21
*b^3*c^7*d^17 - 196*a^22*b^2*c^2*d^22 + 176*a^22*b^2*c^4*d^20 + 3040*a^22*b
^2*c^6*d^18 - 8*a*b^23*c^23*d - 8*a^23*b*c*d^23)^2/4 - (20736*b^18*d^18 - 9
6768*a^2*b^16*d^18 + 173664*a^4*b^14*d^18 - 136032*a^6*b^12*d^18 + 31081*a^
8*b^10*d^18 + 8440*a^10*b^8*d^18 + 400*a^12*b^6*d^18 - 96768*b^18*c^2*d^16
+ 173664*b^18*c^4*d^14 - 136032*b^18*c^6*d^12 + 31081*b^18*c^8*d^10 + 8440*
b^18*c^10*d^8 + 400*b^18*c^12*d^6 - 131328*a*b^17*c^3*d^15 + 216576*a*b^17*
c^5*d^13 - 141104*a*b^17*c^7*d^11 + 20260*a*b^17*c^9*d^9 + 2800*a*b^17*c^11
*d^7 - 131328*a^3*b^15*c*d^17 + 216576*a^5*b^13*c*d^17 - 141104*a^7*b^11*c*
d^17 + 20260*a^9*b^9*c*d^17 + 2800*a^11*b^7*c*d^17 + 495936*a^2*b^16*c^2*d^
16 - 989856*a^2*b^16*c^4*d^14 + 901948*a^2*b^16*c^6*d^12 - 308392*a^2*b^16*
c^8*d^10 - 5260*a^2*b^16*c^10*d^8 + 1600*a^2*b^16*c^12*d^6 + 657408*a^3*b^1
5*c^3*d^15 - 1158992*a^3*b^15*c^5*d^13 + 838256*a^3*b^15*c^7*d^11 - 182200*
a^3*b^15*c^9*d^9 - 3200*a^3*b^15*c^11*d^7 - 989856*a^4*b^14*c^2*d^16 + 2185
654*a^4*b^14*c^4*d^14 - 2218576*a^4*b^14*c^6*d^12 + 900624*a^4*b^14*c^8*d^1
0 - 64720*a^4*b^14*c^10*d^8 + 1600*a^4*b^14*c^12*d^6 - 1158992*a^5*b^13*c^3
*d^15 + 2158808*a^5*b^13*c^5*d^13 - 1641528*a^5*b^13*c^7*d^11 + 406880*a^5*
b^13*c^9*d^9 - 17600*a^5*b^13*c^11*d^7 + 901948*a^6*b^12*c^2*d^16 - 2218576

*a^6*b^12*c^4*d^14 + 2430936*a^6*b^12*c^6*d^12 - 1026928*a^6*b^12*c^8*d^10 + 88720*a^6*b^12*c^10*d^8 + 838256*a^7*b^11*c^3*d^15 - 1641528*a^7*b^11*c^5*d^13 + 1206848*a^7*b^11*c^7*d^11 - 239360*a^7*b^11*c^9*d^9 - 308392*a^8*b^10*c^2*d^16 + 900624*a^8*b^10*c^4*d^14 - 1026928*a^8*b^10*c^6*d^12 + 354016*a^8*b^10*c^8*d^10 - 182200*a^9*b^9*c^3*d^15 + 406880*a^9*b^9*c^5*d^13 - 239360*a^9*b^9*c^7*d^11 - 5260*a^10*b^8*c^2*d^16 - 64720*a^10*b^8*c^4*d^14 + 88720*a^10*b^8*c^6*d^12 - 3200*a^11*b^7*c^3*d^15 - 17600*a^11*b^7*c^5*d^13 + 1600*a^12*b^6*c^2*d^16 + 1600*a^12*b^6*c^4*d^14 + 27648*a*b^17*c*d^17)*(80*a^2*b^28*c^30 - 16*b^30*c^30 - 16*a^30*d^30 - 160*a^4*b^26*c^30 + 160*a^6*b^24*c^30 - 80*a^8*b^22*c^30 + 16*a^10*b^20*c^30 + 16*a^20*b^10*d^30 - 80*a^22*b^8*d^30 + 160*a^24*b^6*d^30 - 160*a^26*b^4*d^30 + 80*a^28*b^2*d^30 + 80*a^30*c^2*d^28 - 160*a^30*c^4*d^26 + 160*a^30*c^6*d^24 - 80*a^30*c^8*d^22 + 16*a^30*c^10*d^20 + 16*b^30*c^20*d^10 - 80*b^30*c^22*d^8 + 160*b^30*c^24*d^6 - 160*b^30*c^26*d^4 + 80*b^30*c^28*d^2 - 320*a*b^29*c^19*d^11 + 1600*a*b^29*c^21*d^9 - 3200*a*b^29*c^23*d^7 + 3200*a*b^29*c^25*d^5 - 1600*a*b^29*c^27*d^3 - 1600*a^3*b^27*c^29*d + 3200*a^5*b^25*c^29*d - 3200*a^7*b^23*c^29*d + 1600*a^9*b^21*c^29*d - 320*a^11*b^19*c^29*d - 320*a^19*b^11*c*d^29 + 1600*a^21*b^9*c*d^29 - 3200*a^23*b^7*c*d^29 + 3200*a^25*b^5*c*d^29 - 1600*a^27*b^3*c*d^29 - 1600*a^29*b*c^3*d^27 + 3200*a^29*b*c^5*d^25 - 3200*a^29*b*c^7*d^23 + 1600*a^29*b*c^9*d^21 - 320*a^29*b*c^11*d^19 + 3040*a^2*b^28*c^18*d^12 - 15280*a^2*b^28*c^20*d^10 + 30800*a^2*b^28*c^22*d^8 - 31200*a^2*b^28*c^24*d^6 + 16000*a^2*b^28*c^26*d^4 - 3440*a^2*b^28*c^28*d^2 - 18240*a^3*b^27*c^17*d^13 + 92800*a^3*b^27*c^19*d^11 - 190400*a^3*b^27*c^21*d^9 + 198400*a^3*b^27*c^23*d^7 - 107200*a^3*b^27*c^25*d^5 + 26240*a^3*b^27*c^27*d^3 + 77520*a^4*b^26*c^16*d^14 - 402800*a^4*b^26*c^18*d^12 + 851360*a^4*b^26*c^20*d^10 - 928000*a^4*b^26*c^22*d^8 + 541200*a^4*b^26*c^24*d^6 - 155120*a^4*b^26*c^26*d^4 + 16000*a^4*b^26*c^28*d^2 - 248064*a^5*b^25*c^15*d^15 + 1331520*a^5*b^25*c^17*d^13 - 2939840*a^5*b^25*c^19*d^11 + 3408640*a^5*b^25*c^21*d^9 - 2184320*a^5*b^25*c^23*d^7 + 736064*a^5*b^25*c^25*d^5 - 107200*a^5*b^25*c^27*d^3 + 620160*a^6*b^24*c^14*d^16 - 3488400*a^6*b^24*c^16*d^14 + 8170000*a^6*b^24*c^18*d^12 - 10229760*a^6*b^24*c^20*d^10 + 7281600*a^6*b^24*c^22*d^8 - 2863760*a^6*b^24*c^24*d^6 + 541200*a^6*b^24*c^26*d^4 - 31200*a^6*b^24*c^28*d^2 - 1240320*a^7*b^23*c^13*d^17 + 7441920*a^7*b^23*c^15*d^15 - 18787200*a^7*b^23*c^17*d^13 + 25721600*a^7*b^23*c^19*d^11 - 20444800*a^7*b^23*c^21*d^9 + 9297920*a^7*b^23*c^23*d^7 - 2184320*a^7*b^23*c^25*d^5 + 198400*a^7*b^23*c^27*d^3 + 2015520*a^8*b^22*c^12*d^18 - 13178400*a^8*b^22*c^14*d^16 + 36434400*a^8*b^22*c^16*d^14 - 55069600*a^8*b^22*c^18*d^12 + 48989680*a^8*b^22*c^20*d^10 - 25575920*a^8*b^22*c^22*d^8 + 7281600*a^8*b^22*c^24*d^6 - 928000*a^8*b^22*c^26*d^4 + 30800*a^8*b^22*c^28*d^2 - 2687360*a^9*b^21*c^11*d^19 + 19638400*a^9*b^21*c^13*d^17 - 60362240*a^9*b^21*c^15*d^15 + 101475200*a^9*b^21*c^17*d^13 - 101172800*a^9*b^21*c^19*d^11 + 60333760*a^9*b^21*c^21*d^9 - 20444800*a^9*b^21*c^23*d^7 + 3408640*a^9*b^21*c^25*d^5 - 190400*a^9*b^21*c^27*d^3 + 2956096*a^10*b^20*c^10*d^20 - 24858080*a^10*b^20*c^12*d^18 + 86150560*a^10*b^20*c^14*d^16 - 162120160*a^10*b^20*c^16*d^14 + 181463680*a^10*b^20*c^18*d^12 - 123188112*a^10*b^20*c^20*d^10 + 48989680*a^10*b^20*c^22*d

^8 - 10229760*a^10*b^20*c^24*d^6 + 851360*a^10*b^20*c^26*d^4 - 15280*a^10*b
^20*c^28*d^2 - 2687360*a^11*b^19*c^9*d^21 + 26873600*a^11*b^19*c^11*d^19 -
106460800*a^11*b^19*c^13*d^17 + 225738240*a^11*b^19*c^15*d^15 - 284331200*a
^11*b^19*c^17*d^13 + 219166080*a^11*b^19*c^19*d^11 - 101172800*a^11*b^19*c^
21*d^9 + 25721600*a^11*b^19*c^23*d^7 - 2939840*a^11*b^19*c^25*d^5 + 92800*a
^11*b^19*c^27*d^3 + 2015520*a^12*b^18*c^8*d^22 - 24858080*a^12*b^18*c^10*d^
20 + 114212800*a^12*b^18*c^12*d^18 - 274937600*a^12*b^18*c^14*d^16 + 390830
000*a^12*b^18*c^16*d^14 - 341426960*a^12*b^18*c^18*d^12 + 181463680*a^12*b^
18*c^20*d^10 - 55069600*a^12*b^18*c^22*d^8 + 8170000*a^12*b^18*c^24*d^6 - 4
02800*a^12*b^18*c^26*d^4 + 3040*a^12*b^18*c^28*d^2 - 1240320*a^13*b^17*c^7*
d^23 + 19638400*a^13*b^17*c^9*d^21 - 106460800*a^13*b^17*c^11*d^19 + 293542
400*a^13*b^17*c^13*d^17 - 472561920*a^13*b^17*c^15*d^15 + 467412160*a^13*b^
17*c^17*d^13 - 284331200*a^13*b^17*c^19*d^11 + 101475200*a^13*b^17*c^21*d^9
 - 18787200*a^13*b^17*c^23*d^7 + 1331520*a^13*b^17*c^25*d^5 - 18240*a^13*b^
17*c^27*d^3 + 620160*a^14*b^16*c^6*d^24 - 13178400*a^14*b^16*c^8*d^22 + 861
50560*a^14*b^16*c^10*d^20 - 274937600*a^14*b^16*c^12*d^18 + 503363200*a^14*
b^16*c^14*d^16 - 563751280*a^14*b^16*c^16*d^14 + 390830000*a^14*b^16*c^18*d
^12 - 162120160*a^14*b^16*c^20*d^10 + 36434400*a^14*b^16*c^22*d^8 - 3488400
*a^14*b^16*c^24*d^6 + 77520*a^14*b^16*c^26*d^4 - 248064*a^15*b^15*c^5*d^25
+ 7441920*a^15*b^15*c^7*d^23 - 60362240*a^15*b^15*c^9*d^21 + 225738240*a^15
*b^15*c^11*d^19 - 472561920*a^15*b^15*c^13*d^17 + 599984128*a^15*b^15*c^15*
d^15 - 472561920*a^15*b^15*c^17*d^13 + 225738240*a^15*b^15*c^19*d^11 - 6036
2240*a^15*b^15*c^21*d^9 + 7441920*a^15*b^15*c^23*d^7 - 248064*a^15*b^15*c^2
5*d^5 + 77520*a^16*b^14*c^4*d^26 - 3488400*a^16*b^14*c^6*d^24 + 36434400*a^
16*b^14*c^8*d^22 - 162120160*a^16*b^14*c^10*d^20 + 390830000*a^16*b^14*c^12
*d^18 - 563751280*a^16*b^14*c^14*d^16 + 503363200*a^16*b^14*c^16*d^14 - 274
937600*a^16*b^14*c^18*d^12 + 86150560*a^16*b^14*c^20*d^10 - 13178400*a^16*b
^14*c^22*d^8 + 620160*a^16*b^14*c^24*d^6 - 18240*a^17*b^13*c^3*d^27 + 13315
20*a^17*b^13*c^5*d^25 - 18787200*a^17*b^13*c^7*d^23 + 101475200*a^17*b^13*c
^9*d^21 - 284331200*a^17*b^13*c^11*d^19 + 467412160*a^17*b^13*c^13*d^17 - 4
72561920*a^17*b^13*c^15*d^15 + 293542400*a^17*b^13*c^17*d^13 - 106460800*a^
17*b^13*c^19*d^11 + 19638400*a^17*b^13*c^21*d^9 - 1240320*a^17*b^13*c^23*d^
7 + 3040*a^18*b^12*c^2*d^28 - 402800*a^18*b^12*c^4*d^26 + 8170000*a^18*b^12
*c^6*d^24 - 55069600*a^18*b^12*c^8*d^22 + 181463680*a^18*b^12*c^10*d^20 - 3
41426960*a^18*b^12*c^12*d^18 + 390830000*a^18*b^12*c^14*d^16 - 274937600*a^
18*b^12*c^16*d^14 + 114212800*a^18*b^12*c^18*d^12 - 24858080*a^18*b^12*c^20
*d^10 + 2015520*a^18*b^12*c^22*d^8 + 92800*a^19*b^11*c^3*d^27 - 2939840*a^1
9*b^11*c^5*d^25 + 25721600*a^19*b^11*c^7*d^23 - 101172800*a^19*b^11*c^9*d^2
1 + 219166080*a^19*b^11*c^11*d^19 - 284331200*a^19*b^11*c^13*d^17 + 2257382
40*a^19*b^11*c^15*d^15 - 106460800*a^19*b^11*c^17*d^13 + 26873600*a^19*b^11
*c^19*d^11 - 2687360*a^19*b^11*c^21*d^9 - 15280*a^20*b^10*c^2*d^28 + 851360
*a^20*b^10*c^4*d^26 - 10229760*a^20*b^10*c^6*d^24 + 48989680*a^20*b^10*c^8*
d^22 - 123188112*a^20*b^10*c^10*d^20 + 181463680*a^20*b^10*c^12*d^18 - 1621
20160*a^20*b^10*c^14*d^16 + 86150560*a^20*b^10*c^16*d^14 - 24858080*a^20*b^
10*c^18*d^12 + 2956096*a^20*b^10*c^20*d^10 - 190400*a^21*b^9*c^3*d^27 + 340

8640*a^21*b^9*c^5*d^25 - 20444800*a^21*b^9*c^7*d^23 + 60333760*a^21*b^9*c^9*d^21 - 101172800*a^21*b^9*c^11*d^19 + 101475200*a^21*b^9*c^13*d^17 - 60362240*a^21*b^9*c^15*d^15 + 19638400*a^21*b^9*c^17*d^13 - 2687360*a^21*b^9*c^19*d^11 + 30800*a^22*b^8*c^2*d^28 - 928000*a^22*b^8*c^4*d^26 + 7281600*a^22*b^8*c^6*d^24 - 25575920*a^22*b^8*c^8*d^22 + 48989680*a^22*b^8*c^10*d^20 - 55069600*a^22*b^8*c^12*d^18 + 36434400*a^22*b^8*c^14*d^16 - 13178400*a^22*b^8*c^16*d^14 + 2015520*a^22*b^8*c^18*d^12 + 198400*a^23*b^7*c^3*d^27 - 2184320*a^23*b^7*c^5*d^25 + 9297920*a^23*b^7*c^7*d^23 - 20444800*a^23*b^7*c^9*d^21 + 25721600*a^23*b^7*c^11*d^19 - 18787200*a^23*b^7*c^13*d^17 + 7441920*a^23*b^7*c^15*d^15 - 1240320*a^23*b^7*c^17*d^13 - 31200*a^24*b^6*c^2*d^28 + 541200*a^24*b^6*c^4*d^26 - 2863760*a^24*b^6*c^6*d^24 + 7281600*a^24*b^6*c^8*d^22 - 10229760*a^24*b^6*c^10*d^20 + 8170000*a^24*b^6*c^12*d^18 - 3488400*a^24*b^6*c^14*d^16 + 620160*a^24*b^6*c^16*d^14 - 107200*a^25*b^5*c^3*d^27 + 736064*a^25*b^5*c^5*d^25 - 2184320*a^25*b^5*c^7*d^23 + 3408640*a^25*b^5*c^9*d^21 - 2939840*a^25*b^5*c^11*d^19 + 1331520*a^25*b^5*c^13*d^17 - 248064*a^25*b^5*c^15*d^15 + 16000*a^26*b^4*c^2*d^28 - 155120*a^26*b^4*c^4*d^26 + 541200*a^26*b^4*c^6*d^24 - 928000*a^26*b^4*c^8*d^22 + 851360*a^26*b^4*c^10*d^20 - 402800*a^26*b^4*c^12*d^18 + 77520*a^26*b^4*c^14*d^16 + 26240*a^27*b^3*c^3*d^27 - 107200*a^27*b^3*c^5*d^25 + 198400*a^27*b^3*c^7*d^23 - 190400*a^27*b^3*c^9*d^21 + 92800*a^27*b^3*c^11*d^19 - 18240*a^27*b^3*c^13*d^17 - 3440*a^28*b^2*c^2*d^28 + 16000*a^28*b^2*c^4*d^26 - 31200*a^28*b^2*c^6*d^24 + 30800*a^28*b^2*c^8*d^22 - 15280*a^28*b^2*c^10*d^20 + 3040*a^28*b^2*c^12*d^18 + 320*a*b^29*c^29*d + 320*a^29*b*c*d^29))^(1/2) + 2*a^24*d^24 + 2*b^24*c^24 + 8*a^2*b^22*c^24 + 8*a^4*b^20*c^24 - 576*a^10*b^14*d^24 + 2784*a^12*b^12*d^24 - 5284*a^14*b^10*d^24 + 4730*a^16*b^8*d^24 - 1780*a^18*b^6*d^24 + 68*a^20*b^4*d^24 + 38*a^22*b^2*d^24 + 8*a^24*c^2*d^22 + 8*a^24*c^4*d^20 - 576*b^24*c^10*d^14 + 2784*b^24*c^12*d^12 - 5284*b^24*c^14*d^10 + 4730*b^24*c^16*d^8 - 1780*b^24*c^18*d^6 + 68*b^24*c^20*d^4 + 38*b^24*c^22*d^2 + 5760*a*b^23*c^9*d^15 - 28224*a*b^23*c^11*d^13 + 54728*a*b^23*c^13*d^11 - 50620*a*b^23*c^15*d^9 + 20360*a*b^23*c^17*d^7 - 1480*a*b^23*c^19*d^5 - 268*a*b^23*c^21*d^3 - 88*a^3*b^21*c^23*d - 160*a^5*b^19*c^23*d + 5760*a^9*b^15*c*d^23 - 28224*a^11*b^13*c*d^23 + 54728*a^13*b^11*c*d^23 - 50620*a^15*b^9*c*d^23 + 20360*a^17*b^7*c*d^23 - 1480*a^19*b^5*c*d^23 - 268*a^21*b^3*c*d^23 - 88*a^23*b*c^3*d^21 - 160*a^23*b*c^5*d^19 - 25920*a^2*b^22*c^8*d^16 + 131904*a^2*b^22*c^10*d^14 - 270604*a^2*b^22*c^12*d^12 + 273544*a^2*b^22*c^14*d^10 - 131660*a^2*b^22*c^16*d^8 + 22060*a^2*b^22*c^18*d^6 - 782*a^2*b^22*c^20*d^4 - 98*a^2*b^22*c^22*d^2 + 69120*a^3*b^21*c^7*d^17 - 379200*a^3*b^21*c^9*d^15 + 860368*a^3*b^21*c^11*d^13 - 1001364*a^3*b^21*c^13*d^11 + 605280*a^3*b^21*c^15*d^9 - 167520*a^3*b^21*c^17*d^7 + 18840*a^3*b^21*c^19*d^5 - 144*a^3*b^21*c^21*d^3 - 120960*a^4*b^20*c^6*d^18 + 756000*a^4*b^20*c^8*d^16 - 1987844*a^4*b^20*c^10*d^14 + 2750664*a^4*b^20*c^12*d^12 - 2073976*a^4*b^20*c^14*d^10 + 793460*a^4*b^20*c^16*d^8 - 138010*a^4*b^20*c^18*d^6 + 10562*a^4*b^20*c^20*d^4 + 88*a^4*b^20*c^22*d^2 + 145152*a^5*b^19*c^5*d^19 - 1116288*a^5*b^19*c^7*d^17 + 3539128*a^5*b^19*c^9*d^15 - 5890780*a^5*b^19*c^11*d^13 + 5437600*a^5*b^19*c^13*d^11 - 2682536*a^5*b^19*c^15*d^9 + 655084*a^5*b^19*c^17*d^7 - 8548

4*a^5*b^19*c^19*d^5 + 4080*a^5*b^19*c^21*d^3 - 120960*a^6*b^18*c^4*d^20 + 1
266048*a^6*b^18*c^6*d^18 - 4977996*a^6*b^18*c^8*d^16 + 10009720*a^6*b^18*c^
10*d^14 - 11209800*a^6*b^18*c^12*d^12 + 6943760*a^6*b^18*c^14*d^10 - 225321
4*a^6*b^18*c^16*d^8 + 396878*a^6*b^18*c^18*d^6 - 36120*a^6*b^18*c^20*d^4 +
1520*a^6*b^18*c^22*d^2 + 69120*a^7*b^17*c^3*d^21 - 1116288*a^7*b^17*c^5*d^1
9 + 5575008*a^7*b^17*c^7*d^17 - 13668308*a^7*b^17*c^9*d^15 + 18576800*a^7*b
^17*c^11*d^13 - 14230520*a^7*b^17*c^13*d^11 + 5889904*a^7*b^17*c^15*d^9 - 1
310504*a^7*b^17*c^17*d^7 + 168344*a^7*b^17*c^19*d^5 - 8960*a^7*b^17*c^21*d^
3 - 25920*a^8*b^16*c^2*d^22 + 756000*a^8*b^16*c^4*d^20 - 4977996*a^8*b^16*c
^6*d^18 + 15144828*a^8*b^16*c^8*d^16 - 25068800*a^8*b^16*c^10*d^14 + 234862
80*a^8*b^16*c^12*d^12 - 12099640*a^8*b^16*c^14*d^10 + 3330518*a^8*b^16*c^16
*d^8 - 529224*a^8*b^16*c^18*d^6 + 36280*a^8*b^16*c^20*d^4 - 379200*a^9*b^15
*c^3*d^21 + 3539128*a^9*b^15*c^5*d^19 - 13668308*a^9*b^15*c^7*d^17 + 276919
52*a^9*b^15*c^9*d^15 - 31562040*a^9*b^15*c^11*d^13 + 19993760*a^9*b^15*c^13
*d^11 - 6731044*a^9*b^15*c^15*d^9 + 1239264*a^9*b^15*c^17*d^7 - 106016*a^9*
b^15*c^19*d^5 + 131904*a^10*b^14*c^2*d^22 - 1987844*a^10*b^14*c^4*d^20 + 10
009720*a^10*b^14*c^6*d^18 - 25068800*a^10*b^14*c^8*d^16 + 34796936*a^10*b^1
4*c^10*d^14 - 26927144*a^10*b^14*c^12*d^12 + 10994964*a^10*b^14*c^14*d^10 -
 2295680*a^10*b^14*c^16*d^8 + 230240*a^10*b^14*c^18*d^6 + 860368*a^11*b^13*
c^3*d^21 - 5890780*a^11*b^13*c^5*d^19 + 18576800*a^11*b^13*c^7*d^17 - 31562
040*a^11*b^13*c^9*d^15 + 29722864*a^11*b^13*c^11*d^13 - 14679348*a^11*b^13*
c^13*d^11 + 3497920*a^11*b^13*c^15*d^9 - 381280*a^11*b^13*c^17*d^7 - 270604
*a^12*b^12*c^2*d^22 + 2750664*a^12*b^12*c^4*d^20 - 11209800*a^12*b^12*c^6*d
^18 + 23486280*a^12*b^12*c^8*d^16 - 26927144*a^12*b^12*c^10*d^14 + 16147404
*a^12*b^12*c^12*d^12 - 4479104*a^12*b^12*c^14*d^10 + 499520*a^12*b^12*c^16*
d^8 - 1001364*a^13*b^11*c^3*d^21 + 5437600*a^13*b^11*c^5*d^19 - 14230520*a^
13*b^11*c^7*d^17 + 19993760*a^13*b^11*c^9*d^15 - 14679348*a^13*b^11*c^11*d^
13 + 4861024*a^13*b^11*c^13*d^11 - 552160*a^13*b^11*c^15*d^9 + 273544*a^14*
b^10*c^2*d^22 - 2073976*a^14*b^10*c^4*d^20 + 6943760*a^14*b^10*c^6*d^18 - 1
2099640*a^14*b^10*c^8*d^16 + 10994964*a^14*b^10*c^10*d^14 - 4479104*a^14*b^
10*c^12*d^12 + 562016*a^14*b^10*c^14*d^10 + 605280*a^15*b^9*c^3*d^21 - 2682
536*a^15*b^9*c^5*d^19 + 5889904*a^15*b^9*c^7*d^17 - 6731044*a^15*b^9*c^9*d^
15 + 3497920*a^15*b^9*c^11*d^13 - 552160*a^15*b^9*c^13*d^11 - 131660*a^16*b
^8*c^2*d^22 + 793460*a^16*b^8*c^4*d^20 - 2253214*a^16*b^8*c^6*d^18 + 333051
8*a^16*b^8*c^8*d^16 - 2295680*a^16*b^8*c^10*d^14 + 499520*a^16*b^8*c^12*d^1
2 - 167520*a^17*b^7*c^3*d^21 + 655084*a^17*b^7*c^5*d^19 - 1310504*a^17*b^7*
c^7*d^17 + 1239264*a^17*b^7*c^9*d^15 - 381280*a^17*b^7*c^11*d^13 + 22060*a^
18*b^6*c^2*d^22 - 138010*a^18*b^6*c^4*d^20 + 396878*a^18*b^6*c^6*d^18 - 529
224*a^18*b^6*c^8*d^16 + 230240*a^18*b^6*c^10*d^14 + 18840*a^19*b^5*c^3*d^21
 - 85484*a^19*b^5*c^5*d^19 + 168344*a^19*b^5*c^7*d^17 - 106016*a^19*b^5*c^9
*d^15 - 782*a^20*b^4*c^2*d^22 + 10562*a^20*b^4*c^4*d^20 - 36120*a^20*b^4*c^
6*d^18 + 36280*a^20*b^4*c^8*d^16 - 144*a^21*b^3*c^3*d^21 + 4080*a^21*b^3*c^
5*d^19 - 8960*a^21*b^3*c^7*d^17 - 98*a^22*b^2*c^2*d^22 + 88*a^22*b^2*c^4*d^
20 + 1520*a^22*b^2*c^6*d^18 - 4*a*b^23*c^23*d - 4*a^23*b*c*d^23)/(16*(5*a^2
*b^28*c^30 - b^30*c^30 - a^30*d^30 - 10*a^4*b^26*c^30 + 10*a^6*b^24*c^30 -

5*a^8*b^22*c^30 + a^10*b^20*c^30 + a^20*b^10*d^30 - 5*a^22*b^8*d^30 + 10*a^24*b^6*d^30 - 10*a^26*b^4*d^30 + 5*a^28*b^2*d^30 + 5*a^30*c^2*d^28 - 10*a^30*c^4*d^26 + 10*a^30*c^6*d^24 - 5*a^30*c^8*d^22 + a^30*c^10*d^20 + b^30*c^20*d^10 - 5*b^30*c^22*d^8 + 10*b^30*c^24*d^6 - 10*b^30*c^26*d^4 + 5*b^30*c^28*d^2 - 20*a*b^29*c^19*d^11 + 100*a*b^29*c^21*d^9 - 200*a*b^29*c^23*d^7 + 200*a*b^29*c^25*d^5 - 100*a*b^29*c^27*d^3 - 100*a^3*b^27*c^29*d + 200*a^5*b^25*c^29*d - 200*a^7*b^23*c^29*d + 100*a^9*b^21*c^29*d - 20*a^11*b^19*c^29*d - 20*a^19*b^11*c*d^29 + 100*a^21*b^9*c*d^29 - 200*a^23*b^7*c*d^29 + 200*a^25*b^5*c*d^29 - 100*a^27*b^3*c*d^29 - 100*a^29*b*c^3*d^27 + 200*a^29*b*c^5*d^25 - 200*a^29*b*c^7*d^23 + 100*a^29*b*c^9*d^21 - 20*a^29*b*c^11*d^19 + 190*a^2*b^28*c^18*d^12 - 955*a^2*b^28*c^20*d^10 + 1925*a^2*b^28*c^22*d^8 - 1950*a^2*b^28*c^24*d^6 + 1000*a^2*b^28*c^26*d^4 - 215*a^2*b^28*c^28*d^2 - 1140*a^3*b^27*c^17*d^13 + 5800*a^3*b^27*c^19*d^11 - 11900*a^3*b^27*c^21*d^9 + 12400*a^3*b^27*c^23*d^7 - 6700*a^3*b^27*c^25*d^5 + 1640*a^3*b^27*c^27*d^3 + 4845*a^4*b^26*c^16*d^14 - 25175*a^4*b^26*c^18*d^12 + 53210*a^4*b^26*c^20*d^10 - 58000*a^4*b^26*c^22*d^8 + 33825*a^4*b^26*c^24*d^6 - 9695*a^4*b^26*c^26*d^4 + 1000*a^4*b^26*c^28*d^2 - 15504*a^5*b^25*c^15*d^15 + 83220*a^5*b^25*c^17*d^13 - 183740*a^5*b^25*c^19*d^11 + 213040*a^5*b^25*c^21*d^9 - 136520*a^5*b^25*c^23*d^7 + 46004*a^5*b^25*c^25*d^5 - 6700*a^5*b^25*c^27*d^3 + 38760*a^6*b^24*c^14*d^16 - 218025*a^6*b^24*c^16*d^14 + 510625*a^6*b^24*c^18*d^12 - 639360*a^6*b^24*c^20*d^10 + 455100*a^6*b^24*c^22*d^8 - 178985*a^6*b^24*c^24*d^6 + 33825*a^6*b^24*c^26*d^4 - 1950*a^6*b^24*c^28*d^2 - 77520*a^7*b^23*c^13*d^17 + 465120*a^7*b^23*c^15*d^15 - 1174200*a^7*b^23*c^17*d^13 + 1607600*a^7*b^23*c^19*d^11 - 1277800*a^7*b^23*c^21*d^9 + 581120*a^7*b^23*c^23*d^7 - 136520*a^7*b^23*c^25*d^5 + 12400*a^7*b^23*c^27*d^3 + 125970*a^8*b^22*c^12*d^18 - 823650*a^8*b^22*c^14*d^16 + 2277150*a^8*b^22*c^16*d^14 - 3441850*a^8*b^22*c^18*d^12 + 3061855*a^8*b^22*c^20*d^10 - 1598495*a^8*b^22*c^22*d^8 + 455100*a^8*b^22*c^24*d^6 - 58000*a^8*b^22*c^26*d^4 + 1925*a^8*b^22*c^28*d^2 - 167960*a^9*b^21*c^11*d^19 + 1227400*a^9*b^21*c^13*d^17 - 3772640*a^9*b^21*c^15*d^15 + 6342200*a^9*b^21*c^17*d^13 - 6323300*a^9*b^21*c^19*d^11 + 3770860*a^9*b^21*c^21*d^9 - 1277800*a^9*b^21*c^23*d^7 + 213040*a^9*b^21*c^25*d^5 - 11900*a^9*b^21*c^27*d^3 + 184756*a^10*b^20*c^10*d^20 - 1553630*a^10*b^20*c^12*d^18 + 5384410*a^10*b^20*c^14*d^16 - 10132510*a^10*b^20*c^16*d^14 + 11341480*a^10*b^20*c^18*d^12 - 7699257*a^10*b^20*c^20*d^10 + 3061855*a^10*b^20*c^22*d^8 - 639360*a^10*b^20*c^24*d^6 + 53210*a^10*b^20*c^26*d^4 - 955*a^10*b^20*c^28*d^2 - 167960*a^11*b^19*c^9*d^21 + 1679600*a^11*b^19*c^11*d^19 - 6653800*a^11*b^19*c^13*d^17 + 14108640*a^11*b^19*c^15*d^15 - 17770700*a^11*b^19*c^17*d^13 + 13697880*a^11*b^19*c^19*d^11 - 6323300*a^11*b^19*c^21*d^9 + 1607600*a^11*b^19*c^23*d^7 - 183740*a^11*b^19*c^25*d^5 + 5800*a^11*b^19*c^27*d^3 + 125970*a^12*b^18*c^8*d^22 - 1553630*a^12*b^18*c^10*d^20 + 7138300*a^12*b^18*c^12*d^18 - 17183600*a^12*b^18*c^14*d^16 + 24426875*a^12*b^18*c^16*d^14 - 21339185*a^12*b^18*c^18*d^12 + 11341480*a^12*b^18*c^20*d^10 - 3441850*a^12*b^18*c^22*d^8 + 510625*a^12*b^18*c^24*d^6 - 25175*a^12*b^18*c^26*d^4 + 190*a^12*b^18*c^28*d^2 - 77520*a^13*b^17*c^7*d^23 + 1227400*a^13*b^17*c^9*d^21 - 6653800*a^13*b^17*c^11*d^19 + 18346400*a^13*b^17*c^13*d^

17 - 29535120*a^13*b^17*c^15*d^15 + 29213260*a^13*b^17*c^17*d^13 - 17770700
*a^13*b^17*c^19*d^11 + 6342200*a^13*b^17*c^21*d^9 - 1174200*a^13*b^17*c^23*
d^7 + 83220*a^13*b^17*c^25*d^5 - 1140*a^13*b^17*c^27*d^3 + 38760*a^14*b^16*
c^6*d^24 - 823650*a^14*b^16*c^8*d^22 + 5384410*a^14*b^16*c^10*d^20 - 171836
00*a^14*b^16*c^12*d^18 + 31460200*a^14*b^16*c^14*d^16 - 35234455*a^14*b^16*
c^16*d^14 + 24426875*a^14*b^16*c^18*d^12 - 10132510*a^14*b^16*c^20*d^10 + 2
277150*a^14*b^16*c^22*d^8 - 218025*a^14*b^16*c^24*d^6 + 4845*a^14*b^16*c^26
*d^4 - 15504*a^15*b^15*c^5*d^25 + 465120*a^15*b^15*c^7*d^23 - 3772640*a^15*
b^15*c^9*d^21 + 14108640*a^15*b^15*c^11*d^19 - 29535120*a^15*b^15*c^13*d^17
 + 37499008*a^15*b^15*c^15*d^15 - 29535120*a^15*b^15*c^17*d^13 + 14108640*a
^15*b^15*c^19*d^11 - 3772640*a^15*b^15*c^21*d^9 + 465120*a^15*b^15*c^23*d^7
 - 15504*a^15*b^15*c^25*d^5 + 4845*a^16*b^14*c^4*d^26 - 218025*a^16*b^14*c^
6*d^24 + 2277150*a^16*b^14*c^8*d^22 - 10132510*a^16*b^14*c^10*d^20 + 244268
75*a^16*b^14*c^12*d^18 - 35234455*a^16*b^14*c^14*d^16 + 31460200*a^16*b^14*
c^16*d^14 - 17183600*a^16*b^14*c^18*d^12 + 5384410*a^16*b^14*c^20*d^10 - 82
3650*a^16*b^14*c^22*d^8 + 38760*a^16*b^14*c^24*d^6 - 1140*a^17*b^13*c^3*d^2
7 + 83220*a^17*b^13*c^5*d^25 - 1174200*a^17*b^13*c^7*d^23 + 6342200*a^17*b^
13*c^9*d^21 - 17770700*a^17*b^13*c^11*d^19 + 29213260*a^17*b^13*c^13*d^17 -
 29535120*a^17*b^13*c^15*d^15 + 18346400*a^17*b^13*c^17*d^13 - 6653800*a^17
*b^13*c^19*d^11 + 1227400*a^17*b^13*c^21*d^9 - 77520*a^17*b^13*c^23*d^7 + 1
90*a^18*b^12*c^2*d^28 - 25175*a^18*b^12*c^4*d^26 + 510625*a^18*b^12*c^6*d^2
4 - 3441850*a^18*b^12*c^8*d^22 + 11341480*a^18*b^12*c^10*d^20 - 21339185*a^
18*b^12*c^12*d^18 + 24426875*a^18*b^12*c^14*d^16 - 17183600*a^18*b^12*c^16*
d^14 + 7138300*a^18*b^12*c^18*d^12 - 1553630*a^18*b^12*c^20*d^10 + 125970*a
^18*b^12*c^22*d^8 + 5800*a^19*b^11*c^3*d^27 - 183740*a^19*b^11*c^5*d^25 + 1
607600*a^19*b^11*c^7*d^23 - 6323300*a^19*b^11*c^9*d^21 + 13697880*a^19*b^11
*c^11*d^19 - 17770700*a^19*b^11*c^13*d^17 + 14108640*a^19*b^11*c^15*d^15 -
6653800*a^19*b^11*c^17*d^13 + 1679600*a^19*b^11*c^19*d^11 - 167960*a^19*b^1
1*c^21*d^9 - 955*a^20*b^10*c^2*d^28 + 53210*a^20*b^10*c^4*d^26 - 639360*a^2
0*b^10*c^6*d^24 + 3061855*a^20*b^10*c^8*d^22 - 7699257*a^20*b^10*c^10*d^20
+ 11341480*a^20*b^10*c^12*d^18 - 10132510*a^20*b^10*c^14*d^16 + 5384410*a^2
0*b^10*c^16*d^14 - 1553630*a^20*b^10*c^18*d^12 + 184756*a^20*b^10*c^20*d^10
 - 11900*a^21*b^9*c^3*d^27 + 213040*a^21*b^9*c^5*d^25 - 1277800*a^21*b^9*c^
7*d^23 + 3770860*a^21*b^9*c^9*d^21 - 6323300*a^21*b^9*c^11*d^19 + 6342200*a
^21*b^9*c^13*d^17 - 3772640*a^21*b^9*c^15*d^15 + 1227400*a^21*b^9*c^17*d^13
 - 167960*a^21*b^9*c^19*d^11 + 1925*a^22*b^8*c^2*d^28 - 58000*a^22*b^8*c^4*
d^26 + 455100*a^22*b^8*c^6*d^24 - 1598495*a^22*b^8*c^8*d^22 + 3061855*a^22*
b^8*c^10*d^20 - 3441850*a^22*b^8*c^12*d^18 + 2277150*a^22*b^8*c^14*d^16 - 8
23650*a^22*b^8*c^16*d^14 + 125970*a^22*b^8*c^18*d^12 + 12400*a^23*b^7*c^3*d
^27 - 136520*a^23*b^7*c^5*d^25 + 581120*a^23*b^7*c^7*d^23 - 1277800*a^23*b^
7*c^9*d^21 + 1607600*a^23*b^7*c^11*d^19 - 1174200*a^23*b^7*c^13*d^17 + 4651
20*a^23*b^7*c^15*d^15 - 77520*a^23*b^7*c^17*d^13 - 1950*a^24*b^6*c^2*d^28 +
 33825*a^24*b^6*c^4*d^26 - 178985*a^24*b^6*c^6*d^24 + 455100*a^24*b^6*c^8*d
^22 - 639360*a^24*b^6*c^10*d^20 + 510625*a^24*b^6*c^12*d^18 - 218025*a^24*b
^6*c^14*d^16 + 38760*a^24*b^6*c^16*d^14 - 6700*a^25*b^5*c^3*d^27 + 46004*a^

25*b^5*c^5*d^25 - 136520*a^25*b^5*c^7*d^23 + 213040*a^25*b^5*c^9*d^21 - 183
740*a^25*b^5*c^11*d^19 + 83220*a^25*b^5*c^13*d^17 - 15504*a^25*b^5*c^15*d^1
5 + 1000*a^26*b^4*c^2*d^28 - 9695*a^26*b^4*c^4*d^26 + 33825*a^26*b^4*c^6*d^
24 - 58000*a^26*b^4*c^8*d^22 + 53210*a^26*b^4*c^10*d^20 - 25175*a^26*b^4*c^
12*d^18 + 4845*a^26*b^4*c^14*d^16 + 1640*a^27*b^3*c^3*d^27 - 6700*a^27*b^3*
c^5*d^25 + 12400*a^27*b^3*c^7*d^23 - 11900*a^27*b^3*c^9*d^21 + 5800*a^27*b^
3*c^11*d^19 - 1140*a^27*b^3*c^13*d^17 - 215*a^28*b^2*c^2*d^28 + 1000*a^28*b
^2*c^4*d^26 - 1950*a^28*b^2*c^6*d^24 + 1925*a^28*b^2*c^8*d^22 - 955*a^28*b^
2*c^10*d^20 + 190*a^28*b^2*c^12*d^18 + 20*a*b^29*c^29*d + 20*a^29*b*c*d^29)
))^(1/2)*(((4*(8*a^2*b^23*c^25 - 32*a^4*b^21*c^25 + 48*a^6*b^19*c^25 - 32*a
^8*b^17*c^25 + 8*a^10*b^15*c^25 + 8*a^25*c^2*d^23 - 32*a^25*c^4*d^21 + 48*a
^25*c^6*d^19 - 32*a^25*c^8*d^17 + 8*a^25*c^10*d^15 - 8*a*b^24*c^16*d^9 + 32
*a*b^24*c^18*d^7 - 48*a*b^24*c^20*d^5 + 32*a*b^24*c^22*d^3 - 72*a^3*b^22*c^
24*d + 368*a^5*b^20*c^24*d - 592*a^7*b^18*c^24*d + 408*a^9*b^16*c^24*d - 10
4*a^11*b^14*c^24*d - 8*a^16*b^9*c*d^24 + 32*a^18*b^7*c*d^24 - 48*a^20*b^5*c
*d^24 + 32*a^22*b^3*c*d^24 - 72*a^24*b*c^3*d^22 + 368*a^24*b*c^5*d^20 - 592
*a^24*b*c^7*d^18 + 408*a^24*b*c^9*d^16 - 104*a^24*b*c^11*d^14 + 104*a^2*b^2
3*c^15*d^10 - 408*a^2*b^23*c^17*d^8 + 592*a^2*b^23*c^19*d^6 - 368*a^2*b^23*
c^21*d^4 + 72*a^2*b^23*c^23*d^2 - 616*a^3*b^22*c^14*d^11 + 2392*a^3*b^22*c^
16*d^9 - 3408*a^3*b^22*c^18*d^7 + 2032*a^3*b^22*c^20*d^5 - 328*a^3*b^22*c^2
2*d^3 + 2184*a^4*b^21*c^13*d^12 - 8536*a^4*b^21*c^15*d^10 + 12272*a^4*b^21*
c^17*d^8 - 7408*a^4*b^21*c^19*d^6 + 1192*a^4*b^21*c^21*d^4 + 328*a^4*b^21*c
^23*d^2 - 5096*a^5*b^20*c^12*d^13 + 20664*a^5*b^20*c^14*d^11 - 31328*a^5*b^
20*c^16*d^9 + 20592*a^5*b^20*c^18*d^7 - 4008*a^5*b^20*c^20*d^5 - 1192*a^5*b
^20*c^22*d^3 + 8008*a^6*b^19*c^11*d^14 - 35672*a^6*b^19*c^13*d^12 + 60768*a
^6*b^19*c^15*d^10 - 46464*a^6*b^19*c^17*d^8 + 11336*a^6*b^19*c^19*d^6 + 400
8*a^6*b^19*c^21*d^4 - 2032*a^6*b^19*c^23*d^2 - 8008*a^7*b^18*c^10*d^15 + 44
408*a^7*b^18*c^12*d^13 - 92512*a^7*b^18*c^14*d^11 + 85536*a^7*b^18*c^16*d^9
 - 24904*a^7*b^18*c^18*d^7 - 11336*a^7*b^18*c^20*d^5 + 7408*a^7*b^18*c^22*d
^3 + 3432*a^8*b^17*c^9*d^16 - 37752*a^8*b^17*c^11*d^14 + 109408*a^8*b^17*c^
13*d^12 - 125472*a^8*b^17*c^15*d^10 + 42696*a^8*b^17*c^17*d^8 + 24904*a^8*b
^17*c^19*d^6 - 20592*a^8*b^17*c^21*d^4 + 3408*a^8*b^17*c^23*d^2 + 3432*a^9*
b^16*c^8*d^17 + 14872*a^9*b^16*c^10*d^15 - 92352*a^9*b^16*c^12*d^13 + 14140
8*a^9*b^16*c^14*d^11 - 59264*a^9*b^16*c^16*d^9 - 42696*a^9*b^16*c^18*d^7 +
46464*a^9*b^16*c^20*d^5 - 12272*a^9*b^16*c^22*d^3 - 8008*a^10*b^15*c^7*d^18
 + 14872*a^10*b^15*c^9*d^16 + 36608*a^10*b^15*c^11*d^14 - 113152*a^10*b^15*
c^13*d^12 + 67008*a^10*b^15*c^15*d^10 + 59264*a^10*b^15*c^17*d^8 - 85536*a^
10*b^15*c^19*d^6 + 31328*a^10*b^15*c^21*d^4 - 2392*a^10*b^15*c^23*d^2 + 800
8*a^11*b^14*c^6*d^19 - 37752*a^11*b^14*c^8*d^17 + 36608*a^11*b^14*c^10*d^15
 + 43264*a^11*b^14*c^12*d^13 - 56256*a^11*b^14*c^14*d^11 - 67008*a^11*b^14*
c^16*d^9 + 125472*a^11*b^14*c^18*d^7 - 60768*a^11*b^14*c^20*d^5 + 8536*a^11
*b^14*c^22*d^3 - 5096*a^12*b^13*c^5*d^20 + 44408*a^12*b^13*c^7*d^18 - 92352
*a^12*b^13*c^9*d^16 + 43264*a^12*b^13*c^11*d^14 + 22464*a^12*b^13*c^13*d^12
 + 56256*a^12*b^13*c^15*d^10 - 141408*a^12*b^13*c^17*d^8 + 92512*a^12*b^13*
c^19*d^6 - 20664*a^12*b^13*c^21*d^4 + 616*a^12*b^13*c^23*d^2 + 2184*a^13*b^

12*c^4*d^21 - 35672*a^13*b^12*c^6*d^19 + 109408*a^13*b^12*c^8*d^17 - 113152
*a^13*b^12*c^10*d^15 + 22464*a^13*b^12*c^12*d^13 - 22464*a^13*b^12*c^14*d^1
1 + 113152*a^13*b^12*c^16*d^9 - 109408*a^13*b^12*c^18*d^7 + 35672*a^13*b^12
*c^20*d^5 - 2184*a^13*b^12*c^22*d^3 - 616*a^14*b^11*c^3*d^22 + 20664*a^14*b
^11*c^5*d^20 - 92512*a^14*b^11*c^7*d^18 + 141408*a^14*b^11*c^9*d^16 - 56256
*a^14*b^11*c^11*d^14 - 22464*a^14*b^11*c^13*d^12 - 43264*a^14*b^11*c^15*d^1
0 + 92352*a^14*b^11*c^17*d^8 - 44408*a^14*b^11*c^19*d^6 + 5096*a^14*b^11*c^
21*d^4 + 104*a^15*b^10*c^2*d^23 - 8536*a^15*b^10*c^4*d^21 + 60768*a^15*b^10
*c^6*d^19 - 125472*a^15*b^10*c^8*d^17 + 67008*a^15*b^10*c^10*d^15 + 56256*a
^15*b^10*c^12*d^13 - 43264*a^15*b^10*c^14*d^11 - 36608*a^15*b^10*c^16*d^9 +
37752*a^15*b^10*c^18*d^7 - 8008*a^15*b^10*c^20*d^5 + 2392*a^16*b^9*c^3*d^2
2 - 31328*a^16*b^9*c^5*d^20 + 85536*a^16*b^9*c^7*d^18 - 59264*a^16*b^9*c^9*
d^16 - 67008*a^16*b^9*c^11*d^14 + 113152*a^16*b^9*c^13*d^12 - 36608*a^16*b^
9*c^15*d^10 - 14872*a^16*b^9*c^17*d^8 + 8008*a^16*b^9*c^19*d^6 - 408*a^17*b
^8*c^2*d^23 + 12272*a^17*b^8*c^4*d^21 - 46464*a^17*b^8*c^6*d^19 + 42696*a^1
7*b^8*c^8*d^17 + 59264*a^17*b^8*c^10*d^15 - 141408*a^17*b^8*c^12*d^13 + 923
52*a^17*b^8*c^14*d^11 - 14872*a^17*b^8*c^16*d^9 - 3432*a^17*b^8*c^18*d^7 -
3408*a^18*b^7*c^3*d^22 + 20592*a^18*b^7*c^5*d^20 - 24904*a^18*b^7*c^7*d^18
- 42696*a^18*b^7*c^9*d^16 + 125472*a^18*b^7*c^11*d^14 - 109408*a^18*b^7*c^1
3*d^12 + 37752*a^18*b^7*c^15*d^10 - 3432*a^18*b^7*c^17*d^8 + 592*a^19*b^6*c
^2*d^23 - 7408*a^19*b^6*c^4*d^21 + 11336*a^19*b^6*c^6*d^19 + 24904*a^19*b^6
*c^8*d^17 - 85536*a^19*b^6*c^10*d^15 + 92512*a^19*b^6*c^12*d^13 - 44408*a^1
9*b^6*c^14*d^11 + 8008*a^19*b^6*c^16*d^9 + 2032*a^20*b^5*c^3*d^22 - 4008*a^
20*b^5*c^5*d^20 - 11336*a^20*b^5*c^7*d^18 + 46464*a^20*b^5*c^9*d^16 - 60768
*a^20*b^5*c^11*d^14 + 35672*a^20*b^5*c^13*d^12 - 8008*a^20*b^5*c^15*d^10 -
368*a^21*b^4*c^2*d^23 + 1192*a^21*b^4*c^4*d^21 + 4008*a^21*b^4*c^6*d^19 - 2
0592*a^21*b^4*c^8*d^17 + 31328*a^21*b^4*c^10*d^15 - 20664*a^21*b^4*c^12*d^1
3 + 5096*a^21*b^4*c^14*d^11 - 328*a^22*b^3*c^3*d^22 - 1192*a^22*b^3*c^5*d^2
0 + 7408*a^22*b^3*c^7*d^18 - 12272*a^22*b^3*c^9*d^16 + 8536*a^22*b^3*c^11*d
^14 - 2184*a^22*b^3*c^13*d^12 + 72*a^23*b^2*c^2*d^23 + 328*a^23*b^2*c^4*d^2
1 - 2032*a^23*b^2*c^6*d^19 + 3408*a^23*b^2*c^8*d^17 - 2392*a^23*b^2*c^10*d^
15 + 616*a^23*b^2*c^12*d^13 - 8*a*b^24*c^24*d - 8*a^24*b*c*d^24))/(a^20*d^2
0 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b
^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d
^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 +
b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a
*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d
^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*
b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19
+ 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*
c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d
^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 -
220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1
168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 224
4*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 15

87*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048
*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744
*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*
a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11
236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 79
2*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27
504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 -
1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 2
0724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8
 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2
 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13
 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*
d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^1
8 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^
8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*
b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*
b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^1
1*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a
^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^1
2*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*
a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495
*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 2157
6*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6
336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 358
8*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 1
3860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 +
1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 -
8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 2
88*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 40
32*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 41
2*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928
*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^1
8*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^
2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19) - (8*tan(e/2 + (f*x)/2)*
(56*a^3*b^22*c^25 - 12*a^25*c*d^24 - 12*a*b^24*c^25 - 104*a^5*b^20*c^25 + 9
6*a^7*b^18*c^25 - 44*a^9*b^16*c^25 + 8*a^11*b^14*c^25 + 56*a^25*c^3*d^22 -
104*a^25*c^5*d^20 + 96*a^25*c^7*d^18 - 44*a^25*c^9*d^16 + 8*a^25*c^11*d^14
+ 16*a*b^24*c^15*d^10 - 76*a*b^24*c^17*d^8 + 144*a*b^24*c^19*d^6 - 136*a*b^
24*c^21*d^4 + 64*a*b^24*c^23*d^2 + 168*a^2*b^23*c^24*d - 784*a^4*b^21*c^24*
d + 1456*a^6*b^19*c^24*d - 1344*a^8*b^17*c^24*d + 616*a^10*b^15*c^24*d - 11
2*a^12*b^13*c^24*d + 16*a^15*b^10*c*d^24 - 76*a^17*b^8*c*d^24 + 144*a^19*b^
6*c*d^24 - 136*a^21*b^4*c*d^24 + 64*a^23*b^2*c*d^24 + 168*a^24*b*c^2*d^23 -
 784*a^24*b*c^4*d^21 + 1456*a^24*b*c^6*d^19 - 1344*a^24*b*c^8*d^17 + 616*a^
24*b*c^10*d^15 - 112*a^24*b*c^12*d^13 - 224*a^2*b^23*c^14*d^11 + 1064*a^2*b
^23*c^16*d^9 - 2016*a^2*b^23*c^18*d^7 + 1904*a^2*b^23*c^20*d^5 - 896*a^2*b^

23*c^22*d^3 + 1456*a^3*b^22*c^13*d^12 - 6992*a^3*b^22*c^15*d^10 + 13464*a^3
*b^22*c^17*d^8 - 13056*a^3*b^22*c^19*d^6 + 6464*a^3*b^22*c^21*d^4 - 1392*a^
3*b^22*c^23*d^2 - 5824*a^4*b^21*c^12*d^13 + 28728*a^4*b^21*c^14*d^11 - 5745
6*a^4*b^21*c^16*d^9 + 59024*a^4*b^21*c^18*d^7 - 32256*a^4*b^21*c^20*d^5 + 8
568*a^4*b^21*c^22*d^3 + 16016*a^5*b^20*c^11*d^14 - 82992*a^5*b^20*c^13*d^12
 + 177048*a^5*b^20*c^15*d^10 - 198696*a^5*b^20*c^17*d^8 + 123584*a^5*b^20*c
^19*d^6 - 40512*a^5*b^20*c^21*d^4 + 5656*a^5*b^20*c^23*d^2 - 32032*a^6*b^19
*c^10*d^15 + 179816*a^6*b^19*c^12*d^13 - 421344*a^6*b^19*c^14*d^11 + 529312
*a^6*b^19*c^16*d^9 - 379008*a^6*b^19*c^18*d^7 + 150024*a^6*b^19*c^20*d^5 -
28224*a^6*b^19*c^22*d^3 + 48048*a^7*b^18*c^9*d^16 - 304304*a^7*b^18*c^11*d^
14 + 805896*a^7*b^18*c^13*d^12 - 1151104*a^7*b^18*c^15*d^10 + 949952*a^7*b^
18*c^17*d^8 - 446736*a^7*b^18*c^19*d^6 + 108136*a^7*b^18*c^21*d^4 - 9984*a^
7*b^18*c^23*d^2 - 54912*a^8*b^17*c^8*d^17 + 412984*a^8*b^17*c^10*d^15 - 126
7344*a^8*b^17*c^12*d^13 + 2077536*a^8*b^17*c^14*d^11 - 1975808*a^8*b^17*c^1
6*d^9 + 1095384*a^8*b^17*c^18*d^7 - 331632*a^8*b^17*c^20*d^5 + 45136*a^8*b^
17*c^22*d^3 + 48048*a^9*b^16*c^7*d^18 - 456456*a^9*b^16*c^9*d^16 + 1657656*
a^9*b^16*c^11*d^14 - 3143504*a^9*b^16*c^13*d^12 + 3453696*a^9*b^16*c^15*d^1
0 - 2247636*a^9*b^16*c^17*d^8 + 831208*a^9*b^16*c^19*d^6 - 151944*a^9*b^16*
c^21*d^4 + 8976*a^9*b^16*c^23*d^2 - 32032*a^10*b^15*c^6*d^19 + 412984*a^10*
b^15*c^8*d^17 - 1812096*a^10*b^15*c^10*d^15 + 4016896*a^10*b^15*c^12*d^13 -
 5121024*a^10*b^15*c^14*d^11 + 3897024*a^10*b^15*c^16*d^9 - 1728832*a^10*b^
15*c^18*d^7 + 404768*a^10*b^15*c^20*d^5 - 38304*a^10*b^15*c^22*d^3 + 16016*
a^11*b^14*c^5*d^20 - 304304*a^11*b^14*c^7*d^18 + 1657656*a^11*b^14*c^9*d^16
 - 4356352*a^11*b^14*c^11*d^14 + 6476288*a^11*b^14*c^13*d^12 - 5745024*a^11
*b^14*c^15*d^10 + 3021984*a^11*b^14*c^17*d^8 - 880256*a^11*b^14*c^19*d^6 +
118032*a^11*b^14*c^21*d^4 - 4048*a^11*b^14*c^23*d^2 - 5824*a^12*b^13*c^4*d^
21 + 179816*a^12*b^13*c^6*d^19 - 1267344*a^12*b^13*c^8*d^17 + 4016896*a^12*
b^13*c^10*d^15 - 7002112*a^12*b^13*c^12*d^13 + 7235136*a^12*b^13*c^14*d^11
- 4480896*a^12*b^13*c^16*d^9 + 1588704*a^12*b^13*c^18*d^7 - 280896*a^12*b^1
3*c^20*d^5 + 16632*a^12*b^13*c^22*d^3 + 1456*a^13*b^12*c^3*d^22 - 82992*a^1
3*b^12*c^5*d^20 + 805896*a^13*b^12*c^7*d^18 - 3143504*a^13*b^12*c^9*d^16 +
6476288*a^13*b^12*c^11*d^14 - 7809984*a^13*b^12*c^13*d^12 + 5666752*a^13*b^
12*c^15*d^10 - 2403856*a^13*b^12*c^17*d^8 + 537264*a^13*b^12*c^19*d^6 - 480
48*a^13*b^12*c^21*d^4 + 728*a^13*b^12*c^23*d^2 - 224*a^14*b^11*c^2*d^23 + 2
8728*a^14*b^11*c^4*d^21 - 421344*a^14*b^11*c^6*d^19 + 2077536*a^14*b^11*c^8
*d^17 - 5121024*a^14*b^11*c^10*d^15 + 7235136*a^14*b^11*c^12*d^13 - 6126848
*a^14*b^11*c^14*d^11 + 3071744*a^14*b^11*c^16*d^9 - 844896*a^14*b^11*c^18*d
^7 + 104104*a^14*b^11*c^20*d^5 - 2912*a^14*b^11*c^22*d^3 - 6992*a^15*b^10*c
^3*d^22 + 177048*a^15*b^10*c^5*d^20 - 1151104*a^15*b^10*c^7*d^18 + 3453696*
a^15*b^10*c^9*d^16 - 5745024*a^15*b^10*c^11*d^14 + 5666752*a^15*b^10*c^13*d
^12 - 3331328*a^15*b^10*c^15*d^10 + 1105104*a^15*b^10*c^17*d^8 - 176176*a^1
5*b^10*c^19*d^6 + 8008*a^15*b^10*c^21*d^4 + 1064*a^16*b^9*c^2*d^23 - 57456*
a^16*b^9*c^4*d^21 + 529312*a^16*b^9*c^6*d^19 - 1975808*a^16*b^9*c^8*d^17 +
3897024*a^16*b^9*c^10*d^15 - 4480896*a^16*b^9*c^12*d^13 + 3071744*a^16*b^9*
c^14*d^11 - 1208064*a^16*b^9*c^16*d^9 + 239096*a^16*b^9*c^18*d^7 - 16016*a^

16*b^9*c^20*d^5 + 13464*a^17*b^8*c^3*d^22 - 198696*a^17*b^8*c^5*d^20 + 9499
52*a^17*b^8*c^7*d^18 - 2247636*a^17*b^8*c^9*d^16 + 3021984*a^17*b^8*c^11*d^
14 - 2403856*a^17*b^8*c^13*d^12 + 1105104*a^17*b^8*c^15*d^10 - 264264*a^17*
b^8*c^17*d^8 + 24024*a^17*b^8*c^19*d^6 - 2016*a^18*b^7*c^2*d^23 + 59024*a^1
8*b^7*c^4*d^21 - 379008*a^18*b^7*c^6*d^19 + 1095384*a^18*b^7*c^8*d^17 - 172
8832*a^18*b^7*c^10*d^15 + 1588704*a^18*b^7*c^12*d^13 - 844896*a^18*b^7*c^14
*d^11 + 239096*a^18*b^7*c^16*d^9 - 27456*a^18*b^7*c^18*d^7 - 13056*a^19*b^6
*c^3*d^22 + 123584*a^19*b^6*c^5*d^20 - 446736*a^19*b^6*c^7*d^18 + 831208*a^
19*b^6*c^9*d^16 - 880256*a^19*b^6*c^11*d^14 + 537264*a^19*b^6*c^13*d^12 - 1
76176*a^19*b^6*c^15*d^10 + 24024*a^19*b^6*c^17*d^8 + 1904*a^20*b^5*c^2*d^23
 - 32256*a^20*b^5*c^4*d^21 + 150024*a^20*b^5*c^6*d^19 - 331632*a^20*b^5*c^8
*d^17 + 404768*a^20*b^5*c^10*d^15 - 280896*a^20*b^5*c^12*d^13 + 104104*a^20
*b^5*c^14*d^11 - 16016*a^20*b^5*c^16*d^9 + 6464*a^21*b^4*c^3*d^22 - 40512*a
^21*b^4*c^5*d^20 + 108136*a^21*b^4*c^7*d^18 - 151944*a^21*b^4*c^9*d^16 + 11
8032*a^21*b^4*c^11*d^14 - 48048*a^21*b^4*c^13*d^12 + 8008*a^21*b^4*c^15*d^1
0 - 896*a^22*b^3*c^2*d^23 + 8568*a^22*b^3*c^4*d^21 - 28224*a^22*b^3*c^6*d^1
9 + 45136*a^22*b^3*c^8*d^17 - 38304*a^22*b^3*c^10*d^15 + 16632*a^22*b^3*c^1
2*d^13 - 2912*a^22*b^3*c^14*d^11 - 1392*a^23*b^2*c^3*d^22 + 5656*a^23*b^2*c
^5*d^20 - 9984*a^23*b^2*c^7*d^18 + 8976*a^23*b^2*c^9*d^16 - 4048*a^23*b^2*c
^11*d^14 + 728*a^23*b^2*c^13*d^12))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^2
0 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a
^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20
*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d
^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^1
3*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a
^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^
19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19
*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11
 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 2
88*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a
^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^
3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^
4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4
*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*
b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*
b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*
b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a
^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7
*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*
a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a
^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300
*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4
032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 404
8*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 4
4936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 +

928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19))*(-(((4*a^24*d^24 + 4*b^24*c^24 + 16*a^2*b^22*c^24 + 16*a^4*b^20*c^24 - 1152*a^10*b^14*d^24 + 5568*a^12*b^12*d^24 - 10568*a^14*b^10*d^24 + 9460*a^16*b^8*d^24 - 3560*a^18*b^6*d^24 + 136*a^20*b^4*d^24 + 76*a^22*b^2*d^24 + 16*a^24*c^2*d^22 + 16*a^24*c^4*d^20 - 1152*b^24*c^10*d^14 + 5568*b^24*c^12*d^12 - 10568*b^24*c^14*d^10 + 9460*b^24*c^16*d^8 - 3560*b^24*c^18*d^6 + 136*b^24*c^20*d^4 + 76*b^24*c^22*d^2 + 11520*a*b^23*c^9*d^15 - 56448*a*b^23*c^11*d^13 + 109456*a*b^23*c^13*d^11 - 101240*a*b^23*c^15*d^9 + 40720*a*b^23*c^17*d^7 - 2960*a*b^23*c^19*d^5 - 536*a*b^23*c^21*d^3 - 176*a^3*b^21*c^23*d - 320*a^5*b^19*c^23*d + 11520*a^9*b^15*c*d^23 - 56448*a^11*b^13*c*d^23 + 109456*a^13*b^11*c*d^23 - 101240*a^15*b^9*c*d^23 + 40720*a^17*b^7*c*d^23 - 2960*a^19*b^5*c*d^23 - 536*a^21*b^3*c*d^23 - 176*a^23*b*c^3*d^21 - 320*a^23*b*c^5*d^19 - 51840*a^2*b^22*c^8*d^16 + 263808*a^2*b^22*c^10*d^14 - 541208*a^2*b^22*c^12*d^12 + 547088*a^2*b^22*c^14*d^10 - 263320*a^2*b^22*c^16*d^8 + 44120*a^2*b^22*c^18*d^6 - 1564*a^2*b^22*c^20*d^4 - 196*a^2*b^22*c^22*d^2 + 138240*a^3*b^21*c^7*d^17 - 758400*a^3*b^21*c^9*d^15 + 1720736*a^3*b^21*c^11*d^13 - 2002728*a^3*b^21*c^13*d^11 + 1210560*a^3*b^21*c^15*d^9 - 335040*a^3*b^21*c^17*d^7 + 37680*a^3*b^21*c^19*d^5 - 288*a^3*b^21*c^21*d^3 - 241920*a^4*b^20*c^6*d^18 + 1512000*a^4*b^20*c^8*d^16 - 3975688*a^4*b^20*c^10*d^14 + 5501328*a^4*b^20*c^12*d^12 - 4147952*a^4*b^20*c^14*d^10 + 1586920*a^4*b^20*c^16*d^8 - 276020*a^4*b^20*c^18*d^6 + 21124*a^4*b^20*c^20*d^4 + 176*a^4*b^20*c^22*d^2 + 290304*a^5*b^19*c^5*d^19 - 2232576*a^5*b^19*c^7*d^17 + 7078256*a^5*b^19*c^9*d^15 - 11781560*a^5*b^19*c^11*d^13 + 10875200*a^5*b^19*c^13*d^11 - 5365072*a^5*b^19*c^15*d^9 + 1310168*a^5*b^19*c^17*d^7 - 170968*a^5*b^19*c^19*d^5 + 8160*a^5*b^19*c^21*d^3 - 241920*a^6*b^18*c^4*

d^20 + 2532096*a^6*b^18*c^6*d^18 - 9955992*a^6*b^18*c^8*d^16 + 20019440*a^6
*b^18*c^10*d^14 - 22419600*a^6*b^18*c^12*d^12 + 13887520*a^6*b^18*c^14*d^10
 - 4506428*a^6*b^18*c^16*d^8 + 793756*a^6*b^18*c^18*d^6 - 72240*a^6*b^18*c^
20*d^4 + 3040*a^6*b^18*c^22*d^2 + 138240*a^7*b^17*c^3*d^21 - 2232576*a^7*b^
17*c^5*d^19 + 11150016*a^7*b^17*c^7*d^17 - 27336616*a^7*b^17*c^9*d^15 + 371
53600*a^7*b^17*c^11*d^13 - 28461040*a^7*b^17*c^13*d^11 + 11779808*a^7*b^17*
c^15*d^9 - 2621008*a^7*b^17*c^17*d^7 + 336688*a^7*b^17*c^19*d^5 - 17920*a^7
*b^17*c^21*d^3 - 51840*a^8*b^16*c^2*d^22 + 1512000*a^8*b^16*c^4*d^20 - 9955
992*a^8*b^16*c^6*d^18 + 30289656*a^8*b^16*c^8*d^16 - 50137600*a^8*b^16*c^10
*d^14 + 46972560*a^8*b^16*c^12*d^12 - 24199280*a^8*b^16*c^14*d^10 + 6661036
*a^8*b^16*c^16*d^8 - 1058448*a^8*b^16*c^18*d^6 + 72560*a^8*b^16*c^20*d^4 -
758400*a^9*b^15*c^3*d^21 + 7078256*a^9*b^15*c^5*d^19 - 27336616*a^9*b^15*c^
7*d^17 + 55383904*a^9*b^15*c^9*d^15 - 63124080*a^9*b^15*c^11*d^13 + 3998752
0*a^9*b^15*c^13*d^11 - 13462088*a^9*b^15*c^15*d^9 + 2478528*a^9*b^15*c^17*d
^7 - 212032*a^9*b^15*c^19*d^5 + 263808*a^10*b^14*c^2*d^22 - 3975688*a^10*b^
14*c^4*d^20 + 20019440*a^10*b^14*c^6*d^18 - 50137600*a^10*b^14*c^8*d^16 + 6
9593872*a^10*b^14*c^10*d^14 - 53854288*a^10*b^14*c^12*d^12 + 21989928*a^10*
b^14*c^14*d^10 - 4591360*a^10*b^14*c^16*d^8 + 460480*a^10*b^14*c^18*d^6 + 1
720736*a^11*b^13*c^3*d^21 - 11781560*a^11*b^13*c^5*d^19 + 37153600*a^11*b^1
3*c^7*d^17 - 63124080*a^11*b^13*c^9*d^15 + 59445728*a^11*b^13*c^11*d^13 - 2
9358696*a^11*b^13*c^13*d^11 + 6995840*a^11*b^13*c^15*d^9 - 762560*a^11*b^13
*c^17*d^7 - 541208*a^12*b^12*c^2*d^22 + 5501328*a^12*b^12*c^4*d^20 - 224196
00*a^12*b^12*c^6*d^18 + 46972560*a^12*b^12*c^8*d^16 - 53854288*a^12*b^12*c^
10*d^14 + 32294808*a^12*b^12*c^12*d^12 - 8958208*a^12*b^12*c^14*d^10 + 9990
40*a^12*b^12*c^16*d^8 - 2002728*a^13*b^11*c^3*d^21 + 10875200*a^13*b^11*c^5
*d^19 - 28461040*a^13*b^11*c^7*d^17 + 39987520*a^13*b^11*c^9*d^15 - 2935869
6*a^13*b^11*c^11*d^13 + 9722048*a^13*b^11*c^13*d^11 - 1104320*a^13*b^11*c^1
5*d^9 + 547088*a^14*b^10*c^2*d^22 - 4147952*a^14*b^10*c^4*d^20 + 13887520*a
^14*b^10*c^6*d^18 - 24199280*a^14*b^10*c^8*d^16 + 21989928*a^14*b^10*c^10*d
^14 - 8958208*a^14*b^10*c^12*d^12 + 1124032*a^14*b^10*c^14*d^10 + 1210560*a
^15*b^9*c^3*d^21 - 5365072*a^15*b^9*c^5*d^19 + 11779808*a^15*b^9*c^7*d^17 -
 13462088*a^15*b^9*c^9*d^15 + 6995840*a^15*b^9*c^11*d^13 - 1104320*a^15*b^9
*c^13*d^11 - 263320*a^16*b^8*c^2*d^22 + 1586920*a^16*b^8*c^4*d^20 - 4506428
*a^16*b^8*c^6*d^18 + 6661036*a^16*b^8*c^8*d^16 - 4591360*a^16*b^8*c^10*d^14
 + 999040*a^16*b^8*c^12*d^12 - 335040*a^17*b^7*c^3*d^21 + 1310168*a^17*b^7*
c^5*d^19 - 2621008*a^17*b^7*c^7*d^17 + 2478528*a^17*b^7*c^9*d^15 - 762560*a
^17*b^7*c^11*d^13 + 44120*a^18*b^6*c^2*d^22 - 276020*a^18*b^6*c^4*d^20 + 79
3756*a^18*b^6*c^6*d^18 - 1058448*a^18*b^6*c^8*d^16 + 460480*a^18*b^6*c^10*d
^14 + 37680*a^19*b^5*c^3*d^21 - 170968*a^19*b^5*c^5*d^19 + 336688*a^19*b^5*
c^7*d^17 - 212032*a^19*b^5*c^9*d^15 - 1564*a^20*b^4*c^2*d^22 + 21124*a^20*b
^4*c^4*d^20 - 72240*a^20*b^4*c^6*d^18 + 72560*a^20*b^4*c^8*d^16 - 288*a^21*
b^3*c^3*d^21 + 8160*a^21*b^3*c^5*d^19 - 17920*a^21*b^3*c^7*d^17 - 196*a^22*
b^2*c^2*d^22 + 176*a^22*b^2*c^4*d^20 + 3040*a^22*b^2*c^6*d^18 - 8*a*b^23*c^
23*d - 8*a^23*b*c*d^23)^2/4 - (20736*b^18*d^18 - 96768*a^2*b^16*d^18 + 1736
64*a^4*b^14*d^18 - 136032*a^6*b^12*d^18 + 31081*a^8*b^10*d^18 + 8440*a^10*b

^8*d^18 + 400*a^12*b^6*d^18 - 96768*b^18*c^2*d^16 + 173664*b^18*c^4*d^14 - 136032*b^18*c^6*d^12 + 31081*b^18*c^8*d^10 + 8440*b^18*c^10*d^8 + 400*b^18*c^12*d^6 - 131328*a*b^17*c^3*d^15 + 216576*a*b^17*c^5*d^13 - 141104*a*b^17*c^7*d^11 + 20260*a*b^17*c^9*d^9 + 2800*a*b^17*c^11*d^7 - 131328*a^3*b^15*c*d^17 + 216576*a^5*b^13*c*d^17 - 141104*a^7*b^11*c*d^17 + 20260*a^9*b^9*c*d^17 + 2800*a^11*b^7*c*d^17 + 495936*a^2*b^16*c^2*d^16 - 989856*a^2*b^16*c^4*d^14 + 901948*a^2*b^16*c^6*d^12 - 308392*a^2*b^16*c^8*d^10 - 5260*a^2*b^16*c^10*d^8 + 1600*a^2*b^16*c^12*d^6 + 657408*a^3*b^15*c^3*d^15 - 1158992*a^3*b^15*c^5*d^13 + 838256*a^3*b^15*c^7*d^11 - 182200*a^3*b^15*c^9*d^9 - 3200*a^3*b^15*c^11*d^7 - 989856*a^4*b^14*c^2*d^16 + 2185654*a^4*b^14*c^4*d^14 - 2218576*a^4*b^14*c^6*d^12 + 900624*a^4*b^14*c^8*d^10 - 64720*a^4*b^14*c^10*d^8 + 1600*a^4*b^14*c^12*d^6 - 1158992*a^5*b^13*c^3*d^15 + 2158808*a^5*b^13*c^5*d^13 - 1641528*a^5*b^13*c^7*d^11 + 406880*a^5*b^13*c^9*d^9 - 17600*a^5*b^13*c^11*d^7 + 901948*a^6*b^12*c^2*d^16 - 2218576*a^6*b^12*c^4*d^14 + 2430936*a^6*b^12*c^6*d^12 - 1026928*a^6*b^12*c^8*d^10 + 88720*a^6*b^12*c^10*d^8 + 838256*a^7*b^11*c^3*d^15 - 1641528*a^7*b^11*c^5*d^13 + 1206848*a^7*b^11*c^7*d^11 - 239360*a^7*b^11*c^9*d^9 - 308392*a^8*b^10*c^2*d^16 + 900624*a^8*b^10*c^4*d^14 - 1026928*a^8*b^10*c^6*d^12 + 354016*a^8*b^10*c^8*d^10 - 182200*a^9*b^9*c^3*d^15 + 406880*a^9*b^9*c^5*d^13 - 239360*a^9*b^9*c^7*d^11 - 5260*a^10*b^8*c^2*d^16 - 64720*a^10*b^8*c^4*d^14 + 88720*a^10*b^8*c^6*d^12 - 3200*a^11*b^7*c^3*d^15 - 17600*a^11*b^7*c^5*d^13 + 1600*a^12*b^6*c^2*d^16 + 1600*a^12*b^6*c^4*d^14 + 27648*a*b^17*c*d^17)*(80*a^2*b^28*c^30 - 16*b^30*c^30 - 16*a^30*d^30 - 160*a^4*b^26*c^30 + 160*a^6*b^24*c^30 - 80*a^8*b^22*c^30 + 16*a^10*b^20*c^30 + 16*a^20*b^10*d^30 - 80*a^22*b^8*d^30 + 160*a^24*b^6*d^30 - 160*a^26*b^4*d^30 + 80*a^28*b^2*d^30 + 80*a^30*c^2*d^28 - 160*a^30*c^4*d^26 + 160*a^30*c^6*d^24 - 80*a^30*c^8*d^22 + 16*a^30*c^10*d^20 + 16*b^30*c^20*d^10 - 80*b^30*c^22*d^8 + 160*b^30*c^24*d^6 - 160*b^30*c^26*d^4 + 80*b^30*c^28*d^2 - 320*a*b^29*c^19*d^11 + 1600*a*b^29*c^21*d^9 - 3200*a*b^29*c^23*d^7 + 3200*a*b^29*c^25*d^5 - 1600*a*b^29*c^27*d^3 - 1600*a^3*b^27*c^29*d + 3200*a^5*b^25*c^29*d - 3200*a^7*b^23*c^29*d + 1600*a^9*b^21*c^29*d - 320*a^11*b^19*c^29*d - 320*a^19*b^11*c*d^29 + 1600*a^21*b^9*c*d^29 - 3200*a^23*b^7*c*d^29 + 3200*a^25*b^5*c*d^29 - 1600*a^27*b^3*c*d^29 - 1600*a^29*b*c^3*d^27 + 3200*a^29*b*c^5*d^25 - 3200*a^29*b*c^7*d^23 + 1600*a^29*b*c^9*d^21 - 320*a^29*b*c^11*d^19 + 3040*a^2*b^28*c^18*d^12 - 15280*a^2*b^28*c^20*d^10 + 30800*a^2*b^28*c^22*d^8 - 31200*a^2*b^28*c^24*d^6 + 16000*a^2*b^28*c^26*d^4 - 3440*a^2*b^28*c^28*d^2 - 18240*a^3*b^27*c^17*d^13 + 92800*a^3*b^27*c^19*d^11 - 190400*a^3*b^27*c^21*d^9 + 198400*a^3*b^27*c^23*d^7 - 107200*a^3*b^27*c^25*d^5 + 26240*a^3*b^27*c^27*d^3 + 77520*a^4*b^26*c^16*d^14 - 402800*a^4*b^26*c^18*d^12 + 851360*a^4*b^26*c^20*d^10 - 928000*a^4*b^26*c^22*d^8 + 541200*a^4*b^26*c^24*d^6 - 155120*a^4*b^26*c^26*d^4 + 16000*a^4*b^26*c^28*d^2 - 248064*a^5*b^25*c^15*d^15 + 1331520*a^5*b^25*c^17*d^13 - 2939840*a^5*b^25*c^19*d^11 + 3408640*a^5*b^25*c^21*d^9 - 2184320*a^5*b^25*c^23*d^7 + 736064*a^5*b^25*c^25*d^5 - 107200*a^5*b^25*c^27*d^3 + 620160*a^6*b^24*c^14*d^16 - 3488400*a^6*b^24*c^16*d^14 + 8170000*a^6*b^24*c^18*d^12 - 10229760*a^6*b^24*c^20*d^10 + 7281600*a^6*b^24*c^22*d^8 - 2863760*a^6*b^24*c^24*

d^6 + 541200*a^6*b^24*c^26*d^4 - 31200*a^6*b^24*c^28*d^2 - 1240320*a^7*b^23
*c^13*d^17 + 7441920*a^7*b^23*c^15*d^15 - 18787200*a^7*b^23*c^17*d^13 + 257
21600*a^7*b^23*c^19*d^11 - 20444800*a^7*b^23*c^21*d^9 + 9297920*a^7*b^23*c^
23*d^7 - 2184320*a^7*b^23*c^25*d^5 + 198400*a^7*b^23*c^27*d^3 + 2015520*a^8
*b^22*c^12*d^18 - 13178400*a^8*b^22*c^14*d^16 + 36434400*a^8*b^22*c^16*d^14
 - 55069600*a^8*b^22*c^18*d^12 + 48989680*a^8*b^22*c^20*d^10 - 25575920*a^8
*b^22*c^22*d^8 + 7281600*a^8*b^22*c^24*d^6 - 928000*a^8*b^22*c^26*d^4 + 308
00*a^8*b^22*c^28*d^2 - 2687360*a^9*b^21*c^11*d^19 + 19638400*a^9*b^21*c^13*
d^17 - 60362240*a^9*b^21*c^15*d^15 + 101475200*a^9*b^21*c^17*d^13 - 1011728
00*a^9*b^21*c^19*d^11 + 60333760*a^9*b^21*c^21*d^9 - 20444800*a^9*b^21*c^23
*d^7 + 3408640*a^9*b^21*c^25*d^5 - 190400*a^9*b^21*c^27*d^3 + 2956096*a^10*
b^20*c^10*d^20 - 24858080*a^10*b^20*c^12*d^18 + 86150560*a^10*b^20*c^14*d^1
6 - 162120160*a^10*b^20*c^16*d^14 + 181463680*a^10*b^20*c^18*d^12 - 1231881
12*a^10*b^20*c^20*d^10 + 48989680*a^10*b^20*c^22*d^8 - 10229760*a^10*b^20*c
^24*d^6 + 851360*a^10*b^20*c^26*d^4 - 15280*a^10*b^20*c^28*d^2 - 2687360*a^
11*b^19*c^9*d^21 + 26873600*a^11*b^19*c^11*d^19 - 106460800*a^11*b^19*c^13*
d^17 + 225738240*a^11*b^19*c^15*d^15 - 284331200*a^11*b^19*c^17*d^13 + 2191
66080*a^11*b^19*c^19*d^11 - 101172800*a^11*b^19*c^21*d^9 + 25721600*a^11*b^
19*c^23*d^7 - 2939840*a^11*b^19*c^25*d^5 + 92800*a^11*b^19*c^27*d^3 + 20155
20*a^12*b^18*c^8*d^22 - 24858080*a^12*b^18*c^10*d^20 + 114212800*a^12*b^18*
c^12*d^18 - 274937600*a^12*b^18*c^14*d^16 + 390830000*a^12*b^18*c^16*d^14 -
 341426960*a^12*b^18*c^18*d^12 + 181463680*a^12*b^18*c^20*d^10 - 55069600*a
^12*b^18*c^22*d^8 + 8170000*a^12*b^18*c^24*d^6 - 402800*a^12*b^18*c^26*d^4
 + 3040*a^12*b^18*c^28*d^2 - 1240320*a^13*b^17*c^7*d^23 + 19638400*a^13*b^17
*c^9*d^21 - 106460800*a^13*b^17*c^11*d^19 + 293542400*a^13*b^17*c^13*d^17 -
 472561920*a^13*b^17*c^15*d^15 + 467412160*a^13*b^17*c^17*d^13 - 284331200*
a^13*b^17*c^19*d^11 + 101475200*a^13*b^17*c^21*d^9 - 18787200*a^13*b^17*c^2
3*d^7 + 1331520*a^13*b^17*c^25*d^5 - 18240*a^13*b^17*c^27*d^3 + 620160*a^14
*b^16*c^6*d^24 - 13178400*a^14*b^16*c^8*d^22 + 86150560*a^14*b^16*c^10*d^20
 - 274937600*a^14*b^16*c^12*d^18 + 503363200*a^14*b^16*c^14*d^16 - 56375128
0*a^14*b^16*c^16*d^14 + 390830000*a^14*b^16*c^18*d^12 - 162120160*a^14*b^16
*c^20*d^10 + 36434400*a^14*b^16*c^22*d^8 - 3488400*a^14*b^16*c^24*d^6 + 775
20*a^14*b^16*c^26*d^4 - 248064*a^15*b^15*c^5*d^25 + 7441920*a^15*b^15*c^7*d
^23 - 60362240*a^15*b^15*c^9*d^21 + 225738240*a^15*b^15*c^11*d^19 - 4725619
20*a^15*b^15*c^13*d^17 + 599984128*a^15*b^15*c^15*d^15 - 472561920*a^15*b^1
5*c^17*d^13 + 225738240*a^15*b^15*c^19*d^11 - 60362240*a^15*b^15*c^21*d^9 +
 7441920*a^15*b^15*c^23*d^7 - 248064*a^15*b^15*c^25*d^5 + 77520*a^16*b^14*c
^4*d^26 - 3488400*a^16*b^14*c^6*d^24 + 36434400*a^16*b^14*c^8*d^22 - 162120
160*a^16*b^14*c^10*d^20 + 390830000*a^16*b^14*c^12*d^18 - 563751280*a^16*b^
14*c^14*d^16 + 503363200*a^16*b^14*c^16*d^14 - 274937600*a^16*b^14*c^18*d^1
2 + 86150560*a^16*b^14*c^20*d^10 - 13178400*a^16*b^14*c^22*d^8 + 620160*a^1
6*b^14*c^24*d^6 - 18240*a^17*b^13*c^3*d^27 + 1331520*a^17*b^13*c^5*d^25 - 1
8787200*a^17*b^13*c^7*d^23 + 101475200*a^17*b^13*c^9*d^21 - 284331200*a^17*
b^13*c^11*d^19 + 467412160*a^17*b^13*c^13*d^17 - 472561920*a^17*b^13*c^15*d
^15 + 293542400*a^17*b^13*c^17*d^13 - 106460800*a^17*b^13*c^19*d^11 + 19638

400*a^17*b^13*c^21*d^9 - 1240320*a^17*b^13*c^23*d^7 + 3040*a^18*b^12*c^2*d^28 - 402800*a^18*b^12*c^4*d^26 + 8170000*a^18*b^12*c^6*d^24 - 55069600*a^18*b^12*c^8*d^22 + 181463680*a^18*b^12*c^10*d^20 - 341426960*a^18*b^12*c^12*d^18 + 390830000*a^18*b^12*c^14*d^16 - 274937600*a^18*b^12*c^16*d^14 + 114212800*a^18*b^12*c^18*d^12 - 24858080*a^18*b^12*c^20*d^10 + 2015520*a^18*b^12*c^22*d^8 + 92800*a^19*b^11*c^3*d^27 - 2939840*a^19*b^11*c^5*d^25 + 25721600*a^19*b^11*c^7*d^23 - 101172800*a^19*b^11*c^9*d^21 + 219166080*a^19*b^11*c^11*d^19 - 284331200*a^19*b^11*c^13*d^17 + 225738240*a^19*b^11*c^15*d^15 - 106460800*a^19*b^11*c^17*d^13 + 26873600*a^19*b^11*c^19*d^11 - 2687360*a^19*b^11*c^21*d^9 - 15280*a^20*b^10*c^2*d^28 + 851360*a^20*b^10*c^4*d^26 - 10229760*a^20*b^10*c^6*d^24 + 48989680*a^20*b^10*c^8*d^22 - 123188112*a^20*b^10*c^10*d^20 + 181463680*a^20*b^10*c^12*d^18 - 162120160*a^20*b^10*c^14*d^16 + 86150560*a^20*b^10*c^16*d^14 - 24858080*a^20*b^10*c^18*d^12 + 2956096*a^20*b^10*c^20*d^10 - 190400*a^21*b^9*c^3*d^27 + 3408640*a^21*b^9*c^5*d^25 - 20444800*a^21*b^9*c^7*d^23 + 60333760*a^21*b^9*c^9*d^21 - 101172800*a^21*b^9*c^11*d^19 + 101475200*a^21*b^9*c^13*d^17 - 60362240*a^21*b^9*c^15*d^15 + 19638400*a^21*b^9*c^17*d^13 - 2687360*a^21*b^9*c^19*d^11 + 30800*a^22*b^8*c^2*d^28 - 928000*a^22*b^8*c^4*d^26 + 7281600*a^22*b^8*c^6*d^24 - 25575920*a^22*b^8*c^8*d^22 + 48989680*a^22*b^8*c^10*d^20 - 55069600*a^22*b^8*c^12*d^18 + 36434400*a^22*b^8*c^14*d^16 - 13178400*a^22*b^8*c^16*d^14 + 2015520*a^22*b^8*c^18*d^12 + 198400*a^23*b^7*c^3*d^27 - 2184320*a^23*b^7*c^5*d^25 + 9297920*a^23*b^7*c^7*d^23 - 20444800*a^23*b^7*c^9*d^21 + 25721600*a^23*b^7*c^11*d^19 - 18787200*a^23*b^7*c^13*d^17 + 7441920*a^23*b^7*c^15*d^15 - 1240320*a^23*b^7*c^17*d^13 - 31200*a^24*b^6*c^2*d^28 + 541200*a^24*b^6*c^4*d^26 - 2863760*a^24*b^6*c^6*d^24 + 7281600*a^24*b^6*c^8*d^22 - 10229760*a^24*b^6*c^10*d^20 + 8170000*a^24*b^6*c^12*d^18 - 3488400*a^24*b^6*c^14*d^16 + 620160*a^24*b^6*c^16*d^14 - 107200*a^25*b^5*c^3*d^27 + 736064*a^25*b^5*c^5*d^25 - 2184320*a^25*b^5*c^7*d^23 + 3408640*a^25*b^5*c^9*d^21 - 2939840*a^25*b^5*c^11*d^19 + 1331520*a^25*b^5*c^13*d^17 - 248064*a^25*b^5*c^15*d^15 + 16000*a^26*b^4*c^2*d^28 - 155120*a^26*b^4*c^4*d^26 + 541200*a^26*b^4*c^6*d^24 - 928000*a^26*b^4*c^8*d^22 + 851360*a^26*b^4*c^10*d^20 - 402800*a^26*b^4*c^12*d^18 + 77520*a^26*b^4*c^14*d^16 + 26240*a^27*b^3*c^3*d^27 - 107200*a^27*b^3*c^5*d^25 + 198400*a^27*b^3*c^7*d^23 - 190400*a^27*b^3*c^9*d^21 + 92800*a^27*b^3*c^11*d^19 - 18240*a^27*b^3*c^13*d^17 - 3440*a^28*b^2*c^2*d^28 + 16000*a^28*b^2*c^4*d^26 - 31200*a^28*b^2*c^6*d^24 + 30800*a^28*b^2*c^8*d^22 - 15280*a^28*b^2*c^10*d^20 + 3040*a^28*b^2*c^12*d^18 + 320*a*b^29*c^29*d + 320*a^29*b*c*d^29))^(1/2) + 2*a^24*d^24 + 2*b^24*c^24 + 8*a^2*b^22*c^24 + 8*a^4*b^20*c^24 - 576*a^10*b^14*d^24 + 2784*a^12*b^12*d^24 - 5284*a^14*b^10*d^24 + 4730*a^16*b^8*d^24 - 1780*a^18*b^6*d^24 + 68*a^20*b^4*d^24 + 38*a^22*b^2*d^24 + 8*a^24*c^2*d^22 + 8*a^24*c^4*d^20 - 576*b^24*c^10*d^14 + 2784*b^24*c^12*d^12 - 5284*b^24*c^14*d^10 + 4730*b^24*c^16*d^8 - 1780*b^24*c^18*d^6 + 68*b^24*c^20*d^4 + 38*b^24*c^22*d^2 + 5760*a*b^23*c^9*d^15 - 28224*a*b^23*c^11*d^13 + 54728*a*b^23*c^13*d^11 - 50620*a*b^23*c^15*d^9 + 20360*a*b^23*c^17*d^7 - 1480*a*b^23*c^19*d^5 - 268*a*b^23*c^21*d^3 - 88*a^3*b^21*c^23*d - 160*a^5*b^19*c^23*d + 5760*a^9*b^15*c*d^23 - 28224*a^11*b^13*c*d^23 + 5472

8*a^13*b^11*c*d^23 - 50620*a^15*b^9*c*d^23 + 20360*a^17*b^7*c*d^23 - 1480*a
^19*b^5*c*d^23 - 268*a^21*b^3*c*d^23 - 88*a^23*b*c^3*d^21 - 160*a^23*b*c^5*
d^19 - 25920*a^2*b^22*c^8*d^16 + 131904*a^2*b^22*c^10*d^14 - 270604*a^2*b^2
2*c^12*d^12 + 273544*a^2*b^22*c^14*d^10 - 131660*a^2*b^22*c^16*d^8 + 22060*
a^2*b^22*c^18*d^6 - 782*a^2*b^22*c^20*d^4 - 98*a^2*b^22*c^22*d^2 + 69120*a^
3*b^21*c^7*d^17 - 379200*a^3*b^21*c^9*d^15 + 860368*a^3*b^21*c^11*d^13 - 10
01364*a^3*b^21*c^13*d^11 + 605280*a^3*b^21*c^15*d^9 - 167520*a^3*b^21*c^17*
d^7 + 18840*a^3*b^21*c^19*d^5 - 144*a^3*b^21*c^21*d^3 - 120960*a^4*b^20*c^6
*d^18 + 756000*a^4*b^20*c^8*d^16 - 1987844*a^4*b^20*c^10*d^14 + 2750664*a^4
*b^20*c^12*d^12 - 2073976*a^4*b^20*c^14*d^10 + 793460*a^4*b^20*c^16*d^8 - 1
38010*a^4*b^20*c^18*d^6 + 10562*a^4*b^20*c^20*d^4 + 88*a^4*b^20*c^22*d^2 +
145152*a^5*b^19*c^5*d^19 - 1116288*a^5*b^19*c^7*d^17 + 3539128*a^5*b^19*c^9
*d^15 - 5890780*a^5*b^19*c^11*d^13 + 5437600*a^5*b^19*c^13*d^11 - 2682536*a
^5*b^19*c^15*d^9 + 655084*a^5*b^19*c^17*d^7 - 85484*a^5*b^19*c^19*d^5 + 408
0*a^5*b^19*c^21*d^3 - 120960*a^6*b^18*c^4*d^20 + 1266048*a^6*b^18*c^6*d^18
- 4977996*a^6*b^18*c^8*d^16 + 10009720*a^6*b^18*c^10*d^14 - 11209800*a^6*b^
18*c^12*d^12 + 6943760*a^6*b^18*c^14*d^10 - 2253214*a^6*b^18*c^16*d^8 + 396
878*a^6*b^18*c^18*d^6 - 36120*a^6*b^18*c^20*d^4 + 1520*a^6*b^18*c^22*d^2 +
69120*a^7*b^17*c^3*d^21 - 1116288*a^7*b^17*c^5*d^19 + 5575008*a^7*b^17*c^7*
d^17 - 13668308*a^7*b^17*c^9*d^15 + 18576800*a^7*b^17*c^11*d^13 - 14230520*
a^7*b^17*c^13*d^11 + 5889904*a^7*b^17*c^15*d^9 - 1310504*a^7*b^17*c^17*d^7
+ 168344*a^7*b^17*c^19*d^5 - 8960*a^7*b^17*c^21*d^3 - 25920*a^8*b^16*c^2*d^
22 + 756000*a^8*b^16*c^4*d^20 - 4977996*a^8*b^16*c^6*d^18 + 15144828*a^8*b^
16*c^8*d^16 - 25068800*a^8*b^16*c^10*d^14 + 23486280*a^8*b^16*c^12*d^12 - 1
2099640*a^8*b^16*c^14*d^10 + 3330518*a^8*b^16*c^16*d^8 - 529224*a^8*b^16*c^
18*d^6 + 36280*a^8*b^16*c^20*d^4 - 379200*a^9*b^15*c^3*d^21 + 3539128*a^9*b
^15*c^5*d^19 - 13668308*a^9*b^15*c^7*d^17 + 27691952*a^9*b^15*c^9*d^15 - 31
562040*a^9*b^15*c^11*d^13 + 19993760*a^9*b^15*c^13*d^11 - 6731044*a^9*b^15*
c^15*d^9 + 1239264*a^9*b^15*c^17*d^7 - 106016*a^9*b^15*c^19*d^5 + 131904*a^
10*b^14*c^2*d^22 - 1987844*a^10*b^14*c^4*d^20 + 10009720*a^10*b^14*c^6*d^18
 - 25068800*a^10*b^14*c^8*d^16 + 34796936*a^10*b^14*c^10*d^14 - 26927144*a^
10*b^14*c^12*d^12 + 10994964*a^10*b^14*c^14*d^10 - 2295680*a^10*b^14*c^16*d
^8 + 230240*a^10*b^14*c^18*d^6 + 860368*a^11*b^13*c^3*d^21 - 5890780*a^11*b
^13*c^5*d^19 + 18576800*a^11*b^13*c^7*d^17 - 31562040*a^11*b^13*c^9*d^15 +
29722864*a^11*b^13*c^11*d^13 - 14679348*a^11*b^13*c^13*d^11 + 3497920*a^11*
b^13*c^15*d^9 - 381280*a^11*b^13*c^17*d^7 - 270604*a^12*b^12*c^2*d^22 + 275
0664*a^12*b^12*c^4*d^20 - 11209800*a^12*b^12*c^6*d^18 + 23486280*a^12*b^12*
c^8*d^16 - 26927144*a^12*b^12*c^10*d^14 + 16147404*a^12*b^12*c^12*d^12 - 44
79104*a^12*b^12*c^14*d^10 + 499520*a^12*b^12*c^16*d^8 - 1001364*a^13*b^11*c
^3*d^21 + 5437600*a^13*b^11*c^5*d^19 - 14230520*a^13*b^11*c^7*d^17 + 199937
60*a^13*b^11*c^9*d^15 - 14679348*a^13*b^11*c^11*d^13 + 4861024*a^13*b^11*c^
13*d^11 - 552160*a^13*b^11*c^15*d^9 + 273544*a^14*b^10*c^2*d^22 - 2073976*a
^14*b^10*c^4*d^20 + 6943760*a^14*b^10*c^6*d^18 - 12099640*a^14*b^10*c^8*d^1
6 + 10994964*a^14*b^10*c^10*d^14 - 4479104*a^14*b^10*c^12*d^12 + 562016*a^1
4*b^10*c^14*d^10 + 605280*a^15*b^9*c^3*d^21 - 2682536*a^15*b^9*c^5*d^19 + 5

889904*a^15*b^9*c^7*d^17 - 6731044*a^15*b^9*c^9*d^15 + 3497920*a^15*b^9*c^1
1*d^13 - 552160*a^15*b^9*c^13*d^11 - 131660*a^16*b^8*c^2*d^22 + 793460*a^16
*b^8*c^4*d^20 - 2253214*a^16*b^8*c^6*d^18 + 3330518*a^16*b^8*c^8*d^16 - 229
5680*a^16*b^8*c^10*d^14 + 499520*a^16*b^8*c^12*d^12 - 167520*a^17*b^7*c^3*d
^21 + 655084*a^17*b^7*c^5*d^19 - 1310504*a^17*b^7*c^7*d^17 + 1239264*a^17*b
^7*c^9*d^15 - 381280*a^17*b^7*c^11*d^13 + 22060*a^18*b^6*c^2*d^22 - 138010*
a^18*b^6*c^4*d^20 + 396878*a^18*b^6*c^6*d^18 - 529224*a^18*b^6*c^8*d^16 + 2
30240*a^18*b^6*c^10*d^14 + 18840*a^19*b^5*c^3*d^21 - 85484*a^19*b^5*c^5*d^1
9 + 168344*a^19*b^5*c^7*d^17 - 106016*a^19*b^5*c^9*d^15 - 782*a^20*b^4*c^2*
d^22 + 10562*a^20*b^4*c^4*d^20 - 36120*a^20*b^4*c^6*d^18 + 36280*a^20*b^4*c
^8*d^16 - 144*a^21*b^3*c^3*d^21 + 4080*a^21*b^3*c^5*d^19 - 8960*a^21*b^3*c^
7*d^17 - 98*a^22*b^2*c^2*d^22 + 88*a^22*b^2*c^4*d^20 + 1520*a^22*b^2*c^6*d^
18 - 4*a*b^23*c^23*d - 4*a^23*b*c*d^23)/(16*(5*a^2*b^28*c^30 - b^30*c^30 -
a^30*d^30 - 10*a^4*b^26*c^30 + 10*a^6*b^24*c^30 - 5*a^8*b^22*c^30 + a^10*b^
20*c^30 + a^20*b^10*d^30 - 5*a^22*b^8*d^30 + 10*a^24*b^6*d^30 - 10*a^26*b^4
*d^30 + 5*a^28*b^2*d^30 + 5*a^30*c^2*d^28 - 10*a^30*c^4*d^26 + 10*a^30*c^6*
d^24 - 5*a^30*c^8*d^22 + a^30*c^10*d^20 + b^30*c^20*d^10 - 5*b^30*c^22*d^8
+ 10*b^30*c^24*d^6 - 10*b^30*c^26*d^4 + 5*b^30*c^28*d^2 - 20*a*b^29*c^19*d^
11 + 100*a*b^29*c^21*d^9 - 200*a*b^29*c^23*d^7 + 200*a*b^29*c^25*d^5 - 100*
a*b^29*c^27*d^3 - 100*a^3*b^27*c^29*d + 200*a^5*b^25*c^29*d - 200*a^7*b^23*
c^29*d + 100*a^9*b^21*c^29*d - 20*a^11*b^19*c^29*d - 20*a^19*b^11*c*d^29 +
100*a^21*b^9*c*d^29 - 200*a^23*b^7*c*d^29 + 200*a^25*b^5*c*d^29 - 100*a^27*
b^3*c*d^29 - 100*a^29*b*c^3*d^27 + 200*a^29*b*c^5*d^25 - 200*a^29*b*c^7*d^2
3 + 100*a^29*b*c^9*d^21 - 20*a^29*b*c^11*d^19 + 190*a^2*b^28*c^18*d^12 - 95
5*a^2*b^28*c^20*d^10 + 1925*a^2*b^28*c^22*d^8 - 1950*a^2*b^28*c^24*d^6 + 10
00*a^2*b^28*c^26*d^4 - 215*a^2*b^28*c^28*d^2 - 1140*a^3*b^27*c^17*d^13 + 58
00*a^3*b^27*c^19*d^11 - 11900*a^3*b^27*c^21*d^9 + 12400*a^3*b^27*c^23*d^7 -
6700*a^3*b^27*c^25*d^5 + 1640*a^3*b^27*c^27*d^3 + 4845*a^4*b^26*c^16*d^14
- 25175*a^4*b^26*c^18*d^12 + 53210*a^4*b^26*c^20*d^10 - 58000*a^4*b^26*c^22
*d^8 + 33825*a^4*b^26*c^24*d^6 - 9695*a^4*b^26*c^26*d^4 + 1000*a^4*b^26*c^2
8*d^2 - 15504*a^5*b^25*c^15*d^15 + 83220*a^5*b^25*c^17*d^13 - 183740*a^5*b^
25*c^19*d^11 + 213040*a^5*b^25*c^21*d^9 - 136520*a^5*b^25*c^23*d^7 + 46004*
a^5*b^25*c^25*d^5 - 6700*a^5*b^25*c^27*d^3 + 38760*a^6*b^24*c^14*d^16 - 218
025*a^6*b^24*c^16*d^14 + 510625*a^6*b^24*c^18*d^12 - 639360*a^6*b^24*c^20*d
^10 + 455100*a^6*b^24*c^22*d^8 - 178985*a^6*b^24*c^24*d^6 + 33825*a^6*b^24*
c^26*d^4 - 1950*a^6*b^24*c^28*d^2 - 77520*a^7*b^23*c^13*d^17 + 465120*a^7*b
^23*c^15*d^15 - 1174200*a^7*b^23*c^17*d^13 + 1607600*a^7*b^23*c^19*d^11 - 1
277800*a^7*b^23*c^21*d^9 + 581120*a^7*b^23*c^23*d^7 - 136520*a^7*b^23*c^25*
d^5 + 12400*a^7*b^23*c^27*d^3 + 125970*a^8*b^22*c^12*d^18 - 823650*a^8*b^22
*c^14*d^16 + 2277150*a^8*b^22*c^16*d^14 - 3441850*a^8*b^22*c^18*d^12 + 3061
855*a^8*b^22*c^20*d^10 - 1598495*a^8*b^22*c^22*d^8 + 455100*a^8*b^22*c^24*d
^6 - 58000*a^8*b^22*c^26*d^4 + 1925*a^8*b^22*c^28*d^2 - 167960*a^9*b^21*c^1
1*d^19 + 1227400*a^9*b^21*c^13*d^17 - 3772640*a^9*b^21*c^15*d^15 + 6342200*
a^9*b^21*c^17*d^13 - 6323300*a^9*b^21*c^19*d^11 + 3770860*a^9*b^21*c^21*d^9
 - 1277800*a^9*b^21*c^23*d^7 + 213040*a^9*b^21*c^25*d^5 - 11900*a^9*b^21*c^

27*d^3 + 184756*a^10*b^20*c^10*d^20 - 1553630*a^10*b^20*c^12*d^18 + 5384410
*a^10*b^20*c^14*d^16 - 10132510*a^10*b^20*c^16*d^14 + 11341480*a^10*b^20*c^
18*d^12 - 7699257*a^10*b^20*c^20*d^10 + 3061855*a^10*b^20*c^22*d^8 - 639360
*a^10*b^20*c^24*d^6 + 53210*a^10*b^20*c^26*d^4 - 955*a^10*b^20*c^28*d^2 - 1
67960*a^11*b^19*c^9*d^21 + 1679600*a^11*b^19*c^11*d^19 - 6653800*a^11*b^19*
c^13*d^17 + 14108640*a^11*b^19*c^15*d^15 - 17770700*a^11*b^19*c^17*d^13 + 1
3697880*a^11*b^19*c^19*d^11 - 6323300*a^11*b^19*c^21*d^9 + 1607600*a^11*b^1
9*c^23*d^7 - 183740*a^11*b^19*c^25*d^5 + 5800*a^11*b^19*c^27*d^3 + 125970*a
^12*b^18*c^8*d^22 - 1553630*a^12*b^18*c^10*d^20 + 7138300*a^12*b^18*c^12*d^
18 - 17183600*a^12*b^18*c^14*d^16 + 24426875*a^12*b^18*c^16*d^14 - 21339185
*a^12*b^18*c^18*d^12 + 11341480*a^12*b^18*c^20*d^10 - 3441850*a^12*b^18*c^2
2*d^8 + 510625*a^12*b^18*c^24*d^6 - 25175*a^12*b^18*c^26*d^4 + 190*a^12*b^1
8*c^28*d^2 - 77520*a^13*b^17*c^7*d^23 + 1227400*a^13*b^17*c^9*d^21 - 665380
0*a^13*b^17*c^11*d^19 + 18346400*a^13*b^17*c^13*d^17 - 29535120*a^13*b^17*c
^15*d^15 + 29213260*a^13*b^17*c^17*d^13 - 17770700*a^13*b^17*c^19*d^11 + 63
42200*a^13*b^17*c^21*d^9 - 1174200*a^13*b^17*c^23*d^7 + 83220*a^13*b^17*c^2
5*d^5 - 1140*a^13*b^17*c^27*d^3 + 38760*a^14*b^16*c^6*d^24 - 823650*a^14*b^
16*c^8*d^22 + 5384410*a^14*b^16*c^10*d^20 - 17183600*a^14*b^16*c^12*d^18 +
31460200*a^14*b^16*c^14*d^16 - 35234455*a^14*b^16*c^16*d^14 + 24426875*a^14
*b^16*c^18*d^12 - 10132510*a^14*b^16*c^20*d^10 + 2277150*a^14*b^16*c^22*d^8
 - 218025*a^14*b^16*c^24*d^6 + 4845*a^14*b^16*c^26*d^4 - 15504*a^15*b^15*c^
5*d^25 + 465120*a^15*b^15*c^7*d^23 - 3772640*a^15*b^15*c^9*d^21 + 14108640*
a^15*b^15*c^11*d^19 - 29535120*a^15*b^15*c^13*d^17 + 37499008*a^15*b^15*c^1
5*d^15 - 29535120*a^15*b^15*c^17*d^13 + 14108640*a^15*b^15*c^19*d^11 - 3772
640*a^15*b^15*c^21*d^9 + 465120*a^15*b^15*c^23*d^7 - 15504*a^15*b^15*c^25*d
^5 + 4845*a^16*b^14*c^4*d^26 - 218025*a^16*b^14*c^6*d^24 + 2277150*a^16*b^1
4*c^8*d^22 - 10132510*a^16*b^14*c^10*d^20 + 24426875*a^16*b^14*c^12*d^18 -
35234455*a^16*b^14*c^14*d^16 + 31460200*a^16*b^14*c^16*d^14 - 17183600*a^16
*b^14*c^18*d^12 + 5384410*a^16*b^14*c^20*d^10 - 823650*a^16*b^14*c^22*d^8 +
 38760*a^16*b^14*c^24*d^6 - 1140*a^17*b^13*c^3*d^27 + 83220*a^17*b^13*c^5*d
^25 - 1174200*a^17*b^13*c^7*d^23 + 6342200*a^17*b^13*c^9*d^21 - 17770700*a^
17*b^13*c^11*d^19 + 29213260*a^17*b^13*c^13*d^17 - 29535120*a^17*b^13*c^15*
d^15 + 18346400*a^17*b^13*c^17*d^13 - 6653800*a^17*b^13*c^19*d^11 + 1227400
*a^17*b^13*c^21*d^9 - 77520*a^17*b^13*c^23*d^7 + 190*a^18*b^12*c^2*d^28 - 2
5175*a^18*b^12*c^4*d^26 + 510625*a^18*b^12*c^6*d^24 - 3441850*a^18*b^12*c^8
*d^22 + 11341480*a^18*b^12*c^10*d^20 - 21339185*a^18*b^12*c^12*d^18 + 24426
875*a^18*b^12*c^14*d^16 - 17183600*a^18*b^12*c^16*d^14 + 7138300*a^18*b^12*
c^18*d^12 - 1553630*a^18*b^12*c^20*d^10 + 125970*a^18*b^12*c^22*d^8 + 5800*
a^19*b^11*c^3*d^27 - 183740*a^19*b^11*c^5*d^25 + 1607600*a^19*b^11*c^7*d^23
 - 6323300*a^19*b^11*c^9*d^21 + 13697880*a^19*b^11*c^11*d^19 - 17770700*a^1
9*b^11*c^13*d^17 + 14108640*a^19*b^11*c^15*d^15 - 6653800*a^19*b^11*c^17*d^
13 + 1679600*a^19*b^11*c^19*d^11 - 167960*a^19*b^11*c^21*d^9 - 955*a^20*b^1
0*c^2*d^28 + 53210*a^20*b^10*c^4*d^26 - 639360*a^20*b^10*c^6*d^24 + 3061855
*a^20*b^10*c^8*d^22 - 7699257*a^20*b^10*c^10*d^20 + 11341480*a^20*b^10*c^12
*d^18 - 10132510*a^20*b^10*c^14*d^16 + 5384410*a^20*b^10*c^16*d^14 - 155363

0*a^20*b^10*c^18*d^12 + 184756*a^20*b^10*c^20*d^10 - 11900*a^21*b^9*c^3*d^2
7 + 213040*a^21*b^9*c^5*d^25 - 1277800*a^21*b^9*c^7*d^23 + 3770860*a^21*b^9
*c^9*d^21 - 6323300*a^21*b^9*c^11*d^19 + 6342200*a^21*b^9*c^13*d^17 - 37726
40*a^21*b^9*c^15*d^15 + 1227400*a^21*b^9*c^17*d^13 - 167960*a^21*b^9*c^19*d
^11 + 1925*a^22*b^8*c^2*d^28 - 58000*a^22*b^8*c^4*d^26 + 455100*a^22*b^8*c^
6*d^24 - 1598495*a^22*b^8*c^8*d^22 + 3061855*a^22*b^8*c^10*d^20 - 3441850*a
^22*b^8*c^12*d^18 + 2277150*a^22*b^8*c^14*d^16 - 823650*a^22*b^8*c^16*d^14
+ 125970*a^22*b^8*c^18*d^12 + 12400*a^23*b^7*c^3*d^27 - 136520*a^23*b^7*c^5
*d^25 + 581120*a^23*b^7*c^7*d^23 - 1277800*a^23*b^7*c^9*d^21 + 1607600*a^23
*b^7*c^11*d^19 - 1174200*a^23*b^7*c^13*d^17 + 465120*a^23*b^7*c^15*d^15 - 7
7520*a^23*b^7*c^17*d^13 - 1950*a^24*b^6*c^2*d^28 + 33825*a^24*b^6*c^4*d^26
- 178985*a^24*b^6*c^6*d^24 + 455100*a^24*b^6*c^8*d^22 - 639360*a^24*b^6*c^1
0*d^20 + 510625*a^24*b^6*c^12*d^18 - 218025*a^24*b^6*c^14*d^16 + 38760*a^24
*b^6*c^16*d^14 - 6700*a^25*b^5*c^3*d^27 + 46004*a^25*b^5*c^5*d^25 - 136520*
a^25*b^5*c^7*d^23 + 213040*a^25*b^5*c^9*d^21 - 183740*a^25*b^5*c^11*d^19 +
83220*a^25*b^5*c^13*d^17 - 15504*a^25*b^5*c^15*d^15 + 1000*a^26*b^4*c^2*d^2
8 - 9695*a^26*b^4*c^4*d^26 + 33825*a^26*b^4*c^6*d^24 - 58000*a^26*b^4*c^8*d
^22 + 53210*a^26*b^4*c^10*d^20 - 25175*a^26*b^4*c^12*d^18 + 4845*a^26*b^4*c
^14*d^16 + 1640*a^27*b^3*c^3*d^27 - 6700*a^27*b^3*c^5*d^25 + 12400*a^27*b^3
*c^7*d^23 - 11900*a^27*b^3*c^9*d^21 + 5800*a^27*b^3*c^11*d^19 - 1140*a^27*b
^3*c^13*d^17 - 215*a^28*b^2*c^2*d^28 + 1000*a^28*b^2*c^4*d^26 - 1950*a^28*b
^2*c^6*d^24 + 1925*a^28*b^2*c^8*d^22 - 955*a^28*b^2*c^10*d^20 + 190*a^28*b^
2*c^12*d^18 + 20*a*b^29*c^29*d + 20*a^29*b*c*d^29)))^(1/2) + (4*(4*a^2*b^20
*c^22 - 12*a^6*b^16*c^22 + 8*a^8*b^14*c^22 + 4*a^22*c^2*d^20 - 12*a^22*c^6*
d^16 + 8*a^22*c^8*d^14 + 48*a*b^21*c^11*d^11 - 212*a*b^21*c^13*d^9 + 360*a*
b^21*c^15*d^7 - 276*a*b^21*c^17*d^5 + 80*a*b^21*c^19*d^3 - 20*a^3*b^19*c^21
*d - 72*a^5*b^17*c^21*d + 204*a^7*b^15*c^21*d - 112*a^9*b^13*c^21*d + 48*a^
11*b^11*c*d^21 - 212*a^13*b^9*c*d^21 + 360*a^15*b^7*c*d^21 - 276*a^17*b^5*c
*d^21 + 80*a^19*b^3*c*d^21 - 20*a^21*b*c^3*d^19 - 72*a^21*b*c^5*d^17 + 204*
a^21*b*c^7*d^15 - 112*a^21*b*c^9*d^13 - 480*a^2*b^20*c^10*d^12 + 2160*a^2*b
^20*c^12*d^10 - 3772*a^2*b^20*c^14*d^8 + 3020*a^2*b^20*c^16*d^6 - 960*a^2*b
^20*c^18*d^4 + 28*a^2*b^20*c^20*d^2 + 2160*a^3*b^19*c^9*d^13 - 10152*a^3*b^
19*c^11*d^11 + 18888*a^3*b^19*c^13*d^9 - 16732*a^3*b^19*c^15*d^7 + 6588*a^3
*b^19*c^17*d^5 - 732*a^3*b^19*c^19*d^3 - 5760*a^4*b^18*c^8*d^14 + 29360*a^4
*b^18*c^10*d^12 - 60792*a^4*b^18*c^12*d^10 + 62708*a^4*b^18*c^14*d^8 - 3189
2*a^4*b^18*c^16*d^6 + 6588*a^4*b^18*c^18*d^4 - 212*a^4*b^18*c^20*d^2 + 1008
0*a^5*b^17*c^7*d^15 - 58860*a^5*b^17*c^9*d^13 + 141880*a^5*b^17*c^11*d^11 -
175592*a^5*b^17*c^13*d^9 + 113748*a^5*b^17*c^15*d^7 - 34492*a^5*b^17*c^17*
d^5 + 3308*a^5*b^17*c^19*d^3 - 12096*a^6*b^16*c^6*d^16 + 87264*a^6*b^16*c^8
*d^14 - 254340*a^6*b^16*c^10*d^12 + 381532*a^6*b^16*c^12*d^10 - 307752*a^6*
b^16*c^14*d^8 + 125568*a^6*b^16*c^16*d^6 - 21232*a^6*b^16*c^18*d^4 + 1068*a
^6*b^16*c^20*d^2 + 10080*a^7*b^15*c^5*d^17 - 99120*a^7*b^15*c^7*d^15 + 3590
64*a^7*b^15*c^9*d^13 - 655076*a^7*b^15*c^11*d^11 + 650108*a^7*b^15*c^13*d^9
- 343368*a^7*b^15*c^15*d^7 + 85760*a^7*b^15*c^17*d^5 - 7652*a^7*b^15*c^19*
d^3 - 5760*a^8*b^14*c^4*d^18 + 87264*a^8*b^14*c^6*d^16 - 402576*a^8*b^14*c^

8*d^14 + 900324*a^8*b^14*c^10*d^12 - 1096236*a^8*b^14*c^12*d^10 + 731392*a^8*b^14*c^14*d^8 - 247352*a^8*b^14*c^16*d^6 + 34548*a^8*b^14*c^18*d^4 - 1612*a^8*b^14*c^20*d^2 + 2160*a^9*b^13*c^3*d^19 - 58860*a^9*b^13*c^5*d^17 + 359064*a^9*b^13*c^7*d^15 - 999816*a^9*b^13*c^9*d^13 + 1494564*a^9*b^13*c^11*d^11 - 1238148*a^9*b^13*c^13*d^9 + 542272*a^9*b^13*c^15*d^7 - 109032*a^9*b^13*c^17*d^5 + 7908*a^9*b^13*c^19*d^3 - 480*a^10*b^12*c^2*d^20 + 29360*a^10*b^12*c^4*d^18 - 254340*a^10*b^12*c^6*d^16 + 900324*a^10*b^12*c^8*d^14 - 1656496*a^10*b^12*c^10*d^12 + 1688232*a^10*b^12*c^12*d^10 - 934868*a^10*b^12*c^14*d^8 + 254492*a^10*b^12*c^16*d^6 - 26952*a^10*b^12*c^18*d^4 + 728*a^10*b^12*c^20*d^2 - 10152*a^11*b^11*c^3*d^19 + 141880*a^11*b^11*c^5*d^17 - 655076*a^11*b^11*c^7*d^15 + 1494564*a^11*b^11*c^9*d^13 - 1870136*a^11*b^11*c^11*d^11 + 1289704*a^11*b^11*c^13*d^9 - 455388*a^11*b^11*c^15*d^7 + 67468*a^11*b^11*c^17*d^5 - 2912*a^11*b^11*c^19*d^3 + 2160*a^12*b^10*c^2*d^20 - 60792*a^12*b^10*c^4*d^18 + 381532*a^12*b^10*c^6*d^16 - 1096236*a^12*b^10*c^8*d^14 + 1688232*a^12*b^10*c^10*d^12 - 1434728*a^12*b^10*c^12*d^10 + 639684*a^12*b^10*c^14*d^8 - 127860*a^12*b^10*c^16*d^6 + 8008*a^12*b^10*c^18*d^4 + 18888*a^13*b^9*c^3*d^19 - 175592*a^13*b^9*c^5*d^17 + 650108*a^13*b^9*c^7*d^15 - 1238148*a^13*b^9*c^9*d^13 + 1289704*a^13*b^9*c^11*d^11 - 715296*a^13*b^9*c^13*d^9 + 186564*a^13*b^9*c^15*d^7 - 16016*a^13*b^9*c^17*d^5 - 3772*a^14*b^8*c^2*d^20 + 62708*a^14*b^8*c^4*d^18 - 307752*a^14*b^8*c^6*d^16 + 731392*a^14*b^8*c^8*d^14 - 934868*a^14*b^8*c^10*d^12 + 639684*a^14*b^8*c^12*d^10 - 211416*a^14*b^8*c^14*d^8 + 24024*a^14*b^8*c^16*d^6 - 16732*a^15*b^7*c^3*d^19 + 113748*a^15*b^7*c^5*d^17 - 343368*a^15*b^7*c^7*d^15 + 542272*a^15*b^7*c^9*d^13 - 455388*a^15*b^7*c^11*d^11 + 186564*a^15*b^7*c^13*d^9 - 27456*a^15*b^7*c^15*d^7 + 3020*a^16*b^6*c^2*d^20 - 31892*a^16*b^6*c^4*d^18 + 125568*a^16*b^6*c^6*d^16 - 247352*a^16*b^6*c^8*d^14 + 254492*a^16*b^6*c^10*d^12 - 127860*a^16*b^6*c^12*d^10 + 24024*a^16*b^6*c^14*d^8 + 6588*a^17*b^5*c^3*d^19 - 34492*a^17*b^5*c^5*d^17 + 85760*a^17*b^5*c^7*d^15 - 109032*a^17*b^5*c^9*d^13 + 67468*a^17*b^5*c^11*d^11 - 16016*a^17*b^5*c^13*d^9 - 960*a^18*b^4*c^2*d^20 + 6588*a^18*b^4*c^4*d^18 - 21232*a^18*b^4*c^6*d^16 + 34548*a^18*b^4*c^8*d^14 - 26952*a^18*b^4*c^10*d^12 + 8008*a^18*b^4*c^12*d^10 - 732*a^19*b^3*c^3*d^19 + 3308*a^19*b^3*c^5*d^17 - 7652*a^19*b^3*c^7*d^15 + 7908*a^19*b^3*c^9*d^13 - 2912*a^19*b^3*c^11*d^11 + 28*a^20*b^2*c^2*d^20 - 212*a^20*b^2*c^4*d^18 + 1068*a^20*b^2*c^6*d^16 - 1612*a^20*b^2*c^8*d^14 + 728*a^20*b^2*c^10*d^12))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^

17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16
*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^1
6*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*
c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*
c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c
^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^1
4*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^1
4*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^1
3*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b
^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^
12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^
8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^
8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^
9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504
*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^
10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 363
00*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^
8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*
d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d
^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^
13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*
d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^
8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8
*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*
c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b
^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6
*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b
^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*
b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*
b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b
^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^
4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^
4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3
*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2
*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^
12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19) - (8*tan(
e/2 + (f*x)/2)*(12*a^5*b^17*c^22 - 4*a^22*c*d^21 - 4*a*b^21*c^22 - 8*a^7*b^
15*c^22 + 12*a^22*c^5*d^17 - 8*a^22*c^7*d^15 - 24*a*b^21*c^12*d^10 + 100*a*
b^21*c^14*d^8 - 164*a*b^21*c^16*d^6 + 120*a*b^21*c^18*d^4 - 28*a*b^21*c^20*
d^2 + 20*a^2*b^20*c^21*d + 72*a^4*b^18*c^21*d - 204*a^6*b^16*c^21*d + 112*a
^8*b^14*c^21*d - 24*a^12*b^10*c*d^21 + 100*a^14*b^8*c*d^21 - 164*a^16*b^6*c
*d^21 + 120*a^18*b^4*c*d^21 - 28*a^20*b^2*c*d^21 + 20*a^21*b*c^2*d^20 + 72*
a^21*b*c^4*d^18 - 204*a^21*b*c^6*d^16 + 112*a^21*b*c^8*d^14 + 216*a^2*b^20*
c^11*d^11 - 908*a^2*b^20*c^13*d^9 + 1540*a^2*b^20*c^15*d^7 - 1200*a^2*b^20*
c^17*d^5 + 332*a^2*b^20*c^19*d^3 - 840*a^3*b^19*c^10*d^12 + 3672*a^3*b^19*c

^12*d^10 - 6788*a^3*b^19*c^14*d^8 + 6132*a^3*b^19*c^16*d^6 - 2388*a^3*b^19*c^18*d^4 + 212*a^3*b^19*c^20*d^2 + 1800*a^4*b^18*c^9*d^13 - 8680*a^4*b^18*c^11*d^11 + 18852*a^4*b^18*c^13*d^9 - 21228*a^4*b^18*c^15*d^7 + 11692*a^4*b^18*c^17*d^5 - 2508*a^4*b^18*c^19*d^3 - 2160*a^5*b^17*c^8*d^14 + 13100*a^5*b^17*c^10*d^12 - 36820*a^5*b^17*c^12*d^10 + 53712*a^5*b^17*c^14*d^8 - 39608*a^5*b^17*c^16*d^6 + 12832*a^5*b^17*c^18*d^4 - 1068*a^5*b^17*c^20*d^2 + 1008*a^6*b^16*c^7*d^15 - 12420*a^6*b^16*c^9*d^13 + 51764*a^6*b^16*c^11*d^11 - 100128*a^6*b^16*c^13*d^9 + 96048*a^6*b^16*c^15*d^7 - 42920*a^6*b^16*c^17*d^5 + 6852*a^6*b^16*c^19*d^3 + 1008*a^7*b^15*c^6*d^16 + 5136*a^7*b^15*c^8*d^14 - 48820*a^7*b^15*c^10*d^12 + 134700*a^7*b^15*c^12*d^10 - 171472*a^7*b^15*c^14*d^8 + 103992*a^7*b^15*c^16*d^6 - 26148*a^7*b^15*c^18*d^4 + 1612*a^7*b^15*c^20*d^2 - 2160*a^8*b^14*c^5*d^17 + 5136*a^8*b^14*c^7*d^15 + 20436*a^8*b^14*c^9*d^13 - 121524*a^8*b^14*c^11*d^11 + 224888*a^8*b^14*c^13*d^9 - 186952*a^8*b^14*c^15*d^7 + 67572*a^8*b^14*c^17*d^5 - 7508*a^8*b^14*c^19*d^3 + 1800*a^9*b^13*c^4*d^18 - 12420*a^9*b^13*c^6*d^16 + 20436*a^9*b^13*c^8*d^14 + 49416*a^9*b^13*c^10*d^12 - 201552*a^9*b^13*c^12*d^10 + 245708*a^9*b^13*c^14*d^8 - 125412*a^9*b^13*c^16*d^6 + 22752*a^9*b^13*c^18*d^4 - 728*a^9*b^13*c^20*d^2 - 840*a^10*b^12*c^3*d^19 + 13100*a^10*b^12*c^5*d^17 - 48820*a^10*b^12*c^7*d^15 + 49416*a^10*b^12*c^9*d^13 + 82088*a^10*b^12*c^11*d^11 - 219092*a^10*b^12*c^13*d^9 + 168468*a^10*b^12*c^15*d^7 - 47152*a^10*b^12*c^17*d^5 + 2832*a^10*b^12*c^19*d^3 + 216*a^11*b^11*c^2*d^20 - 8680*a^11*b^11*c^4*d^18 + 51764*a^11*b^11*c^6*d^16 - 121524*a^11*b^11*c^8*d^14 + 82088*a^11*b^11*c^10*d^12 + 88712*a^11*b^11*c^12*d^10 - 153012*a^11*b^11*c^14*d^8 + 67604*a^11*b^11*c^16*d^6 - 7168*a^11*b^11*c^18*d^4 + 3672*a^12*b^10*c^3*d^19 - 36820*a^12*b^10*c^5*d^17 + 134700*a^12*b^10*c^7*d^15 - 201552*a^12*b^10*c^9*d^13 + 88712*a^12*b^10*c^11*d^11 + 62676*a^12*b^10*c^13*d^9 - 63372*a^12*b^10*c^15*d^7 + 12008*a^12*b^10*c^17*d^5 - 908*a^13*b^9*c^2*d^20 + 18852*a^13*b^9*c^4*d^18 - 100128*a^13*b^9*c^6*d^16 + 224888*a^13*b^9*c^8*d^14 - 219092*a^13*b^9*c^10*d^12 + 62676*a^13*b^9*c^12*d^10 + 26256*a^13*b^9*c^14*d^8 - 12544*a^13*b^9*c^16*d^6 - 6788*a^14*b^8*c^3*d^19 + 53712*a^14*b^8*c^5*d^17 - 171472*a^14*b^8*c^7*d^15 + 245708*a^14*b^8*c^9*d^13 - 153012*a^14*b^8*c^11*d^11 + 26256*a^14*b^8*c^13*d^9 + 5496*a^14*b^8*c^15*d^7 + 1540*a^15*b^7*c^2*d^20 - 21228*a^15*b^7*c^4*d^18 + 96048*a^15*b^7*c^6*d^16 - 186952*a^15*b^7*c^8*d^14 + 168468*a^15*b^7*c^10*d^12 - 63372*a^15*b^7*c^12*d^10 + 5496*a^15*b^7*c^14*d^8 + 6132*a^16*b^6*c^3*d^19 - 39608*a^16*b^6*c^5*d^17 + 103992*a^16*b^6*c^7*d^15 - 125412*a^16*b^6*c^9*d^13 + 67604*a^16*b^6*c^11*d^11 - 12544*a^16*b^6*c^13*d^9 - 1200*a^17*b^5*c^2*d^20 + 11692*a^17*b^5*c^4*d^18 - 42920*a^17*b^5*c^6*d^16 + 67572*a^17*b^5*c^8*d^14 - 47152*a^17*b^5*c^10*d^12 + 12008*a^17*b^5*c^12*d^10 - 2388*a^18*b^4*c^3*d^19 + 12832*a^18*b^4*c^5*d^17 - 26148*a^18*b^4*c^7*d^15 + 22752*a^18*b^4*c^9*d^13 - 7168*a^18*b^4*c^11*d^11 + 332*a^19*b^3*c^2*d^20 - 2508*a^19*b^3*c^4*d^18 + 6852*a^19*b^3*c^6*d^16 - 7508*a^19*b^3*c^8*d^14 + 2832*a^19*b^3*c^10*d^12 + 212*a^20*b^2*c^3*d^19 - 1068*a^20*b^2*c^5*d^17 + 1612*a^20*b^2*c^7*d^15 - 728*a^20*b^2*c^9*d^13))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^

20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19)) + (4*(

288*a*b^18*c^6*d^13 - 1104*a*b^18*c^8*d^11 + 1538*a*b^18*c^10*d^9 - 872*a*b
^18*c^12*d^7 + 108*a*b^18*c^14*d^5 + 40*a*b^18*c^16*d^3 + 8*a^3*b^16*c^18*d
 + 8*a^5*b^14*c^18*d + 288*a^6*b^13*c*d^18 - 1104*a^8*b^11*c*d^18 + 1538*a^
10*b^9*c*d^18 - 872*a^12*b^7*c*d^18 + 108*a^14*b^5*c*d^18 + 40*a^16*b^3*c*d
^18 + 8*a^18*b*c^3*d^16 + 8*a^18*b*c^5*d^14 - 864*a^2*b^17*c^5*d^14 + 3216*
a^2*b^17*c^7*d^12 - 4262*a^2*b^17*c^9*d^10 + 2256*a^2*b^17*c^11*d^8 - 304*a
^2*b^17*c^13*d^6 - 32*a^2*b^17*c^15*d^4 + 8*a^2*b^17*c^17*d^2 + 576*a^3*b^1
6*c^4*d^15 - 3024*a^3*b^16*c^6*d^13 + 6304*a^3*b^16*c^8*d^11 - 7216*a^3*b^1
6*c^10*d^9 + 4944*a^3*b^16*c^12*d^7 - 1664*a^3*b^16*c^14*d^5 - 72*a^3*b^16*
c^16*d^3 + 576*a^4*b^15*c^3*d^16 + 912*a^4*b^15*c^5*d^14 - 8720*a^4*b^15*c^
7*d^12 + 16632*a^4*b^15*c^9*d^10 - 14888*a^4*b^15*c^11*d^8 + 6704*a^4*b^15*
c^13*d^6 - 744*a^4*b^15*c^15*d^4 - 40*a^4*b^15*c^17*d^2 - 864*a^5*b^14*c^2*
d^17 + 912*a^5*b^14*c^4*d^15 + 5140*a^5*b^14*c^6*d^13 - 16080*a^5*b^14*c^8*
d^11 + 23520*a^5*b^14*c^10*d^9 - 20208*a^5*b^14*c^12*d^7 + 7404*a^5*b^14*c^
14*d^5 - 264*a^5*b^14*c^16*d^3 - 3024*a^6*b^13*c^3*d^16 + 5140*a^6*b^13*c^5
*d^14 + 5280*a^6*b^13*c^7*d^12 - 28380*a^6*b^13*c^9*d^10 + 39792*a^6*b^13*c
^11*d^8 - 22728*a^6*b^13*c^13*d^6 + 3096*a^6*b^13*c^15*d^4 - 112*a^6*b^13*c
^17*d^2 + 3216*a^7*b^12*c^2*d^17 - 8720*a^7*b^12*c^4*d^15 + 5280*a^7*b^12*c
^6*d^13 + 15000*a^7*b^12*c^8*d^11 - 40656*a^7*b^12*c^10*d^9 + 40296*a^7*b^1
2*c^12*d^7 - 12984*a^7*b^12*c^14*d^5 + 728*a^7*b^12*c^16*d^3 + 6304*a^8*b^1
1*c^3*d^16 - 16080*a^8*b^11*c^5*d^14 + 15000*a^8*b^11*c^7*d^12 + 16024*a^8*
b^11*c^9*d^10 - 46184*a^8*b^11*c^11*d^8 + 27208*a^8*b^11*c^13*d^6 - 2752*a^
8*b^11*c^15*d^4 - 4262*a^9*b^10*c^2*d^17 + 16632*a^9*b^10*c^4*d^15 - 28380*
a^9*b^10*c^6*d^13 + 16024*a^9*b^10*c^8*d^11 + 22018*a^9*b^10*c^10*d^9 - 301
04*a^9*b^10*c^12*d^7 + 6488*a^9*b^10*c^14*d^5 - 7216*a^10*b^9*c^3*d^16 + 23
520*a^10*b^9*c^5*d^14 - 40656*a^10*b^9*c^7*d^12 + 22018*a^10*b^9*c^9*d^10 +
 13080*a^10*b^9*c^11*d^8 - 8720*a^10*b^9*c^13*d^6 + 2256*a^11*b^8*c^2*d^17
 - 14888*a^11*b^8*c^4*d^15 + 39792*a^11*b^8*c^6*d^13 - 46184*a^11*b^8*c^8*d^
11 + 13080*a^11*b^8*c^10*d^9 + 4360*a^11*b^8*c^12*d^7 + 4944*a^12*b^7*c^3*d
^16 - 20208*a^12*b^7*c^5*d^14 + 40296*a^12*b^7*c^7*d^12 - 30104*a^12*b^7*c^
9*d^10 + 4360*a^12*b^7*c^11*d^8 - 304*a^13*b^6*c^2*d^17 + 6704*a^13*b^6*c^4
*d^15 - 22728*a^13*b^6*c^6*d^13 + 27208*a^13*b^6*c^8*d^11 - 8720*a^13*b^6*c
^10*d^9 - 1664*a^14*b^5*c^3*d^16 + 7404*a^14*b^5*c^5*d^14 - 12984*a^14*b^5*
c^7*d^12 + 6488*a^14*b^5*c^9*d^10 - 32*a^15*b^4*c^2*d^17 - 744*a^15*b^4*c^4
*d^15 + 3096*a^15*b^4*c^6*d^13 - 2752*a^15*b^4*c^8*d^11 - 72*a^16*b^3*c^3*d
^16 - 264*a^16*b^3*c^5*d^14 + 728*a^16*b^3*c^7*d^12 + 8*a^17*b^2*c^2*d^17 -
 40*a^17*b^2*c^4*d^15 - 112*a^17*b^2*c^6*d^13 + 2*a*b^18*c^18*d + 2*a^18*b*
c*d^18))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6
*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d
^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14
 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^
20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5
+ 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^1
3*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 7
2*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5

*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 2
68*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a
^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3
*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b
^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*
b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^
15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^
15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^1
4*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*
b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*
b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*
b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^
7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8
*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156
*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268
*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744
*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27
504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66
*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 -
36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12
*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^
18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^
7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9
*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c
^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8
*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*
b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b
^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^1
3*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*
b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^1
4*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^
14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^
15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^1
5*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16
*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16
*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*
b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*
c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8
*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19) - (8*t
an(e/2 + (f*x)/2)*(a*b^18*c^19 + a^19*c*d^18 + 4*a^3*b^16*c^19 + 4*a^5*b^14
*c^19 + 4*a^19*c^3*d^16 + 4*a^19*c^5*d^14 - 576*a*b^18*c^5*d^14 + 2640*a*b^
18*c^7*d^12 - 4732*a*b^18*c^9*d^10 + 3961*a*b^18*c^11*d^8 - 1344*a*b^18*c^1
3*d^6 + 14*a*b^18*c^15*d^4 + 18*a*b^18*c^17*d^2 + 4*a^2*b^17*c^18*d - 20*a^
4*b^15*c^18*d - 576*a^5*b^14*c*d^18 - 56*a^6*b^13*c^18*d + 2640*a^7*b^12*c*
d^18 - 4732*a^9*b^10*c*d^18 + 3961*a^11*b^8*c*d^18 - 1344*a^13*b^6*c*d^18 +

14*a^15*b^4*c*d^18 + 18*a^17*b^2*c*d^18 + 4*a^18*b*c^2*d^17 - 20*a^18*b*c^4*d^15 - 56*a^18*b*c^6*d^13 + 2304*a^2*b^17*c^4*d^15 - 10944*a^2*b^17*c^6*d^13 + 20720*a^2*b^17*c^8*d^11 - 18788*a^2*b^17*c^10*d^9 + 7392*a^2*b^17*c^12*d^7 - 520*a^2*b^17*c^14*d^5 - 24*a^2*b^17*c^16*d^3 - 3456*a^3*b^16*c^3*d^16 + 20016*a^3*b^16*c^5*d^14 - 48112*a^3*b^16*c^7*d^12 + 58925*a^3*b^16*c^9*d^10 - 36732*a^3*b^16*c^11*d^8 + 9736*a^3*b^16*c^13*d^6 - 760*a^3*b^16*c^15*d^4 - 44*a^3*b^16*c^17*d^2 + 2304*a^4*b^15*c^2*d^17 - 23424*a^4*b^15*c^4*d^15 + 81680*a^4*b^15*c^6*d^13 - 135520*a^4*b^15*c^8*d^11 + 114144*a^4*b^15*c^10*d^9 - 44168*a^4*b^15*c^12*d^7 + 5696*a^4*b^15*c^14*d^5 - 332*a^4*b^15*c^16*d^3 + 20016*a^5*b^14*c^3*d^16 - 99112*a^5*b^14*c^5*d^14 + 213338*a^5*b^14*c^7*d^12 - 235152*a^5*b^14*c^9*d^10 + 130428*a^5*b^14*c^11*d^8 - 31908*a^5*b^14*c^13*d^6 + 3966*a^5*b^14*c^15*d^4 - 140*a^5*b^14*c^17*d^2 - 10944*a^6*b^13*c^2*d^17 + 81680*a^6*b^13*c^4*d^15 - 243832*a^6*b^13*c^6*d^13 + 364608*a^6*b^13*c^8*d^11 - 281736*a^6*b^13*c^10*d^9 + 103104*a^6*b^13*c^12*d^7 - 16860*a^6*b^13*c^14*d^5 + 1660*a^6*b^13*c^16*d^3 - 48112*a^7*b^12*c^3*d^16 + 213338*a^7*b^12*c^5*d^14 - 425832*a^7*b^12*c^7*d^12 + 434414*a^7*b^12*c^9*d^10 - 219064*a^7*b^12*c^11*d^8 + 50732*a^7*b^12*c^13*d^6 - 7220*a^7*b^12*c^15*d^4 + 364*a^7*b^12*c^17*d^2 + 20720*a^8*b^11*c^2*d^17 - 135520*a^8*b^11*c^4*d^15 + 364608*a^8*b^11*c^6*d^13 - 496336*a^8*b^11*c^8*d^11 + 343832*a^8*b^11*c^10*d^9 - 111220*a^8*b^11*c^12*d^7 + 17956*a^8*b^11*c^14*d^5 - 1376*a^8*b^11*c^16*d^3 + 58925*a^9*b^10*c^3*d^16 - 235152*a^9*b^10*c^5*d^14 + 434414*a^9*b^10*c^7*d^12 - 401788*a^9*b^10*c^9*d^10 + 172673*a^9*b^10*c^11*d^8 - 31940*a^9*b^10*c^13*d^6 + 3244*a^9*b^10*c^15*d^4 - 18788*a^10*b^9*c^2*d^17 + 114144*a^10*b^9*c^4*d^15 - 281736*a^10*b^9*c^6*d^13 + 343832*a^10*b^9*c^8*d^11 - 197840*a^10*b^9*c^10*d^9 + 45940*a^10*b^9*c^12*d^7 - 4760*a^10*b^9*c^14*d^5 - 36732*a^11*b^8*c^3*d^16 + 130428*a^11*b^8*c^5*d^14 - 219064*a^11*b^8*c^7*d^12 + 172673*a^11*b^8*c^9*d^10 - 52480*a^11*b^8*c^11*d^8 + 4580*a^11*b^8*c^13*d^6 + 7392*a^12*b^7*c^2*d^17 - 44168*a^12*b^7*c^4*d^15 + 103104*a^12*b^7*c^6*d^13 - 111220*a^12*b^7*c^8*d^11 + 45940*a^12*b^7*c^10*d^9 - 4000*a^12*b^7*c^12*d^7 + 9736*a^13*b^6*c^3*d^16 - 31908*a^13*b^6*c^5*d^14 + 50732*a^13*b^6*c^7*d^12 - 31940*a^13*b^6*c^9*d^10 + 4580*a^13*b^6*c^11*d^8 - 520*a^14*b^5*c^2*d^17 + 5696*a^14*b^5*c^4*d^15 - 16860*a^14*b^5*c^6*d^13 + 17956*a^14*b^5*c^8*d^11 - 4760*a^14*b^5*c^10*d^9 - 760*a^15*b^4*c^3*d^16 + 3966*a^15*b^4*c^5*d^14 - 7220*a^15*b^4*c^7*d^12 + 3244*a^15*b^4*c^9*d^10 - 24*a^16*b^3*c^2*d^17 - 332*a^16*b^3*c^4*d^15 + 1660*a^16*b^3*c^6*d^13 - 1376*a^16*b^3*c^8*d^11 - 44*a^17*b^2*c^3*d^16 - 140*a^17*b^2*c^5*d^14 + 364*a^17*b^2*c^7*d^12))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66

*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a
^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b
^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^
17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^
16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^1
6*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15
*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15
*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14
*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b
^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^1
3*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*
b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b
^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8
*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*
a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^
9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936
*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928
*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 1386
0*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10
 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^1
6*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d
^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^
11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^
17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^
6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^
8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7
*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b
^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b
^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b
^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^1
4*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15
*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15
*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b
^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b
^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^
3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*
c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*
d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 1
2*a^19*b*c*d^19))*1i)/((8*(11700*a*b^15*c^5*d^11 - 7344*a*b^15*c^3*d^13 - 7
829*a*b^15*c^7*d^9 + 1314*a*b^15*c^9*d^7 + 411*a*b^15*c^11*d^5 + 20*a*b^15*
c^13*d^3 - 7344*a^3*b^13*c*d^15 + 11700*a^5*b^11*c*d^15 - 7829*a^7*b^9*c*d^
15 + 1314*a^9*b^7*c*d^15 + 411*a^11*b^5*c*d^15 + 20*a^13*b^3*c*d^15 + 2592*
a^2*b^14*c^2*d^14 - 10944*a^2*b^14*c^4*d^12 + 16014*a^2*b^14*c^6*d^10 - 911
2*a^2*b^14*c^8*d^8 + 1163*a^2*b^14*c^10*d^6 + 170*a^2*b^14*c^12*d^4 + 34776
*a^3*b^13*c^3*d^13 - 62715*a^3*b^13*c^5*d^11 + 50316*a^3*b^13*c^7*d^9 - 147

34*a^3*b^13*c^9*d^7 - 64*a^3*b^13*c^11*d^5 + 80*a^3*b^13*c^13*d^3 - 10944*a
^4*b^12*c^2*d^14 + 48580*a^4*b^12*c^4*d^12 - 75480*a^4*b^12*c^6*d^10 + 4729
4*a^4*b^12*c^8*d^8 - 9176*a^4*b^12*c^10*d^6 - 40*a^4*b^12*c^12*d^4 - 62715*
a^5*b^11*c^3*d^13 + 126404*a^5*b^11*c^5*d^11 - 113533*a^5*b^11*c^7*d^9 + 40
192*a^5*b^11*c^9*d^7 - 3388*a^5*b^11*c^11*d^5 + 80*a^5*b^11*c^13*d^3 + 1601
4*a^6*b^10*c^2*d^14 - 75480*a^6*b^10*c^4*d^12 + 122510*a^6*b^10*c^6*d^10 -
79106*a^6*b^10*c^8*d^8 + 17020*a^6*b^10*c^10*d^6 - 760*a^6*b^10*c^12*d^4 +
50316*a^7*b^9*c^3*d^13 - 113533*a^7*b^9*c^5*d^11 + 108024*a^7*b^9*c^7*d^9 -
38084*a^7*b^9*c^9*d^7 + 3248*a^7*b^9*c^11*d^5 - 9112*a^8*b^8*c^2*d^14 + 47
294*a^8*b^8*c^4*d^12 - 79106*a^8*b^8*c^6*d^10 + 47096*a^8*b^8*c^8*d^8 - 743
2*a^8*b^8*c^10*d^6 - 14734*a^9*b^7*c^3*d^13 + 40192*a^9*b^7*c^5*d^11 - 3808
4*a^9*b^7*c^7*d^9 + 9728*a^9*b^7*c^9*d^7 + 1163*a^10*b^6*c^2*d^14 - 9176*a^
10*b^6*c^4*d^12 + 17020*a^10*b^6*c^6*d^10 - 7432*a^10*b^6*c^8*d^8 - 64*a^11
*b^5*c^3*d^13 - 3388*a^11*b^5*c^5*d^11 + 3248*a^11*b^5*c^7*d^9 + 170*a^12*b
^4*c^2*d^14 - 40*a^12*b^4*c^4*d^12 - 760*a^12*b^4*c^6*d^10 + 80*a^13*b^3*c^
3*d^13 + 80*a^13*b^3*c^5*d^11 + 1728*a*b^15*c*d^15))/(a^20*d^20 + b^20*c^20
- 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^
12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*
c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^
8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^
9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b
^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d -
12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3
*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12
*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*
b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17
*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*
c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^
10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c
^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9
*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^1
5*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*
d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*
c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^
5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*
c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13
*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12
*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*
b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^
11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*
b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^
9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10
*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 4923
6*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6
- 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17

- 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^1
1 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*
d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^1
6 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10
*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^1
6*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^
7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*
c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^
4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6
*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5
*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5
*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c
^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c
^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^
3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9
*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^1
6 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 -
12*a*b^19*c^19*d - 12*a^19*b*c*d^19) + (-(((4*a^24*d^24 + 4*b^24*c^24 + 16
*a^2*b^22*c^24 + 16*a^4*b^20*c^24 - 1152*a^10*b^14*d^24 + 5568*a^12*b^12*d^
24 - 10568*a^14*b^10*d^24 + 9460*a^16*b^8*d^24 - 3560*a^18*b^6*d^24 + 136*a
^20*b^4*d^24 + 76*a^22*b^2*d^24 + 16*a^24*c^2*d^22 + 16*a^24*c^4*d^20 - 115
2*b^24*c^10*d^14 + 5568*b^24*c^12*d^12 - 10568*b^24*c^14*d^10 + 9460*b^24*c
^16*d^8 - 3560*b^24*c^18*d^6 + 136*b^24*c^20*d^4 + 76*b^24*c^22*d^2 + 11520
*a*b^23*c^9*d^15 - 56448*a*b^23*c^11*d^13 + 109456*a*b^23*c^13*d^11 - 10124
0*a*b^23*c^15*d^9 + 40720*a*b^23*c^17*d^7 - 2960*a*b^23*c^19*d^5 - 536*a*b^
23*c^21*d^3 - 176*a^3*b^21*c^23*d - 320*a^5*b^19*c^23*d + 11520*a^9*b^15*c*
d^23 - 56448*a^11*b^13*c*d^23 + 109456*a^13*b^11*c*d^23 - 101240*a^15*b^9*c
*d^23 + 40720*a^17*b^7*c*d^23 - 2960*a^19*b^5*c*d^23 - 536*a^21*b^3*c*d^23
- 176*a^23*b*c^3*d^21 - 320*a^23*b*c^5*d^19 - 51840*a^2*b^22*c^8*d^16 + 263
808*a^2*b^22*c^10*d^14 - 541208*a^2*b^22*c^12*d^12 + 547088*a^2*b^22*c^14*d
^10 - 263320*a^2*b^22*c^16*d^8 + 44120*a^2*b^22*c^18*d^6 - 1564*a^2*b^22*c^
20*d^4 - 196*a^2*b^22*c^22*d^2 + 138240*a^3*b^21*c^7*d^17 - 758400*a^3*b^21
*c^9*d^15 + 1720736*a^3*b^21*c^11*d^13 - 2002728*a^3*b^21*c^13*d^11 + 12105
60*a^3*b^21*c^15*d^9 - 335040*a^3*b^21*c^17*d^7 + 37680*a^3*b^21*c^19*d^5 -
288*a^3*b^21*c^21*d^3 - 241920*a^4*b^20*c^6*d^18 + 1512000*a^4*b^20*c^8*d^
16 - 3975688*a^4*b^20*c^10*d^14 + 5501328*a^4*b^20*c^12*d^12 - 4147952*a^4*
b^20*c^14*d^10 + 1586920*a^4*b^20*c^16*d^8 - 276020*a^4*b^20*c^18*d^6 + 211
24*a^4*b^20*c^20*d^4 + 176*a^4*b^20*c^22*d^2 + 290304*a^5*b^19*c^5*d^19 - 2
232576*a^5*b^19*c^7*d^17 + 7078256*a^5*b^19*c^9*d^15 - 11781560*a^5*b^19*c^
11*d^13 + 10875200*a^5*b^19*c^13*d^11 - 5365072*a^5*b^19*c^15*d^9 + 1310168
*a^5*b^19*c^17*d^7 - 170968*a^5*b^19*c^19*d^5 + 8160*a^5*b^19*c^21*d^3 - 24
1920*a^6*b^18*c^4*d^20 + 2532096*a^6*b^18*c^6*d^18 - 9955992*a^6*b^18*c^8*d
^16 + 20019440*a^6*b^18*c^10*d^14 - 22419600*a^6*b^18*c^12*d^12 + 13887520*
a^6*b^18*c^14*d^10 - 4506428*a^6*b^18*c^16*d^8 + 793756*a^6*b^18*c^18*d^6 -
72240*a^6*b^18*c^20*d^4 + 3040*a^6*b^18*c^22*d^2 + 138240*a^7*b^17*c^3*d^2

1 - 2232576*a^7*b^17*c^5*d^19 + 11150016*a^7*b^17*c^7*d^17 - 27336616*a^7*b^17*c^9*d^15 + 37153600*a^7*b^17*c^11*d^13 - 28461040*a^7*b^17*c^13*d^11 + 11779808*a^7*b^17*c^15*d^9 - 2621008*a^7*b^17*c^17*d^7 + 336688*a^7*b^17*c^19*d^5 - 17920*a^7*b^17*c^21*d^3 - 51840*a^8*b^16*c^2*d^22 + 1512000*a^8*b^16*c^4*d^20 - 9955992*a^8*b^16*c^6*d^18 + 30289656*a^8*b^16*c^8*d^16 - 50137600*a^8*b^16*c^10*d^14 + 46972560*a^8*b^16*c^12*d^12 - 24199280*a^8*b^16*c^14*d^10 + 6661036*a^8*b^16*c^16*d^8 - 1058448*a^8*b^16*c^18*d^6 + 72560*a^8*b^16*c^20*d^4 - 758400*a^9*b^15*c^3*d^21 + 7078256*a^9*b^15*c^5*d^19 - 27336616*a^9*b^15*c^7*d^17 + 55383904*a^9*b^15*c^9*d^15 - 63124080*a^9*b^15*c^11*d^13 + 39987520*a^9*b^15*c^13*d^11 - 13462088*a^9*b^15*c^15*d^9 + 2478528*a^9*b^15*c^17*d^7 - 212032*a^9*b^15*c^19*d^5 + 263808*a^10*b^14*c^2*d^22 - 3975688*a^10*b^14*c^4*d^20 + 20019440*a^10*b^14*c^6*d^18 - 50137600*a^10*b^14*c^8*d^16 + 69593872*a^10*b^14*c^10*d^14 - 53854288*a^10*b^14*c^12*d^12 + 21989928*a^10*b^14*c^14*d^10 - 4591360*a^10*b^14*c^16*d^8 + 460480*a^10*b^14*c^18*d^6 + 1720736*a^11*b^13*c^3*d^21 - 11781560*a^11*b^13*c^5*d^19 + 37153600*a^11*b^13*c^7*d^17 - 63124080*a^11*b^13*c^9*d^15 + 59445728*a^11*b^13*c^11*d^13 - 29358696*a^11*b^13*c^13*d^11 + 6995840*a^11*b^13*c^15*d^9 - 762560*a^11*b^13*c^17*d^7 - 541208*a^12*b^12*c^2*d^22 + 5501328*a^12*b^12*c^4*d^20 - 22419600*a^12*b^12*c^6*d^18 + 46972560*a^12*b^12*c^8*d^16 - 53854288*a^12*b^12*c^10*d^14 + 32294808*a^12*b^12*c^12*d^12 - 8958208*a^12*b^12*c^14*d^10 + 999040*a^12*b^12*c^16*d^8 - 2002728*a^13*b^11*c^3*d^21 + 10875200*a^13*b^11*c^5*d^19 - 28461040*a^13*b^11*c^7*d^17 + 39987520*a^13*b^11*c^9*d^15 - 29358696*a^13*b^11*c^11*d^13 + 9722048*a^13*b^11*c^13*d^11 - 1104320*a^13*b^11*c^15*d^9 + 547088*a^14*b^10*c^2*d^22 - 4147952*a^14*b^10*c^4*d^20 + 13887520*a^14*b^10*c^6*d^18 - 24199280*a^14*b^10*c^8*d^16 + 21989928*a^14*b^10*c^10*d^14 - 8958208*a^14*b^10*c^12*d^12 + 1124032*a^14*b^10*c^14*d^10 + 1210560*a^15*b^9*c^3*d^21 - 5365072*a^15*b^9*c^5*d^19 + 11779808*a^15*b^9*c^7*d^17 - 13462088*a^15*b^9*c^9*d^15 + 6995840*a^15*b^9*c^11*d^13 - 1104320*a^15*b^9*c^13*d^11 - 263320*a^16*b^8*c^2*d^22 + 1586920*a^16*b^8*c^4*d^20 - 4506428*a^16*b^8*c^6*d^18 + 6661036*a^16*b^8*c^8*d^16 - 4591360*a^16*b^8*c^10*d^14 + 999040*a^16*b^8*c^12*d^12 - 335040*a^17*b^7*c^3*d^21 + 1310168*a^17*b^7*c^5*d^19 - 2621008*a^17*b^7*c^7*d^17 + 2478528*a^17*b^7*c^9*d^15 - 762560*a^17*b^7*c^11*d^13 + 44120*a^18*b^6*c^2*d^22 - 276020*a^18*b^6*c^4*d^20 + 793756*a^18*b^6*c^6*d^18 - 1058448*a^18*b^6*c^8*d^16 + 460480*a^18*b^6*c^10*d^14 + 37680*a^19*b^5*c^3*d^21 - 170968*a^19*b^5*c^5*d^19 + 336688*a^19*b^5*c^7*d^17 - 212032*a^19*b^5*c^9*d^15 - 1564*a^20*b^4*c^2*d^22 + 21124*a^20*b^4*c^4*d^20 - 72240*a^20*b^4*c^6*d^18 + 72560*a^20*b^4*c^8*d^16 - 288*a^21*b^3*c^3*d^21 + 8160*a^21*b^3*c^5*d^19 - 17920*a^21*b^3*c^7*d^17 - 196*a^22*b^2*c^2*d^22 + 176*a^22*b^2*c^4*d^20 + 3040*a^22*b^2*c^6*d^18 - 8*a*b^23*c^23*d - 8*a^23*b*c*d^23)^2/4 - (20736*b^18*d^18 - 96768*a^2*b^16*d^18 + 173664*a^4*b^14*d^18 - 136032*a^6*b^12*d^18 + 31081*a^8*b^10*d^18 + 8440*a^10*b^8*d^18 + 400*a^12*b^6*d^18 - 96768*b^18*c^2*d^16 + 173664*b^18*c^4*d^14 - 136032*b^18*c^6*d^12 + 31081*b^18*c^8*d^10 + 8440*b^18*c^10*d^8 + 400*b^18*c^12*d^6 - 131328*a*b^17*c^3*d^15 + 216576*a*b^17*c^5*d^13 - 141104*a*b^17*c^7*d^11 + 20260*a*b^17*c^9*d^9 + 2800*a*b^17*c^11*d^7 -

131328*a^3*b^15*c*d^17 + 216576*a^5*b^13*c*d^17 - 141104*a^7*b^11*c*d^17 + 20260*a^9*b^9*c*d^17 + 2800*a^11*b^7*c*d^17 + 495936*a^2*b^16*c^2*d^16 - 98 9856*a^2*b^16*c^4*d^14 + 901948*a^2*b^16*c^6*d^12 - 308392*a^2*b^16*c^8*d^1 0 - 5260*a^2*b^16*c^10*d^8 + 1600*a^2*b^16*c^12*d^6 + 657408*a^3*b^15*c^3*d ^15 - 1158992*a^3*b^15*c^5*d^13 + 838256*a^3*b^15*c^7*d^11 - 182200*a^3*b^1 5*c^9*d^9 - 3200*a^3*b^15*c^11*d^7 - 989856*a^4*b^14*c^2*d^16 + 2185654*a^4 *b^14*c^4*d^14 - 2218576*a^4*b^14*c^6*d^12 + 900624*a^4*b^14*c^8*d^10 - 647 20*a^4*b^14*c^10*d^8 + 1600*a^4*b^14*c^12*d^6 - 1158992*a^5*b^13*c^3*d^15 + 2158808*a^5*b^13*c^5*d^13 - 1641528*a^5*b^13*c^7*d^11 + 406880*a^5*b^13*c^ 9*d^9 - 17600*a^5*b^13*c^11*d^7 + 901948*a^6*b^12*c^2*d^16 - 2218576*a^6*b^ 12*c^4*d^14 + 2430936*a^6*b^12*c^6*d^12 - 1026928*a^6*b^12*c^8*d^10 + 88720 *a^6*b^12*c^10*d^8 + 838256*a^7*b^11*c^3*d^15 - 1641528*a^7*b^11*c^5*d^13 + 1206848*a^7*b^11*c^7*d^11 - 239360*a^7*b^11*c^9*d^9 - 308392*a^8*b^10*c^2* d^16 + 900624*a^8*b^10*c^4*d^14 - 1026928*a^8*b^10*c^6*d^12 + 354016*a^8*b^ 10*c^8*d^10 - 182200*a^9*b^9*c^3*d^15 + 406880*a^9*b^9*c^5*d^13 - 239360*a^ 9*b^9*c^7*d^11 - 5260*a^10*b^8*c^2*d^16 - 64720*a^10*b^8*c^4*d^14 + 88720*a ^10*b^8*c^6*d^12 - 3200*a^11*b^7*c^3*d^15 - 17600*a^11*b^7*c^5*d^13 + 1600* a^12*b^6*c^2*d^16 + 1600*a^12*b^6*c^4*d^14 + 27648*a*b^17*c*d^17)*(80*a^2*b ^28*c^30 - 16*b^30*c^30 - 16*a^30*d^30 - 160*a^4*b^26*c^30 + 160*a^6*b^24*c ^30 - 80*a^8*b^22*c^30 + 16*a^10*b^20*c^30 + 16*a^20*b^10*d^30 - 80*a^22*b^ 8*d^30 + 160*a^24*b^6*d^30 - 160*a^26*b^4*d^30 + 80*a^28*b^2*d^30 + 80*a^30 *c^2*d^28 - 160*a^30*c^4*d^26 + 160*a^30*c^6*d^24 - 80*a^30*c^8*d^22 + 16*a ^30*c^10*d^20 + 16*b^30*c^20*d^10 - 80*b^30*c^22*d^8 + 160*b^30*c^24*d^6 - 160*b^30*c^26*d^4 + 80*b^30*c^28*d^2 - 320*a*b^29*c^19*d^11 + 1600*a*b^29*c ^21*d^9 - 3200*a*b^29*c^23*d^7 + 3200*a*b^29*c^25*d^5 - 1600*a*b^29*c^27*d^ 3 - 1600*a^3*b^27*c^29*d + 3200*a^5*b^25*c^29*d - 3200*a^7*b^23*c^29*d + 16 00*a^9*b^21*c^29*d - 320*a^11*b^19*c^29*d - 320*a^19*b^11*c*d^29 + 1600*a^2 1*b^9*c*d^29 - 3200*a^23*b^7*c*d^29 + 3200*a^25*b^5*c*d^29 - 1600*a^27*b^3* c*d^29 - 1600*a^29*b*c^3*d^27 + 3200*a^29*b*c^5*d^25 - 3200*a^29*b*c^7*d^23 + 1600*a^29*b*c^9*d^21 - 320*a^29*b*c^11*d^19 + 3040*a^2*b^28*c^18*d^12 - 15280*a^2*b^28*c^20*d^10 + 30800*a^2*b^28*c^22*d^8 - 31200*a^2*b^28*c^24*d^ 6 + 16000*a^2*b^28*c^26*d^4 - 3440*a^2*b^28*c^28*d^2 - 18240*a^3*b^27*c^17* d^13 + 92800*a^3*b^27*c^19*d^11 - 190400*a^3*b^27*c^21*d^9 + 198400*a^3*b^2 7*c^23*d^7 - 107200*a^3*b^27*c^25*d^5 + 26240*a^3*b^27*c^27*d^3 + 77520*a^4 *b^26*c^16*d^14 - 402800*a^4*b^26*c^18*d^12 + 851360*a^4*b^26*c^20*d^10 - 9 28000*a^4*b^26*c^22*d^8 + 541200*a^4*b^26*c^24*d^6 - 155120*a^4*b^26*c^26*d ^4 + 16000*a^4*b^26*c^28*d^2 - 248064*a^5*b^25*c^15*d^15 + 1331520*a^5*b^25 *c^17*d^13 - 2939840*a^5*b^25*c^19*d^11 + 3408640*a^5*b^25*c^21*d^9 - 21843 20*a^5*b^25*c^23*d^7 + 736064*a^5*b^25*c^25*d^5 - 107200*a^5*b^25*c^27*d^3 + 620160*a^6*b^24*c^14*d^16 - 3488400*a^6*b^24*c^16*d^14 + 8170000*a^6*b^24 *c^18*d^12 - 10229760*a^6*b^24*c^20*d^10 + 7281600*a^6*b^24*c^22*d^8 - 2863 760*a^6*b^24*c^24*d^6 + 541200*a^6*b^24*c^26*d^4 - 31200*a^6*b^24*c^28*d^2 - 1240320*a^7*b^23*c^13*d^17 + 7441920*a^7*b^23*c^15*d^15 - 18787200*a^7*b^ 23*c^17*d^13 + 25721600*a^7*b^23*c^19*d^11 - 20444800*a^7*b^23*c^21*d^9 + 9 297920*a^7*b^23*c^23*d^7 - 2184320*a^7*b^23*c^25*d^5 + 198400*a^7*b^23*c^27

*d^3 + 2015520*a^8*b^22*c^12*d^18 - 13178400*a^8*b^22*c^14*d^16 + 36434400*
a^8*b^22*c^16*d^14 - 55069600*a^8*b^22*c^18*d^12 + 48989680*a^8*b^22*c^20*d
^10 - 25575920*a^8*b^22*c^22*d^8 + 7281600*a^8*b^22*c^24*d^6 - 928000*a^8*b
^22*c^26*d^4 + 30800*a^8*b^22*c^28*d^2 - 2687360*a^9*b^21*c^11*d^19 + 19638
400*a^9*b^21*c^13*d^17 - 60362240*a^9*b^21*c^15*d^15 + 101475200*a^9*b^21*c
^17*d^13 - 101172800*a^9*b^21*c^19*d^11 + 60333760*a^9*b^21*c^21*d^9 - 2044
4800*a^9*b^21*c^23*d^7 + 3408640*a^9*b^21*c^25*d^5 - 190400*a^9*b^21*c^27*d
^3 + 2956096*a^10*b^20*c^10*d^20 - 24858080*a^10*b^20*c^12*d^18 + 86150560*
a^10*b^20*c^14*d^16 - 162120160*a^10*b^20*c^16*d^14 + 181463680*a^10*b^20*c
^18*d^12 - 123188112*a^10*b^20*c^20*d^10 + 48989680*a^10*b^20*c^22*d^8 - 10
229760*a^10*b^20*c^24*d^6 + 851360*a^10*b^20*c^26*d^4 - 15280*a^10*b^20*c^2
8*d^2 - 2687360*a^11*b^19*c^9*d^21 + 26873600*a^11*b^19*c^11*d^19 - 1064608
00*a^11*b^19*c^13*d^17 + 225738240*a^11*b^19*c^15*d^15 - 284331200*a^11*b^1
9*c^17*d^13 + 219166080*a^11*b^19*c^19*d^11 - 101172800*a^11*b^19*c^21*d^9
+ 25721600*a^11*b^19*c^23*d^7 - 2939840*a^11*b^19*c^25*d^5 + 92800*a^11*b^1
9*c^27*d^3 + 2015520*a^12*b^18*c^8*d^22 - 24858080*a^12*b^18*c^10*d^20 + 11
4212800*a^12*b^18*c^12*d^18 - 274937600*a^12*b^18*c^14*d^16 + 390830000*a^1
2*b^18*c^16*d^14 - 341426960*a^12*b^18*c^18*d^12 + 181463680*a^12*b^18*c^20
*d^10 - 55069600*a^12*b^18*c^22*d^8 + 8170000*a^12*b^18*c^24*d^6 - 402800*a
^12*b^18*c^26*d^4 + 3040*a^12*b^18*c^28*d^2 - 1240320*a^13*b^17*c^7*d^23 +
19638400*a^13*b^17*c^9*d^21 - 106460800*a^13*b^17*c^11*d^19 + 293542400*a^1
3*b^17*c^13*d^17 - 472561920*a^13*b^17*c^15*d^15 + 467412160*a^13*b^17*c^17
*d^13 - 284331200*a^13*b^17*c^19*d^11 + 101475200*a^13*b^17*c^21*d^9 - 1878
7200*a^13*b^17*c^23*d^7 + 1331520*a^13*b^17*c^25*d^5 - 18240*a^13*b^17*c^27
*d^3 + 620160*a^14*b^16*c^6*d^24 - 13178400*a^14*b^16*c^8*d^22 + 86150560*a
^14*b^16*c^10*d^20 - 274937600*a^14*b^16*c^12*d^18 + 503363200*a^14*b^16*c^
14*d^16 - 563751280*a^14*b^16*c^16*d^14 + 390830000*a^14*b^16*c^18*d^12 - 1
62120160*a^14*b^16*c^20*d^10 + 36434400*a^14*b^16*c^22*d^8 - 3488400*a^14*b
^16*c^24*d^6 + 77520*a^14*b^16*c^26*d^4 - 248064*a^15*b^15*c^5*d^25 + 74419
20*a^15*b^15*c^7*d^23 - 60362240*a^15*b^15*c^9*d^21 + 225738240*a^15*b^15*c
^11*d^19 - 472561920*a^15*b^15*c^13*d^17 + 599984128*a^15*b^15*c^15*d^15 -
472561920*a^15*b^15*c^17*d^13 + 225738240*a^15*b^15*c^19*d^11 - 60362240*a^
15*b^15*c^21*d^9 + 7441920*a^15*b^15*c^23*d^7 - 248064*a^15*b^15*c^25*d^5 +
77520*a^16*b^14*c^4*d^26 - 3488400*a^16*b^14*c^6*d^24 + 36434400*a^16*b^14
*c^8*d^22 - 162120160*a^16*b^14*c^10*d^20 + 390830000*a^16*b^14*c^12*d^18 -
563751280*a^16*b^14*c^14*d^16 + 503363200*a^16*b^14*c^16*d^14 - 274937600*
a^16*b^14*c^18*d^12 + 86150560*a^16*b^14*c^20*d^10 - 13178400*a^16*b^14*c^2
2*d^8 + 620160*a^16*b^14*c^24*d^6 - 18240*a^17*b^13*c^3*d^27 + 1331520*a^17
*b^13*c^5*d^25 - 18787200*a^17*b^13*c^7*d^23 + 101475200*a^17*b^13*c^9*d^21
- 284331200*a^17*b^13*c^11*d^19 + 467412160*a^17*b^13*c^13*d^17 - 47256192
0*a^17*b^13*c^15*d^15 + 293542400*a^17*b^13*c^17*d^13 - 106460800*a^17*b^13
*c^19*d^11 + 19638400*a^17*b^13*c^21*d^9 - 1240320*a^17*b^13*c^23*d^7 + 304
0*a^18*b^12*c^2*d^28 - 402800*a^18*b^12*c^4*d^26 + 8170000*a^18*b^12*c^6*d^
24 - 55069600*a^18*b^12*c^8*d^22 + 181463680*a^18*b^12*c^10*d^20 - 34142696
0*a^18*b^12*c^12*d^18 + 390830000*a^18*b^12*c^14*d^16 - 274937600*a^18*b^12

*c^16*d^14 + 114212800*a^18*b^12*c^18*d^12 - 24858080*a^18*b^12*c^20*d^10 + 2015520*a^18*b^12*c^22*d^8 + 92800*a^19*b^11*c^3*d^27 - 2939840*a^19*b^11*c^5*d^25 + 25721600*a^19*b^11*c^7*d^23 - 101172800*a^19*b^11*c^9*d^21 + 219166080*a^19*b^11*c^11*d^19 - 284331200*a^19*b^11*c^13*d^17 + 225738240*a^19*b^11*c^15*d^15 - 106460800*a^19*b^11*c^17*d^13 + 26873600*a^19*b^11*c^19*d^11 - 2687360*a^19*b^11*c^21*d^9 - 15280*a^20*b^10*c^2*d^28 + 851360*a^20*b^10*c^4*d^26 - 10229760*a^20*b^10*c^6*d^24 + 48989680*a^20*b^10*c^8*d^22 - 123188112*a^20*b^10*c^10*d^20 + 181463680*a^20*b^10*c^12*d^18 - 162120160*a^20*b^10*c^14*d^16 + 86150560*a^20*b^10*c^16*d^14 - 24858080*a^20*b^10*c^18*d^12 + 2956096*a^20*b^10*c^20*d^10 - 190400*a^21*b^9*c^3*d^27 + 3408640*a^21*b^9*c^5*d^25 - 20444800*a^21*b^9*c^7*d^23 + 60333760*a^21*b^9*c^9*d^21 - 101172800*a^21*b^9*c^11*d^19 + 101475200*a^21*b^9*c^13*d^17 - 60362240*a^21*b^9*c^15*d^15 + 19638400*a^21*b^9*c^17*d^13 - 2687360*a^21*b^9*c^19*d^11 + 30800*a^22*b^8*c^2*d^28 - 928000*a^22*b^8*c^4*d^26 + 7281600*a^22*b^8*c^6*d^24 - 25575920*a^22*b^8*c^8*d^22 + 48989680*a^22*b^8*c^10*d^20 - 55069600*a^22*b^8*c^12*d^18 + 36434400*a^22*b^8*c^14*d^16 - 13178400*a^22*b^8*c^16*d^14 + 2015520*a^22*b^8*c^18*d^12 + 198400*a^23*b^7*c^3*d^27 - 2184320*a^23*b^7*c^5*d^25 + 9297920*a^23*b^7*c^7*d^23 - 20444800*a^23*b^7*c^9*d^21 + 25721600*a^23*b^7*c^11*d^19 - 18787200*a^23*b^7*c^13*d^17 + 7441920*a^23*b^7*c^15*d^15 - 1240320*a^23*b^7*c^17*d^13 - 31200*a^24*b^6*c^2*d^28 + 541200*a^24*b^6*c^4*d^26 - 2863760*a^24*b^6*c^6*d^24 + 7281600*a^24*b^6*c^8*d^22 - 10229760*a^24*b^6*c^10*d^20 + 8170000*a^24*b^6*c^12*d^18 - 3488400*a^24*b^6*c^14*d^16 + 620160*a^24*b^6*c^16*d^14 - 107200*a^25*b^5*c^3*d^27 + 736064*a^25*b^5*c^5*d^25 - 2184320*a^25*b^5*c^7*d^23 + 3408640*a^25*b^5*c^9*d^21 - 2939840*a^25*b^5*c^11*d^19 + 1331520*a^25*b^5*c^13*d^17 - 248064*a^25*b^5*c^15*d^15 + 16000*a^26*b^4*c^2*d^28 - 155120*a^26*b^4*c^4*d^26 + 541200*a^26*b^4*c^6*d^24 - 928000*a^26*b^4*c^8*d^22 + 851360*a^26*b^4*c^10*d^20 - 402800*a^26*b^4*c^12*d^18 + 77520*a^26*b^4*c^14*d^16 + 26240*a^27*b^3*c^3*d^27 - 107200*a^27*b^3*c^5*d^25 + 198400*a^27*b^3*c^7*d^23 - 190400*a^27*b^3*c^9*d^21 + 92800*a^27*b^3*c^11*d^19 - 18240*a^27*b^3*c^13*d^17 - 3440*a^28*b^2*c^2*d^28 + 16000*a^28*b^2*c^4*d^26 - 31200*a^28*b^2*c^6*d^24 + 30800*a^28*b^2*c^8*d^22 - 15280*a^28*b^2*c^10*d^20 + 3040*a^28*b^2*c^12*d^18 + 320*a*b^29*c^29*d + 320*a^29*b*c*d^29))^(1/2) + 2*a^24*d^24 + 2*b^24*c^24 + 8*a^2*b^22*c^24 + 8*a^4*b^20*c^24 - 576*a^10*b^14*d^24 + 2784*a^12*b^12*d^24 - 5284*a^14*b^10*d^24 + 4730*a^16*b^8*d^24 - 1780*a^18*b^6*d^24 + 68*a^20*b^4*d^24 + 38*a^22*b^2*d^24 + 8*a^24*c^2*d^22 + 8*a^24*c^4*d^20 - 576*b^24*c^10*d^14 + 2784*b^24*c^12*d^12 - 5284*b^24*c^14*d^10 + 4730*b^24*c^16*d^8 - 1780*b^24*c^18*d^6 + 68*b^24*c^20*d^4 + 38*b^24*c^22*d^2 + 5760*a*b^23*c^9*d^15 - 28224*a*b^23*c^11*d^13 + 54728*a*b^23*c^13*d^11 - 50620*a*b^23*c^15*d^9 + 20360*a*b^23*c^17*d^7 - 1480*a*b^23*c^19*d^5 - 268*a*b^23*c^21*d^3 - 88*a^3*b^21*c^23*d - 160*a^5*b^19*c^23*d + 5760*a^9*b^15*c*d^23 - 28224*a^11*b^13*c*d^23 + 54728*a^13*b^11*c*d^23 - 50620*a^15*b^9*c*d^23 + 20360*a^17*b^7*c*d^23 - 1480*a^19*b^5*c*d^23 - 268*a^21*b^3*c*d^23 - 88*a^23*b*c^3*d^21 - 160*a^23*b*c^5*d^19 - 25920*a^2*b^22*c^8*d^16 + 131904*a^2*b^22*c^10*d^14 - 270604*a^2*b^22*c^12*d^12 + 273544*a^2*b^22*c^14*d^10 - 131660*a^2*b^22 *c^16*d^8 + 22060*a^2*b^22*c^18*d^6 - 782*a^2*b^22*c^20*d^4 - 98*a^2*b^22*c^22*d^2 + 69120*a^3*b^21*c^7*d^17 - 379200*a^3*b^21*c^9*d^15 + 860368*a^3*b^21*c^11*d^13 - 1001364*a^3*b^21*c^13*d^11 + 605280*a^3*b^21*c^15*d^9 - 167520*a^3*b^21*c^17*d^7 + 18840*a^3*b^21*c^19*d^5 - 144*a^3*b^21*c^21*d^3 - 120960*a^4*b^20*c^6*d^18 + 756000*a^4*b^20*c^8*d^16 - 1987844*a^4*b^20*c^10*d^14 + 2750664*a^4*b^20*c^12*d^12 - 2073976*a^4*b^20*c^14*d^10 + 793460*a^4*b^20*c^16*d^8 - 138010*a^4*b^20*c^18*d^6 + 10562*a^4*b^20*c^20*d^4 + 88*a^4*b^20*c^22*d^2 + 145152*a^5*b^19*c^5*d^19 - 1116288*a^5*b^19*c^7*d^17 + 3539128*a^5*b^19*c^9*d^15 - 5890780*a^5*b^19*c^11*d^13 + 5437600*a^5*b^19*c^13*d^11 - 2682536*a^5*b^19*c^15*d^9 + 655084*a^5*b^19*c^17*d^7 - 85484*a^5*b^19*c^19*d^5 + 4080*a^5*b^19*c^21*d^3 - 120960*a^6*b^18*c^4*d^20 + 1266048*a^6*b^18*c^6*d^18 - 4977996*a^6*b^18*c^8*d^16 + 10009720*a^6*b^18*c^10*d^14 - 11209800*a^6*b^18*c^12*d^12 + 6943760*a^6*b^18*c^14*d^10 - 2253214*a^6*b^18*c^16*d^8 + 396878*a^6*b^18*c^18*d^6 - 36120*a^6*b^18*c^20*d^4 + 1520*a^6*b^18*c^22*d^2 + 69120*a^7*b^17*c^3*d^21 - 1116288*a^7*b^17*c^5*d^19 + 5575008*a^7*b^17*c^7*d^17 - 13668308*a^7*b^17*c^9*d^15 + 18576800*a^7*b^17*c^11*d^13 - 14230520*a^7*b^17*c^13*d^11 + 5889904*a^7*b^17*c^15*d^9 - 1310504*a^7*b^17*c^17*d^7 + 168344*a^7*b^17*c^19*d^5 - 8960*a^7*b^17*c^21*d^3 - 25920*a^8*b^16*c^2*d^22 + 756000*a^8*b^16*c^4*d^20 - 4977996*a^8*b^16*c^6*d^18 + 15144828*a^8*b^16*c^8*d^16 - 25068800*a^8*b^16*c^10*d^14 + 23486280*a^8*b^16*c^12*d^12 - 12099640*a^8*b^16*c^14*d^10 + 3330518*a^8*b^16*c^16*d^8 - 529224*a^8*b^16*c^18*d^6 + 36280*a^8*b^16*c^20*d^4 - 379200*a^9*b^15*c^3*d^21 + 3539128*a^9*b^15*c^5*d^19 - 13668308*a^9*b^15*c^7*d^17 + 27691952*a^9*b^15*c^9*d^15 - 31562040*a^9*b^15*c^11*d^13 + 19993760*a^9*b^15*c^13*d^11 - 6731044*a^9*b^15*c^15*d^9 + 1239264*a^9*b^15*c^17*d^7 - 106016*a^9*b^15*c^19*d^5 + 131904*a^10*b^14*c^2*d^22 - 1987844*a^10*b^14*c^4*d^20 + 10009720*a^10*b^14*c^6*d^18 - 25068800*a^10*b^14*c^8*d^16 + 34796936*a^10*b^14*c^10*d^14 - 26927144*a^10*b^14*c^12*d^12 + 10994964*a^10*b^14*c^14*d^10 - 2295680*a^10*b^14*c^16*d^8 + 230240*a^10*b^14*c^18*d^6 + 860368*a^11*b^13*c^3*d^21 - 5890780*a^11*b^13*c^5*d^19 + 18576800*a^11*b^13*c^7*d^17 - 31562040*a^11*b^13*c^9*d^15 + 29722864*a^11*b^13*c^11*d^13 - 14679348*a^11*b^13*c^13*d^11 + 3497920*a^11*b^13*c^15*d^9 - 381280*a^11*b^13*c^17*d^7 - 270604*a^12*b^12*c^2*d^22 + 2750664*a^12*b^12*c^4*d^20 - 11209800*a^12*b^12*c^6*d^18 + 23486280*a^12*b^12*c^8*d^16 - 26927144*a^12*b^12*c^10*d^14 + 16147404*a^12*b^12*c^12*d^12 - 4479104*a^12*b^12*c^14*d^10 + 499520*a^12*b^12*c^16*d^8 - 1001364*a^13*b^11*c^3*d^21 + 5437600*a^13*b^11*c^5*d^19 - 14230520*a^13*b^11*c^7*d^17 + 19993760*a^13*b^11*c^9*d^15 - 14679348*a^13*b^11*c^11*d^13 + 4861024*a^13*b^11*c^13*d^11 - 552160*a^13*b^11*c^15*d^9 + 273544*a^14*b^10*c^2*d^22 - 2073976*a^14*b^10*c^4*d^20 + 6943760*a^14*b^10*c^6*d^18 - 12099640*a^14*b^10*c^8*d^16 + 10994964*a^14*b^10*c^10*d^14 - 4479104*a^14*b^10*c^12*d^12 + 562016*a^14*b^10*c^14*d^10 + 605280*a^15*b^9*c^3*d^21 - 2682536*a^15*b^9*c^5*d^19 + 5889904*a^15*b^9*c^7*d^17 - 6731044*a^15*b^9*c^9*d^15 + 3497920*a^15*b^9*c^11*d^13 - 552160*a^15*b^9*c^13*d^11 - 131660*a^16*b^8*c^2*d^22 + 793460*a^16*b^8*c^4*d^20 - 2253214*a^16*b^8*c^6*d^18 + 3330518*a^16*b^8*c^8*d^16 - 2295680*a^16*b^8*c^10*d^14 + 499520*a^16*b^8*c^12*d^12 - 167

520*a^17*b^7*c^3*d^21 + 655084*a^17*b^7*c^5*d^19 - 1310504*a^17*b^7*c^7*d^17 + 1239264*a^17*b^7*c^9*d^15 - 381280*a^17*b^7*c^11*d^13 + 22060*a^18*b^6*c^2*d^22 - 138010*a^18*b^6*c^4*d^20 + 396878*a^18*b^6*c^6*d^18 - 529224*a^18*b^6*c^8*d^16 + 230240*a^18*b^6*c^10*d^14 + 18840*a^19*b^5*c^3*d^21 - 85484*a^19*b^5*c^5*d^19 + 168344*a^19*b^5*c^7*d^17 - 106016*a^19*b^5*c^9*d^15 - 782*a^20*b^4*c^2*d^22 + 10562*a^20*b^4*c^4*d^20 - 36120*a^20*b^4*c^6*d^18 + 36280*a^20*b^4*c^8*d^16 - 144*a^21*b^3*c^3*d^21 + 4080*a^21*b^3*c^5*d^19 - 8960*a^21*b^3*c^7*d^17 - 98*a^22*b^2*c^2*d^22 + 88*a^22*b^2*c^4*d^20 + 1520*a^22*b^2*c^6*d^18 - 4*a*b^23*c^23*d - 4*a^23*b*c*d^23)/(16*(5*a^2*b^28*c^30 - b^30*c^30 - a^30*d^30 - 10*a^4*b^26*c^30 + 10*a^6*b^24*c^30 - 5*a^8*b^22*c^30 + a^10*b^20*c^30 + a^20*b^10*d^30 - 5*a^22*b^8*d^30 + 10*a^24*b^6*d^30 - 10*a^26*b^4*d^30 + 5*a^28*b^2*d^30 + 5*a^30*c^2*d^28 - 10*a^30*c^4*d^26 + 10*a^30*c^6*d^24 - 5*a^30*c^8*d^22 + a^30*c^10*d^20 + b^30*c^20*d^10 - 5*b^30*c^22*d^8 + 10*b^30*c^24*d^6 - 10*b^30*c^26*d^4 + 5*b^30*c^28*d^2 - 20*a*b^29*c^19*d^11 + 100*a*b^29*c^21*d^9 - 200*a*b^29*c^23*d^7 + 200*a*b^29*c^25*d^5 - 100*a*b^29*c^27*d^3 - 100*a^3*b^27*c^29*d + 200*a^5*b^25*c^29*d - 200*a^7*b^23*c^29*d + 100*a^9*b^21*c^29*d - 20*a^11*b^19*c^29*d - 20*a^19*b^11*c*d^29 + 100*a^21*b^9*c*d^29 - 200*a^23*b^7*c*d^29 + 200*a^25*b^5*c*d^29 - 100*a^27*b^3*c*d^29 - 100*a^29*b*c^3*d^27 + 200*a^29*b*c^5*d^25 - 200*a^29*b*c^7*d^23 + 100*a^29*b*c^9*d^21 - 20*a^29*b*c^11*d^19 + 190*a^2*b^28*c^18*d^12 - 955*a^2*b^28*c^20*d^10 + 1925*a^2*b^28*c^22*d^8 - 1950*a^2*b^28*c^24*d^6 + 1000*a^2*b^28*c^26*d^4 - 215*a^2*b^28*c^28*d^2 - 1140*a^3*b^27*c^17*d^13 + 5800*a^3*b^27*c^19*d^11 - 11900*a^3*b^27*c^21*d^9 + 12400*a^3*b^27*c^23*d^7 - 6700*a^3*b^27*c^25*d^5 + 1640*a^3*b^27*c^27*d^3 + 4845*a^4*b^26*c^16*d^14 - 25175*a^4*b^26*c^18*d^12 + 53210*a^4*b^26*c^20*d^10 - 58000*a^4*b^26*c^22*d^8 + 33825*a^4*b^26*c^24*d^6 - 9695*a^4*b^26*c^26*d^4 + 1000*a^4*b^26*c^28*d^2 - 15504*a^5*b^25*c^15*d^15 + 83220*a^5*b^25*c^17*d^13 - 183740*a^5*b^25*c^19*d^11 + 213040*a^5*b^25*c^21*d^9 - 136520*a^5*b^25*c^23*d^7 + 46004*a^5*b^25*c^25*d^5 - 6700*a^5*b^25*c^27*d^3 + 38760*a^6*b^24*c^14*d^16 - 218025*a^6*b^24*c^16*d^14 + 510625*a^6*b^24*c^18*d^12 - 639360*a^6*b^24*c^20*d^10 + 455100*a^6*b^24*c^22*d^8 - 178985*a^6*b^24*c^24*d^6 + 33825*a^6*b^24*c^26*d^4 - 1950*a^6*b^24*c^28*d^2 - 77520*a^7*b^23*c^13*d^17 + 465120*a^7*b^23*c^15*d^15 - 1174200*a^7*b^23*c^17*d^13 + 1607600*a^7*b^23*c^19*d^11 - 1277800*a^7*b^23*c^21*d^9 + 581120*a^7*b^23*c^23*d^7 - 136520*a^7*b^23*c^25*d^5 + 12400*a^7*b^23*c^27*d^3 + 125970*a^8*b^22*c^12*d^18 - 823650*a^8*b^22*c^14*d^16 + 2277150*a^8*b^22*c^16*d^14 - 3441850*a^8*b^22*c^18*d^12 + 3061855*a^8*b^22*c^20*d^10 - 1598495*a^8*b^22*c^22*d^8 + 455100*a^8*b^22*c^24*d^6 - 58000*a^8*b^22*c^26*d^4 + 1925*a^8*b^22*c^28*d^2 - 167960*a^9*b^21*c^11*d^19 + 1227400*a^9*b^21*c^13*d^17 - 3772640*a^9*b^21*c^15*d^15 + 6342200*a^9*b^21*c^17*d^13 - 6323300*a^9*b^21*c^19*d^11 + 3770860*a^9*b^21*c^21*d^9 - 1277800*a^9*b^21*c^23*d^7 + 213040*a^9*b^21*c^25*d^5 - 11900*a^9*b^21*c^27*d^3 + 184756*a^10*b^20*c^10*d^20 - 1553630*a^10*b^20*c^12*d^18 + 5384410*a^10*b^20*c^14*d^16 - 10132510*a^10*b^20*c^16*d^14 + 11341480*a^10*b^20*c^18*d^12 - 7699257*a^10*b^20*c^20*d^10 + 3061855*a^10*b^20*c^22*d^8 - 639360*a^10*b^20*c^24*d^6 + 53210*a^10*b^20*c^26*d^4 - 955*a^10 *b^20*c^28*d^2 - 167960*a^11*b^19*c^9*d^21 + 1679600*a^11*b^19*c^11*d^19 - 6653800*a^11*b^19*c^13*d^17 + 14108640*a^11*b^19*c^15*d^15 - 17770700*a^11*b^19*c^17*d^13 + 13697880*a^11*b^19*c^19*d^11 - 6323300*a^11*b^19*c^21*d^9 + 1607600*a^11*b^19*c^23*d^7 - 183740*a^11*b^19*c^25*d^5 + 5800*a^11*b^19*c^27*d^3 + 125970*a^12*b^18*c^8*d^22 - 1553630*a^12*b^18*c^10*d^20 + 7138300*a^12*b^18*c^12*d^18 - 17183600*a^12*b^18*c^14*d^16 + 24426875*a^12*b^18*c^16*d^14 - 21339185*a^12*b^18*c^18*d^12 + 11341480*a^12*b^18*c^20*d^10 - 3441850*a^12*b^18*c^22*d^8 + 510625*a^12*b^18*c^24*d^6 - 25175*a^12*b^18*c^26*d^4 + 190*a^12*b^18*c^28*d^2 - 77520*a^13*b^17*c^7*d^23 + 1227400*a^13*b^17*c^9*d^21 - 6653800*a^13*b^17*c^11*d^19 + 18346400*a^13*b^17*c^13*d^17 - 29535120*a^13*b^17*c^15*d^15 + 29213260*a^13*b^17*c^17*d^13 - 17770700*a^13*b^17*c^19*d^11 + 6342200*a^13*b^17*c^21*d^9 - 1174200*a^13*b^17*c^23*d^7 + 83220*a^13*b^17*c^25*d^5 - 1140*a^13*b^17*c^27*d^3 + 38760*a^14*b^16*c^6*d^24 - 823650*a^14*b^16*c^8*d^22 + 5384410*a^14*b^16*c^10*d^20 - 17183600*a^14*b^16*c^12*d^18 + 31460200*a^14*b^16*c^14*d^16 - 35234455*a^14*b^16*c^16*d^14 + 24426875*a^14*b^16*c^18*d^12 - 10132510*a^14*b^16*c^20*d^10 + 2277150*a^14*b^16*c^22*d^8 - 218025*a^14*b^16*c^24*d^6 + 4845*a^14*b^16*c^26*d^4 - 15504*a^15*b^15*c^5*d^25 + 465120*a^15*b^15*c^7*d^23 - 3772640*a^15*b^15*c^9*d^21 + 14108640*a^15*b^15*c^11*d^19 - 29535120*a^15*b^15*c^13*d^17 + 37499008*a^15*b^15*c^15*d^15 - 29535120*a^15*b^15*c^17*d^13 + 14108640*a^15*b^15*c^19*d^11 - 3772640*a^15*b^15*c^21*d^9 + 465120*a^15*b^15*c^23*d^7 - 15504*a^15*b^15*c^25*d^5 + 4845*a^16*b^14*c^4*d^26 - 218025*a^16*b^14*c^6*d^24 + 2277150*a^16*b^14*c^8*d^22 - 10132510*a^16*b^14*c^10*d^20 + 24426875*a^16*b^14*c^12*d^18 - 35234455*a^16*b^14*c^14*d^16 + 31460200*a^16*b^14*c^16*d^14 - 17183600*a^16*b^14*c^18*d^12 + 5384410*a^16*b^14*c^20*d^10 - 823650*a^16*b^14*c^22*d^8 + 38760*a^16*b^14*c^24*d^6 - 1140*a^17*b^13*c^3*d^27 + 83220*a^17*b^13*c^5*d^25 - 1174200*a^17*b^13*c^7*d^23 + 6342200*a^17*b^13*c^9*d^21 - 17770700*a^17*b^13*c^11*d^19 + 29213260*a^17*b^13*c^13*d^17 - 29535120*a^17*b^13*c^15*d^15 + 18346400*a^17*b^13*c^17*d^13 - 6653800*a^17*b^13*c^19*d^11 + 1227400*a^17*b^13*c^21*d^9 - 77520*a^17*b^13*c^23*d^7 + 190*a^18*b^12*c^2*d^28 - 25175*a^18*b^12*c^4*d^26 + 510625*a^18*b^12*c^6*d^24 - 3441850*a^18*b^12*c^8*d^22 + 11341480*a^18*b^12*c^10*d^20 - 21339185*a^18*b^12*c^12*d^18 + 24426875*a^18*b^12*c^14*d^16 - 17183600*a^18*b^12*c^16*d^14 + 7138300*a^18*b^12*c^18*d^12 - 1553630*a^18*b^12*c^20*d^10 + 125970*a^18*b^12*c^22*d^8 + 5800*a^19*b^11*c^3*d^27 - 183740*a^19*b^11*c^5*d^25 + 1607600*a^19*b^11*c^7*d^23 - 6323300*a^19*b^11*c^9*d^21 + 13697880*a^19*b^11*c^11*d^19 - 17770700*a^19*b^11*c^13*d^17 + 14108640*a^19*b^11*c^15*d^15 - 6653800*a^19*b^11*c^17*d^13 + 1679600*a^19*b^11*c^19*d^11 - 167960*a^19*b^11*c^21*d^9 - 955*a^20*b^10*c^2*d^28 + 53210*a^20*b^10*c^4*d^26 - 639360*a^20*b^10*c^6*d^24 + 3061855*a^20*b^10*c^8*d^22 - 7699257*a^20*b^10*c^10*d^20 + 11341480*a^20*b^10*c^12*d^18 - 10132510*a^20*b^10*c^14*d^16 + 5384410*a^20*b^10*c^16*d^14 - 1553630*a^20*b^10*c^18*d^12 + 184756*a^20*b^10*c^20*d^10 - 11900*a^21*b^9*c^3*d^27 + 213040*a^21*b^9*c^5*d^25 - 1277800*a^21*b^9*c^7*d^23 + 3770860*a^21*b^9*c^9*d^21 - 6323300*a^21*b^9*c^11*d^19 + 6342200*a^21*b^9*c^13*d^17 - 3772640*a^21*b^9*c^15*d^15 + 1227400*a^21*b^9*c^17*d^13 - 1679

60*a^21*b^9*c^19*d^11 + 1925*a^22*b^8*c^2*d^28 - 58000*a^22*b^8*c^4*d^26 + 455100*a^22*b^8*c^6*d^24 - 1598495*a^22*b^8*c^8*d^22 + 3061855*a^22*b^8*c^10*d^20 - 3441850*a^22*b^8*c^12*d^18 + 2277150*a^22*b^8*c^14*d^16 - 823650*a^22*b^8*c^16*d^14 + 125970*a^22*b^8*c^18*d^12 + 12400*a^23*b^7*c^3*d^27 - 136520*a^23*b^7*c^5*d^25 + 581120*a^23*b^7*c^7*d^23 - 1277800*a^23*b^7*c^9*d^21 + 1607600*a^23*b^7*c^11*d^19 - 1174200*a^23*b^7*c^13*d^17 + 465120*a^23*b^7*c^15*d^15 - 77520*a^23*b^7*c^17*d^13 - 1950*a^24*b^6*c^2*d^28 + 33825*a^24*b^6*c^4*d^26 - 178985*a^24*b^6*c^6*d^24 + 455100*a^24*b^6*c^8*d^22 - 639360*a^24*b^6*c^10*d^20 + 510625*a^24*b^6*c^12*d^18 - 218025*a^24*b^6*c^14*d^16 + 38760*a^24*b^6*c^16*d^14 - 6700*a^25*b^5*c^3*d^27 + 46004*a^25*b^5*c^5*d^25 - 136520*a^25*b^5*c^7*d^23 + 213040*a^25*b^5*c^9*d^21 - 183740*a^25*b^5*c^11*d^19 + 83220*a^25*b^5*c^13*d^17 - 15504*a^25*b^5*c^15*d^15 + 1000*a^26*b^4*c^2*d^28 - 9695*a^26*b^4*c^4*d^26 + 33825*a^26*b^4*c^6*d^24 - 58000*a^26*b^4*c^8*d^22 + 53210*a^26*b^4*c^10*d^20 - 25175*a^26*b^4*c^12*d^18 + 4845*a^26*b^4*c^14*d^16 + 1640*a^27*b^3*c^3*d^27 - 6700*a^27*b^3*c^5*d^25 + 12400*a^27*b^3*c^7*d^23 - 11900*a^27*b^3*c^9*d^21 + 5800*a^27*b^3*c^11*d^19 - 1140*a^27*b^3*c^13*d^17 - 215*a^28*b^2*c^2*d^28 + 1000*a^28*b^2*c^4*d^26 - 1950*a^28*b^2*c^6*d^24 + 1925*a^28*b^2*c^8*d^22 - 955*a^28*b^2*c^10*d^20 + 190*a^28*b^2*c^12*d^18 + 20*a*b^29*c^29*d + 20*a^29*b*c*d^29)))^(1/2)*((-(((4*a^24*d^24 + 4*b^24*c^24 + 16*a^2*b^22*c^24 + 16*a^4*b^20*c^24 - 1152*a^10*b^14*d^24 + 5568*a^12*b^12*d^24 - 10568*a^14*b^10*d^24 + 9460*a^16*b^8*d^24 - 3560*a^18*b^6*d^24 + 136*a^20*b^4*d^24 + 76*a^22*b^2*d^24 + 16*a^24*c^2*d^22 + 16*a^24*c^4*d^20 - 1152*b^24*c^10*d^14 + 5568*b^24*c^12*d^12 - 10568*b^24*c^14*d^10 + 9460*b^24*c^16*d^8 - 3560*b^24*c^18*d^6 + 136*b^24*c^20*d^4 + 76*b^24*c^22*d^2 + 11520*a*b^23*c^9*d^15 - 56448*a*b^23*c^11*d^13 + 109456*a*b^23*c^13*d^11 - 101240*a*b^23*c^15*d^9 + 40720*a*b^23*c^17*d^7 - 2960*a*b^23*c^19*d^5 - 536*a*b^23*c^21*d^3 - 176*a^3*b^21*c^23*d - 320*a^5*b^19*c^23*d + 11520*a^9*b^15*c*d^23 - 56448*a^11*b^13*c*d^23 + 109456*a^13*b^11*c*d^23 - 101240*a^15*b^9*c*d^23 + 40720*a^17*b^7*c*d^23 - 2960*a^19*b^5*c*d^23 - 536*a^21*b^3*c*d^23 - 176*a^23*b*c^3*d^21 - 320*a^23*b*c^5*d^19 - 51840*a^2*b^22*c^8*d^16 + 263808*a^2*b^22*c^10*d^14 - 541208*a^2*b^22*c^12*d^12 + 547088*a^2*b^22*c^14*d^10 - 263320*a^2*b^22*c^16*d^8 + 44120*a^2*b^22*c^18*d^6 - 1564*a^2*b^22*c^20*d^4 - 196*a^2*b^22*c^22*d^2 + 138240*a^3*b^21*c^7*d^17 - 758400*a^3*b^21*c^9*d^15 + 1720736*a^3*b^21*c^11*d^13 - 2002728*a^3*b^21*c^13*d^11 + 1210560*a^3*b^21*c^15*d^9 - 335040*a^3*b^21*c^17*d^7 + 37680*a^3*b^21*c^19*d^5 - 288*a^3*b^21*c^21*d^3 - 241920*a^4*b^20*c^6*d^18 + 1512000*a^4*b^20*c^8*d^16 - 3975688*a^4*b^20*c^10*d^14 + 5501328*a^4*b^20*c^12*d^12 - 4147952*a^4*b^20*c^14*d^10 + 1586920*a^4*b^20*c^16*d^8 - 276020*a^4*b^20*c^18*d^6 + 21124*a^4*b^20*c^20*d^4 + 176*a^4*b^20*c^22*d^2 + 290304*a^5*b^19*c^5*d^19 - 2232576*a^5*b^19*c^7*d^17 + 7078256*a^5*b^19*c^9*d^15 - 11781560*a^5*b^19*c^11*d^13 + 10875200*a^5*b^19*c^13*d^11 - 5365072*a^5*b^19*c^15*d^9 + 1310168*a^5*b^19*c^17*d^7 - 170968*a^5*b^19*c^19*d^5 + 8160*a^5*b^19*c^21*d^3 - 241920*a^6*b^18*c^4*d^20 + 2532096*a^6*b^18*c^6*d^18 - 9955992*a^6*b^18*c^8*d^16 + 20019440*a^6*b^18*c^10*d^14 - 22419600*a^6*b^18*c^12*d^12 + 13887520*a^6*b^18*c^14*d^10 - 4506428*a^6*b^18 *c^16*d^8 + 793756*a^6*b^18*c^18*d^6 - 72240*a^6*b^18*c^20*d^4 + 3040*a^6*b^18*c^22*d^2 + 138240*a^7*b^17*c^3*d^21 - 2232576*a^7*b^17*c^5*d^19 + 11150016*a^7*b^17*c^7*d^17 - 27336616*a^7*b^17*c^9*d^15 + 37153600*a^7*b^17*c^11*d^13 - 28461040*a^7*b^17*c^13*d^11 + 11779808*a^7*b^17*c^15*d^9 - 2621008*a^7*b^17*c^17*d^7 + 336688*a^7*b^17*c^19*d^5 - 17920*a^7*b^17*c^21*d^3 - 51840*a^8*b^16*c^2*d^22 + 1512000*a^8*b^16*c^4*d^20 - 9955992*a^8*b^16*c^6*d^18 + 30289656*a^8*b^16*c^8*d^16 - 50137600*a^8*b^16*c^10*d^14 + 46972560*a^8*b^16*c^12*d^12 - 24199280*a^8*b^16*c^14*d^10 + 6661036*a^8*b^16*c^16*d^8 - 1058448*a^8*b^16*c^18*d^6 + 72560*a^8*b^16*c^20*d^4 - 758400*a^9*b^15*c^3*d^21 + 7078256*a^9*b^15*c^5*d^19 - 27336616*a^9*b^15*c^7*d^17 + 55383904*a^9*b^15*c^9*d^15 - 63124080*a^9*b^15*c^11*d^13 + 39987520*a^9*b^15*c^13*d^11 - 13462088*a^9*b^15*c^15*d^9 + 2478528*a^9*b^15*c^17*d^7 - 212032*a^9*b^15*c^19*d^5 + 263808*a^10*b^14*c^2*d^22 - 3975688*a^10*b^14*c^4*d^20 + 20019440*a^10*b^14*c^6*d^18 - 50137600*a^10*b^14*c^8*d^16 + 69593872*a^10*b^14*c^10*d^14 - 53854288*a^10*b^14*c^12*d^12 + 21989928*a^10*b^14*c^14*d^10 - 4591360*a^10*b^14*c^16*d^8 + 460480*a^10*b^14*c^18*d^6 + 1720736*a^11*b^13*c^3*d^21 - 11781560*a^11*b^13*c^5*d^19 + 37153600*a^11*b^13*c^7*d^17 - 63124080*a^11*b^13*c^9*d^15 + 59445728*a^11*b^13*c^11*d^13 - 29358696*a^11*b^13*c^13*d^11 + 6995840*a^11*b^13*c^15*d^9 - 762560*a^11*b^13*c^17*d^7 - 541208*a^12*b^12*c^2*d^22 + 5501328*a^12*b^12*c^4*d^20 - 22419600*a^12*b^12*c^6*d^18 + 46972560*a^12*b^12*c^8*d^16 - 53854288*a^12*b^12*c^10*d^14 + 32294808*a^12*b^12*c^12*d^12 - 8958208*a^12*b^12*c^14*d^10 + 999040*a^12*b^12*c^16*d^8 - 2002728*a^13*b^11*c^3*d^21 + 10875200*a^13*b^11*c^5*d^19 - 28461040*a^13*b^11*c^7*d^17 + 39987520*a^13*b^11*c^9*d^15 - 29358696*a^13*b^11*c^11*d^13 + 9722048*a^13*b^11*c^13*d^11 - 1104320*a^13*b^11*c^15*d^9 + 547088*a^14*b^10*c^2*d^22 - 4147952*a^14*b^10*c^4*d^20 + 13887520*a^14*b^10*c^6*d^18 - 24199280*a^14*b^10*c^8*d^16 + 21989928*a^14*b^10*c^10*d^14 - 8958208*a^14*b^10*c^12*d^12 + 1124032*a^14*b^10*c^14*d^10 + 1210560*a^15*b^9*c^3*d^21 - 5365072*a^15*b^9*c^5*d^19 + 11779808*a^15*b^9*c^7*d^17 - 13462088*a^15*b^9*c^9*d^15 + 6995840*a^15*b^9*c^11*d^13 - 1104320*a^15*b^9*c^13*d^11 - 263320*a^16*b^8*c^2*d^22 + 1586920*a^16*b^8*c^4*d^20 - 4506428*a^16*b^8*c^6*d^18 + 6661036*a^16*b^8*c^8*d^16 - 4591360*a^16*b^8*c^10*d^14 + 999040*a^16*b^8*c^12*d^12 - 335040*a^17*b^7*c^3*d^21 + 1310168*a^17*b^7*c^5*d^19 - 2621008*a^17*b^7*c^7*d^17 + 2478528*a^17*b^7*c^9*d^15 - 762560*a^17*b^7*c^11*d^13 + 44120*a^18*b^6*c^2*d^22 - 276020*a^18*b^6*c^4*d^20 + 793756*a^18*b^6*c^6*d^18 - 1058448*a^18*b^6*c^8*d^16 + 460480*a^18*b^6*c^10*d^14 + 37680*a^19*b^5*c^3*d^21 - 170968*a^19*b^5*c^5*d^19 + 336688*a^19*b^5*c^7*d^17 - 212032*a^19*b^5*c^9*d^15 - 1564*a^20*b^4*c^2*d^22 + 21124*a^20*b^4*c^4*d^20 - 72240*a^20*b^4*c^6*d^18 + 72560*a^20*b^4*c^8*d^16 - 288*a^21*b^3*c^3*d^21 + 8160*a^21*b^3*c^5*d^19 - 17920*a^21*b^3*c^7*d^17 - 196*a^22*b^2*c^2*d^22 + 176*a^22*b^2*c^4*d^20 + 3040*a^22*b^2*c^6*d^18 - 8*a*b^23*c^23*d - 8*a^23*b*c*d^23)^2/4 - (20736*b^18*d^18 - 96768*a^2*b^16*d^18 + 173664*a^4*b^14*d^18 - 136032*a^6*b^12*d^18 + 31081*a^8*b^10*d^18 + 8440*a^10*b^8*d^18 + 400*a^12*b^6*d^18 - 96768*b^18*c^2*d^16 + 173664*b^18*c^4*d^14 - 136032*b^18*c^6*d^12 + 31081*b^18*c^8*d^10 + 8440*b^18*c^10*d^8 + 400*b^18*c^12*d^6 - 131328*a *b^17*c^3*d^15 + 216576*a*b^17*c^5*d^13 - 141104*a*b^17*c^7*d^11 + 20260*a*b^17*c^9*d^9 + 2800*a*b^17*c^11*d^7 - 131328*a^3*b^15*c*d^17 + 216576*a^5*b^13*c*d^17 - 141104*a^7*b^11*c*d^17 + 20260*a^9*b^9*c*d^17 + 2800*a^11*b^7*c*d^17 + 495936*a^2*b^16*c^2*d^16 - 989856*a^2*b^16*c^4*d^14 + 901948*a^2*b^16*c^6*d^12 - 308392*a^2*b^16*c^8*d^10 - 5260*a^2*b^16*c^10*d^8 + 1600*a^2*b^16*c^12*d^6 + 657408*a^3*b^15*c^3*d^15 - 1158992*a^3*b^15*c^5*d^13 + 838256*a^3*b^15*c^7*d^11 - 182200*a^3*b^15*c^9*d^9 - 3200*a^3*b^15*c^11*d^7 - 989856*a^4*b^14*c^2*d^16 + 2185654*a^4*b^14*c^4*d^14 - 2218576*a^4*b^14*c^6*d^12 + 900624*a^4*b^14*c^8*d^10 - 64720*a^4*b^14*c^10*d^8 + 1600*a^4*b^14*c^12*d^6 - 1158992*a^5*b^13*c^3*d^15 + 2158808*a^5*b^13*c^5*d^13 - 1641528*a^5*b^13*c^7*d^11 + 406880*a^5*b^13*c^9*d^9 - 17600*a^5*b^13*c^11*d^7 + 901948*a^6*b^12*c^2*d^16 - 2218576*a^6*b^12*c^4*d^14 + 2430936*a^6*b^12*c^6*d^12 - 1026928*a^6*b^12*c^8*d^10 + 88720*a^6*b^12*c^10*d^8 + 838256*a^7*b^11*c^3*d^15 - 1641528*a^7*b^11*c^5*d^13 + 1206848*a^7*b^11*c^7*d^11 - 239360*a^7*b^11*c^9*d^9 - 308392*a^8*b^10*c^2*d^16 + 900624*a^8*b^10*c^4*d^14 - 1026928*a^8*b^10*c^6*d^12 + 354016*a^8*b^10*c^8*d^10 - 182200*a^9*b^9*c^3*d^15 + 406880*a^9*b^9*c^5*d^13 - 239360*a^9*b^9*c^7*d^11 - 5260*a^10*b^8*c^2*d^16 - 64720*a^10*b^8*c^4*d^14 + 88720*a^10*b^8*c^6*d^12 - 3200*a^11*b^7*c^3*d^15 - 17600*a^11*b^7*c^5*d^13 + 1600*a^12*b^6*c^2*d^16 + 1600*a^12*b^6*c^4*d^14 + 27648*a*b^17*c*d^17)*(80*a^2*b^28*c^30 - 16*b^30*c^30 - 16*a^30*d^30 - 160*a^4*b^26*c^30 + 160*a^6*b^24*c^30 - 80*a^8*b^22*c^30 + 16*a^10*b^20*c^30 + 16*a^20*b^10*d^30 - 80*a^22*b^8*d^30 + 160*a^24*b^6*d^30 - 160*a^26*b^4*d^30 + 80*a^28*b^2*d^30 + 80*a^30*c^2*d^28 - 160*a^30*c^4*d^26 + 160*a^30*c^6*d^24 - 80*a^30*c^8*d^22 + 16*a^30*c^10*d^20 + 16*b^30*c^20*d^10 - 80*b^30*c^22*d^8 + 160*b^30*c^24*d^6 - 160*b^30*c^26*d^4 + 80*b^30*c^28*d^2 - 320*a*b^29*c^19*d^11 + 1600*a*b^29*c^21*d^9 - 3200*a*b^29*c^23*d^7 + 3200*a*b^29*c^25*d^5 - 1600*a*b^29*c^27*d^3 - 1600*a^3*b^27*c^29*d + 3200*a^5*b^25*c^29*d - 3200*a^7*b^23*c^29*d + 1600*a^9*b^21*c^29*d - 320*a^11*b^19*c^29*d - 320*a^19*b^11*c*d^29 + 1600*a^21*b^9*c*d^29 - 3200*a^23*b^7*c*d^29 + 3200*a^25*b^5*c*d^29 - 1600*a^27*b^3*c*d^29 - 1600*a^29*b*c^3*d^27 + 3200*a^29*b*c^5*d^25 - 3200*a^29*b*c^7*d^23 + 1600*a^29*b*c^9*d^21 - 320*a^29*b*c^11*d^19 + 3040*a^2*b^28*c^18*d^12 - 15280*a^2*b^28*c^20*d^10 + 30800*a^2*b^28*c^22*d^8 - 31200*a^2*b^28*c^24*d^6 + 16000*a^2*b^28*c^26*d^4 - 3440*a^2*b^28*c^28*d^2 - 18240*a^3*b^27*c^17*d^13 + 92800*a^3*b^27*c^19*d^11 - 190400*a^3*b^27*c^21*d^9 + 198400*a^3*b^27*c^23*d^7 - 107200*a^3*b^27*c^25*d^5 + 26240*a^3*b^27*c^27*d^3 + 77520*a^4*b^26*c^16*d^14 - 402800*a^4*b^26*c^18*d^12 + 851360*a^4*b^26*c^20*d^10 - 928000*a^4*b^26*c^22*d^8 + 541200*a^4*b^26*c^24*d^6 - 155120*a^4*b^26*c^26*d^4 + 16000*a^4*b^26*c^28*d^2 - 248064*a^5*b^25*c^15*d^15 + 1331520*a^5*b^25*c^17*d^13 - 2939840*a^5*b^25*c^19*d^11 + 3408640*a^5*b^25*c^21*d^9 - 2184320*a^5*b^25*c^23*d^7 + 736064*a^5*b^25*c^25*d^5 - 107200*a^5*b^25*c^27*d^3 + 620160*a^6*b^24*c^14*d^16 - 3488400*a^6*b^24*c^16*d^14 + 8170000*a^6*b^24*c^18*d^12 - 10229760*a^6*b^24*c^20*d^10 + 7281600*a^6*b^24*c^22*d^8 - 2863760*a^6*b^24*c^24*d^6 + 541200*a^6*b^24*c^26*d^4 - 31200*a^6*b^24*c^28*d^2 - 1240320*a^7*b^23*c^13*d^17 + 7441920*a^7*b^23*c^15*d^15 - 18787200*a^7*b^23*c^17*d^13 + 25721600*a^7*b^23*c^19 *d^11 - 20444800*a^7*b^23*c^21*d^9 + 9297920*a^7*b^23*c^23*d^7 - 2184320*a^7*b^23*c^25*d^5 + 198400*a^7*b^23*c^27*d^3 + 2015520*a^8*b^22*c^12*d^18 - 13178400*a^8*b^22*c^14*d^16 + 36434400*a^8*b^22*c^16*d^14 - 55069600*a^8*b^22*c^18*d^12 + 48989680*a^8*b^22*c^20*d^10 - 25575920*a^8*b^22*c^22*d^8 + 7281600*a^8*b^22*c^24*d^6 - 928000*a^8*b^22*c^26*d^4 + 30800*a^8*b^22*c^28*d^2 - 2687360*a^9*b^21*c^11*d^19 + 19638400*a^9*b^21*c^13*d^17 - 60362240*a^9*b^21*c^15*d^15 + 101475200*a^9*b^21*c^17*d^13 - 101172800*a^9*b^21*c^19*d^11 + 60333760*a^9*b^21*c^21*d^9 - 20444800*a^9*b^21*c^23*d^7 + 3408640*a^9*b^21*c^25*d^5 - 190400*a^9*b^21*c^27*d^3 + 2956096*a^10*b^20*c^10*d^20 - 24858080*a^10*b^20*c^12*d^18 + 86150560*a^10*b^20*c^14*d^16 - 162120160*a^10*b^20*c^16*d^14 + 181463680*a^10*b^20*c^18*d^12 - 123188112*a^10*b^20*c^20*d^10 + 48989680*a^10*b^20*c^22*d^8 - 10229760*a^10*b^20*c^24*d^6 + 851360*a^10*b^20*c^26*d^4 - 15280*a^10*b^20*c^28*d^2 - 2687360*a^11*b^19*c^9*d^21 + 26873600*a^11*b^19*c^11*d^19 - 106460800*a^11*b^19*c^13*d^17 + 225738240*a^11*b^19*c^15*d^15 - 284331200*a^11*b^19*c^17*d^13 + 219166080*a^11*b^19*c^19*d^11 - 101172800*a^11*b^19*c^21*d^9 + 25721600*a^11*b^19*c^23*d^7 - 2939840*a^11*b^19*c^25*d^5 + 92800*a^11*b^19*c^27*d^3 + 2015520*a^12*b^18*c^8*d^22 - 24858080*a^12*b^18*c^10*d^20 + 114212800*a^12*b^18*c^12*d^18 - 274937600*a^12*b^18*c^14*d^16 + 390830000*a^12*b^18*c^16*d^14 - 341426960*a^12*b^18*c^18*d^12 + 181463680*a^12*b^18*c^20*d^10 - 55069600*a^12*b^18*c^22*d^8 + 8170000*a^12*b^18*c^24*d^6 - 402800*a^12*b^18*c^26*d^4 + 3040*a^12*b^18*c^28*d^2 - 1240320*a^13*b^17*c^7*d^23 + 19638400*a^13*b^17*c^9*d^21 - 106460800*a^13*b^17*c^11*d^19 + 293542400*a^13*b^17*c^13*d^17 - 472561920*a^13*b^17*c^15*d^15 + 467412160*a^13*b^17*c^17*d^13 - 284331200*a^13*b^17*c^19*d^11 + 101475200*a^13*b^17*c^21*d^9 - 18787200*a^13*b^17*c^23*d^7 + 1331520*a^13*b^17*c^25*d^5 - 18240*a^13*b^17*c^27*d^3 + 620160*a^14*b^16*c^6*d^24 - 13178400*a^14*b^16*c^8*d^22 + 86150560*a^14*b^16*c^10*d^20 - 274937600*a^14*b^16*c^12*d^18 + 503363200*a^14*b^16*c^14*d^16 - 563751280*a^14*b^16*c^16*d^14 + 390830000*a^14*b^16*c^18*d^12 - 162120160*a^14*b^16*c^20*d^10 + 36434400*a^14*b^16*c^22*d^8 - 3488400*a^14*b^16*c^24*d^6 + 77520*a^14*b^16*c^26*d^4 - 248064*a^15*b^15*c^5*d^25 + 7441920*a^15*b^15*c^7*d^23 - 60362240*a^15*b^15*c^9*d^21 + 225738240*a^15*b^15*c^11*d^19 - 472561920*a^15*b^15*c^13*d^17 + 599984128*a^15*b^15*c^15*d^15 - 472561920*a^15*b^15*c^17*d^13 + 225738240*a^15*b^15*c^19*d^11 - 60362240*a^15*b^15*c^21*d^9 + 7441920*a^15*b^15*c^23*d^7 - 248064*a^15*b^15*c^25*d^5 + 77520*a^16*b^14*c^4*d^26 - 3488400*a^16*b^14*c^6*d^24 + 36434400*a^16*b^14*c^8*d^22 - 162120160*a^16*b^14*c^10*d^20 + 390830000*a^16*b^14*c^12*d^18 - 563751280*a^16*b^14*c^14*d^16 + 503363200*a^16*b^14*c^16*d^14 - 274937600*a^16*b^14*c^18*d^12 + 86150560*a^16*b^14*c^20*d^10 - 13178400*a^16*b^14*c^22*d^8 + 620160*a^16*b^14*c^24*d^6 - 18240*a^17*b^13*c^3*d^27 + 1331520*a^17*b^13*c^5*d^25 - 18787200*a^17*b^13*c^7*d^23 + 101475200*a^17*b^13*c^9*d^21 - 284331200*a^17*b^13*c^11*d^19 + 467412160*a^17*b^13*c^13*d^17 - 472561920*a^17*b^13*c^15*d^15 + 293542400*a^17*b^13*c^17*d^13 - 106460800*a^17*b^13*c^19*d^11 + 19638400*a^17*b^13*c^21*d^9 - 1240320*a^17*b^13*c^23*d^7 + 3040*a^18*b^12*c^2*d^28 - 402800*a^18*b^12*c^4*d^26 + 8170000*a^18*b^12*c^6*d^24 - 55069600*a^18*b^12*c^8*d^22 + 18

1463680*a^18*b^12*c^10*d^20 - 341426960*a^18*b^12*c^12*d^18 + 390830000*a^1
8*b^12*c^14*d^16 - 274937600*a^18*b^12*c^16*d^14 + 114212800*a^18*b^12*c^18
*d^12 - 24858080*a^18*b^12*c^20*d^10 + 2015520*a^18*b^12*c^22*d^8 + 92800*a
^19*b^11*c^3*d^27 - 2939840*a^19*b^11*c^5*d^25 + 25721600*a^19*b^11*c^7*d^2
3 - 101172800*a^19*b^11*c^9*d^21 + 219166080*a^19*b^11*c^11*d^19 - 28433120
0*a^19*b^11*c^13*d^17 + 225738240*a^19*b^11*c^15*d^15 - 106460800*a^19*b^11
*c^17*d^13 + 26873600*a^19*b^11*c^19*d^11 - 2687360*a^19*b^11*c^21*d^9 - 15
280*a^20*b^10*c^2*d^28 + 851360*a^20*b^10*c^4*d^26 - 10229760*a^20*b^10*c^6
*d^24 + 48989680*a^20*b^10*c^8*d^22 - 123188112*a^20*b^10*c^10*d^20 + 18146
3680*a^20*b^10*c^12*d^18 - 162120160*a^20*b^10*c^14*d^16 + 86150560*a^20*b^
10*c^16*d^14 - 24858080*a^20*b^10*c^18*d^12 + 2956096*a^20*b^10*c^20*d^10 -
190400*a^21*b^9*c^3*d^27 + 3408640*a^21*b^9*c^5*d^25 - 20444800*a^21*b^9*c
^7*d^23 + 60333760*a^21*b^9*c^9*d^21 - 101172800*a^21*b^9*c^11*d^19 + 10147
5200*a^21*b^9*c^13*d^17 - 60362240*a^21*b^9*c^15*d^15 + 19638400*a^21*b^9*c
^17*d^13 - 2687360*a^21*b^9*c^19*d^11 + 30800*a^22*b^8*c^2*d^28 - 928000*a^
22*b^8*c^4*d^26 + 7281600*a^22*b^8*c^6*d^24 - 25575920*a^22*b^8*c^8*d^22 +
48989680*a^22*b^8*c^10*d^20 - 55069600*a^22*b^8*c^12*d^18 + 36434400*a^22*b
^8*c^14*d^16 - 13178400*a^22*b^8*c^16*d^14 + 2015520*a^22*b^8*c^18*d^12 + 1
98400*a^23*b^7*c^3*d^27 - 2184320*a^23*b^7*c^5*d^25 + 9297920*a^23*b^7*c^7*
d^23 - 20444800*a^23*b^7*c^9*d^21 + 25721600*a^23*b^7*c^11*d^19 - 18787200*
a^23*b^7*c^13*d^17 + 7441920*a^23*b^7*c^15*d^15 - 1240320*a^23*b^7*c^17*d^1
3 - 31200*a^24*b^6*c^2*d^28 + 541200*a^24*b^6*c^4*d^26 - 2863760*a^24*b^6*c
^6*d^24 + 7281600*a^24*b^6*c^8*d^22 - 10229760*a^24*b^6*c^10*d^20 + 8170000
*a^24*b^6*c^12*d^18 - 3488400*a^24*b^6*c^14*d^16 + 620160*a^24*b^6*c^16*d^1
4 - 107200*a^25*b^5*c^3*d^27 + 736064*a^25*b^5*c^5*d^25 - 2184320*a^25*b^5*
c^7*d^23 + 3408640*a^25*b^5*c^9*d^21 - 2939840*a^25*b^5*c^11*d^19 + 1331520
*a^25*b^5*c^13*d^17 - 248064*a^25*b^5*c^15*d^15 + 16000*a^26*b^4*c^2*d^28 -
155120*a^26*b^4*c^4*d^26 + 541200*a^26*b^4*c^6*d^24 - 928000*a^26*b^4*c^8*
d^22 + 851360*a^26*b^4*c^10*d^20 - 402800*a^26*b^4*c^12*d^18 + 77520*a^26*b
^4*c^14*d^16 + 26240*a^27*b^3*c^3*d^27 - 107200*a^27*b^3*c^5*d^25 + 198400*
a^27*b^3*c^7*d^23 - 190400*a^27*b^3*c^9*d^21 + 92800*a^27*b^3*c^11*d^19 - 1
8240*a^27*b^3*c^13*d^17 - 3440*a^28*b^2*c^2*d^28 + 16000*a^28*b^2*c^4*d^26
- 31200*a^28*b^2*c^6*d^24 + 30800*a^28*b^2*c^8*d^22 - 15280*a^28*b^2*c^10*d
^20 + 3040*a^28*b^2*c^12*d^18 + 320*a*b^29*c^29*d + 320*a^29*b*c*d^29))^(1/
2) + 2*a^24*d^24 + 2*b^24*c^24 + 8*a^2*b^22*c^24 + 8*a^4*b^20*c^24 - 576*a^
10*b^14*d^24 + 2784*a^12*b^12*d^24 - 5284*a^14*b^10*d^24 + 4730*a^16*b^8*d^
24 - 1780*a^18*b^6*d^24 + 68*a^20*b^4*d^24 + 38*a^22*b^2*d^24 + 8*a^24*c^2*
d^22 + 8*a^24*c^4*d^20 - 576*b^24*c^10*d^14 + 2784*b^24*c^12*d^12 - 5284*b^
24*c^14*d^10 + 4730*b^24*c^16*d^8 - 1780*b^24*c^18*d^6 + 68*b^24*c^20*d^4 +
38*b^24*c^22*d^2 + 5760*a*b^23*c^9*d^15 - 28224*a*b^23*c^11*d^13 + 54728*a
*b^23*c^13*d^11 - 50620*a*b^23*c^15*d^9 + 20360*a*b^23*c^17*d^7 - 1480*a*b^
23*c^19*d^5 - 268*a*b^23*c^21*d^3 - 88*a^3*b^21*c^23*d - 160*a^5*b^19*c^23*
d + 5760*a^9*b^15*c*d^23 - 28224*a^11*b^13*c*d^23 + 54728*a^13*b^11*c*d^23
- 50620*a^15*b^9*c*d^23 + 20360*a^17*b^7*c*d^23 - 1480*a^19*b^5*c*d^23 - 26
8*a^21*b^3*c*d^23 - 88*a^23*b*c^3*d^21 - 160*a^23*b*c^5*d^19 - 25920*a^2*b^

22*c^8*d^16 + 131904*a^2*b^22*c^10*d^14 - 270604*a^2*b^22*c^12*d^12 + 27354
4*a^2*b^22*c^14*d^10 - 131660*a^2*b^22*c^16*d^8 + 22060*a^2*b^22*c^18*d^6 -
782*a^2*b^22*c^20*d^4 - 98*a^2*b^22*c^22*d^2 + 69120*a^3*b^21*c^7*d^17 - 3
79200*a^3*b^21*c^9*d^15 + 860368*a^3*b^21*c^11*d^13 - 1001364*a^3*b^21*c^13
*d^11 + 605280*a^3*b^21*c^15*d^9 - 167520*a^3*b^21*c^17*d^7 + 18840*a^3*b^2
1*c^19*d^5 - 144*a^3*b^21*c^21*d^3 - 120960*a^4*b^20*c^6*d^18 + 756000*a^4*
b^20*c^8*d^16 - 1987844*a^4*b^20*c^10*d^14 + 2750664*a^4*b^20*c^12*d^12 - 2
073976*a^4*b^20*c^14*d^10 + 793460*a^4*b^20*c^16*d^8 - 138010*a^4*b^20*c^18
*d^6 + 10562*a^4*b^20*c^20*d^4 + 88*a^4*b^20*c^22*d^2 + 145152*a^5*b^19*c^5
*d^19 - 1116288*a^5*b^19*c^7*d^17 + 3539128*a^5*b^19*c^9*d^15 - 5890780*a^5
*b^19*c^11*d^13 + 5437600*a^5*b^19*c^13*d^11 - 2682536*a^5*b^19*c^15*d^9 +
655084*a^5*b^19*c^17*d^7 - 85484*a^5*b^19*c^19*d^5 + 4080*a^5*b^19*c^21*d^3
 - 120960*a^6*b^18*c^4*d^20 + 1266048*a^6*b^18*c^6*d^18 - 4977996*a^6*b^18*
c^8*d^16 + 10009720*a^6*b^18*c^10*d^14 - 11209800*a^6*b^18*c^12*d^12 + 6943
760*a^6*b^18*c^14*d^10 - 2253214*a^6*b^18*c^16*d^8 + 396878*a^6*b^18*c^18*d
^6 - 36120*a^6*b^18*c^20*d^4 + 1520*a^6*b^18*c^22*d^2 + 69120*a^7*b^17*c^3*
d^21 - 1116288*a^7*b^17*c^5*d^19 + 5575008*a^7*b^17*c^7*d^17 - 13668308*a^7
*b^17*c^9*d^15 + 18576800*a^7*b^17*c^11*d^13 - 14230520*a^7*b^17*c^13*d^11
+ 5889904*a^7*b^17*c^15*d^9 - 1310504*a^7*b^17*c^17*d^7 + 168344*a^7*b^17*c
^19*d^5 - 8960*a^7*b^17*c^21*d^3 - 25920*a^8*b^16*c^2*d^22 + 756000*a^8*b^1
6*c^4*d^20 - 4977996*a^8*b^16*c^6*d^18 + 15144828*a^8*b^16*c^8*d^16 - 25068
800*a^8*b^16*c^10*d^14 + 23486280*a^8*b^16*c^12*d^12 - 12099640*a^8*b^16*c^
14*d^10 + 3330518*a^8*b^16*c^16*d^8 - 529224*a^8*b^16*c^18*d^6 + 36280*a^8*
b^16*c^20*d^4 - 379200*a^9*b^15*c^3*d^21 + 3539128*a^9*b^15*c^5*d^19 - 1366
8308*a^9*b^15*c^7*d^17 + 27691952*a^9*b^15*c^9*d^15 - 31562040*a^9*b^15*c^1
1*d^13 + 19993760*a^9*b^15*c^13*d^11 - 6731044*a^9*b^15*c^15*d^9 + 1239264*
a^9*b^15*c^17*d^7 - 106016*a^9*b^15*c^19*d^5 + 131904*a^10*b^14*c^2*d^22 -
1987844*a^10*b^14*c^4*d^20 + 10009720*a^10*b^14*c^6*d^18 - 25068800*a^10*b^
14*c^8*d^16 + 34796936*a^10*b^14*c^10*d^14 - 26927144*a^10*b^14*c^12*d^12 +
 10994964*a^10*b^14*c^14*d^10 - 2295680*a^10*b^14*c^16*d^8 + 230240*a^10*b^
14*c^18*d^6 + 860368*a^11*b^13*c^3*d^21 - 5890780*a^11*b^13*c^5*d^19 + 1857
6800*a^11*b^13*c^7*d^17 - 31562040*a^11*b^13*c^9*d^15 + 29722864*a^11*b^13*
c^11*d^13 - 14679348*a^11*b^13*c^13*d^11 + 3497920*a^11*b^13*c^15*d^9 - 381
280*a^11*b^13*c^17*d^7 - 270604*a^12*b^12*c^2*d^22 + 2750664*a^12*b^12*c^4*
d^20 - 11209800*a^12*b^12*c^6*d^18 + 23486280*a^12*b^12*c^8*d^16 - 26927144
*a^12*b^12*c^10*d^14 + 16147404*a^12*b^12*c^12*d^12 - 4479104*a^12*b^12*c^1
4*d^10 + 499520*a^12*b^12*c^16*d^8 - 1001364*a^13*b^11*c^3*d^21 + 5437600*a
^13*b^11*c^5*d^19 - 14230520*a^13*b^11*c^7*d^17 + 19993760*a^13*b^11*c^9*d^
15 - 14679348*a^13*b^11*c^11*d^13 + 4861024*a^13*b^11*c^13*d^11 - 552160*a^
13*b^11*c^15*d^9 + 273544*a^14*b^10*c^2*d^22 - 2073976*a^14*b^10*c^4*d^20 +
 6943760*a^14*b^10*c^6*d^18 - 12099640*a^14*b^10*c^8*d^16 + 10994964*a^14*b
^10*c^10*d^14 - 4479104*a^14*b^10*c^12*d^12 + 562016*a^14*b^10*c^14*d^10 +
605280*a^15*b^9*c^3*d^21 - 2682536*a^15*b^9*c^5*d^19 + 5889904*a^15*b^9*c^7
*d^17 - 6731044*a^15*b^9*c^9*d^15 + 3497920*a^15*b^9*c^11*d^13 - 552160*a^1
5*b^9*c^13*d^11 - 131660*a^16*b^8*c^2*d^22 + 793460*a^16*b^8*c^4*d^20 - 225

3214*a^16*b^8*c^6*d^18 + 3330518*a^16*b^8*c^8*d^16 - 2295680*a^16*b^8*c^10*d^14 + 499520*a^16*b^8*c^12*d^12 - 167520*a^17*b^7*c^3*d^21 + 655084*a^17*b^7*c^5*d^19 - 1310504*a^17*b^7*c^7*d^17 + 1239264*a^17*b^7*c^9*d^15 - 381280*a^17*b^7*c^11*d^13 + 22060*a^18*b^6*c^2*d^22 - 138010*a^18*b^6*c^4*d^20 + 396878*a^18*b^6*c^6*d^18 - 529224*a^18*b^6*c^8*d^16 + 230240*a^18*b^6*c^10*d^14 + 18840*a^19*b^5*c^3*d^21 - 85484*a^19*b^5*c^5*d^19 + 168344*a^19*b^5*c^7*d^17 - 106016*a^19*b^5*c^9*d^15 - 782*a^20*b^4*c^2*d^22 + 10562*a^20*b^4*c^4*d^20 - 36120*a^20*b^4*c^6*d^18 + 36280*a^20*b^4*c^8*d^16 - 144*a^21*b^3*c^3*d^21 + 4080*a^21*b^3*c^5*d^19 - 8960*a^21*b^3*c^7*d^17 - 98*a^22*b^2*c^2*d^22 + 88*a^22*b^2*c^4*d^20 + 1520*a^22*b^2*c^6*d^18 - 4*a*b^23*c^23*d - 4*a^23*b*c*d^23)/(16*(5*a^2*b^28*c^30 - b^30*c^30 - a^30*d^30 - 10*a^4*b^26*c^30 + 10*a^6*b^24*c^30 - 5*a^8*b^22*c^30 + a^10*b^20*c^30 + a^20*b^10*d^30 - 5*a^22*b^8*d^30 + 10*a^24*b^6*d^30 - 10*a^26*b^4*d^30 + 5*a^28*b^2*d^30 + 5*a^30*c^2*d^28 - 10*a^30*c^4*d^26 + 10*a^30*c^6*d^24 - 5*a^30*c^8*d^22 + a^30*c^10*d^20 + b^30*c^20*d^10 - 5*b^30*c^22*d^8 + 10*b^30*c^24*d^6 - 10*b^30*c^26*d^4 + 5*b^30*c^28*d^2 - 20*a*b^29*c^19*d^11 + 100*a*b^29*c^21*d^9 - 200*a*b^29*c^23*d^7 + 200*a*b^29*c^25*d^5 - 100*a*b^29*c^27*d^3 - 100*a^3*b^27*c^29*d + 200*a^5*b^25*c^29*d - 200*a^7*b^23*c^29*d + 100*a^9*b^21*c^29*d - 20*a^11*b^19*c^29*d - 20*a^19*b^11*c*d^29 + 100*a^21*b^9*c*d^29 - 200*a^23*b^7*c*d^29 + 200*a^25*b^5*c*d^29 - 100*a^27*b^3*c*d^29 - 100*a^29*b*c^3*d^27 + 200*a^29*b*c^5*d^25 - 200*a^29*b*c^7*d^23 + 100*a^29*b*c^9*d^21 - 20*a^29*b*c^11*d^19 + 190*a^2*b^28*c^18*d^12 - 955*a^2*b^28*c^20*d^10 + 1925*a^2*b^28*c^22*d^8 - 1950*a^2*b^28*c^24*d^6 + 1000*a^2*b^28*c^26*d^4 - 215*a^2*b^28*c^28*d^2 - 1140*a^3*b^27*c^17*d^13 + 5800*a^3*b^27*c^19*d^11 - 11900*a^3*b^27*c^21*d^9 + 12400*a^3*b^27*c^23*d^7 - 6700*a^3*b^27*c^25*d^5 + 1640*a^3*b^27*c^27*d^3 + 4845*a^4*b^26*c^16*d^14 - 25175*a^4*b^26*c^18*d^12 + 53210*a^4*b^26*c^20*d^10 - 58000*a^4*b^26*c^22*d^8 + 33825*a^4*b^26*c^24*d^6 - 9695*a^4*b^26*c^26*d^4 + 1000*a^4*b^26*c^28*d^2 - 15504*a^5*b^25*c^15*d^15 + 83220*a^5*b^25*c^17*d^13 - 183740*a^5*b^25*c^19*d^11 + 213040*a^5*b^25*c^21*d^9 - 136520*a^5*b^25*c^23*d^7 + 46004*a^5*b^25*c^25*d^5 - 6700*a^5*b^25*c^27*d^3 + 38760*a^6*b^24*c^14*d^16 - 218025*a^6*b^24*c^16*d^14 + 510625*a^6*b^24*c^18*d^12 - 639360*a^6*b^24*c^20*d^10 + 455100*a^6*b^24*c^22*d^8 - 178985*a^6*b^24*c^24*d^6 + 33825*a^6*b^24*c^26*d^4 - 1950*a^6*b^24*c^28*d^2 - 77520*a^7*b^23*c^13*d^17 + 465120*a^7*b^23*c^15*d^15 - 1174200*a^7*b^23*c^17*d^13 + 1607600*a^7*b^23*c^19*d^11 - 1277800*a^7*b^23*c^21*d^9 + 581120*a^7*b^23*c^23*d^7 - 136520*a^7*b^23*c^25*d^5 + 12400*a^7*b^23*c^27*d^3 + 125970*a^8*b^22*c^12*d^18 - 823650*a^8*b^22*c^14*d^16 + 2277150*a^8*b^22*c^16*d^14 - 3441850*a^8*b^22*c^18*d^12 + 3061855*a^8*b^22*c^20*d^10 - 1598495*a^8*b^22*c^22*d^8 + 455100*a^8*b^22*c^24*d^6 - 58000*a^8*b^22*c^26*d^4 + 1925*a^8*b^22*c^28*d^2 - 167960*a^9*b^21*c^11*d^19 + 1227400*a^9*b^21*c^13*d^17 - 3772640*a^9*b^21*c^15*d^15 + 6342200*a^9*b^21*c^17*d^13 - 6323300*a^9*b^21*c^19*d^11 + 3770860*a^9*b^21*c^21*d^9 - 1277800*a^9*b^21*c^23*d^7 + 213040*a^9*b^21*c^25*d^5 - 11900*a^9*b^21*c^27*d^3 + 184756*a^10*b^20*c^10*d^20 - 1553630*a^10*b^20*c^12*d^18 + 5384410*a^10*b^20*c^14*d^16 - 10132510*a^10*b^20*c^16*d^14 + 11341480*a^10*b^20*c^18*d^12 - 7699257*a

^10*b^20*c^20*d^10 + 3061855*a^10*b^20*c^22*d^8 - 639360*a^10*b^20*c^24*d^6
 + 53210*a^10*b^20*c^26*d^4 - 955*a^10*b^20*c^28*d^2 - 167960*a^11*b^19*c^9
*d^21 + 1679600*a^11*b^19*c^11*d^19 - 6653800*a^11*b^19*c^13*d^17 + 1410864
0*a^11*b^19*c^15*d^15 - 17770700*a^11*b^19*c^17*d^13 + 13697880*a^11*b^19*c
^19*d^11 - 6323300*a^11*b^19*c^21*d^9 + 1607600*a^11*b^19*c^23*d^7 - 183740
*a^11*b^19*c^25*d^5 + 5800*a^11*b^19*c^27*d^3 + 125970*a^12*b^18*c^8*d^22 -
 1553630*a^12*b^18*c^10*d^20 + 7138300*a^12*b^18*c^12*d^18 - 17183600*a^12*
b^18*c^14*d^16 + 24426875*a^12*b^18*c^16*d^14 - 21339185*a^12*b^18*c^18*d^1
2 + 11341480*a^12*b^18*c^20*d^10 - 3441850*a^12*b^18*c^22*d^8 + 510625*a^12
*b^18*c^24*d^6 - 25175*a^12*b^18*c^26*d^4 + 190*a^12*b^18*c^28*d^2 - 77520*
a^13*b^17*c^7*d^23 + 1227400*a^13*b^17*c^9*d^21 - 6653800*a^13*b^17*c^11*d^
19 + 18346400*a^13*b^17*c^13*d^17 - 29535120*a^13*b^17*c^15*d^15 + 29213260
*a^13*b^17*c^17*d^13 - 17770700*a^13*b^17*c^19*d^11 + 6342200*a^13*b^17*c^2
1*d^9 - 1174200*a^13*b^17*c^23*d^7 + 83220*a^13*b^17*c^25*d^5 - 1140*a^13*b
^17*c^27*d^3 + 38760*a^14*b^16*c^6*d^24 - 823650*a^14*b^16*c^8*d^22 + 53844
10*a^14*b^16*c^10*d^20 - 17183600*a^14*b^16*c^12*d^18 + 31460200*a^14*b^16*
c^14*d^16 - 35234455*a^14*b^16*c^16*d^14 + 24426875*a^14*b^16*c^18*d^12 - 1
0132510*a^14*b^16*c^20*d^10 + 2277150*a^14*b^16*c^22*d^8 - 218025*a^14*b^16
*c^24*d^6 + 4845*a^14*b^16*c^26*d^4 - 15504*a^15*b^15*c^5*d^25 + 465120*a^1
5*b^15*c^7*d^23 - 3772640*a^15*b^15*c^9*d^21 + 14108640*a^15*b^15*c^11*d^19
 - 29535120*a^15*b^15*c^13*d^17 + 37499008*a^15*b^15*c^15*d^15 - 29535120*a
^15*b^15*c^17*d^13 + 14108640*a^15*b^15*c^19*d^11 - 3772640*a^15*b^15*c^21*
d^9 + 465120*a^15*b^15*c^23*d^7 - 15504*a^15*b^15*c^25*d^5 + 4845*a^16*b^14
*c^4*d^26 - 218025*a^16*b^14*c^6*d^24 + 2277150*a^16*b^14*c^8*d^22 - 101325
10*a^16*b^14*c^10*d^20 + 24426875*a^16*b^14*c^12*d^18 - 35234455*a^16*b^14*
c^14*d^16 + 31460200*a^16*b^14*c^16*d^14 - 17183600*a^16*b^14*c^18*d^12 + 5
384410*a^16*b^14*c^20*d^10 - 823650*a^16*b^14*c^22*d^8 + 38760*a^16*b^14*c^
24*d^6 - 1140*a^17*b^13*c^3*d^27 + 83220*a^17*b^13*c^5*d^25 - 1174200*a^17*
b^13*c^7*d^23 + 6342200*a^17*b^13*c^9*d^21 - 17770700*a^17*b^13*c^11*d^19 +
 29213260*a^17*b^13*c^13*d^17 - 29535120*a^17*b^13*c^15*d^15 + 18346400*a^1
7*b^13*c^17*d^13 - 6653800*a^17*b^13*c^19*d^11 + 1227400*a^17*b^13*c^21*d^9
 - 77520*a^17*b^13*c^23*d^7 + 190*a^18*b^12*c^2*d^28 - 25175*a^18*b^12*c^4*
d^26 + 510625*a^18*b^12*c^6*d^24 - 3441850*a^18*b^12*c^8*d^22 + 11341480*a^
18*b^12*c^10*d^20 - 21339185*a^18*b^12*c^12*d^18 + 24426875*a^18*b^12*c^14*
d^16 - 17183600*a^18*b^12*c^16*d^14 + 7138300*a^18*b^12*c^18*d^12 - 1553630
*a^18*b^12*c^20*d^10 + 125970*a^18*b^12*c^22*d^8 + 5800*a^19*b^11*c^3*d^27
 - 183740*a^19*b^11*c^5*d^25 + 1607600*a^19*b^11*c^7*d^23 - 6323300*a^19*b^1
1*c^9*d^21 + 13697880*a^19*b^11*c^11*d^19 - 17770700*a^19*b^11*c^13*d^17 +
14108640*a^19*b^11*c^15*d^15 - 6653800*a^19*b^11*c^17*d^13 + 1679600*a^19*b
^11*c^19*d^11 - 167960*a^19*b^11*c^21*d^9 - 955*a^20*b^10*c^2*d^28 + 53210*
a^20*b^10*c^4*d^26 - 639360*a^20*b^10*c^6*d^24 + 3061855*a^20*b^10*c^8*d^22
 - 7699257*a^20*b^10*c^10*d^20 + 11341480*a^20*b^10*c^12*d^18 - 10132510*a^
20*b^10*c^14*d^16 + 5384410*a^20*b^10*c^16*d^14 - 1553630*a^20*b^10*c^18*d^
12 + 184756*a^20*b^10*c^20*d^10 - 11900*a^21*b^9*c^3*d^27 + 213040*a^21*b^9
*c^5*d^25 - 1277800*a^21*b^9*c^7*d^23 + 3770860*a^21*b^9*c^9*d^21 - 6323300

*a^21*b^9*c^11*d^19 + 6342200*a^21*b^9*c^13*d^17 - 3772640*a^21*b^9*c^15*d^15 + 1227400*a^21*b^9*c^17*d^13 - 167960*a^21*b^9*c^19*d^11 + 1925*a^22*b^8*c^2*d^28 - 58000*a^22*b^8*c^4*d^26 + 455100*a^22*b^8*c^6*d^24 - 1598495*a^22*b^8*c^8*d^22 + 3061855*a^22*b^8*c^10*d^20 - 3441850*a^22*b^8*c^12*d^18 + 2277150*a^22*b^8*c^14*d^16 - 823650*a^22*b^8*c^16*d^14 + 125970*a^22*b^8*c^18*d^12 + 12400*a^23*b^7*c^3*d^27 - 136520*a^23*b^7*c^5*d^25 + 581120*a^23*b^7*c^7*d^23 - 1277800*a^23*b^7*c^9*d^21 + 1607600*a^23*b^7*c^11*d^19 - 1174200*a^23*b^7*c^13*d^17 + 465120*a^23*b^7*c^15*d^15 - 77520*a^23*b^7*c^17*d^13 - 1950*a^24*b^6*c^2*d^28 + 33825*a^24*b^6*c^4*d^26 - 178985*a^24*b^6*c^6*d^24 + 455100*a^24*b^6*c^8*d^22 - 639360*a^24*b^6*c^10*d^20 + 510625*a^24*b^6*c^12*d^18 - 218025*a^24*b^6*c^14*d^16 + 38760*a^24*b^6*c^16*d^14 - 6700*a^25*b^5*c^3*d^27 + 46004*a^25*b^5*c^5*d^25 - 136520*a^25*b^5*c^7*d^23 + 213040*a^25*b^5*c^9*d^21 - 183740*a^25*b^5*c^11*d^19 + 83220*a^25*b^5*c^13*d^17 - 15504*a^25*b^5*c^15*d^15 + 1000*a^26*b^4*c^2*d^28 - 9695*a^26*b^4*c^4*d^26 + 33825*a^26*b^4*c^6*d^24 - 58000*a^26*b^4*c^8*d^22 + 53210*a^26*b^4*c^10*d^20 - 25175*a^26*b^4*c^12*d^18 + 4845*a^26*b^4*c^14*d^16 + 1640*a^27*b^3*c^3*d^27 - 6700*a^27*b^3*c^5*d^25 + 12400*a^27*b^3*c^7*d^23 - 11900*a^27*b^3*c^9*d^21 + 5800*a^27*b^3*c^11*d^19 - 1140*a^27*b^3*c^13*d^17 - 215*a^28*b^2*c^2*d^28 + 1000*a^28*b^2*c^4*d^26 - 1950*a^28*b^2*c^6*d^24 + 1925*a^28*b^2*c^8*d^22 - 955*a^28*b^2*c^10*d^20 + 190*a^28*b^2*c^12*d^18 + 20*a*b^29*c^29*d + 20*a^29*b*c*d^29)))^(1/2)*(((4*(8*a^2*b^23*c^25 - 32*a^4*b^21*c^25 + 48*a^6*b^19*c^25 - 32*a^8*b^17*c^25 + 8*a^10*b^15*c^25 + 8*a^25*c^2*d^23 - 32*a^25*c^4*d^21 + 48*a^25*c^6*d^19 - 32*a^25*c^8*d^17 + 8*a^25*c^10*d^15 - 8*a*b^24*c^16*d^9 + 32*a*b^24*c^18*d^7 - 48*a*b^24*c^20*d^5 + 32*a*b^24*c^22*d^3 - 72*a^3*b^22*c^24*d + 368*a^5*b^20*c^24*d - 592*a^7*b^18*c^24*d + 408*a^9*b^16*c^24*d - 104*a^11*b^14*c^24*d - 8*a^16*b^9*c*d^24 + 32*a^18*b^7*c*d^24 - 48*a^20*b^5*c*d^24 + 32*a^22*b^3*c*d^24 - 72*a^24*b*c^3*d^22 + 368*a^24*b*c^5*d^20 - 592*a^24*b*c^7*d^18 + 408*a^24*b*c^9*d^16 - 104*a^24*b*c^11*d^14 + 104*a^2*b^23*c^15*d^10 - 408*a^2*b^23*c^17*d^8 + 592*a^2*b^23*c^19*d^6 - 368*a^2*b^23*c^21*d^4 + 72*a^2*b^23*c^23*d^2 - 616*a^3*b^22*c^14*d^11 + 2392*a^3*b^22*c^16*d^9 - 3408*a^3*b^22*c^18*d^7 + 2032*a^3*b^22*c^20*d^5 - 328*a^3*b^22*c^22*d^3 + 2184*a^4*b^21*c^13*d^12 - 8536*a^4*b^21*c^15*d^10 + 12272*a^4*b^21*c^17*d^8 - 7408*a^4*b^21*c^19*d^6 + 1192*a^4*b^21*c^21*d^4 + 328*a^4*b^21*c^23*d^2 - 5096*a^5*b^20*c^12*d^13 + 20664*a^5*b^20*c^14*d^11 - 31328*a^5*b^20*c^16*d^9 + 20592*a^5*b^20*c^18*d^7 - 4008*a^5*b^20*c^20*d^5 - 1192*a^5*b^20*c^22*d^3 + 8008*a^6*b^19*c^11*d^14 - 35672*a^6*b^19*c^13*d^12 + 60768*a^6*b^19*c^15*d^10 - 46464*a^6*b^19*c^17*d^8 + 11336*a^6*b^19*c^19*d^6 + 4008*a^6*b^19*c^21*d^4 - 2032*a^6*b^19*c^23*d^2 - 8008*a^7*b^18*c^10*d^15 + 44408*a^7*b^18*c^12*d^13 - 92512*a^7*b^18*c^14*d^11 + 85536*a^7*b^18*c^16*d^9 - 24904*a^7*b^18*c^18*d^7 - 11336*a^7*b^18*c^20*d^5 + 7408*a^7*b^18*c^22*d^3 + 3432*a^8*b^17*c^9*d^16 - 37752*a^8*b^17*c^11*d^14 + 109408*a^8*b^17*c^13*d^12 - 125472*a^8*b^17*c^15*d^10 + 42696*a^8*b^17*c^17*d^8 + 24904*a^8*b^17*c^19*d^6 - 20592*a^8*b^17*c^21*d^4 + 3408*a^8*b^17*c^23*d^2 + 3432*a^9*b^16*c^8*d^17 + 14872*a^9*b^16*c^10*d^15 - 92352*a^9*b^16*c^12*d^13 + 141408*a^9*b^16*c^14*d^11 - 59264*a^9*b^16*c^16*d

^9 - 42696*a^9*b^16*c^18*d^7 + 46464*a^9*b^16*c^20*d^5 - 12272*a^9*b^16*c^2
2*d^3 - 8008*a^10*b^15*c^7*d^18 + 14872*a^10*b^15*c^9*d^16 + 36608*a^10*b^1
5*c^11*d^14 - 113152*a^10*b^15*c^13*d^12 + 67008*a^10*b^15*c^15*d^10 + 5926
4*a^10*b^15*c^17*d^8 - 85536*a^10*b^15*c^19*d^6 + 31328*a^10*b^15*c^21*d^4
- 2392*a^10*b^15*c^23*d^2 + 8008*a^11*b^14*c^6*d^19 - 37752*a^11*b^14*c^8*d
^17 + 36608*a^11*b^14*c^10*d^15 + 43264*a^11*b^14*c^12*d^13 - 56256*a^11*b^
14*c^14*d^11 - 67008*a^11*b^14*c^16*d^9 + 125472*a^11*b^14*c^18*d^7 - 60768
*a^11*b^14*c^20*d^5 + 8536*a^11*b^14*c^22*d^3 - 5096*a^12*b^13*c^5*d^20 + 4
4408*a^12*b^13*c^7*d^18 - 92352*a^12*b^13*c^9*d^16 + 43264*a^12*b^13*c^11*d
^14 + 22464*a^12*b^13*c^13*d^12 + 56256*a^12*b^13*c^15*d^10 - 141408*a^12*b
^13*c^17*d^8 + 92512*a^12*b^13*c^19*d^6 - 20664*a^12*b^13*c^21*d^4 + 616*a^
12*b^13*c^23*d^2 + 2184*a^13*b^12*c^4*d^21 - 35672*a^13*b^12*c^6*d^19 + 109
408*a^13*b^12*c^8*d^17 - 113152*a^13*b^12*c^10*d^15 + 22464*a^13*b^12*c^12*
d^13 - 22464*a^13*b^12*c^14*d^11 + 113152*a^13*b^12*c^16*d^9 - 109408*a^13*
b^12*c^18*d^7 + 35672*a^13*b^12*c^20*d^5 - 2184*a^13*b^12*c^22*d^3 - 616*a^
14*b^11*c^3*d^22 + 20664*a^14*b^11*c^5*d^20 - 92512*a^14*b^11*c^7*d^18 + 14
1408*a^14*b^11*c^9*d^16 - 56256*a^14*b^11*c^11*d^14 - 22464*a^14*b^11*c^13*
d^12 - 43264*a^14*b^11*c^15*d^10 + 92352*a^14*b^11*c^17*d^8 - 44408*a^14*b^
11*c^19*d^6 + 5096*a^14*b^11*c^21*d^4 + 104*a^15*b^10*c^2*d^23 - 8536*a^15*
b^10*c^4*d^21 + 60768*a^15*b^10*c^6*d^19 - 125472*a^15*b^10*c^8*d^17 + 6700
8*a^15*b^10*c^10*d^15 + 56256*a^15*b^10*c^12*d^13 - 43264*a^15*b^10*c^14*d^
11 - 36608*a^15*b^10*c^16*d^9 + 37752*a^15*b^10*c^18*d^7 - 8008*a^15*b^10*c
^20*d^5 + 2392*a^16*b^9*c^3*d^22 - 31328*a^16*b^9*c^5*d^20 + 85536*a^16*b^9
*c^7*d^18 - 59264*a^16*b^9*c^9*d^16 - 67008*a^16*b^9*c^11*d^14 + 113152*a^1
6*b^9*c^13*d^12 - 36608*a^16*b^9*c^15*d^10 - 14872*a^16*b^9*c^17*d^8 + 8008
*a^16*b^9*c^19*d^6 - 408*a^17*b^8*c^2*d^23 + 12272*a^17*b^8*c^4*d^21 - 4646
4*a^17*b^8*c^6*d^19 + 42696*a^17*b^8*c^8*d^17 + 59264*a^17*b^8*c^10*d^15 -
141408*a^17*b^8*c^12*d^13 + 92352*a^17*b^8*c^14*d^11 - 14872*a^17*b^8*c^16*
d^9 - 3432*a^17*b^8*c^18*d^7 - 3408*a^18*b^7*c^3*d^22 + 20592*a^18*b^7*c^5*
d^20 - 24904*a^18*b^7*c^7*d^18 - 42696*a^18*b^7*c^9*d^16 + 125472*a^18*b^7*
c^11*d^14 - 109408*a^18*b^7*c^13*d^12 + 37752*a^18*b^7*c^15*d^10 - 3432*a^1
8*b^7*c^17*d^8 + 592*a^19*b^6*c^2*d^23 - 7408*a^19*b^6*c^4*d^21 + 11336*a^1
9*b^6*c^6*d^19 + 24904*a^19*b^6*c^8*d^17 - 85536*a^19*b^6*c^10*d^15 + 92512
*a^19*b^6*c^12*d^13 - 44408*a^19*b^6*c^14*d^11 + 8008*a^19*b^6*c^16*d^9 + 2
032*a^20*b^5*c^3*d^22 - 4008*a^20*b^5*c^5*d^20 - 11336*a^20*b^5*c^7*d^18 +
46464*a^20*b^5*c^9*d^16 - 60768*a^20*b^5*c^11*d^14 + 35672*a^20*b^5*c^13*d^
12 - 8008*a^20*b^5*c^15*d^10 - 368*a^21*b^4*c^2*d^23 + 1192*a^21*b^4*c^4*d^
21 + 4008*a^21*b^4*c^6*d^19 - 20592*a^21*b^4*c^8*d^17 + 31328*a^21*b^4*c^10
*d^15 - 20664*a^21*b^4*c^12*d^13 + 5096*a^21*b^4*c^14*d^11 - 328*a^22*b^3*c
^3*d^22 - 1192*a^22*b^3*c^5*d^20 + 7408*a^22*b^3*c^7*d^18 - 12272*a^22*b^3*
c^9*d^16 + 8536*a^22*b^3*c^11*d^14 - 2184*a^22*b^3*c^13*d^12 + 72*a^23*b^2*
c^2*d^23 + 328*a^23*b^2*c^4*d^21 - 2032*a^23*b^2*c^6*d^19 + 3408*a^23*b^2*c
^8*d^17 - 2392*a^23*b^2*c^10*d^15 + 616*a^23*b^2*c^12*d^13 - 8*a*b^24*c^24*
d - 8*a^24*b*c*d^24))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16
*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 +

6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*
a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^
16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b
^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d
 + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b
^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 -
72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*
c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^
16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d
^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^
3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^
8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2
 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9
+ 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3
+ 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10
 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d
^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13
 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*
d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^
16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10
*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c
^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5
*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*
c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*
c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^1
0*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a
^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66
*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 2750
4*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 1
8744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 2
68*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 3
4156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8
 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17
+ 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^1
1 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^
5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^1
4 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12
*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d
^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*
d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^1
6 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^
10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15
 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 +
 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268
*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c

*d^19) - (8*tan(e/2 + (f*x)/2)*(56*a^3*b^22*c^25 - 12*a^25*c*d^24 - 12*a*b^24*c^25 - 104*a^5*b^20*c^25 + 96*a^7*b^18*c^25 - 44*a^9*b^16*c^25 + 8*a^11*b^14*c^25 + 56*a^25*c^3*d^22 - 104*a^25*c^5*d^20 + 96*a^25*c^7*d^18 - 44*a^25*c^9*d^16 + 8*a^25*c^11*d^14 + 16*a*b^24*c^15*d^10 - 76*a*b^24*c^17*d^8 + 144*a*b^24*c^19*d^6 - 136*a*b^24*c^21*d^4 + 64*a*b^24*c^23*d^2 + 168*a^2*b^23*c^24*d - 784*a^4*b^21*c^24*d + 1456*a^6*b^19*c^24*d - 1344*a^8*b^17*c^24*d + 616*a^10*b^15*c^24*d - 112*a^12*b^13*c^24*d + 16*a^15*b^10*c*d^24 - 76*a^17*b^8*c*d^24 + 144*a^19*b^6*c*d^24 - 136*a^21*b^4*c*d^24 + 64*a^23*b^2*c*d^24 + 168*a^24*b*c^2*d^23 - 784*a^24*b*c^4*d^21 + 1456*a^24*b*c^6*d^19 - 1344*a^24*b*c^8*d^17 + 616*a^24*b*c^10*d^15 - 112*a^24*b*c^12*d^13 - 224*a^2*b^23*c^14*d^11 + 1064*a^2*b^23*c^16*d^9 - 2016*a^2*b^23*c^18*d^7 + 1904*a^2*b^23*c^20*d^5 - 896*a^2*b^23*c^22*d^3 + 1456*a^3*b^22*c^13*d^12 - 6992*a^3*b^22*c^15*d^10 + 13464*a^3*b^22*c^17*d^8 - 13056*a^3*b^22*c^19*d^6 + 6464*a^3*b^22*c^21*d^4 - 1392*a^3*b^22*c^23*d^2 - 5824*a^4*b^21*c^12*d^13 + 28728*a^4*b^21*c^14*d^11 - 57456*a^4*b^21*c^16*d^9 + 59024*a^4*b^21*c^18*d^7 - 32256*a^4*b^21*c^20*d^5 + 8568*a^4*b^21*c^22*d^3 + 16016*a^5*b^20*c^11*d^14 - 82992*a^5*b^20*c^13*d^12 + 177048*a^5*b^20*c^15*d^10 - 198696*a^5*b^20*c^17*d^8 + 123584*a^5*b^20*c^19*d^6 - 40512*a^5*b^20*c^21*d^4 + 5656*a^5*b^20*c^23*d^2 - 32032*a^6*b^19*c^10*d^15 + 179816*a^6*b^19*c^12*d^13 - 421344*a^6*b^19*c^14*d^11 + 529312*a^6*b^19*c^16*d^9 - 379008*a^6*b^19*c^18*d^7 + 150024*a^6*b^19*c^20*d^5 - 28224*a^6*b^19*c^22*d^3 + 48048*a^7*b^18*c^9*d^16 - 304304*a^7*b^18*c^11*d^14 + 805896*a^7*b^18*c^13*d^12 - 1151104*a^7*b^18*c^15*d^10 + 949952*a^7*b^18*c^17*d^8 - 446736*a^7*b^18*c^19*d^6 + 108136*a^7*b^18*c^21*d^4 - 9984*a^7*b^18*c^23*d^2 - 54912*a^8*b^17*c^8*d^17 + 412984*a^8*b^17*c^10*d^15 - 1267344*a^8*b^17*c^12*d^13 + 2077536*a^8*b^17*c^14*d^11 - 1975808*a^8*b^17*c^16*d^9 + 1095384*a^8*b^17*c^18*d^7 - 331632*a^8*b^17*c^20*d^5 + 45136*a^8*b^17*c^22*d^3 + 48048*a^9*b^16*c^7*d^18 - 456456*a^9*b^16*c^9*d^16 + 1657656*a^9*b^16*c^11*d^14 - 3143504*a^9*b^16*c^13*d^12 + 3453696*a^9*b^16*c^15*d^10 - 2247636*a^9*b^16*c^17*d^8 + 831208*a^9*b^16*c^19*d^6 - 151944*a^9*b^16*c^21*d^4 + 8976*a^9*b^16*c^23*d^2 - 32032*a^10*b^15*c^6*d^19 + 412984*a^10*b^15*c^8*d^17 - 1812096*a^10*b^15*c^10*d^15 + 4016896*a^10*b^15*c^12*d^13 - 5121024*a^10*b^15*c^14*d^11 + 3897024*a^10*b^15*c^16*d^9 - 1728832*a^10*b^15*c^18*d^7 + 404768*a^10*b^15*c^20*d^5 - 38304*a^10*b^15*c^22*d^3 + 16016*a^11*b^14*c^5*d^20 - 304304*a^11*b^14*c^7*d^18 + 1657656*a^11*b^14*c^9*d^16 - 4356352*a^11*b^14*c^11*d^14 + 6476288*a^11*b^14*c^13*d^12 - 5745024*a^11*b^14*c^15*d^10 + 3021984*a^11*b^14*c^17*d^8 - 880256*a^11*b^14*c^19*d^6 + 118032*a^11*b^14*c^21*d^4 - 4048*a^11*b^14*c^23*d^2 - 5824*a^12*b^13*c^4*d^21 + 179816*a^12*b^13*c^6*d^19 - 1267344*a^12*b^13*c^8*d^17 + 4016896*a^12*b^13*c^10*d^15 - 7002112*a^12*b^13*c^12*d^13 + 7235136*a^12*b^13*c^14*d^11 - 4480896*a^12*b^13*c^16*d^9 + 1588704*a^12*b^13*c^18*d^7 - 280896*a^12*b^13*c^20*d^5 + 16632*a^12*b^13*c^22*d^3 + 1456*a^13*b^12*c^3*d^22 - 82992*a^13*b^12*c^5*d^20 + 805896*a^13*b^12*c^7*d^18 - 3143504*a^13*b^12*c^9*d^16 + 6476288*a^13*b^12*c^11*d^14 - 7809984*a^13*b^12*c^13*d^12 + 5666752*a^13*b^12*c^15*d^10 - 2403856*a^13*b^12*c^17*d^8 + 537264*a^13*b^12*c^19*d^6 - 48048*a^13*b^12*c^21*d^4 + 728*a^13*b^12*c^23*d

^2 - 224*a^14*b^11*c^2*d^23 + 28728*a^14*b^11*c^4*d^21 - 421344*a^14*b^11*c^6*d^19 + 2077536*a^14*b^11*c^8*d^17 - 5121024*a^14*b^11*c^10*d^15 + 7235136*a^14*b^11*c^12*d^13 - 6126848*a^14*b^11*c^14*d^11 + 3071744*a^14*b^11*c^16*d^9 - 844896*a^14*b^11*c^18*d^7 + 104104*a^14*b^11*c^20*d^5 - 2912*a^14*b^11*c^22*d^3 - 6992*a^15*b^10*c^3*d^22 + 177048*a^15*b^10*c^5*d^20 - 1151104*a^15*b^10*c^7*d^18 + 3453696*a^15*b^10*c^9*d^16 - 5745024*a^15*b^10*c^11*d^14 + 5666752*a^15*b^10*c^13*d^12 - 3331328*a^15*b^10*c^15*d^10 + 1105104*a^15*b^10*c^17*d^8 - 176176*a^15*b^10*c^19*d^6 + 8008*a^15*b^10*c^21*d^4 + 1064*a^16*b^9*c^2*d^23 - 57456*a^16*b^9*c^4*d^21 + 529312*a^16*b^9*c^6*d^19 - 1975808*a^16*b^9*c^8*d^17 + 3897024*a^16*b^9*c^10*d^15 - 4480896*a^16*b^9*c^12*d^13 + 3071744*a^16*b^9*c^14*d^11 - 1208064*a^16*b^9*c^16*d^9 + 239096*a^16*b^9*c^18*d^7 - 16016*a^16*b^9*c^20*d^5 + 13464*a^17*b^8*c^3*d^22 - 198696*a^17*b^8*c^5*d^20 + 949952*a^17*b^8*c^7*d^18 - 2247636*a^17*b^8*c^9*d^16 + 3021984*a^17*b^8*c^11*d^14 - 2403856*a^17*b^8*c^13*d^12 + 1105104*a^17*b^8*c^15*d^10 - 264264*a^17*b^8*c^17*d^8 + 24024*a^17*b^8*c^19*d^6 - 2016*a^18*b^7*c^2*d^23 + 59024*a^18*b^7*c^4*d^21 - 379008*a^18*b^7*c^6*d^19 + 1095384*a^18*b^7*c^8*d^17 - 1728832*a^18*b^7*c^10*d^15 + 1588704*a^18*b^7*c^12*d^13 - 844896*a^18*b^7*c^14*d^11 + 239096*a^18*b^7*c^16*d^9 - 27456*a^18*b^7*c^18*d^7 - 13056*a^19*b^6*c^3*d^22 + 123584*a^19*b^6*c^5*d^20 - 446736*a^19*b^6*c^7*d^18 + 831208*a^19*b^6*c^9*d^16 - 880256*a^19*b^6*c^11*d^14 + 537264*a^19*b^6*c^13*d^12 - 176176*a^19*b^6*c^15*d^10 + 24024*a^19*b^6*c^17*d^8 + 1904*a^20*b^5*c^2*d^23 - 32256*a^20*b^5*c^4*d^21 + 150024*a^20*b^5*c^6*d^19 - 331632*a^20*b^5*c^8*d^17 + 404768*a^20*b^5*c^10*d^15 - 280896*a^20*b^5*c^12*d^13 + 104104*a^20*b^5*c^14*d^11 - 16016*a^20*b^5*c^16*d^9 + 6464*a^21*b^4*c^3*d^22 - 40512*a^21*b^4*c^5*d^20 + 108136*a^21*b^4*c^7*d^18 - 151944*a^21*b^4*c^9*d^16 + 118032*a^21*b^4*c^11*d^14 - 48048*a^21*b^4*c^13*d^12 + 8008*a^21*b^4*c^15*d^10 - 896*a^22*b^3*c^2*d^23 + 8568*a^22*b^3*c^4*d^21 - 28224*a^22*b^3*c^6*d^19 + 45136*a^22*b^3*c^8*d^17 - 38304*a^22*b^3*c^10*d^15 + 16632*a^22*b^3*c^12*d^13 - 2912*a^22*b^3*c^14*d^11 - 1392*a^23*b^2*c^3*d^22 + 5656*a^23*b^2*c^5*d^20 - 9984*a^23*b^2*c^7*d^18 + 8976*a^23*b^2*c^9*d^16 - 4048*a^23*b^2*c^11*d^14 + 728*a^23*b^2*c^13*d^12))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 404

8*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 474
4*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676
*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 1
1236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 7
92*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 2
7504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 -
1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 +
20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^
8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^
2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^1
3 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13
*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^
18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c
^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10
*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11
*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^
11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*
a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^
12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300
*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 49
5*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 215
76*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 +
6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 35
88*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 +
13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 +
1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 -
8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 -
288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4
032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 4
12*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 92
8*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^
18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b
^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19))*(-(((4*a^24*d^24 + 4*b
^24*c^24 + 16*a^2*b^22*c^24 + 16*a^4*b^20*c^24 - 1152*a^10*b^14*d^24 + 5568
*a^12*b^12*d^24 - 10568*a^14*b^10*d^24 + 9460*a^16*b^8*d^24 - 3560*a^18*b^6
*d^24 + 136*a^20*b^4*d^24 + 76*a^22*b^2*d^24 + 16*a^24*c^2*d^22 + 16*a^24*c
^4*d^20 - 1152*b^24*c^10*d^14 + 5568*b^24*c^12*d^12 - 10568*b^24*c^14*d^10
+ 9460*b^24*c^16*d^8 - 3560*b^24*c^18*d^6 + 136*b^24*c^20*d^4 + 76*b^24*c^2
2*d^2 + 11520*a*b^23*c^9*d^15 - 56448*a*b^23*c^11*d^13 + 109456*a*b^23*c^13
*d^11 - 101240*a*b^23*c^15*d^9 + 40720*a*b^23*c^17*d^7 - 2960*a*b^23*c^19*d
^5 - 536*a*b^23*c^21*d^3 - 176*a^3*b^21*c^23*d - 320*a^5*b^19*c^23*d + 1152
0*a^9*b^15*c*d^23 - 56448*a^11*b^13*c*d^23 + 109456*a^13*b^11*c*d^23 - 1012
40*a^15*b^9*c*d^23 + 40720*a^17*b^7*c*d^23 - 2960*a^19*b^5*c*d^23 - 536*a^2
1*b^3*c*d^23 - 176*a^23*b*c^3*d^21 - 320*a^23*b*c^5*d^19 - 51840*a^2*b^22*c
^8*d^16 + 263808*a^2*b^22*c^10*d^14 - 541208*a^2*b^22*c^12*d^12 + 547088*a^

2*b^22*c^14*d^10 - 263320*a^2*b^22*c^16*d^8 + 44120*a^2*b^22*c^18*d^6 - 156
4*a^2*b^22*c^20*d^4 - 196*a^2*b^22*c^22*d^2 + 138240*a^3*b^21*c^7*d^17 - 75
8400*a^3*b^21*c^9*d^15 + 1720736*a^3*b^21*c^11*d^13 - 2002728*a^3*b^21*c^13
*d^11 + 1210560*a^3*b^21*c^15*d^9 - 335040*a^3*b^21*c^17*d^7 + 37680*a^3*b^
21*c^19*d^5 - 288*a^3*b^21*c^21*d^3 - 241920*a^4*b^20*c^6*d^18 + 1512000*a^
4*b^20*c^8*d^16 - 3975688*a^4*b^20*c^10*d^14 + 5501328*a^4*b^20*c^12*d^12 -
4147952*a^4*b^20*c^14*d^10 + 1586920*a^4*b^20*c^16*d^8 - 276020*a^4*b^20*c
^18*d^6 + 21124*a^4*b^20*c^20*d^4 + 176*a^4*b^20*c^22*d^2 + 290304*a^5*b^19
*c^5*d^19 - 2232576*a^5*b^19*c^7*d^17 + 7078256*a^5*b^19*c^9*d^15 - 1178156
0*a^5*b^19*c^11*d^13 + 10875200*a^5*b^19*c^13*d^11 - 5365072*a^5*b^19*c^15*
d^9 + 1310168*a^5*b^19*c^17*d^7 - 170968*a^5*b^19*c^19*d^5 + 8160*a^5*b^19*
c^21*d^3 - 241920*a^6*b^18*c^4*d^20 + 2532096*a^6*b^18*c^6*d^18 - 9955992*a
^6*b^18*c^8*d^16 + 20019440*a^6*b^18*c^10*d^14 - 22419600*a^6*b^18*c^12*d^1
2 + 13887520*a^6*b^18*c^14*d^10 - 4506428*a^6*b^18*c^16*d^8 + 793756*a^6*b^
18*c^18*d^6 - 72240*a^6*b^18*c^20*d^4 + 3040*a^6*b^18*c^22*d^2 + 138240*a^7
*b^17*c^3*d^21 - 2232576*a^7*b^17*c^5*d^19 + 11150016*a^7*b^17*c^7*d^17 - 2
7336616*a^7*b^17*c^9*d^15 + 37153600*a^7*b^17*c^11*d^13 - 28461040*a^7*b^17
*c^13*d^11 + 11779808*a^7*b^17*c^15*d^9 - 2621008*a^7*b^17*c^17*d^7 + 33668
8*a^7*b^17*c^19*d^5 - 17920*a^7*b^17*c^21*d^3 - 51840*a^8*b^16*c^2*d^22 + 1
512000*a^8*b^16*c^4*d^20 - 9955992*a^8*b^16*c^6*d^18 + 30289656*a^8*b^16*c^
8*d^16 - 50137600*a^8*b^16*c^10*d^14 + 46972560*a^8*b^16*c^12*d^12 - 241992
80*a^8*b^16*c^14*d^10 + 6661036*a^8*b^16*c^16*d^8 - 1058448*a^8*b^16*c^18*d
^6 + 72560*a^8*b^16*c^20*d^4 - 758400*a^9*b^15*c^3*d^21 + 7078256*a^9*b^15*
c^5*d^19 - 27336616*a^9*b^15*c^7*d^17 + 55383904*a^9*b^15*c^9*d^15 - 631240
80*a^9*b^15*c^11*d^13 + 39987520*a^9*b^15*c^13*d^11 - 13462088*a^9*b^15*c^1
5*d^9 + 2478528*a^9*b^15*c^17*d^7 - 212032*a^9*b^15*c^19*d^5 + 263808*a^10*
b^14*c^2*d^22 - 3975688*a^10*b^14*c^4*d^20 + 20019440*a^10*b^14*c^6*d^18 -
50137600*a^10*b^14*c^8*d^16 + 69593872*a^10*b^14*c^10*d^14 - 53854288*a^10*
b^14*c^12*d^12 + 21989928*a^10*b^14*c^14*d^10 - 4591360*a^10*b^14*c^16*d^8
+ 460480*a^10*b^14*c^18*d^6 + 1720736*a^11*b^13*c^3*d^21 - 11781560*a^11*b^
13*c^5*d^19 + 37153600*a^11*b^13*c^7*d^17 - 63124080*a^11*b^13*c^9*d^15 + 5
9445728*a^11*b^13*c^11*d^13 - 29358696*a^11*b^13*c^13*d^11 + 6995840*a^11*b
^13*c^15*d^9 - 762560*a^11*b^13*c^17*d^7 - 541208*a^12*b^12*c^2*d^22 + 5501
328*a^12*b^12*c^4*d^20 - 22419600*a^12*b^12*c^6*d^18 + 46972560*a^12*b^12*c
^8*d^16 - 53854288*a^12*b^12*c^10*d^14 + 32294808*a^12*b^12*c^12*d^12 - 895
8208*a^12*b^12*c^14*d^10 + 999040*a^12*b^12*c^16*d^8 - 2002728*a^13*b^11*c^
3*d^21 + 10875200*a^13*b^11*c^5*d^19 - 28461040*a^13*b^11*c^7*d^17 + 399875
20*a^13*b^11*c^9*d^15 - 29358696*a^13*b^11*c^11*d^13 + 9722048*a^13*b^11*c^
13*d^11 - 1104320*a^13*b^11*c^15*d^9 + 547088*a^14*b^10*c^2*d^22 - 4147952*
a^14*b^10*c^4*d^20 + 13887520*a^14*b^10*c^6*d^18 - 24199280*a^14*b^10*c^8*d
^16 + 21989928*a^14*b^10*c^10*d^14 - 8958208*a^14*b^10*c^12*d^12 + 1124032*
a^14*b^10*c^14*d^10 + 1210560*a^15*b^9*c^3*d^21 - 5365072*a^15*b^9*c^5*d^19
+ 11779808*a^15*b^9*c^7*d^17 - 13462088*a^15*b^9*c^9*d^15 + 6995840*a^15*b
^9*c^11*d^13 - 1104320*a^15*b^9*c^13*d^11 - 263320*a^16*b^8*c^2*d^22 + 1586
920*a^16*b^8*c^4*d^20 - 4506428*a^16*b^8*c^6*d^18 + 6661036*a^16*b^8*c^8*d^

16 - 4591360*a^16*b^8*c^10*d^14 + 999040*a^16*b^8*c^12*d^12 - 335040*a^17*b^7*c^3*d^21 + 1310168*a^17*b^7*c^5*d^19 - 2621008*a^17*b^7*c^7*d^17 + 2478528*a^17*b^7*c^9*d^15 - 762560*a^17*b^7*c^11*d^13 + 44120*a^18*b^6*c^2*d^22 - 276020*a^18*b^6*c^4*d^20 + 793756*a^18*b^6*c^6*d^18 - 1058448*a^18*b^6*c^8*d^16 + 460480*a^18*b^6*c^10*d^14 + 37680*a^19*b^5*c^3*d^21 - 170968*a^19*b^5*c^5*d^19 + 336688*a^19*b^5*c^7*d^17 - 212032*a^19*b^5*c^9*d^15 - 1564*a^20*b^4*c^2*d^22 + 21124*a^20*b^4*c^4*d^20 - 72240*a^20*b^4*c^6*d^18 + 72560*a^20*b^4*c^8*d^16 - 288*a^21*b^3*c^3*d^21 + 8160*a^21*b^3*c^5*d^19 - 17920*a^21*b^3*c^7*d^17 - 196*a^22*b^2*c^2*d^22 + 176*a^22*b^2*c^4*d^20 + 3040*a^22*b^2*c^6*d^18 - 8*a*b^23*c^23*d - 8*a^23*b*c*d^23)^2/4 - (20736*b^18*d^18 - 96768*a^2*b^16*d^18 + 173664*a^4*b^14*d^18 - 136032*a^6*b^12*d^18 + 31081*a^8*b^10*d^18 + 8440*a^10*b^8*d^18 + 400*a^12*b^6*d^18 - 96768*b^18*c^2*d^16 + 173664*b^18*c^4*d^14 - 136032*b^18*c^6*d^12 + 31081*b^18*c^8*d^10 + 8440*b^18*c^10*d^8 + 400*b^18*c^12*d^6 - 131328*a*b^17*c^3*d^15 + 216576*a*b^17*c^5*d^13 - 141104*a*b^17*c^7*d^11 + 20260*a*b^17*c^9*d^9 + 2800*a*b^17*c^11*d^7 - 131328*a^3*b^15*c*d^17 + 216576*a^5*b^13*c*d^17 - 141104*a^7*b^11*c*d^17 + 20260*a^9*b^9*c*d^17 + 2800*a^11*b^7*c*d^17 + 495936*a^2*b^16*c^2*d^16 - 989856*a^2*b^16*c^4*d^14 + 901948*a^2*b^16*c^6*d^12 - 308392*a^2*b^16*c^8*d^10 - 5260*a^2*b^16*c^10*d^8 + 1600*a^2*b^16*c^12*d^6 + 657408*a^3*b^15*c^3*d^15 - 1158992*a^3*b^15*c^5*d^13 + 838256*a^3*b^15*c^7*d^11 - 182200*a^3*b^15*c^9*d^9 - 3200*a^3*b^15*c^11*d^7 - 989856*a^4*b^14*c^2*d^16 + 2185654*a^4*b^14*c^4*d^14 - 2218576*a^4*b^14*c^6*d^12 + 900624*a^4*b^14*c^8*d^10 - 64720*a^4*b^14*c^10*d^8 + 1600*a^4*b^14*c^12*d^6 - 1158992*a^5*b^13*c^3*d^15 + 2158808*a^5*b^13*c^5*d^13 - 1641528*a^5*b^13*c^7*d^11 + 406880*a^5*b^13*c^9*d^9 - 17600*a^5*b^13*c^11*d^7 + 901948*a^6*b^12*c^2*d^16 - 2218576*a^6*b^12*c^4*d^14 + 2430936*a^6*b^12*c^6*d^12 - 1026928*a^6*b^12*c^8*d^10 + 88720*a^6*b^12*c^10*d^8 + 838256*a^7*b^11*c^3*d^15 - 1641528*a^7*b^11*c^5*d^13 + 1206848*a^7*b^11*c^7*d^11 - 239360*a^7*b^11*c^9*d^9 - 308392*a^8*b^10*c^2*d^16 + 900624*a^8*b^10*c^4*d^14 - 1026928*a^8*b^10*c^6*d^12 + 354016*a^8*b^10*c^8*d^10 - 182200*a^9*b^9*c^3*d^15 + 406880*a^9*b^9*c^5*d^13 - 239360*a^9*b^9*c^7*d^11 - 5260*a^10*b^8*c^2*d^16 - 64720*a^10*b^8*c^4*d^14 + 88720*a^10*b^8*c^6*d^12 - 3200*a^11*b^7*c^3*d^15 - 17600*a^11*b^7*c^5*d^13 + 1600*a^12*b^6*c^2*d^16 + 1600*a^12*b^6*c^4*d^14 + 27648*a*b^17*c*d^17)*(80*a^2*b^28*c^30 - 16*b^30*c^30 - 16*a^30*d^30 - 160*a^4*b^26*c^30 + 160*a^6*b^24*c^30 - 80*a^8*b^22*c^30 + 16*a^10*b^20*c^30 + 16*a^20*b^10*d^30 - 80*a^22*b^8*d^30 + 160*a^24*b^6*d^30 - 160*a^26*b^4*d^30 + 80*a^28*b^2*d^30 + 80*a^30*c^2*d^28 - 160*a^30*c^4*d^26 + 160*a^30*c^6*d^24 - 80*a^30*c^8*d^22 + 16*a^30*c^10*d^20 + 16*b^30*c^20*d^10 - 80*b^30*c^22*d^8 + 160*b^30*c^24*d^6 - 160*b^30*c^26*d^4 + 80*b^30*c^28*d^2 - 320*a*b^29*c^19*d^11 + 1600*a*b^29*c^21*d^9 - 3200*a*b^29*c^23*d^7 + 3200*a*b^29*c^25*d^5 - 1600*a*b^29*c^27*d^3 - 1600*a^3*b^27*c^29*d + 3200*a^5*b^25*c^29*d - 3200*a^7*b^23*c^29*d + 1600*a^9*b^21*c^29*d - 320*a^11*b^19*c^29*d - 320*a^19*b^11*c*d^29 + 1600*a^21*b^9*c*d^29 - 3200*a^23*b^7*c*d^29 + 3200*a^25*b^5*c*d^29 - 1600*a^27*b^3*c*d^29 - 1600*a^29*b*c^3*d^27 + 3200*a^29*b*c^5*d^25 - 3200*a^29*b*c^7*d^23 + 1600*a^29*b*c^9*d^21 - 320*a^29*b*c^11*d^19 + 3040*a^2*b^28

*c^18*d^12 - 15280*a^2*b^28*c^20*d^10 + 30800*a^2*b^28*c^22*d^8 - 31200*a^2*b^28*c^24*d^6 + 16000*a^2*b^28*c^26*d^4 - 3440*a^2*b^28*c^28*d^2 - 18240*a^3*b^27*c^17*d^13 + 92800*a^3*b^27*c^19*d^11 - 190400*a^3*b^27*c^21*d^9 + 198400*a^3*b^27*c^23*d^7 - 107200*a^3*b^27*c^25*d^5 + 26240*a^3*b^27*c^27*d^3 + 77520*a^4*b^26*c^16*d^14 - 402800*a^4*b^26*c^18*d^12 + 851360*a^4*b^26*c^20*d^10 - 928000*a^4*b^26*c^22*d^8 + 541200*a^4*b^26*c^24*d^6 - 155120*a^4*b^26*c^26*d^4 + 16000*a^4*b^26*c^28*d^2 - 248064*a^5*b^25*c^15*d^15 + 1331520*a^5*b^25*c^17*d^13 - 2939840*a^5*b^25*c^19*d^11 + 3408640*a^5*b^25*c^21*d^9 - 2184320*a^5*b^25*c^23*d^7 + 736064*a^5*b^25*c^25*d^5 - 107200*a^5*b^25*c^27*d^3 + 620160*a^6*b^24*c^14*d^16 - 3488400*a^6*b^24*c^16*d^14 + 8170000*a^6*b^24*c^18*d^12 - 10229760*a^6*b^24*c^20*d^10 + 7281600*a^6*b^24*c^22*d^8 - 2863760*a^6*b^24*c^24*d^6 + 541200*a^6*b^24*c^26*d^4 - 31200*a^6*b^24*c^28*d^2 - 1240320*a^7*b^23*c^13*d^17 + 7441920*a^7*b^23*c^15*d^15 - 18787200*a^7*b^23*c^17*d^13 + 25721600*a^7*b^23*c^19*d^11 - 20444800*a^7*b^23*c^21*d^9 + 9297920*a^7*b^23*c^23*d^7 - 2184320*a^7*b^23*c^25*d^5 + 198400*a^7*b^23*c^27*d^3 + 2015520*a^8*b^22*c^12*d^18 - 13178400*a^8*b^22*c^14*d^16 + 36434400*a^8*b^22*c^16*d^14 - 55069600*a^8*b^22*c^18*d^12 + 48989680*a^8*b^22*c^20*d^10 - 25575920*a^8*b^22*c^22*d^8 + 7281600*a^8*b^22*c^24*d^6 - 928000*a^8*b^22*c^26*d^4 + 30800*a^8*b^22*c^28*d^2 - 2687360*a^9*b^21*c^11*d^19 + 19638400*a^9*b^21*c^13*d^17 - 60362240*a^9*b^21*c^15*d^15 + 101475200*a^9*b^21*c^17*d^13 - 101172800*a^9*b^21*c^19*d^11 + 60333760*a^9*b^21*c^21*d^9 - 20444800*a^9*b^21*c^23*d^7 + 3408640*a^9*b^21*c^25*d^5 - 190400*a^9*b^21*c^27*d^3 + 2956096*a^10*b^20*c^10*d^20 - 24858080*a^10*b^20*c^12*d^18 + 86150560*a^10*b^20*c^14*d^16 - 162120160*a^10*b^20*c^16*d^14 + 181463680*a^10*b^20*c^18*d^12 - 123188112*a^10*b^20*c^20*d^10 + 48989680*a^10*b^20*c^22*d^8 - 10229760*a^10*b^20*c^24*d^6 + 851360*a^10*b^20*c^26*d^4 - 15280*a^10*b^20*c^28*d^2 - 2687360*a^11*b^19*c^9*d^21 + 26873600*a^11*b^19*c^11*d^19 - 106460800*a^11*b^19*c^13*d^17 + 225738240*a^11*b^19*c^15*d^15 - 284331200*a^11*b^19*c^17*d^13 + 219166080*a^11*b^19*c^19*d^11 - 101172800*a^11*b^19*c^21*d^9 + 25721600*a^11*b^19*c^23*d^7 - 2939840*a^11*b^19*c^25*d^5 + 92800*a^11*b^19*c^27*d^3 + 2015520*a^12*b^18*c^8*d^22 - 24858080*a^12*b^18*c^10*d^20 + 114212800*a^12*b^18*c^12*d^18 - 274937600*a^12*b^18*c^14*d^16 + 390830000*a^12*b^18*c^16*d^14 - 341426960*a^12*b^18*c^18*d^12 + 181463680*a^12*b^18*c^20*d^10 - 55069600*a^12*b^18*c^22*d^8 + 8170000*a^12*b^18*c^24*d^6 - 402800*a^12*b^18*c^26*d^4 + 3040*a^12*b^18*c^28*d^2 - 1240320*a^13*b^17*c^7*d^23 + 19638400*a^13*b^17*c^9*d^21 - 106460800*a^13*b^17*c^11*d^19 + 293542400*a^13*b^17*c^13*d^17 - 472561920*a^13*b^17*c^15*d^15 + 467412160*a^13*b^17*c^17*d^13 - 284331200*a^13*b^17*c^19*d^11 + 101475200*a^13*b^17*c^21*d^9 - 18787200*a^13*b^17*c^23*d^7 + 1331520*a^13*b^17*c^25*d^5 - 18240*a^13*b^17*c^27*d^3 + 620160*a^14*b^16*c^6*d^24 - 13178400*a^14*b^16*c^8*d^22 + 86150560*a^14*b^16*c^10*d^20 - 274937600*a^14*b^16*c^12*d^18 + 503363200*a^14*b^16*c^14*d^16 - 563751280*a^14*b^16*c^16*d^14 + 390830000*a^14*b^16*c^18*d^12 - 162120160*a^14*b^16*c^20*d^10 + 36434400*a^14*b^16*c^22*d^8 - 3488400*a^14*b^16*c^24*d^6 + 77520*a^14*b^16*c^26*d^4 - 248064*a^15*b^15*c^5*d^25 + 7441920*a^15*b^15*c^7*d^23 - 60362240*a^15*b^15*c^9*d^21 + 22573824

0*a^15*b^15*c^11*d^19 - 472561920*a^15*b^15*c^13*d^17 + 599984128*a^15*b^15*c^15*d^15 - 472561920*a^15*b^15*c^17*d^13 + 225738240*a^15*b^15*c^19*d^11 - 60362240*a^15*b^15*c^21*d^9 + 7441920*a^15*b^15*c^23*d^7 - 248064*a^15*b^15*c^25*d^5 + 77520*a^16*b^14*c^4*d^26 - 3488400*a^16*b^14*c^6*d^24 + 36434400*a^16*b^14*c^8*d^22 - 162120160*a^16*b^14*c^10*d^20 + 390830000*a^16*b^14*c^12*d^18 - 563751280*a^16*b^14*c^14*d^16 + 503363200*a^16*b^14*c^16*d^14 - 274937600*a^16*b^14*c^18*d^12 + 86150560*a^16*b^14*c^20*d^10 - 13178400*a^16*b^14*c^22*d^8 + 620160*a^16*b^14*c^24*d^6 - 18240*a^17*b^13*c^3*d^27 + 1331520*a^17*b^13*c^5*d^25 - 18787200*a^17*b^13*c^7*d^23 + 101475200*a^17*b^13*c^9*d^21 - 284331200*a^17*b^13*c^11*d^19 + 467412160*a^17*b^13*c^13*d^17 - 472561920*a^17*b^13*c^15*d^15 + 293542400*a^17*b^13*c^17*d^13 - 106460800*a^17*b^13*c^19*d^11 + 19638400*a^17*b^13*c^21*d^9 - 1240320*a^17*b^13*c^23*d^7 + 3040*a^18*b^12*c^2*d^28 - 402800*a^18*b^12*c^4*d^26 + 8170000*a^18*b^12*c^6*d^24 - 55069600*a^18*b^12*c^8*d^22 + 181463680*a^18*b^12*c^10*d^20 - 341426960*a^18*b^12*c^12*d^18 + 390830000*a^18*b^12*c^14*d^16 - 274937600*a^18*b^12*c^16*d^14 + 114212800*a^18*b^12*c^18*d^12 - 24858080*a^18*b^12*c^20*d^10 + 2015520*a^18*b^12*c^22*d^8 + 92800*a^19*b^11*c^3*d^27 - 2939840*a^19*b^11*c^5*d^25 + 25721600*a^19*b^11*c^7*d^23 - 101172800*a^19*b^11*c^9*d^21 + 219166080*a^19*b^11*c^11*d^19 - 284331200*a^19*b^11*c^13*d^17 + 225738240*a^19*b^11*c^15*d^15 - 106460800*a^19*b^11*c^17*d^13 + 26873600*a^19*b^11*c^19*d^11 - 2687360*a^19*b^11*c^21*d^9 - 15280*a^20*b^10*c^2*d^28 + 851360*a^20*b^10*c^4*d^26 - 10229760*a^20*b^10*c^6*d^24 + 48989680*a^20*b^10*c^8*d^22 - 123188112*a^20*b^10*c^10*d^20 + 181463680*a^20*b^10*c^12*d^18 - 162120160*a^20*b^10*c^14*d^16 + 86150560*a^20*b^10*c^16*d^14 - 24858080*a^20*b^10*c^18*d^12 + 2956096*a^20*b^10*c^20*d^10 - 190400*a^21*b^9*c^3*d^27 + 3408640*a^21*b^9*c^5*d^25 - 20444800*a^21*b^9*c^7*d^23 + 60333760*a^21*b^9*c^9*d^21 - 101172800*a^21*b^9*c^11*d^19 + 101475200*a^21*b^9*c^13*d^17 - 60362240*a^21*b^9*c^15*d^15 + 19638400*a^21*b^9*c^17*d^13 - 2687360*a^21*b^9*c^19*d^11 + 30800*a^22*b^8*c^2*d^28 - 928000*a^22*b^8*c^4*d^26 + 7281600*a^22*b^8*c^6*d^24 - 25575920*a^22*b^8*c^8*d^22 + 48989680*a^22*b^8*c^10*d^20 - 55069600*a^22*b^8*c^12*d^18 + 36434400*a^22*b^8*c^14*d^16 - 13178400*a^22*b^8*c^16*d^14 + 2015520*a^22*b^8*c^18*d^12 + 198400*a^23*b^7*c^3*d^27 - 2184320*a^23*b^7*c^5*d^25 + 9297920*a^23*b^7*c^7*d^23 - 20444800*a^23*b^7*c^9*d^21 + 25721600*a^23*b^7*c^11*d^19 - 18787200*a^23*b^7*c^13*d^17 + 7441920*a^23*b^7*c^15*d^15 - 1240320*a^23*b^7*c^17*d^13 - 31200*a^24*b^6*c^2*d^28 + 541200*a^24*b^6*c^4*d^26 - 2863760*a^24*b^6*c^6*d^24 + 7281600*a^24*b^6*c^8*d^22 - 10229760*a^24*b^6*c^10*d^20 + 8170000*a^24*b^6*c^12*d^18 - 3488400*a^24*b^6*c^14*d^16 + 620160*a^24*b^6*c^16*d^14 - 107200*a^25*b^5*c^3*d^27 + 736064*a^25*b^5*c^5*d^25 - 2184320*a^25*b^5*c^7*d^23 + 3408640*a^25*b^5*c^9*d^21 - 2939840*a^25*b^5*c^11*d^19 + 1331520*a^25*b^5*c^13*d^17 - 248064*a^25*b^5*c^15*d^15 + 16000*a^26*b^4*c^2*d^28 - 155120*a^26*b^4*c^4*d^26 + 541200*a^26*b^4*c^6*d^24 - 928000*a^26*b^4*c^8*d^22 + 851360*a^26*b^4*c^10*d^20 - 402800*a^26*b^4*c^12*d^18 + 77520*a^26*b^4*c^14*d^16 + 26240*a^27*b^3*c^3*d^27 - 107200*a^27*b^3*c^5*d^25 + 198400*a^27*b^3*c^7*d^23 - 190400*a^27*b^3*c^9*d^21 + 92800*a^27*b^3*c^11*d^19 - 18240*a^27*b^3*c^13*d^17 -

3440*a^28*b^2*c^2*d^28 + 16000*a^28*b^2*c^4*d^26 - 31200*a^28*b^2*c^6*d^24 + 30800*a^28*b^2*c^8*d^22 - 15280*a^28*b^2*c^10*d^20 + 3040*a^28*b^2*c^12*d^18 + 320*a*b^29*c^29*d + 320*a^29*b*c*d^29))^(1/2) + 2*a^24*d^24 + 2*b^24*c^24 + 8*a^2*b^22*c^24 + 8*a^4*b^20*c^24 - 576*a^10*b^14*d^24 + 2784*a^12*b^12*d^24 - 5284*a^14*b^10*d^24 + 4730*a^16*b^8*d^24 - 1780*a^18*b^6*d^24 + 68*a^20*b^4*d^24 + 38*a^22*b^2*d^24 + 8*a^24*c^2*d^22 + 8*a^24*c^4*d^20 - 576*b^24*c^10*d^14 + 2784*b^24*c^12*d^12 - 5284*b^24*c^14*d^10 + 4730*b^24*c^16*d^8 - 1780*b^24*c^18*d^6 + 68*b^24*c^20*d^4 + 38*b^24*c^22*d^2 + 5760*a*b^23*c^9*d^15 - 28224*a*b^23*c^11*d^13 + 54728*a*b^23*c^13*d^11 - 50620*a*b^23*c^15*d^9 + 20360*a*b^23*c^17*d^7 - 1480*a*b^23*c^19*d^5 - 268*a*b^23*c^21*d^3 - 88*a^3*b^21*c^23*d - 160*a^5*b^19*c^23*d + 5760*a^9*b^15*c*d^23 - 28224*a^11*b^13*c*d^23 + 54728*a^13*b^11*c*d^23 - 50620*a^15*b^9*c*d^23 + 20360*a^17*b^7*c*d^23 - 1480*a^19*b^5*c*d^23 - 268*a^21*b^3*c*d^23 - 88*a^23*b*c^3*d^21 - 160*a^23*b*c^5*d^19 - 25920*a^2*b^22*c^8*d^16 + 131904*a^2*b^22*c^10*d^14 - 270604*a^2*b^22*c^12*d^12 + 273544*a^2*b^22*c^14*d^10 - 131660*a^2*b^22*c^16*d^8 + 22060*a^2*b^22*c^18*d^6 - 782*a^2*b^22*c^20*d^4 - 98*a^2*b^22*c^22*d^2 + 69120*a^3*b^21*c^7*d^17 - 379200*a^3*b^21*c^9*d^15 + 860368*a^3*b^21*c^11*d^13 - 1001364*a^3*b^21*c^13*d^11 + 605280*a^3*b^21*c^15*d^9 - 167520*a^3*b^21*c^17*d^7 + 18840*a^3*b^21*c^19*d^5 - 144*a^3*b^21*c^21*d^3 - 120960*a^4*b^20*c^6*d^18 + 756000*a^4*b^20*c^8*d^16 - 1987844*a^4*b^20*c^10*d^14 + 2750664*a^4*b^20*c^12*d^12 - 2073976*a^4*b^20*c^14*d^10 + 793460*a^4*b^20*c^16*d^8 - 138010*a^4*b^20*c^18*d^6 + 10562*a^4*b^20*c^20*d^4 + 88*a^4*b^20*c^22*d^2 + 145152*a^5*b^19*c^5*d^19 - 1116288*a^5*b^19*c^7*d^17 + 3539128*a^5*b^19*c^9*d^15 - 5890780*a^5*b^19*c^11*d^13 + 5437600*a^5*b^19*c^13*d^11 - 2682536*a^5*b^19*c^15*d^9 + 655084*a^5*b^19*c^17*d^7 - 85484*a^5*b^19*c^19*d^5 + 4080*a^5*b^19*c^21*d^3 - 120960*a^6*b^18*c^4*d^20 + 1266048*a^6*b^18*c^6*d^18 - 4977996*a^6*b^18*c^8*d^16 + 10009720*a^6*b^18*c^10*d^14 - 11209800*a^6*b^18*c^12*d^12 + 6943760*a^6*b^18*c^14*d^10 - 2253214*a^6*b^18*c^16*d^8 + 396878*a^6*b^18*c^18*d^6 - 36120*a^6*b^18*c^20*d^4 + 1520*a^6*b^18*c^22*d^2 + 69120*a^7*b^17*c^3*d^21 - 1116288*a^7*b^17*c^5*d^19 + 5575008*a^7*b^17*c^7*d^17 - 13668308*a^7*b^17*c^9*d^15 + 18576800*a^7*b^17*c^11*d^13 - 14230520*a^7*b^17*c^13*d^11 + 5889904*a^7*b^17*c^15*d^9 - 1310504*a^7*b^17*c^17*d^7 + 168344*a^7*b^17*c^19*d^5 - 8960*a^7*b^17*c^21*d^3 - 25920*a^8*b^16*c^2*d^22 + 756000*a^8*b^16*c^4*d^20 - 4977996*a^8*b^16*c^6*d^18 + 15144828*a^8*b^16*c^8*d^16 - 25068800*a^8*b^16*c^10*d^14 + 23486280*a^8*b^16*c^12*d^12 - 12099640*a^8*b^16*c^14*d^10 + 3330518*a^8*b^16*c^16*d^8 - 529224*a^8*b^16*c^18*d^6 + 36280*a^8*b^16*c^20*d^4 - 379200*a^9*b^15*c^3*d^21 + 3539128*a^9*b^15*c^5*d^19 - 13668308*a^9*b^15*c^7*d^17 + 27691952*a^9*b^15*c^9*d^15 - 31562040*a^9*b^15*c^11*d^13 + 19993760*a^9*b^15*c^13*d^11 - 6731044*a^9*b^15*c^15*d^9 + 1239264*a^9*b^15*c^17*d^7 - 106016*a^9*b^15*c^19*d^5 + 131904*a^10*b^14*c^2*d^22 - 1987844*a^10*b^14*c^4*d^20 + 10009720*a^10*b^14*c^6*d^18 - 25068800*a^10*b^14*c^8*d^16 + 34796936*a^10*b^14*c^10*d^14 - 26927144*a^10*b^14*c^12*d^12 + 10994964*a^10*b^14*c^14*d^10 - 2295680*a^10*b^14*c^16*d^8 + 230240*a^10*b^14*c^18*d^6 + 860368*a^11*b^13*c^3*d^21 - 5890780*a^11*b^13*c^5*d^19 + 18576800*a^11*b^13*c^7*d^17 -

31562040*a^11*b^13*c^9*d^15 + 29722864*a^11*b^13*c^11*d^13 - 14679348*a^11
*b^13*c^13*d^11 + 3497920*a^11*b^13*c^15*d^9 - 381280*a^11*b^13*c^17*d^7 -
270604*a^12*b^12*c^2*d^22 + 2750664*a^12*b^12*c^4*d^20 - 11209800*a^12*b^12
*c^6*d^18 + 23486280*a^12*b^12*c^8*d^16 - 26927144*a^12*b^12*c^10*d^14 + 16
147404*a^12*b^12*c^12*d^12 - 4479104*a^12*b^12*c^14*d^10 + 499520*a^12*b^12
*c^16*d^8 - 1001364*a^13*b^11*c^3*d^21 + 5437600*a^13*b^11*c^5*d^19 - 14230
520*a^13*b^11*c^7*d^17 + 19993760*a^13*b^11*c^9*d^15 - 14679348*a^13*b^11*c
^11*d^13 + 4861024*a^13*b^11*c^13*d^11 - 552160*a^13*b^11*c^15*d^9 + 273544
*a^14*b^10*c^2*d^22 - 2073976*a^14*b^10*c^4*d^20 + 6943760*a^14*b^10*c^6*d^
18 - 12099640*a^14*b^10*c^8*d^16 + 10994964*a^14*b^10*c^10*d^14 - 4479104*a
^14*b^10*c^12*d^12 + 562016*a^14*b^10*c^14*d^10 + 605280*a^15*b^9*c^3*d^21
- 2682536*a^15*b^9*c^5*d^19 + 5889904*a^15*b^9*c^7*d^17 - 6731044*a^15*b^9*
c^9*d^15 + 3497920*a^15*b^9*c^11*d^13 - 552160*a^15*b^9*c^13*d^11 - 131660*
a^16*b^8*c^2*d^22 + 793460*a^16*b^8*c^4*d^20 - 2253214*a^16*b^8*c^6*d^18 +
3330518*a^16*b^8*c^8*d^16 - 2295680*a^16*b^8*c^10*d^14 + 499520*a^16*b^8*c^
12*d^12 - 167520*a^17*b^7*c^3*d^21 + 655084*a^17*b^7*c^5*d^19 - 1310504*a^1
7*b^7*c^7*d^17 + 1239264*a^17*b^7*c^9*d^15 - 381280*a^17*b^7*c^11*d^13 + 22
060*a^18*b^6*c^2*d^22 - 138010*a^18*b^6*c^4*d^20 + 396878*a^18*b^6*c^6*d^18
- 529224*a^18*b^6*c^8*d^16 + 230240*a^18*b^6*c^10*d^14 + 18840*a^19*b^5*c^
3*d^21 - 85484*a^19*b^5*c^5*d^19 + 168344*a^19*b^5*c^7*d^17 - 106016*a^19*b
^5*c^9*d^15 - 782*a^20*b^4*c^2*d^22 + 10562*a^20*b^4*c^4*d^20 - 36120*a^20*
b^4*c^6*d^18 + 36280*a^20*b^4*c^8*d^16 - 144*a^21*b^3*c^3*d^21 + 4080*a^21*
b^3*c^5*d^19 - 8960*a^21*b^3*c^7*d^17 - 98*a^22*b^2*c^2*d^22 + 88*a^22*b^2*
c^4*d^20 + 1520*a^22*b^2*c^6*d^18 - 4*a*b^23*c^23*d - 4*a^23*b*c*d^23)/(16*
(5*a^2*b^28*c^30 - b^30*c^30 - a^30*d^30 - 10*a^4*b^26*c^30 + 10*a^6*b^24*c
^30 - 5*a^8*b^22*c^30 + a^10*b^20*c^30 + a^20*b^10*d^30 - 5*a^22*b^8*d^30 +
10*a^24*b^6*d^30 - 10*a^26*b^4*d^30 + 5*a^28*b^2*d^30 + 5*a^30*c^2*d^28 -
10*a^30*c^4*d^26 + 10*a^30*c^6*d^24 - 5*a^30*c^8*d^22 + a^30*c^10*d^20 + b^
30*c^20*d^10 - 5*b^30*c^22*d^8 + 10*b^30*c^24*d^6 - 10*b^30*c^26*d^4 + 5*b^
30*c^28*d^2 - 20*a*b^29*c^19*d^11 + 100*a*b^29*c^21*d^9 - 200*a*b^29*c^23*d
^7 + 200*a*b^29*c^25*d^5 - 100*a*b^29*c^27*d^3 - 100*a^3*b^27*c^29*d + 200*
a^5*b^25*c^29*d - 200*a^7*b^23*c^29*d + 100*a^9*b^21*c^29*d - 20*a^11*b^19*
c^29*d - 20*a^19*b^11*c*d^29 + 100*a^21*b^9*c*d^29 - 200*a^23*b^7*c*d^29 +
200*a^25*b^5*c*d^29 - 100*a^27*b^3*c*d^29 - 100*a^29*b*c^3*d^27 + 200*a^29*
b*c^5*d^25 - 200*a^29*b*c^7*d^23 + 100*a^29*b*c^9*d^21 - 20*a^29*b*c^11*d^1
9 + 190*a^2*b^28*c^18*d^12 - 955*a^2*b^28*c^20*d^10 + 1925*a^2*b^28*c^22*d^
8 - 1950*a^2*b^28*c^24*d^6 + 1000*a^2*b^28*c^26*d^4 - 215*a^2*b^28*c^28*d^2
- 1140*a^3*b^27*c^17*d^13 + 5800*a^3*b^27*c^19*d^11 - 11900*a^3*b^27*c^21*
d^9 + 12400*a^3*b^27*c^23*d^7 - 6700*a^3*b^27*c^25*d^5 + 1640*a^3*b^27*c^27
*d^3 + 4845*a^4*b^26*c^16*d^14 - 25175*a^4*b^26*c^18*d^12 + 53210*a^4*b^26*
c^20*d^10 - 58000*a^4*b^26*c^22*d^8 + 33825*a^4*b^26*c^24*d^6 - 9695*a^4*b^
26*c^26*d^4 + 1000*a^4*b^26*c^28*d^2 - 15504*a^5*b^25*c^15*d^15 + 83220*a^5
*b^25*c^17*d^13 - 183740*a^5*b^25*c^19*d^11 + 213040*a^5*b^25*c^21*d^9 - 13
6520*a^5*b^25*c^23*d^7 + 46004*a^5*b^25*c^25*d^5 - 6700*a^5*b^25*c^27*d^3 +
38760*a^6*b^24*c^14*d^16 - 218025*a^6*b^24*c^16*d^14 + 510625*a^6*b^24*c^1

8*d^12 - 639360*a^6*b^24*c^20*d^10 + 455100*a^6*b^24*c^22*d^8 - 178985*a^6*
b^24*c^24*d^6 + 33825*a^6*b^24*c^26*d^4 - 1950*a^6*b^24*c^28*d^2 - 77520*a^
7*b^23*c^13*d^17 + 465120*a^7*b^23*c^15*d^15 - 1174200*a^7*b^23*c^17*d^13 +
1607600*a^7*b^23*c^19*d^11 - 1277800*a^7*b^23*c^21*d^9 + 581120*a^7*b^23*c
^23*d^7 - 136520*a^7*b^23*c^25*d^5 + 12400*a^7*b^23*c^27*d^3 + 125970*a^8*b
^22*c^12*d^18 - 823650*a^8*b^22*c^14*d^16 + 2277150*a^8*b^22*c^16*d^14 - 34
41850*a^8*b^22*c^18*d^12 + 3061855*a^8*b^22*c^20*d^10 - 1598495*a^8*b^22*c^
22*d^8 + 455100*a^8*b^22*c^24*d^6 - 58000*a^8*b^22*c^26*d^4 + 1925*a^8*b^22
*c^28*d^2 - 167960*a^9*b^21*c^11*d^19 + 1227400*a^9*b^21*c^13*d^17 - 377264
0*a^9*b^21*c^15*d^15 + 6342200*a^9*b^21*c^17*d^13 - 6323300*a^9*b^21*c^19*d
^11 + 3770860*a^9*b^21*c^21*d^9 - 1277800*a^9*b^21*c^23*d^7 + 213040*a^9*b^
21*c^25*d^5 - 11900*a^9*b^21*c^27*d^3 + 184756*a^10*b^20*c^10*d^20 - 155363
0*a^10*b^20*c^12*d^18 + 5384410*a^10*b^20*c^14*d^16 - 10132510*a^10*b^20*c^
16*d^14 + 11341480*a^10*b^20*c^18*d^12 - 7699257*a^10*b^20*c^20*d^10 + 3061
855*a^10*b^20*c^22*d^8 - 639360*a^10*b^20*c^24*d^6 + 53210*a^10*b^20*c^26*d
^4 - 955*a^10*b^20*c^28*d^2 - 167960*a^11*b^19*c^9*d^21 + 1679600*a^11*b^19
*c^11*d^19 - 6653800*a^11*b^19*c^13*d^17 + 14108640*a^11*b^19*c^15*d^15 - 1
7770700*a^11*b^19*c^17*d^13 + 13697880*a^11*b^19*c^19*d^11 - 6323300*a^11*b
^19*c^21*d^9 + 1607600*a^11*b^19*c^23*d^7 - 183740*a^11*b^19*c^25*d^5 + 580
0*a^11*b^19*c^27*d^3 + 125970*a^12*b^18*c^8*d^22 - 1553630*a^12*b^18*c^10*d
^20 + 7138300*a^12*b^18*c^12*d^18 - 17183600*a^12*b^18*c^14*d^16 + 24426875
*a^12*b^18*c^16*d^14 - 21339185*a^12*b^18*c^18*d^12 + 11341480*a^12*b^18*c^
20*d^10 - 3441850*a^12*b^18*c^22*d^8 + 510625*a^12*b^18*c^24*d^6 - 25175*a^
12*b^18*c^26*d^4 + 190*a^12*b^18*c^28*d^2 - 77520*a^13*b^17*c^7*d^23 + 1227
400*a^13*b^17*c^9*d^21 - 6653800*a^13*b^17*c^11*d^19 + 18346400*a^13*b^17*c
^13*d^17 - 29535120*a^13*b^17*c^15*d^15 + 29213260*a^13*b^17*c^17*d^13 - 17
770700*a^13*b^17*c^19*d^11 + 6342200*a^13*b^17*c^21*d^9 - 1174200*a^13*b^17
*c^23*d^7 + 83220*a^13*b^17*c^25*d^5 - 1140*a^13*b^17*c^27*d^3 + 38760*a^14
*b^16*c^6*d^24 - 823650*a^14*b^16*c^8*d^22 + 5384410*a^14*b^16*c^10*d^20 -
17183600*a^14*b^16*c^12*d^18 + 31460200*a^14*b^16*c^14*d^16 - 35234455*a^14
*b^16*c^16*d^14 + 24426875*a^14*b^16*c^18*d^12 - 10132510*a^14*b^16*c^20*d^
10 + 2277150*a^14*b^16*c^22*d^8 - 218025*a^14*b^16*c^24*d^6 + 4845*a^14*b^1
6*c^26*d^4 - 15504*a^15*b^15*c^5*d^25 + 465120*a^15*b^15*c^7*d^23 - 3772640
*a^15*b^15*c^9*d^21 + 14108640*a^15*b^15*c^11*d^19 - 29535120*a^15*b^15*c^1
3*d^17 + 37499008*a^15*b^15*c^15*d^15 - 29535120*a^15*b^15*c^17*d^13 + 1410
8640*a^15*b^15*c^19*d^11 - 3772640*a^15*b^15*c^21*d^9 + 465120*a^15*b^15*c^
23*d^7 - 15504*a^15*b^15*c^25*d^5 + 4845*a^16*b^14*c^4*d^26 - 218025*a^16*b
^14*c^6*d^24 + 2277150*a^16*b^14*c^8*d^22 - 10132510*a^16*b^14*c^10*d^20 +
24426875*a^16*b^14*c^12*d^18 - 35234455*a^16*b^14*c^14*d^16 + 31460200*a^16
*b^14*c^16*d^14 - 17183600*a^16*b^14*c^18*d^12 + 5384410*a^16*b^14*c^20*d^1
0 - 823650*a^16*b^14*c^22*d^8 + 38760*a^16*b^14*c^24*d^6 - 1140*a^17*b^13*c
^3*d^27 + 83220*a^17*b^13*c^5*d^25 - 1174200*a^17*b^13*c^7*d^23 + 6342200*a
^17*b^13*c^9*d^21 - 17770700*a^17*b^13*c^11*d^19 + 29213260*a^17*b^13*c^13*
d^17 - 29535120*a^17*b^13*c^15*d^15 + 18346400*a^17*b^13*c^17*d^13 - 665380
0*a^17*b^13*c^19*d^11 + 1227400*a^17*b^13*c^21*d^9 - 77520*a^17*b^13*c^23*d

^7 + 190*a^18*b^12*c^2*d^28 - 25175*a^18*b^12*c^4*d^26 + 510625*a^18*b^12*c^6*d^24 - 3441850*a^18*b^12*c^8*d^22 + 11341480*a^18*b^12*c^10*d^20 - 21339185*a^18*b^12*c^12*d^18 + 24426875*a^18*b^12*c^14*d^16 - 17183600*a^18*b^12*c^16*d^14 + 7138300*a^18*b^12*c^18*d^12 - 1553630*a^18*b^12*c^20*d^10 + 125970*a^18*b^12*c^22*d^8 + 5800*a^19*b^11*c^3*d^27 - 183740*a^19*b^11*c^5*d^25 + 1607600*a^19*b^11*c^7*d^23 - 6323300*a^19*b^11*c^9*d^21 + 13697880*a^19*b^11*c^11*d^19 - 17770700*a^19*b^11*c^13*d^17 + 14108640*a^19*b^11*c^15*d^15 - 6653800*a^19*b^11*c^17*d^13 + 1679600*a^19*b^11*c^19*d^11 - 167960*a^19*b^11*c^21*d^9 - 955*a^20*b^10*c^2*d^28 + 53210*a^20*b^10*c^4*d^26 - 639360*a^20*b^10*c^6*d^24 + 3061855*a^20*b^10*c^8*d^22 - 7699257*a^20*b^10*c^10*d^20 + 11341480*a^20*b^10*c^12*d^18 - 10132510*a^20*b^10*c^14*d^16 + 5384410*a^20*b^10*c^16*d^14 - 1553630*a^20*b^10*c^18*d^12 + 184756*a^20*b^10*c^20*d^10 - 11900*a^21*b^9*c^3*d^27 + 213040*a^21*b^9*c^5*d^25 - 1277800*a^21*b^9*c^7*d^23 + 3770860*a^21*b^9*c^9*d^21 - 6323300*a^21*b^9*c^11*d^19 + 6342200*a^21*b^9*c^13*d^17 - 3772640*a^21*b^9*c^15*d^15 + 1227400*a^21*b^9*c^17*d^13 - 167960*a^21*b^9*c^19*d^11 + 1925*a^22*b^8*c^2*d^28 - 58000*a^22*b^8*c^4*d^26 + 455100*a^22*b^8*c^6*d^24 - 1598495*a^22*b^8*c^8*d^22 + 3061855*a^22*b^8*c^10*d^20 - 3441850*a^22*b^8*c^12*d^18 + 2277150*a^22*b^8*c^14*d^16 - 823650*a^22*b^8*c^16*d^14 + 125970*a^22*b^8*c^18*d^12 + 12400*a^23*b^7*c^3*d^27 - 136520*a^23*b^7*c^5*d^25 + 581120*a^23*b^7*c^7*d^23 - 1277800*a^23*b^7*c^9*d^21 + 1607600*a^23*b^7*c^11*d^19 - 1174200*a^23*b^7*c^13*d^17 + 465120*a^23*b^7*c^15*d^15 - 77520*a^23*b^7*c^17*d^13 - 1950*a^24*b^6*c^2*d^28 + 33825*a^24*b^6*c^4*d^26 - 178985*a^24*b^6*c^6*d^24 + 455100*a^24*b^6*c^8*d^22 - 639360*a^24*b^6*c^10*d^20 + 510625*a^24*b^6*c^12*d^18 - 218025*a^24*b^6*c^14*d^16 + 38760*a^24*b^6*c^16*d^14 - 6700*a^25*b^5*c^3*d^27 + 46004*a^25*b^5*c^5*d^25 - 136520*a^25*b^5*c^7*d^23 + 213040*a^25*b^5*c^9*d^21 - 183740*a^25*b^5*c^11*d^19 + 83220*a^25*b^5*c^13*d^17 - 15504*a^25*b^5*c^15*d^15 + 1000*a^26*b^4*c^2*d^28 - 9695*a^26*b^4*c^4*d^26 + 33825*a^26*b^4*c^6*d^24 - 58000*a^26*b^4*c^8*d^22 + 53210*a^26*b^4*c^10*d^20 - 25175*a^26*b^4*c^12*d^18 + 4845*a^26*b^4*c^14*d^16 + 1640*a^27*b^3*c^3*d^27 - 6700*a^27*b^3*c^5*d^25 + 12400*a^27*b^3*c^7*d^23 - 11900*a^27*b^3*c^9*d^21 + 5800*a^27*b^3*c^11*d^19 - 1140*a^27*b^3*c^13*d^17 - 215*a^28*b^2*c^2*d^28 + 1000*a^28*b^2*c^4*d^26 - 1950*a^28*b^2*c^6*d^24 + 1925*a^28*b^2*c^8*d^22 - 955*a^28*b^2*c^10*d^20 + 190*a^28*b^2*c^12*d^18 + 20*a*b^29*c^29*d + 20*a^29*b*c*d^29)))^(1/2) - (4*(4*a^2*b^20*c^22 - 12*a^6*b^16*c^22 + 8*a^8*b^14*c^22 + 4*a^22*c^2*d^20 - 12*a^22*c^6*d^16 + 8*a^22*c^8*d^14 + 48*a*b^21*c^11*d^11 - 212*a*b^21*c^13*d^9 + 360*a*b^21*c^15*d^7 - 276*a*b^21*c^17*d^5 + 80*a*b^21*c^19*d^3 - 20*a^3*b^19*c^21*d - 72*a^5*b^17*c^21*d + 204*a^7*b^15*c^21*d - 112*a^9*b^13*c^21*d + 48*a^11*b^11*c*d^21 - 212*a^13*b^9*c*d^21 + 360*a^15*b^7*c*d^21 - 276*a^17*b^5*c*d^21 + 80*a^19*b^3*c*d^21 - 20*a^21*b*c^3*d^19 - 72*a^21*b*c^5*d^17 + 204*a^21*b*c^7*d^15 - 112*a^21*b*c^9*d^13 - 480*a^2*b^20*c^10*d^12 + 2160*a^2*b^20*c^12*d^10 - 3772*a^2*b^20*c^14*d^8 + 3020*a^2*b^20*c^16*d^6 - 960*a^2*b^20*c^18*d^4 + 28*a^2*b^20*c^20*d^2 + 2160*a^3*b^19*c^9*d^13 - 10152*a^3*b^19*c^11*d^11 + 18888*a^3*b^19*c^13*d^9 - 16732*a^3*b^19*c^15*d^7 + 6588*a^3*b^19*c^17*d^5 - 732*a^3*b^19*c^19*d^3 - 576

0*a^4*b^18*c^8*d^14 + 29360*a^4*b^18*c^10*d^12 - 60792*a^4*b^18*c^12*d^10 + 62708*a^4*b^18*c^14*d^8 - 31892*a^4*b^18*c^16*d^6 + 6588*a^4*b^18*c^18*d^4 - 212*a^4*b^18*c^20*d^2 + 10080*a^5*b^17*c^7*d^15 - 58860*a^5*b^17*c^9*d^13 + 141880*a^5*b^17*c^11*d^11 - 175592*a^5*b^17*c^13*d^9 + 113748*a^5*b^17*c^15*d^7 - 34492*a^5*b^17*c^17*d^5 + 3308*a^5*b^17*c^19*d^3 - 12096*a^6*b^16*c^6*d^16 + 87264*a^6*b^16*c^8*d^14 - 254340*a^6*b^16*c^10*d^12 + 381532*a^6*b^16*c^12*d^10 - 307752*a^6*b^16*c^14*d^8 + 125568*a^6*b^16*c^16*d^6 - 21232*a^6*b^16*c^18*d^4 + 1068*a^6*b^16*c^20*d^2 + 10080*a^7*b^15*c^5*d^17 - 99120*a^7*b^15*c^7*d^15 + 359064*a^7*b^15*c^9*d^13 - 655076*a^7*b^15*c^11*d^11 + 650108*a^7*b^15*c^13*d^9 - 343368*a^7*b^15*c^15*d^7 + 85760*a^7*b^15*c^17*d^5 - 7652*a^7*b^15*c^19*d^3 - 5760*a^8*b^14*c^4*d^18 + 87264*a^8*b^14*c^6*d^16 - 402576*a^8*b^14*c^8*d^14 + 900324*a^8*b^14*c^10*d^12 - 1096236*a^8*b^14*c^12*d^10 + 731392*a^8*b^14*c^14*d^8 - 247352*a^8*b^14*c^16*d^6 + 34548*a^8*b^14*c^18*d^4 - 1612*a^8*b^14*c^20*d^2 + 2160*a^9*b^13*c^3*d^19 - 58860*a^9*b^13*c^5*d^17 + 359064*a^9*b^13*c^7*d^15 - 999816*a^9*b^13*c^9*d^13 + 1494564*a^9*b^13*c^11*d^11 - 1238148*a^9*b^13*c^13*d^9 + 542272*a^9*b^13*c^15*d^7 - 109032*a^9*b^13*c^17*d^5 + 7908*a^9*b^13*c^19*d^3 - 480*a^10*b^12*c^2*d^20 + 29360*a^10*b^12*c^4*d^18 - 254340*a^10*b^12*c^6*d^16 + 900324*a^10*b^12*c^8*d^14 - 1656496*a^10*b^12*c^10*d^12 + 1688232*a^10*b^12*c^12*d^10 - 934868*a^10*b^12*c^14*d^8 + 254492*a^10*b^12*c^16*d^6 - 26952*a^10*b^12*c^18*d^4 + 728*a^10*b^12*c^20*d^2 - 10152*a^11*b^11*c^3*d^19 + 141880*a^11*b^11*c^5*d^17 - 655076*a^11*b^11*c^7*d^15 + 1494564*a^11*b^11*c^9*d^13 - 1870136*a^11*b^11*c^11*d^11 + 1289704*a^11*b^11*c^13*d^9 - 455388*a^11*b^11*c^15*d^7 + 67468*a^11*b^11*c^17*d^5 - 2912*a^11*b^11*c^19*d^3 + 2160*a^12*b^10*c^2*d^20 - 60792*a^12*b^10*c^4*d^18 + 381532*a^12*b^10*c^6*d^16 - 1096236*a^12*b^10*c^8*d^14 + 1688232*a^12*b^10*c^10*d^12 - 1434728*a^12*b^10*c^12*d^10 + 639684*a^12*b^10*c^14*d^8 - 127860*a^12*b^10*c^16*d^6 + 8008*a^12*b^10*c^18*d^4 + 18888*a^13*b^9*c^3*d^19 - 175592*a^13*b^9*c^5*d^17 + 650108*a^13*b^9*c^7*d^15 - 1238148*a^13*b^9*c^9*d^13 + 1289704*a^13*b^9*c^11*d^11 - 715296*a^13*b^9*c^13*d^9 + 186564*a^13*b^9*c^15*d^7 - 16016*a^13*b^9*c^17*d^5 - 3772*a^14*b^8*c^2*d^20 + 62708*a^14*b^8*c^4*d^18 - 307752*a^14*b^8*c^6*d^16 + 731392*a^14*b^8*c^8*d^14 - 934868*a^14*b^8*c^10*d^12 + 639684*a^14*b^8*c^12*d^10 - 211416*a^14*b^8*c^14*d^8 + 24024*a^14*b^8*c^16*d^6 - 16732*a^15*b^7*c^3*d^19 + 113748*a^15*b^7*c^5*d^17 - 343368*a^15*b^7*c^7*d^15 + 542272*a^15*b^7*c^9*d^13 - 455388*a^15*b^7*c^11*d^11 + 186564*a^15*b^7*c^13*d^9 - 27456*a^15*b^7*c^15*d^7 + 3020*a^16*b^6*c^2*d^20 - 31892*a^16*b^6*c^4*d^18 + 125568*a^16*b^6*c^6*d^16 - 247352*a^16*b^6*c^8*d^14 + 254492*a^16*b^6*c^10*d^12 - 127860*a^16*b^6*c^12*d^10 + 24024*a^16*b^6*c^14*d^8 + 6588*a^17*b^5*c^3*d^19 - 34492*a^17*b^5*c^5*d^17 + 85760*a^17*b^5*c^7*d^15 - 109032*a^17*b^5*c^9*d^13 + 67468*a^17*b^5*c^11*d^11 - 16016*a^17*b^5*c^13*d^9 - 960*a^18*b^4*c^2*d^20 + 6588*a^18*b^4*c^4*d^18 - 21232*a^18*b^4*c^6*d^16 + 34548*a^18*b^4*c^8*d^14 - 26952*a^18*b^4*c^10*d^12 + 8008*a^18*b^4*c^12*d^10 - 732*a^19*b^3*c^3*d^19 + 3308*a^19*b^3*c^5*d^17 - 7652*a^19*b^3*c^7*d^15 + 7908*a^19*b^3*c^9*d^13 - 2912*a^19*b^3*c^11*d^11 + 28*a^20*b^2*c^2*d^20 - 212*a^20*b^2*c^4*d^18 + 1068*a^20*b^2*c^6*d^16 - 1612*a^20*b^2

*c^8*d^14 + 728*a^20*b^2*c^10*d^12))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^
20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*
a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^2
0*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*
d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^
13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*
a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d
^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^1
9*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11
 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 -
288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*
a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a
^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a
^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^
4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5
*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5
*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6
*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*
a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^
7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576
*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*
a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 3630
0*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 +
4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 40
48*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 -
44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5
+ 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 +
 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10
*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^1
0*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*
c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b
^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b
^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b
^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^
12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^1
3*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a
^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a
^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a
^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 567
6*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744
*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048
*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a
^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a
^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^
17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17

*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2
*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*
d - 12*a^19*b*c*d^19) + (8*tan(e/2 + (f*x)/2)*(12*a^5*b^17*c^22 - 4*a^22*c*
d^21 - 4*a*b^21*c^22 - 8*a^7*b^15*c^22 + 12*a^22*c^5*d^17 - 8*a^22*c^7*d^15
 - 24*a*b^21*c^12*d^10 + 100*a*b^21*c^14*d^8 - 164*a*b^21*c^16*d^6 + 120*a*
b^21*c^18*d^4 - 28*a*b^21*c^20*d^2 + 20*a^2*b^20*c^21*d + 72*a^4*b^18*c^21*
d - 204*a^6*b^16*c^21*d + 112*a^8*b^14*c^21*d - 24*a^12*b^10*c*d^21 + 100*a
^14*b^8*c*d^21 - 164*a^16*b^6*c*d^21 + 120*a^18*b^4*c*d^21 - 28*a^20*b^2*c*
d^21 + 20*a^21*b*c^2*d^20 + 72*a^21*b*c^4*d^18 - 204*a^21*b*c^6*d^16 + 112*
a^21*b*c^8*d^14 + 216*a^2*b^20*c^11*d^11 - 908*a^2*b^20*c^13*d^9 + 1540*a^2
*b^20*c^15*d^7 - 1200*a^2*b^20*c^17*d^5 + 332*a^2*b^20*c^19*d^3 - 840*a^3*b
^19*c^10*d^12 + 3672*a^3*b^19*c^12*d^10 - 6788*a^3*b^19*c^14*d^8 + 6132*a^3
*b^19*c^16*d^6 - 2388*a^3*b^19*c^18*d^4 + 212*a^3*b^19*c^20*d^2 + 1800*a^4*
b^18*c^9*d^13 - 8680*a^4*b^18*c^11*d^11 + 18852*a^4*b^18*c^13*d^9 - 21228*a
^4*b^18*c^15*d^7 + 11692*a^4*b^18*c^17*d^5 - 2508*a^4*b^18*c^19*d^3 - 2160*
a^5*b^17*c^8*d^14 + 13100*a^5*b^17*c^10*d^12 - 36820*a^5*b^17*c^12*d^10 + 5
3712*a^5*b^17*c^14*d^8 - 39608*a^5*b^17*c^16*d^6 + 12832*a^5*b^17*c^18*d^4
 - 1068*a^5*b^17*c^20*d^2 + 1008*a^6*b^16*c^7*d^15 - 12420*a^6*b^16*c^9*d^13
 + 51764*a^6*b^16*c^11*d^11 - 100128*a^6*b^16*c^13*d^9 + 96048*a^6*b^16*c^1
5*d^7 - 42920*a^6*b^16*c^17*d^5 + 6852*a^6*b^16*c^19*d^3 + 1008*a^7*b^15*c^
6*d^16 + 5136*a^7*b^15*c^8*d^14 - 48820*a^7*b^15*c^10*d^12 + 134700*a^7*b^1
5*c^12*d^10 - 171472*a^7*b^15*c^14*d^8 + 103992*a^7*b^15*c^16*d^6 - 26148*a
^7*b^15*c^18*d^4 + 1612*a^7*b^15*c^20*d^2 - 2160*a^8*b^14*c^5*d^17 + 5136*a
^8*b^14*c^7*d^15 + 20436*a^8*b^14*c^9*d^13 - 121524*a^8*b^14*c^11*d^11 + 22
4888*a^8*b^14*c^13*d^9 - 186952*a^8*b^14*c^15*d^7 + 67572*a^8*b^14*c^17*d^5
 - 7508*a^8*b^14*c^19*d^3 + 1800*a^9*b^13*c^4*d^18 - 12420*a^9*b^13*c^6*d^1
6 + 20436*a^9*b^13*c^8*d^14 + 49416*a^9*b^13*c^10*d^12 - 201552*a^9*b^13*c^
12*d^10 + 245708*a^9*b^13*c^14*d^8 - 125412*a^9*b^13*c^16*d^6 + 22752*a^9*b
^13*c^18*d^4 - 728*a^9*b^13*c^20*d^2 - 840*a^10*b^12*c^3*d^19 + 13100*a^10*
b^12*c^5*d^17 - 48820*a^10*b^12*c^7*d^15 + 49416*a^10*b^12*c^9*d^13 + 82088
*a^10*b^12*c^11*d^11 - 219092*a^10*b^12*c^13*d^9 + 168468*a^10*b^12*c^15*d^
7 - 47152*a^10*b^12*c^17*d^5 + 2832*a^10*b^12*c^19*d^3 + 216*a^11*b^11*c^2*
d^20 - 8680*a^11*b^11*c^4*d^18 + 51764*a^11*b^11*c^6*d^16 - 121524*a^11*b^1
1*c^8*d^14 + 82088*a^11*b^11*c^10*d^12 + 88712*a^11*b^11*c^12*d^10 - 153012
*a^11*b^11*c^14*d^8 + 67604*a^11*b^11*c^16*d^6 - 7168*a^11*b^11*c^18*d^4 +
3672*a^12*b^10*c^3*d^19 - 36820*a^12*b^10*c^5*d^17 + 134700*a^12*b^10*c^7*d
^15 - 201552*a^12*b^10*c^9*d^13 + 88712*a^12*b^10*c^11*d^11 + 62676*a^12*b^
10*c^13*d^9 - 63372*a^12*b^10*c^15*d^7 + 12008*a^12*b^10*c^17*d^5 - 908*a^1
3*b^9*c^2*d^20 + 18852*a^13*b^9*c^4*d^18 - 100128*a^13*b^9*c^6*d^16 + 22488
8*a^13*b^9*c^8*d^14 - 219092*a^13*b^9*c^10*d^12 + 62676*a^13*b^9*c^12*d^10
+ 26256*a^13*b^9*c^14*d^8 - 12544*a^13*b^9*c^16*d^6 - 6788*a^14*b^8*c^3*d^1
9 + 53712*a^14*b^8*c^5*d^17 - 171472*a^14*b^8*c^7*d^15 + 245708*a^14*b^8*c^
9*d^13 - 153012*a^14*b^8*c^11*d^11 + 26256*a^14*b^8*c^13*d^9 + 5496*a^14*b^
8*c^15*d^7 + 1540*a^15*b^7*c^2*d^20 - 21228*a^15*b^7*c^4*d^18 + 96048*a^15*
b^7*c^6*d^16 - 186952*a^15*b^7*c^8*d^14 + 168468*a^15*b^7*c^10*d^12 - 63372

*a^15*b^7*c^12*d^10 + 5496*a^15*b^7*c^14*d^8 + 6132*a^16*b^6*c^3*d^19 - 396
08*a^16*b^6*c^5*d^17 + 103992*a^16*b^6*c^7*d^15 - 125412*a^16*b^6*c^9*d^13
+ 67604*a^16*b^6*c^11*d^11 - 12544*a^16*b^6*c^13*d^9 - 1200*a^17*b^5*c^2*d^
20 + 11692*a^17*b^5*c^4*d^18 - 42920*a^17*b^5*c^6*d^16 + 67572*a^17*b^5*c^8
*d^14 - 47152*a^17*b^5*c^10*d^12 + 12008*a^17*b^5*c^12*d^10 - 2388*a^18*b^4
*c^3*d^19 + 12832*a^18*b^4*c^5*d^17 - 26148*a^18*b^4*c^7*d^15 + 22752*a^18*
b^4*c^9*d^13 - 7168*a^18*b^4*c^11*d^11 + 332*a^19*b^3*c^2*d^20 - 2508*a^19*
b^3*c^4*d^18 + 6852*a^19*b^3*c^6*d^16 - 7508*a^19*b^3*c^8*d^14 + 2832*a^19*
b^3*c^10*d^12 + 212*a^20*b^2*c^3*d^19 - 1068*a^20*b^2*c^5*d^17 + 1612*a^20*
b^2*c^7*d^15 - 728*a^20*b^2*c^9*d^13))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*
c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 -
4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a
^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^1
4*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*
c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 7
2*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c
*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a
^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^
11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6
- 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 92
8*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412
*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032
*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*
a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a
^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a
^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a
^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 358
8*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*
a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 215
76*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 49
5*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36
300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6
+ 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 +
4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11
- 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^
5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16
 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^
10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b
^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^
9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11
*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11
*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12
*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*
a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a
^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504

*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792
*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236
*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5
676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 47
44*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 40
48*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587
*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244
*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*
a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^
17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b
^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^1
9*d - 12*a^19*b*c*d^19)) + (4*(288*a*b^18*c^6*d^13 - 1104*a*b^18*c^8*d^11 +
 1538*a*b^18*c^10*d^9 - 872*a*b^18*c^12*d^7 + 108*a*b^18*c^14*d^5 + 40*a*b^
18*c^16*d^3 + 8*a^3*b^16*c^18*d + 8*a^5*b^14*c^18*d + 288*a^6*b^13*c*d^18 -
 1104*a^8*b^11*c*d^18 + 1538*a^10*b^9*c*d^18 - 872*a^12*b^7*c*d^18 + 108*a^
14*b^5*c*d^18 + 40*a^16*b^3*c*d^18 + 8*a^18*b*c^3*d^16 + 8*a^18*b*c^5*d^14
- 864*a^2*b^17*c^5*d^14 + 3216*a^2*b^17*c^7*d^12 - 4262*a^2*b^17*c^9*d^10 +
 2256*a^2*b^17*c^11*d^8 - 304*a^2*b^17*c^13*d^6 - 32*a^2*b^17*c^15*d^4 + 8*
a^2*b^17*c^17*d^2 + 576*a^3*b^16*c^4*d^15 - 3024*a^3*b^16*c^6*d^13 + 6304*a
^3*b^16*c^8*d^11 - 7216*a^3*b^16*c^10*d^9 + 4944*a^3*b^16*c^12*d^7 - 1664*a
^3*b^16*c^14*d^5 - 72*a^3*b^16*c^16*d^3 + 576*a^4*b^15*c^3*d^16 + 912*a^4*b
^15*c^5*d^14 - 8720*a^4*b^15*c^7*d^12 + 16632*a^4*b^15*c^9*d^10 - 14888*a^4
*b^15*c^11*d^8 + 6704*a^4*b^15*c^13*d^6 - 744*a^4*b^15*c^15*d^4 - 40*a^4*b^
15*c^17*d^2 - 864*a^5*b^14*c^2*d^17 + 912*a^5*b^14*c^4*d^15 + 5140*a^5*b^14
*c^6*d^13 - 16080*a^5*b^14*c^8*d^11 + 23520*a^5*b^14*c^10*d^9 - 20208*a^5*b
^14*c^12*d^7 + 7404*a^5*b^14*c^14*d^5 - 264*a^5*b^14*c^16*d^3 - 3024*a^6*b^
13*c^3*d^16 + 5140*a^6*b^13*c^5*d^14 + 5280*a^6*b^13*c^7*d^12 - 28380*a^6*b
^13*c^9*d^10 + 39792*a^6*b^13*c^11*d^8 - 22728*a^6*b^13*c^13*d^6 + 3096*a^6
*b^13*c^15*d^4 - 112*a^6*b^13*c^17*d^2 + 3216*a^7*b^12*c^2*d^17 - 8720*a^7*
b^12*c^4*d^15 + 5280*a^7*b^12*c^6*d^13 + 15000*a^7*b^12*c^8*d^11 - 40656*a^
7*b^12*c^10*d^9 + 40296*a^7*b^12*c^12*d^7 - 12984*a^7*b^12*c^14*d^5 + 728*a
^7*b^12*c^16*d^3 + 6304*a^8*b^11*c^3*d^16 - 16080*a^8*b^11*c^5*d^14 + 15000
*a^8*b^11*c^7*d^12 + 16024*a^8*b^11*c^9*d^10 - 46184*a^8*b^11*c^11*d^8 + 27
208*a^8*b^11*c^13*d^6 - 2752*a^8*b^11*c^15*d^4 - 4262*a^9*b^10*c^2*d^17 + 1
6632*a^9*b^10*c^4*d^15 - 28380*a^9*b^10*c^6*d^13 + 16024*a^9*b^10*c^8*d^11
+ 22018*a^9*b^10*c^10*d^9 - 30104*a^9*b^10*c^12*d^7 + 6488*a^9*b^10*c^14*d^
5 - 7216*a^10*b^9*c^3*d^16 + 23520*a^10*b^9*c^5*d^14 - 40656*a^10*b^9*c^7*d
^12 + 22018*a^10*b^9*c^9*d^10 + 13080*a^10*b^9*c^11*d^8 - 8720*a^10*b^9*c^1
3*d^6 + 2256*a^11*b^8*c^2*d^17 - 14888*a^11*b^8*c^4*d^15 + 39792*a^11*b^8*c
^6*d^13 - 46184*a^11*b^8*c^8*d^11 + 13080*a^11*b^8*c^10*d^9 + 4360*a^11*b^8
*c^12*d^7 + 4944*a^12*b^7*c^3*d^16 - 20208*a^12*b^7*c^5*d^14 + 40296*a^12*b
^7*c^7*d^12 - 30104*a^12*b^7*c^9*d^10 + 4360*a^12*b^7*c^11*d^8 - 304*a^13*b
^6*c^2*d^17 + 6704*a^13*b^6*c^4*d^15 - 22728*a^13*b^6*c^6*d^13 + 27208*a^13
*b^6*c^8*d^11 - 8720*a^13*b^6*c^10*d^9 - 1664*a^14*b^5*c^3*d^16 + 7404*a^14
*b^5*c^5*d^14 - 12984*a^14*b^5*c^7*d^12 + 6488*a^14*b^5*c^9*d^10 - 32*a^15*

b^4*c^2*d^17 - 744*a^15*b^4*c^4*d^15 + 3096*a^15*b^4*c^6*d^13 - 2752*a^15*b
^4*c^8*d^11 - 72*a^16*b^3*c^3*d^16 - 264*a^16*b^3*c^5*d^14 + 728*a^16*b^3*c
^7*d^12 + 8*a^17*b^2*c^2*d^17 - 40*a^17*b^2*c^4*d^15 - 112*a^17*b^2*c^6*d^1
3 + 2*a*b^18*c^18*d + 2*a^18*b*c*d^18))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18
*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 -
4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*
a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^
14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19
*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d -
72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*
c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*
a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d
^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6
- 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 9
28*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 41
2*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 403
2*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288
*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*
a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*
a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*
a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 35
88*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336
*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21
576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 4
95*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 3
6300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6
+ 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 +
4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11
- 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d
^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^1
6 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c
^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*
b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b
^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^1
1*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^1
1*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^1
2*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724
*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*
a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 2750
4*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 79
2*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 1123
6*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 -
5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4
744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4
048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 158

7*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 224
4*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168
*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a
^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*
b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^
19*d - 12*a^19*b*c*d^19) - (8*tan(e/2 + (f*x)/2)*(a*b^18*c^19 + a^19*c*d^18
 + 4*a^3*b^16*c^19 + 4*a^5*b^14*c^19 + 4*a^19*c^3*d^16 + 4*a^19*c^5*d^14 -
576*a*b^18*c^5*d^14 + 2640*a*b^18*c^7*d^12 - 4732*a*b^18*c^9*d^10 + 3961*a*
b^18*c^11*d^8 - 1344*a*b^18*c^13*d^6 + 14*a*b^18*c^15*d^4 + 18*a*b^18*c^17*
d^2 + 4*a^2*b^17*c^18*d - 20*a^4*b^15*c^18*d - 576*a^5*b^14*c*d^18 - 56*a^6
*b^13*c^18*d + 2640*a^7*b^12*c*d^18 - 4732*a^9*b^10*c*d^18 + 3961*a^11*b^8*
c*d^18 - 1344*a^13*b^6*c*d^18 + 14*a^15*b^4*c*d^18 + 18*a^17*b^2*c*d^18 + 4
*a^18*b*c^2*d^17 - 20*a^18*b*c^4*d^15 - 56*a^18*b*c^6*d^13 + 2304*a^2*b^17*
c^4*d^15 - 10944*a^2*b^17*c^6*d^13 + 20720*a^2*b^17*c^8*d^11 - 18788*a^2*b^
17*c^10*d^9 + 7392*a^2*b^17*c^12*d^7 - 520*a^2*b^17*c^14*d^5 - 24*a^2*b^17*
c^16*d^3 - 3456*a^3*b^16*c^3*d^16 + 20016*a^3*b^16*c^5*d^14 - 48112*a^3*b^1
6*c^7*d^12 + 58925*a^3*b^16*c^9*d^10 - 36732*a^3*b^16*c^11*d^8 + 9736*a^3*b
^16*c^13*d^6 - 760*a^3*b^16*c^15*d^4 - 44*a^3*b^16*c^17*d^2 + 2304*a^4*b^15
*c^2*d^17 - 23424*a^4*b^15*c^4*d^15 + 81680*a^4*b^15*c^6*d^13 - 135520*a^4*
b^15*c^8*d^11 + 114144*a^4*b^15*c^10*d^9 - 44168*a^4*b^15*c^12*d^7 + 5696*a
^4*b^15*c^14*d^5 - 332*a^4*b^15*c^16*d^3 + 20016*a^5*b^14*c^3*d^16 - 99112*
a^5*b^14*c^5*d^14 + 213338*a^5*b^14*c^7*d^12 - 235152*a^5*b^14*c^9*d^10 + 1
30428*a^5*b^14*c^11*d^8 - 31908*a^5*b^14*c^13*d^6 + 3966*a^5*b^14*c^15*d^4
- 140*a^5*b^14*c^17*d^2 - 10944*a^6*b^13*c^2*d^17 + 81680*a^6*b^13*c^4*d^15
 - 243832*a^6*b^13*c^6*d^13 + 364608*a^6*b^13*c^8*d^11 - 281736*a^6*b^13*c^
10*d^9 + 103104*a^6*b^13*c^12*d^7 - 16860*a^6*b^13*c^14*d^5 + 1660*a^6*b^13
*c^16*d^3 - 48112*a^7*b^12*c^3*d^16 + 213338*a^7*b^12*c^5*d^14 - 425832*a^7
*b^12*c^7*d^12 + 434414*a^7*b^12*c^9*d^10 - 219064*a^7*b^12*c^11*d^8 + 5073
2*a^7*b^12*c^13*d^6 - 7220*a^7*b^12*c^15*d^4 + 364*a^7*b^12*c^17*d^2 + 2072
0*a^8*b^11*c^2*d^17 - 135520*a^8*b^11*c^4*d^15 + 364608*a^8*b^11*c^6*d^13 -
 496336*a^8*b^11*c^8*d^11 + 343832*a^8*b^11*c^10*d^9 - 111220*a^8*b^11*c^12
*d^7 + 17956*a^8*b^11*c^14*d^5 - 1376*a^8*b^11*c^16*d^3 + 58925*a^9*b^10*c^
3*d^16 - 235152*a^9*b^10*c^5*d^14 + 434414*a^9*b^10*c^7*d^12 - 401788*a^9*b
^10*c^9*d^10 + 172673*a^9*b^10*c^11*d^8 - 31940*a^9*b^10*c^13*d^6 + 3244*a^
9*b^10*c^15*d^4 - 18788*a^10*b^9*c^2*d^17 + 114144*a^10*b^9*c^4*d^15 - 2817
36*a^10*b^9*c^6*d^13 + 343832*a^10*b^9*c^8*d^11 - 197840*a^10*b^9*c^10*d^9
+ 45940*a^10*b^9*c^12*d^7 - 4760*a^10*b^9*c^14*d^5 - 36732*a^11*b^8*c^3*d^1
6 + 130428*a^11*b^8*c^5*d^14 - 219064*a^11*b^8*c^7*d^12 + 172673*a^11*b^8*c
^9*d^10 - 52480*a^11*b^8*c^11*d^8 + 4580*a^11*b^8*c^13*d^6 + 7392*a^12*b^7*
c^2*d^17 - 44168*a^12*b^7*c^4*d^15 + 103104*a^12*b^7*c^6*d^13 - 111220*a^12
*b^7*c^8*d^11 + 45940*a^12*b^7*c^10*d^9 - 4000*a^12*b^7*c^12*d^7 + 9736*a^1
3*b^6*c^3*d^16 - 31908*a^13*b^6*c^5*d^14 + 50732*a^13*b^6*c^7*d^12 - 31940*
a^13*b^6*c^9*d^10 + 4580*a^13*b^6*c^11*d^8 - 520*a^14*b^5*c^2*d^17 + 5696*a
^14*b^5*c^4*d^15 - 16860*a^14*b^5*c^6*d^13 + 17956*a^14*b^5*c^8*d^11 - 4760
*a^14*b^5*c^10*d^9 - 760*a^15*b^4*c^3*d^16 + 3966*a^15*b^4*c^5*d^14 - 7220*

a^15*b^4*c^7*d^12 + 3244*a^15*b^4*c^9*d^10 - 24*a^16*b^3*c^2*d^17 - 332*a^1
6*b^3*c^4*d^15 + 1660*a^16*b^3*c^6*d^13 - 1376*a^16*b^3*c^8*d^11 - 44*a^17*
b^2*c^3*d^16 - 140*a^17*b^2*c^5*d^14 + 364*a^17*b^2*c^7*d^12))/(a^20*d^20 +
b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12
*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20
- 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^
20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^
19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3
+ 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^1
1*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 4
8*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7
*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8
+ 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 22
0*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168
*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a
^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*
a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^
5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^
5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6
*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236
*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a
^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504
*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 151
2*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 2072
4*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 -
17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 -
220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 +
39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7
- 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 -
2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d
^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^1
0*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9
*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b
^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11
*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b
^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^1
2*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^
12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a
^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336
*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a
^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 1386
0*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 116
8*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 834
4*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*
a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*

a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a
^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^
17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b
^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c
^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19)) - (-(((4*a^24*d^24 + 4*b^2
4*c^24 + 16*a^2*b^22*c^24 + 16*a^4*b^20*c^24 - 1152*a^10*b^14*d^24 + 5568*a
^12*b^12*d^24 - 10568*a^14*b^10*d^24 + 9460*a^16*b^8*d^24 - 3560*a^18*b^6*d
^24 + 136*a^20*b^4*d^24 + 76*a^22*b^2*d^24 + 16*a^24*c^2*d^22 + 16*a^24*c^4
*d^20 - 1152*b^24*c^10*d^14 + 5568*b^24*c^12*d^12 - 10568*b^24*c^14*d^10 +
9460*b^24*c^16*d^8 - 3560*b^24*c^18*d^6 + 136*b^24*c^20*d^4 + 76*b^24*c^22*
d^2 + 11520*a*b^23*c^9*d^15 - 56448*a*b^23*c^11*d^13 + 109456*a*b^23*c^13*d
^11 - 101240*a*b^23*c^15*d^9 + 40720*a*b^23*c^17*d^7 - 2960*a*b^23*c^19*d^5
 - 536*a*b^23*c^21*d^3 - 176*a^3*b^21*c^23*d - 320*a^5*b^19*c^23*d + 11520*
a^9*b^15*c*d^23 - 56448*a^11*b^13*c*d^23 + 109456*a^13*b^11*c*d^23 - 101240
*a^15*b^9*c*d^23 + 40720*a^17*b^7*c*d^23 - 2960*a^19*b^5*c*d^23 - 536*a^21*
b^3*c*d^23 - 176*a^23*b*c^3*d^21 - 320*a^23*b*c^5*d^19 - 51840*a^2*b^22*c^8
*d^16 + 263808*a^2*b^22*c^10*d^14 - 541208*a^2*b^22*c^12*d^12 + 547088*a^2*
b^22*c^14*d^10 - 263320*a^2*b^22*c^16*d^8 + 44120*a^2*b^22*c^18*d^6 - 1564*
a^2*b^22*c^20*d^4 - 196*a^2*b^22*c^22*d^2 + 138240*a^3*b^21*c^7*d^17 - 7584
00*a^3*b^21*c^9*d^15 + 1720736*a^3*b^21*c^11*d^13 - 2002728*a^3*b^21*c^13*d
^11 + 1210560*a^3*b^21*c^15*d^9 - 335040*a^3*b^21*c^17*d^7 + 37680*a^3*b^21
*c^19*d^5 - 288*a^3*b^21*c^21*d^3 - 241920*a^4*b^20*c^6*d^18 + 1512000*a^4*
b^20*c^8*d^16 - 3975688*a^4*b^20*c^10*d^14 + 5501328*a^4*b^20*c^12*d^12 - 4
147952*a^4*b^20*c^14*d^10 + 1586920*a^4*b^20*c^16*d^8 - 276020*a^4*b^20*c^1
8*d^6 + 21124*a^4*b^20*c^20*d^4 + 176*a^4*b^20*c^22*d^2 + 290304*a^5*b^19*c
^5*d^19 - 2232576*a^5*b^19*c^7*d^17 + 7078256*a^5*b^19*c^9*d^15 - 11781560*
a^5*b^19*c^11*d^13 + 10875200*a^5*b^19*c^13*d^11 - 5365072*a^5*b^19*c^15*d^
9 + 1310168*a^5*b^19*c^17*d^7 - 170968*a^5*b^19*c^19*d^5 + 8160*a^5*b^19*c^
21*d^3 - 241920*a^6*b^18*c^4*d^20 + 2532096*a^6*b^18*c^6*d^18 - 9955992*a^6
*b^18*c^8*d^16 + 20019440*a^6*b^18*c^10*d^14 - 22419600*a^6*b^18*c^12*d^12
+ 13887520*a^6*b^18*c^14*d^10 - 4506428*a^6*b^18*c^16*d^8 + 793756*a^6*b^18
*c^18*d^6 - 72240*a^6*b^18*c^20*d^4 + 3040*a^6*b^18*c^22*d^2 + 138240*a^7*b
^17*c^3*d^21 - 2232576*a^7*b^17*c^5*d^19 + 11150016*a^7*b^17*c^7*d^17 - 273
36616*a^7*b^17*c^9*d^15 + 37153600*a^7*b^17*c^11*d^13 - 28461040*a^7*b^17*c
^13*d^11 + 11779808*a^7*b^17*c^15*d^9 - 2621008*a^7*b^17*c^17*d^7 + 336688*
a^7*b^17*c^19*d^5 - 17920*a^7*b^17*c^21*d^3 - 51840*a^8*b^16*c^2*d^22 + 151
2000*a^8*b^16*c^4*d^20 - 9955992*a^8*b^16*c^6*d^18 + 30289656*a^8*b^16*c^8*
d^16 - 50137600*a^8*b^16*c^10*d^14 + 46972560*a^8*b^16*c^12*d^12 - 24199280
*a^8*b^16*c^14*d^10 + 6661036*a^8*b^16*c^16*d^8 - 1058448*a^8*b^16*c^18*d^6
 + 72560*a^8*b^16*c^20*d^4 - 758400*a^9*b^15*c^3*d^21 + 7078256*a^9*b^15*c^
5*d^19 - 27336616*a^9*b^15*c^7*d^17 + 55383904*a^9*b^15*c^9*d^15 - 63124080
*a^9*b^15*c^11*d^13 + 39987520*a^9*b^15*c^13*d^11 - 13462088*a^9*b^15*c^15*
d^9 + 2478528*a^9*b^15*c^17*d^7 - 212032*a^9*b^15*c^19*d^5 + 263808*a^10*b^
14*c^2*d^22 - 3975688*a^10*b^14*c^4*d^20 + 20019440*a^10*b^14*c^6*d^18 - 50
137600*a^10*b^14*c^8*d^16 + 69593872*a^10*b^14*c^10*d^14 - 53854288*a^10*b^

14*c^12*d^12 + 21989928*a^10*b^14*c^14*d^10 - 4591360*a^10*b^14*c^16*d^8 + 460480*a^10*b^14*c^18*d^6 + 1720736*a^11*b^13*c^3*d^21 - 11781560*a^11*b^13*c^5*d^19 + 37153600*a^11*b^13*c^7*d^17 - 63124080*a^11*b^13*c^9*d^15 + 59445728*a^11*b^13*c^11*d^13 - 29358696*a^11*b^13*c^13*d^11 + 6995840*a^11*b^13*c^15*d^9 - 762560*a^11*b^13*c^17*d^7 - 541208*a^12*b^12*c^2*d^22 + 5501328*a^12*b^12*c^4*d^20 - 22419600*a^12*b^12*c^6*d^18 + 46972560*a^12*b^12*c^8*d^16 - 53854288*a^12*b^12*c^10*d^14 + 32294808*a^12*b^12*c^12*d^12 - 8958208*a^12*b^12*c^14*d^10 + 999040*a^12*b^12*c^16*d^8 - 2002728*a^13*b^11*c^3*d^21 + 10875200*a^13*b^11*c^5*d^19 - 28461040*a^13*b^11*c^7*d^17 + 39987520*a^13*b^11*c^9*d^15 - 29358696*a^13*b^11*c^11*d^13 + 9722048*a^13*b^11*c^13*d^11 - 1104320*a^13*b^11*c^15*d^9 + 547088*a^14*b^10*c^2*d^22 - 4147952*a^14*b^10*c^4*d^20 + 13887520*a^14*b^10*c^6*d^18 - 24199280*a^14*b^10*c^8*d^16 + 21989928*a^14*b^10*c^10*d^14 - 8958208*a^14*b^10*c^12*d^12 + 1124032*a^14*b^10*c^14*d^10 + 1210560*a^15*b^9*c^3*d^21 - 5365072*a^15*b^9*c^5*d^19 + 11779808*a^15*b^9*c^7*d^17 - 13462088*a^15*b^9*c^9*d^15 + 6995840*a^15*b^9*c^11*d^13 - 1104320*a^15*b^9*c^13*d^11 - 263320*a^16*b^8*c^2*d^22 + 1586920*a^16*b^8*c^4*d^20 - 4506428*a^16*b^8*c^6*d^18 + 6661036*a^16*b^8*c^8*d^16 - 4591360*a^16*b^8*c^10*d^14 + 999040*a^16*b^8*c^12*d^12 - 335040*a^17*b^7*c^3*d^21 + 1310168*a^17*b^7*c^5*d^19 - 2621008*a^17*b^7*c^7*d^17 + 2478528*a^17*b^7*c^9*d^15 - 762560*a^17*b^7*c^11*d^13 + 44120*a^18*b^6*c^2*d^22 - 276020*a^18*b^6*c^4*d^20 + 793756*a^18*b^6*c^6*d^18 - 1058448*a^18*b^6*c^8*d^16 + 460480*a^18*b^6*c^10*d^14 + 37680*a^19*b^5*c^3*d^21 - 170968*a^19*b^5*c^5*d^19 + 336688*a^19*b^5*c^7*d^17 - 212032*a^19*b^5*c^9*d^15 - 1564*a^20*b^4*c^2*d^22 + 21124*a^20*b^4*c^4*d^20 - 72240*a^20*b^4*c^6*d^18 + 72560*a^20*b^4*c^8*d^16 - 288*a^21*b^3*c^3*d^21 + 8160*a^21*b^3*c^5*d^19 - 17920*a^21*b^3*c^7*d^17 - 196*a^22*b^2*c^2*d^22 + 176*a^22*b^2*c^4*d^20 + 3040*a^22*b^2*c^6*d^18 - 8*a*b^23*c^23*d - 8*a^23*b*c*d^23)^2/4 - (20736*b^18*d^18 - 96768*a^2*b^16*d^18 + 173664*a^4*b^14*d^18 - 136032*a^6*b^12*d^18 + 31081*a^8*b^10*d^18 + 8440*a^10*b^8*d^18 + 400*a^12*b^6*d^18 - 96768*b^18*c^2*d^16 + 173664*b^18*c^4*d^14 - 136032*b^18*c^6*d^12 + 31081*b^18*c^8*d^10 + 8440*b^18*c^10*d^8 + 400*b^18*c^12*d^6 - 131328*a*b^17*c^3*d^15 + 216576*a*b^17*c^5*d^13 - 141104*a*b^17*c^7*d^11 + 20260*a*b^17*c^9*d^9 + 2800*a*b^17*c^11*d^7 - 131328*a^3*b^15*c*d^17 + 216576*a^5*b^13*c*d^17 - 141104*a^7*b^11*c*d^17 + 20260*a^9*b^9*c*d^17 + 2800*a^11*b^7*c*d^17 + 495936*a^2*b^16*c^2*d^16 - 989856*a^2*b^16*c^4*d^14 + 901948*a^2*b^16*c^6*d^12 - 308392*a^2*b^16*c^8*d^10 - 5260*a^2*b^16*c^10*d^8 + 1600*a^2*b^16*c^12*d^6 + 657408*a^3*b^15*c^3*d^15 - 1158992*a^3*b^15*c^5*d^13 + 838256*a^3*b^15*c^7*d^11 - 182200*a^3*b^15*c^9*d^9 - 3200*a^3*b^15*c^11*d^7 - 989856*a^4*b^14*c^2*d^16 + 2185654*a^4*b^14*c^4*d^14 - 2218576*a^4*b^14*c^6*d^12 + 900624*a^4*b^14*c^8*d^10 - 64720*a^4*b^14*c^10*d^8 + 1600*a^4*b^14*c^12*d^6 - 1158992*a^5*b^13*c^3*d^15 + 2158808*a^5*b^13*c^5*d^13 - 1641528*a^5*b^13*c^7*d^11 + 406880*a^5*b^13*c^9*d^9 - 17600*a^5*b^13*c^11*d^7 + 901948*a^6*b^12*c^2*d^16 - 2218576*a^6*b^12*c^4*d^14 + 2430936*a^6*b^12*c^6*d^12 - 1026928*a^6*b^12*c^8*d^10 + 88720*a^6*b^12*c^10*d^8 + 838256*a^7*b^11*c^3*d^15 - 1641528*a^7*b^11*c^5*d^13 + 1206848*a^7*b^11*c^7*d^11 - 239360*a^7*b^11*c^9*d^9 - 308392*a^

8*b^10*c^2*d^16 + 900624*a^8*b^10*c^4*d^14 - 1026928*a^8*b^10*c^6*d^12 + 35
4016*a^8*b^10*c^8*d^10 - 182200*a^9*b^9*c^3*d^15 + 406880*a^9*b^9*c^5*d^13
- 239360*a^9*b^9*c^7*d^11 - 5260*a^10*b^8*c^2*d^16 - 64720*a^10*b^8*c^4*d^1
4 + 88720*a^10*b^8*c^6*d^12 - 3200*a^11*b^7*c^3*d^15 - 17600*a^11*b^7*c^5*d
^13 + 1600*a^12*b^6*c^2*d^16 + 1600*a^12*b^6*c^4*d^14 + 27648*a*b^17*c*d^17
)*(80*a^2*b^28*c^30 - 16*b^30*c^30 - 16*a^30*d^30 - 160*a^4*b^26*c^30 + 160
*a^6*b^24*c^30 - 80*a^8*b^22*c^30 + 16*a^10*b^20*c^30 + 16*a^20*b^10*d^30 -
80*a^22*b^8*d^30 + 160*a^24*b^6*d^30 - 160*a^26*b^4*d^30 + 80*a^28*b^2*d^3
0 + 80*a^30*c^2*d^28 - 160*a^30*c^4*d^26 + 160*a^30*c^6*d^24 - 80*a^30*c^8*
d^22 + 16*a^30*c^10*d^20 + 16*b^30*c^20*d^10 - 80*b^30*c^22*d^8 + 160*b^30*
c^24*d^6 - 160*b^30*c^26*d^4 + 80*b^30*c^28*d^2 - 320*a*b^29*c^19*d^11 + 16
00*a*b^29*c^21*d^9 - 3200*a*b^29*c^23*d^7 + 3200*a*b^29*c^25*d^5 - 1600*a*b
^29*c^27*d^3 - 1600*a^3*b^27*c^29*d + 3200*a^5*b^25*c^29*d - 3200*a^7*b^23*
c^29*d + 1600*a^9*b^21*c^29*d - 320*a^11*b^19*c^29*d - 320*a^19*b^11*c*d^29
+ 1600*a^21*b^9*c*d^29 - 3200*a^23*b^7*c*d^29 + 3200*a^25*b^5*c*d^29 - 160
0*a^27*b^3*c*d^29 - 1600*a^29*b*c^3*d^27 + 3200*a^29*b*c^5*d^25 - 3200*a^29
*b*c^7*d^23 + 1600*a^29*b*c^9*d^21 - 320*a^29*b*c^11*d^19 + 3040*a^2*b^28*c
^18*d^12 - 15280*a^2*b^28*c^20*d^10 + 30800*a^2*b^28*c^22*d^8 - 31200*a^2*b
^28*c^24*d^6 + 16000*a^2*b^28*c^26*d^4 - 3440*a^2*b^28*c^28*d^2 - 18240*a^3
*b^27*c^17*d^13 + 92800*a^3*b^27*c^19*d^11 - 190400*a^3*b^27*c^21*d^9 + 198
400*a^3*b^27*c^23*d^7 - 107200*a^3*b^27*c^25*d^5 + 26240*a^3*b^27*c^27*d^3
+ 77520*a^4*b^26*c^16*d^14 - 402800*a^4*b^26*c^18*d^12 + 851360*a^4*b^26*c^
20*d^10 - 928000*a^4*b^26*c^22*d^8 + 541200*a^4*b^26*c^24*d^6 - 155120*a^4*
b^26*c^26*d^4 + 16000*a^4*b^26*c^28*d^2 - 248064*a^5*b^25*c^15*d^15 + 13315
20*a^5*b^25*c^17*d^13 - 2939840*a^5*b^25*c^19*d^11 + 3408640*a^5*b^25*c^21*
d^9 - 2184320*a^5*b^25*c^23*d^7 + 736064*a^5*b^25*c^25*d^5 - 107200*a^5*b^2
5*c^27*d^3 + 620160*a^6*b^24*c^14*d^16 - 3488400*a^6*b^24*c^16*d^14 + 81700
00*a^6*b^24*c^18*d^12 - 10229760*a^6*b^24*c^20*d^10 + 7281600*a^6*b^24*c^22
*d^8 - 2863760*a^6*b^24*c^24*d^6 + 541200*a^6*b^24*c^26*d^4 - 31200*a^6*b^2
4*c^28*d^2 - 1240320*a^7*b^23*c^13*d^17 + 7441920*a^7*b^23*c^15*d^15 - 1878
7200*a^7*b^23*c^17*d^13 + 25721600*a^7*b^23*c^19*d^11 - 20444800*a^7*b^23*c
^21*d^9 + 9297920*a^7*b^23*c^23*d^7 - 2184320*a^7*b^23*c^25*d^5 + 198400*a^
7*b^23*c^27*d^3 + 2015520*a^8*b^22*c^12*d^18 - 13178400*a^8*b^22*c^14*d^16
+ 36434400*a^8*b^22*c^16*d^14 - 55069600*a^8*b^22*c^18*d^12 + 48989680*a^8*
b^22*c^20*d^10 - 25575920*a^8*b^22*c^22*d^8 + 7281600*a^8*b^22*c^24*d^6 - 9
28000*a^8*b^22*c^26*d^4 + 30800*a^8*b^22*c^28*d^2 - 2687360*a^9*b^21*c^11*d
^19 + 19638400*a^9*b^21*c^13*d^17 - 60362240*a^9*b^21*c^15*d^15 + 101475200
*a^9*b^21*c^17*d^13 - 101172800*a^9*b^21*c^19*d^11 + 60333760*a^9*b^21*c^21
*d^9 - 20444800*a^9*b^21*c^23*d^7 + 3408640*a^9*b^21*c^25*d^5 - 190400*a^9*
b^21*c^27*d^3 + 2956096*a^10*b^20*c^10*d^20 - 24858080*a^10*b^20*c^12*d^18
+ 86150560*a^10*b^20*c^14*d^16 - 162120160*a^10*b^20*c^16*d^14 + 181463680*
a^10*b^20*c^18*d^12 - 123188112*a^10*b^20*c^20*d^10 + 48989680*a^10*b^20*c^
22*d^8 - 10229760*a^10*b^20*c^24*d^6 + 851360*a^10*b^20*c^26*d^4 - 15280*a^
10*b^20*c^28*d^2 - 2687360*a^11*b^19*c^9*d^21 + 26873600*a^11*b^19*c^11*d^1
9 - 106460800*a^11*b^19*c^13*d^17 + 225738240*a^11*b^19*c^15*d^15 - 2843312

00*a^11*b^19*c^17*d^13 + 219166080*a^11*b^19*c^19*d^11 - 101172800*a^11*b^1
9*c^21*d^9 + 25721600*a^11*b^19*c^23*d^7 - 2939840*a^11*b^19*c^25*d^5 + 928
00*a^11*b^19*c^27*d^3 + 2015520*a^12*b^18*c^8*d^22 - 24858080*a^12*b^18*c^1
0*d^20 + 114212800*a^12*b^18*c^12*d^18 - 274937600*a^12*b^18*c^14*d^16 + 39
0830000*a^12*b^18*c^16*d^14 - 341426960*a^12*b^18*c^18*d^12 + 181463680*a^1
2*b^18*c^20*d^10 - 55069600*a^12*b^18*c^22*d^8 + 8170000*a^12*b^18*c^24*d^6
 - 402800*a^12*b^18*c^26*d^4 + 3040*a^12*b^18*c^28*d^2 - 1240320*a^13*b^17*
c^7*d^23 + 19638400*a^13*b^17*c^9*d^21 - 106460800*a^13*b^17*c^11*d^19 + 29
3542400*a^13*b^17*c^13*d^17 - 472561920*a^13*b^17*c^15*d^15 + 467412160*a^1
3*b^17*c^17*d^13 - 284331200*a^13*b^17*c^19*d^11 + 101475200*a^13*b^17*c^21
*d^9 - 18787200*a^13*b^17*c^23*d^7 + 1331520*a^13*b^17*c^25*d^5 - 18240*a^1
3*b^17*c^27*d^3 + 620160*a^14*b^16*c^6*d^24 - 13178400*a^14*b^16*c^8*d^22 +
 86150560*a^14*b^16*c^10*d^20 - 274937600*a^14*b^16*c^12*d^18 + 503363200*a
^14*b^16*c^14*d^16 - 563751280*a^14*b^16*c^16*d^14 + 390830000*a^14*b^16*c^
18*d^12 - 162120160*a^14*b^16*c^20*d^10 + 36434400*a^14*b^16*c^22*d^8 - 348
8400*a^14*b^16*c^24*d^6 + 77520*a^14*b^16*c^26*d^4 - 248064*a^15*b^15*c^5*d
^25 + 7441920*a^15*b^15*c^7*d^23 - 60362240*a^15*b^15*c^9*d^21 + 225738240*
a^15*b^15*c^11*d^19 - 472561920*a^15*b^15*c^13*d^17 + 599984128*a^15*b^15*c
^15*d^15 - 472561920*a^15*b^15*c^17*d^13 + 225738240*a^15*b^15*c^19*d^11 -
60362240*a^15*b^15*c^21*d^9 + 7441920*a^15*b^15*c^23*d^7 - 248064*a^15*b^15
*c^25*d^5 + 77520*a^16*b^14*c^4*d^26 - 3488400*a^16*b^14*c^6*d^24 + 3643440
0*a^16*b^14*c^8*d^22 - 162120160*a^16*b^14*c^10*d^20 + 390830000*a^16*b^14*
c^12*d^18 - 563751280*a^16*b^14*c^14*d^16 + 503363200*a^16*b^14*c^16*d^14 -
 274937600*a^16*b^14*c^18*d^12 + 86150560*a^16*b^14*c^20*d^10 - 13178400*a^
16*b^14*c^22*d^8 + 620160*a^16*b^14*c^24*d^6 - 18240*a^17*b^13*c^3*d^27 + 1
331520*a^17*b^13*c^5*d^25 - 18787200*a^17*b^13*c^7*d^23 + 101475200*a^17*b^
13*c^9*d^21 - 284331200*a^17*b^13*c^11*d^19 + 467412160*a^17*b^13*c^13*d^17
 - 472561920*a^17*b^13*c^15*d^15 + 293542400*a^17*b^13*c^17*d^13 - 10646080
0*a^17*b^13*c^19*d^11 + 19638400*a^17*b^13*c^21*d^9 - 1240320*a^17*b^13*c^2
3*d^7 + 3040*a^18*b^12*c^2*d^28 - 402800*a^18*b^12*c^4*d^26 + 8170000*a^18*
b^12*c^6*d^24 - 55069600*a^18*b^12*c^8*d^22 + 181463680*a^18*b^12*c^10*d^20
 - 341426960*a^18*b^12*c^12*d^18 + 390830000*a^18*b^12*c^14*d^16 - 27493760
0*a^18*b^12*c^16*d^14 + 114212800*a^18*b^12*c^18*d^12 - 24858080*a^18*b^12*
c^20*d^10 + 2015520*a^18*b^12*c^22*d^8 + 92800*a^19*b^11*c^3*d^27 - 2939840
*a^19*b^11*c^5*d^25 + 25721600*a^19*b^11*c^7*d^23 - 101172800*a^19*b^11*c^9
*d^21 + 219166080*a^19*b^11*c^11*d^19 - 284331200*a^19*b^11*c^13*d^17 + 225
738240*a^19*b^11*c^15*d^15 - 106460800*a^19*b^11*c^17*d^13 + 26873600*a^19*
b^11*c^19*d^11 - 2687360*a^19*b^11*c^21*d^9 - 15280*a^20*b^10*c^2*d^28 + 85
1360*a^20*b^10*c^4*d^26 - 10229760*a^20*b^10*c^6*d^24 + 48989680*a^20*b^10*
c^8*d^22 - 123188112*a^20*b^10*c^10*d^20 + 181463680*a^20*b^10*c^12*d^18 -
162120160*a^20*b^10*c^14*d^16 + 86150560*a^20*b^10*c^16*d^14 - 24858080*a^2
0*b^10*c^18*d^12 + 2956096*a^20*b^10*c^20*d^10 - 190400*a^21*b^9*c^3*d^27 +
 3408640*a^21*b^9*c^5*d^25 - 20444800*a^21*b^9*c^7*d^23 + 60333760*a^21*b^9
*c^9*d^21 - 101172800*a^21*b^9*c^11*d^19 + 101475200*a^21*b^9*c^13*d^17 - 6
0362240*a^21*b^9*c^15*d^15 + 19638400*a^21*b^9*c^17*d^13 - 2687360*a^21*b^9

*c^19*d^11 + 30800*a^22*b^8*c^2*d^28 - 928000*a^22*b^8*c^4*d^26 + 7281600*a^22*b^8*c^6*d^24 - 25575920*a^22*b^8*c^8*d^22 + 48989680*a^22*b^8*c^10*d^20 - 55069600*a^22*b^8*c^12*d^18 + 36434400*a^22*b^8*c^14*d^16 - 13178400*a^22*b^8*c^16*d^14 + 2015520*a^22*b^8*c^18*d^12 + 198400*a^23*b^7*c^3*d^27 - 2184320*a^23*b^7*c^5*d^25 + 9297920*a^23*b^7*c^7*d^23 - 20444800*a^23*b^7*c^9*d^21 + 25721600*a^23*b^7*c^11*d^19 - 18787200*a^23*b^7*c^13*d^17 + 7441920*a^23*b^7*c^15*d^15 - 1240320*a^23*b^7*c^17*d^13 - 31200*a^24*b^6*c^2*d^28 + 541200*a^24*b^6*c^4*d^26 - 2863760*a^24*b^6*c^6*d^24 + 7281600*a^24*b^6*c^8*d^22 - 10229760*a^24*b^6*c^10*d^20 + 8170000*a^24*b^6*c^12*d^18 - 3488400*a^24*b^6*c^14*d^16 + 620160*a^24*b^6*c^16*d^14 - 107200*a^25*b^5*c^3*d^27 + 736064*a^25*b^5*c^5*d^25 - 2184320*a^25*b^5*c^7*d^23 + 3408640*a^25*b^5*c^9*d^21 - 2939840*a^25*b^5*c^11*d^19 + 1331520*a^25*b^5*c^13*d^17 - 248064*a^25*b^5*c^15*d^15 + 16000*a^26*b^4*c^2*d^28 - 155120*a^26*b^4*c^4*d^26 + 541200*a^26*b^4*c^6*d^24 - 928000*a^26*b^4*c^8*d^22 + 851360*a^26*b^4*c^10*d^20 - 402800*a^26*b^4*c^12*d^18 + 77520*a^26*b^4*c^14*d^16 + 26240*a^27*b^3*c^3*d^27 - 107200*a^27*b^3*c^5*d^25 + 198400*a^27*b^3*c^7*d^23 - 190400*a^27*b^3*c^9*d^21 + 92800*a^27*b^3*c^11*d^19 - 18240*a^27*b^3*c^13*d^17 - 3440*a^28*b^2*c^2*d^28 + 16000*a^28*b^2*c^4*d^26 - 31200*a^28*b^2*c^6*d^24 + 30800*a^28*b^2*c^8*d^22 - 15280*a^28*b^2*c^10*d^20 + 3040*a^28*b^2*c^12*d^18 + 320*a*b^29*c^29*d + 320*a^29*b*c*d^29))^(1/2) + 2*a^24*d^24 + 2*b^24*c^24 + 8*a^2*b^22*c^24 + 8*a^4*b^20*c^24 - 576*a^10*b^14*d^24 + 2784*a^12*b^12*d^24 - 5284*a^14*b^10*d^24 + 4730*a^16*b^8*d^24 - 1780*a^18*b^6*d^24 + 68*a^20*b^4*d^24 + 38*a^22*b^2*d^24 + 8*a^24*c^2*d^22 + 8*a^24*c^4*d^20 - 576*b^24*c^10*d^14 + 2784*b^24*c^12*d^12 - 5284*b^24*c^14*d^10 + 4730*b^24*c^16*d^8 - 1780*b^24*c^18*d^6 + 68*b^24*c^20*d^4 + 38*b^24*c^22*d^2 + 5760*a*b^23*c^9*d^15 - 28224*a*b^23*c^11*d^13 + 54728*a*b^23*c^13*d^11 - 50620*a*b^23*c^15*d^9 + 20360*a*b^23*c^17*d^7 - 1480*a*b^23*c^19*d^5 - 268*a*b^23*c^21*d^3 - 88*a^3*b^21*c^23*d - 160*a^5*b^19*c^23*d + 5760*a^9*b^15*c*d^23 - 28224*a^11*b^13*c*d^23 + 54728*a^13*b^11*c*d^23 - 50620*a^15*b^9*c*d^23 + 20360*a^17*b^7*c*d^23 - 1480*a^19*b^5*c*d^23 - 268*a^21*b^3*c*d^23 - 88*a^23*b*c^3*d^21 - 160*a^23*b*c^5*d^19 - 25920*a^2*b^22*c^8*d^16 + 131904*a^2*b^22*c^10*d^14 - 270604*a^2*b^22*c^12*d^12 + 273544*a^2*b^22*c^14*d^10 - 131660*a^2*b^22*c^16*d^8 + 22060*a^2*b^22*c^18*d^6 - 782*a^2*b^22*c^20*d^4 - 98*a^2*b^22*c^22*d^2 + 69120*a^3*b^21*c^7*d^17 - 379200*a^3*b^21*c^9*d^15 + 860368*a^3*b^21*c^11*d^13 - 1001364*a^3*b^21*c^13*d^11 + 605280*a^3*b^21*c^15*d^9 - 167520*a^3*b^21*c^17*d^7 + 18840*a^3*b^21*c^19*d^5 - 144*a^3*b^21*c^21*d^3 - 120960*a^4*b^20*c^6*d^18 + 756000*a^4*b^20*c^8*d^16 - 1987844*a^4*b^20*c^10*d^14 + 2750664*a^4*b^20*c^12*d^12 - 2073976*a^4*b^20*c^14*d^10 + 793460*a^4*b^20*c^16*d^8 - 138010*a^4*b^20*c^18*d^6 + 10562*a^4*b^20*c^20*d^4 + 88*a^4*b^20*c^22*d^2 + 145152*a^5*b^19*c^5*d^19 - 1116288*a^5*b^19*c^7*d^17 + 3539128*a^5*b^19*c^9*d^15 - 5890780*a^5*b^19*c^11*d^13 + 5437600*a^5*b^19*c^13*d^11 - 2682536*a^5*b^19*c^15*d^9 + 655084*a^5*b^19*c^17*d^7 - 85484*a^5*b^19*c^19*d^5 + 4080*a^5*b^19*c^21*d^3 - 120960*a^6*b^18*c^4*d^20 + 1266048*a^6*b^18*c^6*d^18 - 4977996*a^6*b^18*c^8*d^16 + 10009720*a^6*b^18*c^10*d^14 - 11209800*a^6*b^18*c^12*d^12 + 6943760*a^6*b^18*c^14*d^10 - 22

53214*a^6*b^18*c^16*d^8 + 396878*a^6*b^18*c^18*d^6 - 36120*a^6*b^18*c^20*d^
4 + 1520*a^6*b^18*c^22*d^2 + 69120*a^7*b^17*c^3*d^21 - 1116288*a^7*b^17*c^5
*d^19 + 5575008*a^7*b^17*c^7*d^17 - 13668308*a^7*b^17*c^9*d^15 + 18576800*a
^7*b^17*c^11*d^13 - 14230520*a^7*b^17*c^13*d^11 + 5889904*a^7*b^17*c^15*d^9
 - 1310504*a^7*b^17*c^17*d^7 + 168344*a^7*b^17*c^19*d^5 - 8960*a^7*b^17*c^2
1*d^3 - 25920*a^8*b^16*c^2*d^22 + 756000*a^8*b^16*c^4*d^20 - 4977996*a^8*b^
16*c^6*d^18 + 15144828*a^8*b^16*c^8*d^16 - 25068800*a^8*b^16*c^10*d^14 + 23
486280*a^8*b^16*c^12*d^12 - 12099640*a^8*b^16*c^14*d^10 + 3330518*a^8*b^16*
c^16*d^8 - 529224*a^8*b^16*c^18*d^6 + 36280*a^8*b^16*c^20*d^4 - 379200*a^9*
b^15*c^3*d^21 + 3539128*a^9*b^15*c^5*d^19 - 13668308*a^9*b^15*c^7*d^17 + 27
691952*a^9*b^15*c^9*d^15 - 31562040*a^9*b^15*c^11*d^13 + 19993760*a^9*b^15*
c^13*d^11 - 6731044*a^9*b^15*c^15*d^9 + 1239264*a^9*b^15*c^17*d^7 - 106016*
a^9*b^15*c^19*d^5 + 131904*a^10*b^14*c^2*d^22 - 1987844*a^10*b^14*c^4*d^20
 + 10009720*a^10*b^14*c^6*d^18 - 25068800*a^10*b^14*c^8*d^16 + 34796936*a^10
*b^14*c^10*d^14 - 26927144*a^10*b^14*c^12*d^12 + 10994964*a^10*b^14*c^14*d^
10 - 2295680*a^10*b^14*c^16*d^8 + 230240*a^10*b^14*c^18*d^6 + 860368*a^11*b
^13*c^3*d^21 - 5890780*a^11*b^13*c^5*d^19 + 18576800*a^11*b^13*c^7*d^17 - 3
1562040*a^11*b^13*c^9*d^15 + 29722864*a^11*b^13*c^11*d^13 - 14679348*a^11*b
^13*c^13*d^11 + 3497920*a^11*b^13*c^15*d^9 - 381280*a^11*b^13*c^17*d^7 - 27
0604*a^12*b^12*c^2*d^22 + 2750664*a^12*b^12*c^4*d^20 - 11209800*a^12*b^12*c
^6*d^18 + 23486280*a^12*b^12*c^8*d^16 - 26927144*a^12*b^12*c^10*d^14 + 1614
7404*a^12*b^12*c^12*d^12 - 4479104*a^12*b^12*c^14*d^10 + 499520*a^12*b^12*c
^16*d^8 - 1001364*a^13*b^11*c^3*d^21 + 5437600*a^13*b^11*c^5*d^19 - 1423052
0*a^13*b^11*c^7*d^17 + 19993760*a^13*b^11*c^9*d^15 - 14679348*a^13*b^11*c^1
1*d^13 + 4861024*a^13*b^11*c^13*d^11 - 552160*a^13*b^11*c^15*d^9 + 273544*a
^14*b^10*c^2*d^22 - 2073976*a^14*b^10*c^4*d^20 + 6943760*a^14*b^10*c^6*d^18
 - 12099640*a^14*b^10*c^8*d^16 + 10994964*a^14*b^10*c^10*d^14 - 4479104*a^1
4*b^10*c^12*d^12 + 562016*a^14*b^10*c^14*d^10 + 605280*a^15*b^9*c^3*d^21 -
2682536*a^15*b^9*c^5*d^19 + 5889904*a^15*b^9*c^7*d^17 - 6731044*a^15*b^9*c^
9*d^15 + 3497920*a^15*b^9*c^11*d^13 - 552160*a^15*b^9*c^13*d^11 - 131660*a^
16*b^8*c^2*d^22 + 793460*a^16*b^8*c^4*d^20 - 2253214*a^16*b^8*c^6*d^18 + 33
30518*a^16*b^8*c^8*d^16 - 2295680*a^16*b^8*c^10*d^14 + 499520*a^16*b^8*c^12
*d^12 - 167520*a^17*b^7*c^3*d^21 + 655084*a^17*b^7*c^5*d^19 - 1310504*a^17*
b^7*c^7*d^17 + 1239264*a^17*b^7*c^9*d^15 - 381280*a^17*b^7*c^11*d^13 + 2206
0*a^18*b^6*c^2*d^22 - 138010*a^18*b^6*c^4*d^20 + 396878*a^18*b^6*c^6*d^18 -
 529224*a^18*b^6*c^8*d^16 + 230240*a^18*b^6*c^10*d^14 + 18840*a^19*b^5*c^3*
d^21 - 85484*a^19*b^5*c^5*d^19 + 168344*a^19*b^5*c^7*d^17 - 106016*a^19*b^5
*c^9*d^15 - 782*a^20*b^4*c^2*d^22 + 10562*a^20*b^4*c^4*d^20 - 36120*a^20*b^
4*c^6*d^18 + 36280*a^20*b^4*c^8*d^16 - 144*a^21*b^3*c^3*d^21 + 4080*a^21*b^
3*c^5*d^19 - 8960*a^21*b^3*c^7*d^17 - 98*a^22*b^2*c^2*d^22 + 88*a^22*b^2*c^
4*d^20 + 1520*a^22*b^2*c^6*d^18 - 4*a*b^23*c^23*d - 4*a^23*b*c*d^23)/(16*(5
*a^2*b^28*c^30 - b^30*c^30 - a^30*d^30 - 10*a^4*b^26*c^30 + 10*a^6*b^24*c^3
0 - 5*a^8*b^22*c^30 + a^10*b^20*c^30 + a^20*b^10*d^30 - 5*a^22*b^8*d^30 + 1
0*a^24*b^6*d^30 - 10*a^26*b^4*d^30 + 5*a^28*b^2*d^30 + 5*a^30*c^2*d^28 - 10
*a^30*c^4*d^26 + 10*a^30*c^6*d^24 - 5*a^30*c^8*d^22 + a^30*c^10*d^20 + b^30

*c^20*d^10 - 5*b^30*c^22*d^8 + 10*b^30*c^24*d^6 - 10*b^30*c^26*d^4 + 5*b^30
*c^28*d^2 - 20*a*b^29*c^19*d^11 + 100*a*b^29*c^21*d^9 - 200*a*b^29*c^23*d^7
 + 200*a*b^29*c^25*d^5 - 100*a*b^29*c^27*d^3 - 100*a^3*b^27*c^29*d + 200*a^
5*b^25*c^29*d - 200*a^7*b^23*c^29*d + 100*a^9*b^21*c^29*d - 20*a^11*b^19*c^
29*d - 20*a^19*b^11*c*d^29 + 100*a^21*b^9*c*d^29 - 200*a^23*b^7*c*d^29 + 20
0*a^25*b^5*c*d^29 - 100*a^27*b^3*c*d^29 - 100*a^29*b*c^3*d^27 + 200*a^29*b*
c^5*d^25 - 200*a^29*b*c^7*d^23 + 100*a^29*b*c^9*d^21 - 20*a^29*b*c^11*d^19
+ 190*a^2*b^28*c^18*d^12 - 955*a^2*b^28*c^20*d^10 + 1925*a^2*b^28*c^22*d^8
- 1950*a^2*b^28*c^24*d^6 + 1000*a^2*b^28*c^26*d^4 - 215*a^2*b^28*c^28*d^2 -
 1140*a^3*b^27*c^17*d^13 + 5800*a^3*b^27*c^19*d^11 - 11900*a^3*b^27*c^21*d^
9 + 12400*a^3*b^27*c^23*d^7 - 6700*a^3*b^27*c^25*d^5 + 1640*a^3*b^27*c^27*d
^3 + 4845*a^4*b^26*c^16*d^14 - 25175*a^4*b^26*c^18*d^12 + 53210*a^4*b^26*c^
20*d^10 - 58000*a^4*b^26*c^22*d^8 + 33825*a^4*b^26*c^24*d^6 - 9695*a^4*b^26
*c^26*d^4 + 1000*a^4*b^26*c^28*d^2 - 15504*a^5*b^25*c^15*d^15 + 83220*a^5*b
^25*c^17*d^13 - 183740*a^5*b^25*c^19*d^11 + 213040*a^5*b^25*c^21*d^9 - 1365
20*a^5*b^25*c^23*d^7 + 46004*a^5*b^25*c^25*d^5 - 6700*a^5*b^25*c^27*d^3 + 3
8760*a^6*b^24*c^14*d^16 - 218025*a^6*b^24*c^16*d^14 + 510625*a^6*b^24*c^18*
d^12 - 639360*a^6*b^24*c^20*d^10 + 455100*a^6*b^24*c^22*d^8 - 178985*a^6*b^
24*c^24*d^6 + 33825*a^6*b^24*c^26*d^4 - 1950*a^6*b^24*c^28*d^2 - 77520*a^7*
b^23*c^13*d^17 + 465120*a^7*b^23*c^15*d^15 - 1174200*a^7*b^23*c^17*d^13 + 1
607600*a^7*b^23*c^19*d^11 - 1277800*a^7*b^23*c^21*d^9 + 581120*a^7*b^23*c^2
3*d^7 - 136520*a^7*b^23*c^25*d^5 + 12400*a^7*b^23*c^27*d^3 + 125970*a^8*b^2
2*c^12*d^18 - 823650*a^8*b^22*c^14*d^16 + 2277150*a^8*b^22*c^16*d^14 - 3441
850*a^8*b^22*c^18*d^12 + 3061855*a^8*b^22*c^20*d^10 - 1598495*a^8*b^22*c^22
*d^8 + 455100*a^8*b^22*c^24*d^6 - 58000*a^8*b^22*c^26*d^4 + 1925*a^8*b^22*c
^28*d^2 - 167960*a^9*b^21*c^11*d^19 + 1227400*a^9*b^21*c^13*d^17 - 3772640*
a^9*b^21*c^15*d^15 + 6342200*a^9*b^21*c^17*d^13 - 6323300*a^9*b^21*c^19*d^1
1 + 3770860*a^9*b^21*c^21*d^9 - 1277800*a^9*b^21*c^23*d^7 + 213040*a^9*b^21
*c^25*d^5 - 11900*a^9*b^21*c^27*d^3 + 184756*a^10*b^20*c^10*d^20 - 1553630*
a^10*b^20*c^12*d^18 + 5384410*a^10*b^20*c^14*d^16 - 10132510*a^10*b^20*c^16
*d^14 + 11341480*a^10*b^20*c^18*d^12 - 7699257*a^10*b^20*c^20*d^10 + 306185
5*a^10*b^20*c^22*d^8 - 639360*a^10*b^20*c^24*d^6 + 53210*a^10*b^20*c^26*d^4
 - 955*a^10*b^20*c^28*d^2 - 167960*a^11*b^19*c^9*d^21 + 1679600*a^11*b^19*c
^11*d^19 - 6653800*a^11*b^19*c^13*d^17 + 14108640*a^11*b^19*c^15*d^15 - 177
70700*a^11*b^19*c^17*d^13 + 13697880*a^11*b^19*c^19*d^11 - 6323300*a^11*b^1
9*c^21*d^9 + 1607600*a^11*b^19*c^23*d^7 - 183740*a^11*b^19*c^25*d^5 + 5800*
a^11*b^19*c^27*d^3 + 125970*a^12*b^18*c^8*d^22 - 1553630*a^12*b^18*c^10*d^2
0 + 7138300*a^12*b^18*c^12*d^18 - 17183600*a^12*b^18*c^14*d^16 + 24426875*a
^12*b^18*c^16*d^14 - 21339185*a^12*b^18*c^18*d^12 + 11341480*a^12*b^18*c^20
*d^10 - 3441850*a^12*b^18*c^22*d^8 + 510625*a^12*b^18*c^24*d^6 - 25175*a^12
*b^18*c^26*d^4 + 190*a^12*b^18*c^28*d^2 - 77520*a^13*b^17*c^7*d^23 + 122740
0*a^13*b^17*c^9*d^21 - 6653800*a^13*b^17*c^11*d^19 + 18346400*a^13*b^17*c^1
3*d^17 - 29535120*a^13*b^17*c^15*d^15 + 29213260*a^13*b^17*c^17*d^13 - 1777
0700*a^13*b^17*c^19*d^11 + 6342200*a^13*b^17*c^21*d^9 - 1174200*a^13*b^17*c
^23*d^7 + 83220*a^13*b^17*c^25*d^5 - 1140*a^13*b^17*c^27*d^3 + 38760*a^14*b

^16*c^6*d^24 - 823650*a^14*b^16*c^8*d^22 + 5384410*a^14*b^16*c^10*d^20 - 17
183600*a^14*b^16*c^12*d^18 + 31460200*a^14*b^16*c^14*d^16 - 35234455*a^14*b
^16*c^16*d^14 + 24426875*a^14*b^16*c^18*d^12 - 10132510*a^14*b^16*c^20*d^10
 + 2277150*a^14*b^16*c^22*d^8 - 218025*a^14*b^16*c^24*d^6 + 4845*a^14*b^16*
c^26*d^4 - 15504*a^15*b^15*c^5*d^25 + 465120*a^15*b^15*c^7*d^23 - 3772640*a
^15*b^15*c^9*d^21 + 14108640*a^15*b^15*c^11*d^19 - 29535120*a^15*b^15*c^13*
d^17 + 37499008*a^15*b^15*c^15*d^15 - 29535120*a^15*b^15*c^17*d^13 + 141086
40*a^15*b^15*c^19*d^11 - 3772640*a^15*b^15*c^21*d^9 + 465120*a^15*b^15*c^23
*d^7 - 15504*a^15*b^15*c^25*d^5 + 4845*a^16*b^14*c^4*d^26 - 218025*a^16*b^1
4*c^6*d^24 + 2277150*a^16*b^14*c^8*d^22 - 10132510*a^16*b^14*c^10*d^20 + 24
426875*a^16*b^14*c^12*d^18 - 35234455*a^16*b^14*c^14*d^16 + 31460200*a^16*b
^14*c^16*d^14 - 17183600*a^16*b^14*c^18*d^12 + 5384410*a^16*b^14*c^20*d^10
 - 823650*a^16*b^14*c^22*d^8 + 38760*a^16*b^14*c^24*d^6 - 1140*a^17*b^13*c^3
*d^27 + 83220*a^17*b^13*c^5*d^25 - 1174200*a^17*b^13*c^7*d^23 + 6342200*a^1
7*b^13*c^9*d^21 - 17770700*a^17*b^13*c^11*d^19 + 29213260*a^17*b^13*c^13*d^
17 - 29535120*a^17*b^13*c^15*d^15 + 18346400*a^17*b^13*c^17*d^13 - 6653800*
a^17*b^13*c^19*d^11 + 1227400*a^17*b^13*c^21*d^9 - 77520*a^17*b^13*c^23*d^7
 + 190*a^18*b^12*c^2*d^28 - 25175*a^18*b^12*c^4*d^26 + 510625*a^18*b^12*c^6
*d^24 - 3441850*a^18*b^12*c^8*d^22 + 11341480*a^18*b^12*c^10*d^20 - 2133918
5*a^18*b^12*c^12*d^18 + 24426875*a^18*b^12*c^14*d^16 - 17183600*a^18*b^12*c
^16*d^14 + 7138300*a^18*b^12*c^18*d^12 - 1553630*a^18*b^12*c^20*d^10 + 1259
70*a^18*b^12*c^22*d^8 + 5800*a^19*b^11*c^3*d^27 - 183740*a^19*b^11*c^5*d^25
 + 1607600*a^19*b^11*c^7*d^23 - 6323300*a^19*b^11*c^9*d^21 + 13697880*a^19*
b^11*c^11*d^19 - 17770700*a^19*b^11*c^13*d^17 + 14108640*a^19*b^11*c^15*d^1
5 - 6653800*a^19*b^11*c^17*d^13 + 1679600*a^19*b^11*c^19*d^11 - 167960*a^19
*b^11*c^21*d^9 - 955*a^20*b^10*c^2*d^28 + 53210*a^20*b^10*c^4*d^26 - 639360
*a^20*b^10*c^6*d^24 + 3061855*a^20*b^10*c^8*d^22 - 7699257*a^20*b^10*c^10*d
^20 + 11341480*a^20*b^10*c^12*d^18 - 10132510*a^20*b^10*c^14*d^16 + 5384410
*a^20*b^10*c^16*d^14 - 1553630*a^20*b^10*c^18*d^12 + 184756*a^20*b^10*c^20*
d^10 - 11900*a^21*b^9*c^3*d^27 + 213040*a^21*b^9*c^5*d^25 - 1277800*a^21*b^
9*c^7*d^23 + 3770860*a^21*b^9*c^9*d^21 - 6323300*a^21*b^9*c^11*d^19 + 63422
00*a^21*b^9*c^13*d^17 - 3772640*a^21*b^9*c^15*d^15 + 1227400*a^21*b^9*c^17*
d^13 - 167960*a^21*b^9*c^19*d^11 + 1925*a^22*b^8*c^2*d^28 - 58000*a^22*b^8*
c^4*d^26 + 455100*a^22*b^8*c^6*d^24 - 1598495*a^22*b^8*c^8*d^22 + 3061855*a
^22*b^8*c^10*d^20 - 3441850*a^22*b^8*c^12*d^18 + 2277150*a^22*b^8*c^14*d^16
 - 823650*a^22*b^8*c^16*d^14 + 125970*a^22*b^8*c^18*d^12 + 12400*a^23*b^7*c
^3*d^27 - 136520*a^23*b^7*c^5*d^25 + 581120*a^23*b^7*c^7*d^23 - 1277800*a^2
3*b^7*c^9*d^21 + 1607600*a^23*b^7*c^11*d^19 - 1174200*a^23*b^7*c^13*d^17 +
465120*a^23*b^7*c^15*d^15 - 77520*a^23*b^7*c^17*d^13 - 1950*a^24*b^6*c^2*d^
28 + 33825*a^24*b^6*c^4*d^26 - 178985*a^24*b^6*c^6*d^24 + 455100*a^24*b^6*c
^8*d^22 - 639360*a^24*b^6*c^10*d^20 + 510625*a^24*b^6*c^12*d^18 - 218025*a^
24*b^6*c^14*d^16 + 38760*a^24*b^6*c^16*d^14 - 6700*a^25*b^5*c^3*d^27 + 4600
4*a^25*b^5*c^5*d^25 - 136520*a^25*b^5*c^7*d^23 + 213040*a^25*b^5*c^9*d^21 -
 183740*a^25*b^5*c^11*d^19 + 83220*a^25*b^5*c^13*d^17 - 15504*a^25*b^5*c^15
*d^15 + 1000*a^26*b^4*c^2*d^28 - 9695*a^26*b^4*c^4*d^26 + 33825*a^26*b^4*c^

6*d^24 - 58000*a^26*b^4*c^8*d^22 + 53210*a^26*b^4*c^10*d^20 - 25175*a^26*b^4*c^12*d^18 + 4845*a^26*b^4*c^14*d^16 + 1640*a^27*b^3*c^3*d^27 - 6700*a^27*b^3*c^5*d^25 + 12400*a^27*b^3*c^7*d^23 - 11900*a^27*b^3*c^9*d^21 + 5800*a^27*b^3*c^11*d^19 - 1140*a^27*b^3*c^13*d^17 - 215*a^28*b^2*c^2*d^28 + 1000*a^28*b^2*c^4*d^26 - 1950*a^28*b^2*c^6*d^24 + 1925*a^28*b^2*c^8*d^22 - 955*a^28*b^2*c^10*d^20 + 190*a^28*b^2*c^12*d^18 + 20*a*b^29*c^29*d + 20*a^29*b*c*d^29)))^(1/2)*((-(((4*a^24*d^24 + 4*b^24*c^24 + 16*a^2*b^22*c^24 + 16*a^4*b^20*c^24 - 1152*a^10*b^14*d^24 + 5568*a^12*b^12*d^24 - 10568*a^14*b^10*d^24 + 9460*a^16*b^8*d^24 - 3560*a^18*b^6*d^24 + 136*a^20*b^4*d^24 + 76*a^22*b^2*d^24 + 16*a^24*c^2*d^22 + 16*a^24*c^4*d^20 - 1152*b^24*c^10*d^14 + 5568*b^24*c^12*d^12 - 10568*b^24*c^14*d^10 + 9460*b^24*c^16*d^8 - 3560*b^24*c^18*d^6 + 136*b^24*c^20*d^4 + 76*b^24*c^22*d^2 + 11520*a*b^23*c^9*d^15 - 56448*a*b^23*c^11*d^13 + 109456*a*b^23*c^13*d^11 - 101240*a*b^23*c^15*d^9 + 40720*a*b^23*c^17*d^7 - 2960*a*b^23*c^19*d^5 - 536*a*b^23*c^21*d^3 - 176*a^3*b^21*c^23*d - 320*a^5*b^19*c^23*d + 11520*a^9*b^15*c*d^23 - 56448*a^11*b^13*c*d^23 + 109456*a^13*b^11*c*d^23 - 101240*a^15*b^9*c*d^23 + 40720*a^17*b^7*c*d^23 - 2960*a^19*b^5*c*d^23 - 536*a^21*b^3*c*d^23 - 176*a^23*b*c^3*d^21 - 320*a^23*b*c^5*d^19 - 51840*a^2*b^22*c^8*d^16 + 263808*a^2*b^22*c^10*d^14 - 541208*a^2*b^22*c^12*d^12 + 547088*a^2*b^22*c^14*d^10 - 263320*a^2*b^22*c^16*d^8 + 44120*a^2*b^22*c^18*d^6 - 1564*a^2*b^22*c^20*d^4 - 196*a^2*b^22*c^22*d^2 + 138240*a^3*b^21*c^7*d^17 - 758400*a^3*b^21*c^9*d^15 + 1720736*a^3*b^21*c^11*d^13 - 2002728*a^3*b^21*c^13*d^11 + 1210560*a^3*b^21*c^15*d^9 - 335040*a^3*b^21*c^17*d^7 + 37680*a^3*b^21*c^19*d^5 - 288*a^3*b^21*c^21*d^3 - 241920*a^4*b^20*c^6*d^18 + 1512000*a^4*b^20*c^8*d^16 - 3975688*a^4*b^20*c^10*d^14 + 5501328*a^4*b^20*c^12*d^12 - 4147952*a^4*b^20*c^14*d^10 + 1586920*a^4*b^20*c^16*d^8 - 276020*a^4*b^20*c^18*d^6 + 21124*a^4*b^20*c^20*d^4 + 176*a^4*b^20*c^22*d^2 + 290304*a^5*b^19*c^5*d^19 - 2232576*a^5*b^19*c^7*d^17 + 7078256*a^5*b^19*c^9*d^15 - 11781560*a^5*b^19*c^11*d^13 + 10875200*a^5*b^19*c^13*d^11 - 5365072*a^5*b^19*c^15*d^9 + 1310168*a^5*b^19*c^17*d^7 - 170968*a^5*b^19*c^19*d^5 + 8160*a^5*b^19*c^21*d^3 - 241920*a^6*b^18*c^4*d^20 + 2532096*a^6*b^18*c^6*d^18 - 9955992*a^6*b^18*c^8*d^16 + 20019440*a^6*b^18*c^10*d^14 - 22419600*a^6*b^18*c^12*d^12 + 13887520*a^6*b^18*c^14*d^10 - 4506428*a^6*b^18*c^16*d^8 + 793756*a^6*b^18*c^18*d^6 - 72240*a^6*b^18*c^20*d^4 + 3040*a^6*b^18*c^22*d^2 + 138240*a^7*b^17*c^3*d^21 - 2232576*a^7*b^17*c^5*d^19 + 11150016*a^7*b^17*c^7*d^17 - 27336616*a^7*b^17*c^9*d^15 + 37153600*a^7*b^17*c^11*d^13 - 28461040*a^7*b^17*c^13*d^11 + 11779808*a^7*b^17*c^15*d^9 - 2621008*a^7*b^17*c^17*d^7 + 336688*a^7*b^17*c^19*d^5 - 17920*a^7*b^17*c^21*d^3 - 51840*a^8*b^16*c^2*d^22 + 1512000*a^8*b^16*c^4*d^20 - 9955992*a^8*b^16*c^6*d^18 + 30289656*a^8*b^16*c^8*d^16 - 50137600*a^8*b^16*c^10*d^14 + 46972560*a^8*b^16*c^12*d^12 - 24199280*a^8*b^16*c^14*d^10 + 6661036*a^8*b^16*c^16*d^8 - 1058448*a^8*b^16*c^18*d^6 + 72560*a^8*b^16*c^20*d^4 - 758400*a^9*b^15*c^3*d^21 + 7078256*a^9*b^15*c^5*d^19 - 27336616*a^9*b^15*c^7*d^17 + 55383904*a^9*b^15*c^9*d^15 - 63124080*a^9*b^15*c^11*d^13 + 39987520*a^9*b^15*c^13*d^11 - 13462088*a^9*b^15*c^15*d^9 + 2478528*a^9*b^15*c^17*d^7 - 212032*a^9*b^15*c^19*d^5 + 263808*a^10*b^14*c^2*d^22 - 3975688*a^10*b^14*c^4*d

^20 + 20019440*a^10*b^14*c^6*d^18 - 50137600*a^10*b^14*c^8*d^16 + 69593872*
a^10*b^14*c^10*d^14 - 53854288*a^10*b^14*c^12*d^12 + 21989928*a^10*b^14*c^1
4*d^10 - 4591360*a^10*b^14*c^16*d^8 + 460480*a^10*b^14*c^18*d^6 + 1720736*a
^11*b^13*c^3*d^21 - 11781560*a^11*b^13*c^5*d^19 + 37153600*a^11*b^13*c^7*d^
17 - 63124080*a^11*b^13*c^9*d^15 + 59445728*a^11*b^13*c^11*d^13 - 29358696*
a^11*b^13*c^13*d^11 + 6995840*a^11*b^13*c^15*d^9 - 762560*a^11*b^13*c^17*d^
7 - 541208*a^12*b^12*c^2*d^22 + 5501328*a^12*b^12*c^4*d^20 - 22419600*a^12*
b^12*c^6*d^18 + 46972560*a^12*b^12*c^8*d^16 - 53854288*a^12*b^12*c^10*d^14
+ 32294808*a^12*b^12*c^12*d^12 - 8958208*a^12*b^12*c^14*d^10 + 999040*a^12*
b^12*c^16*d^8 - 2002728*a^13*b^11*c^3*d^21 + 10875200*a^13*b^11*c^5*d^19 -
28461040*a^13*b^11*c^7*d^17 + 39987520*a^13*b^11*c^9*d^15 - 29358696*a^13*b
^11*c^11*d^13 + 9722048*a^13*b^11*c^13*d^11 - 1104320*a^13*b^11*c^15*d^9 +
547088*a^14*b^10*c^2*d^22 - 4147952*a^14*b^10*c^4*d^20 + 13887520*a^14*b^10
*c^6*d^18 - 24199280*a^14*b^10*c^8*d^16 + 21989928*a^14*b^10*c^10*d^14 - 89
58208*a^14*b^10*c^12*d^12 + 1124032*a^14*b^10*c^14*d^10 + 1210560*a^15*b^9*
c^3*d^21 - 5365072*a^15*b^9*c^5*d^19 + 11779808*a^15*b^9*c^7*d^17 - 1346208
8*a^15*b^9*c^9*d^15 + 6995840*a^15*b^9*c^11*d^13 - 1104320*a^15*b^9*c^13*d^
11 - 263320*a^16*b^8*c^2*d^22 + 1586920*a^16*b^8*c^4*d^20 - 4506428*a^16*b^
8*c^6*d^18 + 6661036*a^16*b^8*c^8*d^16 - 4591360*a^16*b^8*c^10*d^14 + 99904
0*a^16*b^8*c^12*d^12 - 335040*a^17*b^7*c^3*d^21 + 1310168*a^17*b^7*c^5*d^19
 - 2621008*a^17*b^7*c^7*d^17 + 2478528*a^17*b^7*c^9*d^15 - 762560*a^17*b^7*
c^11*d^13 + 44120*a^18*b^6*c^2*d^22 - 276020*a^18*b^6*c^4*d^20 + 793756*a^1
8*b^6*c^6*d^18 - 1058448*a^18*b^6*c^8*d^16 + 460480*a^18*b^6*c^10*d^14 + 37
680*a^19*b^5*c^3*d^21 - 170968*a^19*b^5*c^5*d^19 + 336688*a^19*b^5*c^7*d^17
 - 212032*a^19*b^5*c^9*d^15 - 1564*a^20*b^4*c^2*d^22 + 21124*a^20*b^4*c^4*d
^20 - 72240*a^20*b^4*c^6*d^18 + 72560*a^20*b^4*c^8*d^16 - 288*a^21*b^3*c^3*
d^21 + 8160*a^21*b^3*c^5*d^19 - 17920*a^21*b^3*c^7*d^17 - 196*a^22*b^2*c^2*
d^22 + 176*a^22*b^2*c^4*d^20 + 3040*a^22*b^2*c^6*d^18 - 8*a*b^23*c^23*d - 8
*a^23*b*c*d^23)^2/4 - (20736*b^18*d^18 - 96768*a^2*b^16*d^18 + 173664*a^4*b
^14*d^18 - 136032*a^6*b^12*d^18 + 31081*a^8*b^10*d^18 + 8440*a^10*b^8*d^18
+ 400*a^12*b^6*d^18 - 96768*b^18*c^2*d^16 + 173664*b^18*c^4*d^14 - 136032*b
^18*c^6*d^12 + 31081*b^18*c^8*d^10 + 8440*b^18*c^10*d^8 + 400*b^18*c^12*d^6
 - 131328*a*b^17*c^3*d^15 + 216576*a*b^17*c^5*d^13 - 141104*a*b^17*c^7*d^11
 + 20260*a*b^17*c^9*d^9 + 2800*a*b^17*c^11*d^7 - 131328*a^3*b^15*c*d^17 + 2
16576*a^5*b^13*c*d^17 - 141104*a^7*b^11*c*d^17 + 20260*a^9*b^9*c*d^17 + 280
0*a^11*b^7*c*d^17 + 495936*a^2*b^16*c^2*d^16 - 989856*a^2*b^16*c^4*d^14 + 9
01948*a^2*b^16*c^6*d^12 - 308392*a^2*b^16*c^8*d^10 - 5260*a^2*b^16*c^10*d^8
 + 1600*a^2*b^16*c^12*d^6 + 657408*a^3*b^15*c^3*d^15 - 1158992*a^3*b^15*c^5
*d^13 + 838256*a^3*b^15*c^7*d^11 - 182200*a^3*b^15*c^9*d^9 - 3200*a^3*b^15*
c^11*d^7 - 989856*a^4*b^14*c^2*d^16 + 2185654*a^4*b^14*c^4*d^14 - 2218576*a
^4*b^14*c^6*d^12 + 900624*a^4*b^14*c^8*d^10 - 64720*a^4*b^14*c^10*d^8 + 160
0*a^4*b^14*c^12*d^6 - 1158992*a^5*b^13*c^3*d^15 + 2158808*a^5*b^13*c^5*d^13
 - 1641528*a^5*b^13*c^7*d^11 + 406880*a^5*b^13*c^9*d^9 - 17600*a^5*b^13*c^1
1*d^7 + 901948*a^6*b^12*c^2*d^16 - 2218576*a^6*b^12*c^4*d^14 + 2430936*a^6*
b^12*c^6*d^12 - 1026928*a^6*b^12*c^8*d^10 + 88720*a^6*b^12*c^10*d^8 + 83825

6*a^7*b^11*c^3*d^15 - 1641528*a^7*b^11*c^5*d^13 + 1206848*a^7*b^11*c^7*d^11 - 239360*a^7*b^11*c^9*d^9 - 308392*a^8*b^10*c^2*d^16 + 900624*a^8*b^10*c^4*d^14 - 1026928*a^8*b^10*c^6*d^12 + 354016*a^8*b^10*c^8*d^10 - 182200*a^9*b^9*c^3*d^15 + 406880*a^9*b^9*c^5*d^13 - 239360*a^9*b^9*c^7*d^11 - 5260*a^10*b^8*c^2*d^16 - 64720*a^10*b^8*c^4*d^14 + 88720*a^10*b^8*c^6*d^12 - 3200*a^11*b^7*c^3*d^15 - 17600*a^11*b^7*c^5*d^13 + 1600*a^12*b^6*c^2*d^16 + 1600*a^12*b^6*c^4*d^14 + 27648*a*b^17*c*d^17)*(80*a^2*b^28*c^30 - 16*b^30*c^30 - 16*a^30*d^30 - 160*a^4*b^26*c^30 + 160*a^6*b^24*c^30 - 80*a^8*b^22*c^30 + 16*a^10*b^20*c^30 + 16*a^20*b^10*d^30 - 80*a^22*b^8*d^30 + 160*a^24*b^6*d^30 - 160*a^26*b^4*d^30 + 80*a^28*b^2*d^30 + 80*a^30*c^2*d^28 - 160*a^30*c^4*d^26 + 160*a^30*c^6*d^24 - 80*a^30*c^8*d^22 + 16*a^30*c^10*d^20 + 16*b^30*c^20*d^10 - 80*b^30*c^22*d^8 + 160*b^30*c^24*d^6 - 160*b^30*c^26*d^4 + 80*b^30*c^28*d^2 - 320*a*b^29*c^19*d^11 + 1600*a*b^29*c^21*d^9 - 3200*a*b^29*c^23*d^7 + 3200*a*b^29*c^25*d^5 - 1600*a*b^29*c^27*d^3 - 1600*a^3*b^27*c^29*d + 3200*a^5*b^25*c^29*d - 3200*a^7*b^23*c^29*d + 1600*a^9*b^21*c^29*d - 320*a^11*b^19*c^29*d - 320*a^19*b^11*c*d^29 + 1600*a^21*b^9*c*d^29 - 3200*a^23*b^7*c*d^29 + 3200*a^25*b^5*c*d^29 - 1600*a^27*b^3*c*d^29 - 1600*a^29*b*c^3*d^27 + 3200*a^29*b*c^5*d^25 - 3200*a^29*b*c^7*d^23 + 1600*a^29*b*c^9*d^21 - 320*a^29*b*c^11*d^19 + 3040*a^2*b^28*c^18*d^12 - 15280*a^2*b^28*c^20*d^10 + 30800*a^2*b^28*c^22*d^8 - 31200*a^2*b^28*c^24*d^6 + 16000*a^2*b^28*c^26*d^4 - 3440*a^2*b^28*c^28*d^2 - 18240*a^3*b^27*c^17*d^13 + 92800*a^3*b^27*c^19*d^11 - 190400*a^3*b^27*c^21*d^9 + 198400*a^3*b^27*c^23*d^7 - 107200*a^3*b^27*c^25*d^5 + 26240*a^3*b^27*c^27*d^3 + 77520*a^4*b^26*c^16*d^14 - 402800*a^4*b^26*c^18*d^12 + 851360*a^4*b^26*c^20*d^10 - 928000*a^4*b^26*c^22*d^8 + 541200*a^4*b^26*c^24*d^6 - 155120*a^4*b^26*c^26*d^4 + 16000*a^4*b^26*c^28*d^2 - 248064*a^5*b^25*c^15*d^15 + 1331520*a^5*b^25*c^17*d^13 - 2939840*a^5*b^25*c^19*d^11 + 3408640*a^5*b^25*c^21*d^9 - 2184320*a^5*b^25*c^23*d^7 + 736064*a^5*b^25*c^25*d^5 - 107200*a^5*b^25*c^27*d^3 + 620160*a^6*b^24*c^14*d^16 - 3488400*a^6*b^24*c^16*d^14 + 8170000*a^6*b^24*c^18*d^12 - 10229760*a^6*b^24*c^20*d^10 + 7281600*a^6*b^24*c^22*d^8 - 2863760*a^6*b^24*c^24*d^6 + 541200*a^6*b^24*c^26*d^4 - 31200*a^6*b^24*c^28*d^2 - 1240320*a^7*b^23*c^13*d^17 + 7441920*a^7*b^23*c^15*d^15 - 18787200*a^7*b^23*c^17*d^13 + 25721600*a^7*b^23*c^19*d^11 - 20444800*a^7*b^23*c^21*d^9 + 9297920*a^7*b^23*c^23*d^7 - 2184320*a^7*b^23*c^25*d^5 + 198400*a^7*b^23*c^27*d^3 + 2015520*a^8*b^22*c^12*d^18 - 13178400*a^8*b^22*c^14*d^16 + 36434400*a^8*b^22*c^16*d^14 - 55069600*a^8*b^22*c^18*d^12 + 48989680*a^8*b^22*c^20*d^10 - 25575920*a^8*b^22*c^22*d^8 + 7281600*a^8*b^22*c^24*d^6 - 928000*a^8*b^22*c^26*d^4 + 30800*a^8*b^22*c^28*d^2 - 2687360*a^9*b^21*c^11*d^19 + 19638400*a^9*b^21*c^13*d^17 - 60362240*a^9*b^21*c^15*d^15 + 101475200*a^9*b^21*c^17*d^13 - 101172800*a^9*b^21*c^19*d^11 + 60333760*a^9*b^21*c^21*d^9 - 20444800*a^9*b^21*c^23*d^7 + 3408640*a^9*b^21*c^25*d^5 - 190400*a^9*b^21*c^27*d^3 + 2956096*a^10*b^20*c^10*d^20 - 24858080*a^10*b^20*c^12*d^18 + 86150560*a^10*b^20*c^14*d^16 - 162120160*a^10*b^20*c^16*d^14 + 181463680*a^10*b^20*c^18*d^12 - 123188112*a^10*b^20*c^20*d^10 + 48989680*a^10*b^20*c^22*d^8 - 10229760*a^10*b^20*c^24*d^6 + 851360*a^10*b^20*c^26*d^4 - 15280*a^10*b^20*c^28*d^2 - 2687360*a^11*b^19*

c^9*d^21 + 26873600*a^11*b^19*c^11*d^19 - 106460800*a^11*b^19*c^13*d^17 + 2
25738240*a^11*b^19*c^15*d^15 - 284331200*a^11*b^19*c^17*d^13 + 219166080*a^
11*b^19*c^19*d^11 - 101172800*a^11*b^19*c^21*d^9 + 25721600*a^11*b^19*c^23*
d^7 - 2939840*a^11*b^19*c^25*d^5 + 92800*a^11*b^19*c^27*d^3 + 2015520*a^12*
b^18*c^8*d^22 - 24858080*a^12*b^18*c^10*d^20 + 114212800*a^12*b^18*c^12*d^1
8 - 274937600*a^12*b^18*c^14*d^16 + 390830000*a^12*b^18*c^16*d^14 - 3414269
60*a^12*b^18*c^18*d^12 + 181463680*a^12*b^18*c^20*d^10 - 55069600*a^12*b^18
*c^22*d^8 + 8170000*a^12*b^18*c^24*d^6 - 402800*a^12*b^18*c^26*d^4 + 3040*a
^12*b^18*c^28*d^2 - 1240320*a^13*b^17*c^7*d^23 + 19638400*a^13*b^17*c^9*d^2
1 - 106460800*a^13*b^17*c^11*d^19 + 293542400*a^13*b^17*c^13*d^17 - 4725619
20*a^13*b^17*c^15*d^15 + 467412160*a^13*b^17*c^17*d^13 - 284331200*a^13*b^1
7*c^19*d^11 + 101475200*a^13*b^17*c^21*d^9 - 18787200*a^13*b^17*c^23*d^7 +
1331520*a^13*b^17*c^25*d^5 - 18240*a^13*b^17*c^27*d^3 + 620160*a^14*b^16*c^
6*d^24 - 13178400*a^14*b^16*c^8*d^22 + 86150560*a^14*b^16*c^10*d^20 - 27493
7600*a^14*b^16*c^12*d^18 + 503363200*a^14*b^16*c^14*d^16 - 563751280*a^14*b
^16*c^16*d^14 + 390830000*a^14*b^16*c^18*d^12 - 162120160*a^14*b^16*c^20*d^
10 + 36434400*a^14*b^16*c^22*d^8 - 3488400*a^14*b^16*c^24*d^6 + 77520*a^14*
b^16*c^26*d^4 - 248064*a^15*b^15*c^5*d^25 + 7441920*a^15*b^15*c^7*d^23 - 60
362240*a^15*b^15*c^9*d^21 + 225738240*a^15*b^15*c^11*d^19 - 472561920*a^15*
b^15*c^13*d^17 + 599984128*a^15*b^15*c^15*d^15 - 472561920*a^15*b^15*c^17*d
^13 + 225738240*a^15*b^15*c^19*d^11 - 60362240*a^15*b^15*c^21*d^9 + 7441920
*a^15*b^15*c^23*d^7 - 248064*a^15*b^15*c^25*d^5 + 77520*a^16*b^14*c^4*d^26
- 3488400*a^16*b^14*c^6*d^24 + 36434400*a^16*b^14*c^8*d^22 - 162120160*a^16
*b^14*c^10*d^20 + 390830000*a^16*b^14*c^12*d^18 - 563751280*a^16*b^14*c^14*
d^16 + 503363200*a^16*b^14*c^16*d^14 - 274937600*a^16*b^14*c^18*d^12 + 8615
0560*a^16*b^14*c^20*d^10 - 13178400*a^16*b^14*c^22*d^8 + 620160*a^16*b^14*c
^24*d^6 - 18240*a^17*b^13*c^3*d^27 + 1331520*a^17*b^13*c^5*d^25 - 18787200*
a^17*b^13*c^7*d^23 + 101475200*a^17*b^13*c^9*d^21 - 284331200*a^17*b^13*c^1
1*d^19 + 467412160*a^17*b^13*c^13*d^17 - 472561920*a^17*b^13*c^15*d^15 + 29
3542400*a^17*b^13*c^17*d^13 - 106460800*a^17*b^13*c^19*d^11 + 19638400*a^17
*b^13*c^21*d^9 - 1240320*a^17*b^13*c^23*d^7 + 3040*a^18*b^12*c^2*d^28 - 402
800*a^18*b^12*c^4*d^26 + 8170000*a^18*b^12*c^6*d^24 - 55069600*a^18*b^12*c^
8*d^22 + 181463680*a^18*b^12*c^10*d^20 - 341426960*a^18*b^12*c^12*d^18 + 39
0830000*a^18*b^12*c^14*d^16 - 274937600*a^18*b^12*c^16*d^14 + 114212800*a^1
8*b^12*c^18*d^12 - 24858080*a^18*b^12*c^20*d^10 + 2015520*a^18*b^12*c^22*d^
8 + 92800*a^19*b^11*c^3*d^27 - 2939840*a^19*b^11*c^5*d^25 + 25721600*a^19*b
^11*c^7*d^23 - 101172800*a^19*b^11*c^9*d^21 + 219166080*a^19*b^11*c^11*d^19
 - 284331200*a^19*b^11*c^13*d^17 + 225738240*a^19*b^11*c^15*d^15 - 10646080
0*a^19*b^11*c^17*d^13 + 26873600*a^19*b^11*c^19*d^11 - 2687360*a^19*b^11*c^
21*d^9 - 15280*a^20*b^10*c^2*d^28 + 851360*a^20*b^10*c^4*d^26 - 10229760*a^
20*b^10*c^6*d^24 + 48989680*a^20*b^10*c^8*d^22 - 123188112*a^20*b^10*c^10*d
^20 + 181463680*a^20*b^10*c^12*d^18 - 162120160*a^20*b^10*c^14*d^16 + 86150
560*a^20*b^10*c^16*d^14 - 24858080*a^20*b^10*c^18*d^12 + 2956096*a^20*b^10*
c^20*d^10 - 190400*a^21*b^9*c^3*d^27 + 3408640*a^21*b^9*c^5*d^25 - 20444800
*a^21*b^9*c^7*d^23 + 60333760*a^21*b^9*c^9*d^21 - 101172800*a^21*b^9*c^11*d

^19 + 101475200*a^21*b^9*c^13*d^17 - 60362240*a^21*b^9*c^15*d^15 + 19638400
*a^21*b^9*c^17*d^13 - 2687360*a^21*b^9*c^19*d^11 + 30800*a^22*b^8*c^2*d^28
- 928000*a^22*b^8*c^4*d^26 + 7281600*a^22*b^8*c^6*d^24 - 25575920*a^22*b^8*
c^8*d^22 + 48989680*a^22*b^8*c^10*d^20 - 55069600*a^22*b^8*c^12*d^18 + 3643
4400*a^22*b^8*c^14*d^16 - 13178400*a^22*b^8*c^16*d^14 + 2015520*a^22*b^8*c^
18*d^12 + 198400*a^23*b^7*c^3*d^27 - 2184320*a^23*b^7*c^5*d^25 + 9297920*a^
23*b^7*c^7*d^23 - 20444800*a^23*b^7*c^9*d^21 + 25721600*a^23*b^7*c^11*d^19
- 18787200*a^23*b^7*c^13*d^17 + 7441920*a^23*b^7*c^15*d^15 - 1240320*a^23*b
^7*c^17*d^13 - 31200*a^24*b^6*c^2*d^28 + 541200*a^24*b^6*c^4*d^26 - 2863760
*a^24*b^6*c^6*d^24 + 7281600*a^24*b^6*c^8*d^22 - 10229760*a^24*b^6*c^10*d^2
0 + 8170000*a^24*b^6*c^12*d^18 - 3488400*a^24*b^6*c^14*d^16 + 620160*a^24*b
^6*c^16*d^14 - 107200*a^25*b^5*c^3*d^27 + 736064*a^25*b^5*c^5*d^25 - 218432
0*a^25*b^5*c^7*d^23 + 3408640*a^25*b^5*c^9*d^21 - 2939840*a^25*b^5*c^11*d^1
9 + 1331520*a^25*b^5*c^13*d^17 - 248064*a^25*b^5*c^15*d^15 + 16000*a^26*b^4
*c^2*d^28 - 155120*a^26*b^4*c^4*d^26 + 541200*a^26*b^4*c^6*d^24 - 928000*a^
26*b^4*c^8*d^22 + 851360*a^26*b^4*c^10*d^20 - 402800*a^26*b^4*c^12*d^18 + 7
7520*a^26*b^4*c^14*d^16 + 26240*a^27*b^3*c^3*d^27 - 107200*a^27*b^3*c^5*d^2
5 + 198400*a^27*b^3*c^7*d^23 - 190400*a^27*b^3*c^9*d^21 + 92800*a^27*b^3*c^
11*d^19 - 18240*a^27*b^3*c^13*d^17 - 3440*a^28*b^2*c^2*d^28 + 16000*a^28*b^
2*c^4*d^26 - 31200*a^28*b^2*c^6*d^24 + 30800*a^28*b^2*c^8*d^22 - 15280*a^28
*b^2*c^10*d^20 + 3040*a^28*b^2*c^12*d^18 + 320*a*b^29*c^29*d + 320*a^29*b*c
*d^29))^(1/2) + 2*a^24*d^24 + 2*b^24*c^24 + 8*a^2*b^22*c^24 + 8*a^4*b^20*c^
24 - 576*a^10*b^14*d^24 + 2784*a^12*b^12*d^24 - 5284*a^14*b^10*d^24 + 4730*
a^16*b^8*d^24 - 1780*a^18*b^6*d^24 + 68*a^20*b^4*d^24 + 38*a^22*b^2*d^24 +
8*a^24*c^2*d^22 + 8*a^24*c^4*d^20 - 576*b^24*c^10*d^14 + 2784*b^24*c^12*d^1
2 - 5284*b^24*c^14*d^10 + 4730*b^24*c^16*d^8 - 1780*b^24*c^18*d^6 + 68*b^24
*c^20*d^4 + 38*b^24*c^22*d^2 + 5760*a*b^23*c^9*d^15 - 28224*a*b^23*c^11*d^1
3 + 54728*a*b^23*c^13*d^11 - 50620*a*b^23*c^15*d^9 + 20360*a*b^23*c^17*d^7
- 1480*a*b^23*c^19*d^5 - 268*a*b^23*c^21*d^3 - 88*a^3*b^21*c^23*d - 160*a^5
*b^19*c^23*d + 5760*a^9*b^15*c*d^23 - 28224*a^11*b^13*c*d^23 + 54728*a^13*b
^11*c*d^23 - 50620*a^15*b^9*c*d^23 + 20360*a^17*b^7*c*d^23 - 1480*a^19*b^5*
c*d^23 - 268*a^21*b^3*c*d^23 - 88*a^23*b*c^3*d^21 - 160*a^23*b*c^5*d^19 - 2
5920*a^2*b^22*c^8*d^16 + 131904*a^2*b^22*c^10*d^14 - 270604*a^2*b^22*c^12*d
^12 + 273544*a^2*b^22*c^14*d^10 - 131660*a^2*b^22*c^16*d^8 + 22060*a^2*b^22
*c^18*d^6 - 782*a^2*b^22*c^20*d^4 - 98*a^2*b^22*c^22*d^2 + 69120*a^3*b^21*c
^7*d^17 - 379200*a^3*b^21*c^9*d^15 + 860368*a^3*b^21*c^11*d^13 - 1001364*a^
3*b^21*c^13*d^11 + 605280*a^3*b^21*c^15*d^9 - 167520*a^3*b^21*c^17*d^7 + 18
840*a^3*b^21*c^19*d^5 - 144*a^3*b^21*c^21*d^3 - 120960*a^4*b^20*c^6*d^18 +
756000*a^4*b^20*c^8*d^16 - 1987844*a^4*b^20*c^10*d^14 + 2750664*a^4*b^20*c^
12*d^12 - 2073976*a^4*b^20*c^14*d^10 + 793460*a^4*b^20*c^16*d^8 - 138010*a^
4*b^20*c^18*d^6 + 10562*a^4*b^20*c^20*d^4 + 88*a^4*b^20*c^22*d^2 + 145152*a
^5*b^19*c^5*d^19 - 1116288*a^5*b^19*c^7*d^17 + 3539128*a^5*b^19*c^9*d^15 -
5890780*a^5*b^19*c^11*d^13 + 5437600*a^5*b^19*c^13*d^11 - 2682536*a^5*b^19*
c^15*d^9 + 655084*a^5*b^19*c^17*d^7 - 85484*a^5*b^19*c^19*d^5 + 4080*a^5*b^
19*c^21*d^3 - 120960*a^6*b^18*c^4*d^20 + 1266048*a^6*b^18*c^6*d^18 - 497799

6*a^6*b^18*c^8*d^16 + 10009720*a^6*b^18*c^10*d^14 - 11209800*a^6*b^18*c^12*d^12 + 6943760*a^6*b^18*c^14*d^10 - 2253214*a^6*b^18*c^16*d^8 + 396878*a^6*b^18*c^18*d^6 - 36120*a^6*b^18*c^20*d^4 + 1520*a^6*b^18*c^22*d^2 + 69120*a^7*b^17*c^3*d^21 - 1116288*a^7*b^17*c^5*d^19 + 5575008*a^7*b^17*c^7*d^17 - 13668308*a^7*b^17*c^9*d^15 + 18576800*a^7*b^17*c^11*d^13 - 14230520*a^7*b^17*c^13*d^11 + 5889904*a^7*b^17*c^15*d^9 - 1310504*a^7*b^17*c^17*d^7 + 168344*a^7*b^17*c^19*d^5 - 8960*a^7*b^17*c^21*d^3 - 25920*a^8*b^16*c^2*d^22 + 756000*a^8*b^16*c^4*d^20 - 4977996*a^8*b^16*c^6*d^18 + 15144828*a^8*b^16*c^8*d^16 - 25068800*a^8*b^16*c^10*d^14 + 23486280*a^8*b^16*c^12*d^12 - 12099640*a^8*b^16*c^14*d^10 + 3330518*a^8*b^16*c^16*d^8 - 529224*a^8*b^16*c^18*d^6 + 36280*a^8*b^16*c^20*d^4 - 379200*a^9*b^15*c^3*d^21 + 3539128*a^9*b^15*c^5*d^19 - 13668308*a^9*b^15*c^7*d^17 + 27691952*a^9*b^15*c^9*d^15 - 31562040*a^9*b^15*c^11*d^13 + 19993760*a^9*b^15*c^13*d^11 - 6731044*a^9*b^15*c^15*d^9 + 1239264*a^9*b^15*c^17*d^7 - 106016*a^9*b^15*c^19*d^5 + 131904*a^10*b^14*c^2*d^22 - 1987844*a^10*b^14*c^4*d^20 + 10009720*a^10*b^14*c^6*d^18 - 25068800*a^10*b^14*c^8*d^16 + 34796936*a^10*b^14*c^10*d^14 - 26927144*a^10*b^14*c^12*d^12 + 10994964*a^10*b^14*c^14*d^10 - 2295680*a^10*b^14*c^16*d^8 + 230240*a^10*b^14*c^18*d^6 + 860368*a^11*b^13*c^3*d^21 - 5890780*a^11*b^13*c^5*d^19 + 18576800*a^11*b^13*c^7*d^17 - 31562040*a^11*b^13*c^9*d^15 + 29722864*a^11*b^13*c^11*d^13 - 14679348*a^11*b^13*c^13*d^11 + 3497920*a^11*b^13*c^15*d^9 - 381280*a^11*b^13*c^17*d^7 - 270604*a^12*b^12*c^2*d^22 + 2750664*a^12*b^12*c^4*d^20 - 11209800*a^12*b^12*c^6*d^18 + 23486280*a^12*b^12*c^8*d^16 - 26927144*a^12*b^12*c^10*d^14 + 16147404*a^12*b^12*c^12*d^12 - 4479104*a^12*b^12*c^14*d^10 + 499520*a^12*b^12*c^16*d^8 - 1001364*a^13*b^11*c^3*d^21 + 5437600*a^13*b^11*c^5*d^19 - 14230520*a^13*b^11*c^7*d^17 + 19993760*a^13*b^11*c^9*d^15 - 14679348*a^13*b^11*c^11*d^13 + 4861024*a^13*b^11*c^13*d^11 - 552160*a^13*b^11*c^15*d^9 + 273544*a^14*b^10*c^2*d^22 - 2073976*a^14*b^10*c^4*d^20 + 6943760*a^14*b^10*c^6*d^18 - 12099640*a^14*b^10*c^8*d^16 + 10994964*a^14*b^10*c^10*d^14 - 4479104*a^14*b^10*c^12*d^12 + 562016*a^14*b^10*c^14*d^10 + 605280*a^15*b^9*c^3*d^21 - 2682536*a^15*b^9*c^5*d^19 + 5889904*a^15*b^9*c^7*d^17 - 6731044*a^15*b^9*c^9*d^15 + 3497920*a^15*b^9*c^11*d^13 - 552160*a^15*b^9*c^13*d^11 - 131660*a^16*b^8*c^2*d^22 + 793460*a^16*b^8*c^4*d^20 - 2253214*a^16*b^8*c^6*d^18 + 3330518*a^16*b^8*c^8*d^16 - 2295680*a^16*b^8*c^10*d^14 + 499520*a^16*b^8*c^12*d^12 - 167520*a^17*b^7*c^3*d^21 + 655084*a^17*b^7*c^5*d^19 - 1310504*a^17*b^7*c^7*d^17 + 1239264*a^17*b^7*c^9*d^15 - 381280*a^17*b^7*c^11*d^13 + 22060*a^18*b^6*c^2*d^22 - 138010*a^18*b^6*c^4*d^20 + 396878*a^18*b^6*c^6*d^18 - 529224*a^18*b^6*c^8*d^16 + 230240*a^18*b^6*c^10*d^14 + 18840*a^19*b^5*c^3*d^21 - 85484*a^19*b^5*c^5*d^19 + 168344*a^19*b^5*c^7*d^17 - 106016*a^19*b^5*c^9*d^15 - 782*a^20*b^4*c^2*d^22 + 10562*a^20*b^4*c^4*d^20 - 36120*a^20*b^4*c^6*d^18 + 36280*a^20*b^4*c^8*d^16 - 144*a^21*b^3*c^3*d^21 + 4080*a^21*b^3*c^5*d^19 - 8960*a^21*b^3*c^7*d^17 - 98*a^22*b^2*c^2*d^22 + 88*a^22*b^2*c^4*d^20 + 1520*a^22*b^2*c^6*d^18 - 4*a*b^23*c^23*d - 4*a^23*b*c*d^23)/(16*(5*a^2*b^28*c^30 - b^30*c^30 - a^30*d^30 - 10*a^4*b^26*c^30 + 10*a^6*b^24*c^30 - 5*a^8*b^22*c^30 + a^10*b^20*c^30 + a^20*b^10*d^30 - 5*a^22*b^8*d^30 + 10*a^24*b^6*d^30 - 10*a^26*b^4*d^30 +

5*a^28*b^2*d^30 + 5*a^30*c^2*d^28 - 10*a^30*c^4*d^26 + 10*a^30*c^6*d^24 - 5*a^30*c^8*d^22 + a^30*c^10*d^20 + b^30*c^20*d^10 - 5*b^30*c^22*d^8 + 10*b^30*c^24*d^6 - 10*b^30*c^26*d^4 + 5*b^30*c^28*d^2 - 20*a*b^29*c^19*d^11 + 100*a*b^29*c^21*d^9 - 200*a*b^29*c^23*d^7 + 200*a*b^29*c^25*d^5 - 100*a*b^29*c^27*d^3 - 100*a^3*b^27*c^29*d + 200*a^5*b^25*c^29*d - 200*a^7*b^23*c^29*d + 100*a^9*b^21*c^29*d - 20*a^11*b^19*c^29*d - 20*a^19*b^11*c*d^29 + 100*a^21*b^9*c*d^29 - 200*a^23*b^7*c*d^29 + 200*a^25*b^5*c*d^29 - 100*a^27*b^3*c*d^29 - 100*a^29*b*c^3*d^27 + 200*a^29*b*c^5*d^25 - 200*a^29*b*c^7*d^23 + 100*a^29*b*c^9*d^21 - 20*a^29*b*c^11*d^19 + 190*a^2*b^28*c^18*d^12 - 955*a^2*b^28*c^20*d^10 + 1925*a^2*b^28*c^22*d^8 - 1950*a^2*b^28*c^24*d^6 + 1000*a^2*b^28*c^26*d^4 - 215*a^2*b^28*c^28*d^2 - 1140*a^3*b^27*c^17*d^13 + 5800*a^3*b^27*c^19*d^11 - 11900*a^3*b^27*c^21*d^9 + 12400*a^3*b^27*c^23*d^7 - 6700*a^3*b^27*c^25*d^5 + 1640*a^3*b^27*c^27*d^3 + 4845*a^4*b^26*c^16*d^14 - 25175*a^4*b^26*c^18*d^12 + 53210*a^4*b^26*c^20*d^10 - 58000*a^4*b^26*c^22*d^8 + 33825*a^4*b^26*c^24*d^6 - 9695*a^4*b^26*c^26*d^4 + 1000*a^4*b^26*c^28*d^2 - 15504*a^5*b^25*c^15*d^15 + 83220*a^5*b^25*c^17*d^13 - 183740*a^5*b^25*c^19*d^11 + 213040*a^5*b^25*c^21*d^9 - 136520*a^5*b^25*c^23*d^7 + 46004*a^5*b^25*c^25*d^5 - 6700*a^5*b^25*c^27*d^3 + 38760*a^6*b^24*c^14*d^16 - 218025*a^6*b^24*c^16*d^14 + 510625*a^6*b^24*c^18*d^12 - 639360*a^6*b^24*c^20*d^10 + 455100*a^6*b^24*c^22*d^8 - 178985*a^6*b^24*c^24*d^6 + 33825*a^6*b^24*c^26*d^4 - 1950*a^6*b^24*c^28*d^2 - 77520*a^7*b^23*c^13*d^17 + 465120*a^7*b^23*c^15*d^15 - 1174200*a^7*b^23*c^17*d^13 + 1607600*a^7*b^23*c^19*d^11 - 1277800*a^7*b^23*c^21*d^9 + 581120*a^7*b^23*c^23*d^7 - 136520*a^7*b^23*c^25*d^5 + 12400*a^7*b^23*c^27*d^3 + 125970*a^8*b^22*c^12*d^18 - 823650*a^8*b^22*c^14*d^16 + 2277150*a^8*b^22*c^16*d^14 - 3441850*a^8*b^22*c^18*d^12 + 3061855*a^8*b^22*c^20*d^10 - 1598495*a^8*b^22*c^22*d^8 + 455100*a^8*b^22*c^24*d^6 - 58000*a^8*b^22*c^26*d^4 + 1925*a^8*b^22*c^28*d^2 - 167960*a^9*b^21*c^11*d^19 + 1227400*a^9*b^21*c^13*d^17 - 3772640*a^9*b^21*c^15*d^15 + 6342200*a^9*b^21*c^17*d^13 - 6323300*a^9*b^21*c^19*d^11 + 3770860*a^9*b^21*c^21*d^9 - 1277800*a^9*b^21*c^23*d^7 + 213040*a^9*b^21*c^25*d^5 - 11900*a^9*b^21*c^27*d^3 + 184756*a^10*b^20*c^10*d^20 - 1553630*a^10*b^20*c^12*d^18 + 5384410*a^10*b^20*c^14*d^16 - 10132510*a^10*b^20*c^16*d^14 + 11341480*a^10*b^20*c^18*d^12 - 7699257*a^10*b^20*c^20*d^10 + 3061855*a^10*b^20*c^22*d^8 - 639360*a^10*b^20*c^24*d^6 + 53210*a^10*b^20*c^26*d^4 - 955*a^10*b^20*c^28*d^2 - 167960*a^11*b^19*c^9*d^21 + 1679600*a^11*b^19*c^11*d^19 - 6653800*a^11*b^19*c^13*d^17 + 14108640*a^11*b^19*c^15*d^15 - 17770700*a^11*b^19*c^17*d^13 + 13697880*a^11*b^19*c^19*d^11 - 6323300*a^11*b^19*c^21*d^9 + 1607600*a^11*b^19*c^23*d^7 - 183740*a^11*b^19*c^25*d^5 + 5800*a^11*b^19*c^27*d^3 + 125970*a^12*b^18*c^8*d^22 - 1553630*a^12*b^18*c^10*d^20 + 7138300*a^12*b^18*c^12*d^18 - 17183600*a^12*b^18*c^14*d^16 + 24426875*a^12*b^18*c^16*d^14 - 21339185*a^12*b^18*c^18*d^12 + 11341480*a^12*b^18*c^20*d^10 - 3441850*a^12*b^18*c^22*d^8 + 510625*a^12*b^18*c^24*d^6 - 25175*a^12*b^18*c^26*d^4 + 190*a^12*b^18*c^28*d^2 - 77520*a^13*b^17*c^7*d^23 + 1227400*a^13*b^17*c^9*d^21 - 6653800*a^13*b^17*c^11*d^19 + 18346400*a^13*b^17*c^13*d^17 - 29535120*a^13*b^17*c^15*d^15 + 29213260*a^13*b^17*c^17*d^13 - 17770700*a^13*b^17*c^19*d^11 + 6342200*a^

13*b^17*c^21*d^9 - 1174200*a^13*b^17*c^23*d^7 + 83220*a^13*b^17*c^25*d^5 - 1140*a^13*b^17*c^27*d^3 + 38760*a^14*b^16*c^6*d^24 - 823650*a^14*b^16*c^8*d^22 + 5384410*a^14*b^16*c^10*d^20 - 17183600*a^14*b^16*c^12*d^18 + 31460200*a^14*b^16*c^14*d^16 - 35234455*a^14*b^16*c^16*d^14 + 24426875*a^14*b^16*c^18*d^12 - 10132510*a^14*b^16*c^20*d^10 + 2277150*a^14*b^16*c^22*d^8 - 218025*a^14*b^16*c^24*d^6 + 4845*a^14*b^16*c^26*d^4 - 15504*a^15*b^15*c^5*d^25 + 465120*a^15*b^15*c^7*d^23 - 3772640*a^15*b^15*c^9*d^21 + 14108640*a^15*b^15*c^11*d^19 - 29535120*a^15*b^15*c^13*d^17 + 37499008*a^15*b^15*c^15*d^15 - 29535120*a^15*b^15*c^17*d^13 + 14108640*a^15*b^15*c^19*d^11 - 3772640*a^15*b^15*c^21*d^9 + 465120*a^15*b^15*c^23*d^7 - 15504*a^15*b^15*c^25*d^5 + 4845*a^16*b^14*c^4*d^26 - 218025*a^16*b^14*c^6*d^24 + 2277150*a^16*b^14*c^8*d^22 - 10132510*a^16*b^14*c^10*d^20 + 24426875*a^16*b^14*c^12*d^18 - 35234455*a^16*b^14*c^14*d^16 + 31460200*a^16*b^14*c^16*d^14 - 17183600*a^16*b^14*c^18*d^12 + 5384410*a^16*b^14*c^20*d^10 - 823650*a^16*b^14*c^22*d^8 + 38760*a^16*b^14*c^24*d^6 - 1140*a^17*b^13*c^3*d^27 + 83220*a^17*b^13*c^5*d^25 - 1174200*a^17*b^13*c^7*d^23 + 6342200*a^17*b^13*c^9*d^21 - 17770700*a^17*b^13*c^11*d^19 + 29213260*a^17*b^13*c^13*d^17 - 29535120*a^17*b^13*c^15*d^15 + 18346400*a^17*b^13*c^17*d^13 - 6653800*a^17*b^13*c^19*d^11 + 1227400*a^17*b^13*c^21*d^9 - 77520*a^17*b^13*c^23*d^7 + 190*a^18*b^12*c^2*d^28 - 25175*a^18*b^12*c^4*d^26 + 510625*a^18*b^12*c^6*d^24 - 3441850*a^18*b^12*c^8*d^22 + 11341480*a^18*b^12*c^10*d^20 - 21339185*a^18*b^12*c^12*d^18 + 24426875*a^18*b^12*c^14*d^16 - 17183600*a^18*b^12*c^16*d^14 + 7138300*a^18*b^12*c^18*d^12 - 1553630*a^18*b^12*c^20*d^10 + 125970*a^18*b^12*c^22*d^8 + 5800*a^19*b^11*c^3*d^27 - 183740*a^19*b^11*c^5*d^25 + 1607600*a^19*b^11*c^7*d^23 - 6323300*a^19*b^11*c^9*d^21 + 13697880*a^19*b^11*c^11*d^19 - 17770700*a^19*b^11*c^13*d^17 + 14108640*a^19*b^11*c^15*d^15 - 6653800*a^19*b^11*c^17*d^13 + 1679600*a^19*b^11*c^19*d^11 - 167960*a^19*b^11*c^21*d^9 - 955*a^20*b^10*c^2*d^28 + 53210*a^20*b^10*c^4*d^26 - 639360*a^20*b^10*c^6*d^24 + 3061855*a^20*b^10*c^8*d^22 - 7699257*a^20*b^10*c^10*d^20 + 11341480*a^20*b^10*c^12*d^18 - 10132510*a^20*b^10*c^14*d^16 + 5384410*a^20*b^10*c^16*d^14 - 1553630*a^20*b^10*c^18*d^12 + 184756*a^20*b^10*c^20*d^10 - 11900*a^21*b^9*c^3*d^27 + 213040*a^21*b^9*c^5*d^25 - 1277800*a^21*b^9*c^7*d^23 + 3770860*a^21*b^9*c^9*d^21 - 6323300*a^21*b^9*c^11*d^19 + 6342200*a^21*b^9*c^13*d^17 - 3772640*a^21*b^9*c^15*d^15 + 1227400*a^21*b^9*c^17*d^13 - 167960*a^21*b^9*c^19*d^11 + 1925*a^22*b^8*c^2*d^28 - 58000*a^22*b^8*c^4*d^26 + 455100*a^22*b^8*c^6*d^24 - 1598495*a^22*b^8*c^8*d^22 + 3061855*a^22*b^8*c^10*d^20 - 3441850*a^22*b^8*c^12*d^18 + 2277150*a^22*b^8*c^14*d^16 - 823650*a^22*b^8*c^16*d^14 + 125970*a^22*b^8*c^18*d^12 + 12400*a^23*b^7*c^3*d^27 - 136520*a^23*b^7*c^5*d^25 + 581120*a^23*b^7*c^7*d^23 - 1277800*a^23*b^7*c^9*d^21 + 1607600*a^23*b^7*c^11*d^19 - 1174200*a^23*b^7*c^13*d^17 + 465120*a^23*b^7*c^15*d^15 - 77520*a^23*b^7*c^17*d^13 - 1950*a^24*b^6*c^2*d^28 + 33825*a^24*b^6*c^4*d^26 - 178985*a^24*b^6*c^6*d^24 + 455100*a^24*b^6*c^8*d^22 - 639360*a^24*b^6*c^10*d^20 + 510625*a^24*b^6*c^12*d^18 - 218025*a^24*b^6*c^14*d^16 + 38760*a^24*b^6*c^16*d^14 - 6700*a^25*b^5*c^3*d^27 + 46004*a^25*b^5*c^5*d^25 - 136520*a^25*b^5*c^7*d^23 + 213040*a^25*b^5*c^9*d^21 - 183740*a^25*b^5*c^11*d^19 + 83220*a^

25*b^5*c^13*d^17 - 15504*a^25*b^5*c^15*d^15 + 1000*a^26*b^4*c^2*d^28 - 9695*a^26*b^4*c^4*d^26 + 33825*a^26*b^4*c^6*d^24 - 58000*a^26*b^4*c^8*d^22 + 53210*a^26*b^4*c^10*d^20 - 25175*a^26*b^4*c^12*d^18 + 4845*a^26*b^4*c^14*d^16 + 1640*a^27*b^3*c^3*d^27 - 6700*a^27*b^3*c^5*d^25 + 12400*a^27*b^3*c^7*d^23 - 11900*a^27*b^3*c^9*d^21 + 5800*a^27*b^3*c^11*d^19 - 1140*a^27*b^3*c^13*d^17 - 215*a^28*b^2*c^2*d^28 + 1000*a^28*b^2*c^4*d^26 - 1950*a^28*b^2*c^6*d^24 + 1925*a^28*b^2*c^8*d^22 - 955*a^28*b^2*c^10*d^20 + 190*a^28*b^2*c^12*d^18 + 20*a*b^29*c^29*d + 20*a^29*b*c*d^29)))^(1/2)*(((4*(8*a^2*b^23*c^25 - 32*a^4*b^21*c^25 + 48*a^6*b^19*c^25 - 32*a^8*b^17*c^25 + 8*a^10*b^15*c^25 + 8*a^25*c^2*d^23 - 32*a^25*c^4*d^21 + 48*a^25*c^6*d^19 - 32*a^25*c^8*d^17 + 8*a^25*c^10*d^15 - 8*a*b^24*c^16*d^9 + 32*a*b^24*c^18*d^7 - 48*a*b^24*c^20*d^5 + 32*a*b^24*c^22*d^3 - 72*a^3*b^22*c^24*d + 368*a^5*b^20*c^24*d - 592*a^7*b^18*c^24*d + 408*a^9*b^16*c^24*d - 104*a^11*b^14*c^24*d - 8*a^16*b^9*c*d^24 + 32*a^18*b^7*c*d^24 - 48*a^20*b^5*c*d^24 + 32*a^22*b^3*c*d^24 - 72*a^24*b*c^3*d^22 + 368*a^24*b*c^5*d^20 - 592*a^24*b*c^7*d^18 + 408*a^24*b*c^9*d^16 - 104*a^24*b*c^11*d^14 + 104*a^2*b^23*c^15*d^10 - 408*a^2*b^23*c^17*d^8 + 592*a^2*b^23*c^19*d^6 - 368*a^2*b^23*c^21*d^4 + 72*a^2*b^23*c^23*d^2 - 616*a^3*b^22*c^14*d^11 + 2392*a^3*b^22*c^16*d^9 - 3408*a^3*b^22*c^18*d^7 + 2032*a^3*b^22*c^20*d^5 - 328*a^3*b^22*c^22*d^3 + 2184*a^4*b^21*c^13*d^12 - 8536*a^4*b^21*c^15*d^10 + 12272*a^4*b^21*c^17*d^8 - 7408*a^4*b^21*c^19*d^6 + 1192*a^4*b^21*c^21*d^4 + 328*a^4*b^21*c^23*d^2 - 5096*a^5*b^20*c^12*d^13 + 20664*a^5*b^20*c^14*d^11 - 31328*a^5*b^20*c^16*d^9 + 20592*a^5*b^20*c^18*d^7 - 4008*a^5*b^20*c^20*d^5 - 1192*a^5*b^20*c^22*d^3 + 8008*a^6*b^19*c^11*d^14 - 35672*a^6*b^19*c^13*d^12 + 60768*a^6*b^19*c^15*d^10 - 46464*a^6*b^19*c^17*d^8 + 11336*a^6*b^19*c^19*d^6 + 4008*a^6*b^19*c^21*d^4 - 2032*a^6*b^19*c^23*d^2 - 8008*a^7*b^18*c^10*d^15 + 44408*a^7*b^18*c^12*d^13 - 92512*a^7*b^18*c^14*d^11 + 85536*a^7*b^18*c^16*d^9 - 24904*a^7*b^18*c^18*d^7 - 11336*a^7*b^18*c^20*d^5 + 7408*a^7*b^18*c^22*d^3 + 3432*a^8*b^17*c^9*d^16 - 37752*a^8*b^17*c^11*d^14 + 109408*a^8*b^17*c^13*d^12 - 125472*a^8*b^17*c^15*d^10 + 42696*a^8*b^17*c^17*d^8 + 24904*a^8*b^17*c^19*d^6 - 20592*a^8*b^17*c^21*d^4 + 3408*a^8*b^17*c^23*d^2 + 3432*a^9*b^16*c^8*d^17 + 14872*a^9*b^16*c^10*d^15 - 92352*a^9*b^16*c^12*d^13 + 141408*a^9*b^16*c^14*d^11 - 59264*a^9*b^16*c^16*d^9 - 42696*a^9*b^16*c^18*d^7 + 46464*a^9*b^16*c^20*d^5 - 12272*a^9*b^16*c^22*d^3 - 8008*a^10*b^15*c^7*d^18 + 14872*a^10*b^15*c^9*d^16 + 36608*a^10*b^15*c^11*d^14 - 113152*a^10*b^15*c^13*d^12 + 67008*a^10*b^15*c^15*d^10 + 59264*a^10*b^15*c^17*d^8 - 85536*a^10*b^15*c^19*d^6 + 31328*a^10*b^15*c^21*d^4 - 2392*a^10*b^15*c^23*d^2 + 8008*a^11*b^14*c^6*d^19 - 37752*a^11*b^14*c^8*d^17 + 36608*a^11*b^14*c^10*d^15 + 43264*a^11*b^14*c^12*d^13 - 56256*a^11*b^14*c^14*d^11 - 67008*a^11*b^14*c^16*d^9 + 125472*a^11*b^14*c^18*d^7 - 60768*a^11*b^14*c^20*d^5 + 8536*a^11*b^14*c^22*d^3 - 5096*a^12*b^13*c^5*d^20 + 44408*a^12*b^13*c^7*d^18 - 92352*a^12*b^13*c^9*d^16 + 43264*a^12*b^13*c^11*d^14 + 22464*a^12*b^13*c^13*d^12 + 56256*a^12*b^13*c^15*d^10 - 141408*a^12*b^13*c^17*d^8 + 92512*a^12*b^13*c^19*d^6 - 20664*a^12*b^13*c^21*d^4 + 616*a^12*b^13*c^23*d^2 + 2184*a^13*b^12*c^4*d^21 - 35672*a^13*b^12*c^6*d^19 + 109408*a^13*b^12*c^8*d^17 - 113152*a^13*b^12*c^10*d^15 + 22464*a^13

*b^12*c^12*d^13 - 22464*a^13*b^12*c^14*d^11 + 113152*a^13*b^12*c^16*d^9 - 1
09408*a^13*b^12*c^18*d^7 + 35672*a^13*b^12*c^20*d^5 - 2184*a^13*b^12*c^22*d
^3 - 616*a^14*b^11*c^3*d^22 + 20664*a^14*b^11*c^5*d^20 - 92512*a^14*b^11*c^
7*d^18 + 141408*a^14*b^11*c^9*d^16 - 56256*a^14*b^11*c^11*d^14 - 22464*a^14
*b^11*c^13*d^12 - 43264*a^14*b^11*c^15*d^10 + 92352*a^14*b^11*c^17*d^8 - 44
408*a^14*b^11*c^19*d^6 + 5096*a^14*b^11*c^21*d^4 + 104*a^15*b^10*c^2*d^23 -
8536*a^15*b^10*c^4*d^21 + 60768*a^15*b^10*c^6*d^19 - 125472*a^15*b^10*c^8*
d^17 + 67008*a^15*b^10*c^10*d^15 + 56256*a^15*b^10*c^12*d^13 - 43264*a^15*b
^10*c^14*d^11 - 36608*a^15*b^10*c^16*d^9 + 37752*a^15*b^10*c^18*d^7 - 8008*
a^15*b^10*c^20*d^5 + 2392*a^16*b^9*c^3*d^22 - 31328*a^16*b^9*c^5*d^20 + 855
36*a^16*b^9*c^7*d^18 - 59264*a^16*b^9*c^9*d^16 - 67008*a^16*b^9*c^11*d^14 +
113152*a^16*b^9*c^13*d^12 - 36608*a^16*b^9*c^15*d^10 - 14872*a^16*b^9*c^17
*d^8 + 8008*a^16*b^9*c^19*d^6 - 408*a^17*b^8*c^2*d^23 + 12272*a^17*b^8*c^4*
d^21 - 46464*a^17*b^8*c^6*d^19 + 42696*a^17*b^8*c^8*d^17 + 59264*a^17*b^8*c
^10*d^15 - 141408*a^17*b^8*c^12*d^13 + 92352*a^17*b^8*c^14*d^11 - 14872*a^1
7*b^8*c^16*d^9 - 3432*a^17*b^8*c^18*d^7 - 3408*a^18*b^7*c^3*d^22 + 20592*a^
18*b^7*c^5*d^20 - 24904*a^18*b^7*c^7*d^18 - 42696*a^18*b^7*c^9*d^16 + 12547
2*a^18*b^7*c^11*d^14 - 109408*a^18*b^7*c^13*d^12 + 37752*a^18*b^7*c^15*d^10
- 3432*a^18*b^7*c^17*d^8 + 592*a^19*b^6*c^2*d^23 - 7408*a^19*b^6*c^4*d^21
+ 11336*a^19*b^6*c^6*d^19 + 24904*a^19*b^6*c^8*d^17 - 85536*a^19*b^6*c^10*d
^15 + 92512*a^19*b^6*c^12*d^13 - 44408*a^19*b^6*c^14*d^11 + 8008*a^19*b^6*c
^16*d^9 + 2032*a^20*b^5*c^3*d^22 - 4008*a^20*b^5*c^5*d^20 - 11336*a^20*b^5*
c^7*d^18 + 46464*a^20*b^5*c^9*d^16 - 60768*a^20*b^5*c^11*d^14 + 35672*a^20*
b^5*c^13*d^12 - 8008*a^20*b^5*c^15*d^10 - 368*a^21*b^4*c^2*d^23 + 1192*a^21
*b^4*c^4*d^21 + 4008*a^21*b^4*c^6*d^19 - 20592*a^21*b^4*c^8*d^17 + 31328*a^
21*b^4*c^10*d^15 - 20664*a^21*b^4*c^12*d^13 + 5096*a^21*b^4*c^14*d^11 - 328
*a^22*b^3*c^3*d^22 - 1192*a^22*b^3*c^5*d^20 + 7408*a^22*b^3*c^7*d^18 - 1227
2*a^22*b^3*c^9*d^16 + 8536*a^22*b^3*c^11*d^14 - 2184*a^22*b^3*c^13*d^12 + 7
2*a^23*b^2*c^2*d^23 + 328*a^23*b^2*c^4*d^21 - 2032*a^23*b^2*c^6*d^19 + 3408
*a^23*b^2*c^8*d^17 - 2392*a^23*b^2*c^10*d^15 + 616*a^23*b^2*c^12*d^13 - 8*a
*b^24*c^24*d - 8*a^24*b*c*d^24))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 +
6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14
*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^
4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6
+ 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d
^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*
b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19
+ 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*
c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 6
6*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*
a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*
b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b
^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b
^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^
16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^1

5*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19) - (8*tan(e/2 + (f*x)/2)*(56*a^3*b^22*c^25 - 12*a^25*c*d^24 - 12*a*b^24*c^25 - 104*a^5*b^20*c^25 + 96*a^7*b^18*c^25 - 44*a^9*b^16*c^25 + 8*a^11*b^14*c^25 + 56*a^25*c^3*d^22 - 104*a^25*c^5*d^20 + 96*a^25*c^7*d^18 - 44*a^25*c^9*d^16 + 8*a^25*c^11*d^14 + 16*a*b^24*c^15*d^10 - 76*a*b^24*c^17*d^8 + 144*a*b^24*c^19*d^6 - 136*a*b^24*c^21*d^4 + 64*a*b^24*c^23*d^2 + 168*a^2*b^23*c^24*d - 784*a^4*b^21*c^24*d + 1456*a^6*b^19*c^24*d - 1344*a^8*b^17*c^24*d + 616*a^10*b^15*c^24*d - 112*a^12*b^13*c^24*d + 16*a^15*b^10*c*d^24 - 76*a^17*b^8*c*d^24 + 144*a^19*b^6*c*d^24 - 136*a^21*b^4*c*d^24 + 64*a^23*b^2*c*d^24 + 168*a^24*b*c^2*d^23 - 784*a^24*b*c^4*d^21 + 1456*a^24*b*c^6*d^19 - 1344*a^24*b*c^8*d^17 + 616*a^24*b*c^10*d^15 - 112*a^24*b*c^12*d^13 - 224*a^2*b^23*c^14*d^11 + 1064*a^2*b^23*c^16*d^9 - 2016*a^2*b^23*c^18*d^7 + 1904*a^2*b^23*c^20*d^5 - 896*a^2*b^23*c^22*d^3 + 1456*a^3*b^22*c^13*d^12 - 6992*a^3*b^22*c^15*d^10 + 13464*a^3*b^22*c^17*d^8 - 13056*a^3*b^22*c

^19*d^6 + 6464*a^3*b^22*c^21*d^4 - 1392*a^3*b^22*c^23*d^2 - 5824*a^4*b^21*c^12*d^13 + 28728*a^4*b^21*c^14*d^11 - 57456*a^4*b^21*c^16*d^9 + 59024*a^4*b^21*c^18*d^7 - 32256*a^4*b^21*c^20*d^5 + 8568*a^4*b^21*c^22*d^3 + 16016*a^5*b^20*c^11*d^14 - 82992*a^5*b^20*c^13*d^12 + 177048*a^5*b^20*c^15*d^10 - 198696*a^5*b^20*c^17*d^8 + 123584*a^5*b^20*c^19*d^6 - 40512*a^5*b^20*c^21*d^4 + 5656*a^5*b^20*c^23*d^2 - 32032*a^6*b^19*c^10*d^15 + 179816*a^6*b^19*c^12*d^13 - 421344*a^6*b^19*c^14*d^11 + 529312*a^6*b^19*c^16*d^9 - 379008*a^6*b^19*c^18*d^7 + 150024*a^6*b^19*c^20*d^5 - 28224*a^6*b^19*c^22*d^3 + 48048*a^7*b^18*c^9*d^16 - 304304*a^7*b^18*c^11*d^14 + 805896*a^7*b^18*c^13*d^12 - 1151104*a^7*b^18*c^15*d^10 + 949952*a^7*b^18*c^17*d^8 - 446736*a^7*b^18*c^19*d^6 + 108136*a^7*b^18*c^21*d^4 - 9984*a^7*b^18*c^23*d^2 - 54912*a^8*b^17*c^8*d^17 + 412984*a^8*b^17*c^10*d^15 - 1267344*a^8*b^17*c^12*d^13 + 2077536*a^8*b^17*c^14*d^11 - 1975808*a^8*b^17*c^16*d^9 + 1095384*a^8*b^17*c^18*d^7 - 331632*a^8*b^17*c^20*d^5 + 45136*a^8*b^17*c^22*d^3 + 48048*a^9*b^16*c^7*d^18 - 456456*a^9*b^16*c^9*d^16 + 1657656*a^9*b^16*c^11*d^14 - 3143504*a^9*b^16*c^13*d^12 + 3453696*a^9*b^16*c^15*d^10 - 2247636*a^9*b^16*c^17*d^8 + 831208*a^9*b^16*c^19*d^6 - 151944*a^9*b^16*c^21*d^4 + 8976*a^9*b^16*c^23*d^2 - 32032*a^10*b^15*c^6*d^19 + 412984*a^10*b^15*c^8*d^17 - 1812096*a^10*b^15*c^10*d^15 + 4016896*a^10*b^15*c^12*d^13 - 5121024*a^10*b^15*c^14*d^11 + 3897024*a^10*b^15*c^16*d^9 - 1728832*a^10*b^15*c^18*d^7 + 404768*a^10*b^15*c^20*d^5 - 38304*a^10*b^15*c^22*d^3 + 16016*a^11*b^14*c^5*d^20 - 304304*a^11*b^14*c^7*d^18 + 1657656*a^11*b^14*c^9*d^16 - 4356352*a^11*b^14*c^11*d^14 + 6476288*a^11*b^14*c^13*d^12 - 5745024*a^11*b^14*c^15*d^10 + 3021984*a^11*b^14*c^17*d^8 - 880256*a^11*b^14*c^19*d^6 + 118032*a^11*b^14*c^21*d^4 - 4048*a^11*b^14*c^23*d^2 - 5824*a^12*b^13*c^4*d^21 + 179816*a^12*b^13*c^6*d^19 - 1267344*a^12*b^13*c^8*d^17 + 4016896*a^12*b^13*c^10*d^15 - 7002112*a^12*b^13*c^12*d^13 + 7235136*a^12*b^13*c^14*d^11 - 4480896*a^12*b^13*c^16*d^9 + 1588704*a^12*b^13*c^18*d^7 - 280896*a^12*b^13*c^20*d^5 + 16632*a^12*b^13*c^22*d^3 + 1456*a^13*b^12*c^3*d^22 - 82992*a^13*b^12*c^5*d^20 + 805896*a^13*b^12*c^7*d^18 - 3143504*a^13*b^12*c^9*d^16 + 6476288*a^13*b^12*c^11*d^14 - 7809984*a^13*b^12*c^13*d^12 + 5666752*a^13*b^12*c^15*d^10 - 2403856*a^13*b^12*c^17*d^8 + 537264*a^13*b^12*c^19*d^6 - 48048*a^13*b^12*c^21*d^4 + 728*a^13*b^12*c^23*d^2 - 224*a^14*b^11*c^2*d^23 + 28728*a^14*b^11*c^4*d^21 - 421344*a^14*b^11*c^6*d^19 + 2077536*a^14*b^11*c^8*d^17 - 5121024*a^14*b^11*c^10*d^15 + 7235136*a^14*b^11*c^12*d^13 - 6126848*a^14*b^11*c^14*d^11 + 3071744*a^14*b^11*c^16*d^9 - 844896*a^14*b^11*c^18*d^7 + 104104*a^14*b^11*c^20*d^5 - 2912*a^14*b^11*c^22*d^3 - 6992*a^15*b^10*c^3*d^22 + 177048*a^15*b^10*c^5*d^20 - 1151104*a^15*b^10*c^7*d^18 + 3453696*a^15*b^10*c^9*d^16 - 5745024*a^15*b^10*c^11*d^14 + 5666752*a^15*b^10*c^13*d^12 - 3331328*a^15*b^10*c^15*d^10 + 1105104*a^15*b^10*c^17*d^8 - 176176*a^15*b^10*c^19*d^6 + 8008*a^15*b^10*c^21*d^4 + 1064*a^16*b^9*c^2*d^23 - 57456*a^16*b^9*c^4*d^21 + 529312*a^16*b^9*c^6*d^19 - 1975808*a^16*b^9*c^8*d^17 + 3897024*a^16*b^9*c^10*d^15 - 4480896*a^16*b^9*c^12*d^13 + 3071744*a^16*b^9*c^14*d^11 - 1208064*a^16*b^9*c^16*d^9 + 239096*a^16*b^9*c^18*d^7 - 16016*a^16*b^9*c^20*d^5 + 13464*a^17*b^8*c^3*d^22 - 198696*a^17*b^8*c^5*d^20 + 949952*a^17*b^8*c^7*d^18 - 2247636*a^

17*b^8*c^9*d^16 + 3021984*a^17*b^8*c^11*d^14 - 2403856*a^17*b^8*c^13*d^12 + 1105104*a^17*b^8*c^15*d^10 - 264264*a^17*b^8*c^17*d^8 + 24024*a^17*b^8*c^19*d^6 - 2016*a^18*b^7*c^2*d^23 + 59024*a^18*b^7*c^4*d^21 - 379008*a^18*b^7*c^6*d^19 + 1095384*a^18*b^7*c^8*d^17 - 1728832*a^18*b^7*c^10*d^15 + 1588704*a^18*b^7*c^12*d^13 - 844896*a^18*b^7*c^14*d^11 + 239096*a^18*b^7*c^16*d^9 - 27456*a^18*b^7*c^18*d^7 - 13056*a^19*b^6*c^3*d^22 + 123584*a^19*b^6*c^5*d^20 - 446736*a^19*b^6*c^7*d^18 + 831208*a^19*b^6*c^9*d^16 - 880256*a^19*b^6*c^11*d^14 + 537264*a^19*b^6*c^13*d^12 - 176176*a^19*b^6*c^15*d^10 + 24024*a^19*b^6*c^17*d^8 + 1904*a^20*b^5*c^2*d^23 - 32256*a^20*b^5*c^4*d^21 + 150024*a^20*b^5*c^6*d^19 - 331632*a^20*b^5*c^8*d^17 + 404768*a^20*b^5*c^10*d^15 - 280896*a^20*b^5*c^12*d^13 + 104104*a^20*b^5*c^14*d^11 - 16016*a^20*b^5*c^16*d^9 + 6464*a^21*b^4*c^3*d^22 - 40512*a^21*b^4*c^5*d^20 + 108136*a^21*b^4*c^7*d^18 - 151944*a^21*b^4*c^9*d^16 + 118032*a^21*b^4*c^11*d^14 - 48048*a^21*b^4*c^13*d^12 + 8008*a^21*b^4*c^15*d^10 - 896*a^22*b^3*c^2*d^23 + 8568*a^22*b^3*c^4*d^21 - 28224*a^22*b^3*c^6*d^19 + 45136*a^22*b^3*c^8*d^17 - 38304*a^22*b^3*c^10*d^15 + 16632*a^22*b^3*c^12*d^13 - 2912*a^22*b^3*c^14*d^11 - 1392*a^23*b^2*c^3*d^22 + 5656*a^23*b^2*c^5*d^20 - 9984*a^23*b^2*c^7*d^18 + 8976*a^23*b^2*c^9*d^16 - 4048*a^23*b^2*c^11*d^14 + 728*a^23*b^2*c^13*d^12))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300* a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19))*(-(((4*a^24*d^24 + 4*b^24*c^24 + 16*a^2*b^22*c^24 + 16*a^4*b^20*c^24 - 1152*a^10*b^14*d^24 + 5568*a^12*b^12*d^24 - 10568*a^14*b^10*d^24 + 9460*a^16*b^8*d^24 - 3560*a^18*b^6*d^24 + 136*a^20*b^4*d^24 + 76*a^22*b^2*d^24 + 16*a^24*c^2*d^22 + 16*a^24*c^4*d^20 - 1152*b^24*c^10*d^14 + 5568*b^24*c^12*d^12 - 10568*b^24*c^14*d^10 + 9460*b^24*c^16*d^8 - 3560*b^24*c^18*d^6 + 136*b^24*c^20*d^4 + 76*b^24*c^22*d^2 + 11520*a*b^23*c^9*d^15 - 56448*a*b^23*c^11*d^13 + 109456*a*b^23*c^13*d^11 - 101240*a*b^23*c^15*d^9 + 40720*a*b^23*c^17*d^7 - 2960*a*b^23*c^19*d^5 - 536*a*b^23*c^21*d^3 - 176*a^3*b^21*c^23*d - 320*a^5*b^19*c^23*d + 11520*a^9*b^15*c*d^23 - 56448*a^11*b^13*c*d^23 + 109456*a^13*b^11*c*d^23 - 101240*a^15*b^9*c*d^23 + 40720*a^17*b^7*c*d^23 - 2960*a^19*b^5*c*d^23 - 536*a^21*b^3*c*d^23 - 176*a^23*b*c^3*d^21 - 320*a^23*b*c^5*d^19 - 51840*a^2*b^22*c^8*d^16 + 263808*a^2*b^22*c^10*d^14 - 541208*a^2*b^22*c^12*d^12 + 547088*a^2*b^22*c^14*d^10 - 263320*a^2*b^22*c^16*d^8 + 44120*a^2*b^22*c^18*d^6 - 1564*a^2*b^22*c^20*d^4 - 196*a^2*b^22*c^22*d^2 + 138240*a^3*b^21*c^7*d^17 - 758400*a^3*b^21*c^9*d^15 + 1720736*a^3*b^21*c^11*d^13 - 2002728*a^3*b^21*c^13*d^11 + 1210560*a^3*b^21*c^15*d^9 - 335040*a^3*b^21*c^17*d^7 + 37680*a^3*b^21*c^19*d^5 - 288*a^3*b^21*c^21*d^3 - 241920*a^4*b^20*c^6*d^18 + 1512000*a^4*b^20*c^8*d^16 - 3975688*a^4*b^20*c^10*d^14 + 5501328*a^4*b^20*c^12*d^12 - 4147952*a^4*b^20*c^14*d^10 + 1586920*a^4*b^20*c^16*d^8 - 276020*a^4*b^20*c^18*d^6 + 21124*a^4*b^20*c^20*d^4 + 176*a^4*b^20*c^22*d^2 + 290304*a^5*b^19*c^5*d^19 - 2232576*a^5*b^19*c^7*d^17 + 7078256*a^5*b^19*c^9*d^15 - 11781560*a^5*b^19*c^11*d^13 + 10875200*a^5*b^19*c^13*d^11 - 5365072*a^5*b^19*c^15*d^9 + 1310168*a^5*b^19*c^17*d^7 - 170968*a^5*b^19*c^19*d^5 + 8160*a^5*b^19*c^21*d^3 - 241920*a^6*b^18*c^4*d^20 + 2532096*a^6*b^18*c^6*d^18 - 9955992*a^6*b^18*c^8*d^16 + 20019440*a^6*b^18*c^10*d^14 - 22419600*a^6*b^

18*c^12*d^12 + 13887520*a^6*b^18*c^14*d^10 - 4506428*a^6*b^18*c^16*d^8 + 79
3756*a^6*b^18*c^18*d^6 - 72240*a^6*b^18*c^20*d^4 + 3040*a^6*b^18*c^22*d^2 +
138240*a^7*b^17*c^3*d^21 - 2232576*a^7*b^17*c^5*d^19 + 11150016*a^7*b^17*c
^7*d^17 - 27336616*a^7*b^17*c^9*d^15 + 37153600*a^7*b^17*c^11*d^13 - 284610
40*a^7*b^17*c^13*d^11 + 11779808*a^7*b^17*c^15*d^9 - 2621008*a^7*b^17*c^17*
d^7 + 336688*a^7*b^17*c^19*d^5 - 17920*a^7*b^17*c^21*d^3 - 51840*a^8*b^16*c
^2*d^22 + 1512000*a^8*b^16*c^4*d^20 - 9955992*a^8*b^16*c^6*d^18 + 30289656*
a^8*b^16*c^8*d^16 - 50137600*a^8*b^16*c^10*d^14 + 46972560*a^8*b^16*c^12*d^
12 - 24199280*a^8*b^16*c^14*d^10 + 6661036*a^8*b^16*c^16*d^8 - 1058448*a^8*
b^16*c^18*d^6 + 72560*a^8*b^16*c^20*d^4 - 758400*a^9*b^15*c^3*d^21 + 707825
6*a^9*b^15*c^5*d^19 - 27336616*a^9*b^15*c^7*d^17 + 55383904*a^9*b^15*c^9*d^
15 - 63124080*a^9*b^15*c^11*d^13 + 39987520*a^9*b^15*c^13*d^11 - 13462088*a
^9*b^15*c^15*d^9 + 2478528*a^9*b^15*c^17*d^7 - 212032*a^9*b^15*c^19*d^5 + 2
63808*a^10*b^14*c^2*d^22 - 3975688*a^10*b^14*c^4*d^20 + 20019440*a^10*b^14*
c^6*d^18 - 50137600*a^10*b^14*c^8*d^16 + 69593872*a^10*b^14*c^10*d^14 - 538
54288*a^10*b^14*c^12*d^12 + 21989928*a^10*b^14*c^14*d^10 - 4591360*a^10*b^1
4*c^16*d^8 + 460480*a^10*b^14*c^18*d^6 + 1720736*a^11*b^13*c^3*d^21 - 11781
560*a^11*b^13*c^5*d^19 + 37153600*a^11*b^13*c^7*d^17 - 63124080*a^11*b^13*c
^9*d^15 + 59445728*a^11*b^13*c^11*d^13 - 29358696*a^11*b^13*c^13*d^11 + 699
5840*a^11*b^13*c^15*d^9 - 762560*a^11*b^13*c^17*d^7 - 541208*a^12*b^12*c^2*
d^22 + 5501328*a^12*b^12*c^4*d^20 - 22419600*a^12*b^12*c^6*d^18 + 46972560*
a^12*b^12*c^8*d^16 - 53854288*a^12*b^12*c^10*d^14 + 32294808*a^12*b^12*c^12
*d^12 - 8958208*a^12*b^12*c^14*d^10 + 999040*a^12*b^12*c^16*d^8 - 2002728*a
^13*b^11*c^3*d^21 + 10875200*a^13*b^11*c^5*d^19 - 28461040*a^13*b^11*c^7*d^
17 + 39987520*a^13*b^11*c^9*d^15 - 29358696*a^13*b^11*c^11*d^13 + 9722048*a
^13*b^11*c^13*d^11 - 1104320*a^13*b^11*c^15*d^9 + 547088*a^14*b^10*c^2*d^22
- 4147952*a^14*b^10*c^4*d^20 + 13887520*a^14*b^10*c^6*d^18 - 24199280*a^14
*b^10*c^8*d^16 + 21989928*a^14*b^10*c^10*d^14 - 8958208*a^14*b^10*c^12*d^12
+ 1124032*a^14*b^10*c^14*d^10 + 1210560*a^15*b^9*c^3*d^21 - 5365072*a^15*b
^9*c^5*d^19 + 11779808*a^15*b^9*c^7*d^17 - 13462088*a^15*b^9*c^9*d^15 + 699
5840*a^15*b^9*c^11*d^13 - 1104320*a^15*b^9*c^13*d^11 - 263320*a^16*b^8*c^2*
d^22 + 1586920*a^16*b^8*c^4*d^20 - 4506428*a^16*b^8*c^6*d^18 + 6661036*a^16
*b^8*c^8*d^16 - 4591360*a^16*b^8*c^10*d^14 + 999040*a^16*b^8*c^12*d^12 - 33
5040*a^17*b^7*c^3*d^21 + 1310168*a^17*b^7*c^5*d^19 - 2621008*a^17*b^7*c^7*d
^17 + 2478528*a^17*b^7*c^9*d^15 - 762560*a^17*b^7*c^11*d^13 + 44120*a^18*b^
6*c^2*d^22 - 276020*a^18*b^6*c^4*d^20 + 793756*a^18*b^6*c^6*d^18 - 1058448*
a^18*b^6*c^8*d^16 + 460480*a^18*b^6*c^10*d^14 + 37680*a^19*b^5*c^3*d^21 - 1
70968*a^19*b^5*c^5*d^19 + 336688*a^19*b^5*c^7*d^17 - 212032*a^19*b^5*c^9*d^
15 - 1564*a^20*b^4*c^2*d^22 + 21124*a^20*b^4*c^4*d^20 - 72240*a^20*b^4*c^6*
d^18 + 72560*a^20*b^4*c^8*d^16 - 288*a^21*b^3*c^3*d^21 + 8160*a^21*b^3*c^5*
d^19 - 17920*a^21*b^3*c^7*d^17 - 196*a^22*b^2*c^2*d^22 + 176*a^22*b^2*c^4*d
^20 + 3040*a^22*b^2*c^6*d^18 - 8*a*b^23*c^23*d - 8*a^23*b*c*d^23)^2/4 - (20
736*b^18*d^18 - 96768*a^2*b^16*d^18 + 173664*a^4*b^14*d^18 - 136032*a^6*b^1
2*d^18 + 31081*a^8*b^10*d^18 + 8440*a^10*b^8*d^18 + 400*a^12*b^6*d^18 - 967
68*b^18*c^2*d^16 + 173664*b^18*c^4*d^14 - 136032*b^18*c^6*d^12 + 31081*b^18

*c^8*d^10 + 8440*b^18*c^10*d^8 + 400*b^18*c^12*d^6 - 131328*a*b^17*c^3*d^15
 + 216576*a*b^17*c^5*d^13 - 141104*a*b^17*c^7*d^11 + 20260*a*b^17*c^9*d^9 +
 2800*a*b^17*c^11*d^7 - 131328*a^3*b^15*c*d^17 + 216576*a^5*b^13*c*d^17 - 1
41104*a^7*b^11*c*d^17 + 20260*a^9*b^9*c*d^17 + 2800*a^11*b^7*c*d^17 + 49593
6*a^2*b^16*c^2*d^16 - 989856*a^2*b^16*c^4*d^14 + 901948*a^2*b^16*c^6*d^12 -
 308392*a^2*b^16*c^8*d^10 - 5260*a^2*b^16*c^10*d^8 + 1600*a^2*b^16*c^12*d^6
 + 657408*a^3*b^15*c^3*d^15 - 1158992*a^3*b^15*c^5*d^13 + 838256*a^3*b^15*c
^7*d^11 - 182200*a^3*b^15*c^9*d^9 - 3200*a^3*b^15*c^11*d^7 - 989856*a^4*b^1
4*c^2*d^16 + 2185654*a^4*b^14*c^4*d^14 - 2218576*a^4*b^14*c^6*d^12 + 900624
*a^4*b^14*c^8*d^10 - 64720*a^4*b^14*c^10*d^8 + 1600*a^4*b^14*c^12*d^6 - 115
8992*a^5*b^13*c^3*d^15 + 2158808*a^5*b^13*c^5*d^13 - 1641528*a^5*b^13*c^7*d
^11 + 406880*a^5*b^13*c^9*d^9 - 17600*a^5*b^13*c^11*d^7 + 901948*a^6*b^12*c
^2*d^16 - 2218576*a^6*b^12*c^4*d^14 + 2430936*a^6*b^12*c^6*d^12 - 1026928*a
^6*b^12*c^8*d^10 + 88720*a^6*b^12*c^10*d^8 + 838256*a^7*b^11*c^3*d^15 - 164
1528*a^7*b^11*c^5*d^13 + 1206848*a^7*b^11*c^7*d^11 - 239360*a^7*b^11*c^9*d^
9 - 308392*a^8*b^10*c^2*d^16 + 900624*a^8*b^10*c^4*d^14 - 1026928*a^8*b^10*
c^6*d^12 + 354016*a^8*b^10*c^8*d^10 - 182200*a^9*b^9*c^3*d^15 + 406880*a^9*
b^9*c^5*d^13 - 239360*a^9*b^9*c^7*d^11 - 5260*a^10*b^8*c^2*d^16 - 64720*a^1
0*b^8*c^4*d^14 + 88720*a^10*b^8*c^6*d^12 - 3200*a^11*b^7*c^3*d^15 - 17600*a
^11*b^7*c^5*d^13 + 1600*a^12*b^6*c^2*d^16 + 1600*a^12*b^6*c^4*d^14 + 27648*
a*b^17*c*d^17)*(80*a^2*b^28*c^30 - 16*b^30*c^30 - 16*a^30*d^30 - 160*a^4*b^
26*c^30 + 160*a^6*b^24*c^30 - 80*a^8*b^22*c^30 + 16*a^10*b^20*c^30 + 16*a^2
0*b^10*d^30 - 80*a^22*b^8*d^30 + 160*a^24*b^6*d^30 - 160*a^26*b^4*d^30 + 80
*a^28*b^2*d^30 + 80*a^30*c^2*d^28 - 160*a^30*c^4*d^26 + 160*a^30*c^6*d^24 -
 80*a^30*c^8*d^22 + 16*a^30*c^10*d^20 + 16*b^30*c^20*d^10 - 80*b^30*c^22*d^
8 + 160*b^30*c^24*d^6 - 160*b^30*c^26*d^4 + 80*b^30*c^28*d^2 - 320*a*b^29*c
^19*d^11 + 1600*a*b^29*c^21*d^9 - 3200*a*b^29*c^23*d^7 + 3200*a*b^29*c^25*d
^5 - 1600*a*b^29*c^27*d^3 - 1600*a^3*b^27*c^29*d + 3200*a^5*b^25*c^29*d - 3
200*a^7*b^23*c^29*d + 1600*a^9*b^21*c^29*d - 320*a^11*b^19*c^29*d - 320*a^1
9*b^11*c*d^29 + 1600*a^21*b^9*c*d^29 - 3200*a^23*b^7*c*d^29 + 3200*a^25*b^5
*c*d^29 - 1600*a^27*b^3*c*d^29 - 1600*a^29*b*c^3*d^27 + 3200*a^29*b*c^5*d^2
5 - 3200*a^29*b*c^7*d^23 + 1600*a^29*b*c^9*d^21 - 320*a^29*b*c^11*d^19 + 30
40*a^2*b^28*c^18*d^12 - 15280*a^2*b^28*c^20*d^10 + 30800*a^2*b^28*c^22*d^8
- 31200*a^2*b^28*c^24*d^6 + 16000*a^2*b^28*c^26*d^4 - 3440*a^2*b^28*c^28*d^
2 - 18240*a^3*b^27*c^17*d^13 + 92800*a^3*b^27*c^19*d^11 - 190400*a^3*b^27*c
^21*d^9 + 198400*a^3*b^27*c^23*d^7 - 107200*a^3*b^27*c^25*d^5 + 26240*a^3*b
^27*c^27*d^3 + 77520*a^4*b^26*c^16*d^14 - 402800*a^4*b^26*c^18*d^12 + 85136
0*a^4*b^26*c^20*d^10 - 928000*a^4*b^26*c^22*d^8 + 541200*a^4*b^26*c^24*d^6
- 155120*a^4*b^26*c^26*d^4 + 16000*a^4*b^26*c^28*d^2 - 248064*a^5*b^25*c^15
*d^15 + 1331520*a^5*b^25*c^17*d^13 - 2939840*a^5*b^25*c^19*d^11 + 3408640*a
^5*b^25*c^21*d^9 - 2184320*a^5*b^25*c^23*d^7 + 736064*a^5*b^25*c^25*d^5 - 1
07200*a^5*b^25*c^27*d^3 + 620160*a^6*b^24*c^14*d^16 - 3488400*a^6*b^24*c^16
*d^14 + 8170000*a^6*b^24*c^18*d^12 - 10229760*a^6*b^24*c^20*d^10 + 7281600*
a^6*b^24*c^22*d^8 - 2863760*a^6*b^24*c^24*d^6 + 541200*a^6*b^24*c^26*d^4 -
31200*a^6*b^24*c^28*d^2 - 1240320*a^7*b^23*c^13*d^17 + 7441920*a^7*b^23*c^1

5*d^15 - 18787200*a^7*b^23*c^17*d^13 + 25721600*a^7*b^23*c^19*d^11 - 204448
00*a^7*b^23*c^21*d^9 + 9297920*a^7*b^23*c^23*d^7 - 2184320*a^7*b^23*c^25*d^
5 + 198400*a^7*b^23*c^27*d^3 + 2015520*a^8*b^22*c^12*d^18 - 13178400*a^8*b^
22*c^14*d^16 + 36434400*a^8*b^22*c^16*d^14 - 55069600*a^8*b^22*c^18*d^12 +
48989680*a^8*b^22*c^20*d^10 - 25575920*a^8*b^22*c^22*d^8 + 7281600*a^8*b^22
*c^24*d^6 - 928000*a^8*b^22*c^26*d^4 + 30800*a^8*b^22*c^28*d^2 - 2687360*a^
9*b^21*c^11*d^19 + 19638400*a^9*b^21*c^13*d^17 - 60362240*a^9*b^21*c^15*d^1
5 + 101475200*a^9*b^21*c^17*d^13 - 101172800*a^9*b^21*c^19*d^11 + 60333760*
a^9*b^21*c^21*d^9 - 20444800*a^9*b^21*c^23*d^7 + 3408640*a^9*b^21*c^25*d^5
- 190400*a^9*b^21*c^27*d^3 + 2956096*a^10*b^20*c^10*d^20 - 24858080*a^10*b^
20*c^12*d^18 + 86150560*a^10*b^20*c^14*d^16 - 162120160*a^10*b^20*c^16*d^14
 + 181463680*a^10*b^20*c^18*d^12 - 123188112*a^10*b^20*c^20*d^10 + 48989680
*a^10*b^20*c^22*d^8 - 10229760*a^10*b^20*c^24*d^6 + 851360*a^10*b^20*c^26*d
^4 - 15280*a^10*b^20*c^28*d^2 - 2687360*a^11*b^19*c^9*d^21 + 26873600*a^11*
b^19*c^11*d^19 - 106460800*a^11*b^19*c^13*d^17 + 225738240*a^11*b^19*c^15*d
^15 - 284331200*a^11*b^19*c^17*d^13 + 219166080*a^11*b^19*c^19*d^11 - 10117
2800*a^11*b^19*c^21*d^9 + 25721600*a^11*b^19*c^23*d^7 - 2939840*a^11*b^19*c
^25*d^5 + 92800*a^11*b^19*c^27*d^3 + 2015520*a^12*b^18*c^8*d^22 - 24858080*
a^12*b^18*c^10*d^20 + 114212800*a^12*b^18*c^12*d^18 - 274937600*a^12*b^18*c
^14*d^16 + 390830000*a^12*b^18*c^16*d^14 - 341426960*a^12*b^18*c^18*d^12 +
181463680*a^12*b^18*c^20*d^10 - 55069600*a^12*b^18*c^22*d^8 + 8170000*a^12*
b^18*c^24*d^6 - 402800*a^12*b^18*c^26*d^4 + 3040*a^12*b^18*c^28*d^2 - 12403
20*a^13*b^17*c^7*d^23 + 19638400*a^13*b^17*c^9*d^21 - 106460800*a^13*b^17*c
^11*d^19 + 293542400*a^13*b^17*c^13*d^17 - 472561920*a^13*b^17*c^15*d^15 +
467412160*a^13*b^17*c^17*d^13 - 284331200*a^13*b^17*c^19*d^11 + 101475200*a
^13*b^17*c^21*d^9 - 18787200*a^13*b^17*c^23*d^7 + 1331520*a^13*b^17*c^25*d^
5 - 18240*a^13*b^17*c^27*d^3 + 620160*a^14*b^16*c^6*d^24 - 13178400*a^14*b^
16*c^8*d^22 + 86150560*a^14*b^16*c^10*d^20 - 274937600*a^14*b^16*c^12*d^18
+ 503363200*a^14*b^16*c^14*d^16 - 563751280*a^14*b^16*c^16*d^14 + 390830000
*a^14*b^16*c^18*d^12 - 162120160*a^14*b^16*c^20*d^10 + 36434400*a^14*b^16*c
^22*d^8 - 3488400*a^14*b^16*c^24*d^6 + 77520*a^14*b^16*c^26*d^4 - 248064*a^
15*b^15*c^5*d^25 + 7441920*a^15*b^15*c^7*d^23 - 60362240*a^15*b^15*c^9*d^21
 + 225738240*a^15*b^15*c^11*d^19 - 472561920*a^15*b^15*c^13*d^17 + 59998412
8*a^15*b^15*c^15*d^15 - 472561920*a^15*b^15*c^17*d^13 + 225738240*a^15*b^15
*c^19*d^11 - 60362240*a^15*b^15*c^21*d^9 + 7441920*a^15*b^15*c^23*d^7 - 248
064*a^15*b^15*c^25*d^5 + 77520*a^16*b^14*c^4*d^26 - 3488400*a^16*b^14*c^6*d
^24 + 36434400*a^16*b^14*c^8*d^22 - 162120160*a^16*b^14*c^10*d^20 + 3908300
00*a^16*b^14*c^12*d^18 - 563751280*a^16*b^14*c^14*d^16 + 503363200*a^16*b^1
4*c^16*d^14 - 274937600*a^16*b^14*c^18*d^12 + 86150560*a^16*b^14*c^20*d^10
- 13178400*a^16*b^14*c^22*d^8 + 620160*a^16*b^14*c^24*d^6 - 18240*a^17*b^13
*c^3*d^27 + 1331520*a^17*b^13*c^5*d^25 - 18787200*a^17*b^13*c^7*d^23 + 1014
75200*a^17*b^13*c^9*d^21 - 284331200*a^17*b^13*c^11*d^19 + 467412160*a^17*b
^13*c^13*d^17 - 472561920*a^17*b^13*c^15*d^15 + 293542400*a^17*b^13*c^17*d^
13 - 106460800*a^17*b^13*c^19*d^11 + 19638400*a^17*b^13*c^21*d^9 - 1240320*
a^17*b^13*c^23*d^7 + 3040*a^18*b^12*c^2*d^28 - 402800*a^18*b^12*c^4*d^26 +

8170000*a^18*b^12*c^6*d^24 - 55069600*a^18*b^12*c^8*d^22 + 181463680*a^18*b^12*c^10*d^20 - 341426960*a^18*b^12*c^12*d^18 + 390830000*a^18*b^12*c^14*d^16 - 274937600*a^18*b^12*c^16*d^14 + 114212800*a^18*b^12*c^18*d^12 - 24858080*a^18*b^12*c^20*d^10 + 2015520*a^18*b^12*c^22*d^8 + 92800*a^19*b^11*c^3*d^27 - 2939840*a^19*b^11*c^5*d^25 + 25721600*a^19*b^11*c^7*d^23 - 101172800*a^19*b^11*c^9*d^21 + 219166080*a^19*b^11*c^11*d^19 - 284331200*a^19*b^11*c^13*d^17 + 225738240*a^19*b^11*c^15*d^15 - 106460800*a^19*b^11*c^17*d^13 + 26873600*a^19*b^11*c^19*d^11 - 2687360*a^19*b^11*c^21*d^9 - 15280*a^20*b^10*c^2*d^28 + 851360*a^20*b^10*c^4*d^26 - 10229760*a^20*b^10*c^6*d^24 + 48989680*a^20*b^10*c^8*d^22 - 123188112*a^20*b^10*c^10*d^20 + 181463680*a^20*b^10*c^12*d^18 - 162120160*a^20*b^10*c^14*d^16 + 86150560*a^20*b^10*c^16*d^14 - 24858080*a^20*b^10*c^18*d^12 + 2956096*a^20*b^10*c^20*d^10 - 190400*a^21*b^9*c^3*d^27 + 3408640*a^21*b^9*c^5*d^25 - 20444800*a^21*b^9*c^7*d^23 + 60333760*a^21*b^9*c^9*d^21 - 101172800*a^21*b^9*c^11*d^19 + 101475200*a^21*b^9*c^13*d^17 - 60362240*a^21*b^9*c^15*d^15 + 19638400*a^21*b^9*c^17*d^13 - 2687360*a^21*b^9*c^19*d^11 + 30800*a^22*b^8*c^2*d^28 - 928000*a^22*b^8*c^4*d^26 + 7281600*a^22*b^8*c^6*d^24 - 25575920*a^22*b^8*c^8*d^22 + 48989680*a^22*b^8*c^10*d^20 - 55069600*a^22*b^8*c^12*d^18 + 36434400*a^22*b^8*c^14*d^16 - 13178400*a^22*b^8*c^16*d^14 + 2015520*a^22*b^8*c^18*d^12 + 198400*a^23*b^7*c^3*d^27 - 2184320*a^23*b^7*c^5*d^25 + 9297920*a^23*b^7*c^7*d^23 - 20444800*a^23*b^7*c^9*d^21 + 25721600*a^23*b^7*c^11*d^19 - 18787200*a^23*b^7*c^13*d^17 + 7441920*a^23*b^7*c^15*d^15 - 1240320*a^23*b^7*c^17*d^13 - 31200*a^24*b^6*c^2*d^28 + 541200*a^24*b^6*c^4*d^26 - 2863760*a^24*b^6*c^6*d^24 + 7281600*a^24*b^6*c^8*d^22 - 10229760*a^24*b^6*c^10*d^20 + 8170000*a^24*b^6*c^12*d^18 - 3488400*a^24*b^6*c^14*d^16 + 620160*a^24*b^6*c^16*d^14 - 107200*a^25*b^5*c^3*d^27 + 736064*a^25*b^5*c^5*d^25 - 2184320*a^25*b^5*c^7*d^23 + 3408640*a^25*b^5*c^9*d^21 - 2939840*a^25*b^5*c^11*d^19 + 1331520*a^25*b^5*c^13*d^17 - 248064*a^25*b^5*c^15*d^15 + 16000*a^26*b^4*c^2*d^28 - 155120*a^26*b^4*c^4*d^26 + 541200*a^26*b^4*c^6*d^24 - 928000*a^26*b^4*c^8*d^22 + 851360*a^26*b^4*c^10*d^20 - 402800*a^26*b^4*c^12*d^18 + 77520*a^26*b^4*c^14*d^16 + 26240*a^27*b^3*c^3*d^27 - 107200*a^27*b^3*c^5*d^25 + 198400*a^27*b^3*c^7*d^23 - 190400*a^27*b^3*c^9*d^21 + 92800*a^27*b^3*c^11*d^19 - 18240*a^27*b^3*c^13*d^17 - 3440*a^28*b^2*c^2*d^28 + 16000*a^28*b^2*c^4*d^26 - 31200*a^28*b^2*c^6*d^24 + 30800*a^28*b^2*c^8*d^22 - 15280*a^28*b^2*c^10*d^20 + 3040*a^28*b^2*c^12*d^18 + 320*a*b^29*c^29*d + 320*a^29*b*c*d^29))^(1/2) + 2*a^24*d^24 + 2*b^24*c^24 + 8*a^2*b^22*c^24 + 8*a^4*b^20*c^24 - 576*a^10*b^14*d^24 + 2784*a^12*b^12*d^24 - 5284*a^14*b^10*d^24 + 4730*a^16*b^8*d^24 - 1780*a^18*b^6*d^24 + 68*a^20*b^4*d^24 + 38*a^22*b^2*d^24 + 8*a^24*c^2*d^22 + 8*a^24*c^4*d^20 - 576*b^24*c^10*d^14 + 2784*b^24*c^12*d^12 - 5284*b^24*c^14*d^10 + 4730*b^24*c^16*d^8 - 1780*b^24*c^18*d^6 + 68*b^24*c^20*d^4 + 38*b^24*c^22*d^2 + 5760*a*b^23*c^9*d^15 - 28224*a*b^23*c^11*d^13 + 54728*a*b^23*c^13*d^11 - 50620*a*b^23*c^15*d^9 + 20360*a*b^23*c^17*d^7 - 1480*a*b^23*c^19*d^5 - 268*a*b^23*c^21*d^3 - 88*a^3*b^21*c^23*d - 160*a^5*b^19*c^23*d + 5760*a^9*b^15*c*d^23 - 28224*a^11*b^13*c*d^23 + 54728*a^13*b^11*c*d^23 - 50620*a^15*b^9*c*d^23 + 20360*a^17*b^7*c*d^23 - 1480*a^19*b^5*c*d^23 - 268*a^21*b^3*c*d

^23 - 88*a^23*b*c^3*d^21 - 160*a^23*b*c^5*d^19 - 25920*a^2*b^22*c^8*d^16 + 131904*a^2*b^22*c^10*d^14 - 270604*a^2*b^22*c^12*d^12 + 273544*a^2*b^22*c^14*d^10 - 131660*a^2*b^22*c^16*d^8 + 22060*a^2*b^22*c^18*d^6 - 782*a^2*b^22*c^20*d^4 - 98*a^2*b^22*c^22*d^2 + 69120*a^3*b^21*c^7*d^17 - 379200*a^3*b^21*c^9*d^15 + 860368*a^3*b^21*c^11*d^13 - 1001364*a^3*b^21*c^13*d^11 + 605280*a^3*b^21*c^15*d^9 - 167520*a^3*b^21*c^17*d^7 + 18840*a^3*b^21*c^19*d^5 - 144*a^3*b^21*c^21*d^3 - 120960*a^4*b^20*c^6*d^18 + 756000*a^4*b^20*c^8*d^16 - 1987844*a^4*b^20*c^10*d^14 + 2750664*a^4*b^20*c^12*d^12 - 2073976*a^4*b^20*c^14*d^10 + 793460*a^4*b^20*c^16*d^8 - 138010*a^4*b^20*c^18*d^6 + 10562*a^4*b^20*c^20*d^4 + 88*a^4*b^20*c^22*d^2 + 145152*a^5*b^19*c^5*d^19 - 1116288*a^5*b^19*c^7*d^17 + 3539128*a^5*b^19*c^9*d^15 - 5890780*a^5*b^19*c^11*d^13 + 5437600*a^5*b^19*c^13*d^11 - 2682536*a^5*b^19*c^15*d^9 + 655084*a^5*b^19*c^17*d^7 - 85484*a^5*b^19*c^19*d^5 + 4080*a^5*b^19*c^21*d^3 - 120960*a^6*b^18*c^4*d^20 + 1266048*a^6*b^18*c^6*d^18 - 4977996*a^6*b^18*c^8*d^16 + 10009720*a^6*b^18*c^10*d^14 - 11209800*a^6*b^18*c^12*d^12 + 6943760*a^6*b^18*c^14*d^10 - 2253214*a^6*b^18*c^16*d^8 + 396878*a^6*b^18*c^18*d^6 - 36120*a^6*b^18*c^20*d^4 + 1520*a^6*b^18*c^22*d^2 + 69120*a^7*b^17*c^3*d^21 - 1116288*a^7*b^17*c^5*d^19 + 5575008*a^7*b^17*c^7*d^17 - 13668308*a^7*b^17*c^9*d^15 + 18576800*a^7*b^17*c^11*d^13 - 14230520*a^7*b^17*c^13*d^11 + 5889904*a^7*b^17*c^15*d^9 - 1310504*a^7*b^17*c^17*d^7 + 168344*a^7*b^17*c^19*d^5 - 8960*a^7*b^17*c^21*d^3 - 25920*a^8*b^16*c^2*d^22 + 756000*a^8*b^16*c^4*d^20 - 4977996*a^8*b^16*c^6*d^18 + 15144828*a^8*b^16*c^8*d^16 - 25068800*a^8*b^16*c^10*d^14 + 23486280*a^8*b^16*c^12*d^12 - 12099640*a^8*b^16*c^14*d^10 + 3330518*a^8*b^16*c^16*d^8 - 529224*a^8*b^16*c^18*d^6 + 36280*a^8*b^16*c^20*d^4 - 379200*a^9*b^15*c^3*d^21 + 3539128*a^9*b^15*c^5*d^19 - 13668308*a^9*b^15*c^7*d^17 + 27691952*a^9*b^15*c^9*d^15 - 31562040*a^9*b^15*c^11*d^13 + 19993760*a^9*b^15*c^13*d^11 - 6731044*a^9*b^15*c^15*d^9 + 1239264*a^9*b^15*c^17*d^7 - 106016*a^9*b^15*c^19*d^5 + 131904*a^10*b^14*c^2*d^22 - 1987844*a^10*b^14*c^4*d^20 + 10009720*a^10*b^14*c^6*d^18 - 25068800*a^10*b^14*c^8*d^16 + 34796936*a^10*b^14*c^10*d^14 - 26927144*a^10*b^14*c^12*d^12 + 10994964*a^10*b^14*c^14*d^10 - 2295680*a^10*b^14*c^16*d^8 + 230240*a^10*b^14*c^18*d^6 + 860368*a^11*b^13*c^3*d^21 - 5890780*a^11*b^13*c^5*d^19 + 18576800*a^11*b^13*c^7*d^17 - 31562040*a^11*b^13*c^9*d^15 + 29722864*a^11*b^13*c^11*d^13 - 14679348*a^11*b^13*c^13*d^11 + 3497920*a^11*b^13*c^15*d^9 - 381280*a^11*b^13*c^17*d^7 - 270604*a^12*b^12*c^2*d^22 + 2750664*a^12*b^12*c^4*d^20 - 11209800*a^12*b^12*c^6*d^18 + 23486280*a^12*b^12*c^8*d^16 - 26927144*a^12*b^12*c^10*d^14 + 16147404*a^12*b^12*c^12*d^12 - 4479104*a^12*b^12*c^14*d^10 + 499520*a^12*b^12*c^16*d^8 - 1001364*a^13*b^11*c^3*d^21 + 5437600*a^13*b^11*c^5*d^19 - 14230520*a^13*b^11*c^7*d^17 + 19993760*a^13*b^11*c^9*d^15 - 14679348*a^13*b^11*c^11*d^13 + 4861024*a^13*b^11*c^13*d^11 - 552160*a^13*b^11*c^15*d^9 + 273544*a^14*b^10*c^2*d^22 - 2073976*a^14*b^10*c^4*d^20 + 6943760*a^14*b^10*c^6*d^18 - 12099640*a^14*b^10*c^8*d^16 + 10994964*a^14*b^10*c^10*d^14 - 4479104*a^14*b^10*c^12*d^12 + 562016*a^14*b^10*c^14*d^10 + 605280*a^15*b^9*c^3*d^21 - 2682536*a^15*b^9*c^5*d^19 + 5889904*a^15*b^9*c^7*d^17 - 6731044*a^15*b^9*c^9*d^15 + 3497920*a^15*b^9*c^11*d^13 - 552160*a^15*b^9*c^13*d^1

1 - 131660*a^16*b^8*c^2*d^22 + 793460*a^16*b^8*c^4*d^20 - 2253214*a^16*b^8*c^6*d^18 + 3330518*a^16*b^8*c^8*d^16 - 2295680*a^16*b^8*c^10*d^14 + 499520*a^16*b^8*c^12*d^12 - 167520*a^17*b^7*c^3*d^21 + 655084*a^17*b^7*c^5*d^19 - 1310504*a^17*b^7*c^7*d^17 + 1239264*a^17*b^7*c^9*d^15 - 381280*a^17*b^7*c^11*d^13 + 22060*a^18*b^6*c^2*d^22 - 138010*a^18*b^6*c^4*d^20 + 396878*a^18*b^6*c^6*d^18 - 529224*a^18*b^6*c^8*d^16 + 230240*a^18*b^6*c^10*d^14 + 18840*a^19*b^5*c^3*d^21 - 85484*a^19*b^5*c^5*d^19 + 168344*a^19*b^5*c^7*d^17 - 106016*a^19*b^5*c^9*d^15 - 782*a^20*b^4*c^2*d^22 + 10562*a^20*b^4*c^4*d^20 - 36120*a^20*b^4*c^6*d^18 + 36280*a^20*b^4*c^8*d^16 - 144*a^21*b^3*c^3*d^21 + 4080*a^21*b^3*c^5*d^19 - 8960*a^21*b^3*c^7*d^17 - 98*a^22*b^2*c^2*d^22 + 88*a^22*b^2*c^4*d^20 + 1520*a^22*b^2*c^6*d^18 - 4*a*b^23*c^23*d - 4*a^23*b*c*d^23)/(16*(5*a^2*b^28*c^30 - b^30*c^30 - a^30*d^30 - 10*a^4*b^26*c^30 + 10*a^6*b^24*c^30 - 5*a^8*b^22*c^30 + a^10*b^20*c^30 + a^20*b^10*d^30 - 5*a^22*b^8*d^30 + 10*a^24*b^6*d^30 - 10*a^26*b^4*d^30 + 5*a^28*b^2*d^30 + 5*a^30*c^2*d^28 - 10*a^30*c^4*d^26 + 10*a^30*c^6*d^24 - 5*a^30*c^8*d^22 + a^30*c^10*d^20 + b^30*c^20*d^10 - 5*b^30*c^22*d^8 + 10*b^30*c^24*d^6 - 10*b^30*c^26*d^4 + 5*b^30*c^28*d^2 - 20*a*b^29*c^19*d^11 + 100*a*b^29*c^21*d^9 - 200*a*b^29*c^23*d^7 + 200*a*b^29*c^25*d^5 - 100*a*b^29*c^27*d^3 - 100*a^3*b^27*c^29*d + 200*a^5*b^25*c^29*d - 200*a^7*b^23*c^29*d + 100*a^9*b^21*c^29*d - 20*a^11*b^19*c^29*d - 20*a^19*b^11*c*d^29 + 100*a^21*b^9*c*d^29 - 200*a^23*b^7*c*d^29 + 200*a^25*b^5*c*d^29 - 100*a^27*b^3*c*d^29 - 100*a^29*b*c^3*d^27 + 200*a^29*b*c^5*d^25 - 200*a^29*b*c^7*d^23 + 100*a^29*b*c^9*d^21 - 20*a^29*b*c^11*d^19 + 190*a^2*b^28*c^18*d^12 - 955*a^2*b^28*c^20*d^10 + 1925*a^2*b^28*c^22*d^8 - 1950*a^2*b^28*c^24*d^6 + 1000*a^2*b^28*c^26*d^4 - 215*a^2*b^28*c^28*d^2 - 1140*a^3*b^27*c^17*d^13 + 5800*a^3*b^27*c^19*d^11 - 11900*a^3*b^27*c^21*d^9 + 12400*a^3*b^27*c^23*d^7 - 6700*a^3*b^27*c^25*d^5 + 1640*a^3*b^27*c^27*d^3 + 4845*a^4*b^26*c^16*d^14 - 25175*a^4*b^26*c^18*d^12 + 53210*a^4*b^26*c^20*d^10 - 58000*a^4*b^26*c^22*d^8 + 33825*a^4*b^26*c^24*d^6 - 9695*a^4*b^26*c^26*d^4 + 1000*a^4*b^26*c^28*d^2 - 15504*a^5*b^25*c^15*d^15 + 83220*a^5*b^25*c^17*d^13 - 183740*a^5*b^25*c^19*d^11 + 213040*a^5*b^25*c^21*d^9 - 136520*a^5*b^25*c^23*d^7 + 46004*a^5*b^25*c^25*d^5 - 6700*a^5*b^25*c^27*d^3 + 38760*a^6*b^24*c^14*d^16 - 218025*a^6*b^24*c^16*d^14 + 510625*a^6*b^24*c^18*d^12 - 639360*a^6*b^24*c^20*d^10 + 455100*a^6*b^24*c^22*d^8 - 178985*a^6*b^24*c^24*d^6 + 33825*a^6*b^24*c^26*d^4 - 1950*a^6*b^24*c^28*d^2 - 77520*a^7*b^23*c^13*d^17 + 465120*a^7*b^23*c^15*d^15 - 1174200*a^7*b^23*c^17*d^13 + 1607600*a^7*b^23*c^19*d^11 - 1277800*a^7*b^23*c^21*d^9 + 581120*a^7*b^23*c^23*d^7 - 136520*a^7*b^23*c^25*d^5 + 12400*a^7*b^23*c^27*d^3 + 125970*a^8*b^22*c^12*d^18 - 823650*a^8*b^22*c^14*d^16 + 2277150*a^8*b^22*c^16*d^14 - 3441850*a^8*b^22*c^18*d^12 + 3061855*a^8*b^22*c^20*d^10 - 1598495*a^8*b^22*c^22*d^8 + 455100*a^8*b^22*c^24*d^6 - 58000*a^8*b^22*c^26*d^4 + 1925*a^8*b^22*c^28*d^2 - 167960*a^9*b^21*c^11*d^19 + 1227400*a^9*b^21*c^13*d^17 - 3772640*a^9*b^21*c^15*d^15 + 6342200*a^9*b^21*c^17*d^13 - 6323300*a^9*b^21*c^19*d^11 + 3770860*a^9*b^21*c^21*d^9 - 1277800*a^9*b^21*c^23*d^7 + 213040*a^9*b^21*c^25*d^5 - 11900*a^9*b^21*c^27*d^3 + 184756*a^10*b^20*c^10*d^20 - 1553630*a^10*b^20*c^12*d^18 + 5384410*a^10*b^20*c^14*d^16 - 10132510*a

^10*b^20*c^16*d^14 + 11341480*a^10*b^20*c^18*d^12 - 7699257*a^10*b^20*c^20*d^10 + 3061855*a^10*b^20*c^22*d^8 - 639360*a^10*b^20*c^24*d^6 + 53210*a^10*b^20*c^26*d^4 - 955*a^10*b^20*c^28*d^2 - 167960*a^11*b^19*c^9*d^21 + 1679600*a^11*b^19*c^11*d^19 - 6653800*a^11*b^19*c^13*d^17 + 14108640*a^11*b^19*c^15*d^15 - 17770700*a^11*b^19*c^17*d^13 + 13697880*a^11*b^19*c^19*d^11 - 6323300*a^11*b^19*c^21*d^9 + 1607600*a^11*b^19*c^23*d^7 - 183740*a^11*b^19*c^25*d^5 + 5800*a^11*b^19*c^27*d^3 + 125970*a^12*b^18*c^8*d^22 - 1553630*a^12*b^18*c^10*d^20 + 7138300*a^12*b^18*c^12*d^18 - 17183600*a^12*b^18*c^14*d^16 + 24426875*a^12*b^18*c^16*d^14 - 21339185*a^12*b^18*c^18*d^12 + 11341480*a^12*b^18*c^20*d^10 - 3441850*a^12*b^18*c^22*d^8 + 510625*a^12*b^18*c^24*d^6 - 25175*a^12*b^18*c^26*d^4 + 190*a^12*b^18*c^28*d^2 - 77520*a^13*b^17*c^7*d^23 + 1227400*a^13*b^17*c^9*d^21 - 6653800*a^13*b^17*c^11*d^19 + 18346400*a^13*b^17*c^13*d^17 - 29535120*a^13*b^17*c^15*d^15 + 29213260*a^13*b^17*c^17*d^13 - 17770700*a^13*b^17*c^19*d^11 + 6342200*a^13*b^17*c^21*d^9 - 1174200*a^13*b^17*c^23*d^7 + 83220*a^13*b^17*c^25*d^5 - 1140*a^13*b^17*c^27*d^3 + 38760*a^14*b^16*c^6*d^24 - 823650*a^14*b^16*c^8*d^22 + 5384410*a^14*b^16*c^10*d^20 - 17183600*a^14*b^16*c^12*d^18 + 31460200*a^14*b^16*c^14*d^16 - 35234455*a^14*b^16*c^16*d^14 + 24426875*a^14*b^16*c^18*d^12 - 10132510*a^14*b^16*c^20*d^10 + 2277150*a^14*b^16*c^22*d^8 - 218025*a^14*b^16*c^24*d^6 + 4845*a^14*b^16*c^26*d^4 - 15504*a^15*b^15*c^5*d^25 + 465120*a^15*b^15*c^7*d^23 - 3772640*a^15*b^15*c^9*d^21 + 14108640*a^15*b^15*c^11*d^19 - 29535120*a^15*b^15*c^13*d^17 + 37499008*a^15*b^15*c^15*d^15 - 29535120*a^15*b^15*c^17*d^13 + 14108640*a^15*b^15*c^19*d^11 - 3772640*a^15*b^15*c^21*d^9 + 465120*a^15*b^15*c^23*d^7 - 15504*a^15*b^15*c^25*d^5 + 4845*a^16*b^14*c^4*d^26 - 218025*a^16*b^14*c^6*d^24 + 2277150*a^16*b^14*c^8*d^22 - 10132510*a^16*b^14*c^10*d^20 + 24426875*a^16*b^14*c^12*d^18 - 35234455*a^16*b^14*c^14*d^16 + 31460200*a^16*b^14*c^16*d^14 - 17183600*a^16*b^14*c^18*d^12 + 5384410*a^16*b^14*c^20*d^10 - 823650*a^16*b^14*c^22*d^8 + 38760*a^16*b^14*c^24*d^6 - 1140*a^17*b^13*c^3*d^27 + 83220*a^17*b^13*c^5*d^25 - 1174200*a^17*b^13*c^7*d^23 + 6342200*a^17*b^13*c^9*d^21 - 17770700*a^17*b^13*c^11*d^19 + 29213260*a^17*b^13*c^13*d^17 - 29535120*a^17*b^13*c^15*d^15 + 18346400*a^17*b^13*c^17*d^13 - 6653800*a^17*b^13*c^19*d^11 + 1227400*a^17*b^13*c^21*d^9 - 77520*a^17*b^13*c^23*d^7 + 190*a^18*b^12*c^2*d^28 - 25175*a^18*b^12*c^4*d^26 + 510625*a^18*b^12*c^6*d^24 - 3441850*a^18*b^12*c^8*d^22 + 11341480*a^18*b^12*c^10*d^20 - 21339185*a^18*b^12*c^12*d^18 + 24426875*a^18*b^12*c^14*d^16 - 17183600*a^18*b^12*c^16*d^14 + 7138300*a^18*b^12*c^18*d^12 - 1553630*a^18*b^12*c^20*d^10 + 125970*a^18*b^12*c^22*d^8 + 5800*a^19*b^11*c^3*d^27 - 183740*a^19*b^11*c^5*d^25 + 1607600*a^19*b^11*c^7*d^23 - 6323300*a^19*b^11*c^9*d^21 + 13697880*a^19*b^11*c^11*d^19 - 17770700*a^19*b^11*c^13*d^17 + 14108640*a^19*b^11*c^15*d^15 - 6653800*a^19*b^11*c^17*d^13 + 1679600*a^19*b^11*c^19*d^11 - 167960*a^19*b^11*c^21*d^9 - 955*a^20*b^10*c^2*d^28 + 53210*a^20*b^10*c^4*d^26 - 639360*a^20*b^10*c^6*d^24 + 3061855*a^20*b^10*c^8*d^22 - 7699257*a^20*b^10*c^10*d^20 + 11341480*a^20*b^10*c^12*d^18 - 10132510*a^20*b^10*c^14*d^16 + 5384410*a^20*b^10*c^16*d^14 - 1553630*a^20*b^10*c^18*d^12 + 184756*a^20*b^10*c^20*d^10 - 11900*a^21*b^9*c^3*d^27 + 213040*a^21*b^9*c^5*d^25 - 12

77800*a^21*b^9*c^7*d^23 + 3770860*a^21*b^9*c^9*d^21 - 6323300*a^21*b^9*c^11*d^19 + 6342200*a^21*b^9*c^13*d^17 - 3772640*a^21*b^9*c^15*d^15 + 1227400*a^21*b^9*c^17*d^13 - 167960*a^21*b^9*c^19*d^11 + 1925*a^22*b^8*c^2*d^28 - 58000*a^22*b^8*c^4*d^26 + 455100*a^22*b^8*c^6*d^24 - 1598495*a^22*b^8*c^8*d^22 + 3061855*a^22*b^8*c^10*d^20 - 3441850*a^22*b^8*c^12*d^18 + 2277150*a^22*b^8*c^14*d^16 - 823650*a^22*b^8*c^16*d^14 + 125970*a^22*b^8*c^18*d^12 + 12400*a^23*b^7*c^3*d^27 - 136520*a^23*b^7*c^5*d^25 + 581120*a^23*b^7*c^7*d^23 - 1277800*a^23*b^7*c^9*d^21 + 1607600*a^23*b^7*c^11*d^19 - 1174200*a^23*b^7*c^13*d^17 + 465120*a^23*b^7*c^15*d^15 - 77520*a^23*b^7*c^17*d^13 - 1950*a^24*b^6*c^2*d^28 + 33825*a^24*b^6*c^4*d^26 - 178985*a^24*b^6*c^6*d^24 + 455100*a^24*b^6*c^8*d^22 - 639360*a^24*b^6*c^10*d^20 + 510625*a^24*b^6*c^12*d^18 - 218025*a^24*b^6*c^14*d^16 + 38760*a^24*b^6*c^16*d^14 - 6700*a^25*b^5*c^3*d^27 + 46004*a^25*b^5*c^5*d^25 - 136520*a^25*b^5*c^7*d^23 + 213040*a^25*b^5*c^9*d^21 - 183740*a^25*b^5*c^11*d^19 + 83220*a^25*b^5*c^13*d^17 - 15504*a^25*b^5*c^15*d^15 + 1000*a^26*b^4*c^2*d^28 - 9695*a^26*b^4*c^4*d^26 + 33825*a^26*b^4*c^6*d^24 - 58000*a^26*b^4*c^8*d^22 + 53210*a^26*b^4*c^10*d^20 - 25175*a^26*b^4*c^12*d^18 + 4845*a^26*b^4*c^14*d^16 + 1640*a^27*b^3*c^3*d^27 - 6700*a^27*b^3*c^5*d^25 + 12400*a^27*b^3*c^7*d^23 - 11900*a^27*b^3*c^9*d^21 + 5800*a^27*b^3*c^11*d^19 - 1140*a^27*b^3*c^13*d^17 - 215*a^28*b^2*c^2*d^28 + 1000*a^28*b^2*c^4*d^26 - 1950*a^28*b^2*c^6*d^24 + 1925*a^28*b^2*c^8*d^22 - 955*a^28*b^2*c^10*d^20 + 190*a^28*b^2*c^12*d^18 + 20*a*b^29*c^29*d + 20*a^29*b*c*d^29)))^(1/2) + (4*(4*a^2*b^20*c^22 - 12*a^6*b^16*c^22 + 8*a^8*b^14*c^22 + 4*a^22*c^2*d^20 - 12*a^22*c^6*d^16 + 8*a^22*c^8*d^14 + 48*a*b^21*c^11*d^11 - 212*a*b^21*c^13*d^9 + 360*a*b^21*c^15*d^7 - 276*a*b^21*c^17*d^5 + 80*a*b^21*c^19*d^3 - 20*a^3*b^19*c^21*d - 72*a^5*b^17*c^21*d + 204*a^7*b^15*c^21*d - 112*a^9*b^13*c^21*d + 48*a^11*b^11*c*d^21 - 212*a^13*b^9*c*d^21 + 360*a^15*b^7*c*d^21 - 276*a^17*b^5*c*d^21 + 80*a^19*b^3*c*d^21 - 20*a^21*b*c^3*d^19 - 72*a^21*b*c^5*d^17 + 204*a^21*b*c^7*d^15 - 112*a^21*b*c^9*d^13 - 480*a^2*b^20*c^10*d^12 + 2160*a^2*b^20*c^12*d^10 - 3772*a^2*b^20*c^14*d^8 + 3020*a^2*b^20*c^16*d^6 - 960*a^2*b^20*c^18*d^4 + 28*a^2*b^20*c^20*d^2 + 2160*a^3*b^19*c^9*d^13 - 10152*a^3*b^19*c^11*d^11 + 18888*a^3*b^19*c^13*d^9 - 16732*a^3*b^19*c^15*d^7 + 6588*a^3*b^19*c^17*d^5 - 732*a^3*b^19*c^19*d^3 - 5760*a^4*b^18*c^8*d^14 + 29360*a^4*b^18*c^10*d^12 - 60792*a^4*b^18*c^12*d^10 + 62708*a^4*b^18*c^14*d^8 - 31892*a^4*b^18*c^16*d^6 + 6588*a^4*b^18*c^18*d^4 - 212*a^4*b^18*c^20*d^2 + 10080*a^5*b^17*c^7*d^15 - 58860*a^5*b^17*c^9*d^13 + 141880*a^5*b^17*c^11*d^11 - 175592*a^5*b^17*c^13*d^9 + 113748*a^5*b^17*c^15*d^7 - 34492*a^5*b^17*c^17*d^5 + 3308*a^5*b^17*c^19*d^3 - 12096*a^6*b^16*c^6*d^16 + 87264*a^6*b^16*c^8*d^14 - 254340*a^6*b^16*c^10*d^12 + 381532*a^6*b^16*c^12*d^10 - 307752*a^6*b^16*c^14*d^8 + 125568*a^6*b^16*c^16*d^6 - 21232*a^6*b^16*c^18*d^4 + 1068*a^6*b^16*c^20*d^2 + 10080*a^7*b^15*c^5*d^17 - 99120*a^7*b^15*c^7*d^15 + 359064*a^7*b^15*c^9*d^13 - 655076*a^7*b^15*c^11*d^11 + 650108*a^7*b^15*c^13*d^9 - 343368*a^7*b^15*c^15*d^7 + 85760*a^7*b^15*c^17*d^5 - 7652*a^7*b^15*c^19*d^3 - 5760*a^8*b^14*c^4*d^18 + 87264*a^8*b^14*c^6*d^16 - 402576*a^8*b^14*c^8*d^14 + 900324*a^8*b^14*c^10*d^12 - 1096236*a^8*b^14*c^12*d^10 + 731392*a^8*b^14*c^14*d^8 - 247352*a^8*b^14

*c^16*d^6 + 34548*a^8*b^14*c^18*d^4 - 1612*a^8*b^14*c^20*d^2 + 2160*a^9*b^1
3*c^3*d^19 - 58860*a^9*b^13*c^5*d^17 + 359064*a^9*b^13*c^7*d^15 - 999816*a^
9*b^13*c^9*d^13 + 1494564*a^9*b^13*c^11*d^11 - 1238148*a^9*b^13*c^13*d^9 +
542272*a^9*b^13*c^15*d^7 - 109032*a^9*b^13*c^17*d^5 + 7908*a^9*b^13*c^19*d^
3 - 480*a^10*b^12*c^2*d^20 + 29360*a^10*b^12*c^4*d^18 - 254340*a^10*b^12*c^
6*d^16 + 900324*a^10*b^12*c^8*d^14 - 1656496*a^10*b^12*c^10*d^12 + 1688232*
a^10*b^12*c^12*d^10 - 934868*a^10*b^12*c^14*d^8 + 254492*a^10*b^12*c^16*d^6
 - 26952*a^10*b^12*c^18*d^4 + 728*a^10*b^12*c^20*d^2 - 10152*a^11*b^11*c^3*
d^19 + 141880*a^11*b^11*c^5*d^17 - 655076*a^11*b^11*c^7*d^15 + 1494564*a^11
*b^11*c^9*d^13 - 1870136*a^11*b^11*c^11*d^11 + 1289704*a^11*b^11*c^13*d^9 -
 455388*a^11*b^11*c^15*d^7 + 67468*a^11*b^11*c^17*d^5 - 2912*a^11*b^11*c^19
*d^3 + 2160*a^12*b^10*c^2*d^20 - 60792*a^12*b^10*c^4*d^18 + 381532*a^12*b^1
0*c^6*d^16 - 1096236*a^12*b^10*c^8*d^14 + 1688232*a^12*b^10*c^10*d^12 - 143
4728*a^12*b^10*c^12*d^10 + 639684*a^12*b^10*c^14*d^8 - 127860*a^12*b^10*c^1
6*d^6 + 8008*a^12*b^10*c^18*d^4 + 18888*a^13*b^9*c^3*d^19 - 175592*a^13*b^9
*c^5*d^17 + 650108*a^13*b^9*c^7*d^15 - 1238148*a^13*b^9*c^9*d^13 + 1289704*
a^13*b^9*c^11*d^11 - 715296*a^13*b^9*c^13*d^9 + 186564*a^13*b^9*c^15*d^7 -
16016*a^13*b^9*c^17*d^5 - 3772*a^14*b^8*c^2*d^20 + 62708*a^14*b^8*c^4*d^18
 - 307752*a^14*b^8*c^6*d^16 + 731392*a^14*b^8*c^8*d^14 - 934868*a^14*b^8*c^1
0*d^12 + 639684*a^14*b^8*c^12*d^10 - 211416*a^14*b^8*c^14*d^8 + 24024*a^14*
b^8*c^16*d^6 - 16732*a^15*b^7*c^3*d^19 + 113748*a^15*b^7*c^5*d^17 - 343368*
a^15*b^7*c^7*d^15 + 542272*a^15*b^7*c^9*d^13 - 455388*a^15*b^7*c^11*d^11 +
186564*a^15*b^7*c^13*d^9 - 27456*a^15*b^7*c^15*d^7 + 3020*a^16*b^6*c^2*d^20
 - 31892*a^16*b^6*c^4*d^18 + 125568*a^16*b^6*c^6*d^16 - 247352*a^16*b^6*c^8
*d^14 + 254492*a^16*b^6*c^10*d^12 - 127860*a^16*b^6*c^12*d^10 + 24024*a^16*
b^6*c^14*d^8 + 6588*a^17*b^5*c^3*d^19 - 34492*a^17*b^5*c^5*d^17 + 85760*a^1
7*b^5*c^7*d^15 - 109032*a^17*b^5*c^9*d^13 + 67468*a^17*b^5*c^11*d^11 - 1601
6*a^17*b^5*c^13*d^9 - 960*a^18*b^4*c^2*d^20 + 6588*a^18*b^4*c^4*d^18 - 2123
2*a^18*b^4*c^6*d^16 + 34548*a^18*b^4*c^8*d^14 - 26952*a^18*b^4*c^10*d^12 +
8008*a^18*b^4*c^12*d^10 - 732*a^19*b^3*c^3*d^19 + 3308*a^19*b^3*c^5*d^17 -
7652*a^19*b^3*c^7*d^15 + 7908*a^19*b^3*c^9*d^13 - 2912*a^19*b^3*c^11*d^11 +
 28*a^20*b^2*c^2*d^20 - 212*a^20*b^2*c^4*d^18 + 1068*a^20*b^2*c^6*d^16 - 16
12*a^20*b^2*c^8*d^14 + 728*a^20*b^2*c^10*d^12))/(a^20*d^20 + b^20*c^20 - 4*
a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^
8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d
^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4
*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 4
8*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c
^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a
^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^
19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19
*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*
c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*
d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*
d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^

10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19) - (8*tan(e/2 + (f*x)/2)*(12*a^5*b^17*c^22 - 4*a^22*c*d^21 - 4*a*b^21*c^22 - 8*a^7*b^15*c^22 + 12*a^22*c^5*d^17 - 8*a^22*c^7*d^15 - 24*a*b^21*c^12*d^10 + 100*a*b^21*c^14*d^8 - 164*a*b^21*c^16*d^6 + 120*a*b^21*c^18*d^4 - 28*a*b^21*c^20*d^2 + 20*a^2*b^20*c^21*d + 72*a^4*b^18*c^21*d - 204*a^6*b^16*c^21*d + 112*a^8*b^14*c^21*d - 24*a^12*b^10*c*d^21 + 100*a^14*b^8*c*d^21 - 164*a^16*b^6*c*d^21 + 120*a^18*b^4*c*d^21 - 28*a^20*b^2*c*d^21 + 20*a^21*b*c^2*d^20 + 72*a^21*b*c^4*d^18 - 204*a^21*b*c^6*d^16 + 112*a^21*b*c^8*d^14 + 216*a^2*b^20*c^11*d^11 - 908*a^2*b^20*c^13*d^9 + 1540*a^2*b^20*c^15*d^7 - 1200*a^2*b^20*c^17*d^5 + 332*a^2*b^20*c^19*d^3 - 840*a^3*b^19*c^10*d^12 + 3672*a^3*b^19*c^12*d^10 - 6788*a^3*b^19*c^14*d^8 + 6132*a^3*b^19*c^16*d^6 - 2388*a^3*b^19*c^18*d^4 + 212*a^3*b^19*c^20*d^2

+ 1800*a^4*b^18*c^9*d^13 - 8680*a^4*b^18*c^11*d^11 + 18852*a^4*b^18*c^13*d^9 - 21228*a^4*b^18*c^15*d^7 + 11692*a^4*b^18*c^17*d^5 - 2508*a^4*b^18*c^19*d^3 - 2160*a^5*b^17*c^8*d^14 + 13100*a^5*b^17*c^10*d^12 - 36820*a^5*b^17*c^12*d^10 + 53712*a^5*b^17*c^14*d^8 - 39608*a^5*b^17*c^16*d^6 + 12832*a^5*b^17*c^18*d^4 - 1068*a^5*b^17*c^20*d^2 + 1008*a^6*b^16*c^7*d^15 - 12420*a^6*b^16*c^9*d^13 + 51764*a^6*b^16*c^11*d^11 - 100128*a^6*b^16*c^13*d^9 + 96048*a^6*b^16*c^15*d^7 - 42920*a^6*b^16*c^17*d^5 + 6852*a^6*b^16*c^19*d^3 + 1008*a^7*b^15*c^6*d^16 + 5136*a^7*b^15*c^8*d^14 - 48820*a^7*b^15*c^10*d^12 + 134700*a^7*b^15*c^12*d^10 - 171472*a^7*b^15*c^14*d^8 + 103992*a^7*b^15*c^16*d^6 - 26148*a^7*b^15*c^18*d^4 + 1612*a^7*b^15*c^20*d^2 - 2160*a^8*b^14*c^5*d^17 + 5136*a^8*b^14*c^7*d^15 + 20436*a^8*b^14*c^9*d^13 - 121524*a^8*b^14*c^11*d^11 + 224888*a^8*b^14*c^13*d^9 - 186952*a^8*b^14*c^15*d^7 + 67572*a^8*b^14*c^17*d^5 - 7508*a^8*b^14*c^19*d^3 + 1800*a^9*b^13*c^4*d^18 - 12420*a^9*b^13*c^6*d^16 + 20436*a^9*b^13*c^8*d^14 + 49416*a^9*b^13*c^10*d^12 - 201552*a^9*b^13*c^12*d^10 + 245708*a^9*b^13*c^14*d^8 - 125412*a^9*b^13*c^16*d^6 + 22752*a^9*b^13*c^18*d^4 - 728*a^9*b^13*c^20*d^2 - 840*a^10*b^12*c^3*d^19 + 13100*a^10*b^12*c^5*d^17 - 48820*a^10*b^12*c^7*d^15 + 49416*a^10*b^12*c^9*d^13 + 82088*a^10*b^12*c^11*d^11 - 219092*a^10*b^12*c^13*d^9 + 168468*a^10*b^12*c^15*d^7 - 47152*a^10*b^12*c^17*d^5 + 2832*a^10*b^12*c^19*d^3 + 216*a^11*b^11*c^2*d^20 - 8680*a^11*b^11*c^4*d^18 + 51764*a^11*b^11*c^6*d^16 - 121524*a^11*b^11*c^8*d^14 + 82088*a^11*b^11*c^10*d^12 + 88712*a^11*b^11*c^12*d^10 - 153012*a^11*b^11*c^14*d^8 + 67604*a^11*b^11*c^16*d^6 - 7168*a^11*b^11*c^18*d^4 + 3672*a^12*b^10*c^3*d^19 - 36820*a^12*b^10*c^5*d^17 + 134700*a^12*b^10*c^7*d^15 - 201552*a^12*b^10*c^9*d^13 + 88712*a^12*b^10*c^11*d^11 + 62676*a^12*b^10*c^13*d^9 - 63372*a^12*b^10*c^15*d^7 + 12008*a^12*b^10*c^17*d^5 - 908*a^13*b^9*c^2*d^20 + 18852*a^13*b^9*c^4*d^18 - 100128*a^13*b^9*c^6*d^16 + 224888*a^13*b^9*c^8*d^14 - 219092*a^13*b^9*c^10*d^12 + 62676*a^13*b^9*c^12*d^10 + 26256*a^13*b^9*c^14*d^8 - 12544*a^13*b^9*c^16*d^6 - 6788*a^14*b^8*c^3*d^19 + 53712*a^14*b^8*c^5*d^17 - 171472*a^14*b^8*c^7*d^15 + 245708*a^14*b^8*c^9*d^13 - 153012*a^14*b^8*c^11*d^11 + 26256*a^14*b^8*c^13*d^9 + 5496*a^14*b^8*c^15*d^7 + 1540*a^15*b^7*c^2*d^20 - 21228*a^15*b^7*c^4*d^18 + 96048*a^15*b^7*c^6*d^16 - 186952*a^15*b^7*c^8*d^14 + 168468*a^15*b^7*c^10*d^12 - 63372*a^15*b^7*c^12*d^10 + 5496*a^15*b^7*c^14*d^8 + 6132*a^16*b^6*c^3*d^19 - 39608*a^16*b^6*c^5*d^17 + 103992*a^16*b^6*c^7*d^15 - 125412*a^16*b^6*c^9*d^13 + 67604*a^16*b^6*c^11*d^11 - 12544*a^16*b^6*c^13*d^9 - 1200*a^17*b^5*c^2*d^20 + 11692*a^17*b^5*c^4*d^18 - 42920*a^17*b^5*c^6*d^16 + 67572*a^17*b^5*c^8*d^14 - 47152*a^17*b^5*c^10*d^12 + 12008*a^17*b^5*c^12*d^10 - 2388*a^18*b^4*c^3*d^19 + 12832*a^18*b^4*c^5*d^17 - 26148*a^18*b^4*c^7*d^15 + 22752*a^18*b^4*c^9*d^13 - 7168*a^18*b^4*c^11*d^11 + 332*a^19*b^3*c^2*d^20 - 2508*a^19*b^3*c^4*d^18 + 6852*a^19*b^3*c^6*d^16 - 7508*a^19*b^3*c^8*d^14 + 2832*a^19*b^3*c^10*d^12 + 212*a^20*b^2*c^3*d^19 - 1068*a^20*b^2*c^5*d^17 + 1612*a^20*b^2*c^7*d^15 - 728*a^20*b^2*c^9*d^13))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 -

4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19)) + (4*(288*a*b^18*c^6*d^13 - 1104*a*b^18*c^8*d^11 + 1538*a*b^18*c^10*d^9 - 872*a*b^18*c^12*d^7 + 108*a*b^18*c^14*d^

5 + 40*a*b^18*c^16*d^3 + 8*a^3*b^16*c^18*d + 8*a^5*b^14*c^18*d + 288*a^6*b^13*c*d^18 - 1104*a^8*b^11*c*d^18 + 1538*a^10*b^9*c*d^18 - 872*a^12*b^7*c*d^18 + 108*a^14*b^5*c*d^18 + 40*a^16*b^3*c*d^18 + 8*a^18*b*c^3*d^16 + 8*a^18*b*c^5*d^14 - 864*a^2*b^17*c^5*d^14 + 3216*a^2*b^17*c^7*d^12 - 4262*a^2*b^17*c^9*d^10 + 2256*a^2*b^17*c^11*d^8 - 304*a^2*b^17*c^13*d^6 - 32*a^2*b^17*c^15*d^4 + 8*a^2*b^17*c^17*d^2 + 576*a^3*b^16*c^4*d^15 - 3024*a^3*b^16*c^6*d^13 + 6304*a^3*b^16*c^8*d^11 - 7216*a^3*b^16*c^10*d^9 + 4944*a^3*b^16*c^12*d^7 - 1664*a^3*b^16*c^14*d^5 - 72*a^3*b^16*c^16*d^3 + 576*a^4*b^15*c^3*d^16 + 912*a^4*b^15*c^5*d^14 - 8720*a^4*b^15*c^7*d^12 + 16632*a^4*b^15*c^9*d^10 - 14888*a^4*b^15*c^11*d^8 + 6704*a^4*b^15*c^13*d^6 - 744*a^4*b^15*c^15*d^4 - 40*a^4*b^15*c^17*d^2 - 864*a^5*b^14*c^2*d^17 + 912*a^5*b^14*c^4*d^15 + 5140*a^5*b^14*c^6*d^13 - 16080*a^5*b^14*c^8*d^11 + 23520*a^5*b^14*c^10*d^9 - 20208*a^5*b^14*c^12*d^7 + 7404*a^5*b^14*c^14*d^5 - 264*a^5*b^14*c^16*d^3 - 3024*a^6*b^13*c^3*d^16 + 5140*a^6*b^13*c^5*d^14 + 5280*a^6*b^13*c^7*d^12 - 28380*a^6*b^13*c^9*d^10 + 39792*a^6*b^13*c^11*d^8 - 22728*a^6*b^13*c^13*d^6 + 3096*a^6*b^13*c^15*d^4 - 112*a^6*b^13*c^17*d^2 + 3216*a^7*b^12*c^2*d^17 - 8720*a^7*b^12*c^4*d^15 + 5280*a^7*b^12*c^6*d^13 + 15000*a^7*b^12*c^8*d^11 - 40656*a^7*b^12*c^10*d^9 + 40296*a^7*b^12*c^12*d^7 - 12984*a^7*b^12*c^14*d^5 + 728*a^7*b^12*c^16*d^3 + 6304*a^8*b^11*c^3*d^16 - 16080*a^8*b^11*c^5*d^14 + 15000*a^8*b^11*c^7*d^12 + 16024*a^8*b^11*c^9*d^10 - 46184*a^8*b^11*c^11*d^8 + 27208*a^8*b^11*c^13*d^6 - 2752*a^8*b^11*c^15*d^4 - 4262*a^9*b^10*c^2*d^17 + 16632*a^9*b^10*c^4*d^15 - 28380*a^9*b^10*c^6*d^13 + 16024*a^9*b^10*c^8*d^11 + 22018*a^9*b^10*c^10*d^9 - 30104*a^9*b^10*c^12*d^7 + 6488*a^9*b^10*c^14*d^5 - 7216*a^10*b^9*c^3*d^16 + 23520*a^10*b^9*c^5*d^14 - 40656*a^10*b^9*c^7*d^12 + 22018*a^10*b^9*c^9*d^10 + 13080*a^10*b^9*c^11*d^8 - 8720*a^10*b^9*c^13*d^6 + 2256*a^11*b^8*c^2*d^17 - 14888*a^11*b^8*c^4*d^15 + 39792*a^11*b^8*c^6*d^13 - 46184*a^11*b^8*c^8*d^11 + 13080*a^11*b^8*c^10*d^9 + 4360*a^11*b^8*c^12*d^7 + 4944*a^12*b^7*c^3*d^16 - 20208*a^12*b^7*c^5*d^14 + 40296*a^12*b^7*c^7*d^12 - 30104*a^12*b^7*c^9*d^10 + 4360*a^12*b^7*c^11*d^8 - 304*a^13*b^6*c^2*d^17 + 6704*a^13*b^6*c^4*d^15 - 22728*a^13*b^6*c^6*d^13 + 27208*a^13*b^6*c^8*d^11 - 8720*a^13*b^6*c^10*d^9 - 1664*a^14*b^5*c^3*d^16 + 7404*a^14*b^5*c^5*d^14 - 12984*a^14*b^5*c^7*d^12 + 6488*a^14*b^5*c^9*d^10 - 32*a^15*b^4*c^2*d^17 - 744*a^15*b^4*c^4*d^15 + 3096*a^15*b^4*c^6*d^13 - 2752*a^15*b^4*c^8*d^11 - 72*a^16*b^3*c^3*d^16 - 264*a^16*b^3*c^5*d^14 + 728*a^16*b^3*c^7*d^12 + 8*a^17*b^2*c^2*d^17 - 40*a^17*b^2*c^4*d^15 - 112*a^17*b^2*c^6*d^13 + 2*a*b^18*c^18*d + 2*a^18*b*c*d^18))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^

18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19) - (8*tan(e/2 + (f*x)/2)*(a*b^18*c^19 + a^19*c*d^18 + 4*a^3*b^16*c^19 + 4*a^5*b^14*c^19 + 4*a^19*c^3*d^16 + 4*a^19*c^5*d^14 - 576*a*b^18*c^5*d^14 + 2640*a*b^18*c^7*d^12 - 4732*a*b^18*c^9*d^10 + 3961*a*b^18*c^11*d^8 - 1344*a*b^18*c^13*d^6 + 14*a*b^18*c^15*d^4 + 18*a*b^18*c^17*d^2 + 4*a^2*b^17*c^18*d - 20*a^4*b^15*c^18*d - 576*a^5*b^14*c*d^18 - 56*a^6*b^13*c^18*d + 2640*a^7*b^12*c*d^18 - 4732*a^9*b^10*c*d^18 + 3961*a^11*b^8*c*d^18 - 1344*a^13*b^6*c*d^18 + 14*a^15*b^4*c*d^18 + 18*a^17*b^2*c*d^18 + 4*a^18*b*c^2*d^17 - 20*a^18*b*c^4*d^15 - 56*a^18*b*c^6*d^13 + 230

4*a^2*b^17*c^4*d^15 - 10944*a^2*b^17*c^6*d^13 + 20720*a^2*b^17*c^8*d^11 - 1
8788*a^2*b^17*c^10*d^9 + 7392*a^2*b^17*c^12*d^7 - 520*a^2*b^17*c^14*d^5 - 2
4*a^2*b^17*c^16*d^3 - 3456*a^3*b^16*c^3*d^16 + 20016*a^3*b^16*c^5*d^14 - 48
112*a^3*b^16*c^7*d^12 + 58925*a^3*b^16*c^9*d^10 - 36732*a^3*b^16*c^11*d^8 +
 9736*a^3*b^16*c^13*d^6 - 760*a^3*b^16*c^15*d^4 - 44*a^3*b^16*c^17*d^2 + 23
04*a^4*b^15*c^2*d^17 - 23424*a^4*b^15*c^4*d^15 + 81680*a^4*b^15*c^6*d^13 -
135520*a^4*b^15*c^8*d^11 + 114144*a^4*b^15*c^10*d^9 - 44168*a^4*b^15*c^12*d
^7 + 5696*a^4*b^15*c^14*d^5 - 332*a^4*b^15*c^16*d^3 + 20016*a^5*b^14*c^3*d^
16 - 99112*a^5*b^14*c^5*d^14 + 213338*a^5*b^14*c^7*d^12 - 235152*a^5*b^14*c
^9*d^10 + 130428*a^5*b^14*c^11*d^8 - 31908*a^5*b^14*c^13*d^6 + 3966*a^5*b^1
4*c^15*d^4 - 140*a^5*b^14*c^17*d^2 - 10944*a^6*b^13*c^2*d^17 + 81680*a^6*b^
13*c^4*d^15 - 243832*a^6*b^13*c^6*d^13 + 364608*a^6*b^13*c^8*d^11 - 281736*
a^6*b^13*c^10*d^9 + 103104*a^6*b^13*c^12*d^7 - 16860*a^6*b^13*c^14*d^5 + 16
60*a^6*b^13*c^16*d^3 - 48112*a^7*b^12*c^3*d^16 + 213338*a^7*b^12*c^5*d^14 -
 425832*a^7*b^12*c^7*d^12 + 434414*a^7*b^12*c^9*d^10 - 219064*a^7*b^12*c^11
*d^8 + 50732*a^7*b^12*c^13*d^6 - 7220*a^7*b^12*c^15*d^4 + 364*a^7*b^12*c^17
*d^2 + 20720*a^8*b^11*c^2*d^17 - 135520*a^8*b^11*c^4*d^15 + 364608*a^8*b^11
*c^6*d^13 - 496336*a^8*b^11*c^8*d^11 + 343832*a^8*b^11*c^10*d^9 - 111220*a^
8*b^11*c^12*d^7 + 17956*a^8*b^11*c^14*d^5 - 1376*a^8*b^11*c^16*d^3 + 58925*
a^9*b^10*c^3*d^16 - 235152*a^9*b^10*c^5*d^14 + 434414*a^9*b^10*c^7*d^12 - 4
01788*a^9*b^10*c^9*d^10 + 172673*a^9*b^10*c^11*d^8 - 31940*a^9*b^10*c^13*d^
6 + 3244*a^9*b^10*c^15*d^4 - 18788*a^10*b^9*c^2*d^17 + 114144*a^10*b^9*c^4*
d^15 - 281736*a^10*b^9*c^6*d^13 + 343832*a^10*b^9*c^8*d^11 - 197840*a^10*b^
9*c^10*d^9 + 45940*a^10*b^9*c^12*d^7 - 4760*a^10*b^9*c^14*d^5 - 36732*a^11*
b^8*c^3*d^16 + 130428*a^11*b^8*c^5*d^14 - 219064*a^11*b^8*c^7*d^12 + 172673
*a^11*b^8*c^9*d^10 - 52480*a^11*b^8*c^11*d^8 + 4580*a^11*b^8*c^13*d^6 + 739
2*a^12*b^7*c^2*d^17 - 44168*a^12*b^7*c^4*d^15 + 103104*a^12*b^7*c^6*d^13 -
111220*a^12*b^7*c^8*d^11 + 45940*a^12*b^7*c^10*d^9 - 4000*a^12*b^7*c^12*d^7
 + 9736*a^13*b^6*c^3*d^16 - 31908*a^13*b^6*c^5*d^14 + 50732*a^13*b^6*c^7*d^
12 - 31940*a^13*b^6*c^9*d^10 + 4580*a^13*b^6*c^11*d^8 - 520*a^14*b^5*c^2*d^
17 + 5696*a^14*b^5*c^4*d^15 - 16860*a^14*b^5*c^6*d^13 + 17956*a^14*b^5*c^8*
d^11 - 4760*a^14*b^5*c^10*d^9 - 760*a^15*b^4*c^3*d^16 + 3966*a^15*b^4*c^5*d
^14 - 7220*a^15*b^4*c^7*d^12 + 3244*a^15*b^4*c^9*d^10 - 24*a^16*b^3*c^2*d^1
7 - 332*a^16*b^3*c^4*d^15 + 1660*a^16*b^3*c^6*d^13 - 1376*a^16*b^3*c^8*d^11
 - 44*a^17*b^2*c^3*d^16 - 140*a^17*b^2*c^5*d^14 + 364*a^17*b^2*c^7*d^12))/(
a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20
 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^
18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^
8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^
2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^1
9*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d -
 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5
*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48
*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^1
8*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^

18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13
*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^
12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*
d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^1
3 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^
7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14
 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*
d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*
d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d
^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^1
5*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*
d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*
c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*
c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*
c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^
11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10
*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10
*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 138
60*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 9
28*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 4
4936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7
 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 +
4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12
 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8*c^14*d
^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^1
5 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11
*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^
18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*
d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^1
4*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7
*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13
*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d
^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*
d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^
13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 -
 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66
*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19)) + (16*tan(e/2 +
 (f*x)/2)*(864*a*b^15*c^2*d^14 - 3456*a*b^15*c^4*d^12 + 4770*a*b^15*c^6*d^1
0 - 2326*a*b^15*c^8*d^8 + 11*a*b^15*c^10*d^6 + 20*a*b^15*c^12*d^4 + 864*a^2
*b^14*c*d^15 - 3456*a^4*b^12*c*d^15 + 4770*a^6*b^10*c*d^15 - 2326*a^8*b^8*c
*d^15 + 11*a^10*b^6*c*d^15 + 20*a^12*b^4*c*d^15 - 2592*a^2*b^14*c^3*d^13 +
2322*a^2*b^14*c^5*d^11 - 412*a^2*b^14*c^7*d^9 - 37*a^2*b^14*c^9*d^7 + 170*a
^2*b^14*c^11*d^5 - 2592*a^3*b^13*c^2*d^14 + 11052*a^3*b^13*c^4*d^12 - 16128
*a^3*b^13*c^6*d^10 + 7886*a^3*b^13*c^8*d^8 - 64*a^3*b^13*c^10*d^6 + 80*a^3*
b^13*c^12*d^4 + 11052*a^4*b^12*c^3*d^13 - 11374*a^4*b^12*c^5*d^11 + 4534*a^

4*b^12*c^7*d^9 - 1976*a^4*b^12*c^9*d^7 - 40*a^4*b^12*c^11*d^5 + 2322*a^5*b^11*c^2*d^14 - 11374*a^5*b^11*c^4*d^12 + 17846*a^5*b^11*c^6*d^10 - 6488*a^5*b^11*c^8*d^8 - 2188*a^5*b^11*c^10*d^6 + 80*a^5*b^11*c^12*d^4 - 16128*a^6*b^10*c^3*d^13 + 17846*a^6*b^10*c^5*d^11 - 9806*a^6*b^10*c^7*d^9 + 6220*a^6*b^10*c^9*d^7 - 760*a^6*b^10*c^11*d^5 - 412*a^7*b^9*c^2*d^14 + 4534*a^7*b^9*c^4*d^12 - 9806*a^7*b^9*c^6*d^10 + 1976*a^7*b^9*c^8*d^8 + 2448*a^7*b^9*c^10*d^6 + 7886*a^8*b^8*c^3*d^13 - 6488*a^8*b^8*c^5*d^11 + 1976*a^8*b^8*c^7*d^9 - 2632*a^8*b^8*c^9*d^7 - 37*a^9*b^7*c^2*d^14 - 1976*a^9*b^7*c^4*d^12 + 6220*a^9*b^7*c^6*d^10 - 2632*a^9*b^7*c^8*d^8 - 64*a^10*b^6*c^3*d^13 - 2188*a^10*b^6*c^5*d^11 + 2448*a^10*b^6*c^7*d^9 + 170*a^11*b^5*c^2*d^14 - 40*a^11*b^5*c^4*d^12 - 760*a^11*b^5*c^6*d^10 + 80*a^12*b^4*c^3*d^13 + 80*a^12*b^4*c^5*d^11))/(a^20*d^20 + b^20*c^20 - 4*a^2*b^18*c^20 + 6*a^4*b^16*c^20 - 4*a^6*b^14*c^20 + a^8*b^12*c^20 + a^12*b^8*d^20 - 4*a^14*b^6*d^20 + 6*a^16*b^4*d^20 - 4*a^18*b^2*d^20 - 4*a^20*c^2*d^18 + 6*a^20*c^4*d^16 - 4*a^20*c^6*d^14 + a^20*c^8*d^12 + b^20*c^12*d^8 - 4*b^20*c^14*d^6 + 6*b^20*c^16*d^4 - 4*b^20*c^18*d^2 - 12*a*b^19*c^11*d^9 + 48*a*b^19*c^13*d^7 - 72*a*b^19*c^15*d^5 + 48*a*b^19*c^17*d^3 + 48*a^3*b^17*c^19*d - 72*a^5*b^15*c^19*d + 48*a^7*b^13*c^19*d - 12*a^9*b^11*c^19*d - 12*a^11*b^9*c*d^19 + 48*a^13*b^7*c*d^19 - 72*a^15*b^5*c*d^19 + 48*a^17*b^3*c*d^19 + 48*a^19*b*c^3*d^17 - 72*a^19*b*c^5*d^15 + 48*a^19*b*c^7*d^13 - 12*a^19*b*c^9*d^11 + 66*a^2*b^18*c^10*d^10 - 268*a^2*b^18*c^12*d^8 + 412*a^2*b^18*c^14*d^6 - 288*a^2*b^18*c^16*d^4 + 82*a^2*b^18*c^18*d^2 - 220*a^3*b^17*c^9*d^11 + 928*a^3*b^17*c^11*d^9 - 1512*a^3*b^17*c^13*d^7 + 1168*a^3*b^17*c^15*d^5 - 412*a^3*b^17*c^17*d^3 + 495*a^4*b^16*c^8*d^12 - 2244*a^4*b^16*c^10*d^10 + 4032*a^4*b^16*c^12*d^8 - 3588*a^4*b^16*c^14*d^6 + 1587*a^4*b^16*c^16*d^4 - 288*a^4*b^16*c^18*d^2 - 792*a^5*b^15*c^7*d^13 + 4048*a^5*b^15*c^9*d^11 - 8344*a^5*b^15*c^11*d^9 + 8736*a^5*b^15*c^13*d^7 - 4744*a^5*b^15*c^15*d^5 + 1168*a^5*b^15*c^17*d^3 + 924*a^6*b^14*c^6*d^14 - 5676*a^6*b^14*c^8*d^12 + 13860*a^6*b^14*c^10*d^10 - 17164*a^6*b^14*c^12*d^8 + 11236*a^6*b^14*c^14*d^6 - 3588*a^6*b^14*c^16*d^4 + 412*a^6*b^14*c^18*d^2 - 792*a^7*b^13*c^5*d^15 + 6336*a^7*b^13*c^7*d^13 - 18744*a^7*b^13*c^9*d^11 + 27504*a^7*b^13*c^11*d^9 - 21576*a^7*b^13*c^13*d^7 + 8736*a^7*b^13*c^15*d^5 - 1512*a^7*b^13*c^17*d^3 + 495*a^8*b^12*c^4*d^16 - 5676*a^8*b^12*c^6*d^14 + 20724*a^8*b^12*c^8*d^12 - 36300*a^8*b^12*c^10*d^10 + 34156*a^8*b^12*c^12*d^8 - 17164*a^8*b^12*c^14*d^6 + 4032*a^8*b^12*c^16*d^4 - 268*a^8*b^12*c^18*d^2 - 220*a^9*b^11*c^3*d^17 + 4048*a^9*b^11*c^5*d^15 - 18744*a^9*b^11*c^7*d^13 + 39776*a^9*b^11*c^9*d^11 - 44936*a^9*b^11*c^11*d^9 + 27504*a^9*b^11*c^13*d^7 - 8344*a^9*b^11*c^15*d^5 + 928*a^9*b^11*c^17*d^3 + 66*a^10*b^10*c^2*d^18 - 2244*a^10*b^10*c^4*d^16 + 13860*a^10*b^10*c^6*d^14 - 36300*a^10*b^10*c^8*d^12 + 49236*a^10*b^10*c^10*d^10 - 36300*a^10*b^10*c^12*d^8 + 13860*a^10*b^10*c^14*d^6 - 2244*a^10*b^10*c^16*d^4 + 66*a^10*b^10*c^18*d^2 + 928*a^11*b^9*c^3*d^17 - 8344*a^11*b^9*c^5*d^15 + 27504*a^11*b^9*c^7*d^13 - 44936*a^11*b^9*c^9*d^11 + 39776*a^11*b^9*c^11*d^9 - 18744*a^11*b^9*c^13*d^7 + 4048*a^11*b^9*c^15*d^5 - 220*a^11*b^9*c^17*d^3 - 268*a^12*b^8*c^2*d^18 + 4032*a^12*b^8*c^4*d^16 - 17164*a^12*b^8*c^6*d^14 + 34156*a^12*b^8*c^8*d^12 - 36300*a^12*b^8*c^10*d^10 + 20724*a^12*b^8*c^12*d^8 - 5676*a^12*b^8 *c^14*d^6 + 495*a^12*b^8*c^16*d^4 - 1512*a^13*b^7*c^3*d^17 + 8736*a^13*b^7*c^5*d^15 - 21576*a^13*b^7*c^7*d^13 + 27504*a^13*b^7*c^9*d^11 - 18744*a^13*b^7*c^11*d^9 + 6336*a^13*b^7*c^13*d^7 - 792*a^13*b^7*c^15*d^5 + 412*a^14*b^6*c^2*d^18 - 3588*a^14*b^6*c^4*d^16 + 11236*a^14*b^6*c^6*d^14 - 17164*a^14*b^6*c^8*d^12 + 13860*a^14*b^6*c^10*d^10 - 5676*a^14*b^6*c^12*d^8 + 924*a^14*b^6*c^14*d^6 + 1168*a^15*b^5*c^3*d^17 - 4744*a^15*b^5*c^5*d^15 + 8736*a^15*b^5*c^7*d^13 - 8344*a^15*b^5*c^9*d^11 + 4048*a^15*b^5*c^11*d^9 - 792*a^15*b^5*c^13*d^7 - 288*a^16*b^4*c^2*d^18 + 1587*a^16*b^4*c^4*d^16 - 3588*a^16*b^4*c^6*d^14 + 4032*a^16*b^4*c^8*d^12 - 2244*a^16*b^4*c^10*d^10 + 495*a^16*b^4*c^12*d^8 - 412*a^17*b^3*c^3*d^17 + 1168*a^17*b^3*c^5*d^15 - 1512*a^17*b^3*c^7*d^13 + 928*a^17*b^3*c^9*d^11 - 220*a^17*b^3*c^11*d^9 + 82*a^18*b^2*c^2*d^18 - 288*a^18*b^2*c^4*d^16 + 412*a^18*b^2*c^6*d^14 - 268*a^18*b^2*c^8*d^12 + 66*a^18*b^2*c^10*d^10 - 12*a*b^19*c^19*d - 12*a^19*b*c*d^19)))*(-(((4*a^24*d^24 + 4*b^24*c^24 + 16*a^2*b^22*c^24 + 16*a^4*b^20*c^24 - 1152*a^10*b^14*d^24 + 5568*a^12*b^12*d^24 - 10568*a^14*b^10*d^24 + 9460*a^16*b^8*d^24 - 3560*a^18*b^6*d^24 + 136*a^20*b^4*d^24 + 76*a^22*b^2*d^24 + 16*a^24*c^2*d^22 + 16*a^24*c^4*d^20 - 1152*b^24*c^10*d^14 + 5568*b^24*c^12*d^12 - 10568*b^24*c^14*d^10 + 9460*b^24*c^16*d^8 - 3560*b^24*c^18*d^6 + 136*b^24*c^20*d^4 + 76*b^24*c^22*d^2 + 11520*a*b^23*c^9*d^15 - 56448*a*b^23*c^11*d^13 + 109456*a*b^23*c^13*d^11 - 101240*a*b^23*c^15*d^9 + 40720*a*b^23*c^17*d^7 - 2960*a*b^23*c^19*d^5 - 536*a*b^23*c^21*d^3 - 176*a^3*b^21*c^23*d - 320*a^5*b^19*c^23*d + 11520*a^9*b^15*c*d^23 - 56448*a^11*b^13*c*d^23 + 109456*a^13*b^11*c*d^23 - 101240*a^15*b^9*c*d^23 + 40720*a^17*b^7*c*d^23 - 2960*a^19*b^5*c*d^23 - 536*a^21*b^3*c*d^23 - 176*a^23*b*c^3*d^21 - 320*a^23*b*c^5*d^19 - 51840*a^2*b^22*c^8*d^16 + 263808*a^2*b^22*c^10*d^14 - 541208*a^2*b^22*c^12*d^12 + 547088*a^2*b^22*c^14*d^10 - 263320*a^2*b^22*c^16*d^8 + 44120*a^2*b^22*c^18*d^6 - 1564*a^2*b^22*c^20*d^4 - 196*a^2*b^22*c^22*d^2 + 138240*a^3*b^21*c^7*d^17 - 758400*a^3*b^21*c^9*d^15 + 1720736*a^3*b^21*c^11*d^13 - 2002728*a^3*b^21*c^13*d^11 + 1210560*a^3*b^21*c^15*d^9 - 335040*a^3*b^21*c^17*d^7 + 37680*a^3*b^21*c^19*d^5 - 288*a^3*b^21*c^21*d^3 - 241920*a^4*b^20*c^6*d^18 + 1512000*a^4*b^20*c^8*d^16 - 3975688*a^4*b^20*c^10*d^14 + 5501328*a^4*b^20*c^12*d^12 - 4147952*a^4*b^20*c^14*d^10 + 1586920*a^4*b^20*c^16*d^8 - 276020*a^4*b^20*c^18*d^6 + 21124*a^4*b^20*c^20*d^4 + 176*a^4*b^20*c^22*d^2 + 290304*a^5*b^19*c^5*d^19 - 2232576*a^5*b^19*c^7*d^17 + 7078256*a^5*b^19*c^9*d^15 - 11781560*a^5*b^19*c^11*d^13 + 10875200*a^5*b^19*c^13*d^11 - 5365072*a^5*b^19*c^15*d^9 + 1310168*a^5*b^19*c^17*d^7 - 170968*a^5*b^19*c^19*d^5 + 8160*a^5*b^19*c^21*d^3 - 241920*a^6*b^18*c^4*d^20 + 2532096*a^6*b^18*c^6*d^18 - 9955992*a^6*b^18*c^8*d^16 + 20019440*a^6*b^18*c^10*d^14 - 22419600*a^6*b^18*c^12*d^12 + 13887520*a^6*b^18*c^14*d^10 - 4506428*a^6*b^18*c^16*d^8 + 793756*a^6*b^18*c^18*d^6 - 72240*a^6*b^18*c^20*d^4 + 3040*a^6*b^18*c^22*d^2 + 138240*a^7*b^17*c^3*d^21 - 2232576*a^7*b^17*c^5*d^19 + 11150016*a^7*b^17*c^7*d^17 - 27336616*a^7*b^17*c^9*d^15 + 37153600*a^7*b^17*c^11*d^13 - 28461040*a^7*b^17*c^13*d^11 + 11779808*a^7*b^17*c^15*d^9 - 2621008*a^7*b^17*c^17*d^7 + 336688*a^7*b^17*c^19*d^5 - 17920*a^7*b^17*c^21*d^3 - 51840*a^8*b^16*c^2*d^22 + 1512000*a^8*b^16*c^4*d^20 - 9955992*a^8*b^16*c^6*d^18 + 30289

656*a^8*b^16*c^8*d^16 - 50137600*a^8*b^16*c^10*d^14 + 46972560*a^8*b^16*c^12*d^12 - 24199280*a^8*b^16*c^14*d^10 + 6661036*a^8*b^16*c^16*d^8 - 1058448*a^8*b^16*c^18*d^6 + 72560*a^8*b^16*c^20*d^4 - 758400*a^9*b^15*c^3*d^21 + 7078256*a^9*b^15*c^5*d^19 - 27336616*a^9*b^15*c^7*d^17 + 55383904*a^9*b^15*c^9*d^15 - 63124080*a^9*b^15*c^11*d^13 + 39987520*a^9*b^15*c^13*d^11 - 13462088*a^9*b^15*c^15*d^9 + 2478528*a^9*b^15*c^17*d^7 - 212032*a^9*b^15*c^19*d^5 + 263808*a^10*b^14*c^2*d^22 - 3975688*a^10*b^14*c^4*d^20 + 20019440*a^10*b^14*c^6*d^18 - 50137600*a^10*b^14*c^8*d^16 + 69593872*a^10*b^14*c^10*d^14 - 53854288*a^10*b^14*c^12*d^12 + 21989928*a^10*b^14*c^14*d^10 - 4591360*a^10*b^14*c^16*d^8 + 460480*a^10*b^14*c^18*d^6 + 1720736*a^11*b^13*c^3*d^21 - 11781560*a^11*b^13*c^5*d^19 + 37153600*a^11*b^13*c^7*d^17 - 63124080*a^11*b^13*c^9*d^15 + 59445728*a^11*b^13*c^11*d^13 - 29358696*a^11*b^13*c^13*d^11 + 6995840*a^11*b^13*c^15*d^9 - 762560*a^11*b^13*c^17*d^7 - 541208*a^12*b^12*c^2*d^22 + 5501328*a^12*b^12*c^4*d^20 - 22419600*a^12*b^12*c^6*d^18 + 46972560*a^12*b^12*c^8*d^16 - 53854288*a^12*b^12*c^10*d^14 + 32294808*a^12*b^12*c^12*d^12 - 8958208*a^12*b^12*c^14*d^10 + 999040*a^12*b^12*c^16*d^8 - 2002728*a^13*b^11*c^3*d^21 + 10875200*a^13*b^11*c^5*d^19 - 28461040*a^13*b^11*c^7*d^17 + 39987520*a^13*b^11*c^9*d^15 - 29358696*a^13*b^11*c^11*d^13 + 9722048*a^13*b^11*c^13*d^11 - 1104320*a^13*b^11*c^15*d^9 + 547088*a^14*b^10*c^2*d^22 - 4147952*a^14*b^10*c^4*d^20 + 13887520*a^14*b^10*c^6*d^18 - 24199280*a^14*b^10*c^8*d^16 + 21989928*a^14*b^10*c^10*d^14 - 8958208*a^14*b^10*c^12*d^12 + 1124032*a^14*b^10*c^14*d^10 + 1210560*a^15*b^9*c^3*d^21 - 5365072*a^15*b^9*c^5*d^19 + 11779808*a^15*b^9*c^7*d^17 - 13462088*a^15*b^9*c^9*d^15 + 6995840*a^15*b^9*c^11*d^13 - 1104320*a^15*b^9*c^13*d^11 - 263320*a^16*b^8*c^2*d^22 + 1586920*a^16*b^8*c^4*d^20 - 4506428*a^16*b^8*c^6*d^18 + 6661036*a^16*b^8*c^8*d^16 - 4591360*a^16*b^8*c^10*d^14 + 999040*a^16*b^8*c^12*d^12 - 335040*a^17*b^7*c^3*d^21 + 1310168*a^17*b^7*c^5*d^19 - 2621008*a^17*b^7*c^7*d^17 + 2478528*a^17*b^7*c^9*d^15 - 762560*a^17*b^7*c^11*d^13 + 44120*a^18*b^6*c^2*d^22 - 276020*a^18*b^6*c^4*d^20 + 793756*a^18*b^6*c^6*d^18 - 1058448*a^18*b^6*c^8*d^16 + 460480*a^18*b^6*c^10*d^14 + 37680*a^19*b^5*c^3*d^21 - 170968*a^19*b^5*c^5*d^19 + 336688*a^19*b^5*c^7*d^17 - 212032*a^19*b^5*c^9*d^15 - 1564*a^20*b^4*c^2*d^22 + 21124*a^20*b^4*c^4*d^20 - 72240*a^20*b^4*c^6*d^18 + 72560*a^20*b^4*c^8*d^16 - 288*a^21*b^3*c^3*d^21 + 8160*a^21*b^3*c^5*d^19 - 17920*a^21*b^3*c^7*d^17 - 196*a^22*b^2*c^2*d^22 + 176*a^22*b^2*c^4*d^20 + 3040*a^22*b^2*c^6*d^18 - 8*a*b^23*c^23*d - 8*a^23*b*c*d^23)^2/4 - (20736*b^18*d^18 - 96768*a^2*b^16*d^18 + 173664*a^4*b^14*d^18 - 136032*a^6*b^12*d^18 + 31081*a^8*b^10*d^18 + 8440*a^10*b^8*d^18 + 400*a^12*b^6*d^18 - 96768*b^18*c^2*d^16 + 173664*b^18*c^4*d^14 - 136032*b^18*c^6*d^12 + 31081*b^18*c^8*d^10 + 8440*b^18*c^10*d^8 + 400*b^18*c^12*d^6 - 131328*a*b^17*c^3*d^15 + 216576*a*b^17*c^5*d^13 - 141104*a*b^17*c^7*d^11 + 20260*a*b^17*c^9*d^9 + 2800*a*b^17*c^11*d^7 - 131328*a^3*b^15*c*d^17 + 216576*a^5*b^13*c*d^17 - 141104*a^7*b^11*c*d^17 + 20260*a^9*b^9*c*d^17 + 2800*a^11*b^7*c*d^17 + 495936*a^2*b^16*c^2*d^16 - 989856*a^2*b^16*c^4*d^14 + 901948*a^2*b^16*c^6*d^12 - 308392*a^2*b^16*c^8*d^10 - 5260*a^2*b^16*c^10*d^8 + 1600*a^2*b^16*c^12*d^6 + 657408*a^3*b^15*c^3*d^15 - 1158992*a^3*b^15*c^5*d^13 + 838256*a^3*b^

15*c^7*d^11 - 182200*a^3*b^15*c^9*d^9 - 3200*a^3*b^15*c^11*d^7 - 989856*a^4
*b^14*c^2*d^16 + 2185654*a^4*b^14*c^4*d^14 - 2218576*a^4*b^14*c^6*d^12 + 90
0624*a^4*b^14*c^8*d^10 - 64720*a^4*b^14*c^10*d^8 + 1600*a^4*b^14*c^12*d^6 -
 1158992*a^5*b^13*c^3*d^15 + 2158808*a^5*b^13*c^5*d^13 - 1641528*a^5*b^13*c
^7*d^11 + 406880*a^5*b^13*c^9*d^9 - 17600*a^5*b^13*c^11*d^7 + 901948*a^6*b^
12*c^2*d^16 - 2218576*a^6*b^12*c^4*d^14 + 2430936*a^6*b^12*c^6*d^12 - 10269
28*a^6*b^12*c^8*d^10 + 88720*a^6*b^12*c^10*d^8 + 838256*a^7*b^11*c^3*d^15 -
 1641528*a^7*b^11*c^5*d^13 + 1206848*a^7*b^11*c^7*d^11 - 239360*a^7*b^11*c^
9*d^9 - 308392*a^8*b^10*c^2*d^16 + 900624*a^8*b^10*c^4*d^14 - 1026928*a^8*b
^10*c^6*d^12 + 354016*a^8*b^10*c^8*d^10 - 182200*a^9*b^9*c^3*d^15 + 406880*
a^9*b^9*c^5*d^13 - 239360*a^9*b^9*c^7*d^11 - 5260*a^10*b^8*c^2*d^16 - 64720
*a^10*b^8*c^4*d^14 + 88720*a^10*b^8*c^6*d^12 - 3200*a^11*b^7*c^3*d^15 - 176
00*a^11*b^7*c^5*d^13 + 1600*a^12*b^6*c^2*d^16 + 1600*a^12*b^6*c^4*d^14 + 27
648*a*b^17*c*d^17)*(80*a^2*b^28*c^30 - 16*b^30*c^30 - 16*a^30*d^30 - 160*a^
4*b^26*c^30 + 160*a^6*b^24*c^30 - 80*a^8*b^22*c^30 + 16*a^10*b^20*c^30 + 16
*a^20*b^10*d^30 - 80*a^22*b^8*d^30 + 160*a^24*b^6*d^30 - 160*a^26*b^4*d^30
 + 80*a^28*b^2*d^30 + 80*a^30*c^2*d^28 - 160*a^30*c^4*d^26 + 160*a^30*c^6*d^
24 - 80*a^30*c^8*d^22 + 16*a^30*c^10*d^20 + 16*b^30*c^20*d^10 - 80*b^30*c^2
2*d^8 + 160*b^30*c^24*d^6 - 160*b^30*c^26*d^4 + 80*b^30*c^28*d^2 - 320*a*b^
29*c^19*d^11 + 1600*a*b^29*c^21*d^9 - 3200*a*b^29*c^23*d^7 + 3200*a*b^29*c^
25*d^5 - 1600*a*b^29*c^27*d^3 - 1600*a^3*b^27*c^29*d + 3200*a^5*b^25*c^29*d
 - 3200*a^7*b^23*c^29*d + 1600*a^9*b^21*c^29*d - 320*a^11*b^19*c^29*d - 320
*a^19*b^11*c*d^29 + 1600*a^21*b^9*c*d^29 - 3200*a^23*b^7*c*d^29 + 3200*a^25
*b^5*c*d^29 - 1600*a^27*b^3*c*d^29 - 1600*a^29*b*c^3*d^27 + 3200*a^29*b*c^5
*d^25 - 3200*a^29*b*c^7*d^23 + 1600*a^29*b*c^9*d^21 - 320*a^29*b*c^11*d^19
 + 3040*a^2*b^28*c^18*d^12 - 15280*a^2*b^28*c^20*d^10 + 30800*a^2*b^28*c^22*
d^8 - 31200*a^2*b^28*c^24*d^6 + 16000*a^2*b^28*c^26*d^4 - 3440*a^2*b^28*c^2
8*d^2 - 18240*a^3*b^27*c^17*d^13 + 92800*a^3*b^27*c^19*d^11 - 190400*a^3*b^
27*c^21*d^9 + 198400*a^3*b^27*c^23*d^7 - 107200*a^3*b^27*c^25*d^5 + 26240*a
^3*b^27*c^27*d^3 + 77520*a^4*b^26*c^16*d^14 - 402800*a^4*b^26*c^18*d^12 + 8
51360*a^4*b^26*c^20*d^10 - 928000*a^4*b^26*c^22*d^8 + 541200*a^4*b^26*c^24*
d^6 - 155120*a^4*b^26*c^26*d^4 + 16000*a^4*b^26*c^28*d^2 - 248064*a^5*b^25*
c^15*d^15 + 1331520*a^5*b^25*c^17*d^13 - 2939840*a^5*b^25*c^19*d^11 + 34086
40*a^5*b^25*c^21*d^9 - 2184320*a^5*b^25*c^23*d^7 + 736064*a^5*b^25*c^25*d^5
 - 107200*a^5*b^25*c^27*d^3 + 620160*a^6*b^24*c^14*d^16 - 3488400*a^6*b^24*
c^16*d^14 + 8170000*a^6*b^24*c^18*d^12 - 10229760*a^6*b^24*c^20*d^10 + 7281
600*a^6*b^24*c^22*d^8 - 2863760*a^6*b^24*c^24*d^6 + 541200*a^6*b^24*c^26*d^
4 - 31200*a^6*b^24*c^28*d^2 - 1240320*a^7*b^23*c^13*d^17 + 7441920*a^7*b^23
*c^15*d^15 - 18787200*a^7*b^23*c^17*d^13 + 25721600*a^7*b^23*c^19*d^11 - 20
444800*a^7*b^23*c^21*d^9 + 9297920*a^7*b^23*c^23*d^7 - 2184320*a^7*b^23*c^2
5*d^5 + 198400*a^7*b^23*c^27*d^3 + 2015520*a^8*b^22*c^12*d^18 - 13178400*a^
8*b^22*c^14*d^16 + 36434400*a^8*b^22*c^16*d^14 - 55069600*a^8*b^22*c^18*d^1
2 + 48989680*a^8*b^22*c^20*d^10 - 25575920*a^8*b^22*c^22*d^8 + 7281600*a^8*
b^22*c^24*d^6 - 928000*a^8*b^22*c^26*d^4 + 30800*a^8*b^22*c^28*d^2 - 268736
0*a^9*b^21*c^11*d^19 + 19638400*a^9*b^21*c^13*d^17 - 60362240*a^9*b^21*c^15

*d^15 + 101475200*a^9*b^21*c^17*d^13 - 101172800*a^9*b^21*c^19*d^11 + 60333
760*a^9*b^21*c^21*d^9 - 20444800*a^9*b^21*c^23*d^7 + 3408640*a^9*b^21*c^25*
d^5 - 190400*a^9*b^21*c^27*d^3 + 2956096*a^10*b^20*c^10*d^20 - 24858080*a^1
0*b^20*c^12*d^18 + 86150560*a^10*b^20*c^14*d^16 - 162120160*a^10*b^20*c^16*
d^14 + 181463680*a^10*b^20*c^18*d^12 - 123188112*a^10*b^20*c^20*d^10 + 4898
9680*a^10*b^20*c^22*d^8 - 10229760*a^10*b^20*c^24*d^6 + 851360*a^10*b^20*c^
26*d^4 - 15280*a^10*b^20*c^28*d^2 - 2687360*a^11*b^19*c^9*d^21 + 26873600*a
^11*b^19*c^11*d^19 - 106460800*a^11*b^19*c^13*d^17 + 225738240*a^11*b^19*c^
15*d^15 - 284331200*a^11*b^19*c^17*d^13 + 219166080*a^11*b^19*c^19*d^11 - 1
01172800*a^11*b^19*c^21*d^9 + 25721600*a^11*b^19*c^23*d^7 - 2939840*a^11*b^
19*c^25*d^5 + 92800*a^11*b^19*c^27*d^3 + 2015520*a^12*b^18*c^8*d^22 - 24858
080*a^12*b^18*c^10*d^20 + 114212800*a^12*b^18*c^12*d^18 - 274937600*a^12*b^
18*c^14*d^16 + 390830000*a^12*b^18*c^16*d^14 - 341426960*a^12*b^18*c^18*d^1
2 + 181463680*a^12*b^18*c^20*d^10 - 55069600*a^12*b^18*c^22*d^8 + 8170000*a
^12*b^18*c^24*d^6 - 402800*a^12*b^18*c^26*d^4 + 3040*a^12*b^18*c^28*d^2 - 1
240320*a^13*b^17*c^7*d^23 + 19638400*a^13*b^17*c^9*d^21 - 106460800*a^13*b^
17*c^11*d^19 + 293542400*a^13*b^17*c^13*d^17 - 472561920*a^13*b^17*c^15*d^1
5 + 467412160*a^13*b^17*c^17*d^13 - 284331200*a^13*b^17*c^19*d^11 + 1014752
00*a^13*b^17*c^21*d^9 - 18787200*a^13*b^17*c^23*d^7 + 1331520*a^13*b^17*c^2
5*d^5 - 18240*a^13*b^17*c^27*d^3 + 620160*a^14*b^16*c^6*d^24 - 13178400*a^1
4*b^16*c^8*d^22 + 86150560*a^14*b^16*c^10*d^20 - 274937600*a^14*b^16*c^12*d
^18 + 503363200*a^14*b^16*c^14*d^16 - 563751280*a^14*b^16*c^16*d^14 + 39083
0000*a^14*b^16*c^18*d^12 - 162120160*a^14*b^16*c^20*d^10 + 36434400*a^14*b^
16*c^22*d^8 - 3488400*a^14*b^16*c^24*d^6 + 77520*a^14*b^16*c^26*d^4 - 24806
4*a^15*b^15*c^5*d^25 + 7441920*a^15*b^15*c^7*d^23 - 60362240*a^15*b^15*c^9*
d^21 + 225738240*a^15*b^15*c^11*d^19 - 472561920*a^15*b^15*c^13*d^17 + 5999
84128*a^15*b^15*c^15*d^15 - 472561920*a^15*b^15*c^17*d^13 + 225738240*a^15*
b^15*c^19*d^11 - 60362240*a^15*b^15*c^21*d^9 + 7441920*a^15*b^15*c^23*d^7 -
248064*a^15*b^15*c^25*d^5 + 77520*a^16*b^14*c^4*d^26 - 3488400*a^16*b^14*c
^6*d^24 + 36434400*a^16*b^14*c^8*d^22 - 162120160*a^16*b^14*c^10*d^20 + 390
830000*a^16*b^14*c^12*d^18 - 563751280*a^16*b^14*c^14*d^16 + 503363200*a^16
*b^14*c^16*d^14 - 274937600*a^16*b^14*c^18*d^12 + 86150560*a^16*b^14*c^20*d
^10 - 13178400*a^16*b^14*c^22*d^8 + 620160*a^16*b^14*c^24*d^6 - 18240*a^17*
b^13*c^3*d^27 + 1331520*a^17*b^13*c^5*d^25 - 18787200*a^17*b^13*c^7*d^23 +
101475200*a^17*b^13*c^9*d^21 - 284331200*a^17*b^13*c^11*d^19 + 467412160*a^
17*b^13*c^13*d^17 - 472561920*a^17*b^13*c^15*d^15 + 293542400*a^17*b^13*c^1
7*d^13 - 106460800*a^17*b^13*c^19*d^11 + 19638400*a^17*b^13*c^21*d^9 - 1240
320*a^17*b^13*c^23*d^7 + 3040*a^18*b^12*c^2*d^28 - 402800*a^18*b^12*c^4*d^2
6 + 8170000*a^18*b^12*c^6*d^24 - 55069600*a^18*b^12*c^8*d^22 + 181463680*a^
18*b^12*c^10*d^20 - 341426960*a^18*b^12*c^12*d^18 + 390830000*a^18*b^12*c^1
4*d^16 - 274937600*a^18*b^12*c^16*d^14 + 114212800*a^18*b^12*c^18*d^12 - 24
858080*a^18*b^12*c^20*d^10 + 2015520*a^18*b^12*c^22*d^8 + 92800*a^19*b^11*c
^3*d^27 - 2939840*a^19*b^11*c^5*d^25 + 25721600*a^19*b^11*c^7*d^23 - 101172
800*a^19*b^11*c^9*d^21 + 219166080*a^19*b^11*c^11*d^19 - 284331200*a^19*b^1
1*c^13*d^17 + 225738240*a^19*b^11*c^15*d^15 - 106460800*a^19*b^11*c^17*d^13

+ 26873600*a^19*b^11*c^19*d^11 - 2687360*a^19*b^11*c^21*d^9 - 15280*a^20*b
^10*c^2*d^28 + 851360*a^20*b^10*c^4*d^26 - 10229760*a^20*b^10*c^6*d^24 + 48
989680*a^20*b^10*c^8*d^22 - 123188112*a^20*b^10*c^10*d^20 + 181463680*a^20*
b^10*c^12*d^18 - 162120160*a^20*b^10*c^14*d^16 + 86150560*a^20*b^10*c^16*d^
14 - 24858080*a^20*b^10*c^18*d^12 + 2956096*a^20*b^10*c^20*d^10 - 190400*a^
21*b^9*c^3*d^27 + 3408640*a^21*b^9*c^5*d^25 - 20444800*a^21*b^9*c^7*d^23 +
60333760*a^21*b^9*c^9*d^21 - 101172800*a^21*b^9*c^11*d^19 + 101475200*a^21*
b^9*c^13*d^17 - 60362240*a^21*b^9*c^15*d^15 + 19638400*a^21*b^9*c^17*d^13 -
2687360*a^21*b^9*c^19*d^11 + 30800*a^22*b^8*c^2*d^28 - 928000*a^22*b^8*c^4
*d^26 + 7281600*a^22*b^8*c^6*d^24 - 25575920*a^22*b^8*c^8*d^22 + 48989680*a
^22*b^8*c^10*d^20 - 55069600*a^22*b^8*c^12*d^18 + 36434400*a^22*b^8*c^14*d^
16 - 13178400*a^22*b^8*c^16*d^14 + 2015520*a^22*b^8*c^18*d^12 + 198400*a^23
*b^7*c^3*d^27 - 2184320*a^23*b^7*c^5*d^25 + 9297920*a^23*b^7*c^7*d^23 - 204
44800*a^23*b^7*c^9*d^21 + 25721600*a^23*b^7*c^11*d^19 - 18787200*a^23*b^7*c
^13*d^17 + 7441920*a^23*b^7*c^15*d^15 - 1240320*a^23*b^7*c^17*d^13 - 31200*
a^24*b^6*c^2*d^28 + 541200*a^24*b^6*c^4*d^26 - 2863760*a^24*b^6*c^6*d^24 +
7281600*a^24*b^6*c^8*d^22 - 10229760*a^24*b^6*c^10*d^20 + 8170000*a^24*b^6*
c^12*d^18 - 3488400*a^24*b^6*c^14*d^16 + 620160*a^24*b^6*c^16*d^14 - 107200
*a^25*b^5*c^3*d^27 + 736064*a^25*b^5*c^5*d^25 - 2184320*a^25*b^5*c^7*d^23 +
3408640*a^25*b^5*c^9*d^21 - 2939840*a^25*b^5*c^11*d^19 + 1331520*a^25*b^5*
c^13*d^17 - 248064*a^25*b^5*c^15*d^15 + 16000*a^26*b^4*c^2*d^28 - 155120*a^
26*b^4*c^4*d^26 + 541200*a^26*b^4*c^6*d^24 - 928000*a^26*b^4*c^8*d^22 + 851
360*a^26*b^4*c^10*d^20 - 402800*a^26*b^4*c^12*d^18 + 77520*a^26*b^4*c^14*d^
16 + 26240*a^27*b^3*c^3*d^27 - 107200*a^27*b^3*c^5*d^25 + 198400*a^27*b^3*c
^7*d^23 - 190400*a^27*b^3*c^9*d^21 + 92800*a^27*b^3*c^11*d^19 - 18240*a^27*
b^3*c^13*d^17 - 3440*a^28*b^2*c^2*d^28 + 16000*a^28*b^2*c^4*d^26 - 31200*a^
28*b^2*c^6*d^24 + 30800*a^28*b^2*c^8*d^22 - 15280*a^28*b^2*c^10*d^20 + 3040
*a^28*b^2*c^12*d^18 + 320*a*b^29*c^29*d + 320*a^29*b*c*d^29))^(1/2) + 2*a^2
4*d^24 + 2*b^24*c^24 + 8*a^2*b^22*c^24 + 8*a^4*b^20*c^24 - 576*a^10*b^14*d^
24 + 2784*a^12*b^12*d^24 - 5284*a^14*b^10*d^24 + 4730*a^16*b^8*d^24 - 1780*
a^18*b^6*d^24 + 68*a^20*b^4*d^24 + 38*a^22*b^2*d^24 + 8*a^24*c^2*d^22 + 8*a
^24*c^4*d^20 - 576*b^24*c^10*d^14 + 2784*b^24*c^12*d^12 - 5284*b^24*c^14*d^
10 + 4730*b^24*c^16*d^8 - 1780*b^24*c^18*d^6 + 68*b^24*c^20*d^4 + 38*b^24*c
^22*d^2 + 5760*a*b^23*c^9*d^15 - 28224*a*b^23*c^11*d^13 + 54728*a*b^23*c^13
*d^11 - 50620*a*b^23*c^15*d^9 + 20360*a*b^23*c^17*d^7 - 1480*a*b^23*c^19*d^
5 - 268*a*b^23*c^21*d^3 - 88*a^3*b^21*c^23*d - 160*a^5*b^19*c^23*d + 5760*a
^9*b^15*c*d^23 - 28224*a^11*b^13*c*d^23 + 54728*a^13*b^11*c*d^23 - 50620*a^
15*b^9*c*d^23 + 20360*a^17*b^7*c*d^23 - 1480*a^19*b^5*c*d^23 - 268*a^21*b^3
*c*d^23 - 88*a^23*b*c^3*d^21 - 160*a^23*b*c^5*d^19 - 25920*a^2*b^22*c^8*d^1
6 + 131904*a^2*b^22*c^10*d^14 - 270604*a^2*b^22*c^12*d^12 + 273544*a^2*b^22
*c^14*d^10 - 131660*a^2*b^22*c^16*d^8 + 22060*a^2*b^22*c^18*d^6 - 782*a^2*b
^22*c^20*d^4 - 98*a^2*b^22*c^22*d^2 + 69120*a^3*b^21*c^7*d^17 - 379200*a^3*
b^21*c^9*d^15 + 860368*a^3*b^21*c^11*d^13 - 1001364*a^3*b^21*c^13*d^11 + 60
5280*a^3*b^21*c^15*d^9 - 167520*a^3*b^21*c^17*d^7 + 18840*a^3*b^21*c^19*d^5
- 144*a^3*b^21*c^21*d^3 - 120960*a^4*b^20*c^6*d^18 + 756000*a^4*b^20*c^8*d

^16 - 1987844*a^4*b^20*c^10*d^14 + 2750664*a^4*b^20*c^12*d^12 - 2073976*a^4
*b^20*c^14*d^10 + 793460*a^4*b^20*c^16*d^8 - 138010*a^4*b^20*c^18*d^6 + 105
62*a^4*b^20*c^20*d^4 + 88*a^4*b^20*c^22*d^2 + 145152*a^5*b^19*c^5*d^19 - 11
16288*a^5*b^19*c^7*d^17 + 3539128*a^5*b^19*c^9*d^15 - 5890780*a^5*b^19*c^11
*d^13 + 5437600*a^5*b^19*c^13*d^11 - 2682536*a^5*b^19*c^15*d^9 + 655084*a^5
*b^19*c^17*d^7 - 85484*a^5*b^19*c^19*d^5 + 4080*a^5*b^19*c^21*d^3 - 120960*
a^6*b^18*c^4*d^20 + 1266048*a^6*b^18*c^6*d^18 - 4977996*a^6*b^18*c^8*d^16 +
 10009720*a^6*b^18*c^10*d^14 - 11209800*a^6*b^18*c^12*d^12 + 6943760*a^6*b^
18*c^14*d^10 - 2253214*a^6*b^18*c^16*d^8 + 396878*a^6*b^18*c^18*d^6 - 36120
*a^6*b^18*c^20*d^4 + 1520*a^6*b^18*c^22*d^2 + 69120*a^7*b^17*c^3*d^21 - 111
6288*a^7*b^17*c^5*d^19 + 5575008*a^7*b^17*c^7*d^17 - 13668308*a^7*b^17*c^9*
d^15 + 18576800*a^7*b^17*c^11*d^13 - 14230520*a^7*b^17*c^13*d^11 + 5889904*
a^7*b^17*c^15*d^9 - 1310504*a^7*b^17*c^17*d^7 + 168344*a^7*b^17*c^19*d^5 -
8960*a^7*b^17*c^21*d^3 - 25920*a^8*b^16*c^2*d^22 + 756000*a^8*b^16*c^4*d^20
 - 4977996*a^8*b^16*c^6*d^18 + 15144828*a^8*b^16*c^8*d^16 - 25068800*a^8*b^
16*c^10*d^14 + 23486280*a^8*b^16*c^12*d^12 - 12099640*a^8*b^16*c^14*d^10 +
3330518*a^8*b^16*c^16*d^8 - 529224*a^8*b^16*c^18*d^6 + 36280*a^8*b^16*c^20*
d^4 - 379200*a^9*b^15*c^3*d^21 + 3539128*a^9*b^15*c^5*d^19 - 13668308*a^9*b
^15*c^7*d^17 + 27691952*a^9*b^15*c^9*d^15 - 31562040*a^9*b^15*c^11*d^13 + 1
9993760*a^9*b^15*c^13*d^11 - 6731044*a^9*b^15*c^15*d^9 + 1239264*a^9*b^15*c
^17*d^7 - 106016*a^9*b^15*c^19*d^5 + 131904*a^10*b^14*c^2*d^22 - 1987844*a^
10*b^14*c^4*d^20 + 10009720*a^10*b^14*c^6*d^18 - 25068800*a^10*b^14*c^8*d^1
6 + 34796936*a^10*b^14*c^10*d^14 - 26927144*a^10*b^14*c^12*d^12 + 10994964*
a^10*b^14*c^14*d^10 - 2295680*a^10*b^14*c^16*d^8 + 230240*a^10*b^14*c^18*d^
6 + 860368*a^11*b^13*c^3*d^21 - 5890780*a^11*b^13*c^5*d^19 + 18576800*a^11*
b^13*c^7*d^17 - 31562040*a^11*b^13*c^9*d^15 + 29722864*a^11*b^13*c^11*d^13
- 14679348*a^11*b^13*c^13*d^11 + 3497920*a^11*b^13*c^15*d^9 - 381280*a^11*b
^13*c^17*d^7 - 270604*a^12*b^12*c^2*d^22 + 2750664*a^12*b^12*c^4*d^20 - 112
09800*a^12*b^12*c^6*d^18 + 23486280*a^12*b^12*c^8*d^16 - 26927144*a^12*b^12
*c^10*d^14 + 16147404*a^12*b^12*c^12*d^12 - 4479104*a^12*b^12*c^14*d^10 + 4
99520*a^12*b^12*c^16*d^8 - 1001364*a^13*b^11*c^3*d^21 + 5437600*a^13*b^11*c
^5*d^19 - 14230520*a^13*b^11*c^7*d^17 + 19993760*a^13*b^11*c^9*d^15 - 14679
348*a^13*b^11*c^11*d^13 + 4861024*a^13*b^11*c^13*d^11 - 552160*a^13*b^11*c^
15*d^9 + 273544*a^14*b^10*c^2*d^22 - 2073976*a^14*b^10*c^4*d^20 + 6943760*a
^14*b^10*c^6*d^18 - 12099640*a^14*b^10*c^8*d^16 + 10994964*a^14*b^10*c^10*d
^14 - 4479104*a^14*b^10*c^12*d^12 + 562016*a^14*b^10*c^14*d^10 + 605280*a^1
5*b^9*c^3*d^21 - 2682536*a^15*b^9*c^5*d^19 + 5889904*a^15*b^9*c^7*d^17 - 67
31044*a^15*b^9*c^9*d^15 + 3497920*a^15*b^9*c^11*d^13 - 552160*a^15*b^9*c^13
*d^11 - 131660*a^16*b^8*c^2*d^22 + 793460*a^16*b^8*c^4*d^20 - 2253214*a^16*
b^8*c^6*d^18 + 3330518*a^16*b^8*c^8*d^16 - 2295680*a^16*b^8*c^10*d^14 + 499
520*a^16*b^8*c^12*d^12 - 167520*a^17*b^7*c^3*d^21 + 655084*a^17*b^7*c^5*d^1
9 - 1310504*a^17*b^7*c^7*d^17 + 1239264*a^17*b^7*c^9*d^15 - 381280*a^17*b^7
*c^11*d^13 + 22060*a^18*b^6*c^2*d^22 - 138010*a^18*b^6*c^4*d^20 + 396878*a^
18*b^6*c^6*d^18 - 529224*a^18*b^6*c^8*d^16 + 230240*a^18*b^6*c^10*d^14 + 18
840*a^19*b^5*c^3*d^21 - 85484*a^19*b^5*c^5*d^19 + 168344*a^19*b^5*c^7*d^17

- 106016*a^19*b^5*c^9*d^15 - 782*a^20*b^4*c^2*d^22 + 10562*a^20*b^4*c^4*d^20 - 36120*a^20*b^4*c^6*d^18 + 36280*a^20*b^4*c^8*d^16 - 144*a^21*b^3*c^3*d^21 + 4080*a^21*b^3*c^5*d^19 - 8960*a^21*b^3*c^7*d^17 - 98*a^22*b^2*c^2*d^22 + 88*a^22*b^2*c^4*d^20 + 1520*a^22*b^2*c^6*d^18 - 4*a*b^23*c^23*d - 4*a^23*b*c*d^23)/(16*(5*a^2*b^28*c^30 - b^30*c^30 - a^30*d^30 - 10*a^4*b^26*c^30 + 10*a^6*b^24*c^30 - 5*a^8*b^22*c^30 + a^10*b^20*c^30 + a^20*b^10*d^30 - 5*a^22*b^8*d^30 + 10*a^24*b^6*d^30 - 10*a^26*b^4*d^30 + 5*a^28*b^2*d^30 + 5*a^30*c^2*d^28 - 10*a^30*c^4*d^26 + 10*a^30*c^6*d^24 - 5*a^30*c^8*d^22 + a^30*c^10*d^20 + b^30*c^20*d^10 - 5*b^30*c^22*d^8 + 10*b^30*c^24*d^6 - 10*b^30*c^26*d^4 + 5*b^30*c^28*d^2 - 20*a*b^29*c^19*d^11 + 100*a*b^29*c^21*d^9 - 200*a*b^29*c^23*d^7 + 200*a*b^29*c^25*d^5 - 100*a*b^29*c^27*d^3 - 100*a^3*b^27*c^29*d + 200*a^5*b^25*c^29*d - 200*a^7*b^23*c^29*d + 100*a^9*b^21*c^29*d - 20*a^11*b^19*c^29*d - 20*a^19*b^11*c*d^29 + 100*a^21*b^9*c*d^29 - 200*a^23*b^7*c*d^29 + 200*a^25*b^5*c*d^29 - 100*a^27*b^3*c*d^29 - 100*a^29*b*c^3*d^27 + 200*a^29*b*c^5*d^25 - 200*a^29*b*c^7*d^23 + 100*a^29*b*c^9*d^21 - 20*a^29*b*c^11*d^19 + 190*a^2*b^28*c^18*d^12 - 955*a^2*b^28*c^20*d^10 + 1925*a^2*b^28*c^22*d^8 - 1950*a^2*b^28*c^24*d^6 + 1000*a^2*b^28*c^26*d^4 - 215*a^2*b^28*c^28*d^2 - 1140*a^3*b^27*c^17*d^13 + 5800*a^3*b^27*c^19*d^11 - 11900*a^3*b^27*c^21*d^9 + 12400*a^3*b^27*c^23*d^7 - 6700*a^3*b^27*c^25*d^5 + 1640*a^3*b^27*c^27*d^3 + 4845*a^4*b^26*c^16*d^14 - 25175*a^4*b^26*c^18*d^12 + 53210*a^4*b^26*c^20*d^10 - 58000*a^4*b^26*c^22*d^8 + 33825*a^4*b^26*c^24*d^6 - 9695*a^4*b^26*c^26*d^4 + 1000*a^4*b^26*c^28*d^2 - 15504*a^5*b^25*c^15*d^15 + 83220*a^5*b^25*c^17*d^13 - 183740*a^5*b^25*c^19*d^11 + 213040*a^5*b^25*c^21*d^9 - 136520*a^5*b^25*c^23*d^7 + 46004*a^5*b^25*c^25*d^5 - 6700*a^5*b^25*c^27*d^3 + 38760*a^6*b^24*c^14*d^16 - 218025*a^6*b^24*c^16*d^14 + 510625*a^6*b^24*c^18*d^12 - 639360*a^6*b^24*c^20*d^10 + 455100*a^6*b^24*c^22*d^8 - 178985*a^6*b^24*c^24*d^6 + 33825*a^6*b^24*c^26*d^4 - 1950*a^6*b^24*c^28*d^2 - 77520*a^7*b^23*c^13*d^17 + 465120*a^7*b^23*c^15*d^15 - 1174200*a^7*b^23*c^17*d^13 + 1607600*a^7*b^23*c^19*d^11 - 1277800*a^7*b^23*c^21*d^9 + 581120*a^7*b^23*c^23*d^7 - 136520*a^7*b^23*c^25*d^5 + 12400*a^7*b^23*c^27*d^3 + 125970*a^8*b^22*c^12*d^18 - 823650*a^8*b^22*c^14*d^16 + 2277150*a^8*b^22*c^16*d^14 - 3441850*a^8*b^22*c^18*d^12 + 3061855*a^8*b^22*c^20*d^10 - 1598495*a^8*b^22*c^22*d^8 + 455100*a^8*b^22*c^24*d^6 - 58000*a^8*b^22*c^26*d^4 + 1925*a^8*b^22*c^28*d^2 - 167960*a^9*b^21*c^11*d^19 + 1227400*a^9*b^21*c^13*d^17 - 3772640*a^9*b^21*c^15*d^15 + 6342200*a^9*b^21*c^17*d^13 - 6323300*a^9*b^21*c^19*d^11 + 3770860*a^9*b^21*c^21*d^9 - 1277800*a^9*b^21*c^23*d^7 + 213040*a^9*b^21*c^25*d^5 - 11900*a^9*b^21*c^27*d^3 + 184756*a^10*b^20*c^10*d^20 - 1553630*a^10*b^20*c^12*d^18 + 5384410*a^10*b^20*c^14*d^16 - 10132510*a^10*b^20*c^16*d^14 + 11341480*a^10*b^20*c^18*d^12 - 7699257*a^10*b^20*c^20*d^10 + 3061855*a^10*b^20*c^22*d^8 - 639360*a^10*b^20*c^24*d^6 + 53210*a^10*b^20*c^26*d^4 - 955*a^10*b^20*c^28*d^2 - 167960*a^11*b^19*c^9*d^21 + 1679600*a^11*b^19*c^11*d^19 - 6653800*a^11*b^19*c^13*d^17 + 14108640*a^11*b^19*c^15*d^15 - 17770700*a^11*b^19*c^17*d^13 + 13697880*a^11*b^19*c^19*d^11 - 6323300*a^11*b^19*c^21*d^9 + 1607600*a^11*b^19*c^23*d^7 - 183740*a^11*b^19*c^25*d^5 + 5800*a^11*b^19*c^27*d^3 + 125970*a^12*b^18*c^8*d^22 - 1553630*a

^12*b^18*c^10*d^20 + 7138300*a^12*b^18*c^12*d^18 - 17183600*a^12*b^18*c^14*
d^16 + 24426875*a^12*b^18*c^16*d^14 - 21339185*a^12*b^18*c^18*d^12 + 113414
80*a^12*b^18*c^20*d^10 - 3441850*a^12*b^18*c^22*d^8 + 510625*a^12*b^18*c^24
*d^6 - 25175*a^12*b^18*c^26*d^4 + 190*a^12*b^18*c^28*d^2 - 77520*a^13*b^17*
c^7*d^23 + 1227400*a^13*b^17*c^9*d^21 - 6653800*a^13*b^17*c^11*d^19 + 18346
400*a^13*b^17*c^13*d^17 - 29535120*a^13*b^17*c^15*d^15 + 29213260*a^13*b^17
*c^17*d^13 - 17770700*a^13*b^17*c^19*d^11 + 6342200*a^13*b^17*c^21*d^9 - 11
74200*a^13*b^17*c^23*d^7 + 83220*a^13*b^17*c^25*d^5 - 1140*a^13*b^17*c^27*d
^3 + 38760*a^14*b^16*c^6*d^24 - 823650*a^14*b^16*c^8*d^22 + 5384410*a^14*b^
16*c^10*d^20 - 17183600*a^14*b^16*c^12*d^18 + 31460200*a^14*b^16*c^14*d^16
- 35234455*a^14*b^16*c^16*d^14 + 24426875*a^14*b^16*c^18*d^12 - 10132510*a^
14*b^16*c^20*d^10 + 2277150*a^14*b^16*c^22*d^8 - 218025*a^14*b^16*c^24*d^6
+ 4845*a^14*b^16*c^26*d^4 - 15504*a^15*b^15*c^5*d^25 + 465120*a^15*b^15*c^7
*d^23 - 3772640*a^15*b^15*c^9*d^21 + 14108640*a^15*b^15*c^11*d^19 - 2953512
0*a^15*b^15*c^13*d^17 + 37499008*a^15*b^15*c^15*d^15 - 29535120*a^15*b^15*c
^17*d^13 + 14108640*a^15*b^15*c^19*d^11 - 3772640*a^15*b^15*c^21*d^9 + 4651
20*a^15*b^15*c^23*d^7 - 15504*a^15*b^15*c^25*d^5 + 4845*a^16*b^14*c^4*d^26
- 218025*a^16*b^14*c^6*d^24 + 2277150*a^16*b^14*c^8*d^22 - 10132510*a^16*b^
14*c^10*d^20 + 24426875*a^16*b^14*c^12*d^18 - 35234455*a^16*b^14*c^14*d^16
+ 31460200*a^16*b^14*c^16*d^14 - 17183600*a^16*b^14*c^18*d^12 + 5384410*a^1
6*b^14*c^20*d^10 - 823650*a^16*b^14*c^22*d^8 + 38760*a^16*b^14*c^24*d^6 - 1
140*a^17*b^13*c^3*d^27 + 83220*a^17*b^13*c^5*d^25 - 1174200*a^17*b^13*c^7*d
^23 + 6342200*a^17*b^13*c^9*d^21 - 17770700*a^17*b^13*c^11*d^19 + 29213260*
a^17*b^13*c^13*d^17 - 29535120*a^17*b^13*c^15*d^15 + 18346400*a^17*b^13*c^1
7*d^13 - 6653800*a^17*b^13*c^19*d^11 + 1227400*a^17*b^13*c^21*d^9 - 77520*a
^17*b^13*c^23*d^7 + 190*a^18*b^12*c^2*d^28 - 25175*a^18*b^12*c^4*d^26 + 510
625*a^18*b^12*c^6*d^24 - 3441850*a^18*b^12*c^8*d^22 + 11341480*a^18*b^12*c^
10*d^20 - 21339185*a^18*b^12*c^12*d^18 + 24426875*a^18*b^12*c^14*d^16 - 171
83600*a^18*b^12*c^16*d^14 + 7138300*a^18*b^12*c^18*d^12 - 1553630*a^18*b^12
*c^20*d^10 + 125970*a^18*b^12*c^22*d^8 + 5800*a^19*b^11*c^3*d^27 - 183740*a
^19*b^11*c^5*d^25 + 1607600*a^19*b^11*c^7*d^23 - 6323300*a^19*b^11*c^9*d^21
+ 13697880*a^19*b^11*c^11*d^19 - 17770700*a^19*b^11*c^13*d^17 + 14108640*a
^19*b^11*c^15*d^15 - 6653800*a^19*b^11*c^17*d^13 + 1679600*a^19*b^11*c^19*d
^11 - 167960*a^19*b^11*c^21*d^9 - 955*a^20*b^10*c^2*d^28 + 53210*a^20*b^10*
c^4*d^26 - 639360*a^20*b^10*c^6*d^24 + 3061855*a^20*b^10*c^8*d^22 - 7699257
*a^20*b^10*c^10*d^20 + 11341480*a^20*b^10*c^12*d^18 - 10132510*a^20*b^10*c^
14*d^16 + 5384410*a^20*b^10*c^16*d^14 - 1553630*a^20*b^10*c^18*d^12 + 18475
6*a^20*b^10*c^20*d^10 - 11900*a^21*b^9*c^3*d^27 + 213040*a^21*b^9*c^5*d^25
- 1277800*a^21*b^9*c^7*d^23 + 3770860*a^21*b^9*c^9*d^21 - 6323300*a^21*b^9*
c^11*d^19 + 6342200*a^21*b^9*c^13*d^17 - 3772640*a^21*b^9*c^15*d^15 + 12274
00*a^21*b^9*c^17*d^13 - 167960*a^21*b^9*c^19*d^11 + 1925*a^22*b^8*c^2*d^28
- 58000*a^22*b^8*c^4*d^26 + 455100*a^22*b^8*c^6*d^24 - 1598495*a^22*b^8*c^8
*d^22 + 3061855*a^22*b^8*c^10*d^20 - 3441850*a^22*b^8*c^12*d^18 + 2277150*a
^22*b^8*c^14*d^16 - 823650*a^22*b^8*c^16*d^14 + 125970*a^22*b^8*c^18*d^12 +
12400*a^23*b^7*c^3*d^27 - 136520*a^23*b^7*c^5*d^25 + 581120*a^23*b^7*c^7*d

```
^23 - 1277800*a^23*b^7*c^9*d^21 + 1607600*a^23*b^7*c^11*d^19 - 1174200*a^23
*b^7*c^13*d^17 + 465120*a^23*b^7*c^15*d^15 - 77520*a^23*b^7*c^17*d^13 - 195
0*a^24*b^6*c^2*d^28 + 33825*a^24*b^6*c^4*d^26 - 178985*a^24*b^6*c^6*d^24 +
455100*a^24*b^6*c^8*d^22 - 639360*a^24*b^6*c^10*d^20 + 510625*a^24*b^6*c^12
*d^18 - 218025*a^24*b^6*c^14*d^16 + 38760*a^24*b^6*c^16*d^14 - 6700*a^25*b^
5*c^3*d^27 + 46004*a^25*b^5*c^5*d^25 - 136520*a^25*b^5*c^7*d^23 + 213040*a^
25*b^5*c^9*d^21 - 183740*a^25*b^5*c^11*d^19 + 83220*a^25*b^5*c^13*d^17 - 15
504*a^25*b^5*c^15*d^15 + 1000*a^26*b^4*c^2*d^28 - 9695*a^26*b^4*c^4*d^26 +
33825*a^26*b^4*c^6*d^24 - 58000*a^26*b^4*c^8*d^22 + 53210*a^26*b^4*c^10*d^2
0 - 25175*a^26*b^4*c^12*d^18 + 4845*a^26*b^4*c^14*d^16 + 1640*a^27*b^3*c^3*
d^27 - 6700*a^27*b^3*c^5*d^25 + 12400*a^27*b^3*c^7*d^23 - 11900*a^27*b^3*c^
9*d^21 + 5800*a^27*b^3*c^11*d^19 - 1140*a^27*b^3*c^13*d^17 - 215*a^28*b^2*c
^2*d^28 + 1000*a^28*b^2*c^4*d^26 - 1950*a^28*b^2*c^6*d^24 + 1925*a^28*b^2*c
^8*d^22 - 955*a^28*b^2*c^10*d^20 + 190*a^28*b^2*c^12*d^18 + 20*a*b^29*c^29*
d + 20*a^29*b*c*d^29)))^(1/2)*2i)/f
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.723 $\int (a+b\sin(e+fx))(c+d\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=298

$$-\frac{2\left(56acd+15bc^2+25bd^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}-\frac{2\left(c^2-d^2\right)\left(56acd+15bc^2+25bd^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F}{105df\sqrt{c+d\sin(e+fx)}}$$

```
[Out] -2/35*(7*a*d+5*b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/f-2/7*b*cos(f*x+e)*(c
+d*sin(f*x+e))^(5/2)/f-2/105*(56*a*c*d+15*b*c^2+25*b*d^2)*cos(f*x+e)*(c+d*s
in(f*x+e))^(1/2)/f-2/105*(161*a*c^2*d+63*a*d^3+15*b*c^3+145*b*c*d^2)*(sin(1
/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e
+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d/f/((c+d*
sin(f*x+e))/(c+d))^(1/2)+2/105*(c^2-d^2)*(56*a*c*d+15*b*c^2+25*b*d^2)*(sin(
1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*
e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d
/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(56acd+15bc^2+25bd^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}-\frac{2\left(c^2-d^2\right)\left(56acd+15bc^2+25bd^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F}{105df\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*(15*b*c^2 + 56*a*c*d + 25*b*d^2)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])
/(105*f) - (2*(5*b*c + 7*a*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(35*
f) - (2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(7*f) + (2*(15*b*c^3 + 1
61*a*c^2*d + 145*b*c*d^2 + 63*a*d^3)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c
 + d)]*Sqrt[c + d*Sin[e + f*x]])/(105*d*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)
]) - (2*(c^2 - d^2)*(15*b*c^2 + 56*a*c*d + 25*b*d^2)*EllipticF[(e - Pi/2 +
f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(105*d*f*Sqrt[c
+ d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\sin(e+fx))(c+d\sin(e+fx))^{5/2}\,dx = -\frac{2b\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{7f}+\frac{2}{7}\int (c+d\sin(e+fx))^{3/2}$$

$$= -\frac{2(5bc+7ad)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{35f}-\frac{2b\cos(e+fx)}{}$$

$$= -\frac{2\left(15bc^2+56acd+25bd^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}-\frac{2(}{}$$

$$= -\frac{2\left(15bc^2+56acd+25bd^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}-\frac{2(}{}$$

$$= -\frac{2\left(15bc^2+56acd+25bd^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}-\frac{2(}{}$$

$$= -\frac{2\left(15bc^2+56acd+25bd^2\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105f}-\frac{2(}{}$$

Mathematica [A] time = 1.11, size = 275, normalized size = 0.92

$$\frac{-d\cos(e+fx)(c+d\sin(e+fx))\left(6d(7ad+15bc)\sin(e+fx)+154acd+90bc^2-15bd^2\cos(2(e+fx))+65bd^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*d*(5*b*d*(27*c^2 + 5*d^2) + 7*a*(15*c^3 + 17*c*d^2))*EllipticF[(-2*e +
Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - 2*(7*a*d
*(23*c^2 + 9*d^2) + 5*b*(3*c^3 + 29*c*d^2))*((c + d)*EllipticE[(-2*e + Pi -
 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d
)])*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - d*Cos[e + f*x]*(c + d*Sin[e + f*x]
)*(90*b*c^2 + 154*a*c*d + 65*b*d^2 - 15*b*d^2*Cos[2*(e + f*x)] + 6*d*(15*b*
c + 7*a*d)*Sin[e + f*x]))/(105*d*f*Sqrt[c + d*Sin[e + f*x]])
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ac^2+2\,bcd+ad^2-\left(2\,bcd+ad^2\right)\cos\left(fx+e\right)^2-\left(bd^2\cos\left(fx+e\right)^2-bc^2-2\,acd-bd^2\right)\sin\left(fx+e\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a*c^2 + 2*b*c*d + a*d^2 - (2*b*c*d + a*d^2)*cos(f*x + e)^2 - (b*d
^2*cos(f*x + e)^2 - b*c^2 - 2*a*c*d - b*d^2)*sin(f*x + e))*sqrt(d*sin(f*x +
 e) + c), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.51, size = 1839, normalized size = 6.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] 2/105*(-15*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(
-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c
-d)/(c+d))^(1/2))*b*c^5+45*b*c^3*d^2*sin(f*x+e)^2-35*b*c*d^4*sin(f*x+e)^2-9
8*a*c*d^4*sin(f*x+e)-90*b*c^2*d^3*sin(f*x+e)+60*b*c*d^4*sin(f*x+e)^4+98*a*c
*d^4*sin(f*x+e)^3+90*b*c^2*d^3*sin(f*x+e)^3+77*a*c^2*d^3*sin(f*x+e)^2-77*a*
c^2*d^3-45*b*c^3*d^2-25*b*c*d^4+15*b*d^5*sin(f*x+e)^5+21*a*d^5*sin(f*x+e)^4
+10*b*d^5*sin(f*x+e)^3-21*a*d^5*sin(f*x+e)^2-25*b*d^5*sin(f*x+e)+63*((c+d*s
in(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/
(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*
a*d^5-63*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d
*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))*a*d^5+145*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d
/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(
c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c*d^4+105*a*c^4*((c+d*sin(f*x+e))/(c-d))
^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Elli
pticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d+56*a*c^3*((c+d*
sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))
/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))
*d^2-42*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*
(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)
/(c+d))^(1/2))*a*c^2*d^3-56*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)
```

```
*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))
/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*a*c*d^4+15*((c+d*sin(f*x+e))/(c-d))^(1/2
)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF
(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c^4*d+120*((c+d*sin(
f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-
d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c
^3*d^2+10*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-
d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-
d)/(c+d))^(1/2))*b*c^2*d^3-120*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)
-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+
e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c*d^4-161*((c+d*sin(f*x+e))/(c-d))^
(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ellip
ticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*a*c^4*d+98*((c+d*s
in(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/
(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*
a*c^2*d^3-130*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2
)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))*b*c^3*d^2-25*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x
+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f
*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*d^5)/d^2/cos(f*x+e)/(c+d*sin(f*x
+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\sin\left(e+f\,x\right)\right)\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))*(c + d*sin(e + f*x))**(5/2), x)
```

3.724 $\int (a+b\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=235

$$-\frac{2\left(c^2-d^2\right)(5ad+3bc)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15df\sqrt{c+d\sin(e+fx)}}+\frac{2\left(20acd+3b\left(c^2+3d^2\right)\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\right.}{15df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] -2/5*b*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/f-2/15*(5*a*d+3*b*c)*cos(f*x+e)*(c
+d*sin(f*x+e))^(1/2)/f-2/15*(20*a*c*d+3*b*(c^2+3*d^2))*(sin(1/2*e+1/4*Pi+1/
2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*
x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d/f/((c+d*sin(f*x+e))/(c
+d))^(1/2)+2/15*(5*a*d+3*b*c)*(c^2-d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)
/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(
c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(c^2-d^2\right)(5ad+3bc)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15df\sqrt{c+d\sin(e+fx)}}+\frac{2\left(20acd+3b\left(c^2+3d^2\right)\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\right.}{15df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(3*b*c + 5*a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*f) - (2*b*Co
s[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(5*f) + (2*(20*a*c*d + 3*b*(c^2 + 3*
d^2))*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]]
)/(15*d*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*(3*b*c + 5*a*d)*(c^2 - d
^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/
(c + d)])/(15*d*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx = -\frac{2b\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{5f} + \frac{2}{5}\int \sqrt{c+d\sin(e+fx)}\ ($$

$$= -\frac{2(3bc+5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15f} - \frac{2b\cos(e+fx)(c}{}$$

$$= -\frac{2(3bc+5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15f} - \frac{2b\cos(e+fx)(c}{}$$

$$= -\frac{2(3bc+5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15f} - \frac{2b\cos(e+fx)(c}{}$$

$$= -\frac{2(3bc+5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15f} - \frac{2b\cos(e+fx)(c}{}$$

Mathematica [A] time = 0.76, size = 218, normalized size = 0.93

$$\frac{-2d\left(5a\left(3c^2+d^2\right)+12bcd\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)-2\left(20acd+3b\left(c^2+3d^2\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\left((\right.}{15df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*d*(12*b*c*d + 5*a*(3*c^2 + d^2))*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)
/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - 2*(20*a*c*d + 3*b*(c^2 + 3*d
^2))*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF
[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)])*Sqrt[(c + d*Sin[e + f*x])/(c + d)]
- 2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])*(6*b*c + 5*a*d + 3*b*d*Sin[e + f*x]
))/(15*d*f*Sqrt[c + d*Sin[e + f*x]])
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(bd\cos\left(fx+e\right)^2 - ac - bd - (bc+ad)\sin\left(fx+e\right)\right)\sqrt{d\sin\left(fx+e\right)+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*d*cos(f*x + e)^2 - a*c - b*d - (b*c + a*d)*sin(f*x + e))*sqrt(
d*sin(f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2), x)
```

maple [B] time = 1.54, size = 1449, normalized size = 6.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] 2/15*(15*c^3*a*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/
2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2)
,((c-d)/(c+d))^(1/2))*d+5*c^2*a*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e
)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x
+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d^2-15*((c+d*sin(f*x+e))/(c-d))^(1/2
)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF
(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*a*c*d^3-5*((c+d*sin(f*
x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d)
)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*a*d^4
+3*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+si
n(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d
))^(1/2))*b*c^3*d+9*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d)
)^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2))*b*c^2*d^2-3*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(si
n(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*
sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c*d^3-9*((c+d*sin(f*x+e))/(
c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)
*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*d^4-20*((c
+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+
e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/
2))*a*c^3*d+20*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/
2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2)
,((c-d)/(c+d))^(1/2))*a*c*d^3-3*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e
)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x
```

```
+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c^4-6*((c+d*sin(f*x+e))/(c-d))^(1/
2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Elliptic
E(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c^2*d^2+9*((c+d*sin
(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c
-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*
d^4+3*b*d^4*sin(f*x+e)^4+5*a*d^4*sin(f*x+e)^3+9*b*c*d^3*sin(f*x+e)^3+5*a*c*
d^3*sin(f*x+e)^2+6*b*c^2*d^2*sin(f*x+e)^2-3*b*d^4*sin(f*x+e)^2-5*a*d^4*sin(
f*x+e)-9*b*c*d^3*sin(f*x+e)-5*a*c*d^3-6*b*c^2*d^2)/d^2/cos(f*x+e)/(c+d*sin(
f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\sin\left(e+f\,x\right)\right)\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(3/2),x)
[Out] int((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))**(3/2),x)
[Out] Integral((a + b*sin(e + f*x))*(c + d*sin(e + f*x))**(3/2), x)
```

3.725 $\int (a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=181

$$\frac{2(3ad+bc)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2b\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3df\sqrt{c+d\sin(e+fx)}}-\frac{2b\cos}{}$$

```
[Out] -2/3*b*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f-2/3*(3*a*d+b*c)*(sin(1/2*e+1/4*P
i+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/
2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d/f/((c+d*sin(f*x+e)
)/(c+d))^(1/2)+2/3*b*(c^2-d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*
e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/
2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2(3ad+bc)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2b\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3df\sqrt{c+d\sin(e+fx)}}-\frac{2b\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*b*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*f) + (2*(b*c + 3*a*d)*Ellip
ticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*d*f*Sq
rt[(c + d*Sin[e + f*x])/(c + d)]) - (2*b*(c^2 - d^2)*EllipticF[(e - Pi/2 +
f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*d*f*Sqrt[c +
d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx = -\frac{2b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f} + \frac{2}{3}\int \frac{\frac{1}{2}(3ac+bd)+\frac{1}{2}(bc+$$

$$= -\frac{2b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f} + \frac{(bc+3ad)\int\sqrt{c+d\sin(e+}{3d}$$

$$= -\frac{2b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f} + \frac{\left((bc+3ad)\sqrt{c+d\sin(e+}{3d\sqrt{\frac{c+d}{}}}$$

$$= -\frac{2b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f} + \frac{2(bc+3ad)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+f\right.\right.}{3df\sqrt{\frac{c+d}{}}}$$

Mathematica [A] time = 0.63, size = 152, normalized size = 0.84

$$-\frac{2\left((c+d)(3ad+bc)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,E\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)-b\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+}\right.\right.}{3df\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*(b*d*Cos[e + f*x]*(c + d*Sin[e + f*x]) + (c + d)*(b*c + 3*a*d)*Elliptic
E[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)]
- b*(c^2 - d^2)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d
*Sin[e + f*x])/(c + d)]))/(3*d*f*Sqrt[c + d*Sin[e + f*x]])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

maple [B] time = 1.25, size = 862, normalized size = 4.76

$$\frac{2c^2a\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}}\,\sqrt{-\frac{\left(\sin\left(fx+e\right)-1\right)d}{c+d}}\,\sqrt{-\frac{d\left(1+\sin\left(fx+e\right)\right)}{c-d}}\,\mathrm{EllipticF}\left(\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}},\sqrt{\frac{c-d}{c+d}}\right)d-2\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}}\,\sqrt{-\frac{(}{}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/3*(3*c^2*a*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)
*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),(
(c-d)/(c+d))^(1/2))*d-3*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(
c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-
d))^(1/2),((c-d)/(c+d))^(1/2))*a*d^3+((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(
f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticF(((c+d*si
n(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c^2*d-((c+d*sin(f*x+e))/(c-d)
)^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*Ell
ipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*d^3-3*((c+d*si
n(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(
c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*a
*c^2*d+3*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d
*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))*a*d^3-((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+
d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(f*x+e))/(c-d)
)^(1/2),((c-d)/(c+d))^(1/2))*b*c^3+((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*
x+e)-1)*d/(c+d))^(1/2)*(-d*(1+sin(f*x+e))/(c-d))^(1/2)*EllipticE(((c+d*sin(
f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c*d^2+b*d^3*sin(f*x+e)^3+b*c*d^
2*sin(f*x+e)^2-b*d^3*sin(f*x+e)-c*d^2*b)/d^2/cos(f*x+e)/(c+d*sin(f*x+e))^(1
/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\sin\left(e+f\,x\right)\right)\,\sqrt{c+d\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2),x)
[Out] int((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(e+fx\right)\right)\sqrt{c+d\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(c+d*sin(f*x+e))**(1/2),x)
[Out] Integral((a + b*sin(e + f*x))*sqrt(c + d*sin(e + f*x)), x)
```

3.726 $\int \frac{a+b\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=140

$$\frac{2b\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df\sqrt{c+d\sin(e+fx)}}$$

```
[Out] -2*b*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elliptic
E(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)
/d/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+2*(-a*d+b*c)*(sin(1/2*e+1/4*Pi+1/2*f*x)
^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(
1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d/f/(c+d*sin(f*x+e))^(
1/2)
```

Rubi [A] time = 0.12, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2752, 2663, 2661, 2655, 2653}

$$\frac{2b\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (2*b*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])
/(d*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*(b*c - a*d)*EllipticF[(e - P
i/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(d*f*Sqrt[
c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{a+b\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx = \frac{b\int\sqrt{c+d\sin(e+fx)}\,dx}{d} + \frac{(-bc+ad)\int\frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{d}$$

$$= \frac{\left(b\sqrt{c+d\sin(e+fx)}\right)\int\sqrt{\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}}\,dx}{d\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} + \frac{\left((-bc+ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)\int\frac{}{\sqrt{\frac{c}{c+d}}}}{d\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{2bE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{df\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} - \frac{2(bc-ad)F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{}}{df\sqrt{c+d\sin(e+fx)}}$$

Mathematica [A] time = 2.56, size = 101, normalized size = 0.72

$$-\frac{2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\left((ad-bc)F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)+b(c+d)E\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)\right)}{df\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*(b*(c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] + (-(b*c) +
a*d)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)])*Sqrt[(c + d*Sin[e + f
*x])/(c + d)])/(d*f*Sqrt[c + d*Sin[e + f*x]])
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\sin\left(fx+e\right)+a}{\sqrt{d\sin\left(fx+e\right)+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\sin\left(fx+e\right)+a}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

maple [A] time = 1.33, size = 243, normalized size = 1.74

$$-\frac{2\left(c-d\right)\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}}\,\sqrt{-\frac{\left(\sin\left(fx+e\right)-1\right)d}{c+d}}\,\sqrt{-\frac{d\left(1+\sin\left(fx+e\right)\right)}{c-d}}\,\left(\text{EllipticE}\left(\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}},\sqrt{\frac{c-d}{c+d}}\right)bc+\text{EllipticE}\left(\sqrt{\right.\right.}{d^2\cos\left(fx+e\right)\sqrt{c+d\,\mathrm{s}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] -2*(c-d)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d
*(1+sin(f*x+e))/(c-d))^(1/2)*(EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-
d)/(c+d))^(1/2))*b*c+EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))
^(1/2))*b*d-a*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))
```

```
*d-EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*d)/d^2/c
os(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\sin\left(fx+e\right)+a}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

mupad [B] time = 8.58, size = 176, normalized size = 1.26

$$\frac{b\left(2\,c\,\mathrm{F}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin\left(e+f\,x\right)}}{2}\right)\middle|\frac{2\,d}{c+d}\right)-2\,\left(c+d\right)\,\mathrm{E}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin\left(e+f\,x\right)}}{2}\right)\middle|\frac{2\,d}{c+d}\right)\right)\,\sqrt{{\cos\left(e+f\,x\right)}^2}\,\sqrt{\frac{c+d\,\sin\left(e+f\,x\right)}{c+d}}}{d\,f\,\cos\left(e+f\,x\right)\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))/(c + d*sin(e + f*x))^(1/2),x)
```

```
[Out] (b*(2*c*ellipticF(asin((2^(1/2)*(1 - sin(e + f*x))^(1/2))/2), (2*d)/(c + d)
) - 2*(c + d)*ellipticE(asin((2^(1/2)*(1 - sin(e + f*x))^(1/2))/2), (2*d)/(
c + d)))*(cos(e + f*x)^2)^(1/2)*((c + d*sin(e + f*x))/(c + d))^(1/2))/(d*f*
cos(e + f*x)*(c + d*sin(e + f*x))^(1/2)) - (2*a*ellipticF(pi/4 - e/2 - (f*x
)/2, (2*d)/(c + d))*((c + d*sin(e + f*x))/(c + d))^(1/2))/(f*(c + d*sin(e +
 f*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\sin\left(e+fx\right)}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))/sqrt(c + d*sin(e + f*x)), x)
```

3.727 $\int \frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=195

$$-\frac{2(bc-ad)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{c+d\sin(e+fx)}}-\frac{2(bc-ad)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{2b\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+\right.\right.}{df\sqrt{c+d\sin(e}}$$

```
[Out] -2*(-a*d+b*c)*cos(f*x+e)/(c^2-d^2)/f/(c+d*sin(f*x+e))^(1/2)+2*(-a*d+b*c)*(s
in(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1
/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d/(c^2
-d^2)/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-2*b*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1
/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(
d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2(bc-ad)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{c+d\sin(e+fx)}}-\frac{2(bc-ad)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{df\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{2b\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+\right.\right.}{df\sqrt{c+d\sin(e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(b*c - a*d)*Cos[e + f*x])/((c^2 - d^2)*f*Sqrt[c + d*Sin[e + f*x]]) - (2
*(b*c - a*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e
+ f*x]])/(d*(c^2 - d^2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*b*Ellipt
icF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/
(d*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^{3/2}}\,dx = -\frac{2(bc-ad)\cos(e+fx)}{\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} - \frac{2\int \frac{\frac{1}{2}(-ac+bd)+\frac{1}{2}(bc-ad)\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx}{c^2-d^2}$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{b\int \frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{d} - \frac{(bc-ad)\int \sqrt{c+d\sin(e}{d\left(c^2-d^2\right)}$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} - \frac{\left((bc-ad)\sqrt{c+d\sin(e+fx)}\right)\int \sqrt{\frac{c}{c+d}+\frac{d\sin(e+f}{c+d}}}{d\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} - \frac{2(bc-ad)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+}{d\left(c^2-d^2\right)f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Mathematica [A] time = 0.56, size = 159, normalized size = 0.82

$$\frac{2\left(d(ad-bc)\cos(e+fx)+(c+d)(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,E\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)-b\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\right.\right.}{df(c-d)(c+d)\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*(d*(-(b*c) + a*d)*Cos[e + f*x] + (c + d)*(b*c - a*d)*EllipticE[(-2*e + P
i - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - b*(c^2 -
d^2)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*
x])/(c + d)]))/((c - d)*d*(c + d)*f*Sqrt[c + d*Sin[e + f*x]])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)+c}}{d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(d^2*cos(f*x + e)^2
 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \sin\left(fx+e\right)+a}{\left(d \sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(3/2), x)
```

maple [B] time = 3.27, size = 567, normalized size = 2.91

$$\frac{\sqrt{-\left(-d \sin\left(fx+e\right)-c\right)\left(\cos^{2}\left(fx+e\right)\right)}\left(\frac{2b\left(\frac{c}{d}-1\right)\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}}\,\sqrt{\frac{d\left(1-\sin\left(fx+e\right)\right)}{c+d}}\,\sqrt{\frac{\left(-\sin\left(fx+e\right)-1\right)d}{c-d}}\ \mathrm{EllipticF}\left(\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}},\sqrt{\frac{c}{c}}\right.}{d\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^{2}\left(fx+e\right)\right)}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(2*b/d*(c/d-1)*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(
1/2),((c-d)/(c+d))^(1/2))+(a*d-b*c)/d*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d
))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/
2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2
)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(
f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \sin\left(fx+e\right)+a}{\left(d \sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\sin\left(e + f\,x\right)}{\left(c + d\,\sin\left(e + f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))/(c + d*sin(e + f*x))^(3/2),x)
[Out] int((a + b*sin(e + f*x))/(c + d*sin(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))**(3/2),x)
[Out] Timed out
```

3.728 $\int \frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=285

$$\frac{2\left(4acd-b\left(c^2+3d^2\right)\right)\cos(e+fx)}{3f\left(c^2-d^2\right)^2\sqrt{c+d\sin(e+fx)}}-\frac{2(bc-ad)\cos(e+fx)}{3f\left(c^2-d^2\right)(c+d\sin(e+fx))^{3/2}}+\frac{2(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\right.\right.}{3df\left(c^2-d^2\right)\sqrt{c+d\sin(e+fx)}}$$

```
[Out] -2/3*(-a*d+b*c)*cos(f*x+e)/(c^2-d^2)/f/(c+d*sin(f*x+e))^(3/2)+2/3*(4*a*c*d-
b*(c^2+3*d^2))*cos(f*x+e)/(c^2-d^2)^2/f/(c+d*sin(f*x+e))^(1/2)-2/3*(4*a*c*d
-b*(c^2+3*d^2))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*
x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*
x+e))^(1/2)/d/(c^2-d^2)^2/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-2/3*(-a*d+b*c)*(
sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(
1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/
2)/d/(c^2-d^2)/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2754, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(4acd-b\left(c^2+3d^2\right)\right)\cos(e+fx)}{3f\left(c^2-d^2\right)^2\sqrt{c+d\sin(e+fx)}}-\frac{2(bc-ad)\cos(e+fx)}{3f\left(c^2-d^2\right)(c+d\sin(e+fx))^{3/2}}+\frac{2(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\right.\right.}{3df\left(c^2-d^2\right)\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*(b*c - a*d)*Cos[e + f*x])/(3*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^(3/2))
+ (2*(4*a*c*d - b*(c^2 + 3*d^2))*Cos[e + f*x])/(3*(c^2 - d^2)^2*f*Sqrt[c +
d*Sin[e + f*x]]) + (2*(4*a*c*d - b*(c^2 + 3*d^2))*EllipticE[(e - Pi/2 + f*x
)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*d*(c^2 - d^2)^2*f*Sqrt[(c
+ d*Sin[e + f*x])/(c + d)]) + (2*(b*c - a*d)*EllipticF[(e - Pi/2 + f*x)/2,
(2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*d*(c^2 - d^2)*f*Sqrt[
c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{2(bc-ad)\cos(e+fx)}{3\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}(ac-bd)-\frac{1}{2}(bc-ad)\sin(e+fx)}{(c+d\sin(e+fx))^{3/2}}dx}{3\left(c^2-d^2\right)}$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{3\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{2\left(4acd-b\left(c^2+3d^2\right)\right)\cos(e+fx)}{3\left(c^2-d^2\right)^2 f\sqrt{c+d\sin(e+fx)}} + \frac{4\int \cdots}{\cdots}$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{3\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{2\left(4acd-b\left(c^2+3d^2\right)\right)\cos(e+fx)}{3\left(c^2-d^2\right)^2 f\sqrt{c+d\sin(e+fx)}} + \frac{(bc-\cdots}{\cdots}$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{3\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{2\left(4acd-b\left(c^2+3d^2\right)\right)\cos(e+fx)}{3\left(c^2-d^2\right)^2 f\sqrt{c+d\sin(e+fx)}} + \frac{((4a\cdots}{\cdots}$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{3\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{2\left(4acd-b\left(c^2+3d^2\right)\right)\cos(e+fx)}{3\left(c^2-d^2\right)^2 f\sqrt{c+d\sin(e+fx)}} + \frac{2(4a\cdots}{\cdots}$$

Mathematica [A] time = 1.55, size = 199, normalized size = 0.70

$$\frac{2\left(\frac{\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{3/2}\left(\left(b\left(c^2+3d^2\right)-4acd\right)E\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)-(c-d)(bc-ad)F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)\right)}{d(c-d)^2} - \frac{\cos(e+fx)\left(d\left(b\left(c^2+3d^2\right)-4acd\right)\sin(e+fx)+\cdots\right.}{\left(c^2-d^2\right)^2}\cdots\right.}{3f(c+d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*((((-4*a*c*d + b*(c^2 + 3*d^2))*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(
c + d)] - (c - d)*(b*c - a*d)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d
)])*((c + d*Sin[e + f*x])/(c + d))^(3/2))/((c - d)^2*d) - (Cos[e + f*x]*(a*
d*(-5*c^2 + d^2) + 2*b*c*(c^2 + d^2) + d*(-4*a*c*d + b*(c^2 + 3*d^2))*Sin[e
 + f*x]))/(c^2 - d^2)^2))/(3*f*(c + d*Sin[e + f*x])^(3/2))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)+c}}{3\,cd^2\cos\left(fx+e\right)^2-c^3-3\,cd^2+\left(d^3\cos\left(fx+e\right)^2-3\,c^2d-d^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(3*c*d^2*cos(f*x +
e)^2 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e)),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\sin\left(fx+e\right)+a}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(5/2), x)
```

maple [B] time = 5.13, size = 887, normalized size = 3.11

$$\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^{2}\left(fx+e\right)\right)}\left(\frac{(da-cb)\left(\frac{2\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^{2}\left(fx+e\right)\right)}}{3\left(c^{2}-d^{2}\right)d\left(\sin\left(fx+e\right)+\frac{c}{d}\right)^{2}}+\frac{8d\left(\cos^{2}\left(fx+e\right)\right)c}{3\left(c^{2}-d^{2}\right)^{2}\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^{2}\left(fx+e\right)\right)}}+\frac{2\left(3c^{2}+d^{2}\right)\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((a*d-b*c)/d*(2/3/(c^2-d^2)/d*(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/3*d*cos(f*x+e)^2/(c
^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*
c^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+
d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)
^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+8/3*c*
d/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d
))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^
(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2
```

```
))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+b/d*(2*d
*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^
2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((
-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Ellip
ticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d
-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*
x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*El
lipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d
*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e
))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\sin\left(fx+e\right)+a}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\sin\left(e+f\,x\right)}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))/(c + d*sin(e + f*x))^(5/2),x)
[Out] int((a + b*sin(e + f*x))/(c + d*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))**(5/2),x)
[Out] Timed out
```

3.729 $\int \frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=369

$$-\frac{2\left(-8acd+3bc^2+5bd^2\right)\cos(e+fx)}{15f\left(c^2-d^2\right)^2(c+d\sin(e+fx))^{3/2}}-\frac{2(bc-ad)\cos(e+fx)}{5f\left(c^2-d^2\right)(c+d\sin(e+fx))^{5/2}}+\frac{2\left(-8acd+3bc^2+5bd^2\right)\sqrt{\frac{c+d\sin(e+fx}{c+d}}}{15df\left(c^2-d^2\right)^2\sqrt{c+d\,s}}$$

```
[Out] -2/5*(-a*d+b*c)*cos(f*x+e)/(c^2-d^2)/f/(c+d*sin(f*x+e))^(5/2)-2/15*(-8*a*c*
d+3*b*c^2+5*b*d^2)*cos(f*x+e)/(c^2-d^2)^2/f/(c+d*sin(f*x+e))^(3/2)-2/15*(-2
3*a*c^2*d-9*a*d^3+3*b*c^3+29*b*c*d^2)*cos(f*x+e)/(c^2-d^2)^3/f/(c+d*sin(f*x
+e))^(1/2)+2/15*(-23*a*c^2*d-9*a*d^3+3*b*c^3+29*b*c*d^2)*(sin(1/2*e+1/4*Pi+
1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*
f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d/(c^2-d^2)^3/f/((c+d*
sin(f*x+e))/(c+d))^(1/2)-2/15*(-8*a*c*d+3*b*c^2+5*b*d^2)*(sin(1/2*e+1/4*Pi+
1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*
f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d/(c^2-d^2)^2/
f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(-23ac^2d-9ad^3+3bc^3+29bcd^2\right)\cos(e+fx)}{15f\left(c^2-d^2\right)^3\sqrt{c+d\sin(e+fx)}}-\frac{2\left(-8acd+3bc^2+5bd^2\right)\cos(e+fx)}{15f\left(c^2-d^2\right)^2(c+d\sin(e+fx))^{3/2}}-\frac{2(bc-ad)\cos(e}{5f\left(c^2-d^2\right)(c+d\sin}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-2*(b*c - a*d)*Cos[e + f*x])/(5*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^(5/2))
- (2*(3*b*c^2 - 8*a*c*d + 5*b*d^2)*Cos[e + f*x])/(15*(c^2 - d^2)^2*f*(c + d
*Sin[e + f*x])^(3/2)) - (2*(3*b*c^3 - 23*a*c^2*d + 29*b*c*d^2 - 9*a*d^3)*Co
s[e + f*x])/(15*(c^2 - d^2)^3*f*Sqrt[c + d*Sin[e + f*x]]) - (2*(3*b*c^3 - 2
3*a*c^2*d + 29*b*c*d^2 - 9*a*d^3)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c +
d)]*Sqrt[c + d*Sin[e + f*x]])/(15*d*(c^2 - d^2)^3*f*Sqrt[(c + d*Sin[e + f*x
])/(c + d)]) + (2*(3*b*c^2 - 8*a*c*d + 5*b*d^2)*EllipticF[(e - Pi/2 + f*x)/
2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(15*d*(c^2 - d^2)^2*f
*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{a+b\sin(e+fx)}{(c+d\sin(e+fx))^{7/2}}\,dx = -\frac{2(bc-ad)\cos(e+fx)}{5\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} - \frac{2\int \frac{-\frac{5}{2}(ac-bd)-\frac{3}{2}(bc-ad)\sin(e+fx)}{(c+d\sin(e+fx))^{5/2}}dx}{5\left(c^2-d^2\right)}$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{5\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} - \frac{2\left(3bc^2-8acd+5bd^2\right)\cos(e+fx)}{15\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^{3/2}} + \frac{4\int \frac{3}{4}(\cdots}{\cdots}$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{5\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} - \frac{2\left(3bc^2-8acd+5bd^2\right)\cos(e+fx)}{15\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^{3/2}} - \frac{2(3bc\cdots}{\cdots}$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{5\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} - \frac{2\left(3bc^2-8acd+5bd^2\right)\cos(e+fx)}{15\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^{3/2}} - \frac{2(3bc\cdots}{\cdots}$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{5\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} - \frac{2\left(3bc^2-8acd+5bd^2\right)\cos(e+fx)}{15\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^{3/2}} - \frac{2(3bc\cdots}{\cdots}$$

$$= -\frac{2(bc-ad)\cos(e+fx)}{5\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} - \frac{2\left(3bc^2-8acd+5bd^2\right)\cos(e+fx)}{15\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^{3/2}} - \frac{2(3bc\cdots}{\cdots}$$

Mathematica [A] time = 3.06, size = 297, normalized size = 0.80

$$\frac{2\left(\frac{\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{5/2}\left(\left(-23ac^2d-9ad^3+3bc^3+29bcd^2\right)E\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)-(c-d)\left(-8acd+3bc^2+5bd^2\right)F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)\right)}{d(c-d)^3} + \frac{\cos(e+fx)\left(d^2\left(23a\cdots\right.\right.}{\cdots}\right.}{15f(c+d\sin(e+\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (2*((((3*b*c^3 - 23*a*c^2*d + 29*b*c*d^2 - 9*a*d^3)*EllipticE[(-2*e + Pi -
2*f*x)/4, (2*d)/(c + d)] - (c - d)*(3*b*c^2 - 8*a*c*d + 5*b*d^2)*EllipticF[
(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)])*((c + d*Sin[e + f*x])/(c + d))^(5/2)
)/((c - d)^3*d) + (Cos[e + f*x]*(a*d*(34*c^4 - 5*c^2*d^2 + 3*d^4) + b*(-9*c
^5 - 25*c^3*d^2 + 2*c*d^4) + d*(-9*b*c^4 + 54*a*c^3*d - 60*b*c^2*d^2 + 10*a
*c*d^3 + 5*b*d^4)*Sin[e + f*x] + d^2*(-3*b*c^3 + 23*a*c^2*d - 29*b*c*d^2 +
9*a*d^3)*Sin[e + f*x]^2))/(c^2 - d^2)^3))/(15*f*(c + d*Sin[e + f*x])^(5/2))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)+c}}{d^4\cos\left(fx+e\right)^4+c^4+6\,c^2d^2+d^4-2\left(3\,c^2d^2+d^4\right)\cos\left(fx+e\right)^2-4\left(cd^3\cos\left(fx+e\right)^2-c^3d-cd^3\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(d^4*cos(f*x + e)^4
+ c^4 + 6*c^2*d^2 + d^4 - 2*(3*c^2*d^2 + d^4)*cos(f*x + e)^2 - 4*(c*d^3*cos
(f*x + e)^2 - c^3*d - c*d^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\sin\left(fx+e\right)+a}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(7/2), x)
```

maple [B] time = 7.60, size = 1049, normalized size = 2.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(7/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((a*d-b*c)/d*(2/5/(c^2-d^2)/d^2*(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^3+16/15*c/(c^2-d^2)^2
/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+2/15*d*cos(f*
x+e)^2/(c^2-d^2)^3*(23*c^2+9*d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2
*(15*c^3+17*c*d^2)/(15*c^6-45*c^4*d^2+45*c^2*d^4-15*d^6)*(c/d-1)*((c+d*sin(
f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d
))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e)
)/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/15*d*(23*c^2+9*d^2)/(c^2-d^2)^3*(c/d-
1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x
+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*Ell
ipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*
sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+b/d*(2/3/(c^2-d^2)/d*(-(-d*
```

```
sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/3*d*cos(f*x+e)^2/(c^
2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*c
^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d
))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^
(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+8/3*c*d
/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d)
)^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(
1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)
)+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+
e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\sin\left(fx+e\right)+a}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(7/2),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\sin\left(e+f\,x\right)}{\left(c+d\,\sin\left(e+f\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))/(c + d*sin(e + f*x))^(7/2),x)
[Out] int((a + b*sin(e + f*x))/(c + d*sin(e + f*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))/(c+d*sin(f*x+e))**(7/2),x)
[Out] Timed out
```

3.730 $\int (a+b\sin(e+fx))^2(c+d\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=451

$$-\frac{4\left(84a^2cd^2+15abd\left(3c^2+5d^2\right)-\left(b^2\left(5c^3-57cd^2\right)\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{315df}+\frac{4\left(c^2-d^2\right)\left(-84a^2cd^2-\right.}{}$$

```
[Out] -2/315*(7*(9*a^2+7*b^2)*d^2-10*b*c*(-9*a*d+b*c))*cos(f*x+e)*(c+d*sin(f*x+e)
)^(3/2)/d/f+4/63*b*(-9*a*d+b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^(5/2)/d/f-2/9*b
^2*cos(f*x+e)*(c+d*sin(f*x+e))^(7/2)/d/f-4/315*(84*a^2*c*d^2+15*a*b*d*(3*c^
2+5*d^2)-b^2*(5*c^3-57*c*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d/f-2/315*
(21*a^2*d^2*(23*c^2+9*d^2)+30*a*b*d*(3*c^3+29*c*d^2)-b^2*(10*c^4-279*c^2*d^
2-147*d^4))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*E
llipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e)
)^(1/2)/d^2/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-4/315*(c^2-d^2)*(-84*a^2*c*d^2
-45*a*b*c^2*d-75*a*b*d^3+5*b^2*c^3-57*b^2*c*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)
^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(
1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^2/f/(c+d*sin(f*x+e))
^(1/2)
```

Rubi [A] time = 0.95, antiderivative size = 451, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2791, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4\left(84a^2cd^2+15abd\left(3c^2+5d^2\right)+b^2\left(-\left(5c^3-57cd^2\right)\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{315df}+\frac{4\left(c^2-d^2\right)\left(-84a^2cd^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-4*(84*a^2*c*d^2 + 15*a*b*d*(3*c^2 + 5*d^2) - b^2*(5*c^3 - 57*c*d^2))*Cos[
e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(315*d*f) - (2*(7*(9*a^2 + 7*b^2)*d^2 -
10*b*c*(b*c - 9*a*d))*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(315*d*f) +
(4*b*(b*c - 9*a*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(63*d*f) - (2*b
^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(7/2))/(9*d*f) + (2*(21*a^2*d^2*(23*c^
2 + 9*d^2) + 30*a*b*d*(3*c^3 + 29*c*d^2) - b^2*(10*c^4 - 279*c^2*d^2 - 147*
d^4))*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]]
)/(315*d^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*(c^2 - d^2)*(5*b^2*c^
3 - 45*a*b*c^2*d - 84*a^2*c*d^2 - 57*b^2*c*d^2 - 75*a*b*d^3)*EllipticF[(e -
 Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(315*d^2
*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
```

```
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+b\sin(e+fx))^2(c+d\sin(e+fx))^{5/2}\,dx = -\frac{2b^2\cos(e+fx)(c+d\sin(e+fx))^{7/2}}{9df} + \frac{2\int(c+d\sin(e+fx)}{}$$

$$= \frac{4b(bc-9ad)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{63df} - \frac{2b^2\cos(e+f}{}$$

$$= -\frac{2\left(7\left(9a^2+7b^2\right)d^2-10bc(bc-9ad)\right)\cos(e+fx)(c+d\sin(e}{315df}$$

$$= -\frac{4\left(84a^2cd^2+15abd\left(3c^2+5d^2\right)-b^2\left(5c^3-57cd^2\right)\right)\cos(e+f}{315df}$$

$$= -\frac{4\left(84a^2cd^2+15abd\left(3c^2+5d^2\right)-b^2\left(5c^3-57cd^2\right)\right)\cos(e+f}{315df}$$

$$= -\frac{4\left(84a^2cd^2+15abd\left(3c^2+5d^2\right)-b^2\left(5c^3-57cd^2\right)\right)\cos(e+f}{315df}$$

$$= -\frac{4\left(84a^2cd^2+15abd\left(3c^2+5d^2\right)-b^2\left(5c^3-57cd^2\right)\right)\cos(e+f}{315df}$$

Mathematica [A] time = 1.80, size = 382, normalized size = 0.85

$$\frac{8\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\left(\left(-21a^2d^2\left(23c^2+9d^2\right)-30abd\left(3c^3+29cd^2\right)+b^2\left(10c^4-279c^2d^2-147d^4\right)\right)\left((c+d)E\left(\frac{1}{4}(-2e\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (8*(-(d^2*(30*a*b*d*(27*c^2 + 5*d^2) + b^2*c*(155*c^2 + 261*d^2) + 21*a^2*(
15*c^3 + 17*c*d^2))*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]) + (-21
*a^2*d^2*(23*c^2 + 9*d^2) - 30*a*b*d*(3*c^3 + 29*c*d^2) + b^2*(10*c^4 - 279
*c^2*d^2 - 147*d^4))*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d
)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]))*Sqrt[(c + d*Sin[e
+ f*x])/(c + d)] - d*(c + d*Sin[e + f*x])*(2*(924*a^2*c*d^2 + 30*a*b*d*(36*
c^2 + 23*d^2) + b^2*(20*c^3 + 747*c*d^2))*Cos[e + f*x] - 10*b*d^2*(19*b*c +
 18*a*d)*Cos[3*(e + f*x)] + 2*d*(540*a*b*c*d + 126*a^2*d^2 + b^2*(150*c^2 +
 133*d^2))*Sin[2*(e + f*x)] - 35*b^2*d^3*Sin[4*(e + f*x)]))/(1260*d^2*f*Sqr
t[c + d*Sin[e + f*x]])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2 d^2 \cos\left(fx+e\right)^4 + 4\,abcd + \left(a^2+b^2\right)c^2 + \left(a^2+b^2\right)d^2 - \left(b^2c^2 + 4\,abcd + \left(a^2+2\,b^2\right)d^2\right)\cos\left(fx+e\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*d^2*cos(f*x + e)^4 + 4*a*b*c*d + (a^2 + b^2)*c^2 + (a^2 + b^2
)*d^2 - (b^2*c^2 + 4*a*b*c*d + (a^2 + 2*b^2)*d^2)*cos(f*x + e)^2 + 2*(a*b*c
^2 + a*b*d^2 + (a^2 + b^2)*c*d - (b^2*c*d + a*b*d^2)*cos(f*x + e)^2)*sin(f*
x + e))*sqrt(d*sin(f*x + e) + c), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 7.08, size = 2112, normalized size = 4.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b^2*d^3*(-2/9/d*sin(f*x+e)^3*(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+16/63*c/d^2*sin(f*x+e)^2*(-(-d*sin(f*x+e
)-c)*cos(f*x+e)^2)^(1/2)-2/5*(7/9+16/21*c^2/d^2)/d*sin(f*x+e)*(-(-d*sin(f*x
+e)-c)*cos(f*x+e)^2)^(1/2)-2/315*(-64*c^3-62*c*d^2)/d^4*(-(-d*sin(f*x+e)-c)
*cos(f*x+e)^2)^(1/2)+2/315*(32*c^3+36*c*d^2)/d^3*(c/d-1)*((c+d*sin(f*x+e))/
```

```
(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)
/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))
^(1/2),((c-d)/(c+d))^(1/2))+2/315*(128*c^4+108*c^2*d^2+147*d^4)/d^4*(c/d-1)
*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e
)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*Ellip
ticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*si
n(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(2*a*b*d^3+3*b^2*c*d^2)*(-2/7
/d*sin(f*x+e)^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+12/35*c/d^2*sin(f*x
+e)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)-2/3*(5/7+24/35*c^2/d^2)/d*(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(-4/35*c^2/d^2+5/21)*(c/d-1)*((c+d*sin
(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-
d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/105*(-48*c^3-44*c*d^2)/d^3*(c/d-1)*(
(c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-
1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*Ellipti
cE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(
f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(a^2*d^3+6*a*b*c*d^2+3*b^2*c^2*
d)*(-2/5/d*sin(f*x+e)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+8/15*c/d^2*(-
(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+4/15*c/d*(c/d-1)*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(
1/2),((c-d)/(c+d))^(1/2))+2*(3/5+8/15*c^2/d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e)
)/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))))+(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*(-2/3/d*(-(-d*si
n(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*
(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*
x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)
/(c+d))^(1/2))-4/3*c/d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x
+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f
*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c
+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))
+2*(3*a^2*c^2*d+2*a*b*c^3)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin
(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*c
os(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2
)))+2*c^3*a^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d
))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^
(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))/cos(f*
x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)^{2}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + b\,\sin\left(e + f\,x\right)\right)^2\,\left(c + d\,\sin\left(e + f\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sin\left(e + fx\right)\right)^{2}\left(c + d\sin\left(e + fx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**2*(c + d*sin(e + f*x))**(5/2), x)
```

3.731 $\int (a+b\sin(e+fx))^2(c+d\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=347

$$-\frac{2\left(c^2-d^2\right)\left(35a^2d^2+42abcd-\left(b^2\left(6c^2-25d^2\right)\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{105d^2f\sqrt{c+d\sin(e+fx)}}+\frac{4\left(70a^2cd^2+21abd\left(\right.\right.}{}$$

```
[Out] 4/35*b*(-7*a*d+b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/d/f-2/7*b^2*cos(f*x+e
)*(c+d*sin(f*x+e))^(5/2)/d/f-2/105*(5*(7*a^2+5*b^2)*d^2-6*b*c*(-7*a*d+b*c))
*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d/f-4/105*(70*a^2*c*d^2+21*a*b*d*(c^2+3*
d^2)-b^2*(3*c^3-41*c*d^2))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/
4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*
(c+d*sin(f*x+e))^(1/2)/d^2/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+2/105*(c^2-d^2)
*(42*a*b*c*d+35*a^2*d^2-b^2*(6*c^2-25*d^2))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(
1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*
(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^2/f/(c+d*sin(f*x+e))^(1/2
)
```

Rubi [A] time = 0.63, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2791, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(c^2-d^2\right)\left(35a^2d^2+42abcd+b^2\left(-\left(6c^2-25d^2\right)\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{105d^2f\sqrt{c+d\sin(e+fx)}}+\frac{4\left(70a^2cd^2+21abc\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(5*(7*a^2 + 5*b^2)*d^2 - 6*b*c*(b*c - 7*a*d))*Cos[e + f*x]*Sqrt[c + d*S
in[e + f*x]])/(105*d*f) + (4*b*(b*c - 7*a*d)*Cos[e + f*x]*(c + d*Sin[e + f*
x])^(3/2))/(35*d*f) - (2*b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(7*d*
f) + (4*(70*a^2*c*d^2 + 21*a*b*d*(c^2 + 3*d^2) - b^2*(3*c^3 - 41*c*d^2))*El
lipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(105*d
^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*(c^2 - d^2)*(42*a*b*c*d + 35*
a^2*d^2 - b^2*(6*c^2 - 25*d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)
]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(105*d^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
```

```
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+b\sin(e+fx))^2(c+d\sin(e+fx))^{3/2}\,dx = -\frac{2b^2\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{7df} + \frac{2\int(c+d\sin(e+fx)}{}$$

$$= \frac{4b(bc-7ad)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{35df} - \frac{2b^2\cos(e+f}{}$$

$$= -\frac{2\left(5\left(7a^2+5b^2\right)d^2-6bc(bc-7ad)\right)\cos(e+fx)\sqrt{c+d\sin(e}}{105df}$$

$$= -\frac{2\left(5\left(7a^2+5b^2\right)d^2-6bc(bc-7ad)\right)\cos(e+fx)\sqrt{c+d\sin(e}}{105df}$$

$$= -\frac{2\left(5\left(7a^2+5b^2\right)d^2-6bc(bc-7ad)\right)\cos(e+fx)\sqrt{c+d\sin(e}}{105df}$$

$$= -\frac{2\left(5\left(7a^2+5b^2\right)d^2-6bc(bc-7ad)\right)\cos(e+fx)\sqrt{c+d\sin(e}}{105df}$$

Mathematica [A] time = 1.20, size = 292, normalized size = 0.84

$$\frac{4\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\left(-\left(d^2\left(35a^2\left(3c^2+d^2\right)+168abcd+b^2\left(51c^2+25d^2\right)\right)F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)\right)-2\left(70a^2cd^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2),x]

[Out] (4*(-(d^2*(168*a*b*c*d + 35*a^2*(3*c^2 + d^2) + b^2*(51*c^2 + 25*d^2))*Elli
pticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]) - 2*(70*a^2*c*d^2 + 21*a*b*d*(
c^2 + 3*d^2) + b^2*(-3*c^3 + 41*c*d^2))*((c + d)*EllipticE[(-2*e + Pi - 2*f
*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]))
*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - d*(c + d*Sin[e + f*x])*((336*a*b*c*d
+ 140*a^2*d^2 + b^2*(12*c^2 + 115*d^2))*Cos[e + f*x] + 3*b*d*(-5*b*d*Cos[3*
```

```
(e + f*x)] + 4*(4*b*c + 7*a*d)*Sin[2*(e + f*x)])))/(210*d^2*f*Sqrt[c + d*Si
n[e + f*x]])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(2\,abd-\left(b^{2}c+2\,abd\right)\cos\left(fx+e\right)^{2}+\left(a^{2}+b^{2}\right)c-\left(b^{2}d\cos\left(fx+e\right)^{2}-2\,abc-\left(a^{2}+b^{2}\right)d\right)\sin\left(fx+e\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((2*a*b*d - (b^2*c + 2*a*b*d)*cos(f*x + e)^2 + (a^2 + b^2)*c - (b^2
*d*cos(f*x + e)^2 - 2*a*b*c - (a^2 + b^2)*d)*sin(f*x + e))*sqrt(d*sin(f*x +
 e) + c), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 5.14, size = 1575, normalized size = 4.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b^2*d^2*(-2/7/d*sin(f*x+e)^2*(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+12/35*c/d^2*sin(f*x+e)*(-(-d*sin(f*x+e)-
c)*cos(f*x+e)^2)^(1/2)-2/3*(5/7+24/35*c^2/d^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*
x+e)^2)^(1/2)+2*(-4/35*c^2/d^2+5/21)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)
*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))+2/105*(-48*c^3-44*c*d^2)/d^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d
))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/
(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))))+(2*a*b*d^2+2*b^2*c*d)*(-2/5/d*sin(f*x+e)*(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)+8/15*c/d^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(
1/2)+4/15*c/d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d
))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^
```

```
(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2*(3/5+
8/15*c^2/d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d
))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^
(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2
))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(a^2*d^2
+4*a*b*c*d+b^2*c^2)*(-2/3/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(c/
d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f
*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF((
(c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-4/3*c/d*(c/d-1)*((c+d*si
n(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c
-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+
d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))
/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+2*(2*a^2*c*d+2*a*b*c^2)*(c/d-1)*((c+d*
sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/
(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+2*a^2*c^2*(c/d-1)*((c+d*sin(f*x+e))/(
c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2)))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)^{2}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\sin\left(e+f\,x\right)\right)}^2\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^(3/2),x)
[Out] int((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sin\left(e + fx\right)\right)^{2} \left(c + d\sin\left(e + fx\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**2*(c + d*sin(e + f*x))**(3/2), x)
```

3.732 $\int (a+b\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=254

$$\frac{2\left(3d^2\left(5a^2+3b^2\right)-2bc(bc-5ad)\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{4b\left(c^2-d^2\right)(bc-5ad)\sqrt{\frac{c+d\sin(\ldots}{c+\ldots}}}{15d^2f\sqrt{c+d\,s\ldots}}$$

```
[Out] -2/5*b^2*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/d/f+4/15*b*(-5*a*d+b*c)*cos(f*x+
e)*(c+d*sin(f*x+e))^(1/2)/d/f-2/15*(3*(5*a^2+3*b^2)*d^2-2*b*c*(-5*a*d+b*c))
*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(co
s(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^2
/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-4/15*b*(-5*a*d+b*c)*(c^2-d^2)*(sin(1/2*e+
1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*
Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^2/f/(
c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2791, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(3d^2\left(5a^2+3b^2\right)-2bc(bc-5ad)\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{4b\left(c^2-d^2\right)(bc-5ad)\sqrt{\frac{c+d\sin(\ldots}{c+\ldots}}}{15d^2f\sqrt{c+d\,s\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (4*b*(b*c - 5*a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*d*f) - (2*b^2
*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(5*d*f) + (2*(3*(5*a^2 + 3*b^2)*d
^2 - 2*b*c*(b*c - 5*a*d))*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt
[c + d*Sin[e + f*x]])/(15*d^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*b*
(b*c - 5*a*d)*(c^2 - d^2)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt
[(c + d*Sin[e + f*x])/(c + d)])/(15*d^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+b\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}\,dx = -\frac{2b^2\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{5df} + \frac{2\int \sqrt{c+d\sin(e+fx)}}{}$$

$$= \frac{4b(bc-5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15df} - \frac{2b^2\cos(e+fx)(c}{5}$$

$$= \frac{4b(bc-5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15df} - \frac{2b^2\cos(e+fx)(c}{5}$$

$$= \frac{4b(bc-5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15df} - \frac{2b^2\cos(e+fx)(c}{5}$$

$$= \frac{4b(bc-5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15df} - \frac{2b^2\cos(e+fx)(c}{5}$$

Mathematica [A] time = 0.92, size = 214, normalized size = 0.84

$$\frac{2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\left(\left(-15a^2d^2-10abcd+b^2\left(2c^2-9d^2\right)\right)\left((c+d)E\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e-2fx+\pi)\right.\right.\right.}{15d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (2*(-(d^2*(15*a^2*c + 7*b^2*c + 10*a*b*d)*EllipticF[(-2*e + Pi - 2*f*x)/4,
(2*d)/(c + d)]) + (-10*a*b*c*d - 15*a^2*d^2 + b^2*(2*c^2 - 9*d^2))*((c + d)
*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi -
 2*f*x)/4, (2*d)/(c + d)]))*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - 2*b*d*Cos[
e + f*x]*(c + d*Sin[e + f*x])*(b*c + 10*a*d + 3*b*d*Sin[e + f*x]))/(15*d^2*
f*Sqrt[c + d*Sin[e + f*x]])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b^2\cos\left(fx+e\right)^2 - 2\,ab\sin\left(fx+e\right) - a^2 - b^2\right)\sqrt{d\sin\left(fx+e\right)+c}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*sqrt(d*sin(
f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)^{2}\sqrt{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*sqrt(d*sin(f*x + e) + c), x)
```

maple [B] time = 4.43, size = 1100, normalized size = 4.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b^2*d*(-2/5/d*sin(f*x+e)*(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+8/15*c/d^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)
^(1/2)+4/15*c/d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c
+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2
)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2*(3/
5+8/15*c^2/d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c
+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2
)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1
/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(2*a*b
*d+b^2*c)*(-2/3/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(c/d-1)*((c+d
*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d
/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f
*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-4/3*c/d*(c/d-1)*((c+d*sin(f*x+e))/
(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)
/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+
e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1
/2),((c-d)/(c+d))^(1/2))))+2*(a^2*d+2*a*b*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d
))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/
(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2)))+2*a^2*c*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-
sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c
)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d)
)^(1/2)))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin \left(fx + e\right) + a\right)^2 \sqrt{d \sin \left(fx + e\right) + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)^2*sqrt(d*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + b\, \sin\left(e + f\, x\right)\right)^2 \sqrt{c + d\, \sin\left(e + f\, x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^(1/2),x)
[Out] int((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \sin \left(e + fx\right)\right)^2 \sqrt{c + d \sin \left(e + fx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2*(c+d*sin(f*x+e))**(1/2),x)
[Out] Integral((a + b*sin(e + f*x))**2*sqrt(c + d*sin(e + f*x)), x)
```

3.733 $\int \frac{(a+b\sin(e+fx))^2}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=203

$$\frac{2\left(d^2\left(3a^2+b^2\right)+2bc(bc-3ad)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^2f\sqrt{c+d\sin(e+fx)}}-\frac{4b(bc-3ad)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+\right.\right.}{3d^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] -2/3*b^2*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d/f+4/3*b*(-3*a*d+b*c)*(sin(1/2*
e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/
4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^2/f/((c+d*s
in(f*x+e))/(c+d))^(1/2)-2/3*((3*a^2+b^2)*d^2+2*b*c*(-3*a*d+b*c))*(sin(1/2*e
+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4
*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^2/f/
(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2791, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(d^2\left(3a^2+b^2\right)+2bc(bc-3ad)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^2f\sqrt{c+d\sin(e+fx)}}-\frac{4b(bc-3ad)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+\right.\right.}{3d^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*b^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*d*f) - (4*b*(b*c - 3*a*d)
*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*
d^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*((3*a^2 + b^2)*d^2 + 2*b*c*(
b*c - 3*a*d))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[
e + f*x])/(c + d)])/(3*d^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2}{\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{2b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3df} + \frac{2\int \frac{\frac{1}{2}\left(3a^2+b^2\right)d-b(bc-3ad)\sin(e+fx)}{\sqrt{c+d\sin(e+fx)}}\,dx}{3d}$$

$$= -\frac{2b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3df} - \frac{(2b(bc-3ad))\int\sqrt{c+d\sin(e+fx)}\,dx}{3d^2} + \frac{1}{3}\Big($$

$$= -\frac{2b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3df} - \frac{\left(2b(bc-3ad)\sqrt{c+d\sin(e+fx)}\right)\int\sqrt{\frac{c}{c+d}+}}{3d^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

$$= -\frac{2b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3df} - \frac{4b(bc-3ad)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\,\mathrm{si}}}{3d^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Mathematica [A] time = 0.90, size = 173, normalized size = 0.85

$$-\frac{2\left(\left(3a^2d^2-6abcd+b^2\left(2c^2+d^2\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)-2b(c+d)(bc-3ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,E}{3d^2f\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*(b^2*d*Cos[e + f*x]*(c + d*Sin[e + f*x]) - 2*b*(c + d)*(b*c - 3*a*d)*El
lipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c
+ d)] + (-6*a*b*c*d + 3*a^2*d^2 + b^2*(2*c^2 + d^2))*EllipticF[(-2*e + Pi -
 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)]))/(3*d^2*f*Sqr
t[c + d*Sin[e + f*x]])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2}{\sqrt{d\sin\left(fx+e\right)+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)/sqrt(d*sin(
f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^2}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/sqrt(d*sin(f*x + e) + c), x)
```

maple [B] time = 3.00, size = 695, normalized size = 3.42

$$\frac{\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^2\left(fx+e\right)\right)}\left(b^2\left(-\frac{2\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^2\left(fx+e\right)\right)}}{3d}+\frac{2\left(\frac{c}{d}-1\right)\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}}\sqrt{\frac{d\left(1-\sin\left(fx+e\right)\right)}{c+d}}\sqrt{\cdots}}{3\sqrt{-\left(-d\sin\left(f\cdots\right.\right.}}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b^2*(-2/3/d*(-(-d*sin(f*x+e)-c)*co
s(f*x+e)^2)^(1/2)+2/3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+
e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*
x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))
-4/3*c/d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1
/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)
*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+El
lipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+4*a*b*(c/d-1)
*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e
)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*Ellip
ticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*si
n(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+2*a^2*(c/d-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2)))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^2}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)^2/sqrt(d*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\sin\left(e+f\,x\right)\right)^2}{\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/(c + d*sin(e + f*x))^(1/2),x)
[Out] int((a + b*sin(e + f*x))^2/(c + d*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sin\left(e+fx\right)\right)^2}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**(1/2),x)
[Out] Integral((a + b*sin(e + f*x))**2/sqrt(c + d*sin(e + f*x)), x)
```

3.734 $\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=228

$$\frac{2\left(d^2\left(a^2-b^2\right)-2abcd+2b^2c^2\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{d^2f\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{2(bc-ad)^2\cos(e+fx)}{df\left(c^2-d^2\right)\sqrt{c+d\sin(e+fx)}}-\text{4}$$

```
[Out] 2*(-a*d+b*c)^2*cos(f*x+e)/d/(c^2-d^2)/f/(c+d*sin(f*x+e))^(1/2)-2*(2*b^2*c^2
-2*a*b*c*d+(a^2-b^2)*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4
*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(
c+d*sin(f*x+e))^(1/2)/d^2/(c^2-d^2)/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+4*b*(-
a*d+b*c)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elli
pticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/
(c+d))^(1/2)/d^2/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2790, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(d^2\left(a^2-b^2\right)-2abcd+2b^2c^2\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{d^2f\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{2(bc-ad)^2\cos(e+fx)}{df\left(c^2-d^2\right)\sqrt{c+d\sin(e+fx)}}-\text{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x])/(d*(c^2 - d^2)*f*Sqrt[c + d*Sin[e + f*x]]) +
 (2*(2*b^2*c^2 - 2*a*b*c*d + (a^2 - b^2)*d^2)*EllipticE[(e - Pi/2 + f*x)/2,
 (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(d^2*(c^2 - d^2)*f*Sqrt[(c + d*Si
n[e + f*x])/(c + d)]) - (4*b*(b*c - a*d)*EllipticF[(e - Pi/2 + f*x)/2, (2*d
)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(d^2*f*Sqrt[c + d*Sin[e + f*
x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{2\int \frac{\frac{1}{2}d\left(a^2c+b^2c-2abd\right)+\frac{1}{2}\left(2b^2c^2-2abcd+\left(a^2-b^2\right)d^2\right)\sin(e+}{\sqrt{c+d\sin(e+fx)}}}{d\left(c^2-d^2\right)}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} - \frac{(2b(bc-ad))\int \frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{d^2} - \frac{\left(2abcd-a^2\right.}{}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} - \frac{\left(\left(2abcd-a^2d^2-b^2\left(2c^2-d^2\right)\right)\sqrt{c+d\sin(e+\right.}{d^2\left(c^2-d^2\right)\sqrt{\frac{c+d\sin(e}{c+d}}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} - \frac{2\left(2abcd-a^2d^2-b^2\left(2c^2-d^2\right)\right)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+\right.\right.}{d^2\left(c^2-d^2\right)f\sqrt{\frac{c+d\sin(e}{c+d}}}$$

Mathematica [A] time = 0.87, size = 172, normalized size = 0.75

$$\frac{2\left(\frac{\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\left(\left(-a^2d^2+2abcd+b^2\left(d^2-2c^2\right)\right)E\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)+2b(c-d)(bc-ad)F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)\right)}{d(c-d)}+\frac{(bc-ad)^2\cos(e+fx)}{c^2-d^2}\right)}{df\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*(((b*c - a*d)^2*Cos[e + f*x])/(c^2 - d^2) + (((2*a*b*c*d - a^2*d^2 + b^2
*(-2*c^2 + d^2))*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] + 2*b*(c -
 d)*(b*c - a*d)*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)])*Sqrt[(c +
d*Sin[e + f*x])/(c + d)])/((c - d)*d)))/(d*f*Sqrt[c + d*Sin[e + f*x]])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2\right)\sqrt{d\sin\left(fx+e\right)+c}}{d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*sqrt(d*sin(f
*x + e) + c)/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{2}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(3/2), x)
```

maple [B] time = 3.99, size = 888, normalized size = 3.89

$$\frac{\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^{2}\left(fx+e\right)\right)}\left(b\left(\frac{2bd\left(\frac{c}{d}-1\right)\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}}\,\sqrt{\frac{d\left(1-\sin\left(fx+e\right)\right)}{c+d}}\,\sqrt{\frac{\left(-\sin\left(fx+e\right)-1\right)d}{c-d}}\left(\left(-\frac{c}{d}-1\right)\mathrm{EllipticE}\left(\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}}\right.\right.}{\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^{2}\left(fx+e\right)\right)}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b/d^2*(2*b*d*(c/d-1)*((c+d*sin(f*x
+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^
(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin
(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2)))+4*d*a*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)
*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))-2*c*b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+
e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*
x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))
)+(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^2*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x
+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(
1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*s
in(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),(
```

```
(c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d
*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+
e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d
))^(1/2))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{2}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\sin\left(e+f\,x\right)\right)^2}{\left(c+d\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/(c + d*sin(e + f*x))^(3/2),x)
[Out] int((a + b*sin(e + f*x))^2/(c + d*sin(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**(3/2),x)
[Out] Timed out
```

3.735 $\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=329

$$\frac{2\left(-a^2d^2+2abcd+b^2\left(2c^2-3d^2\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^2f\left(c^2-d^2\right)\sqrt{c+d\sin(e+fx)}}+\frac{2(bc-ad)^2\cos(e+fx)}{3df\left(c^2-d^2\right)(c+d\sin(e+fx))^{3/2}}-\frac{4(}{}$$

```
[Out] 2/3*(-a*d+b*c)^2*cos(f*x+e)/d/(c^2-d^2)/f/(c+d*sin(f*x+e))^(3/2)-4/3*(-a*d+
b*c)*(2*a*c*d+b*(c^2-3*d^2))*cos(f*x+e)/d/(c^2-d^2)^2/f/(c+d*sin(f*x+e))^(1
/2)+4/3*(-a*d+b*c)*(2*a*c*d+b*(c^2-3*d^2))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1
/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(
d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^2/(c^2-d^2)^2/f/((c+d*sin(f*x+e))/
(c+d))^(1/2)-2/3*(2*a*b*c*d-a^2*d^2+b^2*(2*c^2-3*d^2))*(sin(1/2*e+1/4*Pi+1/
2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*
x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^2/(c^2-d^2)/f/
(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2790, 2754, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(-a^2d^2+2abcd+b^2\left(2c^2-3d^2\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^2f\left(c^2-d^2\right)\sqrt{c+d\sin(e+fx)}}+\frac{2(bc-ad)^2\cos(e+fx)}{3df\left(c^2-d^2\right)(c+d\sin(e+fx))^{3/2}}-\frac{4(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x])/(3*d*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^(3/2
)) - (4*(b*c - a*d)*(2*a*c*d + b*(c^2 - 3*d^2))*Cos[e + f*x])/(3*d*(c^2 - d
^2)^2*f*Sqrt[c + d*Sin[e + f*x]]) - (4*(b*c - a*d)*(2*a*c*d + b*(c^2 - 3*d^
2))*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/
(3*d^2*(c^2 - d^2)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*(2*a*b*c*d
- a^2*d^2 + b^2*(2*c^2 - 3*d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d
)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*d^2*(c^2 - d^2)*f*Sqrt[c + d*Sin[
e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
```

```
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{2\int \frac{\frac{3}{2}d\left(a^2c+b^2c-2abd\right)+\frac{1}{2}\left(2b^2c^2+2abcd-\left(a^2+3b^2\right)d^2\right)\sin(}{(c+d\sin(e+fx))^{3/2}}}{3d\left(c^2-d^2\right)}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{4(bc-ad)\left(2acd+b\left(c^2-3d^2\right)\right)\cos(e+fx)}{3d\left(c^2-d^2\right)^2 f\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{4(bc-ad)\left(2acd+b\left(c^2-3d^2\right)\right)\cos(e+fx)}{3d\left(c^2-d^2\right)^2 f\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{4(bc-ad)\left(2acd+b\left(c^2-3d^2\right)\right)\cos(e+fx)}{3d\left(c^2-d^2\right)^2 f\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{4(bc-ad)\left(2acd+b\left(c^2-3d^2\right)\right)\cos(e+fx)}{3d\left(c^2-d^2\right)^2 f\sqrt{c+d\sin(e+fx)}}$$

Mathematica [A] time = 2.48, size = 302, normalized size = 0.92

$$\frac{2\left(\frac{(-c-d\sin(e+fx))\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\left(d^2\left(a^2\left(3c^2+d^2\right)-8abcd+b^2\left(c^2+3d^2\right)\right)F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)-2\left(-2a^2cd^2+abd\left(c^2+3d^2\right)+b^2\left(c^3-3cd^2\right)\right)\left((c+d)E\left(\frac{1}{4}(-\right.\right.\right.}{(c-d)^2(c+d)^2}}{3d^2f(c+d\sin(e+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*(((d^2*(-8*a*b*c*d + a^2*(3*c^2 + d^2) + b^2*(c^2 + 3*d^2))*EllipticF[(-
2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - 2*(-2*a^2*c*d^2 + a*b*d*(c^2 + 3*d^2)
 + b^2*(c^3 - 3*c*d^2))*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c
+ d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]))*(-c - d*Sin[e +
 f*x])*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((c - d)^2*(c + d)^2) - (d*(-(b*
c) + a*d)*Cos[e + f*x]*(-(b*c^3) - 5*a*c^2*d + 5*b*c*d^2 + a*d^3 - 2*d*(2*a
```

```
*c*d + b*(c^2 - 3*d^2))*Sin[e + f*x]))/(c^2 - d^2)^2))/(3*d^2*f*(c + d*Sin[
e + f*x])^(3/2))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2\right)\sqrt{d\sin\left(fx+e\right)+c}}{3\,cd^2\cos\left(fx+e\right)^2-c^3-3\,cd^2+\left(d^3\cos\left(fx+e\right)^2-3\,c^2d-d^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*sqrt(d*sin(f
*x + e) + c)/(3*c*d^2*cos(f*x + e)^2 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2
- 3*c^2*d - d^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^2}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(5/2), x)
```

maple [B] time = 6.03, size = 1043, normalized size = 3.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(2*b^2/d^2*(c/d-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))+1/d^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(2/3/(c^2-d
^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/3*d*cos(
f*x+e)^2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2
)/(3*c^4-6*c^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(
f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*co
s(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1
```

```
/2))+8/3*c*d/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f
*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos
(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))
))+2*b/d^2*(a*d-b*c)*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*
x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-si
n(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*
cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^
(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e
))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x
+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d
))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/
cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^2}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\sin\left(e+f\,x\right)\right)^2}{\left(c+d\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^2/(c + d*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.736 $\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=460

$$\frac{2\left(a^2d^2\left(23c^2+9d^2\right)-ab\left(6c^3d+58cd^3\right)-\left(b^2\left(2c^4-19c^2d^2-15d^4\right)\right)\right)\cos(e+fx)}{15df\left(c^2-d^2\right)^3\sqrt{c+d\sin(e+fx)}}+\frac{2\left(a^2d^2\left(23c^2+9d^2\right)-ab\left(\right.\right.}{}$$

```
[Out] 2/5*(-a*d+b*c)^2*cos(f*x+e)/d/(c^2-d^2)/f/(c+d*sin(f*x+e))^(5/2)-4/15*(-a*d
+b*c)*(4*a*c*d+b*(c^2-5*d^2))*cos(f*x+e)/d/(c^2-d^2)^2/f/(c+d*sin(f*x+e))^(
3/2)+2/15*(a^2*d^2*(23*c^2+9*d^2)-a*b*(6*c^3*d+58*c*d^3)-b^2*(2*c^4-19*c^2*
d^2-15*d^4))*cos(f*x+e)/d/(c^2-d^2)^3/f/(c+d*sin(f*x+e))^(1/2)-2/15*(a^2*d^
2*(23*c^2+9*d^2)-a*b*(6*c^3*d+58*c*d^3)-b^2*(2*c^4-19*c^2*d^2-15*d^4))*(sin
(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2
*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^2/(c^2
-d^2)^3/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-4/15*(-a*d+b*c)*(4*a*c*d+b*(c^2-5*
d^2))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Ellipti
cF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+
d))^(1/2)/d^2/(c^2-d^2)^2/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.86, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2790, 2754, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(a^2d^2\left(23c^2+9d^2\right)-ab\left(6c^3d+58cd^3\right)+b^2\left(-\left(-19c^2d^2+2c^4-15d^4\right)\right)\right)\cos(e+fx)}{15df\left(c^2-d^2\right)^3\sqrt{c+d\sin(e+fx)}}+\frac{2\left(a^2d^2\left(23c^2+9d^2\right)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x])/(5*d*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^(5/2
)) - (4*(b*c - a*d)*(4*a*c*d + b*(c^2 - 5*d^2))*Cos[e + f*x])/(15*d*(c^2 -
d^2)^2*f*(c + d*Sin[e + f*x])^(3/2)) + (2*(a^2*d^2*(23*c^2 + 9*d^2) - a*b*(
6*c^3*d + 58*c*d^3) - b^2*(2*c^4 - 19*c^2*d^2 - 15*d^4))*Cos[e + f*x])/(15*
d*(c^2 - d^2)^3*f*Sqrt[c + d*Sin[e + f*x]]) + (2*(a^2*d^2*(23*c^2 + 9*d^2)
- a*b*(6*c^3*d + 58*c*d^3) - b^2*(2*c^4 - 19*c^2*d^2 - 15*d^4))*EllipticE[(
e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(15*d^2*(c^2 -
d^2)^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (4*(b*c - a*d)*(4*a*c*d + b*
(c^2 - 5*d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin
[e + f*x])/(c + d)])/(15*d^2*(c^2 - d^2)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
```

```
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2}{(c+d\sin(e+fx))^{7/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} + \frac{2\int \frac{\frac{5}{2}d\left(\left(a^2+b^2\right)c-2abd\right)+\frac{1}{2}\left(6abcd-3a^2d^2+b^2\left(2c^2-5d^2\right)\right)}{(c+d\sin(e+fx))^{5/2}}}{5d\left(c^2-d^2\right)}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} - \frac{4(bc-ad)\left(4acd+b\left(c^2-5d^2\right)\right)\cos(e+fx)}{15d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^{3/2}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} - \frac{4(bc-ad)\left(4acd+b\left(c^2-5d^2\right)\right)\cos(e+fx)}{15d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^{3/2}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} - \frac{4(bc-ad)\left(4acd+b\left(c^2-5d^2\right)\right)\cos(e+fx)}{15d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^{3/2}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} - \frac{4(bc-ad)\left(4acd+b\left(c^2-5d^2\right)\right)\cos(e+fx)}{15d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^{3/2}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} - \frac{4(bc-ad)\left(4acd+b\left(c^2-5d^2\right)\right)\cos(e+fx)}{15d\left(c^2-d^2\right)^2 f(c+d\sin(e+fx))^{3/2}}$$

Mathematica [A] time = 4.99, size = 424, normalized size = 0.92

$$2\left(\frac{d\cos(e+fx)\left(-2\left(c^2-d^2\right)\left(-4a^2cd^2+abd\left(3c^2+5d^2\right)+b^2\left(c^3-5cd^2\right)\right)(c+d\sin(e+fx))-\left(-a^2d^2\left(23c^2+9d^2\right)+ab\left(6c^3d+58cd^3\right)+b^2\left(2c^4-19c^2d^2-15d^4\right)\right)(c+}{\left(c^2-d^2\right)^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (2*(-(((d^2*(-2*a*b*d*(27*c^2 + 5*d^2) + b^2*c*(7*c^2 + 25*d^2) + a^2*(15*c
^3 + 17*c*d^2))*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - (-(a^2*d^
2*(23*c^2 + 9*d^2)) + a*b*(6*c^3*d + 58*c*d^3) + b^2*(2*c^4 - 19*c^2*d^2 -
15*d^4))*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*Ellip
ticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]))*((c + d*Sin[e + f*x])/(c + d))
^(5/2))/((c - d)^3*(c + d))) + (d*Cos[e + f*x]*(3*(b*c - a*d)^2*(c^2 - d^2)
^2 - 2*(c^2 - d^2)*(-4*a^2*c*d^2 + a*b*d*(3*c^2 + 5*d^2) + b^2*(c^3 - 5*c*d
^2))*(c + d*Sin[e + f*x]) - (-(a^2*d^2*(23*c^2 + 9*d^2)) + a*b*(6*c^3*d + 5
8*c*d^3) + b^2*(2*c^4 - 19*c^2*d^2 - 15*d^4))*(c + d*Sin[e + f*x])^2))/(c^2
 - d^2)^3))/(15*d^2*f*(c + d*Sin[e + f*x])^(5/2))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2\right)\sqrt{d\sin\left(fx+e\right)+c}}{d^4\cos\left(fx+e\right)^4+c^4+6\,c^2d^2+d^4-2\left(3\,c^2d^2+d^4\right)\cos\left(fx+e\right)^2-4\left(cd^3\cos\left(fx+e\right)^2-c^3d-cd^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*sqrt(d*sin(
f*x + e) + c)/(d^4*cos(f*x + e)^4 + c^4 + 6*c^2*d^2 + d^4 - 2*(3*c^2*d^2 +
d^4)*cos(f*x + e)^2 - 4*(c*d^3*cos(f*x + e)^2 - c^3*d - c*d^3)*sin(f*x + e)
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^2}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(7/2), x)
```

maple [B] time = 8.37, size = 1450, normalized size = 3.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(7/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((a^2*d^2-2*a*b*c*d+b^2*c^2)/d^2*(2
/5/(c^2-d^2)/d^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^3
+16/15*c/(c^2-d^2)^2/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+
c/d)^2+2/15*d*cos(f*x+e)^2/(c^2-d^2)^3*(23*c^2+9*d^2)/(-(-d*sin(f*x+e)-c)*c
os(f*x+e)^2)^(1/2)+2*(15*c^3+17*c*d^2)/(15*c^6-45*c^4*d^2+45*c^2*d^4-15*d^6
)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-
sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Ellipt
icF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/15*d*(23*c^2+9*d^
2)/(c^2-d^2)^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+
d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)
^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/
2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+2*b*(a*
d-b*c)/d^2*(2/3/(c^2-d^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*
x+e)+c/d)^2+8/3*d*cos(f*x+e)^2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)
^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*c^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(
c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2))+8/3*c*d/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e)
)/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))))+b^2/d^2*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+
e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1
/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*si
n(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((
c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*
(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e
)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2)
,((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d)
)^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^2}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/(d*sin(f*x + e) + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\sin\left(e+f\,x\right)\right)^2}{\left(c+d\,\sin\left(e+f\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/(c + d*sin(e + f*x))^(7/2),x)
[Out] int((a + b*sin(e + f*x))^2/(c + d*sin(e + f*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**(7/2),x)
[Out] Timed out
```

3.737 $\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=642

$$\frac{2b\left(-297a^2d^2+66abcd-\left(b^2\left(8c^2+81d^2\right)\right)\right)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{693d^2f}-\frac{2\left(693a^3d^3+1485a^2bcd^2-33ab\right.}{}$$

```
[Out] -2/3465*(1485*a^2*b*c*d^2+693*a^3*d^3-33*a*b^2*d*(10*c^2-49*d^2)+5*b^3*(8*c
^3+67*c*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/d^2/f+2/693*b*(66*a*b*c*d-2
97*a^2*d^2-b^2*(8*c^2+81*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^(5/2)/d^2/f+8/99
*b^2*(-6*a*d+b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^(7/2)/d^2/f-2/11*b^2*cos(f*x+
e)*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(7/2)/d/f-2/3465*(1848*a^3*c*d^3+495*a
^2*b*d^2*(3*c^2+5*d^2)-66*a*b^2*d*(5*c^3-57*c*d^2)+5*b^3*(8*c^4+57*c^2*d^2+
135*d^4))*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d^2/f-2/3465*(231*a^3*d^3*(23*c
^2+9*d^2)+495*a^2*b*c*d^2*(3*c^2+29*d^2)-33*a*b^2*d*(10*c^4-279*c^2*d^2-147
*d^4)+5*b^3*(8*c^5+51*c^3*d^2+741*c*d^4))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/
2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d
/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^3/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+
2/3465*(c^2-d^2)*(1848*a^3*c*d^3+495*a^2*b*d^2*(3*c^2+5*d^2)-66*a*b^2*d*(5*
c^3-57*c*d^2)+5*b^3*(8*c^4+57*c^2*d^2+135*d^4))*(sin(1/2*e+1/4*Pi+1/2*f*x)^
2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1
/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^3/f/(c+d*sin(f*x+e))^
(1/2)
```

Rubi [A] time = 1.40, antiderivative size = 642, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2793, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2b\left(-297a^2d^2+66abcd+b^2\left(-\left(8c^2+81d^2\right)\right)\right)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{693d^2f}-\frac{2\left(1485a^2bcd^2+693a^3d^3-33\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*(1848*a^3*c*d^3 + 495*a^2*b*d^2*(3*c^2 + 5*d^2) - 66*a*b^2*d*(5*c^3 - 5
7*c*d^2) + 5*b^3*(8*c^4 + 57*c^2*d^2 + 135*d^4))*Cos[e + f*x]*Sqrt[c + d*Si
n[e + f*x]])/(3465*d^2*f) - (2*(1485*a^2*b*c*d^2 + 693*a^3*d^3 - 33*a*b^2*d
*(10*c^2 - 49*d^2) + 5*b^3*(8*c^3 + 67*c*d^2))*Cos[e + f*x]*(c + d*Sin[e +
f*x])^(3/2))/(3465*d^2*f) + (2*b*(66*a*b*c*d - 297*a^2*d^2 - b^2*(8*c^2 + 8
1*d^2))*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(693*d^2*f) + (8*b^2*(b*c
- 6*a*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(7/2))/(99*d^2*f) - (2*b^2*Cos[e
```

```
 + f*x]*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(7/2))/(11*d*f) + (2*(231
*a^3*d^3*(23*c^2 + 9*d^2) + 495*a^2*b*c*d^2*(3*c^2 + 29*d^2) - 33*a*b^2*d*(
10*c^4 - 279*c^2*d^2 - 147*d^4) + 5*b^3*(8*c^5 + 51*c^3*d^2 + 741*c*d^4))*E
llipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3465
*d^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*(c^2 - d^2)*(1848*a^3*c*d^3
 + 495*a^2*b*d^2*(3*c^2 + 5*d^2) - 66*a*b^2*d*(5*c^3 - 57*c*d^2) + 5*b^3*(8
*c^4 + 57*c^2*d^2 + 135*d^4))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*
Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3465*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))^{5/2}\,dx = -\frac{2b^2\cos(e+fx)(a+b\sin(e+fx))(c+d\sin(e+fx))^{7/2}}{11df} + \frac{2\int(}{}$$

$$= \frac{8b^2(bc-6ad)\cos(e+fx)(c+d\sin(e+fx))^{7/2}}{99d^2f} - \frac{2b^2\cos(e+fx}{}$$

$$= \frac{2b\left(66abcd-297a^2d^2-b^2\left(8c^2+81d^2\right)\right)\cos(e+fx)(c+d\sin(e}{693d^2f}$$

$$= -\frac{2\left(1485a^2bcd^2+693a^3d^3-33ab^2d\left(10c^2-49d^2\right)+5b^3\left(8c^3+\right.\right.}{3465d^2f}$$

$$= -\frac{2\left(1848a^3cd^3+495a^2bd^2\left(3c^2+5d^2\right)-66ab^2d\left(5c^3-57cd^2\right)+\right.}{3465}$$

$$= -\frac{2\left(1848a^3cd^3+495a^2bd^2\left(3c^2+5d^2\right)-66ab^2d\left(5c^3-57cd^2\right)+\right.}{3465}$$

$$= -\frac{2\left(1848a^3cd^3+495a^2bd^2\left(3c^2+5d^2\right)-66ab^2d\left(5c^3-57cd^2\right)+\right.}{3465}$$

$$= -\frac{2\left(1848a^3cd^3+495a^2bd^2\left(3c^2+5d^2\right)-66ab^2d\left(5c^3-57cd^2\right)+\right.}{3465}$$

Mathematica [A] time = 2.61, size = 545, normalized size = 0.85

$$\frac{d(c+d\sin(e+fx))\left(5bd^2\left(1188a^2d^2+2508abcd+b^2\left(452c^2+513d^2\right)\right)\cos(3(e+fx))-4d\left(1386a^3d^3+8910a^2b\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-16*(d^2*(495*a^2*b*d^2*(27*c^2 + 5*d^2) + 231*a^3*c*d*(15*c^2 + 17*d^2) +
 33*a*b^2*d*(155*c^3 + 261*c*d^2) + 5*b^3*(2*c^4 + 663*c^2*d^2 + 135*d^4))*
EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] + (231*a^3*d^3*(23*c^2 + 9*
d^2) + 495*a^2*b*c*d^2*(3*c^2 + 29*d^2) + 33*a*b^2*d*(-10*c^4 + 279*c^2*d^2
```

```
 + 147*d^4) + 5*b^3*(8*c^5 + 51*c^3*d^2 + 741*c*d^4))*((c + d)*EllipticE[(-
2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2
*d)/(c + d)]))*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + d*(c + d*Sin[e + f*x])*
(2*(-20328*a^3*c*d^3 - 990*a^2*b*d^2*(36*c^2 + 23*d^2) - 66*a*b^2*d*(20*c^3
 + 747*c*d^2) + 5*b^3*(32*c^4 - 1866*c^2*d^2 - 1305*d^4))*Cos[e + f*x] + 5*
b*d^2*(2508*a*b*c*d + 1188*a^2*d^2 + b^2*(452*c^2 + 513*d^2))*Cos[3*(e + f*
x)] - 315*b^3*d^4*Cos[5*(e + f*x)] - 4*d*(8910*a^2*b*c*d^2 + 1386*a^3*d^3 +
 33*a*b^2*d*(150*c^2 + 133*d^2) + 5*b^3*(6*c^3 + 619*c*d^2))*Sin[2*(e + f*x
)] + 70*b^2*d^3*(23*b*c + 33*a*d)*Sin[4*(e + f*x)]))/(27720*d^3*f*Sqrt[c +
d*Sin[e + f*x]])
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(\left(2\,b^3cd+3\,ab^2d^2\right)\cos\left(fx+e\right)^4+\left(a^3+3\,ab^2\right)c^2+2\left(3\,a^2b+b^3\right)cd+\left(a^3+3\,ab^2\right)d^2-\left(3\,ab^2c^2+2\left(\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(((2*b^3*c*d + 3*a*b^2*d^2)*cos(f*x + e)^4 + (a^3 + 3*a*b^2)*c^2 +
2*(3*a^2*b + b^3)*c*d + (a^3 + 3*a*b^2)*d^2 - (3*a*b^2*c^2 + 2*(3*a^2*b + 2
*b^3)*c*d + (a^3 + 6*a*b^2)*d^2)*cos(f*x + e)^2 + (b^3*d^2*cos(f*x + e)^4 +
 (3*a^2*b + b^3)*c^2 + 2*(a^3 + 3*a*b^2)*c*d + (3*a^2*b + b^3)*d^2 - (b^3*c
^2 + 6*a*b^2*c*d + (3*a^2*b + 2*b^3)*d^2)*cos(f*x + e)^2)*sin(f*x + e))*sqr
t(d*sin(f*x + e) + c), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 9.40, size = 2728, normalized size = 4.25

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b^3*d^3*(-2/11/d*sin(f*x+e)^4*(-(-
d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+20/99*c/d^2*sin(f*x+e)^3*(-(-d*sin(f*x+
e)-c)*cos(f*x+e)^2)^(1/2)-2/7*(9/11+80/99*c^2/d^2)/d*sin(f*x+e)^2*(-(-d*sin
```

(f*x+e)-c)*cos(f*x+e)^2)^(1/2)-2/3465*(-480*c^3-472*c*d^2)/d^4*sin(f*x+e)*(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)-2/3465*(640*c^4+596*c^2*d^2+675*d^4)
/d^5*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3465*(-320*c^4-348*c^2*d^2+6
75*d^4)/d^4*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))
^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1
/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/3465*(-
1280*c^5-1032*c^3*d^2-1146*c*d^4)/d^5*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2
)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(
f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2))))+(3*a*b^2*d^3+3*b^3*c*d^2)*(-2/9/d*sin(f*x+e)^3*(-(-d*sin(f*
x+e)-c)*cos(f*x+e)^2)^(1/2)+16/63*c/d^2*sin(f*x+e)^2*(-(-d*sin(f*x+e)-c)*co
s(f*x+e)^2)^(1/2)-2/5*(7/9+16/21*c^2/d^2)/d*sin(f*x+e)*(-(-d*sin(f*x+e)-c)*
cos(f*x+e)^2)^(1/2)-2/315*(-64*c^3-62*c*d^2)/d^4*(-(-d*sin(f*x+e)-c)*cos(f*
x+e)^2)^(1/2)+2/315*(32*c^3+36*c*d^2)/d^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^
(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*
sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))+2/315*(128*c^4+108*c^2*d^2+147*d^4)/d^4*(c/d-1)*((c+d*
sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/
(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(3*a^2*b*d^3+9*a*b^2*c*d^2+3*b^3*c^2
d)(-2/7/d*sin(f*x+e)^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+12/35*c/d^
2*sin(f*x+e)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)-2/3*(5/7+24/35*c^2/d^2
)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(-4/35*c^2/d^2+5/21)*(c/d-1)*
((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)
-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*
sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/105*(-48*c^3-44*c*d^2)/d^3*
(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-si
n(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1
)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF((
(c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(a^3*d^3+9*a^2*b*c*d^2
+9*a*b^2*c^2*d+b^3*c^3)*(-2/5/d*sin(f*x+e)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2
)^(1/2)+8/15*c/d^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+4/15*c/d*(c/d-1)
*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e
)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d
*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2*(3/5+8/15*c^2/d^2)*(c/d-1)
*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e
)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*Ellip
ticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*si
n(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(3*a^3*c*d^2+9*a^2*b*c^2*d+3*
a*b^2*c^3)*(-2/3/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(c/d-1)*((c+
d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*
d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(
f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-4/3*c/d*(c/d-1)*((c+d*sin(f*x+e))

```
/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2
)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x
+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(
1/2),((c-d)/(c+d))^(1/2))))+2*(3*a^3*c^2*d+3*a^2*b*c^3)*(c/d-1)*((c+d*sin(f
*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d)
)^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*s
in(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c
-d))^(1/2),((c-d)/(c+d))^(1/2)))+2*c^3*a^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))
^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2)
,((c-d)/(c+d))^(1/2)))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)^3\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\sin\left(e+f\,x\right)\right)^3\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x))^(5/2),x)
[Out] int((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3*(c+d*sin(f*x+e))**(5/2),x)
[Out] Timed out
```

3.738 $\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=496

$$\frac{2b\left(-189a^2d^2+54abcd-\left(b^2\left(8c^2+49d^2\right)\right)\right)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{315d^2f}-\frac{2\left(105a^3d^3+189a^2bcd^2-9ab^2d\left(\right.\right.}{}$$

```
[Out] 2/315*b*(54*a*b*c*d-189*a^2*d^2-b^2*(8*c^2+49*d^2))*cos(f*x+e)*(c+d*sin(f*x
+e))^(3/2)/d^2/f+8/63*b^2*(-5*a*d+b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^(5/2)/d^
2/f-2/9*b^2*cos(f*x+e)*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(5/2)/d/f-2/315*(1
89*a^2*b*c*d^2+105*a^3*d^3-9*a*b^2*d*(6*c^2-25*d^2)+b^3*(8*c^3+39*c*d^2))*c
os(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d^2/f-2/315*(420*a^3*c*d^3+189*a^2*b*d^2*(
c^2+3*d^2)-a*b^2*(54*c^3*d-738*c*d^3)+b^3*(8*c^4+33*c^2*d^2+147*d^4))*(sin(
1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*
e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^3/f/((c
+d*sin(f*x+e))/(c+d))^(1/2)+2/315*(c^2-d^2)*(189*a^2*b*c*d^2+105*a^3*d^3-9*
a*b^2*d*(6*c^2-25*d^2)+b^3*(8*c^3+39*c*d^2))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^
(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)
*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^3/f/(c+d*sin(f*x+e))^(1/
2)
```

Rubi [A] time = 1.03, antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2793, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2b\left(-189a^2d^2+54abcd+b^2\left(-\left(8c^2+49d^2\right)\right)\right)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{315d^2f}-\frac{2\left(189a^2bcd^2+105a^3d^3-9ab^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(189*a^2*b*c*d^2 + 105*a^3*d^3 - 9*a*b^2*d*(6*c^2 - 25*d^2) + b^3*(8*c^
3 + 39*c*d^2))*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(315*d^2*f) + (2*b*(5
4*a*b*c*d - 189*a^2*d^2 - b^2*(8*c^2 + 49*d^2))*Cos[e + f*x]*(c + d*Sin[e +
 f*x])^(3/2))/(315*d^2*f) + (8*b^2*(b*c - 5*a*d)*Cos[e + f*x]*(c + d*Sin[e
 + f*x])^(5/2))/(63*d^2*f) - (2*b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])*(c + d
*Sin[e + f*x])^(5/2))/(9*d*f) + (2*(420*a^3*c*d^3 + 189*a^2*b*d^2*(c^2 + 3*
d^2) - a*b^2*(54*c^3*d - 738*c*d^3) + b^3*(8*c^4 + 33*c^2*d^2 + 147*d^4))*E
llipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(315*
d^3*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*(c^2 - d^2)*(189*a^2*b*c*d^2
 + 105*a^3*d^3 - 9*a*b^2*d*(6*c^2 - 25*d^2) + b^3*(8*c^3 + 39*c*d^2))*Ellip
```

```
ticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])
/(315*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int (a+b\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}\,dx = -\frac{2b^2\cos(e+fx)(a+b\sin(e+fx))(c+d\sin(e+fx))^{5/2}}{9df} + \frac{2\int}{}$$

$$= \frac{8b^2(bc-5ad)\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{63d^2f} - \frac{2b^2\cos(e+f}{}$$

$$= \frac{2b\left(54abcd-189a^2d^2-b^2\left(8c^2+49d^2\right)\right)\cos(e+fx)(c+d\sin}{315d^2f}$$

$$= -\frac{2\left(189a^2bcd^2+105a^3d^3-9ab^2d\left(6c^2-25d^2\right)+b^3\left(8c^3+39c\right.\right.}{315d^2f}$$

$$= -\frac{2\left(189a^2bcd^2+105a^3d^3-9ab^2d\left(6c^2-25d^2\right)+b^3\left(8c^3+39c\right.\right.}{315d^2f}$$

$$= -\frac{2\left(189a^2bcd^2+105a^3d^3-9ab^2d\left(6c^2-25d^2\right)+b^3\left(8c^3+39c\right.\right.}{315d^2f}$$

$$= -\frac{2\left(189a^2bcd^2+105a^3d^3-9ab^2d\left(6c^2-25d^2\right)+b^3\left(8c^3+39c\right.\right.}{315d^2f}$$

Mathematica [A] time = 2.43, size = 410, normalized size = 0.83

$$\frac{d(c+d\sin(e+fx))\left(bd\left(10bd(27ad+10bc)\cos(3(e+fx))-2\sin(2(e+fx))\left(378a^2d^2+432abcd+b^2\left(6c^2+1\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-8*(d^2*(756*a^2*b*c*d^2 + 105*a^3*d*(3*c^2 + d^2) + 9*a*b^2*d*(51*c^2 + 2
5*d^2) + 2*b^3*(c^3 + 93*c*d^2))*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c
+ d)] + (420*a^3*c*d^3 + 189*a^2*b*d^2*(c^2 + 3*d^2) + a*b^2*(-54*c^3*d + 7
38*c*d^3) + b^3*(8*c^4 + 33*c^2*d^2 + 147*d^4))*((c + d)*EllipticE[(-2*e +
Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c
 + d)]))*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + d*(c + d*Sin[e + f*x])*(-2*(1
512*a^2*b*c*d^2 + 420*a^3*d^3 + 9*a*b^2*d*(12*c^2 + 115*d^2) + b^3*(-16*c^3
```

```
 + 402*c*d^2))*Cos[e + f*x] + b*d*(10*b*d*(10*b*c + 27*a*d)*Cos[3*(e + f*x)
] - 2*(432*a*b*c*d + 378*a^2*d^2 + b^2*(6*c^2 + 133*d^2) - 35*b^2*d^2*Cos[2
*(e + f*x)])*Sin[2*(e + f*x)])))/(1260*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3 d\cos\left(fx+e\right)^4-\left(3\,ab^2c+\left(3\,a^2b+2\,b^3\right)d\right)\cos\left(fx+e\right)^2+\left(a^3+3\,ab^2\right)c+\left(3\,a^2b+b^3\right)d-\left(\left(b^3c+\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*d*cos(f*x + e)^4 - (3*a*b^2*c + (3*a^2*b + 2*b^3)*d)*cos(f*x
+ e)^2 + (a^3 + 3*a*b^2)*c + (3*a^2*b + b^3)*d - ((b^3*c + 3*a*b^2*d)*cos(f
*x + e)^2 - (3*a^2*b + b^3)*c - (a^3 + 3*a*b^2)*d)*sin(f*x + e))*sqrt(d*sin
(f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)^3\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^(3/2), x)
```

maple [B] time = 7.03, size = 2112, normalized size = 4.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b^3*d^2*(-2/9/d*sin(f*x+e)^3*(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+16/63*c/d^2*sin(f*x+e)^2*(-(-d*sin(f*x+e
)-c)*cos(f*x+e)^2)^(1/2)-2/5*(7/9+16/21*c^2/d^2)/d*sin(f*x+e)*(-(-d*sin(f*x
+e)-c)*cos(f*x+e)^2)^(1/2)-2/315*(-64*c^3-62*c*d^2)/d^4*(-(-d*sin(f*x+e)-c)
*cos(f*x+e)^2)^(1/2)+2/315*(32*c^3+36*c*d^2)/d^3*(c/d-1)*((c+d*sin(f*x+e))/
(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)
/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))
^(1/2),((c-d)/(c+d))^(1/2))+2/315*(128*c^4+108*c^2*d^2+147*d^4)/d^4*(c/d-1)
*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e
)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*Ellip
```

```
ticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*si
n(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(3*a*b^2*d^2+2*b^3*c*d)*(-2/7
/d*sin(f*x+e)^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+12/35*c/d^2*sin(f*x
+e)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)-2/3*(5/7+24/35*c^2/d^2)/d*(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(-4/35*c^2/d^2+5/21)*(c/d-1)*((c+d*sin
(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-
d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/105*(-48*c^3-44*c*d^2)/d^3*(c/d-1)*(
(c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-
1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*Ellipti
cE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(
f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(3*a^2*b*d^2+6*a*b^2*c*d+b^3*c^
2)*(-2/5/d*sin(f*x+e)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+8/15*c/d^2*(-
(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+4/15*c/d*(c/d-1)*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(
1/2),((c-d)/(c+d))^(1/2))+2*(3/5+8/15*c^2/d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e)
)/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))))+(a^3*d^2+6*a^2*b*c*d+3*a*b^2*c^2)*(-2/3/d*(-(-d*si
n(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*
(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*
x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)
/(c+d))^(1/2))-4/3*c/d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x
+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f
*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c
+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))
+2*(2*a^3*c*d+3*a^2*b*c^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin
(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*c
os(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2
)))+2*c^2*a^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d
))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^
(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))/cos(f*
x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)^{3}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\sin\left(e+f\,x\right)\right)^3\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x))^(3/2),x)
[Out] int((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(e+fx\right)\right)^3\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3*(c+d*sin(f*x+e))**(3/2),x)
[Out] Integral((a + b*sin(e + f*x))**3*(c + d*sin(e + f*x))**(3/2), x)
```

3.739 $\int (a+b\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=375

$$\frac{2b\left(-105a^2d^2+42abcd-\left(b^2\left(8c^2+25d^2\right)\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105d^2f}+\frac{2b\left(c^2-d^2\right)\left(-105a^2d^2+42abcd\right.}{10}$$

```
[Out] 8/35*b^2*(-4*a*d+b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/d^2/f-2/7*b^2*cos(f
*x+e)*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/d/f+2/105*b*(42*a*b*c*d-105*a
^2*d^2-b^2*(8*c^2+25*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d^2/f-2/105*(1
05*a^2*b*c*d^2+105*a^3*d^3-21*a*b^2*d*(2*c^2-9*d^2)+b^3*(8*c^3+19*c*d^2))*(
sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(
1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^3/f
/((c+d*sin(f*x+e))/(c+d))^(1/2)-2/105*b*(c^2-d^2)*(42*a*b*c*d-105*a^2*d^2-b
^2*(8*c^2+25*d^2))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2
*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*si
n(f*x+e))/(c+d))^(1/2)/d^3/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2793, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2b\left(-105a^2d^2+42abcd+b^2\left(-\left(8c^2+25d^2\right)\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{105d^2f}+\frac{2b\left(c^2-d^2\right)\left(-105a^2d^2+42abc\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (2*b*(42*a*b*c*d - 105*a^2*d^2 - b^2*(8*c^2 + 25*d^2))*Cos[e + f*x]*Sqrt[c
 + d*Sin[e + f*x]])/(105*d^2*f) + (8*b^2*(b*c - 4*a*d)*Cos[e + f*x]*(c + d*S
in[e + f*x])^(3/2))/(35*d^2*f) - (2*b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])*(
c + d*Sin[e + f*x])^(3/2))/(7*d*f) + (2*(105*a^2*b*c*d^2 + 105*a^3*d^3 - 21
*a*b^2*d*(2*c^2 - 9*d^2) + b^3*(8*c^3 + 19*c*d^2))*EllipticE[(e - Pi/2 + f*
x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(105*d^3*f*Sqrt[(c + d*Sin[e
 + f*x])/(c + d)]) + (2*b*(c^2 - d^2)*(42*a*b*c*d - 105*a^2*d^2 - b^2*(8*c^
2 + 25*d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e
 + f*x])/(c + d)])/(105*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
```

```
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int (a+b\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}\,dx = -\frac{2b^2\cos(e+fx)(a+b\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{7df}+\frac{2\int}{}$$

$$=\frac{8b^2(bc-4ad)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{35d^2f}-\frac{2b^2\cos(e+fx}{}$$

$$=\frac{2b\left(42abcd-105a^2d^2-b^2\left(8c^2+25d^2\right)\right)\cos(e+fx)\sqrt{c+d\sin(}}{105d^2f}$$

$$=\frac{2b\left(42abcd-105a^2d^2-b^2\left(8c^2+25d^2\right)\right)\cos(e+fx)\sqrt{c+d\sin(}}{105d^2f}$$

$$=\frac{2b\left(42abcd-105a^2d^2-b^2\left(8c^2+25d^2\right)\right)\cos(e+fx)\sqrt{c+d\sin(}}{105d^2f}$$

$$=\frac{2b\left(42abcd-105a^2d^2-b^2\left(8c^2+25d^2\right)\right)\cos(e+fx)\sqrt{c+d\sin(}}{105d^2f}$$

Mathematica [A] time = 1.45, size = 306, normalized size = 0.82

$$\frac{bd(c+d\sin(e+fx))\left(\left(-420a^2d^2-84abcd+b^2\left(16c^2-115d^2\right)\right)\cos(e+fx)+3bd(5bd\cos(3(e+fx))-2(21ad+\right.}{}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]],x]

[Out] (-4*(d^2*(105*a^3*c*d + 147*a*b^2*c*d + 105*a^2*b*d^2 + b^3*(2*c^2 + 25*d^2))*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] + (105*a^2*b*c*d^2 + 105 *a^3*d^3 + 21*a*b^2*d*(-2*c^2 + 9*d^2) + b^3*(8*c^3 + 19*c*d^2))*((c + d)*E llipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2 *f*x)/4, (2*d)/(c + d)]))*Sqrt[(c + d*Sin[e + f*x])/(c + d)] + b*d*(c + d*S in[e + f*x])*((-84*a*b*c*d - 420*a^2*d^2 + b^2*(16*c^2 - 115*d^2))*Cos[e + f*x] + 3*b*d*(5*b*d*Cos[3*(e + f*x)] - 2*(b*c + 21*a*d)*Sin[2*(e + f*x)]))) /(210*d^3*f*Sqrt[c + d*Sin[e + f*x]])

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(3\,ab^2\cos\left(fx+e\right)^2-a^3-3\,ab^2+\left(b^3\cos\left(fx+e\right)^2-3\,a^2b-b^3\right)\sin\left(fx+e\right)\right)\sqrt{d\sin\left(fx+e\right)+c},x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(-(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 - 3 *a^2*b - b^3)*sin(f*x + e))*sqrt(d*sin(f*x + e) + c), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 5.82, size = 1561, normalized size = 4.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(1/2),x)

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b^3*d*(-2/7/d*sin(f*x+e)^2*(-(-d*s
in(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+12/35*c/d^2*sin(f*x+e)*(-(-d*sin(f*x+e)-c)
*cos(f*x+e)^2)^(1/2)-2/3*(5/7+24/35*c^2/d^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+
e)^2)^(1/2)+2*(-4/35*c^2/d^2+5/21)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(
d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x
+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2))+2/105*(-48*c^3-44*c*d^2)/d^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))
^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c
-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((
c-d)/(c+d))^(1/2))))+(3*a*b^2*d+b^3*c)*(-2/5/d*sin(f*x+e)*(-(-d*sin(f*x+e)-
c)*cos(f*x+e)^2)^(1/2)+8/15*c/d^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+4
/15*c/d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/
2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*
EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2*(3/5+8/15*c
^2/d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/
2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*
((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+Ell
ipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(3*a^2*b*d+3*a
*b^2*c)*(-2/3/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(c/d-1)*((c+d*s
in(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(
c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x
+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-4/3*c/d*(c/d-1)*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e)
)/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))))+2*(a^3*d+3*a^2*b*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d
))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/
(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2)))+2*a^3*c*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-
sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c
)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d)
)^(1/2)))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)^{3}\sqrt{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3*sqrt(d*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\sin\left(e+f\,x\right)\right)^3\,\sqrt{c+d\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x))^(1/2),x)
[Out] int((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(e+fx\right)\right)^3\sqrt{c+d\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3*(c+d*sin(f*x+e))**(1/2),x)
[Out] Integral((a + b*sin(e + f*x))**3*sqrt(c + d*sin(e + f*x)), x)
```

3.740 $\int \frac{(a+b\sin(e+fx))^3}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=302

$$-\frac{2b\left(-45a^2d^2+30abcd-\left(b^2\left(8c^2+9d^2\right)\right)\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^3f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{2\left(-15a^3d^3+45a^2bcd^2\right.}{}$$

```
[Out] 8/15*b^2*(-3*a*d+b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d^2/f-2/5*b^2*cos(f
*x+e)*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/d/f+2/15*b*(30*a*b*c*d-45*a^2
*d^2-b^2*(8*c^2+9*d^2))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*P
i+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+
d*sin(f*x+e))^(1/2)/d^3/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+2/15*(45*a^2*b*c*d
^2-15*a^3*d^3-15*a*b^2*d*(2*c^2+d^2)+b^3*(8*c^3+7*c*d^2))*(sin(1/2*e+1/4*Pi
+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2
*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^3/f/(c+d*si
n(f*x+e))^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2793, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(45a^2bcd^2-15a^3d^3-15ab^2d\left(2c^2+d^2\right)+b^3\left(8c^3+7cd^2\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{15d^3f\sqrt{c+d\sin(e+fx)}}-\frac{2b\left(-45a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (8*b^2*(b*c - 3*a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(15*d^2*f) - (2
*b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/(5*d*f) -
(2*b*(30*a*b*c*d - 45*a^2*d^2 - b^2*(8*c^2 + 9*d^2))*EllipticE[(e - Pi/2 +
f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(15*d^3*f*Sqrt[(c + d*Sin[
e + f*x])/(c + d)]) - (2*(45*a^2*b*c*d^2 - 15*a^3*d^3 - 15*a*b^2*d*(2*c^2 +
 d^2) + b^3*(8*c^3 + 7*c*d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]
*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(15*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
```

```
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^3}{\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{2b^2\cos(e+fx)(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{5df} + \frac{2\int \frac{\frac{1}{2}\left(2b^3c+5a^3d+ab^2d\right)-\frac{1}{2}b(2}{}}{}$$

$$= \frac{8b^2(bc-3ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15d^2f} - \frac{2b^2\cos(e+fx)(a+b\sin(e+fx))}{5df}$$

$$= \frac{8b^2(bc-3ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15d^2f} - \frac{2b^2\cos(e+fx)(a+b\sin(e+fx))}{5df}$$

$$= \frac{8b^2(bc-3ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15d^2f} - \frac{2b^2\cos(e+fx)(a+b\sin(e+fx))}{5df}$$

$$= \frac{8b^2(bc-3ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{15d^2f} - \frac{2b^2\cos(e+fx)(a+b\sin(e+fx))}{5df}$$

Mathematica [A] time = 1.20, size = 219, normalized size = 0.73

$$\frac{-2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\left(d^2\left(15a^3d+15ab^2d+2b^3c\right)F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)+b\left(45a^2d^2-30abcd+b^2\left(8c^2+9d^2\right)\right)\right.}{1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-2*(d^2*(2*b^3*c + 15*a^3*d + 15*a*b^2*d)*EllipticF[(-2*e + Pi - 2*f*x)/4,
 (2*d)/(c + d)] + b*(-30*a*b*c*d + 45*a^2*d^2 + b^2*(8*c^2 + 9*d^2))*((c +
d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi
 - 2*f*x)/4, (2*d)/(c + d)]))*Sqrt[(c + d*Sin[e + f*x])/(c + d)] - 2*b^2*d*
Cos[e + f*x]*(c + d*Sin[e + f*x])*(-4*b*c + 15*a*d + 3*b*d*Sin[e + f*x]))/(
15*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{3\,ab^2\cos\left(fx+e\right)^2-a^3-3\,ab^2+\left(b^3\cos\left(fx+e\right)^2-3\,a^2b-b^3\right)\sin\left(fx+e\right)}{\sqrt{d\sin\left(fx+e\right)+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 - 3
*a^2*b - b^3)*sin(f*x + e))/sqrt(d*sin(f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^3}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3/sqrt(d*sin(f*x + e) + c), x)
```

maple [B] time = 4.18, size = 1085, normalized size = 3.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b^3*(-2/5/d*sin(f*x+e)*(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)+8/15*c/d^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(
1/2)+4/15*c/d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d
))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^
(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2*(3/5+
8/15*c^2/d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d
))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^
(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2
))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+3*a*b^2*
(-2/3/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(c/d-1)*((c+d*sin(f*x+e
))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1
/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-
d))^(1/2),((c-d)/(c+d))^(1/2))-4/3*c/d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/
```

```
2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))
^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)
/(c+d))^(1/2))))+6*a^2*b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f
*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos
(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))
)+2*a^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/
2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*
EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))/cos(f*x+e)/(
c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^3}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)^3/sqrt(d*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\sin\left(e+f\,x\right)\right)^3}{\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3/(c + d*sin(e + f*x))^(1/2),x)
[Out] int((a + b*sin(e + f*x))^3/(c + d*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sin\left(e+fx\right)\right)^3}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**(1/2),x)
[Out] Integral((a + b*sin(e + f*x))**3/sqrt(c + d*sin(e + f*x)), x)
```

3.741 $\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=361

$$\frac{2b\left(-3a^2d^2+6abcd-\left(b^2\left(4c^2-d^2\right)\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3d^2f\left(c^2-d^2\right)}-\frac{2b\left(-9a^2d^2+18abcd-\left(b^2\left(8c^2+d^2\right)\right)\right)\sqrt{}}{3d^3f\sqrt{c+d\sin}}$$

```
[Out] 2*(-a*d+b*c)^2*cos(f*x+e)*(a+b*sin(f*x+e))/d/(c^2-d^2)/f/(c+d*sin(f*x+e))^(
1/2)+2/3*b*(6*a*b*c*d-3*a^2*d^2-b^2*(4*c^2-d^2))*cos(f*x+e)*(c+d*sin(f*x+e)
)^(1/2)/d^2/(c^2-d^2)/f+2/3*(9*a^2*b*c*d^2-3*a^3*d^3-9*a*b^2*d*(2*c^2-d^2)+
b^3*(8*c^3-5*c*d^2))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1
/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*s
in(f*x+e))^(1/2)/d^3/(c^2-d^2)/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+2/3*b*(18*a
*b*c*d-9*a^2*d^2-b^2*(8*c^2+d^2))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1
/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^
(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^3/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.62, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2792, 3023, 2752, 2663, 2661, 2655, 2653}

$$\frac{2b\left(-3a^2d^2+6abcd+b^2\left(-\left(4c^2-d^2\right)\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3d^2f\left(c^2-d^2\right)}-\frac{2b\left(-9a^2d^2+18abcd+b^2\left(-\left(8c^2+d^2\right)\right.\right.}{3d^3f\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x]*(a + b*Sin[e + f*x]))/(d*(c^2 - d^2)*f*Sqrt[c
 + d*Sin[e + f*x]]) + (2*b*(6*a*b*c*d - 3*a^2*d^2 - b^2*(4*c^2 - d^2))*Cos[
e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*d^2*(c^2 - d^2)*f) - (2*(9*a^2*b*c*d^
2 - 3*a^3*d^3 - 9*a*b^2*d*(2*c^2 - d^2) + b^3*(8*c^3 - 5*c*d^2))*EllipticE[
(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*d^3*(c^2 -
d^2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*b*(18*a*b*c*d - 9*a^2*d^2 -
 b^2*(8*c^2 + d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c +
d*Sin[e + f*x])/(c + d)])/(3*d^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} - \frac{2\int \frac{\frac{1}{2}\left(2b(bc-ad)^2-ad\left(\left(a^2+b^2\right)c-2abd\right)\right)+\frac{1}{2}\left(a^2bc\cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{2b\left(6abcd-3a^2d^2-b^2\left(4c^2-d^2\right)\right)\cos\cdots}{3d^2\left(c^2-d^2\cdots\right.}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{2b\left(6abcd-3a^2d^2-b^2\left(4c^2-d^2\right)\right)\cos\cdots}{3d^2\left(c^2-d^2\cdots\right.}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{2b\left(6abcd-3a^2d^2-b^2\left(4c^2-d^2\right)\right)\cos\cdots}{3d^2\left(c^2-d^2\cdots\right.}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{2b\left(6abcd-3a^2d^2-b^2\left(4c^2-d^2\right)\right)\cos\cdots}{3d^2\left(c^2-d^2\cdots\right.}$$

Mathematica [A] time = 2.05, size = 311, normalized size = 0.86

$$\frac{2\left(\frac{\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\left(d^2\left(-3a^3cd+9a^2bd^2-9ab^2cd+b^3\left(2c^2+d^2\right)\right)F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)+\left(-3a^3d^3+9a^2bcd^2+9ab^2d\left(d^2-2c^2\right)+b^3\left(8c^3-5cd^2\right)\right)\left((c+d)E\left(\frac{1}{4}(-2\cdots\right.\right.\right.}{(c-d)(c+d)}\cdots\right.}{3d^3f\sqrt{c+d\sin(e+f\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*(((d^2*(-3*a^3*c*d - 9*a*b^2*c*d + 9*a^2*b*d^2 + b^3*(2*c^2 + d^2))*Elli
pticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] + (9*a^2*b*c*d^2 - 3*a^3*d^3 +
```

```
9*a*b^2*d*(-2*c^2 + d^2) + b^3*(8*c^3 - 5*c*d^2))*((c + d)*EllipticE[(-2*e
+ Pi - 2*f*x)/4, (2*d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/
(c + d)]))*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((c - d)*(c + d)) - (d*Cos[e
 + f*x]*(9*a*b^2*c^2*d - 9*a^2*b*c*d^2 + 3*a^3*d^3 + b^3*(-4*c^3 + c*d^2) +
 b^3*d*(-c^2 + d^2)*Sin[e + f*x]))/(-c^2 + d^2)))/(3*d^3*f*Sqrt[c + d*Sin[e
 + f*x]])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(3\,ab^2\cos\left(fx+e\right)^2-a^3-3\,ab^2+\left(b^3\cos\left(fx+e\right)^2-3\,a^2b-b^3\right)\sin\left(fx+e\right)\right)\sqrt{d\sin\left(fx+e\right)+c}}{d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 - 3*
a^2*b - b^3)*sin(f*x + e))*sqrt(d*sin(f*x + e) + c)/(d^2*cos(f*x + e)^2 - 2
*c*d*sin(f*x + e) - c^2 - d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\sin\left(fx+e\right)+a\right)^3}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(3/2), x)
```

maple [B] time = 5.20, size = 1398, normalized size = 3.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(1/d^3*b*(b^2*d^2*(-2/3/d*(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(
d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x
+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2))-4/3*c/d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+
```

```
e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*
x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+
d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+
2*(3*a*b*d^2-b^2*c*d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+
e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*
x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+
d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+6
*a^2*d^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1
/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)
*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-6*a*b*c*d*(c
/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(
f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(
((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2*b^2*c^2*(c/d-1)*((c+d
*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d
/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f
*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^
2*d-b^3*c^3)/d^3*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)
^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*
x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(
f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2
))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(
c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^
2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(
1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(
f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{3}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^3}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3/(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^3/(c + d*sin(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.742 $\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=391

$$-\frac{2\left(-a^2d^2+2abcd+b^2\left(8c^2-9d^2\right)\right)(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^3f\left(c^2-d^2\right)\sqrt{c+d\sin(e+fx)}}+\frac{2\left(4a^3cd^3-3a^2bd^2\left(c^2+3d^2\right.\right.}{}$$

```
[Out] 2/3*(-a*d+b*c)^2*cos(f*x+e)*(a+b*sin(f*x+e))/d/(c^2-d^2)/f/(c+d*sin(f*x+e))
^(3/2)+8/3*(-a*d+b*c)^2*(a*c*d+b*(c^2-2*d^2))*cos(f*x+e)/d^2/(c^2-d^2)^2/f/
(c+d*sin(f*x+e))^(1/2)-2/3*(4*a^3*c*d^3-6*a*b^2*c*d*(c^2-3*d^2)-3*a^2*b*d^2
*(c^2+3*d^2)+b^3*(8*c^4-15*c^2*d^2+3*d^4))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1
/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(
d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^3/(c^2-d^2)^2/f/((c+d*sin(f*x+e))/
(c+d))^(1/2)+2/3*(-a*d+b*c)*(2*a*b*c*d-a^2*d^2+b^2*(8*c^2-9*d^2))*(sin(1/2*
e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/
4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^3/(
c^2-d^2)/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.75, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2792, 3021, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(-a^2d^2+2abcd+b^2\left(8c^2-9d^2\right)\right)(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{3d^3f\left(c^2-d^2\right)\sqrt{c+d\sin(e+fx)}}+\frac{2\left(-3a^2bd^2\left(c^2+3d^2\right)+4a^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x]*(a + b*Sin[e + f*x]))/(3*d*(c^2 - d^2)*f*(c +
 d*Sin[e + f*x])^(3/2)) + (8*(b*c - a*d)^2*(a*c*d + b*(c^2 - 2*d^2))*Cos[e
+ f*x])/(3*d^2*(c^2 - d^2)^2*f*Sqrt[c + d*Sin[e + f*x]]) + (2*(4*a^3*c*d^3
- 6*a*b^2*c*d*(c^2 - 3*d^2) - 3*a^2*b*d^2*(c^2 + 3*d^2) + b^3*(8*c^4 - 15*c
^2*d^2 + 3*d^4))*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Si
n[e + f*x]])/(3*d^3*(c^2 - d^2)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (
2*(b*c - a*d)*(2*a*b*c*d - a^2*d^2 + b^2*(8*c^2 - 9*d^2))*EllipticF[(e - Pi
/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(3*d^3*(c^2
 - d^2)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(2b(bc-ad)^2-3ad\left(\left(a^2+b^2\right)c-2abd\right)\right)-\frac{1}{2}\left(5a^2\right.}{}}{}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{8(bc-ad)^2\left(acd+b\left(c^2-2d^2\right)\right)\cos(e}{3d^2\left(c^2-d^2\right)^2 f\sqrt{c+d\sin(e+fx}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{8(bc-ad)^2\left(acd+b\left(c^2-2d^2\right)\right)\cos(e}{3d^2\left(c^2-d^2\right)^2 f\sqrt{c+d\sin(e+fx}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{8(bc-ad)^2\left(acd+b\left(c^2-2d^2\right)\right)\cos(e}{3d^2\left(c^2-d^2\right)^2 f\sqrt{c+d\sin(e+fx}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{8(bc-ad)^2\left(acd+b\left(c^2-2d^2\right)\right)\cos(e}{3d^2\left(c^2-d^2\right)^2 f\sqrt{c+d\sin(e+fx}}$$

Mathematica [A] time = 3.73, size = 357, normalized size = 0.91

$$\frac{2\left(\frac{(-c-d\sin(e+fx))\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\left(\left(4a^3cd^3-3a^2bd^2\left(c^2+3d^2\right)-6ab^2cd\left(c^2-3d^2\right)+b^3\left(8c^4-15c^2d^2+3d^4\right)\right)\left((c+d)E\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)-cF\left(\frac{1}{4}(-2e-2fx+\right.\right.}{(c-d)^2(c+d)^2}}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*(((d^2*(-12*a^2*b*c*d^2 + a^3*d*(3*c^2 + d^2) + 3*a*b^2*d*(c^2 + 3*d^2)
+ 2*b^3*(c^3 - 3*c*d^2))*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] +
(4*a^3*c*d^3 - 6*a*b^2*c*d*(c^2 - 3*d^2) - 3*a^2*b*d^2*(c^2 + 3*d^2) + b^3*
(8*c^4 - 15*c^2*d^2 + 3*d^4))*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*
d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]))*(-c - d*S
in[e + f*x])*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((c - d)^2*(c + d)^2) - (d
*(b*c - a*d)^2*Cos[e + f*x]*(-4*b*c^3 - 5*a*c^2*d + 8*b*c*d^2 + a*d^3 + d*(
-5*b*c^2 - 4*a*c*d + 9*b*d^2)*Sin[e + f*x]))/(c^2 - d^2)^2))/(3*d^3*f*(c +
d*Sin[e + f*x])^(3/2))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(3\,ab^{2}\cos\left(fx+e\right)^{2}-a^{3}-3\,ab^{2}+\left(b^{3}\cos\left(fx+e\right)^{2}-3\,a^{2}b-b^{3}\right)\sin\left(fx+e\right)\right)\sqrt{d\sin\left(fx+e\right)+c}}{3\,cd^{2}\cos\left(fx+e\right)^{2}-c^{3}-3\,cd^{2}+\left(d^{3}\cos\left(fx+e\right)^{2}-3\,c^{2}d-d^{3}\right)\sin\left(fx+e\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 - 3*
a^2*b - b^3)*sin(f*x + e))*sqrt(d*sin(f*x + e) + c)/(3*c*d^2*cos(f*x + e)^2
 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{3}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(5/2), x)
```

maple [B] time = 6.79, size = 1379, normalized size = 3.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(b^2/d^3*(2*b*d*(c/d-1)*((c+d*sin(f
*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d)
```

```
)^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*s
in(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c
-d))^(1/2),((c-d)/(c+d))^(1/2)))+6*d*a*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/
2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c
-d)/(c+d))^(1/2))-4*c*b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*
x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(
f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2
)))+1/d^3*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*(2/3/(c^2-d^2)/d*(-
(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/3*d*cos(f*x+e)^2
/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4
-6*c^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/
(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)
^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+8/3
*c*d/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(
c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^
2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(
1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+3*b/d
^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)
-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2
)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(
f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-
d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1
-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-
c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),(
(c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^
(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \sin\left(fx+e\right)+a\right)^{3}}{\left(d \sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\sin\left(e+f\,x\right)\right)^3}{\left(c+d\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3/(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^3/(c + d*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.743 $\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=532

$$-\frac{2\left(a^2d^2\left(23c^2+9d^2\right)+2abd\left(7c^3-39cd^2\right)+b^2\left(8c^4-21c^2d^2+45d^4\right)\right)(bc-ad)\cos(e+fx)}{15d^2f\left(c^2-d^2\right)^3\sqrt{c+d\sin(e+fx)}}-\frac{2\left(a^2d^2\left(23c^2+9\right.\right.}{}$$

```
[Out] 2/5*(-a*d+b*c)^2*cos(f*x+e)*(a+b*sin(f*x+e))/d/(c^2-d^2)/f/(c+d*sin(f*x+e))
^(5/2)+8/15*(-a*d+b*c)^2*(2*a*c*d+b*(c^2-3*d^2))*cos(f*x+e)/d^2/(c^2-d^2)^2
/f/(c+d*sin(f*x+e))^(3/2)-2/15*(-a*d+b*c)*(a^2*d^2*(23*c^2+9*d^2)+2*a*b*d*(
7*c^3-39*c*d^2)+b^2*(8*c^4-21*c^2*d^2+45*d^4))*cos(f*x+e)/d^2/(c^2-d^2)^3/f
/(c+d*sin(f*x+e))^(1/2)+2/15*(-a*d+b*c)*(a^2*d^2*(23*c^2+9*d^2)+2*a*b*d*(7*
c^3-39*c*d^2)+b^2*(8*c^4-21*c^2*d^2+45*d^4))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^
(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)
*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/d^3/(c^2-d^2)^3/f/((c+d*sin(f*x+e)
)/(c+d))^(1/2)+2/15*(8*a^3*c*d^3-6*a*b^2*c*d*(c^2-5*d^2)-3*a^2*b*d^2*(3*c^2
+5*d^2)-b^3*(8*c^4-15*c^2*d^2+15*d^4))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/
sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c
+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/d^3/(c^2-d^2)^2/f/(c+d*sin(f*x+e
))^(1/2)
```

Rubi [A] time = 1.11, antiderivative size = 532, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2792, 3021, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(a^2d^2\left(23c^2+9d^2\right)+2abd\left(7c^3-39cd^2\right)+b^2\left(-21c^2d^2+8c^4+45d^4\right)\right)(bc-ad)\cos(e+fx)}{15d^2f\left(c^2-d^2\right)^3\sqrt{c+d\sin(e+fx)}}-\frac{2\left(-3a^2bd^2\left(3c^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x]*(a + b*Sin[e + f*x]))/(5*d*(c^2 - d^2)*f*(c +
 d*Sin[e + f*x])^(5/2)) + (8*(b*c - a*d)^2*(2*a*c*d + b*(c^2 - 3*d^2))*Cos[
e + f*x])/(15*d^2*(c^2 - d^2)^2*f*(c + d*Sin[e + f*x])^(3/2)) - (2*(b*c - a
*d)*(a^2*d^2*(23*c^2 + 9*d^2) + 2*a*b*d*(7*c^3 - 39*c*d^2) + b^2*(8*c^4 - 2
1*c^2*d^2 + 45*d^4))*Cos[e + f*x])/(15*d^2*(c^2 - d^2)^3*f*Sqrt[c + d*Sin[e
 + f*x]]) - (2*(b*c - a*d)*(a^2*d^2*(23*c^2 + 9*d^2) + 2*a*b*d*(7*c^3 - 39*
c*d^2) + b^2*(8*c^4 - 21*c^2*d^2 + 45*d^4))*EllipticE[(e - Pi/2 + f*x)/2, (
2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(15*d^3*(c^2 - d^2)^3*f*Sqrt[(c + d
*Sin[e + f*x])/(c + d)]) - (2*(8*a^3*c*d^3 - 6*a*b^2*c*d*(c^2 - 5*d^2) - 3*
```

```
a^2*b*d^2*(3*c^2 + 5*d^2) - b^3*(8*c^4 - 15*c^2*d^2 + 15*d^4))*EllipticF[(e
 - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(15*d^
3*(c^2 - d^2)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^{7/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} - \frac{2\int \frac{\frac{1}{2}\left(2b(bc-ad)^2-5ad\left(\left(a^2+b^2\right)c-2abd\right)\right)+\frac{1}{2}\left(3a\right.}{}}{}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} + \frac{8(bc-ad)^2\left(2acd+b\left(c^2-3d^2\right)\right)\cos}{15d^2\left(c^2-d^2\right)^2 f(c+d\sin(e+f}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} + \frac{8(bc-ad)^2\left(2acd+b\left(c^2-3d^2\right)\right)\cos}{15d^2\left(c^2-d^2\right)^2 f(c+d\sin(e+f}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} + \frac{8(bc-ad)^2\left(2acd+b\left(c^2-3d^2\right)\right)\cos}{15d^2\left(c^2-d^2\right)^2 f(c+d\sin(e+f}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} + \frac{8(bc-ad)^2\left(2acd+b\left(c^2-3d^2\right)\right)\cos}{15d^2\left(c^2-d^2\right)^2 f(c+d\sin(e+f}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{5d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{5/2}} + \frac{8(bc-ad)^2\left(2acd+b\left(c^2-3d^2\right)\right)\cos}{15d^2\left(c^2-d^2\right)^2 f(c+d\sin(e+f}$$

Mathematica [A] time = 5.30, size = 584, normalized size = 1.10

$$2\left(\frac{d(bc-ad)\cos(e+fx)\left(-d^2\left(a^2d^2\left(23c^2+9d^2\right)+2abd\left(7c^3-39cd^2\right)+b^2\left(8c^4-21c^2d^2+45d^4\right)\right)\cos(2(e+fx))+2d\left(2a^2cd^2\left(27c^2+5d^2\right)+abd\left(27c^4-170c^2d^2+1\right.\right.\right.}{2\left(d^2-c^2\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(7/2),x]
```

```
[Out] (2*(((d^2*(3*a^2*b*d^2*(27*c^2 + 5*d^2) - a^3*c*d*(15*c^2 + 17*d^2) - 3*a*b
^2*d*(7*c^3 + 25*c*d^2) + b^3*(2*c^4 + 15*c^2*d^2 + 15*d^4))*EllipticF[(-2*
e + Pi - 2*f*x)/4, (2*d)/(c + d)] + (-(a^3*d^3*(23*c^2 + 9*d^2)) + 3*a^2*b*
c*d^2*(3*c^2 + 29*d^2) - 3*a*b^2*d*(-2*c^4 + 19*c^2*d^2 + 15*d^4) + b^3*(8*
c^5 - 21*c^3*d^2 + 45*c*d^4))*((c + d)*EllipticE[(-2*e + Pi - 2*f*x)/4, (2*
d)/(c + d)] - c*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]))*((c + d*S
in[e + f*x])/(c + d))^(5/2))/((c - d)^3*(c + d)) + (d*(b*c - a*d)*Cos[e + f
```

```
*x]*(8*b^2*c^6 + 14*a*b*c^5*d + 68*a^2*c^4*d^2 - 2*b^2*c^4*d^2 - 146*a*b*c^
3*d^3 + 13*a^2*c^2*d^4 + 45*b^2*c^2*d^4 - 60*a*b*c*d^5 + 15*a^2*d^6 + 45*b^
2*d^6 - d^2*(a^2*d^2*(23*c^2 + 9*d^2) + 2*a*b*d*(7*c^3 - 39*c*d^2) + b^2*(8
*c^4 - 21*c^2*d^2 + 45*d^4))*Cos[2*(e + f*x)] + 2*d*(2*a^2*c*d^2*(27*c^2 +
5*d^2) + a*b*d*(27*c^4 - 170*c^2*d^2 + 15*d^4) + b^2*(9*c^5 - 20*c^3*d^2 +
75*c*d^4))*Sin[e + f*x]))/(2*(-c^2 + d^2)^3)))/(15*d^3*f*(c + d*Sin[e + f*x
])^(5/2))
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(3\,ab^{2}\cos\left(fx+e\right)^{2}-a^{3}-3\,ab^{2}+\left(b^{3}\cos\left(fx+e\right)^{2}-3\,a^{2}b-b^{3}\right)\sin\left(fx+e\right)\right)\sqrt{d\sin\left(fx+e\right.}}{d^{4}\cos\left(fx+e\right)^{4}+c^{4}+6\,c^{2}d^{2}+d^{4}-2\left(3\,c^{2}d^{2}+d^{4}\right)\cos\left(fx+e\right)^{2}-4\left(cd^{3}\cos\left(fx+e\right)^{2}-c^{3}d-cd^{3}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 - 3
*a^2*b - b^3)*sin(f*x + e))*sqrt(d*sin(f*x + e) + c)/(d^4*cos(f*x + e)^4 +
c^4 + 6*c^2*d^2 + d^4 - 2*(3*c^2*d^2 + d^4)*cos(f*x + e)^2 - 4*(c*d^3*cos(f
*x + e)^2 - c^3*d - c*d^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{3}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(7/2), x)
```

maple [B] time = 9.59, size = 1621, normalized size = 3.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(7/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(2*b^3/d^3*(c/d-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d
```

```
))^(1/2),((c-d)/(c+d))^(1/2))+3*b/d^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(2/3/(c^2
-d^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+8/3*d*co
s(f*x+e)^2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(3*c^2+d
^2)/(3*c^4-6*c^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-si
n(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*
cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^
(1/2))+8/3*c*d/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin
(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*c
os(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2
))))+1/d^3*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*(2/5/(c^2-d^2)/d^2
*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^3+16/15*c/(c^2-d^
2)^2/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+2/15*d*co
s(f*x+e)^2/(c^2-d^2)^3*(23*c^2+9*d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/
2)+2*(15*c^3+17*c*d^2)/(15*c^6-45*c^4*d^2+45*c^2*d^4-15*d^6)*(c/d-1)*((c+d*
sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/
(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*
x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/15*d*(23*c^2+9*d^2)/(c^2-d^2)^3*(
c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin
(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)
*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+3*b^2/d^3*(a*d-b*c)*(2*
d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d
^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*(
(-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Elli
pticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/
d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f
*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*E
llipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+
d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+
e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^3}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\sin\left(e+f\,x\right)\right)^3}{\left(c+d\,\sin\left(e+f\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3/(c + d*sin(e + f*x))^(7/2),x)
[Out] int((a + b*sin(e + f*x))^3/(c + d*sin(e + f*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**(7/2),x)
[Out] Timed out
```

3.744 $\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=716

$$-\frac{2\left(a^2d^2\left(71c^2+25d^2\right)+ab\left(26c^3d-218cd^3\right)+b^2\left(8c^4-17c^2d^2+105d^4\right)\right)(bc-ad)\cos(e+fx)}{105d^2f\left(c^2-d^2\right)^3(c+d\sin(e+fx))^{3/2}}+\frac{2\left(a^2d^2\left(71c^2\right.\right.}{}$$

```
[Out] 2/7*(-a*d+b*c)^2*cos(f*x+e)*(a+b*sin(f*x+e))/d/(c^2-d^2)/f/(c+d*sin(f*x+e))
^(7/2)+8/35*(-a*d+b*c)^2*(3*a*c*d+b*(c^2-4*d^2))*cos(f*x+e)/d^2/(c^2-d^2)^2
/f/(c+d*sin(f*x+e))^(5/2)-2/105*(-a*d+b*c)*(a^2*d^2*(71*c^2+25*d^2)+a*b*(26
*c^3*d-218*c*d^3)+b^2*(8*c^4-17*c^2*d^2+105*d^4))*cos(f*x+e)/d^2/(c^2-d^2)^
3/f/(c+d*sin(f*x+e))^(3/2)+2/105*(16*a^3*c*d^3*(11*c^2+13*d^2)-6*a*b^2*c*d*
(3*c^4-62*c^2*d^2-133*d^4)-9*a^2*b*d^2*(5*c^4+102*c^2*d^2+21*d^4)-b^3*(8*c^
6-23*c^4*d^2+294*c^2*d^4+105*d^6))*cos(f*x+e)/d^2/(c^2-d^2)^4/f/(c+d*sin(f*
x+e))^(1/2)-2/105*(16*a^3*c*d^3*(11*c^2+13*d^2)-6*a*b^2*c*d*(3*c^4-62*c^2*d
^2-133*d^4)-9*a^2*b*d^2*(5*c^4+102*c^2*d^2+21*d^4)-b^3*(8*c^6-23*c^4*d^2+29
4*c^2*d^4+105*d^6))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/
2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*si
n(f*x+e))^(1/2)/d^3/(c^2-d^2)^4/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-2/105*(-a*
d+b*c)*(a^2*d^2*(71*c^2+25*d^2)+a*b*(26*c^3*d-218*c*d^3)+b^2*(8*c^4-17*c^2*
d^2+105*d^4))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)
*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x
+e))/(c+d))^(1/2)/d^3/(c^2-d^2)^3/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.44, antiderivative size = 716, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2792, 3021, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(a^2d^2\left(71c^2+25d^2\right)+ab\left(26c^3d-218cd^3\right)+b^2\left(-17c^2d^2+8c^4+105d^4\right)\right)(bc-ad)\cos(e+fx)}{105d^2f\left(c^2-d^2\right)^3(c+d\sin(e+fx))^{3/2}}+\frac{2\left(-9a^2bd^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(9/2),x]
```

```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x]*(a + b*Sin[e + f*x]))/(7*d*(c^2 - d^2)*f*(c +
 d*Sin[e + f*x])^(7/2)) + (8*(b*c - a*d)^2*(3*a*c*d + b*(c^2 - 4*d^2))*Cos[
e + f*x])/(35*d^2*(c^2 - d^2)^2*f*(c + d*Sin[e + f*x])^(5/2)) - (2*(b*c - a
*d)*(a^2*d^2*(71*c^2 + 25*d^2) + a*b*(26*c^3*d - 218*c*d^3) + b^2*(8*c^4 -
17*c^2*d^2 + 105*d^4))*Cos[e + f*x])/(105*d^2*(c^2 - d^2)^3*f*(c + d*Sin[e
+ f*x])^(3/2)) + (2*(16*a^3*c*d^3*(11*c^2 + 13*d^2) - 6*a*b^2*c*d*(3*c^4 -
```

```
62*c^2*d^2 - 133*d^4) - 9*a^2*b*d^2*(5*c^4 + 102*c^2*d^2 + 21*d^4) - b^3*(8
*c^6 - 23*c^4*d^2 + 294*c^2*d^4 + 105*d^6))*Cos[e + f*x])/(105*d^2*(c^2 - d
^2)^4*f*Sqrt[c + d*Sin[e + f*x]]) + (2*(16*a^3*c*d^3*(11*c^2 + 13*d^2) - 6*
a*b^2*c*d*(3*c^4 - 62*c^2*d^2 - 133*d^4) - 9*a^2*b*d^2*(5*c^4 + 102*c^2*d^2
 + 21*d^4) - b^3*(8*c^6 - 23*c^4*d^2 + 294*c^2*d^4 + 105*d^6))*EllipticE[(e
 - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(105*d^3*(c^2 -
d^2)^4*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*(b*c - a*d)*(a^2*d^2*(71*
c^2 + 25*d^2) + a*b*(26*c^3*d - 218*c*d^3) + b^2*(8*c^4 - 17*c^2*d^2 + 105*
d^4))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x]
)/(c + d)])/(105*d^3*(c^2 - d^2)^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^3}{(c+d\sin(e+fx))^{9/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{7d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{7/2}} - \frac{2\int \frac{\frac{1}{2}\left(2b(bc-ad)^2-7ad\left(\left(a^2+b^2\right)c-2abd\right)\right)+\frac{1}{2}\left(5a(\right.}{}}{}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{7d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{7/2}} + \frac{8(bc-ad)^2\left(3acd+b\left(c^2-4d^2\right)\right)\cos(}{35d^2\left(c^2-d^2\right)^2f(c+d\sin(e+fx)}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{7d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{7/2}} + \frac{8(bc-ad)^2\left(3acd+b\left(c^2-4d^2\right)\right)\cos(}{35d^2\left(c^2-d^2\right)^2f(c+d\sin(e+fx)}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{7d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{7/2}} + \frac{8(bc-ad)^2\left(3acd+b\left(c^2-4d^2\right)\right)\cos(}{35d^2\left(c^2-d^2\right)^2f(c+d\sin(e+fx)}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{7d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{7/2}} + \frac{8(bc-ad)^2\left(3acd+b\left(c^2-4d^2\right)\right)\cos(}{35d^2\left(c^2-d^2\right)^2f(c+d\sin(e+fx)}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{7d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{7/2}} + \frac{8(bc-ad)^2\left(3acd+b\left(c^2-4d^2\right)\right)\cos(}{35d^2\left(c^2-d^2\right)^2f(c+d\sin(e+fx)}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)(a+b\sin(e+fx))}{7d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{7/2}} + \frac{8(bc-ad)^2\left(3acd+b\left(c^2-4d^2\right)\right)\cos(}{35d^2\left(c^2-d^2\right)^2f(c+d\sin(e+fx)}$$

Mathematica [A] time = 6.98, size = 1127, normalized size = 1.57

$$\frac{\sqrt{c+d\sin(e+fx)}\left(-\frac{2\left(-b^3\cos(e+fx)c^3+3ab^2d\cos(e+fx)c^2-3a^2bd^2\cos(e+fx)c+a^3d^3\cos(e+fx)\right)}{7d^2\left(d^2-c^2\right)(c+d\sin(e+fx))^4} - \frac{2\left(8b^3\cos(e+fx)c^6+18ab^2d\cos(e+fx)c^5\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^3/(c + d*Sin[e + f*x])^(9/2),x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*((-2*(-(b^3*c^3*Cos[e + f*x]) + 3*a*b^2*c^2*d*Cos
[e + f*x] - 3*a^2*b*c*d^2*Cos[e + f*x] + a^3*d^3*Cos[e + f*x]))/(7*d^2*(-c^
```

```
2 + d^2)*(c + d*Sin[e + f*x])^4) - (6*(-3*b^3*c^4*Cos[e + f*x] + 2*a*b^2*c^
3*d*Cos[e + f*x] + 5*a^2*b*c^2*d^2*Cos[e + f*x] + 7*b^3*c^2*d^2*Cos[e + f*x
] - 4*a^3*c*d^3*Cos[e + f*x] - 14*a*b^2*c*d^3*Cos[e + f*x] + 7*a^2*b*d^4*Co
s[e + f*x]))/(35*d^2*(-c^2 + d^2)^2*(c + d*Sin[e + f*x])^3) - (2*(-8*b^3*c^
5*Cos[e + f*x] - 18*a*b^2*c^4*d*Cos[e + f*x] - 45*a^2*b*c^3*d^2*Cos[e + f*x
] + 17*b^3*c^3*d^2*Cos[e + f*x] + 71*a^3*c^2*d^3*Cos[e + f*x] + 201*a*b^2*c
^2*d^3*Cos[e + f*x] - 243*a^2*b*c*d^4*Cos[e + f*x] - 105*b^3*c*d^4*Cos[e +
f*x] + 25*a^3*d^5*Cos[e + f*x] + 105*a*b^2*d^5*Cos[e + f*x]))/(105*d^2*(-c^
2 + d^2)^3*(c + d*Sin[e + f*x])^2) - (2*(8*b^3*c^6*Cos[e + f*x] + 18*a*b^2*
c^5*d*Cos[e + f*x] + 45*a^2*b*c^4*d^2*Cos[e + f*x] - 23*b^3*c^4*d^2*Cos[e +
 f*x] - 176*a^3*c^3*d^3*Cos[e + f*x] - 372*a*b^2*c^3*d^3*Cos[e + f*x] + 918
*a^2*b*c^2*d^4*Cos[e + f*x] + 294*b^3*c^2*d^4*Cos[e + f*x] - 208*a^3*c*d^5*
Cos[e + f*x] - 798*a*b^2*c*d^5*Cos[e + f*x] + 189*a^2*b*d^6*Cos[e + f*x] +
105*b^3*d^6*Cos[e + f*x]))/(105*d^2*(-c^2 + d^2)^4*(c + d*Sin[e + f*x]))))/
f - ((-2*(2*b^3*c^5*d - 105*a^3*c^4*d^2 - 153*a*b^2*c^4*d^2 + 720*a^2*b*c^3
*d^3 + 172*b^3*c^3*d^3 - 254*a^3*c^2*d^4 - 894*a*b^2*c^2*d^4 + 432*a^2*b*c*
d^5 + 210*b^3*c*d^5 - 25*a^3*d^6 - 105*a*b^2*d^6)*EllipticF[(-e + Pi/2 - f*
x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/Sqrt[c + d*Sin[e +
 f*x]] - ((8*b^3*c^6 + 18*a*b^2*c^5*d + 45*a^2*b*c^4*d^2 - 23*b^3*c^4*d^2 -
 176*a^3*c^3*d^3 - 372*a*b^2*c^3*d^3 + 918*a^2*b*c^2*d^4 + 294*b^3*c^2*d^4
- 208*a^3*c*d^5 - 798*a*b^2*c*d^5 + 189*a^2*b*d^6 + 105*b^3*d^6)*((2*(c + d
)*EllipticE[(-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(
c + d)])/Sqrt[c + d*Sin[e + f*x]] - (2*c*EllipticF[(-e + Pi/2 - f*x)/2, (2*
d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/Sqrt[c + d*Sin[e + f*x]]))/
d)/(105*(c - d)^4*d^2*(c + d)^4*f)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(3\,ab^{2}\cos\left(fx+e\right)^{2}-a^{3}-3\,ab^{2}+\left(b^{3}\cos\left(fx+e\right)^{2}-3\,a^{2}b-b^{3}\right)\sin}{5\,cd^{4}\cos\left(fx+e\right)^{4}+c^{5}+10\,c^{3}d^{2}+5\,cd^{4}-10\left(c^{3}d^{2}+cd^{4}\right)\cos\left(fx+e\right)^{2}+\left(d^{5}\cos\left(fx+e\right)^{4}+5\,c^{4}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 - 3
*a^2*b - b^3)*sin(f*x + e))*sqrt(d*sin(f*x + e) + c)/(5*c*d^4*cos(f*x + e)^
4 + c^5 + 10*c^3*d^2 + 5*c*d^4 - 10*(c^3*d^2 + c*d^4)*cos(f*x + e)^2 + (d^5
*cos(f*x + e)^4 + 5*c^4*d + 10*c^2*d^3 + d^5 - 2*(5*c^2*d^3 + d^5)*cos(f*x
+ e)^2)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{3}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(9/2), x)
```

maple [B] time = 14.08, size = 2111, normalized size = 2.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(9/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2
*d-b^3*c^3)/d^3*(2/7/(c^2-d^2)/d^3*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/
(sin(f*x+e)+c/d)^4+24/35/(c^2-d^2)^2/d^2*c*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2
)^(1/2)/(sin(f*x+e)+c/d)^3+2/105*(71*c^2+25*d^2)/d/(c^2-d^2)^3*(-(-d*sin(f*
x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+32/105*d*cos(f*x+e)^2/(c^2-d
^2)^4*c*(11*c^2+13*d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*(105*c^4+
254*c^2*d^2+25*d^4)/(105*c^8-420*c^6*d^2+630*c^4*d^4-420*c^2*d^6+105*d^8)*(
c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin
(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF
(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+32/105*c*d*(11*c^2+13*
d^2)/(c^2-d^2)^4*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(
c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^
2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(
1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+3*b^2
*(a*d-b*c)/d^3*(2/3/(c^2-d^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(si
n(f*x+e)+c/d)^2+8/3*d*cos(f*x+e)^2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*
x+e)^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*c^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e
))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1
/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-
d))^(1/2),((c-d)/(c+d))^(1/2))+8/3*c*d/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*
x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2))))+3*b*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^3*(2/5/(c^2-
d^2)/d^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^3+16/15*c
```

```
/(c^2-d^2)^2/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c/d)^2+2
/15*d*cos(f*x+e)^2/(c^2-d^2)^3*(23*c^2+9*d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e
)^2)^(1/2)+2*(15*c^3+17*c*d^2)/(15*c^6-45*c^4*d^2+45*c^2*d^4-15*d^6)*(c/d-1
)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+
e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+
d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/15*d*(23*c^2+9*d^2)/(c^2-
d^2)^3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2
)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(
(-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+Elli
pticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+b^3/d^3*(2*d*co
s(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*
(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-si
n(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Elliptic
F(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)
*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e
)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*Ellip
ticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*si
n(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^
(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^3}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(9/2),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)^3/(d*sin(f*x + e) + c)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\sin\left(e+f\,x\right)\right)^3}{\left(c+d\,\sin\left(e+f\,x\right)\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^3/(c + d*sin(e + f*x))^(9/2),x)
[Out] int((a + b*sin(e + f*x))^3/(c + d*sin(e + f*x))^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

3.745 $\int \frac{(c+d\sin(e+fx))^{5/2}}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=296

$$-\frac{2d\left(-3a^2d^2+6abcd-\left(b^2\left(2c^2+d^2\right)\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|\frac{2d}{c+d}\right)}{3b^3f\sqrt{c+d\sin(e+fx)}}+\frac{2(bc-ad)^3\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\right.}{b^3f(a+b)\sqrt{c+d\sin}}$$

```
[Out] -2/3*d^2*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/b/f-2/3*d*(-3*a*d+7*b*c)*(sin(1/
2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+
1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/b^2/f/((c+d
*sin(f*x+e))/(c+d))^(1/2)+2/3*d*(6*a*b*c*d-3*a^2*d^2-b^2*(2*c^2+d^2))*(sin(
1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*
e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/b
^3/f/(c+d*sin(f*x+e))^(1/2)-2*(-a*d+b*c)^3*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1
/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2*b/(a+b
),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/b^3/(a+b)/f/(c+d*
sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.09, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2793, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{2d\left(-3a^2d^2+6abcd+b^2\left(-\left(2c^2+d^2\right)\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|\frac{2d}{c+d}\right)}{3b^3f\sqrt{c+d\sin(e+fx)}}+\frac{2d(7bc-3ad)\sqrt{c+d\sin(e+f}}{3b^2f\sqrt{\frac{c+d\sin}{c}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (-2*d^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*b*f) + (2*d*(7*b*c - 3*a*
d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(
3*b^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (2*d*(6*a*b*c*d - 3*a^2*d^2 -
 b^2*(2*c^2 + d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c +
d*Sin[e + f*x])/(c + d)])/(3*b^3*f*Sqrt[c + d*Sin[e + f*x]]) + (2*(b*c - a*
d)^3*EllipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c +
 d*Sin[e + f*x])/(c + d)])/(b^3*(a + b)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{a+b\sin(e+fx)}\,dx = -\frac{2d^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3bf} + \frac{2\int \frac{\frac{1}{2}\left(3bc^3+ad^3\right)-\frac{1}{2}d\left(2acd-b\left(9c^2+d^2\right)\right)\sin(e+fx)+\frac{1}{2}d^2}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx}}}{3b}$$

$$= -\frac{2d^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3bf} - \frac{2\int \frac{\frac{1}{2}d\left(acd(7bc-3ad)-b\left(3bc^3+ad^3\right)\right)+\frac{1}{2}d^2\left(6abcd-3a^2d^2\right.}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+f}}}{3b^2d}$$

$$= -\frac{2d^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3bf} + \frac{(bc-ad)^3\int \frac{1}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx}{b^3} -$$

$$= -\frac{2d^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3bf} + \frac{2d(7bc-3ad)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+}}{3b^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

$$= -\frac{2d^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3bf} + \frac{2d(7bc-3ad)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+}}{3b^2f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Mathematica [C] time = 5.77, size = 606, normalized size = 2.05

$$\frac{2i(3ad-7bc)\sec(e+fx)\sqrt{-\frac{d(\sin(e+fx)-1)}{c+d}}\sqrt{\frac{d(\sin(e+fx)+1)}{d-c}}\left(d\left(d\left(b^2-2a^2\right)\Pi\left(\frac{b(c+d)}{bc-ad};i\sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right)|\frac{c+d}{c-d}\right)+2(a+b)(ad-bc)F\left(i\sinh^{-1}\left(\sqrt{-\right.\right.\right.\right.}{b^2\sqrt{-\frac{1}{c+d}}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (((4*I)*(-2*a*c*d + b*(9*c^2 + d^2))*((-(b*c) + a*d)*EllipticF[I*ArcSinh[Sq
rt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] - a*d*Ellipti
cPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e
 + f*x]]], (c + d)/(c - d)])*Sec[e + f*x]*Sqrt[-((d*(-1 + Sin[e + f*x]))/(c
 + d))]*Sqrt[(d*(1 + Sin[e + f*x]))/(-c + d)])/(b*Sqrt[-(c + d)^(-1)]*(b*c
- a*d)) + ((2*I)*(-7*b*c + 3*a*d)*(-2*b*(c - d)*(b*c - a*d)*EllipticE[I*Arc
Sinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(2
*(a + b)*(-(b*c) + a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*
Sin[e + f*x]]], (c + d)/(c - d)] + (-2*a^2 + b^2)*d*EllipticPi[(b*(c + d))/
(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c +
d)/(c - d)]))*Sec[e + f*x]*Sqrt[-((d*(-1 + Sin[e + f*x]))/(c + d))]*Sqrt[(d
```

```
*(1 + Sin[e + f*x]))/(-c + d)])/(b^2*Sqrt[-(c + d)^(-1)]*(b*c - a*d)) - 4*d
^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]] - (2*(6*b*c^3 + 7*b*c*d^2 - a*d^3)
*EllipticPi[(2*b)/(a + b), (-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c +
d*Sin[e + f*x])/(c + d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]))/(6*b*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{b \sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(b*sin(f*x + e) + a), x)
```

maple [B] time = 4.42, size = 1190, normalized size = 4.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(d/b^3*(b^2*d^2*(-2/3/d*(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*
(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e
)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c
+d))^(1/2))-4/3*c/d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e)
)/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+
e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d)
)^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+2*
(-a*b*d^2+3*b^2*c*d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e
))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x
+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d
))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+2*
```

```
a^2*d^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/
2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*
EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-6*a*b*c*d*(c/
d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f
*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF((
(c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+6*b^2*c^2*(c/d-1)*((c+d*
sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/
(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*
x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+2*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*
c^2*d+b^3*c^3)/b^4*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))
/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e
)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-c/d+1)/(-
c/d+a/b),((c-d)/(c+d))^(1/2)))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(b*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c+d\,\sin\left(e+f\,x\right)\right)^{5/2}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(5/2)/(a + b*sin(e + f*x)),x)
[Out] int((c + d*sin(e + f*x))^(5/2)/(a + b*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
[Out] Timed out
```

3.746 $\int \frac{(c+d\sin(e+fx))^{3/2}}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=229

$$\frac{2d(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{b^2f\sqrt{c+d\sin(e+fx)}}+\frac{2(bc-ad)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{b^2f(a+b)\sqrt{c+d\sin(e+fx)}}+2d\sqrt{c}$$

```
[Out] -2*d*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elliptic
E(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)
/b/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-2*d*(-a*d+b*c)*(sin(1/2*e+1/4*Pi+1/2*f*
x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2
^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/b^2/f/(c+d*sin(f*x+e
))^(1/2)-2*(-a*d+b*c)^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*P
i+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2*b/(a+b),2^(1/2)*(d/(c+d))
^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/b^2/(a+b)/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2804, 2655, 2653, 2803, 2663, 2661, 2807, 2805}

$$\frac{2d(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{b^2f\sqrt{c+d\sin(e+fx)}}+\frac{2(bc-ad)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{b^2f(a+b)\sqrt{c+d\sin(e+fx)}}+2d\sqrt{c}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (2*d*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])
/(b*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*d*(b*c - a*d)*EllipticF[(e -
 Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(b^2*f*S
qrt[c + d*Sin[e + f*x]]) + (2*(b*c - a*d)^2*EllipticPi[(2*b)/(a + b), (e -
Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(b^2*(a +
 b)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2804

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[b/d, Int[Sqrt[a + b*Sin[e + f*x]], x], x]
 - Dist[(b*c - a*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{a+b\sin(e+fx)}\,dx = \frac{d\int\sqrt{c+d\sin(e+fx)}\,dx}{b} - \frac{(-bc+ad)\int\frac{\sqrt{c+d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx}{b}$$

$$= \frac{(d(bc-ad))\int\frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{b^2} + \frac{(bc-ad)^2\int\frac{1}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx}{b^2} + \frac{\left(d\sqrt{\right.}}{}$$

$$= \frac{2dE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{bf\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} + \frac{\left(d(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)\int\frac{}{\sqrt{\frac{c}{c+d}}}}{b^2\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{2dE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{bf\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} + \frac{2d(bc-ad)F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right.}{b^2f\sqrt{c+d\sin(e+f}}$$

Mathematica [C] time = 3.96, size = 242, normalized size = 1.06

$$\frac{2i\sec(e+fx)\sqrt{-\frac{d(\sin(e+fx)-1)}{c+d}}\sqrt{-\frac{d(\sin(e+fx)+1)}{c-d}}\left((ad+b(d-2c))F\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right)|\frac{c+d}{c-d}\right)+\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((2*I)*(b*(c - d)*EllipticE[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e
+ f*x]]], (c + d)/(c - d)] + (a*d + b*(-2*c + d))*EllipticF[I*ArcSinh[Sqrt[
-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + (b*c - a*d)*El
lipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*
Sin[e + f*x]]], (c + d)/(c - d)])*Sec[e + f*x]*Sqrt[-((d*(-1 + Sin[e + f*x]
))/(c + d))]*Sqrt[-((d*(1 + Sin[e + f*x]))/(c - d))])/(b^2*Sqrt[-(c + d)^(-
1)]*f)
```

fricas [F] time = 24.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{b\sin\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) + a), x)
```

maple [A] time = 1.52, size = 391, normalized size = 1.71

$$-\frac{2\left(\text{EllipticE}\left(\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}},\sqrt{\frac{c-d}{c+d}}\right)bc+\text{EllipticE}\left(\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}},\sqrt{\frac{c-d}{c+d}}\right)bd+a\,\text{EllipticF}\left(\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}},\sqrt{\frac{c-}{c-}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] -2*(EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*c+Ellip
ticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*b*d+a*EllipticF(((
c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))*d-2*c*b*EllipticF(((c+d*s
in(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-EllipticF(((c+d*sin(f*x+e))/(c
-d))^(1/2),((c-d)/(c+d))^(1/2))*b*d-EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/
2),-(c-d)*b/(a*d-b*c),((c-d)/(c+d))^(1/2))*a*d+EllipticPi(((c+d*sin(f*x+e))
/(c-d))^(1/2),-(c-d)*b/(a*d-b*c),((c-d)/(c+d))^(1/2))*b*c)/b^2*(-d*(1+sin(f
*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*((c+d*sin(f*x+e))/(c-d)
)^(1/2)*(c-d)/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{b \sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c+d\,\sin\left(e+f\,x\right)\right)^{3/2}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(3/2)/(a + b*sin(e + f*x)),x)
[Out] int((c + d*sin(e + f*x))^(3/2)/(a + b*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
[Out] Timed out
```

3.747 $\int \frac{\sqrt{c+d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=153

$$\frac{2(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{bf(a+b)\sqrt{c+d\sin(e+fx)}}+\frac{2d\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{bf\sqrt{c+d\sin(e+fx)}}$$

```
[Out] -2*d*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elliptic
F(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d
))^(1/2)/b/f/(c+d*sin(f*x+e))^(1/2)-2*(-a*d+b*c)*(sin(1/2*e+1/4*Pi+1/2*f*x)
^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2*
b/(a+b),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/b/(a+b)/f/(
c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2803, 2663, 2661, 2807, 2805}

$$\frac{2(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{bf(a+b)\sqrt{c+d\sin(e+fx)}}+\frac{2d\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{bf\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x]),x]
```

```
[Out] (2*d*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])
/(c + d)])/(b*f*Sqrt[c + d*Sin[e + f*x]]) + (2*(b*c - a*d)*EllipticPi[(2*b)
/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c +
 d)])/(b*(a + b)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx = \frac{d\int \frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{b} + \frac{(bc-ad)\int \frac{1}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx}{b}$$

$$= \frac{\left(d\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)\int \frac{1}{\sqrt{\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}}}\,dx}{b\sqrt{c+d\sin(e+fx)}} + \frac{\left((bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\right)\int \frac{1}{(a+b\sin(e+fx))\sqrt{\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}}}\,dx}{b\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{2dF\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{bf\sqrt{c+d\sin(e+fx)}} + \frac{2(bc-ad)\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{b(a+b)f\sqrt{c+d\sin(e+fx)}}$$

Mathematica [A] time = 2.85, size = 114, normalized size = 0.75

$$-\frac{2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\left(d(a+b)F\left(\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)+(bc-ad)\Pi\left(\frac{2b}{a+b};\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)\right)}{bf(a+b)\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x]),x]
```

```
[Out] (-2*((a + b)*d*EllipticF[(-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)] + (b*c - a*d
)*EllipticPi[(2*b)/(a + b), (-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)])*Sqrt[(c
+ d*Sin[e + f*x])/(c + d)])/(b*(a + b)*f*Sqrt[c + d*Sin[e + f*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\sin\left(fx+e\right)+c}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a), x)
```

maple [A] time = 1.42, size = 181, normalized size = 1.18

$$\frac{2\left(\mathrm{EllipticF}\left(\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}},\sqrt{\frac{c-d}{c+d}}\right)-\mathrm{EllipticPi}\left(\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}},-\frac{(c-d)b}{da-cb},\sqrt{\frac{c-d}{c+d}}\right)\right)\sqrt{-\frac{d\left(1+\sin\left(fx+e\right)\right)}{c-d}}\,\sqrt{-\frac{\left(\sin\left(fx+e\right)-\right.}{c+d}}}{b\cos\left(fx+e\right)\sqrt{c+d\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] 2*(EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-EllipticPi
(((c+d*sin(f*x+e))/(c-d))^(1/2),-(c-d)*b/(a*d-b*c),((c-d)/(c+d))^(1/2)))/b*
(-d*(1+sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*((c+d*sin(f
*x+e))/(c-d))^(1/2)*(c-d)/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\sin\left(fx+e\right)+c}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c+d\,\sin\left(e+f\,x\right)}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(1/2)/(a + b*sin(e + f*x)),x)
[Out] int((c + d*sin(e + f*x))^(1/2)/(a + b*sin(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+d\sin\left(e+fx\right)}}{a+b\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
[Out] Integral(sqrt(c + d*sin(e + f*x))/(a + b*sin(e + f*x)), x)
```

3.748 $\int \frac{1}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=75

$$\frac{2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{f(a+b)\sqrt{c+d\sin(e+fx)}}$$

```
[Out] -2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi
(cos(1/2*e+1/4*Pi+1/2*f*x),2*b/(a+b),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x
+e))/(c+d))^(1/2)/(a+b)/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2807, 2805}

$$\frac{2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{f(a+b)\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (2*EllipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d
*Sin[e + f*x])/(c + d)])/((a + b)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx = \frac{\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\int \frac{1}{(a+b\sin(e+fx))\sqrt{\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}}}\,dx}{\sqrt{c+d\sin(e+fx)}}$$
$$= \frac{2\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}{(a+b)f\sqrt{c+d\sin(e+fx)}}$$

Mathematica [A] time = 0.11, size = 74, normalized size = 0.99

$$-\frac{2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{4}(-2e-2fx+\pi)|\frac{2d}{c+d}\right)}{f(a+b)\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (-2*EllipticPi[(2*b)/(a + b), (-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c
 + d*Sin[e + f*x])/(c + d)])/((a + b)*f*Sqrt[c + d*Sin[e + f*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)), x)
```

maple [A] time = 1.55, size = 151, normalized size = 2.01

$$\frac{2\left(c-d\right)\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}}\,\sqrt{-\frac{\left(\sin\left(fx+e\right)-1\right)d}{c+d}}\,\sqrt{-\frac{d\left(1+\sin\left(fx+e\right)\right)}{c-d}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}},-\frac{\left(c-d\right)b}{da-cb},\sqrt{\frac{c-d}{c+d}}\right)}{\left(da-cb\right)\cos\left(fx+e\right)\sqrt{c+d\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] 2*(c-d)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(-(sin(f*x+e)-1)*d/(c+d))^(1/2)*(-d*
(1+sin(f*x+e))/(c-d))^(1/2)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),-(c-d
)*b/(a*d-b*c),((c-d)/(c+d))^(1/2))/(a*d-b*c)/cos(f*x+e)/(c+d*sin(f*x+e))^(1
/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+b\,\sin\left(e+f\,x\right)\right)\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sin\left(e+fx\right)\right)\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/((a + b*sin(e + f*x))*sqrt(c + d*sin(e + f*x))), x)
```

3.749 $\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=220

$$-\frac{2d^2\cos(e+fx)}{f\left(c^2-d^2\right)(bc-ad)\sqrt{c+d\sin(e+fx)}}-\frac{2d\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{f\left(c^2-d^2\right)(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{2b\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+\cdots}\right.}{f(a+b)(bc-ad)\cdots}$$

```
[Out] -2*d^2*cos(f*x+e)/(-a*d+b*c)/(c^2-d^2)/f/(c+d*sin(f*x+e))^(1/2)+2*d*(sin(1/
2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+
1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/(-a*d+b*c)/
(c^2-d^2)/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-2*b*(sin(1/2*e+1/4*Pi+1/2*f*x)^2
)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2*b/
(a+b),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/(a+b)/(-a*d+b
*c)/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2802, 3059, 2655, 2653, 12, 2807, 2805}

$$-\frac{2d^2\cos(e+fx)}{f\left(c^2-d^2\right)(bc-ad)\sqrt{c+d\sin(e+fx)}}-\frac{2d\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{f\left(c^2-d^2\right)(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{2b\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+\cdots}\right.}{f(a+b)(bc-ad)\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (-2*d^2*Cos[e + f*x])/((b*c - a*d)*(c^2 - d^2)*f*Sqrt[c + d*Sin[e + f*x]])
- (2*d*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]
])/((b*c - a*d)*(c^2 - d^2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*b*El
lipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[
e + f*x])/(c + d)])/((a + b)*(b*c - a*d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))^{3/2}}\,dx = -\frac{2d^2\cos(e+fx)}{(bc-ad)\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{2\int \frac{\frac{1}{2}\left(-acd+b\left(c^2-d^2\right)\right)\ldots}{(a+b\sin\ldots}}{(b\ldots}$$

$$= -\frac{2d^2\cos(e+fx)}{(bc-ad)\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} - \frac{2\int -\frac{b^2d\left(c^2\ldots\right.}{2(a+b\sin(e+fx))\ldots}}{bd(bc-ad)\ldots}$$

$$= -\frac{2d^2\cos(e+fx)}{(bc-ad)\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{b\int \frac{1}{(a+b\sin(e+fx))\sqrt{c\ldots}}}{bc-a\ldots}$$

$$= -\frac{2d^2\cos(e+fx)}{(bc-ad)\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} - \frac{2dE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\ldots\right.\right.}{(bc-ad)\left(c^2\ldots\right.}$$

$$= -\frac{2d^2\cos(e+fx)}{(bc-ad)\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} - \frac{2dE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\ldots\right.\right.}{(bc-ad)\left(c^2\ldots\right.}$$

Mathematica [C] time = 6.89, size = 617, normalized size = 2.80

$$-\frac{\frac{4d^2\cos(e+fx)}{\left(c^2-d^2\right)\sqrt{c+d\sin(e+fx)}} + \frac{-\frac{2i\sec(e+fx)\sqrt{-\frac{d(\sin(e+fx)-1)}{c+d}}\sqrt{\frac{d(\sin(e+fx)+1)}{d-c}}\left(d\left(d\left(b^2-2a^2\right)\Pi\left(\frac{b(c+d)}{bc-ad};i\sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right)\middle|\frac{c+d}{c-d}\right)+2(a+b)(ad-bc)F\left(i\ldots\right.\right.\right.}{b\sqrt{-\frac{1}{c+d}}(bc-ad)}\ldots}{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -1/2*((4*d^2*Cos[e + f*x])/((c^2 - d^2)*Sqrt[c + d*Sin[e + f*x]]) + (((4*I)
*(b*c + a*d)*((-(b*c) + a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c
 + d*Sin[e + f*x]]], (c + d)/(c - d)] - a*d*EllipticPi[(b*(c + d))/(b*c - a
*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c -
d)])*Sec[e + f*x]*Sqrt[-((d*(-1 + Sin[e + f*x]))/(c + d))]*Sqrt[(d*(1 + Sin
```

```
[e + f*x]))/(-c + d)])/(b*Sqrt[-(c + d)^(-1)]*(b*c - a*d)) - ((2*I)*(-2*b*(
c - d)*(b*c - a*d)*EllipticE[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e
 + f*x]]], (c + d)/(c - d)] + d*(2*(a + b)*(-(b*c) + a*d)*EllipticF[I*ArcSi
nh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + (-2*a^
2 + b^2)*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)
]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)]))*Sec[e + f*x]*Sqrt[-((d*(-1
+ Sin[e + f*x]))/(c + d))]*Sqrt[(d*(1 + Sin[e + f*x]))/(-c + d)])/(b*Sqrt[-
(c + d)^(-1)]*(b*c - a*d)) + (2*(2*b*c^2 - 2*a*c*d - 3*b*d^2)*EllipticPi[(2
*b)/(a + b), (-2*e + Pi - 2*f*x)/4, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x]
)/(c + d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]))/((c - d)*(c + d)))/((b*c -
a*d)*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2)), x)
```

maple [B] time = 3.77, size = 610, normalized size = 2.77

$$\frac{\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^2\left(fx+e\right)\right)}\left(-\frac{2\left(\frac{c}{d}-1\right)\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}}\sqrt{\frac{d\left(1-\sin\left(fx+e\right)\right)}{c+d}}\sqrt{\frac{\left(-\sin\left(fx+e\right)-1\right)d}{c-d}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}},\frac{-\frac{c}{d}}{-\frac{c}{d}}\right.}{\left(da-cb\right)\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^2\left(fx+e\right)\right)}\left(-\frac{c}{d}+\frac{a}{b}\right)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(-2/(a*d-b*c)*(c/d-1)*((c+d*sin(f*x
+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^
(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*s
in(f*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))+d/(a*d-b*c
)*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(
c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1
/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)
*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*
d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-
sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d
-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF
(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))))/cos(f*x+e)/(c+d*sin
(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \sin\left(fx+e\right)+a\right)\left(d \sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+b\,\sin\left(e+f\,x\right)\right)\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.750 $\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=399

$$\frac{2b^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{f(a+b)(bc-ad)^2\sqrt{c+d\sin(e+fx)}}-\frac{2d^2\left(-4acd+7bc^2-3bd^2\right)\cos(e+fx)}{3f\left(c^2-d^2\right)^2(bc-ad)^2\sqrt{c+d\sin(e+fx)}}-\frac{2d^2\cos(}{3f\left(c^2-d^2\right)(bc-ad)}$$

```
[Out] -2/3*d^2*cos(f*x+e)/(-a*d+b*c)/(c^2-d^2)/f/(c+d*sin(f*x+e))^(3/2)-2/3*d^2*(
-4*a*c*d+7*b*c^2-3*b*d^2)*cos(f*x+e)/(-a*d+b*c)^2/(c^2-d^2)^2/f/(c+d*sin(f*
x+e))^(1/2)+2/3*d*(-4*a*c*d+7*b*c^2-3*b*d^2)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^
(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)
*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/(-a*d+b*c)^2/(c^2-d^2)^2/f/((c+d*s
in(f*x+e))/(c+d))^(1/2)-2/3*d*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e
+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2
))*((c+d*sin(f*x+e))/(c+d))^(1/2)/(-a*d+b*c)/(c^2-d^2)/f/(c+d*sin(f*x+e))^(
1/2)-2*b^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*El
lipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2*b/(a+b),2^(1/2)*(d/(c+d))^(1/2))*((c+d
*sin(f*x+e))/(c+d))^(1/2)/(a+b)/(-a*d+b*c)^2/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.65, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {2802, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{2b^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{f(a+b)(bc-ad)^2\sqrt{c+d\sin(e+fx)}}-\frac{2d^2\left(-4acd+7bc^2-3bd^2\right)\cos(e+fx)}{3f\left(c^2-d^2\right)^2(bc-ad)^2\sqrt{c+d\sin(e+fx)}}-\frac{2d^2\cos(}{3f\left(c^2-d^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (-2*d^2*Cos[e + f*x])/(3*(b*c - a*d)*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^(3/
2)) - (2*d^2*(7*b*c^2 - 4*a*c*d - 3*b*d^2)*Cos[e + f*x])/(3*(b*c - a*d)^2*(
c^2 - d^2)^2*f*Sqrt[c + d*Sin[e + f*x]]) - (2*d*(7*b*c^2 - 4*a*c*d - 3*b*d^
2)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(
3*(b*c - a*d)^2*(c^2 - d^2)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + (2*d*
EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c +
 d)])/(3*(b*c - a*d)*(c^2 - d^2)*f*Sqrt[c + d*Sin[e + f*x]]) + (2*b^2*Ellip
ticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e +
 f*x])/(c + d)])/((a + b)*(b*c - a*d)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{2d^2\cos(e+fx)}{3(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{2\int \frac{-\frac{3}{2}\left(acd-b\left(c^2-\right.\right.}{(a}}{}$$

$$= -\frac{2d^2\cos(e+fx)}{3(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{2d^2\left(7bc^2-4\right.}{3(bc-ad)^2\left(c^2-\right.}$$

$$= -\frac{2d^2\cos(e+fx)}{3(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{2d^2\left(7bc^2-4\right.}{3(bc-ad)^2\left(c^2-\right.}$$

$$= -\frac{2d^2\cos(e+fx)}{3(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{2d^2\left(7bc^2-4\right.}{3(bc-ad)^2\left(c^2-\right.}$$

$$= -\frac{2d^2\cos(e+fx)}{3(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{2d^2\left(7bc^2-4\right.}{3(bc-ad)^2\left(c^2-\right.}$$

$$= -\frac{2d^2\cos(e+fx)}{3(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{2d^2\left(7bc^2-4\right.}{3(bc-ad)^2\left(c^2-\right.}$$

Mathematica [C] time = 7.17, size = 1079, normalized size = 2.70

$$\frac{\sqrt{c+d\sin(e+fx)}\left(\frac{2\left(3b\cos(e+fx)d^4+4ac\cos(e+fx)d^3-7bc^2\cos(e+fx)d^2\right)}{3(bc-ad)^2\left(c^2-d^2\right)^2(c+d\sin(e+fx))} - \frac{2d^2\cos(e+fx)}{3(bc-ad)\left(c^2-d^2\right)(c+d\sin(e+fx))^2}\right)}{f} + \frac{-2\left(6b^2c^4-12abdc^3+6a\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*((-2*d^2*Cos[e + f*x])/(3*(b*c - a*d)*(c^2 - d^2)
*(c + d*Sin[e + f*x])^2) + (2*(-7*b*c^2*d^2*Cos[e + f*x] + 4*a*c*d^3*Cos[e
+ f*x] + 3*b*d^4*Cos[e + f*x]))/(3*(b*c - a*d)^2*(c^2 - d^2)^2*(c + d*Sin[e
 + f*x]))))/f + ((-2*(6*b^2*c^4 - 12*a*b*c^3*d + 6*a^2*c^2*d^2 - 19*b^2*c^2
*d^2 + 8*a*b*c*d^3 + 2*a^2*d^4 + 9*b^2*d^4)*EllipticPi[(2*b)/(a + b), (-e +
 Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a + b)
```

```
*Sqrt[c + d*Sin[e + f*x]]) - ((2*I)*(-12*b^2*c^3*d - 8*a*b*c^2*d^2 + 8*a^2*
c*d^3 + 4*b^2*c*d^3 + 8*a*b*d^4)*Cos[e + f*x]*((b*c - a*d)*EllipticF[I*ArcS
inh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + a*d*E
llipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d
*Sin[e + f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[
-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(
b*d^2*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e +
 f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x]
)^2)/d^2)]) - ((2*I)*(7*b^2*c^2*d^2 - 4*a*b*c*d^3 - 3*b^2*d^4)*Cos[e + f*x]
*Cos[2*(e + f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I*ArcSinh[Sqrt[-(c + d
)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(-2*(a + b)*(-(b*c)
 + a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]],
(c + d)/(c - d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*Ar
cSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)]))*Sqr
t[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*
c) + a*d + b*(c + d*Sin[e + f*x])))/(b^2*d*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*
(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d^2 + 4*c*(c + d*Si
n[e + f*x]) - 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[
e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)]))/(6*(c - d)^2*(c + d)^2*(b*c - a
*d)^2*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate(1/((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2)), x)
```

maple [B] time = 6.20, size = 1072, normalized size = 2.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(2*b/(a*d-b*c)^2*(c/d-1)*((c+d*sin(
f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d
))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+
d*sin(f*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))+d/(a*d-
b*c)*(2/3/(c^2-d^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*x+e)+c
/d)^2+8/3*d*cos(f*x+e)^2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1
/2)+2*(3*c^2+d^2)/(3*c^4-6*c^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^
(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*
sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))+8/3*c*d/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(
1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*s
in(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-
d)/(c+d))^(1/2))))-d*b/(a*d-b*c)^2*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*
x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^
(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*
sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(
d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x
+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/
2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+
d))^(1/2)))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+b\,\sin\left(e+f\,x\right)\right)\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.751 $\int \frac{(c+d\sin(e+fx))^{7/2}}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=534

$$\frac{d\left(-5a^2d^2+6abcd-\left(b^2\left(3c^2-2d^2\right)\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3b^2f\left(a^2-b^2\right)}+\frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx)}{bf\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

```
[Out] (-a*d+b*c)^2*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/b/(a^2-b^2)/f/(a+b*sin(f*x+e
))+1/3*d*(6*a*b*c*d-5*a^2*d^2-b^2*(3*c^2-2*d^2))*cos(f*x+e)*(c+d*sin(f*x+e)
)^(1/2)/b^2/(a^2-b^2)/f-1/3*(29*a^2*b*c*d^2-15*a^3*d^3+b^3*(3*c^3-20*c*d^2)
-a*b^2*(9*c^2*d-12*d^3))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*
Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c
+d*sin(f*x+e))^(1/2)/b^3/(a^2-b^2)/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+1/3*(24
*a^3*b*c*d^3-15*a^4*d^4-12*a*b^3*c*d*(c^2+3*d^2)+2*a^2*b^2*d^2*(c^2+8*d^2)+
b^4*(3*c^4+16*c^2*d^2+2*d^4))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e
+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2
))*((c+d*sin(f*x+e))/(c+d))^(1/2)/b^4/(a^2-b^2)/f/(c+d*sin(f*x+e))^(1/2)-(-
a*d+b*c)^3*(5*a^2*d+2*a*b*c-7*b^2*d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/si
n(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2*b/(a+b),2^(1
/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/(a-b)/b^4/(a+b)^2/f/(c+
d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 2.04, antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {2792, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{d\left(-5a^2d^2+6abcd+b^2\left(-\left(3c^2-2d^2\right)\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3b^2f\left(a^2-b^2\right)}-\frac{\left(2a^2b^2d^2\left(c^2+8d^2\right)+24a^3bcd^3-15a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(7/2)/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (d*(6*a*b*c*d - 5*a^2*d^2 - b^2*(3*c^2 - 2*d^2))*Cos[e + f*x]*Sqrt[c + d*Si
n[e + f*x]])/(3*b^2*(a^2 - b^2)*f) + ((b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin
[e + f*x])^(3/2))/(b*(a^2 - b^2)*f*(a + b*Sin[e + f*x])) + ((29*a^2*b*c*d^2
 - 15*a^3*d^3 + b^3*(3*c^3 - 20*c*d^2) - a*b^2*(9*c^2*d - 12*d^3))*Elliptic
E[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*b^3*(a^2
- b^2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - ((24*a^3*b*c*d^3 - 15*a^4*d^
4 - 12*a*b^3*c*d*(c^2 + 3*d^2) + 2*a^2*b^2*d^2*(c^2 + 8*d^2) + b^4*(3*c^4 +
```

```
 16*c^2*d^2 + 2*d^4))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c
+ d*Sin[e + f*x])/(c + d)])/(3*b^4*(a^2 - b^2)*f*Sqrt[c + d*Sin[e + f*x]])
+ ((b*c - a*d)^3*(2*a*b*c + 5*a^2*d - 7*b^2*d)*EllipticPi[(2*b)/(a + b), (e
 - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a -
b)*b^4*(a + b)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{7/2}}{(a+b\sin(e+fx))^2}\,dx = \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{b\left(a^2-b^2\right)f(a+b\sin(e+fx))} - \frac{\int \frac{\sqrt{c+d\sin(e+fx)}\left(\frac{1}{2}\left(7b^2c^2d+3a^2d^3-2abc\left(c^2+\right.\right.\right.}{}}{}$$

$$= \frac{d\left(6abcd-5a^2d^2-b^2\left(3c^2-2d^2\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3b^2\left(a^2-b^2\right)f} + \frac{(bc-ad)^2\cos}{b\left(a^2-\right.}$$

$$= \frac{d\left(6abcd-5a^2d^2-b^2\left(3c^2-2d^2\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3b^2\left(a^2-b^2\right)f} + \frac{(bc-ad)^2\cos}{b\left(a^2-\right.}$$

$$= \frac{d\left(6abcd-5a^2d^2-b^2\left(3c^2-2d^2\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3b^2\left(a^2-b^2\right)f} + \frac{(bc-ad)^2\cos}{b\left(a^2-\right.}$$

$$= \frac{d\left(6abcd-5a^2d^2-b^2\left(3c^2-2d^2\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3b^2\left(a^2-b^2\right)f} + \frac{(bc-ad)^2\cos}{b\left(a^2-\right.}$$

$$= \frac{d\left(6abcd-5a^2d^2-b^2\left(3c^2-2d^2\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3b^2\left(a^2-b^2\right)f} + \frac{(bc-ad)^2\cos}{b\left(a^2-\right.}$$

Mathematica [C] time = 8.12, size = 1109, normalized size = 2.08

$$\frac{\sqrt{c+d\sin(e+fx)}\left(\frac{-b^3\cos(e+fx)c^3+3ab^2d\cos(e+fx)c^2-3a^2bd^2\cos(e+fx)c+a^3d^3\cos(e+fx)}{b^2\left(b^2-a^2\right)(a+b\sin(e+fx))}-\frac{2d^3\cos(e+fx)}{3b^2}\right)}{f} - \frac{-\frac{2\left(-12ab^2c^4+39b^3dc^3-4\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(7/2)/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*((-2*d^3*Cos[e + f*x])/(3*b^2) + (-(b^3*c^3*Cos[e
 + f*x]) + 3*a*b^2*c^2*d*Cos[e + f*x] - 3*a^2*b*c*d^2*Cos[e + f*x] + a^3*d^
```

```
3*Cos[e + f*x])/(b^2*(-a^2 + b^2)*(a + b*Sin[e + f*x]))))/f - ((-2*(-12*a*b
^2*c^4 + 39*b^3*c^3*d - 45*a*b^2*c^2*d^2 + a^2*b*c*d^3 + 20*b^3*c*d^3 + 5*a
^3*d^4 - 8*a*b^2*d^4)*EllipticPi[(2*b)/(a + b), (-e + Pi/2 - f*x)/2, (2*d)/
(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a + b)*Sqrt[c + d*Sin[e + f*
x]]) - ((2*I)*(-12*a*b^2*c^3*d - 36*a^2*b*c^2*d^2 + 72*b^3*c^2*d^2 + 20*a^3
*c*d^3 - 56*a*b^2*c*d^3 + 8*a^2*b*d^4 + 4*b^3*d^4)*Cos[e + f*x]*((b*c - a*d
)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d
)/(c - d)] + a*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d
)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e + f*
x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d
*Sin[e + f*x])))/(b*d^2*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x]
)*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (
c + d*Sin[e + f*x])^2)/d^2)]) - ((2*I)*(3*b^3*c^3*d - 9*a*b^2*c^2*d^2 + 29*
a^2*b*c*d^3 - 20*b^3*c*d^3 - 15*a^3*d^4 + 12*a*b^2*d^4)*Cos[e + f*x]*Cos[2*
(e + f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I*ArcSinh[Sqrt[-(c + d)^(-1)]
*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(-2*(a + b)*(-(b*c) + a*d)
*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)
/(c - d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[S
qrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)]))*Sqrt[(d -
d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*
d + b*(c + d*Sin[e + f*x])))/(b^2*d*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*
Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d^2 + 4*c*(c + d*Sin[e + f
*x]) - 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x
]) + (c + d*Sin[e + f*x])^2)/d^2)]))/(12*(a - b)*b^2*(a + b)*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*sin(f*x + e) + c)^(7/2)/(b*sin(f*x + e) + a)^2, x)
```

maple [B] time = 7.20, size = 1886, normalized size = 3.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e))^2,x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(d^2/b^4*(b^2*d^2*(-2/3/d*(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(
d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x
+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2))-4/3*c/d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+
e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*
x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+
d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+
2*(-2*a*b*d^2+4*b^2*c*d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f
*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos
(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))
)+6*a^2*d^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))
^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1
/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-16*a*b*c*
d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-
sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Ellipt
icF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+12*b^2*c^2*(c/d-1)*
((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)
-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*
sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))-8/b^5*d*(a^3*d^3-3*a^2*b*c*d
^2+3*a*b^2*c^2*d-b^3*c^3)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(
f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*co
s(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-c/
d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))+1/b^4*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2
*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*(-b^2/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(-(-d*s
in(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-a*d/(a^3*d-a^2*b*c-a*b^2*
d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1
/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)
*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-b*d/(a^3*d-a
^2*b*c-a*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+
e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*
x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+
d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+(
3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)/b*(c/d-1)*((c+d*sin(f*
x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))
^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*
```

```
sin(f*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))))/cos(f*x
+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}{\left(b \sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^(7/2)/(b*sin(f*x + e) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c+d\,\sin\left(e+f\,x\right)\right)^{7/2}}{\left(a+b\,\sin\left(e+f\,x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(7/2)/(a + b*sin(e + f*x))^2,x)
[Out] int((c + d*sin(e + f*x))^(7/2)/(a + b*sin(e + f*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(7/2)/(a+b*sin(f*x+e))**2,x)
[Out] Timed out
```

3.752 $\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=390

$$-\frac{\left(-3a^2d^2+2abcd-\left(b^2\left(c^2-2d^2\right)\right)\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{b^2f\left(a^2-b^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}+\frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin}}{bf\left(a^2-b^2\right)(a+b\sin(e+}$$

```
[Out] (-a*d+b*c)^2*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/b/(a^2-b^2)/f/(a+b*sin(f*x+e
))+(2*a*b*c*d-3*a^2*d^2-b^2*(c^2-2*d^2))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2
)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/
(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/b^2/(a^2-b^2)/f/((c+d*sin(f*x+e))/(c+d
))^(1/2)-(-a*d+b*c)*(2*a*b*c*d+3*a^2*d^2-b^2*(c^2+4*d^2))*(sin(1/2*e+1/4*Pi
+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2
*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/b^3/(a^2-b^2)
/f/(c+d*sin(f*x+e))^(1/2)-(-a*d+b*c)^2*(3*a^2*d+2*a*b*c-5*b^2*d)*(sin(1/2*e
+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2*e+1/
4*Pi+1/2*f*x),2*b/(a+b),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(
1/2)/(a-b)/b^3/(a+b)^2/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.26, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2792, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(3a^2d^2+2abcd+b^2\left(-\left(c^2+4d^2\right)\right)\right)(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{b^3f\left(a^2-b^2\right)\sqrt{c+d\sin(e+fx)}}-\frac{\left(-3a^2d^2+2abcd+b^2\left(-\left(c^2\right.\right.\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((b*c - a*d)^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(b*(a^2 - b^2)*f*(a +
 b*Sin[e + f*x])) - ((2*a*b*c*d - 3*a^2*d^2 - b^2*(c^2 - 2*d^2))*EllipticE[
(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(b^2*(a^2 - b^
2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) + ((b*c - a*d)*(2*a*b*c*d + 3*a^2*
d^2 - b^2*(c^2 + 4*d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[
(c + d*Sin[e + f*x])/(c + d)])/(b^3*(a^2 - b^2)*f*Sqrt[c + d*Sin[e + f*x]])
 + ((b*c - a*d)^2*(2*a*b*c + 3*a^2*d - 5*b^2*d)*EllipticPi[(2*b)/(a + b), (
e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a -
 b)*b^3*(a + b)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+b\sin(e+fx))^2}\,dx = \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f(a+b\sin(e+fx))} - \frac{\int \frac{\frac{1}{2}\left(5b^2c^2d+a^2d^3-2abc\left(c^2+2d^2\right)\right)-d\left(a^2cd-3b^2c\right.}{(a+b}}{}$$

$$= \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{\int \frac{\frac{1}{2}d(bc-ad)\left(abc^2+3a^2cd-5b^2cd+abd^2\right)-\frac{1}{2}d(bc-}{(a+b\sin(e+fx))\sqrt{}}}{b^2\left(a^2-\right.}$$

$$= \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{\left((bc-ad)^2\left(2abc+3a^2d-5b^2d\right)\right)\int}{2b^3\left(a^2-b\right.}$$

$$= \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f(a+b\sin(e+fx))} - \frac{\left(2abcd-3a^2d^2-b^2\left(c^2-2d^2\right)\right)E\left(\frac{1}{2}\right.}{b^2\left(a^2-b^2\right)f}$$

$$= \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f(a+b\sin(e+fx))} - \frac{\left(2abcd-3a^2d^2-b^2\left(c^2-2d^2\right)\right)E\left(\frac{1}{2}\right.}{b^2\left(a^2-b^2\right)f}$$

Mathematica [C] time = 8.18, size = 986, normalized size = 2.53

$$\frac{\sqrt{c+d\sin(e+fx)}\left(-b^2\cos(e+fx)c^2+2abd\cos(e+fx)c-a^2d^2\cos(e+fx)\right)}{b\left(b^2-a^2\right)f(a+b\sin(e+fx))} + \frac{-\frac{2\left(4abc^3-9b^2dc^2+6abd^2c+a^2d^3-2b^2d^3\right.}{(a+b)\sqrt{c}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((-(b^2*c^2*Cos[e + f*x]) + 2*a*b*c*d*Cos[e + f*x] - a^2*d^2*Cos[e + f*x])*
Sqrt[c + d*Sin[e + f*x]])/(b*(-a^2 + b^2)*f*(a + b*Sin[e + f*x])) + ((-2*(4
*a*b*c^3 - 9*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3 - 2*b^2*d^3)*EllipticPi[(2*b
)/(a + b), (-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c
 + d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]) - ((2*I)*(4*a*b*c^2*d + 4*a^2*c*
d^2 - 12*b^2*c*d^2 + 4*a*b*d^3)*Cos[e + f*x]*((b*c - a*d)*EllipticF[I*ArcSi
nh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + a*d*El
lipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*
Sin[e + f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-
((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b
```

```
*d^2*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e +
f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])
^2)/d^2)]) - ((2*I)*(-(b^2*c^2*d) + 2*a*b*c*d^2 - 3*a^2*d^3 + 2*b^2*d^3)*Co
s[e + f*x]*Cos[2*(e + f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I*ArcSinh[Sq
rt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(-2*(a +
b)*(-(b*c) + a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e
+ f*x]]], (c + d)/(c - d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c + d))/(b*c -
a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c -
 d)]))*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c -
d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b^2*d*Sqrt[-(c + d)^(-1)]*(b
*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d^2 + 4*c
*(c + d*Sin[e + f*x]) - 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2 - 2*c*(
c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)]))/(4*(a - b)*b*(a + b)*
f)
```

fricas [F] time = 177.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2\right)\sqrt{d\sin\left(fx+e\right)+c}}{b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)*sqrt(d*sin(f
*x + e) + c)/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(b*sin(f*x + e) + a)^2, x)
```

maple [B] time = 6.70, size = 1363, normalized size = 3.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^2,x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(d^2/b^3*(2*b*d*(c/d-1)*((c+d*sin(f
*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d)
)^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*s
in(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c
-d))^(1/2),((c-d)/(c+d))^(1/2)))-4*d*a*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/
2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c
-d)/(c+d))^(1/2))+6*c*b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*
x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(
f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2
)))+6/b^4*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1
/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*si
n(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-
d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))+1/b^3*(-a^3*d^3+3*a^2*b*
c*d^2-3*a*b^2*c^2*d+b^3*c^3)*(-b^2/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(-(-d*sin(
f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-a*d/(a^3*d-a^2*b*c-a*b^2*d+b
^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)
*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*El
lipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-b*d/(a^3*d-a^2*
b*c-a*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))
/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e
)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))
^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+(3*a
^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)/b*(c/d-1)*((c+d*sin(f*x+e
))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1
/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin
(f*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))))/cos(f*x+e)
/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(b*sin(f*x + e) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c+d\sin\left(e+fx\right)\right)^{5/2}}{\left(a+b\sin\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(5/2)/(a + b*sin(e + f*x))^2,x)
[Out] int((c + d*sin(e + f*x))^(5/2)/(a + b*sin(e + f*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e))**2,x)
[Out] Timed out
```

3.753 $\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=351

$$\frac{\left(a^2d^2+2abcd-\left(b^2\left(c^2+2d^2\right)\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{b^2f\left(a^2-b^2\right)\sqrt{c+d\sin(e+fx)}}+\frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

```
[Out] (-a*d+b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(a^2-b^2)/f/(a+b*sin(f*x+e))-(
-a*d+b*c)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Ell
ipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^
(1/2)/b/(a^2-b^2)/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-(2*a*b*c*d+a^2*d^2-b^2*(
c^2+2*d^2))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*E
llipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e
))/(c+d))^(1/2)/b^2/(a^2-b^2)/f/(c+d*sin(f*x+e))^(1/2)-(-a*d+b*c)*(a^2*d+2*
a*b*c-3*b^2*d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x
)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2*b/(a+b),2^(1/2)*(d/(c+d))^(1/2))*(
(c+d*sin(f*x+e))/(c+d))^(1/2)/(a-b)/b^2/(a+b)^2/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.04, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2799, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(a^2d^2+2abcd+b^2\left(-\left(c^2+2d^2\right)\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{b^2f\left(a^2-b^2\right)\sqrt{c+d\sin(e+fx)}}+\frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+f}}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((b*c - a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/((a^2 - b^2)*f*(a + b*S
in[e + f*x])) + ((b*c - a*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*S
qrt[c + d*Sin[e + f*x]])/(b*(a^2 - b^2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)
]) + ((2*a*b*c*d + a^2*d^2 - b^2*(c^2 + 2*d^2))*EllipticF[(e - Pi/2 + f*x)/
2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(b^2*(a^2 - b^2)*f*Sq
rt[c + d*Sin[e + f*x]]) + ((b*c - a*d)*(2*a*b*c + a^2*d - 3*b^2*d)*Elliptic
Pi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*
x])/(c + d)])/((a - b)*b^2*(a + b)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^2}\,dx = \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{\int \frac{\frac{1}{2}\left(3bcd-a\left(2c^2+d^2\right)\right)-d(ac-bd)\sin(e+fx)-\frac{1}{2}d(bc-}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}}{-a^2+b^2}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{\int \frac{\frac{1}{2}d\left(a^2cd-3b^2cd+ab\left(c^2+d^2\right)\right)+\frac{1}{2}d\left(2abcd+a^2d^2-b^2\right.}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+f}}}{b\left(a^2-b^2\right)d}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{\left((bc-ad)\left(2abc+a^2d-3b^2d\right)\right)\int \frac{}{(a+b\sin}}{2b^2\left(a^2-b^2\right)}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{(bc-ad)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d}}{b\left(a^2-b^2\right)f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{(bc-ad)E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d}}{b\left(a^2-b^2\right)f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Mathematica [C] time = 7.16, size = 891, normalized size = 2.54

$$\frac{\sqrt{c+d\sin(e+fx)}\,(bc\cos(e+fx)-ad\cos(e+fx))}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{-\frac{2\left(4ac^2-5bdc+ad^2\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{(a+b)\sqrt{c+d\sin(e+fx)}} - \frac{2i\left(4acd-4b\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((b*c*Cos[e + f*x] - a*d*Cos[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/((a^2 - b^
2)*f*(a + b*Sin[e + f*x])) + ((-2*(4*a*c^2 - 5*b*c*d + a*d^2)*EllipticPi[(2
*b)/(a + b), (-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/
(c + d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]) - ((2*I)*(4*a*c*d - 4*b*d^2)*C
os[e + f*x]*((b*c - a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d
*Sin[e + f*x]]], (c + d)/(c - d)] + a*d*EllipticPi[(b*(c + d))/(b*c - a*d),
 I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)])
*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(
-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b*d^2*Sqrt[-(c + d)^(-1)]*(b*c - a
*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(
```

```
c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)]) - ((2*I)*(-(b*c*d) + a
*d^2)*Cos[e + f*x]*Cos[2*(e + f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I*Ar
cSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(
-2*(a + b)*(-(b*c) + a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c +
d*Sin[e + f*x]]], (c + d)/(c - d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c + d))
/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c +
 d)/(c - d)]))*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x
])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b^2*d*Sqrt[-(c + d)^
(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d
^2 + 4*c*(c + d*Sin[e + f*x]) - 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2
 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)]))/(4*(a - b)*(a
 + b)*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) + a)^2, x)
```

maple [B] time = 5.11, size = 1027, normalized size = 2.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^2,x)
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(2*d^2/b^2*(c/d-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))-4/b^3*d*(a*d-b*c)*(c/d-1)*((c+d*sin(f*x+e))/(
```

```
c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x
+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))+1/b^2*(a^2*d^2-2
*a*b*c*d+b^2*c^2)*(-b^2/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*c
os(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-a*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(c/d-
1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x
+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c
+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-b*d/(a^3*d-a^2*b*c-a*b^2*d
+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/
2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*
((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+Ell
ipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+(3*a^2*d-2*a*b*
c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)/b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(
1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*s
in(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c
-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))))/cos(f*x+e)/(c+d*sin(f
*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(3/2)/(a + b*sin(e + f*x))^2,x)
```

```
[Out] int((c + d*sin(e + f*x))^(3/2)/(a + b*sin(e + f*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e))**2,x)

[Out] Timed out
```

3.754 $\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=307

$$\frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}-\frac{(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{bf\left(a^2-b^2\right)\sqrt{c+d\sin(e+fx)}}+\frac{\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx\right.\right.}{f\left(a^2-b^2\right)\sqrt{\frac{c+d\sin(e+}{c+d}}}$$

```
[Out] b*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(a^2-b^2)/f/(a+b*sin(f*x+e))-(sin(1/2*e
+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4
*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/(a^2-b^2)/f/((
c+d*sin(f*x+e))/(c+d))^(1/2)+(-a*d+b*c)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)
/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(
c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/b/(a^2-b^2)/f/(c+d*sin(f*x+e))^
(1/2)-(-a^2*d+2*a*b*c-b^2*d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+
1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2*b/(a+b),2^(1/2)*(d/(
c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/(a-b)/b/(a+b)^2/f/(c+d*sin(f*x+
e))^(1/2)
```

Rubi [A] time = 0.87, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2796, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\left(a^2-b^2\right)(a+b\sin(e+fx))}-\frac{(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{bf\left(a^2-b^2\right)\sqrt{c+d\sin(e+fx)}}+\frac{\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx\right.\right.}{f\left(a^2-b^2\right)\sqrt{\frac{c+d\sin(e+}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (b*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/((a^2 - b^2)*f*(a + b*Sin[e + f*x
])) + (EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]
])/((a^2 - b^2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - ((b*c - a*d)*Ellipt
icF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/
(b*(a^2 - b^2)*f*Sqrt[c + d*Sin[e + f*x]]) + ((2*a*b*c - a^2*d - b^2*d)*Ell
ipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e
 + f*x])/(c + d)])/((a - b)*b*(a + b)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
```

```
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^2}\,dx = \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{\int \frac{\frac{1}{2}(-2ac+bd)-ad\sin(e+fx)-\frac{1}{2}bd\sin^2(e+fx)}{(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}\,dx}{-a^2+b^2}$$

$$= \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{\int \sqrt{c+d\sin(e+fx)}\,dx}{2\left(a^2-b^2\right)} + \frac{\int \frac{\frac{1}{2}bd(ac-bd)-\frac{1}{2}bd(bc-a}{(a+b\sin(e+fx))\sqrt{c+}}}{b\left(a^2-b^2\right.}$$

$$= \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} - \frac{(bc-ad)\int \frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{2b\left(a^2-b^2\right)} + \frac{\left(2abc-a^2d-b^2\right.}{}$$

$$= \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{\left(a^2-b^2\right)f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} - \cdots$$

$$= \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\right)\sqrt{c+d\sin(e+fx)}}{\left(a^2-b^2\right)f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}} - \cdots$$

Mathematica [C] time = 6.95, size = 846, normalized size = 2.76

$$-\frac{2(4ac-bd)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{(a+b)\sqrt{c+d\sin(e+fx)}} - \frac{8ia\cos(e+fx)\left((bc-ad)F\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right)|\frac{c+d}{c-d}\right)+ad\Pi\left(\frac{b(c+d)}{bc-ad};i\sinh^{-1}\left(\sqrt{\cdots}\right.\right.\right.}{bd\sqrt{-\frac{1}{c+d}}(bc-ad)(a+b\sin(e+fx))\sqrt{1-\sin^2(e+\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -((b*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/((-a^2 + b^2)*f*(a + b*Sin[e +
f*x]))) + ((-2*(4*a*c - b*d)*EllipticPi[(2*b)/(a + b), (-e + Pi/2 - f*x)/2,
 (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a + b)*Sqrt[c + d*Sin
[e + f*x]]) - ((8*I)*a*Cos[e + f*x]*((b*c - a*d)*EllipticF[I*ArcSinh[Sqrt[-
(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + a*d*EllipticPi[
(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f
*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*S
in[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b*d*Sqrt[-
(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*Sqr
t[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)])
```

```
+ ((2*I)*Cos[e + f*x]*Cos[2*(e + f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I
*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] +
d*(-2*(a + b)*(-(b*c) + a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c
 + d*Sin[e + f*x]]], (c + d)/(c - d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c +
d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (
c + d)/(c - d)]))*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e +
f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b*Sqrt[-(c + d)^(
-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d^
2 + 4*c*(c + d*Sin[e + f*x]) - 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2
- 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)]))/(4*(a - b)*(a
+ b)*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\sin\left(fx+e\right)+c}}{\left(b\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^2,x, algorithm="giac")
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a)^2, x)
```

maple [B] time = 4.82, size = 872, normalized size = 2.84

$$\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^{2}\left(fx+e\right)\right)}\left(\frac{2d\left(\frac{c}{d}-1\right)\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}}\,\sqrt{\frac{d\left(1-\sin\left(fx+e\right)\right)}{c+d}}\,\sqrt{\frac{\left(-\sin\left(fx+e\right)-1\right)d}{c-d}}\;\mathrm{EllipticPi}\left(\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}},\frac{-\frac{c}{d}+}{-\frac{c}{d}+}\right.}{b^{2}\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^{2}\left(fx+e\right)\right)}\,\left(-\frac{c}{d}+\frac{a}{b}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^2,x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(2*d/b^2*(c/d-1)*((c+d*sin(f*x+e))/
(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)
/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*
x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))+(-a*d+b*c)/b*(-
b^2/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(
a+b*sin(f*x+e))-a*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))
/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2
)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d)
)^(1/2),((c-d)/(c+d))^(1/2))-b*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(c/d-1)*((c+
d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*
d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(
((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x
+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*
b*c-a*b^2*d+b^3*c)/b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e
))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x
+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-c/d+1)/
(-c/d+a/b),((c-d)/(c+d))^(1/2))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d \sin\left(fx+e\right)+c}}{\left(b \sin\left(fx+e\right)+a\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c+d\,\sin\left(e+f\,x\right)}}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(1/2)/(a + b*sin(e + f*x))^2,x)
```

```
[Out] int((c + d*sin(e + f*x))^(1/2)/(a + b*sin(e + f*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.755 $\int \frac{1}{(a+b\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=325

$$\frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\left(a^2-b^2\right)(bc-ad)(a+b\sin(e+fx))}-\frac{\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{f\left(a^2-b^2\right)\sqrt{c+d\sin(e+fx)}}+\frac{b\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+f\right.\right.}{f\left(a^2-b^2\right)(bc-ad)\sqrt{\frac{c+d\sin}{c}}}$$

```
[Out] b^2*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(a^2-b^2)/(-a*d+b*c)/f/(a+b*sin(f*x+e
))-b*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elliptic
E(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)
/(a^2-b^2)/(-a*d+b*c)/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+(sin(1/2*e+1/4*Pi+1/
2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*
x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/(a^2-b^2)/f/(c+d
*sin(f*x+e))^(1/2)-(-3*a^2*d+2*a*b*c+b^2*d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(
1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2*b/(a+
b),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/(a-b)/(a+b)^2/(-
a*d+b*c)/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.97, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2802, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\left(a^2-b^2\right)(bc-ad)(a+b\sin(e+fx))}-\frac{\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{f\left(a^2-b^2\right)\sqrt{c+d\sin(e+fx)}}+\frac{b\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+f\right.\right.}{f\left(a^2-b^2\right)(bc-ad)\sqrt{\frac{c+d\sin}{c}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (b^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/((a^2 - b^2)*(b*c - a*d)*f*(a +
 b*Sin[e + f*x])) + (b*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c
+ d*Sin[e + f*x]])/((a^2 - b^2)*(b*c - a*d)*f*Sqrt[(c + d*Sin[e + f*x])/(c
+ d)]) - (EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e +
f*x])/(c + d)])/((a^2 - b^2)*f*Sqrt[c + d*Sin[e + f*x]]) + ((2*a*b*c - 3*a^
2*d + b^2*d)*EllipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*S
qrt[(c + d*Sin[e + f*x])/(c + d)])/((a - b)*(a + b)^2*(b*c - a*d)*f*Sqrt[c
+ d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}\,dx = \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))} - \frac{\int \frac{\frac{1}{2}\left(-2abc+2a^2d-b^2d\right)-abd\,\mathrm{si}\cdots}{(a+b\sin(e+fx))\cdots}}{\left(a^2-b^2\right)\cdots}$$

$$= \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))} + \frac{b\int \sqrt{c+d\sin(e+fx)}\cdots}{2\left(a^2-b^2\right)(bc-ad)\cdots}$$

$$= \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))} - \frac{\int \frac{1}{\sqrt{c+d\sin(e+fx)}}\,dx}{2\left(a^2-b^2\right)} + \frac{(2\cdots}{\cdots}$$

$$= \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))} + \frac{bE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\cdots\right.}{\left(a^2-b^2\right)(bc-ad\cdots}$$

$$= \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))} + \frac{bE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)|\frac{2d}{c+d}\cdots\right.}{\left(a^2-b^2\right)(bc-ad\cdots}$$

Mathematica [C] time = 7.47, size = 871, normalized size = 2.68

$$-\frac{\frac{2\left(4da^2-4bca-3b^2d\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{(a+b)\sqrt{c+d\sin(e+fx)}} + \frac{8ia\cos(e+fx)\left((bc-ad)F\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{c+d}}\sqrt{c+d\sin(e+fx)}\right)|\frac{c+d}{c-d}\right)+ad\Pi\left(\frac{b(c+d)}{bc-ad};i\sin\cdots\right.\right.}{d\sqrt{-\frac{1}{c+d}}(bc-ad)(a+b\sin(e+fx))\sqrt{1-\mathrm{s}\cdots}}}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] -((b^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/((a^2 - b^2)*(-(b*c) + a*d)*f
*(a + b*Sin[e + f*x]))) + ((-2*(-4*a*b*c + 4*a^2*d - 3*b^2*d)*EllipticPi[(2
*b)/(a + b), (-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/
(c + d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]) + ((8*I)*a*Cos[e + f*x]*((b*c
- a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (
c + d)/(c - d)] + a*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(
c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e
 + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(
c + d*Sin[e + f*x])))/(d*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x
])*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) +
```

```
(c + d*Sin[e + f*x])^2)/d^2)]) - ((2*I)*Cos[e + f*x]*Cos[2*(e + f*x)]*(2*b*
(c - d)*(b*c - a*d)*EllipticE[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[
e + f*x]]], (c + d)/(c - d)] + d*(-2*(a + b)*(-(b*c) + a*d)*EllipticF[I*Arc
Sinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + (2*a
^2 - b^2)*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1
)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)]))*Sqrt[(d - d*Sin[e + f*x])/
(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin
[e + f*x])))/(Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 -
 Sin[e + f*x]^2]*(-2*c^2 + d^2 + 4*c*(c + d*Sin[e + f*x]) - 2*(c + d*Sin[e
+ f*x])^2)*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*
x])^2)/d^2)]))/(4*(a - b)*(a + b)*(-(b*c) + a*d)*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{2}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^2*sqrt(d*sin(f*x + e) + c)), x)
```

maple [A] time = 3.82, size = 690, normalized size = 2.12

$$\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^{2}\left(fx+e\right)\right)}\left(-\frac{b^{2}\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^{2}\left(fx+e\right)\right)}}{\left(a^{3}d-a^{2}bc-a\,b^{2}d+b^{3}c\right)\left(a+b\sin\left(fx+e\right)\right)}-\frac{ad\left(\frac{c}{d}-1\right)\sqrt{\frac{c+d\sin\left(fx+e\right)}{c-d}}\,\sqrt{\frac{d\left(1-\sin\left(fx+e\right)\right)}{c+d}}\,\sqrt{(}}{\left(a^{3}d-a^{2}bc-a\,b^{2}d+b^{3}c\right)\sqrt{-(}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(-b^2/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)
*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-a*d/(a^3*d-a^2*b*
c-a*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(
c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^
2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-b*d/
(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-
sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c
)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((
c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(
1/2)))+(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)/b*(c/d-1)*((c+
d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*
d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticP
i(((c+d*sin(f*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2)))/
cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{2}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^2*sqrt(d*sin(f*x + e) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^2\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.756 $\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=449

$$\frac{d\left(2a^2d^2+b^2\left(c^2-3d^2\right)\right)\cos(e+fx)}{f\left(a^2-b^2\right)\left(c^2-d^2\right)(bc-ad)^2\sqrt{c+d\sin(e+fx)}}+\frac{\left(2a^2d^2+b^2\left(c^2-3d^2\right)\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right.\right.}{f\left(a^2-b^2\right)\left(c^2-d^2\right)(bc-ad)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] d*(2*a^2*d^2+b^2*(c^2-3*d^2))*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)^2/(c^2-d^2)/f
/(c+d*sin(f*x+e))^(1/2)+b^2*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(a+b*sin(f*x+
e))/(c+d*sin(f*x+e))^(1/2)-(2*a^2*d^2+b^2*(c^2-3*d^2))*(sin(1/2*e+1/4*Pi+1/
2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*
x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/(a^2-b^2)/(-a*d+b*c)^2/(
c^2-d^2)/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+b*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(
1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*
(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/(a^2-b^2)/(-a*d+b*c)/f/(c+d
*sin(f*x+e))^(1/2)-b*(-5*a^2*d+2*a*b*c+3*b^2*d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^
2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2*b
/(a+b),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/(a-b)/(a+b)^
2/(-a*d+b*c)^2/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.63, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {2802, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{d\left(2a^2d^2+b^2\left(c^2-3d^2\right)\right)\cos(e+fx)}{f\left(a^2-b^2\right)\left(c^2-d^2\right)(bc-ad)^2\sqrt{c+d\sin(e+fx)}}+\frac{\left(2a^2d^2+b^2\left(c^2-3d^2\right)\right)\sqrt{c+d\sin(e+fx)}\,E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right.\right.}{f\left(a^2-b^2\right)\left(c^2-d^2\right)(bc-ad)^2\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (d*(2*a^2*d^2 + b^2*(c^2 - 3*d^2))*Cos[e + f*x])/((a^2 - b^2)*(b*c - a*d)^2
*(c^2 - d^2)*f*Sqrt[c + d*Sin[e + f*x]]) + (b^2*Cos[e + f*x])/((a^2 - b^2)*
(b*c - a*d)*f*(a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]) + ((2*a^2*d^2
+ b^2*(c^2 - 3*d^2))*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c +
d*Sin[e + f*x]])/((a^2 - b^2)*(b*c - a*d)^2*(c^2 - d^2)*f*Sqrt[(c + d*Sin[e
 + f*x])/(c + d)]) - (b*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(
c + d*Sin[e + f*x])/(c + d)])/((a^2 - b^2)*(b*c - a*d)*f*Sqrt[c + d*Sin[e +
 f*x]]) + (b*(2*a*b*c - 5*a^2*d + 3*b^2*d)*EllipticPi[(2*b)/(a + b), (e - P
i/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a - b)*(
a + b)^2*(b*c - a*d)^2*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))^{3/2}}\,dx = \frac{b^2\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}} - \frac{\int}{}$$

$$= \frac{d\left(2a^2d^2+b^2\left(c^2-3d^2\right)\right)\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{}{\left(a^2-b^2\right)(}$$

$$= \frac{d\left(2a^2d^2+b^2\left(c^2-3d^2\right)\right)\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{}{\left(a^2-b^2\right)(}$$

$$= \frac{d\left(2a^2d^2+b^2\left(c^2-3d^2\right)\right)\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{}{\left(a^2-b^2\right)(}$$

$$= \frac{d\left(2a^2d^2+b^2\left(c^2-3d^2\right)\right)\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{}{\left(a^2-b^2\right)(}$$

$$= \frac{d\left(2a^2d^2+b^2\left(c^2-3d^2\right)\right)\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{}{\left(a^2-b^2\right)(}$$

Mathematica [C] time = 8.07, size = 1057, normalized size = 2.35

$$\frac{\sqrt{c+d\sin(e+fx)}\left(\frac{\cos(e+fx)b^3}{\left(a^2-b^2\right)(ad-bc)^2(a+b\sin(e+fx))}+\frac{2d^3\cos(e+fx)}{(bc-ad)^2\left(c^2-d^2\right)(c+d\sin(e+fx))}\right)}{f} + \frac{-\frac{2\left(4cd^2a^3+10bd^3a^2-8bc^2da^2+4b^2c^3a-8b^2cd^2a-\right.}{(a+b)}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*((b^3*Cos[e + f*x])/((a^2 - b^2)*(-(b*c) + a*d)^2
*(a + b*Sin[e + f*x])) + (2*d^3*Cos[e + f*x])/((b*c - a*d)^2*(c^2 - d^2)*(c
 + d*Sin[e + f*x]))))/f + ((-2*(4*a*b^2*c^3 - 8*a^2*b*c^2*d + 7*b^3*c^2*d +
 4*a^3*c*d^2 - 8*a*b^2*c*d^2 + 10*a^2*b*d^3 - 9*b^3*d^3)*EllipticPi[(2*b)/(
a + b), (-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c +
```

```
d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]) - ((2*I)*(4*a*b^2*c^2*d + 4*a^2*b*c
*d^2 - 4*b^3*c*d^2 + 4*a^3*d^3 - 8*a*b^2*d^3)*Cos[e + f*x]*((b*c - a*d)*Ell
ipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c
- d)] + a*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1
)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e + f*x])/(
c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[
e + f*x])))/(b*d^2*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqr
t[1 - Sin[e + f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d
*Sin[e + f*x])^2)/d^2)]) - ((2*I)*(-(b^3*c^2*d) - 2*a^2*b*d^3 + 3*b^3*d^3)*
Cos[e + f*x]*Cos[2*(e + f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I*ArcSinh[
Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(-2*(a
+ b)*(-(b*c) + a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[
e + f*x]]], (c + d)/(c - d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c + d))/(b*c
- a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c
 - d)]))*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c
- d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b^2*d*Sqrt[-(c + d)^(-1)]*
(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d^2 + 4
*c*(c + d*Sin[e + f*x]) - 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2 - 2*c
*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)]))/(4*(a - b)*(a + b)*
(c - d)*(c + d)*(-(b*c) + a*d)^2*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{2}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(3/2)), x)
```

maple [B] time = 6.92, size = 1266, normalized size = 2.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(-2*d/(a*d-b*c)^2*(c/d-1)*((c+d*sin
(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-
d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c
+d*sin(f*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))+d^2/(a
*d-b*c)^2*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/
2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(
c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^
2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c
^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(
1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2
)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+E
llipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))-b/(a*d-b*c)*
(-b^2/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)
/(a+b*sin(f*x+e))-a*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e
))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1
/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-
d))^(1/2),((c-d)/(c+d))^(1/2))-b*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(c/d-1)*((
c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1
)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*Elliptic
E(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f
*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^
2*b*c-a*b^2*d+b^3*c)/b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x
+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f
*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-c/d+1
)/(-c/d+a/b),((c-d)/(c+d))^(1/2))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^2\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^(3/2)),x)
[Out] int(1/((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**(3/2),x)
[Out] Timed out
```

3.757 $\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=661

$$\frac{d\left(2a^2d^2+b^2\left(3c^2-5d^2\right)\right)\cos(e+fx)}{3f\left(a^2-b^2\right)\left(c^2-d^2\right)(bc-ad)^2(c+d\sin(e+fx))^{3/2}}-\frac{\left(2a^2d^2+b^2\left(3c^2-5d^2\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2}{c}\right.}{3f\left(a^2-b^2\right)\left(c^2-d^2\right)(bc-ad)^2\sqrt{c+d\sin(e+fx)}}$$

```
[Out] 1/3*d*(2*a^2*d^2+b^2*(3*c^2-5*d^2))*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)^2/(c^2-
d^2)/f/(c+d*sin(f*x+e))^(3/2)+b^2*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(a+b*si
n(f*x+e))/(c+d*sin(f*x+e))^(3/2)-1/3*(8*a^3*c*d^4-8*a*b^2*c*d^4-4*a^2*b*d^3
*(5*c^2-3*d^2)-b^3*(3*c^4*d-26*c^2*d^3+15*d^5))*cos(f*x+e)/(a^2-b^2)/(-a*d+
b*c)^3/(c^2-d^2)^2/f/(c+d*sin(f*x+e))^(1/2)+1/3*(8*a^3*c*d^3-8*a*b^2*c*d^3-
4*a^2*b*d^2*(5*c^2-3*d^2)-b^3*(3*c^4-26*c^2*d^2+15*d^4))*(sin(1/2*e+1/4*Pi+
1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*
f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/(a^2-b^2)/(-a*d+b*c)^3
/(c^2-d^2)^2/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+1/3*(2*a^2*d^2+b^2*(3*c^2-5*d
^2))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elliptic
F(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d
))^(1/2)/(a^2-b^2)/(-a*d+b*c)^2/(c^2-d^2)/f/(c+d*sin(f*x+e))^(1/2)-b^2*(-7*
a^2*d+2*a*b*c+5*b^2*d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi
+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2*b/(a+b),2^(1/2)*(d/(c+d))^
(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/(a-b)/(a+b)^2/(-a*d+b*c)^3/f/(c+d*sin
(f*x+e))^(1/2)
```

Rubi [A] time = 2.80, antiderivative size = 661, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {2802, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{\left(-4a^2bd^3\left(5c^2-3d^2\right)+8a^3cd^4-8ab^2cd^4-b^3\left(-26c^2d^3+3c^4d+15d^5\right)\right)\cos(e+fx)}{3f\left(a^2-b^2\right)\left(c^2-d^2\right)^2(bc-ad)^3\sqrt{c+d\sin(e+fx)}}+\frac{d\left(2a^2d^2+b^2\left(3c^2\right.\right.}{3f\left(a^2-b^2\right)\left(c^2-d^2\right)(bc}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (d*(2*a^2*d^2 + b^2*(3*c^2 - 5*d^2))*Cos[e + f*x])/(3*(a^2 - b^2)*(b*c - a*
d)^2*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^(3/2)) + (b^2*Cos[e + f*x])/((a^2 -
 b^2)*(b*c - a*d)*f*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2)) - ((8*
a^3*c*d^4 - 8*a*b^2*c*d^4 - 4*a^2*b*d^3*(5*c^2 - 3*d^2) - b^3*(3*c^4*d - 26
*c^2*d^3 + 15*d^5))*Cos[e + f*x])/(3*(a^2 - b^2)*(b*c - a*d)^3*(c^2 - d^2)^
```

```
2*f*Sqrt[c + d*Sin[e + f*x]]) - ((8*a^3*c*d^3 - 8*a*b^2*c*d^3 - 4*a^2*b*d^2
*(5*c^2 - 3*d^2) - b^3*(3*c^4 - 26*c^2*d^2 + 15*d^4))*EllipticE[(e - Pi/2 +
 f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(3*(a^2 - b^2)*(b*c - a*d
)^3*(c^2 - d^2)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - ((2*a^2*d^2 + b^2
*(3*c^2 - 5*d^2))*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*
Sin[e + f*x])/(c + d)])/(3*(a^2 - b^2)*(b*c - a*d)^2*(c^2 - d^2)*f*Sqrt[c +
 d*Sin[e + f*x]]) + (b^2*(2*a*b*c - 7*a^2*d + 5*b^2*d)*EllipticPi[(2*b)/(a
+ b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)]
)/((a - b)*(a + b)^2*(b*c - a*d)^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
```

```
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^2(c+d\sin(e+fx))^{5/2}}\,dx = \frac{b^2\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))(c+d\sin(e+fx))^{3/2}} -$$

$$= \frac{d\left(2a^2d^2+b^2\left(3c^2-5d^2\right)\right)\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{}{\left(a^2-\right.}$$

$$= \frac{d\left(2a^2d^2+b^2\left(3c^2-5d^2\right)\right)\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{}{\left(a^2-\right.}$$

$$= \frac{d\left(2a^2d^2+b^2\left(3c^2-5d^2\right)\right)\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{}{\left(a^2-\right.}$$

$$= \frac{d\left(2a^2d^2+b^2\left(3c^2-5d^2\right)\right)\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{}{\left(a^2-\right.}$$

$$= \frac{d\left(2a^2d^2+b^2\left(3c^2-5d^2\right)\right)\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{}{\left(a^2-\right.}$$

$$= \frac{d\left(2a^2d^2+b^2\left(3c^2-5d^2\right)\right)\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{}{\left(a^2-\right.}$$

Mathematica [C] time = 9.02, size = 1319, normalized size = 2.00

$$\frac{\sqrt{c+d\sin(e+fx)}\left(-\frac{\cos(e+fx)b^4}{(a^2-b^2)(ad-bc)^3(a+b\sin(e+fx))}-\frac{4\left(3b\cos(e+fx)d^5+2ac\cos(e+fx)d^4-5bc^2\cos(e+fx)d^3\right)}{3(bc-ad)^3\left(c^2-d^2\right)^2(c+d\sin(e+fx))}+\frac{2d^3\cos(e+fx)}{3(bc-ad)^2\left(c^2-d^2\right)(c+d\sin\right.}}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*(-((b^4*Cos[e + f*x])/((a^2 - b^2)*(-(b*c) + a*d)
^3*(a + b*Sin[e + f*x]))) + (2*d^3*Cos[e + f*x])/(3*(b*c - a*d)^2*(c^2 - d^
2)*(c + d*Sin[e + f*x])^2) - (4*(-5*b*c^2*d^3*Cos[e + f*x] + 2*a*c*d^4*Cos[
e + f*x] + 3*b*d^5*Cos[e + f*x]))/(3*(b*c - a*d)^3*(c^2 - d^2)^2*(c + d*Sin
[e + f*x]))))/f + ((-2*(-12*a*b^3*c^5 + 36*a^2*b^2*c^4*d - 33*b^4*c^4*d - 3
6*a^3*b*c^3*d^2 + 60*a*b^3*c^3*d^2 + 12*a^4*c^2*d^3 - 104*a^2*b^2*c^2*d^3 +
 86*b^4*c^2*d^3 + 28*a^3*b*c*d^4 - 40*a*b^3*c*d^4 + 4*a^4*d^5 + 44*a^2*b^2*
d^5 - 45*b^4*d^5)*EllipticPi[(2*b)/(a + b), (-e + Pi/2 - f*x)/2, (2*d)/(c +
 d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]])
 - ((2*I)*(-12*a*b^3*c^4*d - 36*a^2*b^2*c^3*d^2 + 36*b^4*c^3*d^2 - 28*a^3*b
*c^2*d^3 + 52*a*b^3*c^2*d^3 + 16*a^4*c*d^4 + 4*a^2*b^2*c*d^4 - 20*b^4*c*d^4
 + 28*a^3*b*d^5 - 40*a*b^3*d^5)*Cos[e + f*x]*((b*c - a*d)*EllipticF[I*ArcSi
nh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + a*d*El
lipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*
Sin[e + f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-
((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b
*d^2*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e +
f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])
^2)/d^2)]) - ((2*I)*(3*b^4*c^4*d + 20*a^2*b^2*c^2*d^3 - 26*b^4*c^2*d^3 - 8*
a^3*b*c*d^4 + 8*a*b^3*c*d^4 - 12*a^2*b^2*d^5 + 15*b^4*d^5)*Cos[e + f*x]*Cos
[2*(e + f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I*ArcSinh[Sqrt[-(c + d)^(-
1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(-2*(a + b)*(-(b*c) + a
*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c +
 d)/(c - d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSin
h[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)]))*Sqrt[(d
 - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) +
 a*d + b*(c + d*Sin[e + f*x]))))/(b^2*d*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a +
 b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d^2 + 4*c*(c + d*Sin[e
+ f*x]) - 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e +
f*x]) + (c + d*Sin[e + f*x])^2)/d^2)])))/(12*(a - b)*(a + b)*(c - d)^2*(c +
d)^2*(-(b*c) + a*d)^3*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{2}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(5/2)), x)
```

maple [B] time = 11.97, size = 1731, normalized size = 2.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(4*b/(a*d-b*c)^3*d*(c/d-1)*((c+d*si
n(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c
-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((
c+d*sin(f*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))+d^2/(
a*d-b*c)^2*(2/3/(c^2-d^2)/d*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(sin(f*
x+e)+c/d)^2+8/3*d*cos(f*x+e)^2/(c^2-d^2)^2*c/(-(-d*sin(f*x+e)-c)*cos(f*x+e)
^2)^(1/2)+2*(3*c^2+d^2)/(3*c^4-6*c^2*d^2+3*d^4)*(c/d-1)*((c+d*sin(f*x+e))/(
c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2))+8/3*c*d/(c^2-d^2)^2*(c/d-1)*((c+d*sin(f*x+e))/(c
-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(
-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e)
)/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2
),((c-d)/(c+d))^(1/2))))-2*d^2/(a*d-b*c)^3*b*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-
(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e
))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1
/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-
d))^(1/2),((c-d)/(c+d))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d
))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/
```

```
(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2))))+b^2/(a*d-b*c)^2*(-b^2/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-a*d/(a^3*d-a^2*b*c
-a*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c
+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2
)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-b*d/(
a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-s
in(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)
*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c
-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1
/2)))+(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)/b*(c/d-1)*((c+d
*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d
/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi
(((c+d*sin(f*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))))/
cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^2/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+b\,\sin\left(e+f\,x\right)\right)^2\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**2/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.758 $\int \frac{(c+d\sin(e+fx))^{9/2}}{(a+b\sin(e+fx))^3}\,dx$

Optimal. Leaf size=816

$$\frac{\left(35d^2a^4+20bcda^3+2b^2\left(4c^2-43d^2\right)a^2-44b^3cda+b^4\left(4c^2+63d^2\right)\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+f}{c+d}}}{4(a-b)^2b^5(a+b)^3f\sqrt{c+d\sin(e+fx)}}$$

```
[Out] 1/4*(-a*d+b*c)^2*(7*a^2*d+6*a*b*c-13*b^2*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(3/
2)/b^2/(a^2-b^2)^2/f/(a+b*sin(f*x+e))+1/2*(-a*d+b*c)^2*cos(f*x+e)*(c+d*sin(
f*x+e))^(5/2)/b/(a^2-b^2)/f/(a+b*sin(f*x+e))^2+1/12*d*(36*a^3*b*c*d^2-35*a^
4*d^3+b^4*d*(45*c^2-8*d^2)-18*a*b^3*c*(c^2+5*d^2)+a^2*b^2*d*(9*c^2+61*d^2))
*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/b^3/(a^2-b^2)^2/f-1/12*(185*a^4*b*c*d^3-
105*a^5*d^4-b^5*c*d*(51*c^2-104*d^2)-15*a^3*b^2*d^2*(3*c^2-13*d^2)-a^2*b^3*
c*d*(21*c^2+361*d^2)+9*a*b^4*(2*c^4+17*c^2*d^2-8*d^4))*(sin(1/2*e+1/4*Pi+1/
2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*
x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/b^4/(a^2-b^2)^2/f/((c+d*
sin(f*x+e))/(c+d))^(1/2)+1/12*(150*a^5*b*c*d^4-105*a^6*d^5-12*a^3*b^3*c*d^2
*(4*c^2+29*d^2)+a^4*b^2*d^3*(26*c^2+223*d^2)-b^6*d*(57*c^4+136*c^2*d^2+8*d^
4)+6*a*b^5*c*(3*c^4+38*c^2*d^2+48*d^4)-a^2*b^4*d*(33*c^4+70*c^2*d^2+128*d^4
))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(
cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))
^(1/2)/b^5/(a^2-b^2)^2/f/(c+d*sin(f*x+e))^(1/2)-1/4*(-a*d+b*c)^3*(20*a^3*b*
c*d-44*a*b^3*c*d+35*a^4*d^2+2*a^2*b^2*(4*c^2-43*d^2)+b^4*(4*c^2+63*d^2))*(s
in(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(
1/2*e+1/4*Pi+1/2*f*x),2*b/(a+b),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/
(c+d))^(1/2)/(a-b)^2/b^5/(a+b)^3/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 3.18, antiderivative size = 816, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {2792, 3047, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(35d^2a^4+20bcda^3+2b^2\left(4c^2-43d^2\right)a^2-44b^3cda+b^4\left(4c^2+63d^2\right)\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)\sqrt{\frac{c+d\sin(e+f}{c+d}}}{4(a-b)^2b^5(a+b)^3f\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(9/2)/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (d*(36*a^3*b*c*d^2 - 35*a^4*d^3 + b^4*d*(45*c^2 - 8*d^2) - 18*a*b^3*c*(c^2
+ 5*d^2) + a^2*b^2*d*(9*c^2 + 61*d^2))*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]
```

```
])/(12*b^3*(a^2 - b^2)^2*f) + ((b*c - a*d)^2*(6*a*b*c + 7*a^2*d - 13*b^2*d)
*Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(4*b^2*(a^2 - b^2)^2*f*(a + b*Sin
[e + f*x])) + ((b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(5/2))/(2*b*
(a^2 - b^2)*f*(a + b*Sin[e + f*x])^2) + ((185*a^4*b*c*d^3 - 105*a^5*d^4 - b
^5*c*d*(51*c^2 - 104*d^2) - 15*a^3*b^2*d^2*(3*c^2 - 13*d^2) - a^2*b^3*c*d*(
21*c^2 + 361*d^2) + 9*a*b^4*(2*c^4 + 17*c^2*d^2 - 8*d^4))*EllipticE[(e - Pi
/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(12*b^4*(a^2 - b^2)^2
*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - ((150*a^5*b*c*d^4 - 105*a^6*d^5 -
12*a^3*b^3*c*d^2*(4*c^2 + 29*d^2) + a^4*b^2*d^3*(26*c^2 + 223*d^2) - b^6*d*
(57*c^4 + 136*c^2*d^2 + 8*d^4) + 6*a*b^5*c*(3*c^4 + 38*c^2*d^2 + 48*d^4) -
a^2*b^4*d*(33*c^4 + 70*c^2*d^2 + 128*d^4))*EllipticF[(e - Pi/2 + f*x)/2, (2
*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(12*b^5*(a^2 - b^2)^2*f*Sq
rt[c + d*Sin[e + f*x]]) + ((b*c - a*d)^3*(20*a^3*b*c*d - 44*a*b^3*c*d + 35*
a^4*d^2 + 2*a^2*b^2*(4*c^2 - 43*d^2) + b^4*(4*c^2 + 63*d^2))*EllipticPi[(2*
b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c
 + d)])/(4*(a - b)^2*b^5*(a + b)^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
```

```
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
```

```
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{9/2}}{(a+b\sin(e+fx))^3}\,dx = \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))^{5/2}}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} - \frac{\int \frac{(c+d\sin(e+fx))^{3/2}\left(\frac{1}{2}\left(5d(bc-ad)^2+4bc\left(2bcd\right.\right.\right.}{}}{}$$

$$= \frac{(bc-ad)^2\left(6abc+7a^2d-13b^2d\right)\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{4b^2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{(bc-ad)^2\,\mathrm{co}}{2b\left(a^2-\right.}$$

$$= \frac{d\left(36a^3bcd^2-35a^4d^3+b^4d\left(45c^2-8d^2\right)-18ab^3c\left(c^2+5d^2\right)+a^2b^2d\left(9c^2+61d^2\right.\right.}{12b^3\left(a^2-b^2\right)^2 f}$$

$$= \frac{d\left(36a^3bcd^2-35a^4d^3+b^4d\left(45c^2-8d^2\right)-18ab^3c\left(c^2+5d^2\right)+a^2b^2d\left(9c^2+61d^2\right.\right.}{12b^3\left(a^2-b^2\right)^2 f}$$

$$= \frac{d\left(36a^3bcd^2-35a^4d^3+b^4d\left(45c^2-8d^2\right)-18ab^3c\left(c^2+5d^2\right)+a^2b^2d\left(9c^2+61d^2\right.\right.}{12b^3\left(a^2-b^2\right)^2 f}$$

$$= \frac{d\left(36a^3bcd^2-35a^4d^3+b^4d\left(45c^2-8d^2\right)-18ab^3c\left(c^2+5d^2\right)+a^2b^2d\left(9c^2+61d^2\right.\right.}{12b^3\left(a^2-b^2\right)^2 f}$$

$$= \frac{d\left(36a^3bcd^2-35a^4d^3+b^4d\left(45c^2-8d^2\right)-18ab^3c\left(c^2+5d^2\right)+a^2b^2d\left(9c^2+61d^2\right.\right.}{12b^3\left(a^2-b^2\right)^2 f}$$

Mathematica [C] time = 8.78, size = 1526, normalized size = 1.87

$$\frac{\sqrt{c+d\sin(e+fx)}\left(-\frac{2\cos(e+fx)d^4}{3b^3}+\frac{-11d^4\cos(e+fx)a^5+27bcd^3\cos(e+fx)a^4+17b^2d^4\cos(e+fx)a^3-15b^2c^2d^2\cos(e+fx)a^3-51b^3cd^3\cos(\right.}{4b^3\left(b^2-a^2\right)^2(a+b\,\mathrm{si}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(9/2)/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*((-2*d^4*Cos[e + f*x])/(3*b^3) + (-(b^4*c^4*Cos[e
 + f*x]) + 4*a*b^3*c^3*d*Cos[e + f*x] - 6*a^2*b^2*c^2*d^2*Cos[e + f*x] + 4*
```

```
a^3*b*c*d^3*Cos[e + f*x] - a^4*d^4*Cos[e + f*x])/(2*b^3*(-a^2 + b^2)*(a + b
*Sin[e + f*x])^2) + (6*a*b^4*c^4*Cos[e + f*x] - 7*a^2*b^3*c^3*d*Cos[e + f*x
] - 17*b^5*c^3*d*Cos[e + f*x] - 15*a^3*b^2*c^2*d^2*Cos[e + f*x] + 51*a*b^4*
c^2*d^2*Cos[e + f*x] + 27*a^4*b*c*d^3*Cos[e + f*x] - 51*a^2*b^3*c*d^3*Cos[e
 + f*x] - 11*a^5*d^4*Cos[e + f*x] + 17*a^3*b^2*d^4*Cos[e + f*x])/(4*b^3*(-a
^2 + b^2)^2*(a + b*Sin[e + f*x]))))/f - ((-2*(-48*a^2*b^3*c^5 - 24*b^5*c^5
+ 306*a*b^4*c^4*d - 177*a^2*b^3*c^3*d^2 - 327*b^5*c^3*d^2 - 105*a^3*b^2*c^2
*d^3 + 501*a*b^4*c^2*d^3 + 13*a^4*b*c*d^4 - 53*a^2*b^3*c*d^4 - 104*b^5*c*d^
4 + 35*a^5*d^5 - 73*a^3*b^2*d^5 + 56*a*b^4*d^5)*EllipticPi[(2*b)/(a + b), (
-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a
+ b)*Sqrt[c + d*Sin[e + f*x]]) - ((2*I)*(-60*a^2*b^3*c^4*d - 12*b^5*c^4*d +
 36*a^3*b^2*c^3*d^2 + 252*a*b^4*c^3*d^2 - 228*a^4*b*c^2*d^3 + 276*a^2*b^3*c
^2*d^3 - 480*b^5*c^2*d^3 + 140*a^5*c*d^4 - 364*a^3*b^2*c*d^4 + 512*a*b^4*c*
d^4 + 56*a^4*b*d^5 - 112*a^2*b^3*d^5 - 16*b^5*d^5)*Cos[e + f*x]*((b*c - a*d
)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d
)/(c - d)] + a*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d
)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e + f*
x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d
*Sin[e + f*x])))/(b*d^2*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x]
)*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (
c + d*Sin[e + f*x])^2)/d^2)]) - ((2*I)*(18*a*b^4*c^4*d - 21*a^2*b^3*c^3*d^2
 - 51*b^5*c^3*d^2 - 45*a^3*b^2*c^2*d^3 + 153*a*b^4*c^2*d^3 + 185*a^4*b*c*d^
4 - 361*a^2*b^3*c*d^4 + 104*b^5*c*d^4 - 105*a^5*d^5 + 195*a^3*b^2*d^5 - 72*
a*b^4*d^5)*Cos[e + f*x]*Cos[2*(e + f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE
[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)]
+ d*(-2*(a + b)*(-(b*c) + a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt
[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c
+ d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]],
 (c + d)/(c - d)]))*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e
+ f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b^2*d*Sqrt[-(c
+ d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^
2 + d^2 + 4*c*(c + d*Sin[e + f*x]) - 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2
- d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)]))/(48*(a -
 b)^2*b^3*(a + b)^2*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e))^3,x, algorithm="giac")
[Out] integrate((d*sin(f*x + e) + c)^(9/2)/(b*sin(f*x + e) + a)^3, x)
```

maple [B] time = 13.69, size = 2775, normalized size = 3.40

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e))^3,x)
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(d^3/b^5*(b^2*d^2*(-2/3/d*(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)+2/3*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(
d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x
+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2))-4/3*c/d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+
e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*
x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+
d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+
2*(-3*a*b*d^2+5*b^2*c*d)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f
*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos
(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))
)+12*a^2*d^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d)
)^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(
1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-30*a*b*c
*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((
-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Ellip
ticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+20*b^2*c^2*(c/d-1)
*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e
)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d
*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))-20/b^6*d^2*(a^3*d^3-3*a^2*b
*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-
sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c
)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),
(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))+1/b^5*(-a^5*d^5+5*a^4*b*c*d^4-10*a
```

```
^3*b^2*c^2*d^3+10*a^2*b^3*c^3*d^2-5*a*b^4*c^4*d+b^5*c^5)*(-1/2*b^2/(a^3*d-a
^2*b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+
e))^2-3/4*b^2*(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-1/4*d*(7*a^3*d-4*a^2*b*
c-a*b^2*d-2*b^3*c)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(c/d-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))-3/4*b*d*(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*
c-a*b^2*d+b^3*c)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))
/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e
)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))
^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+1/4*
(15*a^4*d^2-20*a^3*b*c*d+8*a^2*b^2*c^2-6*a^2*b^2*d^2-4*a*b^3*c*d+4*b^4*c^2+
3*b^4*d^2)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2/b*(c/d-1)*((c+d*sin(f*x+e))/(c-d
))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e)
)/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2)))+5/b^5*d*(a^4*d^4-4
*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*(-b^2/(a^3*d-a^2*b*c-
a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-a*d
/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1
-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-
c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d
))^(1/2))-b*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d)
)^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-
d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(
c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),(
(c-d)/(c+d))^(1/2)))+(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)/
b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-
sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+
a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(
c+d))^(1/2))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c+d\,\sin\left(e+f\,x\right)\right)^{9/2}}{\left(a+b\,\sin\left(e+f\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(9/2)/(a + b*sin(e + f*x))^3,x)
```

```
[Out] int((c + d*sin(e + f*x))^(9/2)/(a + b*sin(e + f*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(9/2)/(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.759 $\int \frac{(c+d\sin(e+fx))^{7/2}}{(a+b\sin(e+fx))^3}\,dx$

Optimal. Leaf size=605

$$\frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2bf\left(a^2-b^2\right)(a+b\sin(e+fx))^2}+\frac{\left(5a^2d+6abc-11b^2d\right)(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4b^2f\left(a^2-b^2\right)^2(a+b\sin(e+fx))}+\frac{(1$$

```
[Out] 1/2*(-a*d+b*c)^2*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)/b/(a^2-b^2)/f/(a+b*sin(f
*x+e))^2+1/4*(-a*d+b*c)^2*(5*a^2*d+6*a*b*c-11*b^2*d)*cos(f*x+e)*(c+d*sin(f*
x+e))^(1/2)/b^2/(a^2-b^2)^2/f/(a+b*sin(f*x+e))+1/4*(8*a^3*b*c*d^2-15*a^4*d^
3+b^4*d*(13*c^2-8*d^2)-2*a*b^3*c*(3*c^2+13*d^2)+a^2*b^2*d*(5*c^2+29*d^2))*(
sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(
1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/b^3/(
a^2-b^2)^2/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-3/4*(-a*d+b*c)*(4*a^3*b*c*d^2+5
*a^4*d^3+a^2*b^2*d*(c^2-11*d^2)-2*a*b^3*c*(c^2+5*d^2)+b^4*d*(5*c^2+8*d^2))*
(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos
(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1
/2)/b^4/(a^2-b^2)^2/f/(c+d*sin(f*x+e))^(1/2)-1/4*(-a*d+b*c)^2*(12*a^3*b*c*d
-36*a*b^3*c*d+15*a^4*d^2+2*a^2*b^2*(4*c^2-19*d^2)+b^4*(4*c^2+35*d^2))*(sin(
1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2
*e+1/4*Pi+1/2*f*x),2*b/(a+b),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+
d))^(1/2)/(a-b)^2/b^4/(a+b)^3/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 2.20, antiderivative size = 605, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {2792, 3047, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{3\left(a^2b^2d\left(c^2-11d^2\right)+4a^3bcd^2+5a^4d^3-2ab^3c\left(c^2+5d^2\right)+b^4d\left(5c^2+8d^2\right)\right)(bc-ad)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}F\left(\frac{1}{2}\left(e+fx\right.\right.}{4b^4f\left(a^2-b^2\right)^2\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(7/2)/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((b*c - a*d)^2*(6*a*b*c + 5*a^2*d - 11*b^2*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e
 + f*x]])/(4*b^2*(a^2 - b^2)^2*f*(a + b*Sin[e + f*x])) + ((b*c - a*d)^2*Cos
[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(2*b*(a^2 - b^2)*f*(a + b*Sin[e + f*x
])^2) - ((8*a^3*b*c*d^2 - 15*a^4*d^3 + b^4*d*(13*c^2 - 8*d^2) - 2*a*b^3*c*(
3*c^2 + 13*d^2) + a^2*b^2*d*(5*c^2 + 29*d^2))*EllipticE[(e - Pi/2 + f*x)/2,
 (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(4*b^3*(a^2 - b^2)^2*f*Sqrt[(c +
```

```
d*Sin[e + f*x])/(c + d)]) + (3*(b*c - a*d)*(4*a^3*b*c*d^2 + 5*a^4*d^3 + a^2
*b^2*d*(c^2 - 11*d^2) - 2*a*b^3*c*(c^2 + 5*d^2) + b^4*d*(5*c^2 + 8*d^2))*El
lipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d
)])/(4*b^4*(a^2 - b^2)^2*f*Sqrt[c + d*Sin[e + f*x]]) + ((b*c - a*d)^2*(12*a
^3*b*c*d - 36*a*b^3*c*d + 15*a^4*d^2 + 2*a^2*b^2*(4*c^2 - 19*d^2) + b^4*(4*
c^2 + 35*d^2))*EllipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]
*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*(a - b)^2*b^4*(a + b)^3*f*Sqrt[c +
d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
```

```
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
```

```
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{7/2}}{(a+b\sin(e+fx))^3}\,dx = \frac{(bc-ad)^2\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} - \frac{\int \frac{\sqrt{c+d\sin(e+fx)}\left(\frac{1}{2}\left(3d(bc-ad)^2+4bc\left(2bcd-\right.\right.\right.}{}}{}$$

$$= \frac{(bc-ad)^2\left(6abc+5a^2d-11b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4b^2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{(bc-ad)^2\cos}{2b\left(a^2-\right.}$$

$$= \frac{(bc-ad)^2\left(6abc+5a^2d-11b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4b^2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{(bc-ad)^2\cos}{2b\left(a^2-\right.}$$

$$= \frac{(bc-ad)^2\left(6abc+5a^2d-11b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4b^2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{(bc-ad)^2\cos}{2b\left(a^2-\right.}$$

$$= \frac{(bc-ad)^2\left(6abc+5a^2d-11b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4b^2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{(bc-ad)^2\cos}{2b\left(a^2-\right.}$$

$$= \frac{(bc-ad)^2\left(6abc+5a^2d-11b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4b^2\left(a^2-b^2\right)^2 f(a+b\sin(e+fx))} + \frac{(bc-ad)^2\cos}{2b\left(a^2-\right.}$$

Mathematica [C] time = 8.35, size = 1323, normalized size = 2.19

$$\frac{\sqrt{c+d\sin(e+fx)}\left(\frac{-b^3\cos(e+fx)c^3+3ab^2d\cos(e+fx)c^2-3a^2bd^2\cos(e+fx)c+a^3d^3\cos(e+fx)}{2b^2\left(b^2-a^2\right)(a+b\sin(e+fx))^2} + \frac{7d^3\cos(e+fx)a^4-8bcd^2\cos(e+fx)a^3-13b^2}{}\right.}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(7/2)/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*((-(b^3*c^3*Cos[e + f*x]) + 3*a*b^2*c^2*d*Cos[e +
 f*x] - 3*a^2*b*c*d^2*Cos[e + f*x] + a^3*d^3*Cos[e + f*x])/(2*b^2*(-a^2 + b
^2)*(a + b*Sin[e + f*x])^2) + (6*a*b^3*c^3*Cos[e + f*x] - 5*a^2*b^2*c^2*d*C
os[e + f*x] - 13*b^4*c^2*d*Cos[e + f*x] - 8*a^3*b*c*d^2*Cos[e + f*x] + 26*a
*b^3*c*d^2*Cos[e + f*x] + 7*a^4*d^3*Cos[e + f*x] - 13*a^2*b^2*d^3*Cos[e + f
*x])/(4*b^2*(-a^2 + b^2)^2*(a + b*Sin[e + f*x]))))/f + ((-2*(16*a^2*b^2*c^4
 + 8*b^4*c^4 - 78*a*b^3*c^3*d + 33*a^2*b^2*c^2*d^2 + 57*b^4*c^2*d^2 + 8*a^3
*b*c*d^3 - 50*a*b^3*c*d^3 + 5*a^4*d^4 - 7*a^2*b^2*d^4 + 8*b^4*d^4)*Elliptic
Pi[(2*b)/(a + b), (-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f
*x])/(c + d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]) - ((2*I)*(20*a^2*b^2*c^3*
d + 4*b^4*c^3*d - 8*a^3*b*c^2*d^2 - 64*a*b^3*c^2*d^2 + 20*a^4*c*d^3 - 12*a^
2*b^2*c*d^3 + 64*b^4*c*d^3 + 8*a^3*b*d^4 - 32*a*b^3*d^4)*Cos[e + f*x]*((b*c
 - a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]],
(c + d)/(c - d)] + a*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-
(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[
e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*
(c + d*Sin[e + f*x])))/(b*d^2*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e
+ f*x])*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x
]) + (c + d*Sin[e + f*x])^2)/d^2)]) - ((2*I)*(-6*a*b^3*c^3*d + 5*a^2*b^2*c^
2*d^2 + 13*b^4*c^2*d^2 + 8*a^3*b*c*d^3 - 26*a*b^3*c*d^3 - 15*a^4*d^4 + 29*a
^2*b^2*d^4 - 8*b^4*d^4)*Cos[e + f*x]*Cos[2*(e + f*x)]*(2*b*(c - d)*(b*c - a
*d)*EllipticE[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c +
 d)/(c - d)] + d*(-2*(a + b)*(-(b*c) + a*d)*EllipticF[I*ArcSinh[Sqrt[-(c +
d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + (2*a^2 - b^2)*d*Elli
pticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Si
n[e + f*x]]], (c + d)/(c - d)]))*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-(
(d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b^
2*d*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f
*x]^2]*(-2*c^2 + d^2 + 4*c*(c + d*Sin[e + f*x]) - 2*(c + d*Sin[e + f*x])^2)
*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2
)]))/(16*(a - b)^2*b^2*(a + b)^2*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin \left(f x + e\right) + c\right)^{\frac{7}{2}}}{\left(b \sin \left(f x + e\right) + a\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(7/2)/(b*sin(f*x + e) + a)^3, x)
```

maple [B] time = 11.60, size = 2237, normalized size = 3.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e))^3,x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(d^3/b^4*(2*b*d*(c/d-1)*((c+d*sin(f
*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d)
)^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*s
in(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c
-d))^(1/2),((c-d)/(c+d))^(1/2)))-6*d*a*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/
2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin
(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c
-d)/(c+d))^(1/2))+8*c*b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*
x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(
f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2
)))+12/b^5*d^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))
^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/
(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))+1/b^4*(a^4*d^4-4*a^3*
b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*(-1/2*b^2/(a^3*d-a^2*b*c-a
*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))^2-3/
4*b^2*(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(-(-d*sin(f*x
+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-1/4*d*(7*a^3*d-4*a^2*b*c-a*b^2*
d-2*b^3*c)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))
^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d
*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2)
,((c-d)/(c+d))^(1/2))-3/4*b*d*(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*
d+b^3*c)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^
(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/
2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+
```

```
EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+1/4*(15*a^4*
d^2-20*a^3*b*c*d+8*a^2*b^2*c^2-6*a^2*b^2*d^2-4*a*b^3*c*d+4*b^4*c^2+3*b^4*d^
2)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2/b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)
*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))
^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2)))-4/b^4*d*(a^3*d^3-3*a^2*b*c
*d^2+3*a*b^2*c^2*d-b^3*c^3)*(-b^2/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(-(-d*sin(f
*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-a*d/(a^3*d-a^2*b*c-a*b^2*d+b^
3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*
((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Ell
ipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-b*d/(a^3*d-a^2*b
*c-a*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/
(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)
^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^
(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+(3*a^
2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)/b*(c/d-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(
f*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))))/cos(f*x+e)/
(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c+d\,\sin\left(e+f\,x\right)\right)^{7/2}}{\left(a+b\,\sin\left(e+f\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(7/2)/(a + b*sin(e + f*x))^3,x)
[Out] int((c + d*sin(e + f*x))^(7/2)/(a + b*sin(e + f*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(7/2)/(a+b*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.760 $\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+b\sin(e+fx))^3}\,dx$

Optimal. Leaf size=549

$$\frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2bf\left(a^2-b^2\right)(a+b\sin(e+fx))^2}+\frac{3\left(a^2d+2abc-3b^2d\right)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4bf\left(a^2-b^2\right)^2(a+b\sin(e+fx))}+\frac{3\left(a^2\right.}{}$$

```
[Out] 1/2*(-a*d+b*c)^2*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/b/(a^2-b^2)/f/(a+b*sin(f
*x+e))^2+3/4*(-a*d+b*c)*(a^2*d+2*a*b*c-3*b^2*d)*cos(f*x+e)*(c+d*sin(f*x+e))
^(1/2)/b/(a^2-b^2)^2/f/(a+b*sin(f*x+e))-3/4*(-a*d+b*c)*(a^2*d+2*a*b*c-3*b^2
*d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE
(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/
b^2/(a^2-b^2)^2/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-1/4*(4*a^3*b*c*d^2+3*a^4*d
^3+a^2*b^2*d*(7*c^2-5*d^2)+b^4*d*(11*c^2+8*d^2)-2*a*b^3*c*(3*c^2+11*d^2))*(
sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(
1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/
2)/b^3/(a^2-b^2)^2/f/(c+d*sin(f*x+e))^(1/2)-1/4*(-a*d+b*c)*(4*a^3*b*c*d-28*
a*b^3*c*d+3*a^4*d^2+2*a^2*b^2*(4*c^2-3*d^2)+b^4*(4*c^2+15*d^2))*(sin(1/2*e+
1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2*e+1/4
*Pi+1/2*f*x),2*b/(a+b),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1
/2)/(a-b)^2/b^3/(a+b)^3/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 2.08, antiderivative size = 549, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {2792, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(a^2b^2d\left(7c^2-5d^2\right)+4a^3bcd^2+3a^4d^3-2ab^3c\left(3c^2+11d^2\right)+b^4d\left(11c^2+8d^2\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\right.}{4b^3f\left(a^2-b^2\right)^2\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((b*c - a*d)^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(2*b*(a^2 - b^2)*f*(a
 + b*Sin[e + f*x])^2) + (3*(b*c - a*d)*(2*a*b*c + a^2*d - 3*b^2*d)*Cos[e +
f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*b*(a^2 - b^2)^2*f*(a + b*Sin[e + f*x])) +
 (3*(b*c - a*d)*(2*a*b*c + a^2*d - 3*b^2*d)*EllipticE[(e - Pi/2 + f*x)/2, (
2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(4*b^2*(a^2 - b^2)^2*f*Sqrt[(c + d*
Sin[e + f*x])/(c + d)]) + ((4*a^3*b*c*d^2 + 3*a^4*d^3 + a^2*b^2*d*(7*c^2 -
5*d^2) + b^4*d*(11*c^2 + 8*d^2) - 2*a*b^3*c*(3*c^2 + 11*d^2))*EllipticF[(e
```

```
- Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*b^3*
(a^2 - b^2)^2*f*Sqrt[c + d*Sin[e + f*x]]) + ((b*c - a*d)*(4*a^3*b*c*d - 28*
a*b^3*c*d + 3*a^4*d^2 + 2*a^2*b^2*(4*c^2 - 3*d^2) + b^4*(4*c^2 + 15*d^2))*E
llipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin
[e + f*x])/(c + d)])/(4*(a - b)^2*b^3*(a + b)^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
```

```
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+b\sin(e+fx))^3}\,dx = \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} - \frac{\int \frac{\frac{1}{2}\left(-4abc^3+9b^2c^2d-6abcd^2+a^2d^3\right)-\left(a^2cd^2+2a\right.}{}}{}$$

$$= \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{3(bc-ad)\left(2abc+a^2d-3b^2d\right)\cos(e}{4b\left(a^2-b^2\right)^2 f(a+b\,\mathrm{s}}$$

$$= \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{3(bc-ad)\left(2abc+a^2d-3b^2d\right)\cos(e}{4b\left(a^2-b^2\right)^2 f(a+b\,\mathrm{s}}$$

$$= \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{3(bc-ad)\left(2abc+a^2d-3b^2d\right)\cos(e}{4b\left(a^2-b^2\right)^2 f(a+b\,\mathrm{s}}$$

$$= \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{3(bc-ad)\left(2abc+a^2d-3b^2d\right)\cos(e}{4b\left(a^2-b^2\right)^2 f(a+b\,\mathrm{s}}$$

$$= \frac{(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2b\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{3(bc-ad)\left(2abc+a^2d-3b^2d\right)\cos(e}{4b\left(a^2-b^2\right)^2 f(a+b\,\mathrm{s}}$$

Mathematica [C] time = 8.07, size = 1149, normalized size = 2.09

$$\frac{\sqrt{c+d\sin(e+fx)}\left(\frac{-b^2\cos(e+fx)c^2+2abd\cos(e+fx)c-a^2d^2\cos(e+fx)}{2b\left(b^2-a^2\right)(a+b\sin(e+fx))^2} - \frac{3\left(d^2\cos(e+fx)a^3+bcd\cos(e+fx)a^2-2b^2c^2\cos(e+fx)a-3b^2d^2\cos\right.}{4b\left(b^2-a^2\right)^2(a+b\sin(e+fx))}\right.}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*((-(b^2*c^2*Cos[e + f*x]) + 2*a*b*c*d*Cos[e + f*x
] - a^2*d^2*Cos[e + f*x])/(2*b*(-a^2 + b^2)*(a + b*Sin[e + f*x])^2) - (3*(-
2*a*b^2*c^2*Cos[e + f*x] + a^2*b*c*d*Cos[e + f*x] + 3*b^3*c*d*Cos[e + f*x]
+ a^3*d^2*Cos[e + f*x] - 3*a*b^2*d^2*Cos[e + f*x]))/(4*b*(-a^2 + b^2)^2*(a
+ b*Sin[e + f*x]))))/f - ((-2*(-16*a^2*b*c^3 - 8*b^3*c^3 + 54*a*b^2*c^2*d -
 15*a^2*b*c*d^2 - 21*b^3*c*d^2 + a^3*d^3 + 5*a*b^2*d^3)*EllipticPi[(2*b)/(a
 + b), (-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d
)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]) - ((2*I)*(-20*a^2*b*c^2*d - 4*b^3*c^
2*d + 4*a^3*c*d^2 + 44*a*b^2*c*d^2 - 8*a^2*b*d^3 - 16*b^3*d^3)*Cos[e + f*x]
*((b*c - a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*
x]]], (c + d)/(c - d)] + a*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[
Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)])*Sqrt[(d -
d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*
d + b*(c + d*Sin[e + f*x])))/(b*d^2*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*
Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e
 + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)]) - ((2*I)*(6*a*b^2*c^2*d - 3*a^2*b
*c*d^2 - 9*b^3*c*d^2 - 3*a^3*d^3 + 9*a*b^2*d^3)*Cos[e + f*x]*Cos[2*(e + f*x
)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c
+ d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(-2*(a + b)*(-(b*c) + a*d)*Ellipti
cF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)
] + (2*a^2 - b^2)*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c
+ d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)]))*Sqrt[(d - d*Sin[e
+ f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c
 + d*Sin[e + f*x])))/(b^2*d*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e +
f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d^2 + 4*c*(c + d*Sin[e + f*x]) - 2
*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c
+ d*Sin[e + f*x])^2)/d^2)]))/(16*(a - b)^2*b*(a + b)^2*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^3,x, algorithm="giac")
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(b*sin(f*x + e) + a)^3, x)
```

maple [B] time = 10.04, size = 1888, normalized size = 3.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^3,x)
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(2*d^3/b^3*(c/d-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d
))^(1/2),((c-d)/(c+d))^(1/2))-6/b^4*d^2*(a*d-b*c)*(c/d-1)*((c+d*sin(f*x+e))
/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2
)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f
*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))+1/b^3*(-a^3*d^
3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)*(-1/2*b^2/(a^3*d-a^2*b*c-a*b^2*d+b^3
*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))^2-3/4*b^2*(3*a
^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(-(-d*sin(f*x+e)-c)*cos
(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-1/4*d*(7*a^3*d-4*a^2*b*c-a*b^2*d-2*b^3*c)
/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*
(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e
)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c
+d))^(1/2))-3/4*b*d*(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2
*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-s
in(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-
1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(
((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+1/4*(15*a^4*d^2-20*a^3
*b*c*d+8*a^2*b^2*c^2-6*a^2*b^2*d^2-4*a*b^3*c*d+4*b^4*c^2+3*b^4*d^2)/(a^3*d-
a^2*b*c-a*b^2*d+b^3*c)^2/b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin
(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*c
os(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-c
/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2)))+3/b^3*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)*
(-b^2/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)
/(a+b*sin(f*x+e))-a*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e
))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1
/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-
d))^(1/2),((c-d)/(c+d))^(1/2))-b*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(c/d-1)*((
c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1
)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*Elliptic
E(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f
*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))))+(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^
2*b*c-a*b^2*d+b^3*c)/b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x
```

```
+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f
*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-c/d+1
)/(-c/d+a/b),((c-d)/(c+d))^(1/2))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
[Out] Timed out
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c+d\,\sin\left(e+f\,x\right)\right)^{5/2}}{\left(a+b\,\sin\left(e+f\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(5/2)/(a + b*sin(e + f*x))^3,x)
[Out] int((c + d*sin(e + f*x))^(5/2)/(a + b*sin(e + f*x))^3, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e))**3,x)
[Out] Timed out
```

3.761 $\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^3}\,dx$

Optimal. Leaf size=472

$$\frac{\left(a^2(-d)+6abc-5b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f\left(a^2-b^2\right)^2(a+b\sin(e+fx))}+\frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f\left(a^2-b^2\right)(a+b\sin(e+fx))^2}-\frac{(bc-ad)\left(a^2\right.}{}$$

```
[Out] 1/2*(-a*d+b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(a^2-b^2)/f/(a+b*sin(f*x+e
))^2+1/4*(-a^2*d+6*a*b*c-5*b^2*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(a^2-b^
2)^2/f/(a+b*sin(f*x+e))-1/4*(-a^2*d+6*a*b*c-5*b^2*d)*(sin(1/2*e+1/4*Pi+1/2*
f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x)
,2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/b/(a^2-b^2)^2/f/((c+d*sin(
f*x+e))/(c+d))^(1/2)+1/4*(-a*d+b*c)*(a^2*d+6*a*b*c-7*b^2*d)*(sin(1/2*e+1/4*
Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1
/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/b^2/(a^2-b^
2)^2/f/(c+d*sin(f*x+e))^(1/2)+1/4*(4*a^3*b*c*d+20*a*b^3*c*d+a^4*d^2-b^4*(4*
c^2+3*d^2)-2*a^2*b^2*(4*c^2+5*d^2))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin
(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2*b/(a+b),2^(1/
2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/(a-b)^2/b^2/(a+b)^3/f/(c
+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.81, antiderivative size = 472, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {2799, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{\left(-2a^2b^2\left(4c^2+5d^2\right)+4a^3bcd+a^4d^2+20ab^3cd-b^4\left(4c^2+3d^2\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{4b^2f(a-b)^2(a+b)^3\sqrt{c+d\sin(e+fx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((b*c - a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(2*(a^2 - b^2)*f*(a + b
*Sin[e + f*x])^2) + ((6*a*b*c - a^2*d - 5*b^2*d)*Cos[e + f*x]*Sqrt[c + d*Si
n[e + f*x]])/(4*(a^2 - b^2)^2*f*(a + b*Sin[e + f*x])) + ((6*a*b*c - a^2*d -
 5*b^2*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f
*x]])/(4*b*(a^2 - b^2)^2*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - ((b*c - a*
d)*(6*a*b*c + a^2*d - 7*b^2*d)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]
*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*b^2*(a^2 - b^2)^2*f*Sqrt[c + d*Sin[
e + f*x]]) - ((4*a^3*b*c*d + 20*a*b^3*c*d + a^4*d^2 - b^4*(4*c^2 + 3*d^2) -
```

```
2*a^2*b^2*(4*c^2 + 5*d^2))*EllipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (
2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*(a - b)^2*b^2*(a + b)^
3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
```

```
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^3}\,dx = \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} - \frac{\int \frac{\frac{1}{2}\left(5bcd-a\left(4c^2+d^2\right)\right)-\left(3acd-b\left(c^2+2d^2\right)\right)\sin(e+fx}{(a+b\sin(e+fx))^2\sqrt{c+d\sin(e+}}}{2\left(a^2-b^2\right)}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{\left(6abc-a^2d-5b^2d\right)\cos(e+fx)\sqrt{c+d}}{4\left(a^2-b^2\right)^2 f(a+b\sin(e+f}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{\left(6abc-a^2d-5b^2d\right)\cos(e+fx)\sqrt{c+d}}{4\left(a^2-b^2\right)^2 f(a+b\sin(e+f}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{\left(6abc-a^2d-5b^2d\right)\cos(e+fx)\sqrt{c+d}}{4\left(a^2-b^2\right)^2 f(a+b\sin(e+f}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{\left(6abc-a^2d-5b^2d\right)\cos(e+fx)\sqrt{c+d}}{4\left(a^2-b^2\right)^2 f(a+b\sin(e+f}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{\left(6abc-a^2d-5b^2d\right)\cos(e+fx)\sqrt{c+d}}{4\left(a^2-b^2\right)^2 f(a+b\sin(e+f}$$

Mathematica [C] time = 7.43, size = 1001, normalized size = 2.12

$$\frac{\sqrt{c+d\sin(e+fx)}\left(\frac{bc\cos(e+fx)-ad\cos(e+fx)}{2\left(a^2-b^2\right)(a+b\sin(e+fx))^2}+\frac{-d\cos(e+fx)a^2+6bc\cos(e+fx)a-5b^2d\cos(e+fx)}{4\left(a^2-b^2\right)^2(a+b\sin(e+fx))}\right)}{f} + \frac{-\frac{2\left(16a^2c^2+8b^2c^2-30abdc+5a^2d^2+b\right.}{(a+b)}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*((b*c*Cos[e + f*x] - a*d*Cos[e + f*x])/(2*(a^2 -
b^2)*(a + b*Sin[e + f*x])^2) + (6*a*b*c*Cos[e + f*x] - a^2*d*Cos[e + f*x] -
```

```
 5*b^2*d*Cos[e + f*x])/(4*(a^2 - b^2)^2*(a + b*Sin[e + f*x]))))/f + ((-2*(1
6*a^2*c^2 + 8*b^2*c^2 - 30*a*b*c*d + 5*a^2*d^2 + b^2*d^2)*EllipticPi[(2*b)/
(a + b), (-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c +
 d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]) - ((2*I)*(20*a^2*c*d + 4*b^2*c*d -
 24*a*b*d^2)*Cos[e + f*x]*((b*c - a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-
1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + a*d*EllipticPi[(b*(c + d)
)/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c
+ d)/(c - d)])*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x
])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x]))))/(b*d^2*Sqrt[-(c + d)^
(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[-((c^2
 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)]) - ((2*I)
*(-6*a*b*c*d + a^2*d^2 + 5*b^2*d^2)*Cos[e + f*x]*Cos[2*(e + f*x)]*(2*b*(c -
 d)*(b*c - a*d)*EllipticE[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e +
f*x]]], (c + d)/(c - d)] + d*(-2*(a + b)*(-(b*c) + a*d)*EllipticF[I*ArcSinh
[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + (2*a^2 -
 b^2)*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*S
qrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)]))*Sqrt[(d - d*Sin[e + f*x])/(c +
 d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e +
 f*x]))))/(b^2*d*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1
 - Sin[e + f*x]^2]*(-2*c^2 + d^2 + 4*c*(c + d*Sin[e + f*x]) - 2*(c + d*Sin[
e + f*x])^2)*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e +
f*x])^2)/d^2)]))/(16*(a - b)^2*(a + b)^2*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^3,x, algorithm="giac")
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) + a)^3, x)
```

maple [B] time = 9.96, size = 1718, normalized size = 3.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^3,x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(2*d^2/b^3*(c/d-1)*((c+d*sin(f*x+e)
)/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/
2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(
f*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))+(a^2*d^2-2*a*
b*c*d+b^2*c^2)/b^2*(-1/2*b^2/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)
-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))^2-3/4*b^2*(3*a^2*d-2*a*b*c-b^2*d)/
(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+
b*sin(f*x+e))-1/4*d*(7*a^3*d-4*a^2*b*c-a*b^2*d-2*b^3*c)/(a^3*d-a^2*b*c-a*b^
2*d+b^3*c)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d)
)^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(
1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-3/4*b*d*
(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(c/d-1)*((c+d*sin(f
*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d)
)^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*s
in(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c
-d))^(1/2),((c-d)/(c+d))^(1/2)))+1/4*(15*a^4*d^2-20*a^3*b*c*d+8*a^2*b^2*c^2
-6*a^2*b^2*d^2-4*a*b^3*c*d+4*b^4*c^2+3*b^4*d^2)/(a^3*d-a^2*b*c-a*b^2*d+b^3*
c)^2/b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2
)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(
-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c
-d)/(c+d))^(1/2)))-2*d*(a*d-b*c)/b^2*(-b^2/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(-
(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-a*d/(a^3*d-a^2*b*c-a
*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d
))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^
(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-b*d/(a^
3*d-a^2*b*c-a*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin
(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*c
os(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d
)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2
)))+(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)/b*(c/d-1)*((c+d*s
in(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(
c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi((
(c+d*sin(f*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))))/co
s(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c+d\,\sin\left(e+f\,x\right)\right)^{3/2}}{\left(a+b\,\sin\left(e+f\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(3/2)/(a + b*sin(e + f*x))^3,x)
[Out] int((c + d*sin(e + f*x))^(3/2)/(a + b*sin(e + f*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e))**3,x)
[Out] Timed out
```

3.762 $\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^3}\,dx$

Optimal. Leaf size=487

$$\frac{b\left(-5a^2d+6abc-b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f\left(a^2-b^2\right)^2(bc-ad)(a+b\sin(e+fx))}+\frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f\left(a^2-b^2\right)(a+b\sin(e+fx))^2}-\frac{3\left(a^2(-d)+2abc-b^2\right.}{4bf\left(a^2\right.}$$

```
[Out] 1/2*b*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(a^2-b^2)/f/(a+b*sin(f*x+e))^2+1/4*
b*(-5*a^2*d+6*a*b*c-b^2*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(a^2-b^2)^2/(-
a*d+b*c)/f/(a+b*sin(f*x+e))-1/4*(-5*a^2*d+6*a*b*c-b^2*d)*(sin(1/2*e+1/4*Pi+
1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*
f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/(a^2-b^2)^2/(-a*d+b*c)
/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+3/4*(-a^2*d+2*a*b*c-b^2*d)*(sin(1/2*e+1/4
*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+
1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/b/(a^2-b^2
)^2/f/(c+d*sin(f*x+e))^(1/2)+1/4*(12*a^3*b*c*d+12*a*b^3*c*d-3*a^4*d^2-b^4*(
4*c^2-d^2)-2*a^2*b^2*(4*c^2+5*d^2))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin
(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2*e+1/4*Pi+1/2*f*x),2*b/(a+b),2^(1/
2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+d))^(1/2)/(a-b)^2/b/(a+b)^3/(-a*d+
b*c)/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.60, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {2796, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{\left(-2a^2b^2\left(4c^2+5d^2\right)+12a^3bcd-3a^4d^2+12ab^3cd-b^4\left(4c^2-d^2\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{4bf(a-b)^2(a+b)^3(bc-ad)\sqrt{c+d\sin(e+fx)}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (b*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(2*(a^2 - b^2)*f*(a + b*Sin[e + f
*x])^2) + (b*(6*a*b*c - 5*a^2*d - b^2*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*
x]])/(4*(a^2 - b^2)^2*(b*c - a*d)*f*(a + b*Sin[e + f*x])) + ((6*a*b*c - 5*a
^2*d - b^2*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e
 + f*x]])/(4*(a^2 - b^2)^2*(b*c - a*d)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]
) - (3*(2*a*b*c - a^2*d - b^2*d)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d
)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*b*(a^2 - b^2)^2*f*Sqrt[c + d*Sin[
e + f*x]]) - ((12*a^3*b*c*d + 12*a*b^3*c*d - 3*a^4*d^2 - b^4*(4*c^2 - d^2)
```

```
- 2*a^2*b^2*(4*c^2 + 5*d^2))*EllipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/2,
(2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*(a - b)^2*b*(a + b)^3
*(b*c - a*d)*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
```

```
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^3}\,dx = \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} - \frac{\int \frac{\frac{1}{2}(-4ac+bd)+(bc-2ad)\sin(e+fx)+\frac{1}{2}bd\sin^2(e+fx)}{(a+b\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}\,dx}{2\left(a^2-b^2\right)}$$

$$= \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{b\left(6abc-5a^2d-b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e}{4\left(a^2-b^2\right)^2(bc-ad)f(a+b\sin(e+fx))}$$

$$= \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{b\left(6abc-5a^2d-b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e}{4\left(a^2-b^2\right)^2(bc-ad)f(a+b\sin(e+fx))}$$

$$= \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{b\left(6abc-5a^2d-b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e}{4\left(a^2-b^2\right)^2(bc-ad)f(a+b\sin(e+fx))}$$

$$= \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{b\left(6abc-5a^2d-b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e}{4\left(a^2-b^2\right)^2(bc-ad)f(a+b\sin(e+fx))}$$

$$= \frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)f(a+b\sin(e+fx))^2} + \frac{b\left(6abc-5a^2d-b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e}{4\left(a^2-b^2\right)^2(bc-ad)f(a+b\sin(e+fx))}$$

Mathematica [C] time = 7.74, size = 1038, normalized size = 2.13

$$\frac{\sqrt{c+d\sin(e+fx)}\left(\frac{b\cos(e+fx)}{2\left(a^2-b^2\right)(a+b\sin(e+fx))^2} - \frac{-d\cos(e+fx)b^3+6ac\cos(e+fx)b^2-5a^2d\cos(e+fx)b}{4\left(a^2-b^2\right)^2(ad-bc)(a+b\sin(e+fx))}\right)}{f} + \frac{-2\left(16cda^3-16bc^2a^2-9bd^2a^2+14b\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*((b*Cos[e + f*x])/(2*(a^2 - b^2)*(a + b*Sin[e + f
*x])^2) - (6*a*b^2*c*Cos[e + f*x] - 5*a^2*b*d*Cos[e + f*x] - b^3*d*Cos[e +
f*x])/(4*(a^2 - b^2)^2*(-(b*c) + a*d)*(a + b*Sin[e + f*x]))))/f + ((-2*(-16
*a^2*b*c^2 - 8*b^3*c^2 + 16*a^3*c*d + 14*a*b^2*c*d - 9*a^2*b*d^2 + 3*b^3*d^
```

```
2)*EllipticPi[(2*b)/(a + b), (-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c +
d*Sin[e + f*x])/(c + d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]) - ((2*I)*(-20*
a^2*b*c*d - 4*b^3*c*d + 16*a^3*d^2 + 8*a*b^2*d^2)*Cos[e + f*x]*((b*c - a*d)
*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)
/(c - d)] + a*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)
^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e + f*x
])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*
Sin[e + f*x])))/(b*d^2*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])
*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c
 + d*Sin[e + f*x])^2)/d^2)]) - ((2*I)*(6*a*b^2*c*d - 5*a^2*b*d^2 - b^3*d^2)
*Cos[e + f*x]*Cos[2*(e + f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I*ArcSinh
[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(-2*(a
 + b)*(-(b*c) + a*d)*EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin
[e + f*x]]], (c + d)/(c - d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c + d))/(b*c
 - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(
c - d)]))*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c
 - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b^2*d*Sqrt[-(c + d)^(-1)]
*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d^2 +
4*c*(c + d*Sin[e + f*x]) - 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2 - 2*
c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)]))/(16*(a - b)^2*(a +
 b)^2*(-(b*c) + a*d)*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d \sin\left(fx+e\right)+c}}{\left(b \sin\left(fx+e\right)+a\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a)^3, x)
```

maple [B] time = 9.44, size = 1525, normalized size = 3.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^3,x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-a*d+b*c)/b*(-1/2*b^2/(a^3*d-a^2*
b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))
^2-3/4*b^2*(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(-(-d*si
n(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-1/4*d*(7*a^3*d-4*a^2*b*c-a
*b^2*d-2*b^3*c)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(c/d-1)*((c+d*sin(f*x+e))/(
c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^
(1/2),((c-d)/(c+d))^(1/2))-3/4*b*d*(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a
*b^2*d+b^3*c)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c
+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2
)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1
/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+1/4*(15
*a^4*d^2-20*a^3*b*c*d+8*a^2*b^2*c^2-6*a^2*b^2*d^2-4*a*b^3*c*d+4*b^4*c^2+3*b
^4*d^2)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2/b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^
(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*
sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/(
c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2)))+d/b*(-b^2/(a^3*d-a^2*
b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))
-a*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(
d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x
+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/
(c+d))^(1/2))-b*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(
c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/
(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e
))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/
2),((c-d)/(c+d))^(1/2)))+(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3
*c)/b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)
*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-
c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-
d)/(c+d))^(1/2))))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d \sin\left(fx+e\right)+c}}{\left(b \sin\left(fx+e\right)+a\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c+d\,\sin\left(e+f\,x\right)}}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(1/2)/(a + b*sin(e + f*x))^3,x)
[Out] int((c + d*sin(e + f*x))^(1/2)/(a + b*sin(e + f*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))**3,x)
[Out] Timed out
```

3.763 $\int \frac{1}{(a+b\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=503

$$\frac{3b^2\left(-3a^2d+2abc+b^2d\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4f\left(a^2-b^2\right)^2(bc-ad)^2(a+b\sin(e+fx))}+\frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f\left(a^2-b^2\right)(bc-ad)(a+b\sin(e+fx))^2}-\frac{\left(-7a^2d+\right.}{4}$$

```
[Out] 1/2*b^2*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(a^2-b^2)/(-a*d+b*c)/f/(a+b*sin(f
*x+e))^2+3/4*b^2*(-3*a^2*d+2*a*b*c+b^2*d)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)
/(a^2-b^2)^2/(-a*d+b*c)^2/f/(a+b*sin(f*x+e))-3/4*b*(-3*a^2*d+2*a*b*c+b^2*d)
*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(co
s(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/(a^
2-b^2)^2/(-a*d+b*c)^2/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+1/4*(-7*a^2*d+6*a*b*
c+b^2*d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elli
pticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/
(c+d))^(1/2)/(a^2-b^2)^2/(-a*d+b*c)/f/(c+d*sin(f*x+e))^(1/2)+1/4*(20*a^3*b*
c*d+4*a*b^3*c*d-15*a^4*d^2-2*a^2*b^2*(4*c^2-3*d^2)-b^4*(4*c^2+3*d^2))*(sin(
1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi(cos(1/2
*e+1/4*Pi+1/2*f*x),2*b/(a+b),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e))/(c+
d))^(1/2)/(a-b)^2/(a+b)^3/(-a*d+b*c)^2/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.68, antiderivative size = 503, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {2802, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{\left(-2a^2b^2\left(4c^2-3d^2\right)+20a^3bcd-15a^4d^2+4ab^3cd-b^4\left(4c^2+3d^2\right)\right)\sqrt{\frac{c+d\sin(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|\frac{2d}{c+d}\right)}{4f(a-b)^2(a+b)^3(bc-ad)^2\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (b^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(2*(a^2 - b^2)*(b*c - a*d)*f*(a
 + b*Sin[e + f*x])^2) + (3*b^2*(2*a*b*c - 3*a^2*d + b^2*d)*Cos[e + f*x]*Sqr
t[c + d*Sin[e + f*x]])/(4*(a^2 - b^2)^2*(b*c - a*d)^2*f*(a + b*Sin[e + f*x]
)) + (3*b*(2*a*b*c - 3*a^2*d + b^2*d)*EllipticE[(e - Pi/2 + f*x)/2, (2*d)/(
c + d)]*Sqrt[c + d*Sin[e + f*x]])/(4*(a^2 - b^2)^2*(b*c - a*d)^2*f*Sqrt[(c
+ d*Sin[e + f*x])/(c + d)]) - ((6*a*b*c - 7*a^2*d + b^2*d)*EllipticF[(e - P
i/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*(a^2 -
b^2)^2*(b*c - a*d)*f*Sqrt[c + d*Sin[e + f*x]]) - ((20*a^3*b*c*d + 4*a*b^3*c
```

```
*d - 15*a^4*d^2 - 2*a^2*b^2*(4*c^2 - 3*d^2) - b^4*(4*c^2 + 3*d^2))*Elliptic
Pi[(2*b)/(a + b), (e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*
x])/(c + d)])/(4*(a - b)^2*(a + b)^3*(b*c - a*d)^2*f*Sqrt[c + d*Sin[e + f*x
]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^3\sqrt{c+d\sin(e+fx)}}\,dx = \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2} - \frac{\int \frac{\frac{1}{2}\left(-4abc+4a^2d-3b^2d\right)+b}{(a+b\sin(e}}{2\left(a^2\right.}$$

$$= \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2} + \frac{3b^2\left(2abc-3a^2d+b\right.}{4\left(a^2-b^2\right)^2(b}$$

$$= \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2} + \frac{3b^2\left(2abc-3a^2d+b\right.}{4\left(a^2-b^2\right)^2(b}$$

$$= \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2} + \frac{3b^2\left(2abc-3a^2d+b\right.}{4\left(a^2-b^2\right)^2(b}$$

$$= \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2} + \frac{3b^2\left(2abc-3a^2d+b\right.}{4\left(a^2-b^2\right)^2(b}$$

$$= \frac{b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2} + \frac{3b^2\left(2abc-3a^2d+b\right.}{4\left(a^2-b^2\right)^2(b}$$

Mathematica [C] time = 7.82, size = 1069, normalized size = 2.13

$$\frac{\sqrt{c+d\sin(e+fx)}\left(\frac{3\left(d\cos(e+fx)b^4+2ac\cos(e+fx)b^3-3a^2d\cos(e+fx)b^2\right)}{4\left(a^2-b^2\right)^2(ad-bc)^2(a+b\sin(e+fx))}-\frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(ad-bc)(a+b\sin(e+fx))^2}\right)}{f}+\frac{-\frac{2\left(16d^2a^4-32bcda^3+16b\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^3*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*(-1/2*(b^2*Cos[e + f*x])/((a^2 - b^2)*(-(b*c) + a
*d)*(a + b*Sin[e + f*x])^2) + (3*(2*a*b^3*c*Cos[e + f*x] - 3*a^2*b^2*d*Cos[
```

```
e + f*x] + b^4*d*Cos[e + f*x]))/(4*(a^2 - b^2)^2*(-(b*c) + a*d)^2*(a + b*Si
n[e + f*x]))))/f + ((-2*(16*a^2*b^2*c^2 + 8*b^4*c^2 - 32*a^3*b*c*d + 2*a*b^
3*c*d + 16*a^4*d^2 - 19*a^2*b^2*d^2 + 9*b^4*d^2)*EllipticPi[(2*b)/(a + b),
(-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a
 + b)*Sqrt[c + d*Sin[e + f*x]]) - ((2*I)*(20*a^2*b^2*c*d + 4*b^4*c*d - 32*a
^3*b*d^2 + 8*a*b^3*d^2)*Cos[e + f*x]*((b*c - a*d)*EllipticF[I*ArcSinh[Sqrt[
-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + a*d*EllipticPi
[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e +
f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*
Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x]))))/(b*d^2*Sqr
t[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*
Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2)
]) - ((2*I)*(-6*a*b^3*c*d + 9*a^2*b^2*d^2 - 3*b^4*d^2)*Cos[e + f*x]*Cos[2*(
e + f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I*ArcSinh[Sqrt[-(c + d)^(-1)]*
Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(-2*(a + b)*(-(b*c) + a*d)*
EllipticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/
(c - d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sq
rt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)]))*Sqrt[(d - d
*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d
 + b*(c + d*Sin[e + f*x]))))/(b^2*d*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*S
in[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d^2 + 4*c*(c + d*Sin[e + f*
x]) - 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]
) + (c + d*Sin[e + f*x])^2)/d^2)]))/(16*(a - b)^2*(a + b)^2*(-(b*c) + a*d)^
2*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{3}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^3*sqrt(d*sin(f*x + e) + c)), x)
```

maple [A] time = 6.34, size = 867, normalized size = 1.72

$$\frac{\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^2\left(fx+e\right)\right)}\left(-\frac{b^2\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos^2\left(fx+e\right)\right)}}{2\left(a^3d-a^2bc-a\,b^2d+b^3c\right)\left(a+b\sin\left(fx+e\right)\right)^2}-\frac{3b^2\left(3a^2d-2abc-b^2d\right)\sqrt{-\left(-d\sin\left(fx+e\right)-c\right)\left(\cos\right.}}{4\left(a^3d-a^2bc-a\,b^2d+b^3c\right)^2\left(a+b\sin\left(fx\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(-1/2*b^2/(a^3*d-a^2*b*c-a*b^2*d+b^
3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))^2-3/4*b^2*(3*
a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(-(-d*sin(f*x+e)-c)*co
s(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-1/4*d*(7*a^3*d-4*a^2*b*c-a*b^2*d-2*b^3*c
)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d
*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+
e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(
c+d))^(1/2))-3/4*b*d*(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^
2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-
sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d
-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF
(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+1/4*(15*a^4*d^2-20*a^
3*b*c*d+8*a^2*b^2*c^2-6*a^2*b^2*d^2-4*a*b^3*c*d+4*b^4*c^2+3*b^4*d^2)/(a^3*d
-a^2*b*c-a*b^2*d+b^3*c)^2/b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-si
n(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*
cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-
c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2)))/cos(f*x+e)/(c+d*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^3\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.764 $\int \frac{1}{(a+b\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=682

$$\frac{b^2\left(-11a^2d+6abc+5b^2d\right)\cos(e+fx)}{4f\left(a^2-b^2\right)^2(bc-ad)^2(a+b\sin(e+fx))\sqrt{c+d\sin(e+fx)}}+\frac{b^2\cos(e+fx)}{2f\left(a^2-b^2\right)(bc-ad)(a+b\sin(e+fx))^2\sqrt{c+d\sin}}$$

```
[Out] -1/4*d*(8*a^4*d^3+a^2*b^2*d*(13*c^2-29*d^2)-b^4*d*(7*c^2-15*d^2)-6*a*b^3*c*
(c^2-d^2))*cos(f*x+e)/(a^2-b^2)^2/(-a*d+b*c)^3/(c^2-d^2)/f/(c+d*sin(f*x+e))
^(1/2)+1/2*b^2*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(a+b*sin(f*x+e))^2/(c+d*si
n(f*x+e))^(1/2)+1/4*b^2*(-11*a^2*d+6*a*b*c+5*b^2*d)*cos(f*x+e)/(a^2-b^2)^2/
(-a*d+b*c)^2/f/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2)+1/4*(8*a^4*d^3+a^2*b
^2*d*(13*c^2-29*d^2)-b^4*d*(7*c^2-15*d^2)-6*a*b^3*c*(c^2-d^2))*(sin(1/2*e+1
/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*P
i+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*(c+d*sin(f*x+e))^(1/2)/(a^2-b^2)^2/(-a*
d+b*c)^3/(c^2-d^2)/f/((c+d*sin(f*x+e))/(c+d))^(1/2)+1/4*b*(-11*a^2*d+6*a*b*
c+5*b^2*d)*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*El
lipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x+e)
)/(c+d))^(1/2)/(a^2-b^2)^2/(-a*d+b*c)^2/f/(c+d*sin(f*x+e))^(1/2)+1/4*b*(28*
a^3*b*c*d-4*a*b^3*c*d-35*a^4*d^2-2*a^2*b^2*(4*c^2-19*d^2)-b^4*(4*c^2+15*d^2
))*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticPi
(cos(1/2*e+1/4*Pi+1/2*f*x),2*b/(a+b),2^(1/2)*(d/(c+d))^(1/2))*((c+d*sin(f*x
+e))/(c+d))^(1/2)/(a-b)^2/(a+b)^3/(-a*d+b*c)^3/f/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 2.77, antiderivative size = 682, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {2802, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{d\left(a^2b^2d\left(13c^2-29d^2\right)+8a^4d^3-6ab^3c\left(c^2-d^2\right)-b^4d\left(7c^2-15d^2\right)\right)\cos(e+fx)}{4f\left(a^2-b^2\right)^2\left(c^2-d^2\right)(bc-ad)^3\sqrt{c+d\sin(e+fx)}}-\frac{\left(a^2b^2d\left(13c^2-29d^2\right)+8a^4d^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -(d*(8*a^4*d^3 + a^2*b^2*d*(13*c^2 - 29*d^2) - b^4*d*(7*c^2 - 15*d^2) - 6*a
*b^3*c*(c^2 - d^2))*Cos[e + f*x])/(4*(a^2 - b^2)^2*(b*c - a*d)^3*(c^2 - d^2
)*f*Sqrt[c + d*Sin[e + f*x]]) + (b^2*Cos[e + f*x])/(2*(a^2 - b^2)*(b*c - a*
d)*f*(a + b*Sin[e + f*x])^2*Sqrt[c + d*Sin[e + f*x]]) + (b^2*(6*a*b*c - 11*
a^2*d + 5*b^2*d)*Cos[e + f*x])/(4*(a^2 - b^2)^2*(b*c - a*d)^2*f*(a + b*Sin[
```

```
e + f*x])*Sqrt[c + d*Sin[e + f*x]]) - ((8*a^4*d^3 + a^2*b^2*d*(13*c^2 - 29*
d^2) - b^4*d*(7*c^2 - 15*d^2) - 6*a*b^3*c*(c^2 - d^2))*EllipticE[(e - Pi/2
+ f*x)/2, (2*d)/(c + d)]*Sqrt[c + d*Sin[e + f*x]])/(4*(a^2 - b^2)^2*(b*c -
a*d)^3*(c^2 - d^2)*f*Sqrt[(c + d*Sin[e + f*x])/(c + d)]) - (b*(6*a*b*c - 11
*a^2*d + 5*b^2*d)*EllipticF[(e - Pi/2 + f*x)/2, (2*d)/(c + d)]*Sqrt[(c + d*
Sin[e + f*x])/(c + d)])/(4*(a^2 - b^2)^2*(b*c - a*d)^2*f*Sqrt[c + d*Sin[e +
 f*x]]) - (b*(28*a^3*b*c*d - 4*a*b^3*c*d - 35*a^4*d^2 - 2*a^2*b^2*(4*c^2 -
19*d^2) - b^4*(4*c^2 + 15*d^2))*EllipticPi[(2*b)/(a + b), (e - Pi/2 + f*x)/
2, (2*d)/(c + d)]*Sqrt[(c + d*Sin[e + f*x])/(c + d)])/(4*(a - b)^2*(a + b)^
3*(b*c - a*d)^3*f*Sqrt[c + d*Sin[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
```

```
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^3(c+d\sin(e+fx))^{3/2}}\,dx = \frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{b^2\cos(e+fx)}{2\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^2\sqrt{c+d\sin(e+fx)}}$$

$$= -\frac{d\left(8a^4d^3+a^2b^2d\left(13c^2-29d^2\right)-b^4d\left(7c^2-15d^2\right)-6ab^3c\left(\right.\right.}{4\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f\sqrt{c+d\sin(e}}$$

$$= -\frac{d\left(8a^4d^3+a^2b^2d\left(13c^2-29d^2\right)-b^4d\left(7c^2-15d^2\right)-6ab^3c\left(\right.\right.}{4\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f\sqrt{c+d\sin(e}}$$

$$= -\frac{d\left(8a^4d^3+a^2b^2d\left(13c^2-29d^2\right)-b^4d\left(7c^2-15d^2\right)-6ab^3c\left(\right.\right.}{4\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f\sqrt{c+d\sin(e}}$$

$$= -\frac{d\left(8a^4d^3+a^2b^2d\left(13c^2-29d^2\right)-b^4d\left(7c^2-15d^2\right)-6ab^3c\left(\right.\right.}{4\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f\sqrt{c+d\sin(e}}$$

$$= -\frac{d\left(8a^4d^3+a^2b^2d\left(13c^2-29d^2\right)-b^4d\left(7c^2-15d^2\right)-6ab^3c\left(\right.\right.}{4\left(a^2-b^2\right)^2(bc-ad)^3\left(c^2-d^2\right)f\sqrt{c+d\sin(e}}$$

Mathematica [C] time = 9.41, size = 1318, normalized size = 1.93

$$\frac{\sqrt{c+d\sin(e+fx)}\left(-\frac{2\cos(e+fx)d^4}{(bc-ad)^3\left(c^2-d^2\right)(c+d\sin(e+fx))}-\frac{7d\cos(e+fx)b^5+6ac\cos(e+fx)b^4-13a^2d\cos(e+fx)b^3}{4\left(a^2-b^2\right)^2(ad-bc)^3(a+b\sin(e+fx))}+\frac{b^3\cos(e+fx)}{2\left(a^2-b^2\right)(ad-bc)^2(a+b\sin\right.}}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (Sqrt[c + d*Sin[e + f*x]]*((b^3*Cos[e + f*x])/(2*(a^2 - b^2)*(-(b*c) + a*d)
^2*(a + b*Sin[e + f*x])^2) - (6*a*b^4*c*Cos[e + f*x] - 13*a^2*b^3*d*Cos[e +
 f*x] + 7*b^5*d*Cos[e + f*x])/(4*(a^2 - b^2)^2*(-(b*c) + a*d)^3*(a + b*Sin[
e + f*x])) - (2*d^4*Cos[e + f*x])/((b*c - a*d)^3*(c^2 - d^2)*(c + d*Sin[e +
 f*x]))))/f + ((-2*(-16*a^2*b^3*c^4 - 8*b^5*c^4 + 48*a^3*b^2*c^3*d - 18*a*b
^4*c^3*d - 48*a^4*b*c^2*d^2 + 95*a^2*b^3*c^2*d^2 - 29*b^5*c^2*d^2 + 16*a^5*
c*d^3 - 80*a^3*b^2*c*d^3 + 34*a*b^4*c*d^3 + 56*a^4*b*d^4 - 95*a^2*b^3*d^4 +
 45*b^5*d^4)*EllipticPi[(2*b)/(a + b), (-e + Pi/2 - f*x)/2, (2*d)/(c + d)]*
Sqrt[(c + d*Sin[e + f*x])/(c + d)])/((a + b)*Sqrt[c + d*Sin[e + f*x]]) - ((
2*I)*(-20*a^2*b^3*c^3*d - 4*b^5*c^3*d + 48*a^3*b^2*c^2*d^2 - 24*a*b^4*c^2*d
^2 + 16*a^4*b*c*d^3 - 12*a^2*b^3*c*d^3 + 20*b^5*c*d^3 + 16*a^5*d^4 - 80*a^3
*b^2*d^4 + 40*a*b^4*d^4)*Cos[e + f*x]*((b*c - a*d)*EllipticF[I*ArcSinh[Sqrt
[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + a*d*EllipticP
i[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e +
 f*x]]], (c + d)/(c - d)])*Sqrt[(d - d*Sin[e + f*x])/(c + d)]*Sqrt[-((d + d
*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b*(c + d*Sin[e + f*x])))/(b*d^2*Sq
rt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e + f*x])*Sqrt[1 - Sin[e + f*x]^2]
*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) + (c + d*Sin[e + f*x])^2)/d^2
)]) - ((2*I)*(6*a*b^4*c^3*d - 13*a^2*b^3*c^2*d^2 + 7*b^5*c^2*d^2 - 6*a*b^4*
c*d^3 - 8*a^4*b*d^4 + 29*a^2*b^3*d^4 - 15*b^5*d^4)*Cos[e + f*x]*Cos[2*(e +
f*x)]*(2*b*(c - d)*(b*c - a*d)*EllipticE[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt
[c + d*Sin[e + f*x]]], (c + d)/(c - d)] + d*(-2*(a + b)*(-(b*c) + a*d)*Elli
pticF[I*ArcSinh[Sqrt[-(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c -
 d)] + (2*a^2 - b^2)*d*EllipticPi[(b*(c + d))/(b*c - a*d), I*ArcSinh[Sqrt[-
(c + d)^(-1)]*Sqrt[c + d*Sin[e + f*x]]], (c + d)/(c - d)]))*Sqrt[(d - d*Sin
[e + f*x])/(c + d)]*Sqrt[-((d + d*Sin[e + f*x])/(c - d))]*(-(b*c) + a*d + b
*(c + d*Sin[e + f*x])))/(b^2*d*Sqrt[-(c + d)^(-1)]*(b*c - a*d)*(a + b*Sin[e
 + f*x])*Sqrt[1 - Sin[e + f*x]^2]*(-2*c^2 + d^2 + 4*c*(c + d*Sin[e + f*x])
- 2*(c + d*Sin[e + f*x])^2)*Sqrt[-((c^2 - d^2 - 2*c*(c + d*Sin[e + f*x]) +
(c + d*Sin[e + f*x])^2)/d^2)]))/(16*(a - b)^2*(a + b)^2*(c - d)*(c + d)*(-(
b*c) + a*d)^3*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{3}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^(3/2)), x)
```

maple [B] time = 13.15, size = 2099, normalized size = 3.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] (-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*(-2*d^2/(a*d-b*c)^3*(c/d-1)*((c+d*s
in(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(
c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi((
(c+d*sin(f*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))-b/(a
*d-b*c)*(-1/2*b^2/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*cos(f*x
+e)^2)^(1/2)/(a+b*sin(f*x+e))^2-3/4*b^2*(3*a^2*d-2*a*b*c-b^2*d)/(a^3*d-a^2*
b*c-a*b^2*d+b^3*c)^2*(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e
))-1/4*d*(7*a^3*d-4*a^2*b*c-a*b^2*d-2*b^3*c)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^
2*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-
sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*Ellipt
icF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-3/4*b*d*(3*a^2*d-2*
a*b*c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2*(c/d-1)*((c+d*sin(f*x+e))/(c-d
))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(
-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/
(c-d))^(1/2),((c-d)/(c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),
((c-d)/(c+d))^(1/2)))+1/4*(15*a^4*d^2-20*a^3*b*c*d+8*a^2*b^2*c^2-6*a^2*b^2*
d^2-4*a*b^3*c*d+4*b^4*c^2+3*b^4*d^2)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)^2/b*(c/d
-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*
x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*E
llipticPi(((c+d*sin(f*x+e))/(c-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^
```

```
(1/2)))+d^3/(a*d-b*c)^3*(2*d*cos(f*x+e)^2/(c^2-d^2)/(-(-d*sin(f*x+e)-c)*cos
(f*x+e)^2)^(1/2)+2*c/(c^2-d^2)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1
-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-
c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d
))^(1/2))+2/(c^2-d^2)*d*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*
x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(
f*x+e)^2)^(1/2)*((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(
c+d))^(1/2))+EllipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))
)-b*d/(a*d-b*c)^2*(-b^2/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(-(-d*sin(f*x+e)-c)*c
os(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e))-a*d/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)*(c/d-
1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x
+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*EllipticF(((c
+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))-b*d/(a^3*d-a^2*b*c-a*b^2*d
+b^3*c)*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/
2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*sin(f*x+e)-c)*cos(f*x+e)^2)^(1/2)*
((-c/d-1)*EllipticE(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2))+Ell
ipticF(((c+d*sin(f*x+e))/(c-d))^(1/2),((c-d)/(c+d))^(1/2)))+(3*a^2*d-2*a*b*
c-b^2*d)/(a^3*d-a^2*b*c-a*b^2*d+b^3*c)/b*(c/d-1)*((c+d*sin(f*x+e))/(c-d))^(
1/2)*(d*(1-sin(f*x+e))/(c+d))^(1/2)*((-sin(f*x+e)-1)*d/(c-d))^(1/2)/(-(-d*s
in(f*x+e)-c)*cos(f*x+e)^2)^(1/2)/(-c/d+a/b)*EllipticPi(((c+d*sin(f*x+e))/(c
-d))^(1/2),(-c/d+1)/(-c/d+a/b),((c-d)/(c+d))^(1/2))))/cos(f*x+e)/(c+d*sin(f
*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^3/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+b\,\sin\left(e+f\,x\right)\right)^3\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + b*sin(e + f*x))^3*(c + d*sin(e + f*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**3/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.765 $\int \sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=888

$$-\frac{\cos(e+fx)(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}\,d^2}{3bf}-\frac{(13bc-3ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(}{12bf}$$

```
[Out] -1/8*(5*a^2*b*c*d^2-a^3*d^3-a*b^2*d*(15*c^2+4*d^2)-5*b^3*(c^3+4*c*d^2))*Ell
ipticPi((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/
2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x
+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*
((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b^3/d/f/(a+b)^(1/2
)+1/24*(c-d)*(14*a*b*c*d-3*a^2*d^2+b^2*(33*c^2+16*d^2))*EllipticE((a+b)^(1/
2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(
a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(a+b)^(1/2)*(c+d)^(1/2)*(-(-
a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*
x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b^2/(-a*d+b*c)/f+1/24*(a+b)^(3/2)*(3*a^
2*d^2-6*a*b*d*(2*c+d)+b^2*(33*c^2+26*c*d+16*d^2))*EllipticF((c+d)^(1/2)*(a+
b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(
c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(
c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^
(1/2)/b^3/f/(c+d)^(1/2)-1/3*d^2*cos(f*x+e)*(a+b*sin(f*x+e))^(3/2)*(c+d*sin(
f*x+e))^(1/2)/b/f-1/24*(14*a*b*c*d-3*a^2*d^2+b^2*(33*c^2+16*d^2))*cos(f*x+e
)*(c+d*sin(f*x+e))^(1/2)/b/f/(a+b*sin(f*x+e))^(1/2)-1/12*d*(-3*a*d+13*b*c)*
cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2)/b/f
```

Rubi [A] time = 3.39, antiderivative size = 888, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2793, 3049, 3061, 3053, 2811, 2998, 2818, 2996}

$$-\frac{\cos(e+fx)(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}\,d^2}{3bf}-\frac{(13bc-3ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(}{12bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(14*a*b*c*d - 3*a^2*d^2 + b^2*(33*c^2 + 16
*d^2))*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]
*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f
*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))
]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a
```

```
+ b*Sin[e + f*x]))/(24*b^2*(b*c - a*d)*f) - (Sqrt[c + d]*(5*a^2*b*c*d^2 - a
^3*d^3 - a*b^2*d*(15*c^2 + 4*d^2) - 5*b^3*(c^3 + 4*c*d^2))*EllipticPi[(b*(c
 + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c +
 d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e
+ f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]
)))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*
(a + b*Sin[e + f*x]))/(8*b^3*Sqrt[a + b]*d*f) - ((14*a*b*c*d - 3*a^2*d^2 +
b^2*(33*c^2 + 16*d^2))*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(24*b*f*Sqrt[
a + b*Sin[e + f*x]]) - (d*(13*b*c - 3*a*d)*Cos[e + f*x]*Sqrt[a + b*Sin[e +
f*x]]*Sqrt[c + d*Sin[e + f*x]])/(12*b*f) - (d^2*Cos[e + f*x]*(a + b*Sin[e +
 f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]])/(3*b*f) + ((a + b)^(3/2)*(3*a^2*d^2
- 6*a*b*d*(2*c + d) + b^2*(33*c^2 + 26*c*d + 16*d^2))*EllipticF[ArcSin[(Sqr
t[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])],
 ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - S
in[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[
e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(24*b^3*S
qrt[c + d]*f)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
```

```
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
```

```
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}\,dx = -\frac{d^2\cos(e+fx)(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}{3bf} + \frac{\int}{}$$

$$= -\frac{d(13bc-3ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+}}{12bf}$$

$$= -\frac{\left(14abcd-3a^2d^2+b^2\left(33c^2+16d^2\right)\right)\cos(e+fx)\sqrt{c+d\sin(e}}{24bf\sqrt{a+b\sin(e+fx)}}$$

$$= -\frac{\left(14abcd-3a^2d^2+b^2\left(33c^2+16d^2\right)\right)\cos(e+fx)\sqrt{c+d\sin(e}}{24bf\sqrt{a+b\sin(e+fx)}}$$

$$= -\frac{\sqrt{c+d}\,\left(5a^2bcd^2-a^3d^3-ab^2d\left(15c^2+4d^2\right)-5b^3\left(c^3+4cd^2\right.\right.}{}$$

$$= \frac{\sqrt{a+b}\,(c-d)\sqrt{c+d}\,\left(14abcd-3a^2d^2+b^2\left(33c^2+16d^2\right)\right)E\Big(}{}$$

Mathematica [B] time = 7.14, size = 1978, normalized size = 2.23

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/48*((-4*(-(b*c) + a*d)*(-48*a*b*c^3 - 59*b^2*c^2*d - 58*a*b*c*d^2 + a^2*
d^3 - 16*b^2*d^3)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Ellip
ticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))
/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f
*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a
+ b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^
2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(-48*b^2*c^3 - 92*a*b*
c^2*d + 4*a^2*c*d^2 - 76*b^2*c*d^2 - 28*a*b*d^3)*((Sqrt[((c + d)*Cot[(-e +
Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2
- f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*
d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c +
d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((
-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/(
(a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[
((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((
a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e +
f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec
[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]
^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*
x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(33*b^2*c^2*d + 14*a*b*c*d^2 - 3*a^2
*d^3 + 16*b^2*d^3)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*S
in[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*Ell
ipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*
Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*S
in[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e +
 f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((
a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))])) - (2*(-(b*c)
+ a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c +
d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e
 + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*
Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)
/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 -
 f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a +
 b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot
[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), A
```

```
rcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*
c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Si
n[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Si
n[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c +
 d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt
[c + d*Sin[e + f*x]])))/(b*d)))/(b*f) + (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c +
d*Sin[e + f*x]]*(-1/12*(d*(13*b*c + a*d)*Cos[e + f*x])/b - (d^2*Sin[2*(e +
f*x)])/6))/f
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sin\left(fx+e\right)+a}\left(d \sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2), x)
```

maple [C] time = 17.17, size = 404501, normalized size = 455.52

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sin\left(fx+e\right)+a}\left(d \sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+b\,\sin\left(e+f\,x\right)}\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.766 $\int \sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=784

$$\frac{\sqrt{c+d}\left(-a^2d^2+6abcd+b^2\left(3c^2+4d^2\right)\right)\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1}{(c-d)(a+b\sin(e+fx)}}}{4b^2df\sqrt{a+b}}$$

```
[Out] 1/4*(6*a*b*c*d-a^2*d^2+b^2*(3*c^2+4*d^2))*EllipticPi((a+b)^(1/2)*(c+d*sin(f
*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d
)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*
(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-
d)/(a+b*sin(f*x+e)))^(1/2)/b^2/d/f/(a+b)^(1/2)+1/4*(c-d)*(a*d+5*b*c)*Ellipt
icE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),(
(a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(a+b)^(1/2)*(c+
d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b
*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b/(-a*d+b*c)/f-1/2*b*cos(f
*x+e)*(c+d*sin(f*x+e))^(3/2)/f/(a+b*sin(f*x+e))^(1/2)+1/4*(a+b)^(3/2)*(-a*d
+5*b*c+2*b*d)*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d
*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x
+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*
(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/b^2/f/(c+d)^(1/2)+1/2*(-a*d+b*
c)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+b*sin(f*x+e))^(1/2)-1/4*(a*d+5*b*
c)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+b*sin(f*x+e))^(1/2)
```

Rubi [A] time = 3.52, antiderivative size = 784, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2821, 3047, 3061, 3053, 2811, 2998, 2818, 2996}

$$\frac{\sqrt{c+d}\left(-a^2d^2+6abcd+b^2\left(3c^2+4d^2\right)\right)\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1}{(c-d)(a+b\sin(e+fx)}}}{4b^2df\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(5*b*c + a*d)*EllipticE[ArcSin[(Sqrt[a + b
]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a -
b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e
+ f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*
x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(4*b*(b*c - a*d)
*f) + (Sqrt[c + d]*(6*a*b*c*d - a^2*d^2 + b^2*(3*c^2 + 4*d^2))*EllipticPi[(
b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt
```

```
[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Se
c[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e +
f*x]))))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]
))]*(a + b*Sin[e + f*x]))/(4*b^2*Sqrt[a + b]*d*f) + ((b*c - a*d)*Cos[e + f*
x]*Sqrt[c + d*Sin[e + f*x]])/(2*f*Sqrt[a + b*Sin[e + f*x]]) - ((5*b*c + a*d
)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*f*Sqrt[a + b*Sin[e + f*x]]) + (
(a + b)^(3/2)*(5*b*c - a*d + 2*b*d)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a +
b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))
/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a
+ b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b
)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(4*b^2*Sqrt[c + d]*f) - (b*
Cos[e + f*x]*(c + d*Sin[e + f*x])^(3/2))/(2*f*Sqrt[a + b*Sin[e + f*x]])
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}\,dx = -\frac{b\cos(e+fx)(c+d\sin(e+fx))^{3/2}}{2f\sqrt{a+b\sin(e+fx)}} + \frac{\int \frac{\sqrt{c+d\sin(e+fx)}\left(\frac{1}{2}d\left(4a^2c-b^2\right.\right.}{}}{}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f\sqrt{a+b\sin(e+fx)}} - \frac{b\cos(e+fx)(c+d\,\mathrm{s}}{2f\sqrt{a+b\sin(}}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f\sqrt{a+b\sin(e+fx)}} - \frac{(5bc+ad)\cos(e+f}{4f\sqrt{a+b\,\mathrm{s}}}$$

$$= \frac{(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{2f\sqrt{a+b\sin(e+fx)}} - \frac{(5bc+ad)\cos(e+f}{4f\sqrt{a+b\,\mathrm{s}}}$$

$$= \frac{\sqrt{c+d}\,\left(6abcd-a^2d^2+b^2\left(3c^2+4d^2\right)\right)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{}}{\sqrt{c+d}\,\sqrt{}}\right.\right.}{}$$

$$= \frac{\sqrt{a+b}\,(c-d)\sqrt{c+d}\,(5bc+ad)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)\middle|\frac{(a-}{(a+}\right.}{}$$

Mathematica [B] time = 9.54, size = 1879, normalized size = 2.40

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/2*(d*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/f +
 ((-4*(-(b*c) + a*d)*(8*a*c^2 + 7*b*c*d + 3*a*d^2)*Sqrt[((c + d)*Cot[(-e +
Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2
- f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*
d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c +
d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((
-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/(
(a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b
*c) + a*d)*(8*b*c^2 + 12*a*c*d + 4*b*d^2)*((Sqrt[((c + d)*Cot[(-e + Pi/2 -
f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/
2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a
 + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[
(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)
*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)
*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d
)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*
d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/
(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*
x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a +
 b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2
*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]
*Sqrt[c + d*Sin[e + f*x]])) + 2*(-5*b*c*d - a*d^2)*((Cos[e + f*x]*Sqrt[c +
d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a +
 b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-
e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d))
/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e +
Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a +
b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a +
b*Sin[e + f*x]))]]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*Co
t[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e
 + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b
*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqr
t[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]
*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) +
a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
- ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Elliptic
Pi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/
2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a
 + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[
(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)
*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)
```

```
*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))/(8*f)
```

fricas [F] time = 100.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2), x)
```

maple [C] time = 6.81, size = 277000, normalized size = 353.32

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+b\,\sin\left(e+f\,x\right)}\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(3/2),x)
[Out] int((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin\left(e+fx\right)}\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e))**(3/2),x)
[Out] Integral(sqrt(a + b*sin(e + f*x))*(c + d*sin(e + f*x))**(3/2), x)
```

3.767 $\int \sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=628

$$-\frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}}+\frac{(a+b)^{3/2}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx}{(a-b)(c+d\sin(e+}}}{bf\sqrt{c+d}}$$

```
[Out] (a*d+b*c)*EllipticPi((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*si
n(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)
*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*
x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b/d/f
/(a+b)^(1/2)+(c-d)*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)
/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*si
n(f*x+e))*(a+b)^(1/2)*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*si
n(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/(
-a*d+b*c)/f+(a+b)^(3/2)*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^
(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c
+d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(
-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/b/f/(c+d)^(1/2)-b*co
s(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+b*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.33, antiderivative size = 628, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2821, 3053, 2811, 12, 2801, 2818, 2996}

$$-\frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}}+\frac{(a+b)^{3/2}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx}{(a-b)(c+d\sin(e+}}}{bf\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*S
in[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((
a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c +
 d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*
(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/((b*c - a*d)*f) + (Sqrt[c + d]
*(b*c + a*d)*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c
 + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c +
d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))
/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c
 - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(b*Sqrt[a + b]*d*f) - (b
```

```
*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(f*Sqrt[a + b*Sin[e + f*x]]) + ((a
+ b)^(3/2)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a
+ b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e
 + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x])
)]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]
*(c + d*Sin[e + f*x]))/(b*Sqrt[c + d]*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2801

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
```

```
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}\,dx = -\frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}} + \frac{\int \frac{\frac{1}{2}d\left(2a^2c-b^2c+abd\right)+ad(bc+ad)\,\mathrm{si}}{(a+b\sin(e+fx))^{3/}}}{}$$

$$= -\frac{b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}} + \frac{\int \frac{-\frac{1}{2}a^2bd(bc+ad)+\frac{1}{2}b^2d\left(2a^2c-b^2c\right.}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+}}}{b^2d}$$

$$= \frac{\sqrt{c+d}\,(bc+ad)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\mathrm{s}}{b\sqrt{}}$$

$$= \frac{\sqrt{c+d}\,(bc+ad)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\mathrm{s}}{b\sqrt{}}$$

$$= \frac{\sqrt{a+b}\,(c-d)\sqrt{c+d}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\mathrm{sec}}{(bc}$$

Mathematica [C] time = 31.71, size = 228392, normalized size = 363.68

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] Result too large to show
```

fricas [F] time = 73.83, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{b\sin\left(fx+e\right)+a}\,\sqrt{d\sin\left(fx+e\right)+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(fx+e\right)+a}\,\sqrt{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

maple [C] time = 1.85, size = 146762, normalized size = 233.70

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sin\left(fx + e\right) + a}\,\sqrt{d \sin\left(fx + e\right) + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a + b\,\sin\left(e + f\,x\right)}\,\sqrt{c + d\,\sin\left(e + f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sin\left(e + fx\right)}\,\sqrt{c + d \sin\left(e + fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(1/2)*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x))*sqrt(c + d*sin(e + f*x)), x)
```

3.768 $\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=198

$$\frac{2\sqrt{c+d}\,\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}\,\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+}}\right.\right.}{df\sqrt{a+b}}$$

```
[Out] 2*EllipticPi((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e)
)^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*si
n(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(
1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/d/f/(a+b)^(1/
2)
```

Rubi [A] time = 0.11, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2811}

$$\frac{2\sqrt{c+d}\,\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}\,\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+}}\right.\right.}{df\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (2*Sqrt[c + d]*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt
[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c
+ d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]
))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/(
(c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(Sqrt[a + b]*d*f)
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx = \frac{2\sqrt{c+d}\,\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(1-\sin(e+}{(c+d)(a+b\sin(e+}}}{\sqrt{a+b}\,df}$$

Mathematica [A] time = 0.22, size = 197, normalized size = 0.99

$$\frac{2\sqrt{c+d}\,\sec(e+fx)(a+b\sin(e+fx))\sqrt{\frac{(ad-bc)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}\,\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right.\right.}{df\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (2*Sqrt[c + d]*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt
[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c
+ d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[((-(b*c) + a*d)*(1 - Sin[e + f*x
]))/((c + d)*(a + b*Sin[e + f*x]))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/(
(c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(Sqrt[a + b]*d*f)
```

fricas [F] time = 25.18, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sin\left(fx+e\right)+a}}{\sqrt{d\sin\left(fx+e\right)+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sin\left(fx+e\right)+a}}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

maple [C] time = 5.86, size = 248299, normalized size = 1254.04

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sin\left(fx+e\right)+a}}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\,\sin\left(e+f\,x\right)}}{\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sin\left(e+fx\right)}}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(1/2),x)
[Out] Integral(sqrt(a + b*sin(e + f*x))/sqrt(c + d*sin(e + f*x)), x)
```

3.769 $\int \frac{\sqrt{a+b\sin(e+fx)}}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=409

$$\frac{2(a-b)\sqrt{a+b}\,\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+}}\right.\right.}{f(c-d)\sqrt{c+d}\,(bc-ad)}$$

```
[Out] -2*(a-b)*EllipticE((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(
f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*
(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*
d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)/(-a*d+b*c)/f/(c+d
)^(1/2)+2*(a-b)*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c
+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f
*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)
*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)/(-a*d+b*c)
/f/(c+d)^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2795, 2818, 2996}

$$\frac{2(a-b)\sqrt{a+b}\,\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+}}\right.\right.}{f(c-d)\sqrt{c+d}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*
x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c
 + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*
Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin
[e + f*x])))]*(c + d*Sin[e + f*x]))/((c - d)*Sqrt[c + d]*(b*c - a*d)*f) + (
2*(a - b)*Sqrt[a + b]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]
])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c +
 d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Si
n[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e
 + f*x])))]*(c + d*Sin[e + f*x]))/((c - d)*Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 2795

```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin
```

```
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), In
t[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sin(e+fx)}}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{(a-b)\int \frac{1}{\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}\,dx}{c-d} + \frac{(bc-ad)\int \frac{1+\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}}\,dx}{c-d}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,E\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(e}{(a+b)(c+d\sin(e}}}{(c-d)\sqrt{c+d}\,(bc-ad)f}$$

Mathematica [A] time = 7.42, size = 263, normalized size = 0.64

$$\frac{2\left(-((bc-ad)\cos(e+fx))-\frac{\sqrt{2}\sqrt{\frac{a-b}{a+b}}(a+b)(c+d)\cos\left(\frac{1}{4}(2e+2fx-\pi)\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\sqrt{\frac{(a+b)(c+d\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{a-b}{a+b}}\cos\left(\frac{1}{4}(2e+2fx+\pi\right)}{\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}\right.\right.}{\sqrt{\frac{(a+b)(\sin(e+fx)+1)}{a+b\sin(e+fx)}}}\right.}{f(c-d)(c+d)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*(-((b*c - a*d)*Cos[e + f*x]) - (Sqrt[2]*Sqrt[(a - b)/(a + b)]*(a + b)*(c
 + d)*Cos[(2*e - Pi + 2*f*x)/4]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Cos
[(2*e + Pi + 2*f*x)/4])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a
*d))/((a - b)*(c + d))]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c
 + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))])/Sqrt[((a + b)*(1 + Sin
[e + f*x]))/(a + b*Sin[e + f*x])]))/((c - d)*(c + d)*f*Sqrt[a + b*Sin[e + f
*x]]*Sqrt[c + d*Sin[e + f*x]])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{b\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+c}}{d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(d^2*cos(f*x +
e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{b\sin\left(fx+e\right)+a}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(3/2), x)
```

maple [B] time = 1.06, size = 46827, normalized size = 114.49

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \sin\left(fx+e\right)+a}}{\left(d \sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,\sin\left(e+f\,x\right)}}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sin\left(e+fx\right)}}{\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x))/(c + d*sin(e + f*x))**(3/2), x)
```

3.770 $\int \frac{\sqrt{a+b\sin(e+fx)}}{(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=489

$$\frac{2d\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f\left(c^2-d^2\right)(c+d\sin(e+fx))^{3/2}}+\frac{2(a-b)\sqrt{a+b}\left(4acd-b\left(3c^2+d^2\right)\right)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)}{(a+b)(c}}}{3f(c-d)^2(c+d}$$

```
[Out] 2/3*d*cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)/(c^2-d^2)/f/(c+d*sin(f*x+e))^(3/2)+
2/3*(a-b)*(4*a*c*d-b*(3*c^2+d^2))*EllipticE((c+d)^(1/2)*(a+b*sin(f*x+e))^(1
/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec
(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*
sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2
)/(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)^2/f+2/3*(a-b)*(3*c+d)*EllipticF((c+d)^(1/2
)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a
-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-si
n(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(
c+d*sin(f*x+e)))^(1/2)/(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)/f
```

Rubi [A] time = 0.87, antiderivative size = 489, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2796, 2998, 2818, 2996}

$$\frac{2d\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f\left(c^2-d^2\right)(c+d\sin(e+fx))^{3/2}}+\frac{2(a-b)\sqrt{a+b}\left(4acd-b\left(3c^2+d^2\right)\right)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)}{(a+b)(c}}}{3f(c-d)^2(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*d*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(3*(c^2 - d^2)*f*(c + d*Sin[e +
 f*x])^(3/2)) + (2*(a - b)*Sqrt[a + b]*(4*a*c*d - b*(3*c^2 + d^2))*Elliptic
E[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin
[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c
- a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a
*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x
]))/(3*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)^2*f) + (2*(a - b)*Sqrt[a + b]*(3
*c + d)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b
]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e +
f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*
Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c
 + d*Sin[e + f*x]))/(3*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)*f)
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sin(e+fx)}}{(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2d\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}(-3ac+bd)-\frac{1}{2}(3bc-ad)\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}}\,dx}{3\left(c^2-d^2\right)}$$

$$= \frac{2d\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{((a-b)(3c+d))\int \frac{1}{\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}}{3(c-d)^2(c+d)}$$

$$= \frac{2d\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{2(a-b)\sqrt{a+b}\left(4acd-b\left(3c^2+d^2\right)\right)E\left(\sin\right.}{}$$

Mathematica [B] time = 6.42, size = 2067, normalized size = 4.23

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((2*d*Cos[e + f*x])/(3*(
c^2 - d^2)*(c + d*Sin[e + f*x])^2) + (2*(3*b*c^2*d*Cos[e + f*x] - 4*a*c*d^2
*Cos[e + f*x] + b*d^3*Cos[e + f*x]))/(3*(b*c - a*d)*(c^2 - d^2)^2*(c + d*Si
n[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(-3*a*b*c^3 + 3*a^2*c^2*d + b^2*c^2*
d - a*b*c*d^2 + a^2*d^3 - b^2*d^3)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2
)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c +
 d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c
 + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/
2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e
+ Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*
Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(-3*b
^2*c^3 + a*b*c^2*d + 4*a^2*c*d^2 - b^2*c*d^2 - a*b*d^3)*((Sqrt[((c + d)*Cot
[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e
+ Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*
c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt
[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*
Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a
*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) -
 (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) +
a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*S
in[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d
```

```
))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 -
f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi
/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b
*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(3*b^2*c^2*d - 4*a*b*c*d^2 +
b^2*d^3)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x
]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[Arc
Sin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*
x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x
]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c
+ d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*(((
(a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Ellipt
icF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/
(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*
x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a +
 b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2
*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/
2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt
[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]
/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi
/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]
))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e +
 f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin
[e + f*x]])))/(b*d)))/(3*(c - d)^2*(c + d)^2*(-(b*c) + a*d)*f)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{b\sin\left(fx+e\right)+a}\,\sqrt{d\sin\left(fx+e\right)+c}}{3\,cd^2\cos\left(fx+e\right)^2-c^3-3\,cd^2+\left(d^3\cos\left(fx+e\right)^2-3\,c^2d-d^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(3*c*d^2*cos(f*
x + e)^2 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e
)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{b\sin\left(fx+e\right)+a}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(5/2), x)
```

maple [B] time = 4.50, size = 196704, normalized size = 402.26

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \sin\left(fx+e\right)+a}}{\left(d \sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,\sin\left(e+f\,x\right)}}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^(1/2)/(c + d*sin(e + f*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sin\left(e+fx\right)}}{\left(c+d\sin\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x))/(c + d*sin(e + f*x))**(5/2), x)
```

3.771 $\int (a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=1080

$$-\frac{d^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}\,(a+b\sin(e+fx))^{5/2}}{4bf}-\frac{d(17bc-3ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}\,(a+b\sin}{24bf}$$

```
[Out] -1/64*(20*a^3*b*c*d^3-3*a^4*d^4-60*a*b^3*c*d*(c^2+4*d^2)-6*a^2*b^2*d^2*(15*
c^2+4*d^2)+b^4*(5*c^4-120*c^2*d^2-48*d^4))*EllipticPi((a+b)^(1/2)*(c+d*sin(
f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+
d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)
*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c
-d)/(a+b*sin(f*x+e)))^(1/2)/b^3/d^2/f/(a+b)^(1/2)+1/192*(c-d)*(57*a^2*b*c*d
^2-9*a^3*d^3+a*b^2*d*(337*c^2+156*d^2)+b^3*(15*c^3+284*c*d^2))*EllipticE((a
+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*
(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(a+b)^(1/2)*(c+d)^(1/
2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1
+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b^2/d/(-a*d+b*c)/f+1/192*(a+b)^(
3/2)*(9*a^3*d^3-3*a^2*b*d^2*(17*c+6*d)+3*a*b^2*d*(73*c^2+36*c*d+28*d^2)+b^3
*(15*c^3+118*c^2*d+284*c*d^2+72*d^3))*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e)
)^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))
*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+
e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/b^3/d/
f/(c+d)^(1/2)-1/24*d*(-3*a*d+17*b*c)*cos(f*x+e)*(a+b*sin(f*x+e))^(3/2)*(c+d
*sin(f*x+e))^(1/2)/b/f-1/4*d^2*cos(f*x+e)*(a+b*sin(f*x+e))^(5/2)*(c+d*sin(f
*x+e))^(1/2)/b/f-1/192*(57*a^2*b*c*d^2-9*a^3*d^3+a*b^2*d*(337*c^2+156*d^2)+
b^3*(15*c^3+284*c*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/b/d/f/(a+b*sin(f*
x+e))^(1/2)-1/96*(54*a*b*c*d-9*a^2*d^2+b^2*(59*c^2+36*d^2))*cos(f*x+e)*(a+b
*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2)/b/f
```

Rubi [A] time = 5.24, antiderivative size = 1080, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2793, 3049, 3061, 3053, 2811, 2998, 2818, 2996}

$$-\frac{d^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}\,(a+b\sin(e+fx))^{5/2}}{4bf}-\frac{d(17bc-3ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}\,(a+b\sin}{24bf}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(57*a^2*b*c*d^2 - 9*a^3*d^3 + a*b^2*d*(337
*c^2 + 156*d^2) + b^3*(15*c^3 + 284*c*d^2))*EllipticE[ArcSin[(Sqrt[a + b]*S
```

```
qrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*
(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f
*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x])
)/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(192*b^2*d*(b*c - a
*d)*f) - (Sqrt[c + d]*(20*a^3*b*c*d^3 - 3*a^4*d^4 - 60*a*b^3*c*d*(c^2 + 4*d
^2) - 6*a^2*b^2*d^2*(15*c^2 + 4*d^2) + b^4*(5*c^4 - 120*c^2*d^2 - 48*d^4))*
EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e +
f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*
(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a
+ b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*
Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(64*b^3*Sqrt[a + b]*d^2*f) - ((57*a^2
*b*c*d^2 - 9*a^3*d^3 + a*b^2*d*(337*c^2 + 156*d^2) + b^3*(15*c^3 + 284*c*d^
2))*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(192*b*d*f*Sqrt[a + b*Sin[e + f*
x]]) - ((54*a*b*c*d - 9*a^2*d^2 + b^2*(59*c^2 + 36*d^2))*Cos[e + f*x]*Sqrt[
a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(96*b*f) - (d*(17*b*c - 3*a*d
)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]])/(24*b*f
) - (d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(5/2)*Sqrt[c + d*Sin[e + f*x]])/
(4*b*f) + ((a + b)^(3/2)*(9*a^3*d^3 - 3*a^2*b*d^2*(17*c + 6*d) + 3*a*b^2*d*
(73*c^2 + 36*c*d + 28*d^2) + b^3*(15*c^3 + 118*c^2*d + 284*c*d^2 + 72*d^3))
*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[
c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sq
rt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(
((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Si
n[e + f*x]))/(192*b^3*d*Sqrt[c + d]*f)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
```

```
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
```

```
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}\,dx = -\frac{d^2\cos(e+fx)(a+b\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}}{4bf} + \cdots$$

$$= -\frac{d(17bc-3ad)\cos(e+fx)(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e \cdots}}{24bf}$$

$$= -\frac{\left(54abcd-9a^2d^2+b^2\left(59c^2+36d^2\right)\right)\cos(e+fx)\sqrt{a+b\sin \cdots}}{96bf}$$

$$= -\frac{\left(57a^2bcd^2-9a^3d^3+ab^2d\left(337c^2+156d^2\right)+b^3\left(15c^3+284 \cdots\right.\right.}{192bdf\sqrt{a+b\sin(e+f \cdots}}$$

$$= -\frac{\left(57a^2bcd^2-9a^3d^3+ab^2d\left(337c^2+156d^2\right)+b^3\left(15c^3+284 \cdots\right.\right.}{192bdf\sqrt{a+b\sin(e+f \cdots}}$$

$$= -\frac{\sqrt{c+d}\,\left(20a^3bcd^3-3a^4d^4-60ab^3cd\left(c^2+4d^2\right)-6a^2b^2d^2\left(\cdots\right.\right.}{\cdots}$$

$$= \frac{\sqrt{a+b}\,(c-d)\sqrt{c+d}\,\left(57a^2bcd^2-9a^3d^3+ab^2d\left(337c^2+156 \cdots\right.\right.}{\cdots}$$

Mathematica [A] time = 7.53, size = 2091, normalized size = 1.94

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/384*((-4*(-(b*c) + a*d)*(-384*a^2*b*c^3 - 133*b^3*c^3 - 971*a*b^2*c^2*d
- 451*a^2*b*c*d^2 - 356*b^3*c*d^2 + 3*a^3*d^3 - 228*a*b^2*d^3)*Sqrt[((c + d
)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc
[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*
(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4
*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a
*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c
```

```
) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]) - 4*(-(b*c) + a*d)*(-532*a*b^2*c^3 - 664*a^2*b*c^2*d - 644*b^3*c^2*d +
12*a^3*c*d^2 - 1160*a*b^2*c*d^2 - 228*a^2*b*d^3 - 144*b^3*d^3)*((Sqrt[((c +
 d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*C
sc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (
2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]
^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) +
 a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b
*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f
*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(
b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(
c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*
(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e +
Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(
-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqr
t[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(15*b^3*c^3 + 337*a*b^
2*c^2*d + 57*a^2*b*c*d^2 + 284*b^3*c*d^2 - 9*a^3*d^3 + 156*a*b^2*d^3)*((Cos
[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a
 - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a -
 b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]],
 (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[
((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*
x]))/((c + d)*(a + b*Sin[e + f*x]))]]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*
d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqr
t[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)
]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + P
i/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x
]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e
+ f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c
+ d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)
/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc
[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*
(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4
*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a
*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c
) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/
(b*d)))/(b*f) + (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*(-1/96*(
(59*b^2*c^2 + 122*a*b*c*d + 3*a^2*d^2 + 42*b^2*d^2)*Cos[e + f*x])/b + (b*d^
2*Cos[3*(e + f*x)])/16 - (d*(17*b*c + 9*a*d)*Sin[2*(e + f*x)])/48))/f
```

fricas [F] time = 81.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(ac^2 + 2\,bcd + ad^2 - \left(2\,bcd + ad^2\right)\cos\left(fx+e\right)^2 - \left(bd^2\cos\left(fx+e\right)^2 - bc^2 - 2\,acd - bd^2\right)\sin\left(fx+e\right)\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral((a*c^2 + 2*b*c*d + a*d^2 - (2*b*c*d + a*d^2)*cos(f*x + e)^2 - (b*d
^2*cos(f*x + e)^2 - b*c^2 - 2*a*c*d - b*d^2)*sin(f*x + e))*sqrt(b*sin(f*x +
 e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(5/2), x)
```

maple [B] time = 27.13, size = 577718, normalized size = 534.92

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(3/2)*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.772 $\int (a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=870

$$-\frac{\left(-\left(\left(3c^2+14dc+16d^2\right)b^2\right)-6ad(4c+d)b+3a^2d^2\right)F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-}{(a+}}}{24b^2d\sqrt{c+d}\,f}$$

```
[Out] 1/8*(a*d+b*c)*(10*a*b*c*d-a^2*d^2-b^2*(c^2-12*d^2))*EllipticPi((a+b)^(1/2)*
(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),b*(c+d)/(a+b)/d,(
(a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(
-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f
*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b^2/d^2/f/(a+b)^(1/2)+1/24*(c-d)*(38*a
*b*c*d+3*a^2*d^2+b^2*(3*c^2+16*d^2))*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))
^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*
sec(f*x+e)*(a+b*sin(f*x+e))*(a+b)^(1/2)*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x
+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*si
n(f*x+e)))^(1/2)/b/d/(-a*d+b*c)/f-1/3*b*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)*(
a+b*sin(f*x+e))^(1/2)/f-1/24*(a+b)^(3/2)*(3*a^2*d^2-6*a*b*d*(4*c+d)-b^2*(3*
c^2+14*c*d+16*d^2))*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2
)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*s
in(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d
+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/b^2/d/f/(c+d)^(1/2)-1/24
*(38*a*b*c*d+3*a^2*d^2+b^2*(3*c^2+16*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2
)/d/f/(a+b*sin(f*x+e))^(1/2)-1/12*(7*a*d+3*b*c)*cos(f*x+e)*(a+b*sin(f*x+e))
^(1/2)*(c+d*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 3.36, antiderivative size = 870, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2821, 3049, 3061, 3053, 2811, 2998, 2818, 2996}

$$-\frac{\left(-\left(3c^2+14dc+16d^2\right)b^2-6ad(4c+d)b+3a^2d^2\right)F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad}{(a+b)(}}}{24b^2d\sqrt{c+d}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(38*a*b*c*d + 3*a^2*d^2 + b^2*(3*c^2 + 16*
d^2))*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*
Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*
x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]
*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a +
```

```
b*Sin[e + f*x]))/(24*b*d*(b*c - a*d)*f) + (Sqrt[c + d]*(b*c + a*d)*(10*a*b
*c*d - a^2*d^2 - b^2*(c^2 - 12*d^2))*EllipticPi[(b*(c + d))/((a + b)*d), Ar
cSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e +
 f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c -
a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)
*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/
(8*b^2*Sqrt[a + b]*d^2*f) - ((38*a*b*c*d + 3*a^2*d^2 + b^2*(3*c^2 + 16*d^2)
)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(24*d*f*Sqrt[a + b*Sin[e + f*x]])
- ((3*b*c + 7*a*d)*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e +
 f*x]])/(12*f) - ((a + b)^(3/2)*(3*a^2*d^2 - 6*a*b*d*(4*c + d) - b^2*(3*c^2
 + 14*c*d + 16*d^2))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]]
)/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c +
d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin
[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e
+ f*x])))]*(c + d*Sin[e + f*x]))/(24*b^2*d*Sqrt[c + d]*f) - (b*Cos[e + f*x]
*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2))/(3*f)
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
```

```
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
```

```
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}\,dx = -\frac{b\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}}{3f} + \frac{\int \frac{\sqrt{c}}{}}{}$$

$$= -\frac{(3bc+7ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}{12f}$$

$$= -\frac{\left(38abcd+3a^2d^2+b^2\left(3c^2+16d^2\right)\right)\cos(e+fx)\sqrt{c+d\sin(e}}{24df\sqrt{a+b\sin(e+fx)}}$$

$$= -\frac{\left(38abcd+3a^2d^2+b^2\left(3c^2+16d^2\right)\right)\cos(e+fx)\sqrt{c+d\sin(e}}{24df\sqrt{a+b\sin(e+fx)}}$$

$$= \frac{\sqrt{c+d}\,(bc+ad)\left(10abcd-a^2d^2-b^2\left(c^2-12d^2\right)\right)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin\right.}{}$$

$$= \frac{\sqrt{a+b}\,(c-d)\sqrt{c+d}\,\left(38abcd+3a^2d^2+b^2\left(3c^2+16d^2\right)\right)E\left(\sin\right.}{}$$

Mathematica [B] time = 6.31, size = 1952, normalized size = 2.24

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((-4*(-(b*c) + a*d)*(48*a^2*c^2 + 17*b^2*c^2 + 82*a*b*c*d + 17*a^2*d^2 + 16
*b^2*d^2)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[Arc
Sin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
 + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[
(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[
e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d
*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*
Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(68*a*b*c^2 + 68*a^2*c*d + 52*
b^2*c*d + 52*a*b*d^2)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]
*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e +
f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec
[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]
^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*
x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*
Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f
*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a
 - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt
[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 -
f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-
(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]
))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e +
f*x]])) + 2*(-3*b^2*c^2 - 38*a*b*c*d - 3*a^2*d^2 - 16*b^2*d^2)*((Cos[e + f*
x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(
a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a
+ b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(
b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b
)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]
]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((
c + d)*(a + b*Sin[e + f*x]))])) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt
[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a
- b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[
2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f
*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(
b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x])
)/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin
[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c +
d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e +
```

```
Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c)
 + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[(
(c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sq
rt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d
)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))
/(48*f) + (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-7*(b*c + a*
d)*Cos[e + f*x])/12 - (b*d*Sin[2*(e + f*x)])/6))/f
```

fricas [F] time = 24.38, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(bd\cos\left(fx+e\right)^{2}-ac-bd-(bc+ad)\sin\left(fx+e\right)\right)\sqrt{b\sin\left(fx+e\right)+a}\,\sqrt{d\sin\left(fx+e\right)+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b*d*cos(f*x + e)^2 - a*c - b*d - (b*c + a*d)*sin(f*x + e))*sqrt(
b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [C] time = 17.43, size = 409354, normalized size = 470.52

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + b \sin\left(e + f x\right)\right)^{3/2} \left(c + d \sin\left(e + f x\right)\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(3/2)*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.773 $\int (a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=740

$$\frac{\sqrt{c+d}\left(3a^2d^2+6abcd-\left(b^2\left(c^2-4d^2\right)\right)\right)\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}}{4bd^2f\sqrt{a+b}}$$

```
[Out] 1/4*(6*a*b*c*d+3*a^2*d^2-b^2*(c^2-4*d^2))*EllipticPi((a+b)^(1/2)*(c+d*sin(f
*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d
)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*
(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-
d)/(a+b*sin(f*x+e)))^(1/2)/b/d^2/f/(a+b)^(1/2)+1/4*(c-d)*(5*a*d+b*c)*Ellipt
icE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),(
(a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(a+b)^(1/2)*(c+
d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b
*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/d/(-a*d+b*c)/f+1/4*(a+b)^(
3/2)*(3*a*d+b*(c+2*d))*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(
1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+
d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-
a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/b/d/f/(c+d)^(1/2)-1/4
*b*(5*a*d+b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d/f/(a+b*sin(f*x+e))^(1/2)
-1/2*b*cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 2.29, antiderivative size = 740, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2821, 3061, 3053, 2811, 2998, 2818, 2996}

$$\frac{\sqrt{c+d}\left(3a^2d^2+6abcd+b^2\left(-\left(c^2-4d^2\right)\right)\right)\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}}{4bd^2f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(b*c + 5*a*d)*EllipticE[ArcSin[(Sqrt[a + b
]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a -
b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e
+ f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*
x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(4*d*(b*c - a*d)
*f) + (Sqrt[c + d]*(6*a*b*c*d + 3*a^2*d^2 - b^2*(c^2 - 4*d^2))*EllipticPi[(
b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt
[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Se
```

```
c[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e +
f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]
))]*(a + b*Sin[e + f*x]))/(4*b*Sqrt[a + b]*d^2*f) - (b*(b*c + 5*a*d)*Cos[e
+ f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*d*f*Sqrt[a + b*Sin[e + f*x]]) - (b*Cos[
e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(2*f) + ((a + b
)^(3/2)*(3*a*d + b*(c + 2*d))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[
e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a -
 b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(
c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c +
 d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(4*b*d*Sqrt[c + d]*f)
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}\,dx = -\frac{b\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}{2f} + \frac{\int \frac{\frac{1}{2}d(4}{}}{}$$

$$= -\frac{b(bc+5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4df\sqrt{a+b\sin(e+fx)}} - \frac{b\cos(e+fx)\sqrt{}}{}$$

$$= -\frac{b(bc+5ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4df\sqrt{a+b\sin(e+fx)}} - \frac{b\cos(e+fx)\sqrt{}}{}$$

$$= \frac{\sqrt{c+d}\left(6ac-\frac{bc^2}{d}+\frac{3a^2d}{b}+4bd\right)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e}}{\sqrt{c+d}\,\sqrt{a+b\sin(e}}\right.\right.}{}$$

$$= \frac{\sqrt{a+b}\,(c-d)\sqrt{c+d}\,(bc+5ad)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a}{(a}\right.}{}$$

Mathematica [B] time = 9.48, size = 1879, normalized size = 2.54

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] -1/2*(b*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/f +
 ((-4*(-(b*c) + a*d)*(8*a^2*c + 3*b^2*c + 7*a*b*d)*Sqrt[((c + d)*Cot[(-e +
Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2
- f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*
d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c +
d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((
-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/(
(a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b
*c) + a*d)*(12*a*b*c + 8*a^2*d + 4*b^2*d)*((Sqrt[((c + d)*Cot[(-e + Pi/2 -
f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/
2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a
 + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[
(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)
*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)
*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d
```

```
)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*
d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/
(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*
x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a +
 b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2
*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]
*Sqrt[c + d*Sin[e + f*x]])) + 2*(-(b^2*c) - 5*a*b*d)*((Cos[e + f*x]*Sqrt[c
+ d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a
 + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[
(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d
))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e
+ Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a
+ b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a
+ b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*
Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(
-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-
(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*S
qrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d
)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
+ a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]
) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Ellipt
icPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x
)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/(
(a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Cs
c[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a -
b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a +
b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))/(8*f)
```

fricas [F] time = 7.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{d\sin\left(fx+e\right)+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c), x)
```

maple [C] time = 8.97, size = 278658, normalized size = 376.56

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\sin\left(e+f\,x\right)\right)}^{3/2}\,\sqrt{c+d\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(e+fx\right)\right)^{\frac{3}{2}}\sqrt{c+d\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(3/2)*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**(3/2)*sqrt(c + d*sin(e + f*x)), x)
```

3.774 $\int \frac{(a+b\sin(e+fx))^{3/2}}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=644

$$\frac{\sqrt{a+b}\,(b(c-d)-2ad)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+}}{\sqrt{a+b}\,\sqrt{c+}}\right.\right.}{d^2f\sqrt{c+d}}$$

```
[Out] (b*(c-d)-2*a*d)*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c
+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f
*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)
*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/d^2/f/(c+d)^(1/2
)-(-3*a*d+b*c)*EllipticPi((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c
+d*sin(f*x+e))^(1/2),(a+b)*d/b/(c+d),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f
*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*si
n(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/
d^2/f/(c+d)^(1/2)-(a-b)*b*EllipticE((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b
)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*
(c+d*sin(f*x+e))*(a+b)^(1/2)*(c+d)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(
c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^
(1/2)/d/(-a*d+b*c)/f-b*cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)/f/(c+d*sin(f*x+e))
^(1/2)
```

Rubi [A] time = 1.54, antiderivative size = 644, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2821, 3053, 2811, 2998, 2818, 2996}

$$\frac{\sqrt{a+b}\,(b(c-d)-2ad)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+}}{\sqrt{a+b}\,\sqrt{c+}}\right.\right.}{d^2f\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(3/2)/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] -((b*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(f*Sqrt[c + d*Sin[e + f*x]])) -
 ((a - b)*b*Sqrt[a + b]*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a +
b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))
/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a
+ b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b
)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(d*(b*c - a*d)*f) + (Sqrt[a
 + b]*(b*(c - d) - 2*a*d)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e +
f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*
```

```
(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c +
d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*S
in[e + f*x])))]*(c + d*Sin[e + f*x]))/(d^2*Sqrt[c + d]*f) - (Sqrt[a + b]*(b
*c - 3*a*d)*EllipticPi[((a + b)*d)/(b*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[a
+ b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d
))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((
a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a -
 b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(d^2*Sqrt[c + d]*f)
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
```

```
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^{3/2}}{\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{b\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{f\sqrt{c+d\sin(e+fx)}} + \frac{\int \frac{\frac{1}{2}d\left(2a^2c+b^2c-abd\right)+ad(bc+ad)\sin(e+fx)-\frac{1}{2}bd(bc-3ad)}{\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}}}{d}$$

$$= -\frac{b\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{f\sqrt{c+d\sin(e+fx)}} + \frac{\int \frac{\frac{1}{2}bc^2d(bc-3ad)+\frac{1}{2}d^3\left(2a^2c+b^2c-abd\right)+d\left(bcd(bc-3ad)+a\right.}{\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/}}}{d^3}$$

$$= -\frac{b\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{f\sqrt{c+d\sin(e+fx)}} - \frac{\sqrt{a+b}\,(bc-3ad)\Pi\left(\frac{(a+b)d}{b(c+d)};\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b}}{\sqrt{a+b}\,\sqrt{c+d}}\right.\right.}{}$$

$$= -\frac{b\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{f\sqrt{c+d\sin(e+fx)}} - \frac{(a-b)b\sqrt{a+b}\,\sqrt{c+d}\,E\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin}}{\sqrt{a+b}\,\sqrt{c+d\sin}}\right.\right.}{}$$

Mathematica [C] time = 32.69, size = 222963, normalized size = 346.22

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(3/2)/Sqrt[c + d*Sin[e + f*x]],x]
[Out] Result too large to show
```

fricas [F] time = 2.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\sqrt{d\sin\left(fx+e\right)+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
[Out] integral((b*sin(f*x + e) + a)^(3/2)/sqrt(d*sin(f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)/sqrt(d*sin(f*x + e) + c), x)
```

maple [B] time = 10.60, size = 529691, normalized size = 822.50

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \sin\left(fx + e\right) + a\right)^{\frac{3}{2}}}{\sqrt{d \sin\left(fx + e\right) + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)/sqrt(d*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{3/2}}{\sqrt{c+d\,\sin\left(e+f\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \sin{\left(e + fx \right)}\right)^{\frac{3}{2}}}{\sqrt{c + d \sin{\left(e + fx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**(3/2)/sqrt(c + d*sin(e + f*x)), x)
```

3.775 $\int \frac{(a+b\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=600

$$-\frac{2\sqrt{a+b}\,(ad+b(c-2d))\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\sin^{-1}\left(\frac{\sqrt{c+d}}{\sqrt{a+b}}\right.\right.}{d^2f(c-d)\sqrt{c+d}}$$

```
[Out] 2*(a-b)*EllipticE((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f
*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(
a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d
+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)/d/f/(c+d)^(1/2)-2*
(b*(c-2*d)+a*d)*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c
+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f
*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)
*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)/d^2/f/(c+d
)^(1/2)+2*b*EllipticPi((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*
sin(f*x+e))^(1/2),(a+b)*d/b/(c+d),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+
e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f
*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/d^2
/f/(c+d)^(1/2)
```

Rubi [A] time = 0.93, antiderivative size = 600, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2798, 2811, 2998, 2818, 2996}

$$-\frac{2\sqrt{a+b}\,(ad+b(c-2d))\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\sin^{-1}\left(\frac{\sqrt{c+d}}{\sqrt{a+b}}\right.\right.}{d^2f(c-d)\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x
]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c
+ d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*S
in[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[
e + f*x])))]*(c + d*Sin[e + f*x]))/((c - d)*d*Sqrt[c + d]*f) - (2*Sqrt[a +
b]*(b*(c - 2*d) + a*d)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x
]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c
+ d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*S
in[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[
```

```
e + f*x]))))]*(c + d*Sin[e + f*x]))/((c - d)*d^2*Sqrt[c + d]*f) + (2*b*Sqrt[
a + b]*EllipticPi[((a + b)*d)/(b*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[a + b*S
in[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((
a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b
)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(
c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(d^2*Sqrt[c + d]*f)
```

Rule 2798

```
Int[((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((a_.) + (b_.)*sin[(e_.) +
 (f_.)*(x_)])^(3/2), x_Symbol] :> Dist[d^2/b^2, Int[Sqrt[a + b*Sin[e + f*x]
]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b^2, Int[Simp[b*c + a
*d + 2*b*d*Sin[e + f*x], x]/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e +
f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
```

```
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{b^2\int \frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}}\,dx}{d^2} - \frac{(bc-ad)\int \frac{bc+ad+2bd\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}}\,dx}{d^2}$$

$$= \frac{2b\sqrt{a+b}\,\Pi\left(\frac{(a+b)d}{b(c+d)};\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(}{(a+b)(c+d\sin(}}}{d^2\sqrt{c+d}\,f}$$

$$= \frac{2(a-b)\sqrt{a+b}\,E\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(e+}{(a+b)(c+d\sin(e+}}}{(c-d)d\sqrt{c+d}\,f}$$

Mathematica [B] time = 9.56, size = 1896, normalized size = 3.16

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(b*c*Cos[e + f*x] - a*d*Cos[e + f*x])*Sqrt[a + b*Sin[e + f*x]])/((c^2 -
 d^2)*f*Sqrt[c + d*Sin[e + f*x]]) + ((-4*(-(b*c) + a*d)*(a^2*c - a*b*d)*Sqr
t[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a
 - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt
[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 -
f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-
(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]
```

```
))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]) - 4*(-(b*c) + a*d)*(a^2*d - b^2*d)*((Sqrt[((c + d)*Cot[(-e + P
i/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 -
 f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d
))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d
)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-
a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((
a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[(
(c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a
 + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f
*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[
e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^
2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x
)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(b^2*c - a*b*d)*((Cos[e + f*x]*Sqrt[c
 + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(
a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin
[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*
d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e
 + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a
 + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a
 + b*Sin[e + f*x]))]]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)
*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[
(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(
-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*
Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*
d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
 + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]
]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Ellip
ticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*
x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/
((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*C
sc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a -
 b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a +
 b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d))/((c - d)
*(c + d)*f)
```

fricas [F] time = 1.82, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(-\frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{d\sin\left(fx+e\right)+c}}{d^{2}\cos\left(fx+e\right)^{2}-2\,cd\sin\left(fx+e\right)-c^{2}-d^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c)/(d^2*cos(f*x
+ e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) + c)^(3/2), x)
```

maple [B] time = 133.54, size = 2626418, normalized size = 4377.36

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \sin\left(e + fx\right)\right)^{\frac{3}{2}}}{\left(c + d \sin\left(e + fx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**(3/2)/(c + d*sin(e + f*x))**(3/2), x)
```

3.776 $\int \frac{(a+b\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=497

$$-\frac{2(bc-ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f\left(c^2-d^2\right)(c+d\sin(e+fx))^{3/2}}+\frac{2(a-b)\sqrt{a+b}\,(a(3c+d)-b(c+3d))\sec(e+fx)(c+d\sin(e+fx)}{3f(c-}$$

```
[Out] -2/3*(-a*d+b*c)*cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)/(c^2-d^2)/f/(c+d*sin(f*x+
e))^(3/2)-8/3*(a-b)*(a*c-b*d)*EllipticE((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/
(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x
+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(
f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c
-d)^2/(c+d)^(3/2)/(-a*d+b*c)/f+2/3*(a-b)*(a*(3*c+d)-b*(c+3*d))*EllipticF((c
+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*
(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*
c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))
/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)/f
```

Rubi [A] time = 0.95, antiderivative size = 497, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2799, 2998, 2818, 2996}

$$-\frac{2(bc-ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f\left(c^2-d^2\right)(c+d\sin(e+fx))^{3/2}}+\frac{2(a-b)\sqrt{a+b}\,(a(3c+d)-b(c+3d))\sec(e+fx)(c+d\sin(e+fx)}{3f(c-}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*(b*c - a*d)*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(3*(c^2 - d^2)*f*(c
+ d*Sin[e + f*x])^(3/2)) - (8*(a - b)*Sqrt[a + b]*(a*c - b*d)*EllipticE[Arc
Sin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e +
f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d
)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(
3*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)*f) + (2*(a - b)*Sqrt[a + b]*(a*(3*c +
 d) - b*(c + 3*d))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/
(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d)
)]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e
 + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e +
f*x])))]*(c + d*Sin[e + f*x]))/(3*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)*f)
```

Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^{3/2}}{(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{2(bc-ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(-3a^2c-b^2c+4abd\right)-\frac{1}{2}\left(4abc-a^2d-3b^2d\right)\sin}{\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}}}{3\left(c^2-d^2\right)}$$

$$= -\frac{2(bc-ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{(4(bc-ad)(ac-bd))\int \frac{1+\sin}{\sqrt{a+b\sin(e+fx)}}}{3(c-d)^2(c+d)}$$

$$= -\frac{2(bc-ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{8(a-b)\sqrt{a+b}\,(ac-bd)E\left(\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{a}}\right.\right.}{}$$

Mathematica [B] time = 6.34, size = 2012, normalized size = 4.05

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(3/2)/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-2*(b*c*Cos[e + f*x] -
 a*d*Cos[e + f*x]))/(3*(c^2 - d^2)*(c + d*Sin[e + f*x])^2) - (8*(-(a*c*d*Co
s[e + f*x]) + b*d^2*Cos[e + f*x]))/(3*(c^2 - d^2)^2*(c + d*Sin[e + f*x]))))
/f + ((-4*(-(b*c) + a*d)*(3*a^2*c^2 + b^2*c^2 - 4*a*b*c*d + a^2*d^2 - b^2*d
^2)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sq
rt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d
)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e +
Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*
x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e
 + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c
 + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(4*a*b*c^2 + 4*a^2*c*d - 4*b^2*c*d -
 4*a*b*d^2)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF
[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(
b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*
Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*
Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c
 + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/
(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[
(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(
-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*
```

```
Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*
d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
 + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) +
2*(-4*a*b*c*d + 4*b^2*d^2)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt
[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x
)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt
[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt
[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b
*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]
*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*
(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2
)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c +
 d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c
 + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/
2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e
+ Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*
Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c
+ d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a +
b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]
))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e +
 f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(
a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2
]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]))))/(b*d)))/(3*(c - d)^2*(c + d)^2*f)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{d\sin\left(fx+e\right)+c}}{3\,cd^{2}\cos\left(fx+e\right)^{2}-c^{3}-3\,cd^{2}+\left(d^{3}\cos\left(fx+e\right)^{2}-3\,c^{2}d-d^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c)/(3*c*d^2*cos(
f*x + e)^2 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x +
 e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) + c)^(5/2), x)
```

maple [B] time = 3.43, size = 190874, normalized size = 384.05

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\sin\left(e+f\,x\right)\right)^{3/2}}{\left(c+d\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^(3/2)/(c + d*sin(e + f*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sin\left(e+fx\right)\right)^{\frac{3}{2}}}{\left(c+d\sin\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**(3/2)/(c + d*sin(e + f*x))**(5/2), x)
```

3.777 $\int (a+b\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=1295

$$-\frac{b^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{7/2}}{5df}+\frac{3b(bc-7ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e}{40df}$$

```
[Out] -1/128*(a*d+b*c)*(28*a^3*b*c*d^3-3*a^4*d^4+28*a*b^3*c*d*(c^2-20*d^2)-2*a^2*
b^2*d^2*(89*c^2+20*d^2)-b^4*(3*c^4+40*c^2*d^2+240*d^4))*EllipticPi((a+b)^(1
/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),b*(c+d)/(a+b)
/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)
*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+s
in(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b^3/d^3/f/(a+b)^(1/2)+1/1920*(c-d)
*(360*a^3*b*c*d^3-45*a^4*d^4+2*a^2*b^2*d^2*(1877*c^2+846*d^2)+8*a*b^3*d*(45
*c^3+791*c*d^2)-b^4*(45*c^4-1692*c^2*d^2-1024*d^4))*EllipticE((a+b)^(1/2)*(
c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)
/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(a+b)^(1/2)*(c+d)^(1/2)*(-(-a*d+
b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e)
)/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b^2/d^2/(-a*d+b*c)/f-1/240*(110*a*b*c*d+93*
a^2*d^2-b^2*(15*c^2-64*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^(3/2)*(a+b*sin(f*x
+e))^(1/2)/d/f+3/40*b*(-7*a*d+b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^(5/2)*(a+b*s
in(f*x+e))^(1/2)/d/f-1/5*b^2*cos(f*x+e)*(c+d*sin(f*x+e))^(7/2)*(a+b*sin(f*x
+e))^(1/2)/d/f+1/1920*(a+b)^(3/2)*(45*a^4*d^4-30*a^3*b*d^3*(11*c+3*d)+30*a^
2*b^2*d^2*(64*c^2+23*c*d+22*d^2)+2*a*b^3*d*(165*c^3+917*c^2*d+2392*c*d^2+51
6*d^3)-b^4*(45*c^4-30*c^3*d-1692*c^2*d^2-1544*c*d^3-1024*d^4))*EllipticF((c
+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*
(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*
x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*
sin(f*x+e)))^(1/2)/b^3/d^2/f/(c+d)^(1/2)-1/1920*(360*a^3*b*c*d^3-45*a^4*d^4
+2*a^2*b^2*d^2*(1877*c^2+846*d^2)+8*a*b^3*d*(45*c^3+791*c*d^2)-b^4*(45*c^4-
1692*c^2*d^2-1024*d^4))*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/b/d^2/f/(a+b*sin(
f*x+e))^(1/2)-1/960*(917*a^2*b*c*d^2+15*a^3*d^3+a*b^2*d*(345*c^2+772*d^2)-b
^3*(45*c^3-516*c*d^2))*cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(
1/2)/b/d/f
```

Rubi [A] time = 7.84, antiderivative size = 1295, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2793, 3049, 3061, 3053, 2811, 2998, 2818, 2996}

$$-\frac{b^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{7/2}}{5df}+\frac{3b(bc-7ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e}{40df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(360*a^3*b*c*d^3 - 45*a^4*d^4 + 2*a^2*b^2*
d^2*(1877*c^2 + 846*d^2) + 8*a*b^3*d*(45*c^3 + 791*c*d^2) - b^4*(45*c^4 - 1
692*c^2*d^2 - 1024*d^4))*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f
*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(
c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a +
 b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*S
in[e + f*x]))]*(a + b*Sin[e + f*x]))/(1920*b^2*d^2*(b*c - a*d)*f) - (Sqrt[c
 + d]*(b*c + a*d)*(28*a^3*b*c*d^3 - 3*a^4*d^4 + 28*a*b^3*c*d*(c^2 - 20*d^2)
 - 2*a^2*b^2*d^2*(89*c^2 + 20*d^2) - b^4*(3*c^4 + 40*c^2*d^2 + 240*d^4))*El
lipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*
x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c
 - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a +
b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Si
n[e + f*x]))]*(a + b*Sin[e + f*x]))/(128*b^3*Sqrt[a + b]*d^3*f) - ((360*a^3
*b*c*d^3 - 45*a^4*d^4 + 2*a^2*b^2*d^2*(1877*c^2 + 846*d^2) + 8*a*b^3*d*(45*
c^3 + 791*c*d^2) - b^4*(45*c^4 - 1692*c^2*d^2 - 1024*d^4))*Cos[e + f*x]*Sqr
t[c + d*Sin[e + f*x]])/(1920*b*d^2*f*Sqrt[a + b*Sin[e + f*x]]) - ((917*a^2*
b*c*d^2 + 15*a^3*d^3 + a*b^2*d*(345*c^2 + 772*d^2) - b^3*(45*c^3 - 516*c*d^
2))*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(960*b*
d*f) + ((a + b)^(3/2)*(45*a^4*d^4 - 30*a^3*b*d^3*(11*c + 3*d) + 30*a^2*b^2*
d^2*(64*c^2 + 23*c*d + 22*d^2) + 2*a*b^3*d*(165*c^3 + 917*c^2*d + 2392*c*d^
2 + 516*d^3) - b^4*(45*c^4 - 30*c^3*d - 1692*c^2*d^2 - 1544*c*d^3 - 1024*d^
4))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sq
rt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]
*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt
[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d
*Sin[e + f*x]))/(1920*b^3*d^2*Sqrt[c + d]*f) - ((110*a*b*c*d + 93*a^2*d^2 -
 b^2*(15*c^2 - 64*d^2))*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e
+ f*x])^(3/2))/(240*d*f) + (3*b*(b*c - 7*a*d)*Cos[e + f*x]*Sqrt[a + b*Sin[e
 + f*x]]*(c + d*Sin[e + f*x])^(5/2))/(40*d*f) - (b^2*Cos[e + f*x]*Sqrt[a +
b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(7/2))/(5*d*f)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
```

```
NeQ[c, 0])))
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
&& NeQ[A, B]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{5/2}\,dx = -\frac{b^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{7/2}}{5df} + \frac{\int \frac{(c}{-}}{}$$

$$= \frac{3b(bc-7ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))}{40df}$$

$$= -\frac{\left(110abcd+93a^2d^2-b^2\left(15c^2-64d^2\right)\right)\cos(e+fx)\sqrt{a+b\sin}}{240df}$$

$$= -\frac{\left(917a^2bcd^2+15a^3d^3+ab^2d\left(345c^2+772d^2\right)-b^3\left(45c^3-51\right.\right.}{960bd}$$

$$= -\frac{\left(360a^3bcd^3-45a^4d^4+2a^2b^2d^2\left(1877c^2+846d^2\right)+8ab^3d\left(4\right.\right.}{1920}$$

$$= -\frac{\left(360a^3bcd^3-45a^4d^4+2a^2b^2d^2\left(1877c^2+846d^2\right)+8ab^3d\left(4\right.\right.}{1920}$$

$$= -\frac{\sqrt{c+d}\,(bc+ad)\left(28a^3bcd^3-3a^4d^4+28ab^3cd\left(c^2-20d^2\right)-\right.}{}$$

$$= \frac{\sqrt{a+b}\,(c-d)\sqrt{c+d}\,\left(360a^3bcd^3-45a^4d^4+2a^2b^2d^2\left(1877c^2\right.\right.}{}$$

Mathematica [**A**] time = 8.47, size = 2276, normalized size = 1.76

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((-4*(-(b*c) + a*d)*(-15*b^4*c^4 + 3840*a^3*b*c^3*d + 4456*a*b^3*c^3*d + 14
702*a^2*b^2*c^2*d^2 + 3236*b^4*c^2*d^2 + 4456*a^3*b*c*d^3 + 10440*a*b^3*c*d
^3 - 15*a^4*d^4 + 3236*a^2*b^2*d^4 + 1024*b^4*d^4)*Sqrt[((c + d)*Cot[(-e +
Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2
```

- f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(-60*a*b^3*c^4 + 6364*a^2*b^2*c^3*d + 2292*b^4*c^3*d + 6364*a^3*b*c^2*d^2 + 17020*a*b^3*c^2*d^2 - 60*a^4*c*d^3 + 17020*a^2*b^2*c*d^3 + 4624*b^4*c*d^3 + 2292*a^3*b*d^4 + 4624*a*b^3*d^4)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(45*b^4*c^4 - 360*a*b^3*c^3*d - 3754*a^2*b^2*c^2*d^2 - 1692*b^4*c^2*d^2 - 360*a^3*b*c*d^3 - 6328*a*b^3*c*d^3 + 45*a^4*d^4 - 1692*a^2*b^2*d^4 - 1024*b^4*d^4)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))/(3840*b*d*f) + (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*(-1/960*((15*b^3*c^3 + 1289*a*b^2*c^2*d + 1289*a^2*b*c*d^2 + 898*b^3*c*d^2 + 15*a^3*d^3 + 898*a*b^2*d^3

```
)*Cos[e + f*x])/(b*d) + (21*b*d*(b*c + a*d)*Cos[3*(e + f*x)])/160 - ((93*b^
2*c^2 + 362*a*b*c*d + 93*a^2*d^2 + 88*b^2*d^2)*Sin[2*(e + f*x)])/480 + (b^2
*d^2*Sin[4*(e + f*x)])/40))/f
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [B] time = 79.80, size = 755109, normalized size = 583.10

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d \sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\sin\left(e+f\,x\right)\right)^{5/2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(5/2),x)
[Out] int((a + b*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(5/2)*(c+d*sin(f*x+e))**(5/2),x)
[Out] Timed out
```

3.778 $\int (a+b\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=1071

$$-\frac{b^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}}{4df}+\frac{b(3bc-17ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,(c+d\sin}{24df}$$

```
[Out] 1/64*(60*a^3*b*c*d^3-5*a^4*d^4-20*a*b^3*c*d*(c^2-12*d^2)+3*b^4*(c^2+4*d^2)^
2+30*a^2*b^2*d^2*(3*c^2+4*d^2))*EllipticPi((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/
2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c
-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x
+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*si
n(f*x+e)))^(1/2)/b^2/d^3/f/(a+b)^(1/2)+1/192*(c-d)*(337*a^2*b*c*d^2+15*a^3*
d^3+a*b^2*d*(57*c^2+284*d^2)-b^3*(9*c^3-156*c*d^2))*EllipticE((a+b)^(1/2)*(
c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)
/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(a+b)^(1/2)*(c+d)^(1/2)*(-(-a*d+
b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e)
)/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b/d^2/(-a*d+b*c)/f+1/24*b*(-17*a*d+3*b*c)*c
os(f*x+e)*(c+d*sin(f*x+e))^(3/2)*(a+b*sin(f*x+e))^(1/2)/d/f-1/4*b^2*cos(f*x
+e)*(c+d*sin(f*x+e))^(5/2)*(a+b*sin(f*x+e))^(1/2)/d/f-1/192*(a+b)^(3/2)*(15
*a^3*d^3-15*a^2*b*d^2*(11*c+2*d)-a*b^2*d*(51*c^2+172*c*d+212*d^2)+b^3*(9*c^
3-6*c^2*d-156*c*d^2-72*d^3))*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(
a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+
e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2
)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/b^2/d^2/f/(c+d)
^(1/2)-1/192*(337*a^2*b*c*d^2+15*a^3*d^3+a*b^2*d*(57*c^2+284*d^2)-b^3*(9*c^
3-156*c*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d^2/f/(a+b*sin(f*x+e))^(1/2
)-1/96*(54*a*b*c*d+59*a^2*d^2-9*b^2*(c^2-4*d^2))*cos(f*x+e)*(a+b*sin(f*x+e)
)^(1/2)*(c+d*sin(f*x+e))^(1/2)/d/f
```

Rubi [A] time = 5.00, antiderivative size = 1071, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2793, 3049, 3061, 3053, 2811, 2998, 2818, 2996}

$$-\frac{b^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}}{4df}+\frac{b(3bc-17ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,(c+d\sin}{24df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(337*a^2*b*c*d^2 + 15*a^3*d^3 + a*b^2*d*(5
7*c^2 + 284*d^2) - b^3*(9*c^3 - 156*c*d^2))*EllipticE[ArcSin[(Sqrt[a + b]*S
```

```
qrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*
(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f
*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x])
)/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(192*b*d^2*(b*c - a
*d)*f) + (Sqrt[c + d]*(60*a^3*b*c*d^3 - 5*a^4*d^4 - 20*a*b^3*c*d*(c^2 - 12*
d^2) + 3*b^4*(c^2 + 4*d^2)^2 + 30*a^2*b^2*d^2*(3*c^2 + 4*d^2))*EllipticPi[(
b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt
[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Se
c[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e +
f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]
))]*(a + b*Sin[e + f*x]))/(64*b^2*Sqrt[a + b]*d^3*f) - ((337*a^2*b*c*d^2 +
15*a^3*d^3 + a*b^2*d*(57*c^2 + 284*d^2) - b^3*(9*c^3 - 156*c*d^2))*Cos[e +
f*x]*Sqrt[c + d*Sin[e + f*x]])/(192*d^2*f*Sqrt[a + b*Sin[e + f*x]]) - ((54*
a*b*c*d + 59*a^2*d^2 - 9*b^2*(c^2 - 4*d^2))*Cos[e + f*x]*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[c + d*Sin[e + f*x]])/(96*d*f) - ((a + b)^(3/2)*(15*a^3*d^3 - 15
*a^2*b*d^2*(11*c + 2*d) - a*b^2*d*(51*c^2 + 172*c*d + 212*d^2) + b^3*(9*c^3
 - 6*c^2*d - 156*c*d^2 - 72*d^3))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*
Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/(
(a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a +
b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*
(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(192*b^2*d^2*Sqrt[c + d]*f) +
 (b*(3*b*c - 17*a*d)*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f
*x])^(3/2))/(24*d*f) - (b^2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Si
n[e + f*x])^(5/2))/(4*d*f)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
```

```
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
```

```
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}\,dx = -\frac{b^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}}{4df} + \frac{\int \frac{(}{-}}{}$$

$$= \frac{b(3bc-17ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx)}{24df}$$

$$= -\frac{\left(54abcd+59a^2d^2-9b^2\left(c^2-4d^2\right)\right)\cos(e+fx)\sqrt{a+b\sin(e}}{96df}$$

$$= -\frac{\left(337a^2bcd^2+15a^3d^3+ab^2d\left(57c^2+284d^2\right)-b^3\left(9c^3-156c\right.\right.}{192d^2f\sqrt{a+b\sin(e+fx)}}$$

$$= -\frac{\left(337a^2bcd^2+15a^3d^3+ab^2d\left(57c^2+284d^2\right)-b^3\left(9c^3-156c\right.\right.}{192d^2f\sqrt{a+b\sin(e+fx)}}$$

$$= \frac{\sqrt{c+d}\left(60a^3bcd^3-5a^4d^4-20ab^3cd\left(c^2-12d^2\right)+3b^4\left(c^2+4\right.\right.}{}$$

$$= \frac{\sqrt{a+b}\,(c-d)\sqrt{c+d}\left(337a^2bcd^2+15a^3d^3+ab^2d\left(57c^2+284\right.\right.}{}$$

Mathematica [A] time = 7.60, size = 2091, normalized size = 1.95

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((-4*(-(b*c) + a*d)*(-3*b^3*c^3 + 384*a^3*c^2*d + 451*a*b^2*c^2*d + 971*a^2
*b*c*d^2 + 228*b^3*c*d^2 + 133*a^3*d^3 + 356*a*b^2*d^3)*Sqrt[((c + d)*Cot[(
-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e +
Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c)
 + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[(
(c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sq
rt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d
```

```
)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4
*(-(b*c) + a*d)*(-12*a*b^2*c^3 + 664*a^2*b*c^2*d + 228*b^3*c^2*d + 532*a^3*
c*d^2 + 1160*a*b^2*c*d^2 + 644*a^2*b*d^3 + 144*b^3*d^3)*((Sqrt[((c + d)*Cot
[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e
+ Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*
c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt
[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*
Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a
*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) -
 (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) +
a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*S
in[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d
))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 -
f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi
/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b
*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(9*b^3*c^3 - 57*a*b^2*c^2*d -
 337*a^2*b*c*d^2 - 156*b^3*c*d^2 - 15*a^3*d^3 - 284*a*b^2*d^3)*((Cos[e + f*
x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(
a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a
+ b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(
b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b
)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]
]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((
c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt
[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a
- b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[
2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f
*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(
b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x])
)/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin
[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c +
d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e +
Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c)
 + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[(
(c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sq
rt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d
)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))
/(384*d*f) + (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*(-1/96*((3*
b^2*c^2 + 122*a*b*c*d + 59*a^2*d^2 + 42*b^2*d^2)*Cos[e + f*x])/d + (b^2*d*C
os[3*(e + f*x)])/16 - (b*(9*b*c + 17*a*d)*Sin[2*(e + f*x)])/48))/f
```

fricas [F] time = 156.25, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(2\,abd - \left(b^2c + 2\,abd\right)\cos\left(fx+e\right)^2 + \left(a^2+b^2\right)c - \left(b^2d\cos\left(fx+e\right)^2 - 2\,abc - \left(a^2+b^2\right)d\right)\sin\left(fx+\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((2*a*b*d - (b^2*c + 2*a*b*d)*cos(f*x + e)^2 + (a^2 + b^2)*c - (b^2
*d*cos(f*x + e)^2 - 2*a*b*c - (a^2 + b^2)*d)*sin(f*x + e))*sqrt(b*sin(f*x +
 e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(3/2), x)
```

maple [B] time = 28.46, size = 577725, normalized size = 539.43

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(5/2)*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.779 $\int (a+b\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=894

$$-\frac{\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}b^2}{3df}+\frac{(3bc-13ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(}{12df}$$

```
[Out] 1/8*(15*a^2*b*c*d^2+5*a^3*d^3-5*a*b^2*d*(c^2-4*d^2)+b^3*(c^3+4*c*d^2))*Elli
pticPi((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2
),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+
e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*(
(-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b/d^3/f/(a+b)^(1/2)
+1/24*(c-d)*(14*a*b*c*d+33*a^2*d^2-b^2*(3*c^2-16*d^2))*EllipticE((a+b)^(1/2
)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a
+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(a+b)^(1/2)*(c+d)^(1/2)*(-(-a
*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x
+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/d^2/(-a*d+b*c)/f-1/3*b^2*cos(f*x+e)*(c+d
*sin(f*x+e))^(3/2)*(a+b*sin(f*x+e))^(1/2)/d/f+1/24*(a+b)^(3/2)*(15*a^2*d^2+
6*a*b*d*(2*c+3*d)-b^2*(3*c^2-2*c*d-16*d^2))*EllipticF((c+d)^(1/2)*(a+b*sin(
f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^
(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*si
n(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/
b/d^2/f/(c+d)^(1/2)-1/24*b*(14*a*b*c*d+33*a^2*d^2-b^2*(3*c^2-16*d^2))*cos(f
*x+e)*(c+d*sin(f*x+e))^(1/2)/d^2/f/(a+b*sin(f*x+e))^(1/2)+1/12*b*(-13*a*d+3
*b*c)*cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^(1/2)/d/f
```

Rubi [A] time = 3.27, antiderivative size = 894, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2793, 3049, 3061, 3053, 2811, 2998, 2818, 2996}

$$-\frac{\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}b^2}{3df}+\frac{(3bc-13ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(}{12df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(5/2)*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[a + b]*(c - d)*Sqrt[c + d]*(14*a*b*c*d + 33*a^2*d^2 - b^2*(3*c^2 - 16
*d^2))*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]
*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f
*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))
]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a
```

```
+ b*Sin[e + f*x]))/(24*d^2*(b*c - a*d)*f) + (Sqrt[c + d]*(15*a^2*b*c*d^2 +
5*a^3*d^3 - 5*a*b^2*d*(c^2 - 4*d^2) + b^3*(c^3 + 4*c*d^2))*EllipticPi[(b*(c
 + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c +
 d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e
+ f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]
)))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*
(a + b*Sin[e + f*x]))/(8*b*Sqrt[a + b]*d^3*f) - (b*(14*a*b*c*d + 33*a^2*d^2
 - b^2*(3*c^2 - 16*d^2))*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(24*d^2*f*S
qrt[a + b*Sin[e + f*x]]) + (b*(3*b*c - 13*a*d)*Cos[e + f*x]*Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(12*d*f) + ((a + b)^(3/2)*(15*a^2*d^2 +
 6*a*b*d*(2*c + 3*d) - b^2*(3*c^2 - 2*c*d - 16*d^2))*EllipticF[ArcSin[(Sqrt
[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])],
((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Si
n[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e
 + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(24*b*d^2*
Sqrt[c + d]*f) - (b^2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e +
f*x])^(3/2))/(3*d*f)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
```

```
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
```

```
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}\,dx = -\frac{b^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}}{3df} + \frac{\int \frac{\sqrt{}}{}}{}$$

$$= \frac{b(3bc-13ad)\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+fx}}{12df}$$

$$= -\frac{b\left(14abcd+33a^2d^2-b^2\left(3c^2-16d^2\right)\right)\cos(e+fx)\sqrt{c+d\sin(}}{24d^2f\sqrt{a+b\sin(e+fx)}}$$

$$= -\frac{b\left(14abcd+33a^2d^2-b^2\left(3c^2-16d^2\right)\right)\cos(e+fx)\sqrt{c+d\sin(}}{24d^2f\sqrt{a+b\sin(e+fx)}}$$

$$= \frac{\sqrt{c+d}\,\left(15a^2bcd^2+5a^3d^3-5ab^2d\left(c^2-4d^2\right)+b^3\left(c^3+4cd^2\right)\right)}{}$$

$$= \frac{\sqrt{a+b}\,(c-d)\sqrt{c+d}\,\left(14abcd+33a^2d^2-b^2\left(3c^2-16d^2\right)\right)E\left(\mathrm{s}\right.}{}$$

Mathematica [B] time = 7.10, size = 1979, normalized size = 2.21

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(5/2)*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] ((-4*(-(b*c) + a*d)*(-(b^3*c^2) + 48*a^3*c*d + 58*a*b^2*c*d + 59*a^2*b*d^2
+ 16*b^3*d^2)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF
[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(
b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*
Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*
Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c
 + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(-4*a*b^2*c^2 + 92*a^2*b*c
*d + 28*b^3*c*d + 48*a^3*d^2 + 76*a*b^2*d^2)*((Sqrt[((c + d)*Cot[(-e + Pi/2
 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*
x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/
((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*C
sc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a -
 b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a +
 b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c
+ d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a +
b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]
))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e +
 f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(
a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2
]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(3*b^3*c^2 - 14*a*b^2*c*d - 33*a^2*b*d^2
 - 16*b^3*d^2)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e
 + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*Ellipti
cE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[
e + f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e
 + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x
])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a +
b)*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]]) - (2*(-(b*c) + a*
d)*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*
EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f
*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[
e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^
2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x
)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*S
in[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e
 + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSi
```

```
n[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) +
 a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-
e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e
+ f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*S
in[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c +
 d*Sin[e + f*x]])))/(b*d)))/(48*d*f) + (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d
*Sin[e + f*x]]*(-1/12*(b*(b*c + 13*a*d)*Cos[e + f*x])/d - (b^2*Sin[2*(e + f
*x)])/6))/f
```

fricas [F] time = 35.76, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2\right)\sqrt{b\sin\left(fx+e\right)+a}\,\sqrt{d\sin\left(fx+e\right)+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*sqrt(b*sin(
f*x + e) + a)*sqrt(d*sin(f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)*sqrt(d*sin(f*x + e) + c), x)
```

maple [C] time = 19.75, size = 410016, normalized size = 458.63

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)*sqrt(d*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\sin\left(e+f\,x\right)\right)^{5/2}\,\sqrt{c+d\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(5/2)*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.780 $\int \frac{(a+b\sin(e+fx))^{5/2}}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=745

$$-\frac{\sqrt{c+d}\left(-15a^2d^2+10abcd-\left(b^2\left(3c^2+4d^2\right)\right)\right)\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin}{(c-d)(a+b\sin}}}{4d^3f\sqrt{a+b}}$$

```
[Out] -1/4*(10*a*b*c*d-15*a^2*d^2-b^2*(3*c^2+4*d^2))*EllipticPi((a+b)^(1/2)*(c+d*
sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)
*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+
b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e)
)/(c-d)/(a+b*sin(f*x+e)))^(1/2)/d^3/f/(a+b)^(1/2)-3/4*b*(c-d)*(-3*a*d+b*c)*
EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(
1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(a+b)^(1/
2)*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((
-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/d^2/(-a*d+b*c)/f-1/4
*(a+b)^(3/2)*(-7*a*d+3*b*c-2*b*d)*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1
/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec
(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))
^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/d^2/f/(c+d
)^(1/2)+3/4*b^2*(-3*a*d+b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d^2/f/(a+b*s
in(f*x+e))^(1/2)-1/2*b^2*cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))
^(1/2)/d/f
```

Rubi [A] time = 2.19, antiderivative size = 745, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2793, 3061, 3053, 2811, 2998, 2818, 2996}

$$-\frac{\sqrt{c+d}\left(-15a^2d^2+10abcd+b^2\left(-\left(3c^2+4d^2\right)\right)\right)\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(}{(c-d)(a+}}}{4d^3f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(5/2)/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (-3*b*Sqrt[a + b]*(c - d)*Sqrt[c + d]*(b*c - 3*a*d)*EllipticE[ArcSin[(Sqrt[
a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], (
(a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - S
in[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e
 + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(4*d^2*(b*c
 - a*d)*f) - (Sqrt[c + d]*(10*a*b*c*d - 15*a^2*d^2 - b^2*(3*c^2 + 4*d^2))*E
```

```
llipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f
*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(
c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a +
 b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*S
in[e + f*x]))]*(a + b*Sin[e + f*x]))/(4*Sqrt[a + b]*d^3*f) + (3*b^2*(b*c -
3*a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*d^2*f*Sqrt[a + b*Sin[e + f
*x]]) - (b^2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]
)/(2*d*f) - ((a + b)^(3/2)*(3*b*c - 7*a*d - 2*b*d)*EllipticF[ArcSin[(Sqrt[c
 + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((
a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[
e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e +
 f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(4*d^2*Sqrt[
c + d]*f)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
```

```
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
```

```
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^{5/2}}{\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{b^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}{2df} + \frac{\int \frac{\frac{1}{2}\left(b^3c+4a^3d+ab^2d\right)-b\left(abc-6a\right.}{\sqrt{a+b\sin}}}{}$$

$$= \frac{3b^2(bc-3ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4d^2f\sqrt{a+b\sin(e+fx)}} - \frac{b^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{2df}$$

$$= \frac{3b^2(bc-3ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4d^2f\sqrt{a+b\sin(e+fx)}} - \frac{b^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{2df}$$

$$= -\frac{\sqrt{c+d}\left(10abcd-15a^2d^2-b^2\left(3c^2+4d^2\right)\right)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\right.}{4\sqrt{a+}}$$

$$= -\frac{3b\sqrt{a+b}\,(c-d)\sqrt{c+d}\,(bc-3ad)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+}{4d^2(bc-ad)f}$$

Mathematica [B] time = 10.25, size = 1894, normalized size = 2.54

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(5/2)/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] -1/2*(b^2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(
d*f) + ((-4*(-(b*c) + a*d)*(-(b^3*c) + 8*a^3*d + 11*a*b^2*d)*Sqrt[((c + d)*
Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(
-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-
(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*S
qrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d
)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
+ a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]
) - 4*(-(b*c) + a*d)*(-4*a*b^2*c + 24*a^2*b*d + 4*b^3*d)*((Sqrt[((c + d)*Co
t[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e
 + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b
```

```
*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqr
t[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]
*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) +
a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
- (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) +
 a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*
Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c +
d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 -
 f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + P
i/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a +
b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(3*b^3*c - 9*a*b^2*d)*((Cos[
e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a
- b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a -
b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]],
(2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[(
(a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x
]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d
)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt
[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]
/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi
/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]
))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e +
 f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c +
 d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/
(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[
(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(
-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*
Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*
d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
 + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(
b*d)))/(8*d*f)
```

fricas [F] time = 13.73, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2\right)\sqrt{b\sin\left(fx+e\right)+a}}{\sqrt{d\sin\left(fx+e\right)+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*sqrt(b*sin(
f*x + e) + a)/sqrt(d*sin(f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)/sqrt(d*sin(f*x + e) + c), x)
```

maple [B] time = 22.14, size = 730813, normalized size = 980.96

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)/sqrt(d*sin(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{5/2}}{\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.781 $\int \frac{(a+b\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=780

$$\frac{b\left(-2a^2d^2+4abcd-\left(b^2\left(3c^2-d^2\right)\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{d^2f\left(c^2-d^2\right)\sqrt{a+b\sin(e+fx)}}-\frac{\sqrt{a+b}\left(-2a^2d^2+4abcd-\left(b^2\left(3c^2-d^2\right)\right)\right)}{}$$

```
[Out] -(4*a*b*c*d-2*a^2*d^2-b^2*(3*c^2-d^2))*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e
))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2)
)*sec(f*x+e)*(a+b*sin(f*x+e))*(a+b)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)
/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))
^(1/2)/d^2/(-a*d+b*c)/f/(c+d)^(1/2)-b*(-5*a*d+3*b*c)*EllipticPi((a+b)^(1/2)
*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),b*(c+d)/(a+b)/d,
((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-
(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(
f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/d^3/f/(a+b)^(1/2)-(a+b)^(3/2)*(2*a*d-
b*(3*c+d))*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*si
n(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e)
)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+
sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/d^2/(c+d)^(3/2)/f+2*(-a*d+b*c)^2*
cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)/d/(c^2-d^2)/f/(c+d*sin(f*x+e))^(1/2)+b*(4
*a*b*c*d-2*a^2*d^2-b^2*(3*c^2-d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/d^2/(
c^2-d^2)/f/(a+b*sin(f*x+e))^(1/2)
```

Rubi [A] time = 2.47, antiderivative size = 780, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2792, 3061, 3053, 2811, 2998, 2818, 2996}

$$\frac{b\left(-2a^2d^2+4abcd+b^2\left(-\left(3c^2-d^2\right)\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{d^2f\left(c^2-d^2\right)\sqrt{a+b\sin(e+fx)}}-\frac{\sqrt{a+b}\left(-2a^2d^2+4abcd+b^2\left(-\left(3c^2-\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((Sqrt[a + b]*(4*a*b*c*d - 2*a^2*d^2 - b^2*(3*c^2 - d^2))*EllipticE[ArcSin
[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x
]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)
*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1
+ Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(d^2
*Sqrt[c + d]*(b*c - a*d)*f)) - (b*Sqrt[c + d]*(3*b*c - 5*a*d)*EllipticPi[(b
```

```
*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[
c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec
[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f
*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x])
)]*(a + b*Sin[e + f*x]))/(Sqrt[a + b]*d^3*f) + (2*(b*c - a*d)^2*Cos[e + f*x
]*Sqrt[a + b*Sin[e + f*x]])/(d*(c^2 - d^2)*f*Sqrt[c + d*Sin[e + f*x]]) + (b
*(4*a*b*c*d - 2*a^2*d^2 - b^2*(3*c^2 - d^2))*Cos[e + f*x]*Sqrt[c + d*Sin[e
+ f*x]])/(d^2*(c^2 - d^2)*f*Sqrt[a + b*Sin[e + f*x]]) - ((a + b)^(3/2)*(2*a
*d - b*(3*c + d))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(
Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))
]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e
+ f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f
*x])))]*(c + d*Sin[e + f*x]))/(d^2*(c + d)^(3/2)*f)
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
```

```
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
```

```
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} - \frac{2\int \frac{\frac{1}{2}\left(b^3c^2-a^3cd-3ab^2cd+3a^2bd^2\right)+\frac{1}{2}\left(a^2bcd-\right.}{}}{}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{b\left(4abcd-2a^2d^2-b^2\left(3c^2-d^2\right)\right)\mathrm{co}}{d^2\left(c^2-d^2\right)f\sqrt{a+}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{d\left(c^2-d^2\right)f\sqrt{c+d\sin(e+fx)}} + \frac{b\left(4abcd-2a^2d^2-b^2\left(3c^2-d^2\right)\right)\mathrm{co}}{d^2\left(c^2-d^2\right)f\sqrt{a+}}$$

$$= -\frac{b\sqrt{c+d}\,(3bc-5ad)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{}}{\sqrt{a+b}\,d^3f}$$

$$= -\frac{\sqrt{a+b}\,\left(4abcd-2a^2d^2-b^2\left(3c^2-d^2\right)\right)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{d^2\sqrt{c+d}\,(bc-}$$

Mathematica [B] time = 6.81, size = 2006, normalized size = 2.57

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(b^2*c^2*Cos[e + f*x] - 2*a*b*c*d*Cos[e + f*x] + a^2*d^2*Cos[e + f*x])*
Sqrt[a + b*Sin[e + f*x]])/(d*(-c^2 + d^2)*f*Sqrt[c + d*Sin[e + f*x]]) - ((-
4*(-(b*c) + a*d)*(-(b^3*c^2) - 2*a^3*c*d - 2*a*b^2*c*d + 4*a^2*b*d^2 + b^3*
d^2)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[S
qrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*
d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e +
 Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f
*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[
e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[
c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(-4*a*b^2*c^2 + 2*a^2*b*c*d - 2*b^3
```

```
*c*d - 2*a^3*d^2 + 6*a*b^2*d^2)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)
/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c +
d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c
+ d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2
 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e +
 Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e
+ Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin
[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) +
a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e
 + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e +
 f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Si
n[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c +
d*Sin[e + f*x]])) + 2*(3*b^3*c^2 - 4*a*b^2*c*d + 2*a^2*b*d^2 - b^3*d^2)*((C
os[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[
(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a
 - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]
], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqr
t[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin
[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e +
f*x]))/((c + d)*(a + b*Sin[e + f*x]))]]) - (2*(-(b*c) + a*d)*((((a + b)*c +
a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[S
qrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*
d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e +
 Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f
*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[
e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[
c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^
2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*C
sc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (
2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]
^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) +
 a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b
*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))
)/(b*d)))/(2*(c - d)*d*(c + d)*f)
```

fricas [F] time = 4.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2\right)\sqrt{b\sin\left(fx+e\right)+a}\,\sqrt{d\sin\left(fx+e\right)+c}}{d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="frica
```

```
s")
```

```
[Out] integral((b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*sqrt(b*sin(f
*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x +
 e) - c^2 - d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c)^(3/2), x)
```

maple [B] time = 51.24, size = 3436958, normalized size = 4406.36

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.782 $\int \frac{(a+b\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=737

$$-\frac{2\sqrt{a+b}\left(a^2d^2(3c+d)+abd\left(3c^2-4cd-7d^2\right)+b^2\left(3c^3-6c^2d-2cd^2+9d^3\right)\right)\sec(e+fx)(c+d\sin(e+fx))}{3d^3f(c-d)^2(c+d)^{3/2}}$$

```
[Out] 2/3*(-a*d+b*c)^2*cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)/d/(c^2-d^2)/f/(c+d*sin(f
*x+e))^(3/2)+2/3*(a-b)*(4*a*c*d+3*b*c^2-7*b*d^2)*EllipticE((c+d)^(1/2)*(a+b
*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c
+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+
e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*si
n(f*x+e)))^(1/2)/(c-d)^2/d^2/(c+d)^(3/2)/f-2/3*(a^2*d^2*(3*c+d)+a*b*d*(3*c^
2-4*c*d-7*d^2)+b^2*(3*c^3-6*c^2*d-2*c*d^2+9*d^3))*EllipticF((c+d)^(1/2)*(a+
b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(
c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x
+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*s
in(f*x+e)))^(1/2)/(c-d)^2/d^3/(c+d)^(3/2)/f+2*b^2*EllipticPi((c+d)^(1/2)*(a
+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),(a+b)*d/b/(c+d),((a
+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*
d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x
+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/d^3/f/(c+d)^(1/2)
```

Rubi [A] time = 1.78, antiderivative size = 737, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2792, 3053, 2811, 2998, 2818, 2996}

$$-\frac{2\sqrt{a+b}\left(a^2d^2(3c+d)+abd\left(3c^2-4cd-7d^2\right)+b^2\left(-6c^2d+3c^3-2cd^2+9d^3\right)\right)\sec(e+fx)(c+d\sin(e+fx}{3d^3f(c-d)^2(c+d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*(b*c - a*d)^2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(3*d*(c^2 - d^2)*f*
(c + d*Sin[e + f*x])^(3/2)) + (2*(a - b)*Sqrt[a + b]*(3*b*c^2 + 4*a*c*d - 7
*b*d^2)*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b
]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e +
f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*
Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c
 + d*Sin[e + f*x]))/(3*(c - d)^2*d^2*(c + d)^(3/2)*f) - (2*Sqrt[a + b]*(a^2
```

```
*d^2*(3*c + d) + a*b*d*(3*c^2 - 4*c*d - 7*d^2) + b^2*(3*c^3 - 6*c^2*d - 2*c
*d^2 + 9*d^3))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqr
t[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*S
ec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f
*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x]
)))]*(c + d*Sin[e + f*x]))/(3*(c - d)^2*d^3*(c + d)^(3/2)*f) + (2*b^2*Sqrt[
a + b]*EllipticPi[((a + b)*d)/(b*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[a + b*S
in[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((
a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b
)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(
c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(d^3*Sqrt[c + d]*f)
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
```

```
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^{5/2}}{(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(b^3c^2-3a^3cd-5ab^2cd+7a^2bd^2\right)-\frac{1}{2}\left(5a^2bcd+\cdots\right.}{\sqrt{a+b\,\mathrm{s}\cdots}}}{\cdots}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{b^3\int \frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}}\,dx}{d^3} - \frac{2\int \frac{\frac{3}{2}b^3c^2\left(c^2-d\cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{2b^2\sqrt{a+b}\,\Pi\left(\frac{(a+b)d}{b(c+d)};\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+\cdots}}{\sqrt{a+b}\,\sqrt{c+\cdots}}\right.\right.}{\cdots}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3d\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{2(a-b)\sqrt{a+b}\,\left(3bc^2+4acd-7bd^2\right)\cdots}{\cdots}$$

Mathematica [**B**] time = 6.94, size = 2169, normalized size = 2.94

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^(5/2)/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-2*(b^2*c^2*Cos[e + f*
x] - 2*a*b*c*d*Cos[e + f*x] + a^2*d^2*Cos[e + f*x]))/(3*d*(-c^2 + d^2)*(c +
 d*Sin[e + f*x])^2) - (2*(3*b^2*c^3*Cos[e + f*x] + a*b*c^2*d*Cos[e + f*x] -
 4*a^2*c*d^2*Cos[e + f*x] - 7*b^2*c*d^2*Cos[e + f*x] + 7*a*b*d^3*Cos[e + f*
x]))/(3*d*(-c^2 + d^2)^2*(c + d*Sin[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(-
(b^3*c^3) + 3*a^3*c^2*d + 2*a*b^2*c^2*d - 8*a^2*b*c*d^2 + b^3*c*d^2 + a^3*d
^3 + 2*a*b^2*d^3)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Ellip
ticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))
/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f
*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a
+ b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^
2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(-4*a*b^2*c^3 + 3*a^2*
b*c^2*d + b^3*c^2*d + 4*a^3*c*d^2 - 7*a^2*b*d^3 + 3*b^3*d^3)*((Sqrt[((c + d
)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc
[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*
(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4
*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a
```

```
*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c
) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*
c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c
+ d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-
c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi
/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e
 + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[
a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(3*b^3*c^3 + a*b^2*c^2*d
 - 4*a^2*b*c*d^2 - 7*b^3*c*d^2 + 7*a*b^2*d^3)*((Cos[e + f*x]*Sqrt[c + d*Sin
[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*C
os[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + P
i/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a
- b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2
- f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin
[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin
[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e
 + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi
/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) +
 a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c
 + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt
[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]
)/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b
*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-
(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*
(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)
*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e +
 Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[
(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))/(3*(c - d)^2*d*(
c + d)^2*f)
```

fricas [F] time = 43.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2\right)\sqrt{b\sin\left(fx+e\right)+a}\,\sqrt{d\sin\left(fx+e\right)+c}}{3\,cd^2\cos\left(fx+e\right)^2-c^3-3\,cd^2+\left(d^3\cos\left(fx+e\right)^2-3\,c^2d-d^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral((b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*sqrt(b*sin(f
*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(3*c*d^2*cos(f*x + e)^2 - c^3 - 3*c*d
^2 + (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c)^(5/2), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sin\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(c+d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^(5/2)/(c + d*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.783 $\int \frac{(c+d\sin(e+fx))^{5/2}}{\sqrt{a+b\sin(e+fx)}}\,dx$

Optimal. Leaf size=772

$$\frac{\sqrt{a+b}\left(3a^2d^2-abd(7c+3d)+b^2\left(8c^2+9cd+2d^2\right)\right)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-a}{(a-b)}}}{4b^3f\sqrt{c+d}}$$

```
[Out] -1/4*(10*a*b*c*d-3*a^2*d^2-b^2*(15*c^2+4*d^2))*EllipticPi((a+b)^(1/2)*(c+d*
sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)
*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+
b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e)
)/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b^3/f/(a+b)^(1/2)+3/4*(c-d)*d*(-a*d+3*b*c)*
EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(
1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(a+b)^(1/
2)*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((
-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b^2/(-a*d+b*c)/f+1/4
*(3*a^2*d^2-a*b*d*(7*c+3*d)+b^2*(8*c^2+9*c*d+2*d^2))*EllipticF((c+d)^(1/2)*
(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b
)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(
f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+
d*sin(f*x+e)))^(1/2)/b^3/f/(c+d)^(1/2)-3/4*d*(-a*d+3*b*c)*cos(f*x+e)*(c+d*s
in(f*x+e))^(1/2)/b/f/(a+b*sin(f*x+e))^(1/2)-1/2*d^2*cos(f*x+e)*(a+b*sin(f*x
+e))^(1/2)*(c+d*sin(f*x+e))^(1/2)/b/f
```

Rubi [A] time = 2.30, antiderivative size = 772, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2793, 3061, 3053, 2811, 2998, 2818, 2996}

$$\frac{\sqrt{a+b}\left(3a^2d^2-abd(7c+3d)+b^2\left(8c^2+9cd+2d^2\right)\right)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-a}{(a-b)}}}{4b^3f\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (3*Sqrt[a + b]*(c - d)*d*Sqrt[c + d]*(3*b*c - a*d)*EllipticE[ArcSin[(Sqrt[a
 + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((
a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Si
n[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e
+ f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(4*b^2*(b*c
- a*d)*f) - (Sqrt[c + d]*(10*a*b*c*d - 3*a^2*d^2 - b^2*(15*c^2 + 4*d^2))*El
```

```
lipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*
x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c
 - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a +
b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Si
n[e + f*x]))]*(a + b*Sin[e + f*x]))/(4*b^3*Sqrt[a + b]*f) - (3*d*(3*b*c - a
*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(4*b*f*Sqrt[a + b*Sin[e + f*x]])
 - (d^2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(2*
b*f) + (Sqrt[a + b]*(3*a^2*d^2 - a*b*d*(7*c + 3*d) + b^2*(8*c^2 + 9*c*d + 2
*d^2))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]
*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f
*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*S
qrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c
+ d*Sin[e + f*x]))/(4*b^3*Sqrt[c + d]*f)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
```

```
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
```

```
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{\sqrt{a+b\sin(e+fx)}}\,dx = -\frac{d^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}{2bf} + \frac{\int \frac{\frac{1}{2}\left(ad^3+bc\left(4c^2+d^2\right)\right)-d\left(acd-\right.}{\sqrt{a+b}}}{}$$

$$= -\frac{3d(3bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4bf\sqrt{a+b\sin(e+fx)}} - \frac{d^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{2bf}$$

$$= -\frac{3d(3bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{4bf\sqrt{a+b\sin(e+fx)}} - \frac{d^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{2bf}$$

$$= -\frac{\sqrt{c+d}\left(10abcd-3a^2d^2-b^2\left(15c^2+4d^2\right)\right)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)\right.}{4b^3\sqrt{}}$$

$$= \frac{3\sqrt{a+b}\,(c-d)d\sqrt{c+d}\,(3bc-ad)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+}{4b^2(bc-ad)f}$$

Mathematica [B] time = 10.39, size = 1894, normalized size = 2.45

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] -1/2*(d^2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(
b*f) + ((-4*(-(b*c) + a*d)*(8*b*c^3 + 11*b*c*d^2 - a*d^3)*Sqrt[((c + d)*Cot
[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e
+ Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*
c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt
[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*
Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a
*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) -
 4*(-(b*c) + a*d)*(24*b*c^2*d - 4*a*c*d^2 + 4*b*d^3)*((Sqrt[((c + d)*Cot[(-
e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + P
i/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c)
```

```
+ a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((
c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqr
t[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)
])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (S
qrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d
)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[
e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]
*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x
)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2
- f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Si
n[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(-9*b*c*d^2 + 3*a*d^3)*((Cos[e +
 f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b
)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/
(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*
(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a
+ b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f
*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))
/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*S
qrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((
-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sq
rt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2
- f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/
(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*
x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*
Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c
 + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e
 + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b
*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqr
t[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]
*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) +
a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d
)))/(8*b*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\sqrt{b\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/sqrt(b*sin(f*x + e) + a), x)
```

maple [B] time = 24.28, size = 731601, normalized size = 947.67

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\sqrt{b\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/sqrt(b*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2}}{\sqrt{a+b\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(5/2)/(a + b*sin(e + f*x))^(1/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^(5/2)/(a + b*sin(e + f*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e))**(1/2),x)
[Out] Timed out
```

3.784 $\int \frac{(c+d\sin(e+fx))^{3/2}}{\sqrt{a+b\sin(e+fx)}}\,dx$

Optimal. Leaf size=644

$$-\frac{\sqrt{a+b}\,(ad-b(2c+d))\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\sin^{-1}\left(\frac{\sqrt{c+d}}{\sqrt{a+b}}\right.\right.}{b^2f\sqrt{c+d}}$$

```
[Out] (-a*d+3*b*c)*EllipticPi((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b
*sin(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x
+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin
(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b^
2/f/(a+b)^(1/2)+(c-d)*d*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^
(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a
+b*sin(f*x+e))*(a+b)^(1/2)*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a
+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1
/2)/b/(-a*d+b*c)/f-(a*d-b*(2*c+d))*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(
1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*se
c(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d
*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/
2)/b^2/f/(c+d)^(1/2)-d*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/f/(a+b*sin(f*x+e))
^(1/2)
```

Rubi [A] time = 1.58, antiderivative size = 644, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2821, 3053, 2811, 2998, 2818, 2996}

$$-\frac{\sqrt{a+b}\,(ad-b(2c+d))\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\sin^{-1}\left(\frac{\sqrt{c+d}}{\sqrt{a+b}}\right.\right.}{b^2f\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[a + b]*(c - d)*d*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d
*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/
((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d
)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(b*(b*c - a*d)*f) + (Sqrt[c
+ d]*(3*b*c - a*d)*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*
Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)
*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e +
```

```
f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]
))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(b^2*Sqrt[a + b]*f
) - (d*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(f*Sqrt[a + b*Sin[e + f*x]])
- (Sqrt[a + b]*(a*d - b*(2*c + d))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b
*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/
((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a +
 b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)
*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(b^2*Sqrt[c + d]*f)
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
```

```
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{\sqrt{a+b\sin(e+fx)}}\,dx = -\frac{d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}} + \frac{\int \frac{-\frac{1}{2}b\left(bcd-a\left(2c^2+d^2\right)\right)+bc(bc+ad)\sin(e+fx)+\frac{1}{2}bd(3bc-ad}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}}{b}$$

$$= -\frac{d\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}} + \frac{\int \frac{-\frac{1}{2}a^2bd(3bc-ad)-\frac{1}{2}b^3\left(bcd-a\left(2c^2+d^2\right)\right)+b\left(-abd(3bc-ad)-\right.}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}}{b^3}$$

$$= \frac{\sqrt{c+d}\,(3bc-ad)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc}{(c+}}}{b^2\sqrt{a+b}\,f}$$

$$= \frac{\sqrt{a+b}\,(c-d)d\sqrt{c+d}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-a}{(c+d)}}}{b(bc-ad)f}$$

Mathematica [C] time = 32.46, size = 222963, normalized size = 346.22

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\sqrt{b\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/sqrt(b*sin(f*x + e) + a), x)
```

maple [B] time = 10.95, size = 544151, normalized size = 844.95

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\sqrt{b \sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/sqrt(b*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}}{\sqrt{a+b\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(3/2)/(a + b*sin(e + f*x))^(1/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^(3/2)/(a + b*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}}{\sqrt{a+b\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e))**(1/2),x)
[Out] Integral((c + d*sin(e + f*x))**(3/2)/sqrt(a + b*sin(e + f*x)), x)
```

3.785 $\int \frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}}\,dx$

Optimal. Leaf size=198

$$\frac{2\sqrt{a+b}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,\Pi\left(\frac{(a+b)d}{b(c+d)};\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+}}{\sqrt{a+b}\sqrt{c+d\sin(e+}}\right.\right.}{bf\sqrt{c+d}}$$

```
[Out] 2*EllipticPi((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e)
)^(1/2),(a+b)*d/b/(c+d),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*si
n(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1
/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/b/f/(c+d)^(1/
2)
```

Rubi [A] time = 0.11, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2811}

$$\frac{2\sqrt{a+b}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,\Pi\left(\frac{(a+b)d}{b(c+d)};\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+}}{\sqrt{a+b}\sqrt{c+d\sin(e+}}\right.\right.}{bf\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (2*Sqrt[a + b]*EllipticPi[((a + b)*d)/(b*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt
[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c
- d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))
/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((
a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(b*Sqrt[c + d]*f)
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}}\,dx = \frac{2\sqrt{a+b}\,\Pi\left(\frac{(a+b)d}{b(c+d)};\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(e+f}{(a+b)(c+d\sin(e+f}}}{b\sqrt{c+d}\,f}$$

Mathematica [A] time = 0.28, size = 197, normalized size = 0.99

$$\frac{2\sqrt{a+b}\,\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{\frac{(ad-bc)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,\Pi\left(\frac{(a+b)d}{b(c+d)};\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right.\right.}{bf\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]],x]
```

```
[Out] (2*Sqrt[a + b]*EllipticPi[((a + b)*d)/(b*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt
[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c
- d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))
/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[((-(b*c) + a*d)*(1 + Sin[e + f*x]))/(
(a - b)*(c + d*Sin[e + f*x]))]*(c + d*Sin[e + f*x]))/(b*Sqrt[c + d]*f)
```

fricas [F] time = 1.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\sin\left(fx+e\right)+c}}{\sqrt{b\sin\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(d*sin(f*x + e) + c)/sqrt(b*sin(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\sin\left(fx+e\right)+c}}{\sqrt{b\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/sqrt(b*sin(f*x + e) + a), x)
```

maple [C] time = 4.64, size = 248841, normalized size = 1256.77

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\sin\left(fx+e\right)+c}}{\sqrt{b\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/sqrt(b*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c+d\,\sin\left(e+f\,x\right)}}{\sqrt{a+b\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(1/2)/(a + b*sin(e + f*x))^(1/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^(1/2)/(a + b*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+d\sin\left(e+fx\right)}}{\sqrt{a+b\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))**(1/2),x)
[Out] Integral(sqrt(c + d*sin(e + f*x))/sqrt(a + b*sin(e + f*x)), x)
```

3.786 $\int \frac{1}{\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=192

$$\frac{2\sqrt{a+b}\,\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\,\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a}{(a}\right.}{f\sqrt{c+d}\,(bc-ad)}$$

```
[Out] 2*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))
^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(
1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*
(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(-a*d+b*c)/f/(c+d)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2818}

$$\frac{2\sqrt{a+b}\,\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\,\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a}{(a}\right.}{f\sqrt{c+d}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (2*Sqrt[a + b]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqr
t[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*S
ec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f
*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x]
)))]*(c + d*Sin[e + f*x]))/(Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx = \frac{2\sqrt{a+b}\,F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc\cdot}{(a+}}}{\sqrt{c+d}\,(bc-a}$$

Mathematica [A] time = 0.24, size = 191, normalized size = 0.99

$$\frac{2\sqrt{a+b}\,\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{\frac{(ad-bc)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)}{(a-b)(}}{f\sqrt{c+d}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (2*Sqrt[a + b]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqr
t[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*S
ec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f
*x]))]*Sqrt[((-(b*c) + a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x
]))]*(c + d*Sin[e + f*x]))/(Sqrt[c + d]*(b*c - a*d)*f)
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{b\sin\left(fx+e\right)+a}\,\sqrt{d\sin\left(fx+e\right)+c}}{bd\cos\left(fx+e\right)^{2}-ac-bd-(bc+ad)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(b*d*cos(f*x +
e)^2 - a*c - b*d - (b*c + a*d)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sin\left(fx+e\right)+a}\,\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)), x)
```

maple [B] time = 0.69, size = 1233, normalized size = 6.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] -4/f*EllipticF((-(cos(f*x+e)*(-c^2+d^2)^(1/2)-c*sin(f*x+e)-d*cos(f*x+e)+(-c
^2+d^2)^(1/2)-d)*(c*(-a^2+b^2)^(1/2)-a*(-c^2+d^2)^(1/2)-d*a+c*b)/(cos(f*x+e
)*(-c^2+d^2)^(1/2)+c*sin(f*x+e)+d*cos(f*x+e)+(-c^2+d^2)^(1/2)+d)/(a*(-c^2+d
^2)^(1/2)+c*(-a^2+b^2)^(1/2)-d*a+c*b))^(1/2),((a*(-c^2+d^2)^(1/2)+c*(-a^2+b
^2)^(1/2)+d*a-c*b)*(a*(-c^2+d^2)^(1/2)+c*(-a^2+b^2)^(1/2)-d*a+c*b)/(a*(-c^2
+d^2)^(1/2)-c*(-a^2+b^2)^(1/2)-d*a+c*b)/(a*(-c^2+d^2)^(1/2)-c*(-a^2+b^2)^(1
/2)+d*a-c*b))^(1/2))*((cos(f*x+e)*(-c^2+d^2)^(1/2)-c*sin(f*x+e)-d*cos(f*x+e
)+(-c^2+d^2)^(1/2)-d)/(cos(f*x+e)*(-c^2+d^2)^(1/2)+c*sin(f*x+e)+d*cos(f*x+e
)+(-c^2+d^2)^(1/2)+d)*(a*(-c^2+d^2)^(1/2)-c*(-a^2+b^2)^(1/2)+d*a-c*b)/(a*(-
c^2+d^2)^(1/2)+c*(-a^2+b^2)^(1/2)-d*a+c*b))^(1/2)*((cos(f*x+e)*(-a^2+b^2)^(
1/2)+a*sin(f*x+e)+b*cos(f*x+e)+(-a^2+b^2)^(1/2)+b)/(cos(f*x+e)*(-c^2+d^2)^(
1/2)+c*sin(f*x+e)+d*cos(f*x+e)+(-c^2+d^2)^(1/2)+d)*(-c^2+d^2)^(1/2)*c/(a*(-
c^2+d^2)^(1/2)+c*(-a^2+b^2)^(1/2)-d*a+c*b))^(1/2)*(-(cos(f*x+e)*(-a^2+b^2)^
(1/2)-a*sin(f*x+e)-b*cos(f*x+e)+(-a^2+b^2)^(1/2)-b)/(cos(f*x+e)*(-c^2+d^2)^
(1/2)+c*sin(f*x+e)+d*cos(f*x+e)+(-c^2+d^2)^(1/2)+d)*(-c^2+d^2)^(1/2)*c/(a*(
-c^2+d^2)^(1/2)-c*(-a^2+b^2)^(1/2)-d*a+c*b))^(1/2)*(a+b*sin(f*x+e))^(1/2)*(
c+d*sin(f*x+e))^(1/2)*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))^2*(c*(-c^2+d^2)^(1/2
)*(-a^2+b^2)^(1/2)*sin(f*x+e)+b*c*(-c^2+d^2)^(1/2)*sin(f*x+e)+c*d*(-a^2+b^2
)^(1/2)*sin(f*x+e)-a*c^2*sin(f*x+e)+b*c*d*sin(f*x+e)+cos(f*x+e)*(-c^2+d^2)^
(1/2)*(-a^2+b^2)^(1/2)*d-cos(f*x+e)*(-c^2+d^2)^(1/2)*a*c+cos(f*x+e)*(-c^2+d
^2)^(1/2)*b*d-cos(f*x+e)*(-a^2+b^2)^(1/2)*c^2+cos(f*x+e)*(-a^2+b^2)^(1/2)*d
^2-cos(f*x+e)*b*c^2+cos(f*x+e)*b*d^2+d*(-c^2+d^2)^(1/2)*(-a^2+b^2)^(1/2)+b*
d*(-c^2+d^2)^(1/2)+d^2*(-a^2+b^2)^(1/2)-a*c*d+d^2*b)/sin(f*x+e)^4/(-cos(f*x
+e)^2*b*d+a*d*sin(f*x+e)+b*c*sin(f*x+e)+c*a+b*d)/(-c^2+d^2)^(1/2)/(a*(-c^2+
d^2)^(1/2)-c*(-a^2+b^2)^(1/2)+d*a-c*b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\,\sin\left(e+f\,x\right)}\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sin\left(e+fx\right)}\,\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sin(e + f*x))*sqrt(c + d*sin(e + f*x))), x)
```

3.787 $\int \frac{1}{\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=405

$$\frac{2\sqrt{a+b}\,\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a}{(a}\right.}{f(c-d)\sqrt{c+d}\,(bc-ad)}$$

```
[Out] 2*(a-b)*d*EllipticE((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin
(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))
*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a
*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)/(-a*d+b*c)^2/f/(
c+d)^(1/2)+2*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*
sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+
e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-
(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)/(-a*d+b*c)/f/
(c+d)^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2801, 2818, 2996}

$$\frac{2\sqrt{a+b}\,\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a}{(a}\right.}{f(c-d)\sqrt{c+d}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*d*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f
*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(
c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d
*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Si
n[e + f*x])))]*(c + d*Sin[e + f*x]))/((c - d)*Sqrt[c + d]*(b*c - a*d)^2*f)
+ (2*Sqrt[a + b]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(S
qrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]
*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e +
 f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*
x])))]*(c + d*Sin[e + f*x]))/((c - d)*Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 2801

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
```

```
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{1}{\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}\,dx}{c-d} - \frac{d\int \frac{1+\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/}}}{c-d}$$

$$= \frac{2(a-b)\sqrt{a+b}\,dE\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+}{(c-d)\sqrt{c+}}$$

Mathematica [B] time = 32.66, size = 90261, normalized size = 222.87

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+c}}{ac^2+2\,bcd+ad^2-\left(2\,bcd+ad^2\right)\cos\left(fx+e\right)^2-\left(bd^2\cos\left(fx+e\right)^2-bc^2-2\,acd-bd^2\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(a*c^2 + 2*b*c*d
 + a*d^2 - (2*b*c*d + a*d^2)*cos(f*x + e)^2 - (b*d^2*cos(f*x + e)^2 - b*c^2
 - 2*a*c*d - b*d^2)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2)), x)
```

maple [B] time = 1.13, size = 41868, normalized size = 103.38

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a+b\,\sin\left(e+f\,x\right)}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sin\left(e+fx\right)}\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sin(e + f*x))*(c + d*sin(e + f*x))**(3/2)), x)
```

3.788 $\int \frac{1}{\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=521

$$-\frac{2d^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f\left(c^2-d^2\right)(bc-ad)(c+d\sin(e+fx))^{3/2}}-\frac{2\sqrt{a+b}\,\left(ad(3c+d)-b\left(3c^2+3cd-2d^2\right)\right)\sec(e+fx)(c+d\sin(}{3f(}$$

```
[Out] -2/3*d^2*cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)/(-a*d+b*c)/(c^2-d^2)/f/(c+d*sin(
f*x+e))^(3/2)-4/3*(a-b)*d*(2*a*c*d-b*(3*c^2-d^2))*EllipticE((c+d)^(1/2)*(a+
b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(
c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x
+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*s
in(f*x+e)))^(1/2)/(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)^3/f-2/3*(a*d*(3*c+d)-b*(3*
c^2+3*c*d-2*d^2))*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/
(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin
(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/
2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)^2/(c+d)^
(3/2)/(-a*d+b*c)^2/f
```

Rubi [A] time = 1.02, antiderivative size = 521, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2802, 2998, 2818, 2996}

$$-\frac{2d^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f\left(c^2-d^2\right)(bc-ad)(c+d\sin(e+fx))^{3/2}}-\frac{2\sqrt{a+b}\,\left(ad(3c+d)-b\left(3c^2+3cd-2d^2\right)\right)\sec(e+fx)(c+d\sin(}{3f(}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (-2*d^2*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(3*(b*c - a*d)*(c^2 - d^2)*f
*(c + d*Sin[e + f*x])^(3/2)) - (4*(a - b)*Sqrt[a + b]*d*(2*a*c*d - b*(3*c^2
 - d^2))*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a +
b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e +
 f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]
*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(
c + d*Sin[e + f*x]))/(3*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)^3*f) - (2*Sqrt[
a + b]*(a*d*(3*c + d) - b*(3*c^2 + 3*c*d - 2*d^2))*EllipticF[ArcSin[(Sqrt[c
 + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((
a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[
e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e +
```

```
 f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(3*(c - d)^2
*(c + d)^(3/2)*(b*c - a*d)^2*f)
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{5/2}}\,dx = -\frac{2d^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} + \frac{2\int \frac{\frac{1}{2}\left(-2bd^2+3c(b\right.}{\sqrt{a+b\sin}}}{3(bc}$$

$$= -\frac{2d^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{\left(ad(3c+d)-b\right.}{}$$

$$= -\frac{2d^2\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}} - \frac{4(a-b)\sqrt{a+b}}{}$$

Mathematica [B] time = 6.78, size = 2102, normalized size = 4.03

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-2*d^2*Cos[e + f*x])/(
3*(b*c - a*d)*(c^2 - d^2)*(c + d*Sin[e + f*x])^2) + (4*(-3*b*c^2*d^2*Cos[e
+ f*x] + 2*a*c*d^3*Cos[e + f*x] + b*d^4*Cos[e + f*x]))/(3*(b*c - a*d)^2*(c^
2 - d^2)^2*(c + d*Sin[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(3*b^2*c^4 - 6*a
*b*c^3*d + 3*a^2*c^2*d^2 - 5*b^2*c^2*d^2 + 2*a*b*c*d^3 + a^2*d^4 + 2*b^2*d^
4)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqr
t[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)
]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + P
i/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x
]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e
+ f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c
+ d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(-6*b^2*c^3*d - 2*a*b*c^2*d^2 + 4*a^2
*c*d^3 + 2*b^2*c*d^3 + 2*a*b*d^4)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^
2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c
+ d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-
c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi
/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e
```

```
 + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)
*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-
e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcS
in[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
+ a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(
-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e
 + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*
Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c
+ d*Sin[e + f*x]])) + 2*(6*b^2*c^2*d^2 - 4*a*b*c*d^3 - 2*b^2*d^4)*((Cos[e +
 f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b
)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/
(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*
(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a
+ b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f
*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))
/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*S
qrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((
-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sq
rt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2
- f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/
(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*
x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*
Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c
 + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e
 + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b
*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqr
t[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]
*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) +
a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d
)))/(3*(c - d)^2*(c + d)^2*(b*c - a*d)^2*f)
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sin\left(fx+e\right)+a}\,\sqrt{d\sin\left(fx+e\right)+c}}{bd^3\cos\left(fx+e\right)^4+ac^3+3\,bc^2d+3\,acd^2+bd^3-\left(3\,bc^2d+3\,acd^2+2\,bd^3\right)\cos\left(fx+e\right)^2+\left(bc^3+3\,ac^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(b*d^3*cos(f*x +
 e)^4 + a*c^3 + 3*b*c^2*d + 3*a*c*d^2 + b*d^3 - (3*b*c^2*d + 3*a*c*d^2 + 2*
b*d^3)*cos(f*x + e)^2 + (b*c^3 + 3*a*c^2*d + 3*b*c*d^2 + a*d^3 - (3*b*c*d^2
 + a*d^3)*cos(f*x + e)^2)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2)), x)
```

maple [B] time = 5.60, size = 218898, normalized size = 420.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/(sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a+b\,\sin\left(e+f\,x\right)}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + b*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sin\left(e+fx\right)}\left(c+d\sin\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**(5/2),x)
[Out] Integral(1/(sqrt(a + b*sin(e + f*x))*(c + d*sin(e + f*x))**(5/2)), x)
```

3.789 $\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+b\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=822

$$\frac{2\cos(e+fx)\sqrt{c+d\sin(e+fx)}\,(bc-ad)^2}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}}+\frac{d\sqrt{c+d}\,(5bc-3ad)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{b^3\sqrt{}}$$

```
[Out] (c-d)*(3*a^2*d^2-4*a*b*c*d+2*b^2*c^2-b^2*d^2)*EllipticE((a+b)^(1/2)*(c+d*si
n(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d)
)^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e)
)/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f
*x+e)))^(1/2)/(a-b)/b^2/(-a*d+b*c)/f/(a+b)^(1/2)+d*(-3*a*d+5*b*c)*EllipticP
i((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),b*(
c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(
c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d
+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b^3/f/(a+b)^(1/2)-(3*a^2
*d^2-2*a*b*d*(c+3*d)-b^2*(2*c^2-6*c*d-d^2))*EllipticF((c+d)^(1/2)*(a+b*sin(
f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^
(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(
a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x
+e)))^(1/2)/(a-b)/b^3/f/(c+d)^(1/2)+2*(-a*d+b*c)^2*cos(f*x+e)*(c+d*sin(f*x+
e))^(1/2)/b/(a^2-b^2)/f/(a+b*sin(f*x+e))^(1/2)+(4*a*b*c*d-3*a^2*d^2-b^2*(2*
c^2-d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/b/(a^2-b^2)/f/(a+b*sin(f*x+e))^
(1/2)
```

Rubi [A] time = 2.65, antiderivative size = 822, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2792, 3061, 3053, 2811, 2998, 2818, 2996}

$$\frac{2\cos(e+fx)\sqrt{c+d\sin(e+fx)}\,(bc-ad)^2}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}}+\frac{d\sqrt{c+d}\,(5bc-3ad)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{b^3\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/(a + b*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((c - d)*Sqrt[c + d]*(2*b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - b^2*d^2)*Elliptic
E[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin
[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*
c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c -
a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x
```

```
]))/((a - b)*b^2*Sqrt[a + b]*(b*c - a*d)*f) + (d*Sqrt[c + d]*(5*b*c - 3*a*d
)*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e
+ f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b
)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(
a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a +
b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(b^3*Sqrt[a + b]*f) + (2*(b*c - a*d
)^2*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(b*(a^2 - b^2)*f*Sqrt[a + b*Sin[
e + f*x]]) + ((4*a*b*c*d - 3*a^2*d^2 - b^2*(2*c^2 - d^2))*Cos[e + f*x]*Sqrt
[c + d*Sin[e + f*x]])/(b*(a^2 - b^2)*f*Sqrt[a + b*Sin[e + f*x]]) - (Sqrt[a
+ b]*(3*a^2*d^2 - 2*a*b*d*(c + 3*d) - b^2*(2*c^2 - 6*c*d - d^2))*EllipticF[
ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e
 + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x])
)/((a - b)*b^3*Sqrt[c + d]*f)
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
```

```
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
```

```
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+b\sin(e+fx))^{3/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}} - \frac{2\int \frac{\frac{1}{2}\left(3b^2c^2d+a^2d^3-abc\left(c^2+3d^2\right)\right)-\frac{1}{2}\left(2a^2cd^2-a\right.}{}}{}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}} + \frac{\left(4abcd-3a^2d^2-b^2\left(2c^2-d^2\right)\right)\cos(}{b\left(a^2-b^2\right)f\sqrt{a+b\sin}}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}} + \frac{\left(4abcd-3a^2d^2-b^2\left(2c^2-d^2\right)\right)\cos(}{b\left(a^2-b^2\right)f\sqrt{a+b\sin}}$$

$$= \frac{d\sqrt{c+d}\,(5bc-3ad)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-}}{b^3\sqrt{a+b}\,f}$$

$$= -\frac{(c-d)\sqrt{c+d}\,\left(4abcd-3a^2d^2-b^2\left(2c^2-d^2\right)\right)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(}{(a+b)(}\right.}{(a-b)b^2\sqrt{a+b}}$$

Mathematica [B] time = 6.84, size = 2005, normalized size = 2.44

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/(a + b*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(b^2*c^2*Cos[e + f*x] - 2*a*b*c*d*Cos[e + f*x] + a^2*d^2*Cos[e + f*x])*
Sqrt[c + d*Sin[e + f*x]])/(b*(-a^2 + b^2)*f*Sqrt[a + b*Sin[e + f*x]]) + ((-
4*(-(b*c) + a*d)*(2*a*b*c^3 - 4*b^2*c^2*d + 2*a*b*c*d^2 + a^2*d^3 - b^2*d^3
)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt
[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]
/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi
/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]
))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e +
```

```
f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c +
d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(2*b^2*c^3 - 2*a*b*c^2*d + 4*a^2*c*d^2
- 6*b^2*c*d^2 + 2*a*b*d^3)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c
+ d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Si
n[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d)
)]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f
*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/
2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[
a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi
/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqr
t[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)
]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + P
i/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x
]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e
+ f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]])) + 2*(-2*b^2*c^2*d + 4*a*b*c*d^2 - 3*a^2*d^3 + b^2*d^3)*((Cos[
e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a
- b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a -
b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]],
(2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[(
(a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x
]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d
)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt
[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]
/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi
/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]
))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e +
f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c +
d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/
(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[
(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(
-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*
Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*
d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
+ a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(
b*d)))/(2*(a - b)*b*(a + b)*f)
```

fricas [F] time = 5.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d^{2} \cos\left(fx+e\right)^{2} - 2\,cd \sin\left(fx+e\right) - c^{2} - d^{2}\right)\sqrt{b \sin\left(fx+e\right) + a}\,\sqrt{d \sin\left(fx+e\right) + c}}{b^{2} \cos\left(fx+e\right)^{2} - 2\,ab \sin\left(fx+e\right) - a^{2} - b^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)*sqrt(b*sin(f
*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x +
 e) - a^2 - b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(b*sin(f*x + e) + a)^(3/2), x)
```

maple [B] time = 59.72, size = 3904542, normalized size = 4750.05

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(b*sin(f*x + e) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(5/2)/(a + b*sin(e + f*x))^(3/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^(5/2)/(a + b*sin(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.790 $\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=600

$$\frac{2\sqrt{a+b}\,(ad+b(c-2d))\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a}}{\sqrt{a+b}\,\sqrt{c}}\right.\right.}{b^2f(a-b)\sqrt{c+d}}$$

```
[Out] 2*(c-d)*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f
*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(
c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d
+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/(a-b)/b/f/(a+b)^(1/2)+2*
d*EllipticPi((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e)
)^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*si
n(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(
1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b^2/f/(a+b)^(
1/2)+2*(b*(c-2*d)+a*d)*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(
1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+
d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e))
)^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(a-b)/b^2
/f/(c+d)^(1/2)
```

Rubi [A] time = 0.93, antiderivative size = 600, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2798, 2811, 2998, 2818, 2996}

$$\frac{2\sqrt{a+b}\,(ad+b(c-2d))\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a}}{\sqrt{a+b}\,\sqrt{c}}\right.\right.}{b^2f(a-b)\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/(a + b*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x
]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c
- d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b
*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin
[e + f*x]))]*(a + b*Sin[e + f*x]))/((a - b)*b*Sqrt[a + b]*f) + (2*d*Sqrt[c
+ d]*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin
[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a
+ b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d
)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a
```

```
 + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(b^2*Sqrt[a + b]*f) + (2*Sqrt[a
+ b]*(b*(c - 2*d) + a*d)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f
*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(
c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d
*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Si
n[e + f*x])))]*(c + d*Sin[e + f*x]))/((a - b)*b^2*Sqrt[c + d]*f)
```

Rule 2798

```
Int[((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((a_.) + (b_.)*sin[(e_.) +
 (f_.)*(x_)])^(3/2), x_Symbol] :> Dist[d^2/b^2, Int[Sqrt[a + b*Sin[e + f*x]
]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b^2, Int[Simp[b*c + a
*d + 2*b*d*Sin[e + f*x], x]/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e +
f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
```

```
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx = \frac{d^2\int \frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx}{b^2} + \frac{(bc-ad)\int \frac{bc+ad+2bd\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx}{b^2}$$

$$= \frac{2d\sqrt{c+d}\,\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(1-\sin}{(c+d)(a+b\sin}}}{b^2\sqrt{a+b}\,f}$$

$$= \frac{2(c-d)\sqrt{c+d}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(1-\sin(e}{(c+d)(a+b\sin(e}}}{(a-b)b\sqrt{a+b}\,f}$$

Mathematica [B] time = 9.45, size = 1896, normalized size = 3.16

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/(a + b*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(-(b*c*Cos[e + f*x]) + a*d*Cos[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/((a^
2 - b^2)*f*Sqrt[a + b*Sin[e + f*x]]) + ((-4*(-(b*c) + a*d)*(a*c^2 - b*c*d)*
Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[(
(-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/S
qrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2
 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))
/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f
```

```
*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d
*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(b*c^2 - b*d^2)*((Sqrt[((c + d)*Cot[(-e
+ Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/
2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) +
a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c
+ d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[
((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])
/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqr
t[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/
((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e
+ f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*S
ec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/
2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 -
f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(-(b*c*d) + a*d^2)*((Cos[e + f*x]*
Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a +
 b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b
)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c
) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*C
os[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*S
qrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c +
 d)*(a + b*Sin[e + f*x]))]]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((
c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b
)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]]
, (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)
/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c
) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(
-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e
+ f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]
*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/
2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) +
a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c
+ d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[
((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])
/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))/((
a - b)*(a + b)*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) + a)^(3/2), x)
```

maple [B] time = 142.72, size = 2948827, normalized size = 4914.71

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(3/2)/(a + b*sin(e + f*x))^(3/2),x)
[Out] int((c + d*sin(e + f*x))^(3/2)/(a + b*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}}{\left(a+b\sin\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e))**(3/2),x)
[Out] Integral((c + d*sin(e + f*x))**(3/2)/(a + b*sin(e + f*x))**(3/2), x)
```

3.791 $\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=409

$$\frac{2\sqrt{a+b}\,(c-d)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right.\right.}{f(a-b)\sqrt{c+d}\,(bc-ad)}$$

```
[Out] 2*(c-d)*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f
*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(
c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d
+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/(a-b)/(-a*d+b*c)/f/(a+b)
^(1/2)+2*(c-d)*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+
d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*
x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*
(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(a-b)/(-a*d+b*c)/
f/(c+d)^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2795, 2818, 2996}

$$\frac{2\sqrt{a+b}\,(c-d)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right.\right.}{f(a-b)\sqrt{c+d}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x
]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c
- d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b
*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin
[e + f*x]))]*(a + b*Sin[e + f*x]))/((a - b)*Sqrt[a + b]*(b*c - a*d)*f) + (2
*Sqrt[a + b]*(c - d)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]]
)/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c +
d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin
[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e
+ f*x])))]*(c + d*Sin[e + f*x]))/((a - b)*Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 2795

```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin
```

```
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), In
t[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx = \frac{(c-d)\int \frac{1}{\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}\,dx}{a-b} - \frac{(bc-ad)\int \frac{1+\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,d}{a-b}$$

$$= \frac{2(c-d)\sqrt{c+d}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)\middle|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(1-\sin}{(c+d)(a+b\sin}}}{(a-b)\sqrt{a+b}\,(bc-ad)f}$$

Mathematica [A] time = 4.16, size = 226, normalized size = 0.55

$$-\frac{2\sqrt{2}\cos\left(\frac{1}{4}(2e+2fx-\pi)\right)\sqrt{\frac{a+b\sin(e+fx)}{a+b}}\sqrt{c+d\sin(e+fx)}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{a-b}{a+b}}\cos\left(\frac{1}{4}(2e+2fx+\pi)\right)}{\sqrt{\frac{a+b\sin(e+fx)}{a+b}}}\right)|\frac{2(ad-bc)}{(a-b)(c+d)}\right)}{f\sqrt{\frac{a-b}{a+b}}\sqrt{\frac{(a+b)(\sin(e+fx)+1)}{a+b\sin(e+fx)}}(a+b\sin(e+fx))^{3/2}\sqrt{\frac{(a+b)(c+d\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[2]*Cos[(2*e - Pi + 2*f*x)/4]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b
)]*Cos[(2*e + Pi + 2*f*x)/4])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*
c) + a*d))/((a - b)*(c + d))]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[c + d
*Sin[e + f*x]])/(Sqrt[(a - b)/(a + b)]*f*Sqrt[((a + b)*(1 + Sin[e + f*x]))/
(a + b*Sin[e + f*x])]*(a + b*Sin[e + f*x])^(3/2)*Sqrt[((a + b)*(c + d*Sin[e
 + f*x]))/((c + d)*(a + b*Sin[e + f*x]))])
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{b\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+c}}{b^{2}\cos\left(fx+e\right)^{2}-2\,ab\sin\left(fx+e\right)-a^{2}-b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(b^2*cos(f*x +
e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{d\sin\left(fx+e\right)+c}}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a)^(3/2), x)
```

maple [B] time = 0.97, size = 47019, normalized size = 114.96

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d \sin \left(fx + e\right) + c}}{\left(b \sin \left(fx + e\right) + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x, algorithm="maxim
a")
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c + d\,\sin\left(e + f\,x\right)}}{{\left(a + b\,\sin\left(e + f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(1/2)/(a + b*sin(e + f*x))^(3/2),x)
[Out] int((c + d*sin(e + f*x))^(1/2)/(a + b*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + d \sin \left(e + fx\right)}}{\left(a + b \sin \left(e + fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))**(3/2),x)
[Out] Integral(sqrt(c + d*sin(e + f*x))/(a + b*sin(e + f*x))**(3/2), x)
```

3.792 $\int \frac{1}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=405

$$\frac{2\sqrt{a+b}\,\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+}{(a-}\right.}{f(a-b)\sqrt{c+d}\,(bc-ad)}$$

```
[Out] 2*b*(c-d)*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin
(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))
*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a
*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/(a-b)/(-a*d+b*c)^2/f/(
a+b)^(1/2)+2*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*
sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+
e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-
(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(a-b)/(-a*d+b*c)/f/
(c+d)^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2801, 2818, 2996}

$$\frac{2\sqrt{a+b}\,\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+}{(a-}\right.}{f(a-b)\sqrt{c+d}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (2*b*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f
*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(
c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a +
 b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*S
in[e + f*x]))]*(a + b*Sin[e + f*x]))/((a - b)*Sqrt[a + b]*(b*c - a*d)^2*f)
+ (2*Sqrt[a + b]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(S
qrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]
*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e +
 f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*
x])))]*(c + d*Sin[e + f*x]))/((a - b)*Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 2801

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
```

```
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx = \frac{\int \frac{1}{\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}\,dx}{a-b} - \frac{b\int \frac{1+\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+f}}}{a-b}$$

$$= \frac{2b(c-d)\sqrt{c+d}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e}{(a-b)\sqrt{a}}$$

Mathematica [B] time = 32.74, size = 90261, normalized size = 222.87

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+c}}{2\,abd-\left(b^2c+2\,abd\right)\cos\left(fx+e\right)^2+\left(a^2+b^2\right)c-\left(b^2d\cos\left(fx+e\right)^2-2\,abc-\left(a^2+b^2\right)d\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(2*a*b*d - (b^2*
c + 2*a*b*d)*cos(f*x + e)^2 + (a^2 + b^2)*c - (b^2*d*cos(f*x + e)^2 - 2*a*b
*c - (a^2 + b^2)*d)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c)), x)
```

maple [B] time = 1.02, size = 40621, normalized size = 100.30

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+b\,\sin\left(e+f\,x\right)\right)^{3/2}\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sin\left(e+fx\right)\right)^{\frac{3}{2}}\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/((a + b*sin(e + f*x))**(3/2)*sqrt(c + d*sin(e + f*x))), x)
```

3.793 $\int \frac{1}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=495

$$-\frac{2\left(a^2d^2+b^2\left(c^2-2d^2\right)\right)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,E\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{}}{\sqrt{a+b}\,\sqrt{}}\right.\right.}{f\sqrt{a+b}\,(c-d)\sqrt{c+d}\,(bc-ad)^3}$$

```
[Out] -2*(a^2*d^2+b^2*(c^2-2*d^2))*EllipticE((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(
a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+
e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2
)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)/(-a*d+b*c
)^3/f/(a+b)^(1/2)/(c+d)^(1/2)+2*(b*(c-2*d)-a*d)*EllipticF((c+d)^(1/2)*(a+b*
sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+
d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+
d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1
/2)/(c-d)/(-a*d+b*c)^2/f/(a+b)^(1/2)/(c+d)^(1/2)+2*b^2*cos(f*x+e)/(a^2-b^2)
/(-a*d+b*c)/f/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.91, antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2802, 2998, 2818, 2996}

$$-\frac{2\left(a^2d^2+b^2\left(c^2-2d^2\right)\right)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,E\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{}}{\sqrt{a+b}\,\sqrt{}}\right.\right.}{f\sqrt{a+b}\,(c-d)\sqrt{c+d}\,(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (2*b^2*Cos[e + f*x])/((a^2 - b^2)*(b*c - a*d)*f*Sqrt[a + b*Sin[e + f*x]]*Sq
rt[c + d*Sin[e + f*x]]) - (2*(a^2*d^2 + b^2*(c^2 - 2*d^2))*EllipticE[ArcSin
[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x
]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(
1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 +
 Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(Sqr
t[a + b]*(c - d)*Sqrt[c + d]*(b*c - a*d)^3*f) + (2*(b*(c - 2*d) - a*d)*Elli
pticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d
*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((
b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c
 - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e +
 f*x]))/(Sqrt[a + b]*(c - d)*Sqrt[c + d]*(b*c - a*d)^2*f)
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2b^2\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}} -$$

$$= \frac{2b^2\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}} -$$

$$= \frac{2b^2\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}} -$$

Mathematica [B] time = 6.92, size = 2082, normalized size = 4.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((2*b^3*Cos[e + f*x])/((
a^2 - b^2)*(-(b*c) + a*d)^2*(a + b*Sin[e + f*x])) + (2*d^3*Cos[e + f*x])/((
b*c - a*d)^2*(c^2 - d^2)*(c + d*Sin[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(a
*b^2*c^3 - 2*a^2*b*c^2*d + 2*b^3*c^2*d + a^3*c*d^2 - 2*a*b^2*c*d^2 + 2*a^2*
b*d^3 - 2*b^3*d^3)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Elli
pticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x])
)/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e +
f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a
 + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]
^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(b^3*c^3 + a*b^2*c^2*
d + a^2*b*c*d^2 - 2*b^3*c*d^2 + a^3*d^3 - 2*a*b^2*d^3)*((Sqrt[((c + d)*Cot[
(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e +
 Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c
) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[
((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*S
qrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*
d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) -
(Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a
```

```
*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Si
n[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d)
)]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f
*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/
2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*
Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(-(b^3*c^2*d) - a^2*b*d^3 + 2*
b^3*d^3)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x
]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[Arc
Sin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*
x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x
]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c
+ d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*(((
(a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Ellipt
icF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/
(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*
x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a +
 b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2
*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/
2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt
[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]
/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi
/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]
))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e +
 f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin
[e + f*x]]))))/(b*d)))/((a - b)*(a + b)*(c - d)*(c + d)*(-(b*c) + a*d)^2*f)
```

fricas [F] time = 1.29, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sin\left(fx+e\right)+a}\,\sqrt{d\sin\left(fx+\right.}}{b^2d^2\cos\left(fx+e\right)^4+4\,abcd+\left(a^2+b^2\right)c^2+\left(a^2+b^2\right)d^2-\left(b^2c^2+4\,abcd+\left(a^2+2\,b^2\right)d^2\right)\cos\left(fx+e\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(b^2*d^2*cos(f*x
 + e)^4 + 4*a*b*c*d + (a^2 + b^2)*c^2 + (a^2 + b^2)*d^2 - (b^2*c^2 + 4*a*b*
c*d + (a^2 + 2*b^2)*d^2)*cos(f*x + e)^2 + 2*(a*b*c^2 + a*b*d^2 + (a^2 + b^2
)*c*d - (b^2*c*d + a*b*d^2)*cos(f*x + e)^2)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(3/2)), x)
```

maple [B] time = 2.08, size = 119964, normalized size = 242.35

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sin\left(e+fx\right)\right)^{\frac{3}{2}}\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(3/2),x)
[Out] Integral(1/((a + b*sin(e + f*x))**(3/2)*(c + d*sin(e + f*x))**(3/2)), x)
```

3.794 $\int \frac{1}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=681

$$\frac{2d\left(a^2d^2+b^2\left(3c^2-4d^2\right)\right)\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f\left(a^2-b^2\right)\left(c^2-d^2\right)(bc-ad)^2(c+d\sin(e+fx))^{3/2}}+\frac{2\left(a^2d^2(3c+d)-6abd\left(c^2-d^2\right)+b^2\left(3c^3-9c^2d-6c\right.\right.}{}$$

```
[Out] 2*b^2*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x
+e))^(1/2)+2/3*d*(a^2*d^2+b^2*(3*c^2-4*d^2))*cos(f*x+e)*(a+b*sin(f*x+e))^(1
/2)/(a^2-b^2)/(-a*d+b*c)^2/(c^2-d^2)/f/(c+d*sin(f*x+e))^(3/2)+2/3*(4*a^3*c*
d^3-4*a*b^2*c*d^3-a^2*b*d^2*(9*c^2-5*d^2)-b^3*(3*c^4-15*c^2*d^2+8*d^4))*Ell
ipticE((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2
),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*
(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a
-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)^4/f/(a+b)^(1/2)+
2/3*(a^2*d^2*(3*c+d)-6*a*b*d*(c^2-d^2)+b^2*(3*c^3-9*c^2*d-6*c*d^2+8*d^3))*E
llipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1
/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c
)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/
(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)^3/f/(a+b)^(1/2
)
```

Rubi [A] time = 2.68, antiderivative size = 681, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2802, 3055, 2998, 2818, 2996}

$$\frac{2d\left(a^2d^2+b^2\left(3c^2-4d^2\right)\right)\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3f\left(a^2-b^2\right)\left(c^2-d^2\right)(bc-ad)^2(c+d\sin(e+fx))^{3/2}}+\frac{2\left(a^2d^2(3c+d)-6abd\left(c^2-d^2\right)+b^2\left(-9c^2d+3c^3-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (2*b^2*Cos[e + f*x])/((a^2 - b^2)*(b*c - a*d)*f*Sqrt[a + b*Sin[e + f*x]]*(c
 + d*Sin[e + f*x])^(3/2)) + (2*d*(a^2*d^2 + b^2*(3*c^2 - 4*d^2))*Cos[e + f*
x]*Sqrt[a + b*Sin[e + f*x]])/(3*(a^2 - b^2)*(b*c - a*d)^2*(c^2 - d^2)*f*(c
+ d*Sin[e + f*x])^(3/2)) + (2*(4*a^3*c*d^3 - 4*a*b^2*c*d^3 - a^2*b*d^2*(9*c
^2 - 5*d^2) - b^3*(3*c^4 - 15*c^2*d^2 + 8*d^4))*EllipticE[ArcSin[(Sqrt[c +
d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a +
 b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e +
 f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*
```

```
x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(3*Sqrt[a + b]*
(c - d)^2*(c + d)^(3/2)*(b*c - a*d)^4*f) + (2*(a^2*d^2*(3*c + d) - 6*a*b*d*
(c^2 - d^2) + b^2*(3*c^3 - 9*c^2*d - 6*c*d^2 + 8*d^3))*EllipticF[ArcSin[(Sq
rt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])]
, ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 -
Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin
[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(3*Sqrt[
a + b]*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)^3*f)
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2b^2\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}} -$$

$$= \frac{2b^2\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}} +$$

$$= \frac{2b^2\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}} +$$

$$= \frac{2b^2\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}} +$$

Mathematica [B] time = 7.62, size = 2350, normalized size = 3.45

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-2*b^4*Cos[e + f*x])/(
(a^2 - b^2)*(-(b*c) + a*d)^3*(a + b*Sin[e + f*x])) + (2*d^3*Cos[e + f*x])/(
3*(b*c - a*d)^2*(c^2 - d^2)*(c + d*Sin[e + f*x])^2) - (2*(-9*b*c^2*d^3*Cos[
e + f*x] + 4*a*c*d^4*Cos[e + f*x] + 5*b*d^5*Cos[e + f*x]))/(3*(b*c - a*d)^3
*(c^2 - d^2)^2*(c + d*Sin[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(-3*a*b^3*c^
5 + 9*a^2*b^2*c^4*d - 9*b^4*c^4*d - 9*a^3*b*c^3*d^2 + 15*a*b^3*c^3*d^2 + 3*
a^4*c^2*d^3 - 20*a^2*b^2*c^2*d^3 + 17*b^4*c^2*d^3 + 5*a^3*b*c*d^4 - 8*a*b^3
*c*d^4 + a^4*d^5 + 7*a^2*b^2*d^5 - 8*b^4*d^5)*Sqrt[((c + d)*Cot[(-e + Pi/2
- f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x
)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/(
(a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Cs
c[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a -
b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a +
b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) +
 a*d)*(-3*b^4*c^5 - 3*a*b^3*c^4*d - 9*a^2*b^2*c^3*d^2 + 15*b^4*c^3*d^2 - 5*
a^3*b*c^2*d^3 + 11*a*b^3*c^2*d^3 + 4*a^4*c*d^4 + a^2*b^2*c*d^4 - 8*b^4*c*d^
4 + 5*a^3*b*d^5 - 8*a*b^3*d^5)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/
(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d
*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c +
 d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2
- f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e +
Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e +
 Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[
Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a
*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e
+ Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e +
f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin
[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d
*Sin[e + f*x]])) + 2*(3*b^4*c^4*d + 9*a^2*b^2*c^2*d^3 - 15*b^4*c^2*d^3 - 4*
a^3*b*c*d^4 + 4*a*b^3*c*d^4 - 5*a^2*b^2*d^5 + 8*b^4*d^5)*((Cos[e + f*x]*Sqr
t[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)
]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*
Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) +
 a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[
(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt
[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)
```

```
*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c +
 d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*C
sc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (
2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]
^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) +
 a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b
*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f
*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*El
lipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 -
 f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d
))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d
)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-
a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((
a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))/(3*(a
 - b)*(a + b)*(c - d)^2*(c + d)^2*(-(b*c) + a*d)^3*f)
```

fricas [F] time = 1.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{}{6\,abc^2d + 2\,abd^3 + \left(3\,b^2cd^2 + 2\,abd^3\right)\cos\left(fx+e\right)^4 + \left(a^2+b^2\right)c^3 + 3\left(a^2+b^2\right)cd^2 - \left(b^2c^3 + 6\,abc^2d + \cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(6*a*b*c^2*d + 2
*a*b*d^3 + (3*b^2*c*d^2 + 2*a*b*d^3)*cos(f*x + e)^4 + (a^2 + b^2)*c^3 + 3*(
a^2 + b^2)*c*d^2 - (b^2*c^3 + 6*a*b*c^2*d + 4*a*b*d^3 + 3*(a^2 + 2*b^2)*c*d
^2)*cos(f*x + e)^2 + (b^2*d^3*cos(f*x + e)^4 + 2*a*b*c^3 + 6*a*b*c*d^2 + 3*
(a^2 + b^2)*c^2*d + (a^2 + b^2)*d^3 - (3*b^2*c^2*d + 6*a*b*c*d^2 + (a^2 + 2
*b^2)*d^3)*cos(f*x + e)^2)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(5/2)), x)
```

maple [B] time = 10.11, size = 415383, normalized size = 609.96

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d \sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="max
ima")
[Out] integrate(1/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(5/2)),x)
[Out] int(1/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(5/2),x)
[Out] Timed out
```

3.795 $\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+b\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=736

$$\frac{2(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3bf\left(a^2-b^2\right)(a+b\sin(e+fx))^{3/2}}+\frac{2(c-d)\sqrt{c+d}\left(3a^2d+4abc-7b^2d\right)\sec(e+fx)(a+b\sin(e+fx))}{3b^2f}$$

```
[Out] 2/3*(c-d)*(3*a^2*d+4*a*b*c-7*b^2*d)*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^
(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*s
ec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a
+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1
/2)/(a-b)^2/b^2/(a+b)^(3/2)/f+2*d^2*EllipticPi((a+b)^(1/2)*(c+d*sin(f*x+e))
^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b
)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin
(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+
b*sin(f*x+e)))^(1/2)/b^3/f/(a+b)^(1/2)+2/3*(3*a^2*b*(c-2*d)*d+3*a^3*d^2+a*b
^2*(3*c^2-4*c*d-2*d^2)+b^3*(c^2-7*c*d+9*d^2))*EllipticF((c+d)^(1/2)*(a+b*si
n(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d)
)^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*
sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2
)/(a-b)^2/b^3/f/(a+b)^(1/2)/(c+d)^(1/2)+2/3*(-a*d+b*c)^2*cos(f*x+e)*(c+d*si
n(f*x+e))^(1/2)/b/(a^2-b^2)/f/(a+b*sin(f*x+e))^(3/2)
```

Rubi [A] time = 1.84, antiderivative size = 736, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2792, 3053, 2811, 2998, 2818, 2996}

$$\frac{2\left(3a^2bd(c-2d)+3a^3d^2+ab^2\left(3c^2-4cd-2d^2\right)+b^3\left(c^2-7cd+9d^2\right)\right)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-}{(a+b)(c+d}}}{3b^3f(a-b)^2\sqrt{a+b}\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(5/2)/(a + b*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*(c - d)*Sqrt[c + d]*(4*a*b*c + 3*a^2*d - 7*b^2*d)*EllipticE[ArcSin[(Sqrt
[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])],
((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 -
Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[
e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(3*(a - b)
^2*b^2*(a + b)^(3/2)*f) + (2*d^2*Sqrt[c + d]*EllipticPi[(b*(c + d))/((a + b
)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b
```

```
*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(
((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*
c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e +
 f*x]))/(b^3*Sqrt[a + b]*f) + (2*(b*c - a*d)^2*Cos[e + f*x]*Sqrt[c + d*Sin[
e + f*x]])/(3*b*(a^2 - b^2)*f*(a + b*Sin[e + f*x])^(3/2)) + (2*(3*a^2*b*(c
- 2*d)*d + 3*a^3*d^2 + a*b^2*(3*c^2 - 4*c*d - 2*d^2) + b^3*(c^2 - 7*c*d + 9
*d^2))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]
*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f
*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*S
qrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c
+ d*Sin[e + f*x]))/(3*(a - b)^2*b^3*Sqrt[a + b]*Sqrt[c + d]*f)
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
```

```
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{5/2}}{(a+b\sin(e+fx))^{5/2}}\,dx = \frac{2(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3b\left(a^2-b^2\right)f(a+b\sin(e+fx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(-3abc^3+7b^2c^2d-5abcd^2+a^2d^3\right)-\frac{1}{2}\left(2a^2c\right.}{(a+b}}{}$$

$$= \frac{2(bc-ad)^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3b\left(a^2-b^2\right)f(a+b\sin(e+fx))^{3/2}} - \frac{2\int \frac{\frac{3}{2}a^2\left(a^2-b^2\right)d^3+\frac{1}{2}b^2\left(-3abc^3+7b^2c^2d-5abc\right.}{}}{}$$

$$= \frac{2d^2\sqrt{c+d}\,\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(bc-ad)(1}{(c+d)(a+b}}}{b^3\sqrt{a+b}\,f}$$

$$= \frac{2(c-d)\sqrt{c+d}\,\left(4abc+3a^2d-7b^2d\right)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(}{3(a-b)^2b^2(a+b)}$$

Mathematica [B] time = 7.09, size = 2172, normalized size = 2.95

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(5/2)/(a + b*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-2*(b^2*c^2*Cos[e + f*
x] - 2*a*b*c*d*Cos[e + f*x] + a^2*d^2*Cos[e + f*x]))/(3*b*(-a^2 + b^2)*(a +
 b*Sin[e + f*x])^2) - (2*(-4*a*b^2*c^2*Cos[e + f*x] + a^2*b*c*d*Cos[e + f*x
] + 7*b^3*c*d*Cos[e + f*x] + 3*a^3*d^2*Cos[e + f*x] - 7*a*b^2*d^2*Cos[e + f
*x]))/(3*b*(-a^2 + b^2)^2*(a + b*Sin[e + f*x]))))/f - ((-4*(-(b*c) + a*d)*(
-3*a^2*b*c^3 - b^3*c^3 + 8*a*b^2*c^2*d - 2*a^2*b*c*d^2 - 2*b^3*c*d^2 + a^3*
d^3 - a*b^2*d^3)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Ellipt
icF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/
(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*
x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a +
 b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2
*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(-4*a*b^2*c^3 - 3*a^2*b
*c^2*d + 7*b^3*c^2*d + 4*a^3*c*d^2 - a^2*b*d^3 - 3*b^3*d^3)*((Sqrt[((c + d)
*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[
(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(
-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*
Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*
```

```
d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
 + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]
]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c
) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c +
 d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c
 + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/
2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e
+ Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a
 + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(4*a*b^2*c^2*d - a^2*b*c*
d^2 - 7*b^3*c*d^2 - 3*a^3*d^3 + 7*a*b^2*d^3)*((Cos[e + f*x]*Sqrt[c + d*Sin[
e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Co
s[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi
/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a -
 b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 -
 f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[
e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[
e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e
+ Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/
2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) +
a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c
+ d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[
((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])
/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*
c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(
b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(
c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*
(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e +
Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(
-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqr
t[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))/(3*(a - b)^2*b*(a
 + b)^2*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(5/2),x, algorithm="frica
s")

[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(b*sin(f*x + e) + a)^(5/2), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(a+b\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(5/2),x)
```

```
[Out] int((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(5/2)/(b*sin(f*x + e) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(5/2)/(a + b*sin(e + f*x))^(5/2),x)
[Out] int((c + d*sin(e + f*x))^(5/2)/(a + b*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e))**(5/2),x)
[Out] Timed out
```

3.796 $\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=497

$$\frac{2(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f\left(a^2-b^2\right)(a+b\sin(e+fx))^{3/2}}+\frac{2(c-d)(3ac-ad+bc-3bd)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)\cdots}{(a+b)(c\cdots}}}{3f(a-b)^2\sqrt{a+\cdots}}$$

```
[Out] 8/3*(c-d)*(a*c-b*d)*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2
)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*s
in(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^
(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/(a-b)^2/(a+b
)^(3/2)/(-a*d+b*c)/f+2/3*(c-d)*(3*a*c-a*d+b*c-3*b*d)*EllipticF((c+d)^(1/2)*
(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b
)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b
)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)
))^(1/2)/(a-b)^2/(-a*d+b*c)/f/(a+b)^(1/2)/(c+d)^(1/2)+2/3*(-a*d+b*c)*cos(f*
x+e)*(c+d*sin(f*x+e))^(1/2)/(a^2-b^2)/f/(a+b*sin(f*x+e))^(3/2)
```

Rubi [A] time = 1.00, antiderivative size = 497, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2799, 2998, 2818, 2996}

$$\frac{2(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f\left(a^2-b^2\right)(a+b\sin(e+fx))^{3/2}}+\frac{2(c-d)(3ac-ad+bc-3bd)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)\cdots}{(a+b)(c\cdots}}}{3f(a-b)^2\sqrt{a+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Sin[e + f*x])^(3/2)/(a + b*Sin[e + f*x])^(5/2),x]
```

```
[Out] (8*(c - d)*Sqrt[c + d]*(a*c - b*d)*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d
*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/
((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d
)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(3*(a - b)^2*(a + b)^(3/2)*(
b*c - a*d)*f) + (2*(b*c - a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(3*(a
^2 - b^2)*f*(a + b*Sin[e + f*x])^(3/2)) + (2*(c - d)*(3*a*c + b*c - a*d - 3
*b*d)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*
Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*
x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sq
rt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c +
 d*Sin[e + f*x]))/(3*(a - b)^2*Sqrt[a + b]*Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{5/2}}\,dx = \frac{2(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3\left(a^2-b^2\right)f(a+b\sin(e+fx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(4bcd-a\left(3c^2+d^2\right)\right)-\frac{1}{2}\left(4acd-b\left(c^2+3d^2\right)\right)\sin\ldots}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\ldots}{3\left(a^2-b^2\right)}$$

$$= \frac{2(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3\left(a^2-b^2\right)f(a+b\sin(e+fx))^{3/2}} + \frac{((c-d)(3ac+bc-ad-3bd))\int \frac{\ldots}{\sqrt{a+\ldots}}}{3(a-b)^2(a+\ldots}$$

$$= \frac{8(c-d)\sqrt{c+d}\,(ac-bd)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{-\frac{(b\ldots}{(a\ldots}}\ldots}{3(a-b)^2(a+b)^{3/2}(bc-ad)\ldots}$$

Mathematica [B] time = 6.35, size = 2012, normalized size = 4.05

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sin[e + f*x])^(3/2)/(a + b*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-2*(-(b*c*Cos[e + f*x]
) + a*d*Cos[e + f*x]))/(3*(a^2 - b^2)*(a + b*Sin[e + f*x])^2) - (8*(-(a*b*c
*Cos[e + f*x]) + b^2*d*Cos[e + f*x]))/(3*(a^2 - b^2)^2*(a + b*Sin[e + f*x])
)))/f + ((-4*(-(b*c) + a*d)*(3*a^2*c^2 + b^2*c^2 - 4*a*b*c*d + a^2*d^2 - b^
2*d^2)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin
[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) +
a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e
 + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e +
 f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Si
n[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqr
t[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(4*a*b*c^2 + 4*a^2*c*d - 4*b^2*c*
d - 4*a*b*d^2)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Ellipt
icF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/
(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*
x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a +
 b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2
*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^
2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*C
sc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (
2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]
```

```
^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) +
 a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b
*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))
 + 2*(-4*a*b*c*d + 4*b^2*d^2)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*S
qrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 -
f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/S
qrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*S
qrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a
+ b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a +
b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) -
(2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2
]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(
c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*
(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e +
Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(
-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c +
d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[(
(c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a
 + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f
*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[
e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^
2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x
)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))/(3*(a - b)^2*(a + b)^2*f)
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{b\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{3\,ab^{2}\cos\left(fx+e\right)^{2}-a^{3}-3\,ab^{2}+\left(b^{3}\cos\left(fx+e\right)^{2}-3\,a^{2}b-b^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(3/2)/(3*a*b^2*cos(
f*x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 - 3*a^2*b - b^3)*sin(f*x +
 e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) + a)^(5/2), x)
```

maple [B] time = 4.10, size = 195220, normalized size = 392.80

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(b \sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(3/2)/(a + b*sin(e + f*x))^(5/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^(3/2)/(a + b*sin(e + f*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+d \sin\left(e+fx\right)\right)^{\frac{3}{2}}}{\left(a+b \sin\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((c + d*sin(e + f*x))**(3/2)/(a + b*sin(e + f*x))**(5/2), x)
```

3.797 $\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=489

$$\frac{2b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f\left(a^2-b^2\right)(a+b\sin(e+fx))^{3/2}}+\frac{2(c-d)\sqrt{c+d}\left(-3a^2d+4abc-b^2d\right)\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-}{(c+d}}}{3f(a-b)^2(a+b}$$

```
[Out] 2/3*(c-d)*(-3*a^2*d+4*a*b*c-b^2*d)*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(
1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*se
c(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+
b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/
2)/(a-b)^2/(a+b)^(3/2)/(-a*d+b*c)^2/f+2/3*(3*a+b)*(c-d)*EllipticF((c+d)^(1/
2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(
a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(
a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x
+e)))^(1/2)/(a-b)^2/(-a*d+b*c)/f/(a+b)^(1/2)/(c+d)^(1/2)+2/3*b*cos(f*x+e)*(
c+d*sin(f*x+e))^(1/2)/(a^2-b^2)/f/(a+b*sin(f*x+e))^(3/2)
```

Rubi [A] time = 0.86, antiderivative size = 489, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2796, 2998, 2818, 2996}

$$\frac{2b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f\left(a^2-b^2\right)(a+b\sin(e+fx))^{3/2}}+\frac{2(c-d)\sqrt{c+d}\left(-3a^2d+4abc-b^2d\right)\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-}{(c+d}}}{3f(a-b)^2(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*(c - d)*Sqrt[c + d]*(4*a*b*c - 3*a^2*d - b^2*d)*EllipticE[ArcSin[(Sqrt[a
 + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((
a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Si
n[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e
+ f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(3*(a - b)^2
*(a + b)^(3/2)*(b*c - a*d)^2*f) + (2*b*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]
])/(3*(a^2 - b^2)*f*(a + b*Sin[e + f*x])^(3/2)) + (2*(3*a + b)*(c - d)*Elli
pticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d
*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((
b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c
 - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e +
 f*x]))/(3*(a - b)^2*Sqrt[a + b]*Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{5/2}}\,dx = \frac{2b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3\left(a^2-b^2\right)f(a+b\sin(e+fx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}(-3ac+bd)+\frac{1}{2}(bc-3ad)\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx}{3\left(a^2-b^2\right)}$$

$$= \frac{2b\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3\left(a^2-b^2\right)f(a+b\sin(e+fx))^{3/2}} + \frac{((3a+b)(c-d))\int \frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}}{3(a-b)^2(a+b)}$$

$$= \frac{2(c-d)\sqrt{c+d}\left(4abc-3a^2d-b^2d\right)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e}{3(a-b)^2(a+b)^{3/2}(bc}$$

Mathematica [B] time = 6.43, size = 2067, normalized size = 4.23

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((2*b*Cos[e + f*x])/(3*(
a^2 - b^2)*(a + b*Sin[e + f*x])^2) + (2*(-4*a*b^2*c*Cos[e + f*x] + 3*a^2*b*
d*Cos[e + f*x] + b^3*d*Cos[e + f*x]))/(3*(a^2 - b^2)^2*(-(b*c) + a*d)*(a +
b*Sin[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(-3*a^2*b*c^2 - b^3*c^2 + 3*a^3*
c*d + a*b^2*c*d - a^2*b*d^2 + b^3*d^2)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/
2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*
(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)
*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e +
 Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[
(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c +
 d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(
-4*a*b^2*c^2 - a^2*b*c*d + b^3*c*d + 3*a^3*d^2 + a*b^2*d^2)*((Sqrt[((c + d)
*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[
(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(
-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*
Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*
d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
 + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]
]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c
) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c +
 d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c
```

```
 + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/
2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e
+ Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a
 + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(4*a*b^2*c*d - 3*a^2*b*d^
2 - b^3*d^2)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e +
 f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE
[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e
+ f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e +
 f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])
]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)
*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)
*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*El
lipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x
]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e
+ f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*
(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/
2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e +
 Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[
Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a
*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e
+ Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e +
f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin
[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d
*Sin[e + f*x]])))/(b*d)))/(3*(a - b)^2*(a + b)^2*(-(b*c) + a*d)*f)
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{b\sin\left(fx+e\right)+a}\,\sqrt{d\sin\left(fx+e\right)+c}}{3\,ab^2\cos\left(fx+e\right)^2-a^3-3\,ab^2+\left(b^3\cos\left(fx+e\right)^2-3\,a^2b-b^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(3*a*b^2*cos(f*
x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 - 3*a^2*b - b^3)*sin(f*x + e
)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\sin\left(fx+e\right)+c}}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a)^(5/2), x)
```

maple [B] time = 4.68, size = 212259, normalized size = 434.07

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d \sin\left(fx+e\right)+c}}{\left(b \sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c+d\,\sin\left(e+f\,x\right)}}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*sin(e + f*x))^(1/2)/(a + b*sin(e + f*x))^(5/2),x)
```

```
[Out] int((c + d*sin(e + f*x))^(1/2)/(a + b*sin(e + f*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + d \sin{\left(e + f x \right)}}}{\left(a + b \sin{\left(e + f x \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(c + d*sin(e + f*x))/(a + b*sin(e + f*x))**(5/2), x)
```

3.798 $\int \frac{1}{(a+b\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=516

$$\frac{2b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f\left(a^2-b^2\right)(bc-ad)(a+b\sin(e+fx))^{3/2}}+\frac{2\left(-3a^2d+3ab(c-d)+b^2(c+2d)\right)\sec(e+fx)(c+d\sin(e+fx))}{3f(a-b)^2}$$

```
[Out] 4/3*b*(c-d)*(-3*a^2*d+2*a*b*c+b^2*d)*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))
^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*
sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(
a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(
1/2)/(a-b)^2/(a+b)^(3/2)/(-a*d+b*c)^3/f+2/3*(3*a*b*(c-d)-3*a^2*d+b^2*(c+2*d
))*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e)
)^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d
+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+
e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(a-b)^2/(-a*d+b*c)^2/f/(a+b)^(1/2)/(c+d)^
(1/2)+2/3*b^2*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/(a^2-b^2)/(-a*d+b*c)/f/(a+b
*sin(f*x+e))^(3/2)
```

Rubi [A] time = 0.99, antiderivative size = 516, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2802, 2998, 2818, 2996}

$$\frac{2b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3f\left(a^2-b^2\right)(bc-ad)(a+b\sin(e+fx))^{3/2}}+\frac{2\left(-3a^2d+3ab(c-d)+b^2(c+2d)\right)\sec(e+fx)(c+d\sin(e+fx))}{3f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^(5/2)*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (4*b*(c - d)*Sqrt[c + d]*(2*a*b*c - 3*a^2*d + b^2*d)*EllipticE[ArcSin[(Sqrt
[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])],
((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 -
Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[
e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(3*(a - b)
^2*(a + b)^(3/2)*(b*c - a*d)^3*f) + (2*b^2*Cos[e + f*x]*Sqrt[c + d*Sin[e +
f*x]])/(3*(a^2 - b^2)*(b*c - a*d)*f*(a + b*Sin[e + f*x])^(3/2)) + (2*(3*a*b
*(c - d) - 3*a^2*d + b^2*(c + 2*d))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a +
b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))
/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a
+ b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b
```

```
)*(c + d*Sin[e + f*x]))))]*(c + d*Sin[e + f*x]))/(3*(a - b)^2*Sqrt[a + b]*Sq
rt[c + d]*(b*c - a*d)^2*f)
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^{5/2}\sqrt{c+d\sin(e+fx)}}\,dx = \frac{2b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(-2b^2d-3a(bc-\cdots}{(a+b\sin(e+\cdots}}{3\left(a^2\cdots\right.}$$

$$= \frac{2b^2\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{3\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^{3/2}} - \frac{\left(2b\left(2abc-3a^2\cdots\right.\right.}{\cdots}$$

$$= \frac{4b(c-d)\sqrt{c+d}\,\left(2abc-3a^2d+b^2d\right)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+\cdots}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+\cdots}}\right.\right.}{3(\cdots}$$

Mathematica [B] time = 6.45, size = 2102, normalized size = 4.07

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^(5/2)*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-2*b^2*Cos[e + f*x])/(
3*(a^2 - b^2)*(-(b*c) + a*d)*(a + b*Sin[e + f*x])^2) + (4*(2*a*b^3*c*Cos[e
+ f*x] - 3*a^2*b^2*d*Cos[e + f*x] + b^4*d*Cos[e + f*x]))/(3*(a^2 - b^2)^2*(
-(b*c) + a*d)^2*(a + b*Sin[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(3*a^2*b^2*
c^2 + b^4*c^2 - 6*a^3*b*c*d + 2*a*b^3*c*d + 3*a^4*d^2 - 5*a^2*b^2*d^2 + 2*b
^4*d^2)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSi
n[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) +
 a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-
e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e
+ f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*S
in[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sq
rt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(4*a*b^3*c^2 - 2*a^2*b^2*c*d + 2
*b^4*c*d - 6*a^3*b*d^2 + 2*a*b^3*d^2)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)
/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2
*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b
)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e
+ Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc
```

```
[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c
+ d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Co
t[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d),
ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b
*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*S
in[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*S
in[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c
+ d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqr
t[c + d*Sin[e + f*x]])) + 2*(-4*a*b^3*c*d + 6*a^2*b^2*d^2 - 2*b^4*d^2)*((Co
s[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(
a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a
- b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]]
, (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt
[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[
e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f
*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a
*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sq
rt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d
)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e +
Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*
x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e
 + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c
 + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2
)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Cs
c[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2
*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^
4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) +
a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*
c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))
/(b*d)))/(3*(a - b)^2*(a + b)^2*(-(b*c) + a*d)^2*f)
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sin\left(fx+e\right)+a}\,\sqrt{d\sin\left(fx+e\right)+}}{b^3d\cos\left(fx+e\right)^4-\left(3\,ab^2c+\left(3\,a^2b+2\,b^3\right)d\right)\cos\left(fx+e\right)^2+\left(a^3+3\,ab^2\right)c+\left(3\,a^2b+b^3\right)d-\left(\left(b^3c+3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(b^3*d*cos(f*x +
 e)^4 - (3*a*b^2*c + (3*a^2*b + 2*b^3)*d)*cos(f*x + e)^2 + (a^3 + 3*a*b^2)*
c + (3*a^2*b + b^3)*d - ((b^3*c + 3*a*b^2*d)*cos(f*x + e)^2 - (3*a^2*b + b^
3)*c - (a^3 + 3*a*b^2)*d)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(5/2)*sqrt(d*sin(f*x + e) + c)), x)
```

maple [B] time = 5.83, size = 242318, normalized size = 469.61

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(5/2)*sqrt(d*sin(f*x + e) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{5/2}\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((a + b*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sin\left(e+fx\right)\right)^{\frac{5}{2}}\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/((a + b*sin(e + f*x))**(5/2)*sqrt(c + d*sin(e + f*x))), x)
```

3.799 $\int \frac{1}{(a+b\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=688

$$\frac{8b^2\left(-2a^2d+abc+b^2d\right)\cos(e+fx)}{3f\left(a^2-b^2\right)^2(bc-ad)^2\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}+\frac{2b^2\cos(e+fx)}{3f\left(a^2-b^2\right)(bc-ad)(a+b\sin(e+fx))^{3/2}\sqrt{c+\ldots}}$$

```
[Out] 2/3*(3*a^4*d^3-b^4*d*(5*c^2-8*d^2)+3*a^2*b^2*d*(3*c^2-5*d^2)-4*a*b^3*c*(c^2
-d^2))*EllipticE((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*
x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((
-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(
f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(a^2-b^2)/(c-d)/(-a*d+b*c)^4/f/(a+b)^
(1/2)/(c+d)^(1/2)-2/3*(3*a^2*b*(2*c-3*d)*d-3*a^3*d^2-3*a*b^2*(c^2-2*d^2)+b^
3*(c^2-6*c*d+8*d^2))*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/
2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*
sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*
d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(a^2-b^2)/(c-d)/(-a*d+b
*c)^3/f/(a+b)^(1/2)/(c+d)^(1/2)+2/3*b^2*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(
a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2)+8/3*b^2*(-2*a^2*d+a*b*c+b^2*d)
*cos(f*x+e)/(a^2-b^2)^2/(-a*d+b*c)^2/f/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+
e))^(1/2)
```

Rubi [A] time = 1.85, antiderivative size = 688, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2802, 3055, 2998, 2818, 2996}

$$-\frac{2\left(3a^2bd(2c-3d)-3a^3d^2-3ab^2\left(c^2-2d^2\right)+b^3\left(c^2-6cd+8d^2\right)\right)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin\ldots}{(a+b)(c+d\sin\ldots}}}{3f\sqrt{a+b}\,\left(a^2-b^2\right)(c-d)\sqrt{c+d}\,(bc-ad)\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (2*b^2*Cos[e + f*x])/(3*(a^2 - b^2)*(b*c - a*d)*f*(a + b*Sin[e + f*x])^(3/2
)*Sqrt[c + d*Sin[e + f*x]]) + (8*b^2*(a*b*c - 2*a^2*d + b^2*d)*Cos[e + f*x]
)/(3*(a^2 - b^2)^2*(b*c - a*d)^2*f*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[
e + f*x]]) + (2*(3*a^4*d^3 - b^4*d*(5*c^2 - 8*d^2) + 3*a^2*b^2*d*(3*c^2 - 5
*d^2) - 4*a*b^3*c*(c^2 - d^2))*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin
[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a
- b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*
(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c
```

```
+ d*Sin[e + f*x]))))]*(c + d*Sin[e + f*x]))/(3*Sqrt[a + b]*(a^2 - b^2)*(c -
d)*Sqrt[c + d]*(b*c - a*d)^4*f) - (2*(3*a^2*b*(2*c - 3*d)*d - 3*a^3*d^2 - 3
*a*b^2*(c^2 - 2*d^2) + b^3*(c^2 - 6*c*d + 8*d^2))*EllipticF[ArcSin[(Sqrt[c
+ d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a
 + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e
 + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e +
f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(3*Sqrt[a + b
]*(a^2 - b^2)*(c - d)*Sqrt[c + d]*(b*c - a*d)^3*f)
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2b^2\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{2b^2\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{2b^2\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}$$

$$= \frac{2b^2\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}$$

Mathematica [B] time = 7.82, size = 2352, normalized size = 3.42

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((2*b^3*Cos[e + f*x])/(3
*(a^2 - b^2)*(-(b*c) + a*d)^2*(a + b*Sin[e + f*x])^2) - (2*(4*a*b^4*c*Cos[e
 + f*x] - 9*a^2*b^3*d*Cos[e + f*x] + 5*b^5*d*Cos[e + f*x]))/(3*(a^2 - b^2)^
2*(-(b*c) + a*d)^3*(a + b*Sin[e + f*x])) - (2*d^4*Cos[e + f*x])/((b*c - a*d
)^3*(c^2 - d^2)*(c + d*Sin[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(-3*a^2*b^3
*c^4 - b^5*c^4 + 9*a^3*b^2*c^3*d - 5*a*b^4*c^3*d - 9*a^4*b*c^2*d^2 + 20*a^2
*b^3*c^2*d^2 - 7*b^5*c^2*d^2 + 3*a^5*c*d^3 - 15*a^3*b^2*c*d^3 + 8*a*b^4*c*d
^3 + 9*a^4*b*d^4 - 17*a^2*b^3*d^4 + 8*b^5*d^4)*Sqrt[((c + d)*Cot[(-e + Pi/2
 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*
x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/
((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*C
sc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a -
 b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a +
 b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c)
+ a*d)*(-4*a*b^4*c^4 + 5*a^2*b^3*c^3*d - 5*b^5*c^3*d + 9*a^3*b^2*c^2*d^2 -
a*b^4*c^2*d^2 + 3*a^4*b*c*d^3 - 11*a^2*b^3*c*d^3 + 8*b^5*c*d^3 + 3*a^5*d^4
- 15*a^3*b^2*d^4 + 8*a*b^4*d^4)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)
/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c +
```

```
d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c
+ d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2
 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e +
 Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e
+ Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin
[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) +
a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e
 + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e +
 f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Si
n[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c +
d*Sin[e + f*x]])) + 2*(4*a*b^4*c^3*d - 9*a^2*b^3*c^2*d^2 + 5*b^5*c^2*d^2 -
4*a*b^4*c*d^3 - 3*a^4*b*d^4 + 15*a^2*b^3*d^4 - 8*b^5*d^4)*((Cos[e + f*x]*Sq
rt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b
)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]
*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c)
+ a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos
[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqr
t[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d
)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c
+ d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*
Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]],
(2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2
]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c)
+ a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(
b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e +
f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*E
llipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2
- f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*
d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c +
d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((
-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/(
(a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))))/(b*d)))/(3*(
a - b)^2*(a + b)^2*(c - d)*(c + d)*(-(b*c) + a*d)^3*f)
```

fricas [F] time = 1.27, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{}{\left(2\,b^{3}cd+3\,ab^{2}d^{2}\right)\cos\left(fx+e\right)^{4}+\left(a^{3}+3\,ab^{2}\right)c^{2}+2\left(3\,a^{2}b+b^{3}\right)cd+\left(a^{3}+3\,ab^{2}\right)d^{2}-\left(3\,ab^{2}c^{2}+2\left(3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/((2*b^3*c*d + 3*
a*b^2*d^2)*cos(f*x + e)^4 + (a^3 + 3*a*b^2)*c^2 + 2*(3*a^2*b + b^3)*c*d + (
a^3 + 3*a*b^2)*d^2 - (3*a*b^2*c^2 + 2*(3*a^2*b + 2*b^3)*c*d + (a^3 + 6*a*b^
2)*d^2)*cos(f*x + e)^2 + (b^3*d^2*cos(f*x + e)^4 + (3*a^2*b + b^3)*c^2 + 2*
(a^3 + 3*a*b^2)*c*d + (3*a^2*b + b^3)*d^2 - (b^3*c^2 + 6*a*b^2*c*d + (3*a^2
*b + 2*b^3)*d^2)*cos(f*x + e)^2)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(3/2)), x)
```

maple [B] time = 10.68, size = 438748, normalized size = 637.72

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + b*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.800 $\int \frac{1}{(a+b\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=941

$$\frac{4\left(-5da^2+2bca+3b^2d\right)\cos(e+fx)b^2}{3\left(a^2-b^2\right)^2(bc-ad)^2f\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}}+\frac{2\cos(e+fx)b^2}{3\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^{3/2}(c+}$$

```
[Out] 2/3*b^2*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f
*x+e))^(3/2)+4/3*b^2*(-5*a^2*d+2*a*b*c+3*b^2*d)*cos(f*x+e)/(a^2-b^2)^2/(-a*
d+b*c)^2/f/(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(1/2)-2/3*d*(a^4*d^3+a^2
*b^2*d*(11*c^2-13*d^2)-b^4*d*(7*c^2-8*d^2)-4*a*b^3*c*(c^2-d^2))*cos(f*x+e)*
(a+b*sin(f*x+e))^(1/2)/(a^2-b^2)^2/(-a*d+b*c)^3/(c^2-d^2)/f/(c+d*sin(f*x+e)
)^(3/2)-8/3*(a^5*c*d^4-2*a^3*b^2*c*d^4+a*b^4*c*(c^4-2*c^2*d^2+2*d^4)+b^5*d*
(2*c^4-7*c^2*d^2+4*d^4)-a^2*b^3*d*(3*c^4-12*c^2*d^2+7*d^4)-a^4*b*(3*c^2*d^3
-2*d^5))*EllipticE((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(
f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*
((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+si
n(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(a^2-b^2)/(c-d)^2/(c+d)^(3/2)/(-a*d
+b*c)^5/f/(a+b)^(1/2)-2/3*(a^4*d^3*(3*c+d)-9*a^3*b*d^2*(c^2-d^2)+a^2*b^2*d*
(9*c^3-18*c^2*d-15*c*d^2+16*d^3)+b^4*(c^4-9*c^3*d+16*c^2*d^2+12*c*d^3-16*d^
4)-3*a*b^3*(c^4-5*c^2*d^2+4*d^4))*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1
/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec
(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))
^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(a^2-b^2)/
(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)^4/f/(a+b)^(1/2)
```

Rubi [A] time = 4.47, antiderivative size = 941, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2802, 3055, 2998, 2818, 2996}

$$\frac{4\left(-5da^2+2bca+3b^2d\right)\cos(e+fx)b^2}{3\left(a^2-b^2\right)^2(bc-ad)^2f\sqrt{a+b\sin(e+fx)}\,(c+d\sin(e+fx))^{3/2}}+\frac{2\cos(e+fx)b^2}{3\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^{3/2}(c+}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (2*b^2*Cos[e + f*x])/(3*(a^2 - b^2)*(b*c - a*d)*f*(a + b*Sin[e + f*x])^(3/2
)*(c + d*Sin[e + f*x])^(3/2)) + (4*b^2*(2*a*b*c - 5*a^2*d + 3*b^2*d)*Cos[e
+ f*x])/(3*(a^2 - b^2)^2*(b*c - a*d)^2*f*Sqrt[a + b*Sin[e + f*x]]*(c + d*Si
n[e + f*x])^(3/2)) - (2*d*(a^4*d^3 + a^2*b^2*d*(11*c^2 - 13*d^2) - b^4*d*(7
```

```
*c^2 - 8*d^2) - 4*a*b^3*c*(c^2 - d^2))*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]
])/(3*(a^2 - b^2)^2*(b*c - a*d)^3*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^(3/2))
 - (8*(a^5*c*d^4 - 2*a^3*b^2*c*d^4 + a*b^4*c*(c^4 - 2*c^2*d^2 + 2*d^4) + b^
5*d*(2*c^4 - 7*c^2*d^2 + 4*d^4) - a^2*b^3*d*(3*c^4 - 12*c^2*d^2 + 7*d^4) -
a^4*b*(3*c^2*d^3 - 2*d^5))*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e +
 f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)
*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c +
 d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*
Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(3*Sqrt[a + b]*(a^2 - b^2)*(c - d)^2
*(c + d)^(3/2)*(b*c - a*d)^5*f) - (2*(a^4*d^3*(3*c + d) - 9*a^3*b*d^2*(c^2
- d^2) + a^2*b^2*d*(9*c^3 - 18*c^2*d - 15*c*d^2 + 16*d^3) + b^4*(c^4 - 9*c^
3*d + 16*c^2*d^2 + 12*c*d^3 - 16*d^4) - 3*a*b^3*(c^4 - 5*c^2*d^2 + 4*d^4))*
EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c
 + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqr
t[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-((
(b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin
[e + f*x]))/(3*Sqrt[a + b]*(a^2 - b^2)*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)^
4*f)
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
```

```
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{(a+b\sin(e+fx))^{5/2}(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2b^2\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx)}$$

$$= \frac{2b^2\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx)}$$

$$= \frac{2b^2\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx)}$$

$$= \frac{2b^2\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx)}$$

$$= \frac{2b^2\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)f(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx)}$$

Mathematica [B] time = 8.99, size = 2669, normalized size = 2.84

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-2*b^4*Cos[e + f*x])/(
3*(a^2 - b^2)*(-(b*c) + a*d)^3*(a + b*Sin[e + f*x])^2) + (8*(a*b^5*c*Cos[e
+ f*x] - 3*a^2*b^4*d*Cos[e + f*x] + 2*b^6*d*Cos[e + f*x]))/(3*(a^2 - b^2)^2
*(-(b*c) + a*d)^4*(a + b*Sin[e + f*x])) - (2*d^4*Cos[e + f*x])/(3*(b*c - a*
d)^3*(c^2 - d^2)*(c + d*Sin[e + f*x])^2) + (8*(-3*b*c^2*d^4*Cos[e + f*x] +
a*c*d^5*Cos[e + f*x] + 2*b*d^6*Cos[e + f*x]))/(3*(b*c - a*d)^4*(c^2 - d^2)^
2*(c + d*Sin[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(3*a^2*b^4*c^6 + b^6*c^6
- 12*a^3*b^3*c^5*d + 8*a*b^5*c^5*d + 18*a^4*b^2*c^4*d^2 - 41*a^2*b^4*c^4*d^
2 + 15*b^6*c^4*d^2 - 12*a^5*b*c^3*d^3 + 48*a^3*b^3*c^3*d^3 - 28*a*b^5*c^3*d
^3 + 3*a^6*c^2*d^4 - 41*a^4*b^2*c^2*d^4 + 74*a^2*b^4*c^2*d^4 - 32*b^6*c^2*d
^4 + 8*a^5*b*c*d^5 - 28*a^3*b^3*c*d^5 + 16*a*b^5*c*d^5 + a^6*d^6 + 15*a^4*b
^2*d^6 - 32*a^2*b^4*d^6 + 16*b^6*d^6)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2
]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(
c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*
```

```
(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e +
Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(
-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c +
d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(4
*a*b^5*c^6 - 8*a^2*b^4*c^5*d + 8*b^6*c^5*d - 12*a^3*b^3*c^4*d^2 - 12*a^4*b^
2*c^3*d^3 + 40*a^2*b^4*c^3*d^3 - 28*b^6*c^3*d^3 - 8*a^5*b*c^2*d^4 + 40*a^3*
b^3*c^2*d^4 - 20*a*b^5*c^2*d^4 + 4*a^6*c*d^5 - 20*a^2*b^4*c*d^5 + 16*b^6*c*
d^5 + 8*a^5*b*d^6 - 28*a^3*b^3*d^6 + 16*a*b^5*d^6)*((Sqrt[((c + d)*Cot[(-e
+ Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/
2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) +
a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c
+ d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[
((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])
/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqr
t[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/
((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e
+ f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*S
ec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/
2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 -
f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(-4*a*b^5*c^5*d + 12*a^2*b^4*c^4*d
^2 - 8*b^6*c^4*d^2 + 8*a*b^5*c^3*d^3 + 12*a^4*b^2*c^2*d^4 - 48*a^2*b^4*c^2*
d^4 + 28*b^6*c^2*d^4 - 4*a^5*b*c*d^5 + 8*a^3*b^3*c*d^5 - 8*a*b^5*c*d^5 - 8*
a^4*b^2*d^6 + 28*a^2*b^4*d^6 - 16*b^6*d^6)*((Cos[e + f*x]*Sqrt[c + d*Sin[e
+ f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[
(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2
 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b
)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f
*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e
+ f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e
+ f*x]))]]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e +
Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2
- f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*
d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c +
d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((
-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/(
(a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c
+ a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*
c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c
+ d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-
c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi
/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e
 + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[
a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))/(3*(a - b)^2*(a + b
)^2*(c - d)^2*(c + d)^2*(-(b*c) + a*d)^4*f)
```

fricas [F] time = 1.18, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{}{b^3d^3\cos\left(fx+e\right)^6-3\left(b^3c^2d+3\,ab^2cd^2+\left(a^2b+b^3\right)d^3\right)\cos\left(fx+e\right)^4-\left(a^3+3\,ab^2\right)c^3-3\left(3\,a^2b+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(b^3*d^3*cos(f*
x + e)^6 - 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + (a^2*b + b^3)*d^3)*cos(f*x + e)^4
 - (a^3 + 3*a*b^2)*c^3 - 3*(3*a^2*b + b^3)*c^2*d - 3*(a^3 + 3*a*b^2)*c*d^2
- (3*a^2*b + b^3)*d^3 + 3*(a*b^2*c^3 + (3*a^2*b + 2*b^3)*c^2*d + (a^3 + 6*a
*b^2)*c*d^2 + (2*a^2*b + b^3)*d^3)*cos(f*x + e)^2 - (3*(b^3*c*d^2 + a*b^2*d
^3)*cos(f*x + e)^4 + (3*a^2*b + b^3)*c^3 + 3*(a^3 + 3*a*b^2)*c^2*d + 3*(3*a
^2*b + b^3)*c*d^2 + (a^3 + 3*a*b^2)*d^3 - (b^3*c^3 + 9*a*b^2*c^2*d + 3*(3*a
^2*b + 2*b^3)*c*d^2 + (a^3 + 6*a*b^2)*d^3)*cos(f*x + e)^2)*sin(f*x + e)), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(5/2)), x)
```

maple [B] time = 30.73, size = 1123217, normalized size = 1193.64

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((b*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+b\,\sin\left(e+f\,x\right)\right)^{5/2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + b*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.801 $\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=28

$$\text{Int}\left((a+b\sin(e+fx))^m(c+d\sin(e+fx))^n,x\right)$$

```
[Out] Unintegrable((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
[Out] Defer[Int][(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x]
```

Rubi steps

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx=\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$$

Mathematica [A] time = 3.25, size = 0, normalized size = 0.00

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
[Out] Integrate[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x]
```

fricas [A] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(fx+e\right)+a\right)^m\left(d\sin\left(fx+e\right)+c\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n, x)
```

maple [A] time = 1.62, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
[Out] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int {\left(a+b\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^n,x)
[Out] int((a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^n, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.802 $\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=311

$$-\frac{\sqrt{2}\cos(e+fx)\left(ad(ad-2bc(m+2))+b^2\left(c^2(m+2)+d^2(m+1)\right)\right)(a+b\sin(e+fx))^m\left(\frac{a+b\sin(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\right.}{b^2f(m+2)\sqrt{\sin(e+fx)+1}}$$

```
[Out] -d^2*cos(f*x+e)*(a+b*sin(f*x+e))^(1+m)/b/f/(2+m)+(a+b)*d*(a*d-2*b*c*(2+m))*
AppellF1(1/2,-1-m,1/2,3/2,b*(1-sin(f*x+e))/(a+b),1/2-1/2*sin(f*x+e))*cos(f*
x+e)*(a+b*sin(f*x+e))^m*2^(1/2)/b^2/f/(2+m)/(((a+b*sin(f*x+e))/(a+b))^m)/(1
+sin(f*x+e))^(1/2)-(a*d*(a*d-2*b*c*(2+m))+b^2*(d^2*(1+m)+c^2*(2+m)))*Appell
F1(1/2,-m,1/2,3/2,b*(1-sin(f*x+e))/(a+b),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(a+
b*sin(f*x+e))^m*2^(1/2)/b^2/f/(2+m)/(((a+b*sin(f*x+e))/(a+b))^m)/(1+sin(f*x
+e))^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2791, 2756, 2665, 139, 138}

$$-\frac{\sqrt{2}\cos(e+fx)\left(ad(ad-2bc(m+2))+b^2\left(c^2(m+2)+d^2(m+1)\right)\right)(a+b\sin(e+fx))^m\left(\frac{a+b\sin(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\right.}{b^2f(m+2)\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -((d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(1 + m))/(b*f*(2 + m))) + (Sqrt[2]
*(a + b)*d*(a*d - 2*b*c*(2 + m))*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Sin[e
 + f*x])/2, (b*(1 - Sin[e + f*x]))/(a + b)]*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m)/(b^2*f*(2 + m)*Sqrt[1 + Sin[e + f*x]]*((a + b*Sin[e + f*x])/(a + b))^
m) - (Sqrt[2]*(a*d*(a*d - 2*b*c*(2 + m)) + b^2*(d^2*(1 + m) + c^2*(2 + m)))
*AppellF1[1/2, 1/2, -m, 3/2, (1 - Sin[e + f*x])/2, (b*(1 - Sin[e + f*x]))/(
a + b)]*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(b^2*f*(2 + m)*Sqrt[1 + Sin[e
+ f*x]]*((a + b*Sin[e + f*x])/(a + b))^m)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
```

```
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2756

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^2\,dx = -\frac{d^2\cos(e+fx)(a+b\sin(e+fx))^{1+m}}{bf(2+m)} + \frac{\int (a+b\sin(e+fx))^m \left(\right.}{}$$

$$= -\frac{d^2\cos(e+fx)(a+b\sin(e+fx))^{1+m}}{bf(2+m)} - \frac{(d(ad-2bc(2+m)))\int (}{b^2(2}$$

$$= -\frac{d^2\cos(e+fx)(a+b\sin(e+fx))^{1+m}}{bf(2+m)} - \frac{(d(ad-2bc(2+m))\cos}{b^2f(2+m)}$$

$$= -\frac{d^2\cos(e+fx)(a+b\sin(e+fx))^{1+m}}{bf(2+m)} + \frac{\left((-a-b)d(ad-2bc(2+\right.}{}$$

$$= -\frac{d^2\cos(e+fx)(a+b\sin(e+fx))^{1+m}}{bf(2+m)} + \frac{\sqrt{2}\,(a+b)d(ad-2bc(2}{}$$

Mathematica [F] time = 18.73, size = 0, normalized size = 0.00

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] Integrate[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2, x]
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(d^2\cos\left(fx+e\right)^2 - 2\,cd\sin\left(fx+e\right) - c^2 - d^2\right)\left(b\sin\left(fx+e\right)+a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)*(b*sin(f*x
+ e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)+c\right)^2\left(b\sin\left(fx+e\right)+a\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*sin(f*x + e) + c)^2*(b*sin(f*x + e) + a)^m, x)
```

maple [F] time = 1.31, size = 0, normalized size = 0.00

$$\int \left(a + b \sin \left(f x + e\right)\right)^{m} \left(c + d \sin \left(f x + e\right)\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x)
[Out] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \sin \left(f x + e\right) + c\right)^{2} \left(b \sin \left(f x + e\right) + a\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^2*(b*sin(f*x + e) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^2,x)
[Out] int((a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m*(c+d*sin(f*x+e))**2,x)
[Out] Timed out
```

3.803 $\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=229

$$-\frac{\sqrt{2}\,(bc-ad)\cos(e+fx)(a+b\sin(e+fx))^m\left(\frac{a+b\sin(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{b(1-\sin(e+fx))}{a+b}\right)}{bf\sqrt{\sin(e+fx)+1}}-$$

```
[Out] -(a+b)*d*AppellF1(1/2,-1-m,1/2,3/2,b*(1-sin(f*x+e))/(a+b),1/2-1/2*sin(f*x+e
))*cos(f*x+e)*(a+b*sin(f*x+e))^m*2^(1/2)/b/f/(((a+b*sin(f*x+e))/(a+b))^m)/(
1+sin(f*x+e))^(1/2)-(-a*d+b*c)*AppellF1(1/2,-m,1/2,3/2,b*(1-sin(f*x+e))/(a+
b),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(a+b*sin(f*x+e))^m*2^(1/2)/b/f/(((a+b*sin
(f*x+e))/(a+b))^m)/(1+sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2756, 2665, 139, 138}

$$-\frac{\sqrt{2}\,(bc-ad)\cos(e+fx)(a+b\sin(e+fx))^m\left(\frac{a+b\sin(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{b(1-\sin(e+fx))}{a+b}\right)}{bf\sqrt{\sin(e+fx)+1}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x]),x]
```

```
[Out] -((Sqrt[2]*(a + b)*d*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Sin[e + f*x])/2,
(b*(1 - Sin[e + f*x]))/(a + b)]*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(b*f*S
qrt[1 + Sin[e + f*x]]*((a + b*Sin[e + f*x])/(a + b))^m)) - (Sqrt[2]*(b*c -
a*d)*AppellF1[1/2, 1/2, -m, 3/2, (1 - Sin[e + f*x])/2, (b*(1 - Sin[e + f*x]
))/(a + b)]*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(b*f*Sqrt[1 + Sin[e + f*x]
]*((a + b*Sin[e + f*x])/(a + b))^m)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 2756

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))\,dx = \frac{d\int (a+b\sin(e+fx))^{1+m}\,dx}{b} + \frac{(bc-ad)\int (a+b\sin(e+fx))^m\,d}{b}$$

$$= \frac{(d\cos(e+fx))\,\mathrm{Subst}\left(\int \frac{(a+bx)^{1+m}}{\sqrt{1-x}\,\sqrt{1+x}}\,dx, x, \sin(e+fx)\right)}{bf\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}} + \frac{((bc-a}{}$$

$$= -\frac{\left((-a-b)d\cos(e+fx)(a+b\sin(e+fx))^m\left(-\frac{a+b\sin(e+fx)}{-a-b}\right)^{-m}\right)\mathrm{S}}{bf\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin}}$$

$$= -\frac{\sqrt{2}\,(a+b)dF_1\left(\frac{1}{2};\frac{1}{2},-1-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{b(1-\sin(e+fx))}{a+b}\right.}{bf\sqrt{1+\sin(e+f}}$$

Mathematica [A] time = 0.57, size = 200, normalized size = 0.87

$$\frac{\sec(e+fx)\sqrt{-\frac{b(\sin(e+fx)-1)}{a+b}}\sqrt{\frac{b(\sin(e+fx)+1)}{b-a}}\,(a+b\sin(e+fx))^{m+1}\left((m+2)(bc-ad)F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{a+b\sin}{a-}\right.\right.}{b^2f(m+1)(m+}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x]),x]
```

```
[Out] (Sec[e + f*x]*Sqrt[-((b*(-1 + Sin[e + f*x]))/(a + b))]*Sqrt[(b*(1 + Sin[e +
 f*x]))/(-a + b)]*(a + b*Sin[e + f*x])^(1 + m)*((b*c - a*d)*(2 + m)*AppellF
1[1 + m, 1/2, 1/2, 2 + m, (a + b*Sin[e + f*x])/(a - b), (a + b*Sin[e + f*x]
)/(a + b)] + d*(1 + m)*AppellF1[2 + m, 1/2, 1/2, 3 + m, (a + b*Sin[e + f*x]
)/(a - b), (a + b*Sin[e + f*x])/(a + b)]*(a + b*Sin[e + f*x])))/(b^2*f*(1 +
 m)*(2 + m))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d\sin\left(fx+e\right)+c\right)\left(b\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e) + c)*(b*sin(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d\sin\left(fx+e\right)+c\right)\left(b\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)*(b*sin(f*x + e) + a)^m, x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e)),x)
```

```
[Out] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d\sin\left(fx+e\right)+c\right)\left(b\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)*(b*sin(f*x + e) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\sin\left(e+f\,x\right)\right)^m\,\left(c+d\,\sin\left(e+f\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^m*(c + d*sin(e + f*x)),x)
[Out] int((a + b*sin(e + f*x))^m*(c + d*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m*(c+d*sin(f*x+e)),x)
[Out] Timed out
```

3.804 $\int (a+b\sin(e+fx))^m\,dx$

Optimal. Leaf size=104

$$-\frac{\sqrt{2}\cos(e+fx)(a+b\sin(e+fx))^m\left(\frac{a+b\sin(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{b(1-\sin(e+fx))}{a+b}\right)}{f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -AppellF1(1/2,-m,1/2,3/2,b*(1-sin(f*x+e))/(a+b),1/2-1/2*sin(f*x+e))*cos(f*x
+e)*(a+b*sin(f*x+e))^m*2^(1/2)/f/(((a+b*sin(f*x+e))/(a+b))^m)/(1+sin(f*x+e)
)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2665, 139, 138}

$$-\frac{\sqrt{2}\cos(e+fx)(a+b\sin(e+fx))^m\left(\frac{a+b\sin(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{b(1-\sin(e+fx))}{a+b}\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^m,x]
```

```
[Out] -((Sqrt[2]*AppellF1[1/2, 1/2, -m, 3/2, (1 - Sin[e + f*x])/2, (b*(1 - Sin[e
+ f*x]))/(a + b)]*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*Sqrt[1 + Sin[e +
f*x]]*((a + b*Sin[e + f*x])/(a + b))^m))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (a+b\sin(e+fx))^m\,dx = \frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{(a+bx)^m}{\sqrt{1-x}\,\sqrt{1+x}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(a+b\sin(e+fx))^m\left(-\frac{a+b\sin(e+fx)}{-a-b}\right)^{-m}\right)\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^m}{\sqrt{1-x}\,\sqrt{1+x}}\,dx, x, \text{si}\right.}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{\sqrt{2}\,F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{b(1-\sin(e+fx))}{a+b}\right)\cos(e+fx)(a+b\sin(e+fx)}{f\sqrt{1+\sin(e+fx)}}$$

Mathematica [A] time = 0.26, size = 120, normalized size = 1.15

$$\frac{\sec(e+fx)\sqrt{-\frac{b(\sin(e+fx)-1)}{a+b}}\sqrt{\frac{b(\sin(e+fx)+1)}{b-a}}\,(a+b\sin(e+fx))^{m+1}F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{a+b\sin(e+fx)}{a-b},\frac{a+b\sin(e+fx)}{a+b}\right)}{bf(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^m,x]
```

```
[Out] (AppellF1[1 + m, 1/2, 1/2, 2 + m, (a + b*Sin[e + f*x])/(a - b), (a + b*Sin[
e + f*x])/(a + b)]*Sec[e + f*x]*Sqrt[-((b*(-1 + Sin[e + f*x]))/(a + b))]*Sq
rt[(b*(1 + Sin[e + f*x]))/(-a + b)]*(a + b*Sin[e + f*x])^(1 + m))/(b*f*(1 +
 m))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(fx+e\right)+a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((b*sin(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e) + a)^m, x)
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m,x)
[Out] int((a+b*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\sin\left(e+f\,x\right)\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^m,x)
[Out] int((a + b*sin(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(e+fx\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m,x)
```

```
[Out] Integral((a + b*sin(e + f*x))**m, x)
```

3.805 $\int \frac{(a+b\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{(a+b\sin(e+fx))^m}{c+d\sin(e+fx)},x\right)$$

```
[Out] Unintegrable((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e)),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]),x]
```

```
[Out] Defer[Int][(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx = \int \frac{(a+b\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx$$

Mathematica [A] time = 2.65, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]),x]
```

```
[Out] Integrate[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\sin\left(fx+e\right)+a\right)^m}{d\sin\left(fx+e\right)+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e)),x, algorithm="fricas")
[Out] integral((b*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \sin\left(fx+e\right)+a\right)^{m}}{d \sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e)),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c), x)
```

maple [A] time = 1.42, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b \sin\left(fx+e\right)\right)^{m}}{c+d \sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e)),x)
[Out] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \sin\left(fx+e\right)+a\right)^{m}}{d \sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^m}{c+d\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^m/(c + d*sin(e + f*x)),x)
```

```
[Out] int((a + b*sin(e + f*x))^m/(c + d*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.806 $\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^2},x\right)$$

```
[Out] Unintegrable((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] Defer[Int][(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^2, x]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^2}\,dx = \int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^2}\,dx$$

Mathematica [A] time = 4.85, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] Integrate[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^2, x]
```

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(b\sin\left(fx+e\right)+a\right)^m}{d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(b*sin(f*x + e) + a)^m/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) -
 c^2 - d^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \sin\left(fx + e\right) + a\right)^{m}}{\left(d \sin\left(fx + e\right) + c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^2, x)
```

maple [A] time = 1.58, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \sin\left(fx + e\right)\right)^{m}}{\left(c + d \sin\left(fx + e\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x)
```

```
[Out] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \sin\left(fx + e\right) + a\right)^{m}}{\left(d \sin\left(fx + e\right) + c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a + b\, \sin\left(e + f\,x\right)\right)^{m}}{\left(c + d\, \sin\left(e + f\,x\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^m/(c + d*sin(e + f*x))^2,x)
```

```
[Out] int((a + b*sin(e + f*x))^m/(c + d*sin(e + f*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.807 $\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^3},x\right)$$

```
[Out] Unintegrable((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^3,x]
[Out] Defer[Int][(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^3, x]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^3}\,dx = \int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^3}\,dx$$

Mathematica [A] time = 15.87, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^3,x]
[Out] Integrate[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^3, x]
```

fricas [A] time = 0.87, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(b\sin\left(fx+e\right)+a\right)^m}{3\,cd^2\cos\left(fx+e\right)^2-c^3-3\,cd^2+\left(d^3\cos\left(fx+e\right)^2-3\,c^2d-d^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(b*sin(f*x + e) + a)^m/(3*c*d^2*cos(f*x + e)^2 - c^3 - 3*c*d^2 +
(d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \sin\left(fx+e\right)+a\right)^{m}}{\left(d \sin\left(fx+e\right)+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^3, x)
```

maple [A] time = 1.91, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b \sin\left(fx+e\right)\right)^{m}}{\left(c+d \sin\left(fx+e\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x)
```

```
[Out] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\sin\left(e+f\,x\right)\right)^{m}}{\left(c+d\,\sin\left(e+f\,x\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^m/(c + d*sin(e + f*x))^3,x)
```

```
[Out] int((a + b*sin(e + f*x))^m/(c + d*sin(e + f*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.808 $\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left((c+d\sin(e+fx))^{5/2}(a+b\sin(e+fx))^m,x\right)$$

```
[Out] Unintegrable((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x)
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(5/2),x]
[Out] Defer[Int][(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(5/2), x]
```

Rubi steps

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{5/2}\,dx=\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{5/2}\,dx$$

Mathematica [A] time = 34.71, size = 0, normalized size = 0.00

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(5/2),x]
[Out] Integrate[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(5/2), x]
```

fricas [A] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2\right)\sqrt{d\sin\left(fx+e\right)+c}\left(b\sin\left(fx+e\right)+a\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)*sqrt(d*sin(
f*x + e) + c)*(b*sin(f*x + e) + a)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\left(b\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate((d*sin(f*x + e) + c)^(5/2)*(b*sin(f*x + e) + a)^m, x)
```

maple [A] time = 0.31, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x)
[Out] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\left(b\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^(5/2)*(b*sin(f*x + e) + a)^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int {\left(a+b\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^(5/2),x)
[Out] int((a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m*(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.809 $\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=30

$$\operatorname{Int}\left((c+d\sin(e+fx))^{3/2}(a+b\sin(e+fx))^m,x\right)$$

```
[Out] Unintegrable((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x)
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] Defer[Int][(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(3/2), x]
```

Rubi steps

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{3/2}\,dx=\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{3/2}\,dx$$

Mathematica [A] time = 13.62, size = 0, normalized size = 0.00

$$\int (a+b\sin(e+fx))^m(c+d\sin(e+fx))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(3/2),x]
```

```
[Out] Integrate[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(3/2), x]
```

fricas [A] time = 0.70, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e) + c)^(3/2)*(b*sin(f*x + e) + a)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((d*sin(f*x + e) + c)^(3/2)*(b*sin(f*x + e) + a)^m, x)
```

maple [A] time = 0.26, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x)
[Out] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\left(b\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^(3/2)*(b*sin(f*x + e) + a)^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int {\left(a+b\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^(3/2),x)
[Out] int((a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.810 $\int (a+b\sin(e+fx))^m\sqrt{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\sqrt{c+d\sin(e+fx)}\,(a+b\sin(e+fx))^m,x\right)$$

```
[Out] Unintegrable((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sin(e+fx))^m\sqrt{c+d\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] Defer[Int][(a + b*Sin[e + f*x])^m*Sqrt[c + d*Sin[e + f*x]], x]
```

Rubi steps

$$\int (a+b\sin(e+fx))^m\sqrt{c+d\sin(e+fx)}\,dx=\int (a+b\sin(e+fx))^m\sqrt{c+d\sin(e+fx)}\,dx$$

Mathematica [A] time = 0.44, size = 0, normalized size = 0.00

$$\int (a+b\sin(e+fx))^m\sqrt{c+d\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] Integrate[(a + b*Sin[e + f*x])^m*Sqrt[c + d*Sin[e + f*x]], x]
```

fricas [A] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{d\sin\left(fx+e\right)+c}\left(b\sin\left(fx+e\right)+a\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sin(f*x + e) + c)*(b*sin(f*x + e) + a)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d \sin\left(fx+e\right)+c}\left(b \sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*sin(f*x + e) + c)*(b*sin(f*x + e) + a)^m, x)
```

maple [A] time = 0.24, size = 0, normalized size = 0.00

$$\int \left(a+b \sin\left(fx+e\right)\right)^{m} \sqrt{c+d \sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x)
[Out] int((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d \sin\left(fx+e\right)+c}\left(b \sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*sin(f*x + e) + c)*(b*sin(f*x + e) + a)^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int {\left(a+b\,\sin\left(e+f\,x\right)\right)}^m\,\sqrt{c+d\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^(1/2),x)
[Out] int((a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \sin{\left(e + f x \right)}\right)^{m} \sqrt{c + d \sin{\left(e + f x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**m*sqrt(c + d*sin(e + f*x)), x)
```

3.811 $\int \frac{(a+b\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{(a+b\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}},x\right)$$

```
[Out] Unintegrable((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] Defer[Int][(a + b*Sin[e + f*x])^m/Sqrt[c + d*Sin[e + f*x]], x]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{(a+b\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}}\,dx$$

Mathematica [A] time = 3.33, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m/Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] Integrate[(a + b*Sin[e + f*x])^m/Sqrt[c + d*Sin[e + f*x]], x]
```

fricas [A] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\sin\left(fx+e\right)+a\right)^m}{\sqrt{d\sin\left(fx+e\right)+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral((b*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \sin\left(fx+e\right)+a\right)^{m}}{\sqrt{d \sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) + c), x)
```

maple [A] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b \sin\left(fx+e\right)\right)^{m}}{\sqrt{c+d \sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x)
[Out] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \sin\left(fx+e\right)+a\right)^{m}}{\sqrt{d \sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) + c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^m}{\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^m/(c + d*sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^m/(c + d*sin(e + f*x))^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sin\left(e+fx\right)\right)^m}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**m/sqrt(c + d*sin(e + f*x)), x)
```

3.812 $\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x)
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(3/2),x]
[Out] Defer[Int][(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(3/2), x]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{3/2}}\,dx=\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{3/2}}\,dx$$

Mathematica [A] time = 4.60, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(3/2),x]
[Out] Integrate[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(3/2), x]
```

fricas [A] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{d\sin\left(fx+e\right)+c}\left(b\sin\left(fx+e\right)+a\right)^m}{d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*sin(f*x + e) + c)*(b*sin(f*x + e) + a)^m/(d^2*cos(f*x + e)
^2 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \sin\left(fx+e\right)+a\right)^{m}}{\left(d \sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^(3/2), x)
```

maple [A] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b \sin\left(fx+e\right)\right)^{m}}{\left(c+d \sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \sin\left(fx+e\right)+a\right)^{m}}{\left(d \sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^m}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^m/(c + d*sin(e + f*x))^(3/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^m/(c + d*sin(e + f*x))^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sin\left(e+fx\right)\right)^{m}}{\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x))**m/(c + d*sin(e + f*x))**(3/2), x)
```

3.813 $\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x)
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] Defer[Int][(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(5/2), x]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{5/2}}\,dx = \int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{5/2}}\,dx$$

Mathematica [A] time = 9.69, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(e+fx))^m}{(c+d\sin(e+fx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(5/2),x]
```

```
[Out] Integrate[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x])^(5/2), x]
```

fricas [A] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{d\sin\left(fx+e\right)+c}\left(b\sin\left(fx+e\right)+a\right)^m}{3\,cd^2\cos\left(fx+e\right)^2-c^3-3\,cd^2+\left(d^3\cos\left(fx+e\right)^2-3\,c^2d-d^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*sin(f*x + e) + c)*(b*sin(f*x + e) + a)^m/(3*c*d^2*cos(f*x
+ e)^2 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e))
, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{m}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^(5/2), x)
```

maple [A] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sin\left(fx+e\right)\right)^{m}}{\left(c+d\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sin\left(fx+e\right)+a\right)^{m}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m/(c+d*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^(5/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^m}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^m/(c + d*sin(e + f*x))^(5/2),x)
```

```
[Out] int((a + b*sin(e + f*x))^m/(c + d*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m/(c+d*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.814 $\int (d\csc(e+fx))^n(a+a\sin(e+fx))^3\,dx$

Optimal. Leaf size=272

$$\frac{a^3d^4(11-4n)\cos(e+fx)(d\csc(e+fx))^{n-4}\,{}_2F_1\left(\frac{1}{2},\frac{4-n}{2};\frac{6-n}{2};\sin^2(e+fx)\right)}{f(2-n)(4-n)\sqrt{\cos^2(e+fx)}}+\frac{a^3d^3(5-4n)\cos(e+fx)(d\csc(e+\ldots}{f(1-n)(3-\ldots}$$

```
[Out] a^3*d^3*(1-2*n)*cot(f*x+e)*(d*csc(f*x+e))^(-3+n)/f/(n^2-3*n+2)+d^3*cot(f*x+
e)*(d*csc(f*x+e))^(-3+n)*(a^3+a^3*csc(f*x+e))/f/(1-n)+a^3*d^3*(5-4*n)*cos(f
*x+e)*(d*csc(f*x+e))^(-3+n)*hypergeom([1/2, 3/2-1/2*n],[5/2-1/2*n],sin(f*x+
e)^2)/f/(n^2-4*n+3)/(cos(f*x+e)^2)^(1/2)+a^3*d^4*(11-4*n)*cos(f*x+e)*(d*csc
(f*x+e))^(-4+n)*hypergeom([1/2, 2-1/2*n],[3-1/2*n],sin(f*x+e)^2)/f/(n^2-6*n
+8)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3814, 3997, 3787, 3772, 2643}

$$\frac{a^3d^4(11-4n)\cos(e+fx)(d\csc(e+fx))^{n-4}\,{}_2F_1\left(\frac{1}{2},\frac{4-n}{2};\frac{6-n}{2};\sin^2(e+fx)\right)}{f(2-n)(4-n)\sqrt{\cos^2(e+fx)}}+\frac{a^3d^3(5-4n)\cos(e+fx)(d\csc(e+\ldots}{f(1-n)(3-\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n*(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (a^3*d^3*(1 - 2*n)*Cot[e + f*x]*(d*Csc[e + f*x])^(-3 + n))/(f*(1 - n)*(2 -
n)) + (d^3*Cot[e + f*x]*(d*Csc[e + f*x])^(-3 + n)*(a^3 + a^3*Csc[e + f*x]))
/(f*(1 - n)) + (a^3*d^3*(5 - 4*n)*Cos[e + f*x]*(d*Csc[e + f*x])^(-3 + n)*Hy
pergeometric2F1[1/2, (3 - n)/2, (5 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*(3 -
 n)*Sqrt[Cos[e + f*x]^2]) + (a^3*d^4*(11 - 4*n)*Cos[e + f*x]*(d*Csc[e + f*x
])^(-4 + n)*Hypergeometric2F1[1/2, (4 - n)/2, (6 - n)/2, Sin[e + f*x]^2])/(
f*(2 - n)*(4 - n)*Sqrt[Cos[e + f*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
```

```
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rubi steps

$$\int (d\csc(e+fx))^n(a+a\sin(e+fx))^3\,dx = d^3\int (d\csc(e+fx))^{-3+n}(a+a\csc(e+fx))^3\,dx$$
$$= \frac{d^3\cot(e+fx)(d\csc(e+fx))^{-3+n}\left(a^3+a^3\csc(e+fx)\right)}{f(1-n)} - \frac{\left(ad^3\right)\int(d$$
$$= \frac{a^3d^3(1-2n)\cot(e+fx)(d\csc(e+fx))^{-3+n}}{f(1-n)(2-n)} + \frac{d^3\cot(e+fx)(d\csc(}{}$$
$$= \frac{a^3d^3(1-2n)\cot(e+fx)(d\csc(e+fx))^{-3+n}}{f(1-n)(2-n)} + \frac{d^3\cot(e+fx)(d\csc(}{}$$
$$= \frac{a^3d^3(1-2n)\cot(e+fx)(d\csc(e+fx))^{-3+n}}{f(1-n)(2-n)} + \frac{d^3\cot(e+fx)(d\csc(}{}$$
$$= \frac{a^3d^3(1-2n)\cot(e+fx)(d\csc(e+fx))^{-3+n}}{f(1-n)(2-n)} + \frac{d^3\cot(e+fx)(d\csc(}{}$$

Mathematica [A] time = 11.57, size = 493, normalized size = 1.81

$$\frac{2^{1-n}\tan\left(\frac{1}{2}(e+fx)\right)(a\sin(e+fx)+a)^3\left(\tan^2\left(\frac{1}{2}(e+fx)\right)+1\right)^{-n}\csc^{-n}(e+fx)(d\csc(e+fx))^n\left(-\frac{15\tan^2\left(\frac{1}{2}(e+fx)\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Csc[e + f*x])^n*(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (2^(1 - n)*(d*Csc[e + f*x])^n*(a + a*Sin[e + f*x])^3*Tan[(e + f*x)/2]*(Cot[
(e + f*x)/2] + Tan[(e + f*x)/2])^n*(Hypergeometric2F1[4 - n, 1/2 - n/2, 3/2
 - n/2, -Tan[(e + f*x)/2]^2]/(1 - n) - (6*Hypergeometric2F1[4 - n, 1 - n/2,
 2 - n/2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(-2 + n) - (15*Hypergeomet
ric2F1[(3 - n)/2, 4 - n, (5 - n)/2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2
)/(-3 + n) - (20*Hypergeometric2F1[4 - n, 2 - n/2, 3 - n/2, -Tan[(e + f*x)/
2]^2]*Tan[(e + f*x)/2]^3)/(-4 + n) - (15*Hypergeometric2F1[4 - n, (5 - n)/2
, (7 - n)/2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^4)/(-5 + n) - (6*Hyperge
ometric2F1[4 - n, 3 - n/2, 4 - n/2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^5
)/(-6 + n) + (Hypergeometric2F1[4 - n, 7/2 - n/2, 9/2 - n/2, -Tan[(e + f*x)
/2]^2]*Tan[(e + f*x)/2]^6)/(7 - n)))/(f*Csc[e + f*x]^n*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^6*(1 + Tan[(e + f*x)/2]^2)^n)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)\right)\left(d\csc\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(
f*x + e))*(d*csc(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^3\left(d\csc\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(d*csc(f*x + e))^n, x)
```

maple [F] time = 5.86, size = 0, normalized size = 0.00

$$\int \left(d\csc\left(fx+e\right)\right)^n\left(a+a\sin\left(fx+e\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n*(a+a*sin(f*x+e))^3,x)
```

```
[Out] int((d*csc(f*x+e))^n*(a+a*sin(f*x+e))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^3\left(d\csc\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(d*csc(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(\frac{d}{\sin\left(e+f\,x\right)}\right)}^n\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(e + f*x))^n*(a + a*sin(e + f*x))^3,x)
```

```
[Out] int((d/sin(e + f*x))^n*(a + a*sin(e + f*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left(\int \left(d \csc \left(e+fx\right)\right)^n \, dx + \int 3 \left(d \csc \left(e+fx\right)\right)^n \sin \left(e+fx\right) dx + \int 3 \left(d \csc \left(e+fx\right)\right)^n \sin^2 \left(e+fx\right) dx + \int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n*(a+a*sin(f*x+e))**3,x)
```

```
[Out] a**3*(Integral((d*csc(e + f*x))**n, x) + Integral(3*(d*csc(e + f*x))**n*sin
(e + f*x), x) + Integral(3*(d*csc(e + f*x))**n*sin(e + f*x)**2, x) + Integr
al((d*csc(e + f*x))**n*sin(e + f*x)**3, x))
```

3.815 $\int (d\csc(e+fx))^n (a+a\sin(e+fx))^2\,dx$

Optimal. Leaf size=203

$$\frac{a^2d^3(3-2n)\cos(e+fx)(d\csc(e+fx))^{n-3}\,{}_2F_1\left(\frac{1}{2},\frac{3-n}{2};\frac{5-n}{2};\sin^2(e+fx)\right)}{f(1-n)(3-n)\sqrt{\cos^2(e+fx)}}+\frac{2a^2d^2\cos(e+fx)(d\csc(e+fx))^{n-2}\,{}_2F_1\left(\ldots\right)}{f(2-n)\sqrt{\cos^2(e+fx)}}$$

```
[Out] a^2*d^2*cot(f*x+e)*(d*csc(f*x+e))^(-2+n)/f/(1-n)+2*a^2*d^2*cos(f*x+e)*(d*cs
c(f*x+e))^(-2+n)*hypergeom([1/2, 1-1/2*n],[2-1/2*n],sin(f*x+e)^2)/f/(2-n)/(
cos(f*x+e)^2)^(1/2)+a^2*d^3*(3-2*n)*cos(f*x+e)*(d*csc(f*x+e))^(-3+n)*hyperg
eom([1/2, 3/2-1/2*n],[5/2-1/2*n],sin(f*x+e)^2)/f/(n^2-4*n+3)/(cos(f*x+e)^2)
^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3238, 3788, 3772, 2643, 4046}

$$\frac{a^2d^3(3-2n)\cos(e+fx)(d\csc(e+fx))^{n-3}\,{}_2F_1\left(\frac{1}{2},\frac{3-n}{2};\frac{5-n}{2};\sin^2(e+fx)\right)}{f(1-n)(3-n)\sqrt{\cos^2(e+fx)}}+\frac{2a^2d^2\cos(e+fx)(d\csc(e+fx))^{n-2}\,{}_2F_1\left(\ldots\right)}{f(2-n)\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n*(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(-2 + n))/(f*(1 - n)) + (2*a^2*d^2*C
os[e + f*x]*(d*Csc[e + f*x])^(-2 + n)*Hypergeometric2F1[1/2, (2 - n)/2, (4
- n)/2, Sin[e + f*x]^2])/(f*(2 - n)*Sqrt[Cos[e + f*x]^2]) + (a^2*d^3*(3 - 2
*n)*Cos[e + f*x]*(d*Csc[e + f*x])^(-3 + n)*Hypergeometric2F1[1/2, (3 - n)/2
, (5 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*(3 - n)*Sqrt[Cos[e + f*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int (d\csc(e+fx))^n(a+a\sin(e+fx))^2\,dx = d^2\int (d\csc(e+fx))^{-2+n}(a+a\csc(e+fx))^2\,dx$$

$$= \left(2a^2d\right)\int (d\csc(e+fx))^{-1+n}\,dx + d^2\int (d\csc(e+fx))^{-2+n}\left(a^2+a^2\right.$$

$$= \frac{a^2d^2\cot(e+fx)(d\csc(e+fx))^{-2+n}}{f(1-n)} + \frac{\left(a^2d^2(3-2n)\right)\int (d\csc(e+f}{1-n}$$

$$= \frac{a^2d^2\cot(e+fx)(d\csc(e+fx))^{-2+n}}{f(1-n)} + \frac{2a^2\cos(e+fx)(d\csc(e+fx)}{f(2}$$

$$= \frac{a^2d^2\cot(e+fx)(d\csc(e+fx))^{-2+n}}{f(1-n)} + \frac{2a^2\cos(e+fx)(d\csc(e+fx)}{f(2}$$

Mathematica [A] time = 6.34, size = 342, normalized size = 1.68

$$2\tan\left(\frac{1}{2}(e+fx)\right)(a\sin(e+fx)+a)^2\sec^2\left(\frac{1}{2}(e+fx)\right)^{-n}(d\csc(e+fx))^n\left(\frac{{}_2F_1\left(3-n,\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}{1-n}+\tan\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[e + f*x])^n*(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (2*(d*Csc[e + f*x])^n*(a + a*Sin[e + f*x])^2*Tan[(e + f*x)/2]*(Hypergeometr
ic2F1[3 - n, 1/2 - n/2, 3/2 - n/2, -Tan[(e + f*x)/2]^2]/(1 - n) + Tan[(e +
f*x)/2]*((-4*Hypergeometric2F1[3 - n, 1 - n/2, 2 - n/2, -Tan[(e + f*x)/2]^2
])/(-2 + n) + Tan[(e + f*x)/2]*((-6*Hypergeometric2F1[(3 - n)/2, 3 - n, (5
- n)/2, -Tan[(e + f*x)/2]^2])/(-3 + n) - (4*Hypergeometric2F1[3 - n, 2 - n/
2, 3 - n/2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(-4 + n) + (Hypergeometr
ic2F1[3 - n, 5/2 - n/2, 7/2 - n/2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)
/(5 - n)))))/(f*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^4)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2\right)\left(d\csc\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*(d*csc(f*x + e)
)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^2\left(d\csc\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(d*csc(f*x + e))^n, x)
```

maple [F] time = 8.41, size = 0, normalized size = 0.00

$$\int\left(d\csc\left(fx+e\right)\right)^n\left(a+a\sin\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n*(a+a*sin(f*x+e))^2,x)
```

```
[Out] int((d*csc(f*x+e))^n*(a+a*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{2}\left(d\csc\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(d*csc(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{d}{\sin\left(e+f\,x\right)}\right)^{n}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(e + f*x))^n*(a + a*sin(e + f*x))^2,x)
```

```
[Out] int((d/sin(e + f*x))^n*(a + a*sin(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int \left(d\csc\left(e+fx\right)\right)^{n}\,dx+\int 2\left(d\csc\left(e+fx\right)\right)^{n}\sin\left(e+fx\right)dx+\int \left(d\csc\left(e+fx\right)\right)^{n}\sin^{2}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n*(a+a*sin(f*x+e))**2,x)
```

```
[Out] a**2*(Integral((d*csc(e + f*x))**n, x) + Integral(2*(d*csc(e + f*x))**n*sin
(e + f*x), x) + Integral((d*csc(e + f*x))**n*sin(e + f*x)**2, x))
```

3.816 $\int (d\csc(e+fx))^n(a+a\sin(e+fx))\,dx$

Optimal. Leaf size=149

$$\frac{ad^2\cos(e+fx)(d\csc(e+fx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\sin^2(e+fx)\right)}{f(2-n)\sqrt{\cos^2(e+fx)}}+\frac{ad\cos(e+fx)(d\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-}{2}\right.}{f(1-n)\sqrt{\cos^2(e+fx)}}$$

```
[Out] a*d*cos(f*x+e)*(d*csc(f*x+e))^(-1+n)*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n]
,sin(f*x+e)^2)/f/(1-n)/(cos(f*x+e)^2)^(1/2)+a*d^2*cos(f*x+e)*(d*csc(f*x+e))
^(-2+n)*hypergeom([1/2, 1-1/2*n],[2-1/2*n],sin(f*x+e)^2)/f/(2-n)/(cos(f*x+e
)^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3238, 3787, 3772, 2643}

$$\frac{ad^2\cos(e+fx)(d\csc(e+fx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\sin^2(e+fx)\right)}{f(2-n)\sqrt{\cos^2(e+fx)}}+\frac{ad\cos(e+fx)(d\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-}{2}\right.}{f(1-n)\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n*(a + a*Sin[e + f*x]),x]
```

```
[Out] (a*d*Cos[e + f*x]*(d*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[1/2, (1 - n)/
2, (3 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*Sqrt[Cos[e + f*x]^2]) + (a*d^2*Co
s[e + f*x]*(d*Csc[e + f*x])^(-2 + n)*Hypergeometric2F1[1/2, (2 - n)/2, (4 -
 n)/2, Sin[e + f*x]^2])/(f*(2 - n)*Sqrt[Cos[e + f*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int (d\csc(e+fx))^n(a+a\sin(e+fx))\,dx &= d\int (d\csc(e+fx))^{-1+n}(a+a\csc(e+fx))\,dx\\ &= a\int (d\csc(e+fx))^n\,dx+(ad)\int (d\csc(e+fx))^{-1+n}\,dx\\ &= \left(a(d\csc(e+fx))^n\left(\frac{\sin(e+fx)}{d}\right)^n\right)\int\left(\frac{\sin(e+fx)}{d}\right)^{-n}\,dx+\left(ad(d\csc\right.\\ &= \frac{a\cos(e+fx)(d\csc(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)\sin(e+f}{f(1-n)\sqrt{\cos^2(e+fx)}}\end{aligned}$$

Mathematica [C] time = 1.66, size = 280, normalized size = 1.88

$$\frac{a2^{n-1}\left(-1+e^{2i(e+fx)}\right)e^{-i(e+fnx)}\left(\frac{ie^{i(e+fx)}}{-1+e^{2i(e+fx)}}\right)^n(\csc(e+fx)+1)\left(e^{ie}(n-1)\left(ne^{i(e+f(n+1)x)}\,{}_2F_1\left(1,\frac{3-n}{2};\frac{n+3}{2};e^{2i(e+fx)}\right)\right.\right.}{f(n-1)n(n+1)\left(\sin\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Csc[e + f*x])^n*(a + a*Sin[e + f*x]),x]
```

```
[Out] (2^(-1 + n)*a*((I*E^(I*(e + f*x)))/(-1 + E^((2*I)*(e + f*x))))^n*(-1 + E^((
2*I)*(e + f*x)))*Csc[e + f*x]^(-1 - n)*(d*Csc[e + f*x])^n*(1 + Csc[e + f*x]
)*(-(E^(I*f*(-1 + n)*x)*n*(1 + n)*Hypergeometric2F1[1, (1 - n)/2, (1 + n)/2
, E^((2*I)*(e + f*x))]) + E^(I*e)*(-1 + n)*(E^(I*(e + f*(1 + n)*x))*n*Hyper
geometric2F1[1, (3 - n)/2, (3 + n)/2, E^((2*I)*(e + f*x))] + (2*I)*E^(I*f*n
*x)*(1 + n)*Hypergeometric2F1[1, 1 - n/2, (2 + n)/2, E^((2*I)*(e + f*x))]))
)/(E^(I*(e + f*n*x))*f*(-1 + n)*n*(1 + n)*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2])^2)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)\left(d\csc\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+a*sin(f*x+e)),x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)*(d*csc(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)\left(d\csc\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+a*sin(f*x+e)),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)*(d*csc(f*x + e))^n, x)
```

maple [F] time = 1.87, size = 0, normalized size = 0.00

$$\int\left(d\csc\left(fx+e\right)\right)^{n}\left(a+a\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n*(a+a*sin(f*x+e)),x)
[Out] int((d*csc(f*x+e))^n*(a+a*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)\left(d\csc\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+a*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)*(d*csc(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(\frac{d}{\sin\left(e+f\,x\right)}\right)}^n\,\left(a+a\,\sin\left(e+f\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(e + f*x))^n*(a + a*sin(e + f*x)),x)
```

```
[Out] int((d/sin(e + f*x))^n*(a + a*sin(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \left(d\csc\left(e+fx\right)\right)^{n}\,dx+\int \left(d\csc\left(e+fx\right)\right)^{n}\sin\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n*(a+a*sin(f*x+e)),x)
```

```
[Out] a*(Integral((d*csc(e + f*x))**n, x) + Integral((d*csc(e + f*x))**n*sin(e +
f*x), x))
```

3.817 $\int \frac{(d\csc(e+fx))^n}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=171

$$\frac{dn\cos(e+fx)(d\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{af(1-n)\sqrt{\cos^2(e+fx)}}+\frac{\cos(e+fx)(d\csc(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\sin^2\right.}{af\sqrt{\cos^2(e+fx)}}$$

```
[Out] -cot(f*x+e)*(d*csc(f*x+e))^n/f/(a+a*csc(f*x+e))+d*n*cos(f*x+e)*(d*csc(f*x+e
))^(-1+n)*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],sin(f*x+e)^2)/a/f/(1-n)/(c
os(f*x+e)^2)^(1/2)+cos(f*x+e)*(d*csc(f*x+e))^n*hypergeom([1/2, -1/2*n],[1-1
/2*n],sin(f*x+e)^2)/a/f/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3238, 3820, 3787, 3772, 2643}

$$\frac{dn\cos(e+fx)(d\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{af(1-n)\sqrt{\cos^2(e+fx)}}+\frac{\cos(e+fx)(d\csc(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\sin^2\right.}{af\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n/(a + a*Sin[e + f*x]),x]
```

```
[Out] -((Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + a*Csc[e + f*x]))) + (d*n*Cos[e
+ f*x]*(d*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/
2, Sin[e + f*x]^2])/(a*f*(1 - n)*Sqrt[Cos[e + f*x]^2]) + (Cos[e + f*x]*(d*C
sc[e + f*x])^n*Hypergeometric2F1[1/2, -n/2, (2 - n)/2, Sin[e + f*x]^2])/(a*
f*Sqrt[Cos[e + f*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3820

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/(a*f*(
a + b*Csc[e + f*x])), x] + Dist[(d*(n - 1))/(a*b), Int[(d*Csc[e + f*x])^(n
- 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d\csc(e+fx))^n}{a+a\sin(e+fx)}\,dx &= \frac{\int \frac{(d\csc(e+fx))^{1+n}}{a+a\csc(e+fx)}\,dx}{d}\\
&= -\frac{\cot(e+fx)(d\csc(e+fx))^n}{f(a+a\csc(e+fx))} + \frac{n\int (d\csc(e+fx))^n(a-a\csc(e+fx))\,dx}{a^2}\\
&= -\frac{\cot(e+fx)(d\csc(e+fx))^n}{f(a+a\csc(e+fx))} + \frac{n\int (d\csc(e+fx))^n\,dx}{a} - \frac{n\int (d\csc(e+fx))^{1+n}\,dx}{ad}\\
&= -\frac{\cot(e+fx)(d\csc(e+fx))^n}{f(a+a\csc(e+fx))} + \frac{\left(n(d\csc(e+fx))^n\left(\frac{\sin(e+fx)}{d}\right)^n\right)\int \left(\frac{\sin(e+fx)}{d}\right)^{-n}dx}{a} - \cdots\\
&= -\frac{\cot(e+fx)(d\csc(e+fx))^n}{f(a+a\csc(e+fx))} + \frac{\cos(e+fx)(d\csc(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\sin^2(e+\cdots\right)}{af\sqrt{\cos^2(e+fx)}}
\end{aligned}$$

Mathematica [F] time = 2.80, size = 0, normalized size = 0.00

$$\int \frac{(d\csc(e+fx))^n}{a+a\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Csc[e + f*x])^n/(a + a*Sin[e + f*x]),x]
```

```
[Out] Integrate[(d*Csc[e + f*x])^n/(a + a*Sin[e + f*x]), x]
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d\csc\left(fx+e\right)\right)^{n}}{a\sin\left(fx+e\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*csc(f*x + e))^n/(a*sin(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\csc\left(fx+e\right)\right)^{n}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*csc(f*x + e))^n/(a*sin(f*x + e) + a), x)
```

maple [F] time = 0.85, size = 0, normalized size = 0.00

$$\int \frac{\left(d\csc\left(fx+e\right)\right)^{n}}{a+a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

```
[Out] int((d*csc(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\csc\left(fx+e\right)\right)^{n}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*csc(f*x + e))^n/(a*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{d}{\sin\left(e+f\,x\right)}\right)^n}{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(e + f*x))^n/(a + a*sin(e + f*x)),x)
[Out] int((d/sin(e + f*x))^n/(a + a*sin(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\left(d\csc\left(e+fx\right)\right)^n}{\sin\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n/(a+a*sin(f*x+e)),x)
[Out] Integral((d*csc(e + f*x))**n/(sin(e + f*x) + 1), x)/a
```

3.818 $\int \frac{(d\csc(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=231

$$\frac{2n\cos(e+fx)(d\csc(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-n-2);-\frac{n}{2};\sin^2(e+fx)\right)}{3a^2d^2f\sqrt{\cos^2(e+fx)}}-\frac{2n\cot(e+fx)(d\csc(e+fx))^{n+2}}{3a^2d^2f(\csc(e+fx)+1)}-\frac{(2n+1)}{}$$

```
[Out] -2/3*n*cot(f*x+e)*(d*csc(f*x+e))^(2+n)/a^2/d^2/f/(1+csc(f*x+e))+1/3*cot(f*x
+e)*(d*csc(f*x+e))^(2+n)/d^2/f/(a+a*csc(f*x+e))^2+2/3*n*cos(f*x+e)*(d*csc(f
*x+e))^(2+n)*hypergeom([1/2, -1-1/2*n],[-1/2*n],sin(f*x+e)^2)/a^2/d^2/f/(co
s(f*x+e)^2)^(1/2)-1/3*(1+2*n)*cos(f*x+e)*(d*csc(f*x+e))^(1+n)*hypergeom([1/
2, -1/2-1/2*n],[1/2-1/2*n],sin(f*x+e)^2)/a^2/d/f/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3817, 4020, 3787, 3772, 2643}

$$\frac{2n\cos(e+fx)(d\csc(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-n-2);-\frac{n}{2};\sin^2(e+fx)\right)}{3a^2d^2f\sqrt{\cos^2(e+fx)}}-\frac{2n\cot(e+fx)(d\csc(e+fx))^{n+2}}{3a^2d^2f(\csc(e+fx)+1)}-\frac{(2n+1)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (-2*n*Cot[e + f*x]*(d*Csc[e + f*x])^(2 + n))/(3*a^2*d^2*f*(1 + Csc[e + f*x]
)) + (Cot[e + f*x]*(d*Csc[e + f*x])^(2 + n))/(3*d^2*f*(a + a*Csc[e + f*x])^
2) + (2*n*Cos[e + f*x]*(d*Csc[e + f*x])^(2 + n)*Hypergeometric2F1[1/2, (-2
- n)/2, -n/2, Sin[e + f*x]^2])/(3*a^2*d^2*f*Sqrt[Cos[e + f*x]^2]) - ((1 + 2
*n)*Cos[e + f*x]*(d*Csc[e + f*x])^(1 + n)*Hypergeometric2F1[1/2, (-1 - n)/2
, (1 - n)/2, Sin[e + f*x]^2])/(3*a^2*d*f*Sqrt[Cos[e + f*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
```

```
!IntegerQ[m] && IntegersQ[n, p]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{(d\csc(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \frac{(d\csc(e+fx))^{2+n}}{(a+a\csc(e+fx))^2}\,dx}{d^2}$$

$$= \frac{\cot(e+fx)(d\csc(e+fx))^{2+n}}{3d^2f(a+a\csc(e+fx))^2} - \frac{\int \frac{(d\csc(e+fx))^{2+n}(a(-1+n)-a(1+n)\csc(e+fx))}{a+a\csc(e+fx)}\,dx}{3a^2d^2}$$

$$= -\frac{2n\cot(e+fx)(d\csc(e+fx))^{2+n}}{3a^2d^2f(1+\csc(e+fx))} + \frac{\cot(e+fx)(d\csc(e+fx))^{2+n}}{3d^2f(a+a\csc(e+fx))^2} - \frac{\int(d\csc(e+fx \ldots}{\ldots}$$

$$= -\frac{2n\cot(e+fx)(d\csc(e+fx))^{2+n}}{3a^2d^2f(1+\csc(e+fx))} + \frac{\cot(e+fx)(d\csc(e+fx))^{2+n}}{3d^2f(a+a\csc(e+fx))^2} - \frac{(2n(2+n))\int(\ldots}{\ldots}$$

$$= -\frac{2n\cot(e+fx)(d\csc(e+fx))^{2+n}}{3a^2d^2f(1+\csc(e+fx))} + \frac{\cot(e+fx)(d\csc(e+fx))^{2+n}}{3d^2f(a+a\csc(e+fx))^2} - \frac{\left(2n(2+n)(d\,c \ldots\right.}{\ldots}$$

$$= -\frac{2n\cot(e+fx)(d\csc(e+fx))^{2+n}}{3a^2d^2f(1+\csc(e+fx))} + \frac{\cot(e+fx)(d\csc(e+fx))^{2+n}}{3d^2f(a+a\csc(e+fx))^2} + \frac{2n\cot(e+fx \ldots}{\ldots}$$

Mathematica [F] time = 4.79, size = 0, normalized size = 0.00

$$\int \frac{(d\csc(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Csc[e + f*x])^n/(a + a*Sin[e + f*x])^2,x]
[Out] Integrate[(d*Csc[e + f*x])^n/(a + a*Sin[e + f*x])^2, x]
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(d\csc\left(fx+e\right)\right)^n}{a^2\cos\left(fx+e\right)^2 - 2\,a^2\sin\left(fx+e\right) - 2\,a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
[Out] integral(-(d*csc(f*x + e))^n/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a
^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\csc\left(fx+e\right)\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*csc(f*x + e))^n/(a*sin(f*x + e) + a)^2, x)
```

maple [F] time = 1.95, size = 0, normalized size = 0.00

$$\int \frac{\left(d\csc\left(fx+e\right)\right)^{n}}{\left(a+a\sin\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
[Out] int((d*csc(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\csc\left(fx+e\right)\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*csc(f*x + e))^n/(a*sin(f*x + e) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{d}{\sin\left(e+f\,x\right)}\right)^{n}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(e + f*x))^n/(a + a*sin(e + f*x))^2,x)
[Out] int((d/sin(e + f*x))^n/(a + a*sin(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\left(d\csc\left(e+fx\right)\right)^{n}}{\sin^{2}\left(e+fx\right)+2\sin\left(e+fx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Integral((d*csc(e + f*x))**n/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1), x)/a**
2
```

3.819 $\int \left(c(d\sin(e+fx))^p\right)^n (a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=113

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\sin^{-np}(e+fx)F_1\left(\frac{1}{2};-np,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx)\right.}{f}$$

```
[Out] -2^(1/2+m)*AppellF1(1/2,-n*p,1/2-m,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos
(f*x+e)*(c*(d*sin(f*x+e))^p)^n*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/f
/(sin(f*x+e)^(n*p))
```

Rubi [A] time = 0.23, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2826, 2787, 2786, 2785, 133}

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\sin^{-np}(e+fx)F_1\left(\frac{1}{2};-np,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx)\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 + m)*AppellF1[1/2, -(n*p), 1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - S
in[e + f*x])/2]*Cos[e + f*x]*(c*(d*Sin[e + f*x])^p)^n*(1 + Sin[e + f*x])^(-
1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*Sin[e + f*x]^(n*p)))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2786

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[((d/b)^IntPart[n]*(d*Sin[e + f*x])^FracPart[n
])/(b*Sin[e + f*x])^FracPart[n], Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 2826

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sin[e + f*x
])^p)^FracPart[n])/(d*Sin[e + f*x])^(p*FracPart[n]), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rubi steps

$$\int \left(c(d\sin(e+fx))^p\right)^n (a+a\sin(e+fx))^m\,dx = \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int (d\sin(e+fx))^{np}(a+$$

$$= \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n(1+\sin(e+fx))^{-m}(a+$$

$$= \left(\sin^{-np}(e+fx)\left(c(d\sin(e+fx))^p\right)^n(1+\sin(e+fx))^{-m}(a+a\,\mathrm{si}$$

$$= -\frac{\left(\cos(e+fx)\sin^{-np}(e+fx)\left(c(d\sin(e+fx))^p\right)^n(1+\sin(e+f\right.}{f}$$

$$= -\frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};-np,\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)}{}$$

Mathematica [B] time = 15.16, size = 2967, normalized size = 26.26

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (-3*AppellF1[1/2, -(n*p), 1 + m + n*p, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Ta
n[(-e + Pi/2 - f*x)/2]^2]*Cos[e + f*x]*Sin[e + f*x]^(n*p)*(c*(d*Sin[e + f*x
])^p)^n*(a + a*Sin[e + f*x])^m)/(f*(Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*Appell
F1[1/2, -(n*p), 1 + m + n*p, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi
/2 - f*x)/2]^2] - 2*((1 + m + n*p)*AppellF1[3/2, -(n*p), 2 + m + n*p, 5/2,
Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + n*p*AppellF1[3/2
, 1 - n*p, 1 + m + n*p, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 -
f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)*((-3*n*p*AppellF1[1/2, -(n*p), 1 +
m + n*p, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Cos[
e + f*x]^2*Sin[e + f*x]^(-1 + n*p))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*Appe
llF1[1/2, -(n*p), 1 + m + n*p, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e +
Pi/2 - f*x)/2]^2] - 2*((1 + m + n*p)*AppellF1[3/2, -(n*p), 2 + m + n*p, 5/2
, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + n*p*AppellF1[3
/2, 1 - n*p, 1 + m + n*p, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2
- f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) + (3*AppellF1[1/2, -(n*p), 1 + m
 + n*p, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sin[e
 + f*x]^(1 + n*p))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -(n*p),
 1 + m + n*p, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]
 - 2*((1 + m + n*p)*AppellF1[3/2, -(n*p), 2 + m + n*p, 5/2, Tan[(-e + Pi/2
- f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + n*p*AppellF1[3/2, 1 - n*p, 1 +
m + n*p, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Tan
[(-e + Pi/2 - f*x)/2]^2)) - (3*m*AppellF1[1/2, -(n*p), 1 + m + n*p, 3/2, Ta
n[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Cos[e + f*x]*Sin[e +
 f*x]^(n*p)*Tan[(-e + Pi/2 - f*x)/2])/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*Ap
pellF1[1/2, -(n*p), 1 + m + n*p, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e
+ Pi/2 - f*x)/2]^2] - 2*((1 + m + n*p)*AppellF1[3/2, -(n*p), 2 + m + n*p, 5
/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + n*p*AppellF1
[3/2, 1 - n*p, 1 + m + n*p, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/
2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) + (3*Cos[e + f*x]*Sin[e + f*x]
^(n*p)*(-1/3*((1 + m + n*p)*AppellF1[3/2, -(n*p), 2 + m + n*p, 5/2, Tan[(-e
 + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^
2*Tan[(-e + Pi/2 - f*x)/2]) - (n*p*AppellF1[3/2, 1 - n*p, 1 + m + n*p, 5/2,
 Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 -
f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/3))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*
AppellF1[1/2, -(n*p), 1 + m + n*p, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-
e + Pi/2 - f*x)/2]^2] - 2*((1 + m + n*p)*AppellF1[3/2, -(n*p), 2 + m + n*p,
 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + n*p*Appell
F1[3/2, 1 - n*p, 1 + m + n*p, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + P
i/2 - f*x)/2]^2])*Tan[(-e + Pi/2 - f*x)/2]^2)) - (3*AppellF1[1/2, -(n*p), 1
 + m + n*p, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*C
os[e + f*x]*Sin[e + f*x]^(n*p)*(-2*((1 + m + n*p)*AppellF1[3/2, -(n*p), 2 +
```

```
 m + n*p, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + n
*p*AppellF1[3/2, 1 - n*p, 1 + m + n*p, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Ta
n[(-e + Pi/2 - f*x)/2]^2])*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)
/2] + 3*(-1/3*((1 + m + n*p)*AppellF1[3/2, -(n*p), 2 + m + n*p, 5/2, Tan[(-
e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]
^2*Tan[(-e + Pi/2 - f*x)/2]) - (n*p*AppellF1[3/2, 1 - n*p, 1 + m + n*p, 5/2
, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 -
 f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/3) - 2*Tan[(-e + Pi/2 - f*x)/2]^2*((1
+ m + n*p)*((-3*(2 + m + n*p)*AppellF1[5/2, -(n*p), 3 + m + n*p, 7/2, Tan[(
-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2
]^2*Tan[(-e + Pi/2 - f*x)/2])/5 - (3*n*p*AppellF1[5/2, 1 - n*p, 2 + m + n*p
, 7/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + P
i/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/5) + n*p*((-3*(1 + m + n*p)*Appel
lF1[5/2, 1 - n*p, 2 + m + n*p, 7/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e +
Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi/2 - f*x)/2])/5 +
(3*(1 - n*p)*AppellF1[5/2, 2 - n*p, 1 + m + n*p, 7/2, Tan[(-e + Pi/2 - f*x)
/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2]*Sec[(-e + Pi/2 - f*x)/2]^2*Tan[(-e + Pi
/2 - f*x)/2])/5))))/((Sec[(-e + Pi/2 - f*x)/2]^2)^m*(3*AppellF1[1/2, -(n*p)
, 1 + m + n*p, 3/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2
] - 2*((1 + m + n*p)*AppellF1[3/2, -(n*p), 2 + m + n*p, 5/2, Tan[(-e + Pi/2
 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2] + n*p*AppellF1[3/2, 1 - n*p, 1 +
 m + n*p, 5/2, Tan[(-e + Pi/2 - f*x)/2]^2, -Tan[(-e + Pi/2 - f*x)/2]^2])*Ta
n[(-e + Pi/2 - f*x)/2]^2)^2)))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral(((d*sin(f*x + e))^p*c)^n*(a*sin(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}\left(a\sin\left(fx+e\right)+a\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate(((d*sin(f*x + e))^p*c)^n*(a*sin(f*x + e) + a)^m, x)
```

maple [F] time = 0.88, size = 0, normalized size = 0.00

$$\int \left(c\left(d\sin\left(fx+e\right)\right)^{p}\right)^{n} \left(a+a\sin\left(fx+e\right)\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^m,x)
[Out] int((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(d\sin\left(fx+e\right)\right)^{p} c\right)^{n} \left(a\sin\left(fx+e\right)+a\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate(((d*sin(f*x + e))^p*c)^n*(a*sin(f*x + e) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c\left(d\,\sin\left(e+f\,x\right)\right)^{p}\right)^{n} \left(a+a\,\sin\left(e+f\,x\right)\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(e + f*x))^p)^n*(a + a*sin(e + f*x))^m,x)
[Out] int((c*(d*sin(e + f*x))^p)^n*(a + a*sin(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m} \left(c\left(d\sin\left(e+fx\right)\right)^{p}\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n*(a+a*sin(f*x+e))**m,x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*(c*(d*sin(e + f*x))**p)**n, x)
```

3.820 $\int \left(c(d\sin(e+fx))^p\right)^n (a+a\sin(e+fx))^3\,dx$

Optimal. Leaf size=299

$$\frac{a^3(4np+11)\sin^2(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+2);\frac{1}{2}(np+4);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{f(np+2)(np+3)\sqrt{\cos^2(e+fx)}}+\frac{a^3(4np+5}{}$$

```
[Out] -a^3*(2*n*p+7)*cos(f*x+e)*sin(f*x+e)*(c*(d*sin(f*x+e))^p)^n/f/(n*p+2)/(n*p+
3)-cos(f*x+e)*sin(f*x+e)*(c*(d*sin(f*x+e))^p)^n*(a^3+a^3*sin(f*x+e))/f/(n*p
+3)+a^3*(4*n*p+5)*cos(f*x+e)*hypergeom([1/2, 1/2*n*p+1/2],[1/2*n*p+3/2],sin
(f*x+e)^2)*sin(f*x+e)*(c*(d*sin(f*x+e))^p)^n/f/(n*p+1)/(n*p+2)/(cos(f*x+e)^
2)^(1/2)+a^3*(4*n*p+11)*cos(f*x+e)*hypergeom([1/2, 1/2*n*p+1],[1/2*n*p+2],s
in(f*x+e)^2)*sin(f*x+e)^2*(c*(d*sin(f*x+e))^p)^n/f/(n*p+2)/(n*p+3)/(cos(f*x
+e)^2)^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2826, 2763, 2968, 3023, 2748, 2643}

$$\frac{a^3(4np+11)\sin^2(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+2);\frac{1}{2}(np+4);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{f(np+2)(np+3)\sqrt{\cos^2(e+fx)}}+\frac{a^3(4np+5}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n*(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -((a^3*(7 + 2*n*p)*Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f*(
2 + n*p)*(3 + n*p))) + (a^3*(5 + 4*n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2,
 (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])
^p)^n)/(f*(1 + n*p)*(2 + n*p)*Sqrt[Cos[e + f*x]^2]) + (a^3*(11 + 4*n*p)*Cos
[e + f*x]*Hypergeometric2F1[1/2, (2 + n*p)/2, (4 + n*p)/2, Sin[e + f*x]^2]*
Sin[e + f*x]^2*(c*(d*Sin[e + f*x])^p)^n)/(f*(2 + n*p)*(3 + n*p)*Sqrt[Cos[e
+ f*x]^2]) - (Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n*(a^3 + a^3
*Sin[e + f*x]))/(f*(3 + n*p))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2826

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sin[e + f*x
])^p)^FracPart[n])/(d*Sin[e + f*x])^(p*FracPart[n]), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \left(c(d\sin(e+fx))^p\right)^n (a+a\sin(e+fx))^3\,dx = \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int (d\sin(e+fx))^{np}(a+$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n\left(a^3+a^3\sin(e+fx\right.}{f(3+np)}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n\left(a^3+a^3\sin(e+fx\right.}{f(3+np)}$$

$$= -\frac{a^3(7+2np)\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(2+np)(3+np)} - \frac{\cos}{}$$

$$= -\frac{a^3(7+2np)\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(2+np)(3+np)} - \frac{\cos}{}$$

$$= -\frac{a^3(7+2np)\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(2+np)(3+np)} + \frac{a^3(5}{}$$

Mathematica [A] time = 1.37, size = 297, normalized size = 0.99

$$-\frac{a^3\sin(e+fx)\cos(e+fx)\sqrt{\cos^2(e+fx)}\left(\frac{1}{2}(np+1)\sin(e+fx)\left(6\left(n^2p^2+7np+12\right){}_2F_1\left(\frac{1}{2},\frac{np}{2}+1;\frac{np}{2}+2;\sin\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n*(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -((a^3*Cos[e + f*x]*Sqrt[Cos[e + f*x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p
)^n*((24 + 26*n*p + 9*n^2*p^2 + n^3*p^3)*Hypergeometric2F1[1/2, (1 + n*p)/2
, (3 + n*p)/2, Sin[e + f*x]^2] + ((1 + n*p)*Sin[e + f*x]*(6*(12 + 7*n*p + n
^2*p^2)*Hypergeometric2F1[1/2, 1 + (n*p)/2, 2 + (n*p)/2, Sin[e + f*x]^2] +
2*(2 + n*p)*Sin[e + f*x]*(3*(4 + n*p)*Hypergeometric2F1[1/2, (3 + n*p)/2, (
5 + n*p)/2, Sin[e + f*x]^2] + (3 + n*p)*Hypergeometric2F1[1/2, 2 + (n*p)/2,
 3 + (n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x])))/2))/(f*(1 + n*p)*(2 + n*p)*(3
 + n*p)*(4 + n*p)*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)\right)\left(\left(d\sin\left(fx+e\right)\right)^p c\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(
f*x + e))*((d*sin(f*x + e))^p*c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{3}\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*((d*sin(f*x + e))^p*c)^n, x)
```

maple [F] time = 1.42, size = 0, normalized size = 0.00

$$\int \left(c\left(d\sin\left(fx+e\right)\right)^{p}\right)^{n}\left(a+a\sin\left(fx+e\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^3,x)
```

```
[Out] int((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)^{3}\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*((d*sin(f*x + e))^p*c)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c\,{\left(d\,\sin\left(e+f\,x\right)\right)}^p\right)}^n\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(e + f*x))^p)^n*(a + a*sin(e + f*x))^3,x)
```

```
[Out] int((c*(d*sin(e + f*x))^p)^n*(a + a*sin(e + f*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int\left(c\left(d\sin\left(e+fx\right)\right)^p\right)^n\,dx+\int 3\left(c\left(d\sin\left(e+fx\right)\right)^p\right)^n\sin\left(e+fx\right)dx+\int 3\left(c\left(d\sin\left(e+fx\right)\right)^p\right)^n\sin^2\left(e+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n*(a+a*sin(f*x+e))**3,x)
```

```
[Out] a**3*(Integral((c*(d*sin(e + f*x))**p)**n, x) + Integral(3*(c*(d*sin(e + f*
x))**p)**n*sin(e + f*x), x) + Integral(3*(c*(d*sin(e + f*x))**p)**n*sin(e +
 f*x)**2, x) + Integral((c*(d*sin(e + f*x))**p)**n*sin(e + f*x)**3, x))
```

3.821 $\int \left(c(d\sin(e+fx))^p\right)^n (a+a\sin(e+fx))^2\,dx$

Optimal. Leaf size=222

$$\frac{2a^2\sin^2(e+fx)\cos(e+fx)\,_2F_1\left(\frac{1}{2},\frac{1}{2}(np+2);\frac{1}{2}(np+4);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{f(np+2)\sqrt{\cos^2(e+fx)}}+\frac{a^2(2np+3)\sin(e}{}$$

```
[Out] -a^2*cos(f*x+e)*sin(f*x+e)*(c*(d*sin(f*x+e))^p)^n/f/(n*p+2)+a^2*(2*n*p+3)*c
os(f*x+e)*hypergeom([1/2, 1/2*n*p+1/2],[1/2*n*p+3/2],sin(f*x+e)^2)*sin(f*x+
e)*(c*(d*sin(f*x+e))^p)^n/f/(n*p+1)/(n*p+2)/(cos(f*x+e)^2)^(1/2)+2*a^2*cos(
f*x+e)*hypergeom([1/2, 1/2*n*p+1],[1/2*n*p+2],sin(f*x+e)^2)*sin(f*x+e)^2*(c
*(d*sin(f*x+e))^p)^n/f/(n*p+2)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2826, 2763, 2748, 2643}

$$\frac{2a^2\sin^2(e+fx)\cos(e+fx)\,_2F_1\left(\frac{1}{2},\frac{1}{2}(np+2);\frac{1}{2}(np+4);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{f(np+2)\sqrt{\cos^2(e+fx)}}+\frac{a^2(2np+3)\sin(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n*(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -((a^2*Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f*(2 + n*p))) +
 (a^2*(3 + 2*n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p
)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f*(1 + n*p)*(2
 + n*p)*Sqrt[Cos[e + f*x]^2]) + (2*a^2*Cos[e + f*x]*Hypergeometric2F1[1/2,
(2 + n*p)/2, (4 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]^2*(c*(d*Sin[e + f*x]
)^p)^n)/(f*(2 + n*p)*Sqrt[Cos[e + f*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2826

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sin[e + f*x
])^p)^FracPart[n])/(d*Sin[e + f*x])^(p*FracPart[n]), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rubi steps

$$\int \left(c(d\sin(e+fx))^p\right)^n (a+a\sin(e+fx))^2\,dx = \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int (d\sin(e+fx))^{np}(a+$$

$$= -\frac{a^2\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(2+np)} + \frac{\left((d\sin(e+f\right.}{}$$

$$= -\frac{a^2\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(2+np)} + \frac{\left(2a^2(d\sin(e\right.}{}$$

$$= -\frac{a^2\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(2+np)} + \frac{a^2(3+2np)\,c}{}$$

Mathematica [A] time = 0.66, size = 222, normalized size = 1.00

$$-\frac{a^2\sin(e+fx)\cos(e+fx)\sqrt{\cos^2(e+fx)}\left(\left(n^2p^2+5np+6\right){}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(e+fx)\right)+(np+\right.}{f(np+1)(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n*(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -((a^2*Cos[e + f*x]*Sqrt[Cos[e + f*x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p
)^n*((6 + 5*n*p + n^2*p^2)*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2,
 Sin[e + f*x]^2] + (1 + n*p)*Sin[e + f*x]*(2*(3 + n*p)*Hypergeometric2F1[1/
2, 1 + (n*p)/2, 2 + (n*p)/2, Sin[e + f*x]^2] + (2 + n*p)*Hypergeometric2F1[
1/2, (3 + n*p)/2, (5 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x])))/(f*(1 + n*p)
*(2 + n*p)*(3 + n*p)*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2\right)\left(\left(d\sin\left(fx+e\right)\right)^p c\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*((d*sin(f*x + e
))^p*c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^2\left(\left(d\sin\left(fx+e\right)\right)^p c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*((d*sin(f*x + e))^p*c)^n, x)
```

maple [F] time = 1.44, size = 0, normalized size = 0.00

$$\int\left(c\left(d\sin\left(fx+e\right)\right)^p\right)^n\left(a+a\sin\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^2,x)
```

```
[Out] int((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)+a\right)^2\left(\left(d\sin\left(fx+e\right)\right)^p c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*((d*sin(f*x + e))^p*c)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c\left(d\,\sin\left(e+f\,x\right)\right)^p\right)^n\,\left(a+a\,\sin\left(e+f\,x\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(e + f*x))^p)^n*(a + a*sin(e + f*x))^2,x)
```

```
[Out] int((c*(d*sin(e + f*x))^p)^n*(a + a*sin(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \left(c\left(d\sin\left(e+fx\right)\right)^p\right)^n\,dx + \int 2\left(c\left(d\sin\left(e+fx\right)\right)^p\right)^n\sin\left(e+fx\right)dx + \int \left(c\left(d\sin\left(e+fx\right)\right)^p\right)^n\sin^2\left(e+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n*(a+a*sin(f*x+e))**2,x)
```

```
[Out] a**2*(Integral((c*(d*sin(e + f*x))**p)**n, x) + Integral(2*(c*(d*sin(e + f*
x))**p)**n*sin(e + f*x), x) + Integral((c*(d*sin(e + f*x))**p)**n*sin(e + f
*x)**2, x))
```

3.822 $\int \left(c(d\sin(e+fx))^p\right)^n (a+a\sin(e+fx))\,dx$

Optimal. Leaf size=163

$$\frac{a\sin^2(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+2);\frac{1}{2}(np+4);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{f(np+2)\sqrt{\cos^2(e+fx)}}+\frac{a\sin(e+fx)\cos(e+$$

```
[Out] a*cos(f*x+e)*hypergeom([1/2, 1/2*n*p+1/2],[1/2*n*p+3/2],sin(f*x+e)^2)*sin(f
*x+e)*(c*(d*sin(f*x+e))^p)^n/f/(n*p+1)/(cos(f*x+e)^2)^(1/2)+a*cos(f*x+e)*hy
pergeom([1/2, 1/2*n*p+1],[1/2*n*p+2],sin(f*x+e)^2)*sin(f*x+e)^2*(c*(d*sin(f
*x+e))^p)^n/f/(n*p+2)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2826, 2748, 2643}

$$\frac{a\sin^2(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+2);\frac{1}{2}(np+4);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{f(np+2)\sqrt{\cos^2(e+fx)}}+\frac{a\sin(e+fx)\cos(e+$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n*(a + a*Sin[e + f*x]),x]
```

```
[Out] (a*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*
x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f*(1 + n*p)*Sqrt[Cos[e + f*x]
^2]) + (a*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n*p)/2, (4 + n*p)/2, Sin
[e + f*x]^2]*Sin[e + f*x]^2*(c*(d*Sin[e + f*x])^p)^n)/(f*(2 + n*p)*Sqrt[Cos
[e + f*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2826

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sin[e + f*x
])^p)^FracPart[n])/(d*Sin[e + f*x])^(p*FracPart[n]), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && !IntegerQ[n]
```

Rubi steps

$$\int \left(c(d\sin(e+fx))^p\right)^n (a+a\sin(e+fx))\,dx = \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int (d\sin(e+fx))^{np}(a+a$$

$$= \left(a(d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int (d\sin(e+fx))^{np}\,dx +$$

$$= \frac{a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(e+fx)\right)\sin(e+f}{f(1+np)\sqrt{\cos^2(e+fx)}}$$

Mathematica [C] time = 1.51, size = 270, normalized size = 1.66

$$\frac{a2^{-np-1}(\sin(e+fx)+1)\left(-ie^{-i(e+fx)}\left(-1+e^{2i(e+fx)}\right)\right)^{np+1}\left(2\left(n^2p^2-1\right)e^{i(e+fx)}\,{}_2F_1\left(1,\frac{np}{2}+1;1-\frac{np}{2};e^{2i(e+fx)}\right)+in}{fnp(np-1)(n}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n*(a + a*Sin[e + f*x]),x]
```

```
[Out] (2^(-1 - n*p)*a*(((-I)*(-1 + E^((2*I)*(e + f*x))))/E^(I*(e + f*x)))^(1 + n*
p)*(2*E^(I*(e + f*x))*(-1 + n^2*p^2)*Hypergeometric2F1[1, 1 + (n*p)/2, 1 -
(n*p)/2, E^((2*I)*(e + f*x))] + I*n*p*((-1 + n*p)*Hypergeometric2F1[1, (1 +
 n*p)/2, (1 - n*p)/2, E^((2*I)*(e + f*x))] - E^((2*I)*(e + f*x))*(1 + n*p)*
Hypergeometric2F1[1, (3 + n*p)/2, (3 - n*p)/2, E^((2*I)*(e + f*x))]))*(c*(d
*Sin[e + f*x])^p)^n*(1 + Sin[e + f*x]))/(f*n*p*(-1 + n*p)*(1 + n*p)*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^2*Sin[e + f*x]^(n*p))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)\left(\left(d\sin\left(fx+e\right)\right)^p c\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e)),x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)*((d*sin(f*x + e))^p*c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e)),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)*((d*sin(f*x + e))^p*c)^n, x)
```

maple [F] time = 0.50, size = 0, normalized size = 0.00

$$\int \left(c\left(d\sin\left(fx+e\right)\right)^{p}\right)^{n}\left(a+a\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e)),x)
[Out] int((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)+a\right)\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+a*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)*((d*sin(f*x + e))^p*c)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,{\left(d\,\sin\left(e+f\,x\right)\right)}^p\right)}^n\,\left(a+a\,\sin\left(e+f\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(e + f*x))^p)^n*(a + a*sin(e + f*x)),x)
[Out] int((c*(d*sin(e + f*x))^p)^n*(a + a*sin(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\left(c\left(d\sin\left(e+fx\right)\right)^{p}\right)^{n}\,dx+\int\left(c\left(d\sin\left(e+fx\right)\right)^{p}\right)^{n}\sin\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n*(a+a*sin(f*x+e)),x)
```

```
[Out] a*(Integral((c*(d*sin(e + f*x))**p)**n, x) + Integral((c*(d*sin(e + f*x))**
p)**n*sin(e + f*x), x))
```

3.823 $\int \frac{(c(d\sin(e+fx))^p)^n}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=189

$$\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{np}{2};\frac{1}{2}(np+2);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{af\sqrt{\cos^2(e+fx)}}-\frac{np\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+$$

```
[Out] -cos(f*x+e)*(c*(d*sin(f*x+e))^p)^n/f/(a+a*sin(f*x+e))+cos(f*x+e)*hypergeom(
[1/2, 1/2*n*p],[1/2*n*p+1],sin(f*x+e)^2)*(c*(d*sin(f*x+e))^p)^n/a/f/(cos(f*
x+e)^2)^(1/2)-n*p*cos(f*x+e)*hypergeom([1/2, 1/2*n*p+1/2],[1/2*n*p+3/2],sin
(f*x+e)^2)*sin(f*x+e)*(c*(d*sin(f*x+e))^p)^n/a/f/(n*p+1)/(cos(f*x+e)^2)^(1/
2)
```

Rubi [A] time = 0.23, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2826, 2769, 2748, 2643}

$$\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{np}{2};\frac{1}{2}(np+2);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{af\sqrt{\cos^2(e+fx)}}-\frac{np\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n/(a + a*Sin[e + f*x]),x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, (n*p)/2, (2 + n*p)/2, Sin[e + f*x]^2]*
(c*(d*Sin[e + f*x])^p)^n)/(a*f*Sqrt[Cos[e + f*x]^2]) - (n*p*Cos[e + f*x]*Hy
pergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]
*(c*(d*Sin[e + f*x])^p)^n)/(a*f*(1 + n*p)*Sqrt[Cos[e + f*x]^2]) - (Cos[e +
f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f*(a + a*Sin[e + f*x]))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2769

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a
*f*(a + b*Sin[e + f*x])), x] + Dist[(d*n)/(a*b), Int[(c + d*Sin[e + f*x])^(
n - 1)*(a - b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (IntegerQ[2*n
] || EqQ[c, 0])
```

Rule 2826

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sin[e + f*x
])^p)^FracPart[n])/(d*Sin[e + f*x])^(p*FracPart[n]), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\left(c(d\sin(e+fx))^p\right)^n}{a+a\sin(e+fx)}\,dx = \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int \frac{(d\sin(e+fx))^{np}}{a+a\sin(e+fx)}\,dx$$

$$= -\frac{\cos(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(a+a\sin(e+fx))} + \frac{\left(dnp(d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int(}{a^2}$$

$$= -\frac{\cos(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(a+a\sin(e+fx))} - \frac{\left(np(d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int(d}{a}$$

$$= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{np}{2};\frac{1}{2}(2+np);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{af\sqrt{\cos^2(e+fx)}} - \frac{np\cos(e+f}{}$$

Mathematica [A] time = 0.27, size = 157, normalized size = 0.83

$$\frac{\sin(e+fx)\cos(e+fx)\sqrt{\cos^2(e+fx)}\left((np+1)\sin(e+fx)\,{}_2F_1\left(\frac{3}{2},\frac{np}{2}+1;\frac{np}{2}+2;\sin^2(e+fx)\right)-(np+2)\,{}_2F_1\left(\frac{3}{2}\right.\right.}{af(np+1)(np+2)(\sin(e+fx)-1)(\sin(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n/(a + a*Sin[e + f*x]),x]
```

```
[Out] (Cos[e + f*x]*Sqrt[Cos[e + f*x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n*(-
((2 + n*p)*Hypergeometric2F1[3/2, (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2]
) + (1 + n*p)*Hypergeometric2F1[3/2, 1 + (n*p)/2, 2 + (n*p)/2, Sin[e + f*x]
^2]*Sin[e + f*x]))/(a*f*(1 + n*p)*(2 + n*p)*(-1 + Sin[e + f*x])*(1 + Sin[e
+ f*x]))
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}}{a\sin\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(((d*sin(f*x + e))^p*c)^n/(a*sin(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(((d*sin(f*x + e))^p*c)^n/(a*sin(f*x + e) + a), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int\frac{\left(c\left(d\sin\left(fx+e\right)\right)^{p}\right)^{n}}{a+a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n/(a+a*sin(f*x+e)),x)
```

```
[Out] int((c*(d*sin(f*x+e))^p)^n/(a+a*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(((d*sin(f*x + e))^p*c)^n/(a*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,{\left(d\,\sin\left(e+f\,x\right)\right)}^p\right)}^n}{a+a\,\sin\left(e+f\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(e + f*x))^p)^n/(a + a*sin(e + f*x)),x)
```

```
[Out] int((c*(d*sin(e + f*x))^p)^n/(a + a*sin(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\left(c \left(d \sin{\left(e + f x \right)}\right)^{p}\right)^{n}}{\sin{\left(e + f x \right)} + 1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n/(a+a*sin(f*x+e)),x)
```

```
[Out] Integral((c*(d*sin(e + f*x))**p)**n/(sin(e + f*x) + 1), x)/a
```

3.824 $\int \frac{\left(c(d\sin(e+fx))^p\right)^n}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=288

$$\frac{2\left(1-n^2p^2\right)\sin^2(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+2);\frac{1}{2}(np+4);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{3a^2f(np+2)\sqrt{\cos^2(e+fx)}}-\frac{np(1-2n}{}$$

```
[Out] 2/3*(-n*p+1)*cos(f*x+e)*sin(f*x+e)*(c*(d*sin(f*x+e))^p)^n/a^2/f/(1+sin(f*x+
e))+1/3*cos(f*x+e)*sin(f*x+e)*(c*(d*sin(f*x+e))^p)^n/f/(a+a*sin(f*x+e))^2-1
/3*n*p*(-2*n*p+1)*cos(f*x+e)*hypergeom([1/2, 1/2*n*p+1/2],[1/2*n*p+3/2],sin
(f*x+e)^2)*sin(f*x+e)*(c*(d*sin(f*x+e))^p)^n/a^2/f/(n*p+1)/(cos(f*x+e)^2)^(
1/2)+2/3*(-n^2*p^2+1)*cos(f*x+e)*hypergeom([1/2, 1/2*n*p+1],[1/2*n*p+2],sin
(f*x+e)^2)*sin(f*x+e)^2*(c*(d*sin(f*x+e))^p)^n/a^2/f/(n*p+2)/(cos(f*x+e)^2)
^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2826, 2766, 2978, 2748, 2643}

$$\frac{2\left(1-n^2p^2\right)\sin^2(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+2);\frac{1}{2}(np+4);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{3a^2f(np+2)\sqrt{\cos^2(e+fx)}}-\frac{np(1-2n}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -(n*p*(1 - 2*n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p
)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(3*a^2*f*(1 + n
*p)*Sqrt[Cos[e + f*x]^2]) + (2*(1 - n^2*p^2)*Cos[e + f*x]*Hypergeometric2F1
[1/2, (2 + n*p)/2, (4 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]^2*(c*(d*Sin[e
+ f*x])^p)^n)/(3*a^2*f*(2 + n*p)*Sqrt[Cos[e + f*x]^2]) + (2*(1 - n*p)*Cos[e
 + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(3*a^2*f*(1 + Sin[e + f*x]))
 + (Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(3*f*(a + a*Sin[e +
 f*x])^2)
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2826

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sin[e + f*x
])^p)^FracPart[n])/(d*Sin[e + f*x])^(p*FracPart[n]), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\left(c(d\sin(e+fx))^p\right)^n}{(a+a\sin(e+fx))^2}\,dx = \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int \frac{(d\sin(e+fx))^{np}}{(a+a\sin(e+fx))^2}\,dx$$

$$= \frac{\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{3f(a+a\sin(e+fx))^2} + \frac{\left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx)\right.\right.}{}$$

$$= \frac{2(1-np)\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{3a^2f(1+\sin(e+fx))} + \frac{\cos(e+fx)\sin(e+fx)\left(c\right.}{3f(a+a\sin(e}$$

$$= \frac{2(1-np)\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{3a^2f(1+\sin(e+fx))} + \frac{\cos(e+fx)\sin(e+fx)\left(c\right.}{3f(a+a\sin(e}$$

$$= -\frac{np(1-2np)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(e+fx)\right)\sin(e+fx)\left(c\right.}{3a^2f(1+np)\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 2.90, size = 195, normalized size = 0.68

$$\frac{\sin(e+fx)\cos(e+fx)\left(c(d\sin(e+fx))^p\right)^n\left(-\frac{2\left(n^2p^2-1\right)\sqrt{\cos^2(e+fx)}\tan(e+fx)\sec(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{np}{2}+1;\frac{np}{2}+2;\sin^2(e+fx)\right)}{np+2}+\frac{np(2}{}\right.}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n*((n*p*(-1 + 2*n*p)*Hype
rgeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2])/((1 + n*p)*Sq
rt[Cos[e + f*x]^2]) + (3 - 2*n*p + (2 - 2*n*p)*Sin[e + f*x])/(1 + Sin[e + f
*x])^2 - (2*(-1 + n^2*p^2)*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/2, 1 +
(n*p)/2, 2 + (n*p)/2, Sin[e + f*x]^2]*Sec[e + f*x]*Tan[e + f*x])/(2 + n*p))
)/(3*a^2*f)
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(\left(d\sin\left(fx+e\right)\right)^p c\right)^n}{a^2\cos\left(fx+e\right)^2 - 2\,a^2\sin\left(fx+e\right) - 2\,a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-((d*sin(f*x + e))^p*c)^n/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e)
 - 2*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(((d*sin(f*x + e))^p*c)^n/(a*sin(f*x + e) + a)^2, x)
```

maple [F] time = 1.43, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(d\sin\left(fx+e\right)\right)^{p}\right)^{n}}{\left(a+a\sin\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n/(a+a*sin(f*x+e))^2,x)
```

```
[Out] int((c*(d*sin(f*x+e))^p)^n/(a+a*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(((d*sin(f*x + e))^p*c)^n/(a*sin(f*x + e) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\left(d\,\sin\left(e+f\,x\right)\right)^{p}\right)^{n}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(e + f*x))^p)^n/(a + a*sin(e + f*x))^2,x)
```

```
[Out] int((c*(d*sin(e + f*x))^p)^n/(a + a*sin(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\left(c\left(d\sin\left(e+fx\right)\right)^{p}\right)^{n}}{\sin^{2}\left(e+fx\right)+2\sin\left(e+fx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Integral((c*(d*sin(e + f*x))**p)**n/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),
 x)/a**2
```

3.825 $\int (d\csc(e+fx))^n(a+b\sin(e+fx))^3\,dx$

Optimal. Leaf size=298

$$\frac{bd^4\left(3a^2(3-n)+b^2(2-n)\right)\cos(e+fx)(d\csc(e+fx))^{n-4}\,{}_2F_1\left(\frac{1}{2},\frac{4-n}{2};\frac{6-n}{2};\sin^2(e+fx)\right)}{f(2-n)(4-n)\sqrt{\cos^2(e+fx)}}+\frac{ad^3\left(a^2(2-n)+3b^2(1\right.}{}$$

```
[Out] a^2*b*d^3*(1-2*n)*cot(f*x+e)*(d*csc(f*x+e))^(-3+n)/f/(n^2-3*n+2)+a^2*d^3*co
t(f*x+e)*(d*csc(f*x+e))^(-3+n)*(b+a*csc(f*x+e))/f/(1-n)+a*d^3*(3*b^2*(1-n)+
a^2*(2-n))*cos(f*x+e)*(d*csc(f*x+e))^(-3+n)*hypergeom([1/2, 3/2-1/2*n],[5/2
-1/2*n],sin(f*x+e)^2)/f/(n^2-4*n+3)/(cos(f*x+e)^2)^(1/2)+b*d^4*(b^2*(2-n)+3
*a^2*(3-n))*cos(f*x+e)*(d*csc(f*x+e))^(-4+n)*hypergeom([1/2, 2-1/2*n],[3-1/
2*n],sin(f*x+e)^2)/f/(n^2-6*n+8)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3842, 4047, 3772, 2643, 4046}

$$\frac{bd^4\left(3a^2(3-n)+b^2(2-n)\right)\cos(e+fx)(d\csc(e+fx))^{n-4}\,{}_2F_1\left(\frac{1}{2},\frac{4-n}{2};\frac{6-n}{2};\sin^2(e+fx)\right)}{f(2-n)(4-n)\sqrt{\cos^2(e+fx)}}+\frac{ad^3\left(a^2(2-n)+3b^2(1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (a^2*b*d^3*(1 - 2*n)*Cot[e + f*x]*(d*Csc[e + f*x])^(-3 + n))/(f*(1 - n)*(2
- n)) + (a^2*d^3*Cot[e + f*x]*(d*Csc[e + f*x])^(-3 + n)*(b + a*Csc[e + f*x]
))/(f*(1 - n)) + (a*d^3*(3*b^2*(1 - n) + a^2*(2 - n))*Cos[e + f*x]*(d*Csc[e
 + f*x])^(-3 + n)*Hypergeometric2F1[1/2, (3 - n)/2, (5 - n)/2, Sin[e + f*x]
^2])/(f*(1 - n)*(3 - n)*Sqrt[Cos[e + f*x]^2]) + (b*d^4*(b^2*(2 - n) + 3*a^2
*(3 - n))*Cos[e + f*x]*(d*Csc[e + f*x])^(-4 + n)*Hypergeometric2F1[1/2, (4
- n)/2, (6 - n)/2, Sin[e + f*x]^2])/(f*(2 - n)*(4 - n)*Sqrt[Cos[e + f*x]^2]
)
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
```

```
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int (d\csc(e+fx))^n(a+b\sin(e+fx))^3\,dx = d^3\int (d\csc(e+fx))^{-3+n}(b+a\csc(e+fx))^3\,dx$$
$$= \frac{a^2d^3\cot(e+fx)(d\csc(e+fx))^{-3+n}(b+a\csc(e+fx))}{f(1-n)} - \frac{d^2\int(d\csc(e$$
$$= \frac{a^2d^3\cot(e+fx)(d\csc(e+fx))^{-3+n}(b+a\csc(e+fx))}{f(1-n)} - \frac{d^2\int(d\csc(e$$
$$= \frac{a^2bd^3(1-2n)\cot(e+fx)(d\csc(e+fx))^{-3+n}}{f(1-n)(2-n)} + \frac{a^2d^3\cot(e+fx)(d\csc$$
$$= \frac{a^2bd^3(1-2n)\cot(e+fx)(d\csc(e+fx))^{-3+n}}{f(1-n)(2-n)} + \frac{a^2d^3\cot(e+fx)(d\csc$$
$$= \frac{a^2bd^3(1-2n)\cot(e+fx)(d\csc(e+fx))^{-3+n}}{f(1-n)(2-n)} + \frac{a^2d^3\cot(e+fx)(d\csc$$

Mathematica [A] time = 0.57, size = 167, normalized size = 0.56

$$-\frac{d\cos(e+fx)\sin^2(e+fx)^{\frac{n-1}{2}}(d\csc(e+fx))^{n-1}\left(a^3\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{3}{2};\cos^2(e+fx)\right)+b\sqrt{\sin^2(e+fx)}\csc(e+fx)\left(\right.\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[e + f*x])^n*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] -((d*Cos[e + f*x]*(d*Csc[e + f*x])^(-1 + n)*(Sin[e + f*x]^2)^((-1 + n)/2)*(
3*a*b^2*Hypergeometric2F1[1/2, (-1 + n)/2, 3/2, Cos[e + f*x]^2] + a^3*Hyper
geometric2F1[1/2, (1 + n)/2, 3/2, Cos[e + f*x]^2] + b*Csc[e + f*x]*(b^2*Hyp
ergeometric2F1[1/2, (-2 + n)/2, 3/2, Cos[e + f*x]^2] + 3*a^2*Hypergeometric
2F1[1/2, n/2, 3/2, Cos[e + f*x]^2])*Sqrt[Sin[e + f*x]^2]))/f)
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(3\,ab^2\cos\left(fx+e\right)^2-a^3-3\,ab^2+\left(b^3\cos\left(fx+e\right)^2-3\,a^2b-b^3\right)\sin\left(fx+e\right)\right)\left(d\csc\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 - 3
*a^2*b - b^3)*sin(f*x + e))*(d*csc(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)^{3}\left(d\csc\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+b*sin(f*x+e))^3,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e) + a)^3*(d*csc(f*x + e))^n, x)
```

maple [F] time = 6.32, size = 0, normalized size = 0.00

$$\int \left(d\csc\left(fx+e\right)\right)^{n}\left(a+b\sin\left(fx+e\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n*(a+b*sin(f*x+e))^3,x)
[Out] int((d*csc(f*x+e))^n*(a+b*sin(f*x+e))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)^{3}\left(d\csc\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)^3*(d*csc(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(\frac{d}{\sin\left(e+f\,x\right)}\right)}^n\,{\left(a+b\,\sin\left(e+f\,x\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(e + f*x))^n*(a + b*sin(e + f*x))^3,x)
[Out] int((d/sin(e + f*x))^n*(a + b*sin(e + f*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\csc\left(e+fx\right)\right)^{n}\left(a+b\sin\left(e+fx\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n*(a+b*sin(f*x+e))**3,x)
[Out] Integral((d*csc(e + f*x))**n*(a + b*sin(e + f*x))**3, x)
```

3.826 $\int (d\csc(e+fx))^n (a+b\sin(e+fx))^2\,dx$

Optimal. Leaf size=213

$$\frac{d^3\left(a^2(2-n)+b^2(1-n)\right)\cos(e+fx)(d\csc(e+fx))^{n-3}\,{}_2F_1\left(\frac{1}{2},\frac{3-n}{2};\frac{5-n}{2};\sin^2(e+fx)\right)}{f(1-n)(3-n)\sqrt{\cos^2(e+fx)}}+\frac{a^2d^2\cot(e+fx)(d\csc(e}{f(1-n)}$$

```
[Out] a^2*d^2*cot(f*x+e)*(d*csc(f*x+e))^(-2+n)/f/(1-n)+2*a*b*d^2*cos(f*x+e)*(d*cs
c(f*x+e))^(-2+n)*hypergeom([1/2, 1-1/2*n],[2-1/2*n],sin(f*x+e)^2)/f/(2-n)/(
cos(f*x+e)^2)^(1/2)+d^3*(b^2*(1-n)+a^2*(2-n))*cos(f*x+e)*(d*csc(f*x+e))^(-3
+n)*hypergeom([1/2, 3/2-1/2*n],[5/2-1/2*n],sin(f*x+e)^2)/f/(n^2-4*n+3)/(cos
(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3238, 3788, 3772, 2643, 4046}

$$\frac{d^3\left(a^2(2-n)+b^2(1-n)\right)\cos(e+fx)(d\csc(e+fx))^{n-3}\,{}_2F_1\left(\frac{1}{2},\frac{3-n}{2};\frac{5-n}{2};\sin^2(e+fx)\right)}{f(1-n)(3-n)\sqrt{\cos^2(e+fx)}}+\frac{a^2d^2\cot(e+fx)(d\csc(e}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(-2 + n))/(f*(1 - n)) + (2*a*b*d^2*C
os[e + f*x]*(d*Csc[e + f*x])^(-2 + n)*Hypergeometric2F1[1/2, (2 - n)/2, (4
- n)/2, Sin[e + f*x]^2])/(f*(2 - n)*Sqrt[Cos[e + f*x]^2]) + (d^3*(b^2*(1 -
n) + a^2*(2 - n))*Cos[e + f*x]*(d*Csc[e + f*x])^(-3 + n)*Hypergeometric2F1[
1/2, (3 - n)/2, (5 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*(3 - n)*Sqrt[Cos[e +
 f*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
```

```
!IntegerQ[m] && IntegersQ[n, p]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (d\csc(e+fx))^n(a+b\sin(e+fx))^2\,dx = d^2\int (d\csc(e+fx))^{-2+n}(b+a\csc(e+fx))^2\,dx$$

$$= (2abd)\int (d\csc(e+fx))^{-1+n}\,dx + d^2\int (d\csc(e+fx))^{-2+n}\left(b^2+a^2\,\text{cs}\right.$$

$$= \frac{a^2d^2\cot(e+fx)(d\csc(e+fx))^{-2+n}}{f(1-n)} + \left(d^2\left(b^2+\frac{a^2(2-n)}{1-n}\right)\right)\int (d\,\text{csc}$$

$$= \frac{a^2d^2\cot(e+fx)(d\csc(e+fx))^{-2+n}}{f(1-n)} + \frac{2ab\cos(e+fx)(d\csc(e+fx))}{f(2-}$$

$$= \frac{a^2d^2\cot(e+fx)(d\csc(e+fx))^{-2+n}}{f(1-n)} + \frac{2ab\cos(e+fx)(d\csc(e+fx))}{f(2-}$$

Mathematica [A] time = 0.39, size = 135, normalized size = 0.63

$$-\frac{d\cos(e+fx)\sin^2(e+fx)^{\frac{n-1}{2}}(d\csc(e+fx))^{n-1}\left(a\left(a\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{3}{2};\cos^2(e+fx)\right)+2b\sqrt{\sin^2(e+fx)}\,\csc(e+f\right.\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[e + f*x])^n*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -((d*Cos[e + f*x]*(d*Csc[e + f*x])^(-1 + n)*(Sin[e + f*x]^2)^((-1 + n)/2)*(
b^2*Hypergeometric2F1[1/2, (-1 + n)/2, 3/2, Cos[e + f*x]^2] + a*(a*Hypergeo
metric2F1[1/2, (1 + n)/2, 3/2, Cos[e + f*x]^2] + 2*b*Csc[e + f*x]*Hypergeom
etric2F1[1/2, n/2, 3/2, Cos[e + f*x]^2]*Sqrt[Sin[e + f*x]^2])))/f)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2\right)\left(d\csc\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*(d*csc(f*x
+ e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)+a\right)^2\left(d\csc\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(d*csc(f*x + e))^n, x)
```

maple [F] time = 7.82, size = 0, normalized size = 0.00

$$\int\left(d\csc\left(fx+e\right)\right)^n\left(a+b\sin\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n*(a+b*sin(f*x+e))^2,x)
```

```
[Out] int((d*csc(f*x+e))^n*(a+b*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)+a\right)^2\left(d\csc\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)^2*(d*csc(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{d}{\sin\left(e+f\,x\right)}\right)^n\,\left(a+b\,\sin\left(e+f\,x\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(e + f*x))^n*(a + b*sin(e + f*x))^2,x)
[Out] int((d/sin(e + f*x))^n*(a + b*sin(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \csc{\left(e + f x \right)}\right)^{n} \left(a + b \sin{\left(e + f x \right)}\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n*(a+b*sin(f*x+e))**2,x)
[Out] Integral((d*csc(e + f*x))**n*(a + b*sin(e + f*x))**2, x)
```

3.827 $\int (d\csc(e+fx))^n(a+b\sin(e+fx))\,dx$

Optimal. Leaf size=149

$$\frac{ad\cos(e+fx)(d\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)\sqrt{\cos^2(e+fx)}}+\frac{bd^2\cos(e+fx)(d\csc(e+fx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\right.}{f(2-n)\sqrt{\cos^2(e+fx)}}$$

```
[Out] a*d*cos(f*x+e)*(d*csc(f*x+e))^(-1+n)*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n]
,sin(f*x+e)^2)/f/(1-n)/(cos(f*x+e)^2)^(1/2)+b*d^2*cos(f*x+e)*(d*csc(f*x+e))
^(-2+n)*hypergeom([1/2, 1-1/2*n],[2-1/2*n],sin(f*x+e)^2)/f/(2-n)/(cos(f*x+e
)^2)^(1/2)
```

Rubi [**A**] time = 0.15, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3238, 3787, 3772, 2643}

$$\frac{ad\cos(e+fx)(d\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)\sqrt{\cos^2(e+fx)}}+\frac{bd^2\cos(e+fx)(d\csc(e+fx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\right.}{f(2-n)\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n*(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*d*Cos[e + f*x]*(d*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[1/2, (1 - n)/
2, (3 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*Sqrt[Cos[e + f*x]^2]) + (b*d^2*Co
s[e + f*x]*(d*Csc[e + f*x])^(-2 + n)*Hypergeometric2F1[1/2, (2 - n)/2, (4 -
 n)/2, Sin[e + f*x]^2])/(f*(2 - n)*Sqrt[Cos[e + f*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int (d\csc(e+fx))^n(a+b\sin(e+fx))\,dx &= d\int (d\csc(e+fx))^{-1+n}(b+a\csc(e+fx))\,dx\\ &= a\int (d\csc(e+fx))^n\,dx+(bd)\int (d\csc(e+fx))^{-1+n}\,dx\\ &= \left(a(d\csc(e+fx))^n\left(\frac{\sin(e+fx)}{d}\right)^n\right)\int\left(\frac{\sin(e+fx)}{d}\right)^{-n}dx+\left(bd(d\csc(\right.\\ &= \frac{a\cos(e+fx)(d\csc(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)\sin(e+fx}{f(1-n)\sqrt{\cos^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.23, size = 105, normalized size = 0.70

$$-\frac{d\cos(e+fx)\sin^2(e+fx)^{\frac{n-1}{2}}(d\csc(e+fx))^{n-1}\left(a\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{3}{2};\cos^2(e+fx)\right)+b\sqrt{\sin^2(e+fx)}\csc(e+fx)\,{}_2F\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[e + f*x])^n*(a + b*Sin[e + f*x]),x]
```

```
[Out] -((d*Cos[e + f*x]*(d*Csc[e + f*x])^(-1 + n)*(Sin[e + f*x]^2)^((-1 + n)/2)*(
a*Hypergeometric2F1[1/2, (1 + n)/2, 3/2, Cos[e + f*x]^2] + b*Csc[e + f*x]*H
ypergeometric2F1[1/2, n/2, 3/2, Cos[e + f*x]^2]*Sqrt[Sin[e + f*x]^2]))/f)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(fx+e\right)+a\right)\left(d\csc\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+b*sin(f*x+e)),x, algorithm="fricas")
[Out] integral((b*sin(f*x + e) + a)*(d*csc(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin \left(fx+e\right)+a\right) \left(d \csc \left(fx+e\right)\right)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+b*sin(f*x+e)),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e) + a)*(d*csc(f*x + e))^n, x)
```

maple [F] time = 2.38, size = 0, normalized size = 0.00

$$\int \left(d \csc \left(fx+e\right)\right)^{n} \left(a+b \sin \left(fx+e\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n*(a+b*sin(f*x+e)),x)
[Out] int((d*csc(f*x+e))^n*(a+b*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin \left(fx+e\right)+a\right) \left(d \csc \left(fx+e\right)\right)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n*(a+b*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)*(d*csc(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{d}{\sin\left(e+f\,x\right)}\right)}^n\,\left(a+b\,\sin\left(e+f\,x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(e + f*x))^n*(a + b*sin(e + f*x)),x)
[Out] int((d/sin(e + f*x))^n*(a + b*sin(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\csc\left(e+fx\right)\right)^{n}\left(a+b\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n*(a+b*sin(f*x+e)),x)
```

```
[Out] Integral((d*csc(e + f*x))**n*(a + b*sin(e + f*x)), x)
```

3.828 $\int \frac{(d\csc(e+fx))^n}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=204

$$\frac{b\sin(e+fx)\cos(e+fx)\sin^2(e+fx)^{n/2}(d\csc(e+fx))^{n+1}F_1\left(\frac{1}{2};\frac{n}{2},1;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{df\left(a^2-b^2\right)}-\frac{a\cos(e+f\ldots}{}$$

```
[Out] b*AppellF1(1/2,1/2*n,1,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b^2))*cos(f*
x+e)*(d*csc(f*x+e))^(1+n)*sin(f*x+e)*(sin(f*x+e)^2)^(1/2*n)/(a^2-b^2)/d/f-a
*AppellF1(1/2,1/2+1/2*n,1,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b^2))*cos
(f*x+e)*(d*csc(f*x+e))^(1+n)*(sin(f*x+e)^2)^(1/2+1/2*n)/(a^2-b^2)/d/f
```

Rubi [A] time = 0.40, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3238, 3869, 2823, 3189, 429}

$$\frac{b\sin(e+fx)\cos(e+fx)\sin^2(e+fx)^{n/2}(d\csc(e+fx))^{n+1}F_1\left(\frac{1}{2};\frac{n}{2},1;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{df\left(a^2-b^2\right)}-\frac{a\cos(e+f\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n/(a + b*Sin[e + f*x]),x]
```

```
[Out] (b*AppellF1[1/2, n/2, 1, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 -
 b^2))]*Cos[e + f*x]*(d*Csc[e + f*x])^(1 + n)*Sin[e + f*x]*(Sin[e + f*x]^2)
^(n/2))/((a^2 - b^2)*d*f) - (a*AppellF1[1/2, (1 + n)/2, 1, 3/2, Cos[e + f*x
]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Csc[e + f*x])^(1
+ n)*(Sin[e + f*x]^2)^((1 + n)/2))/((a^2 - b^2)*d*f)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3869

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d\csc(e+fx))^n}{a+b\sin(e+fx)}\,dx = \frac{\int \frac{(d\csc(e+fx))^{1+n}}{b+a\csc(e+fx)}\,dx}{d}$$

$$= \frac{\left((d\csc(e+fx))^{1+n}\sin^{1+n}(e+fx)\right)\int \frac{\sin^{-n}(e+fx)}{a+b\sin(e+fx)}\,dx}{d}$$

$$= \frac{\left(a(d\csc(e+fx))^{1+n}\sin^{1+n}(e+fx)\right)\int \frac{\sin^{-n}(e+fx)}{a^2-b^2\sin^2(e+fx)}\,dx}{d} - \frac{\left(b(d\csc(e+fx))^{1+n}\sin^{1+n}(e\right.}{d}$$

$$= -\frac{\left(a(d\csc(e+fx))^{1+n}\sin^{1+2\left(-\frac{1}{2}-\frac{n}{2}\right)+n}(e+fx)\sin^2(e+fx)^{\frac{1}{2}+\frac{n}{2}}\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{\frac{1}{2}(-1-n)}}{a^2-b^2+b^2x^2}\,d\right.}{df}$$

$$= \frac{bF_1\left(\frac{1}{2};\frac{n}{2},1;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\csc(e+fx))^{1+n}\sin(e+fx)\sin}{\left(a^2-b^2\right)df}$$

Mathematica [B] time = 16.85, size = 1665, normalized size = 8.16

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Csc[e + f*x])^n/(a + b*Sin[e + f*x]),x]
```

```
[Out] -(((d*Csc[e + f*x])^n*(Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^n*Tan[e + f*x]*(a
*b*(-2 + n)*AppellF1[(1 - n)/2, -1/2*n, 1, (3 - n)/2, -Tan[e + f*x]^2, ((-a
^2 + b^2)*Tan[e + f*x]^2)/a^2] + (-1 + n)*((a^2 - b^2)*AppellF1[1 - n/2, (-
1 - n)/2, 1, 2 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - a^2
*Hypergeometric2F1[1/2 - n/2, 1 - n/2, 2 - n/2, -Tan[e + f*x]^2])*Tan[e + f
*x]))/(a^2*b*f*(-2 + n)*(-1 + n)*(Sec[e + f*x]^2)^(n/2)*(a + b*Sin[e + f*x]
)*(-(((Sec[e + f*x]^2)^(1 - n/2)*(Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^n*(a*b
*(-2 + n)*AppellF1[(1 - n)/2, -1/2*n, 1, (3 - n)/2, -Tan[e + f*x]^2, ((-a^2
 + b^2)*Tan[e + f*x]^2)/a^2] + (-1 + n)*((a^2 - b^2)*AppellF1[1 - n/2, (-1
- n)/2, 1, 2 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - a^2*H
ypergeometric2F1[1/2 - n/2, 1 - n/2, 2 - n/2, -Tan[e + f*x]^2])*Tan[e + f*x
]))/(a^2*b*(-2 + n)*(-1 + n))) - (n*(Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^(-1
 + n)*(Sqrt[Sec[e + f*x]^2] - Csc[e + f*x]^2*Sqrt[Sec[e + f*x]^2])*Tan[e +
f*x]*(a*b*(-2 + n)*AppellF1[(1 - n)/2, -1/2*n, 1, (3 - n)/2, -Tan[e + f*x]^
2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] + (-1 + n)*((a^2 - b^2)*AppellF1[1 -
n/2, (-1 - n)/2, 1, 2 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2
] - a^2*Hypergeometric2F1[1/2 - n/2, 1 - n/2, 2 - n/2, -Tan[e + f*x]^2])*Ta
n[e + f*x]))/(a^2*b*(-2 + n)*(-1 + n)*(Sec[e + f*x]^2)^(n/2)) + (n*(Cot[e +
 f*x]*Sqrt[Sec[e + f*x]^2])^n*Tan[e + f*x]^2*(a*b*(-2 + n)*AppellF1[(1 - n)
/2, -1/2*n, 1, (3 - n)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^
2] + (-1 + n)*((a^2 - b^2)*AppellF1[1 - n/2, (-1 - n)/2, 1, 2 - n/2, -Tan[e
 + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - a^2*Hypergeometric2F1[1/2 - n/2
, 1 - n/2, 2 - n/2, -Tan[e + f*x]^2])*Tan[e + f*x]))/(a^2*b*(-2 + n)*(-1 +
n)*(Sec[e + f*x]^2)^(n/2)) - ((Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^n*Tan[e +
 f*x]*((-1 + n)*((a^2 - b^2)*AppellF1[1 - n/2, (-1 - n)/2, 1, 2 - n/2, -Tan
[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - a^2*Hypergeometric2F1[1/2 - n
/2, 1 - n/2, 2 - n/2, -Tan[e + f*x]^2])*Sec[e + f*x]^2 + a*b*(-2 + n)*(((1
- n)*n*AppellF1[1 + (1 - n)/2, 1 - n/2, 1, 1 + (3 - n)/2, -Tan[e + f*x]^2,
((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 - n) + (
2*(-a^2 + b^2)*(1 - n)*AppellF1[1 + (1 - n)/2, -1/2*n, 2, 1 + (3 - n)/2, -T
an[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*
x])/(a^2*(3 - n))) + (-1 + n)*Tan[e + f*x]*((a^2 - b^2)*(-(((-1 - n)*(1 - n
/2)*AppellF1[2 - n/2, 1 + (-1 - n)/2, 1, 3 - n/2, -Tan[e + f*x]^2, (-1 + b^
2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(2 - n/2)) + (2*(-1 + b
^2/a^2)*(1 - n/2)*AppellF1[2 - n/2, (-1 - n)/2, 2, 3 - n/2, -Tan[e + f*x]^2
, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(2 - n/2)) -
2*a^2*(1 - n/2)*Csc[e + f*x]*Sec[e + f*x]*(-Hypergeometric2F1[1/2 - n/2, 1
```

```
- n/2, 2 - n/2, -Tan[e + f*x]^2] + (1 + Tan[e + f*x]^2)^(-1/2 + n/2)))))/(a
^2*b*(-2 + n)*(-1 + n)*(Sec[e + f*x]^2)^(n/2)))))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d\csc\left(fx+e\right)\right)^{n}}{b\sin\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+b*sin(f*x+e)),x, algorithm="fricas")
[Out] integral((d*csc(f*x + e))^n/(b*sin(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(d\csc\left(fx+e\right)\right)^{n}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+b*sin(f*x+e)),x, algorithm="giac")
[Out] integrate((d*csc(f*x + e))^n/(b*sin(f*x + e) + a), x)
```

maple [F] time = 0.88, size = 0, normalized size = 0.00

$$\int\frac{\left(d\csc\left(fx+e\right)\right)^{n}}{a+b\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n/(a+b*sin(f*x+e)),x)
[Out] int((d*csc(f*x+e))^n/(a+b*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(d\csc\left(fx+e\right)\right)^{n}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(f*x + e))^n/(b*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{d}{\sin\left(e+f\,x\right)}\right)^n}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(e + f*x))^n/(a + b*sin(e + f*x)),x)
[Out] int((d/sin(e + f*x))^n/(a + b*sin(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\csc\left(e+fx\right)\right)^n}{a+b\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n/(a+b*sin(f*x+e)),x)
[Out] Integral((d*csc(e + f*x))**n/(a + b*sin(e + f*x)), x)
```

3.829 $\int \frac{(d\csc(e+fx))^n}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=321

$$-\frac{a^2\sin(e+fx)\cos(e+fx)\sin^2(e+fx)^{\frac{n+1}{2}}(d\csc(e+fx))^{n+2}F_1\left(\frac{1}{2};\frac{n+1}{2},2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{d^2f\left(a^2-b^2\right)^2}+\frac{2ab\cos\cdots}{\cdots}$$

```
[Out] -b^2*AppellF1(1/2,-1/2+1/2*n,2,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b^2)
)*cos(f*x+e)*(d*csc(f*x+e))^(2+n)*sin(f*x+e)^3*(sin(f*x+e)^2)^(-1/2+1/2*n)/
(a^2-b^2)^2/d^2/f-a^2*AppellF1(1/2,1/2+1/2*n,2,3/2,cos(f*x+e)^2,-b^2*cos(f*
x+e)^2/(a^2-b^2))*cos(f*x+e)*(d*csc(f*x+e))^(2+n)*sin(f*x+e)*(sin(f*x+e)^2)
^(1/2+1/2*n)/(a^2-b^2)^2/d^2/f+2*a*b*AppellF1(1/2,1/2*n,2,3/2,cos(f*x+e)^2,
-b^2*cos(f*x+e)^2/(a^2-b^2))*cos(f*x+e)*(d*csc(f*x+e))^(2+n)*(sin(f*x+e)^2)
^(1+1/2*n)/(a^2-b^2)^2/d^2/f
```

Rubi [A] time = 0.54, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3238, 3869, 2824, 3189, 429}

$$-\frac{b^2\sin^3(e+fx)\cos(e+fx)\sin^2(e+fx)^{\frac{n-1}{2}}(d\csc(e+fx))^{n+2}F_1\left(\frac{1}{2};\frac{n-1}{2},2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{d^2f\left(a^2-b^2\right)^2}-\frac{a^2\sin\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -((b^2*AppellF1[1/2, (-1 + n)/2, 2, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x
]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Csc[e + f*x])^(2 + n)*Sin[e + f*x]^3*(Si
n[e + f*x]^2)^((-1 + n)/2))/((a^2 - b^2)^2*d^2*f)) - (a^2*AppellF1[1/2, (1
+ n)/2, 2, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e
+ f*x]*(d*Csc[e + f*x])^(2 + n)*Sin[e + f*x]*(Sin[e + f*x]^2)^((1 + n)/2))/
((a^2 - b^2)^2*d^2*f) + (2*a*b*AppellF1[1/2, n/2, 2, 3/2, Cos[e + f*x]^2, -
((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Csc[e + f*x])^(2 + n)*(
Sin[e + f*x]^2)^((2 + n)/2))/((a^2 - b^2)^2*d^2*f)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2824

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n/((a - b*sin[e +
 f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3869

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d\csc(e+fx))^n}{(a+b\sin(e+fx))^2}\,dx = \frac{\int \frac{(d\csc(e+fx))^{2+n}}{(b+a\csc(e+fx))^2}\,dx}{d^2}$$

$$= \frac{\left((d\csc(e+fx))^{2+n}\sin^{2+n}(e+fx)\right)\int \frac{\sin^{-n}(e+fx)}{(a+b\sin(e+fx))^2}\,dx}{d^2}$$

$$= \frac{\left((d\csc(e+fx))^{2+n}\sin^{2+n}(e+fx)\right)\int \left(-\frac{2ab\sin^{1-n}(e+fx)}{\left(a^2-b^2\sin^2(e+fx)\right)^2}+\frac{a^2\sin^{-n}(e+fx)}{\left(a^2-b^2\sin^2(e+fx)\right)^2}+\frac{b^2\sin}{\left(-a^2+b^2\right.}\right.}{d^2}$$

$$= \frac{\left(a^2(d\csc(e+fx))^{2+n}\sin^{2+n}(e+fx)\right)\int \frac{\sin^{-n}(e+fx)}{\left(a^2-b^2\sin^2(e+fx)\right)^2}\,dx}{d^2} - \frac{\left(2ab(d\csc(e+fx))^{2+n}\right.}{}$$

$$= -\frac{\left(b^2(d\csc(e+fx))^{2+n}\sin^{2+2\left(\frac{1}{2}-\frac{n}{2}\right)+n}(e+fx)\sin^2(e+fx)^{-\frac{1}{2}+\frac{n}{2}}\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{\frac{1-n}{2}}}{\left(-a^2+b^2-b^2\right.}\right.}{d^2 f}$$

$$= -\frac{b^2F_1\left(\frac{1}{2};\frac{1}{2}(-1+n),2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\csc(e+fx))^{2+n}\,\mathrm{s}}{\left(a^2-b^2\right)^2 d^2 f}$$

Mathematica [B] time = 19.06, size = 1872, normalized size = 5.83

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Csc[e + f*x])^n/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((d*Csc[e + f*x])^n*(Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^n*Tan[e + f*x]*(-(a
*(a^2 + b^2)*(-2 + n)*AppellF1[(1 - n)/2, -1/2*n, 1, (3 - n)/2, -Tan[e + f*
x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]) + 2*b*(a*b*(-2 + n)*AppellF1[(1 - n)/
2, -1/2*n, 2, (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] +
(a^2 - b^2)*(-1 + n)*AppellF1[1 - n/2, (-1 - n)/2, 2, 2 - n/2, -Tan[e + f*x
]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x])))/(a^3*(a^2 - b^2)*f*(-2
+ n)*(-1 + n)*(Sec[e + f*x]^2)^(n/2)*(a + b*Sin[e + f*x])^2*(((Sec[e + f*x]
^2)^(1 - n/2)*(Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^n*(-(a*(a^2 + b^2)*(-2 +
n)*AppellF1[(1 - n)/2, -1/2*n, 1, (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2
)*Tan[e + f*x]^2]) + 2*b*(a*b*(-2 + n)*AppellF1[(1 - n)/2, -1/2*n, 2, (3 -
n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + (a^2 - b^2)*(-1 + n
)*AppellF1[1 - n/2, (-1 - n)/2, 2, 2 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)
*Tan[e + f*x]^2]*Tan[e + f*x])))/(a^3*(a^2 - b^2)*(-2 + n)*(-1 + n)) + (n*(
```

```
Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^(-1 + n)*(Sqrt[Sec[e + f*x]^2] - Csc[e +
 f*x]^2*Sqrt[Sec[e + f*x]^2])*Tan[e + f*x]*(-(a*(a^2 + b^2)*(-2 + n)*Appell
F1[(1 - n)/2, -1/2*n, 1, (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e +
 f*x]^2]) + 2*b*(a*b*(-2 + n)*AppellF1[(1 - n)/2, -1/2*n, 2, (3 - n)/2, -Ta
n[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + (a^2 - b^2)*(-1 + n)*AppellF
1[1 - n/2, (-1 - n)/2, 2, 2 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e +
f*x]^2]*Tan[e + f*x])))/(a^3*(a^2 - b^2)*(-2 + n)*(-1 + n)*(Sec[e + f*x]^2)
^(n/2)) - (n*(Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^n*Tan[e + f*x]^2*(-(a*(a^2
 + b^2)*(-2 + n)*AppellF1[(1 - n)/2, -1/2*n, 1, (3 - n)/2, -Tan[e + f*x]^2,
 (-1 + b^2/a^2)*Tan[e + f*x]^2]) + 2*b*(a*b*(-2 + n)*AppellF1[(1 - n)/2, -1
/2*n, 2, (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + (a^2
- b^2)*(-1 + n)*AppellF1[1 - n/2, (-1 - n)/2, 2, 2 - n/2, -Tan[e + f*x]^2,
(-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x])))/(a^3*(a^2 - b^2)*(-2 + n)*(-
1 + n)*(Sec[e + f*x]^2)^(n/2)) + ((Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^n*Tan
[e + f*x]*(-(a*(a^2 + b^2)*(-2 + n)*(((1 - n)*n*AppellF1[1 + (1 - n)/2, 1 -
 n/2, 1, 1 + (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec
[e + f*x]^2*Tan[e + f*x])/(3 - n) + (2*(-1 + b^2/a^2)*(1 - n)*AppellF1[1 +
(1 - n)/2, -1/2*n, 2, 1 + (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e
+ f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 - n))) + 2*b*((a^2 - b^2)*(-1 + n
)*AppellF1[1 - n/2, (-1 - n)/2, 2, 2 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)
*Tan[e + f*x]^2]*Sec[e + f*x]^2 + a*b*(-2 + n)*(((1 - n)*n*AppellF1[1 + (1
- n)/2, 1 - n/2, 2, 1 + (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e +
f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 - n) + (4*(-1 + b^2/a^2)*(1 - n)*Ap
pellF1[1 + (1 - n)/2, -1/2*n, 3, 1 + (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^2/
a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 - n)) + (a^2 - b^2)*(-
1 + n)*Tan[e + f*x]*(-(((-1 - n)*(1 - n/2)*AppellF1[2 - n/2, 1 + (-1 - n)/2
, 2, 3 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^
2*Tan[e + f*x])/(2 - n/2)) + (4*(-1 + b^2/a^2)*(1 - n/2)*AppellF1[2 - n/2,
(-1 - n)/2, 3, 3 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec
[e + f*x]^2*Tan[e + f*x])/(2 - n/2)))))/(a^3*(a^2 - b^2)*(-2 + n)*(-1 + n)*
(Sec[e + f*x]^2)^(n/2))))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(d\csc\left(fx+e\right)\right)^{n}}{b^{2}\cos\left(fx+e\right)^{2}-2\,ab\sin\left(fx+e\right)-a^{2}-b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(d*csc(f*x + e))^n/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2
 - b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \csc\left(fx+e\right)\right)^{n}}{\left(b \sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+b*sin(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*csc(f*x + e))^n/(b*sin(f*x + e) + a)^2, x)
```

maple [F] time = 1.92, size = 0, normalized size = 0.00

$$\int \frac{\left(d \csc\left(fx+e\right)\right)^{n}}{\left(a+b \sin\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n/(a+b*sin(f*x+e))^2,x)
[Out] int((d*csc(f*x+e))^n/(a+b*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \csc\left(fx+e\right)\right)^{n}}{\left(b \sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*csc(f*x + e))^n/(b*sin(f*x + e) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{d}{\sin\left(e+f\,x\right)}\right)^{n}}{\left(a+b\,\sin\left(e+f\,x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(e + f*x))^n/(a + b*sin(e + f*x))^2,x)
[Out] int((d/sin(e + f*x))^n/(a + b*sin(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\csc\left(e+fx\right)\right)^{n}}{\left(a+b\sin\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Integral((d*csc(e + f*x))**n/(a + b*sin(e + f*x))**2, x)
```

3.830 $\int \frac{(d\csc(e+fx))^n}{(a+b\sin(e+fx))^3}\,dx$

Optimal. Leaf size=432

$$-\frac{3ab^2\sin^4(e+fx)\cos(e+fx)\sin^2(e+fx)^{\frac{n-1}{2}}(d\csc(e+fx))^{n+3}F_1\left(\frac{1}{2};\frac{n-1}{2},3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{d^3f\left(a^2-b^2\right)^3}+\frac{3a^2\cdots}{\cdots}$$

```
[Out] -3*a*b^2*AppellF1(1/2,-1/2+1/2*n,3,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-
b^2))*cos(f*x+e)*(d*csc(f*x+e))^(3+n)*sin(f*x+e)^4*(sin(f*x+e)^2)^(-1/2+1/2
*n)/(a^2-b^2)^3/d^3/f+b^3*AppellF1(1/2,-1+1/2*n,3,3/2,cos(f*x+e)^2,-b^2*cos
(f*x+e)^2/(a^2-b^2))*cos(f*x+e)*(d*csc(f*x+e))^(3+n)*sin(f*x+e)^3*(sin(f*x+
e)^2)^(1/2*n)/(a^2-b^2)^3/d^3/f+3*a^2*b*AppellF1(1/2,1/2*n,3,3/2,cos(f*x+e)
^2,-b^2*cos(f*x+e)^2/(a^2-b^2))*cos(f*x+e)*(d*csc(f*x+e))^(3+n)*sin(f*x+e)^
3*(sin(f*x+e)^2)^(1/2*n)/(a^2-b^2)^3/d^3/f-a^3*AppellF1(1/2,1/2+1/2*n,3,3/2
,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b^2))*cos(f*x+e)*(d*csc(f*x+e))^(3+n)*
(sin(f*x+e)^2)^(3/2+1/2*n)/(a^2-b^2)^3/d^3/f
```

Rubi [A] time = 0.70, antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3238, 3869, 2824, 3189, 429}

$$-\frac{3ab^2\sin^4(e+fx)\cos(e+fx)\sin^2(e+fx)^{\frac{n-1}{2}}(d\csc(e+fx))^{n+3}F_1\left(\frac{1}{2};\frac{n-1}{2},3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{d^3f\left(a^2-b^2\right)^3}+\frac{b^3\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^n/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (-3*a*b^2*AppellF1[1/2, (-1 + n)/2, 3, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e +
f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Csc[e + f*x])^(3 + n)*Sin[e + f*x]^4*
(Sin[e + f*x]^2)^((-1 + n)/2))/((a^2 - b^2)^3*d^3*f) + (b^3*AppellF1[1/2, (
-2 + n)/2, 3, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos
[e + f*x]*(d*Csc[e + f*x])^(3 + n)*Sin[e + f*x]^3*(Sin[e + f*x]^2)^(n/2))/(
(a^2 - b^2)^3*d^3*f) + (3*a^2*b*AppellF1[1/2, n/2, 3, 3/2, Cos[e + f*x]^2,
-((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(d*Csc[e + f*x])^(3 + n)*
Sin[e + f*x]^3*(Sin[e + f*x]^2)^(n/2))/((a^2 - b^2)^3*d^3*f) - (a^3*AppellF
1[1/2, (1 + n)/2, 3, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2
))]*Cos[e + f*x]*(d*Csc[e + f*x])^(3 + n)*(Sin[e + f*x]^2)^((3 + n)/2))/((a
^2 - b^2)^3*d^3*f)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2824

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n/((a - b*sin[e +
 f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3869

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d\csc(e+fx))^n}{(a+b\sin(e+fx))^3}\,dx = \frac{\int \frac{(d\csc(e+fx))^{3+n}}{(b+a\csc(e+fx))^3}\,dx}{d^3}$$

$$= \frac{\left((d\csc(e+fx))^{3+n}\sin^{3+n}(e+fx)\right)\int \frac{\sin^{-n}(e+fx)}{(a+b\sin(e+fx))^3}\,dx}{d^3}$$

$$= \frac{\left((d\csc(e+fx))^{3+n}\sin^{3+n}(e+fx)\right)\int \left(-\frac{3a^2b\sin^{1-n}(e+fx)}{\left(a^2-b^2\sin^2(e+fx)\right)^3} + \frac{3ab^2\sin^{2-n}(e+fx)}{\left(a^2-b^2\sin^2(e+fx)\right)^3} + \frac{a^3\sin}{\left(a^2-b^2 s\right.}\right.}{d^3}$$

$$= \frac{\left(a^3(d\csc(e+fx))^{3+n}\sin^{3+n}(e+fx)\right)\int \frac{\sin^{-n}(e+fx)}{\left(a^2-b^2\sin^2(e+fx)\right)^3}\,dx}{d^3} - \frac{\left(3a^2b(d\csc(e+fx))^{3+}\right.}{}$$

$$= -\frac{\left(3ab^2(d\csc(e+fx))^{3+n}\sin^{3+2\left(\frac{1}{2}-\frac{n}{2}\right)+n}(e+fx)\sin^2(e+fx)^{-\frac{1}{2}+\frac{n}{2}}\right)\text{Subst}\left(\int \frac{(1-x^2)}{\left(a^2-b^2+b\right.}\right.}{d^3f}$$

$$= -\frac{3ab^2F_1\left(\frac{1}{2};\frac{1}{2}(-1+n),3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\csc(e+fx))^{3+}}{\left(a^2-b^2\right)^3d^3f}$$

Mathematica [**B**] time = 20.22, size = 2406, normalized size = 5.57

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Csc[e + f*x])^n/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] ((d*Csc[e + f*x])^n*(Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^n*Tan[e + f*x]*(-(a
*(a^2 + 3*b^2)*(-2 + n)*AppellF1[(1 - n)/2, -1 - n/2, 2, (3 - n)/2, -Tan[e
+ f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]) + b*(4*a*b*(-2 + n)*AppellF1[(1 -
 n)/2, -1 - n/2, 3, (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]
^2] + (-1 + n)*((3*a^2 + b^2)*AppellF1[1 - n/2, (-1 - n)/2, 2, 2 - n/2, -Ta
n[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - 4*b^2*AppellF1[1 - n/2, (-1
- n)/2, 3, 2 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2])*Tan[e
+ f*x])))/(a^4*(a^2 - b^2)*f*(-2 + n)*(-1 + n)*(Sec[e + f*x]^2)^(n/2)*(a +
b*Sin[e + f*x])^3*(((Sec[e + f*x]^2)^(1 - n/2)*(Cot[e + f*x]*Sqrt[Sec[e + f
*x]^2])^n*(-(a*(a^2 + 3*b^2)*(-2 + n)*AppellF1[(1 - n)/2, -1 - n/2, 2, (3 -
 n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]) + b*(4*a*b*(-2 + n)
*AppellF1[(1 - n)/2, -1 - n/2, 3, (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2
)*Tan[e + f*x]^2] + (-1 + n)*((3*a^2 + b^2)*AppellF1[1 - n/2, (-1 - n)/2, 2
```

, 2 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - 4*b^2*AppellF1
[1 - n/2, (-1 - n)/2, 3, 2 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f
*x]^2])*Tan[e + f*x])))/(a^4*(a^2 - b^2)*(-2 + n)*(-1 + n)) + (n*(Cot[e + f
*x]*Sqrt[Sec[e + f*x]^2])^(-1 + n)*(Sqrt[Sec[e + f*x]^2] - Csc[e + f*x]^2*S
qrt[Sec[e + f*x]^2])*Tan[e + f*x]*(-(a*(a^2 + 3*b^2)*(-2 + n)*AppellF1[(1 -
 n)/2, -1 - n/2, 2, (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]
^2]) + b*(4*a*b*(-2 + n)*AppellF1[(1 - n)/2, -1 - n/2, 3, (3 - n)/2, -Tan[e
 + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + (-1 + n)*((3*a^2 + b^2)*AppellF
1[1 - n/2, (-1 - n)/2, 2, 2 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e +
f*x]^2] - 4*b^2*AppellF1[1 - n/2, (-1 - n)/2, 3, 2 - n/2, -Tan[e + f*x]^2,
(-1 + b^2/a^2)*Tan[e + f*x]^2])*Tan[e + f*x])))/(a^4*(a^2 - b^2)*(-2 + n)*(
-1 + n)*(Sec[e + f*x]^2)^(n/2)) - (n*(Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^n*
Tan[e + f*x]^2*(-(a*(a^2 + 3*b^2)*(-2 + n)*AppellF1[(1 - n)/2, -1 - n/2, 2,
 (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]) + b*(4*a*b*(-2
 + n)*AppellF1[(1 - n)/2, -1 - n/2, 3, (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^
2/a^2)*Tan[e + f*x]^2] + (-1 + n)*((3*a^2 + b^2)*AppellF1[1 - n/2, (-1 - n)
/2, 2, 2 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - 4*b^2*App
ellF1[1 - n/2, (-1 - n)/2, 3, 2 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[
e + f*x]^2])*Tan[e + f*x])))/(a^4*(a^2 - b^2)*(-2 + n)*(-1 + n)*(Sec[e + f*
x]^2)^(n/2)) + ((Cot[e + f*x]*Sqrt[Sec[e + f*x]^2])^n*Tan[e + f*x]*(-(a*(a^
2 + 3*b^2)*(-2 + n)*((4*(-1 + b^2/a^2)*(1 - n)*AppellF1[1 + (1 - n)/2, -1 -
 n/2, 3, 1 + (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec
[e + f*x]^2*Tan[e + f*x])/(3 - n) - (2*(1 - n)*(-1 - n/2)*AppellF1[1 + (1 -
 n)/2, -1/2*n, 2, 1 + (3 - n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*
x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 - n))) + b*((-1 + n)*((3*a^2 + b^2)*A
ppellF1[1 - n/2, (-1 - n)/2, 2, 2 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Ta
n[e + f*x]^2] - 4*b^2*AppellF1[1 - n/2, (-1 - n)/2, 3, 2 - n/2, -Tan[e + f*
x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2])*Sec[e + f*x]^2 + 4*a*b*(-2 + n)*((6*(
-1 + b^2/a^2)*(1 - n)*AppellF1[1 + (1 - n)/2, -1 - n/2, 4, 1 + (3 - n)/2, -
Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])
/(3 - n) - (2*(1 - n)*(-1 - n/2)*AppellF1[1 + (1 - n)/2, -1/2*n, 3, 1 + (3
- n)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[
e + f*x])/(3 - n)) + (-1 + n)*Tan[e + f*x]*((3*a^2 + b^2)*(-(((-1 - n)*(1 -
 n/2)*AppellF1[2 - n/2, 1 + (-1 - n)/2, 2, 3 - n/2, -Tan[e + f*x]^2, (-1 +
b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(2 - n/2)) + (4*(-1 +
 b^2/a^2)*(1 - n/2)*AppellF1[2 - n/2, (-1 - n)/2, 3, 3 - n/2, -Tan[e + f*x]
^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(2 - n/2))
- 4*b^2*(-(((-1 - n)*(1 - n/2)*AppellF1[2 - n/2, 1 + (-1 - n)/2, 3, 3 - n/2
, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*
x])/(2 - n/2)) + (6*(-1 + b^2/a^2)*(1 - n/2)*AppellF1[2 - n/2, (-1 - n)/2,
4, 3 - n/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*
Tan[e + f*x])/(2 - n/2))))))/(a^4*(a^2 - b^2)*(-2 + n)*(-1 + n)*(Sec[e + f*
x]^2)^(n/2))))

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(d\csc\left(fx+e\right)\right)^{n}}{3\,ab^{2}\cos\left(fx+e\right)^{2}-a^{3}-3\,ab^{2}+\left(b^{3}\cos\left(fx+e\right)^{2}-3\,a^{2}b-b^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(d*csc(f*x + e))^n/(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3
*cos(f*x + e)^2 - 3*a^2*b - b^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\csc\left(fx+e\right)\right)^{n}}{\left(b\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*csc(f*x + e))^n/(b*sin(f*x + e) + a)^3, x)
```

maple [F] time = 2.06, size = 0, normalized size = 0.00

$$\int \frac{\left(d\csc\left(fx+e\right)\right)^{n}}{\left(a+b\sin\left(fx+e\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^n/(a+b*sin(f*x+e))^3,x)
```

```
[Out] int((d*csc(f*x+e))^n/(a+b*sin(f*x+e))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\csc\left(fx+e\right)\right)^{n}}{\left(b\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^n/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((d*csc(f*x + e))^n/(b*sin(f*x + e) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{d}{\sin(e+f\,x)}\right)^n}{\left(a+b\,\sin\left(e+f\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(e + f*x))^n/(a + b*sin(e + f*x))^3,x)
[Out] int((d/sin(e + f*x))^n/(a + b*sin(e + f*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\csc\left(e+fx\right)\right)^n}{\left(a+b\sin\left(e+fx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**n/(a+b*sin(f*x+e))**3,x)
[Out] Integral((d*csc(e + f*x))**n/(a + b*sin(e + f*x))**3, x)
```

3.831 $\int \left(c(d\sin(e+fx))^p\right)^n (a+b\sin(e+fx))^m\,dx$

Optimal. Leaf size=56

$$(d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n \operatorname{Int}\left((a+b\sin(e+fx))^m(d\sin(e+fx))^{np},x\right)$$

```
[Out] (c*(d*sin(f*x+e))^p)^n*Unintegrable((d*sin(f*x+e))^(n*p)*(a+b*sin(f*x+e))^m
,x)/((d*sin(f*x+e))^(n*p))
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(c(d\sin(e+fx))^p\right)^n (a+b\sin(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n*(a + b*Sin[e + f*x])^m,x]
```

```
[Out] ((c*(d*Sin[e + f*x])^p)^n*Defer[Int][(d*Sin[e + f*x])^(n*p)*(a + b*Sin[e +
f*x])^m, x])/(d*Sin[e + f*x])^(n*p)
```

Rubi steps

$$\int \left(c(d\sin(e+fx))^p\right)^n (a+b\sin(e+fx))^m\,dx = \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int (d\sin(e+fx))^{np}(a+$$

Mathematica [A] time = 2.57, size = 0, normalized size = 0.00

$$\int \left(c(d\sin(e+fx))^p\right)^n (a+b\sin(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n*(a + b*Sin[e + f*x])^m,x]
```

```
[Out] Integrate[(c*(d*Sin[e + f*x])^p)^n*(a + b*Sin[e + f*x])^m, x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(d\sin\left(fx+e\right)\right)^p c\right)^n \left(b\sin\left(fx+e\right)+a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral(((d*sin(f*x + e))^p*c)^n*(b*sin(f*x + e) + a)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}\left(b\sin\left(fx+e\right)+a\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate(((d*sin(f*x + e))^p*c)^n*(b*sin(f*x + e) + a)^m, x)
```

maple [A] time = 0.36, size = 0, normalized size = 0.00

$$\int \left(c\left(d\sin\left(fx+e\right)\right)^{p}\right)^{n}\left(a+b\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^m,x)
[Out] int((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^m,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}\left(b\sin\left(fx+e\right)+a\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate(((d*sin(f*x + e))^p*c)^n*(b*sin(f*x + e) + a)^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(c\,{\left(d\,\sin\left(e+f\,x\right)\right)}^p\right)}^n\,{\left(a+b\,\sin\left(e+f\,x\right)\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(e + f*x))^p)^n*(a + b*sin(e + f*x))^m,x)
[Out] int((c*(d*sin(e + f*x))^p)^n*(a + b*sin(e + f*x))^m, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\left(d\sin\left(e+fx\right)\right)^{p}\right)^{n}\left(a+b\sin\left(e+fx\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n*(a+b*sin(f*x+e))**m,x)
```

```
[Out] Integral((c*(d*sin(e + f*x))**p)**n*(a + b*sin(e + f*x))**m, x)
```

3.832 $\int \left(c(d\sin(e+fx))^p\right)^n (a+b\sin(e+fx))^3\,dx$

Optimal. Leaf size=323

$$\frac{b\left(3a^2(np+3)+b^2(np+2)\right)\sin^2(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+2);\frac{1}{2}(np+4);\sin^2(e+fx)\right)\left(c(d\sin(e+fx\right.}{f(np+2)(np+3)\sqrt{\cos^2(e+fx)}}$$

```
[Out] -a*b^2*(2*n*p+7)*cos(f*x+e)*sin(f*x+e)*(c*(d*sin(f*x+e))^p)^n/f/(n*p+2)/(n*
p+3)-b^2*cos(f*x+e)*sin(f*x+e)*(c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))/f/(n
*p+3)+a*(3*b^2*(n*p+1)+a^2*(n*p+2))*cos(f*x+e)*hypergeom([1/2, 1/2*n*p+1/2]
,[1/2*n*p+3/2],sin(f*x+e)^2)*sin(f*x+e)*(c*(d*sin(f*x+e))^p)^n/f/(n*p+1)/(n
*p+2)/(cos(f*x+e)^2)^(1/2)+b*(b^2*(n*p+2)+3*a^2*(n*p+3))*cos(f*x+e)*hyperge
om([1/2, 1/2*n*p+1],[1/2*n*p+2],sin(f*x+e)^2)*sin(f*x+e)^2*(c*(d*sin(f*x+e)
)^p)^n/f/(n*p+2)/(n*p+3)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 303, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2826, 2793, 3023, 2748, 2643}

$$\frac{b\left(\frac{3a^2}{np+2}+\frac{b^2}{np+3}\right)\sin^2(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+2);\frac{1}{2}(np+4);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{f\sqrt{\cos^2(e+fx)}}+\frac{a\left(\frac{a^2}{np+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] -((a*b^2*(7 + 2*n*p)*Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f
*(2 + n*p)*(3 + n*p))) + (a*(a^2/(1 + n*p) + (3*b^2)/(2 + n*p))*Cos[e + f*x
]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2]*Sin[e +
f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f*Sqrt[Cos[e + f*x]^2]) + (b*((3*a^2)/(2 +
n*p) + b^2/(3 + n*p))*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n*p)/2, (4 +
 n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]^2*(c*(d*Sin[e + f*x])^p)^n)/(f*Sqrt[C
os[e + f*x]^2]) - (b^2*Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n*(
a + b*Sin[e + f*x]))/(f*(3 + n*p))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 2826

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sin[e + f*x
])^p)^FracPart[n])/(d*Sin[e + f*x])^(p*FracPart[n]), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \left(c(d\sin(e+fx))^p\right)^n (a+b\sin(e+fx))^3\,dx = \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int (d\sin(e+fx))^{np}(a$$

$$= -\frac{b^2\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n (a+b\sin(e+}{f(3+np)}$$

$$= -\frac{ab^2(7+2np)\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(2+np)(3+np)} -$$

$$= -\frac{ab^2(7+2np)\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(2+np)(3+np)} -$$

$$= -\frac{ab^2(7+2np)\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(2+np)(3+np)} +$$

Mathematica [A] time = 1.04, size = 230, normalized size = 0.71

$$\frac{\sin(e+fx)\cos(e+fx)\left(c(d\sin(e+fx))^p\right)^n\left(\frac{b\left(3a^2(np+3)+b^2(np+2)\right)\sin(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{np}{2}+1;\frac{np}{2}+2;\sin^2(e+fx)\right)}{(np+2)\sqrt{\cos^2(e+fx)}}+\frac{a(np+3)\left(a^2(np+2\right.}{}\right.}{f(np+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n*(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n*(-((a*b^2*(7 + 2*n*p))/
(2 + n*p)) + (a*(3 + n*p)*(3*b^2*(1 + n*p) + a^2*(2 + n*p))*Hypergeometric2
F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2])/((1 + n*p)*(2 + n*p)*Sqr
t[Cos[e + f*x]^2]) + (b*(b^2*(2 + n*p) + 3*a^2*(3 + n*p))*Hypergeometric2F1
[1/2, 1 + (n*p)/2, 2 + (n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x])/((2 + n*p)*Sq
rt[Cos[e + f*x]^2]) - b^2*(a + b*Sin[e + f*x])))/(f*(3 + n*p))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(3\,ab^2\cos\left(fx+e\right)^2-a^3-3\,ab^2+\left(b^3\cos\left(fx+e\right)^2-3\,a^2b-b^3\right)\sin\left(fx+e\right)\right)\left(\left(d\sin\left(fx+e\right)\right)^p c\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 - 3
*a^2*b - b^3)*sin(f*x + e))*((d*sin(f*x + e))^p*c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin\left(fx+e\right)+a\right)^{3} \left(\left(d \sin\left(fx+e\right)\right)^{p} c\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^3,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e) + a)^3*((d*sin(f*x + e))^p*c)^n, x)
```

maple [F] time = 1.27, size = 0, normalized size = 0.00

$$\int \left(c \left(d \sin\left(fx+e\right)\right)^{p}\right)^{n} \left(a+b \sin\left(fx+e\right)\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^3,x)
[Out] int((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin\left(fx+e\right)+a\right)^{3} \left(\left(d \sin\left(fx+e\right)\right)^{p} c\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^3,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)^3*((d*sin(f*x + e))^p*c)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c\,{\left(d\,\sin\left(e+f\,x\right)\right)}^p\right)}^n\,{\left(a+b\,\sin\left(e+f\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(e + f*x))^p)^n*(a + b*sin(e + f*x))^3,x)
[Out] int((c*(d*sin(e + f*x))^p)^n*(a + b*sin(e + f*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c \left(d \sin{\left(e + f x \right)}\right)^{p}\right)^{n} \left(a + b \sin{\left(e + f x \right)}\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n*(a+b*sin(f*x+e))**3,x)
[Out] Integral((c*(d*sin(e + f*x))**p)**n*(a + b*sin(e + f*x))**3, x)
```

3.833 $\int \left(c(d\sin(e+fx))^p\right)^n (a+b\sin(e+fx))^2\,dx$

Optimal. Leaf size=231

$$\frac{\left(a^2(np+2)+b^2(np+1)\right)\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{f(np+1)(np+2)\sqrt{\cos^2(e+fx)}}$$

```
[Out] -b^2*cos(f*x+e)*sin(f*x+e)*(c*(d*sin(f*x+e))^p)^n/f/(n*p+2)+(b^2*(n*p+1)+a^
2*(n*p+2))*cos(f*x+e)*hypergeom([1/2, 1/2*n*p+1/2],[1/2*n*p+3/2],sin(f*x+e)
^2)*sin(f*x+e)*(c*(d*sin(f*x+e))^p)^n/f/(n*p+1)/(n*p+2)/(cos(f*x+e)^2)^(1/2
)+2*a*b*cos(f*x+e)*hypergeom([1/2, 1/2*n*p+1],[1/2*n*p+2],sin(f*x+e)^2)*sin
(f*x+e)^2*(c*(d*sin(f*x+e))^p)^n/f/(n*p+2)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 221, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2826, 2789, 2643, 3014}

$$\frac{\left(\frac{a^2}{np+1}+\frac{b^2}{np+2}\right)\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{f\sqrt{\cos^2(e+fx)}}+\frac{2ab\sin^2(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -((b^2*Cos[e + f*x]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f*(2 + n*p))) +
 ((a^2/(1 + n*p) + b^2/(2 + n*p))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 +
n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)
/(f*Sqrt[Cos[e + f*x]^2]) + (2*a*b*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 +
 n*p)/2, (4 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]^2*(c*(d*Sin[e + f*x])^p)
^n)/(f*(2 + n*p)*Sqrt[Cos[e + f*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
```

```
, f, m}, x]
```

Rule 2826

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sin[e + f*x
])^p)^FracPart[n])/(d*Sin[e + f*x])^(p*FracPart[n]), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \left(c(d\sin(e+fx))^p\right)^n (a+b\sin(e+fx))^2\,dx = \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int (d\sin(e+fx))^{np}(a$$

$$= \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int (d\sin(e+fx))^{np}\left(a^2\right.$$

$$= -\frac{b^2\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(2+np)} + \frac{2ab\cos(e+}{}$$

$$= -\frac{b^2\cos(e+fx)\sin(e+fx)\left(c(d\sin(e+fx))^p\right)^n}{f(2+np)} + \frac{\left(a^2+\frac{b^2(1+n}{2+np}\right.}{}$$

Mathematica [A] time = 0.35, size = 152, normalized size = 0.66

$$-\frac{\cos(e+fx)\sin^2(e+fx)^{\frac{1}{2}(-np-1)}\left(c(d\sin(e+fx))^p\right)^n\left(a\left(a\sin(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{3}{2};\cos^2(e+fx)\right)+2b\right.\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n*(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -((Cos[e + f*x]*(Sin[e + f*x]^2)^((-1 - n*p)/2)*(c*(d*Sin[e + f*x])^p)^n*(b
^2*Hypergeometric2F1[1/2, (-1 - n*p)/2, 3/2, Cos[e + f*x]^2]*Sin[e + f*x] +
```

```
a*(a*Hypergeometric2F1[1/2, (1 - n*p)/2, 3/2, Cos[e + f*x]^2]*Sin[e + f*x]
+ 2*b*Hypergeometric2F1[1/2, -1/2*(n*p), 3/2, Cos[e + f*x]^2]*Sqrt[Sin[e +
f*x]^2])))/f)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2\right)\left(\left(d\sin\left(fx+e\right)\right)^p c\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*((d*sin(f*x
+ e))^p*c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)+a\right)^2\left(\left(d\sin\left(fx+e\right)\right)^p c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*((d*sin(f*x + e))^p*c)^n, x)
```

maple [F] time = 1.42, size = 0, normalized size = 0.00

$$\int\left(c\left(d\sin\left(fx+e\right)\right)^p\right)^n\left(a+b\sin\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^2,x)
```

```
[Out] int((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)+a\right)^2\left(\left(d\sin\left(fx+e\right)\right)^p c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*((d*sin(f*x + e))^p*c)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c\left(d\ \sin\left(e+f\ x\right)\right)^p\right)^n\left(a+b\ \sin\left(e+f\ x\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(e + f*x))^p)^n*(a + b*sin(e + f*x))^2,x)
[Out] int((c*(d*sin(e + f*x))^p)^n*(a + b*sin(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\left(d\sin\left(e+fx\right)\right)^p\right)^n\left(a+b\sin\left(e+fx\right)\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n*(a+b*sin(f*x+e))**2,x)
[Out] Integral((c*(d*sin(e + f*x))**p)**n*(a + b*sin(e + f*x))**2, x)
```

3.834 $\int \left(c(d\sin(e+fx))^p\right)^n (a+b\sin(e+fx))\,dx$

Optimal. Leaf size=163

$$\frac{a\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{f(np+1)\sqrt{\cos^2(e+fx)}}+\frac{b\sin^2(e+fx)\cos(e+\ldots}{}$$

```
[Out] a*cos(f*x+e)*hypergeom([1/2, 1/2*n*p+1/2],[1/2*n*p+3/2],sin(f*x+e)^2)*sin(f
*x+e)*(c*(d*sin(f*x+e))^p)^n/f/(n*p+1)/(cos(f*x+e)^2)^(1/2)+b*cos(f*x+e)*hy
pergeom([1/2, 1/2*n*p+1],[1/2*n*p+2],sin(f*x+e)^2)*sin(f*x+e)^2*(c*(d*sin(f
*x+e))^p)^n/f/(n*p+2)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2826, 2748, 2643}

$$\frac{a\sin(e+fx)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(e+fx)\right)\left(c(d\sin(e+fx))^p\right)^n}{f(np+1)\sqrt{\cos^2(e+fx)}}+\frac{b\sin^2(e+fx)\cos(e+\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n*(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*
x]^2]*Sin[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/(f*(1 + n*p)*Sqrt[Cos[e + f*x]
^2]) + (b*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n*p)/2, (4 + n*p)/2, Sin
[e + f*x]^2]*Sin[e + f*x]^2*(c*(d*Sin[e + f*x])^p)^n)/(f*(2 + n*p)*Sqrt[Cos
[e + f*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2826

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sin[e + f*x
])^p)^FracPart[n])/(d*Sin[e + f*x])^(p*FracPart[n]), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rubi steps

$$\int \left(c(d\sin(e+fx))^p\right)^n (a+b\sin(e+fx))\,dx = \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int (d\sin(e+fx))^{np}(a+$$

$$= \left(a(d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int (d\sin(e+fx))^{np}\,dx$$

$$= \frac{a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(e+fx)\right)\sin(e+}{f(1+np)\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 0.24, size = 129, normalized size = 0.79

$$\frac{\sqrt{\cos^2(e+fx)}\,\tan(e+fx)\left(c(d\sin(e+fx))^p\right)^n\left(a(np+2)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(e+fx)\right)+b(np+}{f(np+1)(np+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n*(a + b*Sin[e + f*x]),x]
```

```
[Out] (Sqrt[Cos[e + f*x]^2]*(c*(d*Sin[e + f*x])^p)^n*(a*(2 + n*p)*Hypergeometric2
F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2] + b*(1 + n*p)*Hypergeomet
ric2F1[1/2, 1 + (n*p)/2, 2 + (n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x])*Tan[e +
 f*x])/(f*(1 + n*p)*(2 + n*p))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(fx+e\right)+a\right)\left(\left(d\sin\left(fx+e\right)\right)^p c\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e) + a)*((d*sin(f*x + e))^p*c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e)),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e) + a)*((d*sin(f*x + e))^p*c)^n, x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \left(c\left(d\sin\left(fx+e\right)\right)^{p}\right)^{n}\left(a+b\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e)),x)
[Out] int((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)+a\right)\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n*(a+b*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)*((d*sin(f*x + e))^p*c)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,{\left(d\,\sin\left(e+f\,x\right)\right)}^p\right)}^n\,\left(a+b\,\sin\left(e+f\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(e + f*x))^p)^n*(a + b*sin(e + f*x)),x)
[Out] int((c*(d*sin(e + f*x))^p)^n*(a + b*sin(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\left(d\sin\left(e+fx\right)\right)^{p}\right)^{n}\left(a+b\sin\left(e+fx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n*(a+b*sin(f*x+e)),x)
```

```
[Out] Integral((c*(d*sin(e + f*x))**p)**n*(a + b*sin(e + f*x)), x)
```

3.835 $\int \frac{\left(c(d\sin(e+fx))^p\right)^n}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=204

$$\frac{b\cos(e+fx)\sin^2(e+fx)^{-\frac{np}{2}}\left(c(d\sin(e+fx))^p\right)^n F_1\left(\frac{1}{2};-\frac{np}{2},1;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)}-\frac{a\cot(e+fx)\sin^2}{}$$

[Out] b*AppellF1(1/2,-1/2*n*p,1,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b^2))*cos(f*x+e)*(c*(d*sin(f*x+e))^p)^n/(a^2-b^2)/f/((sin(f*x+e)^2)^(1/2*n*p))-a*AppellF1(1/2,-1/2*n*p+1/2,1,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b^2))*cot(f*x+e)*(sin(f*x+e)^2)^(-1/2*n*p+1/2)*(c*(d*sin(f*x+e))^p)^n/(a^2-b^2)/f

Rubi [A] time = 0.34, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2826, 2823, 3189, 429}

$$\frac{b\cos(e+fx)\sin^2(e+fx)^{-\frac{np}{2}}\left(c(d\sin(e+fx))^p\right)^n F_1\left(\frac{1}{2};-\frac{np}{2},1;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)}-\frac{a\cot(e+fx)\sin^2}{}$$

Antiderivative was successfully verified.

[In] Int[(c*(d*Sin[e + f*x])^p)^n/(a + b*Sin[e + f*x]),x]

[Out] (b*AppellF1[1/2, -(n*p)/2, 1, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/((a^2 - b^2)*f*(Sin[e + f*x]^2)^((n*p)/2)) - (a*AppellF1[1/2, (1 - n*p)/2, 1, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cot[e + f*x]*(Sin[e + f*x]^2)^((1 - n*p)/2)*(c*(d*Sin[e + f*x])^p)^n)/((a^2 - b^2)*f)

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2826

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sin[e + f*x
])^p)^FracPart[n])/(d*Sin[e + f*x])^(p*FracPart[n]), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && !IntegerQ[n]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] && !IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(c(d\sin(e+fx))^p\right)^n}{a+b\sin(e+fx)}\,dx = \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int \frac{(d\sin(e+fx))^{np}}{a+b\sin(e+fx)}\,dx$$

$$= \left(a(d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int \frac{(d\sin(e+fx))^{np}}{a^2-b^2\sin^2(e+fx)}\,dx - \frac{\left(b(d\sin(e+\right.}{}$$

$$= \frac{\left(b\sin^2(e+fx)^{-\frac{np}{2}}\left(c(d\sin(e+fx))^p\right)^n\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{\frac{np}{2}}}{a^2-b^2+b^2x^2}\,dx, x, \cos(e+fx)\right)}{f} - \cdots$$

$$= \frac{bF_1\left(\frac{1}{2};-\frac{np}{2},1;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\sin^2(e+fx)^{-\frac{np}{2}}\left(c(d\sin(e\right.}{\left(a^2-b^2\right)f}$$

Mathematica [B] time = 18.00, size = 1808, normalized size = 8.86

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((Sec[e + f*x]^2)^((n*p)/2)*(c*(d*Sin[e + f*x])^p)^n*Tan[e + f*x]*(Tan[e +
f*x]/Sqrt[Sec[e + f*x]^2])^(n*p)*((a^2 - b^2)*(1 + n*p)*AppellF1[1 + (n*p)/
```

```
2, (-1 + n*p)/2, 1, 2 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*
x]^2]*Tan[e + f*x] + a*(b*(2 + n*p)*AppellF1[(1 + n*p)/2, (n*p)/2, 1, (3 +
n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] - a*(1 + n*p)*H
ypergeometric2F1[1 + (n*p)/2, (1 + n*p)/2, 2 + (n*p)/2, -Tan[e + f*x]^2]*Ta
n[e + f*x])))/(a^2*b*f*(1 + n*p)*(2 + n*p)*(a + b*Sin[e + f*x])*(((Sec[e +
f*x]^2)^(1 + (n*p)/2)*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^(n*p)*((a^2 - b^2
)*(1 + n*p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 1, 2 + (n*p)/2, -Tan[e + f*
x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x] + a*(b*(2 + n*p)*AppellF1
[(1 + n*p)/2, (n*p)/2, 1, (3 + n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e
 + f*x]^2)/a^2] - a*(1 + n*p)*Hypergeometric2F1[1 + (n*p)/2, (1 + n*p)/2, 2
 + (n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*x])))/(a^2*b*(1 + n*p)*(2 + n*p)) +
(n*p*(Sec[e + f*x]^2)^((n*p)/2)*Tan[e + f*x]^2*(Tan[e + f*x]/Sqrt[Sec[e + f
*x]^2])^(n*p)*((a^2 - b^2)*(1 + n*p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 1,
 2 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]
+ a*(b*(2 + n*p)*AppellF1[(1 + n*p)/2, (n*p)/2, 1, (3 + n*p)/2, -Tan[e + f*
x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] - a*(1 + n*p)*Hypergeometric2F1[1
+ (n*p)/2, (1 + n*p)/2, 2 + (n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*x])))/(a^2*
b*(1 + n*p)*(2 + n*p)) + (n*p*(Sec[e + f*x]^2)^((n*p)/2)*Tan[e + f*x]*(Tan[
e + f*x]/Sqrt[Sec[e + f*x]^2])^(-1 + n*p)*(Sqrt[Sec[e + f*x]^2] - Tan[e + f
*x]^2/Sqrt[Sec[e + f*x]^2])*((a^2 - b^2)*(1 + n*p)*AppellF1[1 + (n*p)/2, (-
1 + n*p)/2, 1, 2 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]
*Tan[e + f*x] + a*(b*(2 + n*p)*AppellF1[(1 + n*p)/2, (n*p)/2, 1, (3 + n*p)/
2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] - a*(1 + n*p)*Hyperg
eometric2F1[1 + (n*p)/2, (1 + n*p)/2, 2 + (n*p)/2, -Tan[e + f*x]^2]*Tan[e +
 f*x])))/(a^2*b*(1 + n*p)*(2 + n*p)) + ((Sec[e + f*x]^2)^((n*p)/2)*Tan[e +
f*x]*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^(n*p)*((a^2 - b^2)*(1 + n*p)*Appel
lF1[1 + (n*p)/2, (-1 + n*p)/2, 1, 2 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a
^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2 + (a^2 - b^2)*(1 + n*p)*Tan[e + f*x]*((2
*(-1 + b^2/a^2)*(1 + (n*p)/2)*AppellF1[2 + (n*p)/2, (-1 + n*p)/2, 2, 3 + (n
*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e
 + f*x])/(2 + (n*p)/2) - ((1 + (n*p)/2)*(-1 + n*p)*AppellF1[2 + (n*p)/2, 1
+ (-1 + n*p)/2, 1, 3 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x
]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(2 + (n*p)/2)) + a*(-(a*(1 + n*p)*Hyperge
ometric2F1[1 + (n*p)/2, (1 + n*p)/2, 2 + (n*p)/2, -Tan[e + f*x]^2]*Sec[e +
f*x]^2) + b*(2 + n*p)*((2*(-a^2 + b^2)*(1 + n*p)*AppellF1[1 + (1 + n*p)/2,
(n*p)/2, 2, 1 + (3 + n*p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)
/a^2]*Sec[e + f*x]^2*Tan[e + f*x])/(a^2*(3 + n*p)) - (n*p*(1 + n*p)*AppellF
1[1 + (1 + n*p)/2, 1 + (n*p)/2, 1, 1 + (3 + n*p)/2, -Tan[e + f*x]^2, ((-a^2
 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 + n*p)) - 2*a*
(1 + (n*p)/2)*(1 + n*p)*Sec[e + f*x]^2*(-Hypergeometric2F1[1 + (n*p)/2, (1
+ n*p)/2, 2 + (n*p)/2, -Tan[e + f*x]^2] + (1 + Tan[e + f*x]^2)^((-1 - n*p)/
2)))))/(a^2*b*(1 + n*p)*(2 + n*p))))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}}{b\sin\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e)),x, algorithm="fricas")
[Out] integral(((d*sin(f*x + e))^p*c)^n/(b*sin(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e)),x, algorithm="giac")
[Out] integrate(((d*sin(f*x + e))^p*c)^n/(b*sin(f*x + e) + a), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(d\sin\left(fx+e\right)\right)^{p}\right)^{n}}{a+b\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e)),x)
[Out] int((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(((d*sin(f*x + e))^p*c)^n/(b*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\left(d\,\sin\left(e+f\,x\right)\right)^p\right)^n}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(e + f*x))^p)^n/(a + b*sin(e + f*x)),x)
[Out] int((c*(d*sin(e + f*x))^p)^n/(a + b*sin(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(d\sin\left(e+fx\right)\right)^p\right)^n}{a+b\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n/(a+b*sin(f*x+e)),x)
[Out] Integral((c*(d*sin(e + f*x))**p)**n/(a + b*sin(e + f*x)), x)
```

3.836 $\int \frac{\left(c(d\sin(e+fx))^p\right)^n}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=322

$$\frac{2ab\cos(e+fx)\sin^2(e+fx)^{-\frac{np}{2}}\left(c(d\sin(e+fx))^p\right)^n F_1\left(\frac{1}{2};-\frac{np}{2},2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^2}-\frac{b^2\sin(e+fx)}{}$$

```
[Out] 2*a*b*AppellF1(1/2,-1/2*n*p,2,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b^2))
*cos(f*x+e)*(c*(d*sin(f*x+e))^p)^n/(a^2-b^2)^2/f/((sin(f*x+e)^2)^(1/2*n*p))
-b^2*AppellF1(1/2,-1/2*n*p-1/2,2,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b^
2))*cos(f*x+e)*sin(f*x+e)*(sin(f*x+e)^2)^(-1/2*n*p-1/2)*(c*(d*sin(f*x+e))^p
)^n/(a^2-b^2)^2/f-a^2*AppellF1(1/2,-1/2*n*p+1/2,2,3/2,cos(f*x+e)^2,-b^2*cos
(f*x+e)^2/(a^2-b^2))*cot(f*x+e)*(sin(f*x+e)^2)^(-1/2*n*p+1/2)*(c*(d*sin(f*x
+e))^p)^n/(a^2-b^2)^2/f
```

Rubi [A] time = 0.51, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2826, 2824, 3189, 429, 16}

$$\frac{2ab\cos(e+fx)\sin^2(e+fx)^{-\frac{np}{2}}\left(c(d\sin(e+fx))^p\right)^n F_1\left(\frac{1}{2};-\frac{np}{2},2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^2}-\frac{b^2\sin(e+fx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (2*a*b*AppellF1[1/2, -(n*p)/2, 2, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^
2)/(a^2 - b^2))]*Cos[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/((a^2 - b^2)^2*f*(S
in[e + f*x]^2)^((n*p)/2)) - (b^2*AppellF1[1/2, (-1 - n*p)/2, 2, 3/2, Cos[e
+ f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*Sin[e + f*x]*(S
in[e + f*x]^2)^((-1 - n*p)/2)*(c*(d*Sin[e + f*x])^p)^n)/((a^2 - b^2)^2*f) -
 (a^2*AppellF1[1/2, (1 - n*p)/2, 2, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x
]^2)/(a^2 - b^2))]*Cot[e + f*x]*(Sin[e + f*x]^2)^((1 - n*p)/2)*(c*(d*Sin[e
+ f*x])^p)^n)/((a^2 - b^2)^2*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2824

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n/((a - b*sin[e +
 f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

Rule 2826

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sin[e + f*x
])^p)^FracPart[n])/(d*Sin[e + f*x])^(p*FracPart[n]), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(c(d\sin(e+fx))^p\right)^n}{(a+b\sin(e+fx))^2}\,dx = \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int \frac{(d\sin(e+fx))^{np}}{(a+b\sin(e+fx))^2}\,dx$$

$$= \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int\left(\frac{a^2(d\sin(e+fx))^{np}}{\left(a^2-b^2\sin^2(e+fx)\right)^2} - \frac{2ab\sin(e+}{\left(a^2-b\right.}\right.$$

$$= \left(a^2(d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int \frac{(d\sin(e+fx))^{np}}{\left(a^2-b^2\sin^2(e+fx)\right)^2}\,dx - \left(2ab(d\,\mathrm{s}\right.$$

$$= \frac{\left(b^2(d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int \frac{(d\sin(e+fx))^{2+np}}{\left(-a^2+b^2\sin^2(e+fx)\right)^2}\,dx}{d^2} - \frac{\left(2ab(d\sin(e+\right.}{}$$

$$= -\frac{a^2F_1\left(\frac{1}{2};\frac{1}{2}(1-np),2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cot(e+fx)\sin^2(e+fx)^{\frac{1}{2}(1-n}}{\left(a^2-b^2\right)^2 f}$$

$$= \frac{2abF_1\left(\frac{1}{2};-\frac{np}{2},2;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\sin^2(e+fx)^{-\frac{np}{2}}\left(c(d\sin\right.}{\left(a^2-b^2\right)^2 f}$$

Mathematica [B] time = 19.02, size = 1970, normalized size = 6.12

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] -(((Sec[e + f*x]^2)^((n*p)/2)*(c*(d*Sin[e + f*x])^p)^n*Tan[e + f*x]*(Tan[e
+ f*x]/Sqrt[Sec[e + f*x]^2])^(n*p)*(-(a*(2 + n*p)*((a^2 + b^2)*AppellF1[(1
+ n*p)/2, (n*p)/2, 1, (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e +
f*x]^2] - 2*b^2*AppellF1[(1 + n*p)/2, (n*p)/2, 2, (3 + n*p)/2, -Tan[e + f*x
]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2])) + 2*b*(a^2 - b^2)*(1 + n*p)*AppellF1[
1 + (n*p)/2, (-1 + n*p)/2, 2, 2 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*
Tan[e + f*x]^2]*Tan[e + f*x]))/(a^3*(a^2 - b^2)*f*(1 + n*p)*(2 + n*p)*(a +
b*Sin[e + f*x])^2*(-(((Sec[e + f*x]^2)^(1 + (n*p)/2)*(Tan[e + f*x]/Sqrt[Sec
[e + f*x]^2])^(n*p)*(-(a*(2 + n*p)*((a^2 + b^2)*AppellF1[(1 + n*p)/2, (n*p)
/2, 1, (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - 2*b^2
*AppellF1[(1 + n*p)/2, (n*p)/2, 2, (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/
a^2)*Tan[e + f*x]^2])) + 2*b*(a^2 - b^2)*(1 + n*p)*AppellF1[1 + (n*p)/2, (-
```

```
1 + n*p)/2, 2, 2 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]
*Tan[e + f*x]))/(a^3*(a^2 - b^2)*(1 + n*p)*(2 + n*p))) - (n*p*(Sec[e + f*x]
^2)^((n*p)/2)*Tan[e + f*x]^2*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^(n*p)*(-(a
*(2 + n*p)*((a^2 + b^2)*AppellF1[(1 + n*p)/2, (n*p)/2, 1, (3 + n*p)/2, -Tan
[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - 2*b^2*AppellF1[(1 + n*p)/2, (
n*p)/2, 2, (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2])) +
 2*b*(a^2 - b^2)*(1 + n*p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 2, 2 + (n*p)
/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]))/(a^3*(a^
2 - b^2)*(1 + n*p)*(2 + n*p)) - (n*p*(Sec[e + f*x]^2)^((n*p)/2)*Tan[e + f*x
]*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^(-1 + n*p)*(-(a*(2 + n*p)*((a^2 + b^2
)*AppellF1[(1 + n*p)/2, (n*p)/2, 1, (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2
/a^2)*Tan[e + f*x]^2] - 2*b^2*AppellF1[(1 + n*p)/2, (n*p)/2, 2, (3 + n*p)/2
, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2])) + 2*b*(a^2 - b^2)*(1 +
n*p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 2, 2 + (n*p)/2, -Tan[e + f*x]^2, (
-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x])*(Sqrt[Sec[e + f*x]^2] - Tan[e +
 f*x]^2/Sqrt[Sec[e + f*x]^2]))/(a^3*(a^2 - b^2)*(1 + n*p)*(2 + n*p)) - ((Se
c[e + f*x]^2)^((n*p)/2)*Tan[e + f*x]*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^(n
*p)*(2*b*(a^2 - b^2)*(1 + n*p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 2, 2 + (
n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2 + 2*
b*(a^2 - b^2)*(1 + n*p)*Tan[e + f*x]*((4*(-1 + b^2/a^2)*(1 + (n*p)/2)*Appel
lF1[2 + (n*p)/2, (-1 + n*p)/2, 3, 3 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a
^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(2 + (n*p)/2) - ((1 + (n*p
)/2)*(-1 + n*p)*AppellF1[2 + (n*p)/2, 1 + (-1 + n*p)/2, 2, 3 + (n*p)/2, -Ta
n[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(
2 + (n*p)/2)) - a*(2 + n*p)*((a^2 + b^2)*((2*(-1 + b^2/a^2)*(1 + n*p)*Appel
lF1[1 + (1 + n*p)/2, (n*p)/2, 2, 1 + (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^
2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 + n*p) - (n*p*(1 + n
*p)*AppellF1[1 + (1 + n*p)/2, 1 + (n*p)/2, 1, 1 + (3 + n*p)/2, -Tan[e + f*x
]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 + n*p))
 - 2*b^2*((4*(-1 + b^2/a^2)*(1 + n*p)*AppellF1[1 + (1 + n*p)/2, (n*p)/2, 3,
 1 + (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f
*x]^2*Tan[e + f*x])/(3 + n*p) - (n*p*(1 + n*p)*AppellF1[1 + (1 + n*p)/2, 1
+ (n*p)/2, 2, 1 + (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]
^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 + n*p)))))/(a^3*(a^2 - b^2)*(1 + n*p)*(
2 + n*p)))))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}}{b^{2}\cos\left(fx+e\right)^{2}-2\,ab\sin\left(fx+e\right)-a^{2}-b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-((d*sin(f*x + e))^p*c)^n/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e)
 - a^2 - b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}}{\left(b\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e))^2,x, algorithm="giac")
[Out] integrate(((d*sin(f*x + e))^p*c)^n/(b*sin(f*x + e) + a)^2, x)
```

maple [F] time = 1.14, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(d\sin\left(fx+e\right)\right)^{p}\right)^{n}}{\left(a+b\sin\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e))^2,x)
[Out] int((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}}{\left(b\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
[Out] integrate(((d*sin(f*x + e))^p*c)^n/(b*sin(f*x + e) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\left(d\,\sin\left(e+f\,x\right)\right)^{p}\right)^{n}}{\left(a+b\,\sin\left(e+f\,x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(e + f*x))^p)^n/(a + b*sin(e + f*x))^2,x)
```

```
[Out] int((c*(d*sin(e + f*x))^p)^n/(a + b*sin(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(d\sin\left(e+fx\right)\right)^{p}\right)^{n}}{\left(a+b\sin\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Integral((c*(d*sin(e + f*x))**p)**n/(a + b*sin(e + f*x))**2, x)
```

3.837 $\int \frac{\left(c(d\sin(e+fx))^p\right)^n}{(a+b\sin(e+fx))^3}\,dx$

Optimal. Leaf size=428

$$\frac{3a^2b\cos(e+fx)\sin^2(e+fx)^{-\frac{np}{2}}\left(c(d\sin(e+fx))^p\right)^n F_1\left(\frac{1}{2};-\frac{np}{2},3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^3}-\frac{3ab^2\sin(e+$$

```
[Out] 3*a^2*b*AppellF1(1/2,-1/2*n*p,3,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b^2
))*cos(f*x+e)*(c*(d*sin(f*x+e))^p)^n/(a^2-b^2)^3/f/((sin(f*x+e)^2)^(1/2*n*p
))+b^3*AppellF1(1/2,-1/2*n*p-1,3,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/(a^2-b^
2))*cos(f*x+e)*(c*(d*sin(f*x+e))^p)^n/(a^2-b^2)^3/f/((sin(f*x+e)^2)^(1/2*n*
p))-3*a*b^2*AppellF1(1/2,-1/2*n*p-1/2,3,3/2,cos(f*x+e)^2,-b^2*cos(f*x+e)^2/
(a^2-b^2))*cos(f*x+e)*sin(f*x+e)*(sin(f*x+e)^2)^(-1/2*n*p-1/2)*(c*(d*sin(f*
x+e))^p)^n/(a^2-b^2)^3/f-a^3*AppellF1(1/2,-1/2*n*p+1/2,3,3/2,cos(f*x+e)^2,-
b^2*cos(f*x+e)^2/(a^2-b^2))*cot(f*x+e)*(sin(f*x+e)^2)^(-1/2*n*p+1/2)*(c*(d*
sin(f*x+e))^p)^n/(a^2-b^2)^3/f
```

Rubi [A] time = 0.65, antiderivative size = 428, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2826, 2824, 3189, 429, 16}

$$\frac{3a^2b\cos(e+fx)\sin^2(e+fx)^{-\frac{np}{2}}\left(c(d\sin(e+fx))^p\right)^n F_1\left(\frac{1}{2};-\frac{np}{2},3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^3}+\frac{b^3\cos(e+f$$

Antiderivative was successfully verified.

```
[In] Int[(c*(d*Sin[e + f*x])^p)^n/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] (3*a^2*b*AppellF1[1/2, -(n*p)/2, 3, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x
]^2)/(a^2 - b^2))]*Cos[e + f*x]*(c*(d*Sin[e + f*x])^p)^n)/((a^2 - b^2)^3*f*
(Sin[e + f*x]^2)^((n*p)/2)) + (b^3*AppellF1[1/2, (-2 - n*p)/2, 3, 3/2, Cos[
e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(c*(d*Sin[e +
 f*x])^p)^n)/((a^2 - b^2)^3*f*(Sin[e + f*x]^2)^((n*p)/2)) - (3*a*b^2*Appell
F1[1/2, (-1 - n*p)/2, 3, 3/2, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 -
 b^2))]*Cos[e + f*x]*Sin[e + f*x]*(Sin[e + f*x]^2)^((-1 - n*p)/2)*(c*(d*Sin
[e + f*x])^p)^n)/((a^2 - b^2)^3*f) - (a^3*AppellF1[1/2, (1 - n*p)/2, 3, 3/2
, Cos[e + f*x]^2, -((b^2*Cos[e + f*x]^2)/(a^2 - b^2))]*Cot[e + f*x]*(Sin[e
+ f*x]^2)^((1 - n*p)/2)*(c*(d*Sin[e + f*x])^p)^n)/((a^2 - b^2)^3*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2824

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n/((a - b*sin[e +
 f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

Rule 2826

```
Int[((c_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sin[e + f*x
])^p)^FracPart[n])/(d*Sin[e + f*x])^(p*FracPart[n]), Int[(a + b*Sin[e + f*x
])^m*(d*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{\left(c(d\sin(e+fx))^p\right)^n}{(a+b\sin(e+fx))^3}\,dx = \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int \frac{(d\sin(e+fx))^{np}}{(a+b\sin(e+fx))^3}\,dx$$

$$= \left((d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int\left(\frac{a^3(d\sin(e+fx))^{np}}{\left(a^2-b^2\sin^2(e+fx)\right)^3} - \frac{3a^2b\sin(e+}{\left(a^2-b\right.}\right.$$

$$= \left(a^3(d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int \frac{(d\sin(e+fx))^{np}}{\left(a^2-b^2\sin^2(e+fx)\right)^3}\,dx - \left(3a^2b(d\right.$$

$$= \frac{\left(b^3(d\sin(e+fx))^{-np}\left(c(d\sin(e+fx))^p\right)^n\right)\int \frac{(d\sin(e+fx))^{3+np}}{\left(-a^2+b^2\sin^2(e+fx)\right)^3}\,dx}{d^3} + \frac{\left(3ab^2(d\sin(e\right.}{}$$

$$= -\frac{a^3F_1\left(\frac{1}{2};\frac{1}{2}(1-np),3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cot(e+fx)\sin^2(e+fx)^{\frac{1}{2}(1-n}}{\left(a^2-b^2\right)^3 f}$$

$$= \frac{3a^2bF_1\left(\frac{1}{2};-\frac{np}{2},3;\frac{3}{2};\cos^2(e+fx),-\frac{b^2\cos^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\sin^2(e+fx)^{-\frac{np}{2}}\left(c(d\,\mathrm{s}\right.}{\left(a^2-b^2\right)^3 f}$$

Mathematica [B] time = 20.51, size = 2570, normalized size = 6.00

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*(d*Sin[e + f*x])^p)^n/(a + b*Sin[e + f*x])^3,x]
```

```
[Out] -(((Sec[e + f*x]^2)^((n*p)/2)*(c*(d*Sin[e + f*x])^p)^n*Tan[e + f*x]*(Tan[e
+ f*x]/Sqrt[Sec[e + f*x]^2])^(n*p)*(-(a*(2 + n*p)*((a^2 + 3*b^2)*AppellF1[(
1 + n*p)/2, -1 + (n*p)/2, 2, (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*T
an[e + f*x]^2] - 4*b^2*AppellF1[(1 + n*p)/2, -1 + (n*p)/2, 3, (3 + n*p)/2,
-Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2])) + b*(3*a^2 + b^2)*(1 + n*
p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 2, 2 + (n*p)/2, -Tan[e + f*x]^2, (-1
 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x] - 4*b^3*(1 + n*p)*AppellF1[1 + (n*
p)/2, (-1 + n*p)/2, 3, 2 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e +
 f*x]^2]*Tan[e + f*x]))/(a^4*(a^2 - b^2)*f*(1 + n*p)*(2 + n*p)*(a + b*Sin[e
 + f*x])^3*(-(((Sec[e + f*x]^2)^(1 + (n*p)/2)*(Tan[e + f*x]/Sqrt[Sec[e + f*
x]^2])^(n*p)*(-(a*(2 + n*p)*((a^2 + 3*b^2)*AppellF1[(1 + n*p)/2, -1 + (n*p)
/2, 2, (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - 4*b^2
```

*AppellF1[(1 + n*p)/2, -1 + (n*p)/2, 3, (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2])) + b*(3*a^2 + b^2)*(1 + n*p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 2, 2 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x] - 4*b^3*(1 + n*p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 3, 2 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]))/(a^4*(a^2 - b^2)*(1 + n*p)*(2 + n*p))) - (n*p*(Sec[e + f*x]^2)^((n*p)/2)*Tan[e + f*x]^2*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^(n*p)*(-(a*(2 + n*p)*((a^2 + 3*b^2)*AppellF1[(1 + n*p)/2, -1 + (n*p)/2, 2, (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - 4*b^2*AppellF1[(1 + n*p)/2, -1 + (n*p)/2, 3, (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2])) + b*(3*a^2 + b^2)*(1 + n*p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 2, 2 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x] - 4*b^3*(1 + n*p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 3, 2 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]))/(a^4*(a^2 - b^2)*(1 + n*p)*(2 + n*p)) - (n*p*(Sec[e + f*x]^2)^((n*p)/2)*Tan[e + f*x]*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^(-1 + n*p)*(-(a*(2 + n*p)*((a^2 + 3*b^2)*AppellF1[(1 + n*p)/2, -1 + (n*p)/2, 2, (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - 4*b^2*AppellF1[(1 + n*p)/2, -1 + (n*p)/2, 3, (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2])) + b*(3*a^2 + b^2)*(1 + n*p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 2, 2 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x] - 4*b^3*(1 + n*p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 3, 2 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x])*(Sqrt[Sec[e + f*x]^2] - Tan[e + f*x]^2/Sqrt[Sec[e + f*x]^2]))/(a^4*(a^2 - b^2)*(1 + n*p)*(2 + n*p)) - ((Sec[e + f*x]^2)^((n*p)/2)*Tan[e + f*x]*(Tan[e + f*x]/Sqrt[Sec[e + f*x]^2])^(n*p)*(b*(3*a^2 + b^2)*(1 + n*p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 2, 2 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2 - 4*b^3*(1 + n*p)*AppellF1[1 + (n*p)/2, (-1 + n*p)/2, 3, 2 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2 + b*(3*a^2 + b^2)*(1 + n*p)*Tan[e + f*x]*((4*(-1 + b^2/a^2)*(1 + (n*p)/2)*AppellF1[2 + (n*p)/2, (-1 + n*p)/2, 3, 3 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(2 + (n*p)/2) - ((1 + (n*p)/2)*(-1 + n*p)*AppellF1[2 + (n*p)/2, 1 + (-1 + n*p)/2, 2, 3 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(2 + (n*p)/2)) - 4*b^3*(1 + n*p)*Tan[e + f*x]*((6*(-1 + b^2/a^2)*(1 + (n*p)/2)*AppellF1[2 + (n*p)/2, (-1 + n*p)/2, 4, 3 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(2 + (n*p)/2) - ((1 + (n*p)/2)*(-1 + n*p)*AppellF1[2 + (n*p)/2, 1 + (-1 + n*p)/2, 3, 3 + (n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(2 + (n*p)/2)) - a*(2 + n*p)*((a^2 + 3*b^2)*((-2*(-1 + (n*p)/2)*(1 + n*p)*AppellF1[1 + (1 + n*p)/2, (n*p)/2, 2, 1 + (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 + n*p) + (4*(-1 + b^2/a^2)*(1 + n*p)*AppellF1[1 + (1 + n*p)/2, -1 + (n*p)/2, 3, 1 + (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3 + n*p)) - 4*b^2*((-2*(-1 + (n*p)/2)*(1 + n*p)*AppellF1[1 + (1 + n*p)/2, (n*p)/2, 3, 1 + (3 + n*p)/2, -

```
Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])
/(3 + n*p) + (6*(-1 + b^2/a^2)*(1 + n*p)*AppellF1[1 + (1 + n*p)/2, -1 + (n*
p)/2, 4, 1 + (3 + n*p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*S
ec[e + f*x]^2*Tan[e + f*x])/(3 + n*p)))))/(a^4*(a^2 - b^2)*(1 + n*p)*(2 + n
*p)))))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}}{3\,ab^{2}\cos\left(fx+e\right)^{2}-a^{3}-3\,ab^{2}+\left(b^{3}\cos\left(fx+e\right)^{2}-3\,a^{2}b-b^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-((d*sin(f*x + e))^p*c)^n/(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2
+ (b^3*cos(f*x + e)^2 - 3*a^2*b - b^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}}{\left(b\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate(((d*sin(f*x + e))^p*c)^n/(b*sin(f*x + e) + a)^3, x)
```

maple [F] time = 1.27, size = 0, normalized size = 0.00

$$\int\frac{\left(c\left(d\sin\left(fx+e\right)\right)^{p}\right)^{n}}{\left(a+b\sin\left(fx+e\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e))^3,x)
```

```
[Out] int((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(d\sin\left(fx+e\right)\right)^{p}c\right)^{n}}{\left(b\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))^p)^n/(a+b*sin(f*x+e))^3,x, algorithm="maxima")
[Out] integrate(((d*sin(f*x + e))^p*c)^n/(b*sin(f*x + e) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\left(d\ \sin\left(e+f\,x\right)\right)^p\right)^n}{\left(a+b\ \sin\left(e+f\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(d*sin(e + f*x))^p)^n/(a + b*sin(e + f*x))^3,x)
[Out] int((c*(d*sin(e + f*x))^p)^n/(a + b*sin(e + f*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(d\sin\left(e+fx\right)\right)^p\right)^n}{\left(a+b\sin\left(e+fx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sin(f*x+e))**p)**n/(a+b*sin(f*x+e))**3,x)
[Out] Integral((c*(d*sin(e + f*x))**p)**n/(a + b*sin(e + f*x))**3, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
   Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
   ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
   func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
   expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
   is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
   leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```